FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
CONCEPT RECAPITULATION TEST – IV
PAPER –1
TEST DATE: 24-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. C

Sol. Let u be the initial speed of the particle

Then $v^2 = u^2 - 2gh$

or $u^2 = v^2 + 2gh$

or $u_x^2 + u_y^2 = v_x^2 + v_y^2 + 2gh \qquad (v_x = u_x)$

or $u_y^2 = v_y^2 + 2gh$

or $u_y^2 = (2)^2 + (2)(10)(0.4) = 12$

$\therefore \quad u_y = \sqrt{12}$ or $2\sqrt{3}$ m / s

and $u_x = v_y = 6$ m/s

$\therefore \quad \tan\theta = \dfrac{u_y}{u_x} = \dfrac{2\sqrt{3}}{6} = \dfrac{1}{\sqrt{3}}$

or $\theta = 30°$

2. B

Sol. $W = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$

$$= \frac{1}{2}m\left[\left(\frac{1}{2}a_2t_2\right)^2 - \left(\frac{1}{2}a_1t_1\right)^2\right]$$

$$= \frac{1}{8}m\left[\frac{t_2^4}{t_1^2}a_1^2 - a_1^2t_1^2\right]\left(\text{Since, } a_2 = \frac{t_2}{t_1}a_1\right)$$

$$= \frac{ma_1^2}{8t_1^2}(t_2^4 - t_1^4)$$

3. B
Sol. Distance of n^{th} minima from central bright fringe

$$x_n = \frac{(2n-1)\lambda D}{2d}$$

For n = 3, i.e. 3^{rd} minima

$$x_3 = \frac{(2\times 3-1)\times 500\times 10^{-9}\times 1}{2\times 1\times 10^{-3}} = 1.25\times 10^{-3}\text{ m} = 1.25\text{ mm}$$

4. C
Sol. $E_K - E_L = \frac{hc}{\lambda} = \frac{(6.6\times 10^{-34})(3\times 10^{8})}{(0.021\times 10^{-9})(1.6\times 10^{-19})}\text{eV} = 59\text{ KeV}.$

5. AC
Sol. $U_{min} = 20 - 5 = 15$ J
$K_{max} = E - U_{min} = 20 - 15 = 5J$
U_{min} is at $\sin(4\pi x) = -1$
$4\pi = \frac{3}{2}\pi, \frac{7}{2}\pi$
$x = \frac{3}{8}, \frac{7}{8}, \ldots..$

6. BD
Sol. $R = \frac{l}{|\alpha_B - \alpha_C|\Delta t}$

7. BCD
Sol. $f = \frac{D^2 - x^2}{4D}$ (Focal length by displacement method)

$\Rightarrow f = \frac{(100)^2 - (40)^2}{4\times 100} = 21\text{cm}$

$\therefore P = \frac{100}{f} = \frac{100}{21} \approx 5D$

$\because x + 40 = 100 - x$

$\Rightarrow x = 30$

$\therefore m_1 = \frac{v}{u} = \frac{70}{-30} = -\frac{7}{3}$

and $m_2 = -\frac{7}{3}$

8. B
Sol. In process I, temperature remains constant, and volume of left parts decreases. Hence the heat is rejected by the gas in the left.
If the piston displaces to the right, then work is done by the gas in the left and change in internal energy is zero.
In process II, dissociation also takes place. As a result, the molar mass becomes half.

9. A

Sol. $A_r = \left(\dfrac{v_2 - v_1}{v_1 + v_2}\right) A_i$

$A_t = \left(\dfrac{2v_2}{v_1 + v_2}\right) A_i$

$v_1 \propto \dfrac{1}{\sqrt{\mu_1}}$; $\mu_1 = \dfrac{0.12}{4.8} = \dfrac{1}{40}$

$v_2 \propto \dfrac{1}{\sqrt{\mu_2}}$; $\mu_2 = \dfrac{0.4}{2.56} = \dfrac{1}{64}$

$A_r = 3.5\left[\dfrac{\sqrt{40} - \sqrt{64}}{\sqrt{40} + \sqrt{64}}\right] = 3.5\left[\dfrac{(8 - 2\sqrt{10})}{8 + 2\sqrt{10}}\right] = 0.84$

$\left(\dfrac{3.5 \times 8}{8 + \sqrt{40}}\right) = 3.9\text{ cm}$

10. B

Sol. Velocity of image of bird w.r.t. water surface = $\dfrac{v_{bs}}{\mu_w}$ (v_{bs} is velocity of bird w.r.t. water surface).

