

The Bardeen-Petterson Effect in accreting supermassive black-hole binaries

Disc breaking and critical obliquity

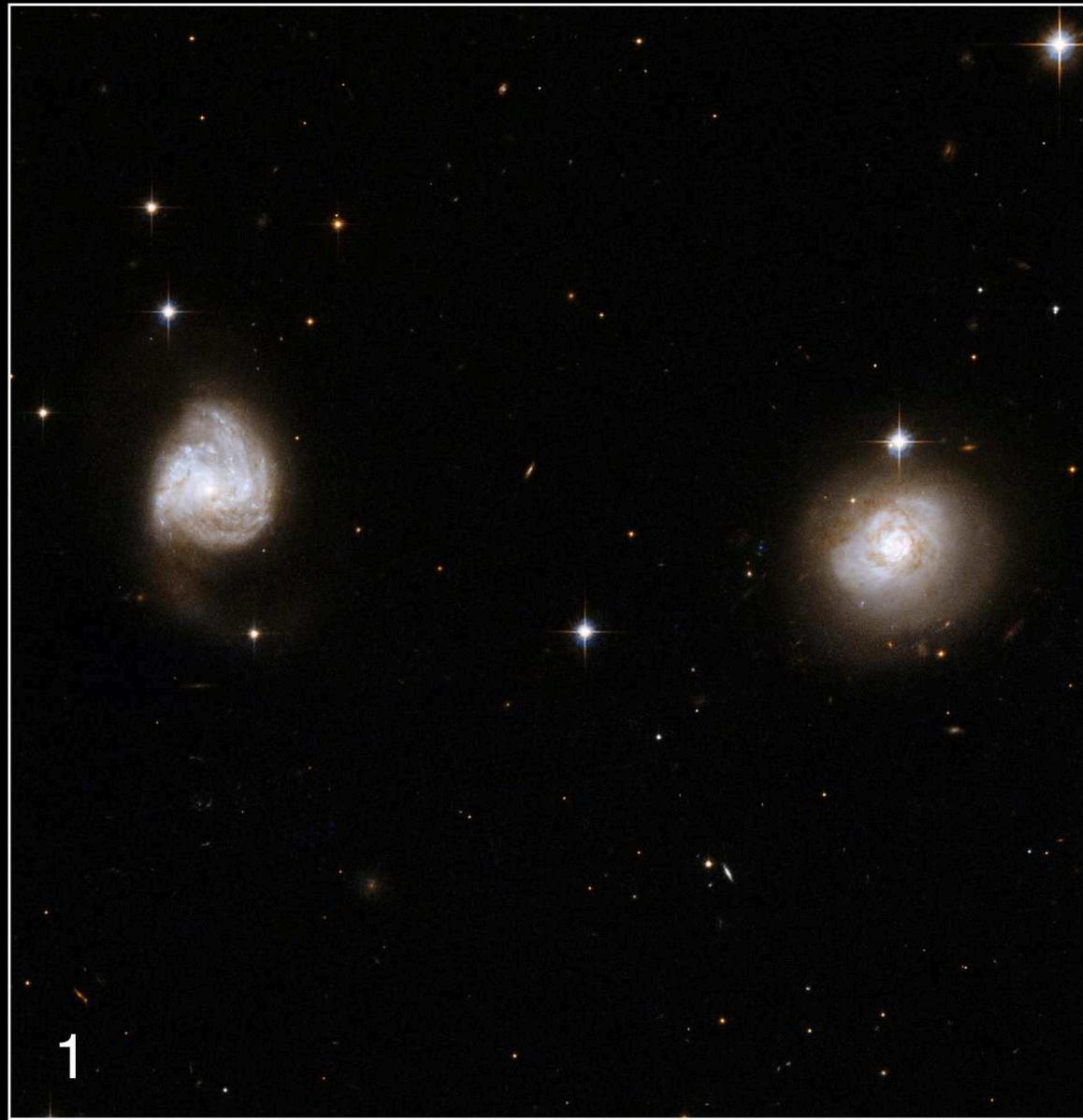


Rebecca Nealon, Stephen Hawking Fellow

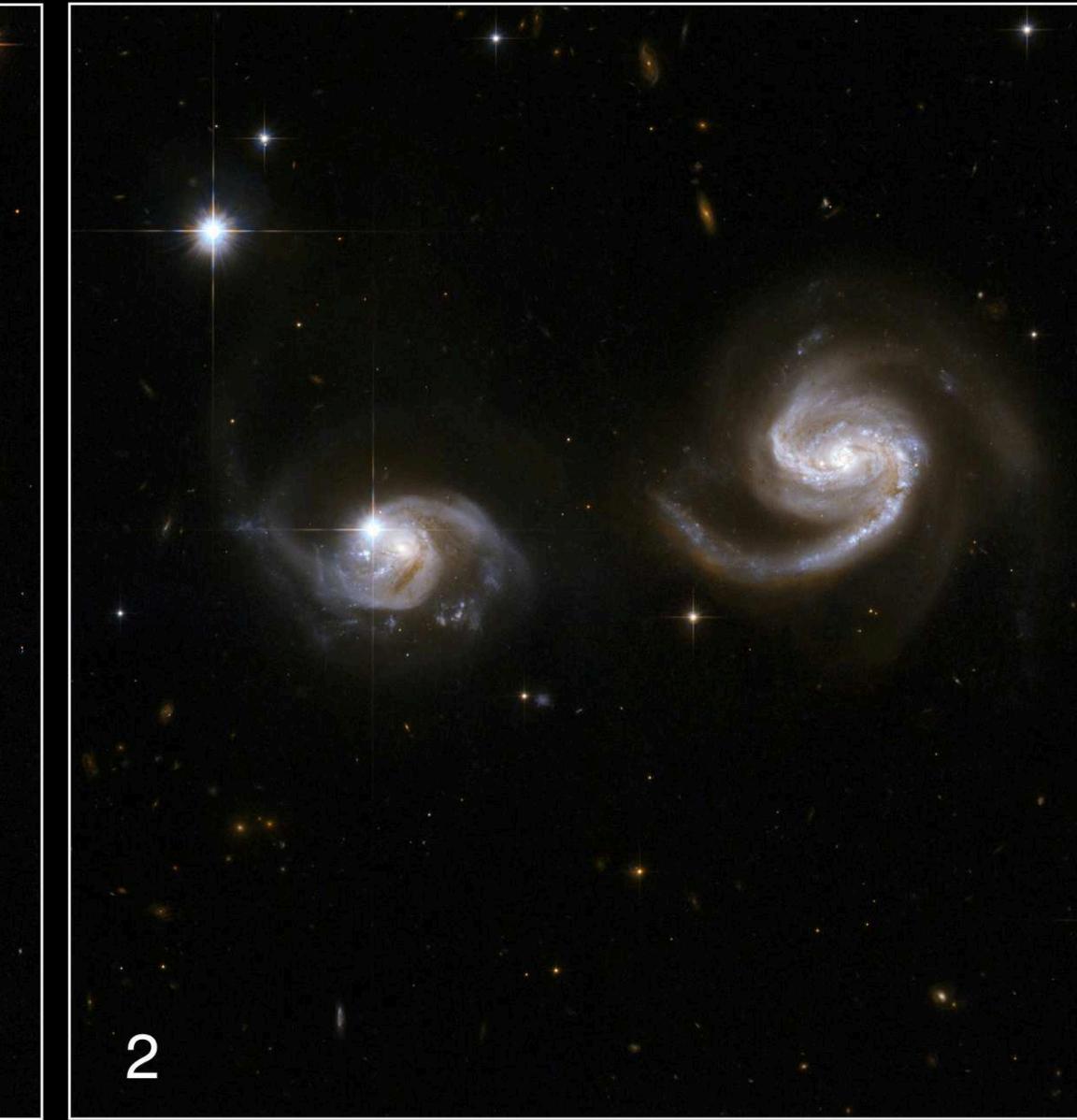
with Enrico Ragusa, Davide Gerosa, Giovanni Rosotti and Riccardo Barbieri



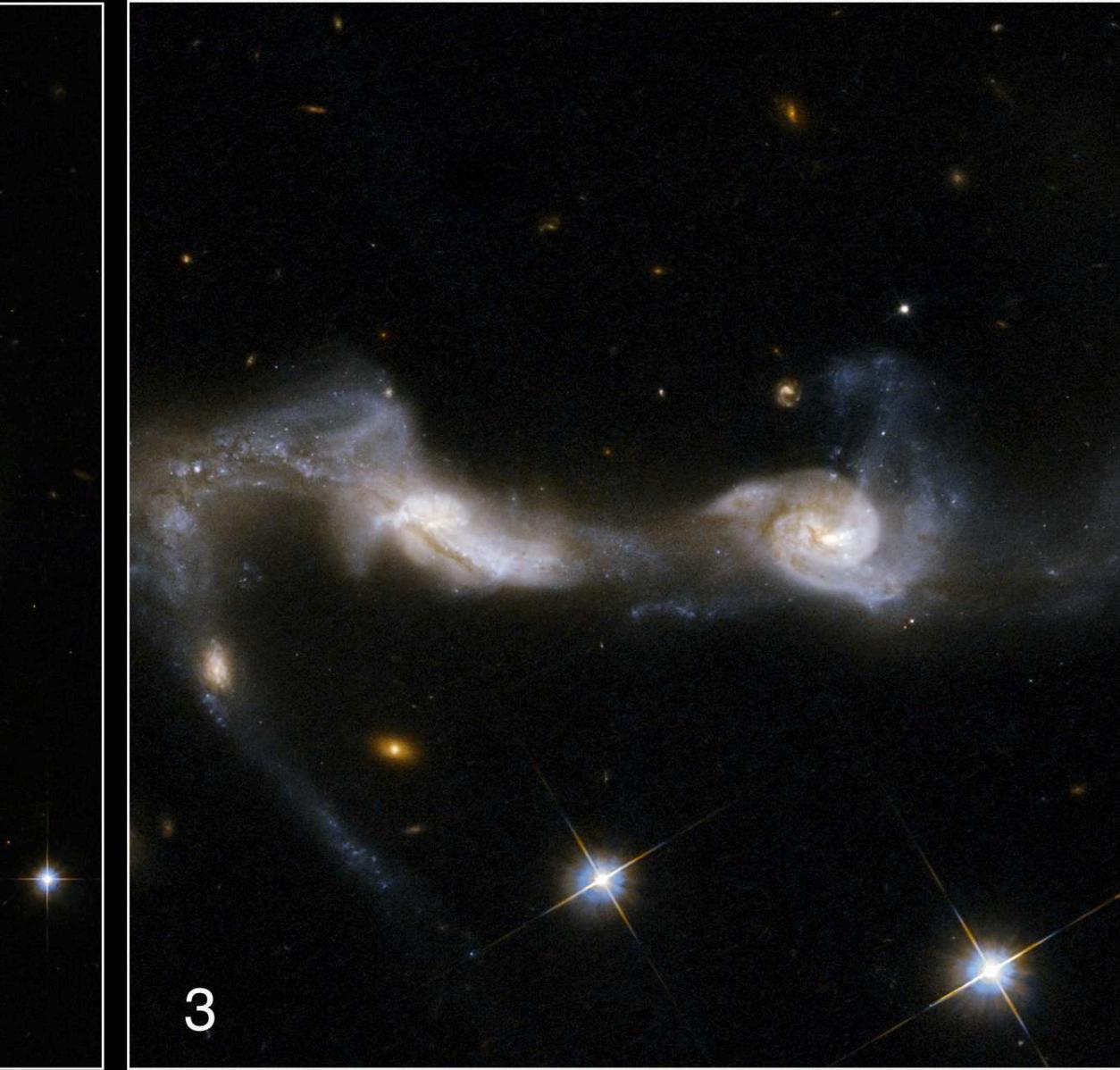
How do supermassive black holes merge?



