



<http://algs4.cs.princeton.edu>

3.3 BALANCED SEARCH TREES

- ▶ *2-3 search trees*
- ▶ *red-black BSTs*
- ▶ *B-trees*

File system model

Page. Contiguous block of data (e.g., a file or 4,096-byte chunk).

Probe. First access to a page (e.g., from disk to memory).



slow



fast

Property. Time required for a probe is much larger than time to access data within a page.

Cost model. Number of probes.

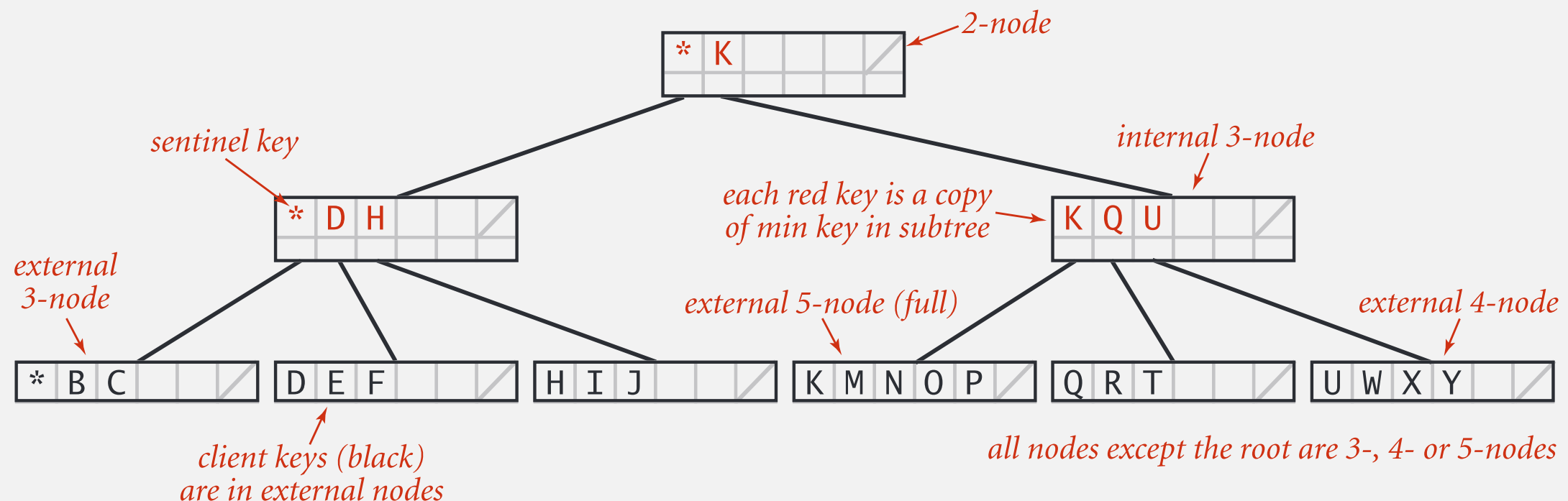
Goal. Access data using minimum number of probes.

B-trees (Bayer-McCreight, 1972)

B-tree. Generalize 2-3 trees by allowing up to $M - 1$ key-link pairs per node.

- At least 2 key-link pairs at root.
- At least $M / 2$ key-link pairs in other nodes.
- External nodes contain client keys.
- Internal nodes contain copies of keys to guide search.

