FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
PART TEST – III
PAPER –2
TEST DATE: 22-12-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. B

Sol. Node can be formed at $x = \frac{L}{2}, \frac{3L}{2}, \frac{5L}{2}, \frac{7L}{2}$ and $\frac{9L}{2}$ which depends on f_0 and other parameter.
Phase difference between any two particle either zero or π at any instant
Potential energy depends on strain.

2. D

Sol. Let x_0 is the distance moved by prism when it just touches the pad
$x_0 \cos 30° = d$

So, time taken by prism $t_1 = \frac{x_0}{v_0} = \frac{2d}{\sqrt{3}\, v_0}$ ...(i)

Let further displacement of prism be x, so compression in the spring will be x cos 30°.
The restoring force will be
$F_r = (kx \cos 30°) \cos 30°$
$= k\cos^2 30° x$

$a = \frac{3k}{4m} x$

$\omega = \sqrt{\frac{3k}{4m}}$

$t_2 = 2\pi\sqrt{\frac{4m}{3k}}$

Total time = $2\pi\sqrt{\frac{4m}{3k}} + \frac{8d}{\sqrt{3}v_0}$

3. B

Sol. $\frac{\lambda}{2} = 3.55$ mm
$\lambda = 7.10$ mm
$v = f\lambda = (2.20 \times 10^{10})\,(7.1 \times 10^{-3}) =)\ 1.56 \times 10^{8}$ m/s

4. C

Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{5/2}{v} - \frac{3/2}{-u_0} = \frac{\left(\frac{5}{2}\right) - \left(\frac{3}{2}\right)}{R}$

$\frac{5}{2v} = \frac{1}{R} - \frac{3}{2u_0}$

$\frac{5}{v} = \frac{2}{R} - \frac{3}{u_0}$

$v \to \infty$ at $u_0 = \frac{3R}{2}$

$u_0 < \frac{3R}{2}$, $v < 0$

5. A, C

Sol. $\frac{x^2}{25} + \frac{y^2}{16} = d^2$

a = 5d, b = 4d

$b^2 = a^2(1 - e^2)$

$16 = 25(1 - e^2)$

$e = \frac{3}{5} = 0.6$

$v_{max} = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)} = \sqrt{\frac{G(100m)}{5d} \times 4} = 20\sqrt{\frac{Gm}{5d}}$

$v_{min} = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)} = \sqrt{\frac{G(100m)}{5d} \times \frac{1}{4}} = \sqrt{\frac{5Gm}{d}}$

$E = \frac{-GMm}{2a} = \frac{-G(100m)m}{2(5d)} = \frac{-10Gm^2}{d}$

$L = mv_{max}a(1-e) = 40md\sqrt{\frac{Gm}{5d}}$

6. B, C

Sol. Intensity of light wave at the mirror

$I = \frac{1}{2}\frac{E_0^2}{\mu_0 C} = \frac{(0.0280)^2}{2(4\pi \times 10^{-7})(3 \times 10^8)} = 1.04 \times 10^{-6}\ W/m^2$

Energy incident per second at the mirror

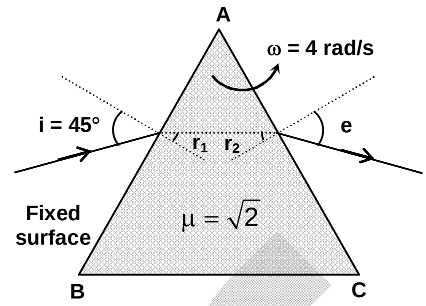
$\frac{dU}{dt} = AI = (5 \times 10^{-4})(1.04 \times 10^{-6}) = 5.2 \times 10^{-10}\ J$

Radiation pressure (p_r) = $\frac{2I}{C} = \frac{2(1.04 \times 10^{-6})}{3 \times 10^8} = 6.93 \times 10^{-15}\ Pa$

Power of source $P = I(4\pi R^2) = (1.04 \times 10^{-6})\left(4 \times 3.14 \times (3.2)^2\right) = 1.34 \times 10^{-4}\ W$

