

Global and Fiscal Uncertainty Shocks: What Matters for Small Open Economies?

Jamie **Cross**¹ ⊗ Joshua **Chan**²
⊗ Timothy **Kam**³ ⊗ Aubrey **Poon**⁴

^{1,3,4}RSE, Australian National University

²University of Technology Sydney

Paper: Coming soon on
github.com/phantomachine/fiscal-global-uncertainty/

Monash, Econometrics and Business Statistics, Mar 24, 2017

Horses for Courses



This paper is about **theory-disciplined** identification of **shocks** ...

... about *quantitative* economic **interpretation** and **business-cycle accounting** of such shocks

not about prediction

Definition

Structural shock uncertainty

Uncertainty Shock definition

Unexpected changes in the (co-)variance of underlying *economic-structural* shocks.

- ▶ “Uncertainty” is from perspective of modeller/statistician (not agents).
- ▶ “Risk” from Model agents’ perspective: Know underlying probability model, including that of random covariance shocks.

Problem Statement

What accounts for *uncertainty shocks* in a small open economy's business cycle?

Shocks to riskiness of distribution of ***global real activity*** and/or ***domestic fiscal policy***?

i.e., “The *world* economy, stupid” or “Blame it on our politicians”?

Why important?

- ▶ Matters for (monetary) policy design
- ▶ How much should policy makers worry about responding to external factors?
- ▶ Or to domestic fiscal stance?

What We Do

Structural time-series modelling approach

Measurement and Inference with Theory

Incomplete-markets small-open-economy DSGE

1. fiscal and monetary policy

- ▶ theory-identified “structural shocks”

2. *stochastic volatility* \equiv “Uncertainty shocks”

- ▶ $\sigma_{Y^*,t}$ in *exogenous world output* $(Y_t^*)_{t \in \mathbb{N}}$ via shocks $(\sigma_{\varepsilon_{Y^*,t}}, \varepsilon_{Y^*,t})_{t \in \mathbb{N}}$
- ▶ $\sigma_{R^*,t}$ in *exogenous world real rate* $(Y_t^*)_{t \in \mathbb{N}}$ via shocks $(\sigma_{\varepsilon_{R^*,t}}, \varepsilon_{R^*,t})_{t \in \mathbb{N}}$
- ▶ $\sigma_{\varepsilon_{\tau_W},t}$ in *domestic fiscal shocks* $(\sigma_{\varepsilon_{\tau_W},t}, \varepsilon_{\tau_W})_{t \in \mathbb{N}}$

Shoulders of Giants

Shoulders of Giants

1. Why not VAR-SV or TVP-VAR-SV, and etc., with loose “theory” or sign restrictions?

- ▶ Difficult to identify dynamic impact of “true” structural shocks even if we “know truth”
 - ▶ VAR/VARMA class: Yao et. al. (*Econ. Let.*, forth.)
- ▶ Similar tautology in generalized classes of TVP-VAR-SV

2. Theory ahead of measurement (and inference):

- ▶ Leap of faith: imposed structure could be “wrong” *vis-à-vis* reality
- ▶ But ... still need for causal and policy *interpretation*
- ▶ Complementary to existing literature in TVP-VAR-SV, VAR-MS:
 - ▶ Bloom; Bloom et. al. (2009 *Ecta*; 2013 *JEL*)
 - ▶ Caggiano et. al.

3. New economic question—a small-open-economy implementation of:

- ▶ Justiniano-Primiceri (2008, *AER*)
- ▶ Fernandez-Villaverde (2011; 2015, *AER*)

Economic Structure

Structural Model

Small open economy, incomplete markets (Alonso-Carrera and Kam, 2015 *MD*):

- ▶ Optimizing households
 - ▶ consume differentiated Home/Foreign goods
 - ▶ work
 - ▶ hold Home/Foreign bonds
- ▶ Firms produce exportable differentiated good
 - ▶ Rotemberg pricing
 - ▶ hire labor
- ▶ Fiscal (labor income tax) rule with SV shocks
- ▶ Monetary policy rule
- ▶ RoW exogenous processes
 - ▶ World output with SV shocks

Structural Model

State vector (in agents' world):

$$\mathbf{s}_t := (A_t, \varepsilon_{R,t}, \sigma_{\tau_W,t}, Y_t^*, \sigma_{Y^*,t}, R_t^*)$$

