Nodal Point Optimization for Vibration Reduction in Rotor Blade

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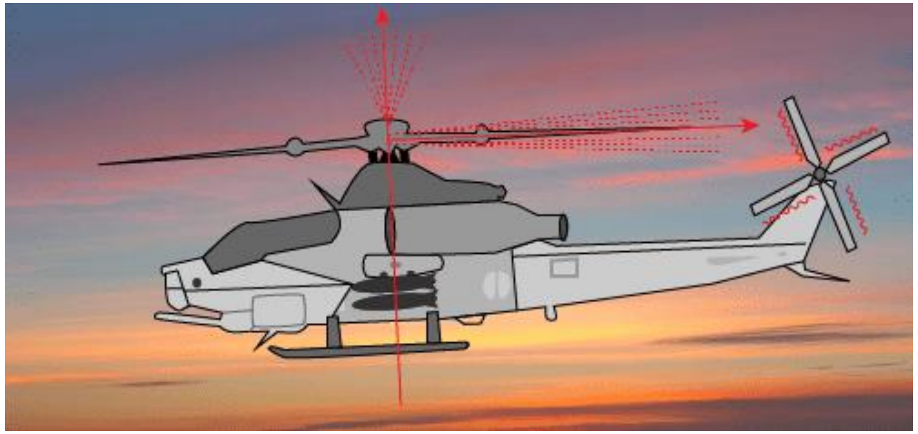
Structural Optimization

The concept of modal shaping has been proposed as a method to reduce structural vibration. [2] [3].

Introduction

- In structural dynamics, modal analysis is one of the key aspects for studying mechanical vibrations and thus it is necessary to include vibration analysis in the design process.
- In helicopters, rotors at high rpm, it is possible to have high vibrations which can lead to self destruction.
- Research done by D.A. Peters et al. [1], it was concluded that nodal point placement has the potential for reducing overall response by placing the node at a strategic location of force distribution to reduce the generalized force. This concept of modal shaping has been proposed as a method to reduce structural vibration. [2] [3]
- Typical candidates for nodal point placement are locations where low response for required, such as pilot or passenger sears, locations of sensitive electronic equipment etc.

