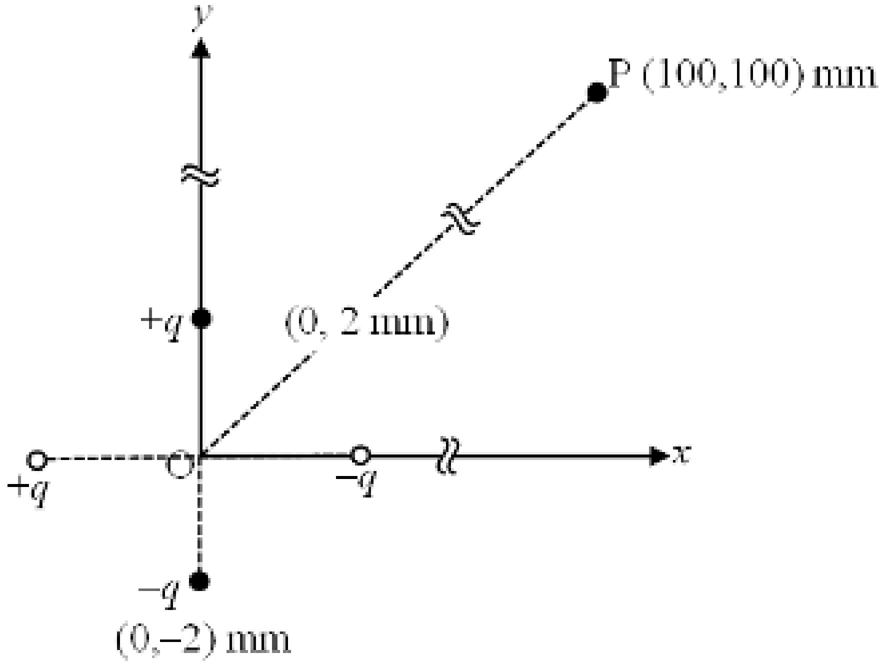
PART-1 : PHYSICS

SECTION-I (i)

1) An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at (0, 2) mm and (0, –2) mm, respectively, as shown in the figure. The electric potential at point P(100, 100) mm due to the dipole is V_0. The charges $+q$ and $-q$ are then moved to the points (–1, 0) mm and (1, 0) mm, respectively. What is the value of electric potential at P due to the new dipole ?



(A) $-\frac{V_0}{4}$

(B) $\frac{V_0}{2}$

(C) $\frac{V_0}{\sqrt{2}}$

(D) $-\frac{V_0}{2}$

2) Density is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $\rho = c^{\alpha} h^{\beta} G^{\gamma}$. Which of the following is the correct option?

(A) $\alpha = 5, \beta = -1, \gamma = -2$

(B) $\alpha = -5, \beta = -1, \gamma = -2$

(C) $\alpha = 5, \beta = -1, \gamma = 2$

(D) $\alpha = -5, \beta = 1, \gamma = -2$

3) A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + x\hat{y})$ where α is a non-zero constant. What is the torque acting on the particle about the origin ?

(A) $\vec{0}$

(B) $2m\alpha xy\hat{k}$

(C) $m\alpha xy$ $\hat{k}$

(D) $-2m\alpha xy$ $\hat{k}$

4) An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas is n. The internal energy of one mole of the gas is U_n and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option ?

(A) $v_3 < v_6$ and $U_3 > U_6$

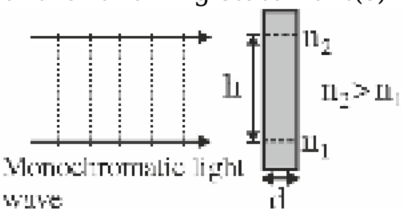
(B) $v_5 > v_3$ and $U_3 > U_5$

(C) $v_5 > v_7$ and $U_5 < U_7$

(D) $v_6 < v_7$ and $U_6 < U_7$

SECTION-I (ii)

1) A monochromatic light wave is incident normally on a thin glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is (are) true about the light wave emerging out of the slab?



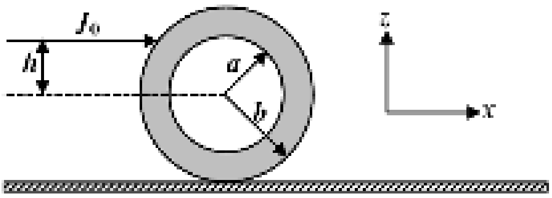
(A) It will deflect up by an angle $\tan^{-1}\left[\dfrac{(n_2^2 - n_1^2)\,d}{2h}\right]$

(B) It will deflect up by an angle $\tan^{-1}\left[\dfrac{(n_2 - n_1)\,d}{h}\right]$

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

2) An spherical shell of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the shell. If h = h_m then the shell rolls without slipping along the x-axis.



Which of the following statement(s) is(are) correct ?

(A) For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$

(B) For $\mu \neq 0$ and $a \to b$, $h_m = 2b/3$

(C) For $h = h_m$ the initial angular velocity is inversely proportional to on the outer radius b.

(D) For $\mu = 0$ and $h = 0$, the shell always slides without rolling.

3) The magnetic field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{B} = 10^{-7}(\hat{x}+\hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ T. Which of the following option(s) is(are) correct? [Given: The speed of light in vacuum, $c = 3 \times 10^8\ ms^{-1}$]

(A) $E_x = -15\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ v/m

(B) $E_y = -15\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ v/m

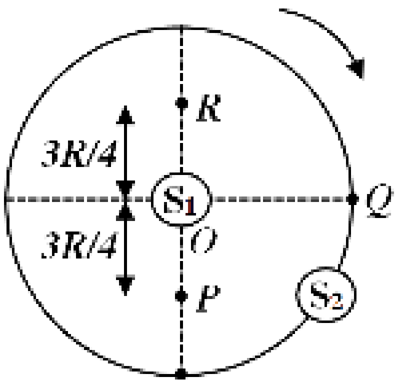
(C) The wave is polarized in the xy-plane with electric field at an angle of 45° with respect to the positive x-axis.

(D) The refractive index of the medium is 2.

SECTION-II (i)

Common Content for Question No. 1 to 2

S_1 and S_2 are two identical sound sources of frequency 536 Hz. The source S_1 is located at O and S_2 moves clockwise with a uniform speed 4 ms^{-1} on a circular path of radius R around O, as shown in the figure. There are three points P, Q and R such that P and R are opposite at a distance of $\frac{3R}{4}$ from O while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP. [Given: The speed of sound in air is 324 ms^{-1}]

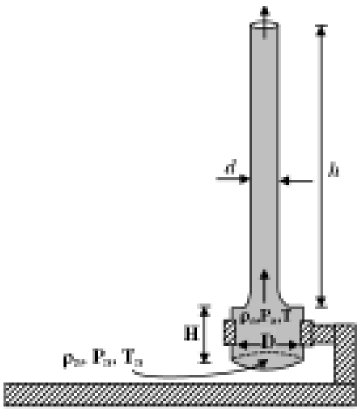


1) When only S_2 is emitting sound and it is at Q, the frequency of sound measured by the detector in Hz is __________.

2) Consider both sources emitting sound. When S_2 is above R and S_1 approaches the detector with a speed 4 ms^{-1}, the beat frequency measured by the detector is ____Hz.

Common Content for Question No. 3 to 4

A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes T = 360 K. The hot air with density rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density ρ_a = 1.2 kg m^{-3}, pressure P_a and temperature T_a = 300 K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects. Assume that just outside the furnace at the ground level pressure P_a is atmospheric. [Given: The acceleration due to gravity g = 10 ms^{-2} and π = 3.14]

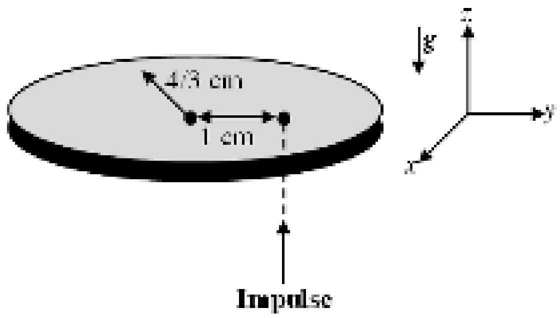


3) Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is _______gm s^{-1}.

4) When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ______ Nm^{-2}.

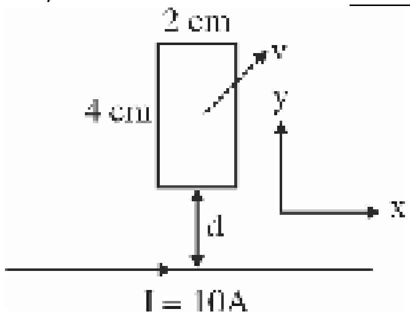
SECTION-II (ii)

1) A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy-plane. The coin is tossed vertically up (+ direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2} N-s$ at a distance 1 cm from its center. The coin spins about its diameter and moves along the + direction. By the time the coin reaches back to its initial position, it completes rotations. The value of n is :- [Given: The



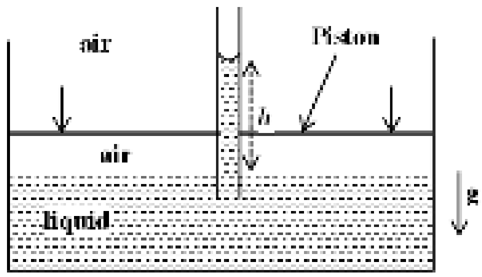
acceleration due to gravity g = 10 ms^{-2}]

2) A rectangular conducting loop of length 4 cm and width 2 cm is in the *xy*-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{4}{5}\hat{x} + \frac{3}{5}\hat{y}$ with a constant speed *v*. The wire is carrying a steady current I = 10 A in the positive *x*-direction. A current of 10 μA flows through the loop when it is at a distance *d* = 8 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of ν is _________ ms^{-1}. [Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$]



3) A string of length 1 m and mass x × 10^{-5} kg is under tension of 5N. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of x is _______.

