On Endogenous Markups Distribution and the Pecuniary Externality of Credit on Monetary Exchange*

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Abstract

We show that competitive banking amplifies retail-goods firms' ability to extract higher markups from ex-post heterogeneous buyers. This works through a new pecuniary-externality channel that is tightly connected to an equilibrium distribution of retail-goods price markups. Our model generates a positive relationship between the consumer credit-to-GDP ratio and retail price markups (and their dispersion). This prediction is consistent with empirical evidence using firm-level data in the United States. The endogeneity in firms' markup responses to the presence of credit renders competitive banking not always and everywhere a welfare-enhancing proposition. Consequently, the welfare-improving role of banking liquidity transformation is ambiguous. Our model also justifies why policymakers should be worried about rising industry markups.

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1 Introduction

In this paper, we are concerned with retail product market power in the form of a distribution of price markups and its interaction with consumer credit. Recent empirical studies find that industry market power—measured in terms of price markups—has been sharply increasing since the 1980s in the United States (see, *e.g.*, Hall, 2018; Rossi-Hansberg, Sarte and Trachter, 2020; De Loecker, Eeckhout and Unger, 2020). This has prompted a literature that investigates the macroeconomic consequences of industry market power (see, *e.g.*, Guerrieri and Lorenzoni, 2017; Autor, Dorn, Katz, Patterson and Reenen, 2020; Edmond, Midrigan and Xu, 2021). Since the 1980s, the U.S. consumer credit-to-GDP ratio has also been accelerating around the same time as the rise in industry market power. The phenomenon of rising industry market power is not only of interest to academics but also to policymakers. For example, U.S. President Biden has recently called for promoting industry competition in the United States (see Executive Order 14036, 2021).

Our contribution. We apply alternative empirical methods on U.S. data to establish two new pieces of evidence. First, we find a positive association between retail-goods–price markups and the consumer credit-to-GDP ratio.[1] Second, we show that there is a statistically-significant and positive relationship between the dispersion in price markups and the consumer credit-to-GDP ratio.

We then construct a model that can rationalize the evidence. The model features decentralized markets where anonymous agents hold money in order to buy goods.[2] Firms post prices and produce on the spot. As in Head, Liu, Menzio and Wright (2012) (HLMW) and Burdett and Judd (1983), buyers observe a random number of price quotes posted by firms and buy at the lowest price they observe. This induces firms to optimally trade off between charging a higher markup on their goods and a lower probability of contact by buyers. Equilibrium in the model results in firms being indifferent between a continuum of these opposing margins of attaining the same maximal expected profit. This renders an equilibrium, realized distribution of posted (and transacted) prices that will depend on monetary policy and the aggregate amount of money.

In equilibrium, some agents will turn out to be own-money constrained, some can top up on their monies by taking out consumer loans, while others are not liquidity constrained. We show that access to bank credit, even within a perfectly competitive sector, may reduce welfare for money-constrained consumers. Overall, the welfare gain from banking for consumers can be negative when inflation is low and positive when inflation is sufficiently high but not infinite.

[1] Although we restrict our attention to the United States, we have also tested the relationship between credit and markup using a panel dataset of advanced economies. We also find a positive correlation between bank credit or household debt and markups. See Online Appendix A for details.

[2] Anonymity here is taken to mean that sellers cannot observe buyers histories and any private promises to repay cannot be enforced. Thus, money is essential, *i.e.*, it has value in equilibrium as a medium of exchange.

Benefit versus cost of bank credit and inflation. The conventional wisdom on "financial inclusion" is that competitive banks ought to play a welfare-improving role of *liquidity transformation*—*i.e.*, facilitating the intermediation between those with excess liquidity and those who need more. This view is in line with the perfectly-competitive banking model of Berentsen, Camera and Waller (2007) (BCW), so long as money has an inferior return to a risk-free outside option (*i.e.*, the economy is away from the Friedman rule).

The novelty of our paper is that there is equilibrium feedback from the ability of some agents to use bank loans, to agents' decision to hold money, to the distribution of goods-price markups. We show that a first-order stochastic dominance result holds: Lower equilibrium real money balance implies firms are more likely to exact higher markups on agents who are liquidity-constrained money-buyers. Thus, the presence of buyers who find it optimal to borrow from banks create a pecuniary externality effect through the pricing-markup distribution. This tends to reduce the consumption level for buyers who do not use banking credit. We identify two opposing benefit and cost effects.

Consider the *benefit* of banking in the model. It comes in two parts. With access to banks, ex-post inactive buyers (those who do not have a trading opportunity) can deposit idle funds with banks to earn interest. In addition, some active buyers (those who have a trading opportunity) may find it optimal to top up their money with bank credit in order to extend their liquidity constraint. In the model, these two forces imply higher consumption and welfare. We call this overall benefit of banking a *composition effect*, which is also present in BCW.

However, unlike BCW, access to bank credit for some agents can create a pecuniary externality *cost* on others. We emphasize that access to banking is costless in the model. In contrast to BCW, goods price dispersion induces (ex-post) heterogeneity in the composition of payments among buyers. Some buyers will use their money balances and bank loans in goods transactions, while others only use their money balances. We call the former credit-buyers and the latter money-buyers. Among the money-buyers there are those who are liquidity constrained and those who are not. Firms with market power anticipate prospective customers to be credit-buyers or money-buyers. Some firms are willing to charge lower prices to trade with the credit-buyers for a larger sales volume. In contrast, other firms trading with money-buyers (who demand fewer goods) will have a lower sales volume. Consequently, these firms need to charge higher price markups on money-buyers to maintain an equal expected profit as the low-price firms in equilibrium. Hence, banking can only benefit those credit-buyers who trade with low-price firms but lowering consumption levels for money-buyers who suffer from high markups. We call this latter negative welfare effect of banking a *price dispersion effect*.

In short order, banking can improve welfare for those with idle money or those who are willing to borrow. However by encouraging less own-money holdings, banking also amplifies goods-price markups' dispersion and average which makes money-constrained buyers worse off. This trade-off, as we will show is sensitive to inflation, and thus, to monetary policy. We discipline the model by calibrating it to the data. We numerically show the following: In contrast to the model without banks (*i.e.*, the HLMW model) average markups under a competitive-banking equilibrium is always higher. Likewise, the dispersion of markups is also higher in the banking equilibrium. The gaps in these measures between the banking equilibrium and the HLMW limit are increasing with inflation. For plausibly low inflation ranges, banking is welfare reducing since for low inflation the gains from banking to depositors of idle money and credit-buyers is small compared to the dispersion effect on money-constrained agents. For sufficiently high inflation, the result reverses.

Related literature. Our result on the negative welfare effect of credit is comparable to that established in Chiu, Dong and Shao (2018). The authors also consider a perfectly competitive banking sector, focusing on banking's role in reallocating idle liquidity, as in Berentsen et al. (2007). In their model, access by borrowers to credit raises the *homogeneous* price level of the goods traded in a decentralized market: more demand for goods by credit-buyers raises the marginal cost of production. With competitive price-taking, this translates to a higher goods price in the authors' model. This pecuniary-externality or feedback-on-higher-price effect tightens the liquidity constraint of money-buyers and reduces their consumption. This is also similar to Berentsen, Huber and Marchesiani (2014). Like us, Chiu et al. (2018) show that even under perfectly-competitive goods and banking markets, credit can induce a pecuniary-externality cost on liquidity-constrained money-buyers. However, their result requires the assumption that there is an exogenous measure of money-constrained buyers and the cost of producing the decentralized-market good is strictly convex.

In contrast, we obtain a negative welfare effect of credit through a channel of endogenous firms' market power in goods price markups and dispersion. Also, in our setting, the measures of money-constrained and other agent types are endogenous. We shut down the possibility of another pecuniary-externality channel like that of Chiu et al. (2018) by assuming that decentralized-market firms have a linear cost of production. Instead, we provide another alternative mechanism for this externality effect. We show that buyers with access to credit can contribute to an increase in the measure of firms charging higher prices and extracting more rent from liquidity constrained money-buyers. Hence, banking can be welfare-reducing in equilibrium.

Dong and Huangfu (2021) present a monetary model in which both money and credit serve as a means of payment. Credit settlement requires money. In their model, the payment instrument involved with money (credit) is subject to the inflation tax (fixed transaction costs). They show that using credit can be welfare-reducing at very low or very high inflation. This is a consequence of having a fixed cost of accessing credit in the model. In contrast, we do not require any cost to accessing bank credit.

There are few other studies incorporating the noisy search process of Burdett and Judd (1983) into a monetary framework for various applications (*see, e.g.*, Head and Kumar, 2005; Head, Kumar and Lapham, 2010; Chen, 2015; Wang, 2016; Wang, Wright and Liu, 2020). Our model setup is similar to Wang et al. (2020) but for a different research question. We focus on the welfare consequences of banking in an economy with endogenous firms' market power. Wang et al. (2020) focus on rationalizing the price-change pattern and cash-credit shares observed at the micro-level data in the United States. In their model, buyers' access to credit incurs a fixed utility cost. In contrast, agents' access to banking is not restricted in our setup, as in Berentsen et al. (2007). It is possible to introduce costly banking in our model but it would not change the basic message in the paper. Boel and Camera (2019) introduce a operating cost for banks in providing loans which will generate a wedge between the lending and deposit rates.

The remainder of the paper is organized as follows. In Section 2, we provide the motivating empirical evidence. In Section 3, we lay out the details of the model, agents' decision problems and characterization of a Stationary Monetary Equilibrium (SME). In Section 4, we provide an analysis of the model mechanism. We then calibrate the model to U.S. data in Section 5. Using numerical results, we illustrate the effects of banking on equilibrium firms' market power and the welfare consequences. We conclude in Section 6.

2 Motivating empirical evidence

In this section, we present two pieces of evidence. We show that there is a positive relationship between the consumer credit-to-GDP ratio and aggregate (and retail) price markup. Also there is a positive relationship between the consumer credit-to-GDP ratio and price dispersion in the United States.

2.1 Data

We put together data on quarterly consumer credit-to-GDP ratio, the aggregate (and retail sector) markup and price dispersion in the United States. The data sample period is from 1980 to 2019. We provide the summary statistics in Table 1.

Table 1: Data sources and summary statistics

Variable	Sample period	Source	Mean	Median	S.D
Aggregate markup	1980 Q1: 2019 Q4	Compustat	1.44	1.44	0.12
Retail markup	1980 Q1: 2019 Q4	Compustat	1.18	1.18	0.14
Markup dispersion	1980 Q1: 2019 Q4	Compustat	0.14	0.05	0.05
Consumer credit-to-GDP (%)	1980 Q1: 2019 Q4	FRED	1.91	1.61	1.03
Relative price variability	1980 Q1: 2019 Q4	BLS	8.12	7.97	0.92
Real GDP	1980 Q1: 2019 Q4	FRED	0.65	0.71	0.7
GDP Deflator (%)	1980 Q1: 2019 Q4	FRED	079	0.93	2.73
Business sector TFP (%)	1980 Q1: 2019 Q4	Fernald (2014)	2.71	2.14	1.84
Unemployment rate (%)	1980 Q1: 2019 Q4	FRED	6.20	5.72	1.68

Note: We obtain the Compustat data from Wharton Research Data Services. BLS refers to the U.S. Bureau of Labor Statistics. FRED is Federal Reserve Economic Data.

Consumer Credit-to-GDP (%) and Real GDP. We obtain total consumer credit owned and securitized by depository institutions from FRED.[3] We use the U.S. real GDP data from FRED and total consumer credit to compute the consumer credit-to-GDP ratio.[4]

Markup. To calculate the aggregate and retail sector markup measures in the United States, we use *Compustat* data on the quarterly balance sheets of publicly listed firms in the United States.[5] Following Hall (1988) and De Loecker et al. (2020), we compute firm-level markups based on the production approach. This approach estimates markup derived from an assumption that firms minimize their cost.

In each period t, an individual firm i markup μ_{it} is defined as

$$\mu_{it} = \theta_{it}^{V} \frac{P_{it}}{P_{it}^{V}} \frac{Q_{it}}{V_{it}}, \tag{2.1}$$

where P_{it}, P_{it}^{V}, V_{it} and Q_{it}, respectively, denote the output price, price of the variable input, the variable input, and output.

According to Equation (2.1), an individual firm's markup comprises two components: (1) The revenue share of the variable input is specified as $\frac{P_{it}}{P_{it}^{V}} \frac{Q_{it}}{V_{it}}$; and (2) the output elasticity of the variable input measured by θ_{it}^{V}.

We fix the output elasticity to be time-invariant (0.85). This assumption is consistent with the empirical evidence documented in De Loecker et al. (2020).[6] Then, we compute the aggregate

[3]Data source: https://fred.stlouisfed.org/series/TOTALDI
[4]Data source: https://fred.stlouisfed.org/series/GDPC1
[5]Data source: Wharton Research Data Services.
[6]The authors document that the pattern of markup with fixed output elasticity (0.85) is similar to that using estimated

markup as

$$\mu_t = \sum m_{it}\mu_{it}, \tag{2.2}$$

where m_{it} is the weight of each firm.[7]

As a robustness check, we also use data from the North American Industry Classification System (NACIS) code $(44-45)$ to calculate the retail sector markup μ^{retail}. The reason for considering this alternative is that the expenditure of households who use banking credit is mostly related to retail-sector final production.

