FIITJEE

ALL INDIA TEST SERIES

JEE (Advanced)-2025

PART TEST – I

PAPER –1

TEST DATE: 17-11-2024

ANSWERS, HINTS & SOLUTIONS

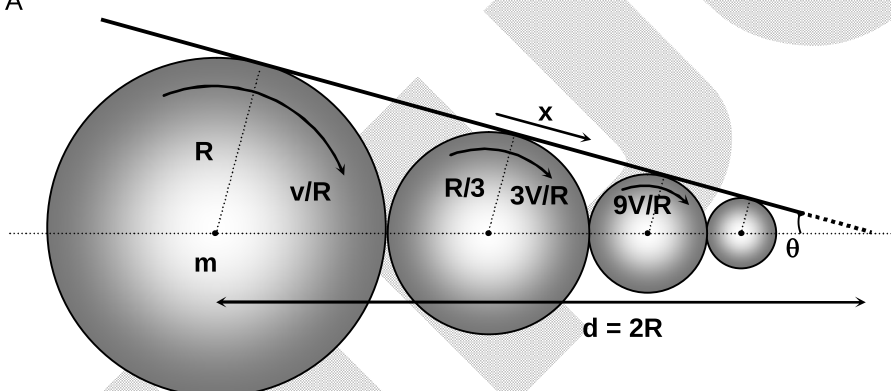
Physics

PART – I

SECTION – A

1. A

Sol.



$$d = R + \frac{2R}{3} + \frac{2R}{9} + ... = R + \frac{2R}{3}\left(1 + \frac{1}{3} + ...\right) = R + \frac{2R}{3}\left(\frac{1}{1-\frac{1}{3}}\right) = 2R$$

$$\sin\theta = \frac{R}{d} = \frac{R}{2R} = \frac{1}{2} \Rightarrow \theta = 30°$$

$$mgx\sin 30° = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 + \frac{1}{2}\left(\frac{m}{9}\right)\left(\frac{R}{3}\right)^2\left(\frac{3v}{R}\right)^2 + \frac{1}{2}\left(\frac{m}{81}\right)\left(\frac{R}{9}\right)^2\left(\frac{9v}{R}\right)^2\right]$$

$$mgx\left(\frac{1}{2}\right) = \frac{mv^2}{2} + \frac{1}{4}mv^2\left[1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + ...\right]$$

$$gx = \frac{25v^2}{16}$$

Differentiate

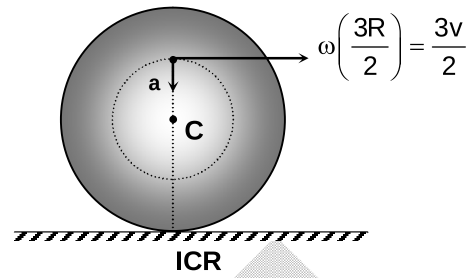
$$g\left(\frac{dx}{dt}\right) = \frac{25v}{8}\left(\frac{dv}{dt}\right) \quad \Rightarrow a = \frac{8g}{25}$$

2. A

Sol. $a = \omega^2\left(\frac{R}{2}\right) = \left(\frac{v}{R}\right)^2\left(\frac{R}{2}\right)$

$a = \frac{v^2}{2R} = \frac{v_{total}^2}{R_C} = \frac{(3v/2)^2}{R_C}$

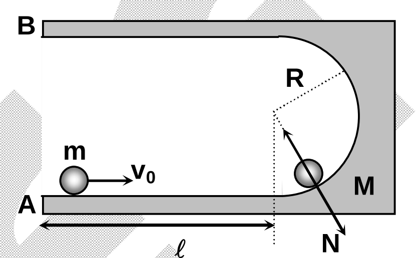
$R_C = \frac{9}{2}R$



3. A

Sol. In the wedge frame speed of particle is always constant

$t = \frac{2\ell + \pi R}{v_0}$



4. C

Sol. $\frac{1}{2}Mv_0^2 = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{M}{m}}v_0$

5. A, C, D

Sol. $\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$ ...(i)

$\vec{\tau} \perp \vec{L}$

$\vec{L} = L\hat{L}$

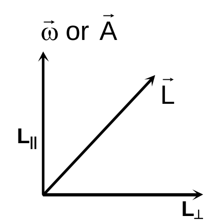
$\frac{d\vec{L}}{dt} = \frac{dL}{dt}\hat{L} + L\frac{d\hat{L}}{dt} = \frac{dL}{dt}\hat{L} + L(\vec{\omega}_L\times\hat{L})$ $\left(\because \frac{d\hat{L}}{dt} = \vec{\omega}_L\times\hat{L}\right)$

$\frac{d\vec{L}}{dt} = \frac{dL}{dt}\hat{L} + \vec{\omega}\times\vec{L}$ ...(ii)

From (i) and (ii)

$\frac{dL}{dt}\hat{L} = 0$

$\vec{A} = \vec{\omega}$

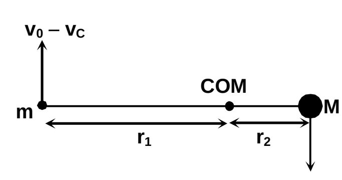


6. A, D

Sol. In COM frame

$a_n = \frac{v^2}{r_1} = \frac{\left(\frac{Mv_0}{m+M}\right)^2}{\left(\frac{MR}{m+M}\right)} = \frac{Mv_0^2}{(m+M)R}$

In ground frame



In COM frame

$$a_n = \frac{Mv_0^2}{(m+M)R} = \frac{v_0^2}{R_C}$$

$$R_C = \frac{(m+M)R}{M}$$

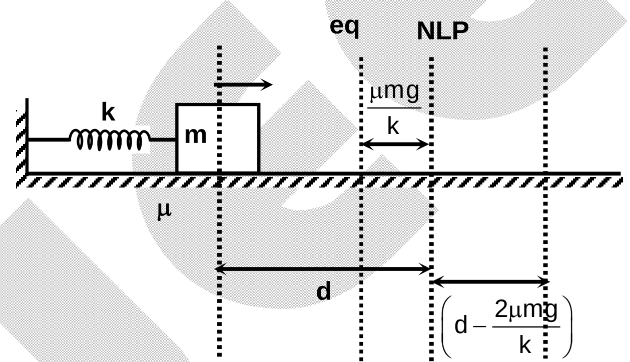
$$N = \frac{m\left(\frac{Mv_0}{m+M}\right)^2}{r_1} = \frac{mMv_0^2}{(m+M)R}$$

If M = 2m

$$N = \frac{2mv_0^2}{3R}$$

7. A, C

Sol. If we compress the spring by d from NLP then it comes to rest first time at a distance $\left(d - \frac{2\mu mg}{k}\right)$ from NLP on the other side.