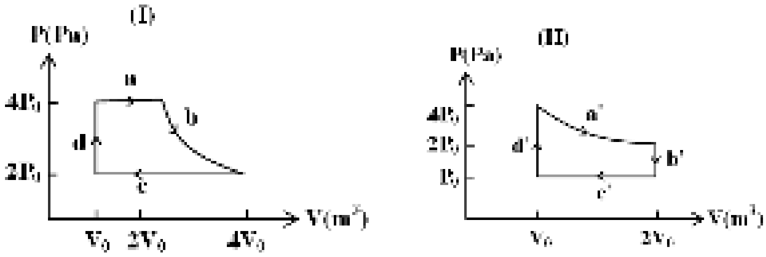
4) An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.3 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (*h*) of the liquid column in the capillary above the liquid level in cm is_______.
[Given: Surface tension of the liquid is 0.075 Nm^{-1}, atmospheric pressure is 10^5 Nm^{-2}, acceleration due to gravity (*g*) is 10 ms^{-2}, density of the liquid is 10^3 kgm^{-3} and contact angle of capillary surface with the liquid is zero]



5) In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where *N*(*t*) is the number of radioactive nuclei at time *t*. Two radioactive sources, S_1 and S_2 have same activity at time *t* = 0. At a

later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. Half life of S_1 is 300 sec and half life of S_2 is 150 sec. At t = 600 sec, the ratio A_1/A_2 is _________.

6) Two mole of an ideal monoatomic gas undergoes two different cyclic processes I and II, as shown in the *P–V* diagrams below. In cycle I, processes *a, b, c* and *d* are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes *a′*, *b′*, *c′* and *d′* are isothermal, isochoric, isobaric and isochoric, respectively. The total heat exchange during cycle I is H_1 and that during cycle II is H_2.



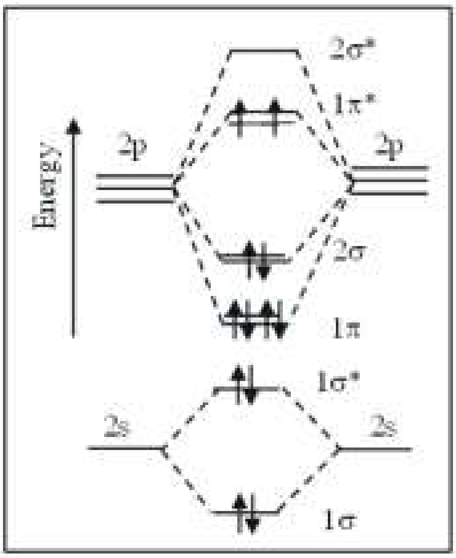
The ratio $\frac{H_1}{H_2}$ is ______.

PART-2 : CHEMISTRY

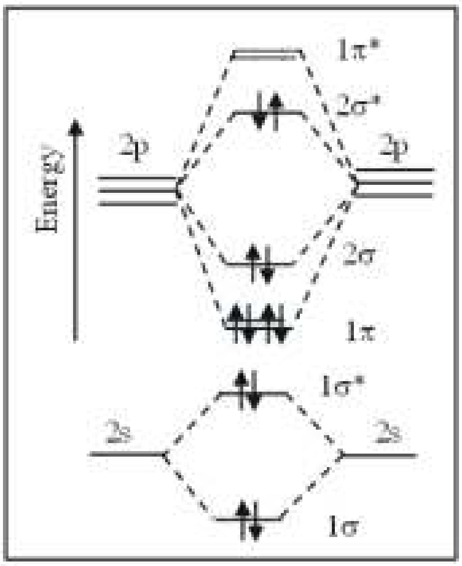
SECTION-I (i)

1) The correct molecular orbital diagram for O_2 molecule (if sp mixing is operative) in the ground state is

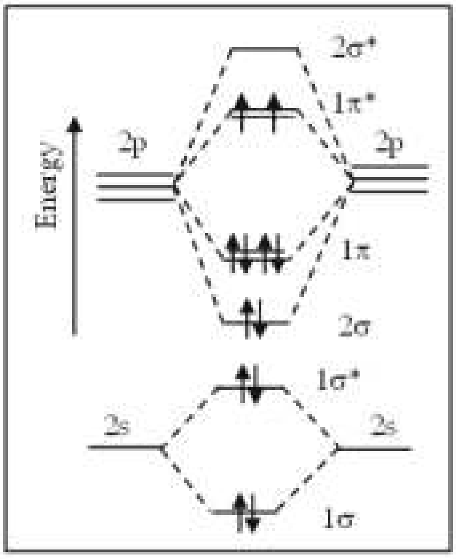
(A)



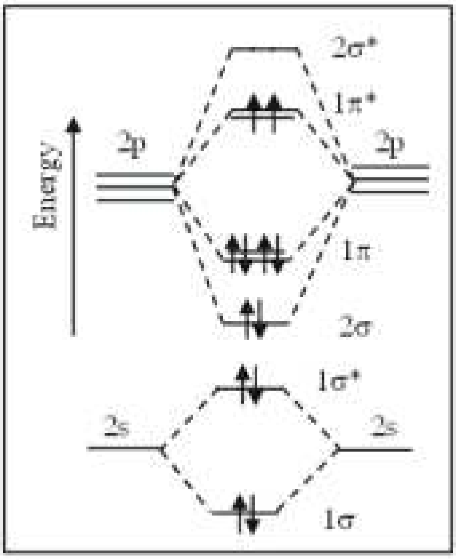
(B)



(C)



(D)



2) Consider the following statements related to colloids.
(I) Lyophobic colloids can not be coagulated.
(II) For gel, both the dispersed phase and the dispersion medium are liquid.
(III) Micelles can not be formed below Kraft temperature.
(IV) Tyndall effect can be observed from a colloidal solution with dispersed phase having the different refractive index as that of the dispersion medium.

The option with the correct set of statements is

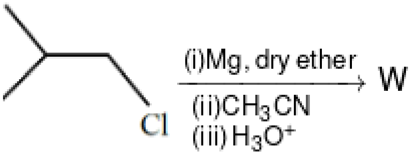
(A) (I) and (II)

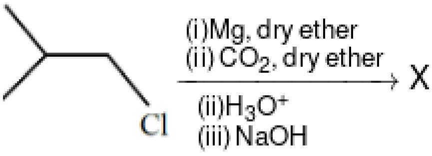
(B) (II) and (III)

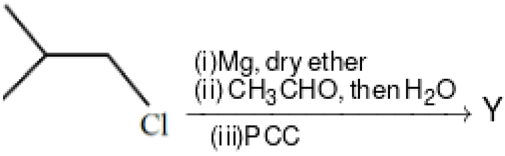
(C) (III) and (IV)

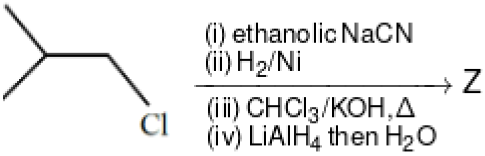
(D) (II) and (IV)

3) In the following reactions, **W**, **X**, **Y**, and **Z** are the major products.









The incorrect statement about **W**, **X**, **Y**, and **Z** is

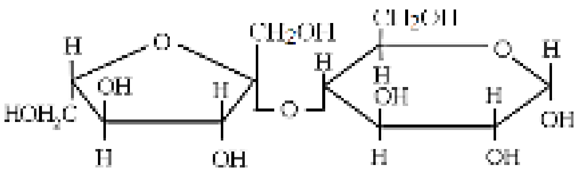
(A) **W** has six carbons and it undergoes Haloform reaction.

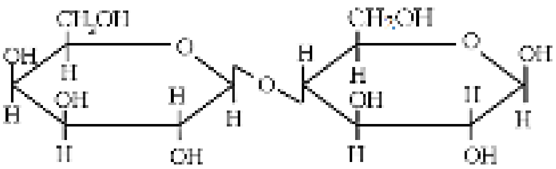
(B) **X** undergoes Kolbe’s electrolysis to give an eight-carbon product.

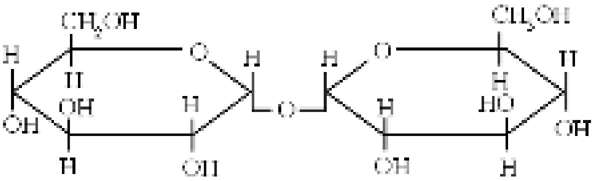
(C) **Y** has six carbons and it undergoes Cannizzaro reaction.

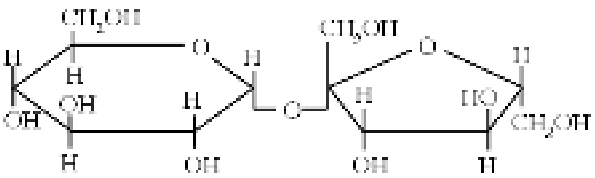
(D) **Z** is a secondary amine with six carbons.

4) A disaccharide **X** cannot be oxidised by Fehling solution. The acid hydrolysis of **X** leads to a laevorotatory solution. The disaccharide **X** is

(A) 

(B) 

(C) 

(D) 

SECTION-I (ii)

1) The complex(es), which can exhibit any type of isomerism shown by $[Co(en)_3]Cl_3$, is(are) [en = $H_2NCH_2CH_2NH_2$] and Gly = NH_2-CH_2-COO^-]

(A) $[Pt(en)(SCN)_2]$

(B) $[Zn(gly)_2]$

(C) $[Pt(NH_3)_2Cl_4]$

(D) $[Cr(en)_2(H_2O)(SO_4)]^+$

2) Atoms of metals x, y, and z form face-centred cubic (fcc) unit cell of edge length L_x, body-centred cubic (bcc) unit cell of edge length L_y, and simple cubic unit cell of edge length L_z, respectively.

