Satellite Communication
Assignment 1

EEE F472

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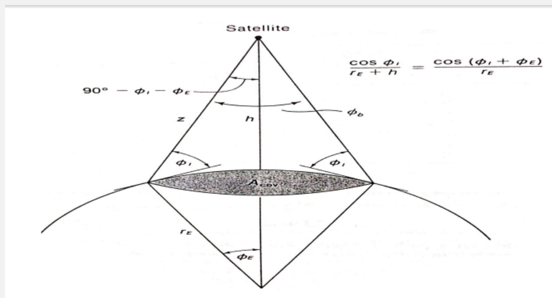
Problem 1

Prove that the coverage area shown in the figure is given by

$$\mathcal{A}_{cov} = 2\pi r_E^2(1 - \cos\phi_E)$$

where $\phi_E = \cos^{-1}\left(\frac{r_E}{r_E+h}\cos\phi_\ell\right) - \phi_\ell$



Solution

The above figure can be equivalently represented as shown below.

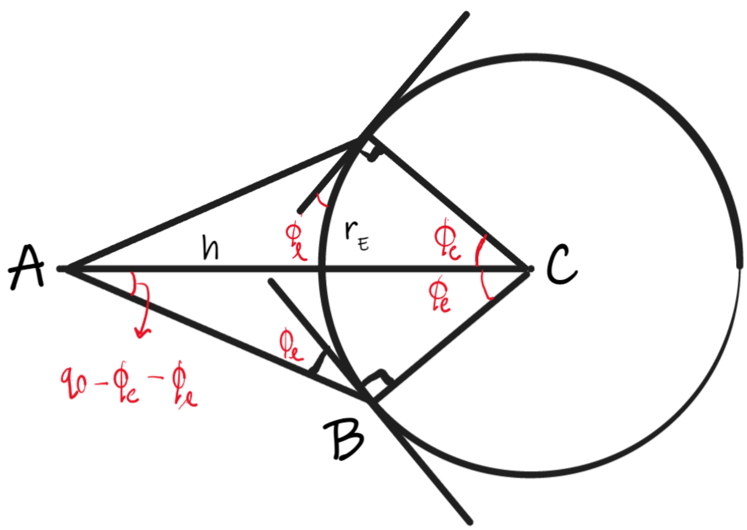


Figure 1: Equivalent representation of the figure

Applying the sine rule on the triangle $\triangle ABC$ we get,

$$\frac{\sin(90 - (\phi_E - \phi_\ell))}{r_E} = \frac{\sin(90 + \phi_\ell)}{r_E + h} \tag{1}$$

$$\implies \frac{\cos(\phi_E - \phi_\ell)}{r_E} = \frac{\cos(\phi_\ell)}{r_E + h} \tag{2}$$

From equation 2 we get,

$$\phi_E = \cos^{-1}\left(\frac{r_E}{r_E + h}\cos\phi_\ell\right) - \phi_\ell \tag{3}$$

To finally find the coverage area, We find the cap area subtended by the solid angle Ω whose corresponding planar angle is ϕ_E.
It is known that:

$$\Omega = \frac{A}{r^2} \tag{4}$$

Where, for the solid angle Ω, A is the cap area (coverage area in our case) and r is the radius of the sphere. We also know the conversion between the said solid angle and the plane angle:

$$\Omega = 2\pi(1 - \cos\phi_E) \tag{5}$$

Here, the radius of the sphere is the radius of the earth, hence we have:

$$\boxed{\mathcal{A}_{cov} = \Omega r_E^2 = 2\pi r_E^2(1 - \cos\phi_E)} \tag{6}$$

where ϕ_E is given by equation 3.

Problem 2

- Show that the slant range is given by

$$z = \sqrt{(r_E \sin\phi_\ell)^2 + 2r_E h + h^2} - r_E \sin\phi_\ell$$

  - verify that $z < r_E + h$

- Suppose the minimum angle of elevation is $5°$ and the carrier frequency is 6GHz. Compute the slant range(in km) and path loss (in dB). Assume $r_E = 6370$ km and orbit radius 42242 km.