1



2



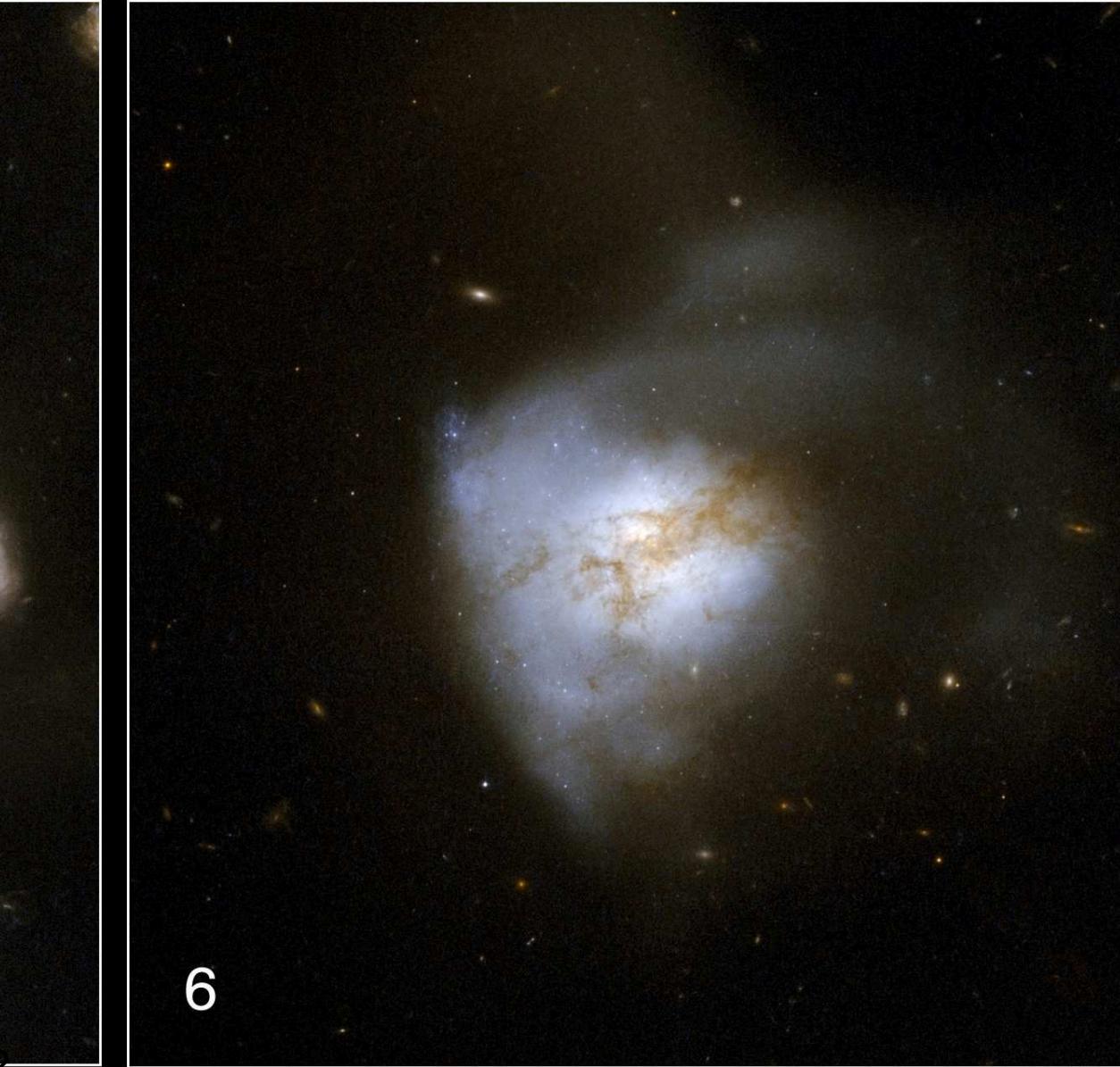
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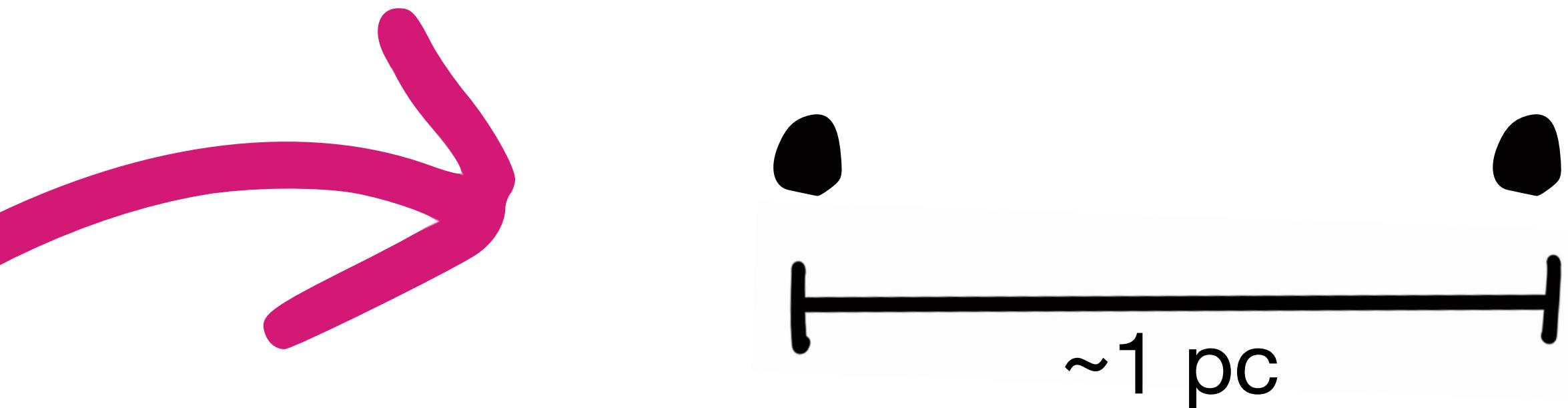


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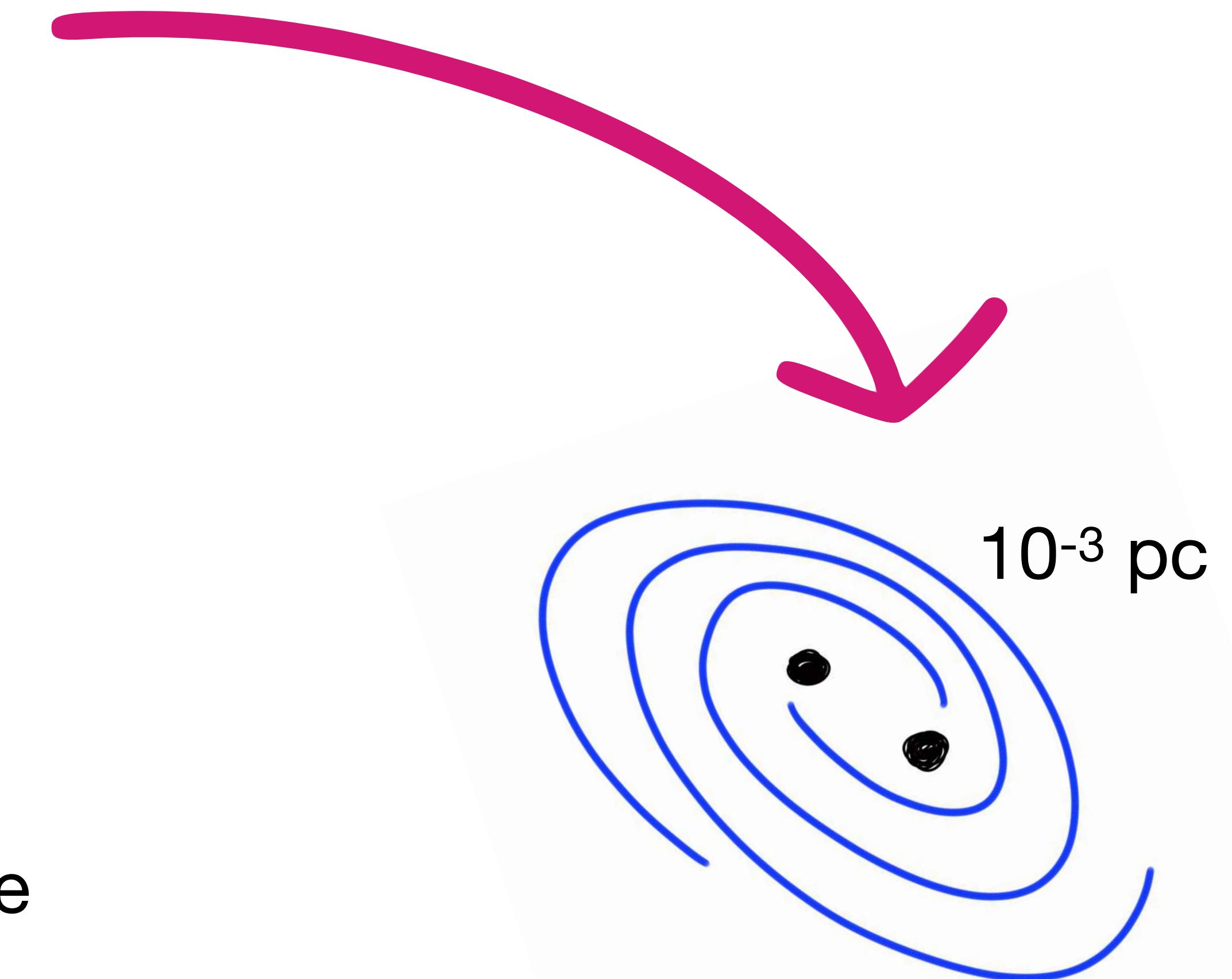
(ESA/NASA)

How do supermassive black holes merge?



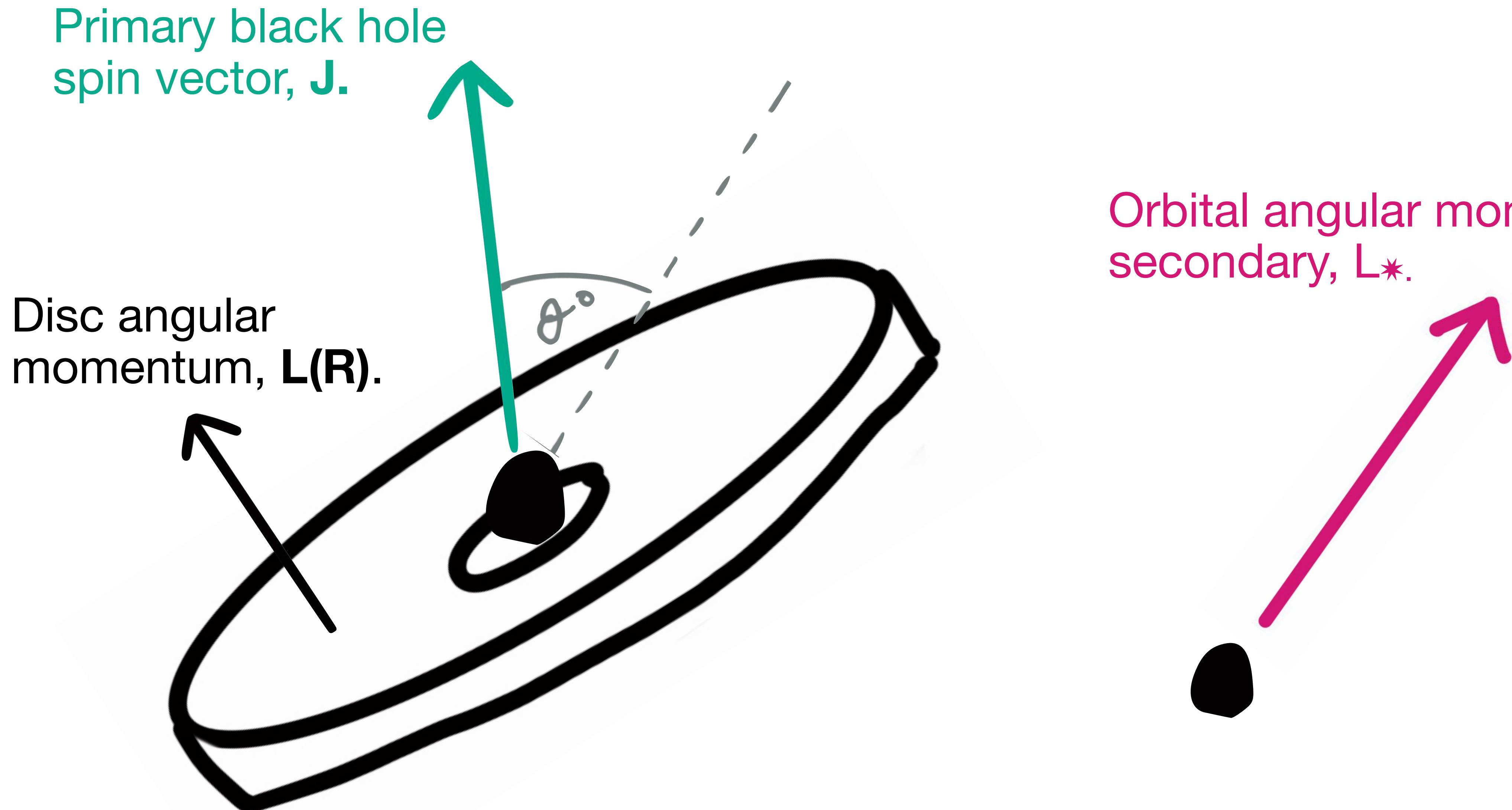
The Final Parsec Problem

- Triaxial galactic potentials (Poon and Merritt 2004)
- Dynamical interactions in supermassive black hole triples (Bonetti et al. 2019)
- Gas accretion (e.g. Armitage and Natarajan 2002 and many others)



The emission of gravitational waves are responsible for the final merge

Disc driven migration

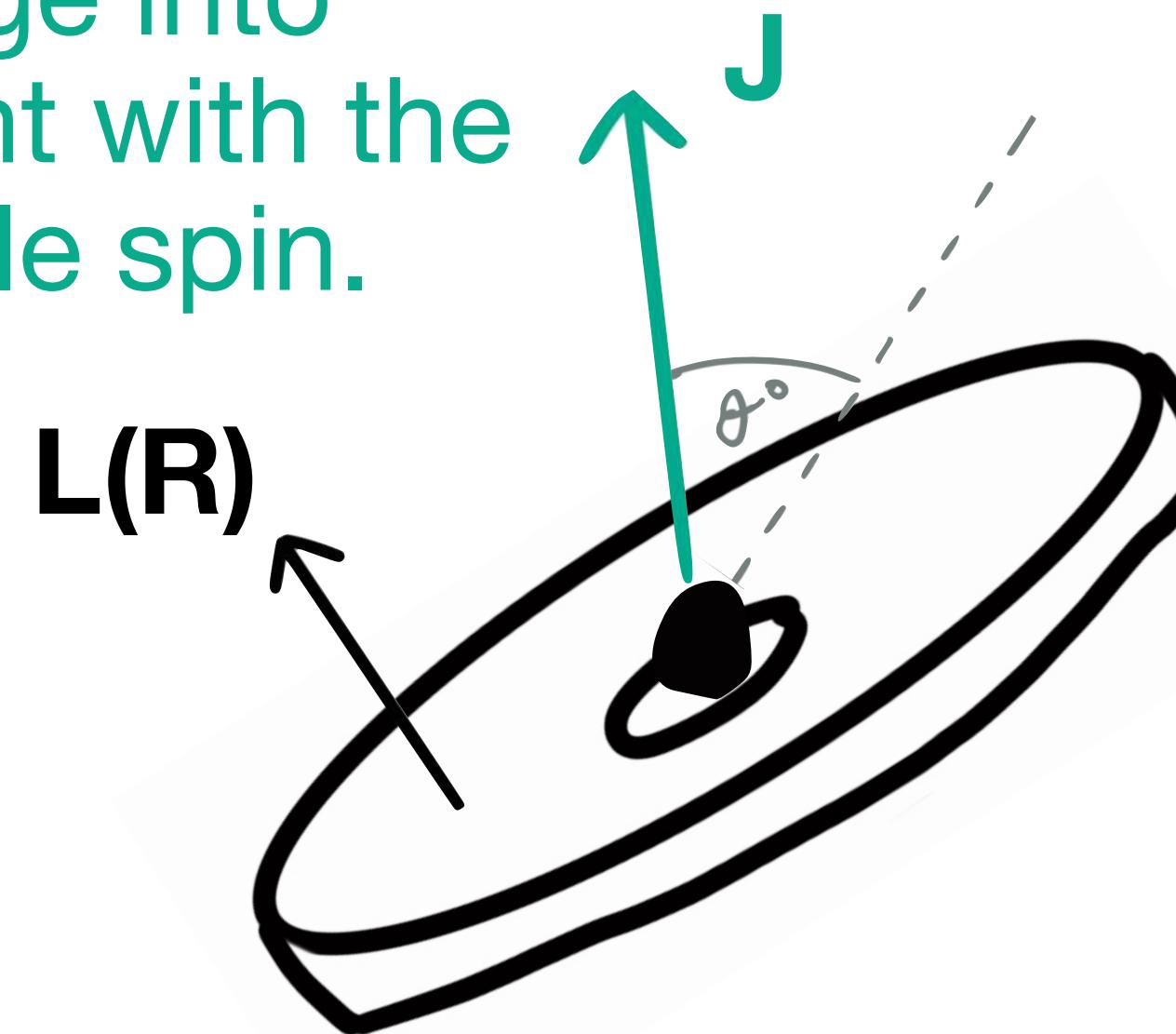


Disc driven migration

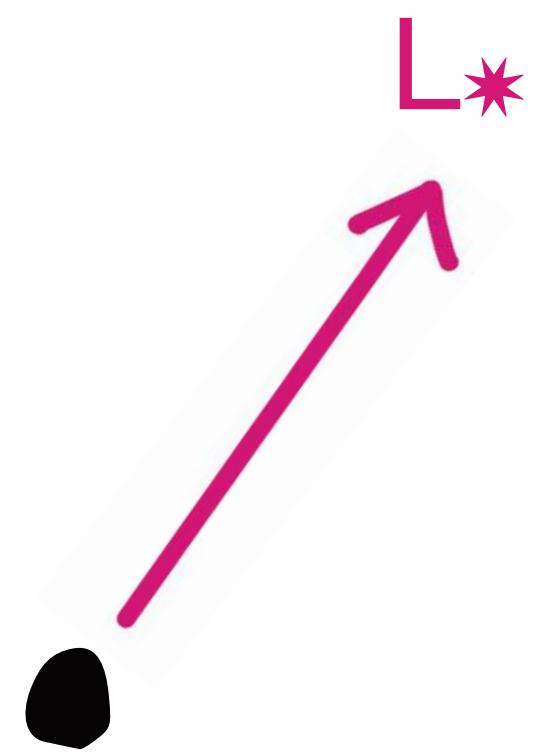
$$\frac{d \cos \theta}{dt} = \frac{d \hat{\mathbf{J}}}{dt} \cdot \hat{\mathbf{L}}_*$$

$$\frac{d \mathbf{J}}{dt} = - \int_{R_{\min}}^{R_{\max}} \frac{2G}{c^2} \frac{\mathbf{J} \times \mathbf{L}}{R^3} 2\pi R dR$$

Tries to pull the inner edge into alignment with the black hole spin.



