Ref: MDY-22-0010, *Macroeconomic Dynamics*
Title: *Inflation, Inequality and Welfare in a Competitive Search Model*
Old Title: *Cost of Inflation and Inequality in a Competitive-search Heterogeneous-agent Model*
Date: December 10, 2024

We thank the Editor, Associate Editor and three anonymous referees for the their time and helpful comments. Below, we reproduce the Associate Editor and Reviewer R2's main comments and provide point-wise responses.

Associate Editor (R1 and R3)

I appreciate that the authors seem to have put in a lot of effort to address my and referees' concerns in this version. In fact, R1 and R3 recommended accepting the paper as it is. However, R2 still has some important comments. I think we should send the paper back one more time to the authors to address R2 comments. At this point I am leaning towards recommending accepting the paper but I'd prefer if the authors address R2's additional points.

Our response.

- We have now re-written the paper addressing all the points raised by Reviewer R2. The presentation of our results and main message significantly improved thanks to R2's questions. We thank R1 and R3 for their assessment and previous comments.

Reviewer R2

This paper builds on Menzio et al. (2013), and extends it by introducing inflation. Through simulation, the authors find that the effects of inflation on money holdings inequality are non monotonic. This is a potentially interesting finding, however it is not always clear what is leading to this result in the model.

Main comments

1. **Unclear Main Mechanism**: I see the major contribution of the paper as being this: "One main result of our model is that we have a non-monotonic relationship between inflation and money-holdings inequality" which the authors suggest is the result of the extensive margin of trade. This seems to be what sets the paper apart from other literature which, as the authors point out, tends to find that inflation is inequality decreasing. However, the exposition of the model does not make it clear why the relationship in the model is non-monotonic. Even though there is no closed-form solution to the model, it should be possible to tell us a story which explains the intuition. This could be along the lines of "increases in inflation lead to y, which causes z, which cause these changes in the distribution of m." Many of the points below relate to this overarching criticism.

   Our response.

   - In this revised version, we have now re-structured the presentation of the mechanism and its narrative.

- The non-monotonic relationship between inflation and money-holdings inequality is a result of the interplay between the intensive margin in terms of trade quantities and extensive margin in terms of trading probabilities. When the inflation rate is low, the intensive margin effect dominates. However, the extensive margin effect is stronger when the inflation rate is sufficiently high. We have emphasized this in the introduction and also worked through the mechanism in more details in Section 4 in the manuscript.
- In Section 4 of the revised manuscript:,
  - We first discuss the straightforward result that agents work less and accumulate less money in the CM (**Figure 5**).
  - Then, we present the DM best responses of agents (i.e., matching probability, payment, and their associated velocity of spending in **Figures 6 to 7**). We also do this with firms (relative pricing behavior) across alternative long-run inflation policies (**now Figure 8**). For clarity of the graphs, we present just two cases of equilibria (at 0% and 10% inflation *per annum*). This allows us to focus on the effects of inflation on the individual agents' and firms' behavior. We continue with observations on how the responses are different, depending on individuals' money holdings positions.
  - We then further clarify the connection between ex-post heterogeneity (households who end up with different money positions) and their typical responses to different inflation policies. To do this, we use 90/10 ratio statistic (of the distribution of *DM-conditional* or *DM-participating agents*). We still think this is a convenient summary of the comparative responses of different groups of households in the distribution of agents *in the DM*.
    - For example, a key intermediate step in the narrative is the observation that the best response in terms of DM payments (and matching probabilities) of all DM buyers shift down with inflation. Coupled with a fall in realizations of money holdings, both payments and matching probability *outcomes* fall across the distribution of DM buyers. However, the interesting aspect here is that the rates at which they fall *across higher inflation* regimes are different. Due to what we label as the *intensive margin* effect of inflation, agents would all, and on average, see a fall in matching rates and payments for goods in the DM (**Figures 6 to 7**) and work less and accumulate less money in the CM (**Figure 5**). Focusing on the top 90% and bottom 10%—which we respectively label as "the rich" and "the poor" so as to economize on sentence lengths—we demonstrate that the rich uniformly are less sensitive than the poor in the decline of their matching probabilities and payments in response to higher inflation (**Figure 10**). However, as a ratio to their own money balances, the rich are transacting at a higher velocity (**Figure 11**).
    - These last two points are symptoms of what we label as the *extensive margin* of inflation on agent behavior, and this margin is the interesting opposing force that works against the compressing effect (on the money distribution) of the intensive margin.
  - The purpose in first focusing on the DM distribution is to highlight the main source of frictions and equilibrium dispersion of agents and pricing outcomes. The extensive margin force arises from here and that also governs how quickly agents return to participate in the labor market in the CM (**Figure 11**). The tension between the intensive and

extensive margin means that there can be interesting non-monotone inequality effects on the DM distribution of money. Having shown this, we then demonstrate that this property of a U-shaped inequality effect gets inherited in the overall (DM and CM) unconditional distribution of money. (We comment on the choice of inequality measures in response to R2's question in Item 4.)

