**Ref**: MDY-22-0010, *Macroeconomic Dynamics*
**Title**: *Inflation, Inequality and Welfare in a Competitive Search Model*
**Old Title**: *Cost of Inflation and Inequality in a Competitive-search Heterogeneous-agent Model*
**Date**: April 15, 2024

*We thank the Editor, Associate Editor and the three anonymous referees for the their time and helpful comments. Below, we reproduce the Associate Editor, Referee R1, R2 and R3's main comments and provide point-wise responses. We would also like to note a third co-author, Tina Kao, had contributed significantly and was added during the revision process.*

## Associate Editor

Menzio, Shi and Sun (2011-JET) (henceforth MSS) construct a tractable directed search model of money with a non-degenerate distribution of money holdings. This paper extends this model by incorporating inflation to study its effects on welfare and the distribution of consumption and money holdings. In the model, buyers direct their search among a continuum of submarkets, each of which specifies the terms of trade and a tightness. Directed search makes the monetary equilibrium tractable that individuals' policy functions, value functions and the market tightness function are all independent of the distribution of individuals over money balances, although the distribution affects the aggregate activity by itself.

MSS model leads to endogenous money-spending cycles with non-degenerate distribution of money holdings in which an individual first works to obtain money to becomes a buyer. The authors show that a higher long-run inflation policy tends to attenuate (exacerbate) money holdings inequality for a sufficient low (high) range of inflation. Also, the welfare cost of inflation is nontrivial, especially when transitional dynamics are considered. Inflation induces a redistributive-tax effect through the intensive margin of trade in all markets. However, inflation also raises individuals' probabilities of not getting to trade in decentralized search markets. These two forces work in the opposite direction.

Even though the MSS version can be solved analytically, the model in this paper requires computational solution, for which the authors develop a new method. They show that the quantitative implications of the model are consistent with some micro empirical evidence on the relationship between inflation and (i) price dispersion, (ii) fraction of money transaction relative to other means of payment, and (iii) speed of payment.

1. Although I commend the authors for rigorous and meticulous work, it is not obvious what the novel contribution is. The paper emphasizes a "new mechanism - an endogenous trade-off between intensive and extensive margins...". However, the trade-off between intensive and extensive margins in a monetary environment is not a new notion. In fact, the trade-off typically exists in a monetary environment with competitive search (see, for example, Sun and Zhou [2018] as cited by the paper). Therefore, I'm not sure what's been theoretically achieved by this paper other than working out an extension of an existing model. Of course, there is nothing wrong with trying out model extensions. But extensions should be applied to help us address an interesting economic issue. Unfortunately, I have difficulty identifying what that is with this paper. I think the authors should try harder in justifying the contribution of their paper.

   **Our response.**

We have now re-written the paper and its introduction. In the new manuscript, we highlight the purpose of the paper in the third paragraph (and outline our contributions from there on to the sixth paragraph) of its introduction. The contributions of the paper are as follows:

(a) **Addressing an open economic problem.** We address the open question regarding the effect of long-run inflation (i.e., an inflation target) on *heterogeneous* trade-offs between trade quantities and matching probabilities (or market tightness). It is not that such a trade-off *per se* is new. We acknowledge in the paper that such a trade-off is a known consequence of competitive search theory. However, what is new and is the focus of the paper is the study of long-run inflation policy on such intensive-versus-extensive margin trade-offs that vary across heterogenous agents, and their implication for the money holdings distribution and welfare.

- There are two reasons why this is worth studying. First, inflation is yet again of concern across many countries. Second, standard monetary policy models and also random matching models are silent on the possibility that markets can have different tightnesses. The heterogeneous-agent setup of Menzio et al. (2013) provides an elegant theory for rationalizing dispersion in market tightness and prices (without having to make ad-hoc cost or direct behavioral restrictions on pricing). How inflation interacts with such a mechanism was not studied in Menzio et al. (2013). In that paper, the authors focused on the zero inflation case to derive a tractable distribution of money holdings. The authors left open the question that we now address in this paper.

