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IIT-JEE
Batch – Growth (May) | Minor Test-5

Time: 3 Hours | **Test Date: 25th August 2024** | **Maximum Marks: 300**

Name of the Candidate: ____Roll No.____________

Centre of Examination (in Capitals):____

Candidate's Signature:________________________Invigilator's Signature:_______________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 90 questions.
3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 30 questions each and every **PART** has two sections.
   (i) **Section-I** contains 20 multiple choice questions with only one correct option. Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
   (ii) **Section-II** contains 10 questions the answer to only 5 questions, is an INTEGERAL VALUE.
   Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For integer-based questions, the answer should be in decimals only not in fraction.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

TEST SYLLABUS

Batch – Growth (May) | Minor Test-5

25th August 2024

Mathematics:	Compound Angle & Trigonometric Eq
Physics:	NLM & Friction
Chemistry:	Chemical Bonding

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821$ Lit atm $K^{-1}mol^{-1}$
		$= 1.987 \approx 2$ Cal $K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}$ Js
		$= 6.25 \times 10^{-27}$ erg.s
1 Faraday		= 96500 Coulomb
1 calorie		= 4.2 Joule
1 amu		$= 1.66 \times 10^{-27}$ kg
1 eV		$= 1.6 \times 10^{-19}$ J

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I

1. If $\tan A = \dfrac{1}{\sqrt{x\left(x^2+x+1\right)}}$, $\tan B = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}, 0 < A, B, C < \dfrac{\pi}{2}$, then A + B is equal to:

(A) C

(B) $\pi - C$

(C) $2\pi - C$

(D) $\dfrac{\pi}{2} - C$

Ans. (A)

Sol. Finding tan(A + B) we get

$$\Rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\dfrac{1}{\sqrt{x\left(x^2+x+1\right)}} + \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \dfrac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B) = \frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{\left(x^2+x\right)(\sqrt{x})}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{\left(x^2+x\right)(\sqrt{x})}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

A + B = C

2. If $\cot\alpha = 1$ and $\sec\beta = -\dfrac{5}{3}$, where $\pi < \alpha < \dfrac{3\pi}{2}$ and $\dfrac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are

(A) $-\dfrac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) -7 and IVth quadrant

(D) $\dfrac{1}{7}$ and Ist quadrant

Ans. (A)

Sol. Given,

$$\cot\alpha = 1, \sec\beta = \frac{-5}{3}$$

So, $\cos\beta = \dfrac{-3}{5}, \tan\beta = \dfrac{-4}{3}$ and $\tan\alpha = 1$

Now using formula $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$$\Rightarrow \tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$$

We know that $\tan\theta$ is negative in 2^{nd} & 4^{th} quadrant but as $\pi < \alpha < \dfrac{3\pi}{2}$, so $\alpha+\beta$ will lie in IV^{th} quadrant.

3. Let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then the value of k is equal to:

(A) $-\dfrac{2}{3}$

(B) -5

(C) $\dfrac{2}{3}$

(D) 5

Ans. (B)

Sol. $3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$

$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$

$5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$

$\tan\beta = -\dfrac{1}{5}\tan\alpha$

$\tan\alpha = -5\tan\beta$

4. The value of $2\sin 12^\circ - \sin 72^\circ$ is

(A) $\dfrac{\sqrt{5}(1-\sqrt{3})}{4}$

(B) $\dfrac{1-\sqrt{5}}{8}$

(C) $\dfrac{\sqrt{3}(1-\sqrt{5})}{2}$

(D) $\dfrac{\sqrt{3}(1-\sqrt{5})}{4}$

Ans. (D)

Sol. To find $\Rightarrow 2\sin 12^\circ - \sin 72$

$\Rightarrow \sin 12^\circ + \sin 12^\circ - \sin 72$

$\Rightarrow \sin 12^\circ + 2\cos 42^\circ \sin\left(-30^\circ\right) \Rightarrow \sin 12^\circ - \cos 42^\circ$

$\Rightarrow \sin 12^\circ - \sin 48^\circ \Rightarrow 2\cos\left(30^\circ\right)\sin\left(-18^\circ\right)$

$\Rightarrow -\sqrt{3}\sin 18^\circ = -\sqrt{3}\left(\dfrac{\sqrt{5}-1}{4}\right) = \sqrt{3}\left(\dfrac{1-\sqrt{5}}{4}\right)$

5. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to:

(A) $\frac{21}{16}$

(B) $\frac{63}{52}$

(C) $\frac{33}{52}$

(D) $\frac{63}{16}$

Ans. (D)

Sol. $\cos(\alpha+\beta)=\frac{3}{5} \Rightarrow \tan(\alpha+\beta)=\frac{4}{3}$

$\sin(\alpha-\beta)=\frac{5}{13} \Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$

Now, $\tan 2\alpha=\tan((\alpha+\beta)+(\alpha-\beta))$

$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}=\frac{63}{16}$

6. If $\sin\theta+\operatorname{cosec}\theta=2$, then $\sin^2\theta+\operatorname{cosec}^2\theta$ is equal to

(A) 1

(B) 4

(C) 2

(D) 0

Ans. (C)

Sol. Conceptual

7. If $\tan\theta=3$ and θ lies in the III quadrant, then the value of $\sin\theta$ is

(A) $\frac{1}{\sqrt{10}}$

(B) $-\frac{1}{\sqrt{10}}$

(C) $\frac{-3}{\sqrt{10}}$

(D) $\frac{3}{\sqrt{10}}$

Ans. (C)

Sol. Conceptual.

8. The sum of the solutions $x \in \mathbb{R}$ of the equation $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

(A) 0

(B) 1

(C) -1

(D) 3

Ans. (C)

Sol. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

$$\Rightarrow \frac{\cos 2x\left(3 + \cos^2 2x\right)}{\cos 2x\left(1 - \sin^2 x \cos^2 x\right)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4\left(3 + \cos^2 2x\right)}{\left(4 - \sin^2 2x\right)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4\left(3 + \cos^2 2x\right)}{\left(3 + \cos^2 2x\right)} = x^3 - x^2 + 6$$

$$x^3 - x^2 + 2 = 0 \Rightarrow (x+1)\left(x^2 - 2x + 2\right) = 0$$

so, sum of real solutions = -1

9. The value of $\sin 15^\circ + \cos 105^\circ$ is

(A) 0

(B) $2\sin 15^\circ$

(C) $\cos 15^\circ + \sin 15^\circ$

(D) $\sin 15^\circ - \cos 15^\circ$

Ans. (A)

Sol. $\sin 15^\circ + \cos 105^\circ = \sin 15^\circ + \cos\left(90^\circ + 15^\circ\right)$

$= \sin 15^\circ - \sin 15^\circ$

$= 0$

10. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \dfrac{n\pi}{2}\right]$, for the least value of $n \in N$ then $\displaystyle\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to:

(A) $\dfrac{1}{2^{15}}\left(2^{14} - 14\right)$

(B) $\dfrac{1}{2^{14}}\left(2^{15} - 15\right)$

(C) $1 - \dfrac{15}{2^{13}}$

(D) $\dfrac{1}{2^{13}}\left(2^{14} - 15\right)$

Ans. (D)

Sol. $2\tan^2\theta - 5\sec\theta - 1 = 0$

$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$

$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = -\frac{1}{2}, 3$

$\Rightarrow \cos\theta = -2, \frac{1}{3}$

$\Rightarrow \cos\theta = \frac{1}{3}$

For 7 solutions n = 13

So, $\sum_{k=1}^{13}\frac{k}{2^k} = S$ (say)

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{13}{2^{13}}$

$\frac{1}{2}S = \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$

$\Rightarrow \frac{S}{2} = \frac{1}{2}\cdot\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}} - \frac{13}{2^{14}} \Rightarrow S = 2\cdot\left(\frac{2^{13}-1}{2^{13}}\right) - \frac{13}{2^{13}}$

