

## PART-1 : PHYSICS

### SECTION-I (i)

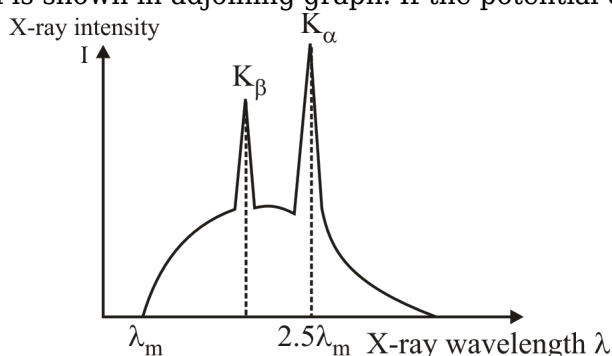
1) In a YDSE both slits produce equal intensities on the screen. A 100% transparent thin glass film is placed in front of one of the slits. Now the intensity of the geometrical centre of system on the screen becomes 75% of the previous intensity. The wavelength of the light in vacuum is  $6000\text{\AA}$  and  $\mu_{\text{glass}} = 1.5$ . The thickness of the film cannot be :-

- (A)  $0.2\text{ }\mu\text{m}$
- (B)  $1.0\text{ }\mu\text{m}$
- (C)  $1.4\text{ }\mu\text{m}$
- (D)  $1.6\text{ }\mu\text{m}$

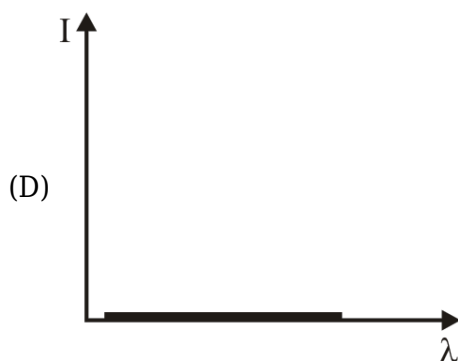
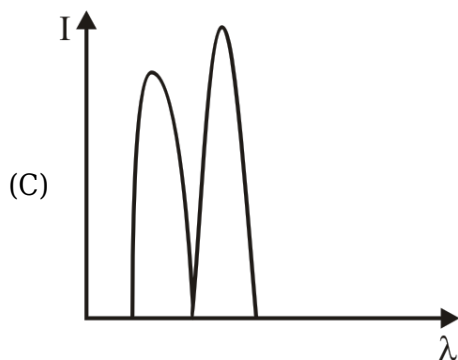
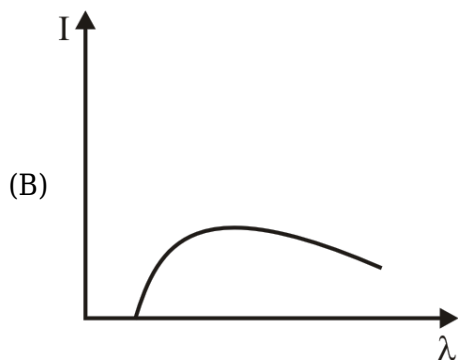
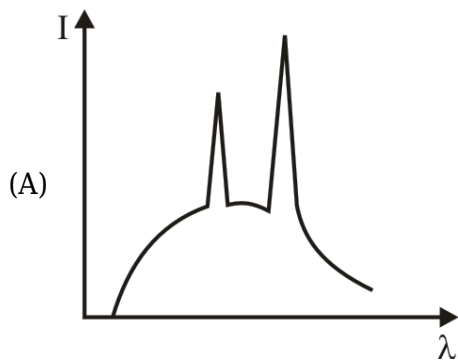
2) A coil having  $N$  turns is wound tightly in the form of a spiral with inner and outer radii  $a$  and  $b$  respectively. When a current  $I$  passes through the coil, the magnetic field at the centre is

- (A)  $\frac{\mu_0 NI}{b}$
- (B)  $\frac{2\mu_0 NI}{a}$
- (C)  $\frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$
- (D)  $\frac{\mu_0 I^N}{2(b-a)} \ln \frac{b}{a}$

3) When an electron accelerated by potential difference  $U$  is bombarded on a specific metal the emitted X-ray spectrum obtained is shown in adjoining graph. If the potential difference is reduced

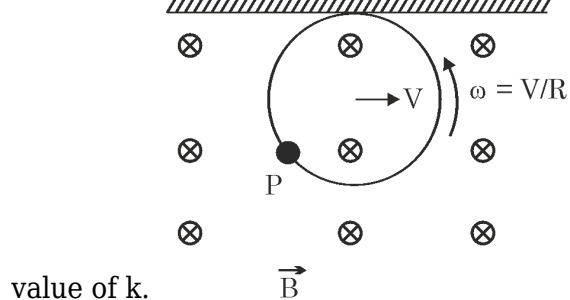


to  $U/3$ , the correct spectrum is :-



4) A rigid ring is made to roll along the ceiling of room, where exists a uniform horizontal magnetic field of induction  $B$  perpendicular to the plane of the ring. The velocity of the centre of the ring is constant and its modulus is  $V$ . A charged particle  $P$  of mass  $m$  is fixed on the ring. The charge  $q$  on

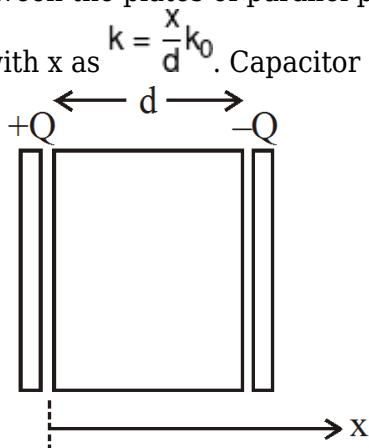
the particle is  $\frac{kmg}{4BV}$  so that there is no force of interaction between the ring and particle. Find out



- (A) 4
- (B) 6
- (C) 1
- (D) 2

# SECTION-I (ii)

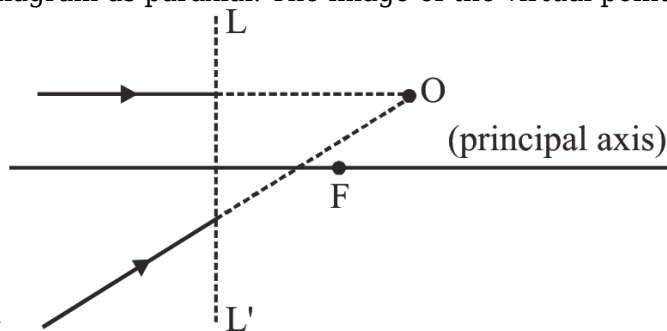
1) A dielectric slab is filled in between the plates of parallel plate charged capacitor. Di-electric constant of the capacitor varies with  $x$  as  $k = \frac{x}{d} k_0$ . Capacitor is not connected with the battery. Then



choose the correct statement(s).

- (A) Net electric field inside the di-electric is uniform throughout the capacitor.
- (B) Net electric field inside the dielectric decreases on moving along  $x$ -direction inside the capacitor.
- (C) Bound charge density inside the di-electric is zero.
- (D) Bound charge density inside the dielectric is non-uniform.

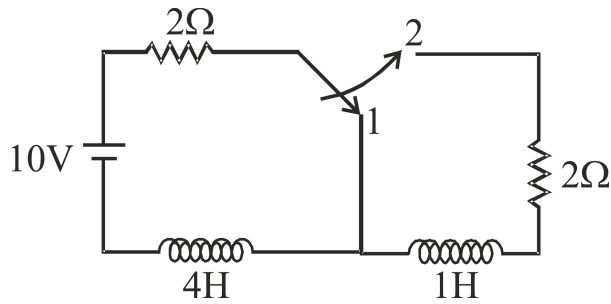
2) Consider the rays shown in the diagram as paraxial. The image of the virtual point object O



formed by the concave lens  $LL'$  is :-

- (A) Virtual
- (B) Real
- (C) Located below the principal axis
- (D) Located left of the lens

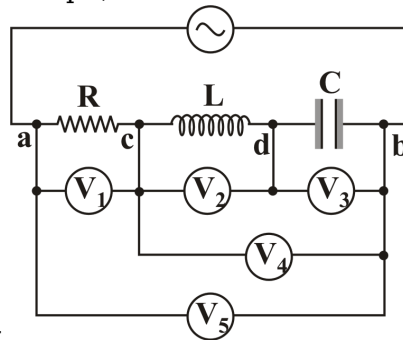
3) A network is given as shown. Switch was at position 1 for long time. at  $t = 0$  it is shifted to



position 2, then

- (A) Current just after switching is  $\sqrt{20}$  Amp  
 (B) Current just after switching is 4 Amp  
 (C) Current in the circuit varies  $i = \frac{5}{2} + \frac{3}{2}e^{-4t/5}$   
 (D) Current just after switching is 0

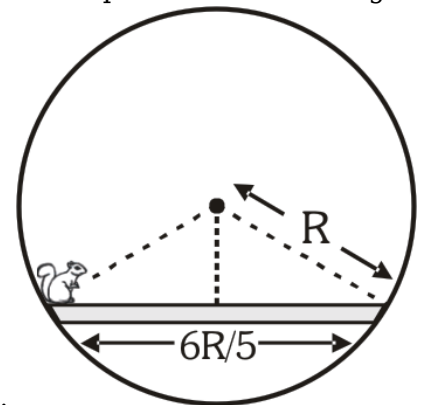
4) Five infinite-impedance voltmeters, calibrated to read rms values, are connected as shown in figure. Let  $R = 400 \Omega$ ,  $L = 3.5 \text{ H}$ ,  $C = 5 \mu\text{F}$ ,  $\omega = 200 \text{ rad/s}$  and AC voltage having rms value of 30 V.



