Breaking the Curse of Kareken and Wallace

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https://github.com/phantomachine/_gkwcurse

Observations

- Multiple currencies can coexist as medium of exchange
- Despite not legal tender, and,
- possibly RoR dominance

Present day (and also late-18th to 19th century U.S.)

Theoretical Conundrum

- Prevailing Theorem:
  - Absent obvious legal restrictions on media of exchange ...
  - circulating currencies must have same rate of return.
  - Implies monetary equilibrium has indeterminate nominal exchange rate.

... Kareken and Wallace (1981, *QJE*).

- Mainstream macro “Band-Aid”:
  - Ad hoc frictions – MIU, CIA, currency specific transactions costs.
  - Essentially assumes coexistence/determinacy as opposed to it being an equilibrium outcome.

Theoretical Conundrum

Wallace's *dictum*

Dictum:

1. Monetary economists should explain why fiat money is essential, not assume it is.
   - This means determining its value without resorting to ad hoc restrictions.
2. A "good" model should always have a non-monetary equilibrium as a possibility.

Theoretical Conundrum

Waller's corollary

Corollary:

- Monetary economists should explain why the nominal exchange rate is determinate, not assume it is.
  - This means determining its value without resorting to ad hoc restrictions.
- A "good" model should always have nominal exchange rate indeterminacy as a possibility.

Questions

1. Can we rationalize coexistence of multiple media of exchange, ...
   - even if they differ in rates of returns ...
   - and if there is no legal requirement to hold certain media of exchange?
2. If yes, what are the properties of equilibrium coexistence of monies and exchange rate?

How we do it

- Integrated world economy setting with some decentralized trading.
- No restrictions on currencies used as media of exchange.
- A pinch of asymmetric information:
  - A seller faces threat that a buyer might produce counterfeit monies;
  - A buyer tries to signal his type if offer of payment is genuine.
  - Li, Rocheteau and Weill (2013, *JPE*).
- Also generalize notion "threat of counterfeiting" to environments with:
  - trade credit; fiat monies as collateral; nominal bonds; fiscal policies; and
  - explicit two-country setting.

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Answers and Lessons

Breaking the Kareken and Wallace (1981, *QJE*) result

1. Information friction equilibria — implying active liquidity constraints
   - induce coexistence of multiple media of exchange
     - even if they differ in rates of returns ...
     - and there is no legal requirement to exchange with certain media of exchange.
2. Coexistence imply determinate nominal exchange rate.
3. Equilibria exchange rate behave similar to that implied by ad-hoc CIA models, but have deeper connections to underlying information and market frictions.

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Answers and Lessons

Moreover ...

There can exist monetary equilibrium where

- a currency is dominated in rate of return; or
- has equal rate of return;
- but there is coexistence of currencies and the nominal exchange rate is determinate.

Equilibrium exchange rate determinacy depends on:

- Relative monetary policies across countries
- Fiscal policies

Model

Markets and IWE

- Discrete-time “integrated world economy” (IWE)
  - Anonymity in *decentralized market* (DM): money is essential
  - sequential submarkets: DM-then-CM
  - new money transfers at start of each *centralized market* CM + quasilinear preferences
    - ★ no need for geographical distinctions (i.e., multi-country setup)
- Equivalent interpretation:
  - a closed economy with multiple money-issuing entities

Model

Some notation in CM(t)

- Value of a unit of m (m^f) in units of CM good: ϕ (ϕ^f)
- Nominal exchange rate (Home price of a unit of Foreign money): e
- Buyer's state:

$$\mathbf{s} := (m + x, m^f + x^f; \phi, e)$$

- Seller's state:

$$\breve{\mathbf{s}} := (\breve{m}, \breve{m}^f; \phi, e)$$

Model

DM-buyers

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^{t}\left[u(q_t)+U(C_t)-N_t\right]\right\},$$

- q_t: DM (equiv. nontradable consumption)
- N_t : CM utility of work
- C_t : CM consumption index. Can introduce international trade here (not necessary)
- Assume: u, U, and D well-behaved preference representations

Model

DM-sellers

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t\left[-c(q_t)+U(C_t)-\breve{N}_t\right]\right\},$$

DM-effort/utility-cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$:

- $c(0) = 0$,
- $c'(q) > 0$, and
- $c''(q) \geq 0$.

