On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion*

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Abstract

We show that even with idealized competitive banks, banking amplifies retail-goods firms' ability to extract higher markups from ex-post heterogeneous buyers. This works through a new pecuniary-externality channel that is tightly connected to an equilibrium distribution of goods-price markups. Our model generates a positive relationship between the consumer credit-to-GDP ratio and goods-price markups (and their dispersion). This prediction is consistent with empirical evidence using firm-level data in the United States. The endogeneity in firms' markup responses to the presence of credit renders banking not always and everywhere a welfare-enhancing proposition. Consequently, the welfare-improving role of banks as intermediaries that help alleviate individual liquidity risk is ambiguous. Our model also justifies why policymakers should be worried about inflation, banking and its connection to rising industry markups.

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1 Introduction

In this paper, we revisit the question of Berentsen, Camera and Waller (2007) on the *essentiality* of banking. We combine the perfectly competitive banks of Berentsen et al. (2007) with a model featuring endogenous goods market power and an equilibrium distribution of posted prices (Head, Liu, Menzio and Wright, 2012).[1] Both models are steeped in the New Monetarist tradition where crucial market frictions are not assumed but are results of deeper informational and contractual environments. This allows the researcher to study questions such as the existence and essentiality of money, banking, financial markets and asset liquidity as equilibrium objects (see, e.g., Williamson and Wright, 2010; Lagos, Rocheteau and Wright, 2017). By combining these two models, we arrive at an insight that might not be so apparent to conventional wisdom: Even with the best-possible case or idealization of perfect competition among banks, banks need not necessarily be a welfare-improving proposition (c.f., Berentsen, Camera and Waller, 2007). One must also worry about the interaction of the bank interest rate with pricing in goods markets where pricing dispersion is endogenous and sellers have market power.

A new equilibrium trade-off and two testable empirical relations.¶ [NEED TO REVISE THIS PART ... REMOVE THE EMPIRICAL PREDICTIONS? MOVE TO APPENDIX. DOWNPLAY CLAIM: R2?]

Competitive banks play a welfare-improving role of facilitating the insurance of idiosyncratic liquidity risks. This is well understood from the Berentsen et al. (2007) (BCW) microfoundation of why banks are essential institutions: They facilitate the intermediation between those with excess liquidity and those who need more, so long as money has an inferior return to a risk-free outside option (i.e., the economy is away from the Friedman rule).

When we have a Head et al. (2012) (HLMW) kind of economy with an endogenous distribution of goods prices—associated with firms in equilibrium possessing varying market power—even having perfect competition among banks is not always and everywhere a socially desirable ideal. In particular, there is now an intricate balance between the *liquidity-risk insurance* benefit of banks and a *pecuniary externality* of bank credit on non-credit buyers of goods. The latter is a novel channel which runs from the anticipation of bank credit by consumers in their ex-ante money holdings decision, through endogenous markup-pricing dispersion responses of goods sellers, to the likelihood of non-credit buyers ending up with more rent being extracted by the goods sellers.

This equilibrium tension renders a non-monotone *ex-ante* welfare implication for competitive

[1] To be able to compare with this existing literature, we restrict attention the Berentsen et al. (2007) definition of banking. That is, we focus on banks solely as vehicles that take deposits of idle money and provide credit to those who turn out to need it. In an environment where private credit contracts are incentive infeasible banks are essential institutions that enable individuals to insure *idiosyncratic liquidity risks*. We also focus on perfect competition in banking as do Berentsen et al. (2007). Some have suggested that we also incorporate market power in banking. We do not do that here since that will only (quantitatively) deepen the welfare cost of banking without changing the basic insights of this paper. In a different paper, Head, Kam, Ng and Pan (2022) study banking with endogenous market power and pricing dispersion in deposit and lending rates.

banking: At sufficiently low inflation, banks need not be *essential* or welfare improving. That is, when inflation is low, the pecuniary externality caused by banks on goods seller's pricing behavior tends to overpower the liquidity-risk insurance benefit coming from banks. However, when inflation is high enough, the liquidity-risk insurance channel dominates the pecuniary externality effect.

As a by-product, the model implies two testable empirical insights. We apply alternative empirical methods on U.S. data to establish two new pieces of evidence that support the model predictions. First, we find a positive association between goods–price markups and the consumer credit-to-GDP ratio.[2] Second, we show that there is a statistically-significant and positive relationship between the dispersion in price markups and the consumer credit-to-GDP ratio, reinforcing similar finding by Chien, Lee and Lee (2022) that uses alternative methods.

On the new trade-off: Benefit versus cost of bank credit and inflation. The novelty of our paper is as follows. Consider the *benefit* of banking in the model. It comes in two parts. With access to banks, ex-post inactive buyers (those who do not have a trading opportunity) can deposit idle funds with banks to earn interest. In addition, some active buyers (those who have a trading opportunity) may find it optimal to top up their money with bank credit in order to relax their liquidity constraint. In the model, these two forces imply higher consumption and welfare. We call this overall benefit of banking a *liquidity-risk insurance effect*, which is also present in BCW.

However, there is equilibrium feedback from the ability of some agents to use bank loans, to agents' ex-ante decision to hold money, to the distribution of goods-price markups. We call this an opposing *pecuniary externality* (through *pricing dispersion*) *effect*. We show that a first-order stochastic dominance result holds: For a given inflation level, lower equilibrium real money balance implies firms are more likely to exact higher markups on agents who are liquidity-constrained and unconstrained money-buyers. Lower real money balance has a direct effect on money-constrained buyers through tightening their ex-post liquidity constraints. Unconstrained money-buyer also suffer lower consumption as their demands for goods are decreasing in price. Thus, the presence of buyers who find it optimal to borrow from banks create a pecuniary externality through the pricing-markup distribution. This tends to reduce the consumption level for buyers who do not use banking credit.

Unlike BCW, access to bank credit for some agents can create a pecuniary externality cost on others even though there is perfect competition among banks and there are no costs to access banking services. In our model, what is sufficient to induce this externality is the Head et al. (2012)-like goods-price distribution that becomes dependent on consumers' ex-ante money balance decision. In turn, this decision is made in anticipation of the possibility of credit-financed events. In short order, banking can improve welfare for those with idle money or those who are willing to borrow. However by encouraging less own-money holdings, banking also amplifies goods-price

[2]Although we restrict our attention to the United States, we have also tested the relationship between credit and markup using a panel dataset of advanced economies. We also find a positive correlation between bank credit or household debt and markups.

markups' dispersion and average which makes non-credit buyers worse off. This trade-off, as we will show is sensitive to inflation, and thus, to monetary policy.

We discipline the model by calibrating it to the data. We numerically show the following: In contrast to the model without banks (i.e., the HLMW model) average markups under a competitive-banking equilibrium is always higher. Likewise, the dispersion of markups is also higher in the banking equilibrium. The gaps in these measures between the banking equilibrium and the HLMW limit are increasing with inflation. For plausibly low inflation ranges, banking is welfare reducing since for low inflation the gains from banking to depositors of idle money and credit-buyers is small compared to the dispersion effect on non-credit agents. For sufficiently high inflation, the result reverses.

Related literature. Head et al. (2012) and Berentsen et al. (2007) both feature decentralized markets where anonymous agents have the incentive to hold money in order to buy goods.[3] Both models are derived from Lagos and Wright (2005). Berentsen et al. (2007) (BCW) introduced perfectly competitive banks into a Lagos and Wright (2005)-type of model to show that banks are welfare improving institutions or are *essential*, in the sense of liquidity transformation or idiosyncratic liquidity risk reallocation. Moreover, in a variation on their model, BCW also consider a decentralized goods markets where there is a (Nash) bargaining friction that also implied market power among sellers. Nevertheless, in their setting bank credit does not induce any pecuniary externality in goods trade. This is because, ex post, in there is no pricing heterogeneity faced by searching buyers. Thus, in BCW, regardless of whether goods sellers in decentralized trades have market power, banks are shown to fully compensate holders the opportunity cost of idle money in terms of deposit interest. In contrast, we show that when there is equilibrium pricing dispersion under heterogeneous market power in the style of Head et al. (2012), this is no longer true because of its pecuniary externality feedback onto ex-post non-credit buyers.[4] (We provide an analytical, comparative-equilibrium study on this point in Section 3.1 in the paper.)

Head et al. (2012) (HLMW) adapt the consumer search model of Burdett and Judd (1983) to rationalize equilibrium price dispersion that is consistent with well-known facts about price stickiness at the micro-level data on product pricing.[5] Their money-neutral model provided an

[3]Anonymity here is taken to mean that sellers cannot observe buyers histories and any private promises to repay cannot be enforced. Thus, money is essential, i.e., it has value in equilibrium as a medium of exchange, just as in Lagos and Wright (2005).

[4]This has a similar flavor to the insights of Geromichalos and Herrenbrueck (2016). Their model has a liquid asset (money) and an illiquid asset that can be liquidated in a frictional over-the-counter (OTC) secondary asset market. Competitive (c.f., frictional OTC) trade in their secondary asset market may not be efficient because an agent's holding of an additional unit of money insures not just their own consumption shock but also that of buyers of the liquid asset in the secondary asset market. However, agents ignore this positive externality on ex-post secondary-market buyers when they make ex-ante money accumulation decisions. In a related sense, we have the pecuniary externality of bank credit on money-buyers arising in a simpler, one-asset model with perfectly competitive banking.

[5]In our model, as in Head et al. (2012) and Burdett and Judd (1983), firms post prices and produce on the spot. Buyers observe a random number of price quotes posted by firms and buy at the lowest price they observe. This induces firms to optimally trade off between charging a higher markup on their goods and a lower probability

important lesson in the spirit of the Lucas critique: Observed price dispersion and stickiness in micro-level price changes do not necessary imply that monetary policy has real effects through these phenomena. Our combination of HLMW with BCW allows us to arrive at a modified statement about the essentiality of banks. Moreover, it also allows to have an equilibrium causal nexus that runs from monetary policy to banking intermediation, which in turn induces a pecuniary externality on agents' allocations through firm's equilibrium pricing-markups and their dispersion.

Our result on the negative welfare effect of credit is comparable to that established in Chiu, Dong and Shao (2018). The authors also consider a perfectly competitive banking sector, focusing on banking's role in reallocating idle liquidity, as in Berentsen et al. (2007). In their model, access by borrowers to credit raises the *homogeneous* price level of the goods traded in a decentralized market: more demand for goods by credit-buyers raises the marginal cost of production. With competitive price-taking, this translates to a higher goods price in the authors' model. This pecuniary-externality or feedback-on-higher-price effect tightens the liquidity constraint of money-buyers and reduces their consumption. This is also similar to Berentsen, Huber and Marchesiani (2014). Like us, Chiu et al. (2018) show that even under perfectly-competitive goods and banking markets, credit can induce a pecuniary-externality cost on liquidity-constrained money-buyers. However, their result requires the assumption that there is an exogenous measure of money-constrained buyers and the cost of producing the decentralized-market good is strictly convex.

In contrast, we obtain a negative welfare effect of credit through a channel of endogenous firms' market power in goods price markups and dispersion. Also, in our setting, the measures of money-constrained and other agent types are endogenous. Moreover, in our model equilibrium, even unconstrained money-buyers can be affected negatively, since there is not just the one goods price in our model and these agents end up drawing higher prices and consuming less as a result. We shut down the possibility of another pecuniary-externality channel like that of Chiu et al. (2018) by assuming that decentralized-market firms have a linear cost of production. Instead, we identify a new and alternative mechanism for this externality effect. We show that buyers with access to credit can contribute to an increase in the measure of firms charging higher prices and extracting more rent from liquidity constrained money-buyers. Hence, banking can be welfare-reducing in equilibrium.

Dong and Huangfu (2021) present a monetary model in which both money and credit serve as a means of payment. Credit settlement requires money. In their model, the payment instrument involved with money (credit) is subject to the inflation tax (fixed transaction costs). They show that using credit can be welfare-reducing at very low or very high inflation. This is a consequence of having a fixed cost of accessing credit in the model. In contrast, we do not require any cost to accessing bank credit.

of contact by buyers. Equilibrium in the model results in firms being indifferent between a continuum of these opposing margins of attaining the same maximal expected profit. This renders an equilibrium, realized distribution of posted (and transacted) prices that will depend on monetary policy and the aggregate amount of money.

There are few other studies incorporating the noisy search process of Burdett and Judd (1983) into a monetary framework for various applications (see, e.g., Head and Kumar, 2005; Head, Kumar and Lapham, 2010; Chen, 2015; Wang, 2016; Wang, Wright and Liu, 2020). Wang et al. (2020) focus on rationalizing the price-change pattern and cash-credit shares observed at the micro-level data in the United States. In their model, buyers' access to credit is costly, so that money and credit are imperfect substitutes as means of payments. In contrast, agents' access to banking is not restricted in our setup, as in Berentsen et al. (2007), and we are not concerned with the question of competing media of exchange. There is just one medium of exchange (money) in decentralized, anonymous trades. It is possible to introduce costly banking in our model but it would not change the basic message in the paper. Boel and Camera (2020) introduce an operating cost for banks in providing loans which will generate a wedge between the lending and deposit rates.

Recent empirical studies find that industry market power, measured in terms of price markups, has been sharply increasing since the 1980s in the United States (see, e.g., Hall, 2018; Rossi-Hansberg, Sarte and Trachter, 2020; De Loecker, Eeckhout and Unger, 2020). This has prompted a literature that investigates the macroeconomic consequences of industry market power (see, e.g., Guerrieri and Lorenzoni, 2017; Autor, Dorn, Katz, Patterson and Reenen, 2020; Edmond, Midrigan and Xu, 2023). Since the 1980s, the U.S. consumer credit-to-GDP ratio has also been accelerating around the same time as the rise in industry market power. The phenomenon of rising industry market power is not only of interest to academics but also to policymakers. For example, U.S. President Biden has recently called for promoting industry competition in the United States (see Executive Order 14036, 2021). Our study complements this literature by highlighting the unexplored nexus between competitive banking and its effect on goods markup-pricing outcomes.

The remainder of the paper is organized as follows. In Section 2, we lay out the details of the model, agents' decision problems and characterization of a Stationary Monetary Equilibrium (SME). In Section 3, we dissect and discuss the new tension underlying the welfare consequences of banking created the new pecuniary externality from banks, even if they are perfectly competitive banks. We provide a set of numerical illustrations to further expound on the model mechanism. We perform these numerical experiments using the baseline model that is calibrated the U.S. data. In Section 4, we provide empirical evidence to support two key predictions of the model to demonstrate that the model, albeit stylized, has empirical relevance. We conclude in Section 5.

2 Model

The model builds on Head et al. (2012) (HLMW) by introducing perfectly competitive markets for bank deposits and loans. As in Berentsen et al. (2007) (BCW), the focus here is on banks' role in terms of intermediating between ex-post heterogeneous liquidity needs of agents.[6] A novelty in our

[6]BCW and our setting abstract from other aspects or functions of banks such as the undertaking of risky investments or bank equity under capital regulation. Also, this nor BCW is a model about different or competing

model will be in the dependence of market power in the DM-good pricing on the price of credit (c.f., Head et al., 2012). This is because, in equilibrium, there may exist a measure of agents who would take out credit from banks. This renders their demand for loans and the DM good dependent on the nominal loan interest rate (i). Thus, agents should anticipate that the equilibrium DM-good pricing distribution would, in general, depend on i. We ~~then~~ use this framework to study the interaction between banking credit and firms' market power in equilibrium.¶

In every period, two markets open sequentially as in Lagos and Wright (2005). First, a decentralized goods market (DM) with trading frictions opens. In the DM, households are anonymous so that private credit arrangements are incentive infeasible. Consequently, fiat money will be valuable as a medium of exchange in equilibrium. The DM will be the source of fundamental frictions in the model. The DM will be followed by a frictionless centralized market (CM) which allows agents to rebalance their asset positions.

2.1 ~~Timing, markets, agents and some related notation¶~~

~~In the model, time is discrete and infinite. Agents discount across period t and $t+1$ by a common discount factor $\beta \in (0,1)$. We will use variables $X \equiv X_t$ and $X_+ \equiv X_{t+1}$ respectively to denote time-dependent outcomes at period t and $t+1$. There are four types of agents: households, firms, banks, and the government. There is a continuum of households and firms, each of measure one. The banking sector is perfectly competitive with free entry. The government supplies fiat money according to the rule $M_+ = \gamma M$, where $\gamma = 1 + \tau$ is money-supply growth factor and $\gamma \in [\beta, \infty)$. Let the variable $\mathbf{a} := (M, \gamma)$ denote the aggregate state of the economy.¶~~

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~~In what follows, we first describe the model primitives. We then describe the sequence of decentralized and centralized markets in each period. Then, we get into the details of the various decisions problems and characterize an equilibrium.¶~~

~~A novelty in our model will be in the dependence of market power in the DM-good pricing on the price of credit (c.f., Head et al., 2012). This is because, in equilibrium, there may exist a measure of agents who would take out credit from banks. This renders their demand for loans and the DM good dependent on the nominal loan interest rate (i). Thus, agents should anticipate that the equilibrium DM-good pricing distribution would, in general, depend on i.~~

payment instruments. Here, credit is a cash top up on a borrower's money holdings extended by a loan contract from a bank.

2.2 Primitives

Preferences. Each household has their per-period utility described by

$$\mathcal{U}(q, x, h) = u(q) + U(x) - h, \tag{2.1}$$

where $u(q)$ is the utility flow from consumption of the goods in the DM, $U(x)$ is the utility flow of consumption goods x in the CM, and $-h$ captures the disutility of labor.

We assume that $u' > 0, u'' < 0$ and u satisfies the standard Inada conditions. Likewise for the CM utility function U. We restrict our attention to the constant-relative-risk-aversion (CRRA) class of functions:

$$u(q) = \frac{q^{1-\sigma} - 1}{1 - \sigma}. \tag{2.2}$$

The risk aversion coefficient $\sigma > 0$ influences the households' price elasticity of demand.

Technologies. In the CM, the general goods x are produced using a technology that is linear in labor input h. Consequently, both real wage and the price of the general goods will be equal to one. In the DM, firms producing one unit of good q requires $h = c \times q$ hours of labor. The parameter $c > 0$ is the constant marginal cost of DM production.¶

2.3 Timing and events in the sequential DM and CM

In the model, time is discrete and infinite. Agents discount across period t and $t+1$ by a common discount factor $\beta \in (0, 1)$. We will use variables $X \equiv X_t$ and $X_+ \equiv X_{t+1}$ respectively to denote time-dependent outcomes at period t and $t+1$. There are four types of agents: households, firms, banks, and the government. There is a continuum of households and firms, each of measure one. The banking sector is perfectly competitive with free entry. The government supplies fiat money according to the rule $M_+ = \gamma M$, where $\gamma = 1 + \tau$ is money-supply growth factor and $\gamma \in [\beta, \infty)$. Let the variable $\mathbf{a} := (M, \gamma)$ denote the aggregate state of the economy.¶

2.4 ~~Events in the sequential DM and CM~~

Decentralized market.¶

- ¶

- At the start of each period (the DM), each household realizes a preference shock that has two possible outcomes: First, a particular household agent turns out to want to consume (the DM good q). This outcome occurs with probability n, and we label the associated agents as *active buyers*. Second, an agent does not wish to consume, and this occurs with probability $1-n$. We label an agent in such an event an *inactive buyer*.[7]¶

- The banking market is open at this stage as well: All DM agents can access a line of credit and can deposit any amount of money that they possess with perfectly competitive banks. Banks charge borrowers a competitive rate of i, and commit to paying depositors at a perfectly-competitive nominal interest rate of i_d.[8]¶

- After realizing their taste shock, the *active buyers* engage in noisy search for goods sellers. As in HLMW, goods trade is modelled as a monetary-exchange version of the Burdett and Judd (1983) noisy search process: ¶

  - Each DM-goods firm posts a price (p) anticipating that buyers with money holdings m will show up. Firms commit to supplying at their posted prices, taking as given the distribution of all posted prices, $J_i(\cdot, m, \mathbf{a})$, and buyers' demand schedule, q_b.[9]¶
  - Ex ante, buyers know the price distribution but not an individual posted price. Hence, this noisy search process rules out that buyers can direct their search to particular sellers with the lowest price. Instead, buyers randomly contact k number of firms. With probability α_k a buyer matches or makes contact with k firms, or equivalently, draws k price quotes. Each price quote is drawn independently from $J_i(\cdot, m, \mathbf{a})$. For simplicity, we assume that buyers either sample one price quote with probability $\alpha_1 \in [0, 1)$ or two independent price quotes with probability $\alpha_2 = 1 - \alpha_1$. (Some of these buyers may turn out to want to borrow from and some may want to deposit excess liquidity with banks.)¶
  - Given p, the agent decides on how much of a good q to purchase and whether to borrow money from banks in addition to their own money holdings. Given these choices, the agent can also deduce whether they will want to deposit any excess money holdings with banks. Thus, the buyer's choices on consumption in the DM, bank credit and deposit depend on the price drawn from the distribution. The (equilibrium) distribution, in

[7] We retain this notation and assumption from Berentsen et al. (2007) for ease of comparison. We will be able to recover a version of their model as a special case of ours when there is no noisy consumer search in the DM goods market and sellers are Walrasian price takers. We also the assumption regarding banking operations as in Berentsen et al. (2007). First, banks operate a financial record-keeping technology at zero cost. Second, banks can perfectly enforce loan repayments. Moreover, agents having access to banks does not rule out the need for money serving as a medium of exchange in the DM. This is because ex-ante agents demand money as a precaution against probable events where they may turn out to optimally not want to borrow from banks, but they still need money in order to buy goods in anonymous DM-good trades.

[8]

- Clearly, the *inactive buyers* will deposit all their idle monies with the banks. We shall also see that it may be optimal for some *active buyers* to deposit their monies as they may not spend it all on DM goods.

[9] It may be profitable for firms to post lotteries over pure linear pricing strategies. See Footnote 18 on page 19 for further explanation.

turn, depends on agents best-response functions and thus will depend on a given lending rate i.

- The banking market closes immediately after all household agents have completed their loan and/or deposit transactions with the banks. ¶
- Goods exchange occurs between the agents and firms in the DM. Buyers face a liquidity constraint consisting of their own money balances m with (or without) loans l and/or deposits d. Buyers then pay the firms to produce the goods for their consumption.

