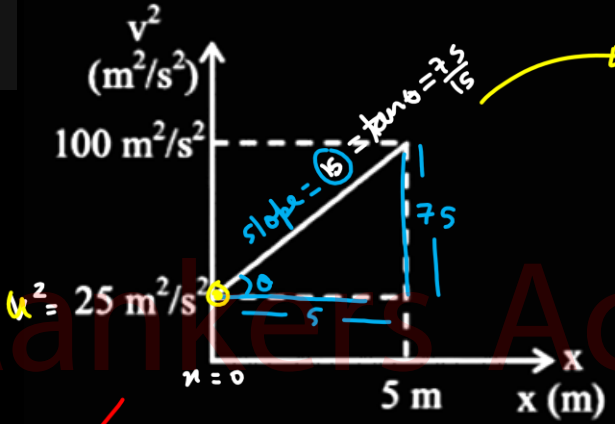

PHYSICS
Rankers Academy JEE

7

A particle is moving along x-axis and graph between square of speed and position of the particle is given in the figure. At t = 0 and x = 0 m, select incorrect statement –



(A) Acceleration of the particle is 15 m/s at t = $\frac{1}{2}$ s FALSE

(B) Acceleration of the particle is 7.5 m/s at t = 1 s TRUE

(C) Acceleration of the particle is constant True.

(D) At t = 1 s, velocity of particle is 12.5 m/s True.

$\text{slope}_{v^2-x} = \frac{d(v^2)}{dx} = \frac{d(v^2)}{dv} \cdot \frac{dv}{dx} = 2v \cdot \frac{dv}{dx} = 2a$

$a = \frac{1}{2} \times \text{slope} = \frac{1}{2} \times 15 = 7.5\ m/s^2$ (constant)

$v = u + at$

$v = 5 + 7.5 \times 1 = 12.5\ m/s$

2

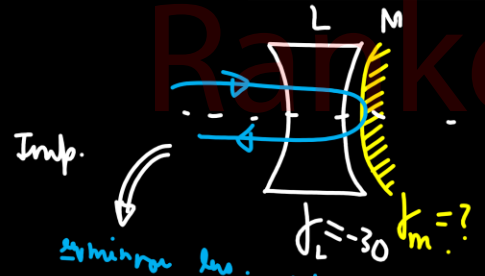
An object is placed at a distance of 20 cm from an equi-concave lens of focal length 30 cm whose one side is silvered. If a virtual image is formed at $\frac{20}{3}$ cm from the lens then what is the radius of curvature of the curved surface?

(A) 30 cm (B) 40 cm

(C) 60 cm (D) 15 cm

$R = 2f_m = 60$

$P_L = +\frac{1}{f_L}$ $P_M = -\frac{1}{f_m}$



Equiconvex lens. mirror

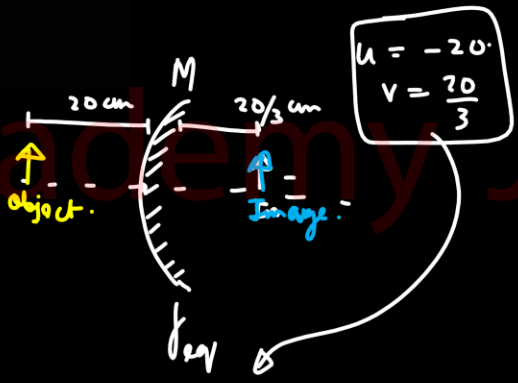
$P_{eq} = 2P_L + 1P_m$

$-\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$

$-\frac{1}{10} = \frac{2}{-30} - \frac{1}{f_m}$

$\frac{1}{f_m} = \frac{3}{30} - \frac{2}{30} = \frac{1}{30}$

$f_m = 30$



$u = -20$

$v = \frac{20}{3}$

$\frac{1}{f_{eq}} = \frac{1}{v} + \frac{1}{u}$

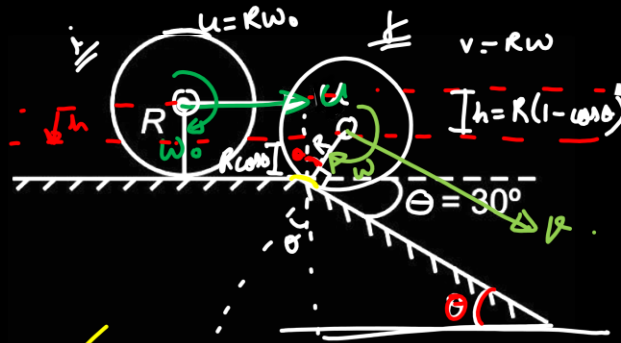
$\frac{1}{f_{eq}} = \frac{1}{\left(\frac{20}{3}\right)} - \frac{1}{20}$

$f_{eq} = 10$ (concave mirror)

3

$I = \frac{1}{2}MR^2$

A uniform solid cylinder of radius R = 15 cm rolls over a horizontal plane passing in to an inclined plane forming an angle $\Theta = 30°$ with the horizontal. Find the maximum value of the velocity u which still permits the cylinder to roll onto the inclined plane section without a jump. The sliding is assumed to be absent.



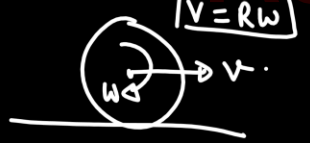
JEE 1

(A) $\frac{7\sqrt{3}-8}{4}$ m/s $\simeq 1$ m/s. (B) 2 m/s

(C) $\sqrt{2}$ m/s (D) $\sqrt{3}$ m/s

Jump

no flying off.



$KE_{cylinder} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$KE_{cylinder} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}MR^2\omega^2$ ($R^2\omega^2 = v^2$)

$KE_{cylinder} = \frac{3}{4}mv^2$.

$E_i = E_f$.

$\frac{3}{4}mu^2 + \frac{4}{3}mgh = \frac{3}{4}mv^2$.

$u^2 = v^2 - \frac{4}{3}gR(1-\cos\theta)$

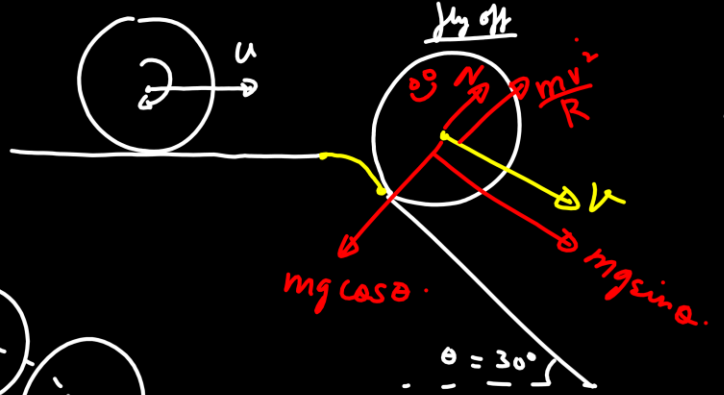
$u^2 = v^2 - \frac{4}{3}\times\frac{3}{2}(1-\cos\theta)$

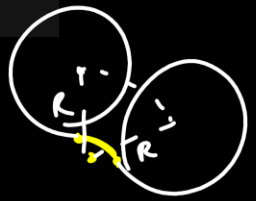
$u^2 = v^2 - 2\left(1-\frac{\sqrt{3}}{2}\right)$ — (1)

$g = 10$

$R = \frac{15}{100}$

$gR = 3/2$





$N + \frac{mv^2}{R} = mg\cos\theta$

$v = \sqrt{gR\cos\theta} =$

$v^2 = \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$ — (2)

① & ②

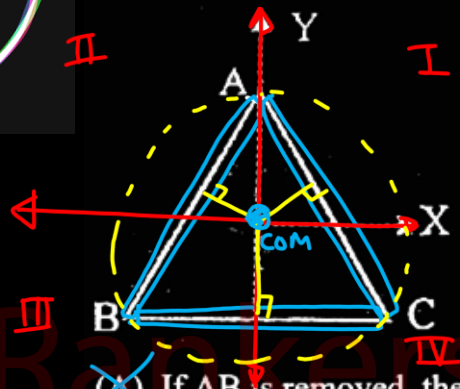
$u^2 = \frac{3\sqrt{3}}{4} - 2\left(1 - \frac{\sqrt{3}}{2}\right)$

$u = \frac{3\sqrt{3}}{4} + \frac{4\sqrt{3}}{4} - 2$

$u = \frac{7\sqrt{3} - 8}{4} \simeq 1$

4

A uniform wire frame ABC is in the shape of an equilateral triangle in xy-plane with centroid at the origin; then point out incorrect statement



(A) If AB is removed, the centre of mass of the remaining figure is in fourth quadrant TRUE

(B) If AC is removed, the centre of mass of the remaining figure is in first quadrant FALSE.

(C) If BC is removed, the centre of mass of the remaining figure is on the positive Y axis TRUE

(D) If AC is removed, the centre of mass of the remaining figure is in third quadrant. TRUE

JEE 1

5

S.T.

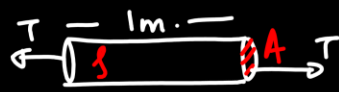
A string will break apart if it is placed under too much tensile stress. One type of steel has density. $\rho_{steel} = 10^4\ kg/m^3$ and breaking stress $\alpha = 9 \times 10^8\ N/m^2$. We make a guitar string from (4π) gram of this type of steel. It should be able to withstand (900π)N without breaking. What is highest possible fundamental frequency (in Hz) of standing waves on the string if the entire length of the string vibrates?

(A) 375 ✓

(B) 225

(C) 350

(D) 425



mass = ρ × vol.

$\mu = \rho A$ ✓

stress = T/A

Ans = $(f_0)_{max}$.

ρ

$stress_{max} = 9 \times 10^8$ Pa.

$m = \frac{4\pi}{1000}$ kg

$\rho = 10^4$ kg/m³

$T_{max} = 900\pi$ N

$m = \rho \times AL$

$L = \frac{m}{\rho A}$

JEE 1

$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$

$f_0 = \frac{1}{2m}\sqrt{\frac{T}{\rho A}}$

$f_0 = \frac{1}{2m}\sqrt{T \rho A}$

$f_0 = \frac{T}{2m}\sqrt{\frac{\rho(A)}{T}} = \frac{T}{2m}\sqrt{\frac{\rho}{(T/A)}}$

$f_0 = \frac{T}{2m}\sqrt{\frac{\rho}{stress}}$

$(f_0)_{max} = \frac{900\pi}{8\pi/1000}\sqrt{\frac{10^4}{9 \times 10^8}}$

$= \frac{9}{8} \times \frac{10^5}{100} = \frac{3}{8} \times 10^3 = \frac{3000}{8} = 375$

6

A particle performs S.H.M of amplitude A along a straight line. When it is at a distance $x = \frac{\sqrt{3}}{2}A$ from mean position, its kinetic energy gets increased by an amount $\frac{1}{2}m\omega^2 A^2$ due to an impulsive force. Then its new amplitude becomes :

(A) $\frac{\sqrt{5}}{2}$ A (B) $\frac{\sqrt{3}}{2}$ A

(C) $\sqrt{2}$ A (D) $\sqrt{5}$ A

$V = \omega\sqrt{A^2 - x^2}$

M-1

E_i extra. new SHM

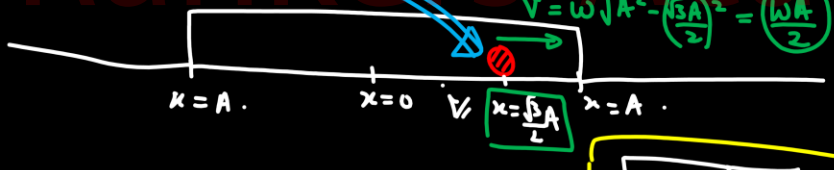
$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = \frac{1}{2}m\omega^2A_1^2$

$2A^2 = A_1^2$ $\quad A_1 = \sqrt{2}A$

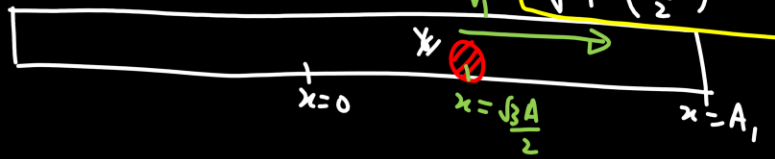
M-2

$KE_{extra} = \frac{1}{2}m\omega^2A^2$

$V = \omega\sqrt{A^2 - \left(\frac{\sqrt{3}A}{2}\right)^2} = \frac{\omega A}{2}$



$V_1 = \omega\sqrt{A_1^2 - \left(\frac{\sqrt{3}}{2}A\right)^2} = \frac{\sqrt{5}}{2}\omega A$



M-2

provided extra KE

$E_i + \frac{1}{2}m\omega^2A^2 = E_f$

$\frac{1}{2}m\left(\frac{\omega A}{2}\right)^2 + \frac{1}{2}m\omega^2A^2 = \frac{1}{2}mV_1^2$

$\frac{5\omega^2A^2}{4} = V_1^2$

$V_1 = \frac{\sqrt{5}}{2}\omega A$

$A_1^2 - \frac{3}{4}A^2 = \frac{5}{4}A^2$

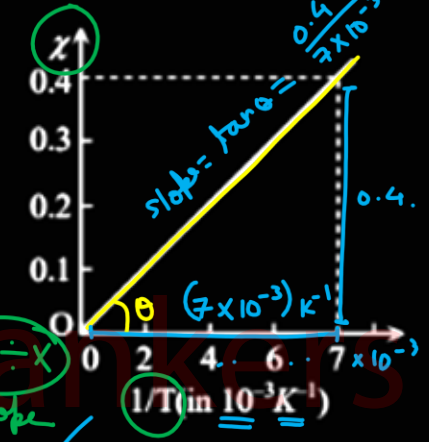
$A_1 = \sqrt{2}A$

7

JEE 1

The $\chi - 1/T$ graph for an alloy of paramagnetic nature is shown in fig. the curie constant is nearly.