Model

Markets and Timing (Li, Rocheteau, Weill, 2013)

Start date $t \in \{0, 1, 2, ...\}$ ...

- DM(t) opens:
  - DM-Sellers randomly matched with buyer w.p. σ, and vice-versa. On-the-spot production.
  - DM-Buyer makes TIOLI offer; any composition, any money; Private information on portfolio.
- DM(t) closes, CM(t) opens.
  - Money suppliers grow monies at factor Π_t and Π_t^f.
  - New monies transferred uniformly to all agents.
  - DM-buyers work and consume, and either:
    - ★ work to accumulate more monies (m, m^f) for next DM and
    - ★ decide to counterfeit m, and/or m^f; per-period fixed costs (κ, κ^f).
    - ★ DM-sellers rebalance portfolio and consume and/or work.

Model

Markets and Timing (Li, Rocheteau, Weill, 2013)

1. *Endogenous buyer type* unobservable by random seller.
   - buyer chooses "type"—asset portfolio—prior to entering DM(t).
   - ex-post match with a seller:
     - buyer sends message—offer $\boldsymbol{\omega} := (d, d^f, q)$
     - seller assigns beliefs regarding probable fraudulence and acts: accept/reject.
2. DM(t) Bargaining Protocol: Buyer makes TIOLI bargaining offer. Seller accepts/reject.

PBE refinement ... Payoff-equivalent reverse-ordered extensive-form game: (Offer, Counterfeit/Not, Nature (σ), Accept/Reject).

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Model

CM$(t-1)$ to DM(t): Pure strategies

A pure strategy of a buyer $\boldsymbol{\sigma}^b$ in the counterfeiting game is a triple $\langle \boldsymbol{\omega}, \boldsymbol{\eta}(\boldsymbol{\omega}), a(\boldsymbol{\omega}) \rangle$:

1. Feasible offer decision rule, $\mathbf{s}_{-1} \mapsto \boldsymbol{\omega} \equiv \langle q, d, d^f \rangle(\mathbf{s}_{-1})$;
2. Binary decision rules on counterfeiting, $\boldsymbol{\chi}(\boldsymbol{\omega}) := \left\langle \chi(\boldsymbol{\omega}), \chi^f(\boldsymbol{\omega}) \right\rangle \in \{0,1\}^2$, for each currency; and
3. Feasible asset accumulation decision, $\boldsymbol{\omega} \mapsto a(\boldsymbol{\omega})$, and, $(d, d^f) \leq a(\boldsymbol{\omega})$.

A pure strategy of a seller $\boldsymbol{\sigma}^s$ is a binary acceptance rule $(\boldsymbol{\omega}, \breve{\mathbf{s}}_{-1}) \mapsto \alpha(\boldsymbol{\omega}, \breve{\mathbf{s}}_{-1}) \in \{0,1\}$.

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Model

CM$(t-1)$ to DM(t): Behavior strategies

DM-Buyer:

$$\tilde{\boldsymbol{\sigma}}^b := \langle \quad \boldsymbol{\omega}, \ G[a(\boldsymbol{\omega})|\boldsymbol{\omega}], \ \boldsymbol{\eta}(\boldsymbol{\chi}|\boldsymbol{\omega}) \quad \rangle$$

- Offer/ToT, (q, d, d^f)
- Lottery over portfolio
- $\eta(\cdot|\omega) := \langle \eta(\cdot|\omega), \eta^f(\cdot|\omega) \rangle$, fraud lottery.

DM-Seller:

- $\tilde{\sigma}^s := \pi(\omega)$ which generates a lottery over $\{0, 1\} \ni \alpha$.

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Model

DM-Buyers total payoffs given $\tilde{\boldsymbol{\sigma}} = (\tilde{\boldsymbol{\sigma}}^b, \tilde{\boldsymbol{\sigma}}^s)$

- CM$(t-1)$ flow cost: portfolio accumulation

$$U^b(\tilde{\boldsymbol{\sigma}}) = -\phi_{-1}(m + e_{-1}m^f)$$

$$-\kappa(1-\eta) - \kappa^f(1-\eta^f)$$

$$+\beta\sigma\hat{\pi}\left[u(q) - \phi\left(\eta d + \eta^f e d^f\right)\right]$$

- CM$(t-1)$ expected cost: portfolio fraud lottery
- Expected DM(t) continuation, given DM-seller strategy $\hat{\pi}$.