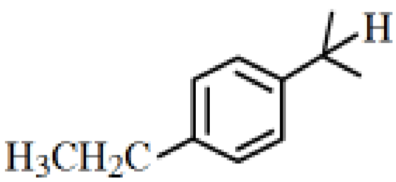
If $r_z = \frac{\sqrt{3}}{2} r_y; r_y = \frac{8}{\sqrt{3}} r_x; M_z = \frac{3}{2} M_y$ and $M_z = 3M_x$, then the correct statement (s) is (are)
[Given : M_x, M_y, and M_z are molar masses of metals x, y, and z, respectively.
r_x, r_y, and r_z are atomic radii of metals x, y, and z, respectively.]

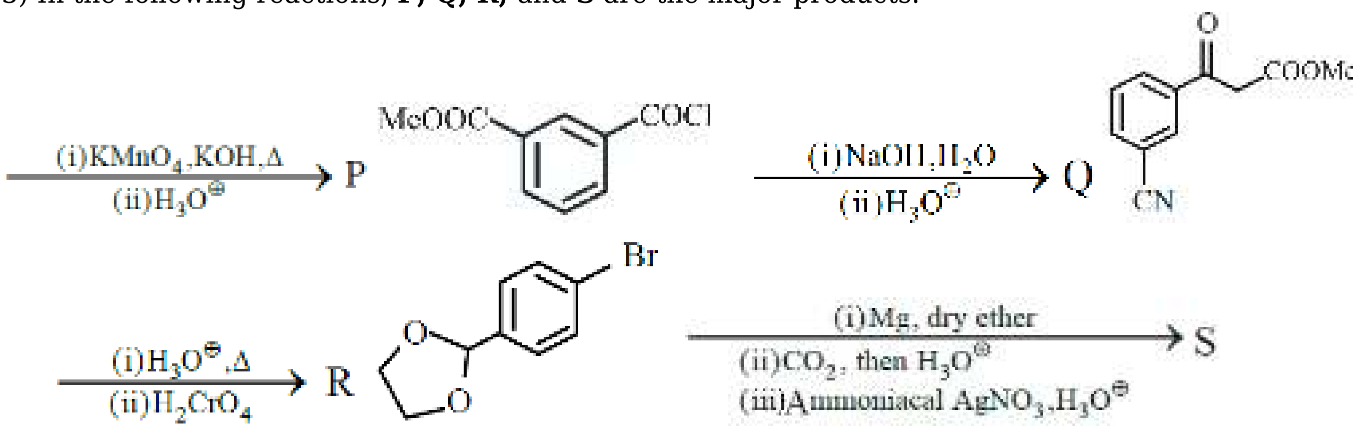
(A) C.N. of x > y > z

(B) $L_x > L_y$

(C) Density of y < density of z

(D) Density of x < Density of y

3) In the following reactions, **P, Q, R,** and **S** are the major products. 



The correct statement (s) about **P, Q, R,** and **S** is (are)

(A) Compounds **P** and **S** are the same.

(B) **P, Q, R** and **S** are dicarboxylic acid.

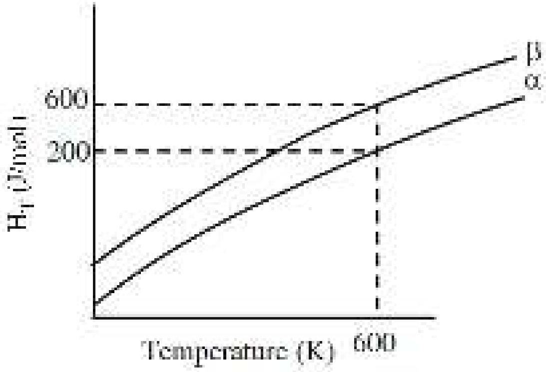
(C) Compounds **Q** and **R** are the same.

(D) **R** does **not** undergo aldol condensation and **S** does **not** undergo Cannizzaro reaction.

SECTION-II (i)

Common Content for Question No. 1 to 2

"PARAGRAPH I" The enthalpy versus temperature plot for phases α and β at 1 bar pressure is given



H_T is enthalpy of phases at temperatures T. The transition temperature for α to β phase change is 600 K and $C_{P,\beta} - C_{P,\alpha} = 1\ J\ mol^{-1}\ K^{-1}$. Assume $(C_{P,\beta} - C_{P,\alpha})$ is independent of temperature in the range of 200 to 700 K. $C_{P,\alpha}$ and $C_{P,\beta}$ are heat capacities of α and β phases, respectively.

1) The value of enthalpy change, $H_\beta - H_\alpha$ (in J mol^{-1}), at 300 K is __.

2) The value of entropy change, $|S_\beta - S_\alpha|$ (in J mol^{-1} K^{-1}), at 300 K is __.
[Use : ln 2 = 0.69
Given : $S_\beta - S_\alpha = 0$ at 0 K]

Common Content for Question No. 3 to 4

"PARAGRAPH II" A trinitro compound, 1, 3,5 tris-(4-nitrophenyl) benzene, on complete reaction with an excess of Fe/HCl gives major product, which on treatment with an excess of $NaNO_2$/HCl at 0°C provides **P** as the product. **P**, upon treatment with excess of H_2O at room temperature, gives the product **Q.** Bromination of **Q** in aqueous medium then treatment with Zn/Δ furnishes the product **R.** The compound **P** upon treatment with an excess of aniline under mild acidic conditions gives the product **S.**

3) The number of heteroatoms present in one molecule of **R** is _____.
[Use: Molar mass (in g mol^{-1}): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5
Atoms other than C and H are considered as heteroatoms]

4) The total number of nitrogen atoms present in one molecule of **S** is **_____.** [Use: Molar mass in g mol^{-1}]: H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

SECTION-II (ii)

1) H_2S (3 moles) reacts completely with acidified aqueous potassium dichromate solution. In

this reaction, the number of moles of water produced is **x**, and the number of moles of electrons involved is **y**. The value of (**x** + **y**) is ____.

2) Among $[I_3]^-$, $[SiO_4]^{4-}$, SO_2Cl_2, XeF_2, SF_4, ClF_3, $Fe(CO)_5$, XeO_2F_2, $[PtCl_4]^{2-}$, XeF_4, and $IO_2F_2^-$, the total number of species having sp^3d hybridised central atom is ______.

3) Consider the following molecules : Br_3O_8, CrO_5, , $H_5P_3O_{10}$, Na_2S_5 , $H_2S_4O_6$, $H_2S_5O_6$, and C_3O_2 , $Mn_2(CO)_{10}$

Count the number of atoms existing in their zero oxidation state in each molecule. Their sum is____.

4) For He^+, a transition takes place from the orbit of radius 26.45 pm to the orbit of radius 105.8 pm. The change in wavelength associated with e^- in transition is ______ pm.

(Round off your answer to nearest integer)

[**Use** : $\pi = 3$, Bohr radius, a = 52.9 pm]

5) 50 mL of 0.2 molal urea solution (density = 1.012 g mL^{-1} at 300 K) is mixed with 500 mL of a solution containing 0.06 g of urea. Both the solutions were prepared in the same solvent. Then ($\Delta T_b \times 100$) is ______ .

[Use : Molar mass of urea = 60 g mol^{-1}; gas constant, R = 62 L Torr K^{-1} mol^{-1} ; Assume, $\Delta_{mix}H = 0$, $\Delta_{mix}V = 0$, K_b = 0.5 K kg mol^{-1}]

ΔT_b = elevation in boiling point of final solution.

6) The reaction of 4-methyloct-1-ene (**P**, 2.52 g) with HBr in the presence of $(C_6H_5CO)_2O_2$ gives two isomeric bromides in a 9 : 1 ratio, with combined yield of 50%. Of these, the entire amount of the primary alkyl bromide was reacted with an appropriate amount of diethylamine followed by treatment with eq. K_2CO_3 to given a non-ionic product **S** in 100% yield. The mass (in mg) of **S** obtained is _______ .

[Use molar mass (in g mol^{-1}) : H = 1, C = 12, N = 14, Br = 80]

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let $f : [1, \infty) \to \mathbb{R}$ be a differentiable function such that $f(1) = \frac{1}{3}$ and 3 $\int_1^x f(t)\,dt = xf(x) - \frac{x^3}{3}, x \in [1, \infty)$. Let *e* denote the base of the natural logarithm. Then

(A) f(x) = 3 has 2 real solution

(B) $f(x) = -\frac{1}{2}$ has 2 real solution

(C) Range of f(x) is $\left[-\frac{1}{2}e^{-5/3}, \infty\right)$

(D) $f(e) = 4e^2/3$

2) Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment

(A) Ends with head given that it starts with tail = $\frac{2}{21}$

(B) Ends with head given that it starts with head = $\frac{1}{14}$

(C) Ends with head given that it starts with tail = $\frac{1}{21}$

(D) Ends with head given that it starts with head = $\frac{2}{7}$

3) Let $M = (a_{ij}),\ i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true ?

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\}$ is singleton where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3×3 identity matrix

4) For $x \in \mathbb{R}$ let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1 + t^{2025}} dt$ is

(A) –1

(B) $-\frac{\pi}{4}$

(C) 0

(D) 1

SECTION-I (ii)

1) Let $f : (0,1) \to \mathbb{R}$ be the function defined as $f(x) = [5x]\left(x - \frac{1}{5}\right)^2\left(x - \frac{2}{5}\right)^3\left(x - \frac{3}{5}\right)$, where [x] denotes the greatest integer less than or equal to x. Then which of the following statements is(are) true?

(A) The function f is discontinuous exactly at one point in (0,1)

(B) There is exactly one point in (0,1) at which the function f is continuous but **NOT** differentiable

(C) The function f is differentiable in $\left(0, \frac{1}{2}\right)$

(D) Function f is differentible in $\left(\frac{2}{5}, \frac{4}{5}\right)$

2) Let S be the set of all twice differentiable functions f from $\mathbb{R}$ to $\mathbb{R}$ such that $\frac{d^2f}{dx^2}(x) > 0$ for all x ∈ (–1,1). For f ∈ S, let X_f be the number of points x ∈ (–1,1) for which f(x) = n(x + 1). Then which of the following statements is(are) true?

