PHYSICS

CGS

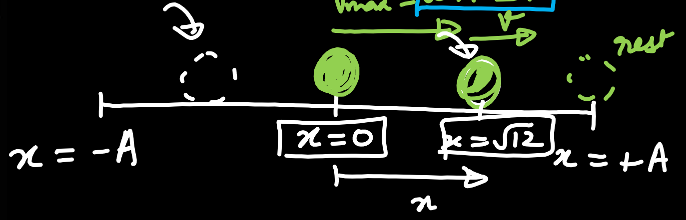
A particle doing SHM with amplitude 4 cm and maximum velocity 10 cm/s (at origin). Find position where velocity is 5 cm/s

(A) $\sqrt{12}$ cm (B) 4 cm

(C) 9 cm (D) $\sqrt{16}$ cm

$\boxed{A = 4}$

$V_{max} = \boxed{\omega A = 10}$



$\boxed{\omega = 2.5}$

$\boxed{A = 4}$

$\boxed{V\big|_{x=?} = 5}$

$$V = \omega\sqrt{A^2 - x^2}$$

$$\overset{1}{\cancel{5}} = \frac{\cancel{5}}{2}\sqrt{16 - x^2}$$

$$4 = 16 - x^2$$

$$\boxed{x = \pm\sqrt{12}}$$ Ans

2

Kinetic energies of two particles A and B of mass 4 and 25 kg are equal. Find ratio of their linear momentum.

(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.8

JEE 1

$\boxed{KE} = \frac{1}{2}mv^2$

$v \rightarrow$

$\boxed{m}$

$\boxed{p}^2 = \frac{1}{2}mv^2(2m)$

$$\boxed{\frac{p^2}{2m} = KE}$$

$$\boxed{\frac{p_1}{p_2} = ?}$$

$m_1 = 4$

$m_2 = 25$

$$\boxed{KE_1 = KE_2 = KE}$$

$$\frac{p_1}{p_2} = \frac{\sqrt{\cancel{2}m_1(\cancel{KE})}}{\sqrt{\cancel{2}m_2(\cancel{KE})}} = \sqrt{\frac{4}{25}} = \frac{2}{5} = \boxed{0.4}$$

3

The speed of sound in oxygen at STP will be approximately?

(Given R = 8.3 J/molK and $\gamma = 1.4$)

(A) 320 m/s (B) 315 m/s

(C) 330 m/s (D) 325 m/s

$f = 5$

$$\gamma = \frac{f+2}{f} = \frac{7}{5}$$

$\gamma = \frac{7}{5}$ $R = 8.3$ $T = 273$ K

$M = 32$ gm.

$$PM = \rho RT$$

$$\frac{P}{\rho} = \frac{RT}{M}$$

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$M = \frac{32}{1000}$$

$$v = \sqrt{\frac{(7/5)(8.3)(273)}{\frac{32}{1000}}} = 314.5 \text{ m/s}$$

4

JEE 1

A Disc of mass m and radius R is rotating with angular speed ω about axis passing through centre of mass. Another identical disc is gently placed on it. Find out loss in Kinetic energy of system

$I = \frac{1}{2} m R^2$

(A) $\frac{1}{2} mR^2\omega^2$

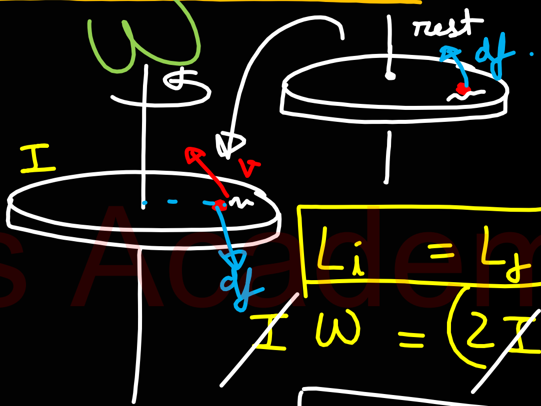
(B) $\frac{1}{4} mR^2\omega^2$

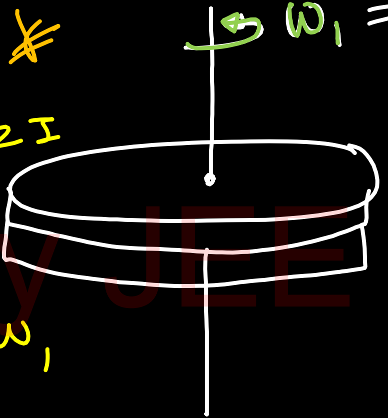
(C) $\frac{1}{6} mR^2\omega^2$

(D) $\frac{1}{8} mR^2\omega^2$ ✓

$\boxed{\textcircled{F} = \frac{dP}{dt}, \quad \textcircled{\tau} = \frac{dL}{dt}}$

$\boxed{\tau_{ext} = 0}$





$\omega_1 = \omega/2$

$\boxed{L_i = L_f}$

$\cancel{I}\,\omega = (2\cancel{I})\,\omega_1$

$\boxed{\frac{\omega}{2} = \omega_1}$

$KE_i = KE_{loss} + KE_f$

$KE_i = \frac{1}{2} I \omega^2$

$KE_f = \frac{1}{\cancel{2}}(\cancel{2}I)\left(\frac{\omega}{2}\right)^2$

$KE_f = \frac{1}{4} I \omega^2$

$KE_{loss} = KE_i - KE_f = \frac{1}{4} I \omega^2$

5

CGS

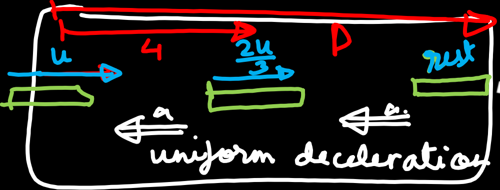
A particle looses $\boxed{1/3^{rd} \text{ of its velocity}}$ when it strikes a block and covers a distance of $\boxed{4\text{ cm}}$ inside the fixed block. Then find D if D is the distance covered by the particle inside block before it stops.

(A) $\frac{63}{5}$ cm

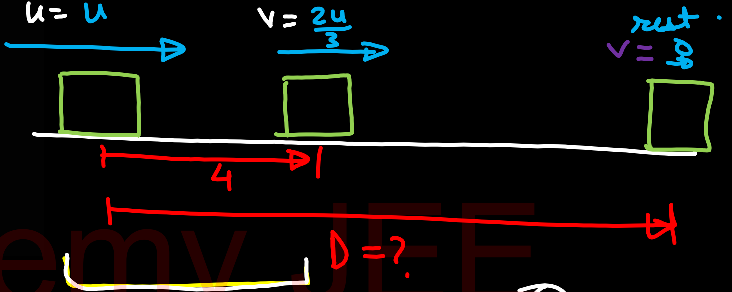
(B) $\frac{36}{5}$ cm ✓

(C) $\frac{54}{5}$ cm

(D) $\frac{21}{5}$ cm



$a = \text{constant } (-ve)$



$\left(\frac{2u}{3}\right)^2 = u^2 + 2a(4)$ — (1)

$0^2 = u^2 + 2aD$ — (2)

$\boxed{v^2 = u^2 + 2as}$

→ (+)

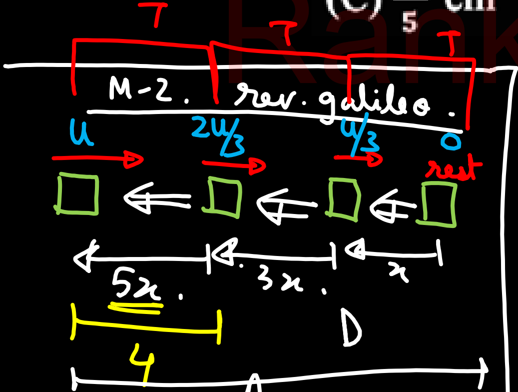
(−) $-\frac{5u^2}{9} = 2a4$

$-u^2 = 2aD$

$\frac{5}{9} = \frac{4}{D}$

$\boxed{D = \frac{36}{5}}$

M-2. rev. galileo.



Ans $= D = 9x$

$= \frac{36}{5}$

$5x = 4$

$\boxed{x = \frac{4}{5}}$

6

Value of gravitational acceleration is same at depth ' d ' and ' h ' from the surface of earth find value of h. (radius of earth $= R_E$, considering $\boxed{h << R}$),

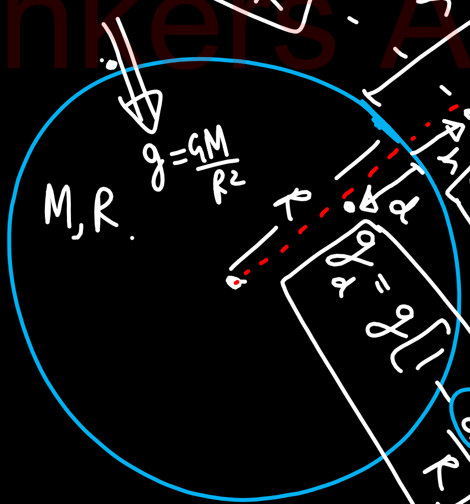
(A) $h = d$

(B) $h = 2d$

(C) $h = \frac{d}{2}$ ✓

(D) $h = \frac{3d}{2}$

$\boxed{h << R}$



$\boxed{g_h = \frac{GM}{(R+h)^2}}$ $\boxed{\text{exact}}$

$\boxed{g_h = g\left[1 - \frac{2h}{R}\right]}$ $\boxed{h << R}$

$\boxed{g_d = g\left[1 - \frac{d}{R}\right]}$ $\boxed{\text{exact}}$

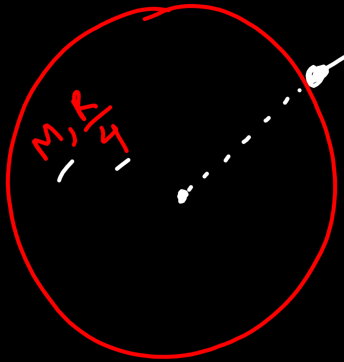
same

$\boxed{d = 2h}$

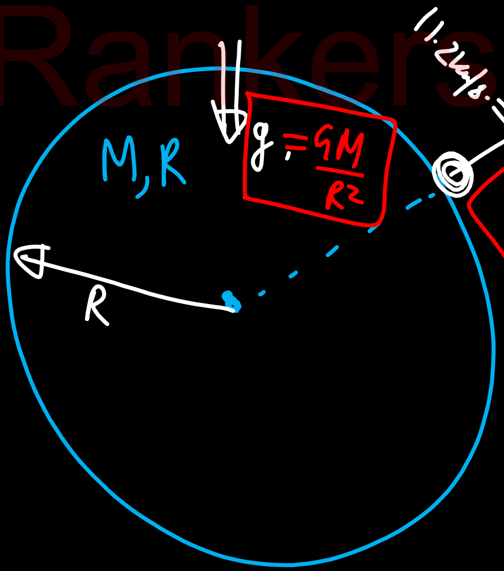
The escape velocity on the surface of the earth is 11.2 km/s. What would be the escape velocity on the surface of another planet of the same mass but 1/4 times the radius of the earth?

(A) 44.8 km/s (B) 22.4 km/s

(C) 5.6 km/s (D) 11.2 km/s



$V_{esc_1} = \sqrt{\frac{2G(M)}{(R/4)}}$



$V_{esc} = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$V_{esc_1} = 2\sqrt{\frac{2GM}{R}}$

$V_{esc_1} = 2(11.2) = 22.4$

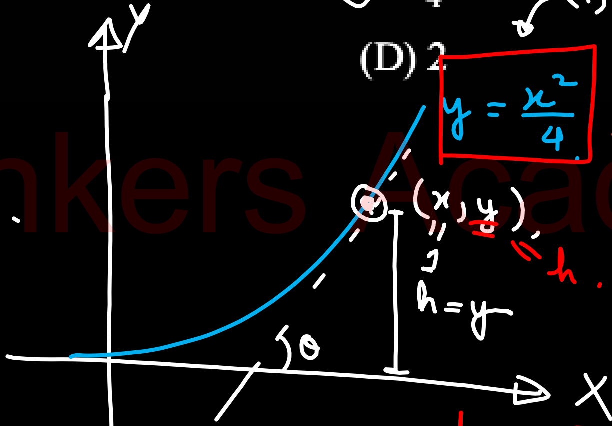
A mass is to be kept on the surface of curve $y = x^2/4$ such that it does not slip. Find the maximum height at witch it should be kept if $\mu = 0.5$

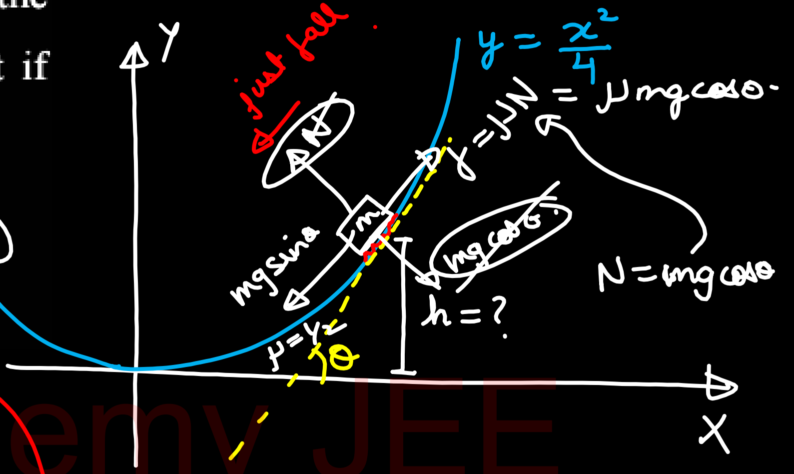
(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 1

(D) 2





$\tan\theta = 1/2$

$y = \frac{1^2}{4}$

$\boxed{y = \frac{1}{4} = h}$

slope $= \tan\theta = \left.\frac{dy}{dx}\right|_{x=x}$

$\frac{dy}{dx} = \boxed{\frac{x}{2}}$

$\cancel{mg}\sin\theta = \mu \cancel{mg}\cos\theta$

$\tan\theta = \mu = \frac{1}{2}$

$\boxed{\tan\theta = 1/2}$

$\frac{1}{\cancel{2}} = \frac{x}{\cancel{2}}$

$\boxed{x = 1}$

9

A simple pendulum have same acceleration at lower position (mean position) & at extreme position. Find its angular amplitude.