Relative price variability. We compute the quarterly measure of relative price variability (RPV) in the United States following Banerjee, Mizen and Russell (2002) and Banerjee, Mizen and Russell (2007). We use the urban consumer price index (CPI-U) data from the Bureau of Labor Statistics (see Appendix B for more details).

We define RPV in period t as

$$\mathrm{RPV}_t = \left[\sum_i w_i(Dp_{it} - Dp_{AIt})^2\right]^{1/2}, \tag{2.3}$$

where w_i is the expenditure weights of each component i index in the all items index, Dp_{it} and Dp_{AIt} are quarterly annualized inflation rates of the i^{th} component indexes and total price index, respectively.[8]

In short, RPV_t denotes the weighted average of the component inflation rates relative to the aggregate measure of annualized inflation.

Markup dispersion. Following Meier and Matthias (2021), we compute aggregate markup dispersion in period t as

$$\nu_t = \sum_i m_{it}\Big[\log(\mu_{it}) - \log(\mu_t)\Big]^2, \tag{2.4}$$

where m_{it} is the weight of each firm, and μ_{it} and μ_t are respectively determined by Equation (2.1) and Equation (2.2).

In short, aggregate markup dispersion is just the weighted average of the log deviation of an individual markup from the aggregate markup.

output elasticities.

[7] We employ the share of sales in the sample as the weight. See De Loecker et al. (2020) for more details.

[8] See also Parks (1978), Blejer and Leiderman (1980), Cukierman and Leiderman (1984), Parsley (1996).

Control variables. Our control variables include: real GDP, GDP deflator, unemployment rate and business sector Total Factor Productivity (TFP) growth rate. We obtain data on real GDP, GDP deflator and unemployment from the FRED.[9] Lastly, we obtain the business sector TFP data from Fernald (2014).[10]

2.2 Empirical model

In this section, we investigate the effect of consumer credit on the aggregate markup and RPV by considering the following linear regression model:

$$y_t = a_0 + a_1 d_t^{CC} + b' \gamma_t + \text{Trend}_t + \epsilon_t, \tag{2.5}$$

where y_t is one of the variables of interest: log of the aggregate markup, retail sector markup, RPV, or markup dispersion. The variables d_t^{CC} and γ_t, respectively, denote consumer credit-to-GDP ratio and the list of control variables previously described. We also include a quadratic trend variable, Trend_t, in our estimation to capture the trend in macro-level time series data. The list of parameters, respectively, for the intercept, the credit-to-GDP ratio and all the controls, (a_0, a_1, b'), and also those implicit in the definition of Trend_t, are estimated by ordinary least squares (OLS). Conditional on the other factors, we are most interested in the various estimates of a_1, which will be presented in the first row of Table 2.

2.3 Empirical results

Table 2 reports the OLS results for Equation (2.5). Columns (1)—(2) indicate that there is a positive and statistically significant relationship between consumer credit-to-GDP and (both the aggregate and retail sector) markup. Columns (3)—(4) indicate that there is a positive and statistically significant relationship between consumer credit-to-GDP and price dispersion.[11]

[9] Data source: Real GDP data from https://fred.stlouisfed.org/series/GDPC1.
GDP deflator from https://fred.stlouisfed.org/series/GDPDEF.
Unemployment rate data from https://fred.stlouisfed.org/series/UNRATE.

[10] See https://www.johnfernald.net/TFP.

[11] We provide more detailed results for a robustness test in Appendix C.

Table 2: OLS results: Markup and dispersion

Dependent Variable:	$\log(\mu_t)$ (1)	$\log(\mu_t^{\text{retail}})$ (2)	$\log(\text{RPV}_t)$ (3)	ν_t (4)
Consumer Credit-to-GDP	1.114**	0.444*	26.890**	1.100**
	(0.410)	(0.184)	(7.960)	(0.324)
Log of real GDP	0.037	0.076	4.223+	0.024
	(0.795)	(0.208)	(0.053)	(0.802)
Business TFP	-0.004	0.034	-3.171**	-0.005
	(0.952)	(0.201)	(0.010)	(0.904)
GDP Deflator	-0.391*	-0.313**	-1.815	-0.213+
	(0.012)	(0.001)	(0.598)	(0.051)
Unemployment rate	-0.132	-0.030	17.860**	-0.059
	(0.378)	(0.644)	(0.001)	(0.542)
R^2	0.928	0.236	0.312	0.895
Observations	160	160	160	160

Note: Robust errors are in parenthesis. $+, *$, and $**$ are respectively at the significance levels of 10 %, 5 % and 1 %. Constant and trend variables are included but not reported.

3 Model

We construct a model that will be consistent with the empirical evidence from Section 2. The model builds on Head et al. (2012) (HLMW) by introducing perfectly competitive markets for bank deposits and loans. As in Berentsen et al. (2007) (BCW), the focus here is on banks' *liquidity transformation* role: They can intermediate between ex-post heterogeneous liquidity needs of agents. (Thus, we do not model other aspects or functions of banks such as the undertaking of risky investments or bank equity and capital regulation.) We then use this framework to study the interaction between banking credit and firms' market power in equilibrium.

3.1 Timing, markets, agents and some related notation

In the model, time is discrete and infinite. Agents discount across period t and $t+1$ by a common discount factor $\beta \in (0,1)$. We will use variables $X \equiv X_t$ and $X_+ \equiv X_{t+1}$ respectively to denote time-dependent outcomes at period t and $t+1$. There are four types of agents: households, firms, banks, and the government. There is a continuum of households and firms, each of measure one. The banking sector is perfectly competitive with free entry. The government supplies fiat money according to the rule $M_+ = \gamma M$, where $\gamma = 1+\tau$ is money-supply growth factor and $\gamma \in [\beta, \infty)$.

In every period, two markets open sequentially as in Lagos and Wright (2005). First, a decentralized goods market (DM) with trading frictions opens. In the DM, households are anonymous so that private credit arrangements are incentive infeasible. Consequently, fiat money will be valuable as a medium of exchange in equilibrium. The DM will be the source of fundamental frictions in the model. The DM will be followed by a frictionless centralized market (CM) which allows agents to rebalance their asset positions.

In what follows, we first describe the model primitives. We then describe the sequence of decentralized and centralized markets in each period. Then, we get into the details of the various decisions problems and characterize an equilibrium.

We should also point out that below, CM and DM value functions are to be denoted by the pair (V, W) and DM consumption and loan decision functions by $(q_b^\star, l^\star)$. A novelty in our model will be in the dependence of market power in the DM-good pricing on the price of credit(*c.f.* Head et al., 2012). This will be because, in equilibrium, there may exist a measure of agents who would take out credit from banks. This renders their demand for the DM good dependent on i through their corresponding demand for bank loans. Thus, agents should anticipate that the equilibrium DM-good pricing distribution would, in general, depend on the nominal loan interest rate (i). We will denote $J_i(\cdot)$ to indicate this.

3.2 Household primitives

Preferences. Each household has their per-period utility described by

$$\mathcal{U}(q, x, h) = u(q) + U(x) - h, \tag{3.1}$$

where $u(q)$ is the utility flow from consumption of the goods in the DM, $U(x)$ is the utility flow of consumption goods x in the CM, and $-h$ captures the disutility of labor.

We assume that $u' > 0, u'' < 0$ and u satisfies the standard Inada conditions. Likewise for the CM utility function U. We restrict our attention to the constant-relative-risk-aversion (CRRA) class of functions:

$$u(q) = \frac{q^{1-\sigma} - 1}{1 - \sigma}. \tag{3.2}$$

The risk aversion coefficient σ influences the households' price elasticity of demand. For our setting, we will restrict $\sigma < 1$. This restriction will be necessary for fitting the historical long-run money demand relation in the United States, as is the case in Wang (2016); Wang et al. (2020). Moreover, this restriction is consistent with the empirical finding in Baker (2018).

Technologies. In the CM, the general goods x are produced using a technology that is linear in labor input h. Consequently, both real wage and the price of the general goods will be equal to one. In the DM, firms producing one unit of good q requires $h = c \times q$ hours of labor. The parameter $c > 0$ is the constant marginal cost of DM production.

3.3 Events in the sequential DM and CM

Decentralized market. As in HLMW, goods trade is modelled as a Burdett and Judd (1983) noisy search process, save for the fact that exchange is monetary. Each DM-goods firm posts a price, p, and commits to supplying at that price, taking as given the distribution of all posted prices, J_i, and buyers' demand schedule, q_b.

From the buyers' ex-ante point of view, they observe the price distribution but not an individual posted price. Hence, this noisy search process rules out that buyers can direct their search to particular sellers with the lowest price. Instead, buyers contact and randomly sample k firms' prices, *i.e.*, independently draw k price quotes from the distribution J_i with probability α_k. For simplicity, we assume that buyers sample zero price quote with probability $\alpha_0 \in (0, 1)$, one price quote with probability $\alpha_1 \in (0, 1 - \alpha_1)$, and two price quotes with probability $\alpha_2 = 1 - \alpha_0 - \alpha_1$. These probabilities are also the matching or contact probabilities between a buyer and a firm. Some buyers (with zero price quotes) are ex-post *inactive* in goods transactions. These inactive buyers are stuck with idle money balances. At the same time, some buyers (with at least one price quote) are ex-post *active* in purchasing and consuming the goods. They may also be ex-post liquidity constrained in their goods transactions depending on the price quote received from firms. Thus, banks have a liquidity-transformation purpose in the model.

Bank deposit-taking and lending activities occur after agents realize their DM types and after ex-post buyers receive price quotes from firms, but before the exchange and production of goods begins. Banks accept nominal deposits from buyers with unproductive idle money balances (*i.e.*, those who have drawn zero price quotes). Banks commit to paying depositors at a perfectly-competitive nominal interest rate of i_d. They then allocate deposits to extend nominal loans to buyers who may need more liquidity at a perfectly-competitive nominal rate, i.[12] We also maintain the assumption regarding banking operations as in Berentsen et al. (2007). First, banks operate a financial record-keeping technology at zero cost. Second, banks can perfectly enforce loan repayments. Moreover, agents having access to banks does not rule out the need for money serving as a medium of exchange in the DM. The reason is that all goods transactions are anonymous in the DM and ex-post inactive agents can earn deposit interest that insures them (to some extent) against inflation tax.

[12]In general, buyers who sample more than zero quotes may or may not borrow additional money balances from banks. Later, we show that their decision on taking out a loan from the bank will depend on the goods price drawn from the distribution.

At this point, the banking sector closes, and the exchange and production of goods happen. Buyers face a liquidity constraint consisting of their own money balances m with (or without) loans l. Buyers then pay the firms to produce the goods for their consumption.

After the DM, agents enter a frictionless CM. Households trade a general good x, supply labor h, settle financial contracts (redeem deposits or repay loans) and accumulate money balances.

Centralized market. An agent entering the CM is denoted by an individual state (m, l, d), *i.e.,* her remaining nominal money balance, outstanding loan and deposit balance. In particular, those who have deposited in the previous DM will earn gross interest $1 + i_d$ on deposits d. Those who have borrowed will need to repay gross interest $1 + i$ on loan l to banks. Households supply labor h to firms for production and consume the general goods x. Households own firms and firms return profits as dividends D to households. Households then accumulate money balances m_+ to carry into the next period.

3.4 Households

To ease notation, we let the variable $\mathbf{a} := (M, \gamma)$ denote the aggregate state of the economy, which consists of total real money stock and monetary policy $\gamma = 1 + \tau$. In what follows, we work backwards from the CM to the DM within the period t.

3.4.1 Households in the CM

An agent beginning the CM with money, loan or deposit balances, (m, l, d), may have been a borrower or a depositor in the previous DM during the first sub-period. Her initial value is

$$W(m, l, d, \mathbf{a}) = \max_{(x,h,m_+) \in \mathbb{R}^3_+} \left\{ U(x) - h + \beta V(m_+, \mathbf{a}_+) \middle| \begin{array}{c} x + \phi(m_+ - m) = \\ h + D + T + \phi(1 + i_d)d - \phi(1 + i)l \end{array} \right\}, \quad (3.3)$$

where V is the value function at the beginning of the next DM, ϕ is the value of money in units of the CM consumption good x, i_d is the deposit interest rate, i is the loan interest rate, h is labor supplied, D is aggregate dividends from firm ownership and T is the lump-sum taxes/transfers from the government.

The first-order conditions with respect to x and m_+ are, respectively, given by

$$U_x(x) = \phi, \quad (3.4)$$

and,

$$\phi = \beta V_m(m_+, \mathbf{a}_+), \quad (3.5)$$

where $V_m(m_+, \mathbf{a}_+)$ captures the marginal value of accumulating an extra unit of money balance taken into the next period $t+1$. The envelope conditions are

$$W_m(m,l,d,\mathbf{a}) = \phi, \;\; W_l(m,l,d,\mathbf{a}) = -\phi(1+i), \;\; \text{and} \;\; W_d(m,l,d,\mathbf{a}) = \phi(1+i_d). \tag{3.6}$$

Note that W is linear in (m,l,d) and the distribution of money balances is degenerate when households exit the CM. As a result, households' optimal choices for CM consumption and money balance are given by Equations (3.4) and (3.5). These equations are independent of the agents' current wealth since per-period preferences are quasilinear.

3.4.2 Households in the DM

We first describe the post-match households problems. We call households who sample at least one price quote in the DM *active buyers*. We label those who sample zero price quotes *inactive buyers*.

