http://algs4.cs.princeton.edu

2.3 Quicksort

- *quicksort*
- *selection*
- *duplicate keys*
- *system sorts*

Sorting applications

Sorting algorithms are essential in a broad variety of applications:

- Sort a list of names.
- Organize an MP3 library.
- Display Google PageRank results.
- List RSS feed in reverse chronological order.

obvious applications

- Find the median.
- Identify statistical outliers.
- Binary search in a database.
- Find duplicates in a mailing list.

problems become easy once items are in sorted order

- Data compression.
- Computer graphics.
- Computational biology.
- Load balancing on a parallel computer.

. . .

non-obvious applications

War story (system sort in C)

A beautiful bug report. [Allan Wilks and Rick Becker, 1991]

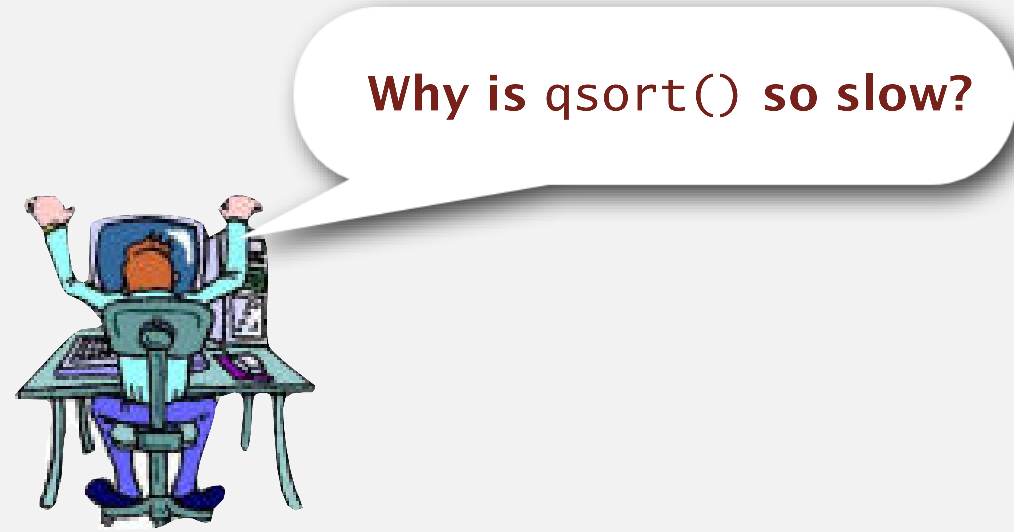
```
We found that qsort is unbearably slow on "organ-pipe" inputs like "01233210":

main (int argc, char**argv) {
   int n = atoi(argv[1]), i, x[100000];
   for (i = 0; i < n; i++)
      x[i] = i;
   for ( ; i < 2*n; i++)
      x[i] = 2*n-i-1;
   qsort(x, 2*n, sizeof(int), intcmp);
}

Here are the timings on our machine:
$ time a.out 2000
real    5.85s
$ time a.out 4000
real   21.64s
$time a.out 8000
real   85.11s
```

War story (system sort in C)

Bug. A `qsort()` call that should have taken seconds was taking minutes.



At the time, almost all `qsort()` implementations based on those in:

- Version 7 Unix (1979): quadratic time to sort organ-pipe arrays.
- BSD Unix (1983): quadratic time to sort random arrays of 0s and 1s.

Engineering a system sort (in 1993)

Bentley-McIlroy quicksort.

samples 9 items

- Cutoff to insertion sort for small subarrays.
- Partitioning item: median of 3 or Tukey's ninther.
- Partitioning scheme: Bentley-McIlroy 3-way partitioning.

similar to Dijkstra 3-way partitioning
(but fewer exchanges when not many equal keys)

Engineering a Sort Function

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SUMMARY

We recount the history of a new `qsort` function for a C library. Our function is clearer, faster and more robust than existing sorts. It chooses partitioning elements by a new sampling scheme; it partitions by a novel solution to Dijkstra's Dutch National Flag problem; and it swaps efficiently. Its behavior was assessed with timing and debugging testbeds, and with a program to certify performance. The design techniques apply in domains beyond sorting.

Very widely used. C, C++, Java 6, ...