~~As in HLMW, goods trade is modelled as a Burdett and Judd (1983) noisy search process, save for the fact that exchange is monetary. Each DM-goods firm, anticipating buyers with money holdings m, posts a price, p, and commits to supplying at that price, taking as given the distribution of all posted prices, $J_i(\cdot, m, \mathbf{a})$, and buyers' demand schedule, q_b.~~¶

~~From the buyers' ex-ante point of view, they observe the price distribution but not an individual posted price. Hence, this noisy search process rules out that buyers can direct their search to particular sellers with the lowest price. Instead, buyers randomly contact k number of firms. With probability α_k a buyer matches or makes contact with k firms, or equivalently, draws k price quotes. Each price quote is drawn independently from $J_i(\cdot, m, \mathbf{a})$. For simplicity, we assume that buyers sample zero price quotes with probability α_0, one price quote with probability $\alpha_1 \in [0, 1)$, and two price quotes with probability $\alpha_2 = 1 - \alpha_0 - \alpha_1$. The buyers with zero price quotes are ex-post *inactive* in goods transactions.[10] These inactive buyers have no immediate use for money and thus consider money to be idle. The buyers with at least one price quote are ex-post *active* in purchasing and consuming goods. Some of these agents may turn out to want to borrow from banks.~~

~~Bank deposit-taking and lending activities occur after agents realize their DM types and after ex-post buyers receive price quotes from firms, but before the exchange and production of goods begins. Banks accept nominal deposits from buyers with unproductive idle money balances (i.e., those who have drawn zero price quotes). Banks commit to paying depositors at a perfectly-competitive nominal interest rate of i_d. They then allocate deposits to extend nominal loans to buyers who may need more liquidity at a perfectly-competitive nominal rate, i.[11] We also maintain the assumption regarding banking operations as in Berentsen et al. (2007). First, banks operate a financial record-keeping technology at zero cost. Second, banks can perfectly enforce loan repayments. Moreover, agents having access to banks does not rule out the need for money serving as a medium of exchange in the DM. This is because ex-ante agents demand money as a precaution against probable events where they may turn out to optimally not want to borrow from banks, but they still need money in order to buy goods in anonymous DM-good trades.~~

[10] ~~These agents are just like the n measure of inactive DM buyers, who are also depositors with banks, in BCW's notation.~~

[11] ~~In general, buyers who sample more than zero quotes may or may not borrow additional money balances from banks. Later, we show that their decision on taking out a loan from the bank will depend on the goods price drawn from the distribution.~~

~~At the end of each DM, the banking sector closes, and exchange and production of goods happen. Buyers face a liquidity constraint consisting of their own money balances m with (or without) loans l. Buyers then pay the firms to produce the goods for their consumption.~~

- After the DM, agents enter a frictionless CM. ~~Households trade a general good x, supply labor h, settle financial contracts (redeem deposits or repay loans) and accumulate money balances.~~

Centralized market. An agent entering the CM is denoted by an individual state (m, l, d), i.e., her remaining nominal money balance, outstanding loan and deposit balance. In particular, those who have deposited in the previous DM will earn gross interest $1 + i_d$ on deposits d. Those who have borrowed ~~will need to~~must repay gross interest $1 + i$ on loan l to banks. Households supply labor h to firms for production and consume the general goods x. Households own firms and firms return profits as dividends D to households. Households then accumulate money balances m_+ to carry into the next period.

2.5 Households

In what follows, we work backwards from the CM to the DM within the period t.

2.5.1 Households in the CM

An agent beginning the CM with money, loan or deposit balances, (m, l, d), may have been a borrower or a depositor in the previous DM during the first sub-period. Her initial value is

$$W(m, l, d, \mathbf{a}) = \max_{(x,h,m_+)\in\mathbb{R}^3_+} \left\{ U(x) - h + \beta V(m_+, \mathbf{a}_+) \middle| \begin{array}{c} x + \phi(m_+ - m) = \\ h + D + T + \phi(1 + i_d)d - \phi(1 + i)l \end{array} \right\}, \quad (2.3)$$

where V is the value function at the beginning of the next DM, ϕ is the value of money in units of the CM consumption good x, i_d is the deposit interest rate, i is the loan interest rate, h is labor supplied, D is aggregate dividends from firm ownership and T is the lump-sum taxes/transfers from the government.

The first-order conditions with respect to x and m_+ are, respectively, given by

$$U_x(x) = 1, \quad (2.4)$$

and,

$$\phi = \beta V_m(m_+, \mathbf{a}_+), \quad (2.5)$$

where $V_m(m_+, \mathbf{a}_+)$ captures the marginal value of accumulating an extra unit of money balance taken into the next period $t+1$. The envelope conditions are

$$W_m(m,l,d,\mathbf{a}) = \phi, \quad W_l(m,l,d,\mathbf{a}) = -\phi(1+i), \quad \text{and} \quad W_d(m,l,d,\mathbf{a}) = \phi(1+i_d). \tag{2.6}$$

Note that W is linear in (m,l,d) and the distribution of money balances is degenerate when households exit the CM. As a result, households' optimal choices for CM consumption and money balance are given by Equations (2.4) and (2.5). These equations are independent of the agents' current wealth since per-period preferences are quasilinear.

2.5.2 Households in the DM

We first describe the post-match household problems. We call households who sample at least one price quote in the DM *active buyers*. We label those who sample zero price quotes *inactive buyers*.

Regarding banking arrangements, it is easy to verify that agents who are active buyers will have no incentive to deposit funds with the bank, whereas inactive buyers will never have an incentive to borrow additional funds from banks. As such, we denote l as the amount of loans an active buyer may take out and d as the amount of money deposited by an inactive buyer throughout the paper.

***Ex-post* inactive buyers.** With probability α_0, a household is inactive. Conditional on being inactive, a household with money holdings, m, can deposit $d \leq m$ of this money with a bank. She has zero utility flow of consuming q and then enters the CM with valuation of $W(m-d,0,d,\mathbf{a})$. Since holding money is subject to inflation tax, it will be optimal for *inactive buyers* to deposit all of their money holdings, i.e., $d^\star(m,\mathbf{a}) = m$.

***Ex-post* active buyer sampling at least one price.** The post-match value of such a buyer is given by:

$$B(m,p,\mathbf{a}) = \max_{q,l} \left\{ u(q) + W(0,l,m+l-pq,\mathbf{a}) \middle| \begin{array}{c} pq \leq m+l, \\ 0 \leq l \leq \bar{l} \end{array} \right\}. \tag{2.7}$$

We assume banks can perfectly enforce loans repayment as in the baseline case of Berentsen et al. (2007). Hence, buyers do not face a borrowing constraint, i.e., $\bar{l} = \infty$.[12]

Taking loan interest rate i as given, we can derive the buyer's demand for DM consumption

[12]Within the same period, given no restrictions to depositing money with banks, *any* active buyer will never want to leave the DM with idle money, as that would be subject to inflation tax. Hence the zero value for the first argument in the continuation value in Equation (2.7). That is, whether a buyer is money-constrained or not, or whether they borrow or not, it is always optimal to not carry excess money out of a DM trade when one can deposit that and be insured or compensated by deposit interest. Notice that a separate/explicit choice variable notation d is redundant here since it must be whatever the residual balance amount $m+l-pq$ is. (We present an equivalent representation of this problem in Online Appendix A.)

goods as:

$$q_b^\star(m,p,i,\mathbf{a}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ \frac{m}{p} & \text{if } \tilde{p}_i < p < \hat{p} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases}, \tag{2.8}$$

where

$$\hat{p} := \hat{p}(m,\mathbf{a}) = \phi^{\frac{1}{\sigma-1}} m^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i := \tilde{p}(i,m,\mathbf{a}) = \hat{p}(1+i)^{\frac{1}{\sigma-1}}. \tag{2.9}$$

The cutoff prices $(\hat{p}, \tilde{p}_i)$ are functions of the state of the economy and monetary policy. Assuming $\sigma < 1$, we can order the cut-off prices as: $0 < \tilde{p}_i < \hat{p} < +\infty$. Later on, when calibrated to data, the DM risk aversion coefficient will turn out to be some number $\sigma < 1$.[13]

For a given loan interest rate i, the buyer's loan demand is:

$$l^\star(m,p,i,\mathbf{a}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}[\phi(1+i)]^{-\frac{1}{\sigma}} - m & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p < \hat{p} \\ 0 & \text{if } p \geq \hat{p} \end{cases}. \tag{2.10}$$

From the liquidity constraint in (2.7), along with the optimal consumption and loan demand schedules—i.e., Equations (2.8) and (2.10)—we can also back out the optimal deposit demands of the various types of *active* DM buyers:

$$d^\star(m,p,i,\mathbf{a}) = \begin{cases} 0 & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p < \hat{p} \\ m - p(p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases}. \tag{2.11}$$

Note that the buyer's demands for goods ~~and~~, loans and deposits will depend on price p. If p turns out to be a random variable drawn from a distribution $J_i(\cdot, m, \mathbf{a})$—and it will be in a certain equilibrium—then, we would observe ex-post heterogeneous consumption ~~and~~, loan and deposit outcomes in the DM. When we present the firms' problem, we will be more explicit about characterizing the distribution of prices.

[13] We will find that $\sigma < 1$ when we calibrate the model such that its implied aggregate money demand is close to the historical long-run money demand relation in the United States. (See Online Appendix C for the details.) Unlike in standard neoclassical and related New Keynesian models where often their centralized market preference CRRA coefficient turns out to be at least unity, here σ corresponds to a frictional, search market for goods. Our calibration of $\sigma < 1$ is consistent with similar findings in other related models (e.g., Wang, 2016; Wang et al., 2020; Head et al., 2012). Moreover, this restriction is consistent with the empirical finding in Baker (2018). The author finds that indebted households face a more elastic demand schedule, which is captured by the first case in Equation (2.12).

Equations (2.8), (2.10) and ~~and~~ (2.11), respectively, imply three possible classes of ex-post heterogeneous demands for goods, loans and deposits. Consider the first case in Equation (2.10). If a buyer draws a p that is sufficiently low, then the buyer optimally borrows money from the bank to top up his initial money holdings. Moreover, the buyer spends all his liquid balances, including his money and bank loan. We call this buyer a *borrower* (or sometimes, a constrained *credit-buyer*). In the intermediate case, p is drawn such that $\tilde{p}_i < p < \hat{p}$. In this event, the buyer prefers not to borrow from the bank but rather to spend all her money. In this case, loan size does not matter for goods demand. We call this type of buyer a *liquidity constrained money-buyer*, or, a *money-constrained buyer*. In the last case, p can be sufficiently high. In that case, the buyer prefers not to borrow and also not to spend all her money balance in the frictional goods market. We call this type of buyer a *liquidity unconstrained money-buyer*. Since the bank-deposit facility is available to everyone, agents with unused money can avoid inflation tax by depositing with banks. Here, only the unconstrained money-buyers and *inactive buyers* will optimally want to deposit. In Equation (2.11), we see that a *liquidity constrained money-buyer* will park their residual amount $m - p(p\phi)^{-1/\sigma}$ with banks to earn deposit interest. (Recall that inactive buyers will deposit all of their unused m.)

It is also worth mentioning the price elasticity of demand for the demand schedule $q_b^\star$ described in Equation (2.8). The buyers' price elasticity of demand is given by

$$\left|\frac{\partial q_b^\star(m,p,i,\mathbf{a})}{\partial p}\frac{p}{q_b^\star(m,p,i,\mathbf{a})}\right| = \begin{cases} \frac{1}{\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ 1 & \text{if } \tilde{p} < p < \hat{p} \\ \frac{1}{\sigma} & \text{if } \hat{p} \leq p \end{cases}. \tag{2.12}$$

This will imply that demand is elastic among buyers other than money-constrained buyers.[14] The implication is that such a buyer cannot spend more than his liquidity constraint at low-enough price levels, $p < \hat{p}$. Above the $\hat{p}$ cut-off price level, a buyer's liquidity constraint does not bind and such buyers will always spend less than their total money holding.

Households in the DM *ex-ante*. Now consider the beginning of period t when households are *ex-ante* homogeneous at the start of the DM (i.e., before exchange and production of the goods). Given an individual real money balance, m, and aggregate state, $\mathbf{a} := (M, \gamma)$, the agent's value is

$$\begin{aligned} V(m,\mathbf{a}) &= (1-n)\,W(m-d,0,d,\mathbf{a}) \\ &\quad + n\left\{\alpha_1\int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}J_i(p,m,\mathbf{a}) + \alpha_2\int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}[1-(1-J_i(p,m,\mathbf{a}))^2]\right\}. \end{aligned}$$

[14]Alternatively, we can show that the elasticity of the buyer's expenditure rule $e(p) := pq_b^\star(m)$ is less than one. Then the buyer's expenditure on the DM goods decreases as he faces a higher price p. We omit the details of its derivation here. Instead, we explain more about how banking credit affects buyers' optimal expenditure rule and firms' pricing strategy in Section 3.1.

(2.13)

In contrast to Head et al. (2012), the value of households entering the DM is different due to the availability of banking services.

~~According to~~Consider Equation (2.13). W~~, w~~ith probability $1-n$, a household is *inactive*, i.e., the household ~~samples zero price quotes from firms~~realizes that they have no desire to consume in the DM. ~~They are the ones stuck with idle money balances. Since banks source deposits to issue loans~~In the aggregate, the measure of $1-n$ *inactive buyers* can deposit their idle money balances at the bank to earn interest i_d.[15] ~~However, in a no-bank economy, this measure of households will enter the subsequent CM while holding their idle money balances subject to an inflation tax. In that case, having unneeded money ex-post can be costly since higher inflation induces a lower value of money.¶~~

Conditional on the being an *active buyer*, w~~W~~ith probability α_1 the household gets to contact~~s~~ one firm posting some price p. For the ex-ante household, this p is perceived as being~~,~~ drawn from the distribution $J_i(\cdot, m, \mathbf{a})$. With probability $\alpha_2 = 1-\alpha_1$, the household ~~contacts and randomly~~gets to sample ~~samples~~ two independent price quotes from firms, and the lower of the two prices is~~are~~ drawn from the distribution $1-(1-J_i(\cdot, m, \mathbf{a}))^2$.~~Moreover, conditional on being an active buyer, he can now choose whether or not to borrow additional money from banks to purchase the goods. The buyer's decision on demanding bank credit depends on the price drawn from the distribution and the market loan interest rate i.~~

Marginal value of money.¶ [LAST: 7 am, 2024-09-25]

To simplify notation, we denote the (equilibrium) cut-off pricing functions by $\underline{p} := \underline{p}(m, \mathbf{a})$, $\tilde{p}_i := \tilde{p}_i(i, m, \mathbf{a})$, $\hat{p} := \hat{p}(m, \mathbf{a})$ and $\overline{p} := \overline{p}(m, \mathbf{a})$. Also, we will use these shorthand notation for probability measures: $\varrho_+ := \left[\alpha_1 + 2\alpha_2\left(1 - J_{i,+}(p, m_+, \mathbf{a}_+)\right)\right]$, $J_{i,+} := J_{i,+}(p, m_+, \mathbf{a}_+)$, and $\mathbf{1}_X$ is the Dirac measure on event X. Differentiating~~e~~ Equation (2.13) with respect to m,~~, and update one period~~we have the following expression for the marginal value of money at the start of a DM one period ahead:~~ to get~~

$$V_m(m_+, \mathbf{a}_+) = \phi_+\left[1 + r_+(m_+, \mathbf{a}_+)\right], \tag{2.14a}$$

[15] In contrast, in a no-bank economy, this measure of households will enter the subsequent CM while holding their idle money balances subject to an inflation tax. In that case, having unneeded money ex-post can be costly since higher inflation induces a lower value of money.

where

$$r_+(m_+, \mathbf{a}_+) := (1-n)\, i_d + n \int_{\underline{p}}^{\tilde{p}_i} \{\varrho_+ \cdot i\}\, \mathrm{d}J_{i,+} + n \int_{\tilde{p}_i}^{\bar{p}} \left\{ \varrho_+ \left[\left(\frac{u_q(m_+/p)}{\phi_+ p} \right) \mathbf{1}_{\{p \in (\tilde{p}_i, \hat{p}]\}} + \left(\frac{u_q\left((p\phi_+)^{-1/\sigma}\right)}{\phi_+ p} \right) \mathbf{1}_{\{p \in (\hat{p}, \bar{p}]\}} - 1 \right] \right\} \mathrm{d}J_{i,+} + n \int_{\hat{p}}^{\bar{p}} \{\varrho_+ \cdot i_d\}\, \mathrm{d}J_{i,+}, \tag{2.14b}$$

and we have made use of the optimal demands for goods, loans and deposits characterized in Equations (2.8) to (2.11).

Equation (2.14a) deserves some commentary. Its right-hand side captures the expected benefit from accumulating an extra unit of money balance to be carried into the DM ~~into~~in the next period. The value of one unit of money balance is captured by ϕ_+ (in units of CM goods). Since money serves as a means of payment in the frictional goods market, it has a liquidity premium captured by ~~the function~~the return $1 + r_+(m_+, \mathbf{a}_+)$. Thus, carrying an extra unit of money has a real marginal benefit of $\phi_+ r_+(m_+, \mathbf{a}_+)$.

In contrast to Head et al. (2012), the liquidity premium on holding money in Equation (2.14b) now depends on the banking arrangement. In particular, the premium comprises four possible terms. ¶

- First, if the household ends up not consuming in the next DM, he can deposit his idle money in the bank to earn interest $i_d > 0$. ~~Hence, the marginal value of money is higher with access to banks.~~ In other words, banks play the same intermediation-of-liquidity-needs role as those in Berentsen et al. (2007). ¶

- Second, i~~I~~f the household samples a low enough price, i.e., $p \in \left(\underline{p}, \tilde{p}_i\right]$, he would take out a bank loan. Thus, t~~T~~he second term on the right of Equation (2.14b) captures the expected, marginal interest-payment liability saved by borrowing one less unit of money. ¶

- Third, ~~The~~there is ~~last term captures~~ the net benefit from being able to spend~~ing~~ an extra unit of money on goods, for the case of the ex-post credit buyer or money-constrained buyer because they draw some price $p \in (\tilde{p}_i, \hat{p}] \cup (\hat{p}, \bar{p}]$. ¶

- Last, we have the benefit arising from the possibility that the buyer is ex-post money-unconstrained so that at the margin, the unspent money can still earn interest i_d. ~~i.e., the expected liquidity premium of carrying one's own money as medium of exchange.~~

Substituting Equation (2.14a) into Equation (2.5), we obtain a money demand Euler equation capturing the households' inter-temporal trade-offs:

$$\phi = \beta\phi_{+}[1 + r_{+}(m_{+}, \mathbf{a}_{+})]. \tag{2.15}$$

The left-hand side of Equation (2.15) captures the cost of accumulating money balance: The household forgoes ϕ units of CM consumption goods in order to carry an extra dollar into the next period. The right-hand side of Equation (2.15) is the expected marginal benefit of accumulating an extra dollar associated with the total liquidity premium captured by $r_{+}(m_{+}, \mathbf{a}_{+})$ in Equation (2.14b).

2.6 Firms

Firms in the Decentralized Market (Overview). A unit measure of firms (or sellers of goods) compete in a price posting environment along the lines of Head et al. (2012). ~~In the DM, the firm posts price p to maximize expected profit and commit to supplying goods at that posted price.~~In our setting, firms may commit to posting lotteries over pure pricing strategies. We will describe the possibility of a lottery over pure pricing contracts shortly in Lemma 2. For now, consider the reference to a posted contract p given consumer demand, as either a lottery indexed by its expected value p, or a degenerate lottery (which is a pure pricing-strategy) at outcome p.

Consider a firm posting ~~price~~ p , given their potential customers' demand schedule $q_b^{\star}$ and the distribution of prices posted by firms J_i. Its expected profit is given by

$$\Pi_i(p) = \underbrace{\Big[\alpha_1 + 2\alpha_2(1 - J_i(p, m, \mathbf{a})) + \alpha_2\nu(p)\Big]}_{\text{extensive margin}} \underbrace{R(p, i, m)}_{\text{intensive margin}}, \tag{2.16}$$

where

$$\nu(p) = \lim_{\epsilon \searrow 0} J_i(p, m, \mathbf{a}) - J_i(p - \epsilon, m, \mathbf{a}),$$

and $R(p, i, m) \equiv R(p, i, m, \mathbf{a})$ is the firm's *effective profit* from expecting to price at p. (We will define this profit function in detail in Lemma 2 below.)

$$\cancel{R_i(p) \equiv R(p, i, m, \mathbf{a}) = q_b(p, i, m, \mathbf{a})(\phi p - c).}$$

The first term in parentheses, labeled *extensive margin*, in Equation (2.16) captures the number of buyers served. With probability α_1, the firm trades with a buyer who has only observed one price quote from this firm and no other. With probability $2\alpha_2[1 - J_i(p, m, \mathbf{a})]$, the buyer purchases the good from this firm because he contacts another firm who has posted a higher price than p. The probability $\alpha_2\nu(p)$ is the measure of buyers that match both this firm and another which has posted the same price (or lottery indexed by)~~,~~ p.[16] The last term, labeled *intensive margin*, captures the firm's profit per customer induced by the firm charging a markup, ~~(~~i.e., posting at an (expected) price above the marginal cost, $p > \phi^{-1}c$).

Observe from Equation (2.16), the firm posting p trades off between an *extensive margin* (i.e., the likelihood of trading with buyers) and an *intensive margin* (i.e., profit per buyer). On the one hand, a firm that posts a higher p can earn a higher profit margin per buyer served. However, on the other hand, a firm that posts a higher p suffers by losing sales to other competitors, i.e., a lower likelihood of trading with buyers.

A hypothetical monopolist. As in Burdett and Judd (1983) and Head et al. (2012), we can characterize the distribution of prices $J_i(p, m, \mathbf{a})$, which is an equilibrium object. The lower and upper bound of the distribution's (connected) support will depend on the description of a monopolist's pricing strategy. We provide its characterization here.¶

Consider a firm serving buyers who have only received one price quote from this one firm. In this case, the firm will behave as a monopolist. The realized profit of a firm setting a monopoly price p^m is

$$\Pi_i^m = \alpha_1 R(p^m, i, m). \tag{2.17}$$

¶

~~We will now describe what p^m can be.~~ A subtlety in our extension of Head et al. (2012) here is that banking outcome i will affect some agents who, ex-post, may demand loans. As a result, i will also condition or "shift" their demand for the DM good. This, along with how much money a buyer carries into the trade, has consequences for the calculation of a firm's profit and also for the equilibrium distribution of DM-good prices. Unlike Head et al. (2012), the effective ex-post profit function can be non-concave, depending on how much money balance DM buyers carry into the match. (This is a result of the possibility of bank credit for buyers.) Despite this seeming complexity, our generalization turns out to be very tractable: We prove these attributes (in Online Appendix B.1) and show that in terms of pricing, we end up with the following modified characterization of the monopoly price. ~~It turns out that this implies two possible cases characterizing the monopoly price. The two cases are determined by the relative orderings between the monopoly price when the firm is faced with credit buyers, money-constrained (non-credit) or money-unconstrained buyers.~~

~~We can rewrite the description of the orderings in terms of a cutoff money balance condition, $\breve{m}_i$. First, we can show that the monopolist would charge (money-constrained and money-unconstrained) buyers who are not sensitive to the loan rate a price equal to $\breve{p}^m = \phi^{-1}c/(1-\sigma)$. Second, we define a marginal buyer named $\breve{m}_i$. This buyer corresponds to the money balance such that if the buyer~~

[16] Suppose two firms post the same price. We assume that prospective buyers use a tie-breaking rule to pick one firm in such a case. This rule incentivizes an individual firm to lower the price to get the sale. In equilibrium, the probability of a buyer contacting two firms that post the same price goes to zero.

