PHYSICS

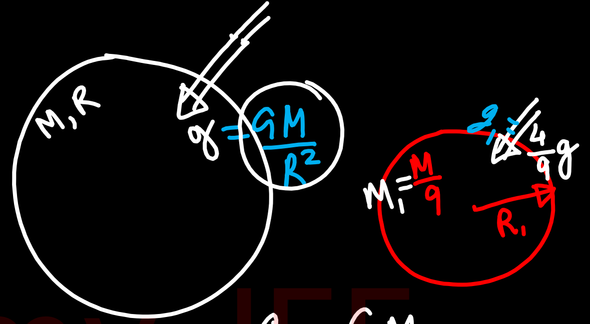
The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is $\boxed{9:4}$. The mass of the planet is $\frac{1}{9}$ that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)

(A) R/3 (B) R/9

(C) R/4 (D) R/2

$\boxed{W = mg}$

$1 : \frac{4}{9}$



$g_1 = \frac{GM_1}{R_1^2} = \frac{4g}{9}$

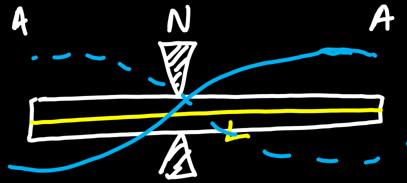
$\frac{G(M/9)}{R_1^2} = \frac{4}{9}\frac{GM}{R^2}$

$\boxed{R_1 = \frac{R}{2}}$

2

A granite rod of 60 cm (0.6 m) length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \text{ kg m}^{-3}$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(A) 5kHz ✓
(B) 2.5kHz
(C) 10kHz
(D) 7.5kHz



$L = \frac{\lambda}{2}$

$\boxed{\lambda = 2L}$

$$\boxed{f_0 = \frac{V}{\lambda} = \frac{V}{2L}}$$

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

$\boxed{L = \frac{3}{5}}$

$\boxed{Y = 9.27 \times 10^{10}}$

$\boxed{\rho = 2.7 \times 10^3}$

$$f_0 = \frac{\sqrt{\frac{Y}{\rho}}}{2L} = 4.8 \times 10^3 \text{ Hz}$$

3

The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants $G, \hbar$ and c. Which of the following correctly gives the Planck length?

(A) $G\hbar^2c^3$

(B) $G^2\hbar c$

(C) $\left(\frac{G\hbar}{c^3}\right)^{1/2}$ ✓

(D) $G^{1/2}\hbar^2 c$

$[L] = G^x h^y c^z$

$M^0L^1T^0 = [M^{-1}L^3T^{-2}]^x [ML^2T^{-1}]^y [LT^{-1}]^z$

$M^0L^1T^0 = M^{-x+y} L^{3x+2y+z} T^{-2x-y-z}$

$\boxed{x = y}$

$5x + z = 1$

$-3x - z = 0$

$2x = 1$

$\boxed{x = 1/2}$ $\boxed{y = 1/2}$

$\boxed{z = -\frac{3}{2}}$

$[G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$[h] = \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

$[c] = LT^{-1}$

4

JEE 1

A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension $\boxed{7.5 \times 10^{-4}\ \mathrm{N\ cm^{-1}}}$. The radius of drop in cm will be $(g = 10\ \mathrm{ms^{-2}})$

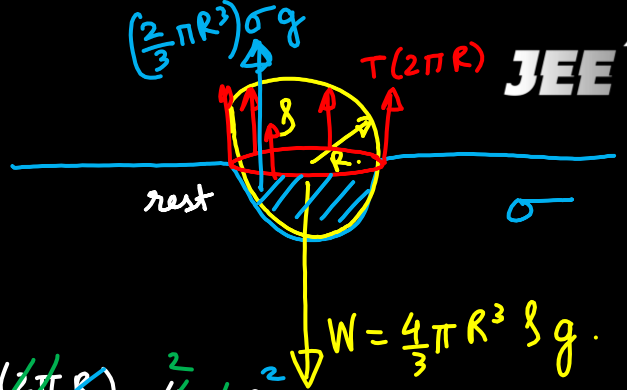
(A) $\frac{15}{\sqrt{(2\rho-\sigma)}}$ cm

(B) $\frac{15}{\sqrt{(\rho-\sigma)}}$ cm.

(C) $\frac{3}{2\sqrt{(\rho-\sigma)}}$ cm.

(D) $\frac{3}{20\sqrt{(2\rho-\sigma)}}$ cm

m.



$$\cancel{\tfrac{2}{3}}\cancel{\pi} R^{\cancel{3}\,2}\sigma g + 3T(\cancel{2}\cancel{\pi}\cancel{R}) = \cancel{\tfrac{4}{3}}^{2}\cancel{\pi} R^{\cancel{3}\,2}\rho g$$

$$R = \sqrt{\frac{3T}{g(2\rho-\sigma)}}$$

$$T = \frac{15}{2}\times \cancel{10^{-4}}^{\,10^{-2}}\ \frac{N}{\cancel{10^{-2}}\,m} = \frac{\cancel{15}\,3}{\cancel{200}\,40} = \frac{3}{40}$$

$$\boxed{R = \frac{3}{20\sqrt{2\rho-\sigma}}}\quad m = \frac{15}{\sqrt{2\rho-\sigma}}\ \text{cm}.$$

5

JEE 1

If two buses, P and Q start from a point at the same time and move in a straight line and their positions are represented by $\boxed{X_P(t) = \alpha t + \beta t^2}$ and $\boxed{X_Q(t) = ft - t^2}$. At what time, both the buses have same velocity?

(A) $\frac{\alpha - f}{1+\beta}$

(B) $\frac{\alpha + f}{2(\beta - 1)}$

(C) $\frac{\alpha + f}{2(1+\beta)}$

(D) $\frac{f - \alpha}{2(1+\beta)}$ ✓

$V_p = \alpha + 2\beta t$

$V_Q = f - 2t$

equate.

$\alpha + 2\beta \boxed{t} = f - 2\boxed{t}$

$t(2\beta + 2) = f - \alpha$

$t = \frac{f - \alpha}{2(\beta + 1)}$

JEE 1

6

Statement I: A cyclist is moving on an unbanked road with a speed of 7 Kmh^{-1} and takes a sharp circular turn along a path of radius of 2 m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve. $(g = 9.8 \text{ m/s}^2)$

Statement II: If the road is banked at an angle of 45°, cyclist can the curve of 2 m radius with speed of 18.5 Kmh^{-1} without slipping.

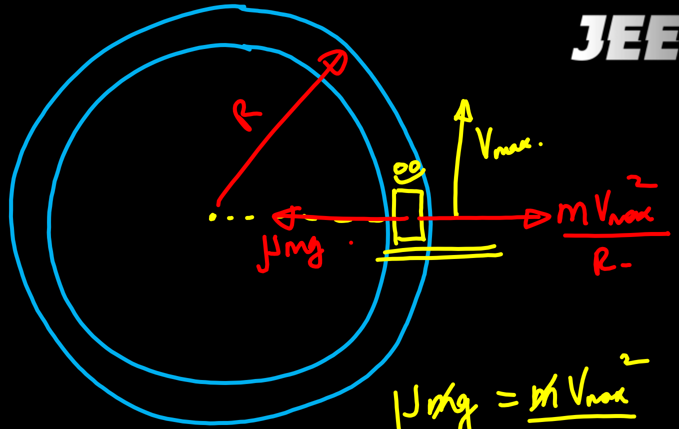
In the light of the above statements, choose the correct answer from the options given below.

(A) Both statement I and statement II are false

(B) Statement I is incorrect and statement II is correct

(C) Statement I is correct and statement II is incorrect

(D) Both statement I and statement II are true



$V_{max} =$

$V_{min} =$

$\mu = \frac{1}{5} \quad g = 9.8$

$R = 2$

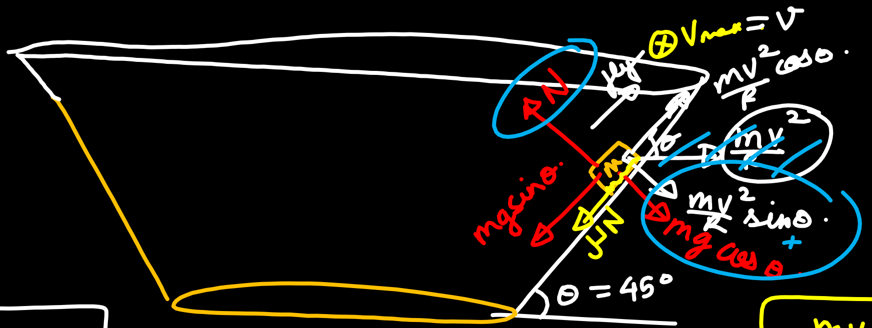
$\mu \cancel{m} g = \frac{\cancel{m} V_{max}^2}{R}$

$V_{max} = \sqrt{\mu g R}$.

$V_{max} = \sqrt{4}$

$V_{max} = 2 \text{ m/s}$

$= 2 \times \frac{18}{5} \simeq 7 \text{ kmh}$



$\cos\theta = 1/\sqrt{2}$

$\sin\theta = 1/\sqrt{2}$

$\tan\theta = 1$

$\boxed{\mu = \frac{1}{5}}$

$$\boxed{N = \frac{mv^2}{R}\sin\theta + mg\cos\theta} \quad (1)$$

$$\boxed{\frac{mv^2}{R}\cos\theta = \mu N + mg\sin\theta} \quad (2)$$

$$\frac{\cancel{m}v^2}{R}\cos\theta = \mu\frac{\cancel{m}v^2}{R}\sin\theta + \mu \cancel{m}g\cos\theta + \cancel{m}g\sin\theta$$

$\boxed{c, s}$

$$V_{max} = v = \sqrt{\frac{gR\left[s/c + \mu\, c/c\right]}{\left[c/c - \mu\, s/c\right]}}$$

$$V_{min} = \sqrt{\frac{g[t - \mu]}{[1 + t]}} = 3.6\,m/s = 12.9\,kmph.$$

$$V_{max} = \sqrt{g\frac{[t + \mu]}{[1 - t]}} = 5.4\,m/s = 19.5\,kmph$$

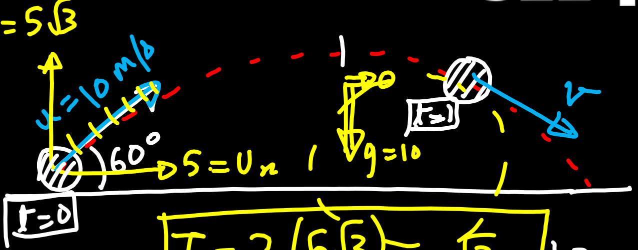
7

A body is projected at $t = 0$ with a velocity $10\ \text{m s}^{-1}$ at an angle of $60°$ with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $\boxed{g = 10\ \text{m/s}^2}$, the value of R is

(A) 2.5 m (B) 2.8 m

(C) 10.3 m (D) 5.1 m

JEE 1

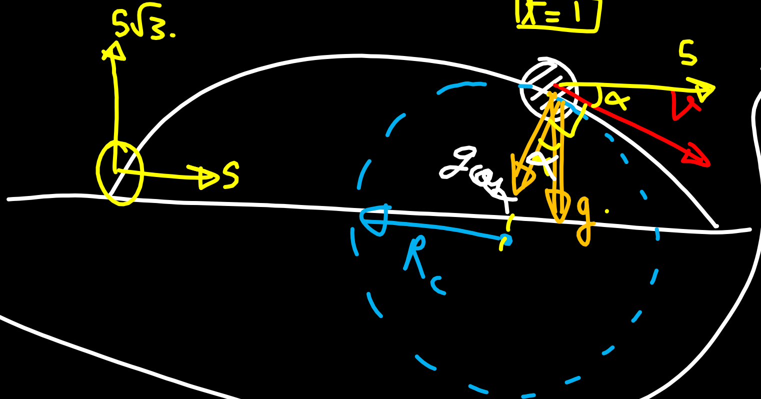


$$\boxed{T = \frac{2(5\sqrt{3})}{10} = \sqrt{3} = 1.7\text{s}}$$

$$\boxed{\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$\boxed{\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\boxed{\tan 15° = 2-\sqrt{3}}$$



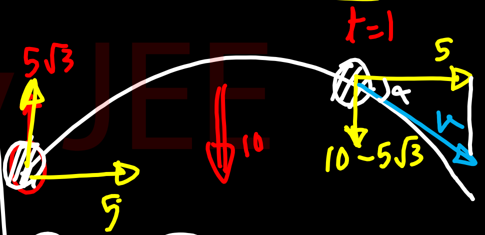
concept

$$\boxed{g\cos\alpha = \frac{V^2}{R_c}}$$

$$R_c = \frac{V^2}{g\cos\alpha}$$

$15°$

$$\boxed{R_c = 2.8\ \text{m}}$$



$$\boxed{V = u + at}$$

$$\boxed{V = \sqrt{5^2 + (10-5\sqrt{3})^2}}$$

$$\tan\alpha = \frac{10-5\sqrt{3}}{5}$$

$$\boxed{\tan\alpha = 2-\sqrt{3}}$$

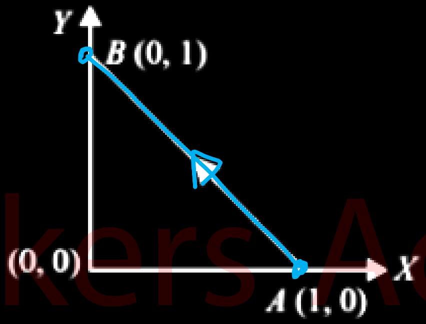
$\alpha = 15°$.