7. B, C

Sol. $r_1 + r_2 = A$



$$\frac{dr_1}{dt} + \frac{dr_2}{dt} = \frac{dA}{dt}$$

$$\frac{dr_2}{dt} = 4 \text{ rad/s} \qquad (\because r_1 = \text{constant})$$

$$1 \times \sin 45° = \sqrt{2}\sin r_1$$

$r_1 = 30° \Rightarrow r_2 = 30°$, when $A = 60°$

$$\Rightarrow \sqrt{2}\sin r_2 = 1 \sin e$$

$$\Rightarrow e = 45°$$

$$\sqrt{2}(\cos r_2)\frac{dr_2}{dt} = (\cos e)\frac{de}{dt}$$

$$\sqrt{2} \times \frac{\sqrt{3}}{2} \times 4 = \frac{1}{\sqrt{2}}\frac{de}{dt}$$

$$\frac{de}{dt} = 4\sqrt{3} \text{ rad/s}$$

$$\delta = i + e - A$$

$$\frac{d\delta}{dt} = \frac{de}{dt} - \frac{dA}{dt} = (4\sqrt{3} - 4) \text{ rad/s}$$

SECTION – B

8. 2000

Sol. $V = Ah = (0.2)(10) = 2 \text{ m}^3$

$\Delta V = A\Delta h = (0.2)(0.2 \times 10^{-3}) = 4 \times 10^{-5} \text{ m}^3$

Volumetric strain, $\dfrac{\Delta V}{V} = \dfrac{4 \times 10^{-5}}{2} = 2 \times 10^{-5}$

Bulk modulus, $B = \dfrac{\Delta P}{\Delta V / V} = \dfrac{Mg/A}{\Delta V/V} = \dfrac{(3000 \times 10)/0.2}{2 \times 10^{-5}} = 7.5 \times 10^9 \text{ N/m}^2$

Speed of sound, $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{7.5 \times 10^9}{1875}} = 2000 \text{ m/s}$

9. 211

Sol. $$V_e = \sqrt{\frac{2GM_e}{R_e}} \qquad \ldots(i)$$

$$\rho = kr^2 = \frac{\rho_0}{R^2}r^2$$

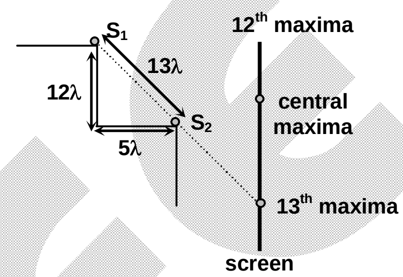
$$M_e = \int \rho dV$$

$$M_e = \int_0^{R_e} \frac{\rho_0}{R_e^2}r^2 \cdot 4\pi r^2 dr = \frac{4\pi\rho_0 R_e^3}{5} \qquad \ldots(ii)$$

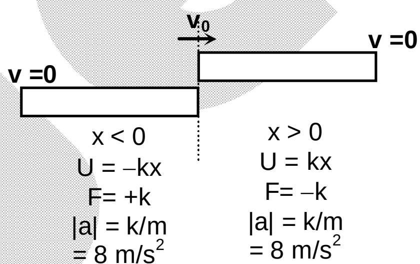
$$V_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{8\pi G\rho_0 R_e^2}{5}}$$

10. 720
Sol. Pitch =0.2 mm
Total no. of divisions on the circular scale = 200
Least count of screw gauge = $\frac{0.2}{200} = 0.001\text{mm}$
$-$Ve zero error = 60×0.001 =0.06 mm
Screw gauge Reading = MSR + CSR =0.6 mm + 60 $\times$ (0.001) mm = 0.66 mm
Diameter of brass = 0.66 + 0.06 =0.72 mm = 720μm

11. 25
Sol. There are 13 maxima below the central maxima and 12 maxima above the central maxima.



12. 12
Sol. $K_0 = \frac{1}{2}mv_0^2$



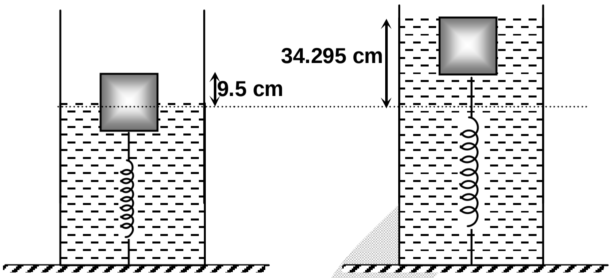
$v_0 = \sqrt{\frac{2K_0}{m}}$
$0 = v_0 - at$
$t = \frac{v_0}{a} = \frac{\sqrt{2K_0m}}{k}$
Time period, T=4t = $\frac{4\sqrt{2K_0m}}{k} = 12\,\text{sec}$