Solution

- In reference to figure we can use the cosine rule on $\triangle ABC$. Assuming the length $AB = z$:

$$\begin{aligned}(r_E + h)^2 &= z^2 + r_E^2 - 2zr_E\cos(90 + \phi_\ell) \\ \implies \cancel{r_E^2} + 2r_E h + h^2 &= z^2 + 2zr_E \sin\phi_\ell + \cancel{r_E^2} \\ \implies z^2 + 2zr_E\sin\phi_\ell - (h^2 + 2r_E h) &= 0 \end{aligned} \tag{7}$$

  Solving the quadriatic equation in 7 we get:

$$\boxed{z = \sqrt{(r_E \sin\phi_\ell)^2 + 2r_E h + h^2} - r_E \sin\phi_\ell} \tag{8}$$

- In a triangle, since sum of 2 sides must be more than the third side, we have:

$$z < r_E + h + r_E \tag{9}$$

  Clearly, $z < 2r_E + h \implies \boxed{z < r_E + h}$

- Given that $\phi_\ell = 5°$, we can find z from equation 8. The orbit radius is given as 42242 km. Hence $h = 42242 - 6370 = 35872$ km.

$$\begin{aligned} z &= \sqrt{(r_E \sin 5°)^2 + 2r_E h + h^2} - r_E \sin 5° \\ &= \sqrt{(6370\sin 5°)^2 + 2(6370)(35872) + (35872)^2} - 6370\sin 5° \\ &= 41207.455 \text{ km} \end{aligned} \tag{10}$$

  The path loss is given by:

$$\text{FSPL} = \left(\frac{4\pi d f}{c}\right)^2 \tag{11}$$

  Where d is the slant range, f is the carrier frequency and c is the speed of light.
  Substituting the value of z from equation 10 we get:

$$\begin{aligned} \text{FSPL} &= \left(\frac{4\pi \times 41207.455 \times 10^3 \times 6 \times 10^6}{3 \times 10^8}\right)^2 \\ &= 1.0726 \times 10^{14} \end{aligned} \tag{12}$$

  In decibels:

$$\begin{aligned} \text{FSPL (dB)} &= 10\log_{10}\left(\frac{4\pi \times 41207.455 \times 10^3 \times 6 \times 10^6}{3 \times 10^8}\right)^2 \\ &= 140.304 \text{ dB} \end{aligned} \tag{13}$$

Problem 3a

Complete the EIRP budget table. (Note: Rounding off the final value to one decimal place)

Region	1	2	3
HPA Output	13.6 dbW	13.6 dbW	13.6 dbW
Losses between HPA & antenna	1.8 dB	0.9 dB	0.6 dB
Antenna gain including pointing error losses, etc.	31.5 dB	27.5 dB	25.5 dB
Min. EIRP (dbW)	?	?	?

Solution

Region 1

$$\begin{aligned}\text{Min. EIRP} &= \text{HPA Output} + \text{Antenna Gain} - \text{Losses}\\ &= 13.6 + 31.5 - 1.8\\ &= 43.3\text{ dBW}\end{aligned}$$

Region 2

$$\begin{aligned}\text{Min. EIRP} &= 13.6 + 27.5 - 0.9\\ &= 40.2\text{ dBW}\end{aligned}$$

Region 3

$$\begin{aligned}\text{Min. EIRP} &= 13.6 + 25.5 - 0.6\\ &= 38.5\text{ dBW}\end{aligned}$$

Final budget table

Region	1	2	3
HPA Output	13.6 dbW	13.6 dbW	13.6 dbW
Losses between HPA & antenna	1.8 dB	0.9 dB	0.6 dB
Antenna gain including pointing error losses, etc.	31.5 dB	27.5 dB	25.5 dB
Min. EIRP (dbW)	43.3 dbW	40.2 dbW	38.5 dbW

Problem 3b

Complete the $\frac{G}{T_e}$ budget table. (Note: Rounding off the final value to one decimal place)

Region	1	2	3
Minimum antenna gain	31.5 db	27.5 db	25.5 db
Transmission Losses between Antenna & preamp	1 dB	0.5 dB	1.5 dB
System noise temperature at preamp input	800 K	900 K	750 K
Min. $\frac{G}{T_e}$ (db/K)	?	?	?

