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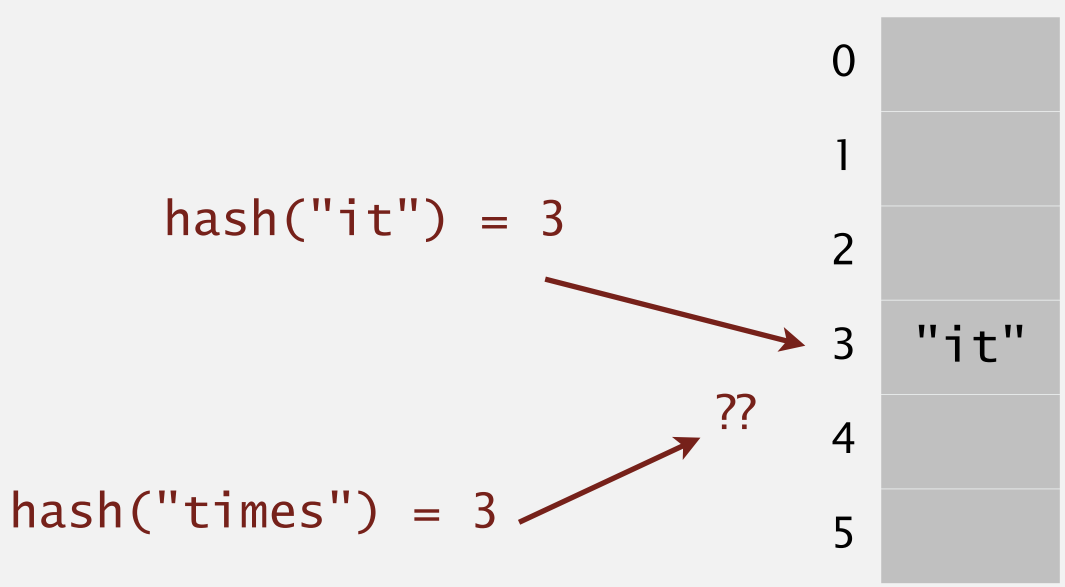
3.4 HASH TABLES

- *hash functions*
- *separate chaining*
- *linear probing*
- *context*

Collisions

Collision. Two distinct keys hashing to same index.

- Birthday problem $\Rightarrow$ can't avoid collisions unless you have a ridiculous (quadratic) amount of memory.
- Coupon collector + load balancing $\Rightarrow$ collisions are evenly distributed.



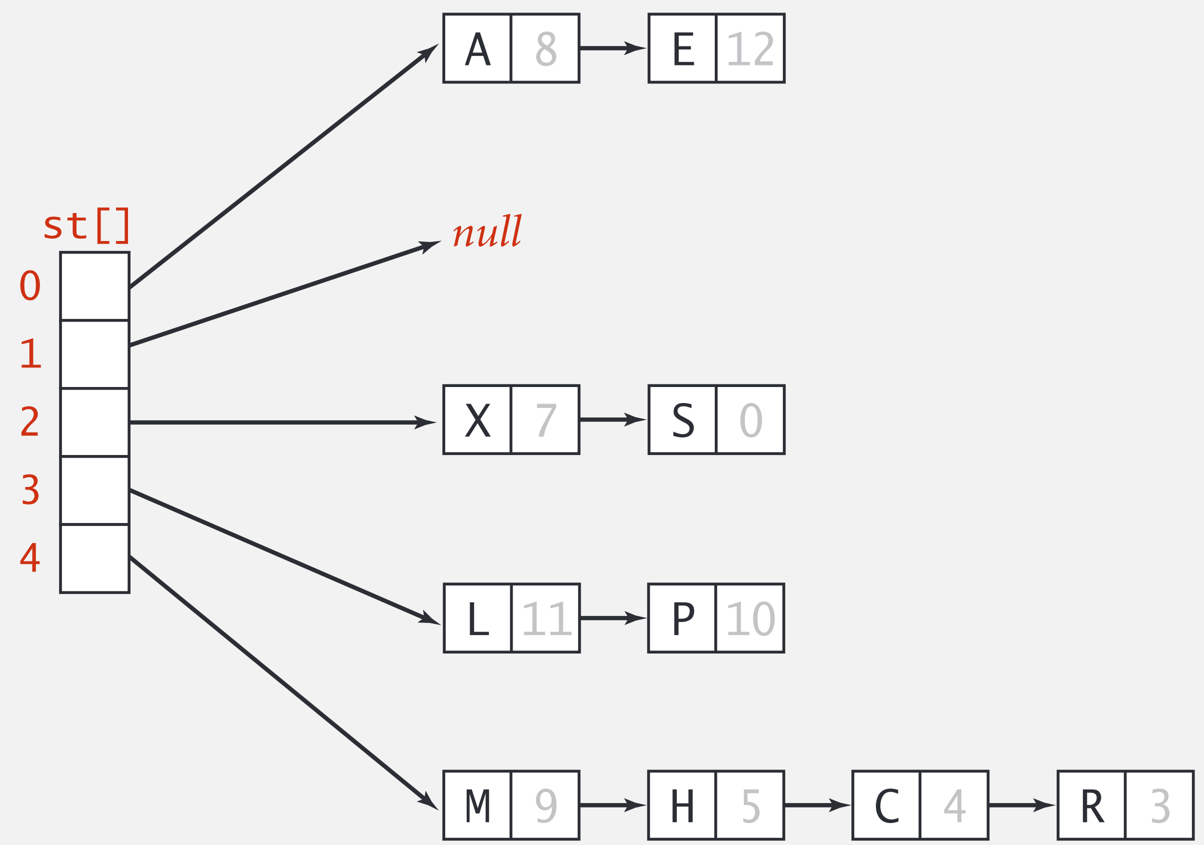
Challenge. Deal with collisions efficiently.

