The Flight of a Golf Ball

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Abstract

The field of projectile motion in classical mechanics is quite vast and has countless applications ranging from modelling the motion of objects in games to the designing of hot air balloons. In this work the principles of projectile motion will be used to model the trajectory of a golf ball and investigate the effects of the backspin and sidespin of the ball on its trajectory. Three models will be presented to model the trajectory of the ball Model I, Model II and Model III respectively. Each model builds up from the previous model and presents a system of differential equations resulting from applying Newton's second law that govern the flight of the ball. In Model II and Model III, the system of differential equations will be solved numerically using the Euler method. Then Model III which the most accurate representation of the flight of a golf ball amongst the three models will be used to investigate the effects of the backspin and sidespin on the ball on its trajectory and to produce a numerical simulation.

Introduction

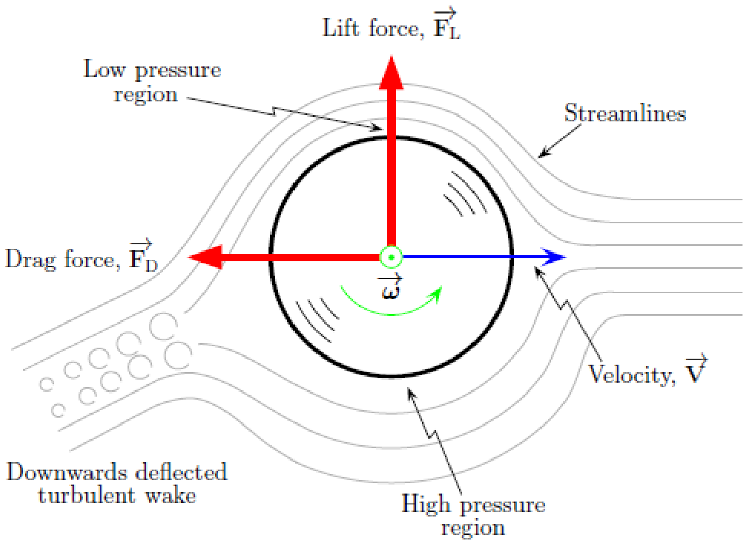
Golf is a very popular sport across the world, a lot of skill and intuition are required from the player for a good strike. The trajectory of the ball has been shown to be affected by the physical composition of the ball with dimpled balls being able to stay in flight longer and travel larger distances when compared to smooth balls. The modern-day golf ball typically has about 400 shallow dimples and if these dimples were not present the range of a long drive would be significantly reduced [1].

Although the physical composition of the ball plays an important role in the trajectory of the ball it is not the only important factor. The lift force resulting from the interaction of the spin of the ball with the surrounding airflow plays an important role in the trajectory of the ball. There are two competing factors due to spin on the trajectory of a golf ball. A large spin results in large drag which makes the ball slow down more rapidly and decrease the distance it covers, however a larger spin also produces a larger lift which would keep the ball in air longer and allow it to cover more distance. Empirical results have shown that the lift force is dominant, hence the ball would travel further and stay in flight longer [2].

A golfer must analyse his shot by considering the slope and condition of where he wants his shot to land. He then makes his judgement on the right amount of spin to impart to the ball, the angle of attack and how hard to hit the ball. If the golfer's swing is out of line then he will impart a side spin to the ball, this side spin results in a sideways force on the ball [1]. If this sideways force results in the ball drifting to the right, the trajectory is called a slice for a right-handed golfer. If the ball drifts left, then the motion is called a hook for a right-handed golfer [2]. The right amount of spin goes a long way in producing the ideal shot.

Literature Review

If a golf ball has no spin as it moves through the air the only forces acting on it would be gravity and air resistance, however if the ball is spinning, then it is subjected to an additional lift force termed the Magnus force. Using three dimensional cartesian coordinates gives the ball six degrees of freedom meaning it can move freely along the three axes and it can change its orientation through rotation about three perpendicular axes. The spin of the golf ball is characterized by its angular velocity [3]. If we consider a golf ball moving to the right that has some velocity $\vec{V}$, angular velocity $\vec{\omega}$, the drag force being $\overrightarrow{F_D}$ and the Magnus force being $\overrightarrow{F_L}$. A schematic presentation of the forces acting on the ball excluding gravity is shown in Figure 1 below.



Streamlines around a rotating golf ball under conditions of a laminar boundary. The ball ideally has backspin only. The Magnus force $\overrightarrow{F_L}$ is perpendicular to both $\vec{V}$ and $\vec{\omega}$.Although dimples are not shown they are assumed to be present.

Figure 1

The ball is spinning anti-clockwise which corresponds to back-spin. Analysing the forces involved and the physics of the scenario in Figure 1 it is identical to a wind tunnel simulation in which it would be ideal to have the spinning ball in a fixed position and the air stream moving in the opposite direction with velocity $\vec{V}$ [3]. Two types of fluid flow can occur in the boundary layer streamline flow (laminar flow) or turbulent flow, in Figure 1 the case for laminar flow is shown with a stable streamline pattern. It can be observed that on the top of the sphere $\vec{\omega}$ is in the same direction as the air velocity relative to the sphere, hence it aids the flow and increases the flow velocity. At the bottom of the sphere $\vec{\omega}$ is in opposite direction of the air velocity and decreases the flow velocity. The Bernoulli equation dictates that in regions where the fluid velocity is low the pressure is high and vice versa. This means that the top of the sphere is a low-pressure region and the bottom is a high-pressure region, this results in a lift force [3].