Finally we want block to stop at NLP

$$d - \left(\frac{2\mu mg}{k}\right)n = 0$$

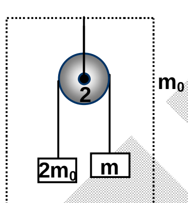
$$d = \left(\frac{2\mu mg}{k}\right)n$$

Here, n = 1, 2, 3, 4, ………

(NLP = natural length position of spring)

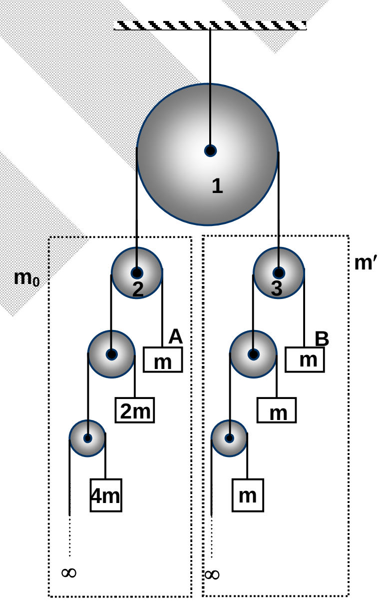
8. C

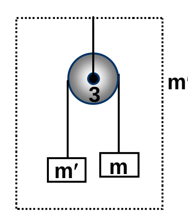
Sol.



$$4\frac{(2m_0)m}{2m_0+m} = m_0$$

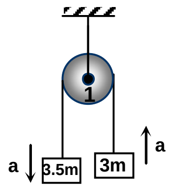
$$m_0 = \frac{7m}{2}$$





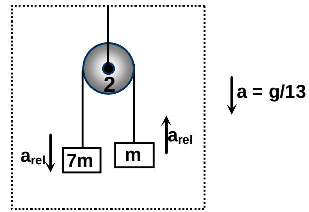
$$\frac{(4m)m'}{m+m'} = m'$$

$$m' = 3m$$



$$T = \frac{2(3m)\left(\frac{7m}{2}\right)}{3m+\frac{7m}{2}}g = \frac{42mg}{13}$$

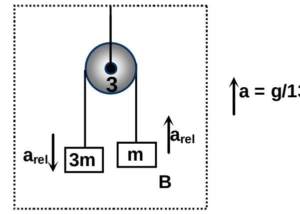
$$a = \left(\frac{\frac{7m}{2}-3m}{\frac{7m}{2}+3m}\right)g = \frac{g}{13}$$



$$g_{eff} = g - \frac{g}{13} = \frac{12g}{13}$$

$$a_{rel} = \left(\frac{7m-m}{7m+m}\right)\left(\frac{12g}{13}\right) = \frac{9g}{13}$$

$$a_A = \frac{9g}{13} - \frac{g}{13} = \frac{8g}{13}$$



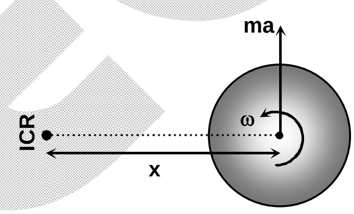
$$g_{eff} = g + \frac{g}{13} = \frac{14g}{13}$$

$$a_{rel} = \left(\frac{3m-m}{3m+m}\right)\left(\frac{14g}{13}\right) = \frac{7g}{13}$$

$$a_B = \frac{7g}{13} + \frac{g}{13} = \frac{8g}{13}$$

9. D

Sol. With respect to the plank, apply pseudo force on the centre of mass of the disc.



(P) $v = \omega x$

$\sqrt{2ay} = \omega x$

$y = 4x^2$ $(x \le 0)$

(Q) $v = \omega x$

$\sqrt{2ay} = \omega x$

$y = 4x^2$ $(x \ge 0)$

(R) $v = \omega y$

$\sqrt{2ax} = \omega y$

$x = 4y^2$ $(y \ge 0)$

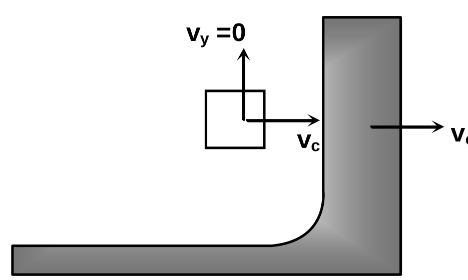
(S) $v = \omega y$

$\sqrt{2ax} = \omega y$

$x = 4y^2$ $(y \le 0)$

10. A

Sol. At maximum height both block and wedge will be moving with the same velocity in the horizontal direction.



$$m_1v_0 = (m_1+m_2)v_c \Rightarrow v_c = \frac{m_1v_0}{m_1+m_2}$$

$$-m_1gh = \frac{1}{2}(m_1+m_2)v_c^2 - \frac{1}{2}m_1v_0^2$$

$$h = \frac{m_2v_0^2}{2(m_1+m_2)g} \qquad \ldots(i)$$

When the block m_1 again comes back on the horizontal surface of the wedge m_2, its velocity relative to wedge is v_0 towards left.

