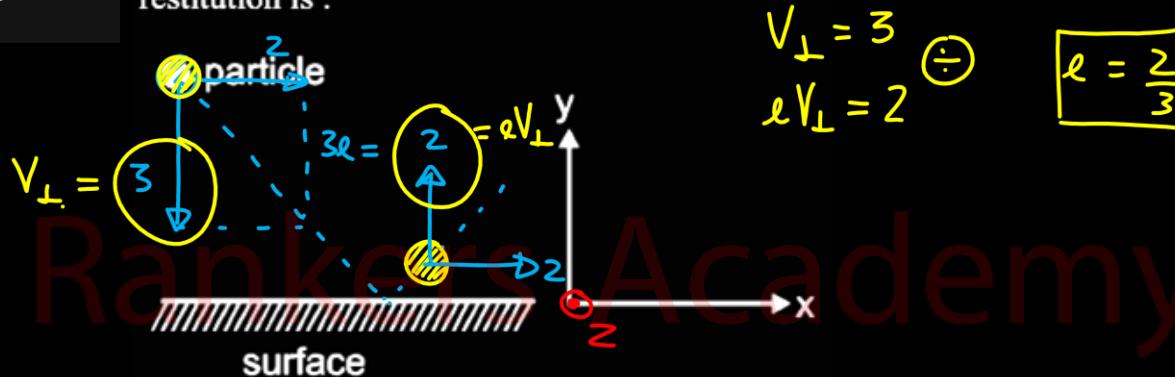


PHYSICS

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A particle moving with velocity $(2\hat{i} - 3\hat{j})$ m/s collides with a surface at rest in xz-plane as shown in figure and moves with velocity $(2\hat{i} + 2\hat{j})$ m/s after collision. Then coefficient of restitution is :



$$\begin{aligned} v_{\perp} &= 3 \\ e v_{\perp} &= 2 \quad \div \\ e &= \frac{2}{3} \end{aligned}$$

(A) $\frac{2}{3}$

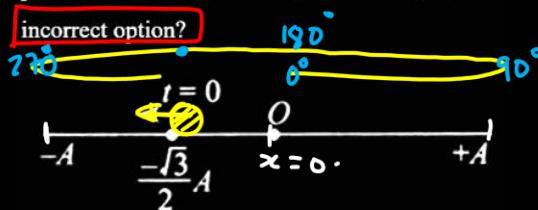
(B) 1

(C) $\sqrt{\frac{8}{13}}$

(D) $\frac{4}{5}$

2

A particle starts from point $x = \frac{-\sqrt{3}}{2}A$ and move towards negative extreme as shown. if the time period of oscillation is T , then which is incorrect option?



(A) The equation of the SHM is $x = A \sin\left(\frac{2\pi t}{T} + \frac{4\pi}{3}\right)$ True.

(B) The time taken by the particle to go directly from its initial position to negative extreme is $\frac{T}{12}$ True.

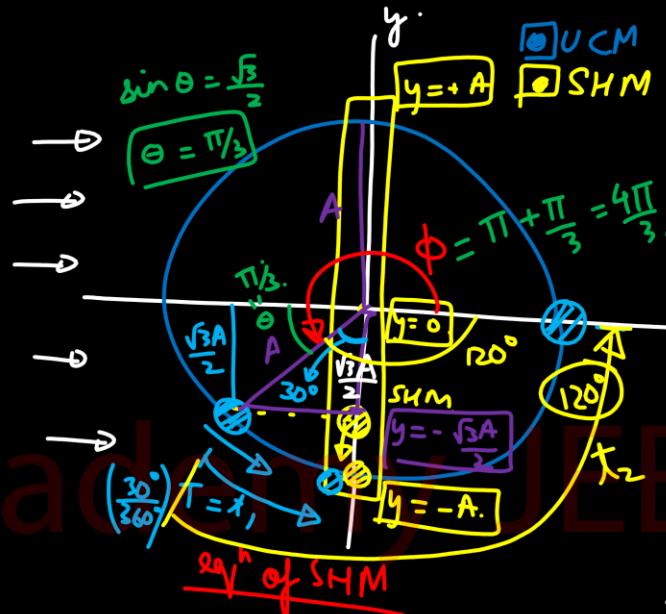
(C) The time taken by the particle to reach at mean position is $\frac{T}{3}$ True.

(D) The equation of the SHM is

$$x = A \sin\left(\frac{2\pi t}{T} + \frac{\pi}{3}\right)$$

Incorrect

Phasor



$$\omega = \frac{2\pi}{T}$$

$$y = A \sin(\omega t + \phi)$$

$$y = A \sin\left(\frac{2\pi}{T} \cdot t + \frac{4\pi}{3}\right)$$

$$t_1 = \left(\frac{30^\circ}{360^\circ}\right) T = \frac{T}{12}$$

$$t_2 = \left(\frac{120^\circ}{360^\circ}\right) T = \frac{T}{3}$$

3

A stone is projected from level ground such that its horizontal and vertical components of initial velocity are $u_x = 10 \text{ m/s}$ and $u_y = 20 \text{ m/s}$ respectively. Then the angle between velocity vector of stone one second before and one second after it attains maximum height is:

- (A) 30°
 (B) 45°
 (C) 60°
 (D) 90°

$$u_y = 20$$

$$T = \frac{2u_y}{g} = 4$$

$t=0$

$t=1$

$t=2$

$t=3$

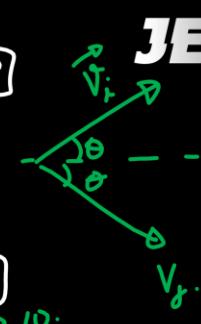
$t=4$

$$V_y = u_x + a_y t$$

$$V_y = u_x - 10 t$$

$$\text{Ans} = 2\theta = 90^\circ$$

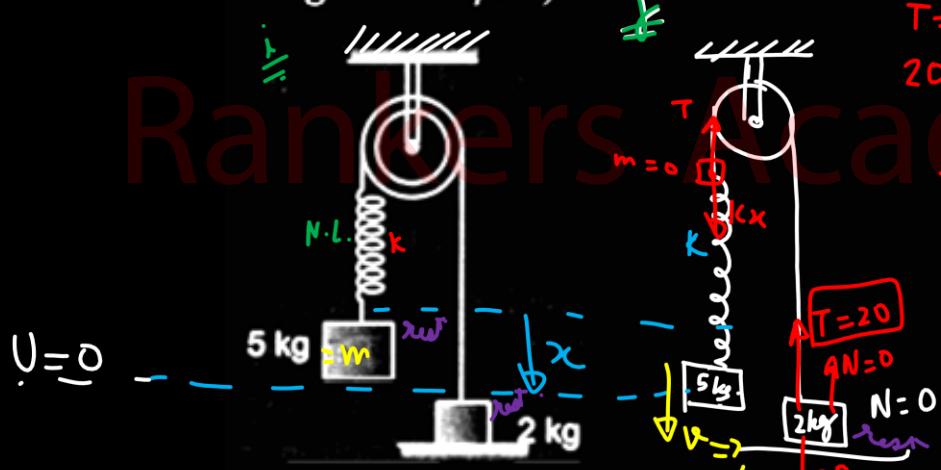
$$\tan \theta = \frac{V_y}{V_x} = \frac{10}{10} = 1$$



JEE 1

4

System shown in figure is released from rest. Pulley and spring is massless and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with ground is (Take force constant of spring $k = 40 \text{ N/m}$ and $g = 10 \text{ m/s}^2$)



- (A) $\sqrt{2} \text{ m/s}$
 (B) $2\sqrt{2} \text{ m/s}$
 (C) 2 m/s
 (D) $4\sqrt{2} \text{ m/s}$

$$T = kx$$

$$20 = 40x$$

$$x = \frac{1}{2}$$

$$E_i = E_f$$

$$[mgx] = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

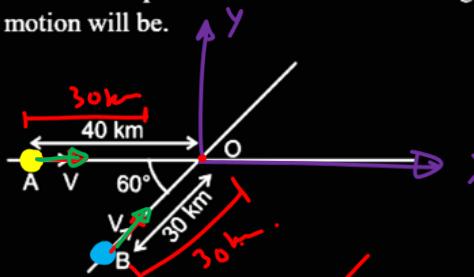
$$\frac{5}{2}g = \frac{20}{4} + \frac{5}{2}v^2$$

$$5 \times 2 = v^2$$

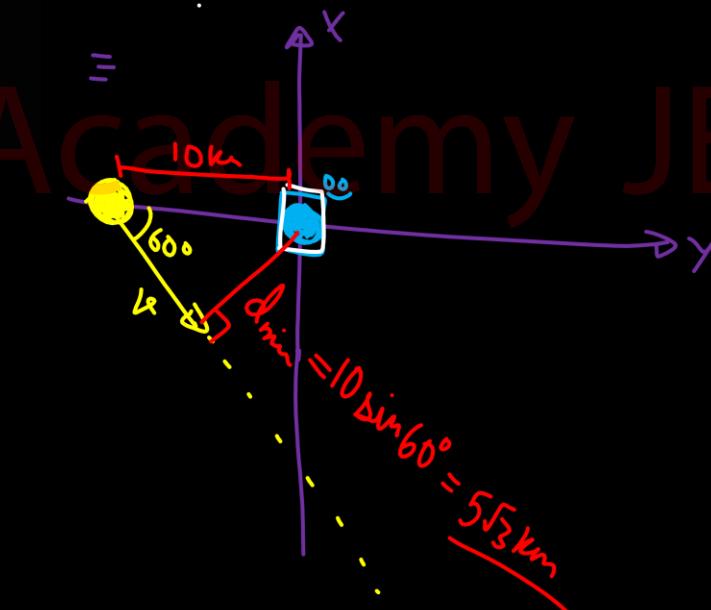
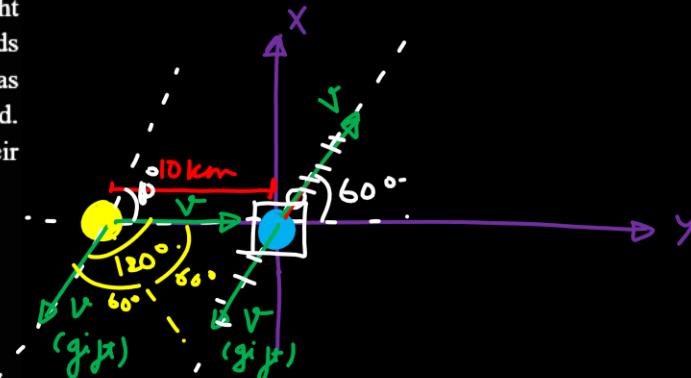
$$V = 2\sqrt{2}$$

5

Two cars A and B moving on two straight tracks inclined at an angle 60° heading towards the crossing initially their positions are as shown in the figure. Both cars have same speed. Minimum separation between them during their motion will be.



- (A) 10 km
 (B) $5\sqrt{3}$ km
 (C) 5 km.
 (D) $\frac{20}{\sqrt{3}}$ km

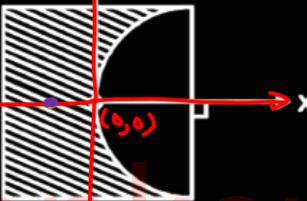


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6

In the figure shown a semicircular area is removed from a uniform square plate of side ' ℓ ' and mass 'm' (before removing). The x -coordinate of centre of mass of remaining portion is (The origin is at the centre of square)

$$\text{Ans} = X_1 = ?$$

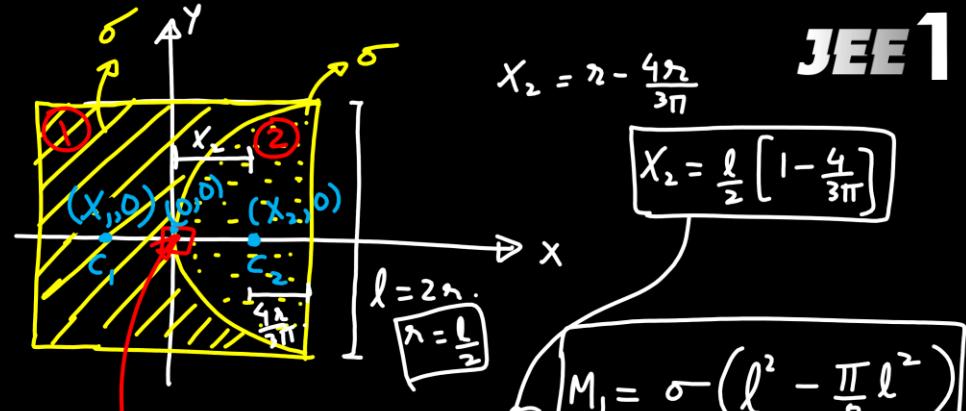


(A) $-\frac{\pi(\pi-2)\ell}{2(8-\pi)}$

(B) $\frac{\pi(\pi-2)\ell}{2(8-\pi)}$

(C) $-\frac{\pi(\pi-2)\ell}{8-\pi}$

(D) $-\frac{\ell(\pi-\frac{4}{3})}{2(8-\pi)}$



$$X_2 = r - \frac{4r}{3\pi}$$

$$X_2 = \frac{l}{2} \left[1 - \frac{4}{3\pi} \right]$$

$$M_1 = \sigma \left(l^2 - \frac{\pi}{8} l^2 \right)$$

$$M_2 = \sigma \left(\frac{\pi}{8} l^2 \right)$$

COM of ① & ②

$$X_{\text{COM}} = \frac{M_1 X_1 + M_2 X_2}{(M_1 + M_2)} = 0$$

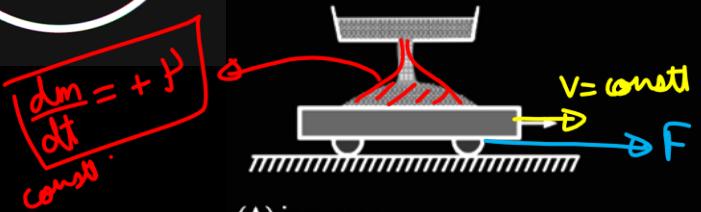
$$\cancel{\frac{1}{2} \left(1 - \frac{\pi}{8} \right) X_1} = - \cancel{\frac{\pi}{8} \ell \cdot \frac{l}{2} \left[1 - \frac{4}{3\pi} \right]}$$

$$\left(\frac{8-\pi}{8} \right) X_1 = - \frac{\ell}{2 \times 8} \left(\pi - \frac{4}{3} \right)$$

$$X_1 = - \frac{\ell}{2} \frac{\left(\pi - \frac{4}{3} \right)}{(8-\pi)}$$

7

Sand is falling on a flat cart being pulled with constant speed. The rate of mass falling on the cart is constant. Then the horizontal component of force exerted by the falling sand on the cart (sand particles sticks to the cart)



- (A) increases
- (B) decreases
- (C) remains constant
- (D) increases and then decreases

$$F = \frac{d(P)}{dt} = \frac{d(mv)}{dt} = v \cdot \frac{dm}{dt}$$

$\frac{dm}{dt}$ is const.

v is var + const.

$$F = v \cdot \frac{dm}{dt}$$

v is const.

$\frac{dm}{dt}$ is const.

$$F = \text{const}$$

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8

A double star consists of two stars having masses M and $2M$. The distance between their centers is equal to r . They revolve under their mutual gravitational attraction. Which is incorrect?