The drag force is assumed to be proportional to the square of the speed of the ball and opposite the direction of $\vec{\boldsymbol{V}}$. This means that if we define $\widehat{\boldsymbol{V}}$ as a unit vector in the direction of $\vec{\boldsymbol{V}}$,then the drag force takes the form

$$\overrightarrow{\boldsymbol{F_D}} = -k\left|\vec{\boldsymbol{V}}\right|^2\widehat{\boldsymbol{V}} \tag{A}$$

The proportionality constant k is determined by means of theory and experiments and is given by

$$k = \frac{1}{2}C_d\rho A \tag{B}$$

Here C_d is a dimensionless quantity called the drag coefficient and is a characteristic of the projectile's surface, it indicates the fraction of drag the projectile will experience depending on an area of the projectile in contact with the fluid , ρ represents the density of air and A represents the cross sectional area of the projectile [4]. In Figure 1 the Magnus force is perpendicular to both $\vec{\boldsymbol{V}}$ and $\vec{\boldsymbol{\omega}}$. Hence the Magnus force is proportional to $\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{V}}$ which means it takes the form

$$\overrightarrow{\boldsymbol{F_L}} = S(\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{V}}) \tag{C}$$

Here S is a constant called the Magnus coefficient and has dimensions of mass in SI units $[S] = kg$. This constant can be thought of as the aggregate of the factors, the lift coefficient, cross sectional area of the ball and the density of the air [5]. When Newton's law is applied to the golf ball with the various forces acting on it, a system of non linear second order differential equations about the three axes will arise. An analytical solution to this system is impossible to obtain while a numerical solution can be obtained by implementing or using a numerical solver given some initial conditions. Numerical solutions will offer a model for the trajectory of the golf ball and dynamic quantities can be extracted from the solution.

The simplest first order numerical integration method is the Euler-method, this method is easy to implement and is not computationally expensive. Although the system of differential we wish to solve is composed of second order nonlinear differential equations the Euler method will still be applicable since any differential equation of order n can always be transformed to a system of n first order differential equations by means of substitution [6].Therefore the system of differential equations will reduce to a system of first order equations in which the Euler method can be used to obtain the position of the golf ball and the velocity of the golf ball at some given time t.

Flight Models

Model I

The simplest model to consider is when the golf ball is treated as if it is moving in a vacuum and it is not spinning. Then there is no air resistance and no Magnus force. The only force acting on the golf ball is gravity. We can consider the motion in x and y axes. The golf ball is launched with a velocity $\overrightarrow{V_0} = (V_{0x}, V_{0y})$ from some position (x_0, y_0). Applying Newton's second law on each axis gives the following

$$m\ddot{y} = -mg \tag{1.1}$$

$$\ddot{y} = -g$$

The solution for $y(t)$ can be trivially obtained with direct integration and it takes the form

$$y(t) = -\frac{1}{2}gt^2 + C_1 t + C_2$$

with the initial conditions $y(0) = y_0$ and $\dot{y}(0) = V_{0y}$ then the exact solution is

$$y(t) = -\frac{1}{2}gt^2 + V_{0y}t + y_0 \tag{1.2}$$

In the x direction there is no force and we can easily obtain a solution

$$m\ddot{x} = 0 \tag{1.3}$$

$$x(t) = C_3 t + C_4$$

$x(0) = x_0$ and $\dot{x} = V_{0x}$ the exact solution is

$$x(t) = V_{0x}t + x_0 \tag{1.4}$$

From equation (1.2) the trajectory of the ball is parabolic. If the launch velocity $\overrightarrow{\boldsymbol{V_0}}$ makes an angle of θ with the x axis, then $V_{\boldsymbol{0x}} = |\overrightarrow{\boldsymbol{V_0}}|\cos(\theta)$ and $V_{\boldsymbol{0y}} = |\overrightarrow{\boldsymbol{V_0}}|\sin(\theta)$. The carry of the ball depends on the launch velocity $\overrightarrow{\boldsymbol{V}}$ and the angle θ.Figure 1 below shows trajectories with the same angle of attack.

