OPTIMIZATION OF GOLF BALLS BASED ON REGRESSION MODELS

By

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ABSTRACT

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This project looks at the purpose of having a dimpled golf ball in golf over a smooth surfaced golf ball. Over the years the golf balls have evolved in many different ways, just like all the sports balls have. The best way to look at what creates a great golf ball would be to examine the aerodynamics and trajectory produced by the way it spins and the amount of drag the golf ball creates. These forces take into account many different things about a golf ball, from its core all the way out to its cover. To model the trajectory of the flight, different differential equations will be used. The purpose is to find out which golf ball may be your best choice whether it is a worn down range ball or brand new chrome soft straight out of the box.

CHAPTER 1

INTRODUCTION

1.1. THE GAME OF GOLF

Golf is a sport in which a player uses various clubs to hit balls into a number of holes on a course in as few shots as possible. Golf is one of the few games that do not require a standardized playing area [B5]. Many golf courses are arranged in their own unique way with their own set of challenges. Golf courses can range from being right by a beautiful ocean to in some of the highest of mountain tops. The game first began in Scotland around the 15th century [B5]. Some of the origins of the game can be traced back to the Roman game of paganica, where players used sticks that were bent to hit what was then a stuffed leather ball [B5]. Imagine playing golf with that nowadays, talk about a struggle. The game has since evolved tremendously, with skill and also technology.

The MacDonald Boys playing golf, 18th Century

1.1.1. GOLF SHOTS

One might say that golf is a simple game. Hit the ball down the course into a hole. No schemes, no defense and no referees. You just have to get the ball into the hole at or under the number of strokes (swings) allowed if you want to be great. But for golfers like me sometimes that does not always happen. Many different shots can be hit in golf.

A *slice* is a shot created when a golf ball curves in flight from left to right for a right handed golfer or right to left for a left handed golfer [B8]. This shot has the same shape as a *fade*shot, the difference would be that a slice is more severe than a fade meaning that it is rarely played intentionally. When a slice is hit the golf ball has a clockwise spin on it around the z-axis(assuming the z-axis is directed vertically), created from the club face hitting the golf ball from the outside and in. (Red in Figure 1)

The next created in golf is called a *hook*, which is the opposite of a slice. A hook is produced when the golf club hits the golf ball from the inside out. This then creates a counter clockwise spin on the ball around the z-axis. This is also another shot that is rarely intentionally hit due to its severe flight pattern [B8]. (Red in Figure 1)A draw is less severe than a hook but has the same type of spin.

You also have two less severe shots called a pull and a push, which are the opposites of each other. These shots, although less severe, are a result of bad contact by only centimeters. These shots start of right (for a push) and left (for a pull) but stay on their initial line [B8]. So unlike the slices and hooks you may only end up a couple feet from your intended target rather than in the woods. (Blue in Figure 1)

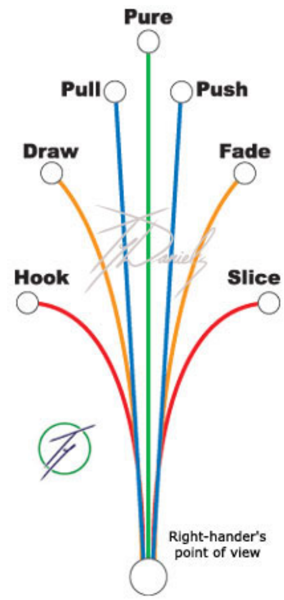

Figure 1

Generally, pushes and pulls have very little sidespin. Those shots are simple hit in the wrong direction. You have the slices, hooks, fades and draws are all created by putting clockwise or counter clockwise spin on the ball from hitting it with an open or closed club face, or by approaching the ball from an angle as opposed to from directly behind the ball. Some of the best golfers can create these shots on purpose to avoid certain obstacles or to simply play the course or wind (fades and draws). Other golfers, who are not the best, create these shots from bad fundamentals. When a golf ball is hit perfectly, backspin is created on the ball around the x-axis to give the ball lift. Many of the best golf balls have been recorded having at least 2000-3000 rotations per minute when hit, which is amazing but also very detrimental [B2]. Think about it, if the ball spins that much when a good shot is hit, then it will surely spin that much when a bad shot is hit. This means for those terrible slices and hooks, off the tee box or fairway, the ball will not only land off target but could easily cause you to lose you a very expensive ball in the woods.