(A) There exists a function f ∈ S such that $X_f = 3$

(B) For every function f ∈ S , we have $X_f \le 2$

(C) There exists a function f ∈ S such that $X_f = 2$

(D) There does **NOT** exist any function f in S such that $X_f = 1$

3) Let the position vectors of the points P,Q,R and S be $\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}$, $\vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$ and $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$, respectively. Then which of the following statements is true?

(A) The points P,Q,R and S are coplanar

(B) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 99

SECTION-II (i)

Common Content for Question No. 1 to 2

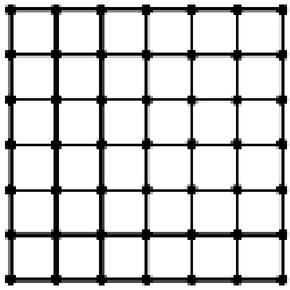
Consider on obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

1) If area of triangle is Δ. Then $64\Delta^2$ =

2) Then the inradius of the triangle ABC is

Common Content for Question No. 3 to 4

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



3) Let p_i be the probability that a randomly chosen point has i many friends, $i = 0,1,2,3,4$. Let X be a random variable such that for $i = 0,1,2,3,4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is

4) Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let p be the probability that they are not friends but have a friend in common . Then the value of $21p$ is

SECTION-II (ii)

1) For $x \in \mathbb{R}$, let y(x) be a solution of the differential equation
$x \sin x \frac{dy}{dx} - (\sin x)\, y = xy \cos x - 2x^3 \sin^2 x$ such that let
$f : (0,\pi) \to \mathbb{R}$
$f(x) = \frac{y(x)}{\sin x}$
$f(1) = 2$
Then the maximum value of the function f(x) is-

2) Let X be the set of all five digit numbers formed using 01122333. For example, 33320 is in X while 02233 and 22233 are not in X . Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 3. Then the value of 61p is equal to

3) Let $A_1, A_2, A_3, \ldots\ldots, A_{16}$ be the vertices of a regular hexadecagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for i = 1,2,....,16. If P varies over the circle, then the maximum value of the product $PA_1\, PA_3.PA_5 \cdot\cdot\cdot\cdot\cdot PA_{15}$, is

4) Let $R = \left\{ \begin{bmatrix} a & b & c \\ b & 3 & d \\ 0 & 4 & 0 \end{bmatrix} ; a,b,c,d \in \{0,1,2,3,6,9\} \right\}$. Then number of invertible matrix in R is

5) Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point A = (4,1) , where 1 < r < 3. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at

T. Mid points of the line segments PQ and ST are joined to form a line which meets the x-axis at a point B. If AB = $\sqrt{5}$, then the value of r^2 is

6) For any $y \in \mathbb{R}$ let $\cot^{-1}(y) \in (\pi, 2\pi)$ and $\tan^{-1}(y) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Then the number of solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{8\pi}{3}$ for $0 < |y| < 3$, is equal to

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	**D**	**A**	**A**	**C**

SECTION-I (ii)

Q.	5	6	7
A.	**B,D**	**B,C,D**	**B,C,D**

SECTION-II (i)

Q.	8	9	10	11
A.	**540.00**	**6.70**	**60.80 to 60.81**	**30.00**

SECTION-II (ii)

Q.	12	13	14	15	16	17
A.	**45**	**10**	**2**	**15**	**4**	**2**

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	18	19	20	21
A.	**A**	**C**	**C**	**D**

SECTION-I (ii)

Q.	22	23	24
A.	**B,D**	**A,C**	**A,B,C,D**

SECTION-II (i)

Q.	25	26	27	28
A.	**100.00**	**0.02 or 0.03**	**6.00**	**9.00**

SECTION-II (ii)

Q.	29	30	31	32	33	34
A.	**13**	**6**	**11**	**159**	**1**	**1791**

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	35	36	37	38
A.	**D**	**A**	**B**	**C**

SECTION-I (ii)

Q.	39	40	41
A.	**A,B,C**	**B,C**	**A,B,D**

SECTION-II (i)

Q.	42	43	44	45
A.	**63.00**	**0.24 to 0.26**	**23.99 to 24.01**	**2.53 to 2.54**

SECTION-II (ii)

Q.	46	47	48	49	50	51
A.	**2**	**3**	**512**	**1110**	**2**	**2**

SOLUTIONS

PART-1 : PHYSICS

1) $V = \frac{kp\cos\theta}{r^2} = \frac{4kq \times 1}{r^2 \times \sqrt{2}}$

$v' = \frac{kq \times 2\cos 135^\circ}{r^2}$

2)

$ML^{-3} = (LT^{-1})^{\alpha}(ML^{2}T^{-1})^{\beta}(M^{-1}L^{3}T^{-2})^{\gamma}$

$0 = -\alpha - \beta - 2\gamma$

$1 = \beta - \gamma$

$-3 = \alpha + 2\beta + 3\gamma$

$\gamma = -2$

$\beta = -1$

$\alpha = 5$

3) $\vec{v} = \alpha\left(y\hat{i} + x\hat{j}\right)$

$\vec{L} = \left(x\hat{i} + y\hat{j}\right) \times \alpha\left(y\hat{i} + x\hat{j}\right)$

$= m\alpha[x^2 - y^2]\hat{k}$

$\vec{\tau} = \frac{d\vec{L}}{dt} = m\alpha\left(2x\frac{dx}{dt} - 2y\frac{dy}{dt}\right)\hat{k}$

$= 2m\alpha(xy - xy)\hat{k}$

$= \vec{0}$

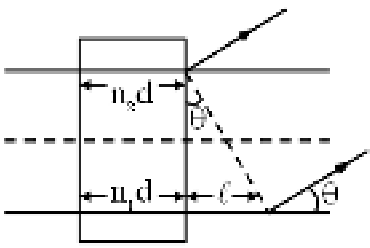
4) $U = \frac{1}{2}fnrT = \frac{frT}{2}$

$\therefore$ A and B are wrong.

$v_{sound} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{f} + 1\right)\frac{RT}{M}}$

$\Rightarrow$ more 'f', less 'v'

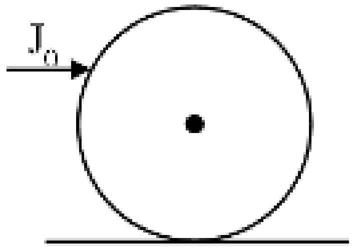
$\therefore v_5 > v_7$



5)

$n_1d + \ell = n_2\,d$

$$\tan\theta = \frac{\ell}{h} = \frac{(n_2 - n_1)d}{h}$$



6)

$J_0 = mv$ ....(1)

$J_0 h_m = I_c \omega$ ....(2)

$v = \omega R$ ....(3)

$\Rightarrow\ h_m = \frac{I_C}{mR}$

(A) If a = 0 $I_C = \frac{2}{5} mb^2$ & R = b □ $h_m = \frac{2b}{5}$

(B) If a = b $I_C = \frac{2}{3} mb^2$ & R = b □ $h_m = \frac{2b}{3}$

(C) $v = \frac{J_0}{m} \Rightarrow \omega = \frac{V}{R} = \frac{J_0}{mR}$

(D) Force is acting on COM □ No rotation.

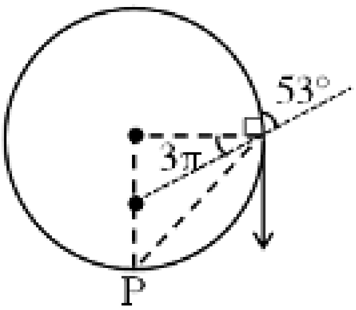
7) $\vec{E} = c^2 \frac{\left(\vec{B} \times \vec{k}\right)}{\omega}$

$C = \frac{\omega}{B} = \frac{5 \times 10^{14}}{10^7} \times 3 = 1.5 \times 10^8$

$\frac{C_0}{C} = \mu = 2$

$$\vec{E} = \frac{2.25 \times 10^{16} \times 10^{-7} \left(\hat{i} + \hat{j}\right) \times \frac{10^7}{3}\hat{k}}{5 \times 10^{14}}$$

$= 15\left(-\hat{j} + \hat{i}\right)$ N/C



8)

$$f = \frac{Cf_0}{C - v\cos(90 + 37)}$$

$$= \frac{C \times f_0}{C - 4 \times \frac{3}{5}} = \frac{324 \times 536}{324 - 2.4}$$

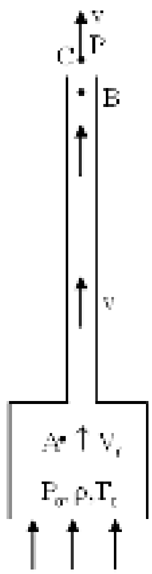
$= \frac{324 \times 536}{321.6} = 540\text{Hz}$

9)

f_P from $S_2 = 536$

f_P from $S_1 = \frac{C}{C-V}f = \frac{536 \times 324}{324-4} = 542.7$

$\Delta f = 6.7$ Hz.