13. 49
Sol. For the vernier callipers P,
7 VSD = 6 MSD
1 VSD = $\frac{6}{7}$ MSD $= \frac{6}{7}$mm
Least count of P, $L_P = 1 - \frac{6}{7} = \frac{1}{7}$mm
For the vernier callipers Q,
7 VSD = 8 MSD
1 VSD = $\frac{8}{7}$ MSD $= \frac{8}{7}$mm
Least count of Q, $L_Q = \frac{8}{7} - 1 = \frac{1}{7}$mm
So, $\frac{1}{L_PL_Q} = 49\ \text{mm}^{-2}$

SECTION – C

14. 24.79
(Range 24.78 to 24.80)

15. 13.71
(Range 13.70 to 13.72)



Sol. Initially $mg = V\rho_\ell g$

$$(V\rho_s)g = \rho_\ell\left(\frac{V}{2}\right)g$$

$$\rho_S = \frac{\rho_\ell}{2} \qquad \ldots(i)$$

$$\rho_S \ell^3 = m$$

$$\ell^3 = \frac{m}{\rho_S} = \frac{2m}{\rho_\ell} = \frac{2(3429.5)\times10^{-3}}{1000}$$

ℓ= 19 cm

After water is poured →
Buoyant force will increase, so the spring will get elongated

$$V\rho_\ell g = mg + f_{sp}$$

Displacement of the cube =34.295 cm – 9.50 cm = 24.795 cm
Volume of water added
= $(500\times34.295)cm^3 - (19\times19\times9.5)cm^3$
= 13718 cm^3 = 13.718 litre

16. 4.00

17. 4.50
Sol. Electric field between the electrodes

$$E = \frac{V_0}{d} \qquad \ldots(i)$$

Acceleration of electron

$$a = \frac{eE}{m} \qquad \ldots(ii)$$

Speed gained by electrons to travel a distance ℓ

$$v^2 = u^2 + 2a\ell$$

$$= 2\left(\frac{eE}{m}\right)\ell = \frac{2eV_0}{md}\ell$$

$$\text{So, K.E.} = \frac{1}{2}mv^2 = \frac{eV_0\ell}{d}$$

Ignition of lamp will take place when

$$\frac{eV_0\ell}{d} = E_0$$

$$V_0 = \frac{E_0 d}{e\ell}$$

Chemistry

PART – II

SECTION – A

18. C

Sol. Lattice energy $\propto \frac{1}{\text{Size}} \propto$ CFSE

CFSE order

$\underset{(d^8)}{Ni^{+2}} > \underset{(d^3)}{V^{+2}} > \underset{(d^5)}{Mn^{+2}} > \underset{(d^0)}{Ca^{+2}}$

19. A

Sol. B, C, D have static effect on microbes.

20. C

Sol. It is according to Hardy-Schulze rule.

21. C

Sol. For 0.001 M CH_3COOH $\alpha = \frac{\Lambda_v}{\Lambda_\infty}$

$$\alpha = \frac{60}{390} = 0.1538 \simeq 0.154$$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$$K_a = \frac{(0.001)(0.154)^2}{(1-0.154)} = 2.80 \times 10^{-5}$$

For 0.05 N acid $K_a = C\alpha^2$

$$\therefore \alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{2.80 \times 10^{-5}}{0.05}}$$

= 0.0236 $\simeq$ 0.024

22. A, B, D

Sol. Sulphide ores are concentrated by Froath-Floatation method.

Pyrolusite is MnO_2.

23. A, B, C

Sol. $N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O$

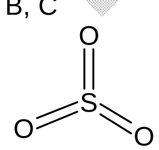
$N_2H_4 + I_2 \longrightarrow N_2 + 4HI$

$2N_2H_4 + O_2 \xrightarrow{\Delta} 2N_2 + O_2 + 4H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

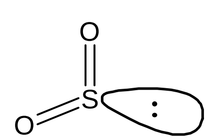
24. B, C

Sol.



