On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion*

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Abstract

We show that even with idealized competitive banks, banking amplifies retail-goods firms' ability to extract higher markups from ex-post heterogeneous buyers. This works through a new pecuniary-externality channel that is tightly connected to an equilibrium distribution of goods-price markups. The endogeneity in firms' markup responses to the presence of credit renders banking not always and everywhere a welfare-enhancing proposition. Consequently, the welfare-improving role of banks as intermediaries that help alleviate individual liquidity risk is ambiguous. Our model also justifies why policymakers should be worried about inflation, banking and its connection to rising industry markups.

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1 Introduction

In this paper, we revisit the question of Berentsen, Camera and Waller (2007) on the *essentiality* of banking. We combine the perfectly competitive banks of Berentsen et al. (2007) with a model featuring endogenous goods market power and an equilibrium distribution of posted prices (Head, Liu, Menzio and Wright, 2012). To be able to compare with this existing literature, we restrict attention to the Berentsen et al. (2007) definition of banking. That is, we focus on banks solely as vehicles that take deposits of idle money and provide credit to those who turn out to need it. In an environment where private credit contracts are incentive infeasible, banks are essential institutions that enable individuals to insure *idiosyncratic liquidity risks*.[1] Both Berentsen et al. (2007) and (Head, Liu, Menzio and Wright, 2012) are steeped in the New Monetarist tradition where crucial market frictions are not assumed but are results of deeper informational and contractual environments. This allows the researcher to study questions such as the existence and essentiality of money, banking, financial markets and asset liquidity as equilibrium objects (see, e.g., Williamson and Wright, 2010; Lagos, Rocheteau and Wright, 2017). By combining these two models, we arrive at an insight that might not be so apparent to conventional wisdom: Even with the best-possible case or idealization of perfect competition among banks, banks need not necessarily be a welfare-improving proposition (c.f., Berentsen, Camera and Waller, 2007). One must also worry about the interaction of the bank interest rate with pricing in goods markets where pricing dispersion is endogenous and sellers have market power.

A new equilibrium trade-off. In the paper, we provide a complete characterization of the dependency of three generic equilibrium classes on long-run inflation. Depending on inflation, and therefore equilibrium real money balance held, there could be a case where substantial measures of self-financed, liquidity constrained and unconstrained money buyers exist alongside buyers who hold both money and take out bank credit. If such an equilibrium emerges, then there is an intricate balance between the *liquidity-risk insurance* benefit of banks and a *pecuniary externality* of bank credit on non-credit buyers of goods: On one hand, competitive banks play a welfare-improving role of facilitating the insurance of idiosyncratic liquidity risks. This is well understood from the Berentsen et al. (2007) (BCW) microfoundation of why banks are essential institutions: They facilitate the intermediation between those with excess liquidity and those who need more, so long as money has an inferior return to a risk-free outside option (i.e., the economy is away from the Friedman rule).

On the other, when we have a Head et al. (2012) (HLMW) kind of economy (with endogenous firm heterogeneity in markup pricing), there can be a novel countervailing channel which runs from the anticipation of bank credit by consumers in their ex-ante money holdings decision, through endogenous markup-pricing dispersion responses of goods sellers, to the likelihood of non-credit buyers ending up with more rent being extracted by the goods sellers. Unlike standard Cournot competition or monopolistic competition models, we focus on HLMW because it provides an

[1] We also focus on perfect competition in banking as do Berentsen et al. (2007). Some have suggested that we also incorporate market power in banking. We do not do that here since that will only (quantitatively) deepen the welfare cost of banking without changing the basic insights of this paper. In a different paper, Head, Kam, Ng and Pan (2022) study banking with endogenous market power and pricing dispersion in deposit and lending rates.

equilibrium mechanism between market power of firms and an empirically-relevant distribution of prices or markups. In our setting, the distribution will become dependent on banking outcomes.

This equilibrium tension renders a non-monotone *ex-ante* welfare implication for competitive banking: At sufficiently low inflation, banks need not be *essential* or welfare improving. That is, when inflation is low, the pecuniary externality caused by banks on goods seller's pricing behavior tends to overpower the liquidity-risk insurance benefit coming from banks. However, when inflation is high enough, the liquidity-risk insurance channel dominates the pecuniary externality effect. This will turn out to be our empirically-relevant equilibrium type.

On the new trade-off: Benefit versus cost of bank credit and inflation. The novelty of our paper is as follows. Consider the *benefit* of banking in the model. It comes in two parts. With access to banks, ex-post inactive buyers (those who do not have a trading opportunity) can deposit idle funds with banks to earn interest. In addition, some active buyers (those who have a trading opportunity) may find it optimal to top up their money with bank credit in order to relax their liquidity constraint. In the model, these two forces imply higher consumption and welfare. We call this overall benefit of banking a *liquidity-risk insurance effect*, which is also present in BCW.

However, there is equilibrium feedback from the ability of some agents to use bank loans, to agents' ex-ante decision to hold money, to the distribution of goods-price markups. We call this an opposing *pecuniary externality* (through *pricing dispersion*) *effect*. We show that a first-order stochastic dominance result holds: For a given inflation level, lower equilibrium real money balance implies firms are more likely to exact higher markups on agents who are liquidity-constrained and unconstrained money-buyers. Lower real money balance has a direct effect on money-constrained buyers through tightening their ex-post liquidity constraints. Unconstrained money-buyer also suffer lower consumption as their demands for goods are decreasing in price. Thus, the presence of buyers who find it optimal to borrow from banks create a pecuniary externality through the pricing-markup distribution. This tends to reduce the consumption level for buyers who do not use banking credit.

Unlike BCW, access to bank credit for some agents can create a pecuniary externality cost on others even though there is perfect competition among banks and there are no costs to access banking services. In our model, what is sufficient to induce this externality is the Head et al. (2012)-like goods-price distribution that becomes dependent on consumers' ex-ante money balance decision. In turn, this decision is made in anticipation of the possibility of credit-financed events. In short order, banking can improve welfare for those with idle money or those who are willing to borrow. However, by encouraging less own-money holdings, banking also amplifies goods-price markups' dispersion and average which makes non-credit buyers worse off. This trade-off, as we will show is sensitive to inflation, and thus, to monetary policy.

We discipline the model by calibrating it to the data. We numerically show the following: In contrast to the model without banks (i.e., the HLMW model) average markups under a competitive-banking equilibrium is always higher. Likewise, the dispersion of markups is also higher in the banking equilibrium. The gaps in these measures between the banking equilibrium and the HLMW limit are increasing with inflation. For plausibly low inflation ranges, banking is welfare reducing since for low inflation the gains from banking to depositors of idle money and credit-buyers is small compared to the dispersion effect on non-credit agents. For sufficiently high inflation, the result reverses.

Related literature. Head et al. (2012) and Berentsen et al. (2007) both feature decentralized markets where anonymous agents have the incentive to hold money in order to buy goods.[2] Both models are derived from Lagos and Wright (2005). Berentsen et al. (2007) (BCW) introduced perfectly competitive banks into a Lagos and Wright (2005)-type of model to show that banks are welfare improving institutions or are *essential*, in the sense of liquidity transformation or idiosyncratic liquidity risk reallocation. Moreover, in a variation on their model, BCW also consider a decentralized goods markets where there is a (Nash) bargaining friction that also implied market power among sellers. Nevertheless, in their setting bank credit does not induce any pecuniary externality in goods trade. This is because, ex post, in there is no pricing heterogeneity faced by searching buyers. Thus, in BCW, regardless of whether goods sellers in decentralized trades have market power, banks are shown to fully compensate holders the opportunity cost of idle money in terms of deposit interest. In contrast, we show that when there is equilibrium pricing dispersion under heterogeneous market power in the style of Head et al. (2012), this is no longer true because of its pecuniary externality feedback onto ex-post non-credit buyers.[3] (We provide an analytical, comparative-equilibrium study on this point in Section 3.1 in the paper.)

Head et al. (2012) (HLMW) adapt the consumer search model of Burdett and Judd (1983) to rationalize equilibrium price dispersion that is consistent with well-known facts about price stickiness at the micro-level data on product pricing.[4] Their money-neutral model provided an important lesson in the spirit of the Lucas critique: Observed price dispersion and stickiness in micro-level price changes do not necessary imply that monetary policy has real effects through these phenomena. Our combination of HLMW with BCW allows us to arrive at a modified statement about the essentiality of banks. Moreover, it introduces or identifies an equilibrium causal nexus that runs from monetary policy to banking intermediation, which in turn induces a pecuniary externality on agents' allocations through firm's equilibrium pricing-markups and their dispersion.

Our result on the negative welfare effect of credit is comparable to that established in Chiu, Dong and Shao (2018). The authors also consider a perfectly competitive banking sector, fo-

[2] Anonymity here is taken to mean that sellers cannot observe buyers histories and any private promises to repay cannot be enforced. Thus, money is essential, i.e., it has value in equilibrium as a medium of exchange, just as in Lagos and Wright (2005).

[3] This has a similar flavor to the insights of Geromichalos and Herrenbrueck (2016). Their model has a liquid asset (money) and an illiquid asset that can be liquidated in a frictional over-the-counter (OTC) secondary asset market. Competitive (c.f., frictional OTC) trade in their secondary asset market may not be efficient because an agent's holding of an additional unit of money insures not just their own consumption shock but also that of buyers of the liquid asset in the secondary asset market. However, agents ignore this positive externality on ex-post secondary-market buyers when they make ex-ante money accumulation decisions. In a related sense, we have the pecuniary externality of bank credit on money-buyers arising in a simpler, one-asset model with perfectly competitive banking.

[4] In our model, as in Head et al. (2012) and Burdett and Judd (1983), firms post prices and produce on the spot. Buyers observe a random number of price quotes posted by firms and buy at the lowest price they observe. This induces firms to optimally trade off between charging a higher markup on their goods and a lower probability of contact by buyers. Equilibrium in the model results in firms being indifferent between a continuum of these opposing margins of attaining the same maximal expected profit. This renders an equilibrium, realized distribution of posted (and transacted) prices that will depend on monetary policy and the aggregate amount of money.

cusing on banking's role in reallocating idle liquidity, as in Berentsen et al. (2007). In their model, access by borrowers to credit raises the *homogeneous* price level of the goods traded in a decentralized market: more demand for goods by credit-buyers raises the marginal cost of production. With competitive price-taking, this translates to a higher goods price in the authors' model. This pecuniary-externality or feedback-on-higher-price effect tightens the liquidity constraint of money-buyers and reduces their consumption. This is also similar to Berentsen, Huber and Marchesiani (2014). Like us, Chiu et al. (2018) show that even under perfectly-competitive goods and banking markets, credit can induce a pecuniary-externality cost on liquidity-constrained money-buyers. However, their result requires the assumption that there is an exogenous measure of money-constrained buyers and the cost of producing the decentralized-market good is strictly convex.

In contrast, we obtain a negative welfare effect of credit through a channel of endogenous firms' market power in goods price markups and dispersion. Also, in our setting, the measures of money-constrained and other agent types are endogenous. Moreover, in our model equilibrium, even unconstrained money-buyers can be affected negatively, since there is not just the one goods price in our model and these agents end up drawing higher prices and consuming less as a result. We shut down the possibility of another pecuniary-externality channel like that of Chiu et al. (2018) by assuming that decentralized-market firms have a linear cost of production. Instead, we identify a new and alternative mechanism for this externality effect. We show that buyers with access to credit can contribute to an increase in the measure of firms charging higher prices and extracting more rent from liquidity constrained money-buyers. Hence, banking can be welfare-reducing in equilibrium.

Dong and Huangfu (2021) present a monetary model in which both money and credit serve as a means of payment. Credit settlement requires money. In their model, the payment instrument involved with money (credit) is subject to the inflation tax (fixed transaction costs). They show that using credit can be welfare-reducing at very low or very high inflation. This is a consequence of having a fixed cost of accessing credit in the model. In contrast, we do not require any cost to accessing bank credit.

There are few other studies incorporating the noisy search process of Burdett and Judd (1983) into a monetary framework for various applications (see, e.g., Head and Kumar, 2005; Head, Kumar and Lapham, 2010; Chen, 2015; Wang, 2016; Wang, Wright and Liu, 2020). Wang et al. (2020) focus on rationalizing the price-change pattern and cash-credit shares observed at the micro-level data in the United States. In their model, buyers' access to credit is costly, so that money and credit are imperfect substitutes as means of payments. In contrast, agents' access to banking is not restricted in our setup, as in Berentsen et al. (2007), and we are not concerned with the question of competing media of exchange. There is just one medium of exchange (money) in decentralized, anonymous trades. It is possible to introduce costly banking in our model but it would not change the basic message in the paper. Boel and Camera (2020) introduce an operating cost for banks in providing loans which will generate a wedge between the lending and deposit rates.

Recent empirical studies find that industry market power, measured in terms of price markups, has been sharply increasing since the 1980s in the United States (see, e.g., Hall, 2018; Rossi-

Hansberg, Sarte and Trachter, 2020; De Loecker, Eeckhout and Unger, 2020). This has prompted a literature that investigates the macroeconomic consequences of industry market power (see, e.g., Guerrieri and Lorenzoni, 2017; Autor, Dorn, Katz, Patterson and Reenen, 2020; Edmond, Midrigan and Xu, 2023). Since the 1980s, the U.S. consumer credit-to-GDP ratio has also been accelerating around the same time as the rise in industry market power. The phenomenon of rising industry market power is not only of interest to academics but also to policymakers. For example, U.S. President Biden has recently called for promoting industry competition in the United States (see Executive Order 14036, 2021). Our study complements this literature by highlighting the unexplored nexus between competitive banking and its effect on goods markup-pricing outcomes.

The remainder of the paper is organized as follows. In Section 2, we lay out the details of the model, agents' decision problems and characterization of a Stationary Monetary Equilibrium (SME). In Section 3, we dissect and discuss the new tension underlying the welfare consequences of banking created the new pecuniary externality from banks, even if they are perfectly competitive banks. We provide a set of numerical illustrations to further expound on the model mechanism. We perform these numerical experiments using the baseline model that is calibrated the U.S. data. We conclude in Section 4.

2 Model

The model builds on Head et al. (2012) (HLMW) by introducing perfectly competitive markets for bank deposits and loans. As in Berentsen et al. (2007) (BCW), the focus here is on banks' role in terms of intermediating between ex-post heterogeneous liquidity needs of agents.[5] A novelty in our model will be in the dependence of market power in the DM-good pricing on the price of credit (c.f., Head et al., 2012). This is because, in equilibrium, there may exist a measure of agents who would take out credit from banks. This renders their demand for loans and the DM good dependent on the nominal loan interest rate (i). Thus, agents should anticipate that the equilibrium DM-good pricing distribution would, in general, depend on i. We use this framework to study the interaction between banking credit and firms' market power in equilibrium.

In every period, two markets open sequentially as in Lagos and Wright (2005). First, a decentralized goods market (DM) with trading frictions opens. In the DM, households are anonymous so that private credit arrangements are incentive infeasible. Consequently, fiat money will be valuable as a medium of exchange in equilibrium. The DM will be the source of fundamental frictions in the model. The DM will be followed by a frictionless centralized market (CM) which allows agents to rebalance their asset positions.

[5] BCW and our setting abstract from other aspects or functions of banks such as the undertaking of risky investments or bank equity under capital regulation. Also, this nor BCW is a model about different or competing payment instruments. Here, credit is a cash top up on a borrower's money holdings extended by a loan contract from a bank.

2.1 Primitives

Preferences. Each household has their per-period utility described by

$$\mathcal{U}(q, x, h) = u(q) + U(x) - h, \tag{2.1}$$

where $u(q)$ is the utility flow from consumption of the goods in the DM, $U(x)$ is the utility flow of consumption goods x in the CM, and $-h$ captures the disutility of labor.

We assume that $u' > 0, u'' < 0$ and u satisfies the standard Inada conditions. Likewise for the CM utility function U. We restrict our attention to the constant-relative-risk-aversion (CRRA) class of functions:

$$u(q) = \frac{q^{1-\sigma} - 1}{1 - \sigma}. \tag{2.2}$$

The risk aversion coefficient $\sigma \in (0, 1)$ influences the households' price elasticity of demand. As in Head et al. (2012), this parameter restriction ensures a well-defined equilibrium (see also Altermatt, 2022; Williamson, 2012).

Technologies. In the CM, the general goods x are produced using a technology that is linear in labor input h. Consequently, both real wage and the price of the general goods will be equal to one. In the DM, firms producing one unit of good q requires $h = c \times q$ hours of labor. The parameter $c > 0$ is the constant marginal cost of DM production.

2.2 Timing and events in the sequential DM and CM

In the model, time is discrete and infinite. Agents discount across period t and $t+1$ by a common discount factor $\beta \in (0, 1)$. We will use variables $X \equiv X_t$ and $X_+ \equiv X_{t+1}$ respectively to denote time-dependent outcomes at period t and $t+1$. There are four types of agents: households, firms, banks, and the government. There is a continuum of households and firms, each of measure one. The banking sector is perfectly competitive with free entry. The government supplies fiat money according to the rule $M_+ = \gamma M$, where $\gamma = 1 + \tau$ is money-supply growth factor and $\gamma \in [\beta, \infty)$. Let the variable $\mathbf{a} := (M, \gamma)$ denote the aggregate state of the economy.

Decentralized market (DM). At the start of each period the DM opens. The following sequence of events arise:

1. Each household realizes a preference shock that has two possible outcomes: First, a particular household agent turns out to want to consume (the DM good q). This outcome occurs with probability n, and we label the associated agents as *active buyers*. Second, an agent does not wish to consume, and this occurs with probability $1 - n$. We label an agent in such an event an *inactive buyer*.[6]

[6] We retain this notation and assumption from Berentsen et al. (2007) for ease of comparison. We will be able to recover a version of their model as a special case of ours when there is no noisy consumer search in the DM goods market and sellers are Walrasian price takers. We also the assumption regarding banking operations as in

The banking market is open at this stage as well: All DM agents can access a line of credit and can deposit any amount of money that they possess with perfectly competitive banks. Banks charge borrowers a competitive rate of i, and commit to paying depositors at a perfectly-competitive nominal interest rate of i_d.[7]

2. After realizing their taste shock, the *active buyers* engage in noisy search for goods sellers. As in HLMW, goods trade is modelled as a monetary-exchange version of the Burdett and Judd (1983) noisy search process: Each DM-goods firm posts a price (p) anticipating that buyers with money holdings m will show up. Firms commit to supplying at the realization of their posted contracts, taking as given the distribution of all posted prices, $J_i(\cdot, m, \mathbf{a})$, and buyers' demand schedule, q_b.[8] Ex ante, buyers know the price distribution but not an individual posted price. Hence, this noisy search process rules out that buyers can direct their search to particular sellers with the lowest price. Instead, buyers randomly contact k number of firms. With probability α_k a buyer matches or makes contact with k firms, or equivalently, draws k price quotes. Each price quote is drawn independently from $J_i(\cdot, m, \mathbf{a})$. For simplicity, we assume that buyers either sample one price quote with probability $\alpha_1 \in [0, 1)$ or two independent price quotes with probability $\alpha_2 = 1 - \alpha_1$. (Some of these buyers may turn out to want to borrow from and some may want to deposit excess liquidity with banks.)

3. Given a draw p from distribution $J_i(\cdot, m, \mathbf{a})$, the agent decides on how much of a good q_b to purchase and whether to borrow money from banks in addition to their own money holdings. Given these choices, the agent can also deduce whether they will want to deposit any excess money holdings with banks.[9]

4. Banking activity (in tems of lending and borrowing) ceases after all household agents have completed their loan and/or deposit transactions with the banks. Goods exchange occurs between the agents and firms in the DM. Buyers face a liquidity constraint consisting of their own money balances m with (or without) loans l and/or deposits d. Buyers then pay the firms to produce the goods for their consumption. After the DM, agents enter a frictionless CM.

Centralized market (CM). An agent entering the CM is denoted by an individual state (m, l, d), i.e., her remaining nominal money balance, outstanding loan and deposit balance. In particular, those who have deposited in the previous DM will earn gross interest $1 + i_d$ on deposits

Berentsen et al. (2007). First, banks operate a financial record-keeping technology at zero cost. Second, banks can perfectly enforce loan repayments. Moreover, agents having access to banks does not rule out the need for money serving as a medium of exchange in the DM. This is because ex-ante agents demand money as a precaution against probable events where they may turn out to optimally not want to borrow from banks, but they still need money in order to buy goods in anonymous DM-good trades.

[7]Clearly, the *inactive buyers* will deposit all their idle monies with the banks. We shall also see that it may be optimal for some *active buyers* to deposit their monies as they may not spend it all on DM goods.

[8]It may be profitable for firms to post lotteries over pure linear pricing strategies. See Footnote 16 on page 15 for further explanation.

[9]Thus, the buyer's choices on consumption in the DM, bank credit and deposit depend on the price drawn from the distribution. The (equilibrium) distribution, in turn, depends on agents best-response functions and thus will depend on a given lending rate i.

d. Those who have borrowed must repay gross interest $1+i$ on loan l to banks. Households supply labor h to firms for production and consume the general goods x. Households own firms and firms return profits as dividends D to households. Households then accumulate money balances m_+ to carry into the next period.

2.3 Households

In what follows, we work backwards from the CM to the DM within the period t.

2.3.1 Households in the CM

An agent beginning the CM with money, loan or deposit balances, (m, l, d), may have been a borrower or a depositor in the previous DM during the first sub-period. Her initial value is

$$W(m,l,d,\mathbf{a}) = \max_{(x,h,m_+)\in\mathbb{R}^3_+} \left\{ U(x) - h + \beta V(m_+, \mathbf{a}_+) : \begin{array}{c} x + \phi(m_+ - m) \\ = h + D + T + \phi(1+i_d)d - \phi(1+i)l \end{array} \right\} \tag{2.3}$$

where V is the value function at the beginning of the next DM, ϕ is the value of money in units of the CM consumption good x, i_d is the deposit interest rate, i is the loan interest rate, h is labor supplied, D is aggregate dividends from firm ownership and T is the lump-sum taxes/transfers from the government.

The first-order conditions with respect to x and m_+ are, respectively, given by

$$U_x(x) = 1, \tag{2.4}$$

and,

$$\phi = \beta V_m(m_+, \mathbf{a}_+), \tag{2.5}$$

where $V_m(m_+, \mathbf{a}_+)$ captures the marginal value of accumulating an extra unit of money balance taken into the next period $t+1$. The envelope conditions are

$$W_m(m,l,d,\mathbf{a}) = \phi, \quad W_l(m,l,d,\mathbf{a}) = -\phi(1+i), \quad \text{and} \quad W_d(m,l,d,\mathbf{a}) = \phi(1+i_d). \tag{2.6}$$

Note that W is linear in (m, l, d) and the distribution of money balances is degenerate when households exit the CM. As a result, households' optimal choices for CM consumption and money balance are given by Equations (2.4) and (2.5). These equations are independent of the agents' current wealth since per-period preferences are quasilinear.

2.3.2 Households in the DM

We first describe the post-match household problems. We call households who sample at least one price quote in the DM *active buyers*. We label those who sample zero price quotes *inactive buyers*.

Regarding banking arrangements, it is easy to verify that agents who are active buyers will have no incentive to deposit funds with the bank, whereas inactive buyers will never have an incentive to borrow additional funds from banks. As such, we denote l as the amount of loans an active buyer may take out and d as the amount of money deposited by an inactive buyer throughout the paper.

***Ex-post* inactive buyers.** With probability $1-n$, a household is inactive. Conditional on being inactive, a household with money holdings, m, can deposit $d \leq m$ of this money with a bank. She has zero utility flow of consuming q_b and then enters the CM with valuation of $W(m-d, 0, d, \mathbf{a})$. Since holding money is subject to inflation tax, it will be optimal for *inactive buyers* to deposit all of their money holdings, i.e., $d^{\star}(m, \mathbf{a}) = m$.