$C = ?$



$\chi = \frac{C}{T}$ → $C = \chi T = \frac{\chi}{\left(\frac{1}{T}\right)}$ ⇒ Slope.

slope $_{\chi - \frac{1}{T}}$ =

$Y = X$

slope

$Y \times X$

Area

C = slope $= \frac{0.4}{7 \times 10^{-3}} = \frac{400}{7} = 57.1428$

(A) 57K

(B) 2.8×10^{-3} K

(C) 570 K

(D) 17.5×10^{-3} K

8

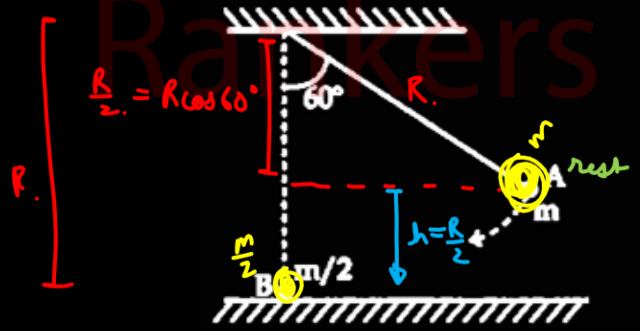
As shown below, bob A of a pendulum having massless string of length ' R ' is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a friction less table in the center. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity.)

$e = 1$

$h = R(1 - \cos 60°)$

$mgh = \frac{1}{2}mu^2$

$u = \sqrt{2gh}$

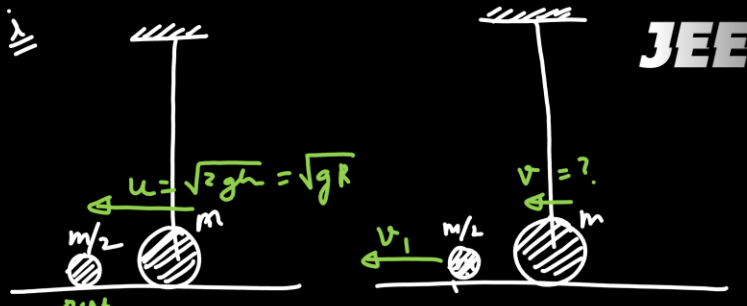


(A) $\frac{4}{3}\sqrt{Rg}$

(B) $\frac{2}{3}\sqrt{Rg}$

(C) $\sqrt{Rg}$

(D) $\frac{1}{3}\sqrt{Rg}$



JEE 1

P

$mu = mv + \frac{m}{2}v_1$

$2u = 2v + v_1$ — ①

C

$e = \frac{V_{sep}}{V_{app}}$ $e = 1$

$1 = \frac{v_1 - v}{u}$

$u = -v + v_1$ — ②

① − ②

$u = 3v$

$v = \frac{u}{3} = \frac{1}{3}\sqrt{gR}$

9

Two merucry drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if $\frac{1}{\pi}$ is the surface tension (in SI unit), is :

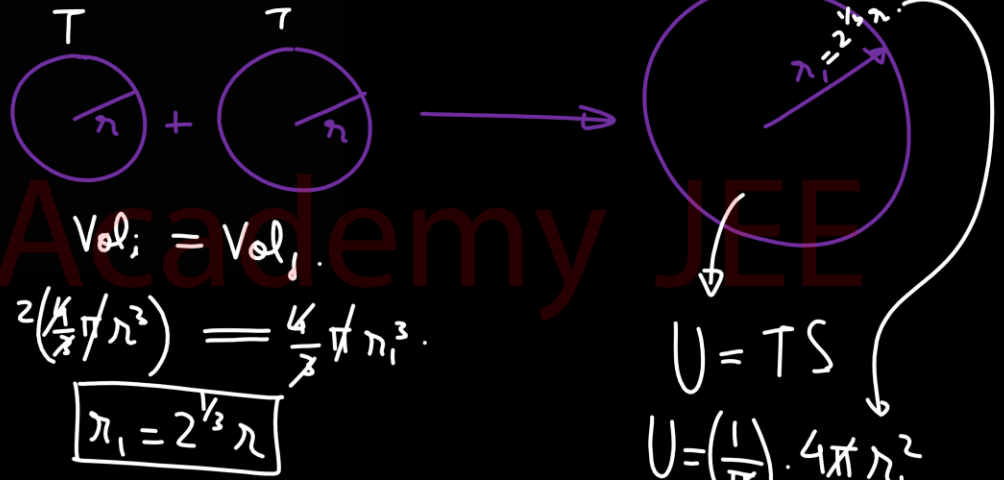
(A) $2^{5/3}r^2$

(B) $4r^2$

(C) $2r^2$

(D) $2^{8/3}r^2$

$T = \frac{1}{\pi}$



$U = TS$

$\Delta U = T\Delta S$

$U_f - U_i = T(S_f - S_i)$

$Vol_i = Vol_f$

$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi r_1^3$

$r_1 = 2^{1/3} r$

$U = TS$

$U = \left(\frac{1}{\pi}\right) \cdot 4\pi r_1^2$

$U = 4 \cdot 2^{2/3} \cdot r^2$

$U = 2^{(2+2/3)} \cdot r^2 = 2^{8/3} r^2$

JEE 1

10

A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is :

(A) a standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, $n = 0,1,2.$

$x = -\frac{\lambda}{4}, \frac{3\lambda}{4}, \cdots$

AN. $R = \pm 2A$ $R = 0$ Node.

(B) a wave travelling along +x direction.

(C) a wave travelling along −x direction

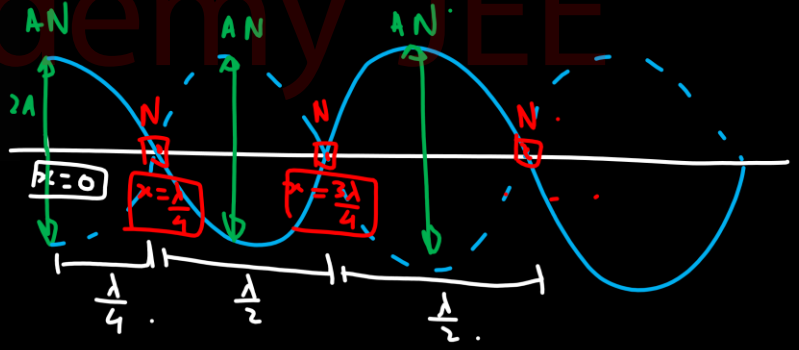
(D) a standing wave having nodes at $x = \frac{n\lambda}{2}$; $n = 0,1,2.$ $x = 0.$

$y_1 = A\sin(\omega t - kx)$ $y_2 = A\sin(\omega t + kx)$ JEE 1

$y_{s/w} = y_1 + y_2$

$y_{s/w} = 2A\cos(kx)\sin(\omega t)$. $x = 0$.

$y_{s/w} = R\sin(\omega t)$.



$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$2\cos B\sin A$

11

The energy of a system is given as $E(t) = \alpha^3 e^{-\beta t}$ where t is the time and $\beta = 0.3\ s^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At $t = 5\ s$, maximum percentage error in the energy is :

(A) 6%

(B) 8.4%

(C) 11.6%

(D) 4%

$y = f(x) = y(x)$

$\frac{dE}{E}$ (hint)

$\left|\frac{\Delta\alpha}{\alpha}\right| \times 100 = 1.2\%$

$\left|\frac{\Delta t}{t}\right| \times 100 = 1.6\%$

Ans $= \left[\left|\frac{\Delta E}{E}\right| \times 100\right]_{t=5} = ?$

I II

$E = \alpha^3\, e^{-0.3t}$

diff. 't'

$\frac{dE}{dt} = 3\alpha^2 \cdot \frac{d\alpha}{dt}\, e^{-0.3t} + \alpha^3\, e^{-0.3t}(-0.3)$

$E \div \Rightarrow \frac{dE}{E} = \frac{3\alpha^2 e^{-0.3t}}{\alpha^3 e^{-0.3t}} \cdot d\alpha - (0.3)\frac{\alpha^3 e^{-0.3t}}{E} \cdot dt$

$\frac{dE}{E} = 3\frac{d\alpha}{\alpha} - (0.3)t\frac{dt}{t}$

error

$\left|\frac{\Delta E}{E}\right| \times 100 = 3\left|\frac{\Delta\alpha}{\alpha}\right| \times 100 + 0.3t\left|\frac{\Delta t}{t}\right| \times 100$

$\%\ \text{error}\ E\big|_{t=5} = 3(1.2\%) + (1.5)(1.6\%) = 3.6\% + 2.4\% = 6\%$

JEE 1

12

A force $\vec{F} = \frac{-k(y\hat{i}-x\hat{j})}{\sqrt{x^2+y^2}}$ N is applied on a block, whose position is P(x, y). The block moves on a circualr path $x^2 + y^2 = r^2$. Find the work done by the force $\vec{F}$ in two complete revolutions.

(A) $k\pi r$

(B) $2k\pi r$

(C) zero

(D) $4k\pi r$

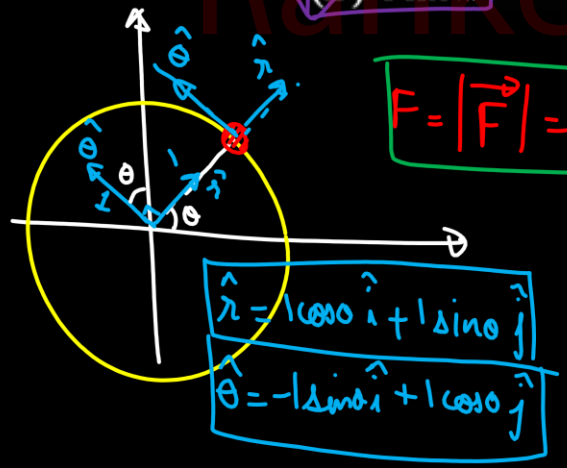
$|\vec{F}| = F = k$

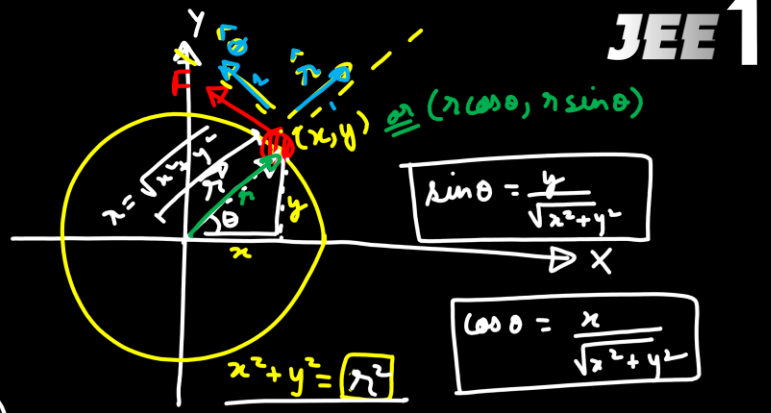
$\vec{F} = \frac{-k}{R}(y\hat{i} - x\hat{j})$

$\vec{F} = -k\left(\frac{y}{\sqrt{x^2+y^2}}\hat{i} - \frac{x}{\sqrt{x^2+y^2}}\hat{j}\right)$

$\vec{F} = -k\underbrace{(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})}_{\text{unit vector}} = -k\,\hat{\theta}$

$F = |\vec{F}| = k$





$W = F(2\pi r \times 2) = 4\pi k r$

13

Statement 1 :When there is a thin layer of water between two glass plates there is a strong attraction between them

Statement 2 :The pressure between the plates become less than atmospheric pressure as pressure difference is created due to surface tension.

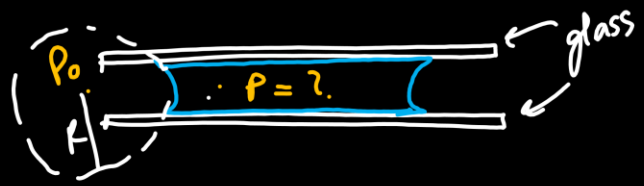
(A) Statement -1 is True, Statement -2 is True; Statement -2 is a correct explanation for Statement -1 .

(B) Statement -1 is True, Statement -2 is True; Statement -2 is not a correct explanation for Statement -1 .

(C) Statement -1 is True, Statement -2 is False.

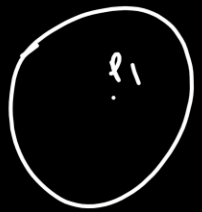
(D) Statement -1 is False, Statement -2 is True.

JEE 1



$P_0 > P \Rightarrow$ Less pressure b/w plates

$P_0 - P = \frac{2T}{R}$



$P_1 - P_2 = \frac{2T}{R}$

14

JEE 1

Statement 1: In an adiabatic process the change in internal energy of a gas is equal to negative of the work done by the gas

Statement 2 :Temperature of the gas remains constant during an adiabatic process

(A) Statement -1 is True, Statement -2 is True; Statement -2 is a correct explanation for Statement - 1 .

(B) Statement -1 is True, Statement -2 is True; Statement -2 is not a correct explanation for Statement -1 .

(C) Statement -1 is True, Statement -2 is False.

(D) Statement -1 is False, Statement -2 is True.

$Q = 0$

First Law of thermodynamics

$\Delta U = Q - W$

$\Delta U = -W =$ – ve of work done 'by' the gas.