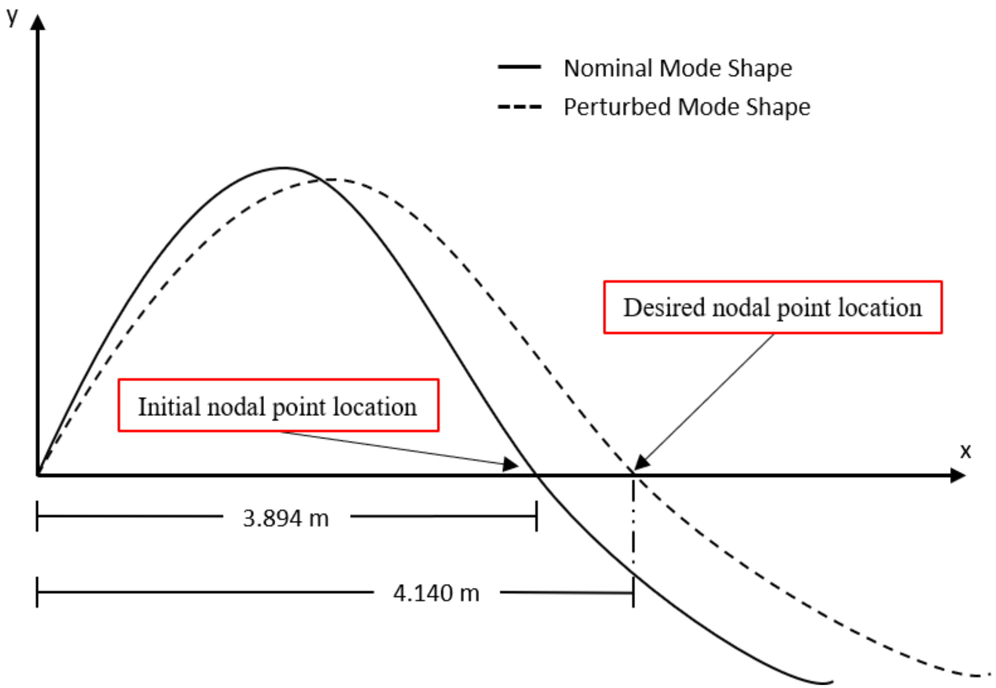


5 STEP PROBLEM

- Problem Description
- Background
- Design Variables
- Objective
- Constraints

Problem Description

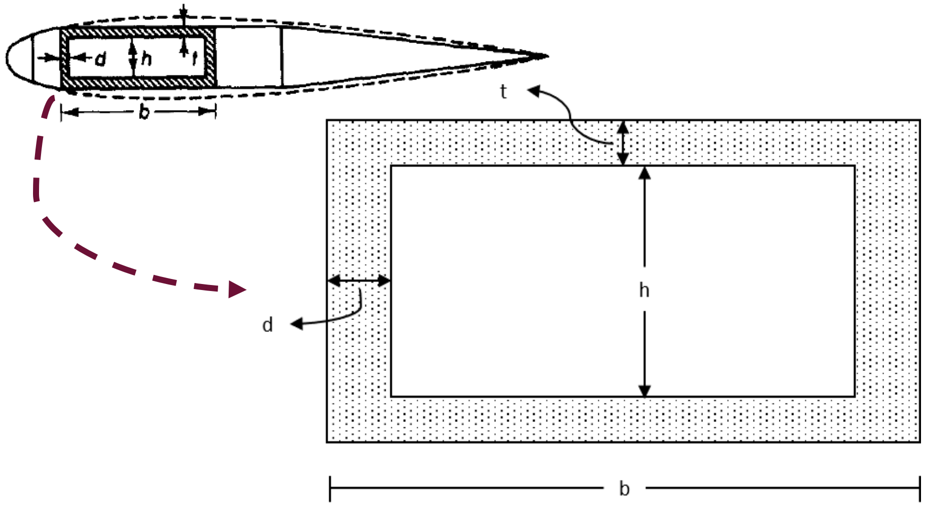
- Aim is to bring the nodal location of the second mode shape for a helicopter rotor blade
- This nodal replacement will effect directly on reducing the generalized force acting on the helicopter blade which would result in the minimization of vibration responses.
- This is successfully done by varying the lumped masses such that nodal location reaches the desired location
- This project is based on Journal Paper by J. I. Pritchard et al. Used as reference and for verifications of results.



Background Information

Element Number	E, Pa	ρ, kg/m³	b, m	h, m	t, m	d, m
1	3.378 E10	1.938 E3	9.530 E-2	6.400 E-2	2.030 E-2	2.540 E-3
2 - 10	4.033 E10	1.938 E3	9.530 E-2	6.400 E-2	2.030 E-2	2.540 E-3

Grid Point Number	3	4	6	7	8	9	10	11
Initial Mass, Kg	1.380	0.757	2.900	3.380	4.880	2.360	2.970	2.990



- Research done in [1] shows that by placing the nodal point at 164.0 in will help in reducing down the generalized force.
- Moving node locations is more effective than other vibration control techniques in certain scenarios
  - Low response amplitude is required at a particular place
  - Make the mode shape orthogonal to force distribution

Identification

- Design Variables
  - Among all eight lumped masses at each node, masses 9, 10, and 11 are considered as my design variables
- Constraint Function
  - constraint function is chosen such that the difference in the distance of the perturbed nodal location from the optimum location should be less than the allowable distance
  - For our analysis we have taken allowable distance $\delta = 1\ in$ and x_{np} is the nodal location computed on a set of design iterations