***Ex-post* active buyer sampling at least one price.** The post-match value of such a buyer is given by:

$$B(m, p, \mathbf{a}) = \max_{q_b \geq 0, l \in [0,\infty)} \{u(q_b) + W(0, l, m + l - pq_b, \mathbf{a}) : pq_b \leq m + l\} \tag{2.7}$$

We assume banks can perfectly enforce loans repayment as in the baseline case of Berentsen et al. (2007). Hence, buyers do not face a borrowing constraint.[10]

Taking loan interest rate i as given, we can derive the buyer's demand for DM consumption goods as:

$$q_b^{\star}(m, p, i, \mathbf{a}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ \frac{m}{p} & \text{if } \tilde{p}_i < p \leq \hat{p}\ , \\ (p\phi)^{-1/\sigma} & \text{if } p > \hat{p} \end{cases} \tag{2.8}$$

where

$$\hat{p} := \hat{p}(m, \mathbf{a}) = \phi^{\frac{1}{\sigma-1}} m^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i := \tilde{p}(i, m, \mathbf{a}) = \hat{p}(1+i)^{\frac{1}{\sigma-1}}. \tag{2.9}$$

The cutoff prices $(\hat{p}, \tilde{p}_i)$ are functions of the state of the economy and monetary policy. Assuming $\sigma < 1$, we can order the cut-off prices as: $0 < \tilde{p}_i < \hat{p} < +\infty$. Later on, when calibrated to data, the DM risk aversion coefficient will turn out to be some number $\sigma < 1$.[11]

[10] Within the same period, given no restrictions to depositing money with banks, *any* active buyer will never want to leave the DM with idle money, as that would be subject to inflation tax. Hence the zero value for the first argument in the continuation value in Equation (2.7). That is, whether a buyer is money-constrained or not, or whether they borrow or not, it is always optimal to not carry excess money out of a DM trade when one can deposit that and be insured or compensated by deposit interest. Notice that a separate/explicit choice variable notation d is redundant here since it must be whatever the residual balance amount $m + l - pq_b$ is. (We present an equivalent representation of this problem in Online Appendix A.)

[11] We will find that $\sigma < 1$ when we calibrate the model such that its implied aggregate money demand is close to the historical long-run money demand relation in the United States. (See Online Appendix C for the details.) Unlike in standard neoclassical and related New Keynesian models where often their centralized market preference CRRA coefficient turns out to be at least unity, here σ corresponds to a frictional, search market for goods. Our calibration of $\sigma < 1$ is consistent with similar findings in other related models (e.g., Wang, 2016; Wang et al., 2020;

For a given loan interest rate i, the buyer's loan demand is:

$$l^{\star}(m,p,i,\mathbf{a}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}[\phi(1+i)]^{-\frac{1}{\sigma}} - m & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \leq \hat{p} \\ 0 & \text{if } p > \hat{p} \end{cases} . \tag{2.10}$$

From the liquidity constraint in (2.7), along with the optimal consumption and loan demand schedules—i.e., Equations (2.8) and (2.10)—we can also back out the optimal deposit demands of the various types of *active* DM buyers:

$$d^{\star}(m,p,i,\mathbf{a}) = \begin{cases} 0 & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \leq \hat{p} \\ m - p(p\phi)^{-1/\sigma} & \text{if } p > \hat{p} \end{cases} . \tag{2.11}$$

Note that the buyer's demands for goods, loans and deposits will depend on price p. If p turns out to be a random variable drawn from a distribution $J_i(\cdot, m, \mathbf{a})$—and it will be in a certain equilibrium—then, we would observe ex-post heterogeneous consumption, loan and deposit outcomes in the DM. When we present the firms' problem, we will be more explicit about characterizing the distribution of prices.

Equations (2.8), (2.10) and (2.11), respectively, imply three possible classes of ex-post heterogeneous demands for goods, loans and deposits. Consider the first case in Equation (2.10). If a buyer draws a p that is sufficiently low, then the buyer optimally borrows money from the bank to top up his initial money holdings. Moreover, the buyer spends all his liquid balances, including his money and bank loan. We call this buyer a (liquidity constrained) *credit-buyer*. In the intermediate case, p is drawn such that $\tilde{p}_i < p \leq \hat{p}$. In this event, the buyer prefers not to borrow from the bank but rather to spend all her money. In this case, loan size does not matter for goods demand. We call this type of buyer a *liquidity constrained money-buyer*, or in short, a *money-constrained buyer*. In the last case, p can be sufficiently high. In that case, the buyer prefers not to borrow and also not to spend all her money balance in the frictional goods market. We call this type of buyer a liquidity *unconstrained money-buyer*. Since the bank-deposit facility is available to everyone, agents with unused money can avoid inflation tax by depositing with banks. Here, only the unconstrained money-buyers and *inactive buyers* will optimally want to deposit. In Equation (2.11), we see that a *liquidity constrained money-buyer* will park their residual amount $m - p(p\phi)^{-1/\sigma}$ with banks to earn deposit interest. (Recall that inactive buyers will deposit all of their unused m.)

It is also worth mentioning the price elasticity of demand for the demand schedule $q_b^{\star}$ described

Head et al., 2012). Moreover, this restriction is consistent with the empirical finding in Baker (2018). The author finds that indebted households face a more elastic demand schedule, which is captured by the first case in Equation (2.12).

in Equation (2.8). The buyers' price elasticity of demand is given by

$$\left|\frac{\partial q_b^\star(m,p,i,\mathbf{a})}{\partial p}\frac{p}{q_b^\star(m,p,i,\mathbf{a})}\right| = \begin{cases} \frac{1}{\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ 1 & \text{if } \tilde{p} < p \leq \hat{p} \\ \frac{1}{\sigma} & \text{if } \hat{p} > p \end{cases}. \tag{2.12}$$

This will imply that demand is elastic among buyers other than money-constrained buyers.[12] The implication is that such a buyer cannot spend more than his liquidity constraint at low-enough price levels, $p < \hat{\rho}$. Above the $\hat{\rho}$ cut-off price level, a buyer's liquidity constraint does not bind and such buyers will always spend less than their total money holding.

Households in the DM *ex-ante*. Now consider the beginning of period t when households are *ex-ante* homogeneous at the start of the DM (i.e., before exchange and production of the goods). Given an individual real money balance, m, and aggregate state, $\mathbf{a} := (M, \gamma)$, the agent's value is

$$\begin{aligned} V(m,\mathbf{a}) = &(1-n)\,W(m-d,0,d,\mathbf{a}) \\ &+ n\left\{\alpha_1 \int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}J_i(p,m,\mathbf{a}) + \alpha_2 \int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}[1-(1-J_i(p,m,\mathbf{a}))^2]\right\}. \end{aligned} \tag{2.13}$$

In contrast to Head et al. (2012), the value of households entering the DM is different due to the availability of banking services.

Consider Equation (2.13). With probability $1-n$, a household is *inactive*, i.e., the household realizes that they have no desire to consume in the DM. In the aggregate, the measure of $1-n$ *inactive buyers* can deposit their idle money balances at the bank to earn interest i_d.[13] Conditional on the being an *active buyer*, with probability α_1 the household gets to contact one firm posting some price p. For the ex-ante household, this p is perceived as being drawn from the distribution $J_i(\cdot, m, \mathbf{a})$. With probability $\alpha_2 = 1 - \alpha_1$, the household gets to sample two independent price quotes from firms, and the lower of the two prices is drawn from the distribution $1-(1-J_i(\cdot, m, \mathbf{a}))^2$. Some buyers may also find it optimal to deposit any idle liquidity with banks, as we will show next.

Marginal value of money. To simplify notation, we denote the (equilibrium) cut-off pricing functions by $\underline{p} := \underline{p}(m,\mathbf{a})$, $\tilde{p}_i := \tilde{p}_i(i,m,\mathbf{a})$, $\hat{p} := \hat{p}(m,\mathbf{a})$ and $\overline{p} := \overline{p}(m,\mathbf{a})$. Also, we will use

[12] Alternatively, we can show that the elasticity of the buyer's expenditure rule $e(p) := pq_b^\star(m)$ is less than one. Then the buyer's expenditure on the DM goods decreases as he faces a higher price p. We omit the details of its derivation here. Instead, we explain more about how banking credit affects buyers' optimal expenditure rule and firms' pricing strategy in Section 3.1.

[13] In contrast, in a no-bank economy, this measure of households will enter the subsequent CM while holding their idle money balances subject to an inflation tax. In that case, having unneeded money ex-post can be costly since higher inflation induces a lower value of money.

these shorthand notation for probability measures: $\varrho_+ := \left[\alpha_1 + 2\alpha_2 \left(1 - J_{i,+}(p, m_+, \mathbf{a}_+)\right)\right]$, $J_{i,+} := J_{i,+}(p, m_+, \mathbf{a}_+)$, and $\mathbf{1}_X$ is the Dirac measure on event X. Differentiating Equation (2.13) with respect to m, we have the following expression for the marginal value of money at the start of a DM one period ahead:

$$V_m(m_+, \mathbf{a}_+) = \phi_+ \left[1 + r_+(m_+, \mathbf{a}_+)\right], \tag{2.14a}$$

where

$$\begin{aligned} r_+(m_+, \mathbf{a}_+) := {} & (1-n)\, i_d \\ & + n \int_{\underline{p}}^{\tilde{p}_i} \left\{\varrho_+ \cdot i\right\} \mathrm{d}J_{i,+} \\ & + n \int_{\tilde{p}_i}^{\bar{p}} \left\{ \varrho_+ \left[\left(\frac{u_q(m_+/p)}{\phi_+ p} \right) \mathbf{1}_{\{p \in (\tilde{p}_i, \hat{p}]\}} - 1 \right] \right\} \mathrm{d}J_{i,+}, \\ & + n \int_{\hat{p}}^{\bar{p}} \left\{\varrho_+ \cdot i_d\right\} \mathrm{d}J_{i,+} \end{aligned} \tag{2.14b}$$

and we have made use of the optimal demands for goods, loans and deposits characterized in Equations (2.8) to (2.11).

Equation (2.14a) deserves some commentary. Its right-hand side captures the expected benefit from accumulating an extra unit of money balance to be carried into the DM in the next period. The value of one unit of money balance is captured by ϕ_+ (in units of CM goods). Since money serves as a means of payment in the frictional goods market, it has a liquidity premium captured by the return $1 + r_+(m_+, \mathbf{a}_+)$. Thus, carrying an extra unit of money has a real marginal benefit of $\phi_+ r_+(m_+, \mathbf{a}_+)$.

In contrast to Head et al. (2012), the liquidity premium on holding money in Equation (2.14b) now depends on the banking arrangement. In particular, the premium comprises four possible terms. First, if the household ends up not consuming in the next DM, he can deposit his idle money in the bank to earn interest $i_d > 0$. In other words, banks play the same intermediation-of-liquidity-needs role as those in Berentsen et al. (2007). Second, if the household samples a low enough price, i.e., $p \in \left(\underline{p}, \tilde{p}_i\right]$, he would take out a bank loan. Thus, the second term on the right of Equation (2.14b) captures the expected marginal interest-payment liability saved by borrowing one less unit of money. Third, there is the net benefit from being able to spend an extra unit of money on goods, for the case of the ex-post credit buyer or money-constrained buyer because they draw some price $p \in (\tilde{p}_i, \hat{p}]$. Last, we have the benefit arising from the possibility that the buyer is ex-post money-unconstrained so that at the margin, the unspent money can still earn interest i_d. Whether these additional terms constitute a benefit will depend on the pricing feedback through $J_i(\cdot, p, \mathbf{a})$ and its endogenous support, namely, $\tilde{p}_i$, $\hat{p}$ and p^m.

Substituting Equation (2.14a) into Equation (2.5), we obtain a money demand Euler equation capturing the households' inter-temporal trade-offs:

$$\phi = \beta\phi_{+}[1 + r_{+}(m_{+}, \mathbf{a}_{+})]. \tag{2.15}$$

The left-hand side of Equation (2.15) captures the cost of accumulating money balance: The household forgoes ϕ units of CM consumption goods in order to carry an extra dollar into the next period. The right-hand side of Equation (2.15) is the expected marginal benefit of accumulating an extra dollar associated with the total liquidity premium captured by $r_{+}(m_{+}, \mathbf{a}_{+})$ in Equation (2.14b).

2.4 Firms

Firms in the Decentralized Market (Overview). A unit measure of firms (or sellers of goods) compete in a price posting environment along the lines of Head et al. (2012). In our setting, firms may commit to posting lotteries over pure pricing strategies. We will describe the possibility of a lottery over pure pricing contracts shortly in Lemma 2. For now, consider the reference to a posted contract p given consumer demand, as either a lottery indexed by its expected value p, or a degenerate lottery (which is a pure pricing-strategy) at outcome p.

Consider a firm posting a contract indexed by p. Given demand schedule $q_b^{\star}$ and the distribution of prices posted by firms J_i, its expected profit is given by

$$\Pi_i(p) = \underbrace{\left[\alpha_1 + 2\alpha_2(1 - J_i(p, m, \mathbf{a})) + \alpha_2\nu(p)\right]}_{\text{extensive margin}} \underbrace{R(p, i, m)}_{\text{intensive margin}}, \tag{2.16}$$

where

$$\nu(p) = \lim_{\epsilon \searrow 0} J_i(p, m, \mathbf{a}) - J_i(p - \epsilon, m, \mathbf{a}),$$

and $R(p, i, m) \equiv R(p, i, m, \mathbf{a})$ is the firm's *effective profit* from expecting to price at p. (We will define this profit function in detail in Lemma 2 below.)

The first term in parentheses, labeled *extensive margin*, in Equation (2.16) captures the number of buyers served. With probability α_1, the firm trades with a buyer who has only observed one price quote from this firm and no other. With probability $2\alpha_2[1 - J_i(p, m, \mathbf{a})]$, the buyer purchases the good from this firm because he contacts another firm who has posted a higher price than p. The probability $\alpha_2\nu(p)$ is the measure of buyers that match both this firm and another which has posted the same price (or lottery indexed by) p.[14] The last term, labeled *intensive margin*, captures the firm's profit per customer induced by the firm charging a markup, i.e., posting at an (expected) price above the marginal cost, $p > \phi^{-1}c$).

Observe from Equation (2.16), the firm posting p trades off between an *extensive margin* (i.e., the likelihood of trading with buyers) and an *intensive margin* (i.e., profit per buyer). On the one

[14] Suppose two firms post the same price. We assume that prospective buyers use a tie-breaking rule to pick one firm in such a case. This rule incentivizes an individual firm to lower the price to get the sale. In equilibrium, the probability of a buyer contacting two firms that post the same price goes to zero.

hand, a firm that posts a higher p can earn a higher profit margin per buyer served. However, on the other hand, a firm that posts a higher p suffers by losing sales to other competitors, i.e., a lower likelihood of trading with buyers.

A hypothetical monopolist. As in Burdett and Judd (1983) and Head et al. (2012), we can characterize the distribution of prices $J_i(p, m, \mathbf{a})$, which is an equilibrium object. The lower and upper bound of the distribution's (connected) support will depend on the description of a monopolist's pricing strategy. We provide its characterization here.

Consider a firm serving buyers who have only received one price quote from this one firm. In this case, the firm will behave as a monopolist. The realized profit of a firm setting a monopoly price p^m is

$$\Pi_i^m = \alpha_1 R(p^m, i, m). \tag{2.17}$$

A subtlety in our extension of Head et al. (2012) here is that banking outcome i will affect some agents who, ex-post, may demand loans. As a result, i will also condition or "shift" their demand for the DM good. This, along with how much money a buyer carries into the trade, has consequences for the calculation of a firm's profit and also for the equilibrium distribution of DM-good prices. Unlike Head et al. (2012), the effective ex-post profit function can be non-concave, depending on how much money balance DM buyers carry into the match. (This is a result of the possibility of bank credit for buyers.) Despite this seeming complexity, our generalization turns out to be very tractable: We prove these attributes (in Online Appendix B.1) and show that in terms of pricing, we end up with the following modified characterization of the monopoly price.

Lemma 1. *Let the realized profit at price outcome p from serving a credit-buyer be $G_1(p; i) := [\phi p\,(1+i)]^{-\frac{1}{\sigma}}(\phi p - c)$, from serving a constrained money-buyer be $G_2(p; m) := \frac{m}{p}(\phi p - c)$, and from selling to an unconstrained money-buyer be $G_3(p) := (\phi p)^{-\frac{1}{\sigma}}(\phi p - c)$. Let $\mathbf{g}(p; i, m) := [G_1(p; i), G_2(p; m), G_3(p)]$. Assume that $\sigma \in (0, 1)$ and $c \in (0, 1]$ such that $0 < \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}$. There exists a set of cut-offs relative to $\hat{z}$ (measurable in units of real money balance or the numeraire), with $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$, $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$ and an endogeous $\breve{z}_i$ defining $\mathring{z}_i := \min\{\breve{z}_i, \tilde{z}_i\}$, such that the cut-offs have the particular ordering: $0 < \mathring{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$. Given these cut-offs, we have the following observations:*

1. *The ex-post profit function is*

$$R^{ex}(p, i, m) = \begin{cases} \langle \mathbf{g}(p; i, m), \mathbf{I}_1(p; i, m) \rangle, & \phi m \in [\hat{z}, \infty) \\ \langle \mathbf{g}(p; i, m), \mathbf{I}_2(p; i, m) \rangle, & \phi m \in [\mathring{z}_i, \hat{z}) \\ \langle \mathbf{g}(p; i, m), \mathbf{I}_3(p; i, m) \rangle, & \phi m \in (0, \mathring{z}_i) \end{cases} \tag{2.18}$$

where, letting $\rho \equiv \phi p$,

$$\mathbf{I}_1(p;i,m) := \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\hat{\rho}\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1(a)}} + \underbrace{\mathbf{1}_{\left\{\hat{\rho}<c\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1(b)}}\right],$$

$$\mathbf{I}_2(\rho;i,z) := \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\tilde{\rho}_i<\rho_0^m<\hat{\rho}\leq\rho\right\}\cap\{z\in[\tilde{z}_i,\hat{z})\}}}_{\text{Case 2(a)}} + \underbrace{\mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\leq\rho\right\}\cap\{z\in[\mathring{z}_i,\tilde{z}_i)\}}}_{\text{Case 2(b)}}\right],$$

$$\mathbf{I}_3(\rho;i,z) := \underbrace{\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \mathbf{1}_{\{c<\hat{\rho}\leq\rho\}}\right] \times \mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\right\}}}_{\text{Case 3}},$$

$\rho_0^m = c/(1-\sigma)$, $\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$, $\tilde{\rho}_i \equiv \tilde{\rho}(i,z) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, $\langle\cdot,\cdot\rangle : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \to \mathbb{R}_+$ is the inner-product operator, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.

2. *The monopoly price and its ex-post profit outcome, respectively, are*

$$p^m = \begin{cases} p_0^m, & \phi m \in [\hat{z}, \infty) \\ \hat{p}(m), & \phi m \in [\mathring{z}_i, \hat{z}) \\ p_0^m, & \phi m \in (0, \mathring{z}_i) \end{cases}, \qquad \textit{and,} \qquad R^{ex}(p^m, i, m) = \begin{cases} G_3(p_0^m), & \phi m \in [\hat{z}, \infty) \\ G_3(\hat{p}(m)), & \phi m \in [\mathring{z}_i, \hat{z}) \\ G_1(p_0^m; i), & \phi m \in (0, \mathring{z}_i) \end{cases}. \tag{2.19}$$

The set of parameters (σ, c) satisfying the inequalities in Lemma 1 is non-empty. For example, if $c = 1$ as in Head et al. (2012), then the sufficient conditions on parameters reduce to $0 < \sigma\left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}} < \left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, which admits any $\sigma \in (0,1)$.

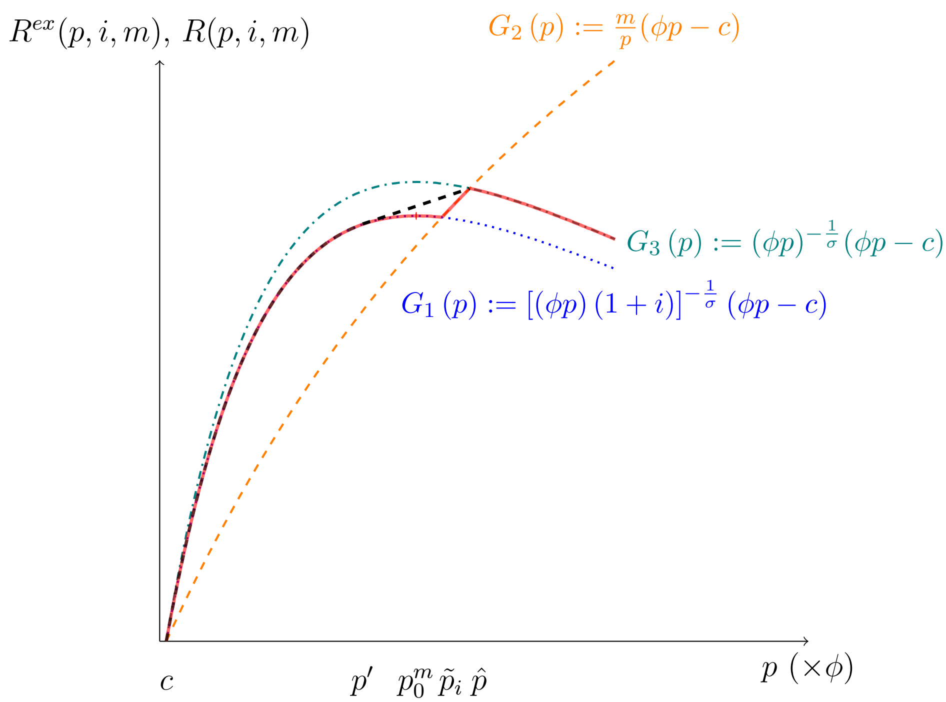
As a consequence of the possibility of earning profit from credit buyers the ex-post per-meeting profit function of firms (in terms of pure pricing strategies) may exhibit segments that are not necessarily monotone increasing up to the monopoly profit point. Also, the profit function may not be strictly concave.[15] Technically, this may pose a problem for the characterization of the equilibrium Burdett-Judd style pricing distribution, as it depends on the monotonicity of $R^{ex}(p, i, m)$ in p. Economically, this also suggests that it may be profitable in some subset(s) of the set of pricing outcomes for firms to be posting random terms of trades—i.e., lotteries over the pure pricing outcomes.[16]

[15]Such non-monotonicities and non-convexities can be interpreted as an artefact of the externality of credit buyers on firms' profitability calculations. We shall see in equilibrium, it is possible (depending on policy and parameters) that such an externality can have a net negative impact on aggregate welfare. In Head et al., 2012, there is no such technical complication since buyers are either cash constrained or not, and there is no feedback between banking and goods-firms pricing.

[16]The idea of posting random contracts already exists in monetary trade, in general equilibrium, or in labor economics (respectively, Berentsen, Molico and Wright, 2002; Chatterjee and Corbae, 1995; Shell and Wright, 1993; Rogerson, 1988), in settings with private information (Prescott and Townsend, 1984) and in models of dynamic price discrimination (Sobel, 1984). In our application, the presence of bank credit and the possibility of demand for goods from buyers with bank credit may render local non-concavity and possibly local non-concave-and-convex

Figure 1 illustrates Case 2(b) from Lemma 1, i.e., where real money balance is low enough $\phi m \in [\mathring{z}_i, \tilde{z}_i)$.[17] In this case, the maximal profit from serving cash-constrained buyers and pricing at such buyers' maximal willingness to pay, $\hat{p}(z)$, can exceed the maximal profits from serving either credit-buyers or unconstrained money buyers. The dashed-black graph in the figure is the convexification of a firm's ex-post profit function via lotteries over pure price posts. (See the proof of Lemma 1 for a complete characterization and graphical illustrations for all the possible cases.) We will refer to a profit function induced by the posting of random contracts (which is relevant for our equilibrium description) as an *effective profit* function.

