



<http://algs4.cs.princeton.edu>

## 1.4 ANALYSIS OF ALGORITHMS

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- ▶ *introduction*
- ▶ *observations*
- ▶ *mathematical models*
- ▶ *order-of-growth classifications*
- ▶ *theory of algorithms*
- ▶ *memory*



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## 1.4 ANALYSIS OF ALGORITHMS

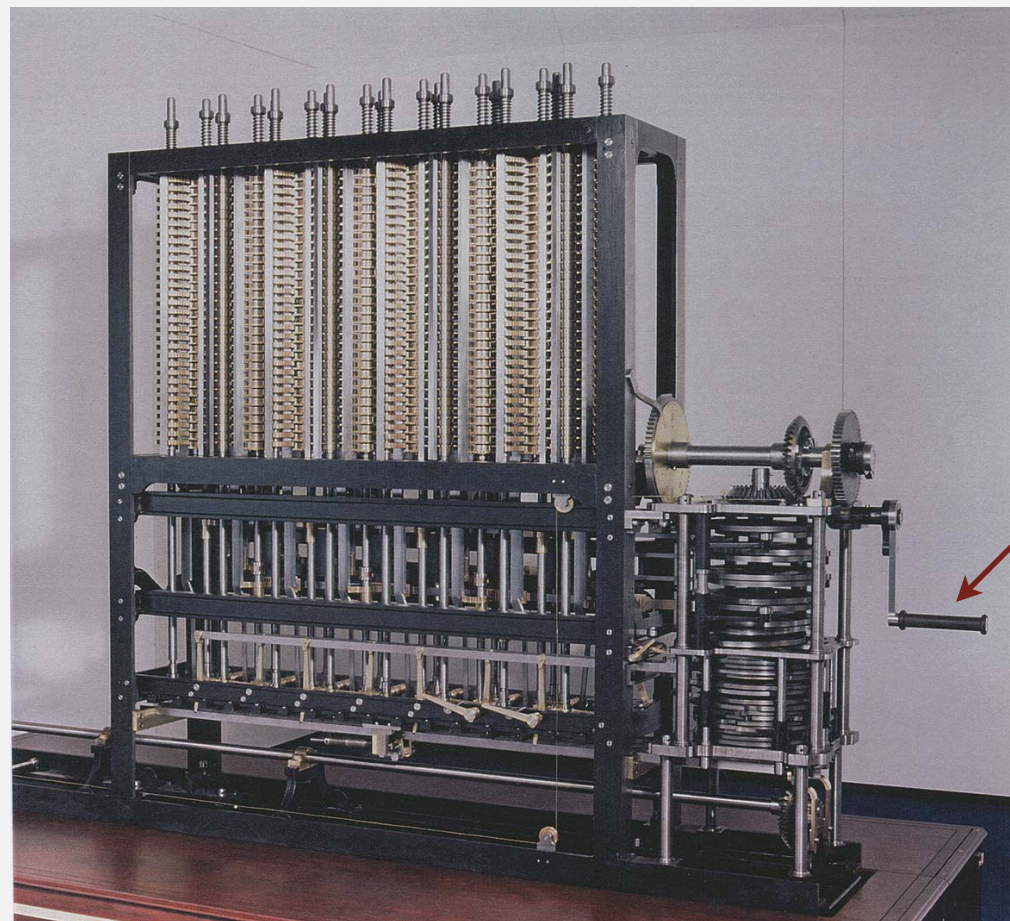
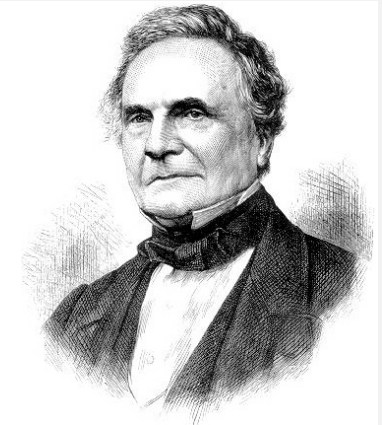
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# Running time

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*“ As soon as an Analytic Engine exists, it will necessarily guide the future course of the science. Whenever any result is sought by its aid, the question will arise—By what course of calculation can these results be arrived at by the machine in the shortest time? ” — Charles Babbage (1864)*



how many times do you have to turn the crank?

**Analytic Engine**

# Cast of characters

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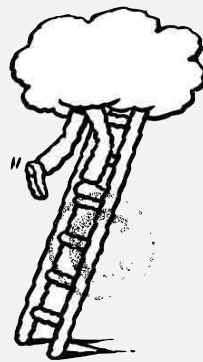
**Programmer** needs to develop a working solution.



