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IIT-JEE (Advanced)
Batch – Growth (May) | Major Test-1 (Paper-II)

Time: 3 Hours | **Test Date: 11th August 2024** | **Maximum Marks: 300**

Name of the Candidate: ____Roll No.____________

Centre of Examination (in Capitals):____

Candidate's Signature:________________________Invigilator's Signature:________________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

2. This Test Booklet consists of 51 questions.

3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 17 questions each and every **PART** has four sections.

(i) **Section-I** contains **4** single choice questions with only one correct option.

Marking scheme: +3 for correct answer, 0 if not attempted and –1 in all other cases.

(ii) **Section-II** contains **3** Question Multiple Choice Option with more than one correct answer.

Marking scheme: (+4 for correct answer, 0 if not attempted and +1 partial marking –2 in all other cases.

(iii) **Section-III** contains **6** Non-Negative Integer Value questions.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

(iv) **Section-IV** contains **4** Paragraph (Numerical Value) questions.

Marking scheme: +3 for correct answer, 0 if not attempted and 0 in all other cases.

4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.

5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.

7. For integer-based questions, the answer should be in decimals only not in fraction.

8. If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.

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TEST SYLLABUS
Batch – IIT JEE Growth May Major-1 P2
11th August 2024

Mathematics: FOM-1 (Real Numbers, Complex Numbers, Even Numbers, Odd Numbers, Composite Numbers, Co-Prime Numbers/ Relatively Prime Numbers, Twin Prime Numbers, LCM and HCF, Indices, Polynomial in One Variable, Degree of Polynomials, Some Special Types of Polynomials, Value and Zeros of a Polynomial, Roots of a Polynomial Equation, Remainder Theorem, Factor Theorem, Factorization, Sets, Types of Sets, Laws of Algebra of Sets (Properties of Sets), INTERVALS AS A SUBSET OF R Venn Diagram), FOM-2 (LINEAR INEQUALITIES, WAVY CURVE, METHOD, Rational Inequalities, IrrationalInequalities, Modulus Inequalities, Logarithmic & Exponential Inequality), Logaritm & (Function - NCERT), Sequence & Series

Physics: Basic Mathematics (Except Vector), Basic Mathematics (Vector) ,Units & Dimension, Kinematics -1D, Kinematics-2D.

Chemistry: Mole Concept & Concentration terms - 1 (Importance of chemistry, Nature of matter, Sig.figure, Laws of chemical combination, Avogadro law, Dalton's atomic theory, Atomic and molecular masses, Till Average/ Mean Atomic Mass), Mole Concept & Concentration terms -2 (Percentage composition,Stoichiometric calculations, Limiting reagent & Concentration, terms Equivalent Concept), Atomic Structure, Periodic Table & Periodic Properties

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821\,Lit\ atm\ K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}\,Js$
		$= 6.25 \times 10^{-27}\,erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66 \times 10^{-27}\,kg$
1 eV		$= 1.6 \times 10^{-19}\,J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I

1. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(A) 80 percent

(B) 40 percent

(C) 60 percent

(D) 70 percent

Ans. (C)

Sol. $n(C) = 20$

$n(B) = 50$

$n(C \cap B) = 10$

$n(C \cup B) = n(C) + n(B) - n(C \cap B)$

$= 20 + 50 - 10 = 60$

2. If the arithmetic progression whose common difference is non-zero, the sum of first 3n terms is equal to the sum of next n terms. Then, the ratio of the sum of the first 2n terms to the sum of next 2n terms is.

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$

Ans. (C)

Sol. Let 'a' be the first term and 'd' be the common difference of A.P.

$\therefore S_{3n} = S_{n(next)}$

$\Rightarrow S_{3n} + S_{3n} = S_{3n} + S_{n(next)}$

$\Rightarrow 2S_{3n} = S_{4n}$

$\Rightarrow 2 \times \frac{3n}{2}[2a + (3n-1)d] = \frac{4n}{2}[2a + (4n-1)d]$

$\Rightarrow 2a = (1 - n)d$ ...(i)

To find: $\frac{S_{2n}}{S_{2n(next)}} = \frac{\frac{2n}{2}[2a+(2n-1)d]}{\frac{2n}{2}[2(a+(2n+1-1)d)+(2n-1)d]}$

$= \frac{2a+(2n-1)d}{2a+(6n-1)d}$

$= \frac{(1-n)+(2n-1)}{(1-n)+(6n-1)}$ {from (i)}

$= \frac{1}{5}$

3. Find the value of $\sqrt[3]{5^{\frac{1}{\log_7 5}} + \frac{1}{(-\log_{10} 0.1)}}$.

(A) 1

(B) 2

(C) 3

(D) 4

Ans. (B)

Sol. Given expression becomes $\sqrt[3]{5^{log_5 7}+1}=\sqrt[3]{8}=2$

4. If $\log_2 x + \log_2 y \geq 6$, then the least value of x + y is:

(A) 2

(B) 4

(C) 8

(D) 16

Ans. (D)

Sol. $\log_2 x + \log_2 y \geq 6$;   x, y > 0

$\Rightarrow \log_2(xy) \geq 6$ $\Rightarrow xy \geq 2^6$ $\Rightarrow xy \geq 64$

$\therefore$ x and y are positive

$\therefore x+y=\left(\sqrt{x}\right)^2+\left(\sqrt{y}\right)^2=\left(\sqrt{x}-\sqrt{y}\right)^2+2\sqrt{xy}$

$\Rightarrow x+y \geq 2\sqrt{xy} \geq 2\times 8$

$\Rightarrow x+y \geq 16$

$\therefore$ least value of x + y = 16

SECTION- II

5. In a survey, it was found that 21 persons liked product A, 26 liked product B and 29 liked product C. If 14 persons liked products A and B, 12 liked products C and A, 13 persons liked products B and C and 8 liked all the three products then which of the following is (are) true?