$2p\pi - d\pi$

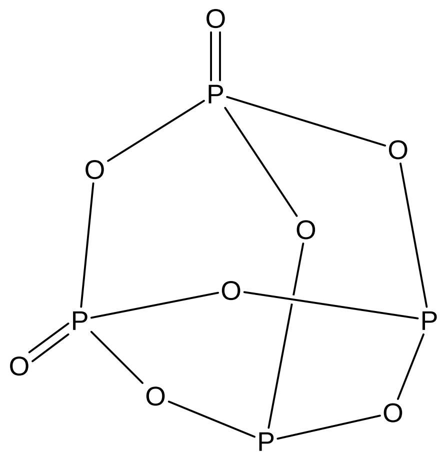
$1p\pi - d\pi$



$1p\pi - d\pi$

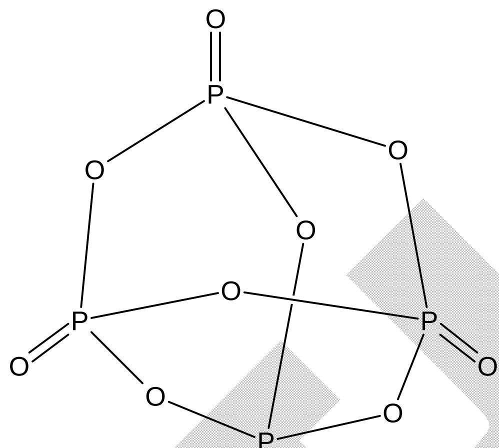
$1p\pi - p\pi$

P_4O_8



$2p\pi - d\pi$

P_4O_9



$3p\pi - d\pi$

SECTION – B

25. 5

Sol. Cerrusite $\rightarrow PbCO_3$

Azurite $\rightarrow Cu(OH)_2.2CuCO_3$

Calamine $\rightarrow ZnCO_3$

Zincite $\rightarrow ZnO$

Siderite $\rightarrow FeCO_3$

Magnetite $\rightarrow Fe_3O_4$

Magnesite $\rightarrow MgCO_3$

Bauxite $\rightarrow Al_2O_3.2H_2O$

26. 40

Sol. $P^o \propto 2.05$

$P_s \propto 2.0$

$$\frac{P^o - P_s}{P_s} = \frac{n_B}{n_A} = \frac{5/m_B}{90/18} \Rightarrow \frac{0.05}{2} = \frac{1}{m_B}$$

$\Rightarrow m_B = 40$

27. 27

Sol. Number of C-atom in one unit cell of diagonal $8\times\frac{1}{8}+6\times\frac{1}{2}+4\times1=8$

If all the atom along one body diagonal and from one face has been removed the remaining atom in each unit cell $=8-5\times\frac{1}{8}-1-1\times\frac{1}{2}=\frac{47}{8}$

$$\therefore \%\ \text{change in density} = \left(\frac{8-\frac{47}{8}}{8}\right)\times100=\left(\frac{64-47}{64}\right)\times100=26.5625$$

28. 24

Sol. $$E_{cell}=\frac{2.303RT}{F}\log\frac{K_{SP}(AgCl)}{K_{SP}(AgBr)}\times\frac{[Br^-]}{[Cl^-]}$$

$$=0.06\log\frac{10^{-10}}{10^{-13}}\times\frac{0.1}{0.01}$$

$$=0.06\log10^4$$

$$=4\times0.06$$

$$=0.24\ V$$

29. 1723

Sol. Suppose weight of Cd required = x

$$\frac{x}{x+20}\times100=20$$

$$\Rightarrow x=5\ gm$$

$$\frac{5}{112/2}=\frac{5\times t}{96488}$$

$$\therefore t=\frac{96488}{56}=1723\ sec.$$

30. 2

Sol. $n_{N_2}=2,\ n_{H_2}=8$

Volume of N_2 compartment $=\frac{2\times R\times300}{1}=600\ R$

Total volume after removing position = 600 R × 5 = 3000 R

At 1000K

$$\begin{array}{ccccc} N_2 & + & 3H_2 & \longrightarrow & 2NH_3 \\ 2 & & 8 & & 0 \\ 0 & & 2 & & 4 \end{array}$$

$\therefore n_{total}=6$ moles

$$P_{total}=\frac{6R\times1000}{3000R}=2\ atm$$

SECTION – C

31. 1.39

Sol. $E^o_{ClO_4^-/Cl_2} = \dfrac{1.19 \times 2 + 1.21 \times 2 + 1.65 \times 2 + 1.61 \times 1}{7}$