Figure 1: Example from Case 2(b) in Lemma 1 where $z \in [\mathring{z}_i, \tilde{z}_i)$. Posting of random contracts is profitable for firms. This yields a strictly increasing and concave *effective profit function* $R(\cdot, i, m)$ (dashed-black graph) on the relevant domain $[c, \phi\hat{p}]$. The ex-post profit function $R^{ex}(\cdot, i, m)$ under pure pricing strategies is illustrated by the solid-red line (whose graph exhibits non-convexity).



In the next result, we show that for all possible cases, the effective profit function $R(\cdot, i, m)$ is monotone increasing and concave in pricing outcomes, so long as firms are allowed to post lotteries over their pricing contracts. We relegate the detailed proof to Online Appendix B.2.

Lemma 2. *Given predetermined and aggregate outcomes (m, i, ϕ), the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, m)$ in Equation (B.1) in Lemma 1, there exists a set of cut-offs*

segments in firm's ex-post profit functions. This possible non-strict-concavity of firms' ex-post profit function does not arise in the monetary setting of Head et al. (2012) where their firms' ex-post profit functions are always strictly concave. We will see that such lotteries can yield firms a weakly welfare-improving payoff, but one that is still no greater than the hypothetical Burdett and Judd (1983) monopoly profit in any setting. The use of lotteries will also ensure a well-behaved equilibrium characterization of the distribution function of pricing outcomes.

[17] The interval $[\mathring{z}_i, \tilde{z}_i)$ can be derived. We relegate the tedious and mechanical details to Online Appendix B.1).

relative to $\hat{z}$ (measurable in units of real money balance or the numeraire), with $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$, $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$ and an endogeous $\breve{z}_i$ defining $\mathring{z}_i := \min\{\breve{z}_i, \tilde{z}_i\}$, such that the cut-offs have the particular ordering: $0 < \mathring{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$.

1. *A firm's **effective profit** at any given reference price p, $R(p,i,m)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

$$
\begin{aligned}
&R(p,i,m) \\
&\quad = \max_{\pi\in[0,1],p_1,p_2} \left\{\pi R^{ex}(p_1,i,m) + (1-\pi)R^{ex}(p_2,i,m) : \pi p_1 + (1-\pi)p_2 = p\right\}. \quad (2.20)
\end{aligned}
$$

The function $R(\cdot,i,m)$ is strictly increasing on $[\phi^{-1}c, p^m)$, and is concave over the firm's effective domain of pricing outcomes $[\phi^{-1}c, p^m]$, where the monopoly price is p^m and its effective profit outcome is $R(p^m,i,m) = R^{ex}(p^m,i,m)$ and these are characterized by

$$
p^m = \begin{cases} p_0^m, & \phi m \in [z', \infty) \\ p_0^m, & \phi m \in [\hat{z}, z') \\ \hat{p}(m), & \phi m \in [\tilde{z}_i, \hat{z}) \\ \hat{p}(m), & \phi m \in [\mathring{z}_i, \tilde{z}_i) \\ p_0^m, & \phi m \in (0, \mathring{z}_i) \end{cases}, \textit{ and, } R(p^m,i,m) = R^{ex}(p^m,i,m) = \begin{cases} G_3(p_0^m), & \phi m \in [z', \infty) \\ G_3(p_0^m), & \phi m \in [\hat{z}, z') \\ G_3(\hat{p}(z)), & \phi m \in [\tilde{z}_i, \hat{z}) \\ G_3(\hat{p}(z)), & \phi m \in [\mathring{z}_i, \tilde{z}_i) \\ G_1(p_0^m), & \phi m \in (0, \breve{z}) \end{cases}. \quad (2.21)
$$

2. *Depending on ϕm, the largest domain containing equlibrium pricing outcomes has the following properties:*

   Case-1(a). If $\phi m \in [z', \infty)$, firms will only have incentive to serve money-unconstrained buyers.

   Case-1(b). If $\phi m \in [\hat{z}, z')$, all three types (credit, money-constrained and money-unconstrained buyers) will be served.

   Case-2(a). If $\phi m \in [\tilde{z}_i, \hat{z})$, only two types—credit and money-constrained buyers—will be served.

   Case-2(b). If $\phi m \in [\mathring{z}_i, \tilde{z}_i)$ (and i is such that this set is non-degenerate), then only two types—credit and money-constrained buyers—will be served.

   Case-3. If $\phi m \in (0, \mathring{z}_i)$, then only credit buyers are served.

Pricing equilibrium. Previewing an equilibrium, firms will earn the same expected profit for any p in the support of the distribution, $\text{supp}(J_i(\cdot,m,\mathbf{a})) = [\underline{p}, \overline{p}]$. That is, they will be indifferent between a continuum of different extensive-intensive margin trade-offs:

$$
\Pi_i^\star = \max_p \Pi_i(p) \quad \text{for all } p \in \text{supp}(J_i(\cdot,m,\mathbf{a})). \quad (2.22)
$$

Lower price firms make up their profit through higher sales volume while higher price firms gain through higher markups.

As in Head et al. (2012), if some buyers observe only one price quote whereas others observe more than one, then this leads to a non-degenerate distribution of prices $J_i(\cdot, m, \boldsymbol{a})$.[18] Since firms expect the same profit outcomes associated with the continuum of markup-versus-trading-probability strategies, then this implies an equal-profit condition. Specifically, equating Equation (2.16) and Equation (2.17), we can derive a closed-form distribution of prices. We summarize this result as follows.[19]

Lemma 3. *Given monetary policy $\gamma > \beta$, aggregate outcomes (m, i, ϕ), and noisy search frictions $\alpha_1, \alpha_2 \in (0, 1)$, the price distribution consistent with profit maximization by all firms is given by*

$$J_i(p, m, \mathbf{a}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{p}, i, m, \mathbf{a})}{R(p, i, m, \mathbf{a})} - 1\right], \tag{2.23}$$

where the lower and upper bounds on the support of $J_i(\cdot, m, \boldsymbol{a})$ are, respectively, determined by $R(\underline{p}, i, m, \mathbf{a}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{p}, i, m, \mathbf{a})$ where $c < \underline{p} < \overline{p} := p^m$, and p^m is governed by Equation (2.19).

Lemma 3 highlights that if some buyers receive only one price (lottery) quote while others receive more than two, the price distribution is continuous and non-degenerate. Moreover, firms are ex post pricing the goods above the marginal cost of production. In contrast to Head et al. (2012), the banking loan interest rate i now matters for determining the good-price distribution. This is a consequence of the buyer's optimal goods demand schedule interacting with credit, which affects the firm's pricing strategy.

Firms in the Centralized Market. In the CM, there is a unit measure of perfectly competitive firms producing the general goods x using a linear production technology in labor h. They then sell the goods to households in the CM. Consequently, both the real wage and price of the DM goods are equal to one.

2.5 Banks

We focus on the liquidity transformation role of banks. The banking sector is perfectly competitive with free entry as in Berentsen et al. (2007). In particular, banks accept deposits d and commit to repaying depositors with interest i_d. Banks then allocate deposits to issue loans l at the interest rate of i to borrowers.[20]

[18]The model has two parametric limits: one with Bertrand pricing (by setting $\alpha_2 = 1$) and one that resembles monopoly (by setting $\alpha_1 = 1$). For our purposes, we focus on cases away from these two parametric limits to rationalize the empirical finding in Section D.

[19]Given Lemma 2, the proof of Lemma 3 follows directly from Head et al. (2012). We omit the details here.

[20]In the equilibrium characterization below, we shall see that the deposit rate will be bid up to the loan rate, $i = i_d = i^\star$, and $i^\star$ is determined by a loan-market-clearing condition where there is perfect competition and free entry. We have assumed that there are no operating costs or reserve requirements in the banking industry. If we relax this assumption, there will be a wedge between the loan rate and the deposit rate. Since we want to focus on the dependency of firms' market power on banking, it suffices to study the case where even perfectly competitive banking can exarcebate goods markup pricing outcomes. One can think of this as a lower-bound case on the severity

2.6 Stationary monetary equilibrium (SME)

We focus on stationary outcomes of the economy. Since the price of the general goods P is used as a unit of account, we then multiply all nominal variables by the value of money balance $\phi = 1/P$ (in units of the CM goods x) from here onward. In particular, we let $z = \phi m$ denote the individual real money balance and $Z = \phi M$ denote the aggregate real money balances; $\rho = \phi p$ denote the real relative price of goods across the DM and the CM; and $\xi = \phi l$ and $\delta = \phi d$ respectively denote the real balances of loans and deposits. For the ease of notation, we also let the variable $\mathbf{s} := (Z, \gamma)$ denote the aggregate state of the economy consisting of total real money stock and monetary policy $\gamma = 1 + \tau$. In a stationary equilibrium, all nominal variables grow at a time-invariant rate according to $\phi/\phi_{+1} = M_{+1}/M = \gamma = 1 + \tau$ and real variables stay constant over time.

Before we provide a summary of the equilibrium characterization, we first present two features that are different in contrast to Head et al. (2012) as follows.

In a stationary monetary equilibrium (SME), the real analog of the price distribution characterized in Lemma 3 is given by:

$$J_i(\rho, z) := J_i(\rho, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z, \mathbf{s})}{R(\rho, i, z, \mathbf{s})} - 1\right] \tag{2.24}$$

where the upper bound on the support of $J_i(\cdot, z)$ was derived in Lemma 2. In stationary variables, this is given as:

$$\bar{\rho} := \rho^m(i, z) = \begin{cases} \rho_0^m, & z \in [\hat{z}, \infty) \\ \hat{\rho}(z), & z \in [\mathring{z}_i, \hat{z}) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases} . \tag{2.25}$$

As in (Burdett and Judd, 1983), the lower bound on the support of $J_i(\cdot, z)$, $\underline{\rho}$, is a solution to the equal expected profit condition:

$$R(\underline{\rho}, i, z) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{\rho}, i, z). \tag{2.26}$$

Observe that in Equation (2.24), the distribution of goods (real) prices now depends on both households' real money holdings z and the competitive loan interest rate $i = i_d = i^\star$ (determined by the loans market clearing condition). Consequently, there are two possible cases regarding the households' optimal demand for real money balances in an SME. We summarize this possibility in the following Lemma.

Lemma 4. *Let monetary policy be such that inflation is away from the Friedman rule, $\gamma > \beta$, and assume that $\alpha_1, \alpha_2 \in (0, 1)$. In a SME, the liquidity premium on holding money over one period*

this problem: If we also make the banking sector non-competitive, our qualitative conclusions would remain but the effects of this new nexus would only be magnified quantitatively.

(2.14b) *is*

$$r(z,\mathbf{s}) := (1-n)\,i_d + n\int_{\underline{\rho}}^{\tilde{\rho}_i} \{\varrho(z,\mathbf{s})\cdot i\}\,dJ_i(\rho,z,\mathbf{s})$$
$$+\,n\int_{\tilde{\rho}_i}^{\bar{\rho}} \left\{\varrho(z,\mathbf{s})\left[\left(\frac{u_q(z/\rho)}{\rho}\right)\mathbf{1}_{\{\rho\in(\tilde{\rho}_i,\hat{\rho}]\}}-1\right]\right\}dJ_i(\rho,z,\mathbf{s}) \tag{2.27}$$
$$+\,n\int_{\hat{\rho}}^{\overline{\rho}} \{\varrho(z,\mathbf{s})\cdot i_d\}\,dJ_i(\rho,z,\mathbf{s}),$$

where $\varrho(z,\mathbf{s}) := \alpha_1 + 2\alpha_2\,(1-J_i(\rho,z,\mathbf{s}))$*, and the demand for real money balance* z *satisfies:*

$$\frac{\gamma}{\beta} = 1 + r(z,\mathbf{s}). \tag{2.28}$$

Lemma 4 reveals the inter-dependency of agent's (ex-ante) precautionary money demand on an endogenous channel between bank credit and non-competitiveness in the DM for goods. The market interest rate, $i = i_d = i^\star$, determines the credit condition. The distribution of goods prices, $J_i(\cdot,z)$, pins down the degree of firms' market power in the goods market.

The equilibrium money demand in Lemma 4, which takes into account equilibrium firm pricing behavior encoded in $J_i(\cdot,z,\mathbf{s})$, admits several possible equilibrium configurations. When read together with Lemma 2, Lemma 4 implies the potential for different configuration of buyer/payment types co-existing in a SME. For example, a parametrization of the model may yield an ex-post mixture of credit–buyers and money–buyers in equilibrium (see Cases 1(b), 2(a) and 2(b), with variations on whether money-buyers include the money-unconstrained ones or not). The credit-buyers are those buyers who draw a sufficiently low price, i.e., $\underline{\rho} \leq \rho \leq \tilde{\rho}_i(z)$. The money-buyers are those who draw a sufficiently high price, i.e., $\tilde{\rho}_i(z) < \rho \leq \overline{\rho}(z)$. The implication is an endogenous channel connecting bank credit condition and firms' market power through the equilibrium pricing distribution. In equilibrium, this channel matters for the agent's (ex-ante) precautionary demand incentives regarding how much real money balances to carry to trade in the following period. Conditional on $\alpha_1, \alpha_2 > 0$—i.e., the distribution $J_i(\cdot,z,\mathbf{s})$ is non-degenerate—there are two interesting extreme possibilities. First, if equilibrium z is high enough (i.e., money is approaching zero holding cost) firms will only want to sell to money-unconstrained agents (see Case 1(a) in Lemma 2). Second, depending on the cut-off value $\mathring{z}_i$ in Lemma 2 that is endogenous to banking market outcome, i, we may have the case that firms will only serve credit buyers (see Case 3 in Lemma 2). Both of the two extreme and theoretically-possible cases imply that there is no pecuniary externality running from credit buyers to money constrained buyers. Theoretically, if $\alpha_1 = 1$ so that $J_i(\cdot,z,\mathbf{s})$ is degenerate at the Bertrand price for i, the liquidity premium term in Equation (2.27) reduces to the same expression as in Berentsen et al. (2007).

Definition 1. Given monetary policy $\gamma \geq \beta$, and taxes/transfers T, a stationary monetary equilibrium is a steady-state allocation $(z^\star, x^\star, h^\star)$ and $\{q_b^\star(\rho,i,z), \xi^\star(\rho,i,z), \delta^\star(\rho,i,z)\}$, and pricing behavior $(J_i^\star(\rho, z), i)$ such that the following conditions are satisfied:

1. The upper bound on the support of $J_i^\star(\cdot, z)$, $\bar{\rho} := \rho^m(z, i)$ satisfies (2.25), and the lower bound on the support of $J_i^\star(\cdot, z)$ solves the equal expected profit condition (2.26).

2. The triple $(h^\star, x^\star, z^\star)$ solves the CM households problem, including the households' ex-ante real money demand decision in Equation (2.28).

   (a) Given $z = z^\star$, $\{q_b^\star(\rho, i, z), \xi^\star(\rho, i, z), \delta^\star(\rho, i, z)\}$ satisfy:

$$q_b^\star(\rho, i, z) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \le \tilde{\rho}_i \\ \frac{z}{\rho} & \text{if } \tilde{\rho}_i < \rho \le \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \rho > \hat{\rho} \end{cases}, \tag{2.29}$$

$$\xi^\star(\rho, i, z) = \begin{cases} \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z & \text{if } 0 < \rho \le \tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i < \rho \le \hat{\rho} \\ 0 & \text{if } \rho > \hat{\rho} \end{cases}, \tag{2.30}$$

   and,

$$\delta^\star(\rho, i, z) = \begin{cases} 0 & \text{if } 0 < \rho \le \tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i < \rho \le \hat{\rho} \\ z - \rho^{\frac{\sigma-1}{\sigma}} & \text{if } \rho > \hat{\rho} \end{cases}, \tag{2.31}$$

   where

$$\hat{\rho} \equiv \hat{\rho}(z, \mathbf{s}) = z^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{\rho}_i \equiv \tilde{\rho}_i(i, z, \mathbf{s}) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}. \tag{2.32}$$

   (b) $J_i^\star(\cdot, z)$ solves the DM firms' problem characterized in Equation (2.24).

   (c) Given $z = z^\star$, $i = i_d = i^\star$ clears the loan market:

$$(1-n)z + n\int_{\hat{\rho}}^{\bar{\rho}} (z - \rho^{1-1/\sigma})\left(\alpha_1 + 2\alpha_2\left[1 - J_i(\rho, z)\right]\right) \mathrm{d}J_i(\rho, z)$$
$$= \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} \xi^\star(\rho, i, z)\left(\alpha_1 + 2\alpha_2\left[1 - J_i(\rho, z)\right]\right) \mathrm{d}J_i(\rho, z). \tag{2.33}$$

The system reduces to finding two unknowns $z^\star$ and $i^\star$ simultaneously. We can back out all the other endogenous variables as a function of $z^\star$ and $i^\star$. The left-hand side of Equation (2.28) captures the opportunity cost of carrying one extra unit of money into the next period. The right-hand side of Equation (2.28) represents the expected net return of holding money that can be decomposed into three terms. The first term reflects the marginal benefit of depositing an extra unit of idle money balances. The second term captures the interest saved by borrowing one less unit of money balances. The last term is the net marginal benefit of spending an extra dollar.

The following observation says that a buyer with a lower real money balance is more likely to draw a higher price from the distribution.

Lemma 5. *Fix a monetary policy at $\gamma > \beta$ and assume $\alpha_1, \alpha_2 \in (0,1)$. Given $i = i_d = i^\star > 0$, consider any two real money balances z and z' such that $z < z'$. The price distribution $J_i(\cdot, z)$ first-order stochastically dominates $J_i(\cdot, z')$. Also, the pricing cutoffs $\tilde{\rho}_i$ (the maximal willingness to pay for a credit buyer) and $\hat{\rho}$ (the maximal willingness to pay of a money-constrained buyer) are decreasing functions of z.*

The proof is in Online Appendix B.3. The reasoning behind Lemma 5 is as follows. Suppose a buyer carries a small amount of real money balance into the goods market. Firms expect to produce and sell a lower quantity of goods. A measure of firms will optimally respond by charging higher prices relative to their marginal cost of production. Consequently, the distribution of goods prices is more dispersed. The buyer with a tighter liquidity constraint is more likely to draw a higher price (or an associated markup) from the distribution. Therefore, the net benefit of banking in equilibrium should be ambiguous in contrast to Berentsen et al. (2007). Here, the gains from accessing a competitive banking sector depend on the interaction between agents' precautionary demand for money holdings and endogenous firms' market power in the goods market.[21]

Using the result established in Lemma 5, we can then show the existence of a stationary monetary equilibrium with both money and credit. Such an equilibrium entails price dispersion in the frictional goods market. We summarize the result in the following Proposition. Details of the proof are in Appendix B.4.[22]

Proposition 1. *Let monetary policy be $\gamma > \beta$ and noisy search frictions be $\alpha_1, \alpha_2 \in (0,1)$. There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

Remark. Recall the discussion earlier surrounding Lemmata 1 to 4 that for generic parameters there can theoretically be different equilibrium configurations where there may exist only money-constrained buyers or some mixtures over the possible types of money and credit buyers. However, we should note that when we discipline the model by calibration to the data later, we shall see that the second case in Lemma 4 (SME with a mixture of constrained- and unconstrained-money buyers and credit-buyers) will be the relevant case—and this is also the most interesting one. In this case, the pecuniary externality is always present. Also, this equilibrium case will always occur for the plausible range of long-run inflation-rate experiments around the empirically calibrated model.

[21] This is the novelty here in contrast to the special case where there is no banking or financial intermediation—i.e., the equivalent Head et al. (2012) setting. In Section 3.1, we will illustrate and decompose the effect of this pecuniary externality channel; and we will show that how severe this effect is in offsetting the liquidity risk insurance role of banks depends on long-run inflation or monetary policy.

[22] This will also help to simplify our characterization of the stationary monetary equilibrium by ruling out the possibility of an extreme equilibrium case—i.e., Case 1(a) of Lemmata 1 and 2—in which agents anticipate only unconstrained money buyers getting served in the DM goods trades.

3 Equilibrium trade-off and welfare effect of banking

In the remainder of the paper, we will focus attention on SMEs where money buyers and credit buyers co-exist in equilibrium. We can see in Definition 1 the equilibrium trade-off between the benefit of banking and its uninternalized social cost on consumer-goods prices (i.e., the *pecuniary externality*): On the one hand, banking is beneficial because it increases the expected net return of holding money. This can be deduced from reading the first and second terms in Equation (2.28). On the other hand, firms' market power (price markups and dispersion) in frictional goods trades can also reduce some of the gains from banking. Banking, through competitive outcome i, affects the agents' precautionary demand for money holdings z, which then feeds back onto the distribution of goods prices $J_i(\rho, z)$, and its support, $\text{supp}(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$. In particular, the integrals on the right-hand side of Equation (2.28) capture the reduction in the expected return on money even though agents have access to a competitive banking sector. This works through the results in Lemma 5: the first-order-stochastic-dominance in $J_i(\cdot, z)$ and the associated increasing pricing-cutoff function $\tilde{\rho}_i$, as z falls.

In the following Section 3.1, we explore this trade-off further. We analytically dissect the model through its special cases in order to identify an equilibrium tension between competitive banks' role in facilitating insurance of individuals' liquidity risks and the externality that such bank credit may have on non-credit users in the economy. The resolution of such a tension ultimately depends on inflation policy. In Section 3.2, we further use the calibrated model to illustrate how the trade-off changes with inflation in the long run and what the resulting welfare implications are for banking in such an economy.

3.1 Inspecting the trade-off

Overview. It is useful to compare our setting to that without banking. In particular, if we remove the banking sector, we get the case of a pure monetary economy with firm market power studied in Head et al. (2012) (HLMW). Let $\bar{z}$ denote the equilibrium real money balance under the HLMW economy. In this special case, Equation (2.28) becomes

$$\frac{\gamma - \beta}{\beta} = n \left[\int_{\underline{\rho}(\overline{z})}^{\overline{\rho}(\overline{z})} \left[\alpha_1 + 2\alpha_2 (1 - \tilde{J}(\rho, \overline{z})) \right] \left(\frac{u_q[q^{no-bank}(\overline{z})]}{\rho} - 1 \right) \mathrm{d}\tilde{J}(\rho, \overline{z}) \right], \tag{3.1}$$

where the price distribution in a no-bank monetary economy is given by

$$\tilde{J}(\rho, \overline{z}) = 1 - \frac{\alpha_1}{2\alpha_2} \left[\frac{\tilde{R}(\overline{\rho})}{\tilde{R}(\rho)} - 1 \right], \tag{3.2}$$

and the bounds are given by $\tilde{R}(\underline{\rho}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \tilde{R}(\overline{\rho})$, and $\overline{\rho} = \max \left\{ \frac{c}{1-\sigma}, \underbrace{\overline{z}^{\frac{\sigma}{\sigma-1}}}_{=:\hat{\rho}} \right\}$, and the real profit per customer served is

$$\tilde{R}(\rho, \overline{z}) = q_b^{no-bank}(\rho, \overline{z})(\rho - c), \tag{3.3}$$

and buyer's optimal demand for goods is given by

$$q_b^{no-bank}(\rho, \overline{z}) = \begin{cases} \frac{\overline{z}}{\rho} & \text{if } 0 < \rho \leq \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \hat{\rho} < \rho \end{cases}. \tag{3.4}$$

Consider anticipated inflation away from the Friedman rule $\gamma > \beta$. In our setting and in that of Head et al. (2012), agents have precautionary demand for money holdings. Anonymity in the goods market gives rise to money as a means of payment. However, inflation ($\gamma > \beta$) induces a rate of return on money that is lower than the risk-free rate or the rate of time preference. Inflation is thus a tax on frictional goods trades. Hence, holding money can be costly when agents (ex-post) are stuck with unproductive idle money balances.

