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Asset Prices and Interest Rates in Cash-in-Advance Models

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We develop a method to solve and simulate cash-in-advance models of money and asset prices. We calibrate the models to U.S. data spanning the period 1890–1987 and study some empirical regularities observed over this period. The phenomena of interest include the average level of stock returns and returns on nominal bonds, the covariation of *realized* real interest rates and real asset returns with inflation, and the ability of nominal interest rates to predict inflation and nominal stock returns.

I. Introduction

Empirical regularities involving nominal interest rates, asset prices, and inflation are determined by the role and effects of money in the economy. In this paper we ask whether or not popular models of money and asset prices can help to interpret those regularities. The models we employ are general equilibrium, stochastic models. They are ideally suited to describe explicitly and to evaluate the distortions produced by monetary policy. They cannot, however, be solved ana-

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lytically, given any realistic assumptions about the stochastic behavior of exogenous shocks. The general Markov structure that we assume for the forcing variables does not allow us to perform comparative dynamics analysis without resorting to numerical simulations. Our strategy is to develop an algorithm for computing equilibria produced by the models, to simulate them, to ask whether or not the simulated data resemble the actual data, and to interpret differences between reality and simulations.

The main virtues of general equilibrium, stochastic models of money and asset prices are their internal consistency and the simplicity of their structure. Their potentially serious drawback, of course, is a lack of "realism." The simple structure of markets and transactions underlying the equations can at best be considered approximations to reality. Our objective in this paper is to determine whether these approximations are acceptable for the purpose of interpreting the comovements of inflation, interest rates, and stock returns. We think that it is useful to evaluate the empirical predictions of these models for two reasons. First, their simplicity allows us to interpret the results of numerical simulations more easily. Second, as Prescott (1986) stresses, once internal consistency is regarded as a necessary condition, it is better to start from simple models: their ability to explain empirical regularities helps to sort out what effects to include in macroeconomic models and what omissions or approximations are acceptable.

The models we simulate are the cash-in-advance models developed by Lucas (1982) and Svensson (1985). This choice is mainly motivated by our intention to provide the closest formal analogue to real models of asset prices commonly used to interpret U.S. data, which employ the infinite-horizon, representative-agent specification. Furthermore, alternatives such as the overlapping-generations models rely on assumptions very similar to those we employ to ensure that agents willingly hold an asset, such as money, that is dominated by other real assets (see Sargent and Wallace 1982). Finally, it is known that cash-in-advance models and money-in-the-utility-function models can be reconciled by appropriately parameterizing tastes and technology.[1]

This paper is complementary to that of Hodrick, Kocherlakota, and Lucas (1991), who ask whether models from the same family as those we study can explain the observed behavior of the velocity of money in the United States, and to those of Kydland (1987) and

[1] This result is due to Feenstra (1986). As Ostroy and Starr (1988) point out, however, that equivalence relies on the assumption that for every dollar received for sales of labor, there is a dollar with which to buy commodities, i.e., that money buys goods and services but goods and services do not buy other goods and services. It thus appears more adequate to make the constraint explicit through cash-in-advance equations.

Cooley and Hansen (1989), who study the effects of introducing money in real business cycle models. Like the papers of LeRoy (1984*a*, 1984*b*), Danthine and Donaldson (1986), and Marshall (1989), our paper discusses the ability of representative-agent monetary models to explain correlations among inflation, nominal interest rates, and stock returns. LeRoy first formally applied general equilibrium models of money in the utility function to analyze the relations between inflation and asset returns. He considers only two-state models in which the source of randomness is either the endowment process or the money supply process. Danthine and Donaldson also study a similar model of money in the utility function with a more general stochastic structure, where inflation uncertainty is a function of endowment uncertainty but money growth is nonstochastic: hence, in their model, all sources of fluctuation in the price level are money demand shocks. Marshall employs a model in which money demand arises from a transactions costs function in the budget constraint. By contrast, we consider an economy in which money supply is stochastic and money demand arises from a cash-in-advance constraint. Whereas Marshall studies his model under simplifying assumptions on the forcing variables' processes, we assess the predictive power of our models by simulating them with processes for the forcing variables that are estimated from U.S. data.[2]

Section II presents the data set and the empirical regularities we choose to discuss. Section III describes the models and establishes the notation. Section IV constructs an algorithm that solves both models. Section V discusses the results of the simulations. Section VI contains a few concluding remarks.

II. Data and Empirical Regularities

Our objective is to point to empirical facts that are sufficiently "general" and, broadly speaking, largely independent of changes in monetary institutions, since the model we use to assess these regularities is clearly ill equipped to deal with the institutional changes underlying many important episodes of U.S. monetary history. For this reason, we update the sample of Grossman and Shiller (1981) and Mehra and Prescott (1985) and augment it with the estimates of the U.S. money stock provided by Friedman and Schwartz (1982).

The data set includes the annual average Standard and Poors composite stock price index, the annual dividends from the Standard and Poors series, per capita consumption of nondurables, the consump-

[2] The basic logic of our algorithm is similar to the one by Danthine and Donaldson.

tion deflator,[3] a nominal yield on short-term securities,[4] and the money stock. This last series was constructed by computing the sums of currency held by the public plus adjusted deposits at all commercial banks less large negotiable certificates of deposit since 1961, divided by population.[5]

The choice of the broader monetary aggregate is justified by two considerations. First, as Friedman and Schwartz (1963, 1982) convincingly argue, M2 is preferred over M1 because it has been redefined fewer times. Second, as Hodrick et al. (1991) (among others) have pointed out, at least in the second postwar period, M1 velocity clearly displays nonstationarity, a feature inconsistent with the basic assumptions of the models we are exploring. The nonstationarity of M1 velocity might be due to the redefinitions of that aggregate, but also to technological progress in the transactions technology, a feature that our model fails to capture.

There is, of course, no firm criterion to determine—in a sample of annual data from 1889 to 1987—*what* represents an empirical regularity. Indeed, it could be argued that, over such a long historical period, very few phenomena of interest involving money and asset prices have maintained the same characteristics from start to end. Our choices do not conform to a single criterion. In general, we focus on empirical regularities that are largely independent of institutional factors and have been extensively discussed in the literature.[6]

[3] These three series come from the following two sources: 1889–1975 data come from the Mehra-Prescott data set (we thank Rajnish Mehra for kindly providing it to us); the 1976–87 data come from the Citibank data base.

[4] These include 60- and 90-day prime commercial paper prior to 1920, Treasury certificates for the 1920–30 period, 90-day Treasury bills for the 1931–75 period (Mehra-Prescott data), and 90-day Treasury bills for the 1976–87 period (Citibank data base, annual averages).

[5] The data for the period 1889–1958 come from Friedman and Schwartz (1982). They are annual averages of monthly data. The data for 1958–87 are constructed following the methods outlined in Friedman and Schwartz (1970). We use the definition of M2 from the statistical releases of the Board of Governors of the Federal Reserve System, which are available in the Citibank data base (series MS2), and subtract from it "small-denomination time deposits at thrift institutions" (series MSTT) and "savings deposits at thrift institutions" (series MSVT). The monthly data are averaged to obtain annual data. As Friedman (1988) points out, in 1983, money supply calculated this way displays an exceptional rate of growth, largely because of shifts out of savings accounts and into money market accounts. These shifts net out in the Federal Reserve definition of M2. Following Friedman, we correct for this problem by updating the net-of-savings-accounts series constructed above with the growth rate of the Fed M2 series from 1983. The rates of growth of our series match very closely those of the corresponding series from Friedman and Schwartz in the years of overlap, 1959–75, but the levels do not. However, in this paper we use only rates of growth of the money stock.

[6] Most of the theories offered to explain the regularities discussed below are independent of institutional factors, with the significant exception of the models of nominal nonneutralities based on features of the U.S. tax system.

TABLE 1

Summary Statistics

	1889–1987		1947–87	
	Mean	Standard Deviation	Mean	Standard Deviation
Nominal interest rate (%)	3.89	2.73	4.79	3.27
Nominal stock return (%)	10.02	16.23	12.26	13.35
Inflation rate (%)	2.86	5.20	4.43	3.02
Equity premium (%)	6.32	16.57	7.60	13.87
Velocity	1.44	.30	1.28	.15

By informally examining the plots of all series (available on request), we let the data suggest the appropriateness of studying subsets of our sample period. While nominal stock returns and the nominal interest rates fail to display major differences over the sample, the variance of consumption growth decreases remarkably after World War II. To some extent, the variance of money supply growth also appears to decrease in the second postwar period. Hence we evaluate the predictions of the monetary models both over the whole sample and for the second postwar period.[7]

Table 1 reports summary statistics from our data. The more recent period is characterized by higher and less volatile inflation, higher and less volatile stock returns, and higher and more volatile nominal interest rates. The mean of consumption velocity has remained approximately unchanged, although the variability of velocity (as measured by the standard deviation) in the second part of the sample is half of that for the whole sample. This decrease in the variability of velocity is consistent with the decrease in volatility of consumption growth and money growth mentioned above. One important feature of the data in table 1 is the difference between the average realized real return on stocks and the average realized real interest rate. This "equity premium," which was first analyzed by Mehra and Prescott (1985), is 6.32 percent in the whole sample and 7.60 percent in the second postwar period.

Table 2 describes the contemporaneous correlations of realized real interest rates and stock returns with inflation. Real interest rates, real stock returns, and inflation are all measured from time t to time $t+1$. The table shows that, when inflation is high, realized real stock returns and interest rates are low, and vice versa. For the real interest rate, this negative relation is more marked over the whole sample,

[7] This strategy is further justified by the work of Romer (1986), suggesting the presence of higher sampling errors in the prewar consumption data.

