FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
CONCEPT RECAPITULATION TEST – II
PAPER –2
TEST DATE: 24-04-2025

ANSWERS, HINTS & SOLUTIONS

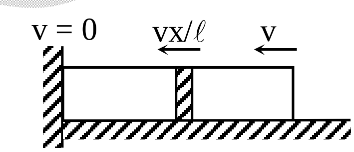
Physics

PART – I

SECTION – A

1. A

Sol. At any moment total energy of rod



$E = K + U$

$= \int dk + \frac{1}{2}\frac{AY}{\ell}x^2$

$= \frac{1}{6}mv^2 + \frac{1}{2}\frac{AY}{\ell}x^2$

$\Rightarrow \frac{dE}{dt} = \frac{ma}{3} + \frac{AY}{\ell}x = 0$

$\Rightarrow \omega = \sqrt{\frac{3AY}{\ell m}}$

2. B

Sol. Similar to gravitational force and orbital motion

$\Rightarrow v_{escape} = \sqrt{2}v_{orbital}$

3. A

Sol. $i_1 = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$

$i_2 = -\frac{2\varepsilon}{R}e^{-\frac{t}{RC}}$

$i_3 = -\frac{\varepsilon}{R}e^{-\frac{t}{RC}}$

4. C

Sol. Frequencey received by approaching car

$f_1 = f_0\left[1 + \frac{2}{330}\right]$

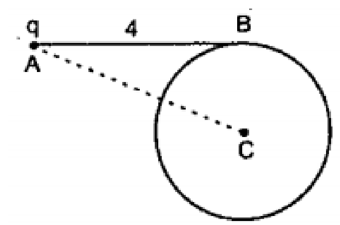
Frequency received by source again

$$f_2 = \frac{f_1}{\left(1-\frac{2}{330}\right)}$$

So, beat frequency $f_B = f_2 - f_0 = 6$ Hz.

5. AC

Sol. AC = 5 m



$$V = \frac{kq}{AC} = \frac{9\times10^9\times1\times10^{-6}}{5}$$

$= 1.8 \times 10^3 = 1.8$ kV

$V_B = (V_B)_{\text{due to q}} + (V_B)_i$

$(V_B)_i = -0.45$ kV

So, (A) and (C) are correct.

6. ABCD

Sol. Let elongation in spring A, B and C be x_1, x_2 and x_3 respectively.

Considering spring forces and constraint relations

$x_2 = 4x_3$ …(i)

$x_2 = 2x_1$ …(ii)

and $x_1 + 2x_2 + x_3 = x$ …(iii)

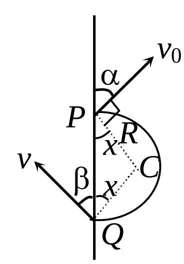
$$\Rightarrow \quad x_1 = \left(\frac{2}{11}\right)x \quad ; \quad x_2 = \left(\frac{4}{11}\right)x \quad ; \quad x_3 = \left(\frac{1}{11}\right)x$$

Also, $F = 2K\left(\frac{x}{11}\right)$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{11m}{2k}}$$

7. ABCD

Sol. $x + \alpha = \beta + x = 90^0 \Rightarrow \alpha = \beta$



Also, $PQ = 2R\cos(90-\alpha) = 2R\sin\alpha = \frac{2mv_0}{qB}\sin\alpha$

$|\vec{v}| =$ constant

$\therefore \quad v = v_0$

For 2π rotation it takes $\frac{2\pi m}{qB}$ time

For $(2\pi - 2\alpha)$ it takes $\frac{2m(\pi-\alpha)}{qB}$

SECTION – B

8. 7

Sol. Given, voltage V = (100 $\pm$ 5)V
Current I = (10 $\pm$ 0.2)A
According to Ohm's law, V = IR or R = V/I
Taking log of both sides,
Log R = log V – log I
Differentiating, we get

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} - \frac{\Delta I}{I}$$

For maximum error, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

Multiplying both sides by 100 for taking percentage, we get

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

Percentage error in resistance R

$$= \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$= \frac{5}{100} \times 100 + \frac{0.2}{10} \times 100 = 7\%.$$