With access to banks, households can now reduce the cost of having unneeded liquidity (via depositing idle funds in the bank to earn interest). In addition, households can borrow extra money balances from the bank. Credit extended by banks helps households to relax their liquidity constraints when they need to make a payment in the goods market. We call this positive welfare effect of banking a *liquidity-risk insurance effect*, which works through an identical mechanism as in Berentsen et al. (2007) (BCW).

To illustrate the potential gains from banking, let us contrast this with a pure monetary economy and BCW, assuming a Walrasian price-taking protocol in the DM goods trades in both economies for the ease of presentation. Moreover, we maintain the assumption of a linear cost of production ($c(q) = q$) in the DM as in BCW. First, a no-bank equilibrium condition for money demand is given by:

$$\frac{\gamma}{\beta} = n\left(\frac{u'(q)}{c'(q)}\right) + (1-n) \times 1, \tag{3.5}$$

where the right-hand side captures the (gross) marginal benefit per extra dollar accumulated, which consists of two components. The first term reflects the benefit goes to agents who get to spend it (with probability n) in the next DM, where the second term reflects those who don't get to spend their money balances (with probability $1-n$) are holding their money balance idle and earning zero interest. Rearrange, it yields

$$1 + \frac{1}{n}\left[\frac{\gamma - \beta}{\beta}\right] = u'(q). \tag{3.6}$$

Conside the left-hand side. Since the measure of active buyers $n \in (0,1)$, then it acts a factor inflating the opportunity cost of money holding. The opportunity cost of holding money is higher the fewer active buyers there are. We refer to this as the *liquidity risk*, which is relevant in a (constrained) monetary equilibria ($\gamma > \beta$). When agents are subject to preference/consumption shocks, ex-post allocations in (constrained) monetary equilibria are typically inefficient since there are some agents who are liquidity constrained and some are holding unproductive idle funds. In contrast, in BCW's banking equilibrium the money demand condition becomes

$$\frac{\gamma}{\beta} = n\underbrace{\frac{u'(q)}{c'(q)}}_{=1+i_l} + (1-n)(1+i_d) \equiv u'(q) \equiv 1+i^{\star}, \tag{3.7}$$

where a loan market clearing condition $i_d = i_l = i^{\star}$ is imposed. This precisely says that the opportunity cost of holding money (borne by what would have been idle-money holders) is fully compensated at the margin by the interest rate earned on deposits. Algebraically, the liquidity risk "inflation factor", $1/n$, is eliminated by the existence of perfectly competitive banks. Such liquidity risk matters and there are gains from banking in equilibria away from the Friedman rule ($\gamma > \beta$).[23] The positive welfare effect of banking here is also robust to an alternative trading protocol with bargaining (see Berentsen et al., 2007).

Now, in our baseline setting with money and credit in equilibrium (cases 1(b), 2(a), 2(b) and 3 in Lemma 2), the liquidity risk "inflation factor" above is not always eliminated by the perfectly competitive banking system. This is due to the fact that noisy search frictions ($0 < \alpha_2 < 1$) induces price competition among firms (an equilibrium non-degenerate distribution of goods prices). The expected marginal benefit of holding money—the integral terms on the right-hand side of Equation (2.28)—depends on the equilibrium distribution $J_i(\cdot, z)$ (an outcome of goods market power) and this depends on equilibrium interest on credit, i. The net benefit of banking here can be ambiguous because of this policy-dependent interaction. This is because even though banks here are perfectly competitive as in Berentsen et al. (2007), imperfect (pricing) competition among firms can give rise to an additional *price dispersion effect* that can contribute to a negative welfare effect of banking. Effectively, the policy-dependent interaction of banking and the search-based market power of firms creates an uninternalized social cost on agents' asset accumulation decision, which has a direct consequence on consumption outcomes in monetary equilibria. Here, imperfect competition among firms in goods trades (arising from search and informational frictions) could potentially hinder an otherwise useful banking system. This possibility is also a novel feature than that in economies with price-taking and bargaining trading protocols. The mechanism is as follows.

Decomposing the welfare effects of banking. To understand the positive and negative welfare effects of banking, we compare Equation (2.28) and Equation (3.1). In our model economy, buyers can deposit funds in the bank to earn interest $i_d > 0$ if they ex-post don't get to spend their money and/or not going to spend all of their money balances (those unconstrained money buyers who sample sufficiently high prices). We label this type of buyers as depositors. The interest paid to depositors increases the expected marginal benefit of accumulating money balances (see the first and the forth terms in Equation (2.28)). This is the same (and sole) benefit of banking in Berentsen et al. (2007).

In addition now, buyers who are liquidity constrained and sample low enough prices of the goods can use bank credit to relax their liquidity constraint (ex-post). The second term on the right-hand side of Equation (2.28) reflect such banking benefits. Due to the liquidity-risk insurance

[23] It is also notice that our baseline model converges to this result when noisy search frictions vanish (i.e., buyers always sample multiple price offers with probability $\alpha_2 = 1$).

effect, banking helps to improve consumption allocation relative to HLMW, on the one hand.

On the other hand, access to credit by buyers can also lower the expected marginal benefit of money when firms can exploit markups in frictional goods trades. In particular, the integrals on the right-hand side of Equation (2.28) capture the negative welfare effects of banking which we label as the *pecuniary externality* or *price dispersion* effect. The reason is as follows.

Firms expect some potential customers to be liquidity constrained by their money balances, and their expenditure rule is inelastic. A measure of firms would then optimally respond by charging higher markups (see Lemma 5). This will affect both the liquidity constrained and unconstrained money-buyers. The former will face a tighter liquidity constraint as the real value of their money goes down so they end up with less goods. The latter, although unconstrained, still best respond by consuming less, since their demands are decreasing in the prices they draw. Moreover, unconstrained money-buyers could now also deposit some of their residual money balances to avoid the inflation tax, since they are not going to spend all of their money. Credit-buyers inadvertently contribute to the bidding up of DM goods prices (both the average level and the dispersion): In the model, this shows up in the form of the support of the goods price distribution, $\text{supp}\,(J(\cdot, z)) = [\underline{\rho}(z), \overline{\rho}(z)]$, being wider than that in HLMW. Specifically, the lowest possible price that a liquidity-constrained (and unconstrained) money buyer can draw becomes higher as ex-ante real balance falls (Lemma 5). In other words, access to credit by buyers amplifies firms' market power in terms of price markups and dispersion.

Recall that banking credit only benefits some buyers but not all. In particular, from Equations (2.29) and (2.30), buyers use credit if they draw a sufficiently low price ρ on goods from the distribution $J_i(\cdot, z)$. However, as we have deduced, banking credit induces higher price dispersion, which implies more high-price firms extracting rent from liquidity constrained money-buyers. Both integrals on the right-hand side of Equation (2.28) capture a reduction in the expected return on holding money along the rising price dispersion. Hence, a distortion will appear in the average interest saved on borrowing an extra dollar for the credit-buyers and the liquidity premium for the money-buyers. Thus, firms' market power in frictional goods trades can potentially reduce gains from a competitive banking sector.

Numerical illustration. Next, we provide a numerical example of the mechanism outlined above by comparing our baseline model economy to that without banking (HLMW), for a given long run monetary policy setting τ.[24]

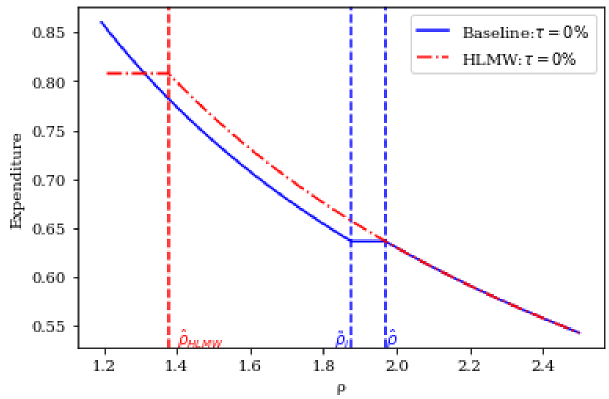
Figure 2 displays the liquidity-risk insurance and price dispersion effects of banking given policy $\tau > \beta - 1$. (Without loss, we plot the case of $\tau = 0\%$.) The dashed-dotted red graph and associated dashed-red pricing cutoff $\hat{\rho}_{HMLW}$ represent the model economy in HLMW. The solid blue graph with dashed blue cutoffs $\tilde{\rho}_i$ and $\hat{\rho}$ represent our baseline model economy (with existence of banking).

Liquidity-risk insurance effect: positive welfare effects of banking. In HLMW, a buyer cannot spend more than his liquidity constraint when faced with a price draw that is at most

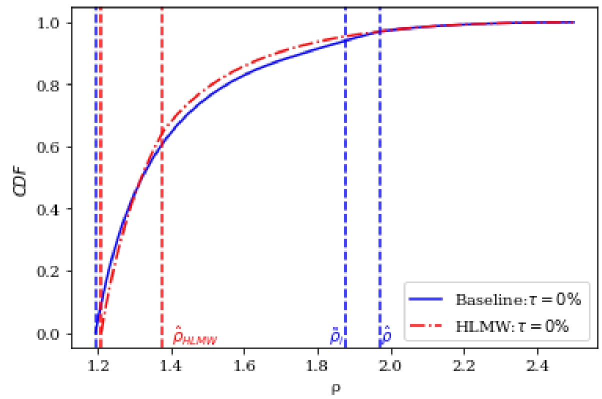
[24] We will focus only on the ex-post buyer types and put aside the obvious Berentsen et al. (2007)-like benefit of banking to the non-consuming depositors. Their surplus will nevertheless be accounted for in our final welfare calculation. The parameter values for this numerical illustration is given by $\beta = 0.98, \tau = 0, \sigma = 0.6, \alpha_1 = 0.6, \alpha_2 = 0.4, n = 0.8, B = 1.8$.

Figure 2: Liquidity-risk insurance and price dispersion effects given policy $\gamma = 1 + \tau > \beta$.

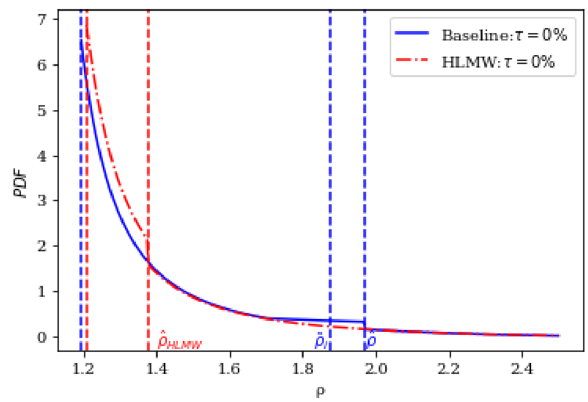
(a) Liquidity-risk insurance effect



(b) Price dispersion effect



(c) Price dispersion effect



$\hat{\rho}_{HLMW}$. The horizontal part of the dashed red graph in Figure 2a reflects the set of expenditure levels of such a type of (ex-post) liquidity constrained buyers. The cut-off $\hat{\rho}_{HMLW}$ is the price level at which the buyer becomes liquidity unconstrained ex-post. Such a buyer spends less than her total money balances if she draws a price higher than $\hat{\rho}_{HMLW}$.

Consider now the solid blue graph in Figure 2a. In contrast to HLMW, there is now a liquidity-risk insurance effect highlighting the benefits of having access to banking credit. A buyer can now borrow additional money balances from the bank to relax his liquidity constraint when $\rho \leq \tilde{\rho}_i$. Thus, the (ex-post) credit-buyer faces a more relaxed liquidity constraint to spend more in the goods market than money-buyers. The (non-credit) money constrained buyers in this case are the ones on the horizontal segment of the solid blue graph — i.e., the ones who draw a ρ from the interval $(\tilde{\rho}_i, \hat{\rho}]$. The (non-credit) money-unconstrained buyers have a downward sloping expenditure function over all $\rho > \hat{\rho}$.

From Figure 2a, we can deduce that credit buyers can potentially benefit from higher expenditures, whereas the money-constrained buyers now can only afford lower expenditures, relative to the HLMW (no-bank) economy. However, this is not the complete picture as, with noisy search, one also has to take into account the equilibrium measure of buyers over each subset of these price-draw intervals. That is, Figure 2a depicts only the individual's *intensive* margin outcomes in terms of possible expenditures as a function of the price draw ρ. A more complete view would have to also factor in effect of banking on the equilibrium distribution of such people. We turn to this *extensive margin* effect next.

Pecuniary externality through price dispersion effect: negative welfare effects. When firms' market power (markup and price dispersion) arises from informational frictions, access to competitive banks can cause an additional negative welfare effect. This is because not all agents can benefit from banking. In particular, those agents who use banking for loans create a price effect in the goods market. This negatively affects agents who do not use banking credit.

Recall that firms expect some prospective customers to be constrained by their money balances, and their expenditure rule to be inelastic. Thus, a measure of firms optimally responds by charging higher prices relative to their marginal cost of production. Consequently, buyers (on average) are more likely to draw a higher price in the sense of first-order stochastic dominance. In Figure 2b, this price dispersion effect of banking externality is reflected in the solid blue distribution function graph over the set $\rho > \tilde{\rho}$ (for the banking equilibrium) is first-order stochastically dominating the dashed-dotted red graph (HMLW, no-banking equilibrium). Also, the support of the price distribution in our baseline model economy is wider than in HLMW and $\tilde{\rho}_i$ is higher than $\hat{\rho}_{HLMW}$. Thus, under our banking equilibrium, each money-constrained and money-unconstrained buyer will tend to draw from a higher range of prices than the no-bank, HLMW economy. Moreover, the equilibrium mass of such buyers is relative higher than that in the HLMW economy (see both Figures 2b and 2c).

Effectively, bank credit induces more price markups on money-buyers. There is also higher price dispersion in frictional goods trades. Consequently, each money-buyer (who draws a high enough price such that $\tilde{\rho}_i \leq \rho \leq \hat{\rho}$) faces a tighter liquidity constraint. In this case, the liquidity constrained money-buyer spends less than the case without access to banking arrangements. For the liquidity unconstrained money-buyer (who draws $\rho > \hat{\rho}_i$), this effect is not there since his liquidity constraint does not bind. Nevertheless, since unconstrained money-buyers' demands are decreasing in ρ, and the corresponding domain for ρ would have shifted up, they would bear the brunt of the externality through lower consumption outcomes (relative to their HLMW counterparts).

In summary, banking affects agents' consumption outcomes differently when firms have market power in frictional goods transactions. In this setting, access to a competitive banking sector can amplify firms' market power, creating an additional welfare-reducing effect of banking. This negative welfare effect, tied to credit-buyers, pushes up price dispersion. This then increases the measure of firms extracting rent from money-buyers. Consequently, the welfare-improving function of banking liquidity transformation is no longer unambiguous, in contrast to Berentsen et al. (2007).[25]

[25]Our result has a similar flavor to the classic pecuniary-externality effect from credit (see, e.g., Chiu et al., 2018). In Chiu et al. (2018), the externality is necessarily dependent on an assumption that the cost of producing goods q is a strictly increasing and convex function. In their competitive price-taking equilibrium, the existence of credit-buyers raises goods quantity, q, which then raises the marginal cost of producing q, $c'(q)$, since $c''(q) > 0$ in their setup. This then raises equilibrium price p and feeds back in the form of tightening money-buyers' liquidity constraints. If $c''(q) = 0$, there is no pecuniary externality in Chiu et al. (2018). In contrast, here, we deliberately shut down the technological avenue necessary in Chiu et al. (2018) to generate the pecuniary externality. Instead, we can still have this effect for a different reason. Here, the pecuniary externality works through market power in

3.2 Trade-off and inflation: experiments on a calibrated economy

In Section 3.1 we identified the benefit and cost of having competitive banking conditional on a particular inflation (policy) setting. We now consider the effect of long-run inflation γ (or monetary policy) on Note that quantitative results are based on the model that is statistically calibrated to U.S. data. See Online Appendix C for further details of the calibration. In particular, our calibration will place the model well away from any pure-money SME—i.e., we will only observe outcomes consistent with SMEs that feature a co-existence of money (constrained and unconstrained) and credit buyers.

Here, we will numerically evaluate this *insurance* versus *price dispersion* tension as a function of inflation (or equivalently, nominal interest policy in the long run). We consider a set of economies, each distinguished by its long-run inflation rates τ from $\tau \in [\beta - 1, \bar{\tau}]$, where we set $\bar{\tau} = 0.05$ (i.e., 5% annual inflation rate).[26]

Overall, whether competitive banks improve welfare in equilibrium is ambiguous. To understand why, we break welfare gains and losses down into the net trading surpluses associated with each ex-post buyer-type events—i.e., events involving credit buyers, money-constrained buyers, and money unconstrained buyers. In Figure 3, these net trading surpluses are measured as the expected utility of each ex-post buyer group net of sellers' expected cost of producing at ex-post different prices.[27]

The solid blue graphs in each panel of Figure 3 correspond to the difference in net DM trading surpluses between banking equilibria in our model and the no-bank HLMW economies for different long run inflation policies. If it is a positive number, it means buyers have a higher net DM trading surplus in our baseline economy than that in the no-bank HLMW economy. (We'll focus only on the ex-post buyers and economize of showing the surplus of depositors, which will just be a constant.) The stark takeaway from these graphs is that *with competitive banking,* there is a positive gap between the net social surplus of credit-buyer events (see Figure 3a), although this gap shrinks with inflation (as to be expected). However, in the following two panels, Figures 3b and 3c, we can see that society is ex-ante worse off if they turn out to be either money-constrained or

the form of price (markups) dispersion. The existence of credit-buyers means that, ex-ante, agents end up carrying (relatively) less real balances, z. By Lemma 5, this tends to shift the distribution $J_i(\cdot, z)$ to the right—i.e., agents are more likely to get squeezed by higher prices and markups. If agents knew for sure they would be money-buyers, they would prefer to have carried more real balance. However, because of the idiosyncratic risk they face, ex-ante, all agents end up creating some pecuniary externality of the ex-post liquidity constrained agents.

[26] It can be verified that price dispersion cannot be sustained at the Friedman rule, i.e., $\tau = \beta - 1$. Moreover, banking is redundant since it is costless for agents to carry money balances. For our purpose, we focus on long-run anticipated inflation away from the Friedman rule. This can be interpreted as some extraneous institutional restrictions that prevent a monetary policy maker from implementing the Friedman rule (see also Berentsen et al., 2007, for the same argument).

[27] For example, in our baseline banking equilibrium we have three ex-post cases with corresponding ex-ante net trading surpluses. The generic formula for the trading surplus measure is

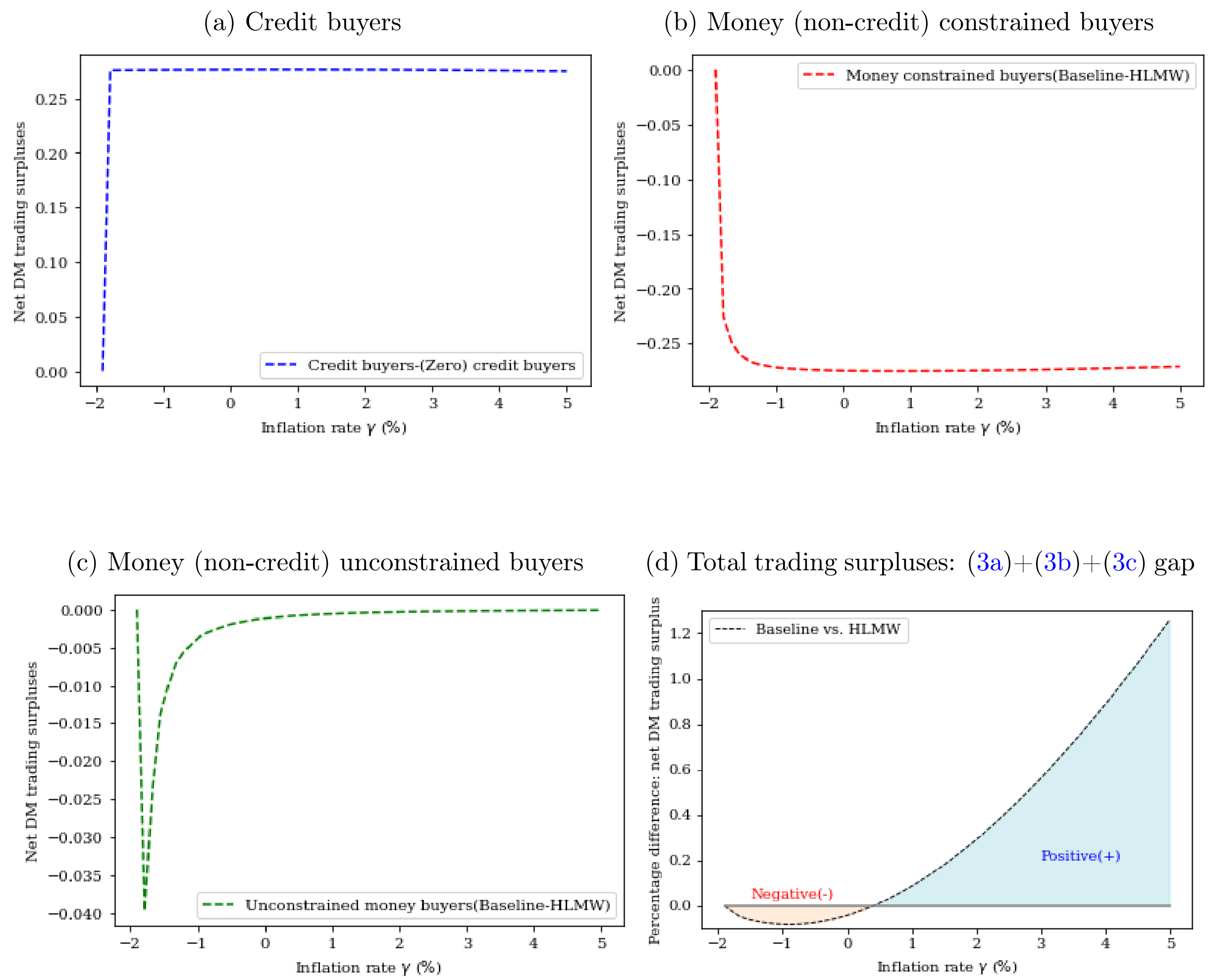
$$\int_{\mathcal{E}} [\alpha_1 + 2\alpha_2(1 - J_i(\rho, z)] u[q_b(\rho, z)] - c[q_b(\rho, z)] \mathrm{d}J_i(\rho, z),$$

where: (a) the credit-buyer cases have $\mathcal{E} := \left[\underline{\rho}(z), \tilde{\rho}_i(z)\right]$ and $q_b(\rho, z) = [\rho(1+i)]^{-1/\sigma}$; (b) the own-money constrained buyer cases have $\mathcal{E} := (\tilde{\rho}_i(z), \hat{\rho}(z)]$ and $q_b(\rho, z) = z/\rho$; and (c) the own-money unconstrained buyer cases have $\mathcal{E} := (\hat{\rho}(z), \bar{\rho}(z)]$ and $q_b(\rho, z) = \rho^{-1/\sigma}$. We can define similar objects for the HLMW no-bank environment except that there will be zero measures of credit buyer events.

money-unconstrained buyers who optimally do not use bank credit.

