Banking Market Power, the Deposits Channel of Monetary Policy and Capital [*]

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Abstract

We propose a deposits channel of monetary policy that works through an endogenous measure of average deposit-rate markdown. Despite there being a homogeneous deposit product, the average deposit-rate markdown depends on an empirically-relevant distribution of heterogeneous deposit rates. In turn, this distribution is an equilibrium object and depends directly on inflation or monetary policy. As anticipated inflation increases, the dispersion and interest-rate spread on deposits rise. These distort the liquidity value of trading in the frictional goods market. Since capital is a productive input in all markets, in a general equilibrium, market power in deposit taking distorts capital accumulation and long-run growth in an otherwise frictionless, neoclassical sector. We also show that competitive banking equilibrium allocations can be restored via an interest-bearing central bank digital currency that serves as an outside option. Our model provides an alternative theory that sheds new light on the deposits channel of monetary policy transmission.

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1 Introduction

In this paper, we show how imperfect competition in deposit markets and ex-post, endogenous heterogeneity in deposits pricing matter for the transmission of monetary policy to the real economy and to its long-run growth prospects. We propose a more nuanced model of the *deposits channel* of monetary policy. The basic proposition of the deposits channel (see Drechsler, Savov and Schnabl, 2017) is as follows: All else equal, if banks do not completely pass on an increase in the monetary policy rate to deposit rates, then this causes an outflow of deposits from the banking system. This then leads to a contraction in lending (*i.e.*, the real economy). In our model, because of consumer search for deposit-takers under imperfect information, identical deposit products may end up with dispersion in posted and transacted prices. We highlight the model's interaction between capital and money accumulation with an equilibrium-determined distribution of deposit-rate markdowns. This has welfare implications for how monetary policy affects banking market power in deposits-taking activity. In turn, this affects the path of capital formation and long-run economic growth.

In our model, an increase in the monetary-policy rate causes average deposit-rate spread (markdown) and its dispersion to increase. That is, deposit customers will be more likely to have their deposit rents extracted by imperfectly-competitive banks. This also renders imperfect pass-through to the real economy: Deposits and lending contract with a higher policy rate. As a result, capital accumulation falls. In a version of our model with long-run growth, this also causes the long-run growth path to be lower, relative to an economy with perfectly competitive banking.

What is novel in this mechanism is that the equilibrium degree of monetary policy pass-through is no longer a fixed mapping that only depends on some parametric measure of the deposit market's demand elasticity (see, *e.g.*, Drechsler et al., 2017). Average deposit-rate spread or markdown is a function of the equilibrium distribution of deposit rates. The distribution varies with monetary policy. Therefore, the mapping underlying the deposits channel (*i.e.*, average deposits spread or markdown) itself is no longer a policy-invariant function.

Banks are essential or welfare-improving institutions in our model. We focus on the *liquidity transformation* role of banks as in Berentsen, Camera and Waller (2007) (hereinafter, BCW): They serve the allocative and welfare-improving role of intermediating ex-post heterogeneous and risky outcomes in the shortage and excess need for liquidity. We show that banking is essential whether there are perfectly- or imperfectly-competitive banks. In the latte case, equilibrium dispersion in deposit-rate spreads and market power means that some of that welfare improvement by banks in facilitation liquidity transformation gets eroded.

Using bank-branch-level data, we consider identical deposit products and control for other factors (*e.g.*, time and geographical fixed effects). We provide new empirical evidence supporting our new aspect of the deposits channel. In particular, we find that residual dispersion is deposit-rate spreads that is positively correlated with average deposit-rate spread. In turn, the latter is positively, but not perfectly associated with the monetary-policy rate.

Some motivating empirical regularities. Figure 1 highlights the relationships between the Federal Funds Rate (FFR) and: (1) the deposit spread as a measure of banking competitiveness; (2) money demand; and (3) capital formation in the United States.[1] In Panel (a) of the figure, we see that aggregate deposit spread is positively associated with the FFR. In Panel (b), we have a standard plot of the aggregate money demand relationship: The inverse velocity of money (M/PY) is negatively correlated with the FFR. In Panel (c), the FFR is negatively associated with the investment-to-GDP (I/Y) ratio. We can think of I/Y as capital formation if we interpret this from a long-run or steady-state perspective of a neoclassical growth model.

These are facts in terms of aggregate ratios. In this paper, in Section 2, we also document new facts at a more microeconomic level. Specifically, we show that there is a positive relationship between deposit-spread dispersion (measured as standard deviation) and the average deposit spread.

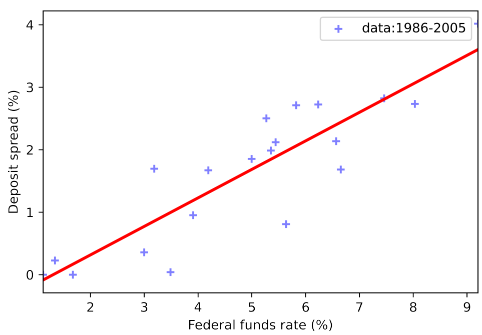
A unified model of money and deposits-side bank market power. We will rationalize the aggregate empirical relationships summarized in Figure 1 and also the micro-level, deposit-rate distributional evidence to be presented in Section 2. We build on the New Monetarist model of Aruoba, Waller and Wright (2011) (AWW) to rationalize these empirical relationships.

The AWW model is a tractable framework where money is essential and nominal outcomes are not decoupled from capital accumulation in the real economy. Our model nests AWW as a special case. Our contribution is to combine banking (with market power on the deposit side) into AWW. In equilibrium, there is a spread between the aggregate deposit rate and cost of funds for banks. Additionally, the model can also rationalize dispersion in such spreads for identical deposit products. Banking has value in equilibrium here because these institutions can intermediate between (ex-post) heterogeneous liquidity risks, as in Berentsen et al. (2007). We will focus on this particular *liquidity transformation* function of banking. Thus, our model nests AWW on one end and BCW on the other as special cases.

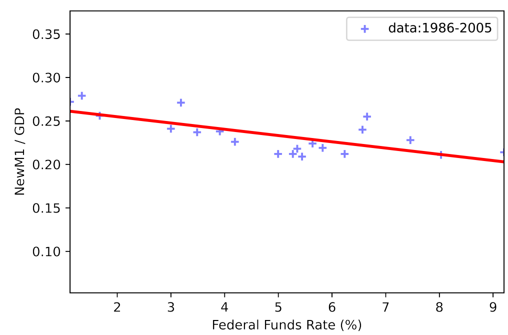
[1]Similar to Drechsler et al. (2017), we use quarterly data from Call Reports to calculate the aggregate deposit spread. The deposit spread is defined as the difference between the FFR and the value-weighted average deposit rate paid by banks. We use the (real) investment-to-GDP ratio as our proxy for capital formation. We obtain the quarterly national accounting data from FRED. The (aggregate) money demand relation uses data from Lucas and Nicolini (2015). The data are from 1986 to 2007. We have also considered a longer time period from 1986 to 2016. The corresponding relationships remain identical.

Figure 1: Monetary policy, bank market power, money and capital

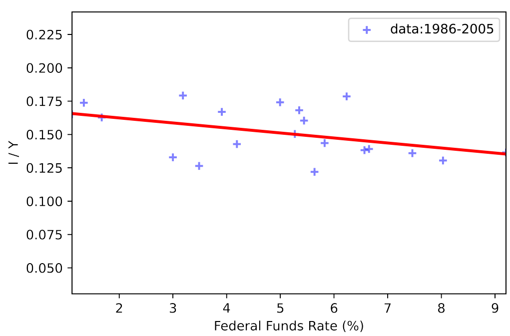
(a) Federal Funds Rate and deposit spread



(b) Federal Funds Rate and M/PY



(c) Federal Funds Rate and capital formation I/Y



The setup of AWW is a two-sector economy as in Lagos and Wright (2005). Some economic activities occur in markets with frictions and some without frictions. The authors show a novel nominal-to-real link regarding the effects of monetary policy transmission relative to the more traditional (and reduced-form) approach in monetary models (*e.g.*, cash-in-advance). The monetary policy transmission mechanism is as follows. First, a higher nominal policy interest rate induces a lower return on money. Hence, agents would optimally accumulate lower money balances. In equilibrium, this is associated with lower quantity of goods traded in the frictional goods market. Second, since capital is an input for producing goods in both sectors of the economy, there is an extra premium in accumulating capital linking from trades in the frictional market to capital accumulation in the frictionless market. This premium on capital is missing in the traditional approach in monetary models. Third, the marginal value of capital falls with the reduction in the frictional goods market trades. Consequently, there is a decrease in capital stock as the policy rate increases. In summary, monetary policy affects goods trades in a market where money is essential. This transmits to capital formation because capital is a complementary input to production in all markets.

Why might banking market power (on the deposits side) and ex-post, endogenous heterogeneity in deposits pricing matter for the transmission of monetary policy to the real economy? In a monetary economy, there is ex-post inefficiency when agents are subject to trading shocks and information frictions result in the incentive-infeasibility of private contracts or promises. Money thus has value for supporting goods exchange. Since inflation acts as a tax on reducing the return on holding money, inefficiency arises as some agents hold idle money balances while others are liquidity constrained. This problem creates a role for banks that take on nominal deposits to make nominal loans. Banks here help to resolve heterogeneous liquidity needs of households similar to that in Berentsen et al. (2007).[2] In BCW, they study a perfectly competitive banking market. We depart from BCW by considering a particular model of imperfect competition in the deposits market.

In our environment, depositors who have idle money balances search among banks for depositing funds. The search process we consider is an adaption of the noisy consumer search process of Burdett and Judd (1983). Banks post deposit rates to attract funds to make loans. Depositors who have contacted more than one bank optimally deposit their funds with the bank that commits to repaying them at a higher rate. When banks decide to post deposit rates, they consider that their potential customers will only receive a random sample of deposit rate quotes. Banks face a trade-off between posting a lower deposit rate to charge a higher interest spread (*an intensive margin*) and posting a higher deposit rate to attract more depositors (*an extensive margin*). Here, the mechanism generates policy-varying dispersion and interest spread in deposit rates as an equilibrium outcome. The reason is that change in the policy interest rate (monetary policy) affects both the opportunity cost of lending for banks and households' incentive to accumulate money balances. Consequently, monetary policy affects banks' pricing on deposits via their intensive-and-extensive margins of trade-off.

The distribution of interest spread in deposit rates implies households have different ex-post degrees of insurance on liquidity risks. Ultimately, the presence of banking market power distorts the return on money, affecting goods trades and capital formation in equilibrium. We show that banking, in general, improves equilibrium allocation and welfare more than a no-bank equilibrium as the nominal policy interest rate goes up. However, bank market power removes some of the gains from financial intermediation. The degree of banking market power distortion depends on the economy's state and monetary policy.

[2] We acknowledge that we do not have banks creating (inside) money that circulates as a means of payment in our model environment (see, *e.g.*, De O. Cavalcanti and Wallace, 1999a,b; Williamson, 1999; He, Huang and Wright, 2005, 2008; Gu, Mattesini, Monnet and Wright, 2013; Chang and Li, 2018). In our model economy, the interest-bearing debt instruments (banks' liabilities) are held by agents who do not want to consume. Hence, for this paper, we focus solely on the monetary policy implications of banking market power (a policy-varying distribution of deposit rates and markdowns) in deposit taking and how this, in turn, affects the real sector of the economy.

How it all works. We now summarize how bank market power distorts the channel from monetary policy to money, goods trades and capital formation. We consider the monetary policy instrument as the long-run inflation target (or equivalently, the policy nominal interest rate). To help with economic intuition, we first compare an economy with perfectly competitive banks and an economy without banks. Then we discuss our benchmark model with endogenous bank market power.

Higher anticipated inflation lowers the rate of return on accumulating money. Banks reduce the cost of households holding idle money balances. With banking, households who do not consume can deposit idle funds to earn interest and avoid inflation tax. Moreover, banks can extend credit (in the form of money balances) to buyers who are liquidity constrained. Banks in intermediating liquidity help improve trades in the frictional goods market, where money serves as a means of payment. Since capital is an input for producing goods in both sectors of the economy, banking also improves the marginal value of capital as captured by the extra premium associated with frictional goods trades. As a result, capital investment in an economy with perfectly competitive banks is higher than in a no-bank economy. Relative to AWW, the novelty here is that agents' decisions on money demand and capital formation will be distorted by a measure of imperfect pass-through of monetary policy (average deposit-rate markdown). In turn, we show that this measure is endogenous: it depends on an equilibrium distribution of deposit-rate markdowns.

A higher policy rate implies that households would need more liquidity insurance from banks to guard against the risk of having idle balances. Moreover, banks' opportunity cost of lending is also higher now. As a result, banks have more incentive to exploit the intensive margin in pricing deposits with a higher interest spread. In equilibrium, both dispersion and interest spread in deposit rates increase as the policy rate increases. Hence, households demand lower money balances than in the case with perfectly competitive banks. The supply of deposits and demand for loans both fall. Consequently, imperfectly competitive banks distort both goods trades and capital formation.

Our model generates an imperfect pass-through of monetary policy to the banking sector consistent with the empirical evidence established in Drechsler et al. (2017). However, the novelty in our noisy-search based model here is that bank market power—measured by the distribution of deposit rate markdowns—is endogenous. That is, the imperfect pass-through in the deposits channel of monetary policy is determined in equilibrium. Moreover, the model also accounts for this through an empirically-relevant dispersion effect in the distribution of heterogenous bank rate pass-throughs. In the model, monetary policy has a direct effect on pass-through via its effect on banks' cost of funds. It also affects pass-through indirectly through general-equilibrium effects on the distribution of deposit rate markdowns.[3]

[3] Our model contains three special cases. In the first case, we have Bertrand pricing as one limit resembling

Related literature. How inflation affects capital formation is one of the classic questions in macroeconomics (see, *e.g.*, Tobin, 1965; Sidrauski, 1967; Stockman, 1981; Cooley and Hansen, 1989; Gomme, 1993). In this paper, we revisit this question. We study how endogenous bank market power distorts the deposits channel of monetary policy and how this, in turn, affects capital accumulation. We also show how market power along this channel can distort the long-run growth path of the real economy.

Other papers also study the connection between money, capital and financial intermediation, see, *e.g.*, Bencivenga and Camera (2011) (BC) and Rocheteau, Wright and Zhang (2018). Similar to the role of banking in BC, banks in our model serve as insurers of liquidity risk to households. However, our paper departs from BC in three main ways. First, banks can also extend credit to consumers. Second, bank market power on deposits arises from search frictions. Third, the degree of banking market power is endogenous to monetary policy in equilibrium. In Rocheteau et al. (2018), they consider the problem from a corporate finance perspective. Firms choose whether to finance their investment project via internal finance (*i.e.*, own money balances), trade credit and bank funding. They show that the pass-through effect of monetary policy depends on the market microstructure and firms' characteristics. The role of financial intermediation in Rocheteau et al. (2018) is to help finance investment. In comparison, we focus on the role of banking in intermediating resources between heterogeneous liquidity needs. Also, capital in our model is dynamic and shares a common ground with standard neoclassical models. In Rocheteau et al. (2018), capital is modelled as a within-period flow variable, one that is more like a firm's working capital.

Recent papers that relate monetary policy to competition in the banking system include Choi and Rocheteau (2021), Dong, Huangfu, Sun and Zhou (2021), Chiu, Davoodalhosseini, Jiang and Zhu (2019) and Wang (2022). Both Choi and Rocheteau (2021), and we focus on the deposits channel of monetary policy. The role of banks in Choi and Rocheteau (2021) serves a different purpose than the one modelled in our paper. Their banks can freely create (inside) money that serves as a means of payment in frictional goods trades. Moreover, banks (but not consumers) have access to an investment technology that yields a higher return than money holdings. As such, the gains from banking in Choi and Rocheteau (2021) are due to consumers' access to a cheaper payment instrument. Choi and Rocheteau (2021) provide a theoretical foundation to capture the imperfect pass-through effects of monetary policy on bank deposits. In their baseline model with complete information, monetary policy transmission on total deposits only works through an extensive margin (i.e., the measure

perfect competition among banks. This case happens when all depositors have at least two depositing opportunities with banks. In the second case, all depositors have an opportunity to deposit funds with one bank. As a result, the bank can charge the monopoly rate. The third case is a no-bank economy in which depositors have no opportunity to deposit funds in the banking system. This third case is identical to AWW. Comparing these three special cases, we can decompose the link between monetary policy and bank market power to see how that affects the equilibrium outcome of the economy.

of deposit contracts offered by bankers). As a result, the bank market power that arises from bargaining is insufficient to rationalize the deposits channel in their baseline model. Consequently, they show that informational frictions (i.e., liquidity needs are the consumers' private information) are crucial to introduce an intensive margin on individual demand for bank deposits. The main message in their extended model is that deposits outflow in response to the higher policy interest rate is concentrated on those with low liquidity needs.

In contrast, the role of banks in our paper is to reallocate liquidity from agents with unproductive idle funds to those who are liquidity constrained. The gains from banking come from generating a positive rate of return on idle funds. Although the role of banks here differs from that considered in Choi and Rocheteau (2021), our main result shares a similar emphasis on the role of informational frictions regarding the deposits channel of monetary policy. Specifically, we highlight that banking market power (due to noisy deposits search) in deposit pricing matters for capital accumulation, which has long-run economic growth consequences. This mechanism works through agents' (ex-ante) precautionary demand for money and capital, all of which depend on policy and (imperfect) competition among banks.

The remaining structure of the paper is as follows. In Section 2, we provide micro-data evidence on the relationship between deposit-rate spreads and the dispersion of these spreads. In Section 3, we discuss the details of the model setup and characterize the stationary monetary equilibrium of the model economy. In Section 4, we provide analytical results of the model. Section 5 illustrates the model insights quantitatively by first calibrating it to the U.S. macro-level data. Using numerical results, we first demonstrate the pass-through of monetary policy to the bank market power in deposits. We also provide an empirically testable prediction on deposit-rate dispersion and spreads. Then we discuss the long run effects of inflation on allocation (money and capital) and economic welfare in equilibrium. In Section 6, we study economic-growth implications of inflation via the lens of banking market power. In Section 7, we study the effects of interest-bearing central bank digital currency (CBDC) on capital accumulation. Finally, we conclude the paper in Section 8.

2 Empirical motivation

Our model of the deposit channel of monetary policy to lending and capital will feature equilibrium banking market power on the deposit side. The equilibrium features an empirically-relevant dispersion in *deposit spreads* between the U.S. federal funds rate and bank-level deposit rates.

In this section, we present new empirical evidence on the deposit channel using bank-branch level data from *RateWatch*. Controlling for other possible sources of variations in deposit-rate pricing, there is a positive relationship between deposit-spread dispersion (standard deviation) and deposit-spread mean.

2.1 Data

Branch-level interest rate data. We obtain weekly interest-rate information on an identical deposit product at each branch from *RateWatch*. Specifically, we use rates for one of the most commonly used time deposit products in the United States known as a twelve-month certificate of deposits.[4] This strategy of focusing on a class of identical deposit products allows us to rule out any observable (and unobservable) pricing heterogeneity across depositors and deposit products.

Our primary sample includes 1,428,900 branch-weekly observations from 12,381 branches, between January 2001 and December 2007.[5] Our sample covers 49 states and the District of Columbia. We drop Hawaii due to insufficient branch-level observations to calculate state-level dispersion. To calculate each branch's deposit spread against the federal funds rate, we collect daily effective federal funds data from the U.S. Federal Reserve H15 report.

Deposit spread. We follow Drechsler et al. (2017) and define the deposit spread as the difference between federal funds rate (FF_t) and branch-level deposit rate ($Rate_{b,s,t}$).[6] Specifically, we calculate each bank branch's deposit spread as

$$Spread_{b,s,t} = FF_t - Rate_{b,s,t}, \tag{2.1}$$

where b denotes the bank branch, s the state, and, t the date that *RateWatch* reports the branch deposit-rate information. We further derive the mean ($\overline{Spread}_{s,t}$) and the standard deviation ($Dispersion_{s,t}$) of branch-level deposit spreads within a particular state s and a time period t.

Figure 2 provides a visual synopsis of our results. Specifically, Figure 2a shows a positive relationship between the standard deviation and the average of deposit spreads at the state level, at a monthly frequency. Figure 2b demonstrates a positive relationship between the average deposit spreads and the federal funds rate, which is consistent with the findings in Drechsler et al. (2017) and Choi and Rocheteau (2021).

[4] We focus on fixed-term time deposits in order to be consistent with our theoretical model. In the model, households use time deposits to save idle money balances in contrast to demand deposits, which helps to smooth out the consumption shocks. Noisy-search frictions induce dispersion for a homogeneous bank deposit product in equilibrium. While we do not report this in the paper, we have also conducted the same empirical analysis on other deposit products and have obtained the same conclusions.

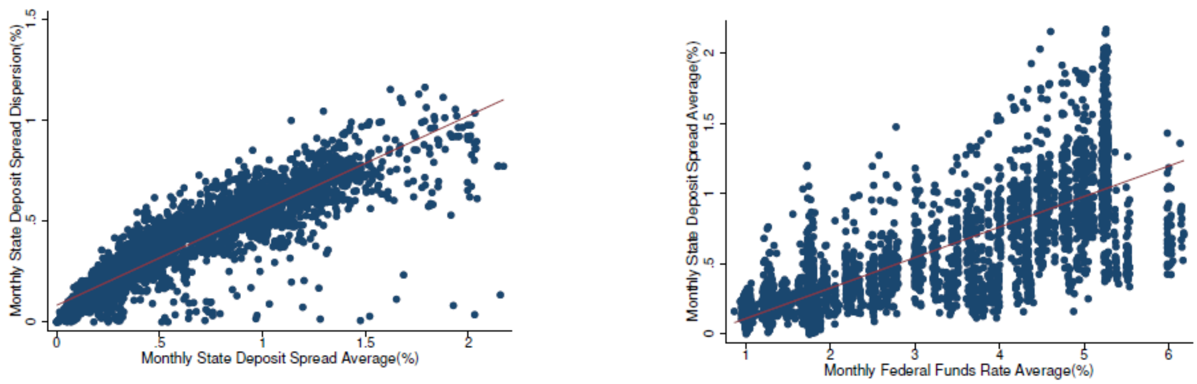
[5] We choose not to include observations beyond 2008 to avoid the near-zero-lower-bound interest rate environment. As Wang (2022) argues, monetary-policy effectiveness could change even before the zero lower bound binds.

[6] We also use an alternative specification of the deposit spread $Spread_{b,s,t} = \frac{FF_t - Rate_{b,s,t}}{FF_t}$ following Wang (2022) and find consistent results.

Figure 2: Dispersion (standard deviation) and average of deposit spreads

(a) Dispersion vs. mean

(b) Mean vs. Federal Funds Rate



2.2 Regression evidence

To formally test the relationship observed in Figure 2a, we run the following OLS regression of deposit-spread standard deviation on deposit-spread average after controlling state fixed effects (Z_s) and time fixed effects (Z_t). Specifically, we estimate b_1 in the following specification.

$$Dispersion_{s,t} = b_0 + b_1 \overline{Spread}_{s,t} + b_2 Z_s + b_3 Z_t + \epsilon_{s,t} \tag{2.2}$$

The index s stands for a particular state and t stands for the month of observation. We cluster standard errors by state.

Table 1 summarizes the regression results for Equation 2.2. All columns show a positive and statistically significant relationship between deposit spread standard deviation and mean. Column (4) suggests 10 basis points increase in the average of deposit spread is associated with a 3.4 basis points increase in the standard deviation of the deposit spread after controlling for state fixed effects and time fixed effects.

Our empirical findings here shed new light on the deposit channel of monetary policy transmission documented in Drechsler et al. (2017) regarding the relationship between monetary policy and deposits spread: Here we provide additional evidence in this relationship regarding its connection to the dispersion in deposits spread. The empirical insights, as summarized in Figure 2a (also, Table 1) and Figure 2b are also consistent with our theoretical model predictions. Next, we present the model.

Table 1: Regression for deposit spread dispersion at state-month level

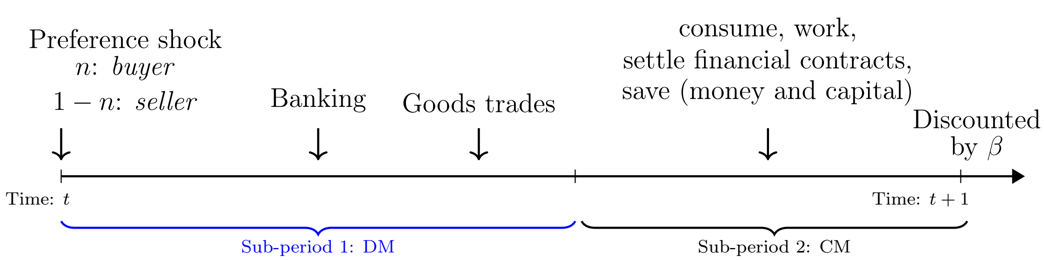
Dependent Variable:	$Dispersion_{s,t}$			
	(1)	(2)	(3)	(4)
$\overline{Spread}_{s,t}$	0.482***	0.360***	0.492***	0.335***
	(0.017)	(0.040)	(0.015)	(0.038)
Constant	0.080***	0.148***	0.075***	0.162***
	(0.006)	(0.021)	(0.008)	(0.021)
Month FEs	No	Yes	No	Yes
State FEs	No	No	Yes	Yes
Observations	4155	4155	4155	4155
Adjusted R^2	0.856	0.894	0.885	0.922

3 Model

In this section, we develop a model that will have predictions that are consistent with the empirical evidence from Section 2. In the model, time is discrete and infinite. Each period divides into two sub-periods as in Lagos and Wright (2005). Figure 3 displays the model timeline. The first sub-period has a decentralized market (DM) with information friction and noisy search for deposits. The main environmental distortion in the DM is one of anonymity: Sellers of DM goods cannot observe individual buyers' histories so that exchange cannot be sustained using private securities. The key insights of the model will emerge from features in the DM. In the second sub-period, a centralized market (CM) operates frictionlessly. The CM is a metaphor for institutions or markets that allow for agents to trade and rebalance their asset positions without any hindrance.