(b) **Open problem, new solution challenges: novel computational technique.** This paper's contribution is to address the above open question. Another contribution here is technical: When one deviates from the theoretically tractable but restrictive case of a zero inflation economy, the Menzio et al. (2013) framework poses new challenges: (1) How to compute non-standard value functions where there are nonconvexities due to interacting intensive-versus-extensive margin effects? (2) How to analyze the monetary equilibrium away from zero inflation when closed-form comparative-static tools can no longer be applied? We advance a solution method that exploits a geometric property of the non-standard equilibrium value function $B$, and hence also $\bar{V}$ in the paper. We show how one can adapt robust computational geometry tools to overcome the computational challenge.

The Associate Editor mentions another paper that embeds the Menzio et al. (2013) competitive search mechanism (Sun and Zhou, 2018) (SZ). However, there is a substantive difference in our setup from that of SZ's. The trade-off we emphasize is absent in their model due to their assumption that all agents who had entered the competitive search DM would exit it after being there for one period. What this does is to then kill off the endogenous spending cycle and hence heterogeneity that would have been present in the original MSS setting. In SZ, the heterogeneity is revived by assuming exogenous idiosyncratic shocks to agents so they enter the DM with different money holdings. We build on the endogenous spending cycle mechanism in Menzio et al. (2013) and study how the endogenous, heterogeneous tradeoffs are further affected by the inflation policy.

We show that the endogenous trade-off has interesting effects on money-holdings inequality and welfare. Existing heterogeneous-agent models with random matching do not feature the competitive search *intensive-extensive margin trade-off*. By extending heterogeneous competitive search

to a world with inflation, we provide an insight into how this trade-off depends on inflation, varies for individuals with different money wealth levels, and show in the aggregate that there is a non-monotone effect of inflation on the inequality of money holdings and consumption. This is due to the additional opposing effect from the extensive margin. It also contributes to staving off a standard redistributive-tax property of inflation. The redistributive-tax property tends to make heterogeneous-agent monetary models (without too many ad-hoc frictions) produce a lower welfare cost of inflation compared to, say, Lucas' original representative-agent conclusion. Thus, our study also highlights a different opposing-effect channel of inflation policy on inequality and welfare.

2. The paper emphasizes that "the details matter". The details matter for what? What do we learn from this model theoretically or quantitatively that is new or significantly different from the predictions of existing models? I don't see any of such answers clearly established in the paper, other than some general descriptions on how this model is different from existing models. The authors should discuss how their new model generates results different from existing models.

   **Our response.**

   - Our original intention was to highlight the new channel of monetary policy on the welfare cost of inflation and money distribution. As mentioned in the previous point, this works through heterogeneous directed search (and thus market tightness and pricing dispersion). This channel provides details that are new or surprising to readers more familiar with standard monetary policy models.
   - *A new Section 4*: We have now done a computational equivalent of a comparative-statics exercise to further explain the effect of inflation on the heterogenous responses of agents associated with this new channel. We demonstrate how this model is different from existing models, and that leads us to the novel distributional and welfare results in Section 5 afterwards. (See also the response above.)
   - We have now eliminated the not-so-precise phrase of "the details matter".
   - R2 pointed out that the literature has also discussed intensive-extensive margin trade-offs in competitive search. However, these papers focus on the degenerate setting of a representative agent (e.g., Rocheteau and Wright, 2009; Lagos and Rocheteau, 2005; Rocheteau and Wright, 2005). Our work complements this by studying the consequence of similar intensive-extensive margin trade-offs in competitive search, in an heterogeneous-agent economy.