Exerts a tidal torque on the outer edge of the disc.



1. What shape does the disc take?
2. How does this affect the alignment of the black holes?

A semi-analytic model

$$\frac{\partial \sigma}{\partial r} = - \left(\beta + \frac{1}{2} \right) \frac{\sigma}{r} - \frac{\xi \sigma \psi^2}{3r} \frac{\tilde{\alpha}_2(\alpha, \psi)}{\tilde{\alpha}_1(\alpha, \psi)} + \frac{r^{-\beta-1}}{3\tilde{\alpha}_1(\alpha, \psi)} - \frac{\sigma}{\tilde{\alpha}_1(\alpha, \psi)} \frac{\partial \tilde{\alpha}_1(\alpha, \psi)}{\partial r},$$

$$\frac{\partial^2 \hat{\mathbf{L}}}{\partial r^2} = \frac{\partial \hat{\mathbf{L}}}{\partial r} \left[- \frac{2r^{-\beta-1}}{\xi \tilde{\alpha}_2(\alpha, \psi) \sigma} + \frac{3}{\xi r} \frac{\tilde{\alpha}_1(\alpha, \psi)}{\tilde{\alpha}_2(\alpha, \psi)} - \left(\beta + \frac{3}{2} \right) \frac{1}{r} - \frac{1}{\sigma} \frac{\partial \sigma}{\partial r} - \frac{1}{\tilde{\alpha}_2(\alpha, \psi)} \frac{\partial \tilde{\alpha}_2(\alpha, \psi)}{\partial r} \right] - \frac{\psi^2}{r^2} \hat{\mathbf{L}}$$

$$- \left(\frac{R_{\text{LT}}}{R_0} \right) \frac{r^{-\beta-3}}{\tilde{\alpha}_2(\alpha, \psi)} (\hat{\mathbf{J}} \times \hat{\mathbf{L}})$$

$$- \left(\frac{R_{\text{tid}}}{R_0} \right)^{-7/2} \frac{r^{-\beta+3/2}}{\tilde{\alpha}_2(\alpha, \psi)} (\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}_\star) (\hat{\mathbf{L}} \times \hat{\mathbf{L}}_\star).$$

Change of variables ...

$$r = \frac{R}{R_0}, \quad \sigma = \frac{2\pi}{\dot{M}} \alpha \nu_0 \Sigma;$$

and using

$$R_{\text{LT}} = \frac{4G^2 M^2 \chi}{c^3 \alpha \nu_0 \xi},$$

$$R_{\text{tid}} = \left(\frac{2}{3} \frac{\sqrt{GM}}{GM_\star} R_\star^3 \alpha \nu_0 \xi \right)^{2/7}$$

Aligns the inner disc with the black hole spin

Aligns the outer disc with the binary orbit

A semi-analytic model

Where the Lense-Thirring torque most strongly affects the warp profile

$$R_{\text{LT}} = \frac{4G^2 M^2 \chi}{c^3 \alpha \nu_0 \zeta},$$

Where the tidal external torque most strongly affects the warp profile

$$R_{\text{tid}} = \left(\frac{2}{3} \frac{\sqrt{GM}}{GM_*} R_*^3 \alpha \nu_0 \zeta \right)^{2/7}$$

Also introduce the convenient parameter:

$$\kappa = \left(\frac{R_{\text{tid}}}{R_{\text{LT}}} \right)^{-7/2}$$

The non-dimensional parameter κ

$$\kappa \simeq 0.66 \left(\frac{M}{10^7 M_\odot} \right)^2 \left(\frac{\chi}{0.5} \right)^2 \left(\frac{M_\star}{10^7 M_\odot} \right) \left(\frac{R_\star}{0.1 \text{pc}} \right)^{-3} \\ \times \left(\frac{H_0/R_0}{0.002} \right)^{-6} \left(\frac{\alpha}{0.2} \right)^{-3} \left[\frac{\xi}{1/(2 \times 0.2^2)} \right]^{-3},$$

Measures the relative importance of the binary.

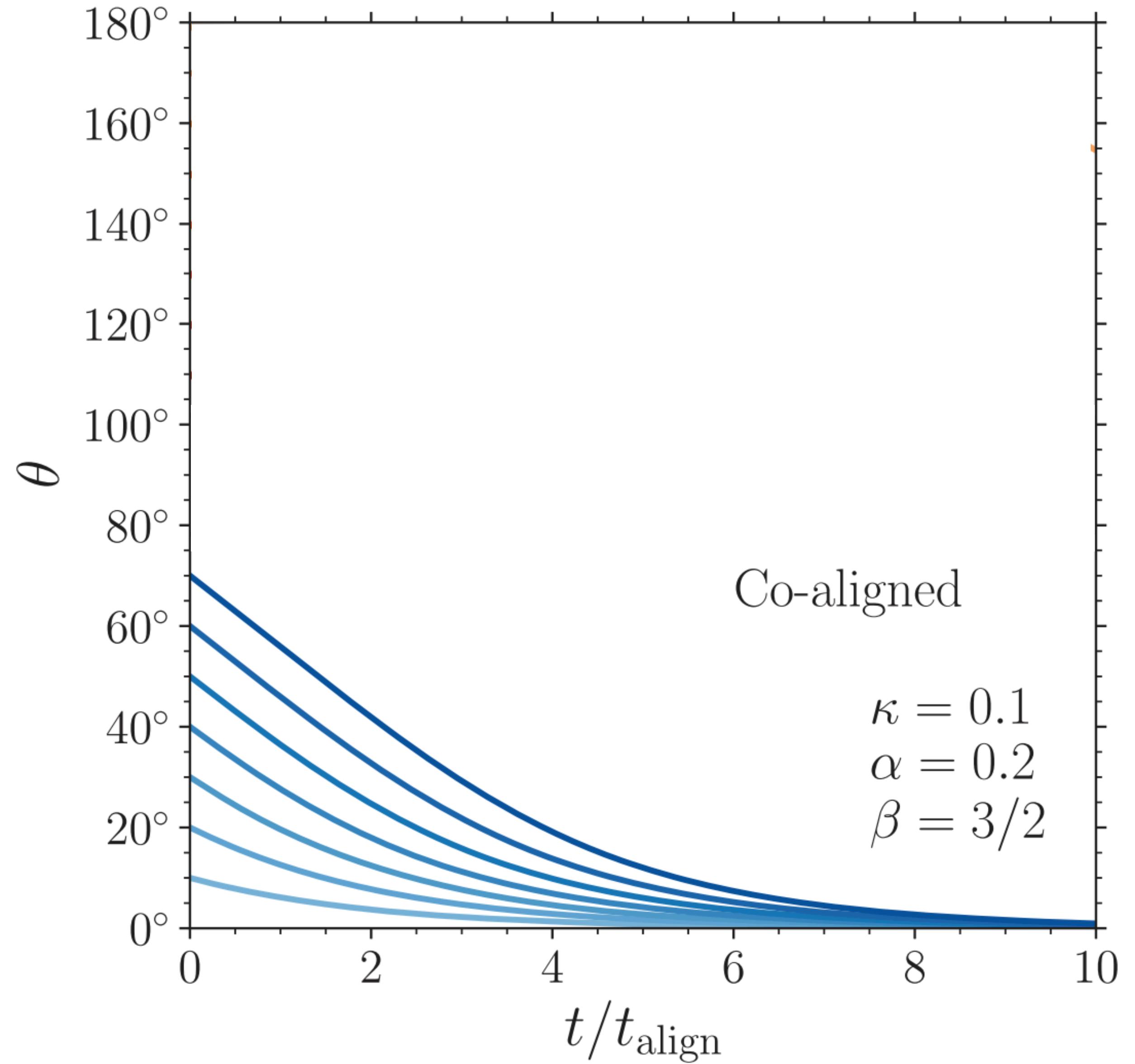
Large κ means the binary is very important.

Small κ means the binary is not important at all.

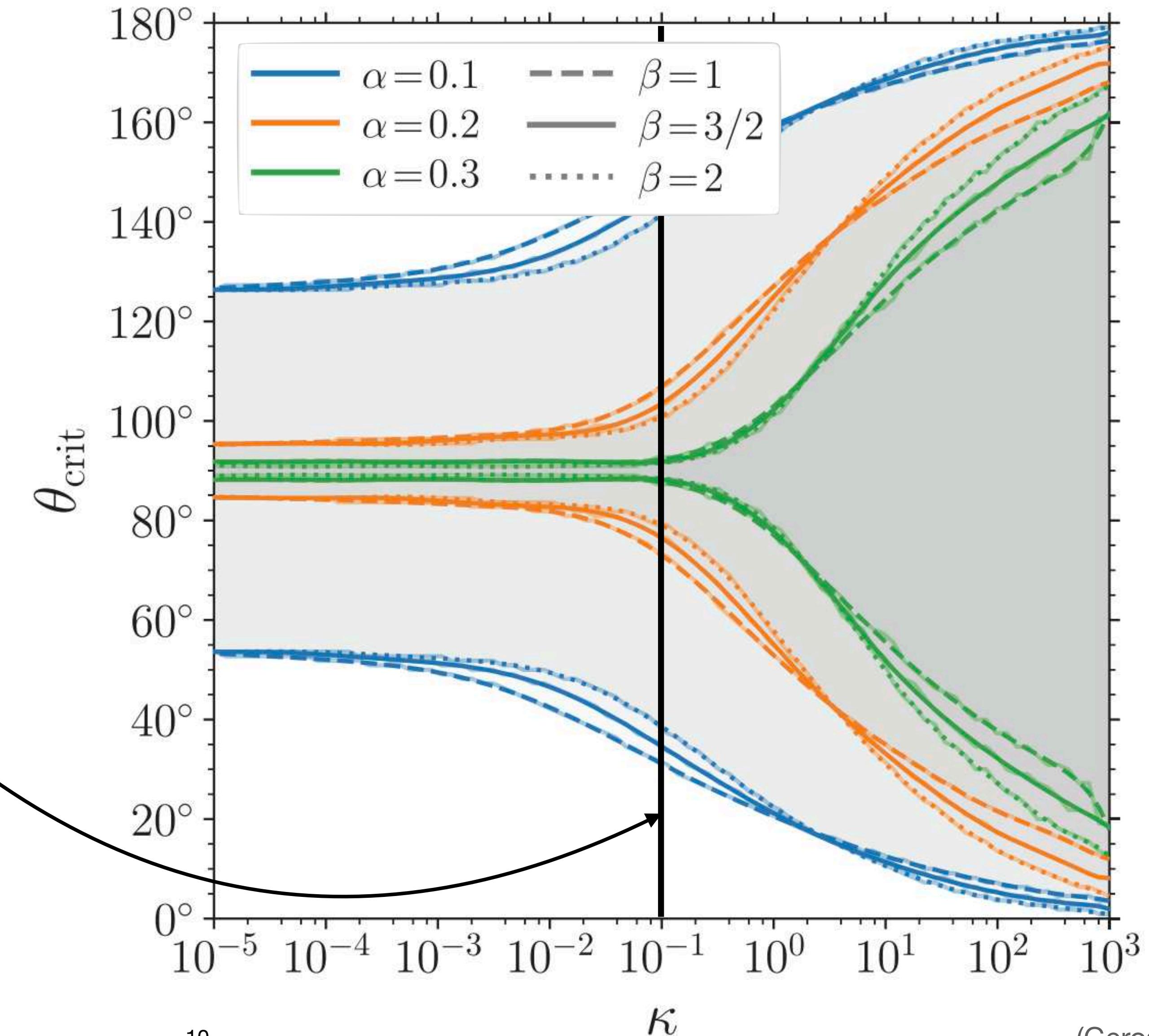
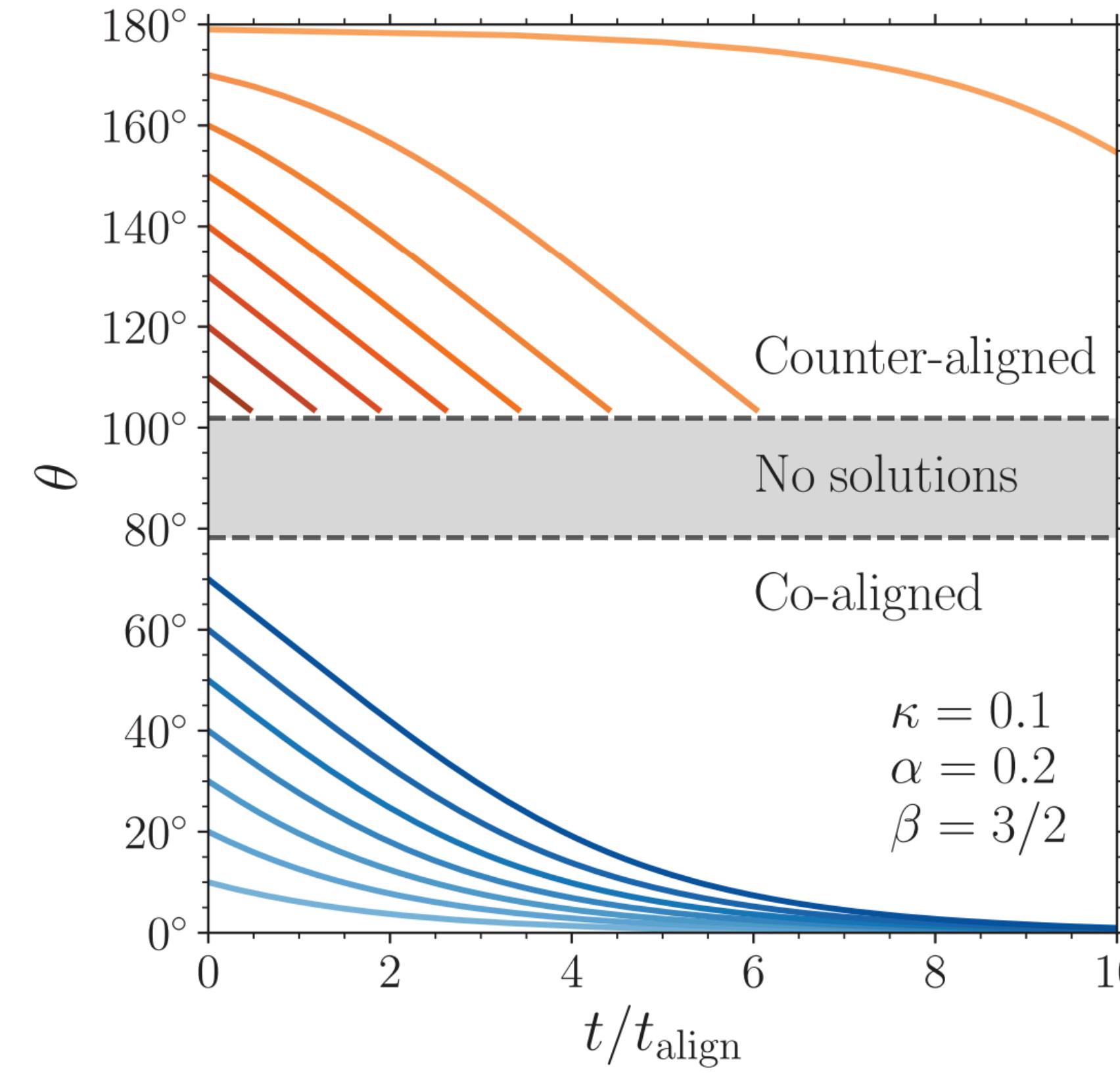
($\kappa=0$ reduces to a single black hole around the primary)