- ▶ A_t : Domestic Total Factor Productivity shock, AR(1)
- ▶ $\varepsilon_{R,t}$: Domestic monetary policy shock, AR(1) or white noise
- ▶ $\sigma_{\tau_W,t}$: **Fiscal Uncertainty shock**, SV
- ▶ Y_t^* : **World output**, AR(1)-SV
 - ▶ $\sigma_{\tau_W,t}$: **World output Uncertainty shock**
- ▶ R_t^* : US Fed Funds Rate, AR(1)

Implicit notation: e.g., $C_t = C(\mathbf{s}_t)$

Structural Model

Consumer problem

Optimal plan on labor, domestic and foreign money claims satisfy FoCs:

$$\underbrace{A_t^{1-\rho} \psi N_t^\varphi C_t^\rho}_{MRS(N,C)} = (1 - \tau_{W,t}) \frac{W_t}{P_t},$$

$$1 = R_t \mathbb{E}_t \left\{ \underbrace{\beta (C_t^a / A_t) \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}}_{MRS(C_t, C_{t+1})} \left(\frac{P_t}{P_{t+1}} \right) \right\},$$

$$1 = R_t^* \mathbb{E}_t \left\{ \beta (C_t^a / A_t) \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \left(\frac{P_t^* Q_{t+1}}{P_{t+1}^* Q_t} \right) \right\},$$

C_t is CES(η) index of Home $C_{H,t}$ and Foreign goods $C_{F,t}$. Each is CES index of differentiated goods, with elasticity of substitution ϵ_H and ϵ_F .

Structural Model

Firms' problems

- Production of differentiated goods

$$Y_{H,t}(i) = A_t N_t(i).$$

Cost minimization implies

$$W_t = MC_t A_t,$$

where MC_t is the nominal marginal cost.

Structural Model

Firms' problems

Convex price-adjustment cost:

$$AC \left(\frac{P_{H,t+s}(i)}{P_{H,t+s-1}(i)}, Y_{H,t+s}(i) \right) := \frac{\varpi}{2} \left(\frac{P_{H,t+s}(i)}{P_{H,t+s-\Pi}(i)} - \Pi \right)^2 \times Y_{H,t+s}(i).$$

Firm i 's optimal pricing strategy $\{P_{H,t}\}$ satisfies:

$$\begin{aligned} 0 = & \underbrace{(1 - \epsilon_H) \frac{Y_{H,t}(i)}{P_{H,t}}}_{MR \text{ wrt } P_{H,t}} - \underbrace{\frac{MC_t}{P_{H,t}} \frac{\partial Y_{H,t}(i)}{\partial P_{H,t}(i)}}_{MC \text{ wrt } P_{H,t}} - \underbrace{\frac{\partial AC \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)}, Y_{H,t}(i) \right)}{\partial P_{H,t}(i)}}_{\text{Curr. marg. price adj. cost}} \\ & - \underbrace{\beta(C_t^a / A_t) \mathbb{E}_t \left\{ \frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{\partial AC \left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)}, Y_{H,t+1}(i) \right)}{\partial P_{H,t}(i)} \right\}}_{\text{PEV future marginal profits (costs) wrt } P_{H,t}}. \end{aligned}$$

Structural Model

Government

- Monetary authority follows a Henderson-McKibbin-Taylor rule:

$$\frac{R_t}{R} = \frac{R_{t-1}}{R} \frac{\Pi_t^{(1-\phi_R)\phi_\Pi}}{\Pi} \frac{Y_t^{(1-\phi_R)\phi_Y}}{Y A_t} \exp \{ \sigma_R \varepsilon_{R,t} \}.$$

- Fiscal authority balances its budget every period

$$G_t = \tau_t^W \frac{W_t N_t}{P_{H,t}}.$$

- Tax smoothing rule responds to output growth via $\phi_{W,Y} > 0$, and, depends on exogenous stochastic volatility (SV) process:

$$\tau_{W,t} - \tau_W = \alpha_W (\tau_{W,t-1} - \tau_W) + \phi_{W,Y} \left(\frac{Y_t}{Y_{t-1}} - 1 \right) + \exp \{ \sigma_{\tau_W,t} \} \varepsilon_{\tau_W,t}$$

Structural Model

Structural shocks

- ▶ Technology A_t follows

$$\Delta \ln(A_t) = g_A + \rho_A \Delta \ln(A_{t-1}) + \sigma_A \varepsilon_{A,t},$$

$$\varepsilon_{A,t} \sim \mathcal{N}(0, 1)$$

$$g_A, \rho_A, \sigma_A > 0$$

- ▶ Monetary policy shock is iid Normal $(0, \sigma_R^2)$
- ▶ World output Y_t^* is AR(1)-SV
- ▶ Fiscal Uncertainty SV and world output SV processes, $i \in \{\tau_W, Y^*\}$

$$\sigma_{i,t} = \sigma_{i,t-1} + \mu_i \nu_{i,t} \quad \nu_{i,t} \sim \mathcal{N}(0, 1),$$

