FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
CONCEPT RECAPITULATION TEST – I
PAPER –1
TEST DATE: 24-04-2025

ANSWERS, HINTS & SOLUTIONS

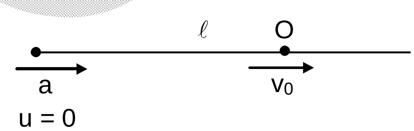
Physics

PART – I

SECTION – A

1. B

Sol. Let first pulse be released at t = 0.



Time when first pulse reaches O = $t_1 = \frac{\ell}{v - v_o}$

Time when second pulse reaches O =

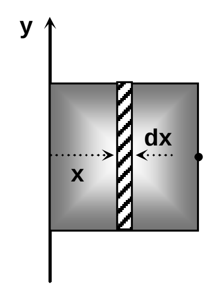
$$t_2 = T + \frac{\ell + v_oT - \frac{1}{2}aT^2}{v - v_o}$$

$$T' = t_2 - t_1 = \frac{vT}{v - v_o} - \frac{aT^2}{2(v - v_o)}$$

$$\therefore \quad f' = \frac{2f^2(v - v_o)}{2fv - a}$$

2. C

Sol. $dM = \sigma_0\left(1 - \frac{x}{a}\right)dx$



$$M = \frac{\sigma_0 a^2}{2}$$

$\Rightarrow$ d(MOI) = dmx^2

$$MOI = \int dMx^2 = \frac{Ma^2}{6}$$

3. A

Sol. $T = kp^x d^y E^z$

$$[T] = [ML^{-1}T^{-2}]^x [ML^{-3}]^y [ML^2T^{-2}]^z$$

$$x + y + z = 0 \qquad \ldots(1)$$

$-x-3y+2z=0$ ...(2)

$-2x-2z=1$ ...(3)

$x+z=-\frac{1}{2}$ ...(4)

$-y=-\frac{1}{2}$

$\Rightarrow \quad y=\frac{1}{2}$

by equation (2) $-x-\frac{3}{2}+2z=0$

$-x+2z=\frac{3}{2}$ ...(5)

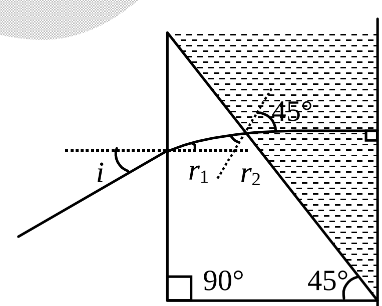
Adding (4) & (5)

$3z=1,\ z=\frac{1}{3}$

$\Rightarrow \quad x=-\frac{1}{2}-\frac{1}{3}=-\frac{5}{6}$

4. B

Sol. Since incident ray retraces its path it must strike the plane mirror perpendicularly.



From Snell's law $\sin i = \mu_1 \sin r_1$

and $\mu_1 \sin r_2 = \mu_2 \sin 45° \Rightarrow \mu_1 \sin r_2 = \frac{\mu_2}{\sqrt{2}}$

$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$

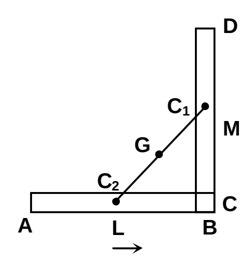
Also, $r_1 + r_2 = \frac{\pi}{4}$

$\therefore \quad r_1 = \frac{\pi}{4}-\sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$

$\therefore \quad i = \sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4}-\sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

5. B, C

Sol. Conservation of angular momentum about G gives:



$$\mathrm{Mv}\cdot\frac{L}{4}=2\times\left[\frac{1}{12}\mathrm{ML}^2+\mathrm{M}\left(\frac{\sqrt{2}L}{4}\right)^2\right]\times\omega$$