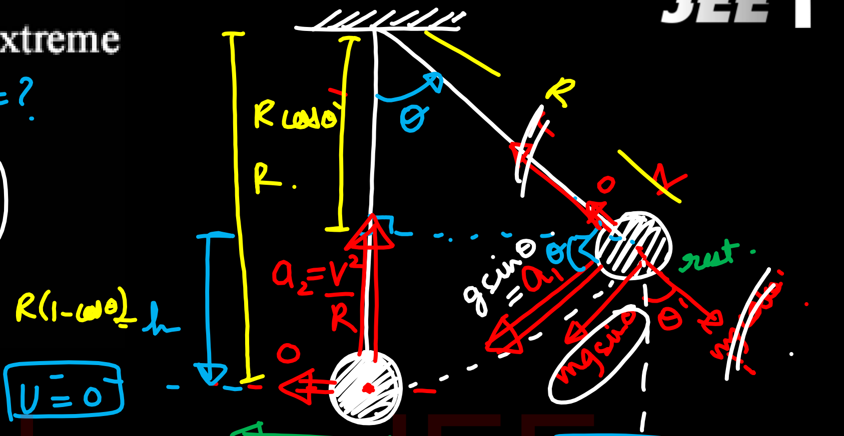
in magnitude.

$\theta = ?$

(A) $2\tan^{-1}\left(\frac{1}{2}\right)$ ✓ (B) $2\cot^{-1}\left(\frac{1}{2}\right)$

(C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\cot^{-1}\left(\frac{1}{2}\right)$

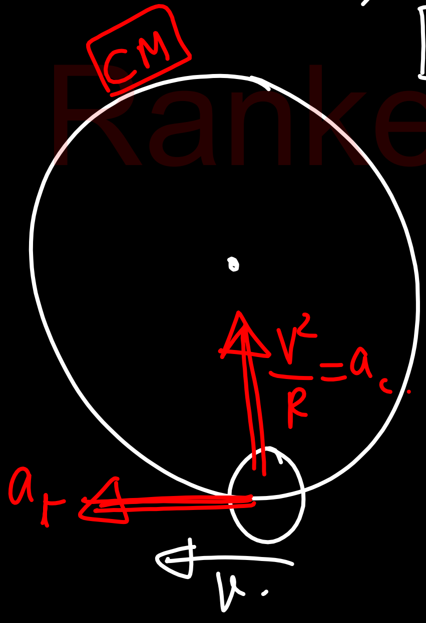
given $\boxed{a_1 = a_2}$



$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

$v = \sqrt{2gR(1-\cos\theta)}$



$\boxed{a_1 = g\sin\theta}$

$\boxed{a_2 = \frac{v^2}{R} = 2g(1-\cos\theta)}$

$\cancel{g}\sin\theta = 2\cancel{g}(1-\cos\theta)$

$\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{1}{2}$

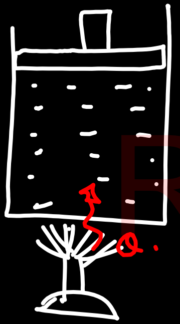
$\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta/2 \cdot \cos\theta/2} = \frac{1}{2}$

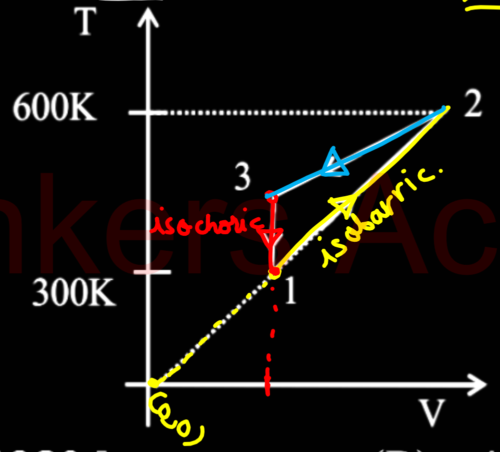
$\tan(\theta/2) = \frac{1}{2}$

$\frac{\theta}{2} = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta = 2\tan^{-1}\left(\frac{1}{2}\right)$

Two moles of an ideal gas is taken through a cyclic process 1-2-3-1 as shown in the $T-V$ diagram. If the heat rejected in the cyclic process is $\boxed{300\text{ J}}$, then work done by the gas in the process 2-3 is (Assume $R = 8.3$ J/molK)





$PV = nRT$

$V = \underbrace{\left(\frac{nR}{P}\right)}_{\text{slope.}} T$

$T = \left(\frac{P}{nR}\right) V$

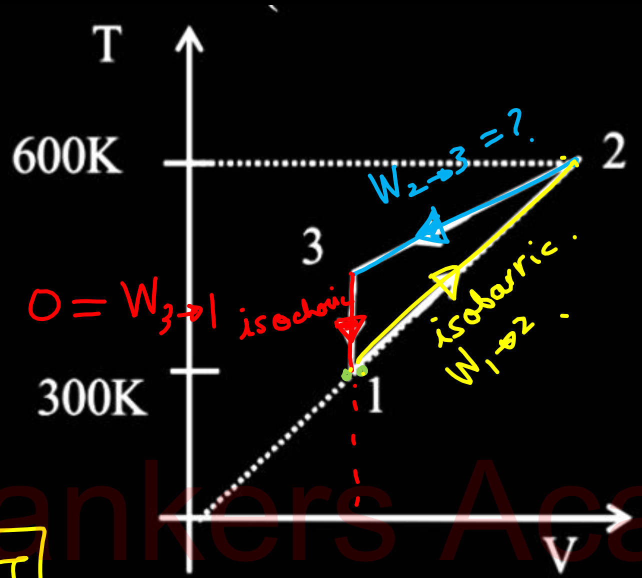
$\boxed{Q = \Delta U + W}$

(A) −4980 J

(B) +4980 J

(C) +5280 J

(D) −5280 J



$1\to2\to3\to1$

For cyclic process

$\boxed{Q} = W + \cancel{\Delta U}$ $\cancel{+nC_v(300-300)}$ 0

Ans

$-300 = \left(W_{1\to2} + W_{2\to3} + W_{3\to1}\right) + 0$

$-300 = \left(4980 + W_{2\to3} + 0\right) + 0.$

$\boxed{W_{2\to3} = -5280}$

$P\Delta V$

$PV = nRT$

$\boxed{P\Delta V} = \boxed{nR\Delta T}$

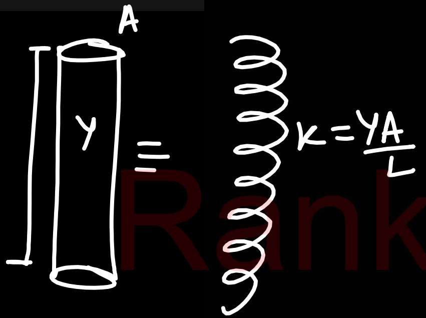
$W_{1\to2} = \boxed{P}\Delta V = nR\Delta T$

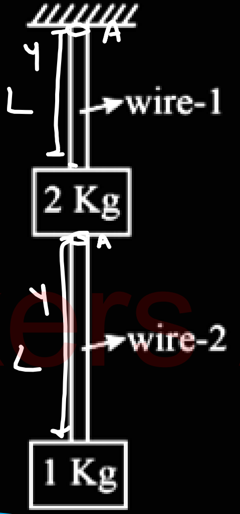
$= 2(8.4)(600-300)$

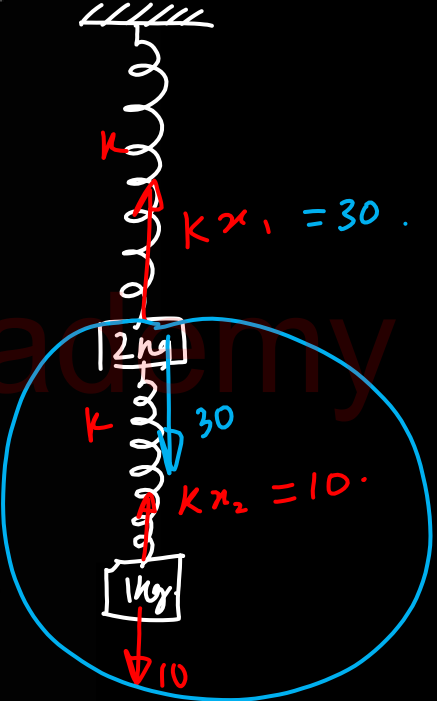
$W_{1\to2} = 4980\,J$

Wire 1 and wire 2 are identical with young's modulus Y, area of cross section-A and length ℓ.

Both the wires are in below given arrangement







$Kx_1 = 30$

$Kx_2 = 10$

$$\boxed{\frac{x_1}{x_2} = \frac{3}{1}}$$

$$\frac{x_1/L}{x_2/L} = \frac{3}{1}$$

Find the ratio of strain in wire- 1 to wire- 2 .

$\frac{\Delta L}{L} = \frac{x}{L}$

(A) 1: 2

(B) 3: 1 ✓

(C) 1: 3

(D) 1: 4

12

JEE 1

(a) Surface Tension S — (i) $[M^1L^2T^{-2}]$

(b) Coefficient of viscosity η — (ii) $[M^1L^2\,T^{-1}]$

(c) Angular momentum L — (iii) $[M^1L^{-1}\,T^{-1}]$

(d) Rotational kinetic energy KE_r — (iv) $[M^1\,L^0\,T^{-2}]$

~~(A) (a) - (iv), b - (ii), c - (iii), d - (i)~~

(B) (a) - (iv), b - (iii), c - (ii), d - (i)

~~(C) (a) - (ii), b - (iii), c - (iv), d - (i)~~

~~(D) (a) - (i), b- (ii), c- (iii), d - (iv)~~

$$[S] = \frac{[F]}{[L]} = \frac{M\cancel{L}T^{-2}}{\cancel{L}} = \boxed{MT^{-2}}$$

$$F = 6\pi\boxed{\eta}rv$$

$$[\eta] = \frac{[F]}{[r][v]} = \frac{M\cancel{L}T^{-2}}{\cancel{L}\,L\,T^{-1}}$$

$$\boxed{[\eta] = ML^{-1}T^{-1}}$$

$$[L] = [m][v][r]$$

$$[L] = M\,LT^{-1}\,L = ML^2T^{-1}$$

$$[KE_R] = ML^2T^{-2}$$

mc^2

13

A particle of mass m is projected at an angle of 30° with initial velocity u. Find its angular momentum about point of projection at maximum height.