3. Related to the above point the authors should do better job explaining the main mechanisms in the introduction. I understand their model is built on MSS so some of the mechanisms are similar (e.g., endogenous money-spending cycles) but most readers may not have read the original paper. Furthermore, *how are these mechanisms changed by the inflation*? For example, they can be clearer about the inflation's opposing effects on intensive and extensive margins.

   **Our response.**

   - We thank the Associate Editor for raising this question. We now explain the basic idea of this model in the introduction. (See also the last response above.)

4. Overall, the referees have some issues with various details but share the same big picture concern about the contribution of the paper. The referees also make important comments. The authors should also address them (if not every single one of them).

   **Our response.**

   - We have now addressed this in the revised manuscript. Please see our itemized responses further below.

5. R3 [*sic*] asks why the authors incorporated some features to the model. The authors should motivate and explain why they need to make these assumptions or what do they gain from them. R3 is the most supportive of the paper among the three referees. S/he has very good suggestions for possible ways to extend the paper to bring out the novel contribution.

   **Our response.**

   - We have now addressed this in the revised manuscript. In particular, Referee R3 questioned about the (previous) assumption of costly participation in the centralized market (CM). We are grateful to R3 for their insight here. This particular feature is now removed and we explain our reasoning in reply to R3 below. We retained a departure of our model from Menzio et al. (2013) in terms of the (CM) preference function specification. This change is remarked in Footnote 14 and it does not change the theory at all but allows us to map the model to data a bit more flexibly (when it comes to calibration). Please see our itemized responses further below.

## Referee 1

The paper studies inequality and cost of inflation in the long-run within a heterogeneous-agent monetary search model. The model features an ex-ante uncertainty on trading opportunities that results in ex-post heterogeneity in money-holdings. Quantitative findings show that in equilibrium, the relationship between long-run inflation rate (arguably can be consider as an inflation target) and inequality (money-holding and consumption) is non-monotonic. Moreover, the model's introduction of competitive-search friction to the otherwise standard heterogenous-agent macro model amplifies the cost of inflation.

1. The paper presents interesting results within a competitive search model. This class of monetary search models provide a nice microfoundation for the existence of money as a medium of exchange however it appears to be a less convincing tool when it comes to quantify the cost of inflation and understand the dynamics of economic inequality. Without explicit modelling of crucial components like idiosyncratic income shocks and endogenous labor supply, the model cannot provide a realistic analysis of inequality. It also cannot adequately explain the cost of inflation in the absence of nominal frictions, interest rate policy, and perhaps housing market. It would be more appropriate to present this work in a condensed form that highlights the new computational algorithm and the new channel (extensive margin) as the paper's main contributions.

   **Our response.**

- We thank Referee R1 and take on board R1's suggestion to condense the paper and focus on the main point of the paper: This is an extension of Menzio et al. (2013)—and therefore the competitive search intensive-extensive margins trade-off—with a purpose to address the open question of what non-zero inflation does to these trade-offs. We focus on dissecting the heterogeneity in this trade-off by summarizing responses of people across each stationary distribution under each inflationary regime. We now add a new Section 4 that provides the explanations using numerical "comparative-statics".
- In contrast to R1's claim, there is endogenous labor supply in this model. It is true that we do not assume exogenous idiosyncratic (income) shocks. The point here is there these shocks are unnecessary for the purposes of understanding the main question we posed. We are not doing a full-fledge quantitative exercise that typically involves assuming more ad-hoc shocks and frictions in order to fit many more micro- and macro-level empirical targets.
- Our purpose in this paper is to address the above-stated open question. While we do discipline the minimal-assumptions model to fit the theory to data, the sole purpose here is to ensure that the numerical comparative statics that we do are not driven by arbitrary parameter choices. Our analysis still gives insightful conclusions in terms of welfare and inequality measures. However, but these are to be taken as qualitative numerical insights as opposed to a large-scale quantitative modeling and accounting of welfare and inequality.
- We think that fleshing out these potential channels of a trade-off that is not always present in standard models may be useful for future and larger-scale quantitative explorations of this literature.

