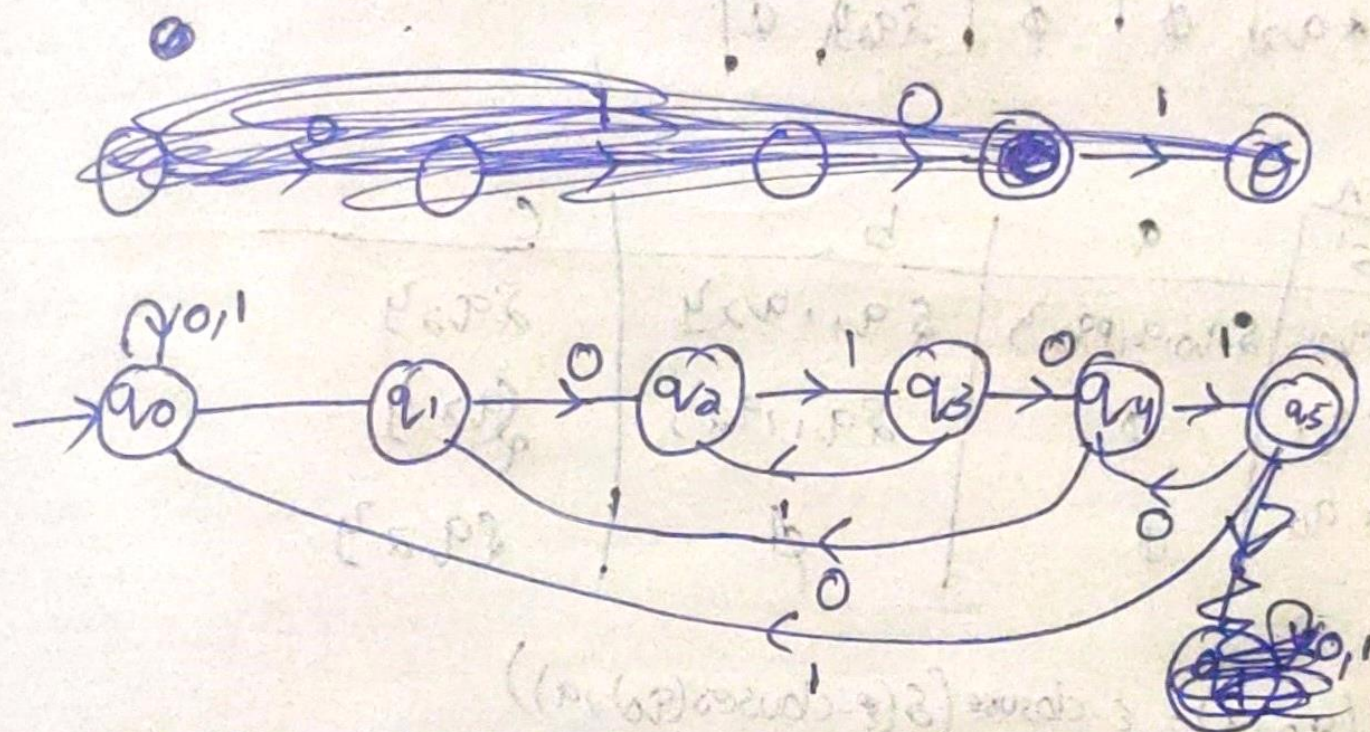


① Demonstrate DFA to accept set of all strings ending with 0101.

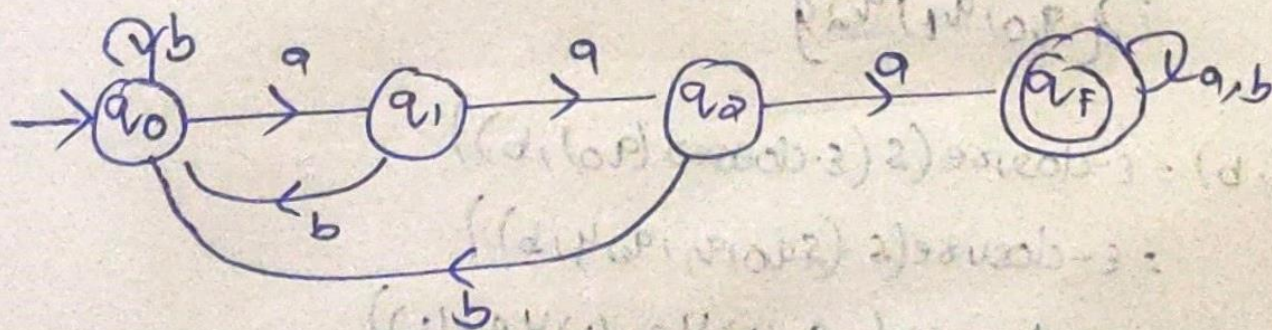
Ans:

Language $(L) = \{ 0101, 00101, 10101, 010101, 100101, \dots \}$



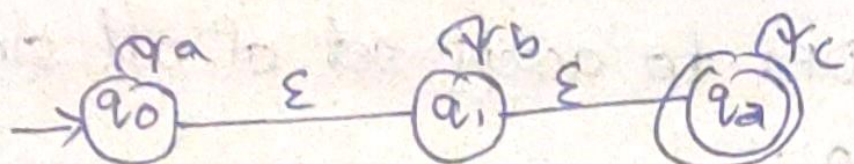
② Describe the DFA with the set of strings having "aaa" as a substring over an alphabet $\Sigma = \{a, b\}$.

Ans: language $(L) = \{ \underline{aaa}, a\underline{aaa}, b\underline{aaa}, ab\underline{aaa}, \dots \}$



④ Describe NFA with ϵ to NFA conversion with an example.

Ans:



ϵ -NFA

δ	a	b	c	ϵ
$\rightarrow q_0$	δq_0	ϕ	ϕ	δq_1
q_1	ϕ	δq_1	ϕ	δq_2
$*q_2$	ϕ	ϕ	δq_2	ϕ

$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

NFA δ'	a	b	c
$* \rightarrow q_0$	$\delta q_0, q_1, q_2$	$\delta q_1, q_2$	δq_2
$* q_1$	ϕ	$\delta q_1, q_2$	δq_2
$* q_2$	ϕ	ϕ	δq_2

$$\delta'(q_0, a) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), a))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, a))$$

