HW3

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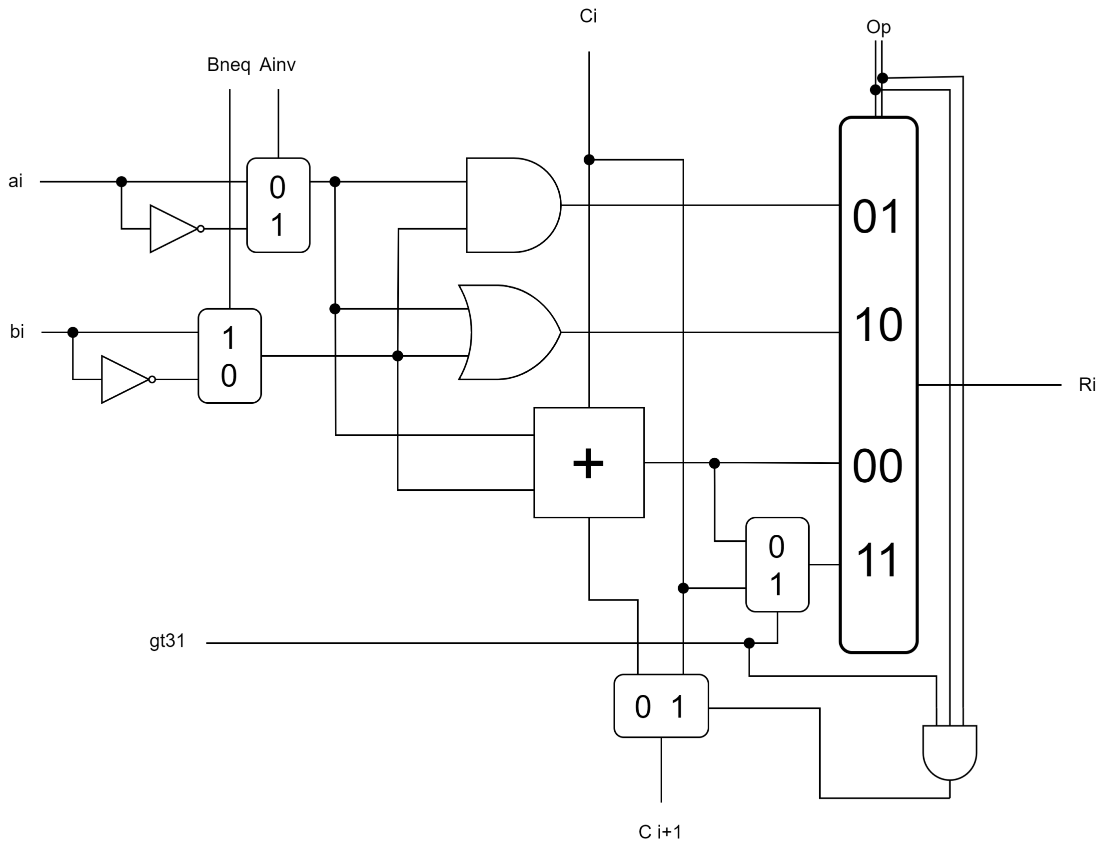
1

Ainvert	Bnegate	Operation	Function
0	1	01	AND
0	1	10	OR
0	1	00	add
0	0	00	sub
1	0	01	NOR
0	1	11	add-ext
0	0	11	sub-ext

