

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 2

QP Code: 100966

PAPER - 2

Time Allotted: 3 Hours

Maximum Marks: 195

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above **MUST** be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with **Blue/Black Ball Point Pen** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For Only One Part.

- (i) **Part-A (01-07)** – Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks: +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened.
Zero Marks: 0 If none of the bubbles is darkened.
Negative Marks: –2 In all other cases.
For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only (A) and (D) will result in **+2 marks**; and darkening (A) and (B) will result in **–2 marks**, as a wrong option is also darkened.
- (ii) **Part-A (08-14)** – Contains seven (07) multiple choice questions which have **ONLY ONE CORRECT** answer. Each question carries **+3 marks** for correct answer and **–1 marks** for wrong answer.
- (iii) **Part-A (15-18)** – This section contains Two paragraphs. Based on each paragraph, there are Two multiple choice questions. Each question has only one correct answer and carries **+4 marks** for the correct answer and **–2 marks** for wrong answer.

Name of the Candidate : _____

Batch : _____ Date of Examination : _____

Enrolment Number : _____

BATCH – Two Year CRP325 batches

SECTION-1 : PHYSICS

PART – A

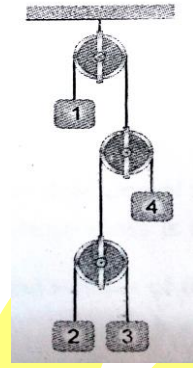
(Multi Correct Choice Type)

This section contains 7 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

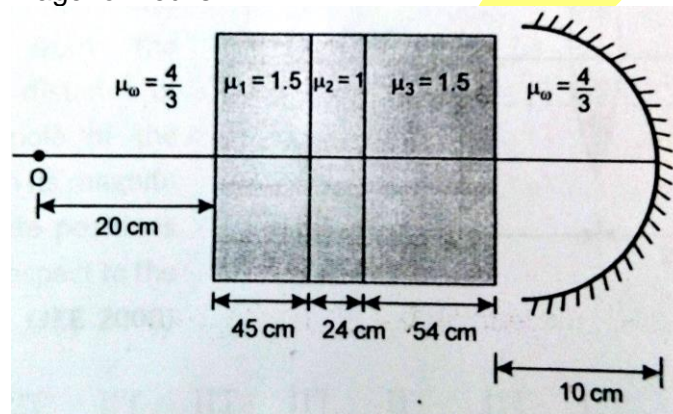
1. The electric field strength in a region is given as $\vec{E} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$. Then select the correct option(s).
 (A) The net charge inside a sphere of radius 'a' with its centre at origin will be $2\pi\epsilon_0 a$
 (B) The net charge inside a sphere of radius 'a' with its centre at origin will be $4\pi\epsilon_0 a$
 (C) The net charge inside a sphere of radius '2a' with its centre at origin will be $4\pi\epsilon_0 a$
 (D) The net charge inside a sphere of radius '2a' with its centre at origin will be $8\pi\epsilon_0 a$
2. A solution contains a mixture of two isotopes A (half life = 10 days) and B (half life = 5 days). Total activity of the mixture is 10^{10} disintegrations per second at time $t = 0$, the activity reduces to 20% in 20 days. Then select correct option(s).
 (A) Initial activity of A is 0.73×10^{10} dps.
 (B) Initial activity of B is 0.27×10^{10} dps.
 (C) The ratio of initial number of their nuclei is $N_A : N_B$ is 5.4
 (D) None of these
3. If $r = 1 \Omega$ and $E = 10 \text{ V}$ in the network shown in figure. Then answer the following:
 (A) The value of current I is 22.85 A.
 (B) The value of current I is 42.85 A.
 (C) The current in branch PQ is 7.62 A.
 (D) The current in branch PQ is 3.8 A.
4. An aluminium wire of cross-sectional area 10^{-6} m^2 is joined to a steel wire of the same cross-sectional area. This compound wire is stretched on a sonometer pulled by a weight of 10 kg. The total length of the compound wire between the bridges is 1.5 m of which the aluminium wire is 0.6 m and the rest is steel wire. Transverse vibrations are set up in the wire by using an external source of variable frequency. The density of aluminium is $2.6 \times 10^3 \text{ kg/m}^3$ and that of steel is $1.04 \times 10^4 \text{ kg/m}^3$. ($g = 10 \text{ m/s}^2$)
 (A) The lowest frequency of excitation for which standing waves are formed such that joint of wires is a node is nearly 82 Hz.
 (B) The lowest frequency of excitation for which standing waves are formed such that joint of wires is a node is nearly 164 Hz.
 (C) A total of 4 nodes are formed on composite wire.
 (D) A total of 5 nodes are formed on composite wire.

Space For Rough Work

5. In the arrangement shown in figure, all pulleys are smooth and massless. When the system is released from rest, accelerations of blocks 2 and 3 relative to 1 are 1 m/s^2 downwards and 5 m/s^2 downwards. Acceleration of block 3 relative to 4 is zero. Then
- (A) The absolute acceleration of block 1 is 2 m/s^2 upward.
 (B) The absolute acceleration of block 2 is 1 m/s^2 downward.
 (C) The absolute acceleration of block 3 is 3 m/s^2 downward.
 (D) The absolute acceleration of block 4 is 3 m/s^2 downward.



6. A composite slab consisting of different media is placed in front of a concave mirror of radius of curvature 150 cm . The whole arrangement is placed in water ($\mu_w = 4/3$). An object O is placed at a distance 20 cm from the slab. The refractive indices are $\mu_1 = 1.5$, $\mu_2 = 1$, $\mu_3 = 1.5$. The position of final image formed is



- (A) 130 cm on left of mirror
 (B) 153 cm on left of mirror
 (C) Final image is real
 (D) Final image is virtual
7. Two identical soap bubbles each of radius 1 cm come close to each other and join together maintaining a common surface between them.
- (A) Their common surface will be curved.
 (B) Their common surface will be flat.
 (C) Area of their common surface will be $\frac{3\pi}{2} \text{ cm}^2$
 (D) Area of their common surface will be $\frac{3\pi}{4} \text{ cm}^2$

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(Single Correct Choice Type)

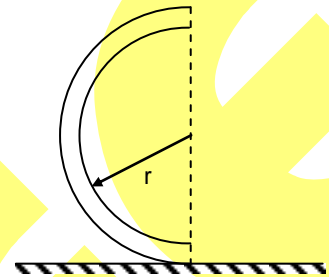
This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

8. A ball of mass "m" moving with kinetic energy 3 J collides with a stationary ball of mass 2m in a head-on elastic collision. During collision, what will be maximum deformation potential energy stored in system?

(A) 1 J (B) 2 J
(C) 2.5 J (D) 1.5 J

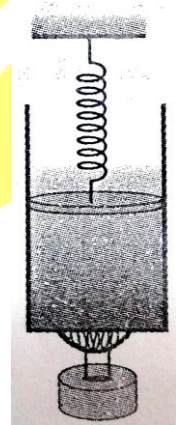
9. A semicircular ring of mass m and radius r is released from rest in the position shown with its lower edge resting on a horizontal surface. Find the minimum coefficient of static friction μ_s which is necessary to prevent any initial slipping of the ring.

(A) 0.2 (B) 0.3
(C) 0.4 (D) 0.5



10. A gas is inside a cylinder closed by a piston. The piston is held from above by a spring whose elastic properties obey Hooke's law. If the gas is heated slowly then determine the work done by the gas in the process if volume of the gas varies from V_1 to V_2 and the pressure varies from P_1 to P_2 .