Regarding banking arrangements, it is easy to verify that agents who are active buyers will have no incentive to deposit funds with the bank, whereas inactive buyers will never have an incentive to borrow additional funds from banks. As such, we denote l as the amount of loans an active buyer may take out and d as the amount of money deposited by an inactive buyer throughout the paper.

***Ex-post* inactive buyers.** With probability α_0, a household is inactive. Conditional on being inactive, a household with money holdings, m, can deposit d of this money with a bank. She has zero utility flow of consuming q and then enters the CM with valuation of $W(m-d, 0, d, \mathbf{a})$.

***Ex-post* active buyer sampling at least one price.** The post-match value of such a buyer is given by:

$$B(m,p,\mathbf{a}) = \max_{q,l} \left\{ u(q) + W(m+l-pq, l, 0, \mathbf{a}) \middle| \begin{array}{c} pq \leq m+l, \\ 0 \leq l \leq \bar{l} \end{array} \right\}. \tag{3.7}$$

We assume banks can perfectly enforce loans repayment as in the baseline case of Berentsen et al. (2007). Hence, buyers do not face a borrowing constraint, *i.e.*, $\bar{l} = \infty$.

Taking loan interest rate i as given, the buyer's demand for DM consumption goods is:

$$q_b^\star(m) := q_b^\star(m,p,i,\mathbf{a}) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ \frac{m}{p} & \text{if } \tilde{p}_i < p < \hat{p}\,, \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases} \tag{3.8}$$

where

$$\hat{p} := \hat{p}(m, \mathbf{a}) = \phi^{\frac{1}{\sigma-1}} m^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i := \tilde{p}(i, m, \mathbf{a}) = \hat{p}(1+i)^{\frac{1}{\sigma-1}}. \tag{3.9}$$

The cutoff prices $(\hat{p}, \tilde{p}_i)$ are functions of the state of the economy and monetary policy. Since $\sigma < 1$, we can order the cut-off prices as: $0 < \tilde{p}_i < \hat{p} < +\infty$.

For a given loan interest rate i, the buyer's loan demand is:

$$l^{\star}(m) := l^{\star}(m, p, i, \mathbf{a}) = \begin{cases} p^{\frac{\sigma-1}{\sigma}} [\phi(1+i)]^{-\frac{1}{\sigma}} - m & \text{if } 0 < p \leq \tilde{p}_i \\ 0 & \text{if } \tilde{p}_i < p < \hat{p} \\ 0 & \text{if } p \geq \hat{p} \end{cases}. \tag{3.10}$$

Previewing an equilibrium, the buyer's ex-post goods and loans demand schedules will depend on the price, p, drawn from the distribution $J_i(p, m, \mathbf{a})$. The key insight is that there is ex-post heterogeneity regarding buyers' consumption outcomes due to the noisy search process in the frictional goods market. When we present the firms' problem, we will be more explicit about characterizing the distribution of prices. For now, we assume the price distribution J_i has a connected support.

From Equations (3.8) and (3.10), we can deduce the following three possible cases of ex-post heterogeneous demands.

Consider the first case in Equation (3.10). If a buyer draws a p that is sufficiently low, then the buyer optimally borrows money from the bank to top up his initial money holdings. Moreover, the buyer spends all his liquid balances, including his money and bank loan. We call this buyer a *borrower* (or sometimes, a *credit-buyer*).

In the intermediate case, p is drawn such that $p_i < p < \hat{p}$. In this event, the buyer prefers not to borrow from the bank but rather to spend all her money. In this case, loan size does not matter for goods demand. We call this type of buyer a *liquidity constrained money-buyer*.

In the last case, p can be sufficiently high. In that case, the buyer prefers not to borrow and also not to spend all her money balance in the frictional goods market. We call this type of buyer a *liquidity unconstrained money-buyer*.

It is also worth mentioning the price elasticity of demand for the demand schedule $q_b^{\star}$ described in Equation (3.8). The buyers' price elasticity of demand is given by

$$\left| \frac{\partial q_b^{\star}(m)}{\partial p} \frac{p}{q_b^{\star}(m)} \right| = \begin{cases} \frac{1}{\sigma} & \text{if } 0 < p \leq \tilde{p}_i \\ 1 & \text{if } \tilde{p} < p < \hat{p} \\ \frac{1}{\sigma} & \text{if } \hat{p} \leq p \end{cases}. \tag{3.11}$$

Later on, when calibrated to data, the DM risk aversion coefficient will turn out to be some number $\sigma < 1$.[13] This will imply that demand is elastic among buyers.[14] The implication is that such a buyer cannot spend more than his liquidity constraint at low-enough price levels, $p < \hat{\rho}$. Above the $\hat{\rho}$ cut-off price level, a buyer's liquidity constraint does not bind and such buyers will always spend less than their total money holding.

Households in the DM *ex-ante*. Now consider the beginning of period t when households are *ex-ante* homogeneous at the start of the DM (*i.e.*, before exchange and production of the goods). Given an individual real money balance, m, and aggregate state, $\mathbf{a} := (M, \gamma)$, the agent's value is

$$\begin{aligned} V(m,\mathbf{a}) = \alpha_0 W(m-d,0,d,\mathbf{a}) + \alpha_1 \int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}J_i(p,m,\mathbf{a}) \\ + \alpha_2 \int_{\underline{p}(m,\mathbf{a})}^{\overline{p}(m,\mathbf{a})} B(m,p,\mathbf{a})\mathrm{d}[1-(1-J_i(p,m,\mathbf{a}))^2]. \end{aligned} \tag{3.12}$$

In contrast to Head et al. (2012), the value of households entering the DM is different due to the availability of banking services.

According to Equation (3.12), with probability α_0, a household is inactive, *i.e.*, the household samples zero price quotes from firms. They are the ones stuck with idle money balances. Since banks source deposits to issue loans, the measure of α_0 buyers can deposit their idle money balances at the bank to earn interest i_d. However, in a no-bank economy, this measure of households will enter the subsequent CM while holding their idle money balances subject to an inflation tax. In that case, having unneeded money ex-post can be costly since higher inflation induces a lower value of money.

With probability α_1 the household contacts one firm posting p, drawn from the distribution $J_i(\cdot, m, \mathbf{a})$. With probability $\alpha_2 = 1 - \alpha_0 - \alpha_1$, the household contacts and randomly samples two price quotes from firms and the lower of the two are drawn from $1 - (1 - J_i(\cdot, m, \mathbf{a}))^2$. Moreover, conditional on being an active buyer, he can now choose whether or not to borrow additional money from banks to purchase the goods. The buyer's decision on demanding bank credit depends on the price drawn from the distribution and the market loan interest rate i.

[13] Moreover, the restriction on $\sigma < 1$ is consistent with empirical evidence in Baker (2018). The author finds that indebted households face a more elastic demand schedule, which is captured by the first case in Equation (3.11).

[14] Alternatively, we can show that the elasticity of the buyer's expenditure rule $e(p) := pq_b^\star(m)$ is less than one. Then the buyer's expenditure on the DM goods decreases as he faces a higher price p. We omit the details of its derivation here. Instead, we explain more about how banking credit affects buyers' optimal expenditure rule and firms' pricing strategy in Section 4.

Marginal value of money. To simplify notation, we denote the cut-off pricing functions by $\underline{p} := \underline{p}(m, \mathbf{a})$, $\tilde{p}_i := \tilde{p}_i(i, m, \mathbf{a})$ and $\overline{p} := \overline{p}(m, \mathbf{a})$. Differentiate Equation (3.12) with respect to m, and update one period to get

$$V_m(m_+, \mathbf{a}_+) = \phi_+ \Big[1 + r_+(m_+, \mathbf{a}_+) \Big], \tag{3.13a}$$

where

$$\begin{aligned} r_+(m_+, \mathbf{a}_+) &:= \alpha_0 i_d + \int_{\underline{p}}^{\tilde{p}_i} i \Big[\alpha_1 + 2\alpha_2 (1 - J_{i,+}(p, m_+, \mathbf{a}_+)) \Big] \mathrm{d} J_{i,+}(p, m_+, a_+) \\ &\quad + \int_{\tilde{p}_i}^{\overline{p}} \Big[\alpha_1 + 2\alpha_2 (1 - J_{i,+}(p, m_+, \mathbf{a}_+)) \Big] \left(\frac{u_q(q_+)}{\phi_+ p} - 1 \right) \mathrm{d} J_{i,+}(p, m_+, a_+). \end{aligned} \tag{3.13b}$$

Equation (3.13a) captures the expected benefit from accumulating an extra unit of money balance into the next period. The value of one unit of money balance is captured by ϕ_+ (in units of CM goods). Since money serves as a means of payment in the frictional goods market, it has a liquidity premium captured by the function $r_+(m_+, \mathbf{a}_+)$. Thus, carrying an extra unit of money has a benefit of $\phi_+ r_+(m_+, \mathbf{a}_+)$.

In contrast to Head et al. (2012), the liquidity premium on holding money in Equation (3.13b) now depends on the banking arrangement. In particular, if the household ends up not consuming in the next DM, he can deposit his idle money in the bank to earn interest $i_d > 0$. Hence, the marginal value of money is higher with access to banks. In other words, banks play the same intermediation-of-liquidity-needs role as those in Berentsen et al. (2007). If the household samples a low enough p, he would take out a bank loan. The second term captures the expected, marginal interest-payment liability saved by borrowing one less unit of money. The last term captures the net benefit from spending an extra unit of money—*i.e.*, the expected liquidity premium of carrying one's own money as medium of exchange.

Substituting Equation (3.13a) into Equation (3.5), we obtain a money demand Euler equation capturing the households' intertemporal trade-offs:

$$\phi = \beta \phi_+ [1 + r_+(m_+, \mathbf{a}_+)]. \tag{3.14}$$

The left-hand side of Equation (3.14) captures the cost of accumulating money balance. The reason is that the household must have foregone ϕ units of CM consumption goods in order to save an extra dollar. The right-hand side of Equation (3.14) is the expected marginal benefit of accumulating an extra dollar associated with the total liquidity premium captured by $r_+(m_+, \mathbf{a}_+)$ in Equation (3.13b).

3.5 Firms

Firms in the Decentralized Market. A unit measure of firms (or sellers of goods) compete in a price posting environment along the lines of Head et al. (2012). In the DM, the firm posts price ρ to maximize expected profit and commit to supplying goods at that posted price.

Consider a firm posting price p, given their potential customers' demand schedule $q_b^\star$ and the distribution of prices posted by firms J_i. Its expected profit is given by

$$\Pi_i(p) = \underbrace{\left[\alpha_1 + 2\alpha_2(1 - J_i(p, m, \mathbf{a})) + \alpha_2 \nu(p)\right]}_{\text{extensive margin}} \underbrace{R_i(p)}_{\text{intensive margin}}, \tag{3.15}$$

where

$$\nu(p) = \lim_{\epsilon \searrow 0} J_i(p, m, \mathbf{a}) - J_i(p - \epsilon, m, \mathbf{a}),$$

and

$$R_i(p) \equiv R(p, i, m, \mathbf{a}) = q_b(p, i, m, \mathbf{a})(p - c).$$

The first term in parentheses, labeled *extensive margin*, in Equation (3.15) captures the number of buyers served. With probability α_1, the firm trades with a buyer who has only observed one price quote from this firm and no other. With probability $2\alpha_2[1 - J_i(p, m, \mathbf{a})]$, the buyer purchases the good from this firm because he contacts another firm who has posted a higher price than p. The probability $\alpha_2 \nu(p)$ is the measure of buyers that match both this firm and another which has posted the same price, p.[15] The last term, labeled *intensive margin*, captures the firm's profit per customer induced by the firm charging a markup (*i.e.,* posting at a price above the marginal cost, $p > c$).

Observe from Equation (3.15), the firm posting p trades off between an *extensive margin* (*i.e.,* the likelihood of trading with buyers) and an *intensive margin* (*i.e.,* profit per buyer). On the one hand, a firm that posts a higher p can earn a higher profit margin per buyer served. However, on the other hand, a firm that posts a higher p suffers by losing sales to other competitors, *i.e.,* a lower likelihood of trading with buyers.

A hypothetical monopolist. Consider a firm serving buyers who have only received one price quote from this one firm. In this case, the firm will behave as a monopolist. The realized profit of

[15] Suppose two firms post the same price. We assume that prospective buyers use a tie-breaking rule to pick one firm in such a case. This rule incentivizes an individual firm to lower the price to get the sale. In equilibrium, the probability of a buyer contacting two firms that post the same price goes to zero.

a firm setting a monopoly price p^m is

$$\Pi_i^m = \alpha_1 R_i(p^m). \tag{3.16}$$

We will now describe what p^m can be. A subtlety in our extension of Head et al. (2012) here is that banking outcome i will affect some agents who, ex-post, may demand loans. As a result, i will also condition or "shift" their demand for the DM good. This, along with how much money a buyer carries into the trade, has consequences for the calculation of a firm's profit and also for the equilibrium distribution of DM-good prices. It turns out that this implies two possible cases characterizing the monopoly price. The two cases are determined by the relative orderings between the monopoly price when the firm is faced with credit buyers, money-constrained (non-credit) or money-unconstrained buyers.