~~were a credit buyer (one whose demand for the DM good is sensitive to i), then his maximal willingness to pay equals that of other i-insensitive buyers, $\breve{p}^m$. That is, $\tilde{p}_i(\breve{m}_i) = \phi^{-1}c/(1-\sigma)$. Using Equation (2.9), we can show that $\breve{m}_i = \phi^{-1}\left(\frac{c}{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}(1+i)^{-1/\sigma}$. Observe that the outcome i can shift this cutoff $\breve{m}_i$.~~¶

~~We summarize the monopoly-price characterization below and relegate its derivation to the Online Appendix.~~

Lemma 1. ~~*Let $\breve{m}_i = \phi^{-1}\left(\frac{c}{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}(1+i)^{-1/\sigma}$ denote a marginal buyer who is indifferent between taking out a loan and not. The monopoly price is*~~

$$\cancel{p^m = \frac{\phi^{-1}c}{1-\sigma} \equiv p_0^m,} \tag{2.18}$$

~~*where $\hat{p}$ is given in Equation (2.9), $\bar{m}_i = \phi^{-1}(\underline{p})^{\frac{\sigma-1}{\sigma}}(1+i)^{-1/\sigma}$ and $\underline{p}$ is a lower bound on p to be determined in equilibrium.*~~*Let the profit at price p from serving a*¶

1. *credit-buyer be $G_1(p) := [\phi p\,(1+i)]^{-\frac{1}{\sigma}}(\phi p - c)$,* ¶
2. *constrained money-buyer be $G_2\,(p;m) := \frac{m}{p}(\phi p - c)$, and* ¶
3. *unconstrained money-buyer be $G_3\,(p) := (\phi p)^{-\frac{1}{\sigma}}(\phi p - c)$.*¶

Assume that $\sigma \in (0,1)$ and $c > 0$ satisfy $0 < \breve{z} := (1-c) + \sigma\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} < \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}$. For any given value of money ϕ, real money balance $\phi m \in (0,\infty)$ and interest rate on loans $i \in [0,\bar{i}]$ where $\bar{i} := \left[\sigma + (1-c)\left(\frac{c}{1-\sigma}\right)^{-(1-\frac{1}{\sigma})}\right]^{-\sigma} - 1 > 0$, there exists a $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-(1-\frac{1}{\sigma})}$ and $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$ such that $0 < \breve{z} < \tilde{z}_i \leq \hat{z} < z' < \infty$. ¶

1. *The ex-post profit function is*

$$R^{ex}\,(p,i,m) := \begin{cases} G_3\,(p)\,, \qquad p \geq c\phi^{-1} > \hat{p}, & \phi m \in [z',\infty) \\ G_1\,(p)\,\mathbf{1}_{\{c<p\leq\tilde{p}_i\}} + G_2\,(p)\,\mathbf{1}_{\{\tilde{p}_i<p<\hat{p}\}} + G_3\,(\rho)\,\mathbf{1}_{\{\hat{p}\leq p\leq p_0^m\}}, & \phi m \in [\hat{z},z') \\ G_1\,(p)\,\mathbf{1}_{\{c<p\leq\tilde{p}_i\}} + G_2\,(p)\,\mathbf{1}_{\{\tilde{p}_i<p<p_0^m\}} + G_3\,(\rho)\,\mathbf{1}_{\{\hat{p}\leq p\}}, & \phi m \in [\tilde{z}_i,\hat{z}) \\ G_1\,(p)\,\mathbf{1}_{\{c<p\leq\tilde{p}_i\}} + G_2\,(p)\,\mathbf{1}_{\{p_0^m<\tilde{p}_i<p<\hat{p}\}} + G_3\,(\rho)\,\mathbf{1}_{\{\hat{p}\leq p\}}, & \phi m \in (0,\breve{z}) \cup [\breve{z},\tilde{z}_i) \end{cases},$$

where $p_0^m = \phi^{-1}c/(1-\sigma)$, $\hat{p} \equiv \hat{p}(m) = \phi^{\frac{1}{\sigma-1}}m^{\frac{\sigma}{\sigma-1}}$, $\tilde{p}_i \equiv \tilde{p}(i,m) = \hat{p}(1+i)^{\frac{1}{\sigma-1}}$, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.¶

2. *The (real) monopoly price and its ex-post profit outcome, respectively, are*

$$p^m = \begin{cases} p_0^m, & \phi m \in [z', \infty) \\ p_0^m, & \phi m \in [\hat{z}, z') \\ \hat{p}(z), & \phi m \in [\tilde{z}_i, \hat{z}) \\ \hat{p}(z), & \phi m \in [\breve{z}, \tilde{z}_i) \\ p_0^m, & \phi m \in (0, \breve{z}) \end{cases}, \text{ and, } R^{ex}(p^m, i, m) = \begin{cases} G_3(p_0^m), & \phi m \in [z', \infty) \quad \text{(Case 1)} \\ G_3(p_0^m), & \phi m \in [\hat{z}, z') \quad \text{(Case 2)} \\ G_3(\hat{p}(m)), & \phi m \in [\tilde{z}_i, \hat{z}) \quad \text{(Case 3)} \\ G_3(\hat{p}(m)), & \phi m \in [\breve{z}, \tilde{z}_i) \quad \text{(Case 4)} \\ G_1(p_0^m), & \phi m \in (0, \breve{z}) \quad \text{(Case 5)} \end{cases}. \tag{2.19}$$

The set of parameters (σ, c) satisfying the inequalities in Lemma 1 is non-empty. For example, if $c = 1$ as in Head et al. (2012), then the sufficient conditions on parameters reduce to $0 < \sigma\left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}} < \left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, which admits any $\sigma \in (0, 1)$.¶

As a consequence of the possibility of earning profit from credit buyers the ex-post per-meeting profit function of firms (in terms of pure pricing strategies) may exhibit segments that are not necessarily monotone increasing up to the monopoly profit point. Also, the profit function may not be strictly concave.[17] Technically, this may pose a problem for the characterization of the equilibrium Burdett-Judd style pricing distribution, as it depends on the monotonicity of $R^{ex}(p, i, m)$ in p. Economically, this also suggests that it may be profitable in some subset(s) of the set of pricing outcomes for firms to be posting random terms of trades—i.e., lotteries over the pure pricing outcomes.[18]¶

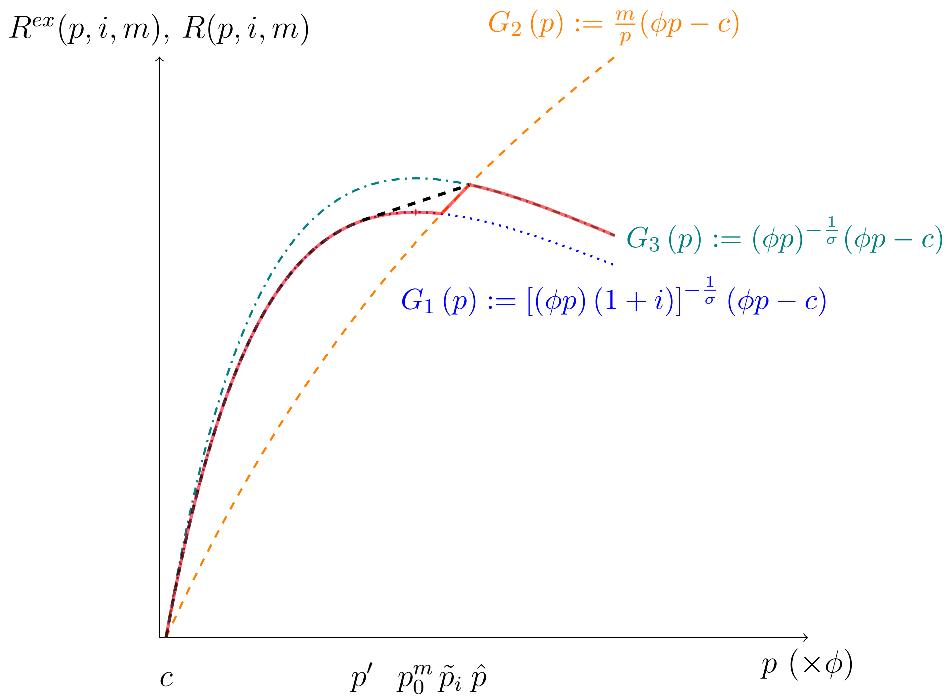
Figure 1 illustrates the fourth generic case from Lemma 1, i.e., where real money balance is low enough $\phi m \in [\breve{z}, \tilde{z}_i)$. In this case, the maximal profit from serving cash-constrained buyers and pricing at such buyers' maximal willingness to pay, $\hat{p}(z)$, can exceed the maximal profits from serving either credit-buyers or unconstrained money buyers. The dashed-black graph in the figure is the convexification of a firm's ex-post profit function via lotteries over pure price posts. (See the proof of Lemma 1 for a complete characterization and graphical illustrations for all the possible

[17]Such non-monotonicities and non-convexities can be interpreted as an artefact of the externality of credit buyers on firms' profitability calculations. We shall see in equilibrium, it is possible (depending on policy and parameters) that such an externality can have a net negative impact on aggregate welfare. In Head et al., 2012, there is no such technical complication since buyers are either cash constrained or not, and there is no feedback between banking and goods-firms pricing.

[18]The idea of posting random contracts already exists in monetary trade, in general equilibrium, or in labor economics (respectively, Berentsen, Molico and Wright, 2002; Chatterjee and Corbae, 1995; Shell and Wright, 1993; Rogerson, 1988), in settings with private information (Prescott and Townsend, 1984) and in models of dynamic price discrimination (Sobel, 1984). In our application, the presence of bank credit and the possibility of demand for goods from buyers with bank credit may render local non-concavity and possibly local non-concave-and-convex segments in firm's ex-post profit functions. This possible non-strict-concavity of firms' ex-post profit function does not arise in the monetary setting of Head et al. (2012) where their firms' ex-post profit functions are always strictly concave. We will see that such lotteries can yield firms a weakly welfare-improving payoff, but one that is still no greater than the hypothetical Burdett and Judd (1983) monopolist in any setting. The use of lotteries will also ensure a well-behaved equilibrium characterization of the distribution function of pricing outcomes.

cases.) We will refer to a profit function induced by the posting of random contracts (which is relevant for our equilibrium description) as an *effective profit* function.¶

Figure 1: Example from Case 4 in Lemma 1 where $z \in [\breve{z}, \tilde{z}_i)$. Posting of random contracts is profitable for firms. This yields a strictly increasing and concanve *effective profit function* $R(\cdot, i, m)$ (dashed-black graph) on the relevant domain $[c, \phi\hat{p}]$. The ex-post profit function $R^{ex}(\cdot, i, m)$ under pure pricing strategies is illustrated by the solid-red line (whose graph exhibits non-convexity).



In the next result, we show that for all possible cases, the effective profit function is monotone increasing and concave in pricing outcomes, so long as firms are allowed to post lotteries over their pricing contracts. We relegate the detailed proof to Online Appendix B.2.

Lemma 2. *Given aggregate outcomes (m, i, ϕ), and the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, m)$ in Equation* (B.1) *in Lemma 1, there exists a $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$ and $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ and $\breve{z} := (1-c) + \sigma\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, such that $0 < \breve{z} < \tilde{z}_i \leq \hat{z} < z' < \infty$. A firm's* ***effective profit*** *at any given reference price p, $R(p, i, m)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

$$R(p, i, m) = \max_{\pi \in [0,1], p_1, p_2} \left\{\pi R^{ex}(p_1, i, m) + (1-\pi) R^{ex}(p_2, i, m) : \pi p_1 + (1-\pi)p_2 = p\right\}. \quad (2.20)$$

The function $R(\cdot, i, m)$ is strictly increasing on $[\phi^{-1}c, p^m)$, and is concave over the firm's effective domain of pricing outcomes $[\phi^{-1}c, p^m]$, where the monopoly price is p^m and its effective profit *outcome is* $R(p^m, i, m) = R^{ex}(p^m, i, m)$ and these are characterized in (2.19).

Pricing equilibrium. Previewing an equilibrium, firms will earn the same expected profit for any p in the support of the distribution, $\text{supp}(J_i(\cdot, m, \mathbf{a})) = [\underline{p}, \overline{p}]$. That is, they will be indifferent between a continuum of different extensive-intensive margin trade-offs~~. The intuition is that~~: ~~lower price firms win on sales volume while higher price firms gain through the profit or markup channel. That is,~~

$$\Pi_i^\star = \max_p \Pi_i(p) \quad \text{for all } p \in \text{supp}(J_i(\cdot, m, \mathbf{a})). \tag{2.21}$$

Lower price firms make up their profit through higher sales volume while higher price firms gain through higher markups.¶

As in Head et al. (2012), if some buyers observe only one price quote whereas others observe more than one, then this leads to a non-degenerate distribution of prices $J_i(\cdot, m, \boldsymbol{a})$.[19] Since firms expect the same profit outcomes associated with the continuum of markup-versus-trading-probability strategies, then this implies an equal-profit condition. Specifically, equating Equation (2.16) and Equation (2.17), we can derive a closed-form distribution of prices. We summarize this result ~~in the following Lemma~~as follows.[20]

Lemma 3. *Given monetary policy* $\gamma > \beta$, ~~-~~ *aggregate outcomes* (m, i, ϕ), *and noisy search frictions* $\alpha_1, \alpha_2 \in (0, 1)$, *the price distribution consistent with profit maximization by all firms is given by*

$$J_i(p, m, \mathbf{a}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R_i(\overline{p}, i, m, \mathbf{a})}{R_i(p, i, m, \mathbf{a})} - 1\right], \tag{2.22}$$

where the lower and upper bounds on the support of $J_i(\cdot, m, \boldsymbol{a})$ *are, respectively, determined by* $R_i(\underline{p}) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R_i(\overline{p})$ *where~~and~~* $c < \underline{p} < \overline{p} := p^m$, *and* p^m *is governed by Equation* (2.19).~~:~~

$$\cancel{\overline{p} := p^m = \begin{cases} \frac{\phi^{-1}c}{1-\sigma} & \text{if } m \leq \breve{m}_i, \\ \max\{\frac{\phi^{-1}c}{1-\sigma}, \hat{p}\}, & \text{if } m > \breve{m}_i \end{cases},} \tag{2.23}$$

~~*where*~~ $\cancel{\breve{m}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]/\phi.}$

Lemma 3 highlights that if some buyers receive only one price (lottery) quote while others receive more than two, the price distribution is continuous and non-degenerate. Moreover, firms ~~can exploit market power in the~~are ex post ~~goods market by~~ pricing the goods above the marginal cost of production. In contrast to Head et al. (2012), the banking loan interest rate i now matters for determining the ~~(monopoly) profit-maximizing price~~good-price distribution~~, as shown in Equation~~

[19]The model has two parametric limits: one with Bertrand pricing (by setting $\alpha_2 = 1$) and one that resembles monopoly (by setting $\alpha_1 = 1$). For our purposes, we focus on cases away from these two parametric limits to rationalize the empirical finding in Section 4.

[20]Given Lemma 2, t~~T~~he proof of Lemma 3 follows directly from Head et al. (2012). We omit the details here.

~~(2.23)~~. This is a consequence of the buyer's optimal goods demand schedule interacting with credit, which affects the firm's pricing strategy.

Firms in the Centralized Market. In the CM, there is a unit measure of perfectly competitive firms producing the general goods x using a linear production technology in labor h. They then sell the goods to households in the CM. Consequently, both the real wage and price of the DM goods are equal to one.

2.7 Banks

We focus on the liquidity transformation role of banks. The banking sector is perfectly competitive with free entry as in Berentsen et al. (2007). In particular, banks accept deposits d and commit to repaying depositors with interest i_d. Banks then allocate deposits to issue loans l at the interest rate of i to borrowers.[21]

2.8 Stationary monetary equilibrium

We focus on stationary outcomes of the economy. Since the price of the general goods P is used as a unit of account, we then multiply all nominal variables by the value of money balance $\phi = 1/P$ (in units of the CM goods x) from here onward. In particular, we let $z = \phi m$ denote the individual real money balance and $Z = \phi M$ denote the aggregate real money balances; $\rho = \phi p$ denote the real relative price of goods across the DM and the CM; and $\xi = \phi l$ and $\delta = \phi d$ respectively denote the real balances of loans and deposits. For the ease of notation, we also let the variable $\mathbf{s} := (Z, \gamma)$ denote the aggregate state of the economy consisting of total real money stock and monetary policy $\gamma = 1 + \tau$. In a stationary equilibrium, all nominal variables grow at a time-invariant rate according to $\phi/\phi_{+1} = M_{+1}/M = \gamma = 1 + \tau$ and real variables stay constant over time.

Before we provide a summary of the equilibrium characterization, we first present two features that are different in contrast to Head et al. (2012) as follows.

In a stationary monetary equilibrium (SME), the real analog of the price distribution characterized in Lemma 3 is given by:

$$J_i(\rho, z) := J_i(\rho, i, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z)}{R(\rho, i, z)} - 1\right] = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{q_b^\star(\overline{\rho}, i, z)(\overline{\rho} - c)}{q_b^\star(\rho, i, z)(\rho - c)} - 1\right], \quad (2.24)$$

[21] In the equilibrium characterization below, we shall see that the deposit rate will be bid up to the loan rate, $i = i_d = i^\star$, and $i^\star$ is determined by a loan-market-clearing condition where there is perfect competition and free entry. We have assumed that there are no operating costs or reserve requirements in the banking industry. If we relax this assumption, there will be a wedge between the loan rate and the deposit rate. Since we want to focus on the dependency of firms' market power on banking, it suffices to study the case where even perfectly competitive banking can exarcebate goods markup pricing outcomes. One can think of this as a lower-bound case on the severity this problem: If we also make the banking sector non-competitive, our qualitative conclusions would remain but the effects of this new nexus would only be magnified quantitatively.

where the upper support of the distribution $J_i(\rho, z)$ is determined by:

$$\overline{\rho} := \overline{\rho}(z, \mathbf{s}) = \begin{cases} \frac{c}{1-\sigma} & \text{if } z \leq \breve{z}_i \\ \max\{c/(1-\sigma), \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}\} & \text{if } z > \breve{z}_i \end{cases}, \tag{2.25}$$

given $\breve{z}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]$, and the lower support of $J_i(\cdot, z)$, $\underline{\rho}$, solves $R(\rho, i, z) = \frac{\alpha_1}{\alpha_1+2\alpha_2} R(\overline{\rho}, i, z)$.

Observe that in Equation (2.24), the distribution of goods (real) prices now depends on both households' real money holdings z and the competitive loan interest rate $i = i_d = i^\star$ (determined by the loans market clearing condition). Consequently, there are two possible cases regarding the households' optimal demand for real money balances in an SME. We summarize this possibility in the following Lemma.

Lemma 4. *Let monetary policy be $\gamma > \beta$ and assume that $\alpha_1, \alpha_2 \in (0, 1)$. Equation (2.15) expressed in real terms, characterizes households' ex-ante demand for real money balances:*

$$\begin{aligned} \frac{\gamma - \beta}{\beta} &= \alpha_0 i_d + \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} i\Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] dJ_i(\rho, z) \\ &\quad + \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] \left(\frac{u_q[q^\star(z)]}{\rho} - 1\right) dJ_i(\rho, z). \end{aligned} \tag{2.26}$$

Given a competitive loan market interest rate $i = i_d = i^\star$, there exists a cut-off value $\breve{z}_i$ such that $0 < \breve{z}_i = [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}] < 1$. There are two possible cases:

1. *If $z \leq \breve{z}_i$, the real-money-demand characterization in Equation (2.26) reduces to:*

$$\frac{\gamma - \beta}{\beta} = \alpha_0 i_d + \int_{\underline{\rho}(z)}^{\overline{\rho}(z)} i\Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] dJ_i(\rho, z) = i, \tag{2.27}$$

   where $\overline{\rho}(z) = c/(1-\sigma)$, and for all price $\rho \in supp\,(J_i(\cdot, z)) = [\underline{\rho}(z), \overline{\rho}(z)]$ satisfies: $\rho \leq \tilde{\rho}_i(z) = z^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}$.

2. *If $\breve{z}_i < z$, real money demand z satisfies (2.26), where $\overline{\rho}(z) = \max\{c/(1-\sigma), \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}\}$, and the ordering of the support of the distribution, $J_i(\cdot, z)$, satisfies: $\underline{\rho}(z) < \tilde{\rho}_i(z) = z^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}} < \overline{\rho}(z)$.*

Lemma 4 reveals the inter-dependency of agent's (ex-ante) precautionary money demand on an endogenous channel between bank credit and non-competitiveness in the DM for goods. The market interest rate, $i = i_d = i^\star$, determines the credit condition. The distribution of goods prices, $J_i(\cdot, z)$, pins down the degree of firms' market power in the goods market.

The first case in Lemma 4 is equivalent to the equilibrium condition for the agent's real money demand decision as in Berentsen et al. (2007). From the agent's ex-ante point of view, he is indifferent between borrowing at the competitive market interest rate and carrying a sufficient amount of real money balances to trade in the following period (DM). In this case, all prices in the equilibrium support of the price distribution will be weakly lower than the agent's maximum willingness to borrow, i.e., for all $\rho \in \text{supp}\,(J_i(\cdot, z)) = [\underline{\rho}(z), \overline{\rho}(z)]$ satisfying that $\rho \leq \tilde{\rho}_i(z)$. Given the agent's optimal goods (and loans) demand schedule, we can verify that there will only be credit–buyers (using both their money balances and bank loans) in ex-post trades. This result is consistent with the firms' pricing strategy. They have no incentive to post a price higher than $\overline{\rho} = c/(1-\sigma)$. Any price higher than $c/(1-\sigma)$ is an off-equilibrium price that makes firms worse off with lower revenue.

The second case in Lemma 4 is one where there will be an ex-post mixture of credit–buyers and money–buyers in equilibrium. The former are those buyers who draw a sufficiently low price, i.e., $\underline{\rho} \leq \rho \leq \tilde{\rho}_i(z)$. The latter occurs when buyers draw a sufficiently high price, i.e., $\tilde{\rho}_i(z) < \rho \leq \overline{\rho}(z)$. The implication is that an endogenous channel works through the connection between the credit condition and firms' market power. In equilibrium, this channel matters for the agent's (ex-ante) precautionary demand incentives regarding how much real money balances to carry to trade in the following period.