choose M as large as possible so
that M links fit in a page, e.g., $M = 1024$

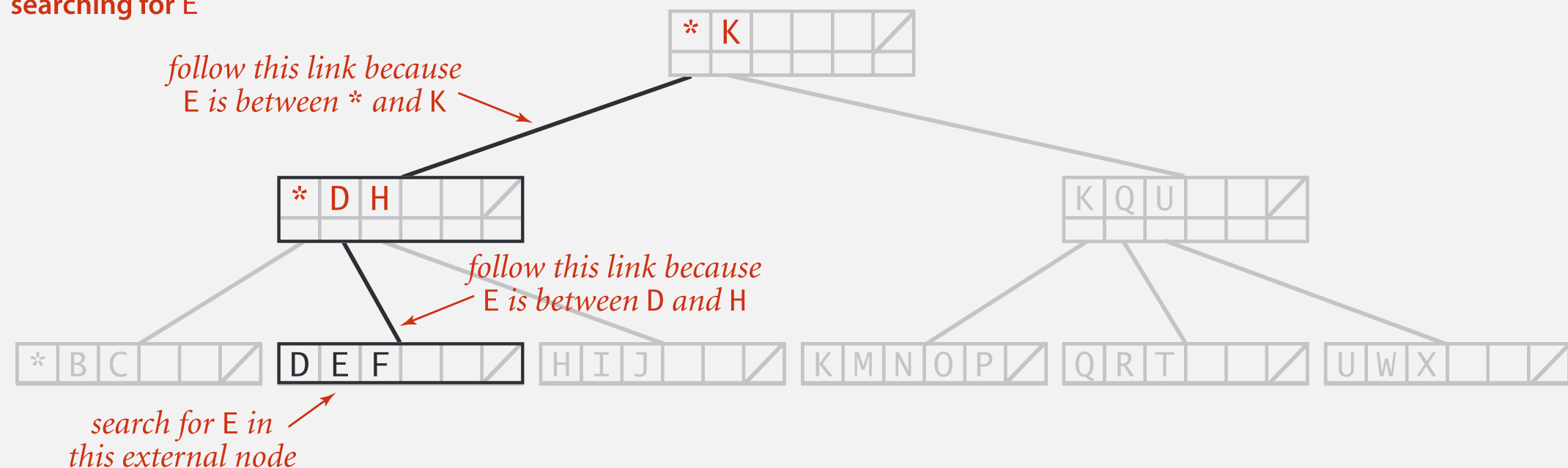


Anatomy of a B-tree set ($M = 6$)

Searching in a B-tree

- Start at root.
- Find interval for search key and take corresponding link.
- Search terminates in external node.

