FIITJEE RBT-3 for (JEE-Advanced)

BATCHES – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 1 | QP Code: 100955 | PAPER - 1

Time Allotted: 3 Hours **Maximum Marks: 234**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-07) –** Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(i) **Part-A (08-13) –** Contains six (06) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **Part-B (01-08)** contains eight (08) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**____________________

Enrolment Number :____

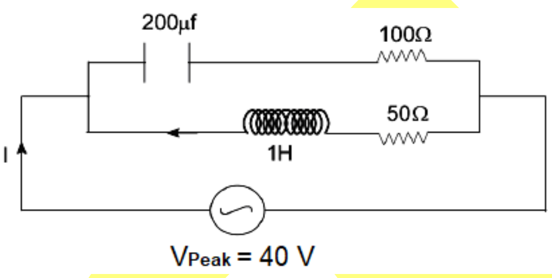
SECTION-1 : PHYSICS

PART – A

(Multi Correct Choice Type)

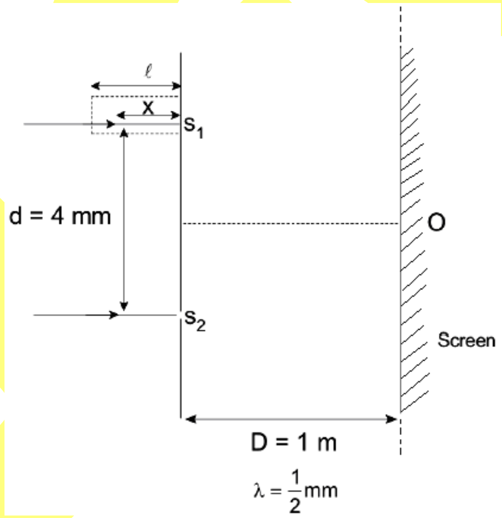
This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. In the given cirucit, the AC source has ω = 50 rad/s considering the inductor and capacitor to be ideal, the correct choice(s) is/are



(A) The voltage across 100 Ω resistor = 20.
(B) The voltage across 50 Ω resistor = 20.
(C) The current through the circuit is $I = \sqrt{5}$ A
(D) the current through the cirucit is I = 1.2 A

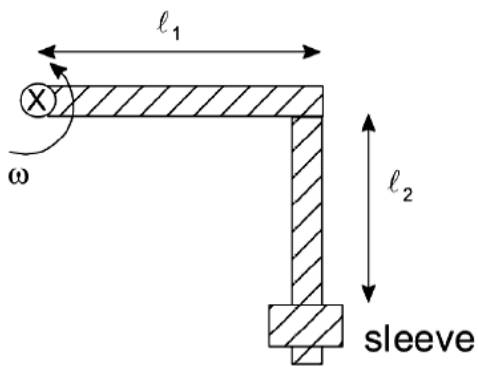
2. In YDSE, D = 1 m, d = 4 mm and $\lambda = \frac{1}{2}$mm in the figure shown, a parallel beam of light incident on the plane of the slits of YDSE. Light incidnet on the slit S_1, passes through a medium of variable refractive index $\mu = (2x + 1)$ (where x is the distance from the plane of slits in mm) up to a distnace ℓ = 1 mm before falling on s_1. Rest of the space filled with air. Then choose the correct option(s):



(A) Position of central bright fringe is at a distance $y = \frac{1}{\sqrt{3}}$m above the central line.
(B) Position of central bright fringe is at a distance $y = \frac{1}{2}$m above the central line.
(C) Number of maxima obtaining on the screen is 7.
(D) Number of maxima obtaining on the screen is 8.

Space For Rough Work

3. A rough L – shaped rod is located in a horizontal plane and a sleeve of mass m is inserted in the rod. The rod is rotated with a cosntant angular velocity ω in the horizontal plane. The lengths ℓ_1 and ℓ_2 are shown in figure. The normal reaction and frictional force acting on the sleeve when it starts slipping are (μ: coefficient of friction between rod and sleeve).



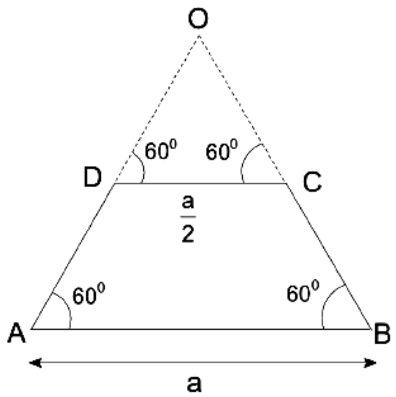
(A) $N = m\omega^2\ell_1$

(B) $\omega^4 = \dfrac{\mu^2 g^2}{\ell_2^2 - \mu^2\ell_1^2}$

(C) $N = m\sqrt{g^2 + \omega^4\ell_1^2}$

(D) $f = \mu N$

4. Consider a uniformly charged sheet ABCD, which is a part of an equilateral triangluar sheet of a side a as shown in the figure. Choose the correct options regarding the electric field E at point O due to this sheet.



(A) Magnitude of $\vec{E}$, increases with the increase in 'a' (Keeping charge density same)

(B) Magnitude of E, decreases with the increase in 'a' (Keeping total charge same)

(C) If charge desnity is σ and a = 1 m, magnitdue of E is equal to $\dfrac{7\sigma}{44\varepsilon_0}\ln\sqrt{2}$

(D) If charge desnity is σ and a = 2 m, magnitdue of E is equal to $\dfrac{7\sigma}{22\varepsilon_0}\ln\sqrt{2}$

5. Which of the folliwng statements is/are correct for mechanical standing wave on a stretched wire?

(A) Elastic potential energy of a small element at antinode is constant and minimum.

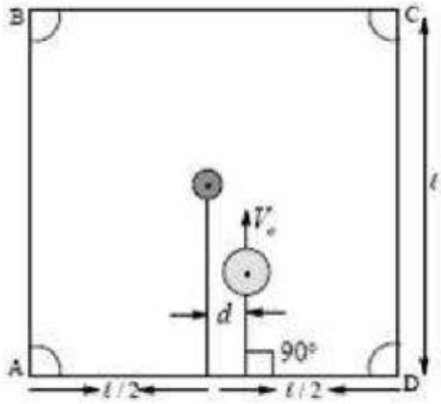
(B) Elastic potential energy of a small element at node is constant and maximum.

(C) Elastic potential energy of a small element at node is constant and minimum.

(D) Total kinetic energy between two consecutive nodes become maximum twice in one-time period.

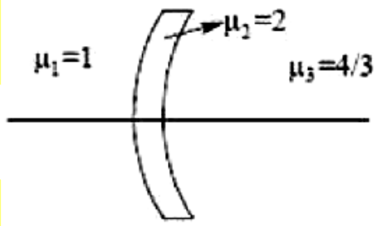
Space For Rough Work

6. The queen is put at the centre of a perfectly smooth carom board (square with side ℓ). The striker strikes the queen with a speed V_e as shown in the figure. Radius of the queen is $\sqrt{10}$ cm and that of the striker is $2\sqrt{10}$ cm. Coefficient of restitution for the collision between the queen and the striker is $\frac{1}{2}$ and that for the collision between the queen and the walls of the board is 1. (Assume ℓ >>> radius of queen)



(A) The value of 'd' for which the queen gets in the hole A is 3 cm.
(B) The value of 'd' for whcih the queen gets in the hole A is 2 cm.
(C) The value of 'd' depends on the coefficient of restitution between the queen and the striker.
(D) The value of 'd' independent of the coefficient of restitution between the queen and the striker.

7. A thin lens of same radius of curvature 20 cm is haivng two different medium on its two sides extending upto infinity as shown in the figure. Then



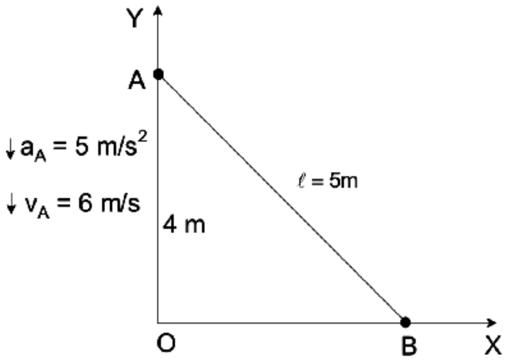
(A) it may behave as a converging lens of focal length 60 cm.
(B) it may behave as a diverging lens of focal elgnth 60 cm.
(C) it may behave as a converging lens of focal length 80 cm.
(D) it may behave as a diverging lens of focal length 80 cm.