Separate-chaining symbol table

Use an array of $M < N$ linked lists. [H. P. Luhn, IBM 1953]

- Hash: map key to integer i between 0 and $M - 1$.
- Insert: put at front of i^{th} chain (if not already there).
- Search: need to search only i^{th} chain.

key	hash	value
S	2	0
E	0	1
A	0	2
R	4	3
C	4	4
H	4	5
E	0	6
X	2	7
A	0	8
M	4	9
P	3	10
L	3	11
E	0	12



Separate-chaining symbol table: Java implementation

```
public class SeparateChainingHashST<Key, Value>
{
   private int M = 97;                 // number of chains
   private Node[] st = new Node[M];    // array of chains

   private static class Node
   {
      private Object key;
      private Object val;
      private Node next;
      ...
   }

   private int hash(Key key)
   {  return (key.hashCode() & 0x7fffffff) % M;  }

   public Value get(Key key) {
      int i = hash(key);
      for (Node x = st[i]; x != null; x = x.next)
         if (key.equals(x.key)) return (Value) x.val;
      return null;
   }

}
```

array doubling and halving code omitted

no generic array creation
(declare key and value of type Object)

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   private int hash(Key key)
   {  return (key.hashCode() & 0x7fffffff) % M;  }

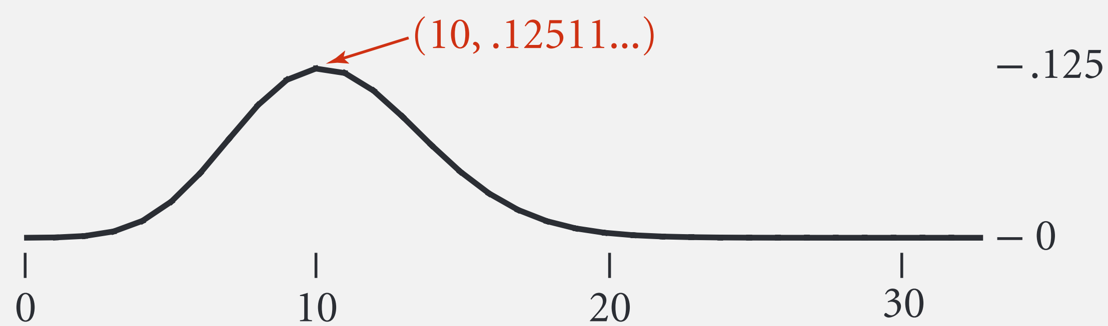
   public void put(Key key, Value val) {
      int i = hash(key);
      for (Node x = st[i]; x != null; x = x.next)
         if (key.equals(x.key)) { x.val = val; return; }
      st[i] = new Node(key, val, st[i]);
   }