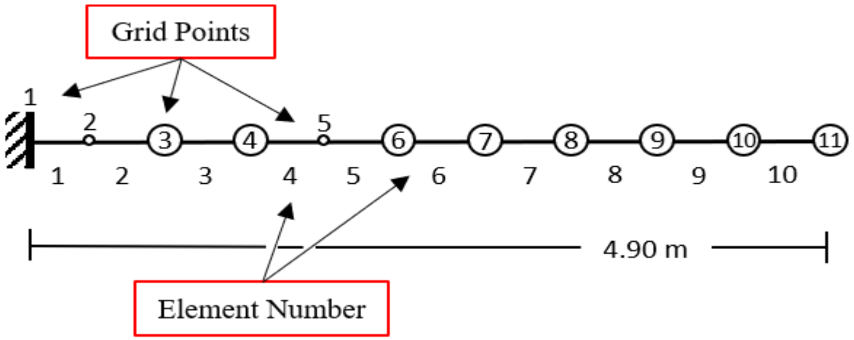
$$g = \left|x_{op} - x_{np}\right| \leq \delta$$

- Objective Function
  - The objective function f is the sum of all the lumped masses, taken as design variables

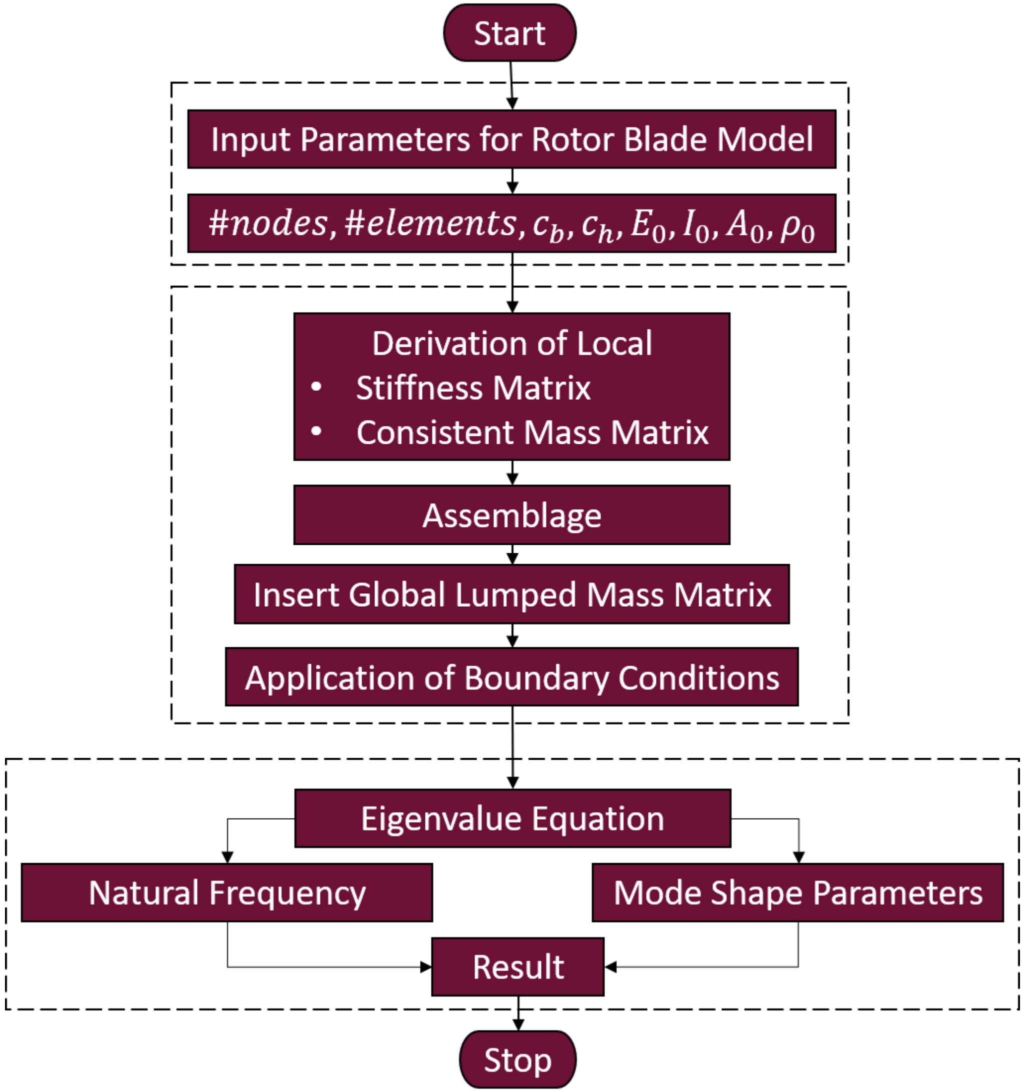
$$f = \sum_{n=1}^{N} M_n$$

Modal Analysis

- FEM Structure
  - A rotor blade or any aircraft wing can be modeled as a 10 elements cantilever beam structure, having a lumped mass placed at nodes = 3, 4, 6, 7, 8, 9, 10
  - Designed own code to implement Finite Element Methods to perform modal analysis



- Eigenvalue Equation
  - Mode Shape and Displacements
  - Natural Frequencies



SENSITIVITY ANALYSIS

- Nodal Point Derivative
- Eigenvector
- Eigenvalue

Nodal Point Derivative

- The formulation of the derivative of the nodal location is based on expanding the perturbed mode in a Taylor series about the nominal nodal point.

$$u(x_{np} + dx_{np}, v + dv) = u(x_{np}, v) + \left.\frac{\partial u}{\partial x}\right|_{x_{np},v} dx_{np} + \left.\frac{\partial u}{\partial v}\right|_{x_{np},v} dv$$

- As we know displacement at nodal location is zero, this means

$$u(x_{np}, v) = 0$$
$$u(x_{np} + dx_{np}, v + dv) = 0$$

- $\frac{\partial u}{\partial x}$: slope of the mode shape at the nodal location.