Space For Rough Work

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. Point A on the rod AB has an acceleration of 5 m/s^2 and a veloicity of 6 m/s at an instant as shown in the figure. The acceleratin of the end B at the same moment is:



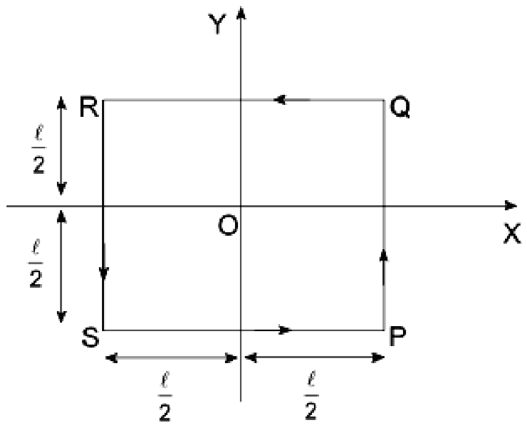
(A) $-\frac{80}{3}\hat{i}\ m/s^2$

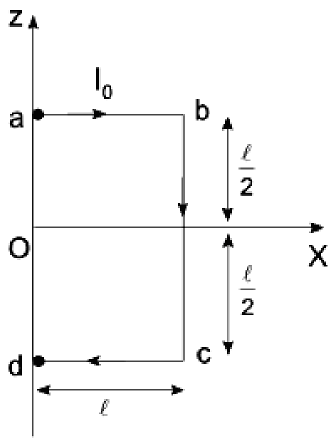
(B) $\frac{80}{3}\hat{i}\ m/s^2$

(C) $-\frac{40}{3}\hat{i}\ m/s^2$

(D) $-\frac{75}{6}\hat{i}\ m/s^2$

9. Current I_0 is flowing through a bent wire $a \to b \to c \to d$ in z – x plane as shown in figure, then $\oint \vec{B}.d\vec{\ell}$ over the loop PQRS lying in the X-Y plane as shown in figure, due to the bent wire abcd



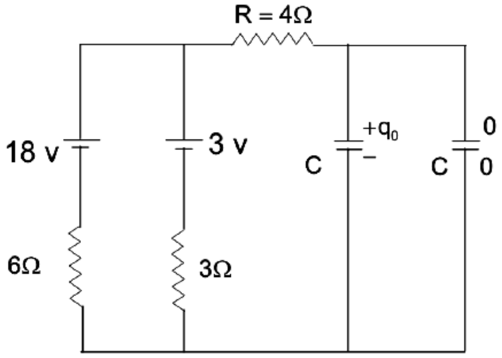


(A) $\frac{\mu_0 I_0}{8}$ (B) $\frac{\mu_0 I_0}{7}$ (C) $\frac{\mu_0 I_0}{5}$ (D) $\frac{\mu_0 I_0}{3}$

Space For Rough Work

10. The circuit is completed at t = 0, then heat loss in R = 4Ω resistance, till the capacitor gets fully charged:

(A) 64 μJ
(B) 384 μJ
(C) 320 μJ
(D) 576 μJ

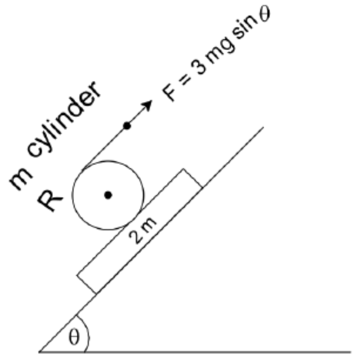


C = 16 μF
$q_0 = 64\mu C$

11. A simple harmonic plane progressive wave is travelling along the +ve x-axis having amplitude A and ω = π rad/s. At t = 1 sec, particle at x = 2 m and x = 3 m are moving in downward and upward direction respectively at the position $y = +\frac{A}{2}$. The time when maximum power transfer will takes place at $x = \frac{11}{4}$m?

(A) $\frac{5}{4}$sec (B) $\frac{5}{3}$sec (C) $\frac{7}{4}$sec (D) $\frac{7}{5}$sec

12. There is sufficient friciton between long plank of mass 2 m and cylinder of mass m and radius R to prevent slipping. Whole system is placed on the smooth inclined surface. Now, force F = 3 mg sin θ is applied on the ideal string wrapped over the cylinder as shown in figure. Choose correct option:



(A) If the plank moves down a distance of S m, work done by the applied force F on the string is 5 mg sin θ S.
(B) If the plank moves down a distance of S m, work done by the applied force F on the string is 6 mg sin θ S.
(C) If the plank moves down a distance of S m, work done by the applied force F on the sstring is 12 mg sinθ S.
(D) If the plank moves down a dsitane of S m, work done by the applied force F on the string is 15 mg sinθ S.

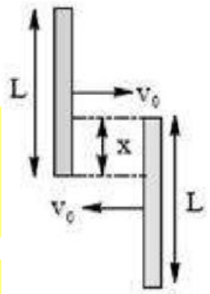
Space For Rough Work

13. A vertical thermally insulated cylinder of volume V contain n moles of an ideal monoatomic gas under a weightless piston. A load of weight W is placed on the piston as a result of which the piston is displaced. If the initial temperature of the gas is 300 K, area of piston is A and atmospheric pressure P_0. (take $W = P_0A$). Determine the value of final temperature of the gas.
(A) 375 K
(B) 425 K
(C) 475 K
(D) 515 K

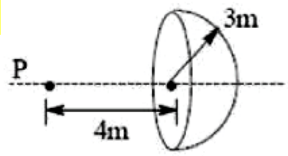
PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

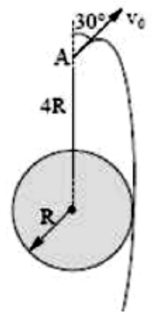
1. Two identical rigid rods, each of mass 1 kg & length $\sqrt{3}$ m, are moving opposite to each other without rotation on a smooth horizontal table as shwon. For what maximum vlue of 'x' (after rouding off), the direciton of motion of each rod would not change after collision, irrespective of the type of collision. Assume no sticking.



2. A point source of power 'P' is placed on th axis of a thin hemispherical shell as shown. The shell has a radius of 3m and behaves like a perfect black body. If the steady state temperature of shell is T. Find T^4 (after rounding off). (Take $P = 2400\pi\,\sigma$ W, σ = Stefan's constant)

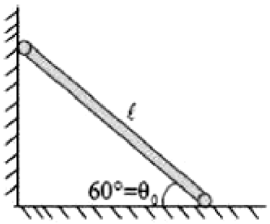


3. A particle is projected from point 'A', that is at a distance 4R from the centre of the Earth, with speed V_0 as shown. If the particle passes grazing the earth's surface, then it is found that $V_0 = 100 \times N$. Find the value of N (after rounding off). [Take $\frac{GM}{R} = 6.4\times10^7 m^2/s^2, \sqrt{2} = 1.414$]



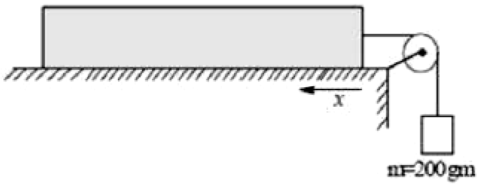
Space For Rough Work

4. A ladder of mass m and length ℓ stands against a frictionless wall with its feet on a fricitonless floor. If it is let go at an initial angle θ_0 = 60° then the angle 'θ' at which the ladder loses contact with the wall is given as $\sin^{-1}\left(1/\sqrt{N}\right)$, find 'N'.

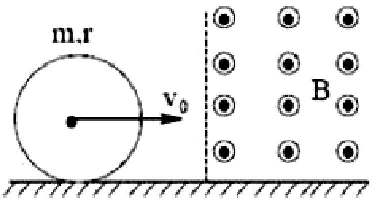


5. An electric charge distribution produces an electric field $\vec{E} = C(1-e^{-\alpha r})\frac{\hat{r}}{r^2}$ where $C = \frac{1}{4\pi\varepsilon_0}$ and α are constant. If the net charge within the radius $r = \frac{1}{\alpha}$ is $(1 - e^{-N})$, then find the value of 'N'?

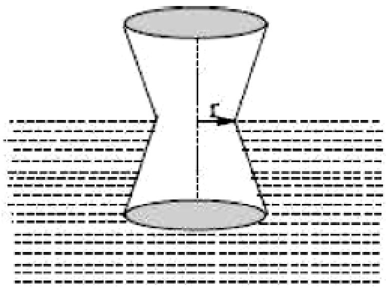
6. Uniform rope of mass = 5 kg and length 1 meter is lying on a rough horizontal surface. Coefficient of static friciton varies from right end of the rope as $\mu = \mu_0 x$ where μ_0 = 0.5 per meter. A block of mass 200 gm is hanging from an ideal string which passes over an ideal pulley as shown in the figure. The minimum value of x (in cm) for which tension at some cross-section of rope becomes zero is 10 × N. Find the value of 'N'.