11. The number of elements in the set $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right\}$ is

(A) 1
(B) 3
(C) 0
(D) infinite

Ans. (A)

Sol. Given,

$2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}$

Now we know that maximum value of cos function is 1, so $2\cos\left(\frac{x^2+x}{6}\right) \le 2$ and using $A.M \ge G.M$

we get $4^x + 4^{-x} \ge 2$

So, $L.H.S. \le 2.\ \&\ R.H.S \ge 2$

Hence $L.H.S. = 2\ \&\ R.H.S. = 2$

Now equating both to 2 we get,

$2\cos\left(\frac{x^2+x}{6}\right) = 2$ and $4^x + 4^{-x} = 2$

Now solving $2\cos\left(\frac{x^2+x}{6}\right) = 2$

$\Rightarrow \cos\left(\frac{x^2+x}{6}\right) = 1$

$\Rightarrow \frac{x^2+x}{6} = 0$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x = 0 \text{ or } -1$ .... (1)

Now solving $4^x + 4^{-x} = 2$ we get x = 0 ......(2)

Now from equation (1) & (2) we can see common solution is x = 0

Hence, possible solution is only one.

12. If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ then θ is equal to -

(A) 30°

(B) 150°

(C) 210°

(D) none of these

Ans. (C)

Sol. Let us first find out θ lying between 0 and 360°.

Since $\sin\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = 210^\circ \text{ or } 330^\circ$ and $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$ or 210°

Hence, $\theta = 210^\circ$ or $\frac{7\pi}{6}$ is the value satisfying both.

13. The value of $\cot\frac{\pi}{24}$ is:

(A) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

(B) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(C) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$

(D) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

Ans. (B)

Sol. We know that $\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta}$

Since, $\theta = \frac{\pi}{24}$

Therefore, $\cot\theta = \dfrac{1+\left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)}{\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \dfrac{1+\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2.$$

14. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of cos 4x is

(A) $-\frac{3}{5}$

(B) $\frac{1}{3}$

(C) $\frac{2}{9}$

(D) $-\frac{7}{9}$

Ans. (D)

Sol. $5\left(\frac{1-\cos 2x}{1+\cos 2x} - \frac{1+\cos 2x}{2}\right) = 2\cos 2x + 9$

Let, cos 2x = y

$\therefore 5\left(\frac{1-y}{1+y} - \frac{1+y}{2}\right) = 2y + 9$

$\Rightarrow 5\left[-y^2 - 4y + 1\right] = 4y^2 + 22y + 18$

$\Rightarrow 9y^2 + 42y + 13 = 0$

$\Rightarrow y = -\frac{1}{3}$ or $y = -\frac{13}{3}$ (Not possible)

Now, $\cos 4x = 2\cos^2 2x - 1$

$\Rightarrow \cos 4x = 2\left(-\frac{1}{3}\right)^2 - 1$

$\Rightarrow \cos 4x = -\frac{7}{9}$

15. Let S be the set od all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to:

(A) [3, 7]

(B) [2, 6]

(C) [1, 4]

(D) R

Ans. (B)

Sol. The given equation can be written as:

$1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\Rightarrow \sin x = \frac{\alpha - 4}{2}, 2(\sin x \neq 2)$

For at least one solution

$$-1 \le \frac{\alpha - 4}{2} \le 1$$

$\Rightarrow \alpha \in [2, 6]$

16. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(A) $\frac{10-\sqrt{10}}{6}$

(B) $\frac{10-\sqrt{10}}{12}$

(C) $\frac{\sqrt{10}-10}{12}$

(D) $\frac{\sqrt{10}-10}{6}$

Ans. (C)

Sol. $4 + 5\tan\theta = \sec\theta$

Squaring : $24\tan^2\theta + 40\tan\theta + 15 = 0$

$$\tan\theta = \frac{-10 \pm \sqrt{10}}{12}$$

17. $16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) 3

(D) $4\sqrt{3}$

Ans. (B)

Sol. We know that $\sin\theta\sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

Now given $16\sin 20^\circ \sin 40^\circ \sin 80^\circ$

Comparing with above formula $\theta = 20^\circ$

we get $16\sin 20^\circ \sin 40^\circ \sin 80^\circ = 16 \times \frac{1}{4} \times \sin(3 \times 20^\circ)$

$$= 16 \times \frac{1}{4} \times \sin 60^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

18. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is:

(A) $\sqrt{19}$

(B) $\sqrt{31}$

(C) $\frac{\sqrt{79}}{2}$

(D) $\sqrt{34}$

Ans. (A)

Sol. Given $f(\theta)=3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$

$$\Rightarrow f(\theta)=3\cos\theta+5\left(\sin\theta\cdot\frac{\sqrt{3}}{2}-\cos\theta\cdot\frac{1}{2}\right)$$

$$\Rightarrow f(\theta)=\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$$

Now using the concept $a\cos x+b\sin x+c\in\left[c-\sqrt{a^2+b^2},c+\sqrt{a^2+b^2}\right]$, we can write

Maximum value of $f(\theta)$ is $\sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

19. For $x\in(0,\pi)$, then equation $\sin x+2\sin 2x-\sin 3x=3$ has

(A) Infinitely solution

(B) Three solutions

(C) One solution

(D) No solution

Ans. (D)

Sol. $\sin x+2\sin 2x-\sin 3x=3$

$\sin x+4\sin x\cos x-3\sin x+4\sin^3 x=3$

$\sin x\left[-2+4\cos x+4\left(1-\cos^2 x\right)\right]=3$

$\sin x\left[2-\left(4\cos^2 x-4\cos x+1\right)+1\right]=3$

$\sin x\left[3-(2\cos x-1)^2\right]=3$

$\Rightarrow \sin x=1$ and $2\cos x-1=0$

$\Rightarrow x=\frac{\pi}{2}$ and $x=\frac{\pi}{3}$

Which is not possible at same time.

Hence, no solution.

20. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}=$

(A) 1

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) 2

Ans. (C)

Sol. Conceptual.

SECTION-II

21. The value of $\tan 9^o - \tan 27^o - \tan 63^o + \tan 81^o$ is ________.

Ans. (4)

Sol. Given

$$\tan 9^o - \tan 27^o - \tan 63^o + \tan 81^o$$

$$= \left(\cot 81^o + \tan 81^o\right) - \left(\tan 27^o + \cot 27^o\right)$$

$$= \left(\tan 9^o + \cot 9^o\right) - \left(\tan 27^o + \cot 27^o\right)$$

$$= \left(\frac{\sin 9^o}{\cos 9^o} + \frac{\cos 9^o}{\sin 9^o}\right) - \left(\frac{\sin 27^o}{\cos 27^o} + \frac{\cos 27^o}{\sin 27^o}\right)$$

$$= \frac{2}{\sin 18^o} - \frac{2}{\sin 54^o}$$

$$= \left(\frac{2 \times 4}{\sqrt{5} - 1} - \frac{2 \times 4}{\sqrt{5} + 1}\right)$$

$$= 8\left(\frac{1}{\sqrt{5} - 1} - \frac{1}{\sqrt{5} + 1}\right)$$

$$= 8\left(\frac{\sqrt{5} + 1 - (\sqrt{5} - 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)}\right)$$

$$= 8\left(\frac{2}{5 - 1}\right) = 4$$

22. If $5\cos\theta = 3,$, then, $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta}$ is equal to

Ans. (4)

Sol. Conceptual.