Agents discount payoff flows across periods t and $t+1$ by $\beta \in (0,1)$. There are four types of agents: government, households, banks and firms. We now describe the sequence of events and actions in the first and second sub-period. We will use the following notation to denote time-dependent variable outcomes: $X \equiv X_t$ and $X_{+1} \equiv X_{t+1}$.

Figure 3: Timing



The first sub-period (DM). At the beginning of the period, the government controls the supply of divisible fiat currency M according to the rule $M_{+1} = \gamma M \equiv (1+\tau)M$, where $\gamma - 1 \geq 0$ is growth rate of money supply.

Households face preference shocks for production and consumption. In particular, an agent discovers she is a buyer with probability n or a seller with probability $1-n$. A buyer cannot produce the special good q but wants to consume it to enjoy a utility flow of $u(q)$ in the DM. In contrast, the seller does not value consumption and has access to a technology $f(e,k)$ to produce the special good on the spot using her effort e and capital k. We assume f is a Cobb-Douglas production function with constant returns to scale.

Anonymity in the DM arises because there is a lack of record-keeping technology among buyers and sellers. Given this friction, buyers lack the commitment to honor any private credit and sellers cannot monitor and enforce such contracts. Hence, money has value in equilibrium for exchange in this model.[7]

We assume that there are institutions called banks. Unlike private individuals, banks have costless access to a superior record-keeping technology, they can perfectly enforce loan contracts and commit to honor deposit liabilities. These assumptions are the same as the baseline setting in Berentsen et al. (2007). Banks can therefore provide financial intermediation between DM sellers and buyers of goods. Banking happens after the realization of households' preference shocks.Banks accept nominal deposits from sellers to make nominal loans to buyers.[8] Banks face perfect competition on loans and compete for deposits in a price posting environment featuring noisy consumer-search of Burdett and Judd (1983). Moreover, we assume that banks can invest b with the central bank to earn a rate of return $i = (\gamma - \beta)/\beta$ if they have any remaining funds.

Banks post their deposit rate i_d to attract deposits in the DM and commit to repaying depositors at that interest rate in the subsequent CM. Depositors search among banks to deposit their idle money balances d. They may receive one or two deposit-rate quotes at a time with probability respectively given by $0 \leq \alpha_1 \leq 1$ and $\alpha_2 = 1 - \alpha_1$.[9] Buyers may or

[7]The focus of this paper is not on how other assets and fiat money compete for the role of medium of exchange. We assume it is costless for agents to counterfeit claims on assets other than fiat money. This assumption ensures that agents cannot use claims on capital as a means of payment in the DM. Hence, we focus on fiat money as the only medium of exchange for simplicity.

[8]We will see that in equilibrium, all agents carry a precautionary amount of money into the DM. Agents who are ex-post buyers will have no incentive to deposit funds with a bank if they are own-liquidity or credit constrained. The buyers who may turn out to be liquidity unconstrained during the DM goods exchange cannot deposit their idle balance since banking activity would have closed prior to DM trades. Ex-post DM sellers will never have an incentive to borrow since they already carry idle money. Sellers also cannot earn deposit interest on additional money paid by buyers since banking activity would have closed at that point. We denote l as the amount of loans a buyer may take out and d as the amount of seller deposits throughout the paper.

[9]We exclude the possible case where a seller receives zero deposit-rate quotes from banks. This assumption maintains the banking interaction as in Berentsen et al. (2007), where sellers can always match with a bank

may not borrow additional money balances l from banks given a loan interest rate i_l.

At this point, all banking (deposit-taking and loan-providing) activity ceases. The DM goods market opens. Buyers and sellers exchange goods via a price-taking protocol. Buyers face a liquidity constraint consisting of their money balances and loans when purchasing goods q in the DM. Sellers produce the q_s on-the-spot using their effort and capital. Let p denote the nominal price of the goods in the DM. Since the banking sector is closed at this point, sellers cannot deposit any revenue pq_s generated from the goods they produce and sell. Likewise, if some buyers are liquidity unconstrained in their DM goods trade, they would not be able to deposit any residual money balance with banks. Loans and deposit interest repayments are settled in the subsequent CM.

The second sub-period (CM). There is general good x split into consumption or investment in the CM. An agent's individual state entering the CM is given by (m, k, l, d), *i.e.*, their remaining money balance, capital, outstanding loan and deposit balance. Those who have deposited in the previous DM will earn interest $1 + i_d$ on deposit d, and those with outstanding debt repay them with interest $1 + i$ on loan l to banks. Each individual holds k claims to the aggregate capital stock K. Each rents capital and supply labor h to a representative competitive firm. The total amount of labor hired by the firm is H. An individual earns rental income on capital and labor, $r(K, H)h$ and $w(K, H)h$, respectively. The representative competitive firm hires capital and labor services to produce $F(K, H)$ units of goods using K and H. We assume F to be a production function with constant returns to scale. We assume the firm does not operate in the DM as in Aruoba et al. (2011). Households then consume and accumulate money and capital to carry into the next period, m_{+1} and k_{+1}.

3.1 Preferences

Households' per-period utility is given by

$$\mathcal{U}(q, k, x, h) = \underbrace{U(x) - \bar{A}h}_{\text{CM}} + \underbrace{u(q) - c(q, k)}_{\text{DM}}. \tag{3.1}$$

In the CM, agents consume good x to enjoy a utility flow of $U(x)$. They encounter $-\bar{A}h$ disutility of working, where $\bar{A}$ is a scaling parameter. We assume a log utility function in the

for depositing their idle funds. The only difference is that sellers now observe a random number of deposit-rate quotes posted by banks. It is also worth mentioning that the maximum number of deposit interest rate quotes can be arbitrarily large. Still, it will not alter the main insights of the model. We could also introduce a fixed deposit-search cost for the sellers in a more general setup. We can then endogenously determine the maximum deposit-rate quotes sampled by depositors. This approach will work through a search-intensity channel along the lines of Head and Kumar (2005) and Wang (2016). For this paper, we maintain the simpler version of Burdett and Judd (1983) where sellers receive, at most, up to two deposit-rate quotes from banks.

CM:

$$U(x) = \bar{B}\ln(x), \tag{3.2}$$

where $\bar{B}$ is a scaling parameter.

In the DM, buyers get utility $u(q)$ from consumption of the special good q. We assume that $u^{'} > 0$, $u^{''} < 0$ and that u satisfies the Inada conditions. We restrict attention to the constant-relative-risk-aversion (CRRA) family of functions:

$$u(q) = \begin{cases} \bar{C}\frac{q^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \bar{C}\ln(q) & \text{if } \sigma = 1 \end{cases}, \tag{3.3}$$

where $\bar{C}$ is a scaling parameter.

DM sellers produce a special good q on the spot using technology $f(e,k) = e^{\psi}k^{1-\psi}$ with own labour (effort) e and capital k, where $\psi \leq 1$. Solving $q = f(e,k)$ for $e = c(q,k)$, then the (utility) cost incurred by sellers in producing q given capital k is captured by the function $c(q,k) = q^{\omega}k^{1-\omega}$, where $\omega := 1/\psi \geq 1$.[10]

3.2 Decentralized market - the first sub-period

We now get into the details of the decision problems within each period. We begin with the first sub-period. To economize on notation, we will use $\mathbf{s} := (m,k)$ to denote the agent's individual state consisting of nominal money balance and capital stock, and $\mathbf{a} := (M,K,\gamma)$ to denote the aggregate state of the economy consisting of total nominal money balance, capital stock and policy.

[10]If we let technology parameter ψ in the DM be $\psi = 1$, then capital does not matter for DM production and the nominal side of the economy is decoupled from the real side (see Aruoba and Wright, 2003; Aruoba et al., 2011). In this case, the special good is produced using a seller's own effort e only. For our baseline analysis, we restrict attention to the case where $\psi < 1$ and so $\omega > 1$ Moreover, the technology f satisfies $f_e > 0, f_{ee} < 0$ and $f_k > 0$ and $f_{kk} < 0$. The cost function $c(\cdot)$ satisfies $c_q(q,k) > 0$, $c_{qq}(q,k) > 0$; $c_k(q,k) < 0$, $c_{kk}(q,k) > 0$. Assume k is a normal input, then it follows that $c_{qk}(q,k) < 0$.

3.2.1 Ex-ante household

At the beginning of the period before the opening of the DM (*i.e.*, before matching, banking and production take place), all individuals have the valuation:

$$\begin{aligned} V(\mathbf{s},\mathbf{a}) = nB(\mathbf{s},\mathbf{a};i_l) + (1-n)\Bigg\{\alpha_1 \int_{\underline{i}_d}^{\bar{i}_d} S(\mathbf{s},\mathbf{a};i_d)\mathrm{d}G(i_d;\gamma) \\ + \alpha_2 \int_{\underline{i}_d}^{\bar{i}_d} S(\mathbf{s},\mathbf{a};i_d)\mathrm{d}[G(i_d;\gamma)]^2\Bigg\}, \end{aligned} \tag{3.4}$$

where $G(i_d;\gamma) := 1 - \bar{G}(i_d;\gamma)$, B and S, respectively, denote the probability of drawing a deposit rate higher than i_d, the post-match value functions of being buyer and seller.

Equation (3.4) says the following. Conditional on being a seller with probability $1-n$ in the DM, she searches for banks to deposit her idle money balances, taking the distribution of posted deposit rates $G(i_d;\gamma)$ as given.[11] With probability $\alpha_1 \in (0,1)$, the seller obtains one deposit-rate quote i_d drawn from the distribution $G(i_d;\gamma)$; With probability $\alpha_2 = 1-\alpha_1$, the seller obtains two deposit-rate quotes and the probability of the higher of the two deposit-rates drawn is captured by $[G(i_d;\gamma)]^2$. With probability n, the post-match valuation of a buyer is given by $B(\mathbf{s},\mathbf{a};i_l)$ taking the bank lending interest rate i_l as given.

We can rewrite Equation (3.4) as

$$V(\mathbf{s},\mathbf{a}) = nB(\mathbf{s},\mathbf{a};i_l) + (1-n)\left\{\int_{\underline{i}_d}^{\bar{i}_d} \Big[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\Big] S(\mathbf{s},\mathbf{a};i_d)\mathrm{d}G(i_d;\gamma)\right\} \tag{3.5}$$

3.2.2 Ex-post DM Sellers

Consider the valuation of an ex-post DM seller in Equation (3.5). The seller takes p, the nominal price of the DM special good as given. Conditional on having observed deposit-rate quote i_d, this valuation is given by:

$$S(\mathbf{s},\mathbf{a};i_d) = \max_{q_s,d}\left\{ -c(q_s,k) + W(\hat{m}_s,k,0,d,\mathbf{a}) \middle| \begin{array}{l} d \le m + \tau_s M, \\ \hat{m}_s = m + \tau_s M - d + pq_s \end{array} \right\}. \tag{3.6}$$

W denotes the value function at the beginning of the subsequent CM. The argument of function W is the seller's individual state vector $(\hat{m}_s,k,0,d)$ upon exiting the DM. This consists of her remaining money balance $\hat{m}_s$, capital k, loan $l_s = 0$ and deposit balance d.

Also, since the individual has realized his type as a DM seller before banking activity begins, then he will deposit his idle, initial money balances, $d = m + \tau_s M$, with the bank as

[11] For now, we assume a compact support of the posted deposit rate distribution. We will be more explicit about the distribution $G(i_d;\gamma)$ and its support when we present the banks problem.

long as the deposit rate i_d is positive. The residual balance at the end of the DM is thus the revenue from having sold goods, $\hat{m} = pq_s$.

The seller will optimally produce the special good up to the point where its marginal cost of production equals its relative price:

$$c_{q_s}(q_s, k) = \frac{p\phi\bar{A}}{w} \tag{3.7}$$

where ϕ is the value of money in units of the general CM good.

3.2.3 Ex-post DM buyers

The buyer chooses how much special good q_b to purchase and whether to borrow l from the bank to top up her initial money holdings m. His post-match valuation is

$$B(\mathbf{s}, \mathbf{a}; i_l) = \max_{q_b, l} \left\{ u(q_b) + W(\hat{m}_b, k, l, 0, \mathbf{a}) \middle| \begin{aligned} &\hat{m}_b = m + \tau_b M + l - pq_b, \\ &\hat{m}_b \geq 0,\ 0 \leq l \leq \bar{l} \leq \infty \end{aligned} \right\}. \tag{3.8}$$

In our baseline model, we assume that the bank can perfectly enforce loan repayments costlessly. In this case, the bank can prevent any default in equilibrium, and the upper borrowing limit is $\bar{l} = \infty$. The buyer's optimal demand strategy for the special good is given by

$$q_b(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w) = \begin{cases} \left[\frac{p\phi\bar{A}}{w}(1 + i_l)\right]^{-\frac{1}{\sigma}} & \text{if } 0 < p \leq \tilde{p}_{i_l} \text{ and } 0 < i_l \leq \hat{i}_l \\ \frac{m + \tau_b M}{p} & \text{if } \tilde{p}_{i_l} < p < \hat{p} \text{ and } \hat{i}_l < i_l \\ \left(\frac{p\phi\bar{A}}{w}\right)^{-\frac{1}{\sigma}} & \text{if } \hat{p} \leq p \text{ and } \hat{i}_l < i_l \end{cases}, \tag{3.9}$$

where

$$\begin{aligned} \hat{p} &:= \hat{p}(\mathbf{s}, \mathbf{a}; p, \phi, w) = (m + \tau_b M)^{\frac{\sigma}{\sigma - 1}} \left(\frac{\phi\bar{A}}{w}\right)^{\frac{1}{\sigma - 1}}, \\ \tilde{p}_{i_l} &:= \tilde{p}_{i_l}(\mathbf{s}, \mathbf{a}; i_l, p, \phi, w) = (m + \tau_b M)^{\frac{\sigma}{\sigma - 1}} \left(\frac{\phi\bar{A}}{w}\right)^{\frac{1}{\sigma - 1}} (1 + i_l)^{\frac{1}{\sigma - 1}} \equiv \hat{p}(1 + i_l)^{-\frac{1}{\sigma}}, \\ \hat{i}_l &:= \hat{i}_l(\mathbf{s}, \mathbf{a}; p, \phi, w) = p^{\sigma - 1} \left(\frac{\phi\bar{A}}{w}\right)^{-1} (m + \tau_b M)^{-\sigma} - 1 > 0. \end{aligned}$$

The buyer's optimal demand for loans is given by

$$l(\mathbf{s},\mathbf{a};i_l,p,\phi,w) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}\left[\frac{\phi\bar{A}}{w}(1+i_l)\right]^{-\frac{1}{\sigma}} - (m+\tau_b M) & \text{if } 0 < p \leq \tilde{p}_{i_l} \text{ and } 0 < i_l \leq \hat{i}_l \\ 0 & \text{if } \tilde{p}_{i_l} < p < \hat{p} \text{ and } \hat{i}_l < i_l \\ 0 & \text{if } \hat{p} \leq p \text{ and } \hat{i}_l < i_l \end{cases} \tag{3.10}$$

If $\sigma < 1$, then we have the ordering $0 < \tilde{p}_{i_l} < \hat{p} < \infty$, and $0 < \hat{i}_l$. We note that when we calibrate the model to data later, this will be the parametric case of interest. For ease of notation, we let $l(\mathbf{s},\mathbf{a};i_l) := l(\mathbf{s},\mathbf{a};i_l,p,\phi,w)$ and $q_b(\mathbf{s},\mathbf{a};i_l) := q_b(\mathbf{s},\mathbf{a};i_l,p,\phi,w)$ denote the buyers' loans and goods demand strategy respectively from here onward.

The buyer is liquidity constrained and prefers to borrow if both relative prices of goods and market interest on loans are sufficiently low, as described by the first case in Equations (3.9) and (3.10). In the remaining cases, respectively, the buyer is and is not liquidity constrained. She does not take out a loan from the bank in both cases. Which of these cases becomes active will depend on policy and the state of the economy.

3.2.4 Depository institutions (banks)

Our focus will be on frictions in the deposit channel of banking. Therefore, we will assume that the lending side of banks is perfectly competitive.[12]

Banks post a deposit-rate (i_d) to attract depositors and they allocate deposits (d) to fund their assets. The asset side of each bank's balance sheet consists of consumer loans (l) and saving of potential excess deposits (b) at the central bank. Consumer loans earn a competitive, nominal interest rate (i_l). Idle deposits saved at the central bank earn the nominal interest rate i. The rate of return i is the central bank policy interest rate.

Each bank's expected profit is given by

$$nl[1+i_l] + b[1+i] - (1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2\eta(i_d)]d[1+i_d], \tag{3.11a}$$

where the last term is the expected deposit liability with respect to the equilibrium distribution of posted deposit rates. This takes into account the likelihood of matching with deposit

[12] We could also allow for two-sided noisy search on loans and deposits in a more general setup. A special case of this—where both borrowers and depositors only receive one interest rate quote in loans and deposits—will turn out to be a monopoly bank problem (as in, *e.g.*, Klein, 1971; Monti, 1972; Andolfatto, 2021). On the other hand, if we take the parametric limit where borrowers and depositors receive two interest rate quotes in loans and deposits, the problem reduces to Berentsen et al. (2007). Since we are not attempting to match with the empirical consumers' loan-rate markups and dispersion as addressed in Head, Kam, Ng and Pan (2021), we keep the asset side of banking operations simple for this paper.

clients who have only contacted one (*i.e.*, the present) bank or two banks. The probability of sellers contacting two banks posting the same deposit-rate i_d is captured by the term

$$\eta(i_d;\gamma) = \lim_{\epsilon\to 0} G(i_d;\gamma) - G(i_d - \epsilon;\gamma), \tag{3.11b}$$

and the profit per deposit customer is given by

$$R(i_d;\gamma) := R(i_d;\mathbf{s},\mathbf{a}) = d[i - i_d]. \tag{3.11c}$$

Each bank faces a balance-sheet and a feasible-lending constraint:

$$\underbrace{nl + b}_{\text{asset}} = \underbrace{(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2\eta(i_d)]d}_{\text{liability}}, \quad \text{and,} \quad b \geq 0. \tag{3.11d}$$

Using Equations (3.11a)-(3.11d), a bank's profit-maximization problem is:

$$\max_l nl[i_l - i] + \max_{i_d}(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2\eta(i_d;\gamma)]R(i_d;\gamma). \tag{3.12}$$

Notice that banks' lending and deposit-taking decisions are independent of each other but both of which dependss on the policy interest rate.[13]

Since banks face perfect competition in the loan market, then the lending rate would always equal the policy interest rate, $i_l = i$. Hence, a bank's problem depends only on profit from deposit-taking activity:

$$\Pi(i_d) = \max_{i_d}(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma) + \alpha_2\eta(i_d)]R(i_d;\gamma). \tag{3.13}$$

Observe from Equation (3.13), each bank faces a trade-off between setting a higher deposit rate to attract more depositors (*i.e.*, an extensive margin) and charging a lower deposit rate to earn a larger spread (*i.e.*, an intensive margin) on deposits.

3.3 Centralized market - the second sub-period

Consider the second sub-period which is the CM. This market is Walrasian and per-se frictionless. It can be interpreted as a stand-in for well-functioning markets that allow agents to all re-balance their asset positions. As in Lagos and Wright (2005), the quasilinearity in CM preferences makes the model tractable by eliminating any ex-post agent heterogeneity at the

[13]Since we have abstracted away from bankruptcy risk and the role of bank capital, we can preserve the well-known result of independence between deposit rate and loan rate along the lines of Klein (1971) and Monti (1972). Andolfatto (2021) is a recent paper that also utilizes a similar independence property. The difference is that the deposits market here features a noisy search process of Burdett and Judd (1983). One can also study the role of bank capital similar to that considered in Dermine (1986).

start of every period.

3.3.1 CM representative firm

Assume there is a perfectly competitive representative firm that has access to a Cobb-Douglas production technology F with constant return to scale. The firm solves:

$$\hat{\Pi} = \max_{K,H} F(K,H) - wH - rK \tag{3.14}$$

Profit maximization is characterized by the firms' first order conditions where capital and labor are paid at a competitive rental rate and wage rate. Respectively, these conditions are

$$r = F_K(K,H) \quad \text{and} \quad w = F_H(K,H), \tag{3.15}$$

where the total labor supplied by households in the CM is

$$H = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)] h^s(m,k;i_d) \mathrm{d}G(i_d;\gamma)}_{\text{DM seller}} + \underbrace{n[h^b(m,k;i_l)]}_{\text{DM buyer}}. \tag{3.16}$$

3.3.2 CM households

An agent with money, capital, loan or deposit balances indexed by (m,k,l,d) entering the CM may have been a buyer or a seller in the DM during the first sub-period. Given her individual state, her maximal lifetime utility is given by

$$W(m,k,l,d,\mathbf{a}) = \max_{\{x,h,m_{+1},k_{+1}\}} \left\{ U(x) - \bar{A}h + \beta V(m_{+1},k_{+1},\mathbf{a}_{+1}) \right\}, \tag{3.17}$$

subject to

$$\begin{aligned} \underbrace{x}_{\text{consumption}} + \underbrace{k_{+1} - (1-\delta)k}_{\text{net investment}} + \underbrace{\phi(m_{+1} - m)}_{\text{real money accumulation}} &= \underbrace{rk}_{\text{real rental income}} + \underbrace{wh}_{\text{real labour income}} + \underbrace{\Pi}_{\text{bank profit}} \\ &+ \underbrace{\phi(1+i_d)d}_{\text{real (gross) deposit return}} - \underbrace{\phi(1+i_l)l}_{\text{real (gross) debt repayment}}. \end{aligned} \tag{3.18}$$

where δ is the depreciation rate of capital stock.[14]

[14] If we set the CM technology parameter $\alpha = 0$, then the CM good is produced one-for-one with labor, and sold at nominal price $1/\phi$. Under this technology, both the real wage and the price of CM goods are equal to one.

Eliminate h by plugging Equation (3.18) into Equation (3.17), we can rewrite the problem as

$$
\begin{aligned}
W(m,k,l,d,\mathbf{a}) = {} & \frac{\phi\bar{A}}{w}\Big[m-(1+i_l)l+(1+i_d)d\Big]+\frac{\bar{A}}{w}\Big[(1+r-\delta)k+\Pi\Big] \\
& + \max_{x,m_{+1},k_{+1}}\left\{U(x)-\frac{\bar{A}}{w}x-\frac{\phi\bar{A}}{w}m_{+1}-\frac{\bar{A}}{w}k_{+1}+\beta V(m_{+1},k_{+1},\mathbf{a}_{+1})\right\}.
\end{aligned}
\tag{3.19}
$$

The first-order conditions with respect to choices: x, m_{+1} and k_{+1}, are respectively given by

$$
U_x(x)=\frac{\bar{A}}{w},\quad \frac{\phi\bar{A}}{w}=\beta V_m(m_{+1},k_{+1},\mathbf{a}_{+1}) \quad \text{and} \quad \frac{\bar{A}}{w}=\beta V_k(m_{+1},k_{+1},\mathbf{a}_{+1}),
\tag{3.20}
$$

where $V_m(m_{+1},k_{+1};\mathbf{a}_{+1})$ ($V_k(m_{+1},k_{+1};\mathbf{a}_{+1})$) is the marginal value of an extra unit of money balance (capital stock) taken into period $t+1$.

The envelope conditions are

$$
\begin{aligned}
W_m(m,k,l,d,\mathbf{a}) &= \frac{\phi\bar{A}}{w},\quad W_k(m,k,l,d,\mathbf{a})=\frac{\bar{A}(1+r-\delta)}{w},\\
W_l(m,k,l,d,\mathbf{a}) &= -\frac{\phi\bar{A}}{w}(1+i_l), \quad \text{and} \quad W_d(m,k,l,d,\mathbf{a})=\frac{\phi\bar{A}}{w}(1+i_d).
\end{aligned}
\tag{3.21}
$$

Note that W is linear in (m,k,l,d) and the distribution of (m_{+1},k_{+1}) exiting the CM is degenerate.

Marginal value of money. Differentiate Equation (3.5) with respect to m_+, and lagged one-period back, we obtain

$$
\begin{aligned}
V_m(m,k,\mathbf{a}) = \phi U_x(x) \Bigg[& \underbrace{\mathbb{I}_{\{i_l\leq\hat{i}\}} n\frac{u_{q_b}(q_b)}{c_{q_s}(q_s,k)}}_{\text{borrow}} + \underbrace{\mathbb{I}_{\{i_l>\hat{i}\}} n\frac{u_{\hat{q}_b}(\hat{q}_b)}{c_{\hat{q}_s}(\hat{q}_s,\hat{k})}}_{\text{no borrow}} \\
& + (1-n)\int_{\underline{i}_d}^{\bar{i}_d}\Big[\alpha_1+2\alpha_2 G(i_d;\gamma)\Big](1+i_d)\mathrm{d}G(i_d;\gamma)\Bigg].
\end{aligned}
\tag{3.22}
$$

The marginal value of money arises because carrying additional money from the end of each CM provides a return in the DM. This return can be decomposed into three terms (on the right-hand side) in Equation (3.22). The first two account for the result that money provides a liquidity premium (*i.e.*, money facilitates trade for DM consumption), whether an ex-post DM buyer decides to take out credit from banks or not. The third and last term is the return to holding money for ex-post DM sellers. A seller is (partially) insured in the sense that her idle initial money balance in the DM will earn some deposit interest. However, with imperfect competition in deposit-taking, some of that benefit will get eroded by banks.