7. A ring of mass m and radius r is made of an insulating material carries uniformly disturbed charge. Initially it rests on a fricionless horizontal table top with its plane vertical. The charge on the ring, such that it starts rolling on entering completely into the region of the magnetic field, is $\frac{\sqrt{N}\,mv_0}{rB}$, then the value of 'N'.



8. A solid object of mass $\frac{22}{7}$kg is in the shape of pellet drum, it is half submerged in water of density 1000 kg/m^3 with dimensions as shown in the figure. Find the time period (in seconds) of small vertical oscillations of the drum. [Take $r = \frac{22}{7}$cm]. If it is displaced slightly and released.



Space For Rough Work

SECTION-2 : CHEMISTRY

PART – A

(Multi Correct Choice Type)

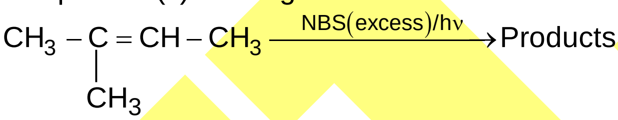
This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. The correct statement(s) regarding the wave function $\psi_{4, 1, -1}$ is/are
(A) it represent the 4p-subshell
(B) orbital associated with the wave function has one nodal plane and two radial nodes
(C) the energy of this orbital is higher than that of 3s orbital
(D) arsenic(Z = 33) contains one electron in this orbital

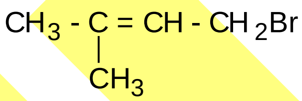
2. The correct statement(s) regarding a complex ion $[MnCl_4]^{2-}$ is/are that
(A) according to crystal field theory, the electronic configuration of the 3d-orbital of Mn^{2+} in the complex is $e^2t_2^3$.
(B) it is a square planar complex
(C) the Cl^- ions acts as weak field ligands
(D) Mn^{2+} undergoes sp^3-hybridization in the complex

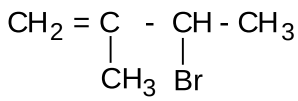
3. The solubility product constant(K_{sp}) of $Be(OH)_2$ is 5×10^{-10} at 298K.
Choose correct statement(s)
(A) the concentration of it's saturated solution is 5×10^{-4} mol L^{-1}
(B) it's solubility increases in a buffer of pH > 11 and decreases in a buffer of pH < 11
(C) addition of NH_4Cl increase it's solubility
(D) addition of NaCl does not affect solubility

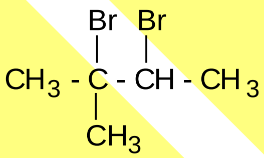
4. The product(s) of the given reaction

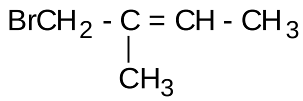


is/are

(A) 

(B) 

(C) 

(D) 

Space For Rough Work

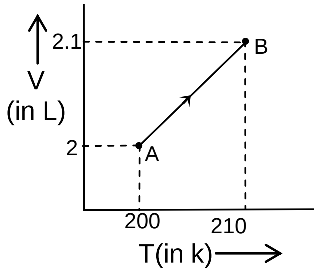
5. Which reaction(s) produce(s) secondary alcohol(s)?

(A) $CH_3-C(Cl)_2-CH_3 \xrightarrow[2.NaBH_4]{1.KOH/H_2O}$

(B) $CH_3-CH_2-CHO \xrightarrow{CH_3MgBr/H_3O^+}$

(C) $CH_3-\overset{\overset{\large O}{||}}{C}-CH_2-COOH \xrightarrow[2.LiAlH_4]{1.Heat}$

(D) $CH_3COOC_2H_5 \xrightarrow{LiAlH_4}$

6. 

One mole of an ideal gas undergoes above thermodynamic process. C_P of the gas is $\frac{5R}{2}$.

[R = 8.3 J K^{-1} mol^{-1}]

Choose correct statement(s) $\left[\ell n\frac{21}{20}=0.048\right]$

(A) The irreversible work done along path A $\rightarrow$ B is -83 J mol^{-1}.
(B) The system absorbs 207.5 J mol^{-1} heat from the surrounding.
(C) The entropy[ΔS(system)] change along the path A $\rightarrow$ B in reversible way is 0.996 J K^{-1} mol^{-1}.
(D) The internal energy of the system will increase to 124.5 J mol^{-1}.

7. The correct hydrolysis order of the given compound(s) is/are
(A) $SiCl_4 > SiF_4$
(B) $NCl_3 > CCl_4$
(C) $BI_3 > BF_3$
(D) $BBr_3 > BCl_3$

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. $BeCl_2$ undergoes polymerization and BCl_3 undergo dimerisation because
(A) $BeCl_2$ is more electron deficient than BCl_3
(B) $BeCl_2$ is linear and BCl_3 is angular
(C) Electronegativity different between Be and Cl is higher than that between B and Cl
(D) $BeCl_2$ is less covalent than BCl_3 molecule

Space For Rough Work

9. $CH_2 = CH - CH = CH_2 \xrightarrow{\text{1, 4-addition of HCl}} (P)$

$CH_2 = CH - CH = CH_2 \xrightarrow{\text{1, 2-addition of HCl}} (Q)$ Major product

Which statement is correct regarding properties of P and Q?
(A) Reactivity with aq.KOH (S_N1 path) P > Q
(B) Number of stereoisomers: P = Q
(C) Rate of reaction with alc.KOH: P > Q
(D) Heat of hydrogenation: P > Q

10. Which solubility order is incorrect?
(A) LiI > NaI > KI > RbI
(B) $BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4$
(C) $Be(OH)_2 > Mg(OH)_2 > Ca(OH)_2 > Sr(OH)_2$
(D) LiF < NaF < KF < RbF

11. Which forms the most stable complex with carbon monoxide?
(A) Fe
(B) Fe^{2+}
(C) Fe^{3+}
(D) $Fe^{+8/3}$

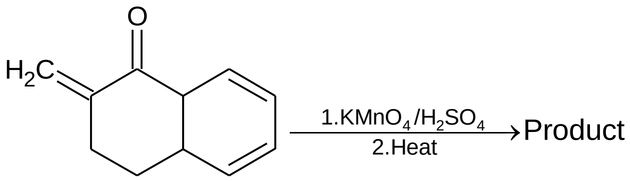
12. Gases having which of the following value of 'a' and 'b'(van der Waal's constant) can be liquefied easily?
[a in atm L^2 mol^{-2}, b in L mol^{-1}]
(A) a = 0.04, b = 0.03
(B) a = 0.08, b = 0.4
(C) a = 0.06, b = 0.03
(D) a = 0.004, b = 0.04

13. Two moles of  forms a single product when they react with
(A) $LiAlH_4$
(B) Zn-Hg/Conc.HCl
(C) NaOH
(D) $C_6H_5NH - NH_2$

Space For Rough Work

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. 

How many maximum number of CO_2 molecule(s) is/are formed in the above reaction?

2. A container contains 200 mL of 0.5 M NH_4NO_3 solution. 200 mL of 0.4 M NaOH solution was added to it. What is the pH of the resulting solution?
[K_b of $NH_4OH = 10^{-5}$] [log 0.25 = -0.6]

3. $$CH_3COCl + CH_3COONa \xrightarrow[-NaCl]{} (P) \xrightarrow{H_2O} (Q) \xrightarrow[\Delta]{NaOH/CaO} (R)$$

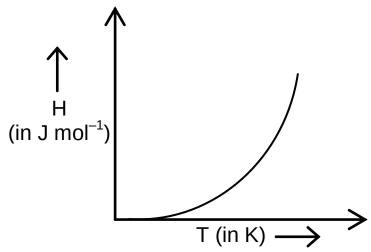
$$(R) + Cl_2 \xrightarrow[-HCl]{h\nu} S + T + U + V$$

Order of molar mass = S < T < U < V
Let x = The number of chlorine atoms present in 'T'.
y = Number of hydrogen atoms present in 'U'
What is the value of $\left(\frac{x+y}{10}\right)$?

4. The crystal field stabilization energy of $[Fe(CN)_6]^{3-}$ is $-x\Delta_o$ and that of $[Fe(CN)_6]^{4-}$ is $-y\Delta_o$. What is the value of (x + y)?