- $\frac{\partial u}{\partial v}$: derivative of the mode shape w.r.t design variable

$$\left.\frac{\partial u}{\partial x}\right|_{x_{np},v} dx_{np} + \left.\frac{\partial u}{\partial v}\right|_{x_{np},v} dv = 0 \quad \Rightarrow \quad \boxed{\frac{dx_{np}}{dv} = -\left[\frac{\frac{\partial u}{\partial v}}{\frac{\partial u}{\partial x}}\right]_{xnp,v}}$$

Eigenvector Derivative

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- Nelson's Method used to solve eigenvectors derivatives with respect to design variables. From the theoretical understanding Nelson's method subroutine is written to compute all the sensitivity values
- Analytical Review
  - Eigenvalue problem

$$(K - \lambda M)\Phi = 0$$

  - Differentiating this Equation

$$(K - \lambda_j M)\Phi_j' = -[K' - \lambda_j' M - \lambda_{jM'}]\Phi_j$$

  - Let, $(K - \lambda_j M) = F_j$ and $[K' - \lambda_j' M - \lambda_{jM'}] = F_j'$ $\Rightarrow$ $F_j \Phi_j' = -F_j' \Phi_j$ $\Rightarrow$ $\Phi_j' = -[F_j]^{-1} F_j' \Phi_j$
  - Since F is a singular matrix, we need to curtail F by removing a row and column

$$\widetilde{V_j} = -[\widetilde{F_j}]^{-1}[F_j{}']\,\Phi_j$$

  - Final Solution becomes

$$\boxed{\Phi_j' = \widetilde{V_j} + c_j \Phi_j \qquad \text{where, } c_j = -\Phi_j^T M \widetilde{V_j} + \frac{\Phi_j^T M' \Phi_j}{2}}$$

