To: Director, Earth Core Mining Operations
From: Pierce Hanlon, Staff Scientist
Date: April 10, 2025
Subject: Evaluation of Depth Measurement via Test Mass Drop in Vertical Shaft

I. Introduction

This report evaluates the feasibility of determining the depth of our 4 km vertical shaft by timing the fall of a 1 kg test mass. While the concept appears straightforward, the fall time is influenced by several physical forces that alter the outcome in measurable ways.

The analysis includes three gravity and drag scenarios, accounts for the Coriolis effect due to Earth's rotation at the equator, and draws comparisons between idealized and realistic planetary models. Simulations also extend to theoretical through-planet drops for Earth and the Moon to contextualize gravity variations and their effects on fall times.

II. Fall Time Under Varying Conditions

Three models were tested to measure fall time under increasingly realistic conditions:

A. Constant Gravity, No Drag

A vacuum environment with unchanging gravitational acceleration yields a fall time of:

28.6 seconds

This scenario provides a reference point based on standard free-fall equations.

B. Variable Gravity

A model where gravitational acceleration decreases with depth results in:

28.4 seconds

The change is minor compared to the constant gravity case but confirms the effect of radial position on gravitational force.

C. With Air Resistance

Adding drag, based on a terminal velocity approximation, increases fall time:

36.9 seconds

Drag primarily affects the final portion of the fall, where deceleration flattens the velocity curve.
(See Figure 1: Position and Velocity vs. Time)

III. Coriolis Effect and Shaft Feasibility

At equatorial latitudes, Earth's rotation produces lateral displacement during vertical free-fall. In both drag and no-drag scenarios, the Coriolis force causes the mass to drift laterally enough to contact the shaft wall before reaching the bottom.

- **Without Drag:**
  Contact occurs at ~3.3 km depth after 27.6 seconds

- **With Drag:**
  Contact occurs at ~3.1 km depth after 35.5 seconds

This deflection is continuous and increases with time, as shown in Figure 2 (Sideways Displacement vs. Depth). In the current shaft design, the deflection exceeds the lateral clearance.

IV. Cross-Planetary Comparison: Earth and Moon Fall Models

To assess the effect of internal density variation, simulated falls through idealized tunnels across Earth and the Moon were conducted using two gravity models: one assuming uniform density and one with a denser core. The following crossing times were recorded:

A. Earth – Constant Density (n = 0)

- Time to center: 1265.0 s

- Max velocity: 7905.7 m/s

B. Earth – Dense Core (n = 9)

- Time to center: 1030.4 s

- Max velocity: 8987.5 m/s

C. Moon – Constant Density

- Time to center: 652.3 s

- Max velocity: 1724.2 m/s

The denser core results in faster acceleration and reduced travel time. The difference in time to center between cases A and B reflects the effect of increasing gravitational force toward the

center.
(See Figure 3: Gravitational Force vs. Radius)

V. Discussion and Recommendations

Based on current shaft dimensions and conditions, the test-mass timing method does not yield usable depth measurements. The Coriolis effect alone results in lateral drift sufficient to prevent the mass from reaching the bottom, and drag compounds the issue. To make this approach viable, modifications would be required, such as:

- Initiating the drop with a calculated lateral offset
- Installing guide rails to restrict horizontal motion

However, these changes introduce complexity and cost. Alternative technologies like laser ranging or sonar mapping are likely to be more practical and reliable.

The extended simulations confirm that gravity's variation with depth — especially with denser cores — significantly affects fall dynamics. These findings are relevant in assessing mining potential or underground exploration methods on other planetary bodies.