Since the cut-off value $\breve{z}_i$ is endogenous to the competitive banking market outcome, i, then we have a theoretically-possible case where there is no pecuniary externality running from credit buyers to money constrained buyers. Money demand in this setting (our first case in Lemma 4) turns out to be identical to that of Berentsen et al. (2007).

However, when calibrated to the data later, we shall see that the second case in Lemma 4 will be the relevant case—and this is also the more interesting one. In this case, the pecuniary externality issue is present. Also, this equilibrium case will always occur for plausible experiments around the empirically calibrated model. Hence, in the remainder of the paper, we focus on the second case in Lemma 4. Next, we summarize the equilibrium characterization and provide further discussions.

Definition 1. Given monetary policy $\gamma \geq \beta$, and taxes/transfers T, a stationary monetary equilibrium co-existing with money and credit in real variables is a steady-state allocation $(z^\star, x^\star, h^\star)$ in the centralized market, decision rules $\{q_b^\star(\rho, z), \xi^\star(\rho, i, z)\}$ in the decentralized market and prices $(J_i^\star(\rho), i)$ such that the following conditions are satisfied:

1. The triple $(h^\star, x^\star, z^\star)$ solves the CM households problem, including the households' ex-ante real money demand decision in Equation (2.26).

(a) Given $z = z^{\star}$, both $\{q_b^{\star}(\rho, z), \xi^{\star}(\rho, i, z)\}$ satisfy:

$$q_b^{\star}(\rho, i, z) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ \frac{z}{\rho} & \text{if } \tilde{\rho}_i < \rho < \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \rho \geq \hat{\rho} \end{cases}, \tag{2.28}$$

and,

$$\xi^{\star}(\rho, i, z) = \begin{cases} \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i < \rho < \hat{\rho} \\ 0 & \text{if } \rho \geq \hat{\rho} \end{cases}, \tag{2.29}$$

where

$$\hat{\rho} \equiv \hat{\rho}(z, \mathbf{s}) = z^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{\rho}_i \equiv \tilde{\rho}_i(i, z, \mathbf{s}) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}. \tag{2.30}$$

(b) $J_i^{\star}(\cdot, z)$ solves the DM firms' problem characterized in Equation (2.24).

(c) Given $z = z^{\star}$, $i = i_d = i^{\star}$ clears the loan market:

$$\alpha_0 z = \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} (\alpha_1 + 2\alpha_2 - 2\alpha_2 J_i(\rho, z)) \cdot \xi^{\star}(\rho, i, z) \mathrm{d}J_i(\rho, z). \tag{2.31}$$

We can back out all the other endogenous variables by solving the money demand Euler Equation (2.26). The left-hand side of Equation (2.26) captures the opportunity cost of carrying one extra unit of money into the next period. The right-hand side of Equation (2.26) represents the expected net return of holding money that can be decomposed into three terms. The first term reflects the marginal benefit of depositing an extra unit of idle money balances. The second term captures the interest saved by borrowing one less unit of money balances. The last term is the net marginal benefit of spending an extra dollar.

The following observation says that a buyer with a lower real money balance is more likely to draw a higher price from the distribution.

Lemma 5. *Fix a monetary policy at $\gamma > \beta$ and assume $\alpha_1, \alpha_2 \in (0, 1)$. Given $i = i_d = i^{\star} > 0$ and $\breve{z}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}] > 0$, consider any two real money balances z and $z^{'}$ such that $\breve{z}_i < z < z^{'}$. The price distribution $J_i(\cdot, z)$ first-order stochastically dominates $J_i(\cdot, z^{'})$. Also, the pricing cutoffs $\tilde{\rho}_i$ (lowest price draw admissible for a constrained money-buyer) and $\hat{\rho}$ (lowest price draw admissible for an unconstrained money buyer) are decreasing functions of z.*

The proof is in Online Appendix B.3. The reasoning behind Lemma 5 is as follows. Suppose a buyer carries a small amount of real money balance into the goods market. Firms expect to produce and sell a lower quantity of goods. A measure of firms will optimally respond by charging higher prices relative to their marginal cost of production. Consequently, the distribution of goods prices is more dispersed. The buyer with a tighter liquidity constraint is more likely to draw a higher price (or an associated markup) from the distribution. Therefore, the net benefit of banking in equilibrium should be ambiguous in contrast to Berentsen et al. (2007). Here, the gains from accessing a competitive banking sector depend on the interaction between agents' precautionary demand for money holdings and endogenous firms' market power in the goods market.[22]

Using the result established in Lemma 5, we can then show the existence of a stationary monetary equilibrium with both money and credit. Such an equilibrium entails price dispersion in the frictional goods market. We summarize the result in the following Proposition. Details of the proof are in Appendix B.4.

Proposition 1. *Let monetary policy be $\gamma > \beta$ and noisy search frictions be $\alpha_1, \alpha_2 \in (0, 1)$. There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

3 Equilibrium trade-off and welfare effect of banking

We can see in Definition 1 the equilibrium trade-off between the benefit of banking and its uninternalized social cost on consumer-goods prices (i.e., the *pecuniary externality*): On the one hand, banking is beneficial because it increases the expected net return of holding money. This can be deduced from reading the first and second terms in Equation (2.26). On the other hand, firms' market power (price markups and dispersion) in frictional goods trades can also reduce some of the gains from banking. Banking, through competitive outcome i, affects the agents' precautionary demand for money holdings z, which then feeds back onto the distribution of goods prices $J_i(\rho, z)$, and its support, $\text{supp}(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$. In particular, the integrals on the right-hand side of Equation (2.26) capture the reduction in the expected return on money even though agents have access to a competitive banking sector. This works through the results in Lemma 5: the first-order-stochastic-dominance in $J_i(\cdot, z)$ and the associated increasing pricing-cutoff function $\tilde{\rho}_i$, as z falls.

In the following Section 3.1, we explore this trade-off further. We analytically dissect the model through its special cases in order to identify an equilibrium tension between competitive banks' role in facilitating insurance of individuals' liquidity risks and the externality that such bank credit may have on non-credit users in the economy. The numerical analyses will be based on the model that is statistically calibrated to U.S. data. See Online Appendix C for further details of the calibration. The resolution of such a tension ultimately depends on inflation policy. In Section

[22] This is the novelty here in contrast to the special case where there is no banking or financial intermediation—i.e., the equivalent Head et al. (2012) setting. In Section 3.1, we will illustrate and decompose the effect of this pecuniary externality channel; and we will show that how severe this effect is in offsetting the liquidity risk insurance role of banks depends on long-run inflation or monetary policy.

3.2, we further use the calibrated model to illustrate how the trade-off changes with inflation in the long run and what the resulting welfare implications are for banking in such an economy.

3.1 Inspecting the trade-off

Overview. It is useful to compare our setting to that without banking. In particular, if we remove the banking sector, we get the case of a pure monetary economy with firm market power studied in Head et al. (2012) (HLMW). As such, Equation (2.26) becomes

$$\frac{\gamma-\beta}{\beta}=\int_{\underline{\rho}(\hat{z})}^{\overline{\rho}(\hat{z})}\left[\alpha_1+2\alpha_2(1-\tilde{J}(\rho,\hat{z}))\right]\left(\frac{u_q[q^{no-bank}(\hat{z})]}{\rho}-1\right)\mathrm{d}\tilde{J}(\rho,\hat{z}), \tag{3.1}$$

where the price distribution in a no-bank monetary economy is given by

$$\tilde{J}(\rho,\hat{z})=1-\frac{\alpha_1}{2\alpha_2}\left[\frac{\tilde{R}(\overline{\rho})}{\tilde{R}(\rho)}-1\right], \tag{3.2}$$

and the bounds are given by $\tilde{R}(\underline{\rho})=\frac{\alpha_1}{\alpha_1+2\alpha_2}\tilde{R}(\overline{\rho})$, and $\overline{\rho}=\max\left\{\frac{c}{1-\sigma},\underbrace{\hat{z}^{\frac{\sigma}{\sigma-1}}}_{=:\hat{\rho}}\right\}$, and the real profit per customer served is

$$\tilde{R}(\rho,\hat{z})=q_b^{no-bank}(\rho,\hat{z})(\rho-c), \tag{3.3}$$

and buyer's optimal demand for goods is given by

$$q_b^{no-bank}(\rho,\hat{z})=\begin{cases}\frac{\hat{z}}{\rho} & \text{if } 0<\rho<\hat{\rho}\\ \rho^{-1/\sigma} & \text{if } \hat{\rho}\leq\rho\end{cases}. \tag{3.4}$$

Consider anticipated inflation away from the Friedman rule $\gamma>\beta$. In our setting and in that of Head et al. (2012), agents have precautionary demand for money holdings. Anonymity in the goods market gives rise to money as a means of payment. However, inflation ($\gamma>\beta$) induces a rate of return on money that is lower than the risk-free rate or the rate of time preference. Inflation is thus a tax on frictional goods trades. Hence, holding money can be costly when agents (ex-post) are stuck with unproductive idle money balances.

With access to banks, households can now reduce the cost of having unneeded liquidity (via depositing idle funds in the bank to earn interest). In addition, households can borrow extra money balances from the bank. Credit extended by banks helps households to relax their liquidity constraints when they need to make a payment in the goods market. We call this positive welfare effect of banking a *liquidity-risk insurance effect*, which works through an identical mechanism as in Berentsen et al. (2007) (BCW).

Let us contrast this with BCW's no-banking and banking equilibria. Consider BCW's no-bank equilibrium condition for money demand which equates an "inflated" opportunity cost of money holding to an individual's ex-post marginal utility of buying goods with money (conditional on them being active buyers): $(\gamma/\beta - 1)/(1 - n) = u'(q)$. On the left of this condition, due to the measure of BCW's inactive buyers $n \in (0, 1)$, the opportunity cost of holding money is higher the more inactive buyers there are (since $1/(1 - n) > 1$). In contrast, in BCW's banking equilibrium the money demand condition becomes $\gamma/\beta - 1 = i$. This precisely says that the opportunity cost of holding money (borne by what would have been idle-money holders) is fully compensated at the margin by the interest rate earned on deposits. Algebraically, the liquidity risk "inflation factor", $1/(1 - n)$, is eliminated by the existence of banks.

Now, in our setting with money and credit in equilibrium, the equivalent of BCW's n measure of potential idle money holders is given by the α_0 measure of households who fail to contact any DM goods seller. Thus, there would also be a similar liquidity risk "inflation factor" here—an "inflated" opportunity cost of holding money. However, in our setting the expected marginal benefit of holding money—the integral terms on the right-hand side of Equation (2.26)—depends on the equilibrium distribution $J_i(\cdot, z)$ (an outcome of goods market power) and this depends on equilibrium interest on credit, i. The net benefit of banking here can be ambiguous because of this policy-dependent interaction. This is because even though banks here are perfectly competitive as in Berentsen et al. (2007), households using bank credit can give rise to an additional *price dispersion effect* that can contribute to a negative welfare effect of banking. The mechanism is as follows.

Decomposing the welfare effects of banking. To understand the positive and negative welfare effects of banking, we compare Equation (2.26) and Equation (3.1). In our model economy, buyers can deposit funds in the bank to earn interest $i_d > 0$ if they (ex-post) fail to match with a firm to trade in the DM. We label this type of buyers as depositors. The interest paid to depositors increases the expected marginal benefit of accumulating money balances. This is the same (and sole) benefit of banking in Berentsen et al. (2007).

In addition now, buyers who are liquidity constrained and sample low enough prices of the goods can use bank credit to relax their liquidity constraint (ex-post). The first and second terms on the right-hand side of Equation (2.26) reflect such banking benefits. Due to the liquidity-risk insurance effect, banking helps to improve consumption allocation relative to HLMW, on the one hand.

On the other hand, access to credit by buyers can also lower the expected marginal benefit of money when firms can exploit markups in frictional goods trades. In particular, the integrals on the right-hand side of Equation (2.26) capture the negative welfare effects of banking which we label as the *pecuniary externality* or *price dispersion* effect. The reason is as follows.

To avoid the inflation tax, buyers would like to carry less own money balances by taking out a loan from the bank (ex-post). However, firms expect some potential customers to be liquidity constrained by their money balances, and their expenditure rule is inelastic. A measure of firms would then optimally respond by charging higher markups (see Lemma 5). This will affect both the liquidity constrained and unconstrained money-buyers. The former will face a tighter liquidity constraint as the real value of their money goes down so they end up with less goods. The latter, although unconstrained, still best respond by consuming less, since their demands are decreasing in the prices they draw. That is, credit-buyers inadvertently contribute to the bidding up of DM goods prices: In the model, this shows in the form of the support of the goods price distribution, $\text{supp}\left(J\left(\cdot, z\right)\right) = [\underline{\rho}(z), \overline{\rho}(z)]$, being wider than that in HLMW. Specifically, the lowest possible price that a liquidity-constrained (and unconstrained) money buyer can draw becomes higher as ex-ante real balance falls (Lemma 5). In other words, access to credit by buyers amplifies firms' market power in terms of price markups and dispersion.

Recall that banking credit only benefits some buyers but not all. In particular, from Equations (2.28) and (2.29), buyers use credit if they draw a sufficiently low price ρ on goods from the distribution $J_i(\cdot, z)$. However, as we have deduced, banking credit induces higher price dispersion, which implies more high-price firms extracting rent from liquidity constrained money-buyers. Both integrals on the right-hand side of Equation (2.26) capture a reduction in the expected return on holding money along the rising price dispersion. Hence, a distortion will appear in the average interest saved on borrowing an extra dollar for the credit-buyers and the liquidity premium for the money-buyers. Thus, firms' market power in frictional goods trades can potentially reduce gains from a competitive banking sector.

Numerical illustration using a calibrated setting. Next, we provide a numerical illustration of the mechanism outlined above by comparing our baseline model economy to that without banking (HLMW), for a given long run monetary policy setting τ.[23]

Figure 2 displays the liquidity-risk insurance and price dispersion effects of banking given policy $\tau > \beta - 1$. (Without loss, we plot the case of $\tau = 0\%$.) The dashed-dotted red graph and associated dashed-red pricing cutoff $\hat{\rho}_{HMLW}$ represent the model economy in HLMW. The solid blue graph with dashed blue cutoffs $\tilde{\rho}_i$ and $\hat{\rho}$ represent our baseline model economy (with existence of banking).

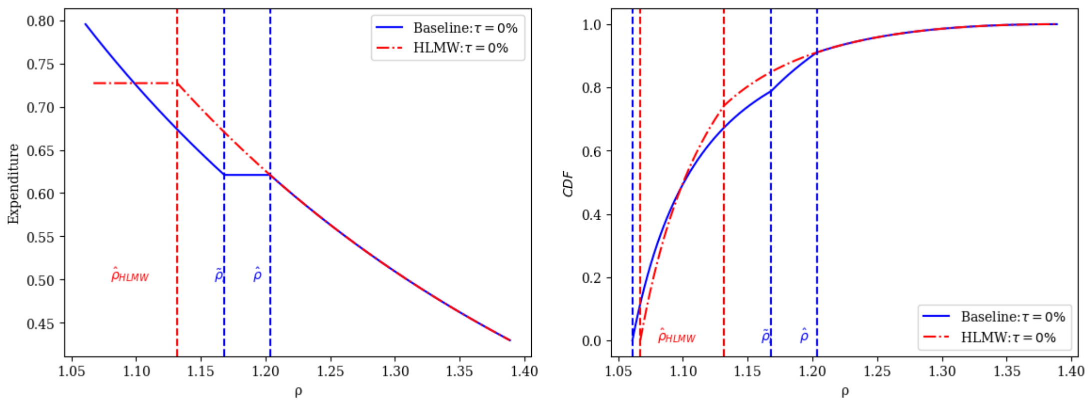
Liquidity-risk insurance effect: positive welfare effects of banking. In HLMW, a buyer cannot spend more than his liquidity constraint when faced with a price draw that is at most $\hat{\rho}_{HLMW}$. The horizontal part of the dashed red graph in Figure 2a reflects the set of expenditure levels of such a type of (ex-post) liquidity constrained buyers. The cut-off $\hat{\rho}_{HMLW}$ is the price level at which the buyer becomes liquidity unconstrained ex-post. Such a buyer spends less than her total money balances if she draws a price higher than $\hat{\rho}_{HMLW}$.

Consider now the solid blue graph in Figure 2a. In contrast to HLMW, there is now a liquidity-risk insurance effect highlighting the benefits of having access to banking credit. A buyer can

[23] we will focus only on the ex-post buyer types and put aside the obvious Berentsen et al. (2007)-like benefit of banking to the non-consuming depositors. Their surplus will nevertheless be accounted for in our final welfare calculation.

Figure 2: Liquidity-risk insurance and price dispersion effects given policy $\gamma = 1 + \tau > \beta$.

(a) Liquidity-risk insurance effect (b) Price dispersion effect



now borrow additional money balances from the bank to relax his liquidity constraint when $\rho \leq \tilde{\rho}_i$. Thus, the (ex-post) credit-buyer faces a more relaxed liquidity constraint to spend more in the goods market than money-buyers. The (non-credit) money constrained buyers in this case are the ones on the horizontal segment of the solid blue graph — i.e., the ones who draw a ρ from the interval $(\tilde{\rho}_i, \hat{\rho}]$. The (non-credit) money-unconstrained buyers have a downward sloping expenditure function over all $\rho > \hat{\rho}$.

From Figure 2a, we can deduce that credit buyers can potentially benefit from higher expenditures, whereas the money-constrained buyers now can only afford lower expenditures, relative to the HLMW (no-bank) economy. However, this is not the complete picture as, with noisy search, one also has to take into account the equilibrium measure of buyers over each subset of these price-draw intervals. That is, Figure 2a depicts only the individual's *intensive* margin outcomes in terms of possible expenditures as a function of the price draw ρ. A more complete view would have to also factor in effect of banking on the equilibrium distribution of such people. We turn to this *extensive margin* effect next.

Pecuniary externality through price dispersion effect: negative welfare effects. When firms' market power (markup and price dispersion) arises from informational frictions, access to competitive banks can cause an additional negative welfare effect. This is because not all agents can benefit from banking. In particular, those agents who use banking for loans create a price effect in the goods market. This negatively affects agents who do not use banking credit.

Recall that firms expect some prospective customers to be constrained by their money balances, and their expenditure rule to be inelastic. Thus, a measure of firms optimally responds by charging higher prices relative to their marginal cost of production. Consequently, buyers (on average) are more likely to draw a higher price in the sense of first-order stochastic dominance. In Figure 2b, this price dispersion effect of banking externality is reflected in the solid blue distribution function graph over the set $\rho > \tilde{\rho}$ (for the banking equilibrium) is first-order stochastically dominating the dashed-dotted red graph (HMLW, no-banking equilibrium). Also, the support of the price distribution in our baseline model economy is wider than in HLMW and $\tilde{\rho}_i$ is higher than $\hat{\rho}_{HLMW}$. Thus, under our banking equilibrium, each money-constrained and money-unconstrained buyer will tend to draw from a higher range of prices than the no-bank, HLMW economy. Moreover, the equilibrium mass of such buyers is relative higher than that in the HLMW economy.

Effectively, bank credit induces more price markups on money-buyers. There is also higher price dispersion in frictional goods trades. Consequently, each money-buyer (who draws a high enough price such that $\tilde{\rho}_i \leq \rho \leq \hat{\rho}$) faces a tighter liquidity constraint. In this case, the liquidity constrained money-buyer spends less than the case without access to banking arrangements. For the liquidity unconstrained money-buyer (who draws $\rho > \hat{\rho}_i$), this effect is not there since his liquidity constraint does not bind. Nevertheless, since unconstrained money-buyers' demands are decreasing in ρ, and the corresponding domain for ρ would have shifted up, they would bear the brunt of the externality through lower consumption outcomes (relative to their HLMW counterparts).

In summary, banking affects agents' consumption outcomes differently when firms have market power in frictional goods transactions. In this setting, access to a competitive banking sector can amplify firms' market power, creating an additional welfare-reducing effect of banking. This negative welfare effect, tied to credit-buyers, pushes up price dispersion. This then increases the measure of firms extracting rent from money-buyers. Consequently, the welfare-improving function of banking liquidity transformation is no longer unambiguous, in contrast to Berentsen et al. (2007).[24]

3.2 Trade-off behavior and inflation

In Section 3.1 we identified the benefit and cost of having competitive banking conditional on a particular inflation (policy) setting. Here, we will numerically evaluate this *insurance* versus *price dispersion* tension as a function of inflation (or equivalently, nominal interest policy in the long run). We consider a set of economies, each distinguished by its long-run inflation rates τ from $\tau \in [\beta - 1, \bar{\tau}]$, where we set $\bar{\tau} = 0.1$ (i.e., 10% annual inflation rate).[25]

[24] Our result has a similar flavor to the classic pecuniary-externality effect from credit (see, e.g., Chiu et al., 2018). In Chiu et al. (2018), the externality is necessarily dependent on an assumption that the cost of producing goods q is a strictly increasing and convex function. In their competitive price-taking equilibrium, the existence of credit-buyers raises goods quantity, q, which then raises the marginal cost of producing q, $c'(q)$, since $c''(q) > 0$ in their setup. This then raises equilibrium price p and feeds back in the form of tightening money-buyers' liquidity constraints. If $c''(q) = 0$, there is no pecuniary externality in Chiu et al. (2018). In contrast, here, we deliberately shut down the technological avenue necessary in Chiu et al. (2018) to generate the pecuniary externality. Instead, we can still have this effect for a different reason. Here, the pecuniary externality works through market power in the form of price (markups) dispersion. The existence of credit-buyers means that, ex-ante, agents end up carrying (relatively) less real balances, z. By Lemma 5, this tends to shift the distribution $J_i(\cdot, z)$ to the right—i.e., agents are more likely to get squeezed by higher prices and markups. If agents knew for sure they would be money-buyers, they would prefer to have carried more real balance. However, because of the idiosyncratic risk they face, ex-ante, all agents end up creating some pecuniary externality of the ex-post liquidity constrained agents.