Solving the semi-analytic model

1. Choose disc parameters and parametrise the result with κ .
2. For a given angle, solve for the evolution of the surface density and angular momentum of the disc.
3. Repeat this for a range of inclinations.



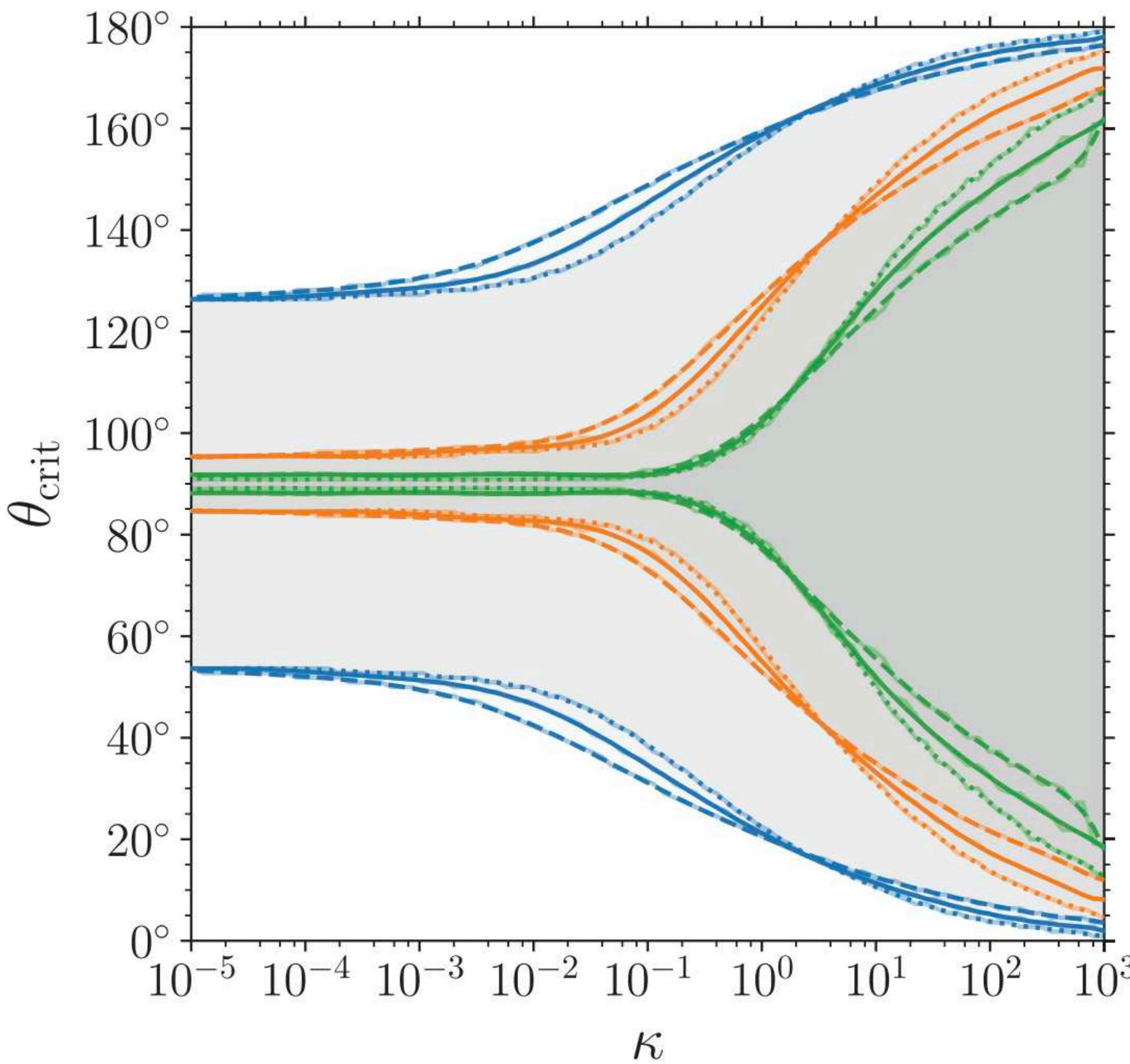
The break-down of the semi-analytic model



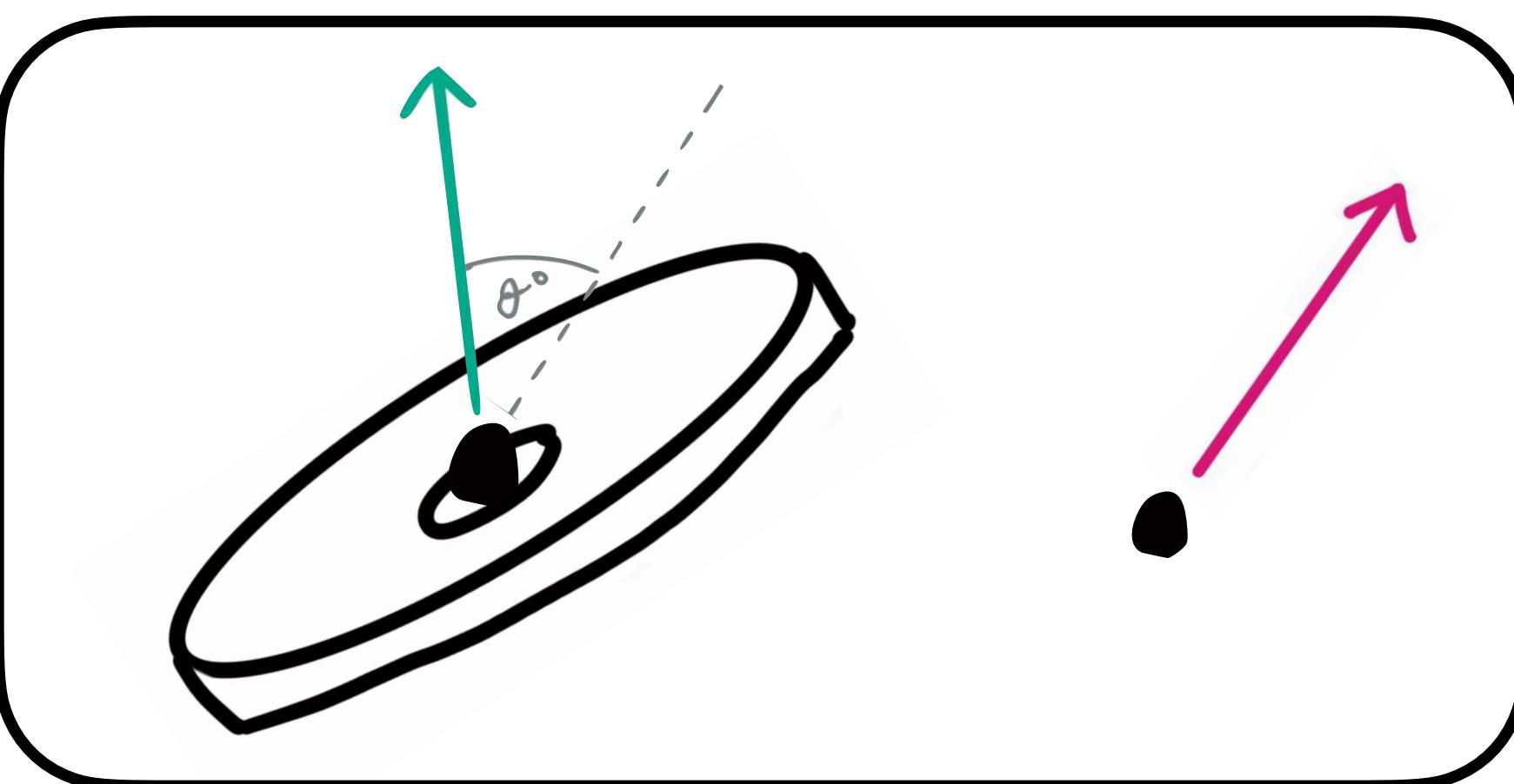
Our simulation suite

143 simulations in total:

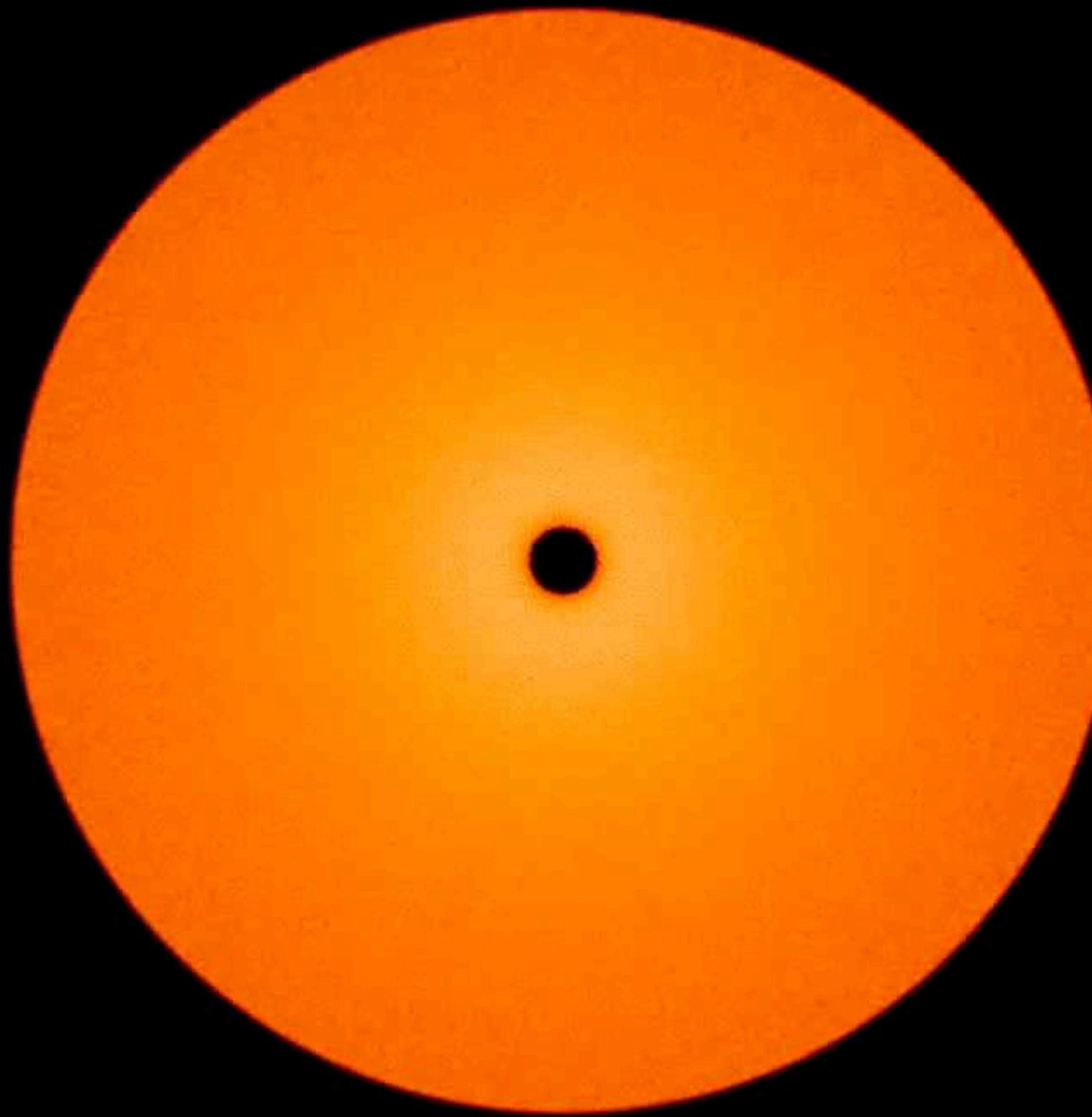
- 2 binary separations
- Four viscosities
- Four aspect ratios
- Inclinations between 20 and 160 degrees
- 12 κ values
- Use a fixed post-Newtonian potential
- Secondary is much lower mass than primary
- Black hole spin of $a=0.9$
- Disc mass is 10^{-6} of the primary black hole
- Minimum time of 150 orbits at R_{ref} (or 3 orbits of the outer binary)