$\therefore$ Similarly velocity of image of fish w.r.t. water surface = $\dfrac{v_{fs}}{\mu_w} \times \mu_w$

11. B

Sol. For (A) $\tau = I\alpha \Rightarrow \alpha = \dfrac{\tau}{I} = \dfrac{F(r+R)}{MR^2 + MR^2} = \dfrac{F(r+R)}{2MR^2}$

$\alpha \neq 0$ and $a_{CM} \neq 0$, so $(A) \to P$

For (B) $\alpha \neq 0$ but $a_{CM} \neq 0$, so $(B) \to S$

(C) $\to$ S

(D) $\alpha \neq 0$ and $a_{Cm} \neq 0$, so (D) $\to$ Q,S

SECTION – B

12. 2

Sol. $P = F\,V = mv\left(\dfrac{dv}{dt}\right) = \dfrac{3t^2}{2}$

$m\int_0^v v\,dv = \int_0^t \dfrac{3t^2}{2}dt$

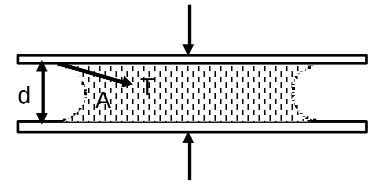
$\Rightarrow$ $V = 2\text{ ms}^{-1}$

13. 2

Sol. Let R be the radius of the circular layer of water.

Then, $\pi R^2 d \times \rho = m$ …(1)

Pressure at $A = p_0 - \dfrac{2T}{d}$ …(2)



Thus pressure between the plates is less than the atmospheric pressure and so the plates are pressed together as though attracted towards each other.

F, force of attraction = $\Delta p \times$ area $\Rightarrow F = \frac{2T}{d} \times \pi R^2$

$$\Rightarrow \quad F = \frac{2T}{d} \times \frac{m}{d \cdot \rho} = \frac{2Tm}{d^2 \cdot \rho} = \frac{2 \times 0.4 \times 10^{-3} \times 0.08}{0.01^2 \times 10^{-4} \times 1000} = 6.4 \text{ N}$$

14. 2

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}$; $q_1 + q_2 = 2Q_0$

$$C_1 = \frac{\varepsilon_0 A}{d_0 + vt};\ C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$$

$$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$$

$$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$$

$$q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$$

$$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 2 \text{ amp}$$

15. 5

Sol. For process AB $\quad T_A = 300 \text{ K},\ T_B = 600 \text{ K}$

$$W = nR\Delta T = nR(T_B - T_A) = 300\ nR = 600R.$$

$$Q = n\,Cp\,\Delta T = 2 \times \frac{5}{2} R\ (300) = 1500R.$$

For process BC $\quad W = nRT\ln\frac{v_f}{v_c} = nRT\ln\frac{p_i}{p_f} = nRT\ln 2 = 1200R\ln 2$

$$Q = W = 1200R\ln 2$$

For process CA $\quad W = \int P\,dV = \int_{600}^{300} \frac{K}{T}\frac{2nRT}{K}dT.$

$$= -2nR(300) = -1200R.$$

$$Q = nC_V\ \Delta T + W$$

$$= 2 \times \frac{3}{2}R(-300) - 1200R.$$

$$= -900R - 1200R = -2100R$$

$$\eta = \frac{600R + 1200R\ln 2 - 1200R}{1500R + 1200R\ln 2}$$

$$= 1 - \frac{21}{12\ln 2 + 15} \quad \Rightarrow \quad x = \frac{21}{4.2} = 5.$$

16. 8

Sol. $D = 1m,\ d = 5\times10^{-3}m$

$$\text{shift} = \frac{D}{d}\left[t_1(\mu_1-1)-t_2(\mu_2-1)\right]$$

$$= \frac{1}{5\times10^{-3}}\left[2(1.5-1)-1.5(1.4-1)\right]\times10^{-3} = \frac{0.4}{5} = 0.08\ m.$$

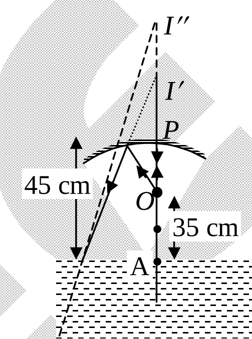
17. 1

Sol. For the reflection at the concave mirror,

u = – 10 cm; v = ?; f = – 15 cm

From the mirror formula, we have

$$v = \frac{uf}{u-f} = \frac{(-10)\times(-15)}{-10+15} = \frac{150}{5} = +30\text{ cm}$$



The positive sign indicates that the image is formed on the other side of the concave mirror,

Now, the image formed by the concave mirror serves as a virtual object for refraction at water surface which takes placed from air to water. So,

$$\mu = \frac{\text{Apparent height}}{\text{Real height}}$$

$\therefore$ AI″ = Apparent height = μ × real height = $\frac{4}{3}\times75$ = 100 cm.