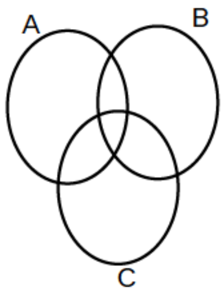
(A) The number of persons who liked the product C only =12

(B) The number of persons who like the products A and B but not C = 16

(C) The number of persons who liked the product C only =6

(D) The number of persons who like the products A and B but not C = 12.

Ans. **(A, B)**



Sol. n(A) = 21, n(B) = 26, n(C) = 29

$n(A \cap B) = 14$, $n(C \cap A) = 12$, $n(B \cap C) = 13$

$n(A \cap B \cap C) = 8$

(A) n(C only) = $n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$

= 29 – 12 – 13 + 8 = 12

(B) n(A and B but not C) = $n(A \cup B) - n(A \cap C) - n(A \cap B) + n(A \cap B \cap C)$

= $n(A) + n(B) - n(A \cap C) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$

= 21 + 26 – 12 – 13 – 14 + 8

= 55 – 39 = 16

6. The number $N = \frac{1+2\log_3 2}{(1+log_3 2)^2} + log_6^2 2$ when simplified reduced to

(A) a prime number

(B) an irrational number

(C) a real number which is less $log_3\pi$

(D) a real number which is greater than $log_7 6$

Ans. (C, D)

Sol. $N = \frac{log_3 3+log_3 4}{(log_3 3+log_3 2)^2} + (log_6 2)^2$

$= \frac{log_3 12}{(log_3 6)^2} + (log_6 2)^2$

$= \frac{(2log_3 2+1)+(log_6 2 log_3 6)^2}{(log_3 6)^2}$

$= \frac{(2log_3 2+1)+(log_3 2)^2}{(log_3 6)^2} = \frac{(log_3 2+1)^2}{(log_3 2+1)^2} = 1$

7. Given that α, γ are the roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation $Bx^2 - 6x + 1 = 0$. If α, β, γ and δ are in H.P. then

(A) A = 3

(B) B = 6

(C) A = 4

(D) B = 8

Ans. (A, D)

Sol. $Ax^2 - 4x + 1 = 0$ (α, γ are its root)

$Bx^2 - 6x + 1 = 0$ (β, δ are its root)

α, β, γ, δ in H.P.

$\frac{1}{\alpha} - \frac{1}{\gamma} = \frac{1}{\beta} - \frac{1}{\delta}$

$\frac{(\gamma-\alpha)^2}{\alpha^2\gamma^2} = \frac{(\delta-\beta)^2}{\beta^2\delta^2}$

$\frac{(\alpha+\gamma)^2-4\alpha\gamma}{\alpha^2\gamma^2} = \frac{(\beta+\delta)^2-4\beta\delta}{\beta^2\delta^2}$

$\frac{\frac{16}{A^2}-\frac{4}{A}}{\frac{1}{A^2}} = \frac{\frac{36}{B^2}-\frac{4}{\beta}}{\frac{1}{B^2}}$

A = 3 B = 8

SECTION-III

8. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is:

Ans. (5)

Sol. $x^2 + 9 < (x + 3)^2 < 8x + 25$

$\Rightarrow$ $x^2 + 9 < (x + 3)^2$ and $(x + 3)^2 < 8x + 25$

$\Rightarrow$ $x^2 + 9 < x^2 + 9 + 6x$ and $x^2 + 9 + 6x < 8x + 25$

$\Rightarrow$ $6x > 0$ and $x^2 - 2x - 16 < 0$

$\Rightarrow$ $x > 0$ and $x \in (1 - \sqrt{17}, 1 + \sqrt{17})$

Intersection gives $x \in (0, 1+\sqrt{17})$

So, the number of integral values satisfying this is 5.

9. Number of non-negative integral values of x satisfying the inequality

$\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

Ans. (2)

Sol. $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$

$\Rightarrow \quad \frac{2(x+1)-(x^2-x+1)-(2x+1)}{(x+1)(x^2-x+1)} \geq 0$

$\Rightarrow \quad \frac{2x+2-x^2+x-1-2x-1}{(x+1)(x^2-x+1)}$

$\Rightarrow \quad \frac{x^2-x}{x+1} \leq 0 \qquad (\because x^2 - x + 1 > 0\ \forall x \in R)$

$\Rightarrow \quad \frac{x(x-1)}{(x+1)} \leq 0$

$x \in (-\infty, -1) \cup [0, 1]$

2 non-negative integral values of 'x'.