10)

$\rho_0 T_0 = \rho T$

$\Rightarrow 1.2 \times 300 = \rho(360) \therefore \rho = 1$

Between A & B

$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho V^2 + \rho gh$ ....(1)

$\frac{\pi D^2}{4}V_0 = \frac{\pi d^2}{4}V$ ....(2)

Between B & C

$P + \frac{1}{2}\rho V^2 = P_0 - \rho_0 g(H+h) + \frac{1}{2}\rho V^2$ .....(3)

from (1) & (2) :

$\Rightarrow P^0 + \frac{1}{2}\rho\left(V\frac{d^2}{D^2}\right)^2 = P + \frac{1}{2}\rho V^2 + \rho gh$

$\Rightarrow \rho_0 g(H+h) = \frac{1}{2}\rho V^2\left[1 - \frac{d^4}{D^4}\right] + \rho gh$

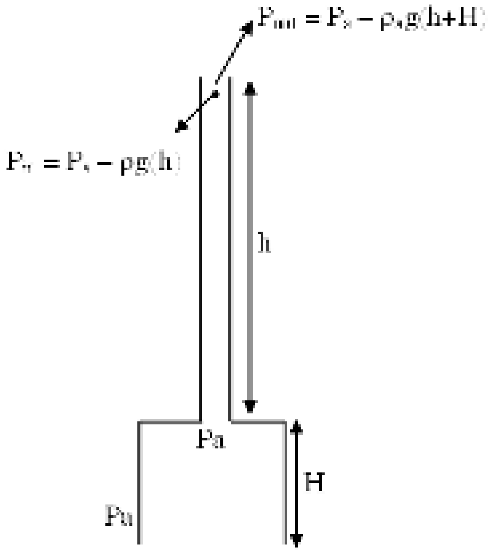
$\Rightarrow V^2 \simeq \frac{2\rho_0}{\rho}g(H+h) - 2gh$

$= 2 \times 1.2 \times 10 \times 10 - 2 \times 10 \times 9$

$= 240 - 180 = 60$ $\therefore V = \sqrt{60}$m/s

$Q_m = \rho\frac{\pi d^2}{4}V = 1 \times \frac{\pi}{4} \times 10^{-2} \times \sqrt{60} \simeq 60.80$

Ans. 60.80 to 60.81



11)

P = constant

$\Rightarrow \rho_a T_a = \rho T$

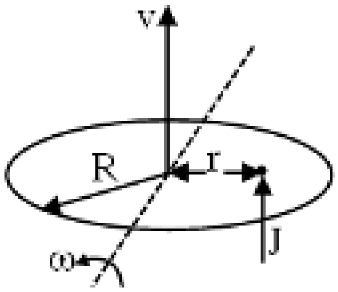
$1.2 \times 300 = \rho \times 360$

$\rho = 1\ kg/m^3$

$\Delta P = \rho_a g\ (h + H) - \rho gh$

$= 1.2 \times 10 \times 10 - 1 \times 10 \times 9$

$= 120 - 90 = 30\ N/m^2$



12)

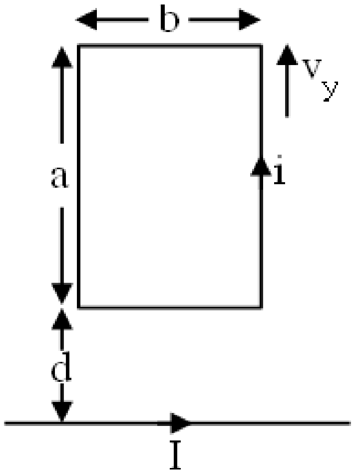
$J = mv$ ......(1)

$Jr = I_c\omega$ ....(2)

$J_C = \frac{1}{4}mR^2$ ....(3)

$t = \frac{2v}{g}$ ....(4)

$\theta = 2\pi N = \omega t$.......(5) $\therefore$ N = 45



13)

R = 0.1Ω

$\varepsilon = (B_1 - B_2)bv_y$

$i = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi R}\left(\frac{1}{d} - \frac{1}{d+a}\right)bv_y$

$\Rightarrow 10^{-5} = \frac{2\times10^{-7}\times10}{0.1}\left[\frac{1}{8} - \frac{1}{12}\right]\times 2.v_y$

 $v_y = 6 = \frac{3v}{5} \Rightarrow v = 10$ m/s

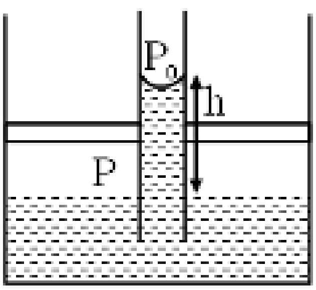
14) $f = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}}$ ....(1)

$1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}}$ ....(2)

$\frac{4}{3} = \frac{P+1}{P}$ P = 3

$\Rightarrow 1000 = \frac{4}{2}\sqrt{\frac{5}{x\times10^{-5}}}$ x = 2



15)

$h_0 = \frac{2T\cos\theta}{\rho gr} = \frac{2\times0.075\times1}{10^3\times10\times10^{-4}\times3} = 5\text{cm}$

$P_0V_0 = P\frac{100V_0}{101} \Rightarrow P = \frac{101}{100}P_0$

$P_0 - \frac{2T\cos\theta}{r} + \rho gh = P = \frac{101}{100}P_0$

$\Rightarrow -\rho g h_0 + \rho g h = \dfrac{P_0}{100}$

$\Rightarrow h = h_0 + \dfrac{P_0}{100\rho g}$

$= 5\text{ cm} + \dfrac{10^5}{100 \times 10^3 \times 10} = 15\text{cm}$

16) $A_1 = \dfrac{A_0}{4}$

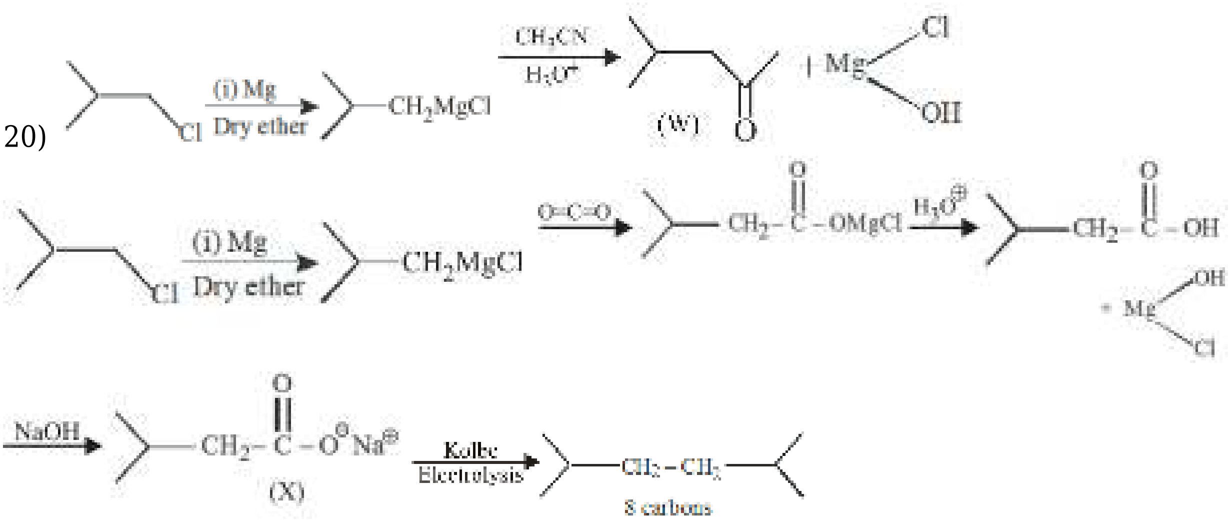
$A_2 = \dfrac{A_0}{16}$

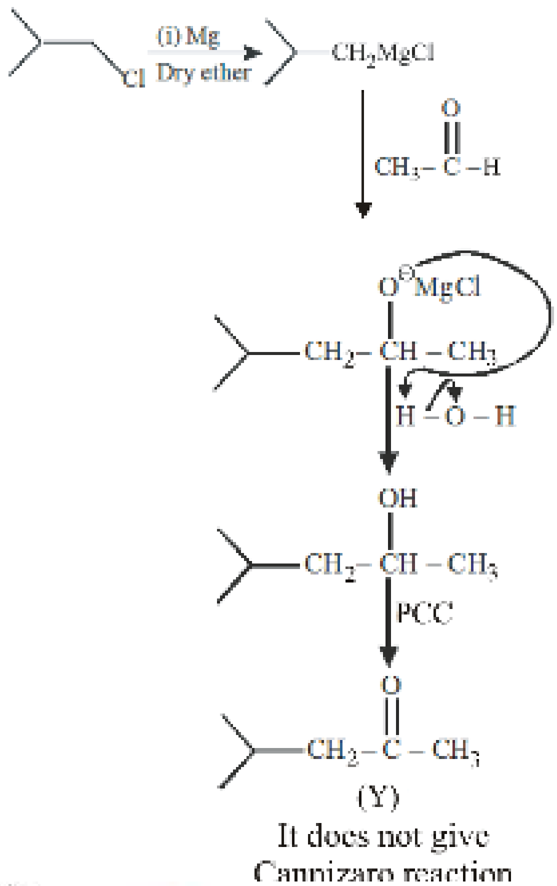
$\dfrac{A_1}{A_2} = 4$

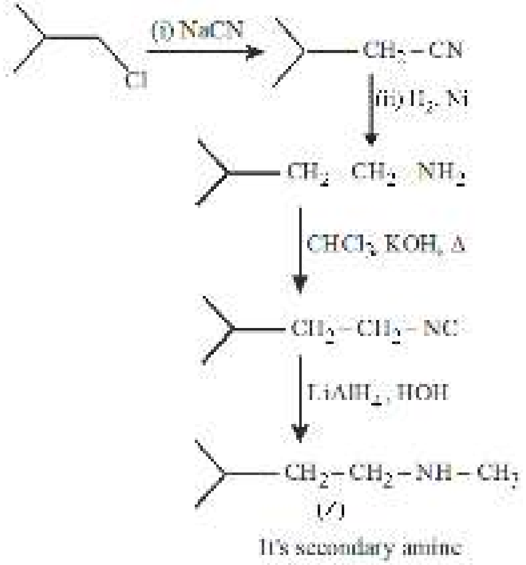
17) $\dfrac{W_I}{W_{II}} = \dfrac{4P_0V_0 + 8P_0V_0\ell n2 - 6P_0V_0 - 0}{4P_0V_0\ell n2 - 0 - P_0V_0 + 0}$ $\Delta U = 0$

$= \dfrac{8\ell n2 - 2}{4\ell n2 - 1} = 2$

PART-2 : CHEMISTRY

20) 





21) Sucrose $\xrightarrow{H_3O^+}$ Glucose + Fructose

Specific rotation + 52.5° –92° (mixture of products is laevorotatory)

Sucrose $\xrightarrow{\text{Fehling solution}}$ No reaction

ABC ⇒ reducing sugars, will get oxidized by Fehling solution.

