FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – VIII
PAPER –1
TEST DATE: 27-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. C

Sol. $(Ndt)R = \frac{7HR^2}{6}\omega$ …(i)

$(Ndt) = M\left(v_{cm} + \sqrt{2gR}\right)$ …(ii)

$e = 1 \Rightarrow v_{CM} + \frac{R\omega}{2} = \sqrt{2gR}$ …(iii)

2. B

Sol. $t_1 + r_1 = 1$

$T = t_1t_2 + t_1r_1r_2t_2 + t_1r_2r_1r_2r_1t_2 ....\infty$

$t_1t_2\left\{1 + r_1r_2 + (r_1r_2)^2 + (r_1r_2)^3 ....\infty\right\}$

$= \frac{t_1t_2}{1-r_1r_2} = \frac{\frac{3}{4}\times\frac{1}{6}}{1-\frac{1}{4}\times\frac{5}{6}} = \frac{\frac{3}{24}}{\frac{19}{24}} = \frac{3}{19}$

3. B

Sol. $F_tv = c$

$m\frac{dv}{dt}v = c$

$\Rightarrow \int_0^t dt = \frac{m}{c}\int_{v_1}^{v_2} vdv$

$t = \frac{m}{2c}(v_2^2 - v_1^2) = \frac{m}{2c}\left(\frac{GM}{R} - \frac{GM}{r}\right) = \frac{GMm}{2c}\left(\frac{1}{R} - \frac{1}{r}\right)$

4. A

Sol. Let the final potential drop across each capacitor is V.

$KCV + CV = CV_0$

$\alpha\, CV^2 + CV = CV_0$
$\Rightarrow \alpha V^2 + V - V_0 = 0$
$V^2 + V - 30 = 0$
$(V + 6)(V - 5) = 0$
$\therefore$ V = 5 volt

5. A, B, D

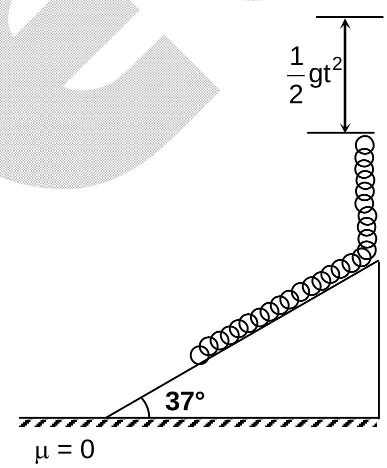
Sol. Pitch of the screw gauge, P = 1 mm

Least count $= \frac{P}{N} = \frac{1}{100} = 0.01\,\text{mm}$

Zero error = – 6 × L.C. = –6 × 0.01 mm = –0.06 mm
The diameter of the wire,
d = 3 mm + 58 × L.C. + 6 × L.C.
d = 3mm + 58 × 0.01 mm + 6 × 0.01 mm
d = 3 mm + 0.58 mm + 0.06 mm
d = 3.64 mm

6. A, B, D

Sol. force as a function of time 't'



$$N = \frac{m}{\ell}\left(\frac{1}{2}gt^2\right)g\cdot\cos 37° + \lambda v^2\cos 37°$$

$$= \frac{2}{5}\frac{mg^2t^2}{\ell} + \frac{m}{\ell}(gt)^2\frac{4}{5} = \frac{6}{5}\frac{mg^2t^2}{\ell}$$

Force when chain falls by $\ell/2$

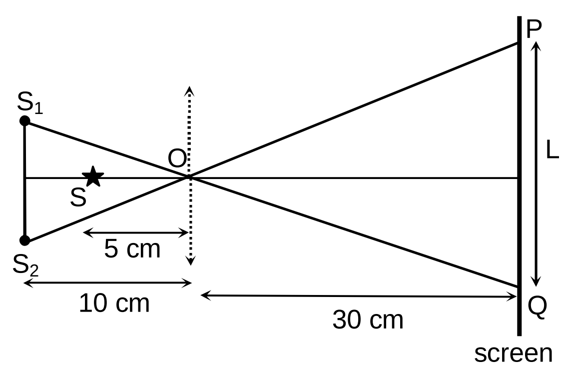
$$N = \frac{m}{\ell}\frac{\ell}{2}g\cdot\cos 37° + \lambda v^2\cos 37°$$

$$= \frac{2}{5}mg + \frac{m}{\ell}\cdot g\ell\frac{4}{5} = \frac{6}{5}mg$$

Total impulse = $\int_0^{\sqrt{2\ell/g}} F_M\cdot dt = \frac{4}{5}m\left(\sqrt{2\ell g}\right)$

7. A, C

Sol. $\frac{1}{v} - \frac{1}{-5} = \frac{1}{10}$



$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{5} = \frac{-1}{10}$

v = –10 cm, $m = \frac{v}{u} = \frac{-10}{-5} = 2$

d = S_1S_2 = 1 mm
D = 10 + 30 = 40 cm = 0.4 m

Fringe width, $\omega = \frac{\lambda D}{d}$

$$\omega = \frac{5\times 10^{-7}\times 0.4}{1\times 10^{-3}} = 2\times 10^{-4}\text{m}$$

ω = 0.2 mm

Now, $\frac{L}{d} = \frac{30}{10} \Rightarrow$ L = 3d = 3mm

The number of interference bands observed on the screen,

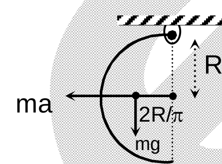
$$n = \frac{L}{\omega} = \frac{3}{0.2} = 15$$
$n = 15$