$$= \epsilon\text{-closure}(q_0(a) \cup q_1(a) \cup q_2(a))$$

$$= \epsilon\text{-closure}(q_0)$$

$$= \{q_0, q_1, q_2\}$$

$$\delta'(q_0, b) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), b))$$

$$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, b))$$

$$= \epsilon\text{-closure}(q_0(b) \cup q_1(b) \cup q_2(b))$$

$$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$$

$$= \epsilon\text{-closure}(q_1)$$

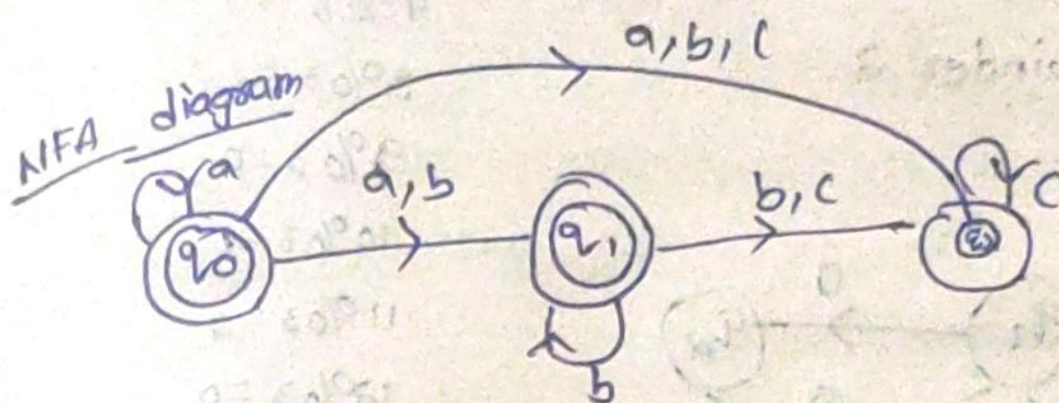
$$= \{q_1, q_2\}$$

Same for

$$\delta'(q_0, c)$$

$$\delta'(q_1, a), \delta'(q_1, b), \delta'(q_1, c)$$

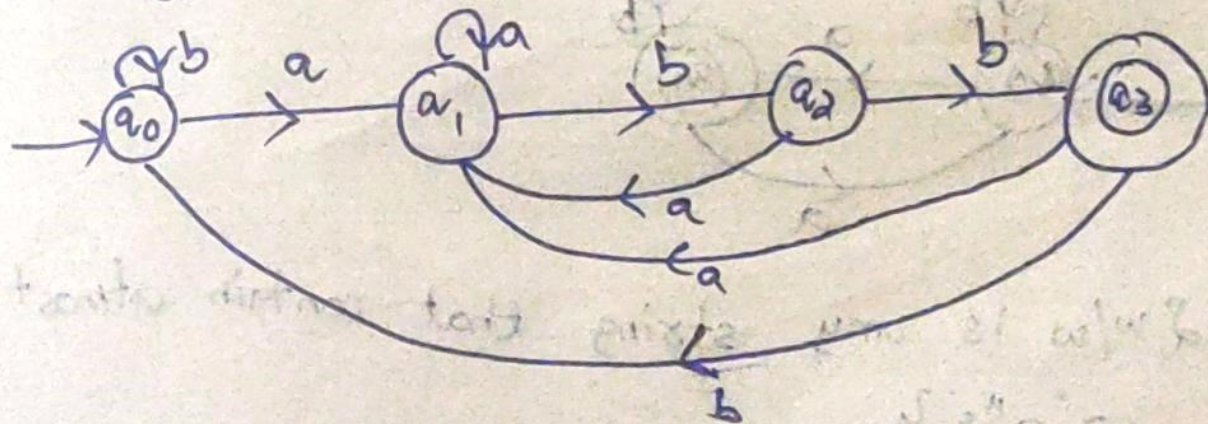
$$\delta'(q_2, a), \delta'(q_2, b), \delta'(q_2, c)$$



5) Describe a DFA to accept the strings a's and b's ending with "abb" over an alphabet

$$\Sigma = \{a, b\}$$

Ans: Language (L) = {abb, aabb, babb, ababb, ...}



⑦ Demonstrate a DFA that any given decimal number is divisible by 3

Ans!

$q_0 \rightarrow$ remainder 0

$q_1 \rightarrow$ remainder 1

$q_2 \rightarrow$ remainder 2

Reminders ± 0 to 2

$$6 \% 3 = 0$$

$$7 \% 3 = 1$$

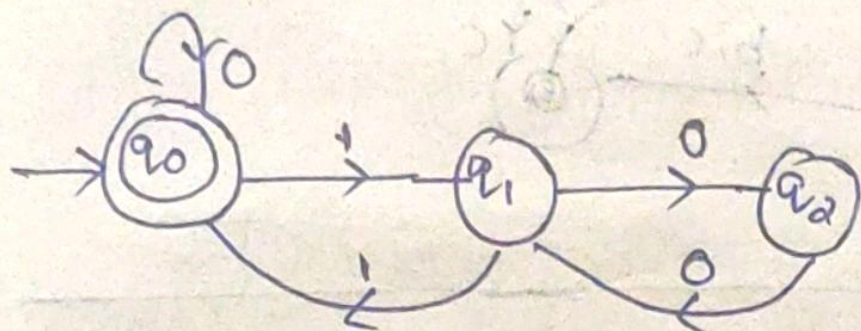
$$8 \% 3 = 2$$

$$9 \% 3 = 0$$

$$10 \% 3 = 1$$

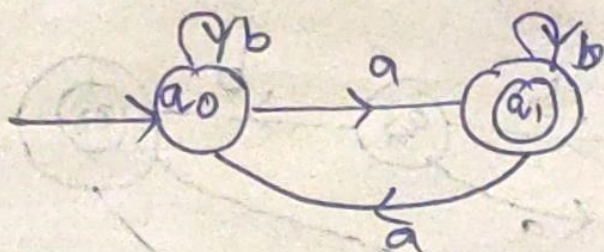
$$11 \% 3 = 2$$

$$12 \% 3 = 0$$



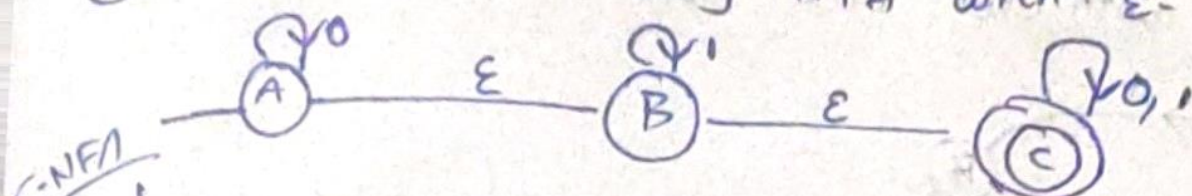
⑧ a) $L = \{ w/w \text{ is any string that doesn't contain exactly two a's} \}$