Using conservation of momentum of the system

$$m_1v_0 = m_1(-v_0+v_2) + m_2v_2$$

Velocity of the wedge relative to ground, $v_2 = \dfrac{2m_1v_0}{m_1+m_2}$

$$v_1 = v_0 - v_2$$

Velocity of the block relative to ground, $v_1 = \frac{(m_2 - m_1)v_0}{m_1 + m_2}$

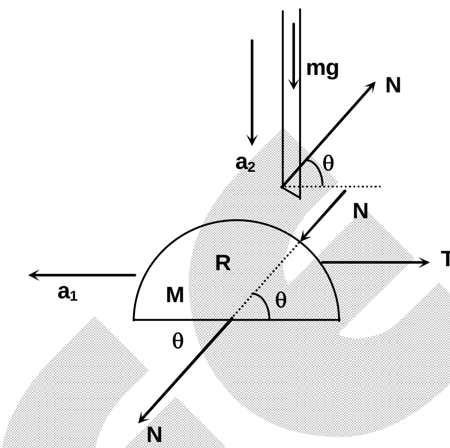
11. A

Sol. Just before the string is cut,

N sin θ = mg

N cos θ = T

$T = \cos\theta\left(\frac{mg}{\sin\theta}\right) \quad \Rightarrow T = mg\cot\theta$



Just after the string is cut,

$N\cos\theta = Ma_1$ ...(i)

$mg - N\sin\theta = ma_2$ ...(ii)

$a_1\cos\theta = a_2\sin\theta$ ...(iii)

$\Rightarrow a_2 = a_1\cot\theta$

From (i) and (ii)

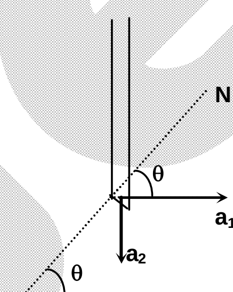
$\left(\frac{mg - ma_2}{\sin\theta}\right)\cos\theta = Ma_1$

$mg\cot\theta - (m\cot\theta)a_1\cot\theta = Ma_1$

$a_1 = \frac{mg\cot\theta}{M + m\cot^2\theta} = \frac{mg}{M\tan\theta + m\cot\theta}$

$a_2 = a_1\cot\theta = \frac{mg\cot\theta}{M\tan\theta + m\cot\theta}$

$N = \frac{Ma_1}{\cos\theta} = \frac{Mmg}{\cos\theta(M\tan\theta + m\cot\theta)}$



SECTION – B

12. 3

Sol. Acceleration of the block with respect to ground just after it starts slipping,

$\alpha = \mu_k g = 0.3 \times 10 = 3\ m/s^2$

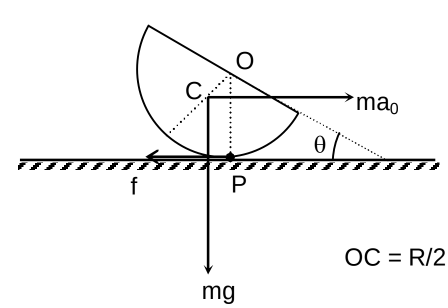
Acceleration of the block with respect to disc just after it starts slipping,

$\beta = \frac{\mu_s mg - \mu_k mg}{m} = (\mu_s - \mu_k)g = (0.4 - 0.3)(10) = 1\ m/s^2$

Hence, $\frac{\alpha}{\beta} = 3$

13. 40

Sol. $mg\frac{R}{2}\sin\theta - ma\left(R - \frac{R}{2}\cos\theta) = I_P\alpha\right)$



$I_0 = I_C + m\left(\frac{R}{2}\right)^2$

$\Rightarrow \frac{2}{3}mR^2 = I_C + \frac{mR^2}{4}$

$I_C = \frac{5}{12}mR^2$

$$I_P = I_C + m\left[\left(\frac{R}{2}\sin\theta\right)^2 + \left(R - \frac{R}{2}\cos\theta\right)^2\right] = mR^2\left(\frac{5}{3} - \cos\theta)\right)$$

From (i)

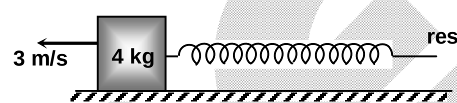
$$mg\frac{R}{2}\sin\theta - maR\left(1 - \frac{1}{2}\cos\theta)\right) = mR^2\left(\frac{5}{3} - \cos\theta\right)\alpha$$

$$\alpha = \frac{27}{13}\ \text{rad/s}^2$$

Hence, (a + b) = 27 + 13 = 40

14. 60

Sol. w.r.t. the point



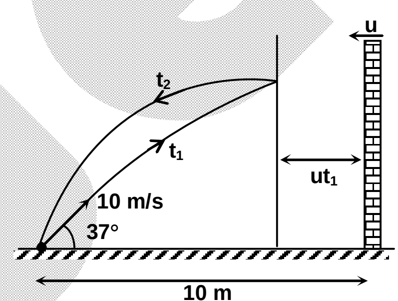
$$-\frac{1}{2}k\left[x^2 - 0^2\right] = 0 - \frac{1}{2}(4)(3)^2$$

$$x = \sqrt{\frac{36}{100}} \Rightarrow x = \frac{6}{10} = 0.6\ \text{m}$$

x = 60 cm

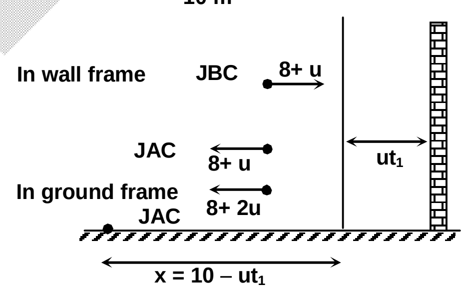
15. 16

Sol. Vertical component of velocity of the particle will not be affected by the wall force. (no effect on time of flight)



$$T = \frac{2\times 6}{10} = 1.2\ \text{sec}$$

In wall frame



$$t_1 = \frac{x}{8},\quad t_2 = \frac{x}{8+2u}$$

x = 10 – ut_1

$$x = 10 - \frac{ux}{8}$$

$$\Rightarrow x = \frac{80}{8+u}$$

$t_1 + t_2 = 1.2$

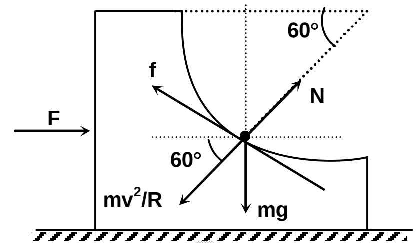
$$\frac{x}{8} + \frac{x}{8+2u} = 1.2$$

$$x\left(\frac{1}{8} + \frac{1}{8+2u}\right) = 1.2$$

$$\left(\frac{80}{8+u}\right)\left(\frac{1}{8} + \frac{1}{8+2u}\right) = 1.2$$

$$\Rightarrow u = \frac{13}{3}\ \text{m/s}$$

16. 25



Sol. $N - mg\cos 30° = \frac{mv^2}{R}$

$N = mg\cos 30° + \frac{mv^2}{R}$ ...(i)

f = mg cos 60° ...(ii)