8. A

Sol. (P) $-mg\frac{R}{2}\theta = \frac{2}{3}mR^2\alpha \Rightarrow \alpha = -\left(\frac{3g}{4R}\right)\theta$

$$T = 2\pi\sqrt{\frac{4R}{3g}}$$

(Q) $maR = mg\frac{2R}{\pi}$



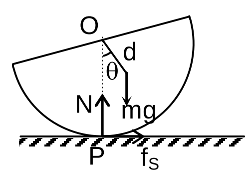
$$a = \frac{2g}{\pi}$$

$$-mg_{eff}\sqrt{R^2 + \frac{4R^2}{\pi^2}}\sin\theta = 2mR^2\alpha$$

$$g_{eff} = \sqrt{a^2 + g^2} = g\sqrt{1 + \frac{4}{\pi^2}}$$

$$\therefore\ \alpha = -\left[\frac{g}{2R}\left(1 + \frac{4}{\pi^2}\right)\right]\theta \Rightarrow T = 2\pi\sqrt{\frac{2R}{g\left(1 + \frac{4}{\pi^2}\right)}}$$

(R) $\tau_P = I_P\alpha$



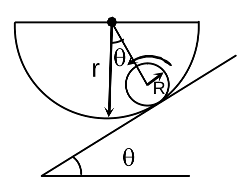
$-mgd\sin\theta = I_P\theta$

$$d = \frac{3R}{8},\ I_P = \frac{13}{20}mR^2$$

$$\alpha = -\left(\frac{mgd}{I_P}\right)\theta$$

$$T = 2\pi\sqrt{\frac{I_P}{mgd}} \Rightarrow T = 2\pi\sqrt{\frac{26R}{15g}}$$

(S) $a = \dfrac{g\sin\theta}{1 + \dfrac{I_C}{mR^2}}$



$$a = -\frac{5g}{7} = -\frac{5g}{7}\frac{x}{r - R} = -\frac{g}{7R}x$$

$$T = 2\pi\sqrt{\frac{7R}{g}}$$

9. C

Sol. $\mu = iA = 0.0550$

$\tau = \vec{\mu} \times \vec{B} = 0.0055$

$|\tau| = I\alpha$

$\alpha = 0.0440$

From conservation of energy

$U_i + K_i = U_f + K_f$

$K_f = -U_f = 0.4400$

10. A

Sol. (P) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{20} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{R}$

$\Rightarrow$ R = 20 cm

For refraction at first surface (air-lens interface)

$\frac{1.5}{v_1} - \frac{1}{-60} = \frac{1.5 - 1}{20}$ …(i)

For refraction at second surface (lens-water interface)

$\frac{4/3}{v_2} - \frac{1.5}{v_1} = \frac{\frac{4}{3} - 1.5}{-20}$ …(ii)

Adding equations (i) and (ii)

$\frac{4}{3v_2} + \frac{1}{60} = \frac{0.5}{20} + \frac{0.5}{60}$

$\Rightarrow v_2 = 80$ cm

(Q) Let F be the focal length of the silvered lens

$\frac{1}{-f} = \frac{2}{f_e} - \frac{1}{f_m 60} = -\frac{1}{-\left(\frac{60}{2}\right)}$

$\Rightarrow$ f = –15 cm

For slivered lens

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{-20} = \frac{1}{-15}$

$\Rightarrow$ v = – 60 cm

(R) The system is equivalent to three thin lens in contact. Let f_2 be the focal length of the concave lens filled with water.

$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(-\frac{1}{15} - \frac{1}{15}\right)$

$\Rightarrow f_2 = -\frac{45}{2}$ cm

The focal length of the equivalent lens from by the three thin lenses in contact

Then, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{30} + \frac{1}{-45/2} + \frac{1}{30}$

$\Rightarrow$ f = 45 cm

For the equivalent lens

$\frac{1}{v} - \frac{1}{-60} = \frac{1}{45}$

$\Rightarrow$ v = 180 cm

(S) For concave lens

$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{-20} \Rightarrow v_1 = -12$ cm