10. Let $a_1, a_2, \ldots, a_{10}$, be in A.P. & $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is:

Ans. (6)

Sol. $a_1, a_2 \ldots, a_{10} \rightarrow$ A.P. $\qquad$ d = common difference

$h_1, h_2, \ldots h_{10} \rightarrow$ H.P.

$\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_{10}} \rightarrow$ A.P. $\qquad$ d_1 = common difference

So, $a_1 = h_1 = 2$

and $a_{10} = h_{10} = 3$

$\Rightarrow a_1 + 9d = 3$ and $\frac{1}{h_{10}} = \frac{1}{3}$

$\Rightarrow 2 + 9d = 3$ and $\frac{1}{h_1} + 9d_1 = \frac{1}{3}$

$\Rightarrow 9d = 1$ and $\frac{1}{2} + 9d_1 = \frac{1}{3}$

$\Rightarrow d = \frac{1}{9}$ $\qquad$ $\Rightarrow 9d_1 = -\frac{1}{6}$

$\Rightarrow d_1 = -\frac{1}{54}$

and $a_4h_7 = (a_1 + 3d)\left(\frac{1}{\frac{1}{h_1}+6d_1}\right)$

$= \left(2 + \frac{3}{9}\right)\frac{1}{\left(\frac{1}{2}-\frac{6}{54}\right)} = \frac{7}{3} \times \frac{18}{7} = 6$

$a_4h_7 = 6$

11. Let 'X' denotes the value of the product

$(1 + a + a^2 + a^3 + \ldots \infty)(1 + b + b^2 + b^3 + \ldots \infty)$

where 'a' and 'b' are the roots of the quadratic equation $11x^2 - 4x - 2 = 0$

and 'Y' denotes the numerical value of the infinite series

$(\log_b 2)^0(\log_b 5^{4^0}) + (\log_b 2)^1(\log_b 5^{4^1}) + (\log_b 2)^2(\log_b 5^{4^2}) + (\log_b 2)^3(\log_b 5^{4^3}) + \cdots \infty$

where b = 2000. Find 15xy.

Ans. (11)

Sol. $11x^2 - 4x - 2 = 0$ (a, b are its roots); $a + b = \frac{4}{11}$, $ab = -\frac{2}{11}$

$x = (1 + a + a^2 + a^3 + \ldots \infty)(1 + b + b^2 + b^3 + \ldots \infty)$

$x = \frac{1}{1-a} \times \frac{1}{1-b} = \frac{1}{1-(a+b)+ab} = \frac{1}{1-\frac{4}{11}-\frac{2}{11}} = \frac{11}{5}$

Also,

$y = (\log_b 2)^0 \cdot 4^0 (\log_b 5) + (\log_b 2)^1 \cdot 4^1 \cdot (\log_b 5) + (\log_b 2)^2 \cdot 4^2 \cdot (\log_b 5) + \cdots \infty$

$\Rightarrow y = (\log_b 5)[1 + 4^1 (\log_b 2)^1 + 4^2 \cdot (\log_b 2)^2 + \cdots \infty]$ $\Rightarrow \infty$ G.P.

$= (\log_b) \times \frac{1}{1-4\cdot(\log_b 2)} = \log_{2000} 5 \times \frac{1}{\log_{2000} 2000 - \log_{2000} 16}$

$= \log_{2000} 5 \times \frac{1}{\log_{2000}\left(\frac{2000}{16}\right)} = \log_{2000} 5 \times \frac{1}{\log_{2000} 125}$

$\Rightarrow y = \frac{1}{3}$ $\therefore xy = \frac{11}{15}$

12. How many positive integers b have the property that $\log_b 729$ is a positive integer?

Ans. (4)

Sol. $\log_b 729 = 6\log_b 3$

If this is an integer, then $b = 3, 3^2, 3^3, 3^6$

13. If mantissa of number N to the base 32 is varying from 0.2 to 0.8 both inclusive, and whose characteristic is 1, then find the number of integral values of N.

Ans. (449)

Sol. $1.2 \le \log_{32} x \le 1.8$

$(32)^{1.2} \le x \le (32)^{1.8}$

$2^6 \le x \le 2^9$

$64 \le x \le 512$

Hence, Number of integral values = 449.

SECTION-IV

PARAGRAPH-I (Question 14 & 15)

$Q(k) = log_3 x + log_{\sqrt{3}} \sqrt{x} + log_{\sqrt[3]{3}} \sqrt[3]{x} + \ldots + log_{\sqrt[k]{3}} \sqrt[k]{x}$

14. If $Q(10) = 5$ and $R^{log_2(Q(20))} = 4$ then $R^{(log_2 Q(20))^2}$ is equal to

Ans. (100)

Sol. $Q(k) = log_3 x + log_{\sqrt{3}} \sqrt{x} + log_{\sqrt[3]{3}} \sqrt[3]{x} + \ldots + log_{\sqrt[10]{3}} \sqrt[10]{x}$