2. The paper matches long-run price dispersion only at the aggregate level, despite having several quantitative implications that can be tested, such as shopping frequency and price dispersion. The Kaplan-Menzio dataset, which is used in this paper's calibration, can be used to compare more specific information, such as the connection between shopping frequency, price dispersion, and money holdings, which is essential for deriving the quantitative results that are currently available in the paper.

   **Our response.**

   - We thank R1 for this interesting suggestion. Related to the last response above, we would like to emphasize that this is not a full-blown model where there is sufficient exogenous sources of structural, individual-level variations. In particular, to understand the new channel we highlight, we chose not to introduce any other ad hoc sources of exogeneous and idiosyncratic risks. Such elements would be needed to map the model to more granular behavioral data such as the ones mentioned by R1. Given the open question that we have highlighted and addressed in the new manuscript, we think that R1's suggestion will be more suitable for a separate, more empirical and structural modeling study.

3. In some cases, the text can be improved by using more formal language and reducing repetition. Examples include "explain a little" instead of "explain," "will pause to discuss" instead of "discuss," "the literature and what we do" instead of "literature review," "computation feasible" instead of "computationally feasible."

   **Our response.**

- We apologize for any tone in our previous writing that was deemed informal. We have now taken R1's suggestion on board and changed the writing style throughout the paper.

4. Authors may want to avoid claims without proper citations such as "During the Great Recession, inflation-targeting policy makers advocated policies that raised longer term inflation rates. Some policy makers in some countries still do." In general, the writing and content of the motivation paragraphs could be improved.

   **Our response.**

   - We thank R1 for this advice. We have now avoided such claims in the current version of the paper.

5. The relationship between long-run inflation policy and inflation target and how it maps to the current model should be clarified.

   **Our response.**

   - In the model, the monetary policy is defined by the parameter $\tau$ which is the growth rate of money supply. Since our focus is on stationary monetary equilibria, by definition, a stationary or long-run inflation rate must equal $\tau$. Therefore, we can interpret $\tau$ as our long run inflation target.

6. The model introduces a new extensive margin through endogenous matching probabilities that is absent in otherwise random matching models. The paper should better motivate why this is an important margin and support this with empirical evidence if possible.

   **Our response.**

   - We now clarify that the idea of endogenous matching probabilities is *per se* not new. What is new is the question we ask: The effect of inflation on heterogeneous trade-offs between the intensive and extensive margins of trade in the monetary competitive search model.
   - While we ourselves do not have direct observations of these margins, we do see that there is measurable price dispersion (among identical products) that can be decomposed into various factors. Among these factors, the behavior of shoppers directing search and taking more or less time searching for certain goods is non-trivial, according to the empirical work in Kaplan and Menzio (2015). In the paper, we take this as a given motivation (see our remark in Footnote 12). Also, in housing markets, we see significant variations in market tightness and delay to trading (see, e.g., Hedlund, 2016; Garriga and Hedlund, 2020).

## Referee 2

### Summary

This paper takes Menzio, Shi and Sun (2013 JET) and introduces inflation, which is a change of an aggregate state variable. Then, they use the model to quantitatively evaluate the welfare and equality implications of monetary policy. I am going to summarize and evaluate the paper from two perspectives, the methodology and the topic.