The reading of voltmeters will be :-

- (A)  $V_2 = 42 \text{ V}$ ,  $V_3 = 64 \text{ V}$ ,  $V_4 = 22 \text{ V}$   
 (B)  $V_1 = 24 \text{ V}$ ,  $V_2 = 42 \text{ V}$ ,  $V_4 = 18 \text{ V}$   
 (C)  $V_1 = 24 \text{ V}$ ,  $V_4 = 6 \text{ V}$ ,  $V_5 = 30 \text{ V}$   
 (D)  $V_2 = 42 \text{ V}$ ,  $V_3 = 60 \text{ V}$ ,  $V_5 = 30 \text{ V}$

5) A small squirrel is put into a circular wheel cage of radius  $R$  which has a frictionless central pivot. A light horizontal platform of length  $6R/5$  is fixed to the wheel below the pivot as shown. Initially squirrel is at rest at one end of the platform. When the platform is released squirrel starts running

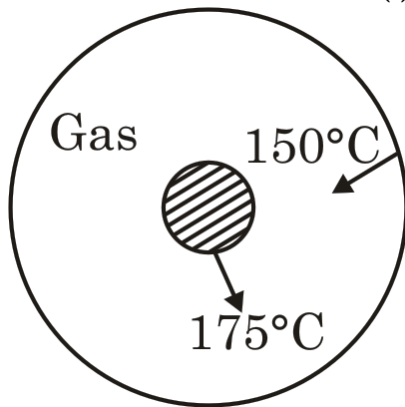


but platform and wheel remain stationary. Choose the correct options.

- (A) Maximum speed of squirrel is  $\sqrt{\frac{Rg}{5}}$

- (B) Maximum speed of squirrel is  $\sqrt{\frac{9Rg}{20}}$
- (C) Maximum acceleration of squirrel is  $\frac{4g}{3}$
- (D) Maximum acceleration of squirrel is  $\frac{3g}{4}$

6) A gas filled tube has 2 mm inside diameter and 25 cm length. The gas is heated by an electrical wire of diameter 50  $\mu\text{m}$  located along the axis of the tube. Current and voltage drop across the heating element are 0.5 A and 4 volts respectively. If the heating wire and inside of tube have temperatures 175°C and 150°C respectively, find the thermal conductivity (k) of the gas filling the tube and heat transfer rate (i). Assume mode of heat transfer as conduction only.

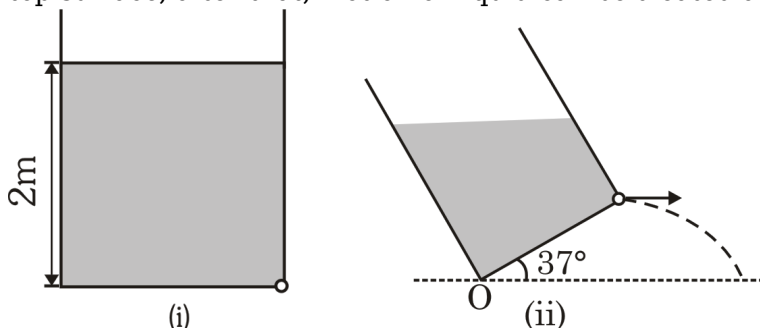


- (A)  $K = 0.188 \text{ W/m}^\circ\text{C}$
- (B)  $i = 2 \text{ W}$
- (C)  $k = 0.567 \text{ W/m}^\circ\text{C}$
- (D)  $i = 6 \text{ W}$

#### SECTION-II (i)

#### Common Content for Question No. 1 to 2

Velocity of efflux in Torricelli's theorem is given by  $v = \sqrt{2gh}$ , here h is the height of hole from the top surface, after that, motion of liquid can be treated as projectile motion. ( $g = 10 \text{ m/s}^2$ )



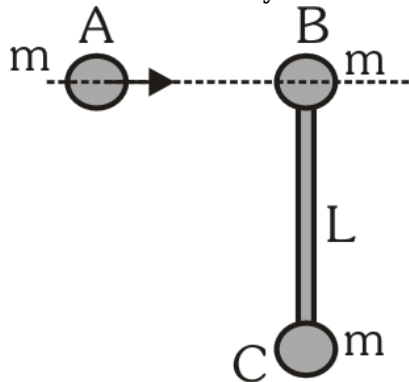
1) Liquid is filled in a vessel of square base (2m  $\times$  2m) upto a height of 2m as shown in figure (i). In figure (ii) the vessel is tilted from horizontal at 37°. If the velocity of efflux in this case (fig. ii) is v m/s. Find value of  $v^2$ . (Liquid does not spill out on tilting.)

2) At what distance (in m) from hole, will the liquid strike on the ground ? (Assume liquid comes out

horizontally.)

### Common Content for Question No. 3 to 4

A dumbbell is made of two small masses B and C each of mass  $m$ , fixed by a massless rod of length  $L$ , lies on a smooth horizontal table. A ball 'A' of mass  $m$  moving with speed  $u = 8 \text{ m/s}$ , strikes the ball B head on. The initial velocity of A is normal to the length of the rod. A and C collide after collision of A



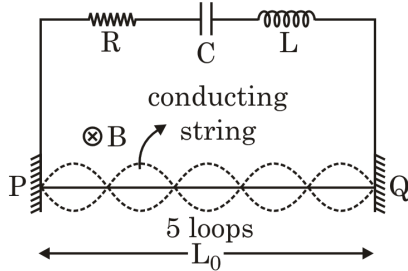
with B.

3) Velocity of A just after the collision with B :-

4) The co-efficient of restitution is - (For collision between A and B)

### Common Content for Question No. 5 to 6

A conducting string is rigidly clamped at the two insulated supports P and Q, separated by a distance ' $L_0$ '. A magnetic field of uniform intensity ' $B_0$ '  $\otimes$  exists in the region. The string is given a slight disturbance due to which standing waves are set in and it vibrates in 5<sup>th</sup> harmonic. The maximum amplitude of oscillations of the string particles is ' $A$ ' and the angular frequency ' $\omega$ '. It is given that at  $t = 0$ , the particles of the string are at the mean position.



In light of the information given above, answer the following questions.

5) If the maximum value of induced Electromotive force in the string is  $\frac{\alpha B_0 A \omega L_0}{\pi}$ , then the value of  $\alpha = \underline{\hspace{2cm}}$ .

6) Assuming  $(x_L - x_C) = \sqrt{3}R$ , if the magnitude of current in the circuit at  $t = \frac{\pi}{\omega}$  is  $\beta \frac{B_0 A \omega L_0}{\pi R}$ , then the value of  $\beta = \underline{\hspace{2cm}}$ .

### SECTION-II (ii)

1) The peak emission from a black body at a certain temperature occurs at a wavelength of  $6000 \text{ \AA}$ .

On increasing its temperature, the total radiation emitted is increased 16 times. These radiations are allowed to fall on a metal surface. Photoelectrons emitted by the peak radiation at higher temperature can be brought to rest by applying a potential equivalent to the excitation potential corresponding to the transition for the level  $n = 4$  to  $n = 2$  in the Bohr's hydrogen atom. The work function of the metal is given by  $\frac{\alpha}{100}$  eV where  $\alpha$  is the numerical constant. Find the value of  $\alpha$ .  
[Take :  $hc = 12420 \text{ eV}\cdot\text{\AA}$  ]

2) A conveyer belt is moving horizontally with speed 3 m/s. A small block enters the belt with velocity 4 m/s perpendicular to velocity of belt. If coefficient of friction between block and belt is 0.5, then the minimum width of belt required so that block remains on belt is (in meter)

3) A conducting sphere of radius ' $3r$ ' having charge ' $Q$ ' is at a great distance from a neutral conducting ball of radius ' $r$ '. They are connected with a thin perfectly conducting wire of inductance ' $L$ ' through a switch. If the maximum current in the wire after the switch is closed is  $kQ\sqrt{\frac{1}{k\pi\epsilon_0 rL}}$ , then find the value of  $k$ .

