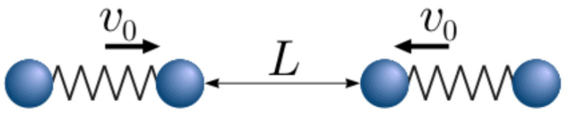
PART-1 : PHYSICS

SECTION-I (i)

1) A "spring-dumbbell" comprises two balls of mass m that are connected with a spring of stiffness k. Two such dumb-bells are sliding toward one another, the velocity of either is v_0. At some point the distance between them is L. After which time is the distance between them equal to L again? The collisions are perfectly elastic.



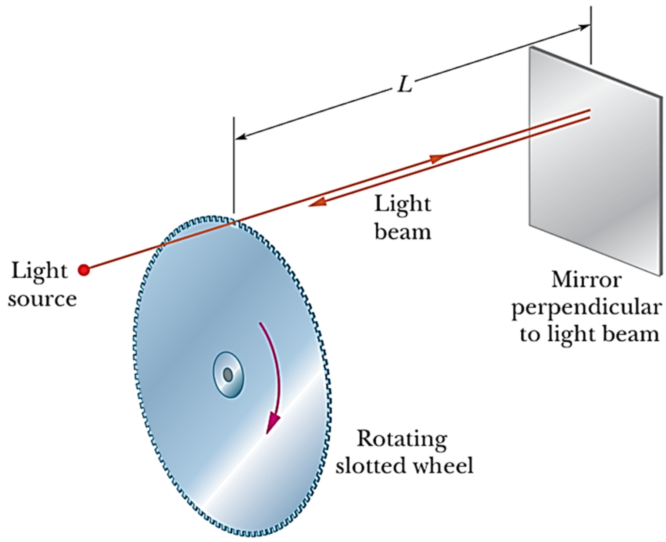
(A) $\frac{L}{v_0} + \pi\sqrt{\frac{m}{k}}$

(B) $\frac{L}{v_0} + \pi\sqrt{\frac{m}{2k}}$

(C) $\frac{2L}{v_0} + \pi\sqrt{\frac{m}{k}}$

(D) $\frac{2L}{v_0} + \pi\sqrt{\frac{m}{2k}}$

2) Figure shows an early method of measuring the speed of light that makes use of a rotating slotted wheel. A beam of light passes through one of the slots at the outside edge of the wheel, travels to a distant mirror, and returns to the wheel just in time to pass through the next slot in the wheel. One such slotted wheel has a radius of 5.0 cm and 500 slots around its edge. Measurements taken when the mirror is L = 500 m from the wheel indicate a speed of light of 3.0×10^5 km/s. What is the (constant) angular speed of the wheel ?



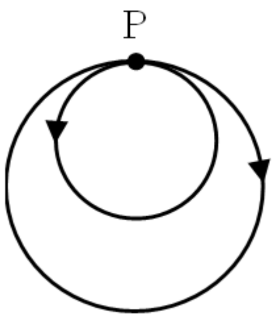
(A) 1.2 π rad/sec

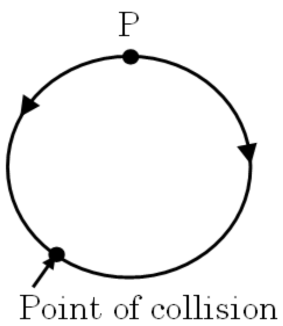
(B) 1.8 π rad/sec

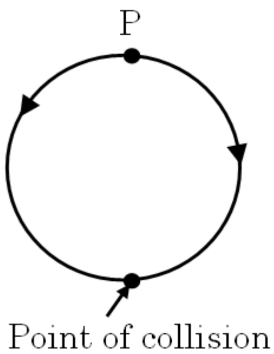
(C) $1.2\pi \times 10^3$ rad/sec

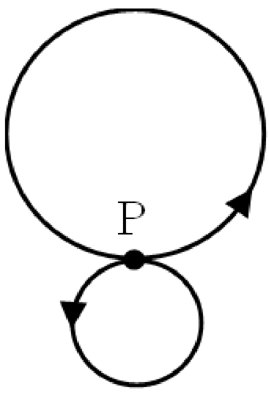
(D) $2.4\pi \times 10^3$ rad/sec

3) A neutral particle at rest in a uniform magnetic field. decays into two charged particles of different masses at point P as shown in the figure. The energy released goes to their kinetic energy and particles move in the plane of the paper. Magnetic field is into the plane of paper. Select the diagram which describes path followed by the particles most appropriately.

(A) 

(B) 

(C) 

(D) 

4) A diatomic ideal gas is heated at constant volume until the pressure is doubled and then heated at constant pressure until volume is doubled. The average molar heat capacity for whole process is :

(A) $\frac{13}{6}R$

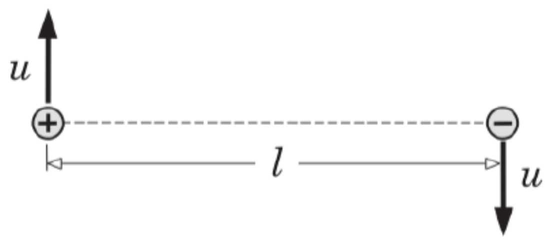
(B) $\frac{19}{6}R$

(C) $\frac{23}{6}R$

(D) $\frac{17}{6}R$

SECTION-I (ii)

1) Two particles each of mass m and charges +q and –q separated by a distance ℓ are given initial velocities u perpendicular to the line joining them as shown in the figure. Neglect gravitational and magnetic forces between the particles in comparison to Coulombian forces between them. Choose



correct options

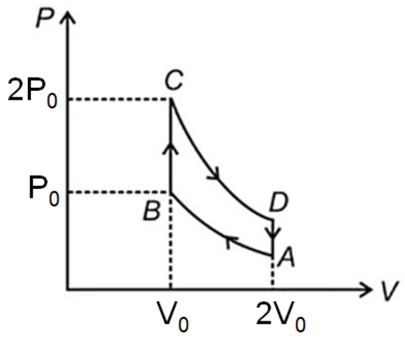
(A) If u = 0, then masses will eventually collide

(B) If $\frac{kq^2}{m\ell u^2} = 2$, then masses follow circular orbits of diameter ℓ

(C) Minimum distance between masses during subsequent motion is ℓ, If $\frac{kq^2}{m\ell u^2} \leqslant 2$

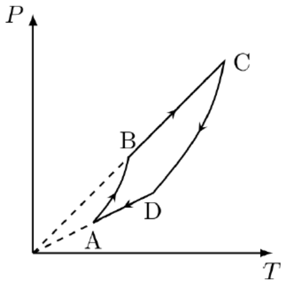
(D) Minimum distance between masses during subsequent motion is ℓ, If $\frac{kq^2}{m\ell u^2} > 2$

2) One mole of an ideal gas ($c_p / c_v = \gamma$ where symbols have their usual meanings) is subjected to an Otto cycle (A-B-C-D) as shown in the following P-V diagram. Path A-B and C-D are adiabats . The temperature at B is $T_B = T_0$. Choose the correct corresponding graph(s). (Symbols have usual

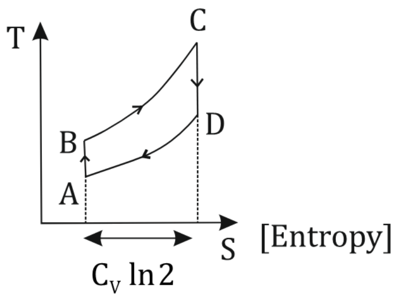


meaning)

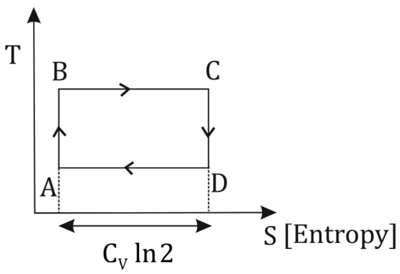
(A)



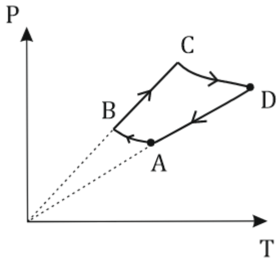
(B)



(C)



(D)



3) Current density in a cylindrical wire is varying with radial distance r as $J = \frac{J_0}{R^2} r(R-r)$, where R is radius of wire. If B is magnetic field due to this wire, then choose the incorrect options.

(A) Maximum magnetic field will be at a distance $\frac{8R}{9}$.

(B) Maximum magnetic field will be at a distance R.

(C) Maximum magnetic field will be $\frac{\mu_0 J_0 R}{12}$.

(D) Maximum magnetic field will be $\frac{64 \mu_0 J_0 R}{729}$.

SECTION-I (iii)

1) An electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume Bohr model to be valid.