We can rewrite the description of the orderings in terms of a cutoff money balance condition, $\check{m}_i$. First, we can show that the monopolist would charge (money-constrained and money-unconstrained) buyers who are not sensitive to the loan rate a price equal to $\check{p}^m = \phi^{-1}c/(1-\sigma)$. Second, we define a marginal buyer named $\check{m}_i$. This buyer corresponds to the money balance such that if the buyer were a credit buyer (one whose demand for the DM good is sensitive to i), then his maximal willingness to pay equals that of other i-insensitive buyers, $\check{p}^m$. That is, $\tilde{p}_i(\check{m}_i) = \phi^{-1}c/(1-\sigma)$. Using Equation (3.9), we can show that $\check{m}_i = \phi^{-1}\left(\frac{c}{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}(1+i)^{-1/\sigma}$. Observe that the outcome i can shift this cutoff $\check{m}_i$.

We summarize the monopoly-price characterization below and relegate its derivation to the Online Appendix.

Lemma 1. *Let $\check{m}_i = \phi^{-1}\left(\frac{c}{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}(1+i)^{-1/\sigma}$ denote a marginal buyer who is indifferent between taking out a loan and not. The monopoly price is*

$$p^m = \begin{cases} \frac{\phi^{-1}c}{1-\sigma}, & \text{if } m \in (0, \check{m}_i) \\ \max\{\frac{\phi^{-1}c}{1-\sigma}, \hat{p}\}, & \text{if } m \in (\check{m}_i, \bar{m}_i) \end{cases}, \tag{3.17}$$

where $\hat{p}$ is given in Equation (3.9), $\bar{m}_i = \phi^{-1}(\underline{p})^{\frac{\sigma-1}{\sigma}}(1+i)^{-1/\sigma}$ and $\underline{p}$ is a lower bound on p to be determined in equilibrium.

Pricing equilibrium. Previewing an equilibrium, firms will earn the same expected profit for any p in the support of the distribution, $\text{supp}(J_i) = [\underline{p}, \overline{p}]$. That is, they will be indifferent between a continuum of different extensive-intensive margin trade-offs. The intuition is that lower price firms win on sales volume while higher price firms gain through the profit or markup channel.

That is,

$$\Pi_i^\star = \max_p \Pi_i(p) \text{ for all } p \in \text{supp}(J_i). \tag{3.18}$$

As in Head et al. (2012), if some buyers observe only one price quote whereas others observe more than one, then this leads to a non-degenerate distribution of prices J_i.[16] Since firms expect the same profit outcomes associated with the continuum of markup-versus-trading-probability strategies, then this implies an equal-profit condition. Specifically, equating Equation (3.15) and Equation (3.16), we can derive a closed-form distribution of prices as in many other applications of Burdett and Judd (1983) (see *e.g.*, Wang, 2016; Wang et al., 2020; Head, Kam, Ng and Pan, 2022). We summarize this result in the following Lemma.[17]

Lemma 2. *Given monetary policy $\gamma > \beta$ and noisy search frictions $\alpha_1, \alpha_2 \in (0,1)$, the price distribution J_i consistent with profit maximization by all firms is*

$$J_i(p) := J_i(p, m, \mathbf{a}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R_i(\overline{p})}{R_i(p)} - 1\right], \tag{3.19}$$

where the lower and upper bounds on the support of J_i are, respectively, determined by $R_i(\underline{p}) = \frac{\alpha_1}{\alpha_1+2\alpha_2}R_i(\overline{p})$ and:

$$\overline{p} := \overline{p}(m, \mathbf{a}) = \begin{cases} \frac{\phi^{-1}c}{1-\sigma} & \text{if } m \leq \check{m}_i, \\ \max\{\frac{\phi^{-1}c}{1-\sigma}, \hat{p}\}, & \text{if } m > \check{m}_i \end{cases}, \tag{3.20}$$

where $\check{m}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]/\phi$.

Lemma 2 highlights that if some buyers receive only one price quote while others receive more than two, the price distribution is continuous and nondegenerate. Moreover, firms can exploit market power in the goods market by pricing the goods above the marginal cost of production. In contrast to Head et al. (2012), the banking loan interest rate i now matters for determining the (monopoly) profit-maximizing price, as shown in Equation (3.20). This is a consequence of the buyer's optimal goods demand schedule interacting with credit, which affects the firm's pricing strategy.

Firms in the Centralized Market. In the CM, there is a unit measure of perfectly competitive firms producing the general goods x using a linear production technology in labor h. They then

[16]The model has two parametric limits: one with Bertrand pricing (by setting $\alpha_2 = 1$) and one that resembles monopoly (by setting $\alpha_1 = 1$). For our purposes, we focus on cases away from these two parametric limits to rationalize the empirical finding in Section 2.

[17] The proof follows directly from Head et al. (2012).We omit the details here.

sell the goods to households in the CM. Consequently, both the real wage and price of the DM goods are equal to one.

3.6 Banks

We focus on the liquidity transformation role of banks. The banking sector is perfectly competitive with free entry as in Berentsen et al. (2007). In particular, banks accept deposits d and commit to repaying depositors with interest i_d. Banks then allocate deposits to issue loans l at the interest rate of i to borrowers. In equilibrium, the deposit rate bids up to the loan rate, $i = i_d = i^\star$, which follows from the free entry condition.[18]

3.7 Stationary monetary equilibrium

We focus on stationary outcomes of the economy. Since the price of the general goods P is used as a unit of account, we then multiply all nominal variables by the value of money balance $\phi = 1/P$ (in units of the CM goods x) from here onward. In particular, we let $z = \phi m$ denote the individual real money balance and $Z = \phi M$ denote the aggregate real money balances; $\rho = \phi p$ denote the real relative price of goods across the DM and the CM; and $\xi = \phi l$ and $\delta = \phi d$ respectively denote the real balances of loans and deposits. For the ease of notation, we also let the variable $\mathbf{s} := (Z, \gamma)$ denote the aggregate state of the economy consisting of total real money stock and monetary policy $\gamma = 1 + \tau$. In a stationary equilibrium, all nominal variables grow at a time-invariant rate according to $\phi/\phi_{+1} = M_{+1}/M = \gamma = 1 + \tau$ and real variables stay constant over time.

Before we provide a summary of the equilibrium characterization, we first present two features that are different in contrast to Head et al. (2012) as follows.

In a stationary monetary equilibrium (SME), the real analog of the price distribution characterized in Lemma 2 is given by:

$$J_i(\rho, z) := J_i(\rho, i, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z)}{R(\rho, i, z)} - 1\right] = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{q_b^\star(\overline{\rho}, i, z)(\overline{\rho} - c)}{q_b^\star(\rho, i, z)(\rho - c)} - 1\right], \tag{3.21}$$

where the upper support of the distribution $J_i(\rho, z)$ is determined by:

$$\overline{\rho} := \overline{p}(z, \mathbf{s}) = \begin{cases} \frac{c}{1-\sigma} & \text{if } z \leq \check{z}_i \\ \max\{c/(1-\sigma), \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}\} & \text{if } z > \check{z}_i \end{cases}, \tag{3.22}$$

[18] We have assumed that there are no operating costs or reserve requirements in the banking industry. If we relax this assumption, there will be a wedge between the loan rate and the deposit rate or tightens the loan sizes extended by banks. Since we want to solely focus on firms' market power in the retail industry, we maintain a simpler structure in terms of banking operations.

given $\breve{z}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]$, and the lower support of J_i, $\underline{\rho}$, solves $R(\rho, i, z) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R(\overline{\rho}, i, z)$.

Observe that in Equation (3.21), the distribution of goods (real) prices now depends on both households' real money holdings z and the competitive loan interest rate $i = i_d = i^\star$ (determined by the loans market clearing condition). Consequently, there are two possible cases regarding the households' optimal demand for real money balances in an SME. We summarize this possibility in the following Lemma.

Lemma 3. *Let monetary policy be $\gamma > \beta$ and assume that $\alpha_1, \alpha_2 \in (0,1)$. Equation (3.14) expressed in real terms, charaterizes households' ex-ante demand for real money balances:*

$$\begin{aligned} \frac{\gamma - \beta}{\beta} = \alpha_0 i_d &+ \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} i \Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] dJ_i(\rho, z) \\ &+ \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] \left(\frac{u_q[q^\star(z)]}{\rho} - 1\right) dJ_i(\rho, z). \end{aligned} \tag{3.23}$$

Given a competitive loan market interest rate $i = i_d = i^\star$, there exists a cut-off value $\breve{z}_i$ such that $0 < \breve{z}_i = [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}] < 1$. There are two possible cases:

1. *If $z \leq \breve{z}_i$, the real-money-demand characterization in Equation (3.23) reduces to:*

$$\frac{\gamma - \beta}{\beta} = \alpha_0 i_d + \int_{\underline{\rho}(z)}^{\overline{\rho}(z)} i \Big[\alpha_1 + 2\alpha_2(1 - J_i(\rho, z))\Big] dJ_i(\rho, z) = i, \tag{3.24}$$

   where $\overline{\rho}(z) = c/(1-\sigma)$, and for all price $\rho \in supp(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$ satisfies: $\rho \leq \tilde{\rho}_i(z) = z^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}}$.

2. *If $\breve{z}_i < z$, real money demand z satisfies (3.23), where $\overline{\rho}(z) = \max\{c/(1-\sigma), \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}\}$, and the ordering of the support of the distribution, $J_i(\rho, z)$, satisfies: $\underline{\rho}(z) < \tilde{\rho}_i(z) = z^{\frac{\sigma}{\sigma-1}}(1+i)^{\frac{1}{\sigma-1}} < \overline{\rho}(z)$.*

Lemma 3 highlights that the agent's (ex-ante) precautionary demand for real money balances depends on an endogenous channel between the credit condition and the competitiveness of firms. The market interest rate, $i = i_d = i^\star$, determines the credit condition. The distribution of goods prices, $J_i(\rho, z)$, pins down the degree of firms' market power in the goods market. Specifically, this interaction determines an ordering of the upper support of the distribution: $\overline{\rho}(z) = \frac{c}{1-\sigma}$ or $\overline{\rho}(z) = \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$. In other words, firms' pricing trade-offs (price markups versus trading probabilities) consider the amount of liquid balance (money/loans) they expect households to carry into trades.

The first case in Lemma 3 is equivalent to the equilibrium condition for the agent's real money demand decision as in Berentsen et al. (2007). From the agent's ex-ante point of view, he is indifferent between borrowing at the competitive market interest rate and carrying a sufficient amount of real money balances to trade in the following period (DM). In this case, all prices in the equilibrium support of the price distribution will be weakly lower than the agent's maximum willingness to borrow, i.e., for all $\rho \in \text{supp}(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$ satisfying that $\rho \leq \tilde{\rho}_i(z)$. Given the agent's optimal goods (and loans) demand schedule, it can verify that there will only be credit–buyers (using both their money balances and bank loans) in ex-post trades. This result is consistent with the firms' pricing strategy. They have no incentive to post a price higher than $\overline{\rho} = c/(1-\sigma)$. Any price higher than $c/(1-\sigma)$ is an off-equilibrium price that makes firms worse off with lower revenue.

The second case in Lemma 3 is one where there will be an ex-post mixture of credit–buyers and money–buyers in equilibrium. The former are those buyers who draw a sufficiently low price, i.e., $\underline{\rho} \leq \rho \leq \tilde{\rho}_i(z)$. The latter occurs when buyers draw a sufficiently high price, i.e., $\tilde{\rho}_i(z) < \rho \leq \overline{\rho}(z)$. The implication is that an endogenous channel works through the connection between the credit condition and firms' market power. In equilibrium, this channel matters for the agent's (ex-ante) precautionary demand incentives regarding how much real money balances to carry to trade in the following period.

As previously discussed, the cut-off value $\check{z}_i$ is endogenous to the competitive banking market outcome, i. This renders a theoretically possible case where there is no pecuniary externality running from credit buyers to money constrained buyers. Money demand in this setting (our first case in Lemma 3) turns out to be identical to that of Berentsen et al. (2007).

Our result shows that there is another more interesting case. When calibrated to the data later, we shall see that the second case in Lemma 3 will be the relevant case. This means that there will be the pecuniary externality issue present. Also, this equilibrium case will always occur for plausible experiments around the empirically calibrated model. Hence, in the remainder of the paper, we focus on the second case in Lemma 3. Next, we summarise the equilibrium characterization and provide further discussions.

Definition 4. Given monetary policy $\gamma \geq \beta$, and taxes/transfers T, a stationary monetary equilibrium co-existing with money and credit in real variables is a steady-state allocation $(z^\star, x^\star, h^\star)$ in the centralized market, decision rules $\{q_b^\star(\rho, z), \xi^\star(\rho, i, z)\}$ in the decentralized market and prices $(J_i^\star(\rho), i)$ such that the following conditions are satisfied:

1. $(h^\star, x^\star, z^\star)$ solves the CM households problem, including the households' ex-ante real money demand decision in Equation (3.23).
2. Given $z = z^\star$, both $\{q_b^\star(\rho, z), \xi^\star(\rho, i, z)\}$ satisfy:

$$q_b^\star(\rho, z) = \begin{cases} [\rho(1+i)]^{-1/\sigma} & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ \frac{z}{\rho} & \text{if } \tilde{\rho}_i < \rho < \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \rho \geq \hat{\rho} \end{cases}, \tag{3.25}$$

and,

$$\xi^{\star}(\rho, i, z) = \begin{cases} \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - z & \text{if } 0 < \rho \leq \tilde{\rho}_i \\ 0 & \text{if } \tilde{\rho}_i < \rho < \hat{\rho}\ , \\ 0 & \text{if } \rho \geq \hat{\rho} \end{cases} \tag{3.26}$$

where

$$\hat{\rho} \equiv \hat{\rho}(z, \mathbf{s}) = z^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{\rho}_i \equiv \tilde{\rho}_i(i, z, \mathbf{s}) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}. \tag{3.27}$$

3. $J_i^{\star}(\rho)$ solves the DM firms' problem characterized in Equation (3.21).