- ▶ Foreign R_t^* is AR(1) and domestic monetary-policy shock is iid $\mathcal{N}(0, \sigma_R^2)$

Structural Model

Competitive equilibrium implications

Zero-arbitrage asset pricing: uncovered interest parity (UIP)

$$R_t \mathbb{E}_t \left\{ \beta (C_t^a / A_t) \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\rho} \right\} = \mathbb{E}_t \left\{ \beta (C_t^a / A_t) \left(\frac{Q_{t+1}}{Q_t} \right) C_{t+1}^{-\rho} \tilde{R}_{t+1}^* \right\},$$

Phillips curve with $\Pi_{H,t} := P_{H,t} / P_{H,t-1}$

$$\begin{aligned} \Pi_{H,t} (\Pi_{H,t} - \Pi) - \frac{\epsilon_H}{2} (\Pi_{H,t} - \Pi)^2 = \\ \beta (C_t^a / A_t) \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} (\Pi_{H,t+1} - \Pi) \Pi_{H,t+1} \cdot \frac{Y_{H,t+1}}{Y_{H,t}} \right\} \\ + \frac{\epsilon_H}{\varpi} \left[mc_{H,t} - \frac{\epsilon_H - 1}{\epsilon_H} \right]. \end{aligned}$$

Competitive equilibrium

Definition

Given policies and exogenous processes, a *recursive competitive equilibrium* is a system of allocation functions

$s_t \mapsto (C_t, N_t, G_t, Y_{H,t}, mc_{H,t})(s_t)$, and, pricing functions

$s_t \mapsto (\Pi_{H,t}, p_{H,t}, \Pi_t, Q_t)(s_t)$, such that:

1. Households optimize
2. Firms optimize
3. Markets clear (given agents optimize)
4. Government budget constraint holds
5. Necc. and Suff. TVC: $\lim_{t \rightarrow \infty} \delta_t R_t \mathbb{E}_t \{ C_{t+1}^{-\rho} \Pi_{t+1}^{-1} B_{t+1} \} = \lim_{t \rightarrow \infty} \delta_t \mathbb{E}_t \{ C_{t+1}^{-\rho} Q_{t+1} \tilde{R}_{t+1}^* B_{t+1}^* \} = 0$

Stochastic trend in A_t

Need to solve in terms of stationary allocation and pricing functions.

Recursive (Markov) Competitive Equilibrium

Approximate Markov Equilibrium

Rational Expectations Equilibrium

First-order perturbation approximation of competitive equilibrium conditions

$$\mathbf{0} = \mathbb{E}_t \left\{ \begin{array}{l} H_{x_{t+1}}(x, y) \mathbf{x}_{t+1} + H_{y_{t+1}}(x, y) \mathbf{y}_{t+1} \\ + H_x(x, y) \mathbf{x}_t + H_y(x, y) \mathbf{y}_t \end{array} \right\}$$

$$\mathbf{0} = \mathbf{x}_{t+1} - J_x(x, y) \mathbf{x}_t - K_e(x, y) \mathbf{e}_t$$

- ▶ \mathbf{x}_t states (percentage deviation from steady state)
- ▶ \mathbf{y}_t co-states (percentage deviation from steady state)
- ▶ Find *stabilizing* linear mapping $(\mathbf{F}_\theta, \mathbf{G}_\theta)$ as fixed point of linearized functionals above.

$$\mathbf{x}_{t+1} = \mathbf{F}_\theta \mathbf{x}_t + \mathbf{G}_\theta \mathbf{e}_t$$

$$\mathbf{y}_t = \mathbf{H}_\theta \mathbf{x}_t$$

Micro parameters of model: θ

Approximate Markov Equilibrium

REE's Econometric Reduced Form

- ▶ *Stabilizing* REE solution

$$\mathbf{x}_{t+1} = \mathbf{F}_{\theta} \mathbf{x}_t + \mathbf{G}_{\theta} \mathbf{e}_t$$

$$\mathbf{y}_t = \mathbf{H}_{\theta} \mathbf{x}_t$$

- ▶ Rewritten as state-space model with data observables:

$$\mathbf{y}_{t+1} = \mathbf{A}_{\theta} \mathbf{y}_t + \mathbf{B}_{\theta} \mathbf{e}_t$$

