Global, Fiscal and Monetary Uncertainty Shocks: What Matters for Small Open Economies?

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Paper: Coming soon on

github.com/phantomachine/fiscal-global-uncertainty/

Horses for Courses

This paper is about **theory-disciplined** identification of **shocks** ...

... about *quantitative* economic **interpretation** and **business-cycle accounting** of such shocks

***not* about prediction**

Definition

Structural shock uncertainty

Uncertainty Shock definition

Unexpected changes in the (co-)variance of underlying *economic-structural* shocks.

- "Uncertainty" is from perspective of modeller/statistician (not agents).
- "Risk" from Model agents' perspective: Know underlying probability model, including that of random covariance shocks.

Problem Statement

What accounts for *uncertainty shocks* in a **small open economy**'s business cycle?

Shocks to riskiness of distribution over ***foreign economic policy/activity*** and/or ***domestic monetary/fiscal policy***?

i.e., "The *world* economy, stupid"[a] or "Blame it on our politicians"?

[a] With apologies to James Carville

Why important?

- Matters for (monetary) policy design
- How much should policy makers worry about responding to external factors?
- Or to domestic fiscal stance?

What We Do

Structural time-series modelling approach

Measurement and Inference with Theory

Incomplete-markets small-open-economy DSGE

1. fiscal and monetary policy
   - theory-identified "structural shocks"
2. ***stochastic volatility*** $\equiv$ "Uncertainty shocks"
   - $\sigma_{Y^*,t}$ in *exogenous* **world output** $(Y_t^*)_{t\in\mathbb{N}}$ via shocks $\left(\sigma_{\varepsilon_{Y^*,t}}\varepsilon_{Y^*,t}\right)_{t\in\mathbb{N}}$
   - $\sigma_{R^*,t}$ in *exogenous* **US inflation** $(\pi_t^*)_{t\in\mathbb{N}}$ via shocks $\left(\sigma_{\varepsilon_{\pi^*,t}}\varepsilon_{\pi^*,t}\right)_{t\in\mathbb{N}}$
   - $\sigma_{R^*,t}$ in *exogenous* **US FFR** $(i_t^*)_{t\in\mathbb{N}}$ via shocks $\left(\sigma_{\varepsilon_{i^*,t}}\varepsilon_{i^*,t}\right)_{t\in\mathbb{N}}$
   - $\sigma_{\varepsilon_{\tau_W},t}$ in **domestic *fiscal shocks*** $\left(\sigma_{\varepsilon_{\tau_W},t}\varepsilon_{\tau_W}\right)_{t\in\mathbb{N}}$

Shoulders

of

Giants

Shoulders of Giants

1. **Why not VAR-SV or TVP-VAR-SV, and etc.**, with loose "theory" or sign restrictions?
    - Difficult to identify dynamic impact of "true" structural shocks even if true DGP known
        - VAR/VARMA class: Yao, Kam and Vahid (*Econ. Let.*, forth.)
    - Similar tautology in generalized classes of TVP-VAR-SV

2. **Theory ahead of measurement (and inference)**:
    - Leap of faith: imposed structure could be "wrong" *vis-à-vis* reality
    - But ... still need for causal and policy *interpretation*
    - Complementary to existing literature in TVP-VAR-SV, VAR-MS:
        - Bloom; Bloom et. al. (2009 *Ecta*; 2013 *JEL)*
        - Caggiano et. al.
3. **New economic question—a small-open-economy implementation of**:
    - Justiniano-Primiceri (2008, *AER*)
    - Fernandez-Villaverde (2011; 2015, *AER*)

Economic Structure

Structural Model

Small open economy, incomplete markets (Alonso-Carrera and Kam, 2015 *MD*):

- Optimizing households
  - consume differentiated Home/Foreign goods
  - work
  - hold Home/Foreign bonds
- Firms produce exportable differentiated good
  - Rotemberg pricing
  - hire labor
- Fiscal (labor income tax) rule with SV shocks
- Monetary policy rule
- RoW exogenous processes
  - World output with SV shocks

Structural Model

State vector (in agents' world):

$$\mathbf{s}_t := (A_t, \varepsilon_{R,t}, \sigma_{\tau_W,t}, Y_t^*, \sigma_{Y^*,t}, \pi_t^*, \sigma_{\pi^*,t}, i_t^*, \sigma_{i^*,t},)$$

- A_t: Domestic Total Factor Productivity shock, AR(1)
- $\varepsilon_{R,t}$: Domestic monetary policy shock, AR(1) or white noise
- $\sigma_{\tau_W,t}$: **Fiscal Uncertainty**, SV
- Y_t^*, π_t^*, i_t^*: **US output/inflation/FFR**, AR(1)-SV$(\sigma_{j,t})$, $j \in \{Y^*, \pi^*, i^*\}$
- $\sigma_{\tau_W,t}$: **Domestic Fiscal Uncertainty**