1.1.3 GOLF BALLS

The first American made golf ball was created in 1895 by Spalding, who is widely known for the making of the National Basketball Association's regulated basketball. Since then, like every other "ball" sport in the world the golf ball has evolved through its generations [B7]. It first started off as a simple wooden sphere, then transformed to what was called the featherie. (Figure 2) This ball was created from tightly packed goose feathers in a horse or cow hide [B7]. This was an ingenious approach, as one might think that feathers will definitely make the ball fly. The next generation of golf balls resembled more of what the golf ball is today. These balls were called gutties. (Figure 2) These balls were created from melting rubber into a spherical form then incasing it in a metal ribbed like casing while it cooled and hardened [B7]. This ball was definitely more aerodynamic than the previous and seemed to be the turning point of why we have dimpled golf balls.

The dimpled golf balls used today are created many different ways, starting with the core which can be made of rubber or even liquid. These cores place the ball into 3 compression ratings (80, 90 or 100) [B7]. The higher compression balls are used by more advanced golfers. The cover of the golf ball is made from either balata (a rubber-like mix) or surlyn (a resin) [B7]. Both covers provide the same kind of feel but the difference between many golf balls are their dimples. It has always been a question for many golfers, "which ball is best for me to play based on my play?" With the following insight on what contributes to the flight of a golf ball, this information should help make that choice in golf ball an easy pick for average golfers.

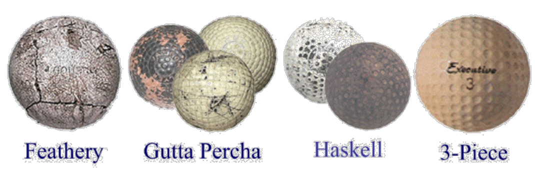

Figure 2

1.3 FLIGHT EQUATIONS

To introduce this project we must first understand how a ball flies. Projectile flights are knowingly not all the same, but when you see any drawing of a projectile whether it is a basketball shot, golf ball hit or batted baseball they all have the same parabolic flight pattern. This parabolic flight pattern is not true to the actual flight of a golf ball or any of those projectiles mentioned. Surprisingly many people do not know the true flight of a golf ball, but it is surely not parabolic. First we will start off by showing the simple parabolic flight pattern then carry on to showing exactly how the flight of a smooth and dimpled golf ball differ.

1.3.1 PARABOLIC FLIGHT

First we can start off with how we acquire a simple parabolic flight pattern from two differential equations found from the research. These equations do not take into account wind, drag, magnus, change in altitude or humidity. All of which have an effect on the flight of a golf ball. These two equations give us what will end up being the initial velocity and position in the x and y directions. In the following discussion, the flight is two dimensional. The y axis is the vertical axis and the x axis is the horizontal axis.

First we begin with the equation in the y direction, which has gravity being the only force acting on it. We use Newton's fundamental law that states that force is equal to mass times acceleration:

$$F = ma. [A1]$$

Hence, we start off by summing all the forces in the y direction and denoting the acceleration by the second derivative.

$$\sum F_y = my'' [A1]$$

$$my'' = -mg [A1]$$

As you can see by the equation above, the mass of the object cancels itself, leaving only gravity to be accounted for in the downward direction of course, which is why there is a negative sign

$$y'' = -g\text{[A1]}$$

$$y' = -gt + C_1\text{[A1]}$$

$$y = -\frac{1}{2}gt^2 + C_1t + C_2.\text{[A1]}$$

To get from the second derivative to the original equation we first took the integral of the second derivative and then took the integral of the first derivative.

When we solve the equation for $C_1$and C_2 we get that

$$C_1 = V_{0y}$$

$$C_2 = y_0.$$

These two variables represent the initial velocity and height in the y direction. When substituted back into the equation, we have

$$y = -\frac{1}{2}gt^2 + V_{0y}t + y_0$$

where t represents time and g represents gravity.

