http://algs4.cs.princeton.edu

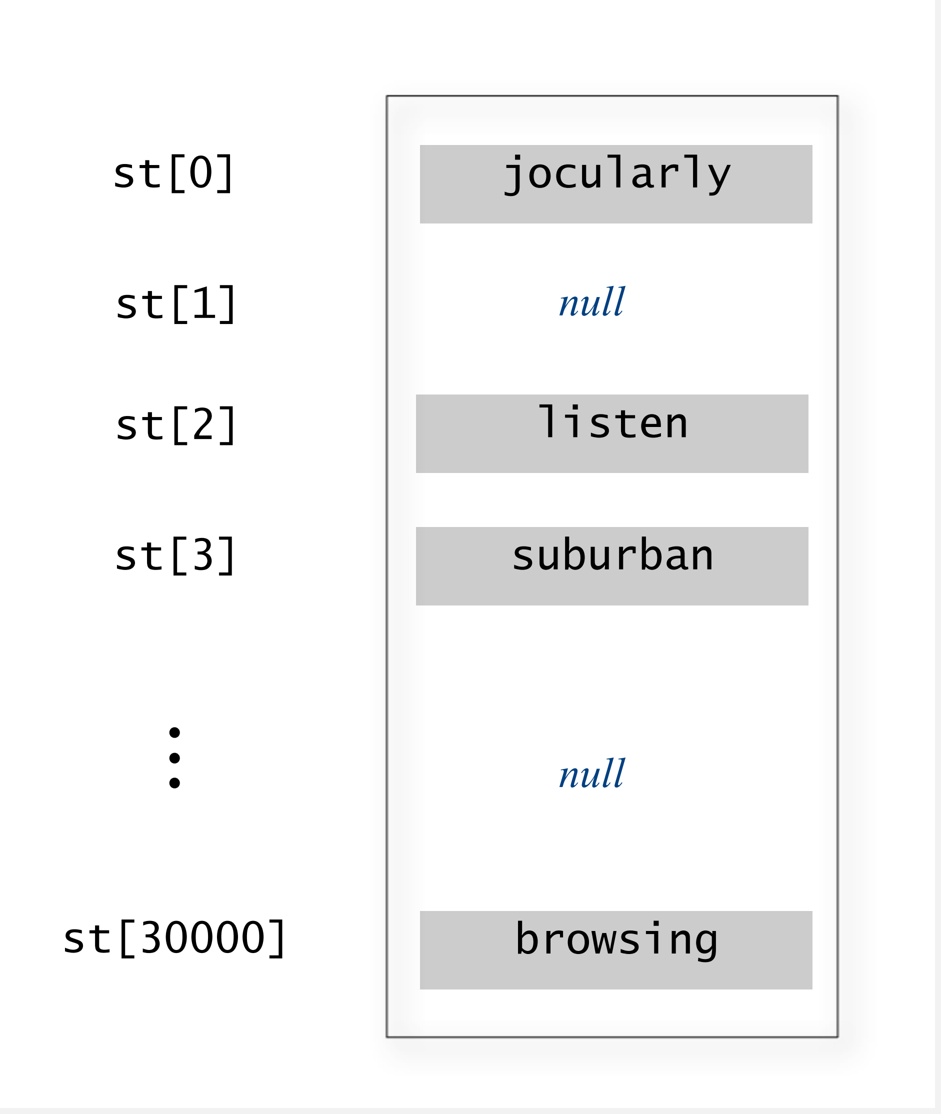
3.4 Hash Tables

- *hash functions*
- *separate chaining*
- *linear probing*
- *context*

Collision resolution: open addressing

Open addressing. [Amdahl-Boehme-Rocherster-Samuel, IBM 1953]

When a new key collides, find next empty slot, and put it there.



linear probing (M = 30001, N = 15000)

Linear-probing hash table demo

Hash. Map key to integer `i` between `0` and `M-1`.

Insert. Put at table index `i` if free; if not try `i+1`, `i+2`, etc.

linear-probing hash table

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
`st[]`																

`M = 16`

Linear-probing hash table demo

Hash. Map key to integer `i` between `0` and `M-1`.

Search. Search table index `i`; if occupied but no match, try `i+1`, `i+2`, etc.

search K

hash(K) = 5

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
`st[]`	P	M			A	C	S	H	L		E				R	X
										K						

`M = 16`

search miss
(return null)

Linear-probing hash table summary

Hash. Map key to integer `i` between `0` and `M-1`.

Insert. Put at table index `i` if free; if not try `i+1`, `i+2`, etc.

Search. Search table index `i`; if occupied but no match, try `i+1`, `i+2`, etc.