Eigenvalue Derivative

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- To successfully obtain eigenvector derivative we need to have K', M', λ'
- We know, $K' = 0$ and $M' = 1$. To compute λ' we will perform eigenvalue derivative
- Analytical Review
  - Differentiating the eigenvalue problem

$$(K - \lambda_j M)\Phi_j' = \lambda_j' M\Phi_j - K'\Phi_j + \lambda_j M'\Phi_j$$

  - On multiplying the above equation with Φ_j^T we get

$$\Phi_j^T(K - \lambda_j M)\Phi_j' = \lambda_j'\Phi_j^T M\Phi_j - \Phi_j^T K'\Phi_j + \lambda_j \Phi_j^T M'\Phi_j$$

  - Since, $\Phi_j^T M\Phi_j = 1$ and we know that $\Phi_j^T(K - \lambda_j M)\Phi_j' = 0$

$$\boxed{\lambda_j' = \Phi_j^T K'\Phi_j - \lambda_j \Phi_j^T M'\Phi_j}$$

Sensitivity Results

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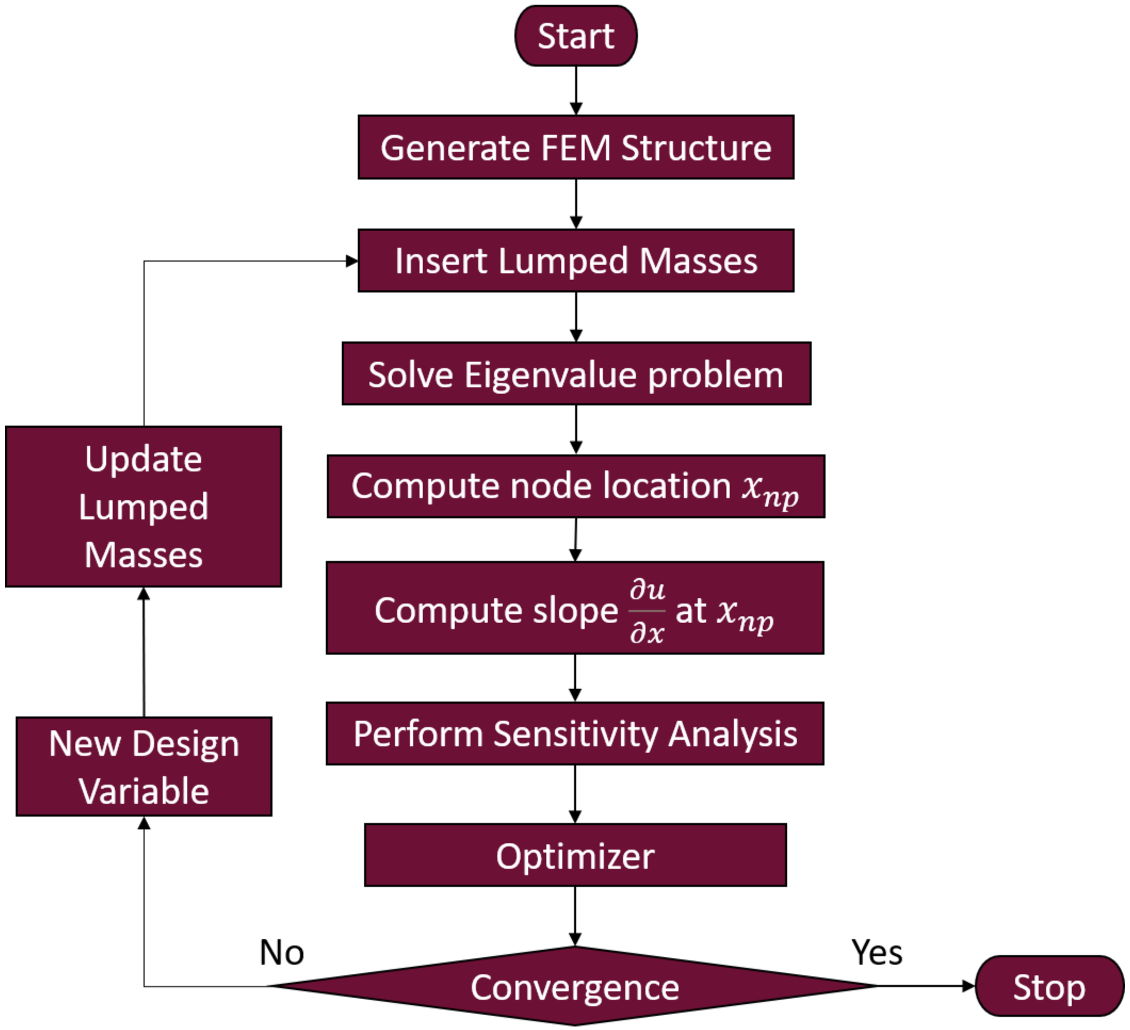
- $\frac{\partial x_{np}}{\partial v} > 0$: mass increase, nodal point moves right
- $\frac{\partial x_{np}}{\partial v} < 0$: mass increase, nodal point moves left
- Mass 10 and 11 → most effective to move the node towards right
- Similarly, decrease the masses at 10, 11 or increase the masses at 6, 7 will have the largest effect on moving nodal point towards left
- Increasing masses at grid points 6 and 7 have highest impact on moving nodal point to the left