$= 1.387 \simeq 1.39$ V

32. 2.00

Sol. $ClO_3^- + 4H^+ + 4e^- \longrightarrow ClO^- + 2H_2O$

$E^o_{ClO_3^-/ClO^-} = \dfrac{1.21 \times 2 + 1.65 \times 2}{4} = 1.43$ V

$E_{ClO_3^-/ClO^-} = E^o_{ClO_3/ClO^-} - \dfrac{0.06}{4}\log\dfrac{[ClO_3^-]}{[ClO^-][H^+]^4}$

$\therefore pH = 2$

33. 82.00

34. 99.00

Sol. (Q. No. 33 and 34).

$C_2H_6 + \dfrac{7}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$

$-372 = 2 \times (-94) + 3(-68) - \Delta H_f^o(C_2H_6)$

$\Delta H_f^o(C_2H_6) = -20$

$C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

$-530 = 3(-94) + 4(-68) - \Delta H_f^o(C_3H_8)$

$\Delta H_f^o(C_3H_8) = -24$

$2C(s) + 3H_2 \longrightarrow C_2H_6$

$-20 = 2 \times 172 + 3 \times (104) - (x + 6y)$

$3C(s) + 4H_2 \longrightarrow C_3H_8$

$-24 = 3 \times 172 + 4(104) - (2x + 8y)$

After solving, we get x and y
x = 82 Kcal/mol
y = 99 Kcal/mol

Mathematics

PART – III

SECTION – A

35. D

Sol. $|z_1 - z_3| + |z_2 - z_3| \geq |(z_1 - z_3) - (z_2 - z_3)|$

$\Rightarrow |z_1 - z_3| + |z_2 - z_3| \geq |z_1 - z_2|$

Let C_1 and C_2 be the centres and r_1 and r_2 be the radii of the circles represented by z_1 and z_2 respectively,

Then $|z_1 - z_2|_{min} = C_1C_2 - (r_1 + r_2) = 10 - (1+3) = 6$

36. B

Sol. $$\sum_{k=0}^{2n} \frac{1}{(2n-k)!(2n+k)!} = \frac{1}{(4n)!} \sum_{k=0}^{2n} \frac{(4n)!}{(2n-k)!(2n+k)!}$$

$$= \frac{1}{(4n)!} \sum_{k=0}^{2n} {}^{4n}C_{2n-k} = \frac{1}{(4n)!} \sum_{k=0}^{2n} {}^{4n}C_k = \frac{1}{(4n)!}\left(\frac{2^{4n} + {}^{4n}C_{2n}}{2}\right)$$

37. C

Sol. $$\frac{a_r^3}{a_r - 1} = \frac{a_r^2}{a_r - 1} + a_r^2 = a_r^2 - \frac{a_r^2}{1 - a_r}$$

$$\therefore S = \left(a_1^2 + a_2^2 + a_3^2 + \ldots\ldots + a_n^2\right) - \left(\frac{a_1^2}{1-a_1} + \frac{a_2^2}{1-a_2} + \frac{a_3^2}{1-a_3} + \ldots\ldots + \frac{a_n^2}{1-a_n}\right) = 900$$

$S = 2^2 3^2 5^2$

38. A

Sol. mean $= \dfrac{2a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 5k}{5} = 2\bar{a} + k$

$$\text{S.D.} = \sqrt{\frac{\sum (2a_i + k)^2}{n} - (2\bar{a} + k)^2} = \sqrt{\frac{\sum\left(4a_i^2 + k^2 + 4ka_i\right)}{5} - \left(4\bar{a}^2 + k^2 + 4\bar{a}k\right)}$$

$$= 2\sqrt{\frac{\sum a_1^2}{5} - \bar{a}^2} = 2s$$

39. A, C

Sol. $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3} \Rightarrow (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 3 \Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$

$(\vec{a} \times \vec{b}).(\vec{b} \times \vec{c}) = (\vec{a}.\vec{b})(\vec{b}.\vec{c}) - \vec{c}.\vec{a}$

$(\vec{b} \times \vec{c}).(\vec{c} \times \vec{a}) = (\vec{b}.\vec{c})(\vec{c}.\vec{a}) - \vec{a}.\vec{b}$