2. **Figure 9**: Figure 9 shows the mean and 10th and 90th percentiles of x and b as inflation increases from 0% to 10%. However, it's unclear what this is meant to show about how inequality responds to changes in inflation. For example, the changes in b are a result of the changing function $b(m)$ (shown in Figure 6) and the new equilibrium distribution of m. This explains why, despite b increasing for every m, the mean b falls, and falls more the higher is inflation. The problem becomes particularly clear when we compare the 90th and 10th percentiles. In a sense, the "rich" in one equilibrium are not the same as the "rich" in another. The higher is inflation, the less money is held by the "rich". Perhaps there's something I'm missing here, but if so, it should be made clear the relationship between this and the ultimate distribution of m.

Our response.

- We thank R2 for raising these questions and helpful remarks.
- We should note that in the previous Figures 9, 10 and 12 (now, respectively **Figures 10, 11, and 13**), the mean and 90/10 ratio statistics pertain to the distribution of agents conditional on them being in the decentralized, competitive search market (DM)—i.e., the DM-conditional distribution (of agents). We apologize for having omitted to make this point clear in the verbal description and mechanism explanation of Section 4. (See also the helpful insight of R2 below—in the *Other Comments* section, Item 1 on page 6—that suggests the same thing.)
  - It is reasonable to focus on the DM-conditional distribution of agents, as we aim to highlight the role of the equilibrium search-and-matching friction and the resulting intensive-extensive margin trade-off, reiterated below.
  - Moreover, as R2 pointed out, if we include the money balance of agents at the start of the CM, individuals in the bottom 10th percentile would have zero money balances. In this case, the unconditional distribution of money holdings would have an ill-defined 90/10 ratio. That is why, when we consider the alternative 90/10 ratio statistic, it is only for the DM-conditional distribution. (This point is related to the next question of R2 in Item 3 on the next page.)
- **Relationship between Figures 9-10 and Figure 11-12:** The main mechanism working through the DM-conditional distribution of agents is the following trade-off that was previously outlined and narrated:
  - **Figure 9 (now 10):** The trade-off is such that the intensive margin of inflation (compressing the distribution or reducing inequality) gets overtaken by the extensive margin (the decline in payments and matching probabilities is slower for the rich than the poor) at some sufficiently high level of inflation.
  - **Figure 10 (now 11):** We also note that, in relation to Figure 9, while the decline in payments and matching probabilities is slower for the rich than the poor, the rich

are actually offloading their money faster in the DM (i.e., the speed or velocity of transactions are higher relative to the poor).

  – **Figure 12 (now 13)**: Having shown the effect of inflation underlying the DM trade-off in distributional outcomes, we then demonstrate that this gets inherited in the overall unconditional distribution of agents' money holdings. This is shown in **Figure 11** (**now 12**). Specifically, the U-shaped 90/10 ratio for the conditional distribution of m (in the DM) gets inherited in the U-shaped Gini measure for the unconditional distribution of all agents (across both DM and CM).

The extensive margin is the offsetting force that clarifies what R2 asks for as "the relationship between this [*i.e., the trade-off* (*sic*)] and the ultimate distribution of m."

- R2 correctly states that "the rich in one equilibrium are not the same as the rich in another." The analysis of comparative steady-state equilibria focuses on the cross-sectional distribution of agents in one equilibrium compared to another. The support of distributions and the measures of individuals over subsets of the supports may vary across different policy-induced steady state equilibria. This is standard in comparative steady states in existing papers with heterogeneous agent models, e.g., if one were to study a textbook Aiyagari model as one varies a parameter such as the ad-hoc borrowing constraint in that model. The 90/10 ratio is just one common statistic used to measure inequality, as is the Gini index.

