

# FIITJEE

## ALL INDIA TEST SERIES

### FULL TEST – VI

**JEE (Main)-2025**

**TEST DATE: 20-01-2025**

**Time Allotted: 3 Hours**

**Maximum Marks: 300**

**General Instructions:**

- The test consists of total 75 questions.
- Each subject (PCM) has 25 questions.
- This question paper contains **Three Parts**.
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.

**Section-A (01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

**Section-B (21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

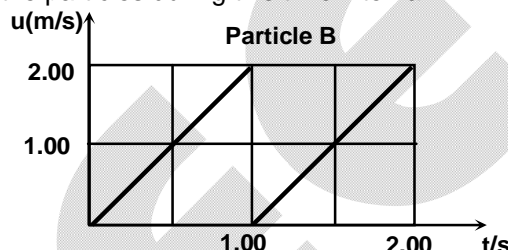
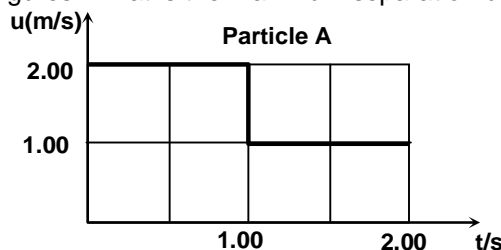
# Physics

## PART – A

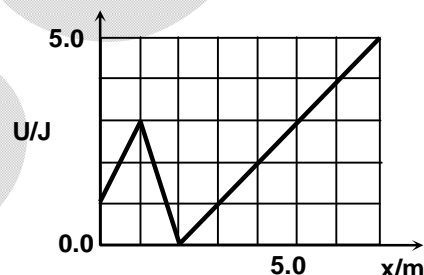
### SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

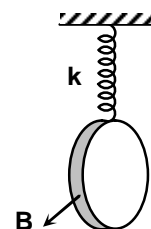
1. Two particles A and B start from the same point and move in the positive x -direction. In a time interval of 2.00 s after they start, their velocities  $v$  vary with time  $t$  as shown in the following figures. What is the maximum separation between the particles during this time interval?



- (A) 1.00 m  
(B) 1.25 m  
(C) 1.50 m  
(D) 2.00 m
2. A potential energy function is shown in the graph. A particle moving in this potential field in the positive x -direction, if possesses 1.0 J of kinetic energy at  $x = 1.0\text{m}$ , where will it reverse its direction of motion?

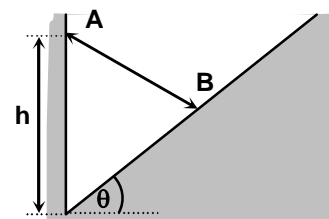


3. A conducting disc of mass  $m$  and volume  $V$  is suspended with the help a light spring of force constant  $k$  from a fixed support. Thickness of the disc is much smaller than its radius. A uniform magnetic field of induction  $B$  parallel to the plane of the disc is established and the disc is pulled down slightly and released. Find period of the small amplitude oscillations of the disc.



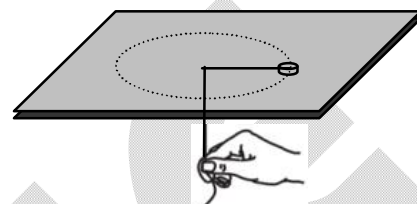
- (A)  $2\pi\sqrt{\frac{m + \epsilon_0 VB^2}{k}}$   
(B)  $2\pi\sqrt{\frac{m + 2\epsilon_0 VB^2}{k}}$   
(C)  $2\pi\sqrt{\frac{m + \epsilon_0 VB^2}{2k}}$   
(D)  $2\pi\sqrt{\frac{2m + \epsilon_0 VB^2}{k}}$
4. A particle of mass  $m$  moving with velocity  $v$  makes a head-on elastic collision with a stationary particle of mass  $2m$ . Kinetic energy lost by the lighter particle during period of deformation is
- (A)  $\frac{1}{2}mv^2$   
(B)  $\frac{1}{18}mv^2$   
(C)  $\frac{4}{9}mv^2$   
(D)  $\frac{8}{9}mv^2$

5. In the setup shown a thread is taut between a nail A on a wall and a nail B on an inclined plane as shown in the figure. You can change the length of the thread by changing location of the nail B. A bead that can slide on the thread without friction is released from the nail A. What should the length of the thread be so that the bead reaches the nail B in shortest time?



- (A)  $h$  (B)  $h \cos \theta$   
(C)  $h \sec(0.5\theta)$  (D)  $h \cos \theta \sec(0.5\theta)$

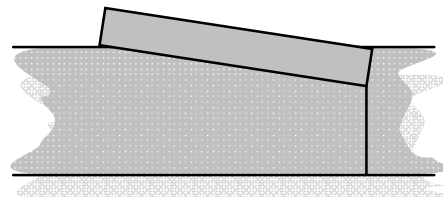
6. A small disc is attached at one end of a light inextensible string that passes through a hole in a frictionless horizontal tabletop. Initially the disc moves on a circle of radius  $R$  with kinetic energy  $K_0$ . The other end of the string is slowly pulled so that the disc finally moves on a circle of radius  $\left(\frac{R}{\eta}\right)$ . What is the work  $W$  done by the pulling agency?



- (A)  $W = 0$  (B)  $W = \eta^2 K_0$   
(C)  $W = (\eta^2 - 1)K_0$  (D)  $W = (\eta - 1)K_0$
7. Consider two identical particles each of mass  $m$  held at a separation  $r_0$  in free space. One of them is given a velocity  $v_0$  perpendicular to  $r_0$  and the other one is simultaneously released. For what range of velocity  $v_0$  will the masses be bound in orbital motion under their mutual gravitational forces.