Match the following column :

List-I		List-II	
(P)	The electron emits an energy of 2.55 eV	(1)	$n_1 = 2, n_2 = 1$
(Q)	Time period of the electron in the initial state is eight times that in the final state	(2)	$n_1 = 4, n_2 = 2$
(R)	Speed of electron become two times	(3)	$n_1 = 5, n_2 = 3$
(S)	Radius of orbit of electron becomes 4.77 Å	(4)	$n_1 = 6, n_2 = 3$
		(5)	$n_1 = 8, n_2 = 4$

(A) P → 2;Q → 1,2,4;R → 1,2,4,5;S → 5

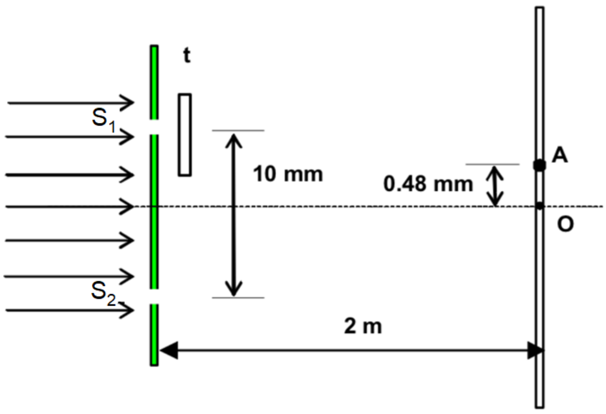
(B) P → 2;Q → 1,2,4,5;R → 1,2,4,5;S → 3,4

(C) P → 3;Q → 1,2,4,5;R → 1,2,4,5;S → 2

(D) P → 5;Q → 1,2,4,5;R → 1,2,4;S → 3,4

2) In a Young's double slit experiment, a plane monochromatic wave of wavelength 6000 Å, is incident normally on the slit plane as shown in the figure. A perfectly transparent film of thickness t, and refractive index 1.5 is placed in front of the slit S_1. The intensity of light on the screen near O is

0



I due to each slit.

List-I		List-II	
(P)	Value of $\frac{I}{I_0}$ at A, if t = 0.6 μm	(1)	4
(Q)	Value of $\frac{I}{I_0}$ at A, if t = 1.2 μm	(2)	1
(R)	The minimum value of t (μm) . (t > 0) for intensity to be maximum at O and match $\frac{10t}{4}$.	(3)	0
(S)	The minimum value of t (μm). (t > 0) for intensity to be the minimum at O and match $\frac{10t}{2}$.	(4)	3
		(5)	6

(A) P → 3;Q → 1;R → 4;S → 5

(B) P → 3;Q → 3;R → 4;S → 5

(C) P → 3;Q → 1;R → 4;S → 2

(D) P → 3;Q → 1;R → 4;S → 4

3) A charged particle with some initial velocity is projected in a region where uniform electric and/or magnetic fields are present. In List-I, information about the existence of a electric and/or magnetic field and direction of initial velocity of charged particle are given, while in List-II the probable path of the charged particle is mentioned. Match the entries of List-I with the entries of List-II.

	List-I		List-II
(P)	$\overline{E} = 0, \overline{B} \neq 0$ and initial velocity may be at any angle with $\overline{B}$.	(1)	Straight line
(Q)	$\overline{E} \neq 0, \overline{B} = 0$ and initial velocity may be at any angle with $\overline{E}$.	(2)	Parabola
(R)	$\bar{E} \neq 0, \bar{B} \neq 0, \bar{E} \parallel \bar{B}$ and initial velocity is perpendicular to both $\overline{E}$ and $\overline{B}$.	(3)	Circular
(S)	$\bar{E} \neq 0, \bar{B} \neq 0, \vec{E} \perp \vec{B}, \bar{v}$ is perpendicular to both $\overline{E}$and $\overline{B}$.	(4)	Helical path
		(5)	Can't be predicted

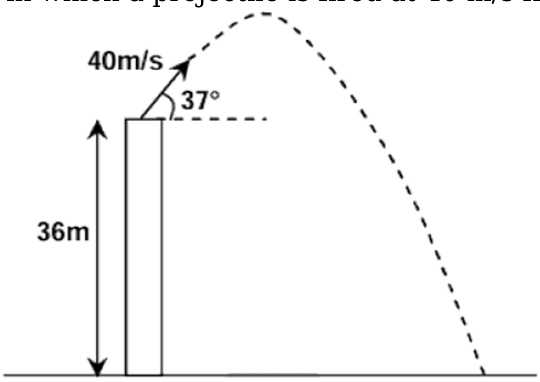
(A) P → 1,3,4;Q → 1,2;R → 4;S → 1

(B) P → 2,3,4;Q → 1,2,5;R → 4;S → 3,4

(C) P → 2,3;Q → 1,2,3,4;R → 1,2;S → 2,4

(D) P → 1,3,4;Q → 1,2;R → 1,5;S → 1,3

4) Consider a situation in which a projectile is fired at 40 m/s from the top of a tower in absence of air resistance at t = 0.



	List-I		List-II
(P)	Time at which particle moves at right angles with the initial direction.	(1)	2.4 s
(Q)	Time at which kinetic energy is least.	(2)	6 s
(R)	Time at which it strikes the ground.	(3)	20/3 s
(S)	Time when the $\vec{V}$ makes 45° with horizontal	(4)	5.6 s
		(5)	7.2 s

(A) P → 3;Q → 1;R → 4;S → 2

(B) P → 1;Q → 3;R → 2;S → 4

(C) P → 1;Q → 2;R → 3;S → 4

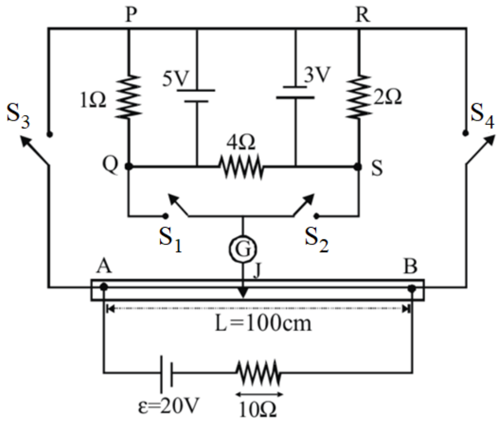
(D) P → 3;Q → 1;R → 2;S → 4

SECTION-II

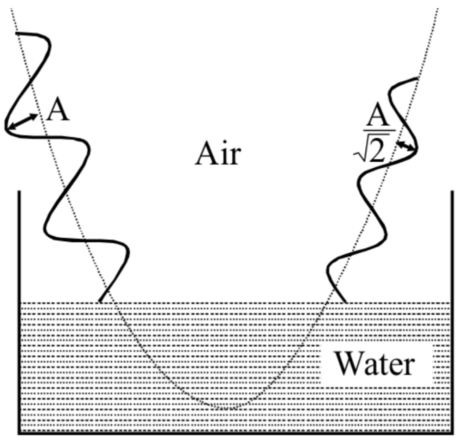
1) A massive star of mass M is in uniform circular orbit around a supermassive black hole of mass M_b. Initially, the radius and angular frequency of the orbit are R and ω respectively. According to Einstein's theory of general relatively the space around the two objects is distorted and gravitational waves are radiated. Energy is lost through this radiation and as a result the orbit of the star shrinks gradually. One may assume, however, that the orbit remains circular throughout and Newtonian mechanics holds.

The power radiated through gravitational wave by this star is given by $L_G = Kc^xG^yM^2R^4\omega^6$ where c is the speed of light, G is the universal gravitational constant, and K is a dimensionless constant. Find value of |x| + |y|.

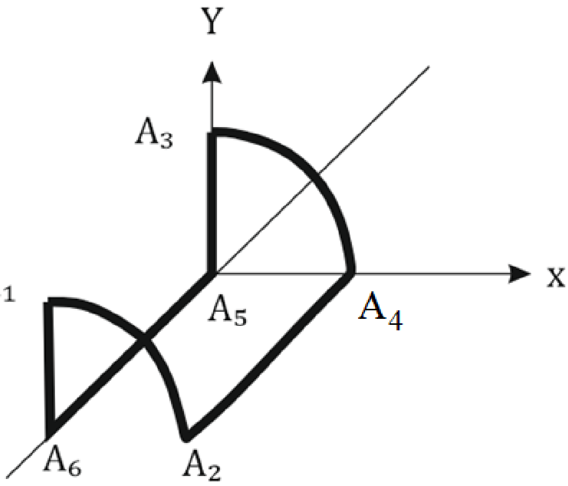
2) Figure shows a potentiometer connected to an external circuit. At an instant either switch S_1 and S_3 is closed or S_2 and S_4 is closed. When switch S_1 and S_3 is closed, null point is attained at J_1 (AJ_1 = ℓ_1) and when S_2 and S_4 is closed, it is attained at J_2 (BJ_2 = ℓ_2). Find the value of $3\left(\frac{\ell_1}{\ell_2}\right)$.



