1.4 ANALYSIS OF ALGORITHMS

Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

▸ *introduction*

▸ *observations*

▸ *mathematical models*

▸ *order-of-growth classifications*

▸ *theory of algorithms*

▸ **memory**
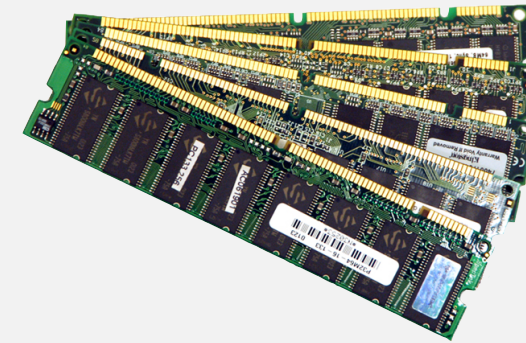
Basics

Bit. 0 or 1.

Byte. 8 bits.

Megabyte (MB). 1 million or 2^{20} bytes.

Gigabyte (GB). 1 billion or 2^{30} bytes.

NIST

most computer scientists

64-bit machine. We assume a 64-bit machine with 8-byte pointers.

- Can address more memory.
- Pointers use more space.

some JVMs "compress" ordinary object pointers to 4 bytes to avoid this cost

Typical memory usage for primitive types and arrays

type	bytes
boolean	1
byte	1
char	2
int	4
float	4
long	8
double	8

primitive types

type	bytes
char[]	$2N + 24$
int[]	$4N + 24$
double[]	$8N + 24$

one-dimensional arrays

type	bytes
char[][]	$\sim 2\,M\,N$
int[][]	$\sim 4\,M\,N$
double[][]	$\sim 8\,M\,N$

two-dimensional arrays

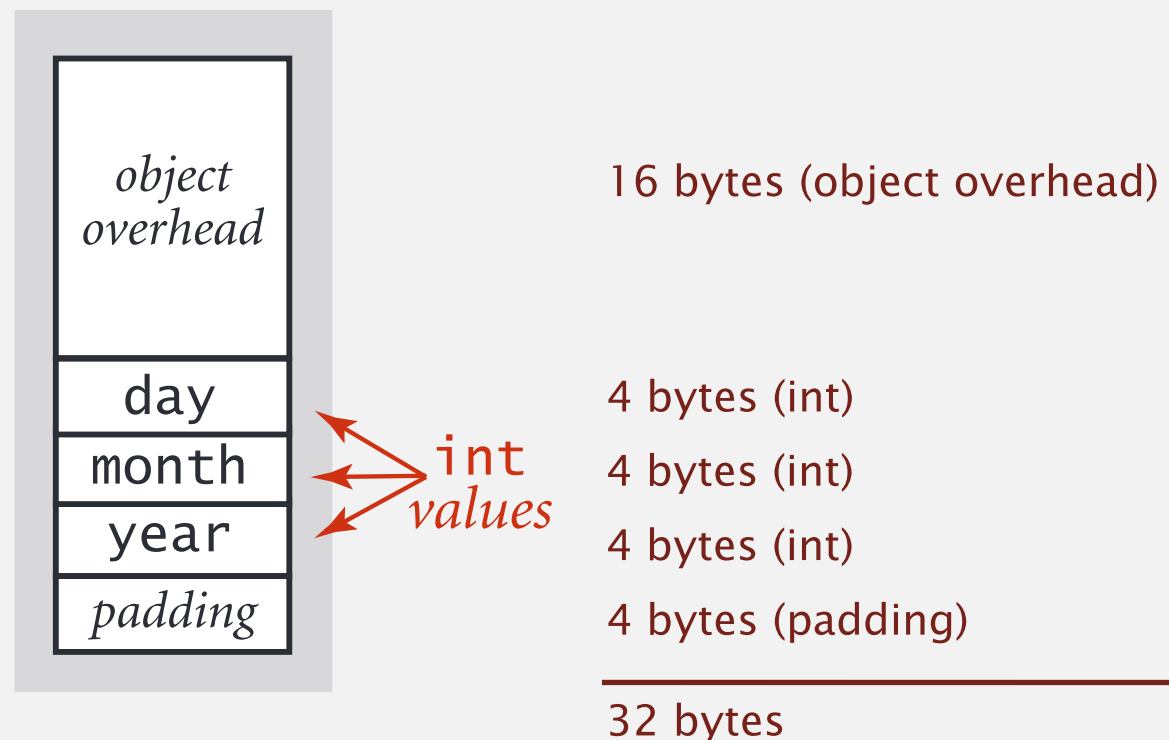# Typical memory usage for objects in Java

Object overhead. 16 bytes.

Reference. 8 bytes.

Padding. Each object uses a multiple of 8 bytes.

Ex 1. A Date object uses 32 bytes of memory.

```
public class Date
{
    private int day;
    private int month;
    private int year;
...
}
```

object overhead	16 bytes (object overhead)
day	4 bytes (int)
month	4 bytes (int)
year	4 bytes (int)
padding	4 bytes (padding)
	32 bytes

int values

Typical memory usage summary

Total memory usage for a data type value:

- Primitive type: 4 bytes for `int`, 8 bytes for `double`, …
- Object reference: 8 bytes.
- Array: 24 bytes + memory for each array entry.
- Object: 16 bytes + memory for each instance variable.
- Padding: round up to multiple of 8 bytes.

+ 8 extra bytes per inner class object
(for reference to enclosing class)

Shallow memory usage: Don't count referenced objects.

Deep memory usage: If array entry or instance variable is a reference, count memory (recursively) for referenced object.

Example

Q. How much memory does `WeightedQuickUnionUF` use as a function of N?
Use tilde notation to simplify your answer.

```
public class WeightedQuickUnionUF
{
    private int[] id;
    private int[] sz;
    private int count;


    public WeightedQuickUnionUF(int N)
    {
        id = new int[N];
        sz = new int[N];
        for (int i = 0; i < N; i++) id[i] = i;
        for (int i = 0; i < N; i++) sz[i] = 1;
    }
    ...
}
```

16 bytes
(object overhead)

8 + (4N + 24) bytes each
(reference + `int[]` array)

4 bytes (`int`)

4 bytes (padding)

8N + 88 bytes
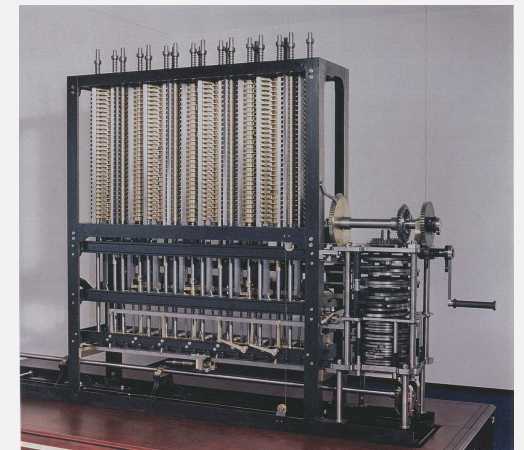
A. $8N + 88 \sim 8N$ bytes.

Turning the crank: summary

Empirical analysis.

- Execute program to perform experiments.
- Assume power law and formulate a hypothesis for running time.
- Model enables us to make predictions.

Mathematical analysis.

- Analyze algorithm to count frequency of operations.
- Use tilde notation to simplify analysis.
- Model enables us to explain behavior.

Scientific method.

- Mathematical model is independent of a particular system; applies to machines not yet built.
- Empirical analysis is necessary to validate mathematical models and to make predictions.