Marginal value of capital. Similarly, differentiate Equation (3.5) with respect to k_+ and lagged one-period back, we have

$$\begin{aligned} V_k(m,k,\mathbf{a}) = U_x(x)\Bigg[& n(1+r-\delta) + (1-n)[1+r(k)+1-\delta] \\ & - (1-n)\mathbb{I}_{\{i_l \le \hat{i}\}}\frac{c_k(q_s,k)}{U_x(x)} - (1-n)\mathbb{I}_{\{i_l > \hat{i}\}}\frac{c_{\hat{k}}(\hat{q}_s,\hat{k})}{U_x(x)}\Bigg], \end{aligned} \tag{3.23}$$

where $c_k(\cdot,k) < 0$ captures the additional capital return (accruing in the DM). This additional return is due to the complementarity of capital in DM production. Having additional capital reduces the marginal cost of production in the DM for an ex-post seller.

This aspect of the DM production technology ensures that the nominal and real sides of the model are not decoupled (Aruoba et al., 2011). The term $1+r-\delta$ is a return on capital in the CM. This would be a familiar object in neoclassical growth models.

3.3.3 Previewing the mechanism

From the last two equations, we can begin to see the new aspect of this model. Specifically, the Aruoba et al. (2011) technological assumption ensures that money and capital outcomes are interdependent. Our addition of imperfect competition in banks' deposit pricing distorts the marginal value of money due to a markdown on deposit rates (*i.e.*, lower than the competitive deposit rate). Bank market power distorts the liquidity premium associated with frictional goods trades. In equilibrium, the extent to which DM goods are consumed and produced is intertwined with ex-ante decisions on how much capital investment to make. This works through a general equilibrium feedback effect via the extra DM return on capital, $c_k(\cdot,k)$. This mechanism highlights that agents' decisions on accumulating money balances and capital stock will ultimately need to consider market power in the deposit channel of monetary policy.

3.4 Stationary monetary equilibrium

In a stationary equilibrium, all nominal variables grow at a time-invariant rate according to $\phi/\phi_{+1} = M_{+1}/M = \gamma = 1+\tau$ and real variables stay constant over time. In what follows, we focus on stationary outcomes of the economy. Since money stock is growing, we multiply all nominal variables by ϕ such that the ratios of nominal variables to price level $1/\phi$ are stationary. Let $z = \phi m$ denote an individual's real money balance, and $Z = \phi M$, the aggregate real money balances. The relative price of goods across the DM and the CM is $\rho = (\phi p)$. The real value of loan and deposit, are respectively $\xi = \phi l$ and $\tilde{d} = \phi d$.

Next, we characterize the components of a stationary monetary equilibrium (SME). We focus on a class of SME with money, capital and credit. Under empirically disciplined calibration of the model, this will be the sort of equilibrium that emerges. We will provide a summary of the equilibrium description at the end.

3.4.1 The distribution of posted deposit interest rates

We can derive an analytical formula for the deposit-rate distribution $G := G(\cdot;\gamma)$. Since the derivation is a well-known result from Burdett and Judd (1983), we relegate the details to Online Appendix A.1. The idea is as follows. From each bank's profit maximization problem in Equation (3.13), there is an intensive (markdown) margin that is traded off with an extensive margin (*i.e.*, influencing the probability of contact with deposit clients). We can show that, in equilibrium, all banks earn positive expected profit (Lemma 6), the probability that two banks quote the same rate vanishes to zero and the equilibrium distribution G is continuous with a connected support (Lemmata 7 and 8). In short order, banks' extensive-versus-intensive profit margin trade-offs will induce dispersion in deposit interest rates. The following summarizes the analytical distribution function G.

Proposition 1. *Assume that the total money stock grows at a factor $1 + \tau \equiv \gamma > \beta$.*

1. *If $\alpha_1 \in (0, 1)$, there is a unique non-degenerate, posted deposit-rate distribution G. This distribution is continuous,*

$$G(i_d;\gamma) = \frac{\alpha_1}{2\alpha_2}\left[\frac{R(i_d^m;\gamma)}{R(i_d;\gamma)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{i - i_d^m}{i - i_d} - 1\right], \tag{3.24}$$

   has connected support, supp $(G) = \left[\underline{i}_d, \bar{i}_d\right]$, where $\underline{i}_d = i_d^m = 0$, $i = (\gamma - \beta)/\beta$ and $\bar{i}_d := \bar{i}_d(\gamma) = \frac{\gamma-\beta}{\beta}\left[1 - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}\right]$ pinned down by the equal-profit condition $(\alpha_1 + 2\alpha_2)\, R(\bar{i}_d;\gamma) = \alpha_1 R\left(i_d^m;\gamma\right)$.

2. *If $\alpha_2 = 1$, then G is degenerate at the central bank policy interest rate $i := i(\gamma)$ such that*

$$G(i_d;\gamma) = \begin{cases} 0 & \text{if } i_d < i \\ 1 & \text{if } i_d \geq i \end{cases}. \tag{3.25}$$

3. *If $\alpha_1 = 1$, G is degenerate at the lowest possible deposit rate (*i.e.*, the monopoly rate)*

$\underline{i}_d$ *such that*

$$G(i_d;\gamma) = \begin{cases} 0 & \textit{if } i_d < \underline{i}_d \\ 1 & \textit{if } i_d \geq \underline{i}_d \end{cases}. \tag{3.26}$$

It is worth noting that the distribution function $G(\cdot;\gamma)$ turns out to be quite simple: It does not depend on state variables other than policy γ. In the proof, one will be able to see that this is a result from ex-post depositors having a predetermined asset position (money balance) by the time they engage in noisy search for deposits. Consequently, the intensive margin of banks' deposit-taking profit is purely accounted for by the markdown of deposit rates from their opportunity cost of lending.[15]

Posted and transacted deposit-rate distributions. The associated density of the posted deposit-rate distribution $G(\cdot;\gamma)$ in Proposition 1 is $\tilde{g}(i_d) = \partial G(i_d;\gamma)/\partial i_d$. Moreover, we can express the cumulative distribution function of transacted deposit rates as

$$J(i_d;\gamma) = \alpha_1 G(i_d;\gamma) + \alpha_2 [G(i_d;\gamma)]^2 \text{ for all } i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d].$$

Its associated density is given by

$$j(i_d;\gamma) = \partial J(i_d;\gamma)/\partial i_d = \alpha_1 \tilde{g}(i_d) + 2\alpha_2 G(i_d;\gamma)\tilde{g}(i_d) = [\alpha_1 + 2\alpha_2 G(i_d;\gamma)]\tilde{g}(i_d).$$

The average posted deposit rate is defined as

$$g(\gamma) = \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G(i_d;\gamma), \tag{3.27}$$

The average transacted deposit rate is

$$\breve{g}(\gamma) := \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}J(i_d;\gamma). \tag{3.28}$$

The empirically relevant case will be the first case in Proposition 1. Assuming this case

[15] The proof for a hypothetical monopoly bank that pays zero deposit interest is summarized in Appendix A.1.2. The reason why $i_d^m = 0$ is as follows. After realizing households' preference shocks at the first sub-period, sellers cannot readjust the amount of money balances they have already carried into the DM. In other words, the amount of money balances that the households have taken into the DM is sunk. In this case, the bank can exercise its full market power to pay zero deposit interest $i_d^m = 0$ since depositors have only one contact with this particular bank. However, this monopoly rate does not rule out funds in the banking system since only a fraction of unlucky sellers obtain zero interest in their idle funds. Other sellers who have contacted two banks will receive positive deposit interest.

of a non-degenerate G, we have the following results regarding the effect of monetary policy (anticipated inflation) on distribution G and on average posted and transacted deposit rates.

Lemma 1. *Let $\alpha_1 \in (0,1)$. Consider two economies that differ in inflation, γ and γ', such that $\gamma' > \gamma > \beta$. The induced posted-deposit-rate distribution $G(\cdot;\gamma')$ first-order stochastically dominates $G(\cdot;\gamma)$.*

Lemma 2. *Assume that $\gamma > \beta$, and $\alpha_1 \in (0,1)$. An increase in the γ leads to:*

1. *An increase in both the average posted and transacted deposit interest rates; and*

2. *An increase in the upper bound of the support of the distribution G, $[\underline{i}_d, \bar{i}_d]$.*

Lemma 1 shows that an increase in inflation γ—or equivalently, the nominal policy rate $i = (\gamma - \beta)/\beta$)—shifts the deposit-rate distribution $G(\cdot;\gamma)$ downward. This result means that depositors are more likely to draw a higher deposit interest rate in the sense of first-order stochastic dominance. Similarly, Lemma 2 highlights that the average posted deposit interest rate is increasing with the anticipated inflation. The upper bound of the support of the distribution shifts to the right and becomes wider in response to higher inflation. The economic intuition is as follows. Higher inflation reduces the value of money balances. As inflation increases, households find lower benefits in carrying money into the DM. Banks need to post a higher deposit rate to attract funds from households as inflation rises. Proofs for these results can be found, respectively, in Online Appendix A.2.1 and A.2.2.

3.4.2 Money, credit and capital

If inflation is above that of the Friedman rule and is not excessively high, there will exist a monetary equilibrium featuring positive bank credit.

Lemma 3. *Let $L(q,K) := u_q(q)/c_q(\frac{n}{1-n}q, K)$ denote the (gross) liquidity premium accruing to DM buyers who carry an incremental amount of money from the CM. If $\alpha_1 \in (0,1)$, $\omega\sigma > \alpha(\omega + \sigma - 1)$, the economy is away from the Friedman rule and inflation is not too high,* i.e., *$\beta < \gamma \leq \bar{\gamma} = \beta L(q,K)$, then:*

1. *Ex-post loan demand is always positive; and*

2. *The steady-state Euler equations for money demand and capital are, respectively, characterized by*

$$\frac{\gamma - \beta}{\beta} = (1-n)\int_{\underline{i}_d}^{\bar{i}_d} \Big[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\Big] i_d dG(i_d;\gamma) + \mathbb{I}_{\{i = i_l \leq \hat{i}\}} n \left[\frac{u_q(q)}{c_q(\frac{n}{1-n}q, K)} - 1\right], \quad (3.29)$$

and,

$$\frac{1}{\beta} = [1 + F_K(K,H) - \delta] - \mathbb{I}_{\{i=i_l \leq \hat{i}\}}(1-n)\left[\frac{c_K(\frac{n}{1-n}q, K)}{U_X(X)}\right]. \tag{3.30}$$

We relegate the proof to Online Appendix A.4. The upper bound on anticipated inflation has a natural interpretation. In particular, for money to be valued, anticipated inflation must not exceed the gross return from holding money (which is the liquidity yield in terms of facilitating DM goods trade). Moreover, this condition also implies that the loan rate does not exceed agents' maximum willingness to pay for a unit of loan. Thus, by Equation (3.10), they would be willing to top up their own money holdings by borrowing from banks.

Consider Equation (3.29). The left-hand side captures the marginal cost of holding an extra unit of money balances. The right-hand side measures the expected (net) marginal benefit of accumulating an extra dollar—derived from Equation (3.22). This has two components regarding different liquidity needs. The first term is the net marginal value of depositing an extra unit of idle balances for an ex-post seller. Banks play the same insurance role as in Berentsen et al. (2007). However, here this marginal value equals an *expected* rate of deposit interest to be earned. We will later see that bank market power distorts the gains from financial intermediation relative to the case of perfectly competitive banks. The second term is the net marginal value of spending an extra dollar for an ex-post buyer. In equilibrium, this accounts for the liquidity premium from holding money as a buyer.

In Equation (3.30), the left-hand side captures the risk-free (gross) real interest rate—*i.e.*, the opportunity cost of foregoing current CM consumption. The right-hand side measures the expected net marginal value of the capital investment, which has two components arising from Equation (3.23). The first term is the return to capital that accrues in the CM production. The second term is an additional return to capital through its cost-reducing complementarity effect in DM product, as in Aruoba et al. (2011).

As highlighted earlier in Section 3.3.3, we now see that in an SME with money, credit and capital, monetary policy has a channel through an imperfectly competitive, bank-deposit channel (with deposit-interest or markdown dispersion) to influence equilibrium money and capital holdings.

3.4.3 Market clearing

Total supply of the DM good must equal total demand:

$$(1-n)q_s = nq \tag{3.31}$$

Feasible banking requires that total assets are balanced with total liabilities among banks:

$$\underbrace{nl(z,k;i)+b}_{\text{total assets}} = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z+\tau_s Z)\mathrm{d}G(i_d;\gamma)}_{\text{total liabilities}}. \tag{3.32}$$

In the CM, goods market clearing requires that:

$$F(K,H) = X + K - (1-\delta)K. \tag{3.33}$$

3.4.4 Summary: SME with money, credit and capital

Definition 1. Given monetary policy γ such that $\beta < \gamma \leq \bar{\gamma}$, a stationary monetary equilibrium (SME) with money, credit and capital is a steady-state allocation $(z^\star, Z^\star, X^\star, k^\star, K^\star, H^\star)$ in the centralized market, and allocation $(q^\star, \xi^\star)$ in the decentralized market, and prices $(i, G(i_d), \rho)$ such that

1. Households optimize: (3.20), (3.29), (3.30);
2. Firm optimizes: (3.15);
3. Banks optimize: (3.24);
4. Banking is feasible (total assets are balanced with total liabilities among banks): (3.32); and
5. Markets clear: (3.31), and (3.33), where $z^\star = Z^\star$, $k^\star = K^\star$.

4 Money, credit and capital: analytical results

We will begin this section by discussing a special limit of the model that yields the first-best allocation in Section 4.1. This is attained under the Friedman rule—*i.e.*, setting the rate of return on money to equal a risk free rate. While the Friedman rule is optimal in theory, in practice there might be other reasons (*e.g.*, unmodelled market or fiscal distortions) for monetary policy to be set above for it. Hence, in the remaining parts we will focus on equilibrium situations that are away from the Friedman rule.

Next, under mild and empirically plausible restrictions, we show the existence and uniqueness of an SME with co-existing money, credit and capital in Section 4.2. Then, we study the allocative and welfare effects of banking in Section 4.2.1. We show how endogenous bank market power distorts the deposits channel of monetary policy and how this, in turn, affects capital formation. Last, we provide some analytical insights on how bank market power responds to the change in monetary policy in Section 4.2.2. We show an imperfect pass-through of monetary policy to deposit interest rates (and associated spreads) in equilibrium. Details of the proofs can be found in Online Appendices A.4, A.5, and A.6.

4.1 Friedman rule and the first best

Consider the case of the Friedman rule at $i = 0$ or equivalently, $\gamma = \beta$. The Friedman rule attains the first-best allocation. Inflation above that of the Friedman rule induces a lower return on money than the risk-free interest rate.[16] The Friedman rule eliminates the distortion on holding idle money and this renders banks (redistributing liquidity across agents) as inessential institutions. This gives us the following insight:

Proposition 2. *If $1 + \tau \equiv \gamma = \beta$, then there is no equilibrium with deposit interest rate dispersion. Moreover, the Friedman rule attains the first-best allocation.*

Details of the proof can be found in Online Appendix A.4.3.

4.2 SME with money, banking and capital

We now restrict attention to cases in which $i > 0$ (or equivalently, $\tau > \beta - 1$). We then study how the equilibrium allocation in an economy with imperfectly competitive banks differs from that with perfect banking competition and no banks.

After some algebra—see Online Appendix A.4.2—solving for an SME can be reduced to one equation in terms of per-capita capital, $\hat{k} := K/H$, for given policy $\gamma = 1 + \tau > \beta$:

$$\frac{1}{\beta} = \underbrace{[1 + \alpha \hat{k}^{\alpha-1} - \delta]}_{=:R_{CM}(\hat{k})} + \underbrace{\tilde{\theta} \left[\hat{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}} \tilde{f}(\hat{k})}_{=:R_{DM}(\hat{k},\gamma)}, \tag{4.1a}$$

where

$$\tilde{\theta} := \frac{1}{\bar{A}} \left[(\omega - 1)(1 - n)(1 - \alpha)\left(\frac{n}{1-n}\right)^{\omega}\right] \left[\omega \left(\frac{n}{1-n}\right)^{\omega}\right]^{\frac{\omega}{1-\omega-\sigma}} > 0,$$

[16] To see this, let the gross return on money be denoted by $R^m = \phi_+/\phi = P/P_+ = 1/(1 + \pi) \equiv 1/(1 + \tau)$, where the inflation rate π equals the money growth rate τ in steady state. At the Friedman rule, it is equivalent to set $\pi \equiv \tau = \beta - 1$ (or $i = 0$). Hence, $\frac{1}{\beta} > \frac{1}{1+\pi}$, *i.e.*, $R^{m,FR} > R^m$, for any $\pi > \beta - 1$. The return on money away from the Friedman rule is inferior to that at the Friedman rule.

$$\hat{C}(\gamma) := 1 + \hat{g}(\gamma) + \frac{1}{n}\left[\underbrace{i(\gamma) - \hat{g}(\gamma)}_{=:s(\gamma)}\right], \tag{4.1b}$$

$$\hat{g}(\gamma) := \int_{\underline{i}_d}^{\bar{i}_d} \left[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\right] i_d \mathrm{d}G(i_d;\gamma),$$

and

$$\tilde{f}(\hat{k}) := \left[\frac{\bar{B}(1-\alpha)}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}\right]^{\frac{\omega\sigma}{1-\omega-\sigma}} \hat{k}^{\frac{\omega\sigma-\alpha(\omega+\sigma-1)}{1-\omega-\sigma}}. \tag{4.1c}$$

We can verify that there is a unique SME with co-existing money, credit and capital, under some regularity conditions. The conditions are easily satisfied in the empirical calibration of the model as well. A proof of this result can be found in Online Appendix A.4.4.

Proposition 3. *Assume loan contracts are perfectly enforceable. If $\omega\sigma > \alpha(\omega+\sigma-1)$ and (gross) inflation rate γ satisfies $\beta < \gamma \leq \bar{\gamma} = \beta L(q,K)$, where the $L(q,K) := q^{-\sigma}/c_q(\frac{n}{1-n}q, K)$ is the gross premium on money as a medium of exchange, then there exists a unique SME co-existing with money, capital and credit.*

4.2.1 Decomposing the mechanism

Consider the right-hand side of Equation (4.1a). We can decompose the expected return on capital investment into two components. The first term $R_{CM}(\hat{k})$ captures the return on capital associated with trades in the CM. Moreover, $R_{CM}(\hat{k})$ is independent of anticipated inflation γ since money is not necessary for facilitating trades in the CM. This part is in common with a standard growth model.

The second term $R_{DM}(\hat{k};\gamma)$ is an additional return on capital associated with trades in the DM. This is decomposable into two parts. First, there is the additional return to capital in the DM because capital is also productive in the DM. This term, originally in Aruoba et al. (2011), is captured by $\tilde{f}(\hat{k})$. Consequently, anticipated inflation γ now not only acts as a tax on goods trades in the DM but also on capital formation in the CM. This connection between monetary policy, inflation and capital formation is the insight from Aruoba et al. (2011).

Second, the DM return on capital is now augmented by the fact that consumers can access bank credit. The basic idea is banking helps economize on the cost of holding money since credit helps facilitate DM consumption. On one hand, this tends to raise the return on capital, since capital is complementary to the production of DM goods. That is, more DM consumption implies a feedback gain to the return on capital in the DM. On the other, there is a deposit-rate markdown distortion that works in opposition to that. The additional deposit channel of monetary policy in our model works through the endogenous deposit-rates distribution and markdown of deposit rates. Specifically, this channel is captured by the term $\hat{C}(\gamma)$ in Equations (4.1a) and (4.1b). The term $\hat{C}(\gamma)$ comes from the money-demand Euler equation: It measures the equilibrium gross cost of holding money. Since $\omega \geq 1$ and $\sigma > 0$ then $\omega/(1-\omega-\sigma) < 0$. Thus the term $[\hat{C}(\gamma)]^{\omega/(1-\omega-\sigma)}$ in Equation (4.1a) is a gross benefit or a boost to the DM return on capital, relative to a no-bank economy, when banking credit exists. In our model, this term contains market-power distortion in that deposit-interest-rate dispersion would imply an average deposit-interest gain that is less than the competitive, risk-free rate: $1 + \hat{g}(\gamma) < 1 + i$. Also, there is an associated policy-dependent interest rate spread on deposits, $s(\gamma)$, as defined in Equation (4.1b). This new distortion term shows up in Equation (4.1a). Further below, we will show that this distorts the return on DM capital downward relative to the same model but with perfectly competitive banks.

As a result, there will be an equilibrium incomplete pass-through of monetary policy to the accumulation of real balances, deposits and hence lending to consumers. Capital complementarity in loan-dependent goods production means that the deposit channel also has an effect on capital accumulation. We highlight this mechanism as the deposit channel of monetary policy and capital. It is helpful first to isolate two special cases: a no-bank economy and a perfectly-competitive banking economy.

An economy with perfectly competitive banks. This case is equivalent to setting $\alpha_2 = 1$ in our model. We can think of this as an Aruoba et al. (2011) model with Berentsen et al. (2007) perfectly competition banks. The deposit interest rate distribution G becomes degenerate at the policy interest rate $i = (\gamma - \beta)/\beta$ (see Proposition 1). Moreover, the spread on deposit interest rates is zero, *i.e.*, $s(\gamma) = 0$. Hence, the SME Equation (4.1a) contains the special case:

$$\frac{1}{\beta} = \underbrace{[1 + \alpha\hat{k}^{\alpha-1} - \delta]}_{=:R_{CM}(\hat{k})} + \underbrace{\tilde{\theta}\left[\tilde{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}} \tilde{f}(\hat{k})}_{=:R_{DM}^{PC}(\hat{k},\gamma)}, \tag{4.2}$$

where

$$\tilde{C}(\gamma) := 1 + \frac{\gamma - \beta}{\beta} = 1 + i.$$

Once we have the solution $\hat{k}^{PC}$ (allocation with perfectly competitive banks) from Equation (4.2), we can recover all other outcomes in the same way as in the baseline setting.

An economy without banks. This case obtains by setting $\alpha_1 = \alpha_2 = 0$. This is a version of the model in Aruoba et al. (2011). In this economy, agents earn zero interest on their idle money balances, *i.e.*, $i_d = 0$. Hence, Equation (4.1a) becomes:

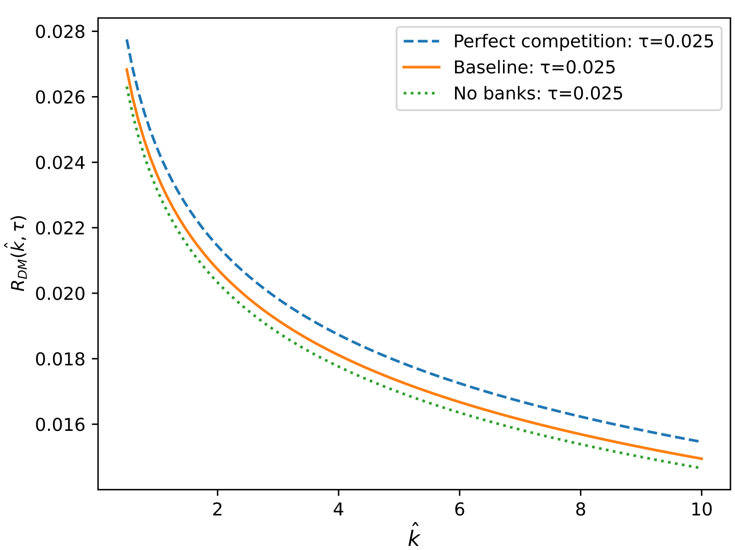
$$\frac{1}{\beta} = \underbrace{[1 + \alpha\hat{k}^{\alpha-1} - \delta]}_{=:R_{CM}(\hat{k})} + \underbrace{\tilde{\theta}\left[\breve{C}(\gamma)\right]^{\frac{\omega}{1-\omega-\sigma}}\tilde{f}(\hat{k})}_{=:R_{DM}^{no-bank}(\hat{k},\gamma)}, \tag{4.3}$$

where

$$\breve{C}(\gamma) := 1 + \frac{1}{n}\left[\frac{\gamma - \beta}{\beta}\right] = 1 + \frac{i}{n}.$$

Next, we compare our baseline economy with these two limits, respectively, an economy without banks and an economy with perfectly competitive banks. It is useful to graph the DM return on capital function $R_{DM}(\cdot, \gamma)$, for some fixed policy $\gamma = 1 + \tau > \beta$ (see Figure 4).

Figure 4: Premium on capital return in the DM



As we explained above, Figure 4 shows that banking, in general, shifts $R_{DM}(\cdot, \gamma)$ up relative to the case in the no-bank economy of Aruoba et al. (2011). This implies that financial intermediation improves consumption and capital formation relative to the no-bank economy.

However, imperfectly competitive banks distort some of the gains from banking, which ultimately reduces consumption and capital investment. As such, $R_{DM}(\cdot, \gamma)$ under imperfectly competitive banking would shift up by less than under perfectly competitive banking.

We first consider the case with perfect competition among banks and no banks, respectively, Equation (4.2) and Equation (4.3). The only difference is the gross cost of accumulating money balances. In particular, n does not appear in Equation (4.2) anymore. Recall that n measure the proportional of agents who become DM buyers (and who get to spend their money holdings). In no-bank equilibrium, agents are left to their own devices: The presence of this risk of each agent ending up with idle money (*i.e.*, $0 < n < 1$) "inflates" the net cost of carry money by a factor of $1/n$ times of that in the economy with perfectly competitive banks. That is competitive banks help insure the risk of carry idle money and thus lower the gross cost of carrying money.