9. 6

Sol. Energy in 2nth state will be $= -\frac{13.6Z^2}{4n^2} eV$

Max energy of photon = $13.6Z^2\left(1 - \frac{1}{4n^2}\right) = 204$ ….(1)

Energy of photon when it makes transition to x in state will be $13.6\,Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) = 40.8$ …..(2)

Dividing equation (1) and (2)

$$\frac{4n^2 - 1}{3} = 5$$

$$\Rightarrow \quad n^2 = 4 \; n = 2$$

Putting of $n = 2$ in equation (2)

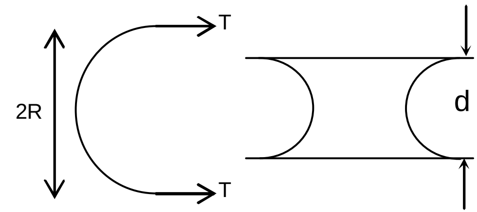
$$13.6\,Z^2\left(\frac{3}{4 \times 4}\right) = 40.8$$

$$Z^2 = \frac{40.8 \times 16}{3 \times 13.6}$$

$$Z = 4$$

10. 1

Sol. The shape of water layer between the two plates is shown in the figure.



Thickness d of the film = 0.12 mm = 0.012 cm.

Radius R of cylindrical face $= \frac{d}{2}$.

$F = T(2l) = P(l \times 2R)$

$P = \frac{T}{R}$

Pressure difference across the cylindrical surface $= \frac{T}{R} = \frac{2T}{d}$.

Area of each plate wetted by water = A.

Force F required to separate the two plates is given by

F = pressure difference × area $= \frac{2T}{d}A$

$= \frac{2 \times 75 \times 8}{0.012} = 1N$

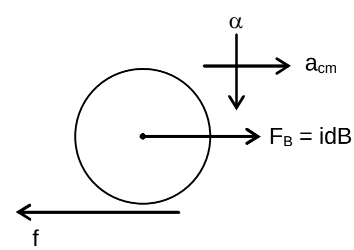
11. 0

Sol. $dw = \vec{E}.d\vec{r} = 0$

12. 2

Sol. $f = \frac{F}{3} = \frac{idB}{3}$

$\Rightarrow \quad \frac{48 \times 0.5 \times 0.25}{3} = 2.00$



13. 2

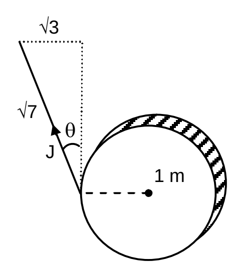
Sol. Impulse equation

$J\cos\theta = m\sqrt{2gR}$

Angular impulse equation

$J\cos\theta R = \frac{mR^2}{2}\omega$

$\omega = 2\sqrt{20}$



SECTION – C

14. 0.80

Sol. ΔQ = (10 g) (80 cal/gm) = 800 cal.

15. 82.50

Sol. ΔQ = (1) (0.5) (5) + (1) (80)

= 82.5 cal

16. 0.52

17. 0.78

Sol. **(for Q. 16-17)**

$R = \frac{2u^2 \sin\theta \cos\theta}{g}$

Impulse of normal on brick at landing = mu sin θ

Impulse due to friction = μmu sin θ = mu cos θ – mv_x

$v_x = u(\cos\theta - \mu\sin\theta)$

Horizontal distance covered = $R + \frac{v_x^2}{2\mu g}$

$= \frac{2u^2 \sin\theta\cos\theta}{g} + \frac{u^2(\cos\theta - \mu\sin\theta)^2}{2\mu g}$

$d = \frac{u^2}{2\mu g}(\cos\theta + \mu\sin\theta)^2$

For maximum distance μ = tan θ

But this is valid only for $v_X > 0$

$\Rightarrow$ cos θ – μ sin θ > 0

$\tan\theta < \frac{1}{\mu}$

Chemistry

PART – II

SECTION – A

18. B

Sol. $LiCl.3NH_{3(s)} \rightleftharpoons LiCl.NH_{3(s)} + 2NH_{3(g)},\ K_p = 9\,atm^2$

$$LiCl.NH_{3(s)} + 2NH_{3(g)} \rightleftharpoons LiCl.3NH_{3(s)},\ K'_p = \frac{1}{9\,atm^2}$$