Implicit notation, Markovian maps: e.g., $C_t = C(\mathbf{s}_t)$

Structural Model

Consumer problem

Optimal plan on labor, domestic and foreign money claims satisfy FoCs:

$$\underbrace{A_t^{1-\rho}\psi N_t^{\varphi} C_t^{\rho}}_{MRS(N,C)} = (1-\tau_{W,t})\frac{W_t}{P_t},$$

$$1 = R_t \mathbb{E}_t \left\{ \underbrace{\beta\left(C_t^a/A_t\right)\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}}_{MRS(C_t,C_{t+1})} \left(\frac{P_t}{P_{t+1}}\right) \right\},$$

$$1 = R_t^* \mathbb{E}_t \left\{ \beta\left(C_t^a/A_t\right)\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t}\right) \right\},$$

C_t is CES(η) index of Home $C_{H,t}$ and Foreign goods $C_{F,t}$. Each is CES index of differentiated goods, with elasticity of subtitution ϵ_H and ϵ_F.

Structural Model

Firms' problems

- Production of differentiated goods

$$Y_{H,t}(i) = A_t N_t(i).$$

Cost minimization implies

$$W_t = MC_t A_t,$$

where MC_t is the nominal marginal cost.

Structural Model

Firms' problems

Convex price-adjustment cost:

$$AC\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i)\right) := \frac{\varpi}{2}\left(\frac{P_{H,t+s}(i)}{P_{H,t+s-\Pi}(i)} - \Pi\right)^2 \times Y_{H,t+s}(i).$$

Firm i's optimal pricing strategy $\{P_{H,t}\}$ satisfies:

$$\begin{aligned} 0 =& \underbrace{(1-\epsilon_H)\frac{Y_{H,t}(i)}{P_{H,t}}}_{MR \text{ wrt } P_{H,t}} - \underbrace{\frac{MC_t}{P_{H,t}}\frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)}}_{MC \text{ wrt } P_{H,t}} - \underbrace{\frac{\partial AC\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i)\right)}{\partial P_{H,t}(i)}}_{\text{Curr. marg. price adj. cost}} \\ & - \underbrace{\beta(C_t^a/A_t)\mathbb{E}_t\left\{\frac{U_C(C_{t+1},N_{t+1})}{U_C(C_t,N_t)}\frac{\partial AC\left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)}, Y_{H,t+1}(i)\right)}{\partial P_{H,t}(i)}\right\}}_{\text{PEV future marginal profits (costs) wrt } P_{H,t}}. \end{aligned}$$

Structural Model

Government

- Monetary authority follows a Henderson-McKibbin-Taylor rule:

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\phi_R} \frac{\Pi_t}{\Pi}^{(1-\phi_R)\phi_\Pi} \frac{Y_t}{Y A_t}^{(1-\phi_R)\phi_Y} \exp\left\{\sigma_R \varepsilon_{R,t}\right\}.$$

- Fiscal authority balances its budget every period

$$G_t = \tau_t^W \frac{W_t N_t}{P_{H,t}}.$$

- Tax smoothing rule responds to output growth via $\phi_{W,Y} > 0$, and, depends on exogenous stochastic volatility (SV) process:

$$\tau_{W,t} - \tau_W = \alpha_W \left(\tau_{W,t-1} - \tau_W\right) + \phi_{W,Y} \left(\frac{Y_t}{Y_{t-1}} - 1\right) + \exp\left\{\sigma_{\tau_W,t}\right\} \varepsilon_{\tau_W,t}$$

Structural Model

Structural shocks

- Technology A_t follows

$$\Delta \ln(A_t) = g_A + \rho_A \Delta \ln(A_{t-1}) + \sigma_A \varepsilon_{A,t},$$
$$\varepsilon_{A,t} \sim \mathcal{N}(0,1)$$
$$g_A, \rho_A, \sigma_A > 0$$

- Monetary policy shock is iid Normal $\left(0, \sigma_R^2\right)$
- Domestic Fiscal and world processes AR(1) with SV component:

$$\sigma_{i,t} = \sigma_{i,t-1} + \mu_i \nu_{i,t} \qquad \nu_{i,t} \sim \mathcal{N}(0,1), \qquad i \in \{\tau_W, Y^*, \pi^*, i^*\}$$