Next we sum forces in the x direction

$$\sum F_x = mx''\text{[A1]}$$

$$mx'' = 0.$$

Since we are ignoring the air resistance and other forces, the sum of these forces equals zero

$$x'' = 0$$

$$x' = C_3$$

$$x = C_3t + C_4.$$

Just like the forces in the y direction, to get to the original x equation we take the integral of the second and first derivatives to get it, then we can solve for both variables to get the following answers

$$C_3 = V_{0x}$$
$$C_4 = x_0.$$

Simple substituting the initial velocity and distance in the x direction in for the variables will give you the complete equation in the x direction

$$x = V_{0x}t + x_0.$$

These following equations give us the simple parabolic flight pattern as follows

1.3.2 SMOOTH BALL FLIGHT

The flight of a smooth ball is very interesting. Although the ball is smooth and has nothing for the air to essentially grip it with, the air resistance from the ball making contact with the air still results in some type of drag [A1]. The air molecules flow around the ball in a parallel fashion which is known as the laminar flow. (Figure 3) This flow creates a large wake behind the ball which in return decelerates the ball causing it not to travel as far, which is why dimpled golf balls were soon created [A1].

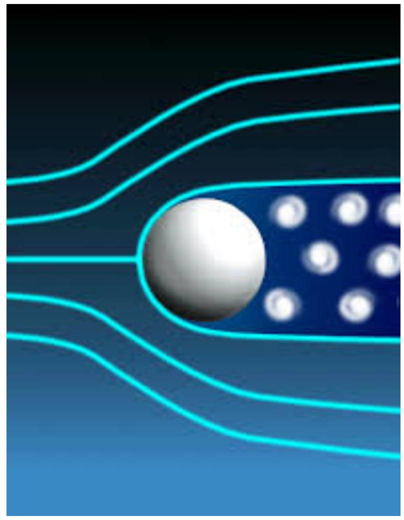

Figure 3

To create the flight pattern of a smooth "golf ball" we first will take into account the air resistance. Air resistance of course is the resistance which the air molecules place on the ball. This can also be known as the aerodynamic drag, which will be discussed later. To resemble the flight pattern of the smooth "golf ball" we use the following equations in the x, y and z directions:

$$\sum F_y = my'' = -mg - R(V_y) \text{ [A1]}$$

$$\sum F_x = mx'' = -R(V_x)\text{[A1]}$$

$$\sum F_z = mz'' = -R(V_z).\text{[A1]}$$

These summation equations take into account the parabolic equations from 1.3.1 but add another force, air resistance/drag. As you can see the R variable is multiplied by each of the initial velocities in each direction. This is done because the air resistance on the ball will affect the balls overall velocity[A1]. Based on the previous equations we are able to get the projection of the smooth "golf ball" to look like so:

As you can see this flight pattern does not match that of the parabolic flight pattern. The ball seems to have the more of a linear path when going up into the air and as it descends, we can see that it has a much steeper angle. This is due to the effect of air resistance, R, on the ball.

1.3.3 DIMPLED BALL FLIGHT

The difference between a dimpled and smooth golf ball is of course the dimples, but these dimples play a huge role in the flight of the golf ball. When dimples are added to a ball the common knowledge thought would be that it would not fly as far as the smooth golf ball. That assumption is wrong. The dimples on the ball actually help it fly further. They create what is called a turbulent flow. (Figure 4)