For refraction at air convex lens interface

$$\frac{1.5}{v_2}-\frac{1}{-40}=\frac{1.5-1}{40}\Rightarrow v_2=-120\text{ cm}$$

For refraction at convex lens-liquid interface

$$\frac{4}{3v_3}-\frac{1.5}{-120}=\frac{\frac{4}{3}-1.5}{-40}$$

$\Rightarrow v_3=-160$ cm

11. B

Sol. (P) $P_N=P_0+\rho g\left(\frac{3H}{2}\right)$

$$P_N-P_0=3\rho g\frac{H}{2}$$

(Q) $V=\sqrt{2g(2H)}=2\sqrt{gH}$

$$P_0+\rho gH=P_N-\rho g\frac{H}{2}+\frac{1}{2}\rho(4gH)$$

$$P_N-P_0=\rho gH+\frac{\rho gH}{2}-2\rho gH=-\frac{\rho gH}{2}$$

(R) $P_N=P_0+\rho gh+\frac{1}{2}\rho\omega^2\left(\frac{H}{2}\right)^2$

$$P_N-P_0=\rho gH+\frac{1}{8}\rho\omega^2H^2$$

$$\omega^2H=2g$$

$$\therefore\ P_N-P_0=\frac{5}{4}\rho gH$$

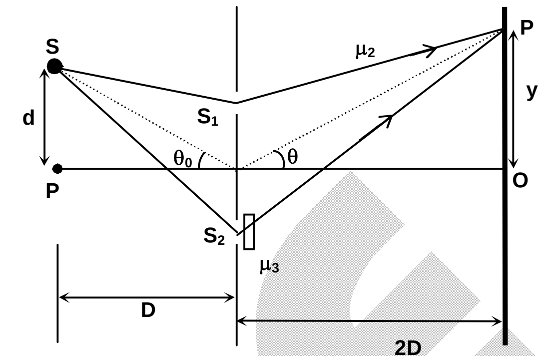
(S) $P_N=P_0+\rho(g\cos 60^\circ)2H$

$$P_N-P_0=\rho gH$$

SECTION – B

12. 7

Sol. Path difference at a point P on the screen



$$\Delta r = \mu_2 d\sin\theta_0 + \mu_2(S_2P - t) + \mu_3 t - \mu_2 S_1P$$
$$= \mu_2 d\tan\theta_0 + \mu_2(S_2P - S_1P) + (\mu_3 - \mu_2)t$$
$$= \mu_2 d\frac{d}{D} + \mu_2 d\frac{y}{2D} + (\mu_3 - \mu_2)t$$

Central bright fringe will be formed at P

If $\Delta r = 0$

$$\Rightarrow \mu_2\frac{d^2}{D} + \mu_2 d\frac{y}{2D} + (\mu_3 - \mu_2)t = 0$$
$$\Rightarrow \mu_2 d\frac{y}{2D} = -\mu_2\frac{d^2}{D} - (\mu_3 - \mu_2)t$$
$$\frac{1.2\times 2\times 10^{-3}}{2}y = \frac{-1.2\times 4\times 10^{-6}}{1} - 0.6\times 6\times 10^{-6}$$
$$\Rightarrow 1.2\times 10^{-3}y = -8.4\times 10^{-6}$$
$$\Rightarrow y = -\frac{8.4}{1.2}\times 10^{-3}\,m = -7\text{ mm}$$

So, distance of the central bright fringe from O is 7 mm

13. 1

Sol. $\left[<\omega> = \frac{\Delta\theta}{\Delta t}\right]$

14. 4

Sol. $Q = Q_0\cos\omega t$

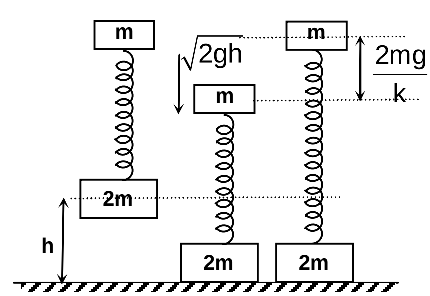
$i = Q_0\,\omega\sin\omega t$

$$\frac{Q_0}{n\sqrt{\pi\varepsilon_0 Lr}} = \frac{Q_0}{\sqrt{L.8\pi\varepsilon_0 r}}.\sin\left(\frac{1}{\sqrt{L.8\pi\varepsilon_0 r}}\times\frac{\pi}{4}\sqrt{8\pi L\varepsilon_0 r}\right)$$
$$= \frac{Q_0}{\sqrt{L.8\pi\varepsilon_0 r}}\times\frac{1}{\sqrt{2}}$$

$n = 4$

15. 4

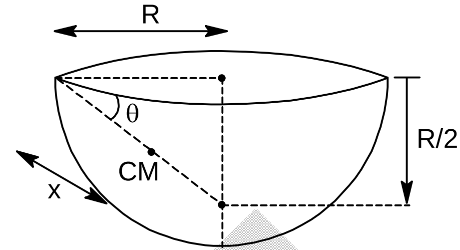
Sol. By the conservation of energy



$$\frac{1}{2}m\left(\sqrt{2gh}\right)^2 = \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 + mg\left(\frac{2mg}{k}\right)$$

16. 3

Sol. $V_y = \omega x \cos\theta = \frac{\omega R}{2}$



Using conservation of angular momentum about centre of mass.

$$mu\frac{R}{4} = \left\{\frac{2}{3}mR^2 - \frac{mR^2}{4} + \frac{5mR^2}{16}\right\}\omega + \frac{5mR^2}{16}\omega$$

$$= \frac{25}{24}mR^2\omega$$

$$\Rightarrow \quad \omega = \frac{6u}{25R}$$

$$\Rightarrow \quad V_y = \frac{3u}{25}$$

17. 2200

Sol. $\Delta\ell_T = \alpha\ell\Delta T$ Extension …(i)

$$\frac{mg}{S} = \frac{F}{S} = Y\frac{\Delta\ell_e}{\ell}$$

$$\Rightarrow \quad \Delta\ell_e = \frac{mg\ell}{SY} \Rightarrow \text{compression} \qquad \text{…(ii)}$$

If there is no change in length, it means extension due to temperature raise must be equal to the elastic compression due to weight. So

$$\alpha\ell\Delta T = \frac{mg\ell}{SY}$$

$$\Rightarrow m = \frac{\alpha SY\Delta T}{g} = \frac{1.1\times10^{-5}\times10\times10^{-4}\times10\times2.0\times10^{11}}{10} = 2.2\times10^3\text{kg}$$

Chemistry

PART – II

SECTION – A

18. D

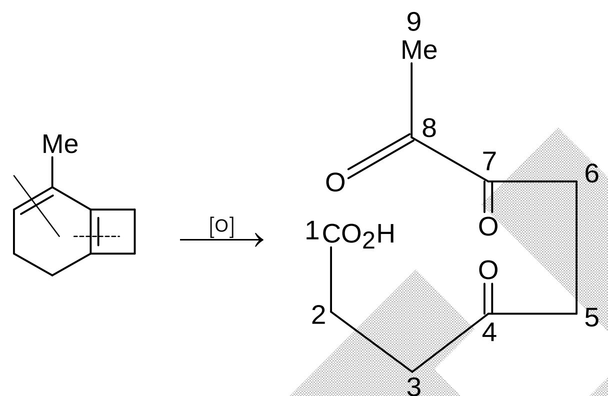
Sol. $K_P = \dfrac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}}\left(\dfrac{P}{\sum n}\right)^{\Delta n}$

$= \dfrac{1\times 1}{4}\times\left(\dfrac{1}{7}\right)^1$

$= \dfrac{1}{28}$

19. C

Sol.