8

Consider a force $\boxed{\vec{F} = -x\hat{i} + y\hat{j}}$. The work done by this force in moving a particle from point A(1,0) to B(0,1) along the line segment is (all quantities are in SI units)

vary.

$$\boxed{\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}}$$



(A) $\frac{3}{2}$

(B) 2

(C) 1 ✓

(D) $\frac{1}{2}$

$$W = \int \vec{F} \cdot \vec{ds}$$

$$W = \int_1^0 -x\,dx + \int_0^1 y\,dy$$

$$W = 2\left[\int_0^1 t\,dt\right] = 1$$

9

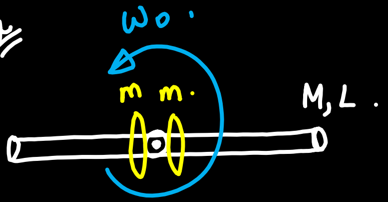
A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be

(A) $\frac{M\omega_0}{M+3m}$ $\boxed{\text{(B) } \frac{M\omega_0}{M+6m}}$ ✓

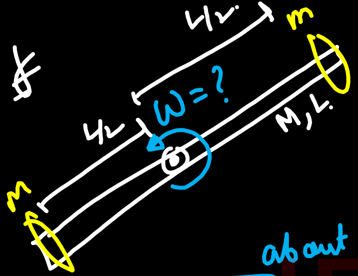
(C) $\frac{M\omega_0}{M+m}$ (D) $\frac{M\omega_0}{M+2m}$

JEE 1

i)



f)



about hinge.

$$\boxed{L_i = L_f}$$

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right)\omega$$

$\omega =$ ______

Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then

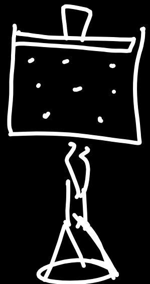
$PV = nRT$

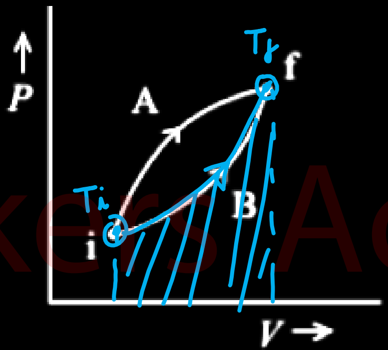
$\boxed{\Delta U} = nC_v \boxed{\Delta T}$

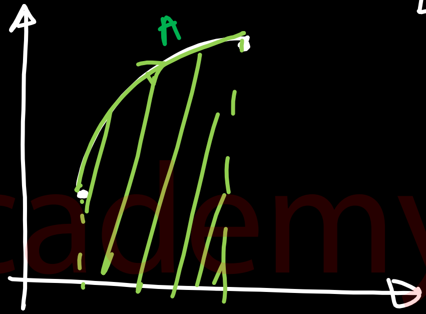
$\boxed{\Delta U_A = \Delta U_B}$

$\boxed{Q = \Delta U + W}$

const.







~~(A)~~ $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$

(B) $\Delta Q_A = \Delta Q_B, \Delta U_A = \Delta U_B$

~~(C)~~ $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$

(D) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$

$W_B < W_A$

$Q_B < Q_A$

11

A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves R in a circular orbit of radius $\frac{R}{2}$ and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is

(A) $+\frac{GMm}{6R}$

(B) $\frac{GMm}{2R}$

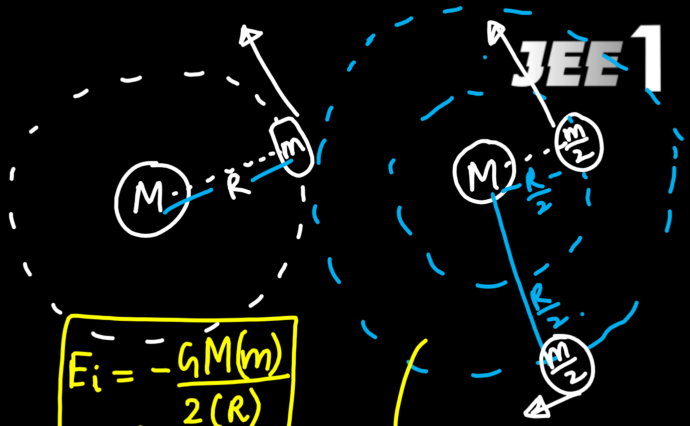
(C) $-\frac{GMm}{2R}$

(D) $-\frac{GMm}{6R}$

$\boxed{U = -\frac{GMm}{r}}$

$\boxed{KE = \frac{GMm}{2r}}$

$\boxed{TE = -\frac{GMm}{2r}}$



$\boxed{E_i = -\frac{GM(m)}{2(R)}}$

$\boxed{E_f = -\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{R}{2}\right)} - \frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{3R}{2}\right)}}$

Ans $= E_f - E_i$

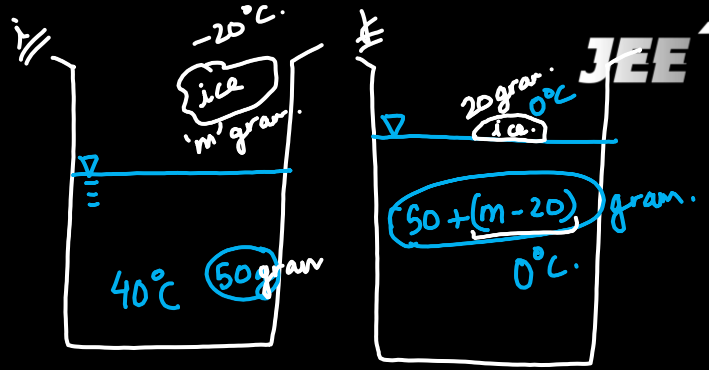
$= -\frac{GMm}{6R}.$

12

Ice at $-20°C$ is added to 50 g of water at $40°C$. When the temperature of the mixture reaches $0°C$, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water $= 4.2\ J/g/°C$ Specific heat of Ice $= 2.1\ J/g$ Heat of fusion of water at $0°C = 334\ J/g$)

(A) 60 g (B) 50 g

(C) 40 g (D) 100 g



Heat Gained = Heat Lost

$$m S_{ice} \Delta T_{ice} + (m-20) L_f = 50\, S_{water} \Delta T_{water}$$

$$m(2.1)(20) + (m-20)334 = 50(4.2)(40)$$

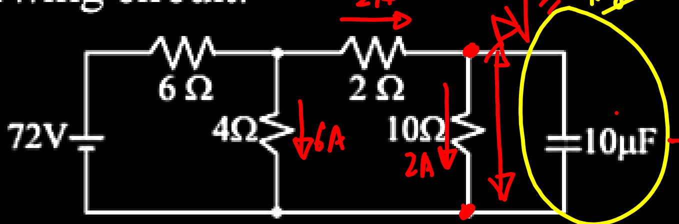
$$\boxed{m = 40.1 g}$$

13

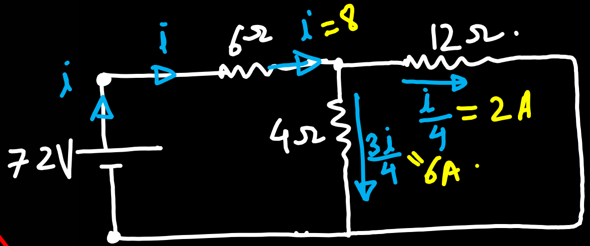
Determine the charge on the capacitor in the following circuit.

1:3

JEE 1



(2) 10 μC (3) 60 μC



(A) 60μC (B) 10μC

(C) 2μC (D) 200μC

$$q = CV$$

$$q = (10\mu)(20)$$

$$\boxed{q = 200\mu}$$

$$\boxed{R_{eq} = 6 + \frac{12\times\cancel{(4)}}{\cancel{(12+4)}_{4}} = 9\Omega}$$

$$\boxed{i = \frac{V_{eq}}{R_{eq}} = \frac{\cancel{72}^{8}}{\cancel{9}} = 8A}$$

14

According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0°C. × OK FALSE

B. The mean free path of gas molecules decreases if the density of molecules is increased. T

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant. T

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

D. Average kinetic energy per molecule per degree of freedom is $\boxed{\frac{3}{2}k_BT}$ (for monoatomic gases). $\boxed{=3}$

$$= \frac{1}{2}N_A k T = \frac{1}{2}RT$$

Choose the most appropriate answer from the options given below

~~(A) A and C only~~ ~~(B)~~ B and C only ✓

~~(C) A and B only~~ ~~(D) C and D only~~

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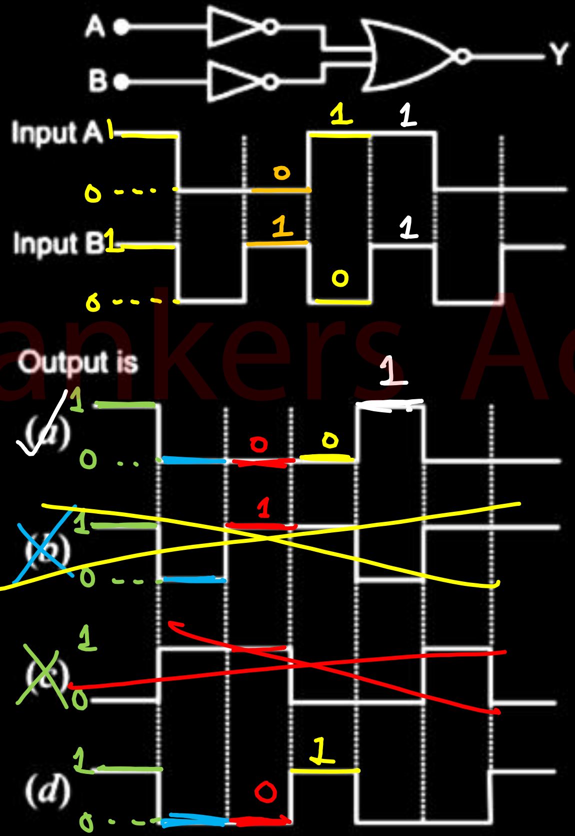
JEE 1

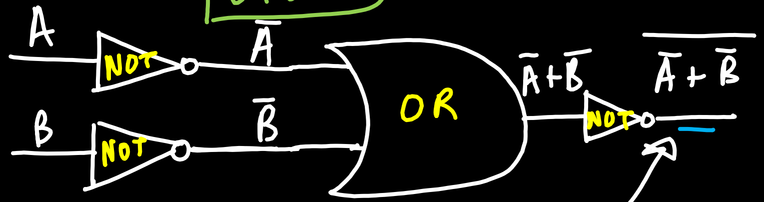
15

The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform.