23. The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is ________.

Ans. (11)

Sol. $3\sin x + 4\cos x = k + 1$

$$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$$

$$\Rightarrow k + 1 \in [-5, 5]$$

$$\Rightarrow k \in [-6, 4]$$

No. of integral values of k = 11

24. The number of intersecting points on the graph for $\sin x = \dfrac{x}{10}$ for $x \in R$ is

Ans. (7)

Sol. Conceptual.

25. If m and n respectively are the numbers of positive and negative value of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to ______.

Ans. (25)

Sol. We have

$$\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$$

$$\Rightarrow 2\cos 2\theta \cos\left(\frac{\theta}{2}\right) = 2\cos\left(\frac{9\theta}{2}\right)\cos 3\theta$$

$$\Rightarrow \cos\left(\frac{5\theta}{2}\right) + \cos\left(\frac{3\theta}{2}\right) = \cos\left(\frac{15\theta}{2}\right) + \cos\left(\frac{3\theta}{2}\right)$$

$$\Rightarrow \cos\left(\frac{15\theta}{2}\right) = \cos\left(\frac{5\theta}{2}\right)$$

$$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}; k \in Z$$

$$\Rightarrow 5\theta = 2k\pi \text{ or } 10\theta = 2k\pi$$

$$\Rightarrow \theta = \frac{2k\pi}{5} \text{ or } \theta = \frac{k\pi}{5}$$

$$\therefore \theta = \left\{-\pi, -\frac{4\pi}{5}, -\frac{3\pi}{5}, -\frac{2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$$

So, $m = 5, n = 5$

$\therefore mn = 25$

26. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to

Ans. (3)

Sol. Given,

Expression $96 \cdot \cos\frac{\pi}{33} \cdot \cos\frac{2\pi}{33} \cdot \cos\frac{4\pi}{33} \ldots\ldots\ldots \cos\frac{16\pi}{33}$

Now we know that,

$$\cos A \cdot \cos 2A \cdot \cos 2^2 A \cdot \cos 2^3 A \ldots. \cdot \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

Now using the above formula in given expression we get,

$$96 \cdot \cos\frac{\pi}{33} \cdot \cos\frac{2\pi}{33} \cdot \cos\frac{4\pi}{33} \ldots\ldots\ldots \cos\frac{16\pi}{33}$$

$$= 96 \times \frac{\sin\frac{32\pi}{33}}{2^5 \sin\frac{\pi}{33}}$$

$$= 96 \times \frac{\sin\left(\pi - \frac{\pi}{33}\right)}{2^5 \sin\frac{\pi}{33}}$$

$$= 96 \times \frac{\sin\left(\frac{\pi}{(3)}\right)}{2^5 \sin\frac{\pi}{33}} \{\text{as } \sin(\pi - \alpha) = \sin\alpha\}$$

$$= 96 \times \frac{1}{32} = 3$$

27. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$, then $\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

Ans. (2)

Sol. Given:

$S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$

So,

$\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\Rightarrow \tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\Rightarrow \pi\cos\theta = n\pi - \pi\sin\theta; n \in Z$

$\Rightarrow \sin\theta + \cos\theta = n$

Now,

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$

$\Rightarrow -\sqrt{2} \le n \le \sqrt{2}$

But $n \in Z$, so $n = -1, 0, 1$

So,

$$\theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$$

So,

$$\sum_{\theta\in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2} + \frac{1}{2} + 0 + 0 + \frac{1}{2} + \frac{1}{2} = 2$$

28. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

Ans. (1)

Sol. If $\cot x > 0 \Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ (rejected as $\cot x < 0$)

So, number of solutions is one.

29. The sum of maximum and minimum values of the expression $5\cos x + 3\sin\left(\frac{\pi}{6} - x\right) + 4$ is

Ans. (8)

Sol. Conceptual.

30. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $2\left(\frac{b}{a}\right)$ is

Ans. (1)

Sol. $\sqrt{3}\cos x + \frac{2b}{a}\sin x = \frac{c}{a}$

Now, $\sqrt{3}\cos\alpha + \frac{2b}{a}\sin\alpha = \frac{c}{a}$ ...(i)

$\sqrt{3}\cos\beta + \frac{2b}{a}\sin\beta = \frac{c}{a}$ ...(ii)

From equations (i) and (ii)

$$\Rightarrow \sqrt{3}[\cos\alpha - \cos\beta] + \frac{2b}{a}(\sin\alpha - \sin\beta) = 0$$

$$\Rightarrow \sqrt{3}\left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right] + \frac{2b}{a}\left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right] = 0$$

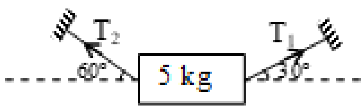
$$\Rightarrow -\sqrt{3} + 2\sqrt{3}\cdot\frac{b}{a} = 0$$

$$\Rightarrow \frac{b}{a} = \frac{1}{2} = 0.5$$

PART-B: PHYSICS

SECTION-I

31. A body of mass 5 kg is suspended by the strings making angles 60º and 30º with the horizontal



(a) T_1 = 25 N (b) T_2 = 25 N (c) $T_1 = 25\sqrt{3}$ N (d) $T_2 = 25\sqrt{3}$ N

(A) a, b

(B) a, d

(C) c, d

(D) b, c

Ans. (B)

Sol. As the mass is at rest the resultant of forces acting on it are equal to zero

so forces in vertical direction are

$T_1\sin30° + T_2\sin60° - mg = 0$

$\frac{T_1}{2} + \frac{\sqrt{3}T_2}{2} = 5g$

$T_1 + \sqrt{3}T_2 = 100$

similarly in horizontal direction

$T_1 \cos 30° - T_2 \cos 60° = 0$

$\frac{T_1}{T_2} + \frac{1}{\sqrt{3}}$

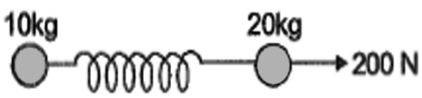
solving above equations will give us

$T_1 + \sqrt{3} \times \sqrt{3}T_1 = 100$

$T_1 = 25N$

$T_2 = 25\sqrt{3}N$

32. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of 12 ms^{-2} towards right, the acceleration of the 20 kg mass is



(A) 2 ms^{-2}
(B) 4 ms^{-2}
(C) 10 ms^{-2}
(D) 20 ms^{-2}

Ans. (B)

Sol. $m_1 = 10kg$; $m_2 = 20kg$; $F_2 = 200N$;

For mass m_1; $F = m_1a_1 = 10 \times 12 = 120N$

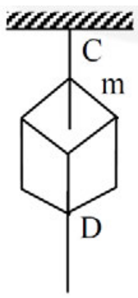
For mass m_2;

$200 - F = 20a_2$

$200-120 = 20a_2$

$a_2 = \frac{80}{20} = 4ms^{-2}$

33. A heavy block of mass m is supported by a cord C from the ceiling, and another cord D is attached to the bottom of the block. If a sudden jerk is given to D, then

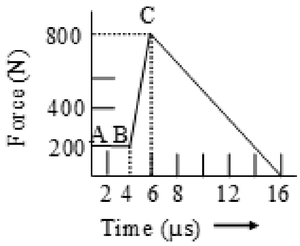


(A) cord C breaks
(B) cord D breaks
(C) cord C and D both break
(D) none of the cords breaks

Ans. (B)

Sol. Conceptual

34. The magnitude of the force(in newtons) acting on a body varies with time t(in microseconds) as shown in the figure. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from t = 4 μs to t = 16 μs is



(A) 0.005 Ns

(B) 0.004 Ns

(C) 0.003 Ns

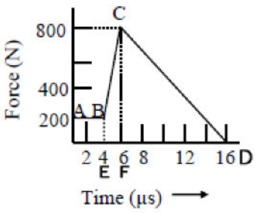
(D) None of these

Ans. (A)