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

- ▶ Micro parameters of model: θ
- ▶ Vector of structural shocks (AR, iid, SV): \mathbf{e}_t

Approximate Markov Equilibrium

REE's Econometric Reduced Form: IVAR-SV

The IVAR-SV $(\theta, \check{\theta})$

$$\mathbf{y}_{t+1} = \mathbf{A}_{\theta} \mathbf{y}_t + \mathbf{B}_{\theta} \mathbf{e}_t$$

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

$$\sigma_{i,t} = \sigma_{i,t-1} + \mu_i \nu_{i,t}, \quad \nu_{i,t} \sim \mathcal{N}(0, 1), \quad i \in \{\tau_W, Y^*\}$$

► $\mathbf{e}_t := (\varepsilon_{A,t}, \varepsilon_{R,t}, \exp(\sigma_{\tau_W,t}) \varepsilon_{\tau_W,t}, \exp(\sigma_{Y^*,t}) \varepsilon_{Y^*,t}, \varepsilon_{R^*,t})$

► Parameters of model:

$$\theta := \left(\underbrace{\rho, \eta, \varpi}_{\text{Micro: taste/production}}, \underbrace{\phi_{\Pi}, \phi_Y}_{\text{Mon. Pol.}}, \underbrace{\alpha_W, \phi_W}_{\text{Fisc. Pol.}}, \underbrace{\sigma_A, g_A, \rho_{g_A}}_{\text{TFP growth AR(1)}}, \underbrace{\sigma_R}_{\text{Mon. Pol. Shock}} \right)$$

► SV parameters: $\check{\theta} := \underbrace{(\mu_i^2)_{i \in \{\tau_W, Y^*\}}}_{\text{SV}}$

Bayesian Estimation

Estimation Method

- Vector of structural SV shocks:

$$\boldsymbol{\eta}_t = (\eta_{i,t})_{i \in \{\tau_W, Y^*\}} = (\exp(\sigma_{i,t}) \varepsilon_{i,t})_{i \in \{\tau_W, Y^*\}}$$

$$T\text{-history: } \boldsymbol{\eta}^T := \{\boldsymbol{\eta}_t\}_{t=1}^T$$

- Vector of log-SVs:

$$\mathbf{h}_t := (\sigma_{i,t})_{i \in \{\tau_W, Y^*\}}$$

$$T\text{-history: } \mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$$

- Parameters of model: $\boldsymbol{\theta} :=$

$$\underbrace{\langle \rho, \eta, \varpi \rangle}_{\text{Micro: taste/production}}, \underbrace{\langle \phi_\Pi, \phi_Y \rangle}_{\text{Mon. Pol.}}, \underbrace{\langle \alpha_W, \phi_W \rangle}_{\text{Fisc. Pol.}}, \underbrace{\langle \sigma_A, g_A, \rho_{g_A} \rangle}_{\text{TFP growth AR(1)}}, \underbrace{\langle \sigma_R \rangle}_{\text{Mon. Pol. Shock}}$$

- SV parameters: $\check{\boldsymbol{\theta}} := \underbrace{(\mu_i^2)_{i \in \{\tau_W, Y^*\}}}_{\text{SV}}$

Estimation Method

Step 0: Initialization

- ▶ Suppose previously accepted (Metropolis-Hastings)
 - ▶ DSGE parameters θ
 - ▶ SV parameters $\check{\theta}$
 - ▶ T -history of SV realizations \mathbf{H}^T

Estimation Method

Step 1: Priors and Markov Equilibrium as IVAR

- Solve for approximate Markov equilibrium:

$$\mathbf{A}_\theta, \mathbf{B}_\theta \leftarrow \text{SolveREE}(\theta)$$

- This gives the (log)-linear IVAR model

$$\mathbf{y}_{t+1} = \mathbf{A}_\theta \mathbf{y}_t + \mathbf{B}_\theta \mathbf{e}_t$$

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

conditional on structural shock process \mathbf{e}_t

Estimation Method

Step 2: Simulate SV sample conditional on IVAR

- ▶ Draw subset of structural shocks \mathbf{e}_t :
 - ▶ SV subset is $\boldsymbol{\eta}_t = (\exp(\sigma_{i,t}) \varepsilon_{i,t})_{i \in \{\tau_W, Y^*\}} \subset \mathbf{e}_t$
- ▶ Apply Durbin and Koopman (2002, *Biometrika*) efficient simulation smoother to linear IVAR (state-space)

$$\boldsymbol{\eta}^T \leftarrow \text{DKsmoother}(\mathbf{A}_\theta, \mathbf{B}_\theta)$$