[25] It can be verified that price dispersion cannot be sustained at the Friedman rule, i.e., $\tau = \beta - 1$. Moreover, banking is redundant since it is costless for agents to carry money balances. For our purpose, we focus on long-run anticipated inflation away from the Friedman rule. This can be interpreted as some extraneous institutional

Overall, whether competitive banks improve welfare in equilibrium is ambiguous. To understand why, we break welfare gains and losses down into the net trading surpluses associated with each ex-post buyer-type events—i.e., events involving credit buyers, money-constrained buyers, and money unconstrained buyers. In Figure 3, these net trading surpluses are measured as the expected utility of each ex-post buyer group net of sellers' expected cost of producing at ex-post different prices.[26]

The solid blue graphs in each panel of Figure 3 correspond to banking equilibria in our model for different long run inflation policies. The corresponding dashed orange graphs are those for the no-bank HLMW economies. (We'll focus only on the ex-post buyers and economize of showing the surplus of depositors, which will just be a constant.) The stark takeaway from these graphs is that *with competitive banking,* there is a positive gap between the net social surplus of credit-buyer events (see Figure 3a), although this gap shrinks with inflation (as to be expected). However, in the following two panels, Figures 3b and 3c, we can see that society is ex-ante worse off if they turn out to be either money-constrained or money-unconstrained buyers who optimally do not use bank credit.

Figure 3d sums up the preceding three graphs vertically to give us the relevant net social surplus across all three ex-post groups. Here, we can already see the symptom of the underlying tension between the liquidity risk insurance benefit of banks (for credit buyers) and the pecuniary-externality cost that operates through the pricing dispersion effect. The resolution is non-monotone with respect to inflation. For low inflation ranges, the latter dominates to create a negative social surplus despite having perfect competition among banks. Only for sufficiently high inflation ranges does the benefit of banking begin to dominate.

Welfare implications of banking. What then of the benefit of banking to the inactive DM buyers (depositors)? We had, thus far, deliberately omitted that in the display and discussions in the previous figures. We now present a complete welfare accounting that includes the ex-ante welfare of ex-post depositor types.[27]

We report the welfare measure in terms of a standard consumption equivalent variation (CEV)

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restrictions that prevent a monetary policy maker from implementing the Friedman rule (see also Berentsen et al., 2007, for the same argument).

[26]For example, in our baseline banking equilibrium we have three ex-post cases with corresponding ex-ante net trading surpluses. The generic formula for the trading surplus measure is

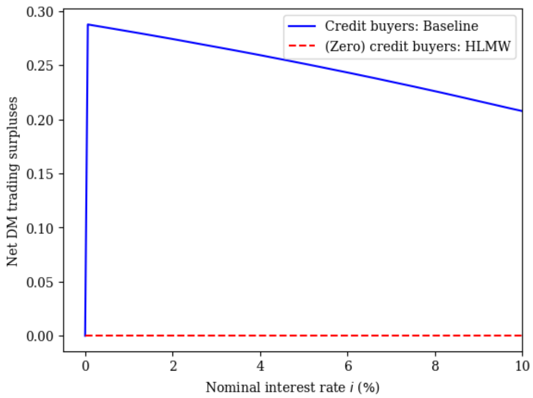
$$\int_{\mathcal{E}} [\alpha_1 + 2\alpha_2(1 - J_i(\rho, z)]u[q_b(\rho, z)] - c[q_b(\rho, z)]\mathrm{d}J_i(\rho, z),$$

where: (a) the own-money constrained buyer cases have $\mathcal{E} := \left[\underline{\rho}(z), \tilde{\rho}_i(z)\right]$ and $q_b(\rho, z) = [\rho(1+i)]^{-1/\sigma}$; (b) the own-money constrained buyer cases have $\mathcal{E} := (\tilde{\rho}_i(z), \hat{\rho}(z)]$ and $q_b(\rho, z) = z/\rho$; and (c) the own-money unconstrained buyer cases have $\mathcal{E} := (\hat{\rho}(z), \bar{\rho}(z)]$ and $q_b(\rho, z) = \rho^{-1/\sigma}$. We can define similar objects for the HLMW no-bank environment except that there will be zero measures of credit buyer events.

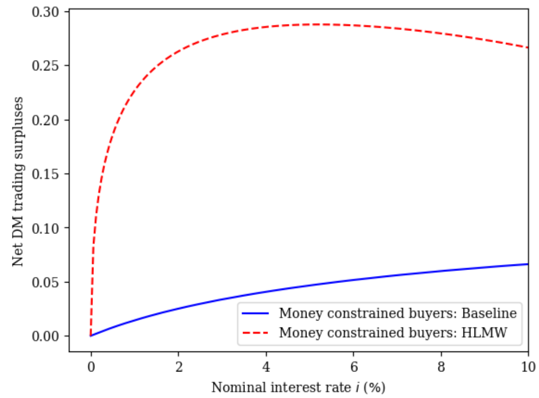
[27]Note that since depositors do not consume in the DM, there is zero net trading surplus emanating from such ex-post events. Depositors' ex-ante welfare are thus solely accounted for by the term $\alpha_0[U(x^\star) - x^\star]$. Together with similar terms accruing to the other ex-post agent groups, the total present-value social welfare in the CM activity is simple: it is just the constant term $(1-\beta)^{-1}[U(x^\star) - x^\star]$.

Figure 3: The effects of inflation on Equilibrium Outcome.

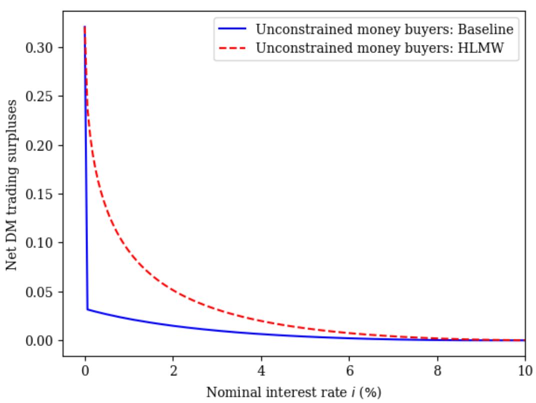
(a) Credit buyers



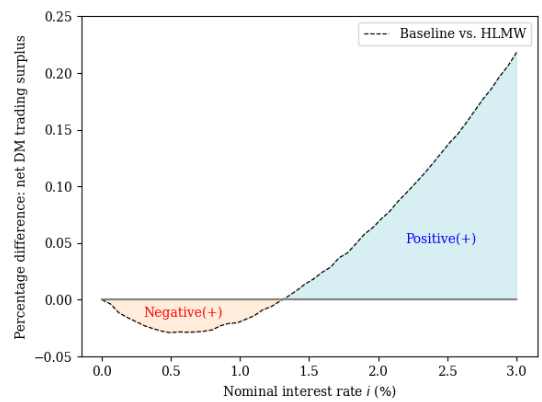
(b) Money (non-credit) constrained buyers



(c) Money (non-credit) unconstrained buyers



(d) Total trading surpluses: (3a)+(3b)+(3c) gap



measure. This captures how much consumption (in units of the CM good) an agent is willing to give up in an economy without banks to live in an economy with banks.

Given $\gamma = 1 + \tau$ policy, the welfare function in an SME is given by

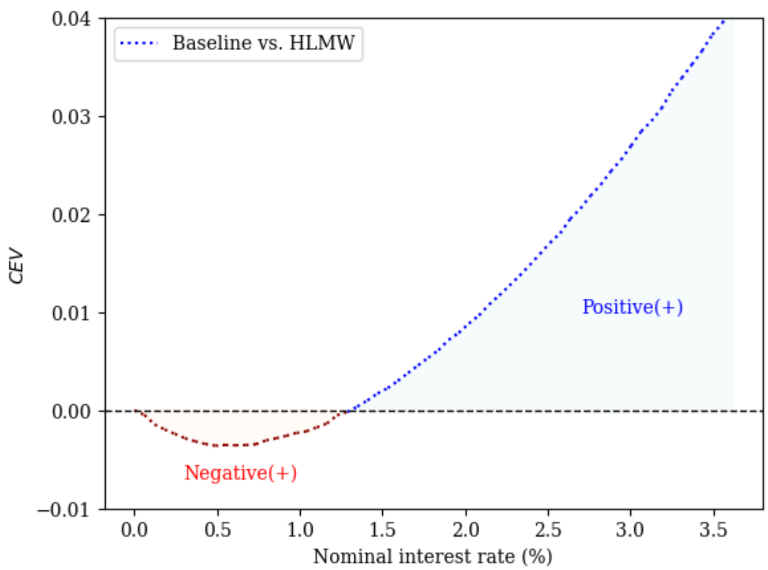
$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(x^\star) - x^\star + \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1 - J_i(\rho, z_e, \gamma))\Big)\Big(u[q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big)\mathrm{d}J_i(\rho, z_e, \gamma)\Bigg], \tag{3.5}$$

where $e \in \{Baseline, HLMW\}$ indexes our baseline model economy or the no-bank economy of HLMW. We can also write the total welfare at a given gross inflation γ with consumption reduced by a factor of $\triangle$ as

$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(\triangle x^\star) - x^\star + \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1-J_i(\rho,z_e,\gamma))\Big)\Big(u[\triangle q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big)\mathrm{d}J_i(\rho,z_e,\gamma)\Bigg]. \tag{3.6}$$

We compute the CEV as the value $1-\triangle$ that solves $W^{Baseline}(\gamma) = W^{HLMW}{}_{\triangle}(\gamma)$ given policy $\gamma = 1+\tau$. This measure says that every agent in the economy with perfectly competitive banks needs to give up $1-\triangle$ percent of his consumption to move to the economy without access to competitive banks at given policy.

Figure 4: Consumption Equivalent Variation (%) of moving from the no-bank HLMW economy to the baseline economy with banking.



According to Figure 4, banking has a non-monotonic welfare consequence when the trend inflation rate varies from just above the Friedman rule $\gamma = 1+\tau = \beta$. In particular, banks are inessential institutions when the trend inflation rate is sufficiently low. This result hinges on the interaction between the liquidity-risk insurance and price dispersion effects discussed earlier in Section 3.1.

On the one hand, positive welfare effects of banking come from the liquidity-risk insurance effect. First, (ex-post) inactive buyers in trading with probability α_0 can deposit their idle money balances to earn an interest i_d. Second, buyers who trade with low-price firms find it worthwhile to borrow additional money balances from the bank. These credit-buyers have more relaxed liquidity constraints. Thus, they can spend more on goods to enjoy a higher utility flow in the DM.

On the other hand, credit can amplify firms' market power (markups and price dispersion), which creates a negative welfare effect of banking on liquidity constrained and unconstrained money-buyers. The reason is as what we had previously discussed.

Summary of insights. Imperfect information through noisy search frictions in the goods market generates a policy-dependent distribution of goods prices (and associated markups), as in Head et al. (2012). The presence of competitive banking benefits only agents who would like to deposit and those who optimally use credit by inducing more firms who serve them to more likely post low prices. In turn, the externality effect is in firms who charge higher prices to money-buyers who do not find it optimal to borrow. These agents' expenditures are either inelastic to the price rise and they end up consuming less (i.e., the money-constrained agents) or they elastically respond to higher price draws by consuming less (i.e., the money-unconstrained agents). When inflation is sufficiently low, the cost of holding money is also low. Thus, the gains from banking along the channel of liquidity-risk insurance effect are also small. The price dispersion effect can easily outweigh such benefits via higher markups distorting the liquidity premium for the money-buyers.

To sum up, firms' price dispersion induces (ex-post) heterogeneous consumption outcomes among credit-buyers and money-buyers. Hence, non-trivial feedback from firms' market power on the welfare consequences of banking. In particular, credit-buyers benefit from banking credit to purchase more goods. However, banking also makes firms extract more rent from money-buyers in goods trades, thus lowering consumption. The essentiality of banks—in terms of helping insure against individual liquidity risks—is no longer unambiguous in our economy with endogenous firms' market power in the goods market.

4 Empirical relevance and supporting evidence

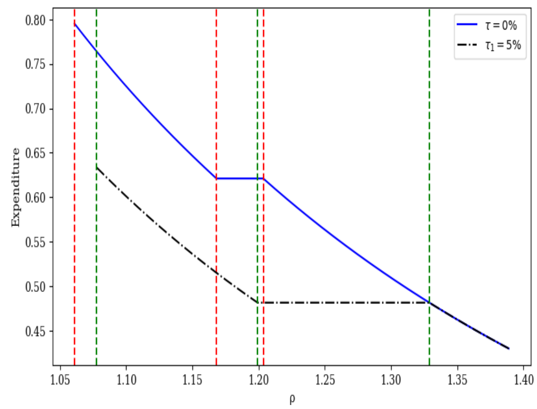
What does the model say about average markups and markup dispersion, in relation to bank credit and inflation? Figure 5 depicts the banking model and the effect of inflation on both an *intensive margin* (summarized by the expenditure of agents as a function of prices drawn) and an *extensive margin* (summarized by the equilibrium distribution of prices and the relevant cutoffs defining the heterogeneous, ex-post buyer types).

Specifically, Figures 5a and 5c show two examples with 0% and 5% inflation. With the higher inflation equilibrium, we see that there is a larger range of prices (and higher price draws) that support the cases of all the non-credit money buyers. That is, the pricing cutoffs demarcating the ranges of price draws consistent with equilibrium non-credit-buyer events rises with inflation. This claim is corroborated further by Figure 5a where we plot the pricing cutoff formulas from Equation (2.9). The $\hat{\rho}$ function for our economy and that for HLMW have the same formula except that the latter's equilibrium real money balance outcome is always dominated by that of the former's, except when inflation is at the Friedman rule. Figure 5a shows that the pricing cutoffs are increasing as a function of inflation.

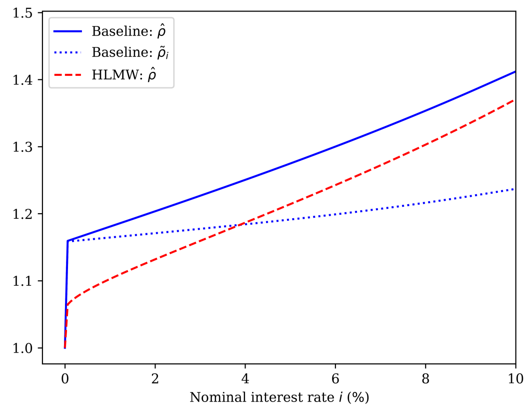
Consider Figure 5c. The solid blue graph is the equilibrium pricing distribution at an example inflation policy of 0% per annum. The dashed-dotted black graph corresponds to an example

Figure 5: Inflation, individual expenditures, equilibrium pricing distribution and cutoffs.

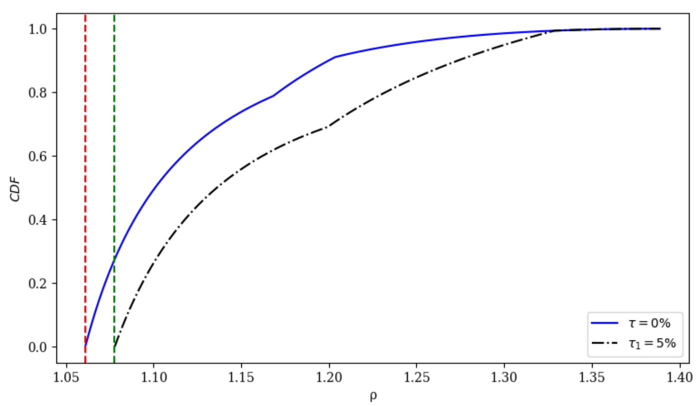
(a) Individual expenditures



(b) Cutoffs (price ranges) and buyer types



(c) Pricing distribution



higher inflation at 5% per annum. Individually, ex-post non-credit buyers are more likely to draw higher goods prices and thus markups. This is due to the firm's optimal responses: On the one hand, with higher inflation, agents economize on holding real balance z. For fixed inflation, by Lemma 5, this has the effect of increasing pricing and markup dispersion.

However, in the economy-wide measure in Figure 6, we see that average markup and the dispersion in markups are falling with inflation. This is because the economy-wide measure of markup and markups dispersion involve an output-weighted average between the two sectors of the economy. In the model, markup in the CM sector is always unity since markets are perfectly competitive. Thus, although the DM average markup and markups dispersion is rising, as we have deduced above, the economy-wide measures of these are falling. This is due to the weight on the DM measures—calculated as the ratio of DM average output to the economy-wide average output—falling faster than the actual rise in the DM markup or markups dispersion.[28]

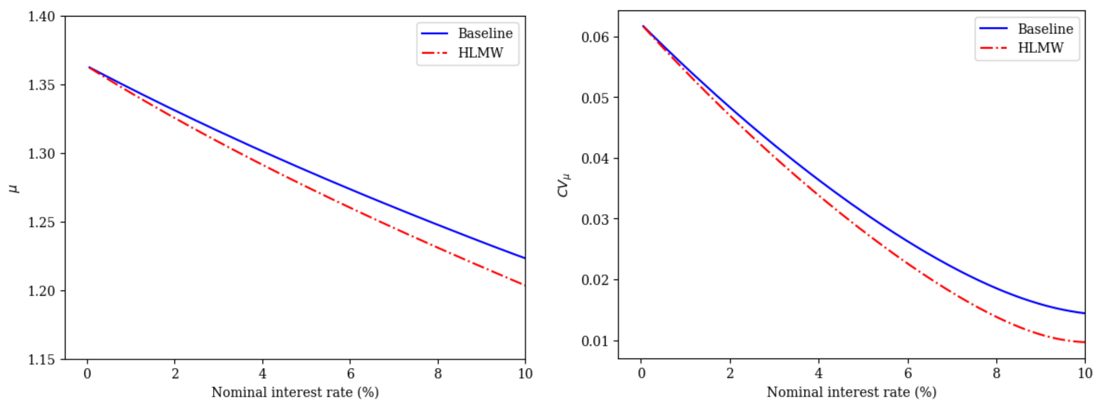
[28] See Online Appendix C for the definition of this measure.

4.1 Two testable model predictions

Figure 6a and 6b, respectively, plot how average markup and markup dispersion (the coefficient of variation in markups) vary with inflation.[29] We do this for both our banking equilibrium (solid blue graph) and the HLMW no-banking equilibrium (dashed-dotted red graph).

Figure 6: The effects of inflation on the aggregate markup and price dispersion given policy $\tau \in (\beta - 1, \bar{\tau}]$.

(a) Average markup and inflation (b) Markup dispersion and inflation



We now focus on the following model prediction regarding the presence of banking in Figure 6:[30]

Observation 1 (Two empirically-relevant predictions)**.** *With access to (or, the existence of) banking credit, the economy will have* higher *average markup and markups dispersion in markups (relative to the economy without banks), at any level of inflation (or nominal interest).*

That average markup being negatively related to inflation in the model (Figure 6a) is consistent with existing empirical findings (see Banerjee and Russell, 2005, 2001). Equivalently, we also show in Figure 7c that average markup is negatively related to the nominal interest rate (proxied by the effective Federal Funds rate). Figure 7b that In the data below (see Figure 7b) we also know that over the relevant sample period, markup dispersion is rising while the nominal interest rate steadily fell (see Figure 7c). The model also produces this stylized negative association between inflation (or the nominal interest rate) and markup dispersion (Figure 6b).

We have previously shown that a perfectly competitive banking sector can amplify firms' market power measured in terms of markup (by Equation (C.2)) and price dispersion (by Equation (C.3)). In Berentsen et al. (2007), the welfare gain of banking liquidity transformation comes from the interest payments on unproductive idle money balances. Households can then accumulate more

[29] In Figure 6a, average markup is reported as a gross factor. For example, a factor of 1 means that there is zero markup, or a factor of 1.3 means an average of 30% price markup on marginal cost.

[30] We can generalize this exercise to a setting with a continuous variation in agents access to bank credit. We omit that exercise here as it provides the same insight as the starker comparison in Figure 6.

money balances to trade in the goods market. Thus, having access to a competitive banking sector is always welfare-improving relative to a pure monetary economy.

Contrast this with a setting with goods market power as in HMLW (the dashed-dotted red graphs in Figure 6). Now, the Berentsen et al. (2007) type of banks involve some banking benefits going to the credit-buyers (by relaxing their liquidity constraint) and inactive buyers (by depositing idle funds). However, banking credit can also distort the liquidity premium for the money-buyers via both higher price dispersion and markups (see the solid blue graphs in Figure 6).

4.2 Empirical evidence

We now present two pieces of reduced-form empirical evidence to support the two stylized model predictions in Observation 1. We show that there is a positive relationship between the consumer credit-to-GDP ratio and aggregate price markup. Also there is a positive relationship between the consumer credit-to-GDP ratio and markup dispersion in the United States.

We put together data on quarterly consumer credit-to-GDP ratio, the aggregate markup and price dispersion in the United States. The data sample period is from 1980Q1 to 2007Q4. More details on the data is available in the Online Appendix D.

In the data (Figure 7a), we can see that there is a positive relationship between our proxy for access to credit and markup. Likewise, in Figure 7b, there is a positive relation between between access to credit and markup dispersion. We can put these casual observations to more formal regression tests.

Empirical model. We now investigate the effect of consumer credit on the aggregate markup and its dispersion by considering the following empirical model specification,

$$y_t = a_0 + a_1 d_t^{CC} + b' \gamma_t + \epsilon_t, \tag{4.1}$$

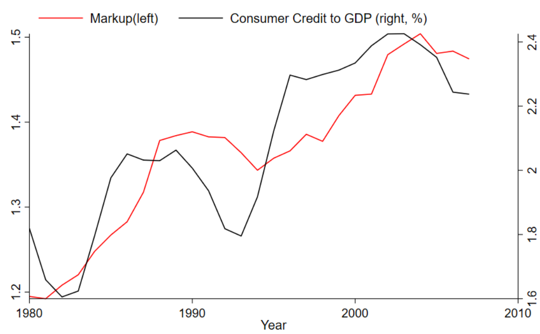
where y_t is one of the variables of interest: log of the aggregate markup, or markup dispersion. The variables d_t^{CC} and γ_t, respectively, denote consumer credit-to-GDP ratio and the list of control variables previously described. The list of parameters, respectively, for the intercept, the credit-to-GDP ratio and all the controls, (a_0, a_1, b') are estimated by ordinary least squares (OLS). Conditional on the other factors, we are most interested in the various estimates of a_1, which will be presented in the first row of Table 1.

Empirical results. Table 1 reports the OLS results for Equation (4.1). Column (1) indicates that there is a positive and statistically significant relationship between consumer credit-to-GDP and aggregate markup. Column (2) shows a positive and statistically significant relationship between consumer credit-to-GDP and price dispersion.[31]

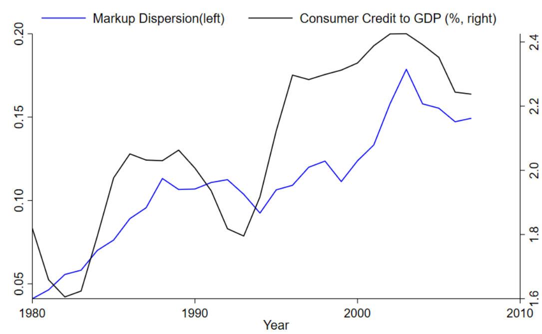
[31] We provide more detailed results for a robustness test in Appendix E.