$1+1=1$
$1+0=1$
$0+1=1$
$0+0=0$

$\equiv$



$$\overline{\left(\bar{A}+\bar{B}\right)} = \boxed{A \cdot B}$$

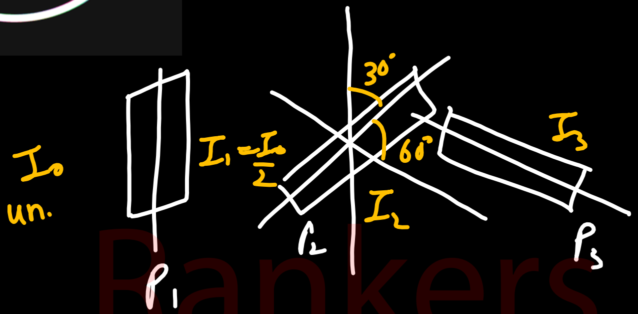
16

JEE 1

A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly)

(A) 10.67 (B) 5.33

(C) 1.80 (D) 16.00



$I_2 = I_1 \cos^2 30°$

$I_3 = I_2 \cos^2 60°$

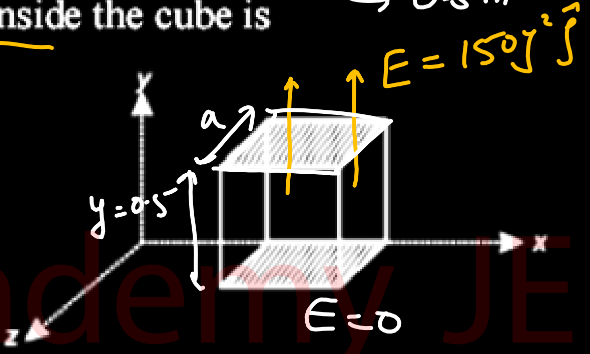
$= I_1 \cos^2 30° \times \cos^2 60°$

$I = \frac{I_0}{2} \frac{3}{4} \times \frac{1}{4}$

$\frac{I_0}{I} = \frac{32}{3} = 10.666..$

A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$ The side of the cube is and is placed in the field as shown in the given figure. The charge inside the cube is

0.5 m



(A) 8.3×10^{-11}C (B) 3.8×10^{-11}C

(C) 3.8×10^{-12}C (D) 8.3×10^{-12}C

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$$E \times a^2 = \text{''}$$

$$(150a^2) \times a^2 = \frac{q_{in}}{\varepsilon_0}$$

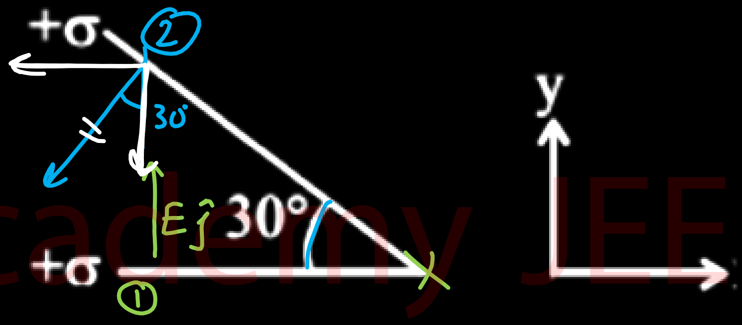
$$q_{in} = 150\left(\frac{1}{2}\right)^4 \times \varepsilon_0$$

$$= \left(150 \times \frac{1}{16}\right) \times 8.854 \times 10^{-\cancel{12}\ -11}$$

18

JEE 1

Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by



(A) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(B) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{y}+\frac{\hat{x}}{2}\right]$

(C) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{y}-\frac{\hat{x}}{2}\right]$

(D) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$ ✓

$\vec{E}_1 = E\hat{j}$ — (1)

$\vec{E}_2 = E\sin 30°(-\hat{i}) + E\cos 30°(-\hat{j})$ — (2)

E

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

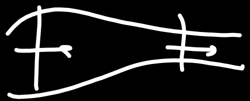
$= E\left[(1-\cos 30°)\hat{j} - \frac{\hat{i}}{2}\right]$

$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{i}}{2} + \left(1-\frac{\sqrt{3}}{2}\right)\hat{j}\right]$

19

$T \uparrow \quad \tau \downarrow$

$v_d = \frac{eE\tau}{m} \downarrow$



$i = neAv_d$

$v_d = \frac{e\left(\frac{V}{l}\right)\tau}{m}$

$v_d \propto \frac{1}{l}$

(A) The drift velocity of electrons decreases with the increase in the temperature of conductor.

(B) The drift velocity is inversely proportional to the area of cross-section of given conductor.

current

(C) The drift velocity does not depend on the applied potential difference to the conductor.

(D) The drift velocity of electron is inversely proportional to the length of the conductor.

(E) The drift velocity increases with the increase in the temperature of conductor.

Choose the correct answer from the options given below.

(A) (A) and (B) only

(B) (A) and (D) only

(C) (B) and (E) only

(D) (B) and (C) only

20

JEE 1

$^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}D + Q$

In the given nuclear reaction, the approximate amount of energy released will be:

[Given, mass of

$^{239}_{92}A = 238.05079 \times 931.5\,MeV/c^2$, mass of

$^{234}_{90}B = 234.04363 \times 931.5\,MeV/c^2$,

mass of $^{4}_{2}D = 4.00260 \times 931.5\,MeV/c^2$]

(A) 3.82MeV (B) 5.9MeV

(C) 2.12MeV (D) 4.25MeV

$Q = \Delta m\, c^2$

$= (m_A - m_B - m_D)c^2$

$= (0.00456) \times 931.5$

$= 4.\ldots$

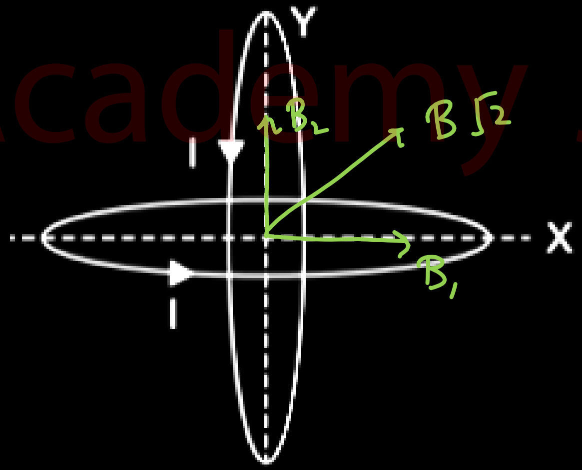
$$\begin{array}{r} 238.05079 \\ -\ 238.04623 \\ \hline .00456 \end{array}$$

21

JEE 1

Two identical circular wires of radius 20 cm and carrying current $\sqrt{2}A$ are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is ______ $\times 10^{-8}$ T. (Take $\pi = 3.14$)

$B = \frac{\mu_0 i}{2R}$



$B_{net} = B\sqrt{2}$

$= \frac{\mu_0 \cancel{i\sqrt{2}}}{\cancel{2}R}$

$= \frac{4\pi \times 10^{-7}}{0.2m}$

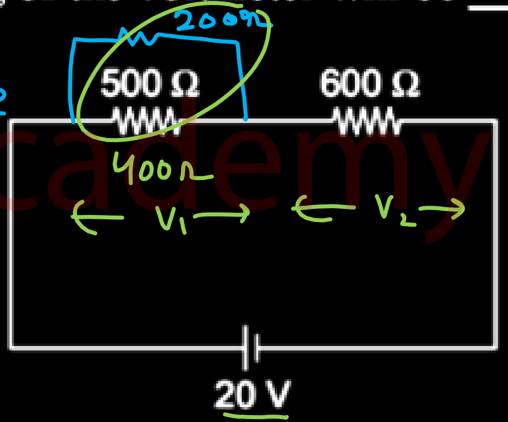
$= 2\pi \times 10^{-6}$

$= 6.28 \times 10^{-6} = \boxed{628} \times 10^{-8}$

22

JEE 1

Two resistor are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000Ω is used to measure the potential difference across 500Ω resistor, the reading of the voltmeter will be _____ V.



$R = \frac{2000 \times 500}{2500}$

$= 400\Omega$

$\frac{V_1}{V_2} = \frac{400\,\Omega}{600\,\Omega}$

$V_1 = \frac{4}{10} \times (20) = \boxed{8}\,V$

23

JEE 1

The surface of a metal is illuminated alternately with photons of energies $E_1 = 4eV$ and $E_2 = 2.5eV$ respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2 . The work function of the metal in (eV) is______.

$K_{max} = \frac{1}{2}mv_m^2 = h\nu - \phi$

$4K = 4eV - \phi$ — (1)

$K = 2.5eV - \phi$ — (2)

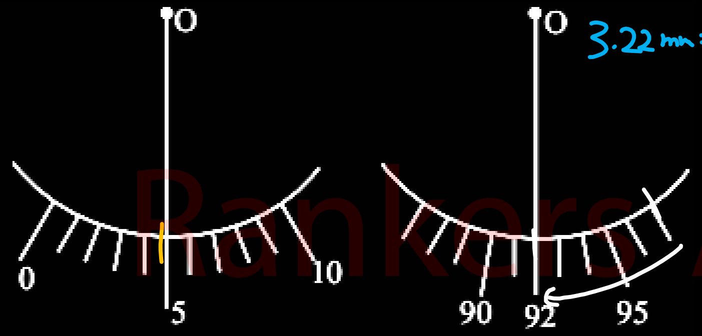
$4(2.5 - \phi) = 4 - \phi$

$10 - 4 = 3\phi \Rightarrow \boxed{\phi = 2eV}$

pitch = 1mm

$L.C. = \frac{1}{100}\,mm = 0.01\,mm$

Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 3.22 mm = $\boxed{0.322\text{ cm.}}$ The absolute value of the difference between the $\boxed{\text{final circular scale}}$ readings observed by the students, A and B is ______. [Figure shows position of reference ' O ' when jaws of screw gauge are closed] Given pitch = 0.1 cm.



Screw gauge (A) | Screw gauge (B)

$\boxed{error_1 = +5}$ 13 $\boxed{error_2 = -8}$

$3.22\,mm = 3\,mm + (CSD_1) \times L.C. - \text{zero error}$

$3.22\,mm = 3 + (CSD_1 - 5) \times 0.01$ — ①

$CSD_1 = 27$ — ①

$3.22\,mm = 3\,mm + CSD_2 \times L.C. - (-8) \times LC$

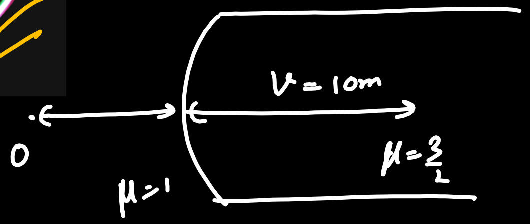
$22 = CSD_2 + 8$

$CSD_2 = 14$ — ②

$\Delta CSD = 27 - 14 = \boxed{13}$

25

The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}^{rd}$ of the distance of the object from the surface. The wavelength of light inside the surface is 2/3 times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of x is ______.



$$\frac{2(u)}{3} = 10 \Rightarrow u = -15\,m$$

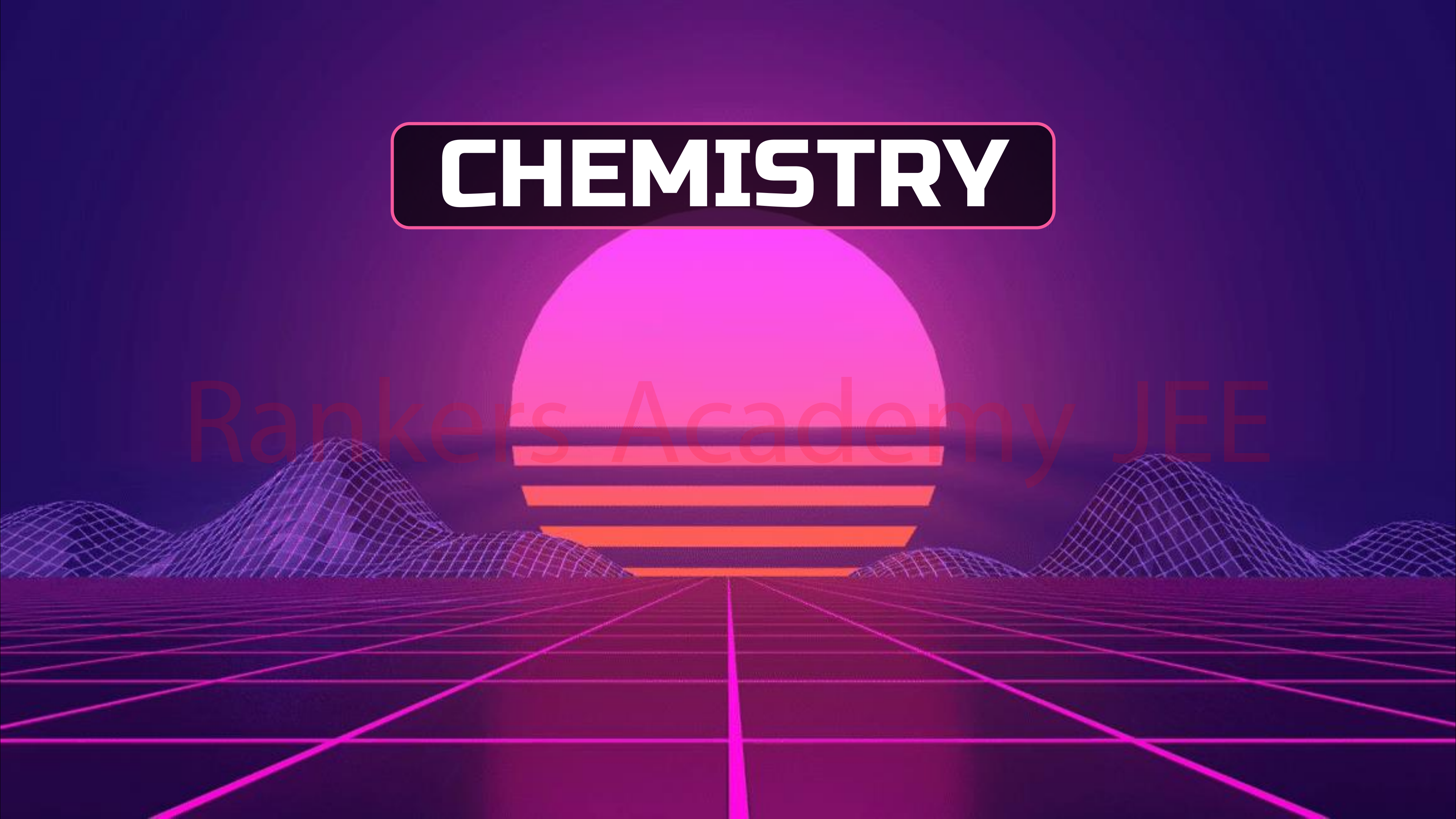
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1.5}{+10} - \frac{1}{-15} = \frac{0.5}{R}$$

$$\frac{3}{20} + \frac{1}{15} = \frac{1}{2R}$$

$$\frac{9+4}{60} = \frac{1}{2R}$$

$$\frac{13}{30} = \frac{1}{R} \Rightarrow \boxed{x = 30}$$


CHEMISTRY

Which optically active compound on reduction with $LiAlH_4$ will give optically inactive compound?