This, intuitively, shows expanded consumption outcomes in the DM in the case of the economy with perfectly competitive banks, relative to its no-bank relative. Since the production of DM consumption is capital-dependent, this raises the return to capital in the DM.

Hence, perfectly-competitive banking helps increase the return on capital associated with DM trades relative to the no-bank economy, *i.e.*, $R_{DM}^{PC}(\hat{k};\gamma) > R_{DM}^{no-bank}(\hat{k};\gamma)$. This result implies that the right-hand side of Equation (4.2) must lie above Equation (4.3) for any $\hat{k}$ given policy $\gamma > \beta$. Since the left-hand side of both equations is constant with respect to $\hat{k}$, and the right-hand side is monotone decreasing in $\hat{k}$, then it follows that $\hat{k}^{\star,no-bank} < \hat{k}^{\star,BCW}$ given policy $\gamma > \beta$. In summary, banking increases the overall return on capital, resulting in a better allocation of both consumption and investment in equilibrium.

Next, we compare Equation (4.1a) and Equation (4.2). The only difference is that noisy search frictions now induce banks to charge a policy-dependent interest rate spread on deposits in equilibrium, *i.e.*, $s(\gamma)$. Hence, households, on average, benefit less from banking with the presence of endogenous market power in the deposits market. In other words, households face a higher cost of accumulating money balances, relative to the perfectly-competitive banking world. As such, bank market power reduces the return on capital associated with trades in the DM such that $R_{DM}(\hat{k};\gamma) < R_{DM}^{PC}(\hat{k};\gamma)$. Following the same logic, we can show that bank market power distorts the allocation of consumption and capital in equilibrium relative to the perfect banking competition economy.

Finally, we compare our baseline economy with the no-bank economy. Imperfectly competitive banks improve goods trades, increasing the value of the capital investment relative to the no-bank equilibrium. The reason is that households still benefit from insurance on liquidity risks through access to banks, even though banks now extract some of the depositors' surplus.

To summarize, the monetary policy transmission mechanism here works through the channel of agents' decisions on accumulating money and capital, both of which interact with bank market power on deposits. We summarize our analysis in the following Proposition. Details of the proof can be found in Appendix A.5.1.

Proposition 4. *Assume $\omega\sigma > \alpha(\omega+\sigma-1)$ and (gross) inflation rate γ satisfies $\beta < \gamma \leq \bar{\gamma} = \beta L(q, K)$, where $L(q, K) := q^{-\sigma}/c_q(\frac{n}{1-n}q, K)$. Financial intermediation improves allocation and welfare relative to a no-bank economy. The economy with perfectly competitive banks Pareto dominates the baseline economy with noisy deposits search:*

$$q^{\star,no-bank} < q^{\star} < q^{\star,PC} \quad \textit{and} \quad K^{\star,no-bank} < K^{\star} < K^{\star,PC},$$

where equilibrium allocation $(q^{\star}, K^{\star})$ approaches $(q^{\star,PC}, K^{\star,PC})$ as the baseline economy tends to its perfect-competition limit, i.e., *as $\alpha_2 \to 1$.*

4.2.2 Imperfect pass-through of monetary policy to deposit-rates

Next, we study the effects of monetary policy pass-through to the banking sector. We show that the average deposit-rates spread is increasing with respect to monetary policy (anticipated inflation).[17]

Proposition 5. *Let the average (posted) interest rate spread on deposits to be defined as*

$$s(\gamma) = i(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d dG(i_d), \tag{4.4}$$

where the distribution G is defined in Proposition 1.

Assume $\bar{\gamma} \geq \gamma > \beta$, and $\alpha_1 \in (0, 1)$. Then, the average posted deposit-rates spread is monotone increasing in inflation γ.

Proposition 5 highlights an imperfect pass-through of monetary policy to interest rate spread on deposits in equilibrium. Ultimately, bank market power affects the monetary policy transmission to the real economy. Moreover, this result highlights that banks become less competitive—they charge a higher spread on deposits—when inflation goes up. In other words, banks, on average, extract more depositors' surplus when depositors' need for liquidity insurance is high. The intuition is as follows.

First, an increase in the anticipated inflation makes it more costly for households to carry money balances across periods. Second, higher inflation induces a fall in the supply of deposits (from the $1 - n$ measure of DM sellers who have unproductive idle funds). Consequently,

[17] Alternatively, we also consider the average deposit-rates markdown as another measure of banking market power in the deposits market. Details for both the proofs of policy-varying average deposit-rates spread and markdown can be found in Appendix A.6.

banks can exploit more of their intensive-margin channel. Banks, on average, charge a higher deposit-rate spread to compensate for the losses from trading with fewer depositors with smaller money balances. Hence, higher inflation essentially gives more market power to the banks. The negative impact of bank market power on welfare is more substantial when households need more liquidity risk insurance through access to banks.

The prediction of Proposition 5 is consistent with Drechsler et al. (2017) and Choi and Rocheteau (2021). The new insight here is that monetary policy affects the degree of banking market power, which affects monetary policy transmission to goods trades and capital investment.

5 Numerical results

In this section, we first calibrate the model to macro-level data in the United States. Second, we illustrate the model mechanism discussed in Section 4 regarding the impact of monetary policy transmission on the equilibrium outcomes of the economy.

5.1 Calibration

We consider the model period at annual frequency and calibrate to annual data in the United States. The model has eleven parameters: $(\tau, i, \beta, \overline{A}, \overline{B}, \sigma, \alpha, \psi, \tilde{n}, \delta, \alpha_1)$.

External calibration. We can directly pin down a few parameters by observable statistics in the United States. By the Fisher equation, we use the long-run inflation rate τ and average effective federal funds rate i to pin down the discount factor β. In the model economy, the measure of DM sellers $\tilde{n} := 1 - n$ corresponds also to that of the depositors. We set $\tilde{n}$ to match the average share of household depositors with commercial banks per thousand adults in the United States.[18] We set δ and α to respectively match the investment-to-capital ratio and the data's share of labor income in total output.

[18]Data source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators".

Table 2: Calibration and targets

Parameter	Value	Empirical Target	Description
$1+\tau$	$(1+0.0303)$	Inflation rate[a]	Inflation rate
$1+i$	$(1+0.0492)$	Effective federal funds rate[a]	Nominal interest rate
β	0.9819	–	Discount factor, $(1+\tau)/(1+i)$
$\overline{A}$	0.9389	Labor hours	CM labor disutility scale
$\overline{B}$	0.3436	Aux reg. $(i, M/PY)^b$	CM preference scale
$\overline{C}$	1	Normalized	DM preference scale
σ	0.2125	Aux reg. $(i, M/PY)^b$	CRRA (DM q)
α	0.33	Labor income share	CM technology
ψ	0.7375	I/Y ratio	DM technology
δ	0.025	I/K ratio	Capital depreciation rate
$\tilde{n}$	0.35	household depositors[c]	Proportion of DM sellers
α_1	0.2602	Average deposit spread	Prob. $k=1$ bank contacts

[a] Annual nominal interest and inflation rates.
[b] Auxiliary statistics (data) via spline function fitted to the annual-data relation between the federal funds rate (i) and Lucas and Nicolini (2015) New-M1-to-GDP ratio (M/PY).
[c] Household depositors with commercial banks per 1000 adults for the United States.

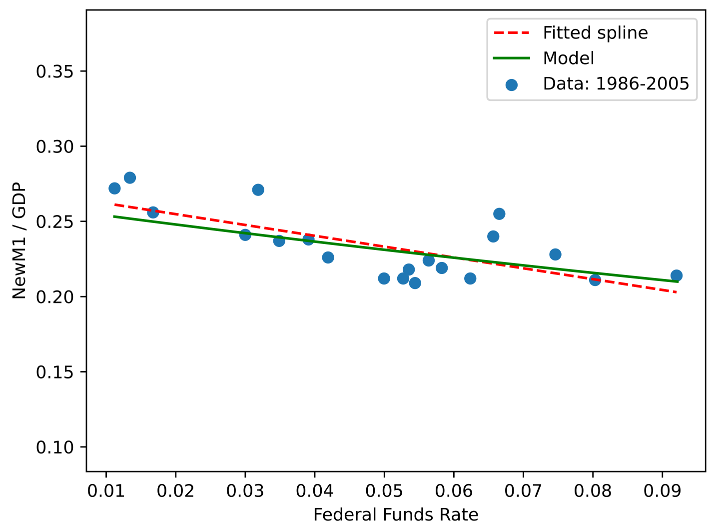
Internal calibration. The remaining parameters are $(\overline{A}, \sigma, \overline{B}, \psi, \alpha_1)$. These parameters are jointly determined to match the following targets. We set $\overline{A}$ to match average labor hours. The DM technology parameter ψ (equivalently, the DM cost parameter $\omega = 1/\psi$) is set to match the investment-to-GDP ratio. We choose the noisy search parameter α_1 to match the average deposit spread. We choose the pair $(\sigma, \overline{B})$ to match the aggregate relationship between nominal interest rate and the inverse of the velocity of money, as in Lucas and Nicolini (2015).

Table 3: Calibration results

Target	Data	Benchmark model
Labor hours	1/3	1/3
Deposit spread	1.646%	1.646%
I/Y	0.151	0.197
I/K	0.025	0.025

Parameter values and internal-calibration results are summarized in Table 2. As part of the internal calibration, we have Figure 5. The figure shows the scatterplot of the empirical money demand relation, a spline-fitted model and our calibrated model's prediction. The rest of the pointwise internal calibration results are summarized in Table 3.

Figure 5: Aggregate money demand —model and data.



5.2 Comparative steady states

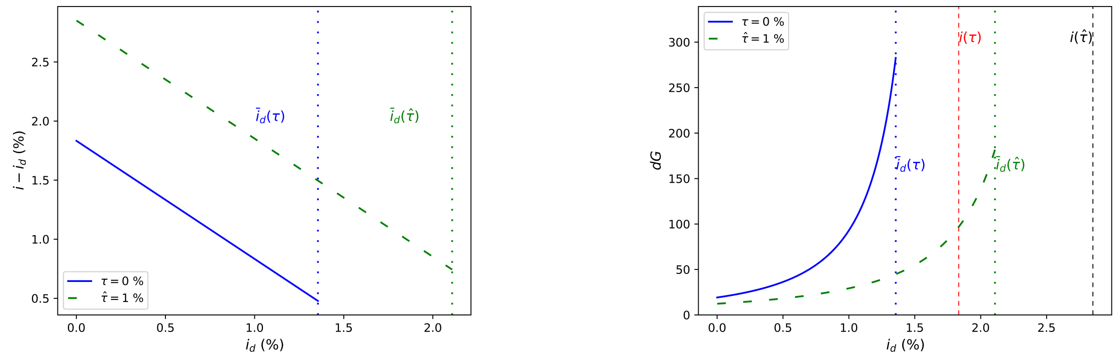
We now study SME away from the first-best ($\tau > \beta - 1$). Note that in an SME, treating money-supply growth or inflation rate τ as the policy variable is the same as interpreting i as a policy rate. We consider a set of economies each distinguished by τ and solve for each SME as a function of $\tau \in (\beta - 1, \bar{\tau}]$. We set $\bar{\tau} = 0.1$ as the exogenous upper bound. We then illustrate an equilibrium nominal-to-real link regarding the effects of monetary policy on banking market power, allocation and economic welfare.

5.2.1 Banks' trade-off in the deposits market

We first discuss a trade-off between intensive-margin (deposit rate spread) and extensive-margin (probability of matching and trading with customers) faced by banks. Figure 6 depicts ex-post deposit-rate spread $i - i_d$ per deposit and posted deposit-rate densities for inflation rates at zero and one percent. The blue and red dashed lines are, respectively, the competitive interest rates at zero percent and one percent inflation. Likewise, the blue and red dotted lines are, respectively, the upper support of the posted deposit-rate distribution G at different inflation rates.

Consider the case of fixing inflation at zero percent, *i.e.*, $\tau = 0.0$. Figure 6a highlights banks' intensive margin channel. The bank can earn a higher ex-post spread per deposit by posting a lower deposit rate i_d. However, there is also an extensive margin channel that banks need to consider. Figure 6b highlights that a bank that posts a lower i_d would suffer from fewer customers depositing funds. Banks face the same equilibrium trade-off as inflation varies. The intensive margin decreases in the posted deposit rate, and the extensive margin increases in the posted deposit rate.

Figure 6: Banks' trade-off given policy, $\gamma = 1 + \tau$.



(a) Intensive margin: (ex-post) deposit spread

(b) Extensive margin: density

Now consider the case with higher inflation, *i.e.*, it increases from $\tau = 0.0$ to $\hat{\tau} = 0.01$. The upper bound of the support of distribution G shifts to the right (denoted by the change in the blue-dotted line to the red-dotted line). This shift means that the banks have to post a higher deposit rate to attract depositor funds. However, the shift in the upper support of the distribution is smaller than the shift of interest rate in an otherwise perfectly competitive banking market, *i.e.*, $\bar{i}_d(\hat{\tau}) - \bar{i}_d(\tau) < i(\hat{\tau}) - i(\tau)$. In other words, the mass of the density becomes more concentrated at "lower" deposit interest rates relative to the competitive deposit interest rate as inflation increases. This result suggests an imperfect pass-through of monetary policy to deposit interest rates. Higher inflation gives more market power to the banks, and they exploit their intensive margin more. Banks have more market power to extract more surplus from depositors when their need for liquidity risk insurance is high. Next, we explain this intuition below.

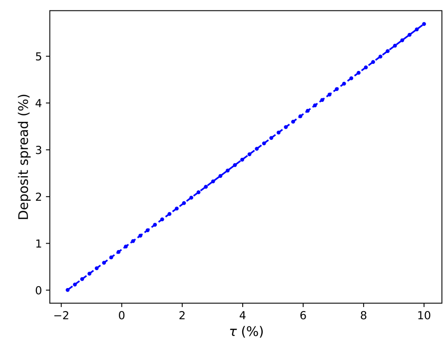
5.2.2 Pass-through of monetary policy to money, capital and banking

Recall that we define the average deposit spread by Equation (4.4). We then measure the spread dispersion by the standard deviation of deposit spread across banks. Figure 7 depicts the effects of monetary policy on bank market power, real money balance ($M/PY = Z/Y$) and capital formation (I/Y).[19] Real money balance and capital are determined by agents' decisions on money demand and capital investment. In our model, the total real output is

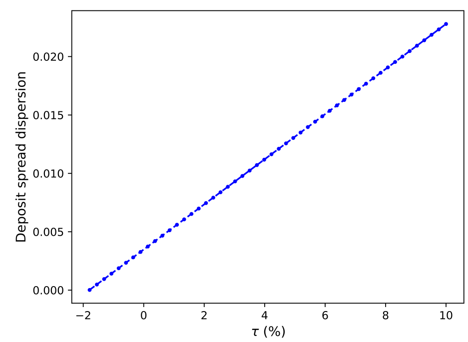
[19]The capital-to-output ratio K/Y works in the same direction as I/Y since investment I is proportional to capital K in a steady state.

$Y = \rho q + F(K, H)$ where the first term captures the total production of the DM goods (in units of CM goods) and the second term captures the total production of the CM goods.

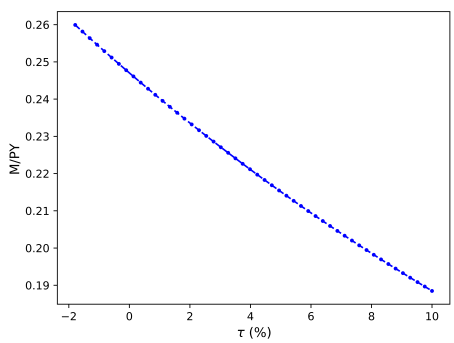
Figure 7: The effects of inflation on money, capital and banks market power for $\tau \in (\beta - 1, \bar{\tau}]$.



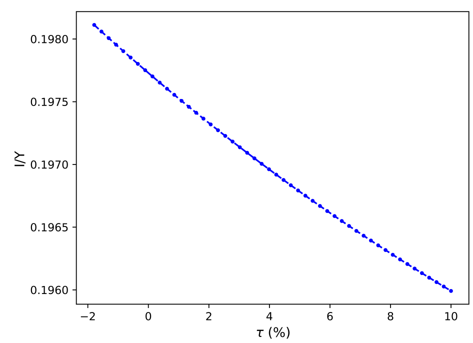
(a) Average deposit spread $s(\tau)$ and τ



(b) Dispersion of deposit spread (SD) and τ



(c) Money demand M/PY and τ



(d) Capital formation I/Y and τ

Observe from Figure 7a and Figure 7b, our model generates positive relationships between the policy interest rate and the standard deviation and the average interest spread in deposit rates. There is an imperfect pass-through of monetary policy to the banking sector, consistent with the empirical analysis in Section 2. Moreover, the results shown in Figure 7c and Figure 7d are consistent with the long-run effects of monetary policy on money and capital formation in the United States, see, *e.g.*, Lucas and Nicolini (2015) and Aruoba et al. (2011).

Our model's novelty is that the pass-through effects of monetary policy on the banking sector (and bank market power) distort agents' incentives to accumulate money balances and capital (see Proposition 4). This channel works through the policy-varying interest rate spread on deposits showing up in Equation (4.1a). The intuition is as follows.

On the one hand, higher inflation induces a higher policy interest rate. This reduces the return on money. As a result, agents carry lower money balances to the DM goods market. Hence, the supply of deposits fall since the value of liquidity trading in the DM is smaller now. Moreover, agents would need more liquidity insurance from banks to guard against the risk of having idle balances. These two effects give banks more market power to exploit their intensive margin on pricing deposits (see Proposition 5). On the other hand, capital formation falls as the policy rate increases. The reason is that the marginal value of capital falls due to a reduction in the quantity of goods traded in the frictional goods market.

This is our addition to the literature's understanding of the deposit channel of monetary policy. In our case, the channel which works through an equilibrium-determined dispersion of deposit-rate markdowns. This distribution-effect channel has empirical support.

5.2.3 Inflation and welfare effects of banking

We now illustrate the impact of monetary policy on equilibrium allocation and economic welfare. The conclusions below are extensions from the analytical insights in Proposition 4 which propose consideration of how the welfare rankings vary with monetary policy (long-run inflation).

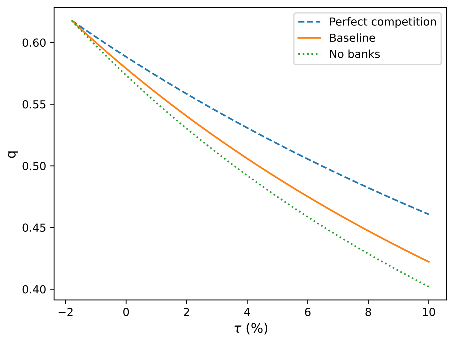
Figure 8 depicts the aggregate allocations (q, K, X, H) as inflation varies away from the Friedman rule. The blue-dashed line is the case with perfectly competitive banks (*i.e.*, $\alpha_2 = 1$). The orange-solid line is our baseline model with imperfect competition among banks (*i.e.*, $\alpha_2 = 0.7398$). The green-dotted line is the case without banks ($\alpha_1 = \alpha_2 = 0$, which resembles the economy in Aruoba et al. (2011)).

Higher inflation induces a lower allocation of q in the DM since inflation acts as a tax on monetary trades. Since capital stock is a productive input in both the DM and the CM, agents' decisions on investing capital will also depend on the DM goods trades. As such, a smaller quantity of goods traded in the DM reduces the value of the capital investment. The reason is that there are smaller gains from investing an extra unit of capital in reducing the marginal cost of DM production. The nominal-to-real link regarding the effects of monetary policy transmission is identical to that in Aruoba et al. (2011).

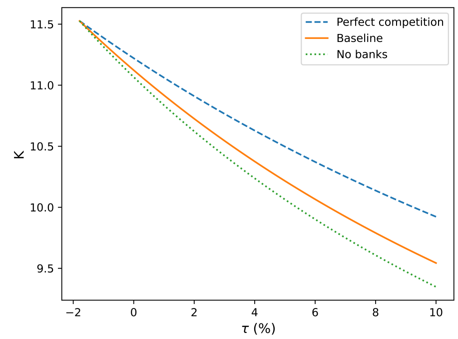
The new insight in our model is the pass-through effects of monetary policy on the banking sector. As a result, bank market power affects the transmission to the real economy in equilibrium (see Proposition 4). The intuition is as follows.

Banking helps households insure against their liquidity risks of having idle balances increasing the marginal value of a dollar. As such, agents have a lower cost of holding money, improving allocation in DM goods trades. Consequently, the value of the capital investment is higher than the no-bank equilibrium. Moreover, the marginal productivity of labor is higher when there is more capital, which increases agents' consumption and incentive to work in the CM. That is why the blue-dashed line lies above the green-dotted line in Figure 8. However, bank market power on deposits essentially extracts surplus from households, which reduces the marginal value of money. Hence, imperfect competition among banks distorts monetary

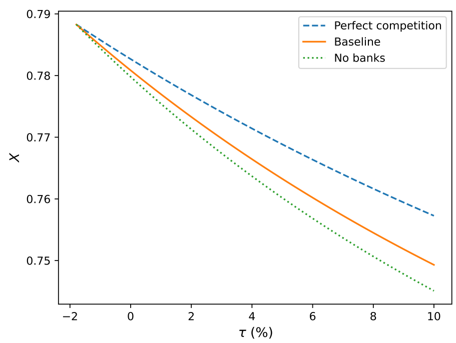
Figure 8: Equilibrium allocation for $\tau \in (\beta - 1, \bar{\tau}]$.



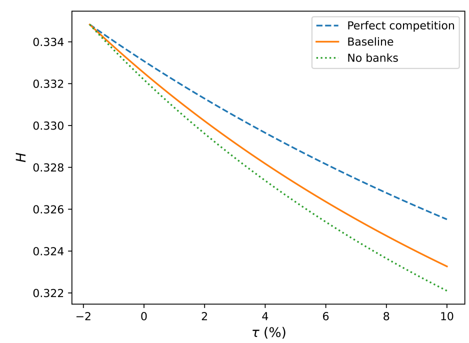
(a) DM consumption q



(b) Capital stock K



(c) CM consumption X



(d) Labor H

trades, which negatively affects the capital stock, consumption and labor in the CM. That is why the orange-solid line (our baseline model economy) falls below the one with perfectly competitive banks.

Welfare effects of banking and inflation. We use the *ex-ante* lifetime utility of homogeneous households as a welfare criterion for a given monetary policy τ:

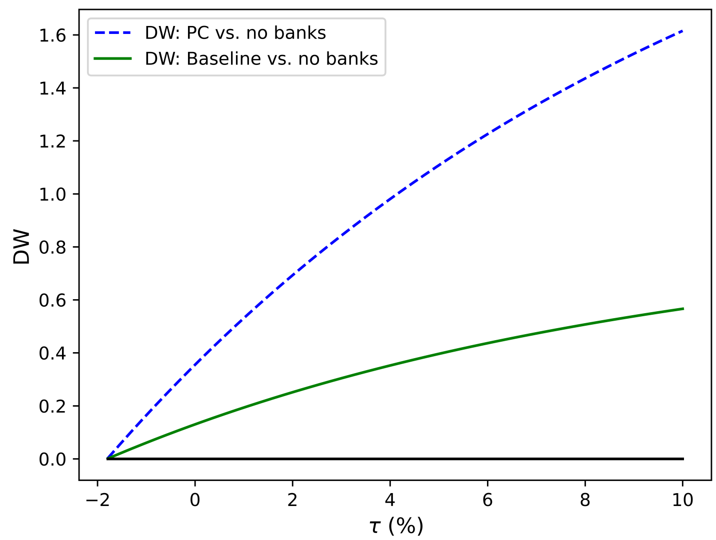
$$W(\tau) = \frac{1}{1-\beta}\Bigg[\underbrace{nu[q(K)] - (1-n)c\bigg(\frac{n}{1-n}q(K), K\bigg)}_{[A]} + \underbrace{U[X(K)] - \bar{A}H(K)}_{[B]}\Bigg], \tag{5.1}$$

where term $[A]$ and term $[B]$, respectively, capture the utility flows derived from the DM and the CM.

Equilibrium for a perfectly competitive banking economy and a no-bank economy are, respectively, pinned down by Equations (4.2) and (4.3). Then we can back out the corresponding equilibrium allocations similar to the baseline model economy. The procedure for calculating welfare for a no-bank equilibrium and a perfectly competitive banking equilibrium is similar to Equation (5.1).

The blue-dashed line is the difference in welfare between an economy with perfectly competitive banks and an economy without banks in Figure 9. Likewise, the green-solid line is the difference in welfare between our baseline (noisy search for deposits) and an economy without banks.

Figure 9: Welfare effects of banking: With banks versus without banks



We first observe little gains from financial intermediation near the Friedman rule. The reason is that the gain from banking to insure liquidity risks is small when it is not so costly for agents to carry money balances. As inflation increases, there are more gains from banking to reduce the cost of agents stuck with idle money balances.[20] Overall, the liquidity insurance role of banks can help improve frictional goods trades by encouraging more capital formation in equilibrium. This is a consequence of the deposit channel of monetary policy and capital in the model. The intuition is as follows.