Sol. As given in the graph,

Impulse is defined as the area under F·t graph

Impulse=area of EBCD



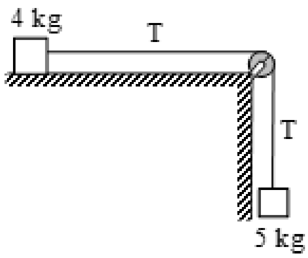
⇒Area EBCD = area EBCF + area of ΔFCD

Impulse I = $\left(\frac{BE+FC}{2}\times EF\right)+\frac{1}{2}FC\times FD$

$\left(\frac{200+1800}{2}\right)\times 2\times 10^{-6}+\left[\frac{1}{2}\times 800\times 10\times 10^{-6}\right]$

Impulse = 5 × 10^{-3}N

35. Two bodies of 5 kg and 4 kg are tied to a string as shown in the figure. If the table and pulley both are smooth, acceleration of 5 kg body will be equal to-



(A) g

(B) $\frac{g}{9}$

(C) $\frac{4g}{9}$

(D) $\frac{5g}{9}$

Ans. (D)

Sol. For 5kg block the equation is

$mg - T = ma \Rightarrow 5g - T = 5a$ (i)

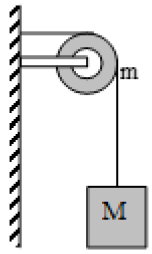
For 4kg block the equation is

$T = ma \Rightarrow T = 4a$ (ii)

Solving (i) and (ii)

$$a = \frac{5g}{9}$$

36. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in figure. The force on the pulley by the clamp is given by



(A) $\sqrt{(M+m)^2 + m^2}$

(B) $\sqrt{2}\,Mg$

(C) $\sqrt{(M+m)^2 + M^2}\;g$

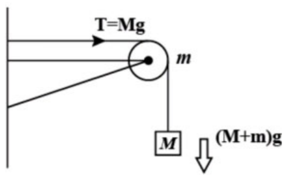
(D) $\sqrt{2}\,mg$

Ans. (C)

Sol. Substituting the values:

$$F_{net} = \sqrt{(Mg)^2 + [(M+m)g]^2}$$

$$g\sqrt{M^2 + (M+m)^2}$$



The horizontal force acting on the clamp is the tension developed in the wire due to the weight of mass M.

Fn = Mg

The Vertical force on the clamp is the mass of the weight acting downward and the weight of the pulley itself.

$F_v = (M + m)g$

Therefore, the resultant force on the clamp is

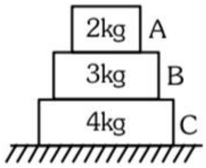
$$F_{net} = \sqrt{F_h^2 + F_v^2}$$

Substituting the values:

$$F_{net} = \sqrt{(Mg)^2 + [(M+m)g]^2}$$

$$= g\sqrt{M + (M+m)^2}$$

37. Three blocks A, B and C are vertically stagged at rest as shown in the figure. Magnitude of contact force between blocks B and C will be equal to



(A) 20 N

(B) 70 N

(C) 30 N

(D) 50 N

Ans. (D)

Sol. Conceptual

38. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. The a is equal to

(A) g

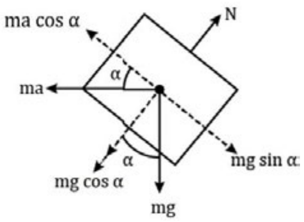
(B) g tan α

(C) g/tan

(D) g cosec α

Ans. (B)

Sol. Analysing from inclined surface frame, FBD of block is

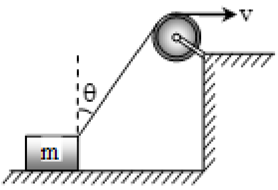


For block to remain stationary,

mg sinα = ma cosα

⇒ a = g tanα

39. A block is dragged on a smooth plane with the help of a rope which moves with a velocity v as shown in figure. The horizontal velocity of the block is



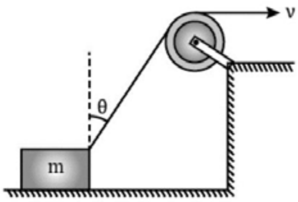
(A) v

(B) $\frac{v}{\sin\theta}$

(C) v sin θ

(D) $\frac{v}{\cos\theta}$

Ans. (B)

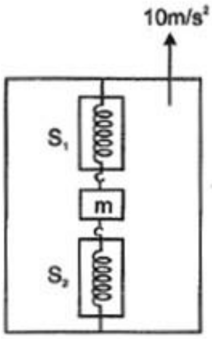
Sol. 

Component of velocity along string must be same,

So, Velocity of m along string = v

So, $V_m \sin\theta = v \Rightarrow v_m = \frac{v}{\sin\theta}$

40. Reading shown in two spring balances S_1 and S_2 is 90 kg and 30 kg respectively when lift is accelerating upwards with acceleration 10 m/s^2. The mass is stationary with respect to lift. Then the mass of the block will be



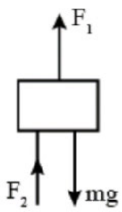
(A) 60 kg

(B) 30 kg

(C) 120 kg

(D) None of these

Ans. (B)

Sol. 

As lift is accelerating upwards from FBD of mass

$F_1 + F_2 - mg = ma$

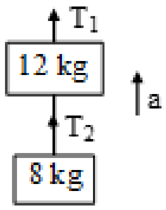
$\therefore\ m = \frac{F_1 + F_2}{(g + a)}$

Here F_1 is force acting on spring balance S_1. So, $F_1 = 90g$ and F_2 is force acting on spring balance S_2. So, $F_2 = 30g$.

Given that a = $10\frac{m}{s^2}$

$\Rightarrow m = \frac{(90 + 30)g}{(g + a)} = \frac{120 \times 10}{10 + 10}$ = 60 kg

41. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled up by a string with an acceleration of $2 m/sec^2$. The tension T_1 will be

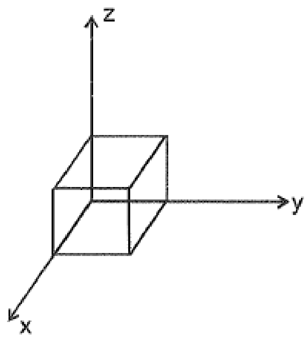


(A) 260 N
(B) 240 N
(C) 220 N
(D) 200 N

Ans. (B)

Sol. $T_1 - (m_1 + m_2)(g) = (m_1 + m_2)a$
$T_1 - (8 + 12)(10 + 2) = 240N$

42. A solid cube of mass 5 kg is placed on a rough horizontal surface, in xy-plane as shown. The friction coefficient between the surface and the cube is 0.4. An external force $\vec{F} = 6\hat{i} + 8\hat{j} + 20\hat{k}$ N is applied on the cube. (Use g = 10 m/s^2)

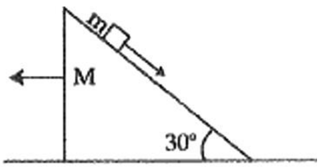


(A) The block starts slipping over the surface.
(B) The friction force on the cube by the surface is 10 N.
(C) The friction force acts in xy-plane at angle 127° with the positive x-axis in clockwise direction.
(D) The contact force exerted by the surface on the cube is $10\sqrt{10}\,N$.

Ans. (B)

Sol. Normal reaction is (50 – 20)N so max friction available is 0.4*30 = 12N
But net for acting along x-y plane is
$\sqrt{6^2 + 8^2} = 10N$
=10N, so only 10N friction will act and body will be on equilibrium.

43. A block of mass m slides on the wooden wedge, which in turn slides backwards on the horizontal surface. The acceleration of the block with respect to the wedge is:
Given m = 8 kg, M = 16 kg Assume all the surfaces shown in the figure to be frictionless.