Cover this
with these



0 orbits



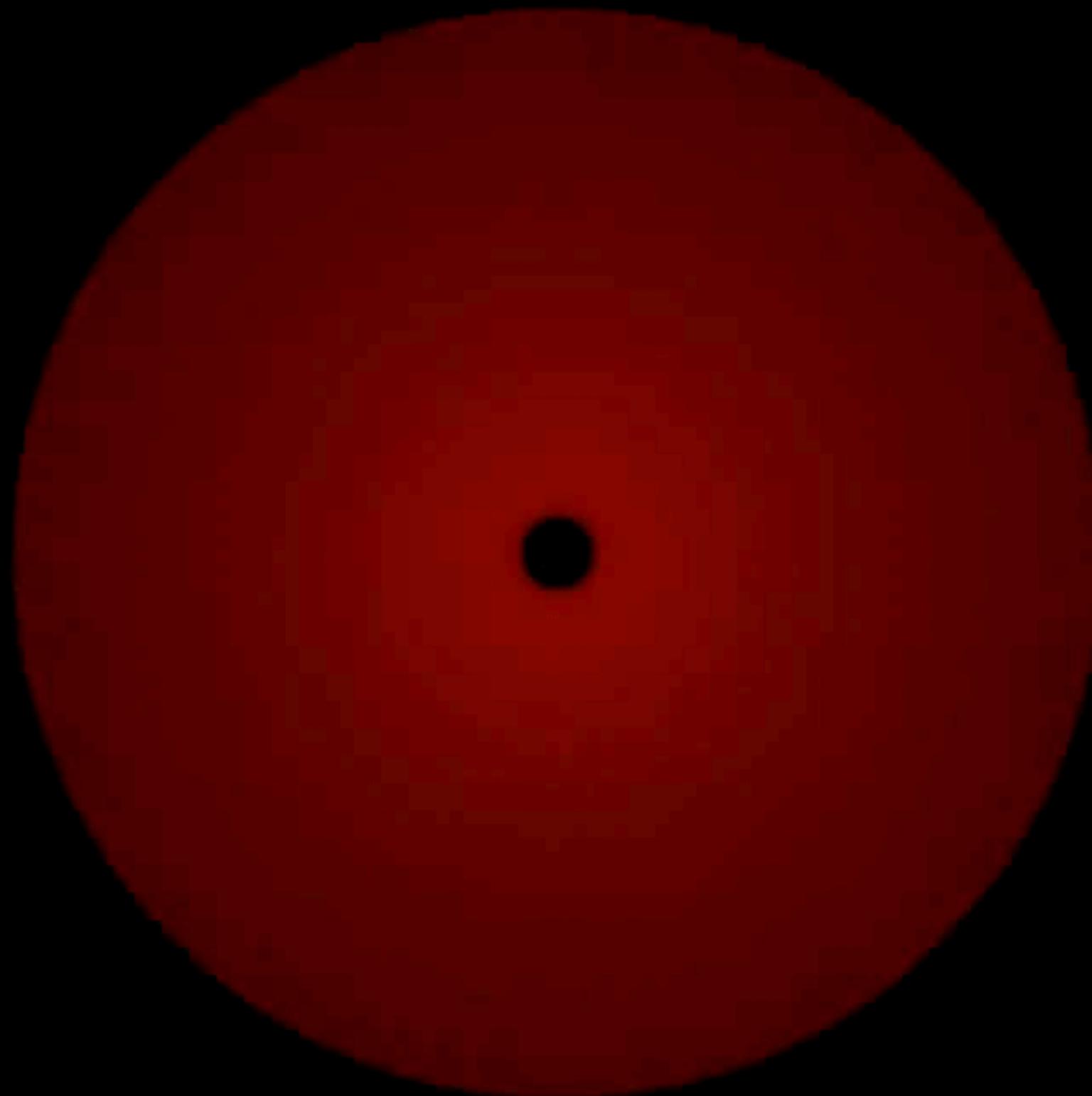
Warping

20 deg



Nealon, Ragusa, Gerosa, Rosotti & Barbieri 2022

0 orbits

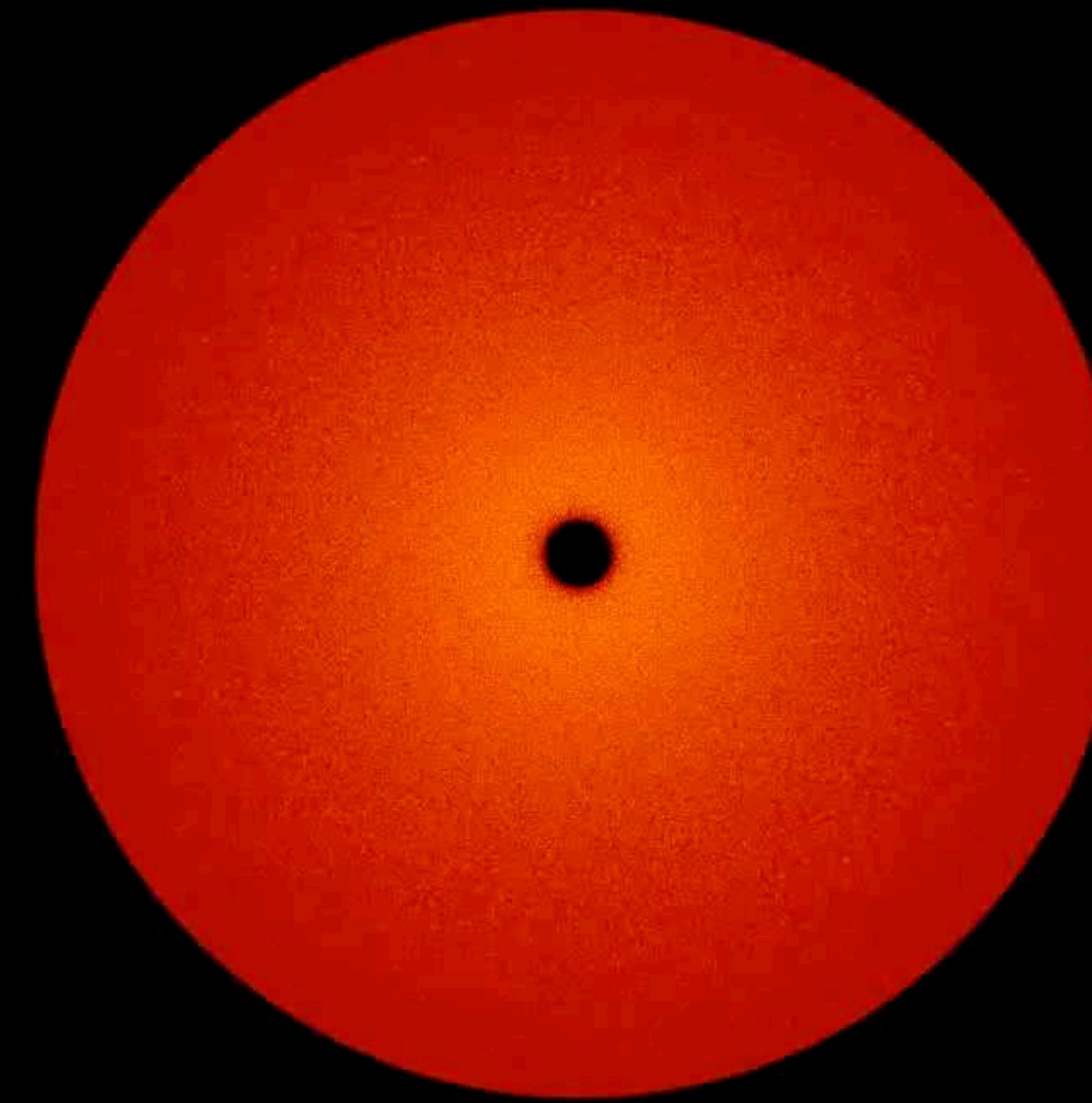


Breaking/tearing

60 deg

Nealon, Ragusa, Gerosa, Rosotti & Barbieri 2022

0 orbits



140 deg

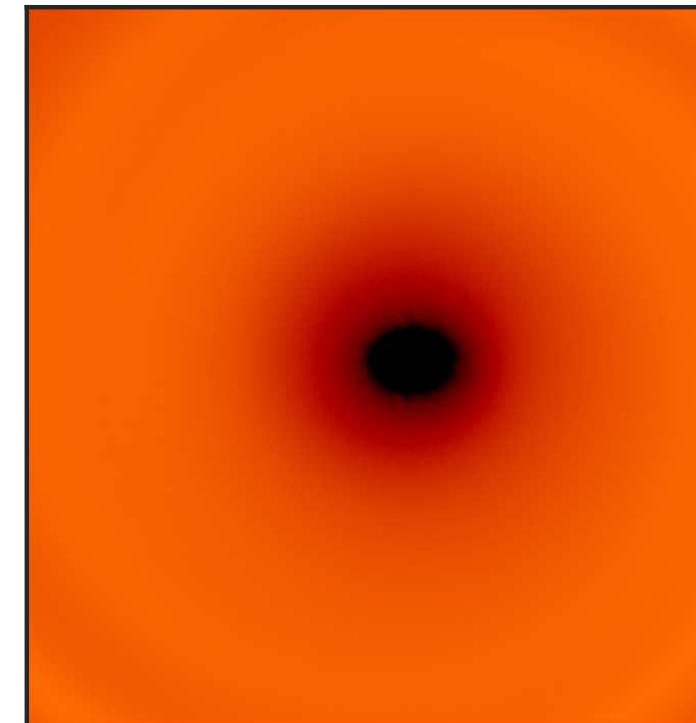


Unsuccessful breaking

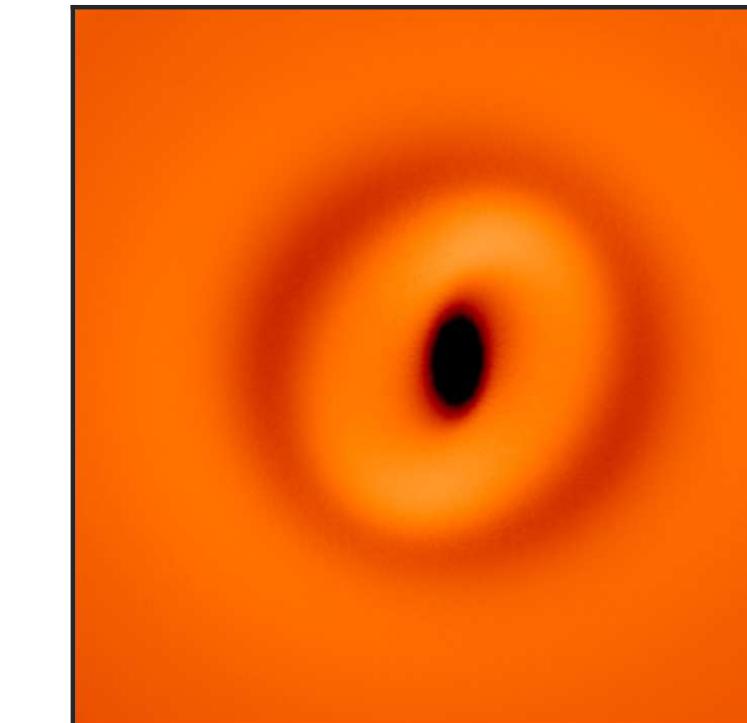
Nealon, Ragusa, Gerosa, Rosotti & Barbieri 2022