4, 7,8-trioxo nonanoic acid

20. B

Sol. In polar protic solvent down the group nucleophilicity increases and when the attacking atom is same generally stronger the base stronger the nucleophile provided there is no unfavourable steric hindrance.

21. B

Sol. Alanine is a neutral amino acid. The isoelectric points of neutral amino acids are in the pH range 5.5 to 6.3

22. A, B

Sol. $$\underset{}{HNO_2 + NH_2-CO-NH_2} \longrightarrow \underset{(X)}{N_2} + \underset{(Y)}{CO_2} + \underset{(Z)}{H_2O}$$

$$Mg + \underset{(X)}{N_2} \longrightarrow \underset{(W)}{Mg_3N_2} \xrightarrow{H_2O} \underset{(P)}{NH_3} \xrightarrow{CuO} \underset{(X)}{N_2}$$

23. A, B, C, D

Sol. Ti^{2+}, V^{2+}, Cr^{2+} acts as reducing agent

$Mn^{3+}, Fe^{3+}, Co^{3+}$ acts as oxidizing agent.

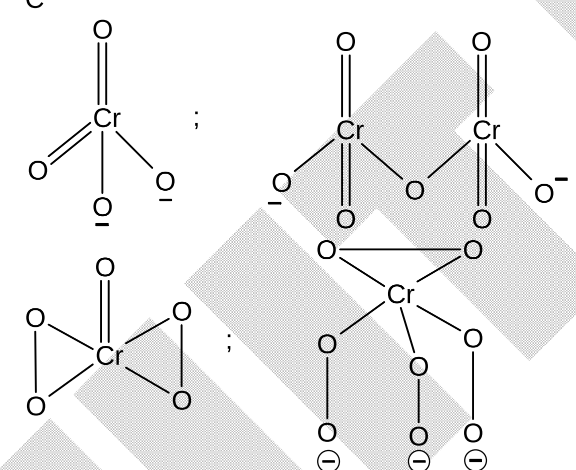
24. A, B, D
Sol. $CH_3-CH_2-CN(P)$
$CH_3-CH_2-CH_2-NH_2\,(Q)$

25. A
Sol. Oxidation always takes place at anode.
In electrolytic cell $\Delta G>0$.
Fuel cell based breath alcohol sensor oxidizes the alcohol in a breath sample and produces an electric current.

26. A
Sol. All Monosaccharides are reducing sugars.
(I) is Mannose and is a C-2 epimer of glucose.
The formation of osazone involves C-1 and C-2.
Glucose, Mannose and Fructose have identical configuration at C-3, C-4 and C-5. Hence, they form the same osazone.

27. B
Sol. (P) is Ascorbic acid (Vitamin C) is a diprotic acid $\left(K_{a_1}=9\times10^{-3}\right)$.
(R) is a Squaric acid and also a diprotic acid. Both are stronger acid than H_2CO_3 and produce effervescence on addition of $NaHCO_3$.

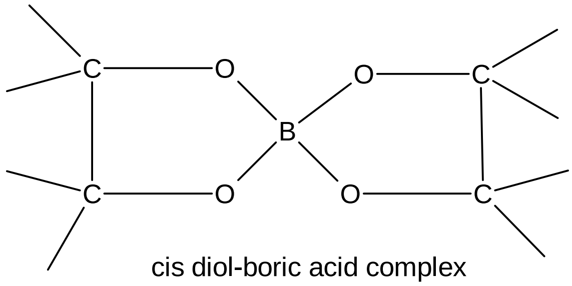
28. C
Sol.



SECTION – B

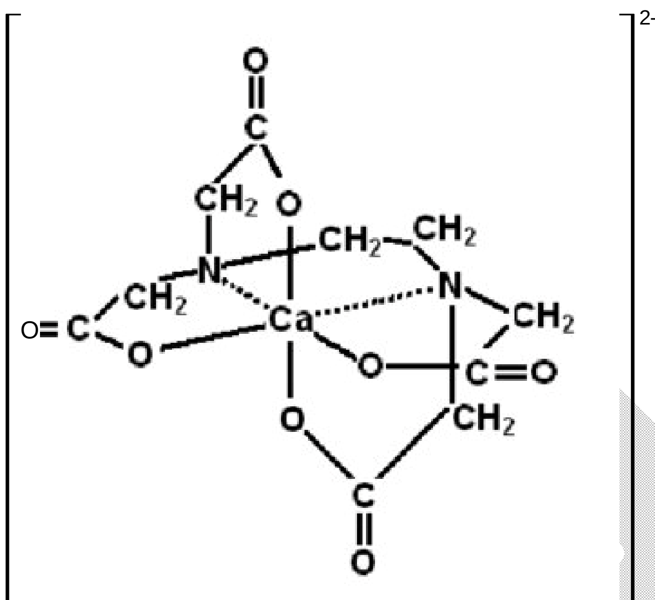
29. 11
Sol. $Na_2CO_3+HCl\longrightarrow NaHCO_3+NaCl$
20 10
10 – 10
$pH=pK_{a_2}=11$