Ans! $L = \{ a, b, ab, abb, aaab, \dots \}$



b) $L = \{ w/w \text{ is any string that contain at most 3 a's} \}$

Q1) Convert the following NFA with ϵ -NFA



ϵ -NFA

δ	0	1	ϵ
A	A	ϕ	B
B	ϕ	B	C
C	C	C	ϕ

$$\epsilon\text{-closure}(A) = \{A, B, C\}$$

$$\epsilon\text{-closure}(B) = \{B, C\}$$

$$\epsilon\text{-closure}(C) = \{C\}$$

NFA

δ'	0	1
$\rightarrow A$	$\{A, B, C\}$	$\{B, C\}$
$* B$	C	$\{B, C\}$
$* C$	C	C

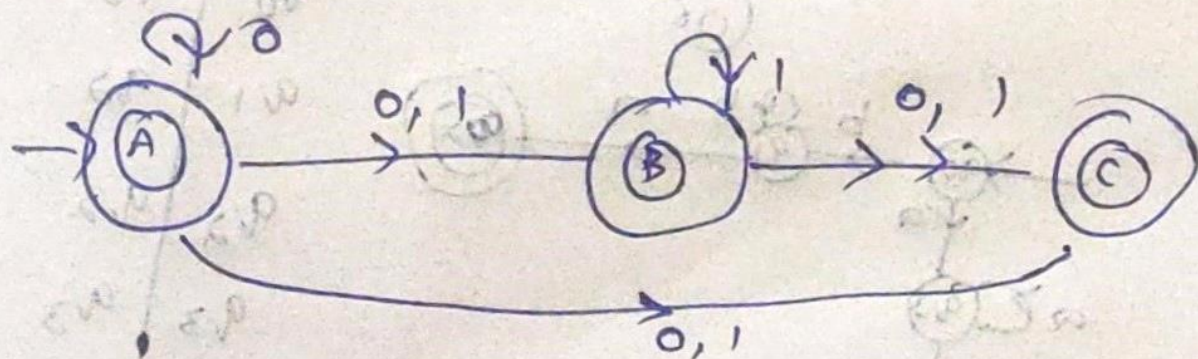
$$\begin{aligned}
 \delta'(A, 0) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(A, 0))) \\
 &= \epsilon\text{-closure}(\delta(\delta(A, 0) \cup \delta(B, 0) \cup \delta(C, 0))) \\
 &= \epsilon\text{-closure}(A \cup \phi \cup C) \\
 &= \epsilon\text{-closure}(A) \cup \epsilon\text{-closure}(C) \\
 &= \{A, B, C\} \cup \{C\} \\
 &= \{A, B, C\}
 \end{aligned}$$

$$\begin{aligned}
 \delta'(A, 1) &= \delta(\delta(\delta(A, 1))) \\
 &= \delta(\delta(\phi \cup B \cup C)) \\
 &= \delta(B, 1) \\
 &= \delta(B \cup C) \\
 &= \{B, C\} \cup \{C\} \\
 &= \{B, C\}
 \end{aligned}$$