- (A)  $v_0 \leq \sqrt{\frac{2Gm}{r_0}}$  (B)  $v_0 < \sqrt{\frac{2Gm}{r_0}}$   
(C)  $v_0 \leq 2\sqrt{\frac{Gm}{r_0}}$  (D)  $v_0 < 2\sqrt{\frac{Gm}{r_0}}$

8. A wooden plank is floating on water in a pool. It is tethered to the bottom of the pool by a string attached at mid-point of an edge of its bottom face, which causes it to float with a diagonal of one of its vertical cross-section coinciding with the level surface of the water, as shown in the figure. What is the specific gravity of the wood?

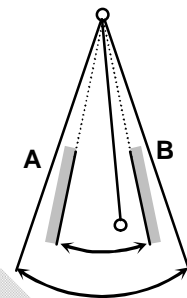


- (A)  $1/5$  (B)  $1/4$   
(C)  $1/3$  (D)  $1/2$
9. In a special quasi-static process, an ideal mono-atomic gas is supplied heat in such a way that its volume increases and frequency of collisions of its atoms on unit area of the walls of the container remains constant. What is the molar heat capacity of the gas?
- (A) 0 (B)  $2R$   
(C)  $3R$  (D)  $4.5R$

10. Volumes  $V_1$  and  $V_2$  of a liquid are maintained in two calorimeters at temperatures  $\theta_1$  and  $\theta_2$  ( $\theta_2 > \theta_1$ ). Coefficient of volume expansion of the liquid is independent of temperature. Now liquids from both the calorimeter are poured into another calorimeter of negligible heat capacity. Heat loss to the surroundings is strictly restricted. Final volume  $V$  of the mixture after they are well mixed is best represented by the equation

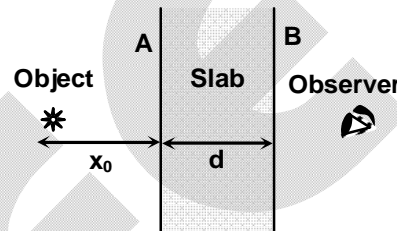
- (A)  $V = V_1 + V_2$  (B)  $V > V_1 + V_2$   
(C)  $V < V_1 + V_2$  (D) Insufficient information

11. A simple pendulum initially oscillating simple harmonically with angular amplitude  $\alpha$  and period  $T_0$  is symmetrically confined between two rigid fixed planes A and B making angle  $\beta < \alpha$  with each other as shown in the figure.



- (A) If collisions at both the walls are elastic, period is  $T_0 \left( \frac{1-\beta}{\alpha} \right)$ .  
 (B) If collisions at both the walls are inelastic, period is  $T_0$ .  
 (C) If collision at one wall is elastic and at the other is inelastic, the period is  $T_0$ .  
 (D) If collision at one wall is elastic and at the other is inelastic, the period is less than  $T_0$ .

12. A self-luminous point object placed at a distance  $x_0$  from a slab of transparent material of negative refractive index is viewed through the slab as shown in the figure. Thickness of the slab is  $d$ , refractive index of material of the slab with respect to the outside medium is  $\mu = -1$ .



For different moduli of values of  $x_0$  and  $d$  the observer may find real or virtual image of the object. Which of the following statements is true?

- (A) If  $x_0 > d$ , image is virtual and for  $x_0 < d$  image is real.  
 (B) If  $x_0 < d$ , image is virtual and for  $x_0 > d$  image is real.  
 (C) If  $x_0 \leq d$ , image is virtual and for  $x_0 > d$  image is real.  
 (D) If  $x_0 > d$ , image is virtual and for  $x_0 \leq d$  image is real.

13. In the figure, a line of electric field created by two point charges  $q_1$  and  $q_2$  is shown. If it is known that  $q_1 = 1 \mu\text{C}$ , the charge  $q_2$  is closest to

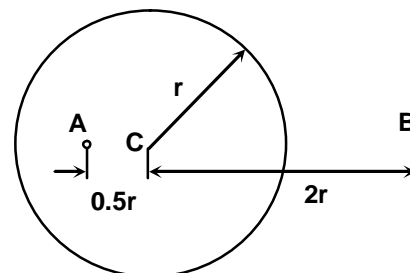


- (A)  $-2\mu\text{C}$   
 (B)  $-4\mu\text{C}$   
 (C)  $-6\mu\text{C}$   
 (D)  $-8\mu\text{C}$

14. Due to a point charge, potential and electric field at a point A are 7 V and 3 V/m respectively and electric field at a point B is less than 3 V/m. Now magnitude of the charge is tripled. If electric field at B becomes 3 V/m, potential at B will become closest to

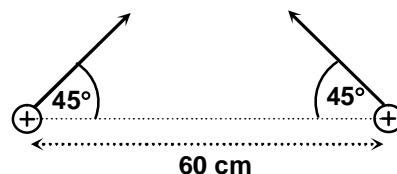
- (A) 7 V  
 (B) 12 V  
 (C) 21 V  
 (D) Insufficient information

15. Consider a thin conducting shell of radius  $r$  carrying total charge  $q$ . Two point charges  $q$  and  $2q$  are placed on points A and B, which are at distances  $0.5r$  and  $2r$  from the centre C of the shell respectively. If the shell is earthed, how much charge will flow to the earth?



- (A)  $2q$   
 (B)  $3q$   
 (C)  $4q$   
 (D) More than  $2q$  and less than  $3q$

16. Two identical point charges are moving in free space, when they are 60 cm apart; their velocity vectors are equal in modulus and make angles of  $45^\circ$  from the line joining them as shown in the figure. If at this instant, their total kinetic energy is equal to their potential energy, what will be the distance of closest approach between them?