Initial moles	0.1	a	0
Final mole	0	a - 0.2	0.1

$$K'_p = \frac{1}{9} = \frac{1}{\left(P'_{NH_3}\right)^2} \Rightarrow P'_{NH_3} = 3atm$$

$$n = \frac{PV}{RT} = \frac{3\times 5}{0.0821\times 310} = 0.59$$

$\therefore$ a – 0.2 = 0.59

or a = 0.79

19. B

Sol.

$$C_2H_5 - \underset{\underset{H}{|}}{\overset{\overset{OH}{|}}{C}} - CH_2Cl \xrightarrow{LiAlH_4} C_2H_5 - \underset{\underset{H}{|}}{\overset{\overset{OH}{|}}{C}} - CH_3$$

R S

20. C

Sol. It is a zero order reaction for which

$$t_{\frac{1}{2}} = \frac{[A_o]}{2k}$$

$$\text{or, } k = \frac{[A_o]}{2\times t_{\frac{1}{2}}} = \frac{4}{2\times 5}$$

$= 4 \times 10^{-1}\ mol\ L^{-1}\ s^{-1}$

21. B

Sol. NO_2BF_4 contains NO_2^+ and BF_4^- ions. The bond angle of NO_2^+ is 180°.

22. ABCD

Sol. The nitrogen atoms which are represented inside the circles do not participate in resonance so they are more basic.

23. BC

Sol. $$h = \sqrt{\frac{K_w}{K_b \times C}}$$

- Dilution decreases ‘C’, hence, ‘h’ increases
- $NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$

Addition of base removes H^+ ion from the solution, so the equilibrium moves towards forward direction.

24. AD

Sol. In (A) and (D), the central atoms of both species undergo sp^3-hybridization.

SECTION – B

25. 3

Sol.
$$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-\underset{\displaystyle Cl}{\underset{|}{CH}}-\underset{\displaystyle Cl}{\underset{|}{CH}}-CH_3 \xrightarrow[\text{ether, }\Delta]{Zn} CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-CH=CH-CH_3 + ZnCl_2$$
$$\downarrow$$
$$CH_3-CH=CH-CH=CH-CH_3$$

The product $ZnCl_2$ is a Lewis acid, which reacts with the enol,

$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-CH=CH-CH_3$ to form the conjugated diene as the final product.

26. 4

Sol. The cell reaction is

$$Ni + 2H^+ \rightleftharpoons Ni^{2+} + H_2$$

$E^o_{cell} = E_C - E_A = 0 -(-0.236) = 0.236$ volt

$$E_{cell} = E^o_{cell} - \frac{0.059}{n}\log\frac{[Ni^{2+}]}{[H^+]^2}$$

At equilibrium, $E_{Cell} = 0$

$$\therefore\ E^o_{cell} = \frac{0.059}{2n}\log\frac{1}{[H^+]^2}$$

or, $0.236 = 0.0295\ \log\dfrac{1}{[H^+]^2}$

or $\log\dfrac{1}{[H^+]^2} = 8 = \log 10^8$

or $\dfrac{1}{[H^+]^2} = 10^8$ or $[H^+]^2 = 10^{-8}$

or $[H^+] = 10^{-4}$

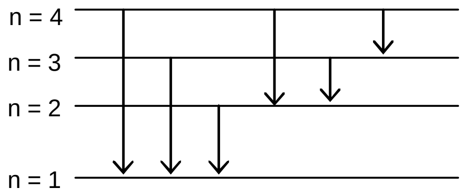
$\therefore$ pH = -log $[H^+]$ = 4

27. 2

Sol. $12.75 = E_n - E_1 = -\dfrac{13.6}{n^2} - (-13.6) = 13.6\left(1 - \dfrac{1}{n^2}\right)$

On solving, n = 4

No. of spectral lines = $\frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$



Visible lines are found in Balmer series for which $n_2 = 2$

$\therefore$ Two spectral lines are visible.

