On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion*

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Abstract

We study the interaction between banking, endogenous market power with price dispersion in goods markets, and reserve requirement regulation. If the reserve requirement never binds, then the economy is a banking generalization of Head, Liu, Menzio and Wright (2012): the addition of banking has no pecuniary externality on goods trades and banking is always welfare improving. If the reserve requirement binds, there is a positive spread between lending and deposit rates. In this empirically-relevant case, there is a pecuniary externality: banking amplifies retail-goods firms' market power. Credit- and policy-dependent heterogeneity in retail-good markups implies a non-monotonicity in the welfare-improving role of banks. We explain the novel opposing forces at work. Our model also justifies why policymakers should be worried about the nexus between inflation, banking and industry markups.

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1 Introduction

In this paper, we revisit the question of Berentsen, Camera and Waller (2007) (BCW) on the *essentiality* of banking. Specifically, we study the interaction between banking, endogenous market power and price dispersion in goods markets, and long-run inflation targeting in the presence of a reserve requirement regulation on banks.

We combine the perfectly competitive banks of BCW with a monetary model featuring endogenous goods market power and an equilibrium distribution of posted prices (Head, Liu, Menzio and Wright, 2012) (HLMW). To be able to compare with this existing literature, we restrict attention to the BCW definition of banking. That is, we focus on banks solely as vehicles that take deposits of idle money and provide credit to those who turn out to need it. In an environment where private credit contracts are incentive infeasible, banks are essential institutions that enable individuals to insure *idiosyncratic liquidity risks*.[1] Both BCW and HLMW are steeped in the New Monetarist tradition where crucial market frictions are not assumed but are results of deeper informational and contractual environments.[2] This allows the researcher to study questions such as the existence and essentiality of money, banking, financial markets and asset liquidity as equilibrium objects (see, e.g., Williamson and Wright, 2010; Lagos, Rocheteau and Wright, 2017). By combining these two models, we arrive at an insight that might not be so apparent to conventional wisdom: Even with the best-possible case or idealization of perfect competition among banks, banks need not necessarily be a welfare-improving proposition (c.f., Berentsen, Camera and Waller, 2007). One must also worry about the interaction of the bank interest rate with pricing in goods markets where pricing dispersion is endogenous and sellers have market power.

A novelty in our model will be in the dependence of market power in the DM-good pricing on the price of credit and deposits (c.f., Head et al., 2012), and this dependency arises when reserve requirement regulation on banks matter (i.e., are binding on banks).

A new equilibrium trade-off. In the paper, we provide a complete characterization of the dependency of generic equilibrium classes on long-run inflation. The additional role of a reserve requirement regulation on banks also matters. If the reserve requirement never binds, then the economy is a generalization of HMLW with BCW banks: Here, banking serves the same role as that in BCW, and the loan interest rate i equals the deposit rate i_d. Despite the existence of ex-post heterogeneous demands for goods being dependent on i, there is no pecuniary externality from banks to the goods market in such an equilibrium configuration.

The more interesting case (that is also the empirically-relevant case), is the equilibrium regime where the reserve requirement in the model binds. When this regulation is binding on banks, there is a positive wedge between i and i_d. As a result of this, there exists heterogeneous buyers of goods: Some buyers would take out credit from banks, some just spend all their money (i.e., money-constrained buyers), and others may deposit and also buy goods. The equilibrium distribution in goods prices also becomes dependent on these interest rates and policy. Goods sellers anticipate that there will be buyers showing up with credit, and in a pricing equilibrium induced by the buyers' noisy search, the random pricing contracts tend to be

[1] We also focus on perfect competition in banking as do Berentsen et al. (2007). Some have suggested that we also incorporate market power in banking. We do not do that here since that will only (quantitatively) deepen the welfare cost of banking without changing the basic insights of this paper. In a different paper, Head, Kam, Ng and Pan (2025) study banking with endogenous market power and pricing dispersion in deposit and lending rates.

[2] For example, markup power by firms and pricing dispersion are both equilibrium results in Head, Liu, Menzio and Wright (2012), and are consistent with microdata on price-change distributions. In contrast, standard New Keynesian models require an assumption of monopolistic competition coupled with either an exogenous price-resetting probability or exogenous shocks to firms' cost functions to engineer price dispersion.

higher for money-constrained agents.

Thus, in this setting there is a pecuniary externality: banking amplifies retail-goods firms' market power and these squeezes the rent from buyers who do not use banking. This negative externality is a new force working against the otherwise positive benefit of banking for intermediating goods trade of BCW. If the externality dominates, then we could see that banking may reduce ex-ante welfare. Whether this arises, depends on the stance of long-run inflation targeting policy. That is, there could be a non-monotonicity in the welfare-improving role of banks and this depends jointly on a monetary policy and prudential regulation policies—the long-run inflation target and reserve-requirement regulation.

This possible equilibrium tension renders a non-monotone *ex-ante* welfare implication for competitive banking: At sufficiently low inflation, banks need not be *essential* or welfare improving. That is, when inflation is low, the pecuniary externality caused by banks on goods seller's pricing behavior tends to overpower the liquidity-risk insurance benefit coming from banks. However, when inflation is high enough, the liquidity-risk insurance channel dominates the pecuniary externality effect. This will turn out to be our empirically-relevant equilibrium type.

On the new trade-off: Benefit versus cost of bank credit and inflation. We explain this novel trade-off in more detail: Assume the case of a binding reserve-requirement regulation and consider the *benefit* of banking in the model. It comes in two parts. With access to banks, ex-post inactive buyers (those who do not have a trading opportunity) can deposit idle funds with banks to earn interest. In addition, some active buyers (those who have a trading opportunity) may find it optimal to top up their money with bank credit in order to relax their liquidity constraint. In the model, these two forces imply higher consumption and welfare. We call this overall benefit of banking a *liquidity-risk insurance effect*, which is also present in BCW.

However, there is equilibrium feedback from the ability of some agents to use bank loans, to agents' ex-ante decision to hold money, to the distribution of goods-price markups. We call this an opposing *pecuniary externality* (through *pricing dispersion*) *effect*. We show that a first-order stochastic dominance result holds: For a given inflation level, lower equilibrium real money balance implies firms are more likely to exact higher markups on agents who are liquidity-constrained and unconstrained money-buyers. Lower real money balance has a direct effect on money-constrained buyers through tightening their ex-post liquidity constraints. Unconstrained money-buyer also suffer lower consumption as their demands for goods are decreasing in price. Thus, the presence of buyers who find it optimal to borrow from banks create a pecuniary externality through the pricing-markup distribution. This tends to reduce the consumption level for buyers who do not use banking credit.

Unlike BCW, access to bank credit for some agents can create a pecuniary externality cost on others even though there is perfect competition among banks and there are no costs to access banking services. In our model, what is sufficient to induce this externality is the Head et al. (2012)-like goods-price distribution that becomes dependent on consumers' ex-ante money balance decision. In turn, this decision is made in anticipation of the possibility of credit-financed events. In short order, banking can improve welfare for those with idle money or those who are willing to borrow. However, by encouraging less own-money holdings, banking also amplifies goods-price markups' dispersion and average which makes non-credit buyers worse off. This trade-off, as we will show is sensitive to inflation, and thus, to monetary policy.

We discipline the model by calibrating it to the data. We numerically show the following: In contrast to the model without banks (i.e., the HLMW model) average markups under a competitive-banking equilibrium is always higher. Likewise, the dispersion of markups is also higher in the banking equilibrium. The gaps in these measures between the banking equilibrium and the HLMW limit are increasing with inflation. For plausibly low inflation ranges, banking is welfare reducing since for low inflation the gains from banking to depositors of idle money and credit-buyers is small compared to the dispersion effect on non-credit agents. For sufficiently high inflation, the result reverses.

Related literature. Head et al. (2012) (HMLW) and Berentsen et al. (2007) (BCW) both feature decentralized markets where anonymous agents have the incentive to hold money in order to buy goods.[3] Both models are derived from Lagos and Wright (2005). BCW introduced perfectly competitive banks into a Lagos and Wright (2005)-type of model to show that banks are welfare improving institutions or are *essential*, in the sense of liquidity transformation or idiosyncratic liquidity risk reallocation. Moreover, in a variation on their model, BCW also consider a decentralized goods markets where there is a (Nash) bargaining friction that also implied market power among sellers. Nevertheless, in their setting bank credit does not induce any pecuniary externality in goods trade. This is because, ex post, in there is no pricing heterogeneity faced by searching buyers. Thus, in BCW, regardless of whether goods sellers in decentralized trades have market power, banks are shown to fully compensate holders the opportunity cost of idle money in terms of deposit interest. In contrast, we show that when there is equilibrium pricing dispersion under heterogeneous market power in the style of HMLW, this is no longer true because of its pecuniary externality feedback onto ex-post non-credit buyers.[4] (We provide an analytical, comparative-equilibrium study on this point in Section 4.1 in the paper.)

HMLW) adapt the consumer search model of Burdett and Judd (1983) to rationalize equilibrium price dispersion that is consistent with well-known facts about price stickiness at the micro-level data on product pricing.[5] Their money-neutral model provided an important lesson in the spirit of the Lucas critique: Observed price dispersion and stickiness in micro-level price changes do not necessary imply that monetary policy has real effects through these phenomena. Our combination of HLMW with BCW allows us to arrive at a modified statement about the essentiality of banks. Moreover, it introduces or identifies an equilibrium causal nexus that runs from monetary policy to banking intermediation, which in turn induces a pecuniary externality on agents' allocations through firm's equilibrium pricing-markups and their dispersion.

Our result on the negative welfare effect of credit is comparable to that established in Chiu, Dong and Shao (2018). The authors also consider a perfectly competitive banking sector, focusing on banking's role in reallocating idle liquidity, as in Berentsen et al. (2007). In their model, access by borrowers to credit raises the *homogeneous* price level of the goods traded in a decentralized market: more demand for goods by credit-buyers raises the marginal cost of production. With competitive price-taking, this translates to a higher goods price in the authors' model. This pecuniary-externality or feedback-on-

[3] Anonymity here is taken to mean that sellers cannot observe buyers histories and any private promises to repay cannot be enforced. Thus, money is essential, i.e., it has value in equilibrium as a medium of exchange, just as in Lagos and Wright (2005).

[4] This has a similar flavor to the insights of Geromichalos and Herrenbrueck (2016). Their model has a liquid asset (money) and an illiquid asset that can be liquidated in a frictional over-the-counter (OTC) secondary asset market. Competitive (c.f., frictional OTC) trade in their secondary asset market may not be efficient because an agent's holding of an additional unit of money insures not just their own consumption shock but also that of buyers of the liquid asset in the secondary asset market. However, agents ignore this positive externality on ex-post secondary-market buyers when they make ex-ante money accumulation decisions. In a related sense, we have the pecuniary externality of bank credit on money-buyers arising in a simpler, one-asset model with perfectly competitive banking.

[5] In our model, as in Head et al. (2012) and Burdett and Judd (1983), firms post prices and produce on the spot. Buyers observe a random number of price quotes posted by firms and buy at the lowest price they observe. This induces firms to optimally trade off between charging a higher markup on their goods and a lower probability of contact by buyers. Equilibrium in the model results in firms being indifferent between a continuum of these opposing margins of attaining the same maximal expected profit. This renders an equilibrium, realized distribution of posted (and transacted) prices that will depend on monetary policy and the aggregate amount of money.

higher-price effect tightens the liquidity constraint of money-buyers and reduces their consumption. This is also similar to Berentsen, Huber and Marchesiani (2014). Like us, Chiu et al. (2018) show that even under perfectly-competitive goods and banking markets, credit can induce a pecuniary-externality cost on liquidity-constrained money-buyers. However, their result requires the assumption that there is an exogenous measure of money-constrained buyers and the cost of producing the decentralized-market good is strictly convex.

In contrast, we obtain a negative welfare effect of credit through a channel of endogenous firms' market power in goods price markups and dispersion. Also, in our setting, the measures of money-constrained and other agent types are endogenous. Moreover, in our model equilibrium, even unconstrained money-buyers can be affected negatively, since there is not just the one goods price in our model and these agents end up drawing higher prices and consuming less as a result. We shut down the possibility of another pecuniary-externality channel like that of Chiu et al. (2018) by assuming that decentralized-market firms have a linear cost of production. Instead, we identify a new and alternative mechanism for this externality effect. We show that buyers with access to credit can contribute to an increase in the measure of firms charging higher prices and extracting more rent from liquidity constrained money-buyers. Hence, banking can be welfare-reducing in equilibrium.

Dong and Huangfu (2021) present a monetary model in which both money and credit serve as a means of payment. In their model, the payment instrument involved with money (credit) is subject to the inflation tax (fixed transaction costs). Moreover, inflation also affects the choice of using credit in transactions and lowers the benefit from using it due to the delayed settlement effect. They show that using credit can be welfare-reducing at very low or very high inflation. In contrast, settlement of credit here is intratemporal as in BCW. We do not require any cost to accessing bank credit.

There are few other studies incorporating the noisy search process of Burdett and Judd (1983) into a monetary framework for various applications (see, e.g., Head and Kumar, 2005; Head, Kumar and Lapham, 2010; Chen, 2015; Wang, 2016; Wang, Wright and Liu, 2020). Wang et al. (2020) focus on rationalizing the price-change pattern and cash-credit shares observed at the micro-level data in the United States. In their model, buyers' access to credit is costly, so that money and credit are imperfect substitutes as means of payments. In contrast, agents' access to banking is not restricted in our setup, as in Berentsen et al. (2007), and we are not concerned with the question of competing media of exchange. There is just one medium of exchange (money) in decentralized, anonymous trades.

Recent empirical studies find that industry market power, measured in terms of price markups, has been sharply increasing since the 1980s in the United States (see, e.g., Hall, 2018; Rossi-Hansberg, Sarte and Trachter, 2020; De Loecker, Eeckhout and Unger, 2020). This has prompted a literature that investigates the macroeconomic consequences of industry market power (see, e.g., Guerrieri and Lorenzoni, 2017; Autor, Dorn, Katz, Patterson and Reenen, 2020; Edmond, Midrigan and Xu, 2023). Since the 1980s, the U.S. consumer credit-to-GDP ratio has also been accelerating around the same time as the rise in industry market power. The phenomenon of rising industry market power is not only of interest to academics but also to policymakers. For example, U.S. President Biden has recently called for promoting industry competition in the United States (see Executive Order 14036, 2021). Our study complements this literature by highlighting the unexplored nexus between competitive banking and its effect on goods markup-pricing outcomes.

The remainder of the paper is organized as follows. In Section 2, we lay out the details of the model, agents' decision problems and characterization of a Stationary Monetary Equilibrium (SME). In Section 4, we dissect and discuss the new tension underlying the welfare consequences of banking created the new pecuniary externality from banks, even if they are perfectly competitive banks. We provide a set of numerical illustrations to further expound on the model mechanism. We perform these numerical experiments using the baseline model that is calibrated the U.S. data. We conclude in Section 5.

2 Model

The model builds on Head et al. (2012) (HLMW) by introducing perfectly competitive markets for bank deposits and loans. There is also a reserve requirement regulation on banks that may or may not be binding. As in Berentsen et al. (2007) (BCW), the focus here is on banks' role in terms of intermediating between ex-post heterogeneous liquidity needs of agents. A novelty in our model will be in the dependence of market power in the DM-good pricing on the price of credit and deposits (c.f., Head et al., 2012), and this depends crucially on the stance of the long-run inflation target (monetary policy) and reserve requirement regulation. This is because, in equilibrium, there may exist heterogeneous buyers of goods—some would take out credit from banks, some just spend all their money, and others may deposit and also buy goods. This renders their heterogeneous demands for the DM good dependent on the nominal loan interest rate (i) or deposit rate (i_d). Goods sellers anticipate this, and in a pricing equilibrium induced by the buyers' noisy search, they end up posting random pricing contracts. The equilibrium distribution in goods prices also becomes dependent on these interest rates and policy.

In every period, two markets open sequentially as in Lagos and Wright (2005). First, a decentralized goods market (DM) with trading frictions opens. In the DM, households are anonymous so that private credit arrangements are incentive infeasible. Consequently, fiat money will be valuable as a medium of exchange in equilibrium. The DM will be the source of fundamental frictions in the model. The DM will be followed by a frictionless centralized market (CM) which allows agents to rebalance their asset positions.

2.1 Primitives

Preferences. Each household has their per-period utility described by

$$\mathcal{U}(q, x, h) = u(q) + U(x) - h, \tag{2.1}$$

where $u(q)$ is the utility flow from consumption of the goods in the DM, $U(x)$ is the utility flow of consumption goods x in the CM, and $-h$ captures the disutility of labor.

We assume that $u' > 0, u'' < 0$ and u satisfies the standard Inada conditions. Likewise for the CM utility function U. We restrict our attention to the constant-relative-risk-aversion (CRRA) class of functions:

$$u(q) = \frac{q^{1-\sigma} - 1}{1 - \sigma}. \tag{2.2}$$

The risk aversion coefficient $\sigma \in (0, 1)$ influences the households' price elasticity of demand. As in Head et al. (2012), this parameter restriction ensures a well-defined equilibrium (see also Altermatt, 2022; Williamson, 2012).

Technologies. In the CM, the general goods x are produced using a technology that is linear in labor input h. Consequently, both real wage and the price of the general goods will be equal to one. In the DM, firms producing one unit of good q requires $h = c \times q$ hours of labor. The parameter $c > 0$ is the constant marginal cost of DM production.

2.2 Timing and events in the sequential DM and CM

In the model, time is discrete and infinite. Agents discount across period t and $t+1$ by a common discount factor $\beta \in (0,1)$. We will use variables $X \equiv X_t$ and $X_+ \equiv X_{t+1}$ respectively to denote time-dependent outcomes at period t and $t+1$.

There are four types of agents: households, firms, banks, and the government. There is a continuum of households and firms, each of measure one. The banking sector is perfectly competitive with free entry. The government supplies fiat money according to the rule $M_+ = \gamma M$, where $\gamma = 1 + \tau$ is money-supply growth factor and $\gamma \in [\beta, \infty)$. The government may also regulate banking through a reserve-requirement ratio instrument $\chi \in [0, 1)$.

Given monetary policy γ, let $\mathbf{a} := (i, i_d, M, \phi; \gamma)$ denote a list of nominal interest rates on loans (i) and deposits (i_d), and the aggregate state of the economy (comprising the total money stock in circulation M, and the value of money or the inverse of the numéraire-good price level ϕ).

Decentralized market (DM). At the start of each period the DM opens. The following sequence of events arise:

1. Each household realizes a preference shock that has two possible outcomes: First, a particular household agent turns out to want to consume (the DM good q). This outcome occurs with probability n, and we label the associated agents as *active buyers*. Second, an agent does not wish to consume, and this occurs with probability $1-n$. We label an agent in such an event an *inactive buyer*.[6]

   The banking market is open at this stage as well: All DM agents can access a line of credit and can deposit any amount of money that they possess with perfectly competitive banks. Banks charge borrowers a competitive rate of i, and commit to paying depositors at a perfectly-competitive nominal interest rate of i_d.[7]

2. After realizing their taste shock, the *active buyers* engage in noisy search for goods sellers. As in HLMW, goods trade is modelled as a monetary-exchange version of the Burdett and Judd (1983) noisy search process: Each DM-goods firm posts a price (p) anticipating that buyers with money holdings m will show up. Firms commit to supplying at the realization of their posted contracts, taking as given the pricing distribution faced by buyers, $\hat{J}(\cdot, m, \mathbf{a})$, and buyers' demand schedule, q_b.[8] Ex ante, buyers know the price distribution but not an individual posted price. Hence, this noisy search process rules out that buyers can direct their search to particular sellers with the lowest

[6] We retain this notation and assumption from Berentsen et al. (2007) for ease of comparison. We will be able to recover a version of their model as a special case of ours when there is no noisy consumer search in the DM goods market and sellers are Walrasian price takers. We also adopt the assumption regarding banking operations as in Berentsen et al. (2007). First, banks operate a financial record-keeping technology at zero cost. Second, banks can perfectly enforce loan repayments. Moreover, agents having access to banks does not rule out the need for money serving as a medium of exchange in the DM. This is because ex-ante agents demand money as a precaution against probable events where they may turn out to optimally not want to borrow from banks, but they still need money in order to buy goods in anonymous DM-good trades.

[7] Clearly, the *inactive buyers* will deposit all their idle monies with the banks. We shall also see that it may be optimal for some *active buyers* to deposit their monies as they may not spend it all on DM goods.

[8] The distribution $\hat{J}(\cdot, m, \mathbf{a})$ is an equilibrium object, and it may possibly represent a compound or reduced-form lottery in the case that each firm may be posting a simple lottery over a simple lottery of pricing outcomes. Such mixed strategies are not welfare-reducing for firms to do. Later, we use the different notation $\left\{ J(\cdot, m, \mathbf{a}), \{\pi_p\}_{p \in P} \right\}$ to denote the component distributions induced by the simple lotteries that compound to the reduced-form lottery with distribution $\hat{J}(\cdot, m, \mathbf{a})$. See Footnote 19 on page 16 for further justifications and Lemma 4 on page 17 for the construction of the set of possible simple lotteries conditional on a draw or outcome of another simpler lottery represented by $J(\cdot, m, \mathbf{a})$. By the Reduction of Compound Lotteries Axiom, we may presume that firms are indifferent between playing the reduced-form lottery with distribution $\hat{J}(\cdot, m, \mathbf{a})$ or take one simple lottery ($J(\cdot, m, \mathbf{a})$) over simple lotteries with distributions $\{\pi_p\}_{p \in P}$, and $p \sim J(\cdot, m, \mathbf{a})$.

price. Instead, buyers randomly contact k number of firms. With probability α_k a buyer matches or makes contact with k firms, or equivalently, draws k price quotes. Each price quote is drawn independently from $\hat{J}(\cdot, m, \mathbf{a})$. For simplicity, we assume that buyers either sample one price quote with probability $\alpha_1 \in [0, 1)$ or two independent price quotes with probability $\alpha_2 = 1 - \alpha_1$. (Some of these buyers may turn out to want to borrow from and some may want to deposit excess liquidity with banks.)

3. Given a draw p from distribution $\hat{J}(\cdot, m, \mathbf{a})$, the agent decides on how much of a good q_b to purchase and whether to borrow money from banks in addition to their own money holdings. Given these choices, the agent can also deduce whether they will want to deposit any excess money holdings with banks.[9]

4. Banking activity (in terms of lending and borrowing) ceases after all household agents have completed their loan and/or deposit transactions with the banks. Goods exchange occurs between the agents and firms in the DM. Buyers face a liquidity constraint consisting of their own money balances m with (or without) loans l and/or deposits d. Buyers then pay the firms to produce the goods for their consumption. After the DM, agents enter a frictionless CM.

Centralized market (CM). An agent entering the CM is denoted by an individual state (m, l, d), i.e., her remaining nominal money balance, outstanding loan and deposit balance. In particular, those who have deposited in the previous DM will earn gross interest $1 + i_d$ on deposits d. Those who have borrowed must repay gross interest $1 + i$ on loan l to banks. Households supply labor h to firms for production and consume the general goods x. Households own firms and firms return profits as dividends $\tilde{D}$ to households. Households then accumulate money balances m_+ to carry into the next period.

2.3 Households

In what follows, we work backwards from the CM to the DM within the period t.

2.3.1 Households in the CM

An agent beginning the CM with money, loan or deposit balances, (m, l, d), may have been a borrower or a depositor in the previous DM during the first sub-period. Her initial value is

$$W(m, l, d, \mathbf{a}) = \max_{(x,h,m_+)\in\mathbb{R}^3_+} \left\{ U(x) - h + \beta V(m_+, \mathbf{a}_+) \;\middle|\; \begin{array}{l} x + \phi(m_+ - m) \\ = h + \tilde{D} + T + \phi(1 + i_d)d - \phi(1 + i)l \end{array} \right\} \tag{2.3}$$

where V is the value function at the beginning of the next DM, ϕ is the value of money in units of the CM consumption good x, i_d is the deposit interest rate, i is the loan interest rate, h is labor supplied, $\tilde{D}$ is aggregate dividends from firm ownership and T is the lump-sum taxes/transfers from the government.

The first-order conditions with respect to x and m_+ are, respectively, given by

$$U_x(x) = 1, \tag{2.4}$$

[9] Thus, the buyer's choices on consumption in the DM, bank credit and deposit depend on the price drawn from the distribution. The (equilibrium) distribution, in turn, depends on agents best-response functions and thus will depend on a given lending rate i.

and,

$$\phi = \beta V_m(m_+, \mathbf{a}_+), \tag{2.5}$$

where $V_m(m_+, \mathbf{a}_+)$ captures the marginal value of accumulating an extra unit of money balance taken into the next period $t+1$. The envelope conditions are

$$W_m(m,l,d,\mathbf{a}) = \phi, \quad W_l(m,l,d,\mathbf{a}) = -\phi(1+i), \quad \text{and} \quad W_d(m,l,d,\mathbf{a}) = \phi(1+i_d). \tag{2.6}$$

Note that W is linear in (m,l,d) and the distribution of money balances is degenerate when households exit the CM. As a result, households' optimal choices for CM consumption and money balance are given by Equations (2.4) and (2.5). These equations are independent of the agents' current wealth since per-period preferences are quasilinear.

2.3.2 Households in the DM

We first describe the post-match household problems. We call households who sample at least one price quote in the DM *active buyers*. We label those who sample zero price quotes *inactive buyers*.

***Ex-post* inactive buyers.** With probability $1-n$, a household is inactive. Conditional on being inactive, a household with money holdings, m, can deposit $d \leq m$ of this money with a bank. She has zero utility flow of consuming q_b and then enters the CM with valuation of $W(m-d, 0, d, \mathbf{a})$. Since holding money is subject to inflation tax, it will be optimal for *inactive buyers* to deposit all of their money holdings, i.e., $d^{\star}(m, \mathbf{a}) = m$.

***Ex-post* active buyer sampling at least one price.** The post-match value of such a buyer is given by:

$$B(m,p,\mathbf{a}) = \max_{q,l,d} \left\{ u(q) + W(\tilde{m}, l, d, \mathbf{a}) \;\middle|\; \begin{array}{l} \tilde{m} := m + l - pq - d \geq 0, \\ d \in [0, m+l], \\ q \geq 0, \\ l \in [0, \infty) \end{array} \right\}. \tag{2.7}$$

We assume banks can perfectly enforce loans repayment as in the baseline case of Berentsen et al. (2007). Hence, buyers do not face a borrowing constraint. In the right-hand-side problem of Equation (2.7), $\tilde{m} := m + l - pq_b - d$ is the residual money balance (i.e., including loan, net of deposit and expenditure on good q) upon entering the CM. This notation implies that there is no *a priori* restriction of who can borrow additional money or deposit idle money.[10] From a buyer's perspective, whether to spend one's total money holdings, to borrow, or to deposit depends on the interest-rate spread $i - i_d \geq 0$ and on the goods price that they face p.[11] We characterize their optimal behavior in the following result.