searching for E

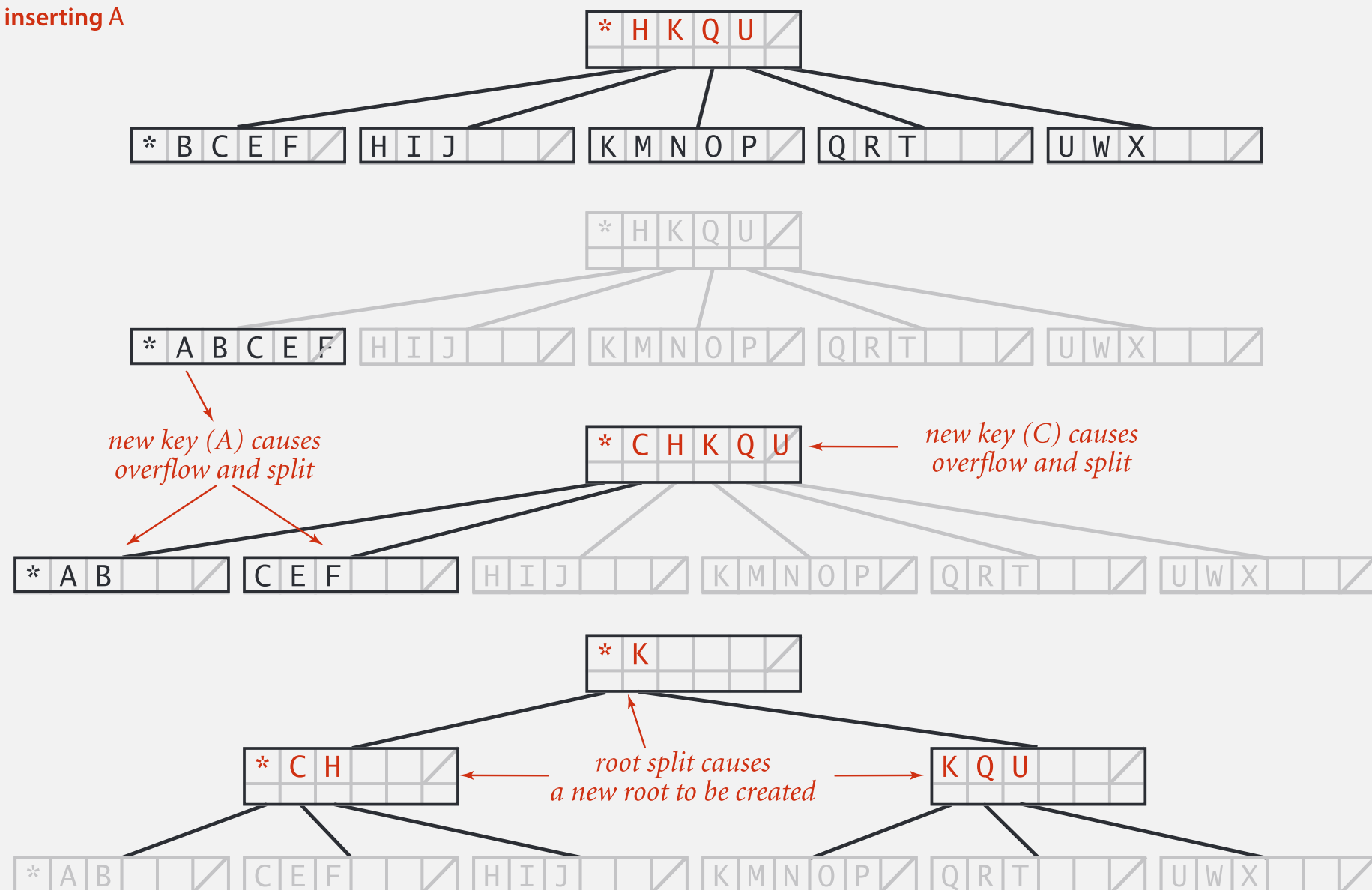


Searching in a B-tree set ($M = 6$)

Insertion in a B-tree

- Search for new key.
- Insert at bottom.
- Split nodes with M key-link pairs on the way up the tree.

inserting A



Inserting a new key into a B-tree set

Balance in B-tree

Proposition. A search or an insertion in a B-tree of order M with N keys requires between $\log_{M-1} N$ and $\log_{M/2} N$ probes.

Pf. All internal nodes (besides root) have between $M/2$ and $M - 1$ links.

In practice. Number of probes is at most 4.  $M = 1024; N = 62 \text{ billion}$
 $\log_{M/2} N \leq 4$

Optimization. Always keep root page in memory.

Building a large B tree



Balanced trees in the wild

Red-black trees are widely used as system symbol tables.