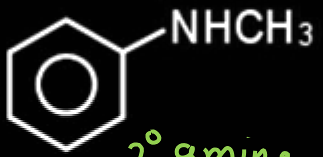
(A) $CH_3-\underset{\underset{OCH_3}{|}}{CH}-COOH$ $\xrightarrow{LAH}$ $CH_3-\overset{\overset{H}{|}}{\underset{\underset{OCH_3}{|}}{C^*}}-CH_2OH$

(B) $CH_3-CH_2-\underset{\underset{OH}{|}}{CH}-COOH$ $\xrightarrow{''}$ $CH_3-CH_2-\underset{\underset{OH}{|}}{\overset{*}{CH}}-CH_2OH$

(C) $CH_3-CH_2-\underset{\underset{CH_2OH}{|}}{CH}-COOH$ $\xrightarrow{''}$ $CH_3CH_2\underset{\underset{CH_2OH}{|}}{CH}-CH_2OH$ (Inactive)

(D) $CH_3-\underset{\underset{OH}{|}}{CH}-CH_2-COOH$

2. Predict the product,



+ $NaNO_2$ + HCl $\longrightarrow$ Product

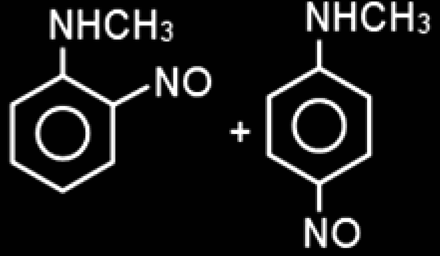
2° amine

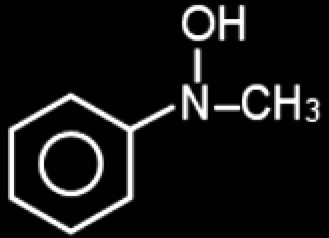
HNO_2

(A) 

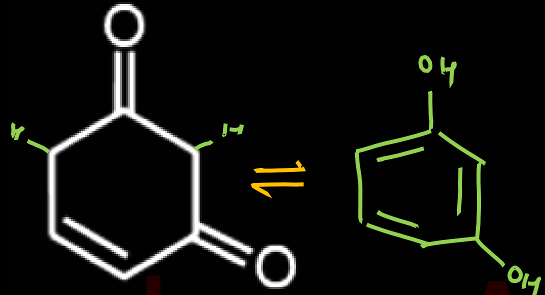
(B) 

yellow oily liquid

(C) 

(D) 

. The order of enol content in the following molecules is

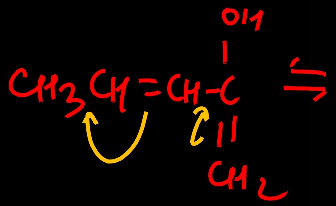


(a)

Aromatic

$$CH_3CH_2\overset{\overset{\displaystyle O}{\|}}{C}CH_2CH_3$$

(b)

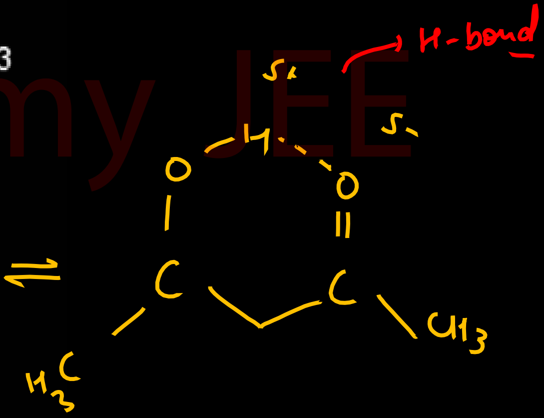


$$CH_3CH{=}CH\overset{\overset{\displaystyle O}{\|}}{C}CH_3$$

(c)

$$CH_3\overset{\overset{\displaystyle O}{\|}}{C}CH_2\overset{\overset{\displaystyle O}{\|}}{C}CH_3$$

(d)

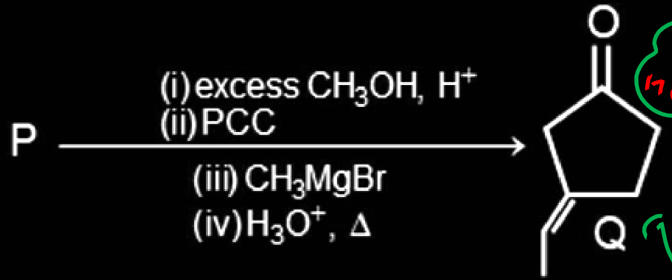


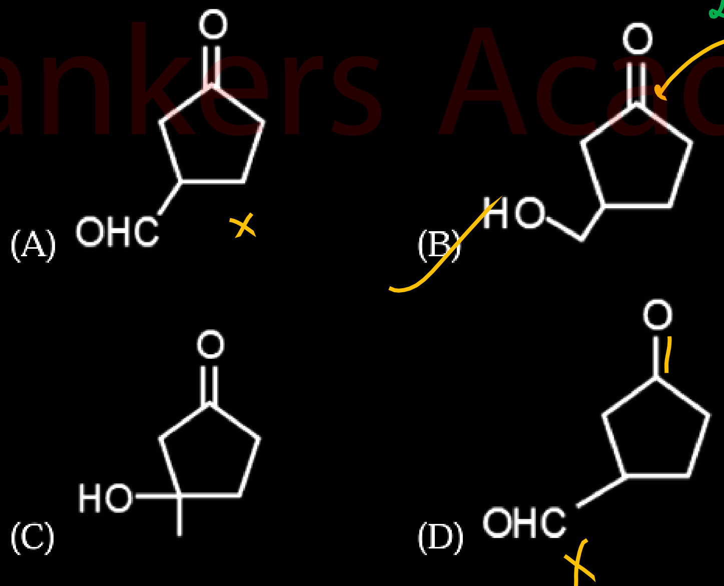
(A) a > d > c > b (B) a > c > d > b

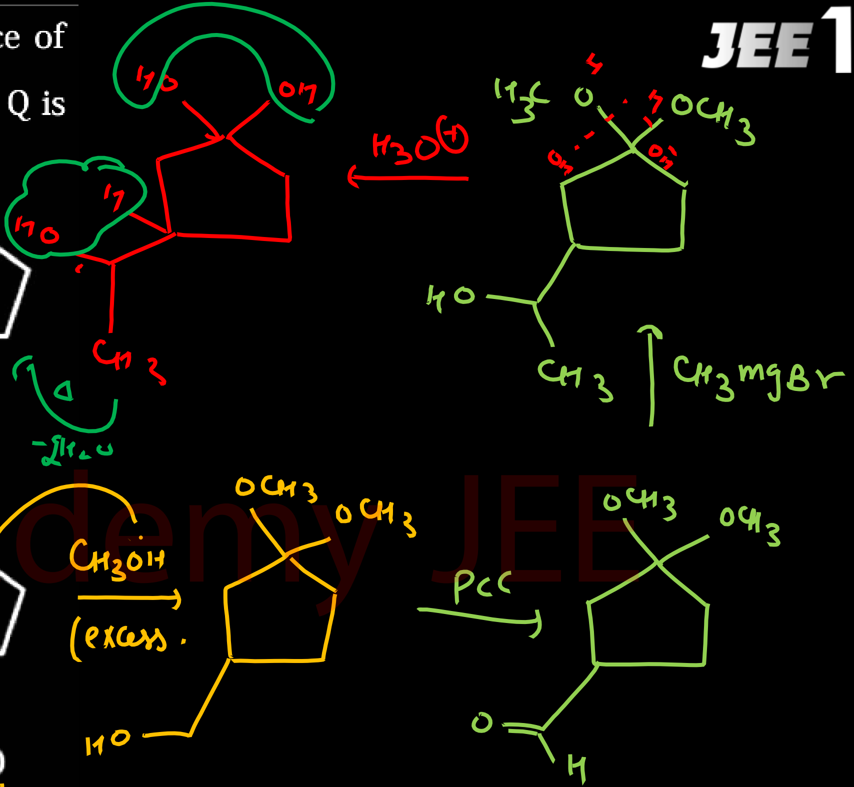
(C) a > c > b > d (D) a > b > c > d

4

Compound ' P ' that undergoes the sequence of reactions given below to give the product Q is







5

JEE 1

. Two elements A and B form compounds having molecular formula AB_2 and AB_4. When dissolve in 20 g of C_6H_6, 1 g of AB_2 lowers the freezing point by 2.55 K, whereas 1.0 g of AB_4 lowers it b 1.7 K. The molar depression constant for benzen is $5.1\ K - kgmol^{-1}$. The atomic masses of A and are (Assume both compound as non electrolyte)

(A) 50,25

(B) 50,50

(C) 25,50

(D) 75,25

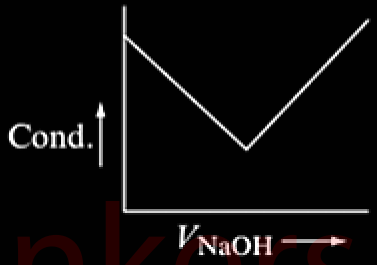
$\Delta T_f = i K_f m$

$2.55 = 1 \times 5.1 \left(\frac{1/M_A + 2M_B}{20/1000} \right)$ — 1

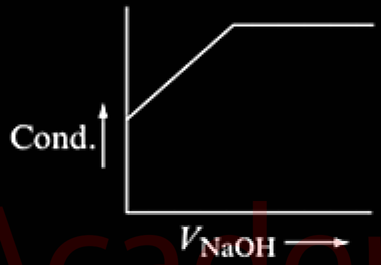
$1.7 = 1 \times 5.1 \left(\frac{1/M_A + 4M_B}{20/1000} \right)$ — 2

$M_A = 50,\ M_B = 25$

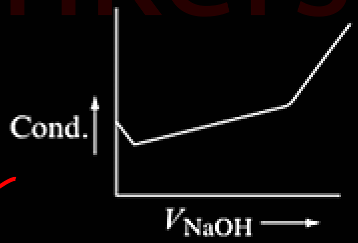
6. Which of the following graph truly represents the titration of CH_3COOH solution against NaOH solution?