T = const for Isothermal Process

15

JEE 1

If the magnetic field of a plane electromagnetic wave is given by

$$B = 200 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right].$$

Then the maximum electric field associated with it is: (The speed of light = 3×10^8 m/s)

(A) 6×10^4 N/C

(B) 3×10^4 N/C

(C) 4×10^4 N/C

(D) 4.5×10^4 N/C

B_0

$E_0 = ?$

$E_0 = B_0 c$ *

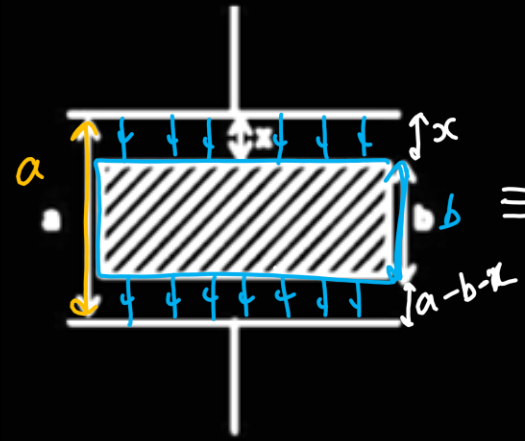
$= 200 \times 10^{-6} \times 3 \times 10^8$

$= 600 \times 10^2$

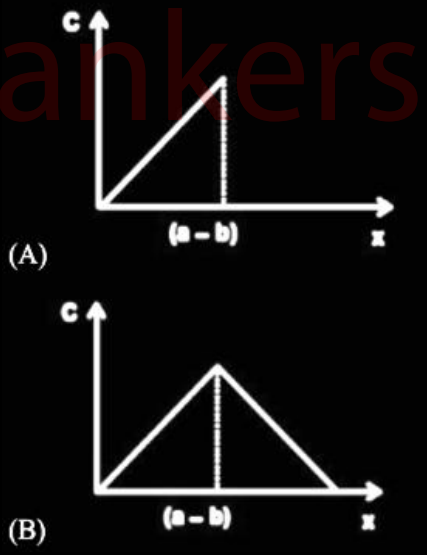
$= 6 \times 10^4$ N/C

16

The distance between two parallel plates of a capacitor is a. A conductor of thickness b (b < a) is inserted between the plates as shown in the figure. The variation of effective capacitance between the plates of the capacitors as a function of the distance (x) is best represented by



JEE 1



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

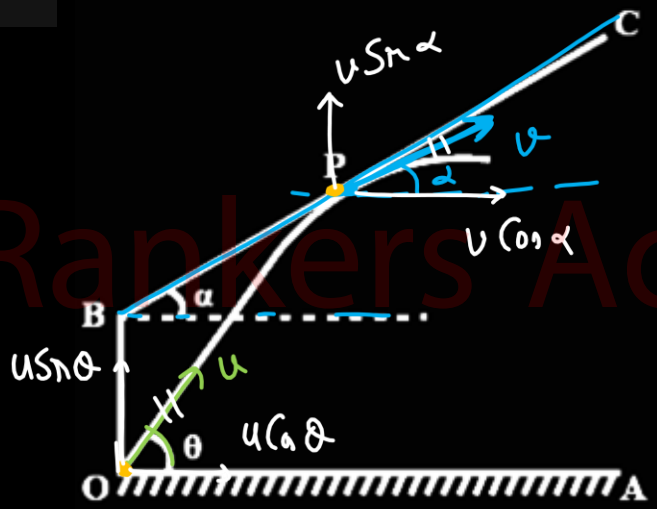
$$\frac{1}{C_q} = \frac{1}{\left(\frac{A\varepsilon_0}{x}\right)} + \frac{1}{\frac{A\varepsilon_0}{(a-b-x)}}$$

$$\frac{1}{C_q} = \frac{x + a - b - x}{A\varepsilon_0} \Rightarrow C_q = \frac{A\varepsilon_0}{(a-b)} = \text{Const.}$$

independent of x

17

A particle projected with speed u, at an angle θ grazes the inclined surface BC at point P as shown. Find the time required to reach P from O.



(A) $\frac{2u \sin \theta}{g \cos \alpha}$

(B) $\frac{u \sin \theta}{g \cos \alpha}$

(C) $\frac{u \sin (\theta-\alpha)}{g \cos \alpha}$

(D) $\frac{2u \sin (\theta-\alpha)}{g \cos \alpha}$

from O to P

JEE 1

during projectile $S_{nx} a_x = 0$

$u_x = u \cos\theta = v \cos\alpha$

$v = \frac{u \cos\theta}{\cos\alpha}$ — (1)

O to P $v_y = u_y + a_y t$

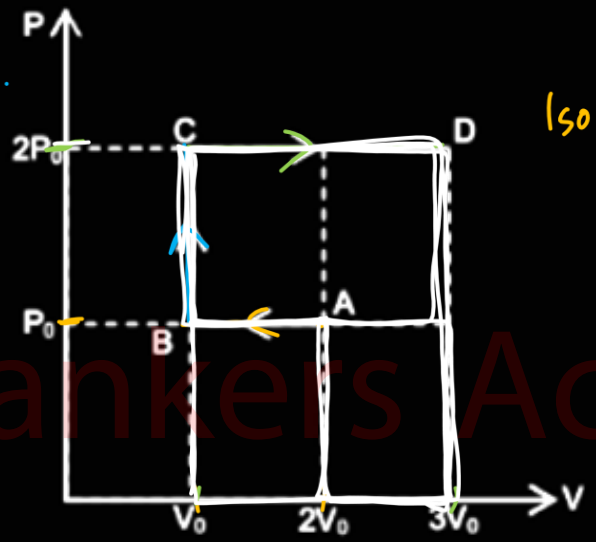
$v \sin\alpha = u \sin\theta - g t$

$g t = u \sin\theta - \left(\frac{u \cos\theta}{\cos\alpha}\right) \sin\alpha$

$t = \frac{u \sin(\theta-\alpha)}{g \cos\alpha}$

18

P – V diagram of an ideal gas is as shown, work done by the gas in the process ABCD is



(A) $4P_0V_0$

(B) $2P_0V_0$

(C) $3P_0V_0$

(D) P_0V_0

Isobaric $W_{AB} = P\Delta V = P_0(V_0 - 2V_0) = -P_0V_0$ — ①

Isochoric $W_{BC} = 0$

Isobaric $W_{CD} = P\Delta V = 2P_0(3V_0 - V_0)$

$= +4P_0V_0$ — ②

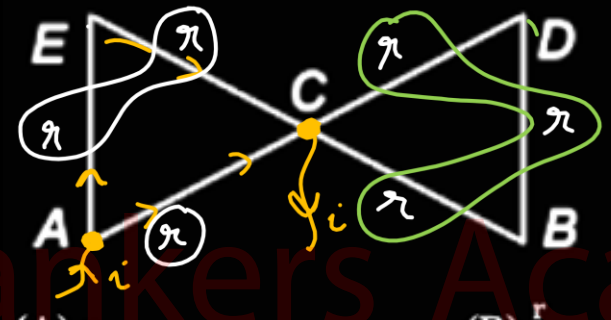
$W_{ABCD} = 4P_0V_0 - P_0V_0$

$= 3P_0V_0$

JEE 1

19

If resistance of each wire in the network shown is r, the equivalent resistance between A & C is equal to



(A) r

(B) $\frac{r}{2}$

(C) $\frac{2r}{3}$

(D) $\frac{3r}{2}$

$R_{CDBC} = 3r$ is shorted by point C.

$R_{AC} = \frac{2r \times r}{2r + r} = \frac{2r}{3}$

20

Figure shows the graph of stopping potential versus the frequency of a photosensitive metal. The plank's constant h and work function W_c of the metal are (V and v_0 are two different constant.)

(A) $W_c = (2\ V)e;\ h = \frac{(3\ V)e}{v_0}$

(B) $W_c = (2\ V)e;\ h = \frac{(2\ V)e}{v_0}$

(C) $W_c = (3\ V)e;\ h = \frac{(3\ V)e}{v_0}$

(D) $W_c = (3\ V)e;\ h = \frac{(2\ V)e}{v_0}$

V_0 } $K_{max} = eV_0 = h\upsilon - \phi$

JEE 1

$eV_0 = h\upsilon - \phi$ *

$10eV = h(4\upsilon_0) - \phi$ — ①

$e(4V) = h(2\upsilon_0) - \phi$

$4eV = h(2\upsilon_0) - \phi$ — ②

① − 2 × ②

$10eV - 8eV = \phi \Rightarrow \phi = 2eV$

$4eV = 2h\upsilon_0 - 2eV$

$\Rightarrow h = \frac{6eV}{2\upsilon_0} = \frac{(3V)e}{\upsilon_0}$

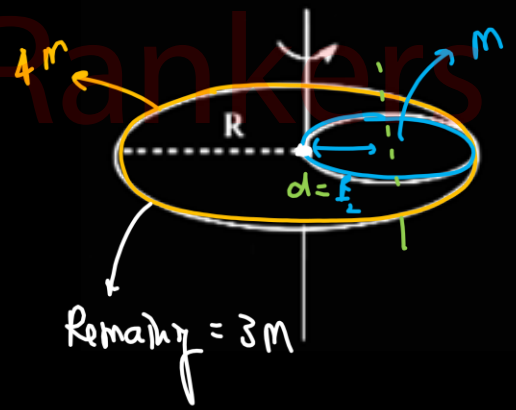
21

JEE 1

A circular hole of radius $\frac{R}{2}$ is cut from a circular disc of radius R. The radius of gyration of this disc about an axis passing through its original centre and normal to its plane is $R\sqrt{\frac{N}{24}}$, find the value of N.

$(3m)K^2 = I$

for $I_{rem} = I_{total} - I_{cavity}$ *



'm' is mass of $\pi\left(\frac{R}{2}\right)^2$

Then '4m' is mass of πR^2

$(3m)K^2 = \frac{(4m)R^2}{2} - I_{cavity}$

Where $I_{cavity} = \frac{mR^2}{2} + md^2$

$= \frac{m\left(\frac{R}{2}\right)^2}{2} + m\left(\frac{R}{2}\right)^2$

$= \frac{3mR^2}{8}$

$3mK^2 = 2mR^2 - \frac{3mR^2}{8}$

$K^2 = \frac{13}{24}R^2$ Ans

22

A charge particle is projected in the magnetic field of $(6\hat{i} - 5\hat{j}) \times 10^{-3}$ T. The acceleration of the particle is found to be $(\alpha\hat{i} + 6\hat{j}) \times 10^{-6}$ ms^{-2}. The value of α is

JEE 1

$$\text{Sin}\alpha \quad \vec{a} = \frac{\vec{F}}{m} = q(\vec{v} \times \vec{B})$$

Here $\vec{a} \perp \vec{v}$

$$\Rightarrow \vec{a} \cdot \vec{v} = 0$$

$$\Rightarrow 6\alpha - 5 \times 6 = 0$$

$$\boxed{\alpha = 5}$$

23

In the circuit shown, the value of L is 5 henry and the power factor of the circuit is 0.8 . It is also given that the voltage drop across capacitor is $\frac{2}{5}$ times the voltage drop across the inductor. Find impendence (in ohm) of the circuit.

JEE 1

$\cos\phi = \frac{R}{Z} = 0.8$ — (1)

$V_C = \frac{2}{5} V_L = 0.4 V_L$

$\Rightarrow X_C = 0.4 X_L$ — (2)

$X_L = \omega L$

$= 314 \times 5$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Eq(1) Eq(2)

$Z^2 = (0.8Z)^2 + (X_L - 0.4X_L)^2$

$\Rightarrow Z^2 - 0.64Z^2 = (0.6X_L)^2$

$\Rightarrow 0.36Z^2 = 0.36X_L$

$Z = X_L = \omega L = 314 \times 5 = 1570\ \Omega$

24

Energy required to remove an electron revolving in orbit with quantum no. ' n ' in a hydrogen like atom is 217.6 eV . Frequency of its revolution is 26.4×10^{15} Hz. If in a time interval of 15 nano-seconds, the electron jumps to orbit with quantum no. '$\frac{n}{4}$', then average torque acting on electron is $K \times 10^{-27}$ N − m. Find ' K '. (Given $h = 2.1\pi \times 10^{-34}$ J − s, $f_0 = 6.6 \times 10^{15}$ Hz is frequency of revolution at ground state in hydrogen atom and $E_0 = 13.6$eV is energy required to remove an e^- in ground state of H − atom respectively.)

$E_n = 13.6\frac{Z^2}{n^2} = 217.6\,eV$

$\frac{Z^2}{n^2} = 16 \quad -①$

$f = \frac{1}{T} = \frac{v}{2\pi r} \propto \frac{\left(\frac{Z}{n}\right)}{\left(\frac{n^2}{Z}\right)}$

$f \propto \frac{Z^2}{n^3}$

$f = f_0\frac{Z^2}{n^3}$

$26.4 \times 10^{15} = (6.6 \times 10^{15})\frac{Z^2}{n^3}$

$\frac{Z^2}{n^3} = 4 \quad -②$

JEE 1

24

Energy required to remove an electron revolving in orbit with quantum no. ' n ' in a hydrogen like atom is 217.6 eV . Frequency of its revolution is 26.4×10^{15} Hz. If in a time interval of 15 nano-seconds, the electron jumps to orbit with quantum no. $'\frac{n}{4}'$, then average torque acting on electron is $K \times 10^{-27}$ N − m. Find ' K '. (Given $h = 2.1\pi \times 10^{-34}$ J − s, $f_0 = 6.6 \times 10^{15}$ Hz is frequency of revolution at ground state in hydrogen atom and $E_0 = 13.6$eV is energy required to remove an e^- in ground state of H − atom respectively.)

JEE 1

from ① ÷ ②

$n = 4$ — ③

& $Z = 16$

$$\tau_{av} = \frac{\Delta L}{\Delta t} = \frac{n\frac{h}{2\pi} - \frac{n}{4}\left(\frac{h}{2\pi}\right)}{\Delta t}$$

$$= \frac{3n}{4}\frac{h}{2\pi} \times \frac{1}{\Delta t}$$

$$= \frac{3 \times 2.1 \times \pi \times 10^{-34} \times 1}{2\pi \times 15 \times 10^{-9}}$$

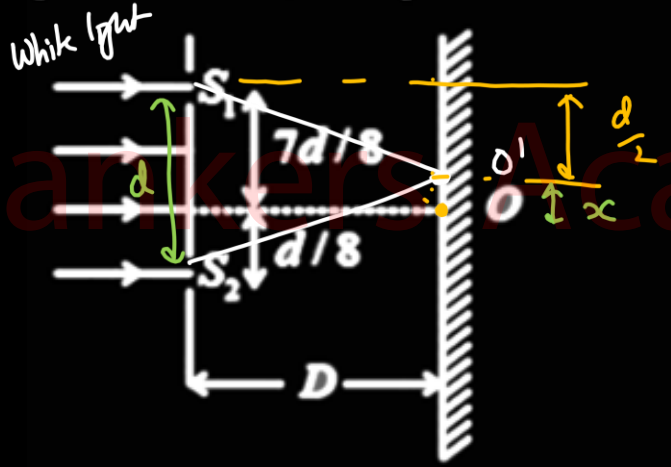
$$= \frac{2.1 \times 10^{-34}}{10^{-8}} = \boxed{21 \times 10^{-27}}$$

5

25

In the figure, if a parallel beam of white light is incident on the plane of the slits S_1 & S_2 then the distance of the central maxima on the screen from O is $\frac{kd}{8}$. Find the value of k.