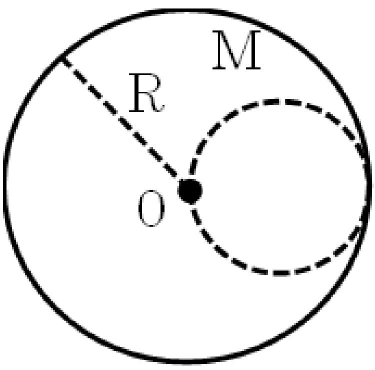
3) A thin string of density 0.1 kg/m is held at one end and the other end is oscillated according to the equation y(x = 0, t) = 20 (cm) sin (40 t) where t is in seconds. The string is under a tension of 10 N. The string passes through a bath filled with $\frac{40}{21}$ kg of water. Due to friction, heat is transferred through the bath and a snapshot at sometime is given below. After 100 α second the temperature of the bath rises by one Kelvin. Then find the value of α. [Specific heat capacity of water = 4.2 kJ/kgK]



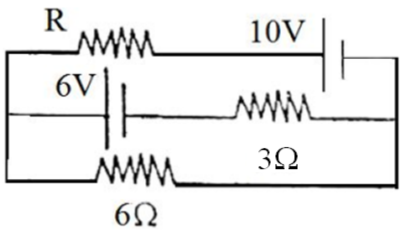
4) A time varying magnetic field $\vec{B} = B_0 t\hat{k}$ is confined in a cylindrical region, cutting the XY plane on a circle of radius $x^2 + y^2 = 4$. (Here x, y are in meter). We have placed a wire frame as shown. Segment A_1A_2 and A_3A_4 are identical quarter circles of radius 1m each. The net emf induced in the wire frame is equal to $\alpha\pi B_0$ volt. Write value of α.



5) Imagine a uniform solid spherical rigid planet of radius R and mass M. Now from it a spherical portion of radius 0.5 R is scooped out as shown in figure. If the scooped out portion is somehow placed back in place, then the contact force between the scooped portion and the rest of the planet is $\frac{GM^2}{2^n R^2}$. Find n.



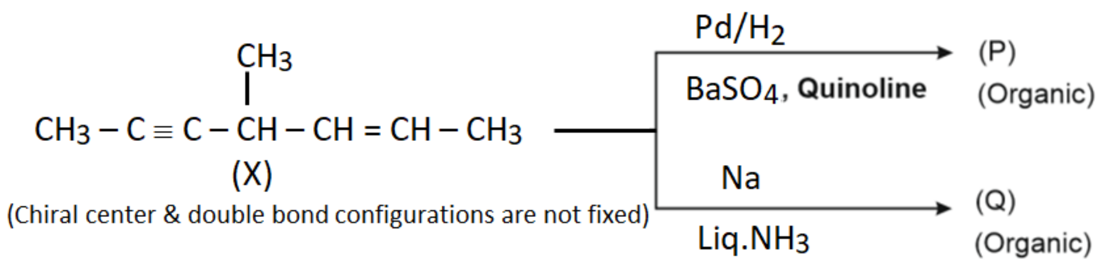
6) In a circuit shown in figure if the internal resistance of the sources are negligible then at what value of resistance R will the thermal power generated in it be the maximum. What is its value in



ohms?

PART-2 : CHEMISTRY

SECTION-I (i)



1)
Which one of the following is **CORRECT** ?

(A) If double bond of 'X' has 'E' configuration, then 'P' is an optically inactive product

(B) If double bond of 'X' has 'Z' configuration, then 'Q' is an optically inactive product

(C) Maximum number of '3' products (including stereo isomers) are formed in each reaction

(D) Formation of 'P' involves non-concerted and 'Q' involves concerted path

2) When 'N_2' is adsorbed on iron metallic surface, an electron transfer occurs from the metal to N_2 . Identify the **INCORRECT** statement regarding this process

(A) Magnetic nature of N_2 changes

(B) Stability of N_2 increases

(C) 'N_2' is chemically adsorbed

(D) $\Delta_r G < 0$

3) Which one of the following statement is **INCORRECT** about 'O_3'

(A) 'O_3' oxidises black PbS into white $PbSO_4$

(B) It tarnishes silver metal

(C) It is a meta stable allotropic form of oxygen

(D) H_2O_2 can oxidise 'O_3'

4) Identify the **CORRECT** statement based on molecular orbital theory

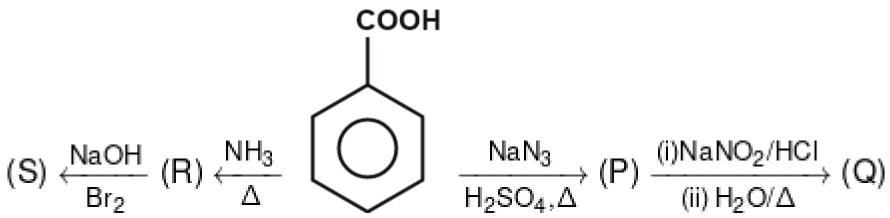
(A) HOMO of O_2 molecule has Ungerade symmetry

(B) HOMO-LUMO energy gap decreases from F_2 to I_2

(C) 'C_2' molecule contains one 'σ' and one 'π' bond

(D) Ionisation energy of 'O_2' is greater than that of oxygen atom

SECTION-I (ii)

1) 

Given : (P), (Q), (R) and (S) are major organic products formed in the above reactions
Identify the **CORRECT** option(s)

(A) (Q) and (S) can be differentiated by neutral $FeC\ell_3$ solution

(B) (S) and (Q) give white ppt with Br_2 / H_2O

(C) (P) and (R) can be differentiated by $CHC\ell_3$ / KOH , Δ

(D) Basicity order : (P) < (R)

2) α-methyl glycine $\xrightarrow{NaOH}\xrightarrow{PtCl_2(aq)}$ Complex ion (P)
Which of the following statement(s) is / are **CORRECT** regarding complex ion (P)

(A) Diamagnetic in nature

(B) It is less stable than $[NiC\ell_4]^{-2}$

(C) Total number of geometrical isomers possible for (P) = 4

(D) Total number of meso isomers possible for (P) = 3

3) The observed density of a gaseous substance at 2 atm and 750 K is 0.20 kg m^{-3} and effuses through a small hole at a rate of two times faster than methane under the same conditions. (Given : R = 0.08 lit atm mol^{-1} K^{-1})
Identify the **CORRECT** statement(s) based on above information

(A) Molar volume of gas is 20 lit $mole^{-1}$

(B) Molecular weight of gas is 4

(C) Repulsive forces are dominating between gas molecules

(D) Expected density of ideal gas is less than 0.2 kg m^{-3} at 2 atm and 750 K

SECTION-I (iii)

1) Match the following :

List-I (Extraction of metal from ore)		**List-II (Required methods)**	
(P)	Chalcopyrite → copper (pure)	(1)	Froth floatation
(Q)	Zinc blende → zinc (pure)	(2)	Hall process

(R)	Bauxite → aluminium (pure)	(3)	Roasting
(S)	Galena → lead (pure)	(4)	Self reduction
		(5)	Carbon reduction

The **CORRECT** option is

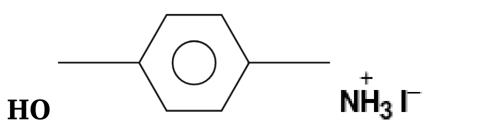
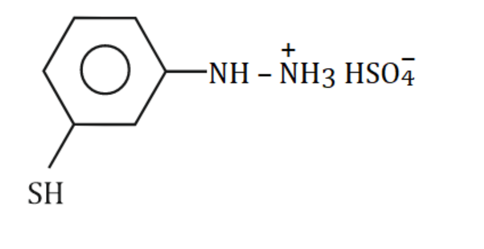
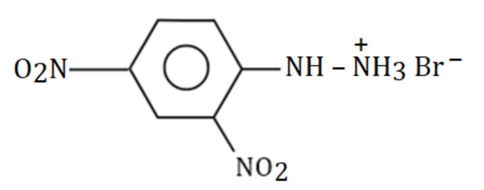
(A) P → 1,3,5 ;Q → 1,3,5 ;R → 2,5 ;S → 1,3,4

(B) P → 1,3,4 ;Q → 1,3,5 ;R → 2 ;S → 1,3,4,5

(C) P → 1,2,3,4 ;Q → 3,5 ;R → 2 ;S → 3,4

(D) P → 1,3,4;Q → 2,3,5 ;R → 2,5 ;S → 1,2,3,5

2) Match the compounds in **List-I** with their characteristics test(s) / reaction(s) given in **List-II**.