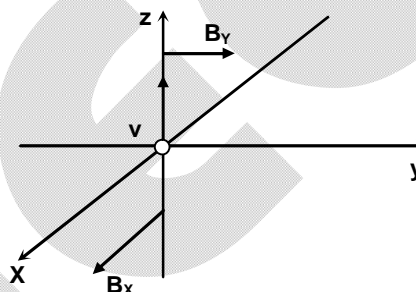


- (A) 20 cm (B) 30 cm  
(C) 40 cm (D) 45 cm

17. A moving neutron collides with a singly ionized stationary helium atom that is in ground state. What should be the minimum speed of the moving neutron for the collision to be perfectly inelastic?

- (A)  $2.50 \times 10^4 \text{ m/s}$  (B)  $4.25 \times 10^4 \text{ m/s}$   
(C)  $6.25 \times 10^4 \text{ m/s}$  (D)  $9.89 \times 10^4 \text{ m/s}$

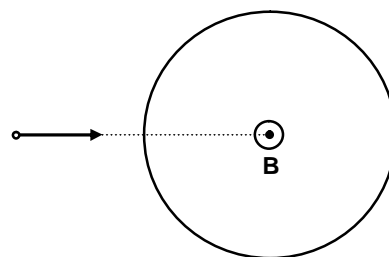
18. A particle of mass  $m$  and charge  $q$  is projected from the origin with a velocity  $v$  in the positive  $z$ -direction. Above the plane  $z = 0$ , a uniform and constant magnetic field of induction  $B_y$  exists in the positive  $y$  direction and below the plane  $z = 0$ , a uniform and constant magnetic field of induction  $B_x$  exists in the positive  $x$ -direction. Determine coordinates of the point where the particle crosses the plane  $z = 0$  third time.



- (A)  $\left( \frac{-4mV}{qB_y}, \frac{-2mV}{qB_x} \right)$  (B)  $\left( \frac{-4mV}{qB_x}, \frac{-2mV}{qB_y} \right)$   
(C)  $\left( \frac{-2mV}{qB_y}, \frac{-4mV}{qB_x} \right)$  (D)  $\left( \frac{-2mV}{qB_x}, \frac{-4mV}{qB_y} \right)$

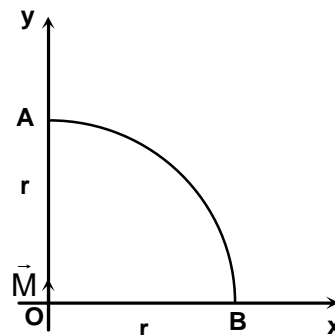
19. Several  $\alpha$ -particles of different speeds enter a uniform magnetic field confined into a cylindrical region. If all the  $\alpha$ -particles enter the field radially, what can you say about time intervals spent by them in the magnetic field?

- (A) Faster is the particle, lesser is the time.  
(B) Angle of deviation remains same for all particles.  
(C) Some particles may deviate by  $180^\circ$ .  
(D) The time is same for all the particles.



20. Consider a quarter circular conducting ring of large radius ' $r$ ' with its centre at the origin, where a magnetic dipole of moment  $\vec{M}$  is placed as shown in the figure. If the ring rotates with a constant angular velocity  $\omega$  about the  $y$ -axis, the emf induced between its ends A and B is

- (A)  $\frac{\mu_0 M \omega}{\pi r}$  (B)  $\frac{\mu_0 M \omega}{2\pi r}$   
(C)  $\frac{\mu_0 M \omega}{4\pi r}$  (D)  $\frac{\mu_0 M \omega}{8\pi r}$

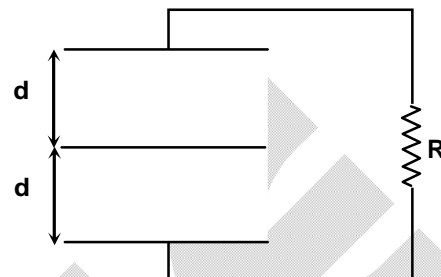


## SECTION – B

## (Numerical Answer Type)

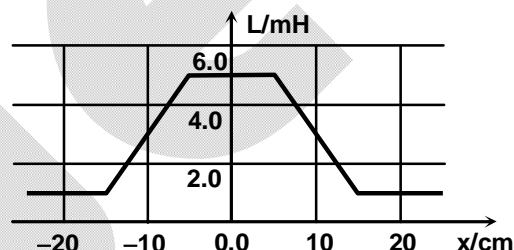
This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. Three identical thin metal plates each of area  $A$  are kept parallel to each other with separation  $d$  between adjacent plates and the outer plates are connected through a resistor  $R$  as shown in the figure. A charge  $Q$  is given to the middle plate and then the middle plate is quickly shifted towards one of the outer plates by a distance  $\left(\frac{d}{2}\right)$ . If the heat dissipated in

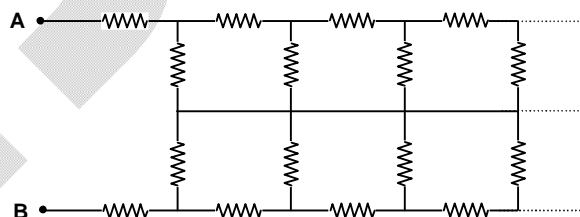


the resistor after this shift is  $\frac{Q^2 d}{k\epsilon_0 A}$ . Find the value of  $k$ . (Neglect the gravity)

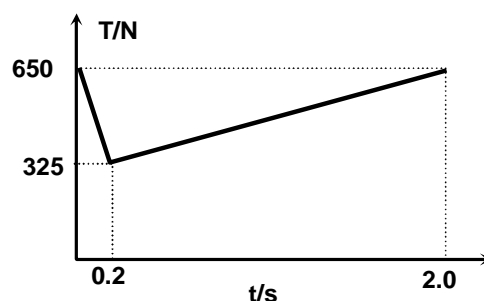
22. A coil is wound on a hollow insulating cylinder, which contains in it a laminated iron core. How the inductance  $L$  of the coil varies with the displacement  $x$  of the iron core is shown in the figure. In the initial state  $x = 0$  (the core is fully inserted into the coil), the current in the coil is 1.0 A. Find current in the coil immediately after the core is quickly taken out of the coil.