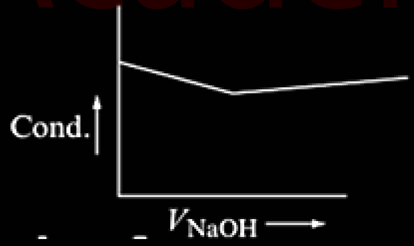
(A)



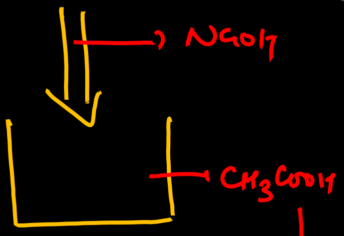
(B)



(C)



(D)



$CH_3COO^- + H^+$

S.B W.A

$NaOH + CH_3COOH$

$\downarrow$

$CH_3COONa + H_2O$

(salt)

7

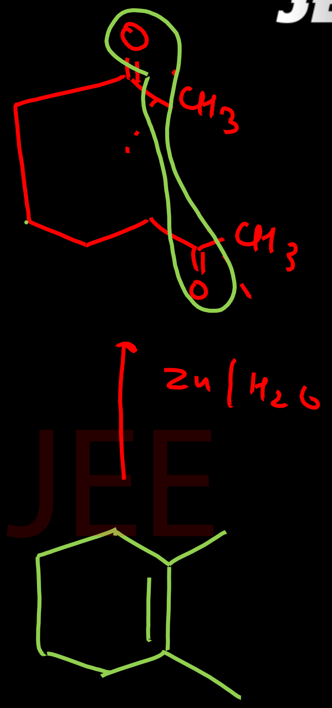
Compound ' X ' in the following reaction is

$$X \xrightarrow[\text{(iii) } Cl_2/NaOH \quad \text{(iv) } H_3O^+]{\text{(i) } O_3 \quad \text{(ii) } Zn/H_2O} HOOC-(CH_2)_4-COOH$$

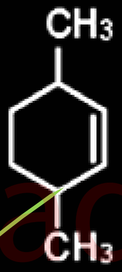
NaOCl

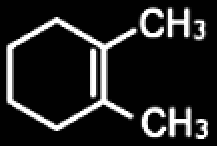
Adipic Acid

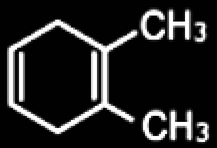
$\xleftarrow[2.\ H_3O^{(+)}]{1.\ Cl_2|NaOH}$



(A) 

(B) 

(C) 

(D) 

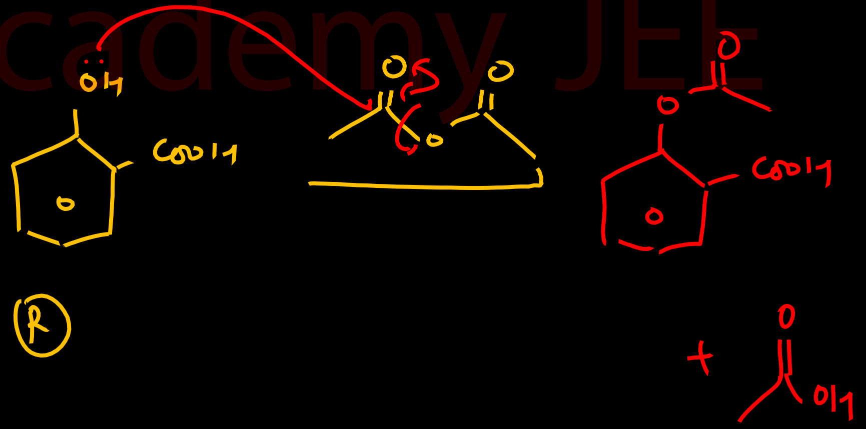
'S' is :

(A) ~~Aspirin~~ (B) Valine

(C) Cumene (D) Salicyclic acid

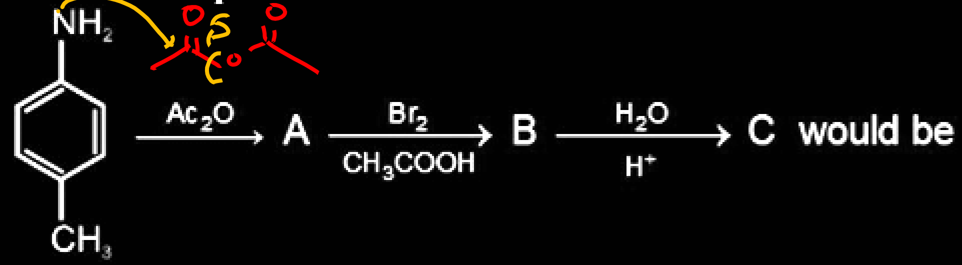
$Ph-O^{\ominus}Na^{\oplus} \xrightarrow[H_3O^+]{CO_2}$

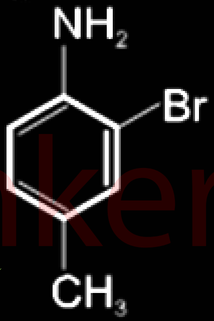
(P)



(R)

The final product C, obtained in this reaction



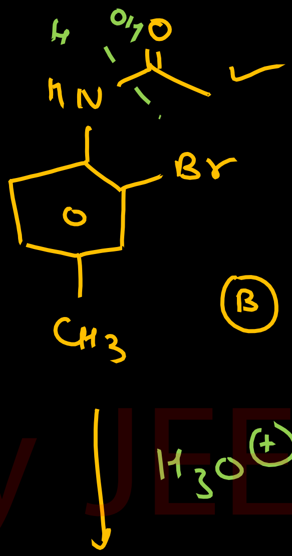


(A)

(B)

(C)

(D)



10

Match the compounds given in List - I with their characteristic reactions given in List - II. Select the correct option.

List - I (Compounds)	List - II (Reactions)
(1) $CH_3CH_2CH_2CH_2NH_2$ (1° amine)	(i) alkaline hydrolysis
(2) $CH_3C \equiv CH$	(ii) with KOH (alcohol) and $CHCl_3$ produces bad smell
(3) $CH_3CH_2COOCH_3$	(iii) gives white ppt. with ammonical $AgNO_3$ (Tollens)
(4) $CH_3CH(OH)CH_3$ 2°	(iv) with Lucas reagent cloudiness appears after 5 minutes

Options :

	(1)	(2)	(3)	(4)
(a)	(iv)	(ii)	(iii)	(i)
(b)	(ii)	(i)	(iv)	(iii)
(c)	(iii)	(ii)	(i)	(iv)
(d)	(ii)	(iii)	(i)	(iv)

11

JEE 1

Select the $\boxed{\text{incorrect statement}}$ among the following with respect to the reducing nature of Cr^{2+} and Fe^{2+}.

(A) Cr^{2+} is a stronger reducing agent than Fe^{2+} in aqueous solutions ✓

(B) Cr^{3+} has a d^3 configuration whereas Fe^{3+} has a d^5 configuration ✓

(C) d^3 has a greater stability than d^5 configuration in aqueous solution because of crystal field stabilisation energy ✓

(D) The stability of d^5 is greater than d^3 in presence of water due to greater number of exchanges ✗

$Cr^{2+} \longrightarrow e^- + Cr^{3+}$

$3d^4 4s^0$ $\quad$ $\boxed{3d^3 4s^0}$ (half-filled t_{2g})

$Fe^{2+} \longrightarrow e^- + Fe^{3+}$

$3d^6 4s^0$ $\quad$ $\boxed{3d^5 4s^0}$ (half-filled d-orb)

12

What is the shape of XeF_3^+ ion?

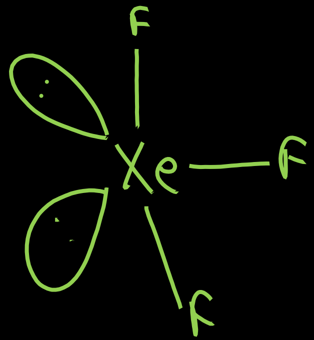
(A) Trigonal planar (B) Trigonal pyramidal

(C) See-Saw (D) Bent-T-shaped

7

XeF_3^{+} 3 Bond Pairs 2 Lone Pairs

Sum : $3 + 2 = 5\ (sp^3d)$



13

1 mole of a gas AB_3 present in 10lt container at pressure $\boxed{2.5 \text{ atm}}$ and 273 K temperature. On increasing the temperature to 546 K, AB_3 dissociates into $AB_2(g)$ and $B_2(g)$. If the degree of dissociation of AB_3 is 80%, then final pressure at 546 K is :

(A) 5 atm
(B) 1.25 atm
(C) 10 atm
(D) 6.5 atm

2.24 atm 0.26 atm

AB_3 + gas.
T = 273 K
P = 2.5 atm
V = 10 lit

$$AB_3 \rightleftharpoons AB_2(g) + \tfrac{1}{2} B_2(g)$$

t = 0: 1 mol, 0, 0
t = eq: 0.2 mol, 0.8 mol, 0.4 mol

$PV = nRT$

$$\boxed{P = \frac{nRT}{V}}$$

$$P = \frac{1\text{ mol} \times 0.0821 \times 273}{10}$$

$= 2.24$ atm

on increasing T to 546 K,

$\underbrace{P_{AB_3} = 4.48 \text{ atm}}$ $P_{gas} = 0.52$ atm

Total moles at eq^m = 0.2 + 0.8 + 0.4 = 1.4 mol

P_{AB_3} at 546 K = 4.48 atm

After decompose = 1.4 × 4.48 = 6.27 atm

P_{gas} at 546 K = 0.52   + 0.52

$\boxed{6.79 \text{ atm}}$

14

JEE 1

Match the following

List -I		List - II	
(I)	$\overset{+3}{Ti}(NO_3)_3$	(P)	Lewis base
(II)	$(SiH_3)_3N$	(Q)	Oxidizing agent
(III)	$SnCl_2$	(R)	Reducing agent
(IV)	$(CH_3)_3\ddot{N}$	(S)	$p\pi - p\pi$ overlap
		(T)	$p\pi - d\pi$ overlap

(A) I → R; II → S; III → Q; IV → P
(B) I → Q; II → S; III → P; IV → T
(C) I → Q; II → T; III → R; IV → P
(D) I → R; II → T; III → P; IV → Q

15

JEE 1

Statement I: $[Co(ox)_3]^{3-}$ shows geometrical as well as optical isomerism. $[M(AA)_3]$ ✗

Statement II: Secondary valency is non-ionisable. ✓ $[ML_n]$ C.I

(A) Statement-I is true, statement-II is true and statement-II is the correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

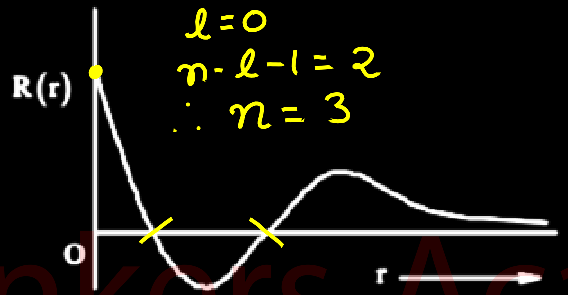
(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true ✓

16

JEE 1

The radial part of wave function of an orbital is plotted against distance from nucleus. Which orbital represent below graph?



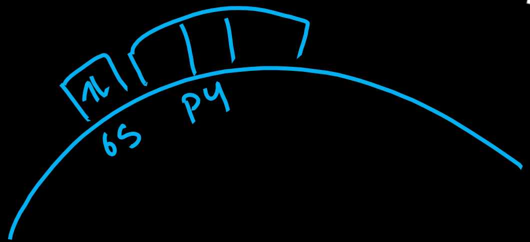
(A) 1 s (B) 2 s

(C) 3 s (D) 2p

When $n = 3$, $l = 0$ $\boxed{3s}$

17

Down the group 13 and group 14 , lower oxidation state compounds are more stable because:



(A) Down the group I.E. decreases

(B) Poor shielding of electrons in s & p - orbitals as compare to d & f-orbitals

(C) Down the group electronegativity decreases

(D) Outer shell electron are less shielded from nucleus due to presence of electrons in dorbitals of penultimate shell

18

$[X] + \text{dil } H_2SO_4 \longrightarrow [Y]$, a colourless gas with irritating smell $[Y] + K_2Cr_2O_7 + H_2SO_4 \longrightarrow$ Green solution The species [X] and [Y] are

(A) SO_3^{2-}, SO_2 (B) Cl^-, HCl

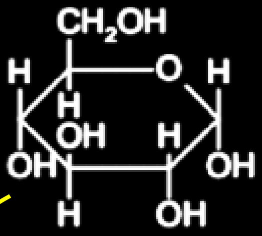
(C) $S^{2-}, H_2 S$ (D) CO_3^{2-}, CO_2

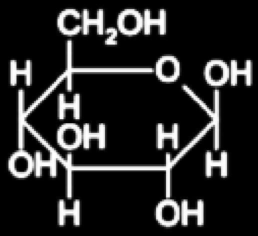
$$\underset{(X)}{SO_3^{2-}} + H^+ \longrightarrow \overset{(Y)}{SO_2}\uparrow$$

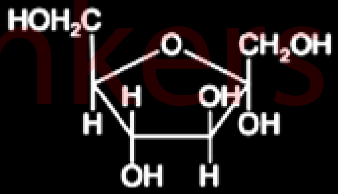
- colourless
- Irritating
- Turns $K_2Cr_2O_7$ from orange to green

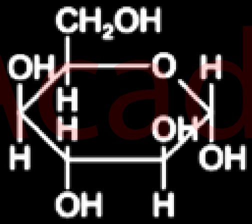
19

Which of the following is correct Haworth projection for α-D glucopyranose?

(A) 

(B) 

(C) 

(D) 

20

JEE1

Heat of the following reaction in bomb calorimeter is -1415 kJ.

$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$

What is the heat released if 1.4 gC_2H_4 is combusted in open atmosphere at 27°C ?