[10] In an earlier version of the paper, we restricted ex-post buyers to not be able to deposit (in a similar spirit to an ad-hoc cash-in-advance constraint assumption). This ad-hoc restriction contributed to our earlier pecuniary externality result. Here, even without any such ad-hockery, we will demonstrate that the pecuniary externality can still exist or be strong enough to render banking intermediation a welfare-reducing institution. This result will depend on the stance of the reserve requirement policy and the long-run inflation policy. We are most grateful to two anonymous referees for suggesting this generalization which has enriched the insight of the paper.

[11] In deriving the result, the assumption is $\sigma < 1$. We will find that $\sigma < 1$ when we calibrate the model such that its implied aggregate money demand is close to the historical long-run money demand relation in the United States. (See Online Appendix C for the details.) Unlike in standard neoclassical and related New Keynesian models where often their centralized

Lemma 1 (Demand equations)**.** *The DM active buyer demands for goods, loans and deposits are as follows.*

1. *If $i = i_d$, then their demand for good q is smooth,*

$$q_b^\star(m,p,\mathbf{a}) = [(1+i)\phi p]^{-\frac{1}{\sigma}}, \tag{2.8}$$

and their demands for loan and deposit are piecewise continuous, depending on the price draw p relative to the endogenous cutoff $\tilde{p}_i$:

$$l^\star(m,p,\mathbf{a}) = \begin{cases} pq^\star(m,p,\mathbf{a}) - m > 0, & \text{if } p \le \tilde{p}_i \\ 0, & \text{otherwise} \end{cases}, \tag{2.9}$$

and,

$$d^\star(m,p,\mathbf{a}) = \begin{cases} 0, & \text{if } p \le \tilde{p}_i \\ m - pq^\star(m,p,\mathbf{a}) > 0, & \text{otherwise} \end{cases}. \tag{2.10}$$

2. *If $i > i_d$, then their demand for good q is:*

$$q_b^\star(m,p,\mathbf{a}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \le \tilde{p}_i \\ \frac{m}{p} & \text{if } \tilde{p}_i < p \le \hat{p}_{i_d} \\ [p\phi(1+i_d)]^{-1/\sigma} & \text{if } \hat{p}_{i_d} < p \end{cases}, \tag{2.11}$$

$$l^\star(m,p,\mathbf{a}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}[\phi(1+i)]^{-\frac{1}{\sigma}} - m & \text{if } 0 < p \le \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \le \hat{p}_{i_d} \\ 0 & \text{if } \hat{p}_{i_d} < p \end{cases}, \tag{2.12}$$

and,

$$d^\star(m,p,\mathbf{a}) = \begin{cases} 0 & \text{if } 0 < p \le \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \le \hat{p}_{i_d} \\ m - p^{\frac{\sigma-1}{\sigma}}[\phi(1+i_d)]^{-\frac{1}{\sigma}} & \text{if } \hat{p}_{i_d} < p \end{cases}, \tag{2.13}$$

where

$$\hat{p} := \hat{p}(m,\mathbf{a}) = \phi^{\frac{1}{\sigma-1}} m^{\frac{\sigma}{\sigma-1}}, \qquad \tilde{p}_i := \tilde{p}_i(m,\mathbf{a}) = \hat{p}(1+i)^{\frac{1}{\sigma-1}}, \qquad \hat{p}_{i_d} := \hat{p}_{i_d}(m,\mathbf{a}) = \hat{p}(1+i_d)^{\frac{1}{\sigma-1}}. \tag{2.14}$$

The cutoff prices $(\tilde{p}_i, \hat{p}_{i_d})$ are functions of the state of the economy and monetary policy. Since $i \ge i_d$ and $\sigma < 1$, we can order the cut-off prices as: $0 < \tilde{p}_i \le \hat{\rho}_{i_d} < \hat{p} < +\infty$.

market preference CRRA coefficient turns out to be at least unity, here σ corresponds to a frictional, search market for goods. Our calibration of $\sigma < 1$ is consistent with similar findings in other related models (e.g., Wang, 2016; Wang et al., 2020; Head et al., 2012). Moreover, this restriction is consistent with the empirical finding in Baker (2018). The author finds that indebted households face a more elastic demand schedule, which is captured by the first case in Equation (2.15).

Note that if p turns out to be a random variable drawn from a distribution $J(\cdot, m, \mathbf{a})$—and it will be in a certain equilibrium—then, we would observe ex-post heterogeneous consumption, loan and deposit outcomes in the DM. When we present the firms' problem, we will be more explicit about characterizing the distribution of prices. Also, note that in the case of $i = i_d$, all active DM buyers are either depositors (not money constrained) or borrowers (who top up their money balance with bank credit), and this depends on their price draw p. The more interesting case is when $i > i_d$. (This is also the empirically-relevant case.)

If $i > i_d$, Equations (2.11), (2.12) and (2.13), respectively, imply three possible classes of ex-post heterogeneous demands for goods, loans and deposits. Consider the first case in Equation (2.12). If a buyer draws a p that is sufficiently low, then the buyer optimally borrows money from the bank to top up his initial money holdings. Moreover, the buyer spends all his liquid balances, including his money and bank loan. We call this buyer a (liquidity constrained) *credit-buyer*. In the intermediate case, p is drawn such that $\tilde{p}_i < p \leq \hat{p}_{i_d}$. In this event, the buyer prefers not to borrow from the bank but rather to spend all her money. In this case, loan size does not matter for goods demand. We call this type of buyer a *liquidity constrained money-buyer*, or in short, a *money-constrained buyer*. In the last case, p can be sufficiently high. In that case, the buyer prefers not to borrow and also not to spend all her money balance in the frictional goods market. We call this type of buyer a liquidity *unconstrained money-buyer* (and depositor). Since the bank-deposit facility is available to everyone, agents with unused money can avoid inflation tax by depositing with banks. Here, only the *unconstrained money-buyers* and *inactive buyers* will optimally want to deposit.

It is also worth mentioning that if $i > i_d$, then the price elasticity of demand for the demand schedule $q_b^\star$ described in Equation (2.11). The buyers' price elasticity of demand is given by

$$\left| \frac{\partial q_b^\star(m,p,\mathbf{a})}{\partial p} \frac{p}{q_b^\star(m,p,\mathbf{a})} \right| = \begin{cases} \frac{1}{\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ 1 & \text{if } \tilde{p}_i < p \leq \hat{p}_{i_d} \\ \frac{1}{\sigma} & \text{if } p > \hat{p}_{i_d} \end{cases}. \tag{2.15}$$

This will imply that demand is elastic among buyers other than money-constrained buyers.[12] The implication is that such a buyer cannot spend more than his liquidity constraint at low-enough price levels, $p < \hat{\rho}_{i_d}$. Above the $\hat{\rho}_{i_d}$ cut-off price level, a buyer's liquidity constraint does not bind and such buyers will always spend less than their total money holding.

Households in the DM *ex-ante*. Now consider the beginning of period t when households are *ex-ante* homogeneous at the start of the DM (i.e., before exchange and production of the goods). Given an individual real money balance, m, and given $\mathbf{a}$, the agent's value is

$$\begin{aligned} V(m,\mathbf{a}) = &(1-n)\, W(m-d, 0, d, \mathbf{a}) \\ &+ n \left\{ \int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a}) \mathrm{d}\hat{J}(p,m,\mathbf{a}) \right\}. \end{aligned} \tag{2.16}$$

That is, with probability $1-n$, the household does not want to consume early (in the DM) and will deposit their idle money with a bank. With probability n, the household is an *active buyer* who may successfully

[12] Alternatively, we can show that the elasticity of the buyer's expenditure rule $e(p) := pq_b^\star(m,p,\mathbf{a})$ is less than one. Then the buyer's expenditure on the DM goods decreases as he faces a higher price p. We omit the details of its derivation here. Instead, we explain more about how banking credit affects buyers' optimal expenditure rule and firms' pricing strategy in Section 4.1.

contact one seller (with probability α_1) or at most two sellers (with probability $\alpha_2 = 1 - \alpha_1$).

In contrast to Head et al. (2012), the value of households entering the DM is different due to the availability of banking services. The distribution function $\hat{J}(\cdot, m, \mathbf{a})$ is determined in equilibrium. Note that $\hat{J}(\cdot, m, \mathbf{a})$ represents a (possible) compound lottery: It is induced by the possibility of meeting more than one seller (and at most two), and each seller possibly offers a lottery over a lottery pricing contract. This depends on the type of equilibrium that we get. In the novel case where $i > i_d$ and there exists an equilibrium featuring a pecuniary externality, it may be profitable for firms to be posting a compound lottery because the graph of their ex-post profit function may exhibit non-convexity.

Previewing the equilibrium description in Section 3, household's perceived pricing distribution must agree with firms' pricing equilibrium. This may involve the possibility of compound pricing lotteries. Let $J(\cdot, m, \mathbf{a})$ denote the distribution function induced by a simple pricing lottery which will be determined in equilibrium (see Lemma 5 on page 18 for its formula). The density over p from having contact with at least one seller or at most two sellers is $\varrho(p, m, \mathbf{a}) := \alpha_1 + 2\alpha_2 (1 - J(p, m, \mathbf{a}))$.[13] Next, we will write $L(p) \neq \emptyset$ to say that there exists a further simple lottery that is expected-profit maximizing with an actuarially fair price that equals some outcome p drawn from $J(\cdot, m, \mathbf{a})$.[14] Thus, the equilibrium (joint) distribution of pricing outcomes can be accounted for as:

$$\hat{J}(B, m, \mathbf{a}) = \int_{[\underline{p}, \overline{p}]} \omega(p, m, \mathbf{a})\, \varrho(p, m, \mathbf{a}) \mathrm{d}J(p, m, \mathbf{a}),$$

where

$$\omega(p, m, \mathbf{a}) := \sum_{\ell=1}^{2} \mathbf{1}_{\left\{L(p) \neq \emptyset \text{ and } \{p\} = \left\{p_{\ell,p}\right\}\right\}} \pi_{\ell,p} + \mathbf{1}_{\{L(p)=\emptyset\}},$$

and B is a Borel subset of the equilibrium's domain of relevant prices $\left[\underline{p}, \overline{p}\right]$. If the compound lottery is a simple lottery itself, $\hat{J}$ is J and has the usual construct as in Head et al. (2012) and (Burdett and Judd, 1983).

Marginal value of money. To lighten notation, we denote the (equilibrium) cut-off pricing functions by $\underline{p} := \underline{p}(m, \mathbf{a})$, $\tilde{p}_i := \tilde{p}_i(m, \mathbf{a})$, $\hat{p}_{i_d} := \hat{p}_{i_d}(m, \mathbf{a})$ and $\overline{p} := \overline{p}(m, \mathbf{a})$. Also, we will use these shorthand notation for probability measures: $\hat{J}_+ := \hat{J}_+(p, m_+, \mathbf{a}_+)$, and $\mathbf{1}_X$ is the Dirac measure on event X. Differentiating Equation (2.16) with respect to m, we have the following expression for the marginal value of money at the start of a DM one period ahead:

$$V_m(m_+, \mathbf{a}_+) = \phi_+ \left[1 + \mathcal{R}(m_+, \mathbf{a}_+)\right] \quad , \tag{2.17a}$$

[13] If $J(\cdot, m, \mathbf{a})$ is the pricing distribution (associated with the "first" simple lottery) with a density representation j such that $j(p, m, \mathbf{a})\mathrm{d}p = \mathrm{d}J(p, m, \mathbf{a})$), then the likelihood of meeting one seller and hence getting to draw a p is $\alpha_1 \mathrm{d}J(p, m, \mathbf{a})$, and the likelihood of meeting two sellers and having two independent draws is $\alpha_2 \frac{\mathrm{d}\left\{1 - [1 - J(p, m, \mathbf{a})]^2\right\}}{\mathrm{d}p} \mathrm{d}J(p, m, \mathbf{a}) = 2\alpha_2 (1 - J(p, m, \mathbf{a}))\, \mathrm{d}J(p, m, \mathbf{a})$.

[14] See Lemma 4 on page 17 ahead for the construction of such additional lotteries given an outcome p drawn from another lottery.

where,

$$\mathcal{R}(m_+, \mathbf{a}_+) = \begin{cases} (1-n)i_d + n\int_{\underline{p}}^{\overline{p}} \underbrace{\left[\left(\frac{u_q[q(m_+, p, \mathbf{a}_+)]}{\phi_+ p}\right) - 1\right]}_{=i} \mathrm{d}\hat{J}_+ = i, & \text{if} \quad i = i_d \\ \\ (1-n)i_d + n\int_{\underline{p}}^{\tilde{p}_i} i\,\mathrm{d}\hat{J}_+ \\ +n\int_{\tilde{p}_i}^{\overline{p}} \left[\left(\frac{u_q(m_+/p)}{\phi_+ p}\right)\mathbf{1}_{\{p\in(\tilde{p}_i, \hat{p}_{i_d}]\}} - 1\right] \mathrm{d}\hat{J}_+, & \text{if} \quad i > i_d \\ +n\int_{\hat{p}_{i_d}}^{\overline{p}} i_d \mathrm{d}\hat{J}_+ \end{cases}, \tag{2.17b}$$

and we have made use of the cases of the demand for q derived in Lemma (1).

Equations (2.17a)-(2.17b) deserve some commentary. The marginal value of money at the start of the DM is given by the right-hand-side of Equation (2.17a). This is the expected benefit from accumulating additional money balance to be carried into the DM in the next period. The value of one unit of money balance is captured by ϕ_+ (in units of CM goods). Since money serves as a means of payment in the frictional goods market, it has a liquidity premium captured by the return $1 + \mathcal{R}(m_+, \mathbf{a}_+)$. Thus, carrying an extra unit of money has a marginal benefit of $\phi_+\mathcal{R}(m_+, \mathbf{a}_+)$, measured in units of the next-period numéraire good.

In contrast to Head et al. (2012), the liquidity premium on holding money in Equation (2.17b) now depends on banking outcomes. Consider the first branch of Equation (2.17b), the case where $i = i_d$. In this case, the liquidity premium consists of two terms. With probability $1-n$, the agent does not want to consume and holds their money balance idle. They can lend an extra dollar to the bank to receive i_d at the margin. With probability n, the agent wants to consume and receives the expected net marginal benefit from spending an extra dollar in the goods market. (This can also be interpreted as the marginal interest saved from borrowing one less unit of money.) Note that the integral on the right-hand side collapses to a number equaling i.[15] Thus, banks raise the marginal value of money because inactive agents can deposit idle money to earn interest. That is, in our setting, if $i = i_d$ then banks play the same intermediation-of-liquidity-needs role as those in BCW.

Next, consider the case where $i > i_d$ in the derivation of the liquidity premium term in Equation (2.17b). In this case, the liquidity premium comprises four possible terms. First, if the household ends up not consuming in the next DM with probability $1-n$, they can also deposit their idle money in the bank to earn interest $i_d > 0$. (The next three terms are derived from Equation (2.11) in Lemma 1.) Second, if the household samples a low enough price, i.e., $p \in (\underline{p}, \tilde{p}_i]$, they would take out a bank loan. Thus, the second term on the right of Equation (2.17b) captures the expected marginal interest-payment liability saved by borrowing one less unit of money. Third, there is the net benefit from being able to spend an extra unit of money on goods, for the case of the ex-post money-constrained buyer because they draw some price $p \in (\tilde{p}_i, \hat{p}_{i_d}]$. Last, we have the benefit arising from the possibility that the buyer is ex-post money-unconstrained so that at the margin, the unspent money can still earn interest i_d. In constrast to the case where $i = i_d$, here, the right-hand side marginal benefit or liquidity premium terms no longer sum up to i. Whether these additional terms constitute a benefit will depend on the pricing feedback through $\hat{J}(\cdot, m, \mathbf{a})$ and its endogenous support. (Anticipating results, it may be the case that there is a

[15] This is because from Equations (2.8)-(2.10) in Lemma 1, the net marginal utility value of consumption always equals i when $i = i_d$. That is, ex-post buyers may draw different prices p and hence consume different quantities of q, but $u_q(q)/p\phi - 1 = i = i_d$. Therefore, regardless of whether the ex-post heterogeneous buyers are borrowers or depositors of different quantities of money, they face the same marginal value of i. Finally, i is a constant with respect to the distribution of prices $\hat{J}(\cdot, m, \mathbf{a})$, so that the integral value is just i.

pecuniary externality from bank credit to firms' pricing behaviour such that some of these benefit terms in the liquidity premium gets eroded or extracted by firms with market power.)[16]

Substituting Equation (2.17a) into Equation (2.5), we obtain a money demand Euler equation capturing the households' inter-temporal trade-off:

$$\phi = \beta \phi_{+}[1 + \mathcal{R}(m_{+}, \mathbf{a}_{+})]. \tag{2.18}$$

The left-hand side of Equation (2.18) captures the cost of accumulating money balance: The household forgoes ϕ units of CM consumption goods in order to carry an extra dollar into the next period. The right-hand side of Equation (2.18) is the expected marginal benefit of accumulating an extra dollar associated with the total liquidity premium captured by $\mathcal{R}(m_{+}, \mathbf{a}_{+})$ in Equation (2.17b).

2.4 Firms

There is production of the numéraire good in the centralized market (CM), and a special good by imperfectly competitive firms in the decentralized market (DM). The key frictions arise in the DM in this model.

Firms in the Centralized Market. In the CM, there is a unit measure of perfectly competitive firms producing the numéraire good x using a linear production technology in labor h. They then sell the goods to households in the CM. Consequently, both the real wage and price of the DM goods are equal to one.

Firms in the Decentralized Market (Overview). A unit measure of firms (or sellers of goods) compete in a price posting environment along the lines of Head et al. (2012). In our setting, firms may commit to posting lotteries over pure pricing strategies. We will describe the possibility of a lottery over pure pricing contracts shortly in Lemma 4. For now, consider the reference to a posted contract p given consumer demand, as either a lottery indexed by its expected value p, or a degenerate lottery (which is a pure pricing-strategy) at outcome p.

Consider a firm posting a contract indexed by p. Given demand schedule $q_b^\star$ and the distribution of prices posted by firms J, its expected profit is given by

$$\Pi_i(p) = \underbrace{\left[\alpha_1 + 2\alpha_2(1 - J(p, m, \mathbf{a})) + \alpha_2 \nu(p)\right]}_{\text{extensive margin}} \underbrace{R(p, i, i_d, m)}_{\text{intensive margin}}, \tag{2.19}$$

where

$$\nu(p) = \lim_{\epsilon \searrow 0} J(p, m, \mathbf{a}) - J(p - \epsilon, m, \mathbf{a}),$$

and $R(p, i, i_d, m) \equiv R(p, m, \mathbf{a})$ is the firm's *effective profit* from expecting to price at p (i.e., that may have been induced by an additional set of simple lotteries given p). (We will define this profit function in detail in Lemma 4 below.)

[16]In our model, a binding reserve requirement makes credit more costly, induces a spread between loan interest and deposit interest (yielding this second case of $i > i_d$), bids up goods prices, and will lower consumption—i.e., the pecuniary externality from credit buyers to money-constrained buyers. Whether this negative externality is sufficiently large to offset the welfare gain from banking (as a liquidity risk insurance vehicle), will depend on government policy. This is a quantitative question that we will answer in the calibrated model later.

The first term in parentheses, labeled *extensive margin*, in Equation (2.19) captures the number of buyers served. With probability α_1, the firm trades with a buyer who has only observed one price quote from this firm and no other. With probability $2\alpha_2[1 - J(p, m, \mathbf{a})]$, the buyer purchases the good from this firm because he contacts another firm who has posted a higher price than p. The probability $\alpha_2 \nu(p)$ is the measure of buyers that match both this firm and another which has posted the same price (or lottery indexed by) p.[17] The last term, labeled *intensive margin*, captures the firm's profit per customer induced by the firm charging a markup, i.e., posting at an (expected) price above the marginal cost, $p > \phi^{-1}c$.

Observe from Equation (2.19), the firm posting p trades off between an *extensive margin* (i.e., the likelihood of trading with buyers) and an *intensive margin* (i.e., profit per buyer). On the one hand, a firm that posts a higher p can earn a higher profit margin per buyer served. However, on the other hand, a firm that posts a higher p suffers by losing sales to other competitors, i.e., a lower likelihood of trading with buyers.

A hypothetical monopolist. As in Burdett and Judd (1983) and Head et al. (2012), we can characterize the distribution of prices $J(\cdot, m, \mathbf{a})$, which is an equilibrium object. The lower and upper bound of the distribution's (connected) support will depend on the description of a monopolist's pricing strategy. We provide its characterization here.

Consider a firm serving buyers who have only received one price quote from this one firm. In this case, the firm will behave as a monopolist. The realized profit of a firm setting a monopoly price p^m is

$$\Pi^m = \alpha_1 R(p^m, i, i_d, m). \tag{2.20}$$

A subtlety in our extension of Head et al. (2012) here is that banking outcomes i and i_d will affect some agents who, ex-post, may demand loans or deposits. As a result, i and i_d also condition, and may "shift" their demand for the DM good in the case where $i > i_d$ (see Lemma 1). This, along with how much money a buyer carries into the trade, has consequences for the calculation of a firm's profit and also for the equilibrium distribution of DM-good prices. Unlike Head et al. (2012), the effective ex-post profit function can be non-concave, depending on how much money balance DM buyers carry into the match. (This is a result of the possibility of bank credit for buyers.) Despite this seeming complexity, our generalization turns out to be very tractable: We prove these attributes (in Online Appendix B.1) and show that in terms of pricing, we end up with the following modified characterization of the monopoly price.

First, we provide an intermediate characterization of equilibrium cutoff real money balances, that will outline the possible cases of a non-concave ex-post profit function whenever $i > i_d$:

Lemma 2 (Loan-deposit rate spread and money-balance cases)**.** *Suppose $i > i_d$. Assume that $\sigma \in (0, 1)$ and $c \in (0, 1]$ such that $0 < (1 + i_d)^{-\frac{1}{\sigma}} \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}_{i_d}$. There exists a set of cut-offs relative to $\hat{z}_{i_d}$ (measurable in units of real money balance or the numéraire), with $\acute{z}_{i_d} = \hat{z}_{i_d} \left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$, $\tilde{z}_{i,i_d} := (1 + i)^{-\frac{1}{\sigma}} \hat{z}_{i_d}$ and there is an endogenous $\breve{z}_{i,i_d}$ such that $\mathring{z}_{i,i_d} := \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$. The cut-offs have the particular ordering: $0 < \mathring{z}_{i,i_d} \leq \tilde{z}_{i,i_d} \leq \hat{z}_{i_d} < \acute{z}_{i_d} < \infty$.*

We ask the reader to take these cutoff functions on faith for now. The statement of Lemma 2 is proved as part of the case-by-case proof towards Lemma 3. The set of parameters (σ, c) satisfying the inequalities

[17] Suppose two firms post the same price. We assume that prospective buyers use a tie-breaking rule to pick one firm in such a case. This rule incentivizes an individual firm to lower the price to get the sale. In equilibrium, the probability of a buyer contacting two firms that post the same price goes to zero.

in Lemma 3 is non-empty. For example, if $c = 1$ as in Head et al. (2012), then the sufficient conditions on parameters reduce to $0 < \left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, which admits any $\sigma \in (0,1)$.

Lemma 3 (Monopoly price and possibly non-concave ex-post profit)**.** *Fix a (pre-determined) real money balance $z \in (0,\infty)$ and a given rate on loans i and deposits i_d. There are two cases to consider:*

1. *If $i = i_d$, then the ex-post profit function at a pure pricing outcome is*

$$R^{ex}(p,i,i_d,m) = G_1(p;i) := [\phi p\,(1+i)]^{-\frac{1}{\sigma}}(\phi p - c),$$

which is strictly concave and non-negative valued, with a unique maximum at the monopoly price p_0^m.

2. *If $i > i_d$, then the ex-post profit function at pure pricing outcomes may not be strictly concave, and is defined as follows: Let the realized profit at price outcome p from serving a credit-buyer be $G_1(p;i) := [\phi p\,(1+i)]^{-\frac{1}{\sigma}}(\phi p - c)$, from serving a constrained money-buyer be $G_2(p;m) := \frac{m}{p}(\phi p - c)$, and from selling to an unconstrained money-buyer be $G_3(p;i_d) := [\phi p\,(1+i_d)]^{-\frac{1}{\sigma}}(\phi p - c)$. Let $\mathbf{g}(p;i,i_d,m) := [G_1(p;i), G_2(p;m), G_3(p;i_d)]$. There exists a set of cut-offs with the particular ordering: $0 < \mathring{z}_{i,i_d} \leq \tilde{z}_{i,i_d} \leq \hat{z}_{i_d} < \acute{z}_{i_d} < \infty$. Given these cut-offs, we have the following observations:*

   (a) *The ex-post profit function at a pure pricing outcome is*

$$R^{ex}(p,i,i_d,m) = \begin{cases} \langle \mathbf{g}(p;i,i_d,m), \mathbf{I}_1(p;i,i_d,\phi m)\rangle, & \phi m \in [\hat{z}_{i_d},\infty) \\ \langle \mathbf{g}(p;i,i_d,m), \mathbf{I}_2(p;i,i_d,\phi m)\rangle, & \phi m \in [\mathring{z}_{i,i_d},\hat{z}_{i_d}) \\ \langle \mathbf{g}(p;i,i_d,m), \mathbf{I}_3(p;i,i_d,\phi m)\rangle, & \phi m \in (0,\mathring{z}_{i,i_d}) \end{cases} \tag{2.21}$$

   where $\rho \equiv \phi p$ and $z = \phi m$, and the vectors of indicator functions are

$$\begin{aligned}
\mathbf{I}_1(p;i,i_d,z) &:= \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}}, \underbrace{\mathbf{1}_{\{c<\hat{\rho}_{i_d}\leq\rho\leq\rho_0^m\}}}_{\text{Case 1(a)}} + \underbrace{\mathbf{1}_{\{\hat{\rho}_{i_d}<c\leq\rho\leq\rho_0^m\}}}_{\text{Case 1(b)}}\right], \\
\mathbf{I}_2(p;i,i_d,z) &:= \begin{bmatrix} \mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} \\ \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}} \\ \underbrace{\mathbf{1}_{\{c<\tilde{\rho}_i<\rho_0^m<\hat{\rho}\leq\rho\}\cap\{z\in[\tilde{z}_{i,i_d},\hat{z}_{i_d})\}}}_{\text{Case 2(a)}} + \underbrace{\mathbf{1}_{\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}_{i_d}\leq\rho\}\cap\{z\in[\mathring{z}_{i,i_d},\tilde{z}_{i,i_d})\}}}_{\text{Case 2(b)}} \end{bmatrix}', \\
\mathbf{I}_3(p;i,i_d,z) &:= \underbrace{\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}}, \mathbf{1}_{\{c<\hat{\rho}_{i_d}\leq\rho\}}\right] \times \mathbf{1}_{\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}_{i_d}\}}}_{\text{Case 3}},
\end{aligned}$$

   $\rho_0^m = c/(1-\sigma)$, $\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$, $\hat{\rho}_{i_d} := \hat{\rho}(1+i_d)^{\frac{1}{\sigma-1}} > \tilde{\rho}_i := \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, $\langle\cdot,\cdot\rangle : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \to \mathbb{R}_+$ is the inner-product operator, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.