$= \log_3 x + \log_3 x + \log_3 x + \ldots + \log_3 x$

$= 10 \log_3 x$

$10 \log_3 x = 5$

$log_3 x = \frac{1}{2}$

$x = \sqrt{3}$

$R^{\left(log_2\left(20log_3\sqrt{3}\right)\right)} = R^{(log_2(10))} = 4$

$R^{(log_2 10)^2} = R^{(log_2 10)(log_2 10)} = 4^{log_2 10}$

= 100

15. Number of real solutions of the equation |x-7| + |2x - Q(20)| = 12 is

(Note: Q(20) is same as obtained in previous question)

Ans. (2)

Sol. $|x - 7| + |2x - 10| = 12$

$|x - 7| + 2|x - 5| = 12$

$x \leq 5$

$- x + 7 - 2x + 10 = 12$

$- 3x = 12 - 17$

$x = 5/3$

$5 < x < 7$

$- x + 7 + 2x - 10 = 12$

$x = 15$ (rejected)

$x \geq 7$

$x - 7 + 2x - 10 = 12$

$3x = 29$

$x = \frac{29}{3}$

2 possible values.

PARAGRAPH-II (Question 16 & 17)

Two consecutive numbers from n natural numbers 1, 2, 3, ..., n are removed. Arithmetic mean of the remaining numbers is $\frac{105}{4}$

16. The value of n is:

Ans. (50)

Sol. Let 2 numbers removed be x, x + 1

$\frac{n(n+1)}{2} - 2x - 1 = \frac{105}{4}(n-2)$

$n^2 + n - 4x - 2 = \frac{105(n-2)}{2}$

$2n^2 + 2n - 8x - 4 = 105n - 210$

$2n^2 - 103n + 206 = 8x$

$x = \frac{2n^2-103n+206}{8}$

(A) n = 48 $\quad x = \frac{-103}{8}$ $\quad$ (B) n = 50 $\quad x = \frac{56}{8} = 7$

(C) n = 52 $\quad x = \frac{258}{8} = 32.25$ $\quad$ (D) n = 49 $\quad x = \frac{-39}{8}$

17. Let removed number are x_1, x_2 then $x_1 + x_2 + n =$

Ans. (65)

Sol. $x_1 + x_2 + n$

$= 7 + 8 + 50$

$= 65$

PHYSICS

SECTION-I

18. If frequency F, velocity V, and density D are considered fundamental units, the dimensional formula for momentum will be

(A) DVF^2

(B) DV^2F^{-1}

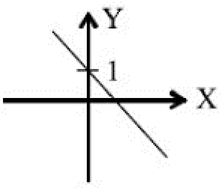
(C) $D^2V^2F^2$

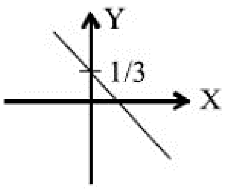
(D) DV^4F^{-3}

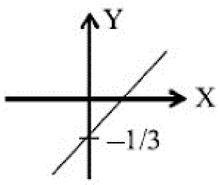
Ans. (D)

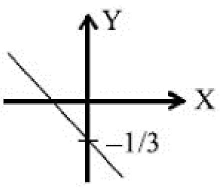
Sol. Momentum, $p = mv = MLT^{-1} = ML^{-3}L^4T^{-4}T^3 = DV^4F^{-3}$

19. Correct graph of 4x + 3y + 1 = 0 is

(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. Conceptual

20. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path.

The relation between H, u and n is

(A) $2gH = n^2u^2$

(B) $gH = (n-2)^2u^2$

(C) $2gH = nu^2(n-2)$

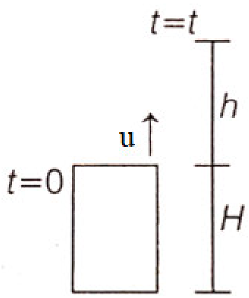
(D) $gH = (n-2)^2u^2$

Ans. (C)

Sol. $t \rightarrow$ time reach to maximum height. nt is total time of motion.

$h = ut - \frac{1}{2}gt^2$ ...(i)

$H + h = \frac{1}{2}g(nt - t)^2$ ...(ii)



On solving these both equations

$H + \frac{u^2}{g} - \frac{gu^2}{g^2} = \frac{gu^2}{2g^2}(n^2 - 2n)$

$= H = \frac{u^2}{2g}n(n-2)$

$= 2gH = u^2n(n-2)$

21. If two vectors $\mathbf{P} = \hat{\mathbf{i}} + 2m\hat{\mathbf{j}} + m\hat{\mathbf{k}}$ and $\mathbf{Q} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + m\hat{\mathbf{k}}$ are perpendicular to each other, then, the value of m will be

(A) 3

(B) -1

(C) 2

(D) 1

Ans. (C)

Sol. Given vectors,

$\mathrm{P} = \hat{\mathbf{i}} + 2m\hat{\mathbf{j}} + m\hat{\mathbf{k}}$

and $\mathrm{Q} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + m\hat{\mathbf{k}}$

As, **P** and **Q** are given perpendicular vectors, so $\mathbf{P} \cdot \mathbf{Q} = 0$