Estimation Method

Step 3a: Draw the SVs

- ▶ Recall $\mathbf{h}_{i,t} = \sigma_{i,t}$ (already defined as log transform)
- ▶ Denote $\tilde{\boldsymbol{\eta}}_{i,t} = \ln \left([\exp (\sigma_{i,t}) \varepsilon_{i,t}]^2 + 0.001 \right)$ and $\mathbf{w}_{i,t} = 2 \ln (\varepsilon_{i,t})$
- ▶ SV process (approx.) re-written as

$$\begin{aligned}\mathbf{h}_t &= \mathbf{h}_{t-1} + \boldsymbol{\nu}_t, & \boldsymbol{\nu}_t &\sim \mathcal{N} \left(0, \text{diag} \left(\check{\boldsymbol{\theta}} \right) \right) \\ \tilde{\boldsymbol{\eta}}_{i,t} &= 2\mathbf{h}_{i,t} + \mathbf{w}_{i,t}, & i &\in \{\tau_W, Y^*\}\end{aligned}$$

Estimation Method

Step 3a: Draw the SVs

- Note: $cov(\boldsymbol{\nu}_t, \mathbf{w}_{i,t}) = 0$, $\boldsymbol{\nu}_t$ Gaussian, but $\mathbf{w}_{i,t} \sim \ln \chi^2(1)$
 - Approximate distribution of $\mathbf{w}_{i,t}$ by mixture of Gaussian densities:

$$f(\mathbf{w}_{i,t}) = \sum_{k=1}^K q_k f_N(\mathbf{w}_{i,t} | \mathbf{s}_{i,t} = k)$$

$q_k = \Pr(\mathbf{s}_{i,t} = k)$ and f_N pdf of Normal distro.

- Draw

$$\begin{aligned} \mathbf{s}_{i,t} &\sim \Pr(\mathbf{s}_{i,t} = k | \boldsymbol{\eta}_t, \mathbf{h}_t) \\ &\propto q_k f_N(\boldsymbol{\eta}_{i,t} | 2\mathbf{h}_{i,t} + m_k - 1.2704, 1.2704 r_k^2) \end{aligned}$$

where $k \in \{1, \dots, 7\}$.

- Kim, Shephard, Chib (1998, *ReStud*): $K = 7$, $f_N(\mathbf{w}_{i,t} | \mathbf{s}_{i,t} = k)$ parametrized by mean-variance (m_k, r_k^2)

$$\mathbf{s}^T \leftarrow \text{KimShephardChib}(\boldsymbol{\eta}^T, \mathbf{H}^T, \check{\theta})$$

Estimation Method

Step 3b: Draw the SVs

- ▶ Given sample \mathbf{s}^T implies $\mathbf{w}_{i,t} \sim f(\mathbf{w}_{i,t})$ for each i , each $t = 1, \dots, T$
- ▶ Then use (approx.) state-space representation of SV:

$$\begin{aligned}\mathbf{h}_t &= \mathbf{h}_{t-1} + \boldsymbol{\nu}_t, & \boldsymbol{\nu}_t &\sim \mathcal{N}\left(0, \text{diag}(\boldsymbol{\theta})\right) \\ \tilde{\boldsymbol{\eta}}_{i,t} &= 2\mathbf{h}_{i,t} + \mathbf{w}_{i,t}, & i &\in \{\tau_W, Y^*\}\end{aligned}$$

to simulate new T -history of SVs, $\mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$:

$$\mathbf{H}^T \leftarrow \text{ChanHsiao}(\mathbf{s}^T) \text{ or } \text{CarterKohn}(\mathbf{s}^T)$$

- ▶ Chan and Hsiao (2014):
 - ▶ banded sparse matrix algebra problem
 - ▶ A faster implementation Carter and Kohn's forward-backward recursion

Estimation Method

Step 4: Update SV coefficients

- ▶ Given new draws $\mathbf{H}^T := \{\mathbf{h}_t\}_{t=1}^T$

- ▶ Note

$$\boldsymbol{\nu}_t = \mathbf{h}_t - \mathbf{h}_{t-1} \quad \boldsymbol{\nu}_t \sim \mathcal{N}\left(0, \text{diag}(\check{\boldsymbol{\theta}})\right)$$