(A) $\frac{4}{3}g$

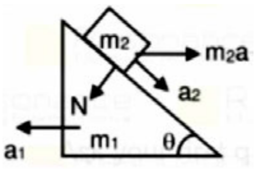
(B) $\frac{6}{5}g$

(C) $\frac{3}{5}g$

(D) $\frac{2}{3}g$

Ans. (D)

Sol.



N cos 60° = $Ma_1 = 16a_1$

$\Rightarrow N = 32\ a_1$

$\perp$ to incline

N = 8g cos 30° – $8a_1$ sin 30° $\Rightarrow 32a_1 = 44$

3g – 4a

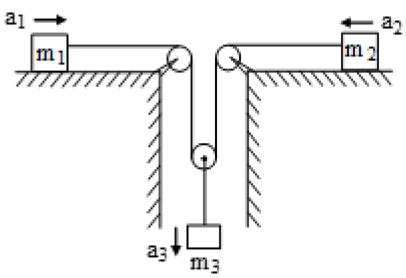
$\Rightarrow a_2 = \frac{3}{9}g$

Along incline

8 g sin 30° + $8a_1$ cos 30° = $ma_2 = 8a_2$

$a_2 = g\times\frac{1}{2}+\frac{3}{9}g\cdot\frac{3}{2}=\frac{2g}{3}$

44. In the figure shown the relation between acceleration is



(A) $a_1 + a_2 + 2a_3 = 0$

(B) $a_1 + a_2 = 2a_3$

(C) $a_1 + a_2 = a_3$

(D) $a_1 + a_3 + a_3 = 0$

Ans. (B)

Sol. Let we consider displacement of m_1 is x_1 and displacement of m_2 is x_2 and vertical displacement of m_3 is y

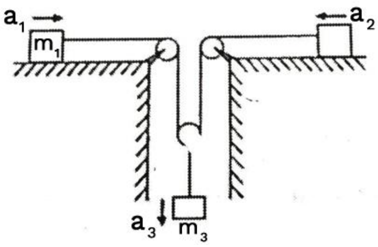
$$y=\frac{x_1+x_2}{2}$$

$$2y = x_1 + x_2$$

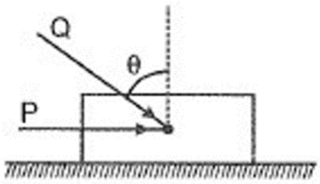
differentiate this $2\frac{dy}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt}$

differentiate again $2\frac{d^2y}{dt^2} = \frac{dx_1^2}{dt} + \frac{dx_2^2}{dt}$

$\Rightarrow 2a_3 = a_1 + a_2$



45. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined at an angle to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is



(A) $\frac{P + Q\sin\theta}{mg + Q\cos\theta}$

(B) $\frac{P\cos\theta + Q}{mg - Q\sin\theta}$

(C) $\frac{P + Q\cos\theta}{mg + Q\sin\theta}$

(C) $\frac{P\sin\theta - Q}{mg - Q\cos\theta}$

Ans. (A)

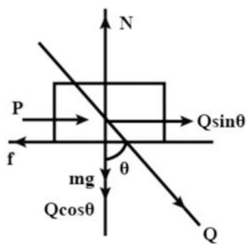
Sol. Correct option is A. $\frac{(P + Q\sin\theta)}{(mg + Q\cos\theta)}$

f = P + Q sin 0, where f is the frictional force

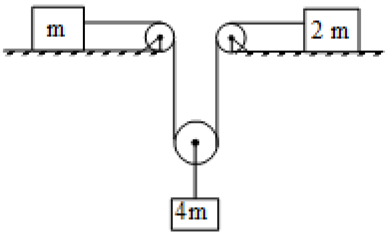
$\mu N = P + Q \sin\theta$

$\mu(mg + Q\cos\theta) = P + Q\sin\theta$

$\therefore \mu = \frac{P + Q\sin\theta}{(mg + Q\cos\theta)}$



46. All surface are frictionless and pulley and strings are light. Acceleration of block of mass 'm' is



(A) $\frac{2}{5}g$

(B) $\frac{4g}{5}$

(C) $\frac{4g}{7}$

(D) $\frac{2g}{7}$

Ans. (B)

Sol. Let acceleration of m, 2 m and 4 m are a_1, a_2 and a_3 respectively.

By constraints;

$a_1 + a_2 = 2a_3$ ...(1)

on m $\Rightarrow T = ma_1$ ...(2)

on 2m $\Rightarrow T = 2ma_2$ ...(3)

on 4 m $\Rightarrow 4mg - 2T = 4ma_3$ ...4)

dividing (2) and (3);

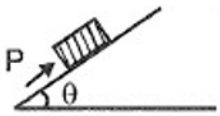
$a_2 = 1/2a_1$

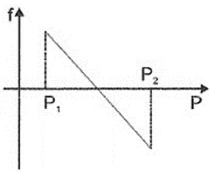
putting values:

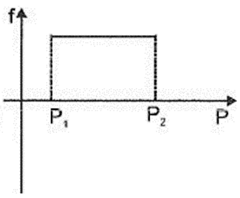
$$4mg - 2(ma_1) = 4m\left(\frac{3}{4}\right)a_1$$

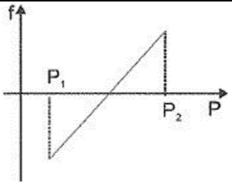
$$\frac{4g}{5} = a_1$$

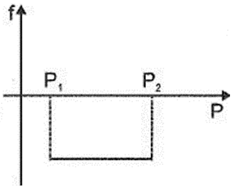
47. A block of mass m is on inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tan θ > μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - u\cos\theta)$ to $P_2\ mg(\sin\theta + u\cos\theta)$, the frictional force f versus P graph will look like



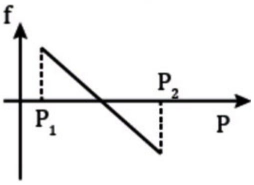
(A)


(B)


(C) 

(D) 

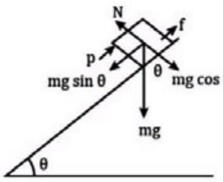
Ans. (A)

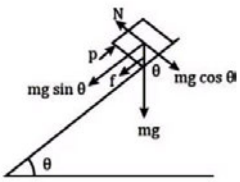
Sol. 

As $\tan\theta > \mu$, the block has a tendency to move down the incline. Therefore a force P is applied upwards along the incline.

Here, at equilibrium

$P + f = mg \sin\theta \Rightarrow f = mg \sin\theta - P$





Now as P increases the value of f decreases linearly with respect to P.

When $P = mg \sin\theta$, $f = 0$

When P is increased further, the block will have a tendency to move upwards along the inclined plane.

Therefore the frictional force acts downwards along the incline in this case.

Here, at equilibrium $P = f + mg$ sine

$\therefore f = P - mg \sin\theta$

Now as P increases, f increases linearly

w.r.t P.

If you consider upward friction as positive and downward negative, this is represented by graph (A).

48. A force of 6.00 N acts on a 3.00-kg object for 10.0 s. What is the object's change in momentum?

(A) 60 Ns

(B) 30 Ns

(C) 15 Ns

(D) 120 Ns

Ans. (A)

Sol. To calculate the change in momentum, we use the formula:

Change in momentum = Force × Time

Given:

- Force (F) = 6.00 N

- Time (t) = 10 s

- Mass (m) = 3.00 kg

Plugging these values into the formula:

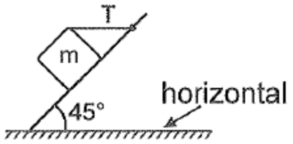
Change in momentum = F × t

= 6.00 N × 10 s

= 60 N · s

Therefore, the object's change in momentum is 60 N · s.