Chemistry

PART – II

SECTION – A

18. C
Sol. Elimination takes place instead of substitution in (C).

19. B
Sol. $HF + NaF \longrightarrow NaHF_2$

20. B
Sol. Phenol is the most acidic among the given compounds.

21. D
Sol. Consider -I effect, back bonding & leaving tendency combinedly to decide the acidic strength.

22. BC
Sol. $3Ba(NH_2)_2 \xrightarrow{\Delta} Ba_3N_2 + 4NH_3$

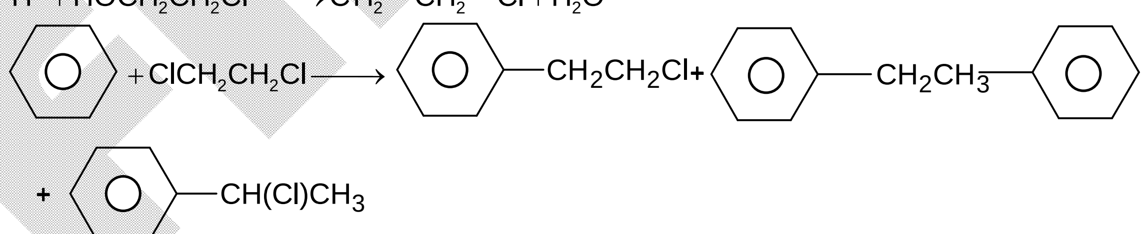
23. AB
Sol. For ideal gas Z = 1 at low pressure and Z > 1 at high pressure.

24. CD
Sol. Tetrapeptide represents 4 amino acid residues not 4 peptide bond. Fructose is a α - hydroxy ketone, therefore it give all the test of aldehyde

25. A
Sol. $2NaNO_3 \longrightarrow 2NaNO_2 + O_2$

$NaCl(aq) \xrightarrow{\text{Electrolysis}} H_2 + Cl_2 + NaOH$

26. C
Sol.
$H^+ + HOCH_2CH_2Cl \longrightarrow \overset{+}{C}H_2 - CH_2 - Cl + H_2O$



27. C

Sol.
$$\begin{array}{c} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \xrightarrow{Br_2/H_2O} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}$$

$$\xrightarrow{(CH_3CO)_2O} \begin{array}{c} CHO \\ | \\ (CHOCCOCH_3)_4 \\ | \\ CH_2OCOCH_3 \end{array}$$

$$\xrightarrow[\text{Red P}]{\text{Conc.HI}} CH_3CH_2CH_2CH_2CH_2CH_3$$

$$\xrightarrow[HCN]{} \begin{array}{c} CH(OH) \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}$$

28. B

Sol.
$$\underset{\text{(Brown colour)}}{NO_2} \xrightarrow{\text{Cooling}} \underset{\text{(Colourless)}}{N_2O_4}$$

$$\underset{\text{(Colourless)}}{2NO} + O_2 \longrightarrow \underset{\text{(Colourful)}}{2NO_2}$$

$$N_2O_3 \xrightarrow{\Delta} NO + NO_2$$

$$N_2O_5 \longrightarrow NO_2^+ + NO_3^-$$

SECTION – B

29. 5

Sol. pOH = 14 – pH = 14 – 12 = 2

$\therefore [OH^-] = 10^{-2}$

$\therefore 2s = 0.01$

$s = 5 \times 10^{-3}$

$$Ca(OH) \rightleftharpoons \underset{s}{Ca^{2+}} + \underset{2s}{2OH^-}$$

$\therefore K_{sp} = s(2s)^2 = 4s^3 = 4(5 \times 10^{-3})^3 = 5 \times 10^{-7}$

$= x \times 10^{-7}$

$\therefore\ x = 5$

30. 5

Sol. $$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_H} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5R}{36}$$

$$\therefore \lambda_H = \frac{36}{5R}$$

Similarly, $$\frac{1}{\lambda_{He^+}} = R(2)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 4R \times \frac{5}{36}$$

or, $\lambda_{He^+} = \dfrac{36}{20R}$

$\therefore \dfrac{\lambda_{He^+}}{\lambda_H} = \dfrac{36}{20R} \times \dfrac{5R}{36} = \dfrac{1}{4} = x : y$