Solving

$\omega=\frac{3v}{5L}$

conservation of momentum gives

$Mv = 2Mv_{CM}$

or, $v_{CM} = v/2$

6. A, C

Sol. $\frac{dv}{dx} = \frac{2}{1} = 2s^{-1}$

$F = \eta A \frac{dv}{dx} = 10^{-3} \times 10 \times 2 = 0.02$ N

7. A, B, C

Sol. The tangential acceleration is $-\mu g$

$v_0 - \mu g t = 0$

$t = \frac{v_0}{\mu g}$

Centripetal acceleration = $\frac{k}{m} v$

$\tan 60 = \frac{a_c}{a_t} = \frac{k}{m} \frac{v_1}{\mu g}, \quad v_1 = \frac{\mu m g \sqrt{3}}{k}$

$\tan 30 = \frac{k v_2}{m \mu g}$

$v_2 = \frac{\mu m g}{\sqrt{3} k}$

$\frac{v_1 - v_2}{\mu g} = t = \frac{m}{k} \frac{2}{\sqrt{3}}$

$\frac{1}{2} m v_0^2 = \mu m g s$

$s = \frac{{v_0}^2}{2\mu g}$

$R = \frac{v^2}{a_c} = \frac{m}{k}(v_0 - \mu g t)$

8. A

Sol. From *A* to *B*

Angular frequency $\omega = \sqrt{K/M}$

$V = \omega\sqrt{A_1^2 - X^2} \Rightarrow V^2 = \frac{K}{M}\left(A_1^2 - L^2\right)$

$\Rightarrow \quad A_1 = \left[\frac{MV^2}{K} + L^2\right]^{1/2} = \sqrt{2}$ ...(i)

Also, $L = A_1 \cos \omega t$, $t_1 = \sqrt{\frac{M}{K}} \cos^{-1}\left[\frac{L}{\left(L^2 + \frac{MV^2}{K}\right)^{1/2}}\right] = \pi/4$

From *B* to *O*

Now angular frequency $\omega_2 = \sqrt{\dfrac{K}{2M}}$, $V = \omega_2\left(A_2^2 - L^2\right)^{1/2}$,

$$A_2 = \left[L^2 + \frac{2MV^2}{K}\right] = \sqrt{3}$$

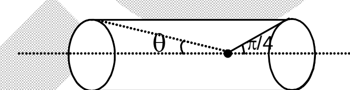
Also $L = A_2 \sin \omega t_2$, $t_2 = \sqrt{\dfrac{2M}{K}}\sin^{-1}\dfrac{L}{\left[L^2 + \dfrac{2MV^2}{K}\right]^{1/2}} = \sqrt{2}\sin^{-1}\dfrac{1}{\sqrt{3}}$

Now V_{cm} at O is $A_2\omega_2 = \sqrt{\dfrac{3}{2}}$

The maximum compression of the spring is $\sqrt{3}$

9. D

Sol. $\tan\theta = \dfrac{R}{2R} = \dfrac{1}{2}$



$\cos\theta = \dfrac{2}{\sqrt{5}}$

Flux through F_1, $\phi_1 = \dfrac{q}{2\varepsilon_0}\left[1-\cos\dfrac{\pi}{4}\right] = \dfrac{q}{2\varepsilon_0}\left[1-\dfrac{1}{\sqrt{2}}\right]$

Flux through F_2, $\phi_2 = \dfrac{q}{2\varepsilon_0}[1-\cos\theta] = \dfrac{q}{2\varepsilon_0}\left[1-\dfrac{2}{\sqrt{5}}\right]$

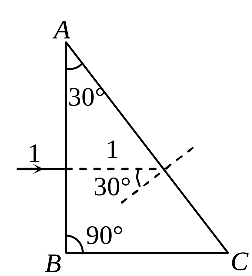
Flux through cylinder $\phi = q/\varepsilon_0$

Flux through curved surface $= \phi - \phi_1 - \phi_2$

$$= \frac{q}{\varepsilon_0}\left[1-\frac{1}{2}+\frac{1}{2\sqrt{2}}-\frac{1}{2}+\frac{1}{\sqrt{5}}\right] = \frac{q}{\varepsilon_0}\left[\frac{1}{2\sqrt{2}}+\frac{1}{\sqrt{5}}\right]$$

10. B

Sol. (A) Refractive index of the prism is the minimum value required for ray (1) to undergo total internal reflection at face AC. Ray (1) falls on face AC at an angle of incidence 30°



∴ 30° > i_C

sin 30° > sin i_C

∴ $\mu > 2$

Minimum value of μ can be taken as 2.

