PHYSICS

The external and internal diameters of a hollow cylinder are measured to be (5.23 ± 0.01)cm and (4.89 ± 0.01)cm. The thickness of the wall of the cylinder is

(A) (0.17 ± 0.02)cm (B) (0.17 ± 0.01)cm

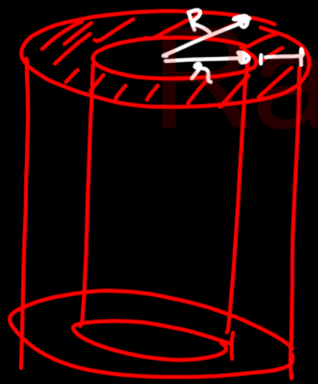
(C) (0.34 ± 0.01)cm (D) (0.34 ± 0.02)cm

JEE 1

$$\boxed{R = \frac{5.23}{2} \pm \frac{0.01}{2}}$$

$$\boxed{r = \frac{4.89}{2} \pm \frac{0.01}{2}}$$

$D \pm \Delta D$



$$\boxed{t = R - r}$$

$$t = \frac{5.23 - 4.89}{2} = 0.17$$

error

$$|\Delta t| = |\Delta R| + |\Delta r|$$

$$|\Delta t| = 0.01$$

Ans $\Rightarrow t \pm \Delta t = (0.17 \pm 0.01)$ cm

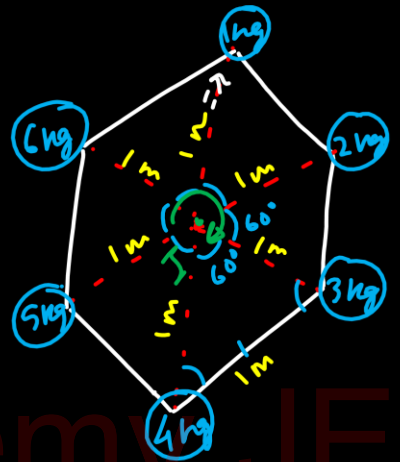
Six particle each of mass 1 kg, 2 kg, 3 kg, 4 kg, 5 kg and 6 kg are kept at the vertex of regular hexagon of side 1 m. The moment of inertia of system passing through its center and perpendicular to its plane is _____ kgm^2

(A) 11

(B) 20

(C) 21

(D) 22



$r = 1$

$I = 1r^2 + 2r^2 + 3r^2 + 4r^2 + 5r^2 + 6r^2$

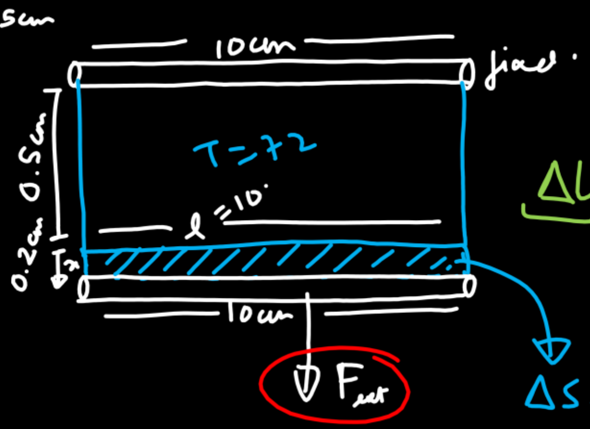
$\boxed{I = 21}$

CGS

A water film is made between two straight parallel wires of length 10 cm separated by $\boxed{5\ \text{mm}}$ (0.5 cm) from each other. If the distance between the wires is increased by $\boxed{2\ \text{mm}}$ (0.2 cm). How much work will be done? Surface tension for water is 72 dyne cm^{-1}

(A) 288 erg ✓ (B) 72 erg

(C) 144 erg ✗ (D) 216 erg



$\Delta U = T \Delta S$

$\Delta S = 2 l x$

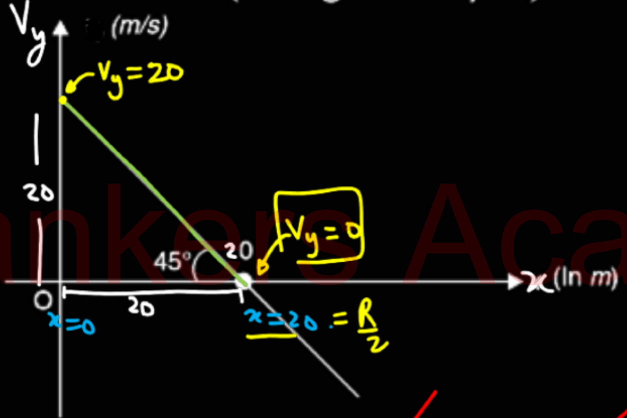
$W = \Delta U = T \cdot \Delta S$

$W = 72(2)(10)(0.2)$

$W = 72 \times 4 = \boxed{288}$

4

A ball is projected from ground making an angle θ with the horizontal (x-direction). The vertical component of its velocity (v_y) changes with its x coordinate according to the graph a shown in the figure. The angle of projection θ with horizontal is (Take $g = 10\ m/s^2$) $\theta = ?$



$$V_y = U_y + a_y t$$

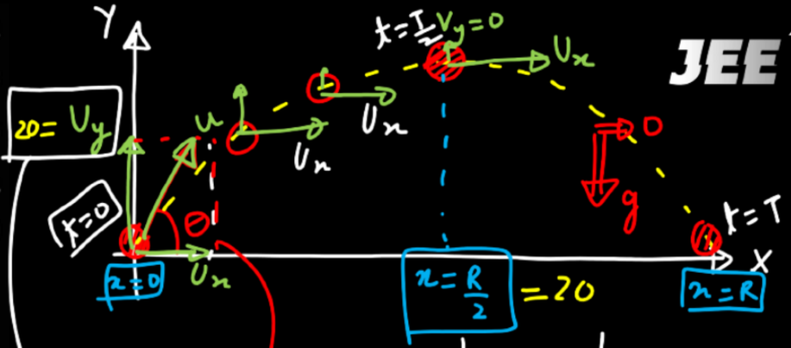
$$\boxed{V_y = 20 - 10t}$$

(A) $\tan^{-1}(4)$

(B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{4}\right)$

(D) $\tan^{-1}\left(\frac{1}{2}\right)$



$$\boxed{20 = U_y}$$

$$R = 40$$

$$T = \frac{2U_y}{g}$$

$$T = \frac{2(20)}{10}$$

$$\boxed{T = 4}$$

$$U_x T = 40$$

$$U_x(4) = 40$$

$$\boxed{U_x = 10}$$

$$\tan\theta = \frac{U_y}{U_x} = \frac{20}{10}$$

$$\tan\theta = 2$$

$$\boxed{\theta = \tan^{-1}(2)}$$

5

A satellite is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be $\boxed{R}$ itself. Then the $\boxed{\text{minimum increase in the speed}}$ required so that the satellite could escape from the gravitation pull of planet is

(A) $\sqrt{\frac{2GM}{R}}$

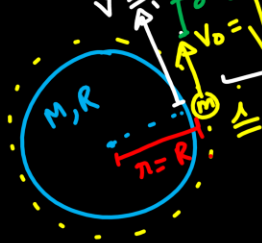
(B) $\sqrt{\frac{GM}{2R}}$

(C) $\sqrt{\frac{GM}{R}}$

(D) $\sqrt{\frac{GM}{R}}(\sqrt{2}-1)$

∞

$\boxed{\begin{matrix} U=0 \\ K=0 \end{matrix}}$



$v = ?$

diff = Ans

$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R}}$

$\boxed{v_0 \rightarrow \sqrt{2}v_0}$

escape.

$E_i = E_f$

$\left(-\frac{GM\cancel{m}}{R}\right) + \frac{1}{2}\cancel{m}v^2 = 0 + 0$

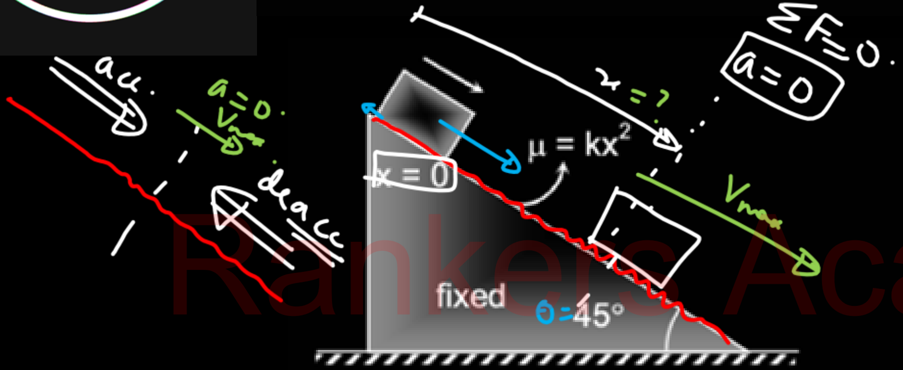
escape $\boxed{\infty}$

$v = \sqrt{\frac{2GM}{R}}$

Ans $\Rightarrow$ inc in speed $= v - v_0 = \sqrt{\frac{GM}{R}}(\sqrt{2}-1)$

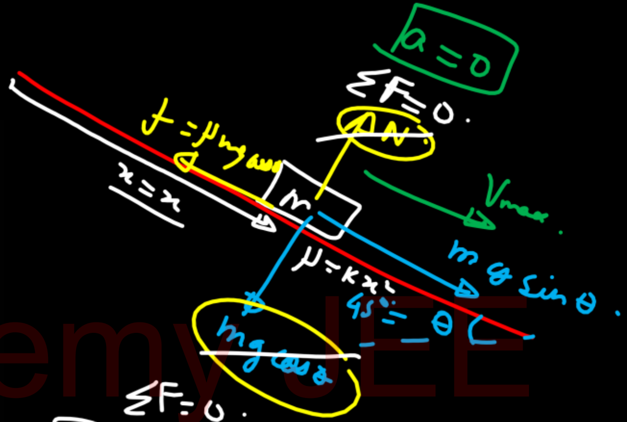
6

A block is released from rest on a rough inclined plane with coefficient of friction varying as $\mu = kx^2$; where k is constant. The maximum velocity of block for



(A) $x = \frac{1}{\sqrt{k}}$ (B) $x = \frac{2}{\sqrt{k}}$

(C) $x = \frac{4}{\sqrt{k}}$ (D) $x = \frac{3}{\sqrt{k}}$



$\Sigma F = 0$

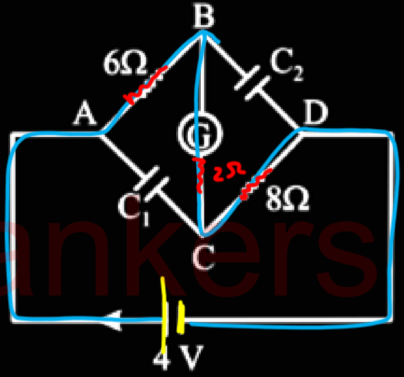
$\boxed{\mu}\cancel{mg}\cancel{\cos\theta} = \cancel{mg}\cancel{\sin\theta}$

$kx^2 = 1$

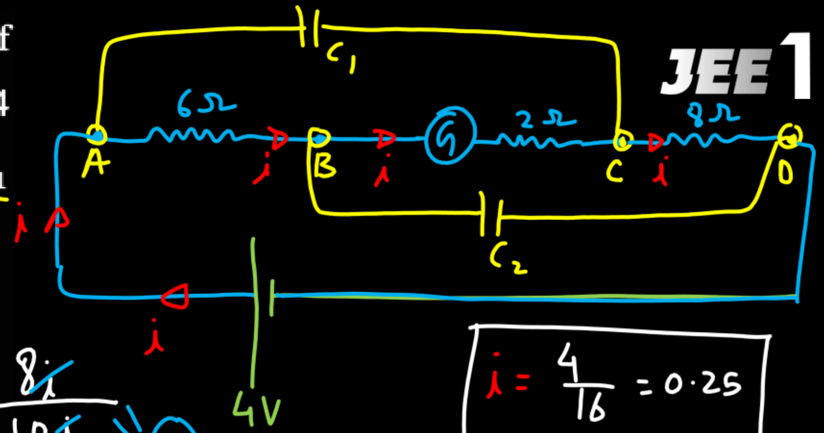
$\boxed{x = \frac{1}{\sqrt{k}}} \leftarrow V_{max}$