30. 12
Sol. Fullerene has 12 five membered rings
x = 12



y = 2

cis diol-boric acid complex

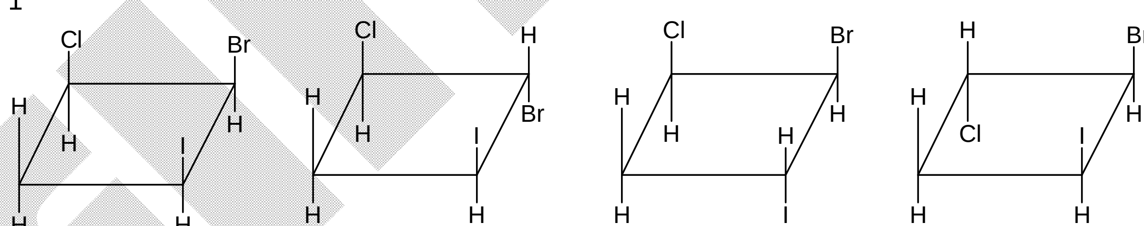


z = 5

$$\therefore \frac{x \times y \times z}{10} = \frac{12 \times 2 \times 5}{10} = 12$$

31. 1

Sol.



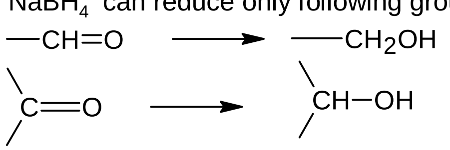
P has four diastereomers and each of the diastereomers exhibits enantiomerism.

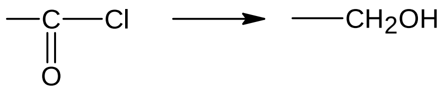
32. 5

Sol. CO_2, CH_3CN, NO_2^+, N_2O & C_3O_2

33. 7

Sol. $NaBH_4$ can reduce only following group.





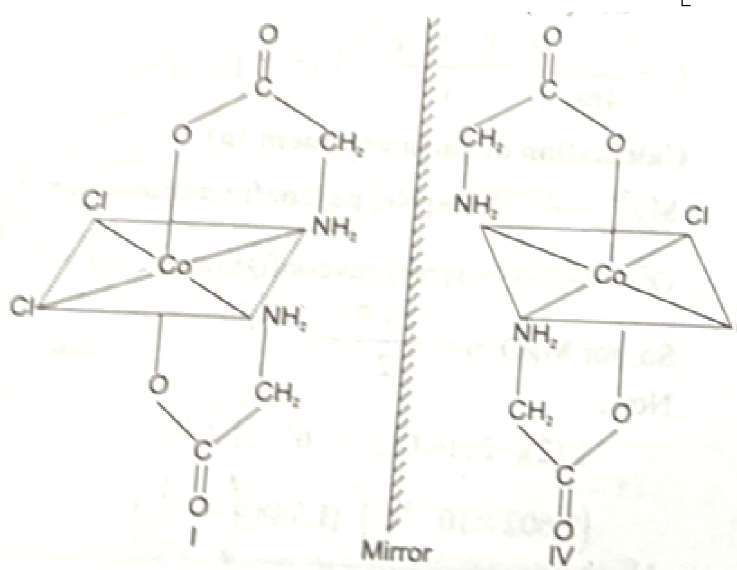
R - X $\longrightarrow$ R - H

($2^o/3^o$ halide)

34. 6

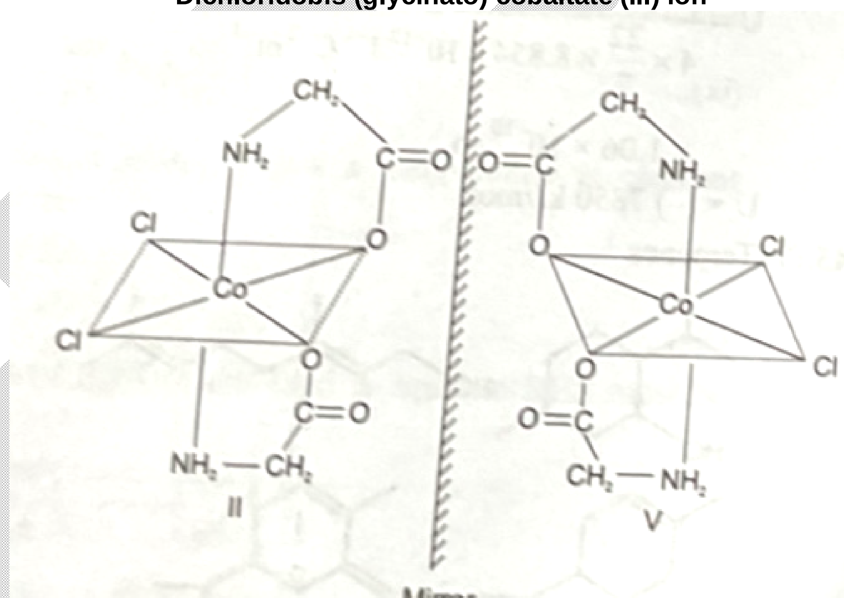
Sol.