3. What drives the 90/10 ratio for m? The simulation results for money distribution (Figure 11) suggest that there are (approximately) 3 masses: (i) around 0, the amount of money at which agents must return to the CM; (ii) around 1.0, the amount of money agents get after exiting the CM; and (iii) an intermediate amount, the result of successfully trading in the DM for the first time.[1] This being the case, the 90/10 ratio should be driven by the values of m represented by the leftmost and rightmost bars. Or, put in the terms of the model, the 90/10 ratio should be given by:

$$90/10 \text{ ratio} \approx \frac{\text{Optimal } m \text{ out of CM}}{\text{Amount of } m \text{ at which agents must return to CM}}.$$

However, perhaps this is not what's being calculated, since I would assume (perhaps wrongly?) that the denominator should be 0, since there's no point in working in the CM to earn money which does not get spent in the DM.

Our response.

- We thank R2 for pointing this out. This is addressed in the last response above.
- Regarding R2's footnote comment, the different masses of individuals over money-balance positions would depend on a particular equilibrium solution. In the model, there is also the ex-ante lottery aspect that determines the recorded money-balance positions. Thus, in some equilibrium, there may be additional lottery segments that can be played and this would explain the holes or missing mass that R2 expected to see. Also, in practice, the histograms were plotted using standard histogram tools. We have experimented with different levels

[1] In the 0.1 inflation case, there is a fourth mass directly above the large intermediate mass. I assume this is the result of agents who, after having traded once in the DM, fail to trade a second time but, with the helicopter drop of money in the second period, see their real money holdings increase. Given that these agents have effectively the same trading probability as agents with $m \approx 1$, I'm surprised there isn't yet another small mass directly below the large mass around 1.0.

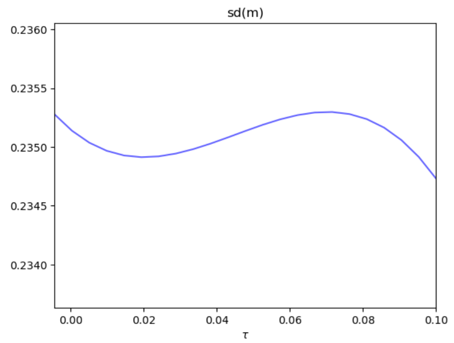

Figure 1: Alternative standard-deviation inequality measure

of granularity (or number of bins) in the definition of the histograms' supports to ensure consistency in plotted outcomes.

4. **How are we measuring inequality?** Increasing inflation leads all agents to want to hold less m, and therefore the distribution of m should become compressed towards 0. Thus, the absolute inequality of m is surely decreasing. Likewise, if we were to measure the inequality of m in terms of the variance, it would also be decreasing. Is there a reason the 90/10 ratio is particularly important? The advantage of the 90/10 ratio and the gini coefficient is that they are scale-invariant – an important feature when differences in scale are driven by prices, for example. However, here, since we are dealing with real values, is scale-invariance important?

Our response.

- We thank the referee for suggesting this.
- Our usage of the 90/10 statistic is just because it is commonly used as one of many ways to characterize dispersion and inequality in empirical distributions. To be sure that the main insight is not an artefact of the use of this statistic, we have previously also plotted the variance statistic as an alternative measure of inequality, but did not mention it.
- As expected, the variance measure also tells us that there is a U-shaped pattern just as in the 90/10 ratio for the DM-conditional distribution or the Gini coefficient for the overall distribution, and for high enough inflation, at some point, the dispersion will go to zero. We mention this expected result—that as inflation becomes arbitrarily high, the monetary equilibrium will tend to disappear—in Footnote 39 now.
- We now mention this in the paper.

5. **Observation 5, the U-Shaped Relationship**: A few questions here. First, is bx/m being divided by aggregate m, or a given individual's m? (I assume the latter!) Second, what drives this relationship? Third, if this relationship exists, it's not evident in Figure 10, where the 90/10 ratio seems to be monotonically increasing.

Our response.

- We simulated the equilibrium outcomes/selections $b(m)$ and $x(m)$, multiplied each correspondingly with their associated individual m outcome. The mean statistic is derived from the simulated series of such outcomes.

- We are terribly embarrassed and sincerely apologize for this glaring typo in the statement of Observation 5. We thank R2 once more for pointing this out. This is now corrected.
- The mechanism is the same as that outlined earlier above: For higher inflation, all agents are realizing a higher velocity of spending in the DM, with the so-called "rich" trading faster (higher per-dollar expenditure in the DM) than the "poor" (see Figure 10, top-right panel). Thu, we also see in Figure 10 (top-left panel) that the average across these is monotone increasing in inflation.