Solution

Region 1

Minimum antenna gain after accounting for losses = (31.5-1) dB = 30.5 dB
Gain in the linear scale = $10^{30.5/10} = 1122.0185$
System noise temperature = 800 K
Gain to noise temperature ratio (linear scale) = $1122.0185/800 = 1.4025$
Gain to noise temperature ratio (dB scale) = $10 \log_{10} 1.4025 = 1.4691 \approx 1.5$ dB/K

Region 2

Minimum antenna gain after accounting for losses = (27.5-0.5) dB = 27 dB
Gain in the linear scale = $10^{27/10} = 501.1872$
System noise temperature = 900 K
Gain to noise temperature ratio (linear scale) = $501.1872/900 = 0.5569$
Gain to noise temperature ratio (dB scale) = $10 \log_{10} 0.5569 = -2.5424 \approx -2.5$ dB/K

Region 3

Minimum antenna gain after accounting for losses = (25.5-1.5) dB = 24 dB
Gain in the linear scale = $10^{24/10} = 251.1886$
System noise temperature = 750 K
Gain to noise temperature ratio (linear scale) = $251.1886/750 = 0.3349$
Gain to noise temperature ratio (dB scale) = $10 \log_{10} 0.3349 = -4.7506 \approx -4.8$ dB/K

Final budget table

Region	1	2	3
Minimum antenna gain	31.5 db	27.5 db	25.5 db
Transmission Losses between Antenna & preamp	1 dB	0.5 dB	1.5 dB
System noise temperature at preamp input	800 K	900 K	750 K
Min. $\frac{G}{T_e}$ (db/K)	1.5 db/K	-2.5 db/K	-4.8 db/K

Problem 4

- Assume that the differences in slant height ranges neglected.
- Let D denote the earth station antenna diameter and let λ denote the operating wavelength.
- An optimum space $\Delta\psi = \frac{\lambda}{D}$ yeilds a transmission capacity(per units of bandwidth and angle) given by $\mathcal{C}_{tr} = \frac{2D}{\lambda}$ bps/Hz/rad (Justify).
- Obtain an expression for the maximum transmission rate (bps). Note that this rate is handled by a satellite system using a bandwidth B and a segment of synchronous orbit spanning ψ radian.
- **Numerical:** Consider a bandwidth of 500 MHz and a 30 m diameter antenna at 4 GHz. What is the theoretical global capacity? If a telephone channel requires 64 Kbps, how many channels can be served approximately?

Solution

- Let the maximum transmission rate be given by $\mathcal{M}_{tr}$. We can use dimensional analysis to derive the formula for $\mathcal{M}_{tr}$.

$$\begin{aligned} \mathcal{M}_{tr} &\propto \frac{2D}{\lambda} \times \Delta\psi \times B \cdot \frac{\text{bps}}{\text{Hz} \cdot \text{rad}} \times \text{Hz} \cdot \text{rad} \\ \implies \mathcal{M}_{tr} &= k \cdot 2B \text{ bps} \end{aligned} \tag{14}$$

  Where k is a constant of proportionality.

- Given B = 500 MHz, $\lambda = 0.075$ m, D = 30 m, $\Delta\psi = \frac{\lambda}{D} = \frac{0.075}{30} = 0.0025$ rad.

$$\begin{aligned} \mathcal{M}_{tr} &= k \cdot 2 \times 500 \times 10^6 \text{ bps} \\ \implies \mathcal{M}_{tr} &= k \cdot 1 \times 10^9 \text{ bps} \end{aligned} \tag{15}$$

  Given that a single telephone channel requires 64 Kbps, we can say that number of channels (N) that the satellite can serve is given by

$$N = k\frac{10^9}{64 \times 10^3} = k \cdot 15625 \tag{16}$$

  Assuming the value of k to be 1 we have:

$$\boxed{N = 15625}$$