(A) $(P_1 + P_2)(V_2 - V_1)$ (B) $\frac{(P_1 + P_2)(V_2 - V_1)}{2}$
(C) $\frac{(P_1 + P_2)(V_2 - V_1)}{4}$ (D) Data insufficient

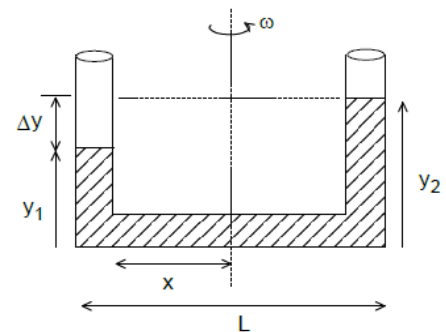


11. An air sealed spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2 . The maximum pressure at any point on the tank is [Assume near vacuum condition]

(A) $4800\sqrt{2} \text{ N/m}^2$ (B) 4800 N/m^2
(C) $3036\sqrt{2} \text{ N/m}^2$ (D) 9600 N/m^2

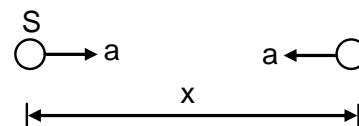
12. An U-shaped tube contains a liquid of density ρ and it rotates about an axis as shown in the figure. Given $L = 2 \text{ m}$, $\Delta y = 1.6 \text{ m}$ and $\omega = 4 \text{ rad/s}$. Then the value of 'x' is

(A) 0.25 m
(B) 0.5 m
(C) 0.75 m
(D) 1 m

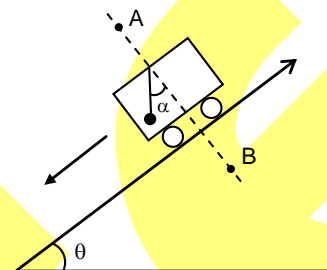


Space For Rough Work

13. A source S and a detector O are initially at a distance of $x = 1$ km. Both start moving towards one another with same acceleration $a = 10 \text{ m/s}^2$. Frequency of source is $f = 2000 \text{ Hz}$. Find the frequency observed by the detector at time $t = 4$ second. Speed of sound in air is $v = 300 \text{ m/s}$.
- (A) 2241 Hz (B) 2341 Hz (C) 2441 Hz (D) 2541 Hz



14. A pendulum hangs from roof of a cart which slides down the smooth incline as shown. Line AB is perpendicular to incline. What is angle " α " made by string of pendulum with line AB in equilibrium position?
- (A) 0°
 (B) θ
 (C) $\frac{\theta}{2}$
 (D) greater than θ

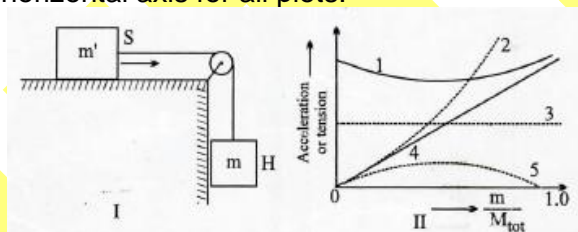


(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Two containers of sand S and H are arranged like the blocks in figure I. The containers alone have negligible mass; the sand in them has a total mass M_{tot} ; the sand in the hanging container H has mass m . You are to measure the magnitude of the acceleration of the system in a series of experiments where m varies from experiment to experiment but M_{tot} does not; that is, you will shift sand between the containers before each trial. $\frac{m}{M_{\text{tot}}}$ is taken on the horizontal axis for all plots.



15. The plot in figure II which gives the acceleration magnitude of the containers (taken on y-axis) against ratio $\left(\frac{m}{M_{\text{tot}}}\right)$ is:
- (A) 1 (B) 3 (C) 4 (D) 5

Space For Rough Work

16. The curve which gives tension in the connecting string (taken on y-axis) against ratio $\left(\frac{m}{M_{\text{tot}}}\right)$ is:
- (A) 1 (B) 2
(C) 4 (D) 5

Paragraph for Question no. 17 to 18

The main scale of a vernier callipers reads in millimetre and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a main scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm; and the fourth vernier division coincides with a scale division. Choose from following the correct option.

17. The zero error in the apparatus is
- (A) -0.07 cm (B) 0.07 cm
(C) 0.03 cm (D) -0.03 cm
18. The measured value of the length of the cylinder is
- (A) 3.14 cm (B) 3.24 cm
(C) 3.07 cm (D) 3.17 cm

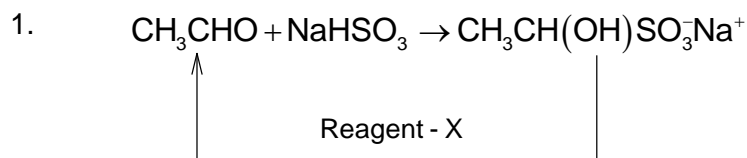
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SECTION-2 : CHEMISTRY

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.



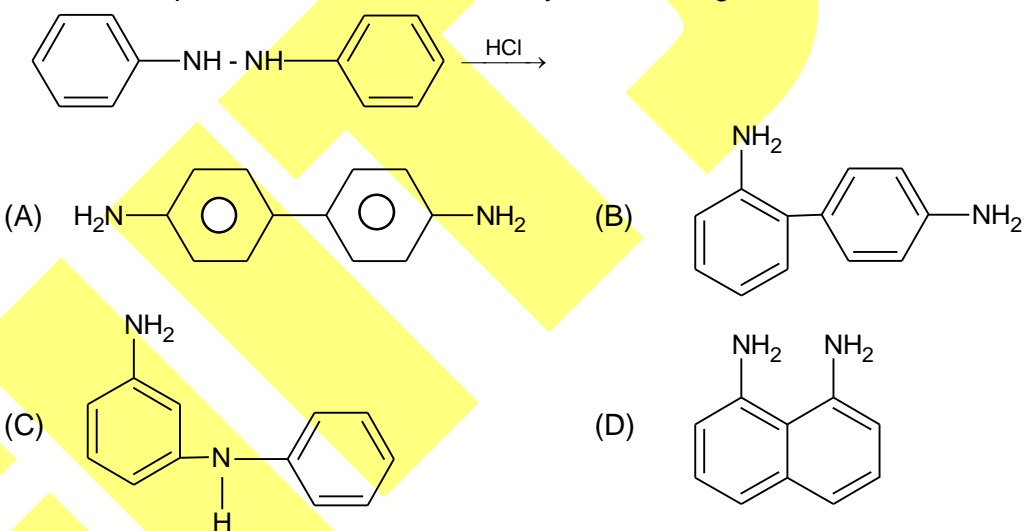
In the above change, the reagent (X) can be

- (A) CCl_4 (B) HCl
(C) NaOH (D) DMF

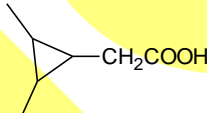
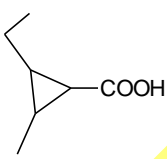
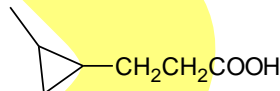
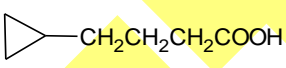
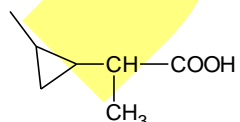
2. Which of the following reaction(s) takes place in presence of NaOH ?