   (b) *The monopoly price and its ex-post profit outcome, respectively, are*

$$p^m = \begin{cases} p_0^m, \\ \hat{p}_{i_d}(m), \\ p_0^m, \end{cases} \quad \text{and, } R^{ex}(p^m,i,i_d,m) = \begin{cases} G_3(p_0^m;i_d), & \phi m \in [\hat{z}_{i_d},\infty) \\ G_3(\hat{p}_{i_d}(m);i_d), & \phi m \in [\mathring{z}_{i,i_d},\hat{z}_{i_d}) \\ G_1(p_0^m;i), & \phi m \in (0,\mathring{z}_{i,i_d}) \end{cases}. \tag{2.22}$$

As a consequence of the possibility of earning profit from credit buyers, the ex-post per-meeting profit function of firms (in terms of pure pricing strategies) may exhibit segments that are not necessarily monotone increasing up to the monopoly profit point. Also, the profit function may not be strictly concave.[18] Technically, this may pose a problem for the characterization of the equilibrium Burdett-Judd style pricing distribution, as it depends on the monotonicity of $R^{ex}(p, i, i_d, m)$ in p. Economically, this also suggests that it may be profitable for firms to further compound the simple lottery implied by the conjectured Burdett-Judd equilibrium distribution of prices with another simple lottery. Here we are merely applying a standard reduction of compound lotteries axiom.[19]

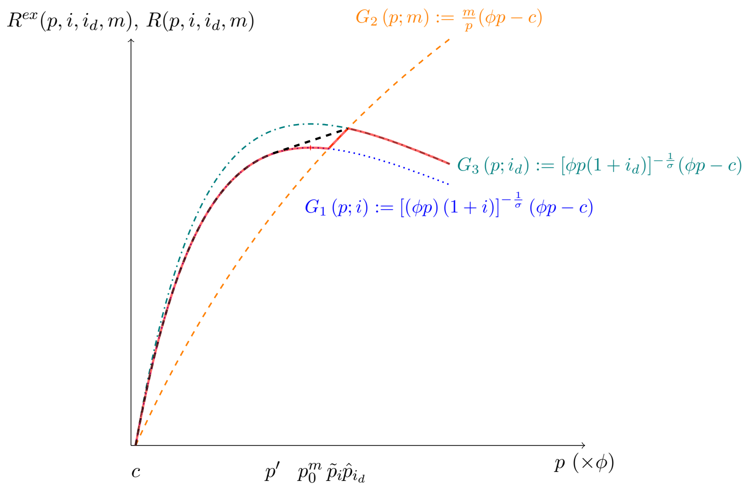
Figure 1 illustrates Case 2(b) from Lemma 3, i.e., where real money balance is low enough $\phi m \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$.[20] In this case, the maximal profit from serving cash-constrained buyers and pricing at such buyers' maximal willingness to pay, $\hat{p}_{i_d}(m)$, can exceed the maximal profits from serving either credit-buyers or unconstrained money buyers. The dashed-black graph in the figure is the convexification of a firm's ex-post profit function via lotteries over pure price posts. (See the proof of Lemma 3 for a complete characterization and graphical illustrations for all the possible cases.) We will refer to a profit function induced by convexification of the graph of the ex-post profit function (over the *effective* or *equilibrium-consistent domain* of pricing) as an *effective profit* function.

[18]Such non-monotonicities and non-convexities can be interpreted as an artefact of the externality of credit buyers on firms' profitability calculations. We shall see in equilibrium, it is possible (depending on policy and parameters) that such an externality can have a net negative impact on aggregate welfare. In Head et al. (2012), there is no such technical complication since buyers are either cash constrained or not, and there is no feedback between banking and goods-firms pricing.

[19]The idea of posting random contracts already exists in monetary trade, in general equilibrium, or in labor economics (respectively, Berentsen, Molico and Wright, 2002; Chatterjee and Corbae, 1995; Shell and Wright, 1993; Rogerson, 1988), in settings with private information (Prescott and Townsend, 1984) and in models of dynamic price discrimination (Sobel, 1984). In our application, the presence of bank credit and the possibility of demand for goods from buyers with bank credit may render local non-concavity and possibly local non-concave-and-convex segments in firm's ex-post profit functions. This possible non-strict-concavity of firms' ex-post profit function does not arise in the monetary setting of Head et al. (2012) where their firms' ex-post profit functions are always strictly concave. We will see that such lotteries can yield firms a weakly welfare-improving payoff, but one that is still no greater than the hypothetical Burdett and Judd (1983) monopoly profit in any setting. The use of lotteries over lotteries will also ensure a well-behaved equilibrium characterization of the distribution function of pricing outcomes. Alternatively, one may invoke the Compound Independence Axiom (Segal, 1990).

[20]The interval $[\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$ depends on the given interest rates i and i_d, and can be derived. We relegate the tedious and mechanical details to Online Appendix B.1).

Figure 1: Example from Case 2(b) in Lemma 4 where $z \in [\mathring{z}_i, \tilde{z}_i)$. Posting of random contracts is profitable for firms. This yields a strictly increasing and concave *effective profit function* $R(\cdot, i, m)$ (dashed-black graph) on the relevant domain $[c, \phi\hat{p}_{i_d}]$. The ex-post profit function $R^{ex}(\cdot, i, m)$ under pure pricing strategies is illustrated by the solid-red line (whose graph exhibits non-convexity).



In the next result, we show that for all possible cases, the effective profit function $R(\cdot, m, \mathbf{a}) \equiv R(\cdot, i, i_d, m)$ is monotone increasing and concave in pricing outcomes, so long as firms are allowed to post lotteries over any hypothetical pure-pricing-strategy contract outcomes. We relegate the detailed proof to Online Appendix B.2.

Lemma 4 (Possibility of simple (sub)-lotteries $\{\pi_p, [p_{1,p}, p_{2,p}]\}$ and effective profit function)**.** *Fix aggregate outcomes (ϕ, i, i_d) and m, and consider the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, i_d, m)$ in Equation (B.1) in Lemma 3. Let $z := \phi m$.*

1. *If $i = i_d$, then*

   (a) *a firm's effective profit is the same as its ex-post profit,*

   $$R(p, i, i_d, m) = R^{ex}(p, i, i_d, m) = [\phi p (1 + i)]^{-\frac{1}{\sigma}}(\phi p - c),$$

   and this function is strictly concave and non-negative valued, with a unique maximum at the monopoly price p_0^m.

   (b) *Depending on the demand for loan and deposit functions in (2.9) and (2.10), firms either serve credit buyers or unconstrained buyer-depositors. There are no money-constrained buyers.*

2. *If $i > i_d$, then we have the following: There exists a set of cut-offs with the particular ordering: $0 < \mathring{z}_{i,i_d} \leq \tilde{z}_{i,i_d} \leq \hat{z}_{i_d} < \acute{z}_{i_d} < \infty$.*

   (a) *A firm's* ***effective profit*** *at any given reference price p, $R(p, i, i_d, m)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

$$R(p, i, i_d, m) = \max_{\pi_{1,p} \in [0,1], p_{1,p}, p_{2,p}} \left\{ \begin{array}{r} \pi_{1,p} R^{ex}(p_{1,p}, i, i_d, m) + (1 - \pi_{1,p}) R^{ex}(p_{2,p}, i, i_d, m) : \\ \pi_{1,p} p_{1,p} + (1 - \pi_{1,p}) p_{2,p} = p \end{array} \right\}. \quad (2.23)$$

The function $R(\cdot, i, i_d, m)$ is strictly increasing on $\left[\phi^{-1}c, p^m\right)$, and is concave over the firm's effective domain of pricing outcomes $\left[\phi^{-1}c, p^m\right]$, where the monopoly price is p^m and its effective profit outcome is $R(p^m, i, i_d, m) = R^{ex}(p^m, i, i_d, m)$ and these are characterized by

$$p^m = \begin{cases} p_0^m, & z \in [\acute{z}_{i_d}, \infty) \\ p_0^m, & z \in [\hat{z}_{i_d}, \acute{z}_{i_d}) \\ \hat{p}_{i_d}(z), & z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d}) \\ \hat{p}_{i_d}(z), & z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d}) \\ p_0^m, & z \in (0, \mathring{z}_{i,i_d}) \end{cases}, \qquad \textit{and}, \tag{2.24}$$

$$R(p^m, i, i_d, m) = R^{ex}(p^m, i, i_d, m) = \begin{cases} G_3(p_0^m; i_d), & z \in [\acute{z}_{i_d}, \infty) \\ G_3(p_0^m; i_d), & z \in [\hat{z}_{i_d}, \acute{z}_{i_d}) \\ G_3(\hat{p}_{i_d}(m); i_d), & z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d}) \\ G_3(\hat{p}_{i_d}(m); i_d), & z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d}) \\ G_1(p_0^m; i), & z \in (0, \mathring{z}_{i,i_d}) \end{cases}.$$

(b) Depending on z the largest domain containing equlibrium pricing outcomes has the following properties:

Case-1(a). *If $z \in [\acute{z}_{i_d}, \infty)$, firms will only have incentive to serve money-unconstrained buyers (who also deposit any residual money balance).*

Case-1(b). *If $z \in [\hat{z}_{i_d}, \acute{z}_{i_d})$, all (credit, money-constrained, and money-unconstrained) buyers will be served.*

Case-2(a). *If $z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d})$, only two types—credit and money-constrained buyers—will be served.*

Case-2(b). *If $z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$ and this interval is non-degenerate, then only two types—credit and money-constrained buyers—will be served.*

Case-3. *If $z \in (0, \mathring{z}_{i,i_d})$, then only credit buyers are served.*

Pricing equilibrium. As in Head et al. (2012), firms will earn the same expected profit for any p in the support of the equilibrium distribution, $\operatorname{supp}(J(\cdot, m, \mathbf{a})) = [\underline{p}, \overline{p}]$. That is, they will be indifferent between a continuum of different extensive-intensive margin trade-offs:

$$\Pi^\star = \max_p \Pi(p) \quad \text{for all } p \in \operatorname{supp}(J(\cdot, m, \mathbf{a})). \tag{2.25}$$

Lower price firms make up their profit through higher sales volume while higher price firms gain through higher markups. If some buyers observe only one "price quote" whereas others observe more than one, then this leads to a non-degenerate distribution of prices $J(\cdot, m, \boldsymbol{a})$. [21] Since firms expect the same profit outcomes associated with the continuum of markup-versus-trading-probability strategies, then this implies an equal-profit condition. Specifically, equating Equation (2.19) and Equation (2.20), we can derive a closed-form distribution of prices. We summarize this result as follows.[22]

[21] The model has two parametric limits: one with Bertrand pricing (by setting $\alpha_2 = 1$) and one that resembles monopoly (by setting $\alpha_1 = 1$).

[22] Given Lemma 4, the proof of Lemma 5 follows directly from Head et al. (2012). We omit the details here.

Lemma 5. *Given monetary policy $\gamma > \beta$, aggregate outcomes (m, i, ϕ), and noisy search frictions $\alpha_1, \alpha_2 \in (0, 1)$, the price distribution consistent with profit maximization by all firms is given by*

$$J(p, m, \mathbf{a}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{p}, m, \mathbf{a})}{R(p, m, \mathbf{a})} - 1\right], \tag{2.26}$$

where the lower and upper bounds on the support of $J(\cdot, m, \boldsymbol{a})$ are, respectively, determined by an equal effective profit condition

$$R(\underline{p}, m, \mathbf{a}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{p}, m, \mathbf{a}) \tag{2.27}$$

where $c < \underline{p} < \overline{p} := p^m$, and p^m is governed by Equation (2.22).

Recall from Lemma 4 that when we refer to a "price quote", we mean that firms can also commit to a (possibly-degenerate) lottery around a reference price quote (in an equilibrium where $i > i_d$). Thus, in Lemma 5, the distribution of posted prices faced by potential buyers represents a simple lottery which gets compounded with the the set of simple lotteries defined in Problem (2.23) (Lemma 4, case of $i > i_d$). From the buyers' point of view, they anticipate all these (in equilibrium) so all they care about is the resulting distribution $J(\cdot, m, \mathbf{a})$. Note that, if the additional simple lotteries as defined by Problem (2.23) were all degenerate (which is precisely the $i = i_d$ case in Lemma 4), then the solution for $J(\cdot, m, \mathbf{a})$ is as in Head et al. (2012) or Burdett and Judd (1983).[23]

In contrast to Head et al. (2012), the banking loan and deposit interest rates (i and i_d), through $\mathbf{a}$, now matter for determining the goods-price distribution faced by consumers. This is a consequence of the buyer's optimal goods demand schedule interacting with credit (see Lemma 1), which then affects the firm's pricing strategy (Lemmata 4 and 5). As we shall see, in the case where $i > i_d$, this credit channel may induce a pecuniary externality on money-constrained buyers if the measure of such agents and their responsiveness to the externality is large enough.

2.5 Banks

As in Berentsen et al. (2007), we focus on the liquidity transformation role of banks and abstract from inside money creation. The banking sector is perfectly competitive with free entry. In the aggregate, banks have (total) deposits D and commit to repaying depositors with interest i_d. Banks issue loans at the interest rate of i. Let the total quantity of loans issued be L. Following Jiang and Zhu (2021), banks face a reserve requirement policy: They are required to hold a fraction χ of deposits in reserves r with the central bank, which can earn an interest i_r, so that $r \geq \chi D$. The banks' problem is

$$\max_{L,D,r}\left\{ iL + i_r r - i_d D \;\middle|\; \begin{array}{l} \underbrace{L + r}_{\text{assets}} = \underbrace{D}_{\text{liabilities}} \\ r \geq \chi D \end{array} \right\}. \tag{2.28}$$

There are two possible cases: Either the reserve requirement binds, or it is slack. One can check that the first case occurs if $i_r \leq \bar{i}_r$, where the endogenous cutoff interest rate $\bar{i}_r$ is determined by the competitive

[23] We thank an anonymous referee for raising this point.

credit market clearing condition:

$$\begin{aligned}\underbrace{(1-\chi)\left[(1-n)M + n\int_{\breve{p}_{i_d}}^{\overline{p}} d^{\star}(m,p,\mathbf{a})\,\mathrm{d}\hat{J}(p,m,\mathbf{a})\right]}_{=:D(\bar{i}_r),\text{ total deposits}} \\ = \underbrace{n\int_{\underline{p}}^{\tilde{p}_i} l^{\star}(m,p,\mathbf{a})\,\mathrm{d}\hat{J}(p,m,\mathbf{a})}_{=:L(\bar{i}_r),\text{ total loans}}.\end{aligned} \tag{2.29}$$

Equation (2.29) determines the cutoff interest rate $\bar{i}_r$. It is the interest rate such that banks are indifferent between lending to consumers and holding reserves. Since Equation (2.29) is required to clear any excess loan demand, this means that the cutoff interest rate $\bar{i}_r$ is also the market rate for loans, i. If the central bank pays a lower return than loans ($i_r < \bar{i}_r \equiv i$), the reserve requirement is binding because rational banks would lend out exactly the amount of $L = (1-\chi)D$ and hold just enough reserve to satisfy the requirement, so $r = \chi D$. In terms of interest rates, to induce banks to hold a fraction of less productive reserves, there must be a positive wedge that compensates the bank for the cost of putting deposits into the lower return assets, so then $i > i_d > i_r$.[24]

If the reserve requirement is not binding, $i_r > \bar{i}_r$. Interest on reserve is more attractive, so banks will hold more reserves than those required by the central bank. In this case, the equilibrium loan rate, reserve rate, and deposit rate must be equal to each other. Otherwise, the bank can allocate more resources to the asset that yields a higher return. Under perfectly competitive banking with free entry, banks earn zero profit. Thus, the zero profit condition implies that banks' marginal benefit from reserves must equal their marginal cost of servicing deposit claims, and since there is abundant source of funds, competition must drive the deposit rate up to the loan rate. That is, $i = i_d = i_r$ has to hold.

Proposition 1 (Determination of market interest rates)**.** *There are two cases to consider:*

1. *If the reserve requirement binds, $i_r < \bar{i}_r \equiv i$. The equilibrium market interest rates for loans and deposits, i and i_d, are jointly determined by*

$$\underbrace{(1-\chi)i + \chi i_r}_{\textit{marginal revenue of deposits}} = \underbrace{i_d}_{\textit{marginal cost of deposits}} \tag{2.30a}$$

$$L(i) = (1-\chi)D(i_d) \tag{2.30b}$$

   Moreover, $i > i_d > i_r$.

2. *If the reserve requirement is not binding, $i_r \geq \bar{i}_r$. In this case, it must be that $i = i_d = i_r = \bar{i}_r$.*

We relegate the proof of Proposition 1 to Online Appendix B.3. We close this section by noting that Proposition 1 provides a banking-regulation rationale for considering the cases of either $i > i_d$ or $i = i_d$, when we characterized the DM buyers' behavior (in Lemma 1) and firms' pricing behavior (Lemmata 3, 4 and 5).

[24] Another way to generate $i > i_d$ is to assume costly banking (see, e.g., Boel and Camera, 2020) in our model, but it would not change the basic message in the paper regarding the novel case with pecuniary externality. We chose to model the wedge between i and i_d as an equilibrium feature because we can then relate that to reserve requirement regulation.

3 Stationary monetary equilibrium (SME)

We now focus on stationary outcomes of the economy. Since the price of the general goods P is used as a unit of account, we then multiply all nominal variables by the value of money balance $\phi = 1/P$ (in units of the CM goods x) from here onward. In particular, we let $z = \phi m$ denote the individual real money balance and $Z = \phi M$ denote the aggregate real money balances; $\rho = \phi p$ denote the real relative price of goods across the DM and the CM; and $\xi = \phi l$ and $\delta = \phi d$ respectively denote the real balances of loans and deposits. For the ease of notation, we define the state vector now as $\mathbf{s} := (i, i_d, Z; \gamma)$. In a stationary equilibrium, all nominal variables grow at a time-invariant rate according to $\phi/\phi_{+1} = M_{+1}/M = \gamma = 1+\tau$ and real variables stay constant over time.

Before we provide a summary of the equilibrium characterization, we first present two features that are different in contrast to Head et al. (2012) as follows.

Simple lottery over ρ. In a stationary monetary equilibrium (SME), the real analog of the price distribution characterized in Lemma 5 is given by:

$$J(\rho, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, z, \mathbf{s})}{R(\rho, z, \mathbf{s})} - 1\right], \tag{3.1}$$

where the upper bound on the support of $J(\cdot, z, \mathbf{s})$ was derived in Lemma 4. In stationary variables, this is given as:

$$\bar{\rho} := \rho^m(z, \mathbf{s}) = \begin{cases} \rho_0^m, & z \in [\hat{z}, \infty) \\ \hat{\rho}_{i_d}(z), & z \in [\mathring{z}_i, \hat{z}) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases}. \tag{3.2}$$

The lower bound on the support of $J(\cdot, z, \mathbf{s})$, $\underline{\rho}$, is a solution to the equal expected profit condition:

$$R(\underline{\rho}, z, \mathbf{s}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{\rho}, z, \mathbf{s}). \tag{3.3}$$

Compound lottery representation: from J and R to $\hat{J}$. Now we circle back to connect J again to the joint distribution $\hat{J}$ first used in describing household expectations in Section 2.2, in Equation (2.16). In Lemma 3 in the paper, if there is a draw ρ from the distribution $J(\cdot, z, \mathbf{s})$ such that a firm can increase expected profit by further playing a lottery around ρ, then the firm solves

$$R(\rho, i, i_d, z) = \max_{\pi_{1,p} \in [0,1], \rho_{1,\rho}, \rho_{2,\rho}} \left\{ \begin{array}{r} \pi_{1,\rho} R^{ex}(\rho_{1,\rho}, i, i_d, z) + (1 - \pi_{1,\rho}) R^{ex}(\rho_{2,\rho}, i, i_d, z) : \\ \pi_{1,\rho}\rho_{1,\rho} + (1 - \pi_{1,\rho})\rho_{2,\rho} = \rho \end{array} \right\}. \tag{3.4}$$

For a given outcome ρ, let $L(\rho)$ denote a non-denegerate lottery solving (3.4). Specifically,

$$L(\rho) := \left\{ (\{\pi_{1,\rho}, \pi_{2,\rho}\}, \{\rho_{1,\rho}, \rho_{2,\rho}\}) \,\middle|\, \begin{array}{l} (3.4) \text{ holds}, \\ \pi_{2,\rho} = 1 - \pi_{1,\rho} \in (0,1), \\ \pi_{1,\rho} R^{ex}(\rho_{1,\rho}, i, i_d, z) + \pi_{2,\rho} R^{ex}(\rho_{2,\rho}, i, i_d, z) = R(\rho) \end{array} \right\}.$$

We will write $L(\rho) \neq \emptyset$ to say that there exists a further simple lottery that is profit-maximizing at ρ.

In equilibrium household's perceived pricing distribution must agree with firms' pricing equilibrium with the possibility of compound pricing lotteries. Thus, households in the CM (or ex-ante at the start of the DM) face a (joint) distribution of pricing outcomes:

$$\hat{J}(B,z,\mathbf{s}) = \int_{[\underline{\rho},\overline{\rho}]} \left\{ \sum_{\ell=1}^{2} \mathbf{1}_{\{L(\rho)\neq\emptyset \text{ and } \{\rho\}=\{\rho_{\ell,\rho}\}\}} \pi_{\ell,\rho} + \mathbf{1}_{\{L(\rho)=\emptyset\}} \right\} \varrho(\rho,z,\mathbf{s}) \mathrm{d}J(\rho,z,\mathbf{s}),$$

where B is a Borel subset of the equilibrium's domain of relevant prices $[\underline{\rho},\overline{\rho}]$, and, $\varrho(\rho,z,\mathbf{s}) := \alpha_1 + 2\alpha_2(1-J(\rho,z,\mathbf{s}))$. Note that this is induced by the possibility of meeting more than one seller and each seller possibly offering a lottery contract over a lottery. Observe that in Equation (3.1), the joint distribution of goods (real) prices now depends on both households' real money holdings z and the interest rates i and i_d (embedded in $\mathbf{s}$). At a fixed ρ, let the indicator-weighting function be:

$$\omega(\rho,z,\mathbf{s}) := \left(\mathbf{1}_{\{L(\rho)\neq\emptyset \text{ and } \{\rho\}=\{\rho_{\ell,\rho}\}\}} \times \pi_{\ell,\rho}\right) + \mathbf{1}_{\{L(\rho)=\emptyset\}}, \qquad ,\ell\in\{1,2\}, \qquad \rho\sim J(\rho,z,\mathbf{s}). \tag{3.5}$$

Now that this is understood, in the SME characterization below, we may interchange $\hat{J}(\cdot,z,\mathbf{s})$ with $J(\cdot,z,\mathbf{s})$ and $\omega(\cdot,z,\mathbf{s})$.

There are two possible cases regarding the households' optimal demand for real money balances in an SME, depending on the reserve requirement policy. We summarize this possibility in the following Lemma.

Lemma 6. *Let monetary policy be such that inflation is away from the Friedman rule, $\gamma>\beta$, and assume that $\alpha_1,\alpha_2\in(0,1)$. In a SME, the liquidity premium on holding money over one period (2.17b) is*

$$\mathcal{R}(z,\mathbf{s}) = \begin{cases} i \equiv \int_{\underline{\rho}}^{\overline{\rho}} \left\{ \left[\left(\frac{u_q[q(z,\rho,\mathbf{s})]}{\rho} \right) - 1 \right] \right\} \varrho(\rho,z,\mathbf{s})\, dJ(\rho,z,\mathbf{s}) & \text{if} \quad i=i_d \\ (1-n)i_d + n\int_{\underline{\rho}}^{\tilde{\rho}_i} \left\{ \underbrace{\sum_{\rho_{\ell,\rho}\in\{\rho_{1,\rho},\rho_{2,\rho}\}} \omega(\rho_{\ell,\rho},z,\mathbf{s})\cdot i}_{\equiv i} \right\} \varrho(\rho,z,\mathbf{s})\, dJ(\rho,z,\mathbf{s}) & \\ +n\int_{\tilde{\rho}_i}^{\bar{\rho}} \left\{ \sum_{\rho_{\ell',\rho}\in\{\rho_{1,\rho},\rho_{2,\rho}\}} \omega(\rho_{\ell,\rho},z,\mathbf{s}) \cdot \left[\left(\frac{u_q(z/\rho_{\ell,\rho})}{\rho_{\ell,\rho}} \right) \mathbf{1}_{\{\rho\in(\tilde{\rho}_i,\hat{\rho}_{i_d}]\}} - 1 \right] \right\} & \\ \qquad \times \varrho(\rho,z,\mathbf{s})\, dJ(\rho,z,\mathbf{s}) & \text{if} \quad i>i_d \\ +n\int_{\hat{\rho}_{i_d}}^{\overline{\rho}} \left\{ \underbrace{\sum_{\rho_{\ell,\rho}\in\{\rho_{1,\rho},\rho_{2,\rho}\}} \omega(\rho_{\ell,\rho},z,\mathbf{s})\cdot i_d}_{\equiv i_d} \right\} \varrho(\rho,z,\mathbf{s}) dJ(\rho,z,\mathbf{s}) & \end{cases}, \tag{3.6}$$

where $\varrho(\rho,z,\mathbf{s}) := \alpha_1 + 2\alpha_2(1-J(\rho,z,\mathbf{s}))$, and the demand for real money balance z satisfies:

$$\frac{\gamma}{\beta} = 1 + \mathcal{R}(z,\mathbf{s}). \tag{3.7}$$

Lemma 6 reveals the inter-dependency of agent's (ex-ante) precautionary money demand on an endogenous channel between bank credit and non-competitiveness in the DM for goods. The distribution of goods prices, $\hat{J}(\cdot,z)$, encodes the degree of firms' market power in the goods market.

The equilibrium money demand in Lemma 6, which takes into account equilibrium firm pricing behavior encoded in $\hat{J}(\cdot,z,\mathbf{s})$, admits several possible equilibrium configurations. When read together with Lemma

1, Lemma 4 and Proposition 1, Lemma 6 implies the potential for different configuration of buyer/payment types co-existing in a SME.

For example, if the reserve requirement is slack and $i = i_d$, then the money demand condition looks just like that in Berentsen et al. (2007). Nothing interesting here happens in terms of any pricing externality from banking to the frictional goods market.