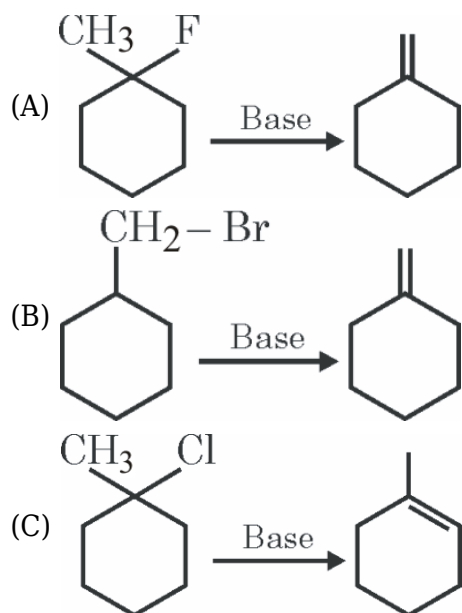
## PART-2 : CHEMISTRY

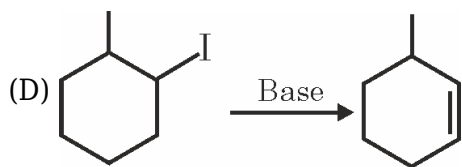
### SECTION-I (i)

1) Which of the following mixtures is obtained by the reaction of conc. HCl and  $\text{KClO}_3$ ?

- (A)  $\text{O}_2 + \text{Cl}_2\text{O}$
- (B)  $\text{O}_2 + \text{ClO}_2$
- (C)  $\text{Cl}_2 + \text{ClO}_2$
- (D)  $\text{Cl}_2\text{O} + \text{Cl}_2$

2) Incorrectly matched to their major product on treatment with sodium tert-Butoxide as base are





3) 2 moles of phenol reacts with phthalic anhydride to give compound 'X'. In basic medium, compound 'X' gets converted to compound 'Y'. Degree of unsaturation in compound 'Y' is

- (A) 12
- (B) 15
- (C) 13
- (D) 14

4) A 150 ml of solution of  $I_2$  is divided into two parts. Part 1<sup>st</sup> reacts with hypo solution in acidic medium. 15 ml of 0.4 M hypo was consumed. Part 2<sup>nd</sup> was added with 100 ml of 0.3 M NaOH. Residual base required 10 ml of 0.3 M  $H_2SO_4$  for complete neutralization. Then the incorrect statement is

- (A) No. of moles of  $I_2$  reacted in part 1<sup>st</sup> is  $3 \times 10^{-3}$
- (B) Initial conc. of  $I_2$  solution is 0.1 M
- (C) No. of moles of  $I_2$  reacted in part 2<sup>nd</sup> is  $15 \times 10^{-3}$
- (D) No. of moles of  $I_2$  reacted in part 2<sup>nd</sup> is  $12 \times 10^{-3}$

#### SECTION-I (ii)



80%

1) 'Anti infective agent'

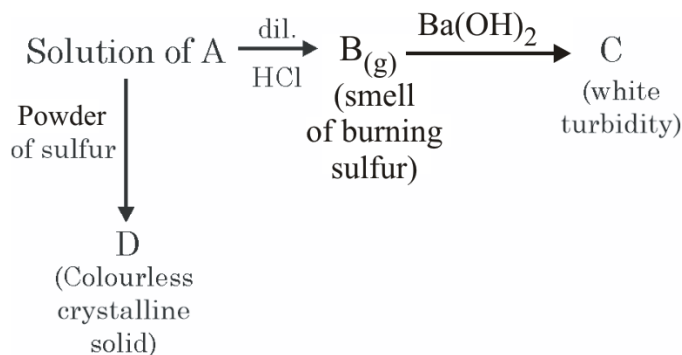
Correct statement for 'X' is

- (A) Maximum carbon atom present in a plane is 6
- (B) N-N bond present in X is 0
- (C) Methylene ( $-\text{CH}_2-$ ) group is present in 'X' is 6.
- (D) All atoms in 'X' are  $sp^3$  hybridised and total 4 lone pair is present.

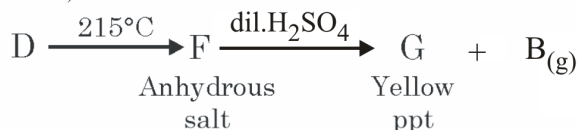
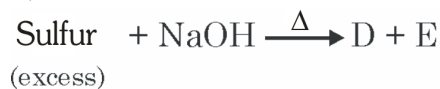
2) White phosphorus on reaction with thionyl chloride  $\text{PCl}_3$ ,  $\text{SO}_2$  and a compound 'X' an analogous (same as 'X'). Compound of Se undergoes disproportionation to form Y and Z, if central atom of Y has higher oxidation state than central atom of Z, then which of the following statement is **CORRECT**?

- (A) 'X' has restricted rotation around single bond.
- (B) Y has 4 '90°' angle in its structure.
- (C) The oxidation state of Z is zero.
- (D) The compound Y is  $\text{Se}_2\text{Cl}_2$





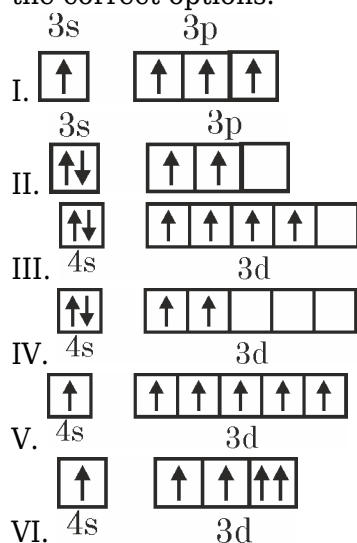
3)



D is used to estimate  $\text{I}_2$  quantitatively and during the reaction forms  $\text{NaI}$  and a compound H. Which of following is/are correct?

- (A) F on reaction with  $\text{Br}_2$  can produces  $\text{Na}_2\text{SO}_4$ .
- (B) F is used in photography as toner
- (C) E is  $\text{Na}_2\text{S}_3$ .
- (D) The number of water of crystallisation in D is 5.

4) In the following six electronic configuration (remaining inner orbitals are completely filled) mark the correct options.



- (A) Configuration VI violates all the three rule (Aufbau, Hund's, Pauli) rules of electronic configuration.
- (B) Spin multiplicity for V is maximum.
- (C) If configuration IV represent element X, then  $\text{X}^{2+}$  ion will be diamagnetic in nature.
- (D) If configuration I represent excited state of an element Y, then it can show four covalency.

5) In a 100 ml 0.1 M  $\text{CH}_3\text{COOH}$  solution, 0.1 M  $\text{Ca(OH)}_2$  is slowly added. Then which one of the following statement(s) is/are correct ( $K_a$  of  $\text{CH}_3\text{COOH} = 10^{-5}$ ,  $\log 3 = 0.5$ ,  $\log 2 = 0.3$ )

- (A) pH of the solution will be 5 after adding 25 ml of the base.

- (B) pH of the solution will be nearly 8.9 after adding 50 ml of the base.  
 (C) Before adding the base, initial pH of the solution was nearly 3.  
 (D) After adding 100 ml of the base pH of the solution becomes 13.

6) Which are incorrectly matched?

- (A) **Penicillin** : 3 chiral centre compound giving positive  $\text{NaHCO}_3$  test and Lassign test give purple colour with sodium nitropruside test.  
 (B) **Azodye** : Used as antibiotic and formed when benzene diazonium ion reacts with phenol at  $\text{pH} = 8.5 - 9.5$ .  
 (C) **Codeine** : Having phenolic OH and ether group and soluble in acid, with  $\text{DOU} = 11$   
 (D) **Aspirin** : It is used as nonnarcotic analgesic. It prevents blood clotting and it is formed when salicylic acid reacts with methanol.

## SECTION-II (i)

### Common Content for Question No. 1 to 2

Equilibrium of complex formation is of much interest for chemists. Complex formation finds potential applications in qualitative analysis, medicinal chemistry, metallurgy, electroplating etc. Theoretically the formation constant of the complex can be used to infer the stability of the complex. The overall formation constant for the reaction of  $\text{CN}^-$  with  $\text{Co}^{2+}$  at 298 K is  $10^{19}$  and the rate constant for the formation of  $[\text{Co}(\text{CN})_6]^{3-}$  is  $10^3 \text{ M}^{-5} \text{ s}^{-1}$ . The standard reduction potential for the reaction  $[\text{Co}(\text{CN})_6]^{3-} + \text{e}^- \rightarrow [\text{Co}(\text{CN})_6]^{4-}$  is  $-0.835 \text{ V}$ . Standard reduction potential of  $\text{Co}^{3+}|\text{Co}^{2+}$  couple is  $1.82 \text{ V}$ .

$$\left[ \frac{2.303RT}{F} = 0.059 \right]$$

- 1) Rate constant for the formation of  $[\text{Co}(\text{CN})_6]^{4-}$  is  $10^x$  times greater than the rate constant for the dissociation of the same complex. Then  $x =$   
 2) The magnitude of standard Gibbs energy change for formation of complex  $[\text{Co}(\text{CN})_6]^{4-}$  (in kJ/mol) is :