~~$2M$~~

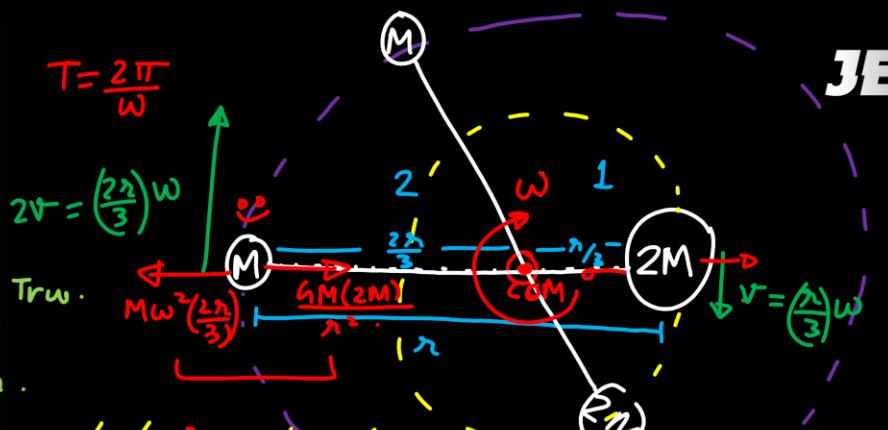
- (A) Heavier star revolves in an orbit of radius $\frac{r}{3}$
 (B) Both the stars have same time period of revolution which is equal to $\frac{2\pi}{\sqrt{3GM}} r^{3/2} = T$

(C) Kinetic energy of heavier star is twice that of the lighter star ~~FALSE~~

(D) KE of lighter star is twice that of heavier star

~~Ans. C~~

ans. is
one of these.



$$\frac{1}{3} M \omega^2 r = \frac{1}{3} G M^2 \cdot \frac{1}{r^2}$$

$$\omega T = 2\pi$$

$$\omega = \sqrt{\frac{3GM}{r^3}} = \sqrt{\frac{3GM}{r^3/2}}$$

heavy

$$KE_{2M} = \frac{1}{2}(2M)v^2 = MW^2$$

light

$$KE_M = \frac{1}{2}(M)(2v)^2 = 2Mv^2$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3GM}} r^{3/2}$$

$$\frac{KE_{\text{heavy}}}{KE_{\text{light}}} = \frac{1}{2}$$

$$KE_{\text{light}} = (2) KE_{\text{heavy}}$$

9

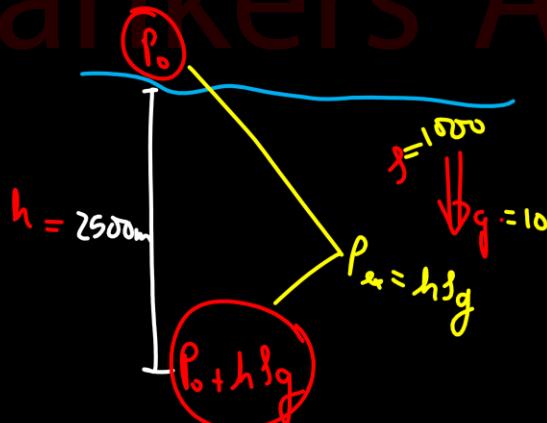
The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of 2500 m below the sea level is _____ %.

Given, the Bulk modulus of water = $2 \times 10^9 \text{ N m}^{-2}$, density of water = 10^3 kg m^{-3} , acceleration due to gravity = $g = 10 \text{ m s}^{-2}$.

- (A) 1.25 (B) 1.0
 (C) 1.75 (D) 1.5

$$\underline{\Delta V} = \left| \frac{\Delta V}{V} \right| \times 100 = ?$$

$$B = 2 \times 10^9$$



$$B = - \frac{P_{ext} \cdot \Delta V}{V} = \left| \frac{P_{ext} \cdot \Delta V}{V} \right|$$

ΔV

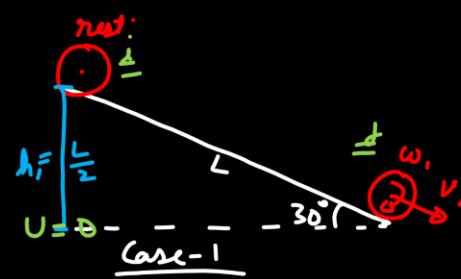
$$2 \times 10^1 = \frac{250}{10} \times 100 \times 10.$$

$$\left| \frac{\Delta V}{V} \right| \times 100 = \frac{25}{20} = \frac{5}{4} = 1.25\%$$

10

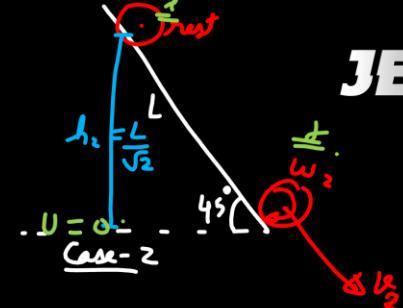
A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1 . If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2 . The ratio $v_1^2 : v_2^2$ is

- (A) $1:\sqrt{2}$ (B) $1:\sqrt{3}$
 (C) $1:3$ (D) $1:2$



$$E_i = E_f$$

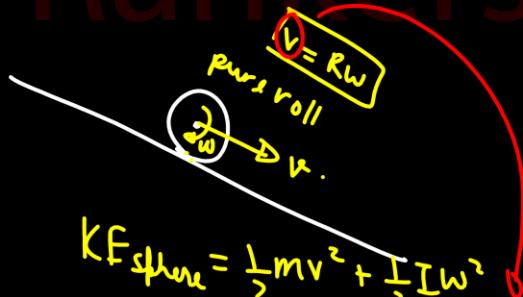
$$Mg\left(\frac{L}{2}\right) = 0.7mv_1^2$$



$$E_i = E_f$$

$$Mg \frac{L}{\sqrt{2}} = 0.7mv_2^2$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{v_1^2}{v_2^2}$$



$$K_{\text{Sphere}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = 0.7mv^2$$

JEE 1

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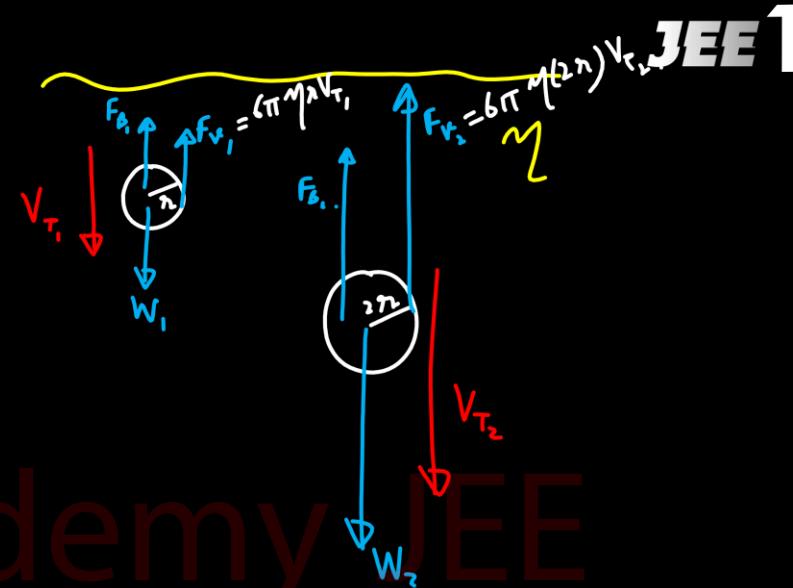
77

Two copper balls of radius r and $2r$ are released at rest in a long tube filled with liquid of uniform viscosity. After some time when both the spheres acquire critical velocity (terminal velocity) then ratio of viscous force on the balls is :

- (A) 1: 2
- (B) 1: 4
- (C) 1: 8
- (D) 1: 18

$$F_v = 6\pi\eta A v$$

$$V_T = \frac{2}{9} \frac{\pi^2 g}{\eta} [8 - 8_L]$$



$$\text{Ans} = \frac{F_{v_1}}{F_{v_2}} = \frac{6\pi\eta(4\pi r^2) V_{T_1}}{6\pi\eta(4\pi (2r)^2) V_{T_2}} = \frac{1}{2} \frac{\frac{2}{9} \frac{\pi^2 g}{\eta} [8 - 8_L]}{\frac{2}{9} \frac{\pi^2 (2r)^2 g}{\eta} [8 - 8_L]} = \boxed{\frac{1}{8}}$$

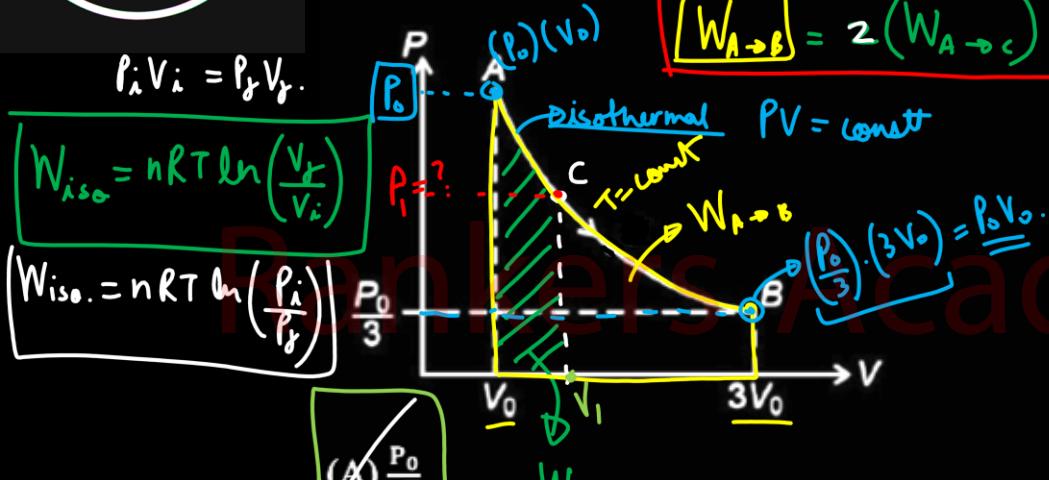
12

An ideal monatomic gas undergoes isothermal expansion from state A to B. Work done by the gas from $A \rightarrow B$ is double of work done by the gas from $A \rightarrow C$. Pressure at point C is

$$P_0 V_0 = n R T.$$

$$P_0 V_0 = P_1 V_1$$

JEE 1



$$n R T \ln\left(\frac{3V_0}{V_0}\right) = 2 \left[n R T \ln\left(\frac{V_1}{V_0}\right) \right]$$

$$\frac{1}{2} \ln(3) = \ln\left(\frac{P_0}{P_1}\right)$$

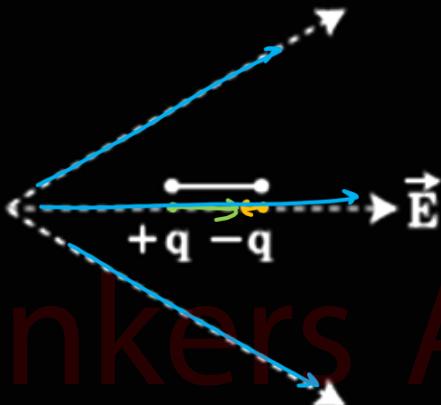
$$\ln(\sqrt{3}) = \ln\left(\frac{P_0}{P_1}\right)$$

$$P_1 = \frac{P_0}{\sqrt{3}}$$

13

A dipole is placed in an electric field as shown.

In which direction will it move?



(A) towards the left as its potential energy will increase.

~~(B) towards the right as its potential energy will decrease.~~

(C) towards the left as its potential energy will decrease.

~~(D) towards the right as its potential energy will increase.~~

$$\begin{array}{c} \rightarrow \leftarrow \\ F_+ F_- \\ \rightarrow \\ F_{\text{net}} \end{array}$$

Along \vec{E} Potential reduces
 \Rightarrow P.E. decrease.

$$\begin{aligned} V &= -\vec{p} \cdot \vec{E} \\ &= -pE \cos 180^\circ \\ &= +pE \\ \text{as } E \downarrow 0 \downarrow \end{aligned}$$

14

A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0 , which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly.

What will be the ratio of the magnetic field at a distance x from the axis when (i) $x < a$ and (ii)

$a < x < b$?

(A) $\frac{x^2}{a^2}$

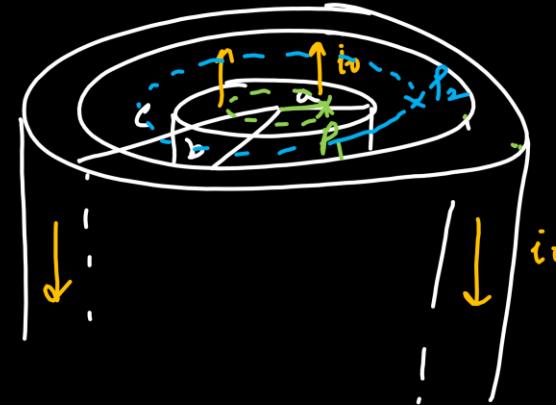
(B) $\frac{a^2}{x^2}$

(C) $\frac{x^2}{b^2-a^2}$

(D) $\frac{b^2-a^2}{x^2}$

$$\text{at } P_1, \quad B_1 = \frac{\mu_0 i_0}{2\pi a} x \quad \text{(i)}$$

$$\text{or } B_1 = \frac{\mu_0 i_0}{2\pi x} x \quad \text{(ii)}$$



$$\text{at } P_1, \quad B_1 = \frac{\mu_0 i_0}{2\pi a} x \quad \text{(i)}$$

$$\text{at } P_2, \quad B_2 = \frac{\mu_0 i_0}{2\pi x} x \quad \text{(ii)}$$

$$\frac{B_1}{B_2} = \frac{x^2}{a^2}$$

15

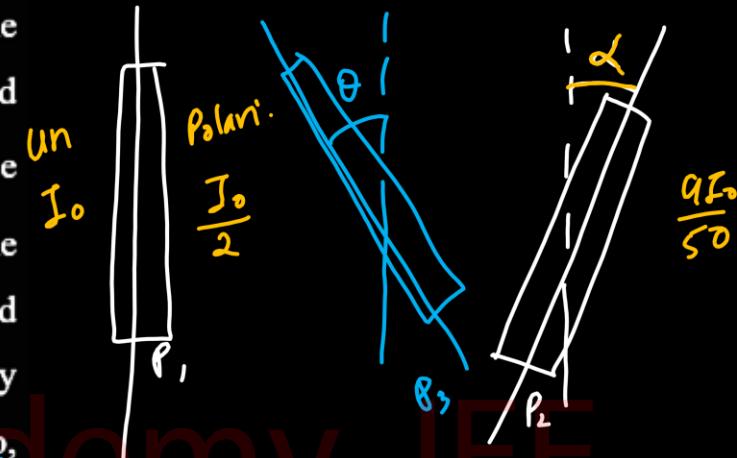
Two polaroids are placed in the path of unpolarized beam of intensity I_0 such that the intensity of light emitted from the second polaroid is $\frac{9}{50}I_0$. If a third polaroid whose polarization axis makes an angle θ with the polarization axis of first polaroid, is placed between these polaroids, such that the intensity of light emerging from the last polaroid is zero, then angle θ is

