



<http://algs4.cs.princeton.edu>

2.2 MERGESORT

- ▶ *mergesort*
- ▶ *bottom-up mergesort*
- ▶ *sorting complexity*
- ▶ *comparators*
- ▶ *stability*

Bottom-up mergesort

Basic plan.

- Pass through array, merging subarrays of size 1.
- Repeat for subarrays of size 2, 4, 8,

	a[i]															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
sz = 1	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 0, 0, 1)	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 2, 2, 3)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 4, 5)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 6, 6, 7)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 8, 8, 9)	E	M	G	R	E	S	O	R	E	T	X	A	M	P	L	E
merge(a, aux, 10, 10, 11)	E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
merge(a, aux, 12, 12, 13)	E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
merge(a, aux, 14, 14, 15)	E	M	G	R	E	S	O	R	E	T	A	X	M	P	E	L
sz = 2	E	G	M	R	E	S	O	R	E	T	A	X	M	P	E	L
merge(a, aux, 0, 1, 3)	E	G	M	R	E	O	R	S	E	T	A	X	M	P	E	L
merge(a, aux, 4, 5, 7)	E	G	M	R	E	O	R	S	A	E	T	X	M	P	E	L
merge(a, aux, 8, 9, 11)	E	G	M	R	E	O	R	S	A	E	T	X	E	L	M	P
merge(a, aux, 12, 13, 15)	E	G	M	R	E	O	R	S	A	E	T	X	E	L	M	P
sz = 4	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, aux, 0, 3, 7)	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge(a, aux, 8, 11, 15)	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
sz = 8	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X
merge(a, aux, 0, 7, 15)	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Bottom-up mergesort: Java implementation

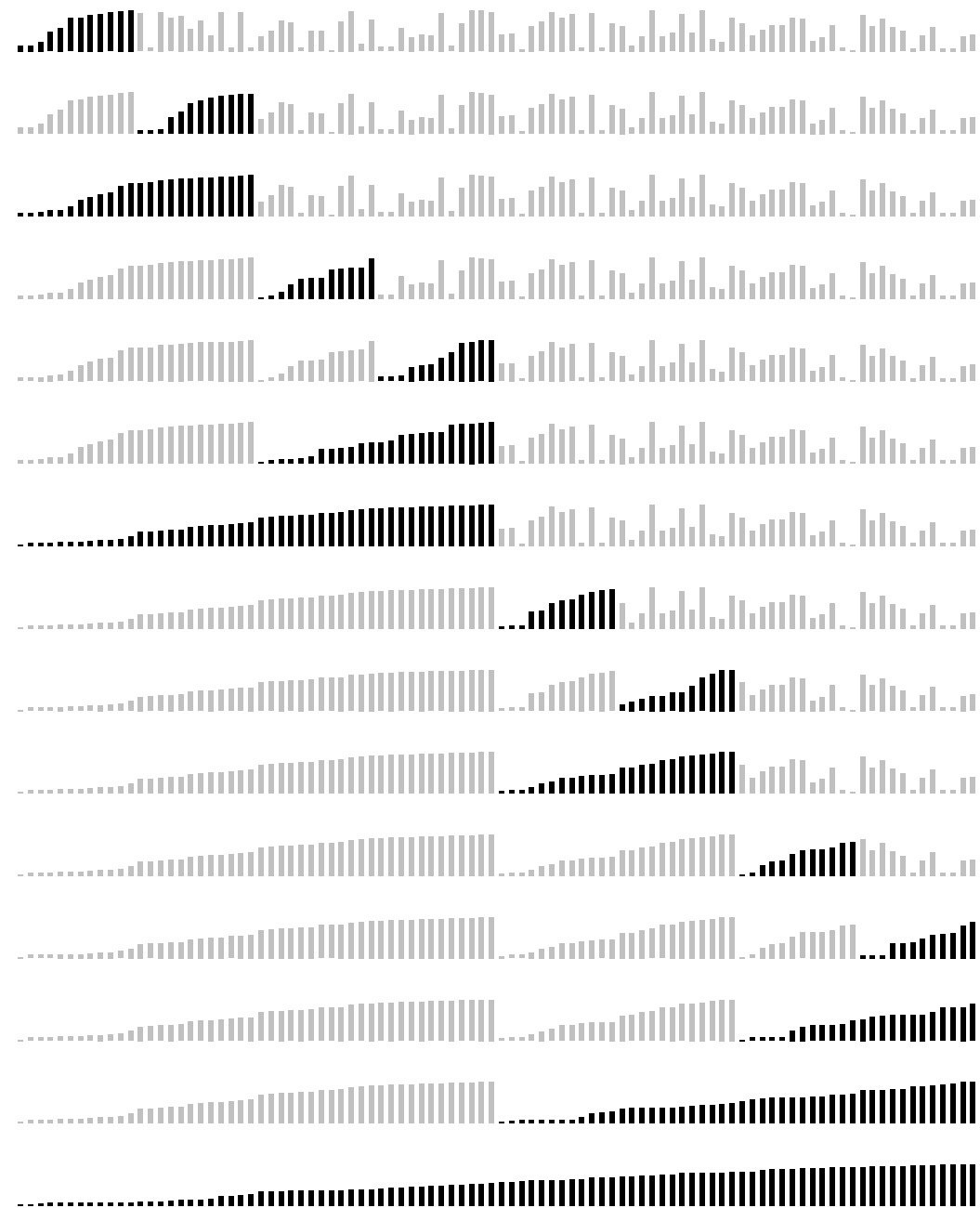
```
public class MergeBU
{
    private static void merge(...)
    { /* as before */ }

    public static void sort(Comparable[] a)
    {
        int N = a.length;
        Comparable[] aux = new Comparable[N];
        for (int sz = 1; sz < N; sz = sz+sz)
            for (int lo = 0; lo < N-sz; lo += sz+sz)
                merge(a, aux, lo, lo+sz-1, Math.min(lo+sz+sz-1, N-1));
    }
}
```