First is about methodology. This paper builds on the New Monetarist literature starting from Lagos and Wright (2005 JPE). To avoid keeping track of agents' money holding distributions or a Krusell-Smith type problem, they use an alternating market setup and quasi-linear preferences. The cost is to lose the income effect of assets/money on consumption and the distributional effects of monetary policy. Many attempts have been made to put the ex-ante non-degenerate distribution of money holdings back to the framework, such as Molico (2006 IER), Chiu and Molico (2010, 2011, 2020), Wong (2016 RES), and Rocheteau, Weill, and Wong (2021 JME). This paper takes a different approach to use the block-recursive structure of competitive search models. The notion of Block Recursive Equilibrium was first used on labor search papers by Menzio and Shi (2010, 2011). It builds on lattice-theoretic techniques, which were first used by Lagos and Rocheteau (2005) in this literature. Later, Menzio, Shi, and Sun (2013) introduce the block-recursive structure to monetary search models. One major drawback is that the analytical results will not hold when the aggregate environment evolves across time, e.g., when money supply changes and inflation is not zero. Then, we have to go to numerical methods. That is exactly what this paper is doing. The authors also develop a new and more accurate method to solve the non-concavity problem of value functions due to competitive search.

Since the key departure of this paper from the original MSS (2013) is due to positive inflation, the authors then naturally apply this framework and the new quantitative techniques to explore the distributional effects of monetary policy and inequality, an old and classical topic. The new thing here is the trade-off between intensive and extensive margins. The intensive margin refers to changes in real money balances and hence consumption, while the extensive margin refers to changes in matching/consumption probabilities. These two margins are automatically linked in general equilibrium due to competitive search, but such a link has been briefly discussed in Lagos and Rocheteau (2005) and Rocheteau and Wright (2009). I think the novelty here lies in the interaction of such a link to individual agents' money holding, as the authors put it, What is different here is that the endogenous matching probabilities are dependent on heterogeneous agent states. This is identified by the authors as the fourth channel in the paper. Basically, consumers at the top and the bottom part of the liquidity/money distribution respond differently to inflation, and their response at the intensive margin are mitigated by adjustments of trading probabilities. This intensive-extensive margin trade-off is key to generate the non-monotonic inequality result.

## Comments

The theoretical and quantitative analysis in the paper has been done rigorously and thoroughly. The paper is carefully written and has been polished completely. Hence, I only provide some minor comments.

1. I think the main contribution of the paper is meaningful and useful in the literature for future studies. While the main topic of distributional effects of inflation is a bit traditional and not so popular these days, the methodological contribution is significant. I wonder if the authors may be able to apply the current framework in their paper to address other timely and important topics. For example, I assume they can address similar questions as in Kaplan, Moll, and Violante (2018 AER) after introducing 2 multiple assets, and explore other topics in those papers following HANK. Although the current paper focuses on long-run or expected inflation, the trade-off in the competitive search framework can also operate in the short run. Well, of course, this is for future research.

**Our response.**

- We thank R2 for the suggestion. Indeed, as R2 has noted, to address a question such as was done in HANK one needs a multiple asset model. We note that this has been addressed recently by Bethune and Rocheteau (2023) in a random-matching, Bewley-style model.
- We agree with R2 that their proposal above is for future work. We are interested in investigating the additional channel with competitive search trade-offs instead of random matching in future work.

2. Related to Kaplan, Moll, and Violante (2018 AER), I wonder whether the current paper can say something on the indirect and direct channels of monetary policy transmission on consumption. Although the current paper has only one liquid asset, the labor decision is nontrivial and unlike Lagos and Wright (2005), the intertemporal substitution exists. I hope the author can provide some comparison or discussion, at least in the introduction.

**Our response.**

- We discussed in the introduction the difference between (Kaplan et al., 2018) (HANK) and our paper. Here without an illiquid asset we cannot directly compare with or comment on the "intertemporal substitution" channel that a permanent-income-hypothesis agent would face in HANK versus its hand-to-mouth type of agent.
- In HANK models, a modeler has an easier task in deriving and quantifying "direct" and "indirect" effects because the essential frictions in the model are parametrized through smooth-function assumptions (e.g., an ad-hoc menu cost of changing prices and asset-liquidation cost). These allowed the original authors to derive elasticity terms due to direct and indirect effects of policy.
- In monetary search models, where frictions and heterogeneity (as is our case here) are equilibrium objects, the insights are more intricate. As discussed, with one liquid asset only, both effects are interwind and cannot be decomposed analytically.