- (A) 37° (B) 53°
 (C) 60° (D) 30°

$$I = I_0 \cos^2 \alpha$$

$$\text{For } \alpha \quad \frac{9I_0}{50} = \left(\frac{I_0}{2}\right) \cos^2 \alpha$$

$$\Rightarrow \cos^2 \alpha = \frac{3}{5} \Rightarrow \alpha = 53^\circ$$



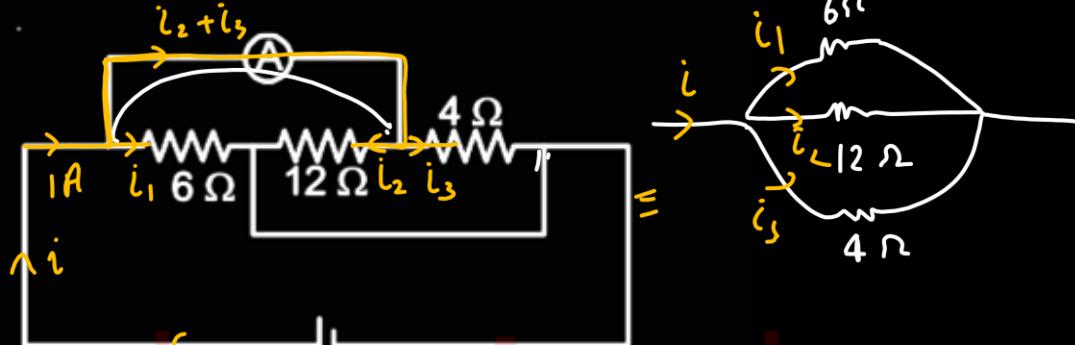
Since $I = 0$, P_2, P_3 must be crossed
 $\theta + \alpha = 90^\circ$

$$\theta = 90^\circ - 53^\circ \\ = 37^\circ$$

16

The reading of ammeter in following circuit is

(resistance of ammeter is negligible)



$$i_1 : i_2 : i_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

$$= \frac{1}{6} : \frac{1}{12} : \frac{1}{4}$$

$$i_1 : i_2 : i_3 = 2 : 1 : 3$$

(1) $\frac{1}{3}$ A

(2) $\frac{2}{3}$ A

(3) 1 A

(4) $\frac{1}{6}$ A

$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$$

$$\frac{1}{R_{eq}} = \frac{2+1+3}{12} = \frac{1}{2}$$

$$i = \frac{\epsilon_e}{R_{eq}} = \frac{2}{2} = 1 \text{ A}$$

then $i_1 = \frac{1}{3}$ $i_2 = \frac{1}{6}$ $i_3 = \frac{1}{2}$

(A) Readig $i_2 + i_3 = \frac{1}{6} + \frac{1}{2}$

$$= \frac{4}{6} = \frac{2}{3} \text{ A}$$



An ideal gas is expanded so that amount of heat given is equal to the decrease in internal energy.

The gas undergoes the process $TV^{1/5} =$

constant. The adiabatic compressibility of gas

when pressure is P , is –

$$K = ?$$

$$\rightarrow PV^{\frac{C}{S}} = \text{const}$$

$$C_V - \frac{R}{(g-1)} = -C_V$$

$$2C_r = \frac{R}{n-1}$$

$$\frac{2K}{Y-1} = \frac{R}{n-1}$$

$$r-1 = 2n-2$$

$$y = 2x - 1$$

$$= \frac{7}{5}$$

- (A) $\frac{7}{5P}$

20

(C) $\frac{2}{5P}$

(D) $\frac{7}{3P}$

$$\begin{aligned} \text{Adiabatic} \quad & P V^r = \text{const} \\ \Rightarrow P(rV^{r-1}dV) + V^r dP &= 0 \\ \Rightarrow B = -\frac{dP}{dV} &= rP \end{aligned}$$

18

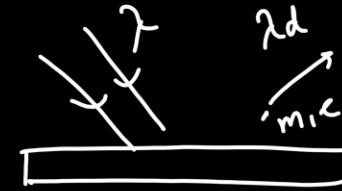
An electromagnetic wave of wavelength ' λ ' is incident on a photosensitive surface of negligible work function. If 'm' mass is of photoelectron emitted from the surface has de-Broglie wavelength λ_d , then:

A) $\lambda = \left(\frac{2m}{hc}\right) \lambda_d^2$

B) $\lambda_d = \left(\frac{2mc}{h}\right) \lambda^2$

C) $\lambda = \left(\frac{2mc}{h}\right) \lambda_d^2$

D) $\lambda = \left(\frac{2h}{mc}\right) \lambda_d^2$



$$\lambda_d = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

$$\lambda_d = \frac{h}{\sqrt{2mhc}} \frac{1}{\lambda}$$

$$\lambda_d^2 = \frac{h}{2mc} \lambda$$

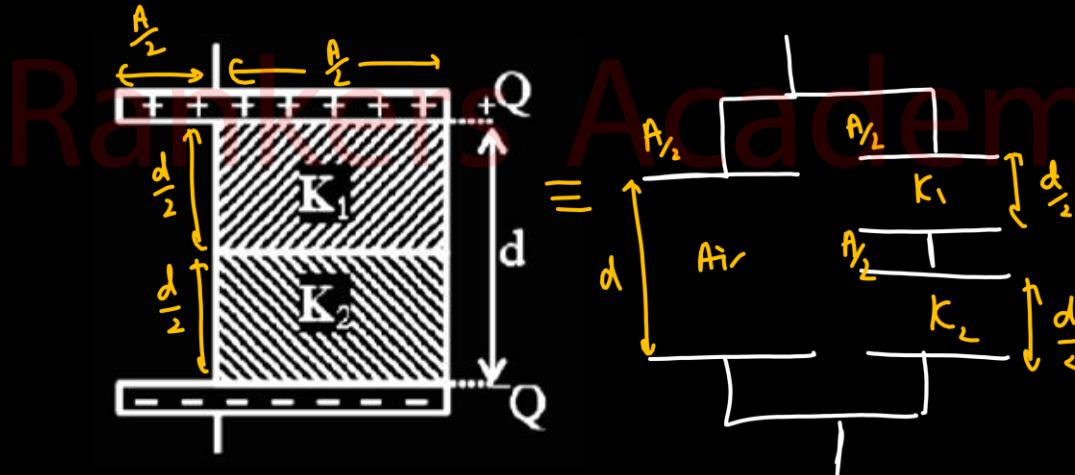
$$\lambda = \left(\frac{2mc}{h}\right) \lambda_d^2$$

$$K_{max} = E - \varphi$$

$$K = \frac{hc}{\lambda} - O \quad ①$$

19

A parallel - plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area $A/2$ and thickness $d/2$ are inserted in the space between the plates. The capacitance of the capacitor will be given by:



$$(A) \frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$$

$$(B) \frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{2(K_1 + K_2)} \right)$$

$$(C) \frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 + K_2}{K_1 K_2} \right)$$

$$(D) \frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{2(K_1 + K_2)}{K_1 K_2} \right)$$

$$C_1 = K_1 \frac{A}{\frac{d}{2}} \frac{\epsilon_0}{\frac{d}{2}} = K_1 \frac{A \epsilon_0}{d}$$

$$C_2 = K_2 \frac{A}{\frac{d}{2}} \frac{\epsilon_0}{\frac{d}{2}} = K_2 \frac{A \epsilon_0}{d}$$

$$C_{\text{eff}} = \frac{\frac{A}{2} \epsilon_0}{d} = \frac{A \epsilon_0}{2d}$$

19

A parallel - plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area $A/2$ and thickness $d/2$ are inserted in the space between the plates. The capacitance of the capacitor will be given by:

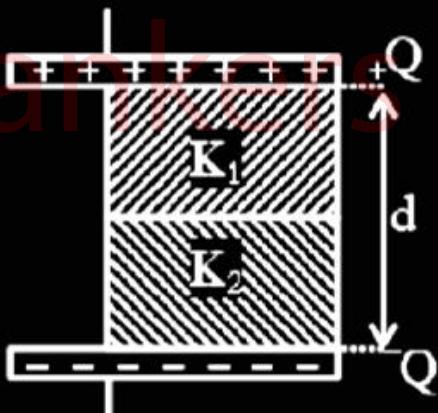
(A) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$

(B) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{2(K_1 + K_2)} \right)$

(C) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{K_1 + K_2}{K_1 K_2} \right)$

(D) $\frac{\epsilon_0 A}{d} \left(\frac{1}{2} + \frac{2(K_1 + K_2)}{K_1 K_2} \right)$

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$$C_{\text{eq}} = C_{A, \infty} + \frac{C_1 C_2}{C_1 + C_2}$$

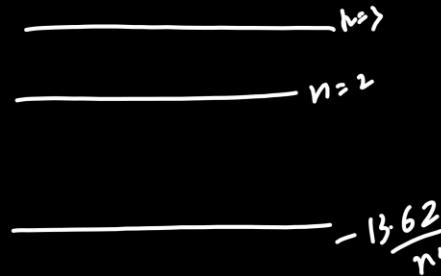
$$= \frac{A \epsilon_0}{2d} + \left(\frac{K_1 K_2}{K_1 + K_2} \right) \frac{A \epsilon_0}{d}$$

$$= \frac{A \epsilon_0}{d} \left(\frac{1}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$$



As an electron makes a transition from an excited state to the ground state of a hydrogen like atom ion

- (A) Kinetic energy, potential energy and total energy decrease
- (B) Kinetic energy decreases, potential energy increases and total energy remains same
- (C) Kinetic energy and total energy decrease but potential energy increases
- (D) its Kinetic energy increases but potential energy and total energy decrease



$$K \propto V^2 \propto \left(\frac{Z^2}{r^n}\right)^2 \quad n \downarrow \quad K \uparrow$$

$$U = -\frac{kze^2}{r} \quad E = -\frac{kze^2}{2\pi}$$

As $n \downarrow$ $U \downarrow E \downarrow$

Stability ↑

21

In 'an organ pipe' successive resonance are obtained at 250 Hz, 350 Hz and 450 Hz. If the speed of sound is 300 m/s, then length of organ pipe (in m) is x, then the value of $10x$ is _____. (ignore end correction)

$$f_0 = 50 \text{ Hz}$$

$$\frac{V}{4l} = 50$$

$$\Rightarrow l = \frac{300}{4 \times 50} = 1.5 \text{ m}$$

$$10x = 15 \text{ m}$$



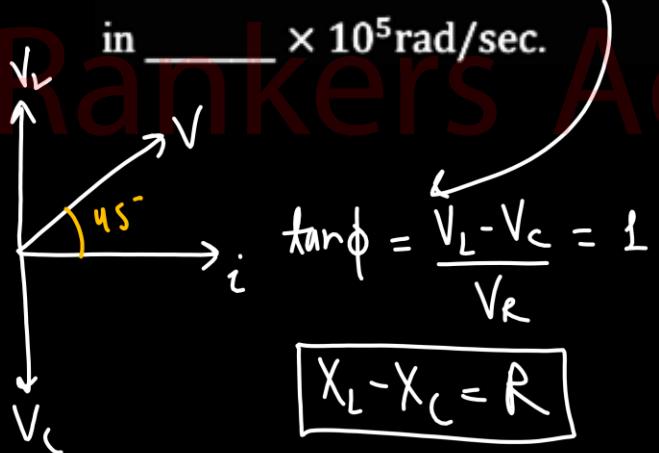
Closed organ pipe

$$\text{odd } \frac{\lambda}{4} = l$$

$$f = \text{odd} \left(\frac{\lambda}{4l} \right)^*$$



A series LCR circuit containing a resistance of 120Ω has angular resonance frequency $4 \times 10^5 \text{ rads}^{-1}$. At resonance the voltage across resistance and inductance are 60 V & 40 V respectively. At what frequency the current in the circuit lags the voltage by 45° . Give answer in $\text{---} \times 10^5 \text{ rad/sec.}$



At Resonance

$$\omega_0 = 4 \times 10^{+5} \quad R = 120\Omega$$

$$V_R = 60 \text{ V} \quad \& \quad V_L = V_C = 40 \text{ V}$$

$$iR = 60$$

$$i = \frac{1}{2} \text{ A}$$

$$X_L = X_C = \frac{40}{i}$$

$$\omega_0 L = \frac{1}{\omega_0 C} = 80\Omega$$

$$L = \frac{80}{4 \times 10^5} = 2 \times 10^{-4} \text{ H}$$

$$\& \quad C = \frac{1}{\omega_0 \times 80} = \frac{1}{32 \times 10^{-6}}$$

A series LCR circuit containing a resistance of 120Ω has angular resonance frequency $4 \times 10^5 \text{ rads}^{-1}$. At resonance the voltage across resistance and inductance are 60 V & 40 V respectively. At what frequency the current in the circuit lags the voltage by 45° ? Give answer



$$\tan \phi = \frac{V_L - V_C}{R} = 1$$

$$X_L - X_C = R$$

$$\omega L - \frac{1}{\omega C} = R$$

$$\omega^2 L - \omega R - \frac{1}{C} = 0$$

$$\omega^2 (2 \times 10^4) - (20\omega) - 80 \times 4 \times 10^5 = 0$$

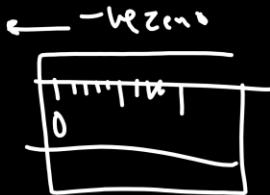
$$\omega = +120 \pm \sqrt{120^2 + 4 \times 2 \times 80 \times 4 \times 10^5}$$

$$= \frac{120 + \sqrt{120^2 + 25600}}{4 \times 10^4}$$

$$= \frac{120 + 200}{4 \times 10^4} = \boxed{8} \times 10^5$$

23

Pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between studs of the screw gauge, zero of circular scale is 8 divisions above the reference line and zero of the main scale is not visible. Now, when a cylindrical wire is placed between its studs the main scale reading is 2 divisions and 15th division of circular scale coincides with reference line. Diameter of the wire is x mm . Find the value of $100x$.



$$L.C. = \frac{1\text{ mm}}{100 \text{ div}} = 0.01 \text{ mm} \quad \text{--- (1)}$$

$$\text{Zero error} = 8 \times L.C. \quad \text{--- (2)}$$

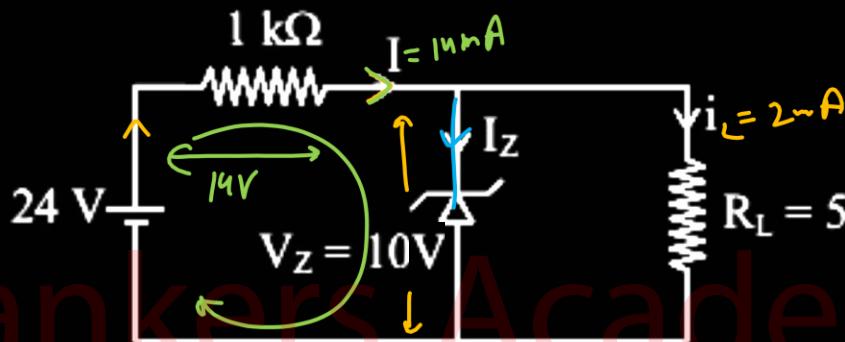
$$\text{Reading} = \text{MSR} + CCGD \times L.C. - \text{zero error}$$

$$\text{Diameter} = 2 \times (1\text{ mm}) + 15 \times L.C. - (-8 \times L.C.)$$

$$= 2.23 \text{ mm}$$

$$= 2.23 \times 10^{-2} \text{ m}$$

For the given circuit, the power across zener diode is mW.



$$\textcircled{1} \quad i_L = \frac{V_Z}{R_L} = \frac{10V}{5k\Omega} = 2mA$$

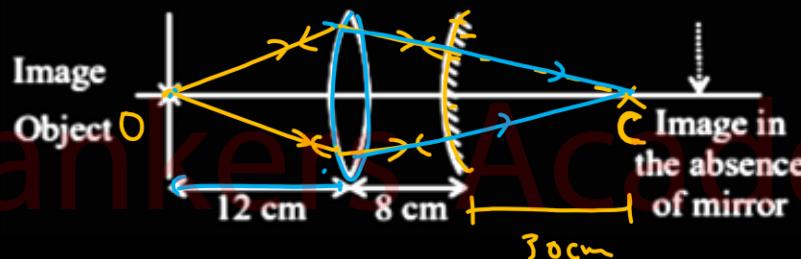
$$\textcircled{2} \quad i = \frac{V - V_Z}{R} = \frac{24 - 10}{1k\Omega} = 14mA$$

$$\textcircled{3} \quad i_Z = i - i_L \\ = 14 - 2 = 12mA$$

$$P = i_Z V_Z = (12mA) \times 10V = \boxed{120} \text{ mWatt}$$

25

An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.