(B) For ray 2, refractive angle of prism is 30°. Apply Snell's law for refraction at face AB.

1 sin i = μ sin r

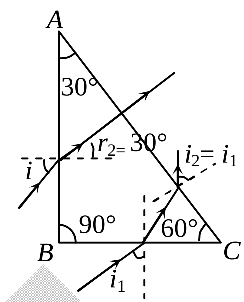
i = 90°

(C) Using the relation $i_1 + i_2 = A + \delta$ for ray 2.

$90° + 0° = 30° + \delta$

$\delta = 60°$

(D) $\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{2}\right)}{\sin\dfrac{A}{2}} \Rightarrow \delta m = 120°$



11. D

Sol. $w_g = \vec{F}.\vec{S} = (mg\sin\theta)S = 2\times10\times\frac{1}{2}\times\frac{20}{100} = 2\text{ J}$

$w_s = -\frac{1}{2}kx^2 = -\frac{1}{2}(1000)\left(\frac{20}{100}\right)^2 = -20\text{ J}$

$w_N = 0$

From work energy theorem $w_g + w_{sp} + w_N + w_{ex} = \Delta k$ or $2 - 20 + 0 + w_{ex} = 0$

$w_{ex} = 18$ J

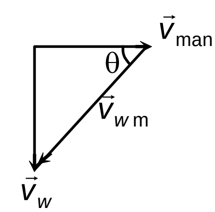
SECTION – B

12. 100

Sol. Let velocity of wind $= v_w$

Initially velocity of man = 2.5 π m/s

Velocity of wind with respect to man makes an angle 45^0 with track initially



$\tan\theta = \dfrac{v_w}{v_{man}}$

$\theta = 45^0$ when $v_{man} = 2.5\pi$ m/s $\Rightarrow$ $v_w = 2.5\ \pi$ m/s

Let when velocity of man is v_m the wind appears to make an angle θ with track, so

$v_m = v_w\cot\theta$

$v_m = 2.5\pi\cot\theta$

$\dfrac{dx}{dt} = 2.5\pi\cot\theta$ [where x is position of man w.r.t. origin at time t]

$\Rightarrow \dfrac{dx}{d\theta}.\dfrac{d\theta}{dt} = 2.5\pi\cot\theta$

but $\dfrac{d\theta}{dt} = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\pi}{4\left(\dfrac{10}{\ln\sqrt{2}}\right)} = \dfrac{\pi\ln\sqrt{2}}{40}$

$\dfrac{dx}{d\theta} = \dfrac{100\pi}{\pi\ln\sqrt{2}}\cot\theta$

$x = \dfrac{100\pi}{\pi\ln\sqrt{2}}[\ln\sin\theta]_{\pi/4}^{\pi/2} = \dfrac{100\pi}{\pi\ln\sqrt{2}}\left[\ln\sqrt{2}\right] = 100$ m

So length of race is 100 m.

13. 350

Sol. Time period of oscillation of *R*

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \times \frac{10}{\pi} = 20s$$

At a time t = 10 s, *R* will be at mean position and moving along negative x-axis.

$$v_R = A\omega = 15 \text{ m/s}$$

The sound which is received at t = 10s, is emitted at $t = t_0$ s.

$$\frac{1}{2}at_0^2 = v(10 - t_0)$$

$$\frac{1}{2} \times 18.75 t_0^2 = 300(10 - t_0) \Rightarrow t_0 = 8\,\text{s}$$

$$v_s = at = 18.75 \times 8 = 150 \text{ m/s}$$

$$f' = 500\left(\frac{300+15}{300+150}\right) = 350\,\text{Hz}$$

14. 1

Sol. $$P = \frac{1}{2}\rho\omega^2 A^2 sV$$

Since $\frac{\lambda_1}{\lambda_2} = \frac{1}{2}, \frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \frac{2}{1}$

Since $P_1 = P_2, \omega_1 A_1 = \omega_2 A_2$,

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{1}{2}$$

Pressure amplitude, $P_0 = B_0 Ak$

$$(P_0)_1/(P_0)_2 = \left(\frac{A_1}{A_2}\right)\left(\frac{k_1}{k_2}\right) = \left(\frac{A_1}{A_2}\right)\left(\frac{\lambda_2}{\lambda_1}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$$