7

In this figure the resistance of the coil of galvanometer G is 2Ω. The emf of the cell is 4 V . The ratio of potential difference across C_1 and C_2 is:



$$\text{Ans} = \frac{V_{AC}}{V_{BD}} = \frac{8\cancel{i}}{10\cancel{i}} = \frac{4}{5}$$



$$\boxed{i = \frac{4}{16} = 0.25}$$

(A) 1

(B) $\frac{4}{5}$

(C) $\frac{3}{4}$

(D) $\frac{5}{4}$

8

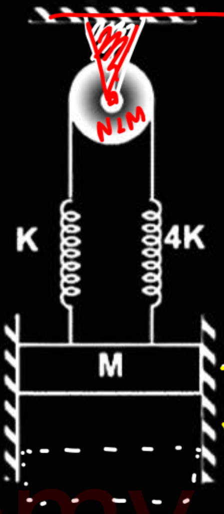
GOOD ADV

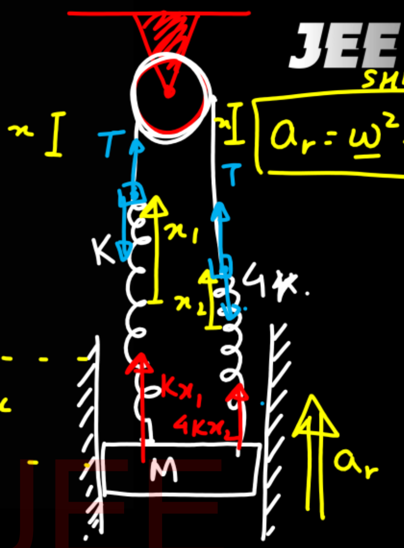
A block of mass M is suspended with the help of two light springs and light inextensible string that passes over an ideal pulley. Force constant of the spring are K and 4 K respectively as shown. (walls are frictionless). The time period for small oscillations is

remove $\vec{g}$

(A) $\pi\sqrt{\frac{5M}{K}}$ (B) $\frac{\pi}{2}\sqrt{\frac{5M}{K}}$

(C) $\pi\sqrt{\frac{M}{5K}}$ (D) $\pi\sqrt{\frac{2M}{5K}}$





JEE 1 SHM

$\boxed{a_r = \omega^2 x}$

$T = Kx_1 = 4Kx_2$

$\boxed{x_1 = 4x_2}$ — ②

$\boxed{Kx_1 + 4Kx_2 = Ma_r}$ — ①

inc $\boxed{2x = x_1 + x_2}$ — ③

② & ③ $\boxed{\begin{matrix} x_2 = 0.4x \\ x_1 = 1.6x \end{matrix}}$

$1.6Kx + 1.6Kx = Ma_r$

$\frac{3.2K}{M}x = a_r = \omega^2 x$ SHM

$\omega = \sqrt{\frac{\cancel{32}16K}{\cancel{10}5M}}$ $T = \frac{2\pi}{\omega}$

Ans.

$T = 2\pi\sqrt{\frac{5M}{16K}}$

$\boxed{T = \frac{\pi}{2}\sqrt{\frac{5M}{K}}}$

9

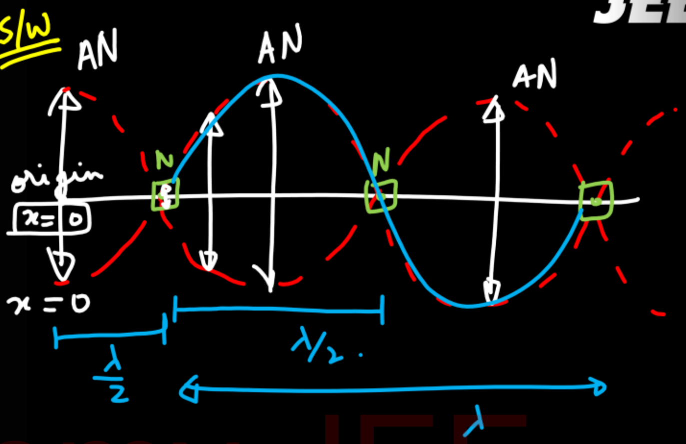
Two travelling wave produces standing wave represented by equation

$$y = \boxed{2\,(\text{mm})}\cos\left[\left(\frac{\pi}{4}\,\text{cm}^{-1}\right)x\right]\sin\left[\left(78.5\,\text{s}^{-1}\right)t\right]$$

k, ω, S/W

The node closest to the origin in the region $x > 0$ will be at $x =$ ___2___ cm

(A) 4
(B) 2
(C) 1
(D) 8



S/W

$$y_{s/w} = \boxed{2A\cdot\cos(kx)}\,\sin(\omega t)$$

$$y_{s/w} = \boxed{R}\,\sin(\omega t).$$

CGS

$k = \frac{2\pi}{\lambda}$

$\pi/4 = 2\pi/\lambda$

$\boxed{\frac{\lambda}{4} = 2}$

Ans $= \frac{\lambda}{4} = 2\,\text{cm}$

10

Two coherent sources of light interfere. The intensity ratio of two sources is $\boxed{1:4}$. For this interference pattern if the value of

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}}\left(\begin{array}{l} I_{max} = \text{Maximum intensity value} \\ I_{min} = \text{Minimum intensity value} \end{array}\right)$$

(A) 1.5

(B) 1.25

(C) 0.5

(D) 1

I_1

ϕ

I_2

Theory

$$\boxed{I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi}$$

max = 1

min = -1

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$\frac{I_1}{I_2} = \frac{1I}{4I}$

$\boxed{\begin{array}{l} I_1 = I \\ I_2 = 4I \end{array}}$

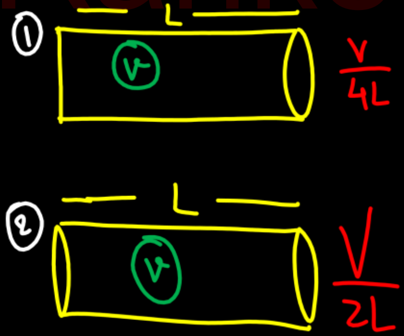
$\boxed{\begin{array}{l} I_{max} = 9I \\ I_{min} = I \end{array}}$

Ans $= \frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{10}{8} = \frac{5}{4} = 1.25$

A closed organ pipe and an open pipe of same length produce $\boxed{6 \text{ beats per sec}}$ when they are set into vibrations simultaneously with their fundamental frequency. If the length of each pipe is doubled, then the number of beats produced is

(A) 4 (B) 3

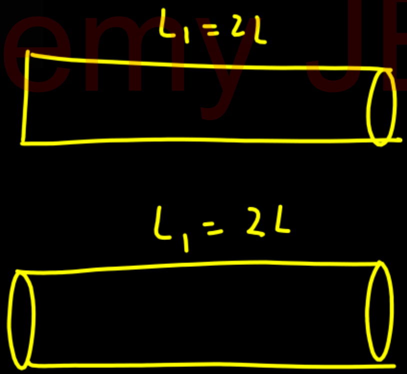
(C) 5 (D) 7



$f_B = |f_1 - f_2|$

$f_B = 6 = \frac{V}{2L} - \frac{V}{4L}$

$f_B = \frac{V}{4L} = 6$

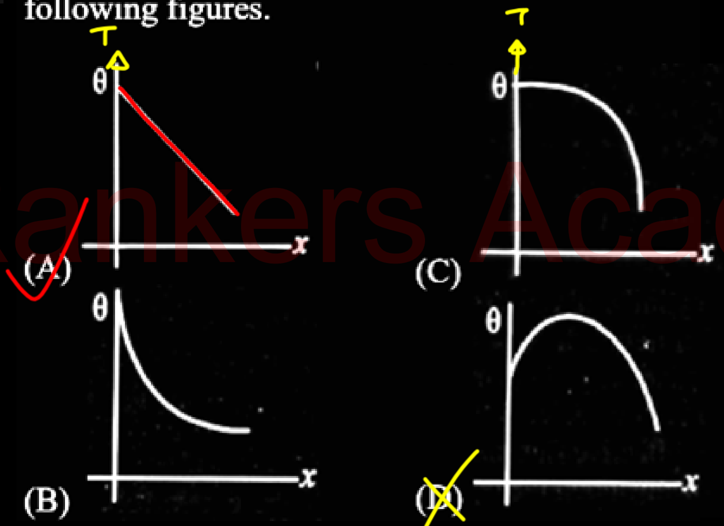


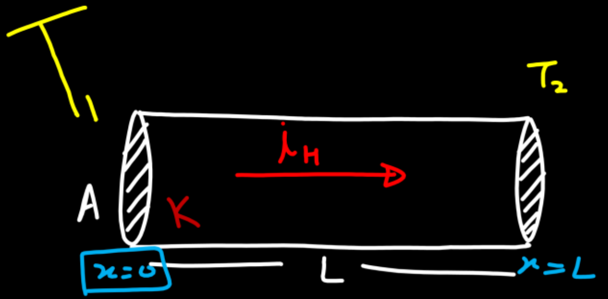
$f_B = \frac{V}{4L_1} = \frac{V}{4(2L)} = \frac{V}{8L} = 3$

12

A long metallic bar of uniform cross section is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures.

$i_H = \text{const}$





$$\boxed{i_H = \frac{KA}{L}(T_1 - T_2)}$$

$\boxed{\Delta T = T_2 - T_1}$

$$i_H = -KA\left(\frac{\Delta T}{L}\right)$$

const   const

$$i_H = -KA\left(\frac{dT}{dx}\right)$$

$$\boxed{\frac{dT}{dx} = -(\text{const})}$$

13

JEE1

Infinite number of charges of equal magnitude q, but alternate charge of opposite sign are placed along the x-axis at $x = 1, x = 2, x = 4, x = 8, \ldots$ and so on. The electric potential at the point $x = 0$ due to all these charges will be (If first charge is positive) –

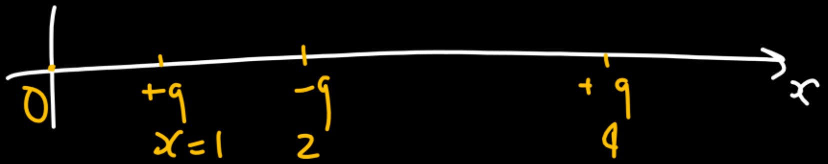
(A) $kq/2$ (B) $kq/3$

(C) $2kq/3$ (D) $3kq/2$

$$V = \frac{kq}{1} + \frac{k(-q)}{2} + \frac{kq}{4} - \frac{kq}{8} + \ldots -$$

$$= kq\left[1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \ldots\right]$$

$$= \frac{kq}{1-\left(-\frac{1}{2}\right)} = \frac{2kq}{3}$$



14

JEE 1

In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4^{th} division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1cm?

(A) 10 (B) 5

(C) 40 (D) 20

zero error = $n \times L.C.$

$0.04\,mm = 4 \times L.C.$

$L.C. = 0.01\,mm$ — ①

$50\,VSD = 49\,MSD$ — ②

$L.C. = MSD - VSD$

$= MSD - \frac{49}{50} MSD$

$0.01\,mm = \frac{1}{50} MSD$

$MSD = 0.5\,mm$ — ③

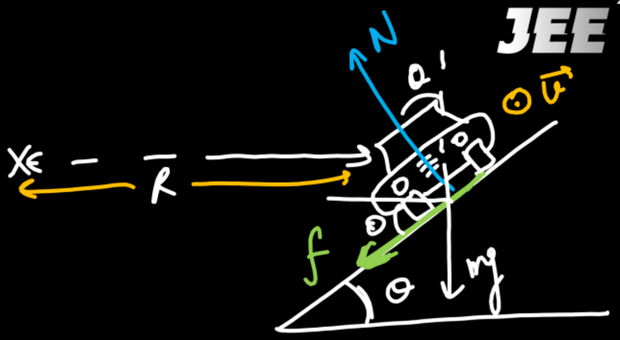
$N = \frac{1cm}{MSD} = \frac{10\,mm}{0.5\,mm} = 20$

15

A bike of mass 300 kg taking turn on a banked road of radius 100 m and banking angle 37^0 If co efficient of static friction is 0.25 . Then the maximum speed with which car can negotiate the turn safely: ($g = 10\ m/s^2$)

(A) 28.5 m/s (B) 35 m/s

(C) 40 m/s (D) 42.5 m/s



$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \quad \text{— (1)}$$

$$N\cos\theta - f\sin\theta = mg \quad \text{— (2)}$$

$$(1)\times\sin\theta + (2)\times\cos\theta \Rightarrow N = \frac{mv^2}{R}\sin\theta + mg\cos\theta \quad \text{— (3)}$$

$$(1)\times\cos\theta - (2)\times\sin\theta \Rightarrow f = \frac{mv^2}{R}\cos\theta - mg\sin\theta \quad \text{— (4)}$$

Put $\boxed{f \le \mu N}$ $\longrightarrow$

15

A bike of mass 300 kg taking turn on a banked road of radius 100 m and banking angle 37^0 If co efficient of static friction is 0.25 . Then the maximum speed with which car can negotiate the turn safely: ($g = 10\ m/s^2$)

(A) 28.5 m/s
(B) 35 m/s
(C) 40 m/s
(D) 42.5 m/s

$$\left(\frac{mv^2}{R}\cos\theta - mg\sin\theta\right) \le \mu\left(\frac{mv^2}{R}\sin\theta + mg\cos\theta\right)$$

$$(\cos\theta - \mu\sin\theta)\frac{mv^2}{R} \le mg(\mu\cos\theta + \sin\theta)$$

$$v \le \sqrt{\frac{gR(\mu\cos\theta + \sin\theta)}{(\cos\theta - \mu\sin\theta)}}$$

$$\boxed{v \le \sqrt{\frac{gR(\mu + \tan\alpha)}{(1 - \mu\tan\alpha)}}}$$

$$v_{max} = \sqrt{\frac{10\times100[0.25 + 0.75]}{(1 - 0.25\times0.75)}}$$

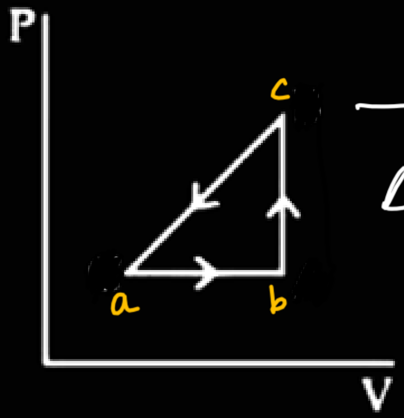
$$= \sqrt{\frac{16000}{13}}$$

$$= 35.08$$

$$\mu \to (-\mu) \quad v_{min} = \sqrt{\frac{gR(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$$