$F = N\cos 60° - F\cos 30°$

$F = mg\cos 30°\cos 60° + \frac{mv^2}{R}\cos 60° - mg\cos 60°\cos 30°$

$F = \frac{mv^2}{R}\cos 60°$

$50\times 10^{-3}\times\frac{1}{4\times 0.25}\times\frac{1}{2} = 25\times 10^{-3}\text{N} = 25$ millinewton

17. 96

Sol. $\frac{dx}{dt} = v_x = 24\cos 6t$

$\frac{dy}{dt} = v_y = 24\sin 6t$

Speed, $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 24$ m/s (constant)

Distance covered in first 4 seconds, S = 24 × 4 = 96 m

Chemistry

PART – II

SECTION – A

18. D

Sol. $\frac{T_1}{T_2} = \frac{n_1^3}{Z_1^2} \times \frac{Z_2^2}{n_2^3}$

$\frac{x}{T_2} = \frac{2^3}{2^2} \times \frac{3^2}{3^3}$

$\frac{x}{T_2} = \frac{2}{3}$

$T_2 = \frac{3x}{2}\text{sec}$

19. A

Sol.

	$2NO(g) + Cl_2(g) \rightleftharpoons$		$2NOCl(g)$
Initial partial pressure	P^o	P^o	0
Partial pressure at equilibrium	$P^o - 2x$	$P^o - x$	$2x$

P_{total} at equilibrium $= P^o - 2x + P^o - x + 2x$

$2P^o - x = 1$

Given, $2x = \frac{1}{2}(P^o - x)$

$4x = P^o - x, \qquad P^o = 5x$

$\therefore 2P^o - x = 1$

$2 \times 5x - x = 1$

$9x = 1$

$x = \frac{1}{9}$

$P_{NO} = P^o - 2x = 5x - 2x = 3x$

$P_{Cl_2} = P^o - x = 5x - x = 4x$

$P_{NOCl} = 2x$

$K_P = \frac{(P_{NOCl})^2}{(P_{NO})^2 \times (P_{Cl_2})}$

$K_P = \frac{(2x)^2}{(3x)^2 \times 4x} = \frac{1}{9x}$

$K_P = \frac{1}{9 \times \frac{1}{9}} = 1$

K_P for the reaction is $1\,\text{atm}^{-1}$.

20. B

Sol. (A) Sodium does not react with N_2.

(C) KO_2 is paramagnetic.

(D) Dilute solution of sodium metal in liquid ammonia is deep blue in colour.

21. D

Sol. $K = \frac{1}{t}\ell n\frac{V_\infty}{V_\infty - V_t}$

$\frac{1}{10}\ell n\frac{16.8}{16.8-8.4} = \frac{1}{20}\ell n\frac{16.8}{16.8-V_t}$

V_t = 12.6 ml

22. A, B, D

Sol. (A) The number of photoelectrons ejected is directly proportional to the intensity of incident light.

(B) The kinetic energy of the photoelectrons is directly proportional to the frequency of incident light.

(D) The work function of a metal is independent of the frequency of incident light.

23. A, D

Sol. (B) $\Delta_{eg}He = +48\ \text{kJ mol}^{-1}$

$\Delta_{eg}Ne = +116\ \text{kJ mol}^{-1}$

(C) Covalency of Al in $\left[AlCl(H_2O)_5\right]^{2+}$ is 6.

24. A, D

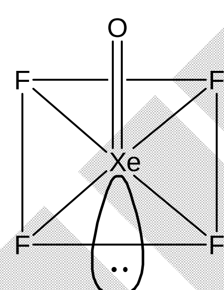
Sol. Structure of XeF_5^- is pentagonal planar.

Structure of PF_5 is trigonal bipyramidal.

25. B

Sol.

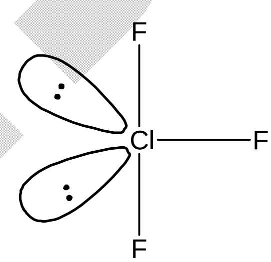
(P)



$XeOF_4$

Square pyramidal, lone pair = 1

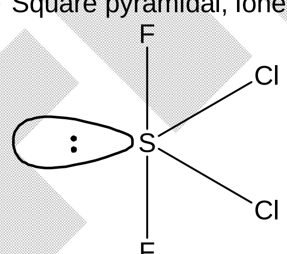
(Q)



ClF_3

T-shaped, lone pair =2

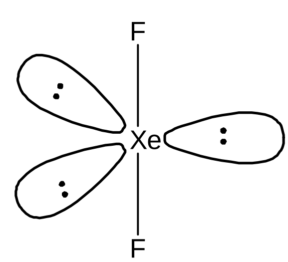
(R)



SF_2Cl

See-saw, lone pair = 1

(S)



Linear, lone pair =3

26. C
Sol. (P) NF_3, SO_2 and H_2S are polar molecules.
(Q) $BeCl_2$, BCl_3 and BeH_2 are electron-deficient molecules.
(R) BF_3, CS_2 and PCl_5 have zero dipole moment.
(S) Hybridisation of NH_3, CH_4 and XeO_4 is sp^3.