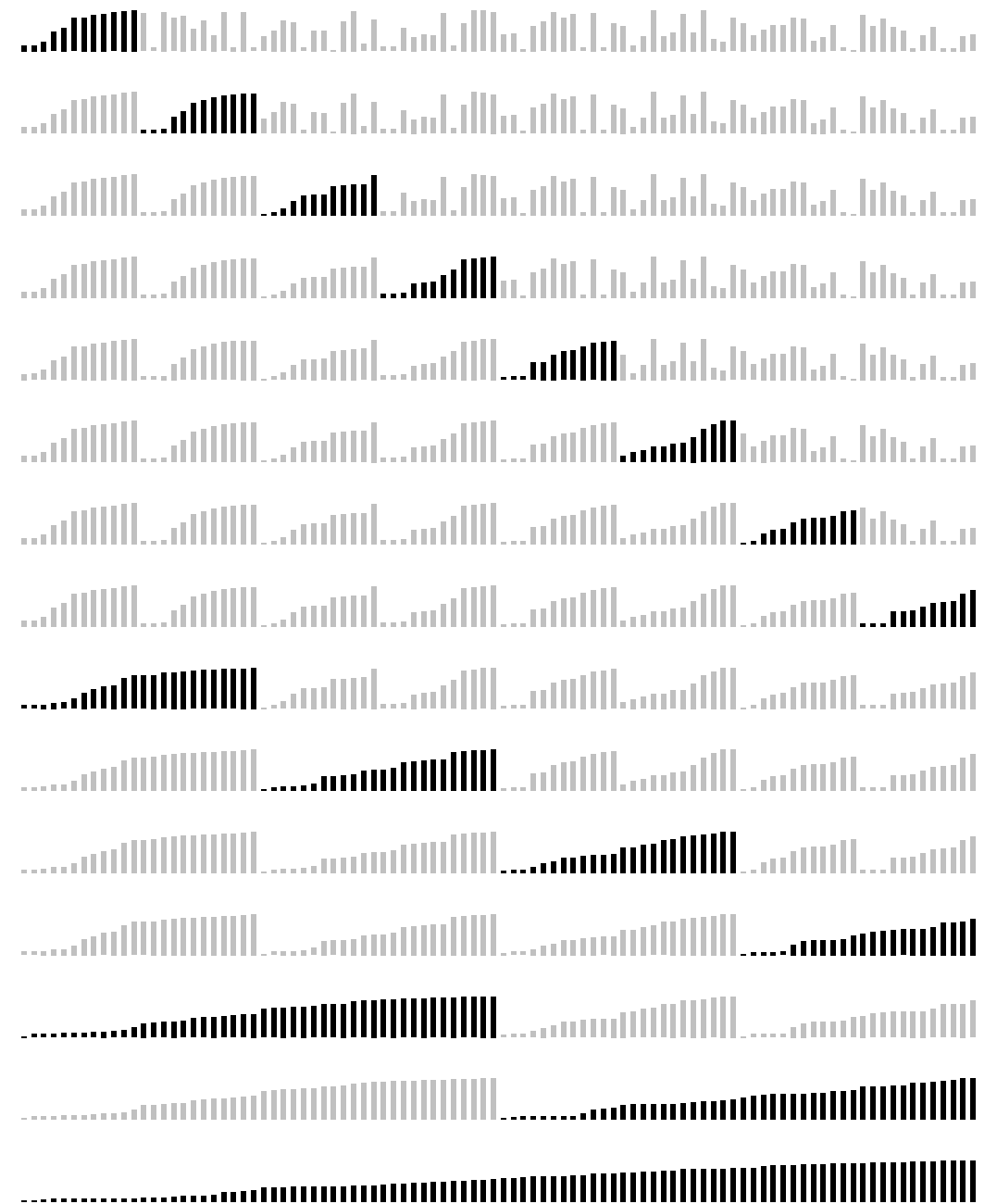
but about 10% slower than recursive,
top-down mergesort on typical systems