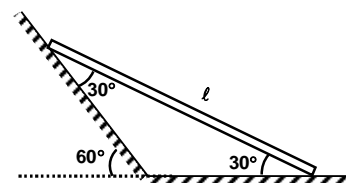
23. The circuit shown in the diagram extends to right into infinity. Each resistance is denoted by  $r$ . If the equivalent resistance between the points A and B is  $(\sqrt{a} + b)r$ , then find the value of  $(a + b)$ .



24. A mountain climber is sliding down a vertical rope. Her total mass, including equipment, is 65 kg. By adjusting frictional force of the rope, she controls the force that the rope exerts on her. For a 2.0 s interval, this force  $T$  is shown as a function of time  $t$  in the given graph. Find the change in her speed (in m/s) in this interval. Assume acceleration of free fall to be  $10 \text{ m/s}^2$  and neglect air resistance.



25. A thin uniform rod of length  $\ell$  is released from rest from the position shown in the figure. All contact surfaces are smooth. If the initial angular acceleration of the rod is  $\left(\frac{k\sqrt{3}g}{20\ell}\right)$ . Find the value of  $k$ .





# Chemistry

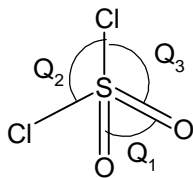
## PART – B

### SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

26. The atomic weight of two element A and B are 40 and 80 respectively. If x gm of A contains Y atom, how many atoms are present in 2x gm of B?  
 (A)  $\frac{Y}{2}$  (B)  $\frac{Y}{4}$   
 (C) Y (D) 2Y
27. A 5 ml solution of  $H_2O_2$  liberates 0.508 gm of iodine from an acidified KI solution. What is the volume strength of  $H_2O_2$  solution?  
 (A) 2.2% (B) 3.8%  
 (C) 4.48% (D) 6.58%
28. How many electrons in an atom with atomic number is 105, can have  $(n + \ell) = 8$ ?  
 (A) 30 (B) 17  
 (C) 15 (D) 20
29. When two moles of an ideal gas  $\left(C_p = \frac{5}{2}R\right)$ , heated from  $27^\circ\text{C}$  to  $327^\circ\text{C}$  at constant pressure. The entropy change will be  
 (A)  $\frac{3}{2}R\ln 2$  (B)  $-\frac{3}{2}R\ln 2$   
 (C)  $5R\ln 2$  (D)  $\frac{5}{2}R\ln 2$
30. At  $1000^\circ\text{K}$  for the equilibrium  
 (i)  $\text{CaCO}_3(s) \rightleftharpoons \text{CaO}(s) + \text{CO}_2$   $K_p = 4 \times 10^{-2} \text{ atm}$   
 (ii)  $\text{C} + \text{CO}_2(g) \rightleftharpoons 2\text{CO}(g)$   $K'_p = 2.0 \text{ atm}$   
 Solid  $\text{CaCO}_3$ , solid 'C' and solid  $\text{CaO}$  are mixed and allowed to attain equilibrium at  $1000^\circ\text{K}$ . The partial pressure of CO at equilibrium is  
 (A) 0.14 atm (B) 0.28 atm  
 (C) 0.56 atm (D) 0.70 atm
31. The solution containing 4.13 gm LiCl per litre freezes at  $-0.343^\circ\text{C}$ . Calculate van't Hoff's factor [Given  $K_f = 1.86$ ]  
 (A) 0.9 (B) 1.9  
 (C) 2.9 (D) 0.79
32. The decomposition of azomethane at certain temperature is first order:  
 $\text{CH}_3 - \text{N} = \text{N} - \text{CH}_3 \longrightarrow \text{C}_2\text{H}_6 + \text{N}_2$   
 After 40 minutes from the start, the total pressure develop is found to be 350 mm of Hg. If the initial pressure of azomethane be taken at 200 mm of Hg. Find the rate constant of the reaction?  
 (A)  $2.88 \times 10^{-4} \text{ sec}^{-1}$  (B)  $5.77 \times 10^{-4} \text{ sec}^{-1}$   
 (C)  $3.88 \times 10^{-4} \text{ sec}^{-1}$  (D)  $1.25 \times 10^{-4} \text{ sec}^{-1}$

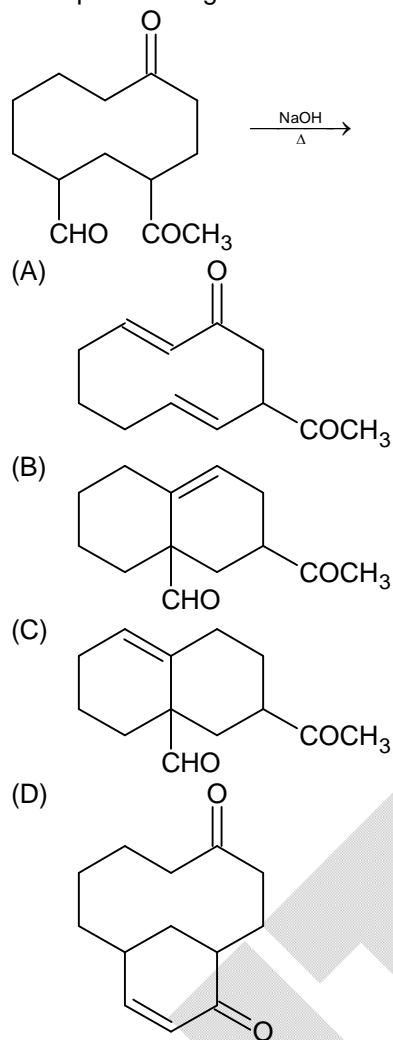
33. Which of the following statement is correct for