[Assume $D \gg d, d \gg \lambda$].



$$\frac{d}{2} + x = \frac{7d}{8}$$

$$x = \left(\frac{7}{8} - \frac{1}{2}\right) d$$

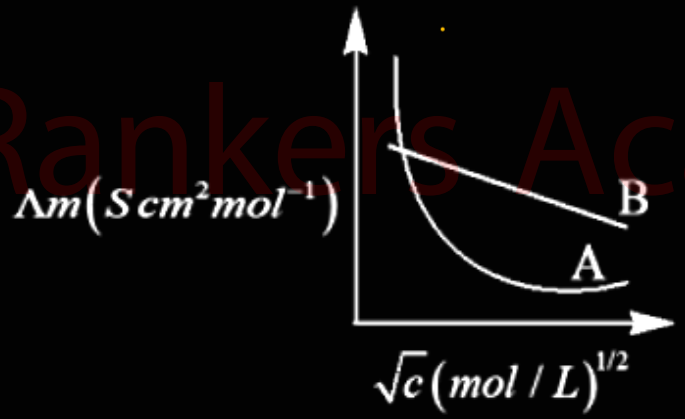
$$= \frac{3}{8} d$$

JEE 1

CHEMISTRY

7

Following figure shows dependence of molar conductance of two electrolytes on concentration. Λ°_m is the limiting molar conductivity.



Which one of the following statements is incorrect regarding above graph?

(A) A represents the molar conductance's variation with concentration for weak electrolyte like acetic acid.

(B) B represents the molar conductance's variation with concentration for strong electrolyte like potassium chloride.

(C) For weak electrolyte Λ°_m can not be obtained by extrapolation of graph.

(D) Λ°_m for weak electrolyte can not be calculated by using λ°_m for individual ions.

2

Choose the correct statement among the following?

(A) $[FeCl_4]^-$ has square planar geometry ✗

(B) $[Fe(C_2O_4)_3]^{3-}$ has one unpaired electrons ✗

(C) $[Co(C_2O_4)_3]^{3-}$ has zero unpaired electrons ✓

(D) $[Ni(CO)_4]$ has square palanar geometry ✗

$3d^8 4s^2 \Rightarrow sp^3 \Rightarrow Td$

Fe^{+3} : d^5 weak L

Co^{+3} : ox behaves str. L

$3d^6$

3

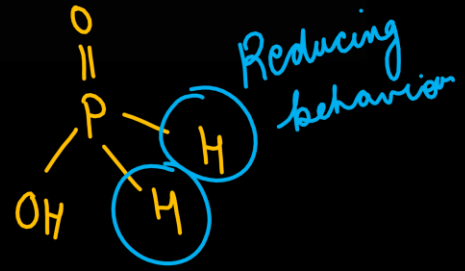
JEE 1

Given below are two statements, one is labelled Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Hypophosphorous acid is a reducing agent and converts silver nitrate into silver metal. $Ag^+ \longrightarrow Ag$

Reason (R) : Hypophosphorous acid contains two P - H bonds. In the light of the above statements, choose the correct answer from the options given below:

H_3PO_2



(A) A is not correct but R is correct.

(B) Both A and R are correct and R is the correct explanation of A.

(C) Both A and R are correct but R is not the correct explanation of A .

(D) A is correct but R is not correct.

4

When 40 ml of 0.2 M silver nitrate solution is mixed with 20 ml of 0.12 M sodium chromate solution, moles of silver chromate precipitated out is _____

(A) 8×10^{-3}
(B) 2.4×10^{-3}
(C) 4×10^{-3}
(D) 1.2×10^{-3}

L.R

$2AgNO_3 + Na_2CrO_4 \longrightarrow Ag_2CrO_4 + 2NaNO_3$

8 m mol 2.4 m mol

2.4 m mmol
$= 2.4 \times 10^{-3}$ mol

5

Which one of the following metal ions forms blue coloured bead in the borax bead test?

(A) Cr^{3+}

(B) Fe^{2+}

(C) Co^{2+}

(D) Ni^{2+}

JEE 1

Cr Mn Fe Co Ni Cu

Green Red Yellow Blue violet Green Brown.

6

Given below are two statements:

Statement-I: Higher the value of K_H at a given temperature, higher the solubility of the gas in a solvent. $P = K_H \uparrow C_B \downarrow$ X

Statement-II: The value of K_H increases with increasing temperature. ✓

In the light of above statements, choose the correct answer from the options given below:

(A) Statement-I is true but Statement-II is false.

(B) Statement-I is false but Statement-II is true. ✓

(C) Both Statement-I and Statement-II are true.

(D) Both Statement-I and Statement-II are false.

7

The correct order of metallic radius of Sc, Ti, Cu and Zn is _____

(A) Sc > Ti > Cu > Zn

(B) Zn > Cu > Ti > Sc

(C) Sc > Ti > Zn > Cu

(D) Zn > Sc > Ti > Cu

JEE 1

Element		Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
Atomic number		21	22	23	24	25	26	27	28	29	30
Electronic configuration	M	$3d^1 4s^2$	$3d^2 4s^2$	$3d^3 4s^2$	$3d^5 4s^1$	$3d^5 4s^2$	$3d^6 4s^2$	$3d^7 4s^2$	$3d^8 4s^2$	$3d^{10} 4s^1$	$3d^{10} 4s^2$
	M^+	$3d^1 4s^1$	$3d^2 4s^1$	$3d^3 4s^1$	$3d^5$	$3d^5 4s^1$	$3d^6 4s^1$	$3d^7 4s^1$	$3d^8 4s^1$	$3d^{10}$	$3d^{10} 4s^1$
	M^{2+}	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	$3d^8$	$3d^9$	$3d^{10}$
	M^{3+}	[Ar]	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	–	–
Enthalpy of atomisation, $\Delta_a H^\ominus$/kJ mol^{-1}		326	473	515	397	281	416	425	430	339	126
Ionisation enthalpy/$\Delta_i H^\ominus$/kJ mol^{-1}											
$\Delta_i H^\ominus$	I	631	656	650	653	717	762	758	736	745	906
$\Delta_i H^\ominus$	II	1235	1309	1414	1592	1509	1561	1644	1752	1958	1734
$\Delta_i H^\ominus$	III	2393	2657	2833	2990	3260	2962	3243	3402	3556	3837
Metallic/ionic radii/pm	M	164	147	135	129	137	126	125	125	128	137
	M^{2+}	–	–	79	82	82	77	74	70	73	75
	M^{3+}	73	67	64	62	65	65	61	60	–	–
Standard electrode potential $E^\ominus$/V	M^{2+}/M	–	−1.63	−1.18	−0.90	−1.18	−0.44	−0.28	−0.25	+0.34	-0.76
	M^{3+}/M^{2+}	–	−0.37	−0.26	−0.41	+1.57	+0.77	+1.97	–	–	–
Density/g cm^{-3}		3.43	4.1	6.07	7.19	7.21	7.8	8.7	8.9	8.9	7.1

8

Fusion of MnO_2 with KOH in the presence of oxygen produces the dark green solution which on acidification undergoes disproportionation reaction which one of the following statements is correct regarding the process _____

(A) Green colour is due to MnO_4^{2-} which is paramagnetic.

(B) Green colour is due to MnO_4^- which is diamagnetic.

(C) MnO_4^{2-} undergoes disproportionation in acidic medium to give MnO_4^- and MnO .

(D) Green colour is due to MnO_4^- in which all the $Mn - O$ bond length are identical.

Potassium permanganate $KMnO_4$

Potassium permanganate is prepared by fusion of MnO_2 with an alkali metal hydroxide and an oxidising agent like KNO_3. This produces the dark green K_2MnO_4 which disproportionates in a neutral or acidic solution to give permanganate.

$$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$$

$$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

Commercially it is prepared by the alkaline oxidative fusion of MnO_2 followed by the electrolytic oxidation of manganate (Vl).

$$MnO_2 \xrightarrow[\text{with air or } KNO_3]{\text{Fused with KOH, oxidised}} MnO_4^{2-}$$

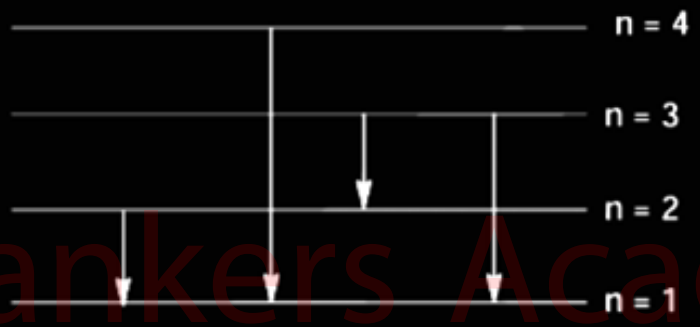
manganate ion

$$MnO_4^{2-} \xrightarrow[\text{alkaline solution}]{\text{Electrolytic oxidation in}} MnO_4^-$$

manganate permanganate ion

9

A hypothetical single electronic atom gives a red, green, blue and violet line spectrum as given in the following diagram



What is the jump of electron corresponding to red line

(A) $3 \rightarrow 1$
(B) $2 \rightarrow 1$
(C) $4 \rightarrow 1$
(D) $3 \rightarrow 2$

10

Which one of the following statements is incorrect?

(A) The actinoids show a greater range of oxidation states than the lanthanoids because the 5f, 6 d and 7 s levels are of comparable energies.

(B) 5f electrons can participate in bonding to a far greater extent than that 4f orbitals because unlike 4f orbitals, 5f orbitals are not as buried as 4f orbirtals.

(C) The lanthanoid contraction is more important than the actinoid contraction because the chemistry of elements succeeding the actinoids are much less known at the present time.

(D) The magnetic properties of the actinoids are less complex than the lanthanoids.

JEE 1

The magnetic properties of the actinoids are more complex than those of the lanthanoids. Although the variation in the magnetic susceptibility of the actinoids with the number of unpaired 5 *f* electrons is roughly parallel to the corresponding results for the lanthanoids, the latter have higher values.

L.R

$H_2A + 2KOH \rightarrow K_2A + 2H_2O$

$\frac{300 \times 0.2}{1000}$ $\frac{700 \times 0.1}{100}$

60 m mol 70 m moles.

70 m moles of H_2O is formed.

$70 \times 10^{-3} \times 13.5 = m\,S\,\Delta T$

$\Delta T = 2.25 \times 10^{-1}$

$= 225 \times 10^{-3}$

When 300 ml of 0.2M H_2A (diprotic strong acid) solution is mixed with 700 ml of 0.1 M KOH solution, the increase in temperature of the final solution is _____ $\times 10^{-3}$ K.

[Given $H^+(aq) + OH^-(aq) \longrightarrow H_2O$

$\Delta_r H = -13.5$ Kcalmol^{-1}]

Specific heat of water = 4.2 cal K^{-1} g^{-1}

Density of water = 1 g cm^{-3}

Assume there is no change in volume of solution on mixing.

(A) 32 (B) 192

(C) 964 (D) 225

JEE 1

12

The nature of oxide MO_2 and physical property of the element which has the lowest 1st ionisation energy among C, Si, Ge, Sn, Pb is

(A) Acidic & soft metal

(B) Amphoteric & metalloid

(C) Amphoteric & soft metal

(D) Basic and metalloid

C > Si > Ge > Sn < Pb

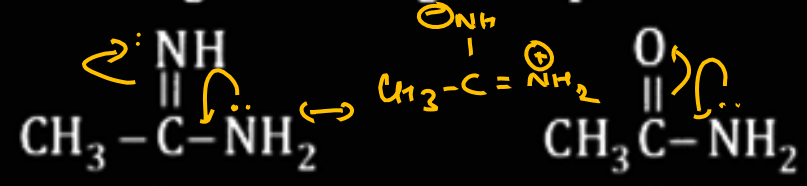
JEE 1

whereas tin and lead are soft metals with low melting points. Melting points and boiling points of group 14 elements are much higher than those of corresponding elements of group 13.

— CO_2, SiO_2 and GeO_2 are acidic, whereas SnO_2 and PbO_2 are amphoteric in nature. Among monoxides, CO is neutral, GeO is distinctly acidic whereas SnO and PbO are amphoteric.

13

Arrange the following compounds in their decreasing basic strength in aq. Solution.