A beautiful mailing list post (Yaroslavskiy, September 2009)

Replacement of quicksort in java.util.Arrays with new dual-pivot quicksort

```
Hello All,

I'd like to share with you new Dual-Pivot Quicksort which is faster than the
known implementations (theoretically and experimental). I'd like to propose
to replace the JDK's Quicksort implementation by new one.

...

The new Dual-Pivot Quicksort uses *two* pivots elements in this manner:

1. Pick an elements P1, P2, called pivots from the array.
2. Assume that P1 <= P2, otherwise swap it.
3. Reorder the array into three parts: those less than the smaller pivot,
   those larger than the larger pivot, and in between are those elements
   between (or equal to) the two pivots.
4. Recursively sort the sub-arrays.

The invariant of the Dual-Pivot Quicksort is:

[ < P1 | P1 <= & <= P2 } > P2 ]

...
```

http://mail.openjdk.java.net/pipermail/core-libs-dev/2009-September/002630.html

A beautiful mailing list post (Yaroslavskiy-Bloch-Bentley, October 2009)

Replacement of quicksort in java.util.Arrays with new dual-pivot quicksort

```
Date: Thu, 29 Oct 2009 11:19:39 +0000
Subject: Replace quicksort in java.util.Arrays with dual-pivot implementation

Changeset: b05abb410c52
Author:    alanb
Date:      2009-10-29 11:18 +0000
URL:       http://hg.openjdk.java.net/jdk7/tl/jdk/rev/b05abb410c52

6880672: Replace quicksort in java.util.Arrays with dual-pivot implementation
Reviewed-by: jjb
Contributed-by: vladimir.yaroslavskiy at sun.com, joshua.bloch at google.com,
jbentley at avaya.com

! make/java/java/FILES_java.gmk
! src/share/classes/java/util/Arrays.java
+ src/share/classes/java/util/DualPivotQuicksort.java
```

http://mail.openjdk.java.net/pipermail/compiler-dev/2009-October.txt

Dual-pivot quicksort

Use two partitioning items p_1 and p_2 and partition into three subarrays:

- Keys less than p_1.
- Keys between p_1 and p_2.
- Keys greater than p_2.

$< p_1$	p_1	$\geq p_1$ and $\leq p_2$	p_2	$> p_2$
↑ lo	↑ lt		↑ gt	↑ hi

Recursively sort three subarrays.

degenerates to Dijkstra's 3-way partitioning

Note. Skip middle subarray if $p_1 = p_2$.

Dual-pivot partitioning demo

Initialization.

- Choose `a[lo]` and `a[hi]` as partitioning items.
- Exchange if necessary to ensure `a[lo] ≤ a[hi]`.

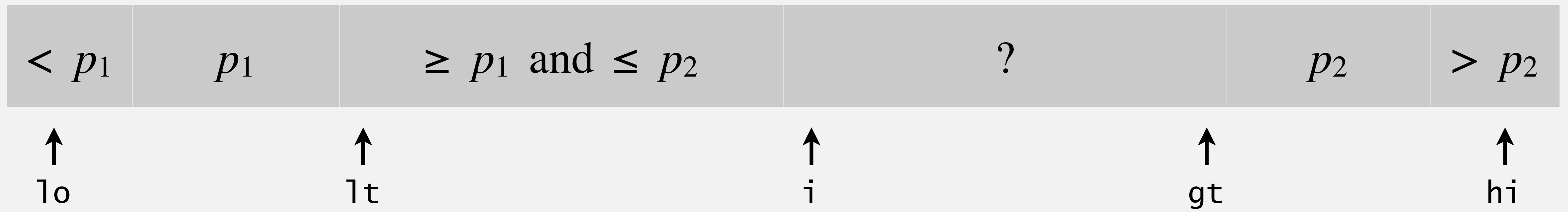
S	E	A	Y	R	L	F	V	Z	Q	T	C	M	K
↑ lo													↑ hi

exchange a[lo] and a[hi]