$\therefore$ x + y = 5

31. 17
Sol. a = 4, b = 8, c = 1 & d = 4

32. 6
Sol. AB is a salt of weak acid and weak base.

$$pH = \frac{1}{2}\left[p^{K_w} + p^{K_a} - p^{K_b}\right] = \frac{1}{2}[14 + 6 - 8] = 6$$

33. 10
Sol. A is H_3PO_4, B is $H_4P_2O_7$, x = 5, y = 5

34. 31
Sol. x = 16, y = 14, z = 1

Mathematics

PART – III

SECTION – A

35. C

Sol. $(1-x)^n = \sum_{r=0}^{n} {}^nC_r (-1)^r x^r$

Or $x(1-x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^{r+1}$

Integrating both sides within the limits 0 to 1, we get $\int_0^1 x(1-x)^n dx = \sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{r+2}$

Or $\sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{r+2} = \int_0^1 x(1-x)^n dx$

$= \int_0^1 (1-x)x^n dx$ (Replace x by $1-x$)

$= \left.\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right|_0^1$

$= \frac{1}{n+1} - \frac{1}{n+2}$

$= \frac{1}{(n+1)(n+2)}$

Now put $n = 50$.

36. C

Sol. Let $x_n = \tan\theta_n$

Now, $\tan\theta_{n+1} = x_{n+1} = \frac{x_n}{1+\sqrt{1+x_n^2}}$

$= \frac{\tan\theta}{1+\sqrt{1+\tan^2\theta_n}}$

$\Rightarrow \tan\theta_{n+1} = \frac{\tan\theta_n}{1+\sec\theta_n}$

$= \frac{\sin\theta_n}{1+\cos\theta_n} = \tan\frac{\theta_n}{2}$

$\Rightarrow \theta_{n+1} = \frac{\theta_n}{2} \Rightarrow \theta_n = \frac{\theta_{n-1}}{2}$

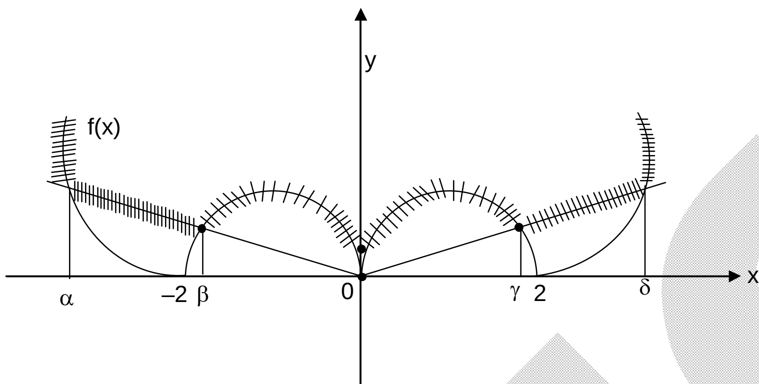
Now, $\theta_1 = \frac{\pi}{3} \Rightarrow \theta_n = \frac{\pi}{3.2^{n-1}}$

$\Rightarrow x_n = \tan\left(\frac{2\pi}{3.2^n}\right)$

$$\Rightarrow \lim_{n\to\infty} 2^n x_n = \lim_{n\to\infty} \frac{\tan\left(\frac{2\pi}{3.2^n}\right)}{\left(\frac{1}{2^n}\right)} = \frac{2\pi}{3}$$

37. B

Sol. $f(x)$ is non – differentiable at $x = \alpha, \beta, 0, \gamma, \delta$ and $g(x)$ is non – differentiable at $x = \alpha, \beta, 0, -2, 2, \gamma, \delta$



38. C

Sol. $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right)\right)$

$= \lim_{n\to\infty}\left(\sum_{r=1}^{n}\sin^{-1}\left(\frac{1}{\sqrt{r}}\sqrt{1-\frac{1}{r+1}}-\frac{1}{\sqrt{r+1}}\sqrt{1-\frac{1}{r}}\right)\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(\sin^{-1}\frac{1}{\sqrt{r}}-\sin^{-1}\frac{1}{\sqrt{r+1}}\right)$

