ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)

FULL SYLLABUS

12-02-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE ALL STAR BATCH (SCORE-II)

ANSWER KEY

PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	B	B	A	B	A	A
SECTION-I (ii)	Q.	7	8	9	10	11	12
	A.	A,C	A,B,C,D	A,C,D	B,C	A,B,D	A,D
SECTION-II	Q.	1	2	3	4	5	6
	A.	5.00	5.00	83.33	1.68 to 1.70	2.00	0.75

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	D	C	C	C	D	C
SECTION-I (ii)	Q.	7	8	9	10	11	12
	A.	A,C	B,C,D	B,D	A,C,D	A,B,C	A,B,C
SECTION-II	Q.	1	2	3	4	5	6
	A.	1.25	1.20	2.80 to 2.84	6.00	37.50	600.00

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	A	C	D	A	B	A
SECTION-I (ii)	Q.	7	8	9	10	11	12
	A.	A,B,D	A	B,C,D	C,D	A,B,C	A,B,C,D
SECTION-II	Q.	1	2	3	4	5	6
	A.	3.00	1.50	0.70	1.70	2.00	4.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

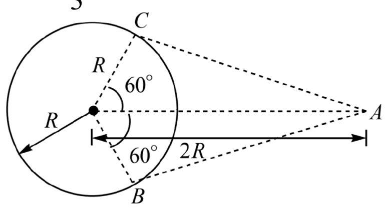
2. **Ans (B)**

$$\frac{3}{2L}v = \frac{5}{4L'}v$$

$$\frac{L'}{L} = \frac{5}{6}$$

4. **Ans (B)**

'A' records zero magnetic field when α particle is moving on a line which passes through A, which happen when tangent to path passes through A. Angle covered between two such point B and C $= \frac{2\pi}{3}$ as shown in figure.



Time taken to go from B to C is given as

$$t = \frac{T}{3} = \frac{2\pi}{\omega}\left(\frac{1}{3}\right)$$

$$\Rightarrow \omega = \frac{2\pi}{3t} = 2\pi f \Rightarrow f = \frac{1}{3t}$$

5. Ans (A)

$\frac{9}{4}$ intensity of $S_1 = 9$ times of intensity S_2

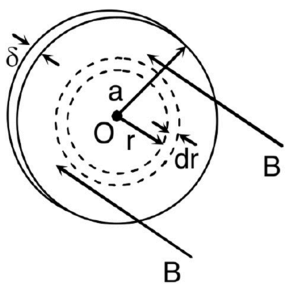
Intensity of maxima $= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I_2$

$\Rightarrow S_2P - S_1P = n\lambda$ for maxima

$\sqrt{d^2 + x^2} - x = n\lambda \qquad x = \frac{d^2 - n^2\lambda^2}{2n\lambda}$

6. Ans (A)

Consider two circles of radii r and r + dr concentric with the disc (0 < r < a) (figure). The induced e.m.f. in the circular path of radius r is



$$\varepsilon = -\frac{d}{dt}(\pi r^2 B) = -\pi r^2 \frac{dB}{dt} = -\pi r^2 R$$

The resistance of the circular path between radii r and r + dr is

$\frac{1}{\sigma}\frac{2\pi r}{\delta dr}$,

The length of the path being $2\pi r$ and the cross sectional area of currrent flow being δdr. The power dissipated inside this path is

$$dP = \frac{\varepsilon^2}{R} = \frac{\pi\delta\sigma}{2}(R)^2 r^3 dr$$

The total dissipated power P is

$$P = \frac{\pi\delta\sigma}{2}(R)^2 \int_0^a r^3 dr = \frac{\pi\delta\sigma a^4}{8}(R)^2$$

PART-1 : PHYSICS

SECTION-I (ii)

7. Ans (A,C)

$$-nKx + Kx = m\frac{d^2x}{dt^2}$$

$$-Kx = m\frac{d^2(2x)}{dt^2}$$

$$\Rightarrow \frac{(n-1)K}{m} = \frac{K}{2m}$$

$$\Rightarrow n = \frac{3}{2}$$

8. Ans (A,B,C,D)

For any physical quantity;

numerical value × unit = constant

For (A) $n_1u_1 = n_2u_2 \Rightarrow n_2 = \left(\frac{u_1}{u_2}\right) n_1 = \left(\frac{L_1}{L_2}\right)$

