TEST CODE : 113011

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JEE-JEE
Batch-Growth (July) | Minor Test - 11

Time: 3 Hours **Date: 19th January 2025** **Maximum Marks:300**

Name of the Candidate: ____________________________________Roll No.____________

Centre of Examination (in Capitals):____

Candidate's Signature:___________________ Invigilator's Signature:____________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 75 questions.
3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 25 questions each and every **PART** has two sections.
   (i) **Section-I** contains 20 multiple choice questions with only one correct option. Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
   (ii) **Section-II** contains 5 questions with INTEGERAL VALUE.
   Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For the numerical based questions in Section-II of Mathematics, Physics, Chemistry, the answer should be in whole number only.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

TEST SYLLABUS

Batch-Growth (July) | Minor Test – 11

Test Date-19th Jan. 2025

Mathematics : Parabola

Physics : KTG & Thermodynamics

Chemistry : GOC

Useful Data Chemistry:

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's Constant	h	=	6.626×10^{-34} Js
		=	6.25×10^{-27} erg.s
1 Faraday		=	96500 Coulomb
1 calorie		=	4.2 Joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No : H=1, D=1, Li=3, Na=11, K=19, Rb=37, Cs=55, F=9, Ca=20, He=2, O=8, Au=79.

Atomic Masses: He=4, Mg=24, C=12, O=16, N=14, P=31, Br=80, Cu=63.5, Fe=56, Mn=55, Pb=207, Au=197, Ag=108, F=19, H=2, Cl=35.5, Sn=118.6

Useful Data Physics:

Acceleration due to gravity g = 10 m/s^2

PART-A: MATHEMATICS

SECTION-I

1. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)$ $(\beta, 0)$ both to the right of the origin, A circle also passes through these two points. The length of a tangent from the origin to the circle is:

(A) $\sqrt{\frac{bc}{a}}$

(B) ac^2

(C) $\frac{b}{a}$

(D) $\sqrt{\frac{c}{a}}$

Ans. (D)

Sol. $y = ax^2 + bx + c$

Equation of circle $S + \lambda L = 0$

$(x-\alpha)(x-\beta) + y^2 + \lambda(y) = 0$

$(0, 0)$ length of tangent $\sqrt{S_1} = \sqrt{\alpha\beta + 0}$

$= \sqrt{\frac{c}{a}}$

2. If AFB is a focal chord of the parabola $y^2 = 4ax$ and $AF = 4$, $FB = 5$, then length of latus rectum of parabola is:

(A) $\frac{81}{12}$

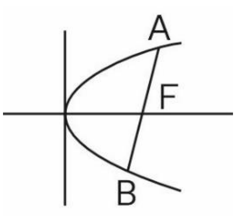
(B) $\frac{80}{9}$

(C) $\frac{81}{10}$

(D) 9

Ans. (B)

Sol.



$2a = \frac{2AF \times FB}{AF + FB}$

$4a = \frac{2 \times 2 \times 4 \times 5}{9} = \frac{80}{9}$

3. Length of the latus rectum of the parabola $25\left[(x-2)^2 + (y-3)^2\right] = (3x - 4y + 7)^2$ is

(A) 4

(B) 2

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

Ans. (D)

Sol. $(x-2)^2+(y-3)^2=\left(\frac{3x-4y+7}{\sqrt{25}}\right)^2$

$2a=\left|\frac{3(2)-4(3)+7}{\sqrt{(3)^2+(-4)^2}}\right|=\frac{1}{5}$

$4a=\frac{2}{5}$

4. The Angle between the tangents drawn from a point $(-a,\ 2a)$ to $y^2=4ax$ is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{6}$

Ans. (C)

Sol. Point (–a, 2a) lies on directrix.

5. If $y=2x+\lambda$ is normal to parabola $y^2=8x$, then λ is

(A) –25

(B) –24

(C) –18

(D) –19

Ans. (B)

Sol. $y=2x+\lambda$

$y=mx-2am-am^3$

$\lambda=-2am-am^3=-2\times2\times2-2\times(2)^3=-24$

6. The length of chord of the parabola $x^2=4y$ passing through the vertex and having slope as $\cot\alpha$ is:

(A) $4\cos\alpha\ \text{cosec}^2\alpha$

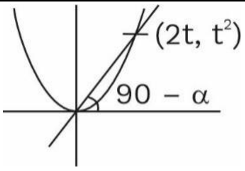
(B) $4\tan\alpha\ \sec\alpha$

(C) $4\sin\alpha\ \sec^2\alpha$

(D) None of these

Ans. (A)

Sol.



$$\frac{t^2}{2t} = \cot\alpha$$

$$t = 2\cot\alpha$$

$$L = \sqrt{4t^2 + t^4} = (2\cot\alpha)\sqrt{4 + 4\cot^2\alpha}$$

$$L = 2\cot\alpha(2\operatorname{cosec}\alpha)$$

$$L = 4\cos\alpha\operatorname{cosec}^2\alpha$$

7. The minimum distance between $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$

(A) $\sqrt{6}$

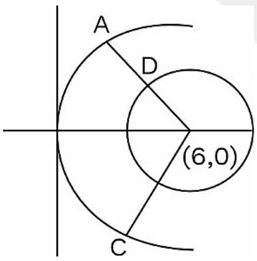
(B) $\sqrt{7}$

(C) 2

(D) $\sqrt{5}$

Ans. (D)

Sol.



The shortest distance between the two curves will be along common normal.

$\therefore$ Equation of normal at $(t^2, 2t)$ is

$$y = -tx + 2t + at^3$$

But it passes through centre (6, 0) of circle.

$\therefore$ $t^3 - 4t = 0 \Rightarrow t = 0,\ t = \pm 2$

$\therefore$ $A = (4, 4),\ C = (4, -4)$

$\Rightarrow$ $AD = AP - PD = 2\sqrt{5} - \sqrt{5} = \sqrt{5}$

8. y-intercept of the common tangent to the parabola $y^2 = 32x$ and $x^2 = 108y$.

(A) –18

(B) –12

(C) –9

(D) 6

Ans. (B)

Sol. $y = mx + \dfrac{8}{m}$ $\qquad y = mx - 27m^2$

$$\frac{8}{m} = -27m^2$$

$$m^3 = -\frac{8}{27}$$

$$m = -\frac{2}{3}$$

$$y = mx + \frac{8}{m} \qquad y = -\frac{2}{3}x - 12.$$

9. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola to the point (1, 2) is:

(A) $(x-1)^2 = 4(y+1)$

(B) $(x-1)^2 = 4(y+1)^2$

(C) $(x+1)^2 = 4(y-1)$

(D) $(x-1)^2 = 4(y-1)$

Ans. (C)

Sol. Equation of tangent at (1, 2) is, $y - 2 = 1(x-1)$

$\Rightarrow x - y + 1 = 0$

Let (h, k) is the mirror image of point $(t^2, 2t)$ for the tangent $x - y + 1 = 0$.

