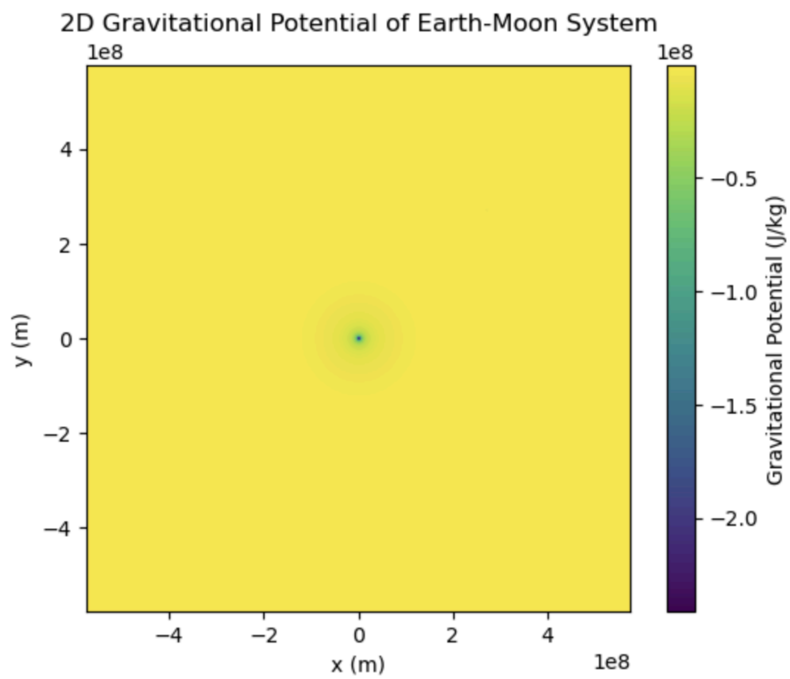
Report to Director Gene Kranz and Congressional Committee Members On the Gravitational and Performance Analysis of the Saturn V Rocket

Introduction

The Apollo program is a massive undertaking, one that requires us to have a deep understanding of the physics involved in getting to the Moon. The calculations we do here aren't just numbers on a page. They determine whether or not we can make this mission a reality. This report breaks down the gravitational environment the spacecraft will deal with and takes a hard look at how the first stage of the Saturn V rocket actually performs.

We've done the math, and we've also compared it to real test data. While there are minor discrepancies, we'll talk about why, and what needs to be improved moving forward.

The Gravitational Potential of the Earth-Moon System



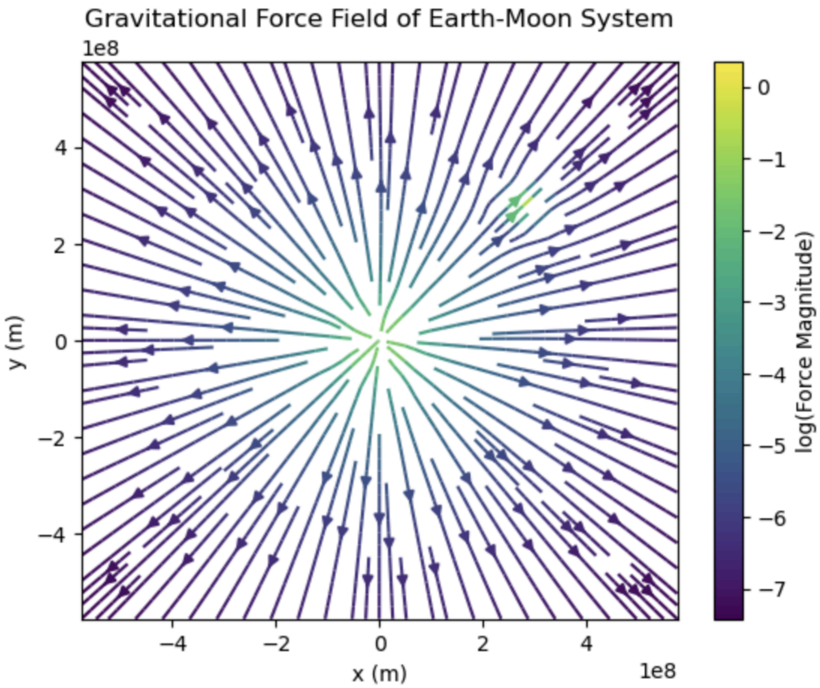
When we talk about gravity, we're talking about how much energy it takes to move something within a gravitational field. The equation for gravitational potential is:

Phi (r) = - (G * M) / r

where G is the gravitational constant.

For our purposes, we treated the Earth and Moon as point masses. This simplifies things, but obviously isn't perfect. When we plotted out the gravitational potential from Earth's surface outward, we saw a sharp drop-off. But, when we mapped the gravitational field in two dimensions, we saw the key balancing points where forces cancel out. These are critical for Apollo's trajectory planning.

The Gravitational Force of the Earth-Moon System



Newton's Law of Gravitation tells us the force between two masses is:

F = - (G * M1 * M2) / r^2

We calculated the net gravitational influence from both Earth and Moon at different points in space, which gave us a force field map. This visualization helps us determine how much thrust is needed at different stages of the journey.

Projected Performance of the Saturn V Stage 1

Now, onto the rocket itself. Its first stage operates based on a simple principle: conservation of momentum. The change in velocity as fuel burns off follows this equation:

Delta v (t) = ve * ln (m0 / m (t)) - g * t

where:
ve = 2400 m/s (exhaust velocity),
m0 = 2.8 * 10^6 kg (wet mass),
mf = 7.5 * 10^5 kg (dry mass),
m_dot = 1.3 * 10^4 kg/s (burn rate),
g = 9.81 m/s^2 (gravitational acceleration).

Using these values, we calculated the burn time:

T = (m0 - mf) / m_dot = 157.69 seconds

By integrating velocity over time, we estimated the rocket's altitude at burnout:

h = 74093.98 m (approximately 74.1 km)

Discussion and Future Work

Now, about those discrepancies I mentioned. NASA's test data showed a burn time of about 160 seconds and a peak altitude of roughly 70 km. Our numbers don't match exactly, so let me explain why that is.

A few reasons:
- We ignored atmospheric drag. In reality, air resistance eats away at altitude gains.
- We assumed gravity was constant, even though it weakens as altitude increases.
- We didn't factor in structural flexing, slight variations in fuel burn rates, or potential engine inefficiencies.

These seem like small things in isolation, but they add up. The bottom line is: our model is solid for a first pass, but it needs fine-tuning. More accurate calculations would require drag modeling, non-uniform gravity considerations, and real-world engine performance data.

Conclusion

At the end of the day, these numbers mean everything. They determine whether Apollo gets off the ground or not. While our theoretical results are close, reality is always more complicated. But that's exactly why we test, refine, and test again. With continued work, we'll bridge the gap between expectation and reality, ensuring Apollo reaches its goal: landing humans on the Moon and bringing them home safely.