16

A sample of an ideal gas is taken through cyclic process abca as shown in the figure. The change in internal energy of the gas along the path ca is -180 J . The gas absorbs 250 J of heat along the path ab and 60 J along the path bc . The work done by the gas along the path abc is



JEE 1

(A) 120 J
(B) 130 J
(C) 140 J
(D) 100 J

$(\Delta U)_{ca} = -180$

$(\Delta U)_{ac} = +180$ — 1

$Q_{abc} = Q_{ab} + Q_{bc}$

$= 250 + 60$

$= 310\,J$ — 2

$\Delta U_{ac} = Q_{abc} - W_{abc}$

$W_{abc} = Q_{abc} - \Delta U_{ac}$

$= 310 - 180$

$= 130$

17

JEE1

The energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(A) 12.1 eV

(B) 36.3 eV

(C) 108.8 eV

(D) 122.4 eV

$$\Delta E = 13.6\, Z^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$= 13.6 \times 3^2 \left[\frac{8}{9}\right]$$

$$= 13.6 \times 8$$

$$= 108.8\, eV$$

18

JEE1

An ideal gas has an adiabatic exponent r. In some process its molar heat capacity varies as $\boxed{C = \frac{\alpha}{T}}$, where α is a constant. The work done by one mole of the gas during its heating from the temperature T_0 to the temperature ηT_0

(A) $\alpha \ell n\eta - \frac{RT_0(\eta-1)}{r-1}$ (B) $\alpha \ln \frac{1}{\eta} - \frac{RT_0(\eta-1)}{r-1}$

(C) $\alpha \ell n\eta + \frac{RT_0(\eta-1)}{r-1}$ (D) $\alpha \ln \frac{1}{\eta} + \frac{RT_0(\eta-1)}{r-1}$

$\gamma = r$ — (1)

$\boxed{W = Q - \Delta U}$

$= n\alpha \ln \frac{T_f}{T_i} - nC_v \Delta T$

$= \alpha \ln\left(\frac{\eta T_0}{T_0}\right) - \frac{R(\eta T_0 - T_0)}{(\gamma - 1)}$

$= \alpha \ln \eta - \frac{RT_0(\eta-1)}{\gamma - 1}$

$dQ = nC\,dT$

$= n\frac{\alpha}{T}dT$

$Q = n\alpha \ln \frac{T_f}{T_i}$ — (2)

18

JEE 1

An ideal gas has an adiabatic exponent r. In some process its molar heat capacity varies as $C = \frac{\alpha}{T}$, where α is a constant. The work done by one mole of the gas during its heating from the temperature T_0 to the temperature ηT_0

(A) $\alpha \ell n \eta - \frac{RT_0(\eta-1)}{r-1}$

(B) $\alpha \ln \frac{1}{\eta} - \frac{RT_0(\eta-1)}{r-1}$

(C) $\alpha \ell n \eta + \frac{RT_0(\eta-1)}{r-1}$

(D) $\alpha \ln \frac{1}{\eta} + \frac{RT_0(\eta-1)}{r-1}$

19

STATEMENT - 1: Energy is released in nuclear fission.

STATEMENT - 2: Total binding energy of the fission fragments is larger than the total binding energy of parent nucleus.

(A) Statement -1 is True, Statement -2 is True; Statement -2 is a correct explanation for Statement -1 .

(B) Statement -1 is True, Statement -2 is True; Statement -2 is NOT a correct explanation for Statement -1 .

(C) Statement -1 is True, Statement -2 is False.

(D) Statement -1 is False, Statement -2 is True.

$$\boxed{Q = B.E._p - B.E._R}$$

for $\boxed{Q > 0}$

$B.E_p > B.E_R$

fission fragment

A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is

(A) 2: 5
(B) 6: 5
(C) 4: 5
(D) 6: 4

$$\boxed{B_{arc} = \frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right)}$$

$$B_A = \frac{\mu_0 (2A)}{2(2cm)}\left(\frac{3\pi/2}{2\pi}\right) \quad \text{— (1)}$$

$$B_B = \frac{\mu_0 (3A)}{2(4cm)}\left(\frac{5\pi/3}{2\pi}\right) \quad \text{— (2)}$$

$$\frac{B_A}{B_B} = \frac{1}{\frac{2}{4}} \cdot \frac{\frac{3}{2}}{\frac{5}{3}} = 6:5$$

21

JEE1

Binding energy per nucleons of a nucleus ${}^{4}X$ is 8 MeV . It absorbs a neutron moving with KE 2 MeV and converts into Y , emitting a photon of energy 5 MeV . The binding energy per nucleus of Y (in MeV) is _____ .

$${}^{4}X + {}^{1}_{0}n \longrightarrow {}^{5}Y + h\nu$$

2 MeV

5 MeV

$$Q = B.E_Y - B.E_X$$

$$\Rightarrow 5 - 2 = 5 \times y - 4 \times 8$$

$$y = \frac{35}{5} = 7\,\text{MeV}$$

22

An alternating voltage $V(t) = 220\sin\left(\frac{50}{3}\pi t\right)$ volt here t is in seconds is applied to a purely resistive load of 50Ω The time taken (in ms) for the current to rise from half of the peak value to the peak value is:

JEE 1

$\frac{i_0}{2} \longrightarrow i_0$

$\phi = \frac{\pi}{6} \longrightarrow \frac{\pi}{2}$

$\Delta\phi = \omega \Delta t$

$\frac{\pi}{2} - \frac{\pi}{6} = \frac{50\pi}{3}\Delta t$

$\Rightarrow \Delta t = \frac{1}{50}\,s$

$= \frac{1000}{50}\,ms$

$= 20\,ms$

23

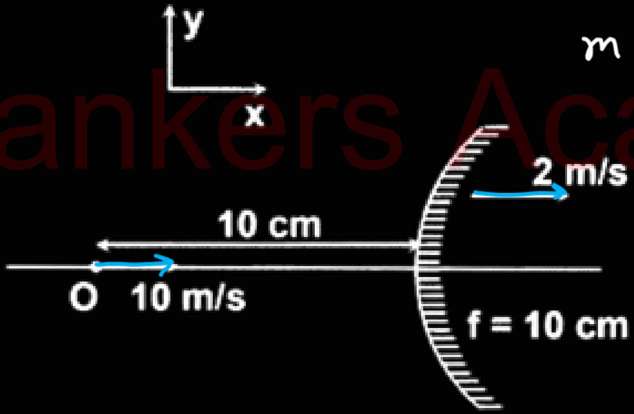
A boy can throw a stone up to a maximum height of 10 m . The maximum horizontal distance that the boy can throw the same stone up to will be ______ (in m)

$$h = \frac{u^2}{2g} \Rightarrow 10m = \frac{u^2}{2g} \quad \text{--- (1)}$$

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g} = 20\,m$$

24

Consider a particle moving towards convex mirror along its principal axis with velocity 10 m/s as shown in figure. The velocity of the mirror is 2 m/s. The velocity of particle with respect ground is _____ m/s.



$$m = \frac{f}{f-u} = \frac{+10}{+10-(-10)} = +\frac{1}{2} \quad (1)$$

$$\boxed{\vec{v}_{I,m} = -m^2\, \vec{v}_{O,m}}$$

$$\vec{v}_i - \vec{v}_m = -\left(\frac{1}{2}\right)^2 \left[\vec{v}_o - \vec{v}_m\right]$$

$$\vec{v}_i - 2 = -\frac{1}{4}\left[+10-(+2)\right]$$

$$\vec{v}_i - 2 = -\frac{1}{4} \times 8$$

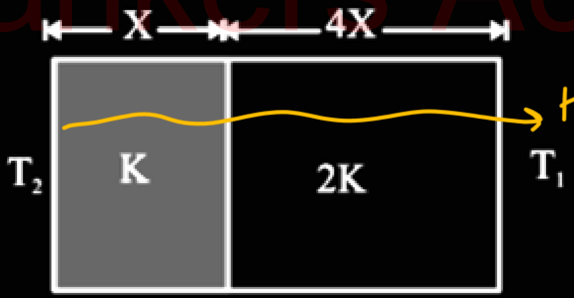
$$v_i - 2 = -2$$

$$\vec{v}_i = 0$$

w.r.t ground

25

The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficient of thermal conductivity K and 2K and thickness x and 4x respectively are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2-T_1)K}{x}\right) f$, then the value of $\frac{1}{f}$.



$$H = \frac{\Delta T}{R_{eq}}$$

$$R_{eq} = R_1 + R_2$$

$$\boxed{R_{th} = \frac{l}{KA}}$$

$$R_{eq} = \frac{x}{KA} + \frac{4x}{2KA}$$

$$= \frac{3x}{KA} \quad \text{—(1)}$$

$$H = \frac{T_2 - T_1}{\frac{3x}{KA}}$$

$$= \frac{KA(T_2 - T_1)}{\boxed{3}x}$$

CHEMISTRY

The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Mn(H_2O)_6]Cl_2$ respectively are :

SL

+2 WL

(Atomic No: –Ru = 44, Mn = 25)

(A) $t_{2g}^6 e_g^0$ and $t_{2g}^5 e_g^0$

(B) $t_{2g}^6 e_g^0$ and $t_{2g}^3 e_g^2$

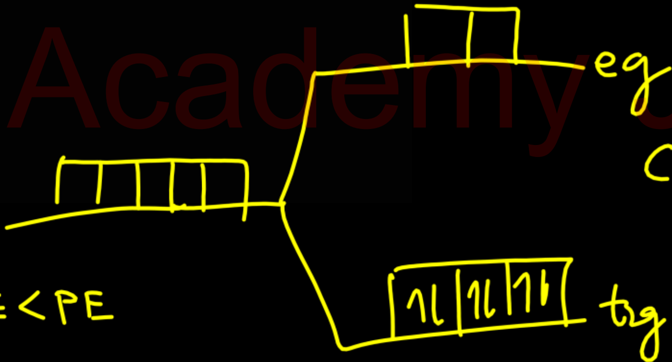
(C) $t_{2g}^4 e_g^2$ and $t_{2g}^3 e_g^0$

(D) $t_{2g}^4 e_g^2$ and $t_{2g}^3 e_g^2$

Fe(26)

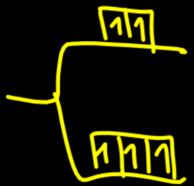
$Ru^{+2}(44) : 3d^6 4s^0$

$= t_{2g}^6$



CFSE > PE

$Mn^{+2} : 3d^5$



CFSE < PE

$t_{2g}^3 e_g^2$

Which of the following characteristic of the elements of boron family increases on moving down the group?

(A) Stability of +3 oxidation state

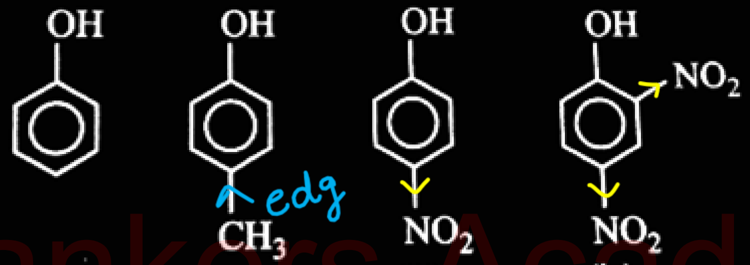
(B) Number of electrons in the outermost orbits of the atoms

(C) Nuclear charge of the atoms due to ↑ in no. of protons

(D) Acidic nature of the oxides of the elements

3

In the following compounds order of acidic strength is



(i) (ii) (iii) (iv)

(A) III > IV > I > II

(B) I > IV > III > II

(C) II > I > III > IV

(D) IV > III > I > II

NO_2 = ewg

ewg ↑ ses acidity

edg ↓ acidity

IV > III > I > II

4

In a 0.2 molal aqueous solution of a weak acid HX , the degree of ionization is 0.3 . Taking K_f for water as 1.85 , the freezing point of the solution will be nearest to :

(A) $-0.360°C$

(B) $-0.260°C$

(C) $+0.480°C$

(D) $-0.480°C$

$HX \rightleftharpoons H^+ + X^-$

$n = 2$

For dissociating electrolyte

$\alpha = \frac{i-1}{n-1}$

$0.3 = \frac{i-1}{1}$

$i = 0.3 + 1 = 1.3$

$\Delta T_f = i m k_f = 1.3 \times 0.2 \times 1.85$

$= 0.480°C$

$m = 0.2$

$k_f = 1.85$

$T_f = 0 - 0.48 = -0.480°C$

5

JEE 1

Assertion (A) : 1 mole of FeC_2O_4 is oxidised by 0.6 mole of MnO_4^- in acidic medium

Reason (R) : MnO_4^- oxidises both Fe^{2+} as well as $C_2O_4^{2-}$

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

$\overset{+2}{Fe}\overset{+3}{C_2}O_4 + MnO_4^-$

v.f = 1 + 2 = 3

$\longrightarrow \overset{+3}{Fe} + \overset{+4}{C}O_2$

Applying Law of equivalence

$N_1 V_1 = N_2 V_2$

$v.f \times M_1 \times V_1 = v.f \times M_2 \times V_2$

$n_1 \times (\text{no. of moles})_{FeC_2O_4} = n_2 \times (\text{no of moles})_{MnO_4^-}$

$\Rightarrow 3 \times 1 = 5 \times x$

$x = \frac{3}{5} = 0.6$ moles

6

JEE 1

Which of the following electronic configurations of valence electrons is in correct decreasing order of $\Delta_{eg}H^{\ominus}$?