(A) $\frac{mu^3}{4g}$

(B) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$

(C) $\frac{\sqrt{2}mu^3}{2g}$

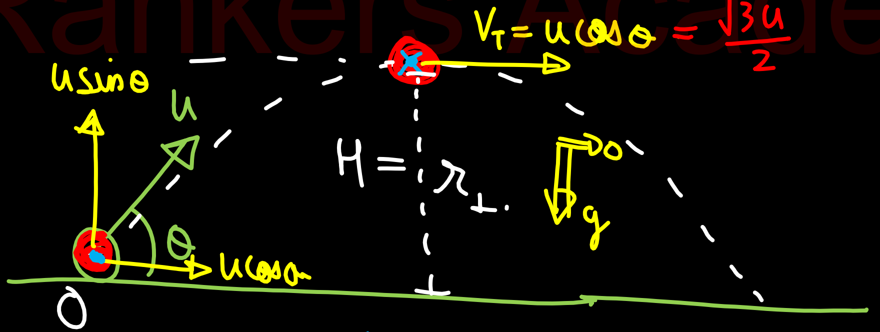
(D) $\frac{\sqrt{3}mu^3}{2g}$

$\vec{L} = m(\vec{r} \times \vec{v})$

JEE1

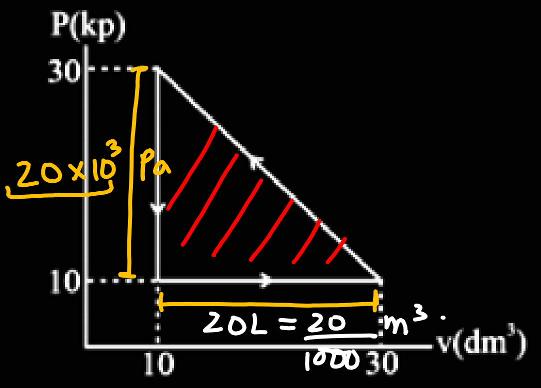
$$\boxed{L = m v r_\perp = m v_\perp r}$$

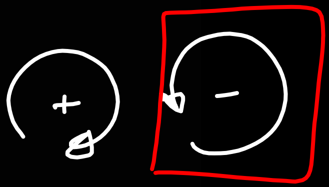
$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{8g}$$

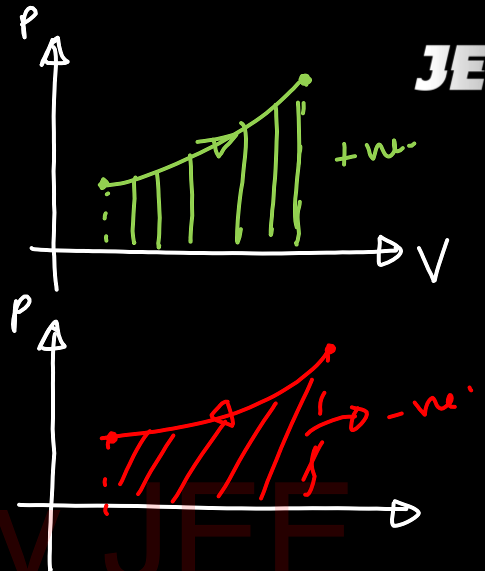


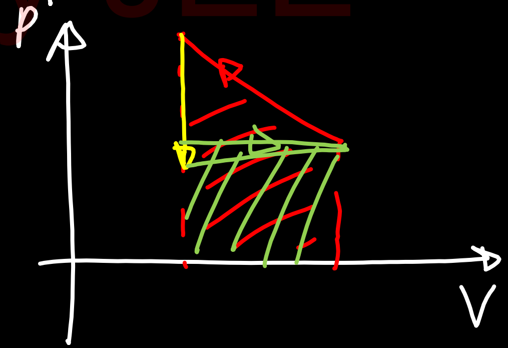
$$L_0 = m V_T r_\perp = m V_T H = \frac{\sqrt{3}}{16}\frac{mu^3}{g}$$

Find the work done by gas in cyclic process.









$1\,dm^3 = 1L$

$1\,m^3 = 1000L$

(A) ~~200 J~~ (B) ~~200KJ~~

(C) −200 J (D) −200KJ

$$W = -\frac{1}{2}\left(\frac{\cancel{20}^{\,10}}{\cancel{1000}}\right)\left(20 \times \cancel{1000}\right) J$$

$$\boxed{W = -200J}$$

15

A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of big drop and that of all small drops will be.

(A) 1:10 (B) 10:1

(C) 100:1 (D) 1:100

$\boxed{\Delta U = T\Delta S}$

JEE 1

$(U_f - U_i) = T(S_f - S_i)$

$\boxed{U = TS}$

① ②

1000 drops

$Vol_i = Vol_f$

$1000\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$\boxed{R = 10r}$

$$\frac{U_2}{U_1} = \frac{\cancel{T}(\cancel{4\pi}R^2)}{\cancel{T}(1000 \times \cancel{4\pi} r^2)}$$

$$\frac{U_2}{U_1} = \frac{100\cancel{r^2}}{1000\cancel{r^2}} = \frac{1}{10}$$

16

JEE 1

There is a prism of apex angle of ' A '. Its refractive index is equal to cot A/2 then find minimum angle of deviation?

(A) $\frac{\pi}{2} - A$

(B) $\pi - 2A$

(C) $\pi + A$

(D) $\frac{\pi}{2} + A$

$$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

$$\cot\frac{A}{2} = \frac{(\quad)}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta_{min}}{2}$$

$$\Rightarrow \boxed{\delta_{min} = \pi - 2A} *$$

17

JEE 1

If path difference between two rays is $\frac{7\lambda}{4}$ (where λ is wave length of light ray) and both rays have same intensity find the ratio of net resultant intensity for a given path difference versus maximum possible net intensity

(A) 1

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{\sqrt{2}}$

$\Delta x = \frac{7\lambda}{4}$

$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}}$

$\Delta\phi = 2\pi \times \frac{7\lambda}{4\lambda} = \frac{7\pi}{2}$

$\boxed{I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)}$

$I_{max} = 4I_0$ — (2)

$I = 4I_0 \cos^2\left(\frac{7\pi}{4}\right)$

$= 4I_0 \left(\frac{1}{\sqrt{2}}\right)^2$

$= 2I_0$ — (1)

$\frac{I_1}{I_{max}} = \frac{2I_0}{4I_0} = \frac{1}{2}$

Ans

18

JEE 1

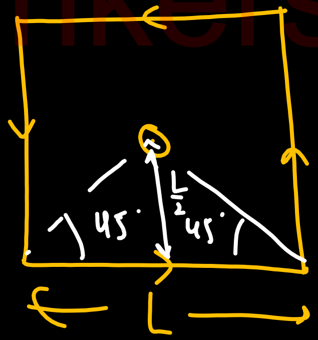
5A current is passing through a square of side 1 m then find the magnetic field at the centre of this square.

(A) $8\sqrt{2} \times 10^{-6}$ T

(B) $4\sqrt{2} \times 10^{-6}$ T

(C) $2\sqrt{2} \times 10^{-6}$ T

(D) $6\sqrt{2} \times 10^{-6}$ T



$$\boxed{B = \frac{2\sqrt{2}\,\mu_0 i}{\pi L}} = \frac{\cancel{2}\sqrt{2} \times 4\pi \times 10^{\cancel{-7}\,-6} \times \cancel{5}}{\pi\,(1\text{m})}$$

$$= 4\sqrt{2} \times 10^{-6}\ \text{T}$$

$$B_{sq} = 4\,B_{side} = 4 \times \frac{\mu_0 i}{4\pi\left(\frac{L}{2}\right)}\left(2\cos 45\right)$$

H.W. also find B_{Δ} $B_{hexagon}$

19

JEE 1

$\vec{E} = E_0(\hat{i})\sin[\underbrace{(\omega t - kz)}_{+z\text{-direction}}]$, then B will be

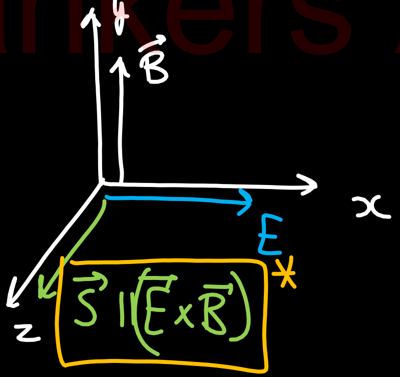
(A) $B = (E_0 c)\sin(\omega t - kz)\hat{j}$

(B) $B = (E_0/c)\sin(\omega t - kz)\hat{j}$

~~(C)~~ $B = (E_0 c)\sin(\omega t - kz)\hat{i}$

~~(D)~~ $B = (E_0/c)\sin(\omega t - kz)\hat{i}$

$\boxed{B_0 = \frac{E_0}{c}}$ *



20

JEE 1

3 moles of oxygen gas and 2 moles of argon gas are mixed together. If the total internal energy of mixture is xRT. Find the value of x.

(A) $\frac{19}{2}$

(B) 10

(C) 11

(D) $\frac{21}{2}$

$$U_{mix} = U_1 + U_2$$

$$\boxed{U_{mix} = n_1 C_{v_1} T + n_2 C_{v_2} T}$$

$$= 3 \times \left(\frac{5R}{2}\right) \times T + 2 \times \left(\frac{3R}{2}\right) \times T$$

$$= \left(\frac{15+6}{2}\right) RT = \left(\frac{21}{2}\right) RT$$

21

JEE 1

A nucleus x has mass number 192 and there is a second nucleus y having radius half of radius of x. Find mass number of y nucleus.

$$\boxed{R = R_0 A^{\frac{1}{3}}}$$

$$R = R_0 (192)^{\frac{1}{3}} \quad \text{—} ①$$

$$\frac{R}{2} = R_0 (y)^{\frac{1}{3}} \quad \text{—} ②$$

② ÷ ①

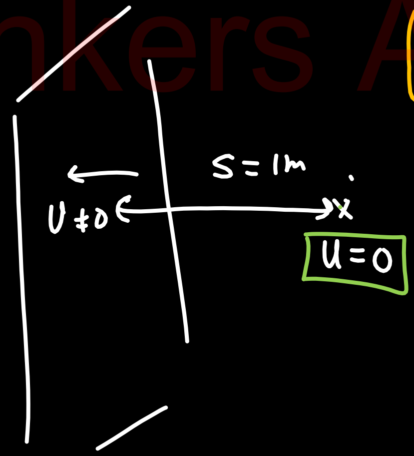
$$\frac{y}{192} = \left(\frac{1}{2}\right)^3$$

$$y = \frac{192}{8} = 24 \quad \text{Ans}$$

22

JEE 1

An electron is placed 1 meter away in front of uniformly charged sheet and now electron is released from rest and strikes after 1 second to charged sheet. If charge density of sheet $x\left(\frac{m\varepsilon_0}{e}\right)$ then value of x will be:



$$\boxed{s = ut + \frac{1}{2}at^2}$$

u = -1 (Jan 2024)

$$1m = 0 + \frac{1}{2}a(1)^2$$

$$a = 2\,m/s^2$$

$$a = \frac{eE}{m} = \frac{e}{m}\left(\frac{\sigma}{2\varepsilon_0}\right)$$

$$\Rightarrow 2 = \frac{e\sigma}{m \times 2\varepsilon_0}$$

$$\sigma = \boxed{4}\left(\frac{m\varepsilon_0}{e}\right)$$

Ans

23

JEE 1

Length of a pendulum is 20 cm and error in its measurement is 2 mm. If it completes 50 oscillations in 40sec. and time was measured by a watch of resolution 1sec. Find % error in calculation of acceleration due to gravity

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}\ *$$

$$g = (4\pi^2)\frac{l}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$= \frac{2\times10^{-3}\,m}{0.2\,m} + 2\frac{\left(\frac{1}{50}\,sec\right)}{\left(\frac{40}{50}\,sec\right)}$$

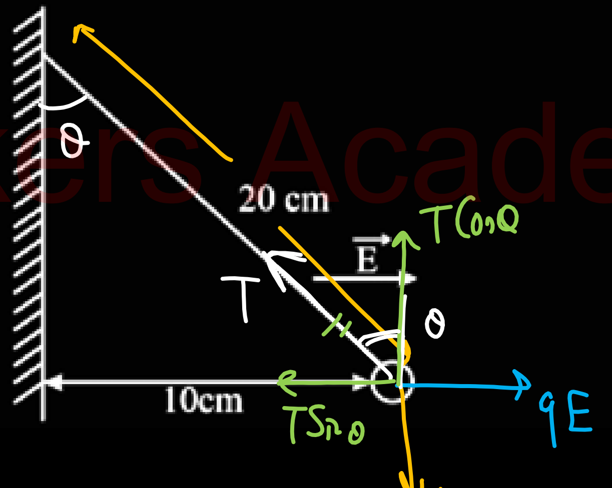
$$\%\,err = \left(10^{-2} + \frac{1}{20}\right)\times 100\% = 1 + 5 = 6\%$$

Ans

24

JEE 1

A charge of particle of mass 2gm and charge $\frac{1}{\sqrt{x}}\,\mu C$ is suspended by a thread of length 20 cm as shown. Find the value of x if magnitude of uniform horizontal electric field is 2×10^4 N/C



$\sin\theta = \frac{10}{20} = \frac{1}{2}$

$$\left.\begin{aligned} T\sin\theta &= qE \\ T\cos\theta &= mg \end{aligned}\right\} \quad \tan\theta = \frac{qE}{mg}$$

$$\tan 30^\circ = \frac{qE}{mg}$$

$$q = \frac{mg}{\sqrt{3}E} = \frac{2\times10^{-3}\times 10}{\sqrt{3}\,2\times10^{4}}$$

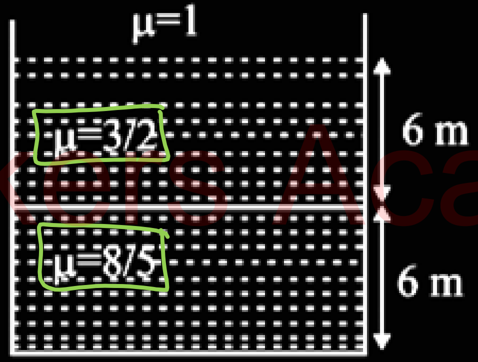
$$q = \frac{1}{\sqrt{3}}\,\mu C$$

Ans

25

JEE 1

If a beaker is filled with immiscible transparent liquid 1 and 2 of refractive index μ_1 and μ_2 having depth 6m each then the apparent depth of the bottom of beaker is $\boxed{\frac{n}{4}\text{m}}$. Find n