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

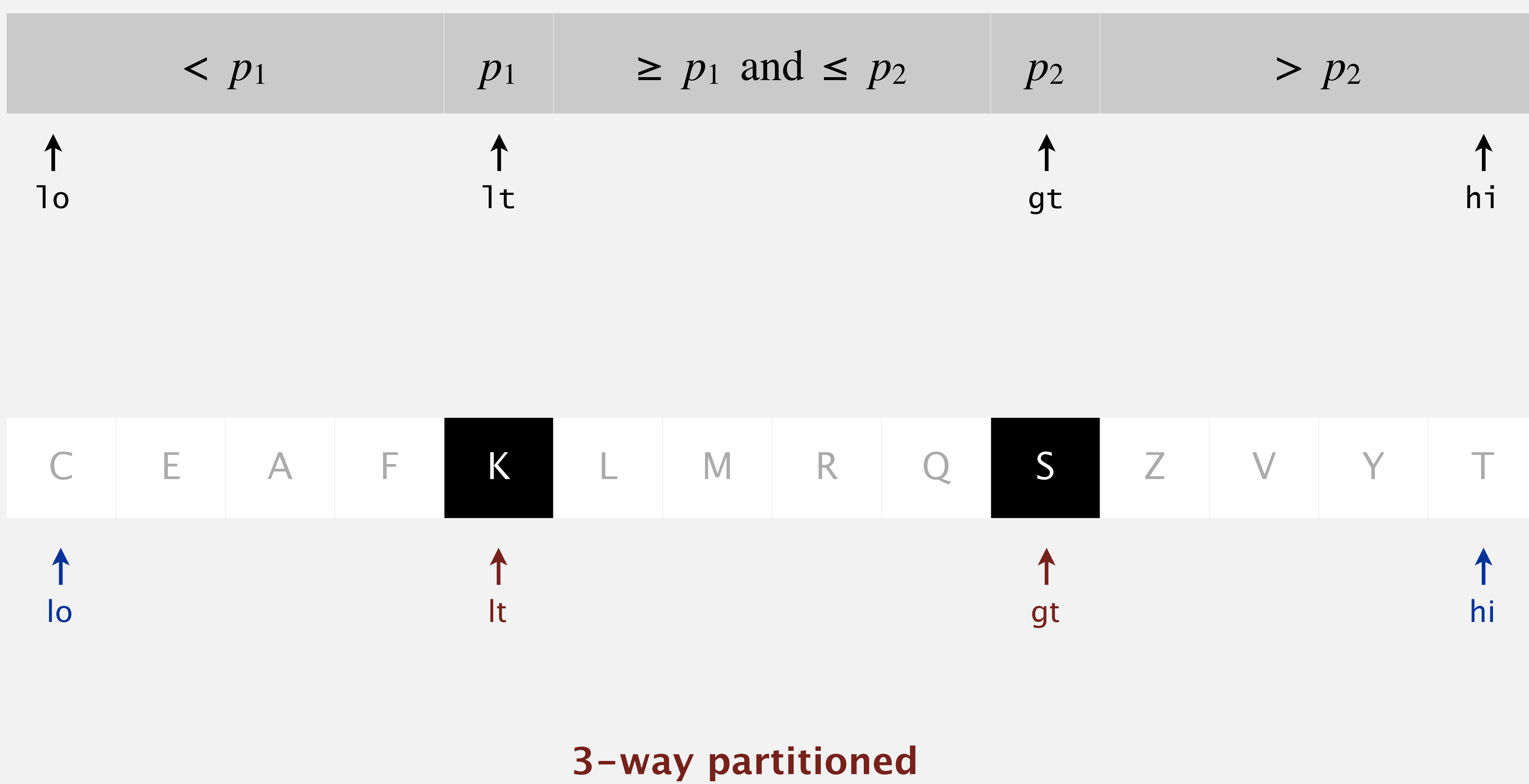
- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.



Dual-pivot partitioning demo

Finalize.

- Exchange `a[lo]` with `a[--lt]`.
- Exchange `a[hi]` with `a[++gt]`.



Dual-pivot quicksort

Use two partitioning items p_1 and p_2 and partition into three subarrays:

- Keys less than p_1.
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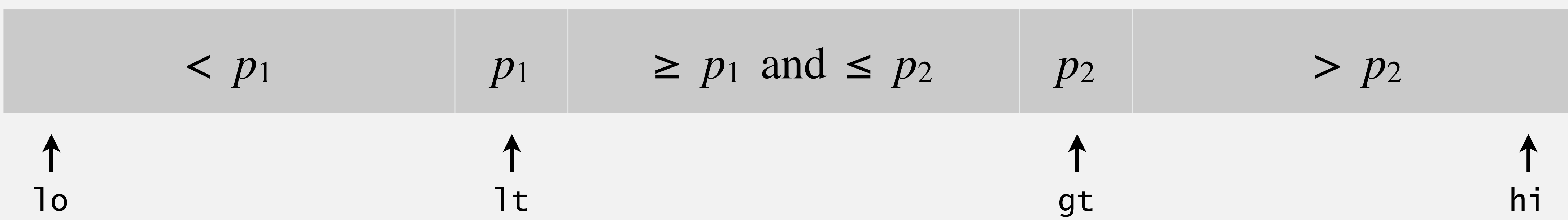
- Keys greater than p_2.



Now widely used. Java 7, Python unstable sort, Android, …

Three-pivot quicksort

Use three partitioning items p_1, p_2, and p_3 and partition into four subarrays:

- Keys less than p_1.
- Keys between p_1 and p_2.
- Keys between p_2 and p_3.
- Keys greater than p_3.

$< p_1$	p_1	$\geq p_1$ and $\leq p_2$	p_2	$\geq p_2$ and $\leq p_3$	p_3	$> p_3$
↑ `lo`	↑ `a1`		↑ `a2`		↑ `a3`	↑ `hi`

Multi-Pivot Quicksort: Theory and Experiments

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Quicksort quiz 4

Why do 2-pivot (and 3-pivot) quicksort perform better than 1-pivot?

A. Fewer compares.

B. Fewer exchanges.

C. Fewer cache misses.

D. *I don't know.*

Quicksort quiz 4

Why do 2-pivot (and 3-pivot) quicksort perform better than 1-pivot?