TABLE 2

Comovements of Inflation and Ex Post Real Stock Returns and Real Interest Rates

Regression Equation and Sample Period	Constant Term	Slope Coefficient	R^2	Durbin-Watson Statistic	Standard Error of Estimate
Real stock returns on inflation:					
1890–1987	.10	−.78	.06	1.76	.16
	(.02)	(.32)			
1947–87	.19	−2.48	.26	1.87	.13
	(.04)	(.67)			
Real interest rate on inflation:					
1890–1987	.04	−.93	.76	.24	.03
	(.003)	(.05)			
1947–87	.02	−.45	.18	.37	.03
	(.01)	(.15)			

Note.—Standard errors are in parentheses.

whereas for the real stock return the relation is more marked in the second postwar period. The relationship between realized real interest rates and inflation is documented, for the postwar period, by Fama and Schwert (1977) and Mishkin (1981), among others. Ibbotson and Sinquefield (1976) and Summers (1983) report it for the periods 1926–74 and 1860–1979, respectively. The relationship between realized stock returns and inflation is documented by Jaffe and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), Schwert (1981), and Summers (1983) for the second postwar period. Summers (1983) also looks at the period 1870–1979, and Kaul (1987) studies the 1930s.

Table 3 reports the relation between nominal interest rates and *subsequent* inflation and nominal stock returns. We regress inflation and nominal stock returns from year t to year $t + 1$ on the nominal interest rate in year t. These regressions measure the forecasting ability of nominal interest rates, and under the assumption of rational expectations, they provide information about the comovements of ex ante real interest rates and stock returns and expected inflation. The relation between interest rates and subsequent inflation is discussed by Fama (1975), Mishkin (1981, 1989), Fama and Gibbons (1982), and Summers (1983). The prewar evidence is studied by Barsky (1987) (who emphasizes, like Mishkin [1989], the shifts in the stochastic properties of inflation) and Summers (1983). Our evidence is consistent with the findings in the literature: the coefficient of the nominal interest rate is significantly less than one, and the low values of

TABLE 3

NOMINAL INTEREST RATES AS PREDICTORS OF INFLATION AND NOMINAL STOCK RETURNS

Regression Equation and Sample Period	Constant Term	Slope Coefficient	R^2	Durbin-Watson Statistic	Standard Error of Estimate
Inflation on nominal interest rate:					
1890–1987	.02	.24	.02	1.00	.05
	(.01)	(.19)			
1947–87	.02	.45	.25	.75	.03
	(.04)	(.13)			
Nominal stock returns on nominal interest rate:					
1890–1987	.11	−.20	.001	1.72	.16
	(.03)	(.61)			
1947–87	.13	−.13	.001	1.75	.14
	(.04)	(.64)			

NOTE.—Standard errors are in parentheses.

the Durbin-Watson statistics suggest the presence of in-sample instability of the equation. Under rational expectations, this evidence indicates that real interest rates fluctuate over time and might be negatively correlated with expected inflation.[8] The bottom panel of the table reports regressions of nominal stock returns (from time t to time $t + 1$) on the nominal interest rate at time t. The estimated coefficients are negative but insignificant. The R^2 statistics indicate that nominal interest rates are very poor predictors of stock returns. Similar results have been reported by Fama and Schwert (1977) and Giovannini and Jorion (1987).

III. The Models

The two monetary models we simulate are Lucas's (1982) and Svensson's (1985). The models are characterized by representative agents maximizing a time-separable isoelastic utility function subject to a liquidity constraint, which compels them to buy goods with money, and a wealth constraint. Endowment is stochastic, exogenous, and growing over time. There is no storage or investment technology available. Money supply is also exogenous and stochastically growing and is distributed lump-sum to the agent. The crucial difference be-

[8] In Sec. V we identify and discuss all the components in the slope coefficients of these regressions.

tween the two models lies in the timing of transactions in goods and asset markets.

In Lucas's model, individuals can acquire money after observing the state of the economy but before purchasing the consumption good; hence, given their risk and return characteristics, money and assets are equally suitable to intertemporal consumption smoothing. In Svensson's model, by contrast, individuals begin the period with predetermined money balances, which they need to purchase the consumption good. This feature introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. For the purpose of intertemporal consumption smoothing, money is imperfectly substitutable with other assets and is more "liquid" than other assets.[9]

Formally, the consumer's problem in the Lucas model is

$$\max_{\{c_t, z_t, M_t^d\}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \frac{1}{1-\gamma} c_t^{1-\gamma} \right] \tag{1}$$

subject to

$$c_t \leq M_t^d \pi_t \tag{2}$$

and

$$\begin{aligned} M_t^d \pi_t + z_t q_t \leq \left(\frac{\pi_t}{\pi_{t-1}} y_{t-1} + q_t \right) z_{t-1} + (\omega_t - 1) M_{t-1} \pi_t \\ + \left(M_{t-1}^d \pi_t - c_{t-1} \frac{\pi_t}{\pi_{t-1}} \right). \end{aligned} \tag{3}$$

The notation is standard: notice, in particular, that we use π to indicate the inverse of the price level, that is, the purchasing power of money; z stands for the shares of the productive asset—a claim to future dividends—held and demanded by the consumer. Equation (2) is the liquidity constraint and equation (3) is the wealth constraint. The evolution of exogenous variables is

$$y_t = \eta_t y_{t-1} \tag{4}$$

and

$$M_t = \omega_t M_{t-1}. \tag{5}$$

The timing of transactions is as follows. At the beginning of a period, individuals learn the realizations of the monetary and endowment

[9] See Giovannini (1989). For this reason the model gives rise to a form of "precautionary" money demand.

shocks, ω and η, respectively, and receive their monetary transfer, which in real terms is $(\omega_t - 1)M_{t-1}\pi_t$. In the assets market, they obtain dividend payments from "firms," $z_{t-1}(\pi_t/\pi_{t-1})y_{t-1}$, and the value of their stockholdings, $q_t z_{t-1}$.[10]

Agents use these resources to purchase money balances and stocks. In the goods market they use currency to purchase the consumption good. Notice that the money balances turned over by consumers to the firm are held by it until the asset market opens at the beginning of the next period. Hence the inflation tax is levied *directly* on the firm.

The market-clearing conditions are

$$c_t = y_t, \tag{6}$$

$$M_t^d = M_t, \tag{7}$$

and

$$z_t = 1. \tag{8}$$

In the Svensson model, the monetary transfer, which is observed at the beginning of each period, is received in the assets market. However, goods trade occurs before asset trade. Consumers use their money balances at the beginning of the period to purchase the consumption good and then enter the assets market with any remaining cash balances. The money stock evolves as

$$M_t = \omega_{t-1}M_{t-1}, \tag{9}$$

which implies that at time t agents know the nominal stock of money available to purchase goods at time $t + 1$. The consumer's problem is

$$\max_{\{c_t, z_t, M_{t+1}^d\}} E_0\left[\sum_{t=0}^{\infty} \delta^t \frac{1}{1-\gamma} c_t^{1-\gamma}\right] \tag{10}$$

subject to

$$c_t \leq M_t^d \pi_t \tag{11}$$

and

$$M_{t+1}^d \pi_t + z_t q_t \leq [(y_t + q_t)z_{t-1}] + (\omega_t - 1)M_t\pi_t + (M_t^d \pi_t - c_t). \tag{12}$$

The sequence of market equilibrium conditions is, of course, identical to equations (6)–(8).

[10] This interpretation differs from that of Lucas, where equities' dividend payments occur in the same period, although they can be spent only in the next period.

IV. Solution Methods

In this section we derive the first-order necessary conditions and sufficient conditions for existence and uniqueness of the equilibrium. These conditions are the basis for the algorithm used to compute the equilibrium realizations of the model-determined variables. Both versions of the model can be formulated as a dynamic program with unbounded returns.

Since the endowment is growing over time and the isoelastic utility function is unbounded, the maximization problem posed in equations (1) and (10) may not be well defined because total expected utility may be infinite. To ensure finite expected utility, the following condition has to be satisfied:

$$E_0\eta^{1-\gamma} < \frac{1}{\delta}; \tag{13}$$

that is, the expected rate of growth of utility has to be less than the utility discount factor.

For both versions of the model, the state is represented by a realization of the variables η_t, ω_t, M_t, and y_t. An equilibrium is a set of functions of the state: q, π, a value function, and associated multiplier functions μ (the liquidity constraint multiplier) and λ (the wealth constraint multiplier). Given the π and q functions, a representative agent can solve his maximization problem.

We consider only stationary equilibria, that is, fixed functions of the state. Nonstationary equilibria may exist, but we do not consider them here because we wish to develop a model that is suited to study empirical regularities.[11] For each set of price functions, standard arguments can be used to show that there is a unique, continuous, and bounded value function, which can then be used to construct the multiplier functions associated with the problem. The main task is then to compute the pair of equilibrium price functions.

For notational consistency, we denote the state of the economy at time t as s_t. For the Lucas version of the model, any pair of equilibrium price functions must satisfy the first-order conditions of the individual optimization problem together with the market equilibrium conditions. Substituting market equilibrium conditions into the first-order conditions with respect to c_t, M_t^d, and z_t, we have

$$y_t^{-\gamma} = \lambda(s_t), \tag{14}$$

[11] An early discussion of nonstationary equilibria in cash-in-advance models appears in Scheinkman (1980).

$$\lambda(s_t)\pi(s_t) = \delta E_t[\lambda(s_{t+1})\pi(s_{t+1})] + \mu(s_t)\pi(s_t), \tag{15}$$

and

$$\lambda(s_t)q(s_t) = \delta E_t\left[\lambda(s_{t+1})\left[q(s_{t+1}) + y_t\frac{\pi_{t+1}}{\pi_t}\right]\right]. \tag{16}$$

Since λ is determined by (14), we use (15) and the cash-in-advance constraint to compute π. This information is then used to determine the equilibrium equity price function from (16).