Bottom line. Simple and non-recursive version of mergesort.

Mergesort: visualizations



top-down mergesort (cutoff = 12)



bottom-up mergesort (cutoff = 12)

Natural mergesort

Idea. Exploit pre-existing order by identifying naturally-occurring runs.

input

1	5	10	16	3	4	23	9	13	2	7	8	12	14
---	---	----	----	---	---	----	---	----	---	---	---	----	----

first run

1	5	10	16	3	4	23	9	13	2	7	8	12	14
---	---	----	----	---	---	----	---	----	---	---	---	----	----

second run

1	5	10	16	3	4	23	9	13	2	7	8	12	14
---	---	----	----	---	---	----	---	----	---	---	---	----	----

merge two runs

1	3	4	5	10	16	23	9	13	2	7	8	12	14
---	---	---	---	----	----	----	---	----	---	---	---	----	----

Tradeoff. Fewer passes vs. extra compares per pass to identify runs.

Timsort

- Natural mergesort.
- Use binary insertion sort to make initial runs (if needed).
- A few more clever optimizations.



Tim Peters

Intro

This describes an adaptive, stable, natural mergesort, modestly called timsort (hey, I earned it <wink>). It has supernatural performance on many kinds of partially ordered arrays (less than $\lg(N!)$ comparisons needed, and as few as $N-1$), yet as fast as Python's previous highly tuned samplesort hybrid on random arrays.

In a nutshell, the main routine marches over the array once, left to right, alternately identifying the next run, then merging it into the previous runs "intelligently". Everything else is complication for speed, and some hard-won measure of memory efficiency.

...

Consequence. Linear time on many arrays with pre-existing order.

Now widely used. Python, Java 7, GNU Octave, Android,

The Zen of Python



python™

<http://www.python.org/dev/peps/pep-0020/>

<http://westmarch.sjsoft.com/2012/11/zen-of-python-poster/>