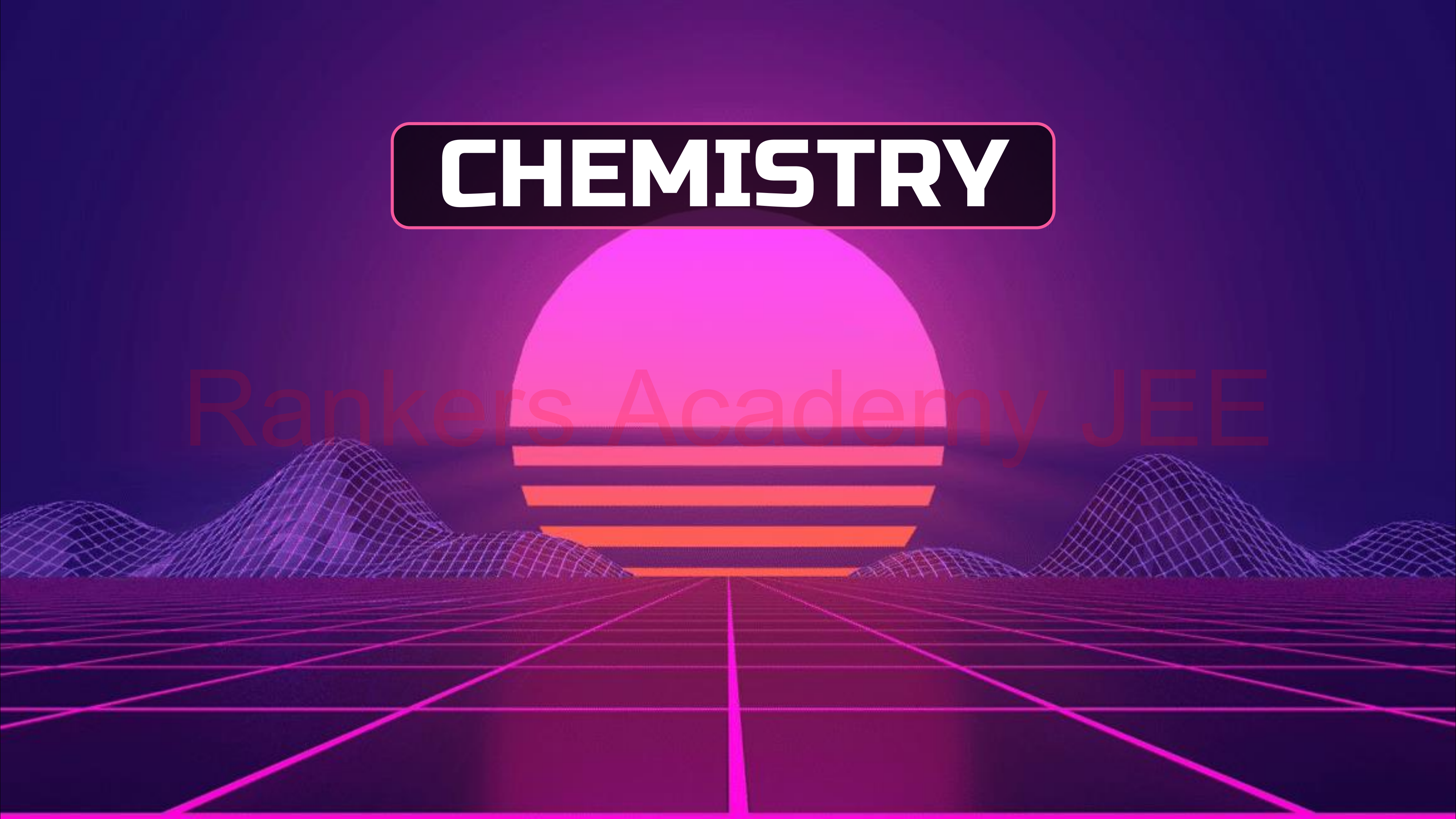


$$h = \left(\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}\right)$$

$$= \frac{6}{3/2} + \frac{6}{8/5}$$

$$= 6\left[\frac{2}{3} + \frac{5}{8}\right]$$

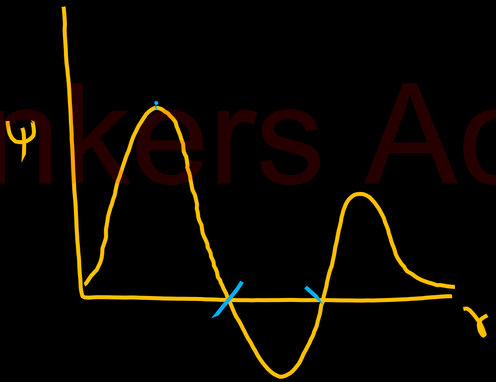
$$= 6\left(\frac{16+15}{24}\right) = \boxed{\frac{31}{4}} \text{ Ans}$$

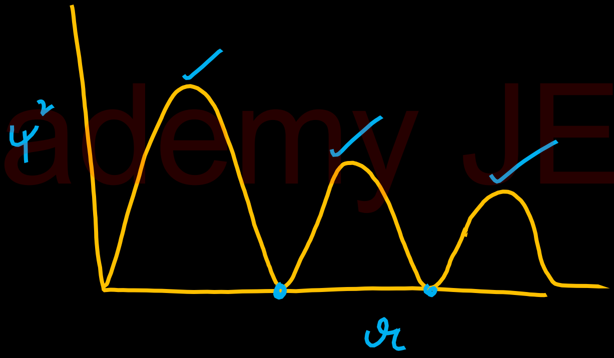

CHEMISTRY

The radial probability distribution curve of an orbital of hydrogen has 3 local maxima and the orbital has 2 angular node, the orbital will be

(A) 4p (B) 5 d

(C) 5f (D) 7 s





radial nodes $= 2 = n - l - 1$

$$\boxed{\begin{array}{c} l = 2 \\ n = 5 \end{array}}$$

2

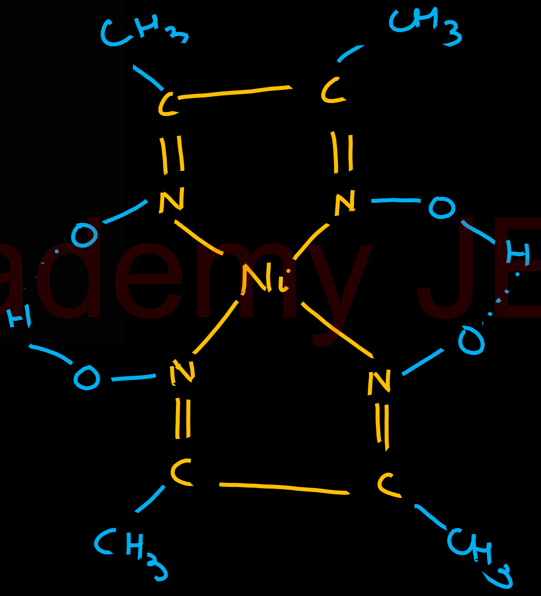
During the qualitative analysis of salt with cation y^{2+}, addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler"s reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}



Rosy Red complex

Match the molecules in List - I with their magnetic property and bond order respectively in List - II and choose the correct option

List - I		List - II	
(P)	C_2	(1)	Diamagnetic , 3
(Q)	N_2	(2)	Paramagnetic, 1
(R)	B_2	(3)	Diamagnetic , 1
(S)	O_2	(4)	Diamagnetic , 2
		(5)	Paramagnetic, 2

(A) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 5; S \rightarrow 1$

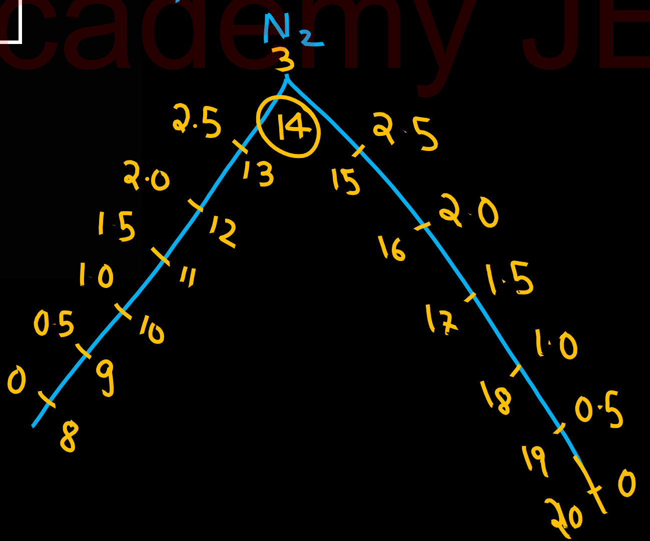
(B) $P \rightarrow 5; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 1$

(C) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 5; S \rightarrow 2$

(D) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 5$

Trick for Paramagnetic molecules

10, 16, odd number of electrons.



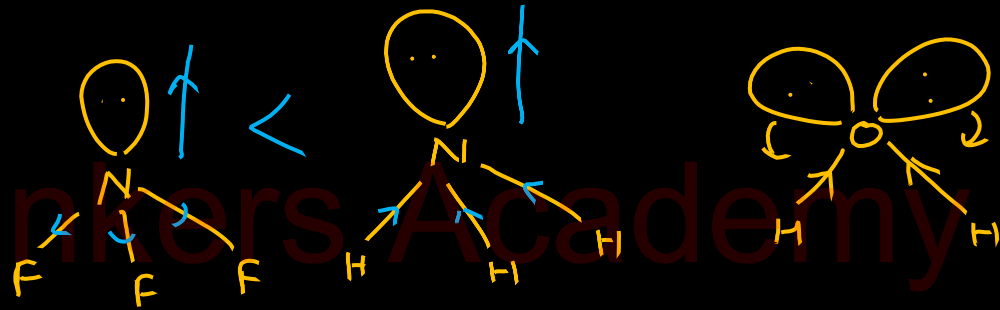
Increasing order of dipole moments is given by

(A) $CF_4 < NH_3 < NF_3 < H_2O$

(B) $CF_4 < NH_3 < H_2O < NF_3$

(C) $CF_4 < NF_3 < H_2O < NH_3$

(D) $CF_4 < NF_3 < NH_3 < H_2O$



5

One mole of an ideal gas at 300 K expands isothermally and reversibly from 10 atm to 1 atm. Then $\Delta U + \Delta H + W_{max} + q$ is equal to-

(A) 1 (B) 2

(C) 0 (D) -1

$\underset{0}{\Delta U} + \underset{0}{\Delta H} + \underbrace{W_{max} + q}_{0} = 0$

$\Delta U = 0$

$\Delta H = 0$

$\boxed{\Delta U = q + W}$

6

Stability of α -Helix structure of proteins depends upon

(A) dipolar interaction

(B) H-bonding interaction

(C) van der Waals forces

(D) π-stacking interaction

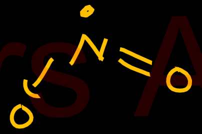
The oxide which contains an odd electron at the nitrogen atom is

(A) N_2O (B) NO_2

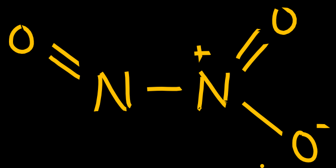
(C) N_2O_3 (D) N_2O_5

(A) N_2O $N\equiv\overset{+}{N}-O^-$

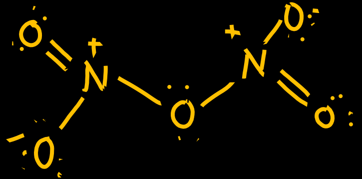
(B) NO_2



(C) N_2O_3



(D) N_2O_5



8

The most common oxidation state of Lanthanoid elements is +3 . Which of the following is likely to deviate easily from +3 oxidation state?

(A) Ce(At.No.58) (B) La(At.No.57)

(C) Lu (At.No.71) (D) Gd (At No.64)

1. Ce $= [Xe]\,4f^1\,5d^1\,6s^2$
2. La $= [Xe]\,4f^0\,5d^1\,6s^2$
3. Lu $= [Xe]\,4f^{14}\,5d^1\,6s^2$
4. Gd $= [Xe]\,4f^7\,5d^1\,6s^2$

Match List-I with List-II.