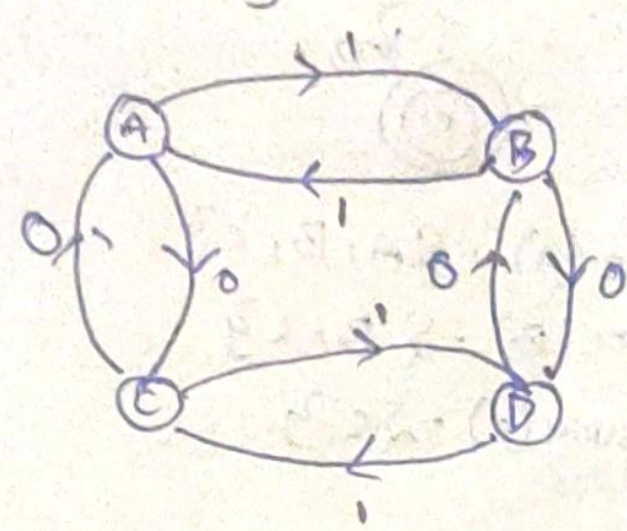
Same For

$$\delta'(B, 0), \delta'(B, 1)$$

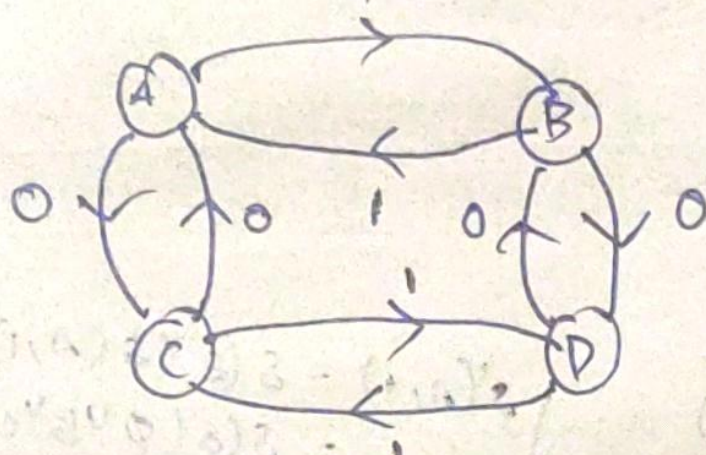
$$\delta'(C, 0), \delta'(C, 1)$$



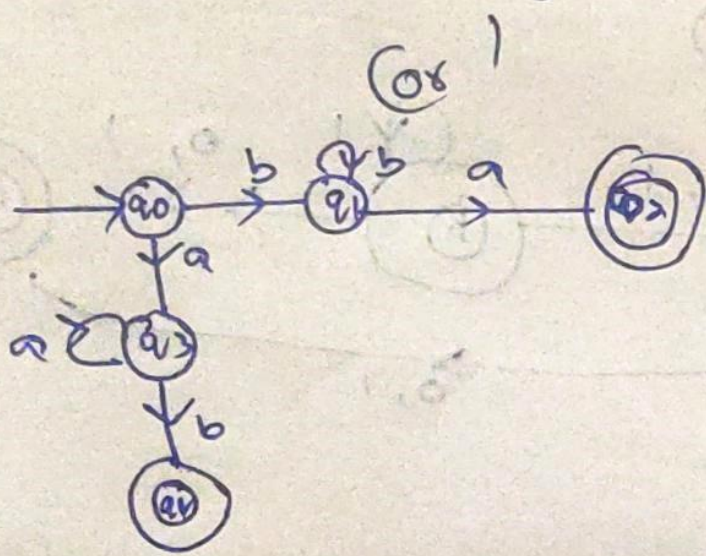
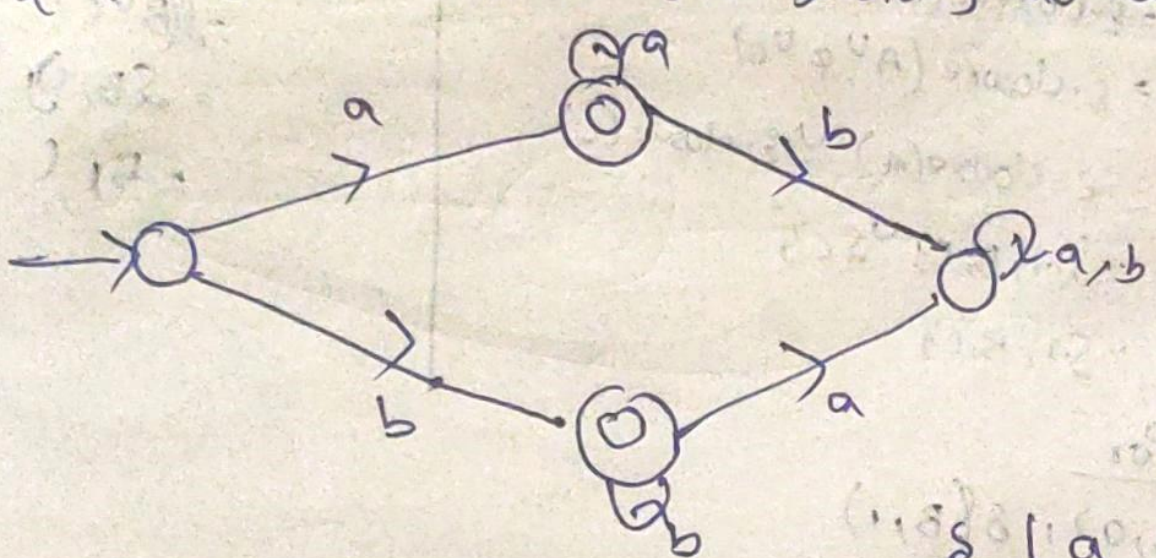
10) a) The string with even no. of 0's and odd no. of 1's