Figure 7: Time series of credit-to-GDP ratio and markup statistics

(a) Credit and average markup



(b) Credit and markups dispersion



(c) Average markup and the effective Federal Funds rate

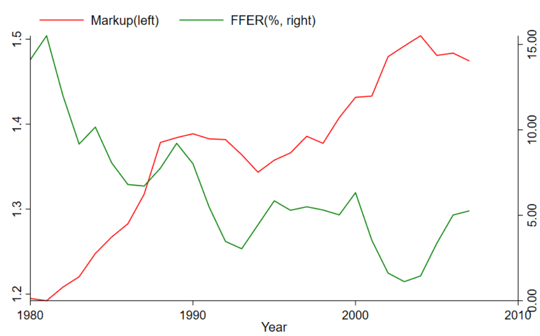


Table 1: OLS results: Markup and dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	4.335***	3.337***
	(1.571)	(1.001)
CPI Inflation	-0.559***	-0.331***
	(0.0928)	(0.0626)
Log of real GDP	0.163***	0.0671***
	(0.0165)	(0.0111)
Business TFP	0.0427	0.0689
	(0.0693)	(0.0488)
Real wage	-0.252***	-0.169***
	(0.0871)	(0.0642)
log of Real Exchange rate	-0.111***	-0.0172
	(0.0257)	(0.0160)
Real interest rate	-0.172	-0.177**
	(0.140)	(0.0839)
R^2	0.915	0.853
Observations	112	112

Note: Robust errors are in parenthesis, with $*, **$ and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

5 Conclusion

We construct a model of money, credit and endogenous retail market power where informational frictions induce a policy-dependent distribution of goods prices and associated markups in equilibrium. We show that access by borrowers to credit can contribute to amplifying firms' market power, reducing the welfare gains from banking. The increased demand for goods by credit-buyers expands the measure of firms charging higher prices, extracting rent from money-buyers. The latter comes in two ways: First, higher price draws affect agents who turn out to be liquidity constrained money-buyers by squeezing their liquidity constraints and thus lowering their consumption. Second, higher price draws also reduce unconstrained money-buyers' consumption even though there is no binding liquidity constraint on them. This is simply because their consumption demands are decreasing functions of the relevant prices they draw. As a result, market power in the retail industry can make an otherwise competitive banking sector less efficient in reallocating liquidity in equilibrium.

Thus, the welfare-improving role of banking liquidity transformation is no longer unambiguous in a monetary economy with endogenous firms' market power. Our model also generates a positive relationship between the consumer credit-to-GDP ratio and firms' market power (measured by price markups and dispersion), consistent with the empirical observation using firm-level data. Our model highlights a new channel that can be surprising if policymakers attempt to regulate banking competition without taking into account its externality on consumers in non-competitive goods markets that have, evidently, price dispersion.

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Online Appendix

On a Pecuniary Externality of Competitive Banking through Goods Pricing Dispersion

Omitted Proofs and Other Results

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¶

A Two-stage decision representation of Problem (2.7).¶

Here is another way to rationalize this problem by breaking up the within-period DM buyers' problem in two virtual stages: Consider the last stage for any active buyer at the end of the DM. The value of any excess money balance, $\tilde{m}$, is given by

$$D\left(\tilde{m}, l, \mathbf{a}\right) = \max_{d}\left\{W\left(0, l, d, \mathbf{a}\right) : d \leq \tilde{m}\right\}. \tag{A.1}$$

Since we have shown (in the paper) that W_d is increasing and linear in d with slope $(1 + i_d)$, then we have $d^{\star}(m, p, i, \mathbf{a}) = \tilde{m}$ and its induced value is

$$D\left(\tilde{m}, l, \mathbf{a}\right) = W\left(0, l, \tilde{m}, \mathbf{a}\right).$$

Now, step back to the stage in the DM where the DM buyers are ones who have drawn at least one price quote. Their value is:

$$B(m, p, \mathbf{a}) = \max_{q,l}\left\{u(q) + D\left(m + l - pq, l, \mathbf{a}\right)\middle| \begin{array}{c} pq \leq m + l, \\ 0 \leq l < \infty \end{array}\right\}. \tag{A.2}$$

Note that if we write $\tilde{m} := m + l - pq$, then Problem (A.2) and (A.1) together is equivalent to the reduced-form Problem (2.7). ¶

¶

B Omitted proofs

B.1 Proof of Lemma 1 (Monopoly pricing)

The following is a partial equilibrium result, taking as parametric the pre-determined money holding of agents z, and the rate of interest on loans i. It provides for a complete characterization of what would determine the upper bound ($\bar{p} = p^m$) on the equilibrium support of the DM-good price distribution. Below, we rewrite nominal variables in terms of stationary variables: Measured in units of the CM numéraire good, real money balance is $z := \phi m$ and the relative price of a DM good is $\rho := \phi p$. (Dividing the results through with the value of money ϕ will yield the result in Lemma 1, which was presented in nominal terms.) Thus, Lemma 1 re-stated in equivalent stationary-variable terms is:

Lemma. *Fix a (pre-determined) real money balance $z \in (0, \infty)$ and a given rate on loans i. Let the profit at price ρ from serving*

1. *a credit-buyer be $G_1(\rho; i) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$,*
2. *a constrained money-buyer be $G_2(\rho; z) := \frac{z}{\rho}(\rho - c)$,*
3. *an unconstrained money-buyer be $G_3(\rho) := (\rho)^{-\frac{1}{\sigma}}(\rho - c)$,*

Let $\mathbf{g}(\rho; i, z) := [G_1(\rho; i), G_2(\rho; z), G_3(\rho)]$. Assume that $\sigma \in (0,1)$ and $c \in (0,1]$ such that $0 < \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} =: \hat{z}$ and there is an $\bar{i} := \left[\sigma\left(\frac{1}{1-\sigma}\right)^{1-\frac{1}{\sigma}} + c^{-(1-\frac{1}{\sigma})}\right]^{-\sigma(1-\sigma)} - 1 > 0$. There exists a $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z} < \hat{z}$. Furthermore, if $i \in [0, \bar{i}]$ then there is also a $\breve{z}_i := \left(\frac{1+\sigma\tilde{z}_i}{c}\right)^{1-\sigma} \leq \tilde{z}_i$ such that $0 < \breve{z}_i \leq \tilde{z}_i \leq \hat{z} < z' < \infty$. Denote $\mathring{z}_i := \min\{\breve{z}_i, \tilde{z}_i\}$.

1. *The ex-post profit function is*

$$R^{ex}(\rho, i, z) = \begin{cases} \langle \mathbf{g}(\rho; i, z), \mathbf{I}_1(\rho; i, z) \rangle, & z \in [\hat{z}, \infty) \\ \langle \mathbf{g}(\rho; i, z), \mathbf{I}_2(\rho; i, z) \rangle, & z \in [\mathring{z}_i, \hat{z}) \\ \langle \mathbf{g}(\rho; i, z), \mathbf{I}_3(\rho; i, z) \rangle, & z \in (0, \mathring{z}_i) \end{cases} \tag{B.1}$$

where

$$\mathbf{I}_1(\rho;i,z) := \left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\hat{\rho}\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1b}} + \underbrace{\mathbf{1}_{\left\{\hat{\rho}<c\leq\rho\leq\rho_0^m\right\}}}_{\text{Case 1a}}\right],$$

$$\mathbf{I}_2(\rho;i,z) :=$$

$$\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \underbrace{\mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\leq\rho\right\}\cap\{z\in[\mathring{z}_i,\tilde{z}_i)\}}}_{\text{Case 2b}} + \underbrace{\mathbf{1}_{\left\{c<\tilde{\rho}_i<\rho_0^m<\hat{\rho}\leq\rho\right\}\cap\{z\in[\mathring{z}_i,\hat{z})\}}}_{\text{Case 2a}}\right],$$

$$\mathbf{I}_3(\rho;i,z) := \underbrace{\left[\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}}, \mathbf{1}_{\{c<\tilde{\rho}_i<\rho<\hat{\rho}\}}, \mathbf{1}_{\{c<\hat{\rho}\leq\rho\}}\right] \times \mathbf{1}_{\left\{c<\rho_0^m<\tilde{\rho}_i<\hat{\rho}\right\}}}_{\text{Case 3}},$$

$\rho_0^m = c/(1-\sigma)$, $\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$, $\tilde{\rho}_i \equiv \tilde{\rho}(i,z) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$, $\langle\cdot,\cdot\rangle : \mathbb{R}_+^3 \times \mathbb{R}_+^3 \to \mathbb{R}_+$ is the inner-product operator, and $\mathbf{1}_{\{X\}}$ is the Dirac delta function on event X.

2. *The (real) monopoly price and its ex-post profit outcome, respectively, are*

$$\rho^m = \begin{cases} \rho_0^m, & z \in [\hat{z}, \infty) \\ \hat{\rho}(z), & z \in [\mathring{z}_i, \hat{z}) \\ \rho_0^m, & z \in (0, \mathring{z}_i) \end{cases}, \qquad \textit{and}, \qquad R^{ex}(\rho^m, i, z) = \begin{cases} G_3(\rho_0^m), & z \in [\hat{z}, \infty) \\ G_3(\hat{\rho}(z)), & z \in [\mathring{z}_i, \hat{z}) \\ G_1(\rho_0^m; i), & z \in (0, \mathring{z}_i) \end{cases}. \tag{B.2}$$

Proof. The demand is classified by Equation (2.8) in the paper. In terms of the stationary variables, this is equivalently given as:

$$q_b^\star(z,\rho,i,\mathbf{a}) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0<\rho\leq\tilde{\rho}_i \\ z & \text{if } \tilde{\rho}_i<\rho<\hat{\rho} \\ (\rho)^{-1/\sigma} & \text{if } \rho\geq\hat{\rho} \end{cases}, \tag{B.3}$$

where we recall the definitions

$$\hat{\rho} \equiv \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{\rho}_i \equiv \tilde{\rho}(i,z) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}, \tag{B.4}$$

and z and i are taken as fixed (parametric) by both buyers and firms. Consider the case of a firm that posts a monopoly price ρ^m in a monetary equilibrium where $0 < z < \infty$ and $i > 0$. Given buyers' demand structure $q_b^\star(z,\rho,i,\mathbf{a})$ derived in Equation (B.3), a firm's ex-post profit per trade, $R^{ex}(\rho,i,z)$ is $R^{ex}(\rho,i,z) = q_b^\star(z,\rho,i,\mathbf{a})(\rho - c)$. Note that the relevant domain for pricing has a lower bound of c since no firm will want to price below marginal cost. (In equilibrium, we can show that the lower bound on the support the price distribution is bounded below by c.)

Properties of ex-post profit function components. The components G_1, G_2 and G_3 have the following geometric properties:

1. P1. Since $\sigma < 1$ then $(1+i)^{-\frac{1}{\sigma}} < 1$, so that it is always the case that $\tilde{\rho}_i < \hat{\rho}$ and the function value $G_1(\rho) := [\rho(1+i)]^{-\frac{1}{\sigma}}(\rho - c)$ is everywhere dominated by that of $G_3(\rho) := \rho^{-\frac{1}{\sigma}}(\rho - c)$ (i.e., the third case).

2. We can also verify that $G_1(c) = G_2(c; z) = G_3(c) = 0$ and $\lim_{\rho \nearrow \infty} G_1(\rho) = \lim_{\rho \nearrow \infty} G_3(\rho) = 0$. These two functions are strictly positive-valued on (c, ∞), and have unique maxima in ρ: If ρ is not constrained anywhere on the feasible domain $[c, \infty)$, then the unique maximum for the function G_3 solves first-order condition $\frac{\partial G_3}{\partial \rho}(\rho) = \frac{\partial G_3}{\partial \rho}\left[\rho^{-\frac{1}{\sigma}}(\rho - c)\right] = 0$, which yields $\rho_0^m = \frac{c}{1-\sigma}$. In nominal terms, this is $p_0^m = \frac{\phi^{-1}c}{1-\sigma}$.

3. Since $\sigma < 1$ and $i \geq 0$, then $(1+i)^{-\frac{1}{\sigma}} < 1$, so that $G_1(\rho) \leq G_3(\rho)$ for all ρ—i.e., $G_1(\rho)$ is a constant factor $(1+i)^{-\frac{1}{\sigma}} < 1$ smaller than $G_3(\rho)$. We can also deduce that there is a unique unconstrained maximizer for G_1, i.e., $\arg\max_\rho G_1(\rho) = \arg\max_\rho G_3(\rho) = p_0^m$.

4. Neither G_1 nor G_3 depend on z. The function G_2 is the only piece that depends on z, is such that $G_2(c; z) = 0$, $\lim_{\rho \nearrow \infty} G_2(\rho; z) = \infty$, and its image is strictly increasing and strictly concave in ρ.

Properties 1 to 4 imply that for all $\rho > c$, there can only be the following five generic cases, depending on the size of real money balance that buyers carry, z.

Case 1. **Real money balance z is sufficiently high,** $z \in (z', \infty) \equiv \left(c^{1-\frac{1}{\sigma}}, \infty\right)$**.** In such a case, graph $(G_2(\cdot; z))$ never intersects the graphs of G_1 and G_3 at any $\rho > \rho_0^m > c$. This case exists if $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c)$ and $\frac{\partial G_2}{\partial \rho}(\rho_0^m; z) \geq \frac{\partial G_3}{\partial \rho}(\rho_0^m) = 0$. The second restriction is always satisfied since $\frac{\partial G_2}{\partial \rho}(\cdot; z) > 0$ everywhere. We can check that $\frac{\partial G_2}{\partial \rho}(c; z) \geq \frac{\partial G_3}{\partial \rho}(c)$ if and only if $z > \acute{z} \equiv c^{1-\frac{1}{\sigma}}$. Thus, $\hat{\rho}(z) \equiv z^{\frac{\sigma}{\sigma-1}}$ does not exist if $z > \acute{z}$, since $\hat{\rho}(z) < \hat{\rho}(\acute{z}) = c$. That is, no firm will face constrained money buyers, or credit buyers, by the fact that $\tilde{\rho}_i(z) < \hat{\rho}(z) < c$ whenever $z > \acute{z}$. This implies that: ***(i)*** only *unconstrained money buyers* are served; ***(ii)*** the effective profit function for a firm is that which is associated with the demand from unconstrained money buyers, $R^{ex}(\rho, i, z)|_{z \in (z', \infty)} = G_3(\rho) := \rho^{-\frac{1}{\sigma}}(\rho - c)$; and ***(iii)*** the monopoly pricing outcome is $\rho_0^m = \frac{c}{1-\sigma}$, with its induced profit being $G_3(\rho_0^m) := (\rho_0^m)^{-\frac{1}{\sigma}}(\rho_0^m - c) > 0$. (Figure 8 illustrates an example of such a generic case where the solid-red line is the effective profit function.)

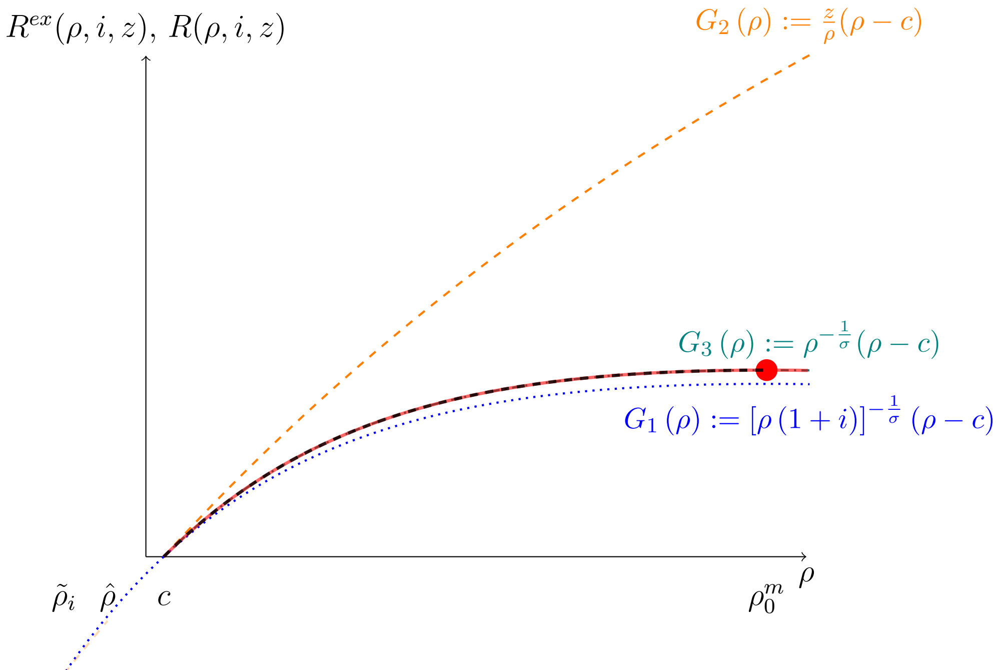

Figure 8: Case 1. Real money balance is sufficiently high, $z \in [z', \infty)$.

Case 2. **Real money balance z is intermediate (I),** $z \in [\hat{z}, z') \equiv \left[\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}, c^{1-\frac{1}{\sigma}}\right)$. Let $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ be the cutoff real balance where graph $(G_2(\cdot; z))$ intersects graph (G_3) uniquely at $\rho = \hat{\rho}(\hat{z}) = \rho_0^m > c$. Thus, consider real money balance at any given $z \in [\hat{z}, \acute{z})$, where $\hat{z} :< \acute{z} \equiv c^{1-\frac{1}{\sigma}}$, since $\sigma \in (0, 1)$. We can check that $c < \tilde{\rho}_i(z) < \hat{\rho}(z) \leq \rho_0^m$. Also, $G_2(\rho; z) < G_3(\rho)$ for all $\rho \in (c, \hat{\rho}(z))$, $G_2(\rho; z) = G_3(\rho)$ only if $\rho = c$ or $\rho = \hat{\rho}(z) \leq \rho_0^m$, and $G_2(\rho; z) > G_3(\rho)$ for all $\rho > \rho_0^m$. Thus, graph $(G_2(\cdot; z))$ and graph (G_3) uniquely intersect at $\rho = \hat{\rho}(z) \leq \rho_0^m$. Since G_1 is always dominated by G_3 (Property 1), and $G_2(\rho; z)$ is increasing in ρ, then graph $(G_2(\cdot; z))$ can only uniquely intersect graph (G_1) at some at a unique point $\tilde{\rho}_i(z)$. This implies that: ***(i)*** each firm's effective profit function is given by

$$R^{ex}(\rho, i, z)\big|_{z\in[\breve{z},z')} = G_1(\rho)\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\mathbf{1}_{\{\tilde{\rho}_i<\rho<\hat{\rho}\}} + G_3(\rho)\mathbf{1}_{\{\hat{\rho}\leq\rho\leq\rho_0^m\}},$$

whenever $z \in [\breve{z}, z')$; ***(ii)*** the maximal price that can exist is the Ramsey monopoly price ρ_0^m; and ***(iii)*** its associated profit outcome is $G_3(\rho_0^m)$ since

$$\rho_0^m = \arg\max_{\rho}\left\{R^{ex}(\rho, i, z)\big|_{z\in[\breve{z},z')}\right\} = \arg\max_{\rho}\{G_3(\rho)\}.$$

(Figure 9 illustrates an example of such a generic case where the solid-red line is the effective profit function.)

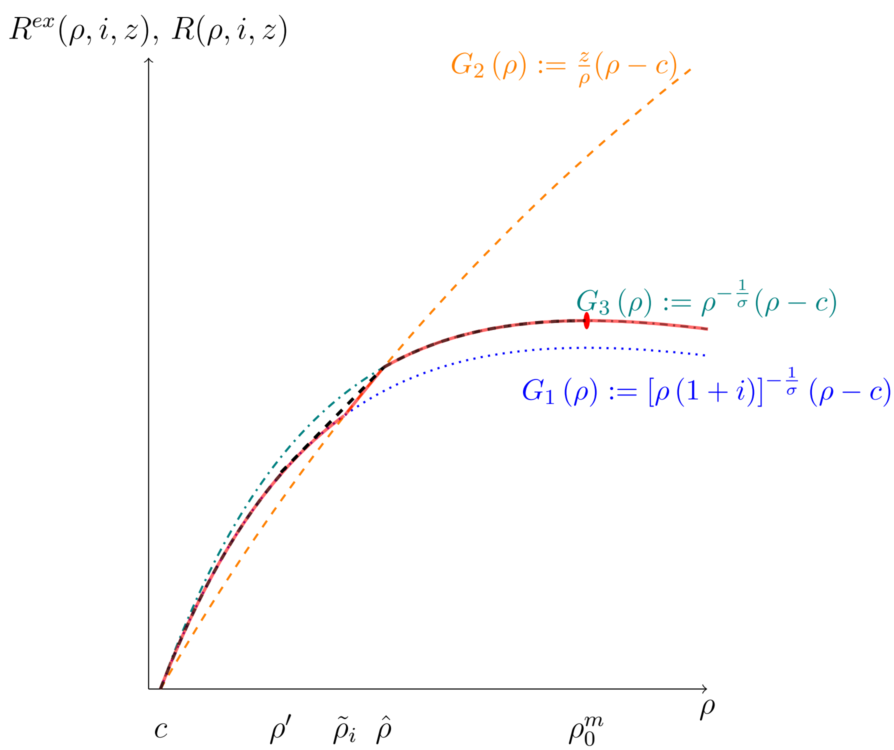

Figure 9: Case 2. Intermediate level of real money balance (I), $[\hat{z}, z')$.

Case 3. **Real money balance z is intermediate (II), $z \in [\tilde{z}_i, \hat{z})$.** Consider next the possible case where $z \in [\tilde{z}_i, \hat{z}) \equiv \left[(1+i)^{-\frac{1}{\sigma}} \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}, \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}\right)$. The cutoff value $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}} \hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}} > \tilde{z}_i$, is such that $\tilde{\rho}_i(\tilde{z}_i) = \rho_0^m$ and $G_2(\tilde{\rho}_i(\tilde{z}_i), \tilde{z}_i) = G_1(\rho_0^m)$. At the given z and i, we have $\tilde{\rho}_i(z) \leq \rho_0^m < \hat{\rho}(z)$ in this case. We can check that $G_2(\rho; z) < G_1(\rho)$ for all $\rho \in (c, \tilde{\rho}_i(z))$, $G_2(\rho; z) = G_3(\rho)$ only if $\rho = c$ or $\rho = \hat{\rho}(z) > \rho_0^m$, and $G_2(\rho; z) > G_3(\rho)$ for all $\rho > \rho_0^m$. This implies that: ***(i)*** each firm's effective profit function is given by

$$G_1(\rho) \mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho) \mathbf{1}_{\{\tilde{\rho}_i<\rho<\rho_0^m\}} + G_3(\rho) \mathbf{1}_{\{\hat{\rho}\leq\rho\}},$$

whenever $z \in [\tilde{z}_i, \hat{z})$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}(z)$; and ***(iii)*** its associated profit outcome is $G_3(\hat{\rho}(z))$ since

$$\hat{\rho}(z) = \arg\max_{\rho} \left\{ R^{ex}(\rho, i, z)\big|_{z\in[\tilde{z}_i,\hat{z})} \right\} = \arg\max_{\rho} \{G_3(\rho)\}.$$

(Figure 12 illustrates an example of such a generic case where the solid-red line is the effective profit function.)

