Dynamic Optimality and the Second Welfare Theorem in OLG

The Big Picture. This week we reinforce what we have studied thus far:

1. Pareto optimal allocations and optimal steady states
2. Relation to competitive equilibrium; failure of first welfare theorem
3. Competitive equilibrium with transfer: second welfare theorem

These are groundwork which pave the way for analyses on the role of government (redistributive) policies in the OLG environment.

Please prepare you answers before attending sessions.

Homework 1. Consider the last exercise, and assume $\delta = 1$.

1. Set up a Pareto planner's optimal allocation problem, where the planner is indexed by the discount factor γ^t on the total welfare of each date-t generation, with $\gamma \in (0, 1)$.
2. Characterize the γ-planner's optimal allocation.
3. Verify that there exists a competitive equilibrium with appropriate transfers which replicates the optimal allocation.
4. Provide a necessary and sufficient condition on the particular type of γ-planner such that the efficient competitive equilibrium with transfers features positive transfers from the young to the old, each period.
5. Discuss and provide economic intuition for the property of your last result.

Homework 2. *Just as you might master the writings of Immanuel Kant if you could read them back in Swahili to someone else, this exercise further strengthens your grasp of the material by getting you to implement solutions in* `Python`. *This will also sharpen your computational skills towards your second-year Master's program and beyond.*

Go back to the setting in Homework 3 of T05, but use the preference function from Homework 1. Write a set of Python programs to compute the RCE trajectory of the economy, assuming some constant tax rate, i.e., $\tau = \tau_t$ for all $t \geq 0$.

Hints: You can re-purpose most tricks or ideas from the Solow-Swan example. Try this workflow below.

- Create a fresh `Jupyter Notebook`.
- Define and bind these parameters to some values of your choosing:
  - preferences: β, θ
  - production technology: α, δ
  - fiscal policy: τ
- Set up `def` modules or functions to describe U, f, $\tilde{f}$, and the tax function τw_t. It might be convenient to also construct `def` modules to define equilibrium objects like w_t and R_t.

- Use a `for` loop to recursively construct the RCE outcomes.

Consider the following:

1. Compute the RCE allocation from an initial k_0 (of your choosing) when $\tau = 0$.
2. Compute the RCE allocation from an initial k_0 (of your choosing) when $\tau > 0$.
3. Comment on the dynamics and steady states of these two economies. If you can vary the experiment for different τ levels. What pattern do you see? Provide some explanation for your results, as per our earlier lectures.

Note: It may be that for some k_0 and some τ there may exist no RCE. A good code would have safeguards against these non-existence of equilibrium.