### Common Content for Question No. 3 to 4

An optically active organic compound **A** has molecular formula ( $\text{C}_7\text{H}_{12}\text{O}_3$ ) and found to produce an yellow precipitate with alkaline solution of Iodine. Also **A** neither decolourised brown colour of bromine water nor evolved any gas with aqueous solution of  $\text{NaHCO}_3$  but evolved a gas on heating with sodium metal. On hydrolysing in acidic medium, **A** produced another optically active compound **B** ( $\text{C}_7\text{H}_{14}\text{O}_4$ ) which also produced yellow ppt with  $\text{NaOH}/\text{I}_2$ . Also **B** evolved a colourless, acidic gas on treatment with aqueous  $\text{NaHCO}_3$  solution. **A** on treating with  $\text{CrO}_3/\text{HCl}/\text{Pyridine}$  in  $\text{CH}_2\text{Cl}_2$  produced another optically active compound **C** ( $\text{C}_7\text{H}_{10}\text{O}_4$ ). **C** on refluxing with aqueous  $\text{Ag}_2\text{O}$  produced an optically inactive compound **D** ( $\text{C}_7\text{H}_{10}\text{O}_5$ ). **D** does not decarboxylate (does not evolve  $\text{CO}_2$ ) on simple heating. However, heating **D** with sodalime gave **E** ( $\text{C}_5\text{H}_{10}\text{O}$ ). In a separate analysis, ethyl 3-oxobutanoate was heated with excess of sodium metal and then with  $\text{CH}_3\text{I}$  to produce **X** ( $\text{C}_8\text{H}_{14}\text{O}_3$ ). **X** on hydrolysis followed by simple heating of product yielded **E**.

3) Identify number of correct statements:

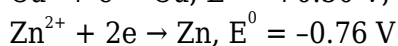
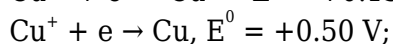
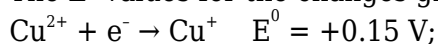
- (1) '**A**' gives red colour with C.A.N.

- (2) Number of different functional groups in 'D' is 3  
 (3) 'E' gives yellow precipitate with NaOI  
 (4) 'C' on heating gives 2 moles CO<sub>2</sub>  
 (5) 'B' on heating gives cyclic ester  
 (6) 'X' on reaction with excess D<sub>2</sub>O/OD<sup>-</sup> gives product containing 5 deuterium in all the products obtained.

4) DOU of 'C' = P, DOU of 'A' = Q, find out  $\frac{P}{Q}$  ?

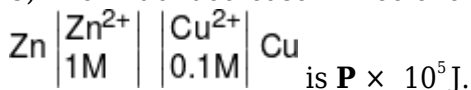
### Common Content for Question No. 5 to 6

The E<sup>0</sup> values for the changes given below are measured against SHE at 25°C .



$$2.303 \text{ RT/F} = 0.059$$

5) The initial decrease in free energy during the cell reaction in the Galvanic cell :



The value of |P| is :

6) The cell potential (in volts) for the cell (in volts) Zn | Zn<sup>2+</sup> (1M) || Cu<sup>2+</sup> (0.1 M) | Cu is :

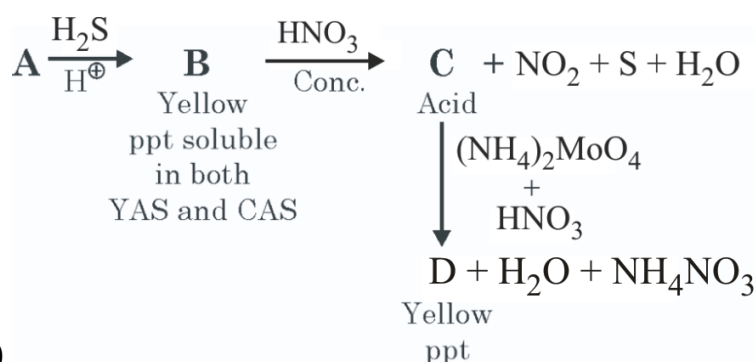
### SECTION-II (ii)

1) In a FCC unit cell of 'A' atoms (At wt = 150) having side length 10Å, the number of atom per unit cell is Z, number of next nearest neighbour is X & packing efficiency is y %. Find the value of  $\frac{yz}{(x^2 + 1)}$ . In the nearest integer.

2) The rate law for the reaction of OH<sup>-</sup> with tert-butyl bromide to form an elimination product in 75% ethanol and 25% water at 30°C in the sum of rate laws for the E<sub>2</sub> and E<sub>1</sub> reactions.

$$r = (7.2 \times 10^{-5} \text{ M}^{-1}\text{s}^{-1}) [\text{tert-butyl bromide}] [\text{OH}^-] + (4 \times 10^{-5} \text{ s}^{-1}) [\text{tert-butyl bromide}].$$

What % of reaction takes place by E<sub>2</sub> pathway, when concentration of OH<sup>-</sup> is 5 M?



3)

(I) D contains 'x' number of  $\text{MoO}_4$  units.

(II) Atomicity in C is 'y'.

Calculate  $|y - x|$

## PART-3 : MATHEMATICS

### SECTION-I (i)

1) If  $x, y, z > 0$  and  $x + y + z = 1$ , then least value of the expression  $\left(\frac{4x+2}{2-x} + \frac{4y+2}{2-y} + \frac{4z+2}{2-z}\right)$  is

- (A) 4
- (B) 5
- (C) 6
- (D) 7

2) If  $\sin x + \sin y \geq \cos \alpha \cdot \cos x$ ,  $\forall x \in \mathbb{R}$ , then  $\sin y + \cos \alpha$  is equal to

- (A) -2
- (B)  $-\frac{3}{2}$
- (C) 0
- (D) 1

3) If  $xf(x) = 3(f(x))^2 + 2$ , then  $\int \frac{2x^2 - 12x \cdot f(x) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} dx$   
(Where 'c' is the constant of integration)

- (A)  $\frac{1}{x^2 - f(x)} + c$
- (B)  $\frac{-1}{x^2 - f(x)} + c$
- (C)  $\frac{1}{(6f(x) - x)(x^2 - f(x))} + c$
- (D)  $\frac{-1}{(6f(x) - x)(x^2 - f(x))} + c$

4) For positive t; let  $\alpha$  and  $\beta$  be the roots of  $x^2 + t^2x - 2t = 0$ . If minimum value of

$\int_{-1}^2 \left( \left(x + \frac{1}{\alpha^2}\right) \left(x + \frac{1}{\beta^2}\right) + \frac{1}{\alpha\beta} \right) dx$  is  $\sqrt{p} + q$ , where  $p \in \text{Rational number}$  and  $q \in \text{Natural number}$ , then value of  $[p] + q$  is (where  $[ \cdot ]$  denotes greatest integer function)

- (A) 1
- (B) 2

- (C) 3  
(D) 4

SECTION-I (ii)

1) Let  $P \equiv \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) : Q \equiv \left( -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$  be the vertices of regular polygon having 12 sides such that PQ is a diameter of the circle circumscribing the polygon. Which of the following points can be the vertex of this polygon

- (A)  $\left( \frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}} \right)$   
 (B)  $\left( \frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{2\sqrt{2}} \right)$   
 (C)  $\left( \frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{1-\sqrt{3}}{2\sqrt{2}} \right)$   
 (D)  $\left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$

2) Let  $P(x) = x^3 + ax^2 + bx + c$  be a polynomial, where  $a, b, c \in I$  and  $c$  is an odd integer. Let  $P_i$  be the value of  $P(x)$  at  $x = i$ , given that  $P_1^3 + P_2^3 + P_3^3 = 3P_1P_2P_3$ . then  
(Where  $I$  is the set of integers)

- (A)  $P(x)$  is increases in  $(3, \infty)$   
 (B)  $P(x)$  is decreases in  $(-1, 2)$   
 (C)  $P(x)$  can't have local maxima or local minima.  
 (D)  $P(x)$  has local maxima and local minima

3) Let  $f : \mathbb{R} - \left\{ -\frac{1}{2} \right\} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \frac{4x^2 + 12x + 3}{4x^2 + 4x + 1}.$$

Then which of the following options is (are) **TRUE** ?

(A)  $f(x)$  is onto function.

(B) If  $A = \left\{ a : \frac{4x^2 + 12x + 3}{4x^2 + 4x + 1} = a \text{ has two solution} \right\}$  then the set  $A \cap \mathbb{N}$  has at least 3 elements. ( $\mathbb{N}$  is set of natural numbers)

(C) If  $B = \left\{ b : \frac{4x^2 + 12x + 3}{4x^2 + 4x + 1} = b \text{ has one solution} \right\}$  then  $|B| \geq 2$ .