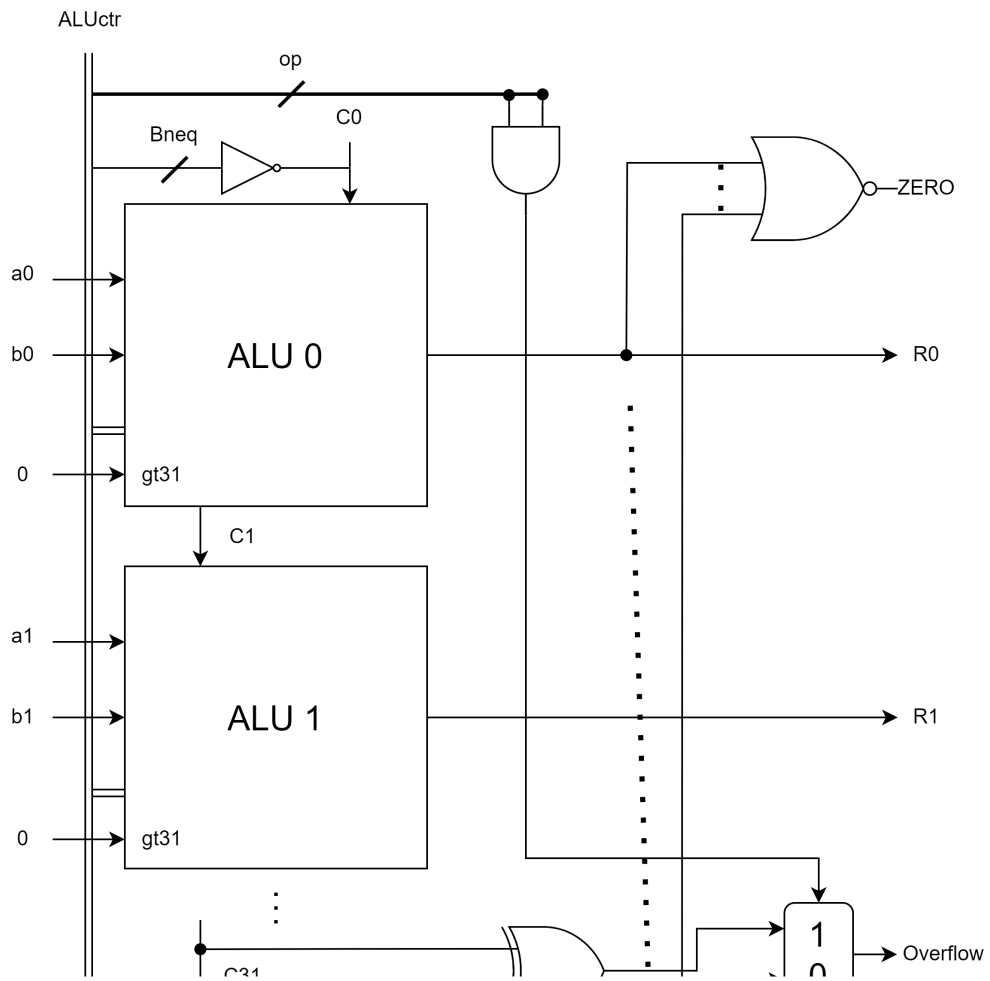
In the 1-bit ALU, I use C_i to propagate the sign bit when sign extension is needed (when op = 11 and gt31 = 1); with signal `gt31` indicates that whether the ALU sequence number is greater than 31.

