Sequences of given length where every element is more than or equal to twice of previous

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***Abstract*—In this paper, we have devised 2 algorithms to find the number of possible sequences of length n such that each of the next element is greater than or equal to twice of the previous element but less than or equal to m, while optimizing time and space complexity. We have determined both the time, and the space complexity of our algorithm by illustrating through some graphs.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

For finding the number of possible sequences of length n such that each of the next element is greater than or equal to twice of the previous element but less than or equal to m can be solved using recursion. Recursion is the process of repeating items in a self-similar way. In programming languages, if a program allows you to call a function inside the same function, then it is called a recursive call of the function.

As we know wherever we see a recursive solution that has repeated calls for the same inputs, we can optimize it using Dynamic Programming. The idea is to simply store the results of sub-problems, so that we do not have to re-compute them when needed later. This simple optimization reduces time complexities from exponential to polynomial.

As per the given condition the n-th value of the sequence can be at most m. There can be two cases for n-th element.

1) If it is m, then the (n-1)th element is at most m/2. We recur for m/2 and n-1.
2) If it is not m, then it is at most m-1. We recur for (m-1) and n.

The total number of sequences is the sum of the number of sequences including m and the number of sequences where m is not included. Thus the original problem of finding number of sequences of length n with max value m can be subdivided into independent sub-problems of finding number of sequences of length n with max value m-1 and number of sequences of length n-1 with max value m/2.

II. ALGORITHM 1

A. ALGORITHM DESIGN

1) Take input of integers n and m in variables n and m respectively.
2) Call for the solve function with parameters as n and m which will give the answer.
3) Solve function is using a recursive approach to find the answer.
4) Base cases for this function is when m becomes zero then we will return zero and also when n will become one then we will return m.
5) Create a new variable ret initialized with zero and looping from 1 to m and adding answers from 1 to m in ret by recursively calling the solve function for smaller input.
6) Return the value of ret which has the final answer.

B. ALGORITHM

Input: N(length of sequence) and M(upper bound on sequence elements)

Output: Number of possible sequences of length n such that each of the next element is greater than or equal to twice of the previous element but less than or equal to m.

Method:

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1) call the solve function with parameters as n and m: solve(n,m);
2) if $m <= 0$ return 0
3) if $n == 1$ return m
4) Declare and Initialise ret with 0.
5) Loop the current element(i) of the sequence from 1 to m.
        6) call the solve function with parameters n-1 and i/2. Add the returned value to ret
7) return ret;
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C. APRIORI ANALYSIS

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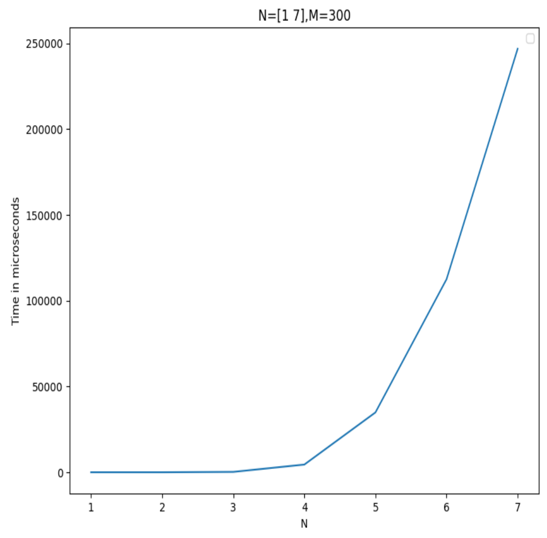
D. APOSTERIORI ANALYSIS

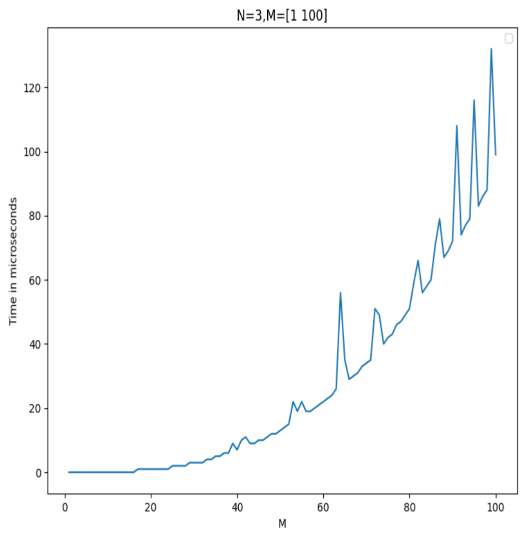
First we Fix M=300 and vary N from 1 to 7

N	Time
1	0
2	3
3	224
4	4493
5	34892
6	112558
7	246904



Now we Fix n=4 and vary M from 1 to 100

N	Time
1	0
16	0
30	3
40	7
50	13
60	22
100	99



From these two graphs we can conclude that complexity is exponential in 'n' and polynomial in 'm'