Note. Array size `M` must be greater than number of key-value pairs `N`.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
`st[]`	P	M			A	C	S	H	L		E				R	X

`M = 16`

Linear-probing symbol table: Java implementation

```
public class LinearProbingHashST<Key, Value>
{
   private int M = 30001;
   private Value[] vals = (Value[]) new Object[M];
   private Key[]   keys = (Key[])   new Object[M];

   private int hash(Key key)             {  /* as before */   }

   private void put(Key key, Value val) {  /* next slide */  }

   public Value get(Key key)
   {
      for (int i = hash(key); keys[i] != null; i = (i+1) % M)
         if (key.equals(keys[i]))
             return vals[i];
      return null;
   }

}
```

array doubling and halving code omitted

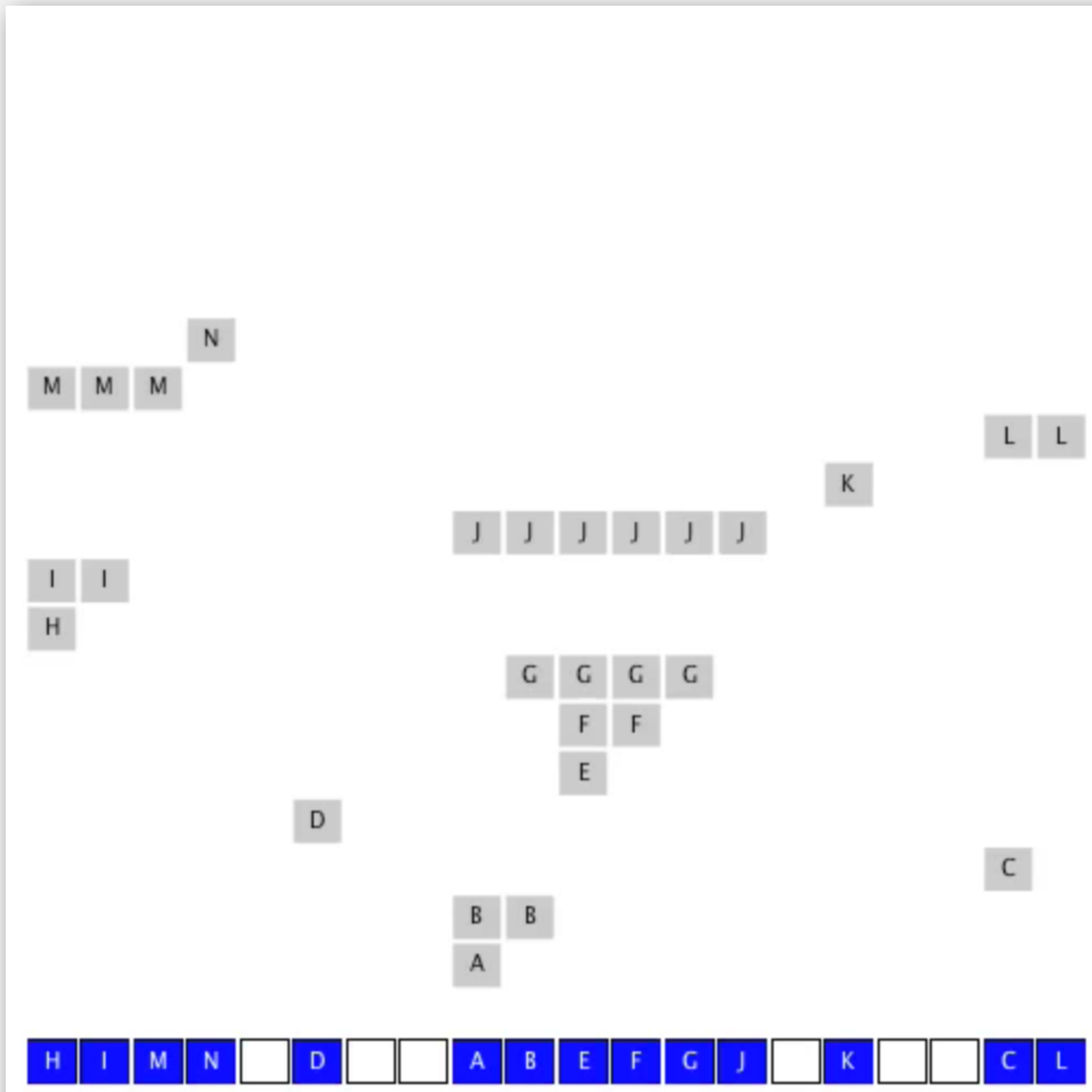
Linear-probing symbol table: Java implementation

```
public class LinearProbingHashST<Key, Value>
{
   private int M = 30001;
   private Value[] vals = (Value[]) new Object[M];
   private Key[]   keys = (Key[])   new Object[M];

   private int hash(Key key)   { /* as before      */  }

   private Value get(Key key)  { /* previous slide */  }

   public void put(Key key, Value val)
   {
      int i;
      for (i = hash(key); keys[i] != null; i = (i+1) % M)
         if (keys[i].equals(key))
             break;
      keys[i] = key;
      vals[i] = val;
   }

}
```

Clustering

Cluster. A contiguous block of items.