28. 41

Sol. $\frac{r_{SO_3}}{r_{Ne}} = \frac{p_{SO_3}}{p_{Ne}}\sqrt{\frac{M_{Ne}}{M_{SO_3}}}$

or, $\frac{r_{SO_3}}{r_{Ne}} = \frac{\frac{4}{5}}{\frac{5}{8}}\sqrt{\frac{20}{80}} = \frac{32}{25} \times \frac{1}{2} = \frac{16}{25} = x : y$

$\therefore$ x + y = 16 + 25 = 41

29. 9

Sol. x = 2, y = 1 and z = 6

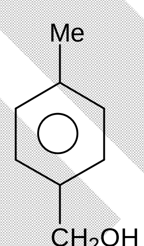
The oxide is Tl_2O

30. 3

Sol. The products are $NC - CH_2 - CH_2 - CN$, $CN - CH_2 - CH_2 - NC$ and $CN - CH_2 - CH_2 - CN$.

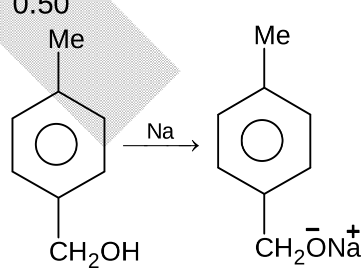
SECTION – C

31. 2.50

Sol. P is HCHO, Q is , R is $HCO\bar{O}K^+$

32. 0.50

Sol.



33. 11.25

Sol. Let the amount of water present in the radiator at -6°C be W.

$\Delta T_f = K_f \text{ x } m = K_f \times \frac{w}{m} \times \frac{1000}{W}$

or, 0-(-6) = 1.86 x $\frac{620}{62} \times \frac{1000}{W}$

$\therefore$ W = 3100 g

Mass of water present in the radiator before freezing = 4 Kg = 4000g

Mass of water left in the radiator after freezing = 3100 g

$\therefore$ Mass of ice separated = 4000 – 3100 = 900 g = x

$\therefore \frac{x}{80} = \frac{900}{80} = 11.25$

34. 1.86

Sol. $\Delta T_f = K_f m = 1.86 \times 1 \times \frac{1000}{1000} = 1.86$

$T_f^o - T_f = 1.86$

$T_f = T_f^o - 1.86 = 0 - 1.86 = -1.86^oC = -x^oC$

$\therefore$ x = 1.86

Mathematics

PART – III

SECTION – A

35. A

Sol. $\int \frac{\sec^2 x\,dx}{\tan^{101} x\left(1+(\tan x)^{-100}\right)}$

Let $1+(\tan x)^{-100}=t$

$-100(\tan x)^{-101}\sec^2 x\;dx=dt$

$-\int \frac{dt}{100t}=-\frac{1}{100}\ell nt+c$

$=-\frac{1}{100}\ell n\left(1+\frac{1}{(\tan x)^{100}}\right)+c$

$\because g\left(\frac{\pi}{4}\right)=\frac{-\ell n x}{100}\therefore c=0$

$\therefore \lim\limits_{x\to\frac{\pi}{2}}-\frac{1}{100}\ell n\left(1+\frac{1}{(\tan x)^{100}}\right)=0$

36. A

Sol. Let the third root be c

$\therefore \quad abc=1$

$\Rightarrow \quad c=\frac{1}{ab}$

But c is root of $x^3+3x^2-1=0$

$c^3+3c^2-1=0$

$\frac{1}{(ab)^3}+\frac{3}{(ab)^2}-1=0$

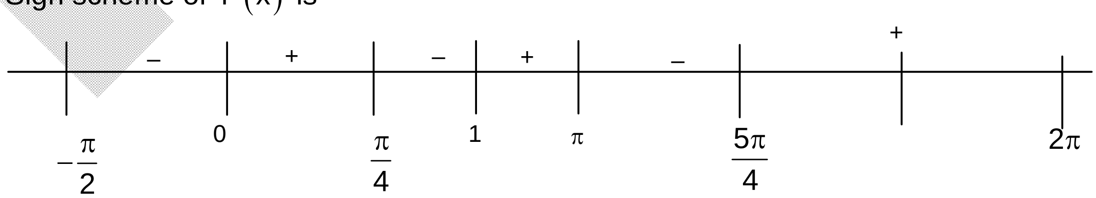
$\Rightarrow -(ab)^3+3(ab)+1=0$

or $y^2-3y-1=0$

37. C

Sol. $f'(x)=(e^x-1)(x-1)(\sin x-\cos x)\sin x$

Sign scheme of $f'(x)$ is



Clearly, $f(x)$ is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right) \cup (1, \pi) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ and decreasing in $\left(\frac{\pi}{4}, 1\right) \cup \left(\pi, \frac{5\pi}{4}\right)$.