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Model

Total payoffs: DM-Sellers

$$U^{s}(\tilde{\boldsymbol{\sigma}}) = \beta\sigma\pi(\boldsymbol{\omega})\left[-c(q) + W^{s}(\hat{\eta}d, \hat{\eta}^{f}d^{f}; \phi, e)\right]$$

$$+ \beta\left[\sigma\left(1 - \pi(\boldsymbol{\omega})\right) + (1 - \sigma)\right]\left[-c(0) + W^{s}(0, 0; \phi, e)\right]$$

$$= \beta\sigma\pi(\boldsymbol{\omega})\left[\phi\left(\hat{\eta}d + e\hat{\eta}^{f}d^{f}\right) - c(q)\right],$$

Signalling-Bargaining Game

Solution concept: RI-Equilibrium

- Perfect Bayesian Equilibrium (PBE)
- In-Wright reordering refinement

a.k.a. Reordering-Invariant- or RI-equilibrium.

Signalling-Bargaining Game

RI-Equilibrium

A PBE $\tilde{\boldsymbol{\sigma}} := (\tilde{\boldsymbol{\sigma}}^b, \tilde{\boldsymbol{\sigma}}^s) = \langle \boldsymbol{\omega}, \boldsymbol{\eta}(\boldsymbol{\omega}), \pi(\boldsymbol{\omega}) \rangle$ of the reordered game is such that

1. Players' beliefs are consistent with behavior strategies;
2. Seller's mixed strategy over accept/reject π, given offer $\boldsymbol{\omega}$, is profit maximizing (SPC);
3. Buyer's mixed strategy over counterfeit/not either/both assets, (η, η^f), following $\boldsymbol{\omega}$ is incentive compatible (BIC); and
4. And buyer's offer $\boldsymbol{\omega}$ is surplus maximizing given Items 1-3 (TIOLI).

Signalling-Bargaining Game

Backward induction

Characterizing RI-equilibrium of game

- Backward induction on reordered extensive form
  - BW3. Sellers decision: acceptance mixed strategy π
  - BW2. Buyer's counterfeiting mixed strategy $\boldsymbol{\eta} = (\eta, \eta^f)$
  - BW1. Buyer's message: optimal offer $\boldsymbol{\omega} \equiv (q, d, d^f)$
- Check candidate behavior strategy profile $\tilde{\sigma}$ if PBE

Signalling-Bargaining Game

Backward induction

BW3. Seller's Problem

$$\pi(\boldsymbol{\omega}) \in \left\{ arg \max_{\pi' \in [0,1]} \pi' \left[\phi \left(\hat{\eta} d + e \hat{\eta}^f d^f \right) - c(q) \right] \right\},$$

... given history $\boldsymbol{\omega} = (d, d^f, q)$ and seller's belief $\hat{\boldsymbol{\eta}} = (\hat{\eta}, \hat{\eta}^f)$.

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Signalling-Bargaining Game

Backward induction

BW2. Buyer's Counterfeiting Problem

$$\boldsymbol{\eta}(\boldsymbol{\omega}) \in \left\{ arg \max_{\boldsymbol{\eta}(\boldsymbol{\omega}) \in [0,1]^2} \left[-\kappa(1-\eta) - \kappa^f \left(1-\eta^f\right) \right.\right.$$
$$- \left(\frac{\phi_{-1}}{\phi} - \beta\right) \phi m - \left(\frac{\phi_{-1} e_{-1}}{\phi e} - \beta\right) \phi e m^f$$
$$\left.\left. - \beta\sigma\hat{\pi}\phi\left(\eta d + e\eta^f d^f\right) \right] \right\}.$$

given offer history $\boldsymbol{\omega}$, and belief about the seller's best response, $\hat{\pi}$.