- (A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{OH} \longrightarrow \text{CH}_3\text{CH}=\text{CH}_2 + \text{H}_2\text{O}$
(B) $\text{CH}_3\text{CHO} + \text{CH}_3\text{CHO} \longrightarrow \text{CH}_3\text{CH}=\text{CHCHO} + \text{H}_2\text{O}$
(C) $\text{CH}_3\text{C}(=\text{O})-\text{O}-\text{C}(=\text{O})-\text{CH}_3 \longrightarrow \text{CH}_3\text{COO}^-$
(D) $\text{CH}_3\text{CONH}_2 \longrightarrow \text{CH}_3\text{CN} + \text{H}_2\text{O}$

3. Find out the products which are formed by the following reaction:



Space For Rough Work

4. Which statement is/are correct?
 (A) Final temperature in reversible adiabatic expansion is lesser than in irreversible adiabatic expansion.
 (B) When heat is supplied to an ideal gas in an isothermal process, kinetic energy of gas will increase.
 (C) When an ideal gas is subjected to adiabatic expansion it gets cooled
 (D) Entropy increases in atomization of dihydrogen
5. Choose the correct statement(s):
 (A) Methyl alcohol and water form minimum boiling azeotrope.
 (B) At the b.p. the molar free energy of each phase must be equal
 (C) For an ideal solution of two liquids, $\Delta S_{\text{system}} = +\text{Ve}$ and $\Delta S_{\text{surr}} = 0$
 (D) Minimum boiling azeotrope results when mixture of two volatile liquids show negative deviation from Raoult's law
6. Which of the following statements is/are correct?
 (A) Boiling point of liquid racemic mixture is same as that of pure enantiomer.
 (B) Melting point of solid racemic mixture is same as that of pure enantiomer.
 (C) In osmosis there is movement of solvent molecules from higher concentration (i.e. solvent) to lower concentration.
 (D) In osmosis there is movement of solvent molecules from lower concentration (i.e. solute) to higher concentration.
7. Which of the following acid(s) is are more acidic than  ?
- (A)  (B) 
 (C)  (D) 

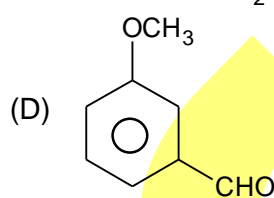
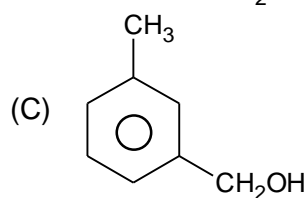
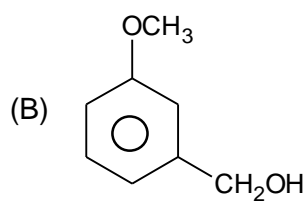
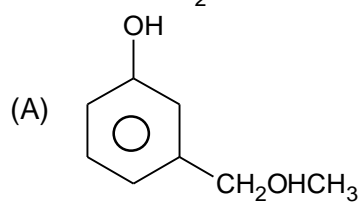
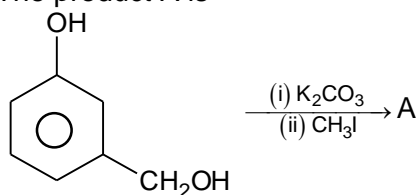
(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

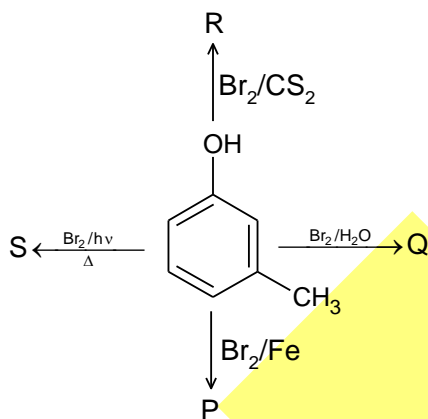
8. The compound that will react most readily with NaOH to form methanol is:-
 (A) $(\text{CH}_3)_4\text{NI}^+$ (B) $\text{CH}_3 - \text{O} - \text{CH}_3$
 (C) $(\text{CH}_3)_3\text{SI}^+$ (D) $(\text{CH}_3)_3\text{C} - \text{Cl}$

Space For Rough Work

9. The product A is



10.

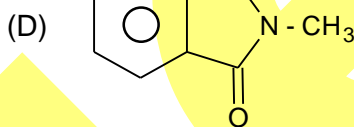
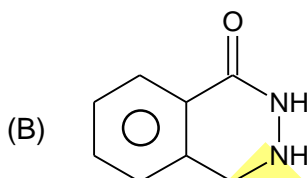
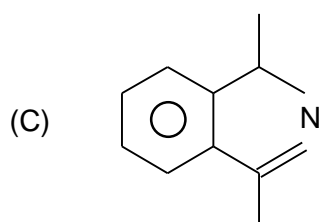
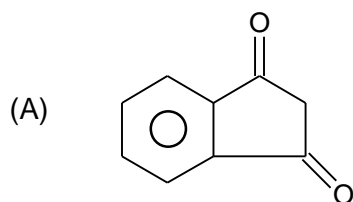
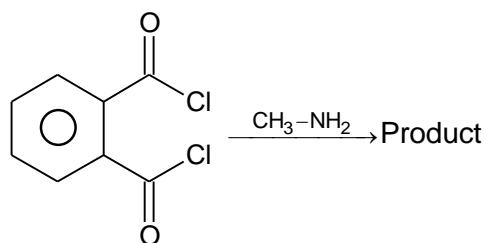


Which of the following undergoes the easiest S_N1 reaction with KCN?

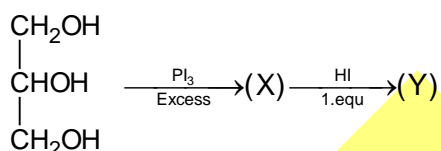
- (A) P (B) Q
(C) R (D) S

Space For Rough Work

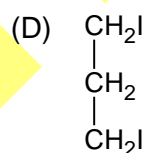
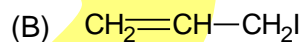
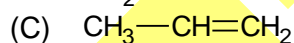
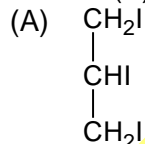
11.



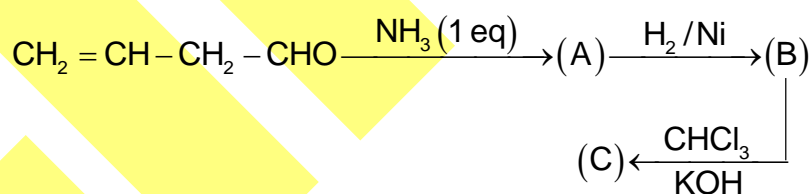
12.



Product (Y) in the above reaction is :



13.

How many pi(π) bond(s) is/are present in one molecule of (C)?

- (A) 1
(C) 4

- (B) 2
(D) 3

Space For Rough Work

14. Which of the following resonating structure is most stable?

- (A) $\overset{\oplus}{\text{CH}_2} - \text{CH} = \text{CH} - \overset{\ominus}{\text{CH}_2}$ (B) $\text{CH}_2 = \text{CH} - \overset{\oplus}{\text{CH}} - \overset{\ominus}{\text{CH}_2}$
 (C) $\overset{\oplus}{\text{CH}_2} - \overset{\ominus}{\text{CH}} - \text{CH} = \text{CH}_2$ (D) $\overset{\ominus}{\text{CH}_2} - \overset{\oplus}{\text{CH}} - \overset{\ominus}{\text{CH}} - \overset{\oplus}{\text{CH}_2}$

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

An inorganic salt on treatment with Nessler's reagent give brown ppt. in alkaline medium. Same salt with conc. HNO_3 and ammonium molybdate solution, a canary yellow ppt. is formed

15. In Nessler's reagent reaction the active species is:

- (A) HgI_2^- (B) Hg^{2+}
 (C) HgI_4^{2-} (D) Hg_2I_2

16. The formula of the yellow precipitate is:

- (A) $(\text{NH}_4)_3\text{PO}_4$ (B) $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_4$
 (C) $(\text{NH}_4)_3\text{PO}_4 \cdot 12\text{MoO}_3$ (D) $(\text{NH}_4)_3\text{PO}_4 \cdot (\text{NH}_4)_2 \cdot \text{MoO}_4$

Paragraph for Question no. 17 to 18

An aromatic tertiary alcohol upon acid catalysed dehydration gives a product (I). Reductive ozonolysis of (I) forms compounds (J) and (K). Compound (J) upon reaction with NaOH gives benzyl alcohol and compound (L), whereas (K) on heating with NaOH gives only (M)

Answer the following questions on the basis of the above write up.