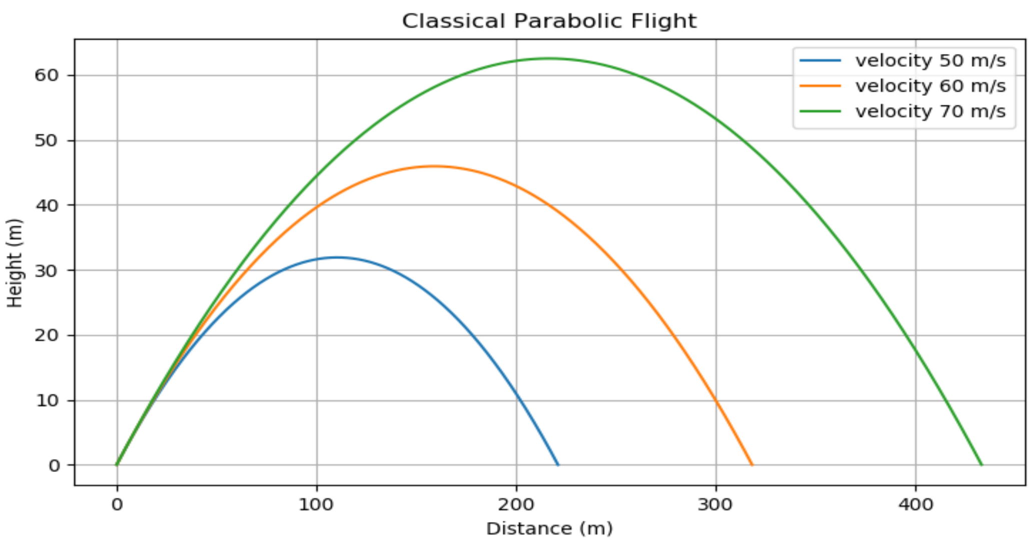


Figure 2

Model II

The previous model is not quite accurate and deviates greatly from real life observations. We now drop the assumption that the golf ball moves in a vacuum and account for the air resistance. Although we have incorporated the air resistance, the ball still has no spin, there is no wind and atmospheric conditions remain uniform. We now use three dimensional cartesian coordinates where the ball moves up and down along the y axis, moves forwards and backwards along the x axis and moves left and right along the z axis. The ball is launched with velocity $\overrightarrow{\boldsymbol{V}_0} = (V_{0x}, V_{0y}, V_{0z})$ from some position (x_0, y_0, z_0). Using the components of equation (A) for the expression of the air resistance in an axis and applying Newton's second we get.

$$\sum F_x = -R_x$$

$$m\ddot{x} = -\frac{1}{2}k(\dot{x})^2 \tag{2.1}$$

$$\sum F_y = -mg - R_y$$

$$m\ddot{y} = -mg - \frac{1}{2}k(\dot{y})^2 \tag{2.2}$$

$$\sum F_z = -R_z$$

$$m\ddot{z} = -\frac{1}{2}k(\dot{z})^2 \tag{2.3}$$

Now we have three independent second order nonlinear differential equations. To make use of the Euler-method we will first need to transform each equation to 2 first order differential equations. First consider equation (2.1) it can be written in the form

$$\ddot{x} = -\frac{1}{2m}k(\dot{x})^2$$

We then make the following substitutions

$$\alpha = \frac{k}{2m}$$

$$x_1(t) = x(t)$$

$$x_2(t) = \dot{x}(t) = \dot{x}_1$$

We then have a system of first order differential equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\alpha(x_2)^2$$

$$x_1(0) = x_0\ ,\ x_2(0) = V_{0x}$$

A similar procedure can be applied to equation (2.2) noting we have an extra constant

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -g - \alpha(y_2)^2$$

$$y_1(0) = y_0 \text{ , } y_2(0) = V_{0y}$$

Equation (2.3) is analogous to equation (2.1) and we have

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\alpha(z_2)^2$$

$$z_1(0) = z_0 \text{ , } z_2(0) = V_{0z}$$

The Euler method can then be applied to each of the system of first order equations to give numerical solutions that model the trajectory of the golf ball. Figure 3 below shows the resulting trajectories where the launch velocities are different, but all other parameters are the same.

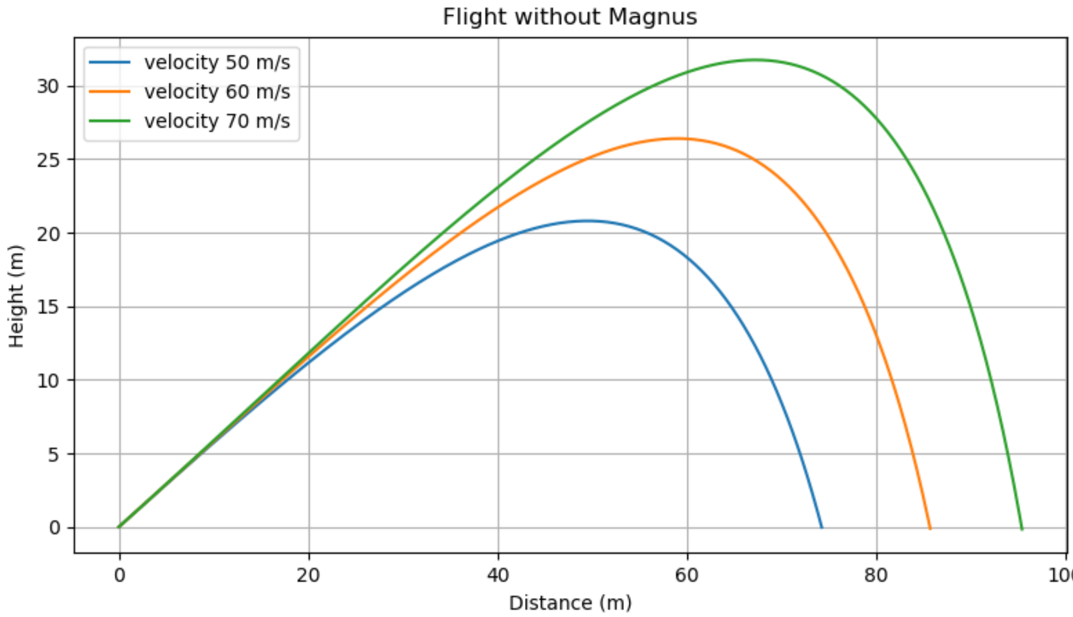


Figure 3

The numerical solutions from the Euler method of equations (2.1) and (2.2) show that the trajectory of the golf ball is not parabolic. The curves in Figure 2 and Figure 2 all have the same angle of attack. If we compare curves in Figure 2 and Figure 3 with the same launch velocity, we observe that the distance covered, and maximum height reached by the ball in the presence of air resistance are significantly lower than the distance covered, and maximum height reached by the golf ball when air resistance is ignored.