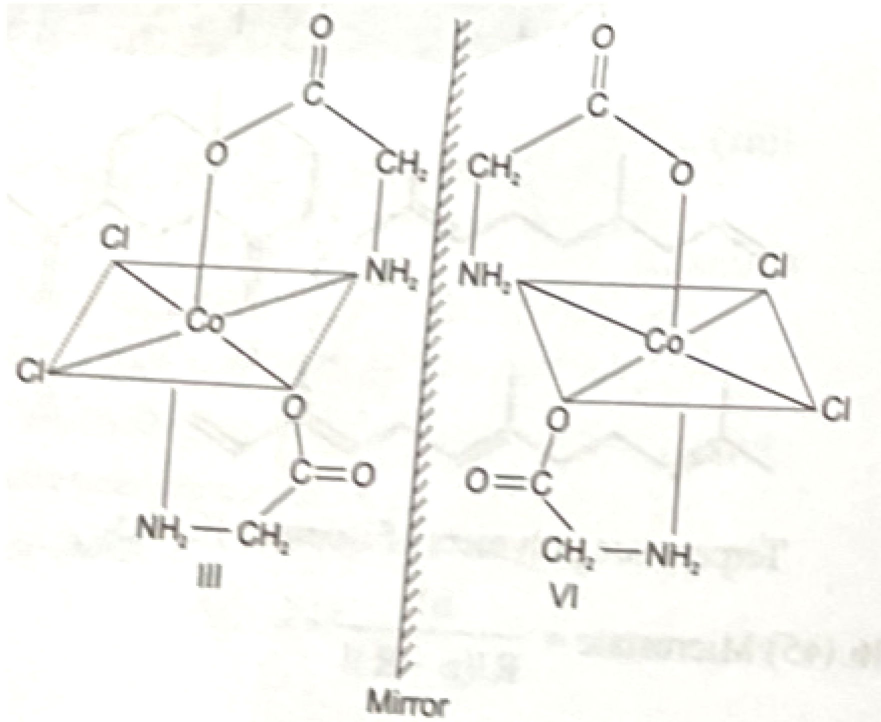
Dichloridobis (glycinato) cobaltate (III) ion $\left[CoCl_2\left(gly\right)_2\right]^{\ominus}$



Cl

Dichloridobis (glycinato) cobaltate (III) ion





Total number of optical active isomers = 6(six)

Mathematics

PART – III

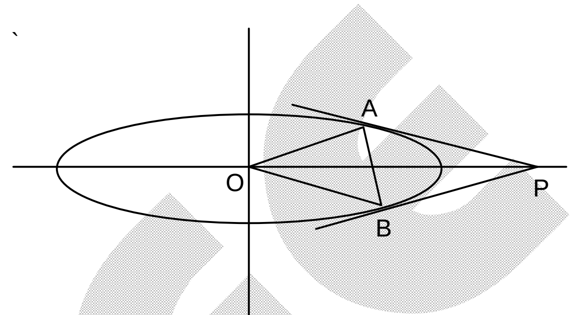
SECTION – A

35. D

Sol. Let the point of intersection of tangents A and B be P(h, k), then equation of AB is

$$\frac{xh}{4}+\frac{yk}{1}=1 \quad \ldots(1)$$

Homogenizing the ellipse using (1)



$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2\left(k^2-1\right)+\frac{2hk}{4}xy=0 \quad \ldots(2)$$

Given equation of OA and OB is

$x^2 + 4y^2 + \alpha\, xy = 0$ …(3)

$\because$ (2) and (3) represent same line

Hence $\frac{h^2-4}{16}=\frac{k^2-1}{4}=\frac{hk}{2\alpha}$

$h^2 - 4 = 4(k^2 - 1)$

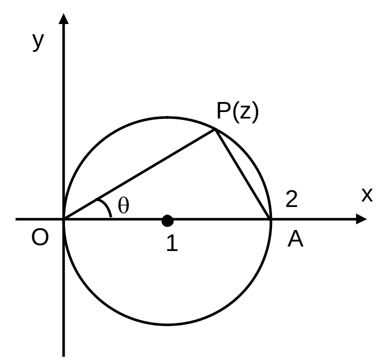
$\Rightarrow h^2 - 4k^2 = 0$

$(h - 2k)(h + 2k) = 0$

$\therefore$ locus $(x - 2y)(x + 2y) = 0$.

36. B

Sol. $|z - 1| = 1$ represents a circle with centre at 1 and radius equal to 1.



We have $\angle OPA = \frac{\pi}{2}$

$\Rightarrow \arg\left(\frac{2-z}{0-z}\right)=\frac{\pi}{2} \Rightarrow \frac{z-2}{z}=\frac{AP}{OP}i.$

Now in ΔOAP

$\tan\theta=\frac{AP}{OP}$.

Thus $\frac{z-2}{z}=i\tan\theta$.

37. A

Sol. $f(x) = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)+2\sqrt{2}$

or $f(x) = \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)+2\sqrt{2}$

$\Rightarrow Y = \left[\sqrt{2},\ 3\sqrt{2}\right]$

and $X = \left[-\frac{3\pi}{4},\ \frac{\pi}{4}\right]$ or $\left[\frac{\pi}{4},\ \frac{5\pi}{4}\right]$.

38. A

Sol. $g(x) = x + x\int_0^1 y^2 g(y)\,dy + x^2\int_0^1 y g(y)\,dy$

$= x + \alpha x + \beta x^2$

$\alpha = \int_0^1 y^2 g(y)\,dy = \int_0^1 y^2\left(y + \alpha y + \beta y^2\right)dy$

$= \frac{3\alpha}{4} - \frac{\beta}{5} = \frac{1}{4}$ …(1)

$\beta = \int_0^1 y\left(y + \alpha y + \beta y^2\right)dy$

$= \frac{3\beta}{4} - \frac{\alpha}{3} = \frac{1}{3}$ …(2)

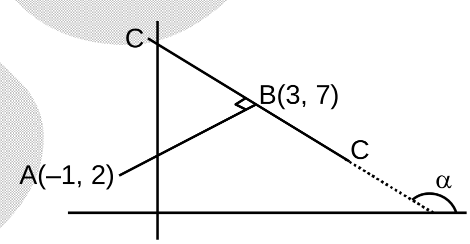
$\alpha = \frac{61}{119}$, $\beta = \frac{80}{119}$.