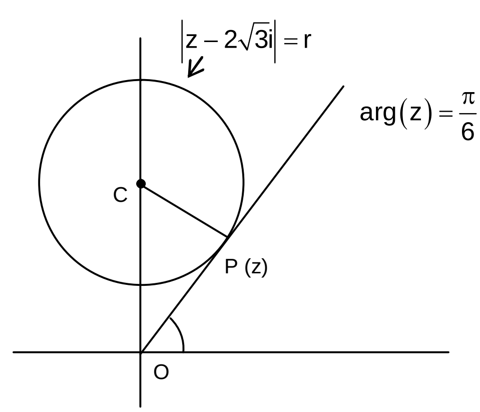
$= \lim_{n\to\infty}\left(\sin^{-1}1-\sin^{-1}\frac{1}{\sqrt{n+1}}\right) = \frac{\pi}{2}$

39. AD

Sol. $CP = r,\ OC = 2\sqrt{3}, \angle COP = \frac{\pi}{3}$

$\Rightarrow CP = OC\sin\frac{\pi}{3} = 2\sqrt{3}\,\frac{\sqrt{3}}{2} = 3$

Thus when $r = 3$, the circle touches the line. Hence, for two distinct points of intersection, $3 < r < 2\sqrt{3}$.



40. CD

Sol. $f(x)=\int_0^{\pi}\cos t\cos(x-t)dt$ ....(i)

$=\int_0^{\pi}-\cos t.\cos(x-\pi+t)dt$

$f(x)=\int_0^{\pi}\cos t.\cos(x+t)dt$ ....(ii)

On adding equations (i) and (ii), we get

$2f(x)=\int_0^{\pi}\cos t(2\cos x.\cos t)dt$

$2f(x)=2\cos x\int_0^{\pi/2}\cos^2 t\,dt$

$f(x)=\frac{\pi\cos x}{2}$ Now, verify.

41. AB

Sol. $I=\int_0^{\alpha}\frac{dx}{1-\cos\alpha\cos x}$

$=\int_0^{\alpha}\frac{dx}{\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)-\cos\alpha\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$

$\int_0^{\alpha}\frac{dx}{(1-\cos\alpha)\cos^2\frac{x}{2}+(1+\cos\alpha)\sin^2\frac{x}{2}}$

$=\int_0^{\alpha}\frac{dx}{2\sin^2\left(\frac{\alpha}{2}\right)\cos^2\frac{x}{2}+2\cos^2\left(\frac{\alpha}{2}\right)\sin^2\frac{x}{2}}$

$=\frac{1}{2}\int_0^{\alpha}\frac{\sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{\alpha}{2}\right)+\tan^2\left(\frac{x}{2}\right)}dx$

Putting $\tan\frac{x}{2}=t$, we get

$I=\int_0^{\tan\frac{\alpha}{2}}\frac{\sec^2\left(\frac{\alpha}{2}\right)dt}{t^2+\tan^2\left(\frac{\alpha}{2}\right)}$

$=\sec^2\frac{\alpha}{2}\cot\frac{\alpha}{2}\left[\tan^{-1}\left(\frac{t}{\tan\frac{\alpha}{2}}\right)\right]_0^{\tan\frac{\alpha}{2}}=\frac{2}{\sin\alpha}.\frac{\pi}{4}=\frac{\pi}{2\sin\alpha}$

Thus, $\frac{A}{\sin\alpha}+B=\frac{\pi}{2\sin\alpha}$ $\therefore A=\frac{\pi}{2}$ and $B=0$ or $A=\frac{\pi}{4}$ and $B=\frac{\pi}{4\sin\alpha}$

42. B

Sol. (P) $\because z^n - 1 = \prod_{r=0}^{n-1}(z - \alpha^r)$

$\Rightarrow \ln(z^n - 1) = \sum_{r=0}^{n-1} \ln(z - \alpha^r)$

Differentiate both sides w.r.t. z; then

$\Rightarrow \frac{nz^{n-1}}{z^n - 1} = \sum_{r=0}^{n-1} \frac{1}{(z - \alpha^r)}$

Put $z = 2$, then $\frac{n(2)^{n-1}}{2^n - 1} = \sum_{r=0}^{n-1} \frac{1}{(2 - \alpha^r)}$

$= \frac{1}{1} + \sum_{r=1}^{n-1} \frac{1}{(2 - \alpha^r)}$

$\therefore \sum_{r=1}^{n-1} \frac{1}{(2 - \alpha^r)} = \frac{n(2)^{n-1}}{2^n - 1} - 1$