15. 100

Sol. $B = -\frac{P}{\Delta V/V}$ or $P = -B.\frac{\Delta V}{V}$

$\therefore$ $$P = 9.8 \times 10^8 \times \left(\frac{0.1}{100}\right)$$

or $$h\rho g = 9.8 \times 10^8 \times \left(\frac{0.1}{100}\right)$$

$\therefore$ $h = \frac{9.8 \times 10^8}{1000 \times 9.8} \times \frac{0.1}{100}$ m = 100 m

16. 6

Sol. As source (horn of bus) is approaching stationary wall (say, listener), therefore, apparent frequency striking the wall is

$$v' = \frac{\upsilon\, v}{\upsilon - \upsilon_s} \quad \ldots(1)$$

Sound of this frequency will be reflected by the wall (now, source). The passenger is the listener moving towards source. Therefore, frequency heard by the listener $v'' = \frac{(\upsilon + \upsilon_L)v'}{\upsilon}$

Using (1)

$$v'' = \frac{\upsilon + \upsilon_L}{\upsilon} \times \frac{\upsilon v}{\upsilon - \upsilon_s} = \frac{(\upsilon + \upsilon_L)v}{\upsilon - \upsilon_s}$$

$$= \frac{(330+5)\times 200}{330-5} = \frac{335}{325} \times 200$$

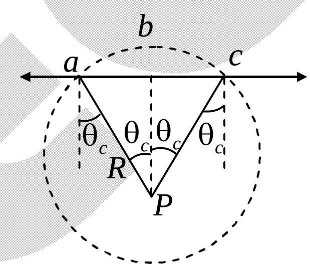
$v'' = 206$ Hz

$\therefore$ Beat frequency = $(v'' - v)$

$= 206 - 200 = 6$ Hz

17. 1

Sol. The light escape is confined within a cone of apex angle '$2\theta_c$' where θ_c is the critical angle. Imagine a sphere with source of light as its centre and the surface area *abc* is *A*.



here $A = \int_0^{\theta_c} 2\pi R^2 \sin\theta \, d\theta = 2\pi R^2 (1 - \cos\theta_c)$

$$= \pi R^2 \qquad \left[\therefore \theta_c = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ\right]$$

$\therefore$ Power transfer $= P \times \frac{A}{4\pi R^2} = = 4 \times \frac{1}{4} = 1\,\text{W}$

Chemistry PART – II

SECTION – A

18. C

Sol. At 2nd equation point:

$$P^H = \frac{1}{2}\left(PK_{a_2} + PK_{a_3}\right) = \frac{8+12}{12} = 10$$

Now, $K_{a_1}.K_{a_2}.K_{a_3} = \frac{\left[H^+\right]\left[A^{3-}\right]}{\left[H_3A\right]}$

or $7.5\times10^{-4}\times10^{-8}\times10^{-12} = \frac{\left(10^{-10}\right)\times\left[A^{3-}\right]}{\left[H_3A\right]}$

$$\therefore \frac{\left[H_3A\right]}{\left[A^{3-}\right]} = \frac{10^{-6}}{7.5} = 1.33\times10^{-7}$$

19. B

Sol The mole fraction of A in distillate,

$$X_A^{'} = Y_A = \frac{X_A.P_A^{\circ}}{P_{total}} = \frac{\frac{1}{4}\times100}{\frac{1}{4}\times100+\frac{3}{4}\times80} = \frac{5}{17}$$

Now, V.P. of distillate, $P = X_A^{'}.P_A^{\circ} + X_B^{'}.P_B^{\circ}$

$$= \frac{5}{17}\times100 + \frac{12}{17}\times80 = 85.88\,\text{mm Kg}$$

20. D

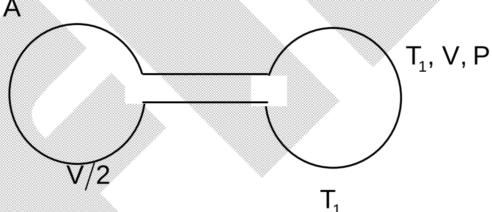
Sol. Using MOT of O_2 molecule.