List-I		**List-II**	
(P)		(1)	Sodium fusion extract of the compound gives blood red solution with $FeCl_3$ aqueous solution
(Q)	$H_2N-\overset{+}{N}H_3Cl^-$	(2)	Gives white ppt with $Pb(NO_3)_2$ aqueous solution
(R)		(3)	Gives coloured ppt (non white) with $AgNO_3$ aqueous solution
(S)		(4)	Sodium fusion extract of the compound obtained from either limited sodium or with excess sodium gives prussian blue colour test
		(5)	Gives coloured ppt (non white) with aldehyde

Identify the **CORRECT** option in the following

(A) P → 2,3,5 ;Q → 2,4 ;R → 1,4,5 ;S → 2,5

(B) P → 3,4 ;Q → 2,5 ;R → 2 ;S → 1,5

(C) P → 3,4 ;Q → 2 ;R → 1,2,4 ;S → 2,3,4,5

(D) P → 3,5 ;Q → 1,2,5 ;R → 3,4 ;S → 1,2,3,4,5

3)

List-I		**List-II**	
(P)	Radius of 2nd orbit of Li^{2+} ion	(1)	$\frac{K\pi e^2}{h}$
(Q)	Speed of electron in 4th orbit of He^+ ion	(2)	$\frac{-13.6 \times 9}{4}$eV

(R)	Total energy of electron in 2nd orbit of electron in Li^{2+} ion	(3)	$\frac{h^2}{\pi^2 mKe^2 \times 3}$
(S)	Frequency of revolution of electron in 2nd orbit of electron in Li^{2+} ion	(4)	$\frac{-13.6 \times 4}{9}eV$
		(5)	$\frac{9\pi^2 mK^2e^4}{2h^3}$

The **CORRECT** option (from Bohr's atomic model) is :

(A) P → 1 ;Q → 5 ;R → 4 ;S → 3

(B) P → 5 ;Q → 1 ;R → 2 ;S → 3

(C) P → 3 ;Q → 1 ;R → 2 ;S → 5

(D) P → 3 ;Q → 5 ;R → 4 ;S → 1

4) Some unit cell parameters are given in List-I and some elements or compounds are given in List-II. Match correctly the element or compound (in List-II) with their correct unit cell parameter (in List-I).

List-I		List-II	
(P)	$a = b \neq c$; $\alpha = \beta = \gamma = 90°$	(1)	H_3BO_3
(Q)	$a = b \neq c$; $\alpha = \beta = 90°$; $\gamma = 120°$	(2)	Mg
(R)	$a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90°$	(3)	$Na_2SO_4.10H_2O$
(S)	$a \neq b \neq c$; $\alpha = \gamma = 90°$; $\beta \neq 90°$	(4)	$CaSO_4$
		(5)	ZnO

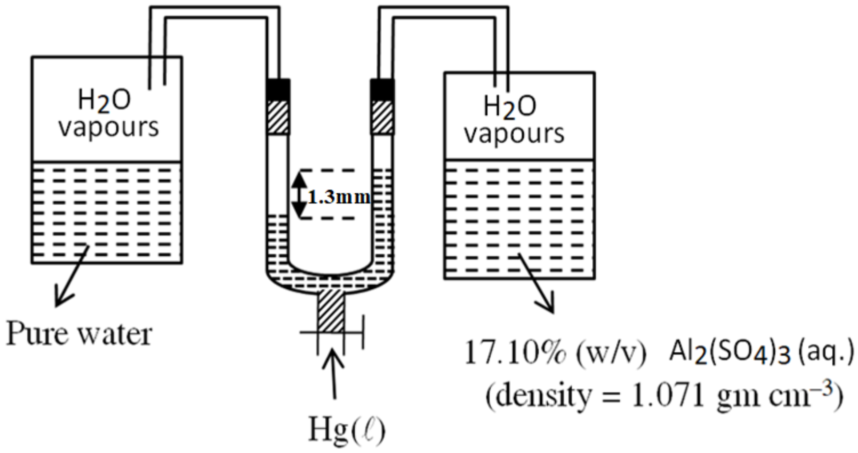
(A) P → 3;Q → 1;R → 2,5;S → 4

(B) P → 4;Q → 5;R → 3;S → 1

(C) P → 4;Q → 2,5;R → 1;S → 3

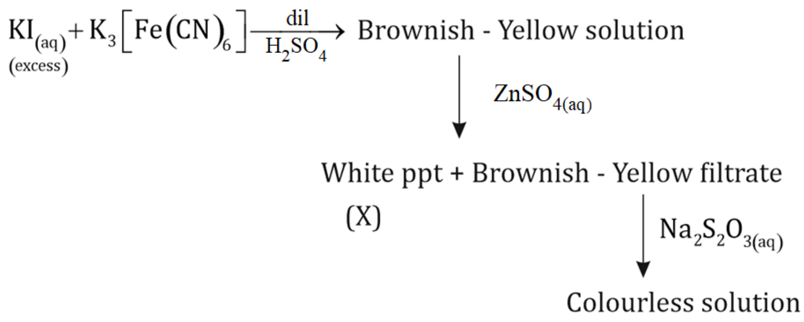
(D) P → 4;Q → 3;R → 2,5;S → 1

SECTION-II



1)
At the temperature of experiment, the vapour pressure of pure water is 51.3 mm Hg. The percentage dissociation of $Al_2(SO_4)_3$ into Al^{3+} and SO_4^{2-} ions in the aqueous solution, is .... %.

2) $\Delta_f G°_{1000}$ for n-pentane (g) and isopentane (g) are 85.04 and 82.84 kcal mol^{-1}, respectively. The mole percent of n-pentane (g) in the equilibrium mixture of these two gases at 1000 K and 0.50 bar pressure is ______ %.
[Given : ln2 = 0.7, ln3 = 1.1, ln5 = 1.6, R = 2 cal/K-mol]



3)
Then, the spin-only magnetic moment of (X) is .... B.M.

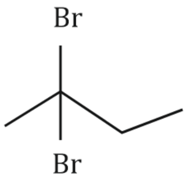
4) $Na_2Cr_2O_{7(s)} \xrightarrow[\text{conc. } H_2SO_4,\ \Delta]{NaCl(s)}$ Coloured product **(P)**

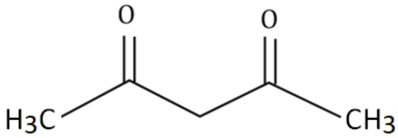
n-heptane $\xrightarrow[\text{at High temperature \& pressure}]{Cr_2O_3}$ **(Q)**

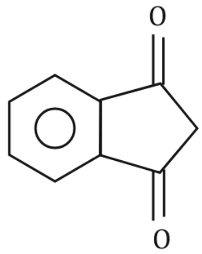
(P) + **(Q)** $\xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CS_2}$ **(R)**
Then total number of sp^2 hybridized carbons present in one molecule of compound **(R)** is

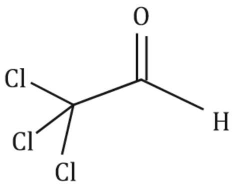
5) In 500 mL solution of 0.2 M NaOH and 0.3 M NH_4OH, 1000 mL of 0.2 M $HC\ell$ is added at 25°C. Determine pH of the resulting solution
[Given : K_b $(NH_4OH) = 2 \times 10^{-5}$; $\log_{10}(2) = 0.3$]

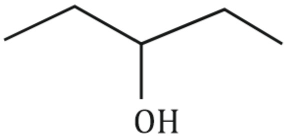
6) How many of following can give yellow ppt with I_2 / NaOH

(i) 

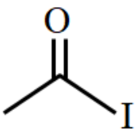
(ii) 

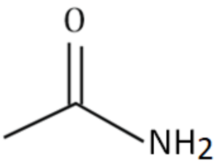
(iii) 

(iv) 

(v) 

(vi) 

(vii) 

(viii) 

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let R^2 denotes R × R, Let line $L : \vec{r} = \left(-\hat{i} - 6\hat{j}\right) + \lambda\left(a\hat{i} + b\hat{j}\right)$, where λ is real parameter and curve C : $y^2 = 8x$. If line L touches curve C, then sum of all possible values of $\frac{a}{b}$ is -

(A) 3

(B) 6

(C) -6

(D) –3

2) Let f(x) be a differentiable function satisfying $\sqrt[3]{f(x+y)} = \sqrt[3]{f(x)} + \sqrt[3]{f(y)} + 1 \ \ \forall x, y \in R$ and f '(0) = 3,

h(x) = f(x) – x^3 and A = { □ : □ is a point where h(|x|) is non-derivable}.

Let x^0 is solution of the equation $f(x) = f^{-1}(x)$, then $\frac{1}{\pi}\left(\cos^{-1}(\cos 2x_0) + 4\tan^{-1}\left(\tan \frac{x_0}{2}\right)\right) + |A|$ is equal to :

(|A| = No.of elements in A)

(A) 0

(B) 1

(C) 3

(D) 5

3) Three circles each of radius 3 are drawn with centers at (0, 16), (3, 0) and (5, 8). A line of slope m passing through (3, 0) is such that the total area of the part of the three circles to one side of the line

is equal to the total area of the three circles to the other side of it. Then $\left|\frac{m}{8}\right|$ is equal to

(A) 1
(B) 2
(C) 3
(D) 4

4) If x, y, z, w ∈ R satisfy the following system of equations $x + y + z + w = 1$; $x + 2y + 4z + 8w = 16$; $x + 3y + 9z + 27w = 81$ and $x + 4y + 16z + 64w = 256$, then the pair which has H.C.F as 2 is

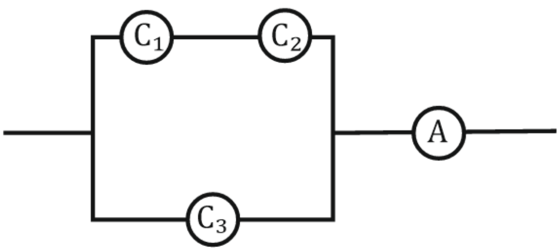
(A) $(|w|, |z|)$
(B) $(|z|, |y|)$
(C) $(|y|, |x|)$
(D) $(|z|, |x|)$

SECTION-I (ii)

1) If $I = \sum_{k=1}^{998} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$, then

(A) $I < \ln 999$
(B) $I > \ln 999$
(C) $I < 6$
(D) $I > 6$

2) An electrical apparatus A with three components C_1, C_2 and C_3 as in the diagram. The components works independently of each other with probabilities $\frac{8}{10}$, $\frac{9}{10}$ and $\frac{3}{4}$. For apparatus to be operational current flows from left to right. Let X denote the event that apparatus is operational and X_1, X_2, X_3 denotes respectively the events that the component C_1, C_2 and C_3 are functioning. Which of the following is (are) true ?