49. A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact may be ($g = 10\ m/s^2$)



(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{1}{4}$

Ans. (A)

Sol. In horizontal direction,

$\Sigma F_x = 0$ given

$mg \sin\theta = f + T\cos\theta$ (f= frictional force)

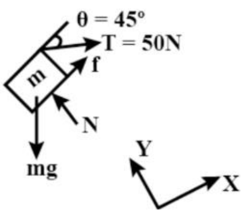
$\Rightarrow f = \mu N = mg \sin\theta - T \cos\theta$ ...(1)

and Fy = 0 given: -

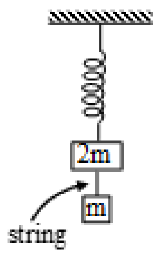
$mg \cos\theta + T \sin\theta = N$ ...(2)

Divide (1) by (2) to get, Put values :-

$$\mu = \frac{mg\sin\theta - T\cos\theta}{mg\cos\theta + T\sin\theta} = \frac{150\sin(45°) - 50\cos(45°)}{150\cos(45°) + 50\sin(45°)}$$



50. In the system shown if the inextensible string connecting 2 m and m is cut, the accelerations of mass m and 2 m are



(A) $\frac{g}{2}, \frac{g}{2}$

(B) g, $\frac{g}{2}$

(C) $\frac{g}{2}$, g

(D) g, g

Ans. (B)

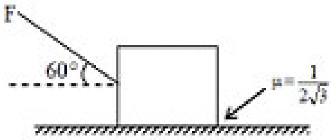
Sol. Immediately after cutting, the below mass m will be free falling. so the acceleration of mass m is g. But mass 2m goes to upward direction.

for mass 2m, 2ma = 2mg - mg = mg

$$a = \frac{g}{2}$$

SECTION-II

51. A force F is applied to a block of mass $2\sqrt{3}$ kg as shown in the diagram. What should be the maximum value of force (in newton) so that the block does not move?



Ans. (40)

Sol. F force will be having 2 components: Horizontal component will be F cos 60° and the veritical component will be F sin 60°

Balancing vertical forces, we have

N = F sin 60° + mg (1)

Balancing Horizontal forces

For the block not to move, we must have

F cos 60° = μN

taking value of N from equation 1

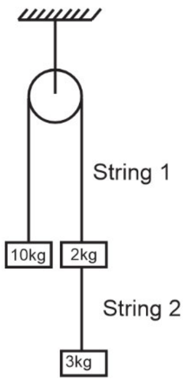
i.e., F cos 60° = μ(F sin 60° + mg)

$$F\left(\frac{1}{2}\right) = \frac{1}{2\sqrt{3}}\left[F\left(\frac{\sqrt{3}}{2}\right) + (2\sqrt{3})(10)\right]N$$

$$\text{or } F\left(\frac{1}{2} - \frac{1}{4}\right) = 10N$$

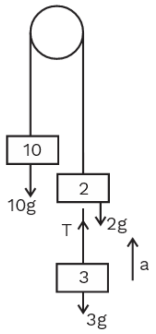
or F = 4 × 10 = 40 N

52. Find tension in string 2 (in newton), when system is released. (g = 10 m/s^2)



Ans. (40)

Sol.

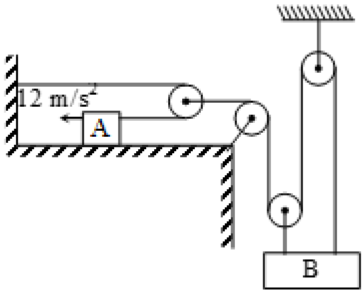


$$\frac{10g-5g}{15}=a\,,\ a=\frac{g}{3}$$

$$T-3g=3\times\frac{g}{3}$$

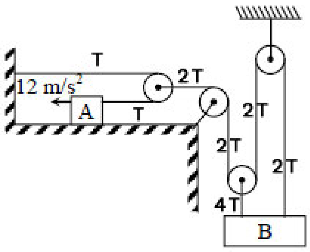
$T = 4g = 40N$

53. Assuming that the block is always remains horizontal, hence the acceleration (in m/s^2) of B is



Ans. (2)

Sol.



By Constraint

$6Ta_B - 12T = 0$

$a_B = 2m/s^2$

54. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is $5m/s^2$, then what is the frictional force (in newton) acting on the block? (Take $g = 10\ m/s^2$)

Ans. (5)

Sol. As the truck is accelerating, let it be in the forward direction the block will experience a pseudo force in the backward direction. In order to oppose the motion of acceleration of the block, the frictional force acts on the block.

Let us take $g = 10m/s^2$.

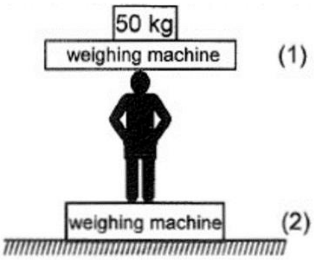
$F_s = \mu N$ where μ is the coefficient of friction and N is the normal force = mg.

Therefore, $F_s = \mu mg = 0.6 \times 1 \times 10 = 6N$.

Also, F_s = ma = 1 × 5 = 5N.

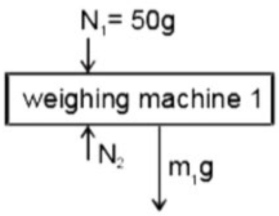
As 5 N is required to oppose the motion of the block, so the frictional force acting on the block is 5 N.

55. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another similar weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1). Calculate the readings of weighing machines (2) (in newton) (g = 10 m/s^2)



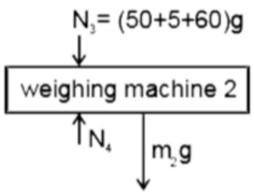
Ans. (1150 N)

Sol.



$R_1 = N_1 = 50 \times g = 500N$

where R1 = reading in weighing machine1

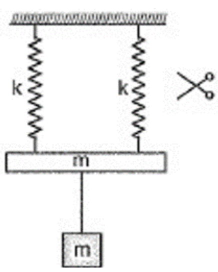


$R_1 = N_1 = 50 \times g = 500N$

$= 115 \times 10 = 1150N$

where R_2 = reading in weighing machine 2

56. System shown in figure is in equilibrium. What is the magnitude of change in tension (in newton) in the string just before and just after, when one of the spring is cut? Mass of both the blocks is same and equal to m = 3 kg and spring constant of both springs is k. (Neglect any effect of rotation) (g = 10 m/s^2)



Ans. (15)

Sol. $T_1 = mg$

$2kx = 2mg$

$\therefore \quad kx = mg$

One kx force (acting in upward direction) is suddenly removed. So net downward force on system will be kx or mg. Therefore net downward acceleration of system,

$$a = \frac{mg}{2m} = \frac{g}{2}$$

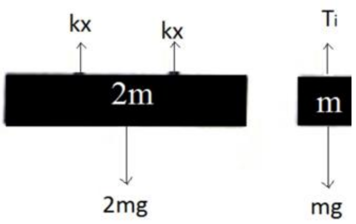
Free body diagram of lower block gives the equation,

$$mg - T_f = ma = \frac{mg}{2}$$

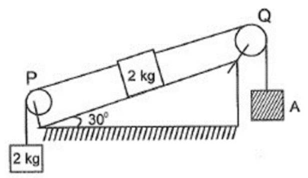
$$\therefore \quad T_f = \frac{mg}{2} \quad ...(ii)$$

From these two equations we get,

$$\Delta T = \frac{mg}{2}$$



57. In the arrangement shown in figure, what should be the mass of block A in kg, so that the system remains at rest ($g = 10\ m/s^2$)



Ans. (3)