Structural Model

Competitive equilibrium implications

Zero-arbitrage asset pricing: uncovered interest parity (UIP)

$$R_t \mathbb{E}_t \left\{ \beta \left(C_t^a / A_t \right) \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\rho} \right\} = \mathbb{E}_t \left\{ \beta \left(C_t^a / A_t \right) \left(\frac{Q_{t+1}}{Q_t} \right) C_{t+1}^{-\rho} \tilde{R}_{t+1}^{*} \right\},$$

Phillips curve with $\Pi_{H,t} := P_{H,t} / P_{H,t-1}$

$$\begin{aligned} \Pi_{H,t}(\Pi_{H,t} - \Pi) - \frac{\epsilon_H}{2}(\Pi_{H,t} - \Pi)^2 = & \\ & \beta (C_t^a / A_t) \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \left(\Pi_{H,t+1} - \Pi \right) \Pi_{H,t+1} \cdot \frac{Y_{H,t+1}}{Y_{H,t}} \right\} \\ & + \frac{\epsilon_H}{\varpi} \left[mc_{H,t} - \frac{\epsilon_H - 1}{\epsilon_H} \right]. \end{aligned}$$

Competitive equilibrium

Definition

Given policies and exogenous processes, a *recursive competitive equilibrium* is a system of allocation functions $\mathbf{s}_t \mapsto (C_t, N_t, G_t, Y_{H,t}, mc_{H,t})(\mathbf{s}_t)$, and, pricing functions $\mathbf{s}_t \mapsto (\Pi_{H,t}, p_{H,t}, \Pi_t, Q_t)(\mathbf{s}_t)$, such that:

1. Households optimize
2. Firms optimize
3. Markets clear (given agents optimize)
4. Government budget constraint holds
5. Necc. and Suff. TVC: $\lim_{t\to\infty} \delta_t R_t \mathbb{E}_t \left\{ C_{t+1}^{-\rho} \Pi_{t+1}^{-1} B_{t+1} \right\} = \lim_{t\to\infty} \delta_t \mathbb{E}_t \left\{ C_{t+1}^{-\rho} Q_{t+1} \tilde{R}_{t+1}^{*} B_{t+1}^{*} \right\} = 0$

Stochastic trend in A_t

Need to solve in terms of stationary allocation and pricing functions.

Recursive (Markov) Competitive Equilibrium

Approximate Markov Equilibrium

Rational Expectations Equilibrium

First-order perturbation aproximation of competitive equilibrium conditions

$$\mathbf{0} = \mathbb{E}_t \left\{ \begin{array}{c} H_{x_{+1}}(x, y)\,\mathbf{x}_{t+1} + H_{y_{+1}}(x, y)\,\mathbf{y}_{t+1} \\ + H_x(x, y)\,\mathbf{x}_t + H_y(x, y)\,\mathbf{y}_t \end{array} \right\}$$

$$\mathbf{0} = \mathbf{x}_{t+1} - J_x(x, y)\,\mathbf{x}_t - K_e(x, y)\,\mathbf{e}_t$$

- $\mathbf{x}_t$ states (percentage deviation from steady state)
- $\mathbf{y}_t$ co-states (percentage deviation from steady state)
- Find *stabilizing* linear mapping $(\mathbf{F}_\theta, \mathbf{G}_\theta)$ as fixed point of linearized functionals above.

$$\mathbf{x}_{t+1} = \mathbf{F}_\theta \mathbf{x}_t + \mathbf{G}_\theta \mathbf{e}_t$$
$$\mathbf{y}_t = \mathbf{H}_\theta \mathbf{x}_t$$

Micro parameters of model: θ

Approximate Markov Equilibrium

REE's Econometric Reduced Form

- *Stabilizing* REE solution

$$\mathbf{x}_{t+1} = \mathbf{F}_\theta \mathbf{x}_t + \mathbf{G}_\theta \mathbf{e}_t$$
$$\mathbf{y}_t = \mathbf{H}_\theta \mathbf{x}_t$$

- Rewritten as state-space model with data observables:

$$\mathbf{y}_{t+1} = \mathbf{A}_\theta \mathbf{y}_t + \mathbf{B}_\theta \mathbf{e}_t$$
$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

- Micro parameters of model: θ
- Vector of structural shocks (AR, iid, SV): $\mathbf{e}_t$

Approximate Markov Equilibrium

REE's Econometric Reduced Form: IVAR-SV

The IVAR-SV$\left(\theta, \breve{\theta}\right)$

$$\mathbf{y}_{t+1} = \mathbf{A}_\theta \mathbf{y}_t + \mathbf{B}_\theta \mathbf{e}_t$$
$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$
$$\sigma_{i,t} = \sigma_{i,t-1} + \mu_i \nu_{i,t},\ \nu_{i,t} \sim \mathcal{N}(0,1),\ i \in \{\tau_W, Y^*\}$$