Model III

The previous model would be a good representation if golf balls did not possess any spin, but in reality some spin is imparted on a golf ball from a swing and this results in the ball experiencing an additional lift force which changes the governing flight equations. In this model the golf ball has some spin characterized by its angular velocity $\vec{\boldsymbol{\omega}} = (\omega_x, \omega_y, \omega_z)$ which results in the ball experiencing the Magnus force in addition to the air resistance and force of gravity. The spin considered here will only be the backspin and the sidespin. In the case of backspin, the ball has anticlockwise rotation about the z axis quantified by the z component of $\vec{\boldsymbol{\omega}}$ .In the case of sidespin the ball has either clockwise or anticlockwise rotation about the y axis and is quantified by the y component of $\vec{\boldsymbol{\omega}}$ .We assume that the atmospheric condition remain uniform and there is no wind. The golf ball is launched with a velocity $\overrightarrow{\boldsymbol{V}_0} = (V_{0x}, V_{0y}, V_{0z})$ from some position (x_0, y_0, z_0). The flight equations are as follows

$$\sum F_x = -R_x + F_{Mx}$$
$$m\ddot{x} = -\frac{1}{2}k(\dot{x})^2 + F_{Mx} \tag{3.1}$$
$$\sum F_y = -R_y + F_y$$
$$m\ddot{y} = -g - \frac{1}{2}k(\dot{y})^2 + F_{My} \tag{3.2}$$
$$\sum F_z = -R_z + F_{Mz}$$
$$m\ddot{z} = -\frac{1}{2}k(\dot{z})^2 + F_{Mz} \tag{3.3}$$

The relevant expressions for F_{Mx} , F_{My} and F_{Mz} can be obtained as follows from equation (C)

$$\overrightarrow{\boldsymbol{F_L}} = S(\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{V}})$$

$$\overrightarrow{\boldsymbol{F_L}} = S\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \omega_x & \omega_y & \omega_z \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix}$$

This gives the following expression

$$F_{Mx} = S(\omega_y\,\dot{z} - \omega_z\,\dot{y})\ , F_{My} = S(\omega_z\,\dot{x} - \omega_x\,\dot{z}), F_{Mx} = S(\omega_x\,\dot{y} - \omega_y\,\dot{x})$$

The substitution of the relevant forms of F_{Mx} , F_{My} and F_{Mz} into equations (3.1), (3.2) and (3.3) and defining a constant $s = \frac{S}{m}$ the flight equations become

$$\ddot{x} = -\alpha(\dot{x})^2 + s(\omega_y\,\dot{z} - \omega_z\,\dot{y}) \tag{4.1}$$

$$\ddot{y} = -g - \alpha(\dot{y})^2 + s(\omega_z\,\dot{x} - \omega_x\,\dot{z}) \tag{4.2}$$

$$\ddot{z} = -\alpha(\dot{z})^2 + s(\omega_x\,\dot{y} - \omega_y\,\dot{x}) \tag{4.3}$$

Where the constant α is as defined in the previous model

These equations can be reduced to systems of first order equations through substitutions as previously done in Model II. The substitutions are $x_1(t) = x(t)$, $x_2(t) = \dot{x}(t)$ and $y_1(t) = y(t)$, $y_2(t) = \dot{y}(t)$ and finally $z_1(t) = z(t)$, $z_2(t) = \dot{z}(t)$.

After the substitutions equations (4.1), (4.2) and (4.3) respectively produce the following system of equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\alpha(x_2)^2 + s(\omega_y z_2 - \omega_z y_2)$$
$$x_1(0) = x_0 \, , x_2(0) = V_{0x}$$
$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = -g - \alpha(y_2)^2 + s(\omega_z x_2 - \omega_x z_2)$$
$$y_1(0) = y_0 \, , y_2(0) = V_{0y}$$
$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = -\alpha(z_2)^2 + s(\omega_x y_2 - \omega_y x_2)$$
$$z_1(0) = z_0 \, , z_2(0) = V_{0z}$$

The Euler method can then be used to solve the above system of first order differential equations. The trajectory of a golf ball using Model II and the trajectory of a ball using Model III is shown in Figure 4 below. The two balls have the same launch parameters except that one ball has backspin while the other has no spin.