(D)  $f(x)$  is decreasing in the interval  $(-\infty, -1]$

4) If  $a_1 = 1$  and for  $n > 1$ ,  $a_n = a_{n-1} + \frac{1}{a_{n-1}}$  then  $[\sin^{-1}(a_{75} - 12)]$  can't be equal to where  $[ ]$  is greatest integer function

- (A) -2
- (B) -1
- (C) 2
- (D) -3

5) The value of  $\sum_{0 \leq p \leq q \leq r < s = t \leq n} 1$

- (A)  ${}^{n+1}C_4 + 2 {}^{n+1}C_3 + {}^{n+1}C_2$
- (B)  ${}^{n+3}C_4$
- (C)  ${}^{n+1}C_5 + 2 {}^{n+1}C_4 + {}^{n+1}C_3$
- (D)  ${}^{n+3}C_5$

6) Given that  $\cot x = \frac{a_0}{x} + a_1 x + a_2 x^3 + \dots + a_k x^{2k-1} + \dots$   $\infty$  terms  $\forall 0 < |x| < \pi$ , then which of the following are correct?

- (A)  $a_0 = 1$
- (B)  $a_0 = 3|a_1|$
- (C)  $a_0 + a_1 = \frac{4}{3}$
- (D)  $a_0 + 9a_1^2 = 2$

## SECTION-II (i)

### Common Content for Question No. 1 to 2

If  $f(x)$  is twice differentiable function wherever it is continuous and  $f'(c_1) = f'(c_2) = 0$ .  $f''(c_1) \cdot f''(c_2) < 0$ ,  $f(c_1) = 5$ ,  $f(c_2) = 0$  and  $(c_1 < c_2)$ .

- 1) If  $f(x)$  is continuous in  $[c_1, c_2]$  and  $f''(c_1) - f''(c_2) > 0$ , then minimum number of solution of  $f'(x) = 0$  in  $[c_1 - 1, c_2 + 1]$  is
- 2) If  $f(x)$  is continuous in  $[c_1, c_2]$  and  $f''(c_1) - f''(c_2) < 0$ , then minimum number of solution of  $f'(x) = 0$  in  $[c_1 - 1, c_2 + 1]$  is

### Common Content for Question No. 3 to 4

Let a function is defined by  $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{0\}$ . A line  $\frac{x}{-2} = -\frac{f(t)y}{t} = t^2 z$  is perpendicular to the line of intersection of the planes

$$P_1 : t f(t)x + f\left(\frac{1}{t^2}\right)z + f(-t) = 0$$

$$P_2 : t y + f(-t)z + f(t^2) = 0$$

3) Let  $t = \tan \theta$  where  $\theta \in \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2}, n\pi, n\pi \pm \frac{\pi}{4} \right\}$ , where  $n$  is integer such that  $f(\tan \theta) + \tan(\lambda \theta) f(\cot^2 \theta) = 0$ . Then  $\frac{1}{2\lambda}$  is equal to  
(Where  $\lambda \in \mathbb{N}$ )

4) Let  $f(t)$  is an identity function in the domain. If a plane is obtained by rotating  $P_2$  about its line of intersection with  $P_1$  by  $90^\circ$  and this new plane passes through origin then sum of all values of  $t$  is equal to

### Common Content for Question No. 5 to 6

If the area bounded by the

$$a \left( \pi^3 + 1 \right)^{f(x)} = \left( \cos^{-1}(\cos x) \right)^2 \quad g(x) = \cos^{-1}(\cos(x)) \quad x = \pm 2\pi$$

function and is equal to  $\frac{c}{b\pi^2}$ , where  $a, b, c \in \mathbb{N}$ , where  $a$  &  $c$  are co-prime.

5) The value of  $\frac{1}{12} \left( a + \frac{b}{c} \right)$  is

6) Three dice are thrown simultaneously and  $p$  is the probability that sum of numbers appearing on each of the three dice is divisible by  $a + b + c$ . Then  $36p$  is equal to

### SECTION-II (ii)

1) The number of the roots  $\omega$  of the equation  $\omega^{2025} = 1$  such that  $|\omega + 1| \geq \sqrt{2 + \sqrt{2}}$  are

2) Let  $\alpha$  and  $\beta$  ( $\alpha < \beta$ ) are the solution of the equation,  $x^9 - 81x - 62 = 18x^3(3x + 2)^{1/3}$ . If

$A = \begin{bmatrix} -\alpha & \beta + 1 & 1 \\ 0 & \beta & 2 \\ 0 & 0 & |\alpha| \end{bmatrix}$ ;  $B = \begin{bmatrix} -\alpha & 1 & \beta \\ 1 & \beta + 1 & 4 \\ 1 & -\alpha & 3 \end{bmatrix}$ , then the value of  $|\text{adj}(\text{adj}(-\alpha A))| + |\text{adj}(B)^8|$  is equal to.

3) The number of permutations  $x_1, x_2, x_3, x_4, x_5$  of numbers 1, 2, 3, 4, 5 taken all at a time such that  $(x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5 + x_4x_5x_1 + x_5x_1x_2)$  is divisible by 3 is

## ANSWER KEYS

### PART-1 : PHYSICS

#### SECTION-I (i)

Q.	1	2	3	4
A.	D	C	B	A

#### SECTION-I (ii)

Q.	5	6	7	8	9	10
A.	B,D	A,C,D	B,C	B,D	B,D	A,B

#### SECTION-II (i)

Q.	11	12	13	14	15	16
A.	20.00	2.48 to 2.52	2.66 to 2.67	0.33	0.40	0.20

#### SECTION-II (ii)

Q.	17	18	19
A.	159	2	48

### PART-2 : CHEMISTRY

#### SECTION-I (i)

Q.	20	21	22	23
A.	C	C	D	C

#### SECTION-I (ii)

Q.	24	25	26	27	28	29
A.	B,C,D	A,C	A,D	A,B,D	A,B,C	A,C,D

#### SECTION-II (i)

Q.	30	31	32	33	34	35
A.	19.00	108.00 to 109.00	3.00	1.50	2.03 or 2.04	1.05 or 1.06

#### SECTION-II (ii)

Q.	36	37	38
A.	8	90	4

### PART-3 : MATHEMATICS



## SECTION-I (i)

Q.	39	40	41	42
A.	C	D	A	D

## SECTION-I (ii)

Q.	43	44	45	46	47	48
A.	A,B,C	A,D	C,D	A,B,C,D	A,B	A,B,D

## SECTION-II (i)

Q.	49	50	51	52	53	54
A.	4.00	2.00	0.25	0.00	0.50	3.50

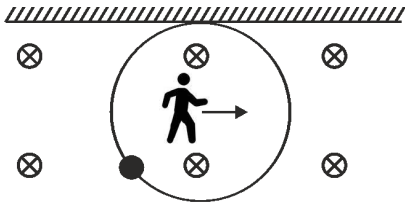
## SECTION-II (ii)

Q.	55	56	57
A.	507	32	80

## SOLUTIONS

### PART-1 : PHYSICS

- 3) Since potential difference is reduced to  $\frac{U}{3} \Rightarrow$  no characteristic X-ray [ $K_\alpha$  or  $K_\beta$ ] is observed



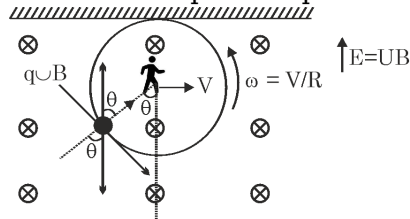
- 4)  $\otimes \quad \otimes \quad \otimes$

Take a frame moving rightward with velocity  $\vec{V}$ .

$$-a\vec{V} \times \vec{B} + a\vec{E} = 0$$

$$\vec{E} = \vec{V} \times \vec{B}$$

In this frame particle perform uniform circular motion.



$$mg \sin \theta = qvB \sin \theta \text{ (Tangential force must be zero)}$$

$$mg = qvB$$

$$mg = \frac{kmg}{4BV} BV \Rightarrow k = 4$$

- 7) Flux should remain constant just before and after the switch closed.

$$\text{So } L_{\text{net}} i_{\text{final}} = L i_{\text{initial}}$$

$$5 \times i_f = 4 \times 5$$

$$i_f = 4 \text{ Amp}$$

$$\text{Now using charging L-R circuit equation } i \text{ varies as } = \frac{5}{2} + \frac{3}{2} e^{-4t/5}$$

$$8) I = \frac{V}{Z}, Z = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2}$$

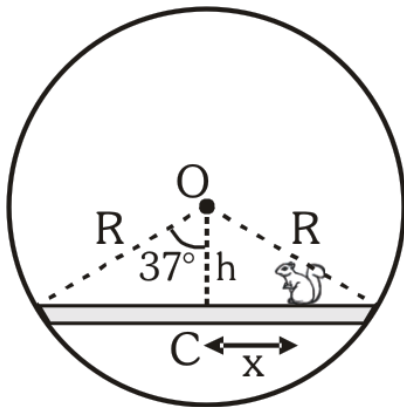
$$V_1 = IR$$

$$V_2 = I(\omega L)$$

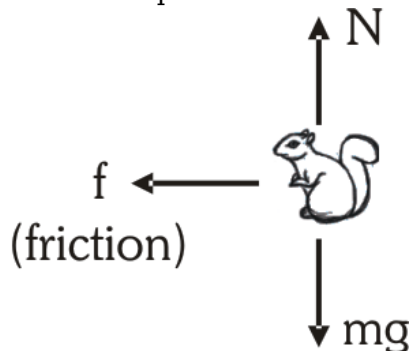
$$V_3 = I \left( \frac{1}{\omega C} \right)$$

$$V_4 = V_2 \sim V_3$$

- 9) Consider squirrel at distance  $x$  from midpoint  $C$  of platform.