- $\mathbf{e}_t := \left(\begin{array}{c} \varepsilon_{A,t}, \varepsilon_{R,t}, \exp\left(\sigma_{\tau_W,t}\right)\varepsilon_{\tau_W,t}, \exp\left(\sigma_{Y^*,t}\right)\varepsilon_{Y^*,t}, \\ \exp\left(\sigma_{\pi^*,t}\right)\varepsilon_{\pi^*,t}, \varepsilon_{\pi^*,t}, \exp\left(\sigma_{i^*,t}\right)\varepsilon_{i^*,t}, \varepsilon_{i^*,t} \end{array} \right)$
- Parameters of model:
  $\theta := \left\langle \underbrace{\rho, \eta, \varpi}_{\text{Micro: taste/production}}, \underbrace{\phi_\Pi, \phi_Y}_{\text{Mon. Pol.}}, \underbrace{\alpha_W, \phi_W}_{\text{Fisc. Pol.}}, \underbrace{\sigma_A, g_A, \rho_{g_A}}_{\text{TFP growth AR(1)}}, \underbrace{\sigma_R}_{\text{Mon. Pol. Shock}} \right\rangle$
- SV parameters: $\breve{\theta} := \underbrace{\left(\mu_i^2\right)_{i \in \{\tau_W, Y^*, \pi^*, i^*\}}}_{\text{SV}}$

Bayesian Estimation

Estimation Method

- Vector of structural SV shocks:

$$\boldsymbol{\eta}_t = (\eta_{i,t})_{i\in\{\tau_W, Y^*\}} = (\exp(\sigma_{i,t})\,\varepsilon_{i,t})_{i\in\{\tau_W, Y^*, \pi^*, i^*\}}$$

  T-history: $\boldsymbol{\eta}^T := \{\boldsymbol{\eta}_t\}_{t=1}^T$

- Vector of log-SVs:

$$\mathbf{h}_t := (\sigma_{i,t})_{i\in\{\tau_W, Y^*, \pi^*, i^*\}}$$

  T-history: $\mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$

- Parameters of model: $\theta :=$

$$\langle \underbrace{\rho, \eta, \varpi}_{\text{Micro: taste/production}}, \underbrace{\phi_\Pi, \phi_Y}_{\text{Mon. Pol.}}, \underbrace{\alpha_W, \phi_W}_{\text{Fisc. Pol.}}, \underbrace{\sigma_A, g_A, \rho_{g_A}}_{\text{TFP growth AR(1)}}, \underbrace{\sigma_R}_{\text{Mon. Pol. Shock}} \rangle$$

- SV parameters: $\breve{\theta} := \underbrace{(\mu_i^2)_{i\in\{\tau_W, Y^*, \pi^*, i^*\}}}_{\text{SV}}$

Estimation Method

Step 0: Initialization

- Suppose previously accepted (Metropolis-Hastings)
  - DSGE parameters θ
  - SV parameters $\breve{\theta}$
  - T-history of SV realizations $\mathbf{H}^T$

Estimation Method

Step 1: Priors and Markov Equilibrium as IVAR

- Solve for approximate Markov equilibrium:

$$\mathbf{A}_\theta, \mathbf{B}_\theta \leftarrow \mathsf{SolveREE}(\theta)$$

- This gives the (log)-linear IVAR model

$$\mathbf{y}_{t+1} = \mathbf{A}_\theta \mathbf{y}_t + \mathbf{B}_\theta \mathbf{e}_t$$
$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

  conditional on structural shock process $\mathbf{e}_t$

Estimation Method

Step 2: Simulate SV sample conditional on IVAR

- Draw subset of structural shocks $\mathbf{e}_t$:
  - SV subset is $\boldsymbol{\eta}_t = (\exp(\sigma_{i,t})\,\varepsilon_{i,t})_{i\in\{\tau_W, Y^*, \pi^*, i^*\}} \subset \mathbf{e}_t$
- Apply Durbin and Koopman (2002, *Biometrika*) efficient simulation smoother to linear IVAR (state-space)

$$\boldsymbol{\eta}^T \leftarrow \mathsf{DKsmoother}(\mathbf{A}_\theta, \mathbf{B}_\theta)$$

Estimation Method

Step 3a: Draw the SVs

- Recall $\mathbf{h}_{i,t} = \sigma_{i,t}$ (already defined as log transform)
- Denote $\widetilde{\boldsymbol{\eta}}_{i,t} = \ln\left(\left[\exp\left(\sigma_{i,t}\right)\varepsilon_{i,t}\right]^2 + 0.001\right)$ and $\mathbf{w}_{i,t} = 2\ln\left(\varepsilon_{i,t}\right)$
- SV process (approx.) re-written as