}
```

Analysis of separate chaining

Proposition. Under uniform hashing assumption, prob. that the number of keys in a list is within a constant factor of N / M is extremely close to 1.

Pf sketch. Distribution of list size obeys a binomial distribution.



Binomial distribution $(N = 10^4, M = 10^3, \alpha = 10)$

equals() and hashCode()

Consequence. Number of probes for search/insert is proportional to N / M.

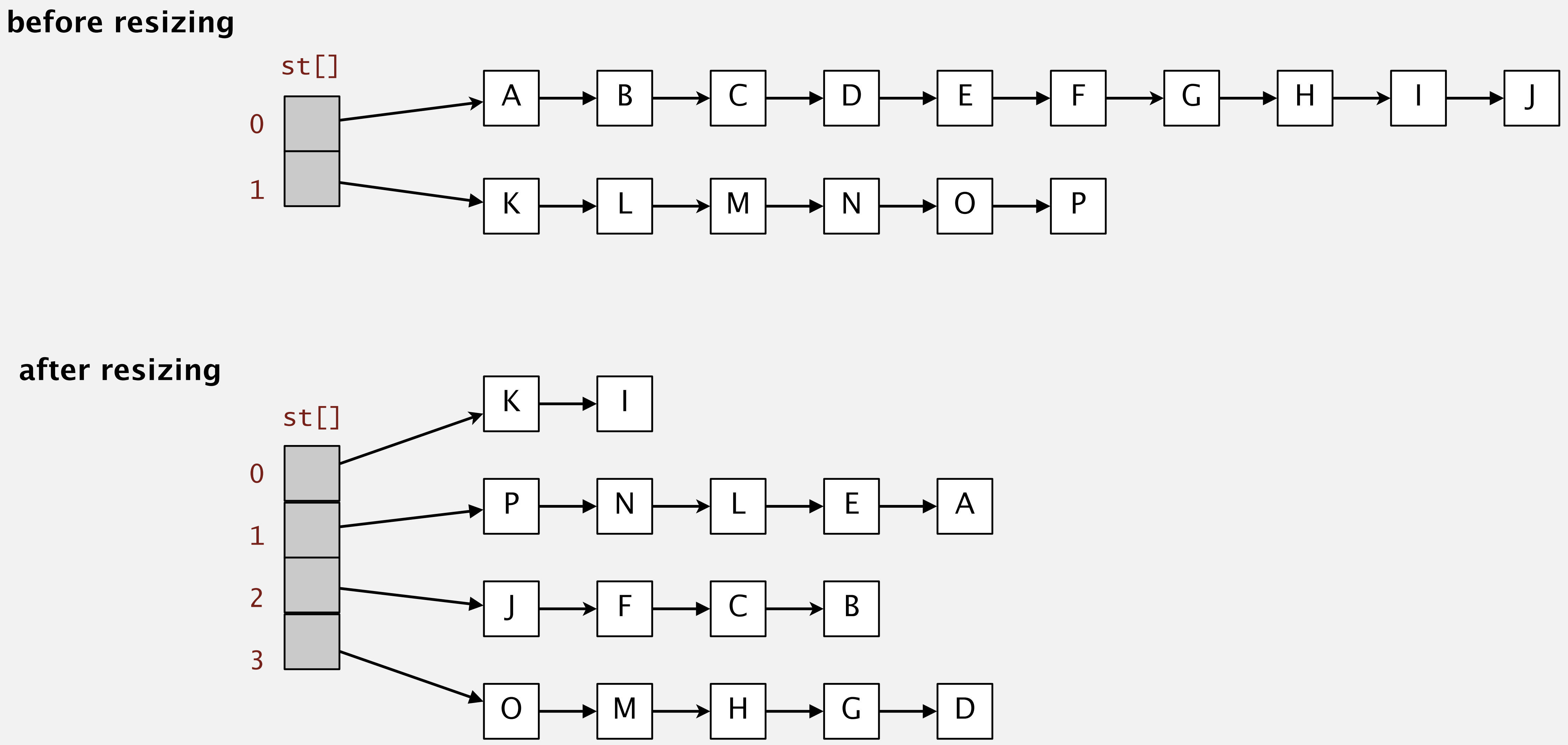
- M too large $\Rightarrow$ too many empty chains.
- M too small $\Rightarrow$ chains too long.
- Typical choice: $M \sim N / 4 \Rightarrow$ constant-time ops.

M times faster than sequential search

Resizing in a separate-chaining hash table

Goal. Average length of list N / M = constant.

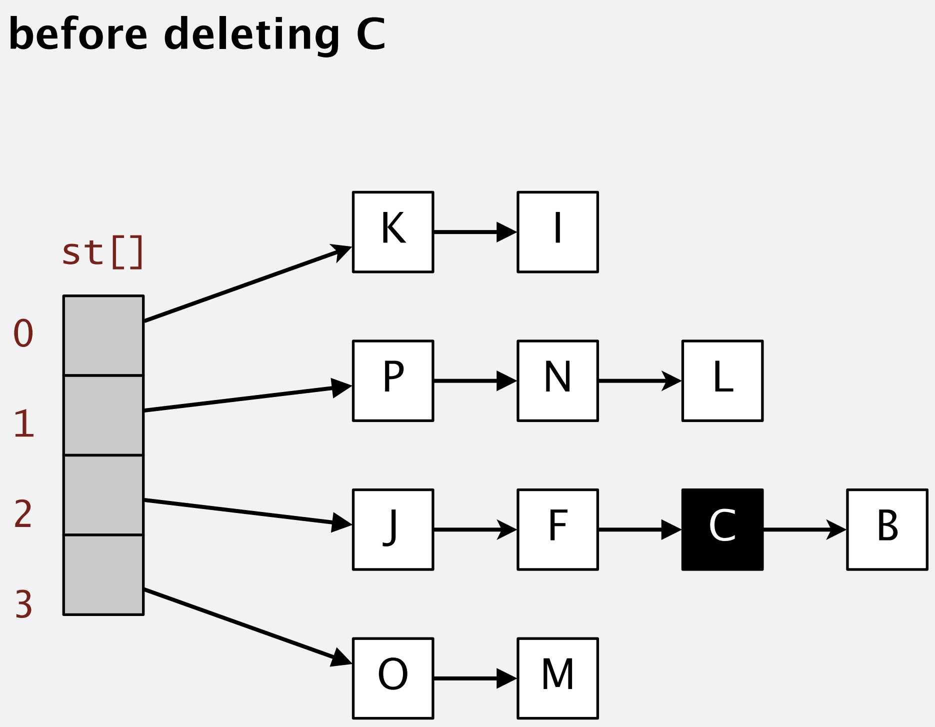
- Double size of array M when $N / M \geq 8$.
- Halve size of array M when $N / M \leq 2$.
