Taxes, Transfers and RCE in OLG

The Big Picture. This week we reinforce what we have studied thus far:

1. Pareto optimal allocations and optimal steady states
2. Relation to competitive equilibrium; failure of first welfare theorem
3. Competitive equilibrium with transfer: second welfare theorem

These are groundwork which pave the way for analyses on the role of government (redistributive) policies in the OLG environment.

Please prepare you answers before attending sessions.

Homework 1. Consider a partial order (preferences) over the consumption set $\mathbb{R}_{++} := \{c \in \mathbb{R} | c > 0\}$, represented by the iso-elastic family of utility functions $U(\cdot;\theta)$:

$$U(c;\theta) = \frac{c^{1-\theta}}{1-\theta} - \frac{1}{1-\theta}, \qquad \theta > 0.$$

1. One version of this utility function (an all-time favorite in macro applications), is the special case of $\lim_{\theta\to1} U(c;\theta) = \ln(c)$. Verify that this is true.

2. If U is interpreted as a per-period utility function, then, verify that these functions exhibit the property of a "constant intertemporal elasticity of substitution", i.e. $\sigma(c) = U'(c)/[U''(c)c] = \sigma$. What is the expression for the constant σ here?

Homework 2. Consider agents with lifetime preference representations:

$$U(c_t) + \beta U(d_{t+1}),$$

where $U(x) = \ln(x)$, for $x > 0$, $\beta \in (0,1)$, c_t is young-age consumption, and d_t is consumption of the date-t old. Each young agent is endowed with 1 unit of time. Define total resources at time-t, given state k_t, as

$$\tilde{f}(k_t) = k_t^\alpha + (1-\delta)k_t,$$

where $\alpha \in (0,1)$ and $\delta \in (0,1]$. Assume initial per-worker capital stock k_0 is given. The population of young agents, of measure N_t, grow at a constant rate $n > -1$. Assume N_0 is given. All agents are prices takers—i.e. they take the real wage w_t and gross rental rate R_t as given. Denote R_{t+1}^e as agents' subjective expectation of next period capital return, where the expectation is formed at each date t. Old agents own the capital stock at time t.

There is lump-sum tax/transfer, $a_t = \tau_t w_t \in \mathbb{R}$, where $\tau_t < 1$ for all $t \in \mathbb{N}$. The transfer to (tax on) the old is $z_t \in \mathbb{R}$. The transfer system is such that, for each period, the budget for the system is balanced.

1. Now characterize a (perfect-foresight) competitive equilibrium and lump sum taxes/transfers. To do so, follow these steps:

   (a) Set up the consumer's decision problem for every generation $t \geq 0$. Derive exactly the optimal decision rule(s) for the consumer, given (w_t, R_{t+1}^e).

(b) Set up the firm's decision problem. [Hint: denote k_t^d and N_t^d respectively, as the firm's demand of per-worker capital and labor services at time t.] Derive exactly the firm's best response functions, given w_t and R_t.

(c) Using the final goods market clearing condition and consumers' budget constraint, verify that the capital market clears. Derive what this condition is.

(d) Now state, precisely, what a *perfect-foresight recursive competitive equilibrium with transfers* is, in this example economy.

Homework 3. Consider the last exercise, and assume $\delta = 1$.

1. Set up a Pareto planner's optimal allocation problem, where the planner is indexed by the discount factor γ^t on the total welfare of each date-t generation, with $\gamma \in (0,1)$.

2. Characterize the γ-planner's optimal allocation.

3. Verify that there exists a competitive equilibrium with appropriate transfers which replicates the optimal allocation.

4. Provide a necessary and sufficient condition on the particular type of γ-planner such that the efficient competitive equilibrium with transfers features positive transfers from the young to the old, each period.

5. Discuss and provide economic intuition for the property of your last result.

Homework 4. *Just as you might master the writings of Immanuel Kant if you could read them back in Swahili to someone else, this exercise further strengthens your grasp of the material by getting you to implement solutions in* `Python`. *This will also sharpen your computational skills towards your second-year Master's program and beyond.*

Go back to the setting in Homework 3, but use the preference function from Homework 1. Write a set of Python programs to compute the RCE trajectory of the economy, assuming some constant tax rate, i.e., $\tau = \tau_t$ for all $t \geq 0$.

Hints: You can re-purpose most tricks or ideas from the Solow-Swan example. Try this workflow below.

- Create a fresh `Jupyter Notebook`.
- Define and bind these parameters to some values of your choosing:
  - preferences: β, θ
  - production technology: α, δ
  - fiscal policy: τ
- Set up `def` modules or functions to describe U, f, $\tilde{f}$, and the tax function τw_t. It might be convenient to also construct `def` modules to define equilibrium objects like w_t and R_t.
- Use a `for` loop to recursively construct the RCE outcomes.

Consider the following:

1. Compute the RCE allocation from an initial k_0 (of your choosing) when $\tau = 0$.

2. Compute the RCE allocation from an initial k_0 (of your choosing) when $\tau > 0$.

3. Comment on the dynamics and steady states of these two economies. If you can vary the experiment for different τ levels. What pattern do you see? Provide some explanation for your results, as per our earlier lectures.

Note: It may be that for some k_0 and some τ there may exist no RCE. A good code would have safeguards against these non-existence of equilibrium.