Signalling-Bargaining Game

Backward induction

BW1. Buyer's Offer/Message

$$
\begin{aligned}
\boldsymbol{\omega} \in \Bigg\{ arg \max_{\boldsymbol{\omega}' \in \Omega(\phi,e)} & \{ -\kappa\,(1-\hat{\eta}) - \kappa^{f}\left(1-\hat{\eta}^{f}\right) \\
& - \left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi m - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e m^{f} \\
& + \beta\sigma\hat{\pi}\left[u(q) - \phi\left(\hat{\eta}\hat{d} + \hat{\eta}^{f} e \hat{d}^{f}\right)\right] \Bigg\}.
\end{aligned}
$$

Signalling-Bargaining Game

RI-Equilibrium Characterization

1. Each seller accepts with probability $\hat{\pi} = \pi(\boldsymbol{\omega}) = 1$;
2. Each buyer does not counterfeit: $\left(\hat{\eta}, \hat{\eta}^f\right) = \left(\eta, \eta^f\right) = (1, 1)$; and
3. Each buyer's TIOLI offer $\boldsymbol{\omega}$ is such that:

$$
\begin{aligned}
\boldsymbol{\omega} \in \Bigg\{ \text{arg} \max_{\boldsymbol{\omega} \in \Omega(\phi, e)} & \left[- \left(\frac{\phi_{-1}}{\phi} - \beta \right) \phi m - \left(\frac{\phi_{-1} e_{-1}}{\phi e} - \beta \right) \phi e m^f \right. \\
& \left. + \beta \sigma \hat{\pi} \left[u(q) - \phi \left(d + e d^f \right) \right] \right] : \\
& (SPC, \zeta) : \quad \phi \left(d + e \right) - c(q) = 0, \\
& (BIC, \lambda) : \quad \phi d \leq \frac{\kappa}{\phi_{-1}/\phi - \beta(1 - \sigma)}, \\
& (BIC^f, \lambda^f) : \quad \phi e d^f \leq \frac{\kappa^f}{\phi_{-1} e_{-1}/\phi e - \beta(1 - \sigma)} \Bigg\}
\end{aligned}
$$

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General Monetary Equilibrium

Steady State Version

- Game's RI-equilibrium embeddable into infinite-horizon general equilibrium. Note: $d = m = M$ and $d = m^f = M^f$.

- Stationary monetary equilibrium (SME), for all t:

$$\phi_{-1} M_{-1} = \phi M \tag{\$}$$

$$e_{-1} \phi_{-1} M^f_{-1} = e \phi M^f \tag{€}$$

$$q_{-1} = q \tag{☕}$$

- Steady-state equilibria money supplies grow at some constant rates Π and Π^f.

- (\$) and (€) imply

$$\frac{e_{t+1}}{e_t} = \frac{\Pi}{\Pi^f}$$

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General Monetary Equilibrium

Steady State Version

Buyers' Euler (weak) inequalities:

$$\beta\sigma\left[\frac{u'(q)}{c'(q)} - 1\right] = \lambda - \nu + (\Pi - \beta) = \lambda^f - \nu^f + (\Pi^f - \beta). \quad (⚒.a)$$

DM-sellers' participation constraint binding ($\zeta > 0$):

$$c(q) = \phi d + \phi^f d^f, \quad (⚒.b)$$

KKT conditions are:

$$\begin{aligned}
\lambda \cdot [\bar{\kappa}(\Pi) - \phi d] &= 0, & \lambda &\geq 0, & \phi d &\leq \bar{\kappa}(\Pi), \\
\lambda^f \cdot \left[\bar{\kappa}^f(\Pi^f) - \phi^f d^f\right] &= 0, & \lambda^f &\geq 0, & \phi^f d^f &\leq \bar{\kappa}^f(\Pi^f), \\
-\nu \cdot d &= 0, & d &\geq 0, & \nu &\geq 0, \\
-\nu^f \cdot d^f &= 0, & d^f &\geq 0, & \nu^f &\geq 0.
\end{aligned} \quad (⚒.c)$$

General Monetary Equilibrium

Steady State Version

Cases:

A. Foreign money dominates in rate of return: $\Pi^f < \Pi$.

B. Home money dominates in rate of return: $\Pi^f > \Pi$.

C. Both monies have equal rate of return: $\Pi^f = \Pi$.

General Monetary Equilibrium

Proposition: Coexistence and the KW Curse Breaks

Case A. $\Pi > \Pi^f$

1. Exists equilibria s.t. $(\lambda = \lambda^f = 0)$, or $(\lambda > 0, \lambda^f = 0)$. Then only low inflation money circulates; or,
2. Exists equilibria s.t $(\lambda^f > \lambda = 0)$. Then exists unique SME with coexistence, and unique

$$e = \frac{M}{M^f} \frac{\kappa^f}{c(q)\left[\Pi^f - \beta(1-\sigma)\right] - \kappa^f};$$

   where q unique ...
3. Exists equilibria s.t. $(\lambda^f > \lambda > 0)$, then unique SME with coexistence and unique

$$e = \frac{\kappa^f M}{\kappa M^f} \frac{\Pi - \beta(1-\sigma)}{\Pi^f - \beta(1-\sigma)}.$$

General Monetary Equilibrium

Proposition: Coexistence and the KW Curse Breaks

Case C. $\Pi = \Pi^f$

1. neither liquidity constraints bind $(\lambda = \lambda^f = 0)$, then the fiat monies coexist but the individual's currency portfolio composition and the nominal exchange rate are indeterminate (the KW equilibrium); or
2. both liquidity constraints bind $(\lambda = \lambda^f > 0)$, then the currencies coexist. The individual's currency portfolio composition is unique, and thus there is a unique nominal exchange rate
$$e = \frac{\kappa^f M}{\kappa M^f}.$$

Case B. $\Pi < \Pi^f$. Just read Case A upside down.

General Monetary Equilibrium

Coexistence: properties

(Cases A and B): Coexistence of currencies when one currency RoR-dominates:

- Occurs because of threat of counterfeiting; LC on the currency binds.
- Hold LC'ed dominating currency up to LC limit; then top up with lower return money.
- Topping up because equilibrium DM q is inefficient.
  - By offering to pay with another currency, buyer can consume more;
  - Seller willing to accept since his participation constraints continue to hold.
- Not possible if no private information problem and no currency controls.

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General Monetary Equilibrium

Coexistence equilibria: properties

In Cases A and B:

- Higher Π causes a depreciation of the domestic currency. Obvious, ... but for different reasons.
  - In Case A, the domestic-money LC is inactive, foreign-money is LC'ed. However, higher $\Pi \Rightarrow$ economize on d, reduces q, and thus increases e.
  - In Case B, the domestic money LC'ed, LC tightens with Π. Thus, the marginal liquidity value of an additional unit of domestic currency goes to zero sooner, hence it is less valuable relative to a unit of foreign currency.

(Case C): Equal rates of return

- Possible for determinate portfolio when both LCs bind.
- When one LC binds, all must bind. Otherwise, all do not bind: indeterminacy.

General Monetary Equilibrium

Coexistence: properties

(Case C): Equal rates of return ...

- The shocking result is when $\lambda = \lambda^f > 0$ and we have identical currencies $\Pi = \Pi^f$ and $\kappa = \kappa^f$
- In this case we have
$$e = \frac{M}{M^f}$$
- This is the standard solution for the nominal exchange rate coming out of a symmetric two country CIA model.
- Is this a micro-foundation for that model?
  - NO!!! This only holds for certain parameter values – not a general result. Hence beware of the monetary economist with free parameters.
  - Also, c.f. CIA, here both currencies are traded in every match.

First Best

Proposition

When both the domestic and foreign inflation rates are set at the Friedman rule, $\Pi = \Pi^f = \beta$, the first best quantity q^* **may not** be attainable, Case (3b), and the nominal exchange rate may not be determinate, Case (3a).

First Best

Counterexample

- Assume Friedman rule attains first best for all possible monetary equilibria.
- Case 3(b) at Friedman rule: $\Pi = \beta$ and $\Pi^f = \beta$. Then

$$\sigma\beta\frac{u'(q) - c'(q)}{c'(q)} = \lambda.$$

- First best q^* iff $\lambda = 0$.
- But Case 3(b) has binding liquidity constraints, $\lambda > 0$.