3. The following papers also discuss inflation and welfare in a monetary search environment with competitive search, although they do not have ex-ante heterogeneous agents. In particular, Rocheteau and Wright (2009) quantitatively evaluate the welfare implications of inflation in a competitive search model. The authors might consider them relevant to the current paper.

- Inflation, Output, and Welfare by Ricardo Lagos and Guillaume Rocheteau, 2005, IER.
- Inflation and Welfare in Models with Trading Frictions by Guillaume Rocheteau and Randall Wright, 2009, in Monetary Policy in Low Inflation Economies, Ed Nosal and Dave Altig, Cambridge University Press.

**Our response.**

- We thank R2 for the suggestions. We have now included these references and discussed them in the paper.

4. There are some errors and typos to be corrected. For example, on page 5, in the third paragraph, on line 11, it says "Also, there are less agents at the upper end...," while it is better to say "fewer

agents" instead. Another example, on page 33, it says "Figure 10 (top-left and top-right panels) shows that...," but Figure 10 on the next page has only one panel (a).

**Our response.**

- We thank R2 for the suggestions and apologize for having overlooked these typos earlier. The paper has been re-written accordingly.

# Reviewer 3

## Summary

This paper extends the model of Menzio et al. (2013) in at least three ways:

1. Includes inflation.
2. Includes a good which can be consumed in the complete market.
3. Includes a cost of joining the complete market.

The authors show that, whereas in some other heterogeneous agent models inflation tends to be inequality-reducing, the framework of Menzio et al. (2013) introduces a channel operating in the opposite direction. In particular, the authors (like Menzio et al.) show that in equilibrium, agents with lower money balances will search in trading posts with lower matching probabilities. The authors also show that this relationship is concave – the effect of decreasing money balance on trading probabilities is higher for poor households than for rich households. The authors suggest that both of these can result in higher inequality of money balances.

The authors also show that the model is consistent with several real world features of inflation: higher inflation implies higher price dispersion, inflation and frequency of money payments are negatively correlated, and inflation and the value of money payments per transaction are positively correlated.

Finally, the authors show that their model implies welfare costs of inflation in line with other papers.

## Major Comments

1. **Cost of CM ($\chi$):** I'm not sure this is a useful device. First, it's unclear what its real-world counterpart is. Second, I was confused by the way it is modelled. Is it a monetary cost? It seems to be, but if so, to whom is it paid? Absent this, the model is not stock-flow consistent – that is, there is a flow of money, $\chi$, which does not add to anyone's stock. The result is that simulation may lead to decreasing money stocks over time.

   Moreover, I'm confused by the timing and the ability of the agent to borrow. For sake of argument, imagine that recipient of $\chi$ allows the individual to pay it in the second period, after participation in the CM. Then, any agent participating in the CM will have the budget constraint

   $$C + y = l + m - \chi$$

   The quasilinear utility means we can easily find optimal choices $C^*$ and $y^*$. Since m and $\chi$ are given, we can then solve for $l^*$ as:

   $$l^* = C^* + y^* - m + \chi$$

Since there is no physical bound on $l$, there is no value of $\chi$ which poses a constraint on the individual's choice of $C$, $y$ or their ability to participate in the CM. Agents always pay for $\chi$ simply by working additional hours.

The same argument applies if the agent issues an I.O.U. that he commits to repaying after participating in the CM. The I.O.U. simply becomes an additional cost which he must pay, and is subtracted from his budget constraint.

It seems, then, that if one wanted to keep this constraint in the model, one could either (A) introduce limited commitment, or (B) impose a maximum physical bound on $l$ (i.e. the total number of hours in the day).

The authors insist numerous times that the inclusion of $\chi$ (and therefore, I assume also the participation constraint) does not change the main results. I'm confused, therefore, about why it is included at all.