Figure 3d sums up the preceding three graphs vertically to give us the relevant net social surplus across all three ex-post groups. Here, we can already see the symptom of the underlying tension between the liquidity risk insurance benefit of banks (for credit buyers) and the pecuniary-externality cost that operates through the pricing dispersion effect. The resolution is non-monotone with respect to inflation. For low inflation ranges, the latter dominates to create a negative social surplus despite having perfect competition among banks. Only for sufficiently high inflation ranges does the benefit of banking begin to dominate.

Figure 3: The effects of inflation on Equilibrium Outcome.



(a) Credit buyers

(b) Money (non-credit) constrained buyers

(c) Money (non-credit) unconstrained buyers

(d) Total trading surpluses: (3a)+(3b)+(3c) gap

Welfare implications of banking. What then of the benefit of banking to the inactive DM buyers (depositors)? We had, thus far, deliberately omitted that in the display and discussions in the previous figures. We now present a complete welfare accounting that includes the ex-ante welfare of ex-post depositor types.[28]

[28]Note that since inactive buyers do not consume in the DM, there is zero net trading surplus emanating from such ex-post events. These inactive buyer's ex-ante welfare are thus solely accounted for by the term $(1 - n)\,[U(x^{\star}) - x^{\star}]$.

We report the welfare measure in terms of a standard consumption equivalent variation (CEV) measure. This captures how much consumption (in units of the CM good) an agent is willing to give up in an economy without banks to live in an economy with banks.

Given $\gamma = 1 + \tau$ policy, the welfare function in an SME is given by

$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(x^\star) - x^\star + n \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1 - J_i(\rho, z_e, \gamma))\Big)\Big(u[q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big) \mathrm{d}J_i(\rho, z_e, \gamma)\Bigg], \tag{3.8}$$

where $e \in \{Baseline, HLMW\}$ indexes our baseline model economy or the no-bank economy of HLMW. We can also write the total welfare at a given gross inflation γ with consumption reduced by a factor of $\triangle$ as

$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(\triangle x^\star) - x^\star + n \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1 - J_i(\rho, z_e, \gamma))\Big)\Big(u[\triangle q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big) \mathrm{d}J_i(\rho, z_e, \gamma)\Bigg]. \tag{3.9}$$

We compute the CEV as the value $1 - \triangle$ that solves $W^{Baseline}(\gamma) = W^{HLMW}{}_{\triangle}(\gamma)$ given policy $\gamma = 1 + \tau$. This measure says that every agent in the economy with perfectly competitive banks needs to give up $1 - \triangle$ percent of his consumption to move to the economy without access to competitive banks at given policy. Note that the integral term already includes the measure of active buyers who would optimally chosen to deposit some of their idle monies.

According to Figure 4, banking has a non-monotonic welfare consequence when the trend inflation rate varies from just above the Friedman rule $\gamma = 1 + \tau = \beta$. In particular, banks are inessential institutions when the trend inflation rate is sufficiently low. This result hinges on the interaction between the liquidity-risk insurance and price dispersion effects discussed earlier in Section 3.1.

On the one hand, positive welfare effects of banking come from the liquidity-risk insurance effect. First, both the (ex-post) inactive buyers in trading with probability $(1 - n)$ and unconstrained money-buyers can deposit their idle money balances to earn an interest i_d. Second, buyers who trade with low-price firms find it worthwhile to borrow additional money balances from the bank. These credit-buyers have more relaxed liquidity constraints. Thus, they can spend more on goods to enjoy a higher utility flow in the DM.

On the other hand, credit can amplify firms' market power (markups and price dispersion), which creates a negative welfare effect of banking on liquidity constrained and unconstrained money-buyers via higher prices (both the average level and dispersion). The reason is as what we

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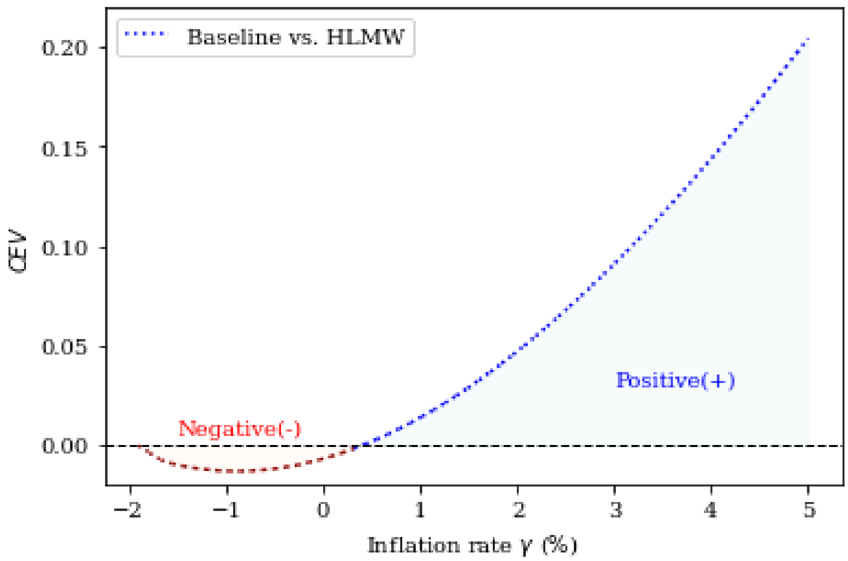
Together with similar terms accruing to the other ex-post agent groups, the total present-value social welfare in the CM activity is simple: it is just the constant term $(1-\beta)^{-1}[U(x^\star) - x^\star]$.

Figure 4: Consumption Equivalent Variation (%) of moving from the no-bank HLMW economy to the baseline economy with banking.



had previously discussed.

Summary of insights. Imperfect information through noisy search frictions in the goods market generates a policy-dependent distribution of goods prices (and associated markups), as in Head et al. (2012). The presence of competitive banking benefits only agents who would like to deposit and those who optimally use credit by inducing more firms who serve them to more likely post low prices. In turn, the externality effect is in firms who charge higher prices to money-buyers who do not find it optimal to borrow. These agents' expenditures are either inelastic to the price rise and they end up consuming less (i.e., the money-constrained agents) or they elastically respond to higher price draws by consuming less (i.e., the money-unconstrained agents). When inflation is sufficiently low, the cost of holding money is also low. Thus, the gains from banking along the channel of liquidity-risk insurance effect are also small. The price dispersion effect can easily outweigh such benefits via higher markups distorting the liquidity premium for the money-buyers.

To sum up, firms' price dispersion induces (ex-post) heterogeneous consumption outcomes among credit-buyers and money-buyers. Hence, non-trivial feedback from firms' market power on the welfare consequences of banking. In particular, credit-buyers benefit from banking credit to purchase more goods. However, banking also makes firms extract more rent from money-buyers in goods trades, thus lowering consumption. The essentiality of banks—in terms of helping insure against individual liquidity risks—is no longer unambiguous in our economy with endogenous firms' market power in the goods market.

4 Conclusion

We construct a model of money, bank credit and endogenous retail market power where informational frictions induce a policy-dependent distribution of goods prices and associated markups in equilibrium. We show that access by borrowers to credit can contribute to amplifying firms' market power, reducing the welfare gains from banking. The increased demand for goods by credit-buyers expands the measure of firms charging higher prices, extracting rent from money-buyers. The latter comes in two ways: First, higher price draws affect agents who turn out to be liquidity constrained money-buyers by squeezing their liquidity constraints and thus lowering their consumption. Second, higher price draws also reduce unconstrained money-buyers' consumption even though there is no binding liquidity constraint on them. This is simply because their consumption demands are decreasing functions of the relevant prices they draw. As a result, market power in the retail industry can make an otherwise competitive banking sector less efficient in reallocating liquidity in equilibrium.

Thus, the welfare-improving role of banking liquidity transformation is no longer unambiguous in a monetary economy with endogenous firms' market power. Our model highlights a new channel that can be surprising if policymakers attempt to regulate banking competition without taking into account its externality on consumers in non-competitive goods markets that have, evidently, price dispersion.

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Online Appendix

On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion

Omitted Proofs and Other Results

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A Two-stage decision representation of Problem (2.7).

Here is another way to rationalize this problem by breaking up the within-period DM buyers' problem in two virtual stages: Consider the last stage for any active buyer at the end of the DM. The value of any excess money balance, $\tilde{m}$, is given by

$$D\left(\tilde{m}, l, \mathbf{a}\right) = \max_{d} \left\{W\left(0, l, d, \mathbf{a}\right) : d \leq \tilde{m}\right\}. \tag{A.1}$$

Since we have shown (in the paper) that W_d is increasing and linear in d with slope $(1 + i_d)$, then we have $d^{\star}(m, p, i, \mathbf{a}) = \tilde{m}$ and its induced value is

$$D\left(\tilde{m}, l, \mathbf{a}\right) = W\left(0, l, \tilde{m}, \mathbf{a}\right).$$

Now, step back to the stage in the DM where the DM buyers are ones who have drawn at least one price quote. Their value is:

$$B(m, p, \mathbf{a}) = \max_{q,l} \left\{ u(q) + D\left(m + l - pq, l, \mathbf{a}\right) \middle| \begin{matrix} pq \leq m + l, \\ 0 \leq l < \infty \end{matrix} \right\}. \tag{A.2}$$

Note that if we write $\tilde{m} := m + l - pq$, then Problem (A.2) and (A.1) together is equivalent to the reduced-form Problem (2.7).

B Omitted proofs

B.1 Proof of Lemma 1 (Monopoly pricing)

The following is a partial equilibrium result, taking as parametric the pre-determined money holding of agents z, and the rate of interest on loans i. It provides for a complete characterization of what would determine the upper bound ($\bar{p} = p^m$) on the equilibrium support of the DM-good price distribution. Below, we rewrite nominal variables in terms of stationary variables: Measured in units of the CM numéraire good, real money balance is $z := \phi m$ and the relative price of a DM good is $\rho := \phi p$. (Dividing the results through with the value of money ϕ will yield the result in Lemma 1, which was presented in nominal terms.) Thus, Lemma 1 re-stated in equivalent stationary-variable terms is:

Lemma (1). *Fix a (pre-determined) real money balance $z \in (0, \infty)$ and a given rate on loans i. Let the realized profit at price outcome ρ from serving*

1. *a credit-buyer be $G_1(\rho; i) := [\rho\,(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$,*
2. *a constrained money-buyer be $G_2\,(\rho; z) := \frac{z}{\rho}(\rho - c)$,*
3. *an unconstrained money-buyer be $G_3\,(\rho) := (\rho)^{-\frac{1}{\sigma}}(\rho - c)$,*

Let $\mathbf{g}(\rho; i, z) := [G_1\,(\rho; i)\,, G_2\,(\rho; z)\,, G_3\,(\rho)]$. Assume that $\sigma \in (0,1)$ and $c \in (0,1]$ such that $0 < \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}$ There exists a $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$ and $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$. Furthermore, there is a $\breve{z}_i$ where $\mathring{z}_i := \min\{\breve{z}_i, \tilde{z}_i\}$ and $0 < \mathring{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$.

1. *The ex-post profit function is*

$$R^{ex}\,(\rho, i, z) = \begin{cases} \langle \mathbf{g}(\rho; i, z), \mathbf{I}_1(\rho; i, z)\rangle\,, & z \in [\hat{z}, \infty) \\ \langle \mathbf{g}(\rho; i, z), \mathbf{I}_2(\rho; i, z)\rangle\,, & z \in [\mathring{z}_i, \hat{z}) \\ \langle \mathbf{g}(\rho; i, z), \mathbf{I}_3(\rho; i, z)\rangle\,, & z \in (0, \mathring{z}_i) \end{cases} \tag{B.1}$$

where

$$\begin{aligned}
\mathbf{I}_1(p; i, m) &:= \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\hat{\rho}\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1(a)}} + \underbrace{\mathbf{1}_{\left\{\hat{\rho}<c\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1(b)}}\right], \\
\mathbf{I}_2(\rho; i, z) &:= \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\tilde{\rho}_i<\rho_0^m<\hat{\rho}\leq\rho\right\}\cap\{z\in[\tilde{z}_i,\hat{z})\}}}_{\text{Case 2(a)}} + \underbrace{\mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\leq\rho\right\}\cap\{z\in[\mathring{z}_i,\tilde{z}_i)\}}}_{\text{Case 2(b)}}\right], \\
\mathbf{I}_3(\rho; i, z) &:= \underbrace{\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \mathbf{1}_{\{c<\hat{\rho}\leq\rho\}}\right] \times \mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\right\}}}_{\text{Case 3}},
\end{aligned}$$

$\rho_0^m = c/(1-\sigma)$, $\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$, $\tilde{\rho}_i \equiv \tilde{\rho}(i, z) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, $\langle \cdot, \cdot \rangle : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \to \mathbb{R}_+$ is the inner-product operator, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.

2. The (real) monopoly price and its ex-post profit outcome, respectively, are

$$\rho^m = \begin{cases} \rho_0^m, & z \in [\hat{z}, \infty) \\ \hat{\rho}(z), & z \in [\mathring{z}_i, \hat{z}) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases} \quad \textit{and,} \quad R^{ex}(\rho^m, i, z) = \begin{cases} G_3(\rho_0^m), & z \in [\hat{z}, \infty) \\ G_2(\hat{\rho}(z)) = G_3(\hat{\rho}(z)), & z \in [\mathring{z}_i, \hat{z}) \\ G_1(\rho_0^m; i), & z \in (0, \mathring{z}_i) \end{cases}. \tag{B.2}$$

Proof. The demand is classified by Equation (2.8) in the paper. In terms of the stationary variables, this is equivalently given as:

$$q_b^\star(z, \rho, i, \mathbf{a}) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ z & \text{if } \tilde{\rho}_i < \rho \leq \hat{\rho}, \\ (\rho)^{-1/\sigma} & \text{if } \rho > \hat{\rho} \end{cases} \tag{B.3}$$

where we recall the definitions

$$\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{\rho}_i \equiv \tilde{\rho}(i, z) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}, \tag{B.4}$$

and z and i are taken as fixed (parametric) by both buyers and firms. Consider the case of a firm that posts a monopoly price ρ^m in a monetary equilibrium where $0 < z < \infty$ and $i > 0$. Given buyers' demand structure $q_b^\star(z, \rho, i, \mathbf{a})$ derived in Equation (B.3), a firm's ex-post profit per trade, $R^{ex}(\rho, i, z)$ is $R^{ex}(\rho, i, z) = q_b^\star(z, \rho, i, \mathbf{a})(\rho - c)$. Note that the relevant domain for pricing has a natural lower bound of c since no firm would ever want to price below marginal cost in a one-shot market. (In equilibrium, we can show that the lower bound on the support of the price distribution is bounded below by c.)

Properties of ex-post profit function components. The components G_1, G_2 and G_3 have the following geometric properties:

1. Since $\sigma < 1$ then $(1+i)^{-\frac{1}{\sigma}} < 1$, so that it is always the case that $\tilde{\rho}_i < \hat{\rho}$ and the function value $G_1(\rho) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$ is everywhere dominated by that of $G_3(\rho) := \rho^{-\frac{1}{\sigma}}(\rho - c)$ (i.e., the third case).

2. We can also verify that $G_1(c) = G_2(c; z) = G_3(c) = 0$ and $\lim_{\rho \nearrow \infty} G_1(\rho) = \lim_{\rho \nearrow \infty} G_3(\rho) = 0$. These two functions are strictly positive-valued on (c, ∞), and have unique maxima in ρ: If ρ is not constrained anywhere on the feasible domain $[c, \infty)$, then the unique maximum for the function G_3 solves first-order condition $\frac{\partial G_3}{\partial \rho}(\rho) = \frac{\partial G_3}{\partial \rho}\left[\rho^{-\frac{1}{\sigma}}(\rho - c)\right] = 0$, which yields $\rho_0^m = \frac{c}{1-\sigma}$. In nominal terms, this is $\rho_0^m = \frac{\phi^{-1}c}{1-\sigma}$.

3. Since $\sigma < 1$ and $i \geq 0$, then $(1+i)^{-\frac{1}{\sigma}} < 1$, so that $G_1(\rho) \leq G_3(\rho)$ for all ρ—i.e., $G_1(\rho)$ is a constant factor $(1+i)^{-\frac{1}{\sigma}} < 1$ smaller than $G_3(\rho)$. We can also deduce that there is a unique unconstrained maximizer for G_1, i.e., $\arg\max_\rho G_1(\rho) = \arg\max_\rho G_3(\rho) = \rho_0^m$.

4. Neither G_1 nor G_3 depend on z. The function G_2 is the only piece that depends on z, is such that $G_2(c; z) = 0$, $\lim_{\rho \nearrow \infty} G_2(\rho; z) = \infty$, and its image is strictly increasing and strictly concave in ρ.

Properties 1 to 4 imply that for all $\rho > c$, there can only be the following three generic cases, depending on the range of sizes of real money balance that buyers carry, z.

Case 1. **Real money balance is "sufficiently high".** Consider a given $i > 0$ and $z \in [\hat{z}, \infty)$. Let $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ be the cutoff real balance where graph $(G_2(\cdot; z))$ intersects graph (G_3) uniquely at $\rho = \hat{\rho}(\hat{z}) = \rho_0^m > c$. For $z > \hat{z}$, there are two sub-cases to consider:

(a) There is a $z' > \hat{z}$ and $z \in (z', \infty) \equiv \left(c^{1-\frac{1}{\sigma}}, \infty\right)$. Here, graph $(G_2(\cdot; z))$ never intersects the graphs of G_1 and G_3 at any $\rho > \rho_0^m > c$. This case exists if $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c)$ and $\frac{\partial G_2}{\partial \rho}(\rho_0^m; z) \geq \frac{\partial G_3}{\partial \rho}(\rho_0^m) = 0$. The second restriction is always satisfied since $\frac{\partial G_2}{\partial \rho}(\cdot; z) > 0$ everywhere. We can check that $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c)$ if and only if $z > \acute{z} \equiv c^{1-\frac{1}{\sigma}}$. Thus, $\hat{\rho}(z) \equiv z^{\frac{\sigma}{\sigma-1}}$ does not exist if $z > \acute{z}$, since $\hat{\rho}(z) < \hat{\rho}(\acute{z}) = c$. That is, no firm will face constrained money buyers, or credit buyers, by the fact that $\tilde{\rho}_i(z) < \hat{\rho}(z) < c$ whenever $z > \acute{z}$. This implies that: ***(i)*** only *unconstrained money buyers* are served; ***(ii)*** the effective profit function for a firm is that which is associated with the demand from unconstrained money buyers, $R^{ex}(\rho, i, z)|_{z \in (z', \infty)} = G_3(\rho) := \rho^{-\frac{1}{\sigma}}(\rho - c)$; and ***(iii)*** the monopoly pricing outcome is $\rho_0^m = \frac{c}{1-\sigma}$, with its induced profit being $G_3(\rho_0^m) := (\rho_0^m)^{-\frac{1}{\sigma}}(\rho_0^m - c) > 0$. (Figure 5 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

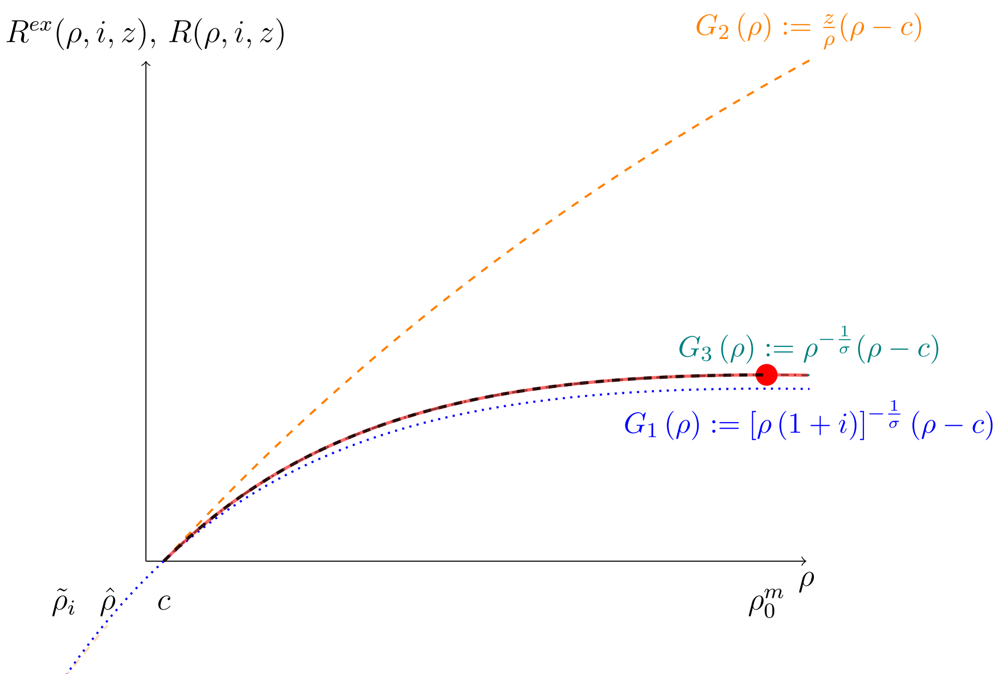


Figure 5: Case 1(a). Real money balance is sufficiently high, $z \in [z', \infty)$.

(b) Real money balance z is such that $z \in [\hat{z}, z') \equiv \left[\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}, c^{1-\frac{1}{\sigma}}\right)$. Thus, consider real money balance at any given $z \in [\hat{z}, \acute{z})$, where $\hat{z} :< \acute{z} \equiv c^{1-\frac{1}{\sigma}}$, since $\sigma \in (0,1)$. We can check that $c < \tilde{\rho}_i(z) < \hat{\rho}(z) \leq \rho_0^m$. Also, $G_2(\rho; z) < G_3(\rho)$ for all $\rho \in (c, \hat{\rho}(z))$, $G_2(\rho; z) = G_3(\rho)$ only if $\rho = c$ or $\rho = \hat{\rho}(z) \leq \rho_0^m$, and $G_2(\rho; z) > G_3(\rho)$ for all $\rho > \rho_0^m$. Thus, graph$(G_2(\cdot; z))$ and graph(G_3) uniquely intersect at $\rho = \hat{\rho}(z) \leq \rho_0^m$. Since G_1 is always dominated by G_3 (Property 1), and $G_2(\rho; z)$ is increasing in ρ, then graph$(G_2(\cdot; z))$ can only uniquely intersect graph(G_1) at some at a unique point $\tilde{\rho}_i(z)$. This implies that: ***(i)*** each firm's effective profit function is given by

$$R^{ex}(\rho, i, z)\big|_{z \in [\breve{z}, z')} = G_1(\rho)\,\mathbf{1}_{\{c < \rho \leq \tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\{\tilde{\rho}_i < \rho < \hat{\rho}\}} + G_3(\rho)\,\mathbf{1}_{\{\hat{\rho} \leq \rho \leq \rho_0^m\}},$$

whenever $z \in [\breve{z}, z')$; ***(ii)*** the maximal price that can exist is the Ramsey monopoly price ρ_0^m; and ***(iii)*** its associated profit outcome is $G_3(\rho_0^m)$ since

$$\rho_0^m = \arg\max_{\rho} \left\{ R^{ex}(\rho, i, z)\big|_{z \in [\breve{z}, z')} \right\} = \arg\max_{\rho} \{G_3(\rho)\}.$$

(Figure 6 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

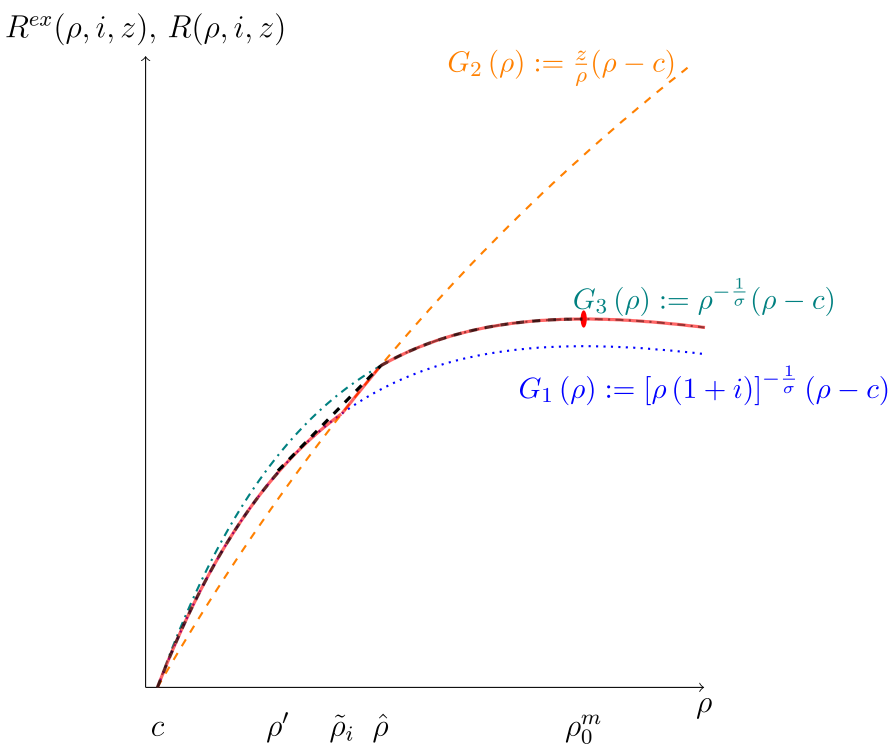


Figure 6: Case 1(b). Real money balance is sufficiently high, $[\hat{z}, z')$.