List-I (Oxide)		List-II (Nature)	
(A)	Cl_2O_7	(I)	Amphoteric
(B)	Na_2O	(II)	Basic
(C)	Al_2O_3	(III)	Neutral
(D)	N_2O	(IV)	Acidic

Choose the correct answer from the options given bib:

(A) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(B) (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

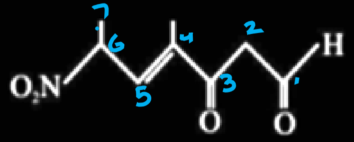
(C) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

(D) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

$$\underbrace{NO \quad CO \quad N_2O \quad H_2O}_{\text{Neutral oxides.}}$$

10

The correct IUPAC name of the following compound is :



(A) 4-methyl-2-nitro-5-oxo hept-3-enal

(B) 4-methyl-5-oxo-2-nitro hept-3-enal

(C) 4-methyl-6-nitro-3-oxo hept-4-enal

(D) 6-formyl-4-methyl-2-nitro hex-3-enal

Most reactive for S_N2 reaction :-

(A) $CH_3-\underset{\underset{O}{\|}}{C}-\overset{\alpha}{\underset{\delta+}{CH_2}}-Cl$ α- halogenated ketone.

(B) $Ph-CH_2-Cl$

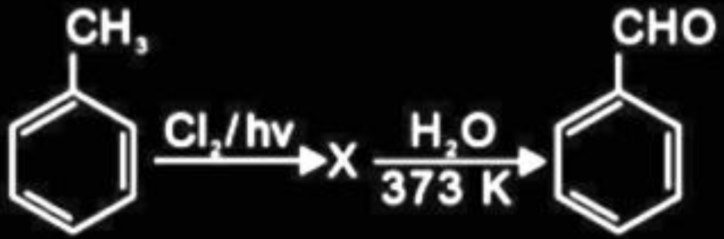
(C) $CH_3-\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}}-Cl$

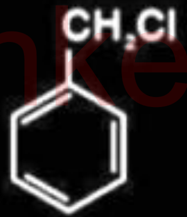
(D) $CH_3-\underset{\underset{CH_3}{|}}{CH}-Cl$

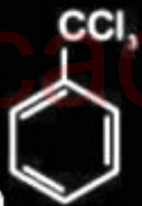
rate of S_N2

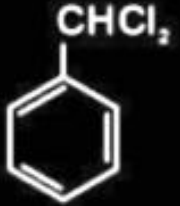
12

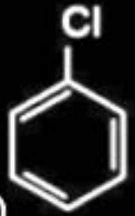
Identify compound X in the following sequence of reactions:

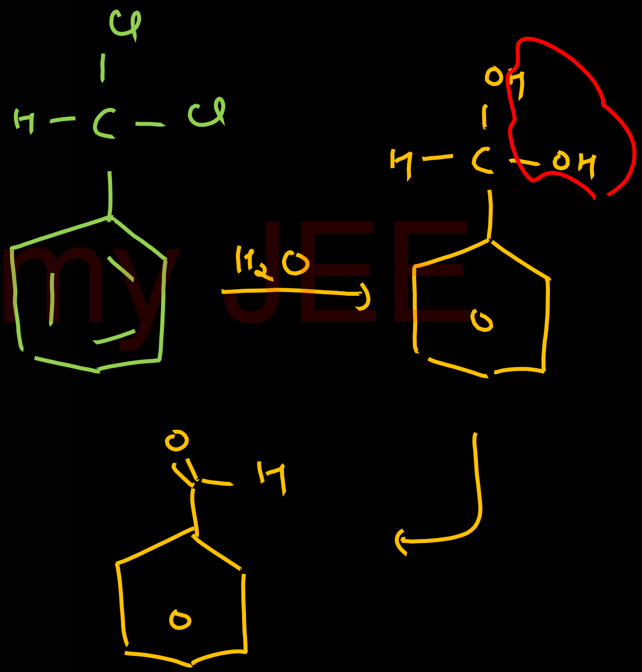


(A) 

(B) 

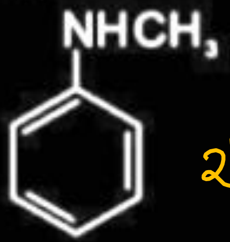
(C) 

(D) 

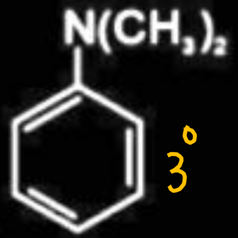


13

Which of the following amine will give the carbylamine test?

(A) 

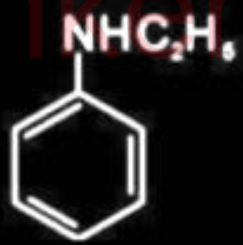
2°

(B) 

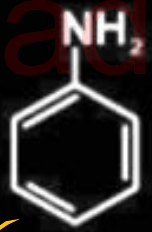
3°

$R-NH_2 \longrightarrow R-N\equiv C$

1° amine

(C) 

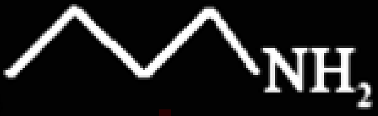
2°

(D) 

1°

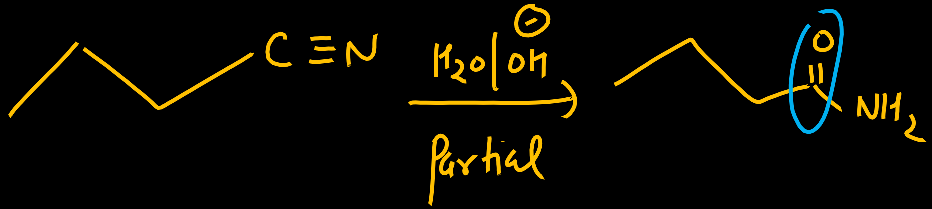
Product B is :-

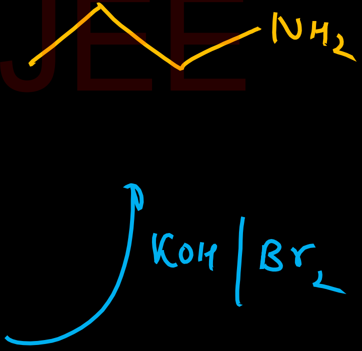
(A) 

(B) 

(C) 

(D) 





15

JEE 1

The standard enthalpy of formation of C_7H_{16} is -200 kJ/mole. The enthalpy of formation of CO_2 and H_2O are -394 kJ/mol and -286 kJ/mol respectively. The ennthalpy of combustion of C_7H_{16} is :-

(A) -5476 kJ/mol (B) -4846 kJ/mol

(C) -5200 kJ/mol (D) -5726 kJ/mol

$$C_7H_{16} + 11\,O_2 \longrightarrow 7CO_2 + 8H_2O \quad \Delta_c H$$

$$\Delta_c H = 7\,\Delta_f^{\circ}H(CO_2) + 8\,\Delta_f^{\circ}H(H_2O) - \Delta_f^{\circ}H(C_7H_{16})$$

$$= 7(-394) + 8(-286) - (-200) = \underline{-4846}$$

16

JEE 1

If 15 mL of H_2 and 10 mL of O_2 reacts to form water, what is left at the end of the reaction (Assume all volumes are measured under same temperature and pressure) :-

$V \propto n$

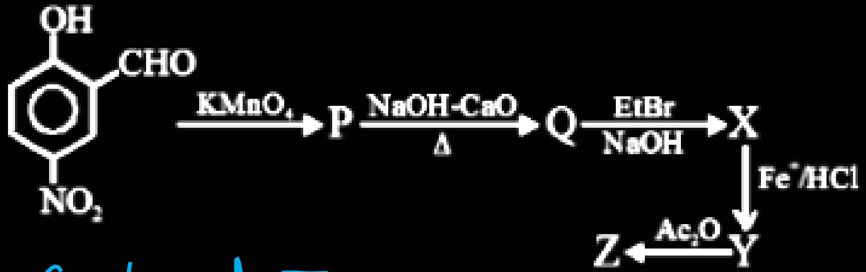
(A) 5 mLH_2 (B) 10 mLO_2

(C) 2.5 mLO_2 (D) 10 mLH_2

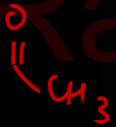
$$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$$

$t=0$ 15 ml 10 ml –

$t=t$ – $(10-7.5)$ ml $\underline{15 \text{ ml}}$

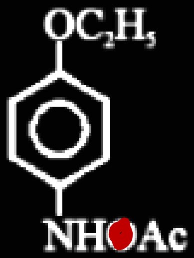


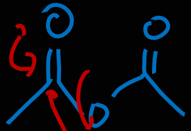
Compound Z



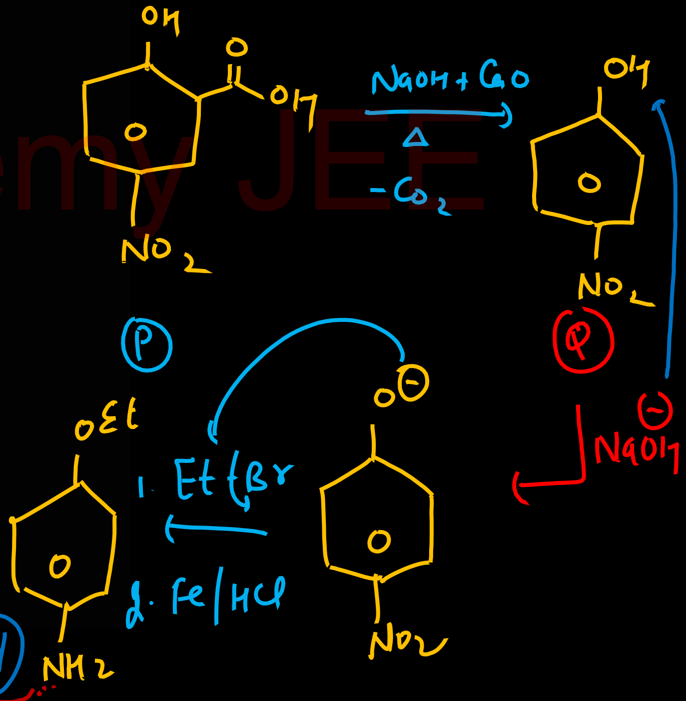
(A) 

(B) 

(C) 



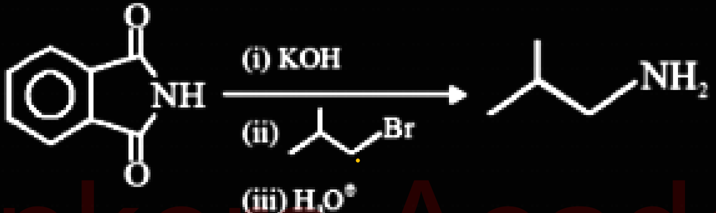
(D) 

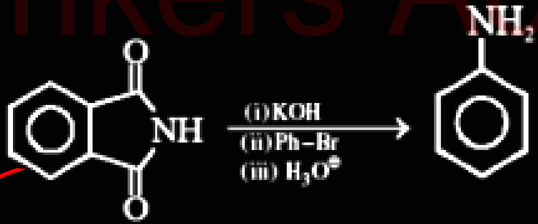


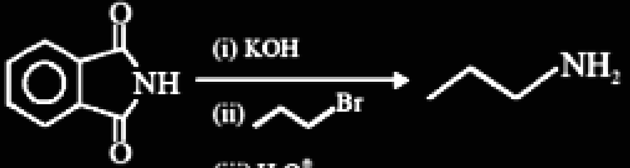
18

Which of the following reaction is incorrect :

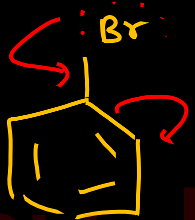
(A) 

(B) 

(C) 

(D) 

S_N2



Partial double bond character.