- ▶ Back out $\check{\boldsymbol{\theta}}$ from inverse Gamma distro of $\boldsymbol{\nu}_t$

$$\check{\boldsymbol{\theta}} \leftarrow \text{InvGamma}(\mathbf{H}^T)$$

Estimation Method

Step 5: Propose competing model θ^{alt}

- Draw alternative DSGE model

$$\theta^{alt} \leftarrow \text{RandomWalk}(\theta; V, \theta_0 \sim p(\theta_0))$$

Scaling covariance V (jump size); $p(\theta_0)$ prior density

- Solve for approximate Markov equilibrium:

$$\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}} \leftarrow \text{SolveREE}(\theta^{alt})$$

- This gives the (log)-linear IVAR model

$$\mathbf{y}_{t+1} = \mathbf{A}_{\theta^{alt}} \mathbf{y}_t + \mathbf{B}_{\theta^{alt}} \mathbf{e}_t$$

$$\mathbf{y}_t^o = \mathbf{H}^o \mathbf{y}_t$$

conditional on structural shock process \mathbf{e}_t

Estimation Method

Step 6: Metropolis-Hastings

Conditional on SVs:

- Construction data likelihood function of competing models:

$$p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T) \leftarrow \text{KalmanFilter}(\mathbf{A}_\theta, \mathbf{B}_\theta, \mathbf{H}^T)$$

$$p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T) \leftarrow \text{KalmanFilter}(\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}}, \mathbf{H}^T)$$

- Metropolis-Hastings decision:

- Acceptance ratio

$$a \leftarrow \min \left\{ 1, \frac{p(\mathbf{y}^{o,T}|\theta^{alt}, \mathbf{H}^T)p(\theta^{alt})}{p(\mathbf{y}^{o,T}|\theta, \mathbf{H}^T)p(\theta)} \right\}$$

- Accept new proposal $\theta^{alt}|\mathbf{H}^T$ with probability a

Estimation Method

Algorithm 1 Stochastic optimization of model posterior

```
1: Initialize  $\theta \leftarrow p(\theta), \check{\theta} \leftarrow p(\check{\theta}), \mathbf{H}^T, \mathbf{m} = \text{list}[]$ 
2:  $\mathbf{A}_\theta, \mathbf{B}_\theta \leftarrow \text{SolveREE}(\theta)$ 
3: for all  $n$  in  $\text{range}(N)$  do
4:    $\boldsymbol{\eta}^T \leftarrow \text{DKsmoother}(\mathbf{A}_\theta, \mathbf{B}_\theta)$ 
5:    $\mathbf{s}^T \leftarrow \text{KimShephardChib}(\boldsymbol{\eta}^T, \mathbf{H}^T, \check{\theta})$ 
6:    $\mathbf{H}^T \leftarrow \text{ChanHsiao}(\mathbf{s}^T)$  or  $\text{CarterKohn}(\mathbf{s}^T)$ 
7:    $\check{\theta}^{new} \leftarrow \text{InvGamma}(\mathbf{H}^T)$ 
8:    $\theta^{alt} \leftarrow \text{RandomWalk}(\theta; V, \theta_0 \sim p(\theta_0))$ 
9:    $\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}} \leftarrow \text{SolveREE}(\theta^{alt})$ 
10:   $p(\mathbf{y}^{o,T} | \theta, \mathbf{H}^T) \leftarrow \text{KalmanFilter}(\mathbf{A}_\theta, \mathbf{B}_\theta, \mathbf{H}^T)$ 
11:   $p(\mathbf{y}^{o,T} | \theta^{alt}, \mathbf{H}^T) \leftarrow \text{KalmanFilter}(\mathbf{A}_{\theta^{alt}}, \mathbf{B}_{\theta^{alt}}, \mathbf{H}^T)$ 
12:   $a \leftarrow \min \left\{ 1, \frac{p(\mathbf{y}^{o,T} | \theta^{alt}, \mathbf{H}^T) p(\theta^{alt})}{p(\mathbf{y}^{o,T} | \theta, \mathbf{H}^T) p(\theta)} \right\}$ 
13:   $u \sim \mathbf{U}([0, 1])$ 
14:  if  $a > u$  then
15:     $(\theta, \check{\theta}, \mathbf{H}^T) \leftarrow (\theta^{alt}, \check{\theta}^{new}, \mathbf{H}^T)$ 
16:  end if
17:   $\mathbf{m.append}(\theta, \check{\theta}, \mathbf{H}^T)$ 
18: end for
```