Case 2. **Real money balance z is intermediate, $z \in [\mathring{z}_i, \hat{z})$.** There is a cutoff value $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} > \tilde{z}_i$, is such that $\tilde{\rho}_i(\tilde{z}_i) = \rho_0^m$ and $G_2(\tilde{\rho}_i(\tilde{z}_i), \tilde{z}_i) = G_1(\rho_0^m)$. In Case 2(b) below, we wil show that there may exist another cutoff $\breve{z}_i \gtreqless \tilde{z}_i$, so that we may define the lower bound z for this case as $\mathring{z}_i = \min\{\breve{z}_i, \tilde{z}_i\}$.

(a) Consider $z \in [\tilde{z}_i, \hat{z}) \equiv \left[(1+i)^{-\frac{1}{\sigma}}\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}, \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}\right)$. At the given z and i, we have $\tilde{\rho}_i(z) \leq \rho_0^m < \hat{\rho}(z)$ in this case. We can check that $G_2(\rho; z) < G_1(\rho)$ for all $\rho \in (c, \tilde{\rho}_i(z))$, $G_2(\rho; z) = G_3(\rho)$ only if $\rho = c$ or $\rho = \hat{\rho}(z) > \rho_0^m$, and $G_2(\rho; z) > G_3(\rho)$ for all $\rho > \rho_0^m$. This implies that: ***(i)*** each firm's effective profit function is given by

$$G_1(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\left\{\tilde{\rho}_i<\rho<\rho_0^m\right\}} + G_3(\rho)\,\mathbf{1}_{\{\hat{\rho}\leq\rho\}},$$

whenever $z \in [\tilde{z}_i, \hat{z})$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}(z)$; and ***(iii)*** its associated profit outcome is $G_3(\hat{\rho}(z))$ since

$$\hat{\rho}(z) = \arg\max_{\rho}\left\{R^{ex}(\rho, i, z)\big|_{z\in[\tilde{z}_i,\hat{z})}\right\} = \arg\max_{\rho}\{G_3(\rho)\}.$$

(Figure 9 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

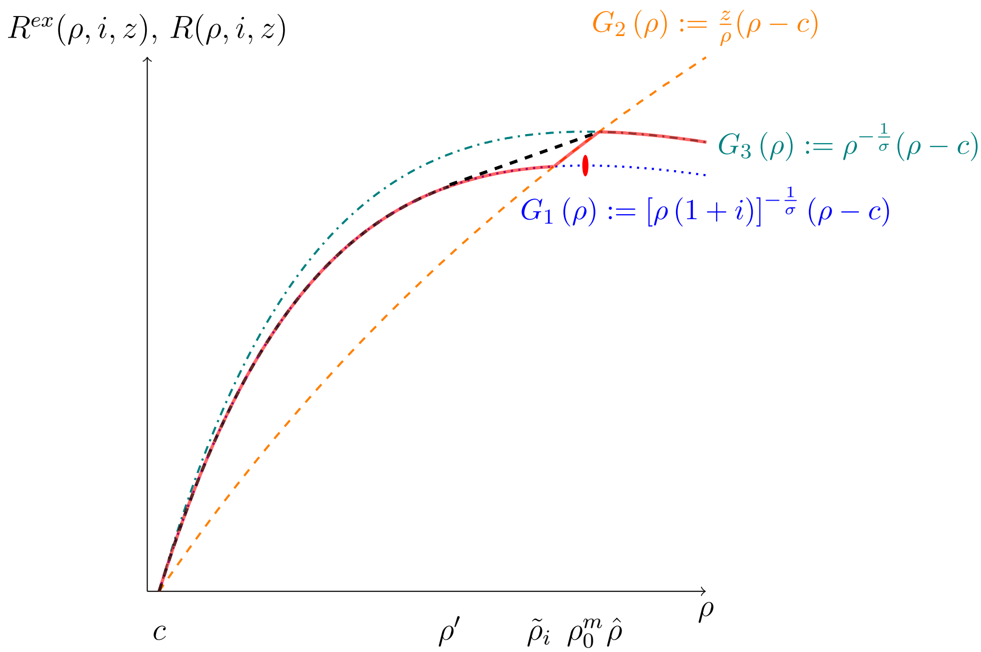


Figure 7: Case 2(a). Intermediate level of real money balance, $z \in [\tilde{z}_i, \hat{z})$.

(b) Now we derive the possibility of $\breve{z}_i$, where the case that $z \in [\mathring{z}_i, \tilde{z}_i)$. Assume for now that the cutoff $\breve{z}_i$ exists (at least for some z and i). (It may be that such that $\breve{z}_i \gtreqless \tilde{z}_i$, so a natural lower bound will be $\mathring{z}_i = \min\{\breve{z}_i, \tilde{z}_i\}$.) From Property 2 and the previous case, we can also deduce that there can be a $z \geq \mathring{z}_i$ where graph $(G_2(\cdot; z))$ and graph (G_3) uniquely intersect at $\rho = \hat{\rho}(z) > \rho_0^m$ and $G_3(\hat{\rho}(z); z) > G_1(\rho_0^m)$. Following from the last case, we can see that when $z < \tilde{z}_i$, then $\tilde{\rho}_i(z) > \rho_0^m$, and from Properties 1 and 4 it must be that $G_2(\tilde{\rho}_i(z), z) < G_1(\rho_0^m)$. However, in this sub-case, $G_2(\hat{\rho}(z), z) \geq G_1(\rho_0^m)$ and $c < \rho_0^m < \tilde{\rho}_i(z) < \hat{\rho}(z)$. Next, we derive the unknown candidate for the lower bound on z, i.e., $\breve{z}_i$ such that $G_2(\hat{\rho}(z), z) \geq G_1(\rho_0^m)$ holds. If

$$
\begin{aligned}
& G_2(\hat{\rho}(z), z) \geq G_1(\rho_0^m) \\
\Leftrightarrow\ & \frac{z}{\hat{\rho}(z)}\left[\hat{\rho}(z) - c\right] \geq \left[\rho_0^m(1+i)\right]^{-\frac{1}{\sigma}}\left(\rho_0^m - c\right) \\
\Leftrightarrow\ & z - c \cdot z^{\frac{1}{1-\sigma}} \geq \frac{\sigma}{c}\tilde{z}_i.
\end{aligned}
$$

Consider the situation where the above inequality just binds. Observe that the RHS term is strictly positive valued, and is constant with respect to z since $\tilde{z}_i$ depends only on parameters and i (which is fixed). The LHS, has the following properties: (i) $G_2(\hat{\rho}, 0) = 0$, (ii) $\lim_{z\to\infty} G_2(\hat{\rho}, z) = -\infty$, (iii) $G_2(\hat{\rho}, \cdot)$ is "hump shaped": $G_2(\hat{\rho}, \cdot)$ is strictly increasing on $[0, \check{z})$ and strictly decreasing on $[\check{z}, \infty)$, with $\check{z} = \left(\frac{c}{1-\sigma}\right)^{1-1/\sigma} \equiv \hat{z}$ being the unique maximizer. Thus when $G_2(\hat{\rho}(z), z) = G_1(\rho_0^m)$, there can be (i) at most two solutions, (ii) one solution, or (iii) no solution (in particular, if i is too large).

Suppose we have possibility (iii) that there is no solution. Then sub-case 2(b) is moot. Now suppose that possibility (ii) arises. In this case, we have an impossibility too, so again, sub-case 2(b) is non-existent. That leaves possibility (i) where there are two distinct roots, say, z_0 and z_1, where $z_0 < z_1$. Since $G_2(\hat{\rho}(z), z) = G_1(\rho_0^m)$ has two roots and given the geometric properties of the LHS and RHS terms, and they cannot be $\hat{z}$, then it can only be possible that $z_0 < \hat{z} < z_1$. However, $z_1 > \hat{z}$ would not be a feasible selection since all $z < \hat{z}$ here. Therefore if there are two distinction solutions, it must always be the smaller of the two, z_0, and $z_0 < \hat{z}$. thus, $z_0 \equiv \breve{z}_i$. Last, it is possible that $\breve{z}_i \gtreqless \tilde{z}_i$. If $\breve{z}_i \geq \tilde{z}_i$, then it must be that $\breve{z}_i = \tilde{z}_i$ (degenerate case). Otherwise there is a unique number $\breve{z}_i < \tilde{z}_i$. Altogher, we can conclude that a natural lower bound for this case is $\mathring{z}_i = \min\{\breve{z}_i, \tilde{z}_i\}$.

This implies that: ***(i)*** each firm's effective profit function is given by

$$
G_1(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\left\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}\right\}} + G_3(\rho)\,\mathbf{1}_{\{\hat{\rho}\leq\rho\}},
$$

whenever $z \in [\mathring{z}_i, \tilde{z}_i)$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}(z)$; and ***(iii)*** its associated profit outcome is $G_3(\hat{\rho}(z))$ since

$$\hat{\rho}(z) = \arg\max_{\rho} \left\{ R^{ex}(\rho, i, z)\big|_{z\in[\mathring{z}_i, \tilde{z}_i)} \right\} = \arg\max_{\rho} \{G_3(\rho)\}.$$

(Figure 8 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

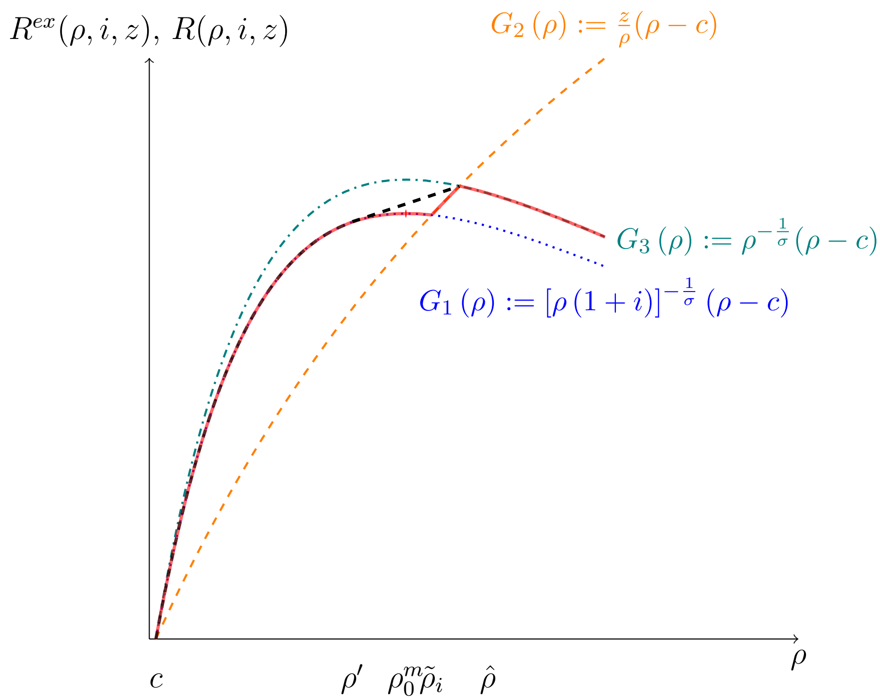


Figure 8: Case (2b). Intermediate level of real money balance, $z \in [\mathring{z}_i, \tilde{z}_i)$.

Case 3. **Real money balance z is sufficiently low, $z \in (0, \mathring{z}_i)$.** From Property 2 and the previous case, we can also deduce that if $z < \mathring{z}_i$, then graph$\,(G_2(\cdot; z))$ and graph$\,(G_3)$ uniquely intersect at $\rho = \hat{\rho}\,(z) > \rho_0^m$ so that $G_3\,(\hat{\rho}(z); z) < G_1\,(\rho_0^m)$. Thus, we have: ***(i)*** the effective profit function is given by

$$R^{ex}\,(\rho, i, z)\big|_{z\in(0,\breve{z})} = G_1\,(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2\,(\rho)\,\mathbf{1}_{\left\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}\right\}} + G_3\,(\rho)\,\mathbf{1}_{\{\hat{\rho}\leq\rho\}};$$

(ii) the maximal price that can exist is the Ramsey monopoly pricing outcome ρ_0^m, and ***(iii)*** its associated profit outcome is $G_1\,(\rho_0^m)$. (Figure 9 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

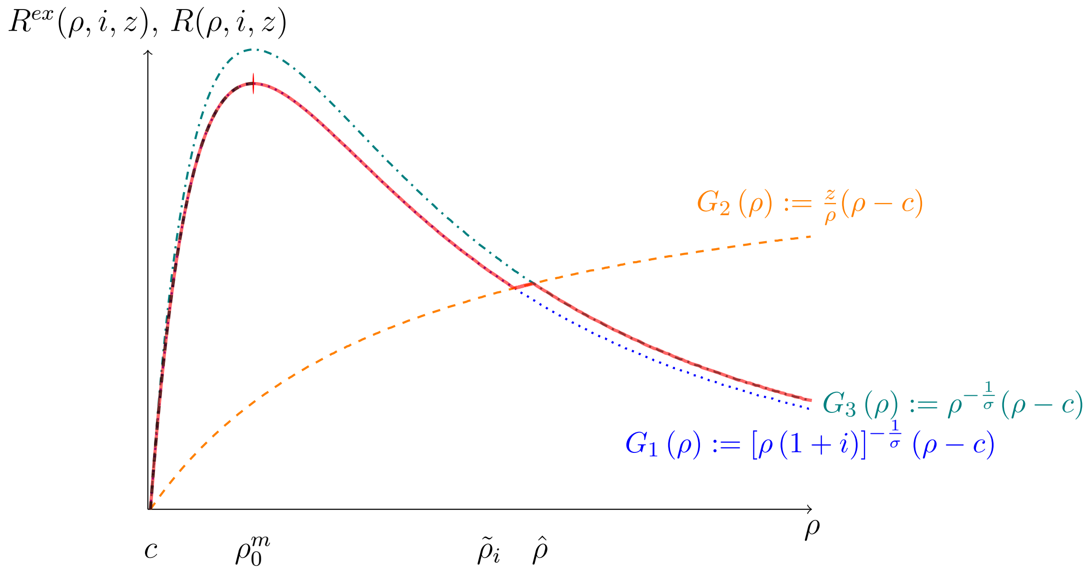


Figure 9: Case 3. Real balance is sufficiently low, $z \in (0, \mathring{z}_i)$.

□

B.2 Proof of Lemma 2

In Burdett and Judd (1983) and Head et al. (2012), firms are assumed to commit to implementing the outcomes of their posted (pure) prices. In our extension, as Lemma 2 implies, there may be states of the world where it is (weakly) profitable for firms to commit to posting lotteries over prices (i.e., terms of trade, given buyer demand). Proposition 2 provides the characterization of the *effective (per-trade) profit function* for a firm committed to posting such random contracts. From each buyer's perspective, the possible lotteries are already compounded or internalized in their perceived (and actual) equilibrium distribution of prices $J(\cdot, i, z)$. Thus buyers in equilibrium will be drawing from the distribution $J(\cdot, i, z)$, just as in Burdett and Judd (1983) and Head et al. (2012).

In the proof below, we re-write Lemma 2 in terms of stationary variables.

Lemma (2)**.** *Given aggregate outcomes (z, i), and the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, z)$ in Equation (B.1) in Lemma 1, there exists a $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$ and $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ and an endogenous $\mathring{z}_i$, such that $0 < \mathring{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$. A firm's* ***effective profit*** *at any given reference price ρ, $R(\rho, i, z)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

$$R(\rho, i, z) = \max_{\pi \in [0,1], \rho_1, \rho_2} \left\{ \pi R^{ex}(\rho_1, i, z) + (1-\pi) R^{ex}(\rho_2, i, z) : \pi\rho_1 + (1-\pi)\rho_2 = \rho \right\}. \tag{B.5}$$

1. The function $R(\cdot, i, z)$ is strictly increasing on $[c, \rho^m)$, and is concave over the firm's effective domain of pricing outcomes $[c, \rho^m]$, where the (real) monopoly price and its effective profit outcome, respectively, are

$$\rho^m = \begin{cases} \rho_0^m, & z \in [z', \infty) \\ \rho_0^m, & z \in [\hat{z}, z') \\ \hat{\rho}(z), & z \in [\tilde{z}_i, \hat{z}) \\ \hat{\rho}(z), & z \in [\mathring{z}_i, \tilde{z}_i) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases}, \text{ and, } R(\rho^m, i, z) = R^{ex}(\rho^m, i, z) = \begin{cases} G_3(\rho_0^m), & z \in [z', \infty) \\ G_3(\rho_0^m), & z \in [\hat{z}, z') \\ G_3(\hat{\rho}(z)), & z \in [\tilde{z}_i, \hat{z}) \\ G_3(\hat{\rho}(z)), & z \in [\mathring{z}_i, \tilde{z}_i) \\ G_1(\rho_0^m), & z \in (0, \mathring{z}_i) \end{cases}. \tag{B.6}$$

2. Depending on z the largest domain containing equlibrium pricing outcomes has the following properties:

   Case-1(a). If $z \in [z', \infty)$, firms will only have incentive to serve money-constrained buyers.

   Case-1(b). If $z \in [\hat{z}, z')$, all three types—credit, money-constrained and money-unconstrained buyers—will be served.

   Case-2(a). If $z \in [\tilde{z}_i, \hat{z})$, only two types—credit and money-constrained buyers—will be served.

   Case-2(b). If $z \in [\mathring{z}_i, \tilde{z}_i)$ (and i is such that this set is non-degenerate), then only two types—credit and money-constrained buyers—will be served.

Case-3. If $z \in (0, \mathring{z}_i)$, then only credit buyers are served.

Proof. From (the proof of) Lemma 2, we have deduced the cut-offs in z and their ordering: $0 < \mathring{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$.

1. We also have shown that the unique maximizer ρ^* for the ex-post profit function $R^{ex}(\cdot, i, z)$ exists. Moreover, the maximum value $R^{ex}(\rho^*, i, z)$ only arises at the upper bound of the feasible-pricing domain $[c, \rho^m]$, i.e., $\rho^* = \rho^m$, and ρ^m is characterized by Equation (B.2). By definition of the lottery problem in (B.5), it is immediate that $R(\rho, i, z) = R^{ex}(\rho, i, z)$ if there is no neighborhood $[\rho_1, \rho_2]$ containing ρ, such that $\pi R^{ex}(\rho_1, i, z) + (1 - \pi) R^{ex}(\rho_2, i, z) > R^{ex}(\rho, i, z)$. That is, any lottery would be (locally) degenerate whenever $R^{ex}(\cdot, i, z)$ is already strictly concave on any such subdomains $[\rho_1, \rho_2]$. Otherwise, $R(\rho, i, z)$ is given by the right-hand-side operator in Equation (B.5). Since $R^{ex}(\cdot, i, z)$ has a minimum at c and a unique maximum at $R^{ex}(\rho^m, i, z)$, then the convexification through (B.5) implies that $R(\rho, i, z)$ is strictly increasing in $\rho \in [c, \rho^m)$ and it is concave over $[c, \rho^m]$.

2. Finally, the types of buyers that will be served were also enumerated in the (the proof of each case) in Lemma 2.

□

B.2.1 Efficient representation of the effective profit function

Given aggregate outcomes (i, z), we can define

$$\text{co}R^{ex}_{i,z} = \text{co}\left\{(\rho, R^{ex}(\rho, i, z)) : \rho \in [c, \rho^m]\right\},$$

i.e., the convex hull of the graph of $R^{ex}(\cdot, i, z)$ restricted to the feasible pricing domain of $[c, \rho^m]$, where R^{ex} is defined in (B.1) and ρ^m is governed by (B.6). The set of points in $\text{co}R^{ex}_{i,z}$ other than those in the set Graph$\{R(\cdot, i, z)\}$ can be defined in advance as

$$U_{i,z} := \text{int}\left(\text{co}R^{ex}_{i,z}\right) \cup \text{int}\left\{(\rho^m, r) : r \in [0, R(\rho^m, i, z)]\right\},$$

where $R(\rho^m, i, z)$ is pinned down by (B.6). The effective profit function in (B.5) can be equivalently represented as

$$\text{Graph}\{R(\cdot, i, z)\} = \text{co}R^{ex}_{i,z} \setminus U_{i,z}. \tag{B.7}$$

This equivalent representation of (B.5) will be computationally convenient since the function domain is closed and bounded, and its graph is always convex. What this means is that open-source and industry-standard convex-hull algorithms, in combination with shape-preserving spline approximants and set-valued logical operations can be employed to represent (B.7) precisely, efficiently, and continuously. Crucially, we can avoid having to solve the brute-force optimization in the representation (B.5), and its associated tangent-search problem (which in practice would involve imprecise discretized approximations). Moreover, using shape-preserving splines as the basis of an approximant to the final endogenous object $R(\cdot, i, z)$ allows us to efficiently compute its derivative.

Its derivative with respect to ρ is used in composing the density representation of the equilibrium pricing distribution, $\mathrm{d}J_i(\cdot, z)$.

B.3 Proof of Lemma 5

In Section B.3, we study how the price distribution J_i changes with respect to the asset position of the households. We then establish the existence of a stationary monetary equilibrium with both money and credit in Section B.4. Fix the trend inflation rate away from the Friedman rule $\tau > \beta - 1$. Assume $\alpha_1 \in (0, 1)$. Let $i = i_d = i^\star$ be the market loan interest rate. By Lemma 3, the analytical formula for the real price distribution $J_i(\rho, z)$ is given by

$$J_i(\rho, z) := J_i(\rho, i, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z)}{R(\rho, i, z)} - 1\right] \tag{B.8}$$

where the upper bound on the support of the distribution $J_i(\cdot, z)$ is determined by (2.25), repeated here as:

$$\bar{\rho} := \rho^m(i, z) = \begin{cases} \rho_0^m, & z \in [z', \infty) \\ \rho_0^m, & z \in [\hat{z}, z') \\ \hat{\rho}(z), & z \in [\tilde{z}_i, \hat{z}) \\ \hat{\rho}(z), & z \in [\mathring{z}_i, \tilde{z}_i) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases} . \tag{B.9}$$

and the lower bound on the support of $J_i(\cdot, z)$, $\underline{\rho}$, solves $R(\rho, i, z) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{\rho}, i, z)$.