$(500) = \left(\frac{1m}{1000m}\right)(500) = 0.5$

For (B) $n_2 = \left(\frac{u_1}{u_2}\right) n_1 = \left(\frac{T_1}{T_2}\right)(n_1)$

$= \left(\frac{1s}{3600s}\right)(7200) = 2$

For (C) $n_2 = \left(\frac{M_1L_1^2T_1^{-2}}{M_2L_2^2T_2^{-2}}\right)(n_1)$

$= \left[\frac{(1kg)(1m)^2(1s)^{-2}}{(1000kg)(1000m)^2(3600s)^{-2}}\right]\left(\frac{1}{36}\right)$

$= 3.6 \times 10^{-4}$

For (D) $n_2 = \left(\frac{M_1L_1T_1^{-2}}{M_2L_2T_2^{-2}}\right)(n_1)$

$= \left[\frac{(1kg)(1m)(1s)^{-2}}{(1000kg)(1000m)(3600s)^{-2}}\right]\left(\frac{1}{36}\right) = 0.36$

9. Ans (A,C,D)

$\lambda = \frac{3500}{1 - \frac{1}{p^2}} \qquad \lambda_{max} = \frac{3500}{1 - \frac{1}{4}} = 466.67 \text{ nm}$

11. Ans (A,B,D)

We have half life of the decay is 3.8 day.

$\therefore$ Amount left after 11.4 days = 3 half life

$= \frac{N_0}{2^3} = \frac{N_0}{8}$

Activity of radon after 7.6 days = 2 half life $= \frac{A_0}{4}$.

Rules of radioactive decay applies for large number of sample product.

Po will be more stable

12. Ans (A,D)

$O = \Delta U_{Cycle} = \Delta U_{12} + \Delta U_{23} + \Delta U_{31}$

$O = O + \Delta U_{23} + \Delta U_{31}$

$\Delta U_{31} = -\Delta U_{23}$

$= -(-40)$

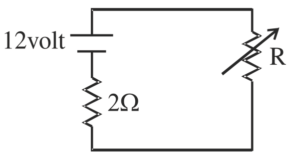
$\Delta U_{31} = 40$ J

PART-1 : PHYSICS

SECTION-II

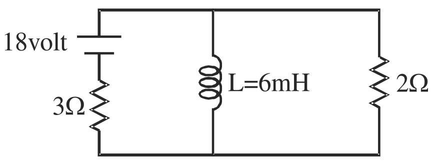
1. Ans (5.00)

The given circuit (1) can be drawn as

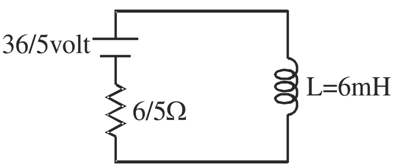


When power dissipated in R is max then R = 2 Ω.

Now circuit (2) will appear



This current can be drawn as



So, $c = \frac{L}{R} = \frac{6}{6/5}$ ms = 5ms

2. Ans (5.00)

$I_1^2R + I_2^2R = (I')^2 R$

$I' = \sqrt{I_1^2 + I_2^2}$

$I' = \sqrt{3^2 + 4^2} = 5$

4. Ans (1.68 to 1.70)

$\frac{x}{R} = \frac{\ell}{1-\ell} \Rightarrow x = \frac{40}{60} \times 50 = \frac{100}{3}\Omega$

$\frac{\Delta x}{x} = \frac{\Delta \ell}{\ell(1-\ell)} \Rightarrow \frac{\Delta x}{x} = \frac{0.1}{40 \times 60} = \frac{1}{240}\%$

$x = \frac{\rho \ell}{\frac{\pi d^2}{4}} \Rightarrow \rho = \frac{\pi x d^2}{4\ell}$

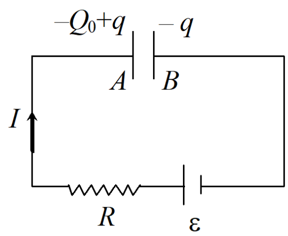
$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta x}{x} + \frac{2\Delta d}{d} + \frac{\Delta \ell}{\ell}$

$= \frac{1}{240} + 2 \times \frac{0.01}{1.6} \times 100 + \frac{0.01}{2.25} \times 100$

$\frac{\Delta \rho}{\rho} = \frac{2}{1.6} + \frac{1}{4.25} = 1.25 + 0.44 + 0.0042$

$= 1.69$

5. Ans (2.00)



Let at any time t charge flown through the plate B to plate A is q and instantaneous current is I.