27. D
Sol. (P) Shortest wavelength in Balmer series of Li^{2+}
$$\frac{1}{\lambda} = R \times 3^2\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{9R}{4}, \lambda = \frac{4x}{9}$$
(Q) Longest wavelength in Balmer series of He^+
$$\frac{1}{\lambda} = R \times 2^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{9}, \lambda = \frac{9x}{5}$$
(R) Longest wavelength in Lyman series of He^+
$$\frac{1}{\lambda} = R \times 2^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 3R, \qquad \lambda = \frac{x}{3}$$
(S) Shortest wavelength in Lyman series of Li^{2+}
$$\frac{1}{\lambda} = R \times 3^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 9R, \qquad \lambda = \frac{x}{9}$$

28. A
Sol. Electronegativity Br < Cl < F

Electron affinity	Br	<	F	<	Cl
(kJ / mol)	–325		–328		–349

Atomic radius F < Cl < Br

SECTION – B

29. 5
Sol. The following species are diamagnetic
H_2, Li_2, C_2, N_2, F_2

30. 5
Sol. $$S_2O_3^{2-}(aq) + 2Br_2(\ell) + 5H_2O(\ell) \longrightarrow 2SO_4^{2-}(aq) + 4Br^-(aq) + 10H^+(aq)$$
x = 1, y = 2, z = 5, a = 2, b = 4, c = 10
x + y + z = 1 + 2 + 5 = 8
x + b = 1 + 4 = 5

31. 128
Sol. $$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$
$$2(1-0.8) \qquad 2\times 0.8 \qquad 0.8$$
$$K_P = \frac{\left(P_{SO_2}\right)^2 \times P_{O_2}}{\left(P_{SO_3}\right)^2}$$
$$K_P = \frac{(2\times 0.8)^2 \times 0.8}{(2\times 0.2)^2}$$
$$K_P = 12.8$$
x = 12.8
10x = 12.8 × 10 = 128

32. 40

Sol. $pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$4.5 = 4.2 + \log\frac{x \times 1.2}{48 \times 0.5}$

$0.3 = \log\frac{x \times 1.2}{48 \times 0.5}$

$\log 2 = \log\frac{x \times 1.2}{48 \times 0.5}$

x = 40

33. 75

Sol. $x = kt, \quad t_{1/2} = \frac{a}{2k}$

$\frac{3a}{4} = \frac{a}{2 \times 50} \times t$

t = 75 sec.

34. 108

Sol. $(NH_4)_2S(s) \rightleftharpoons 2NH_3(g) + H_2S(g)$

2P P

At equilibrium, 3P = 9 atm

$\therefore P = 3$ atm

$P_{NH_3} = 6$ atm $\quad P_{H_2S} = 3$ atm

$K_P = (P_{NH_3})^2 \times P_{H_2S}$

$K_P = (6)^2 \times 3$

$K_P = 108$

Mathematics PART – III

SECTION – A

35. D

Sol. For any $m \in R$ we have $M \cap N \neq \phi$

Which means point (0, c) is on or in the ellipse

$\frac{x^2}{4/3} + \frac{y^2}{4} = 1$. Therefore $\frac{c^2}{4} \leq 1 \Rightarrow -2 \leq c \leq 2$ hence correct answer is D

Alternative

Our aim is find range of c such that intersection.

$M \cap N \neq \phi \ \forall\ m \in R$.

Put $y = mx + c$ in ellipse $3x^2 + y^2 = 4$

$\Rightarrow\ 3x^2 + (mx + c)^2 - 4 = 0$

$\Rightarrow\ (3 + m^2)x^2 + 2mxc + (c^2 - 4) = 0$

$D = B^2 - 4AC,\quad D \geq 0$

$(2mc)^2 - 4(3 + m^2)(c^2 - 4) \geq 0$

$\Rightarrow\ -12c^2 + 48 + 16m^2 \geq 0 \quad \forall\ m \in R$

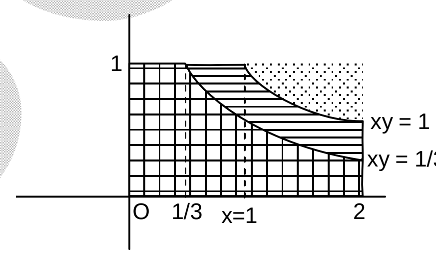
$\Rightarrow\ -12c^2 + 48 \geq 0,\ c^2 \leq 4 \Rightarrow |c| \leq 2,\ c \in [-2, 2]$ hence correct answer is D.

36. A

Sol. Required area

$$\int_{1/3}^{1}\left(1-\frac{1}{3x}\right)dx + \int_{1}^{2}\left(\frac{1}{x}-\frac{1}{3x}\right)dx$$

$$= \frac{2}{3} - \frac{1}{3}\ln 3 + \frac{2}{3}\ln 2$$



37. D

Sol. (A) By rolles theorem option (A) is correct

(B) $h_3\left(\frac{\pi}{2}\right) = ?$

for n = 1, $h_1(x) = \frac{\sin x}{1+\cos x}$

n = 2, $h_2(x) = \frac{\sin x}{1+\cos x} + \frac{\sin 2x}{2+\cos 2x}$

n = 3, $h_3(x) = \frac{\sin x}{1+\cos x} + \frac{\sin 2x}{2+\cos 2x} + \frac{\sin 3x}{3+\cos 3x}$

$h_3\left(\frac{\pi}{2}\right) = 1 + 0 - \frac{1}{3} = \frac{2}{3}$.

(C) $h_n(x) + h_n(-x) = 0 \Rightarrow h_n(x)$ is an odd function.