Defining a break

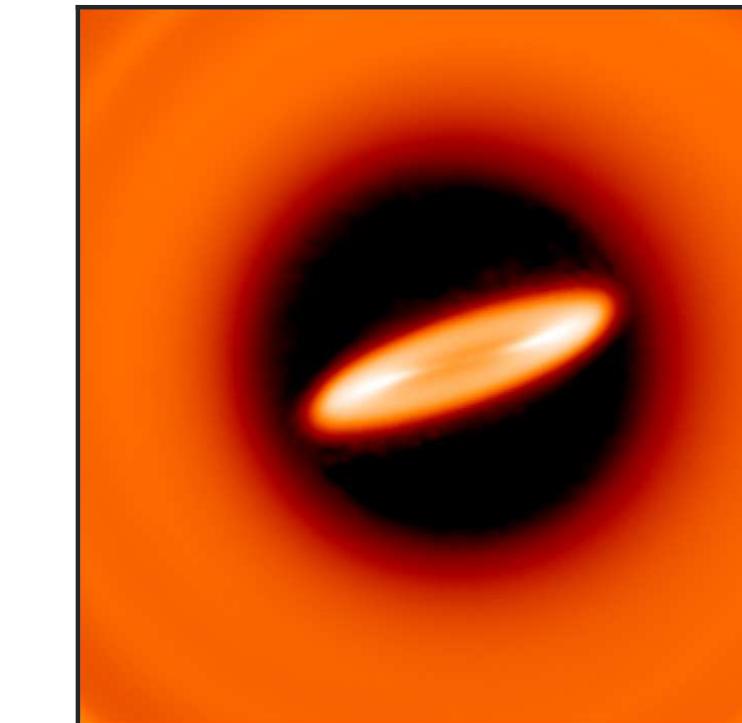
Warping



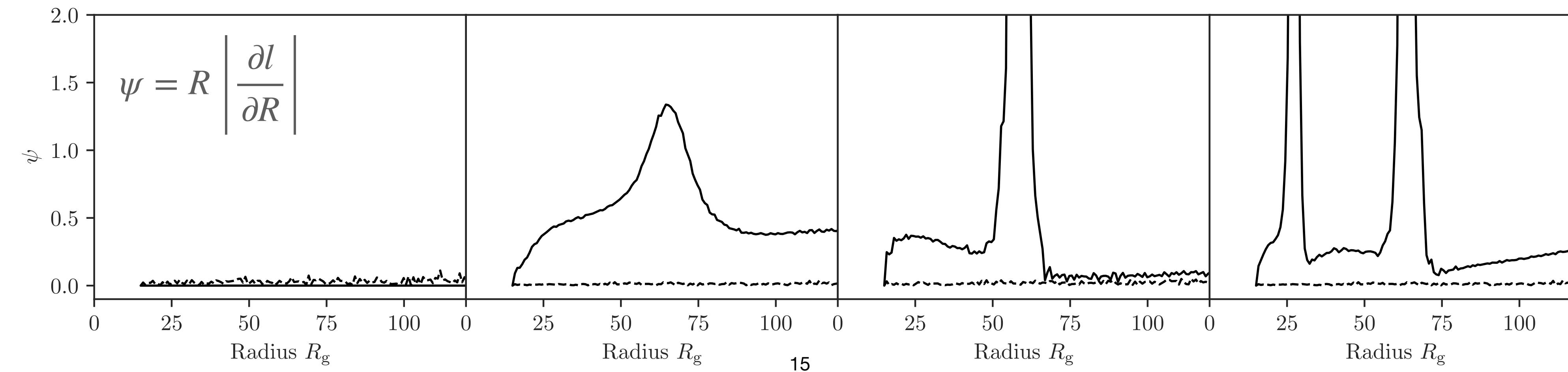
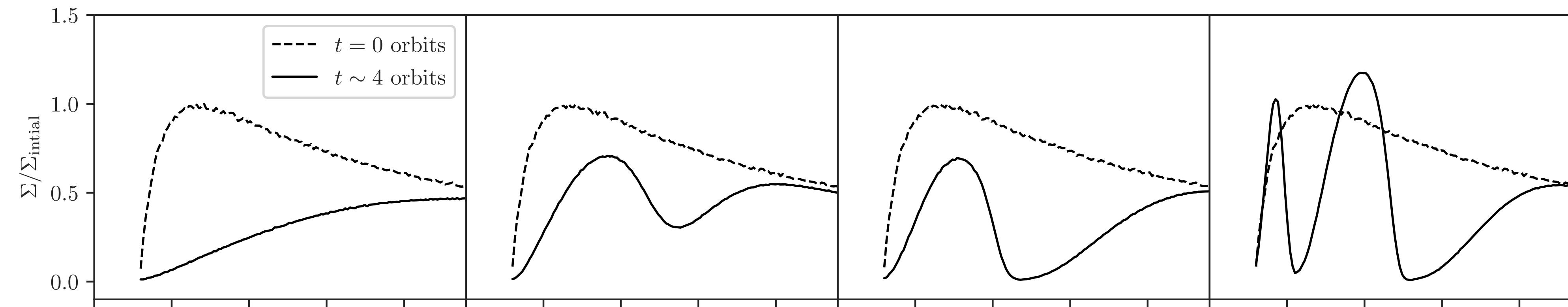
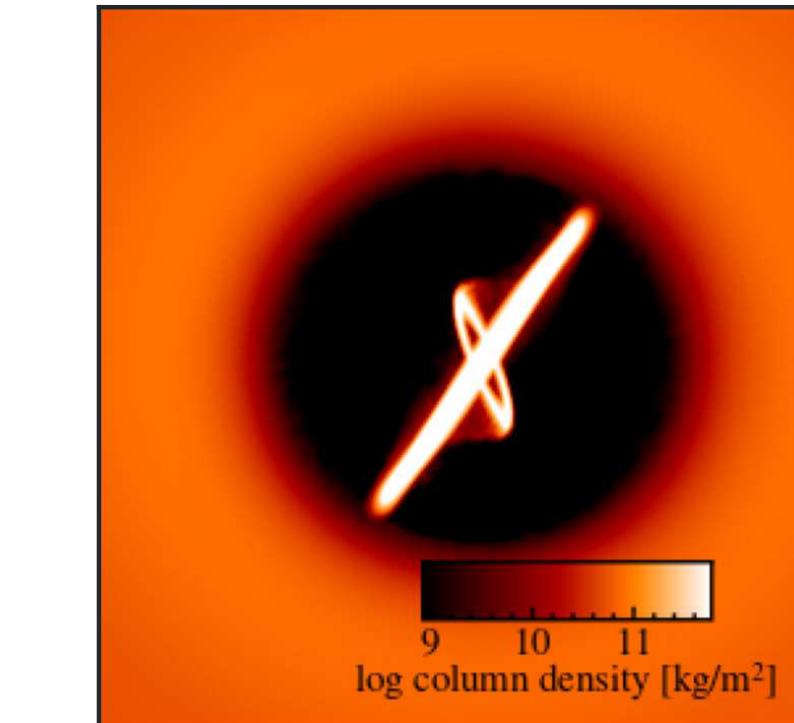
Unsuccessful breaking



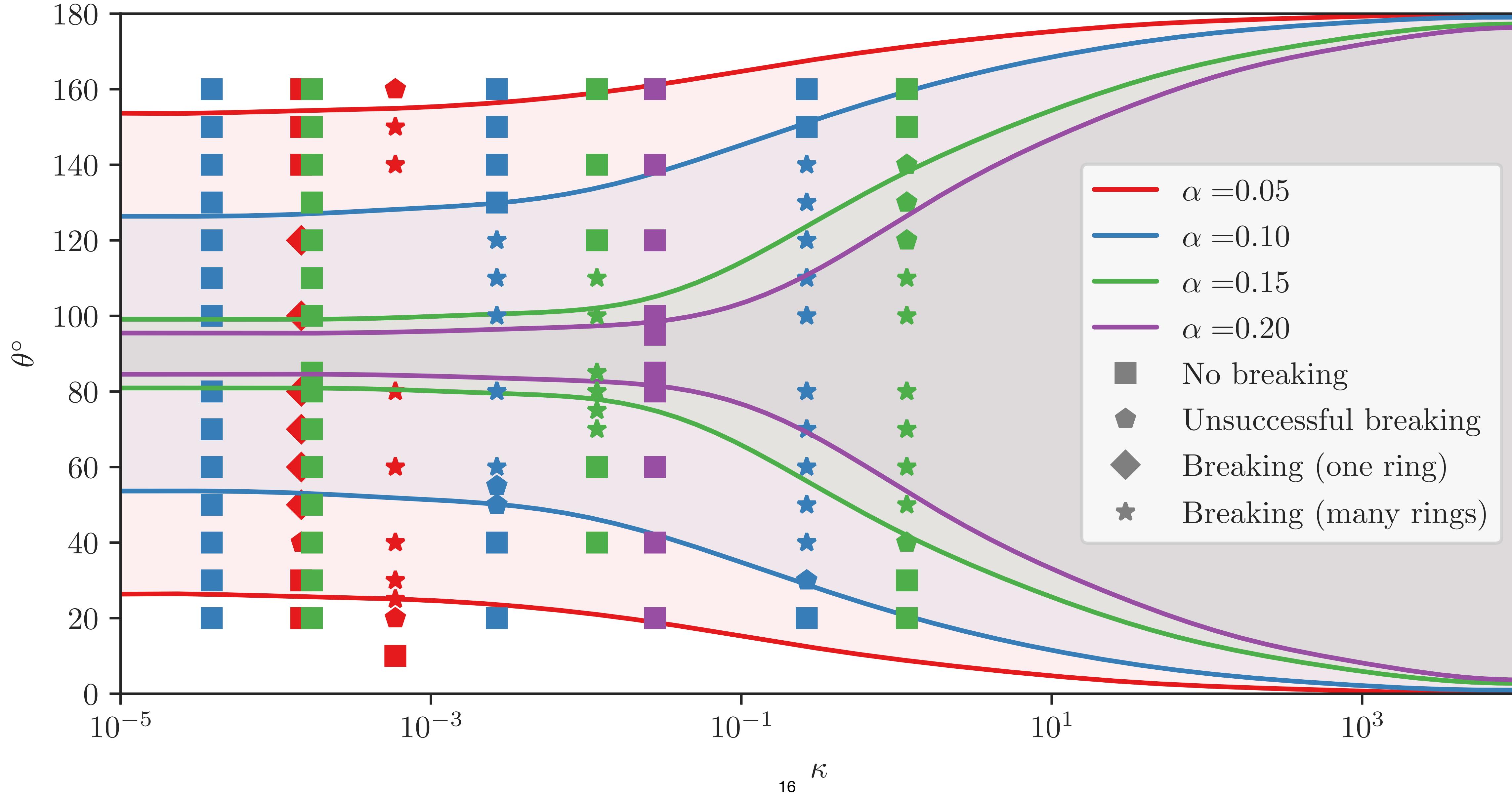
Breaking/tearing (single)



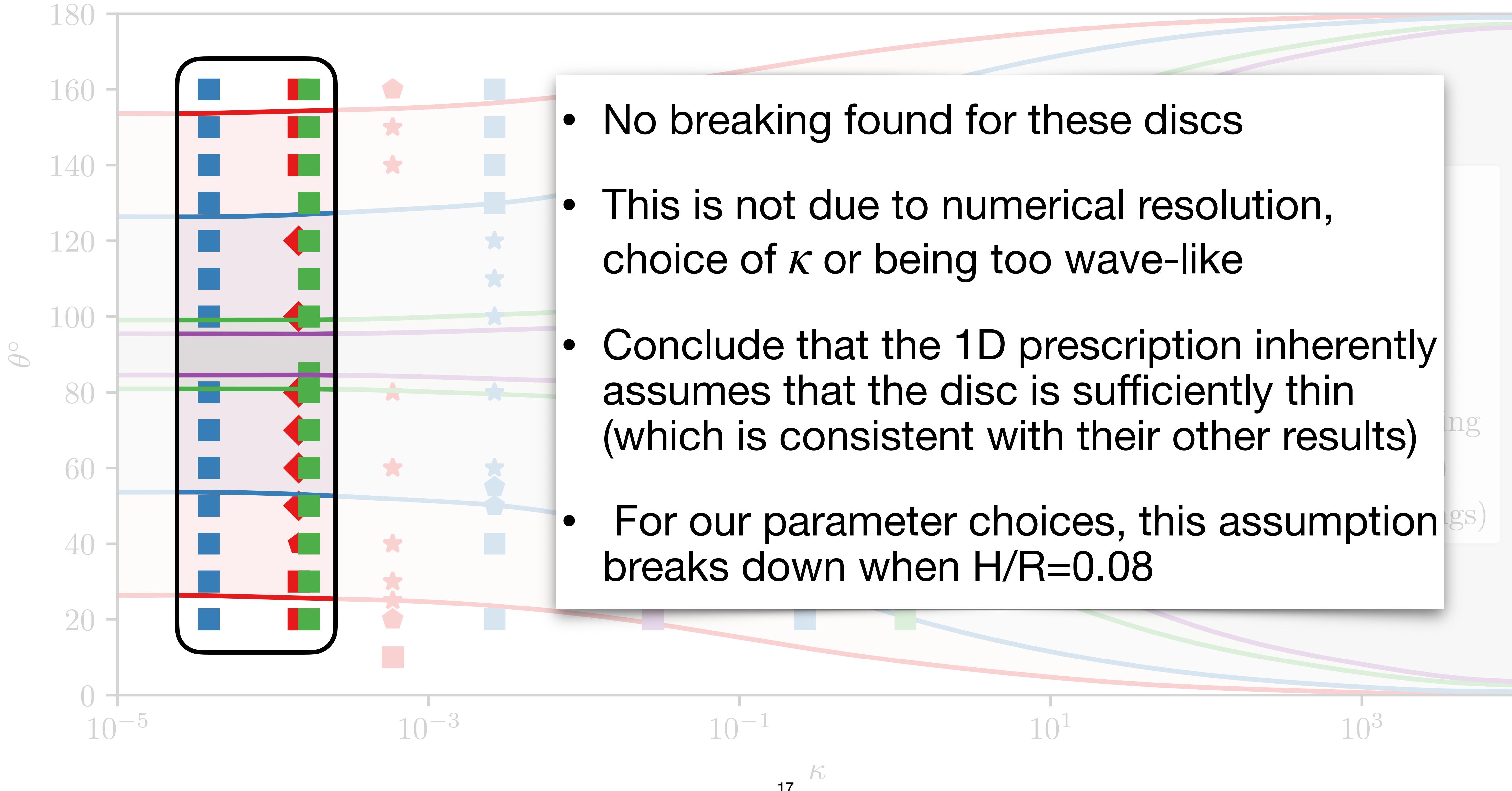
Breaking/tearing (multiple)