F.B.D. of squirrel :



$$f = ma \quad \dots(1)$$

$$N = mg \quad \dots(2)$$

F.B.D. of platform + Wheel cage

Platform + wheel cage remain at equilibrium only if

$\tau_{\text{pivot}} = 0$ , which is possible according to following F.B.D. only.

$$\Rightarrow fh = Nx$$

$$\Rightarrow mah = mgx \quad (\text{from (1) \& (2)})$$

$$\boxed{a = \frac{gx}{h}}$$

towards C (restoring acceleration)

□ Squirrel performs SHM about mean position C, angular frequency  $\omega = \sqrt{\frac{g}{h}}$  and amplitude  $A = \frac{3R}{5}$  (half the platform's length).

In SHM, velocity at extreme should be zero. Squirrel is at rest at one end of the platform at  $t = 0$ .

$$\text{Maximum velocity} = A\omega = \frac{3R}{5} \sqrt{\frac{g}{h}}$$

$$V_{\text{max}} = \frac{3R}{5} \sqrt{\frac{g}{4R/5}} = \sqrt{\frac{9Rg}{25} \cdot \frac{5}{4}} = \sqrt{\frac{9Rg}{20}}$$

$$\text{Maximum acceleration} = \omega^2 A$$

$$a_{\text{max}} = \frac{gA}{h} = \frac{g \cdot 3R/5}{4R/5} = \frac{3g}{4}$$

$$10) \quad i = \frac{2\pi kL(\Delta T)}{\ln\left(\frac{R_2}{R_1}\right)} = VI$$

11) The volume of liquid should remain unchanged

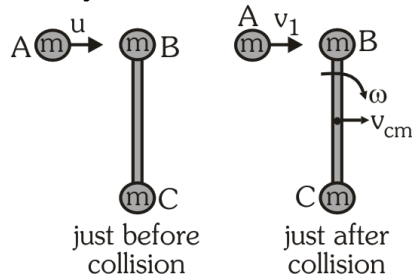
$$\text{Hence, } 2 \times 2 \times 2 = \frac{1}{2} [x + x + 1.5] \times 2 \times 2$$

$$\Rightarrow x \approx 1.25 \text{ m}$$

$$\text{Now } h = x \sin 53^\circ = 1 \text{ m}$$

$$\therefore v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1} = \sqrt{20} \text{ m/s}$$

13) By conservation of linear momentum  $mu = 2mv_{cm} + mv_1$



By conservation of angular momentum

$$mu \left( \frac{L}{2} \right) = \frac{mL^2}{2} \omega + mv_1 \left( \frac{L}{2} \right)$$

For collision of A and C :

$$v_{cm} = v_1 \text{ therefore } mu = 3mv_1$$

$$\Rightarrow v_1 = \frac{u}{3} \text{ and } \omega L = \frac{2u}{3}$$

$$14) e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{u}{3} + \frac{u}{3} - \frac{u}{3}}{u} = \frac{1}{3}$$

15) Since the string starts oscillating from the mean position at  $t = 0$ , and the ends are always the nodes, the equation of the standing wave in the string can be written as

$$y = A \sin kx \cdot \sin \omega t$$

The induced Electromotive force in the string,  $\epsilon_{ind}$  can be given by

$$\epsilon_{ind} = \int d\epsilon_{ind} = \int d\vec{r} \cdot (\vec{v} \times \vec{B})$$

$$= \int B_0 \cdot dx \cdot v_p$$

$$\text{where, } v_p = \frac{2y}{2t} = A\omega \cdot \sin kx \cos \omega t$$

$$\therefore \epsilon_{ind} = B_0 A \omega \cos \omega t \int_0^{\pi/2} \sin kx \cdot dx$$

( $\square$  the  $\epsilon_{ind}$  in one complete sine cycle, i.e.,  $0 \rightarrow \lambda = 0$ ).

$$\therefore \epsilon_{ind} = B_0 A \omega \cos \omega t \cdot \frac{2}{k} = \frac{2B_0 A \omega L_0}{n\pi} \sin \left( \omega t + \frac{\pi}{2} \right)$$

$$\text{for } n = 5, \text{ we get } \epsilon_{ind}|_{\max} = 0.4 = \frac{B_0 A \omega t L_0}{\pi}$$

$$\therefore \alpha = 0.4$$

16) Say, the current (i) in the circuit lags behind Electromotive force by a phase of  $\phi$ ,

$$\therefore i = \frac{\epsilon_0}{Z} \sin \left( \omega t + \frac{\pi}{2} - \phi \right)$$

where,  $z = \sqrt{R^2 + (x_L - x_C)^2}$  and  
 $\phi = \tan^{-1} \left( \frac{x_L - x_C}{R} \right)$   
 $\therefore (x_L - x_C) = \sqrt{3}R, \phi = \frac{\pi}{3}$  and  $z = 2R$   
 $\therefore$  at  $t = \frac{\pi}{\omega}, i = \frac{1}{5} \frac{B_0 A \omega L_0}{\pi R} \sin \left( \pi + \frac{\pi}{6} \right)$   
 $\therefore i|_{t=\frac{\pi}{\omega}} = \frac{-1}{10} \frac{B_0 A \omega L_0}{\pi R}$   
 $\beta = 0.1$

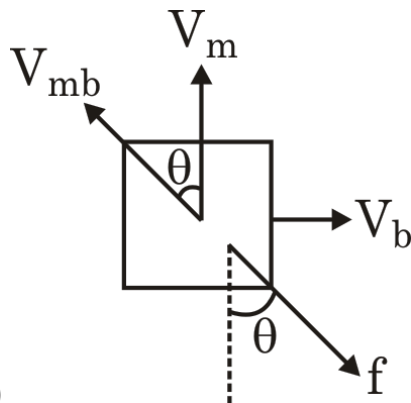
17) Radiation  $\propto T^4$ .

So  $T_2 = 2T_1$  and by Wein's displacement law  $\lambda \propto \frac{1}{T}$

So  $\lambda_2 = \frac{\lambda_1}{2} = 3000 \text{ \AA}$ ; by Einstein's photoelectric equation  $\frac{hc}{\lambda} = eV_s + \phi$

$$f = \frac{hc}{\lambda} - eV_s = \frac{hc}{3000 \text{ \AA}} - (13.6 \text{ eV}) \left( \frac{1}{2^2} - \frac{1}{4^2} \right) = 4.14 - 2.55$$

$$\phi = 1.59 \text{ eV} = 1.59 = \left( \frac{\alpha}{100} \right) \text{ or } \alpha = 159$$



18)

Velocity of belt :  $\vec{V}_b = 3\hat{i}$

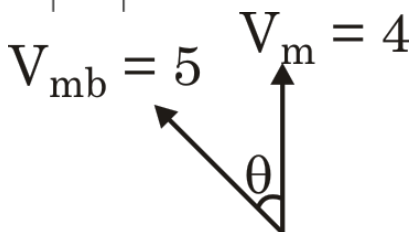
Velocity of mass :  $\vec{V}_m = 4\hat{j}$

$$\vec{V}_{mb} = \vec{V}_m - \vec{V}_b$$

$$= 4\hat{i} - 3\hat{j}$$

$$= -3\hat{i} + 4\hat{j}$$

$$\Rightarrow |\vec{V}_{mb}| = 5$$



$$\cos \theta = \frac{4}{5}$$

Time taken by block to come to rest

$$V = u + at$$

$$0 = V_{mb} - \mu g.t.$$

$$t = \frac{V_{mb}}{\mu g} = \frac{5}{0.5 \times 10}$$

$$a = \mu g$$

$$= 0.5 \times 10$$

$$a = 5 \text{ m/s}^2$$

$$t = 1 \text{ sec}$$

For width :

Retardation along y.

$$a_y = a \cos \theta$$

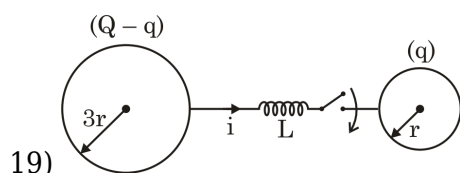
$$= 5 \times \frac{4}{5}$$

$$a_y = 4 \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 4(1) - \frac{1}{2} \times 4(t)^2$$

$$\text{Width of belt} = \theta = 2\text{m}$$



$$\text{At } i_{\max}, \frac{di}{dt} = 0 \Rightarrow \Delta V_L = L \cdot \frac{di}{dt} = 0.$$

□ the two spheres are equipotential

$$\Rightarrow \frac{(Q-q)}{4\pi\epsilon_0 (3r)} = \frac{q}{4\pi\epsilon_0 (r)} \Rightarrow q = \frac{Q}{4}.$$