38. C

Sol. $\frac{d}{dx}g(x) = \frac{d}{dx}\frac{1}{x}\int_0^x f(t)dt$

$$\frac{d}{dx}g(x)=\frac{1}{x}f(x)+\int_0^x f(t)dt\left(-\frac{1}{x^2}\right) \qquad \text{..... (1)}$$

and as g′(2) = 0, we must have $-\frac{1}{4}\int_0^2 f(t)dt+\frac{f(2)}{2}=0$

$$\Rightarrow \int_0^2 f(t)dt=2f(2)=10$$

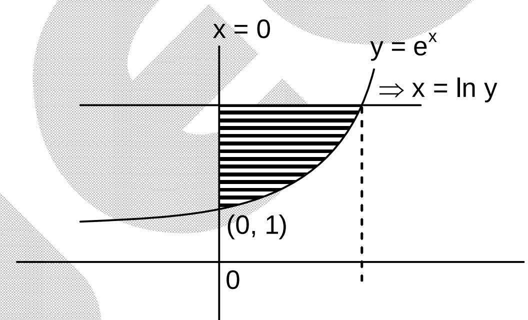
Now, $\frac{d^2}{dx^2}g(x)=\frac{d}{dx}\left(-\frac{1}{x^2}\int_0^x f(t)dt+\frac{1}{x}f(x)\right) = \frac{2}{x^3}\int_0^x f(t)dt-\frac{1}{x^2}f(x)-\frac{1}{x^2}f(x)+\frac{1}{x}f'(x)$

$$\frac{d^2}{dx^2}g(2)=\frac{1}{4}\int_0^2 f(t)dt-\frac{f(2)}{2}+\frac{f'(2)}{2}=-\frac{3}{2}<0$$

Hence, g(x) has a local maximum at x = 2

39. A, D

Sol. $f^2(x)=\int_0^x\left[(f(t))^2+(f'(t))^2\right]dt+2025$



Differentiate w.r.t x

$2f(x)f'(x) = f(x)^2 + f'(x)^2$

$\Rightarrow (f(x) - f'(x))^2 = 0 \Rightarrow f(x) = ce^x$

$f(0) = c = \pm\sqrt{2025}$

$f(x)=\sqrt{2025}\,e^x$ (f(x) is strictly increasing function)

$$\Rightarrow \frac{f(x)}{\sqrt{2025}}=e^x$$

Area = $e-\int_0^1 e^x dx=1$ and area = $\int_1^e \ln y\, dy$ apply Kings property area = $\int_1^e \ln(1+e-y)dy$

and $\frac{3}{\pi^3}\int_{-\pi}^{\pi}\frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}dx = \frac{3}{\pi^3}I$

$$I=\int_{-\pi}^{\pi}\frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}dx = \int_0^{\pi}\frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}+\frac{x^2}{1-\sin x+\sqrt{1+\sin^2 x}}dx$$

$\int_0^{\pi}x^2dx=\frac{\pi^3}{3}$ hence $\frac{3}{\pi^3}I=1$

40. A, B

Sol. Let $\log_{10} x = t$

$\Rightarrow \quad 1 - 8t^2 = t - 2t^2$

$\Rightarrow \quad 6t^2 + t - 1 = 0 \Rightarrow t=-\frac{1}{2}$ or $t=\frac{1}{3}$

$\log_{10} x = -\frac{1}{2} \Rightarrow x=10^{-1/2}=\frac{1}{10^{1/2}}=\alpha$

and $\log_{10} x=\frac{1}{3}\Rightarrow x=10^{1/3}=\beta$

Then $\frac{1}{\alpha^4}+\beta^3=110$ and $\alpha^2\beta^3=1$.

$$I = \int_{-\pi}^{\pi} \frac{(2x)(1+\sin x)}{1+\cos^2 x}dx$$

Using king property and add

$$2I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)+2(-x)(1-\sin x)}{1+\cos^2 x}dx$$

$$2I = 4\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \Rightarrow I = 2\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \;= 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$$

Using king property then add.

$$2I = 4\pi\int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Put cosx $= t \Rightarrow$ sinx dx = – dt

$$I = -2\pi\int_{1}^{-1} \frac{dt}{1+t^2} = -2\pi\tan^{-1} t\,\Big|_{1}^{-1}$$

$$= -2\pi\left(-\frac{\pi}{4}-\frac{\pi}{4}\right) = \pi^2.$$

41. A, C, D

Sol. $\lim\limits_{t\to x}\dfrac{f(x)\cos t - f(t)\cos x}{t-x} = \dfrac{\cos^2 x}{x^2}$

$$= -f(x)\sin x - f'(x)\cos x = \frac{\cos^2 x}{x^2}$$

$$\Rightarrow \quad y\sin x + \frac{dy}{dx}\cos x = -\frac{\cos^2 x}{x^2}$$

$\Rightarrow \quad \dfrac{dy}{dx} + y\tan x = -\dfrac{\cos x}{x^2}$ (which is a linear diff. equation)

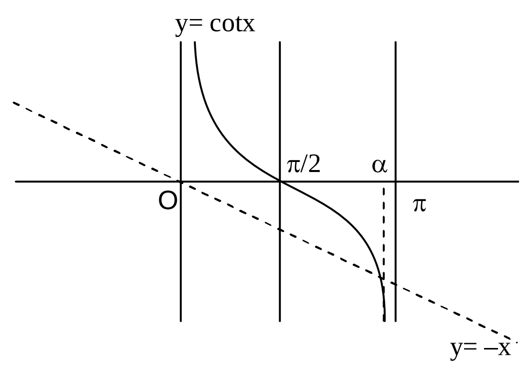
I.F. = $e^{\int \tan x\, dx} = \sec x$

$$y \cdot \sec x = -\int \frac{1}{x^2}dx$$

$$y\sec x = \frac{1}{x} + c$$

given $f(\pi) = -\dfrac{1}{\pi} \Rightarrow c = 0$

$$f(x) = \frac{\cos x}{x}$$



$$f'(x) = \frac{(x)(-\sin x)-(\cos x)}{x^2} = \frac{-(\sin x)(x+\cot x)}{x^2}$$

(A) sign scheme of f′(x)

$\underset{\pi/2}{|} \overset{-}{\quad} \underset{\alpha}{|} \overset{+}{\quad} \underset{\pi}{|}$ $\quad \alpha \in (\pi/2, \pi)$

(B) Incorrect $f'(x) = -\dfrac{\sin x}{x^2}(x+\cot x)$ – ve $\forall\ x \in \left(0, \dfrac{\pi}{3}\right)$.