We start by defining implicitly a function K as the inverse of velocity:

$$\pi(s_t) = \frac{y_t K(s_t)}{M_t}.$$

By the implicit function theorem, we can study the properties of the function K as well as the function π since y and M are strictly positive and are determined exogenously. The cash-in-advance constraint (2) implies that the value of the function K for any state cannot fall below unity (since this violates the lower bound on the inverse of the price level imposed by the constraint). For each s_t, if $K(s_t)$ exceeds one, the cash-in-advance constraint is nonbinding; if $K(s_t)$ equals one, the constraint is binding.

Substituting for π in (15) and using (14), we obtain after some simplification

$$K(s_t)\mu(s_t) = y_t^{-\gamma}\left\{K(s_t) - \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma}K(s_{t+1})}{\omega_{t+1}}\right]\right\}. \tag{17}$$

Our iterative procedure is based on equation (17). If the cash-in-advance constraint is nonbinding, $\mu_t = 0$ and

$$K(s_t) = \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma}K(s_{t+1})}{\omega_{t+1}}\right]. \tag{18}$$

On the other hand, if the cash-in-advance constraint is binding at s_t, $\mu_t > 0$, $K(s_t) = 1$, and

$$\mu(s_t) = y_t^{-\gamma}\left\{1 - \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma}K(s_{t+1})}{\omega_{t+1}}\right]\right\}. \tag{19}$$

This reasoning suggests that the function K at s_t is

$$K(s_t) = \max\left\{1, \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma}K(s_{t+1})}{\omega_{t+1}}\right]\right\}. \tag{20}$$

To study the properties of the function K, we start by defining the operator S_1 by

$$S_1 K(s_t) = \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}}\right]. \tag{21}$$

If K is a continuous, bounded, and nonnegative function, then the operator in (21) is well defined. We define a second operator T by

$$TK(s_t) = \max\{1, K(s_t)\} \tag{22}$$

so that the composite operator is

$$T \cdot S_1 K(s_t) = \max\{1, S_1 K(s_t)\} = \max\left\{1, \delta E_t\left[\frac{\eta_{t+1}^{1-\gamma} K(s_{t+1})}{\omega_{t+1}}\right]\right\}. \tag{23}$$

Let H_1 denote the composite operator $T \cdot S$. The following theorem shows that, under certain conditions, the composite operator H_1^n (the operator applied n times) is a contraction mapping. Let ζ denote the space of continuous, bounded functions that are defined over the state space, and let ζ have the supremum norm. The properties of the solution to the functional equation (23) are described by the theorem.

THEOREM 1. If $E_0[\eta_t^{1-\gamma}/\omega_t] < 1/\delta$, there exists exactly one continuous bounded function K that solves equation (23).

Proof. See the Appendix.

In the proof we show that H_1^n is a contraction; hence we can use the method of successive approximations by applying H_1 to find the fixed point K. Once the fixed point is found, the equilibrium purchasing power of money is determined as $\pi(s_t) = y_t K(s_t)/M_t$, where K is the fixed point of H_1. The function π is used in (15) to determine the equilibrium multiplier function μ.

An important property of the solution is that the fixed point K is a function of a subset of the state space, namely (η, ω), and is not a function of the levels of the endowment or of the money stock (see the proof in the Appendix). This property greatly simplifies the computation of the equilibrium. Furthermore, the condition used to establish the existence and uniqueness of the fixed point is equivalent to the sufficient condition for the existence of an equilibrium in which money is valued. If $E_t[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] < 1/\delta$ for all states, the cash-in-advance constraint is always binding. If the unconditional expectation $E_0[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] > 1/\delta$, a monetary equilibrium does not exist because the cash-in-advance constraint is never binding. For an equilibrium to display variable velocity (alternating between a binding and a nonbinding cash-in-advance constraint), the argument above suggests that $E_0[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] < 1/\delta$ must be true, but $E_t[\eta_{t+1}^{1-\gamma}/\omega_{t+1}] > 1/\delta$ should hold in some states.

The equilibrium K function is then used to construct the equilibrium equity price. The equity price function q must satisfy the functional equation

$$y_t^{-\gamma} q(s_t) = \delta E_t \left[y_{t+1}^{-\gamma} \left[q(s_{t+1}) + y_t \frac{\pi_{t+1}}{\pi_t} \right] \right]. \tag{24}$$

From the definition of K,

$$y_t^{-\gamma} q(s_t) = \delta E_t[y_{t+1}^{-\gamma} q(s_{t+1})] + \delta E_t \left[\left(\frac{y_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right]. \tag{25}$$

Factoring out $y_t^{1-\gamma}$ from both sides of the equation and letting $\xi_t = q_t/y_t$ (the price/earnings ratio) result in

$$\xi(s_t) = \delta E_t[\eta_{t+1}^{1-\gamma} \xi(s_{t+1})] + \delta E_t \left[\left(\frac{\eta_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right]. \tag{26}$$

We iterate on the function ξ. Define the operator T_1 as

$$T_1 \xi(s_t) = \delta E_t[\eta_{t+1}^{1-\gamma} \xi(s_{t+1})] + \delta E_t \left[\left(\frac{\eta_{t+1}^{1-\gamma}}{\omega_{t+1}} \right) \frac{K(s_{t+1})}{K(s_t)} \right]. \tag{27}$$

The mapping T_1 takes continuous bounded functions into continuous bounded functions. If we start with an initial guess that is in the space of continuous bounded functions, the application of the operator T_1 results in a function that is an element of the same space. The operator T_1 is monotone but does not display the discounting property when applied to any arbitrary function in ζ. When T_1 is applied repeatedly, there is some positive integer n such that T_1^n is a contraction mapping. The stock price function can now be solved in two steps. In the first step we use equation (26) to solve for the price/earnings ratio, the function ξ. Notice that, once again, we are able to limit the domain of ξ to a subset of the state space. Finally, we obtain the stock price simply by multiplying ξ times y_t.

For the Svensson version of the model, the first-order conditions and the market equilibrium conditions imply

$$y_t^{-\gamma} = \mu(s_t) + \lambda(s_t), \tag{28}$$

$$\lambda(s_t)\pi(s_t) = \delta E_t[[\lambda(s_{t+1}) + \mu(s_{t+1})]\pi(s_{t+1})], \tag{29}$$

and

$$\lambda(s_t)q(s_t) = \delta E_t[\lambda(s_{t+1})[q(s_{t+1}) + y_{t+1}]]. \tag{30}$$

Using the arguments developed above, we obtain, in equilibrium, the following functional equation:

$$K(s_t) = \max\left\{1, \frac{\delta}{\omega_t} E_t[\eta_{t+1}^{1-\gamma} K(s_{t+1})]\right\}. \quad (31)$$

A comparison of equations (31) and (20) demonstrates that we apply an essentially identical solution procedure to both models. The only difference between the two equations lies in the timing of the nominal shock ω: this difference reflects the basic assumption that, in the Svensson model, individuals start every period with a predetermined money stock and must acquire money balances for purchases in the future periods.

If the existence condition (13) is satisfied, the proof of theorem 1 (in the Appendix) can be applied to verify that the recursive solution procedure outlined above will also obtain the unique equilibrium. Notice that the condition for the existence of a monetary equilibrium is identical to that for the Lucas model. By contrast, the condition for the cash-in-advance constraint to be binding for a particular s_t is $E_t[\eta_{t+1}^{1-\gamma}]/\omega_t < 1/\delta$. The computation of stock prices follows the steps described above.

V. Simulation Results

We use the consumption series and money stock series described in Section II to compute the growth rates of the endowment (η) and of the money supply (ω). As equations (5) and (9) indicate, the two models require different timing of the processes for the growth rate of money. In the Lucas model, ω_t is the ratio of the stock of money at time t to the stock at time $t - 1$. By contrast, in Svensson's model, ω_t is the ratio of the stock of money at time $t + 1$ to the stock at time t. This difference is reflected in the bivariate autoregressive systems that we estimate. We estimate first-order and second-order autoregressive processes for the two versions of the model over both the full sample and the second postwar sample, and we apply a likelihood ratio test to determine the order of the system: in all cases we fail to reject the hypothesis that the bivariate autoregressions are first-order.

We then fit a Markov process by discretizing the state space (the space of η and ω) using Tauchen's quadrature method.[12] This is a method to construct a discrete probability model that approximates the conditional density function over the state space. The grid points in the state space and the discrete probability weights are chosen

[12] The algorithm was kindly supplied to us by George Tauchen.

optimally,[13] using the method of moments. In Tauchen's procedure the approximation is formed up through moments $2N - 1$.[14] In particular, the law of motion of the simulated discrete process is constructed to approximate closely the estimated law of motion. Tauchen's quadrature method results in conditional transition probabilities and a state-space grid for η and ω. In our simulations there are 64 possible realizations of these two variables.