($R = 8.3 JK^{-1} mol^{-1}$)

(A) -71.0 kJ (B) -1415 kJ

(C) -710 kJ (D) -1420 kJ

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta n_g = 2-4 = -2$

$\Delta n_g RT = -2 \times 8.314 \times 10^{-3} \times 300 k$

$= -5 kJ$

$\Delta H = -1415 kJ - 5 kJ$

$= -1420 kJ$

28 g, $\longrightarrow$ $-1420 kJ$

1.4g $\longrightarrow$ $\frac{-1420}{28} \times 1.4$

$= -71.0 kJ$

21

JEE 1

How many grams of solute should be added in 100 g water to get a solution of density 1.2 g/ml and strength 5% (w/v)?

(Nearest integer)

$d = \frac{m}{V}$

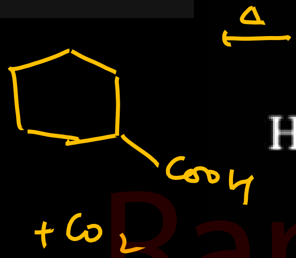
$V = \frac{m}{d}$

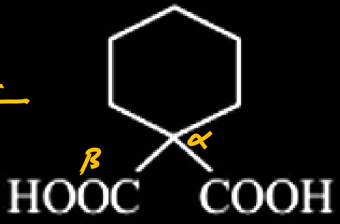
5% (w/v) → 5 g solute in 100 ml solution

$$\frac{5}{100} = \frac{\text{wt. of solute}}{\text{volume of solution}} = \frac{w}{\frac{(w+W)}{d}}$$

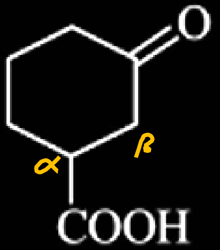
$$\frac{5}{100} = \frac{w}{(w+100)} \times 1.2$$

22\. Number of compounds that will undergo decarboxylation on heating

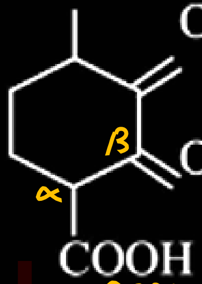




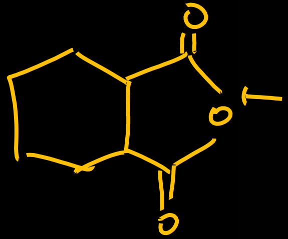
(I)

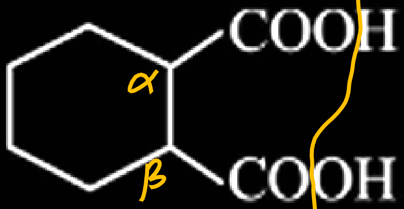


(II)

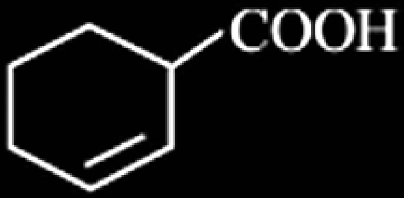


(III)





(IV)



(V)

23

Consider two first-order reactions I and II. The frequency factor of I is 100 times that of II, the activation energy of I is 4.606kcal higher than that of II. If the ratio of rate constants for the reactions I and II is x: 1 at 227°C, then the value of x is

(use R = 2 Cal)

Ans.

$A_I = 100 A_{II}$

$E_I = 4.606 + E_{II}$

$\therefore E_I - E_{II} = 4.606$

$\frac{1}{x} = \frac{K_{II}}{K_I} = \frac{A_{II} e^{-E_{II}/RT}}{A_I e^{-E_I/RT}} = \frac{1}{100} e^{\frac{E_I - E_{II}}{RT}} = \frac{1}{100} e^{\frac{+4.606 \times 10^3}{2 \times 500}}$

$= \frac{1}{100} e^{4.606}$

$e^{+4.606} = e^{2.303 \log 100}$

$= e^{+\ln 100}$

$e^{4.606} = 100$

24

JEE 1

A quantity of 1 g of metal carbonate was dissolved in 25ml of normal HCl. The resulting liquid requires 50ml of N/10 caustic soda solution to neutralize it completely. The equivalent weight of metal carbonate is (nearest integer).

one.

NaOH

eq. of Acid = eq. of M.C + eq. of NaOH

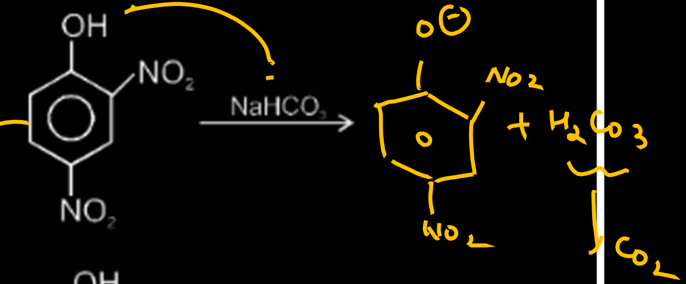
$$\frac{1 \times 25}{1000} = \frac{1}{E} + \frac{\frac{1}{10} \times 50}{1000}$$

$$\frac{25-5}{1000} = \frac{1}{E} \quad \therefore E = \frac{1000}{20} = 50$$

25

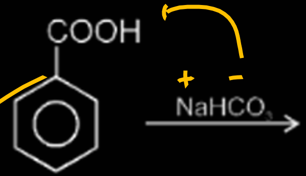
How many of the following reactions is/are feasible :

(i)

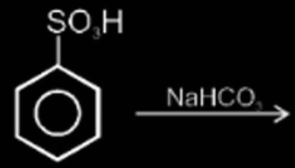


(ii)

(iii)

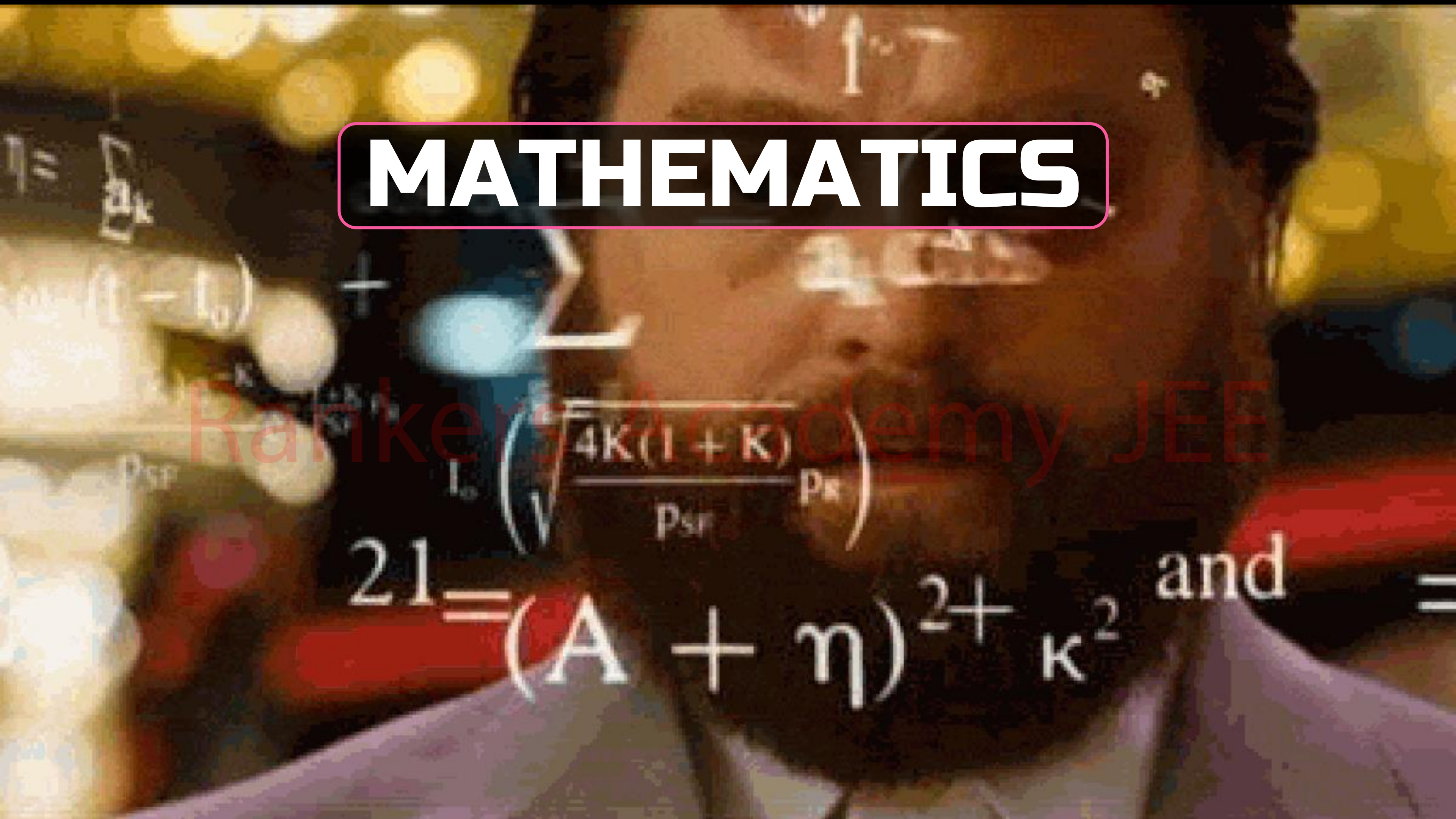


(iv)



3

MATHEMATICS



1

If ω is one of the imaginary cube root of unity then the value of the expression, $(1+2\omega+2\omega^2)^{10}+(2+\omega+2\omega^2)^{10}+(2+2\omega+\omega^2)^{10}$ is : $= 1+\omega+\omega^2=0$

(A) 0 (B) 1

(C) ω (D) ω^2

$\omega^3=1$

$\omega^{3n}=1$

$1+\omega+\omega^2=0$

$(1+2\omega+2\omega^2)^{10} = (2+\cancel{2\omega+2\omega^2}-1)^{10}$

$= (-1)^{10} = 1$

$(2+\omega+2\omega^2)^{10} = (2(1+\cancel{\omega}+\omega^2)-\omega)^{10}$

$= \omega^{10} = (\omega^3)^3.\omega = \omega$

$(2+2\omega+\omega^2)^{10} = (2(1+\cancel{\omega}+\omega^2)-\omega^2)^{10}$

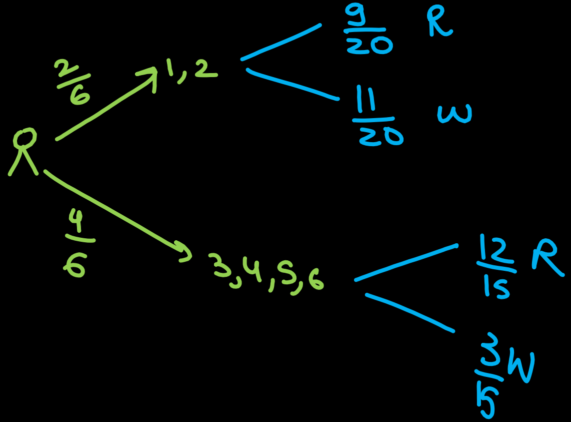
$= \omega^{20} = (\omega^3)^6\,\omega^2 = \omega^2$

2

Urn A contains 9 red balls and 11 white balls. Urn B contains 12 red balls and 3 white balls. A person is to roll a single fair die. If the result is a one or a two, then he is to randomly select a ball from urn A. Otherwise he is to randomly select a ball form urn B. The probability of obtaining a red ball, is

(A) $\frac{41}{60}$ ✓ (B) $\frac{19}{60}$

(C) $\frac{21}{35}$ (D) $\frac{35}{60}$



$$P(\text{red}) = \frac{2}{6}\cdot\frac{9}{20} + \frac{4}{6}\cdot\frac{12}{15}$$

$$= \frac{3}{20} + \frac{8}{15}$$

$$= \frac{9+32}{60} = \frac{41}{60}.$$

3

If the equations $ax^2+bx+c=0$ and $x^3+6x^2+12x+9=0$ have two common roots, $(a, b, c \in R)$, then

(A) $a=3b=3c$ (B) $a=b=c$

(C) $a=-3b=c$ (D) $3a=b=c$

JEE 1

$$x^3+6x^2+12x+9=0$$

$$\Rightarrow x^3+6x^2+12x+8+1=0$$

$$\Rightarrow (x+2)^3+1=0$$

$$\Rightarrow (x+2)^3=-1$$

$$\Rightarrow x+2=-1, -\omega, -\omega^2$$

$$\Rightarrow x=-3, -2-\omega, -2-\omega^2$$

$\therefore$ common roots : $-2-\omega$, $-2-\omega^2$

$$\text{Sum}=(-2-\omega)+(-2-\omega^2)$$
$$=-4-(\omega+\omega^2)$$
$$=-4-(-1)=-3=-\frac{b}{a}$$

$$\boxed{b=3a}$$

$$\text{Prod}=\frac{c}{a}=(-2-\omega)(-2-\omega^2)$$
$$=(2+\omega)(2+\omega^2)$$
$$=4+2(\omega+\omega^2)+\omega^3=3$$

$$\boxed{c=3a}$$

If the point (2,3) lies inside the circle $x^2 + y^2 - 6x - 10y + k = 0$ and the circle neither touches nor intersect the line $y = 1$. Then the set of all possible value of k lies in the interval