23)

Element	X	Y	Z
Packing	FCC	BCC	Primitive

Edge	L_x	L_y	L_z
Relation between edge length and radius	$L_x = 2\sqrt{2}r_x$	$L_y = \frac{4}{\sqrt{3}}r_y$	$L_z = 2r_z$
C.N.	12	8	6

Now, $r_y = \frac{8}{\sqrt{3}} r_x$ & $r_z = \frac{\sqrt{3}}{2} r_y = \frac{\sqrt{3}}{2} \times \frac{8}{\sqrt{3}} r_x \Rightarrow r_z = 4r_x$

So, $L_x = 2\sqrt{2} r_x$, $L_y = \frac{4}{\sqrt{3}} \times \frac{8}{\sqrt{3}} r_x$, $L_z = 8r_x$

$L_x = 2\sqrt{2} r_x$, $L_y = \frac{32}{3} r_x$, $L_z = 8r_x$

So $L_y > L_z > L_x$

Density $\frac{4M_x}{L_x^3}$, $\frac{2 \times M_y}{L_y^3}$, $\frac{1 \times M_z}{L_2^3}$

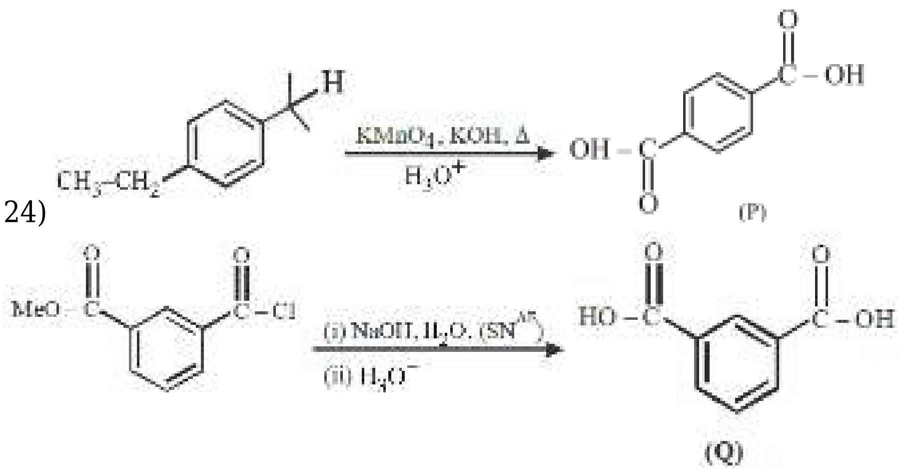
Now, $3M_x = \frac{3M_y}{2}$ or $M_x \times 2 = M_y$

$$\frac{\text{density (x)}}{\text{density (y)}} = \frac{4M_x}{2M_y} \times \frac{L_y^3}{L_x^3}$$

$$= \frac{4M_x}{4M_x} \times \frac{\left(\frac{32}{3}\right)^3}{\left(2\sqrt{2}\right)^3}$$

$$\frac{dy}{dz} = \frac{2M_y}{M_z} \cdot \left(\frac{L_z}{L_y}\right)^3 = 2 \times \frac{2}{3} \times \left(\frac{3}{4}\right) = \frac{27}{48}$$

24)



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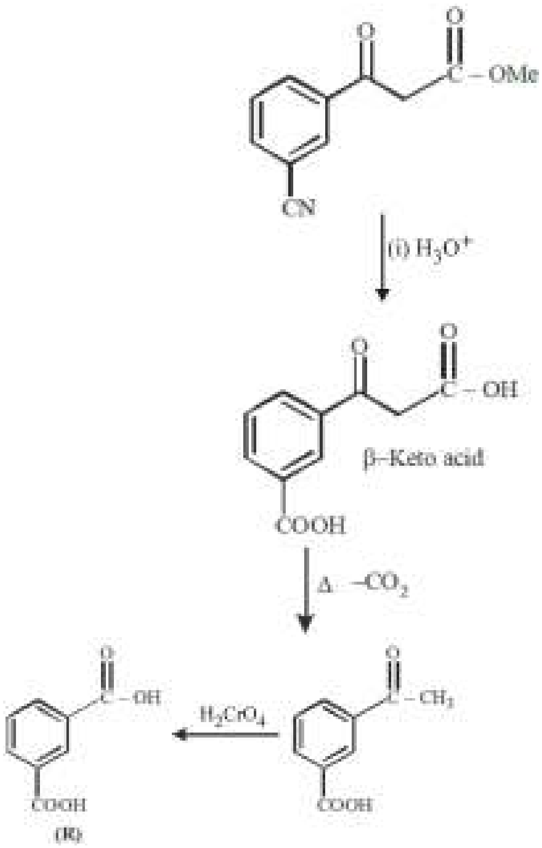
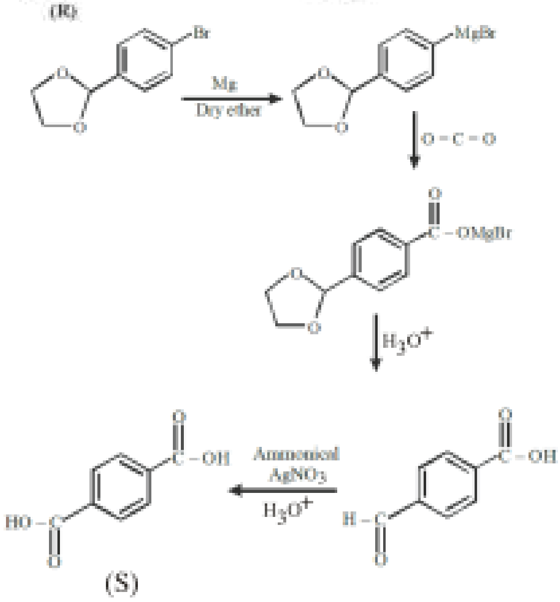




25) As the phase transition temperature is 600 K

$\Delta H_{600K} = 400$ J

So $\Delta H_{600} - \Delta H_{300} = \Delta C_P (T_2 - T_1)$

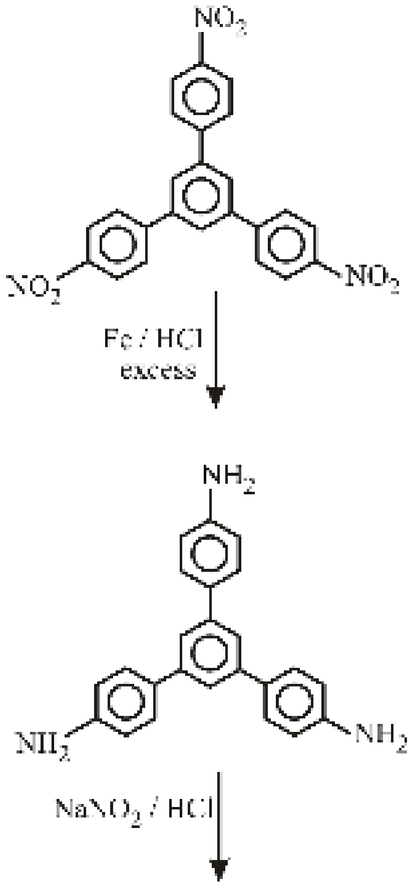
$\Delta H_{600} - \Delta H_{300} = 1 \times 300$

$\Delta H_{300} = \Delta H_{600} - 300 = 400 - 300 = 100$ Joule/mole.

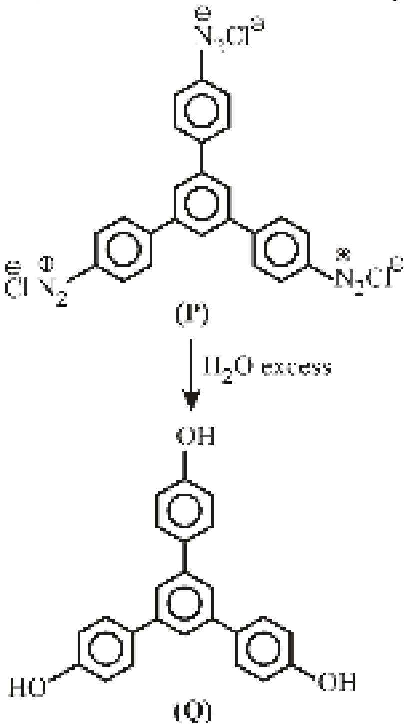
26) $(\Delta S)_{600\,K} = \dfrac{\Delta H}{600} = \dfrac{400}{600} = \dfrac{2}{3}$ J/K

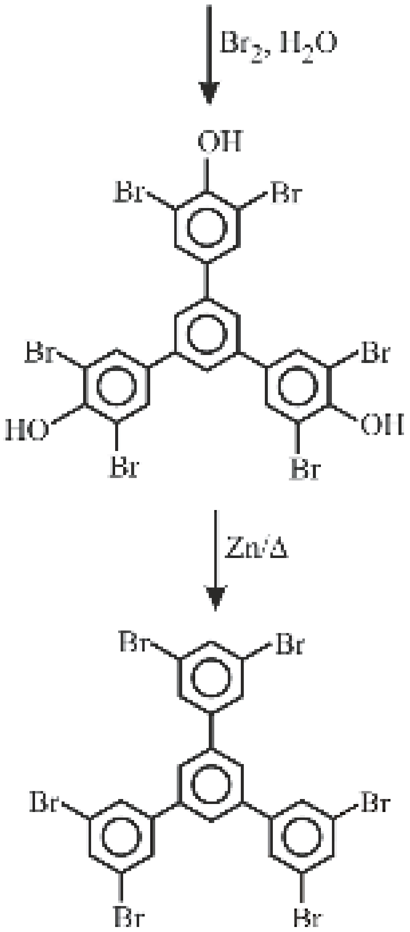
$(\Delta S)_{600\,K} = (\Delta S)_{300\,K} + (\Delta C_P)_r \ln\left(\dfrac{600}{300}\right)$