For both models, using the transition probabilities matrices and the state-space vectors, we then apply the solution algorithm developed in Section IV to solve for the K and ξ functions. In practice, these functions are vectors of size 64 (the elements of the vectors are the functions K and ξ evaluated at the elements of the state-space vectors). The functions are evaluated under different assumptions about the taste parameters: we vary the utility discount rate from 1 percent a year to 3 percent a year and the elasticity parameter from 0.5 to 10. The program implementing our algorithm was written in Gauss. Two facts about the speed of convergence are worth noting: first, convergence on the ξ function was slower than convergence on the K function, and, second, the speed of convergence was insignificantly affected by the value of the utility discount factor β. This is not surprising, however, since as we pointed out above, the condition for the existence of a solution to our functional equation involves β together with the expectation of a function of the forcing variables. This condition is satisfied for all parameter combinations, but convergence tends to be much slower for low values of γ.

Finally, for every combination of taste parameters (a total of 15) and each model, we use the state-space vectors, the functions, and the transition probabilities to generate 100 realizations of the money stock, the endowment, and all endogenous variables. These samples of size 100 are then employed to compute the same statistics as those reported in tables 1, 2, and 3. A subset of those statistics is reported in tables 4–9. We limit ourselves to samples of size 100 since that

[13] The criterion is optimal in the following sense. Let the grid points of an n-point quadrature be $(s_0^n, \ldots, s_n^n)$ (a set of n abscissas). Define an nth-order polynomial of the abscissas as

$$x_n(s) = \sum_{k=0}^{n} \lambda_k^n s^k.$$

The abscissas are chosen so that the polynomials x_n are orthogonal with respect to the conditional density function. A polynomial is orthogonal with weight f when

$$(x_m, x_n) = \int x_m(s)x_n(s)f(s)ds = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n. \end{cases}$$

See Kantorovich and Akilov (1982) for a discussion of the choice of λ.

[14] See Tauchen (1986) for additional details.

is the approximate size of the U.S. data samples. Therefore, any small-sample biases present in the statistics computed over U.S. data should also be present in the statistics computed over simulated data.

A. *Average Returns and Inflation*

Tables 4 and 5 report summary statistics for inflation, velocity, real stock returns, and real interest rates for the two models and the two sample periods. The first column in these tables is the coefficient of variation of velocity. In the simulations, we find that velocity is much less volatile, relative to its average, than in the actual data. This just confirms the findings of Hodrick et al. (1991), who study the Svensson model (and others) over the second postwar period. For the full-sample calibrations, part A of tables 4 and 5 shows that velocity displays greater variability than in the calibrations over the second postwar period. In other words, the liquidity constraint is binding more frequently in the simulations based on the second postwar period. Furthermore, the liquidity constraint is binding more frequently in the Lucas model than in the Svensson model. This difference is consistent with the models' predictions about nominal interest rates. The Lucas model implies that only when the liquidity constraint is binding are nominal interest rates positive. Since the model is calibrated using consumption and money data exclusively and since nominal interest rates are positive in the data, the model's predictions are, in this sense, correct.

The second column of these tables reports simulated means and standard deviations of the inflation rate. Notice that these statistics are largely independent of the values of the preference parameters. While there are differences in the means (ranging from 1 percent to 2 percent), the standard deviations of inflation for the Lucas model closely match those of the U.S. data and those for the Svensson model are slightly higher.

The last three columns in the tables are real returns on stocks, real returns on one-period bonds, and the equity premium. To interpret the results, recall that, in these models, both nominal bonds and stocks are risky assets (in real terms). As we showed in Section IV, the equilibrium expected returns on these assets satisfy the following conditions:

$$1 = \delta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} R_{t+1}\right], \tag{32}$$

$$1 = \delta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} R^n_{t+1}\right], \tag{33}$$

where R is the real return on stocks and R^n the real return on nominal bonds. Similarly, we can derive the rate of return on an indexed bond (the risk-free rate), which equals the reciprocal of the marginal rate of substitution of present and future wealth. Using the definition of R^n in (33) and rearranging, we obtain the familiar asset-pricing equations

$$E_t[R_{t+1}] - R^f_t = -\frac{\operatorname{cov}_t\left(\frac{\lambda_{t+1}}{\lambda_t}, R_{t+1}\right)}{E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]}, \tag{34}$$

$$E_t[R^n_{t+1}] - R^f_t = -(1 + i_t)\frac{\operatorname{cov}_t\left(\frac{\lambda_{t+1}}{\lambda_t}, \frac{\pi_{t+1}}{\pi_t}\right)}{E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]}, \tag{35}$$

where i represents the nominal interest rate. In the Lucas model, the risk-free rate depends only on the distribution of real shocks; in the Svensson model, the marginal utility of wealth and, hence, the risk-free rate are affected by monetary shocks. Stochastic inflation affects risk premia in the Lucas model by affecting payoffs to stocks and bonds and through the covariance of nominal and real disturbances. By contrast, in the Svensson model, nominal shocks change the marginal rate of substitution between present and future wealth even if they are independent of real disturbances.[15]

[15] Stochastic inflation also affects the equilibrium stock prices. In the Lucas model, solving eq. (16) recursively and using (14), we obtain

$$q_t = \frac{1}{y_t^{-\gamma}} E_t \sum_{j=0}^{\infty} \delta^{j+1} y_{t+j+1}^{-\gamma} y_{t+j} \frac{\pi_{t+j+1}}{\pi_{t+j}}.$$

In the Svensson model, solving eq. (30) recursively and substituting for λ_{t+i} from eq. (29), we obtain

$$q_t = \frac{1}{\delta E_t y_{t+1}^{-\gamma}(\pi_{t+1}/\pi_t)} E_t \sum_{i=1}^{\infty} \delta^{i+1}\left(y_{t+i+1}^{-\gamma}\frac{\pi_{t+i+1}}{\pi_{t+i}}\right) y_{t+i}.$$

In both models, real stock prices are affected by the discounted future path of the rate of deflation. As we pointed out above, the difference between the two models lies in the timing of transactions in the money market. This difference is highlighted by the effects of next-period expected inflation on current stock prices. In the Lucas model, inflation levies a direct tax on dividend payments. In particular, expected inflation, other things equal, lowers the expected return on stocks. In the Svensson model, inflation does not affect dividend payments directly, but affects the marginal utility of wealth. In particular, other things equal, expected inflation increases the marginal rate of substitution between present and future wealth (since current wealth is, in part, accounted for by money balances that will be given at the beginning of next period); hence the expected return on stocks is lowered, through an increase in the current stock prices. By contrast, the marginal rate of substitution is unchanged in the Lucas model.

TABLE 4

Sample Means and Standard Deviations Obtained from the Lucas Version of the Cash-in-Advance Model (%)

	Velocity (Coefficient of Variation)	Inflation Rate		Returns: Stock Returns (Real)		Returns: Interest Rate (Real)		Returns: Equity Premium	
				A. Simulations over the Whole Sample					
Data	20.45	2.86	(5.20)	7.34	(16.71)	1.02	(5.56)	6.32	(16.57)
Simulations (ρ, γ):									
(.01, .5)	.63	5.06	(5.63)	2.42	(3.22)	1.25	(5.05)	1.17	(6.05)
(.01, 1)	.66	5.06	(5.64)	3.37	(3.32)	2.34	(5.24)	1.03	(5.01)
(.01, 2)	.78	5.07	(5.68)	5.26	(4.86)	4.43	(5.89)	.83	(3.30)
(.01, 5)	1.31	5.09	(5.99)	10.81	(11.81)	10.33	(9.23)	.49	(4.36)
(.01, 10)	2.90	5.21	(7.45)	19.08	(23.87)	18.66	(17.10)	.42	(11.86)
(.03, .5)	.19	5.06	(5.50)	4.64	(3.19)	3.41	(4.89)	1.23	(5.76)
(.03, 1)	.23	5.06	(5.51)	5.62	(3.38)	4.50	(5.10)	1.12	(4.76)
(.03, 2)	.36	5.06	(5.54)	7.57	(4.97)	6.64	(5.77)	.94	(3.08)
(.03, 5)	.89	5.07	(5.72)	13.31	(11.89)	12.69	(9.18)	.62	(4.27)
(.03, 10)	2.26	5.14	(6.80)	21.92	(23.97)	21.26	(17.15)	.67	(11.90)
(.05, .5)	.00	5.06	(5.50)	7.05	(3.74)	5.88	(5.10)	1.17	(5.84)
(.05, 1)	.00	5.06	(5.51)	3.54	(3.87)	2.49	(5.47)	1.05	(5.17)

(.05, 2)	.00	5.06	(5.54)	5.20	(5.22)	4.16	(6.26)	1.04	(3.62)
(.05, 5)	.52	5.07	(5.72)	9.62	(12.09)	8.43	(10.07)	1.18	(3.79)
(.05, 10)	1.77	5.14	(6.80)	14.36	(24.82)	12.45	(19.28)	1.91	(10.55)
	B. Simulations over the Second Postwar Sample								
Data	11.49	4.43	(3.02)	7.83	(14.64)	.23	(3.20)	7.60	(13.87)
Simulations (ρ, γ):									
(.01, .5)	.21	5.36	(3.53)	2.04	(1.59)	1.43	(2.64)	.62	(3.25)
(.01, 1)	.05	5.35	(3.49)	3.00	(1.42)	2.36	(2.58)	.64	(2.94)
(.01, 2)	.00	5.35	(3.48)	4.93	(1.22)	4.24	(2.62)	.69	(2.50)
(.01, 5)	.00	5.35	(3.48)	10.80	(1.83)	9.92	(2.91)	.88	(1.51)
(.01, 10)	.00	5.35	(3.48)	20.77	(4.02)	19.59	(3.84)	1.18	(1.22)
(.03, .5)	.00	5.35	(3.48)	4.28	(1.56)	3.55	(2.55)	.72	(3.05)
(.03, 1)	.00	5.35	(3.48)	5.26	(1.42)	4.50	(2.57)	.75	(2.84)
(.03, 2)	.00	5.35	(3.48)	7.23	(1.27)	6.41	(2.63)	.82	(2.44)
(.03, 5)	.00	5.35	(3.48)	13.23	(1.93)	12.20	(2.94)	1.04	(1.52)
(.03, 10)	.00	5.35	(3.48)	23.43	(4.10)	22.06	(3.90)	1.36	(1.25)
(.05, .5)	.00	5.35	(3.48)	6.51	(1.55)	5.67	(2.56)	.84	(2.97)
(.05, 1)	.00	5.35	(3.48)	7.52	(1.43)	6.64	(2.58)	.88	(2.77)
(.05, 2)	.00	5.35	(3.48)	9.54	(1.33)	8.58	(2.64)	.96	(2.40)
(.05, 5)	.00	5.35	(3.48)	15.67	(2.03)	14.48	(2.98)	1.19	(1.54)
(.05, 10)	.00	5.35	(3.48)	26.08	(4.20)	24.54	(3.97)	1.54	(1.30)

NOTE.—$\rho = (1/\delta) - 1$. Standard deviations are in parentheses.