(a) $3s^2 3p^4$ (S)

(b) $2s^2 2p^4$ (O)

(c) $2s^2 2p^3$ (N)

(d) $2s^2 2p^5$ (F)

(A) C > A > B > D

(B) C > B > A > D

(C) B > C > D > A

(D) A > B > C > D

N < O < F

S

F < S < O < N

-333 +31

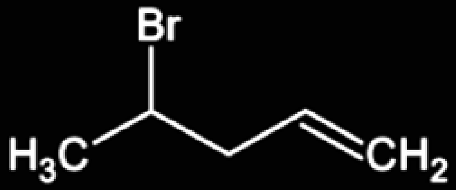
7

Which of the following will be the correct product (P) of the reaction?

Allylic Bromination

(A)


(B)


(C)


(D)

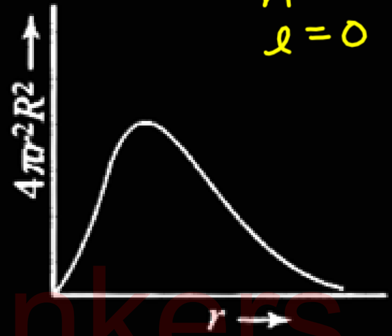

JEE 1

Select the correct plot of radial probability function ($4\pi r^2R^2$) for 2s-orbital.

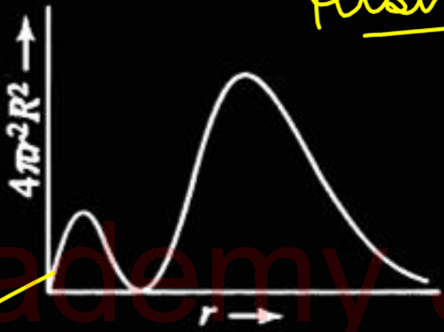
$n=2$
$l=0$

radial nodes $= n-l-1$
$= 2-0-1 = 1$

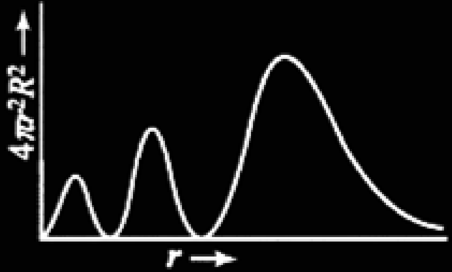
Rising Peaks



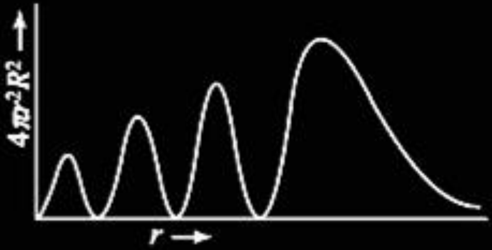
(A)



(C)



(B)

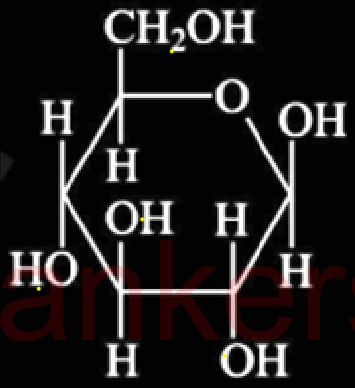


(D)

9

JEE 1

The incorrect statements about above structure of glucose are



(A) It is a pyranose form ✓

(B) It is a furanose form ✗

(C) It is a β-anomer ✓

(D) It is a D-sugar ✓

10

JEE 1

$\overset{+2}{[Cr(H_2O)_6]Cl_2}$ and $\overset{+2}{[Fe(H_2O)_6]Cl_2}$ are reducing in nature because

(A) Cr^{+2} and Fe^{+2} will form stable Cr^{+3} and Fe^{+3} stable ion

(B) Both Cr and Fe will form +3 and +4 oxidation states respectively

(C) Due to Cl^- both will be oxidised

(D) Due to mixture of H_2O and Cl^-

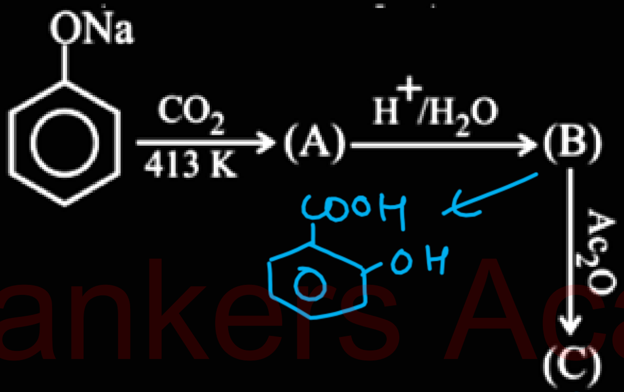
$$Cr^{+2} \longrightarrow Cr^{+3}$$

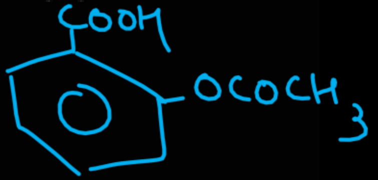
$3d^4$ $\qquad$ $3d^3$ (t_{2g} half filled)

$$Fe^{+2} \longrightarrow Fe^{+3}$$

$3d^6$ $\qquad$ $3d^5$ (half-filled stability)

The end product of the following sequence of reactions is :-





(A) Salicylic acid

(B) Salicyladehyde

(C) Phenyl acetate

(D) Aspirin

Which of the following option is CORRECT?

(A) $3p - 3p > 3p - 3d > 3d - 3d$ (π bond strength)

(B) Ethanol < Glycerol < ethylene glycol (Viscosity)

$\underset{OH}{CH_2}-\underset{OH}{CH}-\underset{OH}{CH_2}$

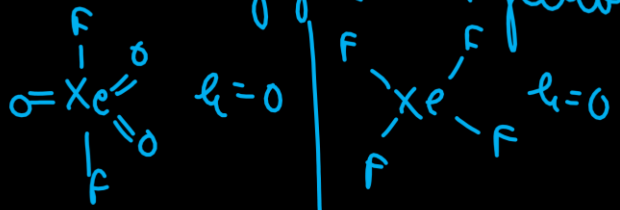
(C) $XeO_3F_2 > XeF_4 = XeF_2$ (Dipole moment)

(D) In 13[th] group Al has least standard reduction potential

(A) $p\pi - d\pi$ more stronger due to effective overlap

(B) Ethanol < glycol < Glycerol [Extensive H-bonding]

(C)



$F-Xe-F$ $\mu = 0$

Table 11.2 Atomic and Physical Properties of Group 13 Elements

Property		Element				
		Boron B	Aluminium Al	Gallium Ga	Indium In	Thallium Tl
Atomic number		5	13	31	49	81
Atomic mass(g mol^{-1})		10.81	26.98	69.72	114.82	204.38
Electronic Configuration		$[He]2s^22p^1$	$[Ne]3s^23p^1$	$[Ar]3d^{10}4s^24p^1$	$[Kr]4d^{10}5s^25p^1$	$[Xe]4f^{14}5d^{10}6s^26p^1$
Atomic radius/pm[a]		(88)	143	135	167	170
Ionic radius M^{3+}/pm[b]		(27)	53.5	62.0	80.0	88.5
Ionic radius M^+/pm		-	-	120	140	150
Ionization enthalpy ($kJ\ mol^{-1}$)	Δ_iH_1	801	577	579	558	589
	Δ_iH_2	2427	1816	1979	1820	1971
	Δ_iH_3	3659	2744	2962	2704	2877
Electronegativity[c]		2.0	1.5	1.6	1.7	1.8
Density /g cm^{-3} at 298 K		2.35	2.70	5.90	7.31	11.85
Melting point / K		2453	933	$\boxed{303}$	430	576
Boiling point / K		3923	2740	2676	2353	1730
$E^\ominus$/ V for (M^{3+}/M)		-	–1.66	–0.56	–0.34	+1.26
$E^\ominus$ / V for (M^+/M)		-	+0.55	-0.79(acid) –1.39(alkali)	–0.18	–0.34

13

JEE1

Which of the following reaction does not form ethylene?

(A) $I-CH_2-CH_2-I \xrightarrow[\Delta]{Zn}$ $CH_2=CH_2 + ZnI_2$

Zn

(B) $HC\equiv CH + H_2 \xrightarrow{Ni}$ CH_3-CH_3 (Alkane)

(C) $C_2H_5OH \xrightarrow[\text{High temp}]{\text{Conc.}H_2SO_4}$ $H_2C=CH_2 + H_2O$

$-H_2O$

(D) $C_2H_5Cl + KOH \xrightarrow[\Delta]{\text{Alcohol}}$ $H_2C=CH_2$

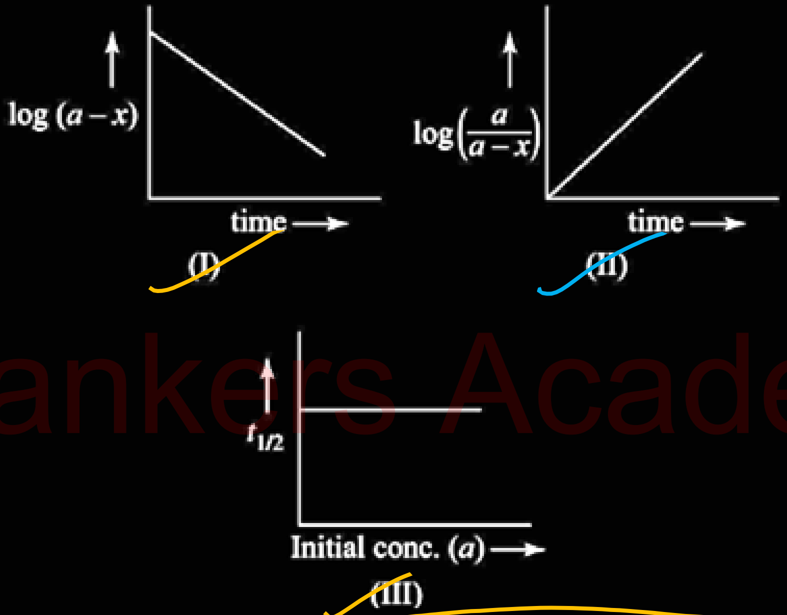
$CH_2-CH_2-Cl \xrightarrow[\ominus OH]{\text{Alc.}}$

H

$OH^{\ominus}$

14

Which of the following graphs is/are correct for the first order reaction?



(A) I, II only

(B) II, III only

(C) I, II, III

(D) I, III only.

$$\boxed{t_{1/2} = \frac{0.693}{K} = \text{const}}$$

$$K = \frac{2.303}{t}\log\left(\frac{a}{a-x}\right)$$

$$\log\left(\frac{a}{a-x}\right) = \frac{K}{2.303}t$$

$$\boxed{y = mx}$$

$$\log a - \log(a-x) = \frac{K}{2.303}t$$

$$\log(a-x) = \frac{-K}{2.303}t + \log a$$

$$y = mx + c$$

15

Identify the alkyne in the following sequence of reactions Alkyne

$$\text{Alkyne} \xrightarrow[\text{Lindlar's Catalyst}]{H_2} A \xrightarrow[H_2O]{O_3/Zn} \underset{\text{Only}}{B} \xleftarrow[\text{Process}]{\text{Wacker}} H_2C=CH_2$$

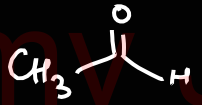
(A) $CH_3-C\equiv C-CH_3$

(B) $CH_3-CH_2-C\equiv CH$

(C) $CH_2=CH-C\equiv CH$

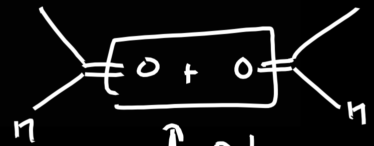
(D) $CH\equiv C-CH_2-C\equiv CH$

$\downarrow PdCl_2 \,|\, CuCl_2$



B

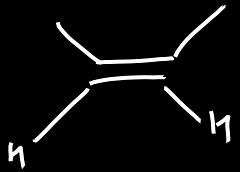
\#



$\Updownarrow O_3/Zn$

$H_3C-C\equiv C-CH_3 \xrightarrow[CaCO_3]{H_2/Pd}$

P



A

16

Statement-1: In complex $[Cr(NH_3)_4BrCl]Cl$, the 'spin only' magnetic moment is close to 1.73 B.M. ×

Statement-2: All known mononuclear octahedral complexes of chromium(III), irrespective of the strength of the ligand field, must have three unpaired electrons. ✓

(A) Statement-1 is True, Statement- 2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement- 1 is True, Statement- 2 is True; Statement- 2 is NOT a correct explanation for Statement- 1.