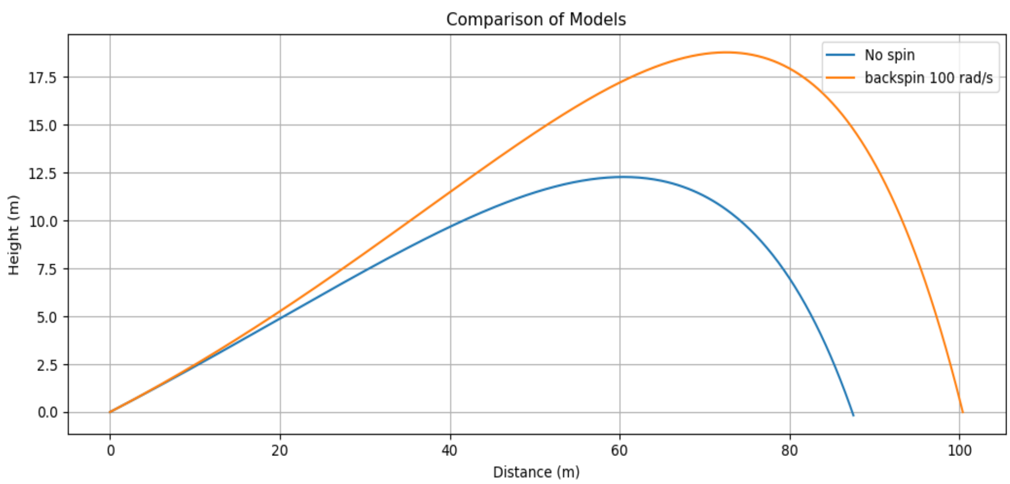


Figure 4

From Figure 4 the distance covered, and maximum height reached by golf ball that is spinning and subjected to the Magnus force are significantly greater than the distance covered, and maximum height reached by a golf ball that has no spin and is not subjected to the Magnus force.

Effects of Spin

Backspin

From the resulting trajectory of a golf ball in Model III the spin has a significant effect on the trajectory of the ball. We first consider backspin in which we check the effects of increasing the backspin on the carry of the ball and the maximum height it reaches. All other launch parameters are kept the same while the spin is varied from between 100 radians per second to 350 radians per second. The resulting trajectories are shown in Figure 5 below

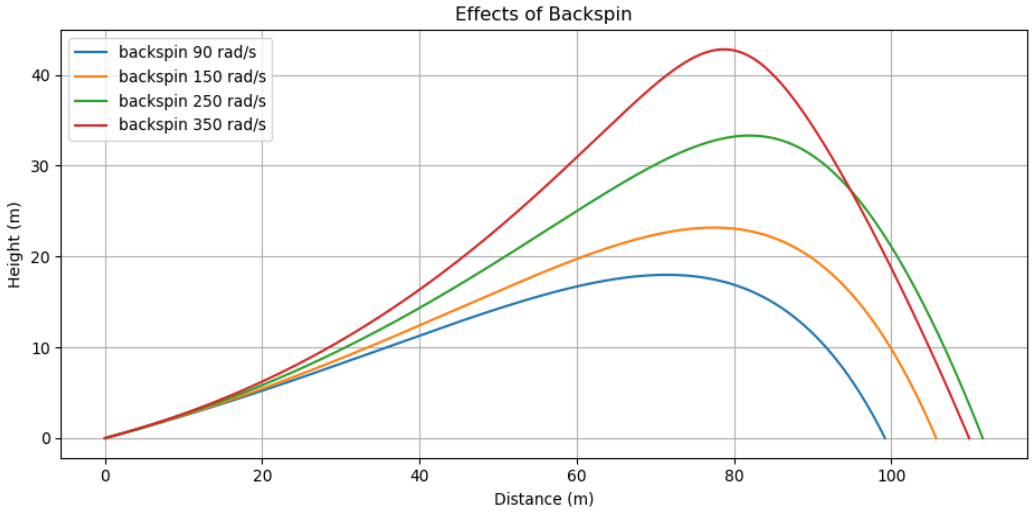


Figure 5

The curves in Figure 5 show that increasing the backspin means that the golf ball will stay in the air longer and will reach greater heights, as when the backspin gets larger the distance covered by the golf ball increases and the maximum height reached increases. The amount of backspin needs to be optimal as too much backspin results in the ball having a steep ascent which would increase the maximum height but reduce the distance the ball covers, this occurs when the backspin exceeds 300 radians per second, the red curve in Figure shows the reduction in the carry of the golf ball that would occur when the backspin is too high.

Sidespin

A golfer may unintentionally give the ball excess sidespin due to his swing being out of line, the sidespin may result in an undesirable left or right drift in the resulting trajectory. Highly skilled players may deliberately impart excess sidespin onto the golf ball in order to achieve a certain trajectory to manoeuvre around the course. To demonstrate the effects of sidespin on the trajectory of the golf ball, the sidespin is varied between -20 radians per second and 20 radians per second. A top view of the resulting trajectories is shown in Figure 6 below.