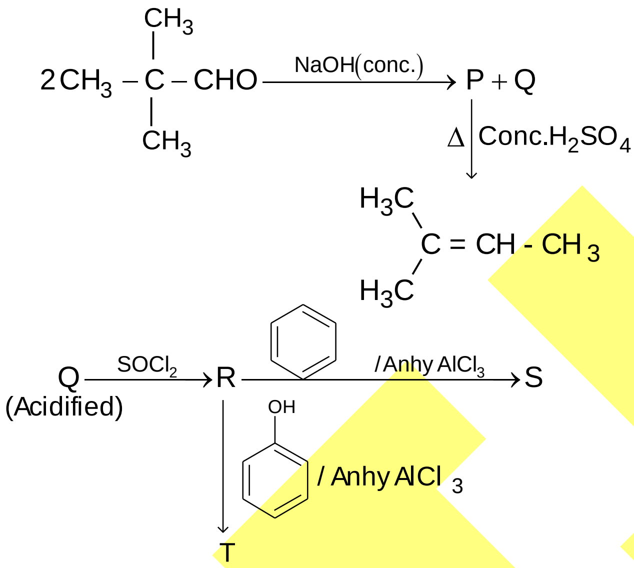
5. In a first order reaction $A(g) \longrightarrow Product$. What is the value of $\frac{[A_t]}{[A_o]}$ after two half-lives.

Space For Rough Work

6. 

If the slope of the above curve is 20.75 J K^{-1} mol^{-1}, what is the value of C_v in J K^{-1} mol^{-1} unit? [R = 8.3 J K^{-1} mol^{-1}]

7. A tetrahedral chloro complex of cobalt contains same number of electrons in the 'e' and t_2 orbitals according to crystal field theory. If the oxidation number of cobalt in the complex is +x, what is the value of $\frac{x}{2}$ is

8. 

If the molar mass difference between the organic product S and T is X, what is $\frac{X}{10}$?

Space For Rough Work

SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. Consider the equation $z^6 + 6z + 20 = 0$, Then which of following can be true?
(A) The number of the roots in the first quadrant can be 1or 2
(B) The number of the roots in the second quadrant can be 1or 2
(C) The number of the roots in the third quadrant can be 1or 2
(D) The number of the roots in the fourth quadrant can be 1or 2

2. A function f from integers to integers is defined as

$$f(x) = \begin{cases} n+3 & n \in \text{odd} \\ n/2 & n \in \text{even} \end{cases}$$

suppose k $\in$ odd and f(f(f(k))) = 27 then

(A) k can be 45
(B) sum of the digit of k is 6
(C) sum of the digit of k is 9
(D) k can be 105

3. Let α and $f(\alpha)$ be the eccentricity of the ellipse $\frac{x^2}{3b^2 - 2a^2} + \frac{y^2}{2b^2 - a^2} = 1, \left(3b^2 > 2a^2\right)$ and $\frac{x^2}{2b^2 - a^2} + \frac{y^2}{b^2} = 1, \left(2b^2 > a^2\right)$ then

(A) $f(\alpha) = \frac{\alpha}{\sqrt{1-\alpha^2}}, b \in R - \{0\}$

(B) $\int_0^{1/2} ffff(\alpha)\, d\alpha = \frac{1}{4}$

(C) $\int e^{\alpha}\left(f(\alpha) - f''(\alpha)\right) d\alpha = e^{\alpha}\left[\frac{\alpha}{\sqrt{1-\alpha^2}} - \frac{1}{\left(1-\alpha^2\right)^{3/2}}\right] + C$

(D) $f(\alpha) = \frac{\alpha}{2\sqrt{1-\alpha^2}}, b \in R - \{0\}$

Space For Rough Work

4. If $f(x)=(a^2+a+1)x^2+bx+c$ and $f(x)$ is symmetrical about the line x = 1, then

(A) $f(1-\sqrt{3})<f(\sqrt{3})<f(2+\sqrt{3})<f(4)$ (B) $f(\sqrt{3})<f(1-\sqrt{3})<f(2+\sqrt{3})<f(4)$

(C) $f(2+\sqrt{3})>f(4)>f(\sqrt{3})>f(1-\sqrt{3})$ (D) $f(4)>f(2+\sqrt{3})>f(-\sqrt{2})>f(\sqrt{2})$

5. If $\int_0^1 (4x^3-f(x))f(x)dx=\frac{4}{7}$, then which of the following is/are true

(A) $f(x)$ has a point of inflexion

(B) $\left|f(x)-\frac{1}{4}\right|$ has exactly one point of non-differentiability

(C) area bounded by $y=f(x)$, x – axis, x = 1 and x =2 is $\frac{15}{2}$

(D) The function $f(x)$ is concave upward for $\forall x\in R$

6. In triangle ABC, a=4 and $b=c=2\sqrt{2}$. A point P moves with the triangle such that the square of its distance from BC is half the rectangle contained by its distances from the other two sides. If D be the centre of P, then

(A) locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

(B) locus of P is a hyperbola with eccentricity $\sqrt{\frac{3}{2}}$

(C) area of the quadrilateral ABCD= $\frac{16}{3}$ sq.units

(D) area of the quadrilateral ABCD= = $\frac{32}{3}$ sq. units

7. If $f:R\to[2,4]$ be a periodic function such that the equation $f(x)=g(x)$ has a unique solution, if $g(x)=1-\cos\pi x$ then period of $f(x)$ can be

(A) e (B) $\sqrt{3}$

(C) π (D) 2020

Space For Rough Work

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. A boy have 32 cards out of which 10 cards, each of blue, green and red colours, have denominations as $\{2^1, 2^2, ..., 2^{10}\}$ and one black and one white each has value unity. The number of ways in which the boy can get a sum of 2012 if he can choose any number of cards, is

(A) $(1001)^2$
(B) $(1002)^2$
(C) $(1007)^2$
(D) $(1111)^2$

9. The minimum value of $(\sin\alpha - \cot\beta)^2 + (\tan\beta - \cos\alpha)^2$ for all admissible real Values of α and β is

(A) $3 - 2\sqrt{2}$
(B) 0
(C) $3 + 2\sqrt{2}$
(D) $2 - 3\sqrt{3}$

10. On the circle $|z - i\sqrt{3}| = 1$ points z_1, z_2, z_3 are so located that $3z_1 = 2z_2 + 2z_3 - i\sqrt{3}$. Then which is true ?

(A) $|z_1 - z_2| = \frac{1}{\sqrt{2}}$

(B) $|z_2 - z_3| = \frac{\sqrt{7}}{4}$

(C) The value of $\frac{z_2 - i\sqrt{3}}{z_3 - i\sqrt{3}} = \frac{1 \pm i\sqrt{63}}{4}$

(D) $|z_2 - z_3| = \frac{\sqrt{7}}{2}$

11. If f(x) be a positive, continuous and differentiable on the interval (a,b). If $\lim_{x\to a^+} f(x) = 1$ and $\lim_{x\to b^-} f(x) = 3^{1/4}$. Also $f'(x) + \frac{1}{f(x)}$, then

(A) $b - a \geq \frac{\pi}{24}$
(B) $b - a \leq \frac{\pi}{24}$
(C) $b - a = \frac{\pi}{12}$
(D) none of these

Space For Rough Work

12. If P be a point inside and equilateral triangle ABC such that PA=3,PB=4 and PC=5, then the side length of the equilateral triangle ABC is

(A) $\sqrt{25-12\sqrt{3}}$
(B) 13
(C) $\sqrt{25+12\sqrt{3}}$
(D) 17

13. If $f(x)$ is continuous function and satisfying $f(x)+f(1+x)=\left|2^x-1\right|+\left|x-1\right|$ in $0\le x\le 2$, then the value of $\int_0^2 f(x)dx$ is equal to

(A) $\frac{1}{\ln 2}-\frac{1}{2}$
(B) $\frac{1}{\ln 2}-1$
(C) $\frac{2}{\ln 2}-1$
(D) none of these

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. Let f be a continuous function and $f(x)=f(2x)$, $x\in R$. If f(x) = 3 then

$$\int_{-1}^{+1} f(f(f(x)))\,dx$$ is equal to

2. The number of ordered triplet (x,y,z) such that LCM(x,y)=3375, LCM(y,z)=1125, LCM(z,x)=3375 is equal to

3. Let $A=\begin{bmatrix}1&0&2\\2&0&1\\1&1&2\end{bmatrix}$, then find the value of determinant $\left((A-I)^3-4A\right)$. [Note I is an identity matrix of order 3] .

Space For Rough Work

4. A biased coin shows head with probability of $\frac{3}{4}$ and tail with a probability of $\frac{1}{4}$. Let 'A' be the event that three or more heads occurs in four tosses and 'B' be the event that three heads do not occur in first three tosses. If $P\left(\frac{A}{B}\right)=\frac{m}{n}$ where m, n $\in$ N then find the least value of (m + n – 225) .