When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be

_____ (cm)

$$12 + 8 + 30 = \underline{50 \text{ cm}}$$

CHEMISTRY

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Assertion : Treatment of chloroethane with saturated solution of AgCN give ethyl isocyanide as major product.

AgCN: Covalent

Reason : Cyanide ion (CN^-) is an ambident nucleophile.

(A) Both assertion and reason are Correct, and

reason is the correct explanation of the assertion.

(B) Both assertion and reason are True, but

reason is not the correct explanation of the assertion.

(C) Assertion is Incorrect, but reason is Correct.

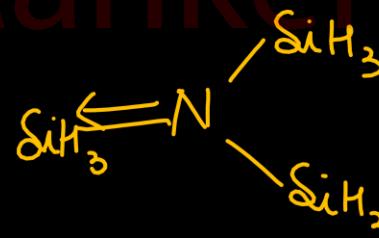
(D) Both assertion and reason are Incorrect.

2

The shape with respect to central atom of the following molecules are in the order



- (A) Planar, Pyramidal, Planar
- ~~(B) Planar, Pyramidal, Pyramidal~~
- (C) Pyramidal, Pyramidal, Tetrahedral
- (D) Pyramidal, Planar, Pyramidal



$\text{N}(\text{SiH}_3)_3 = \text{Back bonding}$

$\text{N}(\text{CH}_3)_3 = \text{No Back bonding bcz}$
of absence of vacant orbital
in C



$3p \text{ } \text{P}(\text{SiH}_3)_3 = \text{No BB both orb. are } 3p$



3

Which of the following statement is incorrect?

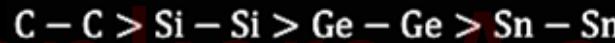
- (A) The melting point of hydrides of Group 15:



- (B) ΔH_{eg} : S > Se > O > Te > Po

- (C) Melting point: B > Al > Tl > In > Ga

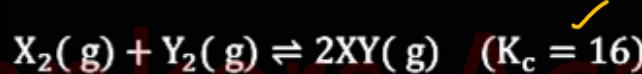
- (D) Bond enthalpy:



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4

2 mol of $X_2(g)$ contained in a flask of volume 2 L and at $27^\circ C$ is allowed to react with 2 mol of $Y_2(g)$ contained in another flask of volume 6 L and at $27^\circ C$, after connecting the two flasks by a narrow glass tube, the reaction proceeds as follows:



The concentration of $XY(g)$ at equilibrium is

- (A) 0.33 M
- (B) 0.44 M
- (C) 0.66 M
- (D) 1.00 M



$$\begin{array}{ccc} 2 & 2 & - \\ \frac{2-x}{8} & \frac{2-x}{8} & \frac{2x}{8} \end{array}$$

$$K_c = 16 = \frac{\left(\frac{2x}{8}\right)^2}{\left(\frac{2-x}{8}\right)^2}$$

$$\frac{2x}{8} = 4$$

$$\frac{2-x}{8} \quad \text{so } x = 4/3$$

$$[XY] = 0.33M$$

5

The transition metal (M) complex that can have all isomers (geometric, linkage, and ionization) is

- (A) $[M(NH_3)_4Br_2]SCN$
- (B) $[M(NH_3)_4Cl_2]Br$
- (C) $[M(NH_3)_4(H_2O)_2]Cl_3$
- (D) $[M(NH_3)_4(H_2O)_2](SCN)$

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For linkage iso : Ambidentate ligand has to be there .

6

An atomic orbital is given by

$nr^2 e^{-\frac{r}{3a_0}} (3\cos^2 \theta - 1)$ in hydrogen atom. The orbital represents:

(A) 2p

(B) 3p

(C) 3d

(D) 4d

$$\ell = 2$$

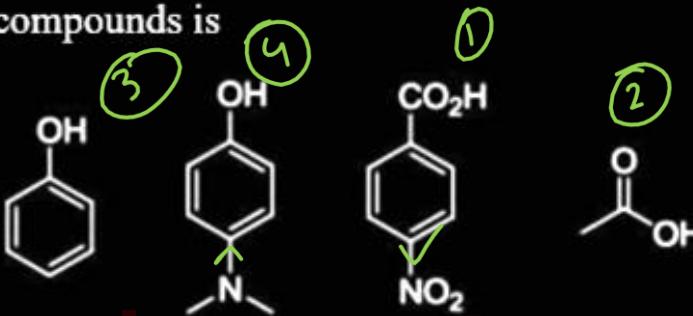
$$n - \ell - 1 = 0$$

$$n = 3$$

3d

7

The correct order of pKa for the following compounds is



$$\text{Acidity} \propto \frac{1}{P_{\text{Ka}}}.$$

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- (A) II > I > III > IV
 (B) II > I > IV > III
 (C) III > IV > I > II
 (D) IV > II > I > III

$$\begin{aligned} K_a: & \text{ III} > \text{IV} > \text{I} > \text{II} \\ P_{K_a}: & \text{ II} > \text{I} > \text{IV} > \text{III} \end{aligned}$$

8

The first ionization energy for which of the following elements is highest?

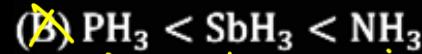
- (A) $T\ell$ (B) Ga
 (C) $A\ell$ (D) In

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B > Te > Zn > Al > In

9

Which is the correct order of bond angle?



→ B.A. Increases down the

group because of increase
in electronegativity.



10

The maximum oxidation state of an element among Cr, Mn, Fe, CO, Ni which has the most negative standard electrode potential M^{2+}/M

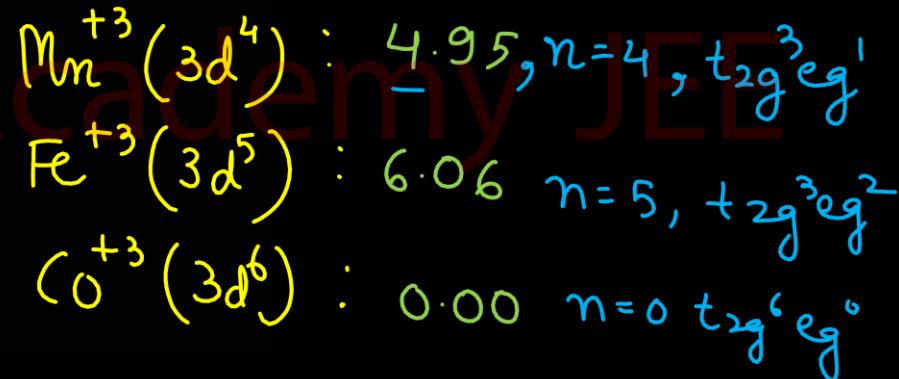
will be

Element	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	
Atomic number	21	22	23	24	25	26	27	28	29	30	
Electronic configuration											
M	$3d^1 4s^2$	$3d^2 4s^2$	$3d^3 4s^2$	$3d^5 4s^1$	$3d^5 4s^2$	$3d^6 4s^2$	$3d^7 4s^2$	$3d^8 4s^2$	$3d^{10} 4s^1$	$3d^{10} 4s^2$	
M^+	$3d^1 4s^1$	$3d^2 4s^1$	$3d^3 4s^1$	$3d^5$	$3d^5 4s^1$	$3d^6 4s^1$	$3d^7 4s^1$	$3d^8 4s^1$	$3d^{10}$	$3d^{10} 4s^1$	
M^{2+}	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	$3d^8$	$3d^9$	$3d^{10}$	
M^{3+}	[Ar]	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	-	-	
Enthalpy of atomisation, $\Delta_a H^\ominus / \text{kJ mol}^{-1}$	326	473	515	397	281	416	425	430	339	126	
Ionisation enthalpy/ $\Delta_i H^\ominus / \text{kJ mol}^{-1}$											
$\Delta_i H^\ominus$	I	631	656	650	653	717	762	758	736	745	906
$\Delta_i H^\ominus$	II	1235	1309	1414	1592	1509	1561	1644	1752	1958	1734
$\Delta_i H^\ominus$	III	2393	2657	2833	2990	3260	2962	3243	3402	3556	3837
Metallic/ionic radii/pm	M	164	147	135	129	137	126	125	125	128	137
M ²⁺	-	-	79	82	82	77	74	70	73	75	
M ³⁺	73	67	64	62	65	65	61	60	-	-	
Standard electrode potential E^\ominus / V	M^{2+}/M	-	-1.63	-1.18	-0.90	-1.18	-0.44	-0.28	-0.25	+0.34	-0.76
	M^{3+}/M^{2+}	-	-0.37	-0.26	-0.41	+1.57	+0.77	+1.97	-	-	-
Density/g cm ⁻³	3.43	4.1	6.07	7.19	7.21	7.8	8.7	8.9	8.9	7.1	

11

The observed magnetic moment of octahedral Mn^{3+} , Fe^{3+} and Co^{3+} complexes are 4.95, 6.06 and 0.00 BM , respectively. The correct option for the electronic configuration of Mn^{3+} , Fe^{3+} and Co^{3+} metal ion in these complexes, respectively, is

- (A) $t_{2g}^4 e_g^0$, $t_{2g}^3 e_g^2$ and $t_{2g}^4 e_g^2$
- (B) $t_{2g}^3 e_g^1$, $t_{2g}^5 e_g^0$ and $t_{2g}^6 e_g^0$
- ~~(C) $t_{2g}^3 e_g^1$, $t_{2g}^3 e_g^2$ and $t_{2g}^6 e_g^0$~~
- (D) $t_{2g}^2 e_g^1$, $t_{2g}^3 e_g^2$ and $t_{2g}^4 e_g^2$

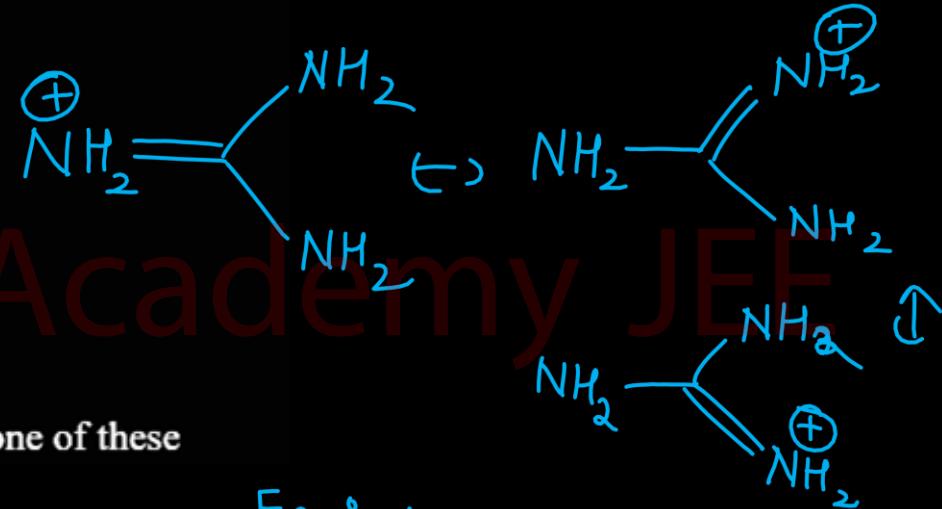


12

Which nitrogen is protonated readily in the Guanidine?

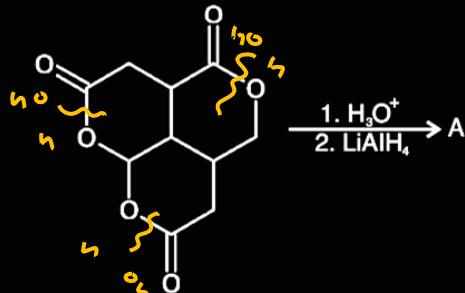


- (A) 1
- (B) 2
- (C) 3
- (D) None of these

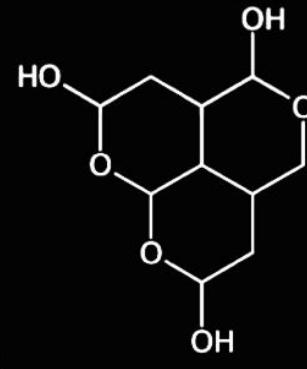
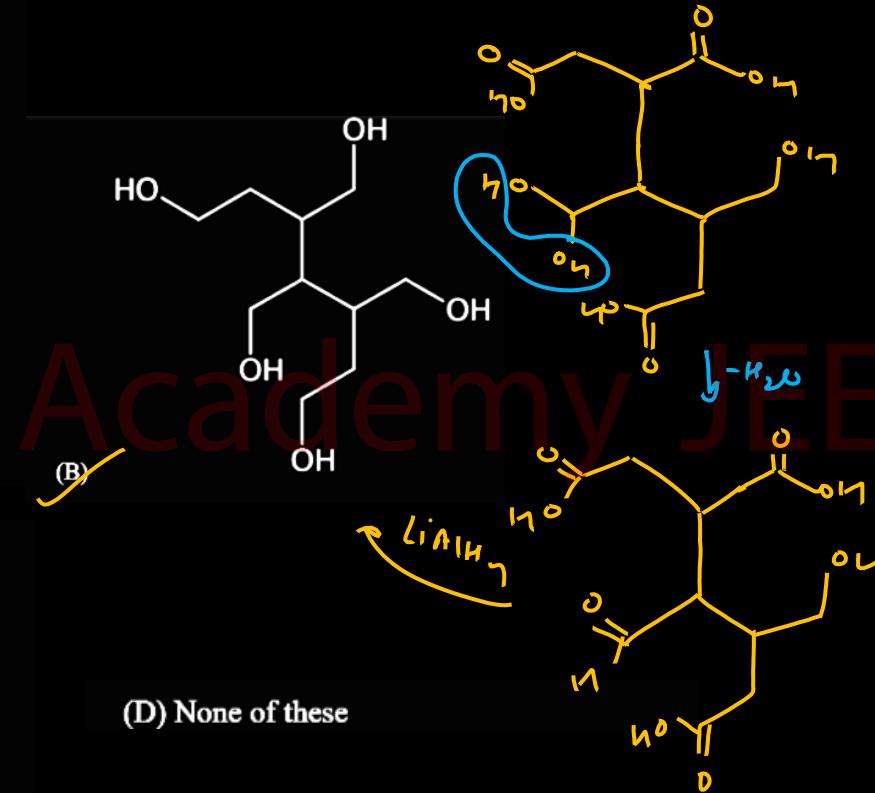
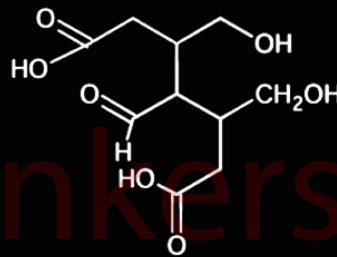