39. B, C

Sol. Solving the differential equation we get

$\left(\sin^2 x\right)y + (\sin x)y^2 + \left(\cos x + \cos^2 x\right) = 0$

40. A, C



Sol. Slope of AB = $\frac{7-2}{3-(-1)} = \frac{5}{4}$

$\Rightarrow$ Slope of BC = $-\frac{4}{5}$ = tan α

$\Rightarrow \sin\alpha = \frac{4}{\sqrt{41}}$; $\cos\alpha = \frac{-5}{\sqrt{41}}$

Now, $AB = \sqrt{16+25} = \sqrt{41}$

$\Rightarrow BC = \frac{3}{4}\sqrt{41}$

Let, C $\equiv$ (h, k)

$\Rightarrow \frac{h-3}{\cos\alpha} = \frac{k-7}{\sin\alpha} = \pm\frac{3}{4}\sqrt{41}$

$\Rightarrow \frac{h-3}{-5/\sqrt{41}} = \frac{k-7}{4/\sqrt{41}} = \pm\frac{3}{4}\sqrt{41}$

$\Rightarrow (h, k) \equiv \left(\frac{27}{4}, 4\right)$ or $\left(-\frac{3}{4}, 10\right)$

$\Rightarrow$ Mid–point of rectangle $\equiv \left(\frac{23}{8}, 3\right)$ or $\left(\frac{-7}{8}, 6\right)$

$\Rightarrow$ Distance from origin = $d = \sqrt{\left(\frac{23}{8}\right)^2 + 3^2}$ or $d = \sqrt{\left(\frac{-7}{8}\right)^2 + 6^2}$

$\Rightarrow$ [d] = 4 or 6

41. A, B

Sol. $x^{1/3} = t$

$x = t^3$

$\int e^t \cdot 3t^2 \cdot dt = 3e^{x^{1/3}}\left(x^{2/3} - 2x^{1/3} + 2\right) + c$

42. B

Sol. L_1 and L_2 are coplanar

So, they are intersecting $\Rightarrow 6 - \lambda = 1 + \mu$ and $-1 + 2\lambda = -1 + 3\mu$

$\Rightarrow \lambda = 3$ and $\mu = 2$

$$\cos\theta = \frac{(2\vec{b} - \vec{a})\cdot(\vec{a} + 2\vec{b})}{|2\vec{b} - \vec{a}||\vec{a} + 3\vec{c}|} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \text{ so, } [\tan\theta] + 3 = 4$$

Point of intersection is $3\vec{a} + 5\vec{b}$. It's distance from origin is $\sqrt{34} = d$

So, [d] = 5

43. C

Sol. (P) $2024^{2024} = 2^{6072} \cdot 11^{2024} \cdot 23^{2024}$

$$f_1(2024^{2024}) = (2025)^2 \cdot 6073 = 3^8 \cdot 5^4 \cdot 6073$$
$$f_2(2024^{2024}) = f_1(3^8 \cdot 5^4 \cdot 6073) = 9 \cdot 5 \cdot 2 = 90$$
$$f_3(2024^{2024}) = f_1(90) = 12$$
$$f_4(2024^{2024}) = f_1(12) = 6$$
$$f_5(2024^{2024}) = f_1(6) = 4$$
$$f_6(2024^{2024}) = f_1(4) = 3$$

(Q) $|A^{-1}| = -2 \Rightarrow |A| = -\frac{1}{2}$

$|adj(2A)| = |2A|^2 = (2^3|A|)^2 = 16$

$|2A| = -4$

$\Rightarrow \frac{1}{3}(|adj(2A)| + |2A|) = 4$

(R) A(a, b). The centre of the given circle (1, –2)

Equation of circumcircle of ΔABC is $(x - a)(x - 1) + (y - b)(y + 2) = 0$

$x^2 + y^2 - (a + 1)x + (2 - b)y - a - 2b = 0$ ..... (1)

$x^2 + y^2 - x + y - 2 = 0$ ..... (2)

Comparing equation (1) and (2), we get

$a + 1 = 1 \Rightarrow a = 0$, $2 - b = 1 \Rightarrow b = 1 \Rightarrow a + b = 1$

(S) $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7} = \frac{\sqrt{7}}{8}$

$\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = \frac{1}{8}$

$\Rightarrow \tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{3\pi}{7} = \sqrt{7}$

44. D

Sol. (P) $\because$ α, β, γ are roots of $x^3 - x^2 - 2x - 3 = 0$

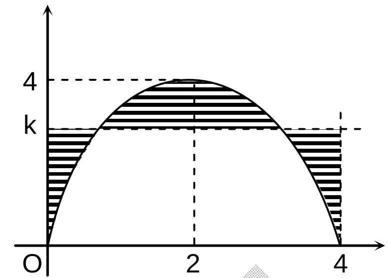
$\sum\alpha = 1$, $\sum\alpha\beta = -2$, $\alpha\beta\gamma = 3$

$$\Delta = \begin{vmatrix} 2\beta\gamma - \alpha^2 & \gamma^2 & \beta^2 \\ \gamma^2 & 2\alpha\gamma - \beta^2 & \alpha^2 \\ \beta^2 & \alpha^2 & 2\alpha\beta - \gamma^2 \end{vmatrix} = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}^2$$

$$= \left(\sum\alpha\right)^2 \cdot \left(\sum\alpha^2 - \sum\alpha\beta\right)^2 = 1\cdot\left(\left(\sum\alpha\right)^2 - 3\sum\alpha\beta\right)^2 = 49$$