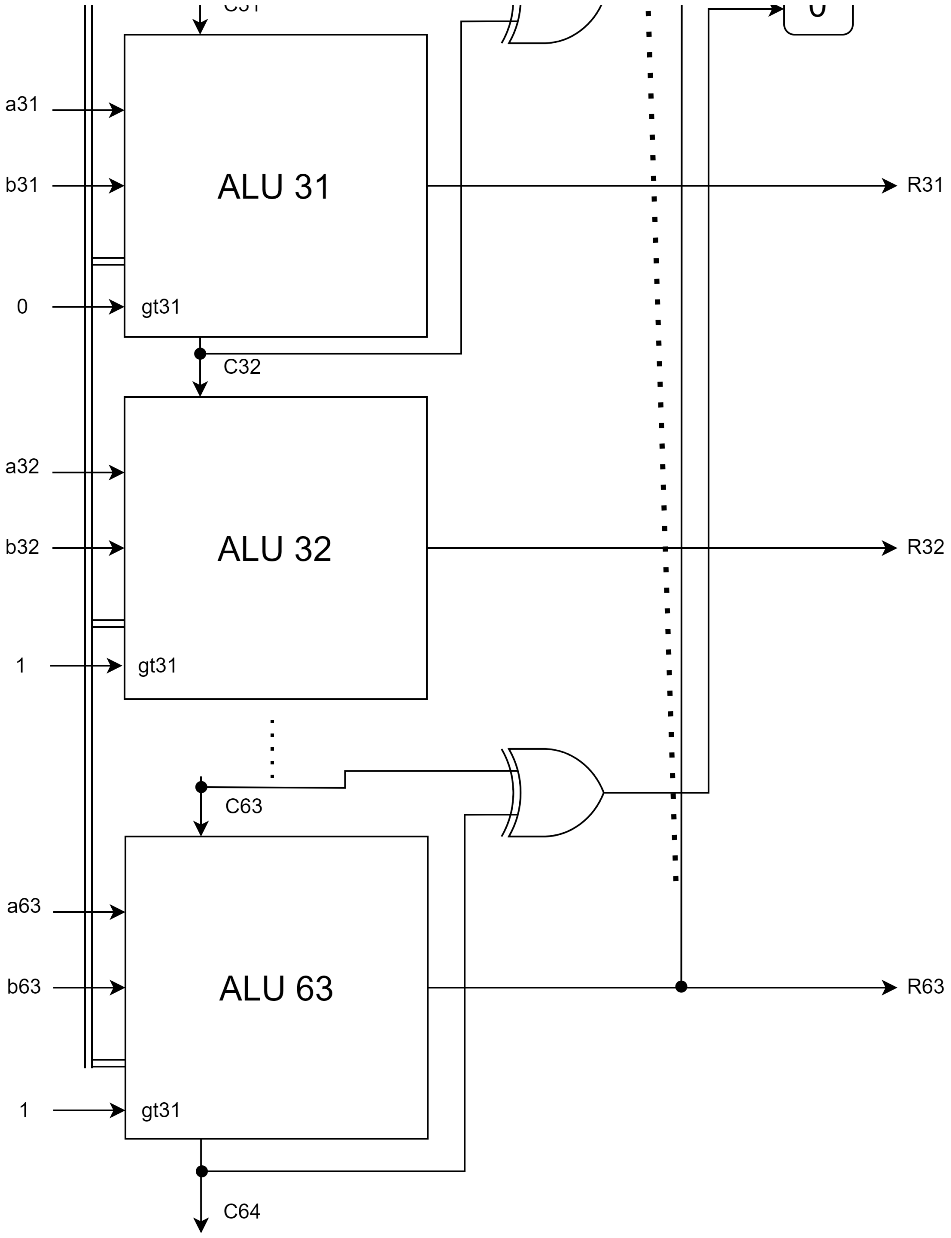
For the 64-bit ALU, the dotted line indicates that each R_i is connected to the multi-OR gate for the `ZERO` output; and `op` is used to determine which version (32 or 64) of overflow is outputted as the `Overflow` output.

1-bit ALU:


Ci
Op
Bneq Ainv
ai
0
1
01
bi
1
0
10
Ri
+
00
0
1
11
gt31
0 1
C i+1

64-bit ALU:


ALUctr
op
Bneq
C0
ZERO
a0
b0
ALU 0
R0
0
gt31
C1
a1
b1
ALU 1
R1
0
gt31
1
Overflow
C31



2

a

multiplier	mul.cand	prod.
1001	0000 1110	0000 0000
0100	0001 1100	0000 1110
0010	0011 1000	..
0001	0111 0000	..
0000	1110 0000	0111 1110

b

mul.cand	prod, multiplier
1110	0000 1001
	1110 1001
1110	0111 0100
	..
1110	0011 1010
	..
1110	0001 1101
	1111 1101
1110	0111 1110

3

a

quotient	divisor	remainder
0000	0101 0000	0000 0111
		1011 0111
0000	0010 1000	0000 0111
		1001 1111
0000	0001 0100	0000 0111
		1111 0011
0000	0000 1010	0000 0111
		1111 1101
0000	0000 0101	0000 0111
		0000 0010
0001	0000 0101	0000 0010

b

remainder, quo	divisor	test pass
0000 0111	0101	
0000 1110		

remainder, quo	divisor	test pass
1011 1110		F
0001 1100		
1100 1100		F
0011 1000		
1110 1000		F
0111 0000		
0010 0000		T
0100 0001		
0010 0001		

4

a

0x05948DEC = $101100101001000110111101100_2$

signed: $2^{26} + 2^{24} + 2^{23} + 2^{20} + 2^{18} + 2^{15} + 2^{11} + 2^{10} + 2^8 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2$

un-signed: same as signed

hex	un-signed	signed
05948DEC	$93{,}621{,}740_{10}$	$93{,}621{,}740_{10}$

Since left-most bit = 0 (5 = 0101), they're the same.