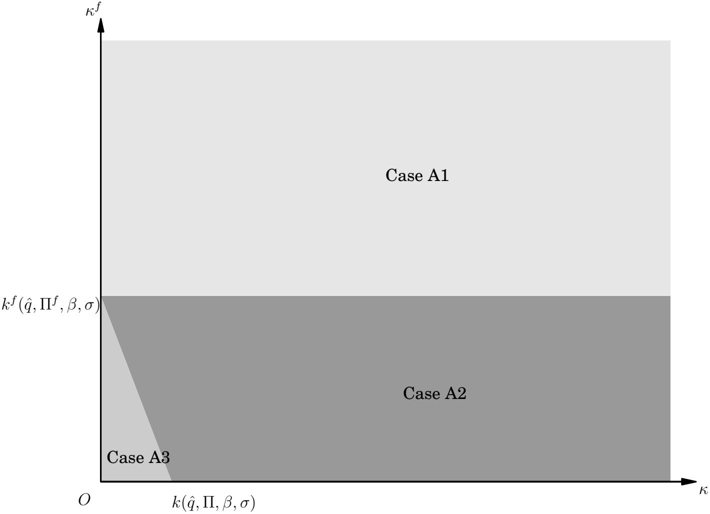
Comparative Statics

Case A: $\Pi > \Pi^f$

- Focus on this case with coexistence equilibria as example.
- What are (κ, κ^f) required to sustain coexistence in this Case?
- How do they vary with policy or fundamentals?