(A) P(exactly two components are functioning / X) = 1
(B) If P(X) is $\frac{a}{b}$ (a, b are co-prime) then b – a is 7
(C) $P(X/X_3) = 1$
(D) $P(X/X_2) = 19/20$

3) For the all function $f(x)=\int_0^{x^2}\left(\cos\frac{1}{\sqrt{t}}+1\right)dt,\ x\geqslant 1$

(A) f(x) is an increasing function in the interval [1, ∞)

(B) $\lim_{x\to\infty} f''(x) = 4$

(C) f(2) + f(4) > 2f(3)

(D) For all x in [1, ∞), f '(x + 3) – f '(x) < 6

SECTION-I (iii)

1) Match the following :
Let ABC be an isosceles triangle with AB = AC. If AB lies along x + y = 10 and AC lies along 7x – y = 30 and area of triangle be 20 sq. units.

	List-I		List-II
(P)	Coordinates of point B can be	(1)	(10, 0)
(Q)	Coordinates of point C can be	(2)	(4, –2)
(R)	Centroid of ΔABC can be	(3)	$\left(\frac{-5}{2},\frac{5}{2}\right)$
(S)	Circumcentre of ΔABC can be	(4)	$\left(3,\frac{13}{3}\right)$
		(5)	(0, 10)

(A) P → 5;Q → 2;R → 4;S → 1

(B) P → 1;Q → 2;R → 3;S → 4

(C) P → 1;Q → 2;R → 4;S → 3

(D) P → 5;Q → 2;R → 3;S → 4

2)

	List-I		List-II
(P)	The vector equation of the plane perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$ and passing through the point (3, 6, 2) is	(1)	$\vec{r}\cdot\left(\hat{i}-3\hat{j}-2\hat{k}\right)=3$
(Q)	The vector equation of the plane through the point (5, –2, 4) and parallel to the plane 4x – 12y – 8z = 7 is	(2)	$\vec{r}\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)=2$
(R)	The vector equation of the plane containing the line $\vec{r}=2\hat{i}+\lambda\left(\hat{j}-\hat{k}\right)$ and perpendicular to the plane $\vec{r}\cdot\left(\hat{i}+\hat{k}\right)=3$ is	(3)	$\vec{r}\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)=0$

(S)	The vector equation of the plane containing the lines $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z}{-1}$; $\frac{x-1}{-1}=\frac{y-1}{1}=\frac{z}{-2}$ is	(4)	$\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=1$
		(5)	$\vec{r}\cdot(\hat{i}-\hat{j}-\hat{k})=1$

(A) P → 1;Q → 4;R → 2;S → 5
(B) P → 4;Q → 1;R → 2;S → 3
(C) P → 1;Q → 4;R → 3;S → 5
(D) P → 4;Q → 1;R → 5;S → 2

3) If 3 sin x + 12 cos y + 15 sin z + 5 sin y + 4 cos x + 8 cos z ≥ 35 where x, y, z ∈ (0, π)

	List-I		**List-II**
(P)	sin x	(1)	$\frac{5}{12}$
(Q)	tan y	(2)	$\frac{3}{5}$
(R)	tan z	(3)	$\frac{4}{5}$
(S)	cos x	(4)	$\frac{15}{8}$
		(5)	$\frac{6}{7}$

(A) P → 3;Q → 4;R → 5;S → 1
(B) P → 2;Q → 1;R → 4;S → 3
(C) P → 2;Q → 3;R → 1;S → 4
(D) P → 1;Q → 2;R → 4;S → 5

4) Let $f(x)=\left|1+e^{|x|}-e^{-x}\right|$ and $g(x)=\left|e^{|x|}-2\right|$
Match each entry in List-I to the correct entry in List-II

	List-I		**List-II**
(P)	Area bounded between f(x) and g(x) is	(1)	$e+\frac{1}{e}$
(Q)	Area bounded by y = f(x) , x = –1, x = 1 and the x-axis is	(2)	$\ln\frac{16}{e^2}$
(R)	Area bounded by y = g(x) and the x-axis is	(3)	$\ln\left(\frac{27}{16}\right)$
(S)	Area bounded by max {f(x), g(x)}, x-axis, x = -2 and y-axis is	(4)	$\ln 27+e^2-7$
		(5)	4 ln 2 - 1

The correct option is

(A) P → 3;Q → 1;R → 2;S → 4
(B) P → 5;Q → 1;R → 3;S → 4
(C) P → 3;Q → 5;R → 2;S → 1
(D) P → 5;Q → 3;R → 2;S → 4

SECTION-II

1) Let $f(x) = \left(1 + x + x^2 + \dots\dots\dots + x^n\right)^2$. If C(r, n) denotes coefficient of x^r in the polynomial expansion of $f'(x)$.

Then $2025\left(\sum_{r=0}^{2023} \frac{1}{C(r, 2025)}\right)$ is
{ Note : f '(x) denotes derivate of f(x)}

2) 14 teams participated in ALLEN CRICKET TOURNMENT. Each team plays the other exactly once. There are no draws. Assume the teams are labelled by j, 1 ≤ j ≤ 14. Denote x_j and y_j as follows :
x_j : Number of games team j wins
y_j : Number of games team j lost

Then $\left|\frac{\sum_{j=1}^{14} (x_j)^2}{\sum_{j=1}^{14} (y_j)^2}\right| + \left|\sum_{j=1}^{14} x_j - \sum_{j=1}^{14} y_j\right|$ is

3) Let $a = \log_3 5, b = \log_3 4$ and $c = -\log_3 20$. Then the value of
$\frac{a^2+b^2}{a^2+b^2+ab} + \frac{b^2+c^2}{b^2+c^2+bc} + \frac{c^2+a^2}{c^2+a^2+ac}$ is

4) Let $A = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ where a, b, c ∈ R and $a^2 + b^2 + c^2 = 1$. If $A^8 - 2A^6 + A^3 + I = B$ then the value of $b_{11}^2 + b_{22}^2 + b_{33}^2$ is,
(where $B = [b_{ij}]_{3\times 3}$)

5) Let $f(x) = \sin^{-1}(2x-1) + \cos^{-1}\left(2\sqrt{x - x^2}\right) + \tan^{-1}\left(\frac{1}{1+[x^2]}\right)$ where [k] denotes greatest integer less than or equal to k then $\frac{12}{\pi}\left(f\left(\frac{3}{4}\right)\right)$ is equal to

6) Value of $\left|6\int_0^1\left(\sqrt{\frac{1}{4x^2}+\frac{1}{x}-x}-\sqrt{\frac{x^4}{4}-x+1}-\frac{1}{2x}\right)dx\right|$ is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	**B**	**C**	**B**	**B**

SECTION-I (ii)

Q.	5	6	7
A.	**A,B,C**	**A,B**	**B,C**

SECTION-I (iii)

Q.	8	9	10	11
A.	**B**	**D**	**A**	**D**

SECTION-II

Q.	12	13	14	15	16	17
A.	**6**	**5**	**5**	**0**	**4**	**2**

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	18	19	20	21
A.	**C**	**B**	**D**	**B**

SECTION-I (ii)

Q.	22	23	24
A.	**A,B,C**	**A,C**	**A,B,D**

SECTION-I (iii)

Q.	25	26	27	28
A.	**B**	**C**	**C**	**C**

SECTION-II

Q.	29	30	31	32	33	34
A.	**40**	**25**	**0**	**7**	**9**	**4**

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	35	36	37	38
A.	**D**	**C**	**C**	**C**

SECTION-I (ii)

Q.	39	40	41
A.	**A,D**	**B,C,D**	**A,B,C**

SECTION-I (iii)