From loop theorem

$\left(\frac{2q - Q_0}{2C}\right) + IR - \varepsilon = 0$

$\Rightarrow R\frac{dq}{dt} = \frac{-2q + 2\varepsilon C + Q_0}{2C}$

$\Rightarrow \frac{dq}{2\varepsilon C + Q_0 - 2q} = \frac{dt}{2RC}$

Now for charge on plate A to be zero $q = Q_0$.

Integrating $= \int_0^{Q_0} \frac{dq}{2\varepsilon C + Q_0 - 2q} = \int_0^t \frac{dt}{2RC}$

$t = RC \ln\left[\frac{2\varepsilon C + Q_0}{2\varepsilon C - Q_0}\right]$

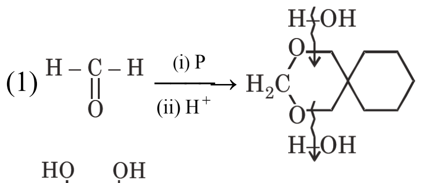
Putting the value of C, Q_0, ε and R .

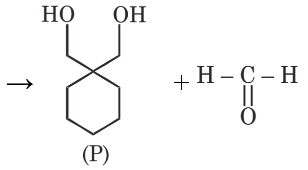
We get t = 2 seconds.

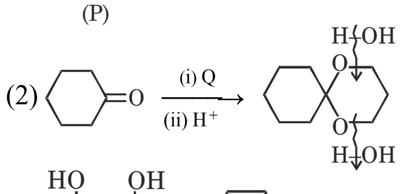
PART-2 : CHEMISTRY

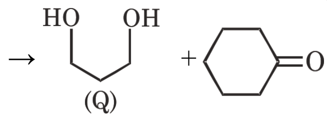
SECTION-I (i)

1. **Ans (D)**









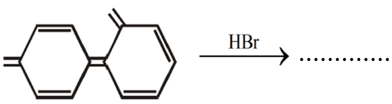
(3) Rate of NAR $\propto$ δ positive on carbonyl carbon

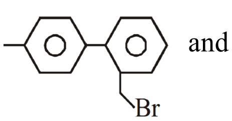
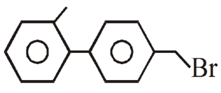
$$\propto \frac{1}{\text{Steric hindrance factor}}$$

(4) Correct IUPAC name is

2-Amino-3-formyl cyclopropane carboxylic acid

2. **Ans (C)**

In the reaction 

compounds  and 

are obtained as major product

3. **Ans (C)**

$Sn + \text{Conc. } HNO_3 \longrightarrow H_2SnO_3 + NO_2 + H_2O$

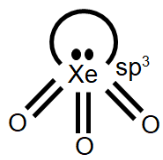
$Li_2C_2 + H_2O \longrightarrow LiOH + C_2H_2$

$Al_4C_3 + H_2O \longrightarrow Al(OH)_3 + CH_4$

4. **Ans (C)**

$XeF_2 + H_2O \longrightarrow Xe + O_2 + HF$

$XeF_4 + H_2O \longrightarrow Xe + O_2 + HF + XeO_3$



Trigonal pyramidal

5. **Ans (D)**

Methyleneblue Sol is (+)vely charged Sol

6. **Ans (C)**

1st process : Isothermal reversible

$W = -2.303nRT \log \frac{V_2}{V_1}$

$= -2.303(P_1V_1)\log \frac{V_2}{V_1}$

$= -2.303 \times 5 \times 2 \log \frac{10}{2}$

$= -\ 16.12$ atm Lit.

$Q = -W = 16.12$ atm $-$ Lit

2nd process : Isoboric (single step)

$\Rightarrow P_{ext} \Rightarrow$ Constant $\Rightarrow$ 5 atm

$W = -\ P_{ext.}(V_2 - V_1) = -5(2 - 10) = +\ 40$ atm-Lit

$\because T_2 = T_1 \Rightarrow \Delta U = 0$

$Q = -W = -40$ atm-Lit

Net heat gained by surrounding = $(-16.12 + 40)$

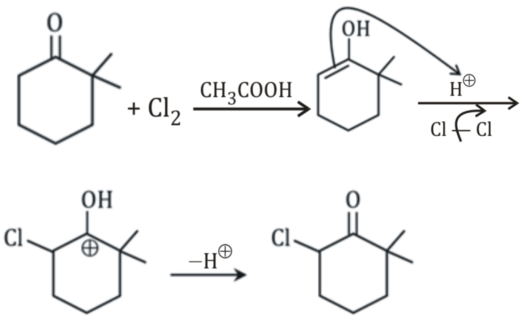
= 23.88 atm-Lit

$\Delta S_{System} = 0$ (Process is cyclic)