5. Let foci of the conic represented by the equation $ax^2+2(a+2)xy+ay^2+2fy=0$, where $a<-1$ and $f\neq0$ be $F_1(\alpha,-3)$ and $F_2(\beta,-5)$. If the feet of perpendicular from F_1 and F_2 upon x-axis be M & N and accentricity of the conic given by the equation $\sqrt{10}\,e^2-7e+\sqrt{10}=0$ then the square of the radius of director circle of the conic is

6. If [sinx]+[cosx]+2=0, then number of integral values in the range of f(x)=sinx-cosx+3, corresponding to the solution set of the given equation is (where [.] denotes the greatest integer function)

7. $f(n)\sum_{r=1}^{n}\left[r^2\left({}^nC_r-{}^nC_{r-1}\right)+(2r+1)\left({}^nC_r\right)\right]$, then f(30) is

8. If p is a positive integer and 'f' be a function defined for positive numbers and attains only positive values such that $f(xf(y))=x^py^4$, then p=

Space For Rough Work

Q.P. Code: 100955

Answers

SECTION-1 : PHYSICS

PART – A

1.	AB	2.	AD	3.	BCD	4.	BC
5.	AD	6.	AD	7.	AC	8.	A
9.	D	10.	B	11.	B	12.	D
13.	A						

PART – B

1.	0.73	2.	8.89	3.	56.56	4.	3.00
5.	1.00	6.	4.00	7.	2.00	8.	2.00

SECTION-2 : CHEMISTRY

PART – A

1.	BCD	2.	ACD	3.	ACD	4.	ABCD
5.	ABC	6.	ABCD	7.	ABCD	8.	A
9.	C	10.	C	11.	A	12.	C
13.	C						

PART – B

1.	4	2.	9.6	3.	0.3	4.	4.4
5.	0.25	6.	12.45	7.	1.5	8.	4.4

SECTION-3 : MATHEMATICS

PART – A

1.	ABCD	2.	BD	3.	ABC	4.	BD
5.	ABC	6.	AC	7.	ABC	8.	C
9.	A	10.	A	11.	B	12.	C
13	A						

PART – B

1.	6	2.	50	3.	8	4.	4
5	40	6.	1	7.	960	8.	2

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **AB**

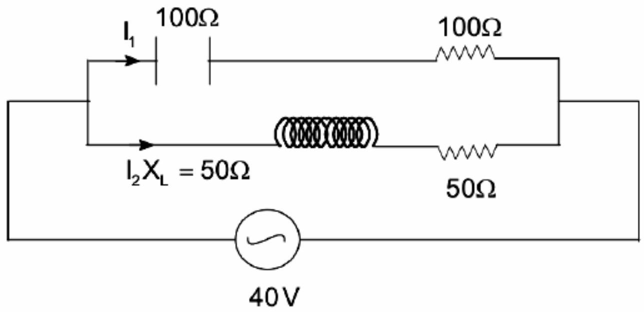
Sol. $I_1 = \frac{40}{z_1}$

$= \frac{2}{5\sqrt{2}}$ A at 45° leading

$I_2 = \frac{40}{z_2} = \frac{2\sqrt{2}}{5}$ A at 45° lagging

$I_{1_{RMS}} = \frac{1}{5}A$

$I_{2_{RMS}} = \frac{2}{5}A$



2. **AD**

Sol. For central fringe

d sin θ = 2 mm

4 mm sin θ = 2 mm

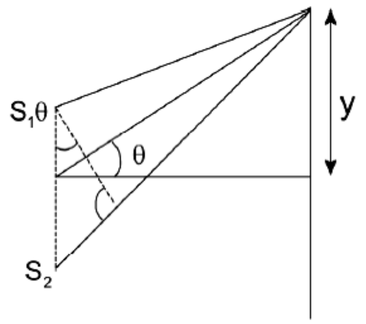
θ = 30°

tan 30° = $\frac{y}{1}$

$y = \frac{1}{\sqrt{3}}m$

For bright fringe (d sin θ ±2) = nλ

Whre n is 0, 1, 2, 3 ....



3. **BCD**

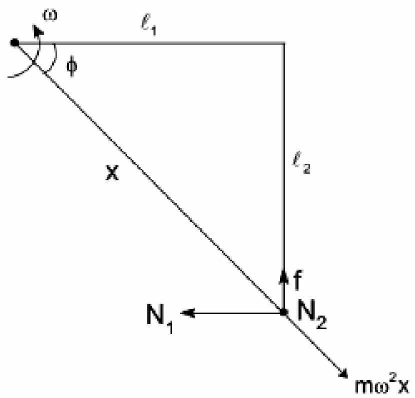
Sol. $N_1^2 + N_2^2 = N^2$

$N_2 = mg$

$m\omega^2 \times \sin\phi = f$

$f = m\omega^2\ell_2$

$N_1 = m\omega^2\ell_1$

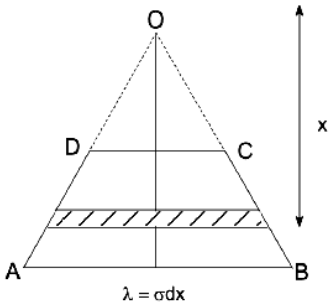


4. **BC**

Sol. $dE = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\sigma dx}{x}\left(\frac{1}{2}\right)$

$dE = \frac{\sigma}{4\pi\varepsilon_0}\frac{dx}{x}$

$E = \frac{7\sigma}{44\varepsilon_0}\ln\sqrt{2}$



5. **AD**

Sol. Elastic potential energy $\propto \left(\frac{\partial y}{\partial x}\right)^2$, kinetic energy $\propto \left(\frac{\partial y}{\partial t}\right)^2$. So for antinode elastic potential energy is constant & minimum. $\frac{\partial y}{\partial x}$ always changes for all other points.

6. **AD**

Sol. Let's assume, time taken by the queen to get into the hole is t

$t = \frac{\ell}{2V\sin\theta}$ & also $t = \frac{\ell}{2V\cos\theta} + \frac{\ell}{eV\cos\theta}$

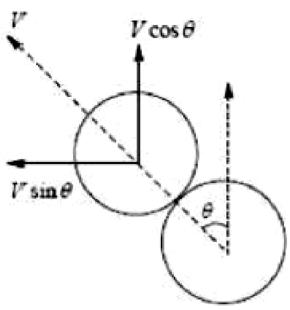
Therefore

$\frac{\ell}{2V\sin\theta} = \frac{\ell}{2V\cos\theta} + \frac{\ell}{eV\cos\theta}$

$\therefore \quad \tan\theta = \frac{1}{3}$

$\therefore \quad \frac{d}{\sqrt{(r_1+r_2)^2 - d^2}} = \frac{1}{3}$

$d^2 = 9 \Rightarrow d = 3$ cm.



7. **AC**

Sol. When parallel light falls from left side

$\frac{\mu_3}{f} = \frac{(\mu_3 - \mu_2)}{R} + \frac{(\mu_2 - \mu_1)}{R}$

$\frac{4/3}{f_1} = \frac{(4/3 - 2)}{R} + \frac{(2-1)}{R} \Rightarrow f_1 = 80$ cm

When parallel light falls from right side

$\frac{1}{f_2} = \frac{(1-2)}{-R} + \frac{(2-4/3)}{-R} \Rightarrow f_2 = 60$ cm

8. **A**

Sol. $\vec{a}_B = \vec{a}_A + \left(\vec{\alpha} \times \vec{r}_{B,A}\right) - \omega^2 \vec{r}_{B,A}$

$= -5j + 4\alpha i + 3\alpha j - (12i - 16j)$

a_B along y-axis should be zero.