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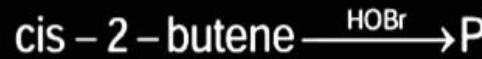
13



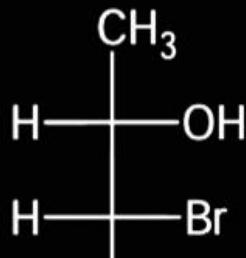
Product A is



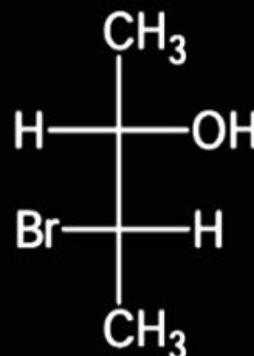
14



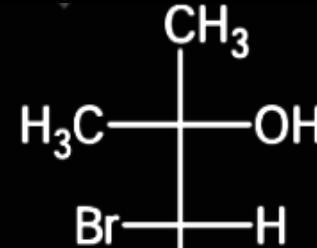
Product P is



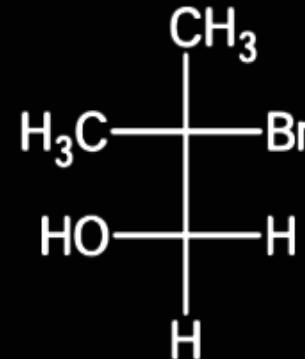
(A)



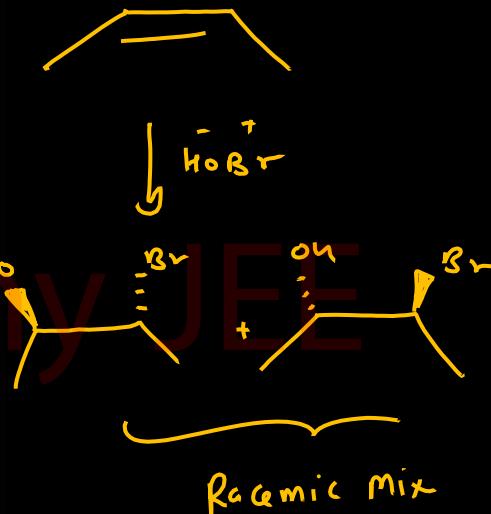
(B) ✓



(C)

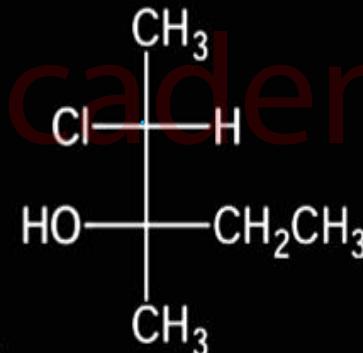
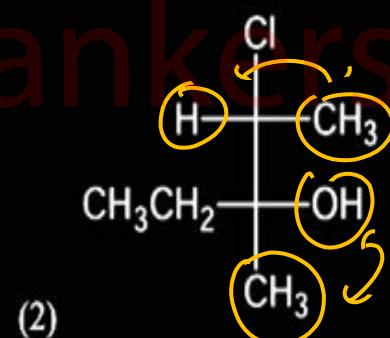
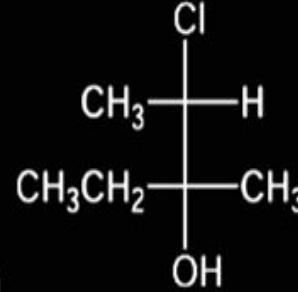
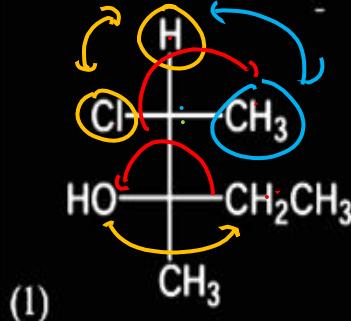


(D)



15

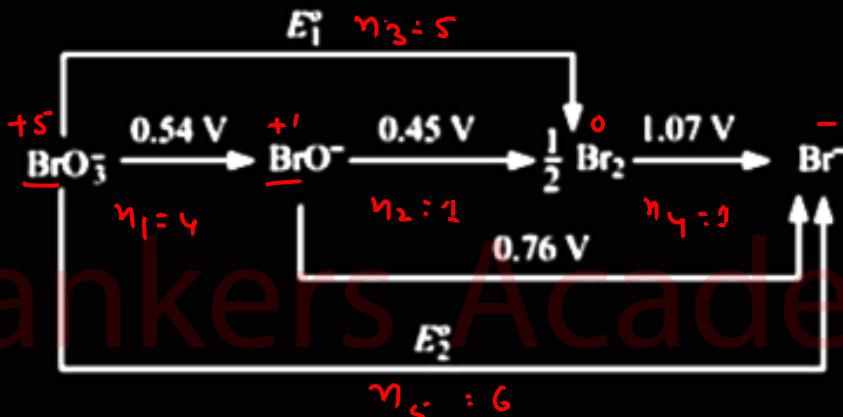
Which of the following structure are
superimposable?



- (A) 1 and 2 (*enantiomer*) (B) 2 and 3 (*enantiomer*)
 (C) 1 and 4 (*diastereomer*) (D) 1 and 3

16

From the standard potential shown in the following diagram, calculate the potentials E_1° and E_2°



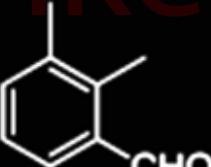
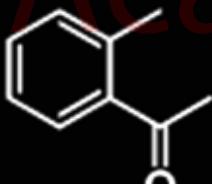
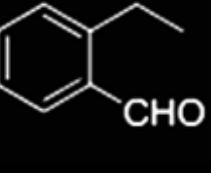
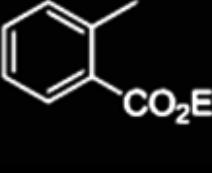
- (A) $E_1^\circ = 0.52 \text{ V}, E_2^\circ = 0.61 \text{ V}$
 (B) $E_1^\circ = 0.52 \text{ V}, E_2^\circ = 0.52 \text{ V}$
 (C) $E_1^\circ = 0.61 \text{ V}, E_2^\circ = 0.79 \text{ V}$
 (D) $E_1^\circ = 0.44 \text{ V}, E_2^\circ = 0.88 \text{ V}$

$$\begin{aligned} E_1^\circ &= \frac{(\eta_1 \times 0.54) + (\eta_2 \times 0.45)}{\eta_3} \\ &= \frac{(4 \times 0.54) + (1 \times 0.45)}{5} \\ &= 0.52 \text{ V} \end{aligned}$$

$$\begin{aligned} E_2^\circ &= \frac{(0.54 \times 4) + (0.45 \times 1) + (1.07 \times 1)}{6} \\ E_2^\circ &= 0.61 \text{ V} \end{aligned}$$

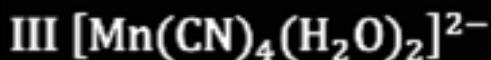
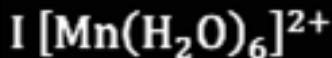
17

An organic compound on reaction with 2,4-dinitrophenylhydrazine (2,4-DNP) gives a yellow precipitate. It also gives silver mirror on reaction with ammoniacal AgNO_3 . It gives an alcohol and sodium salt of a carboxylic acid on reaction with concentrated NaOH . It yields benzene-1,2dicarboxylic acid on heating with alkaline KMnO_4 . The structure of the compound among the following is

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- (A) 
- (B) 
- (C) 
- (D) 

18

Predict the order of Δ_0 for the following compounds



(A) Δ_0 (I) < Δ_0 (II) < Δ_0 (III)

(B) Δ_0 (II) < Δ_0 (I) < Δ_0 (III)

(C) Δ_0 (III) < Δ_0 (II) < Δ_0 (I)

(D) Δ_0 (I) < Δ_0 (III) < Δ_0 (II)

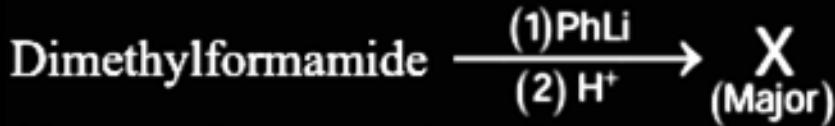
$\Delta_0 \propto \Sigma_{\text{eff}} \propto \text{charge}$

\propto strength of ligand.

($\text{CN} > \text{H}_2\text{O}$)

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19



(degree of unsaturation=5)

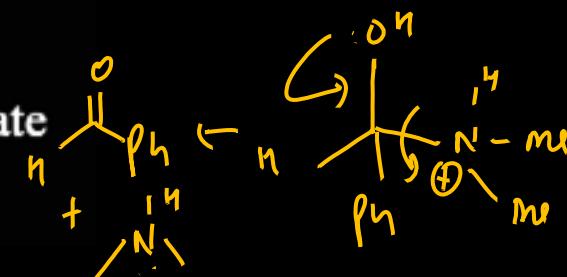
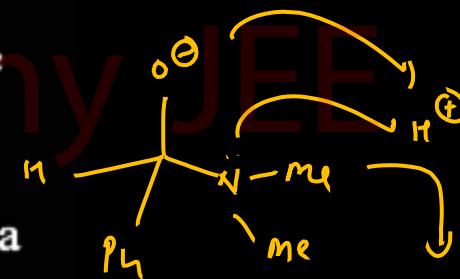
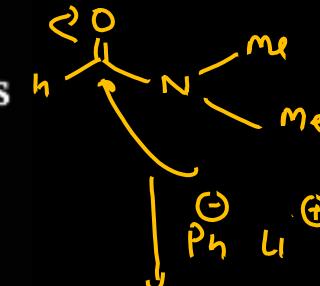
Which of the following statement about X is correct?

(A) X on oxidation gives ketone.

(B) X reacts with iodoform reagent to produce an intense yellow precipitate.

(C) X reacts with Tollen's reagent, producing a silver mirror as an observable product.

(D) X on reaction with ceric ammonium nitrate forms a red colour precipitate.



20

When 2 g non-volatile hydrocarbon containing

94.4 % carbon by mass is dissolved in 100 g of benzene, the vapour pressure of benzene at 30°C is lowered from 89.78 mm Hg to 89.0 mm Hg . The molecular formula of the hydrocarbon is

- (A) $C_{12}H_{34}$
 (C) $C_{14}H_{12}$

(B) $C_{13}H_{22}$

\checkmark (D) $C_{14}H_{10}$

$$\frac{0.78}{89.78} = \frac{78n}{78n + 100}$$

$$\frac{1}{89.78} = \frac{n}{78n + 100}$$

$$78n + 100 = 89.78n$$

$$100 = 89.78n$$

$$n = \frac{1}{89}$$

$$\text{Mol. wt } C_xH_y = 89 \times 2 = 178$$

$$x = \frac{178 \times 94.4}{100} = 14$$

$$\frac{1}{T_2}$$

$$y = \frac{178 \times 5.6}{100} = 10$$

$$\frac{P^o - P_s}{P^o} = \frac{n}{n+N}$$

$$\frac{(89.78 - 89)}{89.78} = \frac{n}{n + \frac{100}{78}}$$

21

In Duma's method for estimation of nitrogen, 0.1 gram an organic compound gave 76 ml of nitrogen collected at 700 K and 714 mm of Hg pressure. The percentage composition of nitrogen in the compound is ____

[Round off the nearest integer] [Given aqueous

tension at 700 K is 14 mm of Hg] [R = 0.08 L atm mol⁻¹ K⁻¹]

$$\text{Mole of N}_2 \text{ gas} = \frac{P_{N_2} V_{N_2}}{RT} = \left\{ \frac{(714 - 14)}{760} \right\} \times \left(\frac{76}{1000} \right) \times \frac{1}{0.08 \times 700}$$

$$\text{wt of Nitrogen} = \frac{28 \text{ Mole}}{\omega} \times 100 = \frac{1 \times 10^{-4}}{0.08} \times \frac{28}{0.1} \times 100$$

$\eta \rightarrow$ mole of N_2 gas

$$= \frac{28}{8} \times 10^{-4}$$

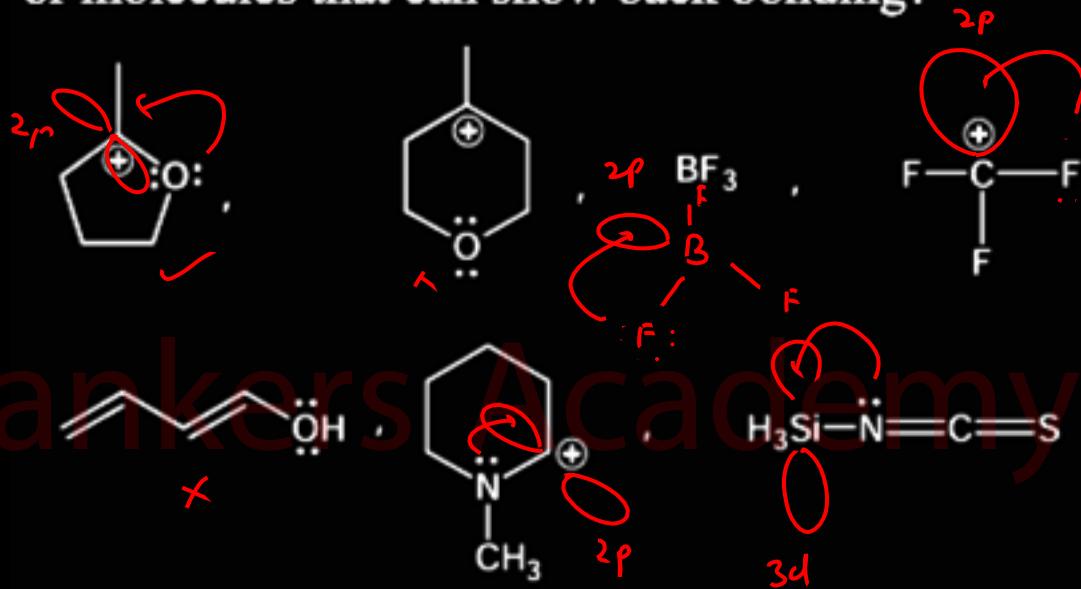
$$\text{Mole of } N_2 = \frac{1}{0.08} \times 10^{-4}$$

$\omega \rightarrow$ wt of O.C.

$$\approx 35.1$$

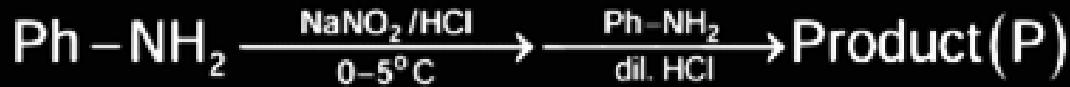
22

Among the following, find out number of ions or molecules that can show back bonding?