Mass at # Grid Point	Sensitivity Analysis					
	$\frac{\partial x_{np}}{\partial v}$			$\frac{\partial u}{\partial x}$	$\frac{\partial u}{\partial v}$	$\frac{\partial \phi}{\partial v}$
	My Solution	Verification from paper				
	Nelson Method	Analytical Method	Finite Difference	Slope	Eigenvector Sensitivity	Eigenvalue Sensitivity
3	-0.0253	-0.0278	-0.0277	-0.0036	-0.0001	-0.0995
4	-0.0824	-0.0881	-0.088		-0.0003	-0.3015
6	-0.2211	-0.231	-0.23		-0.0008	-0.6047
7	-0.2294	-0.237	-0.236		-0.0008	-0.4555
8	-0.1636	-0.166	-0.165		-0.0006	-0.1683
9	-0.0056	-0.0038	-0.00361		0	-1.29E-04
10	0.3048	0.309	0.309		0.0011	-0.2226
11	0.8229	0.828	0.826		0.003	-0.9775

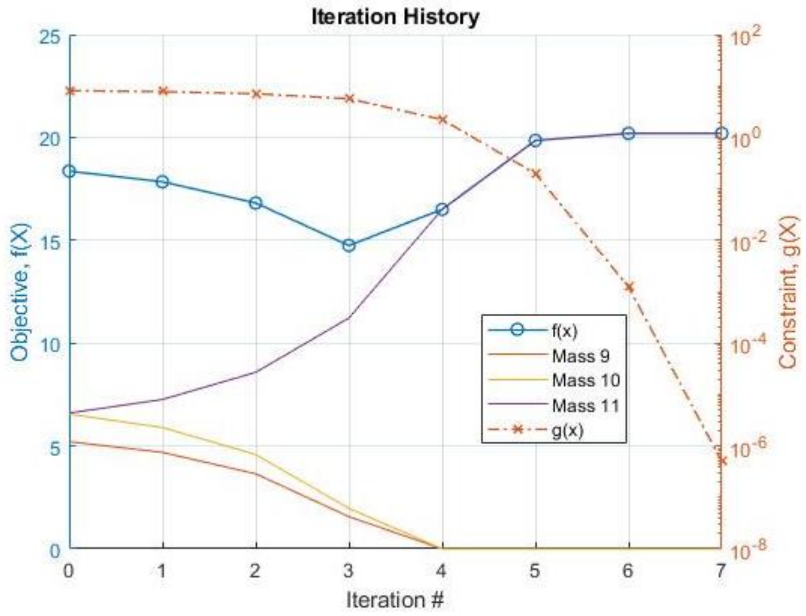
Optimization Approach

- The first computation is to generate the structural model and introducing lumped masses to the system.
- Perform vibrational analysis, and obtain
  - x_{np} : Nodal location
  - $\left.\frac{\partial u}{\partial x}\right|_{x_{np},v}$: Slope of mode shape at nodal location
  - $\left.\frac{\partial u}{\partial v}\right|_{x_{np},v}$: Eigenvector derivative at nodal location
- Compute nodal point sensitivity
- Pass it to Optimizer (slp_trust – Dr. Robert Canfield)
  - Script that returns, objective and constraint
  - Script that returns gradients of objective and constraints
  - Initial design variables, and bounds for design variables
- If converged stop, otherwise change design variables and update lumped mass matrix and reiterate.