17. The structure of the compound (I) is:

- (A) $\text{CH}_3\text{CH} = \underset{\text{Ph}}{\text{C}} - \text{Ph}$ (B) $\text{PhCH} = \underset{\text{Ph}}{\text{C}} - \text{CH}_3$
 (C) $\text{PhCH} = \text{CH} - \text{CH}_3$ (D) $\text{PhCH}_2\text{CH} = \text{CH}(\text{Ph})$

18. The structure of compounds J, K and L respectively are:

- (A) PhCHO , PhCH_2CHO and $\text{PhCH}_2\text{COONa}$
 (B) PhCHO , PhCOCH_3 and PhCOONa
 (C) PhCHO , PhCOCH_3 and $\text{PhCH}_2\text{COONa}$
 (D) PhCHO , PhCOPh and PhCOONa

Space For Rough Work

SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. A line L passing through the point P (1, 4, 3) is perpendicular to both the lines $\frac{x-1}{2} = \frac{y+3}{1} = -\frac{z-2}{4}$ and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$.
If the position vector of point Q on L is (a_1, a_2, a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be:
(A) 16 (B) 15
(C) 2 (D) 1
2. If $\lim_{x \rightarrow 0} \frac{a \sin x - bx + cx^2 + x^3}{2x^2 \log(1+x) - 2x^3 + x^4}$ exists and is finite, then
(A) $a = b$ (B) $c = 0$
(C) $a = 6$ (D) $c = 2$
3. Which of the following definite integral(s) vanishes?
(A) $\int_0^{\pi/2} \ln(\cot x) dx$ (B) $\int_0^{2\pi} \sin^3 x dx$
(C) $\int_{1/e}^e \frac{dx}{x(\ln x)^{1/3}}$ (D) $\int_0^{\pi} \sqrt{\frac{1 + \cos 2x}{2}} dx$
4. Let f be a real valued function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $\lim_{x \rightarrow 0} \frac{f(1+x)}{x} = 3$. The area bounded by the curve $y = f(x)$, the y-axis and the line $y = 3$ is
(A) e (B) 2e
(C) 3e (D) 4e
5. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5, & x \leq 1 \\ -2x + \log_2(b^2 - 2), & x > 1 \end{cases}$. If $f(x)$ has greatest value at $x = 1$, then $b^2 \in (2, \lambda]$, where λ is
(A) multiple of 13 (B) even number
(C) divisible by 5 (D) odd number

Space For Rough Work

6. The area of the region bounded by the curve $y = \frac{16 - x^2}{4}$ and $y = \sec^{-1}[-\sin^2 x]$ where $[.]$ denotes the greatest integer function) is
- (A) $\frac{1}{3}(4 - \pi)^{3/2}$ (B) $8(4 - \pi)^{3/2}$
- (C) $\frac{8}{3}(4 - \pi)^{3/2}$ (D) $\left(\frac{3}{8(4 - \pi)^{3/2}}\right)^{-1}$
7. If $x = 2 + 5i$ (where $i^2 = -1$) and $2\left(\frac{1}{1!9!} + \frac{1}{3!7!}\right) + \frac{1}{5!5!} = \frac{2^a}{b!}$, then the value of $(x^3 - 5x^2 + 33x - 19)$ is equal to:
- (A) $a + 1$ (B) b
- (C) $a - b + 11$ (D) $a + b - 9$

(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

8. A square, of each side 2, lies above the x - axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x - axis, then the sum of the x - coordinates of the vertices of the square is
- (A) $\sqrt{3} - 2$ (B) $2\sqrt{3} - 1$
- (C) $\sqrt{3} - 1$ (D) $2\sqrt{3} - 2$
9. All the chords of the hyperbola $3x^2 - y^2 - 2x + 4y = 0$ subtending a right angle at the origin pass through the fixed points
- (A) $(1, -2)$ (B) $(-1, 2)$
- (C) $(1, 2)$ (D) none of these
10. The sum of two natural numbers n_1 and n_2 is known to be equal to 100. The probability that their product being greater than 1600, is equal to:
- (A) $\frac{20}{33}$ (B) $\frac{58}{99}$
- (C) $\frac{13}{33}$ (D) $\frac{59}{99}$

Space For Rough Work

11. The reflection of the complex number $\frac{4+3i}{1+2i}$, in the straight line $iz = \bar{z}$ is
 (A) $2+i$ (B) $2-i$
 (C) $1+2i$ (D) $1-2i$
12. Let a, b, c be distinct positive numbers such that each of the quadratics ax^2+bx+c, bx^2+cx+a and cx^2+ax+b is non-negative for all $x \in \mathbb{R}$. If $R = \frac{a^2+b^2+c^2}{ab+bc+ca}$, then
 (A) $1 \leq R < 4$ (B) $1 < R \leq 4$
 (C) $1 \leq R \leq 4$ (D) $1 < R < 4$
13. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$, where $r, s \in \{1, 2, 3\}$ and $z = \frac{-1+i\sqrt{3}}{2}$. If $P^2 = -I_2$, then total number of ordered pairs (r, s) is
 (A) 0 (B) 1
 (C) 2 (D) 3
14. Let $S_n = \cot^{-1}\left(3x + \frac{2}{x}\right) + \cot^{-1}\left(6x + \frac{2}{x}\right) + \cot^{-1}\left(10x + \frac{2}{x}\right) + \dots + n \text{ terms}$, where $x > 0$. If $\lim_{n \rightarrow \infty} S_n = 1$, then x equals
 (A) $\frac{\pi}{4}$ (B) 1 (C) $\tan 1$ (D) $\cot 1$

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Let f be an even function satisfying $f(x-2) = f\left(x + \left[\frac{6x^2+13}{x^2+2}\right]\right) \forall x \in \mathbb{R}$ and

$$f(x) = \begin{cases} 3x, & 0 \leq x < 1 \\ 4-x, & 1 \leq x \leq 4 \end{cases}$$

[Note : $[y]$ denotes greatest integer function of y .]

15. The area bounded by the graph of $f(x)$ and the x -axis from $x = -1$ to $x = 9$ is:
 (A) $\frac{31}{2}$ (B) 15 (C) 12 (D) $\frac{15}{2}$

Space For Rough Work

16. The value of $f(-89) - f(-67) + f(46)$ is equal to :
- (A) 4 (B) 5
(C) 6 (D) 7

Paragraph for Question no. 17 to 18

Let a differentiable function 'f' satisfies the functional rule
 $f(xy) = f(x) + f(y) + xy - x - y \forall x, y > 0$ and $f'(1) = 4$.