PART-2 : CHEMISTRY

SECTION-I (ii)

7. **Ans (A,C)**



9. **Ans (B,D)**

$NH_4NO_2(s) \xrightarrow[\Delta]{} N_2(g) + H_2O(g)$

$SiO_2 + NaOH \xrightarrow[\Delta]{\text{fusion}} Na_2SiO_3$

$\xrightarrow{NaOH(aq.)} Na_2SiO_3$

11. **Ans (A,B,C)**

$2A(s) + B^{2x+}(aq.) \longrightarrow 2A^{x+}(aq.) + B(s)$

$\Rightarrow 0 = \Delta G^\circ + RT\,\ell n\dfrac{[A^{x+}]^2}{[B^{2x+}]^1}$

$\Delta G^\circ = -RT\,\ell n\dfrac{1^2}{0.4} = -RT\,\ell n\dfrac{10}{4}$

$= -\,2.303\ RT\ (\log 10 - \log 4)$

$= -2.303\ RT \times 0.4 = (-)ve$

$\Rightarrow \Delta_r H^\circ = 2\Delta_r G^\circ = (-)ve$

$E^\circ_{cell} = \dfrac{-\Delta_r G^\circ}{nF} \Rightarrow (+)ve$

$\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$

$= 2\Delta_r G^\circ - T\Delta_r S^\circ$

$\Delta_r S^\circ = -2.303 \times 8.3 \times 0.4$

$= -7.65\ JK^{-1}mol^{-1}$

$\dfrac{dE^\circ_{cell}}{dT} = \dfrac{\Delta_r S^\circ}{nF} = (-)ve$

12. **Ans (A,B,C)**

Let the organic base is $R - NH_2$

$\dfrac{w}{M} \times n - \text{factor} = 0.07 \times 2 \times 40 \times 10^{-3}$

$\dfrac{0.252}{M} \times 1 = 5.6 \times 10^{-3}$

$\Rightarrow M = 45$ g/mole

On adding 20 ml of H_2SO_4

	$2R\text{-}NH_2$	$+\ H_2SO_4 \longrightarrow$	$(RNH_3)_2SO_4$	$+\ H_2O$
millimoles:	5.6	0.07×20	—	—
	2.8	0	1.4	—

Resultant solution: Basic buffer of $(R\text{-}NH_2 + RNH_3^+)$

$pOH = pK_b + \log\dfrac{[RNH_3^+]}{[RNH_2]}$

$\Rightarrow (14 - 10.7) = {}_pK_b + \log\dfrac{2.8}{28}$

$\Rightarrow pK_b = 3.3$

$K_b = \text{antilog}(-3.3) = 5 \times 10^{-4}$

At equivalence point: Solution is salt of S.A & W.B

$\Rightarrow$ Acidic due to cationic hydrolysis

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (1.25)**

(i) $Ph\text{-}\underset{\underset{CH_3}{|}}{C}\text{=}O \xrightarrow[(2)\ H^\oplus]{(1)\ NaOI} CHI_3 + Ph\text{-}COOH\ (B)$

DOU of compound B = 5

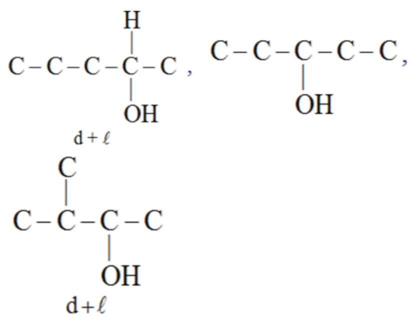
(ii) $\underset{\text{Sod.Nitroprusside}}{Na_2S + Na_2[Fe(CN)_5(NO)]} \rightarrow \underset{\substack{\text{sod.Sulphonitroprusside}\\ \text{violet colour.}}}{Na_4[Fe(CN)_5(NOS)]}$

2. **Ans (1.20)**

X $\Rightarrow$ LAH reduces aldehyde, ketone acid and ester. (4 groups)

Y $\Rightarrow$ SBH can't reduce ester and acid. It reduces only aldehyde and ketone. (2 groups).