Future models can increase fidelity by incorporating:

- Altitude-dependent drag coefficients
- Ellipsoidal Earth geometry
- Realistic density distributions (e.g., PREM)

Prepared by:
Pierce Hanlon
Staff Scientist, Earth Core Mining
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Figures:

Figure 1: Position and velocity vs. time

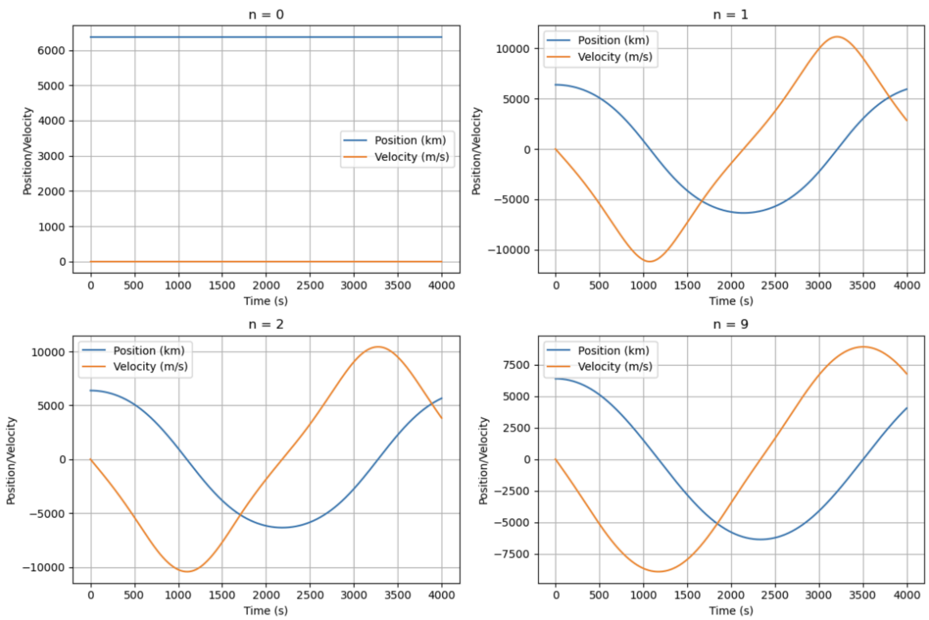

Figure 2: Sideways displacement due to Coriolis effect

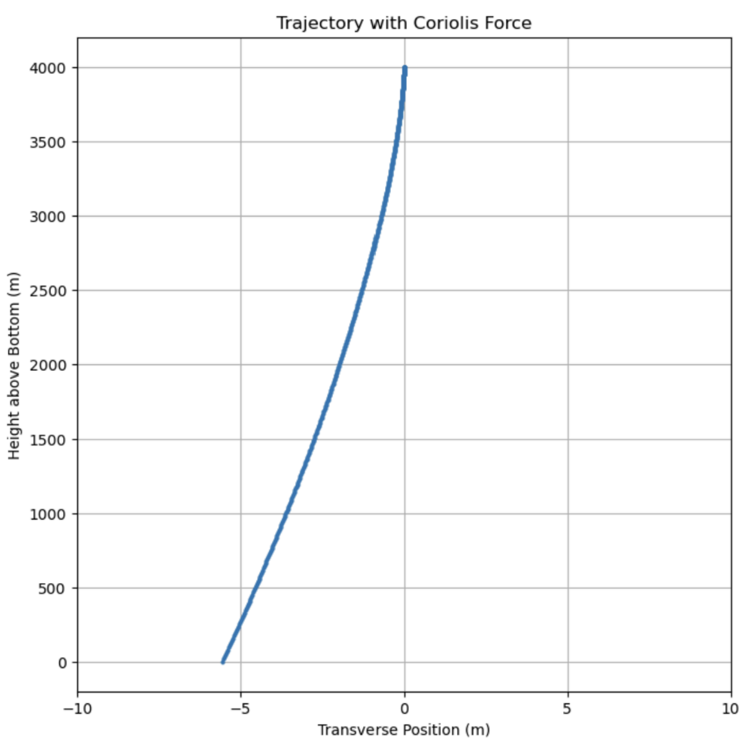

Figure 3: Gravitational force profile vs. planetary radius