17. If $f(x_0) = 0$, then x_0 lies in the interval
- (A) $(0, 1)$ (B) $(1, e)$
(C) (e, e^2) (D) (e^2, e^3)
18. If $\int e^{f(x)} dx = e^x (ax^2 + cx + d) + \lambda$, then the value of $(a + b + c + d)$ is equal to:
- (A) -1 (B) -2
(C) 3 (D) 6

Space For Rough Work

QP Code: 100966**Answers****SECTION-1 : PHYSICS****PART – A**

- | | | | |
|--------|--------|-------|-------|
| 1. BD | 2. ABC | 3. AC | 4. BD |
| 5. ACD | 6. BC | 7. BD | 8. B |
| 9. C | 10. B | 11. C | 12. B |
| 13. B | 14. A | 15. C | 16. D |
| 17. B | 18. C | | |

SECTION-1 : CHEMISTRY**PART – A**

- | | | | |
|--------|--------|-------|--------|
| 1. BC | 2. BC | 3. AB | 4. ACD |
| 5. ABC | 6. ACD | 7. A | 8. A |
| 9. B | 10. D | 11. D | 12. C |
| 13. B | 14. B | 15. C | 16. C |
| 17. B | 18. B | | |

SECTION-1 : MATHEMATICS**PART – A**

- | | | | |
|--------|--------|---------|-------|
| 1. BD | 2. ABC | 3. ABC | 4. C |
| 5. ABC | 6. CD | 7. ABCD | 8. D |
| 9. A | 10. D | 11. D | 12. D |
| 13. B | 14. D | 15. B | 16. A |
| 17. A | 18. B | | |

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **BD**

Sol. At a point P(x, y, z) on sphere of radius 'a' unit vector \perp to surface:

$$\hat{n} = \frac{x}{a}\hat{i} + \frac{y}{a}\hat{j} + \frac{z}{a}\hat{k}$$

$$\vec{E} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$$

$$\vec{E} \cdot \hat{n} = \frac{1}{a} = \text{Independent of point P.}$$

$$\Rightarrow \phi = \frac{1}{a} \times 4\pi a^2 = \frac{q}{\epsilon_0}$$

2. **ABC**

Sol. Initial activities:

$$A_{O1} + A_{O2} = 10^{10}$$

After 20 days:

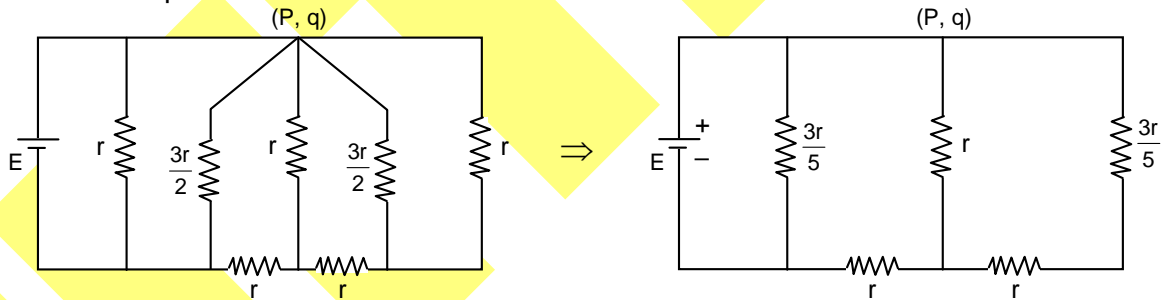
$$A_1 = \left(\frac{1}{2}\right)^2 A_{O1},$$

$$A_2 = \left(\frac{1}{2}\right)^4 A_{O2}$$

$$A_1 + A_2 = 0.2 \times 10^{10}.$$

3. **AC**

Sol. Circuit simplification:



4. **BD**

Sol. Let $n_a \rightarrow$ Number of loops on Al wire.

$N_s \rightarrow$ Number of loops on steel wire.

$$f_a = f_s$$

$$n_a \left(\frac{V_a}{2\ell_a} \right) = n_s \left(\frac{V_s}{2\ell_s} \right)$$

$$\Rightarrow \frac{n_a}{n_s} = \left(\frac{V_s}{V_a} \right) \left(\frac{\ell_a}{\ell_s} \right) ; \quad \frac{V_s}{V_a} = \sqrt{\frac{\rho_a}{\rho_s}} \quad \left(\text{Using } V = \sqrt{\frac{T}{\mu}} \right)$$

5. **ACD**

Sol. String constrains and $\sum_{i=1}^n \vec{T}_i \cdot \vec{a}_i = 0$

6. **BC**

Sol. Shift produced by three slabs:

$$\Delta x_1 = \left(1 - \frac{\mu_w}{\mu_1}\right)(45) = 5 \text{ cm}$$

$$\Delta x_2 = \left(\frac{\mu_w}{\mu_2} - 1\right)(24) = 8 \text{ cm}$$

$$\Delta x_3 = \left(1 - \frac{\mu_w}{\mu_3}\right)(54) = 6 \text{ cm}$$

 $(\Delta x_1, \Delta x_3) \longrightarrow$ Towards mirror

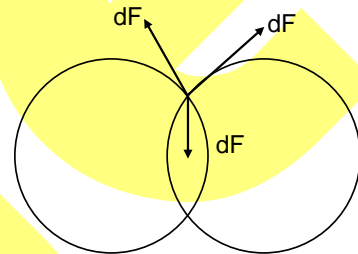
 $\Delta x_2 \longrightarrow$ Away from mirror.
Net shift = $5 + 6 - 8 = 3 \text{ cm}$

Now, solution can be proceeded with.

7. **BD**

Sol. As pressure on both sides of common surface is equal, hence it will be flat.

By symmetry of surface tension forces (dF) on common interface, angle between any two surfaces will be 120° .

8. **B**

Sol. Conservation of momentum:

$$mu = (3m)V$$

$$V = \frac{u}{3} = \text{Common speed at maximum deformation.}$$

Maximum deformation energy = Maximum loss of kinetic energy during impact.

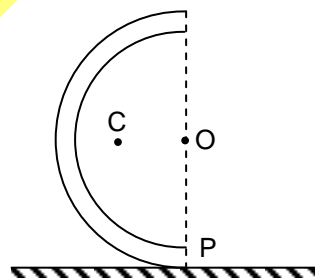
$$= \frac{1}{2}mu^2 - \frac{1}{2}(3m)V^2$$

9. **C**

Sol. As there is no slipping I.C.R is at point P.

C \longrightarrow Centre of mass

$$OC = \frac{2r}{\pi}$$

10. **B**Sol. When piston moves up by distance Δx :

$$\text{Gas pressure} = P = P_0 + \frac{mg}{A} + \frac{k\Delta x}{A}$$

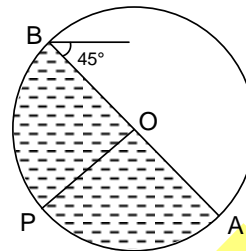
$$P = P_0 + \frac{mg}{A} + \frac{k}{A^2}(V - V_i)$$

Draw P-V graph which will be straight line.

Work done = Area under P-V graph.

11. **C**

- Sol. In frame of reference of tank, g_{eff} will be along OP where $OP \perp AB$. Maximum pressure will be at point P.



12. **B**

Sol. Centrifugal force on horizontal liquid makes the liquid rise in vertical section.

13. **B**

Sol. Let t be the time when sound is emitted which is received at $t = 4$ sec.

$$\text{Then: } \frac{1}{2}at^2 + \frac{1}{2}a(4)^2 + v(4-t) = 1000$$

$$\frac{1}{2}(10)t^2 + \frac{1}{2}(10)(16) + (300)(4-t) = 1000$$

$$t = 0.95 \text{ sec.}$$

Hence, pulse emitted at 0.95 sec will be received at $t = 4$ sec.

14. **A**

Sol. Due to effect of pseudo force, g_{eff} in frame of cart will be along line AB. Hence, $\alpha = 0^\circ$.

15. **C**

Sol. Let acceleration of system = a

$$mg - T = ma \quad \dots(1)$$

$$(M - m)a = T \quad \dots(2)$$

16. **D**

Sol. As in problem 15, tension (T) can also be calculated.