19

JEE1

The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction of these oils from the flowers is:

(A) Distillation

(B) Steam distillation

(C) Distillation under reduced

(D) None of the above

20

JEE 1

The molar conductivities at infinite dilution (Λ_m^0) of NH_4Cl, KOH and KCl are 150, 270 and 140 $S\,cm^2\,mol^{-1}$ respectively. K_b of 0.02M NH_4OH with $\Lambda_m = 25\ S\,cm^2\,mol^{-1}$ at the same temperature is :-

(A) 1.5×10^{-5} (B) 1.7×10^{-4}

(C) 1.8×10^{-10} (D) 2×10^{-5}

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$c - c\alpha \quad c\alpha \quad c\alpha$

$Ka = \frac{c\alpha^2}{1-\alpha}$

$Ka = \frac{0.02(0.09)^2}{1-0.09}$

$\simeq \underline{1.7 \times 10^{-4}}$

$\Lambda_m^0(NH_4OH) = \Lambda_m^0(NH_4Cl) + \Lambda_m^0(KOH) - \Lambda_m^0(KCl)$

$= 150 + 270 - 140 = 280$

$\alpha = \frac{\Lambda_m}{\Lambda_m^0} = \frac{25}{280} = 0.09 \simeq \underline{9\%}$

21

The speed of an electron in an orbit of hydrogen atom is 4.36×10^5 m/sec. Total number of waves formed by the electron in one complete revolution in this orbit is 5

$$v = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}$$

$$n = \frac{2.18 \times 10^6 \times 1}{4.36 \times 10^5} = 5$$

22

JEE 1

40ml of 0.2M NH_4Cl solution was added to 'x' ml of 0.2M NH_4OH solution and the pH of the resulting was found to be 8.56. The value of 'x' is 8 (pK_b NH_4OH = 4.74, log 5 = 0.7)

Basic Buffer

$$pOH = pK_b + \log \frac{Salt}{Base}$$

$$5.44 = 4.74 + \log \frac{40 \times \cancel{0.2}}{x \times \cancel{0.2}}$$

$$0.7 = \log \frac{40}{x}$$

$$\log 5 = \log \frac{40}{x}$$

$$5 = \frac{40}{x}$$

$$\boxed{x = 8}$$

23

JEE 1

Among

$SF_3Cl, XeO_2F_2, [ICl_4]^-, [NH_4]^+, [ICl_6]^-, XeO_3,$
$SF_4, [XeF_5]^+$ and $[XeO_6]^{4-}$, the total number of species having sp^3d hybridized central atom is

3

$\boxed{SF_3Cl}$ ✓	sp^3d
$\boxed{XeO_2F_2}$ ✓	sp^3d
$[ICl_4]^-$	sp^3d^2
NH_4^+	sp^3
$[ICl_6]^-$	sp^3d^3
XeO_3	sp^3
$\boxed{SF_4}$ ✓	sp^3d
XeF_5^+	sp^3d^2
XeO_6^{4-}	sp^3d^2

24

Among $N_2^-, O_2^{2-}, H_2^-, Li_2, F_2, He_2, N_2^+, O_2^+, Be_2$ the number of species which are paramagnetic is 4

$\boxed{10, 16, \text{odd}}$

$\boxed{N_2^-}$	Para
O_2^{2-}	Dia
$\boxed{H_2^-}$	Para
Li_2	Dia
F_2	Dia
He_2	Dia
$\boxed{N_2^+}$	Para
$\boxed{O_2^+}$	Para
Be_2	Dia

25

How many of the following are low spin complexes with para magnetic behaviour

+2 SF SF SF

$K_4[Fe(CN)_6], K_3[Fe(CN)_6], [Cu(NH_3)_4]^{+2}$

SF SF SF

$[Ni(NH_3)_6]^{+2}, [Ni(CN)_4]^{-2} [Mn(CN)_6]^{-3}$

3

$K_4[Fe(CN)_6]$ Fe^{+2} | ↑↓ | ↑↓ | ↑↓ | | | Di

$K_3[Fe(CN)_6]$ Fe^{+3} | ↑↓ | ↑↓ | ↑ | | Para ✓

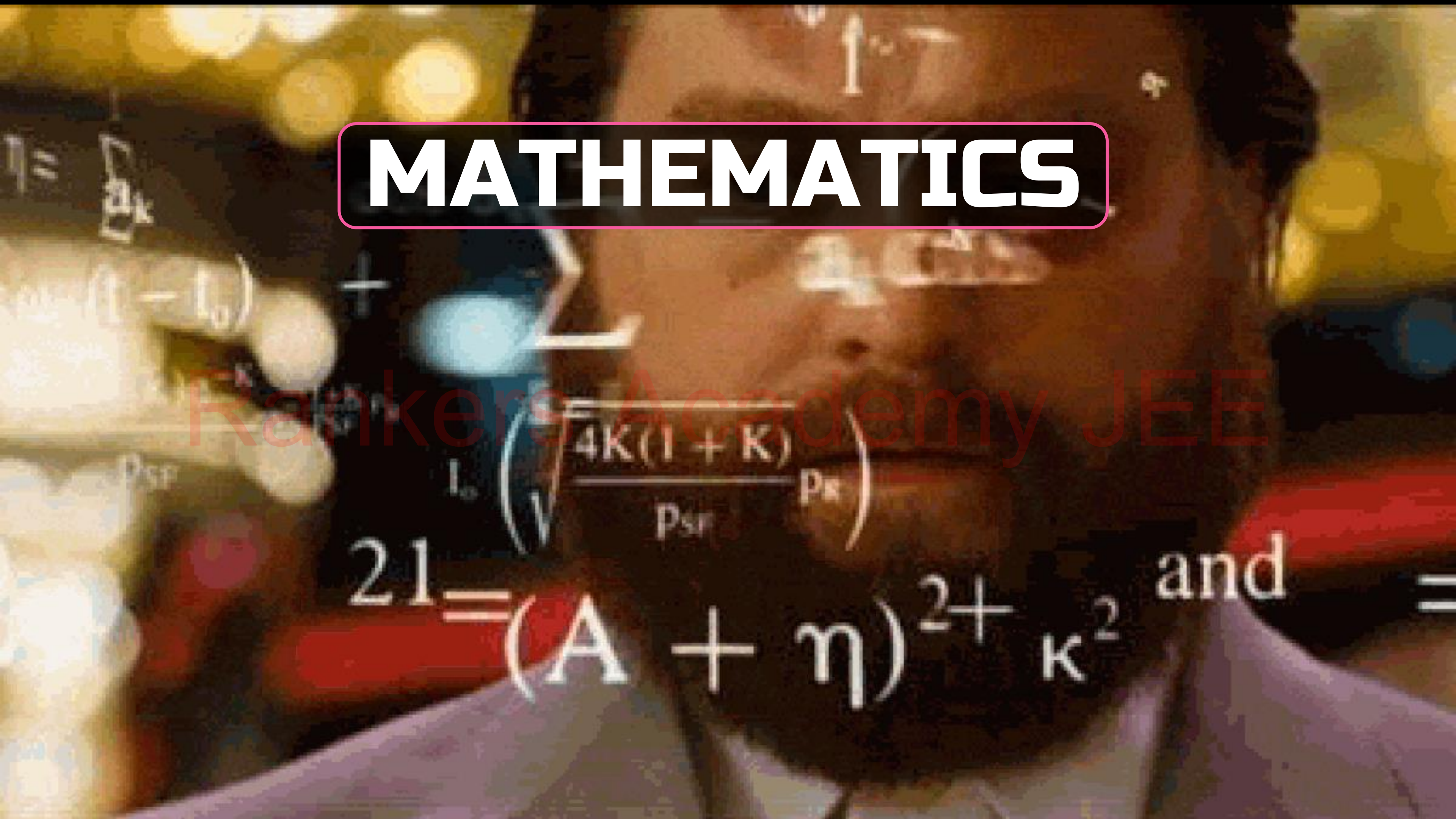
$[Cu(NH_3)_4]^{+3}$ Para ✓

$[Ni(NH_3)_6]^{+2}$ Para High spin

$[Ni(CN)_4]^{-2}$ +2 Dimag

$[\overset{+3}{Mn}(CN)_6]^{-3}$ +3 Para ✓

MATHEMATICS



If $f(x) + f(\sqrt{1-x^2}) = 2$, $\int_0^1 \frac{f(x)dx}{\sqrt{1-x^2}}$ is equal to -

(A) 0 (B) $\frac{\pi}{2}$

(C) π (D) 2π

$$I = \int_0^1 \frac{f(x)\,dx}{\sqrt{1-x^2}} \quad \text{--- (1)}$$

Let $1 - x^2 = t^2 \Rightarrow x = \sqrt{1-t^2}$

$-\cancel{2}x\,dx = \cancel{2}t\,dt$

$$I = \int_1^0 \frac{f(\sqrt{1-t^2})}{\cancel{t}}\left(\frac{-\cancel{t}\,dt}{\sqrt{1-t^2}}\right) = \int_0^1 \frac{f(\sqrt{1-t^2})\,dt}{\sqrt{1-t^2}} = \int_0^1 \frac{f(\sqrt{1-x^2})\,dx}{\sqrt{1-x^2}} \quad \text{--- (2)}$$

JEE 1

$$\Rightarrow 2I = \int_0^1 \frac{f(x) + f(\sqrt{1-x^2})}{\sqrt{1-x^2}}\, dx$$

$$\Rightarrow \cancel{2} I = \int_0^1 \frac{\cancel{2}}{\sqrt{1-x^2}}\, dx$$

$$I = \sin^{-1} x \Big|_0^1$$

$$= \pi/2$$

2

Let $S = \frac{8}{5} + \frac{16}{65} + \frac{24}{325} \dots \dots + \frac{128}{2^{18}+1}$, then

(A) $S = \frac{1088}{545}$ ✓

(B) $S = \frac{545}{1088}$

(C) $S = \frac{1056}{545}$

(D) $S = \frac{545}{1056}$

JEE 1

$T_n = \frac{8n}{(2n^2)^2+1}$

2^2+1

8^2+1

18^2+1

$T_n = \frac{8n}{4n^4+1+4n^2-4n^2}$

$= \frac{8n}{(2n^2+1)^2-(2n)^2}$

$= \frac{8n}{(2n^2+1+2n)(2n^2+1-2n)}$

$\Rightarrow 2\left(\frac{(2n^2+1+2n)-(2n^2+1-2n)}{(2n^2+1+2n)(2n^2+1-2n)}\right)$

$T_n \Rightarrow 2\left(\frac{1}{2n^2+1-2n} - \frac{1}{2n^2+1+2n}\right)$

2

JEE 1

$$S = \sum_{n=1}^{16} T_n$$

$$8n = 128 = 2^7$$

$$n = \frac{2^7}{2^3} = 16$$

$$= 2\left[1 - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cdots - \frac{1}{545} \right]$$

$$\begin{array}{r} 513 \\ 32 \\ \hline 545 \end{array}$$

$$\Rightarrow 2\left(1 - \frac{1}{545}\right)$$

$$\Rightarrow \frac{2 \times 544}{545}$$

3

JEE 1

Let $f(x) = x + x^2 + x^4 + x^8 + x^{16} + x^{32} + \cdots$.

the coefficient of x^{10} in $f(f(x))$ is

(A) 28

(B) 40

(C) 52

(D) none of these

$$f(f(x)) = f(x) + (f(x))^2 + (f(x))^4 + (f(x))^8 + (f(x))^{16} + \cdots$$

x

$(x + x^2 + x^4 + x^8 + \ldots)^2$

$\frac{2!}{1!\,1!} = 2$

$(x + x^2 + x^4 + x^8 + \ldots)^4$

$(x^1)^2 (x^4)^2$ or $(x^2)^3 (x^4)^1$

$\frac{4!}{2!\,2!} + \frac{4!}{3!\,1!}$

$6 + 4$

$= 10$

$(x + x^2 + x^4 + x^8 + \ldots)^8$

$(x)^6 (x^2)^2$

$\frac{8!}{6!\,2!} = 28$

$x^{16}(1 + x + x^3 \ldots)^{16}$ ✗

4

The domain of the function

$$f(x) = \sqrt{\frac{(2x-1)}{(2x^3+3x^2+x)}} + \sqrt{\sin^{-1}(\log_2 x)} \text{ is}$$

~~(A)~~ $\left(\frac{1}{2}, \infty\right)$

~~(B)~~ $\left(\frac{1}{2}, 2\right]$

(C) $[1,2]$ ✓

~~(D)~~ $(1, \infty)$

① $x > 0$

② $-1 \le \log_2 x \le 1 \Longrightarrow 0 \le \log_2 x \le 1$

$\Rightarrow \frac{1}{2} \le x \le 2$

$1 \le x \le 2$

③ $2x^3 + 3x^2 + x \ne 0$

$x(2x^2 + 3x + 1) \ne 0$

$x(2x+1)(x+1) \ne 0$

$x \ne 0, -1/2, -1$

④ $\dfrac{2x-1}{x(2x+1)(x+1)} \ge 0$

$+ \quad - \quad + \quad - \quad +$

$-1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$x \in (-\infty, -1) \cup \left(-\frac{1}{2}, 0\right) \cup \left[\frac{1}{2}, \infty\right)$

5

If $f''(x) > 0, \forall x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then $g(x)$ is increasing in

(A) $\left(0, \frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(C) $\left(0, \frac{\pi}{3}\right)$ (D) none of these

JEE 1

$\uparrow$: $g'(x) > 0$

$\Rightarrow f'(\tan^2 x - 2\tan x + 4) \cdot (2\tan x \sec^2 x - 2\sec^2 x) > 0$

$\Rightarrow f'((\tan x - 1)^2 + 3) \cdot 2\sec^2 x(\tan x - 1) > 0$

$\Rightarrow \underbrace{f'((\tan x - 1)^2 + 3)}_{+ve} \cdot (\tan x - 1) > 0$

$x \in (0, \pi/4)$ $-ve$ $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $+ve$

$\Rightarrow \tan x - 1 > 0$

$\Rightarrow \tan x > 1$

5

JEE 1

$\Rightarrow f'' > 0$

$\Rightarrow (f')' > 0$

$\Rightarrow f' : \uparrow f^n$

i/p $\uparrow$ o/p $\uparrow$

$(\tan x - 1)^2 < (\tan x - 1)^2 + 3$

or

$\boxed{3}$

$f'(3) < f'((\tan x - 1)^2 + 3)$

$0 < f'((\tan x - 1)^2 + 3)$

Let $f(x) = (x^2 - 4)|(x^3 - 6x^2 + 11x - 6)| + \frac{x}{1+|x|}$. The set of points at which the function f(x) is not differentiable is