Banking increases the return on capital associated with trading in the frictional goods market (see Proposition 4). The additional gain from capital investment increases the liquidity premium associated with frictional goods trades due to the general equilibrium feedback effect. In particular, more capital stock leads to a lower marginal cost of production, which

[20] We have also considered a hyperinflationary regime for robustness checks, *e.g.*, we set $\bar{\tau} = 3$. In this case, the gains from banking DW will increase, decrease, and eventually approach zero as inflation increases. The reason is that there are little gains from banks in redistributing liquidity when the value of money goes to zero in a hyperinflationary economy. This non-monotone welfare effect of banking in response to inflation is consistent with the insights from Berentsen et al. (2007).

reduces the price of goods in the frictional goods market at equilibrium. Thus, financial intermediation improves allocation and welfare relative to a no-bank economy. However, banks with market power can distort the deposit channel of monetary policy, affecting capital formation. The reason is as follows.

In our baseline model economy, there is an added dimension of friction featuring a noisy search process of Burdett and Judd (1983) in the deposits market. As a result, deposit interest rate dispersion and the associated policy-dependent interest rate spread on deposits occur at equilibrium. The reason is that there is a positive probability that depositors only receive one deposit interest rate quote from banks. Consequently, banks can exploit market power to offer a lower deposit interest rate than the competitive interest rate. As such, positive economic profits earned by banks reduce the surplus of households participating in the banking system. Hence, banking market power reduces the value of money, which distorts goods trades and capital formation more than the economy with perfect competition among banks.

Moreover, as discussed earlier, banks tend to charge a higher interest rate spread on deposits as inflation increases (see Proposition 5). Thus, agents benefit less from banking when they need more liquidity risks insurance. That is why the green-solid line lies below the blue-dashed line. The result here suggests that one ought to have a more "competitive" banking market along the deposit channel of monetary and capital formation.

6 Long-run growth consequences

We have established a novel deposit channel running from monetary policy to *dispersion in deposit-rate markdowns*. This translates to policy's imperfect pass-through to investment or capital accumulation. It is natural to then ask what this implies for long-run growth.

To answer this question, consider the same model augmented with exogenous growth in total factor productivity. The exercise here is similar in spirit to Waller (2011). The difference here is in the implications of the imperfect monetary-policy pass-through on banking and on growth. By our extension of Waller (2011), we show how banking in general and bank market power affect the real economy along the balanced growth path.

The basic structure of the model remains the same as in Section 3. We only lay out the new features here to avoid repetition.[21] The difference here is that the aggregate production function in the centralized market is given by $Y = F(K, AH) = K^{\alpha}(AH)^{1-\alpha}$ where $\alpha < 1$, and A is a labor-augmenting technology factor. Now, A evolves according to the process $A_{+} = (1 + \mu)A$. In the decentralized market, output is given by $q^s = f(k, Ae) = k^{\psi}(Ae)^{1-\psi}$ where $\psi < 1$. Sellers produce q^s using capital k and effort e. The disutility cost of production

[21] For more details, please see Appendix B.

for the sellers can also be expressed as $c(\frac{q^s}{A}, \frac{k}{A}) = (\frac{q^s}{A})^{\omega}(\frac{k}{A})^{1-\omega}$, where $\omega := 1/\psi > 1$.

6.1 Equilibrium

Again, we restrict attention to equilibrium featuring money, credit and capital. As such, we need to rely on mild restrictions like those discussed in Lemma 3. In order to ensure there is a balanced growth path, we now consider logarithmic utility functions in both the DM and CM similar to Waller (2011). The only restriction here is to have a positive nominal policy interest rate that is not too high. This is because the restriction on parameter $\omega\sigma > \alpha(\omega + \sigma - 1)$ reduces to $\alpha < 1$. This is satisfied automatically given the CM production technology that we consider. At equilibrium, the system of equations consists of:

1. Money demand Euler equation:

$$\begin{aligned}\frac{\phi U_x(x)}{\phi_+ U_x(x_+)} = \beta \Bigg\{ & (1-n) \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G_+(i_d)](1+i_d)\mathrm{d}G_+(i_d) \\ & + n \left[\frac{u^{'}(q_+)}{c_q\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}} \right] \Bigg\},\end{aligned} \tag{6.1}$$

   where the DM goods market clearing condition $(1-n)q^s = nq$ is imposed.

2. Capital investment Euler equation:

$$U_x(x) = \beta U_x(x_+)\left[1 + F_K(K_+, A_+H_+) - \frac{1}{U_x(x_+)}(1-n)c_k\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}\right]. \tag{6.2}$$

3. CM consumption-labor trade off:

$$U_x(X) = \frac{1}{F_H(K, AH)A}. \tag{6.3}$$

4. CM goods market clearing condition:

$$F(K, AH) = X + K_+ - (1-\delta)K. \tag{6.4}$$

An equilibrium with money, credit and capital satisfies Equations (6.1)—(6.4) given initial capital stock K_0 and money stock M_0.

6.2 Balanced growth

Following from Waller (2011), we also assume constant labor hours, and the real variables $(q, X, K_+, \phi M)$ all grow at the rate of $1+\mu$. The growth of real money stock along the balanced growth path satisfies

$$\underbrace{1+\tau}_{\text{gross growth of money stock}} = \underbrace{(1+\pi)}_{\text{gross inflation rate}} (1+\mu), \tag{6.5}$$

and the nominal policy interest rate i satisfies

$$1+i = \frac{(1+\pi)(1+\mu)}{\beta}, \tag{6.6}$$

where $1/\beta$ is the gross risk-free real interest rate.[22]

Let $1+\varphi := (1+\pi)(1+\mu)$. We assume the deposit rate distribution is adjusted for growth such that $G_+([1+\varphi]i_d) = G(i_d)$. Let $\hat{K} := K/AH$ denote the capital-to-effective-labor ratio.

The system of equations can be reduced to one equation in terms of $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1+\alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K}), \tag{6.7}$$

where

$$\bar{\theta} := n(1-\alpha)\left(\frac{\omega-1}{\omega}\right), \quad \omega > 1 \quad \text{and } 0 < n < 1,$$

$$\tilde{C}(i) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \frac{1}{n}\left[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)}_{\text{average interest rate spread on deposits}}\right],$$

and,

$$\tilde{f}(\hat{K}) := \hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}, \quad 0 < \alpha < 1.$$

The left-hand side of Equation (6.7) captures the (gross) risk-free real interest rate adjusted for growth. The right-hand side of Equation (6.7) captures the (gross) return on capital which can decompose into two components relating to trades in the CM and the DM.

[22]Note: As before, banks face perfect competition in the loan market but not in the deposit market. Hence, the zero-profit condition on loans induces the lending rate to equal the policy interest rate $i^l = i$.

Since the left-hand side is constant with respect to $\hat{K}$, and the right-hand side is monotone decreasing in $\hat{K}$, there exists a unique $\hat{K}^\star$ given policy i. Once we solve for $\hat{K}^\star$, we can use the following equations to back out the other endogenous variables $\{q, X, K, H\}$, *i.e.*, DM consumption, CM consumption, capital and labor along the balanced growth path.

$$X = (1-\alpha)\hat{K}^\alpha A,$$

$$K = \frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}A,$$

$$H = \frac{1-\alpha}{1-(\delta+\mu)\hat{K}^{1-\alpha}},$$

and

$$q = A\left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\tilde{C}(i)\right]^{-\frac{1}{\omega}}\left[\frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}\right]^{\frac{\omega-1}{\omega}}.$$

6.2.1 Discussion

Once more, we restrict attention to cases away from the Friedman rule, $i > 0$.[23] We then show how banking market power (along the deposits channel of monetary policy and capital) can distort the balanced growth path of the real economy. It is helpful first to consider three special cases: the case with perfect competition among banks; the case without banks; and the neoclassical economy. Details of the characterization can be found in Appendix B.1.2.

An economy with perfect competition among banks. Similar to our decomposition earlier in Section 4.2.1, this case is equivalent to setting $\alpha_2 = 1$. The deposit-rate distribution is degenerate at the policy interest rate i and the interest rate spread on deposits is zero. As

[23] Similar to Waller (2011), the Friedman rule achieves the first-best allocation when the policy interest rate is equal to zero, $i = 0$. We sketch the idea of the proof as follows. The only distortion is associated with the inflation tax on holding money, *i.e.*, a cost on liquidity. As such, inflation (away from the Friedman rule) induces the return on money to lower than the time discount rate adjusted for growth along the balanced growth path. To see this, let the gross return on money be denoted by $R^m \equiv \phi_+/\phi = P/P_+ = 1/(1+\pi)$. Use this in Equation (6.6), it follows that $\frac{1+\mu}{\beta} > \frac{1+\mu}{\beta(1+i)}$, *i.e.*, $R^{m,FR} > R^m$, for any $i > 0$. Setting the nominal policy rate to zero $i = 0$ coincides with efficient trades in the DM such that $u_q(q) = c_q(q,k)$ holds at equilibrium. Thus, the Friedman rule eliminates the distortion associated with inflation acting as a cost on liquidity along the balanced growth path. Moreover, banks generate no additional gain in intermediating liquidity across agents at the Friedman rule.

such, Equation (6.7) becomes:

$$\begin{aligned}\frac{1+\mu}{\beta} &= (1+\alpha\hat{K}^{\alpha-1}-\delta) \\ &\quad + \left[n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\right]\left[1+i\right]^{-1}\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right],\end{aligned} \tag{6.8}$$

where $\omega > 1$, $0 < n < 1$ and $0 < \alpha < 1$.

An economy without banks. To obtain this case, we set $\alpha_1 = \alpha_2 = 0$. Equation (6.7) becomes:

$$\begin{aligned}\frac{1+\mu}{\beta} &= (1+\alpha\hat{K}^{\alpha-1}-\delta) \\ &\quad + \left[n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\right]\left[1+\frac{i}{n}\right]^{-1}\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right].\end{aligned} \tag{6.9}$$

Neoclassical growth model. This arises by setting $n = 0$ or $\omega = 1$. This restriction means that we shut down the DM goods trades or the additional benefit of capital associated with DM goods trades. In particular, money plays no role in such an economy. The capital-per-effective-labor ratio is then given by

$$\frac{1+\mu}{\beta} = 1+\alpha\hat{K}^{\alpha-1}-\delta. \tag{6.10}$$

First, we compare the two monetary economies: one with access to perfectly competitive banks and one without banking. On the left of either Equation (6.8) or Equation (6.9), the (gross) risk-free real interest rate adjusted for growth is identical in both economies. However, we can deduce that the right-hand-side of Equation (6.8) is higher than the right-hand-side of Equation (6.9) for any policy $i > 0$. Hence, banking improves the capital-to-effective-labor ratio $\hat{K}$ along the balanced growth path relative to the economy without banks.

The intuition is as follows. Distortion in a monetary economy occurs due to the inflation tax inducing a lower return on money than the time rate of discount adjusted for growth. With perfectly competitive banks, the equilibrium deposit rate must equal the lending rate, $i_d = i_l = i$. Similar to Berentsen et al. (2007), the gains from banking come from paying interest to depositors (*i.e.*, the DM sellers) who have idle money balances. Essentially, banking helps reduce the cost of agents stuck with unneeded liquidity when it is costly to hold money. Banks, in intermediating liquidity that resolve heterogeneous liquidity needs among buyers and sellers, help to improve the DM goods trades. As a result, the capital-to-effective-labor ratio in an economy with perfect competition among banks is higher than the economy without banks.

Comparing Equation (6.7) and Equation (6.8), we can also deduce that banks with market power distort the capital-to-effective-labor ratio along the balanced growth path more than that with perfect competition among banks. The reason is as follows. In our baseline model, there is a positive probability that agents only get one deposit interest rate quote which gives the market power to banks to offer a lower deposit interest rate than the competitive interest rate. Consequently, deposit interest rate dispersion and an associated policy-dependent interest rate spread on deposits show up in the equilibrium condition as captured in Equation (6.7). Agents (on average) benefit less from banking with the presence of market power at equilibrium. Overall, bank market power distorts the deposits channel of monetary policy. In turn, this lowers the capital-to-effective-labor ratio along the balanced growth path of the economy with deposit market power relative to that with perfectly competitive banks.

Consider Equations (6.7) and (6.10). The capital-per-effective-labor ratio in a monetary economy (with or without banks) is always higher than that in a neoclassical growth model. The reason is that capital has an additional value from reducing the cost of DM production. This insight is consistent with Waller (2011). The difference in our model is now, there is an imperfect pass-through effect of monetary policy working through deposit markdowns dispersion. As we showed earlier, this distorts the capital-to-effective-labor ratio downwards, along the balanced growth path.

6.3 Dynamics

To obtain an analytical understanding of how monetary policy affects growth dynamics, we also set $\delta = 1$ and focus on constant hours along the transition path. This special-case permits analytical insights in the same way as in Waller (2011). Full details of the characterization can be found in Appendix B.1.3. The corresponding growth rate of $\hat{K}_+$ in our baseline model is given by

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{6.11}$$

where

$$\tilde{C}(i) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \frac{1}{n}\Bigg[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average interest rate spread on deposits}}\Bigg].$$

Once again, it is helpful to consider three special cases: the case with perfect competition among banks, the case without banks and the neoclassical economy.

Neoclassical growth model. In this case, the growth rate of capital in such an economy is given by

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\alpha\beta\hat{K}^{\alpha-1}\right]. \tag{6.12}$$

Perfect competition among banks. In this case, the growth rate of capital is given by

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}{1+n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{6.13}$$

An economy without banks. The growth rate of capital is given by

$$\frac{\hat{K}_+}{\hat{K}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}{1+n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{6.14}$$

6.3.1 Discussion

From Equations (6.11) through to (6.14), we can deduce the following: First, monetary policy does not affect capital accumulation in a neoclassical growth model. The reason is that money plays no role in such an economy. Consequently, nominal outcomes are decoupled from its neoclassical real sector with capital accumulation.

Second, capital accumulation in a monetary economy (where money is essential in equilibrium) grows faster than in the neoclassical growth model. This result is due to the additional premium on the investment associated with the DM goods trades.

Third, banking liquidity transformation generally improves capital growth rate relative to the monetary economy without banks. The reason is that access to banks can help households to reduce the cost of accumulating money balances. This improves DM goods trades. As a result, there is a higher return on capital investment since capital is also productive in the DM.

Last, monetary policy has an imperfect pass-through effect on the banking sector due to a distribution of interest rate spreads on deposits. Hence, banking market power removes some of the benefits of financial intermediation, reducing the capital growth rate compared to when there are perfectly competitive banks.

7 Central bank digital currency and long-run growth

Section 6 highlights how informational frictions in the private banking deposit market can contribute to lower capital accumulation in a monetary economy. It is then natural to ask under what policy tool can the central bank restore competitive banking equilibrium allocations.

To answer this question, we consider having an interest-bearing CBDC made available to the public along the lines of Andolfatto (2021).[24] For our paper, we focus solely on the impact of CBDC on the effectiveness of banking liquidity transformation via the lens of deposit pricing.[25]

In an economy with noisy deposits search, the central bank now has two separate policy tools. One that targets the trend inflation γ (or equivalently, the nominal policy rate i) and one that controls the interest rate on CBDC, i^{CBDC}. Both i and i^{CBDC} are exogenous parameters. The nominal policy rate i and the CBDC rate i^{CBDC} can differ.

The basic structure of the model remains the same as we have discussed thus far. The only differences are the (ex-post) sellers' problem and (private) banks' profit-maximization problem in the DM. We highlight the implications of these differences as follows.

The presence of CBDC as an alternative depository facility competes with the deposits-taking by private banks. Specifically, the measure of $(1-n)$ sellers with idle money balances can now choose to deposit with the private or central bank (or both). Since sellers can voluntarily hold bank deposits and CBDC, they optimally deposit their idle funds with the one that offers a higher interest rate. Given the sellers' optimal deposit schedule, the equilibrium distribution of deposit rates (and associated markdowns) now depends on both policy rate i and the CBDC rate i^{CBDC}.

Introducing CBDC as an outside option can discipline bank market power in the deposit market.[26] We summarize our main result regarding the effects of CBDC on capital accumulation in the following Proposition. Proof of this result is in Appendix C.4.

Proposition 6. *Assume $0 < i^{CBDC} \leq i$ and $\alpha_1 \in (0,1)$. In an economy with noisy deposits search, interest-bearing CBDC as an alternative depository facility improves capital growth rate than the economy without CBDC:*

$$g_k < g_k^{CBDC} \leq g_k^{PC},$$

where g_k^{CBDC} approaches to g_k^{PC} as the CBDC rate tends to the policy rate, i.e., $i^{CBDC} \to i$.

[24]For more details, please see Appendix C.

[25]While the main focus of this paper is studying the connection between the role of banking liquidity transformation and capital formation in a monetary economy, our discussion on the effects of CBDC on capital accumulation contributes to a growing literature on CBDC and its policy implications, (see, *e.g.*, Engert and Fung, 2017; Chapman and Wilkins, 2019; Chiu et al., 2019; Jiang and Zhu, 2021; Fernández-Villaverde, Sanches, Schilling and Uhlig, 2021; Wang and Rahman, 2022; Dong and Xiao, 2022; Keister and Sanches, 2022).

[26]In Appendix C.3, we show how interest-bearing CBDC affects the distribution of deposit rates (and associated markdowns).

The economic intuition of Proposition 6 is as follows. In an economy with noisy deposits search, interest-bearing CBDC (as an outside option for depositors) promotes more competition among private banks on deposits-taking activity. Effectively, a higher CBDC rate limits the private banks' incentive to exploit their intensive margin (i.e., markdown on deposit rates). The reason is that private banks must at least match their depositors' outside offer to maintain a depositors base to fund its assets. Consequently, private banks compete more along the extensive margin (i.e., posting higher deposit rates). This mechanism works through the effect of the CBDC rate that pushes up the support of the distribution of deposit rates posted by private banks. Hence, an overall reduction in the average interest spread on deposits charged by private banks. As a result, households accumulate more money balances to trade in the frictional goods market, improving capital accumulation more than the economy without CBDC.

When the CBDC rate equals the policy rate, the distribution of deposit rates offered by private banks degenerates at $i^{CBDC} = i = i_d$. Hence, the introduction of CBDC can restore competitive banking equilibrium allocations.

In summary, the prediction of Proposition 6 complements the work done by Chiu et al. (2019) and Andolfatto (2021). We focus on how CBDC may affect long-run economic growth in a monetary economy featuring imperfectly competitive banks. We highlight that having an interest-bearing CBDC in such an economy can induce more capital accumulation. Furthermore, our mechanism is also related to the insight of *latent medium of exchange* studied in Lagos and Zhang (2021). They show that even if the use of money in the frictional goods market goes to zero, monetary policy that governs the value of the outside option can still be effective in disciplining equilibrium allocations. Here, even if depositors choose to deposit all their idle funds with the private banks, the central bank can still use the CBDC rate as an extra policy tool to discipline the distribution of deposit-rate markdowns in equilibrium. Effectively, CBDC helps to remove (private) bank market power that arises from informational frictions. Consequently, it restores the effectiveness of banking liquidity transformation, improving capital accumulation in equilibrium.

8 Conclusion

We have constructed a microfounded model of money, capital and imperfectly competitive banking. Our model sheds new light on the deposit channel of monetary policy recently popularized by Drechsler et al. (2017). Specifically, our version of the channel is one that flows through policy-dependent deposits markdown dispersion to affect money and capital demand.

Our model generates a positive relationship between the interest rate spread on deposits and the Federal Funds Rate, which is consistent with the new evidence documented in Drechsler et al. (2017). Moreover, our new dispersion channel also has empirical support. Our model generates a positive relationship between the standard deviation and average interest rate spread on deposits, consistent with empirical evidence using the bank–branch level data in the United States.

In addition, the resulting imperfect pass through of monetary policy to investment has consequences for the long-run growth path of the economy. We find that when informational frictions distort the gains from banking liquidity transformation, having interest-bearing CBDC as an alternative depository facility can help restore competitive banking equilibrium allocations.

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A Omitted proofs

In this online appendix, we provide the intermediate results and proofs for the main results of the model. We lay out the structure of this online appendix as follows.

First, we provide the details for characterizing the posted deposit-rate cumulative distribution function $G(i_d;\gamma)$ in Section A.1. We also provide a discussion on the transacted deposit-rate cumulative distribution function $J(i_d;\gamma)$ afterwards.

Second, we characterize both the steady-state Euler equations for the money demand and the investment demand in Section A.3.

Third, given mild restrictions on the model, we provide intermediate results and proofs for the existence and uniqueness of a stationary monetary equilibrium (SME) co-existing with money, capital and credit in Section A.4.

Fourth, we also provide a proof for the first best allocation result in Section A.4.3. In such a regime, banks generate no additional gains in redistributing liquidity among households.

Fifth, we study how the allocation in an economy with imperfectly competitive banks differs from that with perfect banking competition and no-banks in Section A.5. We then prove that allocation and welfare in a banking equilibrium (with and without market power) always dominate more than that in a no-bank equilibrium. However, allocation and welfare in an imperfectly competitive banking equilibrium are always lower than the equilibrium with perfectly competitive banks.

Sixth, we define measures of deposit-side bank market power by the average deposit-to-policy-rate spreads and the deposit-rate markdown. We then ask how the degree of banking market power on deposits responds to changes in monetary policy. The intermediate results and proofs pertaining to this question are contained in Section A.6.

Seventh, we consider an exogenous growth version of the model for analyses similar to those considered in Waller (2011). We lay out the characterizations of various cases of this model in Section B.

Last, we consider having an interest-bearing central bank digital currency (CBDC) along the lines of Andolfatto (2021). We then study the effects of CBDC on capital accumulation in Section C.

A.1 Deposit interest rate distribution

In this section, we provide the intermediate results and proofs from Section A.1.1 to Section A.1.5. Then we incorporate these results into the characterization of an analytical formula for the equilibrium posted-deposit-rate distribution G. The derived formula G is summarized in Section A.1.6.

A.1.1 Positive monopoly bank profit from deposits

Lemma 4. *$\Pi^m(i_d) > 0$ for any $i - i_d > 0$.*

Proof. Consider a bank's ex-ante problem defined in Equations (3.11a) and (3.11d). Hypothetically, a monopolist bank's profit can be derived as

$$\Pi^m(i_d) = \underbrace{nR^l(i_l)}_{\text{profit from loans}} + \underbrace{(1-n)\alpha_1 R(i_d)}_{\text{profit from deposits}} = n\underbrace{l[i_l - i]}_{=:\pi_l} + (1-n)\alpha_1 \underbrace{d[i - i_d]}_{=:\pi_d},$$

where l is the amount of loans, d is the amount of (inelastic) deposits, i_l is the loan interest rate, i is the central bank policy interest rate and i_d is the deposit interest rate.

First, consider the profit from loans. The interest rate that the lending bank would have earned by investing funds (with the central bank) is the policy rate i. Since banks are perfectly competitive on the loan side then the equilibrium loan interest rate must equal the opportunity cost of lending, $i_l = i$. It follows that the bank profit from making loans is zero in equilibrium, *i.e.*, $\pi_l = 0$.

We now consider the profit from deposits. Since the bank has market power in the deposits market, it can charge an interest spread on deposits. For any positive spread (or markdown) on deposit, *i.e.*, $0 < i - i_d$, then $\pi_d > 0$ and therefore $\Pi^m(i_d) > 0$. □

Remark. An opportunity for the bank to invest (idle) funds with the central bank at a competitive rate is a convenient modelling choice. This modelling strategy allows us to separate the bank lending and deposit-taking decision independent of each other, but both depend on the policy interest rate. Given the assumption that banks face perfect competition in the loans market, we drop the term π_l for the ease of notation when expressing banks' profit from here onward.

In what follows, we will denote $G(\cdot, \gamma)$ by $G(\cdot)$ or just G.

A.1.2 Monopoly deposit rate

Lemma 5. *The lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$.*

Proof. Consider a hypothetical bank serving depositors who have contacted only this one bank. The bank chooses deposit rate i_d to maximize profit, and the first-order condition is

$$\frac{\partial \Pi^m(i_d)}{\partial i_d} = (1-n)\alpha_1 \left[(i(\gamma) - i_d)\frac{\partial d^\star}{\partial i_d} - d^\star \right] = 0 \implies i_d = \mathcal{M}(i_d) i(\gamma), \tag{A.1}$$

where

$$\mathcal{M}(i_d) = \frac{\epsilon(i_d)}{1 + \epsilon(i_d)} \text{ and } \epsilon(i_d) = \frac{i_d}{d^\star}\frac{\partial d^\star}{\partial i_d}.$$

We can think of $\epsilon(i_d)$ in Equation (A.1) as the elasticity of deposit supply, and $\mathcal{M}(i_d)$ captures the markdown for monopoly pricing on deposits. From here, we can see that the monopoly deposit interest rate is proportional to the policy rate depending on $\mathcal{M}(i_d)$. However, the term $\partial d^\star/\partial i_d = 0$ implies $\epsilon(i_d) = 0$ and therefore $i_d^m = 0$. Since the nominal deposit rate cannot go negative, it follows that the lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$. □

Remark. The reason why a hypothetical monopoly bank pays zero deposit interest is as follows. After realizing the households' preference shocks for consumption and production, sellers cannot readjust the amount of money balances they have already carried into the DM. In other words, the households' decision to bring money balances into the DM is sunk because the decision has already been made in the previous CM. Since the bank matches with depositors who have contacted only this one particular bank, the bank acts as a monopoly bank. Consequently, the bank can exercise its full market power to pay no interest on deposits $i_d^m = 0$. The zero–monopoly interest rate does not induce a zero–deposit supply in the banking system. Only a fraction of sellers who happen to be unlucky obtain zero interest on their idle funds.

A.1.3 All banks earn positive expected profit

Lemma 6. $\Pi^\star > 0$.