17. **B**

Sol. Zero error = M.S.R + (V.S.R) (L.C) (with closed scale)

$$= 0 + (7) \left(\frac{1}{10} \text{ mm} \right)$$

$$= 0.07 \text{ cm.}$$

18. **C**

Sol. Length = M.S.R + (V.S.R) (L.C) - (Z.E.)

$$= 3.1 \text{ cm} + 4 \times (0.01 \text{ cm}) - (0.07) \text{ cm}$$

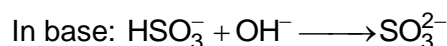
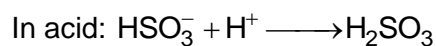
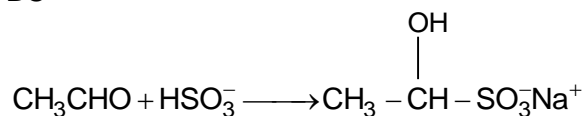
$$\text{Length} = 3.07 \text{ cm.}$$

SECTION-2 : CHEMISTRY

PART – A

1. BC

Sol.



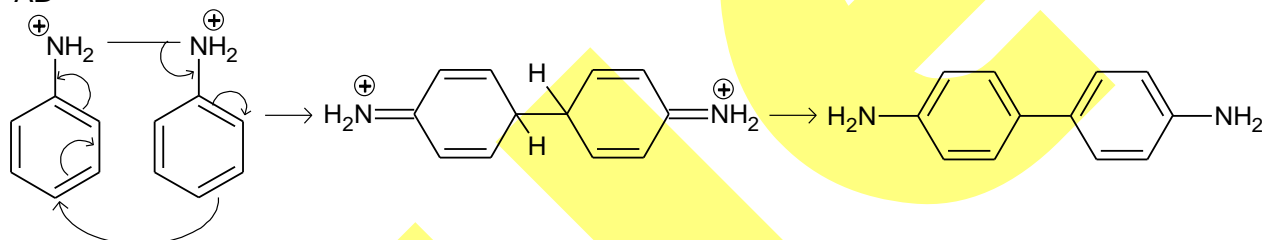
In both cases backward reaction takes place.

2. BC

Sol. Aldol & I hydrolysis

3. AB

Sol.



4. ACD

Sol.

(A) The final temperature is higher for an irreversible adiabatic process than for a reversible adiabatic process

$$\text{As, } \Delta U = Q + W, \quad Q = 0$$

$$\Delta U = W \Rightarrow C\Delta T = -P\Delta V$$

$|W_{\text{irreversible}}|$ is lesser than $|W_{\text{reversible}}|$

(B) K.E depends upon temperature only

(C) Adiabatic, $Q = 0$, T decreases

5. ABC

Sol.

$$K_f = \frac{MR(T_b^0)^2}{\Delta H_{\text{vap}}}, \text{ at equilibrium } \Delta G = 0$$

6. ACD

Sol.

Theoretical

7. A

Sol.

Only acid in option (A) will be more acidic than given acid.

8. A

Sol.

Due to greater electronegativity of N over S, +ve charge on N will make CH_3 group more electrons deficient than +ve charge on S.

$\therefore (\text{CH}_3)_4\text{N}^+\text{I}^-$ will undergo nucleophilic substitution reaction more readily

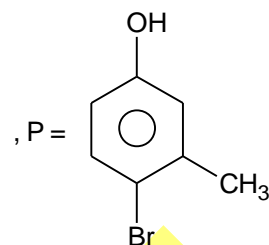
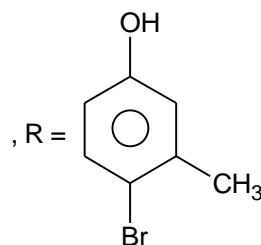
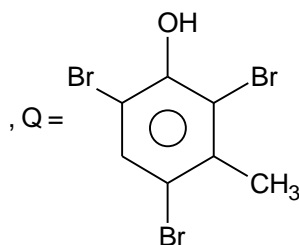
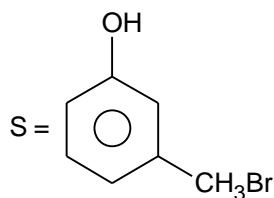
9. B

Sol.

Acidity Phenol > Alcohol

10. D

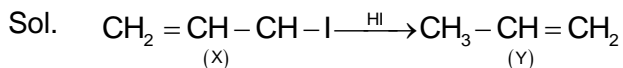
Sol.

S gives easiest S_N1

11. D

Sol. $S_{N_{AE}}$ of acid halide.

12. C



13. B

Sol. $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{N} \equiv \text{C}$ (imine followed by reduction followed by carbylamine)

14. B

Sol. Theoretical, opposite charge close

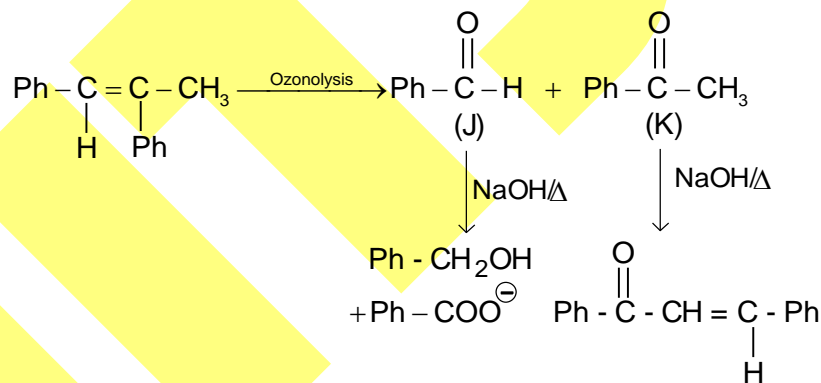
15. Nessler's reagent $[\text{HgI}_4]^{2-}$

16. Fact based

17. B

18. B

Sol for 17 & 18.



SECTION-3 : MATHEMATICS

PART – A

1. BD

Sol. Equation of the line passing through P(1, 4, 3) is: $\frac{x-1}{a} = \frac{y-4}{b} = \frac{z-3}{c}$ (i)

Since equation (i) is perpendicular to $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{4}$ and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$

Hence $2a+b+4c=0$ and $3a+2b-2c=0$

$$\therefore \frac{a}{-2-8} = \frac{b}{12+4} = \frac{c}{4-3}$$

$$\Rightarrow \frac{a}{-10} = \frac{b}{16} = \frac{c}{1}$$

Hence the equation of the lines is $\frac{x-1}{-10} = \frac{y-4}{16} = \frac{z-3}{1}$ (ii)

Now any point Q on (2) can be taken as $(1-10\lambda, 16\lambda+4, \lambda+3)$

$$\therefore \text{Distance of Q from P (1, 4, 3)} = (10\lambda)^2 + (16\lambda)^2 + \lambda^2 = 357$$

$$\Rightarrow (100+256+1)\lambda^2 = 357$$

$$\Rightarrow \lambda = 1 \text{ or } -1$$

$$\therefore \text{Q is } (-9, 20, 4) \text{ or } (11, -12, 2)$$

Hence $a_1 + a_2 + a_3 = 15$ or 1

2. ABC

Sol.
$$\lim_{x \rightarrow 0} \frac{a \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \dots \right) - bx + cx^2 + x^3}{2x^2 \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right) - 2x^3 + x^4}$$

$$= \lim_{x \rightarrow 0} \frac{(a-b)x + cx^2 + (1-a/6)x^3 + ax^5/120 + \dots}{2x^5/3 - x^6/2 + \dots}$$

For this limit to exist we must have $a-b=0$, $c=0$, and $1-a/6=0$, that is, $a=b=6$ and $c=0$

3. ABC

Sol.