$= \frac{n(2)^{n-1} - 2^n + 1}{(2^n - 1)}$

$= \frac{(n-2)2^{n-1} + 1}{(2^n - 1)}$

(Q) $\because z^n - 1 = \prod_{r=0}^{n-1}(z - \alpha^r)$

$(z^n - 1) = (z - 1)\prod_{r=1}^{n-1}(z - \alpha^r)$

Or $\frac{(z^n - 1)}{(z - 1)} = \prod_{r=1}^{n-1}(z - \alpha^r)$ ....(i)

Put $n = 5$ and $z = -\omega$, then $\frac{-\omega^5 - 1}{-\omega - 1} = \prod_{r=1}^{4}(-\omega - \alpha^r)$

$\frac{\omega^2 + 1}{\omega + 1} = \prod_{r=1}^{4}(\omega + \alpha^r)$

Or $\frac{-\omega}{-\omega^2} = \prod_{p=1}^{4}(\omega + \alpha^p)$ ....(ii)

i.e. $\prod_{p=1}^{4}(\omega + \alpha^p) = \frac{1}{\omega}$

Also in (i) put $z = \omega^2, n = 8, r = q$ then $\frac{\omega^{16} - 1}{\omega^2 - 1} = \prod_{q=1}^{7}(\omega^2 - \alpha^q)$

$\Rightarrow \frac{1}{\omega + 1} = \prod_{q=1}^{7}(\omega^2 - \alpha^q)$

$\Rightarrow -\frac{1}{\omega^2} = \prod_{q=1}^{7}(\omega^2 - \alpha^q)$ ....(iii) from (ii) and (iii),

$$\frac{\prod_{p=1}^{4}\left(\omega+\alpha^{p}\right)}{\prod_{q=1}^{7}\left(\omega^{2}-\alpha^{q}\right)}=\frac{\frac{1}{\omega}}{-\frac{1}{\omega^{2}}}=-\omega=-\left(\frac{-1+i\sqrt{3}}{2}\right)$$

$$=\frac{1-i\sqrt{3}}{2}$$

(R) According to question $\sum_{r=0}^{n-1}\frac{\bar{a}(\alpha)^{r}+a(\bar{\alpha})^{r}+b}{|\bar{a}|+|a|}$

$$\Rightarrow \sum_{r=0}^{n-1}\frac{\bar{a}(\alpha)^{r}+a(\bar{\alpha})^{r}+b}{2|a|}$$

$$\Rightarrow \frac{1}{2|a|}\left\{\bar{a}\sum_{r=0}^{n-1}(\alpha)^{r}+a\sum_{r=0}^{n-1}(\bar{\alpha})^{r}+b\sum_{r=0}^{n-1}1\right\}$$

$$=\frac{1}{2|a|}(0+0+nb)$$

$=\frac{nb}{2|a|}$ ($\because$ sum of n, nth roots of unity is zero)

(S) $\because \alpha^{7}=1$ and $1+\alpha+\alpha^{2}+\alpha^{3}+\alpha^{4}+\alpha^{5}+\alpha^{6}=0$

$\Rightarrow 1+\left(\alpha+\alpha^{2}+\alpha^{4}\right)+\left(\alpha^{3}+\alpha^{5}+\alpha^{6}\right)=0$

Or $\left(\alpha+\alpha^{2}+\alpha^{4}\right)+\left(\alpha^{3}+\alpha^{5}+\alpha^{6}\right)=-1$ and $\left(\alpha+\alpha^{2}+\alpha^{4}\right)\left(\alpha^{3}+\alpha^{5}+\alpha^{6}\right)$

$=\alpha^{4}+\alpha^{6}+\alpha^{7}+\alpha^{5}+\alpha^{7}+\alpha^{8}+\alpha^{7}+\alpha^{9}+\alpha^{10}$

$=\alpha^{4}+\alpha^{6}+1+\alpha^{5}+1+\alpha+1+\alpha^{2}+\alpha^{3}$

$=2+\left(1+\alpha+\alpha^{2}+\alpha^{3}+\alpha^{4}+\alpha^{5}+\alpha^{6}\right)$