Estimation Method

Implied Gibbs sampler with correct posterior densities

Algorithm (Step 1-6): a Gibbs sampler with conditional blocks

B1. $\mathbf{H}^T \sim \tilde{p}(\mathbf{H}^T | \mathbf{y}^{o,T}, \theta, \mathbf{s}^T)$

- ▶ (posterior from last Metropolis Hastings step as proposal density)

B2. $(\theta, \mathbf{s}^T) \sim \tilde{p}(\theta, \mathbf{s}^T | \mathbf{y}^{o,T}, \mathbf{H}^T)$ via marginals:

- ▶ $\theta \sim p(\theta | \mathbf{y}^{o,T}, \mathbf{H}^T)$ (convenient, does not depend on \mathbf{s}^T)
- ▶ $\mathbf{s}^T \sim p(\mathbf{s}^T | \mathbf{y}^{o,T}, \theta, \mathbf{H}^T)$ (via Kim-Shephard-Chib)

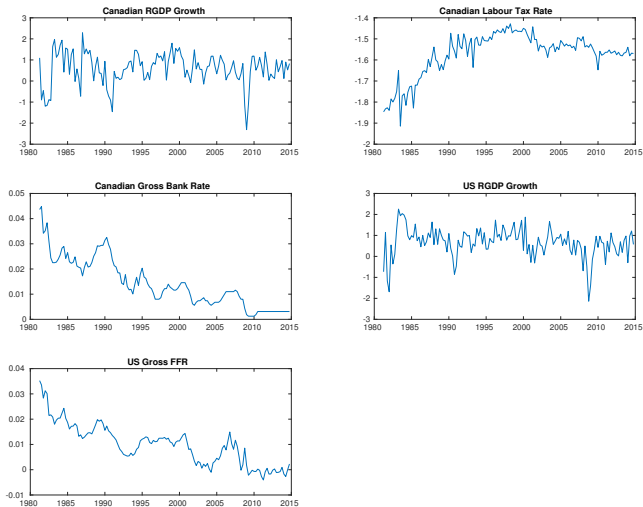
Turns out, this sampler eventually induces the correct posterior density of (θ, \mathbf{H}^T) or equivalently $(\theta, \check{\theta})$

- ▶ Stroud, Müller, Poulson (2003, *JASSA*)
- ▶ Del Negro and Primiceri (2015, *ReStud*, corrigendum's online appendix)

Results

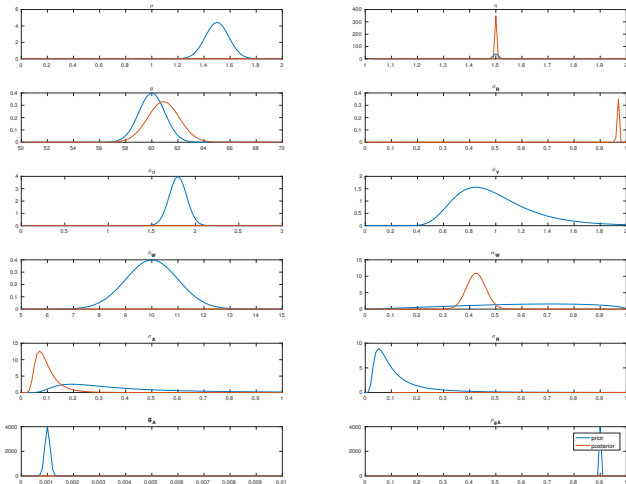
Observed data

Figure: Data (Canada)



Results

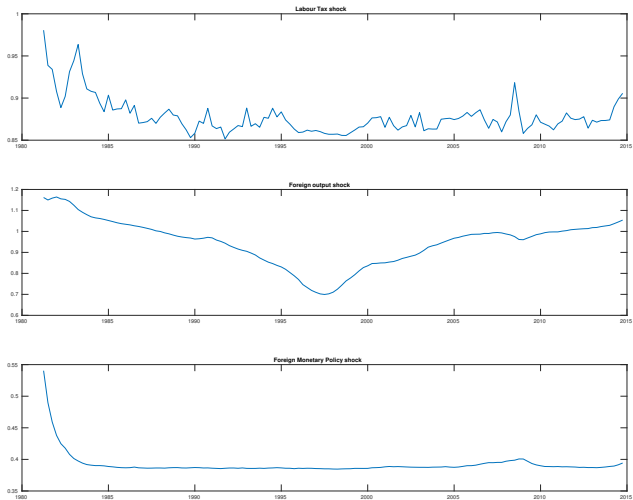
Figure: Prior and Posterior parameter densities



Results

Estimated SV shocks (Top 2 panels)

Figure: SV of (Y^*, R^*, τ_W)



Which Uncertainty
Shocks?