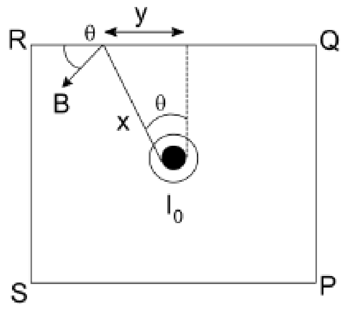
$\Rightarrow \quad 11 + 3\alpha = 0$

$\Rightarrow \quad \alpha = -\frac{11}{13}\text{rad/s}^2$

$a_B = (4\alpha - 12)i = \frac{-80}{3} i\ \text{m/s}^2$

9. **D**

Sol. $y = \frac{\ell}{2}\tan\theta$



$dy = \frac{\ell}{2}\sec^2\theta d\theta$

$$8\int_0^{\pi/4} B dy \cos\theta + \oint_{a\to b\to c\to d} \vec{B}\cdot\vec{d\ell} = \mu_0 I_0$$

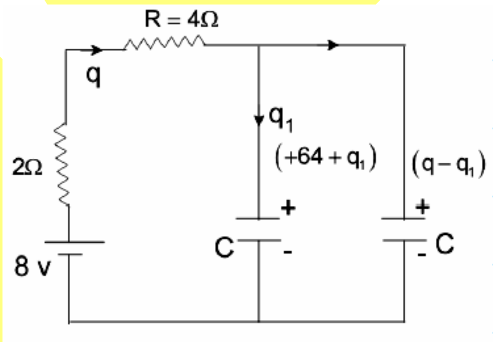
$$\int_0^{\pi/4} \frac{8\mu_0 I_0}{4\pi x}\cdot\frac{2\frac{\ell}{2}}{\sqrt{x^2+\frac{\ell^2}{4}}}\cdot\frac{\ell}{2}\sec^2 d\theta\cos\theta + \oint_{a\to b\to c\to d} \vec{B}\cdot\vec{d\ell} = \mu_0 I_0$$

$$\frac{2}{3}\mu_0 I_0 + \oint_{a\to b\to c\to d} \vec{B}\cdot\vec{d\ell} = \mu_0 I_0$$

$$\oint_{a\to b\to c\to d} \vec{B}\cdot\vec{d\ell} = \mu_0 I_0 - \frac{2}{3}\mu_0 I_0 = \frac{\mu_0 I_0}{3}$$

10. **B**

Sol. Let $R + 2 = R_1$



$$q(t) = \frac{3CE}{2}\left(1 - e^{\frac{t}{2R_1C}}\right)$$

$$I = \frac{dq}{dt} = \frac{3E}{4R_1}e^{-\frac{1}{2R_1C}}$$

Heat loss in $R_1 = \int I^2 R_1 dt = 576\ \mu J$

So, heat loss in $R = 4\Omega$ is $= 576 \times \frac{2}{3} = 384\ \mu J$.

11. **B**

Sol. Maximum power transfer will takes place when v_P is maximum.

12. **D**

Sol. $a_C = R\alpha$ (for no slipping)

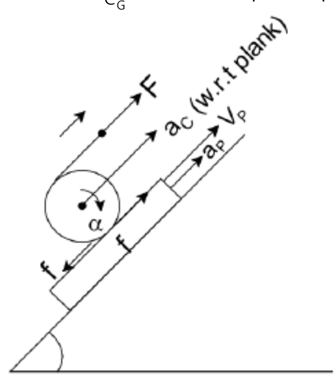
$a_{net} = a_{C_G} + R\alpha = 2a_p + 3a_p$

$V_c = R\omega$ (V_C is the speed of cylinder w.r.t plank)

$a_C + 3a_P = 0$

$V_C + 3V_P = 0$

$S_C + 3S_P = 0$



13. **A**

Sol. Let T_1 and T_2 be the initial and final temperature, then

$P_0V = nRT_1$ …(i)

$\left(P_0 + \frac{W}{A}\right)(V - Ah) = nRT_2$ …(ii)

Where h is the displacement of piston.

Also, work done on gas = ΔU

$$Wh = nC_v\Delta T = \frac{3}{2}nR(T_2 - T_1) \quad \ldots(iii)$$

From equation (i), (ii) and (iii)

$$\frac{2}{3}Wh = \frac{WV}{A} - Wh - P_0Ah$$

or, $$Ah = \frac{WV}{P_0A + \frac{5}{3}W} \quad \ldots(iv)$$

From equation (ii) and (iv)

$$T_2 = \frac{I}{nR}\left(P_0 + \frac{W}{A}\right)\left(V - \frac{WV}{P_0A + \frac{5}{3}W}\right) = \frac{1}{nR}[P_0 + P_0]\ V\left[1 - \frac{W}{W + \frac{5}{3}W}\right]$$

$$= \frac{2P_0V}{nR}\frac{5}{8} = \frac{5}{4}T_1 = 375$$

PART – B

1. **0.73**

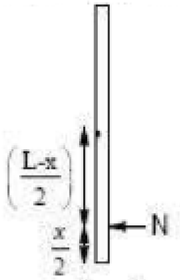
Sol. Impulse due to 'N' would act at the centre of the colliding parts.

Applying impulse momentum theorem

$$-\int N\,dt = M(v - v_0) \quad \ldots(1)$$

& Applying angular impulse, angular momentum theorem

$$\int N\left(\frac{L-x}{2}\right)dt = \frac{ML^2}{12}\omega \quad \ldots(2)$$



Also $$e = \frac{\omega\left(\frac{L-x}{2}\right) - v}{v_0} \quad \ldots(3)$$

From (1), (2) & (3)

$$v\left[1 + 3\left(\frac{L-x}{2}\right)^2\right] - v_0\left[3\frac{(L-x)^2}{L^2} - e\right]$$

$\therefore$ for $v > 0$

$$x < L\left[1 - \sqrt{\frac{e}{3}}\right]$$

For inequality to hold true for all 'e'

$$x < L\left[1 - \frac{1}{\sqrt{3}}\right]$$

or $x < (\sqrt{3} - 1)$, $\therefore\ x_{max} = 0.732$

2. **8.89**

Sol. Power absorbed by shell

$$\Rightarrow \frac{P \times 2\pi(1 - \cos 37)}{4\pi}$$

$$\Rightarrow \frac{P}{10}$$

For equilibrium

$$\Rightarrow \quad \frac{P}{10} = 3\pi R^2 \sigma T^4$$

$$\therefore \quad T^4 = \frac{P}{30\pi R^2 \sigma} = \frac{2400\ \sigma\pi}{30\pi \times 9 \times \sigma} \quad \Rightarrow \quad 8.89.$$

3. **56.56**

Sol. Applying conservation of angular momentum w.r.t. the centre of earth,

$4R \times mv_0 \sin 30 = mvR$, $v = 2v_0$

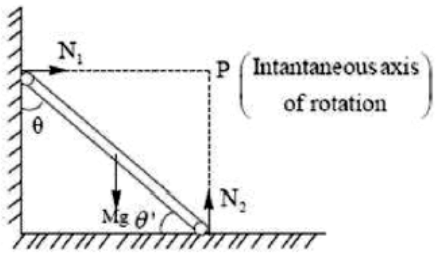
Applying energy conversation

$$\frac{1}{2}m(2v_0)^2 - \frac{GMm}{R} = -\frac{GMm}{4R} + \frac{1}{2}mv_0^2$$

$$\therefore \quad v_0 = 40\sqrt{2} \quad \Rightarrow \quad 56.56$$

4. **3.00**

Sol. x coordinate of centre of mass of the rod



$$x = \frac{\ell}{2}\sin\theta$$

$$v_x = \frac{\ell}{2}\cos\theta\,\omega \text{ and } a_x = \frac{\ell}{2}(\cos\theta\alpha - \sin\theta\omega^2)$$

When $N_1 = 0 \Rightarrow \cos\theta\,\alpha = \sin\theta\,\omega^2$ ...(1)

Using COME:

$$mg\frac{\ell}{2}(\cos\beta - \cos\theta) = \frac{1}{2}m\frac{\ell^2}{4}\omega^2 + \frac{1}{2}m\frac{\ell^2}{12}\omega^2$$

Where β is angle of θ from vertical wall i.e. 30°

$$\left[v = \frac{\ell}{2}\omega \text{ from IAOR}\right]$$

$$\omega^2 = \frac{3g}{\ell}(\cos\beta - \cos\theta) \qquad \ldots(2)$$

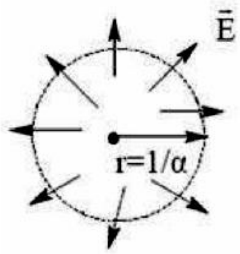
$$\alpha = \frac{3g}{2\ell}\sin\theta \qquad \ldots(3)$$

Substituting the value of ω^2 and α in equation (1) gives

$$\Rightarrow \quad \cos\theta = \frac{2}{3}\cos\beta \Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \sin\theta' = \frac{1}{\sqrt{3}} \text{ So } N = 3.$$

5. **1.00**

Sol. As the electric field is radial, by applying gauss law, we can write



$$\int \vec{E}\cdot\vec{ds} = \frac{Q}{\varepsilon_0}$$

For $r = \frac{1}{\alpha}$, $\vec{E} = C(1 - e^{-a\times 1/\alpha})\frac{\hat{r}}{(1/\alpha)^2}$

$$\therefore \quad \oint \vec{E}\cdot\vec{ds} = C(1 - e^{-1})\alpha^2 \times 4\pi(1/\alpha)^2$$

$$\Rightarrow \quad \frac{Q}{\varepsilon_0} = 4\pi C(1 - e^{-1}) \Rightarrow Q = (1 - e^{-1}) \quad \Rightarrow \quad \therefore N = 1$$

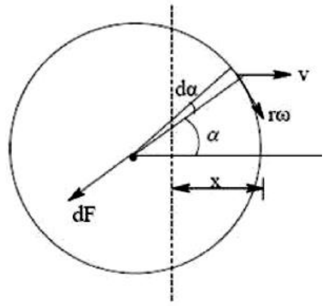
6. **4.00**

Sol. $$\int_0^{x_{min}} 0.5x\frac{mg}{\ell}dx = 2$$

$\Rightarrow \quad x_{min} = 40$ cm.