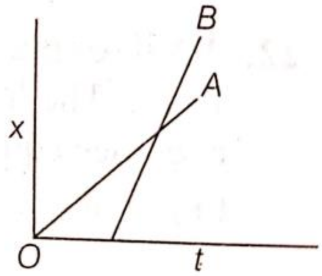
$\Rightarrow (\hat{\mathbf{i}} + 2m\hat{\mathbf{j}} + m\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + m\hat{\mathbf{k}}) = 0$

$\Rightarrow 4 - 4m + m^2 = 0$

$\Rightarrow m = 2$

SECTION-II

22. The position-time graphs for two students A and B returning from the school to their homes are shown in figure.



(a) A lives closer to the school

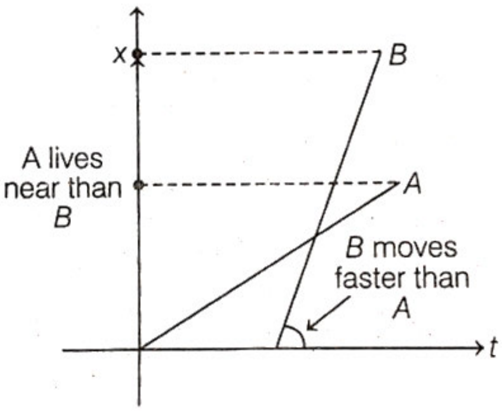
(b) B lives closer to the school

(c) A travels faster than B

(d) B travels faster than A

Ans. (A, D)

Sol.



Given, slope of $B >$ slope of A. So, B is fast.

Projection of A on X-axis is less than projection of B. So, A lives near to school.

23. A particle is moving along a curve. Then

(A) if its speed is constant it has no acceleration

(B) if its speed is increasing the acceleration of the particle is along its direction of motion

(C) if its speed is constant the magnitude of its acceleration is also constant.

(D) the direction of its acceleration cannot be along the tangent.

Ans. (C, D)

Sol. Conceptual

24. The displacement x of a particle depend on time t as $x = \alpha t^2 - \beta t^3$

(A) particle will return to its starting point after time $\frac{\alpha}{\beta}$.

(B) the particle will come to rest after time $\frac{2\alpha}{3\beta}$

(C) the initial velocity of the particle was zero but its initial acceleration was not zero.

(D) no net force act on the particle at time $\frac{\alpha}{3\beta}$

Ans. (A, B, C, D)

Sol. (A) $x = 0 = \alpha t^2 - \beta t^3$

$t = \frac{\alpha}{\beta}$

$v = 2\alpha t - 3\beta t^2$

(B) $v = 0$

$t = \frac{2\alpha}{3\beta}$

$a = 2\alpha - 6\beta t$

at $t = 0$, $v = 0$

$a = 2\alpha$ (C)

at $t = \frac{\alpha}{3\beta}$, $a = 0$ (D)

SECTION-III

25. Value of $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]$ will be equal to

Ans. (1)

Sol. $\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$

$\cos(2\pi + \theta) = \cos\theta$

$\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta$

$\cot(2\pi + \theta) = \cot\theta$

Hence given expression becomes

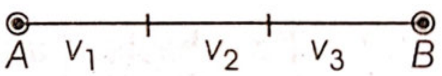
$\sin\theta\cos\theta\,[\tan\theta + \cot\theta] = \sin\theta\cos\theta\left[\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right] = 1$

26. The heat generated in a circuit is given by $Q = I^2Rt$, where I is current R is resistance, and t is time. If the percentage errors in measuring I, R, and t are 2%, 1% and 1% respectively, then the maximum percentange error in measuring heat will be

Ans. (6%)

Sol. **Required error is** $2\times 2\% + 1\% + 1\%$, *i.e.*, 6%.

27. A car covers AB distance with first one-third at velocity v_1 ms^{-1}, second one-third at v_2 ms^{-1} and last one-third at v_3 ms^{-1}. If $v_3 = 3v_1, v_2 = 2v_1$ and $v_1 = 11$ ms^{-1}, then the average velocity of the car is .... ms^{-1}.



Ans. (18)

Sol. Let the total distance be l

$$v_{avg} = \frac{l_{total}}{t_{total}} = \frac{l/3 + l/3 + l/3}{\frac{l}{3v_1} + \frac{l}{3v_2} + \frac{l}{3v_3}}$$

[$\because v_1, v_2, v_3$ are three speeds]

$$= \frac{l}{\frac{l}{3}\left(\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}\right)} = \frac{1}{\frac{1}{3}\left(\frac{1}{11}+\frac{1}{22}+\frac{1}{33}\right)} \quad \begin{bmatrix} v_1 = 11\text{m/s} \\ v_2 = 2v_1 \\ v_3 = 3v_1 \end{bmatrix}$$

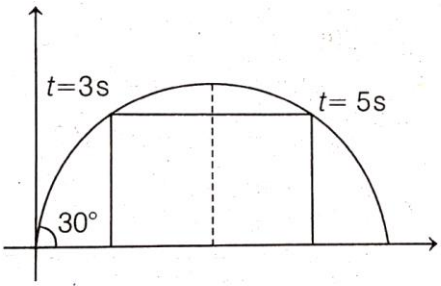
$$= \frac{33}{1+\frac{1}{2}+\frac{1}{3}} = \frac{198}{6+3+2} = \frac{198}{11} = 18\text{m/s}$$