4. Given $z = z^{\star}$, $i = i_d = i^{\star}$ clears the loan market:

$$\alpha_0 z = \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} (\alpha_1 + 2\alpha_2 - 2\alpha_2 J_i(\rho, z)) \cdot \xi^{*}(\rho, i, z) \mathrm{d}J_i(\rho, z). \tag{3.28}$$

We can back out all the other endogenous variables by solving the money demand Euler Equation (3.23). The left-hand side of Equation (3.23) captures the opportunity cost of carrying one extra unit of money into the next period. The right-hand side of Equation (3.23) represents the expected net return of holding money that can be decomposed into three terms. The first term reflects the marginal benefit of depositing an extra unit of idle money balances. The second term captures the interest saved by borrowing one less unit of money balances. The last term is the net marginal benefit of spending an extra dollar.

In contrast to Head et al. (2012), the novelty here is that banking arrangements affect the agents' precautionary demand for money holdings z, which then feeds back onto the distribution of goods prices $J_i(\rho, z)$, and its support, $\text{supp}(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$.

On the one hand, banking increases the expected net return of holding money captured by the first and second terms in Equation (3.23). On the other hand, firms' market power (price markups and dispersion) in frictional goods trades can also reduce some of the gains from banking. In particular, the integrals on the right-hand side of Equation (3.23) capture the reduction in the expected return on money even though agents have access to a competitive banking sector. This works through the first-order-stochastic-dominance result in Lemma 5 below. Also, the pricing cutoff function $\tilde{\rho}_i$ is a decreasing function of z.

Lemma 5. *Let monetary policy be $\gamma > \beta$ and noisy search frictions be $\alpha_1, \alpha_2 \in (0,1)$. Given $i = i_d = i^{\star} > 0$, and $\breve{z}_i := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}] > 0$, consider any two real money balances z and z' such that $\breve{z}_i < z < z'$. The price distribution $J_i(\cdot\,, z, \gamma)$ first-order stochastically dominates $J_i(\cdot\,, z', \gamma)$.*

The proof is in Online Appendix D.2. Lemma 5 highlights that a buyer with a lower real money balance, z, will be more likely to draw a higher price from the distribution. The intuition is as follows.

Suppose a buyer carries a small amount of real money balance into the goods market. Firms expect to produce and sell a lower quantity of goods. Given the buyer has an inelastic expenditure rule, a measure of firms will optimally respond by charging higher prices relative to their marginal cost of production. Consequently, the distribution of goods prices is more dispersed. The buyer with a tighter liquidity constraint is more likely to draw a higher price (or an associated markup) from the distribution. Therefore, the net benefit of banking in equilibrium should be ambiguous in contrast to Berentsen et al. (2007). Here, the gains from accessing a competitive banking sector depend on the interaction between agents' precautionary demand for money holdings and endogenous firms' market power in the goods market.

Using the result established in Lemma 5, we can then show the existence of a stationary monetary equilibrium with both money and credit. Such an equilibrium entails price dispersion in the frictional goods market. We summarize the result in the following Proposition. Details of the proof are in Appendix D.3.

Proposition 6. *Let monetary policy be $\gamma > \beta$ and noisy search frictions be $\alpha_1, \alpha_2 \in (0,1)$. There exists a stationary monetary equilibrium with both money and credit. Moreover, such an equilibrium entails price dispersion.*

4 Discussion

Overview. In this section, we discuss our model's mechanism. It is useful to compare our setting to that without banking. In particular, if we remove the banking sector, the model economy is equivalent to a pure monetary economy featuring firms market power studied in Head et al. (2012) (HLMW). As such, Equation (3.23) becomes

$$\frac{\gamma-\beta}{\beta} = \int_{\underline{\rho}(\hat{z})}^{\overline{\rho}(\hat{z})} \left[\alpha_1 + 2\alpha_2(1-\tilde{J}(\rho,\hat{z}))\right]\left(\frac{u_q[q^{no-bank}(\hat{z})]}{\rho} - 1\right) \mathrm{d}\tilde{J}(\rho,\hat{z}), \tag{4.1}$$

where the price distribution in a no-bank monetary economy is given by

$$\tilde{J}(\rho,\hat{z}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\tilde{R}(\overline{\rho})}{\tilde{R}(\rho)} - 1\right], \tag{4.2}$$

and the bounds are given by $\tilde{R}(\underline{\rho}) = \frac{\alpha_1}{\alpha_1+2\alpha_2}\tilde{R}(\overline{\rho})$, and $\overline{\rho} = \max\left\{\frac{c}{1-\sigma}, \underbrace{\hat{z}^{\frac{\sigma}{\sigma-1}}}_{=:\hat{\rho}}\right\}$, and the real profit per customer served is

$$\tilde{R}(\rho,\hat{z}) = q_b^{no-bank}(\rho,\hat{z})(\rho - c), \tag{4.3}$$

and buyer's optimal demand for goods is given by

$$q_b^{no-bank}(\rho,\hat{z}) = \begin{cases} \frac{\hat{z}}{\rho} & \text{if } 0 < \rho < \hat{\rho} \\ \rho^{-1/\sigma} & \text{if } \hat{\rho} \leq \rho \end{cases}. \tag{4.4}$$

Consider anticipated inflation away from the Friedman rule $\gamma > \beta$. In our setting and in that of Head et al. (2012), agents have precautionary demand for money holdings. Anonymity in the goods market gives rise to money as a means of payment. However, inflation ($\gamma > \beta$) induces the rate of return on money lower than the time discount rate, which acts as a tax on frictional goods trades. Hence, holding money can be costly when agents (ex-post) are stuck with unproductive idle money balances.

With access to banks, households can now reduce the cost of having unneeded liquidity (via depositing idle funds in the bank to earn interest). In addition, households can borrow extra money balances from the bank. Credit extended by banks helps households to relax their liquidity constraints when they need to make a payment in the goods market. We call this positive welfare effect of banking a *composition effect*, which works through an identical mechanism as in Berentsen et al. (2007).

Even though banks here are perfectly competitive as in Berentsen et al. (2007), households using bank credit can create an additional price effect in frictional goods transactions. We highlight a *price dispersion effect* that can contribute to a negative welfare effect of banking. The mechanism is as follows.

According to Equation (3.25), only some buyers use money balances and bank loans in goods transactions. Whereas others only use their money balances. We call the former credit-buyers and the latter money-buyers. In contrast to BCW, here, only some buyers (ex-post) borrow extra money balances from the bank because of equilibrium price dispersion in the goods market.

Firms expect some prospective customers to be money-buyers, and their expenditure rule is inelastic. Consequently, some firms desire to charge higher price markups to trade with money-buyers who demand fewer goods. At the same time, some firms would like to sell at a lower price to trade with the credit-buyers who demand more goods. Effectively, banking credit here can only benefit those credit-buyers who trade with low-price firms.

Therefore, the presence of banking credit can contribute to more high-price firms extracting rent from the liquidity constrained money-buyers. Thus, firms' market power can potentially reduce some of the gains from banking in equilibrium, even though buyers have access to a competitive banking market. This result has a similar flavor to the classic pecuniary-externality effect from credit (see, *e.g.*, Chiu et al., 2018). In Chiu et al. (2018), the externality is necessarily dependent on an assumption that the cost of producing goods q is a strictly increasing and convex function. In their competitive price-taking equilibrium, the existence of credit-buyers raises q which then raises the marginal cost of producing q, $c'(q)$, since $c''(q) > 0$ in their setup. This then raises equilibrium price p and feeds back in the form of tightening money-buyers' liquidity constraints.

If $c''(q) = 0$, there is no pecuniary externality in Chiu et al. (2018). In contrast, here, we have shut down the technological avenue necessary to generate the pecuniary externality of Chiu et al. (2018). Instead, we still have this effect for a different reason. Here, the pecuniary externality works through market power in the form of price (markups) dispersion. The existence of credit-buyers means ex-ante, agents end up carrying (relatively) less real balances, z. By Lemma 5, this tends to shift the distribution J to the right—*i.e.*, agents are more likely to get squeezed by higher prices and markups. If agents knew for sure they would be money-buyers, they would prefer to have carried more real balance. However, because of the idiosyncratic risk they face, ex-ante, all agents end up creating some pecuniary externality of the ex-post liquidity constrained agents.

Decomposing the welfare effects of banking. To understand the positive and negative welfare effects of banking, we compare Equation (3.23) and Equation (4.1). In our model economy, buyers can deposit funds in the bank to earn interest $i_d > 0$ if they (ex-post) fail to match with a firm to trade in the DM. We call this type of buyer depositors. The interest paid to depositors increases the expected marginal benefit of accumulating money balances. Moreover, buyers who are liquidity constrained and sample low enough prices of the goods can use bank credit to relax their liquidity constraint (ex-post). The first and second terms on the right-hand side of Equation (3.23) reflect such banking benefits. Due to the composition effect, banking helps to improve consumption allocation relative to HLMW, on the one hand.

On the other hand, access to credit by buyers can also lower the expected marginal benefit of money when firms can exploit markups in frictional goods trades. In particular, the integrals on the right-hand side of Equation (3.23) capture the negative welfare effects of banking. The reason is as follows.

To avoid the inflation tax, buyers would like to carry less own money balances by taking out a loan from the bank (ex-post). However, firms expect some potential customers to be liquidity constrained by their money balances, and their expenditure rule is inelastic. A measure of firms would then optimally respond by charging higher markups on those liquidity constrained money-buyers (see Lemma 5).

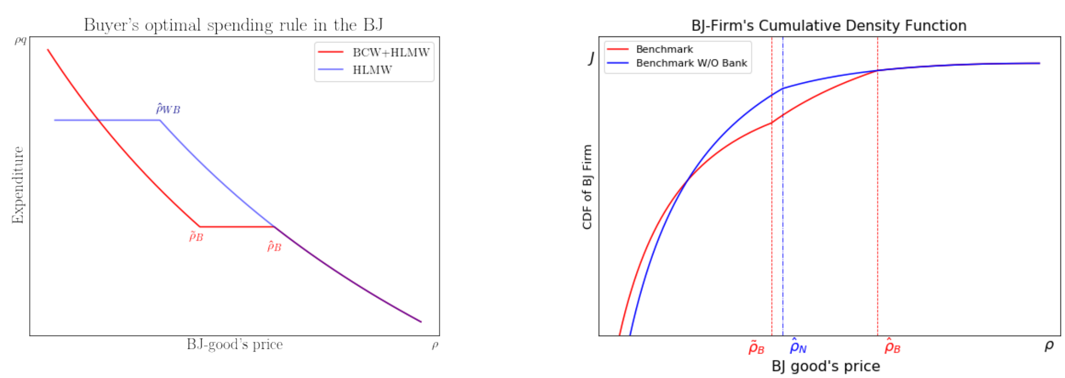
Effectively, credit-buyers can contribute to bidding up the price of DM goods, in the sense that the support of the goods price distribution, $\text{supp}(J_i) = [\underline{\rho}(z), \overline{\rho}(z)]$, to be wider than that in

HLMW. Access to credit by buyers amplifies firms' market power in terms of price markups and dispersion. This effect works through firms' trade-off between intensive margin (profit margin) and extensive margin (likelihood of trading).

Recall that banking credit only benefits some buyers but not all. In particular, buyers use credit if they draw a sufficiently low price ρ on goods from the distribution $J_i(\rho, z)$ (see Equation (3.25) and Equation (3.26)). However, banking credit induces higher price dispersion, which implies more high-price firms extracting rent from liquidity constrained money-buyers. Both integrals on the right-hand side of Equation (3.23) capture a reduction in the expected return on holding money along the rising price dispersion. Hence, a distortion will appear on the average interest saved on borrowing an extra dollar for the credit-buyers and the liquidity premium for the money-buyers. We highlight that firms' market power in frictional goods trades can potentially reduce gains from a competitive banking sector.

Numerical illustration. Next, we provide a numerical illustration of our mechanism above by comparing our baseline model economy to that without banking (HLMW). Figure 1 displays the composition and price dispersion effects of banking given policy $\tau > \beta - 1$. The blue line represents the model economy in HLMW. The red line represents our baseline model economy (with banking arrangement).

Figure 1: Composition and price dispersion effects given policy $\gamma = 1 + \tau > \beta$.



(a) Composition effect

(b) Price dispersion effect

Composition effect: positive welfare effects of banking. In HLMW, a buyer cannot spend more than his liquidity constraint at a low enough price level $\hat{\rho}_{WB}$, even though he would like to do so. The horizontal blue line reflects this type of (ex-post) liquidity constrained buyer in Figure 1a. The cut-off $\hat{\rho}_{WB}$ is the price level at which the buyer becomes liquidity unconstrained ex-post. The buyer spends less than his total money balances if he draws a price higher than $\hat{\rho}_{WB}$.

In contrast to HLMW, there is now a composition effect highlighting the benefits of having access to banking credit. A buyer can now borrow additional money balances from the bank to relax his liquidity constraint when $\rho \leq \tilde{\rho}_B$. Thus, the (ex-post) credit-buyer faces a more relaxed liquidity constraint to spend more in the goods market than money-buyers.