Q.	42	43	44	45
A.	**C**	**B**	**B**	**A**

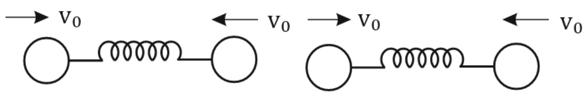
SECTION-II

Q.	46	47	48	49	50	51
A.	**2024**	**1**	**4**	**3**	**7**	**1**

PART-1 : PHYSICS

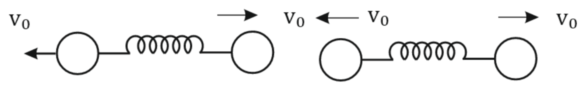
1) $t_1 = \frac{L}{2v_0}$

after collision



$t_2 = \pi\sqrt{\frac{m}{2k}}$

after t > t_2



$t_3 = \frac{L}{2v_0}$

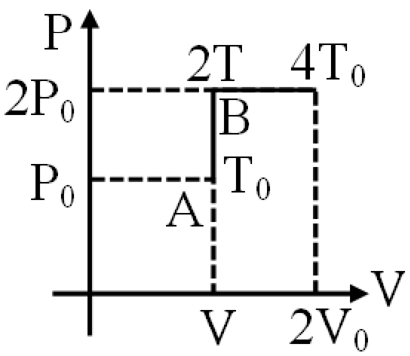
Total time $= \frac{L}{v_0} + \pi\sqrt{\frac{m}{2k}}$

2) $\frac{2\pi}{500} = \omega \times \frac{1000}{3 \times 10^8}$

$\omega = \frac{6 \times \pi \times 10^8}{5 \times 10^5} = \frac{6\pi}{5} \times 10^3$

$= 1.2\pi \times 10^3$ rad/sec

4)



Total heat absorbed by 1 mole of gas

$\Delta Q = C_v(2T_0 - T_0) + C_P(4T_0 - 2T_0)$

$= \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0 = \frac{19}{2}RT_0$

Total change in temperature from state A to C is $\Delta T = 3T_0$

 Molar heat capacity

$= \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$

5) System can be modified as

AMC $\frac{m}{2} \times v_1 \times r = \frac{m}{2} \times 2u \times \ell$

$v_1 r = 2u\ell$

Energy

$$mu^2 - \frac{kq^2}{\ell} = \frac{1}{2}\frac{m}{2}v_1^2 - \frac{kq^2}{r}$$

$$mu^2 - \frac{kq^2}{\ell} = \frac{1}{2}\frac{m}{2} \times \frac{4u^2\ell^2}{r^2} - \frac{kq^2}{r}$$

$$mu^2\left[1 - \frac{\ell^2}{r^2}\right] = kq^2\left[\frac{1}{\ell} - \frac{1}{r}\right]$$

$$mu^2\left[\frac{r+\ell}{r^2}\right] = kq^2\left[\frac{1}{r\ell}\right]$$

$$\left(r\ell + \ell^2\right) = \frac{kq^2}{mu^2} \times r$$

$$\frac{kq^2}{mu^2}r - r\ell = \ell^2$$

$$r = \frac{\ell^2}{\frac{kq^2}{mu^2} - \ell}$$

6) $\Delta S = \int \frac{d\theta R}{T} \Rightarrow nC_V\frac{dT}{T}$

$\Rightarrow nC_V \ln\frac{T_g}{T_\ell}$

$\Delta S = C_V \ln 2$

8) $E_4 = \frac{-13.6}{16}eV \quad E_2 = \frac{-13.6}{4}eV$

$|E_2 - E_4| = 2.55\,eV$

$T = \frac{2\pi r}{v} \propto \frac{\eta^2}{z \times z} \times \eta \propto \frac{\eta^3}{z^2}$

$T \propto \frac{\eta^3}{z^2}$

13) Let potential drop per unit length in the potentiometer wire be ρ.

$\therefore \rho\ell_1 = 5$ and $\therefore \rho\ell_2 = 3$

$\therefore \frac{\ell_1}{\ell_2} = \frac{5}{3}$

14) $P = \frac{TA^2\omega k}{2}$

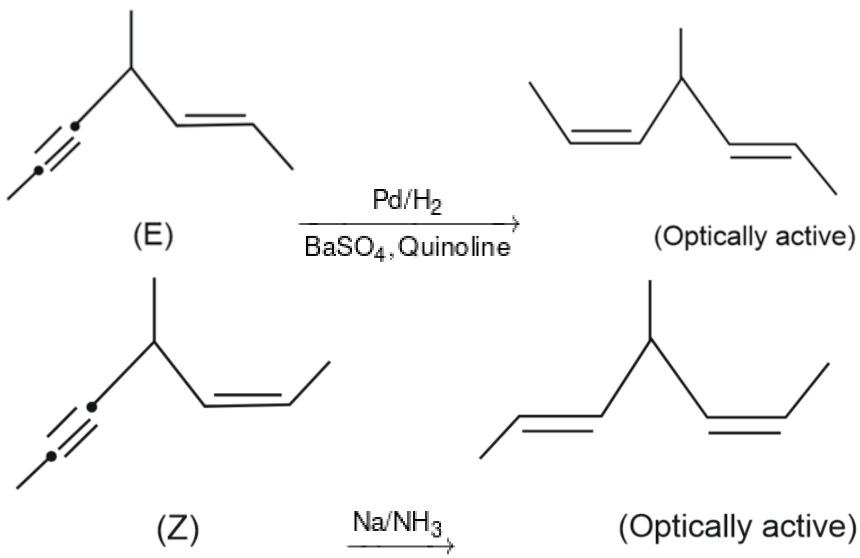
$$P_{in} - P_{out} = ms\frac{dT}{dt}$$

$$\frac{TA^2\omega k}{4} = ms\frac{dT}{dt}$$

t = 500 sec

PART-2 : CHEMISTRY

18) Catalytic hydrogenation follows concerted and Birch reduction follows non-concerted mechanism

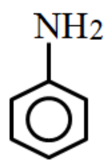


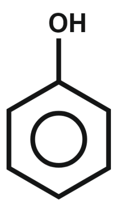
19) Bond order decreases due to increase in number of antibonding electrons.
□ Stability decreases

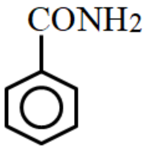
20) Oxidising ability of 'O_3' is greater than that of H_2O_2

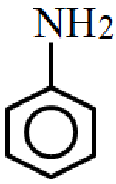
21)

A. π* has gerade symmetry
B. HOMO-LUMO energy gap decreases from F_2 to I_2 due to decrease in electronic repulsion
C. 'C_2' contains 2π bonds
D. I.E for removal of electron from different orbitals : B.M.O > A.O > A.B. M.O

22) (P) is 

; (Q) is 

(R) is 

; (S) is 

23) * "P" is $Pt\left(H_2N-\underset{CH_3}{CH}-\overset{O}{\overset{||}{C}}-\bar{O}\right)_2$

* dsp^2 hybridisation and square planar complex
* Geometrical isomers are 4 and meso compounds are 2

24) $\dfrac{r_{gas}}{r_{CH_4}} = \sqrt{\dfrac{M_{CH_4}}{M_{gas}}} \Rightarrow 4 = \dfrac{16}{M_{gas}} \Rightarrow M_{gas} = 4$

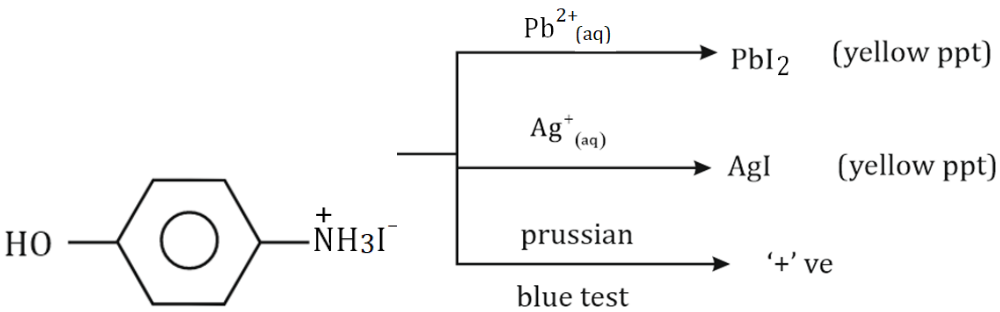
$V_m = \dfrac{4}{200} = 0.02\,m^3mol^{-1}$