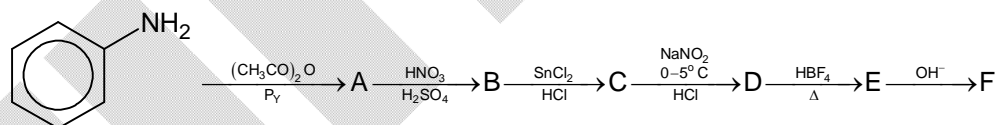
- (A) It contains  $p\pi - p\pi$  and  $p\pi - d\pi$   
 (B) It has regular tetrahedral geometry.  
 (C)  $Q_1 > Q_3$ .  
 (D) Plane which contains maximum number of atom is 4.
34. The chain controlling unit for Silicone is  
 (A)  $R_3SiCl$   
 (B)  $R_2SiCl_2$   
 (C)  $RSiCl_3$   
 (D)  $SiCl_4$
35. Identify P and Q product in the given reaction  
 $KI + MnO_4^- \xrightarrow{H^+} P + Mn^{2+}$   
 $KI + MnO_4^- \xrightarrow{OH^-} IO_3^- + Q$   
 (A)  $IO_3^-$ ,  $MnO_2$  respectively  
 (B)  $I_2$ ,  $Mn^{2+}$  respectively  
 (C)  $IO_3^-$ ,  $Mn^{2+}$  respectively  
 (D)  $I_2$ ,  $MnO_2$  respectively
36. John Teller theorem is not applicable to  
 (A)  $[Cu(H_2O)_6]^{2+}$   
 (B)  $FeO_4^{4-}$   
 (C)  $[Cu(CN)_5]^{3-}$   
 (D)  $[Ag(NH_3)_2]^+$
37. A tetrapeptide has  $-COOH$  group of alanine. This produce gly, ala, phe, val on complete hydrolysis. For this tetrapeptide the number of possible sequence with  $NH_2$  – group attached to a chiral centre is  
 (A) 6  
 (B) 5  
 (C) 4  
 (D) 8
38. Which of the following are correct statements?  
 (1) Solid  $PCl_5$  exist as  $[PCl_4]^+$  and  $[PCl_6]^-$  ions.  
 (2) Solid  $PBr_5$  exist as  $[PBr_4]^+$  and  $Br^-$  ions.  
 (3) Solid  $N_2O_5$  exist as  $[NO_2]^+$  and  $[NO_3]^-$  ions.  
 (4) Oxide of phosphorus  $P_2O_3$  and  $P_2O_5$  exists as monomer.  
 (A) 1, 3  
 (B) 2, 3, 4  
 (C) 1, 3, 4  
 (D) 1, 2, 3



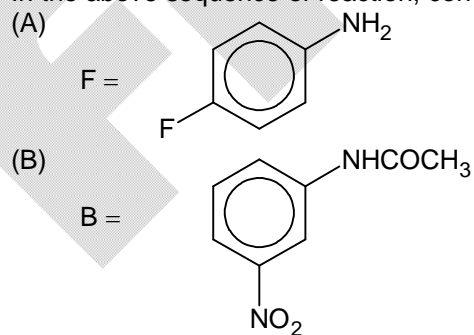
39. Final product of given reaction is:

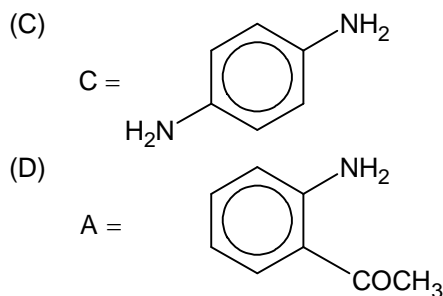


40.



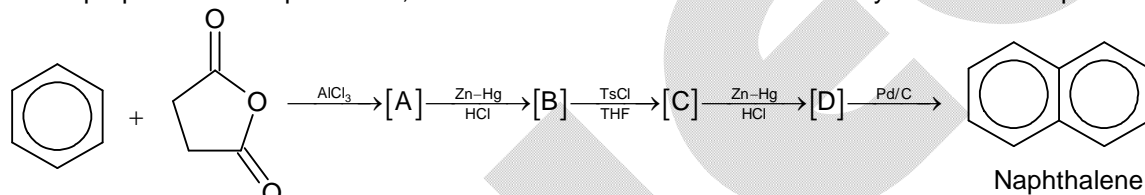
In the above sequence of reaction, correct product is



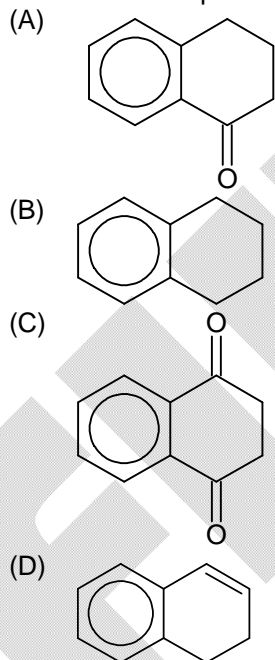


41. Assertion: 2,2-Dimethyl propanal undergo Cannizzaro reaction with NaOH.  
Reason: Cannizzaro reaction is a disproportionation reaction.  
(A) Assertion and reason both are correct and reason is the correct explanation of assertion.  
(B) Assertion and reason both are correct and two reason is not correct explanation of assertion.  
(C) Assertion is correct but reason is incorrect.  
(D) Assertion and reason both are incorrect.