In this proof, we want to show the relationship of how the price distribution $J_i(\cdot, z)$ changes with respect to the change in the real money holdings z. Note: For the ease of notation, we will denote $\overline{\rho}(z)$ and $\underline{\rho}(z)$ respectively by $\overline{\rho}$ and $\underline{\rho}$ occasionally. Likewise, we denote the cut-off prices by

$$\hat{\rho} := \hat{\rho}(z) = z^{\frac{\sigma}{\sigma - 1}} \tag{B.10}$$

and

$$\tilde{\rho}_i := \tilde{\rho}_i(z, i) = \hat{\rho}(z)(1 + i)^{\frac{1}{\sigma - 1}}. \tag{B.11}$$

It should be kept in mind that all these cut-off prices and bounds of the price distribution depend on the state of the economy z and policy τ in general. Recall that the CRRA risk aversion parameters requires to be $\sigma < 1$, and from the result established earlier in Section B.1, we then have the following order:

$$\underline{\rho} < \tilde{\rho}_i < \hat{\rho} \leq \overline{\rho}.$$

Proof. Consider two real money holdings z and $z^{'}$ such that $\breve{z} < z < z^{'} < \overline{z}$. We want to determine whether $J_i(\cdot, z)$ is lying on top or below for z relative to $z^{'}$.

Observe from the upper bound of the price distribution B.9, there are only two possible cases: either $\overline{\rho} = \hat{\rho}$ or $\overline{\rho} = c/(1-\sigma)$.

Case 1. Suppose $\overline{\rho} = \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$.

Consider any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$. First, it is clear that $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. Using this result and the equal profit condition of the firms, we can then deduce the lower support also satisfies that $\underline{\rho}(z_0) > \underline{\rho}(z_1)$. Thus, we have $[\overline{\rho}(z_0) - \underline{\rho}(z_0)] - [\overline{\rho}(z_1) - \underline{\rho}(z_1)] > 0$. In words, the support of the price distribution with lower real money balance is wider than that with higher real money balance.

Second, for $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$, then $J_i(\rho, z_0) < J_i(\rho, z_1)$ because of $\overline{\rho}(z_0) > \overline{\rho}(z_1)$ and from Lemmata 2 and 3 we have that the function $R(\cdot, i, z)$ is strictly increasing on the equilibrium domain of prices, and thus $J_i(\cdot, z_1)$ is a non-decreasing function. That is, buyers with lower money holdings are more likely to be liquidity constrained, and that pushes up the measure of firms posting higher prices. Thus, $J_i(\rho, z_0)$ falls below $J_i(\rho, z_1)$ for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$.

Next, by the fact that the price distribution J_i is a cumulative distribution function, it then follows that $J_i(\rho, z_0) = J_i(\rho, z_1) = 1$ for some $\rho \geq \overline{\rho}(z_0)$. Likewise, we have $J_i(\rho, z_0) = J_i(\rho, z_1) = 0$ for some $\rho \leq \overline{\rho}(z_1)$.

Therefore $J_i(\rho, z_0)$ first-order stochastically dominates $J_i(\rho, z_1)$. That is, $J_i(\rho, z_0) \leq J_i(\rho, z_1)$ within the interval $[\underline{\rho}(z_1), \overline{\rho}(z_0)]$, and strict inequality for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$ given any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$.

Case 2. Suppose $\overline{\rho} = c/(1-\sigma)$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \hat{\rho}(z) < \overline{\rho}$. The reasoning for this case is similar to Case 1 above. The only difference is that the upper support of the price distribution is independent of real money holding z, i.e., $\overline{\rho}(z_0) = \overline{\rho}(z_1)$ where $z_0 \neq z_1$. However, we can deduce the following order for the cut-off prices, $\tilde{\rho}_i(z_0) > \tilde{\rho}_i(z_1)$ and $\hat{\rho}(z_0) > \hat{\rho}(z_1)$, and the lower bound of the support of $J_i(\cdot, z)$ satisfies $\underline{\rho}(z_0) > \underline{\rho}(z_1)$ given any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$. For $\rho \in (\tilde{\rho}_i(z_1), \hat{\rho}(z_0))$, then $J(\rho, z_0) < J(\rho, z_1)$ because $z_0 < z_1$. Hence, $J(\rho, z_0)$ first order stochastically dominates $J(\rho, z_1)$ given two real money holdings z_0, z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

Finally, from (B.10) and (B.11), since $\sigma < 1$, we can deduce that the maximal admissible price draw, respectively, for a money-constrained buyer ($\hat{\rho}$) and for a credit-buyer ($\tilde{\rho}_i$) are decreasing functions of z. Hence as z falls, these pricing cutoff functions increase in value. □

B.4 Proof of Proposition 1

Proposition (1)**.** *Let monetary policy be $\gamma > \beta$ and noisy search frictions be $\alpha_1, \alpha_2 \in (0,1)$. There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

Proof. We suppress the notation of aggregate dependency by writing $J_i(\cdot, z) \equiv J_i(\cdot, z, \mathbf{s})$. Such an equilibrium requires finding a fixed point in two numbers, the SME real money balance and the competitive loan/deposit interest, $z^\star$ and $i^\star$.

Pure-money SME. If Case 1(a) of Lemma 2 were to emerge, the maximal willingness to pay, $\hat{\rho}(z)$, for money-constrained agents is below all firms' marginal cost c. Also, since $\tilde{\rho}_i(z) < \hat{\rho}(z) < c$, then no credit- nor money-constrained buyers will be served by firms. Since there is no demand for loans, banks will not take on any deposit liabilities, and so no loans and deposits are traded. The only class of active buyers are the money-constrained agents. In this case, by Lemmata 2 and 3, the distribution of prices $J_i(\cdot, z) = J(\cdot)$ (is independent of both z and i), and its support will be $\left[\underline{\rho}, \rho_0^m\right] \subseteq [c, \rho_0^m]$, since in this case, $\rho_0^m = \frac{c}{1-\sigma}$, and by the equal-expected-profit condition (2.26), it will be immediate that $\underline{\rho}$ is also independent of z and i. That is, the SME reduces to a special characterization

$$\frac{\gamma}{\beta} - 1 := (1-n)i. \tag{B.12}$$

Thus this special case of the money-demand Euler equation does not determine the level of z. However, the RHS of (B.12) concerns the marginal benefit of being able to deposit with banks. Although there are no loans to be made so there are no deposits to be taken by banks (i.e., the loan market clearing condition is redundant) we can still price the competitive deposit rate (and hence loan rate) as i solving (B.12). The rest of the equilibrium system in 1 *sans* the credit market clearing condition, when evaluated under Case 1(a) of Lemma 2, is independent of z. Thus any z satisfying the condition for Case 1(a) of Lemma 2 constitutes a pure-money SME.

Mixed money-and-credit SME. For all other possible cases that could emerge as an SME—i.e., Cases 1(b), 2(a)-(b) and 3 in Lemma 2—we know that the distribution $J_i(\cdot, z)$ will always have some positive measure over money- and credit-buyers. In these cases, the generic characterization in Definition 1 applies. We have a fixed point problem in (z, i). Fixing $i = i_d$, consider the solution for z, which is determined by the Euler condition (2.27) and (2.28). This can be re-written, using $\mathrm{d}H_i(\rho, z) := \mathrm{d}\big(1 - [1 - J_i(\rho, z)]^2\big) = 2\alpha_2(1 - J_i(\rho, z))\mathrm{d}J_i(\rho, z)$, as:

$$\frac{\gamma}{\beta} - 1 = r(z)$$

where

$$\begin{aligned} r(z) := {} & (1-n)\, i \\ & + n\alpha_1 \left[\int_{\underline{\rho}}^{\tilde{\rho}_i} i \mathrm{d}J_i(\rho, z) + \alpha_2 \int_{\underline{\rho}}^{\tilde{\rho}_i} i \mathrm{d}H_i(\rho, z) \right] \\ & + n \left[\alpha_1 \int_{\tilde{\rho}_i}^{\hat{\rho}} \left(\frac{u_q(z/\rho)}{\rho} - 1 \right) \mathrm{d}J_i(\rho, z) + \alpha_2 \int_{\tilde{\rho}_i}^{\hat{\rho}} \left(\frac{u_q(z/\rho)}{\rho} - 1 \right) \mathrm{d}H_i(\rho, z) \right] \\ & + n \left[\alpha_1 \int_{\hat{\rho}}^{\overline{\rho}} i \mathrm{d}J_i(\rho, z) + \alpha_2 \int_{\hat{\rho}}^{\overline{\rho}} i \mathrm{d}H_i(\rho, z) \right], \end{aligned}$$

and, we have again suppressed dependency of variables on $\mathbf{s}$, the aggregate state.

First, consider the right-hand-side (benefit of holding money) terms, $r(z)$. Consider any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < z'$, where z' was defined in Lemma 1. The result in Lemma 5 establishes that $J_i(\cdot, z_0)$ first-order stochastically dominates $J_i(\cdot, z_1)$, and consequently, $1 - [1 - J_i(\cdot, z_0)]^2$ also first-order stochastically dominates $1 - [1 - J_i(\cdot, z_1)]^2$. Also, the marginal utility of consumption (of money-constrained agents) terms, $u_q(z/\rho)$, are diminising in z. Thus, $r(z)$ is monotone decreasing in z. The left-hand-side term, $\gamma/\beta - 1$, is a constant with respect to z.

Second, if $z \searrow 0$ (denoted as $z = 0^+$), then $\hat{\rho}(z) \nearrow \hat{\rho}\left(0^+\right) := \min\{\bar{\rho}, \infty\}$ and $\tilde{\rho}_i(z) \nearrow \hat{\rho}\left(0^+\right)$, so that the domain of the integrals $\left[\hat{\rho}\left(0^+\right), \bar{\rho}\right] \to \{\bar{\rho}\}$. This implies that the measure over this singleton set $\{\bar{\rho}\}$ is zero, and so the last two lines of $r(z) = r\left(0^+\right)$ converges to zero, and

$$r\left(0^+\right) = (1-n)i + I_i\left(0^+\right),$$

where $I_i\left(0^+\right) := n\alpha_1 \left[\int_{\underline{\rho}}^{\bar{\rho}} i \mathrm{d}J_i(\rho, 0^+) + \alpha_2 \int_{\underline{\rho}}^{\bar{\rho}} i \mathrm{d}H_i(\rho, 0^+)\right]$. If $z \nearrow \infty$ (denoted as $z = \infty^-$), then $\hat{\rho}(z) \searrow \max\left\{\underline{\rho}, 0\right\} = \underline{\rho} > 0$. Likewise, $\tilde{\rho}_i(z) \searrow \underline{\rho}$. In this limit,

$$r\left(\infty^-\right) = (1-n)i + I_i\left(\infty^-\right),$$

where $I_i\left(\infty^-\right) := n\left[\alpha_1 \int_{\underline{\rho}}^{\bar{\rho}} i \mathrm{d}J_i(\rho, \infty^-) + \alpha_2 \int_{\underline{\rho}}^{\bar{\rho}} i \mathrm{d}H_i(\rho, \infty^-)\right]$. Observe that $I_i(0^+) > I_i\left(\infty^-\right)$ by again, an application of the first-order stochastic dominance result from Lemma 5 and the first argument above. Thus, if the cost of holding money net of the expected benefit of being an ex-post inactive agent is neither too small nor too large, i.e.,

$$I_i\left(\infty^-\right) < \frac{\gamma}{\beta} - 1 - (1-n)i < I_i(0^+),$$

then there exists a unique solution for z, for a given i.

Next, consider the determination of i, for a given candidate z and aggregate state $\mathbf{s}$. Recall, loan demand at a given price draw $\rho \in \left[\underline{\rho}, \tilde{\rho}_i\right]$ is given by the function of i: $\xi(i; \rho) = \rho^{1-\frac{1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z$. Loan demand has the following properties:

1. $\xi(i; \rho)$ is strictly decreasing in i.
2. $\lim_{i\to\infty} \xi(i; \rho) = 0$
3. $\lim_{i\to 0} \xi(i; \rho) = \rho^{1-\frac{1}{\sigma}} - z > 0$, so long as there is demand for credit.

Let

$$L^d(i; z) := n \int_{\underline{\rho}}^{\tilde{\rho}_i} \left[\alpha_1 + 2\alpha_2\left(1 - J_i(\rho, z)\right)\right] \xi(i; \rho) \mathrm{d}J_i(\rho, z)$$

be the aggregate demand for loans. The aggregate supply of loans (deposits) is

$$L^s(i; z) := (1-n)z + n \int_{\hat{\rho}}^{\bar{\rho}} \left[\alpha_1 + 2\alpha_2\left(1 - J_i(\rho, z)\right)\right] \left(z - \rho^{1-\frac{1}{\sigma}}\right) \mathrm{d}J_i(\rho, z).$$

Thus, the excess demand for loans is:

$$
\begin{aligned}
e\left(i;z\right) &= L^{d}\left(i;z\right) - L^{s}\left(i;z\right) \\
&= -(1-n)z + n\int_{\underline{\rho}}^{\tilde{\rho}_i}\left[\alpha_1 + 2\alpha_2\left(1 - J_i(\rho,z)\right)\right]\xi(i;\rho)\mathrm{d}J_i(\rho,z) \\
&\qquad - n\int_{\hat{\rho}}^{\bar{\rho}}\left[\alpha_1 + 2\alpha_2\left(1 - J_i(\rho,z)\right)\right]\left[z - \rho^{1-\frac{1}{\sigma}}\right]\mathrm{d}J_i(\rho,z).
\end{aligned}
$$

First, we want to show that the excess demand switches signs at the extreme limits of i. If $i \nearrow \infty$, $e\left(i;z\right) \to b := -(1-n)z < 0$. If $i \searrow 0$ (denoted as $i = 0^{+}$), $\tilde{\rho}_i \nearrow \hat{\rho}$, and $e\left(i;z\right) \to a(0^{+};z)$, where

$$
\begin{aligned}
a(0^{+};z) &:= -(1-n)z - z \times \left(\underbrace{n\int_{\underline{\rho}}^{\overline{\rho}}\left[\alpha_1 + 2\alpha_2\left(1 - J_{0^{+}}(\rho,z)\right)\right]\mathrm{d}J_{0^{+}}(\rho,z)}_{=1}\right) \\
&\qquad + n\int_{\underline{\rho}}^{\overline{\rho}}\left[\alpha_1 + 2\alpha_2\left(1 - J_{0^{+}}(\rho,z)\right)\right]\rho^{1-\frac{1}{\sigma}}\mathrm{d}J_{0^{+}}(\rho,z) \\
&= -z + n\int_{\underline{\rho}}^{\overline{\rho}}\left[\alpha_1 + 2\alpha_2\left(1 - J_{0^{+}}(\rho,z)\right)\right]\left(\rho^{1-\frac{1}{\sigma}} - z\right)\mathrm{d}J_{0^{+}}(\rho,z).
\end{aligned}
$$

Generically, $a(0;z) \lesseqqgtr 0$. If $n\int_{\underline{\rho}}^{\overline{\rho}}\left[\alpha_1 + 2\alpha_2\left(1 - J_{0^{+}}(\rho,z)\right)\right]\left(\rho^{1-\frac{1}{\sigma}} - z\right)\mathrm{d}J_{0^{+}}(\rho,z) > z$, then for fixed z, there is a unique solution for i.

Finally, in both cases above, since $\alpha_1, \alpha_2 \in (0,1)$, then by Lemma 3 the pricing distribution is alway non-degenerate. □

C Statistical calibration of model

We perform the numerical analyses based on the model that is disciplined by calibration to relevant macro-level statistics in the United States.

C.1 Baseline calibration

We interpret one period in the model to be a year. Our calibration strategy is to match the empirical money demand and the firms' average (percentage) markup in the United States.

The aggregate output in the DM is $q_{DM} := n\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)}\left[\alpha_1 + 2\alpha_2\left(1 - J_i(\rho,z,\gamma)\right)\right]\rho q_b^{\star}(\rho,z)\mathrm{d}J_i(\rho,z,\gamma)$. The aggregate output in our economy is given by:

$$
Y = q_{DM} + x^{\star}. \tag{C.1}
$$

Given policy $\gamma = 1+\tau$, we measure the model's aggregate (percentage) markup as by the weighted average of percentage markups in both markets in the model:

$$\mu(\gamma) = \omega_{DM} \underbrace{\left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \frac{\rho - c}{c} \mathrm{d}J_i(\rho, z, \gamma) \right]}_{\mu_{DM}(\gamma)} + (1 - \omega_{DM}) \cdot 0 \equiv \omega_{DM}\mu_{DM}(\gamma), \tag{C.2}$$

where the weight on DM is $\omega_{DM} := q_{DM}/Y$. The gross markup in the CM is unity (or its percentage markup is zero) since firms are perfectly competitive there. Price dispersion (coefficient of variation) is defined as:

$$CV(\gamma) = \frac{1}{\mu(\gamma)} \left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} (\rho - \breve{\rho})^2 \mathrm{d}J_i(\rho, z, \gamma) \right]^{\frac{1}{2}}, \tag{C.3}$$

where $\breve{\rho} = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho \mathrm{d}J_i(\rho, z, \gamma)$.

We assume a log-utility function in the CM, $U(x) = B\ln(x)$, where B is a scaling parameter that determines the relative importance of CM and DM consumption. With quasi-linear preferences, real CM consumption is determined by $x^\star = (U')^{-1}(B)$. The noisy-search probabilities in the DM are α_1, $\alpha_2 (= 1 - \alpha_1)$, respectively. We normalize the cost of DM production to one ($c = 1$) as in Head et al. (2012). The DM utility function is given by Equation (2.2).

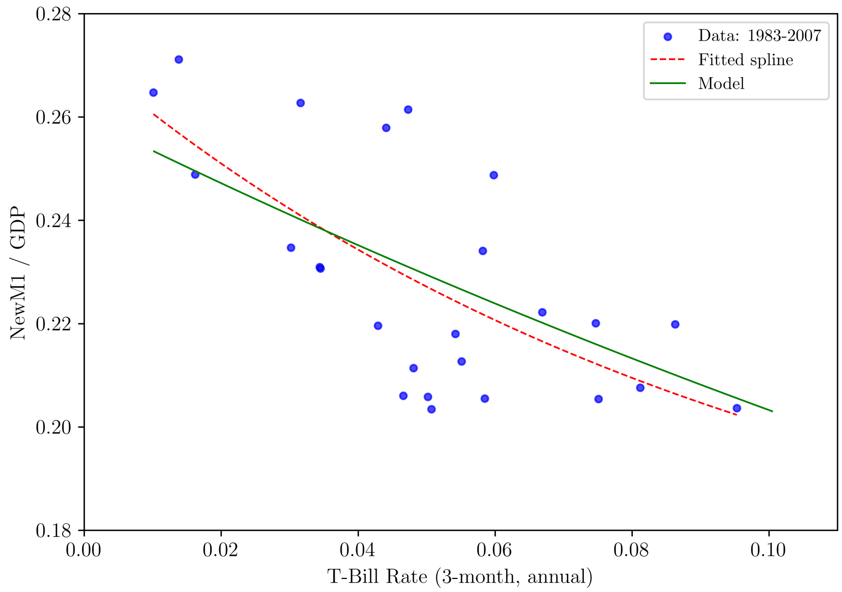
Sample period and data. Our model is fitted to long-run data spanning from 1983 to 2007 to avoid the Great Recession period where the nominal interest rate is at the zero lower bound. We use the New M1-to-GDP ratio defined in Lucas and Nicolini (2015) as a measure of the money demand M/PY in the United States. We employ the U.S. markup data from De Loecker et al. (2020). We obtain the U.S. three-month T-bill interest rate data from the FRED.

Identification and calibration. The parameters that need to determined are: $\beta, \tau, \sigma, \alpha_1, \alpha_2, n$, and B. The parameter β is the time discount factor. The CM utility scaling parameter B affects the average of money demand M/PY. This is because the parameter B affects CM consumption x and thus output Y. The CRRA risk aversion parameter σ pins down the price elasticity of demand for the DM consumption goods, which affects the elasticity of money demand with respect to the nominal interest rate i. The noisy-search probabilities directly affect the price distribution J_i, and thus the aggregate markup. From the Fisher equation, we use both the average interest rate of the three-month T-bill, $i = 0.051$, and the long-run inflation rate, $\tau = 0.031$, to pin down the discount factor $\beta = 0.98$. We set the fraction of passive depositors $(1 - n)$ to 0.35, which corresponds to the mean ratio of household deposit accounts at commercial banks per thousand adults in the U.S.[29]

The remaining parameters $(\sigma, B, \alpha_1, \alpha_2)$ are calibrated internally. We jointly choose $(\sigma, B, \alpha_1, \alpha_2)$ to match the point elasticity of money demand, the average of money demand M/PY, and aggregate markup μ, all of which are with respect to the nominal interest rate i. Figure 10 depicts

[29] Source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators"

Figure 10: Aggregate money demand calibration (result)



a reasonable fit between the calibrated model's implied aggregate money demand curve (*i.e.*, the green-solid graph) and that of the data (blue dots). (The data observations are the blue-circled markers and an empirical spline-model best fit of these sample points is given by the dashed-red graph.)

We summarize the value of jointly calibrated parameters and calibration results in Table 1. Given a reasonable fit of our model to the empirical targets, we can use the calibration above as a benchmark model.

Table 1: Calibration targets and results

Parameter	**Value**	**Empirical Targets**	**Model**
σ	0.32	Elasticity of $M/PY = -0.13^a$	-0.12
B	1.8	Mean of $M/PY = 0.23$	0.22
α_1, α_2	0.047,0.953	Markup = 30%	25%

[a] The point elasticity refers to the elasticity of M/PY with respect to the nominal interest rate i, evaluated at the data mean of i.

D Empirics and supporting evidence (extra material not for review)

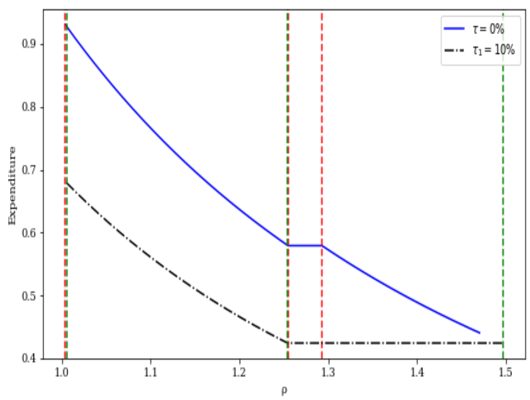
This section contains miscellaneous empirical results that appeared in earlier working-paper versions of this paper. We have decided to relegate such material to this Online Appendix following a suggestion of the referees.

What does the model say about average markups and markup dispersion, in relation to bank credit and inflation? Figure 11 depicts the banking model and the effect of inflation on both an *intensive margin* (summarized by the expenditure of agents as a function of prices drawn) and an *extensive margin* (summarized by the equilibrium distribution of prices and the relevant cutoffs defining the heterogeneous, ex-post buyer types).