Then $\frac{h-t^2}{1} = \frac{k-2t}{-1} = -2\frac{(t^2-2t+1)}{1+1}$

$\Rightarrow \quad h = 2t - 1 \quad \Rightarrow t = \frac{h+1}{2}$

$\Rightarrow k = t^2 + 1$

$$k = \left(\frac{h+1}{2}\right)^2 + 1$$

$$4k = (h+1)^2 + 4$$

Replacing (h, k) with (x, y)

$$4y = (x+1)^2 + 4$$

$$(x+1)^2 = 4(y-1)$$

10. The length of focal chord of parabola $y^2 = 12x$ which makes an angle 60° with positive x-axis is:

(A) 18

(B) 16

(C) 20

(D) 15

Ans. (B)

Sol. Length of focal chord = $4a\,\mathrm{cosec}^2\alpha$

Length of focal chord = $4 \times 3 \times \frac{4}{3} = 16$

11. If three distinct and real normal can be drawn to $y^2 = 8x$ from the point (a, 0), then $a \in$

(A) [0, 4]

(B) $(-\infty, 4)$

(C) $(4, \infty)$

(D) ϕ

Ans. (C)

Sol. If $y = mx - 2am - am^3$ passes through (a, 0), then $am - 4m - 2m^3 = 0$

$\Rightarrow m(a - 4 - m^2) = 0$

$\Rightarrow m = 0, m^2 = \dfrac{a-4}{2}$

For three distinct normal, $a - 4 > 0 \Rightarrow a > 4$

12. The co-ordinate of a point on the parabola $y^2 = 8x$, whose focal distance is 4 unit is:

(A) (2, 4)

(B) (4, 2)

(C) (–2, –4)

(D) (4, –2)

Ans. (A)

Sol. $a = 2$ focus $S(2, 0)$

$\Rightarrow$ focal distance of point P = SP

$= \sqrt{(x_1 - 2)^2 + (y_1 - 0)^2}$ $y_1^2 = 8x_1$

$= |x_1 + 2| = 4$ $x_1 = 2, -6$

(2, 4)

13. If (2, –8) is one end of a focal chord of the parabola $y^2 = 32x$, then the other end of the focal chord is-

(A) (32, 32)

(B) (32, –32)

(C) (–2, 8)

(D) (2, 8)

Ans. (A)

Sol. $P(8t_1^2, 16t_1)$ $Q(8t_2^2, 16t_2)$

For focal chord $t_1t_2 = -1$

$8t_1^2 = 2$ $16t_1 = -8$

$t_1 = \dfrac{-1}{2}$

$t_2\left(\dfrac{-1}{2}\right) = -1$

$t_2 = 2$

$(8\times(2)^2, 16\times 2)$ i.e. (32, 32)

14. A line from (–1, 0) intersects the parabola $x^2 = 4y$ at A and B. Then the locus of centroid of ΔOAB is:

(A) $3x^2 - 2x = 4y$

(B) $3y^2 - 2y = 4x$

(C) $3x^2 + 2x = 4y$

(D) None of these

Ans. (C)

Sol. Let $A(2t_1, t_1^2)$ and $B(2t_2, t_2^2)$ P(–1, 0), A and B are collinear.

$$\Rightarrow \frac{t_2^2 - t_1^2}{2(t_2 - t_1)} = \frac{t_2^2}{2t_2 + 1}$$

$$\Rightarrow (t_1 + t_2) = -2t_1t_2$$

Let centroid be (h, k)

$$\therefore\ h = \frac{2(t_1 + t_2)}{3}$$

$$\therefore\ k = \frac{t_1^2 + t_2^2}{3}$$

$$3h^2 + 2h = 4k$$

15. The locus of vertices of family of parabolas $y = ax^2 + 2a^2x + 1$ $(a \neq 0)$ is a curve passing through

(A) (1, 0)

(B) (1, 1)

(C) (0, 1)

(D) (0, 0)

Ans. (C)

Sol. $y = ax^2 + 2a^2x + 1$

$$\frac{y}{a} = x^2 + 2ax + \frac{1}{a}$$

$$\frac{y}{a} = (x + a)^2 + \frac{1}{a} - a^2$$

$$\frac{y}{a} = (x + a)^2 + \frac{1 - a^3}{a}$$

$$(y - (1 - a^3)) = a(x + a)^2$$

$$h = -a \quad k = 1 - a^3$$

$$k = 1 + h^3$$

$$x^3 - y + 1 = 0$$

16. The Area (in sq. unit) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola is:

(A) $256\sqrt{3}$

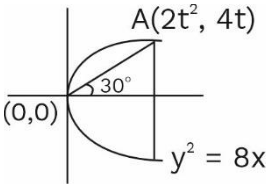
(B) $64\sqrt{3}$

(C) $128\sqrt{3}$

(D) $192\sqrt{3}$

Ans. (D)

Sol.



$$\tan 30^\circ = \frac{4t}{2t^2} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$AB = 8t = 16\sqrt{3}$$

$$\text{Area} = 256.3.\frac{\sqrt{3}}{4} = 192\sqrt{3}$$

17. The locus of the centre of the circle described on by focal chord of a parabola $y^2 = 4ax$ as diameter is

(A) $x^2 = 2a(y-a)$

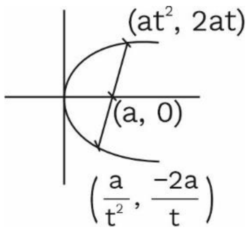
(B) $x^2 = -2a(y-a)$

(C) $y^2 = 2a(x-a)$

(D) $y^2 = -2a(x-a)$

Ans. (C)

Sol.



$$\frac{a\left(t^2 + \frac{1}{t^2}\right)}{2} = h$$

$$\frac{2a\left(t - \frac{1}{t}\right)}{2} = k$$

$$y^2 = 2a(x-a)$$

18. The director circle of the parabola $(y-2)^2 = 16(x+7)$ touches the circle $(x-1)^2 + (y+1)^2 = r^2$, then r is equal to:

(A) 10

(B) 11

(C) 12

(D) None of these

Ans. (C)

Sol. Director circle of $(y-2)^2 = 16(x+7)$

$$x+7=-4$$

$$x=-11 \qquad x+11=0$$

$$\frac{1+11}{1}=r \qquad r=12$$

19. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x+y=1$. Which of the following point does not lie on it?

(A) (0, 3)

(B) (–6, 0)

(C) (4, 5)

(D) (5, 4)

Ans. (D)

Sol. $y = mx + \frac{a}{m}$

$$y = \frac{x}{2}+3$$

20. Which one of the following equations represented parametrically, represent equation to a parabolic profile?

(A) $x = 3\cos t$; $y = 4\sin t$

(B) $x^2 - 2 = -2\cos t$; $y = 4\cos^2\left(\frac{t}{2}\right)$

(C) $\sqrt{x} = \tan t$, $\sqrt{y} = \sec t$

(D) $x = \sqrt{1-\sin t}$, $y = \sin\frac{t}{2} + \cos\left(\frac{t}{2}\right)$

Ans. (B)

Sol. $x^2 = 2(1-\cos t) \qquad y = 4\cos^2\left(\frac{t}{2}\right)$

$$x^2 = 2\times 2\times \sin^2\frac{t}{2} \qquad y = 4\cos^2\left(\frac{t}{2}\right)$$

$$x^2 = 2\times 2\times \sin^2\frac{t}{2} \qquad y = 4-4\sin^2\frac{t}{2}$$

$$y = 4-4\times\left(\frac{x^2}{4}\right)$$

$$x^2+y=4$$

SECTION-II

21. If the angle between the two tangents drawn from the point (1, 4) to the parabola $y^2 = 12x$ is $\tan^{-1}(k)$, then 2k is equal to:

Ans. (1)