$(\vec{c} \times \vec{a}).(\vec{a} \times \vec{b}) = (\vec{c}.\vec{a})(\vec{a}.\vec{b}) - \vec{b}.\vec{c}$

$\Rightarrow \lambda = (\vec{a}.\vec{b})(\vec{b}.\vec{c}) + (\vec{b}.\vec{c})(\vec{c}.\vec{a}) + (\vec{c}.\vec{a})(\vec{a}.\vec{b})$

$\Rightarrow \lambda \leq 0 \; \{\because x + y + z = 0 \Rightarrow xy + yz + zx \leq 0\}$

and $\lambda_{max} = 0$ only when $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\Rightarrow \vec{a} \perp \vec{b},\ \vec{b} \perp \vec{c}$ and $\vec{c} \perp \vec{a}$

$\Rightarrow \left(2\vec{a}+3\vec{b}+4\vec{c}\right).\left(\vec{a}\times\vec{b}+5\vec{b}\times\vec{c}+6\vec{c}\times\vec{a}\right) = 10\vec{a}.\left(\vec{b}\times\vec{c}\right)+18\vec{b}.\left(\vec{c}\times\vec{a}\right)+4\vec{c}.\left(\vec{a}\times\vec{b}\right) = 32$

40. B, D

Sol. $\left|z+\frac{1}{z}\right|^2 = 4 \Rightarrow \left(z+\frac{1}{z}\right)\left(\bar{z}+\frac{1}{\bar{z}}\right) = 4$

$\Rightarrow (z\bar{z})^2 - 4(z\bar{z}) + z^2 + \bar{z}^2 + 1 = 0$

$\Rightarrow (z\bar{z}-1)^2 + (z-\bar{z})^2 = 0 \Rightarrow (z\bar{z}-1)^2 - i^2(z-\bar{z})^2 = 0$

$\Rightarrow \left(z\bar{z}-1+i(z-\bar{z})\right)\left(z\bar{z}-1-i(z-\bar{z})\right) = 0$

Each of the factors in the above equation represents circles with centres at $(0,1)$ & $(0,-1)$ and radii equal to $\sqrt{2}$ for both

41. A, C

Sol. $P^T - adj(Q) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow P - (adj(Q))^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow P - adj(Q^T) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

adding above results to get $P + adj(Q^T) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

we get, $2P = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \Rightarrow P = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow |P| = 0$

now, $P^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2P$

similarly,

$P^n = 2^{n-1}P$

$\left|P + P^2 + P^3 + P^4 + P^5\right| = |31P| = 0$

also, $2adj(Q^T) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$

$\Rightarrow (\text{adjQ})^T = I$

$\Rightarrow adjQ = I \Rightarrow Q = I$

SECTION – B

42. 8

Sol. Centre of the hexagon is at origin (O) as O lies on the perpendicular bisector of AC and $\angle AOC = \frac{2\pi}{3}$

43. 70

Sol. $pqrrqp = p\left(10^5+1\right)+q\left(10^4+10\right)+r\left(10^3+10^2\right)$

$= p\ (1001 - 1)\ 100 + p + 10q(1001) + (100)\ (11)\ r$

$= (7.11.13.100)p - 99p + 10q(7.11.13) + (98 + 2)(11)r$

$= 7x + (r - p)$ where x is an integer

Now if $r - p$ is a multiple of 7 then $r - p = 7, 0, -7$

Hence number of ordered pairs of (p, r) is 14.

44. 12

Sol. $\dfrac{\left(a^3+b^3+c^3\right)}{3} \geq \left(\dfrac{a+b+c}{3}\right)^3 \Rightarrow \dfrac{(a+b+c)^2}{2} \geq \dfrac{(a+b+c)^3}{27} \Rightarrow a+b+c \leq \dfrac{27}{2}$

Now, $a+b+c=13$ does not satisfy the given relation

$a+b+c=12$ satisfies the given relation for $a = 3,\ b = 4$ and $c = 5$

45. 81

Sol. $(p+q)^2+p(p+q)+q=0 \Rightarrow 2p^2+3qp+q^2+q=0$

Discriminant must be a perfect square (since p is an integer)

Let $D = 9q^2-8\left(q^2+q\right)=k^2$

$\Rightarrow q^2-8q=k^2 \Rightarrow (q-4)^2-16=k^2 \Rightarrow (q-4-k)(q-4+k)=16$