(A) $I = \int_0^{\pi/2} \ln(\cot x) dx \Rightarrow I = \int_0^{\pi/2} \ln(\tan x) dx$

$$I = - \int_0^{\pi/2} \ln(\cot x) dx \Rightarrow I = -I \Rightarrow I = 0$$

(B) $I = \int_0^{2\pi} \sin^3 x dx = - \int_0^{2\pi} \sin^3 x dx \Rightarrow I = 0$

(C) $I = \int_e^{1/e} \frac{-1}{t} \left(\ln t \right)^{1/3} dt = \int_{1/e}^e \frac{dt}{t (\ln t)^{1/3}}$

$$I = -I$$

$$I = 0$$

(D) $\sqrt{\frac{1+\cos 2x}{2}} > 0$
 $\int_0^\pi \sqrt{\frac{1+\cos 2x}{2}} dx > 0$

4. C

Sol. Given $f\left(\frac{x}{y}\right) = f(x) - f(y)$... (1)

Putting $x = y = 1$, we get

Now, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f\left(1 + \frac{h}{x}\right)}{\frac{h}{x}}$ (From (1))

$= \lim_{h \rightarrow 0} \frac{f\left(1 + \frac{h}{x}\right)}{\left(\frac{h}{x}\right)x}$

$\Rightarrow f'(x) = \frac{3}{x}$

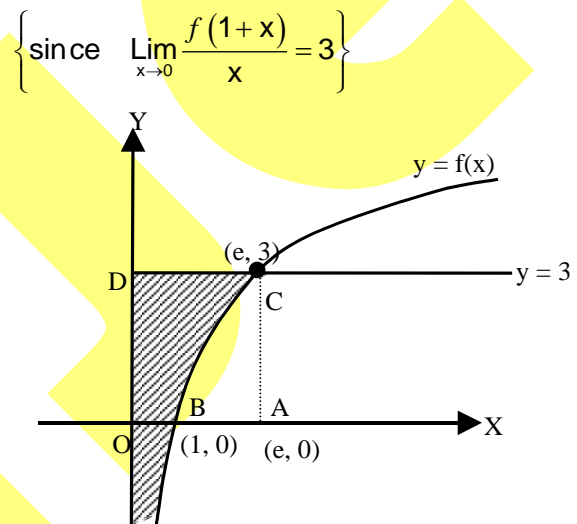
$\Rightarrow f(x) = 3 \ln x + c$

Putting $x = 1$

$\Rightarrow c = 0$

$\Rightarrow f(x) = 3 \ln x = y$ (say)

\therefore Required area $= \int_{-\infty}^3 x dy$
 $= \int_{-\infty}^3 e^{y/3} dy$
 $= 3 \left[e^{y/3} \right]_{-\infty}^3$
 $= 3(e - 0)$



5. ABC

Sol. For $x \leq 1$ $f'(x) = 3x^2 - 2x + 10$

$= 3 \left\{ \left(x - \frac{1}{3} \right)^2 + \frac{29}{9} \right\} > 0$

$\therefore f(x)$ is increasing function for $x \leq 1$ Now, for $x > 1$,

$f'(x) = -2 < 0$

So, $f(x)$ is decreasing function, for $x > 1$

Now, $f(x)$ will have greatest value at $x = 1$ if

$\Rightarrow \lim_{x \rightarrow 1^+} f(x) \leq f(1)$

$\Rightarrow \lim_{h \rightarrow 0} f(1+h) \leq 5$

$\Rightarrow \lim_{h \rightarrow 0} -2(1+h) + \log_2(b^2 - 2) \leq 5$

$\Rightarrow -2 + \log_2(b^2 - 2) \leq 5$

$\Rightarrow \log_2(b^2 - 2) \leq 7$

$$\Rightarrow b^2 - 2 \leq 2^7$$

$$\Rightarrow b^2 \leq 130$$

but $b^2 - 2 > 0$

$$\therefore 2 < b^2 \leq 130$$

Hence, $\lambda = 130$

6. CD

Sol. $0 \leq \sin^2 x \leq 1$

$$\Rightarrow -1 \leq -\sin^2 x \leq 0$$

$$\therefore [\sin^2 x] = 0 \text{ or } -1$$

but $\sec^{-1}(0)$ is not defined

hence $y = \sec^{-1}[-\sin^2 x] = \sec^{-1}(-1) = \pi$

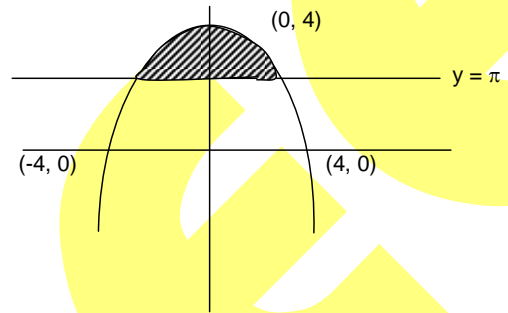
now $\pi = \frac{16 - x^2}{4}$

$$\Rightarrow x^2 = 16 - 4\pi = 4(4 - \pi)$$

$$x = \pm 2\sqrt{(4 - \pi)}$$

The required area =

$$\int_{-2\sqrt{4-\pi}}^{2\sqrt{4-\pi}} \left(\frac{16 - x^2}{4} - \pi \right) dx = \frac{8}{3} (4 - \pi)^{3/2}$$



7. ABCD

Sol. $2\left(\frac{1}{1!9!} + \frac{1}{3!7!}\right) + \frac{1}{5!5!} = \frac{2^a}{b!}$

$$\Rightarrow \frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!} \left(\frac{2!10!}{1!9!} + \frac{2!10!}{3!7!} + \frac{10!}{5!5!} \right) = \frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!} (2 \cdot {}^{10}C_1 + 2 \cdot {}^{10}C_3 + {}^{10}C_5) = \frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!} ({}^{10}C_1 + {}^{10}C_1 + {}^{10}C_3 + {}^{10}C_3 + {}^{10}C_5) = \frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!} ({}^{10}C_1 + {}^{10}C_9 + {}^{10}C_3 + {}^{10}C_7 + {}^{10}C_5) = \frac{2^a}{b!}$$

$$\Rightarrow \frac{1}{10!} ({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9) = \frac{2^a}{b!}$$

$$\Rightarrow \frac{2^{10-1}}{10!} = \frac{2^a}{b!}$$

$$\therefore a = 9, b = 10$$

Also, $x = 2 + 5i$

$$\therefore (x - 2)^2 = (5i)^2$$

$$\Rightarrow x^2 - 4x + 29 = 0$$

$$\therefore x^3 - 5x^2 + 33x - 19 = (x - 1)(x^2 - 4x + 29) + 10$$

$$= 0 + 10$$

$$(\because x^2 - 4x + 29 = 0)$$

$$= 10 = b$$

8. D

Sol. Side of square = 2

$$\frac{x}{\cos 30^\circ} = \frac{y}{\sin 30^\circ} = 2$$

$$\Rightarrow x = \frac{2\sqrt{3}}{2} = \sqrt{3} \text{ and } y = 1$$

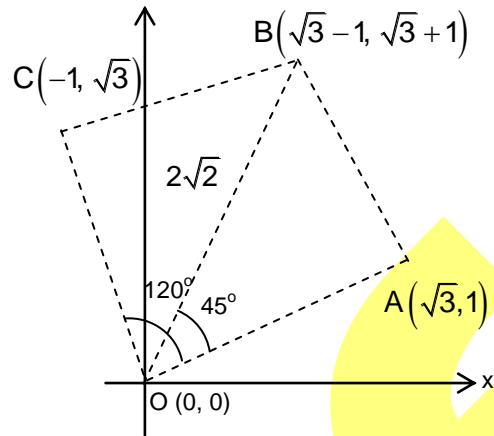
$$\frac{x}{\cos 120^\circ} = \frac{y}{\sin 120^\circ} = 2$$

$$\Rightarrow x = -1, y = \sqrt{3}$$

$$\frac{x}{\cos 75^\circ} = \frac{y}{\sin 75^\circ} = 2\sqrt{2}$$

$$\Rightarrow x = \sqrt{3} - 1 \text{ and } y = \sqrt{3} + 1$$

$$\therefore \text{Required sum} = 0 + \sqrt{3} + \sqrt{3} - 1 + (-1) \\ = 2\sqrt{3} - 2$$