(5)

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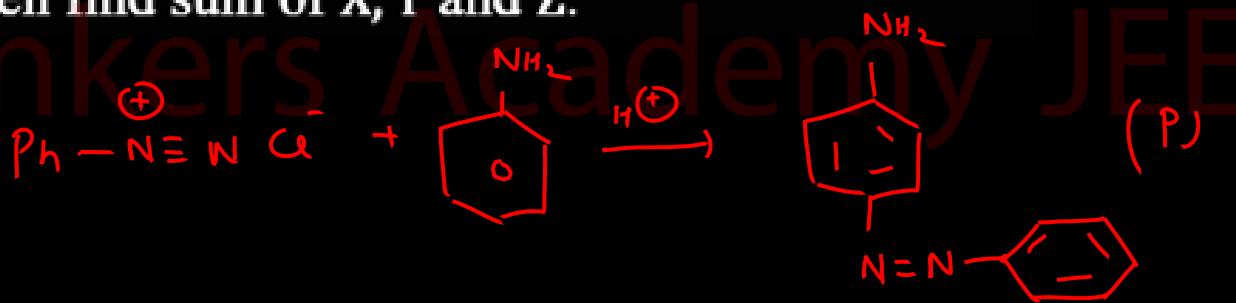


If X = Number of nitrogen atoms in the P

Y = Number of stereoisomers of P

Z = Degree of unsaturation of P

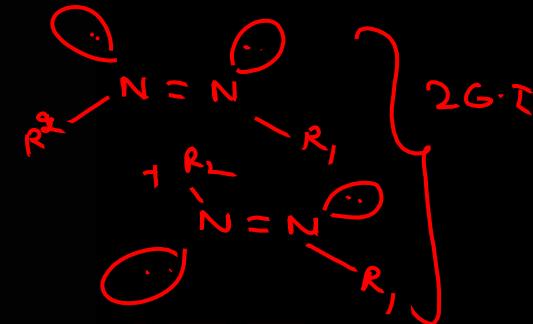
Then find sum of X, Y and Z.



$$x = 3 \quad y = 2$$

z = 9

14



24

Carbon monoxide reacts with O_2 to form CO_2 : $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$ information on this reaction is given in the table below.

$[CO]$ mol/L	$[O_2]$ mol/L	Rate of reaction (mol/L. min)
0.02	0.02	4×10^{-5}
0.04	0.02	1.6×10^{-4}
0.02	0.04	8×10^{-5}

What is the value for the rate of constant for the reaction in proper related unit?

$$\text{rate} = K [CO]^x [O_2]^y$$

$$x=2$$

$$y=1$$

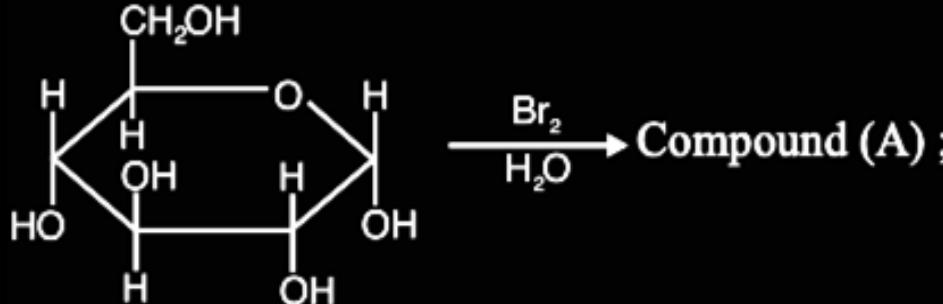
$$\therefore \text{rate} = K [CO]^2 [O_2]$$

$$4 \times 10^{-5} = K [0.02]^2 [0.02]$$

$$K = \frac{4 \times 10^{-5}}{8 \times 10^{-6}} = \frac{40}{8} = 5$$

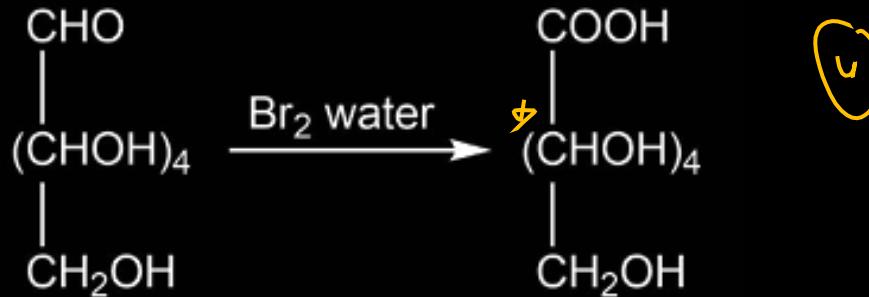
25

Number of chiral centre in compound A is



Sol.

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MATHEMATICS

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$$21 = (A + \eta)^2 + \kappa^2 \text{ and}$$

Let $f(x) = \frac{1}{\sin^2 x - (x-a)^2}$, $x \in [0,1]$ and $a \in [2,3]$,

then which of the following is correct?

- (A) $f(x)$ always increases in $(0,1)$
- (B) $f(x)$ always decreases in interval $(0,1)$
- (C) there exists a point of local minima in $(0,1)$
- (D) there exists a point of local maxima in $(0,1)$

$$f'(x) = \frac{-1}{(\sin^2 x - (x-a)^2)^2} \cdot [2 \sin x \cos x - 2(x-a)]$$

$$\begin{aligned}
 &= (-ve) \left[\underbrace{\sin 2x - 2x}_{-ve} + \underbrace{\frac{2a}{1}}_{\in [4,6]} \right] \\
 &\quad \underbrace{+ ve}_{+ve}
 \end{aligned}$$

2

Let A and B be 3×3 matrices and P =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, B = PAP, \text{ then } \lim_{n \rightarrow \infty} \frac{\text{tr}(A^n + PB^nP)}{\text{tr}(A^n)}$$

is equal to (where $|A^n| \neq 0$, tr represents trace of square matrix)

- (A) 1
(C) 3

- ~~(B) 2~~
(D) 4

$$\begin{aligned} B^n &= (PAP)(PAP)(PAP)\dots(PAP) \\ &= P \boxed{A P^2} A \boxed{P^2} \dots \boxed{P^2} A P \\ \boxed{B^n = P A^n P} \\ P^{-1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \\ P B^n P &= \underline{P} \underline{(P A^n P)} \underline{P} = A^n \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{\text{tr}(A^n + A^n)}{\text{tr}(A^n)} = \frac{\text{tr}(2A^n)}{\text{tr}(A^n)} = 2 \frac{\text{tr}(A^n)}{\text{tr}(A^n)} = 2$$

3

Given the A.P sequence of numbers $a_1, a_2, \dots, a_{1013}$ which satisfy $\frac{a_1}{a_1+1} = \frac{a_2}{a_2+3} = \frac{a_3}{a_3+5} = \dots = \frac{a_{1013}}{a_{1013}+2025}$ also $\underline{a_1 + a_2 + \dots + a_{1013} = 2026}$, then 41th term of the sequence is equal to

(A) $\frac{166}{1013}$

(B) $\frac{162}{1013}$

(C) $\frac{160}{1013}$

(D) $\frac{157}{1013}$

$$\frac{a_1}{a_1+1} = \frac{a_2}{a_2+3} = \frac{a_3}{a_3+5} = \dots = \frac{a_{1013}}{a_{1013}+2025} = K$$

$$\Rightarrow \frac{a_1}{a_1+1} = K$$

$$\Rightarrow \frac{a_1+1}{a_1} = \frac{1}{K}$$

$$\Rightarrow 1 + \frac{1}{a_1} = \frac{1}{K}$$

$$\Rightarrow \frac{1}{a_1} = \frac{1}{K} - 1$$

$$a_1 = \frac{K}{1-K}$$

$$a_2 = \frac{3K}{1-K}$$

$$a_3 = \frac{5K}{1-K}$$

$$a_{1013} = \frac{2025K}{1-K}$$

$$a_{41} = 81 \left(\frac{K}{1-K} \right)$$

$$= 81 \left(\frac{4}{2026} \right) = \frac{162}{1013}$$

~~$$2026 = \frac{1013}{2} (a_1 + a_{1013})$$~~

$$4 = a_1 + a_{1013}$$

$$4 = \frac{K}{1-K} + \frac{2025K}{1-K}$$

$$\Rightarrow 4 = \frac{2026K}{1-K} \Rightarrow \frac{K}{1-K} = \frac{4}{2026}$$

$$\Rightarrow 4 - 4K = 2026K$$

$$\Rightarrow 4 = 2030K$$

$$\Rightarrow K = \frac{2}{1015}$$

Number of solution of the equation

$$\tan^{-1} \left(\frac{x}{1+\sqrt{1-x^2}} \right) + \sin \left(2\tan^{-1} \sqrt{\frac{1-x}{1+x}} \right) = \sqrt{1-x^2}$$

is

- (A) 0
(B) 1
(C) 2
(D) 3

$$\tan \theta = \sqrt{\frac{1-x}{1+x}}$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$= \frac{2 \sqrt{\frac{1-x}{1+x}}}{1 + \frac{1-x}{1+x}}$$

$$= \sqrt{1-x^2}$$

$\underbrace{}_{\theta}$

(B)

(D)

$$\tan^{-1} \left(\frac{x}{1+\sqrt{1-x^2}} \right) + \cancel{\sqrt{1-x^2}} = \cancel{\sqrt{1-x^2}}$$

$$\tan^{-1} \left(\frac{x}{1+\sqrt{1-x^2}} \right) = 0$$

$$\Rightarrow \frac{x}{1+\sqrt{1-x^2}} = 0$$

\Rightarrow

$$\boxed{x=0}$$

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5

The value of

$$\lim_{n \rightarrow \infty} \left(\frac{\sin \left\{ \frac{2}{n} \right\}}{\left[2 \tan \frac{1}{n} \right] \left(\tan \frac{1}{n} \right)} + \frac{1}{n^2 + \cos n} \right)^{n^2} \text{ where } [.] = \text{GIF and } \{ \} = \text{FPF, is}$$

(A) 1

(B) 2

(C) 3

(D) 0

$$\left(\frac{\left\{ \frac{2}{n} \right\} = \frac{2}{n}}{\left(2 \tan \frac{1}{n} \right) \left(\frac{1}{n} \right)} + 0 \right)^{\infty} \equiv 1^{\infty}$$

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$$\lim_{n \rightarrow \infty} n^2 \left[\frac{\sin \left(\frac{2}{n} \right)}{\left[2 \tan \frac{1}{n} \right] \tan \frac{1}{n}} + \frac{1}{n^2 + \cos n} - 1 \right]$$

$$= e^{n^2 \left[\frac{1}{n^2 + \cos n} + \frac{2 \sin \left(\frac{2}{n} \right) \cos^2 \left(\frac{1}{n} \right)}{\left[2 \tan \frac{1}{n} \right] \sin^2 \left(\frac{1}{n} \right)} - 1 \right]}$$

$$= e^0 = 1$$

6

If $f(x) = ||\sin(|x| - 1)| - 2|$ then

- (A) $f(x)$ is continuous at $x = 2$
- (B) ~~$f(x)$ is differentiable at $x = 2$~~
- (C) $f(x)$ is non-differentiable $x = 2$
- (D) $f(x)$ is non-differentiable at $x = 0$

$$|x| = x$$

$$f(x) = \left| \underbrace{|\sin(x-1)|}_{\in (0,1)} - \frac{2}{2} \right|$$

$$f(x) = 2 - |\sin(x-1)|$$

$$\text{At } x=2 \longrightarrow$$

$$2 - \sin 1$$

$$\text{At } x=0 \longrightarrow$$

$$2 - \sin 1$$

At $x=2$

$$f(x) = 2 - \sin(x-1)$$

$$f'(x) = -\cos(x-1)$$

$$= -\cos 1$$

7

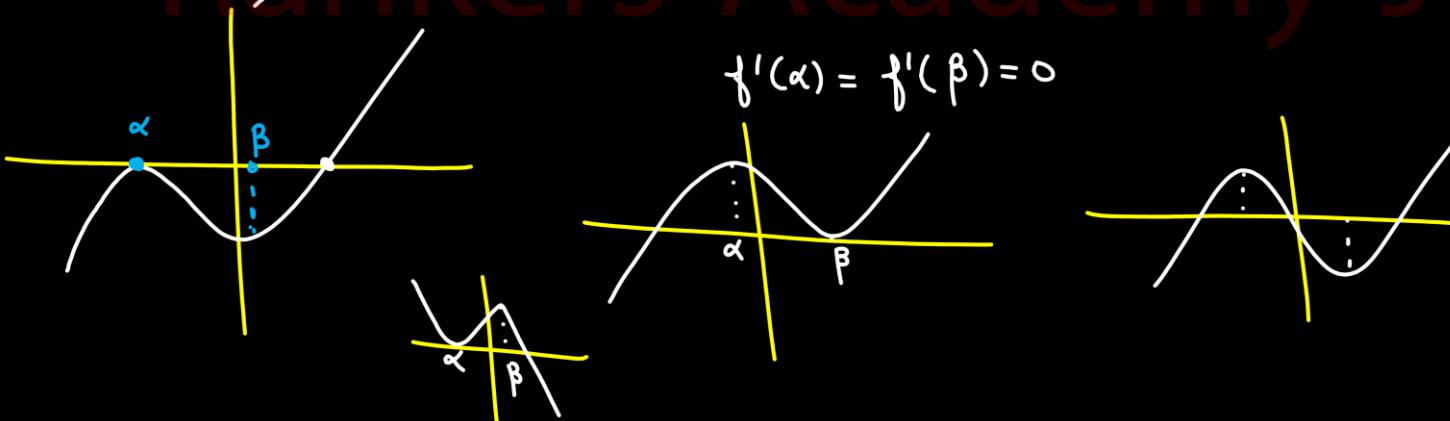
Let $f(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial ($a, b, c, d \in \mathbb{R}$). If $f(\alpha)f(\beta) = 0$ where α and β are the distinct real roots of $f'(x) = 0$, then

(A) $f(x) = 0$ has all three different real roots

~~(B)~~ $f(x) = 0$ has three real roots but two of them are equal

~~(C)~~ $f(x) = 0$ has only one real root

~~(D)~~ all three roots of $f(x) = 0$ are real and equal



8

The value of $\sum_{r=0}^{20} r(20-r)({}^{20}C_r)^2$ is equal to

- (A) $400 \cdot {}^{39}C_{20}$ (B) $400 \cdot {}^{40}C_{19}$
 (C) $400 \cdot {}^{39}C_{19}$ (D) $400 \cdot {}^{38}C_{20}$