Sol. Given,

2Blocks having mass = 2kg, Angle = 30°

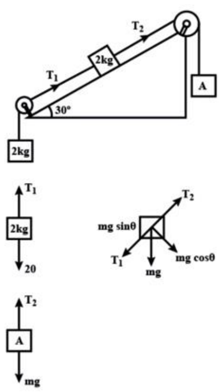
So,

$T_2 = T_1 + mg\sin 30°$

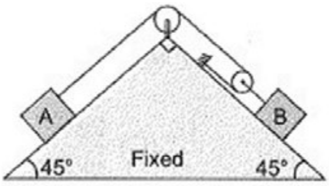
$$= 20 \times \frac{1}{2} + 20$$

$T_2 = 30$

So the mass of the block is $M = \dfrac{30}{10} = 3$ kg



58. Two blocks A and B of mass 10 kg and 40 kg are connected by an ideal string as shown in the figure. Neglect the masses of the pulleys and effect of friction. If the value of tension in larger string is T newton then what is the value of $\sqrt{2}\,T$ (g = 10 m/s^2)



Ans. (150)

Sol. As per constant relation if acceleration of block B is 'a' then acceleration of block A is '2a'

Now as per force equation of B

$m_b \sin 45 - 2T = m_b a$ ...(i)

$T - m_a g \sin 45 = m_a \times 2a$ ...(ii)

now solving above two equations for acceleration of the blocks

$m_b \sin 45 - 2m_a \sin 45 = (m_b + 4m_a) \times a$

$$a = \frac{(m_b \sin 45 - 2m_a \sin 45)g}{m_b + 4m_a}$$

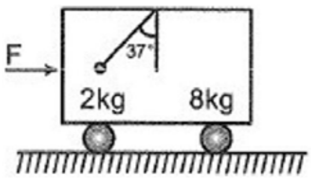
now plug in all values

$$a = \frac{(20\sqrt{2} - 10\sqrt{2})}{40 + 40}$$

Put a in equation (ii)

$T = 75\sqrt{2}$

59. A trolley of mass 8 kg is standing on a frictionless surface inside which an object of mass 2 kg is suspended. A constant force F starts acting on the trolley as a result of which the string stood at an angle of 37° from the vertical (bob at rest relative to trolley) Then what is the tension in string in Newton.



Ans. (25)

Sol. We know that the angle made by string is $\theta = \tan^{-1}\left(\frac{a}{g}\right)$

where a is the acceleration of the frame in which the string is hanging in our case it is a trolley.

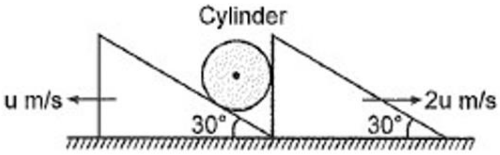
putting θ = 37° or tan37° = $\frac{3}{4}$

we get a = $\frac{3g}{4}$ = 30/4 = 7.5m/s^2

$T = m\sqrt{g^2 + a^2}$

T = 25N

60. System is shown in the figure. Assume that cylinder remains in contact with the two wedges. Find the velocity of cylinder in m/s if $u = 2\sqrt{7}\,m/s$.

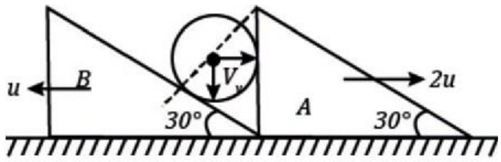


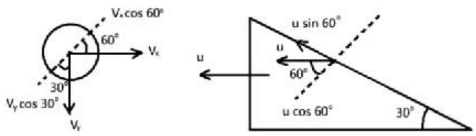
Ans. (14)

Sol. Whose x and y components are Vx and V_y respectively.

As cylinder will remain in contact with wedge A and B

$\Rightarrow V_x = 24$





And, from the FBDs

$u\cos60° = V_y\cos30° - V_x\cos30°$

$\Rightarrow u\cos60° + V_x\cos60° = V_y\cos30°$

$\Rightarrow u\cos60° + 2u\cos60° = V_y\cos30°$

$\Rightarrow 3u\left(\frac{1}{2}\right) = V_y\left(\frac{\sqrt{3}}{2}\right)$

$V_y = \sqrt{3}u$

$\therefore \quad V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u = 14$

PART-C: CHEMISTRY

SECTION-I

61. The molecule which contain both polar and non-polar covalent bond present in its structure?

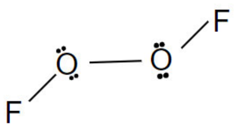
(A) H_2F_2

(B) O_2F_2

(C) O_3

(D) All of these

Ans. (B)

Sol. O_2F_2:  has both polar and non polar bond.

62. Which of the following ion do not have bond order of 2.5?

(A) O_2^-

(B) O_2^+

(C) N_2^+

(D) N_2^-

Ans. (A)

Sol. $O_2^- \Rightarrow$ No. of e^- = 17

B. O. of O_2^- = 1.5

63. Which of the following molecules/species has the minimum number of lone pairs on central atom?

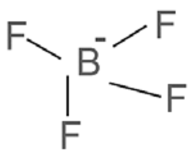
(A) ICl_3

(B) BF_4^-

(C) $SnCl_2$

(D) XeF_4

Ans. (B)

Sol. $BF_4^- \Rightarrow$  (No. of lone pairs on B = O)

64. Which of the following molecule involve d_{z^2} orbital in it's hybridisation

(A) XeF_4

(B) $XeOF_4$

(C) XeO_2F_2

(D)All of these

Ans. (D)

Sol. $\left.\begin{array}{l} XeF_4 \Rightarrow sp^3d^2 \\ XeO_2F_2 \Rightarrow sp^3d^2 \\ XeO_2F_2 \Rightarrow sp^3d \end{array}\right\}$ All the molecule involved dz^2 orbital in it's hybridisation

65. Correct order of melting point is :-

(A) $NaF < MgF_2 < AlF_3$

(B) $AlF_3 > NaF > MgF_2$

(C) $MgF_2 > NaF > AlF_3$

(D) None

Ans. (A)

Sol. Melting point depends on lattice energy.

hence, correct order of melting point is $NaF < MgF_2 < AlF_3$.

66. Which of the following molecule have zero dipole moment:-

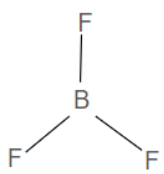
(A) BF_3

(B) CH_2Cl_2

(C) NF_3

(D) SO_2

Ans. (A)

Sol. $BF_3 \Rightarrow$  $M_{net} = 0$

67. The pair of species with similar shape is?

(A) XeF_4 & SF_4

(B) PF_5 & IF_5

(C) XeO_2F_2 & SeF_4

(D) All pairs are iso-structural

Ans. (C)

Sol. $XeO_2F_2 \Rightarrow sp^3d \Rightarrow$ see – saw

$SeF_4 \Rightarrow sp^3d \Rightarrow$ see – saw

68. Hybridisation of ClF_3, SF_4 & SOF_4 respectively will be

(A) sp^3, sp^3d, sp^3

(B) sp^3, sp^3, sp^3

(C) sp^3, sp^3d^2, sp^3d^2

(D) All sp^3d

Ans. (D)

Sol. $ClF_3 \Rightarrow sp^3d$

$SF_4 \Rightarrow sp^3d$

$SOF_4 \Rightarrow sp^3d$

69. Choose the incorrect order of bond strength :-

(A) $3p\pi - 3p\pi < 2p\pi - 3d\pi$

(B) $3p\pi - 3p\pi < 3d\pi - 3d\pi$

(C) $3p\pi - 3d\pi < 2p\pi - 3d\pi$

(D) $3p\pi - 3d\pi < 3p\pi - 3p\pi$

Ans. (D)

Sol. Bond strength $\propto \dfrac{1}{(n_1 + n_2)}$

All the options are correct except (D).