The more interesting case is when the reserve requirement is binding and $i > i_d$: A parametrization of the model may yield an ex-post mixture of credit–buyers and money–buyers in equilibrium (see Cases 1(b), 2(a) and 2(b), with variations on whether money-buyers include the money-unconstrained ones or not). The credit-buyers are those buyers who draw a sufficiently low price, i.e., $\underline{\rho} \leq \rho \leq \tilde{\rho}_i(z)$. The money-buyers are those who draw a sufficiently high price, i.e., $\tilde{\rho}_i(z) < \rho \leq \overline{\rho}(z)$. The implication is an endogenous channel connecting bank credit condition and firms' market power through the equilibrium pricing distribution. In equilibrium, this channel matters for the agent's (ex-ante) precautionary demand incentives regarding how much real money balances to carry to trade in the following period. Conditional on $\alpha_1, \alpha_2 > 0$—i.e., the distribution $J(\cdot, z, \mathbf{s})$ is non-degenerate—there are two interesting extreme possibilities. First, if equilibrium z is high enough (i.e., money is approaching zero holding cost) firms will only want to sell to money-unconstrained agents (see Case 1(a) in Lemma 4). Second, depending on the cut-off value $\mathring{z}_i$ in Lemma 4 that is endogenous to banking market outcome, i, we may have the case that firms will only serve credit buyers (see Case 3 in Lemma 4). Both of the two extreme and theoretically-possible cases imply that there is no pecuniary externality running from credit buyers to money constrained buyers. Theoretically, if $\alpha_1 = 1$ so that $J(\cdot, z, \mathbf{s})$ is degenerate at the Bertrand price for i, the liquidity premium term in Equation (3.6) reduces to the same expression as in Berentsen et al. (2007).

Definition 1. Given monetary policy $\gamma \geq \beta$ and reserve-requirement regulation (i_r, χ), and taxes/transfers T, a stationary monetary equilibrium is a steady-state allocation $(z^\star, x^\star, h^\star)$ and $\{q_b^\star(\rho, i, z), \xi^\star(\rho, i, z), \delta^\star(\rho, i, z)\}$, and pricing behavior $(J^\star(\rho, z, \mathbf{s}), i, i_d)$ such that the following conditions are satisfied:

1. The upper bound on the support of $J^\star(\cdot, z, \mathbf{s})$, $\bar{\rho} := \rho^m(z, \mathbf{s})$ satisfies (3.2), and the lower bound on the support of $J^\star(\cdot, z, \mathbf{s})$ solves the equal expected profit condition (3.3).

2. The triple $(h^\star, x^\star, z^\star)$ solves the CM households problem, including the households' ex-ante real money demand decision in Equations (3.6) and (3.7).

   (a) Given $z = z^\star = Z$, $\{q_b^\star(\rho, z, \mathbf{s}), \xi^\star(\rho, z, \mathbf{s}), \delta^\star(\rho, z, \mathbf{s})\}$ satisfy one of these two cases:

      i. If the reserve requirement binds ($i > i_d > i_r$), then

$$q_b^\star(\rho, z, \mathbf{s}) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ \frac{z}{\rho} & \text{if } \tilde{\rho}_i < \rho \leq \hat{\rho}_{i_d} \\ [\rho(1+i_d)]^{-1/\sigma} & \text{if } \rho > \hat{\rho}_{i_d} \end{cases}, \tag{3.8}$$

$$\xi^\star(\rho, z, \mathbf{s}) = \begin{cases} \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i < \rho \leq \hat{\rho}_{i_d} \\ 0 & \text{if } \rho > \hat{\rho}_{i_d} \end{cases}, \tag{3.9}$$

and,

$$\delta^{\star}(\rho,z,\mathbf{s}) = \begin{cases} 0 & \text{if } 0<\rho\leq\tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i<\rho\leq\hat{\rho}_{i_d} \\ z-\rho^{\frac{\sigma}{\sigma-1}}(1+i_d)^{-\frac{1}{\sigma}} & \text{if } \rho>\hat{\rho}_{i_d} \end{cases}, \tag{3.10}$$

where

$$\hat{\rho} := \hat{\rho}(z,\mathbf{s}) = z^{\frac{\sigma}{\sigma-1}} \qquad \tilde{\rho}_i \equiv \tilde{\rho}_i(z,\mathbf{s}) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}} \quad \text{and} \quad \hat{\rho}_{i_d} := \hat{\rho}_{i_d}(z,\mathbf{s}) = \hat{\rho}(1+i_d)^{\frac{1}{\sigma-1}}. \tag{3.11}$$

ii. If the reserve requirement is slack ($i=i_d=i_r$), then $q_b^{\star}(\rho,z,\mathbf{s}) = [\rho(1+i)]^{-1/\sigma}$ and

$$\xi^{\star}(\rho,z,\mathbf{s}) = \begin{cases} \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}-z & \text{if } 0<\rho\leq\tilde{\rho}_i \\ 0 & \text{otherwise} \end{cases}, \tag{3.12}$$

and,

$$\delta^{\star}(\rho,z,\mathbf{s}) = \begin{cases} 0 & \text{if } 0<\rho\leq\tilde{\rho}_i \\ z-\rho^{\frac{\sigma}{\sigma-1}}(1+i)^{-\frac{1}{\sigma}} & \text{otherwise} \end{cases}. \tag{3.13}$$

(b) $J^{\star}(\cdot,z,\mathbf{s})$ solves the DM firms' problem characterized in Equation (3.1).

(c) Given $z=z^{\star}=Z$, $i=\bar{i}_r=i^{\star}$ clears the loan market:

$$\underbrace{(1-\chi\cdot\mathbf{1}_{\{i>i_d>i_r\}})\begin{bmatrix} (1-n)z \\ +n\int_{\underline{\rho}_i}^{\bar{\rho}}\delta^{\star}(\rho,i_d,z)\,\mathrm{d}\hat{J}(\rho,z,\mathbf{s}) \end{bmatrix}}_{=:D(\bar{i}_r),\text{ total deposits}} = \underbrace{n\int_{\underline{\rho}}^{\tilde{\rho}_i}\xi^{\star}(\rho,i,z)\,\mathrm{d}\hat{J}(\rho,z,\mathbf{s})}_{=:L(\bar{i}_r),\text{ total loans}}, \tag{3.14}$$

where $\underline{\rho}_i := \mathbf{1}_{\{i>i_d>i_r\}}\cdot\hat{\rho}_{i_d}+\mathbf{1}_{\{i=i_d=i_r\}}\cdot\tilde{\rho}_i$; and if the reserve requirement is binding, i_d solves (2.30a). Otherwise, $i=i_d$.

The system reduces to finding three unknowns, $z^{\star}$, $i^{\star}$ and $i_d^{\star}$, simultaneously.

The following observation is straightforward. From Proposition 1, if the reserve-requirement regulation is not binding, $i_r>\bar{i}_r$ so that $i=i_d$. From Lemma 4, we know that there will be no money-constrained buyers in equilibrium. In this case, banking is always beneficial—see the liquidity premium term in Equation (3.6) which is identical to the result in Berentsen et al. (2007)—and there are no externality effects since there are no money-constrained ex-post buyers of goods. We can immediately deduce that banks in this case are always welfare improving over the original Head et al. (2012) economy without the liquidity-risk insurance role of banking, when holding money is costly ($\gamma>\beta$).

Proposition 2 (Banks improve welfare if reserve-requirements are slack)**.** *If $i_r>\bar{i}_r$ so that $i=i_d$, then banks are always welfare improving in this economy.*

Looking ahead. In the rest of the paper, we will focus on equilibria where the reserve requirement ratio is binding, as this will be the case that is empirically consistent and it possesses the novel externality effect. In the data's sample period used in our model calibration (pre-2008), the interest paid on reserves in the data is zero, so our assumption here satisfies this easily.[25]

Assumption 1. *Let monetary policy be $\gamma > \beta$, the reserve-requirement regulation (i_r, χ) be such that $i_r < i$, and noisy search frictions be $\alpha_1, \alpha_2 \in (0, 1)$.*

The following observation says that a buyer with a lower real money balance is more likely to draw a higher price from the distribution.

Lemma 7 (Stochastic dominance and z)**.** *Let Assumption 1 hold. Consider any two real money balances z and $z^{'}$ such that $z < z^{'}$. The price distribution $J(\cdot, z, \mathbf{s})$ first-order stochastically dominates $J(\cdot, z^{'}, \mathbf{s})$. Also, the pricing cutoffs $\tilde{\rho}_i$ (the maximal willingness to pay for a credit buyer) and $\hat{\rho}_{i_d}$ (the maximal willingness to pay of a money-constrained buyer) are decreasing functions of z.*

The proof is in Online Appendix B.4. The reasoning behind Lemma 7 is as follows. Suppose a buyer carries a small amount of real money balance into the goods market. Firms expect to produce and sell a lower quantity of goods. A measure of firms will optimally respond by charging higher prices relative to their marginal cost of production. Consequently, the distribution of goods prices is more dispersed. The buyer with a tighter liquidity constraint is more likely to draw a higher price (or an associated markup) from the distribution. Therefore, the net benefit of banking in equilibrium should be ambiguous in contrast to Berentsen et al. (2007). Here, the gains from accessing a competitive banking sector depend on the interaction between agents' precautionary demand for money holdings and endogenous firms' market power in the goods market.[26]

Using the result established in Lemma 7, we can then show the existence of a stationary monetary equilibrium with both money and credit. Such an equilibrium entails price dispersion in the frictional goods market. We summarize the result in Proposition 3. Details of the proof are in Appendix B.5.[27]

Proposition 3. *Let Assumption 1 hold. There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

Remark. Recall the discussion earlier surrounding Lemmata 3 to 6 that for generic parameters there can theoretically be different equilibrium configurations where there may exist only money-constrained buyers or some mixtures over the possible types of money and credit buyers. However, we should note that when we discipline the model by calibration to the data later, we shall see that the second case in Lemma 6 (SME with a mixture of constrained- and unconstrained-money buyers and credit-buyers) will be the relevant case—and this is also the most interesting one. In this case, the pecuniary externality is always present. Also, this equilibrium case will always occur for the plausible range of long-run inflation-rate experiments around the empirically calibrated model.

[25] The US Fed's *Interest on Reserve Balances* series is available from https://www.federalreserve.gov/monetarypolicy/reserve-balances.htm. See also the same discussion in Niepelt (2024).

[26] This is the novelty here in contrast to the special case where there is no banking or financial intermediation—i.e., the equivalent Head et al. (2012) setting. In Section 4.1, we will illustrate and decompose the effect of this pecuniary externality channel; and we will show that how severe this effect is in offsetting the liquidity risk insurance role of banks depends on long-run inflation or monetary policy.

[27] This will also help to simplify our characterization of the stationary monetary equilibrium by ruling out the possibility of an extreme equilibrium case—i.e., Case 1(a) of Lemmata 3 and 4—in which agents anticipate only unconstrained money buyers getting served in the DM goods trades.

4 Equilibrium trade-off and welfare effect of banking

We continue to focus attention on SMEs where the reserve requirement constraint is binding, and, money-constrained buyers and credit buyers co-exist in equilibrium. We can see in Definition 1 the equilibrium trade-off between the benefit of banking and its uninternalized social cost on consumer-goods prices (i.e., the *pecuniary externality*): On the one hand, banking is beneficial because it increases the expected net return of holding money. This can be deduced from reading the first and second terms in Equation (3.7). On the other hand, firms' market power (price markups and dispersion) in frictional goods trades can also reduce some of the gains from banking. Banking, through competitive outcomes i and i_d (embedded in the vector $\mathbf{s}$), affects the agents' precautionary demand for money holdings z, which then feeds back onto the distribution of goods prices $\hat{J}(\cdot, z, \mathbf{s})$, and its support, $[\underline{\rho}(z, \mathbf{s}), \overline{\rho}(z, \mathbf{s})]$. In particular, the integrals on the right-hand side of Equation (3.7) capture the reduction in the expected return on money even though agents have access to a competitive banking sector. This works through the results in Lemma 7: the first-order-stochastic-dominance in $J(\cdot, z, \mathbf{s})$ and the associated increasing pricing-cutoff functions $\tilde{\rho}_i$ and $\hat{\rho}_{i_d}$ as z falls.

In the following Section 4.1, we explore this trade-off further. We analytically dissect the model through its special cases in order to identify an equilibrium tension between competitive banks' role in facilitating insurance of individuals' liquidity risks and the externality that such bank credit may have on non-credit users in the economy. The resolution of such a tension ultimately depends on inflation policy. In Section 4.2, we further use the calibrated model to illustrate how the trade-off changes with inflation in the long run and what the resulting welfare implications are for banking in such an economy.

4.1 Inspecting the trade-off

Overview. It is useful to compare our setting to that without banking. In particular, if we remove the banking sector, we get the case of a pure monetary economy with firm market power studied in Head et al. (2012) (HLMW). Let $\bar{z}$ denote the equilibrium real money balance under the HLMW economy. In this special case, Equation (3.7) becomes

$$\frac{\gamma - \beta}{\beta} = n \left[\int_{\underline{\rho}(\overline{z})}^{\overline{\rho}(\overline{z})} \left[\alpha_1 + 2\alpha_2 (1 - \tilde{J}(\rho, \overline{z})) \right] \left(\frac{u_q[q^{no-bank}(\overline{z})]}{\rho} - 1 \right) \mathrm{d}\tilde{J}(\rho, \overline{z}) \right], \tag{4.1}$$

where the price distribution in a no-bank monetary economy is given by

$$\tilde{J}(\rho, \overline{z}) = 1 - \frac{\alpha_1}{2\alpha_2} \left[\frac{\tilde{R}(\overline{\rho})}{\tilde{R}(\rho)} - 1 \right], \tag{4.2}$$

and the bounds are given by $\tilde{R}(\underline{\rho}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} \tilde{R}(\overline{\rho})$, and $\overline{\rho} = \max \left\{ \frac{c}{1-\sigma}, \underbrace{\overline{z}^{\frac{\sigma}{\sigma-1}}}_{=:\hat{\rho}} \right\}$, and the real profit per customer served is

$$\tilde{R}(\rho, \overline{z}) = q_b^{no-bank}(\rho, \overline{z})(\rho - c), \tag{4.3}$$

and buyer's optimal demand for goods is given by

$$q_b^{no-bank}(\rho, \overline{z}) = \begin{cases} \frac{\overline{z}}{\rho} & \text{if } 0 < \rho \leq \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \hat{\rho} < \rho \end{cases}. \tag{4.4}$$

Consider anticipated inflation away from the Friedman rule $\gamma > \beta$. In our setting and in that of Head et al. (2012), agents have precautionary demand for money holdings. Anonymity in the goods market gives rise to money as a means of payment. However, inflation ($\gamma > \beta$) induces a rate of return on money that is lower than the risk-free rate or the rate of time preference. Inflation is thus a tax on frictional goods trades. Hence, holding money can be costly when agents (ex-post) are stuck with unproductive idle money balances.

With access to banks, households can now reduce the cost of having unneeded liquidity (via depositing idle funds in the bank to earn interest). In addition, households can borrow extra money balances from the bank. Credit extended by banks helps households to relax their liquidity constraints when they need to make a payment in the goods market. We call this positive welfare effect of banking a *liquidity-risk insurance effect*, which works through an identical mechanism as in BCW.

To illustrate the potential gains from banking, let us contrast this with a pure monetary economy and BCW, assuming a Walrasian price-taking protocol in the DM goods trades in both economies for the ease of presentation. Moreover, we maintain the assumption of a linear cost of production ($c(q) = q$) in the DM as in BCW. First, a no-bank equilibrium condition for money demand is given by:

$$\frac{\gamma}{\beta} = n\left(\frac{u'(q)}{c'(q)}\right) + (1-n) \times 1, \tag{4.5}$$

where the right-hand side captures the (gross) marginal benefit per extra dollar accumulated, which consists of two components. The first term reflects the benefit goes to agents who get to spend it (with probability n) in the next DM, where the second term reflects those who don't get to spend their money balances (with probability $1-n$) are holding their money balance idle and earning zero interest. Rearrange, it yields

$$1 + \frac{1}{n}\left[\frac{\gamma - \beta}{\beta}\right] = u'(q). \tag{4.6}$$

Conside the left-hand side. Since the measure of active buyers $n \in (0, 1)$, then it acts a factor inflating the opportunity cost of money holding. The opportunity cost of holding money is higher the fewer active buyers there are. We refer to this as the *liquidity risk*, which is relevant in a (constrained) monetary equilibria ($\gamma > \beta$). When agents are subject to preference/consumption shocks, ex-post allocations in (constrained) monetary equilibria are typically inefficient since there are some agents who are liquidity constrained and some are holding unproductive idle funds. In contrast, in BCW's banking equilibrium the money demand condition becomes

$$\frac{\gamma}{\beta} = n\underbrace{\frac{u'(q)}{c'(q)}}_{=1+i_l} + (1-n)(1+i_d) \equiv u'(q) \equiv 1 + i^{\star}, \tag{4.7}$$

where a loan market clearing condition $i_d = i_l = i^{\star}$ is imposed. This precisely says that the opportunity cost of holding money (borne by what would have been idle-money holders) is fully compensated at the margin by the interest rate earned on deposits. Algebraically, the liquidity risk "inflation factor", $1/n$, is eliminated by the existence of perfectly competitive banks. Such liquidity risk matters and there are gains from banking in equilibria away from the Friedman rule ($\gamma > \beta$).[28] The positive welfare effect of banking here is also robust to an alternative trading protocol with bargaining (see Berentsen et al., 2007).

Now, in our baseline setting with money and credit in equilibrium (see Cases 1(b), 2(a), 2(b) and 3 in Lemma 4), the liquidity risk "inflation factor" above is not always eliminated by the perfectly competitive banking system. This is due to the fact that noisy search frictions ($0 < \alpha_2 < 1$) induces price competition among firms (an equilibrium non-degenerate distribution of goods prices). The expected marginal benefit of holding money—the integral terms on the right-hand side of Equation (3.7)—depends on the equilibrium distribution $J(\cdot, z, \mathbf{s})$ (an outcome of goods market power) and this depends on equilibrium interest on credit, i. The net benefit of banking here can be ambiguous because of this policy-dependent interaction. This is because even though banks here are perfectly competitive as in Berentsen et al. (2007), imperfect (pricing) competition among firms can give rise to an additional *price dispersion effect* that can contribute to a negative welfare effect of banking. Effectively, the policy-dependent interaction of banking and the search-based market power of firms creates an uninternalized social cost on agents' asset accumulation decision, which has a direct consequence on consumption outcomes in monetary equilibria. Here, imperfect competition among firms in goods trades (arising from search and informational frictions) could potentially hinder an otherwise useful banking system. This possibility is also a novel feature than that in economies with price-taking and bargaining trading protocols. The mechanism is as follows.

Decomposing the welfare effects of banking. To understand the positive and negative welfare effects of banking, we compare Equation (3.7) and Equation (4.1). In our model economy, buyers can deposit funds in the bank to earn interest $i_d > 0$ if they ex-post don't get to spend their money and/or not going to spend all of their money balances (those unconstrained money buyers who sample sufficiently high prices). We label this type of buyers as depositors. The interest paid to depositors increases the expected marginal benefit of accumulating money balances (see the first and the forth terms in Equation (3.7)). This is the same (and sole) benefit of banking in Berentsen et al. (2007).

In addition now, buyers who are liquidity constrained and sample low enough prices of the goods can use bank credit to relax their liquidity constraint (ex-post). The second term on the right-hand side of Equation (3.7) reflect such banking benefits. Due to the liquidity-risk insurance effect, banking helps to improve consumption allocation relative to HLMW, on the one hand.

On the other hand, access to credit by buyers can also lower the expected marginal benefit of money when firms can exploit markups in frictional goods trades. In particular, the integrals on the right-hand side of Equation (3.7) capture the negative welfare effects of banking which we label as the *pecuniary externality* or *price dispersion* effect. The reason is as follows.

Firms expect some potential customers to be liquidity constrained by their money balances, and their expenditure rule is inelastic. A measure of firms would then optimally respond by charging higher markups (see Lemma 7). This will affect both the liquidity constrained and unconstrained money-buyers. The former will face a tighter liquidity constraint as the real value of their money goes down so they end up with less goods. The latter, although unconstrained, still best respond by consuming less, since their demands are decreasing in the prices they draw. Moreover, unconstrained money-buyers could now also deposit some of their residual money balances to avoid the inflation tax, since they are not going to spend all of their money. Credit-buyers inadvertently contribute to the bidding up of DM goods prices (both the average level and the dispersion): In the model, this shows up in the form of the support of the goods

[28] It is also notice that our baseline model converges to this result when noisy search frictions vanish (i.e., buyers always sample multiple price offers with probability $\alpha_2 = 1$).

price distribution, supp $(J(\cdot, z, \mathbf{s})) = [\underline{\rho}(z), \overline{\rho}(z)]$, being wider than that in HLMW. Specifically, the lowest possible price that a liquidity-constrained (and unconstrained) money buyer can draw becomes higher as ex-ante real balance falls (Lemma 7). In other words, access to credit by buyers amplifies firms' market power in terms of price markups and dispersion.

Recall that banking credit only benefits some buyers but not all. In particular, from Equations (3.8) and (3.9), buyers use credit if they draw a sufficiently low price ρ on goods from the distribution $J(\cdot, z, \mathbf{s})$. However, as we have deduced, banking credit induces higher price dispersion, which implies more high-price firms extracting rent from liquidity constrained money-buyers. Both integrals on the right-hand side of Equation (3.7) capture a reduction in the expected return on holding money along the rising price dispersion. Hence, a distortion will appear in the average interest saved on borrowing an extra dollar for the credit-buyers and the liquidity premium for the money-buyers. Thus, firms' market power in frictional goods trades can potentially reduce gains from a competitive banking sector.

Numerical illustration. Next, we provide a numerical example of the mechanism outlined above by comparing our baseline model economy to that without banking (HLMW), for a given long run monetary policy setting τ.[29]

Figure 2 displays the liquidity-risk insurance and price dispersion effects of banking given policy $\tau > \beta - 1$. (Without loss, we plot the case of $\tau = 0.05\%$.) The dashed-dotted red graph and associated dashed-red pricing cutoff $\hat{\rho}_{HMLW}$ represent the model economy in HLMW. The solid blue graph with dashed blue cutoffs $\tilde{\rho}_i$ and $\hat{\rho}$ represent our baseline model economy (with existence of banking).

Liquidity-risk insurance effect: positive welfare effects of banking. In HLMW, a buyer cannot spend more than his liquidity constraint when faced with a price draw that is at most $\hat{\rho}_{HLMW}$. The horizontal part of the dashed red graph in Figure 2a reflects the set of expenditure levels of such a type of (ex-post) liquidity constrained buyers. The cut-off $\hat{\rho}_{HMLW}$ is the price level at which the buyer becomes liquidity unconstrained ex-post. Such a buyer spends less than her total money balances if she draws a price higher than $\hat{\rho}_{HMLW}$.

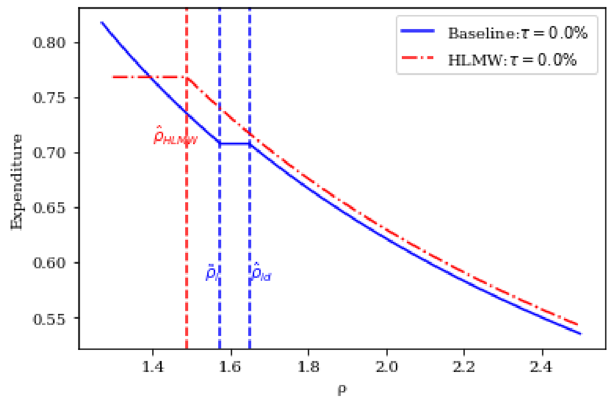
Consider now the solid blue graph in Figure 2a. In contrast to HLMW, there is now a liquidity-risk insurance effect highlighting the benefits of having access to banking credit. A buyer can now borrow additional money balances from the bank to relax his liquidity constraint when $\rho \leq \tilde{\rho}_i$. Thus, the (ex-post) credit-buyer faces a more relaxed liquidity constraint to spend more in the goods market than money-buyers. The (non-credit) money constrained buyers in this case are the ones on the horizontal segment of the solid blue graph — i.e., the ones who draw a ρ from the interval $(\tilde{\rho}_i, \hat{\rho}_{i_d}]$. The (non-credit) money-unconstrained buyers have a downward sloping expenditure function over all $\rho > \hat{\rho}_{i_d}$. Since these unconstrained buyers can also deposit their residual money balances, each dollar the active buyer deposits will reduce the amount she can use to purchase goods. Here, we can see that the unconstrained buyers trade-off their consumption demand (hence, expenditure) in return for interest earned on their residual balances. In comparison with HLMW (the dashed-red line), our unconstrained buyers now uniformly expend less on goods.

From Figure 2a, we can deduce that credit buyers can potentially benefit from higher expenditures, whereas the money-constrained buyers now can only afford lower expenditures, relative to the HLMW (no-bank) economy. However, this is not the complete picture as, with noisy search, one also has to take into account the equilibrium measure of buyers over each subset of these price-draw intervals. That is,

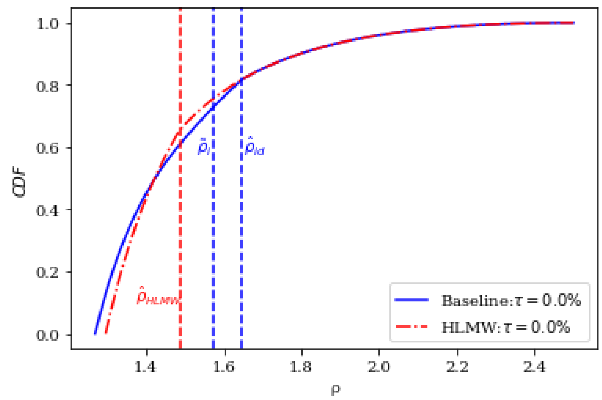
[29] The parameter values for this numerical illustration is given by $\beta = 0.98, \tau = 0, \sigma = 0.6, \alpha_1 = 0.7, \alpha_2 = 0.3, n = 0.8, B = 1.8$, $i_r = 0$, $\chi = 0.7$.

Figure 2: Liquidity-risk insurance and price dispersion effects given policy $\gamma = 1 + \tau > \beta$.

(a) Liquidity-risk insurance effect



(b) Price dispersion effect



(c) Price dispersion effect

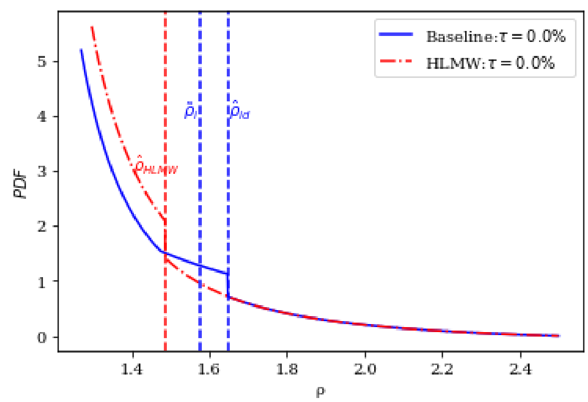


Figure 2a depicts only the individual's *intensive* margin outcomes in terms of possible expenditures as a function of the price draw ρ. A more complete view would have to also factor in effect of banking on the equilibrium distribution of such people. We turn to this *extensive margin* effect next.