Comparing simulations and semi-analytic theory

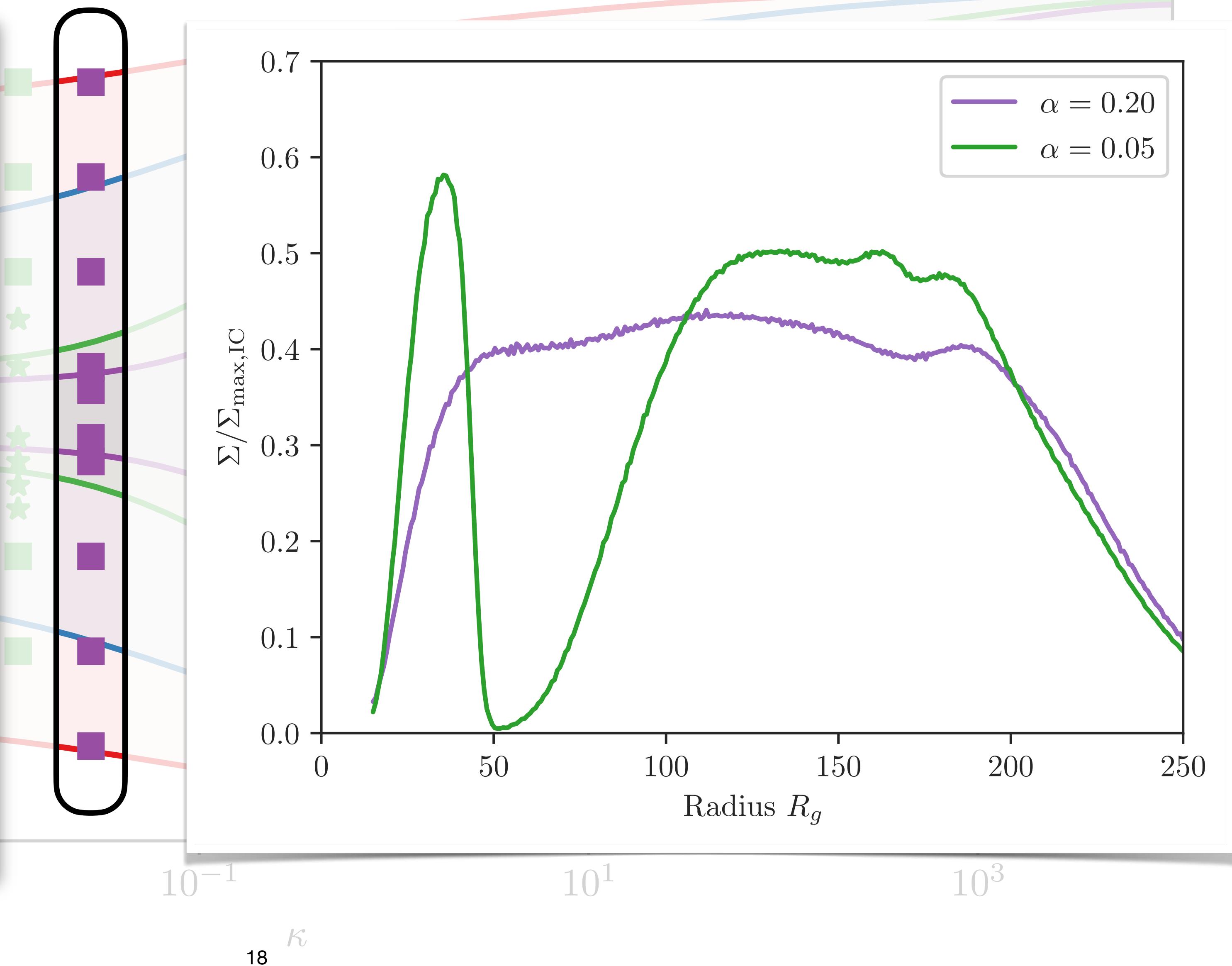


Thick discs don't break



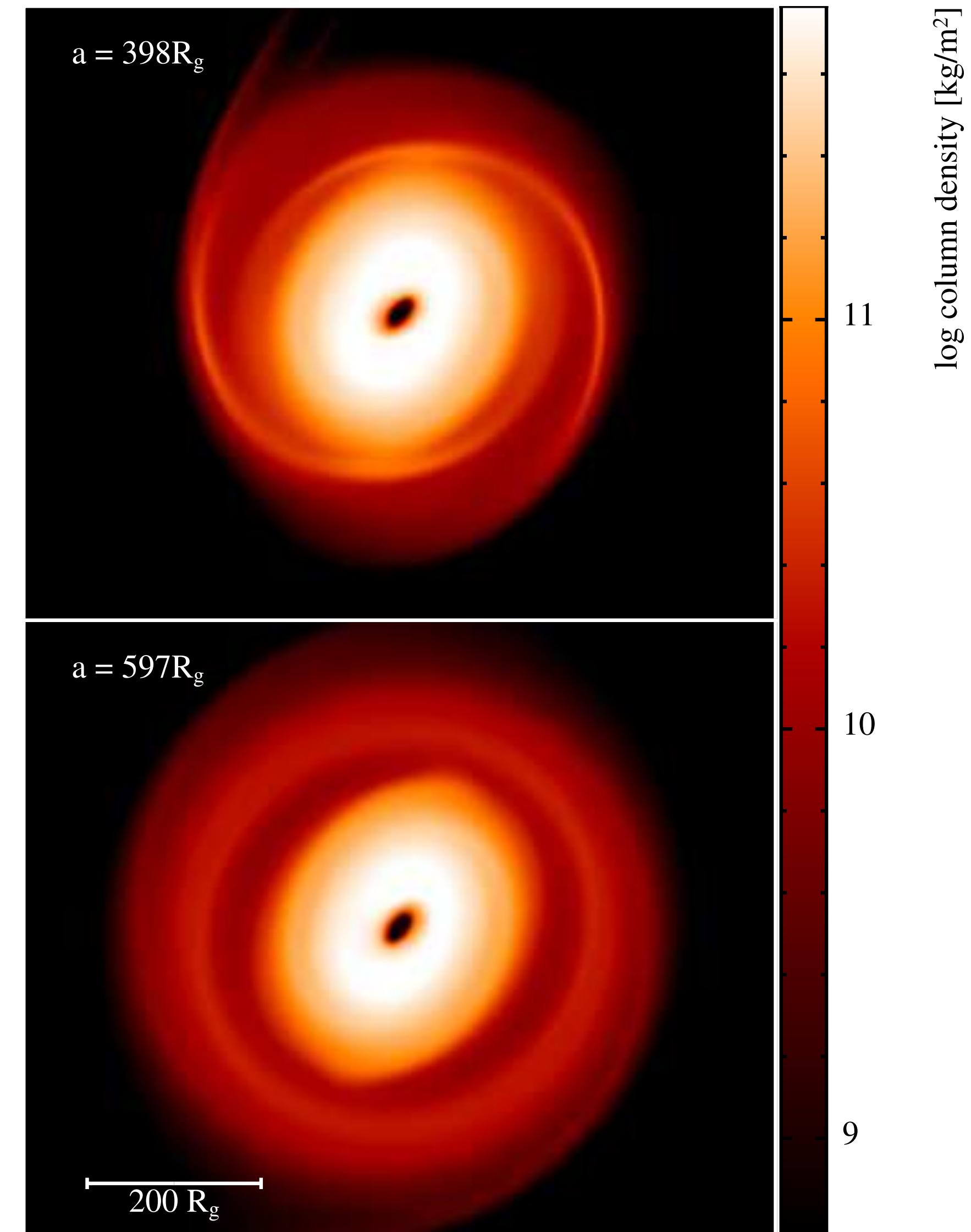
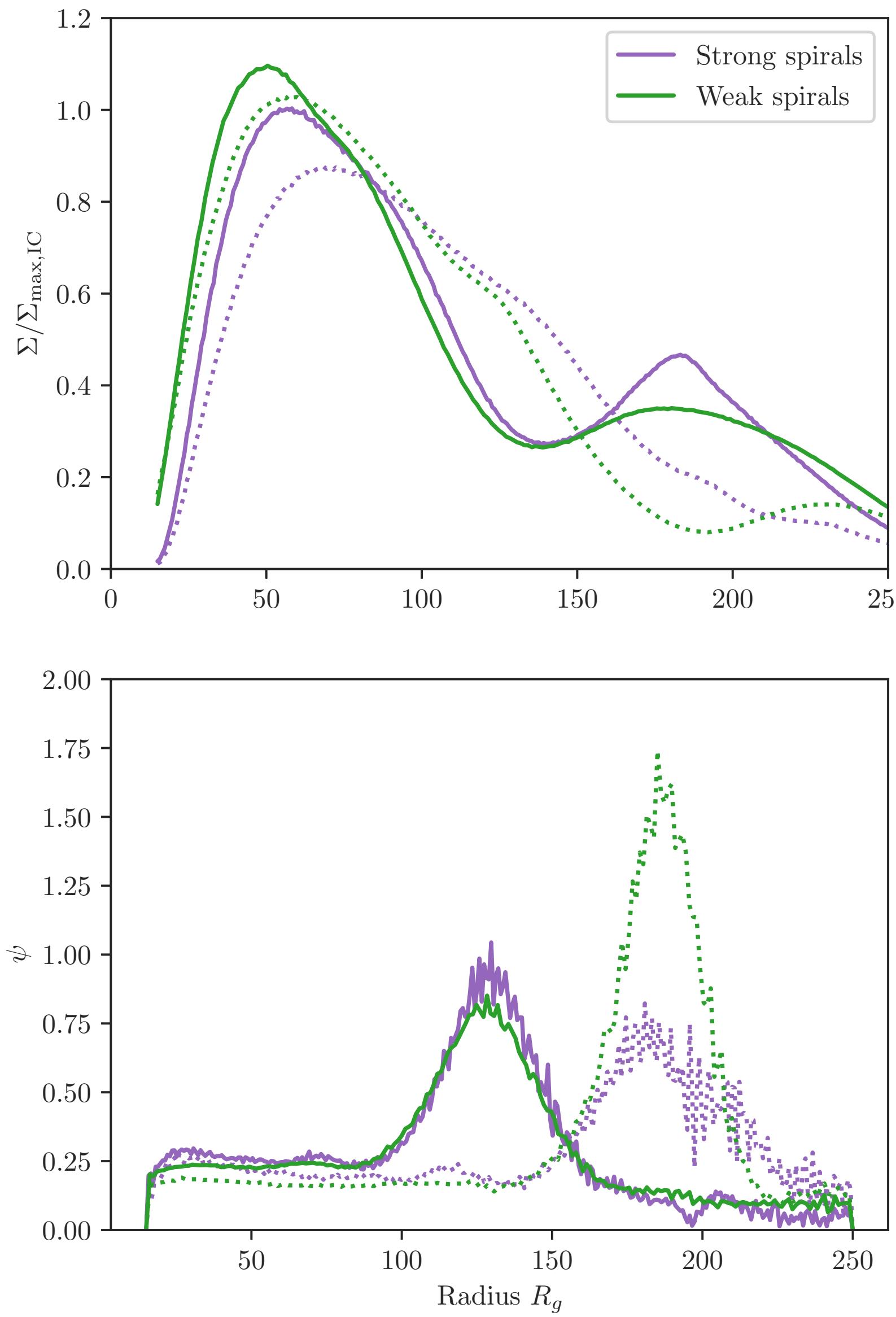
High viscosity discs don't break

- Again, not likely due to limitations of the simulations
- But looking at the surface density profile, we find that the inner disc accretes rapidly
- If breaking were to occur, this is where the inner ring would form
- Not enough material to make that inner ring -> no breaking



Spiral arms can prevent disc breaking

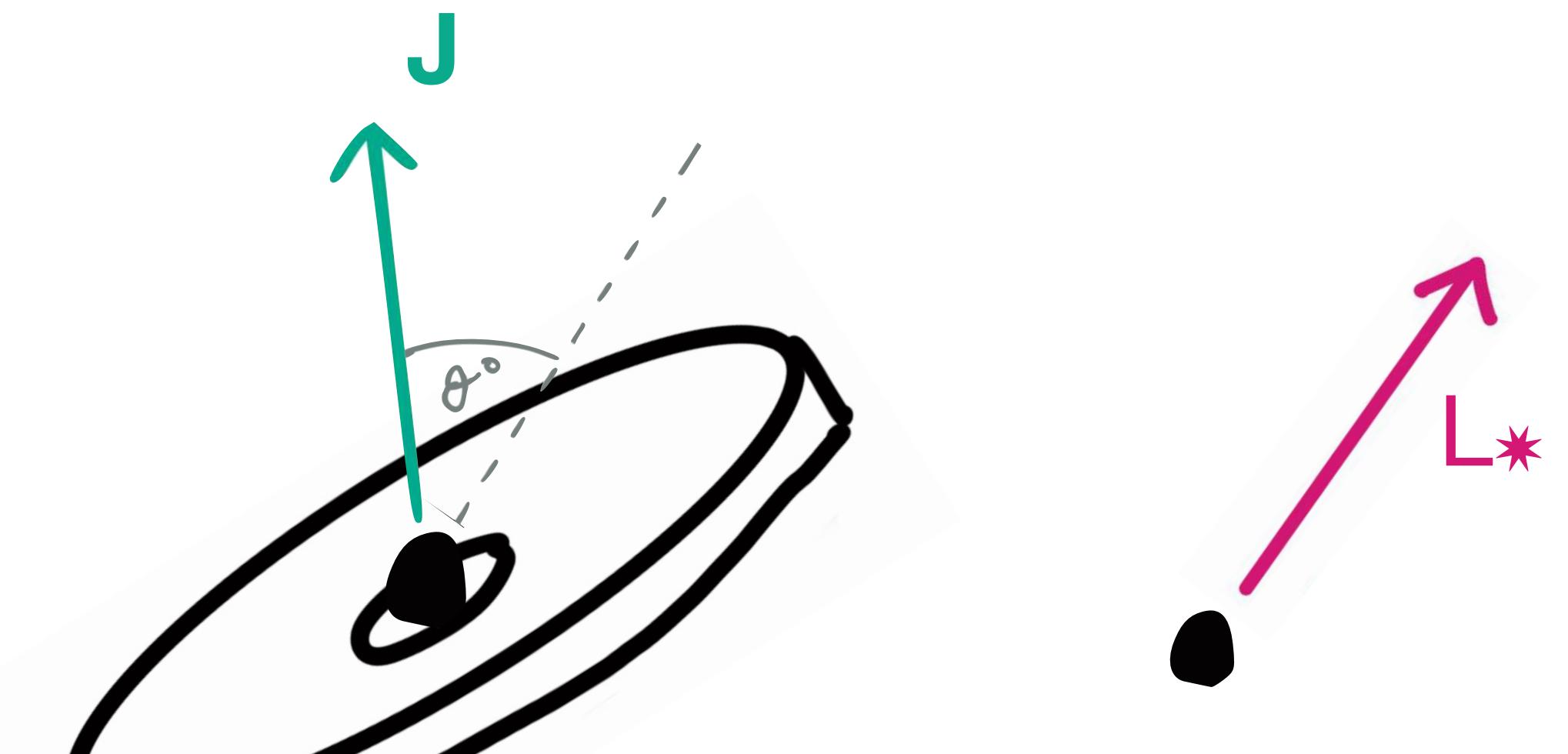
- We identify this one as unsuccessful breaking
- As the warp propagates out, it increases in amplitude until it hits the spiral arms
- Spiral arms increase the local disc viscosity, which in turn makes it harder to break the disc



Disc driven migration

$$\frac{d \cos \theta}{dt} = \frac{d \hat{\mathbf{J}}}{dt} \cdot \hat{\mathbf{L}}_*$$

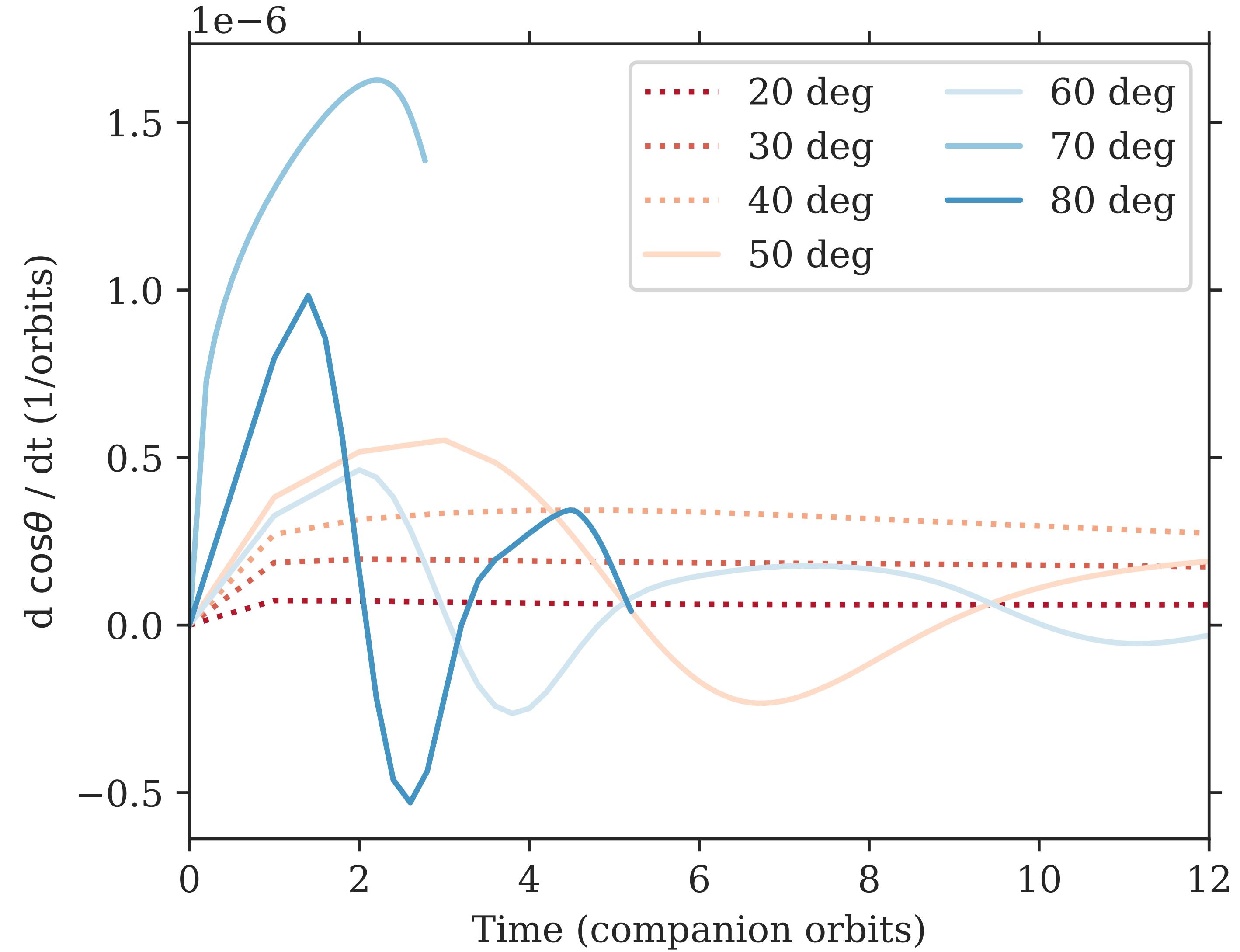
$$\frac{d \mathbf{J}}{dt} = - \int_{R_{\min}}^{R_{\max}} \frac{2G}{c^2} \frac{\mathbf{J} \times \mathbf{L}}{R^3} 2\pi R dR$$