(A) no value of k

(B) (9,29)

(C) (18,29)

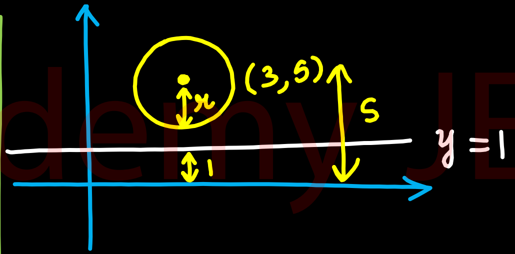
(D) (25,31)

$S : x^2 + y^2 - 6x - 10y + k = 0$

$S_1 < 0$ (inside)

$4 + 9 - 12 - 30 + k < 0$

$\boxed{k < 29}$



$r < 4$

$\sqrt{3^2 + 5^2 - k} < 4$

$\Rightarrow 9 + 25 - k < 16$

$\boxed{k > 18}$

5

The maximum value of $y = 4\cos 2x + 3\sin x + 5$ is equal to

(A) 10
(B) $\frac{297}{32}$ ✓
(C) 0
(D) None of these

$$y = 4(1 - 2\sin^2 x) + 3\sin x + 5$$

$$= -8\sin^2 x + 3\sin x + 9$$

$$= -8\left[\sin^2 x - \frac{3}{8}\sin x - \frac{9}{8}\right]$$

$$= -8\left[\left(\sin x - \frac{3}{16}\right)^2 - \frac{9}{8} - \left(\frac{3}{16}\right)^2\right]$$

$$= -8\left[\left(\sin x - \frac{3}{16}\right)^2 - \frac{9}{8} - \frac{9}{256}\right]$$

$$= -8\left[\left(\sin x - \frac{3}{16}\right)^2 - \frac{9}{8}\left(\frac{33}{32}\right)\right]$$

$\sin x = \frac{3}{16} \longrightarrow -8\left[0 - \frac{9}{8}\cdot\frac{33}{32}\right]$

$= \frac{9 \cdot 33}{32} = \frac{297}{32}$ max

$\sin x = 1 \longrightarrow -8\left[\left(\frac{13}{16}\right)^2 - \frac{9}{8}\cdot\frac{33}{32}\right]$

$-\frac{(169-297)}{32}$

$\sin x = -1 \longrightarrow -8\left[\left(\frac{-19}{16}\right)^2 - \frac{9}{8}\cdot\frac{33}{32}\right]$

$-\frac{(361-297)}{32}$

JEE 1

M2 $f'(x) = -8\sin 2x + 3\cos x = 0$

$\Rightarrow 16\sin x\cancel{\cos x} = 3\cancel{\cos x}$

$\Rightarrow \sin x = \frac{3}{16}$

$\sin x = 3/16$ (Max)

$\sin x = 1$

$\sin x = -1$ (Min)

6

The sum of all integral values of $a \in [1,10]$ for which $f(x) = x^3 - 3x^2 + ax + 2\cos x$ is increasing.

(A) 10 (B) 15

(C) 40 (D) 45

$f'(x) > 0$

$\Rightarrow 3x^2 - 6x + a - 2\sin x \geq 0$

$\Rightarrow 3x^2 - 6x + a \geq 2\sin x$

$\Rightarrow 3x^2 - 6x + a \geq 2$

$\Rightarrow 3x^2 - 6x + (a-2) \geq 0$

$D \leq 0$

$36 - 12(a-2) \leq 0$

$(3 - a + 2) \leq 0$

$a \geq 5$

$\Rightarrow a = 5, 6, 7, 8, 9, 10$

$\Rightarrow \frac{6}{2}(5+10)$

$= 3(15)$

$= 45.$

7

JEE 1

Let $S_n = \cot^{-1}\left(6x + \frac{2}{x}\right) + \cot^{-1}\left(10x + \frac{2}{x}\right) + \cot^{-1}\left(15x + \frac{2}{x}\right) + \cdots \ldots + n$ terms where $x > 0$. If $\lim_{n\to\infty} S_n = 1$, then x equals

(A) $\cot 1$

(B) $\frac{2}{3}\cot 1$

(C) $\frac{3}{2}\tan 1$

(D) None of these

$$\Rightarrow \tan^{-1}\left(\frac{x}{2+6x^2}\right) + \tan^{-1}\left(\frac{x}{2+10x^2}\right) + \tan^{-1}\left(\frac{x}{2+15x^2}\right) + - -$$

$$\Rightarrow \tan^{-1}\left(\frac{x/2}{1+3x^2}\right) + \tan^{-1}\left(\frac{x/2}{1+5x^2}\right) + \tan^{-1}\left(\frac{x/2}{1+\frac{15}{2}x^2}\right) + - -$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{4x}{2} - \frac{3x}{2}}{1+\frac{3x}{2}\cdot\frac{4x}{2}}\right) + \tan^{-1}\left(\frac{\frac{5}{2}x - \frac{4}{2}x}{1+\frac{5x}{2}\cdot\frac{4x}{2}}\right) + \tan^{-1}\left(\frac{\frac{6x}{2} - \frac{5x}{2}}{1+\frac{5x}{2}\cdot\frac{6x}{2}}\right) + - -$$

$\Rightarrow \cancel{\tan^{-1}\frac{4x}{2}} - \tan^{-1}\frac{3x}{2}$

$+ \cancel{\tan^{-1}\frac{5x}{2}} - \cancel{\tan^{-1}\frac{4x}{2}}$

$\tan^{-1}(\infty) -$

$1 \Rightarrow \frac{\pi}{2} - \tan^{-1}\left(\frac{3x}{2}\right)$

$1 = \cot^{-1}\left(\frac{3x}{2}\right)$

$\cot 1 = \frac{3x}{2}$

$\boxed{x = \frac{2}{3}\cot 1}$

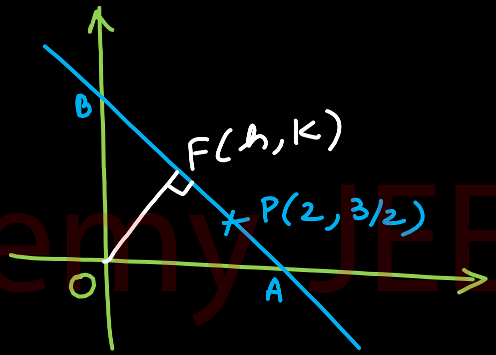
A variable line passing through the point $P\left(2, \frac{3}{2}\right)$ meets co-ordinate axes at points A&B, then locus of the foot of perpendicular from origin on the line is

(A) $x^2 + y^2 - 4x - 3y = 0$

(B) $2x^2 + 2y^2 - 4x - 3y = 0$ ✓

(C) $x^2 + y^2 - 3x + 4y = 0$

(D) $2x^2 + 2y^2 + 4x + 3y = 0$



$m_{OF} = \frac{k-0}{h-0} = \frac{k}{h}$

$OF \perp AB \Rightarrow m_{AB} = -\frac{h}{k}$

Line: $y - k = -\frac{h}{k}(x - h)$

$\frac{3}{2} - k = -\frac{h}{k}(2 - h)$

JEE 1

$$\Rightarrow \frac{3-2k}{2} = \frac{-(2h-h^2)}{k}$$

$$\Rightarrow 3k-2k^2 = -2(2h-h^2)$$

$$\Rightarrow$$

$$2h^2+2k^2-4h-3k=0$$

$$2x^2+2y^2-4x-3y=0$$

9

JEE 1

Number of terms free from radical sign in the expansion of $(\sqrt{5}+\sqrt[4]{11})^{100}$ is

(A) 24 (B) 25

(C) 26 (D) 27

$$\left(5^{\frac{1}{2}}+11^{\frac{1}{4}}\right)^{100}$$

$$\text{LCM}(2,4)=4$$

Ans: $\left[\frac{100}{4}\right]+1=26.$ (Shortcut)

10

If $f(x)=\int\sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}}\,dx$ and $f\left(\frac{-\pi}{2}\right)=0$,

then $f\left(\frac{-\pi}{3}\right)$ is equal to

(A) $\frac{-\pi}{3}$

(B) $\frac{-\pi}{6}$

(C) $\frac{-2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$

(D) $\frac{2}{3}\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)$

$$\Rightarrow \int_{-\pi/2}^{-\pi/3}\sqrt{\frac{\cos x(1-\cos^2 x)}{1-\cos^3 x}}\,dx$$

$$\Rightarrow \int_{-\pi/2}^{-\pi/3}\sqrt{\frac{\cos x}{1-\cos^3 x}}\,|\sin x|\,dx = f(-\pi/3)-\cancel{f(-\pi/2)}$$

$$f\left(-\frac{\pi}{3}\right) = \int_{-\pi/2}^{-\pi/3} \sqrt{\frac{\cos x}{1-\cos^3 x}} \cdot (-\sin x)\,dx$$

Let $\cos x = t$

$$\Rightarrow \int_0^{\frac{1}{2}} \sqrt{\frac{t}{1-t^3}}\,dt$$

$$t^{3/2} = z$$

$$\frac{3}{2}\sqrt{t}\,dt = dz$$

$$\Rightarrow \frac{2}{3}\int_0^{\frac{1}{\sqrt{8}}} \frac{dz}{\sqrt{1-z^2}}$$

$$= \frac{2}{3}\sin^{-1} z \Big|_0^{1/\sqrt{8}}$$

$$= \frac{2}{3}\sin^{-1}\frac{1}{\sqrt{8}}$$

11

JEE 1

$\lim_{x \to \frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln(\sin x)}$ is equal to

(A) 2 (B) 1

(C) 3 (D) 0

$t = \sin x - 1$

$t \to 0$

$$\lim_{t \to 0} \frac{(1+t) - (1+t)^{(1+t)}}{-t + \ln(1+t)} \quad \left(\frac{0}{0}\right)$$

$\frac{d}{dt} e^{(t+1)\ln(t+1)} \longrightarrow e^{(t+1)\ln(t+1)} \left[1 \cdot \ln(t+1) + \frac{(t+1)}{t+1}\right]$

$$\Rightarrow \lim_{t \to 0} \frac{1 - (t+1)^{t+1} \left(\ln(t+1) + 1\right)}{-1 + \frac{1}{t+1}}$$

$$\Rightarrow \lim_{t\to 0} \frac{\left[1-(t+1)^{t+1}\left(1+\ln(t+1)\right)\right](t+1)}{-t}$$

$$\Rightarrow \lim_{t\to 0} \frac{\left[1-(t+1)^{t+1}\left(1+\ln(t+1)\right)\right](1)+(\overset{=1}{t+1})\left[-(t+1)^{t+1}\left(1+\ln(t+1)\right)^2-(t+1)^{t+1}\left(\frac{1}{t+1}\right)\right]}{-1}$$

$$= \frac{-2}{-1}$$

M2

$$\lim_{\substack{x \to \frac{\pi}{2} \\ \sin x \to 1}} \frac{\sin x \left[1 - (\sin x)^{\sin x - 1}\right]}{(1 - \sin x) + \underset{(\sin x - 1)}{\cancel{\ln(\sin x - 1 + 1)}}}$$

12

If $y = 3[x] + 1 = 4[x-1] - 10$, then $[x+2y]$ is equal to (where [.] is G.I.F.)