(C) Statement- 1 is True, Statement- 2 is False

(D) Statement- 1 is False, Statement- 2 is True ✓

Cr^{+3}: $3d^3$

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}$

3d

$n = 3$

$\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15}$ B.M.

17

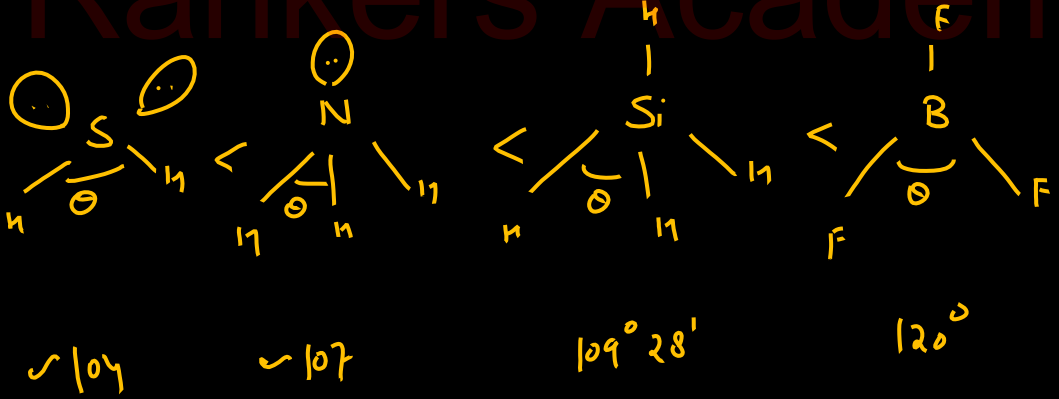
The correct order of bond angles in H_2S, NH_3, BF_3 and SiH_4 is :-

(A) $H_2S < NH_3 < SiH_4 < BF_3$

(B) $NH_3 < H_2S < SiH_4 < BF_3$

(C) $H_2S < SiH_4 < NH_3 < BF_3$

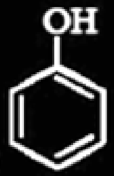
(D) $H_2S < NH_3 < BF_3 < SiH_4$



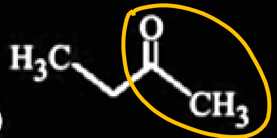
Match the coloumn

List-I (Compound) List-II (Test)

(P)  (1) +ve Iodoform test

(Q)  (2) +ve Isocynide test

(R)  (3) +ve Liebermann nitroso test

(S)  (4) +ve Tollen's test

(A) P → 1, Q → 3, R → 2, S → 4

(B) P → 4, Q → 2, R → 3, S → 1

(C) P → 1, Q → 2, R → 3, S → 4

(D) P → 4, Q → 3, R → 2, S → 1

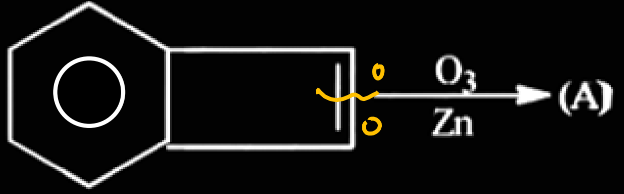
(P) → (4)

(Q) → 3

(R) 2

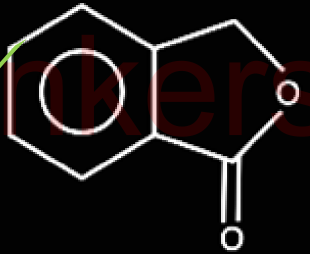
(S) → 1

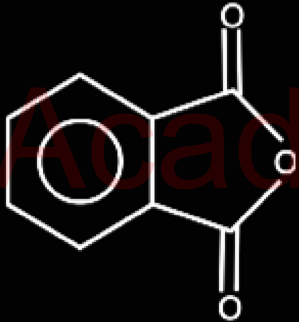
19

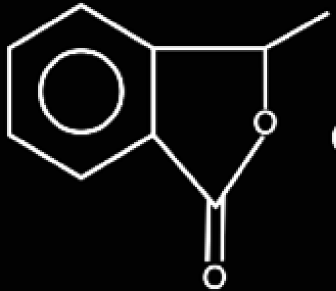


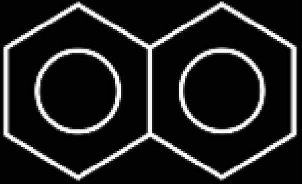
$\xrightarrow{\text{conc. KOH}}$ (B) $\xrightarrow[\Delta]{H^+}$ (C);

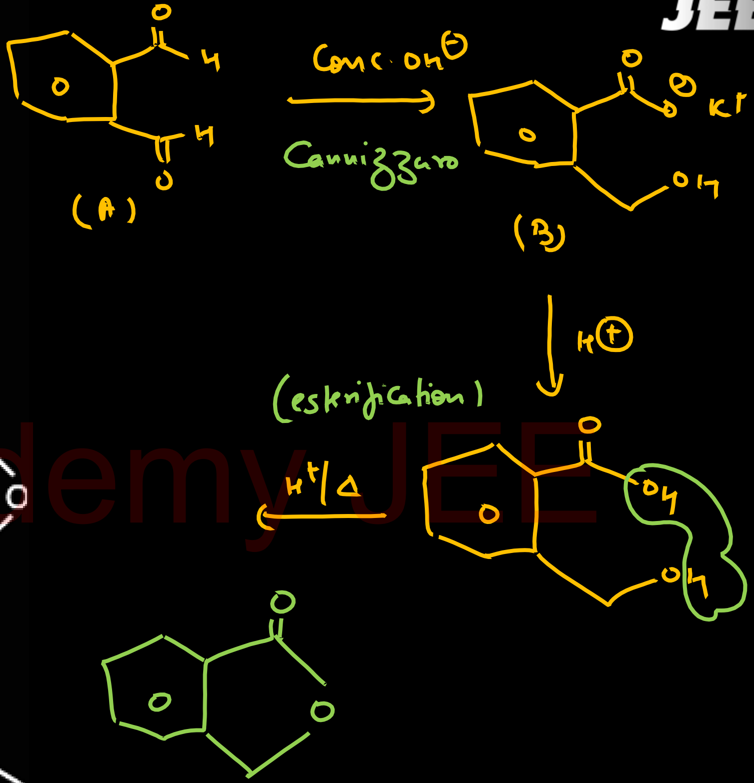
Product (C) is

(A) 

(B) 

(C) 

(D) 



20

When sodium nitroprusside is added to Na_2S (alkaline) then complex compound solution is formed whose colour is

(A) Red

(B) Black

(C) violet

(D) White

On treating sodium fusion extract with sodium nitroprusside, appearance of a violet colour further indicates the presence of sulphur.

$$S^{2-} + [Fe(CN)_5NO]^{2-} \longrightarrow \underset{\text{Violet}}{[Fe(CN)_5NOS]^{4-}}$$

+1

21

JEE 1

2 mole of compound A (contain C & H only) required 480 gm of oxygen for complete combustion at standard state. If standard enthalpy of combustion of A(ℓ) is $\boxed{3400\ kJ/}$ mole, while enthalpy of formation $CO_2(g)$, $H_2O(\ell)$ and A(ℓ) are −400, −300 and 100 kJ/ mole respectively. Then molar mass (gm/mole) of compound A is (nearest integer)

$-ve$

$\Delta_c H$ = Exothermic

$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

2 mole. $n_{O_2} = \frac{480}{32} = 15$ mole.

$\Delta_c A = x\,\Delta_f^\circ(CO_2) + \frac{y}{2}\Delta_f^\circ(H_2O) - \Delta_f^\circ(A)$

$2\left(x+\frac{y}{4}\right) = 15$

$2x + \frac{y}{2} = 15$

$4x + y = 30$ —— (1)

$x(-400) + \frac{y}{2}(-300) - 100$

$= -3400$

$40x + 15y + 10 = 340$

$40x + 15y = 330$

$8x + 3y = 66$ —— (2)

$\boxed{x = 6,\ y = 6}$

A = C_6H_6

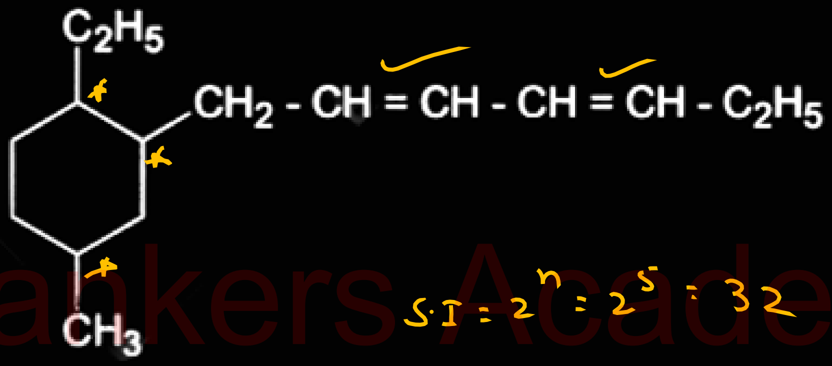
Mol.wt = [12×6] + [6×1]

$\boxed{= 78}$

22

JEE1

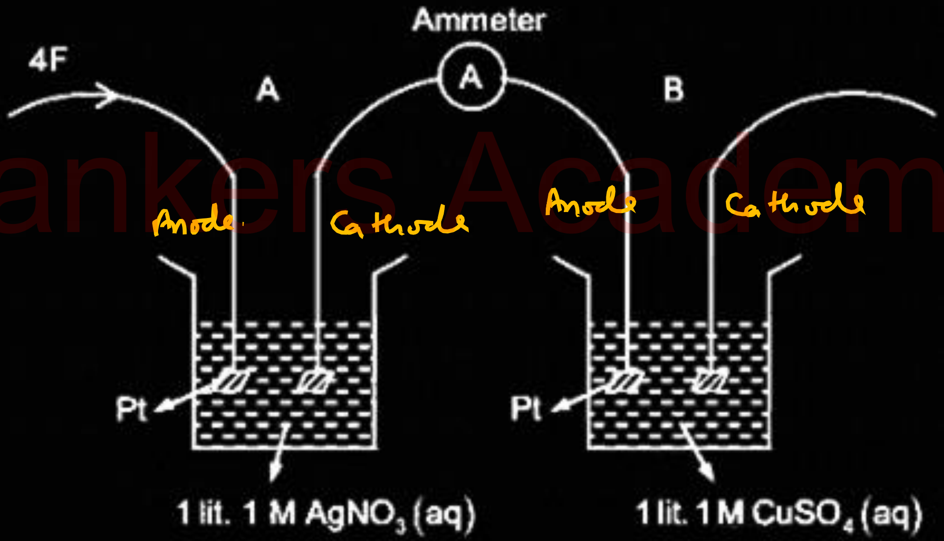
How many maximum number of stereoisomer(s) is/are possible for the above compound?