38. B

Sol. $\boxed{\text{INDIA}}$ N I D O L $\boxed{\text{INDIAN}}$ I D O I

$\therefore\ A = 6! - 5!$ and

$\boxed{\text{INDIAN}}$ I D O L $\boxed{\text{INDIAN}}$ $\boxed{\text{IDOL}}$

$b = 5! - 2!$

39. AC

Sol. $f(x) = (|2x-1| + |x| + |2x+1|)\left[\frac{1}{4}\left\{(x^2-1)^2 + 3\right\}\right]$

Let $t = \frac{1}{4}\left\{(x^2-1)^2 + 3\right\} \Rightarrow$ for $x \in \left[\frac{-3}{2}, \frac{3}{2}\right], t \in \left[\frac{3}{4}, \frac{73}{64}\right]$

Take $\frac{1}{4}\left((x^2-1)^2 + 3\right) = 1 \Rightarrow (x^2-1)^2 = 1 \Rightarrow x^2 - 1 = \pm 1 \Rightarrow x = \pm\sqrt{2}, 0$

$$\therefore [t] = \left[\frac{1}{4}\left\{(x^2-1)^2 + 3\right\}\right] = \begin{cases} 0, & x \in (-\sqrt{2}, 0) \cup (0, \sqrt{2}) \\ 1, & x \in \left[\frac{-3}{2}, -\sqrt{2}\right] \cup \{0\} \cup \left[\sqrt{2}, \frac{3}{2}\right] \end{cases}$$

Clearly, $f(x)$ is discontinuous as well as non – differentiable at $x = -\sqrt{2}, 0, \sqrt{2}$

40. AC

Sol. Given $\angle APB = \frac{\pi}{4} \Rightarrow \angle ACB = \frac{\pi}{2}$

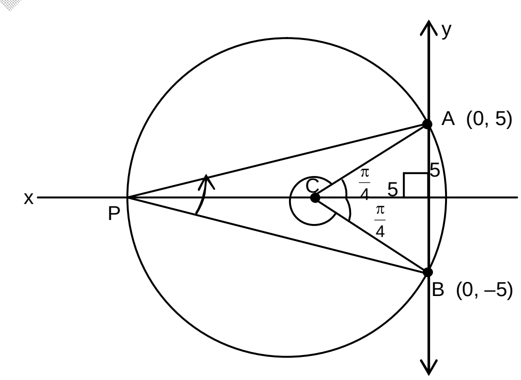
Let $CA = CB = R =$ radius of the circle

$\Rightarrow 2R^2 = 10^2 \Rightarrow R = 5\sqrt{2}$

length of the arc $= 5\sqrt{2} \times \frac{3\pi}{2}$

$\therefore$ Equation of the circle $(x+5)^2 + y^2 = 50$

$y = \pm\sqrt{50 - (x+5)^2}$ ……(i)

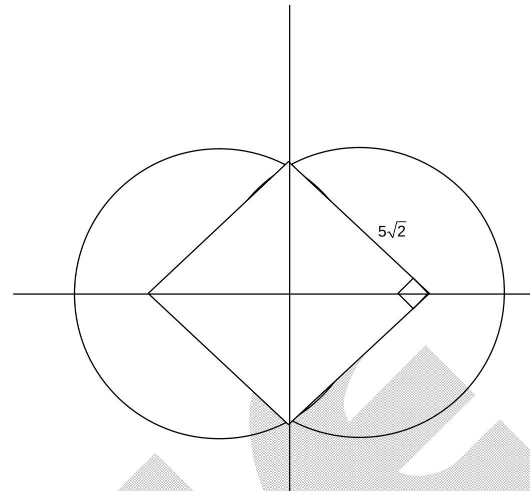


According to the question put $x = 0, -1, -2, \ldots, -12$ we get the number of integral points inside the circle and lies on the imaginary line