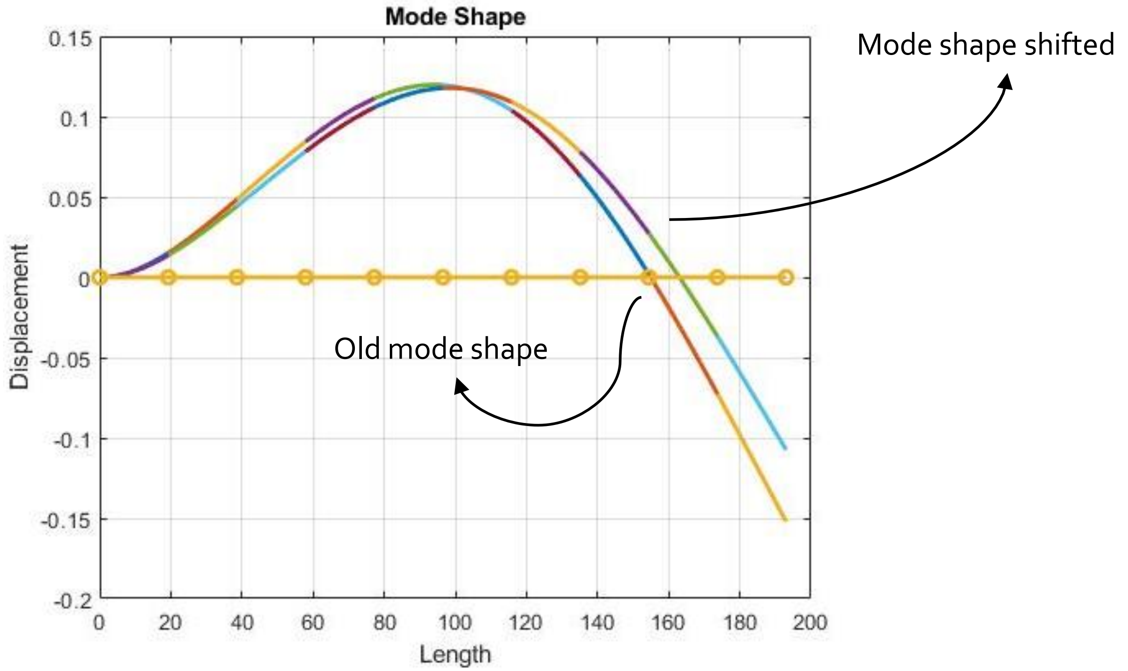
Optimization Results

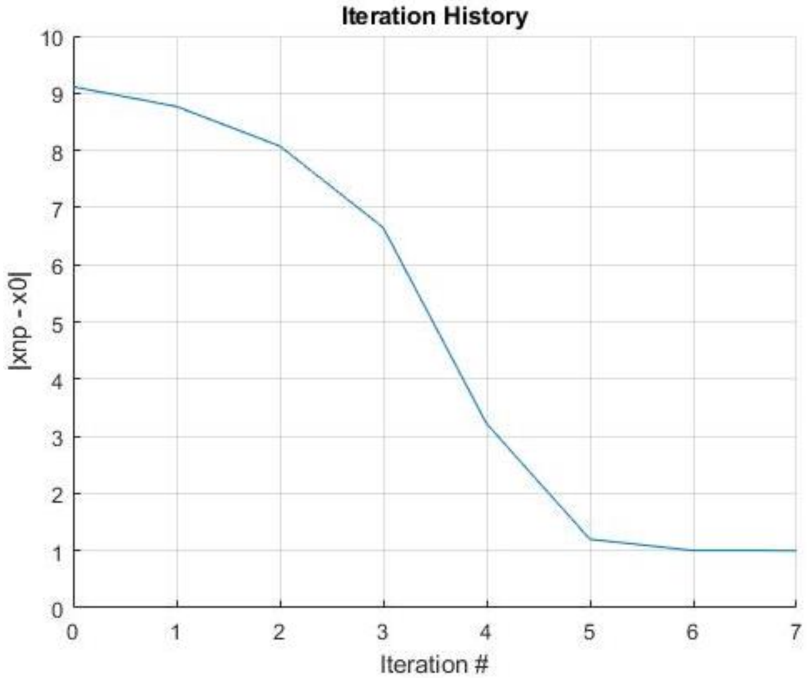
- Initial Masses are chosen as the given values of lumped masses
- Results are shown in table, and it was found that mass 11 which had the highest sensitive drives the nodal point location.
- Mass 11 is increased to 20.198 whereas Mass 9 and Mass 10 are reduced down to zero. This leads to the total mass of design variables equals to 20.198 lbs



Parameter	Design	
	Initial	Final
Mass 9	5.21	5.44E-11
Mass 10	6.55	5.28E-11
Mass 11	6.60	20.198
Total Mass	18.36	20.198
Nodal Location	154.8873	163.00

Optimization Results





- We can see that now after choosing the new masses, our mode shape is changed, and the nodal point is shifted to the desired location
- Right side, the iterations history shows the convergence for the constraint function.

Conclusion

- Nodal point placement method for controlling vibration at desired location is demonstrated
- Using slp with trust regions, the solutions get converged in less than 10 iterations.
- Having 0, 0, 20.198 lbs at grid point 9, 10 and 11 respectively will result in nodal location as 163.0
- Placing the nodal point at the centroid of load distribution minimizes the generalized load
- Future Work:
  - Adding rotational velocity of 425 rpm to the system and reanalyzing the entire optimization.
  - Creating current problem into multi-objective problem by minimizing the generalized force.

Possible Questions

- Why the second mode shape?
  - The only reason to take second mode shape is to deal only with one nodal location. Thus avoiding extensive calculation for eigenvalue sensitivity for every nodal location and thus making the optimization simpler.