(A) 107 (B) 92

(C) 46 (D) 61

JEE 1

$[x + \text{int}] = [x] + \text{int}$

$y = 3[x] + 1$

$y = 4[x] - 4 - 10 = 4[x] - 14$

$3[x] + 1 = 4[x] - 14$

$[x] = 15$

$y = 46$

$[x + 2y] = [x + 92] = [x] + 92$

$= 15 + 92$

$= 107$

13

If $g(x^3+1) = x^6 + x^3 + 2$, then the value of $g(x^2-1)$ is

(A) $x^4 - 3x^2 + 3$ (B) $x^4 + x^2 + 4$

(C) $x^4 - 3x^2 + 4$ (D) $x^4 + x^2 + 2$

$$g(x^3+1) = (x^3+1)^2 - x^3 + 1$$

$$= (x^3+1)^2 - (x^3+1) + 2$$

$$g(\square) = \square^2 - \square + 2$$

$$g(x^2-1) = (x^2-1)^2 - (x^2-1) + 2$$

$$= x^4 - 3x^2 + 4$$

14

If the function $f(x) = axe^{-bx}$ has a local maximum at the point (2,10), then

(A) $a = 5; b = 0$

(B) $a = 5e, b = 1/2$

(C) $a = 5e^2, b = 1$

(D) none

JEE 1

$(2,10) \longrightarrow f(x) = axe^{-bx}$

$10 = 2ae^{-2b}$

$\boxed{ae^{-2b} = 5}$ — 1

$f'(x) = a\left[e^{-bx} + xe^{-bx}(-b)\right]$

$0 = a\left[e^{-2b} - 2be^{-2b}\right]$

$0 = ae^{-2b}(1-2b)$

$\boxed{b = 1/2}$

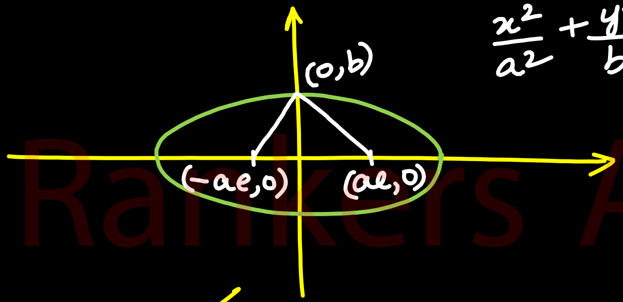
$ae^{-2(1/2)} = 5$

$\boxed{a = 5e}$

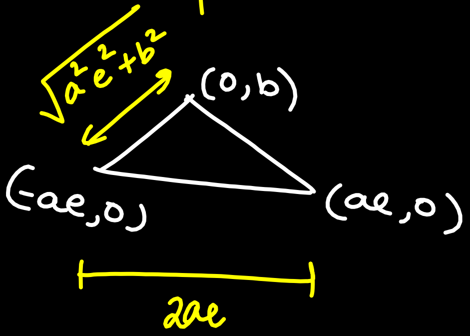
In an ellipse, the triangle formed by end point of minor axis with focii is equilateral triangle, then reciprocal of eccentricity, is

(A) 2 ✓ (B) ~~1/2~~

(C) 3 (D) $\sqrt{3}$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$e^2 = 1-\frac{b^2}{a^2} \quad \text{—— (1)}$$

$$\sqrt{a^2e^2+b^2} = 2ae$$

$$a^2e^2+b^2 = 4a^2e^2$$

$$b^2 = 3a^2e^2$$

$$\frac{b^2}{a^2} = 3e^2$$

15

JEE 1

$e^2 = 1 - 3e^2$

$4e^2 = 1$

$e = \frac{1}{2}$

Ans: $\frac{1}{e} = 2.$

16

Quadratic equation such that sum of its roots is $\left(\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}\right)$ and product of its roots is $\left(\frac{\sin 85^\circ - \sin 35^\circ}{\cos 65^\circ}\right)$ has

(A) Imaginary roots

(B) Real & equal roots

(C) Rational roots

(D) Irrational roots

$$S = \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$$

$$= \left(\frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}\right)$$

$$= \frac{4\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{2\sin 10^\circ \cos 10^\circ}$$

$$= \frac{4\,\cancel{(\cos(70^\circ))}}{\cancel{(\sin 20^\circ)}} = 4$$

$$P = \frac{2\,\cancel{\sin(25^\circ)}\cos(60^\circ)}{\cancel{(\cos 65^\circ)}} = 1$$

QE : $x^2 - 4x + 1 = 0$; $D = 16 - 4 = 12$

17

JEE 1

Let $P = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ & P^T is transpose of P and $Q = P \cdot A \cdot P^T$ where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then trace of matrix $(P^T \cdot Q^{2018} \cdot P)$ is

(A) 0

(B) 1

(C) 2

(D) 2018

$$P^T(PAP^T)(PAP^T)\text{---}(PAP^T)P$$

$$\boxed{A^{2018}}$$

$$P \cdot P^T = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

18

JEE 1

The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. It is known that one of the students got ' 12 ' instead of 8 . If the new mean of the marks is 10.2 then the new variance is equal to

(A) 4.04 (B) 4.08

(C) 3.96 (D) 3.92

$$\bar{X}_1 = 10 = \frac{\sum x_i + 8}{n}$$

$$\bar{X}_2 = 10.2 = \frac{\sum x_i + 12}{n}$$

$$0.2 = \frac{4}{n} \Rightarrow \boxed{n = 20}$$

$$\sigma_1^2 = 4 = \frac{\sum x_i^2 + 8^2}{20} - (10)^2$$

$$\sigma_2^2 = \frac{\sum x_i^2 + 12^2}{20} - (10.2)^2$$

$$\sigma_2^2 - 4 = \frac{12^2 - 8^2}{20} - \left[(10.2)^2 - 10^2\right]$$

$$\sigma_2^2 = 4 + 4 - (20.2 \times 0.2)$$

19

Let $A = \{1,2,3,4,5,6,7\}$. The number of surjective functions defined from A to A such that $f(i) = i$ for atleast four values of i from $i = 1,2,\ldots,7$, is:

(A) 7 !

(B) 92

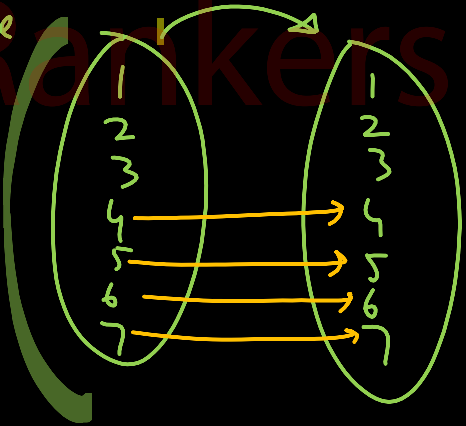
(C) 126

(D) 407

onto

$\Downarrow$

one-one



(i) Exactly 4 $\boxed{f(i)=i}$

$^7C_4\left[3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)\right]$

(ii) Exactly 5 $\boxed{f(i)=i}$

$^7C_5\,[1]$

(iii) Exactly 7 $f(i)=i$

$\boxed{1}$

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JEE 1

Let a line l pass through the origin and be perpendicular to the lines

$l_1: \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in R$ and l_2:

$\vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in R$.

If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $(\alpha + \beta + \gamma)$ is

(A) $\frac{5}{9}$ (B) $\frac{9}{5}$

(C) $\frac{19}{5}$ (D) $\frac{5}{3}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(-4) - \hat{j}(-5) + \hat{k}(-2)$$

$$l: \frac{x}{-4} = \frac{y}{5} = \frac{z}{-2}$$

$$l_1: \frac{x-1}{1} = \frac{y+11}{2} = \frac{z+7}{3} = \lambda$$

$$l_2: \frac{x+1}{2} = \frac{y}{2} = \frac{z-1}{1} = \mu$$

$\boxed{P}$: $(\lambda+1, 2\lambda-11, 3\lambda-7)$

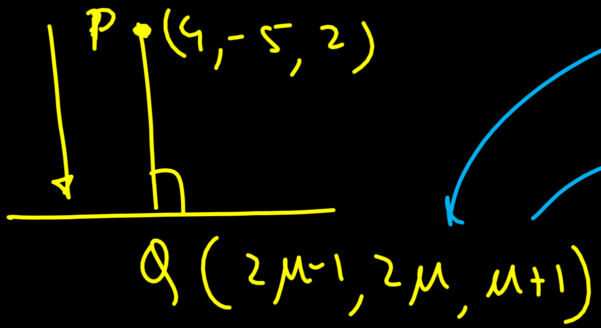
$$\Rightarrow \frac{\lambda+1}{-4} = \frac{2\lambda-11}{5} = \frac{3\lambda-7}{-2}$$

$5\lambda + 5 = -8\lambda + 44$

$13\lambda = 39 \Rightarrow \boxed{\lambda = 3}$

$P \equiv (4, -5, 2)$

$L_2 \equiv \dfrac{x+1}{2} = \dfrac{y}{2} = \dfrac{z-1}{1} = \mu$



$2(2\mu-5)+2(2\mu+5)+1(\mu-1)=0$

$4\mu-\cancel{10}+4\mu+\cancel{10}+\mu-1=0$

$9\mu=1$

$\boxed{\mu=1/9}$

$\left(-\dfrac{7}{9}, \dfrac{2}{9}, \dfrac{10}{9}\right)$

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For $y > 0$ and $x \in R$, $ydx + y^2dy = xdy$ where $y = f(x)$. If $f(1) = 1$, then the value of $f(-3)$ is

$$\frac{y\,dx - x\,dy}{y^2} + \frac{\cancel{y^2}\,dy}{\cancel{y^2}} = 0$$

$$\int d\left(\frac{x}{y}\right) + \int dy = 0$$

$$\boxed{\frac{x}{y} + y = c}$$

$(1,1)$: $\boxed{c = 2}$

$$\frac{-3}{y} + y = 2$$

$$-3 + y^2 = 2y$$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$\boxed{y = 3}$ ✓ & $y = -1$ ✗

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JEE 1

If x, y, z satisfy the system of equations $\tan^2 x + \cot^2 x = 2\cos^2 y$ and $\cos^2 y + \sin^2 z = 1$, then the value of $\int_{\cos^2 y}^{\sec^2 x + \sec^2 z} \frac{t^2}{t^2-4t+8}\,dt$ is equal to

$$\underbrace{\frac{\left(\tan^2 x + \frac{1}{\tan^2 x}\right)}{2}}_{\geq 1} = \underbrace{\cos^2 y}_{\leq 1}$$

$$\cos^2 y = 1 \;;\quad \tan^2 x = 1$$

$$\sec^2 x = 2$$

$$\sin^2 z = 0 \Rightarrow \cos^2 z = 1$$

$$\int_1^3 \frac{t^2}{t^2-4t+8}\,dt$$

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JEE 1

$$I = \int_1^3 \left[\frac{\cancel{(t^2-4t+8)}}{\cancel{(t^2-4t+8)}} + \frac{(4t-8)}{(t^2-4t+8)} \right] dt$$

$$I = (t)_1^3 + 2\left(\ln(t^2-4t+8)\right)_1^3$$

$$I = 2 + 2\ln\left(\frac{9-12+8}{1-4+8}\right)$$

$$= \boxed{2}$$

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JEE 1

Let $f(x) = \begin{cases} \cos(x^3); & -\infty < x < 0 \\ \sin(x^3) - |x^3 - 1|; & 0 \le x < \infty \end{cases}$

then number of points where $g(x) = f(|x|)$ is non-differentiable is

2 : Ans

$$f(|x|) = \begin{cases} -\sin x^3 - |-x^3 - 1| ; & (-\infty, 0) \\ \sin x^3 - |x^3 - 1| ; & (0, \infty) \end{cases}$$

$$= \begin{cases} -\sin x^3 - |x^3 + 1| ; & (-\infty, 0) \\ \sin x^3 - |x^3 - 1| ; & (0, \infty) \end{cases}$$

$\boxed{\text{N.D. at } x = -1 \ \& +1}$

Eg:

$f(x) \rightarrow$

$f(|x|) \rightarrow$

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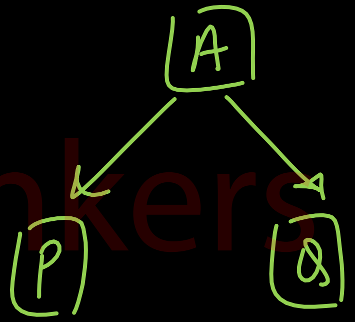
Let A = {1,2,3,4,5}. The number of unordered pairs of subsets P and Q of A such that $P \cap Q = \phi$ is 'n'. Then the sum of digits of n is



$(3)(3)(3)(3)(3)$

Eg:

$P = \{1\}$, $Q = \{\ \}$

same

$P = \{\ \}$, $Q = \{1\}$

$$\frac{3^5+1}{2} = \frac{243+1}{2} = 122$$

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JEE 1

The total number of positive integral solution of

$15 < x_1 + x_2 + x_3 \le 20$ is equal to

$$x_1 + x_2 + x_3 = 16$$

$$\begin{cases} x_1 = 1 + y_1 \ ; \ y_1 \geq 0 \\ x_2 = 1 + y_2 \ ; \ y_2 \geq 0 \\ x_3 = 1 + y_3 \ ; \ y_3 \geq 0 \end{cases}$$

$$y_1 + y_2 + y_3 = 13$$

$16 : \quad {}^{15}C_2 \quad + {}^{15}C_3 - {}^{15}C_3$

$17 : \quad {}^{16}C_2$

$18 : \quad {}^{17}C_2$

$19 : \quad {}^{18}C_2$

$20 : \quad {}^{19}C_2$

$${}^{20}C_3 - {}^{15}C_3 = 685$$