# Assignment 1 Solutions

## MapReduce and PageRank

**Question 1**:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer *i* and produces the list of pairs (*p*,*i*) such that *p* is a prime divisor of *i*. For example, map (12) = [(2,12), (3,12)]. The reduce function is addition. That is, reduce (*p*, [$i_1$, $i_2$, ..., $i_k$]) is ($p$,$i_1$+$i_2$+...+$i_k$). Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

**Answer 1: The output of map function is**

**map (15) = [(3, 15), (5, 15)]**

**map (21) = [(3, 21), (7, 21)]**

**map (24) = [(2, 24), (3, 24)]**

**map (30) = [(2, 30), (3, 30), (5, 30)]**

**map (49) = [(7, 49)]**

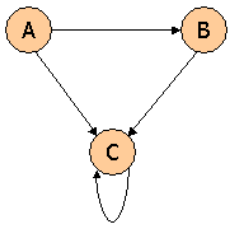
**These are the respective prime divisors of inputs**

**The output of reduce function is**

**reduce (2, 54), reduce (3,90), reduce (5,45), reduce (7, 70).**

**Question 2**:

Consider three Web pages with the following links:




Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the Page Ranks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the Page Ranks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Q2.

* Formula:-

$a = \beta(0) + (1-\beta)$

$b = \beta\left(\frac{a}{2}\right) + (1-\beta)$

$c = \beta\left(\frac{a}{2} + b + c\right) + (1-\beta)$

Here $\beta = 0.7$ and $a+b+c = 3$

$a = 0.7(0) + (1-0.7) = 0.3$ $\boxed{a = 0.3}$

$b = 0.7\left(\frac{0.3}{2}\right) + (1-0.7)$ $\boxed{b = 0.405}$

$= \frac{0.21}{2} + 0.3 = 0.405$

$c = 0.7\left(\frac{0.3}{2} + 0.405 + c\right) + (1-0.7)$

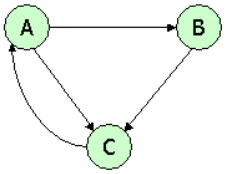
$c = 0.7\left(\frac{0.3}{2}\right) + 0.7(0.405) + 0.7(c) + 0.3$

$c = 0.405 + 0.7(0.405) + 0.7(c)$

$\Rightarrow c = 1.7(0.405) + 0.7(c) \Rightarrow (1-0.7)c = 1.7(0.405)$

$\Rightarrow 0.3c = 0.6885 \Rightarrow c = \frac{0.6885}{0.3} = 2.295 \Rightarrow \boxed{c = 2295}$

**Question 3**:




Suppose we compute PageRank with β=0.85. Write the equations for the Page Ranks *a*, *b*, and *c* of the three pages A, B, and C, respectively.

Q3

* Formula:-

$a = \beta * c + (1-\beta)\frac{1}{3}$

$b = \beta * \frac{a}{2} + (1-\beta)\frac{1}{3}$

$C = \beta * \left(\frac{a}{2} + b\right) + (1-\beta)\frac{1}{3}$

Here $\beta = 0.85$

$a = 0.85 * c + (1-0.85)\frac{1}{3}$

$a = 0.85c + 0.05$

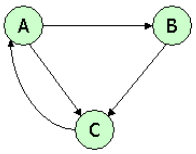
$b = 0.85 * 0.5 * a + 0.05$

$b = 0.425a + 0.05$

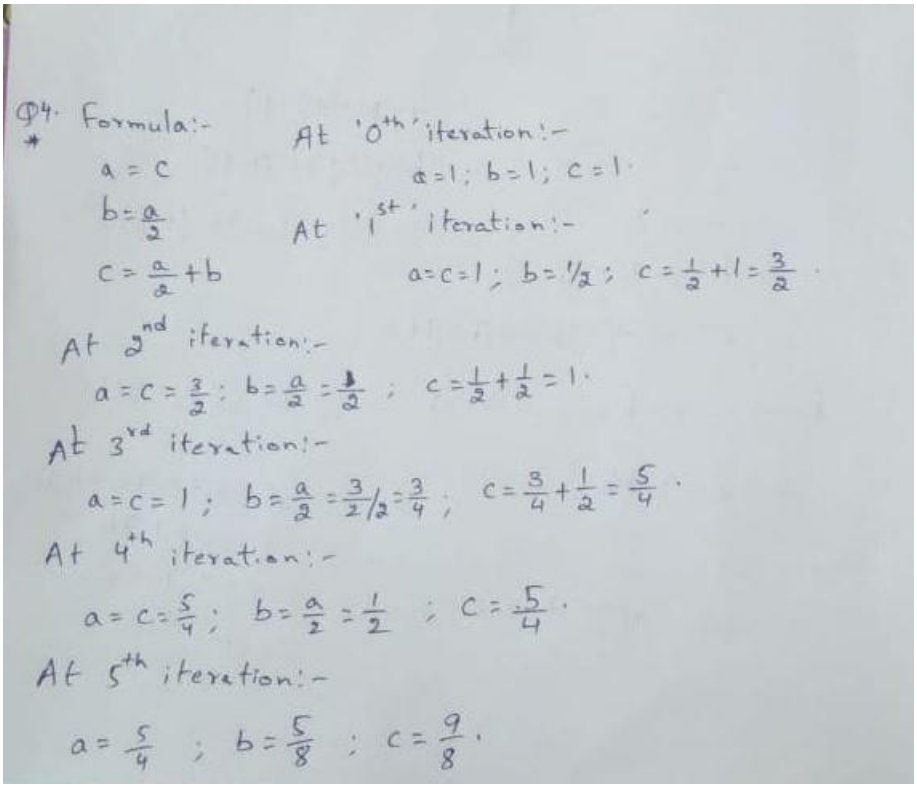
$c = 0.85 * [0.5 * a + b] + 0.05$

$= 0.425a + 0.85b + 0.05$

**Question 4**:



Assuming no "taxation," compute the Page Ranks *a*, *b*, and *c* of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the Page Ranks are in the limit.



Q4. Formula:-

$a = c$

$b = \frac{a}{2}$

$c = \frac{a}{2} + b$

At '0th' iteration:-

$a = 1;\ b = 1;\ c = 1$

At '1st' iteration:-

$a = c = 1;\ b = 1/2;\ c = \frac{1}{2} + 1 = \frac{3}{2}$

At 2nd iteration:-

$a = c = \frac{3}{2};\ b = \frac{a}{2} = \frac{1}{2};\ c = \frac{1}{2} + \frac{1}{2} = 1$

At 3rd iteration:-

$a = c = 1;\ b = \frac{a}{2} = \frac{3}{2}/2 = \frac{3}{4};\ c = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$

At 4th iteration:-

$a = c = \frac{5}{4};\ b = \frac{a}{2} = \frac{1}{2};\ c = \frac{5}{4}$

At 5th iteration:-

$a = \frac{5}{4};\ b = \frac{5}{8};\ c = \frac{9}{8}$