b) The string with odd no. of 0's and odd no. of 1's

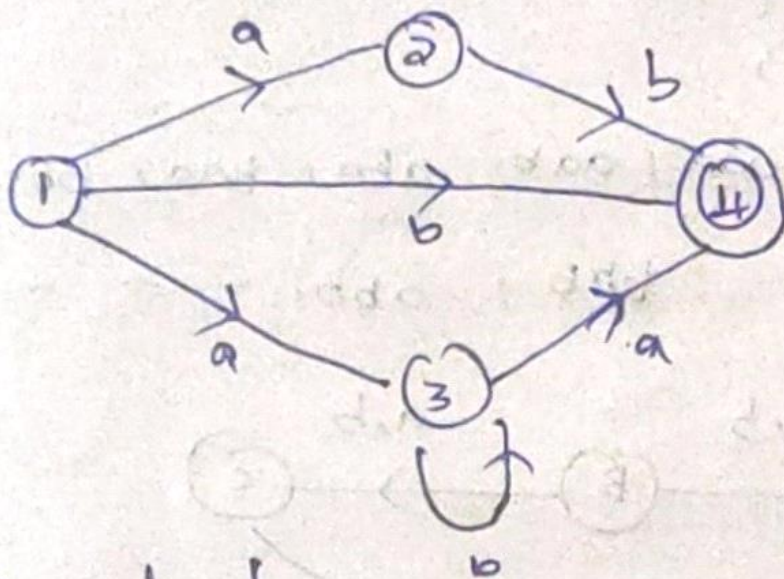


11) $L = \{ w \mid w \text{ contains neither the substring } ab \text{ nor } ba \}$

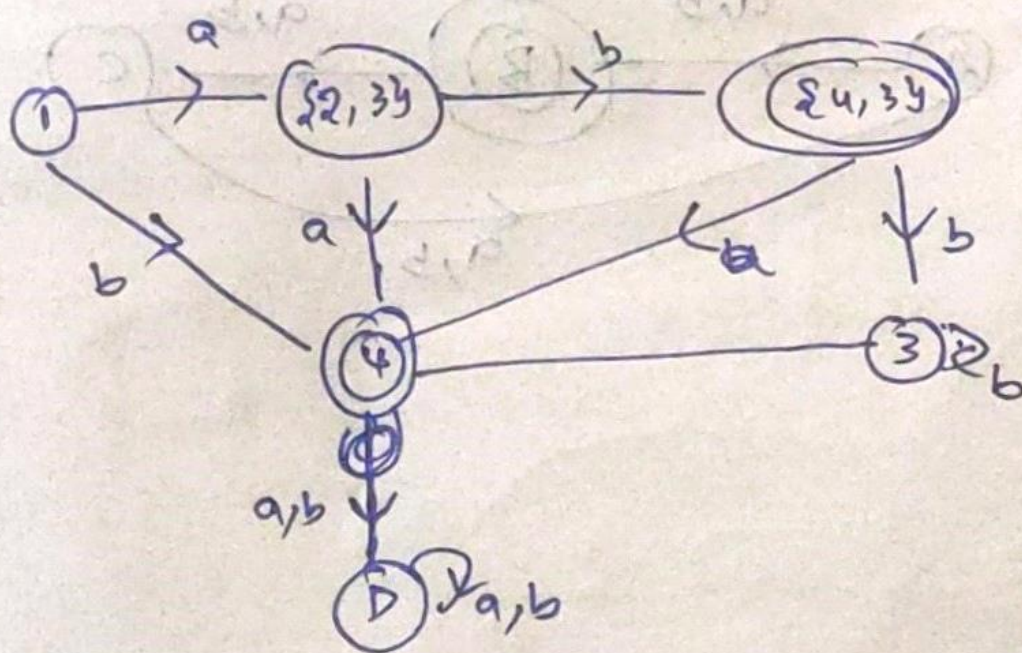


δ	a	b
q_0	q_3	q_1
q_1	q_2	q_1
q_2	q_2	q_3
q_3	q_3	q_4

12 Convert the following NFA into DFA



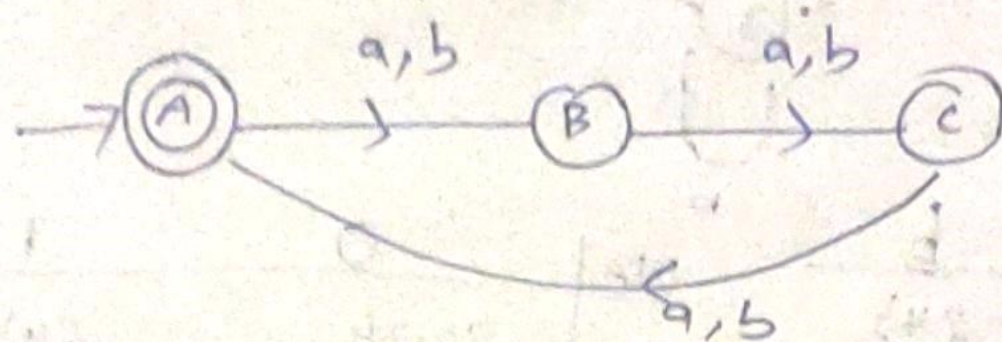
NFA	a	b	DFA	0	1
→ 1	{2, 3}	{4}	→ 1	{2, 3}	{4}
2	∅	{4}	{2, 3}	{4}	{4, 3}
3	{4}	{3}	{2, 3}	{4}	{4, 3}
*4	∅	∅	*{4, 3}	{4}	{3}
			{3}	{4}	{3}
			*{4}	∅	∅