Proof. The expected profit from posting a deposit interest rate i_d is given by

$$\begin{aligned}\Pi(i_d) &= (1-n)[\alpha_1 + 2\alpha_2 - 2\alpha_2 G(i_d) + \alpha_2\eta(i_d)]R(i_d)\\ &= (1-n)\alpha_1 R(i_d^m)\\ &= \Pi^m(i_d^m) > 0,\end{aligned}$$

where $R(i_d) = \underbrace{d^\star}_{\text{deposit}}\underbrace{(i - i_d)}_{\text{deposit spread}}$, the first two lines are implied by equilibrium equal profit condition, and the last line follows from Lemma 4. In particular, we have

$$\Pi^\star = \max_{i_d} \Pi(i_d) = \Pi^m(i_d^m) > 0 \text{ for all } i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d].$$

□

A.1.4 Distribution is continuous

Lemma 7. *$G(\cdot, \gamma)$ is a continuous distribution function.*

Proof. Suppose there is a $i_d^0 \in \text{supp}(G)$ such that $\eta\left(i_d^0\right) > 0$ and

$$\Pi\left(i_d^0\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^0\right) + \alpha_2 \eta\left(i_d^0\right)\right] R\left(i_d^0\right).$$

Given the per-deposit profit function R is continuous in deposit rate i_d, there is a $i_d^1 < i_d^0$ such that $R\left(i_d^1\right) > 0$ and $\Delta \equiv R\left(i_d^0\right) - R\left(i_d^1\right) < \frac{\alpha_2 \eta\left(i_d^0\right) R\left(i_d^0\right)}{\alpha_1 + 2\alpha_2}$. Then

$$\begin{aligned}
\Pi\left(i_d^1\right) &= (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^1\right) + \alpha_2 \eta\left(i_d^1\right)\right] R\left(i_d^1\right) \\
&\geq (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d^0\right) + \alpha_2 \eta\left(i_d^0\right)\right]\left[R\left(i_d^0\right) - \Delta\right] \\
&\geq \Pi(i) + (1-n)\left\{\alpha_2 \eta\left(i_d^0\right)\left[R\left(i_d^0\right) - \Delta\right] - (\alpha_1 + 2\alpha_2)\Delta\right\},
\end{aligned}$$

where the second line follows from $G(i_0^d) - G(i_1^d) \geq \eta(i_0^1)$. Since $R(i_d^0) > \Delta$ and $\Delta < \alpha_2 \eta(i_d^0) R(i_d^0)/(\alpha_1 + 2\alpha_2)$, then the last line implies $\Pi\left(i_d^1\right) > \Pi\left(i_d^0\right)$. This contradicts $i_d^0 \in \text{supp}(G)$. □

A.1.5 Support of distribution is connected

Lemma 8. *The support of G, $\text{supp}(G)$, is a connected set.*

Proof. Suppose deposit rates $i_d, i_d^{'} \in \text{supp}(G)$ with $i_d^{'} < i_d$ and $G(i_d) = G(i_d^{'})$. The bank's expected profit evaluated at these two deposits are respectively given by

$$\Pi\left(i_d\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G\left(i_d\right)\right] R\left(i_d\right),$$

and,

$$\Pi\left(i_d^{'}\right) = (1-n)\left[\alpha_1 + 2\alpha_2 G(i_d^{'})\right] R\left(i_d^{'}\right).$$

Since by assumption we have $G\left(i_d\right) = G\left(i_d^{'}\right)$, then the probability weighting function in the two profit evaluations above are identical, *i.e.*,

$$(1-n)\left[\alpha_1 + 2\alpha_2 G(i_d)\right] = (1-n)\left[\alpha_1 + 2\alpha_2 G(i_d^{'})\right].$$

Since $i_d, i_d^{'} \in \text{supp}(G)$ and we have $i_d^m \leq i_d^{'} < i_d \leq \bar{i}_d < i \equiv \frac{\gamma - \beta}{\beta}$. The bank's profit-margin per deposit is then strictly decreasing for all $i_d \in [i_d^m, \bar{i}_d]$, meaning that the bank earns a lower deposit spread if they price closer to the central bank policy interest rate i. Hence, we have $R(i_d^{'}) > R(i_d)$ and therefore $\Pi(i_d^{'}) > \Pi(i_d)$, which violates the equal profit condition $(\Pi(i_d^{'}) = \Pi(i_d) = \Pi^\star)$ for all deposit rates chosen from the support of the distribution in equilibrium. □

A.1.6 Proof of Proposition 1: Deposit-rate distribution

Proof. The probability of obtaining one deposit-rate quote is $\alpha_1 \in (0,1)$. Since G has no mass points from Lemma 8,

$$\Pi = (1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma)]R(i_d;\gamma),$$

and profit is maximized at $i_d^m = \underline{i}_d$ from Lemma 4,

$$\Pi^\star = (1-n)\alpha_1 R(i_d^m;\gamma).$$

By equal profit condition, for any $i_d \in \text{supp}(G) = [i_d^m, \bar{i}_d]$, we have $\Pi(i_d) = \Pi(i_d^m)$ such that

$$(1-n)[\alpha_1 + 2\alpha_2 G(i_d;\gamma)]R(i_d;\gamma) = (1-n)\alpha_1 R(i_d^m;\gamma). \tag{A.2}$$

Solving Equation (A.2) for the cumulative distribution function G, we have an analytical expression in Proposition (1). Specifically, the analytical formula for the case with non-degenerate distribution $G(\cdot,\gamma)$ is:

$$G(i_d;\gamma) = \frac{\alpha_1}{2\alpha_2}\left[\frac{R(i_d^m;\gamma)}{R(i_d;\gamma)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{d^\star(i-i_d^m)}{d^\star(i-i_d)} - 1\right] = \frac{\alpha_1}{2\alpha_2}\left[\frac{i-i_d^m}{i-i_d} - 1\right], \tag{A.3}$$

where $i := i(\gamma) = (\gamma-\beta)/\beta$.

Finally, given the lower support of the distribution G, $\underline{i}_d = i_d^m = 0$ from Lemma 5, and the fact that G is a cumulative distribution function, we can then back out the upper support of G by using the equal profit condition (A.2), and we obtain $\bar{i}_d = i - \frac{\alpha_1}{\alpha_1+2\alpha_2}[i - \underline{i}_d] < i$.

This establishes the first case in Proposition 1. Proofs for the remaining cases follow directly from Burdett and Judd (1983).

□

Remark. Note that the associate density of the distribution G is characterized by $\tilde{g}(i_d;\gamma) = \partial G(i_d;\gamma)/\partial i_d$. Moreover, a depositor randomly receives deposit-rates quote from banks, which can be one quote or two quotes with probability α_1 and $\alpha_2 = 1-\alpha_1$ respectively. So, the cumulative distribution function of transacted deposit rates can then be described by

$$J(i_d;\gamma) = \alpha_1 G(i_d;\gamma) + \alpha_2[G(i_d;\gamma)]^2 \text{ for all } i_d \in \text{supp}(G),$$

and the associate density of $J(i_d;\gamma)$ is given by

$$j(i_d;\gamma) \equiv \partial J(i_d;\gamma)/\partial i_d = \alpha_1\tilde{g}(i_d;\gamma) + 2\alpha_2 G(i_d;\gamma)\tilde{g}(i_d;\gamma) = [\alpha_1 + 2\alpha_2 G(i_d;\gamma)]\tilde{g}(i_d;\gamma).$$

A.2 Deposit-rates distribution and inflation

A.2.1 Proof of Lemma 1: First-order stochastic dominance and inflation

Proof. Consider the economy away from the Friedman rule: $\gamma > \beta$. The analytical formula for the deposit-rate distribution $G(i_d;\gamma)$ is characterized in Proposition 1. Let $i_\gamma := \partial i(\gamma)/\partial\gamma$ denote the partial derivative of the policy rate with respect to inflation γ.

Now consider how the value of G varies with γ at each fixed i_d such that $0 = \underline{i}_d < i_d < \bar{i}_d$. We have that

$$\frac{\partial G(i_d;\gamma)}{\partial\gamma} = \frac{\alpha_1}{2\alpha_2}\left[\frac{i_\gamma(i-i_d) - i(i_\gamma - i_{d,\gamma})}{(i-\bar{i}_d)^2}\right] = -\frac{\alpha_1}{2\alpha_2}\left[\frac{i_\gamma i_d}{(i-i_d)^2}\right],$$

where $i_\gamma = 1/\beta > 1$ and the second equality obtains since for fixed i_d, $i_{d,\gamma} = 0$.

Since all the other terms are strictly positive, we therefore have, for every fixed $i_d \in (\underline{i}_d, \bar{i}_d) = \text{supp}(G)$, $\partial G(i_d,\gamma)/\partial\gamma < 0$. Thus, we establish that the posted-deposit-rate distribution $G(i_d;\gamma')$ first-order stochastically dominates $G(i_d;\gamma)$ for $\gamma' > \gamma > \beta$.

□

Remark. We have now characterized the relationship between the posted deposit interest rates distribution G and anticipated inflation in Section A.2.1. Since the transacted deposit interest rates distribution J is just a probability re-weighting of the distribution G, the conclusions above regarding inflation and G also apply to J. Hence, we leave out the details here. Instead, we use distribution G for the proof below.

A.2.2 Proof of Lemma 2: Average deposit rate and inflation

Proof. Given monetary policy γ, the nominal policy rate is determined by $i := i(\gamma) = (\gamma-\beta)/\beta$. We first consider the first statement in Lemma 2. First, apply integration by parts to Equation (3.27). This yields

$$g(\gamma) = \left[i_d G(i_d;\gamma)\right]_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} \frac{\partial i_d}{\partial i_d} G(i_d;\gamma)\mathrm{d}i_d = \bar{i}_d(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G(i_d;\gamma)\mathrm{d}i_d.$$

We want to show that $\partial g(\gamma)/\partial\gamma > 0$. Using Leibniz' rule, we have

$$g_\gamma(\gamma) = \frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} - \left[\frac{\partial\bar{i}_d(\gamma)}{\partial\gamma} + \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d\right] = -\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma)\mathrm{d}i_d > 0, \tag{A.4}$$

where $G_\gamma(i_d;\gamma) < 0$ follows from the result in Lemma 1.

Observe that the only difference between the average posted deposit rate and the average transacted deposit rate is that an additional probability weighting function appears in the latter. Hence, we can deduce that $\hat{g}(\gamma) \leq g(\gamma)$ holds since the average transacted rate cannot exceed the average posted rate. It follows that the transacted rate cannot grow faster than the posted rate. Therefore, we have $0 < \hat{g}_\gamma(\gamma) \leq g_\gamma(\gamma)$.

Next, we consider the second statement in Lemma 2. Recall that the lower support of the distribution G is given by $\underline{i}_d = i_d^m = 0$, which is invariant to inflation change since the "hypothetical" monopoly bank can always pay zero deposit interest. Using the equal profit condition: $i - \bar{i}_d = \frac{\alpha_1}{\alpha_1+2\alpha_2}[i - i_d^m]$, we can back out the upper support of the distribution by $\bar{i}_d = i[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$. Differentiate the upper bound of the support of distribution G with respect to inflation γ. We obtain $\frac{\partial \bar{i}_d(\gamma)}{\partial \gamma} = \frac{1}{\beta}[1 - \frac{\alpha_1}{\alpha_1+2\alpha_2}]$ and it satisfies that $0 < \frac{\partial \bar{i}_d(\gamma)}{\partial \gamma} < \frac{1}{\beta}$.

All together, it establishes that the upper bound of the support of the distribution, $\text{supp}(G) = [\underline{i}_d, \bar{i}_d]$ shifts to the right and it becomes wider at a rate less than $1/\beta$ as inflation γ goes up. □

A.3 Money and Capital

Steady-state money demand Euler equation. Take the partial derivative of the DM value function in Equation (3.5) with respect to money balance. Evaluate this marginal value of money one period ahead, and combine this with the CM first order condition in Equation (3.20). Then rewrite this in terms of stationary variables to obtain the steady-state money demand Euler equation:

$$\begin{aligned}\frac{\gamma - \beta}{\beta} = (1-n) \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)] i_d \mathrm{d}G(i_d;\gamma) \\ + \underbrace{n\mathbb{I}_{\{i_l \leq \hat{i}\}}\left[\frac{u_q(q)}{c_{q_s}(q_s, K)} - 1\right]}_{\text{borrow}} + \underbrace{n\mathbb{I}_{\{i_l > \hat{i}\}}\left[\frac{u_{\hat{q}}(\hat{q})}{c_{\hat{q}_s}(\hat{q}_s, \hat{K})} - 1\right]}_{\text{no borrow}},\end{aligned} \tag{A.5}$$

where $q = (z + \tau_b Z + \xi)/\rho$, $\hat{q} = (z + \tau_b z)/\rho$, $q_s = \frac{n}{1-n} q$ and $\hat{q}_s = \frac{n}{1-n}\hat{q}$.

The left-hand side of Equation (A.5) measures the marginal cost of carrying money balances. The right-hand side of Equation (A.5) measures the marginal value of bringing one extra unit of money balance into the DM, and it has two components. The first term captures the marginal value of depositing an additional unit of idle money balance when a seller does not want to consume in the DM. The second term is the net benefit (marginal utility minus marginal cost) of spending an extra unit money balance when a buyer wants to consume in the DM. However, the buyer may or may not take out a loan from the bank depending on whether her maximum willingness to borrow $\hat{i}$ exceeds the market interest rate i_l. Hence, the liquidity premium is associated with or without bank credit.[27]

Steady-state investment Euler equation. Following a similar procedure for deriving Equation (A.5), the investment Euler equation is characterized by

$$\frac{1}{\beta} = [1 + F_K(K,H) - \delta] - (1-n)\mathbb{I}_{\{i_l \leq i\}}\left[\frac{c_K(q_s, K)}{U_X(X)}\right] - (1-n)\mathbb{I}_{\{i_l > i\}}\left[\frac{c_{\hat{K}}(\hat{q}_s, \hat{K})}{U_{\hat{X}}(\hat{X})}\right] \tag{A.6}$$

where $c_K(\cdot; K) < 0$, $U_x(\cdot) = \overline{A}/F_H(K,H)$, $q_b = (z + \tau_b Z + \xi)/\rho$, $\hat{q} = (z + \tau_b Z)/\rho$, $q_s = \frac{n}{1-n} q$ and $\hat{q}_s = \frac{n}{1-n}\hat{q}$.

The left-hand side of Equation (A.6) captures the (gross) real interest rate. The right-hand side of Equation (A.6) captures the (gross) value of investing an extra unit of capital. The first component is the return incurred in CM production. The second component is the return incurred in a DM. This term reflects the additional gains from investing capital in reducing the ex-post marginal cost of production in the DM when a seller produces the goods. Also, the DM goods allocation will depend on whether it is associated with or without bank credit.

A.4 Stationary Monetary Equilibrium

In this section, we restrict attention to an SME with money, credit and capital. We prove its existence and uniqueness. The proof for this relies on two mild restrictions regarding parameters and anticipated inflation to be not too high.

A.4.1 Proof of Lemma 3: Money, credit and capital

Proof. First, recall that the technology in the CM is given by $F(K,H) = K^{\alpha}H^{1-\alpha}$, and the technology in the DM is given by $f(e,k) = e^{1-\psi}k^{\psi}$. Also, we can transform the DM production into a (utility) cost representation of the sellers such that $c(q,k) = q^{\omega}k^{1-\omega},\ \omega := 1/\psi$.

Given a restriction on both technology parameters in the CM and the DM, such that $0 < \alpha < 1$ and $0 < \psi < 1$, households always have an incentive to accumulate a positive

[27] Notice that the banking structure here is slightly different to the one in Berentsen et al. (2007). In a banking equilibrium of BCW, all deposit funds sourced from sellers have to be loaned out to borrowers by banks. Hence, lending is essential there to support a feasible deposit interest payment. Here, even if the banks make zero loans to the buyers in DM, they can invest all of their remaining funds with the central bank to earn a rate of return i. Moreover, the deposit rate $i_d \in \text{supp}(G)$ is always bounded above by the policy rate i. Feasible deposit interest paid to depositors is less of a concern here. However, we need to be careful under what condition bank credit exists in equilibrium and how the banks allocate their deposits between consumer loans and funds they invest with the central bank.

amount of capital in the economy.[28] Next, we want to show positive ex-ante money demand and ex-post loan demand in the economy.

1. Recall that the buyer takes out a loan from the bank as long as the market interest rate on loans satisfy $0 < i_l \leq \hat{i}_l = \rho^{\sigma-1}(z+\tau_b Z)^{-\sigma}\frac{w(K,H)}{\bar{A}} - 1$. Also, recall that we have $c_q(q,k) = (\rho\bar{A})/w(K,H)$ where $w(K,H) = F_H(K,H)$ from the seller's optimization problem. Hence, we can rewrite the buyer's maximum willingness to borrow, $\hat{i}_l$ as

$$\hat{i}_l = \frac{[(z+\tau_b Z)/\rho]^{-\sigma}}{c_{q_s}(q_s,K)} - 1,$$

where $q_s = (\frac{n}{1-n})[\frac{z+\tau_b Z}{\rho}] \equiv (\frac{n}{1-n})\hat{q}$. Note that the buyer's maximum willingness to borrow $\hat{i}$ is equivalent to the value of an extra dollar spent (*i.e.*, a liquidity premium) in an otherwise pure-monetary economy.

   Since banks face perfect competition in the loans market, then $i_l = i = (\gamma-\beta)/\beta$. Hence, we can bound gross inflation γ (or equivalently the policy interest rate) by

$$\frac{\gamma-\beta}{\beta} \leq \hat{i}_l \implies \gamma \leq \beta\left[\frac{[(z+\tau_b Z)/\rho]^{-\sigma}}{c_{q_s}(q_s,K)}\right]. \tag{A.7}$$

   Hence, if condition (A.7) holds, then there must be ex-post positive loan demand. In other words, if inflation is weakly smaller than the discounted gross value of a dollar spent in an otherwise pure-monetary economy, there is ex-post positive loan demand.

2. Given positive loan demand, combining the agent's first-order condition for money demand and investment, the steady-state money demand Euler Equation (A.5) becomes

$$\frac{\gamma-\beta}{\beta} = (1-n)\int_{\underline{i}_d}^{\bar{i}_d}\left[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\right] i_d \mathrm{d}G(i_d;\gamma) + n\mathbb{I}_{\{i_l\leq\hat{i}_l\}}\left[\frac{u_q(q)}{c_q(\frac{n}{1-n}q,K)} - 1\right], \tag{A.8}$$

   where the goods allocation in the DM is supported by both real money balances and real loans.

   Also, the steady-state investment Euler Equation (A.6) is given by

$$\frac{1}{\beta} = [1 + F_K(K,H) - \delta] - (1-n)\mathbb{I}_{\{i_l\leq\hat{i}_l\}}\left[\frac{c_K(\frac{n}{1-n}q,K)}{U_X(X)}\right]. \tag{A.9}$$

□

[28]Note: if $\alpha = 0$ and $\psi = 1$, then capital is irrelevant for production in both the CM and DM. Hence, what matters in equilibrium is the money demand Euler equation (A.5). In this case, we are back to a model of money and banking credit.

A.4.2 Equilibrium

To restrict attention to the case with co-existing money, credit and capital, we need to rely on mild restrictions on parameters and anticipated inflation not to be too high. In particular, using the result in Lemma 3, the system of equations consists of:

1. Money demand Euler equation:

$$\frac{\gamma-\beta}{\beta} = (1-n)\underbrace{\int_{\underline{i}_d}^{\bar{i}_d}\Big[\alpha_1 + 2\alpha_2 G(i_d;\gamma)\Big] i_d \mathrm{d}G(i_d;\gamma)}_{=:\hat{g}(\gamma)} + n\left[\frac{u_q(q)}{c_q(\frac{n}{1-n}q, K)} - 1\right], \tag{A.10}$$

   where the DM goods market clearing condition $(1-n)q_s = nq$ is imposed.

2. Capital investment Euler equation:

$$\frac{1}{\beta} = [1 + F_K(K,H) - \delta] - (1-n)\left[\frac{c_K(\frac{n}{1-n}q, K)}{U_X(X)}\right]. \tag{A.11}$$

3. CM labor market clearing:

$$\underbrace{\frac{\bar{B}}{X}}_{=U_X(X)} = \frac{\bar{A}}{F_H(K,H)}, \tag{A.12}$$

   since we have assumed log-utility for CM consumption.

4. CM goods market clearing condition:

$$F(K,H) = X + K - (1-\delta)K. \tag{A.13}$$

Monetary policy works through the channel of agents' money demand and capital investment decisions. These, respectively, are governed by Equation (A.10) and Equation (A.11). This nominal-to-real link regarding the effects of monetary policy transmission is identical to that in Aruoba et al. (2011). The new feature here is the effect of monetary policy pass-through to the banking sector, captured by $\hat{g}(\gamma)$ in Equation (A.10). Hence, banking market power alters agents' incentives on accumulating money and capital.

Let $\hat{k} := K/H$, the system of equations down to one equation in terms of per-capita variable. To do this, we need to use the functional forms: $U(x) = \bar{B}\ln(x)$; $u(q) = \bar{C}\frac{q^{1-\sigma}-1}{1-\sigma}$; $F(K,H) = K^{\alpha}H^{1-\alpha}$ and $c(q,k) = q^{\omega}k^{1-\omega}$, where $\bar{B} > 0, \bar{C} = 1, \sigma < 1, \alpha < 1,$ and $\omega = \frac{1}{\psi} > 1$.

First, we combine Equation (A.12) with Equation (A.13) to obtain

$$K = \frac{\bar{B}(1-\alpha)\hat{k}}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}. \tag{A.14}$$

Next, recall that we have $F_K(K,H) = \alpha\hat{k}^{\alpha-1}$, $F_H(K,H) = (1-\alpha)\hat{k}^{\alpha}$ and $c_K(\frac{n}{1-n}q, K) = (1-\omega)[\frac{n}{1-n}q]^{\omega}K^{-\omega}$. Applying these equations in Equation (A.11). The capital investment Euler equation becomes

$$\frac{1}{\beta} = [1+\alpha\hat{k}^{\alpha-1}-\delta] - \left[(1-n)(1-\omega)\left(\frac{n}{1-n}\right)^{\omega}(1-\alpha)\overline{A}^{-1}\right]q^{\omega}K^{-\omega}\hat{k}^{\alpha}, \tag{A.15}$$

Next, recall that we have the functional form for the marginal utility of DM consumption and the marginal cost of production. These, respectively, are $u_q(q) = q^{-\sigma}$, and $c_q(\frac{n}{1-n}q, K) = \omega(\frac{n}{1-n})^{\omega-1}q^{\omega-1}K^{1-\omega}$. Substitute these two equations in Equation (A.10) and rearrange, Then, we can express the money demand Euler equation as:

$$q = \left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\hat{C}(\gamma)\right]^{\frac{1}{1-\sigma-\omega}}K^{\frac{1-\omega}{1-\omega-\sigma}}, \tag{A.16}$$

where

$$\hat{C}(\gamma) := 1 + \frac{1}{n}\Big[i(\gamma) - (1-n)\hat{g}(\gamma)\Big] = 1 + \hat{g}(\gamma) + \frac{1}{n}\Big[\underbrace{i(\gamma) - \hat{g}(\gamma)}_{=:s(\gamma)}\Big], \tag{A.17}$$

and the average deposit interest rate $\hat{g}(\gamma)$ is given by Equation (A.10).

Next, we combine Equation (A.16) and Equation (A.14), and then substitute that into Equation (A.15) to get

$$\frac{1}{\beta} = \underbrace{1+\alpha\hat{k}^{\alpha-1}-\delta}_{=:R^{CM}(\hat{k})} + \underbrace{\tilde{\theta}\Big[\hat{C}(\gamma)\Big]^{\frac{\omega}{1-\omega-\sigma}}\tilde{f}(\hat{k})}_{=:R^{DM}(\hat{k};\gamma)}, \tag{A.18}$$

where

$$\tilde{\theta} := \frac{1}{\bar{A}}\left[(\omega-1)(1-n)(1-\alpha)\left(\frac{n}{1-n}\right)^{\omega}\right]\left[\omega\left(\frac{n}{1-n}\right)^{\omega}\right]^{\frac{\omega}{1-\omega-\sigma}} > 0,$$

$$\tilde{f}(\hat{k}) := \left[\frac{\bar{B}(1-\alpha)}{\bar{A}(1-\delta\hat{k}^{1-\alpha})}\right]^{\frac{\omega\sigma}{1-\omega-\sigma}}\hat{k}^{\frac{\omega\sigma-\alpha(\omega+\sigma-1)}{1-\omega-\sigma}},$$

and $\hat{C}(\gamma)$ is given by Equation (A.17).

In steady-state, the system of equations reduces to one equation in terms of $\hat{k}$ governed by Equation (A.18). This can be decomposed into two components reflecting the return on investing capital associated with the CM and DM trades. Also, the nominal policy interest rate satisfies $i(\gamma) = (\gamma - \beta)/\beta$. The new insight is that there is a policy-dependent interest rate spread on deposits captured by the term $s(\gamma)$ showing up in Equation (A.18). In equilibrium, the effects of monetary policy transmission matter for banking, goods trades and capital formation.

A.4.3 Proof of Proposition 2: First-best allocation

Proof. Suppose the economy is at the Friedman rule such that the real money stock grows at $1 + \tau \equiv \gamma = \beta$. By the Fisher equation $1 + i = \gamma/\beta$, it is then equivalently set to the nominal policy interest rate of zero at the Friedman rule, $i = 0$.

Also, suppose that the probability of obtaining two deposit interest rate quotes is less than one, $\alpha_2 < 1$. From Proposition 1, every deposit rate i_d in the support of the deposit interest rate distribution is lower than the policy interest rate i, *i.e.*, $\underline{i}_d \leq i_d \leq \bar{i}_d < i$. Since $i = 0$ holds at the Friedman rule, and the nominal interest rate cannot go negative, it follows that the integral term of Equation (A.18) collapses to zero. That is, the Friedman rule cannot support deposit interest dispersion in equilibrium.