9. A

Sol. Let $ax + by = 1$ be the chord (1)

Making the equation of hyperbola homogeneous using (1),

We get

$$3x^2 - y^2 + (-2x + 4y)(ax + by) = 0$$

$$\text{or, } (3 - 2a)x^2 + (-1 + 4b)y^2 + (-2b + 4a)xy = 0$$

Since the angle subtended at the origin is a right angle, so, coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow (3 - 2a) + (-1 + 4b) = 0 \Rightarrow a = 2b + 1$$

$$\therefore \text{The chords are } (2b + 1)x + by - 1 = 0$$

$$\text{Or, } b(2 + y) + (x - 1) = 0,$$

which, clearly, pass through the fixed point (1, -2)

10. D

Sol. Total number of ways in which $n_1 + n_2 = 100$ is equal to 99.

Now, $n_1, n_2 > 1600$

$$\Rightarrow n_1(100 - n_1) > 1600$$

$$\Rightarrow n_1^2 - 100n_1 + 1600 < 0$$

$$\Rightarrow (n_1 - 80)(n_1 - 20) < 0$$

$$\Rightarrow 20 < n_1 < 80$$

$$\Rightarrow 21 \leq n_1 \leq 79$$

Thus, number of favourable ways

$$= 79 - 21 + 1 = 59$$

$$\text{Hence, required probability} = \frac{59}{99},$$

11. D

Sol. We have,

$$z_1 = \frac{4 + 3i}{1 + 2i} = \frac{(4 + 3i)(1 - 2i)}{(1 + 2i)(1 - 2i)}$$

$$= \frac{10 - 5i}{5} = 2 - i \text{ which represents the point whose coordinates are } (2, -1)$$

Also, we have,

$$iz = \bar{z}$$

$$\Rightarrow i(x + iy) - (x - iy) = 0 \quad [\text{Putting } z = x + iy]$$

$$\Rightarrow i(x + y) - (x + y) = 0$$

$$\Rightarrow (i - 1)(x + y) = 0 \text{ which represents the line } y = -x$$

Hence, reflection of the point (2, -1) in the line $y = -x$ gives the point (1, -2) which is equivalent to $1 - 2i$ in the argand plane.

The correct option is (D)

12. D

Sol. Given, $b^2 \leq 4ac$, $c^2 \leq 4ab$ and $a^2 \leq 4ac$

Equality cannot hold simultaneously

[\because a, b, c are different]

$$\therefore a^2 + b^2 + c^2 < 4(ab + bc + ca) \Rightarrow R < 4$$

$$\text{Also, } a^2 + b^2 + c^2 - ab - bc - ca$$

$$\Rightarrow \frac{1}{2}[(b-c)^2 + (c-a)^2 + (a-b)^2] > 0 \Rightarrow R > 1$$

The correct option is (D)

13. B

Sol. $z = \omega$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} = \begin{bmatrix} \omega^{2r} + \omega^{4s} & \omega^{r+2s}(1 + (-1)^r) \\ \omega^{r+2s}(1 + (-1)^r) & \omega^{2r} + \omega^{4s} \end{bmatrix} = -I_2$$

$$\Rightarrow 1 + (-1)^r = 0 \Rightarrow r = 1 \text{ or } 3 \text{ and } \omega^{2r} + \omega^{4s} = -1 \quad \dots\dots\dots(i)$$

Possible options are (r, s) = (1, 1), (1, 2), (1, 3), (3, 1), (3, 2), (3, 3) of which one (1, 1) satisfies (i).

14. D

$$\text{Sol. As, } T_n = \cot^{-1} \left[\frac{(n+1)(n+2)}{2} x + \frac{2}{x} \right]$$

$$\Rightarrow T_n = \tan^{-1} \left(\frac{2x}{(n+2)(n+1)x^2 + 4} \right)$$

$$\therefore T_n = \tan^{-1} \left(\left(\frac{n+2}{2} \right) x \right) - \tan^{-1} \left(\left(\frac{n+1}{2} \right) x \right)$$

$$\text{So, } S_n = \tan^{-1} \left(\left(\frac{n+2}{2} \right) x \right) - \tan^{-1} x$$

$$\Rightarrow \lim_{n \rightarrow \infty} S_n = \frac{\pi}{2} - \tan^{-1} x$$

$$= \cot^{-1} x = 1 \quad (\text{given})$$

$$\Rightarrow x = \cot 1$$

15. B

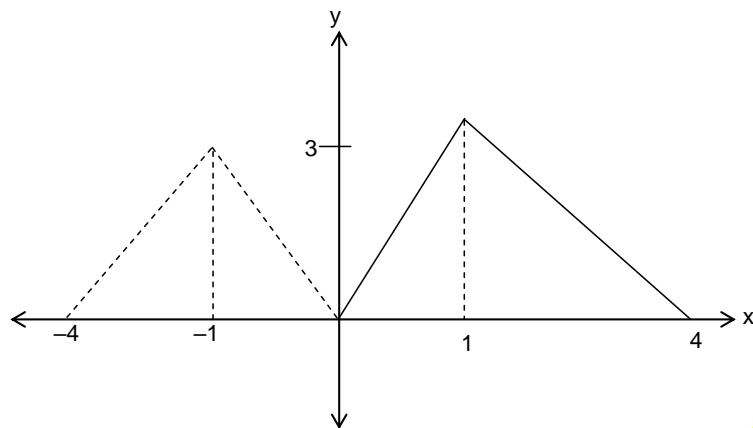
$$\text{Sol. Required area} = 2 \times \frac{1}{2} \times 4 \times 3 + 2 + 2 \times \frac{1}{2} \times 1 \times 3 = 15$$

16. A

$$\text{Sol. } f(-89) - f(-67) + f(46) = f(-1) - f(-3) + f(-2) \\ = 3 - 1 + 2 = 4$$

$$f(x-2) = f(x+6)$$

$$\Rightarrow f(x) = f(x+8)$$



17. A

18. B

(Sol. 17 to 18)

$$f(xy) = f(x) + f(y) + xy - x - y$$

Partially differentiating w.r.t. y (taking x as a constant)

$$f'(xy) \cdot x = f'(y) + x - 1$$

Putting $y = 1$

$$f'(x) = x = f'(1) + x - 1$$

$$f'(x) \cdot x = 3 + x$$

$$f'(x) = \frac{3}{x} + 1$$

Integrating both sides.

$$f(x) = 3 \ln x + x + c$$

From the given functional rule

$$x = y = 1, f(1) = 1$$

$$f(x) = 3 \ln x + x + c$$

$$\therefore f(x) = 3 \ln x + x$$

$$(i) f(x) = 0 \Rightarrow 3 \ln x + x = 0$$

$$\Rightarrow \ln x = \frac{-x}{3}$$

Clearly, x_0 lies in the interval $(0, 1)$.

$$(ii) \int e^{f(x)} dx = e^x (ax^3 + bx^2 + cx + d) + \lambda$$

$$\Rightarrow$$

$$\int e^{3 \ln x + x} dx = e^x (ax^3 + bx^2 + cx + d) + \lambda$$