28. A projectile fired at 30° to the ground is observed to be at same height at time 3s and 5s after projection, during its flight. The speed (in ms^{-1}) of projection of the projectile is ms^{-1}

(Given, $g = 10\ \text{ms}^{-2}$)

Ans. (80)

Sol. Time of flight $= 5 + 3 = 8\ s$



$T = \frac{2u\sin\theta}{g} \Rightarrow 8 = \frac{2u\sin 30^\circ}{10}$

$\Rightarrow u\sin 30^\circ = 40$

$\Rightarrow u \times \frac{1}{2} = 40$

$\Rightarrow u = 80\ \text{m/s}$

29. A ball is thrown vertically upwards from the ground. It crosses a point at the height of 25 m twice at an interval of 4 sec. The ball was thrown with the velocity (in ms^{-1}) of

Ans. (30.00)

Sol. Let initial velocity of ball = u

$$\therefore \quad 25 = ut - \frac{1}{2}gt^2 = ut - 5t^2$$

$$\Rightarrow \quad 5t^2 - ut + 25 = 0$$

If t_1 and t_2 are the two solutions,

Then, $t_2 - t_1 = \sqrt{(t_2+t_1)^2 - 4t_1t_2}$

$$\Rightarrow 4^2 = \frac{u^2}{25} - 4 \times \frac{25}{5}$$

$$\Rightarrow \quad 16 = \frac{u^2}{25} - 4 \times \frac{25}{5}$$

$$\Rightarrow u = 30 m/s$$

30. If $y = [3x + 2]\ [2x - 1]$, then $\frac{dy}{dx}$ at x = 1 is

Ans. (13)

Sol. Differentiating both sides, we get

$$\frac{dy}{dx} = \frac{d}{dx}[3x+2][2x-1]$$

Using product rule, we get

$$\frac{d[uv]}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = [3x+2]\frac{d}{dx}[2x-1] + [2x-1]\frac{d}{dx}[3x+2]$$

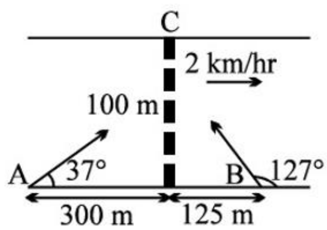
$= [3x + 2]\ [2] + [2x - 1]3$

$= 6x + 4 + 6x - 3 = 12x + 1$

SECTION-IV

Paragraph-I (31 & 32)

Two swimmers start a race. One who reaches the point C first on the other bank wins the race. A makes his strokes in a direction of 37^0 to the river flow with velocity 5 km/hr relative to water. B makes his strokes in a direction 127^0 to the river flow with same relative velocity. River is flowing with speed of 2 km/hr and is 100 m wide. Speeds of A and B on the ground are 8 km/hr and 6 km/hr respectively.



31. The time taken by A to reach the point C is (in seconds)

Ans. (165)

Sol. $t_1 = \dfrac{100}{3 \times \dfrac{5}{18}} = 120\,\text{sec}$

$$x = 6 \times \frac{5}{18} \times 120 = 200\,m$$

$$t_2 = \frac{100}{8 \times \frac{5}{18}} = 45 \text{ sec}$$

$$T = t_1 + t_2 = 165\,sec$$

32. The time taken by B to reach the point C is (in seconds)

Ans. (150)

Sol. $t_1 = \frac{100}{4 \times \frac{5}{18}} = 90 \text{ sec}$

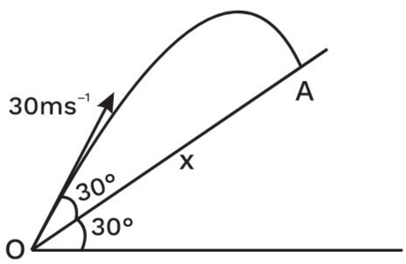
$$x = 1 \times \frac{5}{18} \times 90 = 25\,m$$

$$t_2 = \frac{125 - 25}{6 \times \frac{5}{18}} = 60\,sec$$

$$T = t_1 + t_2 = 150\,sec$$

Paragraph-II (33 & 34)

An object is projected up the incline at the angle shown in figure with an initial velocity of 30 ms^{-1}.



33. If T is the time of flight of the projectile then find T^2

Ans. (12)

Sol. $T = \frac{2 \times 30 \times \sin 30°}{10 \times \cos 30°} = 2\sqrt{3}$

34. The distance (in meter) x up the incline at which the object lands is

Ans. (60)

Sol.

$$x = \frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta} \text{ where } \alpha = 60° \text{ and } \beta = 30°$$

$$\Rightarrow x = \frac{2(30)^2 \sin(30°)\cos(60°)}{g\cos^2(30°)}$$

$$\Rightarrow x = \frac{(2)(900)(0.5)}{(10)\left(\frac{3}{4}\right)} = 60 \text{ m}$$

PART-: CHEMISTRY

SECTION-I

35. Arrange $Ce^{+3}, La^{+3}, Pm^{+3}$ and Yb^{+3} in increasing order of their ionic radii.

(A) $Yb^{+3} < Pm^{+3} < Ce^{+3} < La^{+3}$

(B) $Ce^{+3} < Yb^{+3} < Pm^{+3} < La^{+3}$

(C) $Yb^{+3} < Pm^{+3} < La^{+3} < Ce^{+3}$

(D) $Pm^{+3} < La^{+3} < Ce^{+3} < Yb^{+3}$

Ans (A)

Sol. Size of lanthanides progressively decreases as atomic number ↑.