Sol. $y = mx + \frac{a}{m}$

$4 = m + \frac{3}{m}$

$m^2 - 4m + 3 = 0$

$m = 1, m = 3$

$\tan\theta = \left|\frac{3-1}{1+3}\right| = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$k = \frac{1}{2} \quad 2k = 1$

22. The straight line $3x - 4y + C = 0$ intersect the parabola $x^2 = 4y$ at the point A & B, if the lines joining points A & B to the vertex of parabola are perpendicular, then C is equal to

Ans. (16)

Sol. $3x - 4y = -C$

$\frac{4y - 3x}{C} = 1$

$x^2 - 4y = 0$

$x^2 - 4y\left(\frac{4y - 3x}{C}\right) = 0$

$x^2 - \frac{16y^2}{C} + \frac{12xy}{C} = 0$

$1 - \frac{16}{C} = 0$

$C = 16$

23. If $y = px + q$ is the common normal of parabola $y^2 = 4x$ and $(y-2)^2 = 4(x-1)$ then |p + q| is equal to:

Ans. (10)

Sol. Equation of normal

$y = mx - 2am - am^3$ of parabola $y^2 = 4x$, $a = 1$

Equation of normal $y - 2 = m(x-1) - 2m - m^3$ of second parabola

$y = mx - 3m - m^3 + 2$

Both respect the same line

$\frac{1}{1} = \frac{-2m - m^3}{-3m - m^3 + 2} \Rightarrow m = 2$

$\therefore$ Equation of normal $y = 2x - 12$

$P = 2, q = -12$

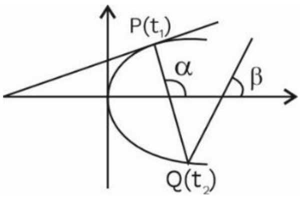
$|p+q| = 10$

24. If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angle α and β, respectively with the x-axis, then $|\tan\alpha(\tan\alpha + \tan\beta)|$ is equal to

Ans. (2)

Sol. Slope of normal at $t_1 = \tan\alpha = -t_1$

Slope of normal at $t_2 = \tan\beta = -t_2$



As normal at t_1 meets parabola at t_2,

Then $t_2 = -t_1 - \frac{2}{t_1}$

Or $t_1t_2 + t_1^2 = -2$

$\Rightarrow \tan\alpha(\tan\beta + \tan\alpha) = -2$

$|\tan\alpha(\tan\beta + \tan\alpha)| = 2$

25. The midpoint of the chord $x + y = 2$ of parabola $y^2 = 4x$ is (p, q), then p + q is equal to:

Ans. (2)

Sol. Let the mid-point (p, q)

$T = S_1$

$yq - 2(x + P) = q^2 - 4P$

$2x - qy + q^2 - 2p = 0$

$x + y = 2$

$\frac{2}{1} = \frac{-q}{1} = \frac{q^2 - 2p}{-2}$

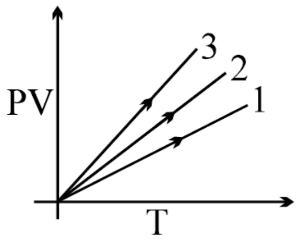
$p = 4, q = -2$ $\qquad$ $p + q = 2$

PART-B: PHYSICS

SECTION-I

26. PV versus T graph of equal masses of H_2, He and CO_2 is shown in figure.

Choose the correct alternative.



(A) 3 correspond to H_2, 2 to He and 1 to CO_2.

(B) 1 corresponds to He, 2 to H_2 and 3 to CO_2.

(C) 1 corresponds to He, 3 to H_2 and 2 to CO_2.

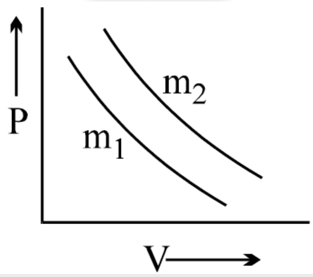
(D) 1 corresponds to CO_2, 2 to H_2 and 3 to He

Ans. (A)

Sol. $\frac{PV}{T} = \tan\theta = nR$

$\therefore$ slop α no. of moles.

27. Two different isotherms representing the relationship between pressure P and volume V at a same temperature of the same ideal gas are shown for masses m_1 and m_2 of the gas respectively in the figure given, then:



(A) $m_1 > m_2$

(B) $m_1 = m_2$

(C) $m_1 < m_2$

(D) All of the above are possible

Ans. (C)

Sol. $PV = \frac{m}{M}RT$

$V \alpha m$ (for given pressure)

$V_1 < V_2 \Rightarrow m_1 < m_2$

28. Let v denote the root mean square speed of the molecules in an ideal diatomic gas at absolute temperature T. the mass of a molecule is m. Neglecting vibration energy terms, which is false?

(A) a molecule can have a speed greater than $\sqrt{2}\,v$

(B) v is proportional to $\sqrt{T}$

(C) the average rotational kinetic energy of a molecule is $\frac{mv^2}{4}$

(D) the average kinetic energy of a molecule is $\frac{5mv^2}{6}$

Ans. (C)

Sol. Avg. K. E. $= \frac{5}{2}kT = \frac{5}{6}mv^2 \left[\therefore \frac{1}{2}mv^2 = \frac{3}{2}kT\right]$.

So rotational $K.E. = 2\times\frac{1}{2}kT$

$$= 2\times\frac{1}{6}mv^2$$

$$= \frac{mv^2}{3}$$

(C) is false.

29. An isolated container holds 4 moles of hydrogen and 2 moles of helium at 400 K. The ratio of the total kinetic energy of the hydrogen molecules to the total kinetic energy of the helium molecules is:

(A) 4:1

(B) 2:1

(C) 1:2

(D) 10:3

Ans. (D)

Sol. $k = \left(\frac{1}{2}RT\right)(n)f$

$$k_{H_2} = \left(\frac{1}{2}RT\right)\times 4(5) = 10RT$$

$$k_{He} = \left(\frac{1}{2}RT\right)\times 2(3) = 3RT$$

30. Which of the following sample of gases has the same internal energy as the energy of one mole of hydrogen gas at 27°C.

(A) 1 mole of Helium gas at 327°C

(B) 3 mole of Helium gas at 27°C

(C) $\frac{1}{2}$ mole of O_2 gas at 327°C

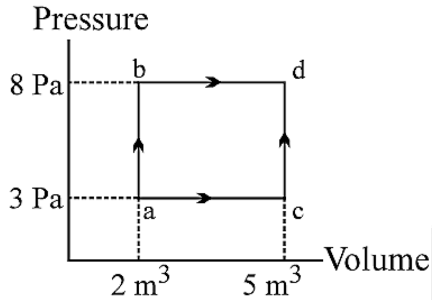
(D) 2 mole of O_2 gas at 27°C

Ans. (C)

Sol. $U = nC_vT = 1 \times \frac{5}{2}R \times 300 = 750R$ for H_2.

$U = nC_vT = \frac{1}{2} \times \frac{5R}{2} \times 600 = 750R$ for O_2

31. In the pressure versus Volume graph shown, in the process of going from a to b, 60 J of heat is added, and in the process of going from b to d, 20 J of heat is added. In the process of going from a to c to d, what is the total heat (in Joule) added?