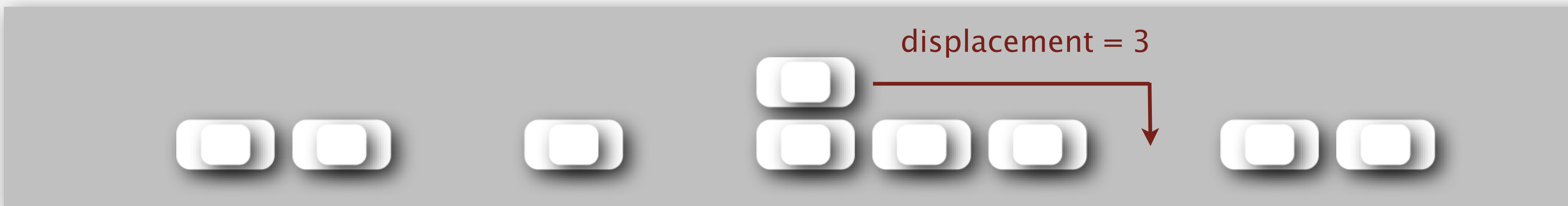
Observation. New keys likely to hash into middle of big clusters.



Knuth's parking problem

Model. Cars arrive at one-way street with M parking spaces.
Each desires a random space i : if space i is taken, try $i+1$, $i+2$, etc.

Q. What is mean displacement of a car?



Half-full. With $M / 2$ cars, mean displacement is $\sim 3 / 2$.
Full. With M cars, mean displacement is $\sim \sqrt{\pi M / 8}$.

Analysis of linear probing

Proposition. Under uniform hashing assumption, the average # of probes in a linear probing hash table of size M that contains $N = \alpha M$ keys is:

$$\sim \frac{1}{2}\left(1 + \frac{1}{1-\alpha}\right) \qquad \sim \frac{1}{2}\left(1 + \frac{1}{(1-\alpha)^2}\right)$$

search hit | **search miss / insert**

Pf.

[My first analysis of an algorithm, originally done during summer 1962 in Madison.]

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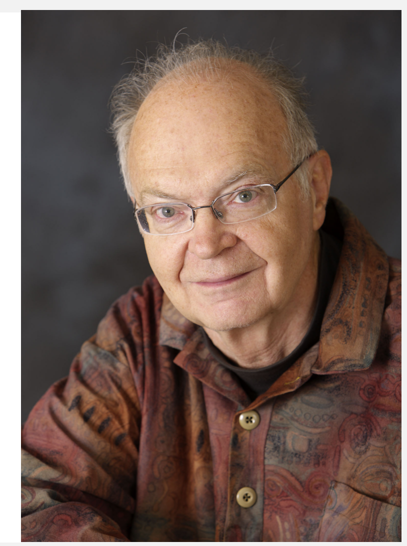
NOTES ON "OPEN" ADDRESSING. D. Knuth 7/22/63

1. Introduction and Definitions. Open addressing is a widely-used technique for keeping "symbol tables." The method was first used in 1954 by Samuel, Amdahl, and Boehme in an assembly program for the IBM 701. An extensive discussion of the method was given by Peterson in 1957 [1], and frequent references have been made to it ever since (e.g. Schay and Spruth [2], Iverson [3]). However, the timing characteristics have apparently never been exactly established, and indeed the author has heard reports of several reputable mathematicians who failed to find the solution after some trial. Therefore it is the purpose of this note to indicate one way by which the solution can be obtained.



Parameters.

- M too large $\Rightarrow$ too many empty array entries.
- M too small $\Rightarrow$ search time blows up.
- Typical choice: $\alpha = N / M \sim \frac{1}{2}$. ← # probes for search hit is about 3/2 / # probes for search miss is about 5/2

Resizing in a linear-probing hash table

Goal. Average length of list $N / M \leq ½$.

- Double size of array M when $N / M \geq ½$.
- Halve size of array M when $N / M \leq ⅛$.
- Need to rehash all keys when resizing.

before resizing

	0	1	2	3	4	5	6	7
keys[]		E	S			R	A	
vals[]		1	0			3	2	

after resizing

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]					A		S				E				R	
vals[]					2		0				1				3	

Deletion in a linear-probing hash table

Q. How to delete a key (and its associated value)?

A. Requires some care: can't just delete array entries.

before deleting S

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]	P	M			A	C	S	H	L		E				R	X
vals[]	10	9			8	4	0	5	11		12				3	7

doesn't work, e.g., if hash(H) = 4

after deleting S ?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
keys[]	P	M			A	C		H	L		E				R	X
vals[]	10	9			8	4		5	11		12				3	7

ST implementations: summary

implementation	guarantee			average case			ordered ops?	key interface
	search	insert	delete	search hit	insert	delete		
sequential search (unordered list)	N	N	N	½ N	N	½ N		`equals()`
binary search (ordered array)	lg N	N	N	lg N	½ N	½ N	✔	`compareTo()`
BST	N	N	N	1.39 lg N	1.39 lg N	$\sqrt{N}$	✔	`compareTo()`
red-black BST	2 lg N	2 lg N	2 lg N	1.0 lg N	1.0 lg N	1.0 lg N	✔	`compareTo()`
separate chaining	N	N	N	3-5 *	3-5 *	3-5 *		`equals()` `hashCode()`
linear probing	N	N	N	3-5 *	3-5 *	3-5 *		`equals()` `hashCode()`

* under uniform hashing assumption