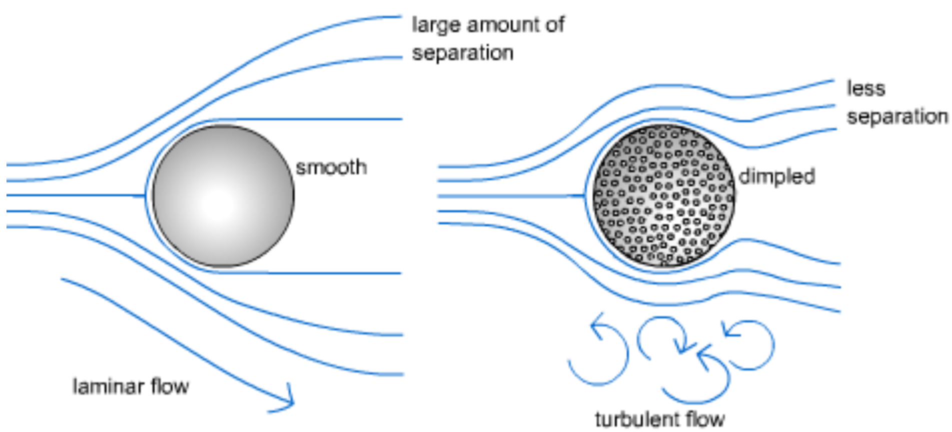

Figure 4: Here you can see the difference between the laminar, and turbulent flow. Notice that the turbulent flow adds lift to the golf ball unlike the laminar flow.

This flow is created when the air molecules are able to huge closer to the ball because the boundary layer created by the dimples, are turbulent [B6]. So with this in mind the summation equations of the forces in the x,y and z direction are modified. This modification is called the Magnus Force, F_M.

$$\sum F_y = my'' = -mg - R(V_y) + F_M(w, v_x, v_y)[\mathrm{A1}]$$

$$y'' = -g - \frac{R}{m}\left(V_y\right) + \frac{F_M}{m}(w, v_x, v_z)[\text{A1}]$$

$$\sum F_x = mx'' = -R(V_x) + F_M(w, v_x, v_z)[\text{A1}]$$

$$x'' = -g - \frac{R}{m}(V_x) + \frac{F_M}{m}(w, v_y, v_z)[\text{A1}]$$

$$\sum F_z = mz^n = -R(V_z) + F_M(w, v_x, v_y)[\text{A1}]$$

$$z'' = -\frac{R}{m}(V_z) + \frac{F_M}{m}(w, v_x, v_y)[\text{A1}]$$

With these equations we are able to see that the flight of a dimpled golf ball is indeed different than that of the smooth and simple parabolic flight.

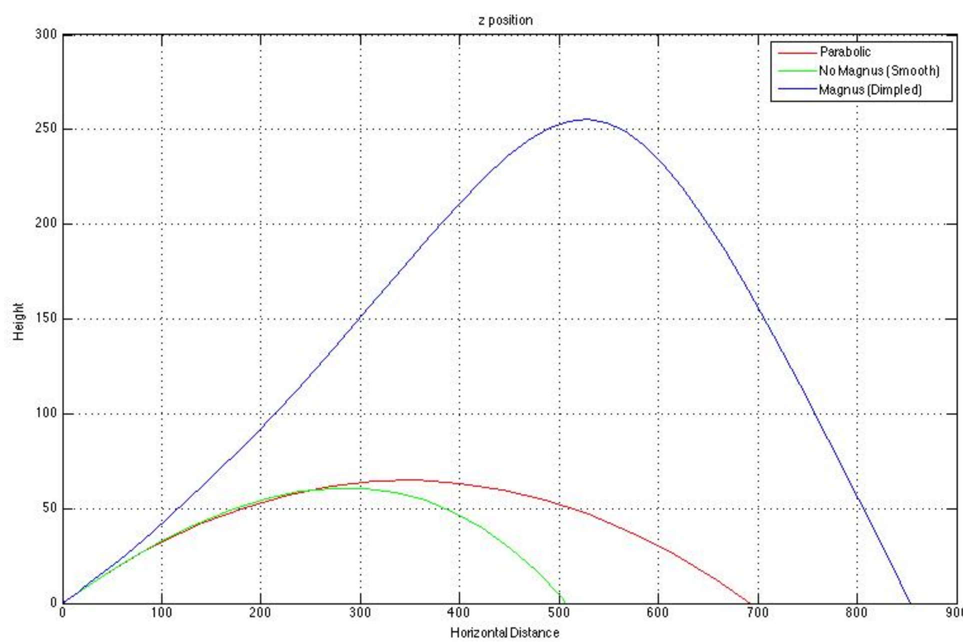

Figure 5

Depending on the amount of backspin added to the equation, which relates to the loft of the club and the w in the equations above, we get different amounts of lift on the ball. This is because the dimples on the ball create a smaller amount of low pressure giving higher air pressure on the top of the ball allowing it to go up.