TABLE 5

SAMPLE MEANS AND STANDARD DEVIATIONS OBTAINED FROM THE SVENSSON VERSION OF THE CASH-IN-ADVANCE MODEL (%)

	Velocity (Coefficient of Variation)	Inflation Rate	Returns: Stock Returns (Real)	Returns: Interest Rate (Real)	Returns: Equity Premium
			A. Simulations over the Whole Sample		
Data	20.45	2.86 (5.20)	7.34 (16.71)	1.02 (5.56)	6.32 (16.57)
Simulations (ρ, γ):					
(.01, .5)	2.62	4.61 (6.12)	2.67 (5.71)	1.44 (3.77)	1.23 (6.99)
(.01, 1)	2.64	4.61 (6.12)	3.55 (4.90)	2.45 (3.67)	1.10 (6.14)
(.01, 2)	2.74	4.61 (6.15)	5.30 (3.60)	4.46 (3.67)	.85 (4.58)
(.01, 5)	3.54	4.63 (6.46)	10.35 (4.64)	10.22 (5.32)	.12 (1.56)
(.01, 10)	6.12	4.80 (8.19)	17.69 (12.66)	18.80 (11.18)	−1.11 (5.42)
(.03, .5)	1.16	4.60 (6.02)	4.71 (5.59)	3.65 (3.20)	1.05 (6.04)
(.03, 1)	1.24	4.60 (6.01)	5.62 (4.83)	4.69 (3.08)	.94 (5.26)
(.03, 2)	1.44	4.60 (6.02)	7.44 (3.58)	6.73 (3.05)	.71 (3.80)
(.03, 5)	2.37	4.61 (6.15)	12.67 (4.41)	12.64 (4.63)	.03 (1.23)
(.03, 10)	4.91	4.70 (7.31)	20.32 (12.14)	21.49 (10.41)	−1.17 (5.61)
(.05, .5)	.58	4.60 (6.05)	6.77 (5.50)	5.80 (3.08)	.97 (5.64)
(.05, 1)	.62	4.60 (6.05)	7.71 (4.75)	6.85 (2.92)	.86 (4.86)

(.05, 2)	.75	4.60	(6.04)	9.58	(3.53)	8.94	(2.81)	.64	(3.41)
(.05, 5)	1.54	4.60	(6.05)	14.97	(4.36)	15.00	(4.31)	−.03	(1.09)
(.05, 10)	3.82	4.65	(6.75)	22.91	(11.79)	24.14	(9.89)	−1.23	(5.87)
	B. Simulations over the Second Postwar Sample								
Data	11.49	4.43	(3.02)	7.83	(14.64)	.23	(3.20)	7.60	(13.87)
Simulations (ρ, γ):									
(.01, .5)	1.17	5.47	(4.33)	2.30	(3.61)	1.61	(2.00)	.69	(3.47)
(.01, 1)	1.02	5.47	(4.32)	3.21	(3.17)	2.62	(1.85)	.59	(2.96)
(.01, 2)	.75	5.47	(4.32)	5.04	(2.44)	4.64	(1.61)	.41	(2.01)
(.01, 5)	.47	5.47	(4.31)	10.73	(2.13)	10.77	(1.65)	−.04	(.76)
(.01, 10)	.21	5.47	(4.30)	20.63	(5.12)	21.23	(3.51)	−.60	(3.49)
(.03, .5)	.49	5.47	(4.30)	4.38	(3.42)	3.75	(1.77)	.62	(3.06)
(.03, 1)	.45	5.47	(4.31)	5.31	(3.02)	4.76	(1.65)	.54	(2.62)
(.03, 2)	.38	5.47	(4.30)	7.20	(2.38)	6.83	(1.47)	.38	(1.81)
(.03, 5)	.20	5.47	(4.30)	13.04	(2.16)	13.07	(1.67)	−.03	(7.48)
(.03, 10)	.00	5.47	(4.31)	23.16	(5.12)	23.74	(3.65)	−.58	(3.41)
(.05, .5)	.21	5.47	(4.30)	6.47	(3.33)	5.88	(1.69)	.59	(2.88)
(.05, 1)	.18	5.47	(4.30)	7.43	(2.97)	6.92	(1.59)	.51	(2.47)
(.05, 2)	.11	5.47	(4.30)	9.37	(2.37)	9.01	(1.44)	.36	(1.70)
(.05, 5)	.00	5.47	(4.31)	15.34	(2.21)	15.37	(4.31)	−.03	(.74)
(.05, 10)	.00	5.47	(4.31)	25.69	(5.15)	26.25	(3.78)	−.56	(3.32)

NOTE.—$\rho = (1/\delta) - 1$. Standard deviations are in parentheses.

The estimated equity premia in part A of table 4 range from 0.42 percent to 1.91 percent. Simulating over the second postwar sample (pt. B), we find that equity premia are virtually unchanged, ranging from 0.62 percent to 1.54 percent. These values are much larger than those reported by Mehra and Prescott (1985) but fall well below the data (the tables report the U.S. statistics in the first row). Table 5 shows that equity premia can be negative in the Svensson model.[16] They range from -1.23 percent to 1.23 percent.

Equations (34) and (35) indicate that the equity premium ($R - R^n$) is "small" when the risk premia on stocks ($R - R^f$) and bonds ($R^n - R^f$) are of comparable size, either large or small. To study the relation between the measured equity premia and the risk premia, we compute both premia choosing $\gamma = 2$, a utility discount factor equal to 3 percent, and the processes for the forcing variables corresponding to the full sample.[17] Figures 1 and 2 plot the ex ante real returns on stocks, nominal bonds, and indexed bonds in the Lucas and Svensson models. The figures show that the expected returns on the three assets are nearly identical. To highlight the behavior of risk premia, we plot them in figures 3 and 4. The striking implication of the four figures is that the only important factor in the fluctuation of ex ante asset returns appears to be the marginal rate of substitution. Fluctuations in expected returns generated by these models are generally not due to fluctuations in risk premia. Shiller (1982), among others, has emphasized that asset-pricing models like those we study need to explain the wide fluctuations in ex ante returns of different assets (see also Barsky 1986). Our results confirm the observation of Weil (1988), who simulates a model without money and concludes that the "equity premium puzzle" is really a puzzle about the differences between the behavior of real interest rates and the marginal rates of substitution generated by asset-pricing models (the "risk-free rate puzzle"). The mean equity returns and real interest rates reported in tables 4 and 5 illustrate this observation. For example, for the Lucas model over the full sample (pt. A of table 4) with $\gamma = 2$ and the utility discount factor equal to 3 percent, the mean of the realized real stock return is 7.57 percent, and the actual figure is within one standard deviation of the simulated mean (the standard deviation of the mean, not reported in the table, is 0.499 percent); by contrast, the mean of the realized real interest rate is 6.64 percent, but the actual figure is 1.02 percent and the standard deviation of the simulated mean is 0.58 percent.

[16] This does not occur, as eqq. (34) and (35) demonstrate, when the equity premium is computed using indexed bonds (see Labadie 1989).

[17] Other parameter combinations do not change the evidence in any appreciable way.

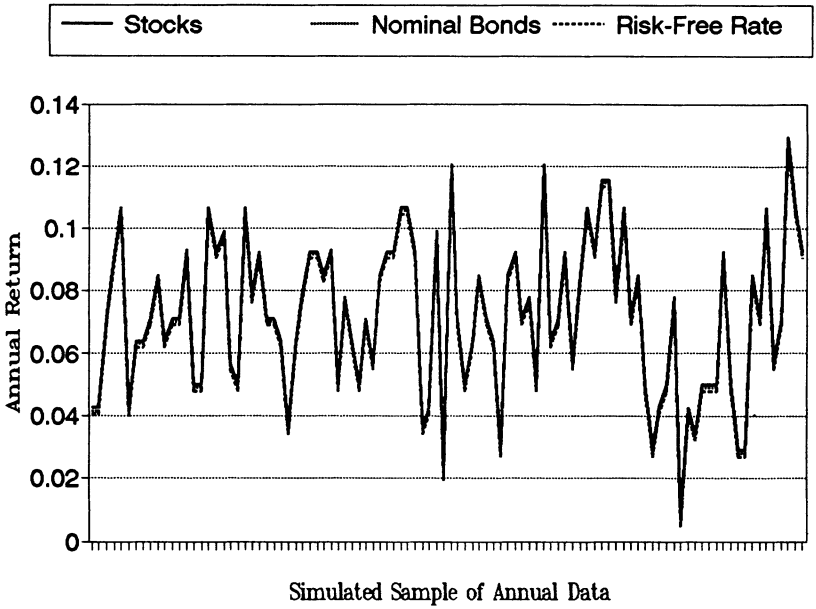

FIG. 1.—Expected returns on stocks, nominal bonds, and real bonds in the Lucas model.