~~(A) $\{-2,2,1,3\}$~~ ~~(B) $\{-2,0,3\}$~~

~~(C) $\{-2,2,0\}$~~ (D) $\{1,3\}$

$$f(x) = (x+2)(x-2)\,|(x-1)(x-2)(x-3)| + \frac{x}{|x|+1}$$

$x = 1$, $x = 2$, $x = 3$

$$f(x) = \begin{cases} -(x+2)(x-2)^2(x-1)(x-3) + \frac{x}{x+1}; & 3 > x > 2 \\ (x+2)(x-2)^2(x-1)(x-3) + \frac{x}{x+1}; & x < 2 \end{cases}$$

RHD = LHD

$-\ 1\ +\ 2\ -\ 3\ +$

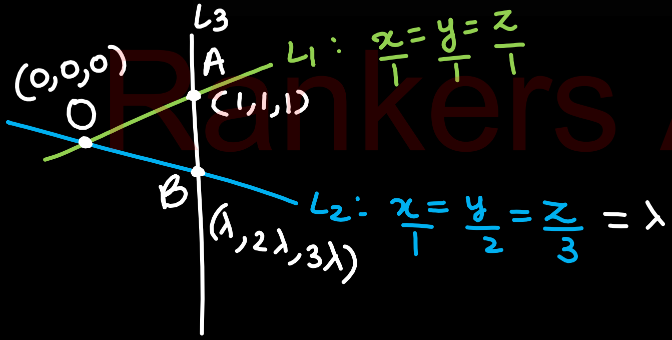
7

JEE 1

If lines $x = y = z, x = \frac{y}{2} = \frac{z}{3}$ and third line passing through (1,1,1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line will be

(A) (1,2,3) (B) (2,4,6)

(C) $\left(\frac{4}{3}, \frac{8}{3}, \frac{12}{3}\right)$ (D) none of these



ar(ΔAOB) = $\sqrt{6}$

$\frac{1}{2}|\vec{OA} \times \vec{OB}| = \sqrt{6}$

$\Rightarrow \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \lambda & 2\lambda & 3\lambda \end{vmatrix} = \sqrt{6}$

$\Rightarrow \frac{1}{2}|\lambda\hat{i} - 2\lambda\hat{j} + \lambda\hat{k}| = \sqrt{6}$

$= \frac{1}{2}\sqrt{\lambda^2 + 4\lambda^2 + \lambda^2} = \sqrt{6}$

$\Rightarrow \frac{1}{2}\cdot\cancel{\sqrt{6}}|\lambda| = \cancel{\sqrt{6}}$

$\Rightarrow \lambda = \pm 2.$

B: (2,4,6)

or

(−2,−4,−6)

Rankers Academy JEE

8

The sum

$\binom{100}{98} + \binom{99}{97} + \binom{98}{96} + \cdots \ldots + \binom{3}{1} + \binom{2}{0}$ is

equal to, where $\binom{n}{r} = {}^nC_r$

(A) $\binom{100}{98}$ (B) $\binom{101}{99}$

(C) $\binom{100}{99}$ (D) $\binom{101}{98}$

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$\Rightarrow {}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 - - - + {}^{98}C_{96} + {}^{99}C_{97} + {}^{100}C_{98}$$

$${}^2C_0 \to 1 \to {}^3C_0$$

$$\underbrace{\underbrace{\underbrace{{}^3C_0 + {}^3C_1}_{{}^4C_1} + {}^4C_2}_{{}^5C_2} + {}^5C_3}_{{}^6C_3}$$

$${}^{100}C_{97} + {}^{100}C_{98} = {}^{101}C_{98}$$

The vertices of a triangle in the argand plane are $3+4i, 4+3i$ and $2\sqrt{6}+i$, then distance between orthocentre and circumcentre of the triangle is equal to,

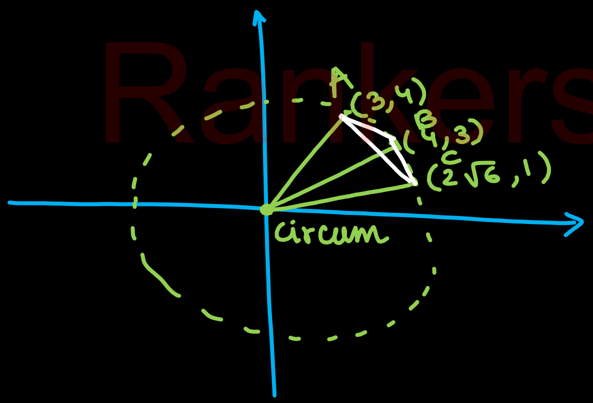
$OC = ? = 3\,GC$

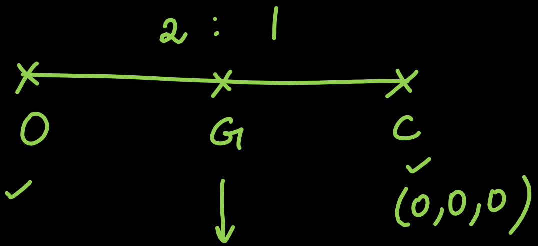
(A) $\sqrt{137-28\sqrt{6}}$

(B) $\sqrt{137+28\sqrt{6}}$

(C) $\frac{1}{2}\sqrt{137+28\sqrt{6}}$

(D) $\frac{1}{3}\sqrt{137+28\sqrt{6}}$





$$\frac{3+4+2\sqrt{6}}{3}, \frac{4+3+1}{3}$$

$$\left(\frac{7+2\sqrt{6}}{3}, \frac{8}{3}\right)$$

$$|\vec{GC}| = \sqrt{\left(\frac{7+2\sqrt{6}}{3}\right)^2+\left(\frac{8}{3}\right)^2}$$

$$OC = 3GC = \sqrt{(7+2\sqrt{6})^2+8^2}$$

10

JEE1

Let $f: X \to Y$ be a function such that $f(x) = \sqrt{x-2} + \sqrt{4-x}$, then the set of X and Y for which f(x) is both injective as well as surjective, is

1-1

M-1

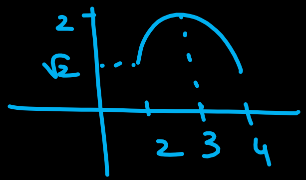
(A) $[2,4]$ and $[\sqrt{2}, 2]$

(B) $[3,4]$ and $[\sqrt{2}, 2]$

(C) $[2,4]$ and $[1,2]$

(D) $[2,3]$ and $[1,2]$

$4 \geq x \geq 2$



$f(2) = \sqrt{2} = f(4)$ min

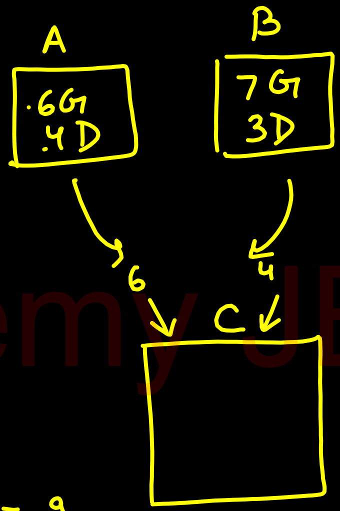
$f\left(\frac{2+4}{2}\right) = f(3) = 2$ max

Range $= [\sqrt{2}, 2] = Y$

surj

Lot A consists of 6 good and 4 defective articles. Lot B consists of 7 good and 3 defective articles. A new lot C is formed by taking 6 articles from lot A and 4 articles from lot B. The probability that an articles chosen at random from the lot C is defective, is $\frac{k}{25}$ then k is

(A) 144
(B) 9
(C) 18
(D) 16



$$P(\text{def.}) = \underset{A}{\frac{6}{10}} \cdot \underset{D}{\frac{4}{10}} + \underset{B}{\frac{4}{10}} \cdot \underset{D}{\frac{3}{10}} = \frac{36}{100} = \frac{9}{25}$$

12

Given a function g continuous on R such that $\int_0^1 g(t)dt = 2$ and $g(1) = 5$. If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)dt$, then the value of $(f'''(1) - f''(1))$ is equal to:

$= 5 - 2 = 3.$

(A) 0
(B) 3
(C) 5
(D) 7

JEE 1

$$f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\,dt$$

$$= \frac{1}{2}\int_0^x (x^2+t^2-2xt)\,g(t)\,dt$$

$$f(x) = \frac{1}{2}\left[x^2 \underbrace{\int_0^x g(t)\,dt} + \int_0^x t^2 g(t)\,dt - 2x \int_0^x t\,g(t)\,dt\right]$$

Diff wrt x

$$f' = \frac{1}{2}\left[2x\int_0^x g(t)\,dt + \cancel{x^2 g(x)} + \cancel{x^2 g(x)} - 2\left(\int_0^x t\,g(t)\,dt + \cancel{x \cdot x g(x)}\right)\right]$$

$$\Rightarrow f'(x) = \frac{1}{\cancel{2}}\left[\cancel{2}x\int_0^x g(t)\,dt - \cancel{2}\int_0^x t\,g(t)\,dt\right]$$

$$\Rightarrow f''(x) \Rightarrow \left[\int_0^x g(t)\,dt + x\cancel{g(x)} - \cancel{x g(x)}\right]$$

$$\Rightarrow f''(x) \Rightarrow \int^x g(t)\,dt \longrightarrow f'''(x) = g(x)$$

$$f''(1) = \int_0^1 g(t)\,dt = 2$$

$$f'''(1) = g(1) = 5$$

13

JEE 1

Let $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ where $x, y, z \in N$ if $|\text{adj}(\text{adj}(\text{adj}(\text{adj } A)))| = 4^8 \cdot 5^{16}$ then the number of such matrix is p and value of $\int_{-\pi/2}^{\pi/2} 2^{\sin x}dx + \int_{5/2}^{4} \sin^{-1}(\log_2(x-2))dx = \frac{q\pi}{4}$, then $p + q$ is

(A) 40
(B) 41
(C) 42
(D) 44

$|A|^{(n-1)^4} = 4^8 \cdot 5^{16}$

$|A|^{(3-1)^4}$

$|A|^{16} = 2^{16} 5^{16}$

$|A| = \pm 10$

$|A| = \begin{vmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} = x + y + z = 10$

$x, y, z \in N$

$x, y, z \geq 1$

${}^{7+3-1}C_{3-1} = {}^9C_2 = 36$

$\therefore p = 36$

13

JEE 1

$$9\frac{\pi}{4} = \int_{-\pi/2}^{\pi/2} 2^{\sin x}\,dx + \int_{5/2}^{4} \sin^{-1}\left(\log_2(x-2)\right) dx$$

Let $x-2=t$
$dx = dt$

$$9\frac{\pi}{4} = \underbrace{\int_{-\pi/2}^{\pi/2} 2^{\sin x}\,dx}_{f} + \underbrace{\int_{1/2}^{2} \sin^{-1}(\log_2 t)\,dt}_{f^{-1}}$$

$f(x) = y = 2^{\sin x}$

$\log_2 y = \sin x$

$x = \sin^{-1}(\log_2 y)$

$$a\frac{\pi}{4} = \frac{\pi}{2}(2) - \left(-\frac{\pi}{2}\right)\left(\frac{1}{2}\right)$$

$$= \pi + \frac{\pi}{4}$$

$$a\frac{\pi}{4} = \frac{5\pi}{4}$$

$$\therefore \boxed{a=5}$$

14

The quadratic polynomial $P(x) = ax^2 + bx + c$ has two different zeroes including -2 . The quadratic polynomial $Q(x) = ax^2 + cx + b$ has two different zeroes including 3 . If α and β be the other zeroes of $P(x)$ and $Q(x)$ respectively, then find the value of $\frac{\alpha}{\beta}$.