21. B

Sol. $Cu^{+2} + 2CN^{-1} \rightarrow CuCN + \frac{1}{2}(CN)_2\,\text{gas}$

22. A

Sol.



$PV = nRT$

$$n_1 = \frac{PV}{RT_1}$$

$$n_2 = \frac{PV/2}{RT_2}$$

$$P_1\left(\frac{V}{RT_1} + \frac{V}{2RT_2}\right) = \frac{P(V)}{RT_1}$$

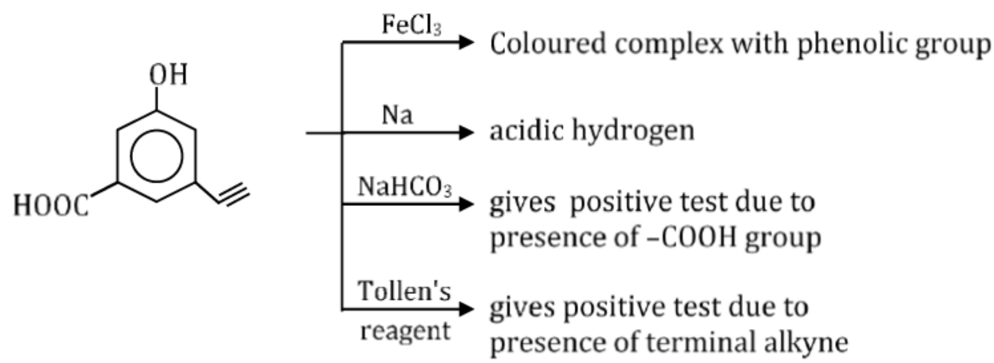
$$P_1\left(1+\frac{T_1}{2T_2}\right)=P$$

$$P_1\left(\frac{2T_2+T_1}{2T_2}\right)=P$$

$$P_1=\frac{2PT_2}{T_1+2T_2}$$

23. A, B, C, D

Sol.



24. A,B,C,D

Sol. factual

25. A

Sol. (P) $[Cr(H_2O)_4Br_2]^+ \Rightarrow$ Paramagnetic, d^2sp^3, show geometrical isomerism

(Q) $[Cu(NH_2CH_2CH_2NH_2)(CN)_2Cl]^{2-} \Rightarrow$ Paramagnetic, sp^3d^2, show Geometrical isomerism

(R) $[Pt(ox)_2]^{2-} \Rightarrow$ Diamagnetic, dsp^2

(S) $[Fe(OH)_4]^- \Rightarrow$ Paramagnetic, sp^3

26. A

Sol. HCO_3^-, CO_3^{2-} - CO_2 type gas with dil. H_2SO_4

Both Can't react with $K_2Cr_2O_7$ solution (acidic) due to + 4 oxidation of C in HCO_3^-, CO_3^{2-}

27. A

Sol. Cr_2O_3- Amphoteric oxide

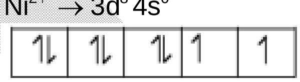
N_2O- Neutral oxide

Fe_3O_4- Mix oxide

CrO_3- Acidic oxide

28. C

Sol. $Ni^{2+} \rightarrow 3d^8 4s^0$



$n \rightarrow 2$

$\mu=\sqrt{n(n+2)}$

$Cr^{2+} \rightarrow 3d^4 \rightarrow d-d$ transition because in aq. medium it exists as $[Cr(H_2O)_6]^{2+}$

$V^{2+} \rightarrow d^3 \rightarrow t_{2g}^3 e_g^0$

$Ti^{4+} \rightarrow [Ar]3d^0 4s^0 \rightarrow$ no unpaired electron

SECTION – B

29. 5

Sol. Using MOT

2,1,2 Pia (π) bond in given C_2, O_2, N_2

30. 5

Sol. Carbon already exit in higher oxidation state (+ 4)

31. 2

Sol. $n = 1 \Rightarrow t_{100\%} = \frac{1}{K}.\ln\frac{[A_0]}{0} = \text{Infinite}$