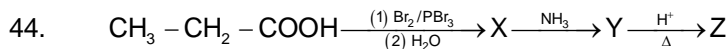
42. In the preparation of naphthalene, clemmensen reduction is used effectively in different steps.



In the above sequence of reaction, product (D) is



43. Statement I: Finkelstein reaction is a halogen exchange equation.  
Statement II: In Finkelstein reaction driven forward by using differential solubility of sodium halides in dry acetone.  
Choose the correct option:  
(A) Both the statement are correct.  
(B) Statement I is correct and statement II is incorrect.  
(C) Statement I is incorrect and statement II is correct.  
(D) Both statement I and statement II are incorrect.



The product Z in the reaction is

- (A)  $\begin{array}{c} \text{Br} \\ | \\ \text{H}_3\text{C}-\text{CH}-\text{COOH} \end{array}$
- (B)  $\begin{array}{c} \text{NH}_2 \\ | \\ \text{H}_3\text{C}-\text{CH}-\text{COOH} \end{array}$
- (C)  $\begin{array}{c} \text{CH}_3 \\ | \\ \text{O}=\text{C}-\text{CH}-\text{NH} \\ | \quad \quad | \\ \text{HN} \quad \quad \text{C}=\text{O} \\ | \\ \text{CH}_3 \end{array}$
- (D)  $\text{H}_3\text{C}-\text{CH}_2-\text{CONH}_2$

45. Which of the following molecules does not undergo Iodoform reaction

- (A)  $\text{H}_3\text{C}-\text{CH}(\text{OH})-\text{CH}_3$
- (B)  $\text{CH}_3-\text{CH}_2-\text{OH}$
- (C)  $\begin{array}{c} \text{H}_3\text{C}-\text{C}(=\text{O})-\text{CH}_3 \\ | \\ \text{C}_6\text{H}_4 \end{array}$
- (D)  $\text{H}_3\text{C}-\text{C}(=\text{O})-\text{CH}_3$

### SECTION – B

#### (Numerical Answer Type)

This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

46. The value of n in molecular formula  $\text{Be}_n\text{Al}_2\text{Si}_6\text{O}_{18}$  is.....
47. Calculate the work done by the reaction in (lit-atom)  
 $\text{Fe(s)} + \text{H}_2\text{SO}_4(\text{aq}) \longrightarrow \text{FeSO}_4(\text{aq}) + \text{H}_2(\text{g})$   
 When 0.3125 mole of  $\text{H}_2$  gas is collected at 273 K at 1.0 atm.
48. In the following non-redox reaction find the value of sum of x and y is....  
 $[\text{Fe}(\text{CO})_5] + x\text{NO} \longrightarrow [\text{Fe}(\text{CO})_y(\text{NO})_x] + \text{CO} \uparrow$
49.  $\text{M}[\text{OH}]_x$  has  $K_{\text{SP}} = 4 \times 10^{-12}$  and solubility  $10^{-4}$  M. Hence, x is.....
50. Number of visible lines when an electron returns from 5<sup>th</sup> orbit to ground state in H-spectrum is.....

# Mathematics

## PART – C

### SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

51. Let  $I = \int_0^2 \left[ x^2 - 5x + 4 \right] + \left[ \sin \frac{3\pi}{2} x \right] dx$ ; (where  $[.]$  represents greatest integer function) then  $I + 2/3$  is  
 (A)  $\frac{13 - \sqrt{17} - \sqrt{21} - 2\sqrt{5} - \sqrt{13}}{2}$  (B)  $\frac{13 + \sqrt{17} - \sqrt{21} - \sqrt{5} - \sqrt{13}}{2}$   
 (C)  $\frac{14 - \sqrt{17} - \sqrt{21} + \sqrt{5} - \sqrt{13}}{2}$  (D)  $\frac{14 - \sqrt{17} - \sqrt{21} + 2\sqrt{5} - \sqrt{13}}{2}$
52. A normal with slope  $\frac{1}{m}$  is drawn from the point  $P(0, -k)$  to the parabola  $x^2 = -12y$ . Also the line through the point  $Q\left(0, \frac{-k}{506}\right)$  parallel to the tangent at vertex intersects the parabola at two distinct points R and S. If the area of triangle ORS; (O being the vertex) is 144 sq. unit then  $m^2$  is equal to  
 (A) 2022 (B) 2024  
 (C) 2025 (D) 2026
53. If sum of the series  $1 + \frac{\sqrt{5} - \sqrt{3}}{2\sqrt{5}} + \frac{8 - 2\sqrt{15}}{30} + \frac{14\sqrt{5} - 18\sqrt{3}}{60\sqrt{5}} + \frac{124 - 32\sqrt{15}}{500} + \dots$   
 $= 2 + \frac{a + \sqrt{15}}{b} \log_e \left( \frac{a}{c} \right)$ ;  $a, b, c \in \mathbb{N}$  and H.C.F( $a, b, c$ ) = 1, then the value of  $2(a^2 + b^2 + c^2)$  is  
 (A) 25 (B) 50  
 (C) 100 (D) 150
54. Consider two lines  $L_1$  and  $L_2$  in space defined by  
 $L_1 = \{3\sqrt{3}x = 4\sqrt{\sin \theta}y + (2\sqrt{\sin \theta} - 3)3\sqrt{3}, 3\sqrt{3}z = (2\sqrt{\sin \theta} - 3)y + 3\sqrt{3}\sqrt{\sin \theta}\}$   
 $L_2 = \{\sqrt{3}x = -2\sqrt{\cos \theta}y + \sqrt{3}(3 - 2\sqrt{\cos \theta}), \sqrt{3}z = (3 - 2\sqrt{\cos \theta})y + \sqrt{3}\sqrt{\cos \theta}\}$   
 If  $L_1$  and  $L_2$  are perpendicular then  
 (A)  $\theta = n\pi + \frac{\pi}{4}; n \in \mathbb{I}$  (B)  $\theta = n\pi + \frac{\pi}{4}; n$  is even  
 (C)  $\theta = n\pi + \frac{\pi}{4}; n$  is odd (D)  $\theta = n\pi; n \in \mathbb{I}$