(C) $\int_{1/2}^{1}(x^3)\left(-\frac{\sin x}{x^2}\right)(x+\cot x)\ dx$

$= -\int_{1/2}^{1} x\sin x(x+\cot x)dx < 0$, True

(D) $x^3 \cdot f(x) = \frac{x^3 \cdot \cos x}{x} = x^2\cos x$

$x^2\cos x = x^2\left(1-\frac{x^2}{2!}+\frac{x^2}{4!}-\frac{x^6}{6!}+....\right)$

$= x^2 - \frac{x^4}{2} + \frac{x^6}{24} - .....$

$x^2 - \frac{x^4}{2} < x^2\cos x < x^2$

$\int_0^1 x^2 - \frac{x^4}{2}dx < \int_0^1 x^2\cos x\ dx < \int_0^1 x^2\,dx$

$\frac{7}{30} < \int_0^1 x^2\cos x\,dx < \frac{1}{3}$.

42. C

Sol. $I_{n+2} = \int_0^1 x^{n+2}\tan^{-1}x\,dx = \frac{\pi}{4(n+3)} - \frac{1}{n+3}\int_0^1 \frac{x^{n+3}}{1+x^2}\ dx$

$\Rightarrow\ (n+3)I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{1+x^2}\ dx$

similarly $(n+1)I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2}\ dx$

$\Rightarrow\ (n+3)I_{n+2} + (n+1)I_n = \frac{\pi}{2} - \frac{1}{n+2}$

$\Rightarrow\ a_n = n+3,\ b_n = n+1,\ c_n = \frac{\pi}{2} - \frac{1}{n+2}$

43. B

Sol. Given g(f(x)) = x

$\Rightarrow$ g(x) is inverse of f(x)

g(f(x)) = x

$\Rightarrow\ g'(f(x))f'(x) = 1$

$\Rightarrow\ g'(f(x)) = \frac{1}{f'(x)}$ ... (i)

$g'(f(0)) = \frac{1}{f'(0)} = \frac{1}{3}$

$\Rightarrow\ g'(1) = \frac{1}{3}$

Now h(g(g(x)) = x

$\Rightarrow$ h(g(g(f(x)))) = f(x)

$\Rightarrow$ h(g(x)) = f(x) ... (ii)

$\Rightarrow$ h(g(1)) = f(1) = 5

Now from equation (ii)

$h(g(x)) = f(x)$

$\Rightarrow$ $h(g(f(x))) = f(f(x))$

$\Rightarrow$ $h(x) = f(f(x))$ ... (iii)

$\Rightarrow$ $h'(x) = f'(f(x)) \cdot f'(x)$

$\Rightarrow$ $h'(0) = f'(f(0))\, f'(0) = f'(1) \cdot 3 = 18$

and $g(h(g(x))) = g(f(x)) = x$

$\Rightarrow$ $g(h(g(7)) = 7$.

44. D

Sol. The given functional equation along with the same equation but with x replaced by $\frac{x-1}{x}$ and $\frac{1}{1-x}$ respectively, yields:

$$f(x)+f\left(1-\frac{1}{x}\right)=\tan^{-1}(x)$$

$$f\left(\frac{x-1}{x}\right)+f\left(\frac{1}{1-x}\right)=\tan^{-1}\left(\frac{x-1}{x}\right)$$

$$f\left(\frac{1}{1-x}\right)+f(x)=\tan^{-1}\left(\frac{1}{1-x}\right).$$

Adding the first and third equations and subtracting the second gives:

$$2f(x)=\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{1-x}\right)-\tan^{-1}\left(\frac{x-1}{x}\right)$$

now $\tan^{-1}(t)+\tan^{-1}\left(\frac{1}{t}\right) = \frac{\pi}{2}$ if $t > 0$ and $-\frac{\pi}{2}$ if $t < 0$; it follows that for $x\in(0,1)$,

$$2(f(x)+f(1-x))=\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)+\left(\tan^{-1}(1-x)+\tan^{-1}\left(\frac{1}{1-x}\right)\right)$$

$$-\left(\tan^{-1}\left(\frac{x-1}{x}\right)+\tan^{-1}\left(\frac{x}{x-1}\right)\right)$$

$$=\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2}=\frac{3\pi}{2}.$$

Thus, $4\int_0^1 f(x)dx=2\int_0^1 (f(x)+f(1-x))\,dx=\frac{3\pi}{2}$.

and $4.\lim_{x\to1^-} f(x)=4\times\frac{1}{2}\lim_{x\to1^-}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{1-x}\right)-\tan^{-1}\left(\frac{x-1}{x}\right)\right)=2\left(\frac{\pi}{4}+\frac{\pi}{2}-0\right)=\frac{3\pi}{2}$

and $g(x)=2f(x)-\tan^{-1}x=\tan^{-1}\left(\frac{1}{1-x}\right)-\tan^{-1}\left(\frac{x-1}{x}\right)$

$$g\left(\frac{1}{x}\right)=\tan^{-1}\left(\frac{1}{1-\frac{1}{x}}\right)-\tan^{-1}\left(\frac{\frac{1}{x}-1}{\frac{1}{x}}\right) = \tan^{-1}\frac{x}{x-1}-\tan^{-1}(1-x)$$

$$\Rightarrow \left|g(x)-g\left(\frac{1}{x}\right)\right|=\left|\tan^{-1}\left(\frac{1}{1-x}\right)+\tan^{-1}(1-x)-\left(\tan^{-1}\frac{x-1}{x}+\tan^{-1}\frac{x}{x-1}\right)\right|$$

$$=\left|\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right|=\pi \text{ \{when } x\in(0,1)\}.$$

45. A

Sol. $f(f(2)) - f(f(1)) = 0$

$\Rightarrow$ $(3a + b)(5a^2 + 3ab + 2ac + b) = 0$

$\Rightarrow$ $3a + b = 0$ ... (i)
or $5a^2 + 3ab + 2ac + b = 0$ ... (ii)
$f(f(3)) - f(f(1)) = 0$
$\Rightarrow$ $2(4a + b)(10a^2 + 4ab + 2ac + b) = 0$
$\Rightarrow$ $4a + b = 0$ ... (iii)
or $10a^2 + 4ab + 2ac + b = 0$ ... (iv)

Case I : When $b = -3a$ put it in IV we get $c = a + \frac{3}{2}$ (not possible)