$=2+0=2$

Then required equation is $z^{2}-(-1)z+2=0$ i.e. $z^{2}+z+2=0$

43. D

Sol. (P) $\because 16m^{2}=8l+1$

$\Rightarrow 16\left(l^{2}+m^{2}\right)=16l^{2}+8l+1$

$=(4l+1)^{2}$

Or $4\sqrt{\left(l^{2}+m^{2}\right)}=|4l+1|$

Or $\frac{|4l+1|}{\sqrt{\left(l^{2}+m^{2}\right)}}=4$

$\therefore$ Centre $\equiv(4, 0)$ and radius $\equiv 4$

Equation of circle is $(x-4)^{2}+(y-0)^{2}=4^{2}$

$\Rightarrow \quad x^{2}+y^{2}-8x=0$

(Q) $\because 9l^2 + 16m^2 + 1 + 24lm + 6l + 8m$
$= 25\left(l^2 + m^2\right)$
$\Rightarrow (3l + 4m + 1)^2 = 25\left(l^2 + m^2\right)$
$\Rightarrow \dfrac{|3l + 4m + 1|}{\sqrt{\left(l^2 + m^2\right)}} = 5$
$\therefore$ Centre $\equiv (3, 4)$ and radius $\equiv 5$
$\Rightarrow$ Equation of circle is $(x-3)^2 + (y-4)^2 = 5^2 = 25$
$\therefore$ Equation of director circle is $(x-3)^2 + (y-4)^2 = 50$
or $x^2 + y^2 - 6x - 8y = 25$

(R) $\because 4l^2 - 5m^2 + 6l + 1 = 0$
$\Rightarrow 4l^2 + 6l + 1 = 5m^2$
$\Rightarrow 9l^2 + 6l + 1 = 5\left(l^2 + m^2\right)$
$\Rightarrow (3l + 1)^2 = 5\left(l^2 + m^2\right)$
$\Rightarrow \dfrac{|3l + 1|}{\sqrt{\left(l^2 + m^2\right)}} = \sqrt{5}$
$\therefore$ Centre $\equiv (3, 0)$ and radius $\equiv \sqrt{5}$

(S). $\because x + 2y - 3 = 0$ and $2x + y - 3 = 0$ are non parallel
Bisector of lines $\dfrac{(x + 2y - 3)}{\sqrt{5}} = \pm\dfrac{(2x + y - 3)}{\sqrt{5}}$
$\Rightarrow (x + 2y - 3) = (2x + y - 3)$
$\Rightarrow x - y = 0$ ……..(i)
and $(x + 2y - 3) = -(2x + y - 3)$
$\Rightarrow x + y = 2$ …….(ii)
$\because$ Centre of circle lies on angle bisector of lines.
Also centre lies on a line $3x + 4y = 5$ ……..(iii)
Solving (i) and (iii), we get $\left(\dfrac{5}{7}, \dfrac{5}{7}\right)$ and solving equation (ii) and (iii), we get $(3, -1)$

44. C
Sol. (P) Consider CC as single object, U, CC, E can be arranged in $3!$ ways
$\times U \times CC \times E \times$
Now the three Ss are to be placed in four available places.
Hence required number of ways $= {}^4C_3 \times 3! = 24$

(Q) Let us first find the words in which no two Ss are together.
(i) Arrange the remaining letters $= \dfrac{4!}{2!} = 12$ ways
(ii) $\times U \times C \times C \times E \times$
There are five available places for three SSS.

Hence, total number of ways no two Ss together
$z = 12 \times {}^5C_3 = 120$
$\therefore$ Hence, number of words having CC separated and SSS separated $= 120 - 24 = 96$.

(R) Total number of ways $= \frac{7!}{2!3!} = 420$
consonants in SUCCESS are S, C, C, S, S
Number of ways arranging consonants $= \frac{5!}{2!3!} = 10$
Hence, number of words in which the consonants appear in alphabetic order $= \frac{420}{10} = 42$

(S) The alphabetic order is C, E, S, U. The number of words beginning with C is $\frac{6!}{3!} = 120$ ways and those beginning with E is $\frac{6!}{2!3!} = 60$ ways
Then come words beginning with SC, numbering $\frac{5!}{2!} = 60$, SE, numbering $\frac{5!}{2!2!} = 30$ and SS, numbering $\frac{5!}{2!} = 60$.
After which come the word SUCCESS.
Thus the rank of SUCCESS is $120 + 60 + 60 + 30 + 60 + 1 = 331$

45. A

Sol. (P) $\because \frac{dT}{dt} = -k(T - 290)$

$\Rightarrow \frac{dT}{(T-290)} = -k\ dt$

$\Rightarrow \ln(T - 290) = -kt + c$ ....(i)