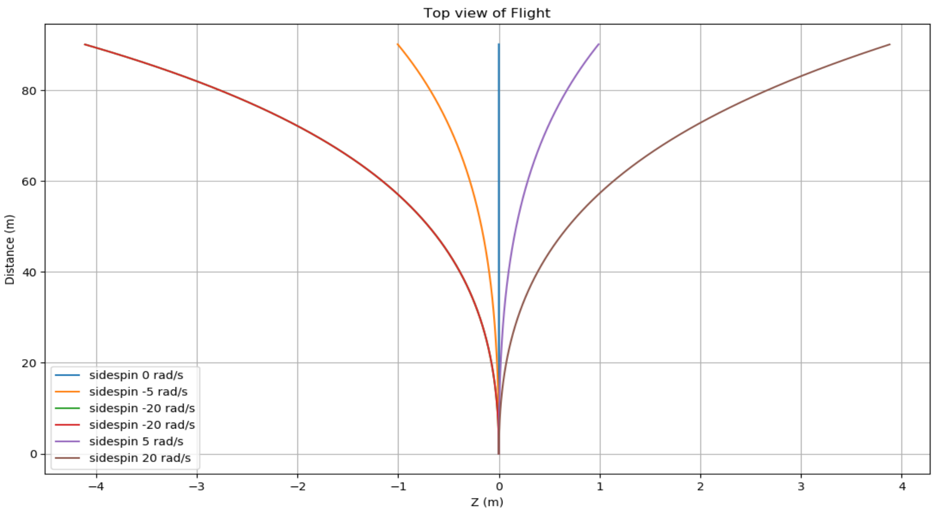


Figure 6

From left to right Figure 6 shows the Hook, Pull, Push and Slice shots. The Hook and Slice have the greatest amount of sideways drift. The Hook occurs when there is great deal of anticlockwise sidespin and the Slice results when there is a large amount of clockwise sidespin. The Push and Pull have less severe drift and occur for considerably low amounts of sidespin. The shots shown above may be done intentionally or by accident depending on the skill of the golfer. A three-dimensional view of the Hook, Slice and an ideal shot is shown in Figure 7 below

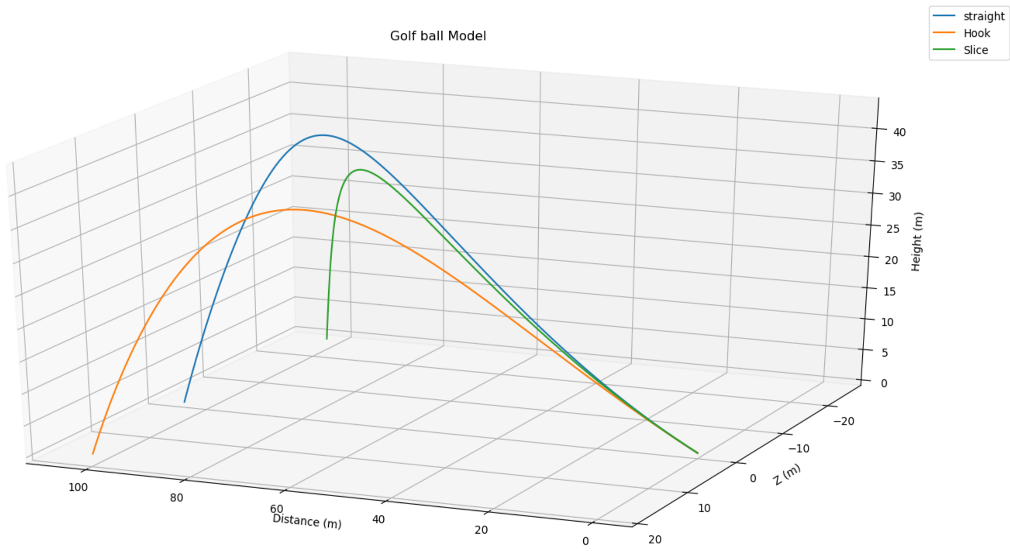


Figure 7

The three-dimensional view clearly shows the trajectory of a golf ball and the influences of sidespin are clearly visible. The straight shot has optimal amounts of spin and covers a great distance without drifting of left or right.

Numerical Analysis and Simulation

Numerical integration

In Model II and Model III analytical solutions to the flight equations were impossible to obtain and we resorted to numerical methods, particularly the Euler method. The Euler method was chosen for its simplicity and computational efficiency. To demonstrate the simplicity of this numerical integration method it will be applied to one of the flight equations in Model II. The transformation of equation (2.2) into a system of first order equations resulted in the following equations

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = -g - \alpha(y_2)^2$$
$$y_1(0) = y_0\ , y_2(0) = V_{0y}$$

The Euler method approximates the solution of an initial value problem of the form $y' = f(x,y)$, $y(x_0) = y_0$ with the formula $y_{n+1} = y_n + hf(x,y)$. The recursive application of this formula produces approximations to the values of the function $y(x)$ at successive points $x_n = x_0 + nh$. For the golf ball we want the position $y_1(t)$ and the velocity $y_2(t)$ at successive time steps $t_n = t_0 + n(\Delta t)$. Using the recursive formula, the position and velocity would be given by the following

$$y_1(t_{n+1}) = y_1(t_n) + \Delta t y_2(t_n)$$
$$y_2(t_{n+1}) = y_2(t_n) + \Delta t(-g - \alpha(y_2(t_n))^2)$$

If $y_0 = 0$ and $V_{0y} = 30\ ms^{-1}$ and the time step $\Delta t = 0.01$.Then the first few values of the position and velocity would be

t	$y_1(t)$	$y_2(t)$
0.001	0.3	29.7268
0.002	0.59726	29.4568
0.003	0.89184	29.1899

This will be continued till the golf ball returns to its initial positions and we would have the trajectory of the ball. The simplicity of the Euler method comes at the cost of accuracy, even though the flight equations in Model III incorporate all the forces acting on the ball, the numerical integration scheme has a truncation error as the generated values from the Euler method will not correspond to the actual function values, which means the resulting trajectories are not quite accurate when the truncation error is large. The local truncation error of the Euler method is of order $O(h^2)$ and the accuracy improves with a reduction in the step size. One of the more popular and most accurate numerical procedures used to obtain approximate solutions to first order initial value problems is the fourth-order Runge Kutta method. More precise trajectories in which the distance covered and maximum height reached by the golf ball are accurate could have been achieved by using the fourth-order Runge Kutta method instead of the Euler method.