(A) 80

(B) 60

(C) 65

(D) 56

Ans. (C)

Sol. $Q_{abd} - Q_{acd} = (W_{abd} - W_{acd}) + (\Delta U_{abd} - \Delta U_{acd})$

$= W_{abd} - W_{acd} + 0$ (internal energy change is same for two paths)

$=$ area of abdca $= 15$ J

$Q_{abd} = Q_{ab} + Q_{bd} = 60 + 20 = 80\,J$

$Q_{acd} = Q_{abd} - 15 = 65$ J

32. The efficienty of a carnot engine is 0.6. It rejects total 20 J of heat. Find work done (in joule) by the engine.

(A) 30

(B) 40

(C) 50

(D) 20

Ans. (A)

Sol. $0.6 = \frac{\text{workdone}}{Q_{input}} = \frac{Q_{input} - Q_{reject}}{Q_{input}}$

$= 1 - \frac{Q_r}{Q_i}$, $0.6 = 1 - \frac{20}{Q_i}$, $Q_1 = 50$

$W = Q_i - Q_r = 30$ J

33. The heat capacity at constant volume of a sample of an ideal monoatomic gas is 35 J/K. Following are given three statements. State in order form (i) to (iii) whether they are are true (T) or False (F). Taking $R = \frac{25}{3} J/mole\ K$.

(i) Number of moles of the gas are 1.5

(ii) Internal energy of the gas at 0°C is approx $35 \times 273 J$

(iii) Molar heat capacity at constant pressure is approx 125/6 J/mol-K

(A) TTT

(B) TFT

(C) FTT

(D) TFF

Ans. (C)

Sol. (i) $C = nC_v, C_v = \frac{f}{2}R = \frac{3}{2}R, n = \frac{C}{C_v} = \frac{35}{(3/2)R} = \frac{35}{3} \times \frac{2}{R}$

$$n = \frac{35}{3} \times \frac{2}{25} \times 3 = \frac{14}{5} = 2.8, n = 2.8$$

(ii) $V = \frac{f}{2}nRT = \frac{3}{2} \times \frac{14}{5} \times \frac{25}{3} \times 273 = 35 \times 273 J$

(iii) $C_p - C_v = R, C_p = C_v + R, C_p = \frac{25}{2} + \frac{25}{3} = \frac{125}{6}$

34. If one mole of an ideal monoatomic gas $(\gamma = 5/3)$ is mixed with one mole of ideal diatomic gas $(\gamma = 7/5)$, the find the value of γ for the mixture.

(A) 1.5

(B) 1.4

(C) 1.53

(D) 3.07

Ans. (A)

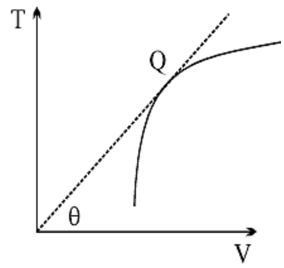
Sol. $U_{mixture} = U_{monoatomic} + U_{diatomic}$

$$\therefore \frac{(n_1 + n_2)RT}{\gamma_{mix} - 1} = \frac{n_1RT}{\gamma_1 - 1} + \frac{n_2RT}{\gamma_2 - 1}$$

$$\therefore \frac{2}{\gamma_{mix} - 1} = \frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{1}{\left(\frac{7}{5} - 1\right)}$$

$$\therefore \gamma_{mix} = 1.5$$

35. Consider the TV diagram of a thermodynamic process. Using the details shown in the figure, find out the molar heat capacity of the gas at the point Q. Consider the gas to be ideal and monoatmic.



(A) 2.5R

(B) 3R

(C) 4R

(D) 5.2R

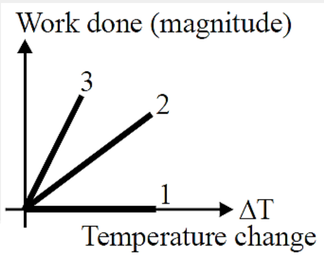
Ans. (A)

Sol. $CdT = PdV + 1.5\ RdT$

$$C = \frac{P}{(dT/dV)} + 1.5R = \frac{P}{(T/V)} + 1.5R = \frac{PV}{T} + 1.5R$$

$C = 2.5R$

36. For an ideal gas graph is shown for three processes. Processes 1, 2 and 3 respectively are.



(A) Isobaric, adiabatic, isochoric

(B) Adiabatic, isobaric, isochoric

(C) Isochoric, adiabatic, isobaric

(D) Isochoric, isobaric, adiabatic

Ans. (D)

Sol. Isochoric process dv = 0

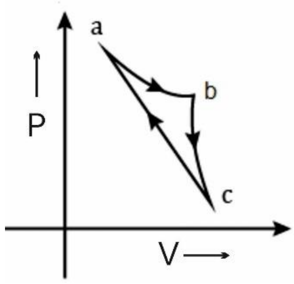
W = 0

Isobaric : $W = P\Delta V = nR\Delta T$

Adiabatic : $W = \dfrac{nR(T_i - T_f)}{\gamma - 1}$

$$|W| = \frac{nR\Delta T}{\gamma - 1} \quad \Rightarrow 0 < \gamma - 1 < 1$$

37. In the P-V diagram shown, the gas does 5 J of work in isothermal process ab and 4 J in adiabatic process bc. What will be the magnitude of change in internal energy (in Joule) of the gas in straight path c to a?



(A) 9

(B) 1

(C) 5

(D) 4

Ans. (D)

Sol. $\Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$

$\Delta U_{ab} = 0$ Isothermal

$\Delta U_{bc} = -\Delta w_{bc}$ adiabatic

Hence $\Delta U_{ca} = -4\,J$

38. The temperature of an ideal gas is increased from 27°C to 927°C. Then how many times rms speed will increase?

(A) twice

(B) half

(C) four times

(D) one fourth

Ans. (A)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1200}{300}} = 2$

39. Under constant temperature, graph between P and $\frac{1}{V}$ is a-

(A) Parabola

(B) Hyperbola

(C) Straight Line

(D) Circle

Ans. (C)

Sol. From ideal gas equation $PV = nRT \Rightarrow P = (nRT)\frac{1}{V}$

$y = mx$ (straight line)

40. At constant temperature on increasing the pressure of a gas by 5%. Then find percentage decreases in its volume.

(A) 5.26%

(B) 4.26%

(C) 4.76%

(D) 5

Ans. (D)

Sol. at constant temperature PV = constant

$$\Rightarrow P \alpha \frac{1}{V} \quad \Rightarrow \frac{\Delta P}{P} = 1 \times \frac{\Delta V}{V} \quad \Rightarrow \frac{\Delta P}{P} \times 100 = \frac{\Delta V}{V} \times 100$$

$$\Rightarrow \frac{\Delta V}{V} \times 100 = 5\%$$

41. If the molecular weight of two gases are M_1 and M_2, then at a temperature the ratio of root mean square velocity (v_1) and (v_2) will be-

(A) $\sqrt{\frac{M_1}{M_2}}$

(B) $\sqrt{\frac{M_2}{M_1}}$

(C) $\sqrt{\frac{M_1+M_2}{M_1-M_2}}$

(D) $\sqrt{\frac{M_1-M_2}{M_1+M_2}}$

Ans. (B)

Sol. $V = \sqrt{\frac{3RT}{M}} \Rightarrow V \alpha \frac{1}{\sqrt{M}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}}$

42. A perfect gas is compressed adiabatically. In that state the value of $\frac{\Delta P}{P}$ will be-

(A) $\frac{1}{\gamma}\frac{\Delta V}{V}$

(B) $-\frac{\Delta V}{V}$

(C) $-\gamma\frac{\Delta V}{V}$

(D) $\gamma\frac{\Delta V}{V}$

Ans. (C)