Z $\Rightarrow$ 5



3. **Ans (2.80 to 2.84)**

$MnSO_4(aq.) + Pb_3O_4 \xrightarrow[\Delta]{H^+} \underset{(A)}{[MnO_4^-]};\ \mu_A = \sqrt{n(n+2)}$ BM
$\mu_A = 0$

$NiCl_2(aq.) + \text{ethylenediamine} \rightarrow \underset{(B)}{[Ni(en)_3]^{2+}};$

$\mu_B = \sqrt{2(2+2)} = 2.828\,BM$

$CuSO_4(aq.) + KCN \rightarrow \underset{(C)}{[Cu(CN)_4]^{3-}};\ \mu_C = 0$ BM

$\mu_{net} = 2.828$ BM

4. **Ans (6.00)**

Mg^{2+}, Zn^{2+} Pb^{+2}, Hg^{2+}, Cu^{2+} with Na_2CO_3,

& Hg_2^{2+} nitrate with NH_4OH

$Mg^{2+} + Na_2CO_3(aq) \rightarrow \underset{\text{Basic magnesium carbonate}}{4MgCO_3.Mg(OH)_2.5H_2O} \downarrow$

$Hg_2^{2+} + NO_3^- + NH_4OH \rightarrow \underset{\text{Black}}{Hg} \downarrow + \underset{\text{(white)}}{HgO.Hg(NH_2)NO_3} \downarrow$

whereas

$Hg_2^{2+} + Na_2CO_3(aq.) \rightarrow \underset{\text{yellow}}{Hg_2CO_3} \downarrow$

5. **Ans (37.50)**

$n_C = 0.2$ mole, $n_{CO} = x$, $n_{CO_2} = (0.2 - x)$

$$\underset{x\ \text{mole}}{5CO} + I_2O_5 \rightarrow 5CO_2 + \underset{\frac{x}{5}\text{mole}}{I_2}$$

$$I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$$

$$\frac{x}{5}\text{mole} \Rightarrow \frac{2x}{5}\text{mole}$$

$$\Rightarrow \frac{2x}{5} = 300 \times 0.1 \times 10^{-3}$$

$$\Rightarrow x = 0.075 \text{ mole}$$

$$n_{CO_2} = (0.2 - 0.075) = 0.125 \text{ mole}$$

$$\% \text{ CO (by mole)} = \frac{0.075}{0.2} \times 100 = 37.50\%$$

6. **Ans (600.00)**

1 m, 1060 kg solution

$W_{solvent} = 1000$ g

$\Delta T_f = K_f \times 1$ -----------(i)

$(\Delta T_f + 10) = K_f \times \frac{1}{500} \times 1000 = 2\,K_f$ ---------(ii)

$\Rightarrow K_f = 10\ \&\ \Delta T_f = 10$

When solution is placed at temperature 15°C below its freezing point

$\Rightarrow (\Delta T_f + 15) = K_f \times m$

$$\Rightarrow 25 = 10 \times \frac{1 \times 1000}{W_1}$$

$\Rightarrow W_1 = 400$ gm

$\therefore$ mass of solvent freezes out $= (1000 - 400) = 600$ gm

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (A)**

$$fof(x) = \frac{1}{2} - 2\left(\frac{1}{2} - f\right)^2$$

$$= \frac{1}{2} - 2^3\left(\frac{1}{2} - x\right)^4$$

$$fofof(x) = \frac{1}{2} - 2^7\left(\frac{1}{2} - x\right)^8$$

$$\text{and } f_n = \frac{1}{2} - 2^{2^n-1}\left(\frac{1}{2} - x\right)^{2^n}$$

Let $2^n = k$, $\frac{1}{2} - x = t$

$$\int_0^1 f(x)dx = \frac{1}{2} - 2^{k-1}\int_{-1/2}^{1/2} t^k dt$$

$$= \frac{k}{2(k+1)} = \frac{2^{n-1}}{2^n + 1}$$

$\Rightarrow q = n, p = n - 1 \Rightarrow q - p = 1$

2. **Ans (C)**

$g(x) = kx - 1,\ g'(0) = k$

$\Rightarrow g(x)$ is one-one function.