Pecuniary externality through price dispersion effect: negative welfare effects. When firms' market power (markup and price dispersion) arises from informational frictions, access to competitive banks can cause an additional negative welfare effect. This is because not all agents can benefit from banking. In particular, those agents who use banking for loans create a price effect in the goods market. This negatively affects agents who do not use banking credit.

Recall that firms expect some prospective customers to be constrained by their money balances, and their expenditure rule to be inelastic. Thus, a measure of firms optimally responds by charging higher prices relative to their marginal cost of production. Consequently, buyers (on average) are more likely to draw a higher price in the sense of first-order stochastic dominance. In Figure 2b, this price dispersion effect of banking externality is reflected in the solid blue distribution function graph over the set $\rho > \tilde{\rho}_i$ (for the banking equilibrium) is first-order stochastically dominating the dashed-dotted red graph (HMLW, no-banking equilibrium). Also, the support of the price distribution in our baseline model economy is wider than in HLMW and $\tilde{\rho}_i$ is higher than $\hat{\rho}_{HLMW}$. Thus, under our banking equilibrium, each money-constrained and money-unconstrained buyer will tend to draw from a higher range of prices than the no-bank, HLMW economy. Moreover, the equilibrium mass of such buyers is relative higher than that in the HLMW economy (see both Figures 2b and 2c).

Effectively, bank credit induces more price markups on money-buyers. There is also higher price dispersion in frictional goods trades. Consequently, each money-buyer (who draws a high enough price such that $\tilde{\rho}_i \leq \rho \leq \hat{\rho}_{i_d}$) faces a tighter liquidity constraint. In this case, the liquidity constrained money-buyer spends less than the case without access to banking arrangements. For the liquidity unconstrained money-buyer (who draws $\rho > \hat{\rho}_{i_d}$), this effect is not there since his liquidity constraint does not bind. Nevertheless, since unconstrained money-buyers' demands are decreasing in ρ, and the corresponding domain for ρ would have shifted up, they would bear the brunt of the externality through lower consumption outcomes (relative to their HLMW counterparts).

In summary, banking affects agents' consumption outcomes differently when firms have market power in frictional goods transactions. In this setting, access to a competitive banking sector can amplify firms' market power, creating an additional welfare-reducing effect of banking. This negative welfare effect, tied to credit-buyers, pushes up price dispersion. This then increases the measure of firms extracting rent from money-buyers. Consequently, the welfare-improving function of banking liquidity transformation is no longer unambiguous, in contrast to Berentsen et al. (2007).[30]

4.2 Trade-off and inflation: experiments on a calibrated economy

In Section 4.1 we identified the benefit and cost of having competitive banking conditional on a particular inflation (policy) setting. We now consider the effect of long-run inflation γ (or monetary policy) on Note that quantitative results are based on the model that is statistically calibrated to U.S. data. See Online Appendix C for further details of the calibration. In particular, our calibration will place the model well away from any pure-money SME—i.e., we will only observe outcomes consistent with SMEs that feature a co-existence of money (constrained and unconstrained) and credit buyers.

Here, we will numerically evaluate this *insurance* versus *price dispersion* tension as a function of inflation (or equivalently, nominal interest policy in the long run). We consider a set of economies, each distinguished by its long-run inflation rates τ from $\tau \in [\beta - 1, \bar{\tau}]$, where we set $\bar{\tau} = 0.1$ (i.e., 10% annual inflation rate).[31]

Overall, whether competitive banks improve welfare in equilibrium is ambiguous. To understand why, we break welfare gains and losses down into the net trading surpluses associated with each ex-post buyer-type events—i.e., events involving credit buyers, money-constrained buyers, and money unconstrained buyers. In Figure 3, these net trading surpluses are measured as the expected utility of each ex-post buyer

[30] Our result has a similar flavor to the classic pecuniary-externality effect from credit (see, e.g., Chiu et al., 2018). In Chiu et al. (2018), the externality is necessarily dependent on an assumption that the cost of producing goods q is a strictly increasing and convex function. In their competitive price-taking equilibrium, the existence of credit-buyers raises goods quantity, q, which then raises the marginal cost of producing q, $c'(q)$, since $c''(q) > 0$ in their setup. This then raises equilibrium price p and feeds back in the form of tightening money-buyers' liquidity constraints. If $c''(q) = 0$, there is no pecuniary externality in Chiu et al. (2018). In contrast, here, we deliberately shut down the technological avenue necessary in Chiu et al. (2018) to generate the pecuniary externality. Instead, we can still have this effect for a different reason. Here, the pecuniary externality works through market power in the form of price (markups) dispersion. The existence of credit-buyers means that, ex-ante, agents end up carrying (relatively) less real balances, z. By Lemma 7, this tends to shift the distribution $J(\cdot, z, \mathbf{s})$ to the right—i.e., agents are more likely to get squeezed by higher prices and markups. If agents knew for sure they would be money-buyers, they would prefer to have carried more real balance. However, because of the idiosyncratic risk they face, ex-ante, all agents end up creating some pecuniary externality of the ex-post liquidity constrained agents.

[31] It can be verified that price dispersion cannot be sustained at the Friedman rule, i.e., $\tau = \beta - 1$. Moreover, banking is redundant since it is costless for agents to carry money balances. For our purpose, we focus on long-run anticipated inflation away from the Friedman rule. This can be interpreted as some extraneous institutional restrictions that prevent a monetary policy maker from implementing the Friedman rule (see also Berentsen et al., 2007, for the same argument).

group net of sellers' expected cost of producing at ex-post different prices.[32]

The solid blue graphs in each panel of Figure 3 correspond to the difference in net DM trading surpluses between banking equilibria in our model and the no-bank HLMW economies for different long run inflation policies. If it is a positive number, it means buyers have a higher net DM trading surplus in our baseline economy than that in the no-bank HLMW economy. (We'll focus only on the ex-post buyers and economize of showing the surplus of depositors, which will just be a constant.) The stark takeaway from these graphs is that *with competitive banking,* there is a positive gap between the net social surplus of credit-buyer events (see Figure 3a), although this gap shrinks with inflation (as to be expected). However, in the following two panels, Figures 3b and 3c, we can see that society is ex-ante worse off if they turn out to be either money-constrained buyers who optimally do not use bank credit nor deposit with banks.

Figure 3d sums up the preceding three graphs vertically to give us the relevant net social surplus across all three ex-post groups. Here, we can already see the symptom of the underlying tension between the liquidity risk insurance benefit of banks (for credit buyers) and the pecuniary-externality cost that operates through the pricing dispersion effect. The resolution is non-monotone with respect to inflation. For low inflation ranges, the latter dominates to create a negative social surplus despite having perfect competition among banks. Only for sufficiently high inflation ranges does the benefit of banking begin to dominate.

Welfare implications of banking. What then of the benefit of banking to the inactive DM buyers (depositors)? We had, thus far, deliberately omitted that in the display and discussions in the previous figures. We now present a complete welfare accounting that includes the ex-ante welfare of ex-post depositor types.[33]

We report the welfare measure in terms of a standard consumption equivalent variation (CEV) measure. This captures how much consumption (in units of the CM good) an agent is willing to give up in an economy without banks to live in an economy with banks.

Given $\gamma = 1 + \tau$ policy, the welfare function in an SME is given by

$$W^e(\gamma) = \frac{1}{1-\beta}\left[U(x^\star) - x^\star + n \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(u[q_b^\star(z_e)] - c[q_b^\star(z_e)] \Big) \mathrm{d}\hat{J}(\rho, z_e, \mathbf{s}) \right], \tag{4.8}$$

where $e \in \{Baseline, HLMW\}$ indexes our baseline model economy or the no-bank economy of HLMW. We can also write the total welfare at a given gross inflation γ with consumption reduced by a factor of

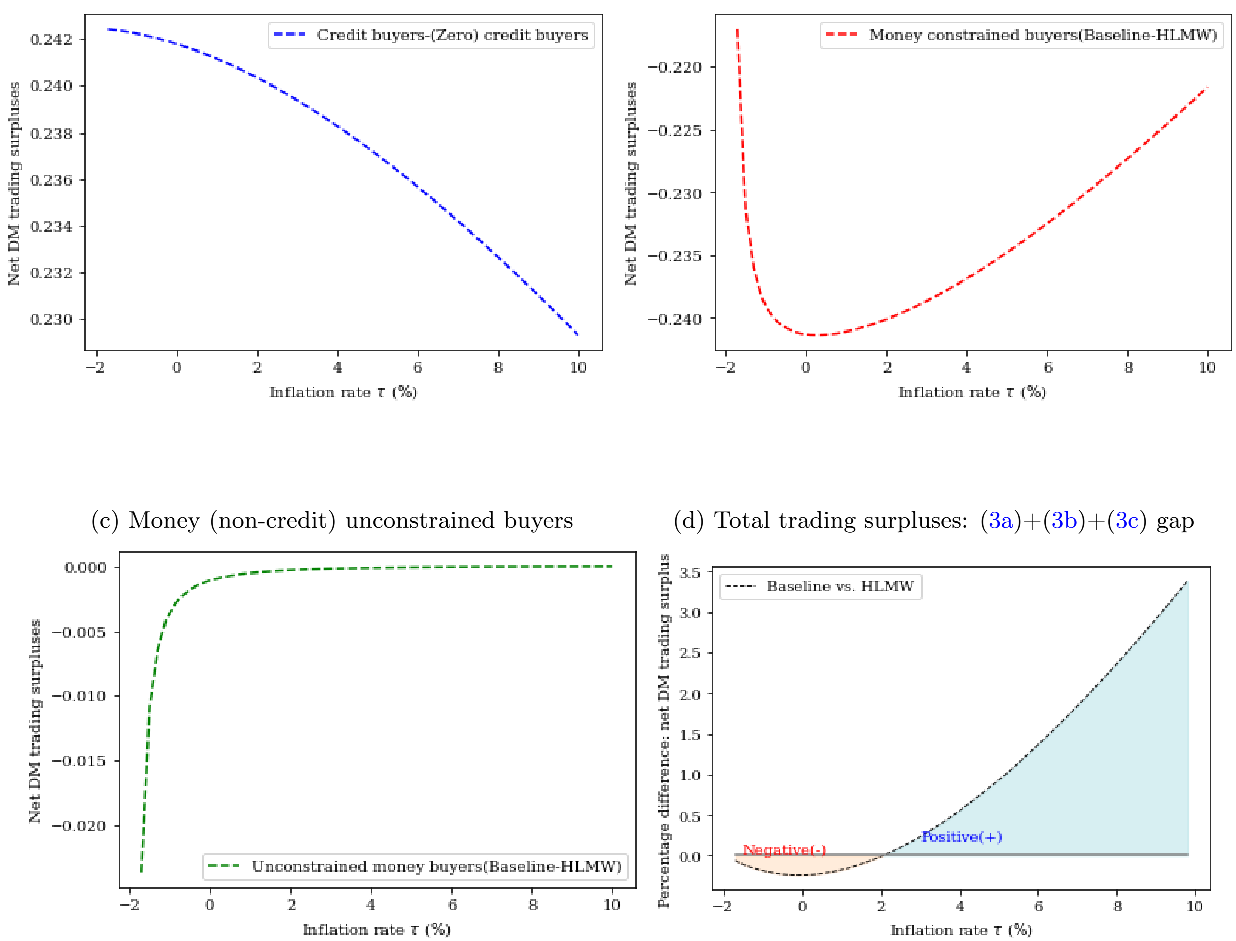
[32]For example, in our baseline banking equilibrium we have three ex-post cases with corresponding ex-ante net trading surpluses. The generic formula for the trading surplus measure is

$$\int_{\mathcal{E}} \{u[q_b(z,\rho,\mathbf{s})] - c[q_b(z,\rho,\mathbf{s})]\} \, \mathrm{d}\hat{J}(\rho, z, \mathbf{s}),$$

where: (a) the credit-buyer cases have $\mathcal{E} := \left[\underline{\rho}(z), \tilde{\rho}_i(z)\right]$ and $q_b(\rho, z) = [\rho(1+i)]^{-1/\sigma}$; (b) the own-money constrained buyer cases have $\mathcal{E} := (\tilde{\rho}_i(z), \hat{\rho}_{i_d}(z)]$ and $q_b(\rho, z) = z/\rho$; and (c) the own-money unconstrained buyer cases have $\mathcal{E} := (\hat{\rho}_{i_d}(z), \bar{\rho}(z)]$ and $q_b(\rho, z) = [\rho(1+i_d)]^{-1/\sigma}$. We can define similar objects for the HLMW no-bank environment except that there will be zero measures of credit buyer events.

[33]Note that since inactive buyers do not consume in the DM, there is zero net trading surplus emanating from such ex-post events. These inactive buyer's ex-ante welfare are thus solely accounted for by the term $(1-n)[U(x^\star) - x^\star]$. Together with similar terms accruing to the other ex-post agent groups, the total present-value social welfare in the CM activity is simple: it is just the constant term $(1-\beta)^{-1}[U(x^\star) - x^\star]$.

Figure 3: The effects of inflation on Equilibrium Outcome.



(a) Credit buyers

(b) Money (non-credit) constrained buyers

(c) Money (non-credit) unconstrained buyers

(d) Total trading surpluses: (3a)+(3b)+(3c) gap

$\triangle$ as

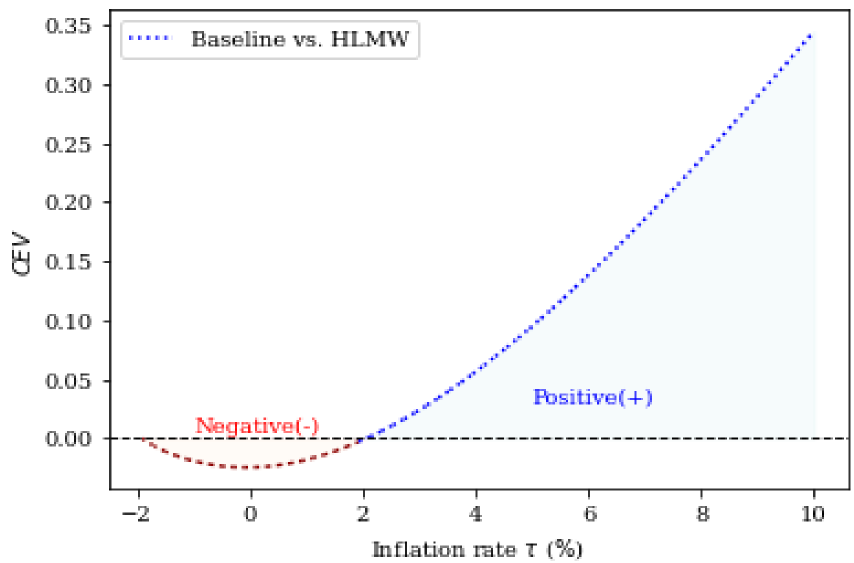
$$W^e(\gamma) = \frac{1}{1-\beta}\left[U(\triangle x^\star) - x^\star + n\int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(u[\triangle q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big)\mathrm{d}\hat{J}(\rho, z_e, \mathbf{s})\right]. \tag{4.9}$$

We compute the CEV as the value $1-\triangle$ that solves $W^{Baseline}(\gamma) = W^{HLMW}{}_{\triangle}(\gamma)$ given policy $\gamma = 1+\tau$. This measure says that every agent in the economy with perfectly competitive banks needs to give up $1-\triangle$ percent of his consumption to move to the economy without access to competitive banks at given policy. Note that the integral term already includes the measure of active buyers who would optimally chosen to deposit some of their idle money.

According to Figure 4, banking has a non-monotonic welfare consequence when the trend inflation rate varies from just above the Friedman rule $\gamma = 1+\tau = \beta$. In particular, banks are inessential institutions when the trend inflation rate is sufficiently low. This result hinges on the interaction between the liquidity-risk insurance and price dispersion effects discussed earlier in Section 4.1.

On the one hand, positive welfare effects of banking come from the liquidity-risk insurance effect. First, both the (ex-post) inactive buyers in trading with probability $(1-n)$ and unconstrained money-buyers

Figure 4: Consumption Equivalent Variation (%) of moving from the no-bank HLMW economy to the baseline economy with banking.



can deposit their idle money balances to earn an interest i_d. Second, buyers who trade with low-price firms find it worthwhile to borrow additional money balances from the bank. These credit-buyers have more relaxed liquidity constraints. Thus, they can spend more on goods to enjoy a higher utility flow in the DM.

On the other hand, credit can amplify firms' market power (markups and price dispersion), which creates a negative welfare effect of banking on liquidity constrained and unconstrained money-buyers via higher prices (both the average level and dispersion). The reason is as what we had previously discussed.

Summary of insights. Imperfect information through noisy search frictions in the goods market generates a policy-dependent distribution of goods prices (and associated markups), as in Head et al. (2012). The presence of competitive banking benefits only agents who would like to deposit and those who optimally use credit by inducing more firms who serve them to more likely post low prices. In turn, the externality effect is in firms who charge higher prices to money-buyers who do not find it optimal to borrow. These agents' expenditures are either inelastic to the price rise and they end up consuming less (i.e., the money-constrained agents) or they elastically respond to higher price draws by consuming less (i.e., the money-unconstrained agents). When inflation is sufficiently low, the cost of holding money is also low. Thus, the gains from banking along the channel of liquidity-risk insurance effect are also small. The price dispersion effect can easily outweigh such benefits via higher markups distorting the liquidity premium for the money-buyers.

To sum up, firms' price dispersion induces (ex-post) heterogeneous consumption outcomes among credit-buyers and money-buyers. Hence, non-trivial feedback from firms' market power on the welfare consequences of banking. In particular, credit-buyers benefit from banking credit to purchase more goods. However, banking also makes firms extract more rent from money-buyers in goods trades, thus lowering consumption. The essentiality of banks—in terms of helping insure against individual liquidity risks—is no longer unambiguous in our economy with endogenous firms' market power in the goods market.

5 Conclusion

We construct a model of money, bank credit and endogenous retail market power where informational frictions induce a policy-dependent distribution of goods prices and associated markups in equilibrium. We show that access by borrowers to credit can contribute to amplifying firms' market power, reducing the welfare gains from banking, if there is a wedge between loan and deposit rates induced by a binding reserve requirement regulation on banks. In our quantitative discipline, this is indeed the case. If the reserve requirement is not binding, banks are always welfare improving as they support trade and insure against inflation tax on idle money.

When the reserve requirment is binding, the increased demand for goods by credit-buyers expands the measure of firms charging higher prices, extracting rent from money-buyers. The latter comes in two ways: First, higher price draws affect agents who turn out to be liquidity constrained money-buyers by squeezing their liquidity constraints and thus lowering their consumption. Second, higher price draws also reduce unconstrained money-buyers' consumption even though there is no binding liquidity constraint on them. This is simply because their consumption demands are decreasing functions of the relevant prices they draw. As a result, market power in the retail industry can make an otherwise competitive banking sector less efficient in reallocating liquidity in equilibrium.

Thus, the welfare-improving role of banking liquidity transformation is no longer unambiguous in a monetary economy with endogenous firms' market power. Our model highlights a new channel that can be surprising if policymakers attempt to regulate banking competition without taking into account its externality on consumers in non-competitive goods markets that have, evidently, price dispersion. While our result is predicated on equilibria where the reserve regulation policy is binding, it also suggests the following conjecture: Paying interest on reserves can complement the traditional long-run inflation targeting policy. From this model's perspective, paying interest on reserves can help slacken the reserve requirement constraint on banks and indirectly, help weaken the pecuniary externality problem. This question is interesting but outside the scope of the present paper, but we are investigating this conjecture in follow-up research.

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Online Appendix

On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion

Omitted Proofs and Other Results

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A Demand solution to Problem (2.7).

Definition 2. The first-best allocation is a DM outcome $q = q^{FR} \in (0, \infty)$ satisfying $u_q(q)/p = 1$.

A.1 Proof of Lemma 1

Proof. An *ex-post* DM good buyer with state $(m, p, \mathbf{a})$, who gets to sample at least one price, has the value

$$B(m, p, \mathbf{a}) = \max_{q_b, l, d} \left\{ u(q) + W(\tilde{m}, l, d, \mathbf{a}) \;\middle|\; \begin{array}{l} \tilde{m} := m + l - pq - d \geq 0, \\ d \in [0, m+l], \\ q \geq 0, \\ l \in [0, \infty) \end{array} \right\}. \tag{A.1}$$

Since u is strictly concave and satisfies the Inada conditions, any optimal selection will involve $q \in (0, \infty)$. In any monetary equilibrium, we will require $p, \phi \in (0, \infty)$. Hence, for any amount $m + l$, the constraint $q \geq 0$ is never binding, and the requirement $d \leq m + l$ is redundant (as it will be implied by the constraint $\tilde{m} \geq 0$). The Lagrangian for the right-hand-side problem in Equation (A.1) can thus be simplied as

$$L(q, l, d, \lambda, \mu, \theta | m, p, \mathbf{a}) = u(q) + W(\tilde{m}, l, d, \mathbf{a}) - \lambda [0 - \tilde{m}] - \mu [0 - d] - \theta [0 - l],$$

where (λ, μ, θ), respectively, are the non-negative Lagrange multipliers on the buyer's liquidity constraint, deposit non-negativity constraint, and loan non-negativity constraint.

The first-order conditions for maximizing $L(q, l, d, \lambda, \mu, \theta | m, p, \mathbf{a})$, in the direction of the choice variables (q, l, d), respectively, are

$$\frac{\partial L}{\partial q}(q, l, d, \lambda, \mu, \theta | m, p, \mathbf{a}) = u(q) - \lambda p - pW_m(\tilde{m}, l, d, \mathbf{a}) = 0,$$

$$\frac{\partial L}{\partial l}(q, l, d, \lambda, \mu, \theta | m, p, \mathbf{a}) = W_m(\tilde{m}, l, d, \mathbf{a}) + W_l(\tilde{m}, l, d, \mathbf{a}) + \lambda + \theta = 0,$$

and,

$$\frac{\partial L}{\partial d} = -W_m(\tilde{m}, l, d, \mathbf{a}) + W_d(\tilde{m}, l, d, \mathbf{a}) - \lambda + \mu = 0.$$

The Karush-Kuhn-Tucker (KKT) conditions are:

$$\lambda \tilde{m} = 0, \qquad \mu d = 0, \text{ and}, \qquad \theta l = 0.$$

The envelope conditions for W are given by the marginal value of money at the start of the CM,

$$W_m(\tilde{m}, l, d, \mathbf{a}) = \phi,$$

of loan liability,

$$W_l(\tilde{m}, l, d, \mathbf{a}) = -(1 + i)\phi,$$

and, of deposit claim,

$$W_d(\tilde{m}, l, d, \mathbf{a}) = (1 + i_d)\,\phi.$$

Combining the first-order, KKT and envelope conditions, we have the following system of equations:

$$\begin{aligned}
\lambda &= \left[\frac{u_q(q)}{p} - \phi\right], \\
\phi i &= \lambda + \theta, \\
\phi i_d &= \lambda - \mu, \\
\lambda \tilde{m} &= 0, \\
\mu d &= 0, \\
\theta l &= 0.
\end{aligned} \tag{A.2}$$

Conditional on the reserve-requirement constraint case (i.e., on whether $i = i_d$ or $i > i_d$), there are eight cases to verify for this system of equations. First, we consider the obvious case where the reserve-requirement constraint is slack and derive the demand functions for good q, loan l, and deposit d. Then, we characterize these for the case where the reserve-requirement constraint is binding.

Slack reserve-requirement constraint $(i = i_d)$**.** In this case, it will be apparent that the only case consistent with such an equilibrium (with $i = i_d$) is one where $\lambda > 0$ and $\mu = \theta = 0$. From the system (A.2), given $i > 0$ and $p, \phi \in (0, \infty)$, we can deduce that the optimal $q^\star$ satisfies $u_q(q)/p = (1+i)\phi$, and is such that $q^\star < q^{FR}$ (where q^{FR} is the first-best allocation). Also, we can derive $i = i_d > 0$. Since $i = i_d$, the ex-post DM buyer is indifferent between borrowing and depositing. Thus, without loss, we set net borrowing as $\tilde{l} = l - d$, so that the optimal net borrowing is $\tilde{l}^\star = -(m - pq^\star)$.

Define the cutoff $\tilde{l}_c = 0 = -\left(m - pu_q^{-1}\left[(1+i)\,\phi p\right)\right.$ where demand for q is equivalent to a money-constrained buyer (i.e., all of m is spent on q). Assume u is CRRA(σ). Associated with this cutoff is the pricing cutoff $\tilde{p}_i := \hat{p}\left[(1+i)\phi\right]^{\frac{1}{\sigma-1}}$, where $\hat{p} := m^{\frac{\sigma}{\sigma-1}}$. Thus, we may set $l^\star = \tilde{l}_c = -\left(m - pu_q^{-1}\left[(1+i)\,\phi p\right)\right. > 0$ and $d^\star = 0$, if $p \leq \tilde{p}_i$. Otherwise, $l^\star = 0$ and $d^\star = m - pu_q^{-1}\left[(1+i)\,\phi p > 0\right.$. That is, for pricing outcomes that are sufficiently low (at most $\tilde{p}_i$), the buyer will only take out a loan. Otherwise, the buyer does not spend all their money balance on goods and deposits the remainder.

For completeness, we summarize the other cases here that yield results that contradict a monetary equilibrium, and so cannot exist:

1. **Case** $\lambda = 0, \mu > 0, \theta > 0$**.** This case implies that $i = \theta/\phi > 0$ but $i_d = -\mu/\phi < 0$.

2. **Case** $\lambda = 0, \mu = 0, \theta = 0$**.** This results in $i = i_d = 0$, which is not a consistent with the requirement that $i > 0$.

3. **Case** $\lambda = 0, \mu = 0, \theta > 0$**.** Here, we end up with $i_d = 0$, and $i = \theta/\phi > 0$, but this is inconsistent with the fact that banks are perfectly competitive in deposit taking and loans and there is no binding reserve requirement.

4. **Case** $\lambda = 0, \mu > 0, \theta = 0$**.** Here, $i = 0$ but $i_d = -\mu/\phi < 0$.

5. **Case** $\lambda > 0, \mu > 0, \theta > 0$**.** Here, $i = (\lambda + \theta)/\phi > 0$ but $i_d = (\lambda - \mu)/\phi \gtreqless 0$, and $i \neq i_d$. Since $i = i_d$ in an equilibrium with unconstrained, perfectly-competitive banks, we have a contradiction.

6. **Case $\lambda > 0, \mu > 0, \theta = 0$.** Here, $i = \lambda/\phi > 0$ and $i_d = i - \mu/\phi$, so that $i \neq i_d$. Again, this is inconsistent with the fact that banking is perfectly competitive in deposits and loans, and the reserve requirement is not binding.

7. **Case $\lambda > 0, \mu = 0, \theta > 0$.** Here, $i = i_d + \theta/\phi > 0$ and $i_d = \lambda/\phi > 0$, but $i \neq i_d$. Again, this is inconsistent with the fact that banking is perfectly competitive in deposits and loans, and the reserve requirement is not binding.