Numerical Simulation

A simulation was produced in order to demonstrate the different trajectories of a golf ball under various launch conditions. While running the simulation users are able to set launch parameters such as the launch velocity, angle of attack, backspin and sidespin. After the desired parameters have been set a shot can be taken. When a shot is taken trail is shown which traces out the trajectory of the ball and all relevant flight data such as the current position the distance covered, the velocity and time of flight are shown. At the end of the shot a graph with a curve showing the trajectory is generated and the user can compare different trajectories from different shots using this graph. The simulation was implemented in python and can be run by following the instructions at https://github.com/phantom820/Physics.

Conclusion

In this work the equations governing the flight of golf ball have been presented and numerical solutions to the equations presented .To build up to the general flight equations three models were presented Model I, Model II and Model III respectively. In Model I the golf ball was assumed to have no spin and to be moving in a vacuum, the only force acting on it was gravity, the relevant flight equations under these assumptions were solved analytically and the trajectory was obtained. In Model II air resistance was incorporated into the flight equations while still the ball was assumed to have no spin, then the flight equations were numerically solved using the Euler method. The resulting trajectory showed that air resistance significantly reduces the carry of the ball. Then finally in Model III the spin of the ball was accounted for and an additional lift force was added to the flight equations in Model II. The resulting trajectories in Model III showed that a golf ball with spin has more carry than a golf ball that has no spin.

The effects of spin on the trajectory of the ball was then examined using Model III. Comparing trajectories with different amounts of backspin revealed that increasing the amount of backspin on the ball increases the carry of the ball, but there a critical value to the amount of backspin that can be put into the ball. Once this critical value is exceeded then the trajectory of the ball becomes steep and the carry is reduced, which would not be an ideal shot. When the golf ball is given sidespin it will drift left or right depending on whether the sidespin is anticlockwise or clockwise. Comparing various trajectories with different amounts of sidespin shows that the undesirable hook and slice result when the ball has been given too much sidespin and it drifts too far lift or too far right. When deployed correctly and deliberately the spin of a golf ball will allow the golfer to take great shots and manoeuvre around the golf course. The overall effects of spin can be summed as follows

• Optimal amounts of backspin increases the carry, while too much of it reduces the carry

• Too much sidespin results in an undesirable left or right drift, the sidespin should be minimized to achieve a straight shot.

Possible extensions

In this work the effects of wind were completely ignored, in reality wind conditions are rarely ever ideal and there is usually a cross wind. The spin was also treated as fixed throughout the flight of the ball which is not the case in the presence of wind. Possible extensions to this work include the following

- The effects of wind on the flight of a golf ball
- The decay of spin as a function of time
- Application of flight equations to other sports such as cricket and soccer.

Appendix A

Symbols and constants used in this work are listed below, numerical values of constants pertaining to physical composition of the golf ball were adapted from [7]

Symbol	Meaning
A	Cross sectional area of projectile
C_D	Dimensionless drag coefficient
ρ	Density of air taken to be $1.22\ kg\ m^{-3}$
k	Proportionality constant for drag force given by $A\rho C_d$
g	Acceleration due to gravity with value $9.8\ ms^2$
m	Mass of the golf ball
α	Defined for convenience in flight equations given by $\frac{k}{2m}$
$\vec{\boldsymbol{V}}$	Golf ball velocity
V_x, V_y, V_z	Magnitude and x, y, z components of $\vec{\boldsymbol{V}}$
$\overrightarrow{\boldsymbol{F_D}}$	Drag force
$\overrightarrow{\boldsymbol{F_L}}$	Magnus force or Lift force
F_x, F_y, F_z	Components of a resultant force in Cartesian coordinates
$\vec{\boldsymbol{\omega}}$	Golf ball angular velocity vector
$\omega_x, \omega_y, \omega_z$	Magnitude and x, y, z component of $\vec{\boldsymbol{\omega}}$ $(rad\ s^{-1})$