36. IE(I) and IE(II) of Mg are 178 and 348kcalmol^{-1}. The energy required for the reaction $Mg \rightarrow Mg^{2+} + 2e^-$is :

(A) +170 kcal

(B) +526 kcal

(C) -170 kcal

(D) -526 kcal

Ans. (B)

Sol. $E = IE_1 + IE_2 = 178 + 348 = +526 \text{kcal}$

37. If electronegativity of X be 3.2 and that of Y be 2.2, the percentage ionic character of XY is :

(A) 19.5

(B) 18.5

(C) 9.5

(D) 29.5

Ans. (A)

Sol. $\text{EN}_x - \text{EN}_y = 3.2 - 2.2 = 1$

$\Delta = 1$

[Δ = difference of electronegativity values between x and y]

% ionic character $= 16\Delta + 3.5\Delta^2 = 19.5$

38. The correct order of decreasing electron affinity of $\text{B}, \text{C}, \text{N}$ and O is:

(A) $O > C > N > B$

(B) $B > N > C > O$

(C) $O > C > B > N$

(D) $O > B > C > N$

Ans. (C)

Sol. Conceptual

SECTION-II

39. Uncertainty in position is twice the uncertainty in momentum, uncertainty in velocity is

(A) $\sqrt{\frac{h}{\pi}}$

(B) $\frac{1}{2m}\sqrt{\frac{h}{\pi}}$

(C) $\frac{1}{2m}\sqrt{h}$

(D) $\frac{1}{2m}\sqrt{\frac{h}{2\pi}}$

Ans. (D)

Sol. $\Delta x = 2\Delta p$

Now $\Delta x.\Delta p \geq \frac{h}{4\pi}$

$2\Delta p^2 \geq \frac{h}{4\pi}$; $2(m\Delta v)^2 \geq \frac{h}{4\pi}$; $\Delta v \geq \frac{1}{2m}\sqrt{\frac{h}{2\pi}}$

40. Which of the following is/are characteristic/s of wave function ψ ?

(A) ψ must be single valued

(B) ψ must be finite

(C) $\int_{-\infty}^{+\infty} \psi^2 dx\,dydz = 1$

(D) ψ must be continuous

Ans. (A, B, C, D)

Sol. Conceptual

41. Ground state configuration of Nitrogen atom can be represented by:

(A) 

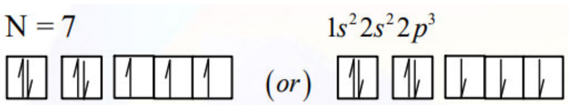
(B) 

(C) 

(D) 

Ans. (A, D)

Sol.



SECTION-III

42. 10 mL of a mixture of $CO(g)$ and $CH_4(g)$ was mixed with 22 mL of O_2 gas and subjected to sparking. The contraction observed when the residual gases are passed through alc. KOH is given by x mL. Find x ?

[All volumes are measured at same temperature and pressure]

Ans. (10)

Sol. $CO \xrightarrow{+O_2} CO_2$

$CH_4 \xrightarrow{+O_2} CO_2 + H_2O(l)$

volume of $(CO + CH_4) = 10$ mL Since 1mol CO gives

1 molCO_2 and 1 molCH_4 gives 1 mol CO_2

$\therefore$ Volume of CO_2 released $= 10$ mL

$\therefore$ Volume contraction $= 10$ mL

(since all CO_2 absorbed by KOH)

43. What would be the maximum volume (in mL) of 3M HCl solution that can be prepared by using 2M, 1L, HCl and 5M, 2L, HCl kept in separate vessels.

Ans. (1500)

Sol. $5 \times x + 2 \times 1 = 3(1 + x)$

$5x - 3x = 1$

$x = \frac{1}{2}$ L

$\therefore$ Total volume $= 1000$ mL $+ 500$ mL $= 1500$ mL.

44. In 1200 g solution, 12 g urea (NH_2CONH_2) is present. If density of the solution is 1.2 g/mL, then the molarity of the solution is xM. Find $10x$?

Ans. (2)

Sol. Wt of solution $= 1200$ g

wt of solute $= 12$ g

density solution $= 1.2$ g/mL

$\therefore$ Volume of solution $= \frac{1200}{1.2} = 1000$ mL

$M = \frac{\text{No of moles of solute}}{\text{volume of solution}} = \frac{12/60}{1\ L}$

$x = \frac{1}{5}$

$\therefore\ 10x = \frac{1}{5} \times 10 = 2$

45. 10 mL of a gaseous organic compound $C_xH_yO_zN_p$ (molar mass = 61) is taken in an eudiometry tube and mixed with sufficient oxygen gas such that volume becomes 42.5 mL. On sparking, some contraction was observed. On passing the residual gas through alcoholic KOH, a contraction of 20 mL was observed. The volume of the residual gas is 5 mL. All volumes are measured at room temperature and pressure. Calculate the value of the four-digit number '$xyzp$'.