Price dispersion effect: negative welfare effects of banking. When firms' market power (markup and price dispersion) arises from informational frictions, access to competitive banks can cause an additional negative welfare effect. This result is because not all agents can benefit from banking. In particular, those agents who use bank for loans, create a price effect in the goods market. This negatively affects agents who do not use banking credit.

Recall that firms expect some prospective customers to be constrained by their money balances, and their expenditure rule to be inelastic. Thus, a measure of firms optimally responds by charging higher prices relative to their marginal cost of production. Consequently, buyers (on average) are more likely to draw a higher price in the sense of first-order stochastic dominance. This price dispersion effect is reflected by some parts of the red line falling below the blue line in Figure 1b. Moreover, the support of the price distribution in our baseline model economy is wider than in HLMW and $\tilde{\rho}_i$ is increasing.

Effectively, bank credit induces firms to exploit more price markups in extracting surplus from the (ex-post) liquidity constrained money-buyers. In other words, the credit-buyers increase the price markups and dispersion in frictional goods trades. Consequently, each money-buyer (who draws a high enough price such that $\tilde{\rho}_B \leq \rho \leq \hat{\rho}_B$) faces a tighter liquidity constraint. In this case, the liquidity constrained money-buyer spends less than the case without access to banking arrangements. However, this negative effect does not matter for the liquidity unconstrained money-buyer (who draws $\rho > \hat{\rho}_B$). The reason is that his liquidity constraint does not bind.

In summary, banking affects agents' consumption outcomes differently when firms have market power in frictional goods transactions. In this setting, access to a competitive banking sector can amplify firms' market power, creating an additional welfare-reducing effect of banking. This negative welfare effect, tied to credit-buyers, pushes up price dispersion. This then increases the measure of firms extracting rent from liquidity constrained money-buyers. Consequently, the welfare-improving function of banking liquidity transformation is no longer unambiguous, in contrast to Berentsen et al. (2007).

5 Quantitative evaluation

In this section, we calibrate our model to the macro-level data in the United States. Then, we analyze the model numerically regarding the composition and price dispersion effects discussed in Section 4. Finally, we present the welfare implications of banking in an economy with endogenous firms' market power in the DM goods trades.

5.1 Baseline calibration

We interpret one period in the model to be a year. Our calibration strategy is to match the empirical money demand and the firms' average (percentage) markup in the United States.

Given policy $\gamma = 1 + \tau$, we measure the aggregate (percentage) markup and price dispersion (relative price variability) respectively by:

$$\mu(\gamma) = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \frac{\rho - c}{c} \mathrm{d}J_i(\rho, z, \gamma), \tag{5.1}$$

and,

$$\mathrm{RPV}(\gamma) = \left[\int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} (\rho - \breve{\rho})^2 \mathrm{d}J_i(\rho, z, \gamma) \right]^{\frac{1}{2}}, \tag{5.2}$$

where $\breve{\rho} = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho \mathrm{d}J_i(\rho, z, \gamma)$.

The aggregate output in our economy is given by:

$$Y = \int_{\underline{\rho}(z,\gamma)}^{\overline{\rho}(z,\gamma)} \rho q_b^{\star}(\rho, z) \mathrm{d}J_i(\rho, z, \gamma) + x^{\star}. \tag{5.3}$$

We assume a log-utility function in the CM, $U(x) = B\ln(x)$, where B is a scaling parameter that determines the relative importance of CM and DM consumption. With quasi-linear preferences, real CM consumption is determined by $x^{\star} = (U')^{-1}(B)$. The noisy-search probabilities in the DM can be re-parametrized by a number λ. That is, we can set $\alpha_0 = (1-\lambda)^2$, $\alpha_1 = 2(1-\lambda)\lambda$, and $\alpha_2 = \lambda^2$. We normalize the cost of DM production to one ($c = 1$) as in Head et al. (2012). The DM utility function is given by Equation (3.2).

Sample period and data. Our model is fitted to long-run data spanning from 1960 to 2007 to avoid the Great Recession period where the nominal interest rate is at the zero lower bound. We use the NewM1-to-GDP ratio defined in Lucas and Nicolini (2015) as a measure of the money demand M/PY in the United States. We employ the U.S. markup data from De Loecker et al. (2020). We obtain the U.S. three-month T-bill interest rate data from the FRED.

Identification and calibration. The parameters that need to determined are: β, τ, σ, B, and λ. The parameter β is the time discount factor. The CM utility scaling parameter B affects the average of money demand M/PY. This is because the parameter B affects CM consumption x and thus output Y. The CRRA risk aversion parameter σ pins down the price elasticity of demand for the

DM consumption goods, which affects the elasticity of money demand with respect to the nominal interest rate i. The noisy-search probabilities, via λ, directly affect the price distribution J_i, and thus the aggregate markup.

From the Fisher equation, we use both the average interest rate of the three-month T-bill, $i = 0.063$, and the long-run inflation rate, $\tau = 0.042$, to pin down the discount factor $\beta = 0.98$. The remaining parameters (σ, B, λ) are calibrated internally. We jointly choose (σ, B, λ) to match the point elasticity of money demand, the average of money demand M/PY, and aggregate markup μ, all of which are with respect to the nominal interest rate i.

We summarize the value of jointly calibrated parameters and calibration results in Table 3. Given a reasonable fit of our model to the empirical targets, we can use the calibration above as a benchmark model.

Table 3: Calibration targets and results

Parameter	Value	Empirical Targets	Model
σ	0.5	Elasticity of $M/PY = -0.25$	-0.2
B	2.3	Mean of $M/PY = 0.22^a$	0.2
$\{\alpha_n\}_{n\in\{0,1,2\}}$	$\lambda = 0.6^b$	Markup =35%	35%

[a] The point elasticity refers to the elasticity of M/PY with respect to the nominal interest rate i, evaluated at the data mean of i.
[b] $\alpha_0 = (1-\lambda)^2$, $\alpha_1 = 2(1-\lambda)\lambda$, and $\alpha_2 = \lambda^2$.

5.2 Comparative steady states

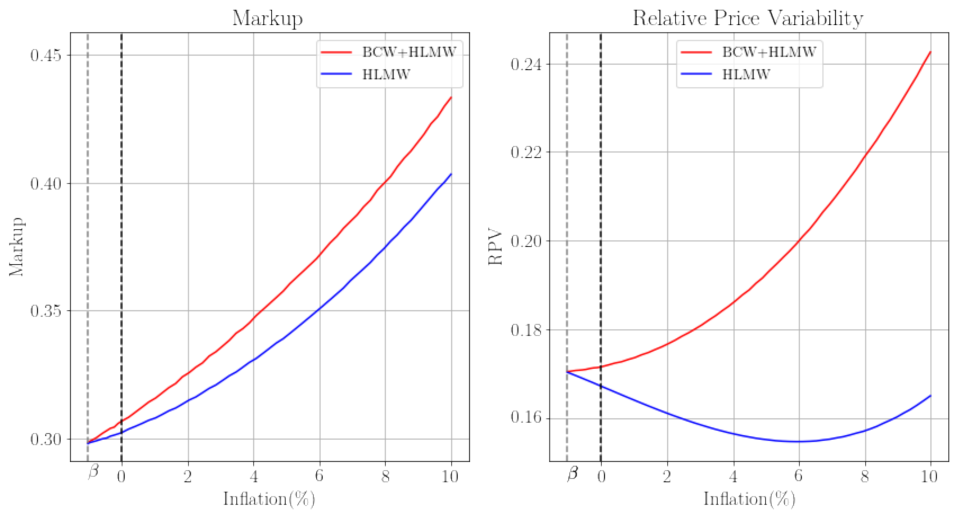
In this section, we study the welfare effects of banking in an economy with endogenous firms' market power in goods trades. Then, we consider a set of economies, each distinguished by its long-run inflation rates τ from $\tau \in (\beta - 1, \bar{\tau}]$, where we set $\bar{\tau} = 0.1$ (*i.e.,* 10% annual inflation rate) as the exogenous upper bound.[19]

Figure 2 further illustrates our model mechanism discussed in Section 4. In particular, we have shown that a perfectly competitive banking sector can amplify firms' market power measured in terms of markup (by Equation (5.1)) and price dispersion (by Equation (5.2)). Consequently, the presence of banking has different welfare consequences relative to Berentsen et al. (2007).

In Berentsen et al. (2007), the welfare gain of banking liquidity transformation comes from the interest payments on unproductive idle money balances. Households can then accumulate more money balances to trade in the goods market. Thus, having access to a competitive banking sector is always welfare-improving relative to a pure monetary economy.

[19]Note: It can be verified that price dispersion cannot be sustained at the Friedman rule, *i.e.,* $\tau = \beta - 1$. Moreover, banking is redundant since it is costless for agents to carry money balances. For our purpose, we focus on long-run anticipated inflation away from the Friedman rule.

Figure 2: The effects of inflation on the aggregate markup and price dispersion given policy $\tau \in (\beta - 1, \bar{\tau}]$.



Here, some banking benefits go to the credit-buyers (by relaxing their liquidity constraint) and inactive buyers (by depositing idle funds). However, banking credit can also distort the liquidity premium for the liquidity constrained money-buyers via higher price dispersion and markups. Overall, whether competitive banks, in transforming liquidity, improve welfare in equilibrium (with the endogenous market power of firms arising from informational frictions) is ambiguous.

Welfare implications of banking. We measure welfare by the consumption equivalent variation (CEV), capturing how much consumption an agent is willing to give up in an economy without banks to live in an economy with banks.

Given $\gamma = 1 + \tau$ policy, the welfare function in an SME is given by

$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(x^\star) - x^\star + \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1 - J_i(\rho, z_e, \gamma))\Big)\Big(u[q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big)\mathrm{d}J_i(\rho, z_e, \gamma)\Bigg], \tag{5.4}$$

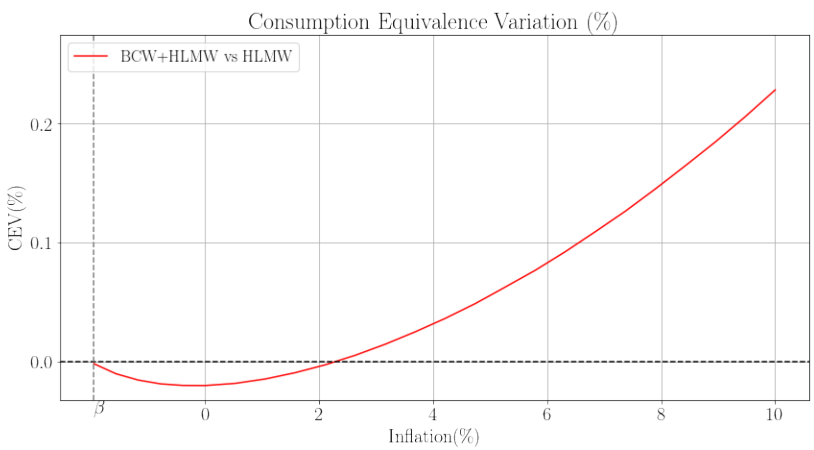
where $e \in \{Baseline, HLMW\}$ indexes our baseline model economy or the no-bank economy of HLMW.

We can also write the total welfare at given γ inflation rate with consumption reduced by a factor of $\triangle$ as

$$W^e(\gamma) = \frac{1}{1-\beta}\Bigg[U(\triangle x^\star) - x^\star + \int_{\underline{\rho}(z_e,\gamma)}^{\overline{\rho}(z_e,\gamma)} \Big(\alpha_1 - 2\alpha_2(1 - J_i(\rho, z_e, \gamma))\Big)\Big(u[\triangle q_b^\star(z_e)] - c[q_b^\star(z_e)]\Big)\mathrm{d}J_i(\rho, z_e, \gamma)\Bigg]. \tag{5.5}$$

We compute the CEV as the value $1-\triangle$ that solves $W^{Baseline}(\gamma) = W^{HLMW}{}_{\triangle}(\gamma)$ given policy $\gamma = 1+\tau$. This measure says that every agent in the economy with perfectly competitive banks needs to give up $1-\triangle$ percent of his consumption to move to the economy without access to competitive banks at given policy.

Figure 3: Consumption Equivalent Variation (%): With banks versus without banks



According to Figure 3, banking has a non-monotonic welfare consequence when the trend inflation rate varies from just above the Friedman rule $\gamma = 1+\tau = \beta$ to 10% annual inflation. In particular, banks are inessential institutions when the trend inflation rate is sufficiently low. This result hinges on the interaction between the composition and price dispersion effects discussed earlier in Section 4.

On the one hand, positive welfare effects of banking come from the composition effect. First, (ex-post) inactive buyers in trading with probability α_0 can deposit their idle money balances to earn an interest i_d. Second, buyers who trade with low-price firms find it worthwhile to borrow additional money balances from the bank. These credit-buyers have more relaxed liquidity constraints. Thus, they can spend more on goods to enjoy a higher utility flow in the DM.

On the other hand, credit can amplify firms' market power (markups and price dispersion), which creates a negative welfare effect of banking on liquidity constrained money-buyers. The reason is as follows.

Some firms would find it optimal to trade more frequently with credit-buyers (who demand more goods) at lower prices. Recall that in equilibrium, all firms earn equal expected profit for any price ρ in the support of the price distribution, $\text{supp}(J_i) = [\underline{\rho}, \overline{\rho}]$. To maintain the high-price firms ($\tilde{\rho}_i < \rho \leq \overline{\rho}$) indifferent to the expected profit earned by the low-price firms, they will need to charge a higher markup over the marginal cost of production. Effectively, the presence of credit-buyers shifts the probability mass up (or right) over the range of prices faced by the money-constrained buyers. The availability of banking credit induces more market power to the high-price firms in extracting rent from the (ex-post) liquidity constrained money-buyers.