7. **2.00**

Sol. $mr^2\beta = \int_0^\theta \frac{2q}{2\pi} d\alpha\, \nu Br \cos\alpha$



x=r-rcosθ

dx=rsinθdθ

$\beta = \frac{dw}{dt} = \frac{q\nu B \sin\theta}{\pi\ mr}$ ...(1)

$-ma = \int_0^\theta 2\frac{q}{2\pi} d\,\alpha r\omega B \cos\alpha$

$a = \frac{dv}{dt} = \frac{qr\omega B \sin\theta}{\pi\, m}$ ...(2)

$= \frac{d\omega}{d\nu} = \frac{\nu}{r^2\omega} \Rightarrow r^2\int_0^\omega wd\omega = -\int_{\nu_0}^{\nu} \nu d\nu$

$\Rightarrow \quad \nu = \frac{\nu_0}{\sqrt{2}}$

$\frac{\nu d\nu}{dx} = \frac{qr\omega B\sin\theta}{\pi\, m}$; $\int_{\nu_0}^{\nu_0/\sqrt{2}} \frac{\nu d\nu}{\sqrt{\nu_0^2 - \nu^2}} = \frac{qrB}{m\pi}\int_0^\pi \sin^2\theta d\theta$

$\Rightarrow \quad q = \frac{\sqrt{2}\, mv_0}{Br}$

8. **2.00**

Sol. For slight displacement $(\pi r^2 x)\rho g = ma$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{\pi r^2 \rho g}}$

$\Rightarrow$ T = 2 sec.

SECTION-2 : CHEMISTRY

PART – A

1. BCD

Sol. $\psi_{4, 1, -1}$ is an orbital of 4p-subshell. It may be $4p_x$ or $4p_y$ or $4p_z$.

2. ACD

Sol. $[MnCl_4]^{2-}$ is a tetrahedral complex.

3. ACD

Sol. $K_{sp} = 4s^3 = 5 \times 10^{-10}$

$\therefore s = 5 \times 10^{-4}$

$\therefore$ Conc. of saturated solution = 5×10^{-4} M

pOH of saturated solution = $-\log[OH^-]$

$= -\log[2 \times 5 \times 10^{-4}] = -\log 10^{-3} = 3$

$\therefore$ pH = 14 – 3 = 11

$\therefore$ Solubility decreases in a solution of pH > 11 and increases in a solution of pH < 11.

4. ABCD

Sol. With excess of NBS, addition as well as allylic substitution reactions take place.

5. ABC

Sol.

$$CH_3-\overset{\overset{\large O}{||}}{C}-CH_2-COOH \xrightarrow{\Delta} CH_3COCH_3 \xrightarrow{LiAlH_4} CH_3-\overset{\overset{\large OH}{|}}{CH}-CH_3$$

$CH_3COOC_2H_5$ forms primary alcohols.

6. ABCD

Sol. It is an isobaric process(pressure remains constant)

$W_{irr} = -P(V_2 - V_1)$

$q = nC_P\Delta T$

$\Delta S = nC_P \,\ell n \dfrac{T_2}{T_1}$

$\Delta U = nC_v\Delta T$

7. ABCD

8. A

Sol. $BeCl_2$ and BCl_3 remove electron deficiency by forming polymers and dimers respectively.

9. C

Sol. P will show geometrical isomerism(cis and trans) and Q will show optical isomerism (R and S).

10. C

Sol. For group-1 iodides and fluorides, mismatch of ionic radius favours solubility.

11. A

Sol. Metals in lower oxidation state forms stable complexes with CO.

12. C

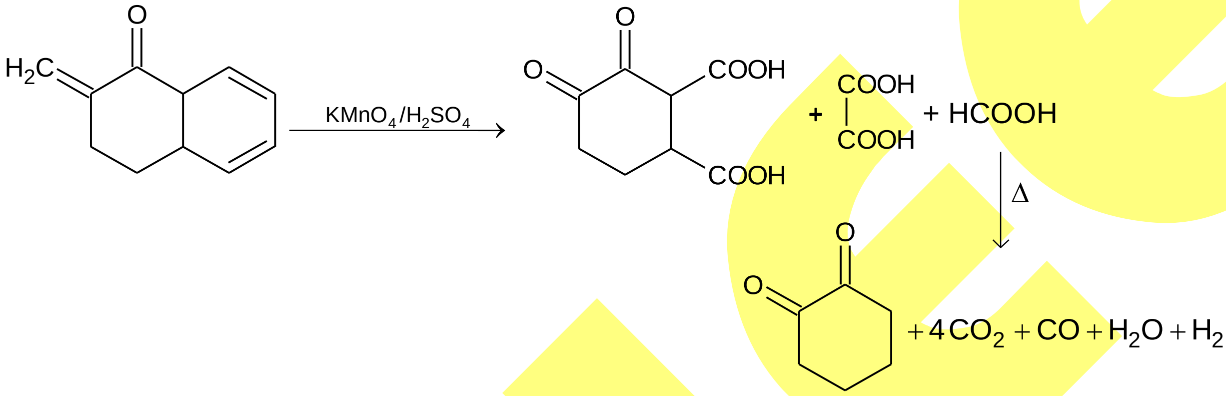
Sol. $T_C = \dfrac{8a}{27Rb}$

$\therefore$ Liquification α $\dfrac{a}{b}$ ratio.

13. C
Sol. Aldol condensation will take place.

PART – B

1. 4
Sol.



2. 9.6

Sol.	NH_4NO_3 +	$NaOH \rightleftharpoons$	NH_4OH +	$NaNO_3$
Before reac^n M_{eq}	$200 \times 0.5 = 100$	$200 \times 0.4 = 80$	0	0
After reac^n M_{eq}	$100 - 80 = 20$	$80 - 80 = 0$	80	80

$\therefore pOH = p^{K_b} + \log\dfrac{[NH_4NO_3]}{[NH_4OH]}$

$= 5 + \log\dfrac{20}{80} = 5 - 0.6 = 4.4$

pH = 14 – 4.4 = 9.6

3. 0.3
Sol. R = CH_4, x = 2, y = 1

4. 4.4
Sol. $Fe^{3+}\left(t_{2g}^5 e_g^0\right)$: CFSE = 5 × -0.4$\Delta_o$ = -2Δ_o

$Fe^{2+}\left(t_{2g}^6 e_g^0\right)$: CFSE = 6 × -0.4$\Delta_o$ = -2.4Δ_o

$\therefore$ x + y = 2 + 2.4 = 4.4

5. 0.25
Sol. $[A_t] = [A_o](½)^n$

or, $\dfrac{[A_t]}{[A_o]} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4} = 0.25$

6. 12.45
Sol. Slope = C_P = 20.75 JK^{-1} mol^{-1}.
$C_P - C_V = R$
$\therefore$ $C_V = C_P - R$ = 20.75 – 8.3 = 12.45

7. 1.5
Sol. The oxidation number of cobalt in the complex is +3.