Ans. (2711)

Sol. $C_xH_YO_ZN_P \xrightarrow[\text{combution}]{+O_2} xco_2 + \frac{y}{2}H_2O + \frac{P}{2}N_2$

$12x + y + 16z + 14P = 61$

Compound (10 mL) on combustion gives 20mL CO_2 means

$\therefore 10x = 20$

$x = 2$

Volume of residual gas $N_2 = 5$ mL

$\therefore \frac{P}{2} \times 10 = 5$

$P = 1$

Y and Z can not be - Ve or fraction

$\therefore y = 7$ and $z = 1$

$\therefore 12 \times 2 + 7 + 16 + 14 = 61\text{M}$ mass

$\therefore$ correct and 2711

46. Three substances A, B and C can react to form C and D as shown below.

$2\text{ A} + 3\text{ B} + \text{C} \rightarrow 4\text{D} + 2\text{E}$

If molar masses of A, B, C and D are 40,30, 20 and 15 respectively and 570 g of mixture of A, B, and C is reacted then calculate maximum mass of E which can be obtained (in gram).

Ans. (390)

Sol. $\underset{2a\text{ moles}}{2\text{A}} + \underset{3a\text{ moles}}{3\text{B}} + \underset{a\text{ moles}}{\text{C}} \rightarrow 4\text{D} + 2\text{E}$

for obtaining maximum mass of E there should be no reactant after reaction

$\therefore 2a \times 40 + 3a \times 30 + a \times 20 = 570$

$80a + 90a + 20a = 570$

$190a = 570$

$a = 3$

$\therefore n_A = 6$ mole

$n_B = 9$ mole

$n_C = 3$ mole

gives 4×3 mole of D and 2×3 moles of E

$\therefore$ Mass of $D = 12 \times 15 = 180$ g

Mass of $A + B + C =$ mass of (D) + (E) (mass conservation)

$570 = 180 +$ mass of E

mass of $\text{E} = 390\text{g}$

47. 10^{X} number of photons of radiation of frequency $5 \times 10^{13}\ \text{s}^{-1}$ that must be absorbed in order to melt one g ice when the latent heat of fusion of ice is 330 J/g. calculate value of x

use h = 6.6×10^{-34}

Ans. (22)

Sol. 330 J = n(h ν)

$330\text{ J} = n[6.6 \times 10^{-34} \times 5 \times 10^{13}]$

$\frac{330}{6.6\times10^{-34}\times5\times10^{13}} = n = 10^{22}$

SECTION-IV

PARAGRAPH-I (Question 48 & 49)

A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 and 17.00 eV, respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 and 5.95 eV, respectively.

48. The value of 'n' is

Ans. (6)

Sol. $$\Delta E = 13.6z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\text{eV}$$

$$10.2+17.0 = 13.6z^2\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

$$4.25+5.95 = 13.6z^2\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$$

n = 6

49. The value of Z is

Ans. (3)

Sol. $$\Delta E = 13.6z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\text{eV}$$

$$10.2+17.0 = 13.6z^2\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

$$4.25+5.95 = 13.6z^2\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$$

Z = 3

PARAGRAPH-II (Question 50 & 51)

The French physicist Louis de Broglie in 1924 postulated that matter, like radiation, should exhibit a dual behaviour. He proposed the following relationship between the wavelength lambda of a material particle, its linear momentum p and Planck constant h. $\lambda = \frac{h}{p} = \frac{h}{mv}$

The de Broglie relation implies that the wavelength of a particle should decrease as its velocity increases. It also implies that for a given velocity heavier particles should have shorter wavelength than lighter particles. The waves associated with particles in motion are called matter waves or de Broglie waves. These waves differ from the electromagnetic waves as they

(i) have lower velocities

(ii) have no electrical and magnetic fields and

(iii) are not emitted by the particle under consideration.

The experimental confirmation of the de Broglie's relation was obtained when Davisson and Germer, in 1927, observed that a beam of electrons is diffracted by a nickel crystal. As diffraction is a characteristic property of waves, hence the beam of electron behaves as a wave, as proposed by de Broglie.

50. The de Broglie wavelength of a vehicle moving with velocity v is λ. Its load is changed so that the velocity as well as kinetic energy is doubled. The new de Broglie wavelength is x × λ value of x is.

Ans. (1)

Sol. $p = mv = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}v} = \text{const} \Rightarrow \lambda = \text{Constant}$

51. An electron is continuously accelerated in a vacuum tube by the application of a potential difference. If its de Broglie wavelength decreases by 1% over a path length of l cm, then its kinetic energy is increases about y percent the value of y is.

Ans. (2)

Sol. $\lambda = \dfrac{h}{\sqrt{2mE}} \Rightarrow \dfrac{E_2}{E_1} = \left(\dfrac{\lambda_1}{\lambda_2}\right)^2 = \left(\dfrac{100}{99}\right)^2 \approx 1.02$

$\therefore E_2$ is about 2% greater than E_1.

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