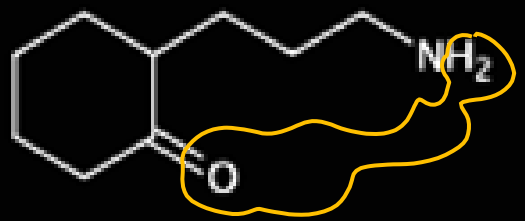
(I)

(III)

(IV)

(A) IV > II > III > I
(B) II > I > IV > III
(C) I > IV > III > II
(D) IV > I > II > III

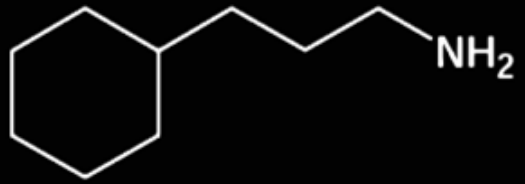
14



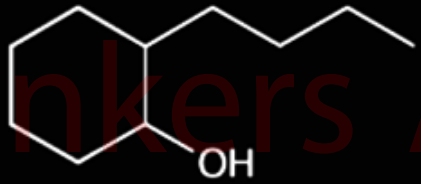
pH (4 - 5) → X → H_2 / Ni → Y. Here Y is:

JEE 1

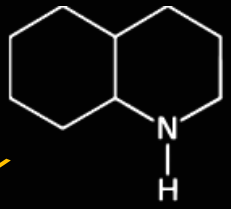
(A)



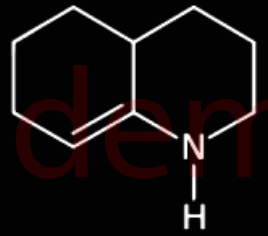
(B)



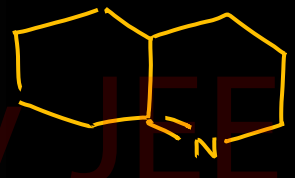
(C)



(D)



(X)

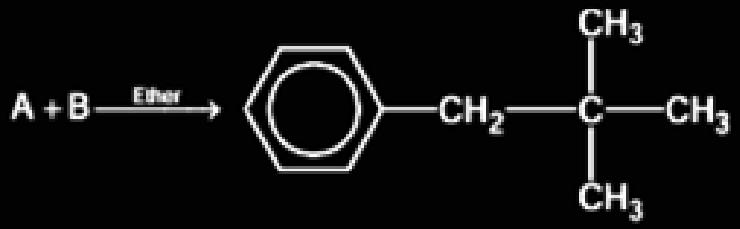


↓ H_2 / Ni

(Y)

15

The best yield of given product can be obtained by using which set of reactants A and B respectively:

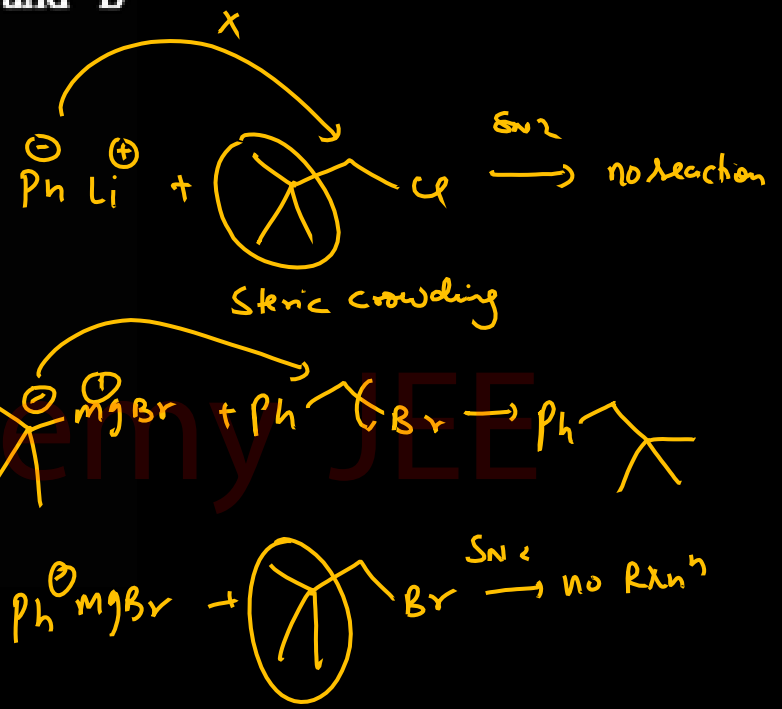


(A) PhLi + Neopentyl chloride

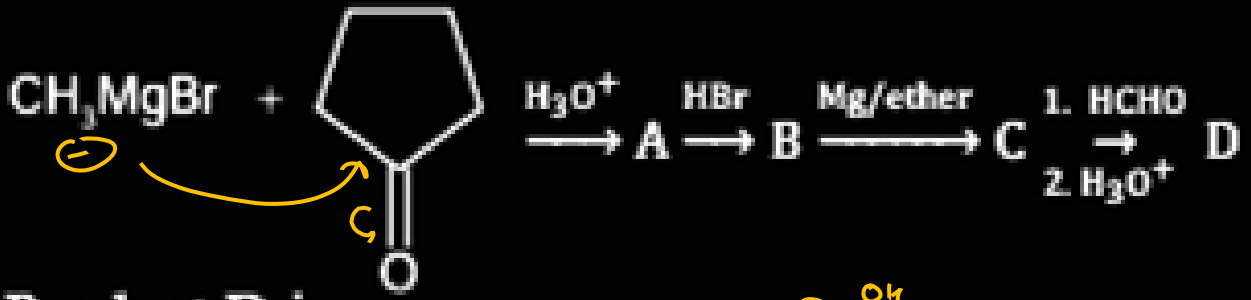
(B) t – BuMgBr + Benzyl bromide

(C) PhMgBr + Neopentyl bromide

(D) Benzyl chloride + t-Butyl chloride

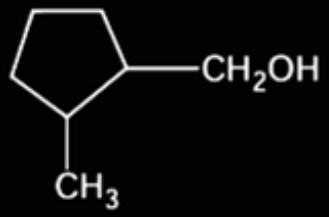


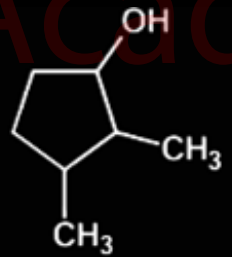
16

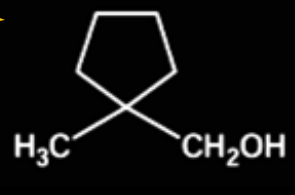


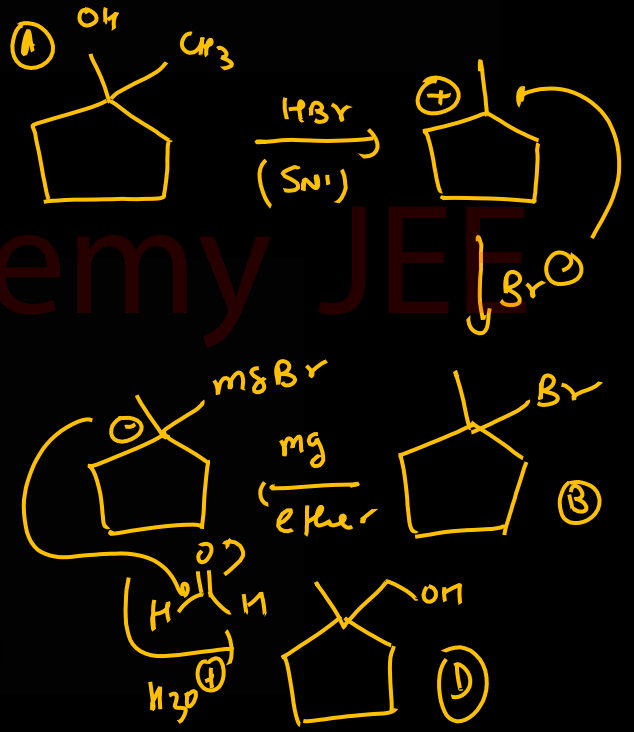
Product D is :

(A) 

(B) 

(C) 

(D) 

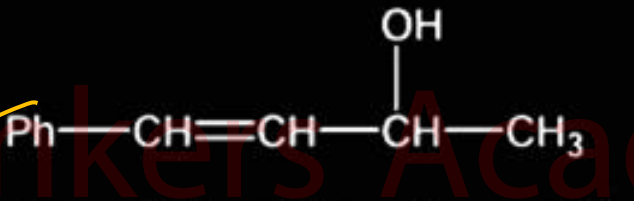


17

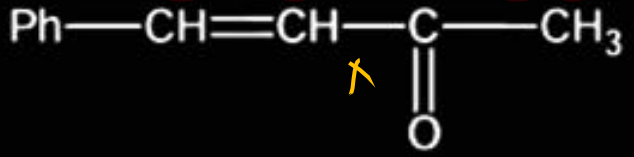
An organic compound containing one oxygen gives red colour with ceric ammonium nitrate, decolourises cold dilute alkaline $KMnO_4$, responds iodoform test and shows geometrical isomerism. It should be:

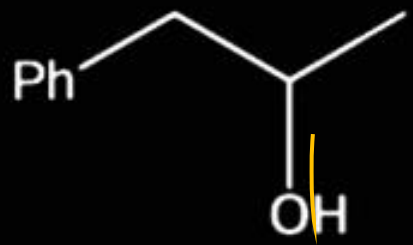
CAN Test → Alcohol.
Unsaturation.

(A) $Ph - CH = CH - CH_2OH$ CAN ✓, Alkali $KMnO_4$ ✓, Iodoform ✗

(B) 

→ 

(C) 

(D) 

18

For the reaction $X(g) \longrightarrow 2Y(g) + \frac{1}{2}Z(g)$

If the initial pressure of the system is 600 mm Hg and pressure at any time is 900 mm Hg, then the mole fraction of X(g) decomposed at constant volume and temperature is _____

(Assume ideal gas behaviour)

(A) 0.197

(B) 0.333

(C) 0.157

(D) 0.147

JEE 1

$PV = nRT$

Const T & V

$P \propto n$

$X(g)$ decomposed $= \frac{200}{600} = \frac{1}{3} = 0.33$

$X(g) \longrightarrow 2Y(g) + \frac{1}{2}Z(g)$

t=0	600 mm	—	—	
t=t	$(600-x)$	$2x$	$\frac{x}{2}$	
	400	400	100 mm Hg.	

$(600-x) + 2x + \frac{x}{2} = 900$

$\frac{3x}{2} = 300 \Rightarrow \boxed{x = 200}$

19

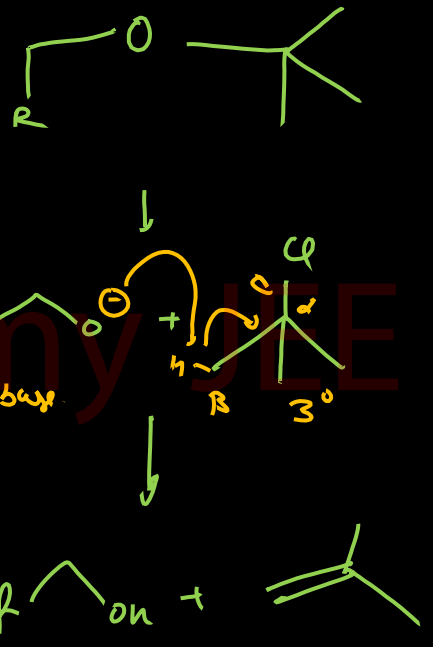
In Williamson synthesis of mixed ether having a primary alcohol and a tertiary alkyl group if tertiary halide is used then:

(A) Rate of reaction will be slow due to slow cleavage of carbon-halogen bond

(B) Alkene will be the main product

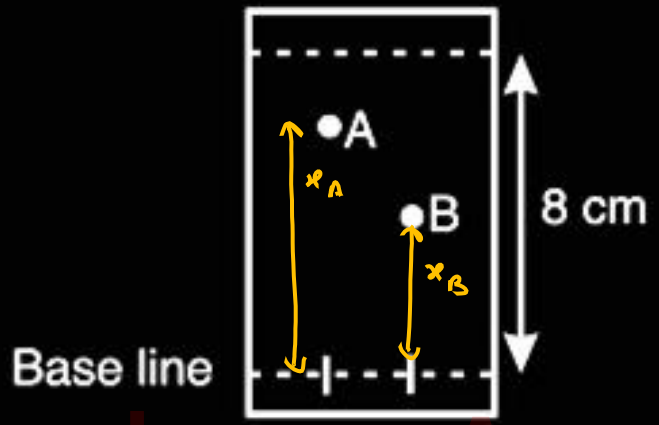
(C) Simple ether will form instead of mixed ether

(D) Expected mixed ether will be formed



20

From the following TLC plate



if the R_f values of A and B are 0.75 and 0.5 respectively, then the difference in the height of A and B on TLC plate is

(A) 1 cm (B) 1.5 cm

(C) 2 cm (D) 2.5 cm

$\frac{x_A}{8} = 0.75$

$x_A = 0.75 \times 8$ — (1)

$\frac{x_B}{8} = 0.5$

$x_B = 0.5 \times 8$ — (2)

$x_A - x_B = (0.75 - 0.5) \times 8$

$= 0.25 \times 8$

$= \frac{1}{4} \times 8 = 2\text{cm}$

21

JEE 1

For the electrochemical cell

$Ca(s) | Ca^{2+}(aq, 1M) \| Cu^{2+}(aq., 1M) | Cu(s)$

The standard emf of the cell is 3.20 V at 300 K .

When the concentration of Ca^{2+} is changed to x M, the cell potential changes to 3.185 V at 300 K . The value of 100 x is _____

[Given: $\frac{F}{R} = 11500 KV^{-1}$, where F is the Faraday constant and R is the gas constant, $\ell n 3.15 = 1.15$]

$3.18 = 3.20 - \frac{RT}{nF} \ln x$

$3.18 = 3.20 - \frac{300}{2 \times 11500} \ln x$

$\ln x = 1.15$

$x = 3.15$

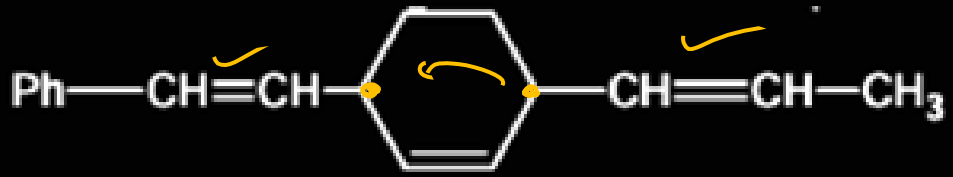
Ans. $3.15 \times 100 = 315$

$Ca(s) + Cu^{+2} \rightarrow Ca^{+2} + Cu(s)$

$1\,m \qquad x\,m$

22

The total number of geometrical isomers possible in following compound is:



$G.I = 2^n = 2^3 = 8$

23

Find out the number of dimerize products obtained by following reaction

$H_3C-Cl + CH_3-CH_2-Cl + CH_3-CH_2-CH_2-Cl$

Dry ether | Na

$CH_3-Cl + Cl-CH_3 \xrightarrow[ether]{Na} CH_3-CH_3$ (ethane)

$CH_3CH_2Cl + ClCH_2CH_3 \xrightarrow{\cdots}$ (butane)

$2\,CH_3CH_2CH_2Cl \xrightarrow{\cdots}$ (n-hexane)

$CH_3-Cl + CH_3CH_2Cl \xrightarrow{\cdots}$ (Propane)

$CH_3CH_2Cl + CH_3CH_2CH_2Cl \xrightarrow{\cdots}$ (n-Pentane)

5

24

A certain volume of 0.2 M benzoic acid (pKa = 4.2) solution is added to a 0.4 M potassium benzoate solution and the resultant volume was found to be 600 mL . If pH of the solution is found to be 4.5 then find the volume of benzoic acid solution added (in ml) (log 2 = 0.3)

WA + Salt

$PhCOOH$ + $PhCOO^{-}K^{+}$

$(0.2M, V)$ $(0.4M, (600-V)ml)$

Acidic Buffer

$pH = pK_a + \log \frac{[PhCOO^{-}K^{+}]}{[PhCOOH]}$ ($PhCOO^{-}K^{+} \rightarrow N_1V_1$, $PhCOOH \rightarrow N_2V_2$)

$4.5 = 4.2 + \log \frac{0.4(600-V)}{0.2 \times V}$

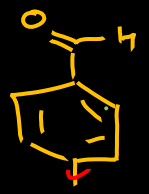
$\log 2 = 0.3 = \log \left[2\left(\frac{600-V}{V}\right)\right]$

$V = 300\ ml$

25

Examine the structural formula of following compounds and find out the number of compounds which show higher rate of nucleophilic addition than

+I
EWG.