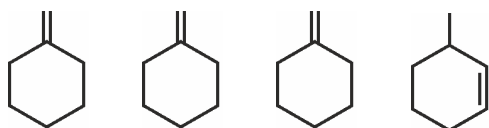
From the conservation of energy,  $U_i = U_f$ .

$$\Rightarrow \frac{Q^2}{2C_1} = \left( \frac{(Q-q)^2}{2C_1} \right) + \left( \frac{q^2}{2C_2} \right) + \frac{1}{2}Li^2$$

using  $q = \frac{Q}{4}$  and  $i = i_{\max}$ , we get

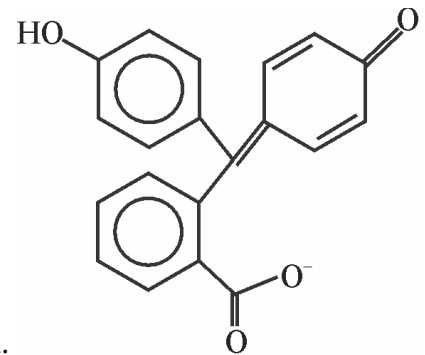
$$i_{\max} = Q \sqrt{\frac{1}{48\pi\epsilon_0 rL}}$$

## PART-2 : CHEMISTRY

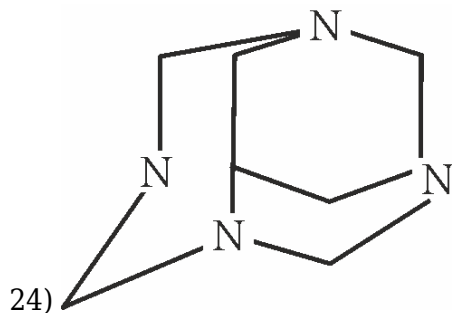


21) (1) Major (2) Major (3) Major (4) Major

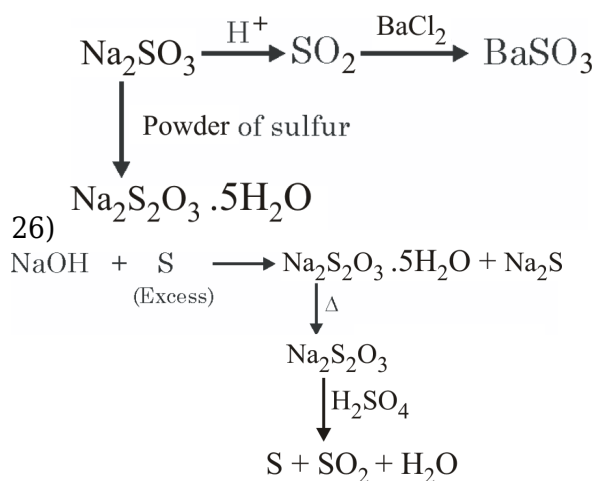
22) 2 moles of phenol reacts with phthalic anhydride to give phenolphthalein which is



colourless. In basic medium it gets converted to quinonoid form.



It reacts as a anti-infective agent which is most commonly used to treat urinary tract infections.



28)

After adding 25 ml solution acts as buffer solution and  $\text{pH} = \text{pK}_a + \log \frac{\text{salt}}{\text{base}} = 5$

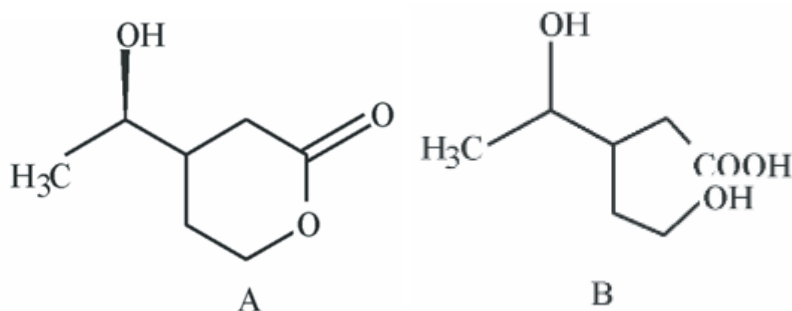
After adding 50 ml hydrolysis of salt occurs and  $\text{pH} = \frac{1}{2} \left[ 14 + 5 + \log \frac{1}{15} \right] = 8.9$

Initially  $\text{pH} = \sqrt{\text{cK}_a} = 3$

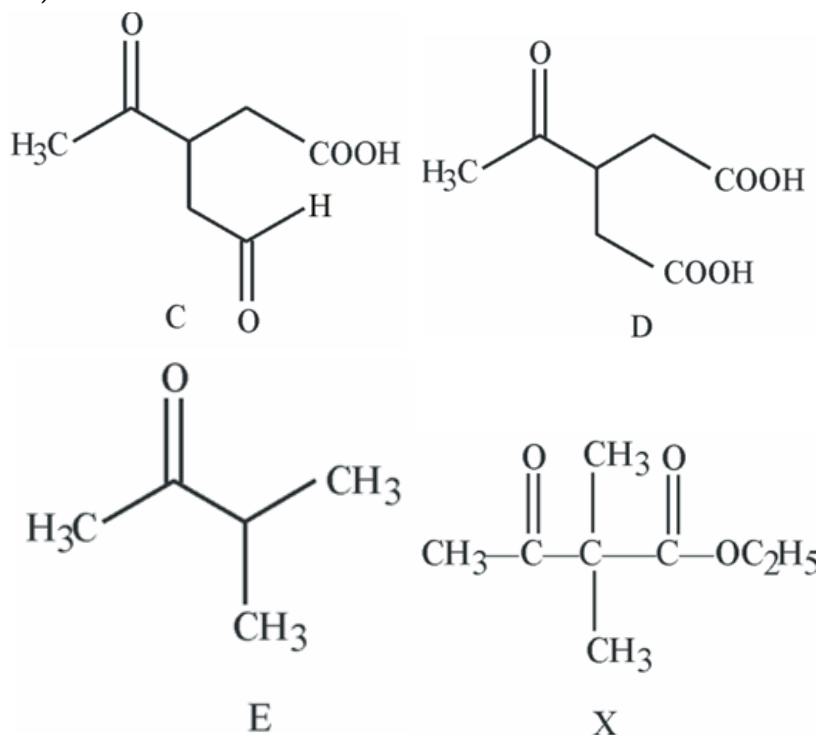
30)

$$K_{\text{formation}} = \frac{k_{\text{formation}}}{k_{\text{dissociation}}} = 10^{19}$$

32) 1, 3, 5 are correct statements.



33)



$$\begin{aligned}
 34) E &= 1.085 + \frac{0.059}{2} \log 10^{-1} \\
 &= 1.085 - 0.0295 = 1.0555 \\
 \Delta G^\circ &= -nFE^\circ = 2.037 \times 10^5 \text{ J}
 \end{aligned}$$

### PART-3 : MATHEMATICS

$$\begin{aligned}
 39) \quad & \frac{3+5 \left[ \frac{2}{2-x} + \frac{2}{2-y} + \frac{2}{2-z} - 3 \right]}{\frac{2-x+2-y+2-z}{3}} \geq \frac{3}{\frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z}} \\
 \text{i.e., } & \geq 6
 \end{aligned}$$

$$\begin{aligned}
 40) \quad & x = -\frac{\pi}{2}; \sin y \geq 1; 1 + \sin x \geq \cos \alpha \cdot \cos x \\
 \text{LHS} & \geq 1 : \text{RHS} \leq 1 \\
 \text{i.e., } & \sin x + 1 = \cos \alpha \cdot \cos x : \sqrt{1 + \cos^2 \alpha} \leq 1
 \end{aligned}$$



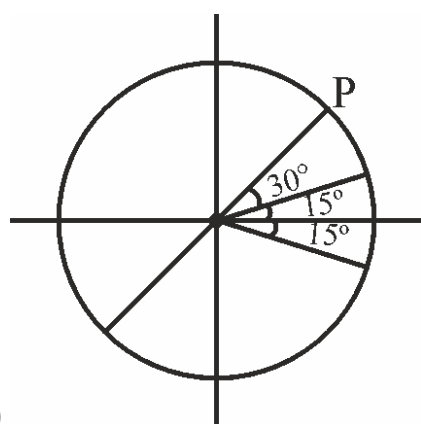
$$\Rightarrow \cos \alpha = 0.$$

$$41) \frac{2x(x - 6f(x)) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} = -\frac{2x}{(x^2 - f(x))^2} + \frac{f(x)}{(6f(x) - x)(x^2 - f(x))^2}$$

$$\Rightarrow \int \frac{f'(x) - 2x}{(x^2 - f(x))^2} dx = \frac{1}{x^2 - f(x)} + c$$

$$42) \int_{-1}^2 x^2 dx + \frac{3}{2} \left( \frac{t^2}{4} + \frac{1}{t} \right) + 3 \left( \frac{1}{4t^2} - \frac{1}{2t} \right)$$

$$f'(t) = 0 \Rightarrow t = \pm (2)^{1/4} \quad f(t)_{\min} = 3 + \sqrt{\frac{9}{8}}$$