$n \neq t_{100\%} = \frac{[A_0]^{1-n} - (0)^{1-n}}{K(1-n)} = \frac{[A_0]^{1-n}}{K(1-n)} \text{if } n < 1$

Infinite if n > 1

32. 5

Sol. $t_{1/2} = \frac{0.693}{6.93 \times 10^{-4}} = 1000 \text{ sec}$

$$\begin{array}{lcc} & A \longrightarrow & nB \\ t = 0 & a\,\text{mole} & 0 \\ t = 1000\,\text{sec} & \frac{a}{2}\text{mole} & \frac{n.a}{2}\text{mole} \end{array}$$

$\text{Now,} \frac{\frac{a}{2} + \frac{n.a}{2}}{a} = 3 \Rightarrow n = 5$

33. 1520

Sol. $SnCl_2 \longrightarrow Sn^{2+} + 2Cl^-$

Cathode: $Sn^{2+} + 2e^- \longrightarrow Sn$

Anode: $2Cl^- \longrightarrow Cl_2 + 2e-$

$Cl_2 + SnCl_2 \longrightarrow SnCl_4$

Moles of $SnCl_2$ taken = $\frac{19}{190} = 0.1$

Moles of Sn produce = $\frac{1.19}{119} = 0.01$

= moles of Cl_2 produced

= moles of $SnCl_4$ formed

and moles of $SnCl_4$ left = 0.1 – (0.01 + 0.01) = 0.08

$\therefore \frac{m_{SnCl_2}}{m_{SnCl_4}} = \frac{0.08 \times 190}{0.01 \times 261} = \frac{1520}{261}$

34. 108

Sol. $\Lambda^{\circ}_{eq}\left[Ba_3(PO_4)_2\right] = 160 + 140 - 100$

$= 200 \text{ Ohm}^{-1} \text{ cm}^2 \text{ eq}^{-1}$

Now, $\wedge^{\circ}_{eq} = \wedge_{eq} = \frac{\kappa}{C} \Rightarrow 200 = \frac{1.2 \times 10^{-5}}{S}$

$\therefore S = 6 \times 10^{-8} \text{eq}/\text{cm}^3 = 6 \times 10^{-5} N = 10^{-5} M$

$\therefore K_{sp} = 108\, S^5 = 108 \times 10^{-25} M^5$

Mathematics PART – III

SECTION – A

35. B

Sol. The locus will be circle with diameter with (1, 2) and (2, 3)

36. C

Sol. $V = \frac{1}{6} abd \sin\theta$

37. D

Sol. $f'(x) = -(2\cos(\sin x)\sin\sin(x)\cos x + 2\sin(\cos)\cos(\cos x)\sin x) \le 0$

$\Rightarrow$ Range is $\left[f\left(\frac{\pi}{2}\right), f(0)\right]$ as f(x) is periodic with period $\frac{\pi}{2}$

38. D

Sol. As $(a + b + c)^3 = a^3 + b^3 + c^3 \Rightarrow (a + b)(b + c)(c + a) = 0$

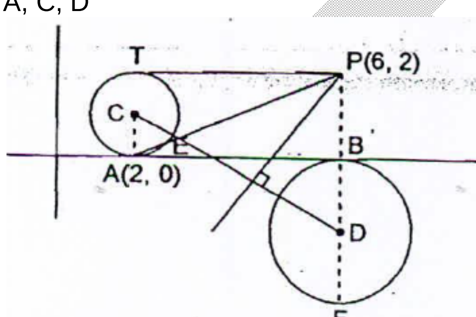
$\Rightarrow (\sin x + \sin 2x)(\sin 2x + \sin 3x)(\sin 3x + \sin x) = 0$

$\Rightarrow 2\sin\frac{3x}{2}\cos\frac{x}{2}$ or $2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$ or $2\sin 2x \cos x = 0$

$\Rightarrow$ On solving least positive angle is $\frac{2\pi}{5}$.

39. A, C, D

Sol.