Next, given that the integral term of Equation (A.18) collapses to zero and $i = 0$, it follows that we have a condition of $\hat{C}(\gamma) = 1$. This condition reflects that it is costless for agents to carry money balances across periods at the Friedman rule. Equivalently, this condition coincides with efficient trades in the DM such that $u_q(q) = c_q(q, k)$ holds at equilibrium.

Since $\hat{C}(\gamma) = 1$ holds at $\gamma = \beta$, then Equation (A.18) becomes:

$$\frac{1}{\beta} = [1 + \alpha \hat{k}^{\alpha-1} - \delta] + \tilde{\theta}\tilde{f}(\hat{k}), \tag{A.19}$$

where $\tilde{\theta}$ and $\tilde{f}(\hat{k})$ are identical to that shown in Equation (A.18).

Next, let $\tilde{\gamma} > \gamma = \beta$. Evaluate Equation (A.18) at $\tilde{\gamma}$, and compare this with Equation (A.19), we can deduce that $\hat{k}^{\star,FB} > \hat{k}^{\star}$ for any policy $\tilde{\gamma} > \beta$. From Equation (A.14) and Equation (A.16), both capital stock K and DM consumption q are positively related to $\hat{k}$. It follows that $q^{\star,FB} > q^{\star}$ and $K^{\star,FB} > K^{\star}$ for any policy $\tilde{\gamma} > \beta$. □

A.4.4 Proof of Proposition 3: Unique SME with money and credit

We are now ready to establish the existence and uniqueness of a class of SME featuring the co-existence of money, credit and capital.

Proof. Fix long-run inflation target γ such that $\bar{\gamma} \geq \gamma > \beta$ where $\bar{\gamma}$ is defined in the proof of

Lemma 3 in Section A.4.1.

Observe the first term on the right-hand-side of Equation (A.18) is continuous and monotone decreasing in $\hat{k}$ since $\alpha - 1 < 0$. Second, the parameter in the DM cost function is assumed to be $\omega > 1$. As such, the product of the second term is non-negative. Third, observe that the term C is a constant with respect to $\hat{k}$. Forth, it is assumed that $\omega\sigma > \alpha(\omega + \sigma - 1)$ holds. It follows that $\tilde{f}(\hat{k})$ is monotone decreasing in $\hat{k}$ since $1 - \omega - \sigma < 0$. Fifth, the left-hand-side of Equation (A.18) is a constant with respect to $\hat{k}$. Therefore, there exists a unique solution $\hat{k}^\star$ to Equation (A.18).

Given $\hat{k}^\star$, Equation (A.14) pins down $K^\star$. Given $K^\star$, Equation (A.16) pins down $q^\star$. Given $\hat{k}^\star$ and $K^\star$, we can back out $H^\star$ using the definition of $\hat{k} = K/H$. Finally, we can back out $X^\star$ using the CM goods market clearing condition. Likewise, we can back out the other variables including real money balances $z^\star$, and real loans $\xi^\star$. Moreover $z^\star = Z^\star$ and $k^\star = K^\star$ in equilibrium. Details are omitted here.

Next, we want to establish the uniqueness of an SME with co-existing money, capital and credit. What remains is to show that aggregate banking feasibility is satisfied such that

$$\underbrace{n\xi^\star(z^\star; i_l, Z^\star, \gamma)}_{\text{total loans}} + b = \underbrace{(1-n)\int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z^\star + \tau_b Z^\star)\mathrm{d}G(i_d;\gamma)}_{\text{total deposits}} .$$

First, notice that banks cannot lend more to borrowers than the amount they have sourced from depositors. That is, the total amount of loans demanded by borrowers (*i.e.*, buyers) cannot exceed the total amount of deposits supplied by depositors (*i.e.*, sellers) such that

$$n\xi^\star(z; i_l, Z, \gamma) \leq (1-n)\int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} [\alpha_1 + 2\alpha_2 G(i_d;\gamma)](z^\star + \tau_b Z^\star)\mathrm{d}G(i_d;\gamma).$$

If the above condition does not hold with equality, then the banks can invest any remaining funds b with the central bank to earn a rate of return i. As such, the aggregate banking-feasibility constraint is always balanced. Note: In an otherwise perfectly competitive banking sector, the above condition will always hold at equality given a market clearing interest rate. In such a case, the extra channel of investing idle funds with the central bank is redundant. The reason is that the amount of deposits will eventually be loaned out to borrowers in a perfectly competitive banking market.

Finally, we need to check whether a deposit interest payment is feasible. First, the loan market's zero-profit condition implies that the equilibrium loan interest rate is equal to the policy interest rate, $i_l = i$. Second, all deposit rate i_d in the distribution G, *i.e.*, $i_d \in \text{supp}(G) = [\underline{i}_d, \bar{i}_d]$ is strictly less than policy interest rate i. Third, banks source deposit funds to fund their assets. The cost of funds is lower than what the banks can earn via their assets. Hence, deposit interest paid to the depositors in the CM is always feasible. □

A.5 Equilibrium with banks versus without banks

In this section, we study how the allocation in an economy with banks differs from that without banks. We first discuss two special cases: (1) an economy with perfectly competitive banks and (2) an economy without banks. Then we provide the proof for Proposition 4 in Section A.5.1.

An economy with perfectly competitive banks. This case is equivalent to setting $\alpha_2 = 1$, so that G is degenerate on the singleton set $\{i = i_d = (\gamma - \beta)/\beta\}$ by Proposition 1. We then obtain

$$\frac{1}{\beta} = [1 + \alpha\hat{k}^{\alpha-1} - \delta] + \tilde{\theta}\left[\tilde{C}(\gamma)\right]^{\frac{\omega}{1-\sigma-\omega}} \tilde{f}(\hat{k}), \tag{A.20}$$

where $\tilde{C}(\gamma) := 1 + \frac{\gamma-\beta}{\beta} \equiv 1 + i(\gamma)$, and the rest of the terms are identical to Equation (A.18).

An economy without banks. In this case, we have $\alpha_1 = \alpha_2 = 0$ in which agents earn zero interest on their idle money balances, *i.e.*, $i_d = 0$. The system of equations is then reduced to

$$\frac{1}{\beta} = [1 + \alpha\hat{k}^{\alpha-1} - \delta] + \tilde{\theta}\left[\breve{C}(\gamma)\right]^{\frac{\omega}{1-\sigma-\omega}} \tilde{f}(\hat{k}), \tag{A.21}$$

where $\breve{C}(\gamma) := 1 + \frac{1}{n}\left(\frac{\gamma-\beta}{\beta}\right) \equiv 1 + \frac{i(\gamma)}{n}$, and the rest of the terms are identical to Equation (A.18).

A.5.1 Proof of Proposition 4: Banking versus no-banking allocations

Proof. Fix an anticipated inflation γ such that $\beta < \gamma \leq \bar{\gamma}$, where $\bar{\gamma}$ is defined in Section A.4.1. Given inflation $\gamma > \beta$, the nominal policy interest rate in steady state satisfies $i := i(\gamma) = (\gamma - \beta)/\beta > 0$.

Case 1. We first compare a perfectly competitive banking equilibrium to a no-bank equilibrium. Observe that from Equation (A.20) and Equation (A.21), the only difference across these two economies is due to the gross cost of accumulating money balances.

Since the measure of DM buyers satisfies $0 < n < 1$, then it follows that $\breve{C}(\gamma) = 1 + \frac{i}{n} > 1 + i = \tilde{C}(\gamma)$. This follows that the right-hand side of Equation (A.21) must be smaller than the right-hand side of Equation (A.20) due to these terms are raised to a negative power, $1-(\sigma+\omega)<0$. In words, the extra return on capital associated with DM trades is smaller in an economy without access to banks. Moreover, the right-hand side of Equations (A.20) and (A.21) are both monotonically decreasing in $\hat{k}$ using the result established in Proposition 3. Hence, the following order must hold

$$\hat{k}^{\star,No-bank} < \hat{k}^{\star,PC}, \tag{A.22}$$

for any given policy $\beta < \gamma$.

Recall that both DM consumption q and capital stock K are both positively related to $\hat{k}$. Given Condition (A.22), we can deduce that the following ranking holds:

$$q^{\star,No-bank} < q^{\star,PC} \quad \text{and} \quad K^{\star,No-bank} < K^{\star,PC}, \tag{A.23}$$

for any given policy $\beta < \gamma$.

In summary, Case 1 establishes that banking improves goods trades by increasing the additional return on capital associated with DM trades, relative to the economy without banks. The reason is that banking reduces the gross cost of accumulating money balances for households.

Case 2. Now we compare our baseline economy $\alpha_2 \in (0,1)$ to an economy with a perfectly competitive banking sector ($\alpha_2 = 1$).

We will show the following: As long as $\alpha_2 \in (0,1)$, we can show that the gross cost of accumulating money balances is higher in our baseline economy than that with perfectly competitive banks, *i.e.*, $\hat{C}(\gamma) > \tilde{C}(\gamma)$.

Suppose to the contrary that $\hat{C}(\gamma) \leq \tilde{C}(\gamma)$. Using the expressions of $\hat{C}(\gamma)$ and $\tilde{C}(\gamma)$, respectively, from Equation (A.18) and (A.20), we have

$$\begin{aligned}
&\hat{C}(\gamma) = 1 + \frac{1}{n}[i - (1-n)\hat{g}(i_d;\gamma)] \leq 1 + i = \tilde{C}(\gamma) \\
&\implies i - \hat{g}(i_d;\gamma) \leq n[i - \hat{g}(i_d;\gamma)].
\end{aligned}$$

Since it is required that the n measure of DM buyers satisfies that $0 < n < 1$, there is a contradiction to the weak inequality. Thus, we have $\hat{C}(\gamma) > \tilde{C}(\gamma)$.

Applying a similar reasoning as in Case 1, we can conclude that

$$\hat{k}^{\star} < \hat{k}^{\star,PC}, \quad \text{and} \quad \hat{k}^{\star} \to \hat{k}^{\star,PC} \quad \text{as } \alpha_2 \to 1, \tag{A.24}$$

for any given policy $\beta < \gamma$.

Likewise, we can deduce that the following order must also hold:

$$q^{\star} \leq q^{\star,PC} \quad \text{and} \quad K^{\star} \leq K^{\star,PC}, \tag{A.25}$$

given policy $\beta < \gamma$. The equality in Condition (A.25) holds when $\alpha_2 = 1$.

Case 3. Finally, we compare our baseline economy to a no-bank economy. From Equation (A.18) and Equation (A.21), it follows immediately that households face a higher gross cost of accumulating money balances in a no-bank equilibrium than the case with imperfectly competitive banks. The reason is that depositors, on average, can still benefit from (imperfectly competitive) banks by receiving a positive interest on idle money balances. This is better than being stuck with idle balances being subject to inflation tax. Using similar reasoning as above, we can also verify that the following relationships also hold

$$q^{\star,No-bank} \leq q^{\star} \quad \text{and } K^{\star,No-bank} \leq K^{\star}, \tag{A.26}$$

for any given policy γ such that $\beta < \gamma \leq \bar{\gamma}$. The equality in Condition (A.26) holds when $\alpha_1 = \alpha_2 = 0$. In other words, Case 3 says that having imperfectly competitive banks is welfare-improving relative to the no-bank equilibrium. This is because households still receive liquidity risk insurance through banking. Hence, imperfectly competitive banks can still improve goods trades, augmenting capital investment value relative to the no-bank equilibrium.

In summary, by Conditions (A.23)-(A.26), we have established the following order

$$q^{\star,No-bank} < q^{\star} < q^{\star,PC} \text{ and } K^{\star,No-bank} < K^{\star} < K^{\star,PC},$$

for any given policy γ such that $\beta < \gamma \leq \bar{\gamma}$. Moreover, we have $(q^{\star}, k^{\star}) \rightarrow (q^{\star,PC}, K^{\star,PC})$ as $\alpha_2 \rightarrow 1$ by Proposition 1. □

A.6 Deposit-rates spread, markdown and inflation

In this section, we consider two measures of bank market power in the deposit market: the average interest rate spread on deposits and the average deposit-rates markdown. We then study how bank market power responds to the change in the anticipated inflation, γ. We provided intermediate results and proofs in Section A.2.1 and Section A.2.2. We will apply these results in the proofs in Section A.6.1 and Section A.6.2 to see how monetary policy affects the degree of banking market power in deposits.

A.6.1 Proof of Proposition 5: Deposit-rates spread and inflation

Deposit-rates spread. Let the average interest rate spread on deposits be defined as the difference between the central bank policy interest rate and the average of deposit interest rates across banks. As such, the average posted interest rate spread on deposits is defined as

$$s(\gamma) = i(\gamma) - \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma), \tag{A.27}$$

where the distribution G is characterized in Proposition 1.

Proof. We first consider the average posted deposit-rates spread and make a few observations before we show how it changes with respect to the change in inflation. Recall that all deposit interest rate i_d in the support of the distribution G must be smaller than the policy interest rate i in a banking equilibrium. This means that $i(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$, since all banks earn positive expected profit in equilibrium by marking down the deposit rate that they post from the result in Lemma 6.

Bank market power arises from the noisy search frictions in the deposit market. Therefore, we can first establish that the deposit spread is positive. That is, $i(\gamma) > \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$ implies that $s(\gamma) > 0$, for any γ such that $\bar{\gamma} \geq \gamma > \beta$ and $\bar{\gamma}$ is defined in Section A.4.1.

Next, we consider how the average posted deposit-rates spread $s(\gamma)$ moves with respect to the change in inflation. Let the function $\hat{g}(\gamma)$ to denote the average posted deposit rates, *i.e.*, $\hat{g}(\gamma) := \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d;\gamma)$.

Differentiate Equation (A.27) with respect to γ, we obtain

$$s_\gamma(\gamma) = i_\gamma(\gamma) - \hat{g}_\gamma(\gamma) \equiv i_\gamma(\gamma) - \left[- \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} G_\gamma(i_d;\gamma) \mathrm{d}i_d \right]. \tag{A.28}$$

We show that the average deposit rate is increasing with respect to inflation from the result in Lemma 2, *i.e.*, $\hat{g}_\gamma(\gamma) > 0$ since $G_\gamma(\cdot) < 0$. We also show that the growth rate of the support of the distribution G is less than $1/\beta$ in Lemma 2. It follows that the integral function $\hat{g}_\gamma(\gamma)$ must be also less than $1/\beta$. Hence, we have $\frac{1}{\beta} > \hat{g}_\gamma(\gamma) > 0$.

Next, recall that the growth rate of the policy interest rate is given by $i_\gamma(\gamma) = 1/\beta$. Combining this result with the inequality above, then $i_\gamma(\gamma) > \hat{g}_\gamma(\gamma)$ implies that $s_\gamma(\gamma) = i_\gamma(\gamma) - \hat{g}_\gamma(\gamma) > 0$. This establishes that the average posted deposit-rates spread is increasing with inflation. Moreover, it follows that growth rate of the average posted deposit-rates spread is also bounded such that $\frac{1}{\beta} > s_\gamma(\gamma) > 0$ since $i_\gamma(\gamma) > i_\gamma(\gamma) - \hat{g}_\gamma(\gamma)$ holds.

□

Remark. Proposition 5 shows that:

1. There is an imperfect pass-through of monetary policy to the deposit rates; and
2. Banks are less competitive as they charge a higher deposit-rates spread when inflation goes up.

In other words, banks with market power in the deposits market extract more surplus from depositors when their need for liquidity insurance is high.

The intuition is as follows. First, an increase in the anticipated inflation (*i.e.*, equivalent to a rise in the nominal risk-free policy interest rate in a stationary equilibrium) makes it more costly for households to carry money balances across periods. As such, households need more banking to insure against the risk of holding idle balances. Second, the supply of deposits (from $1-n$ measure of DM sellers who have idle money balances) falls as inflation goes up. Consequently, banks can exploit their intensive-margin channel more (*i.e.*, a higher deposit-rate spread) to compensate for the losses from trading with fewer depositors with smaller money balances. Hence, higher inflation gives more market power to the banks in pricing their deposit rates. This distorts the gains from financial intermediation by more.

A.6.2 Proof of Proposition 7: Deposit-rate markdown and inflation

Alternatively, we can consider deposit-rates markdown as another measure of bank market power in the deposits market. Following the notation from previous sections, we first let the average posted-deposit rates be denoted by $\hat{g}(\gamma) = \int_{\underline{i}_d(\gamma)}^{\bar{i}_d(\gamma)} i_d \mathrm{d}G(i_d, \gamma)$.

Since we focus on linear pricing strategies, then we can define a (gross) markdown $\mathcal{M}$ on average (gross) posted deposit-rates $1+\hat{g}(\gamma)$ over the (gross) policy interest rate $1+i(\gamma)$:

$$\mathcal{M} := \mathcal{M}(\gamma) = \frac{1+\hat{g}(\gamma)}{1+i(\gamma)}, \tag{A.29}$$

In this section, we study the relationship between $\mathcal{M}$and anticipated inflation γ. We now summarize our discussion formally in Proposition 7 and provide the proof below.

Proposition 7. *Assume $\bar{\gamma} \geq \gamma > \beta$, and $\alpha_1 \in (0,1)$. Then, both the average posted- and transacted-deposit-markdowns are monotonically decreasing in inflation γ.*

Proof. Assume anticipated inflation γ satisfies $\bar{\gamma} \geq \gamma > \beta$. From the result in Proposition 5, we show that the increase in the average posted deposit rate is always less than the increase in the policy interest rate as inflation goes up, *i.e.*, $i_\gamma(\gamma) > \hat{g}_\gamma(\gamma) > 0$. It follows that the increase in the numerator of Equation (A.29) must be smaller than that in the denominator given an increase in inflation γ. Hence, the gross markdown on the average posted deposit rates is monotonically decreasing in inflation γ. □

Remark. The average posted deposit-rates markdown $\mathcal{M}$ measures the deviation away from the policy interest rate. As such, banks are more competitive if $\mathcal{M}$ is closer to one (*i.e.*, banks markdown less on deposits in which the average posted deposit rates are closer to the policy rate). Conversely, they are less competitive if $\mathcal{M}$ is closer to zero (*i.e.*, markdown more).

Proposition 7 highlights that banks tend to markdown more on deposit rates as inflation increases. The intuition is that the supply of deposits falls when it is more costly for households to carry money balances across periods. So banks exploit more on their intensive-margin channel to make up the trading loss with fewer depositors with lower money balances as inflation increases.

In summary, Proposition 5 and Proposition 7 both measure the degree of banking market power responding to the change in monetary policy. It has the following implications for the welfare effects of banking. As inflation increases, the need for liquidity insurance against the risk of holding idle money balances is high. This effect gives more market power to the banks in which they can charge a larger deposit-rates spread (or markdown more). As such, banks extract more surplus from depositors, further distorting the gains from financial intermediation. Moreover, this distortion induces a lower allocation in the goods market as agents carry fewer money balances to trade, negatively impacting capital investment. Overall, the degree of banking market power distorts equilibrium allocation, and economic welfare varies with monetary policy changes (*i.e.*, anticipated inflation, or equivalently, the risk-free nominal policy interest rate).

B Long-run growth path

Here, we provide the details of the equilibrium description under exogenous growth found in Section 6. The basic structure of the model remains the same as in Section 3. We only lay out the new features here to avoid repetition. The difference here is that the aggregate production function in the CM is given by $Y = F(K, AH) = K^{\alpha}(AH)^{1-\alpha}$ where $\alpha < 1$, and A is a labor-augmenting technology factor. Moreover, A evolves according to the process $A_{+} = (1 + \mu)A$. In the DM, output is given by $q^s = f(k, Ae) = k^{\psi}(Ae)^{1-\psi}$ where $\psi < 1$. Sellers produce q^s using capital k and effort e. The disutility cost of production for the sellers can also be expressed as $c(\frac{q^s}{A}, \frac{k}{A}) = (\frac{q^s}{A})^{\omega}(\frac{k}{A})^{1-\omega}$, where $\omega := 1/\psi > 1$.

B.1 SME with growth

To restrict attention to an equilibrium with money, credit and capital, we need to rely on mild restrictions on parameters, *i.e.*, $\omega\sigma > \alpha(\omega + \sigma - 1)$, and, the requirement that the nominal policy interest rate be positive and not too high. This restriction is similar to that discussed in Lemma 3. Since we consider log utility ($\sigma = 1$) in both the DM and the CM in order to compare results with Waller (2011) (and also for existence of a balanced growth path), the only restriction is to have a positive nominal policy interest rate that is not too high. This is because the restriction on parameters, $\omega\sigma > \alpha(\omega+\sigma-1)$, simplifies to the requirement $\alpha < 1$ when $\sigma = 1$. This requirement is automatically satisfied by the Cobb-Douglas CM production technology. For the ease of presentation, we normalize the preferences scale parameters to $\overline{A} = \overline{B} = \overline{C} = 1$ in what follows.

Given the formula for G, the equilibrium system of equations are as follows.

1. Money demand Euler equation:

$$\phi U_x(x) = \beta\phi_+ U_x(x_+)\left\{(1-n)\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G_+(i_d)](1+i_d)\mathrm{d}G_+(i_d) + n\left[\frac{u^{'}(q_+)}{c_q\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}}\right]\right\}, \tag{B.1}$$

   where the DM goods market clearing condition $(1-n)q^s = nq$ is imposed.

2. Capital investment Euler equation:

$$U_x(x) = \beta U_x(x_+)\left[1 + F_K(K_+, A_+H_+) - \frac{1}{U_x(x_+)}(1-n)c_k\left(\frac{n}{1-n}\frac{q_+}{A_+}, \frac{k_+}{A_+}\right)\frac{1}{A_+}\right]. \tag{B.2}$$

3. CM labor market clearing:

$$U_x(X) = \frac{1}{F_H(K, AH)A}. \tag{B.3}$$

4. CM goods market clearing condition:

$$F(K, AH) = X + K_+ - (1-\delta)K. \tag{B.4}$$

An equilibrium with money, credit and capital solves Equations (B.1)-(B.4) given initial capital stock K_0 and money stock M_0.

B.1.1 Balanced-growth steady state: Baseline model

We now derive the conditions describing a steady state in our model. As in Waller (2011), we assume constant labor hours and the real variables, $(q, X, K_+, \phi M)$, all grow at the constant rate of $1+\mu$. The growth of real money stock satisfies

$$1+\tau = (1+\pi)(1+\mu), \tag{B.5}$$

and the nominal policy interest rate i satisfies the growth-adjusted Fisher equation:

$$1+i = \frac{(1+\pi)(1+\mu)}{\beta}. \tag{B.6}$$

Let $1+\varphi := (1+\pi)(1+\mu)$. In a steady state equilibrium under balanced growth, we have the deposit-rate distribution be such that $G_{+1}([1+\varphi]i_d) = G(i_d)$. Let $\hat{K} := K/AH$ denote the capital-to-effective-labor ratio.

Next, we derive the balanced-growth steady state using similar steps to that for the model without growth (see Appendix A.4.4). Given policy i, we can reduce the system of equations down to one equation solving for the steady-state point $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1+\alpha\hat{K}^{\alpha-1}-\delta) + \bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K}), \tag{B.7}$$

where

$$\tilde{C}(i) := 1 + \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d) + \frac{1}{n}\left[i - \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)\right],$$

$\bar{\theta} := n(1-\alpha)[(\omega-1)/\omega]$, and $\tilde{f}(\hat{K}) := \hat{K}^{\alpha-1}[(1-\alpha)/(1-[\delta+\mu]\hat{K}^{1-\alpha})]^{-1}$.

The left-hand side of Equation (B.7) captures the (gross) risk-free interest rate adjusted for growth. The two terms on the right-hand side of Equation (B.7), respectively, capture the (gross) return on capital used in the CM and that in the DM. Again, note that the return to capital in the DM is augmented by the deposit-side market power distortion term $\tilde{C}(i)$.

Proposition 8. *There is a unique strictly-positive, balanced-growth steady state equilibrium $\hat{K}^\star$.*

Proof. The left-hand side of Equation (B.7) is constant with respect to $\hat{K}$. The right-hand side is monotone decreasing in $\hat{K}$: First, CM production exhibits diminishing returns to capital—*i.e.*, the term $\alpha\hat{K}^{\alpha-1} + (1-\delta)$ is a continuous and strictly decreasing function of $\hat{K}$. Second, the additional return to capital in the DM, $\bar{\theta}\left[\tilde{C}(i)\right]^{-1}\tilde{f}(\hat{K})$ is strictly decreasing in $\hat{K}$. Note that $\tilde{C}(i)$ is independent of $\hat{K}$ by virtue of Proposition 1. It is straightforward to verify that $\tilde{f}' < 0$. Therefore, there is a unique $\hat{K}^\star > 0$ satisfying Condition (B.7). □

We can then back out the other endogenous variables (q, K, X, H) at the balanced-growth steady state $\hat{K}^\star$. In particular, we have

$$X = (1-\alpha)\hat{K}^\alpha A, \quad K = \frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}A, \quad H = \frac{1-\alpha}{1-(\delta+\mu)\hat{K}^{1-\alpha}},$$

and $q = A\left[\omega\left(\frac{n}{1-n}\right)^{\omega-1}\tilde{C}(i)\right]^{-\frac{1}{\omega}}\left[\frac{(1-\alpha)\hat{K}}{1-(\delta+\mu)\hat{K}^{1-\alpha}}\right]^{\frac{\omega-1}{\omega}}$.