$g(f^2(x) + 2 + x^2f^2(x)) = g(x^2f(x) + 3f(x))$

$\Rightarrow f^2(x) + 2 + x^2f^2(x) = x^2f(x) + 3f(x)$

$(f^2(x))\,(1 + x^2) - (x^2 + 3)\,f(x) + 2 = 0$

$\Rightarrow$ for $x \in R$, $f(x) = 1$ or $f(x) = \frac{2}{1 + x^2}$

f is continuous on R and $f(2023) = f(2024) \neq f(0)$

$$\text{then } f(x) = \begin{cases} \frac{2}{1+x^2}, & x \leqslant 1 \\ 1, & x > 1 \end{cases}$$

$$f\left(\frac{1}{2}\right) + f(2) = \frac{8}{5} + 1 = \frac{13}{5}$$

3. **Ans (D)**

$$[f(x)\ g(x)\ h(x)] = \begin{vmatrix} \cos x & \sin x & \cos 2x \\ \tan x & \sin 3x & \cos 4x \\ \cos 3x & \sin 5x & \cos 6x \end{vmatrix}$$

$$\therefore \int_0^{\pi} [f,\ g,\ h](x)dx = \int_0^{\pi} [f,\ g,\ h](\pi - x)dx$$

$$\Rightarrow \int_0^{\pi} [f,\ g,\ h]\,dx = -\int_0^{\pi} [f,\ g,\ h]dx$$

$(\because C_1 \to -C_1)$

$$\Rightarrow \int_0^{\pi} [f,\ g,\ h]dx = 0$$

4. **Ans (A)**

Let $D(\vec{O}),\ A(\vec{a}),\ B(\vec{b}),\ C(\vec{c})$

given volume of tetrahedron is $\left|\frac{1}{6}[\bar{a}\,\bar{b}\,\bar{c}]\right| = \frac{81}{2}$

$|[\bar{a}\ \bar{b}\ \bar{c}]| = 3^5$

Centroids by faces are

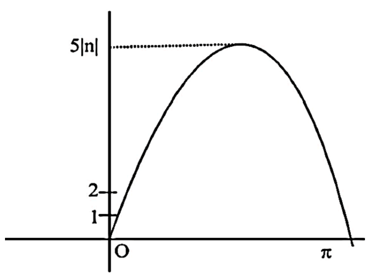
$$G_1\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right), G_2\left(\frac{\vec{a}+\vec{b}}{3}\right), G_3\left(\frac{\vec{b}+\vec{c}}{3}\right), G_4\left(\frac{\vec{c}+\vec{a}}{3}\right)$$

Volume of parallelopiped $= \left|\left[\overrightarrow{G_1G_2}\ \overrightarrow{G_1G_3}\ \overrightarrow{G_1G_4}\right]\right|$

$$= \left|\left[-\frac{\vec{c}}{3}\ -\frac{\vec{a}}{3}\ -\frac{\vec{b}}{3}\right]\right| = \left|\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{3^3}\right| = 9$$

5. **Ans (B)**

$[2 + 5|n||\sin x|] = 2 + [5|n|\sin x]$



Number of points of non dierivability

$= 2(5\,|n| - 1) + 1 = 10\,|n| - 1 = 19$

$\Rightarrow |n| = 2$

6. **Ans (A)**

Case I : each gets 1R $\Rightarrow 1.4^3 = 64$

Case II : one gets 2R, one get 1R, one get 1R

$^4C_3.3$

then remaining get 1 or 2 or 3 balls

$^4C_3.3(^3C_3 + {}^3C_2.3 + {}^3C_1.3^2) = 444$

Case III : 2R + 2R or 3R + 1R

$^4C_2(1 + 2)$, then remaining two get at least one.

$^4C_2.3(^3C_2.(2!).2 + {}^3C_2.(2!)) = 324$

Total ways $\Rightarrow 832$

PART-3 : MATHEMATICS

SECTION-I (ii)

7. **Ans (A,B,D)**

$\because$ Finding minimum value of

$$PA + PB = \sqrt{\lambda^2 + 100} + \sqrt{(\lambda - 15)^2 + 400}$$

is same as finding minimum value of P'A' + P'B,

where $P'(\alpha, 0), A'(0, 10)$ and $B'(15, -20)$

which is possible only when P', A', B' are collinear.