70. What is the order of boiling point of the following compounds? HF, NH_3, H_2O, CH_4

(A) $CH_4 > NH_3 > H_2O > CH_4$

(B) $HF > H_2O > NH_3 > CH_4$

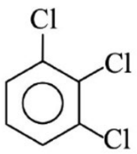
(C) $H_2O > HF > NH_3 > CH_4$

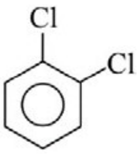
(D) $H_2O > NH_3 > HF > CH_4$

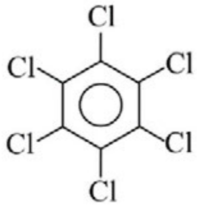
Ans. (C)

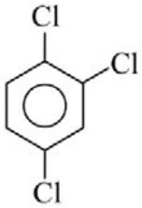
Sol. Correct order, of boiling point is $H_2O > HF > NH_3 > CH_4$ due to presence of Hydrogen bond.

71. Which of the following have maximum dipole moment?

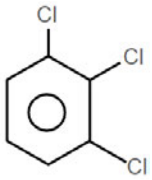
(A) 

(B) 

(C) 

(D) 

Ans. (A)

Sol. Net dipole moment of  is maximum.

72. The number of I – F bonds having the longer and shorter lengths are respectively in IF_7

(A) 5 and 2

(B) 2 and 5

(C) 5 and 5

(D) 2 and 2

Ans. (A)

Sol. IF_7 has 5 equitorial bond & two axial bond. In sp^3d^3 hydridisation pentagonal bipyramidal has longer equitorial bond.

73. Electron geometry of the molecule XeF_2 & ICl_2^- are respectively?

(A) square bipyramidal, tetrahedral

(B) linear & linear

(C) Trigonal bipyramidal & tetrahedral

(D) Both Trigonal bipyramidal

Ans. (D)

Sol. ICl_2^- & $XeF_2 \Rightarrow sp^3d \Rightarrow$ trigonal bipyramidal

74. CORRECT order of bond length is

(A) Si – O, < P – O < S – O < Cl – O

(B) Cl – O < S – O < Si – O < P – O

(C) CI – O <S – O < P – O < Si – O

(D) S – O < P – O < Cl – O < Si – O

Ans. (C)

Sol. Correct order of bond strength is Cl – O < S – O < P – O < Si – O.

$$\text{Bond strength} \propto \frac{1}{\text{Bond length}}$$

75. If x is internuclear axis, δ (delta) bond can be formed by

(A) $d_{x^2-y^2} + d_{x^2-y^2}$

(B) $d_{xy} + d_{yz}$

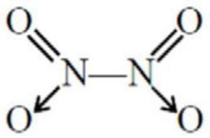
(C) $d_{yz} + d_{yz}$

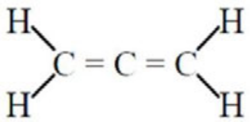
(D) $d_{xz} + d_{xy}$

Ans. (C)

Sol. If x is the internuclear axis ____ $dy_z + dy_z$ from a delta bond.

76. Which of the following molecule have all atoms sp^2 Hybridised ?

(A) 

(B) 

(C) Benzene

(D) None of these

Ans. (C)

Sol. Conceptual

77. Conditions for ionic bond formation is/are:

(a) Small cation, large anion

(b) Low IP of cation, high electron affinity of anion

(c) Large cation, small anion and less charge

(d) Less lattice energy

Correct answer is:

(A) a, d

(B) b, c and d

(C) b and c

(D) a, b

Ans. (C)

Sol. $A \rightarrow A^+ + e$ (Ionisation energy should be low)

$B + e^+ \rightarrow B^-$ (electron affinity should be high)

& for Ionic bond, cation should be large & anion should be small.

78. A sigma bond is formed by the overlapping of:

(A) s-s orbital alone

(B) s and p orbitals alone

(C) s-s, s-p or p-p orbitals along internuclear axis

(D) p-p orbital along the sides

Ans. (C)

Sol. Conceptual

79. Which of the following cation is having maximum polarizing power?

(A) Mg^{+2}

(B) Al^{3+}

(C) Na^{+}

(D) Ca^{2+}

Ans. (B)

Sol. Polarizing power $\propto$ charge $\Rightarrow$ Al^{+3} have maximum polarizing power.

80. Which one is only V-shaped molecule or ion?

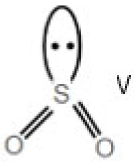
(A) I_3^-

(B) N_3^-

(C) SO_2

(D) C_3^{4-}

Ans. (C)

Sol. SO_2 :  V-shaped

SECTION-II

81. How many of the given compound (s) is/are not having zero dipole moment?

BF_3, CO_2, BeF_2, $BeCl_2$, SO_2, H_2O

Ans. (2)

Sol. SO_2 and H_2O are polar molecule they are not having zero dipole moment.

82. How many of the following compound(s) is/are Ionic in nature?

NaCl, AlF_3, KCl, KF, MgF_2

Ans. (5)

Sol. All the given compounds are Ionic in nature.

83. Total number of molecules which contain any $F\text{-}\hat{X}\text{-}F$ bond angle which is less than 90° ?

(X = Central atom)

IF_7, BrF_3, PF_5, SF_4, $XeOF_4$, SF_6

Ans. (4)

Sol. Except PF_5 & SF_6, All the molecules have bond angle less than 90°.

84. How many sets of given orbitals can from π bond?

(z – axis is internuclear axis)

$p_x + p_y$, $p_y + p_y$, $p_z + p_x$, $p_z + p_z$, $d_{x^2-y^2} + d_{z^2}$,

$p_z + d_{xy}$, $p_z + d_{xz}$, $p_z + d_{yz}$, $d_{xz} + d_{yz}$, $d_{yz} + d_{yz}$,

$d_{xy} + d_{xy}$, $d_{xy} + d_{x^2-y^2}$

Ans. (2)

Sol. On z-axis, $(p_y + p_y)$ and $(dy_z + dy_z)$ formed $\pi -$ bond.

85. Find the number of molecules having sp^3d hybridisation

I_3^- , SF_4, $BeCl_2$, XeF_4

Ans. (2)

Sol. $I_3^- \rightarrow sp^3d$

$SF_4 \rightarrow sp^3d$

$BeCl_2 \rightarrow sp$

$XeF_4 \rightarrow sp^3d^2$

86. Number of species having bond order 2 will be:

$O_2^{+2}, N_2^{+2}, N_2^{-2}, O_2^{+}, N_2^{+}, C_2, B_2^{-2}$

Ans. (4)

Sol.

O_2^{+2}	B · O
N_2^{+2}	2
N_2^{2-}	2
$O_2^{\oplus}$	2.5
$N_2^{\oplus}$	2.5
C_2^{+}	2
B_2^{2-}	2

87. Number of molecules having all bond angles equal are

CH_2F_2, BHF_2, NF_3, XeF_5^-

Ans. (2)

Sol. NF_3 & XeF_5^- have all Equal bonds.

88. Find total number of orbitals in which electron density is observed along any of the axis (x, y or z).

p_x, p_y, p_z, d_{xy}, d_{xz}, d_{yz}, d_{z^2}, $d_{x^2-y^2}$

Ans. (5)

Sol. Except d_{xy}, d_{xz} and d_{yz}, All the orbitals have electron density along the axes.

89. The number of unpaired electron present in O_2 molecule according to M.O.T. is

Ans. (2)

Sol. According to M.O.T, O_2 is paramagnet & It has two unpaired electron.

90. In the molecular orbital diagram for the molecular ion, N_2^+, the number of electrons in the σ_{2p} molecular orbital is:

Ans. (1)

Sol. Conceptual

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