Clearly P lies on radical axis

$\Rightarrow PE \cdot PA = PB \cdot PF$

$\Rightarrow \frac{PE}{PF} = \frac{PB}{PA} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$

Again PE . PA = PT^2

$\Rightarrow PE = \frac{4^2}{2\sqrt{5}} = \frac{8}{\sqrt{5}}$

40. A, B, D

Sol. Given $a \ne b \ne c$, $a, b, c \in R$

Now $ax^2 + bx + c \ge 0 \Rightarrow b^2 - 4ac \le 0$ and $a > 0$ ....(1)

$bx^2 + cx + a \ge 0 \Rightarrow c^2 - 4ab \le 0$ and $b > 0$ ....(2)

$cx^2 + ax + b \le 0 \Rightarrow a^2 - 4bc \le 0$ and $c > 0$ ....(3)

Equality cannot hold simultaneously ($\because a \ne b \ne c$)

$\therefore \frac{a^2+b^2+c^2}{ab+bc+ca} < 4$

and since $a \neq b \neq c$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 > 0$

$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} > 1$

$\Rightarrow \frac{a^2+b^2+c^2}{ab+bc+ca} \in (1, 4)$

41. A, B

Sol. Required probability = $\frac{{}^5C_4(2)^4}{{}^{10}C_4} + \frac{\left({}^5C_1 \times 1\right)\times\left({}^4C_2 \times 2^2\right)}{{}^{10}C_4} \times \frac{\left({}^2C_2 \times 2^2\right)}{{}^6C_4} = \frac{8}{15}$

42. D

Sol. $P \to 2$; $Q \to 1$; $R \to 4$; $S \to 3$

43. D

Sol. (P) no. of digits of form $a_1 > a_2 > a_3 > a_4 = {}^{10}C_4 \times 1$

no. of digits of form $a_1 > a_2 = a_3 > a_4 = {}^{10}C_3 \times 1$

no. of digits = ${}^{10}C_4 + {}^{10}C_3 = 330$

(Q) $4\lambda+2 = 2(2\lambda+1) = 2 \times$ odd no

$\Rightarrow$ no. of division = $(8+1)\times(6+1)-1 = 62$

(R) Let E_i is the set which contains all possible function in which $x_i = y_i$ from inclusion exclusion

$n(UE_i) = \sum(nE_i) - \sum n(E_i \cap E_j) + \sum n(E_i \cap E_j \cap E_k) - \ldots\ldots\ldots$

$n(E_i) = 5^9$ $n(E_i \cap E_j) = 5^8$ $n(E_i \cap E_j \cap E_k) = 5^7$.& so on

$n(UE_i) = {}^5C_1 5^9 - {}^5C_2 5^8 + {}^5C_3 5^7 - {}^5C_4 5^6 + {}^5C_5 5^5$

no. of desired fn. = 5^{10}-n (UE^i).

(S) Let the sides of polygon are n.

So no. of diagonals ${}^nC_2 - n = 35$

$\Rightarrow$ n= 10

$\Rightarrow$ no. of triangles can be formed is = ${}^{10}C_3 = 120$

44. D

Sol. (P) $\alpha + \beta = \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$

(Q) $\alpha\beta = (\omega + \omega^2 + \omega^4)(\omega^3 + \omega^5 + \omega^6) = 3\omega$

$(\alpha\beta)^3 = 27\omega^3 = 27$

(R) $\alpha = -1 + \omega = -1 + \frac{\left(-1 \pm \sqrt{3}i\right)}{2}$

(S) $\sum_{k=0}^{6} \omega^{k^2} = 1 + \omega^2 + \omega^4 + \omega^9 + \omega^{16} + \omega^{25} + \omega^{36}$

$= 3 + 4\omega = 3 + 4\frac{\left(-1 \pm \sqrt{3}i\right)}{2} = 1 \pm 2\sqrt{3}i = 1 \pm \sqrt{8}i$

45. B

Sol. (P) $\left(t_1^2, 2t_1\right), Q\left(t_2^2, 2t_2\right)$ and $t_2 = -t_1 - \frac{2}{t_1}$