$\Rightarrow \alpha = 5$

$\therefore$ Equation of plane passing through

$P(5,\ 0,\ 0), A(0,\ 6,\ 8)$ and $B(15,\ 20,\ 0)$

is $2x - y + 2z = 10$

$$\Rightarrow d = \frac{10}{3} \text{ and } (\alpha, \beta, \gamma) = \left(\frac{40}{9}, \frac{-20}{9}, \frac{40}{9}\right)$$

8. **Ans (A)**

We have,

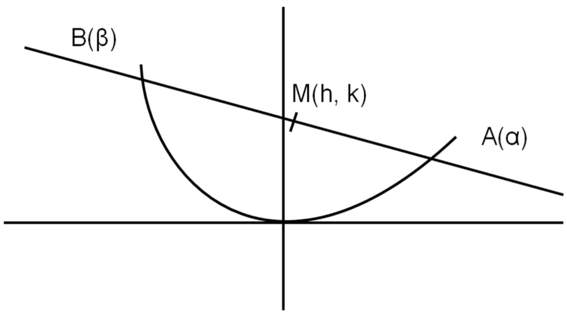
$$F(x) = \begin{cases} -(x+1) & , \quad x < -1 \\ (x+1) & , \quad -1 \leqslant x \leqslant 0 \\ x & , \quad x > 0 \end{cases}$$

$$\text{and } G(x) = \begin{cases} -x+1 & , \quad -\infty < x \leqslant 0 \\ x-2 & , \quad 0 < x < 2 \\ 2-x & , \quad x \geqslant 2 \end{cases}$$

Now,

$$H(x) = F(x) + G(x) = \begin{cases} -2x & , \quad -\infty < x < -1 \\ 2 & , \quad -1 \leqslant x \leqslant 0 \\ 2x-2 & , \quad 0 < x < 2 \\ 2 & , \quad x \geqslant 2 \end{cases}$$

9. **Ans (B,C,D)**



$A(\alpha, \alpha^2), B(\beta, \beta^2)$

equation of PQ

$y - \alpha^2 = (\alpha + \beta)(x - \alpha)$

$$\int_\alpha^\beta \left((\alpha+\beta)x - \alpha\beta - x^2\right)dx = \frac{4}{3}$$

$\beta - \alpha = 2$

Let M be (h, k) locus is $T = S_1$

$2xh - y = 2x^2 - k$

$x^2 - 2hx + 2h^2 - k = 0$

$x = h \pm \sqrt{k - h^2}$

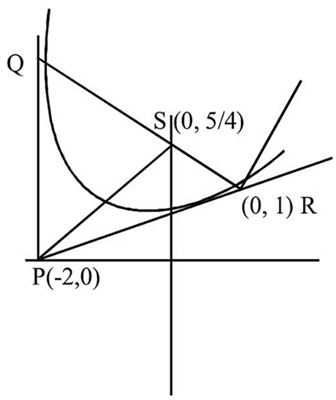
$\beta - \alpha = 2$

$f(x, y) = 0 \Rightarrow y = 1 + x^2$

Since $\Delta PQS \sim \Delta RPS$

$\therefore SQ.\,SR = SP^2$

$= 4 + \frac{25}{16} = \frac{89}{16}$



10. **Ans (C,D)**

After simplification

$D = 2\tan A \tan B \tan C (\tan A + \tan B + \tan C)^3$

$= 2(\tan A + \tan B + \tan C)^4$

as $\tan A + \tan B + \tan C \geqslant 3\sqrt{3}$

so, $\frac{D}{1000} \geqslant 1.458$

so least integral value is 2

11. **Ans (A,B,C)**

Here, $b = \frac{2ac}{a+c}$ ...(1)

and $b^2 = -2ac$ ...(2)

$\Rightarrow b = \frac{-b^2}{a+c} \Rightarrow a + b + c = 0$

Now, verify it

12. **Ans (A,B,C,D)**

Given, $|2z + 3i| = |z^2|$

$|2z + 3i| \leqslant 2|z| + 3$

$\Rightarrow |z|^2 \leqslant 2|z| + 3$

$\Rightarrow 0 \leqslant |z| \leqslant 3$ .....(1)

Again, $|2z + 3i| \geqslant |2|z|-3|$

$\Rightarrow |z|^2 \geqslant |2|z|-3|$

$\therefore |z| \geqslant 1$ .....(2)