Φ EWG (-M, -I)

+M

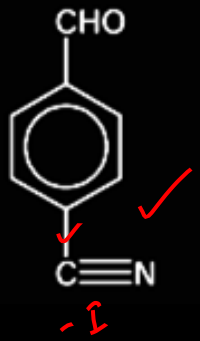
-I > +M

-M

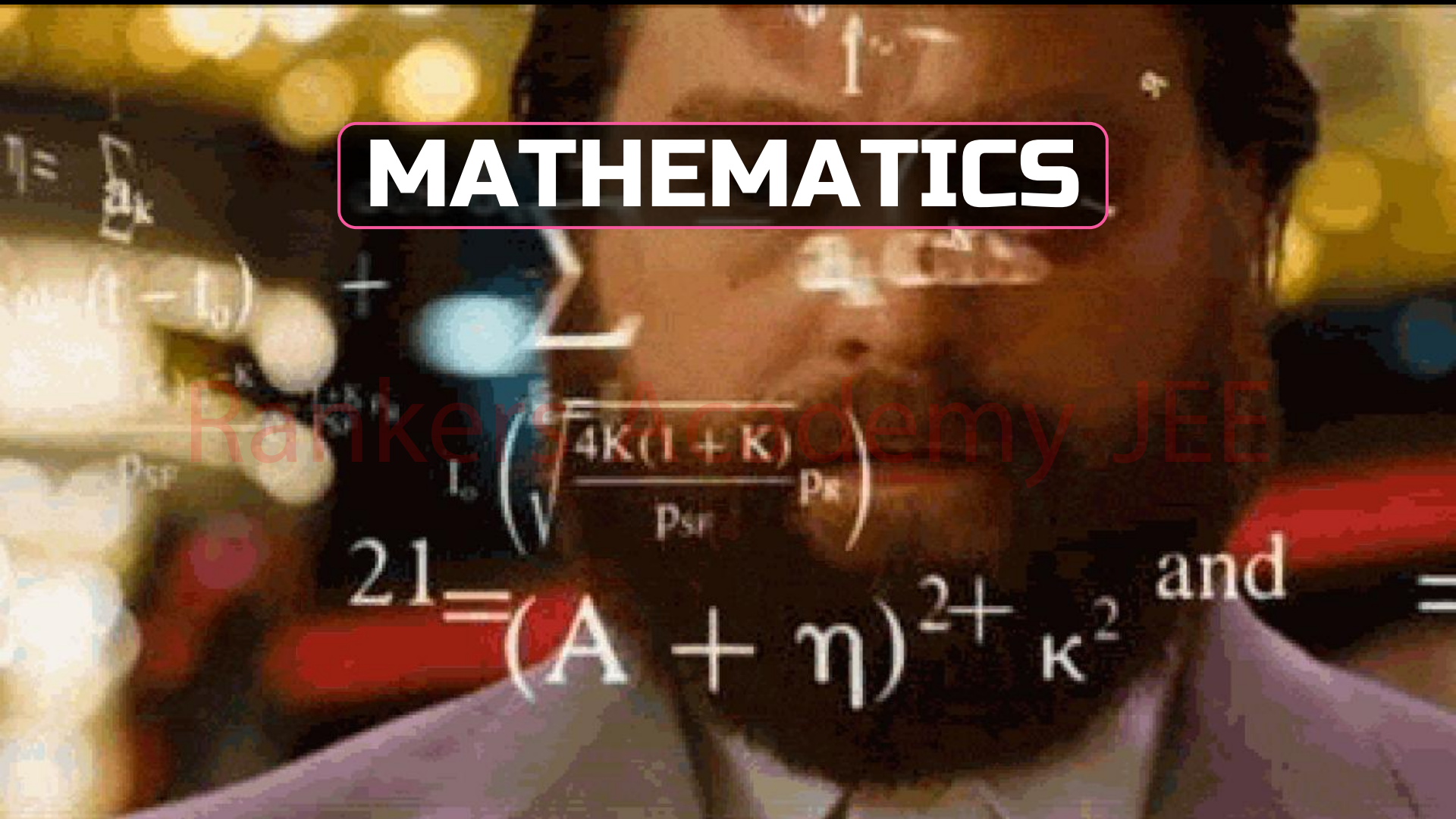
3

+M

+M



-I


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

The number of ways of arranging the letters of the word ASSOCIATION so that vowels are in alphabetical order is

(A) $\frac{11!}{(2!)^4}$

(B) $\frac{11!}{2!6!}$

(C) $\frac{(6!)^2}{(2!)^4}$

(D) $\frac{11!}{2!}$

A O I A I O → A A I I O O

S S C T N

$\Rightarrow {}^{11}C_6 \times 1 \times \frac{5!}{2!}$

$\Rightarrow \frac{11!}{5!6!} \frac{5!}{2!}$

2

A line cuts the x-axis at A(7,0) and the y-axis at B(0,−5). A variable line is drawn perpendicular to AB, cutting the x-axis at P (P,0) and the y-axis at Q. If AQ and BP intersect at R, the locus of R is (0,q)

(A) $x^2 + y^2 + 7x - 5y = 0$

(B) $x^2 + y^2 - 7x + 5y = 0$ ✓

(C) $5x - 7y = 35$

(D) $7x + 5y = 49$

JEE 1

AB: $\frac{x}{7} + \frac{y}{-5} = 1$

$m_{AB} = \frac{-\frac{1}{7}}{\frac{1}{-5}} = \frac{5}{7}$

$m_{line} = -\frac{7}{5}$ ✓

PQ: $\frac{x}{P} + \frac{y}{q} = 1$

$\frac{-\frac{1}{P}}{\frac{1}{q}} = -\frac{q}{P} = -\frac{7}{5}$

$\frac{q}{P} = \frac{7}{5}$

AQ: $\frac{x}{7} + \frac{y}{q} = 1 \Rightarrow \frac{y}{1-\frac{x}{7}} = q = \frac{7y}{7-x}$

BP: $\frac{x}{P} + \frac{y}{-5} = 1 \Rightarrow \frac{x}{1+\frac{y}{5}} = P = \frac{5x}{y+5}$

$\Rightarrow \frac{\frac{7y}{7-x}}{\frac{5x}{y+5}} = \frac{7}{5}$

$\Rightarrow \frac{y(y+5)}{x(7-x)} = 1$

$\Rightarrow y^2 + 5y = -x^2 + 7x$

$\Rightarrow x^2 + y^2 - 7x + 5y = 0$

3

For $k \in N$, let $b_k = \int_1^e \frac{1}{(k+\log x)(k+1+\log x)} \frac{dx}{x}$

and $a_n = \sum_{k=1}^{n} b_k$. Then find $\lim_{n\to\infty} a_n$

(A) $\ell n 2$ (B) 1

(C) $\ell n 10$ (D) 0

JEE 1

$$b_k = \int_1^e \frac{dx}{(k+\ln x)(k+1+\ln x)x}$$

Let $k+\ln x = t$

$\frac{1}{x}dx = dt$

$$b_k = \int_k^{k+1} \frac{dt}{t(t+1)} = \int \frac{1}{t} - \frac{1}{t+1} dt$$

$$= \ln \frac{t}{t+1}\Big|_k^{k+1} = \ln \frac{k+1}{k+2} - \ln \frac{k}{k+1}$$

$$a_n = \sum_{k=1}^{n} b_k = \ln\frac{2}{3} - \ln\frac{1}{2} + \ln\frac{3}{4} - \ln\frac{2}{3} + \ln\frac{n+1}{n+2} -$$

$$= \ln \frac{n+1}{n+2} - \ln\frac{1}{2}$$

$$= \ln\left(2\left(\frac{n+1}{n+2}\right)\right)$$

$$\lim_{n\to\infty} a_n = \ln\left(\lim_{n\to\infty} 2\frac{(n+1)}{n+2}\right) = \ln 2(1)$$

$$= \ln 2$$

4

If $f(x) = \tan^{-1} x - (1/2)\log x$. then which of the following options is incorrect

(A) The greatest value of $f(x)$ on $[1/\sqrt{3}, \sqrt{3}]$ is $\pi/6 + (1/4)\log 3$

(B) The least value of $f(x)$ on $[1/\sqrt{3}, \sqrt{3}]$ is $\pi/3 - (1/4)\log 3$.

(C) $f(x)$ decreases on $(0, \infty)$ (T)

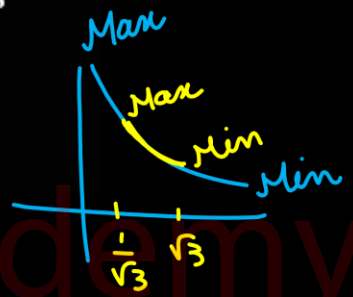
(D) $f(x)$ increases on $(-\infty, 0)$

$f(x) = \tan^{-1} x - \frac{1}{2}\ln x \quad ; x > 0$

$x \in (0, \infty)$

$f'(x) = \frac{1}{1+x^2} - \frac{1}{2x}$

$= \frac{2x-(1+x^2)}{2x(1+x^2)} = -\frac{(x-1)^2}{2x(1+x^2)} < 0$



$f_{max} = f\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} - \frac{1}{2}\ln 3^{-1/2} = \frac{\pi}{6} + \frac{1}{4}\ln 3$

$f_{min} = f(\sqrt{3}) = \frac{\pi}{3} - \frac{1}{2}\ln 3^{1/2} = \frac{\pi}{3} - \frac{1}{4}\ln 3$

5

The value of

$4\cos\left(\frac{\pi}{10}\right) - 3\sec\left(\frac{\pi}{10}\right) - 2\tan\left(\frac{\pi}{10}\right)$ is equal to

(A) $\sqrt{5} - 1$ (B) $\sqrt{5} + 1$

(C) 0 (D) 1

JEE 1

$= \left(4\cos 18^\circ - 3\sec 18^\circ\right) - 2\tan 18^\circ$

$= \left(4\cos 18^\circ - \frac{3}{\cos 18^\circ}\right) - 2\tan 18^\circ$

$= \frac{4\cos^2 18^\circ - 3}{\cos 18^\circ} - 2\frac{\sin 18^\circ}{\cos 18^\circ}$

$= \frac{\left(4\cos^3 18^\circ - 3\cos 18^\circ\right) - 2\sin 18^\circ \cos 18^\circ}{\cos^2 18^\circ}$

$= \frac{\cos 54^\circ - \sin 36^\circ}{\cos^2 18^\circ} = 0$

6

A polynomial function satisfies $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right), x \neq 0$ and $f(3) = -26$, then value of $f(4)$ is-

(A) -15 (B) -63

(C) -47 (D) -255

$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

$f(x) = 1 \pm x^n$

$f(3) = 1 \pm 3^n = -26$

$1 - 3^3 = -26$

$\therefore f(x) = 1 - x^3$

$f(4) = 1 - 4^3 = -63.$

7

The sum of the series

$$\frac{13}{1\cdot 2\cdot 3\cdot 2}+\frac{26}{2\cdot 3\cdot 4\cdot 4}+\frac{43}{3\cdot 4\cdot 5\cdot 8}+\frac{64}{4\cdot 5\cdot 6\cdot 16}+\cdots\infty$$

(A) 3/4 (B) 7/4

(C) 3/2 (D) 4/3

$T_r = 13 + 13(r-1) + 4\frac{(r-1)(r-2)}{2!}$

$= 13r + 2(r^2-3r+2)$

$= 2r^2+7r+4$

$T_r = \frac{2r^2+7r+4}{r(r+1)(r+2)\cdot 2^r}$

$= \frac{2(2r+1)(r+2)-(2r+3)(r)}{r(r+1)(r+2)\cdot 2^r}$

$= \frac{2r+1}{r(r+1)\cdot 2^{r-1}} - \frac{2r+3}{(r+1)(r+2)\cdot 2^r}$

JEE 1

$$S_\infty = \frac{3}{1 \cdot 2 \cdot 2^0} - \frac{5}{2 \cdot 3 \cdot 2^1} + \frac{5}{2 \cdot 3 \cdot 2^1} - \frac{7}{3 \cdot 4 \cdot 2^2} + \frac{7}{3 \cdot 4 \cdot 2^2} - \cdots$$

$$= \frac{3}{2} - 0 = \frac{3}{2}$$

$$\frac{-2n+3}{(n+1)(n+2)2^{n-1}}$$

Let $x_1, x_2 \ldots x_n$ be n observations such that Ind $\Sigma x_i^2 = 200$ and $\Sigma x_i = 40$ then a possible value of n the following is

(A) 4 (B) 5

(C) 6 (D) 10

$\Sigma x_i^2 = 200$

$\Sigma x_i = 40$

$n = ?$

$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} \geq 0$

$\frac{\Sigma x_i^2}{n} \geq \left(\frac{\Sigma x_i}{n}\right)^2$

$\Rightarrow \frac{200}{n} \geq \frac{40^2}{n^2}$

$\Rightarrow 200n \geq 1600$

$\Rightarrow \boxed{n \geq 8}$

9

Let f be a polynomial of degree 5 such that $\lim_{x\to 0}\left(\frac{f(-x)}{2x^4}\right)^{1/x} = \frac{1}{e^4}$, then the value of f(1) is