How Important?

Importance of Uncertainty Shocks

Historical Decomposition of Shocks

Impulse response function (\mathbf{y}_1 given) to shock $\mathbf{e}_{m,t-j}$:

$$\mathbf{y}_t^o(m) = \mathbf{H}^o \left[(\mathbf{A}_\theta)^{t-1} \mathbf{y}_1 + \sum_{j=0}^{t-2} (\mathbf{A}_\theta)^j \mathbf{B}_\theta \mathbf{S}_m \mathbf{e}_{t-j} \right], \quad \forall t > 1$$

- \mathbf{S}_m selection matrix on a particular structural shock $\mathbf{e}_{m,t-j}$

Historical decomposition at date t , of shock m 's impact on (observable) variable n :

$$HD_{n,t}(m) := \frac{|\mathbf{y}_{n,t}^o(m)|}{\sum_{m=1}^M |\mathbf{y}_{n,t}^o(m)|}$$

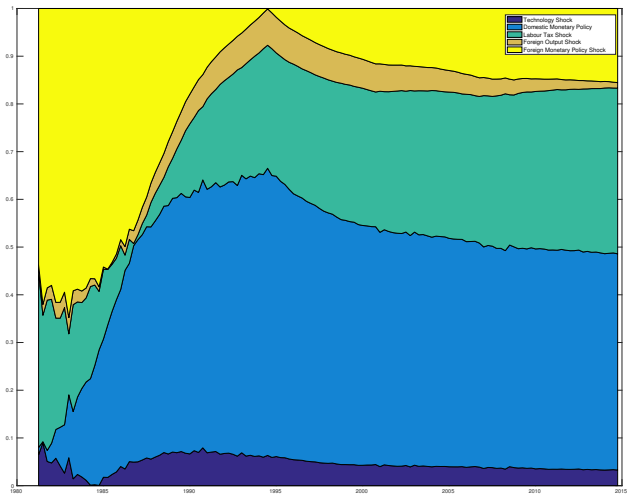
Given posterior estimate of model:

- Get simulated sample $\{\mathbf{e}_{m,t}\}_{t=2}^T$ of structural shocks (which contains SVs), for each $m = 1, \dots, M$
- Given focus on variable $\mathbf{y}_{n,t}^o$, evaluate accounting transforms $\{HD_{n,t}(m) : m = 1, \dots, M\}_{t=2}^T$

Importance of Uncertainty Shocks

Historical Decomposition of Shocks

Figure: Historical Decomposition



Importance of Uncertainty Shocks

Discussion

Canadian shock history (conditional on structural model and identified structural shocks):

- ▶ Over the last 30 years, innovations in US monetary policy dominant, 50-60%
- ▶ Dropped to less than 20% in 1990s, increasing again post-GFC (2008)
- ▶ Interestingly, US output variations not so dominant
- ▶ **Domestic Fiscal Uncertainty** has dominated the historical influence of structural shocks (35% recent)
- ▶ **Domestic Monetary Policy** shocks too (40% recent)

Summary

Global or Fiscal Uncertainty?

Method: How to identify structural (demand/supply/policy) shocks?

- ▶ Reduced-form with loose theory not palatable—difficulty of interpretation and shown to be empirically misleading
- ▶ Strong theoretical restrictions—Markov equilibrium as time-series representation
 - ▶ but, possibly wrong cross-equation restrictions (alternative structures, Bayesian model comparison)
 - ▶ still, a need for plausible and internally consistent interpretation of “shocks” and “uncertainty shocks”

Application: Historical Shock Decomposition Exercise

- ▶ **Domestic Fiscal Uncertainty** has dominated the historical influence of structural shocks (35% recent)
- ▶ **Domestic Monetary Policy** shocks too (40% recent)

To Do List

1. Alternative data sets

- ▶ Define “world output” to be more than just US output!
- ▶ Australia
- ▶ New Zealand

2. Bayesian model comparison

- ▶ Alternative DSGE structures and mechanisms
- ▶ Alternative SV sources of structural shocks:
 - ▶ Energy Economics

3. China and the small open economies

- ▶ Australia
- ▶ New Zealand
- ▶ East Asia

Job Candidates you should meet



Jamie Cross

<https://sites.google.com/site/jamiecross89/>

Time series, Macroeconomics, Energy Economics



Aubrey Poon

<https://sites.google.com/site/aubreybcpoon/>

Time series, Macroeconomics, ECB consultant