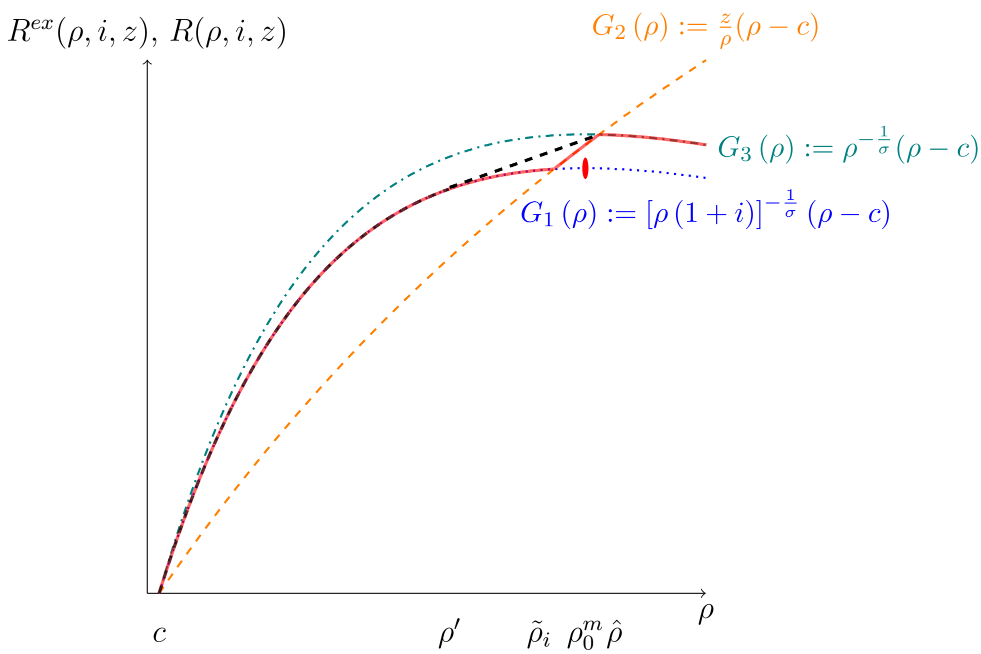

Figure 10: Case 3. Intermediate level of real money balance (II), $z \in [\tilde{z}_i, \hat{z})$.

Case 4. **Real money balance z is intermediate (III), $z \in [\breve{z}, \tilde{z}_i)$.** Consider the case where $z \in [\breve{z}, \tilde{z}_i) \equiv \left[(1-c) + \sigma\hat{z}, (1+i)^{-\frac{1}{\sigma}}\hat{z}\right)$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$. The parameter restriction (sufficient condition) $0 < \breve{z} := (1-c) + \sigma\hat{z} < \hat{z}$ also implies that $[(1-c) + \sigma\hat{z}]\,\hat{z}^{-1} < 1$, and therefore, $\bar{i} := [\sigma + (1-c)\hat{z}^{-1}]^{-\sigma} - 1 > 0$. If $i \in [0, \bar{i}]$, then the interval $[\breve{z}, \tilde{z}_i)$ has positive measure. From Property 2 and the previous case, we can also deduce that if $z \geq \breve{z} := (1-c) + \sigma\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, then graph$\,(G_2(\cdot; z))$ and graph$\,(G_3)$ uniquely intersect at $\rho = \hat{\rho}(z) > \rho_0^m$ and $G_3(\hat{\rho}(z); z) > G_1(\rho_0^m)$. Following from the last case, we can see that when $z < \tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, then $\tilde{\rho}_i(z) > \rho_0^m$, and from Properties 1 and 4 it must be that $G_2(\tilde{\rho}_i(z), z) < G_1(\rho_0^m)$. In this case we have $c < \rho_0^m < \tilde{\rho}_i(z) < \hat{\rho}(z)$. This implies that: ***(i)*** each firm's effective profit function is given by

$$G_1(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\left\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}\right\}} + G_3(\rho)\,\mathbf{1}_{\{\hat{\rho}\leq\rho\}},$$

whenever $z \in [\breve{z}, \tilde{z}_i)$; ***(ii)*** the maximal price that can exist is the the maximal willingness to pay of the money-constrained buyer, $\hat{\rho}(z)$; and ***(iii)*** its associated profit outcome is $G_3(\hat{\rho}(z))$ since

$$\hat{\rho}(z) = \arg\max_{\rho}\left\{ R^{ex}(\rho, i, z)\big|_{z\in[\breve{z},\tilde{z}_i)}\right\} = \arg\max_{\rho}\{G_3(\rho)\}.$$

(Figure 11 illustrates an example of such a generic case where the solid-red line is the effective profit function.)

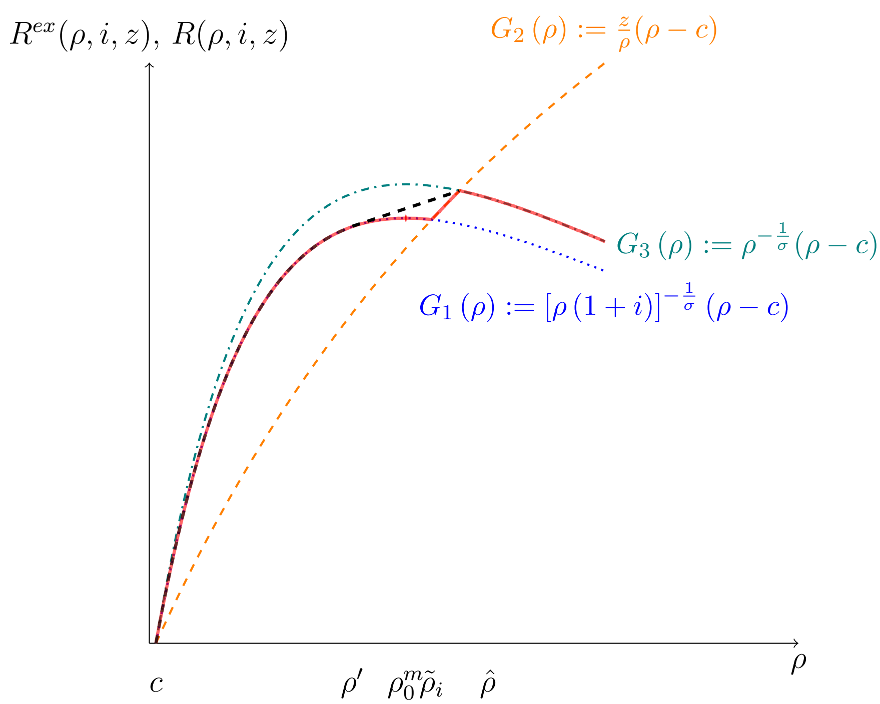

Figure 11: Case 4. Intermediate level of real money balance (III), $z \in [\breve{z}, \tilde{z}_i)$.

Case 5. **Real money balance z is sufficiently low, $z \in (0, \breve{z}) \equiv \left(0, (1-c) + \sigma\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}\right)$.** From Property 2 and the previous case, we can also deduce that if $z < \breve{z}$, then graph $(G_2(\cdot; z))$ and graph (G_3) uniquely intersect at $\rho = \hat{\rho}(z) > \rho_0^m$ so that $G_3(\hat{\rho}(z); z) < G_1(\rho_0^m)$. Thus, we have: ***(i)*** the effective profit function is given by

$$R^{ex}(\rho, i, z)\big|_{z\in(0,\breve{z})} = G_1(\rho)\,\mathbf{1}_{\{c<\rho\leq\tilde{\rho}_i\}} + G_2(\rho)\,\mathbf{1}_{\left\{\rho_0^m<\tilde{\rho}_i<\rho<\hat{\rho}\right\}} + G_3(\rho)\,\mathbf{1}_{\{\hat{\rho}\leq\rho\}};$$

(ii) the maximal price that can exist is the Ramsey monopoly pricing outcome ρ_0^m, and ***(iii)*** its associated profit outcome is $G_1(\rho_0^m)$. (Figure 12 illustrates an example of such a generic case where the solid-red line is the effective profit function.)

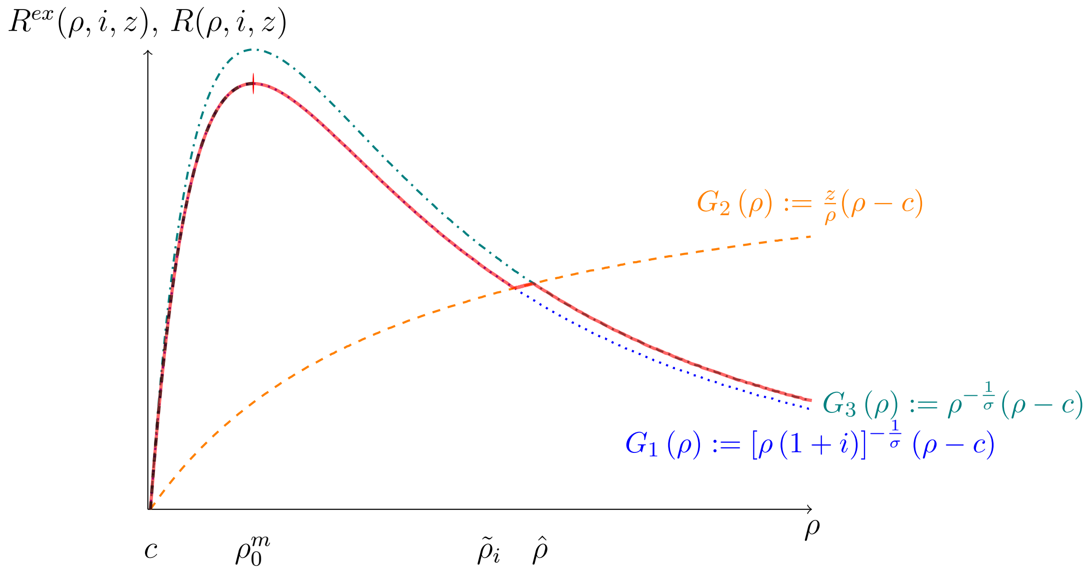

Figure 12: Case 5. Real balance is sufficiently low, $z \in (0, \breve{z})$.

□

B.2 Proof of Lemma 2

In Burdett and Judd (1983) and Head et al. (2012), firms are assumed to commit to implementing the outcomes of their posted (pure) prices. In our extension, as Lemma 2 implies, there may be states of the world where it is (weakly) profitable for firms to commit to posting lotteries over prices (i.e., terms of trade, given buyer demand). Proposition 2 provides the characterization of the *effective (per-trade) profit function* for a firm committed to posting such random contracts. From each buyer's perspective, the possible lotteries are already compounded or internalized in their perceived (and actual) equilibrium distribution of prices $J(\cdot, i, z)$. Thus buyers in equilibrium will be drawing from the distribution $J(\cdot, i, z)$, just as in Burdett and Judd (1983) and Head et al. (2012).

In the proof below, we re-write Proposition 2 in terms of stationary variables.

Lemma. *Given aggregate outcomes (z, i), and the parametric assumptions and ex-post profit function $R^{ex}(\cdot, i, z)$ in Equation (B.1) in Lemma 1, there exists a $z' = \hat{z}\left(\frac{1}{1-\sigma}\right)^{-\left(1-\frac{1}{\sigma}\right)}$ and $\tilde{z}_i := (1+i)^{-\frac{1}{\sigma}}\hat{z}$, where $\hat{z} := \left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$ and $\breve{z} := (1-c)+\sigma\left(\frac{c}{1-\sigma}\right)^{1-\frac{1}{\sigma}}$, such that $0 < \breve{z} < \tilde{z}_i \leq \hat{z} < z' < \infty$. A firm's **effective profit** at any given reference price ρ, $R(\rho, i, z)$, is the value induced by its commitment to ex-ante posted lotteries over prices:*

$$R(\rho, i, z) = \max_{\pi\in[0,1],\rho_1,\rho_2} \left\{\pi R^{ex}(\rho_1, i, z) + (1-\pi) R^{ex}(\rho_2, i, z) : \pi\rho_1 + (1-\pi)\rho_2 = \rho\right\}. \tag{B.5}$$

The function $R(\cdot, i, z)$ is strictly increasing on $[c, \rho^m)$, and is concave over the firm's effective domain of pricing outcomes $[c, \rho^m]$, where the (real) monopoly price and its effective profit outcome, respectively, are

$$\rho^m = \begin{cases} \rho_0^m, & z \in [z', \infty) \\ \rho_0^m, & z \in [\hat{z}, z') \\ \hat{\rho}(z), & z \in [\tilde{z}_i, \hat{z}) \\ \hat{\rho}(z), & z \in [\breve{z}, \tilde{z}_i) \\ \rho_0^m, & z \in (0, \breve{z}) \end{cases}, \text{ and, } R(\rho^m, i, z) = R^{ex}(\rho^m, i, z) = \begin{cases} G_3(\rho_0^m), & z \in [z', \infty) \\ G_3(\rho_0^m), & z \in [\hat{z}, z') \\ G_3(\hat{\rho}(z)), & z \in [\tilde{z}_i, \hat{z}) \\ G_3(\hat{\rho}(z)), & z \in [\breve{z}, \tilde{z}_i) \\ G_1(\rho_0^m), & z \in (0, \breve{z}) \end{cases}. \tag{B.6}$$

Proof. From (the proof of) Lemma 2, we have deduced that the unique maximizer ρ^* for the ex-post profit function $R^{ex}(\cdot, i, z)$ exists. Moreover, the maximum value $R^{ex}(\rho^*, i, z)$ only arises at the upper bound of the feasible-pricing domain $[c, \rho^m]$, i.e., $\rho^* = \rho^m$, and ρ^m is characterized by Equation (B.2). By definition of the lottery problem in (B.5), it is immediate that $R(\rho, i, z) = R^{ex}(\rho, i, z)$ if there is no neighborhood $[\rho_1, \rho_2]$ containing ρ, such that $\pi R^{ex}(\rho_1, i, z) + (1-\pi) R^{ex}(\rho_2, i, z) > R^{ex}(\rho, i, z)$. That is, any lottery would be (locally) degenerate whenever $R^{ex}(\cdot, i, z)$ is already strictly concave on any such subdomains $[\rho_1, \rho_2]$. Otherwise, $R(\rho, i, z)$ is given by the right-hand-side operator in Equation (B.5). Since $R^{ex}(\cdot, i, z)$ has a minimum at c and a unique maximum at $R^{ex}(\rho^m, i, z)$, then the convexification through (B.5) implies that $R(\rho, i, z)$ is strictly increasing in

$\rho \in [c, \rho^m)$ and it is concave over $[c, \rho^m]$. $\square$

B.2.1 Efficient representation of the effective profit function

Given aggregate outcomes (i, z), we can define

$$\text{co}R^{ex}_{i,z} = \text{co}\left\{(\rho, R^{ex}(\rho, i, z)) : \rho \in [c, p^m]\right\},$$

i.e., the convex hull of the graph of $R^{ex}(\cdot, i, z)$ restricted to the feasible pricing domain of $[c, p^m]$, where R^{ex} is defined in (B.1) and p^m is governed by (B.6). The set of points in $\text{co}R^{ex}_{i,z}$ other than those in the set Graph $\{R(\cdot, i, z)\}$ can be defined in advance as

$$U_{i,z} := \text{int}\left(\text{co}R^{ex}_{i,z}\right) \cup \text{int}\left\{(\rho^m, r) : r \in [0, R(\rho^m, i, z)]\right\},$$

where $R(\rho^m, i, z)$ is pinned down by (B.6). The effective profit function in (B.5) can be equivalently represented as

$$\text{Graph}\{R(\cdot, i, z)\} \ = \text{co}R^{ex}_{i,z} \setminus U_{i,z}. \tag{B.7}$$

This equivalent representation of (B.5) will be computationally convenient since the function domain is closed and bounded, and its graph is always convex. What this means is that open-source and industry-standard convex-hull algorithms, in combination with shape-preserving spline approximants and set-valued logical operations can be employed to represent (B.7) precisely, efficiently, and continuously. Crucially, we can avoid having to solve the brute-force optimization in the representation (B.5), and its associated tangent-search problem (which in practice would involve imprecise discretized approximations).

XXXXX NOT DONE BELOW XXXXXXXXXXXXXXX

B.3 Proof of Lemma 5

In Section B.3, we study how the price distribution J_i changes with respect to the asset position of the households. We then establish the existence of a stationary monetary equilibrium with both money and credit in Section B.4.

Proof. Fix the trend inflation rate away from the Friedman rule $\tau > \beta - 1$. Assume $\alpha_1 \in (0, 1)$. Let $i = i_d = i^\star$ be the market loan interest rate. By Lemma 3, the analytical formula for the real price distribution $J_i(\rho, z)$ is given by

$$J_i(\rho, z) := J_i(\rho, i, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z)}{R(\rho, i, z)} - 1\right] = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{q^\star_b(\overline{\rho}, i, z)(\overline{\rho} - c)}{q^\star_b(\rho, i, z)(\rho - c)} - 1\right], \tag{B.8}$$

where the upper bound on the support of the distribution $J_i(\cdot, z)$ is determined by:

$$\overline{\rho} := \overline{p}(z, \mathbf{s}) = \begin{cases} \max\{c/(1-\sigma), \underbrace{z^{\sigma/(\sigma-1)}}_{=\hat{\rho}}\} & \text{if } z > \breve{z} \\ \frac{c}{1-\sigma} & \text{if } z \leq \breve{z} \end{cases}, \tag{B.9}$$

given $\breve{z} := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]$, and the lower bound on the support of $J_i(\cdot, z)$, $\underline{\rho}$, solves $R(\rho, i, z) = \frac{\alpha_1}{\alpha_1+2\alpha_2} R(\overline{\rho}, i, z)$.

In this proof, we want to show the relationship of how the price distribution $J_i(\cdot, z)$ changes with respect to the change in the real money holdings z. Consider two real money holdings z and $z^{'}$ such that $\breve{z} < z < z^{'} < \overline{z}$. Then we want to check whether $J_i(\cdot, z)$ is lying on top or below for z relative to $z^{'}$.

Note: For the ease of notation, we will denote $\overline{\rho}(z)$ and $\underline{\rho}(z)$ respectively by $\overline{\rho}$ and $\underline{\rho}$ occasionally. Likewise, we denote the cut-off prices by

$$\hat{\rho} := \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}} \tag{B.10}$$

and

$$\tilde{\rho}_i := \tilde{\rho}_i(z, i) = \hat{\rho}(z)(1+i)^{\frac{1}{\sigma-1}}. \tag{B.11}$$

It should be kept in mind that all these cut-off prices and bounds of the price distribution depend on the state of the economy z and policy τ in general.

Suppose a real money balance z that satisfies: $\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} = \breve{z} < z < \overline{z} = \underline{\rho}^{\frac{\sigma-1}{\sigma}}(1+i)^{\frac{-1}{\sigma}}$. Recall that the CRRA risk aversion parameters requires to be $\sigma < 1$, and from the result established earlier in Section B.1, we then have the following order:

$$\underline{\rho} < \tilde{\rho}_i < \hat{\rho} \leq \overline{\rho}.$$

Observe from the upper bound of the price distribution, it has two possible cases either $\overline{\rho} = \hat{\rho}$ or $\rho = c/(1-\sigma)$ when $z \in (\breve{z}, \overline{z})$. We need to check for each case. □

Case 1.

Proof. Suppose $\overline{\rho} = \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$.

Given the buyer's optimal demand schedule defined in Equation (2.28) and $z \in (\breve{z}, \overline{z})$, we can write out the price distribution $J_i(\cdot, z)$ more explicitly by

$$J_i(\rho, z) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1}z(\overline{\rho}(z)-c)}{\rho^{-1}z(\rho-c)} - 1\right] & \text{if } \rho \in (\tilde{\rho}_i(z), \overline{\rho}(z)] \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1}z(\overline{\rho}(z)-c)}{[\rho(1+i)]^{-1/\sigma}(\rho-c)} - 1\right] & \text{if } \rho \in [\underline{\rho}(z), \tilde{\rho}_i(z)] \end{cases}, \tag{B.12}$$

with support $[\underline{\rho}(z), \overline{\rho}(z)]$.

Consider any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$. First, it is clear that $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. Using this result and the equal profit condition of the firms, we can then deduce the lower support also satisfies that $\underline{\rho}(z_0) > \underline{\rho}(z_1)$. Thus, we have $[\overline{\rho}(z_0) - \underline{\rho}(z_0)] - [\overline{\rho}(z_1) - \underline{\rho}(z_1)] > 0$. In words, the support of the price distribution with lower real money balance is wider than that with higher real money balance.

Second, for $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$, then $J_i(\rho, z_0) < J_i(\rho, z_1)$ because of $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. The intuition is that buyers with lower money holdings are more likely to be liquidity constrained, and that pushes up the measure of firms posting higher prices. Thus, $J_i(\rho, z_0)$ falls below $J_i(\rho, z_1)$ for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$.

Next, by the fact that the price distribution J_i is a cumulative distribution function, it then follows that $J_i(\rho, z_0) = J_i(\rho, z_1) = 1$ for some $\rho \geq \overline{\rho}(z_0)$. Likewise, we have $J_i(\rho, z_0) = J_i(\rho, z_1) = 0$ for some $\rho \leq \overline{\rho}(z_1)$.

Collect these results, we have then established that $J_i(\rho, z_0)$ first-order stochastically dominates $J_i(\rho, z_1)$. That is, $J_i(\rho, z_0) \leq J_i(\rho, z_1)$ within the interval $[\underline{\rho}(z_1), \overline{\rho}(z_0)]$, and strict inequality for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$ given any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$. $\square$

Case 2.

Proof. Suppose $\overline{\rho} = c/(1-\sigma)$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \hat{\rho}(z) < \overline{\rho}$.

Likewise, we can write out the price distribution $J_i(\rho, z)$ explicitly by

$$J_i(\rho, z) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{\rho^{-1/\sigma}(\rho-c)} - 1\right] & \text{if } \rho \in [\hat{\rho}(z), \overline{\rho}(z)] \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{\rho^{-1}z(\rho-c)} - 1\right] & \text{if } \rho \in (\tilde{\rho}_i(z), \hat{\rho}(z)) \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{[\rho(1+i)]^{-1/\sigma}(\rho-c)} - 1\right] & \text{if } \rho \in [\underline{\rho}(z), \tilde{\rho}_i(z)] \end{cases}, \tag{B.13}$$

with support $[\underline{\rho}(z), \overline{\rho}(z)]$.