$= 9 + 11 + 13 + 13 + 13 + 15 + 13 + 13 + 13 + 11 + 9 + 7 + 1 = 141$

Required area

$$= 2\left[\frac{1}{2}\times(5\times 10)+\frac{1}{2}\left(5\sqrt{2}\right)^2\times\frac{3\pi}{2}\right] = 50+75\pi$$



41. BD

Sol. $P_1 : x+3y+5z=2,\ P_2 : x-2y+z=5$

P_1 and P_2 are perpendicular

If directional ratios of line represented by $P_1 = 0\ P_2 = 0$ are (a, b, c)

$$\Rightarrow a+3b+5c=0,\ a-2b+c=0 \Rightarrow \frac{a}{13}=\frac{b}{4}=\frac{c}{-5}$$

dr's may be (13, 4, –5)

option A is wrong

Now length of $\perp^r$ from (–4, 7, 11) to the plane $P_1 = \dfrac{|-4+21+55-2|}{\sqrt{1+9+25}} = \dfrac{70}{\sqrt{35}}$

Length of $\perp^r$ from (–4, 7, 11) to $P_2 \_ \dfrac{|-4-14+11-5|}{\sqrt{1+2^2+1}} = \dfrac{12}{\sqrt{6}}$

Area of $\Delta PQR = \dfrac{1}{2}\times\dfrac{70}{\sqrt{35}}\times\dfrac{12}{\sqrt{6}} = 2\sqrt{210}$

Foot of $\perp^r$ from P to P_1 is (x_2, y_2, z_2)

$$\frac{x_2+4}{1}=\frac{y_2-7}{3}=\frac{z_2-11}{5}=-\left(\frac{-4+21+55-2}{35}\right)=-2 \Rightarrow x_2=-6 y_2=1\ z_2=1$$

If foot of perpendicular from P to P_3 is (x_3, y_3, z_3)

$$\frac{x_3+4}{1}=\frac{y_3-7}{3}=\frac{z_2-11}{1}=-\left(\frac{-4+14+11-5}{6}\right)=2 \Rightarrow x_3=-2,\ y_3=3,\ z_3=13$$

$Q=(-6, 1, 1),\ R=(-2, 3, 13)$

Equation of plane through $(-4, 7, 11), (-6, 1, 1)$ and $(-2, 3, 13)$

$$\begin{vmatrix} x+4 & y-7 & z-11 \\ -2 & -6 & -10 \\ 2 & -4 & 2 \end{vmatrix} = 0 \Rightarrow 13x+4y-5z+79=0$$

SECTION – B

42. 4

Sol. $\sum_{r=0}^{10} \frac{3^r \left(r!\left(3r^2+5r+1\right)\right)}{r^2+3r+2}$

$= \sum_{r=0}^{10} \frac{3^r r!\left\{3(r+1)^2-(r+2)\right\}}{(r+1)(r+2)} = \sum_{r=0}^{10}\left\{\frac{3^{r+1}(r+1)!}{r+2} - \frac{3^r r!}{r+1}\right\}$

$= \frac{3^{11}(11!)-12}{12} = \frac{3^m.(n!)-12}{12} \Rightarrow m = n = 11$

$T_r = \frac{r+2}{(r+1)!} - \frac{(r+3)}{(r+2)!} \Rightarrow \sum_{r=1}^{100} T_r = \frac{3}{2!} - \frac{103}{102!}$

$\Rightarrow \left[\sum_{r=1}^{100}\left(\frac{r^2+3r+1}{(r+2)!}\right) + \frac{103}{102!}\right] \times \frac{8}{3} = 4$

43. 3

Sol. $I_n = \int_0^{\frac{3\pi}{2}} \underbrace{\ln|\sin x|}_{I} \underbrace{\cos(2nx)}_{II} dx$

applying integration by parts

$I_n = \left\{\ln|\sin x| . \frac{\sin 2nx}{2n}\right\}_0^{\frac{3\pi}{2}} - \int_0^{\frac{3\pi}{2}} \frac{\cot x . \sin 2nx}{2n}$

$I_n = 0 - \frac{1}{2n} J_n$, where $J_n = \int_0^{\frac{3\pi}{2}} \frac{\cos x . \sin 2x}{\sin x} dx$

$J_n - J_{n-1} = \int_0^{\frac{3\pi}{2}} \frac{\cos x\left(\sin 2nx - \sin(2n-2)x\right)}{\sin x} dx$