**Student** might play any or all of these roles someday.



**Client** wants to solve problem efficiently.



**Theoretician** wants to understand.



# Reasons to analyze algorithms

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Predict performance.

Compare algorithms.

Provide guarantees.

Understand theoretical basis.

this course (COS 226)

theory of algorithms (COS 423)

**Primary practical reason:** avoid performance bugs.



**client gets poor performance because programmer  
did not understand performance characteristics**



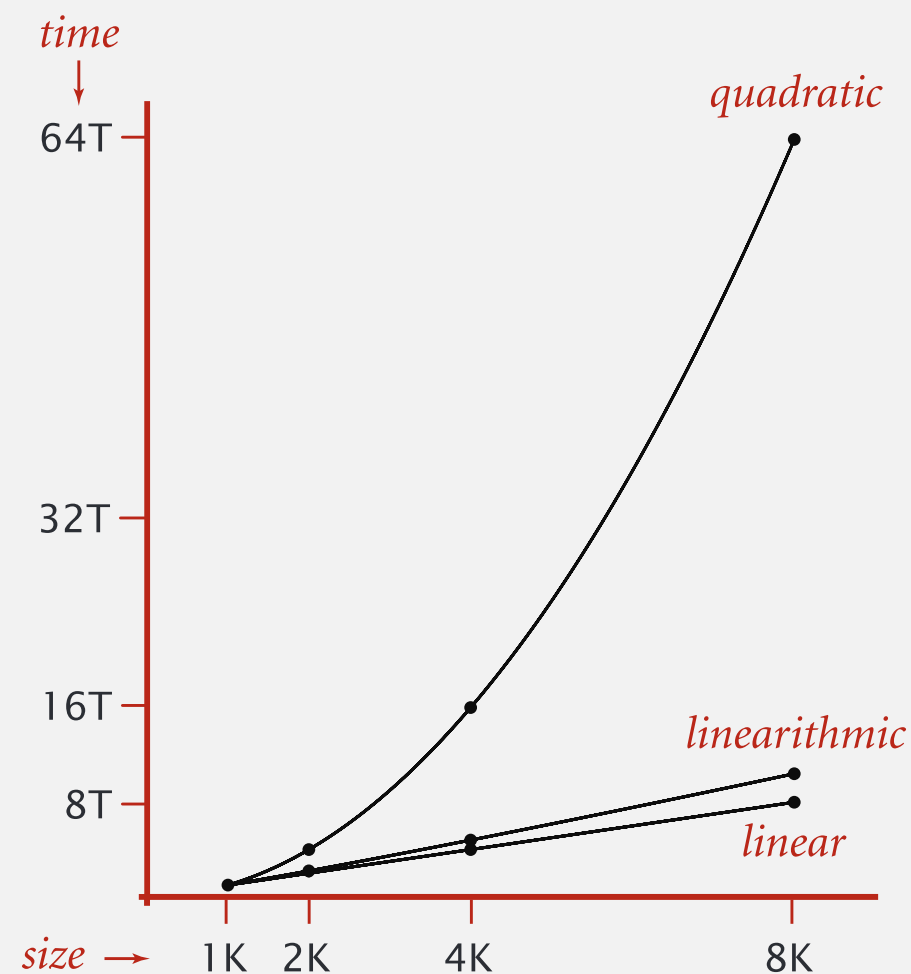
# Some algorithmic successes

## Discrete Fourier transform.

- Break down waveform of  $N$  samples into periodic components.
- Applications: DVD, JPEG, MRI, astrophysics, ....
- Brute force:  $N^2$  steps.
- FFT algorithm:  $N \log N$  steps, **enables new technology.**