**Our response.**

- We thank R3 for their insight. We agree and in the new manuscript, the model now does not have this $\chi$ assumption. It is not needed. There is still mixing in equilibrium in the sense that agents would still participate in both markets asynchronously. Intuitively, this is because agents have a preference for variety of the CM and DM goods. Therefore, there is always a non-zero probability for any agent to be in each market at any time.
- We apologize for the previous unnecessary and cumbersome setup. The paper has drastically improved from R3's insightful suggestion.

2. **Consumption in the CM:** I'm not sure I understand what purpose this serves. Is it necessary for the results of the model? If so, why? Otherwise, perhaps it is better to leave it out.

**Our response.**

- We thank R3 for pointing this out. We have added an explanation in the paper. Having utility over CM consumption ($C$) serves no purpose for the theoretical equilibrium description. The equilibrium trade-off is not affected by $C$ since it is a constant by virtue of quasilinearity in the preference function. However, from a numerical (and quantitative-calibration perspective), having a non-zero $C$ allows us to calculate well-defined sector shares for the CM and DM.

3. **The Extensive-Margin Effect:** It's unclear to me that the increasing dispersion of matching probability should lead to increasing asset inequality. Are there not reasons to think this should go in the opposite direction? High-money balance households are more likely to spend their money quickly, which should decrease their money balances towards those of the poor, thus decreasing inequality all else equal. I see this as the major contribution of the paper, and therefore I think it requires more fleshing out and explanation.

**Our response.**

- We now explain this point further using numerical comparative statics in Section 4 of the paper. In particular, Figures 6 to 8 show that "richer" agents match faster, but are less sensitive in their matching probabilities (and volume of trade per dollar) compared to the

"poorer" ones. Inflation uniformly lowers these individual elasticities. However, the equilibrium distributional feedback means that, on average these can still be higher.

- Our analysis in Section 4 helps one to better appreciate the distributional outcomes. In Figure 9 (third and bottom panel), the 90/10 ratio summarizes the information that for given inflation policy higher money wealth agents tend to match faster than lower money wealth ones.

4. For instance, is there not a way to mathematically link the distribution of money balances to their flows (determined by trading probabilities, entry/exit from the CM, and $y^*$)? I'm thinking in the same way that a Markov chain leads to a steady-state distribution. This could prove enlightening here.

**Our response.**

- R3's conjecture here is correct when one studies this model at zero inflation. In that case, Menzio et al. (2013) provide a closed-form, stock-and-flow accounting derivation of the equilibrium distribution of agents' money holdings. This is their Markov chain characterization of the equilibrium Markov map underlying the distribution of agents. This Markov-chain accounting works because with zero inflation, the equilibrium response functions, including the matching probability function, are independent of the aggregate variable $\omega$ (relative price).
  - For a different reason (i.e., agents synchronously enter and exit DM and CM), one can also construct a steady-state distribution of agents by means of Markov-chain accounting ideas in a Bewley-style model such as Bethune and Rocheteau (2023). (We are grateful to Zach Bethune for providing his Julia codes publicly, which verifies our point here.)
- Away from zero inflation, in this model with endogenous and stochastic market participations, this is no longer true. Hence our explicit notation in Equation (2.16) in the manuscript. What this means, practically, is that how often an agent stays in or visits a market is now nonlinearly and simultaneously determined with $\omega$ (or its inflation, along a transition path). Menzio et al. (2013) previously sketched out this prospect and termed it a "partial block recursivity" property. This is also the reason why we had to use Monte Carlo methods to simulate a steady state distribution under non-zero inflation.

5. **Mechanical effects of inflation on inequality:** The way that money growth/inflation is introduced in the paper will mechanically lead to lower inequality in money balances. Because nominal money balances are added to each agent arithmetically, but inflation reduces the value of real money balances geometrically, the inflation process will lead to a growth in real money balances for poor households, and a fall in real money balances for rich households. I think this is a different channel than what you highlight as leading to lower money-inequality in the face of higher inflation. For this reason, it would be interesting to see how the results change if agents receive nominal money balances in proportion to their current stock.