$S.I = 2^n = 2^5 = 32$

23

4 Faraday charge is passed through electrode A and B which are connected in series. Find out net mass (gm) deposited at electrodes A and B. (At. Mass Ag = 108, Cu = 63.5)



A

$Ag^{+} + e^{-} \longrightarrow Ag$

$1F \longrightarrow 108\ g$

$4F \longrightarrow [108 \times 4]$ gram

B

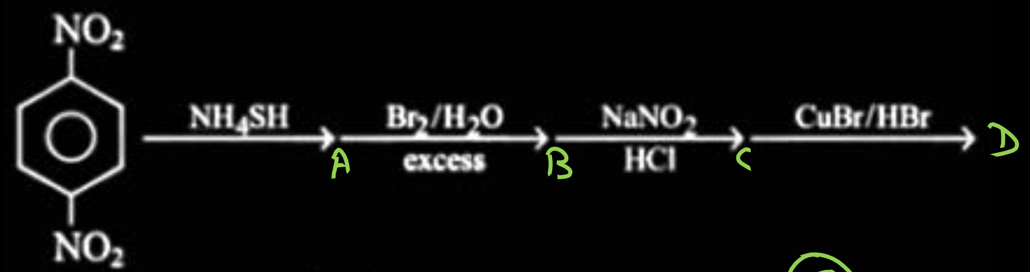
$Cu^{+2} + 2e^{-} \longrightarrow Cu(s)$

$2F \longrightarrow 63.5$

$4F \longrightarrow [63.5 \times 2]\ g.$

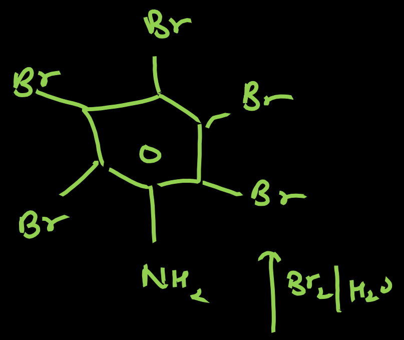
Net mass : $432 + 127 = \underline{559\ g}$

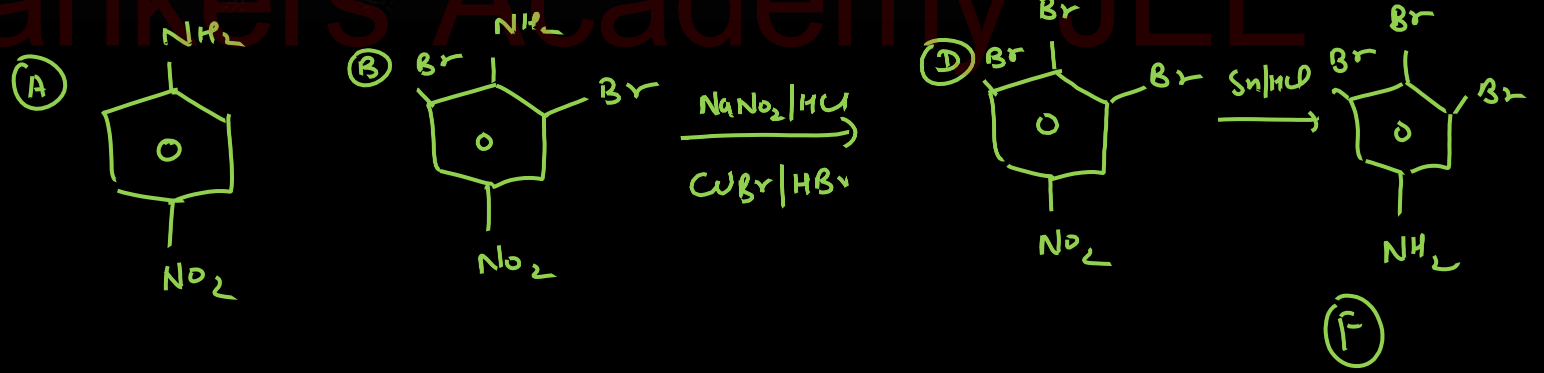
24



$E \xrightarrow{Sn/HCl} F \xrightarrow{Br_2/H_2O}$ major end product. 5

Find the total number of halogen atoms present in the major end product.





25

Consider the reaction:

$AB_2(g) \rightleftharpoons AB(g) + B(g)$. If the initial pressure of AB_2 is 500 torr and equilibrium pressure is 600 torr, equilibrium constant K_p in terms of torr is (nearest integer)

JEE 1

$$AB_2(g) \rightleftharpoons AB(g) + B(g)$$

$t=0$ 500 torr – –

$t=t$ $(500-x)$ x x

$$(500-x) + x + x = 600$$

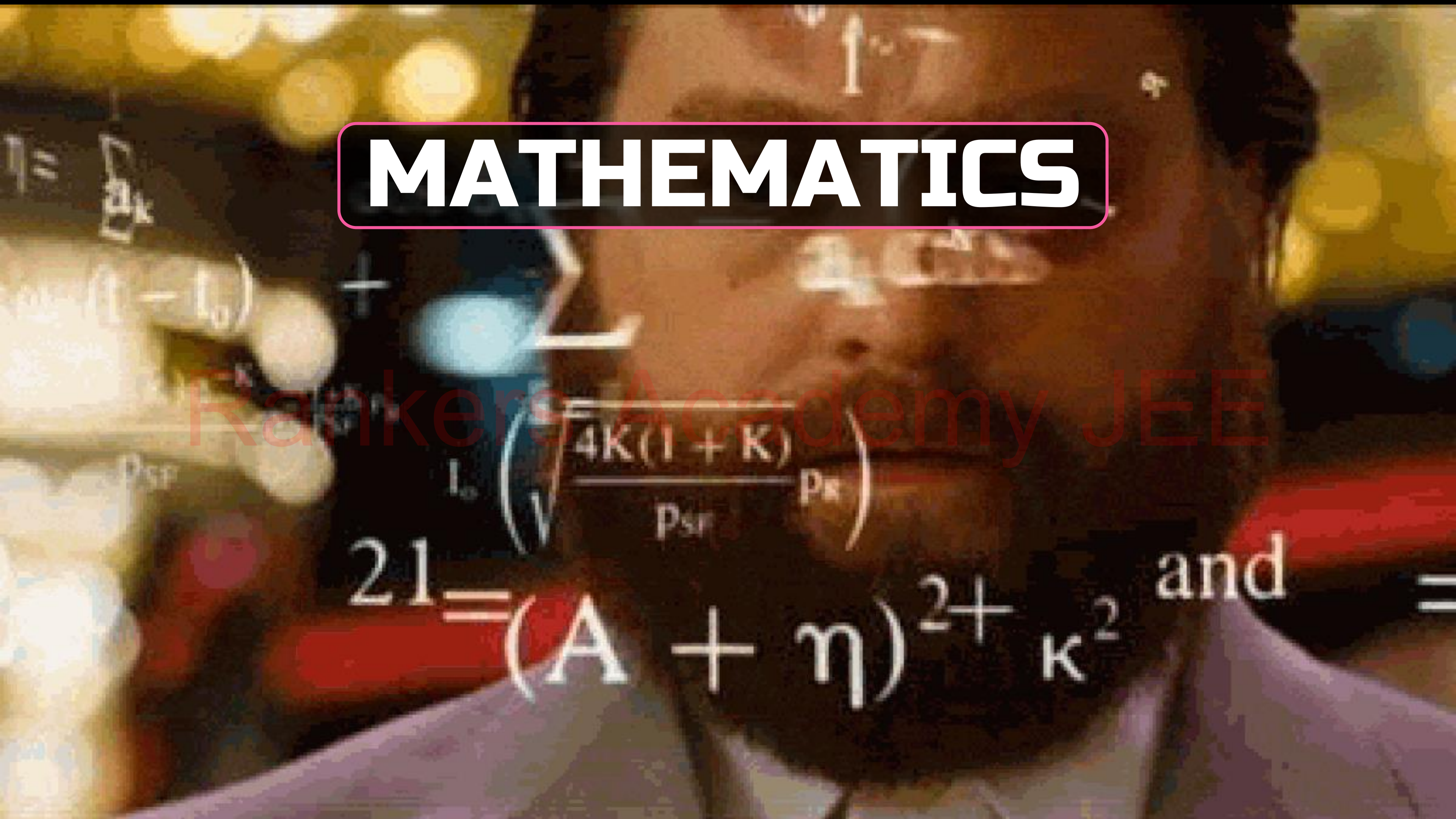
$$\boxed{x = 100 \text{ torr}}$$

$$K_p = \frac{x \cdot x}{(500-x)}$$

$$= \frac{100 \times 100}{(500-100)} = \frac{100 \times 100}{400}$$

$$\boxed{K_p = 25}$$

MATHEMATICS



Let $P = \{2,4,6,8\}$ and R be a relation on P defined as $R = \{(x,y): x+y \le 10\}$. If m is number of elements in R and n is minimum number of elements to be added to make 'R' is equivalence then 'mn' is

(A) 60 ✓ (B) 48

(C) 20 (D) 24

$m = 10$

$\boxed{(2,2)}$ $\boxed{(2,4)}$ $\boxed{(2,6)}$ $\boxed{(2,8)}$

$\boxed{(4,2)}$ $\boxed{(4,4)}$ $\boxed{(4,6)}$ $\boxed{(4,8)}$

$\boxed{(6,2)}$ $\boxed{(6,4)}$ $\boxed{(6,6)}$ $\boxed{(6,8)}$

$\boxed{(8,2)}$ $\boxed{(8,4)}$ $\boxed{(8,6)}$ $\boxed{(8,8)}$

Ref : $\longrightarrow 2 \longrightarrow (6,6), (8,8)$

Sym : $2 \longrightarrow (8,4), (8,6)$

Trans: $2 \longrightarrow (4,8), (6,8)$

$(4,2)$
$(2,8)$
$\overline{(4,8)}$

$6\ 2$
$2\ 8$
$\overline{(6,8)}$

$\boxed{n = 6}$

Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors having magnitudes 1,2,3 respectively satisfy the relation $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = 6$. If $\vec{d}$ is a unit vector coplanar with $\vec{b}$ and $\vec{c}$ such that that $\vec{b} \cdot \vec{d} = 1$ then the value of $|(\vec{a} \times \vec{c}) \cdot \vec{d}|^2 + |(\vec{a} \times \vec{c}) \times \vec{d}|^2$ is

(A) 9   (B) 3

(C) 27   (D) $\frac{9}{2}$

Ans: $|\vec{a} \times \vec{c}|^2 |\vec{d}|^2$

$(1)^2 (3)^2 (1)^2 (1)^2 = 9$

$|\vec{a}| = 1$

$|\vec{b}| = 2$

$|\vec{c}| = 3$

$|(\vec{a} \times \vec{b}) \cdot \vec{c}| = 6$

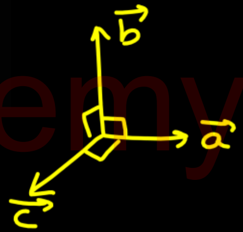
$|\vec{a} \times \vec{b}| |\vec{c}| \cos\theta = 6$ where $\vec{a} \times \vec{b} \wedge \vec{c} = \theta$

$|\cancel{\vec{a}}| |\cancel{\vec{b}}| \sin\phi \, |\cancel{\vec{c}}| \cos\theta = \cancel{6}\ 1$ $\vec{a} \wedge \vec{b} = \phi$

$\sin\phi \cos\theta = 1$

$\sin\phi = \cos\theta = 1$

$\phi = \frac{\pi}{2}$, $\theta = 0$



3

If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ then:

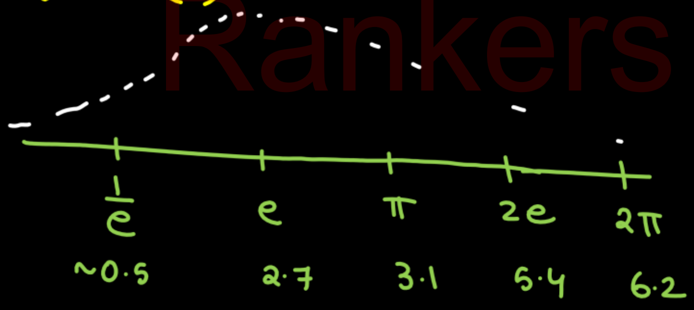
~~(A)~~ $(2e)^{\pi} > \pi^{(2e)}$ (B) $e^{\pi} < \pi^{e}$

(C) $e^{\pi} > \pi^{e}$ ✓ ~~(D)~~ $e^{2\pi} < (2\pi)^{e}$

$f(x) = x^{\frac{1}{x}} \longrightarrow$ Max at $x = e$

$f(x) = \left(\frac{1}{x}\right)^{x} \longrightarrow$ " " $x = \frac{1}{e}$

♡



(A) check $2e^{\pi} > \pi^{2e} \Rightarrow 2e^{\frac{1}{2e}} \cancel{>} \pi^{\frac{1}{\pi}}$

$2e > \pi$ ↓

$2e^{\frac{1}{2e}} < \pi^{\frac{1}{\pi}}$

(D) $e < 2\pi$

$e^{\frac{1}{e}} > 2\pi^{\frac{1}{2\pi}}$

$e^{2\pi} > 2\pi^{e}$

(B)/(C)

$e < \pi$

$e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$

$e^{\pi} > \pi^{e}$

4

The number of three term increasing geometrical progressions comprising distinct natural numbers less than or equal to 100, with common ratio as a natural number is

(A) 106 (B) 53

(C) 47 (D) 104

JEE 1

Ans : 25 + 11 + 6 + 4 + 2 + 2 + 1 + 1 + 1

a, ar, ar^2 ; $r > 1$ $\boxed{a \in N \\ r \in N}$

$ar^2 \le 100$

$\boxed{a \le \frac{100}{r^2}}$

$r = 2 \rightarrow$ 1, 2, 4; 2, 4, 8; 3, 6, 12; ⋮ 25, 50, 100 $\} = 25 = \left[\frac{100}{2^2}\right]$

$r = 3 \rightarrow$ 1, 3, 9; 2, 6, 18; ⋮ 11, 33, 99 $\} \Rightarrow 11 = \left[\frac{100}{3^2}\right]$

$r = 4 \rightarrow$ 1, 4, 16; 2, 8, 32; ⋮ 6, 24, 96 $\} \Rightarrow 6 = \left[\frac{100}{4^2}\right]$

$r = 5 \rightarrow \left[\frac{100}{5^2}\right] = 4$

$r = 6 \rightarrow \left[\frac{100}{6^2}\right] = 2$

$r = 7 \rightarrow \left[\frac{100}{7^2}\right] = 2$

$r = 8 \rightarrow \left[\frac{100}{8^2}\right] = 1$

$r = 9 \rightarrow \left[\frac{100}{9^2}\right] = 1$

$r = 10 \rightarrow \left[\frac{100}{10^2}\right] = 1$

$r = 11 \rightarrow \left[\frac{100}{11^2}\right] = 0$

5

JEE 1

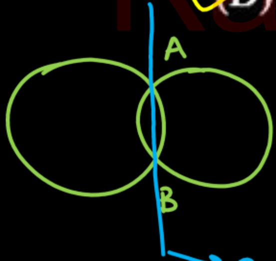
If the circles $x^2 + y^2 + 10\alpha x + \beta y + \alpha = 0$ and $x^2 + y^2 - 5\alpha x + \gamma y - 1 = 0$ intersect in two distinct points A and B, then the line $15x + \delta y - \alpha = 0$ passes through A and B for