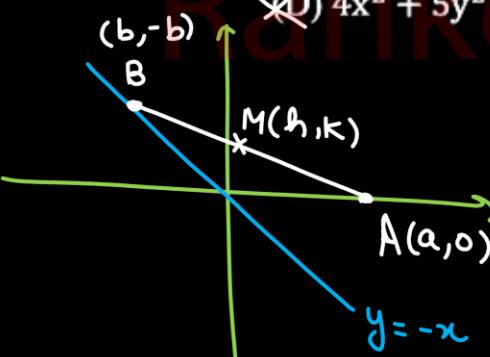
$$\begin{aligned}
 & \sum_{r=0}^{20} r(20-r) \cdot ({}^{20}C_r)^2 \\
 &= \sum_{r=0}^{20} r(20-r) \cdot {}^{20}C_r \cdot {}^{20}C_r \\
 &= \sum_{r=0}^{20} r \cdot (20-r) \cdot \frac{20}{r} {}^{19}C_{r-1} \cdot {}^{20}C_{20-r} \\
 &= \sum_{r=0}^{20} (20-r) \cdot 20 \cdot {}^{19}C_{r-1} \cdot \frac{20}{20-r} {}^{19}C_{19-r}
 \end{aligned}$$

$$\begin{aligned}
 &= 400 \sum {}^{19}C_{r-1} \cdot {}^{19}C_{19-r} \\
 &= 400 \cdot {}^{38}C_{18} \\
 &= 400 \cdot {}^{38}C_{20}
 \end{aligned}$$

9

A line of fixed length 2 units moves so that its one end is on the positive x-axis and other end on that part of the line $x + y = 0$ which lies in the second quadrant. The locus of the mid-point of the line is given by

- (A) $x^2 + 5y^2 + 4xy - 1 = 0$
 (B) $x^2 + 5y^2 + 4xy + 1 = 0$
 (C) $x^2 + 5y^2 - 4xy - 1 = 0$
 (D) ~~$4x^2 + 5y^2 + 4xy + 1 = 0$~~



$$\begin{aligned} (a-b)^2 + b^2 &= 2^2 \\ (2h+2k+2k)^2 + 4k^2 &= 4 \quad | \quad \text{Divide by } 4 \\ (h+2k)^2 + k^2 &= 1 \\ h^2 + 5k^2 + 4hk - 1 &= 0 \end{aligned}$$

$$\begin{aligned} \frac{a+b}{2} &= h \Rightarrow a = 2h - b \\ a &= 2h + k \end{aligned}$$

$$\frac{-b+0}{2} = k \Rightarrow b = -2k$$

$$x^2 + 5y^2 + 4xy - 1 = 0$$

10

If $\alpha \neq \beta$ but $\alpha^2 = 2\alpha - 3$; $\beta^2 = 2\beta - 3$ then the

equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

- (A) $2x^2 + 3x + 2 = 0$
- ~~(B)~~ $3x^2 + 2x + 3 = 0$
- (C) $2x^2 - 3x + 2 = 0$
- (D) $3x^2 - 2x + 3 = 0$

$$\alpha^2 - 2\alpha + 3 = 0$$

$$\beta^2 - 2\beta + 3 = 0$$

$$\boxed{\alpha^2 - 2\alpha + 3 = 0 < \beta^2 - 2\beta + 3 = 0}$$

$$\alpha\beta = 3$$

Roots $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$\frac{\alpha^2}{\alpha\beta}, \frac{\beta^2}{\alpha\beta}$$

Roots $= \frac{\alpha^2}{3}, \frac{\beta^2}{3}$

$$y = \frac{x^2}{3} \Rightarrow x = \sqrt{3y}$$

$$3y - 2\sqrt{3y} + 3 = 0$$

$$\Rightarrow 3y + 3 = 2\sqrt{3y}$$

$$\Rightarrow 9y^2 + 9 + 18y = 4(3y)$$

$$\Rightarrow 9y^2 + 6y + 9 = 0$$

$$\Rightarrow 3y^2 + 2y + 3 = 0$$

$$\Rightarrow 3x^2 + 2x + 3 = 0$$

11

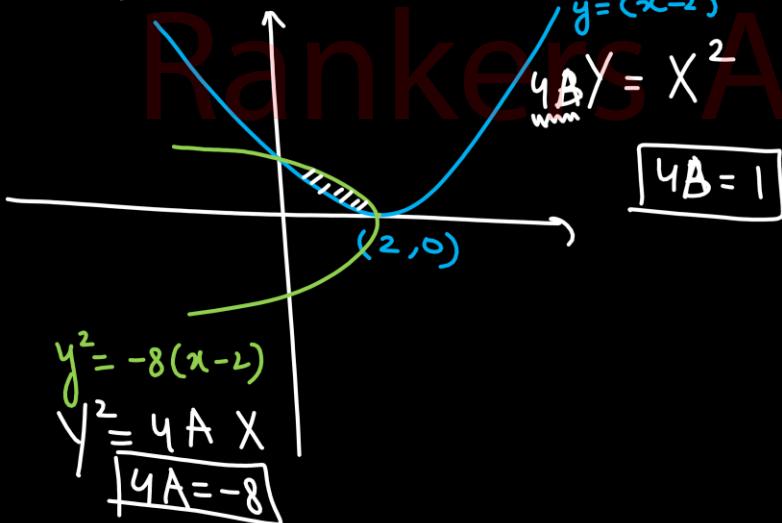
The area of the region enclosed by the curves

$y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is :

- (A) $\frac{8}{3}$
- (B) $\frac{4}{3}$
- (C) 8
- (D) 5

$$y = (x-2)^2$$

$$y^2 = -8(x-2)$$



$$\text{Ans: } \left| \frac{(4A)(4B)}{3} \right| \\ = \frac{8 \times 1}{3} = \frac{8}{3}$$

12

Let $I(x) = \int \frac{dx}{(x-11)^{11/13}(x+15)^{15/13}}$. If $I(37) - I(24)$

$$I(24) = \frac{1}{4} \left(\frac{1}{b^{1/13}} - \frac{1}{c^{1/13}} \right), b, c \in \mathbb{N}, \text{ then } 3(b+c)$$

is

- (A) 22
 (B) 39
 (C) 40
 (D) 26

$$I = \int \frac{dx}{\left(\frac{x-11}{x+15}\right)^{11/13} (x+15)^{15/13} (x+15)^{11/13}}$$

$$= \int \frac{dx}{\left(\frac{x-11}{x+15}\right)^{11/13} \left(\frac{x+15}{x-11}\right)^{15/13}}$$

$$\text{Let } \frac{x-11}{x+15} = t$$

$$\frac{(x+15)-(x-11)}{(x+15)^2} dx = dt$$

- ~~(B) 39~~
 (D) 26

$$\frac{26}{(x+15)^2} dx = dt$$

$$I = \frac{1}{26} \int \frac{dt}{t^{11/13}} = \frac{1}{26} \int t^{-11/13} dt$$

$$= \frac{1}{26} \frac{t^{2/13}}{\frac{2}{13}} + C$$

$$= \frac{1}{4} \left(\frac{t^{2/13}}{t+15} \right) + C$$

$$I(37) - I(24)$$

$$= \frac{1}{4} \left(\frac{37-11}{37+15} \right)^{2/13} - \frac{1}{4} \left(\frac{24-11}{24+15} \right)^{2/13}$$

$$= \frac{1}{4} \left[\left(\frac{1}{2} \right)^{2/13} - \left(\frac{1}{3} \right)^{2/13} \right]$$

$$= \frac{1}{4} \left[\left(\frac{1}{4} \right)^{1/13} - \left(\frac{1}{9} \right)^{1/13} \right]$$

$$b = 4$$

$$c = 9$$

$$3(b+c) = 39$$

13

Let all letters of word 'MATHEMATICS' are arranged in all possible order. Three events A, B and C are defined as :

A : Both M are together

$$P(A) = \frac{\frac{10!}{2!2!} \times 1}{\frac{11!}{2!2!2!}} = \frac{2}{11} = P(B) = P(C)$$

B: Both T are together

$$P(A \cap B) = \frac{\frac{9!}{2!} \times 1 \times 1}{\frac{11!}{2!2!2!}} = \frac{4}{10 \times 11} = \frac{2}{55}$$

$$= P(B \cap C) = P(A \cap C)$$

C: Both A are together

Which of the following does not hold good?

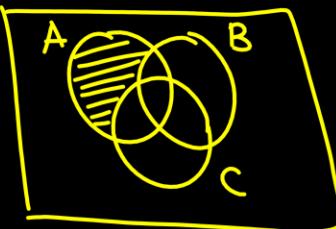
(A) $P(A) = P(B) = \frac{2}{11}$ ✓

(B) $P(A \cap B) = P(B \cap C) = P(C \cap A) = \frac{2}{55}$ ✓

(C) $P(A \cap B \cap C) = \frac{2}{495}$ ✗

(D) $P((A \cap \bar{B}) \mid \bar{C}) = \frac{58}{405}$ ✓

M	M
A	A
↑	T
H	
E	
I	
C	
S	
/ /	



$$P(A \cap \bar{B} \mid \bar{C}) = \frac{P(A \cap \bar{B} \cap \bar{C})}{P(\bar{C})} = \frac{P(A) - P(A \cap B)}{1 - \frac{2}{11}}$$

$$\frac{58}{405} = \frac{(90 - 18 - 18 + 4)}{\frac{495 \times 9}{45}} \times \frac{1}{11}$$

$$= \frac{2}{11} - \frac{2}{55} - \frac{2}{55} + \frac{4}{495}$$

$$= \frac{1}{11}$$

14

A differentiable function $f(x)$ satisfies the

equation $(x+1)f'(x) - 2(x^2+x)f(x) = \frac{e^{x^2}}{x+1}$. If

$f(0) = 5$, then $f(x) =$

(A) $\left(\frac{3x+5}{x+1}\right)e^{x^2}$

(B) $\left(\frac{6x+5}{x+1}\right)e^{x^2}$

(C) $\left(\frac{6x+5}{(x+1)^2}\right)e^{x^2}$

(D) $\left(\frac{5-6x}{x+1}\right)e^{x^2}$

~~$$(x+1) \frac{dy}{dx} - 2x(x+1)y = \frac{e^{-x^2}}{(x+1)^2}$$~~

$$\text{L.F.} = e^{\int -2x dx} = e^{-x^2}$$

$$J(e^{-x^2}) = \int \frac{x^2}{(1+x)^2} \times e^{-x^2} dx$$

$$J(e^{-x^2}) = \frac{-1}{(x+1)} + C$$

$$(0, 5) \uparrow$$

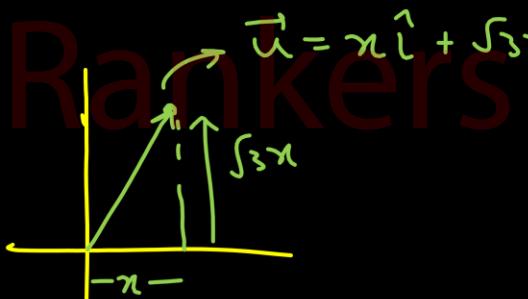
$$5(1) = -1 + C$$

$$\Rightarrow C = 6$$

15

Let \vec{u} be a vector in the $x - y$ plane with slope $\sqrt{3}$. Further $|\vec{u}|, |\vec{u} - \hat{i}|, |\vec{u} - 2\hat{i}|$ are in G.P., \hat{i} being the unit vector along positive x -axis, then $|\vec{u}|$ is equal to

- (A) $\sqrt{3 + 2\sqrt{2}}$ (B) $\sqrt{6 + 2\sqrt{2}}$
✓ (C) $\tan \frac{9\pi}{8}$ (D) $\cot \frac{5\pi}{8}$



$$|\vec{u}| = \sqrt{x^2 + (\sqrt{3}x)^2} = \sqrt{4x^2} = 2x$$



$$|\vec{u} - \hat{i}| = \sqrt{(x-1)^2 + 3x^2} \\ = \sqrt{4x^2 - 2x + 1}$$

$$|\vec{u} - 2\hat{i}| = \sqrt{(x-2)^2 + 3x^2} \\ = \sqrt{4x^2 - 4x + 4}$$

∴ in G.P.

$$(4x^2 - 2x + 1) = (2x)\sqrt{4x^2 - 4x + 4}$$

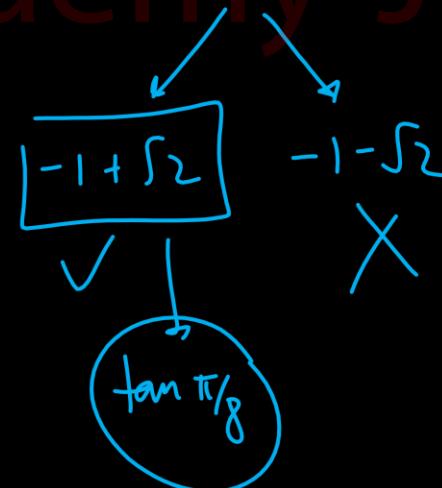
$$(4x^2 - 2x + 1)^2 = (4x^2)(4x^2 - 4x + 4)$$

$$\left(\begin{matrix} 16x^4 + 4x^2 + 1 \\ -16x^3 - 4x + 8x^2 \end{matrix} \right) = \left(\begin{matrix} 16x^4 - 16x^3 + 16x^2 \end{matrix} \right)$$

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$$x = \frac{-4 \pm \sqrt{16+16}}{4 \times 2}$$

$$x = \frac{-1 \pm \sqrt{2}}{2}$$



16

If z_1 & z_2 are two complex numbers satisfying $|z - 4| = \operatorname{Re}(z)$ and having greatest and least argument respectively, then area of triangle formed by origin, z_1 & z_2 is

- (A) 10 sq. units (B) 12 sq. units
~~(C) 16 sq. units~~ (D) 20 sq. units

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$$|(x-4) + iy| = x$$

$$\sqrt{(x-4)^2 + y^2} = x$$

$$x^2 + 16 - 8x + y^2 = x^2$$

$$y^2 = 8x - 16$$

$$y^2 = 8(x-2)$$

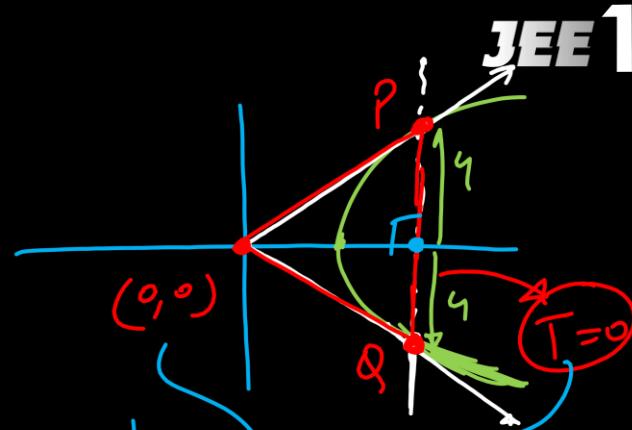
$$y^2 = 8(x-2)$$

$$0 = 8\left(\frac{x}{2} - 2\right)$$

$$x = 4$$

$$\Delta = \frac{1}{2}(8)(4)$$

$$= 16$$



JEE 1

17

Let $f(x)$ be a differentiable function such that

$f(0) = 0$ and $\int_0^2 f'(2t) e^{f(2t)} dt = 5$, then the value of $f(4)$ equals

(A) $2 \ln 3$

(B) $\ln 10$

(C) ~~$\ln 11$~~

(D) $3 \ln 2$

Let:

$$\left\{ \begin{array}{l} f(2t) = u \\ f'(2t) \times 2dt = du \end{array} \right.$$

$$\left\{ \begin{array}{l} e^u \frac{du}{2} = 5 \\ f(0) \end{array} \right.$$

$$(e^u)^{f(4)} = 10$$

$$e^{f(4)} - e^{f(0)} = 10$$

$$e^{f(4)} - 1 = 10$$

$$f(4) = \ln(11)$$

18

$$\text{If lines } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \text{ and } \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

intersect, then the value of k is

(A) $\frac{3}{2}$

(B) $\frac{9}{2}$

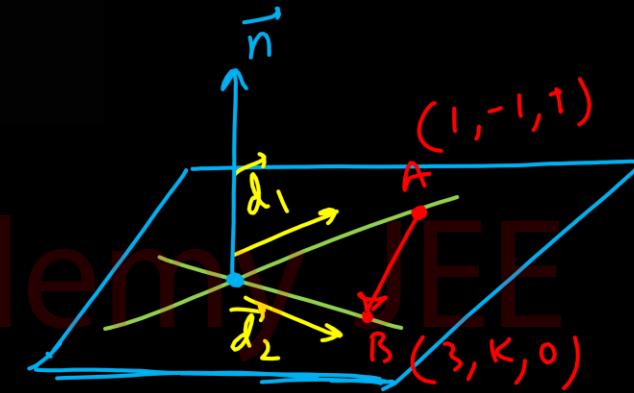
(C) $-\frac{2}{9}$

(D) $-\frac{3}{2}$

M-1: $\vec{n} = \vec{d}_1 \times \vec{d}_2$

$$\overrightarrow{AB} = 2\hat{i} + (k+1)\hat{j} - k\hat{k}$$

$$\overrightarrow{AB} \cdot \vec{n} = 0$$



$$[\overrightarrow{AB} \quad \vec{d}_1 \quad \vec{d}_2] = 0$$

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & \{- \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow k = 9$$

19

C_1 and C_2 are two circles of radii a and b ($a < b$) touching both the coordinate axes and have their centres in the first quadrant. Then which of the following is false?