(A) 10 (B) 11

(C) 12 (D) 15

JEE 1

$\lim_{x\to 0}\left(\frac{f(-x)}{2x^4}\right)^{1/x} = e^{-4}$ (1^∞)

$\lim_{x\to 0}\frac{1}{x}\left[\frac{f(-x)}{2x^4}-1\right}$ $= e^{-4}$

$\Rightarrow e$

$\Rightarrow \lim_{x\to 0}\frac{f(-x)-2x^4}{2x^5} = -4$

$f(x) = 8x^5 + 2x^4$

$f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$f(-x) = -ax^5 + bx^4 - cx^3 + dx^2 - ex + f$

$\lim_{x\to 0}\frac{-ax^5 + (b-2)x^4 - cx^3 + dx^2 - ex + f}{2x^5} = -4$

$-\frac{a}{2} = -4$

$a = 8$

#Barabari for limit

Chhoti power = 0

$b - 2 = 0 \Rightarrow b = 2$

$-c = 0 \Rightarrow c = 0$

$d = 0$

$e = 0$

$f = 0$

Bade log = Ignore

10

JEE 1

Let $I_1 = \int_0^{\pi/3} (\sec x)^{100} dx$, $I_2 =$

$\int_0^{\ln(2+\sqrt{3})} (e^x + e^{-x})^{99} dx$ then $\frac{I_1}{I_2}$ is

(A) 1 (B) 2

(C) 1/2 (D) 2^{99}

$e^x = \sec t + \tan t$

$e^x dx = (\sec t \tan t + \sec^2 t)\, dt$

$e^x dx = \sec t\,(\sec t + \tan t)\, dt$

$dx = \sec t\, dt$

$e^{-x} = \frac{1}{e^x} = \frac{1}{\sec t + \tan t} = \sec t - \tan t$

$e^x + e^{-x} = 2\sec t$

$I_2 = \int_0^{\pi/3} (2\sec t)^{99} \cdot \sec t\, dt$

$I_2 = 2^{99} \int_0^{\pi/3} \sec^{100} t\, dt$

$I_1 = 2^{100} \int_0^{\pi/3} (\sec x)^{100} dx$

$\frac{I_1}{I_2} = 2$

11

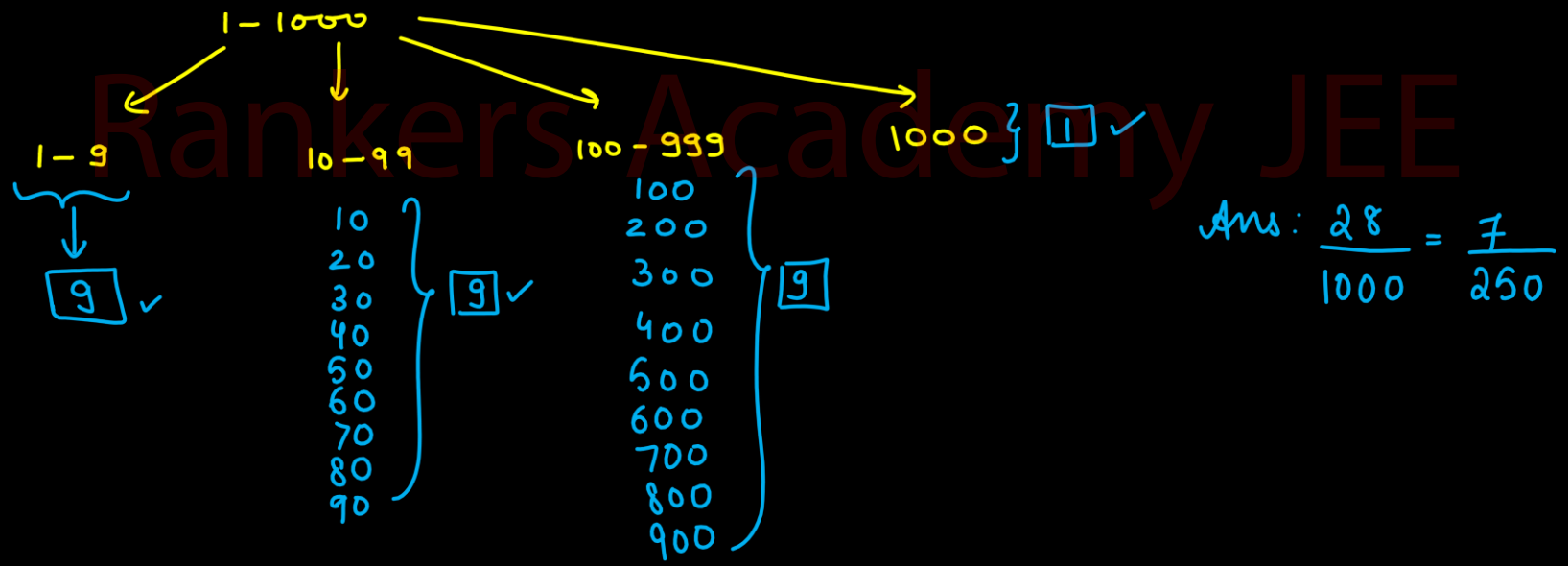
A natural number is selected from 1 to 1000 at random, such that only one non zero digit can appear in the number, find its probability.

(A) $\frac{2}{250}$

(B) $\frac{143}{250}$

(C) $\frac{243}{250}$

(D) $\frac{7}{250}$



12

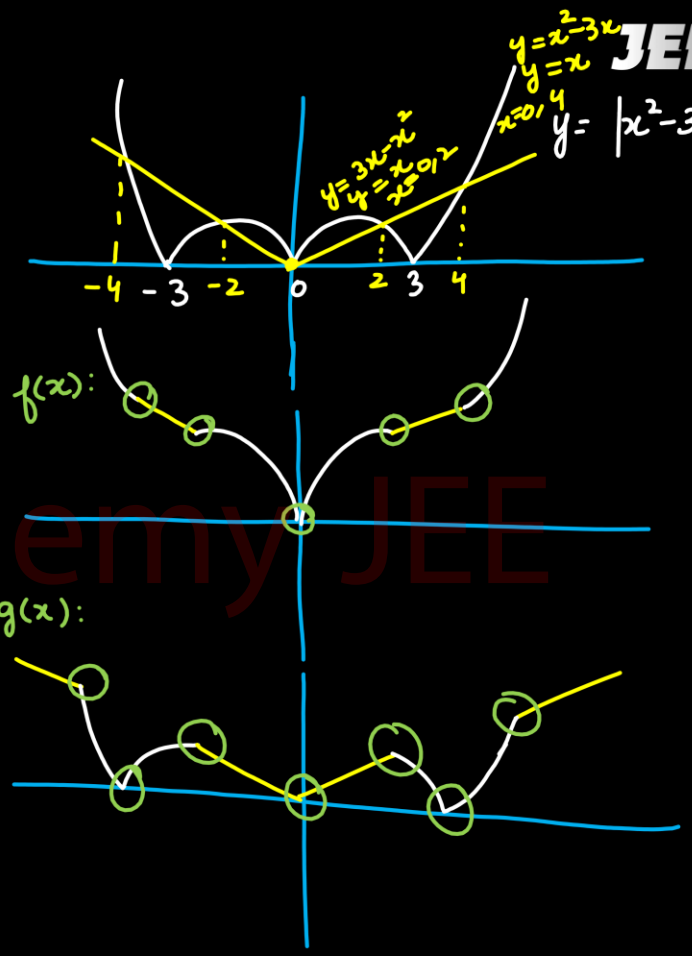
Let $f(x) = \max\{|x|^2 - 3|x||, |x|\}$ and $g(x) = \min\{|x^2 - 3|x||, |x|\}$, then

(A) f(x) and g(x) not differentiable at 5 and 7 points respectively

(B) f(x) and g(x) not differentiable at 7 and 5 points respectively

(C) f(x) and g(x) not differentiable at 5 and 6 points respectively

(D) f(x) and g(x) not differentiable at 6 and 5 points respectively



13

If z_1, z_2, z_3 are the roots of the equation $z^3 - z^2(1+3i) + z(3i-2) + 2 = 0$. then $\text{Im}(z_1) + \text{Im}(z_2) + \text{Im}(z_3)$ is

(A) 0 (B) 1

(C) 2 (D) 3

$z^3 - (1+3i)z^2 + (3i-2)z + 2 = 0$

$z_1 = a_1 + ib_1$

$z_2 = a_2 + ib_2$

$z_3 = a_3 + ib_3$

Ans $= b_1 + b_2 + b_3$

$= 3$

$z_1 + z_2 + z_3 = (1+3i)$

$a_1 + ib_1 + a_2 + ib_2 + a_3 + ib_3$

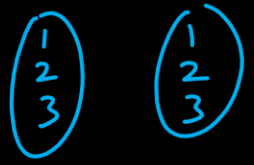
$(a_1 + a_2 + a_3) + i(b_1 + b_2 + b_3) = 1 + 3i$

14

The number of the relations, on the set {1,2,3} containing (1,2) and (2,3). which are reflexive and transitive but not symmetric, is equal to

(A) 3 (B) 4

(C) 6 (D) 8



$R_1 = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$

Remaining

$\{(2,1),(3,2),(3,1)\}$

$R_2 = R_1 \cup \{(2,1)\}$

$R_3 = R_1 \cup \{(3,2)\}$

15

JEE 1

Let α, β are the roots of the equation $x^2 + ax + b = 0$ and γ, δ are the roots of $x^2 - ax + b - 2 = 0$. If $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + \left(\frac{1}{\gamma} + \frac{1}{\delta}\right) = \frac{5}{12}$ and $\alpha\beta\gamma\delta = 24$, then the value of ' a ' is

(A) 4 (B) -4

(C) 5 (D) -5

$(\alpha\beta)(\gamma\delta) = 24$

$(b)(b-2) = 24$

$b = 6$

$\frac{\alpha+\beta}{\alpha\beta} + \frac{\gamma+\delta}{\gamma\delta} = \frac{5}{12}$

$\frac{-a}{6} + \frac{a}{4} = \frac{5}{12}$

$\frac{-2a+3a}{12} = \frac{5}{12} \Rightarrow a = 5$

16

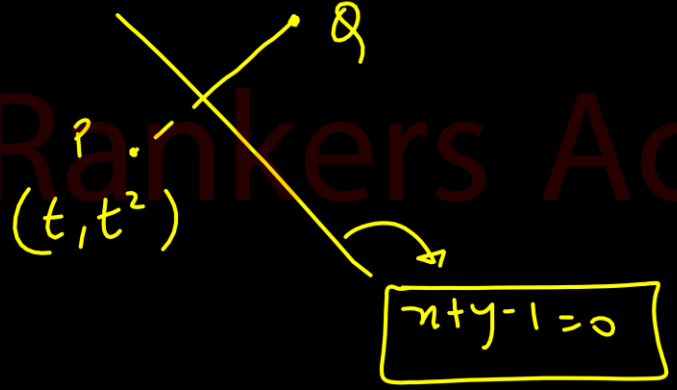
If the image of the parabola $y = x^2$ in the line $x + y = 1$ is $x + y^2 = ky$, then k is equal to

(A) 2 (B) 4

(C) 1 (D) 3

JEE 1

$$\frac{x-t}{1} = \frac{y-t^2}{1} = \frac{-2(t+t^2-1)}{2}$$



$$x - t = -t - t^2 + 1$$

$$x = 1 - t^2$$

$$y - t^2 = -t - t^2 + 1$$

$$y = 1 - t$$

$$x = 1 - (1-y)^2$$

$$x = \cancel{1} - (\cancel{1} + y^2 - 2y)$$

$$x = -y^2 + 2y$$

$$\boxed{x + y^2 = 2y}$$

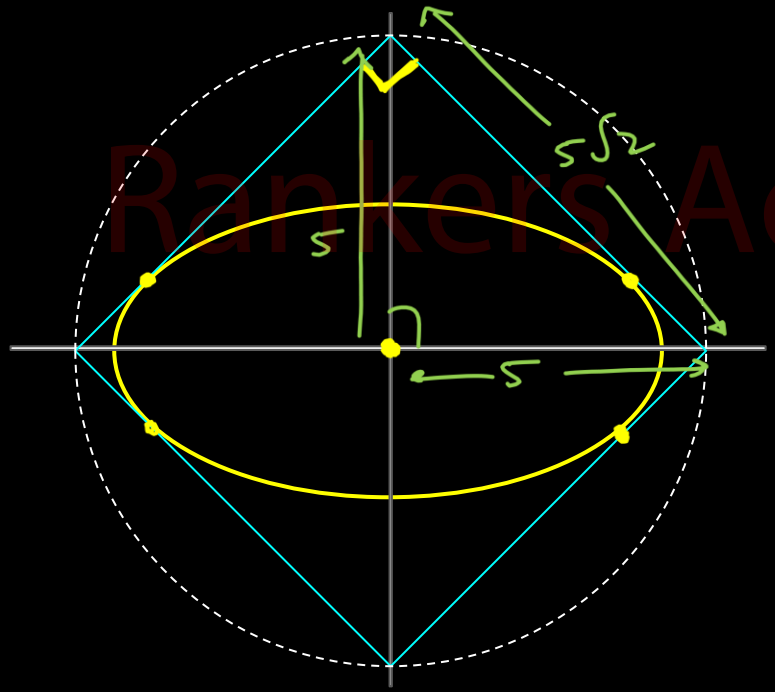
17

If the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ is inscribed in a square of side length $5\sqrt{2}$, then area of ellipse is –

(A) 4π (B) 8π

(C) 10π (D) 12π

JEE 1



$E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$DC: x^2+y^2=a^2+b^2$

$\sqrt{a^2+b^2}=5$

$16+b^2=25$

$b^2=9$

$b=3$

$\Delta=\pi ab$

$=\pi(4)(3)$

$=12\pi$

18

Given $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and L be a line whose equation is $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$. The shortest distance of the point P(0,1,2) from the line L is equal to –