55. There are two sections A and B in class XII of a school containing 40% and 60% of the total students respectively. 20% of the students get selected in IIT.  $P$  (student selected in IIT from section A) = 5  $P$  (student selected in IIT from section B); where  $P(E)$  denotes the probability of an event  $E$ . A student randomly selected was found to be not selected in IIT then the probability that he is from section B is
- (A)  $\frac{4}{13}$  (B)  $\frac{5}{13}$   
(C)  $\frac{9}{13}$  (D)  $\frac{10}{13}$
56. Consider a triangle ABC having its vertices  $A(2025\omega + 2024i)$ ,  $B(2024i\omega^2 + 2025)$  and  $C(2024\omega^2 - 2025i)$ . where  $\omega$  represents non real cube root of unity whose imaginary part is positive. If the internal bisector of  $\angle ACB$  meets side AB at D and  $\angle BDC = \frac{k\pi}{24}$ , then the value of  $k$  is
- (A) 6 (B) 12  
(C) 14 (D) 16
57. If one of the roots of the equation  $\tan^{-1} \cot \left( \frac{3x^2 + 3|x| + 1}{x^2 + |x| + 1} \right) = \frac{\pi}{2} - \operatorname{cosec} \operatorname{cosec}^{-1} \left( \frac{3|x| + 2}{|x| + 1} \right)$  is  $2\sin\theta$ ,  $\theta \in \left(0, \frac{\pi}{2}\right)$ , then the value of  $\tan\frac{\theta}{9} \tan\frac{7\theta}{9} \tan\frac{11\theta}{9} \tan\frac{13\theta}{9}$  is
- (A) 0 (B) -1  
(C) 1 (D)  $\frac{1}{3}$
58. The equation of two sides of a triangle are  $3x + 4y - 24 = 0$  and  $2x + y - 16 = 0$ . If the circumcentre of the triangle lies at  $(0, 6)$  then the inradius of the triangle is
- (A)  $6\sqrt{5}$  (B)  $6(\sqrt{5} - 1)$   
(C)  $6\sqrt{5} - 10$  (D) 10
59. A curve  $y = f(x)$  passing through origin satisfies the differential equation  $\frac{7dy}{dx} + \frac{28x^3y}{1+x^4} = \frac{40x^4}{1+x^4}$ . If the area enclosed by the curve  $y = f^{-1}(x)$ , (Inverse of  $f(x)$ ),  $x$ -axis and the ordinate  $x = 4/7$  in 1<sup>st</sup> quadrant is  $A$  then the value of  $42\pi$  is
- (A) 4 (B) 5  
(C) 6 (D)  $15/2$
60. Number of natural numbers having 5 distinct digits that can be formed using the digits 1, 2, 3, 5, 6, 7, 9 and which are divisible by 6 are
- (A) 72 (B) 192  
(C) 216 (D) 240
61. Let  $L_1 = \lim_{x \rightarrow 0} \frac{\tan 3x - \tan x}{2\cos 4x \sin 3x + \sin 5x - (2 - 2\cos 4x + \sin 3x)}$   
 $L_2 = \lim_{x \rightarrow 0^+} \frac{e^{\sec(\tan x^{1012n})} - e}{\sin(x^{2025m})} = \frac{e}{2}$ ;  $n, m$  are natural numbers, then the value of  $\frac{1}{4L_1 - \frac{m}{n}}$  is
- (A) 1012 (B) 2024  
(C) 2025 (D)  $\frac{1}{2025}$

