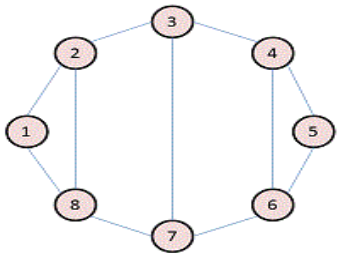
Communities

Question 1:

For the following graph:



Write the adjacency matrix A, the degree matrix D, and the Laplacian matrix L. For each, find the sum of all entries and the number of nonzero entries.

Answer:

Adjacency Matrix (A):

	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	1
2	1	0	1	0	0	0	0	1
3	0	1	0	1	0	0	1	0
4	0	0	1	0	1	1	0	0
5	0	0	0	1	1	0	0	0
6	0	0	0	1	1	0	1	0
7	0	0	1	0	0	1	0	1
8	1	1	0	0	0	0	1	0

No of non-zero entries = 22; Sum of all entries = 22

Degree Matrix (D):

	1	2	3	4	5	6	7	8
1	2	0	0	0	0	0	0	0
2	0	3	0	0	0	0	0	0
3	0	0	3	0	0	0	0	0
4	0	0	0	3	0	0	0	0
5	0	0	0	0	2	0	0	0
6	0	0	0	0	0	3	0	0
7	0	0	0	0	0	0	3	0
8	0	0	0	0	0	0	0	3

No of non-zero entries = 8; Sum of all entries = 8

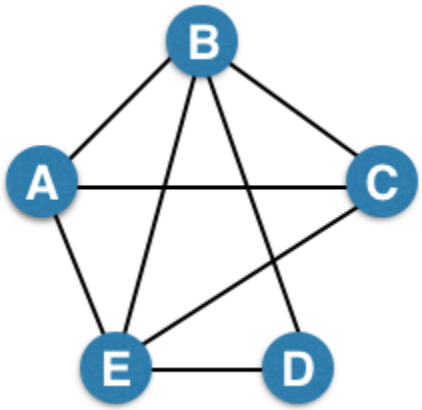
Laplacian Matrix is L = D – A

	1	2	3	4	5	6	7	8
1	2	-1	0	0	0	0	0	-1
2	-1	3	-1	0	0	0	0	-1
3	0	-1	3	-1	0	0	-1	0
4	0	0	-1	3	-1	-1	0	0
5	0	0	0	-1	2	-1	0	0
6	0	0	0	-1	-1	3	-1	0
7	0	0	-1	0	0	-1	3	-1
8	-1	-1	0	0	0	0	-1	3

No of non-zero entries = 30; Sum of all entries = 0

Question 2:

Consider the following undirected graph (i.e., edges may be considered bidirectional):

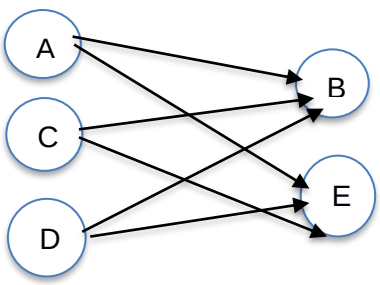


Run the "trawling" algorithm for finding dense communities on this graph and find all complete bipartite subgraphs of types $K_{3,2}$ and $K_{2,2}$. Note: In the case of $K_{2,2}$, we consider {{W, X}, {Y, Z}} and {{Y, Z}, {W, X}} to be identical.

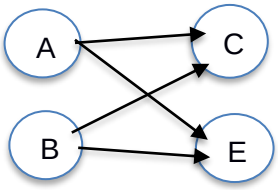
Answer:

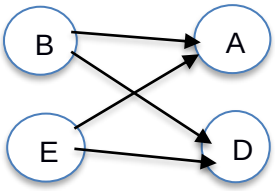
From the given graph, A = {B, C, E}; B = {A, C, D, E}; C = {A, B, E}; D = {B, E}; E = {A, B, C, D}. Here B and E are having support more than A, C, D.

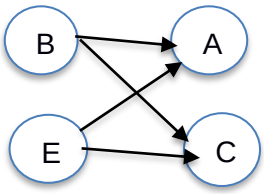
Bipartite subgraph of $K_{3,2}$ is:

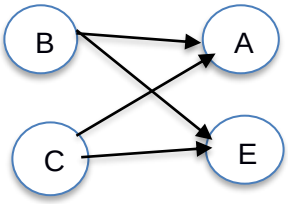


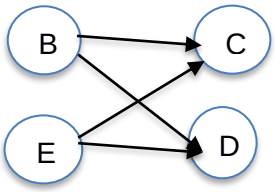
Bipartite subgraph of $K_{2,2}$ is:





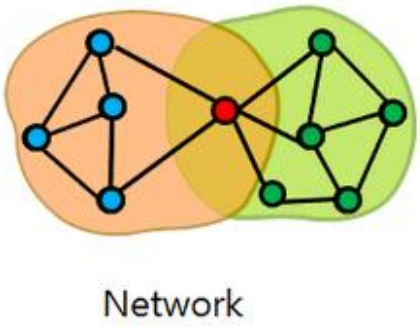


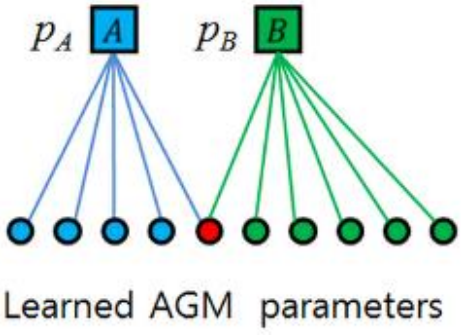




Question 3:

We fit AGM to the network on the left, and found the parameters on the right:





Find the optimal values for p_A and p_B.

Answer:

Pa = Number of edges in the network / Total possible number of edges = 7/5c2 = 7/10.

Pb = Number of edges in the network / Total possible number of edges = 9/6c2 = 9/15.