$= \int_0^{\frac{3\pi}{2}} \frac{\cos x . \cos(2n-1)x \sin x}{\sin x} dx$

$J_n - J_{n-1} = \int_0^{\frac{3\pi}{2}} 2\cos(2n-1)x . \cos dx = 0$

$J_n = J_{n-1} = J_{n-2} = ... = J_1$

$J_1 = \int_0^{\frac{3\pi}{2}} \frac{\sin 2x \cos x}{\sin x} dx = \int_0^{\frac{3\pi}{2}} 2\cos^2 x\, dx = \int_0^{\frac{3\pi}{2}} (1+\cos 2x) dx = \frac{3\pi}{2}$

$\therefore I_n = \frac{3\pi}{4n}$

$12I_3 = -3\pi, 16I_2 = -6\pi$

$\Rightarrow 12I_3 - 16I_2 = 3\pi$

44. 2

Sol. Tangent to parabola $y = mx - \frac{a}{4m}$ ....(1)

It is passes through the point (1, 0) $\Rightarrow$ $a = 4m^2$

(1) $\Rightarrow$ the equation of the tangent is $y = mx - m$

Since the portion of the tangent between point of contact and the directrix always subtends a right angle at focus.

i.e. $(1, 0)$ lies on the directrix of hyperbola, $\frac{2}{e} = 1 \Rightarrow e = 2$

$b^2 = a^2(e^2 - 1) = 4(4-1) = 12$

The equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

The equation of tangent to hyperbola $y = mx \pm \sqrt{4m^2 - 12}$ ....(2)

(1) and (2) represents the same line

$-m = \sqrt{4m^2 - 12} \Rightarrow 3m^2 = 12 \Rightarrow m^2 = 4$

$\therefore \frac{a+b^2}{14} = \frac{4m^2+12}{14} = 2$

45. 20

Sol. $\bar{x} = \frac{\sum_1^5 x_1}{5} = 150$ $\Rightarrow \sum_1^5 x_i = 750$

Variance $= \frac{\sum x_i^2}{5} - (\bar{x})^2 = 18$ ...(i)

$\Rightarrow \sum_1^5 x_i^2 = 112590$ ...(ii)

Height of new students = 156,

$\therefore$ $x_6 = 156$

$\bar{x}_{new} = \frac{750+156}{6} = 151$

$\therefore$ New variance $= \frac{\sum_1^6 x_i^2}{6} - (\bar{x}_{new})^2$

$= \frac{112590 + (156)^2}{6} - (151)^2$

$= 22821 - 22801 = 20$

46. 5

Sol. Total ways of $A \times B = 5 \times 4 = 20$

Favourble cases = (1, 8), (3, 6), (5, 4), (7, 2) $(\because a + b = 9)$

$\therefore$ Number of favourable cases = 4

$\therefore$ Required probability $= \frac{4}{20} = \frac{1}{5}$.

47. 2

Sol. The centre of the circle is $(1, 1)$ and radius $= 2\sqrt{2}$.

As $M(a, a)$ lies outside the circle,

So, $2a^2 - 4a - 6 > 0$

$\Rightarrow a < -1$ or $a > 3$ ....(i)

$\Rightarrow a^2 - 2a - 3 > 0 \Rightarrow (a-3)(a+1) > 0$

Now, $\tan\frac{\theta}{2} = \frac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}}$