Sol. $PV^{\gamma} = \text{constant} \Rightarrow P\gamma V^{\gamma-1}\frac{dV}{dx} + V^{\gamma}\frac{dP}{dx} = 0 \Rightarrow \gamma PdV = VdP$

$$\Rightarrow -\frac{\gamma dV}{V} = \frac{dp}{P}$$

43. The ratio of the slope of adiabatic and isothermal (P, V) curves will be-

(A) γ

(B) $\frac{1}{\gamma}$

(C) $\gamma+1$

(D) $\gamma-1$

Ans. (A)

Sol. Slope of adiabatic curve $= -\gamma\dfrac{P}{V}$

Slope of isothermal curve = $= -\dfrac{P}{V}$

44. On mixing 1 gm mole of a monoatomic with 1 gm mole of a diatomic gas the specific heat of mixture at constant volume will be-

(A) R

(B) $\dfrac{3}{2}$ R

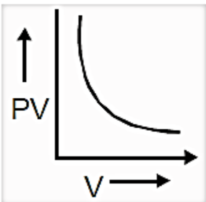
(C) 2R

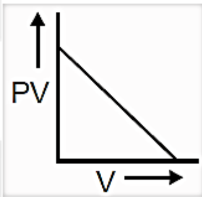
(D) $\dfrac{5}{2}$ R

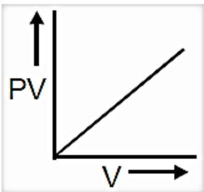
Ans. (C)

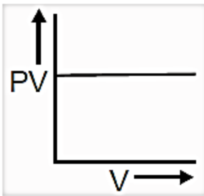
Sol. $(C_v)_{mix} = \dfrac{n_1Cv_1 + n_2Cv_2}{n_1 + n_2} = \dfrac{1\times 3R/2 + 1\times 5R/2}{1+1} = 2R$

45. Which of the following curves is isothermal-

(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. Process equation of isothermal curve PV = constant.

SECTION-II

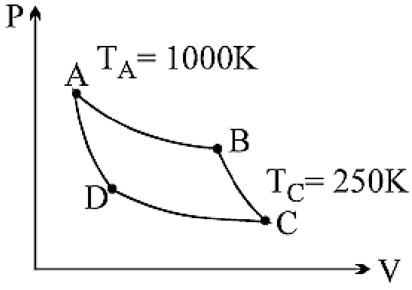
46. A monoatomic ideal gas expands at constant pressure, with heat Q supplied. If X is the percentage of heat supplied which goes as work done by the gas then find X.

Ans. (40)

Sol. For isobaric process, $\dfrac{W}{Q} = \dfrac{nR(\Delta T)}{nC_p(\Delta T)} = \dfrac{R}{C_P}$

$\therefore \frac{W}{C_P} = \frac{2}{5}$ (for monoatomic gas)

47. A monatomic ideal gas is used as the working substance for the Carnot cycle shown in the figure. Processes AB and CD are isothermal, while processes BC and DA are adiabatic. During process AB, 400 J of work is done by the gas on the surroundings. If heat is expelled by the gas during process CD is Q (in joule) then find $\frac{Q}{10}$.



Ans. (10)

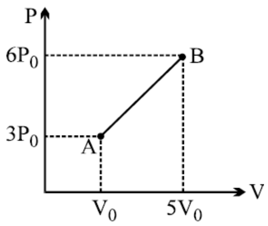
Sol. $Q_{AB} = W_{AB} = nRT_A \ell n\left(\frac{V_B}{V_A}\right)$

$Q_{CD} = W_{CD} = nRT_C \ell n\left(\frac{V_D}{V_C}\right) = -nRT_C \ell n\left(\frac{V_B}{V_A}\right)$

$\frac{Q_{AB}}{Q_{CD}} = \frac{W_{AB}}{W_{CD}} = -4 \Rightarrow Q_{CD} = -\frac{W_{AB}}{4} = -100$

Heat rejected during $C \to D = 100\,J$

48. One mole of a monoatomic ideal gas undergoes the process $A \to B$ in the given P-V diagram. If the specific heat for this process is mR/n then find m + n.



Ans. (19)

Sol. $W = \int_{V_i}^{V_f} PdV$ equation of line $P = \left[\frac{3P_0}{4V_0}V + \frac{9P_0}{4}\right]$

$W = \int_{V_0}^{5V_0} \left(\frac{3P_0}{4V_0}V + \frac{9P_0}{4}\right)dV$

$W = \frac{3P_0}{4V_0}\left(\frac{V^2}{2}\right)_{V_0}^{5V_0} + \frac{9P_0}{4}[V]_{V_0}^{5V_0}$, $W = 18P_0V_0$

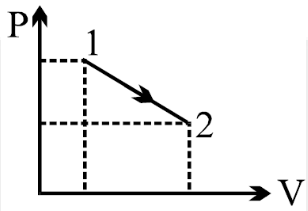
$\Delta U = \frac{P_fV_f - P_iV_i}{(5/3-1)} = \frac{30P_0V_0 - 3P_0V_0}{2/3} = \frac{81\,P_0V_0}{2}$

$$\Delta Q = W + \Delta U = \frac{117}{2}P_0V_0,\ nC(T_f - T_i) = \frac{117}{2}P_0V_0$$

$$C\left(\frac{6P_0 \times 5V_0}{R} - \frac{3P_0V_0}{R}\right) = \frac{117}{2}P_0V_0,\ C = \frac{\frac{117}{2}P_0V_0}{\frac{27P_0V_0}{R}} = \frac{13R}{6}$$

$$C = \frac{13R}{6}$$

49. A process $1 \to 2$ using monoatomic gas is shown on the P-V diagram on the right. $P_1 = 2P_2 = 10^6 N/m^2$, $V_2 = 4V_1 = 0.4 m^3$. If the heat absorbed (in kJ) by the gas in this process is Q then find the value of Q/10:



Ans. (375)

Sol. $W = \frac{1}{2} \times 3v_0 \times P_0 + 3v_0 \times P_0$

$W = \frac{3}{2}P_0V_0 + 3v_0P_0$

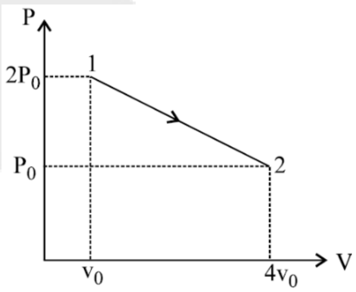
$W = \frac{9}{2}P_0V_0$

$\Delta U = nC_v\Delta T = n\left(\frac{3R}{2}\right)(T_f - T_1)$

$\Delta U = \frac{3}{2}nR(T_f - T_i) = \frac{3}{2}[P_fV_f - P_iV_i]$

$\Delta U = \frac{3}{2}[4P_0V_0 - 2P_0V_0]$

$\Delta U = 3P_0V_0$



$\Delta Q = \Delta U + W = \frac{9}{2}P_0V_0 + 3P_0V_0 = \frac{15}{2}P_0V_0$

$P_0 = \frac{10^6}{2},\ V_0 = 0.1$

$\Delta Q = \frac{15}{2} \times \frac{10^6}{2} \times 0.1 = 375000\,J$

$\Delta Q = 375$ kJ

50. If velocities of 5 molecules of certain gas are −7, 5, 4, −3 and 1 m/sec respectively then find mean speed of molecules is (m/sec)-