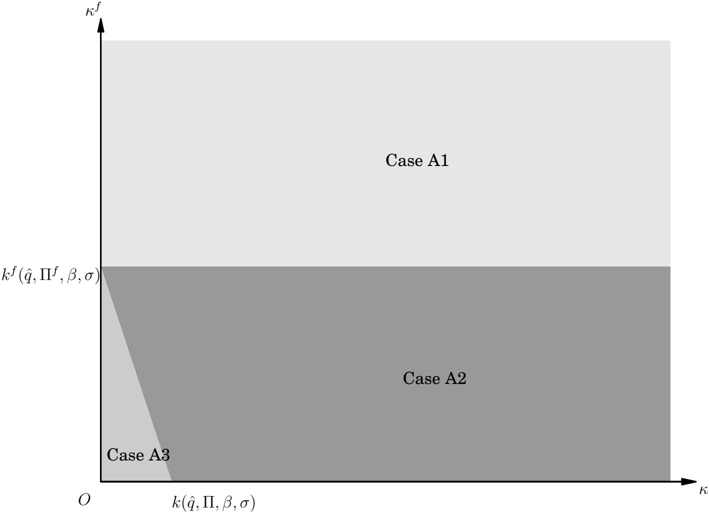
Comparative Statics

Case A: $\Pi > \Pi^f$ ... baseline



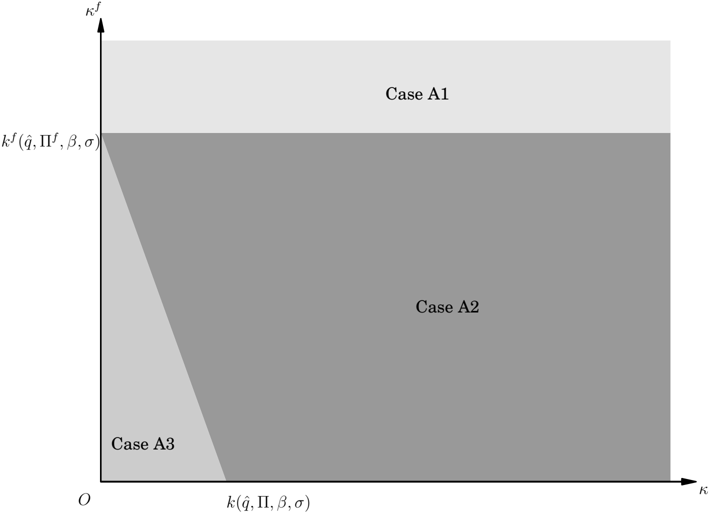
Comparative Statics

Case A: $\Pi > \Pi^f$ ... Higher Π^f



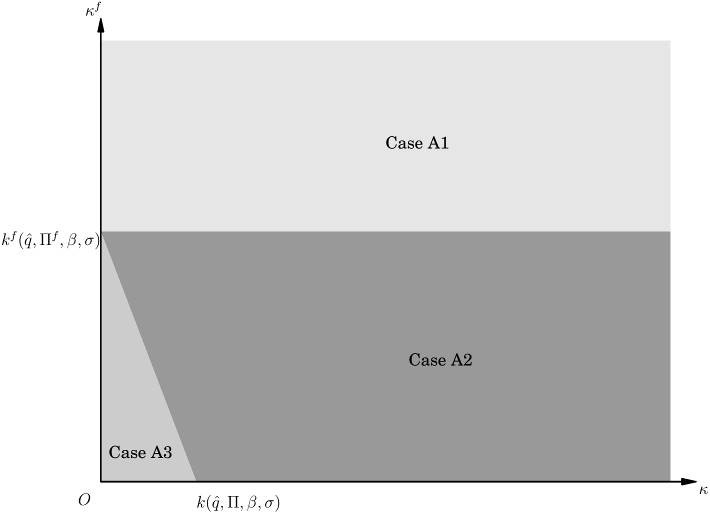
Comparative Statics

Case A: $\Pi > \Pi^f$ ... Higher matching probability σ



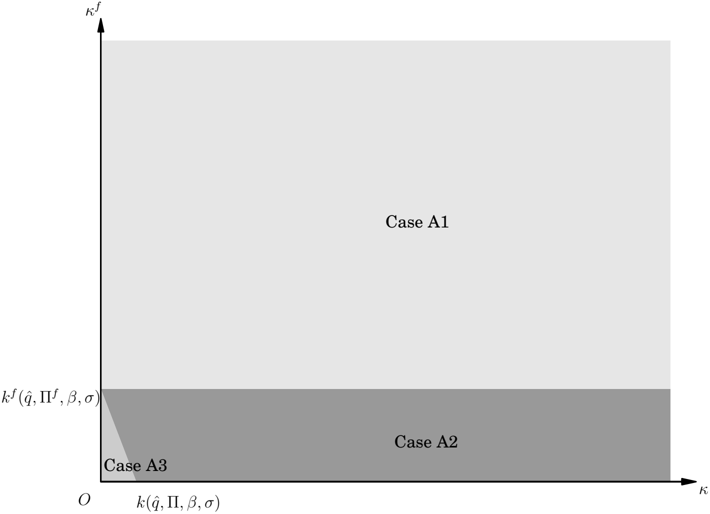
Comparative Statics

Case A: $\Pi > \Pi^f$ ... Lower IES $1/\theta$



Comparative Statics

Case A: $\Pi > \Pi^f$ ... more convex $c(q)$



Comparative Statics

Coexistence: comparative statics (Case A: $\Pi > \Pi^f$)

If a currency

- has low enough inflation relative to its foreign counterpart,
- circulates in DM with lower matching frictions,
- is demanded by agents with low intertemporal elasticity of substitution (in q), or that
- DM production cost $c(q)$ is not too convex,

then to sustain coexistence (determinate e) equilibria, the costs of counterfeiting must be relatively higher.

More in the Paper

Extension 1: Fiscal Policies

- Income tax on CM output, results in:

$$\phi m \leq \frac{\kappa(1-\tau)}{\phi_{-1}/\phi - \beta(1-\sigma)},$$

and

$$e\phi m^f \leq \frac{\kappa^f(1-\tau^f)}{\phi_{-1}e_{-1}/\phi e - \beta(1-\sigma)}$$

Assume income taxation in CM and lump-sum transfers of fiscal revenue to all agents. There exists income tax rates τ and τ^f such that the two monies coexist, and e is made determinate.

More in the Paper

More Extensions

Main coexistence and determinacy result survives environments with:

- Trade Credit in DM
- Fiat monies as collateral
- Nominal bonds (and fraud in assets)

More in the Paper

Timing of government transfers

Consider alternative:

- Seigniorage revenue transferred to DM-buyers after CM closes
- Vital who gets which new money
- Implies explicit modelling of country-specific DMs
- Two-country model suffices

Main coexistence and determinacy result survives, but now 3×16 cases of equilibrium configurations to consider.

Conclusion

- Standard international monetary economics presume "currency-in-advance" to pin down nominal exchange rates, e.

- Kareken-Wallace (QJE 81): Absent this assumption, e is everywhere indeterminate.

- This paper: Shows connection between threat of counterfeiting (private info) and
  - phenomena of partial and complete dollarization; and
  - equilibrium e determinacy,

  without ad-hoc restrictions on payment instruments.