(A) 7
(B) 77
(C) 11
(D) 49

JEE 1

$P(x) = ax^2+bx+c$ — -2, α

$\alpha - 2 = -b/a$

$-2\alpha = c/a$

$Q(x) = ax^2+cx+b$ — 3, β

$\beta + 3 = -c/a$

$3\beta = b/a$

$-2\alpha = -\beta - 3 \Rightarrow -2\alpha + \beta = -3$ — (1)

$2 - \alpha = 3\beta$ — (2)

$4 - 2\alpha = 6\beta$ — (3)

$\beta - 4 = -3 - 6\beta$

$\beta = \frac{1}{7}$

$\alpha = 2 - \frac{3}{7} = \frac{11}{7}$

15

The value of $\sum_{r=1}^{5} \cos(2r-1)\frac{\pi}{11}$ is :

(A) $\frac{1}{2}$ ✓ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$

$$\Rightarrow \frac{\sin(nd/2)\cos\left(a+(n-1)\frac{d}{2}\right)}{\sin(d/2)}$$

$n = 5$

$a = \pi/11$

$d = 2\pi/11$

$$\Rightarrow \frac{\sin(5\pi/11)\cos\left(\frac{\pi}{11}+4\frac{\pi}{11}\right)}{\sin(\pi/11)} = \frac{2\sin 5\pi/11 \cos 5\pi/11}{2\sin\pi/11}$$

$$= \frac{\sin(10\pi/11)}{2\sin(\pi/11)} = \frac{1}{2}.$$

16

On the portion of the straight line $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the coordinates of point of intersection of its diagonals is

(A) (2,3)

(B) (3,2)

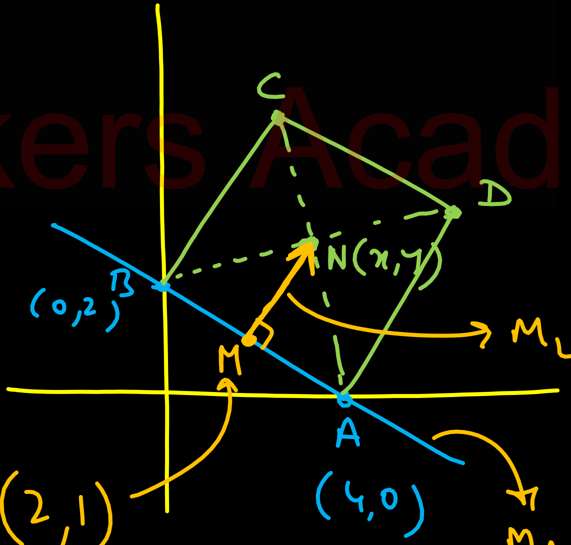
(C) (3,3)

(D) (6,4)

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$\frac{x-2}{1/\sqrt{5}} = \frac{y-1}{2/\sqrt{5}} = \sqrt{5}$$

$$\boxed{x = 3;\ y = 3}$$



$AB = \sqrt{16+4}$

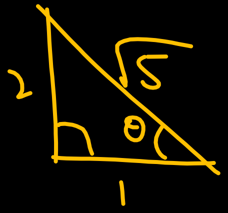
$= 2\sqrt{5}$

$MN = \sqrt{5} = r$

$(2,1)$

$M_L = 2 = \tan\theta$

$M_{AB} = \frac{2}{-4} = -\frac{1}{2}$



17

JEE 1

If $f(x) = \begin{cases} x+2, & \text{when } x < 1 \\ 4x-1, & \text{when } 1 \le x \le 3 \\ x^2+5, & \text{when } x > 3 \end{cases}$, then

correct statement is -

(A) $\lim_{x\to1} f(x) = \lim_{x\to3} f(x)$

(B) f(x) is continuous at $x = 3$

(C) f(x) is continuous at $x = 1$

(D) f(x) is continuous at $x = 1$ and 3

$x = 1 \rightarrow \left.\begin{matrix} LHL = 3 \\ RHL = 3 \end{matrix}\right\} \checkmark$

18

$$\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$$

(A) $\sin x-6\tan^{-1}(\sin x)+c$

(B) $\sin x-2\sin^{-1}x+c$

(C) $\sin x-2(\sin x)^{-1}-6\tan^{-1}(\sin x)+c$

(D) $\sin x-2(\sin x)^{-1}+5\tan^{-1}(\sin x)+c$

JEE 1

$$\int \frac{(1+\cos^2 x)(\cos^2 x)(\cos x)\,dx}{\sin^2 x+\sin^4 x}$$

Let: $\sin x = t$

$\cos x\, dx = dt$

$$\int \frac{(2-t^2)(1-t^2)}{t^2+t^4}dt$$

$$\int \frac{t^4-3t^2+2}{t^4+t^2}dt$$

$$\int \frac{t^4+t^2}{t^4+t^2}dt+\int \frac{-4t^2+2}{t^4+t^2}dt$$

18

JEE 1

$$\int dt + \int \frac{-4t^2-4}{t^2(t^2+1)}\,dt + \int \frac{6}{t^2(t^2+1)}\,dt$$

$$\int dt - 4\int \frac{1}{t^2}\,dt + 6\int\left(\frac{1}{t^2} - \frac{1}{(t^2+1)}\right)dt$$

$$t - 4\left(-\frac{1}{t}\right) + 6\left(-\frac{1}{t} - \tan^{-1}t\right) + c$$

$$t - \frac{2}{t} - 6\tan^{-1}t + c$$

$$(\sin x) - \frac{2}{(\sin x)} - 6\tan^{-1}(\sin x) + c$$

19

Area bounded by the curves

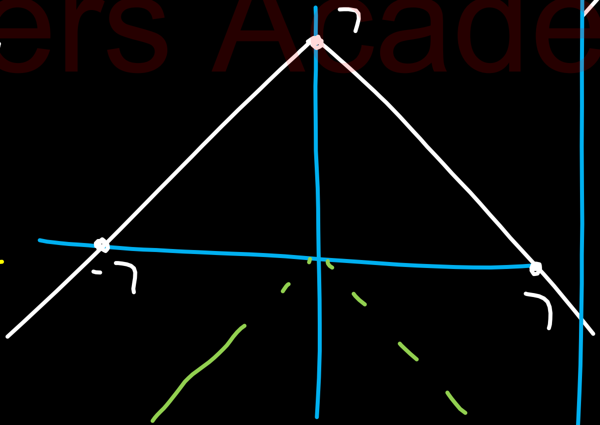
$4y = |x^2 - 4|$ and $y + |x| = 7$, is equal to

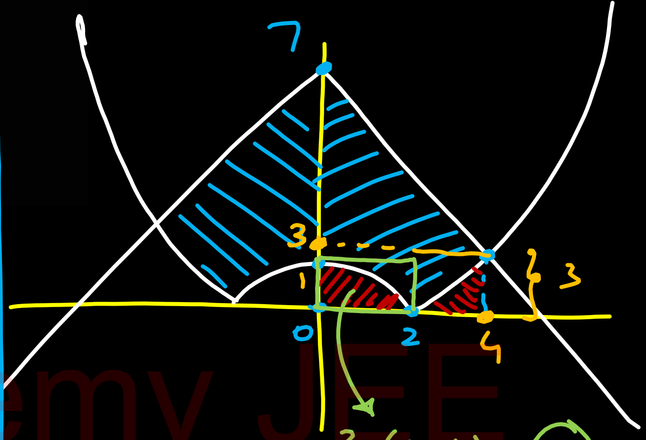
(A) 8 sq. units (B) 16 sq. units

(C) 4 sq. units ~~(D)~~ 32 sq. units

$y = \frac{1}{4}|x^2-4|$

$y = -|x|+7$





$\frac{2}{3}(2\times 1) = \frac{4}{3}$

$\frac{x^2-4}{4} = -x+7$

$x^2-4 = -4x+28$

$x^2+4x-32=0$

$(x+8)(x-4)=0 \Rightarrow \boxed{x=4}$

JEE 1

$$\int_2^4 \frac{x^2-4}{4}\,dx$$

$$= \frac{1}{4}\left(\frac{x^3}{3}-4x\right)_2^4$$

$$= \frac{8}{3}$$

Area of trap $= \frac{1}{2}(7+3)(4) = 20$

Final Area $= 2\left[20-\left(\frac{4}{3}+\frac{8}{3}\right)\right]$

$$= 2\left[20-4\right]$$

$$= 32$$

20

JEE 1

The slope of the tangnet at (x, y) to a curve passing through a point (2,1) is $\frac{x^2+y^2}{2xy}$, then the equation of the curve is

(A) $2(x^2-y^2)=3x$ (B) $2(x^2-y^2)=6y$

(C) $x(x^2-y^2)=6$ (D) $x(x^2+y^2)=10$

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$$

$$\frac{dy}{dx}=\frac{1+(y/x)^2}{2(y/x)}$$

Let: $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$$

$$x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$$

$$x\frac{dv}{dx}=\frac{1-v^2}{2v}$$

$$\int\frac{2v}{1-v^2}dv=\int\frac{dx}{x}$$

$$\int\frac{2v\,dv}{v^2-1}=-\int\frac{dx}{x}$$

$\ln|v^2-1| = -\ln|cx|$

$\ln\left|\left(\frac{y^2}{x^2}-1\right)(cx)\right| = 0$

$\left(\frac{y^2-x^2}{x^2}\right)(cx) = 1$

$\boxed{(y^2-x^2)\,c = x}$

$x=2\,;\,y=1$

$(1-4)\,c = 2$

$c = -\frac{2}{3}$

$(y^2-x^2)\left(-\frac{2}{3}\right) = x$

$\boxed{2(x^2-y^2) = 3x}$

21

JEE 1

The mean and variance of 8 observations are 10 and 13.5 respectively. If 6 of these observations are 5,7,10, 12,14,15 , then the absolute difference of the remaining two observations is____ 7

$x = 5$

$y = 12$

Diff. 7

$$\text{Mean} = 10 = \frac{(5+7+10+12+14+15+x+y)}{8}$$

$$x^2 + y^2 = 169$$

$$\boxed{x+y=17}$$

$$\sigma^2 = 13.5 = \frac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$$

22

JEE 1

If $S = \frac{1}{2}\sum_{n=2}^{\infty} \frac{2(3n^2+1)}{(n^2-1)^3}$ then $\frac{9}{S} =$ $\frac{9}{9/16} = \boxed{16}$

$(n+1)^3(n-1)^3$

$(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$(n-1)^3 = n^3 - 3n^2 + 3n - 1$

$(n+1)^3 - (n-1)^3 = 6n^2 + 2$

$= 2(3n^2+1)$

$S = \frac{1}{2}\sum_{n=2}^{\infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^3(n-1)^3}$

$= \frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{1}{(n-1)^3} - \frac{1}{(n+1)^3}\right]$

$= \frac{1}{2}\left\{\begin{array}{l} \frac{1}{1^3} - \cancel{\frac{1}{3^3}} \\ \frac{1}{2^3} - \cancel{\frac{1}{4^3}} \\ \cancel{\frac{1}{3^3}} - \cancel{\frac{1}{5^3}} \\ \vdots \quad \vdots \end{array}\right\}$

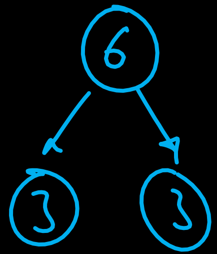
$S = \frac{1}{2}\left(1 + \frac{1}{8}\right)$

$= \frac{9}{16}$

23

Consider seven digit number $x_1x_2, \ldots, x_7$, where $x_1, x_2 \ldots \ldots, x_7 \neq 0$ having the property that x_4 is the greatest digit and all digits towards the left and right of x_4 are in decreasing order. Then total number of such number in which all digits are distinct is a, then find the sum of digits of a.

$\_\ \_\ \_\ x_4\ \_\ \_\ \_$



Case-1: 9

$$^8C_6\left(\frac{6!}{3!\,3!\,\cancel{2!}}\right)(\cancel{2!})$$

Case-2: $x_4 = 8$

$$^7C_6\left(\frac{6!}{3!\,3!\,\cancel{2!}}\times\cancel{2!}\right)$$

Case-3: $x_4 = 7$

$$^6C_6\left(\frac{6!}{3!\,3!\,\cancel{2!}}\times\cancel{2!}\right)$$

JEE 1

Final Ans:

$$\left({}^{8}C_{2} + {}^{7}C_{1} + {}^{6}C_{6}\right)\left(\frac{6!}{6\times 6}\right)$$

$$\left(\frac{\overset{4}{\cancel{8}}\times 7}{\cancel{2}} + 7 + 1\right)\left(\frac{\overset{20}{\cancel{120}}}{\cancel{6}}\right)$$

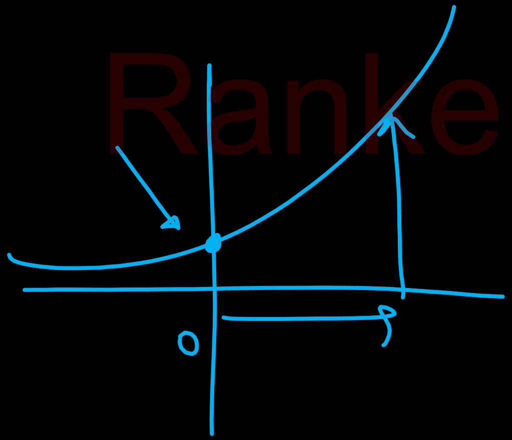
$$= \boxed{720}$$

24

JEE 1

Find the minimum value of $e^{(2x^2-2x+1)\sin^2 x}$.

1



$(\underbrace{2x^2-2x+1}_{D<0})(\sin^2 x)$

(always +ve)

25

JEE 1

If $f(x)+f\left(1-\frac{1}{x}\right)=\boxed{1+x}\ \forall x\in R-\{0,1\}$.

Then find the value of $\boxed{4f(2)}$

$\boxed{x=2}$: $f(2)+f\left(\frac{1}{2}\right)=3$

$\boxed{x=\frac{1}{2}}$: $f\left(\frac{1}{2}\right)+f(-1)=\frac{3}{2}$

$\boxed{x=-1}$: $f(-1)+f(2)=0$

$\rightarrow f\left(\frac{1}{2}\right)-f(2)=\frac{3}{2}$

$2f(2)=\frac{3}{2}$

$\Rightarrow \boxed{4f(2)=3}$