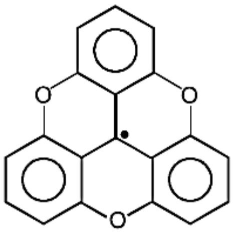
Ans. (4)

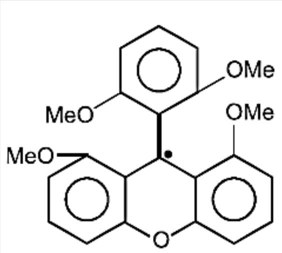
Sol. $V_{rms} = \frac{7+5+4+3+1}{5} = 4\,m/s$

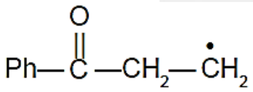
PART-C: CHEMISTRY

SECTION-I

51. Which is most stable?

(A) 

(B) 

(C) 

(D) $CH_2{=}CH\bullet$

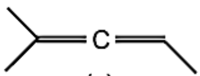
Ans. (A)

Sol. Due to planarity of (A), the free radical is stabilized but in (B) steric inhibition of resonance the free radical is less stable.

52. The correct stability order of following species is

(x)

(y)



(z)

(w)

(A) x > y > w > z

(B) y > x > w > z

(C) x > w > z > y

(D) z > x > y > w

Ans. (C)

Sol. X is a conjugated diene system.

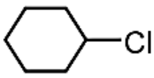
W is an isolated diene system

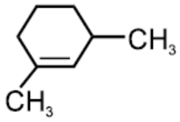
Z is a cumulated diene system

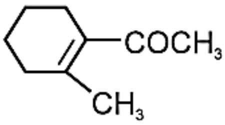
Y is antiaromatic system

Hence stability order is X > W > Z > Y

53. In which of the following molecules all the effects namely inductive, mesomeric and hyperconjugation operate?

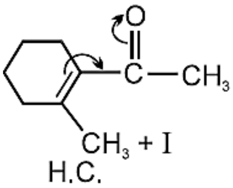
(A) 

(B) 

(C) 

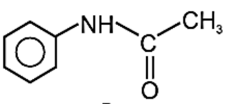
(D) 

Ans. (C)

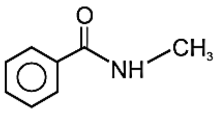
Sol. 

has all effect inductive, mesomeric & hyperconjugation.

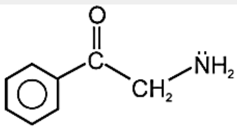
54. The correct basic strength order is



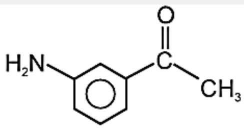
I



II



III



IV

(A) I > II > IV > III

(B) IV > III > II > I

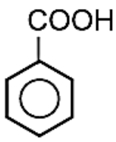
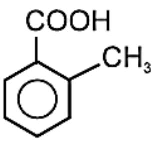
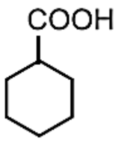
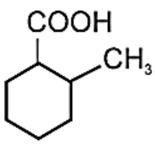
(C) III > II > IV > I

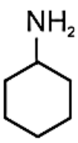
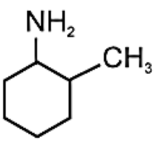
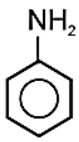
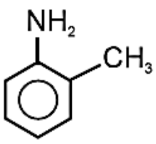
(D) III > IV > II > I

Ans. (D)

Sol. The basically order will be inversally proportional to resonance stability of lone pair.

55. The correct order of acid and basic strength for the following pair of compounds should be?

Acid strength :  (I) and  (II) ;  (III) and  (IV)

Basic strength :  (V) and  (VI) ;  (VII) and  (VIII)

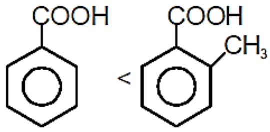
(A) I > II ; III > IV ; V < VI ; VII < VIII

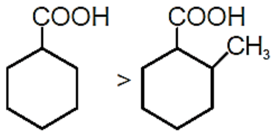
(B) I < II ; III > IV ; V < VI ; VII > VIII

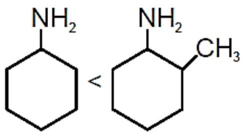
(C) I > II ; III > IV ; V > VI ; VII > VIII

(D) I < II ; III > IV ; V < VI ; VII < VIII

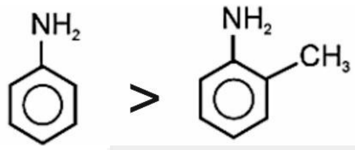
Ans. (B)

Ka :  (due to ortho effect)

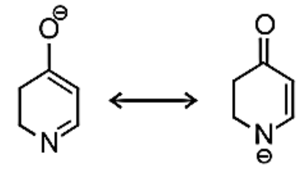
Ka :  (due to +I effect of $-CH_3$)

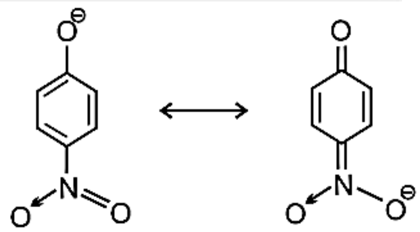
Kb :  (due to +I effect of $-CH_3$)

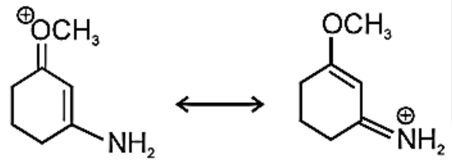
Sol.

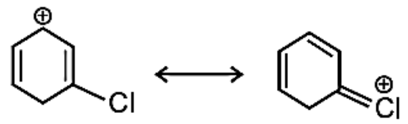
 (Due to ortho effect)

56. In the following set of resonating structures which sets have the second resonating structure more contributing first:

I : 

III : 

II : 

IV : 

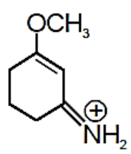
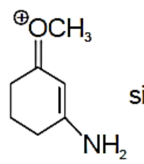
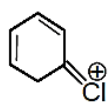
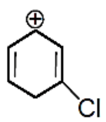
(A) I and III

(B) II and IV

(C) I, II and IV

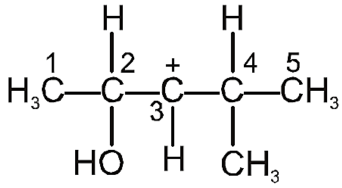
(D) II, III and IV

Ans. (B)

In II,  more stable than  since +ve charge is located on less electronegative atom Nitrogen. And in IV,  is more stable than  since octet of all atoms is complete.

Sol.

57. In the following carbocation; H/CH_3 that is most likely to migrate to the positively charged carbon is



(A) CH_3 at C-4

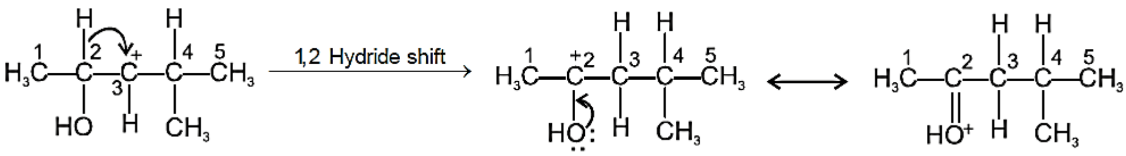
(B) H at C-4

(C) CH_3 at C-2

(D) H at C-2

Ans. (D)

Sol.