So, (1) ∩ (2) gives

$1 \leqslant |z| \leqslant 3$

Also, $|z|_{\text{max imum}} \Rightarrow z = 3i$

So, $\alpha = 0, \beta = 3$

& $|z|_{\text{mininum}} \Rightarrow z = -i$

so, $x = 0, y = -1$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (3.00)**

$f(x) + x^3 - 1 = \alpha x$

$\Rightarrow f(x) = -x^3 - x - 1$

Since $f'(x) = -1 - 3x^2 < 0 \ \forall\ x \in R$

Hence f is decreasing

2. Ans (1.50)

There can be four such numbers i.e. 43, 34, 62, 26

Whose prduct of digit is 12

$\Rightarrow$ Probability that the man will laugh by seeing the chosen numbers $= \frac{4}{90} = \frac{2}{45}$

$\Rightarrow$ Required probability $= 1 - \left(1 - \frac{2}{45}\right)^3$

$= 1 - \left(\frac{43}{45}\right)^3$

3. Ans (0.70)

$f(x) = \frac{1}{x}$

$$\int_2^3 \frac{\frac{3}{x^5} - \frac{1}{x}}{1 - \frac{1}{x^4}} dx = \int_2^3 \frac{\frac{3}{x^7} - \frac{1}{x^3}}{\frac{1}{x^2} - \frac{1}{x^6}} dx$$

$$\frac{1}{x^2} - \frac{1}{x^6} = t \Rightarrow \left(\frac{-2}{x^3} + \frac{6}{x^7}\right) dx = dt$$

$$\int_{\frac{15}{2^6}}^{\frac{80}{3^6}} \frac{dt}{2.t} = \frac{1}{2}\ell n\left(\frac{80}{3^6} . \frac{2^6}{15}\right) = \frac{1}{2}\ell n \frac{2^{10}}{3^7}$$

$\alpha = 10,\ \beta = 7$

4. Ans (1.70)

$f(x) = x^3 - 2x^2 + 4x - 1$

$f'(x) = 3x^2 - 6x + 4$

$D < 0 \Rightarrow$ Strictly increasing

$$\int_{4/27}^{50/27} f(f(x))dx = \int_{4/27}^{50/27} f(f(2-x))dx$$

$$= \int_{4/27}^{50/27} f(2 - f(x))dx \Rightarrow \int_{4/27}^{50/27} dx = \frac{46}{27}$$

5. Ans (2.00)

$$p = \sum_{r=1}^{50} (-1)^{r-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{r}\right) . {}^{50}C_r$$

$$= \int_0^1 \sum_{r=1}^{50} (-1)^{r-1} \left(1 + x + x^2 + \ldots + x^{r-1}\right) . {}^{50}C_r$$

$$= \int_0^1 \sum_{r=1}^{50} (-1)^{r-1} . {}^{50}C_r . \frac{(1-x^r)}{1-x} dx$$

$$= \int_0^1 \frac{1}{1-x} \sum_{r=1}^{50} \left\{(-1)^{r-1} . {}^{50}C_r - (-1)^{r-1} . {}^{50}C_r\right\} dx$$

$$= \int_0^1 \frac{1}{1-x} \left\{1 + ((1-x)^{50} - 1)\right\} dx$$

$$= \int_0^1 (1-x)^{49} dx = \frac{1}{50} = p$$

6. Ans (4.00)

Let $u = |z^2| \left| z^2 + \frac{1}{z^2} + z + \frac{1}{2} - 2i \right|$

$= \left|(z^2 + \bar{z}^2) + (z + \bar{z}) - 2i\right|$

$= \left|(z + \bar{z})^2 - 2z\bar{z} + (z + \bar{z}) - 2i\right|$

let $z = x + iy \quad \therefore u = \left|(2x)^2 - 2 + 2x - 2i\right|$

$u = 2\left|2x^2 + x - 1 - i\right|$

$u^2 = 4((2x^2 + x - 1)^2 + 1)$

$\because |z| = 1 \quad \therefore x^2 + y^2 = 1 \quad \therefore -1 \le x \le 1$

Now

$t = 2x^2 + x - 1 = 2\left(x^2 + \frac{1}{2}x - \frac{1}{2}\right)$

$= 2\left(\left(x + \frac{1}{4}\right)^2 - \frac{9}{16}\right)$

$\frac{-9}{8} \leqslant t \leqslant 2 \quad \therefore u^2_{max} = 20$