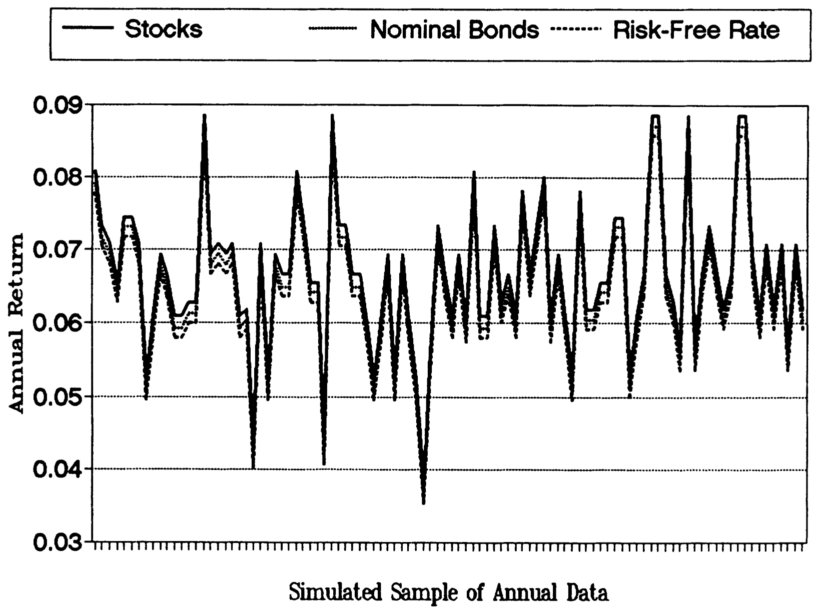

FIG. 2.—Expected returns on stocks, nominal bonds, and real bonds in the Svensson model.

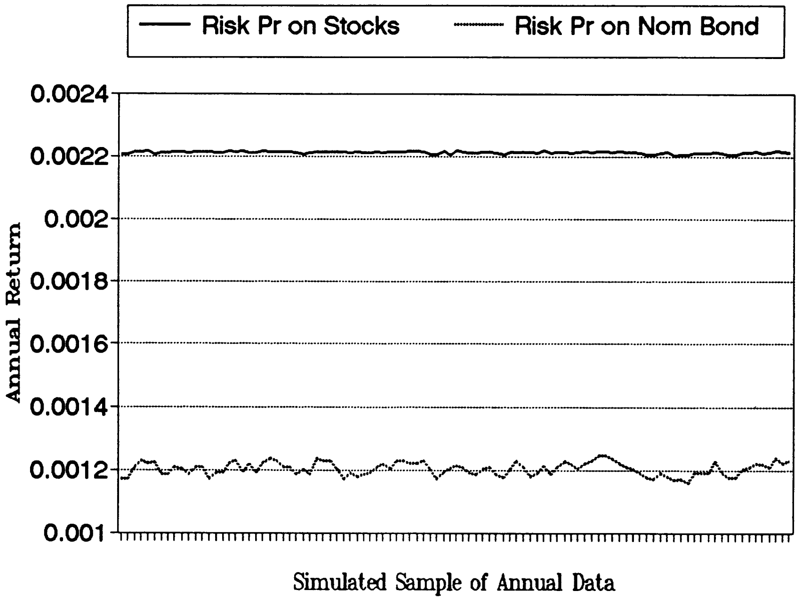

FIG. 3.—Risk premia in the Lucas model

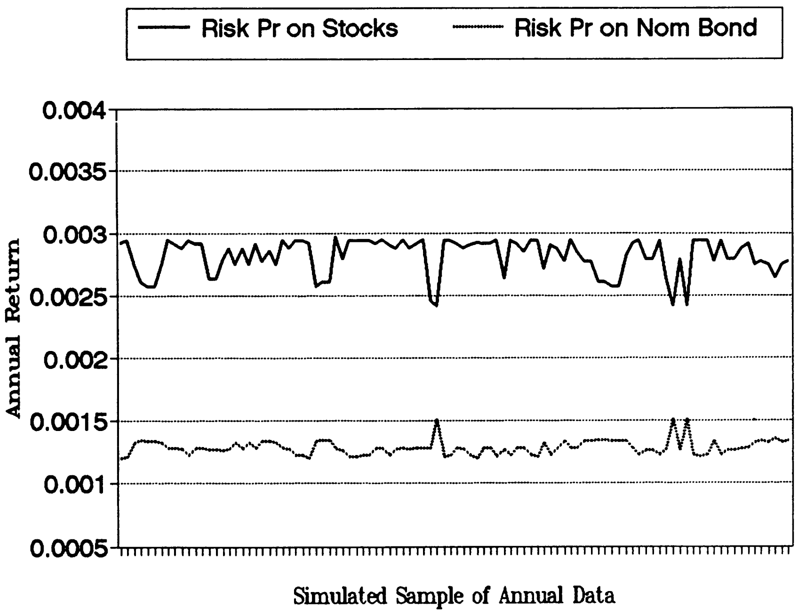

FIG. 4.—Risk premia in the Svensson model

B. *Inflation, Stock Returns, and Real Interest Rates*

Tables 6 and 7 contain the regression coefficients of realized real returns on stocks and one-period bonds on contemporaneous inflation. The slope coefficient of stock returns on inflation is equal to, by definition, the covariance between ex ante real stock returns and expected inflation, plus the covariance between innovations in stock returns and inflation innovations, divided by the variance of inflation. Stocks are good "inflation hedges" whenever the covariance between innovations in stock returns and inflation innovations is either positive or zero. This occurs in the Lucas model, when simulated using postwar data, and in the Svensson model, for high values of γ. The results of the simulations in the Lucas model contrast with the U.S. data, where the negative relation between realized real stock returns and inflation is stronger, especially in the second postwar period. In table 7 the positive relation between inflation and real stock returns observed in the simulations of the Svensson model corresponds to the cases in which the equity premium becomes negative, explaining in part that phenomenon. Since stocks are good hedges against inflation, they command a very small or negative inflation premium, and, as a consequence, their ex ante returns closely match the ex ante real returns on nominal bonds. Our results should be compared with those reported by LeRoy (1984*b*) and Danthine and Donaldson (1986). They discuss the covariations of stock returns and inflation in models in which there are only endowment shocks, that is, in which the supply of money balances is constant.[18] Hence fluctuations in inflation are due, loosely speaking, to fluctuations in money demand. These authors conclude that the negative correlation between real stock returns and inflation can be reproduced by their general equilibrium models. Their results are explained by the effects described in Fama (1981): if the nominal money supply is constant, real activity tends to be positively correlated with stock returns but negatively correlated with inflation. Hence the negative correlation between stock returns and inflation. By contrast, taking explicitly into account fluctuations in money *supply*—and their covariation with consumption—that resemble those in the U.S. economy, we find that general equilibrium models fail to produce these results. This suggests the important role of money supply volatility.

The last two columns in tables 6 and 7 contain the estimated covariations between ex post real interest rates and inflation. By definition

[18] Both papers assume that money enters the representative agent's utility function and do not simulate the model using actual data on the forcing variables' processes, as we do.

TABLE 6

COMOVEMENTS OF INFLATION AND EX POST REAL STOCK RETURNS AND REAL INTEREST RATES OBTAINED FROM THE LUCAS VERSION OF THE CASH-IN-ADVANCE MODEL

	Real Stock Returns		Real Interest Rate	
	Slope Coefficient	R^2	Slope Coefficient	R^2
	A. Simulations over the Whole Sample			
Data	−.78	.06	−.93	.76
Simulations (ρ, γ):				
(.01, .5)	−.02	.00	−.72	.65
(.01, 1)	−.11	.03	−.69	.56
(.01, 2)	−.35	.17	−.64	.38
(.01, 5)	−.96	.24	−.52	.11
(.01, 10)	−1.60	.25	−.61	.07
(.03, .5)	.01	.00	−.70	.62
(.03, 1)	−.11	.03	−.67	.52
(.03, 2)	−.35	.15	−.60	.34
(.03, 5)	−.95	.21	−.44	.08
(.03, 10)	−1.63	.21	−.46	.03
(.05, .5)	.01	.00	−.69	.60
(.05, 1)	−.11	.03	−.65	.50
(.05, 2)	−.33	.13	−.57	.30
(.05, 5)	−.93	.19	−.38	.05
(.05, 10)	−1.65	.18	−.30	.01
	B. Simulations over the Second Postwar Sample			
Data	−2.48	.26	−.45	.18
Simulations (ρ, γ):				
(.01, .5)	.02	.00	−.47	.40
(.01, 1)	.00	.00	−.43	.34
(.01, 2)	−.00	.00	−.40	.29
(.01, 5)	.01	.00	−.31	.13
(.01, 10)	.11	.01	−.13	.01
(.03, .5)	.02	.00	−.43	.35
(.03, 1)	.01	.00	−.42	.33
(.03, 2)	.01	.00	−.39	.27
(.03, 5)	.03	.00	−.29	.12
(.03, 10)	.16	.02	−.11	.01
(.05, .5)	.03	.00	−.43	.34
(.05, 1)	.02	.00	−.41	.31
(.05, 2)	.03	.00	−.38	.25
(.05, 5)	.06	.01	−.28	.11
(.05, 10)	.20	.03	−.09	.01

NOTE.—$\rho = (1/\delta) - 1$.