(More stable carbocation due to +m effect of – OH group and + I and hyperconjugative effect of $-CH_3$ group)

58. The correct heat of hydrogenation order is

(p) 1, 3-Pentadiene (q) 1, 3-butadiene

(r) 2, 3-Dimethyl-1, 3-butadiene (s) Propadiene

(A) p > q > r > s

(B) s > q > p > r

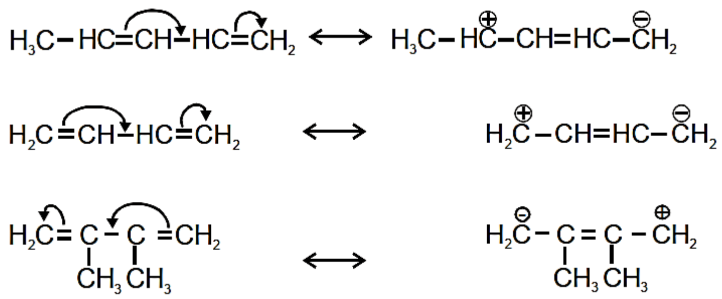
(C) q > s > p > r

(D) s > p > q > r

Ans. (B)

Sol. stability $\alpha \dfrac{1}{\text{heat of hydrogenation}}$

(p) (q) (r)



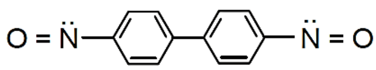
(p) H.C. + Resonance

(q) Resonance

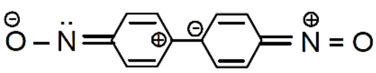
(r) 6H – H.C. Resonance

(s) $H_2C = C = CH_2$ then the order of heat of hydrogenation is
s > q > p > r

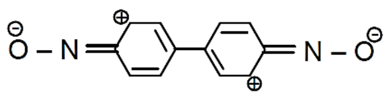
59. The most stable resonating structure of following compound is

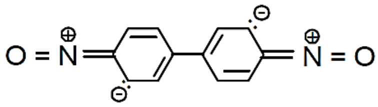


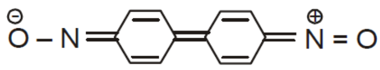
(A)



(B)

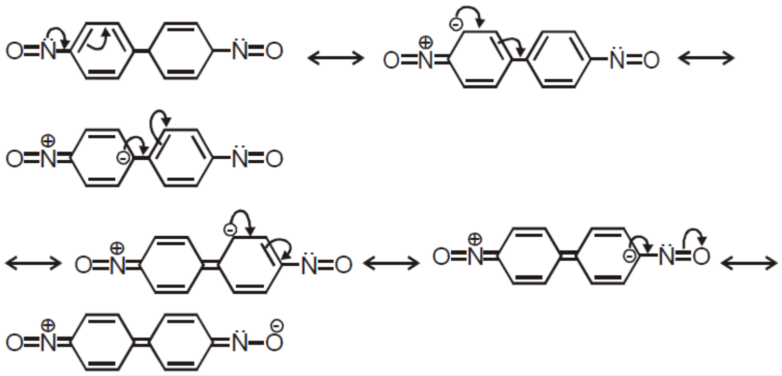


(C) 

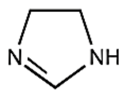
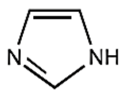
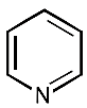
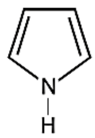
(D) 

Ans. (D)

Sol. Maximum charge separation stability increases



60. Acid strength of the conjugate acids of the following are-

(I) 
(II) 
(III) 
(IV) 

(A) I > II > III > IV

(B) III > II > I > IV

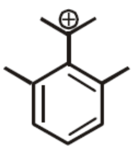
(C) IV > III > II > I

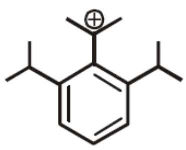
(D) None of these

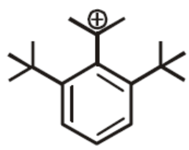
Ans. (C)

Sol. Stronger the base weaker is the acidic strength of conjugate acid.

61. The stability order of following carbocation is


(I)


(II)


(III)

(A) I > II > III

(B) II > I > III

(C) III > I > II

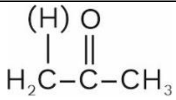
(D) II > III > I

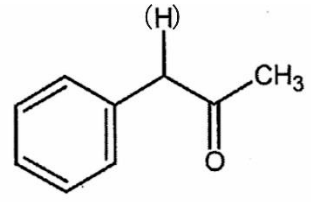
Ans. (A)

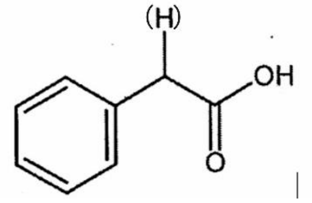
Sol. On the basis of SIR effect

62. Among the following, marked proton of which compound shows lowest pK_a value?

(A) $\overset{(H)}{\overset{|}{H_2C}}{-}COOH$

(B) 

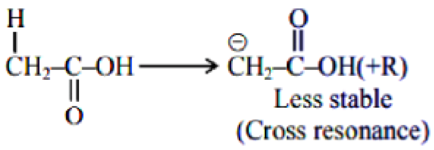
(C) 

(D) 

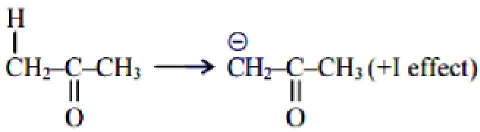
Ans. (C)

Sol.

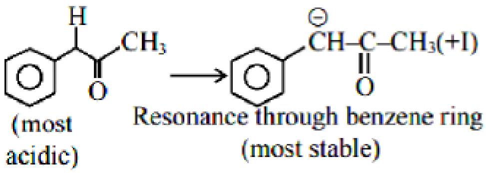
(A)



(B)



(C)



So it has least pK_a value.

63. The increasing order of stability of the following free radicals is:

(A) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$

(B) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$

(C) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$

(D) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$

Ans. (D)

Sol. The order of stability of free radical is as follows:

Teritary > secondary > primary.