In short order, if the reserve requirement policy is not binding (so that $i = i_d$), an active DM buyer's demand for good q is smooth,

$$q_b^\star(m, p, \mathbf{a}) = [(1+i)\phi p]^{-\frac{1}{\sigma}},$$

and their demands for loan and deposit are piecewise continuous, depending on the price draw p relative to the endogenous cutoff $\tilde{p}_i$:

$$l^\star(m, p, \mathbf{a}) = \begin{cases} pq^\star(m, p, \mathbf{a}) - m > 0, & \text{if } p \leq \tilde{p}_i \\ 0, & \text{otherwise} \end{cases},$$

and,

$$d^\star(m, p, \mathbf{a}) = \begin{cases} 0, & \text{if } p \leq \tilde{p}_i \\ m - pq^\star(m, p, \mathbf{a}) > 0, & \text{otherwise} \end{cases}.$$

Binding reserve-requirement constraint ($i > i_d$). If the reserve requirement binds, then there are only three of eight possibilities that are consistent with such an equilibrium configuration. We derive these case by case and then characterize the resulting demand system.

1. **Case $\lambda > 0, \mu > 0, \theta = 0$ (Credit buyer).** Consider the system (A.2). Note that $\mu > 0$ if and only if the deposit non-negativity constraint binds, $d^\star = 0$. Likewise, $\theta = 0$ if and only if the loan non-negativity constraint is slack, $l^\star > 0$. Also we can derive $i = \lambda/\phi > 0$ and $i_d = i - \mu/\phi$. In the equilibrium, for banks to take deposits (from other non-buyer-type depositors), it must be that $i_d > 0$. From this, we can derive the demand for q as $q^\star = u_q^{-1}[(1+i)\phi p] < q^{FR}$. Also, $\lambda > 0$ if and only if the liquidity constraint binds: $l^\star = m - pq^\star$. This and $l^\star > 0$ imply that $p < m^{\frac{\sigma}{1-\sigma}}[(1+i)\phi]^{\frac{1}{\sigma-1}} =: \tilde{p}_i$. Using the CRRA($\sigma$) form of u, we can derive explicitly $q^\star = [(1+i)\phi p]^{-1/\sigma}$ and also back out the expression for $l^\star$. To sum up, if the buyer draws a sufficiently low price, $p < \tilde{p}_i$, the buyer tops up their money holdings by taking out bank credit. The buyer does not have any liquidity left for depositing.

2. **Case $\lambda > 0, \mu > 0, \theta > 0$ (Money-constrained buyer).** From the system (A.2), we can derive the following: $l^\star = d^\star = 0$ and $q^\star = m/p$. Since $\theta > 0$, we also have $u_q(q)/p < \phi(1+i)$—i.e., the marginal utility of consumption is lower than the real marginal cost of servicing a loan. This inequality implies that $p > m^{\frac{\sigma}{1-\sigma}}[(1+i)\phi]^{\frac{1}{\sigma-1}} =: \tilde{p}_i$. Furthermore, since $\mu > 0$, we can also derive that $u_q(q)/p > \phi(1+i_d)$—i.e., the marginal utility of consumption is higher than the real marginal benefit of depositing. This inequality implies that $p < m^{\frac{\sigma}{1-\sigma}}[(1+i_d)\phi]^{\frac{1}{\sigma-1}} =: \hat{p}_{i_d}$. Note that since $\sigma < 1$, the pricing cutoff functions uniformly (i.e., for all m and $i > i_d$) satisfy the ordering $0 < \tilde{p}_i < \hat{p}_{i_d}$. Hence, when $p \in (\tilde{p}_i, \hat{p}_{i_d})$, the buyer is a money-constrained buyer who demands no loan nor deposit.

3. **Case $\lambda > 0, \mu = 0, \theta > 0$ (Unconstrained depositor-buyer).** Consider the system (A.2). Note that $\mu = 0$ if and only if the deposit non-negativity constraint is slack, $d^\star > 0$. Likewise, $\theta > 0$ if and only if the loan non-negativity constraint binds, $l^\star = 0$. Also we can derive $i_d = \lambda/\phi > 0$ and $i = i_d + \theta/\phi > i_d$, consistent with the putative equilibrium's configuration of the interest spread. From this, we can derive the demand for q as $q^\star = u_q^{-1}[(1+i_d)\phi p] < q^{FR}$. Also, $\lambda > 0$ if and only if the liquidity constraint binds: $d^\star = m - pq^\star$. This and $d^\star > 0$ imply that $p > m^{\frac{\sigma}{1-\sigma}}[(1+i_d)\phi]^{\frac{1}{\sigma-1}} =: \hat{p}_{i_d}$. Using the CRRA($\sigma$) form of u, we can derive explicitly $q^\star = [(1+i_d)\phi p]^{-1/\sigma}$. That is, if the buyer draws a sufficiently high price, $p > \hat{p}_{i_d} > \tilde{p}_i$, the buyer optimally does not spend all their money holdings on goods. The buyer deposits the remainder money balance.

For completeness of this proof, we enumerate the rest of the cases that contradict a monetary equilibrium under $i > i_d$:

1. **Case $\lambda > 0, \mu = 0, \theta = 0$.** This case implies $i = i_d = \lambda/\phi > 0$. However, since $i > i_d$, we can rule this case out.
2. **Case $\lambda = 0, \mu > 0, \theta > 0$.** This case implies that $i = \theta/\phi > 0$ but $i_d = -\mu/\phi < 0$.
3. **Case $\lambda = 0, \mu = 0, \theta = 0$.** This results in $i = i_d = 0$, which is not a consistent with the requirement that $i > 0$.
4. **Case $\lambda = 0, \mu = 0, \theta > 0$.** Here, we end up with $i_d = 0$, and $i = \theta/\phi > 0$, but this is inconsistent with the fact that banks are perfectly competitive in deposit taking and that $i_d > 0$.
5. **Case $\lambda = 0, \mu > 0, \theta = 0$.** Here, $i = 0$ but $i_d = -\mu/\phi < 0$. No bank will be operational. This contradicts an equilibrium with banking.

To summarize, if the reserve requirement policy is binding (so that $i > i_d$), an active DM buyer's demand for good q, loan l, and deposit d, are piewise smooth (depending on p):

$$q_b^\star(m, p, \mathbf{a}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ \frac{m}{p} & \text{if } \tilde{p}_i < p \leq \hat{p}_{i_d} \\ [p\phi(1+i_d)]^{-1/\sigma} & \text{if } \hat{p}_{i_d} < p \end{cases},$$

$$l^\star(m, p, \mathbf{a}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}[\phi(1+i)]^{-\frac{1}{\sigma}} - m & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \leq \hat{p}_{i_d} \\ 0 & \text{if } \hat{p}_{i_d} < p \end{cases},$$

and,

$$d^\star(m, p, \mathbf{a}) = \begin{cases} 0 & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p \leq \hat{p}_{i_d} \\ m - p^{\frac{\sigma-1}{\sigma}}[\phi(1+i_d)]^{-\frac{1}{\sigma}} & \text{if } \hat{p}_{i_d} < p \end{cases},$$

where

$$\hat{p} := \hat{p}(m, \mathbf{a}) = \phi^{\frac{1}{\sigma-1}} m^{\frac{\sigma}{\sigma-1}},$$
$$\tilde{p}_i := \tilde{p}_i(m, \mathbf{a}) = \hat{p}(1+i)^{\frac{1}{\sigma-1}},$$

and,

$$\hat{p}_{i_d} := \hat{p}_{i_d}(m, \mathbf{a}) = \hat{p}(1 + i_d)^{\frac{1}{\sigma-1}}.$$

The cutoff prices $(\tilde{p}_i, \hat{p}_{i_d})$ are functions of the state of the economy and monetary policy. Since $i \geq i_d$ and $\sigma < 1$, we can order the cut-off prices as: $0 < \tilde{p}_i \leq \breve{p}_{i_d} < \hat{p} < +\infty$. $\square$

B Omitted proofs

B.1 Proof of Lemma 3 (Monopoly pricing)

The following is a partial equilibrium result, taking as parametric the pre-determined money holding of agents z, and the rate of interest on loans i and deposits i_d. It provides for a complete characterization of what would determine the upper bound ($\bar{p} = p^m$) on the equilibrium support of the DM-good price distribution. Below, we rewrite nominal variables in terms of stationary variables: Measured in units of the CM numéraire good, real money balance is $z := \phi m$ and the relative price of a DM good is $\rho := \phi p$. (Dividing the results through with the value of money ϕ will yield the result in Lemma 3, which was presented in nominal terms.) Thus, Lemma 3 re-stated in equivalent stationary-variable terms is:

Lemma (3)**.** *Fix a (pre-determined) real money balance $z \in (0, \infty)$ and a given rate on loans i and deposits i_d. There are two cases to consider:*

1. *If $i = i_d$, then the ex-post profit function is*

$$G_1(\rho; i) := [\rho\,(1 + i)]^{-\frac{1}{\sigma}}(\rho - c),$$

which is strictly concave and non-negative valued, with a unique maximum at the monopoly price ρ_0^m.

2. *If $i > i_d$, then the ex-post profit function may not be strictly concave, and is defined as follows: Let the realized profit at price outcome ρ from serving*

   - *a credit-buyer be $G_1(\rho; i) := [\rho\,(1 + i)]^{-\frac{1}{\sigma}}(\rho - c)$,*
   - *a constrained money-buyer be $G_2(\rho; z) := \frac{z}{\rho}(\rho - c)$, and,*
   - *an unconstrained money-buyer be $G_3(\rho; i_d) := [\rho\,(1 + i_d)]^{-\frac{1}{\sigma}}(\rho - c)$.*

   Let $\mathbf{g}(\rho; i, i_d, z) := [G_1(\rho; i), G_2(\rho; z), G_3(\rho; i_d)]$. Assume that $\sigma \in (0, 1)$ and $c \in (0, 1]$ such that $0 < (1 + i_d)^{-\frac{1}{\sigma}}\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}_{i_d}$. There exists a $\acute{z}_{i_d} = \hat{z}_{i_d}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$ and $\tilde{z}_{i,i_d} := (1 + i)^{-\frac{1}{\sigma}}\hat{z}_{i_d}$. Furthermore, there is a $\breve{z}_{i,i_d}$ where $\mathring{z}_{i,i_d} := \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$ and $0 < \mathring{z}_{i,i_d} \leq \tilde{z}_{i,i_d} \leq \hat{z}_{i_d} < \acute{z}_{i_d} < \infty$.

   (a) *The ex-post profit function is*

$$R^{ex}(\rho, i, z) = \begin{cases} \langle \mathbf{g}(\rho; i, i_d, z), \mathbf{I}_1(\rho; i, i_d, z)\rangle\,, & z \in [\hat{z}_{i_d}, \infty) \\ \langle \mathbf{g}(\rho; i, i_d, z), \mathbf{I}_2(\rho; i, i_d, z)\rangle\,, & z \in [\mathring{z}_{i,i_d}, \hat{z}_{i_d}) \\ \langle \mathbf{g}(\rho; i, i_d, z), \mathbf{I}_3(\rho; i, i_d, z)\rangle\,, & z \in (0, \mathring{z}_{i,i_d}) \end{cases} \tag{B.1}$$

where

$$\mathbf{I}_1(p;i,i_d,m) := \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}}, \underbrace{\mathbf{1}_{\{c<\hat{\rho}_{i_d}\leq\rho\leq\rho_0^m\}}}_{\text{Case 1(a)}} + \underbrace{\mathbf{1}_{\{\hat{\rho}_{i_d}<c\leq\rho\leq\rho_0^m\}}}_{\text{Case 1(b)}}\right],$$

$$\mathbf{I}_2(\rho;i,i_d,z) := \begin{bmatrix} \mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} \\ \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}} \\ \underbrace{\mathbf{1}_{\{c<\tilde{\rho}_i<\rho_0^m<\hat{\rho}_{i_d}\leq\rho\}\cap\{z\in[\tilde{z}_i,\hat{z})\}}}_{\text{Case 2(a)}} + \underbrace{\mathbf{1}_{\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}_{i_d}\leq\rho\}\cap\{z\in[\mathring{z}_i,\tilde{z}_i)\}}}_{\text{Case 2(b)}} \end{bmatrix}',$$

$$\mathbf{I}_3(\rho;i,i_d,z) := \underbrace{\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}}, \mathbf{1}_{\{c<\hat{\rho}_{i_d}\leq\rho\}}\right] \times \mathbf{1}_{\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}_{i_d}\}}}_{\text{Case 3}},$$

$\rho_0^m = c/(1-\sigma)$, $\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$, $\hat{\rho}_{i_d} := \hat{\rho}(1+i_d)^{\frac{1}{\sigma-1}} > \tilde{\rho}_i := \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, $\langle\cdot,\cdot\rangle : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \to \mathbb{R}_+$ is the inner-product operator, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.

(b) The (real) monopoly price and its ex-post profit outcome, respectively, are

$$\rho^m = \begin{cases} \rho_0^m, \\ \hat{\rho}_{i_d}(z), \\ \rho_0^m, \end{cases} \quad \textit{and, } R^{ex}(\rho^m, i, i_d, z) = \begin{cases} G_3(\rho_0^m; i_d), & z \in [\hat{z}_{i_d}, \infty) \\ G_2(\hat{\rho}(z); z) = G_3(\hat{\rho}(z); i_d), & z \in [\mathring{z}_{i,i_d}, \hat{z}_{i_d}) \\ G_1(\rho_0^m; i), & z \in (0, \mathring{z}_{i,i_d}) \end{cases}. \tag{B.2}$$

Proof. The first case with $i = i_d$ is straightforward, so we will dispense with it first. Then we will consider the case where $i > i_d$.

Case of $i = i_d$. The demand for q is given by Equation (2.8). This is a smooth, downward-sloping function of the relative price ρ, for fixed i. The ex-post profit is thus $G_1(\rho;i) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$. It is easy to check that there is a unique maximizer for the strictly concave $G_1(\cdot;i)$, i.e., $\arg\max_\rho G_1(\rho;i) = \rho_0^m$.

Case of $i > i_d$. The demand for q when $i > i_d$ is classified by Equation (2.11) in the paper. In terms of the stationary variables, this is equivalently given as:

$$q_b^\star(z,\rho,\mathbf{a}) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ z & \text{if } \tilde{\rho}_i < \rho \leq \hat{\rho}_{i_d} \\ [\rho(1+i_d)]^{-1/\sigma} & \text{if } \rho > \hat{\rho}_{i_d} \end{cases}, \tag{B.3}$$

where we recall the definitions

$$\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}, \qquad \text{and} \qquad \hat{\rho}_{i_d} := \hat{\rho}(1+i_d)^{\frac{1}{\sigma-1}} > \tilde{\rho}_i := \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}, \tag{B.4}$$

and z and i are taken as fixed (parametric) by both buyers and firms. Consider the case of a firm that posts a monopoly price ρ^m in a monetary equilibrium where $0 < z < \infty$ and $i > 0$. Given buyers' demand structure $q_b^\star(z,\rho,\mathbf{a})$ derived in Equation (B.3), a firm's ex-post profit per trade, $R^{ex}(\rho,i,i_d,z)$ is $R^{ex}(\rho,i,i_d,z) = q_b^\star(z,\rho,\mathbf{a})(\rho - c)$. Note that the relevant domain for pricing has a natural lower bound of c since no firm would ever want to price below marginal cost in a one-shot market. (In equilibrium, we can show that the lower bound on the support of the price distribution is bounded below by c.)

Properties of ex-post profit function components. The components G_1, G_2 and G_3 have the following geometric properties:

1. Since $\sigma < 1$ and $i > i_d$, then $0 < (1+i)^{-\frac{1}{\sigma}} < (1+i_d)^{-\frac{1}{\sigma}} < 1$, so that it is always the case that $\tilde{\rho}_i < \hat{\rho}_{i_d}$ and the function value $G_1(\rho; i) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$ is everywhere dominated by that of $G_3(\rho; i_d) := [\rho(1+i_d)]^{-\frac{1}{\sigma}}(\rho - c)$ (i.e., the third case).

2. We can also verify that $G_1(c; i) = G_2(c; z) = G_3(c; i_d) = 0$ and $\lim_{\rho \nearrow \infty} G_1(\rho; i) = \lim_{\rho \nearrow \infty} G_3(\rho; i_d) = 0$. These two functions are strictly positive-valued on (c, ∞), and have unique maxima in ρ: If ρ is not constrained anywhere on the feasible domain $[c, \infty)$, then the unique maximum for the function G_3 solves first-order condition $\frac{\partial G_3}{\partial \rho}(\rho; i_d) = \frac{\partial G_3}{\partial \rho}\left[[\rho(1+i_d)]^{-\frac{1}{\sigma}}(\rho - c)\right] = 0$, which yields $\rho_0^m = \frac{c}{1-\sigma}$. In nominal terms, this is $\rho_0^m = \frac{\phi^{-1}c}{1-\sigma}$.

3. We can also deduce that there is a unique unconstrained maximizer for G_1, i.e., $\arg\max_\rho G_1(\rho; i) = \arg\max_\rho G_3(\rho; i_d) = \rho_0^m$.

4. Neither G_1 nor G_3 depend on z. The function G_2 is the only piece that depends on z, is such that $G_2(c; z) = 0$, $\lim_{\rho \nearrow \infty} G_2(\rho; z) = \infty$, and its image is strictly increasing and strictly concave in ρ.

Properties 1 to 4 imply that for all $\rho > c$, there can only be the following three generic cases, depending on the range of sizes of real money balance that buyers carry, z.

Case 1. **Real money balance is "sufficiently high".** Consider a given $i > 0$ and $z \in [\hat{z}_{i_d}, \infty)$. Let $\hat{z}_{i_d} := (1+i_d)^{-\frac{1}{\sigma}} \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ be the cutoff real balance if and when graph $(G_2(\cdot; z))$ intersects graph $(G_3(\cdot; i_d))$ uniquely at $\rho = \hat{\rho}_{i_d}(\hat{z}_{i_d}) = \rho_0^m > c$. For $z > \hat{z}_{i_d}$, there are two sub-cases to consider:

(a) There is a $\acute{z}_{i_d} > \hat{z}_{i_d}$ and $z \in (\acute{z}_{i_d}, \infty) \equiv \left((1+i_d)^{-\frac{1}{\sigma}} c^{1-\frac{1}{\sigma}}, \infty\right)$ where graph $(G_2(\cdot; z))$ never intersects the graphs of $G_1(\cdot; i)$ and $G_3(\cdot; i_d)$ at any $\rho > \rho_0^m > c$. This case exists if $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c; i_d)$ and $\frac{\partial G_2}{\partial \rho}(\rho_0^m; z) \geq \frac{\partial G_3}{\partial \rho}(\rho_0^m; i_d) = 0$. The second restriction is always satisfied since $\frac{\partial G_2}{\partial \rho}(\cdot; z) > 0$ everywhere. We can check that $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c; i_d)$ if and only if $z > \acute{z}_{i_d} \equiv (1+i_d)^{-\frac{1}{\sigma}} c^{1-\frac{1}{\sigma}}$. Thus, $\rho = \hat{\rho}_{i_d}(z)$ does not exist if $z > \acute{z}_{i_d}$, since $\hat{\rho}_{i_d}(z) < \hat{\rho}_{i_d}(\acute{z}_{i_d}) = c$. That is, no firm will face constrained money buyers, or credit buyers, by the fact that $\tilde{\rho}_i(z) < \hat{\rho}_{i_d}(z) < c$ whenever $z > \acute{z}_{i_d}$. This implies that: ***(i)*** only *unconstrained money buyers* are served; ***(ii)*** the effective profit function for a firm is that which is associated with the demand from unconstrained money buyers, $R^{ex}(\rho, i, i_d, z)|_{z \in \left(z'_{i_d}, \infty\right)} = G_3(\rho; i_d) := [\rho(1+i_d)]^{-\frac{1}{\sigma}}(\rho - c)$; and ***(iii)*** the monopoly pricing outcome is $\rho_0^m = \frac{c}{1-\sigma}$, with its induced profit being $G_3(\rho_0^m; i_d) := [\rho_0^m(1+i_d)]^{-\frac{1}{\sigma}}(\rho_0^m - c) > 0$. (Figure 5 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

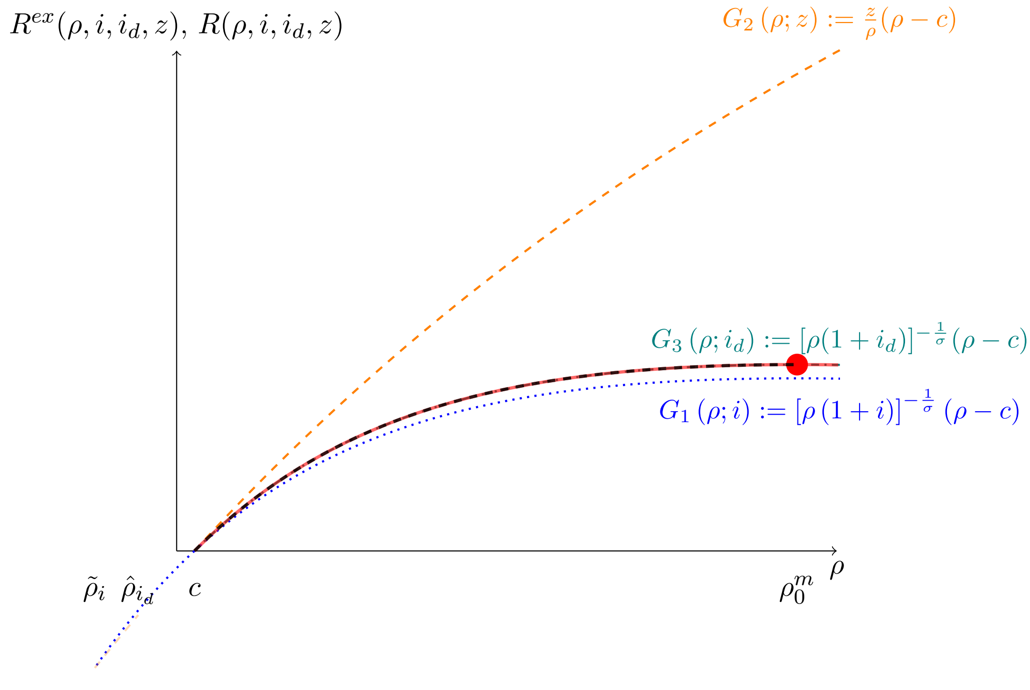

Figure 5: Case 1(a). Real money balance is sufficiently high, $z \in [\acute{z}_{i_d}, \infty)$.

(b) Real money balance z is such that $z \in [\hat{z}_{i_d}, \acute{z}_{i_d})$. Recall that $[\hat{z}_{i_d}, \acute{z}_{i_d}) = \left[(1+i_d)^{-\frac{1}{\sigma}} \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}, (1\right.$

where $\hat{z}_{i_d} < \acute{z}_{i_d} \equiv (1+i_d)^{-\frac{1}{\sigma}} c^{1-\frac{1}{\sigma}}$, since $\sigma \in (0,1)$. Thus, consider real money balance at any given $z \in [\hat{z}_{i_d}, \acute{z}_{i_d})$. We can check that $c < \tilde{\rho}_i(z) < \hat{\rho}_{i_d}(z) \leq \rho_0^m$. Also, $G_2(\rho; z) < G_3(\rho; i_d)$ for all $\rho \in (c, \hat{\rho}_{i_d}(z))$, $G_2(\rho; z) = G_3(\rho; i_d)$ only if $\rho = c$ or $\rho = \hat{\rho}_{i_d}(z) \leq \rho_0^m$, and $G_2(\rho; z) > G_3(\rho; i_d)$ for all $\rho > \rho_0^m$. Thus, graph $(G_2(\cdot; z))$ and graph $(G_3(\cdot; i_d))$ uniquely intersect at $\rho = \hat{\rho}_{i_d}(z) \leq \rho_0^m$. Since $G_1(\cdot; i)$ is always dominated by $G_3(\cdot; i_d)$ (Property 1), and $G_2(\rho; z)$ is increasing in ρ, then graph $(G_2(\cdot; z))$ can only uniquely intersect graph $(G_1(\cdot; i))$ at some at a unique point $\tilde{\rho}_i(z)$. This implies that: ***(i)*** each firm's effective profit function is given by

$$\begin{aligned} &R^{ex}(\rho, i, i_d, z)\big|_{z \in [\hat{z}_{i_d}, \acute{z}_{i_d})} \\ &\qquad = G_1(\rho; i)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho; z)\,\mathbf{1}_{\{\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}} + G_3(\rho; i_d)\,\mathbf{1}_{\{\hat{\rho}_{i_d}\leq\rho\leq\rho_0^m\}}, \end{aligned}$$

whenever $z \in [\hat{z}_{i_d}, \acute{z}_{i_d})$; ***(ii)*** the maximal price that can exist is the Ramsey monopoly price ρ_0^m; and ***(iii)*** its associated profit outcome is $G_3(\rho_0^m; i_d)$ since

$$\rho_0^m = \arg\max_{\rho} \left\{ R^{ex}(\rho, i, i_d, z)\big|_{z \in [\hat{z}_{i_d}, \acute{z}_{i_d})} \right\} = \arg\max_{\rho} \{G_3(\rho; i_d)\}.$$

(Figure 6 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

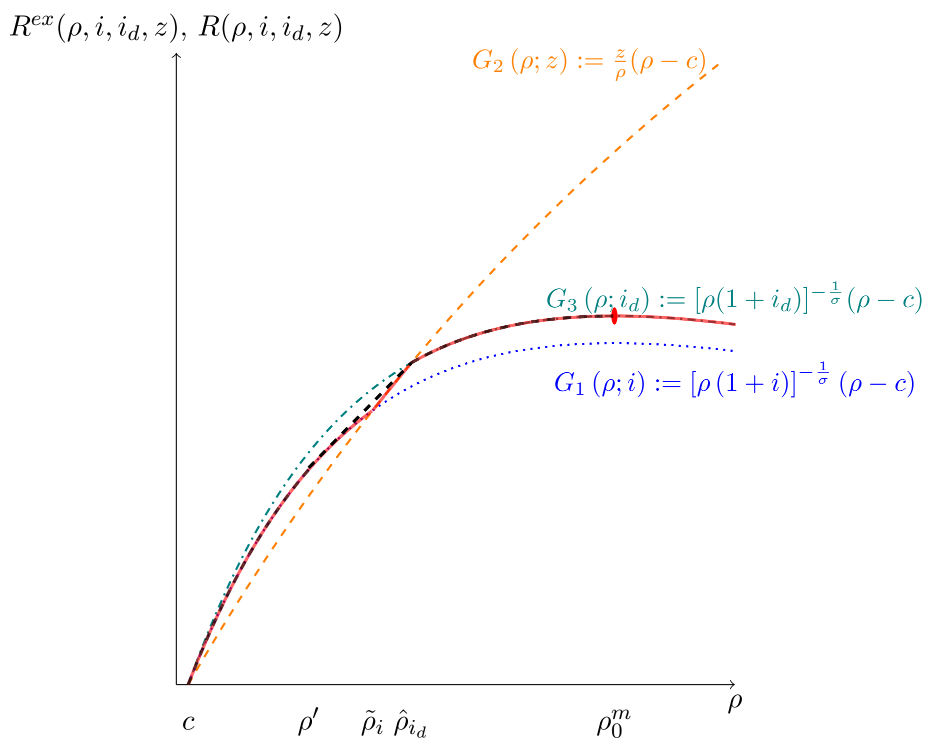

Figure 6: Case 1(b). Real money balance is sufficiently high, $[\hat{z}_{i_d}, \acute{z}_{i_d})$.