13) a) mod 3 = 0

$\Sigma = \{a, b\}$

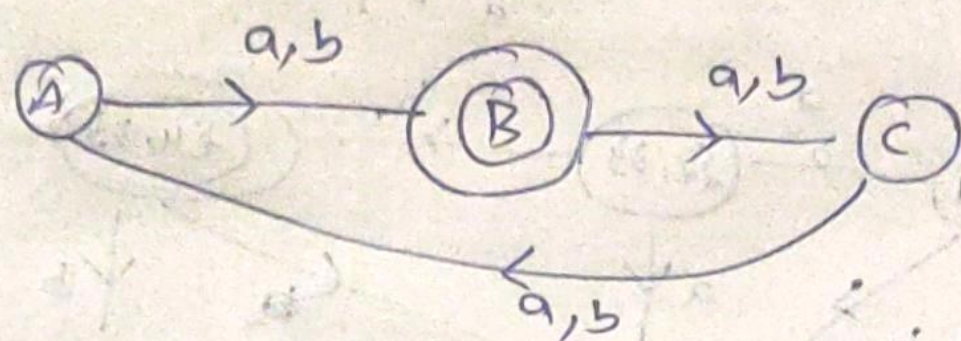
$L = \{ \epsilon, aaa, aab, aba, baa, bbb, bba, bab, abb, \dots \}$



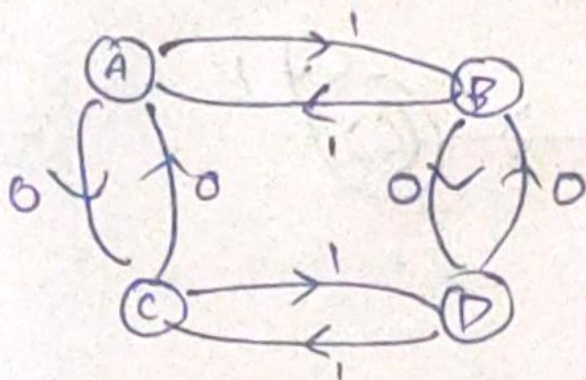
b) mod 3 = 1

$\Sigma = \{a, b\}$

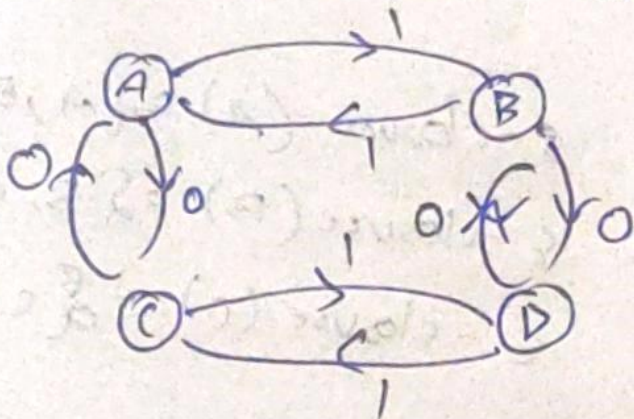
$L = \{ aaaa, aabb, \dots \}$



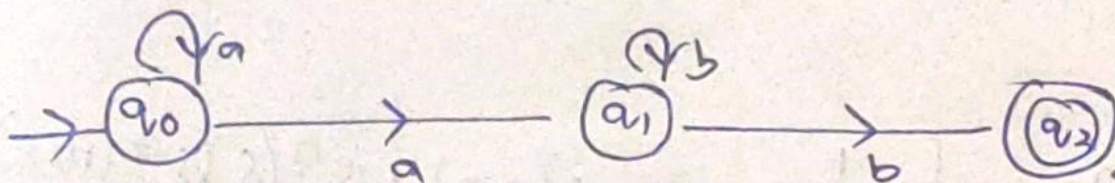
(14) a) even no. of 0's and even no. of 1's