1.4 MAGNUS AND DRAG

1.4.1 DRAG

When any projectile flies through the air a drag force is created from the resistance of the air also known as air resistance. This drag is different depending on the projectile flying through the air. In the earlier sections we showed how the flight equations of a dimpled golf ball have to take into account the air resistance on the ball. So what exactly is that air resistance? It is represented by the variable R in our previous equations but here is exactly what the R consist of. The following equation takes into account the drag coefficient,C_D, air density, p, surface area, A, and the velocity of the golf ball in the equations respective direction, V.

$$R = \frac{1}{2} C_D p A V^2 \text{[A1]}$$

Here you can see the equations for air resistance for each direction:

$$R_x = \frac{1}{2} C_D p A {V_x}^2 \text{[A1]}$$

$$R_y = \frac{1}{2} C_D p A {V_y}^2 \text{[A1]}$$

$$R_z = \frac{1}{2} C_D p A {V_z}^2 \text{.[A1]}$$

Something that will be focused on and used later in the paper will be the drag coefficient. This coefficient seems to be something that will help a golfer pick out his/her ball. The less drag the higher ranking the ball shall get.

1.4.2 MAGNUS

The Magnus effect or, Magnus force in this case, was discovered by Gustav Magnus in 1852 [B4]. This force occurs when a spinning ball through the air creates a boundary layer of air that clings to the balls' surface as it travels. The boundary layer travels along the ball on both sides when it is rotating. On one side the boundary layer will collide with the wind causing the air to decelerate, which then creates an area of high pressure[B4]. On the opposite side the layer will move the same direction as the wind, helping the air move collectively faster which sets up an area of low pressure. The difference in pressure is what causes the ball to move. Depending on the spin, whether it is sideways or end over end, the ball will move from an area of high pressure to an area of low pressure[B4].

The calculation of the Magnus force goes as follows. We take into account three factors: velocity, angular velocity and a magnus coefficient. The magnus force,F_M, is calculated using the cross product of the velocity and angular velocity. This is because the rotation caused by air disturbances is perpendicular to the direction of the angular velocity, which causes the flight of the ball to be altered perpendicular to the linear velocity. The following work will show you how the equations are assembled [A1]:

$$\mathbf{F_M} = S(\omega \times V)$$

$$S\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_\mathbf{i} & \omega_\mathbf{j} & \omega_\mathbf{k} \\ V_i & V_j & V_k \end{vmatrix} = S[(\omega_j V_k - \omega_i V_j)\mathbf{i} - (\omega_i V_k - \omega_k V_i)\mathbf{j} + (\omega_i V_j - \omega_j V_i)\mathbf{k}]$$

$$\mathbf{F_{Mx}} = S(\omega_j V_k - \omega_k V_j)$$
$$\mathbf{F_{My}} = S(\omega_k V_i - \omega_i V_k)$$
$$\mathbf{F_{Mz}} = S(\omega_i V_j - \omega_j V_i)$$

1.5 NUMERICAL SOLUTIONS

Recreating some code from my research, I was able to come up with some solutions in matlab that would help show the flight of a dimpled golf ball. The numbers used were pulled from a ball data website that gave some numbers recorded from test done on the golf balls.

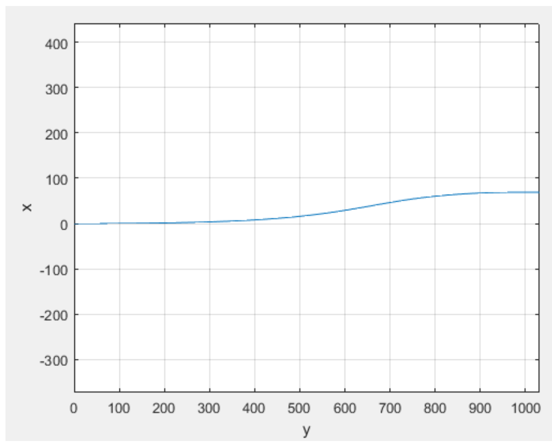

Figure 6 shows what happens when counter clockwise spin about the z-axis is placed on a ball