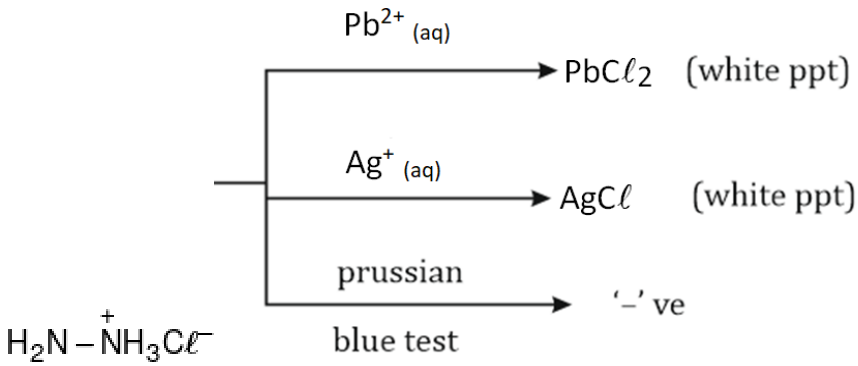
$= 20$ lit mol^{-1}

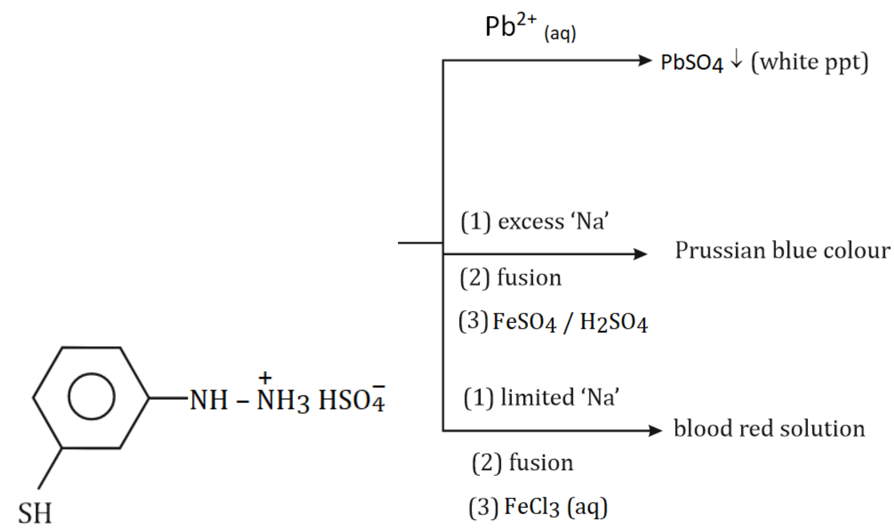
$2 \times 20 = Z \times 0.08 \times 750$

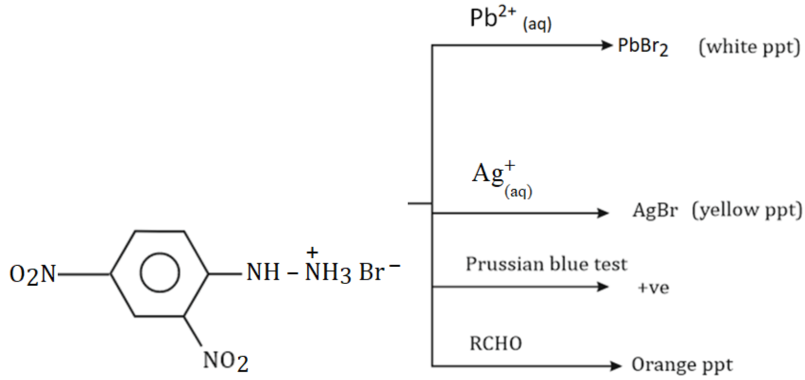
$Z < 1$

$d = \dfrac{P.M}{RT} < 0.2$ kg m^{-3}

26) 







27) (A) $r_n = \frac{n^2h^2}{4\pi^2 mKZe^2}$

$(r_2)_{Li^{2+}} = \frac{h^2}{3\pi^2 mKe^2}$

(B) $V_n = \frac{2\pi KZe^2}{nh}$

$(V_4)_{He^+} = \frac{K\pi e^2}{h}$

(C) $E_n = -13.6 \times \frac{Z^2}{n^2} eV$

$(E_2)_{Li^{2+}} = \frac{-13.6 \times 9}{4} eV$

(D) $f_n = \frac{4\pi^2 mK^2 Z^2 e^4}{n^3h^3} \Rightarrow (f_2)_{Li^{3+}} = \frac{9\pi^2 mK^2 e^4}{2h^3}$

28) Theory based.

29) 100 ml solution contains 17.10 gm $Al_2(SO_4)_3$
(= 107.1 gm) (= 0.05 mole)
∴ mass of water = 107.1 – 17.10 = 90.0 gm

$Al_2(SO_4)_3 \rightleftharpoons 2Al^{3+} + 3SO_4^{2-}$

0.05 (1–α) 0.05 × 2α 0.05 × 3α

Effective moles of solute = 0.05 (1 + 4α)

Now, $\frac{P^o - P}{P^0} = X_1 \Rightarrow \frac{1.3}{51.3} = \frac{0.05(1+4\alpha)}{0.05(1+4\alpha)+\frac{90}{18}}$

$\therefore \alpha = 0.4 \Rightarrow$ 40% dissociation.

30) n-pentane (g) $\rightleftharpoons$ isopentane (g)
(A) (B)

$\Delta_f G°_{1000} = \Delta_f G°_B - \Delta_f G°_A = 82.84 - 85.04 = -2.2$ kcal mol^{-1}

Now, $\Delta_r G° = -RT \ln K°_P$

or, $-2.2 \times 10^3 = -2 \times 1000 \times \ln \frac{P_B}{P_A}$

or, $\frac{P_B}{P_A} = \frac{3}{1}$

As the equilibrium is independent from pressure change, the ratio at 1.0 bar and 0.5 bar will remain same.

Hence, mole percent of n-pentane (A) = 25%.

$$\underset{(\text{excess})}{KI} + K_3[Fe(CN)_6] \xrightarrow{H^+} \underbrace{KI_3 + K_4[Fe(CN)_6]}$$

$$\downarrow Zn^{2+}$$

$$\underset{(\text{white ppt})}{K_2Zn_3[Fe(CN)_6]_2} + KI_3 \xrightarrow{S_2O_3^{2-}} KI$$

31)

'X' is $K_2Zn_3[Fe(CN)_6]_2$

32) (P) is $CrO_2C\ell_2$; (Q) is Toluene ; (R) is Benzaldehyde

33) m.mol of HCl = 200

m. mol of NaOH = 100

m. mol of NH_4OH = 150

$$NH_4OH + HCl \rightleftharpoons NH_4Cl + H_2O$$

At eq : m. moles : ~ 50 ~ 100

$p^{OH} = p^{K_b} + \log \frac{[NH_4^+]}{[NH_4OH]}$

$= 4.70 + \log\left(\frac{100}{50}\right) = 5.00$

$\therefore$ pH = 9.00

34) (i), (ii), (iii), (vi)

PART-3 : MATHEMATICS

35) L : Passes through (–1, –6)

Let slope $m\left(m=\frac{b}{a}\right)$

line : y = mx + m – 6

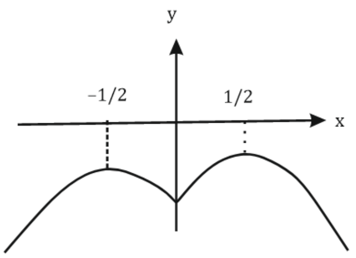
for tangent $m-6=\frac{2}{m}$

$m^2-6m-2=0 \left\langle \begin{matrix} m_1 \\ m_2 \end{matrix}\right. \; ; 2m^2+6m-1=0 \left\langle \begin{matrix} 1/m_1 \\ 1/m_2 \end{matrix}\right.$

$\therefore \sum \frac{a}{b}=-3$

36) $f(x)=(x-1)^3$

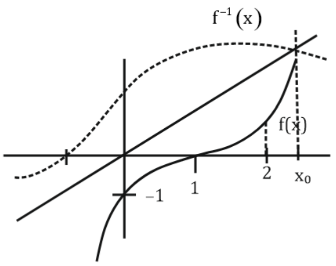
(i) Clearly, $y=h(|x|)$ is non-derivable at exactly one point



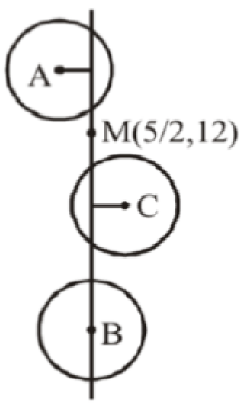
| A | = 1

(ii) f(x) and $f^{-1}(x)$ meet on the line y = x at x = x_0 where 2 < x_0 < 3.