b) odd no. of 0's and even no. of 1's



(15) NFA to DFA

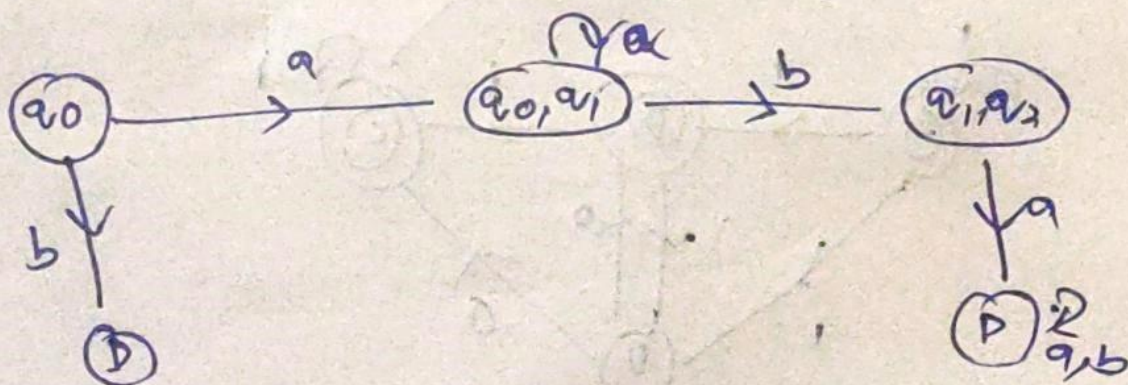


NFA

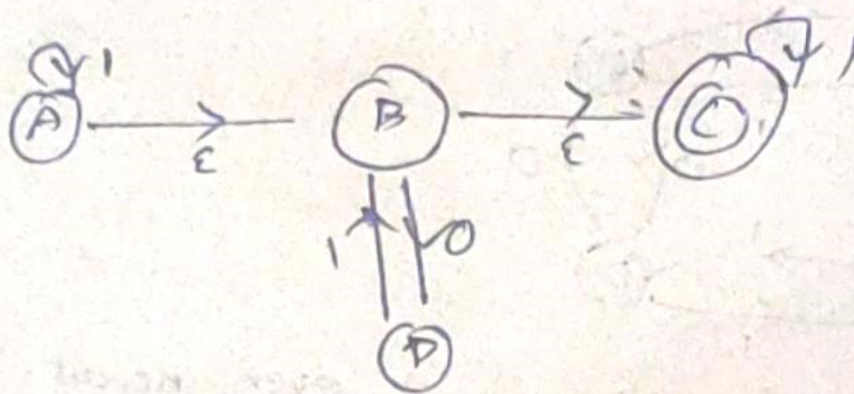
δ	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	\emptyset
q_1	\emptyset	$\{q_1, q_2\}$
q_2	\emptyset	\emptyset

DFA

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	\emptyset
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	\emptyset	$\{q_1, q_2\}$



16) ϵ -NFA to NFA



ϵ -NFA

S	0	1	ϵ
A	\emptyset	A	B
B	\emptyset	D	C
C	\emptyset	C	\emptyset
D	B	\emptyset	\emptyset

$$\epsilon\text{-closure}(A) = \{A, B, C\}$$

$$\epsilon\text{-closure}(B) = \{B, C\}$$

$$\epsilon\text{-closure}(C) = \{C\}$$

NFA

S'	0	1
$\rightarrow A$	$\{\emptyset\}$	$\{A, B, C, D\}$
B	$\{\emptyset\}$	$\{C, D\}$
C	$\{\emptyset\}$	$\{C\}$
D	$\{B, C\}$	$\{\emptyset\}$

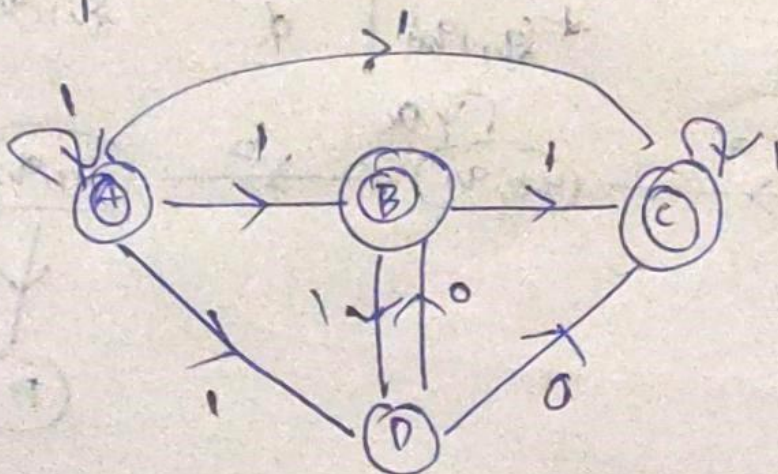
$$S'(A, 0) = S^*(S(S^*(A, \epsilon)0)\epsilon)$$

$$S^*(S(A, B, C)0)\epsilon)$$

$$S^*(S(A, 0) \cup S(B, 0) \cup S(C, 0))\epsilon)$$

$$S^*(\emptyset \cup \emptyset \cup \emptyset)C$$

\emptyset



(17) Same as (13) one

(18) Same as (16) one

(19) starts with 01 and ends with 01