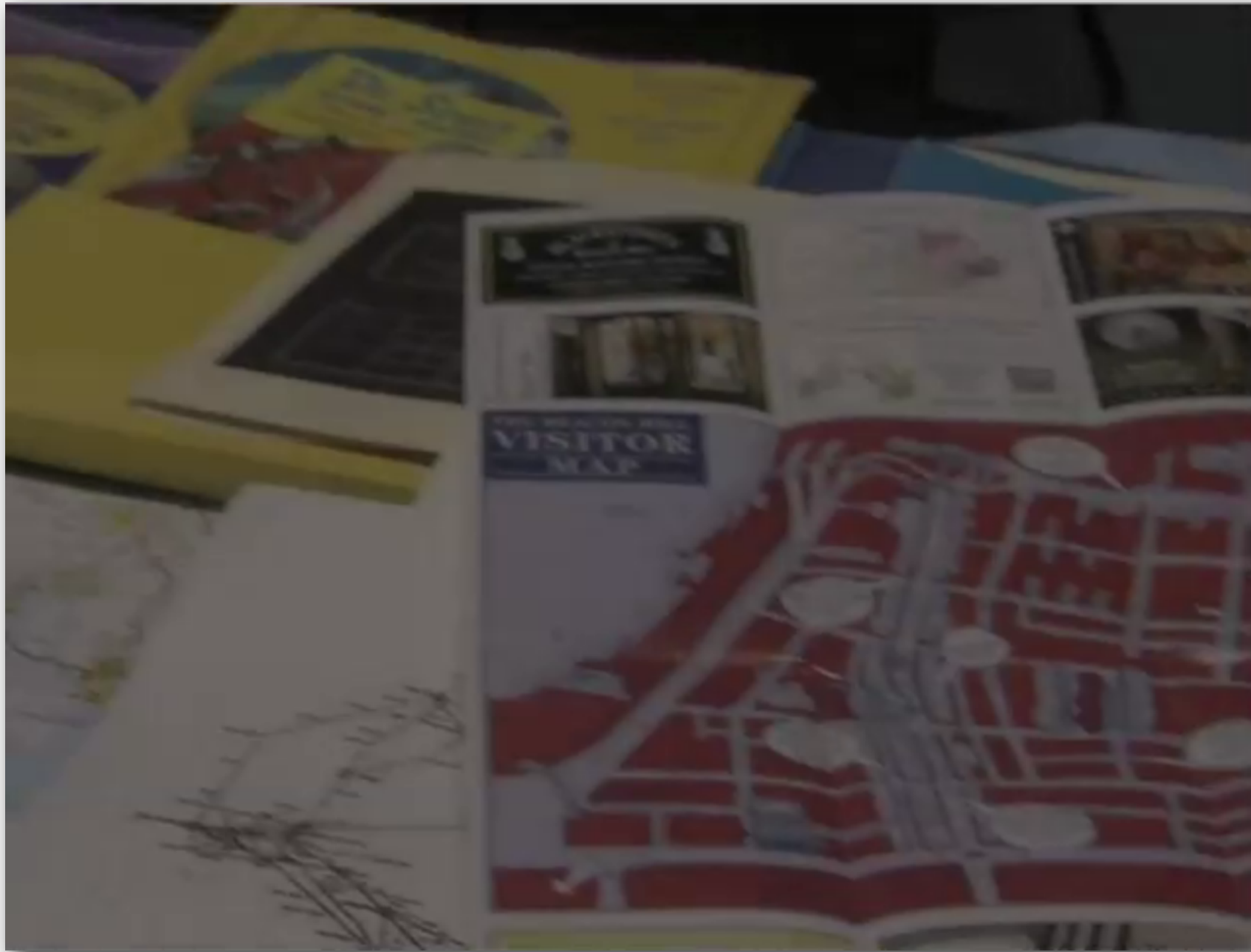
- Java: `java.util.TreeMap`, `java.util.TreeSet`.
- C++ STL: `map`, `multimap`, `multiset`.
- Linux kernel: completely fair scheduler, `linux/rbtree.h`.
- Emacs: conservative stack scanning.

B-tree variants. B+ tree, B*tree, B# tree, ...

B-trees (and variants) are widely used for file systems and databases.

- Windows: NTFS.
- Mac: HFS, HFS+.
- Linux: ReiserFS, XFS, Ext3FS, JFS.
- Databases: ORACLE, DB2, INGRES, SQL, PostgreSQL.

Red-black BSTs in the wild



*Common sense. Sixth sense.
Together they're the
FBI's newest team.*

Red-black BSTs in the wild

ACT FOUR

FADE IN:

48 INT. FBI HQ - NIGHT

48

Antonio is at THE COMPUTER as Jess explains herself to Nicole and Pollock. The CONFERENCE TABLE is covered with OPEN REFERENCE BOOKS, TOURIST GUIDES, MAPS and REAMS OF PRINTOUTS.

JESS

It was the red door again.

POLLOCK

I thought the red door was the storage container.

JESS

But it wasn't red anymore. It was black.

ANTONIO

So red turning to black means... what?

POLLOCK

Budget deficits? Red ink, black ink?

NICOLE

Yes. I'm sure that's what it is. But maybe we should come up with a couple other options, just in case.

Antonio refers to his COMPUTER SCREEN, which is filled with mathematical equations.

ANTONIO

It could be an algorithm from a binary search tree. A red-black tree tracks every simple path from a node to a descendant leaf with the same number of black nodes.

JESS

Does that help you with girls?