B.1.2 Balanced-growth steady state: Three special limits

Our steady-state characterization from Section B.1.1 nests three specials cases: A neoclassical growth model; a version of our monetary environment with perfectly competitive banks, *i.e.*, a combination of Aruoba et al. (2011) and Berentsen et al. (2007); and, a version of the monetary model without banks (Waller, 2011).

1. **Neoclassical growth model.** To obtain this case, we can set $n = 0$ or $\omega = 1$. Setting $n = 0$ shuts down the DM. Alternatively, setting $\omega = 1$ eliminates the additional return to capital associated with DM goods production. In the first setting, money plays no role in the CM and that makes the limit economy a neoclassical one. In the second, we have the result akin to Aruoba and Wright (2003) where nominal activity is decoupled from the real economy. Either way, the limit economies imply that money is inconsequential to capital accumulation and growth. Either case imply the neoclassical growth model's steady state condition:

$$\frac{1+\mu}{\beta} = 1 + \alpha\hat{K}^{\alpha-1} - \delta. \tag{B.8}$$

2. **Perfect competition among banks.** This case, is equivalent to requiring $\alpha_2 = 1$ (depositors contact at most one bank) in our model. By Proposition 1 the deposit rate distribution is degenerate at the policy interest rate i and the interest rate spread on deposits is zero. The steady state fixed-point condition becomes

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\Big[1+i\Big]^{-1}\tilde{f}(\hat{K}), \tag{B.9}$$

where $\bar{\theta}$ and $\tilde{f}(\hat{K})$ are identical as in Equation (B.7).

3. **No-bank.** This case obtains if we set $\alpha_1 = \alpha_2 = 0$. It reduces to the (price-taking) setup in Waller (2011). In particular, we have

$$\frac{1+\mu}{\beta} = (1 + \alpha\hat{K}^{\alpha-1} - \delta) + \bar{\theta}\left[1+\frac{i}{n}\right]^{-1}\tilde{f}(\hat{K}), \tag{B.10}$$

where $\bar{\theta}$ and $\tilde{f}(\hat{K})$ are identical as in Equation (B.7).

From the right-hand-side of Equations (B.7)—(B.10), we can deduce the following order: $\hat{K}^{\text{neoclassical},\star} \underbrace{\leq}_{\text{“}=\text{''if } n=0 \text{ or } \omega=1} \hat{K}^{\text{no-bank},\star} < \hat{K}^{\star} < \hat{K}^{\text{PC},\star}$, given policy $i > 0$.

Suppose we let $n > 0$ and $\omega > 1$. In that case, we can see that the capital-per-effective-labor ratio in a monetary economy (with or without banks) is always higher than that in a neoclassical growth model. The reason is that capital has an additional value from reducing the cost of DM production. Moreover, having access to banks, in general, improves such benefits relative to the setting without banks. However, bank market power distorts some of these gains and this reduces the capital-to-effective-labor ratio in equilibrium relative to the case with perfect competition among banks.

B.1.3 Dynamics: Baseline model

We derive an analytical special case for the model's balanced-growth-path dynamics. This case will be comparable to the results in Waller (2011). To do so, we set the rate of depreciation for capital as $\delta = 1$. The equilibrium dynamical system simplifies to

$$\frac{q_{+1}}{\hat{K}_{+1}} = \left(\frac{1}{A_{+1}}\right)^{-\frac{1}{\omega}} \left[\omega \left(\frac{n}{1-n}\right)^{\omega-1} \tilde{C}(i)\right]^{-\frac{1}{\omega}} \hat{K}_{+1}^{-\frac{1}{\omega}}, \tag{B.11}$$

$$\left(\frac{X_{+1}}{X}\right)\frac{1}{\beta} = \alpha \hat{K}_{+1}^{\alpha-1} + X_{+1}(1-n)\left(\frac{\omega-1}{A_{+1}}\right)\left(\frac{n}{1-n}\right)\left(\frac{q_{+1}}{\hat{K}_{+1}}\right)^{\omega}, \tag{B.12}$$

and,

$$\hat{K}_{+1} = \frac{1}{1+\mu}\left[\frac{H}{H_{+1}} - \frac{1-\alpha}{H_{+1}}\right]\hat{K}^{\alpha}, \tag{B.13}$$

where

$$\tilde{C}(i) := 1 + \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d) + \frac{1}{n}\left[\, i - \int_{\underline{i}_d}^{\bar{i}_d} [\alpha_1 + 2\alpha_2 G(i_d)] i_d \mathrm{d}G(i_d)\right].$$

Equation (B.11) is obtained by rearranging the money demand Euler equation. Equation (B.12) is the capital Euler equation. We then combine the CM labor- and goods-market-clearing conditions—Equation (B.3) and Equation (B.4)—to obtain Equation (B.13).

Combining Equation (B.11) and Equation (B.12) yields

$$\hat{K}_{+1} = \frac{\beta}{1+\mu}\left[\alpha + \frac{n(1-\alpha)(\frac{\omega-1}{\omega})}{H_{+1}}[\tilde{C}(i)]^{-1}\right]\hat{K}^{\alpha}. \tag{B.14}$$

Equations (B.13) and (B.14) jointly pin down the transitional dynamics of capital ($\hat{K}$) and labor (H).

Following Waller (2011), we consider constant hours (H) along a balanced growth path. The constant labor hours can be derived as

$$H = \frac{1-\alpha}{1-\alpha\beta}\left[1 + n\beta\left(\frac{\omega-1}{\omega}\right)\left[\tilde{C}(i)\right]^{-1}\right]. \tag{B.15}$$

This further yield the restriction on the dynamics of capital per effective worker:

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i)]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{B.16}$$

where

$$\tilde{C}(i) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average deposit rate}} + \frac{1}{n}\left[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G(i_d)]i_d \mathrm{d}G(i_d)}_{\text{average interest rate spread on deposits}}\right]$$

is the price-dispersion distortion effect in our model. This is the additional feature affecting capital growth dynamics relative to Waller (2011). Recall that this distortion arises from endogenous market power on the deposit side of banking.

B.1.4 Dynamics: Three special limits

As before, we can also derive three special parametric limits of our model:

1. **Neoclassical growth model.** This case obtains in our model if we set $n = 0$ or $\omega = 1$. The corresponding growth rate of capital in such an economy is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{neoclassical}}} = \frac{1}{1+\mu}\left[\alpha\beta\hat{K}^{\alpha-1}\right]. \tag{B.17}$$

2. **Perfect competition among banks.** Set $\alpha_2 = 1$. The corresponding growth rate of

capital is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{PC}}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}{1+n\beta(\frac{\omega-1}{\omega})[1+i]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{B.18}$$

3. **No-bank.** Set $\alpha_1 = \alpha_2 = 0$. The corresponding growth rate of capital reduces to the (price-taking) setup in Waller (2011), which is given by

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g^{\text{no-bank}}{}_k} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}{1+n\beta(\frac{\omega-1}{\omega})[1+\frac{i}{n}]^{-1}}\right]\hat{K}^{\alpha-1}. \tag{B.19}$$

From the right-hand-side of Equations (B.16)-(B.19), and given policy $i > 0$, we can deduce the following order: $g_k^{\text{neoclassical}} \underbrace{\leq}_{\text{``=''if } n=0 \text{ or } \omega=1} g_k^{\text{no-bank}} < g_k < g_k^{\text{PC}}$. This ranks the growth rate of capital across the different economies.

The following is similar to the reasoning in Section B.1.2. If $n > 0$ and $\omega > 1$, then starting from the same value of $\hat{K}$, capital in a monetary economy (with or without banks) is always accumulated at a faster rate than the neoclassical growth model. The reason is that capital investment in a monetary economy has an additional value in reducing the DM cost of production. Banking, in general, improves such benefits than the economy without banks. However, bank market power lowers the growth rate of capital relative to the economy with perfectly competitive banks.

C Central bank digital currency (CBDC)

In this section, we consider having an interest-bearing central bank digital currency (CBDC) made available to the public along the lines of Andolfatto (2021). The central bank now has two separate policy tools. One that targets the trend inflation γ (equivalently, the nominal rate $i = (\gamma - \beta)/\beta$) and one that controls the interest rate on CBDC, i^{CBDC}. Both γ and i^{CBDC} are exogenous parameters. The nominal policy rate i and the CBDC rate i^{CBDC} can differ.

The purpose here is to study how the presence of CBDC affects the endogenous market power on the deposit side of (private) banking arising from informational frictions. We then study the implications for deposit rate markdowns (and dispersion), capital formation and long-run growth.

For our purpose, we assume that private bank deposits and CBDC have no technological advantage over each other, as in Andolfatto (2021). The central bank provides lending and deposit facilities that private banks can borrow and lend at the same policy rate. Consequently, private bank deposit and lending decisions are separate, as presented in the main text. We also keep private banks' lending side of operations competitive. As before, the loan rate equals the policy rate and is not affected by the CBDC rate. However, the deposit rate is affected by both the policy rate and CBDC rate, when CBDC serves as an alternative depository facility for the households.

C.1 Overview

The model's basic structure remains the same as we have discussed thus far. The only differences are the (ex-post) sellers' problem and (private) banks' profit-maximization problem. We lay out the implications as follows.

First, $(1-n)$ sellers can now choose where to deposit their unproductive idle funds m. Sellers can deposit their idle money balances m at the private bank saving account or the central bank CBDC account (or both). The such decision depends on the deposit rate offered by private banks and the CBDC rate set by the central bank. Since households can choose to deposit at the private or central banks, they optimally deposit at the one that offers a higher interest rate. That is, households optimally deposit zero (all) idle funds at the private banks (central bank) if $i_d < i^{CBDC}$, and vice versa. They are indifferent between saving at the private banking system and the central bank if $i_d = i^{CBDC}$.

Second, the central bank can use interest-bearing CBDC to discipline the distribution of deposit interest rates and associated markdowns arising from informational frictions in the private banking deposit market. Next, we briefly discuss the intuition behind this effect.

Recall that depositors can now switch to depositing their idle funds with CBDC at a rate of i^{CBDC} offered by the central bank. Suppose the depositor has only one contact with the private bank (in the event of α_1). In this case, the private monopoly bank has to match their interest rate i_d^m to the CBDC rate. Otherwise, the private bank cannot source resources to fund its assets. As a consequence, the lower support of the distribution G^{CBDC} is now disciplined by the CBDC rate such that $\underline{i}_d = i_d^m = i^{CBDC} \geq 0$. As before, we can back out the upper support of the distribution G^{CBDC} using the equal profit condition. In this case, the upper support is determined by $\bar{i}_d = i(\gamma) - \frac{\alpha_1}{\alpha_1+2\alpha_2}[i(\gamma) - \underline{i}_d]$, depending on both trend inflation γ and CBDC rate i^{CBDC}. Consequently, the CBDC rate (as an additional policy tool) matters for the distribution $G^{CBDC}(i_d; \gamma, i^{CBDC})$ and equilibrium allocations.

In summary, an interest-bearing CBDC as an outside option can help to discipline the market power (both markdowns and dispersion) of private banks in the deposit market, improving capital accumulation. This mechanism is related to the insight of *latent medium of exchange* shown in Lagos and Zhang (2021).

In what follows, we use $G^{CBDC}(i_d; i, i^{CBDC})$, and, $G(i_d; i)$ to respectively denote the posted deposit interest rate distribution in an economy with CBDC and without CBDC. For shorthand notation, we use $G^{CBDC}(\cdot)$, $G(\cdot)$ or just G^{CBDC} and G.

C.2 Baseline model with interest-bearing CBDC

We derive the balanced-growth steady state in an economy with interest-bearing CBDC using similar steps to that for the model without CBDC (see Appendix B). Given policies $i = [\gamma(1+\mu)]/\beta$ and i^{CBDC}, we can reduce the system of equations down to one equation solving for the steady-state point $\hat{K}$:

$$\frac{1+\mu}{\beta} = (1+\alpha\hat{K}^{\alpha-1}-\delta) + n(1-\alpha)\left(\frac{\omega-1}{\omega}\right)\left[\hat{K}^{\alpha-1}\left(\frac{1-\alpha}{1-[\delta+\mu]\hat{K}^{1-\alpha}}\right)^{-1}\right]\left[\tilde{C}(i, i^{CBDC})\right]^{-1}, \tag{C.1}$$

where

$$\tilde{C}(i, i^{CBDC}) := 1 + \underbrace{\int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G^{CBDC}(i_d; i, i^{CBDC})]i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})}_{\text{average deposit rate}}$$
$$+ \frac{1}{n}\Bigg[\underbrace{i - \int_{\underline{i}_d}^{\bar{i}_d}[\alpha_1 + 2\alpha_2 G^{CBDC}(i_d; i, i^{CBDC})]i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})}_{\text{average interest rate spread on deposits}}\Bigg].$$

Similar to Section B.1.3 and setting $\delta = 1$, dynamics of capital per effective worker in an economy with CBDC is given by:

$$\underbrace{\frac{\hat{K}_{+1}}{\hat{K}}}_{=:1+g_k^{\text{CBDC}}} = \frac{1}{1+\mu}\left[\frac{\alpha\beta + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i, i^{CBDC})]^{-1}}{1 + n\beta(\frac{\omega-1}{\omega})[\tilde{C}(i, i^{CBDC})]^{-1}}\right]\hat{K}^{\alpha-1}, \tag{C.2}$$

where the central bank can use i^{CBDC} to affect the posted deposit rates distribution G^{CBDC}, and hence the term $\tilde{C}(i, i^{CBDC})$. This is the additional feature affecting capital growth dynamics relative to Waller (2011) and discussion in Section B.1.3. Recall that endogenous market power on the deposit side of private banking arises from informational frictions.

Note: If we set the CBDC rate to be $i^{CBDC} = 0$, then Equations (C.1) and (C.2) are identical to the baseline model discussed in Section B.1.3.

C.3 Analysis

In this section, we study the effects of interest-bearing CBDC on the equilibrium outcome of the economy.

Posted deposit-rate distribution. Assuming $\alpha_1 \in (0, 1)$, the analytical formula for the distribution of deposit rates posted by private banks in an economy with CBDC is characterized by:

$$G^{CBDC}(i_d; i, i^{CBDC}) = \frac{\alpha_1}{2\alpha_2}\left[\frac{i - \underline{i}_d}{i - i_d} - 1\right], \tag{C.3}$$

where the lower support of the distribution is $\underline{i}_d = i_d^m = i^{CBDC}$ and the upper support of the distribution is $\bar{i}_d$ and $\bar{i}_d := \bar{i}_d(i, i^{CBDC}) = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$.

The associated density of the posted deposit-rate distribution $G^{CBDC}(i_d; i, i^{CBDC})$ is $\tilde{g}^{CBDC}(i_d) = \partial G^{CBDC}(i_d; i, i^{CBDC})/\partial i_d = \frac{\alpha_1}{2\alpha_2}[\frac{i - \underline{i}_d}{(i - i_d)^2}]$.

Remark.

- In general, the nominal policy rate i and the CBDC rate i^{CBDC} can be different.
- If $0 < i^{CBDC} < i$, then G^{CBDC} is non-degenerate with a connected support $[\underline{i}_d, \bar{i}_d]$, and every $i_d \in [\underline{i}_d, \bar{i}_d]$ is below i.
- If $0 < i^{CBDC} = i$, then G^{CBDC} is degenerate at $i = i^{CBDC} = i_d$.
- If $0 < i < i^{CBDC}$, then the private banks' profits earned from deposits is negative, $\pi^d < 0$. Since we have focused on perfect competition on the loans side, then profit is zero, $\pi^l = 0$. Overall, the private banks' profit is $\pi = \pi^l + \pi^d < 0$. In this case, private banks will not operate, and there is no such G in this particular economy.

In what follows, we exclude the possibility that $0 < i < i^{CBDC}$.

First-order stochastic dominance and CBDC

Lemma 9. *Consider the economy away from the Friedman rule: $i > 0$. Assume $\alpha_1 \in (0, 1)$. Consider any two CBDC interest rates i_1^{CBDC} and i_2^{CBDC} such that $0 < i_1^{CBDC} < i_2^{CBDC} < i$. The induced deposit rate distribution $G^{CBDC}(i_d; i, i_2^{CBDC})$ first-order stochastically dominates $G^{CBDC}(i_d; i, i_1^{CBDC})$.*

Proof. Now consider how the value of G^{CBDC} varies with i^{CBDC} at each fixed i_d such that $i^{CBDC} = \underline{i}_d < i_d < \bar{i}_d$. We have that $\partial G^{CBDC}(i_d; i, i^{CBDC})/\partial i^{CBDC} = -\frac{\alpha_1}{2\alpha_2(i - i_d)} < 0$ since all the other terms are strictly positive. Hence, $G^{CBDC}(i_d; i, i_2^{CBDC})$ first-order stochastically dominates $G^{CBDC}(i_d; i, i_1^{CBDC})$ for $0 < i_1^{CBDC} < i_2^{CBDC} < i$. □

Average posted deposit rates and CBDC

Lemma 10. *Fix $i > i^{CBDC} > 0$. Assume $\alpha_1 \in (0,1)$. An increase in the CBDC rate leads to:*

1. *an increase in the average deposit interest rates posted by private banks;*
2. *an increase in the lower and upper bound of the support of the distribution G^{CBDC}, $[\underline{i}_d, \bar{i}_d]$.*

Proof. Suppose $i > i^{CBDC} > 0$. Let $g^{CBDC}(i, i^{CBDC}) = \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC})$ to denote the average deposit interest rates posted by private banks in an economy with CBDC. We first consider the first statement in Lemma 10. Apply integration by parts to $g^{CBDC}(i, i^{CBDC})$, and this yields

$$g^{CBDC}(i, i^{CBDC}) = [i_d G(i_d; i, i^{CBDC})]_{\underline{i}_d}^{\bar{i}_d} - \int_{\underline{i}_d}^{\bar{i}_d} \frac{\partial i_d}{\partial i_d} G(i_d; i, i^{CBDC}) \mathrm{d}i_d = \bar{i}_d - \int_{\underline{i}_d}^{\bar{i}_d} G(i_d; i, i^{CBDC}) \mathrm{d}i_d.$$

Next, we want to show that $\partial g^{CBDC}(i, i^{CBDC})/\partial i^{CBDC} > 0$. Using Leibniz' rule, we have

$$\begin{aligned} \frac{\partial g^{CBDC}(i, i^{CBDC})}{\partial i^{CBDC}} &= \frac{\partial \bar{i}_d}{\partial i^{CBDC}} - \left[\frac{\partial \bar{i}_d}{\partial i^{CBDC}} + \int_{\underline{i}_d}^{\bar{i}_d} G_{i^{CBDC}}(i_d; i, i^{CBDC}) \mathrm{d}i_d \right] \\ &= - \int_{\underline{i}_d}^{\bar{i}_d} \underbrace{G_{i^{CBDC}}(i_d; i, i^{CBDC})}_{<0} \mathrm{d}i_d > 0, \end{aligned} \tag{C.4}$$

where the last equality follows from the result in Lemma 9 meaning that higher CBDC rate shifts the distribution G^{CBDC} downward.

Next, we consider the second statement in Lemma 10. Recall that the lower support of the distribution G^{CBDC} is given by $\underline{i}_d = i_d^m = i^{CBDC} > 0$. The lower support is a one-to-one to change in the CBDC rate since the monopoly private banks has to match their deposit rate up to the CBDC rate.

Using equal profit condition, $R(\bar{i}_d; i, i^{CBDC}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\underline{i}_d; i, i^{CBDC})$, we can back out the upper support of the distribution G^{CBDC} by $\bar{i}_d := \bar{i}_d(i, i^{CBDC}) = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$. Differentiate $\bar{i}_d$ with respect to i^{CBDC}, we have $\frac{\partial \bar{i}_d}{\partial i^{CBDC}} = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} > 0$ and it is less than one. Hence, all together establishes that the support of the distribution, $\mathrm{supp}(G^{CBDC}) = [\underline{i}_d, \bar{i}_d]$ shifts to the right in response to a higher CBDC rate. □

Deposit-rates spread and CBDC As before, the average interest rate spread on deposits to be defined as the difference between the central bank policy interest rate and the average of deposit interest rates across banks. As such, the average posted interest rate spread on deposits in an economy with interest-bearing CBDC is defined as

$$\tilde{s}(i, i^{CBDC}) = i - \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC}), \tag{C.5}$$

where the distribution G^{CBDC} is determined by Equation (C.3).

If we set $i^{CBDC} = 0$, then $\tilde{s}(i, i^{CBDC})$ is identical to the baseline model without CBDC, i.e., $s(i)$ defined in Equation (A.27).

Lemma 11. *Suppose the nominal policy interest rate, $i > 0$, and $\alpha_1 \in (0,1)$, are fixed in both economies (with CBDC and without CBDC) featuring noisy deposit search. Then, we have*

1. *If $0 = i^{CBDC} < i$, then $0 < \tilde{s}(i, i^{CBDC}) = s(i)$.*
2. *If $0 < i^{CBDC} < i$, then $0 < \tilde{s}(i, i^{CBDC}) < s(i)$.*
3. *If $0 < i^{CBDC} = i$, then $0 = \tilde{s}(i, i^{CBDC}) < s(i)$.*

Proof. We first consider the first statement in Lemma 11. Suppose the central bank sets the CBDC rate to be zero, $i^{CBDC} = 0$. It then follows that the support of the deposit rate distribution in an economy with CBDC is identical to that without CBDC, i.e., $\mathrm{supp}(G^{CBDC}) = \mathrm{supp}(G)$. Hence, $\int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC}) = \int_{\underline{i}_d}^{\bar{i}_d(i)} i_d \mathrm{d}G(i_d; i)$. Since i and $\alpha_1 \in (0,1)$ are fixed the same in these two economies, it then follows that $\tilde{s}(i, i^{CBDC}) = s(i) > 0$. Hence, the conclusion of the first statement in Lemma 11.

Next, we consider the second statement in Lemma 11. Suppose the central bank sets the CBDC rate above zero but below the nominal policy rate, $0 < i^{CBDC} < i$. In an economy with CBDC, the lower and upper bound of the support of the distribution G^{CBDC} are respectively given by $\underline{i}_d = i_d^m = i^{CBDC} > 0$ and $\bar{i}_d = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2}[i - \underline{i}_d]$. Let $\underline{i}_d^{'}$, and, $\bar{i}_d^{'}$ to respectively denote the lower and upper bound of the support of the distribution G in an economy without CBDC. In this case, we have $\underline{i}_d^{'} = i_d^m = 0$, and $\bar{i}_d^{'} = i - \frac{\alpha_1}{\alpha_1 + 2\alpha_2} i$. Since both economies have an identical nominal policy rate i, and the same degree of noisy search frictions, we have the ordering: $\underline{i}_d^{'} < \underline{i}_d < \bar{i}_d^{'} < \bar{i}_d$. It follows that $\int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G^{CBDC}(i_d; i, i^{CBDC}) > \int_{\underline{i}_d}^{\bar{i}_d} i_d \mathrm{d}G(i_d; i)$. That is, the average deposit rate posted by private banks in an economy with CBDC is higher than the economy without CBDC. Also, $\alpha_1 \in (0,1)$ implies each i_d drawn from G^{CBDC} (or G) must be lower than the policy rate, implying a positive deposit spread. All together, we then have $0 < \tilde{s}(i, i^{CBDC}) < s(i)$ as stated in the second statement in Lemma 11.

Finally, we consider the third statement in Lemma 11. Suppose the central bank sets the CBDC rate equals to the nominal policy rate, $0 < i^{CBDC} = i$. In this case, the distribution

G^{CBDC} is degenerate at $i = i^{CBDC} = i_d$. Hence, it follows that the average deposit spread collapse to zero in this economy, i.e., $\tilde{s}(i, i^{CBDC}) = 0$. For the case without CBDC, using result established above, we have $s(i) > 0$. Hence, we have the conclusion of the third statement in Lemma 11.

□

C.4 Proof of Proposition 6: Effects of CBDC on capital growth

Proof. Recall that the growth rate of capital per effective worker in (1) an economy with CBDC and noisy deposits search, (2) an economy without CBDC and noisy deposits search, and, (3) an economy with perfectly competitive banks but no CBDC, are respectively determined by Equations (C.2), (B.16) and (B.18). Let g_k^{CBDC}, g_k, and g_K^{PC} to respectively denote the corresponding growth rate.

Suppose $0 < i^{CBDC} < i$ and $\alpha_1 \in (0, 1)$. We then have $0 < \tilde{s}(i, i^{CBDC}) < s(i)$ by the result established in Lemma 11. Comparing Equations (C.2), (B.16) and (B.18), we have that $(1 + i)^{-1} > [\tilde{C}(i, i^{CBDC})]^{-1} > [\tilde{C}(i)]^{-1}$. Hence, the right-hand side of Equation (C.2) must be lower that the right-hand side of Equation (B.18) and higher than the right-hand side of Equation (B.16). All together, it establishes that $g_k < g_k^{CBDC} < g^{PC}$.

Next, suppose $0 < i^{CBDC} = i$ and $\alpha_1 \in (0, 1)$. By the result established in Lemma 11, the distribution G^{CBDC} degenerates at $i = i^{CBDC} = i_d$ in this case. It then follows that $(1 + i)^{-1} = [\tilde{C}(i, i^{CBDC})]^{-1} > [\tilde{C}(i)]^{-1}$. Thus, we have that $g_k < g_k^{CBDC} = g_k^{PC}$. □

In summary, Proposition 6 highlights that having an interest-bearing CBDC (as an alternative depository facility) can help to reduce the positive deposit spread that arises from informational frictions in the private banking deposit market. When the central bank ties the CBDC rate to the nominal policy rate, the effectiveness of banking liquidity transformation and capital accumulation can be restored as in a perfectly competitive banking equilibrium.