6. **For high enough inflation, shouldn't inequality go to 0?** As inflation increases, the cost of holding money increases, and relatively more economic activity takes place in the CM. With $\tau = \infty$, no one would want to hold money outside of the CM, and inequality should go to 0.

Our response.

- Correct. It kicks in for sufficiently high inflation. If we had plotted the variance figure as above in Figure 1, we could also see this showing up "earlier" along the inflation axis. We have now explicitly stated this in the paper.

Other Comments

1. **Figures 5-8**: Figures 5 through 8 are a little strange. First, the "average agent" is somewhat misleading. Take Figure 6, which shows the function $b(m)$. The average agent has the average m, but such an agent doesn't necessarily exist. Moreover, the average is being pulled down by a large mass of agents with low m ($m \approx 0$) who don't trade in the DM. Thus, it's possible that the average m is lower than the m of any agent who actually trades in the DM, and for whom b is actually relevant. It's unclear why the average b, for example, is a useful statistic (how does it relate to the overarching story of the paper?) but if it is necessary, perhaps it makes more sense to have the average b, conditional on participation in the DM. Similarly, for Figure 5, I would be more interested in average l, conditional on participation in the CM. Another concern is the fluctuating bounds on the x-axis of these figures, which don't necessarily accommodate the actions of actual agents in the model. According to Figure 11, there are a large number of agents with $m \approx 1$, and yet none of these figures include 1 in the domain. What is happening for these agents? I understand that a problem might be that if one extends the axis to include all m between 0 and 1, it might be difficult to see the changes occurring in the model following the changes in inflation. If this is a concern, you could always have two figures: one which includes a wide domain, and the other which is "zoomed in" to see the changes.

Our response.

- We thank R2 for making this suggestion. We agree with R2's intuition that it may be possible the average m is lower than the m of any agent who actually trades in the DM, and for whom b is actually relevant. In the revised figures, we make it clear that when we plot mean outcomes along respective best response functions, these are the means of agents in the DM, following R2's suggestion.
- For labor supply (Figure 5) we have indicated the average labor supply markers as what R2 expected—i.e., as averages conditional on agents participating in the CM.

- Regarding the different truncations of the domain of these figures, we now incorporate R2's suggestion to show the full graph (complete domain) with particular zoom-in or figure insets

2. **Figure 15**: Given that there is only one DM market open when inflation is 0, shouldn't the 90/10 percentile of prices in Figure 15 be equal to 1 for this inflation rate?

   Our response.

   - We agree with and thank R2 for pointing this out. It should be and the figure contained a coding error. Given that the left panel of (the previous) Figure 15 contains the same (and correct information), we have now decided to discard what was its right panel. The updated figure is now Figure 14.

3. What is the chosen function for $c(q)$? Perhaps I'm wrong, but it seems that it should matter. For a large enough c(0), there could be instances where agents never go to the DM. Isn't this correct? I'm not sure I'm convinced by the claim that agents having preferences over DM and CM goods guarantees that they will necessarily participate in both markets. Imagine a two-period case, where both markets are complete and there is no inflation. Furthermore, imagine the following functions:

$$
\begin{aligned}
u(z) &= \frac{z^{1-\sigma}}{1-\sigma}; \quad \text{for } z = C, q \\
c(q) &= 1 \\
C &= l \\
U &= u(C_1) + u(q_1) - l_1 + \beta\left(U(C_2) + u(q_2) - l_2\right)
\end{aligned}
$$

   The agent chooses between two possibilities: stay in CM for both periods, or go to CM in period 1, then go to DM in period 2.

   Case 1: If the agent spends both periods in the CM, her optimal choices are $C_1 = C_2 = 1$ and $l_1 = l_2 = 1$, which yields.

$$U = (1+\beta)\frac{\sigma}{1-\sigma}$$

   Case 2: If the agent spends the first period in the CM, her optimal choice is still $C_1 = 1$. She also has to consider the relative costs and benefits of working today

   to generate income to purchase goods in the DM next period. This yields optimal choices: $C_1 = 1$, $l_1 = 1 + \beta^{1/\sigma}$, $q_2 = \beta^{1/\sigma}$ and therefore:

$$U = (1+\beta^{1/\sigma})\frac{\sigma}{1-\sigma}$$

   which, if $\beta, \sigma < 1$, is strictly less than the utility of staying the CM for both periods. Is this not correct? If $c(0) = c'(0) = 0$, then I believe we are guaranteed agent participation in the DM, since the first little bit of utility in the DM is effectively free.