$\{\because f(2)=1 \text{ and } f(3)=8\}$



$\therefore \cos^{-1}(\cos 2x_0)+4\tan^{-1}\left(\tan\frac{x_0}{2}\right)=(2\pi-2x_0)+4\frac{x_0}{2}=2\pi$



37)

A (0, 16),

B (3,0), C (5, 8)

line will pass through the mid point of A & C

$m_{AB}=\frac{12}{-1/2}=-24$

$$\left|\frac{m}{8}\right| = 3$$

38) Observe that 1, 2, 3, 4 are roots of
$n^4 - wn^3 - zn^2 - yn - x = 0$
$w = 10$, $z = -35$, $y = 50$, $x = -24$

39) For $x \in [k, k+1], x+1 \geqslant k+1$
Hence $\frac{k+1}{x(x+1)} \leqslant \frac{1}{x}$

$$\therefore \sum_{k=1}^{998} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$$

$$\leqslant \sum_{k=1}^{998} \int_{k}^{k+1} \frac{1}{x} dx$$

$$= \int_{1}^{999} \frac{1}{x} dx = \ln 999$$

Hence $I < \ln 999$
Also, $x \leq k + 1$

$$\therefore I \geqslant \sum_{k=1}^{998} \int_{k}^{k+1} \frac{1}{x+1} dx$$

$$= \int_{1}^{999} \ln (x+1) dx$$

$= \ln 500 > 6$

40) $P(X) = P(X_3) + P\left(\overline{X_3} \cap X_1 \cap X_2\right)$

$$= \frac{3}{4} + \frac{1}{4} \cdot \frac{8}{10} \cdot \frac{9}{10} = \frac{93}{100}$$

$$P\left(\frac{\text{Exactly two working}}{X}\right) \neq 1$$

$$P\left(\frac{X}{X_2}\right) = \frac{\frac{9}{10} \cdot \frac{8}{10} + \frac{9}{10} \cdot \frac{2}{10} \cdot \frac{3}{4}}{\frac{9}{10}} = \frac{19}{20}$$

41) $f'(x) = 2x \cos \frac{1}{x} + 2x > 0 \quad \forall x \in [1, \infty)$
$\Rightarrow$ f is increasing

$$f''(x) = 2 \cos \frac{1}{x} + 2x \left(-\sin \frac{1}{x} \left(-\frac{1}{x^2}\right)\right) + 2$$

$$= 2 \cos \frac{1}{x} + \frac{2}{x} \sin \frac{1}{x} + 2 > 0$$

(B) $\lim_{x \to \infty} f''(x) = 4$
(C) $f(2) + f(4) > f(3)$
(D) LMVT for $f'(t)$ in $[x, x + 3]$

as $f''' = +\frac{2}{x^2}\sin\frac{1}{x} - \frac{2}{x^2}\sin\frac{1}{x} + \frac{2}{x}\cos\frac{1}{x}\left(-\frac{1}{x^2}\right)$

$= -\frac{2}{x^3}\cos\frac{1}{x} < 0$

$\therefore f'' > 4$

$f'(\alpha) = \frac{f'(x+3) - f'(x)}{3} > 4$

$\Rightarrow f'(x+3) - f'(x) > 12 \quad \forall x \in [1, \infty)$

42) $\sin A = \frac{4}{5}, AB = AC = 5\sqrt{2}$

So, point B (0, 10) or (10, 0) and point C (4, –2) or (6, 12)

Centroid $\left(3, \frac{13}{3}\right)$ ....

Circumcentre $\left(-\frac{5}{2}, \frac{5}{2}\right)$

43) (P) $\left((x-3)\hat{i} + (y-6)\hat{j} + (z-2)\hat{k}\right) \cdot \left(\hat{i} - \hat{j} + 2\hat{k}\right) = 0$

(Q) 4 (x – 5) – 12 (y + 2) – 8(2 – 4) = 0

x – 3y – 2z = 3

44) $x_1 = 3\sin x + 4\cos x$

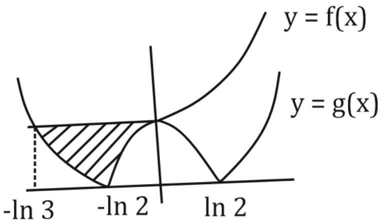
$x_2 = 12\cos y + 5\sin y$

$x_3 = 15\sin z + 8\cos z$

$\Rightarrow x_1 + x_2 + x_3 \leqslant 35$

$\Rightarrow \sin x = \frac{3}{5}, \cos x = \frac{4}{5}, \tan y = \frac{5}{12}, \tan z = \frac{15}{8}$

45)



(P)

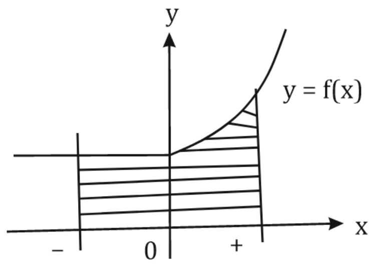
$A = \int_{-\ln 3}^{0} \left(1 - \left|e^{|x|} - 2\right|\right) dx$

$= \ln 3 - \int_{-\ln 3}^{-\ln 2} (e^{-x} - 2)\, dx - \int_{-\ln 2}^{0} (2 - e^{-x})\, dx$

$= \ln 3 + (2-3) + 2\left(\ln\frac{3}{2}\right) - 2\ln 2 - (1-2)$

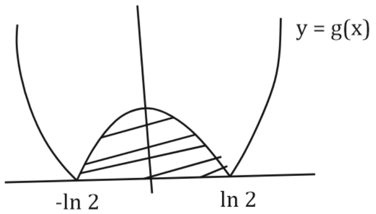
$= \ln\left(\frac{27}{16}\right)$

(Q)



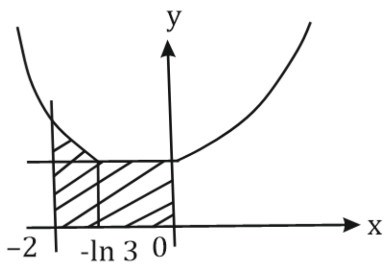
$$A = 1 + \int_0^1 \left(1 + e^x - e^{-x}\right) dx$$
$$= 2 + (e-1) + \left(e^{-1} - 1\right)$$
$$= e + e^{-1}$$

(R)



$$A = 2\int_0^{\ln 2} \left(2 - e^x\right) dx$$
= 4 ln 2 - 2

(S)



$$A = \int_{-2}^{-\ln 3} \left(e^{-x} - 2\right) dx + \ln 3$$
$$= e^2 - 3 - 2(2 - \ln 3) + \ln 3$$
$$= e^2 + 3\ln 3 - 7$$

46) Since $f(x) = \left(1 + x + x^2 + \ldots\ldots\ldots + x^n\right)^2$

$$f'(x) = 2\left(1 + 2x + \ldots\ldots\ldots\ldots + nx^{n-1}\right)\left(1 + x + x^2 + \ldots. + x^n\right)$$

So, $\begin{cases} C(r, n) = (r+1)(r+2) & \text{if } 0 \leqslant r \leqslant n-1,\ r \in Z \\ C(r, n) = (r+1)(2n - r) & \text{if } n \leqslant r \leqslant 2n-1, r \in Z \end{cases}$

This can be easily checked with induction or otherwise.
Particularly in our case pattern helps as well :

$$f(x) = \left(1 + x + \ldots\ldots. + x^{2025}\right)^2$$
$$f'(x) = 2\left(1 + x + x^2 + \ldots\ldots.x^{2025}\right)\left(1 + 2x + 3x^2 + \ldots\ldots\ldots + 2025x^{2024}\right)$$

⇒ C(0, 2025) = 2.1
C(1, 2025) = 2.3
C (2, 2025) = 2(1.3 + 1.2 + 1.1) = 3.4
.
.
.

an so on

Hence $\sum_{r=0}^{2023} \frac{1}{C(r, 2025)} = \frac{1}{1.2} + \frac{1}{2.3} + ... + \frac{1}{2024.2025}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + ... + \frac{1}{2024} - \frac{1}{2025}$

$= 1 - \frac{1}{2025} = \frac{2024}{2025}$

47) The logic works for any n.

Note that $\sum x_j = \sum y_j = \binom{14}{2}$ as every game is won and lost by some body.

Also $x_j + y_j = 13$

Now $\sum_j x_j^2 = \sum_j \left(13 - y_j\right)^2$

$= 13^2 . 14 - 2.13 \sum_j y_j + \sum_j y_j^2$

$= \sum_j y_j^2$

Since, $\sum_j y_j = \frac{14.13}{2}$

48) c = – (a + b)

$\therefore b^2 + c^2 + bc = b^2 + (a+b)^2 - b(a+b)$

$= a^2 + b^2 + ab$

Similarly, $c^2 + a^2 + ac = a^2 + b^2 + ab$

Hence the given expression reduces to $\frac{2\left(a^2+b^2+c^2\right)}{a^2+b^2+ab} = \frac{2\left(a^2+b^2+(a+b)^2\right)}{a^2+b^2+ab}$

= 4

49) $A^2 = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$= \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix} = A$

 $A^8 - 2A^6 + A^3 + I = I$

Hence $\sum_{i,j=1}^{3} b_{ij}^2 = 3$

50) $f(x) = \sin^{-1}(2x-1) + \cos^{-1}\left(2\sqrt{x - x^2}\right) + \tan^{-1}\left(\frac{1}{1 + [x^2]}\right)$

$$f\left(\frac{3}{4}\right) = \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{4} = \frac{7\pi}{12}$$

51)

$$I = \int_0^1 \left(\underbrace{\frac{\sqrt{1+4x-4x^3}-1}{2x}}_{f(x)} - \frac{\sqrt{x^4-4x+4}}{2} \right) dx$$

$$f^{-1}(x) = \frac{-x^2 + \sqrt{x^4-4x+4}}{2}$$

$$\int_0^1 f(x)\,dx + \int_{f(0)}^{f(1)} f^{-1}(x)\,dx + \int_1^0 \frac{x^2}{2} dx$$

$$= 1f(1) - 0f(0) + \frac{1}{6}(-1) = -\frac{1}{6}$$