Appendix B

An extract from the python code used for illustrative examples and numerical simulation

```
#euler method that incorporates Magnus force
def eulerMagnus(launchData):
        #retrieve data form list [y0,v0y,x0,v0x,z0,v0z,dt,omega_i,omega_j,omega_k]
        y0=launchData[0]
        yprime0=launchData[1]
        x0=launchData[2]
        xprime0=launchData[3]
        z0=launchData[4]
        zprime0=launchData[5]
        dt=launchData[6]
        #spin values
        omega_i=launchData[7]
        omega_j=launchData[8]
        omega_k=launchData[9]
        #the coefficient of square veocity term cd=0.25,p=1.225, A = pir^2 , r=42.67/1000and m= 0.045
        alpha=((0.25)*1.225*math.pi*(42.67/1000)**2)/(2*0.045)
        #the  coefficient of magnus terms S=s/m where s=.000005
        S=s=0.00005/0.045
        y=[y0]
        yprime=[yprime0]
        x=[x0]
        xprime=[xprime0]
        z=[z0]
        zprime=[zprime0]
        t=[0]
        i=0;
        time=0
        while y[i]>=0:
                #y and y' update
                y.append(y[i]+dt*yprime[i])
                yprime.append(yprime[i]+dt*(-9.8-alpha*(yprime[i])**2+S*(omega_k*xprime[i]-omega_i*zprime[i])))
                #x and x' update
                x.append(x[i]+dt*xprime[i])
                xprime.append(xprime[i]+dt*(-alpha*(xprime[i])**2+S*(omega_j*zprime[i]-omega_k*yprime[i])))
                #z and z' updates
                z.append(z[i]+dt*zprime[i])
                zprime.append(zprime[i]+dt*(-alpha*(zprime[i])**2+S*(omega_i*yprime[i]-omega_j*xprime[i])))
                t.append(t[i]+dt)
                i=i+1
        #matrix with position and velocity information
        matrix=[x,xprime,y,yprime,z,zprime]

        return matrix
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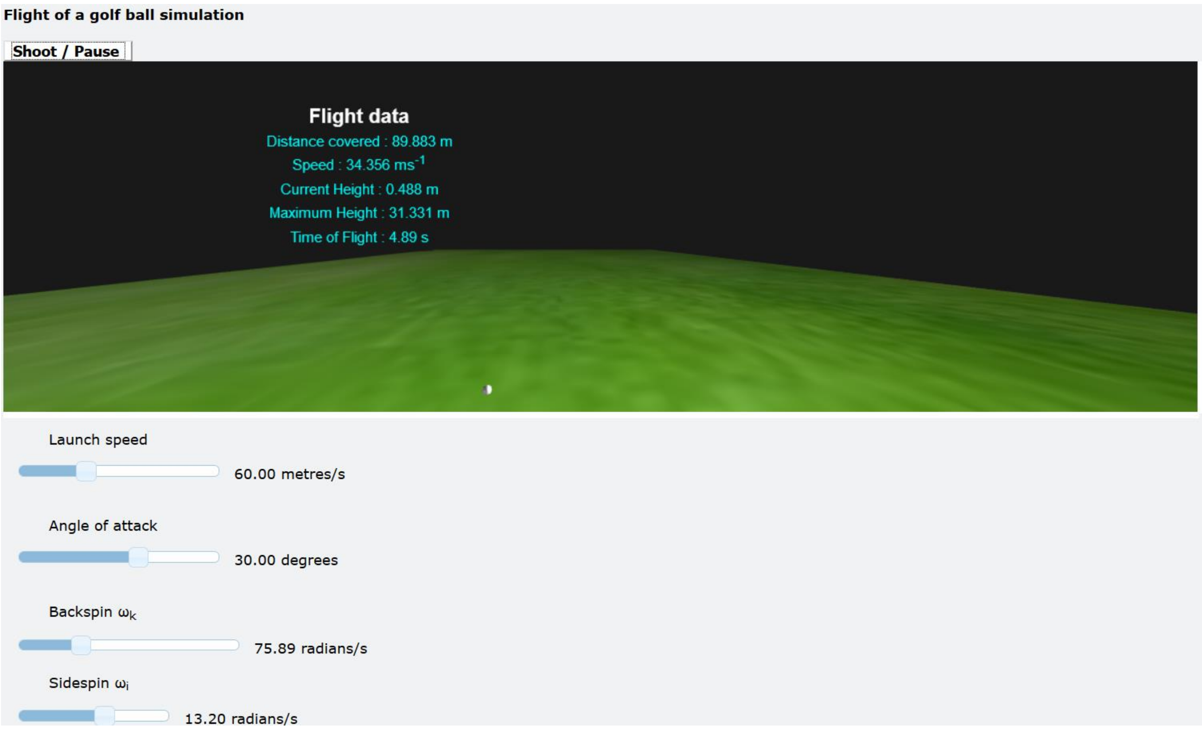

References

[1] Wesson, J. (2012). *The science of golf*. Oxford: Oxford University Press.

[2] Jorgensen, T. P. (T. P. (1999). *The physics of golf*. United States: Springer.

[3] G. Robinson and I. Robinson, “The motion of an arbitrarily rotating spherical projectile and its application to ball games,” *Phys. Scr.*, vol. 88, no. 1, p. 018101, 2013.

[4] V. Pagonis, D. Guerra, S. Chauduri, B. Hornbecker, and N. Smith, “Effects of air resistance,” *The Physics Teacher*, vol. 35, pp. 364–368, Sep. 1997.

[5] H. Sarafian, “Impact of the Drag Force and the Magnus Effect on the Trajectory of a Baseball,” *World J. Mech.*, vol. 5, no. 04, p. 49, 2015.

[6] Zill, D. G., Wright, W. S., & Cullen, M. R. (2013). *Differential equations with boundary value problems*. Boston: PWS Publ.

[7] F. Alam, H. Chowdhury, H. Moria, R. L. Brooy, and A. Subic, “A Comparative Study of Golf Ball Aerodynamics,” p. 4.