Case II : When $b = -4a$ put it in II we get $c = \frac{7a}{2} + 2$ $\Rightarrow a = 2\lambda,\ c = 7\lambda + 2\ \&\ b = -8\lambda$

Case III : When $5a^2 + 3ab + 2ac + b = 0\ \&\ 10a^2 + 4ab + 2ac + b = 0$
$\Rightarrow$ $5a^2 + ab = 0 \Rightarrow b = -5a$ (not possible)
So $a = 2\lambda,\ c = 7\lambda + 2\ \&\ b = -8\lambda$
Now $-20 \le -8\lambda \le 20$ $(\lambda \ne 0)$
$\Rightarrow$ $-2.5 \le \lambda \le 0.25 \Rightarrow \lambda = -2, -1, 1, 2.$
when $\lambda = 1$, $a = 2$, $b = -8$, $c = 9 \Rightarrow f(x) = 2x^2 - 8x + 9$
when $\lambda = 2$, $a = 4$, $b = -16$, $c = 16 \Rightarrow f(x) = 4x^2 - 16x + 16$
when $\lambda = -1$, $a = 2$, $b = 8$, $c = -5 \Rightarrow f(x) = -2x^2 + 8x - 5$
when $\lambda = -2$, $a = -4$, $b = 16$, $c = -12 \Rightarrow f(x) = -4x^2 + 16x - 12$

SECTION – B

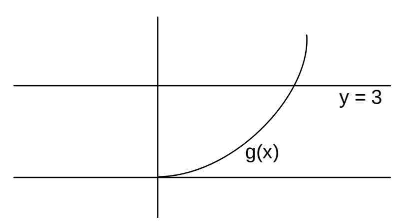
46. 625

Sol. $$125 = \frac{\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{r=1}^{n}\left(\frac{r}{n}\right)^{\frac{1}{4}}}{\lim\limits_{n\to\infty} \frac{1}{n}\sum\limits_{r=1}^{n}\frac{1}{\left(a+\frac{r}{n}\right)^3}} = \frac{\int_0^1 x^{\frac{1}{4}}dx}{\int_0^1 \frac{1}{(a+x)^3}dx}$$

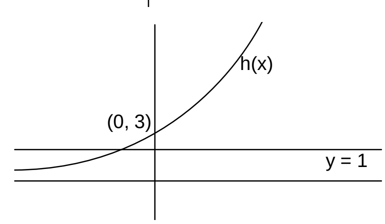
simplifying gives $\frac{4a^2(a+1)^2}{0.5+a} = 625$

47. 2

Sol. $(e^{7x}-1)^2 - (e^{7x}-1)\cdot e^{3x} - 6\cdot(e^{3x})^2 = 0$
let $e^{7x} - 1 = t,\ e^{3x} = s$
$t^2 - ts - 6s^2 = 0$
$\Rightarrow$ $(t-3s)(t+2s) = 0$
$\Rightarrow$ $t = 3s$, or $t + 2s = 0$
$\Rightarrow$ $e^{7x} - 1 = 3e^{3x}$ or $e^{7x} - 1 + 2e^{3x} = 0$
$e^{4x} - e^{-3x} = 3$
Let $g(x) = e^{4x} - e^{-3x}$
$g'(x) > 0$
Only one solution



or $e^{7x} + 2e^{3x} = 1$
$h(x) = e^{7x} + 2e^{3x}$
$h'(x) > 0$
Only one solution
Hence total number of real solutions two



48. 111

Sol. Let $n = \frac{1}{h}$,

$$\lim_{h\to 0} \frac{\int_0^h x^{2025x+2}dx}{h^3} \quad (0/0 \text{ type})$$

Apply L. H rule

$$= \lim_{h\to 0} \frac{h^{2025h+2}}{3h^2} = \frac{1}{3}\lim_{h\to 0} h^{2025h} = \frac{1}{3}.$$

49. 4

Sol. Simplify $f(x) = 2\left(x^2 + \frac{1}{x^2}\right)$

Now using AM $\geq$ GM inequality min value of f(x) is 4.

50. 16

Sol. Given g(x) = g(8 – x) $\Rightarrow$ g(x) is symmetric about x = 4

$\Rightarrow \quad g'(x) = -g'(8-x)$

$\Rightarrow \quad g'(4) = 0,\ g'(0) = g'(8);\ g'(2) = g'(6);\ g'(5) = g'(3);\ g'(7) = g'(1)$

$\Rightarrow \quad g'(0) = g'(1) = g'(2) = g'(3) = g'(4)$

$\quad = g'(5) = g'(6) = g'(7) = g'(8) = 0$

Hence g′(x) = 0 has min 9 roots in [0, 8]

Now using rolle's then $g''(x) = 0$ having at least 8 roots in [0, 8]

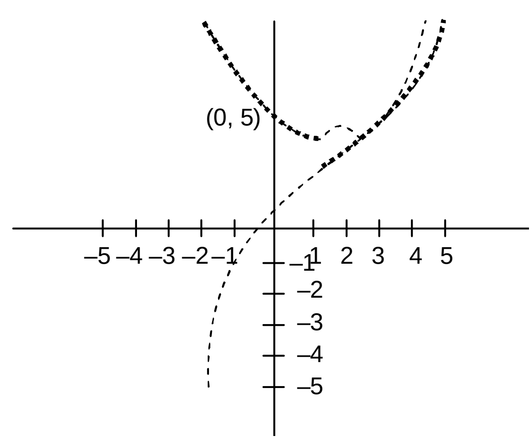
Now given equation $g''(x)^2 + g'(x)g''(x) = 0$

$$\frac{d}{dx}\underset{\text{min }9}{g'(x)}\underset{\text{min }8}{g''(x)} = 0$$

according to rolles theorem min roots of equation is 16.

51. 11

Sol. The points of non–differentiability are at x = 1 and x = 2 and the point of discontinuity is at x = 1



$$\int_1^2 \left|x^2 - 3x + 2\right| dx = -\frac{1}{6}$$

$$\left|66\int_\ell^m \left|x^2 - 3x + 2\right| dx\right| = 11.$$