$\Rightarrow q=9,-1,8,0$

46. 88

Sol. Let $S=\sum_{r=1}^{8} ra_r = a_1+2a_2+3a_3+\ldots\ldots+8a_8$

$\Rightarrow S=(a_1+a_4+a_7)+2(a_2+a_5+a_8)+3(a_3+a_4+a_5+2a_6+2a_7+2a_8)$

$\Rightarrow S=(a_1+a_4+a_7)+2(a_2+a_5+a_8)+3m$

$\Rightarrow S=(a_1-a_2)+(a_4-a_5)+(a_7-a_8)+3n$

let $a_1-a_2=b_1,\ a_4-a_5=b_2$ and $a_7-a_8=b_3$

Possible values of b_i are $-2,0,2$

$b_1=b_2=b_3=0$ is possible in 8 ways

$b_1=b_2=b_3=2$ is possible in 1 way

$b_1=b_2=b_3=-2$ is possible in 1 way

One out of b_1,b_2,b_3 can be 2, second -2 and the last one 0 in a total of $6\times 2=12$ ways

For each pf the above ways a_3 and a_6 can be 1 or -1 in 4 ways

$\therefore$ Total $(8+1+1+12)\times 4 = 88$ ways

47. 6

Sol. $(\text{Area of } \Delta ABC)^2 = (\text{Area of } \Delta OAB)^2 + (\text{Area of } \Delta OBC)^2 + (\text{Area of } \Delta OAC)^2$

$\therefore A_4^{\,2}=A_1^{\,2}+A_2^{\,2}+A_3^{\,2}$

Let d be the common difference of the AP

$\Rightarrow (1+3d)^2 = 1^2 + (1+d)^2 + (1+2d)^2 \quad \Rightarrow d = \frac{1}{\sqrt{2}}$

$\therefore$ Area of $\Delta ABC = \frac{3+\sqrt{2}}{\sqrt{2}}$

Volume of tetrahedron $= \frac{1}{3} \times (\text{Area of } \Delta ABC) \times (\text{Length of perpendicular from origin})$

SECTION – C

48. 0.67

Sol. $P = \{1, 2, 3, ..., 50\}$

In terms of divisibility by 5, any number x can be of one of the following 5 forms-

i) $x = 5c+1 \Rightarrow x^4$ is also of $5c+1$ form

ii) $x = 5c+2 \quad \Rightarrow x^4$ is of $5c+1$ form

iii) $x = 5c+3 \quad \Rightarrow x^4$ is of $5c+1$ form

iv) $x = 5c+4 \quad \Rightarrow x^4$ is of $5c+1$ form

v) $x = 5c \quad \Rightarrow x^4$ is also of $5c$ form

$\therefore$ a and b should both be of $5c$ or both should not be of $5c$ form.

$P(\text{both of } 5c \text{ form}) = \frac{{}^{10}C_2}{{}^{50}C_2}$

$P(\text{both not of } 5c \text{ form}) = \frac{{}^{40}C_2}{{}^{50}C_2}$

49. 0.58

Sol. $P = \{1, 2, 3, 4, 5, 6\}$

$x + y + z = a \quad \Rightarrow z = a - (x+y)$

$xy + yz + zx = b \quad \Rightarrow xy + (x+y)\{a - (x+y)\} = b$

$\Rightarrow y^2 + (x-a)y + x^2 - ax + b = 0$

Since y is real $\Rightarrow D \geq 0 \quad \Rightarrow (x-a)^2 - 4(x^2 - ax + b) \geq 0$

$\Rightarrow 3x^2 - 2ax - a^2 + 4b \leq 0$

Since x is also some real number $\Rightarrow D \geq 0 \quad \Rightarrow a^2 \geq 3b$

50. 9.00

Sol. $|A - xI| = 0 \quad \Rightarrow \begin{vmatrix} 1-x & 2 & 0 \\ 2 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = 0 \quad \Rightarrow \quad (1-x)^3 - 4(1-x) = 0$

$\Rightarrow x = -1, 1, 3$

$\Rightarrow D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \Rightarrow \text{adj}(D) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

51. 0.00

Sol. $ABA^T = D \quad \Rightarrow \quad (ABA^T)^T = D^T \quad \Rightarrow (A^T)^T B^T A^T = D$

$\Rightarrow AB^T A^T = D \quad \Rightarrow B^T = B$