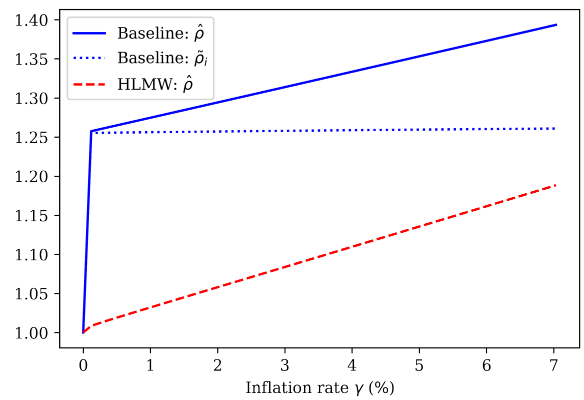
Specifically, Figures 11a and 11c show two examples with 0% and 5% inflation. With the higher inflation equilibrium, we see that there is a larger range of prices (and higher price draws) that support the cases of all the non-credit money buyers. That is, the pricing cutoffs demarcating the ranges of price draws consistent with equilibrium non-credit-buyer events rises with inflation. This claim is corroborated further by Figure 11a where we plot the pricing cutoff formulas from Equation (2.9). The $\hat{\rho}$ function for our economy and that for HLMW have the same formula except that the latter's equilibrium real money balance outcome is always dominated by that of the former's, except when inflation is at the Friedman rule. Figure 11a shows that the pricing cutoffs are increasing as a function of inflation.

Figure 11: Inflation, individual expenditures, equilibrium pricing distribution and cutoffs.

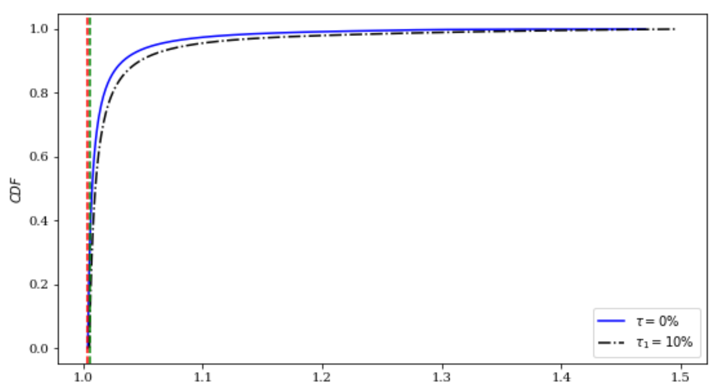
(a) Individual expenditures



(b) Cutoffs (price ranges) and buyer types



(c) Pricing distribution



Consider Figure 11c. The solid blue graph is the equilibrium pricing distribution at an example inflation policy of 0% per annum. The dashed-dotted black graph corresponds to an example higher inflation at 10% per annual. Individually, ex-post non-credit buyers are more likely to draw higher goods prices and thus markups. This is due to the firm's optimal responses: On the one hand, with higher inflation, agents economize on holding real balance z. For fixed inflation, by Lemma 5, this has the effect of increasing pricing and markup dispersion.

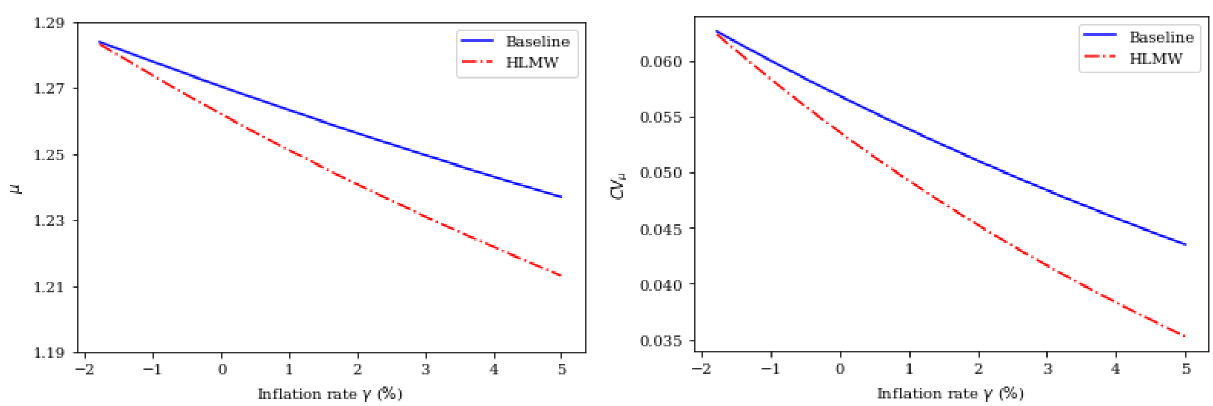
However, in the economy-wide measure in Figure 12, we see that average markup and the dispersion in markups are falling with inflation. This is because the economy-wide measure of markup and markups dispersion involve an output-weighted average between the two sectors of the economy. In the model, markup in the CM sector is always unity since markets are perfectly competitive. Thus, although the DM average markup and markups dispersion is rising, as we have deduced above, the economy-wide measures of these are falling. This is due to the weight on the DM measures—calculated as the ratio of DM average output to the economy-wide average output—falling faster than the actual rise in the DM markup or markups dispersion measure.[30]

D.1 Two testable model predictions

Figure 12a and 12b, respectively, plot how average markup and markup dispersion (the coefficient of variation in markups) vary with inflation.[31] We do this for both our banking equilibrium (solid blue graph) and the HLMW no-banking equilibrium (dashed-dotted red graph).

Figure 12: The effects of inflation on the aggregate markup and price dispersion given policy $\tau \in (\beta - 1, \bar{\tau}]$.

(a) Average markup and inflation (b) Markup dispersion and inflation



We now focus on the following model prediction regarding the presence of banking in Figure 12:[32]

Observation 1 (Two empirically-relevant predictions)**.** *With access to (or, the existence of) banking credit, the economy will have* higher *average markup and markups dispersion in markups (relative to the economy without banks), at any level of inflation (or nominal interest).*

That average markup being negatively related to inflation in the model (Figure 12a) is consistent with existing empirical findings (see Banerjee and Russell, 2005, 2001). Equivalently, we also

[30]See Online Appendix C for the definition of this measure.

[31]In Figure 12a, average markup is reported as a gross factor. For example, a factor of 1 means that there is zero markup, or a factor of 1.3 means an average of 30% price markup on marginal cost.

[32]We can generalize this exercise to a setting with a continuous variation in agents access to bank credit. We omit that exercise here as it provides the same insight as the starker comparison in Figure 12.

show in Figure 13c that average markup is negatively related to the nominal interest rate (proxied by the effective Federal Funds rate). Figure 13b that In the data below (see Figure 13b) we also know that over the relevant sample period, markup dispersion is rising while the nominal interest rate steadily fell (see Figure 13c). The model also produces this stylized negative association between inflation (or the nominal interest rate) and markup dispersion (Figure 12b).

We have previously shown that a perfectly competitive banking sector can amplify firms' market power measured in terms of markup (by Equation (C.2)) and price dispersion (by Equation (C.3)). In Berentsen et al. (2007), the welfare gain of banking liquidity transformation comes from the interest payments on unproductive idle money balances. Households can then accumulate more money balances to trade in the goods market. Thus, having access to a competitive banking sector is always welfare-improving relative to a pure monetary economy.

Contrast this with a setting with goods market power as in HMLW (the dashed-dotted red graphs in Figure 12). Now, the Berentsen et al. (2007) type of banks involve some banking benefits going to the credit-buyers (by relaxing their liquidity constraint) and inactive buyers (by depositing idle funds). However, banking credit can also distort the liquidity premium for the money-buyers via both higher price dispersion and markups (see the solid blue graphs in Figure 12).

D.2 Empirical evidence

We now present two pieces of reduced-form empirical evidence to support the two stylized model predictions in Observation 1. We show that there is a positive relationship between the consumer credit-to-GDP ratio and aggregate price markup. Also there is a positive relationship between the consumer credit-to-GDP ratio and markup dispersion in the United States.

We put together data on quarterly consumer credit-to-GDP ratio, the aggregate markup and price dispersion in the United States. The data sample period is from 1980Q1 to 2007Q4. More details on the data is available in the Online Appendix D.3.

In the data (Figure 13a), we can see that there is a positive relationship between our proxy for access to credit and markup. Likewise, in Figure 13b, there is a positive relation between between access to credit and markup dispersion. We can put these casual observations to more formal regression tests.

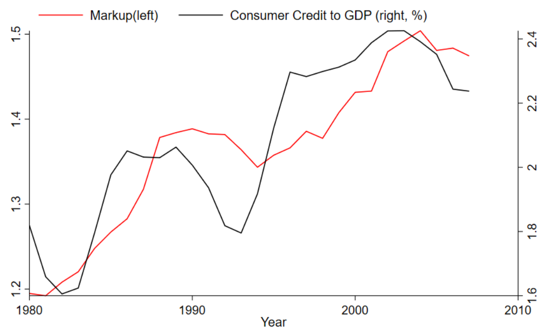
Empirical model. We now investigate the effect of consumer credit on the aggregate markup and its dispersion by considering the following empirical model specification,

$$y_t = a_0 + a_1 d_t^{CC} + b'\gamma_t + \epsilon_t, \tag{D.1}$$

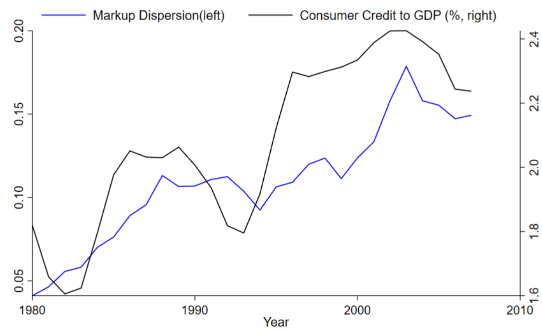
where y_t is one of the variables of interest: log of the aggregate markup, or markup dispersion. The variables d_t^{CC} and γ_t, respectively, denote consumer credit-to-GDP ratio and the list of control variables previously described. The list of parameters, respectively, for the intercept, the credit-to-GDP ratio and all the controls, (a_0, a_1, b') are estimated by ordinary least squares (OLS). Conditional on the other factors, we are most interested in the various estimates of a_1, which will be presented in the first row of Table 2.

Figure 13: Time series of credit-to-GDP ratio and markup statistics

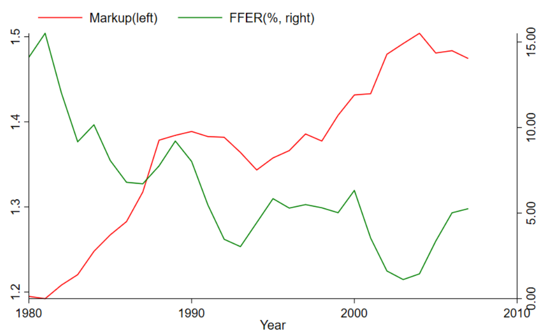
(a) Credit and average markup



(b) Credit and markups dispersion



(c) Average markup and the effective Federal Funds rate



Empirical results. Table 2 reports the OLS results for Equation (D.1). Column (1) indicates that there is a positive and statistically significant relationship between consumer credit-to-GDP and aggregate markup. Column (2) shows a positive and statistically significant relationship between consumer credit-to-GDP and price dispersion.[33]

D.3 Data and measurement

We put together data on quarterly consumer credit-to-GDP ratio, the aggregate markup and its dispersion in the United States. The data sample period is from 1980Q1 to 2007Q4. The summary statistics concerning the data are in Table 3.

Consumer Credit-to-GDP (%) and Real GDP. We obtain total consumer credit owned and securitized by depository institutions from FRED.[34] We use the U.S. nominal GDP data from FRED and total consumer credit to compute the consumer credit-to-GDP ratio.

[33]We provide more detailed results for a robustness test in Appendix D.4.

[34]Data source: https://fred.stlouisfed.org/series/TOTALDI

Table 2: OLS results: Markup and dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	4.335***	3.337***
	(1.571)	(1.001)
CPI Inflation	-0.559***	-0.331***
	(0.0928)	(0.0626)
Log of real GDP	0.163***	0.0671***
	(0.0165)	(0.0111)
Business TFP	0.0427	0.0689
	(0.0693)	(0.0488)
Real wage	-0.252***	-0.169***
	(0.0871)	(0.0642)
log of Real Exchange rate	-0.111***	-0.0172
	(0.0257)	(0.0160)
Real interest rate	-0.172	-0.177**
	(0.140)	(0.0839)
R^2	0.915	0.853
Observations	112	112

Note: Robust errors are in parenthesis, with $*, **$ and $* * *$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

Markup. To calculate the aggregate markup measures, we use *Compustat* data on the quarterly balance sheets of publicly listed firms in the United States.[35] We use quarterly firm-level balance sheet data of listed U.S. firms for the period 1980Q1 to 2007Q4 from Compustat North America. Following De Loecker et al. (2020), our industry classification is on the basis of the North American Industry Classification System (NAICS). Particularly, we observe measures of input expenditure, sales, detailed industry activity classifications, and capital stock information. The item from the financial statement of the firm that we will utilize to measure the variable input is the cost of goods sold (COGS). It bundles all expenses directly attributable to the production of the goods sold by the firm and includes materials and intermediate inputs, labor cost, energy, and so on.

Following Hall (1988) and De Loecker et al. (2020), we compute firm-level markups based on the production approach. This approach estimates markup derived from an assumption that firms minimize their cost.

In each period t, an individual firm i markup μ_{it} is defined as

$$\mu_{it} = \theta_{it}^V \frac{P_{it}}{P_{it}^V} \frac{Q_{it}}{V_{it}}, \tag{D.2}$$

[35]Data source: Wharton Research Data Services.

Table 3: Data sources and summary statistics

Variable	**Source**	**Mean**	**Median**	**S.D**
Aggregate markup	Compustat	1.37	1.38	0.09
Markup dispersion	Compustat	0.11	0.11	0.04
Consumer credit-to-GDP (%)	FRED	2.08	2.06	0.26
CPI inlfation rate (%)	FRED	3.86	3.19	2.57
Real GDP	FRED	0.74	0.80	0.71
Business sector TFP (%)	Fernald (2014)	0.86	1.11	2.78
Real Wage	FRED	2.23	2.92	2.90
Real Exchange rate	BIS	87.44	84.82	9.52
Real Interest rate	FRED	2.48	2.80	2.38

Note: All data series are from 1980 (Q1) to 2007 (Q4). Compustat data is from the Wharton Research Data Services. FRED stands for Federal Reserve Economic Data, while BIS refers Bank for International Settlements. Real GDP is calculated as the growth rate from the previous period.

where P_{it}, P_{it}^V, V_{it} and Q_{it}, respectively, denote the output price, price of the variable input, the variable input, and output.

According to Equation (D.2), an individual firm's markup comprises two components: (1) The revenue share of the variable input is specified as $\frac{P_{it}}{P_{it}^V}\frac{Q_{it}}{V_{it}}$; and (2) the output elasticity of the variable input measured by θ_{it}^V.

We fix the output elasticity to be time-invariant (0.85). This assumption is consistent with the empirical evidence documented in De Loecker et al. (2020).[36] Then, we compute the aggregate markup as

$$\mu_t = \sum m_{it}\mu_{it}, \tag{D.3}$$

where m_{it} is the weight of each firm.[37]

Markup dispersion. Following Meier and Reinelt (2022), we compute aggregate markup dispersion in period t as

$$\nu_t = \sum_i m_{it}\Big[\log(\mu_{it}) - \log(\mu_t)\Big]^2, \tag{D.4}$$

where m_{it} is the weight of each firm, and μ_{it} and μ_t are respectively determined by Equation (D.2) and Equation (D.3).

In short, aggregate markup dispersion is just the weighted average of the log deviation of an individual markup from the aggregate markup.

Control variables. Our control variables include: log of real GDP, CPI(Consumer Price Index) inflation rate, business sector, real wage, real interest rate, real effect exchange rate, and Total

[36]The authors document that the pattern of markup with fixed output elasticity (0.85) is similar to that using estimated output elasticities.

[37]We employ the share of sales in the sample as the weight. See De Loecker et al. (2020) for more details.

Factor Productivity (TFP) growth rate. Note that real interest rate is calculated by subtracting the inflation rate from the Federal funds effective rate, and real wage is calculated by deducting the inflation rate from the nominal growth wage rate. We obtain data on real GDP, CPI inflation rate, federal fund effective rate, and wage growth rate from the FRED.[38] We obtain the business sector TFP data from Fernald (2014).[39] The real effective exchange rate data is from the BIS.

D.4 Robustness test

In this section, we present our robustness tests regarding the empirical results discussed in Section D. We implement the following alternative empirical tests or measurements as a robustness check: (1) vector error correction model (VECM); (2) we include the lag term; (3) we use different credit measure, (4) dummy variable for the Volcker period (1979 to 1987).

VECM. Prior to estimation, we use a unit root test to investigate integration properties of the data. We find that the log of markup, markup dispersion, consumer credit–to–GDP, log of real GDP, and log of real exchange rate as $I(1)$.[40] The vector error correction model is given by

$$\Delta X_t = \gamma + \sum_{j=1}^{p-1} \Gamma_j \Delta X_{t-1} + \epsilon_t, \tag{D.5}$$

where γ is a vector of intercepts, ϵ_t is a vector of contemporaneous errors, X_t is the three–dimensional vector containing the variables, and Γ_j is a set of matrices of short-run coefficients.

Since there is a positive relationship between markup and markup dispersion and the sample is relatively small, then we need to minimize the number of variables. Then, we utilize the two VECM specifications: 1) log of markup, log of real GDP, consumer credit–to–GDP, and log of real exchange rate; and 2) markup dispersion, log of real GDP, consumer credit–to–GDP, and log of real exchange rate. We set the $p(lag) = 5$ based on information criteria in both empirical specifications.

First, we employ the four–dimensional vector including the variables: log of markup, consumer credit–to–GDP (%), the log of real GDP, and log of real exchange rate to investigate the long-run relationship between log of markup and consumer credit–to–GDP. The Johansen Trace indicates one cointegrating vector among four variables at the 1% level of significance. (Below, we denote an estimated coefficient with such a level of statistical significance with a three-asterisk supercript or ***.) Based on the VECM analysis, we show that the estimated long-run relationship is

$$\log(\mu_t) = 61.07^{***} \cdot d_t^{CC} - 0.36^{***} \cdot log(RGDP_t) - 0.12 \cdot log(REER_t) + 293.74. \tag{D.6}$$

Equation (D.6) suggests that there is a positive long-run relationship between markup and

[38]Real GDP data comes from https://fred.stlouisfed.org/series/GDPC1. We can get CPI inflation rate from https://fred.stlouisfed.org/series/CPIAUCSL. Wage and salary comes from https://fred.stlouisfed.org/series/A132RC1Q027SBEA. Federal fund effective rate can be obtained from https://fred.stlouisfed.org/series/FEDFUNDS.

[39]See https://www.johnfernald.net/TFP.

[40]The CPI Inflation rate is a stationary variable according to unit root test results.

consumer credit–to–GDP d_t^{CC}. The estimate is statistically significant.

To estimate the long-run effect of consumer credit on markup dispersion, we utilize the four–dimensional vector including the variables: markup dispersion, consumer credit–to–GDP (%), the log of real GDP, and log of real exchange rate. The Johansen Trace indicates one cointegrating vector among three variables at the 1% level of significance. We can obtain the estimated long-run equation as

$$\nu_t = 14.6^{***} \cdot d_t^{CC} - 0.02 \cdot log(RGDP_t) - 0.06 \cdot log(REER_t) + 30.7. \tag{D.7}$$

Equation (D.7) suggests that the long run relationship between markup dispersion and consumer credit–to–GDP (%) is positive, and the estimate is statistically significant. These results support our empirical validity for the result of OLS in Section D.

Including the Lagged Dependent Variable. By utilizing the Durbin-Watson test, we find that there is a serial correlation in the residuals from OLS in Section D. Then, we include the lagged dependent variable to reduce the serial correlation. Based on the information criteria, we decide to include the one lag terms of dependent variable. The empirical specification is almost identical to Equation (D.1) except it includes the lag terms. As shown in Table 4, the results of the robustness test suggest that the positive association between consumer credit and markup/markup dispersion is valid.

Table 4: OLS results: Markup and Markup Dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	1.870*	2.446***
	(1.100)	(0.925)
CPI Inflation	-0.169*	-0.209**
	(0.0766)	(0.0723)
Log of real GDP	0.0333*	0.0404**
	(0.0154)	(0.0129)
Business TFP	0.0297	0.0564
	(0.0429)	(0.0464)
Real wage	-0.0770	-0.125*
	(0.0658)	(0.0646)
log of Real Exchange rate	-0.0253	-0.00894
	(0.0162)	(0.0150)
Real interest rate	-0.00862	-0.106
	(0.0960)	(0.0794)
R^2	0.962	0.870
Observations	111	111

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

Different Measure: Consumer Credit and Markup Dispersion For robustness check, we use other measure of consumer credit and markup dispersion. First, we utilize different consumer credit measures: (1) the percentage ratio of total consumer credit to GDP or (2) the percentage ratio of consumer loans from commercial banks to GDP. The empirical specifications are similar to Equation (D.1) except for replacing consumer credit–to–GDP with total consumer credit to GDP and consumer loans from commercial banks to GDP.[41]

Table 5 reports the empirical results. Columns (1)—(2) show that the relationship between total credit and markup(and its dispersion) is positive, and the estimates are statistically significant. Furthermore, the association between bank credit on aggregate markup and its dispersion is statistically significant and positive as shown in Columns (3)—(4). These results support the validity of our main empirical results presented in Section D.

[41] Total consumer credit owned and securitized comes from https://fred.stlouisfed.org/series/TOTALSL. Consumer loans from all commercial banks is from https://fred.stlouisfed.org/series/CONSUMER.

Table 5: OLS results: Markup and Markup Dispersion

Dependent Variable:	$\log(\mu_t)$ (1)	ν_t (2)	Dependent Variable:	$\log(\mu_t)$ (3)	ν_t (4)
Total Consumer Credit-to-GDP	3.844***	3.640***	Consumer Loan-to-GDP	9.320***	5.517***
	(1.100)	(0.714)		(1.922)	(1.125)
CPI Inflation	-0.652***	-0.423***	CPI Inflation	-0.513***	-0.299***
	(0.010)	(0.069)		(0.093)	(0.0605)
Log of real GDP	0.116***	0.0153	Log of real GDP	0.238***	0.118***
	(0.027)	(0.0182)		(0.0120)	(0.0075)
Business TFP	0.0429	0.0663	Business TFP	0.0635	0.0840+
	(0.070)	(0.046)		(0.066)	(0.048)
Real wage	-0.194**	-0.119*	Real wage	-0.318***	-0.204***
	(0.0875)	(0.0608)		(0.0808)	(0.0628)
log of Real Exchange rate	-0.140***	-0.0477***	log of Real Exchange rate	-0.066***	0.012
	(0.029)	(0.017)		(0.021)	(0.016)
Real interest rate	-0.198	-0.197**	Real interest rate	-0.160	-0.174**
	(0.138)	(0.0821)		(0.128)	(0.083)
R^2	0.919	0.870	R^2	0.926	0.863
Observations	112	112	Observations	112	112

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

Dummy Variables (Volcker period) We conducted our analysis by incorporating a dummy variable for the Volcker period (1979-1987). This accounts for to the high inflation rates experienced during Paul Volcker's tenure as the Chairman of the Federal Reserve. Even with the inclusion of the dummy variable for the Volcker period, we find a statistically-significant negative relationship between markup (and its dispersion) and inflation. This confirms that our main findings in the paper are robust.

Table 6: OLS results: Markup and Markup Dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	3.714**	3.242***
	(1.493)	(0.993)
CPI Inflation	-0.405***	-0.307***
	(0.0947)	(0.0646)
Log of real GDP	0.137***	0.0630***
	(0.0181)	(0.0126)
Business TFP	0.0436	0.0690
	(0.0643)	(0.0489)
Real wage	-0.183**	-0.159**
	(0.0831)	(0.0658)
log of Real Exchange rate	-0.0371	-0.006
	(0.0275)	(0.0200)
Real interest rate	-0.209*	-0.182**
	(0.126)	(0.0835)
Volcker dummy	-3.413***	-0.524
	(0.988)	(0.619)
R^2	0.924	0.854
Observations	111	111

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.