As $\frac{\pi}{3} < \theta < \pi$

$\Rightarrow \frac{\pi}{6} < \frac{\theta}{2} < \pi$

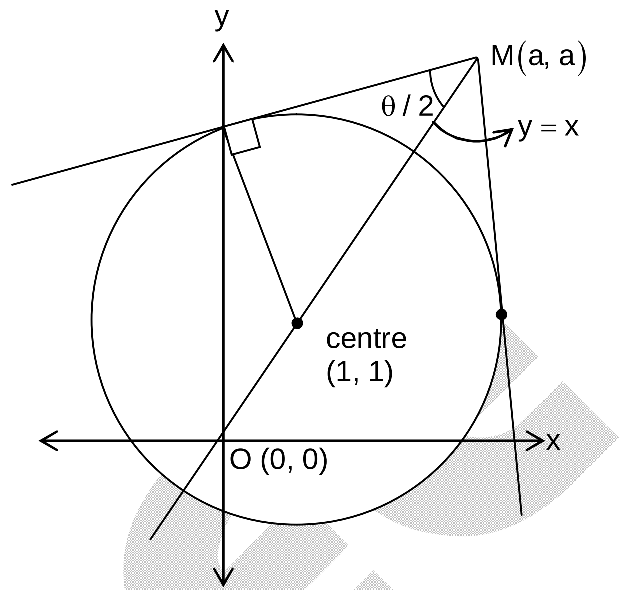
$\Rightarrow \frac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}} > \frac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{a^2 - 2a - 3} < 2\sqrt{3}$

$\Rightarrow a^2 - 2a - 15 < 0$

$\Rightarrow (a-5)(a+3) < 0$

$\Rightarrow -3 < a < 5$ ...(ii)



SECTION – C

48. 1.25

49. 4.00

Sol. **(for Q. 48 & 49):**

The tangent to $y = f(x)$ at (x, f) is $Y - f = f'(X - x)$ which meets the x – axis at $\left(x - \frac{f}{f'}, 0\right)$

The tangent to $y = g(x)$ at (x, g) is $Y - g = g'(X - x)$

$\Rightarrow Y - g = f(X - x)$ since $g' = f$. The tangent meets the x – axis at $\left(x - \frac{g}{f'}, 0\right)$

$\therefore x - \frac{f}{f'} = x - \frac{g}{f} \Rightarrow g = \frac{f^2}{f'}$

Differentiating $f = 2f - \frac{f^2 f''}{(f')^2} \Rightarrow f''f = (f')^2 \Rightarrow \frac{f''}{f'} = \frac{f'}{f}$

Now integrating on both sides we get $\ln f' = \ln f + \ln c \Rightarrow f' = cf \Rightarrow f = Ae^{cx}$, $f(0) = 1$

Then $f(x) = e^{cx}$

$g(x) = \int_{-\infty}^{x} e^{ct} dt = \frac{e^{cx}}{c}$, $g(0) = \frac{1}{2} \Rightarrow g(x) = \frac{e^{2x}}{2}$, $f(x) = e^{2x}$

$f'(0)=2$ the tangent at (0, 1) is $y-1=2x$
The normal is $x+2y-2=0$.

These lines meets the x – axis at $\left(-\frac{1}{2},0\right)$ and (2, 0)

The area $=\frac{1}{2}\left(2+\frac{1}{2}\right).1=\frac{5}{4},\lim_{x\to 0}\frac{e^{4x}-1}{x}=4$

50. 5.00

Sol. $f(x)=|A-xI|$

$f(A)=A^2+aA+3I$

$\Rightarrow tr(A)$ = sum of roots $=-a\Rightarrow a=-5;\ |A|=$ product of roots = 3

$A^2-5A+3I=0\Rightarrow 3A^{-1}=5I-A$ ...(1)

Again multiplied A^{-1} on both sides

$\Rightarrow 3\left(A^{-1}\right)^2=5A^{-1}-I$ ...(2)

$\left(3A^{-1}\right)^2=(a+1)3A^{-1}+3I=15A^{-1}-3I-12A^{-1}+3I\ \ (\because \text{from }(2))$

$=3A^{-1}=5I-A$

$Tr\left(\left(3A^{-1}\right)^2+(a+1)3A^{-1}+3I\right)=Tr(5I-A)=5Tr(I)-Tr(A)=5(2)-5=5$

51. 6.00

Sol. If $a_{ij}=\{1,2,3,4\},A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$tr(A)=a+d=5$ and $ad-bc=3$

$(a,d)=(2,3),(3,2),(1,4),(4,1)$

Therefore required matrices are

$\left\{\begin{bmatrix}2&1\\3&3\end{bmatrix},\begin{bmatrix}2&3\\1&3\end{bmatrix},\begin{bmatrix}3&1\\3&2\end{bmatrix},\begin{bmatrix}3&3\\1&2\end{bmatrix},\begin{bmatrix}1&1\\1&4\end{bmatrix},\begin{bmatrix}4&1\\1&1\end{bmatrix}\right\}$