(A) infinity many values of α

(B) exactly two values of α

(C) exactly one value of α

(D) no value of α



$\rightarrow$ common chord

$S_1 - S_2 = 0$

$15\alpha x + (\beta - \gamma)y + (\alpha + 1) = 0$

$15x + \delta y - \alpha = 0$

compare

$\Rightarrow \dfrac{15\alpha}{15} = \dfrac{\beta - \gamma}{\delta} = \dfrac{\alpha + 1}{-\alpha}$

$\Rightarrow \alpha = \dfrac{\alpha + 1}{-\alpha}$

$\Rightarrow \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \boxed{x^2 + x + 1 = 0}$ ω, ω^2

no real value of α

6

JEE1

The maximum value of the function $f(x) = \frac{4\cot^{-1} x}{\pi} - \frac{\pi}{4\cot^{-1}(-x)}$ occurs at x equals to

(A) -1 (B) 0

(C) 1 (D) none of these

$$f(x) = \frac{4}{\pi}\cot^{-1} x - \frac{\pi}{4\cot^{-1}(-x)}$$

$$= \frac{4}{\pi}(\pi - \cot^{-1}(-x)) - \frac{\pi}{4\cot^{-1}(-x)}$$

$$= 4 - \left[\frac{4}{\pi}\cot^{-1}(-x) + \frac{\pi}{4\cot^{-1}(-x)}\right]$$

$$= \left(\alpha + \frac{1}{\alpha}\right) \geq 2 \; ; \alpha > 0$$

$$f(x)_{max} = 4 - 2 = 2$$

happen when $\frac{4}{\pi}\cot^{-1}(x) = 1$

$\cot^{-1}(-x) = \frac{\pi}{4}$

$-x = 1$

$\boxed{x = -1}$

7

JEE 1

Let f(x) be a cubic polynomial on R which increases in interval $(-\infty, 0)$ and in $(1, \infty)$ and decreases in interval (0,1). If $f'(2) = 6$ and $f(2) = 2$, then the value of

$$\tan^{-1}(f(1)) + \tan^{-1}\left(f\left(\frac{3}{2}\right)\right) + \tan^{-1}(f(0))$$ is

equal to

(A) $\tan^{-1} 2$ (B) $\cot^{-1} 2$

(C) $-\tan^{-1} 2$ (D) $-\cot^{-1} 2$ ✓

$f \to$ cubic

$f' \to$ quad

f': $+$ (0) $-$ (1) $+$

$f'(x) = a(x-0)(x-1)$; $a \neq 0$

$\underline{\underline{x=2}}$ $f'(2) = 6 = a(2-0)(2-1) \Rightarrow a = 3$

$f'(x) = 3x(x-1) = 3(x^2 - x)$

Integrate wrt x

$f(x) = x^3 - 3\frac{x^2}{2} + c$

$\underline{\underline{x=2}}$ $2 = 8 - 6 + c$

$c = 0$

$$\boxed{f(x) = x^3 - \frac{3}{2}x^2}$$

$f(1) = -\frac{1}{2}$

$f\left(\frac{3}{2}\right) = 0$

$f(0) = 0$

Ans: $\tan^{-1}\left(-\frac{1}{2}\right) + \tan^{-1} 0 + \tan^{-1} 0$

$= \tan^{-1}\left(-\frac{1}{2}\right)$

$= -\tan^{-1}\left(\frac{1}{2}\right)$

$= -\cot^{-1} 2$

8

If A and B are square matrices of order 3 where $|A| = -2$ and $|B| = 1$ then,

$|(A^{-1})\text{adj}\,(B^{-1})\text{adj}\,(2A^{-1})|$ is equal to

(A) 8 (B) -8

(C) 1 (D) -1

JEE1

✓ $|A^{-1}| = \frac{1}{|A|}$

✓ $|\text{adj}\,A| = |A|^{n-1}$

$|2A| = 2^3|A|$

✓ $|ABC| = |A||B||C|$

$|A^{-1}|\,|\text{adj}(B^{-1})|\,|\text{adj}(2A^{-1})|$

$= \frac{1}{|A|}\quad |B^{-1}|^{3-1}\quad |2A^{-1}|^{3-1}$

$= \frac{1}{|A|}\cdot\left(\frac{1}{|B|}\right)^2\left(2^3|A^{-1}|\right)^2$

$= \frac{1}{|A||B|^2}\cdot 64\cdot\frac{1}{|A|^2}$

$= \frac{1}{(-2)^3}\cdot 64 = -8$

If $\Delta(x) = \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A +$

$Bx + Cx^2 + \cdots$, then B is equal to

(A) 0 (B) 1

(C) 2 (D) none of these

$\Delta' = 0 + B + 2Cx + \cdots\cdots$

$x = 0 \rightarrow \Delta' = B = \text{Ans.}$

$$\Delta' = \begin{vmatrix} e^x & 2\cos 2x & 2x\sec^2(x^2) \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \frac{1}{x+1} & -\sin x & \cos x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ -2x\sin x^2 & e^x & 2x\cos x^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

10

Let f be a polynomial function such that for all real x, $f(x^2+1) = x^4 + 5x^2 + 2$, then $\int f(x)dx$ is

(A) $\frac{x^3}{3} + \frac{3x^2}{2} - 2x + c$ (B) $\frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$

(C) $\frac{x^3}{3} - \frac{3x^2}{2} - 2x + c$ (D) $\frac{x^3}{3} - \frac{3x^2}{2} + 2x + c$

JEE 1

$$f(x^2+1) = x^4 + 5x^2 + 2$$

$$= (x^2+1)^2 + 3x^2 + 1$$

$$= (x^2+1)^2 + 3(x^2+1) - 2$$

$$f(\text{input}) = (\text{input})^2 + 3(\text{input}) - 2$$

$$\boxed{f(t) = t^2 + 3t - 2}$$

$$\int f(t)\,dt = \int t^2 + 3t - 2\,dt$$

$$= \frac{t^3}{3} + \frac{3t^2}{2} - 2t + c$$

11

JEE 1

If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, the k can have:

(A) exactly two values ✓

(B) exactly three values

(C) no values

(D) exactly one value

s.d. = 0

$[\vec{a_2}-\vec{a_1} \quad \vec{b_1} \quad \vec{b_2}] = 0$

$\vec{a_1} \equiv (2,3,4)$ | $\vec{b_1} \equiv (1,1,-k)$

$\vec{a_2} \equiv (1,4,5)$ | $\vec{b_2} \equiv (k,2,1)$

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow -1(\cancel{1}+2k) - 1(\cancel{1}+k^2) + 1(\cancel{2}-k) = 0$

$\Rightarrow -k^2 - 3k = 0$

$\Rightarrow k = 0, -3$

12

If $\sin 18^\circ + \dfrac{1}{\underbrace{\sin 72^\circ}_{90-18}} - \dfrac{1}{\underbrace{\sin 108^\circ}_{90+18}} + \underbrace{\sin 162^\circ}_{180-18} = \alpha$,

then the value of $\left(\alpha + \dfrac{1}{\alpha}\right)$ is

(A) $2\sqrt{5}$ (B) $\dfrac{2-\sqrt{5}}{3}$

(C) $\sqrt{5}$ ✓ (D) $\dfrac{\sqrt{3}-1}{2}$

$$\alpha = \sin 18^\circ + \frac{1}{\cos 18^\circ} - \frac{1}{\cos 18^\circ} + \sin 18^\circ$$

$$\alpha = 2\sin 18^\circ$$

$$\alpha = 2\left(\frac{\sqrt{5}-1}{4}\right) = \frac{\sqrt{5}-1}{2}$$

Ans: $\alpha + \dfrac{1}{\alpha} = \dfrac{\sqrt{5}-1}{2} + \dfrac{2}{\sqrt{5}-1}$

$$= \frac{\sqrt{5}-1}{2} + \frac{\cancel{2}(\sqrt{5}+1)}{\cancel{4}\,2}$$

$$= \sqrt{5}.$$

13

Let f: R → R be a function defined by f(x) = [x + 2] + $[x]^2$ − 8 (where [.] denotes GIF)

(A) ~~One-one onto~~ (B) Many-one onto

(C) ~~One-one into~~ (D) Many-one into ✓

JEE 1

$[x+n] = [x] + n \;;\; n \in \mathbb{Z}$

$f(x) = [x] + 2 + [x]^2 - 8$

$= [x]^2 + [x] - 6$

$x = 2.1$

$x = 2.2$

$\vdots$

$x = 2.99$

$\to [x] = 2$

$\boxed{\text{M-1}}$

Co-dom : R

: $(-\infty, \infty)$

$f(x) = \underbrace{[x]^2}_{\text{int}} + \underbrace{[x]}_{\text{int}} - \underbrace{6}_{\text{int}} = \text{int}$

Range : $\mathbb{Z}$

$\therefore$ Codom $\neq$ Range

into.

14

The area of the shorter region bounded by $|y| = 4 - x^2$ and $|y| = 3x$ is given by $\left(3k + \frac{1}{3}\right)$ sq-unit where K is equal to :-

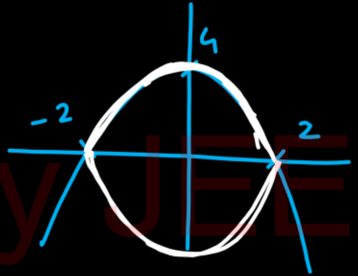
(A) 1

~~(B)~~ 2

(C) 3

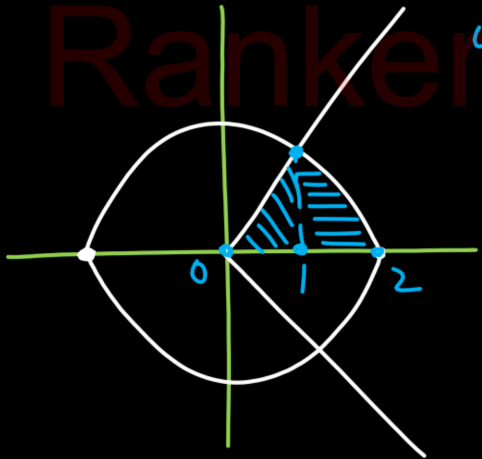
(D) $3\frac{1}{3}$

$|y| = (4 - x^2)$



$4 - x^2 = 3x$

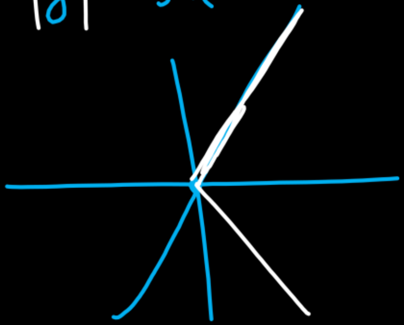
$x = 1$



$$2\left[\int_0^1 3x\,dx + \int_1^2 (4 - x^2)\,dx\right]$$

$$= \boxed{6 + \frac{1}{3}}$$

$|y| = 3x$



15

Number of values of x in $[-4,4]$ where $f(x) = [3x + 14] + |4x^2 - 1|(2x^2 + 3x - 2) + \sin\left(\frac{\pi x}{2}\right)$ is non - derivable, is equal to

[Note: [k] denotes the largest integer less than or equal to k.]

(A) 23 (B) 24

(C) 25 (D) 26

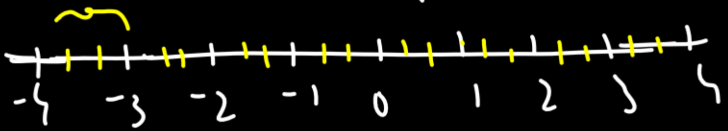
$2x^2+3x-2$

$2x^2+4x-x-2$

$(2x-1)(x+2)$

$$f(x) = [3x]+14 + |(2x-1)(2x+1)|\underbrace{\left((2x-1)(x+2)\right)} + \sin\frac{\pi x}{2}$$

$x=\frac{1}{2}$ $\quad$ $x=-\frac{1}{2}$



24 point + 1 points

16

JEE1

Four digit numbers are formed using the digits from the set {0,1,2,3,4,5} repetition of digits is allowed then :

Statement (S_1) : The number of such numbers formed that are odd is 480 . → False.

Statement (S_2) : The number of such numbers formed such that it contains exactly three different digits is 360 . False

(A) S_1, S_2 are true

(B) S_1, S_2 are false

(C) S_1 is true and S_2 is false

(D) S_1 is false and S_2 is true

$5 \times 6 \times 6 \;\; 3 = 30 \times 18$

Case-1 : zero not included

_ _ _ _

$({}^5C_3)({}^3C_1)\left(\frac{4!}{2!}\right)$

Case-2 : zero is selected.