E. TIME COMPLEXITY

Worst case : O(m^n)

Best case : Ω(1) when n=1

III. ALGORITHM 2

A. ALGORITHM DESIGN

1) Take input of integers n and m in variables n and m respectively.
2) Call for the isZero function with parameters as n and m which will check whether the answer exists or not.
3) If $n >= 60$ then return true as our algorithm works till long long int.Also if m ¡ 2 to the power n-1 then we cannot have the sequence of length n having max value upto m so return true. And if the return value is true or it does not exist then we exit after printing 0.
4) Else using we create a vector of vectors mem of n+1 rows and m+1 columns and initializing it to -1.
5) Call the solve function with parameters n,m,mem which uses recursion with memoization (Dynamic Programming) to find the answer.
6) Base cases for this function is when the isZero function with n,m as parameters return true then return zero and also when n will become one then we will return m.
7) Check if the value at n,m in the mem vector is not -1 then we return mem[n][m] as answer for this n,m is stored in the previous call of the solve function on n,m.
8) As per the given condition the n-th value of the sequence can be at most m. There can be two cases for n-th element.
   (i) If it is m, then the (n-1)th element is at most m/2. We recur for m/2 and n-1.
   (ii) If it is not m, then it is at most m-1. We recur for (m-1) and n.
   So we recursive call for the solve function with parameters n,m-1,mem and n-1,m/2,mem and add the return values from both and store it in mem[n][m] so if further this n,m is called then we can return its value without computation as we have stored.
9) Final answer is returned from the solve function and displayed.

B. ALGORITHM

Input: N(length of sequence) and M(upper bound on sequence elements)

Output: Number of possible sequences of length n such that each of the next element is greater than or equal to twice of the previous element but less than or equal to m.

Method of isZero(n,m)

1) if $n >= 60$ return false;
2) if $m < 2^{n-1}$ return false;

Method of solve(n,m)

1) Check if the answer is 0. If found 0 return.
2) if n==1 return m
3) if mem[n][m]!=-1 return mem[n][m]
4) Call solve for n,m-1 and n-1,m/2 and assign their sum in mem[n][m]

C. APRIORI ANALYSIS

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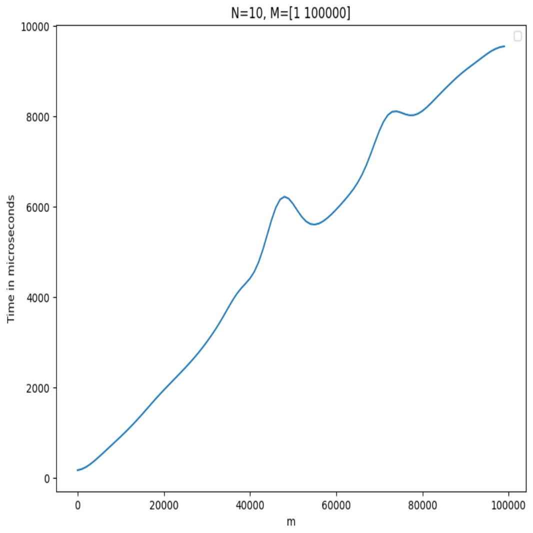
D. APOSTERIORI ANALYSIS

First we Fix N=10 and vary M from 1 to 100000

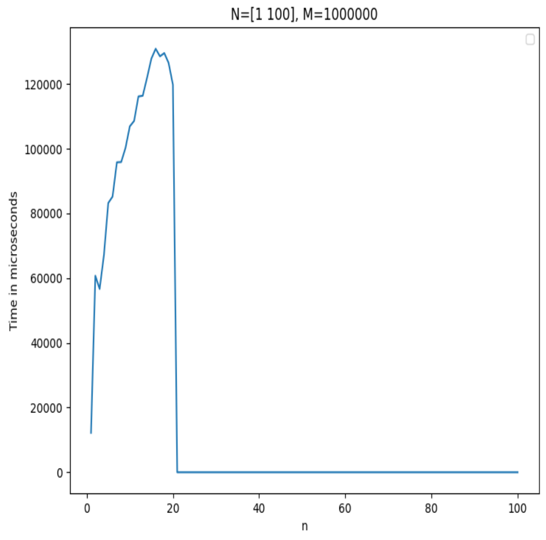
M	Time
1	0
2001	180
9001	224
43001	4282
78001	7857
88001	8701
99001	9704



From above graph we can conclude that complexity is linear in 'm'.

Now we Fix m=1000000 and vary n from 1 to 100

N	Time
1	12205
16	130887
30	0
40	0
50	0
60	0
100	0



In above graph complexity first increases linearly with 'n', but as 'n' increases the isZero() function calculates the answer in O(1) and graph gets merged with the x-axis.

E. TIME COMPLEXITY

Worst case : O(min(n,$log_2(MAX_M)$)*m)
Best case : Ω(1) when answer is 0

CONCLUSION

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

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