TABLE 7

COMOVEMENTS OF INFLATION AND EX POST REAL STOCK RETURNS AND REAL INTEREST RATES OBTAINED FROM THE SVENSSON VERSION OF THE CASH-IN-ADVANCE MODEL

	Real Stock Returns		Real Interest Rate	
	Slope Coefficient	R^2	Slope Coefficient	R^2
	A. Simulations over the Whole Sample			
Data	−.78	.06	−.93	.76
Simulations (ρ, γ):				
(.01, .5)	−.12	.02	−.51	.69
(.01, 1)	−.12	.02	−.46	.59
(.01, 2)	−.11	.03	−.36	.37
(.01, 5)	−.12	.03	−.16	.04
(.01, 10)	−.52	.11	−.31	.05
(.03, .5)	−.22	.06	−.42	.64
(.03, 1)	−.19	.05	−.37	.52
(.03, 2)	−.13	.04	−.26	.26
(.03, 5)	.01	.00	.03	.00
(.03, 10)	.28	.03	−.01	.00
(.05, .5)	−.25	.08	−.40	.62
(.05, 1)	−.21	.07	−.34	.49
(.05, 2)	−.13	.05	−.21	.21
(.05, 5)	.09	.01	.14	.04
(.05, 10)	.01	.00	.30	.04
	B. Simulations over the Second Postwar Sample			
Data	−2.48	.26	−.45	.18
Simulations (ρ, γ):				
(.01, .5)	−.32	.15	−.38	.67
(.01, 1)	−.30	.16	−.33	.59
(.01, 2)	−.24	.17	−.23	.37
(.01, 5)	−.01	.00	.07	.03
(.01, 10)	−.46	.15	.61	.56
(.03, .5)	−.33	.17	−.33	.66
(.03, 1)	−.30	.18	−.29	.56
(.03, 2)	−.22	.17	−.19	.32
(.03, 5)	.02	.00	.10	.07
(.03, 10)	.50	.18	.64	.57
(.05, .5)	−.33	.18	−.31	.63
(.05, 1)	−.29	.18	−.27	.53
(.05, 2)	−.22	.16	−.17	.27
(.05, 5)	.03	.00	.12	.09
(.05, 10)	.53	.20	.67	.58

NOTE.—$\rho = (1/\delta) - 1$.

these are equal to the covariance between the ex ante real interest rate and expected inflation, minus the variance of the inflation innovation, divided by the variance of inflation. If the covariance between ex ante real rates and expected inflation is positive and does not exceed the variance of inflation innovations, the estimates in the last two columns of tables 6 and 7 are negative. This occurs in both models, in the majority of cases.

C. *Interest Rates as Predictors of Inflation and Nominal Stock Returns*

Finally, tables 8 and 9 contain regressions of inflation on nominal interest rates and of realized nominal stock returns on nominal interest rates. The coefficient of the interest rates in the inflation equation equals the covariance between the ex ante real interest rate and expected inflation, plus the variance of expected inflation, divided by the variance of the nominal interest rate. It tends to one whenever the variance of expected inflation accounts for the largest fraction of the total variation of the nominal interest rate; that is, the variance of the real rate is small (hence its covariance with the expected rate of inflation is also small). In the data, it appears that if the rational expectations hypothesis is true, either the variance of the real interest rate is significant or the covariance between the ex ante real interest rate and expected inflation is negative, or both propositions are true. In the simulations, most regression coefficients tend to cluster around the values estimated by Fama (1975) and others using data from the early 1950s until the late 1970s. As figures 1 and 2 indicate, however, ex ante real interest rates vary significantly. The estimated coefficients are then due to very low correlations between ex ante real interest rates and expected inflation.

Finally, probably the most striking results of our simulations appear in the regressions of *ex post* nominal stock returns on the nominal interest rate. We find the nominal interest rate to be an extremely good predictor of subsequent nominal stock returns. All estimated coefficients are very close to unity, and, surprisingly, the R^2 statistics in the regressions are as high as 97 percent. These results just confirm our observations above: most of the covariation between interest rates and stock returns is driven by their common factor: the reciprocal of the marginal rate of substitution of wealth (the risk-free real interest rate). Any time-varying risk premia between the two financial assets are far too small to explain the data.

TABLE 8

NOMINAL INTEREST RATES AS PREDICTORS OF INFLATION AND NOMINAL STOCK RETURNS IN THE LUCAS VERSION OF THE CASH-IN-ADVANCE MODEL

	INFLATION		NOMINAL STOCK RETURNS	
	Slope Coefficient	R^2	Slope Coefficient	R^2
	A. Simulations over the Whole Sample			
Data	.24	.02	−.20	.00
Simulations (ρ, γ):				
(.01, .5)	.77	.21	.80	.17
(.01, 1)	.64	.20	.86	.31
(.01, 2)	.45	.16	.93	.68
(.01, 5)	.21	.10	1.02	.82
(.01, 10)	.08	.03	1.05	.69
(.03, .5)	.77	.23	.84	.20
(.03, 1)	.64	.21	.89	.36
(.03, 2)	.45	.18	.96	.72
(.03, 5)	.21	.12	1.03	.84
(.03, 10)	.08	.04	1.05	.70
(.05, .5)	.76	.23	.86	.23
(.05, 1)	.63	.22	.91	.39
(.05, 2)	.45	.19	.89	.76
(.05, 5)	.21	.13	1.04	.85
(.05, 10)	.08	.06	1.06	.71
	B. Simulations over the Second Postwar Sample			
Data	.45	.25	−.13	.00
Simulations (ρ, γ):				
(.01, .5)	.86	.45	.82	.33
(.01, 1)	.83	.47	.84	.41
(.01, 2)	.79	.47	.89	.54
(.01, 5)	.64	.44	1.01	.86
(.01, 10)	.45	.39	1.12	.96
(.03, .5)	.85	.48	.85	.39
(.03, 1)	.83	.48	.88	.45
(.03, 2)	.77	.47	.92	.58
(.03, 5)	.63	.44	1.03	.86
(.03, 10)	.44	.39	1.13	.97
(.05, .5)	.84	.48	.88	.43
(.05, 1)	.81	.48	.90	.75
(.05, 2)	.76	.47	.94	.60
(.05, 5)	.61	.44	1.04	.87
(.05, 10)	.43	.39	1.14	.97

NOTE.—$\rho = (1/\delta) - 1$.

TABLE 9

NOMINAL INTEREST RATES AS PREDICTORS OF INFLATION AND NOMINAL STOCK RETURNS IN THE SVENSSON VERSION OF THE CASH-IN-ADVANCE MODEL

	Inflation		Nominal Stock Returns	
	Slope Coefficient	R^2	Slope Coefficient	R^2
	A. Simulations over the Whole Sample			
Data	.24	.02	−.20	.00
Simulations (ρ, γ):				
(.01, .5)	1.37	.67	.96	.20
(.01, 1)	1.23	.66	.96	.29
(.01, 2)	1.01	.64	.96	.51
(.01, 5)	.62	.52	.95	.96
(.01, 10)	.31	.21	.93	.82
(.03, .5)	1.33	.76	.98	.29
(.03, 1)	1.20	.76	.98	.40
(.03, 2)	1.00	.74	.97	.64
(.03, 5)	.63	.65	.96	.98
(.03, 10)	.33	.32	.94	.82
(.05, .5)	1.31	.79	.99	.34
(.05, 1)	1.19	.79	.99	.46
(.05, 2)	1.00	.78	.98	.71
(.05, 5)	.64	.73	.97	.98
(.05, 10)	.35	.45	.94	.82
	B. Simulations over the Second Postwar Sample			
Data	.45	.25	−.13	.00
Simulations (ρ, γ):				
(.01, .5)	1.36	.85	.95	.39
(.01, 1)	1.28	.86	.96	.51
(.01, 2)	1.13	.87	.96	.75
(.01, 5)	.83	.89	.96	.98
(.01, 10)	.56	.90	.96	.80
(.03, .5)	1.33	.89	.97	.48
(.03, 1)	1.25	.89	.97	.59
(.03, 2)	1.11	.89	.97	.79
(.03, 5)	.82	.90	.96	.98
(.03, 10)	.55	.90	.97	.82
(.05, .5)	1.30	.89	.97	.52
(.05, 1)	1.22	.89	.97	.63
(.05, 2)	1.08	.89	.97	.82
(.05, 5)	.80	.89	.97	.98
(.05, 10)	.54	.90	.97	.83

NOTE.—$\rho = (1/\delta) - 1$

D. Alternative Specifications

The results may be sensitive to the choice of Tauchen's method to discretize the state space. To check this we construct transition probabilities with an alternative method. Using the estimates of the vector autoregression (VAR) coefficients and residuals, we generate, via a bootstrapping technique (Efron 1982), a sample of 3,000 realizations of (η, ω). Unlike Tauchen, we construct a grid of equally spaced values of the forcing variables based on the simulated series of 3,000 elements. Transition probabilities are then obtained by computing the frequency with which the process moves from state i to state j.

This alternative method does not alter our main results. For example, when we simulate the Lucas model over the full sample with $\gamma = 2$ and the utility discount factor equal to 3 percent (see pt. A of table 4), the mean of real stock returns is 7.28 percent (compared to 7.57 percent under the Tauchen method), and the mean of the real interest rate is 6.56 percent (compared to 6.64 percent under the Tauchen method). Therefore, the mean equity premium is still small.[19] One different result appears in the inflation rate. When the bootstrapping method is used to simulate the model, the mean of inflation is 2.97 percent; the Tauchen method results in an inflation rate of 5.06 percent. Our interpretation of this discrepancy is that the bootstrapping method gives more weight to large and infrequent realizations of η and ω, like those associated with the Depression, which might be smoothed out by fitting normal distributions.[20] The regression of inflation on the nominal interest rate gives a slope coefficient of .38 (compared to .45 under the Tauchen method). The regression of the stock return on the interest rate gives a slope coefficient of .96 (identical to that obtained under Tauchen's method).