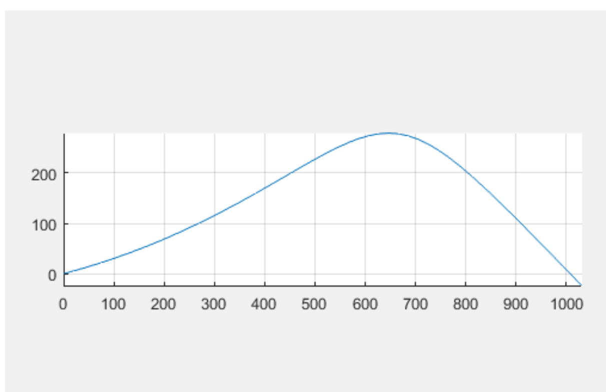

Figure 7 shows the flight of a golf ball at an initial velocity of 144 mph with a launch angle of 14.7 degress

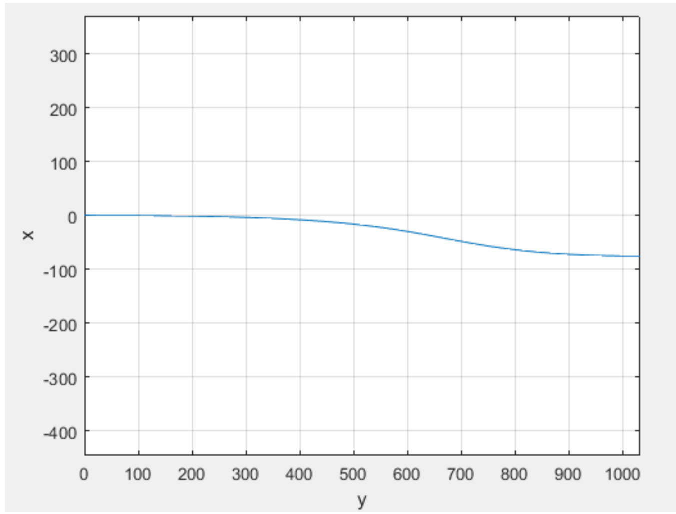

Figure 8 shows what happens when clockwise spin about the z-axis is placed on a ball

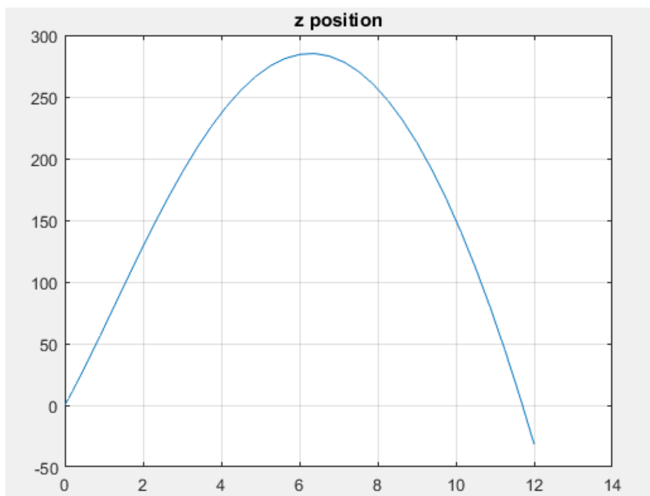

Figure 9 shows the height of a golf ball, throughout flight, hit with a 14.9 degree launch angle

1.6 CONCLUSION

In this chapter we were able to describe and recreate the flight of a standard dimpled golf ball, the most basic but yet important steps for going forward with this project. We found some ball data that we were able to collect numbers from to build different graphs, but the numbers also helped us know what we needed to fix within our equations. So the next step is to find ways to take this information further to answer our research question. This ball data used will be something we keep looking back on to help create the regression models that will help rate the golf balls.

CHAPTER 2
FUTURE

2.1 REGRESSION

Regression analysis is a statistical tool for the investigation of relationships between factors [B9]. The investigator seeks to ascertain the causal effect of one variable upon another—the effect of a price increase upon demand, for example, or the effect of dimples and the flight distance of a golf ball. To explore such issues, the investigator assembles data on the underlying variables of interest and employs regression to estimate the quantitative effect of the causal variables upon the variable that they influence [B9]. The investigator also typically assesses the "statistical significance" of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship.