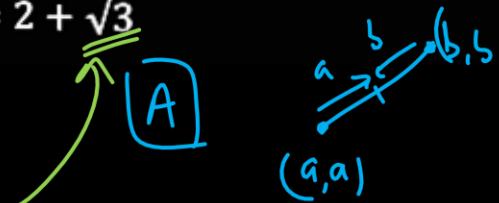
(A) If C_1, C_2 touch each other then $\frac{b}{a} = 3 + 2\sqrt{2}$

(B) If C_1, C_2 are orthogonal then $\frac{b}{a} = 2 + \sqrt{3}$

(C) If C_1, C_2 intersect in such a way that their common chord has maximum length, then $\frac{b}{a} = 3$.

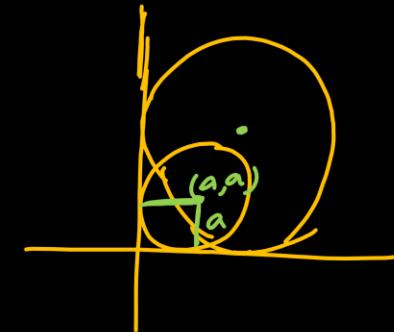
(D) If C_2 passes through the centre of C_1 , then

$$\frac{b}{a} = 2 + \sqrt{3}$$



$$\sqrt{(a-b)^2 + (a-b)^2} = (a+b)$$

$$\sqrt{2}|a-b| = (a+b)$$



$$C_1: (x-a)^2 + (y-a)^2 = a^2$$

$$\left\{ \begin{array}{l} C_1: x^2 + y^2 - 2ax - 2ay + a^2 = 0 \\ C_2: x^2 + y^2 - 2bx - 2by + b^2 = 0 \end{array} \right.$$

$$\left. \begin{array}{l} C_1: x^2 + y^2 - 2ax - 2ay + a^2 = 0 \\ C_2: x^2 + y^2 - 2bx - 2by + b^2 = 0 \end{array} \right\}$$

$$\sqrt{2}(b-a) = a+b$$

$$\cancel{\sqrt{2}b - \sqrt{2}a = a+b}$$

$$(\sqrt{2}-1)b = (\sqrt{2}+1)a$$

$$\frac{b}{a} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$\boxed{\frac{b}{a} = (\sqrt{2}+2\sqrt{2})}$$

B

	a	b	c
C_1	$-a$	$-a$	a^2
C_2	$-b$	$-b$	b^2

$$2a_1a_2 + 2b_1b_2 = c_1 + c_2$$

$$2ab + 2ab = a^2 + b^2$$

$$4ab = a^2 + b^2$$

$$4\left(\frac{b}{a}\right) = 1 + \left(\frac{b}{a}\right)^2$$

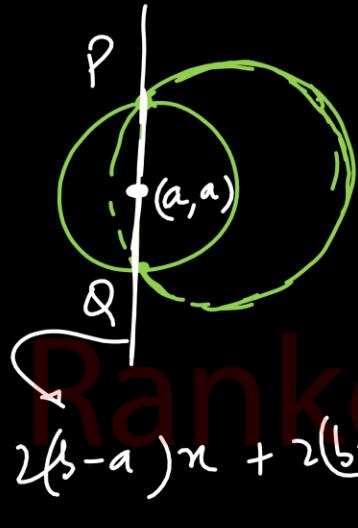
$$t^2 - 4t + 1 = 0$$

$$t = \frac{4 \pm \sqrt{16-4}}{2}$$

$$t = 2 \pm \sqrt{3}$$

$2 - \sqrt{3} \times \quad \quad \quad 2 + \sqrt{3} \checkmark$

C



PQ = ?

$$2(b-a)x + 2(b-a)y + a^2 - b^2 = 0$$

$$2(b-a)x + 2(b-a)y = (b^2 - a^2)$$

$$\boxed{2x + 2y = a + b}$$

$$(a, a) : \sqrt{a} = a + b$$

$$3a = b$$

$$\frac{b}{a} = 3$$

D

$$2a^2 - 4ab + b^2 = 0$$

$$2 - 4\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 = 0$$

$$t^2 - 4t + 2 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$2 \pm \sqrt{2}$$



Let x_1, x_2, \dots, x_{10} be ten observations such that

$$\sum_{i=1}^{10} (x_i - 2) = 30, \sum_{i=1}^{10} (x_i - \beta)^2 = 98, \beta > 2,$$

and their variance is $\frac{4}{5}$. If μ and σ^2 are

respectively the mean and the variance of
 $2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, \dots, 2(x_{10} - 1) + 4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

- (A) 100
 (B) 120
 (C) 110
 (D) 90

$$\sum_{i=1}^{10} (x_i - 2) = 30$$

$$\sum_{i=1}^{10} [(x_i - \beta) + (\beta - 2)] = 30$$

$$\sum_{i=1}^{10} (x_i - \beta) + \sum_{i=1}^{10} (\beta - 2) = 30$$

$$\sum_{i=1}^{10} (x_i - \beta) + 10(\beta - 2) = 30$$

$$\sum_{i=1}^{10} (x_i - \beta) = [50 - 10\beta]$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N} \right)^2$$

$$\sigma^2 = \frac{\sum (x_i - \beta)^2}{N} - \left(\frac{\sum (x_i - \beta)}{N} \right)^2$$

$$\frac{4}{5} = \frac{98}{10} - \left(\frac{(\sum x - 10\beta)}{10} \right)^2$$

$$(\sum x - \beta)^2 = \frac{49}{5} - \frac{4}{5}$$

$$25 + \beta^2 - 10\beta = 9$$

$$\beta^2 - 10\beta + 16 = 0$$

$$(\beta - 2)(\beta - 8) = 0$$

$$\beta = 2 ; 8 \quad (\because \beta > 0) \\ \Rightarrow \beta = 8$$

	Mean	Variance
$x_i - \beta$ \downarrow $(x_i - 8)$	-3	$4/5$
x_i	5	$4/5$
$(2x_i + 20)$	$10 + 30$ = 40	$\frac{16}{5}$

$$\frac{\beta M}{\sigma^2} = \frac{8 \times \frac{20}{20}}{\frac{16}{5}} \times 5 = 100$$

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If the system of equations $x + 4y - z = \alpha$, $7x + 9y + \beta z = -3$ and $5x + y + 2z = -1$ has infinitely many solution, then $4\beta - 3\alpha$ is

$$\begin{aligned} \text{LHS} & \left\{ \alpha = 0 \rightarrow \beta = ? \right. \\ & \left. \beta = 0 \rightarrow \alpha = ? \right\} \end{aligned}$$

 ~~$\alpha = 2$~~

$$x + 4y - z = \alpha \rightarrow P_1$$

$$7x + 9y + \beta z = -3 \rightarrow P_2$$

$$5x + y + 2z = -1 \rightarrow P_3$$

$$P_2 = P_3 + 2P_1$$

$$\beta = 2 - 2 = 0$$

$$-3 = -1 + 2\alpha$$

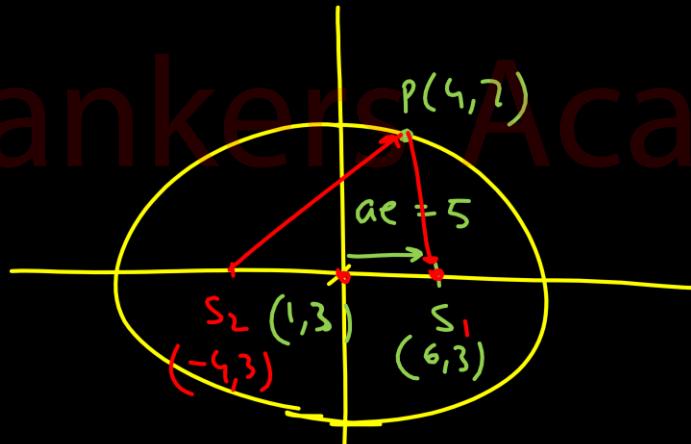
$$\alpha = -1$$

$$\therefore 4\beta - 3\alpha = \boxed{3}$$



An ellipse E has its centre C(1,3), focus at S(6,3) and passing through the point P(4,7), then If a and b is length of semi-major axis and semi minor axis of ellipse E, then $a^2 - b^2$ is

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$$\zeta^2 = a^2(1-e^2)$$

$$\zeta^2 = a^2 - a^2 e^2$$

$$\underline{a^2 e^2 = (a^2 - \zeta^2)}$$

25

23

If $y = \int_x^{x^2} \frac{dt}{t+\sqrt{t}}$, then least value of $\frac{e^y}{(\sqrt{2}-1)^2}$ is

$$\frac{dy}{dx} = \frac{1}{x^2+1} \times 2x - \frac{1}{x+\sqrt{x}} \times 1 = 0$$

$$\frac{e^y \left(\frac{dy}{dx} \right)}{(\sqrt{2}-1)^2} = 0$$

$$\frac{dy}{dx} = 0$$

$$2x + 2\sqrt{x} = x + 1$$

$$x - 1 = -2\sqrt{x}$$

$$x^2 - 2x + 1 = 4x$$

$$x^2 - 6x + 1 = 0$$

$$2\sqrt{x} = 1-x$$

$$x = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$$

$$\therefore \boxed{n = 3 - 2\sqrt{2}}$$

Now:

$$J = \int_{\sqrt{2}}^{n^2} \frac{dt}{\sqrt{t(\sqrt{t}+1)}}$$

Let: $t = u^2$
 $dt = 2u du$

$$\therefore J = \int_{\sqrt{2}}^n \frac{2u du}{u(u+1)}$$

$$J = 2 \left(\ln(u+1) \right) \Big|_{\sqrt{2}}^n$$

$$J = 2 \left(\ln \left(\frac{n+1}{\sqrt{n+1}} \right) \right)$$

$$e^J = \left(\frac{n+1}{\sqrt{n+1}} \right)^2$$

$$\begin{cases} n = 3 - 2\sqrt{2} \\ \sqrt{n} = \sqrt{3 - 2\sqrt{2}} \\ = \sqrt{(\sqrt{2}-1)^2} \\ = (\sqrt{2}-1) \end{cases}$$

$$e^J = \left(\frac{3 - 2\sqrt{2} + 1}{\sqrt{2} - 1 + 1} \right)^2$$

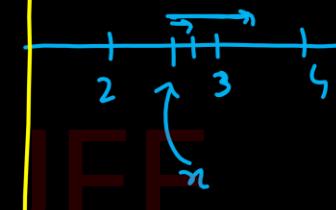
$$e^J = \left(\frac{4 - 2\sqrt{2}}{\sqrt{2}} \right)^2$$

$$\frac{e^J}{(\sqrt{2}-1)^2} = 1 //$$

24

Let x be a real number for which $\left[x + \frac{19}{100}\right] + \left[x + \frac{20}{100}\right] + \left[x + \frac{21}{100}\right] + \dots + \left[x + \frac{91}{100}\right] = 546$,
 then find $[100x]$

$$\underbrace{\left\lfloor x \right\rfloor + \left\lfloor x \right\rfloor + \left\lfloor x \right\rfloor + \dots + \left(\left\lfloor x \right\rfloor + 1 \right) + \dots + \left(\left\lfloor x \right\rfloor + 1 \right)}_{n+1 \text{ terms}} = 546$$



$$73(\left\lfloor x \right\rfloor) + n = 546$$

\downarrow
7

$$511 + n = 546$$

$$n = 35$$

$$91, 90, 89, - \sim T_{35}$$

$$T_{35} = 91 + (34)(-1) = 57$$

$$x + \frac{57}{100} = 741$$

$$x = \left(7 + \frac{57}{100} \right)$$

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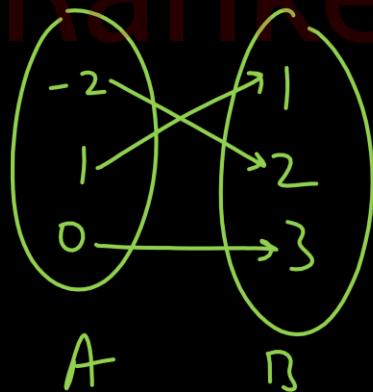
$$100x = 700 + 57$$

$$= 757$$

25

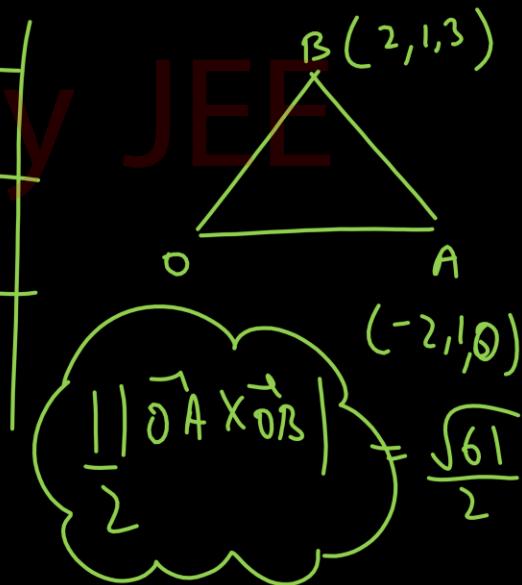
Consider f be a injective function from set $A\{-2,1,0\}$ to set $B\{1,2,3\}$ such that exactly one of the following statement is true : $f(-2) = 1, f(1) \neq 1, f(0) \neq 2$ and the remaining two are false. If the area of the triangle formed by $(-2,1,0), (f(-2), f(1), f(0))$ and origin is given

by $\frac{\sqrt{k}}{2}$, then k is



$f(-2) = 1$	T	F	F
$f(1) \neq 1$	F	T	F
$f(0) \neq 2$	F	F	T

$$\left\{ \begin{array}{l} (-2, 1, 0) \\ (2, 1, 3) \end{array} \right\}$$



आप हो तो हम हैं