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim \mathcal{N}\left(0, \mathsf{diag}\left(\breve{\theta}\right)\right)$$

$$\widetilde{\boldsymbol{\eta}}_{i,t} = 2\mathbf{h}_{i,t} + \mathbf{w}_{i,t}, \qquad i \in \{\tau_W, Y^*, \pi^*, i^*\}$$

Estimation Method

Step 3a: Draw the SVs

- Note: $cov\left(\boldsymbol{\nu}_t, \mathbf{w}_{i,t}\right) = 0$, $\boldsymbol{\nu}_t$ Gaussian, but $\mathbf{w}_{i,t} \sim \ln \chi^2 (1)$
  - Approximate distribution of $\mathbf{w}_{i,t}$ by mixture of Gaussian densities:

$$f\left(\mathbf{w}_{i,t}\right) = \sum_{k=1}^{K} q_k f_N\left(\mathbf{w}_{i,t} | \mathbf{s}_{i,t} = k\right)$$

    $q_k = \Pr\left(\mathbf{s}_{i,t} = k\right)$ and f_N pdf of Normal distro.
  - Draw

$$\begin{aligned} \mathbf{s}_{i,t} &\sim \Pr\left(\mathbf{s}_{i,t} = k | \boldsymbol{\eta}_t, \mathbf{h}_t\right) \\ &\propto q_k f_N\left(\boldsymbol{\eta}_{i,t} | 2\mathbf{h}_{i,t} + m_k - 1.2704, 1.2704 r_k^2\right) \end{aligned}$$

    where $k \in \{1, ..., 7\}$.
  - Kim, Shephard, Chib (1998, *ReStud*): $K = 7$, $f_N\left(\mathbf{w}_{i,t} | \mathbf{s}_{i,t} = k\right)$ parametrized by mean-variance $\left(m_k, r_k^2\right)$

$$\mathbf{s}^T \leftarrow \text{KimShephardChib}(\boldsymbol{\eta}^T, \mathbf{H}^T, \breve{\theta})$$

Estimation Method

Step 3b: Draw the SVs

- Given sample $\mathbf{s}^T$ implies $\mathbf{w}_{i,t} \sim f(\mathbf{w}_{i,t})$ for each i, each $t = 1, ..., T$
- Then use (approx.) state-space representation of SV:

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim \mathcal{N}\left(0, \mathrm{diag}\left(\breve{\theta}\right)\right)$$

$$\widetilde{\boldsymbol{\eta}}_{i,t} = 2\mathbf{h}_{i,t} + \mathbf{w}_{i,t}, \qquad i \in \{\tau_W, Y^*, \pi^*, i^*\}$$

to simulate new T-history of SVs, $\mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$:

$$\mathbf{H}^T \leftarrow \mathrm{ChanHsiao}(\mathbf{s}^T) \text{ or } \mathrm{CarterKohn}(\mathbf{s}^T)$$

- Chan and Hsiao (2014):
  - banded sparse matrix algebra problem
  - A faster implementation Carter and Kohn's forward-backward recursion

Estimation Method

Step 4: Update SV coefficients

- Given new draws $\mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$
- Note

$$\boldsymbol{\nu}_t = \mathbf{h}_t - \mathbf{h}_{t-1} \qquad \boldsymbol{\nu}_t \sim \mathcal{N}\left(0, \mathsf{diag}\left(\breve{\theta}\right)\right)$$

- Back out $\breve{\theta}$ from inverse Gamma distro of $\boldsymbol{\nu}_t$

$$\breve{\theta} \leftarrow \mathsf{InvGamma}\left(\mathbf{H}^T\right)$$

Estimation Method

Step 5: Propose competing model θ^{alt}

- Draw alternative DSGE model

$$\theta^{alt} \leftarrow \mathsf{RandomWalk}\left(\theta; V, \theta_0 \sim p(\theta_0)\right)$$

Scaling covariance V (jump size); $p(\theta_0)$ prior density

- Solve for approximate Markov equilibrium:

$$\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}} \leftarrow \mathsf{SolveREE}(\theta^{alt})$$

- This gives the (log)-linear IVAR model

$$\mathbf{y}_{t+1} = \mathbf{A}_{\theta^{alt}} \mathbf{y}_t + \mathbf{B}_{\theta^{alt}} \mathbf{e}_t$$
$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

conditional on structural shock process $\mathbf{e}_t$

Estimation Method

Step 6: Metropolis-Hastings

Conditional on SVs:

- Construction data likelihood function of competing models:

$$p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T) \leftarrow \mathsf{KalmanFilter}(\mathbf{A}_\theta, \mathbf{B}_\theta, \mathbf{H}^T)$$
$$p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T) \leftarrow \mathsf{KalmanFilter}(\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}}, \mathbf{H}^T)$$

- Metropolis-Hastings decision:
  - Acceptance ratio

$$a \leftarrow \min\left\{1, \frac{p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T)p(\theta^{alt})}{p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T)p(\theta)}\right\}$$

  - Accept new proposal $\theta^{alt}|\mathbf{H}^T$ with probability a

Estimation Method

Algorithm 1 Stochastic optimization of model posterior

Initialize $\theta \leftarrow p(\theta), \breve{\theta} \leftarrow p(\breve{\theta}), \mathbf{H}^T, \mathbf{m} =$ list[ ]
$\mathbf{A}_\theta, \mathbf{B}_\theta \leftarrow$ SolveREE(θ)
for all n in range(N) **do**
    $\boldsymbol{\eta}^T \leftarrow$ DKsmoother($\mathbf{A}_\theta, \mathbf{B}_\theta$)
    $\mathbf{s}^T \leftarrow$ KimShephardChib($\boldsymbol{\eta}^T, \mathbf{H}^T, \breve{\theta}$)
    $\mathbf{H}^T \leftarrow$ ChanHsiao($\mathbf{s}^T$) or CarterKohn($\mathbf{s}^T$)
    $\breve{\theta}^{new} \leftarrow$ InvGamma($\mathbf{H}^T$)
    $\theta^{alt} \leftarrow$ RandomWalk $(\theta; V, \theta_0 \sim p(\theta_0))$
    $\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}} \leftarrow$ SolveREE(θ^{alt})
    $p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T) \leftarrow$ KalmanFilter($\mathbf{A}_\theta, \mathbf{B}_\theta, \mathbf{H}^T$)
    $p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T) \leftarrow$ KalmanFilter($\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}}, \mathbf{H}^T$)
    $a \leftarrow \min\left\{1, \frac{p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T)p(\theta^{alt})}{p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T)p(\theta)}\right\}$
    $u \sim \mathbf{U}([0,1])$
    if $a > u$ **then**
        $(\theta, \breve{\theta}, \mathbf{H}^T) \leftarrow (\theta^{alt}, \breve{\theta}^{new}, \mathbf{H}^T)$
    end if
    $\mathbf{m}$.append($\theta, \breve{\theta}, \mathbf{H}^T$)
end for

Estimation Method

Implied Gibbs sampler with correct posterior densities

Algorithm (Step 1-6): a Gibbs sampler with conditional blocks

B1. $\mathbf{H}^T \sim \tilde{p}\left(\mathbf{H}^T | \mathbf{y}^{o,T}, \theta, \mathbf{s}^T\right)$

- (posterior from last Metropolis Hastings step as proposal density)

B2. $\left(\theta, \mathbf{s}^T\right) \sim \tilde{p}\left(\theta, \mathbf{s}^T | \mathbf{y}^{o,T}, \mathbf{H}^T\right)$ via marginals:

- $\theta \sim p\left(\theta | \mathbf{y}^{o,T}, \mathbf{H}^T\right)$ (convenient, does not depend on $\mathbf{s}^T$)
- $\mathbf{s}^T \sim p\left(\mathbf{s}^T | \mathbf{y}^{o,T}, \theta, \mathbf{H}^T\right)$ (via Kim-Shephard-Chib)

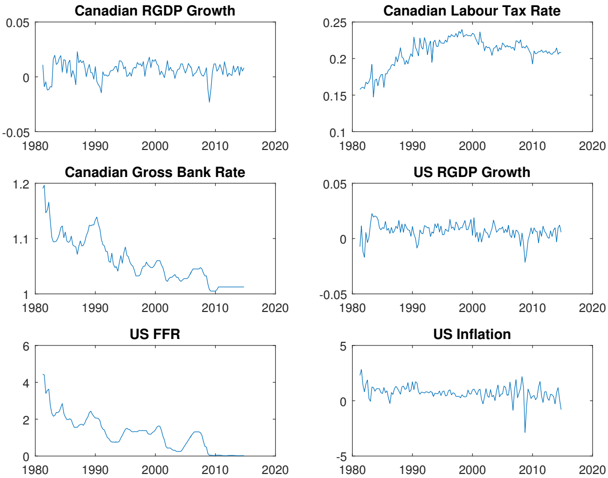
This sampler (in the limit) induces the correct posterior density of $\left(\theta, \mathbf{H}^T\right)$ or equivalently $\left(\theta, \breve{\theta}\right)$