Friedrich Gauss  
1805



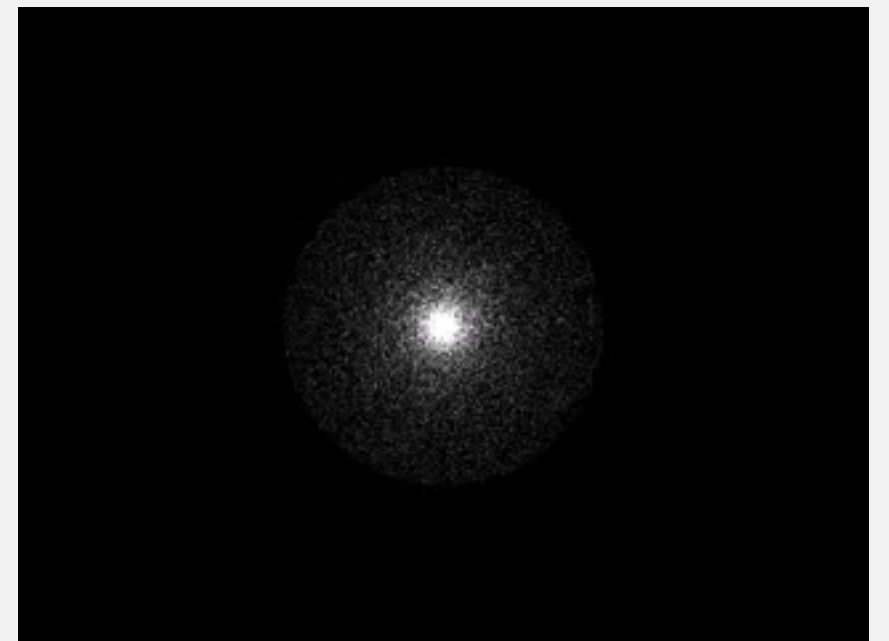
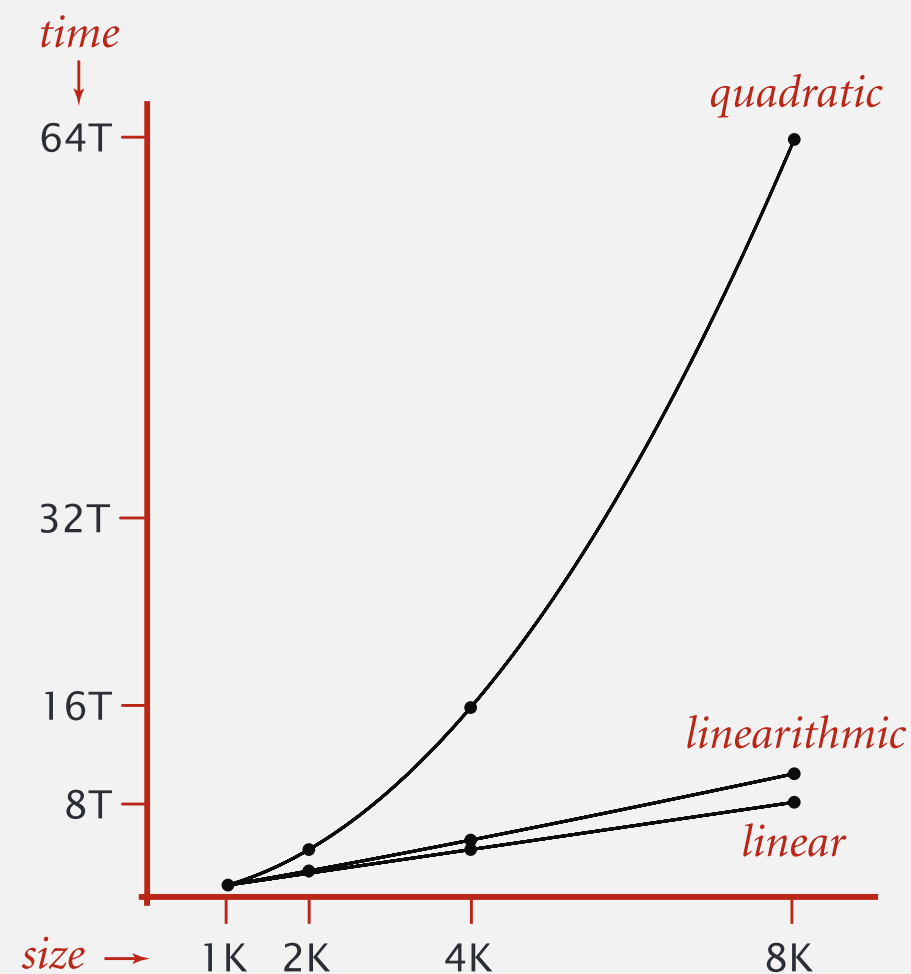
# Some algorithmic successes

## N-body simulation.

- Simulate gravitational interactions among  $N$  bodies.
- Brute force:  $N^2$  steps.
- Barnes-Hut algorithm:  $N \log N$  steps, **enables new research.**



Andrew Appel  
PU '81



# The challenge

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Q. Will my program be able to solve a large practical input?

Why is my program so slow ?

Why does it run out of memory ?



Insight. [Knuth 1970s] Use **scientific method** to understand performance.



# Scientific method applied to analysis of algorithms

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A framework for predicting performance and comparing algorithms.

## Scientific method.

- **Observe** some feature of the natural world.
- **Hypothesize** a model that is consistent with the observations.
- **Predict** events using the hypothesis.
- **Verify** the predictions by making further observations.
- **Validate** by repeating until the hypothesis and observations agree.

## Principles.

- Experiments must be **reproducible**.
- Hypotheses must be **falsifiable**.

Feature of the natural world. Computer itself.