  - The results from consideration of second mode can be considered as proof-of-concept study to reduce the generalized force over the rotor blade.

- In Sensitivity Analysis for computing Eigenvector sensitivity, where we curtail F matrix to remove the singularity, how to choose which row and column. Is it random, first or last?
  - It is based on the highest value of eigenvector, location/index at which there is highest eigenvector, will be removed.
  - I tried removing random row and respective column, but it did not make much difference. But the recommendation is to remove the max(eigenvector)

References

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[2] R. B. Taylor, Helicopter rotor blade design for minimum vibration, Vols. NASA CR-3825, Anaheim: National Aeronautics and Space Administration, Scientific and Technical~..., 1984.

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[4] H. A. R. H. J.I. Pritchard, "Sensitivity analysis and optimization of nodal point placement for vibration reduction," *Journal of Sound and Vibration,*, vol. 2, no. 119, pp. 277-289, 1987.

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Lessons Learned

- Course AOE 5064 – Structural Optimization has been found very useful to study and learn each and every concepts for the completion of this project.
- Learned to design a 5-step optimization problem,
- Popular approaches such as Nelson's Method for eigenvector derivatives, sequential linear programming (slp), sequential quadratic programming (sqp), finite difference and complex step methods.
- Analytical and computer programming techniques to solve for optimization problem.
- Advantages of technique such as Trust Region strategy used for faster convergence.

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