- Need to rehash all keys when resizing. ← `x.hashCode()` does not change but `hash(x)` can change

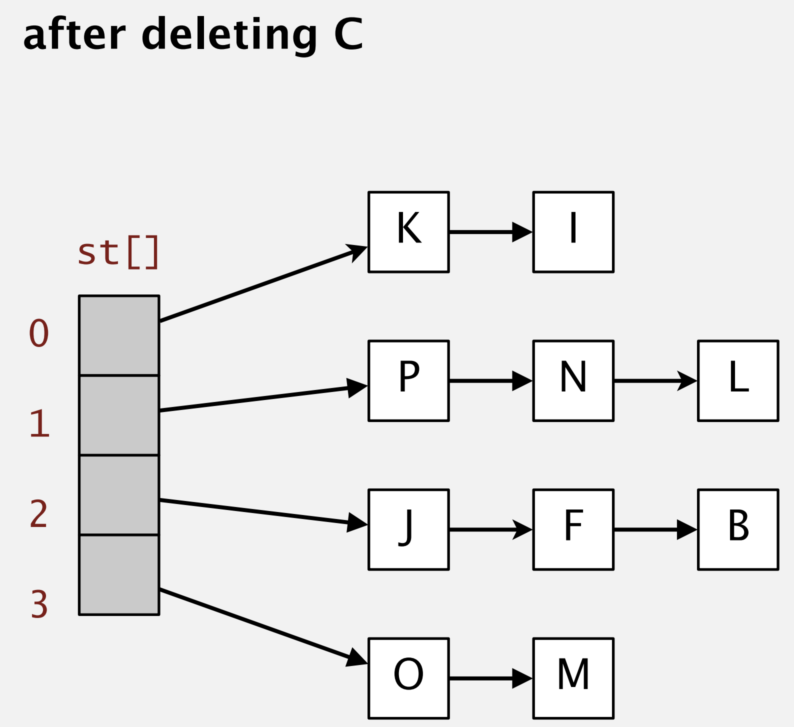


Deletion in a separate-chaining hash table

Q. How to delete a key (and its associated value)?

A. Easy: need only consider chain containing key.





Symbol table implementations: summary

implementation	guarantee			average case			ordered ops?	key interface
	search	insert	delete	search hit	insert	delete		
sequential search (unordered list)	N	N	N	½ N	N	½ N		`equals()`
binary search (ordered array)	$\lg N$	N	N	$\lg N$	½ N	½ N	✔	`compareTo()`
BST	N	N	N	$1.39 \lg N$	$1.39 \lg N$	$\sqrt{N}$	✔	`compareTo()`
red-black BST	$2 \lg N$	$2 \lg N$	$2 \lg N$	$1.0 \lg N$	$1.0 \lg N$	$1.0 \lg N$	✔	`compareTo()`
separate chaining	N	N	N	3-5 *	3-5 *	3-5 *		`equals()` `hashCode()`

* under uniform hashing assumption