Benzyl free radicals are stabilised by resonance and hence are more stable than alkyl free radicals. Further as the number of phenyl group attached to the carbon atom holding the odd electron increases, the stability of a free radical increases accordingly i.e.

$$(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$$

64. Arrange the carbanions, $(CH_3)_3\overline{C}$, $\overline{C}Cl_3$, $(CH_3)_2\overline{C}H$, $C_6H_5\overline{C}H_2$ in order of their decreasing stability

(A) $(CH_3)_2\overline{C}H > \overline{C}Cl_3 > C_6H_5\overline{C}H_2 > (CH_3)_3\overline{C}$

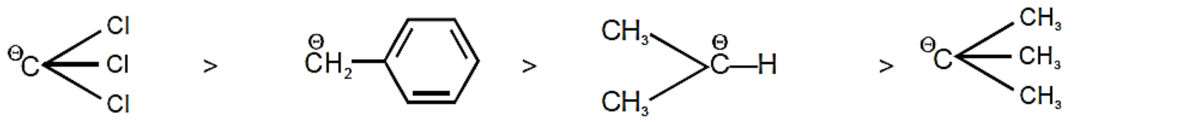
(B) $\overline{C}Cl_3 > C_6H_5\overline{C}H_2 > (CH_3)_2\overline{C}H > (CH_3)_3\overline{C}$

(C) $(CH_3)_3\overline{C} > (CH_3)_2\overline{C}H > C_6H_5\overline{C}H_2 > \overline{C}Cl_3$

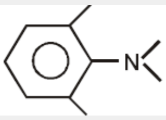
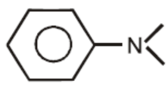
(D) $C_6H_5\overline{C}H_2 > \overline{C}Cl_3 > (CH_3)_3\overline{C} > (CH_3)_2\overline{C}H$

Ans. (B)

Sol.



pπ-dπ back bonding > delocalisation > +I effect of two CH_3 > +I effect of three CH_3

65. **Assertion:** C – N bond length in  is more than C – N bond length in 

Reason: This is due to steric inhibition of resonance in former which does not appear in the later.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

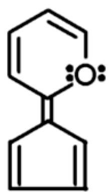
(C) If Assertion is true, but Reason is false.

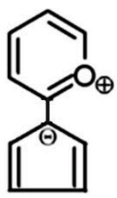
(D) If Assertion is false, but the Reason is true.

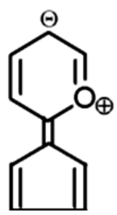
Ans. (A)

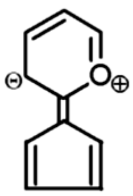
Sol. On the basis of SIR Effect.

66. The most stable canonical structure of this molecule is:



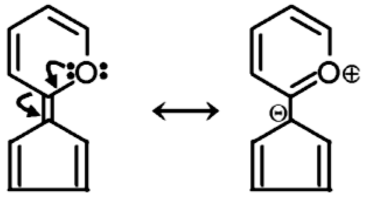
(A) 

(B) 

(C) 

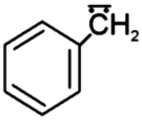
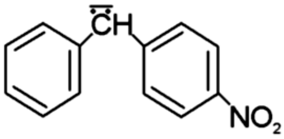
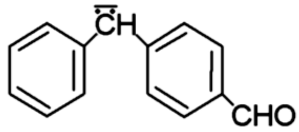
(D) All structures are equal contribution

Ans. (A)

Sol. 

Both ring are aromatic

67. Select the correct stability order of the following carbanions.

(i)  (ii)  (iii) 

(A) (i) > (ii) > (iii)

(B) (ii) > (iii) > (i)

(C) (iii) > (ii) > (i)

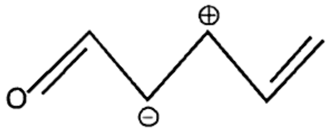
(D) (ii) > (i) > (iii)

Ans. (B)

Sol. –M effect increases stability of carbanion.

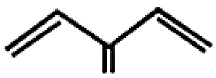
68. The least stable resonating structure is

(A) .  (B) 

(C)  (D) 

Ans. (C)

Sol. Positive charge of present on more electronegative atom.

69. 
I


II


III

Among these compounds, the correct order of resonance energy is:

(A) I > II > III

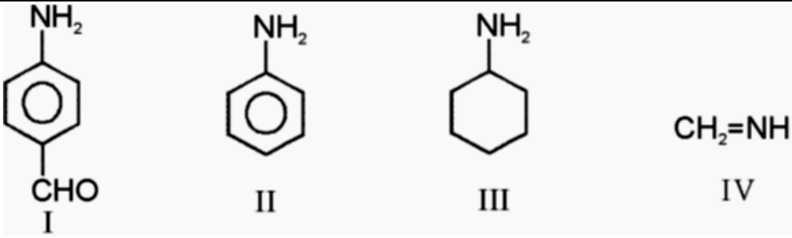
(B) II > I > III

(C) III > I > II

(D) I > III > II

Ans. (B)

Sol. Resonance energy α Resonating Structure

70. 

Among these compounds the correct order of C-N bond length is

(A) IV > I > II > III

(B) III > I > II > IV

(C) III > II > I > IV

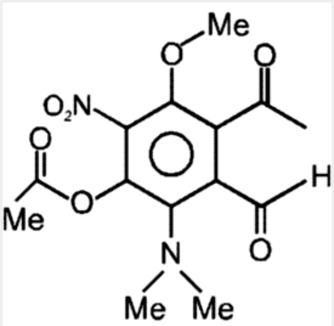
(D) III > I > IV > II

Ans. (C)

Sol. Delocalisation of multiple bonds increases bond length increases.

SECTION-II

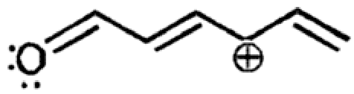
71. Find the total + m groups attached to the benzene ring in the given compound?



Ans. (3)

Sol. $-O-COCH_3$, $-OCH_3$, $-N(CH_3)_2$

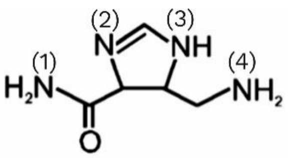
72. Number of p electrons in resonance in the following structure is



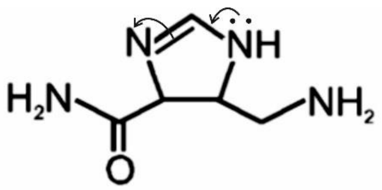
Ans. (6)

Sol. It has only 6 p-electron in conjugation.

73. In the given compound which is the most basic N–atom.



Ans. (2)



Sol.

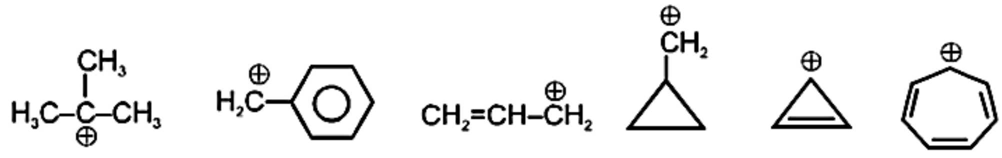
74. How many carbocations given below are more stable than sec. butyl carbocation

t-butyl carbocation Benzyl carbocation Allyl carbocation Cyclopropenyl cation

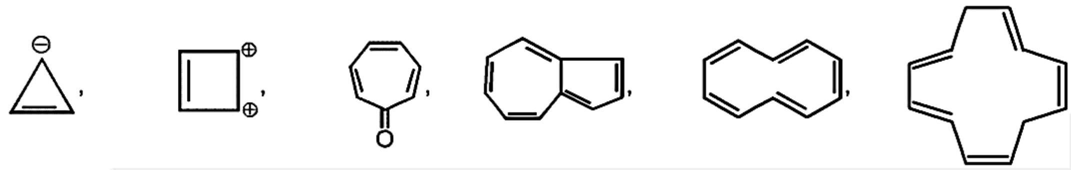
Tropylium cation n-butyl carbocation cyclopropylmethyl carbocation

Ans. (6)

Sol.

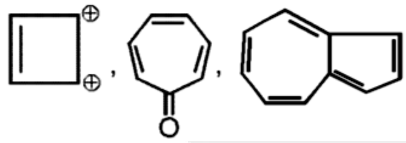


75. How many species out of the following are aromatic?



Ans. (3)

Sol.


are Aromatic species.