   Our response.

   - In the earlier model description part of the paper, we assume that $c(q)$ is a continuous convex function: The theory part of the paper was derived with this assumption. For the carlibration, we assume that $c(q) = q$. We thank R2's question here, and have now emphasized this in the calibration section of the paper, along with the description of the other primitive functions used.

- We also agree with R2 that having convex preferences over DM and CM goods is not sufficient to guarantee participation in both markets (specifically DM), We apologize for our imprecise statement earlier. We have changed the description in the paper.
  - We can already see this in a special case of our model under zero inflation, which is similar to the original Menzio, Sun and Shi (MSS) paper. MSS discuss some sufficient conditions (but for the case of a zero-inflation, analytical equilibrium only). They show that all else equal, if agents are not too impatient, or if agents' disutility of labor is not too convex and if firms' DM fixed cost of setting up shop (and hence producing)—which R2 had identified as $c(0)$ or k in the model—then there will be a monetary equilibrium with non-degenerate distribution of agents (i.e., agents will participate in both markets).
    * All these conditions are very intuitive: If agents are too impatient, then the marginal benefit of accumulating money and going shopping in the DM may not be large enough. Hence agents may just live in the CM where money is not required. If firm's fixed cost of posting a submarket k is too large, no firm will want to produce there and even if agents may want to, there won't be an equilibrium where DM trade exists. Likewise, if the marginal cost of work rises too fast, it may overwhelm the marginal benefit of accumulating money to go to the DM.
- It is however not of immediate importance for our analyses to theoretically characterize the generic set of parameters that ensure the existence of a monetary equilibrium with non-degeneracy in the money distribution. (With non-zero inflation, this task is prohibitively non-analytical.) In our calibrated model and all the experiments around it, we have checked that the associated equilibria are always monetary with a non-degenerate distribution of of money, and hence, involve probabilistic participation in both markets.

Minor Comments

Just a couple of typos:

- **Page 2**: "higher inflation rates due to the increase in money supply in response to the COVID-19 pandemic." This seems like a contentious claim given that there are many competing views about the causes of inflation during the pandemic.

  Our response.

  - We have now removed the language that may trigger any suspicion that we are attempting to provide a causal explanation for inflation post-COVID. That was never our intention. We now simply state that most advanced economies are facing higher inflation rates in recent times.

- **Page 3 (and elsewhere)**: "...high money-balance agents decrease their money holding at a slower rate than agents with low money balances." This is somewhat unclear and misleading. In all equilibria, rich households are always spending their money faster than poor households (i.e. b is higher, as is bx/m). What I believe you're talking about here is the change in these variables as we vary inflation.

Our response.

- We thank R2 for helping to clarify this. We agree and have now made the relevant changes in our writing.

- **Footnote 7**: "they reported only have one lottery segment" I believe should be "having one..."

  Our response.

  - We thank R2 for helping to clarify this. We have now made the relevant changes in our writing.

- **Page 4, second-last paragraph**: "There is a similar effect in our model, but with an different effect." Should be "a different". "Higher inflation exacts a greater downsize risk..." should be "downside risk" I believe.

  Our response.

  - We thank R2 for this. We have now made the relevant changes in our writing.

- **Page 6, second last paragraph**: "There are one general" should be "is".

  Our response.

  - We thank R2 for this. We have now made the relevant changes in our writing.

- **Page 7, second paragraph**: "...and transforms it linearly into the same amount of DM good" should be "CM good".

  Our response.

  - We thank R2 for asking this.The statement was correct. The production functions are both linear for the CM and DM goods.

- **Page 14, final line**: "see part 3(d) in proposition 2", there is no part 3(d) in proposition 2.

  Our response.

  - We are terribly sorry for this slip-up in the cross-referencing, and thank R2 for noticing this. This is now fixed.

- **Page 26, final line**: "If we ran an experiments" should be "If we run experiments".

  Our response.

  - We thank R2 for this. We have now made the relevant changes in our writing.

- **Figure 13, panel 2**: The inflation rates displayed in the legend should be 0 and 0.1, I believe, not 0.025 (the quarterly inflation rate).

  Our response.

  - We thank R2 for this. We have now made this labelling change in the figure.