$(\Delta S)_{300\,K} = \frac{2}{3} - 0.69 = -0.024$



28) **Common solution for Q.no. 16 and 17**

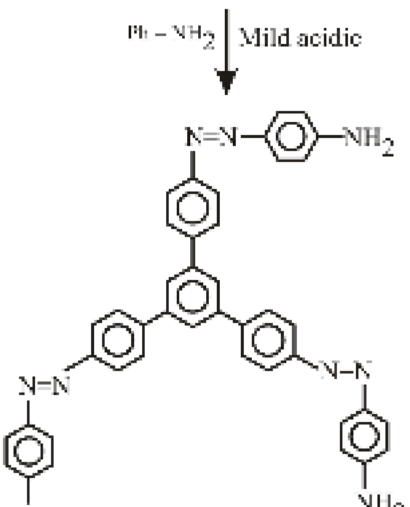




(R)

Number of hetroatom = 6

(P)



(S)

Number of N-atom in S = 9

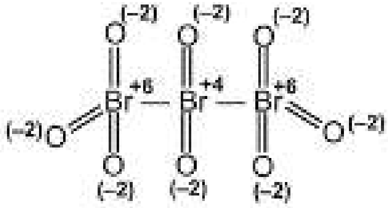
29)

$Cr_2O_7^{2-} + 3H_2S + 8H^+ \rightarrow 3s + 2Cr^{+3} + 7H_2O$

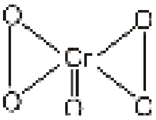
x = 7

y = 6

31) Br_3O_8

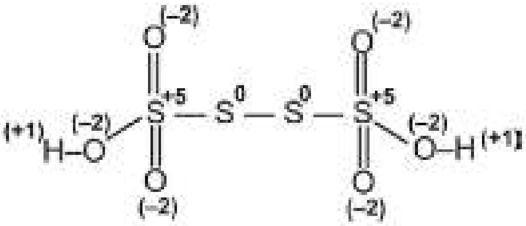


Number of atoms with zero oxidation state = 0



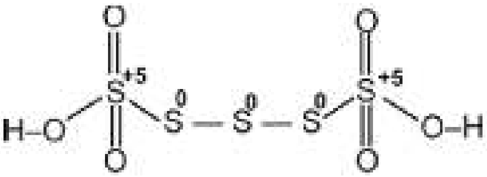
Number of atom with zero oxidation state = 0

$H_2S_4O_6$



Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



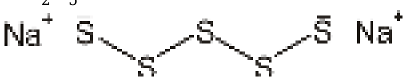
Number of atoms where zero oxidation state = 3

C_3O_2

$$O = C = \overset{(0)}{C} = C = O$$

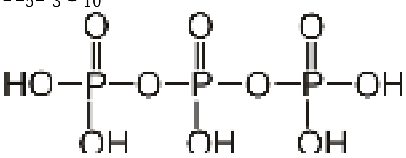
Number of atoms with zero oxidation state = 1

Na_2S_5



Number of atoms where zero oxidation state = 3

$H_5P_3O_{10}$



Number of atoms where zero oxidation state = 0

$Mn_2(CO)_{10}$

Number of Mn with zero oxidation state = 2

32) For single electron system

$r = 52.9 \times \frac{n^2}{Z}$ pm

Given Z = 2 for He^+

$r_2 = 105.8$ pm

So $105.8 = 52.9 \times \frac{n_2^2}{2}$

$n_2 = 2$

$r_1 = 26.45$

So $26.45 = 52.9 \times \frac{n_1^2}{2}$

$n_1 = 1$

So transition is from 1 to 2.

$\lambda = \frac{2\pi r}{n}$

$\Delta\lambda = 2\pi\left(\frac{r_2}{n_2} - \frac{r_1}{n_1}\right)$

$= 2\pi \times (52.9 - 26.45) = 158.7$

33) Weight of 50 ml 0.2 molal urea = V × d = 50 × 1.012 = 50.6 gm

Given 0.2 molal implies

1000 gm solvent has 0.2 moles urea

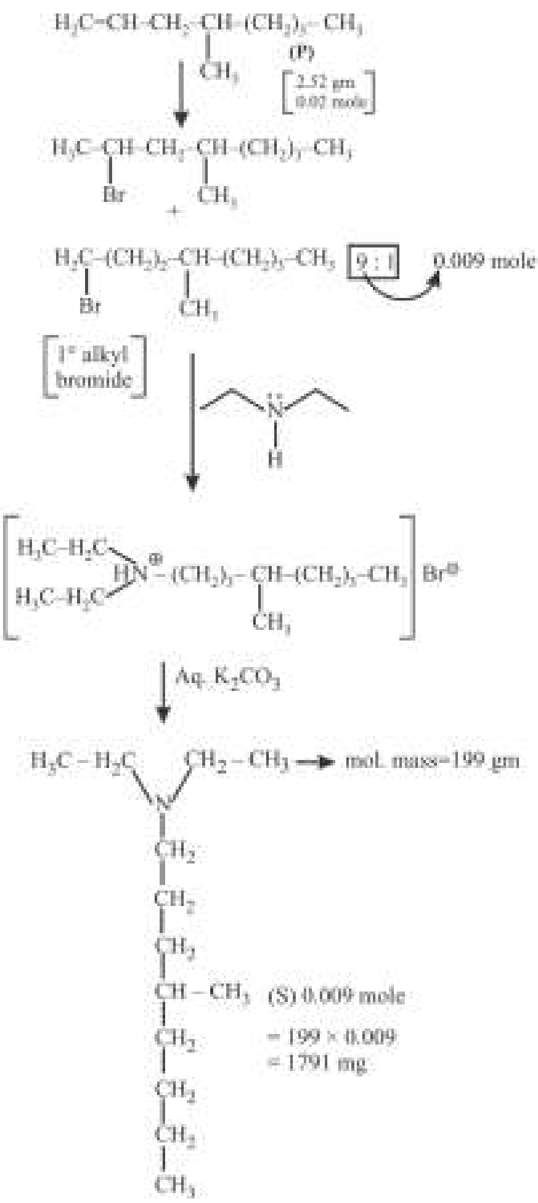
So weight of solution = 1000 + 0.2 × 60 = 1012 gm.

So wt. of urea in 50.6 gm solution = $\frac{12 \times 50.6}{1012}$ = 0.6 gm

Total urea = 0.6 + 0.06 = 0.66 gm

Total volume = 550 ml

$\Delta T_b = K_b.m = 0.5 \times \frac{0.66 \times 1000}{60 \times 550} = 10^{-2}$

34) 

PART-3 : MATHEMATICS

35)

Diff. wr.t 'x'

$3f(x) = f(x) + xf'(x) - x^2$

$\frac{dy}{dx} - \left(\frac{2}{x}\right) y = x$

$IF = e^{-2\ell nx} = \frac{1}{x^2}$

$y\left(\frac{1}{x^2}\right) = \int x.\frac{1}{x^2}dx$

$y = x^2 \ln x + cx^2$

$\because y(1) = \frac{1}{3} \Rightarrow c = \frac{1}{3}$

$y(e) = \frac{4e^2}{3}$

36) $P(H) = \frac{1}{3}; P(T) = \frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or ....)

+ P(THH or THTHH or THTHTHH or ....)

$= \frac{\frac{1}{3}\cdot\frac{1}{3}}{1-\frac{2}{3}\cdot\frac{1}{3}} + \frac{\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}{1-\frac{2}{3}\cdot\frac{1}{3}} = \frac{5}{21}$

37)

$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

|M| = –1 + 1 = 0 ⇒ M is singular so non-invertible

(B) $M\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$\left.\begin{array}{l} a_1 + a_2 + a_3 = -a_1 \\ a_1 + a_3 = -a_2 \\ a_2 = -a_3 \end{array}\right\} \Rightarrow$ $a_1 = 0$ and $a_2 + a_3 = 0$ infinite solutions exists [B] is correct.

Option (D)

$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

|M – 2I| = 0 ⇒ [D] is wrong

Option (C) :

$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

x + y + z = 0

x + z = 0

y = 0

 Infinite solution

[C] is correct

38) $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{t-\cos t}}{1+t^{2023}}dt$

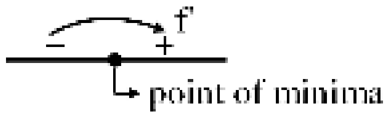
$f'(x) = \frac{e^{x\tan^{-1}x - \cos(x\tan^{-1}x)}}{1+(x\tan^{-1}x)^{2023}} \cdot \left(\frac{x}{1+x^2} + \tan^{-1}x\right)$

For x < 0, $\tan^{-1}x \in \left(-\frac{\pi}{2}, 0\right)$

For $x \geq 0$, $\tan^{-1}x \in \left[0, \frac{\pi}{2}\right)$

$\Rightarrow x\tan^{-1}x \geq 0 \ \forall\ x \in R$

And $$\frac{x}{1+x^2} + \tan^{-1}x = \begin{cases} > 0 & \text{For } x > 0 \\ < 0 & \text{For } x < 0 \\ 0 & \text{For } x = 0 \end{cases}$$



Hence minimum value is $f(0) = \int_0^0 = 0$

39)

f(x) is discontinuous at $x = \frac{4}{5}$ only

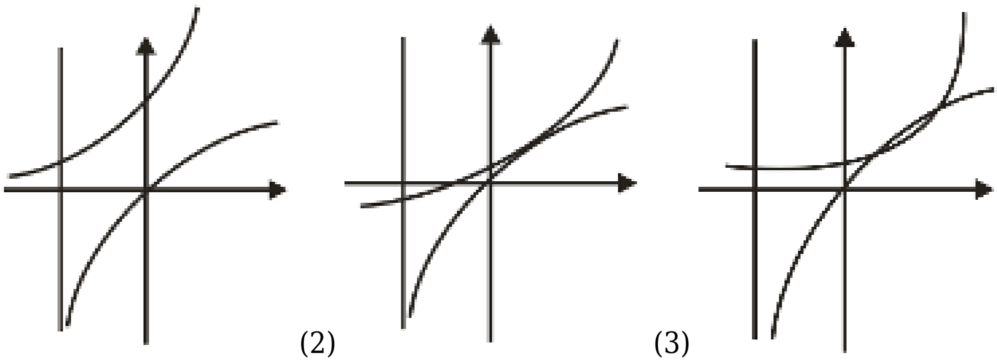
f(x) is non differentible at $x = \frac{3}{5}$ and $x = \frac{4}{5}$

40) S = Set of all twice differentiable functions $f : R \to R$

$\frac{d^2f}{dx^2} > 0$ in (–1, 1)

Graph 'f' is Concave upward.