1. What shape does the disc take?
2. How does this affect the alignment of the black holes?

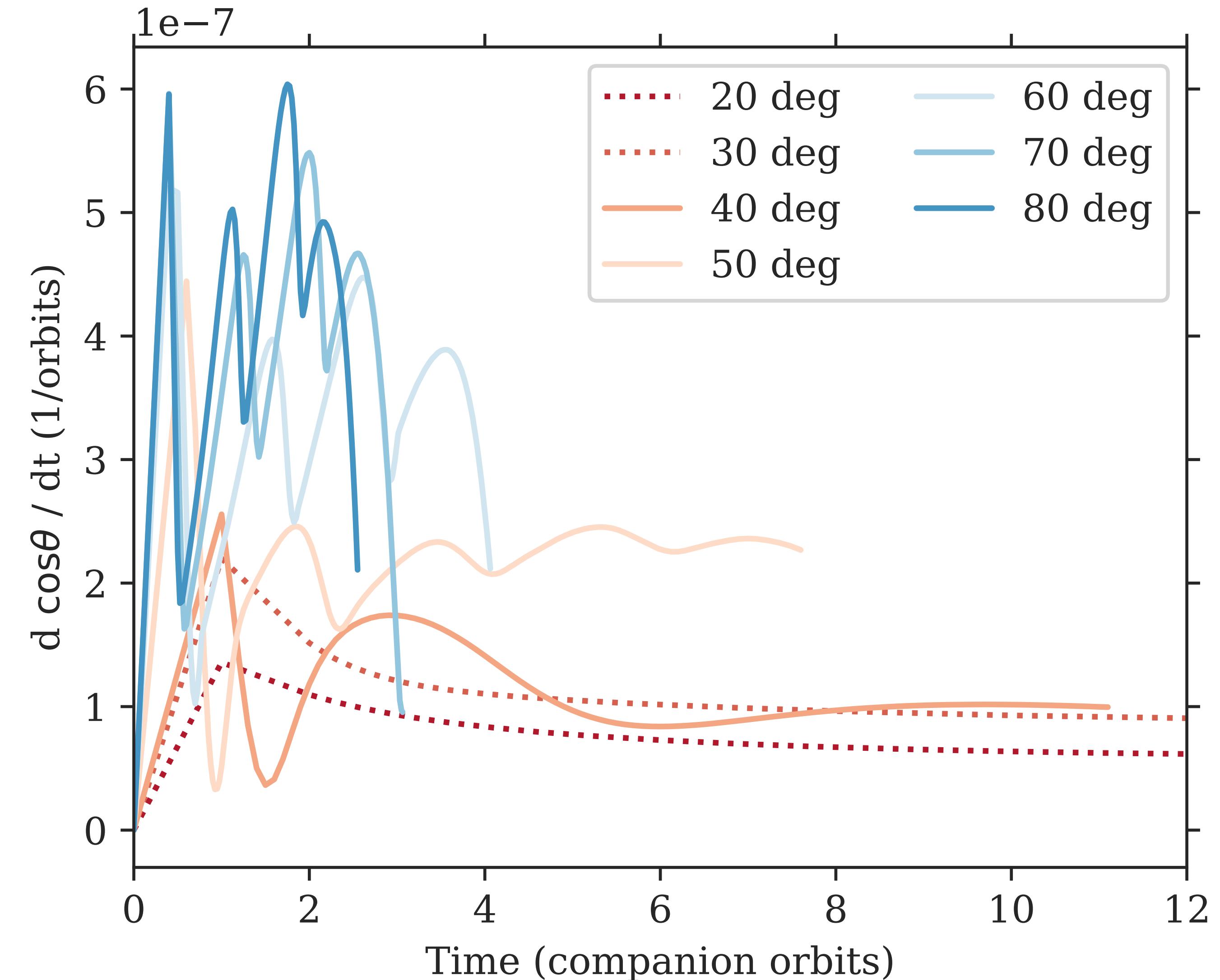
Disc-black hole misalignment

- Warping results in alignment
- Breaking with one ring results in oscillations, with occasional anti-alignment



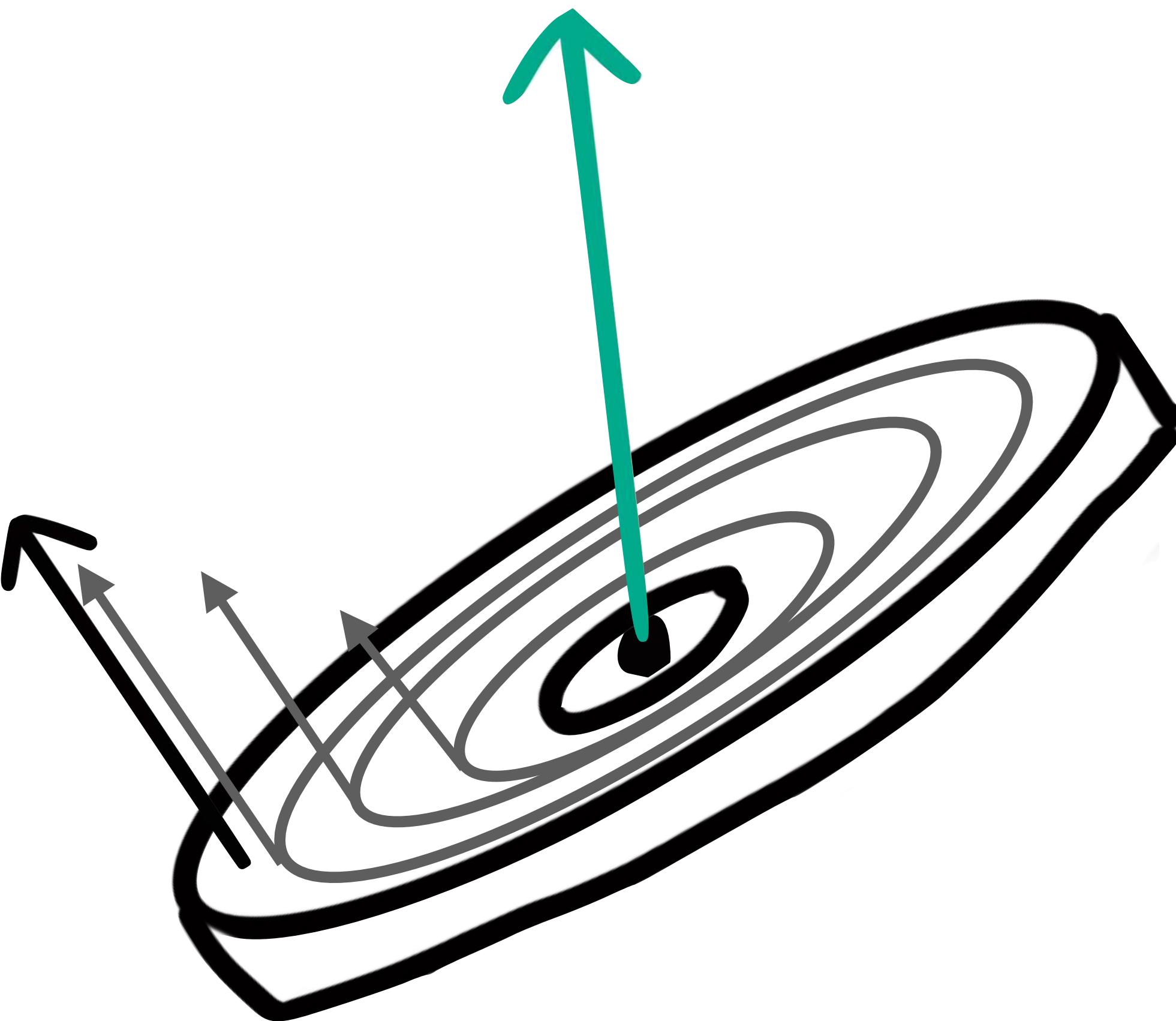
Disc-black hole misalignment

- Warping results in alignment
- Breaking with one ring results in oscillations, with occasional anti-alignment
- Breaking with multiple rings results in unpredictable oscillations



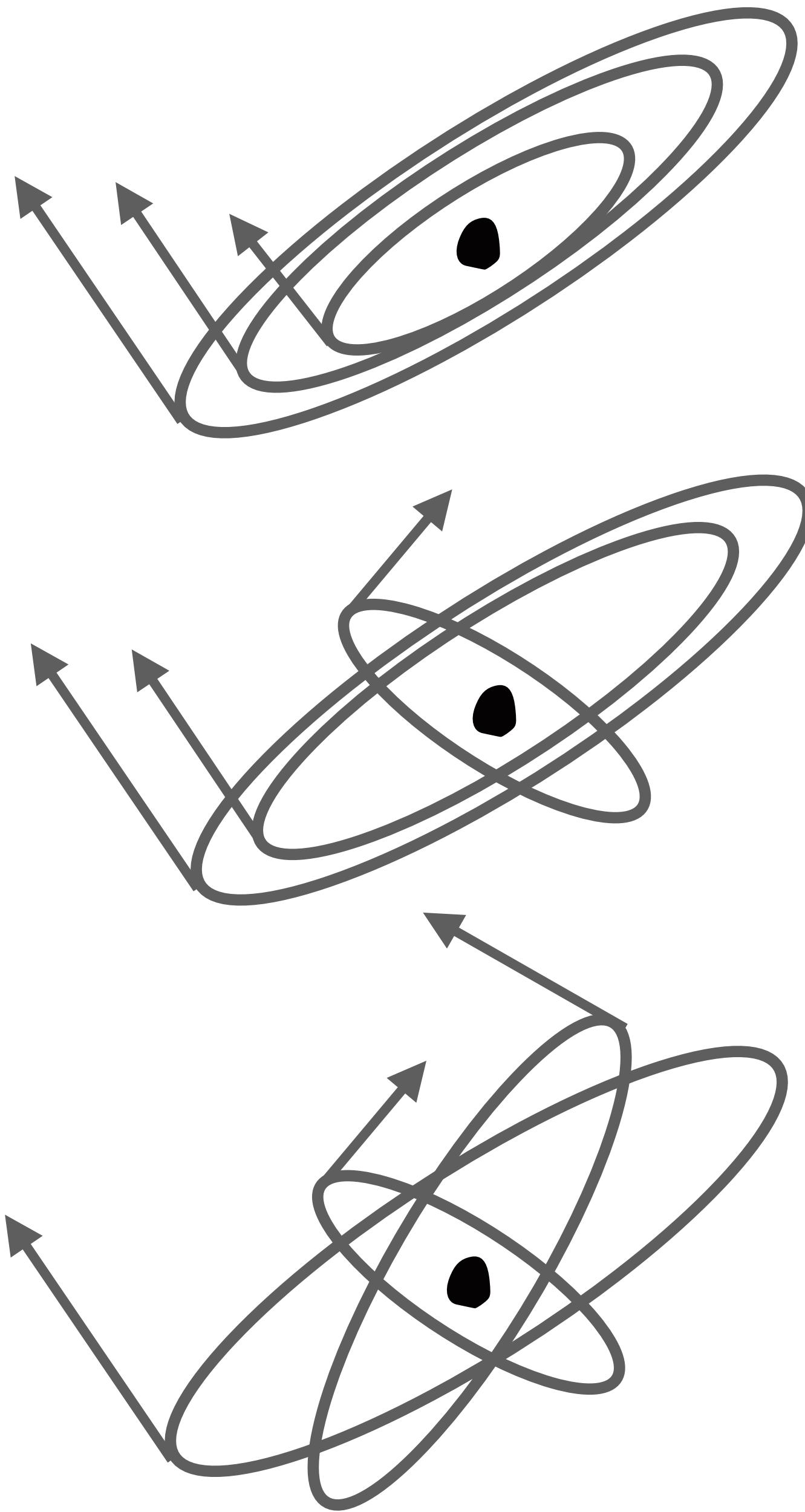
$$\frac{d\mathbf{J}}{dt} = - \int_{R_{\min}}^{R_{\max}} \frac{2G}{c^2} \frac{\mathbf{J} \times \mathbf{L}}{R^3} 2\pi R dR$$

- Warping results in alignment
- Breaking with one ring results in oscillations, with occasional anti-alignment
- Breaking with multiple rings results in unpredictable oscillations



$$\frac{d\mathbf{J}}{dt} = - \int_{R_{\min}}^{R_{\max}} \frac{2G}{c^2} \frac{\mathbf{J} \times \mathbf{L}}{R^3} 2\pi R dR$$

- Only warping results in alignment
- Breaking with one ring results in oscillations, with occasional anti-alignment
- Breaking with multiple rings results in unpredictable oscillations



Our results

- We find excellent agreement with the semi-analytic model of Gerosa et al. 2020
 - In the region where the semi-analytic model breaks down, we demonstrate that the discs are breaking
 - We have explained exceptions for very thick discs, large alpha and very large κ
- Spiral arms can stabilise the disc against disc breaking
- Disc breaking hinders (and in some cases prevents) alignment between the disc and the black hole



- Followed up by Steinle & Gerosa et al. 2023, showing that there are likely to be distinct populations that should be observable