2.1.1 WHY REGRESSION?

We will consider two of the biggest factors in ball flight and performance, drag and magnus coefficients. These factors take into account a lot of what the golf ball is made of. Certain golf balls are rated higher based on things such as distance in flight and the amount of spin that can be created, all of which are dependent on the magnus and drag coefficients. With the drag multiple regression model, we will look at things that this coefficient would be dependent upon such as weight and the cover. The cover of a golf ball is made with different materials as is the core of a golf ball. The magnus multiple regression model would be dependent upon variables such as ball dimples and rotations per minute that the ball creates. With these models in mind we would be able to create some type of ranking system for golf balls. The main idea is to be able to take ball data and regress it so that we can use the two coefficients and model their flight. This will then allow us to rank the golf balls based on a flight ratio, such as height to distance.

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APPENDIX

Solver equations from Matlab

```
%solve flighteqans
%initial conditions
bspeed = 142.9;   %mph
bspeed2 = bspeed*5280/(60*60); %ft/s
langle = 15.8;  %degrees
langle2 = langle*pi/180;
x0 = 0; %initial x
vx0 = 0;  %initial velocity in x direction
y0 = 0; %initial y
vy0 = bspeed2*cos(langle2)  %initial velocity in y direction
z0 = 0; %initial z
vz0 = bspeed2*sin(langle2)  %initial velocity in z direction
[t,f] = ode45(@flighteqns,[0 15],[x0, vx0, y0, vy0, z0, vz0]);
z = f(:,5);
ztest = z(2);
n = 2;
while ztest > 0
    n = n+1;
    ztest = z(n);
end
m = (z(n) - z(n-1))/(t(n) - t(n-1));
tfinal = t(n) - z(n)/m
y = f(:,3);
my = (y(n)-y(n-1))/(t(n) - t(n-1));
yfinal = my*(tfinal - t(n))+y(n);
yfinal/3
close all;
plot(t(1:n), f(1:n,1));
title('x position');
grid on;
figure;
plot(t(1:n), f(1:n,3));
title('y position');
grid on;
figure;
plot(t(1:n), f(1:n,5));
title('z position');
grid on;
figure;
plot(f(1:n,3),f(1:n,1));
xlabel('y');
ylabel('x');
axis equal;
grid on;
figure;
plot3(f(1:n,1),f(1:n,3),f(1:n,5));
grid on;
axis equal;
```

Flight equations from Matlab

```
function dydt = flighteqns(t,f)
%y is the vector of functions
%f1 = position in x directoin, f2 = velocity in x direction
%f3 = position in y directoin, f4 = velocity in y direction
%f5 = position in z directoin, f6 = velocity in z direction
%f1' = f2;
%f2' = -g - R/m*f2^2+MagnusF/m = -g - R/m*f2+*S/m*(omegay*f6 -
omegaz*f4)
%f3' = f4
%f4' = -g - R/m*f4^2+*S/m*(omegaz*f2 - omegax*f6)
%f5' = f6
%f6' = -g - R/m*f6^2+*S/m*(omegax*f4 - omegay*f2)
g = 32.2;
Cd = .15;
rho = .002378;
A = .25*pi*(1.75/12)^2;
m = (1.5/(16*32.2));
s = .000005;
omegax = 150;
omegay = 70;
omegaz = 0;
dydt = [f(2);
     -0.5/m*Cd*rho*A.*f(2).^2+s/m*(omegay*f(6)-omegaz*f(4));
    f(4);
     -0.5/m*Cd*rho*A.*f(4).^2+s/m*(omegaz*f(2)-omegax*f(6));
    f(6);
     -g-0.5/m*Cd*rho*A.*f(6).^2+s/m*(omegax*f(4)-omegay*f(2))];
% dydt = [f(2);
%       -0.5/m*Cd*rho*A.*f(2).^2;
%      f(4);
%       -0.5/m*Cd*rho*A.*f(4).^2;
%      f(6);
%       -g-0.5/m*Cd*rho*A.*f(6).^2];
end
```

BIOGRAPHICAL SKETCH