(A) $\frac{\sqrt{119}}{3}$

(B) 2

(C) $\frac{\sqrt{219}}{3}$

(D) $\frac{\sqrt{13}}{3}$

$\vec{r} \times \vec{b} - \vec{a} \times \vec{b} = 0$

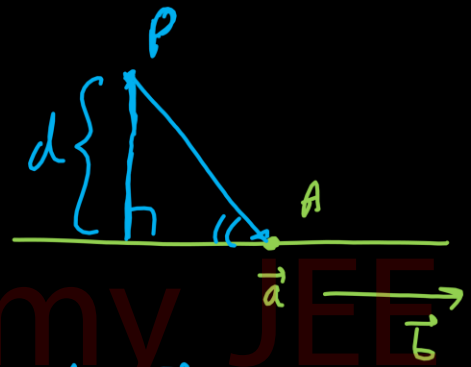
$(\vec{r} - \vec{a}) \times \vec{b} = 0$

$(\vec{r} - \vec{a}) \parallel \vec{b}$

$\vec{r} - \vec{a} = \lambda \vec{b}$

$\vec{r} = \vec{a} + \lambda \vec{b}$

Point Direction



$d = |\vec{PA}| \sin\theta$

$= \frac{|\vec{PA} \times \vec{b}|}{|\vec{b}|}$

$P \equiv (0,1,2)$

$A \equiv (1,2,-3)$

$$\begin{cases} \vec{PA} = \hat{i} + \hat{j} - 5\hat{k} \\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \end{cases}$$

$$\vec{PA} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -5 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i}(-4) - \hat{j}(11) + \hat{k}(-3)$$

$$d = \frac{\sqrt{16+121+9}}{\sqrt{4+1+1}} = \sqrt{\frac{146}{6}} = \sqrt{\frac{73}{3}}$$

19

The constant term in the expansion of

$\left(\left(x + x^2\right) + \left(\frac{1}{x} + \frac{1}{x^2}\right)\right)^{15}$ is

(A) $\left({}^{30}C_{15}\right)^2$

(B) ${}^{30}C_{15}$

(C) $(2) \cdot \sum_{r=0}^{15} \left({}^{15}c_r\right)^2$

(D) $\sum_{r=0}^{5} \left({}^{15}C_{5+r}\right)\left({}^{15}C_{3r}\right)$

$\left[(x + x^2) + \frac{(x^2 + x)}{x^3}\right]^{15}$

$\left[(x + x^2)\left(1 + \frac{1}{x^3}\right)\right]^{15}$

$x^{15}(1+x)^{15}\frac{(1+x^3)^{15}}{x^{45}}$

$\frac{(1+x)^{15}(1+x^3)^{15}}{x^{30}}$

$\Rightarrow$ x^{30} in $(1+x^3)^{15}(1+x)^{15}$

$15C_{10} \cdot 15C_0 + 15C_9 \cdot 15C_3 + 15C_8 \cdot 15C_6$

$+ \cdots + 15C_5 \cdot 15C_{15}$

20

JEE 1

Let $M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ be a matrix and

$A - 8M = M^4$ and $B + 5M^3 = 8M^2 - 10M$,

then the value of $\left(\frac{\det(A+B)}{\text{tr}(A+B)}\right)$ is

(A) $\frac{16}{5}$ (B) $\frac{8}{5}$

(C) $\frac{4}{5}$ (D) $\frac{32}{5}$

$A+B-8M+5M^3 = M^4+8M^2-10M$

$(A+B) = M^4-5M^3+8M^2-2M$ — (1)

$|M-\lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(2-\lambda)^2 = 0$

$(1-\lambda)(4+\lambda^2-4\lambda) = 0$

$4+\lambda^2-4\lambda-4\lambda-\lambda^3+4\lambda^2 = 0$

$$-\lambda^3 - 5\lambda^2 + 8\lambda = 4$$

$$\hookrightarrow M^3 - 5M^2 + 8M = 4I$$

$$M^4 - 5M^3 + 8M^2 = 4M$$

Use in Eq^n (1)

$\therefore A + B = 2M$

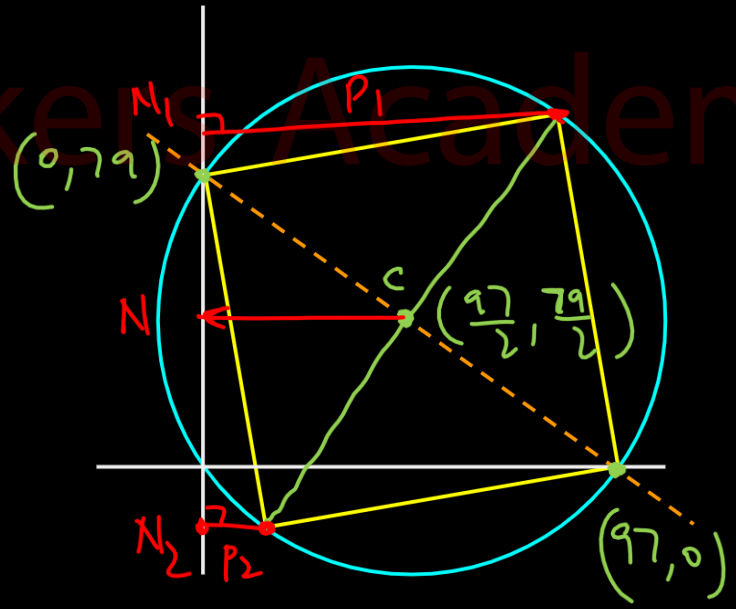
$$|A+B| = |2M| = 8|M| = 8(4) = 32$$

$$tr(A+B) = tr(2M) = 2\,tr(M) = 2(5) = 10$$

$$\therefore \frac{|A+B|}{tr(A+B)} = \frac{32}{10} = \frac{16}{5}$$

21

One diagonal of a square is the portion of the line $\frac{x}{97}+\frac{y}{79}=1$ intercepted between the axes. p_1, p_2 are the lengths of the perpendiculars from the vertices of the other diagonal on the axis y then $p_1 + p_2 =$



$CN = \frac{P_1+P_2}{2}$

$\frac{97}{2} = \frac{P_1+P_2}{2}$

$P_1+P_2=97$

22

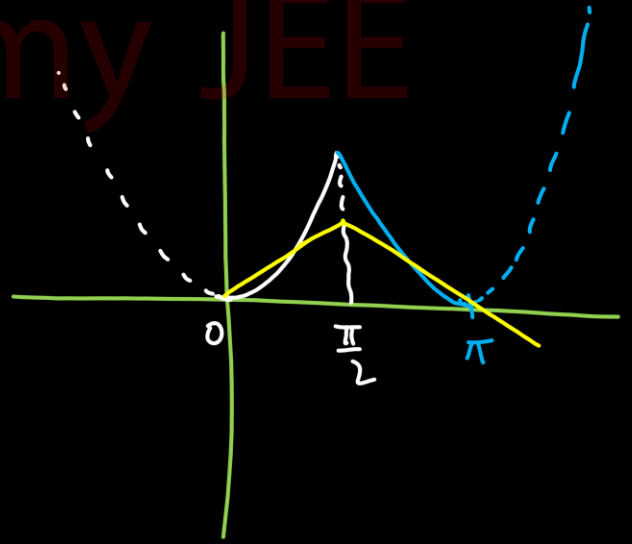
If the area bounded by the curves $f(x) = (\cos^{-1}|\cos x|)^2, g(x) = (\cos^{-1}|\cos x|)$ for $x \in [0, \pi]$ is $a\pi^3 + b\pi^2 + c$, then $2|a + 5b + c|$ equals

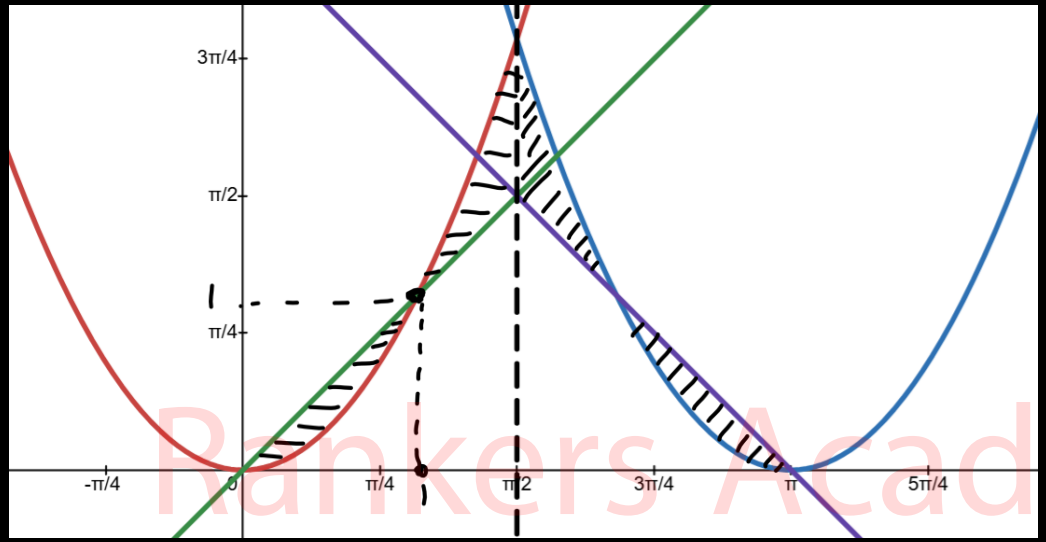
JEE 1

$\cos^{-1}(-\cos x)$
$= \pi - \cos^{-1}(\cos x)$
$= (\pi - x)$

$$f(x) = \begin{cases} x^2 & ; x \in (0, \frac{\pi}{2}) \\ (\pi - x)^2 & ; x \in (\frac{\pi}{2}, \pi) \end{cases}$$

$$g(x) = \begin{cases} x & ; x \in (0, \frac{\pi}{2}) \\ (\pi - x) & ; x \in (\frac{\pi}{2}, \pi) \end{cases}$$





Ans: (1)

$$2\left[\int_0^1 (x-x^2)\,dx + \int_1^{\pi/2} (x^2-x)\,dx\right]$$

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JEE 1

Let M be the greatest and m be the smallest value of $\sqrt{\sin^{-1} x} + \sqrt{\cos^{-1} x}$ then the value of $\left(\frac{M}{m}\right)^4$ is

Domain $\equiv [0, 1]$

$$f(x) = \sqrt{\sin^{-1}x} + \sqrt{\cos^{-1}x}$$

$$f'(x) = \frac{1}{2\sqrt{\sin^{-1}x}}\left(\frac{1}{\sqrt{1-x^2}}\right) + \frac{1}{2\sqrt{\cos^{-1}x}}\left(\frac{-1}{\sqrt{1-x^2}}\right) = 0$$

$$\sin^{-1}x = \cos^{-1}x \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$f(0) = \sqrt{\frac{\pi}{2}} \; ; \; f(1) = \sqrt{\frac{\pi}{2}} \; ; \; f\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{\frac{\pi}{4}} = \sqrt{\pi}$$

$$\frac{M}{m} = \frac{\sqrt{\pi}}{\sqrt{\pi/2}} = \sqrt{2}$$

$$\left(\frac{M}{m}\right)^4 = 4$$

24

JEE 1

If $I = \int \frac{\sin x(\cos x)^{-5/2} dx}{\sqrt{\sin x + 3\cos x} + \sqrt{\sin x + 4\cos x}}$

$= A\left((\tan x + 4)^{5/2} - (\tan x + 3)^{5/2}\right)$

$+ B\left[4(\tan x + 4)^{3/2} - 3(\tan x + 3)^{3/2}\right] + C,$

then $15(A - B) =$

$$I = \int \frac{\tan x \cdot \sec^2 x}{\sqrt{\tan x + 3} + \sqrt{\tan x + 4}} dx$$

Let: $\tan x = t$

$$I = \int \frac{t\, dt}{\sqrt{t+3} + \sqrt{t+4}}$$

$$I = \int t\left(\sqrt{t+4} - \sqrt{t+3}\right) dt$$

$$I = \int t\sqrt{t+4}\, dt - \int t\sqrt{t+3}\, dt$$

$$I_a = \int t\sqrt{t+a}\,dt$$

$$\begin{cases} t+a = u^2 \\ dt = 2u\,du \end{cases}$$

$$I_a = \int (u^2-a)(u)(2u)\,du$$

$$= 2\int (u^4 - au^2)\,du$$

$$= 2\left[\frac{u^5}{5} - a\frac{u^3}{3}\right]$$

$$I_a = 2\left[\frac{1}{5}(t+a)^{5/2} - \frac{a}{3}(t+a)^{3/2}\right]$$

$$I = I_4 - I_3$$

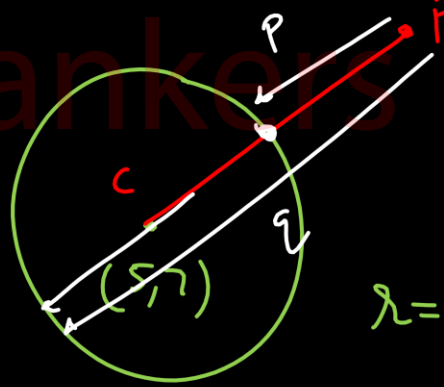
$$= \left[\frac{2}{5}(t+4)^{5/2} - \frac{2(4)}{3}(t+4)^{3/2}\right]$$

$$-\left[\frac{2}{5}(t+3)^{5/2} - \frac{2(3)}{3}(t+3)^{3/2}\right]$$

$A = \frac{2}{5}$; $B = -\frac{2}{3}$; $(A-B) = \frac{16}{15}$ ✓

25

If p and q be the longest and the shortest distances respectively of the point $(-7,2)$ from any point (α, β) on the circle $x^2 + y^2 - 10x - 14y - 51 = 0$. Then the geometric mean of p and q is equal to $2\sqrt{n}$, then find n



$r = \sqrt{25+49+51}$

$= 5\sqrt{5}$

$\therefore CP = \sqrt{12^2 + 5^2} = 13$

$P = 13 - 5\sqrt{5}$

$Q = 13 + 5\sqrt{5}$

$GM(P,Q) = \sqrt{169 - 125}$

$= \sqrt{44} = 2\sqrt{11}$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