Number of solutions of $f(x) = x \to x_f$



(1) (2) (3)

$\Rightarrow$ Graph of $y = f(x)$ can intersect graph of $y = x$ at atmost two points $\Rightarrow 0 \leq x_f \leq 2$

Aliter

$\frac{d^2f(x)}{dx^2} > 0$

Let $\phi(x) = f(x) - $□$n(x + 1)$

$\phi''(x) > 0$

$\therefore\ \phi'(x) = 0$ has atmost 1 root in $x \in (-1, 1)$

$\therefore\ \phi(x) = 0$ has atmost 2 roots in $x \in (-1, 1)$

□ $x_f \leq 2$

41) $P\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) = P\left(\vec{a}\right)$

$Q\left(3\hat{i} + 6\hat{j} + 3\hat{k}\right) = Q\left(\vec{b}\right)$

$R\left(\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}\right)=R(\vec{c})$

$S\left(2\hat{i}+\hat{j}+\hat{k}\right)=S\left(\vec{d}\right)$

$\frac{\vec{b}+2\vec{d}}{3}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}$

$\frac{5\vec{c}+4\vec{a}}{9}=\frac{21\hat{i}+24\hat{j}+15\hat{k}}{9}$

$\Rightarrow \frac{\vec{b}+2\vec{d}}{3}=\frac{5\vec{c}+4\vec{a}}{9}$

so [B] is correct.

option –D

$\left|\vec{b}\times\vec{d}\right|^2=\left|\vec{b}\right|\left|\vec{d}\right|^2-\left(\vec{b}.\vec{d}\right)^2$

$= (9 + 36 + 9) (4 + 1 + 1) – (6 + 6 + 3)^2$

$= 54 \times 6 – (15)^2$

$= 324 – 225$

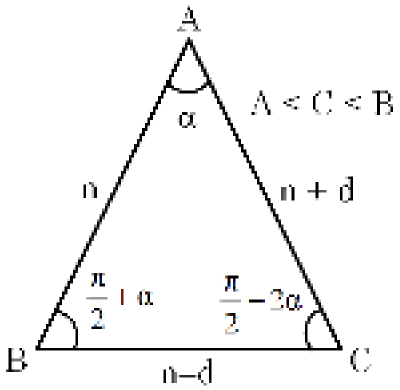
$= 99$

42) Distance between

INcentre and circumcentre

is $= \sqrt{R^2-2Rr}$

$D=\sqrt{1-\left(\frac{1}{4}\right)}=\sqrt{\frac{1}{2}}$



$n - d = 2 \sin\alpha$ ...(1)

$n + d = 2 \sin\left(\frac{\pi}{2}+\alpha\right)$

$\Rightarrow n + d = 2 \cos\alpha$ ....(2)

$n=2\sin\left(\frac{\pi}{2}-2\alpha\right)$

$\Rightarrow n=2\cos 2\alpha$ ...(3)

$\Rightarrow 2 \cos2\alpha = \sin\alpha + \cos\alpha$

$\Rightarrow 2(\cos\alpha - \sin\alpha) = 1$

$\Rightarrow \sin 2\alpha=\frac{3}{4}$

43)

From equation in above Ques.

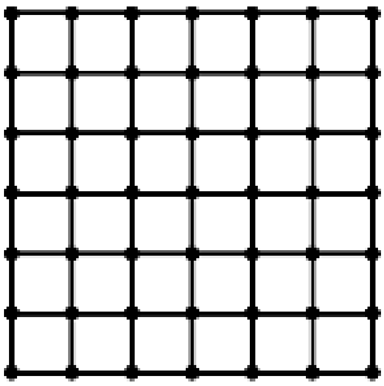
$$r = \frac{\Delta}{s} = \frac{1}{2}\frac{n(n+d)\sin\alpha}{\left(\frac{3n}{2}\right)}$$

$$= \frac{(n+d).\sin\alpha}{3}$$

$$= \frac{2\cos\alpha.\sin\alpha}{3} \text{ (from (2))}$$

$$r = \frac{\sin 2\alpha}{3} = \frac{1}{4}$$

44)



P_i = Probability that randomly selected points has friends

$P_0 = 0$ (0 friends)

$P_1 = 0$ (exactly 1 friends)

$P_2 = \frac{^4C_1}{^{49}C_1} = \frac{4}{9}$ (exactly 2 friends)

$P_3 = \frac{^{20}C_1}{^{49}C_1} = \frac{20}{49}$ (exactly 3 friends)

$P_4 = \frac{^{25}C_1}{^{49}C_1} = \frac{25}{49}$ (exactly 4 friends)

x	0	1	2	3	4
P(x)	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

Mean = E(x) = $\sum x_i P_i = 0 + 0 + \frac{8}{49} + \frac{60}{49} + \frac{100}{49} = \frac{168}{49}$

$7(E(x)) = \frac{168}{49} \times 7 = 24$

45) Total number of ways of selecting 2 persons = $^{49}C_2$

Number of ways in which 2 friends the selected $= \frac{12+32+60+24+84+72}{2} = 142$

$21P = 21 \times \frac{142 \times 2}{49 \times 48} = \frac{71}{28}$

No. of points	Mutual Friends

4 8 12 4 12 9	3 4 5 6 7 8

46) $x\sin x\frac{dy}{dx}-(\sin x+x\cos x)y=-2x^3\sin^2x$

$d\left(\frac{y}{x\sin x}\right)=-2xdx$

$\frac{y}{x\sin x}=-x^2+c$

$y = x\sin x(c - x^2)$

$f(x) = x(c - x^2)$

$f(1) = 2 \Rightarrow c = 3$

$f(x) = 3x - x^3$

47)

No. of elements in X which are multiple of 3 are formed using

1,2,3,3,3 or

1,2,3,3,0

11220 or

11223

Total = 122

No. of element in X which are multiple of 20 = 3 + 3 = 6

$P=\frac{6}{122}=\frac{3}{61}$

48)

$z^8 - 2^8 = (z - 2)(z - \alpha)(z - \alpha^2)\ldots(z - \alpha^7)$

Put $z = 2e^{i}\theta$

$2^8(e^{i8}\theta - 1) = (2e^{i\theta} - 2)(2e^{i\theta} - a)\ldots\ldots(2e^{i\theta} - a^7)$

Take mod

$2^8|e^{i8\theta} - 1| = PA_1\ PA_3\ PA_5\ldots\ PA_{15}$

$2^8|2\sin4\theta| = PA_1\ PA_3\ PA_5\ldots\ PA_{15}$

$(PA_1\ PA_3\ PA_5\ldots\ PA_{15})_{max} = 512$

49)

Case I : ad = bc = 0

Total ways = 11 × 11 = 121

Case II : ad = bc ≠ 0

(i) a = b = c = d

$^5C_1$

(ii) $ad = bc$

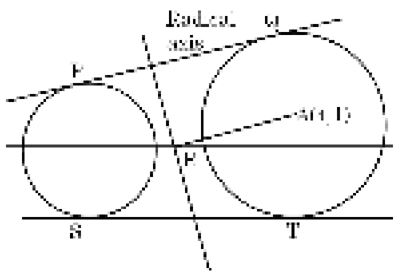
No. of ways = 60

Total = 5 + 60 = 65

$|R| = 0$ for total = 65 + 121 = 186

$|R| \neq 0$ for 1296 – 186 = 1110

50)



Let C_2 $(x - 4)^2 + (y - 1)^2 = r^2$

radical axis $8x + 2y - 17 = 1 - r^2$

$8x + 2y = 18 - r^2$

$B\left(\frac{18 - r^2}{8}, 0\right)$ A(4,1)

$AB = \sqrt{5}$

$$\sqrt{\left(\frac{18 - r^2}{8} - 4\right)^2 + 1} = \sqrt{5}$$

$r^2 = 2$

51) Case-I : $y \in (-3,0)$

$$\tan^{-1}\left(\frac{6y}{9 - y^2}\right) + \pi + \tan^{-1}\left(\frac{6y}{9 - y^2}\right) = \frac{8\pi}{3}$$

$$2\tan^{-1}\left(\frac{6y}{9 - y^2}\right) = \frac{5\pi}{3}$$

$y^2 - 6\sqrt{3}y - 9 = 0 \Rightarrow y = 3\sqrt{3} - 6$ ($\because y \in (-3,0)$)

Case-I : $y \in (0,3)$

$$2\tan^{-1}\left(\frac{6y}{9 - y^2}\right) = \frac{8\pi}{3} \Rightarrow \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$y = \sqrt{3}$ or $y = -3\sqrt{3}$ (rejected)