**Our response.**

- We thank R3 for the suggestion. We have worked out results with proportional transfers

instead of lump sum new money transfers. We can report that proportional transfers would not have any consequence in our setting.

  - Consider our notation of $y$ as the end of CM residual money balance.
  - If an agent carries any $y$ into the DM at the start of a period $t+1$ and new money transfers are implemented proportionally as $\tau y$, then her money holding in DM is

$$m_{+1} = \frac{\omega(1+\tau)y}{\omega_{+1}(1+\tau)} = \frac{\omega y}{\omega_{+1}}.$$

    That is a *proportional-tranfers* inflation policy $\tau$ has *no real effect*.

- Alternative if we change the timing of money transfers to the start of CM then again it does not matter as agents in the CM have linear preference over labor/leisure—there is no wealth effect.

### Minor Comments

- Figure 3 is perhaps a little messy. Also, is $W$ a flat horizontal line? Does this make sense? The value of participating in the CM should be increasing in $m$ since it will mean lower amounts of work, no?
- A few times you use the word "envelop" when what you mean is "envelope".
- The first statement of 2.4.3 is a tautology. I think there's a typo there.
- P.15, shortly after equation 2.10: "Likewise, the buyer's probability of matching..." is a sentence fragement.
- P.10, 7th line in section 2.2.3, there is a closed bracket missing: "...taking the probability of being matched with a buyer as given.)"
- P. 27 Second line in section "How might inflation...", should be "inflation has a redistributive-tax effect".
- P. 9 "and transforms it (linearly) into the same amount of DM good i". I believe that should be CM good i.

**Our response.**

- We now re-plot Figure 3 and show more clearly that $W$ is not flat. It is increasing in $m$ as R3 stated. (The old figure's relative scaling meant that $W$ had appeared flat to a reader.)
- We thank R3 and apologize for these typographical irritations.
- We have fixed all the issues raised in R3's minor comments in the new manuscript.

## References

**Bethune, Zachary and Guillaume Rocheteau**, "Unemployment and the Distribution of Liquidity," *Journal of Political Economy Macroeconomics*, 2023, *1* (4), 742–787.

**Garriga, Carlos and Aaron Hedlund**, "Mortgage Debt, Consumption, and Illiquid Housing Markets in the Great Recession," *American Economic Review*, June 2020, *110* (6), 1603–34.

**Hedlund, Aaron**, "Illiquidity and its discontents: Trading delays and foreclosures in the housing market," *Journal of Monetary Economics*, 2016, *83*, 1–13.

**Kaplan, Greg and Guido Menzio**, "The Morphology Of Price Dispersion," *International Economic Review*, November 2015, *56*, 1165–1206.

_ **, Benjamin Moll, and Giovanni L. Violante**, "Monetary Policy According to HANK," *American Economic Review*, March 2018, *108* (3), 697–743.

**Lagos, Ricardo and Guillaume Rocheteau**, "Inflation, Output, and Welfare," *International Economic Review*, 2005, *46* (2), 495–522.

**Menzio, Guido, Shouyong Shi, and Hongfei Sun**, "A Monetary Theory with Non-degenerate Distributions," *Journal of Economic Theory*, 2013, *148* (6), 2266–2312.

**Rocheteau, Guillaume and Randall Wright**, "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 2005, *73* (1), 175–202.

_ **and** _ , "Inflation and Welfare in Models with Trading Frictions," in Ed Nosal and Dave Altig, eds., *Monetary Policy in Low Inflation Economies*, Cambridge University Press 2009.

**Sun, Hongfei and Chenggang Zhou**, "Monetary and fiscal policies in a heterogeneous-agent economy," *Canadian Journal of Economics*, 2018, *51* (3), 747–783.