$({}^5C_2)\left\{\left(\frac{4!}{2!}-3!\right)+\left(\frac{4!}{2!}-\frac{3!}{2!}\right)\times 2\right\}$

{0, 1, 2}

0, 0, 1, 2 : - - - -

0, 1, 1, 2 : - - - -

0, 1, 2, 2

17

JEE 1

Let $f(x) = \text{Lim}_{n\to\infty} \tan^{-1}\left(4n^2\left(1-\cos\frac{x}{n}\right)\right)$

and $g(x) = \text{Lim}_{n\to\infty} \frac{n^2}{2}\ell n\cos\left(\frac{2x}{n}\right)$. Then

$\text{Lim}_{x\to 0}\left(\frac{e^{-2g(x)}-e^{f(x)}}{x^6}\right)$ is equal to

(A) $\frac{-5}{3}$ (B) $\frac{5}{3}$

(C) $\frac{-8}{3}$ (D) $\frac{8}{3}$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$

$f(x) = \tan^{-1}\left(\lim_{n\to\infty} \cancel{4n^2}\boxed{\left(\frac{1-\cos(x/n)}{(x/n)^2}\right)}\times\frac{x^2}{\cancel{n^2}}\right)$ $\frac{1}{2}$

$= \boxed{\tan^{-1}(2x^2)}$

$g(x) = \lim_{n\to\infty} \frac{n^2}{2}\ln\left(1-2\sin^2\frac{x}{n}\right)$

$g(x) = \lim_{n\to\infty} \frac{n^2}{2}\boxed{\frac{\ln(1-2\sin^2 x/n)}{\left(-2\sin^2\frac{x}{n}\right)}}\times\left(\;\right)$ 1

$= \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{2}}\left(-\cancel{2}\frac{\sin^2 x/n}{x^2/n^2}\right)\times\frac{x^2}{\cancel{n^2}}$ 1

$= -x^2$

Rankers Academy JEE

$$\lim_{x \to 0} \frac{e^{+2x^2} - e^{\tan^{-1}2x^2}}{x^6}$$

$$\lim_{x \to 0} \frac{\underbrace{e^{\tan^{-1}2x^2}}_{1}\left[e^{\boxed{2x^2 - \tan^{-1}2x^2}} - 1\right]}{x^6 \underbrace{\left(2x^2 - \tan^{-1}2x^2\right)}_{1}} \times \left(2x^2 - \tan^{-1}2x^2\right)$$

$$\lim_{x \to 0} \frac{2x^2 - \tan^{-1}(2x^2)}{x^6}$$

$$\lim_{x \to 0} \frac{\cancel{2x^2} - \left(\cancel{2x^2} - \frac{8x^6}{3}\right)}{x^6} = \frac{8}{3}$$

$$\boxed{\tan^{-1}x = x - \frac{x^3}{3}}$$

18. Let z_1, z_2 be two fixed points in the Argand plane and z be a variable complex number satisfying $\left|\frac{z-z_1}{z-z_2}\right| = 3$, then the locus of z is a circle with

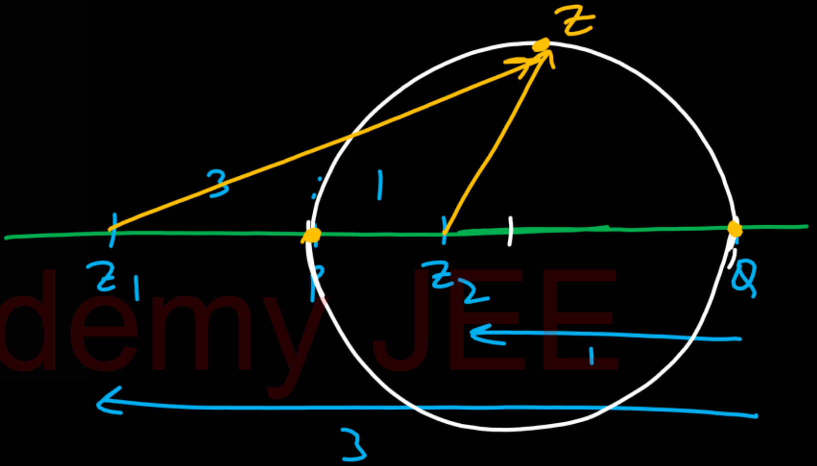
(A) z_1 as an interior point

(B) z_2 as an interior point

(C) z_1 and z_2 as interior points

(D) None of z_1, z_2 be in the interior

$|z-z_1| = 3|z-z_2|$



$\left|\frac{z-z_1}{z-z_2}\right| = K \quad (\text{where} : K \neq 1)$

is a circle

19

JEE 1

Sum of coefficients of all terms containing integral power of x in expansions of $(1-2\sqrt{x})^{20}$ is

(A) $3^{20}-1$

(B) $\frac{3^{20}+1}{3}$

(C) $\frac{3^{20}-1}{2}$

(D) $\frac{3^{20}+1}{2}$ ✓

$$(1-2\sqrt{x})^{20} = {}^{20}C_0(1)^{20}(-2\sqrt{x})^0 + {}^{20}C_1(1)^{19}(-2\sqrt{x})^1 + {}^{20}C_2(1)^{18}(-2\sqrt{x})^2 + \cdots$$

$$(1+2\sqrt{x})^{20} = {}^{20}C_0(1)^{20}(2\sqrt{x})^0 + {}^{20}C_1(1)^{19}(2\sqrt{x})^1 + {}^{20}C_2(1)^{18}(2\sqrt{x})^2 + \cdots$$

$$(1-2\sqrt{x})^{20} + (1+2\sqrt{x})^{20} = 2\left[{}^{20}C_0 + {}^{20}C_2(2\sqrt{x})^2 + {}^{20}C_4(2\sqrt{x})^4 + \cdots\right]$$

$$1+3^{20} = 2(\text{Requirement})$$

20

JEE 1

Solution of the differential equation

$$\left(x\frac{dy}{dx}+y\right)=e^{xy-\ln x^2}\left(x\frac{dy}{dx}-y\right)$$

(A) $\frac{y}{x}-e^{-xy}=c$

(B) $\frac{x}{y}+e^{-xy}=c$

(C) $\frac{y}{x}+e^{-xy}=c$

(D) $-\frac{x}{y}+e^{-xy}=c$

$$\left(x\frac{dy}{dx}+y\right)=\frac{e^{xy}}{e^{\ln x^2}}\left(x\frac{dy}{dx}-y\right)$$

$$\left(\frac{x\,dy+y\,dx}{\cancel{dx}}\right)=\frac{e^{xy}}{x^2}\left(\frac{x\,dy-y\,dx}{\cancel{dx}}\right)$$

$$d(xy)=e^{xy}\left(\frac{x\,dy-y\,dx}{x^2}\right)$$

$$\int\frac{d(xy)}{e^{(xy)}}=\int d(y/x)$$

$$-e^{-xy}=\frac{y}{x}+c$$

21

JEE 1

The coefficient of x^{13} in

$(1-x)^5\underbrace{(1+x+x^2+x^3)}^4$

$$(1-x)^5\left(\frac{1(1-x^4)^4}{(1-x)^4}\right)$$

$$(1-x)\underbrace{(1-x^4)^4}$$

$$(1-x^4)^4 - x\underbrace{(1-x^4)^4}$$

$$0 - {}^4C_3(-1)^3 = 4$$

$$(1-x^4)^4$$

$$GT = {}^4C_r(1)^{n-r}(-x^4)^r$$

$${}^4C_r(-x^4)^r$$

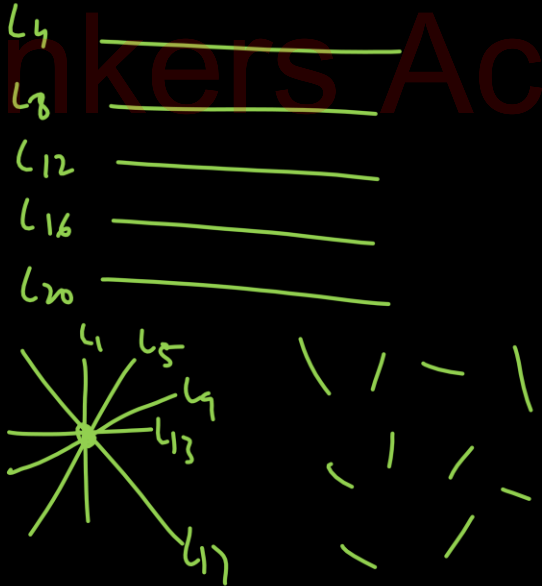
$$\left[{}^4C_r(-1)^r\right]x^{4r}$$

22

The lines $L_1, L_2, L_3, \ldots L_{20}$ are distinct. All the lines L_4, L_8, L_{12}, L_{16} and L_{20} are parallel. All the lines $L_1, L_5, L_9, L_{13}, L_{17}$ pass through a given point A. The maximum number of points of intersection of these 20 lines is

$$1 + {}^{10}C_2 + {}^{10}C_1 \times {}^{5}C_1 + {}^{10}C_1\,{}^{5}C_1$$

$${}^{5}C_1 \cdot {}^{5}C_1$$



$$1 + \frac{\cancel{10}^{\,5} \times 9}{\cancel{2}} + 50 + 50 + 25$$

$$= 171$$

23

JEE 1

Given the equation of the ellipse $\frac{(x-3)^2}{16} + \frac{(y-4)^2}{49} = 1$, a parabola is such that its vertex is the lowest point of the ellipse and it passes through the ends of the minor axis of the ellipse. The equation of the parabola is in the form $16y = A(x-H)^2 - K$. Determine the value of $\frac{A}{7} + \frac{H}{3} + \frac{K}{16}$ is equal to

Ans: 5

$A \to 7$, $H \to 3$, $K \to 48$

$(x-3)^2 = \lambda(y+3)$

$(7,4)$:

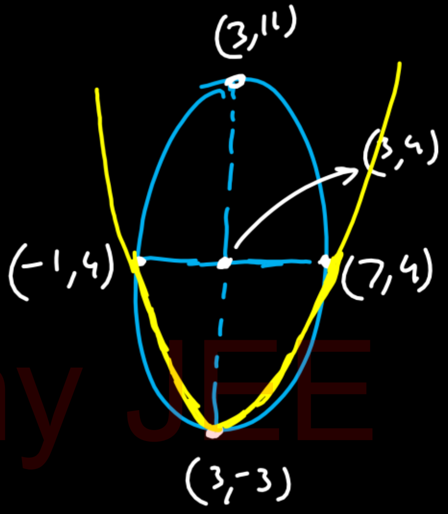
$16 = \lambda(7)$

$\lambda = 16/7$

$(x-3)^2 = \frac{16}{7}(y+3)$

$7(x-3)^2 = 16y + 48$

$\boxed{16y = 7(x-3)^2 - 48}$



$X^2 = 4AY$

24

JEE 1

If m arithmetic (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to:

$3, A_1, A_2, \ldots, A_m, 243$

$d = \frac{243-3}{m+1} = \frac{240}{m+1}$

$A_4 = 3 + 4d = \left(3 + \frac{240 \times 4}{m+1}\right)$

$3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{1/4} = 3$

$G_2 = 27$

$3 + \frac{240 \times 4}{m+1} = 27$

$m = 39$

25

Let E_1, E_2, E_3 be three independent events associated with a random experiment such that

$$3P(E_1 \cap \overline{E_2} \cap \overline{E_3}) = P(\overline{E_1} \cap E_2 \cap \overline{E_3}) = 9P(\overline{E_1} \cap \overline{E_2} \cap E_3) = 3 - 3P(E_1 \cup E_2 \cup E_3),$$

where $P(E_1)$, $P(E_2), P(E_3) \neq 1$ and $P(A)$ denotes probability of event A. If absolute value of

$$\begin{vmatrix} P(E_1) & P(E_2) & P(E_3) \\ P(E_2) & P(E_3) & P(E_1) \\ P(E_3) & P(E_1) & P(E_2) \end{vmatrix} = \frac{a}{b}$$

where $a, b \in N$, then least value of $a + b$ is

$p(E_1) = p \;;\; p(\bar{E_1}) = (1-p)$

$p(E_2) = q \;;\; p(\bar{E_2}) = (1-q)$

$p(E_3) = r \;;\; p(\bar{E_3}) = (1-r)$

$$3(p)(1-q)(1-r) = (1-p)q(1-r) = 9(1-p)(1-q)(r) = 3(1-p)(1-q)(1-r)$$

$$\Rightarrow \frac{3p}{(1-p)} = \frac{q}{1-q} = \frac{9r}{1-r} = 3$$

$3p = 3 - 3p$, $p = 1/2$

$q = 3 - 3q$, $q = 3/4$

$3r = 1 - r$, $r = 1/4$

$$\begin{vmatrix} 1/2 & 3/4 & 1/4 \\ 3/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{vmatrix}$$

$$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\left(\frac{1}{4\times4\times4}\right)\left(2(-1)-3(7)+1(+5)\right)$$

$$\frac{1}{4\times4\times4}(-18) = -\frac{9}{32}$$

$$\therefore \frac{a}{b} = \frac{9}{32}$$

$$a+b = 41$$

$$1-P(A) = P(\bar{A})$$

$$1-P(E_1\cup E_2\cup E_3)$$

$$= P(\overline{E_1\cup E_2\cup E_3})$$

$$= P(\bar{E}_1\cap\bar{E}_2\cap\bar{E}_3)$$