Summary of insights. Imperfect information through noisy search frictions in the goods market generates a policy-dependent distribution of goods prices (and associated markups), as in Head et al. (2012). The presence of competitive banking benefits only agents who would like to deposit and those who optimally use credit by inducing more firms who serve them to more likely post low prices. In turn, the externality effect is in firms who charge higher prices to money-constrained agents who do not find it optimal to borrow. These agents' expenditures are inelastic to the price rise and they end up consuming less. When inflation is sufficiently low, the cost of holding money is also low. Thus, the gains from banking along the channel of composition effect are also small. The price dispersion effect can easily outweigh such benefits via higher markups distorting the liquidity premium for the liquidity constrained money-buyers.

To sum up, firms' price dispersion induces (ex-post) heterogeneous consumption outcomes among credit-buyers and money-buyers. Hence, non-trivial feedback from firms' market power on the welfare consequences of banking. In particular, credit-buyers benefit from banking credit to purchase more goods. However, banking also makes firms extract more rent from liquidity constrained money-buyers in goods trades, thus lowering consumption. The essentiality of banks in liquidity transformation is no longer unambiguous in our economy with endogenous firms' market power in the goods market.

6 Conclusion

We construct a model of money, credit and endogenous retail market power where informational frictions induce a policy-dependent distribution of goods prices and associated markups in equilibrium.

We show that access by borrowers to credit can contribute to amplifying firms' market power, reducing the welfare gains from banking. Effectively, more goods demanded by credit-buyers increases the measure of firms charging higher prices, extracting rent from liquidity constrained money-buyers. As a result, market power in the retail industry can make an otherwise competitive banking sector less productive in reallocating liquidity in equilibrium.

Thus, the welfare-improving role of banking liquidity transformation is no longer unambiguous in a monetary economy with endogenous firms' market power. Our model speaks to why policymakers are worrying about the rising industry market power in the United States and calling for promoting more competition for better allocation of resources.

Our model generates a positive relationship between the consumer credit-to-GDP ratio and firms' market power (measured by price markups and dispersion), consistent with the empirical observation using firm-level data.

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Online Appendix

On Endogenous Markups Distribution and the Pecuniary Externality of Credit on Monetary Exchange

Omitted Proofs and Other Results

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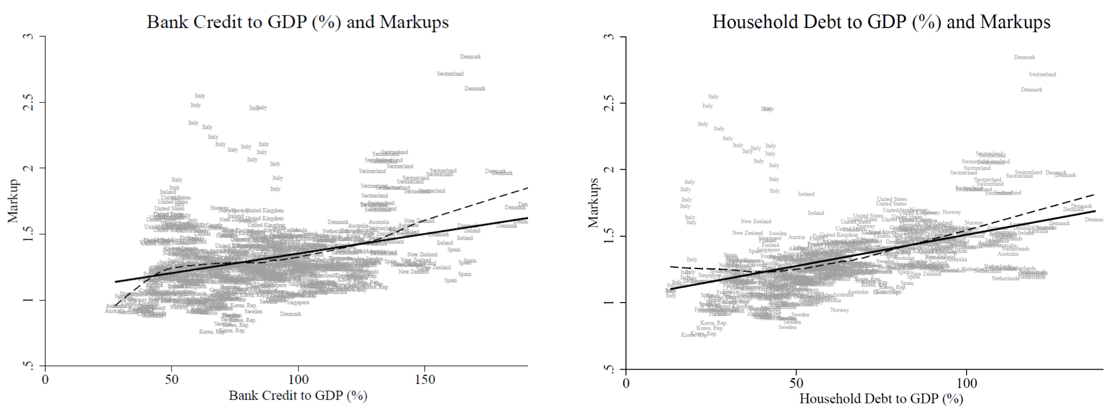
A Cross-country Panel Data

Using the dataset from 19 advanced countries, we find that there is a positive correlation between bank credit/household debt and markups during the periods between 1980 and 2016. The magnitude of the correlation between bank credit/household debt and markups is about 0.34 and 0.47, respectively.

The bank credit and household debt data come from the Bank for International Settlements (BIS), Long series on the total credit to the non-financial Sectors. We get the country specific markups from De Loecker, Eeckhout and Mongey (2021).

We follow the World Bank definition on advanced countries based on the gross national income in 2000. The 19 advanced countries which we use include: Australia, Austria, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, New Zealand, Norway, Singapore, Korea Rep, Spain, Sweden, Switzerland, United Kingdom, and United States.

Figure 4: Markups and Credit



B U.S. Data

Markup. We use quarterly firm-level balance sheet data of listed U.S. firms for the period 1980Q1 to 2019Q4 from Compustat North America. Following De Loecker et al. (2020), our industry classification is on the basis of the North American Industry Classification System (NAICS). Particularly, we observe measures of input expenditure, sales, detailed industry activity classifications, and capital stock information. The item from the financial statement of the firm that we will utilize to measure the variable input is the cost of goods sold (COGS). It bundles all expenses directly attributable to the production of the goods sold by the firm and includes materials and intermediate inputs, labor cost, energy, and so on.

Relative Price Variability (RPV). Following Banerjee et al. (2002) and Banerjee et al. (2007), we compute the relative price variability (RPV) for the period from 1980Q1 to 2019Q4 by using the Bureau of Labor Statistics (BLS) data.[20] The monthly data is converted to quarterly data by averaging the monthly levels of the respective indexes.

Table 4 shows the component series in CPI-U (Consumer Price Index — All Urban Consumers). The weights are provided from Archived Relative Importance of Components in the Consumer Price.[21] Note that we employ the Entertainment component to compute the RPV until 1998, but then we instead utilize the Recreation component. The reason is that BLS change the Entertainment component to Recreation after 1998.

Table 4: Data sources

Name of Index	BLS Series Code	Note
All Items	CUUR0000SA0	-
Food and Beverages	CUUR0000SAF	-
Housing	CUUR0000SAH	-
Apparel	CUUR0000SAA	-
Transportation	CUUR0000SAT	-
Medical Care	CUUR0000SAM	-
Recreation	CUUR0000SAR	1997 – 2020
Entertainment	-	1980 – 1996
Education and Communication	CUUR0000SAE	1997 – 2020
Other Goods and Services	CUUR0000SAG	-

Note: We obtain the data from the U.S. Bureau of Labor Statistics (BLS).

C Robustness test - Alternative ECM method

In this section, we present our robustness tests regarding the empirical results discussed in Section 2. We implement robustness tests as follows: 1) vector error correction model (VECM); 2) we include the lag term; and 3) we use bank credit-to-GDP (from the Bank for International Settlements) as another proxy.

VECM. Prior to estimation, we use a unit root test to investigate integration properties of the data. We find that the log of markup, consumer credit–to–GDP, and log of real GDP are as $I(1)$,

[20] Data source: https://www.bls.gov/cpi/data.htm

[21] Data source: https://www.bls.gov/cpi/tables/relative-importance/home.html

but the log of RPV is $I(0)$. The vector error correction model is given by

$$\Delta X_t = \gamma + \sum_{j=1}^{p-1} \Gamma_j \Delta X_{t-1} + \epsilon_t, \tag{C.1}$$

where γ is a vector of intercepts, ϵ_t is a vector of contemporaneous errors, X_t is the three–dimensional vector containing the variables, and Γ_j is a set of matrices of short-run coefficients.[22]

Since there is a positive relationship between markup and RPV and the sample is relatively small, then we need to minimize the number of variables. Then, we utilize the two VECM specifications: 1) log of markup, log of real GDP, and consumer credit–to–GDP; and 2) log of RPV, log of real GDP, and consumer credit–to–GDP. We set the $p(lag) = 4$ based on information criteria in both empirical specifications.

First, we employ the three–dimensional vector including the variables: log of markup, consumer credit–to–GDP $(\%)$, and the log of real GDP to investigate the long-run relationship between log of markup and consumer credit–to–GDP. The Johansen Trace indicates one cointegrating vector among three variables at the 5% level of significance. Based on the VECM analysis, we show that the estimated long-run relationship is

$$\log(\mu_t) = 23.02^{**} \cdot d_t^{CC} - 0.11^{**} \cdot \log(\text{RGDP}_t). \tag{C.2}$$

Equation (C.2) suggests that there is a positive long-run relationship between markup and consumer credit–to–GDP d_t^{CC}. The estimate is statistically significant.

To estimate the long-run effect of consumer credit on RPV, we utilize the three–dimensional vector including the variables: log of RPV, consumer credit–to–GDP $(\%)$, and the log of real GDP. The Johansen Trace indicates one cointegrating vector among three variables at the 1% level of significance. We can obtain the estimated long-run equation as

$$\log(\text{RPV}_t) = 17.015^{*} \cdot d_t^{CC} + 0.036 \cdot \log(\text{RGDP}_t). \tag{C.3}$$

Equation (C.3) suggests that the long run relationship between RPV and consumer credit–to–GDP $(\%)$ is positive, and the estimate is statistically significant. These results support our empirical validity for the result of OLS in Section 2.

Including the Lagged Dependent Variable. By utilizing the Durbin-Watson test, we find that there is a serial correlation in the residuals from OLS in Section 2. Then, we include the lagged dependent variable to reduce the serial correlation. Based on the information criteria, we decide to include the one–to–four lag terms of dependent variable. The empirical specification is almost

[22]Note: we do not include the constant term.

identical to Equation (2.5) except it includes the lag terms. As shown in Table 5, the results of the robustness test suggest that the positive association between consumer credit and markup/price dispersion is positive. Even though the estimate for RPV is less significant, there is a positive correlation between RPV and consumer credit.

Table 5: OLS results: Markup and RPV

Dependent Variable:	$\log(\mu_t)$ (1)	$\log(\mu_t^{\text{retail}})$ (2)	$\log(\text{RPV}_t)$ (3)	ν_t (4)
Consumer Credit-to-GDP	0.582^{*}	0.159^{*}	8.326^{+}	0.519^{*}
	(0.316)	(0.072)	(4.687)	(0.252)
Log of real GDP	0.019	0.017	1.298	0.004
	(0.807)	(0.421)	(0.377)	(0.960)
Business TFP	-0.053	-0.023^{*}	-2.383^{**}	-0.0321
	(0.173)	(0.042)	(0.005)	(0.335)
GDP Deflator	-0.279^{**}	-0.065^{*}	-0.170	-0.082
	(0.005)	(0.037)	(0.952)	(0.388)
Unemployment rate	-0.022	-0.002	5.137^{*}	-0.064
	(0.781)	(0.934)	(0.016)	(0.419)
R^2	0.973	0.914	0.715	0.929
Observations	156	156	156	156

Note: Robust errors are in parenthesis. $+, *,$ and $**$ are respectively at the significance level of 10 %, 5 % and 1 %. Constant and trend variables are included but not reported.

Different Measure: Bank Credit. We utilize a different measure: bank credit–to–GDP (%) from the BIS. The empirical specifications are similar to Equation (2.5) except for replacing consumer credit–to–GDP with bank credit–to–GDP.

Table 6 reports the empirical results by utilizing the bank credit–to–GDP. Columns (1)—(3) show that the relationship between bank credit and markup is positive, and the estimates are statistically significant. Furthermore, the relationship between bank credit on RPV is statistically significant and positive as shown in Columns (4)—(6). These results support the validity of our main empirical results presented in Section 2.

Table 6: OLS results: Markup and RPV

Dependent Variable:		$\log(\mu_t)$			$\log(\mathrm{RPV}_t)$	
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Credit-to-GDP	0.312**	0.318**	0.300**	4.081**	3.924**	4.038**
	(0.488)	(0.048)	(0.049)	(1.225)	(1.262)	(1.257)
Log of real GDP	-0.025	-0.028	0.061	-1.737	-1.670	-2.230
	(0.731)	(0.701)	(0.415)	(0.343)	(0.372)	(0.277)
Business TFP		0.0734	0.0384		-1.697	-1.476
		(0.231)	(0.529)		(0.221)	(0.289)
GDP Deflator			-0.415**			2.625
			(0.002)			(0.442)
R^2	0.935	0.936	0.938	0.121	0.131	0.134
Observations	160	160	160	160	160	160

Note: Robust errors are in parenthesis. $+, *$, and $**$ are respectively at the significance level of 10 %, 5 % and 1 %. Constant and trend variables are included but not reported.

D Omitted proofs

In Section D.2, we study how the price distribution J_i changes with respect to the asset position of the households. We then establish the existence of a stationary monetary equilibrium with both money and credit in Section D.3.

D.1 Proof of Lemma 1

Below, we rewrite nominal variables in real terms. Multiplying the results through with the value of money ϕ yields the result in Lemma 1, which was presented in nominal terms.

Lemma 7. *Given the buyer's real money holding z and the buyer's optimal demand for q_b^*, the relative monopoly price ρ^m is defined as*

$$\rho^m = \begin{cases} \frac{c}{1-\sigma} & \text{if } z \in (0, \breve{z}) \\ \max\left(\frac{c}{1-\sigma}, \hat{\rho}\right) & \text{if } z \in (\breve{z}, \bar{z}) \end{cases}$$

where $\breve{z} = \left(\frac{c}{1-\sigma}\right)^{\frac{\sigma-1}{\sigma}} (1+i)^{-\frac{1}{\sigma}}$, $\bar{z} = \underline{\rho}^{\frac{\sigma-1}{\sigma}} (1+i)^{-\frac{1}{\sigma}}$, *and* $\hat{\rho} := \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$.