A. ~~Fewer compares.~~

B. ~~Fewer exchanges.~~

C. Fewer cache misses.

entries scanned is a good proxy for cache performance when comparing quicksort variants

partitioning	compares	exchanges	entries scanned
1-pivot	$2\ N \ln N$	$0.333\ N \ln N$	$2\ N \ln N$
median-of-3	$1.714\ N \ln N$	$0.343\ N \ln N$	$1.714\ N \ln N$
2-pivot	$1.9\ N \ln N$	$0.6\ N \ln N$	$1.6\ N \ln N$
3-pivot	$1.846\ N \ln N$	$0.616\ N \ln N$	$1.385\ N \ln N$

Reference: Analysis of Pivot Sampling in Dual-Pivot Quicksort by Wild-Nebel-Martínez

Bottom line. Caching can have a significant impact on performance.

beyond scope of this course

Which sorting algorithm to use?

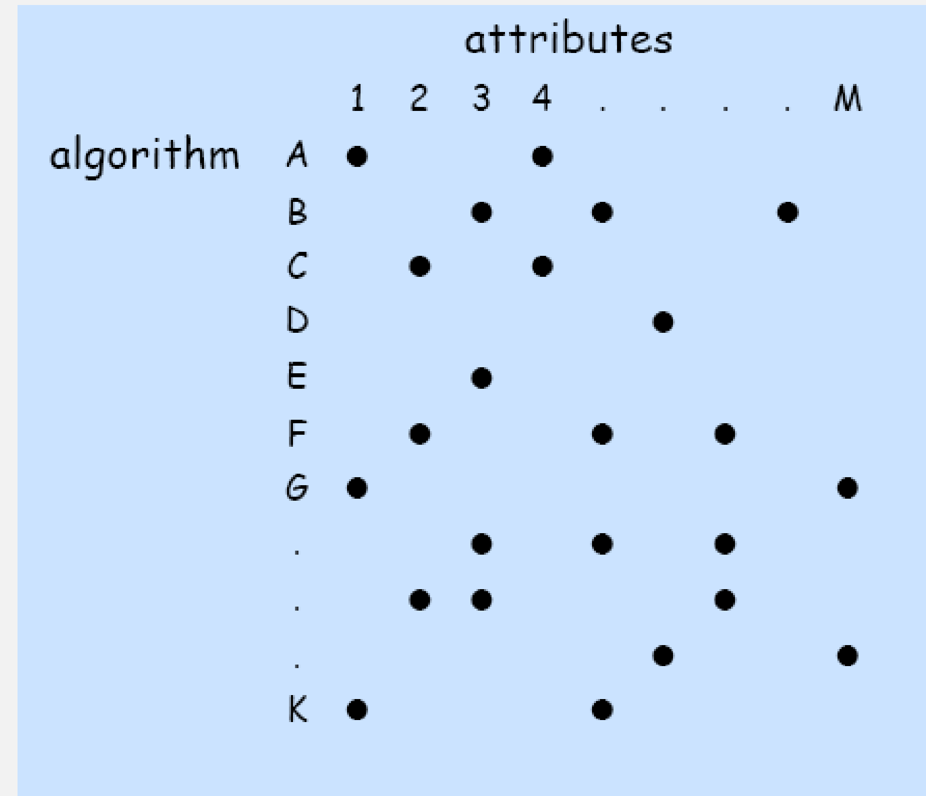
Many sorting algorithms to choose from:

sorts	algorithms
elementary sorts	insertion sort, selection sort, bubblesort, shaker sort, ...
subquadratic sorts	quicksort, mergesort, heapsort, shellsort, samplesort, ...
system sorts	dual-pivot quicksort, timsort, introsort, ...
external sorts	Poly-phase mergesort, cascade-merge, psort, ...
radix sorts	MSD, LSD, 3-way radix quicksort, ...
parallel sorts	bitonic sort, odd-even sort, smooth sort, GPUsort, ...

Which sorting algorithm to use?

Applications have diverse attributes.

- Stable?
- Parallel?
- In-place?
- Deterministic?
- Duplicate keys?
- Multiple key types?
- Linked list or arrays?
- Large or small items?
- Randomly-ordered array?
- Guaranteed performance?



many more combinations of attributes than algorithms

Q. Is the system sort good enough?

A. Usually.

System sort in Java 7

`Arrays.sort()`.

- Has method for objects that are `Comparable`.
- Has overloaded method for each primitive type.
- Has overloaded method for use with a `Comparator`.
- Has overloaded methods for sorting subarrays.

Algorithms.

- Dual-pivot quicksort for primitive types.
- Timsort for reference types.

Q. Why use different algorithms for primitive and reference types?

Ineffective sorts

http://xkcd.com/1185