- Stroud, Müller, Poulson (2003, *JASSA*)
- Del Negro and Primiceri (2015, *ReStud*, corrigendum's online appendix)

Results

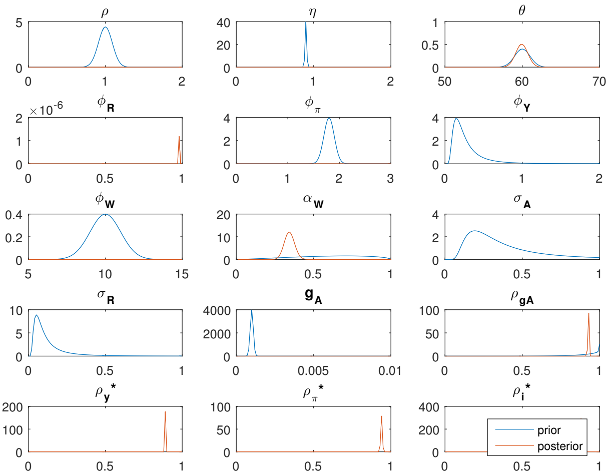
Observed data

Figure: Data (Canada)



Results

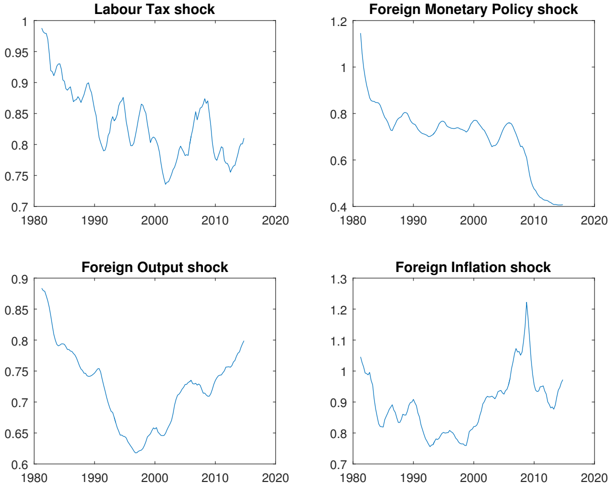
Figure: Prior and Posterior parameter densities



Results

Estimated SV shocks

Figure: SV of $(\tau_W, i^*, Y^*, \pi^*)$



Which Uncertainty Shocks?

How Important?

Importance of Uncertainty Shocks

Historical Decomposition of Shocks

Impulse response function ($\mathbf{y}_1$ given) to shock $\mathbf{e}_{m,t-j}$:

$$\mathbf{y}_t^o(m) = \mathbf{H}^o \left[(\mathbf{A}_\theta)^{t-1} \mathbf{y}_1 + \sum_{j=0}^{t-2} (\mathbf{A}_\theta)^j \mathbf{B}_\theta \mathbf{S}_m \mathbf{e}_{t-j} \right], \qquad \forall t > 1$$

- $\mathbf{S}_m$ selection matrix on a particular structural shock $\mathbf{e}_{m,t-j}$

Historical decomposition at date t, of shock m's impact on (observable) variable n:

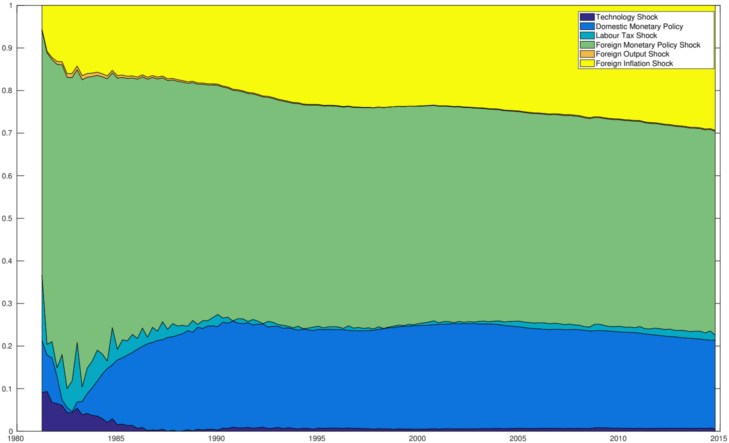
$$HD_{n,t}(m) := \frac{\left|\mathbf{y}_{n,t}^o(m)\right|}{\sum_{m=1}^{M} \left|\mathbf{y}_{n,t}^o(m)\right|}$$

Given posterior estimate of model:

- Get simulated sample $\{\mathbf{e}_{m,t}\}_{t=2}^{T}$ of structural shocks (which contains SVs), for each $m = 1, ..., M$
- Given focus on variable $\mathbf{y}_{n,t}^o$, evaluate accounting transforms $\{HD_{n,t}(m) : m = 1, ..., M\}_{t=2}^{T}$

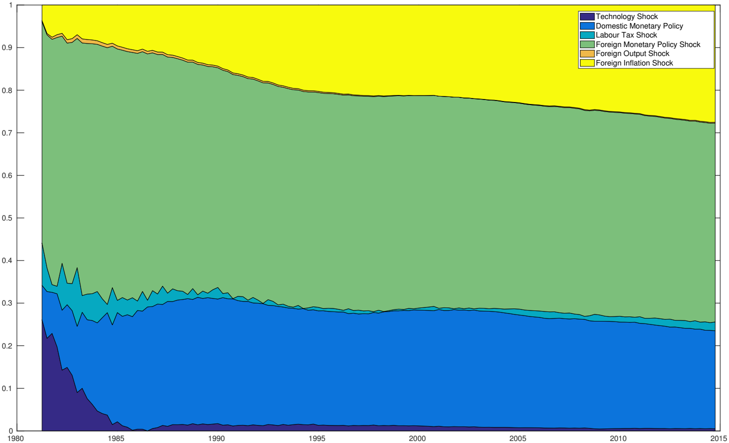
Importance of Uncertainty Shocks

Figure: Historical Decomposition—posterior mean



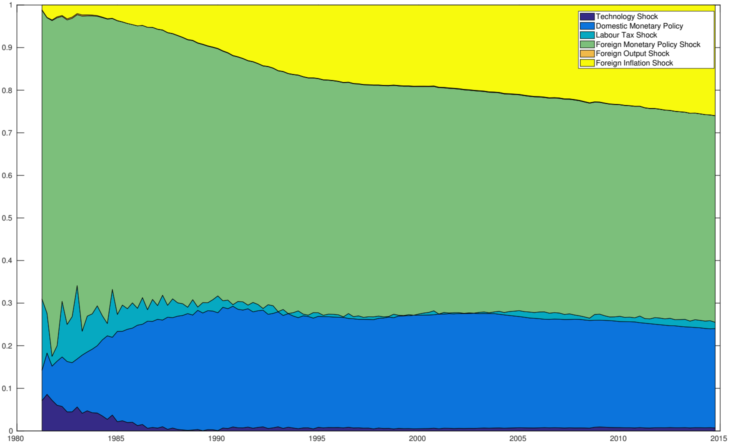
Importance of Uncertainty Shocks

Figure: Historical Decomposition—posterior median



Importance of Uncertainty Shocks

Figure: Historical Decomposition—posterior mode



Importance of Uncertainty Shocks

1981 to 2015: Discussion

Canadian shock history (conditional on structural model and identified structural shocks):

1. **US monetary policy Uncertainty** still important
   - 1980s - dominant, 50-60%
   - Recent decade $\sim$50%
2. **US Inflation Uncertainty** increased importance
   - 1980s, 10%
   - Recent decade, $\sim$20%
3. Interestingly, **US output uncertainty** not so dominant
4. **Domestic Fiscal Uncertainty**
   - 1980s, 10-20%
   - Recent decade,$\sim$2%
5. **Domestic Monetary Policy** sizable, accounts 20%
   - small exception around Volker disinflation era

Summary

Global or Fiscal Uncertainty?

Method: How to identify structural (demand/supply/policy) shocks?

- Reduced-form with loose theory not palatable—interpretational issue; empirically misleading
- Strong theoretical restrictions—Markov equilibrium's IVAR-SV
  - possibly wrong cross-equation restrictions
  - a need for plausible and internally consistent interpretation of "shocks" and "uncertainty shocks"

Application: Historical Shock Decomposition Exercise

- **US Monetary Policy and Inflation Uncertainty** has dominated the historical influence of structural shocks (80% recent)
- **Domestic Monetary Policy** shocks too (20% recent)
- **Policy Takeaway**:
  - The **global economy**, ...
  - don't blame it parliament

To Do List

1. **Alternative data sets**
   - Define "world output" to be more than just US output!
   - Australia
   - New Zealand

2. **Bayesian model comparison**
   - Alternative DSGE structures and mechanisms
   - Alternative SV sources of structural shocks:
     - Energy Economics

3. **China and the small open economies**
   - Australia
   - New Zealand
   - East Asia

Job Candidates you should meet

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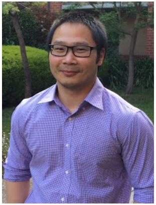

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