The proof strategy for this case is similar to Case 1 above. The only difference is that the upper support of the price distribution is independent of real money holding z, i.e., $\overline{\rho}(z_0) = \overline{\rho}(z_1)$

where $z_0 \neq z_1$. However, we can deduce the following order for the cut-off prices

$$\tilde{\rho}_i(z_0) > \tilde{\rho}_i(z_1),$$

$$\hat{\rho}(z_0) > \hat{\rho}(z_1),$$

and the lower support satisfies

$$\underline{\rho}(z_0) > \underline{\rho}(z_1),$$

given any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

For $\rho \in (\tilde{\rho}_i(z_1), \hat{\rho}(z_0))$, then $J(\rho, z_0) < J(\rho, z_1)$ because $z_0 < z_1$. Hence, $J(\rho, z_0)$ first order stochastically dominates $J(\rho, z_1)$ given two real money holdings z_0, z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

Finally, from (B.10) and (B.11), since $\sigma < 1$, we can deduce that the lowest admissible price draws for a money unconstrained money-buyer ($\hat{\rho}$) and for a constrained money-buyer ($\tilde{\rho}_i$) are decreasing functions of z. Hence as z falls, these pricing cutoff functions increase in value. □

Remark. The proof for the first-order stochastic dominance result when $0 < z \leq \breve{z}$ is similar to the case shown above. We leave out the details of this case here.

B.4 Proof of Proposition 1

Proof. Given policy $\gamma = 1 + \tau > \beta$ and the distribution $J_i(\cdot, z)$, the equilibrium condition for optimal real money demand z is:

$$\begin{aligned}\frac{\gamma-\beta}{\beta} - \alpha_0 i_d = \alpha_1 i \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} i \mathrm{d}J_i(\rho, z) + \alpha_2 i \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} \mathrm{d}(1 - [1 - J_i(\rho, z)]^2) \\ + \alpha_1 \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \left[\left(\frac{z}{\rho}\right)^{-\sigma} \frac{1}{\rho} - 1\right] \mathrm{d}J_i(\rho, z) \\ + \alpha_2 \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \left[\left(\frac{z}{\rho}\right)^{-\sigma} \frac{1}{\rho} - 1\right] \mathrm{d}(1 - [1 - J_i(\rho, z)]^2).\end{aligned} \tag{B.14}$$

The left-hand side of Equation (B.14) is constant with respect to z. To establish existence of optimal money holdings, it remains to verify whether the right-hand side is monotone increasing/decreasing in z. Let the function $\chi(z)$ denote the right-hand side of Equation (B.14).

Consider any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$. The result in Lemma 5 establishes that $J_i(\cdot, z_0)$ first order stochastically dominates $J_i(\cdot, z_1)$, and consequently, $1 - [1 - J_i(\cdot, z_0)]^2$ also first order stochastically dominates $1 - [1 - J_i(\cdot, z_1)]^2$.

From this result and the fact that $(z/\rho)^{-\sigma}/\rho - 1$ is monotone decreasing in z, it then follows that $\chi(z)$ is monotone decreasing in z. Hence, there exists a unique $z = z^\star$ that solves Equation (B.14).

Next, we want to verify the loans market clearing condition. The cut-off price $\tilde{\rho}_i(z)$ (when buyers borrow additional funds from the bank) satisfies $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$. Then, there is always positive loan demand by a measure of buyers who draw low enough prices such that $\underline{\rho}(z) \leq \rho \leq \tilde{\rho}_i(z)$. Since the credit market is perfectly competitive, then the total loans has to equal to total deposits in equilibrium:

$$\alpha_0 z = \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} (\alpha_1 + 2\alpha_2 - 2\alpha_2 J_i(\rho, z)) \cdot \xi^*(\rho, i, z) \mathrm{d}J_i(\rho, z), \tag{B.15}$$

given $z = z^\star$ determined by Equation (B.14). □

Remark. The proof for the existence of real money holdings when $0 < z \leq \breve{z}$ is similar to the case shown above. We omit the discussion of this case.

C Statistical calibration of model

We perform the numerical analyses based on the model that is disciplined by calibration to relevant macro-level statistics in the United States.

C.1 Baseline calibration

We interpret one period in the model to be a year. Our calibration strategy is to match the empirical money demand and the firms' average (percentage) markup in the United States.

The aggregate output in the DM is $q_{DM} := \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho q_b^\star(\rho, z) \mathrm{d}J_i(\rho, z, \gamma)$. The aggregate output in our economy is given by:

$$Y = q_{DM} + x^\star. \tag{C.1}$$

Given policy $\gamma = 1+\tau$, we measure the model's aggregate (percentage) markup as by the weighted average of percentage markups in both markets in the model:

$$\mu(\gamma) = \omega_{DM} \underbrace{\left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \frac{\rho - c}{c} \mathrm{d}J_i(\rho, z, \gamma) \right]}_{\mu_{DM}(\gamma)} + (1 - \omega_{DM}) \cdot 0 \equiv \omega_{DM} \mu_{DM}(\gamma), \tag{C.2}$$

where the weight on DM is $\omega_{DM} := q_{DM}/Y$. The gross markup in the CM is unity (or its percentage markup is zero) since firms are perfectly competitive there. Price dispersion (coefficient of variation) is defined as:

$$CV(\gamma) = \frac{1}{\mu(\gamma)} \left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} (\rho - \breve{\rho})^2 \mathrm{d}J_i(\rho, z, \gamma) \right]^{\frac{1}{2}}, \tag{C.3}$$

where $\breve{\rho} = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho \mathrm{d}J_i(\rho, z, \gamma)$.

We assume a log-utility function in the CM, $U(x) = B\ln(x)$, where B is a scaling parameter that determines the relative importance of CM and DM consumption. With quasi-linear preferences, real CM consumption is determined by $x^{\star} = (U')^{-1}(B)$. The noisy-search probabilities in the DM can be re-parametrized by a number λ. That is, we can set $\alpha_0 = (1-\lambda)^2$, $\alpha_1 = 2(1-\lambda)\lambda$, and $\alpha_2 = \lambda^2$. We normalize the cost of DM production to one ($c = 1$) as in Head et al. (2012). The DM utility function is given by Equation (2.2).

Sample period and data. Our model is fitted to long-run data spanning from 1980 to 2007 to avoid the Great Recession period where the nominal interest rate is at the zero lower bound. We use the New M1-to-GDP ratio defined in Lucas and Nicolini (2015) as a measure of the money demand M/PY in the United States. We employ the U.S. markup data from De Loecker et al. (2020). We obtain the U.S. three-month T-bill interest rate data from the FRED.

Identification and calibration. The parameters that need to determined are: β, τ, σ, B, and λ. The parameter β is the time discount factor. The CM utility scaling parameter B affects the average of money demand M/PY. This is because the parameter B affects CM consumption x and thus output Y. The CRRA risk aversion parameter σ pins down the price elasticity of demand for the DM consumption goods, which affects the elasticity of money demand with respect to the nominal interest rate i. The noisy-search probabilities, via λ, directly affect the price distribution J_i, and thus the aggregate markup.

From the Fisher equation, we use both the average interest rate of the three-month T-bill, $i = 0.058$, and the long-run inflation rate, $\tau = 0.038$, to pin down the discount factor $\beta = 0.98$. The remaining parameters (σ, B, λ) are calibrated internally. We jointly choose (σ, B, λ) to match the point elasticity of money demand, the average of money demand M/PY, and aggregate markup μ, all of which are with respect to the nominal interest rate i. Figure 13 depicts a reasonable fit between the calibrated model's implied aggregate money demand curve (*i.e.*, the green-solid graph) and that of the data (blue dots). (The data observations are the blue-circled markers and an empirical spline-model best fit of these sample points is given by the dashed-red graph.)

We summarize the value of jointly calibrated parameters and calibration results in Table 2. Given a reasonable fit of our model to the empirical targets, we can use the calibration above as a benchmark model.

Figure 13: Aggregate money demand calibration (result)

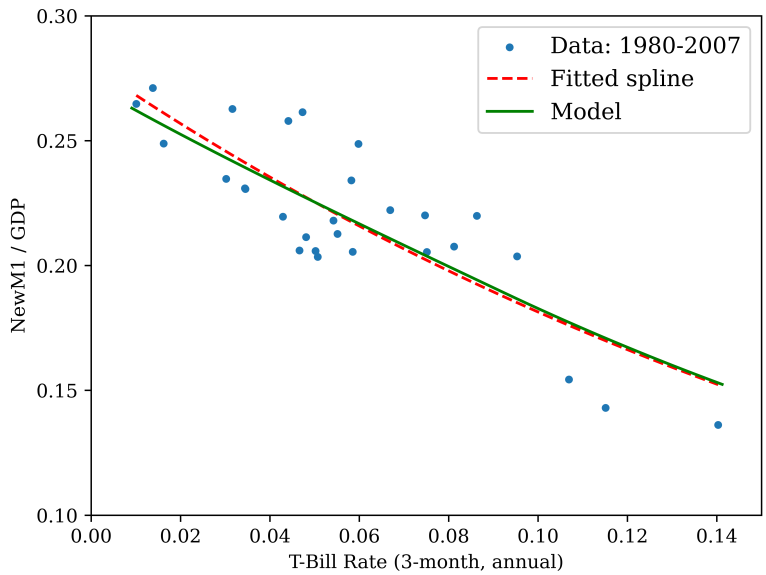

Table 2: Calibration targets and results

Parameter	Value	Empirical Targets	Model
σ	0.28	Elasticity of $M/PY = -0.25^a$	-0.2
B	1.9	Mean of $M/PY = 0.22$	0.2
$\{\alpha_n\}_{n\in\{0,1,2\}}$	$\lambda = 0.62^b$	Markup = 30%	28%

[a] The point elasticity refers to the elasticity of M/PY with respect to the nominal interest rate i, evaluated at the data mean of i.
[b] $\alpha_0 = (1-\lambda)^2$, $\alpha_1 = 2(1-\lambda)\lambda$, and $\alpha_2 = \lambda^2$.

D Data and measurement

We put together data on quarterly consumer credit-to-GDP ratio, the aggregate markup and its dispersion in the United States. The data sample period is from 1980Q1 to 2007Q4. The summary statistics concerning the data are in Table 3.

Consumer Credit-to-GDP (%) **and Real GDP.** We obtain total consumer credit owned and securitized by depository institutions from FRED.[32] We use the U.S. nominal GDP data from FRED and total consumer credit to compute the consumer credit-to-GDP ratio.

[32] Data source: https://fred.stlouisfed.org/series/TOTALDI

Table 3: Data sources and summary statistics

Variable	**Source**	**Mean**	**Median**	**S.D**
Aggregate markup	Compustat	1.37	1.38	0.09
Markup dispersion	Compustat	0.11	0.11	0.04
Consumer credit-to-GDP (%)	FRED	2.08	2.06	0.26
CPI inlfation rate (%)	FRED	3.86	3.19	2.57
Real GDP	FRED	0.74	0.80	0.71
Business sector TFP (%)	Fernald (2014)	0.86	1.11	2.78
Real Wage	FRED	2.23	2.92	2.90
Real Exchange rate	BIS	87.44	84.82	9.52
Real Interest rate	FRED	2.48	2.80	2.38

Note: All data series are from 1980 (Q1) to 2007 (Q4). Compustat data is from the Wharton Research Data Services. FRED stands for Federal Reserve Economic Data, while BIS refers Bank for International Settlements. Real GDP is calculated as the growth rate from the previous period.

Markup. To calculate the aggregate markup measures, we use *Compustat* data on the quarterly balance sheets of publicly listed firms in the United States.[33] We use quarterly firm-level balance sheet data of listed U.S. firms for the period 1980Q1 to 2007Q4 from Compustat North America. Following De Loecker et al. (2020), our industry classification is on the basis of the North American Industry Classification System (NAICS). Particularly, we observe measures of input expenditure, sales, detailed industry activity classifications, and capital stock information. The item from the financial statement of the firm that we will utilize to measure the variable input is the cost of goods sold (COGS). It bundles all expenses directly attributable to the production of the goods sold by the firm and includes materials and intermediate inputs, labor cost, energy, and so on.

Following Hall (1988) and De Loecker et al. (2020), we compute firm-level markups based on the production approach. This approach estimates markup derived from an assumption that firms minimize their cost.

In each period t, an individual firm i markup μ_{it} is defined as

$$\mu_{it} = \theta_{it}^{V} \frac{P_{it}}{P_{it}^{V}} \frac{Q_{it}}{V_{it}}, \tag{D.1}$$

where P_{it}, P_{it}^{V}, V_{it} and Q_{it}, respectively, denote the output price, price of the variable input, the variable input, and output.

According to Equation (D.1), an individual firm's markup comprises two components: (1) The revenue share of the variable input is specified as $\frac{P_{it}}{P_{it}^{V}} \frac{Q_{it}}{V_{it}}$; and (2) the output elasticity of the variable input measured by θ_{it}^{V}.

We fix the output elasticity to be time-invariant (0.85). This assumption is consistent with the empirical evidence documented in De Loecker et al. (2020).[34] Then, we compute the aggregate

[33] Data source: Wharton Research Data Services.

[34] The authors document that the pattern of markup with fixed output elasticity (0.85) is similar to that using estimated output elasticities.

markup as

$$\mu_t = \sum m_{it}\mu_{it}, \tag{D.2}$$

where m_{it} is the weight of each firm.[35]

Markup dispersion. Following Meier and Reinelt (2022), we compute aggregate markup dispersion in period t as

$$\nu_t = \sum_i m_{it}\left[\log(\mu_{it}) - \log(\mu_t)\right]^2, \tag{D.3}$$

where m_{it} is the weight of each firm, and μ_{it} and μ_t are respectively determined by Equation (D.1) and Equation (D.2).

In short, aggregate markup dispersion is just the weighted average of the log deviation of an individual markup from the aggregate markup.

Control variables. Our control variables include: log of real GDP, CPI(Consumer Price Index) inflation rate, business sector, real wage, real interest rate, real effect exchange rate, and Total Factor Productivity (TFP) growth rate. Note that real interest rate is calculated by subtracting the inflation rate from the Federal funds effective rate, and real wage is calculated by deducting the inflation rate from the nominal growth wage rate. We obtain data on real GDP, CPI inflation rate, federal fund effective rate, and wage growth rate from the FRED.[36] We obtain the business sector TFP data from Fernald (2014).[37] The real effective exchange rate data is from the BIS.

E Robustness test

In this section, we present our robustness tests regarding the empirical results discussed in Section 4. We implement the following alternative empirical tests or measurements as a robustness check: (1) vector error correction model (VECM); (2) we include the lag term; (3) we use different credit measure, (4) dummy variable for the Volcker period (1979 to 1987).

VECM. Prior to estimation, we use a unit root test to investigate integration properties of the data. We find that the log of markup, markup dispersion, consumer credit–to–GDP, log of real

[35] We employ the share of sales in the sample as the weight. See De Loecker et al. (2020) for more details.

[36] Real GDP data comes from https://fred.stlouisfed.org/series/GDPC1. We can get CPI inflation rate from https://fred.stlouisfed.org/series/CPIAUCSL. Wage and salary comes from https://fred.stlouisfed.org/series/A132RC1Q027SBEA. Federal fund effective rate can be obtained from https://fred.stlouisfed.org/series/FEDFUNDS.

[37] See https://www.johnfernald.net/TFP.

GDP, and log of real exchange rate as $I(1)$.[38] The vector error correction model is given by

$$\Delta X_t = \gamma + \sum_{j=1}^{p-1} \Gamma_j \Delta X_{t-1} + \epsilon_t, \tag{E.1}$$

where γ is a vector of intercepts, ϵ_t is a vector of contemporaneous errors, X_t is the three–dimensional vector containing the variables, and Γ_j is a set of matrices of short-run coefficients.

Since there is a positive relationship between markup and markup dispersion and the sample is relatively small, then we need to minimize the number of variables. Then, we utilize the two VECM specifications: 1) log of markup, log of real GDP, consumer credit–to–GDP, and log of real exchange rate; and 2) markup dispersion, log of real GDP, consumer credit–to–GDP, and log of real exchange rate. We set the $p(lag) = 5$ based on information criteria in both empirical specifications.

First, we employ the four–dimensional vector including the variables: log of markup, consumer credit–to–GDP (%), the log of real GDP, and log of real exchange rate to investigate the long-run relationship between log of markup and consumer credit–to–GDP. The Johansen Trace indicates one cointegrating vector among four variables at the 1% level of significance. (Below, we denote an estimated coefficient with such a level of statistical significance with a three-asterisk supercript or ***.) Based on the VECM analysis, we show that the estimated long-run relationship is

$$\log(\mu_t) = 61.07^{***} \cdot d_t^{CC} - 0.36^{***} \cdot log(RGDP_t) - 0.12 \cdot log(REER_t) + 293.74. \tag{E.2}$$

Equation (E.2) suggests that there is a positive long-run relationship between markup and consumer credit–to–GDP d_t^{CC}. The estimate is statistically significant.

To estimate the long-run effect of consumer credit on markup dispersion, we utilize the four–dimensional vector including the variables: markup dispersion, consumer credit–to–GDP (%), the log of real GDP, and log of real exchange rate. The Johansen Trace indicates one cointegrating vector among three variables at the 1% level of significance. We can obtain the estimated long-run equation as

$$\nu_t = 14.6^{***} \cdot d_t^{CC} - 0.02 \cdot log(RGDP_t) - 0.06 \cdot log(REER_t) + 30.7. \tag{E.3}$$

Equation (E.3) suggests that the long run relationship between markup dispersion and consumer credit–to–GDP (%) is positive, and the estimate is statistically significant. These results support our empirical validity for the result of OLS in Section 4.

Including the Lagged Dependent Variable. By utilizing the Durbin-Watson test, we find that there is a serial correlation in the residuals from OLS in Section 4. Then, we include the lagged dependent variable to reduce the serial correlation. Based on the information criteria, we

[38]The CPI Inflation rate is a stationary variable according to unit root test results.

decide to include the one lag terms of dependent variable. The empirical specification is almost identical to Equation (4.1) except it includes the lag terms. As shown in Table 4, the results of the robustness test suggest that the positive association between consumer credit and markup/markup dispersion is valid.

Table 4: OLS results: Markup and Markup Dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	1.870*	2.446***
	(1.100)	(0.925)
CPI Inflation	-0.169*	-0.209**
	(0.0766)	(0.0723)
Log of real GDP	0.0333*	0.0404**
	(0.0154)	(0.0129)
Business TFP	0.0297	0.0564
	(0.0429)	(0.0464)
Real wage	-0.0770	-0.125*
	(0.0658)	(0.0646)
log of Real Exchange rate	-0.0253	-0.00894
	(0.0162)	(0.0150)
Real interest rate	-0.00862	-0.106
	(0.0960)	(0.0794)
R^2	0.962	0.870
Observations	111	111

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

Different Measure: Consumer Credit and Markup Dispersion For robustness check, we use other measure of consumer credit and markup dispersion. First, we utilize different consumer credit measures: (1) the percentage ratio of total consumer credit to GDP or (2) the percentage ratio of consumer loans from commercial banks to GDP. The empirical specifications are similar to Equation (4.1) except for replacing consumer credit–to–GDP with total consumer credit to GDP and consumer loans from commercial banks to GDP.[39]

Table 5 reports the empirical results. Columns (1)—(2) show that the relationship between total credit and markup(and its dispersion) is positive, and the estimates are statistically significant. Furthermore, the association between bank credit on aggregate markup and its dispersion

[39] Total consumer credit owned and securitized comes from https://fred.stlouisfed.org/series/TOTALSL. Consumer loans from all commercial banks is from https://fred.stlouisfed.org/series/CONSUMER.

is statistically significant and positive as shown in Columns (3)—(4). These results support the validity of our main empirical results presented in Section 4.

Table 5: OLS results: Markup and Markup Dispersion

Dependent Variable:	$\log(\mu_t)$ (1)	ν_t (2)	Dependent Variable:	$\log(\mu_t)$ (3)	ν_t (4)
Total Consumer Credit-to-GDP	3.844***	3.640***	Consumer Loan-to-GDP	9.320***	5.517***
	(1.100)	(0.714)		(1.922)	(1.125)
CPI Inflation	-0.652***	-0.423***	CPI Inflation	-0.513***	-0.299***
	(0.010)	(0.069)		(0.093)	(0.0605)
Log of real GDP	0.116***	0.0153	Log of real GDP	0.238***	0.118***
	(0.027)	(0.0182)		(0.0120)	(0.0075)
Business TFP	0.0429	0.0663	Business TFP	0.0635	0.0840+
	(0.070)	(0.046)		(0.066)	(0.048)
Real wage	-0.194**	-0.119*	Real wage	-0.318***	-0.204***
	(0.0875)	(0.0608)		(0.0808)	(0.0628)
log of Real Exchange rate	-0.140***	-0.0477***	log of Real Exchange rate	-0.066***	0.012
	(0.029)	(0.017)		(0.021)	(0.016)
Real interest rate	-0.198	-0.197**	Real interest rate	-0.160	-0.174**
	(0.138)	(0.0821)		(0.128)	(0.083)
R^2	0.919	0.870	R^2	0.926	0.863
Observations	112	112	Observations	112	112

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.

Dummy Variables (Volcker period) We conducted our analysis by incorporating a dummy variable for the Volcker period (1979-1987). This accounts for to the high inflation rates experienced during Paul Volcker's tenure as the Chairman of the Federal Reserve. Even with the inclusion of the dummy variable for the Volcker period, we find a statistically-significant negative relationship between markup (and its dispersion) and inflation. This confirms that our main findings in the paper are robust.

Table 6: OLS results: Markup and Markup Dispersion

Dependent Variable:	Log of Markup: $\log(\mu_t)$ (1)	Markup Dispersion: ν_t (2)
Consumer Credit-to-GDP	3.714**	3.242***
	(1.493)	(0.993)
CPI Inflation	-0.405***	-0.307***
	(0.0947)	(0.0646)
Log of real GDP	0.137***	0.0630***
	(0.0181)	(0.0126)
Business TFP	0.0436	0.0690
	(0.0643)	(0.0489)
Real wage	-0.183**	-0.159**
	(0.0831)	(0.0658)
log of Real Exchange rate	-0.0371	-0.006
	(0.0275)	(0.0200)
Real interest rate	-0.209*	-0.182**
	(0.126)	(0.0835)
Volcker dummy	-3.413***	-0.524
	(0.988)	(0.619)
R^2	0.924	0.854
Observations	111	111

Note: Robust errors are in parenthesis, with $*, **$, and $***$, respectively, denoting a statistical significance level of 10%, 5% and 1%. Constant is included but not reported.