$\sqrt{\Delta} = 7$

(Q) $f(x) = x(4 - x)$ and $y = k$
F(k) is the shaded regions area

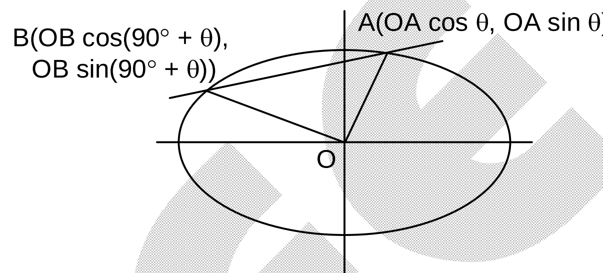


Area $A = 2\left[\int_0^k f^{-1}(x)dx + \int_k^4 \left(2 - f^{-1}(x)\right)dx\right]$

$\frac{dA}{dk} = 2\left(f^{-1}(k) - 2 + f^{-1}(k)\right)$

$\frac{dA}{dk} = 0 \Rightarrow f^{-1}(k) = 1 \Rightarrow k = 3$

(R) $\frac{1}{OA^2} = \frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{\lambda^2}$ ..... (1)



$\frac{1}{OB^2} = \frac{\sin^2\theta}{9} + \frac{\cos^2\theta}{\lambda^2}$ ..... (2)

$\because \frac{1}{OA^2} + \frac{1}{OB^2} = \frac{14}{45}$

$\Rightarrow \frac{1}{\lambda^2} = \frac{14}{45} - \frac{1}{9} = \frac{1}{5} \Rightarrow \lambda^2 = 5$

(S) Hints: Use Sandwich Theorem

45. A

Sol. $g(x) = f^{-1}(x) \Rightarrow f(g(x)) = x$

$f'(g(x)) \cdot g'(x) = 1 \Rightarrow g'(x) = \frac{1}{f'(g(x))}$

and $g''(x) = -\frac{1}{\left(f'(g(x))\right)^2} \cdot f''(g(x)) \cdot g'(x) = -\frac{f''(g(x))}{\left(f'(g(x))\right)^3}$

(P) $f(x) = 2x + \cos x$ $\quad f(0) = 1 \quad g(1) = 0$
$f'(x) = 2 - \sin x$ $\quad f'(0) = 2$
$f''(x) = -\cos x$ $\quad f''(0) = -1$

$g''(1) = -\frac{f''(0)}{(f'(0))^3} = \frac{1}{8}$

$\Rightarrow \frac{4}{g''(1)} = 2$

(Q) $f(x) = x^x$; $f(1) = 1 \Rightarrow g(1) = 1$
$f'(x) = x^x(1 + \log_e x) \Rightarrow f'(1) = 1$
$f''(x) = x^x(1 + \log_e x)^2 + x^{x-1} \Rightarrow f''(1) = 2$

$g''(1) = -\frac{f''(1)}{(f'(1))^3} = -2$

(R) $f(x) = \tan^{-1} x$; $f(0) = 0 \Rightarrow g(0) = 0$

$f'(x) = -\frac{1}{1+x^2}$; $f'(0) = 1$

$f''(x) = -\frac{2x}{(1+x^2)^2} \Rightarrow f''(0) = 0$

$g''(0) = -\frac{f''(0)}{(f'(0))^3} = 0$

(S) $f(x)=e^{x^3+x}$ f(0) = 1 $\Rightarrow$ g(1) = 0

$f'(x)=e^{x^3+x}\left(3x^2+1\right)$ $\Rightarrow$ f′(0) = 1

$f''(x)=e^{x^3+x}\left(3x^2+1\right)+6xe^{x^3+x}$ $\Rightarrow$ f″(0) = 1

$\Rightarrow g''(1)=-\dfrac{f''(0)}{(f'(0))^3}=-1$

SECTION – B

46. 7

Sol. Let [x] = p; [y] = q; [z] = r

$$\Delta=\begin{vmatrix} p+2 & q & r \\ p & q+1 & r \\ p & q & r+1 \end{vmatrix} = 2(q + r + 1) + p$$

max Δ = 17

max (Δ – 10) = 7

47. 4

Sol. 4 $\sin^2\theta$ = 1

$\sin^2\theta=\dfrac{1}{4}$

2 $\sin^2\theta$ + 3 $|\sin\theta|$ – 2 = 0

$|\sin\theta|$ = –2; $|\sin\theta|$ = $\dfrac{1}{2}$

Hence 4

48. 8

Sol. HCF of 6174, 57624, 6048 is 42 hence terms common will be 1, 2, 3, 6, 7, 14, 21, 42 hence 8 terms.

49. 4

Sol. $P_1 \equiv x + y = 0$

$P_2 \equiv y + z = 0$

$P_3 \equiv z + x = 0$

$P_4 \equiv x + y + z = 1$

P_1, P_2, P_3 intersect at origin P_1 P_2 P_4 intersect at (1, –1, 1) and so on

Hence volume = $\dfrac{1}{6}[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ = $\dfrac{2}{3}$

6V = 4

50. 5

Sol. $\log S=\sum_{r=1}^{2n}\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)=\int_0^2\log\left(1+x^2\right)dx$

51. 2

Sol. Let f(x) = $x^3 - 3x^2 + 5x = (x-1)^3 + 2(x-1) + 3$

$g(y) = y^3 + 2y \Rightarrow g'(y) = 3y^2 + 2 > 0\ \forall\ y \in R$

$\Rightarrow g(\alpha - 1) = -2$ and $g(\beta - 1) = 2$ and g(y) is odd

$\Rightarrow (\alpha + \beta) = 2$