Case 2. **Real money balance z is intermediate, $z \in [\mathring{z}_{i,i_d}, \hat{z}_{i_d})$.** There is a cutoff value $\tilde{z}_{i,i_d} := (1+i)^{-\frac{1}{\sigma}}\hat{z}_{i_d}$ such that $\tilde{\rho}_i(\tilde{z}_{i,i_d}) = \rho_0^m$ and $G_2(\tilde{\rho}_i(\tilde{z}_{i,i_d}), \tilde{z}_{i,i_d}) = G_1(\rho_0^m; i)$. Note that $(1+i_d)^{-\frac{1}{\sigma}}\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} \equiv \hat{z}_{i_d} > \tilde{z}_{i,i_d}$. In Case 2(b) below, we will show that there may exist another cutoff $\breve{z}_{i,i_d} \gtreqless \tilde{z}_{i,i_d}$, so that we may define the lower bound z for this case as $\mathring{z}_{i,i_d} = \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$. For now, we assert this as a given.

(a) Consider $z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d})$. At the given z, i_d and i, we have $\tilde{\rho}_i(z) \leq \rho_0^m < \hat{\rho}_{i_d}(z)$ in this case. We can check that $G_2(\rho; z) < G_1(\rho; i)$ for all $\rho \in (c, \tilde{\rho}_i(z))$, $G_2(\rho; z) = G_3(\rho; i_d)$ only if $\rho = c$ or $\rho = \hat{\rho}_{i_d}(z) > \rho_0^m$, and $G_2(\rho; z) > G_3(\rho; i_d)$ for all $\rho > \rho_0^m$. This implies that: ***(i)*** each firm's effective profit function is given by

$$R^{ex}(\rho, i, i_d, z)\big|_{z\in[\tilde{z}_i,\hat{z})} = G_1(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\left\{\tilde{\rho}_i<\rho<\rho_0^m\right\}} + G_3(\rho)\,\mathbf{1}_{\left\{\hat{\rho}_{i_d}\leq\rho\right\}},$$

whenever $z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d})$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}_{i_d}(z)$; and ***(iii)*** its associated profit outcome is $G_3(\hat{\rho}_{i_d}(z); i_d)$ since

$$\hat{\rho}(z) = \arg\max_{\rho}\left\{ R^{ex}(\rho, i, i_d, z)\big|_{z\in[\tilde{z}_i,\hat{z})}\right\}.$$

(Figure 9 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

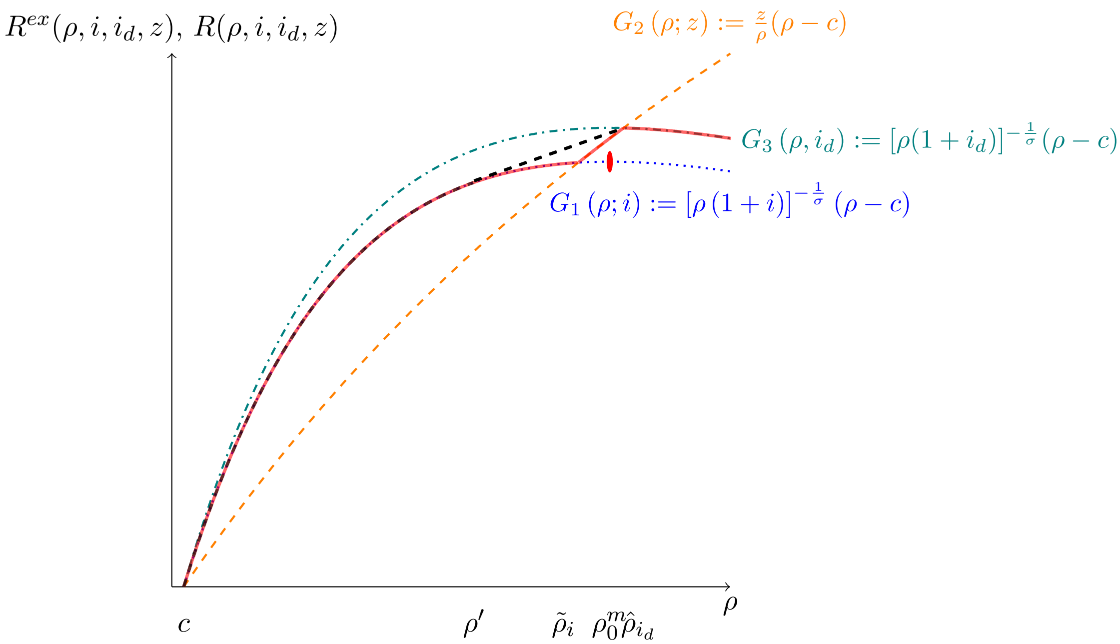

Figure 7: Case 2(a). Intermediate level of real money balance, $z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d})$.

(b) Now we derive the possibility of $\breve{z}_{i,i_d}$, where the case that $z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$. Assume for now that the cutoff $\breve{z}_{i,i_d}$ exists (at least for some z, i_d and i). (It may be that such that $\breve{z}_{i,i_d} \gtreqless \tilde{z}_{i,i_d}$, so a natural lower bound will be $\mathring{z}_{i,i_d} = \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$.) From Property 2 and the previous case, we can also deduce that there can be a $z \geq \mathring{z}_{i,i_d}$ where graph $(G_2(\cdot; z))$ and graph $(G_3(\cdot; i_d))$ uniquely intersect at $\rho = \hat{\rho}_{i_d}(z) > \rho_0^m$ and $G_3(\hat{\rho}(z); i_d) > G_1(\rho_0^m; i)$. Following from the last case, we can see that when $z < \tilde{z}_{i,i_d}$, then $\tilde{\rho}_i(z) > \rho_0^m$, and from Properties 1 and 4 it must be that $G_2(\tilde{\rho}_i(z), z) < G_1(\rho_0^m; i)$. However, in this sub-case, $G_2(\hat{\rho}(z), z) \geq G_1(\rho_0^m; i)$ and $c < \rho_0^m < \tilde{\rho}_i(z) < \hat{\rho}_{i_d}(z)$. Next, we derive the unknown candidate for the lower bound on z, i.e., $\breve{z}_{i,i_d}$ such that $G_2(\hat{\rho}_{i_d}(z), z) \geq G_1(\rho_0^m; i)$ holds. Note that

$$\begin{aligned} & G_2(\hat{\rho}_{i_d}(z), z) \geq G_1(\rho_0^m; i) \\ \Leftrightarrow\ & \frac{z}{\hat{\rho}_{i_d}(z)}\left[\hat{\rho}_{i_d}(z) - c\right] \geq \left[\rho_0^m(1+i)\right]^{-\frac{1}{\sigma}}(\rho_0^m - c) \\ \Leftrightarrow\ & z - c \cdot (1+i_d)^{-\frac{1}{\sigma}} z^{\frac{1}{1-\sigma}} \geq \frac{\sigma}{c}\tilde{z}_{i,i_d}. \end{aligned}$$

Consider the situation where the above inequality just binds. Observe that the RHS term is strictly positive valued, and is constant with respect to z since $\tilde{z}_{i,i_d}$ depends only on parameters, i_d and i (which are fixed). The LHS, has the following properties: (i) $G_2(\hat{\rho}, 0) = 0$, (ii) $\lim_{z\to\infty} G_2(\hat{\rho}, z) = -\infty$, (iii) $G_2(\hat{\rho}, \cdot)$ is "hump shaped": $G_2(\hat{\rho}, \cdot)$ is strictly increasing on $[0, \check{z}_{i_d})$ and strictly decreasing on $[\check{z}_{i_d}, \infty)$, with $\check{z}_{i_d} = (1+i_d)^{-\frac{1}{\sigma}}\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} \equiv \hat{z}_{i_d}$ being the unique maximizer. Thus when $G_2(\hat{\rho}_{i_d}(z), z) = G_1(\rho_0^m; i)$, there can be (i) at most two solutions, (ii) one solution, or (iii) no solution (in particular, if i is too large).

Suppose we have possibility (iii) that there is no solution. Then sub-case 2(b) is moot. Now suppose that possibility (ii) arises. In this case, we have an impossibility too, so again, sub-case 2(b) is non-existent. That leaves possibility (i) where there are two distinct roots, say, z_0 and z_1, where $z_0 < z_1$. Since $G_2(\hat{\rho}_{i_d}(z), z) = G_1(\rho_0^m; i)$ has two roots and given the geometric properties of the LHS and RHS terms, and they cannot be $\hat{z}_{i_d}$, then it can only be possible that $z_0 < \hat{z}_{i_d} < z_1$. However, $z_1 > \hat{z}_{i_d}$ would not be a feasible selection since all $z < \hat{z}_{i_d}$ here. Therefore if there are two distinction solutions, it must always be the smaller of the two, z_0, and $z_0 < \hat{z}_{i_d}$. thus, $z_0 \equiv \breve{z}_{i,i_d}$. Last, it is possible that $\breve{z}_{i,i_d} \gtreqless \tilde{z}_{i,i_d}$. If $\breve{z}_{i,i_d} \geq \tilde{z}_{i,i_d}$, then it must be that $\breve{z}_{i,i_d} = \tilde{z}_{i,i_d}$ (degenerate case). Otherwise there is a unique number $\breve{z}_{i,i_d} < \tilde{z}_{i,i_d}$. Altogher, we can conclude that a natural lower bound for this case is $\mathring{z}_{i,i_d} = \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$.

This implies that: ***(i)*** each firm's effective profit function is given by

$$\begin{aligned} & R^{ex}(\rho, i, i_d, z)| \\ & \qquad = G_1(\rho; i)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho; z)\,\mathbf{1}_{\left\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\right\}} + G_3(\rho; i_d)\,\mathbf{1}_{\left\{\hat{\rho}_{i_d}\leq\rho\right\}}, \end{aligned}$$

whenever $z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}_{i_d}(z)$; and ***(iii)*** its associated profit outcome is $G_3\left(\hat{\rho}\left(z\right);i_d\right)$ since

$$\hat{\rho}_{i_d}\left(z\right)=\arg\max_{\rho}\left\{\left.R^{ex}\left(\rho,i,i_d,z\right)\right|_{z\in\left[\mathring{z}_{i,i_d},\tilde{z}_{i,i_d}\right)}\right\}.$$

(Figure 8 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

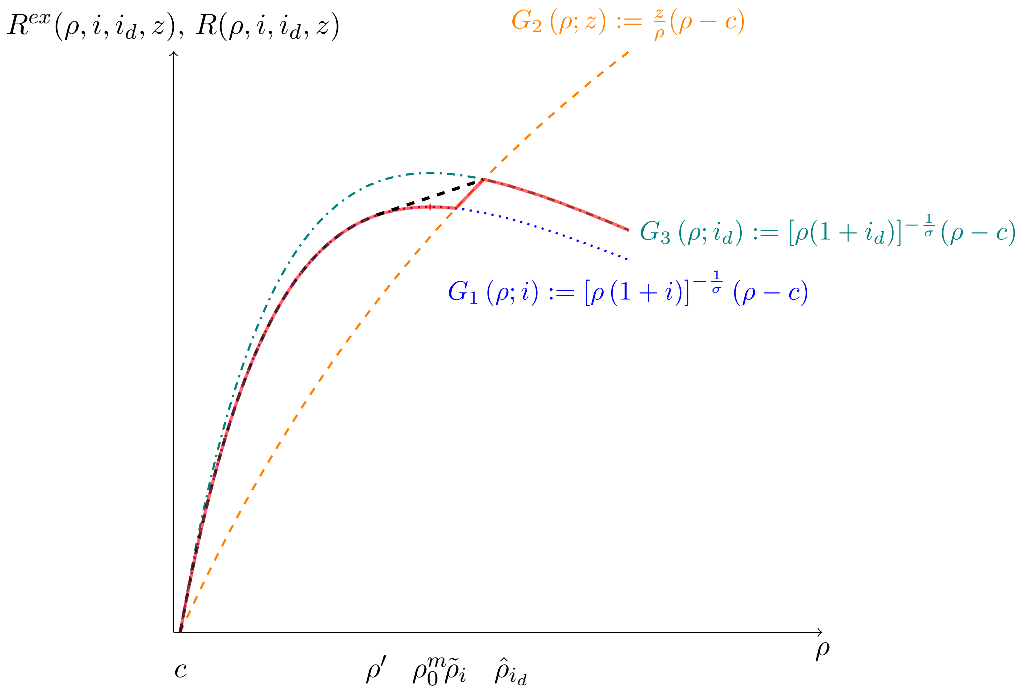

Figure 8: Case (2b). Intermediate level of real money balance, $z\in[\mathring{z}_{i,i_d},\tilde{z}_{i,i_d})$.

Case 3. **Real money balance z is sufficiently low, $z \in (0, \mathring{z}_{i,i_d})$.** From Property 2 and the previous case, we can also deduce that if $z < \mathring{z}_{i,i_d}$, then graph $(G_2(\cdot; z))$ and graph $(G_3(\cdot; i_d))$ uniquely intersect at $\rho = \hat{\rho}_{i_d}(z) > \rho_0^m$ so that $G_3(\hat{\rho}_{i_d}(z); i_d) < G_1(\rho_0^m; i)$. Thus, we have: ***(i)*** the effective profit function is given by

$$R^{ex}(\rho, i, i_d, z)\big|_{z \in (0, \mathring{z}_{i,i_d})} = G_1(\rho; i)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho; z)\,\mathbf{1}_{\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}_{i_d}\}} + G_3(\rho; i_d)\,\mathbf{1}_{\{\hat{\rho}_{i_d}\leq\rho\}};$$

(ii) the maximal price that can exist is the Ramsey monopoly pricing outcome ρ_0^m, and ***(iii)*** its associated profit outcome is $G_1(\rho_0^m; i)$. (Figure 9 illustrates an example of such a generic case where the solid-red line is the ex-post profit function, R^{ex}.)

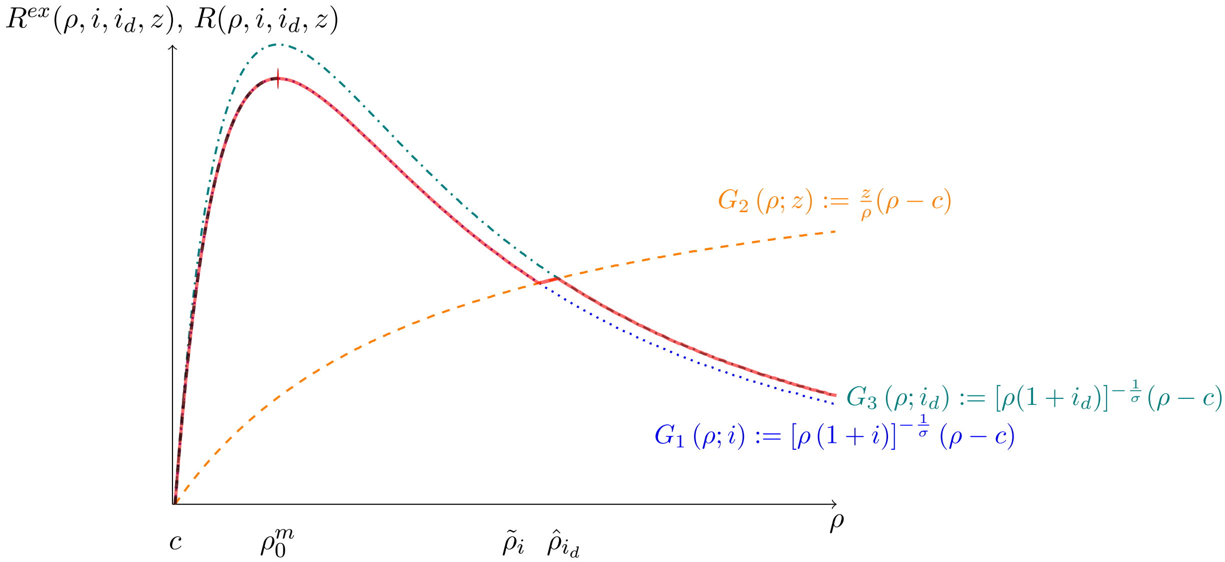

Figure 9: Case 3. Real balance is sufficiently low, $z \in (0, \mathring{z}_{i,i_d})$.

$\square$

B.2 Proof of Lemma 4

In Burdett and Judd (1983) and Head et al. (2012), firms are assumed to commit to implementing the outcomes of their posted (pure) prices. In our extension, as Lemma 4 implies, there may be states of the world where it is (weakly) profitable for firms to commit to posting lotteries over prices (i.e., terms of trade, given buyer demand). Proposition 4 provides the characterization of the *effective (per-trade) profit function* for a firm committed to posting such random contracts. From each buyer's perspective, the possible lotteries are already compounded or internalized in their perceived (and actual) equilibrium distribution of prices $J(\cdot, z, \mathbf{a})$. Thus buyers in equilibrium will be drawing from the distribution $J(\cdot, z, \mathbf{a})$, just as in Burdett and Judd (1983) and Head et al. (2012).

In the proof below, we re-write Lemma 4 in terms of stationary variables.

Lemma (4)**.** *Fix z and aggregate outcomes (i, i_d), and the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, i_d, z)$ in Equation (B.1) in Lemma 3.*

1. *If $i = i_d$, then*

   (a) *a firm's effective profit is the same as its ex-post profit,*

   $$R(\rho, i, i_d, z) = R^{ex}(\rho, i, i_d, z) = [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c),$$

   as all pricing lotteries are degenerate.

   (b) *Depending on the demand for loan and deposit functions in (2.9) and (2.10), firms either serve credit buyers or unconstrained buyer-depositors. There are no money-constrained buyers.*

2. *If $i > i_d$, then we have the following: There exists a set of cut-offs relative to $\hat{z}_{i_d}$ (measurable in units of real money balance or the numeraire), with $\acute{z}_{i_d} = \hat{z}_{i_d}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$, $\tilde{z}_{i,i_d} := (1+i)^{-\frac{1}{\sigma}}\hat{z}_{i_d}$ and an endogeous $\breve{z}_{i,i_d}$ defining $\mathring{z}_{i,i_d} := \min\{\breve{z}_{i,i_d}, \tilde{z}_{i,i_d}\}$, such that the cut-offs have the particular ordering: $0 < \mathring{z}_{i,i_d} \leq \tilde{z}_{i,i_d} \leq \hat{z}_{i_d} < \acute{z}_{i_d} < \infty$. A firm's* ***effective profit*** *at any given reference price ρ, $R(\rho, i, i_d, z)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

   $$R(\rho, i, i_d, z) = \max_{\pi\in[0,1],\rho_1,\rho_2}\left\{\pi R^{ex}(\rho_1, i, i_d, z) + (1-\pi)R^{ex}(\rho_2, i, i_d, z) : \pi\rho_1 + (1-\pi)\rho_2 = \rho\right\}. \tag{B.5}$$

   (a) *The function $R(\cdot, i, i_d, z)$ is strictly increasing on $[c, \rho^m)$, and is concave over the firm's effective domain of pricing outcomes $[c, \rho^m]$, where the (real) monopoly price and its effective profit outcome, respectively, are*

   $$\rho^m = \begin{cases} \rho_0^m, \\ \rho_0^m, \\ \hat{\rho}_{i_d}(z), \\ \hat{\rho}_{i_d}(z), \\ \rho_0^m, \end{cases} \quad \text{and, } R(\rho^m, i, i_d z) = R^{ex}(\rho^m, i, i_d z) = \begin{cases} G_3(\rho_0^m; i_d), & z \in [\acute{z}_{i_d}, \infty) \\ G_3(\rho_0^m; i_d), & z \in [\hat{z}_{i_d}, \acute{z}_{i_d}) \\ G_3(\hat{\rho}(z); i_d), & z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d}) \\ G_3(\hat{\rho}(z); i_d), & z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d}) \\ G_1(\rho_0^m; i), & z \in (0, \mathring{z}_{i,i_d}) \end{cases}. \tag{B.6}$$

   (b) *Depending on z the largest domain containing equlibrium pricing outcomes has the following properties:*

Case-1(a). *If $z \in [\acute{z}_{i_d}, \infty)$, firms will only serve money-unconstrained buyers (who also deposit any residual money balance).*

Case-1(b). *If $z \in [\hat{z}_{i_d}, \acute{z}_{i_d})$, all three types—credit, money-constrained and money-unconstrained buyers—will be served.*

Case-2(a). *If $z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d})$, only two types—credit and money-constrained buyers—will be served.*

Case-2(b). *If $z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d})$ (and i is such that this set is non-degenerate), then only two types—credit and money-constrained buyers—will be served.*

Case-3. *If $z \in (0, \mathring{z}_{i,i_d})$, then only credit buyers are served.*

Proof. We first prove the case of $i = i_d$. Then we consider $i > i_d$.

Case of $i = i_d$. The demand for q is given by Equation (2.8).

Part (a): As shown in the proof of Lemma 3, this is a smooth, downward-sloping function of the relative price ρ, for fixed i. The ex-post profit is thus $G_1(\rho; i) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$. It is easy to check that this profit function is strictly concave and there is a unique maximizer for G_1, i.e., $\arg\max_\rho G_1(\rho; i) = \rho_0^m$.

Part (b): Since the profit function in this case is strictly concave, any non-degenerate lottery about a given pure pricing outcome, ρ, would yield a strictly inferior expected profit to that under $G_1(\rho; i)$ itself. From the demand for loan and deposit functions in (2.9) and (2.10), respectively, firms that end up charging below a cutoff price of $\tilde{\rho}_i$ will serve buyers who optimally take out a loan. Firms that end up charging a price of at least $\tilde{\rho}_i$ will serve buyers who optimally do not spend all their money balance and who deposit the remainder with a bank.

Case of $i > i_d$. The demand for q is given by Equation (2.8). This is a smooth, downward-sloping function of the relative price ρ, for fixed i. The ex-post profit is thus $G_1(\rho; i) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$. It is easy to check that there is a unique maximizer for G_1, i.e., $\arg\max_\rho G_1(\rho; i) = \rho_0^m$.

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1. We also have shown that the unique maximizer ρ^* for the ex-post profit function $R^{ex}(\cdot, i, i_d, z)$ exists. Moreover, the maximum value $R^{ex}(\rho^*, i, i_d, z)$ only arises at the upper bound of the feasible-pricing domain $[c, \rho^m]$, i.e., $\rho^* = \rho^m$, and ρ^m is characterized by Equation (B.2). By definition of the lottery problem in (B.5), it is immediate that $R(\rho, i, i_d, z) = R^{ex}(\rho, i, i_d, z)$ if there is no neighborhood $[\rho_1, \rho_2]$ containing ρ, such that $\pi R^{ex}(\rho_1, i, i_d, z) + (1-\pi) R^{ex}(\rho_2, i, i_d, z) > R^{ex}(\rho, i, i_d, z)$. That is, any lottery would be (locally) degenerate whenever $R^{ex}(\cdot, i, i_d, z)$ is already strictly concave on any such subdomains $[\rho_1, \rho_2]$. Otherwise, $R(\rho, i, i_d, z)$ is given by the right-hand-side operator in Equation (B.5). Since $R^{ex}(\cdot, i, i_d, z)$ has a minimum at c and a unique maximum at $R^{ex}(\rho^m, i, i_d, z)$, then the convexification through (B.5) implies that $R(\rho, i, i_d, z)$ is strictly increasing in $\rho \in [c, \rho^m)$ and it is concave over $[c, \rho^m]$.

2. Finally, the types of buyers that will be served were also enumerated in the (the proof of each case) in Lemma 4.

□

B.2.1 Efficient representation of the effective profit function

Given aggregate outcomes (i, i_d, z), we can define

$$\text{co}R^{ex}_{i,i_d,z} = \text{co}\left\{(\rho, R^{ex}(\rho, i, i_d, z)) : \rho \in [c, \rho^m]\right\},$$

i.e., the convex hull of the graph of $R^{ex}(\cdot, i, i_d, z)$ restricted to the feasible pricing domain of $[c, \rho^m]$, where R^{ex} is defined in (B.1) and ρ^m is governed by (B.6). The set of points in $\text{co}R^{ex}_{i,i_d,z}$ other than those in the set Graph $\{R(\cdot, i, i_d, z)\}$ can be defined in advance as

$$U_{i,i_d,z} := \text{int}\left(\text{co}R^{ex}_{i,z}\right) \cup \text{int}\left\{(\rho^m, r) : r \in [0, R(\rho^m, i, z)]\right\},$$

where $R(\rho^m, i, i_d, z)$ is pinned down by (B.6). The effective profit function in (B.5) can be equivalently represented as

$$\text{Graph}\{R(\cdot, i, i_d, z)\} = \text{co}R^{ex}_{i,i_d,z} \setminus U_{i,i_d,z}. \tag{B.7}$$

This equivalent representation of (B.5) will be computationally convenient since the function domain is closed and bounded, and its graph is always convex. What this means is that open-source and industry-standard convex-hull algorithms, in combination with shape-preserving spline approximants and set-valued logical operations can be employed to represent (B.7) precisely, efficiently, and continuously. Crucially, we can avoid having to solve the brute-force optimization in the representation (B.5), and its associated tangent-search problem (which in practice would involve imprecise discretized approximations). Moreover, using shape-preserving splines as the basis of an approximant to the final endogenous object $R(\cdot, i, i_d, z)$ allows us to efficiently compute its derivative. Its derivative with respect to ρ is used in composing the density representation of the equilibrium pricing distribution, $\text{d}J(\cdot, z, \mathbf{a})$.

B.3 Proof of Proposition 1

Proof. Consider the two cases:

1. **The reserve requirement binds,** $i_r < \bar{i}_r \equiv i$**.** First, we show that $i_d > i_r$ if and only if $i > i_r$. Second, we show that $i > i_r$ if and only if $i > i_d$. Together, these give the interest rate ordering $i > i_d > i_r$. Recall the interest rate pricing condition from Equation (2.30a), which equates marginal revenue and marginal cost of deposits under competitive banking: $i_d = (1-\chi)i + \chi i_r$. Subtracting i_r from both sides gives:

   $$i_d - i_r = (1-\chi)i + \chi i_r - i_r \implies i_d - i_r = (1-\chi)[i - i_r].$$

   Since $1 - \chi > 0$, it follows that $i_d > i_r$ if and only if $i > i_r$. This establishes the first step. Second, from the pricing condition (2.30a), we can rearrange to obtain

   $$i - i_d = \chi(i - i_r).$$

   Since $\chi > 0$, it then follows that $i > i_d$ if and only if $i > i_r$. Thus, $i > i_d > i_r$ in equilibrium if and only if the reserve requirement is binding.