43)

$$\frac{2\pi}{12} = \frac{\pi}{6}$$

$$\frac{\pi}{4} \pm \frac{k\pi}{6}$$

i.e vertices will be at eccentric angle  $\frac{\pi}{4} \pm \frac{k\pi}{6}$   
where  $k \in \mathbb{I}$

$$\text{for D : } \frac{2\pi}{3} = \frac{\pi}{4} + \frac{\pi k}{6} \Rightarrow k = \frac{5}{2}$$

$$44) P_1 = P_2 = P_3 = k; P_1 + P_2 + P_3 \neq 0$$

$$P(x) - k = (x - 1)(x - 2)(x - 3)$$

$$45) f(x) = \frac{4x^2 + 12x + 3}{4x^2 + 4x + 1}$$

$$f: \mathbb{R} - \left\{ \frac{-1}{2} \right\} \rightarrow \mathbb{R}$$

$$f'(x) = \frac{(2x+1)^2(8x+12) - (4x^2+12x+3)(8x+4)}{(2x+1)^4}$$

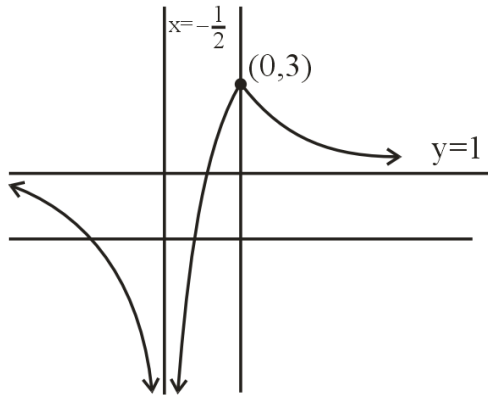
$$f'(x) = \frac{4(2x+1)((2x+1)(2x+3) - (4x^2+12x+3))}{(2x+1)^4}$$

$$f(x) = \frac{-16x}{(2x+1)^3},$$

$$f \uparrow \forall x \in \left(-\frac{1}{2}, 0\right),$$

$$f \downarrow \forall x \in \left(-\infty, -\frac{1}{2}\right) \cup (0, \infty)$$

$$f''(x) = \frac{-16}{(2x+1)^3} + \frac{48x}{(2x+1)^4} = \frac{16(x-1)}{(2x+1)^4}$$



$a \in (-\infty, 1) \cup (1, 3) \rightarrow$  two solution.  
 $A \cap N = (1, 3) \rightarrow a = 2$ , only one element.  
 $b = \{1, 3\}$ ,  $|B| = 2$

$$46) \quad a_n^2 - a_{n-1}^2 = 2 + \frac{1}{a_{n-1}^2}$$

$$0 < \frac{1}{a_{n-1}^2} \leq 1$$

$$2 < a_n^2 - a_{n-1}^2 < 3$$

$$\sqrt{149} < a_{75} < \sqrt{223}$$

$$47) \quad p < q < r < s = t$$

$$p = 1 < r < s = t$$

$$p < q = r < s = t$$

$$p = q = r < s = t$$

$$48) \quad x \cot x = a_0 + a_1 x^2 + a_2 x^4 + \dots + a_k x^{2k} + \dots \text{ take } \lim_{x \rightarrow 0^+} \quad 1 = a_0$$

$$\lim_{x \rightarrow 0^+} \frac{x \cot x - a_0}{x^2} = \lim_{x \rightarrow 0^+} (a_1 + a_2 x^2 + \dots)$$

$$a_1 = -\frac{1}{3}$$

$$49) \quad f''(c_1) > f''(c_2) = f''(c_1) > 0 \text{ and } f''(c_2) < 0 \quad x = c_1 \text{ local minima and } x = c_2 \text{ local maxima}$$

$$50) \quad f''(c_1) < f''(c_2) = f''(c_1) < 0 \text{ and } f''(c_2) > 0 \quad x = c_1 \text{ local maxima and } x = c_2 \text{ local minima}$$

$$51) \begin{vmatrix} 0 & t & f(-t) \\ tf(t) & 0 & f\left(\frac{1}{t^2}\right) \\ -2 & \frac{-1}{f(t)} & \frac{1}{t^2} \end{vmatrix} = 0$$

$$2tf\left(\frac{1}{t^2}\right) + f(t) + t^2f(-t) = 0 \quad \dots (i)$$

$$-2tf\left(\frac{1}{t^2}\right) + f(-t) + t^2f(t) = 0 \quad \dots (ii)$$

(i) + (ii)

$$f(t) + f(-t) = 0$$

$$P_1 + \lambda P_2 = 0$$

$$\Rightarrow \lambda = \frac{1}{2t^3}$$

$$\text{Also, } \lambda = \frac{1}{t}, \quad t^2 = \frac{1}{2}$$

$$52) \begin{vmatrix} 0 & t & f(-t) \\ tf(t) & 0 & f\left(\frac{1}{t^2}\right) \\ -2 & \frac{-1}{f(t)} & \frac{1}{t^2} \end{vmatrix} = 0$$

$$2tf\left(\frac{1}{t^2}\right) + f(t) + t^2f(-t) = 0 \quad \dots (i)$$

$$-2tf\left(\frac{1}{t^2}\right) + f(-t) + t^2f(t) = 0 \quad \dots (ii)$$

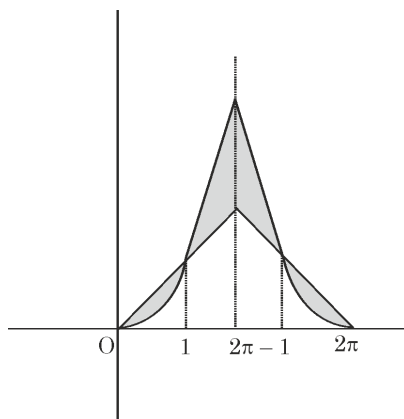
$$P_1 + \lambda P_2 = 0$$

$$\Rightarrow \lambda = \frac{1}{2+3}$$

$$\text{Also, } \lambda = \frac{1}{t}, \quad t^2 = \frac{1}{2}$$

(i) + (ii)

$$f(t) + f(-t) = 0$$



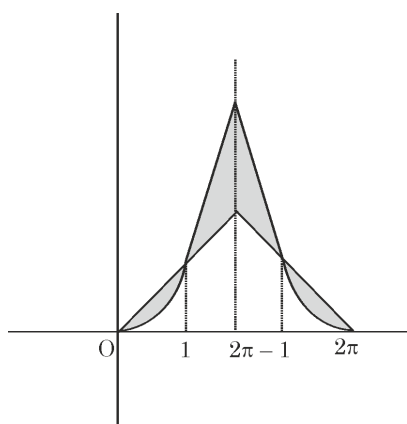
53)

Required area

$$= 4 \left[ \int_0^1 (x - x^2) dx + \int_1^\pi (x^2 - x) dx \right]$$

$$= \frac{4(\pi^3 + 1) - 6\pi^2}{3}$$

$$p = \frac{21}{216}$$



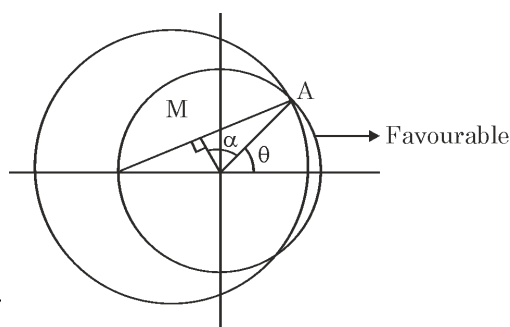
54)

Required area

$$= 4 \left[ \int_0^1 (x - x^2) dx + \int_1^\pi (x^2 - x) dx \right]$$

$$= \frac{4(\pi^3 + 1) - 6\pi^2}{3}$$

$$p = \frac{21}{216}$$



$$55) \sqrt{2 + \sqrt{2}} = 2 \cos \frac{\pi}{8}$$

$$AM = \cos \frac{\pi}{8} \Rightarrow \sin \alpha = \cos \frac{\pi}{8}$$

$$\alpha = \frac{3\pi}{8}; \theta = \frac{\pi}{4}, \frac{2\pi r}{2025} \frac{\pi}{4}$$

$$r \leq \frac{2025}{8} = 253.125$$

$$r = 0, 1, 2, \dots, 253$$

$$56) \text{ Let } a = x^3; b^3 = 3x + 2 \quad a^3 - 27b^3 - 8 = 18ab$$

$$\Rightarrow a - 3b - 2 = 0$$

$$\frac{x^3 - 2}{3} = (3x + 2)^{\frac{1}{3}}$$

$$f(x) = f^{-1}(x)$$

$$x^3 - 3x - 2 = 0 \Rightarrow x = -1, 2,$$

$$\alpha = -1; \beta = 2$$

$$57) 5 \times 8 \times 2 = 80$$