$\tan\alpha = \dfrac{2}{t_1+t_2} = \dfrac{2}{-\dfrac{2}{t_1}} = -t_1$

$\tan\beta = -t_2 = t_1 + \dfrac{2}{t_1}$

(Q) $\left(\dfrac{x+y+1}{\sqrt{2}}\right)^2 = \dfrac{1}{\sqrt{2}}\left(\dfrac{y-x}{\sqrt{2}}\right)$

$\Rightarrow 4a = \dfrac{1}{\sqrt{2}} \Rightarrow a = \dfrac{1}{4\sqrt{2}}$

Therefore shortest normal chord = $6.\dfrac{1}{4\sqrt{2}}\sqrt{3}$

$\Rightarrow p - q = 1$
(S) $y^2 = 4\,(ay^2+2y+1)$
$y^2\,(1-4a) - 8\,y - 4 = 0$
$D = 0$
$64 + 16\,(1-4a) = 0$
$\Rightarrow a = \dfrac{5}{4}$

SECTION – B

46. 3

Sol. $2 - a = SP$
$= \dfrac{1}{2}$
$\Rightarrow a = \dfrac{3}{2} \Rightarrow 2a = 3.$

47. 2

Sol. Points on the first and second lines are (–1, 2, 0) and (3, –4, 1).
Equation of the plane is $8(x + 1) + (y - 2) - 13k(z) = 0$
(3, –4, 1) will also lie on it
$\Rightarrow 8(4) - 6 - 13k = 0$
$\Rightarrow 13k = 26 \Rightarrow k = 2.$

48. 2

Sol. $\dfrac{dy}{dx} + x\left(\dfrac{dy}{dx}\right)^2 - y = 0$ ……(1)
Let $y = mx + c$
$m + xm^2 = mx + c$
$\Rightarrow m = c,\ m^2 - m = 0$
$\Rightarrow m = 0, 1$
$y = 0,\ x + 1$

49. 2

Sol. Let $f(x) = x^3 - 3x^2 + 5x = (x-1)^3 + 2(x-1) + 3$
$g(y) = y^3 + 2y \Rightarrow g'(y) = 3y^2 + 2 > 0\ \forall y \in R$
$\Rightarrow g(\alpha - 1) = -2$ and $g(\beta - 1) = 2$ and g(y) is odd
$\Rightarrow (\alpha + \beta) = 2$

50. 3

Sol. f(x) = sin x + $\int_{-\pi/2}^{\pi/2}(\sin x + t\cos x)\left(-\frac{1}{k}\sin t - \frac{2}{k}\cos t\right)dt$

$= \sin x + I_1 + I_2 + I_3 + I_4$

Where $I_1 = -\frac{\sin x}{k}\int_{-\pi/2}^{\pi/2}\sin t dt = 0$

$I_2 = -\frac{2\sin x}{k}\int_{-\pi/2}^{\pi/2}\cot dt = -\frac{4\sin x}{k}$

$I_3 = -\frac{\cos x}{k}\int_{-\pi/2}^{\pi/2}t\sin t dt = -\frac{2\cos x}{k}$

$I_4 = \frac{4\cos x}{k}\int_{-\pi/2}^{\pi/2}t\cos t dt = 0$ $(\because$ tcost is odd $)$

$\Rightarrow -\frac{1}{k}\sin x - \frac{2}{k}\cos x = \sin x - \frac{4\sin x}{k} - \frac{2\cos x}{k}$

$\Rightarrow -\frac{1}{k} = 1 - \frac{4}{k} \Rightarrow k = 3.$

51. 1

Sol. $4\int_1^x f(t)dt = 2xf(x) - x^2$

Differentiating both sides f(x) = xf′(x) – x

or $y = x\frac{dy}{dx} - x$

or $\frac{dy}{dx} - \frac{y}{x} = 1$ (Linear form)

On solving $\frac{y}{x} = \ell nx + c$

$\because f(e) = 1 \Rightarrow \frac{1}{e} = 1 + c \Rightarrow c = \frac{1}{e} - 1$

$\frac{y}{x} = \ell nx + \frac{1}{e} - 1$

$y = x\ell nx + \frac{1}{e} - 1$

$y = x\ell nx + \frac{x}{e} - x$

$f(3) = 3\ell n3 + \frac{3}{e} - 3 = 1.394$

$\therefore [f(3)] = 1$