A second alternative specification we explore entails an increase in the state space. We add to the exogenous processes a variable that under the assumptions of the model is endogenous and check whether the results are significantly affected. If the model were strictly true, adding to the state space an endogenous variable would not affect the results since that endogenous variable does not expand the information set.

The variable chosen is the change in the nominal interest rate.[21] We estimate a first-order VAR that includes money growth, consumption

[19] The standard deviations for these variables are also quite similar for both methods.

[20] When large negative realizations of ω are allowed for, the velocity function is much more variable even in the Lucas model, and excess cash holdings are more frequent; hence the lower realized rate of inflation.

[21] The choice of the *change* in the nominal interest rate was dictated by the failure of a VAR that includes the level of interest rates to produce a stationary covariance matrix.

growth, and the change in the nominal interest rate. We then compute transition probabilities with Tauchen's method, assuming a state space that admits eight realizations of η, eight realizations of ω, and four realizations of the change in the interest rate, for a total of 256 possible states.

Adding a forcing variable alters the simulation results in a number of ways. First, the algorithm does not converge for several combinations of the parameters.[22] Adding an additional state variable changes the transition probabilities in such a way that the condition for convergence of the algorithm presented above is no longer satisfied. Second, in the cases in which the algorithm converges, some of the simulated results differ. For example, when we simulate as above the Lucas model over the full sample with $\gamma = 2$ and the utility discount factor equal to 3 percent, the mean of real stock returns is 9.95 percent, the mean of the real interest rate is 9.68 percent, and hence the mean equity premium is even smaller than that reported earlier. The mean of inflation is 1.68 percent. The regression of inflation on the nominal interest rate gives a slope coefficient of $-.10$.

In general, the results of these experiments (which we do not report fully to economize space) suggest that changing the state space or the calibration method for the Markov processes does not change the central result of this paper: in cash-in-advance models the comovement between bond and stock returns is much higher than in the U.S. data.

Given these negative results, questions are raised about the specification of preferences and trading structure. Hansen and Jagannathan (1991) propose a test that exploits exclusively the first-order conditions of our models but does not depend on the assumption that the dividend process equals the consumption process. This test amounts to verifying an inequality that involves moments of the marginal rate of substitution of wealth and the rate of return on any asset.[23] To perform the test, we compute unconditional population moments for the marginal rate of substitution of wealth, $\delta\lambda_{t+1}/\lambda_t$, and

[22] In particular, convergence is not achieved for the high and the low values of γ.

[23] From eq. (32),

$$\operatorname{cov}\left(\delta\frac{\lambda_{t+1}}{\lambda_t}, R_{t+1}\right) = 1 - E\left[\delta\frac{\lambda_{t+1}}{\lambda_t}\right]E[R_{t+1}].$$

Applying the triangular inequality and after some manipulation, Hansen and Jagannathan (1991) obtain

$$\frac{\mathrm{SD}\left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right)}{E\left[\delta\frac{\lambda_{t+1}}{\lambda_t}\right]} \geq \frac{\left|ER_{t+1} - \frac{1}{E[\delta(\lambda_{t+1}/\lambda_t)]}\right|}{\mathrm{SD}(R_{t+1})},$$

where SD indicates standard deviation.

use the sample moments for the real rate of return on stocks obtained from the data and reported in tables 4 and 5. For both models and under all parameter combinations considered above, this inequality is not satisfied.[24] The result indicates that the empirical failures of the models do not depend exclusively on our equilibrium assumptions.

VI. Concluding Remarks

We have calibrated two standard cash-in-advance models to U.S. data to determine their ability to reproduce important empirical regularities affecting asset returns and inflation. The most important result is that the models predict a very high covariation between ex ante returns on stocks and nominal bonds. Accounting for money supply uncertainty does not add significantly to the variation of conditional risk premia. In the data generated by the model, the equity premium is only a small fraction of that observed in the U.S. data. This is a consequence of the high covariation of ex ante returns characterizing the models. This feature also explains why nominal interest rates implied by the models turn out to predict nominal stock returns extremely well. Finally, we find that the real returns on stocks are only occasionally negatively related to inflation, as they are in the data (especially in the more recent years), and that ex ante real interest rates are uncorrelated with expected inflation.

Since we consider a representative-agent economy and, by the cash-in-advance constraint, allow for a relatively large and stable inflation-tax base, our results suggest that the pure inflation-tax effects, measured by the area under the money demand function, are likely to play a minor role in explaining the empirical regularities illustrated here. This suggests two possible extensions of the type of analysis we undertook in this paper, only one of which we believe is feasible given the current state of knowledge of solution methods of stochastic general equilibrium models. The first should aim at exploring the interactions between nonneutralities in the system of taxation (especially taxation of firms) and inflation. The second extension could allow for heterogeneous agents and incomplete insurance markets. These extensions might improve our understanding of phenomena that, to date, we are still unable to explain fully.

Appendix

Proof of Theorem 1

We shall show first that H_1 takes bounded continuous functions into bounded continuous functions and, second, that H_1^n is a contraction.

[24] Results of these tests are not reported but are available from us on request.

Let $s_t = (\eta_t, \omega_t)$ and let S denote the state space of all the elements s_t. Assume that the shocks to the system form a Markov process with a stationary transition function Q. Then S and Q satisfy the following assumption.

ASSUMPTION 1. S is compact and Q has the Feller property.

This assumption is needed to ensure that the expectations operator maps continuous bounded functions into continuous bounded functions.

Both T and S_1 are linear operators. If $K^0 \in \zeta$, then S_1K^0 is an element of ζ since E is a continuous linear operator. The operator T is bounded, and because a linear operator is bounded if and only if it is continuous (Luenberger 1969, p. 144), it is also continuous. This establishes that H_1 takes bounded continuous functions into bounded continuous functions.

The next step is to verify Blackwell's sufficient conditions (monotonicity and the discounting property) for a contraction mapping. For notational convenience, let x_t denote $\eta_{t+1}^{1-\gamma}/\omega_{t+1}$. To determine whether the composite operator is monotone, notice that, for any $f > g$, $S_1f(s) > S_1g(s)$ for all s. When T is applied,

$$(T \cdot S_1)f(s_t) = \max\{1, S_1f(s_{t+1})\} \geq (T \cdot S_1)g(s_t) = \max\{1, S_1g(s_{t+1})\};$$

hence the composite operator is monotone.

To determine whether or not the composite mapping has the discounting property, notice that, because $\delta E_t x_{t+1}$ may exceed unity, application of the composite operator to an arbitrary function in ζ generally will not have the discounting property because

$$\begin{aligned} H_1(f + a)(s_t) &= \max\{1, S_1(f + a)(s_t)\} = \max\{1, \delta E_t x_{t+1}[f(s_{t+1}) + a]\} \\ &\leq \max\{1, \delta E_t x_{t+1} f(s_{t+1})\} + \max\{1, a\delta E_t x_{t+1}\}, \end{aligned}$$

where the coefficient for a in the last term may exceed unity. Since x is by assumption a stationary process, there exists a j_t such that $\delta E_t x_{t+j_t} < 1$. Define N_t such that

$$\delta^{N_t} E_t \prod_{\iota=1}^{N_t} x_{t+\iota} < 1.$$

Let N denote the maximum over the N_t. Start with an initial guess $K^0 \in \zeta$ and define

$$\begin{aligned} K^1(s_t) &= H_1K^0(s_t) = (T \cdot S_1)K^0(s_t) \\ &= \max\{1, \delta E_t x_{t+1}[K^0(s_{t+1})]\}. \end{aligned}$$

When H_1 is applied again,

$$\begin{aligned} H_1K^1(s_t) &= \max\{1, \delta E_t x_{t+1}[K^1(s_{t+1})]\} \\ &= \max\{1, \delta E_t x_{t+1} T[\delta E_{t+1} x_{t+2} K^0(s_{t+2})]\}. \end{aligned}$$

When H_1 is applied M times,

$$\begin{aligned} H_1^M K^0(s_t) &= (T \cdot S_1)^M K^0(s_t) = \max\{1, \delta E_t x_{t+1}[K^{M-1}(s_{t+1})]\} \\ &= (T \cdot S_1)^{M-1} \max\{1, \delta E_t x_{t+1}[K^0(s_{t+1})]\}. \end{aligned}$$

To determine whether H_1^M has the discounting property, define

$$\begin{aligned} H_1^M(K^0 + a)(s_t) &= (T \cdot S_1)^M (K^0 + a)(s_t) \\ &= (T \cdot S_1)^{M-1} \max\{1, \delta E_t x_{t+1}[K^0(s_{t+1}) + a]\} \end{aligned}$$

$$\leq (T \cdot S_1)^{M-1} \max\{1, \delta E_t x_{t+1}[K^0(s_{t+1})]\} + \left(\delta^N E_t \prod_{i=1}^{N} x_{t+i}\right)a.$$

If $M > N$, then $(\delta^N E_t \Pi_{i=1}^{N} x_{t+i})a = \beta a$, where $0 < \beta < 1$, and hence H_1^M has the discounting property. By theorem 2 of Luenberger (1969, p. 275), if H_1^M is a continuous mapping from a closed subset of a Banach space into itself and if H_1^M is a contraction for some positive integer M, then the fixed point of H_1 can be found by the method of successive approximations. Q.E.D.

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