2. **The reserve requirement is slack,** $i_r \geq \bar{i}_r \equiv i$**.** Recall $L + r = D$ (from the balance sheet identity) and $r > \chi D$ (under slack reserve requirement) in this case. Then, we have $(1-\chi)D > L$.

First, we rewrite the bank's profit using the balance sheet identity, we get

$$\pi^{bank} = iL + i_r r - i_d D = (i - i_r)L + (i_r - i_d)D.$$

Next, given the pricing condition, we have $i_d = (1-\chi)i + \chi i_r$, then substitute i_d into the last interest term above, we can rewrite it as

$$i_r - i_d = i_r - (1-\chi)i - \chi i_r = (1-\chi)(i_r - i).$$

Substituting this into the bank's profit, we get

$$\begin{aligned}\pi^{bank} &= (i - i_r)L + [(1-\chi)(i_r - i)]D \\ &= (i - i_r)[L - (1-\chi)D].\end{aligned}$$

Notice that the slack reserve requirement induces $L - (1-\chi)D < 0$. If $i < i_r$, then $\pi^{bank} > 0$. If $i > i_r$, then $\pi^{bank} < 0$. Both violate the zero profit condition. Thus, the only feasible solution (in equilibrium where the reserve requirement is slack) is $i = i_r$, which also implies $i_d = (1-\chi)i + \chi i_r = i$. So, $\bar{i}_r = i = i_d = i_r$ must hold.

□

Remark. Even though there is a wedge $i > i_d > i_r$ when the reserve requirement binds, the zero profit condition still holds in equilibrium. Recall that $L + r = D$ (from the balance sheet identity) and $r = \chi D$ (from the binding reserve requirement) in this case. Then $L = (1-\chi)D$. From the interest rate pricing condition in Equation (2.30a), we can rewrite the loan rate as

$$i = \frac{i_d - \chi i_r}{1-\chi}.$$

The bank's profit can then be rewritten as

$$\begin{aligned}\pi^{bank} &= iL + i_r r - i_d D \\ &= \frac{i_d - \chi i_r}{1-\chi}(1-\chi)D + i_r \chi D - i_d D \\ &= i_d D - \chi i_r D + i_r \chi D - i_d D = 0.\end{aligned}$$

If the reserve requirement is slack, $i_r \geq \bar{i}_r$. Interest on reserve is more attractive, so banks will hold more reserves than those required by the government. In this case, the equilibrium loan rate, reserve rate, and deposit rate must be equal to each other. Otherwise, the bank can allocate more resources to the asset that yields a higher return. Under competitive banking, the zero profit condition requires the marginal benefit of deposits to equal their marginal cost. Thus, $i = i_d = i_r$ has to hold.

B.4 Proof of Lemma 7

In Section B.4, we study how the price distribution $J(\cdot, z, \mathbf{s})$ changes with respect to the asset position of the households. We then establish the existence of a stationary monetary equilibrium with both money and credit in Section B.5. Fix the trend inflation rate away from the Friedman rule $\tau > \beta - 1$. Assume

$\alpha_1 \in (0,1)$. By Lemma 5, the analytical formula for the real price distribution $J(\cdot, z, \mathbf{s})$ is given by

$$J(\rho, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, z, \mathbf{s})}{R(\rho, z, \mathbf{s})} - 1\right] \tag{B.8}$$

where the upper bound on the support of the distribution $J(\cdot, z, \mathbf{s})$ is determined by (3.2), repeated here as:

$$\bar{\rho} := \rho^m(i, z) = \begin{cases} \rho_0^m, & z \in [\acute{z}_{i_d}, \infty) \\ \rho_0^m, & z \in [\hat{z}_{i_d}, \acute{z}_{i_d}) \\ \hat{\rho}_{i_d}(z), & z \in [\tilde{z}_{i,i_d}, \hat{z}_{i_d}) \\ \hat{\rho}_{i_d}(z), & z \in [\mathring{z}_{i,i_d}, \tilde{z}_{i,i_d}) \\ \rho_0^m, & z \in (0, \mathring{z}_{i,i_d}) \end{cases} . \tag{B.9}$$

and the lower bound on the support of $J(\cdot, z, \mathbf{s})$, $\underline{\rho}$, solves $R(\rho, z, \mathbf{s}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{\rho}, z, \mathbf{s})$.

In this proof, we want to show the relationship of how the price distribution $J(\cdot, z, \mathbf{s})$ changes with respect to the change in the real money holdings z. Note: For the ease of notation, we will denote $\overline{\rho}(z)$ and $\underline{\rho}(z)$ respectively by $\overline{\rho}$ and $\underline{\rho}$ occasionally. Likewise, we denote the cut-off prices by

$$\hat{\rho}_{i_d} := \hat{\rho}_{i_d}(z) = \hat{\rho}(z)(1 + i_d)^{\frac{1}{\sigma - 1}} \tag{B.10}$$

and

$$\tilde{\rho}_i := \tilde{\rho}_i(z, i) = \hat{\rho}(z)(1 + i)^{\frac{1}{\sigma - 1}}, \tag{B.11}$$

where $\hat{\rho}(z) = z^{\frac{\sigma}{\sigma - 1}}$. Recall that the CRRA risk aversion is $\sigma < 1$, and from the result established earlier in Section B.1, we then have the following order:

$$\underline{\rho} < \tilde{\rho}_i < \hat{\rho}_{i_d} \leq \overline{\rho}.$$

Proof. Consider two real money holdings z and $z^{'}$ such that $0 < z < z^{'} < \overline{z}$. We want to determine whether $J(\cdot, z, \mathbf{s})$ is lying on top or below for z relative to $z^{'}$.

Observe from the upper bound of the price distribution B.9, there are only two possible cases: either $\overline{\rho} = \hat{\rho}$ or $\overline{\rho} = c/(1 - \sigma)$.

Case 1. Suppose $\overline{\rho} = \hat{\rho}_{i_d}(z)$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$.

Consider any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$. First, it is clear that $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. Using this result and the equal profit condition of the firms, we can then deduce the lower support also satisfies that $\underline{\rho}(z_0) > \underline{\rho}(z_1)$. Thus, we have $[\overline{\rho}(z_0) - \underline{\rho}(z_0)] - [\overline{\rho}(z_1) - \underline{\rho}(z_1)] > 0$. In words, the support of the price distribution with lower real money balance is wider than that with higher real money balance.

Second, for $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$, then $J(\rho, z_0, \mathbf{s}) < J_i(\rho, z_1, \mathbf{s})$ because of $\overline{\rho}(z_0) > \overline{\rho}(z_1)$ and from Lemmata 4 and 5 we have that the function $R(\cdot, z, \mathbf{s})$ is strictly increasing on the equilibrium domain of prices, and thus $J(\cdot, z_1, \mathbf{s})$ is a non-decreasing function. That is, buyers with lower money holdings are more likely to be liquidity constrained, and that pushes up the measure of firms posting higher prices. Thus, $J(\rho, z_0, \mathbf{s})$ falls below $J(\rho, z_1, \mathbf{s})$ for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$.

Next, by the fact that the price distribution $J(\cdot, z, \mathbf{s})$ is a cumulative distribution function, it then follows that $J(\rho, z_0, \mathbf{s}) = J(\rho, z_1, \mathbf{s}) = 1$ for some $\rho \geq \overline{\rho}(z_0)$. Likewise, we have $J(\rho, z_0, \mathbf{s}) = J(\rho, z_1, \mathbf{s}) = 0$ for some $\rho \leq \overline{\rho}(z_1)$.

Therefore $J(\rho, z_0, \mathbf{s})$ first-order stochastically dominates $J(\rho, z_1, \mathbf{s})$. That is, $J(\rho, z_0, \mathbf{s}) \leq J(\rho, z_1, \mathbf{s})$ within the interval $[\underline{\rho}(z_1), \overline{\rho}(z_0)]$, and strict inequality for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$ given any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$.

Case 2. Suppose $\overline{\rho} = c/(1-\sigma)$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \hat{\rho}_{i_d}(z) < \overline{\rho}$. The reasoning for this case is similar to Case 1 above. The only difference is that the upper support of the price distribution is independent of real money holding z, i.e., $\overline{\rho}(z_0) = \overline{\rho}(z_1)$ where $z_0 \neq z_1$. However, we can deduce the following order for the cut-off prices, $\tilde{\rho}_i(z_0) > \tilde{\rho}_i(z_1)$ and $\hat{\rho}_{i_d}(z_0) > \hat{\rho}_{i_d}(z_1)$, and the lower bound of the support of $J(\cdot, z, \mathbf{s})$ satisfies $\underline{\rho}(z_0) > \underline{\rho}(z_1)$ given any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < \overline{z}$. For $\rho \in (\tilde{\rho}_i(z_1), \hat{\rho}_{i_d}(z_0))$, then $J(\rho, z_0, \mathbf{s}) < J(\rho, z_1, \mathbf{s})$ because $z_0 < z_1$. Hence, $J(\cdot, z_0, \mathbf{s})$ first order stochastically dominates $J(\cdot, z_1, \mathbf{s})$ given two real money holdings z_0, z_1 such that $0 < z_0 < z_1 < \overline{z}$.

Finally, from (B.10) and (B.11), since $\sigma < 1$, we can deduce that the maximal admissible price draw, respectively, for a money-constrained buyer ($\hat{\rho}$) and for a credit-buyer ($\tilde{\rho}_i$) are decreasing functions of z. Hence as z falls, these pricing cutoff functions increase in value. □

B.5 Proof of Proposition 3

Proposition (3)**.** *There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

Proof. We suppress the notation of aggregate dependency by writing $\hat{J}(\cdot, z) \equiv \hat{J}(\cdot, z, \mathbf{s})$. Such an equilibrium requires finding a fixed point in two numbers, the SME real money balance and the competitive loan/deposit interest, $z^\star$ and $i^\star$.

Pure-money SME. If Case 1(a) of Lemma 4 were to emerge, the maximal willingness to pay, $\hat{\rho}(z)$, for money-constrained agents is below all firms' marginal cost c. Also, since $\tilde{\rho}_i(z) < \hat{\rho}(z) < c$, then no credit- nor money-constrained buyers will be served by firms. Since there is no demand for loans, banks will not take on any deposit liabilities, and so no loans and deposits are traded. The only class of active buyers are the money-constrained agents. In this case, by Lemmata 4 and 5, the distribution of prices $\hat{J}(\cdot, z)$ (is independent of both z and i), and its support will be $\left[\underline{\rho}, \rho_0^m\right] \subseteq [c, \rho_0^m]$, since in this case, $\rho_0^m = \frac{c}{1-\sigma}$, and by the equal-expected-profit condition (3.3), it will be immediate that $\underline{\rho}$ is also independent of z and i. That is, the SME reduces to a special characterization

$$\frac{\gamma}{\beta} - 1 := (1-n)i. \tag{B.12}$$

Thus this special case of the money-demand Euler equation does not determine the level of z. However, the RHS of (B.12) concerns the marginal benefit of being able to deposit with banks. Although there are no loans to be made so there are no deposits to be taken by banks (i.e., the loan market clearing condition is redundant) we can still price the competitive deposit rate (and hence loan rate) as i solving (B.12). The rest of the equilibrium system in 1 *sans* the credit market clearing condition, when evaluated under Case 1(a) of Lemma 4, is independent of z. Thus any z satisfying the condition for Case 1(a) of Lemma 4 constitutes a pure-money SME.

Mixed money-and-credit SME. For all other possible cases that could emerge as an SME—i.e., Cases 1(b), 2(a)-(b) and 3 in Lemma 4—we know that the distribution $J(\cdot, z, \mathbf{s})$ will always have some positive measure over money- and credit-buyers. In these cases, the generic characterization in Definition 1 applies. We have a fixed point problem in (z, i, i_d). Consider the solution for z, which is determined by the Euler condition (3.6) and (3.7). This can be re-written, using $\mathrm{d}H(\rho, z, \mathbf{s}) := \mathrm{d}\Big(1 - [1 - J(\rho, z, \mathbf{s})]^2\Big) = 2\alpha_2(1 - J(\rho, z, \mathbf{s}))\mathrm{d}J(\rho,$ as:

$$\frac{\gamma}{\beta} - 1 = \mathcal{R}(z, \mathbf{s}),$$

where $\mathcal{R}(z, \mathbf{s})$ is defined in the paper.

First, consider the right-hand-side (benefit of holding money) terms, $\mathcal{R}(z, \mathbf{s})$. Consider any two real money holdings z_0 and z_1 such that $0 < z_0 < z_1 < z'$, where z' was defined in Lemma 3. The result in Lemma 7 establishes that $J(\cdot, z_0, \mathbf{s})$ first-order stochastically dominates $J(\cdot, z_1, \mathbf{s})$, and consequently, $1 - [1 - J(\cdot, z_0, \mathbf{s})]^2$ also first-order stochastically dominates $1 - [1 - J(\cdot, z_1, \mathbf{s})]^2$. Also, the marginal utility of consumption (of money-constrained agents) terms, $u_q(z/\rho)$, are diminising in z. Thus, $\mathcal{R}(z, \mathbf{s})$ is monotone decreasing in z. The left-hand-side term, $\gamma/\beta - 1$, is a constant with respect to z.

Second, if $z \searrow 0$ (denoted as $z = 0^+$), then $\hat{\rho}_{i_d}(z) \nearrow \hat{\rho}_{i_d}(0^+) := \min\{\bar{\rho}, \infty\}$ and $\tilde{\rho}_i(z) \nearrow \hat{\rho}_{i_d}(0^+)$, so that the domain of the integrals $[\hat{\rho}_{i_d}(0^+), \bar{\rho}] \to \{\bar{\rho}\}$. This implies that the measure over this singleton set $\{\bar{\rho}\}$ is zero, and so the last two lines of $\mathcal{R}(z, \mathbf{s}) = \mathcal{R}(0^+, \mathbf{s})$ converges to zero, and

$$\mathcal{R}(0^+, \mathbf{s}) = (1 - n)i_d + I\left(0^+\right),$$

where $I(0^+) := n\alpha_1 \left[\int_{\underline{\rho}}^{\bar{\rho}} i\omega(\rho, z, \mathbf{s})\,\mathrm{d}J(\rho, 0^+, \mathbf{s}) + \alpha_2 \int_{\underline{\rho}}^{\bar{\rho}} i\omega(\rho, z, \mathbf{s})\,\mathrm{d}H(\rho, 0^+, \mathbf{s})\right]$. If $z \nearrow \infty$ (denoted as $z = \infty^-$), then $\hat{\rho}_{i_d}(z) \searrow \max\{\underline{\rho}, 0\} = \underline{\rho} > 0$. Likewise, $\tilde{\rho}_i(z) \searrow \underline{\rho}$. In this limit,

$$\mathcal{R}\left(\infty^-, \mathbf{s}\right) = (1 - n)i_d + I\left(\infty^-\right),$$

where $I(\infty^-) := n\left[\alpha_1 \int_{\underline{\rho}}^{\bar{\rho}} i\omega(\rho, z, \mathbf{s})\,\mathrm{d}J(\rho, \infty^-, \mathbf{s}) + \alpha_2 \int_{\underline{\rho}}^{\bar{\rho}} i\omega(\rho, z, \mathbf{s})\,\mathrm{d}H(\rho, \infty^-, \mathbf{s})\right]$. Observe that $I(0^+) > I(\infty^-)$ by again, an application of the first-order stochastic dominance result from Lemma 7 and the first argument above. Thus, if the cost of holding money net of the expected benefit of being an ex-post inactive agent is neither too small nor too large, i.e.,

$$I\left(\infty^-\right) < \frac{\gamma}{\beta} - 1 - (1 - n)i_d < I(0^+),$$

then there exists a unique solution for z, for a given i_d.

Next, consider the determination of i, for a given candidate z and aggregate state $\mathbf{s}$. Recall, loan demand at a given price draw $\rho \in \left[\underline{\rho}, \tilde{\rho}_i\right]$ is given by the function of i: $\xi(z, \rho, \mathbf{s}) = \rho^{1-\frac{1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z$. Loan demand has the following properties:

1. $\xi(z, \rho, \mathbf{s})$ is strictly decreasing in i.
2. $\lim_{i \to \infty} \xi(z, \rho, \mathbf{s}) = 0$
3. $\lim_{i \to 0} \xi(z, \rho, \mathbf{s}) = \rho^{1-\frac{1}{\sigma}} - z > 0$, so long as there is demand for credit.

Let

$$L^d(i; z) := n \int_{\underline{\rho}}^{\tilde{\rho}_i} \left[\alpha_1 + 2\alpha_2 (1 - J(\rho, z, \mathbf{s}))\right] \xi(i; \rho) \mathrm{d}J_i(\rho, z)$$

be the aggregate demand for loans. The aggregate supply of loans (deposits) is

$$L^s(i;z) := (1-\chi)\left[(1-n)z + n\int_{\hat{\rho}_{i_d}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J(\rho,z,\mathbf{s}))\right]\left(z-\rho^{1-\frac{1}{\sigma}}\right)\mathrm{d}J(\rho,z,\mathbf{s})\right].$$

Thus, the excess demand for loans is:

$$\begin{aligned}
e(i;z) &= L^d(i;z) - L^s(i;z) \\
&= -(1-n)(1-\chi)z + n\int_{\underline{\rho}}^{\tilde{\rho}_i} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J(\rho,z,\mathbf{s}))\right]\xi(z,\rho,\mathbf{s})\mathrm{d}J(\rho,z,\mathbf{s}) \\
&\qquad - (1-\chi)n\int_{\hat{\rho}_{i_d}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J(\rho,z,\mathbf{s}))\right]\left[z-\rho^{1-\frac{1}{\sigma}}\right]\mathrm{d}J(\rho,z,\mathbf{s}).
\end{aligned}$$

First, we want to show that the excess demand switches signs at the extreme limits of i. If $i \nearrow \infty$, $e(i;z) \to b := -(1-n)(1-\chi)z < 0$. If $i \searrow i_d \searrow 0$ (denoted as $i = 0^+$), $\tilde{\rho}_i \nearrow \hat{\rho}_{i_d}$, and $e(i;z) \to a(0^+;z)$, where

$$\begin{aligned}
a(0^+;z) &:= -(1-\chi)\left\{(1-n)z - z\left(\underbrace{n\int_{\underline{\rho}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J_{0^+}(\rho,z,\mathbf{s}))\right]\mathrm{d}J_{0^+}(\rho,z,\mathbf{s})}_{=1}\right)\right\} \\
&\qquad + (1-\chi)n\int_{\underline{\rho}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J_{0^+}(\rho,z,\mathbf{s}))\right]\rho^{1-\frac{1}{\sigma}}\mathrm{d}J_{0^+}(\rho,z,\mathbf{s}) \\
&= (1-\chi)\left[-z + n\int_{\underline{\rho}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J_{0^+}(\rho,z,\mathbf{s}))\right]\left(\rho^{1-\frac{1}{\sigma}} - z\right)\mathrm{d}J_{0^+}(\rho,z,\mathbf{s})\right].
\end{aligned}$$

Generically, $a(0;z) \lesseqqgtr 0$. If $n\int_{\underline{\rho}}^{\bar{\rho}} \omega(\rho,z,\mathbf{s})\left[\alpha_1 + 2\alpha_2(1-J_{0^+}(\rho,z,\mathbf{s}))\right]\left(\rho^{1-\frac{1}{\sigma}} - z\right)\mathrm{d}J_{0^+}(\rho,z,\mathbf{s}) > z$, then for fixed z, there is a unique solution for i.

Finally, in both cases above, since $\alpha_1, \alpha_2 \in (0,1)$, then by Lemma 5 the pricing distribution is alway non-degenerate. □

C Statistical calibration of model

We perform the numerical analyses based on the model that is disciplined by calibration to relevant macro-level statistics in the United States.

C.1 Baseline calibration

We interpret one period in the model to be a year. Our calibration strategy is to match the empirical money demand and the firms' average (percentage) markup in the United States.

The aggregate output in the DM is $q_{DM} := n\int_{\underline{\rho}(z,\gamma)}^{\bar{\rho}(z,\gamma)} \rho q_b^\star(\rho,z)\mathrm{d}\hat{J}(\rho,z,\mathbf{s})$. The aggregate output in our economy is given by:

$$Y = q_{DM} + x^\star. \tag{C.1}$$

Given policy $\gamma = 1+\tau$, we measure the model's aggregate (percentage) markup as by the weighted average of percentage markups in both markets in the model:

$$\mu(\gamma) = \omega_{DM} \underbrace{\left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \frac{\rho - c}{c} \mathrm{d}\hat{J}(\rho, z, \mathbf{s}) \right]}_{\mu_{DM}(\gamma)} + (1 - \omega_{DM}) \cdot 0 \equiv \omega_{DM} \mu_{DM}(\gamma), \tag{C.2}$$

where the weight on DM is $\omega_{DM} := q_{DM}/Y$. The gross markup in the CM is unity (or its percentage markup is zero) since firms are perfectly competitive there. Price dispersion (coefficient of variation) is defined as:

$$CV(\gamma) = \frac{1}{\mu(\gamma)} \left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} (\rho - \breve{\rho})^2 \mathrm{d}\hat{J}(\rho, z, \mathbf{s}) \right]^{\frac{1}{2}}, \tag{C.3}$$

where $\breve{\rho} = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho \mathrm{d}\hat{J}(\rho, z, \mathbf{s})$.

We assume a log-utility function in the CM, $U(x) = B\ln(x)$, where B is a scaling parameter that determines the relative importance of CM and DM consumption. With quasi-linear preferences, real CM consumption is determined by $x^\star = (U')^{-1}(B)$. The noisy-search probabilities in the DM are α_1, $\alpha_2 (= 1 - \alpha_1)$, respectively. We normalize the cost of DM production to one ($c = 1$) as in Head et al. (2012). The DM utility function is given by Equation (2.2).

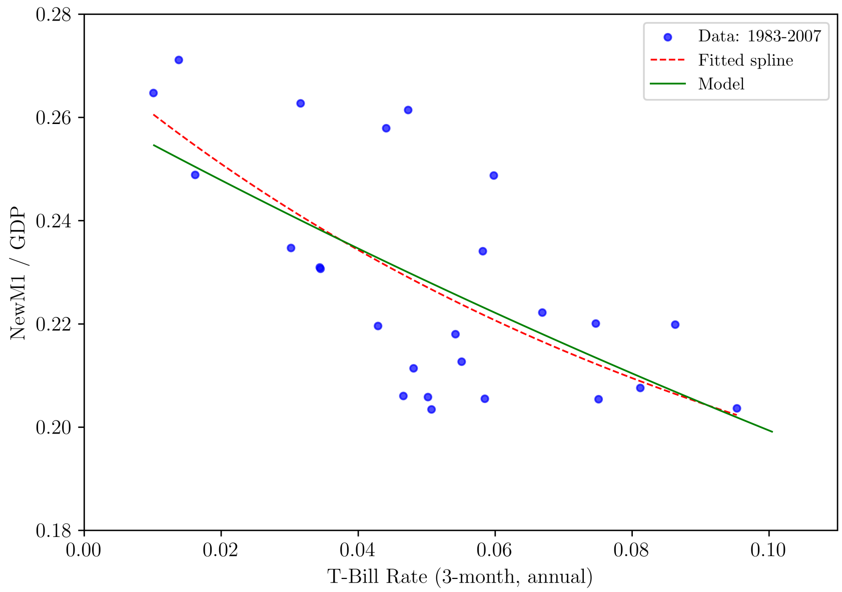
Sample period and data. Our model is fitted to long-run data spanning from 1983 to 2007 to avoid the Great Recession period where the nominal interest rate is at the zero lower bound. We use the New M1-to-GDP ratio defined in Lucas and Nicolini (2015) as a measure of the money demand M/PY in the United States. We employ the U.S. markup data from De Loecker et al. (2020). We obtain the U.S. three-month T-bill interest rate data from the FRED.

Identification and calibration. The parameters that need to determined are: $\beta, \tau, \sigma, \alpha_1, \alpha_2, n, i_r, \chi$, and B. The parameter β is the time discount factor. The CM utility scaling parameter B affects the average of money demand M/PY. This is because the parameter B affects CM consumption x and thus output Y. The CRRA risk aversion parameter σ pins down the price elasticity of demand for the DM consumption goods, which affects the elasticity of money demand with respect to the nominal interest rate i. The noisy-search probabilities directly affect the price distribution J_i, and thus the aggregate markup. From the Fisher equation, we use both the average interest rate of the three-month T-bill, $i = 0.051$, and the long-run inflation rate, $\tau = 0.031$, to pin down the discount factor $\beta = 0.98$. We set the fraction of passive depositors $(1 - n)$ to 0.35, which corresponds to the mean ratio of household deposit accounts at commercial banks per thousand adults in the U.S.[34] We set $i_r = 0$ since no interest on reserves was paid before 2008, as observed in the data series, *Federal Reserve Bank's Interest on Reserve Balances* (also, see Niepelt, 2024).[35] For deposit and reserve data, we use quarterly averages from several FRED series. Checkable deposits are sourced from TCDSL (Total Checkable Deposits, Billions of Dollars, Seasonally Adjusted), while savings deposits utilize SAVINGSL (Savings Deposits - Total, Billions of Dollars, Seasonally Adjusted). For reserves, we employ the quarterly average of RESBALNS (Total

[34] Source: FRED Series USAFCDODCHANUM, "Use of Financial Services—key indicators".

[35] See https://www.federalreserve.gov/monetarypolicy/reserve-balances.htm

Figure 10: Aggregate money demand calibration (result)



Reserve Balances Maintained with Federal Reserve Banks, Billions of Dollars, Not Seasonally Adjusted). The required reserves to transaction balances ratio χ is set to 0.007 for the period 1980-2007.

The remaining parameters $(\sigma, B, \alpha_1, \alpha_2)$ are calibrated internally. We jointly choose $(\sigma, B, \alpha_1, \alpha_2)$ to match the point elasticity of money demand, the average of money demand M/PY, and aggregate markup μ, all of which are with respect to the nominal interest rate i. Figure 10 depicts a reasonable fit between the calibrated model's implied aggregate money demand curve (*i.e.*, the green-solid graph) and that of the data (blue dots). (The data observations are the blue-circled markers and an empirical spline-model best fit of these sample points is given by the dashed-red graph.)

We summarize the value of jointly calibrated parameters and calibration results in Table 1. Given a reasonable fit of our model to the empirical targets, we can use the calibration above as a benchmark model.

Table 1: Calibration targets and results

Parameter	**Value**	**Empirical Targets**	**Model**
σ	0.28	Elasticity of $M/PY = -0.13^a$	-0.14
B	1.8	Mean of $M/PY = 0.22$	0.23
α_1, α_2	0.05,0.95	Markup = 30%	27%

[a] The point elasticity refers to the elasticity of M/PY with respect to the nominal interest rate i, evaluated at the data mean of i.