Proof. A monopoly firm posts the monopoly price ρ^m, considering a buyer's optimal demand for

BJ good. Given a buyer's optimal demand for q_b^*, a firm's profit per trade, $R(\rho; z, i)$ is

$$R(\rho, z) = \begin{cases} (\rho(1+i))^{-\frac{1}{\sigma}}(\rho - c) & \text{if } 0 < \rho \leq \tilde{\rho} \\ \frac{z}{\rho}(\rho - c) & \text{if } \tilde{\rho} < \rho < \hat{\rho} \\ \rho^{-\frac{1}{\sigma}}(\rho - c) & \text{if } \hat{\rho} \leq \rho \end{cases}$$

where $\hat{\rho} := \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$ and, $\tilde{\rho}(z, i) = \hat{\rho}(1+i)^{\frac{1}{\sigma-1}}$.

First, when a monopoly firm contacts a borrower (first case), the monopoly price (ρ^m) satisfies

$$\frac{\partial R(\rho, z)}{\partial \rho} = -\frac{1}{\sigma}(1+i)\,(\rho(1+i))^{-\frac{1}{\sigma}-1}(\rho - c) + (\rho(1+i))^{-\frac{1}{\sigma}} = 0.$$

Therefore, $\rho^m = \frac{c}{1-\sigma}$.

Second, when a monopoly firm contacts a constrained money user (second case), the monopoly price (ρ^m) is $\rho^m = \hat{\rho}(z; i, \rho) = z^{\frac{\sigma}{\sigma-1}}$ since $\partial R(\rho)/\partial \rho > 0$.

Last, when a monopoly firm contacts a non-constrained money user (last case), the monopoly price (ρ^m) solves

$$\frac{\partial R(\rho, z)}{\partial \rho} = \left(1 - \frac{1}{\sigma}\right)\rho^{-\frac{1}{\sigma}} + \frac{c}{\sigma}\rho^{-\frac{1}{\sigma}-1} = 0$$

Therefore, $\rho^m = \frac{c}{1-\sigma}$.

If $z < \breve{z}_i$, *i.e.*, $\frac{c}{1-\sigma} < \tilde{\rho}$, then $\rho^m = \frac{c}{1-\sigma}$, where $\breve{z}_i = \left(\frac{c}{1-\sigma}\right)^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}$. If $z > \breve{z}_i$, $\rho^m = max\left\{\frac{c}{1-\sigma}, \hat{\rho}\right\}$. Therefore, we can summarize ρ^m as stated in the Lemma above. Rewriting for nominal variables, we have Lemma 1. □

D.2 Proof of Lemma 5

Proof. Fix the trend inflation rate away from the Friedman rule $\tau > \beta - 1$. Assume $\alpha_1 \in (0, 1)$. Let $i = i_d = i^\star$ be the market loan interest rate. By Lemma 2, the analytical formula for the real price distribution $J_i(\rho, z)$ is given by

$$J_i(\rho, z) := J_i(\rho, i, z, \mathbf{s}) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R(\overline{\rho}, i, z)}{R(\rho, i, z)} - 1\right] = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{q_b^\star(\overline{\rho}, i, z)(\overline{\rho} - c)}{q_b^\star(\rho, i, z)(\rho - c)} - 1\right], \tag{D.1}$$

where the upper support of the distribution $J_i(\rho, z)$ is determined by:

$$\overline{\rho} := \overline{p}(z, \mathbf{s}) = \begin{cases} \max\{c/(1-\sigma), \underbrace{z^{\sigma/(\sigma-1)}}_{=\hat{\rho}}\} & \text{if } z > \breve{z} \\ \frac{c}{1-\sigma} & \text{if } z \leq \breve{z} \end{cases}, \tag{D.2}$$

given $\check{z} := [\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}]$, and the lower support of J_i, $\underline{\rho}$, solves $R(\rho, i, z) = \frac{\alpha_1}{\alpha_1+2\alpha_2} R(\overline{\rho}, i, z)$.

In this proof, we want to show the relationship of how the price distribution $J_i(\rho, z)$ changes with respect to the change in the real money holdings z. Consider two real money holdings z and $z^{'}$ such that $\check{z} < z < z^{'} < \overline{z}$. Then we want to check whether $J_i(\rho, z)$ is lying on top or below for z relative to $z^{'}$.

Note: For the ease of notation, we will denote $\overline{\rho}(z)$ and $\underline{\rho}(z)$ respectively by $\overline{\rho}$ and $\underline{\rho}$ occasionally. Likewise, we denote the cut-off prices by $\hat{\rho} := \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$ and $\tilde{\rho}_i := \tilde{\rho}_i(z, i) = \hat{\rho}(z)(1+i)^{\frac{1}{\sigma-1}}$. It should be kept in mind that all these cut-off prices and bounds of the price distribution depend on the state of the economy z and policy τ in general.

Suppose a real money balance z that satisfies: $\frac{c}{1-\sigma}^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} = \check{z} < z < \overline{z} = \underline{\rho}^{\frac{\sigma-1}{\sigma}}(1+i)^{\frac{-1}{\sigma}}$. Recall that the CRRA risk aversion parameters requires to be $\sigma < 1$, and from the result established earlier in Section D.1, we then have the following order:

$$\underline{\rho} < \tilde{\rho}_i < \hat{\rho} \leq \overline{\rho}.$$

Observe from the upper bound of the price distribution, it has two possible cases either $\overline{\rho} = \hat{\rho}$ or $\rho = c/(1-\sigma)$ when $z \in (\check{z}, \overline{z})$. We need to check for each case.

Case 1. Suppose $\overline{\rho} = \hat{\rho}(z) = z^{\frac{\sigma}{\sigma-1}}$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$.

Given the buyer's optimal demand schedule defined in Equation (3.25) and $z \in (\check{z}, \overline{z})$, we can write out the price distribution $J_i(\rho, z)$ more explicitly by

$$J_i(\rho, z) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1} z(\overline{\rho}(z)-c)}{\rho^{-1} z(\rho - c)} - 1\right] & \text{if } \rho \in (\tilde{\rho}_i(z), \overline{\rho}(z)] \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1} z(\overline{\rho}(z)-c)}{[\rho(1+i)]^{-1/\sigma}(\rho - c)} - 1\right] & \text{if } \rho \in [\underline{\rho}(z), \tilde{\rho}_i(z)] \end{cases}, \tag{D.3}$$

with support $[\underline{\rho}(z), \overline{\rho}(z)]$.

Consider any two real money holdings z_0 and z_1 such that $\check{z} < z_0 < z_1 < \overline{z}$. First, it is clear that $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. Using this result and the equal profit condition of the firms, we can then deduce the lower support also satisfies that $\underline{\rho}(z_0) > \underline{\rho}(z_1)$. Thus, we have $[\overline{\rho}(z_0) - \underline{\rho}(z_0)] - [\overline{\rho}(z_1) - \underline{\rho}(z_1)] > 0$. In words, the support of the price distribution with lower real money balance is wider than that with higher real money balance.

Second, for $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$, then $J_i(\rho, z_0) < J_i(\rho, z_1)$ because of $\overline{\rho}(z_0) > \overline{\rho}(z_1)$. The intuition is that buyers with lower money holdings are more likely to be liquidity constrained, and that pushes up the measure of firms posting higher prices. Thus, $J_i(\rho, z_0)$ falls below $J_i(\rho, z_1)$ for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$.

Next, by the fact that the price distribution J_i is a cumulative distribution function, it then follows that $J_i(\rho, z_0) = J_i(\rho, z_1) = 1$ for some $\rho \geq \overline{\rho}(z_0)$. Likewise, we have $J_i(\rho, z_0) = J_i(\rho, z_1) = 0$ for some $\rho \leq \overline{\rho}(z_1)$.

Collect these results, we have then established that $J_i(\rho, z_0)$ first-order stochastically dominates $J_i(\rho, z_1)$. That is, $J_i(\rho, z_0) \leq J_i(\rho, z_1)$ within the interval $[\underline{\rho}(z_1), \overline{\rho}(z_0)]$, and strict inequality for some $\rho \in (\underline{\rho}(z_1), \overline{\rho}(z_0))$ given any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

Case 2. Suppose $\overline{\rho} = c/(1-\sigma)$. We have the following order: $\underline{\rho}(z) < \tilde{\rho}_i(z) < \hat{\rho}(z) < \overline{\rho}$.

Likewise, we can write out the price distribution $J_i(\rho, z)$ explicitly by

$$J_i(\rho, z) = \begin{cases} 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{\rho^{-1/\sigma}(\rho-c)} - 1\right] & \text{if } \rho \in [\hat{\rho}(z), \overline{\rho}(z)] \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{\rho^{-1}z(\rho-c)} - 1\right] & \text{if } \rho \in (\tilde{\rho}_i(z), \hat{\rho}(z))\ , \\ 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{\overline{\rho}(z)^{-1/\sigma}(\overline{\rho}(z)-c)}{[\rho(1+i)]^{-1/\sigma}(\rho-c)} - 1\right] & \text{if } \rho \in [\underline{\rho}(z), \tilde{\rho}_i(z)] \end{cases} \tag{D.4}$$

with support $[\underline{\rho}(z), \overline{\rho}(z)]$.

The proof strategy for this case is similar to Case 1 above. The only difference is that the upper support of the price distribution is independent of real money holding z, i.e., $\overline{\rho}(z_0) = \overline{\rho}(z_1)$ where $z_0 \neq z_1$. However, we can deduce the following order for the cut-off prices

$$\tilde{\rho}_i(z_0) > \tilde{\rho}_i(z_1),$$

$$\hat{\rho}(z_0) > \hat{\rho}(z_1),$$

and the lower support satisfies

$$\underline{\rho}(z_0) > \underline{\rho}(z_1),$$

given any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

For $\rho \in (\tilde{\rho}_i(z_1), \hat{\rho}(z_0))$, then $J(\rho, z_0) < J(\rho, z_1)$ because $z_0 < z_1$. Hence, $J(\rho, z_0)$ first order stochastically dominates $J(\rho, z_1)$ given two real money holdings z_0, z_1 such that $\breve{z} < z_0 < z_1 < \overline{z}$.

□

Remark. The proof for the first-order stochastic dominance result when $0 < z \leq \breve{z}$ is similar to the case shown above. We leave out the details of this case here.

D.3 Proof of Proposition 6

Proof. Given policy $\gamma = 1 + \tau > \beta$ and the distribution $J_i(\rho, z)$, the equilibrium condition for optimal real money demand z is:

$$
\begin{aligned}
\frac{\gamma - \beta}{\beta} - \alpha_0 i_d = \alpha_1 i \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} i \mathrm{d} J_i(\rho, z) + \alpha_2 i \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} \mathrm{d}(1 - [1 - J_i(\rho, z)]^2) \\
+ \alpha_1 \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \left[\left(\frac{z}{\rho} \right)^{-\sigma} \frac{1}{\rho} - 1 \right] \mathrm{d} J_i(\rho, z) \\
+ \alpha_2 \int_{\tilde{\rho}_i(z)}^{\overline{\rho}(z)} \left[\left(\frac{z}{\rho} \right)^{-\sigma} \frac{1}{\rho} - 1 \right] \mathrm{d}(1 - [1 - J_i(\rho, z)]^2).
\end{aligned}
\tag{D.5}
$$

The left-hand side of Equation (D.5) is constant with respect to z. To establish existence of optimal money holdings, it remains to verify whether the right-hand side is monotone increasing/decreasing in z. Let the function $\chi(z)$ denote the right-hand side of Equation (D.5).

Consider any two real money holdings z_0 and z_1 such that $\breve{z} < z_0 < z_1 < \bar{z}$. The result in Lemma 5 establishes that $J_i(\rho, z_0)$ first order stochastically dominates $J_i(\rho, z_1)$, and consequently, $1 - [1 - J_i(\rho, z_0)]^2$ also first order stochastically dominates $1 - [1 - J_i(\rho, z_1)]^2$.

From this result and the fact that $(z/\rho)^{-\sigma}/\rho - 1$ is monotone decreasing in z, it then follows that $\chi(z)$ is monotone decreasing in z. Hence, there exists a unique $z = z^\star$ that solves Equation (D.5).

Next, we want to verify the loans market clearing condition. The cut-off price $\tilde{\rho}_i(z)$ (when buyers borrow additional funds from the bank) satisfies $\underline{\rho}(z) < \tilde{\rho}_i(z) < \overline{\rho}(z)$. Then, there is always positive loan demand by a measure of buyers who draw low enough prices such that $\underline{\rho}(z) \leq \rho \leq \tilde{\rho}_i(z)$. Since the credit market is perfectly competitive, then the total loans has to equal to total deposits in equilibrium:

$$
\alpha_0 z = \int_{\underline{\rho}(z)}^{\tilde{\rho}_i(z)} (\alpha_1 + 2\alpha_2 - 2\alpha_2 J_i(\rho, z)) \cdot \xi^*(\rho, i, z) \mathrm{d} J_i(\rho, z), \tag{D.6}
$$

given $z = z^\star$ determined by Equation (D.5). □

Remark. The proof for the existence of real money holdings when $0 < z \leq \breve{z}$ is similar to the case shown above. We omit the discussion of this case.