62. A circle  $x^2 + y^2 = r^2$  intersects the ellipse  $16x^2 + 25y^2 = 400$ ;  $4 < r < 5$ . Common tangent of the two curves having slope  $m$ ;  $m > 0$  in second quadrant intersects the coordinate axis at points P and Q respectively. If area of  $\triangle OPQ$ ; (O being origin) is minimum, then  $m$  is
- (A)  $\frac{2}{\sqrt{5}}$  (B)  $\frac{3}{5}$   
(C)  $\frac{4}{5}$  (D)  $\frac{5}{3}$
63. The equation  $1012x^{2023} - 12138x^{2022} - 119x + 714 = 0$  has a root lying in the interval  $(a^{1/2022}, b^{1/3})$ ;  $a$  and  $b \in \mathbb{N} \geq 2$ , then the value of  $4 \int_a^{b^{1/3}} \frac{x \cos x^2}{\cos(x^2) + \cos(263 - x^2)}$  is
- (A) 25 (B) 35  
(C) 40 (D) 45
64. Let three distinct chords drawn to the ellipse  $x^2 + 4y^2 = 2000$  from the point  $P(0, a)$  are bisected by the parabola  $x^2 = 20y$ . If exhaustive set of  $a$  is  $(k_1, k_2)$  then the number of positive integral solution of the equation  $x + y = k_2 - k_1$  is
- (A) 169 (B) 19  
(C) 51 (D) 969
65. Consider two circles  $C_1 : x^2 + y^2 = 625$  and  $C_2 : (x - a)^2 + y^2 = 576$ ;  $a \in (1, 49)$  having their centres at Q and R respectively. Let P be one of the point of intersection of the two circles such that  $\angle QPR = \cos^{-1}\left(\frac{\sqrt{481}}{25}\right)$ . If the length of common tangent of the circle is  $\sqrt{1295}$ , then the length of common chord is equal to
- (A) 8 (B) 16  
(C) 24 (D) 28
66. Let  $P = \begin{bmatrix} 0 & 2 & \lambda \\ 2 & 3 & 1 \\ 1 & \mu & 3 \end{bmatrix}$  and  $\text{Adj}(P) = \begin{bmatrix} 10 & -7 & -1 \\ -5 & -1 & 2 \\ -5 & 2 & -4 \end{bmatrix}$ , then  $|(\text{adj}P)^{-1} + 14\text{adj}(P^{-1})|$  is equal to
- (A) 15 (B) -15  
(C)  $15^2$  (D)  $-15^3$
67. Let  $f(x) = \frac{1}{2} \begin{vmatrix} 1 & \tan x & 1 \\ -\tan x & 1 & \tan x \\ -1 & -\tan x & 1 \end{vmatrix} + \begin{vmatrix} \cot \frac{\pi}{2} & \sec\left(x + \frac{\pi}{3}\right) & \sec\left(x + \frac{\pi}{12}\right) \\ \text{cosec}\left(x - \frac{\pi}{6}\right) & \sin 2024\pi & e^{i2024\pi} \\ \text{cosec}\left(x - \frac{5\pi}{12}\right) & e^{i2025\pi} & \tan(2025)\pi \end{vmatrix}$
- and  $g : \left(0, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$  be a function defined by  $g(x) = \sqrt{f(x) - 1} + \sqrt{f\left(2025\frac{\pi}{2} - x\right) - 1}$ . If  $m$  be the minimum value of  $f(x)$  and  $M$  be the minimum value of  $g(x)$  such that  $h(x) = (x - m)(x - M)$  then range of  $h(x)$  in  $[0, 3]$  is
- (A)  $\left[-\frac{1}{4}, \infty\right)$  (B)  $\left[-\frac{1}{4}, 2\right]$   
(C)  $[-2, 4]$  (D)  $\left[\frac{3}{2}, 4\right]$



68. Let  $I = \int \frac{\sin^2 \theta \log_e (e \cot \theta) d\theta}{\cos^4 \theta ((\log_e \tan \theta)^4 - (\tan \theta)^4)} = \frac{1}{4} \log_e |(f(\theta))| - \frac{1}{2} \tan^{-1}(g(\theta)) + c$ , then the value of  $f(\theta) - g(\theta)$  at  $\theta = \frac{\pi}{3}$  is
- (A) 0 (B)  $\frac{\ln 3 + 2\sqrt{3}}{\ln 3 - 2\sqrt{3}} - \frac{\ln 3}{2\sqrt{3}}$
- (C)  $\frac{\ln 3 - 2\sqrt{3}}{\ln 3 + 2\sqrt{3}} - \frac{\ln 3}{2\sqrt{3}}$  (D)  $\frac{\ln 3}{2\sqrt{3}}$
69. Let  $S : \{(x, y) \in \mathbb{R} \times \mathbb{R} : y^2 \leq 8x, y^2 \geq 32 - 8x, x + y - 6 \geq 0, 4x - 3y - 8 \leq 0, x \geq 0, y \geq 0\}$ . If area of region S is A, then the value of  $21A + 792$  is
- (A) 2024 (B) 1012
- (C) 1000 (D) 343
70. Consider three sets  $A = \{n \in \mathbb{N} : n^2 \leq n + 15000\}$ ,  $B = \{5k + 4 ; k \in \mathbb{N}\}$  and  $C = \{3k ; k \in \mathbb{N}\}$ . If the sum of all the elements of the set  $A \cap (B - C)$  is equal to N, then number of proper divisors of N are
- (A) 14 (B) 15
- (C) 18 (D) 30

### SECTION – B

#### (Numerical Answer Type)

This section contains 05 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}$  and  $h : \mathbb{R} \rightarrow \mathbb{R}$  be differentiable function such that  $f(x) = x^5 + x^3 + 3x + 7$ ,  $g(f(x)) = x$  and  $h(g(g(x))) = x \quad \forall x \in \mathbb{R}$ , then the value of  $h'(-1)$  is
72. Let  $X = (23\sqrt{3} + 39)^{2025}$ ,  $Y = X - [X]$  ; (where  $[.]$  represents greatest integer function) then the remainder obtained if  $XY$  is divided by 31 is
73. Let  $\alpha$  and  $\beta$  be the roots of the equation  $x^2 - 10x + 2 = 0$ , then the value of  $\frac{\alpha^{2028} + \beta^{2028} + 8\alpha^{2022} + 8\beta^{2022}}{\alpha^{2025} + \beta^{2025}}$  is
74. Consider the following frequency distribution
- | Class     | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|-----------|--------|---------|---------|---------|---------|---------|
| Frequency | 3      | c       | d       | 11      | 5       | 5       |
- If the mean is 31 and median is  $\frac{340}{11}$ , then the value of  $\left[ \tan^{-1} \frac{2cd}{d^2 - c^2} \right]$  (where  $[.]$  represents greatest integer function) is
75. Let  $\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$  and  $\vec{c} = 5\hat{i} - 12\hat{j} - 13\hat{k}$ . If  $\vec{r}$  is a vector such that  $\vec{r} \times \vec{b} = \vec{b} \times (\vec{a} - \vec{c})$  and  $\vec{r} \cdot (\vec{a} + \vec{c}) = 0$  then the value of  $|\vec{r} - 36\vec{b}|^2$  is