8. 4.4
Sol. Cannizzaro Reaction takes place

SECTION-3 : MATHEMATICS
PART – A

1. ABCD

Sol. Consider a function f(x)= $x^6+6x+20$ and then use monotonicity to plot the graph and nature of the function

2. BD

Sol. $\therefore k \in \text{odd}\,(\text{even})$

f(k) = k + 3

$f(f(k)) = \frac{k+3}{2}$

if $\frac{k+3}{2} \in \text{odd} \Rightarrow 27 = \frac{k+3}{2} + 3 \Rightarrow k = 45$ not possible

Now let $\frac{k+3}{2} \in$ even

$\therefore 27 = f(f(f(k))) = f\left(\frac{k+3}{2}\right) = \frac{k+3}{4}$

$\therefore k = 105$

verifying $f(f(f(105))) = f(f(108)) = f(54) = 27$

3. ABC

Sol: eliminate a and b by using the formulae for eccentricity

4. BD

Sol f(x) is vertical upward parabola & is sym.about its axis x=1 , so points situated at greater distance from axis have greater outputs

5. ABC

Sol Make integrand perfect square

6. AC

Sol. PM=k

Equation of $AB \equiv -x + y = 2$

Equation of $AC \equiv x + y = 2$

According to question

$\left(\frac{2-h-k}{\sqrt{2}}\right)\left(\frac{2+h+k}{\sqrt{2}}\right) = 2k^2$

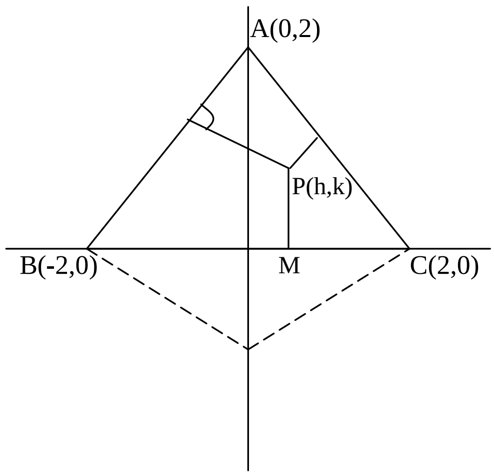
$\Rightarrow h^2 + 3k^2 + 4k = 4$

$\Rightarrow h^2 + 3\left(k^2 + \frac{4}{3}k + \frac{4}{9}\right) = 4 + \frac{4}{3}$

$\Rightarrow h^2 + 3\left(k + \frac{2}{3}\right)^2 = \frac{16}{3}$

$\Rightarrow \frac{h^2}{16/3} + \frac{\left(k+\frac{2}{3}\right)^2}{16/9} = 1$

$\Rightarrow$ellipse with e= $\sqrt{\frac{2}{3}}$ and $D \equiv \left(0, -\frac{2}{3}\right)$



7. ABC
Sol. Period of f(x) must be rational

8. C
Sol. Coefficient of x^{2012} in $(1+x)^2\left((1+x^2)..............(1+x^{2^{10}})\right)^3$

$\Rightarrow$ Coefficient of x^{2012} in $\dfrac{\left(1-x^{2^{11}}\right)^3}{(1+x)(1-x)^3}$

Coefficient of x^{2012} in $(1+x^2+x^4+......)(1+2x+3x^2+.....)$

$\Rightarrow 1.(2013)+1.(2011)+1.(2009)+..........+1$

$\Rightarrow 2k+1=2013 \Rightarrow k=1006$

We know that $1+3+5+.......+(2k+1)=(k+1)^2$

9. A
Sol. Minimise the distance btw a circle and rectangular hyp which is along the common normal

10. A
Sol. The mid – point of z_2 & z_3 divides the line joining the points z_1 & $i\sqrt{3}$ in the ratio 1:3 internally.

11. B
Sol. Since $f'(x)\ge f^3(x)+\dfrac{1}{f(x)}$

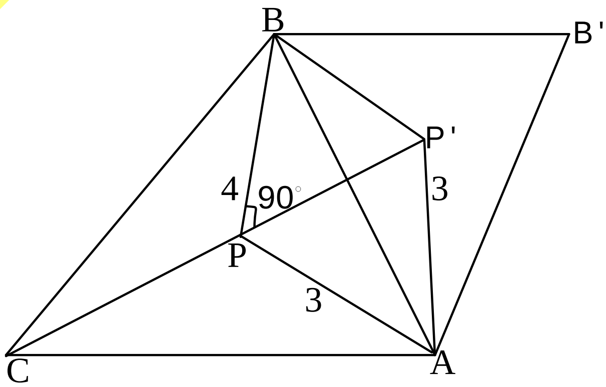
$f(x)f'(x)\ge f^4(x)+1 \Rightarrow \dfrac{f(x)f'(x)}{1+f^4(x)}\ge 1$

On integrating w.r.t.x from x=a to b

$\dfrac{1}{2}\left(\tan^{-1}f^2(x)\right)_a^b \ge b-a$

Or $b-a\le \dfrac{1}{2}\left[\lim\limits_{x\to b^-}\tan^{-1}\left(f^2(x)\right)-\lim\limits_{x\to a^+}\tan^{-1}\left(f^2(x)\right)\right]$ or $b-a\le\dfrac{\pi}{24}$

12. C
Sol. Rotate the triangle in clockwise direction through an angle 60°. Let the points A,B,C and P will be A,B', B and P' respectively after the rotation.
We have PA=P'A=3
And $\angle PAP'=60^\circ \Rightarrow PP'=3$
Also CP=BP'=5



So $\Delta BPP'$ is right angle triangle which $\angle BPP'=90^\circ$.
Now apply cosine rule in triangle BPA because
$\angle BPA=90^\circ+60^\circ=150^\circ$, PA=3 and BP=4, we can get AB

13 A
Sol. Use $\int_0^2 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx=\int_0^1 f(x)dx+\int_0^1 f(x+1)dx$

PART – B

1. 6

Sol. $f(2x)=f(x)=f(x/2)=f(x/2^2)=f\left(\frac{x}{2^n}\right)$

f is continuous, when $n\to\infty$, $f(2x)=f(0)$

$\Rightarrow$ $f(x)$ is constant

2. 50

Sol. $1125=5^3.3^2, 3375=5^3.3^3$

Clearly 3^3 is a factor of x and 3^2 is a factor of atleast one of y and z. This can be done in 5 ways. Also 5^3 is a factor of atleast two of the numbers x,y,z which can be done in $\left({}^3C_2\times 4-2\right)=10$

3. 8

Sol. We have $|A-\lambda I|=0$

$$\Rightarrow\begin{vmatrix}1-\lambda & 0 & 2\\ 2 & -\lambda & 1\\ 1 & 1 & 2\end{vmatrix}=0$$

$\Rightarrow \lambda^3-3\lambda^2-\lambda-3=0$

So, $A^3-3A^2-A=3I_3$

Now $\left|(A-I)^3-4A\right|$

$=\left|A^3-3A^2+3A-I-4A\right|$

$=\left|A^3-3A^2-A-I\right|=|2I_3|=8$

4. 4

Sol $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{{}^3C_1.\frac{1}{4}\left(\frac{3}{4}\right)^2.\frac{3}{4}}{1-\left[\left(\frac{3}{4}\right)^4.\frac{1}{4}+\left(\frac{3}{4}\right)^4\right]}$

$=\frac{81}{148}$

$(m+n)_{least}=81+148=229$

5 40

Sol. Given conic is an ellipse and x – axis being the tangent to it.

6. 1

Sol. $[\sin x]+[\cos x]+2=0$ is possible only when

$[\sin x]=-1$ and $[\cos x]=-1$

Or $-1\le \sin x<0$ and $-1\le\cos x<0\Rightarrow x\in\left(\pi,\frac{3\pi}{2}\right)$,

Now f(x)=sinx-cpsx+3= $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)+3$

For $x\in\left(\pi,\frac{3\pi}{2}\right),\sin\left(x-\frac{\pi}{4}\right)\in\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$$\Rightarrow y = f(x) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) + 3, \text{then } f(x) \in (2,4)$$

7. 960

Sol. $f(n)\sum_{r=1}^{n}\left[r^2\left({}^nC_r - {}^nC_{r-1}\right) + (2r)^n C_r + {}^nC_r\right] = \sum_{r=1}^{n}\left[\left(r^2+2r+1\right)^n C_r - r^2\ {}^nC_{r-1}\right]$

$$= \sum_{r=1}^{n}\left[(r+1)^2\ {}^nC_r - r^2\ {}^nC_{r-1}\right] = \sum_{r=1}^{n}\left[V_{r+1} - V_r\right]$$

$$= V_2 - V_1 + V_3 - V_2 + ... + V_{n+1} - V_n$$

$$V_{n+1} - V_1 = (n+1)^2\ {}^nC_{n+1} - 1 = n^2 + 2n$$

8. 2

Sol. $f(x(y)) = x^p y^4$, put $x = \frac{1}{f(y)}$

$$f(1) = \left(\frac{1}{f(y)}\right)^p y^4 = \frac{y^4}{(f(y))^p} \text{ for } y=1, f(1) = \frac{1}{(f(1))^p} \Rightarrow f(1) = 1$$

So $f(y) = y^{4/p}$ ....(1)

Hence $f\left(xy^{4/p}\right) = x^p y^4$

Put $y = z^{p/4}$

$f(xz) = x^p z^p \Rightarrow f(x) = x^p$ ...(2)

From (1) and (2) $\frac{4}{p} = p \Rightarrow p = 2$