b

NOT(0xFA6B7214) + 1 = NOT($11111010011010110111001000010100_2$) + 1 = $00000101100101001000110111101100_2$

signed: $-(2^{26} + 2^{24} + 2^{23} + 2^{20} + 2^{18} + 2^{15} + 2^{11} + 2^{10} + 2^8 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2)$

un-signed: $2^{31} + 2^{30} + 2^{29} + 2^{28} + 2^{27} + 2^{25} + 2^{22} + 2^{21} + 2^{19} + 2^{17} + 2^{16} + 2^{14} + 2^{13} + 2^{12} + 2^9 + 2^4 + 2^2$

hex	un-signed	signed

hex	un-signed	signed
FA6B7214	$4{,}201{,}345{,}556_{10}$	$-93{,}621{,}740_{10}$

Since left-most bit = 1 (F = 1111), they're not the same.

c

0x05948DEC = 0, 0000 1011, 0010 1001 0001 1011 1101 100_2 = $+1.00101001000110111101100 \cdot 2^{11-127}$ = $1.3969987 \cdot 10^{-35}$

0xFA6B7214 = 1, 1111 0100, 1101 0110 1110 0100 0010 100_2 = $-1.11010110111001000010100 \cdot 2^{244-127}$ = $-3.0562589 \cdot 10^{35}$

hex	IEEE 754 floating point (decimal)
05948DEC	1.3969987E-35
FA6B7214	-3.0562589E35

5

bias = 127

a

88.4375

88.4375_{10} = 1011000.0111_2 = $1.0110000111 \cdot 2^6$; 6+127 = 10000101_2

=> = 0, 1000 0101, 0110 0001 1100 0000 000

-7.3125

-7.3125_{10} = -111.0101_2 = $-1.110101 \cdot 2^2$; 2+127 = 10000001_2

=> = 1, 1000 0001, 1101 0100 0000 0000 000

b

Step1: multiply significand

1.0110000111 * 1.110101 = 10.1000 0110 1011 0011 $\overset{\text{normalize}}{=}$ $1.01000011010110011000 \cdot 2^1$

Step2: calc exp: 127 + (6 + 2) + 1 = 10001000_2

Step3: calc sign: 1 * -1 = -1

=> = 1, 1000 1000, 0100 0011 0101 1001 1000 000

6

a

ans: 0, 0 0000 0001, 0000 00 = $1.0 \cdot 2^{-254} = a_0$

b

1st biggest: 0, 0 0000 0000, 1111 11 = $0.111111 \cdot 2^{-254} = a_1$

2nd biggest: 0, 0 0000 0000, 1111 10 = $0.111110 \cdot 2^{-254} = a_2$

c

$|a_0 - a_1| = 0.000001 \cdot 2^{-254}$

$|a_1 - a_2| = 0.000001 \cdot 2^{-254}$

d

1, 0 1111 0110, 1001 11 => $1.100111 \cdot 2^{246-255} = -1.100111 \cdot 2^{-9}$ = -0.000000001100111_2

e

$1.31_{10} = 1.0100111101011100011 \cdots_2$

=> $0, 011111111, 010100 = 1.010100 \cdot 2^{255-255} = \underbrace{1.010100_2}_{\text{U}} = 1.3125_{10}$

7

`(X + a) >> 2` $\equiv \lfloor \frac{X+a}{4} \rfloor$

a

```
(X + 3) >> 2
```

testing X = 2:

=> $\lfloor \frac{2+3}{4} \rfloor = 1 \neq 0 = (2/4)$ => false

Ans: not equivalent

b

```
((X >= 0) ? X >> 2 : (X + 3) >> 2)
```

Obviously true for $X \geq 0$,

for $X < 0,\ f(X) = \lfloor \frac{X+3}{4} \rfloor$

testing a full period (-1 ~ -5):

$f(-1) = 0, f(-2) = 0, f(-3) = 0, f(-4) = -1, f(-5) = -1, \ldots$ => true

Ans: equivalent

c

```
X >> 2
```

Obviously true for $X \geq 0$,

for $X < 0,\ f(X) = \lfloor \frac{X}{4} \rfloor$

testing X = -1:

$f(-1) = -1 \neq 0$ => false

Ans: not equivalent

d

```
(X + ((X >> 31) & 3 )) >> 2
```

if $X \geq 0$,

> (X >> 31) & 3 = (0..0) & (10) = 0

if $X < 0$,

> (X >> 31) & 3 = (1..1) & (10) = 3

by conclusion from (a), (b), (c),

if $X \geq 0$: `X >> 2` => true

if $X < 0$: `(X + 3) >> 2` => true

Ans: equivalent