Initially, T = 370K and t = 0, then $\ln(80) = c$

From equation (i) $\ln(T - 290) = -kt + \ln 80$

$\ln\left(\frac{T-290}{80}\right) = -kt$

$\Rightarrow \frac{T-290}{80} = e^{-kt}$

Or $T = 290 + 80e^{-kt}$

(Q) For $t = 10$ min, $T = 330$

Then, $330 - 290 = 80e^{-10k}$

$\Rightarrow \frac{1}{2} = e^{-10k}$

$\Rightarrow e^{10k} = 2$

$\Rightarrow 10k = \ln 2$

Or $k = \frac{\ln 2}{10}$

(R) At $T = 295K$

$295 - 290 = 80e^{-kt}$

$\Rightarrow 16 = e^{kt}$

$\Rightarrow\ 4\ln 2 = kt = \dfrac{\ln 2}{10}\times t$ $\left(\because k = \dfrac{\ln 2}{10}\right)$

$\therefore\ t = 40$ min

(S) $\because T = f(t) = 290 + 80e^{-kt}$

$\therefore\ f\left(-\dfrac{2}{k}\right) = 290 + 80e^{2}$

SECTION – B

46. 4

Sol. Given that $AA^T = 4I$

$\Rightarrow |A|^2 = 4$

Or $|A| = \pm 2$

So $A^T = 4A^{-1} = 4\dfrac{\text{adj}\,A}{|A|}$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} = \frac{4}{|A|}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

Now, $a_{ij} = \dfrac{4}{|A|}c_{ij}$

$\Rightarrow -2c_{ij} = \dfrac{4}{|A|}c_{ij}\ \left(\text{as } a_{ij} + 2c_{ij} = 0\right)$

$\Rightarrow |A| = -2$

Now $|A + 4I| = \left|A + AA^T\right|$

$= |A|\left|I + A^T\right|$

$= -2\left|(I + A)^T\right|$

$= -2|I + A|$

$\Rightarrow |A + 4I| + 2|A + I| = 0,$

So on comparing, we get $5\lambda = 2 \Rightarrow \lambda = \dfrac{2}{5}$

Hence, $10\lambda = 4$

47. 1

Sol. Put $x = 1$ in $2f(x) = f(xy) + f\left(\dfrac{x}{y}\right)$ ……..(i)

$2f(1) = f(y) + f\left(\dfrac{1}{y}\right)$ ……..(ii)

$\Rightarrow f(y) = -f\left(\dfrac{1}{y}\right)$

Replacing x by y and y by x in equation (i), we get

$2f(y) = f(yx) + f\left(\frac{y}{x}\right)$ ....(iii)

Equation (i) and (iii),

$2\{f(x) - f(y)\} = f\left(\frac{x}{y}\right) - \left\{-f\left(\frac{x}{y}\right)\right\} = 2f\left(\frac{x}{y}\right)$ ....(iv)

$f(x) - f(y) = f\left(\frac{x}{y}\right)$ ....(v)

$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = f'(1) = 1,\qquad \lim_{h\to 0}\frac{f(1+h)}{h} = 1,$ as $f(1) = 0$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)}{h} = \frac{1}{x}$

$f(x) = \log|x| + c$

$f(1) = 0 \Rightarrow c = 0$

$f(e) = 1$

48. 8

Sol. $I = \int (x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3}dx$

$= \int x^2(x^6 + x^3 + 1)(2x^9 + 3x^6 + 6x^3)^{1/3}dx$

Put $2x^9 + 3x^6 + 6x^3 = t,\ 18(x^8 + x^5 + x^2)dx = dt$

$\Rightarrow 18x^2(x^6 + x^3 + 1)dx = dt$

$\therefore I = \frac{1}{24}(2x^9 + 3x^6 + 6x^3)^{4/3} + K \qquad \therefore \frac{AB}{4} = 8$

49. 60

Sol. Total number of ways in which papers of 4 students, can be checked by seven teachers $= 7^4$

Now, choosing two teachers out of 7 is ${}^7C_2 = 21$

The number of ways in which 4 papers can be checked by exactly two teachers $= 2^4 - 2 = 14$

$\therefore$ Favourable ways $= (21)(14)$

$\therefore$ Required probability $= \frac{(21)(14)}{7^4}$

$= \frac{6}{49}$

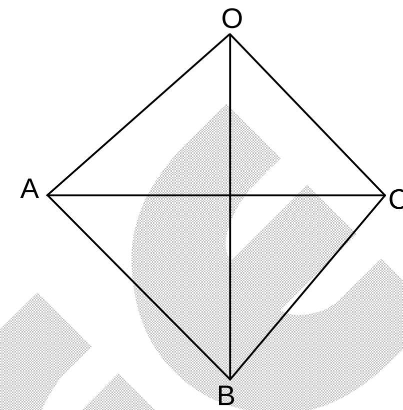
$= A$

$\therefore 490A = 490 \times \frac{6}{49} = 60$

50. 1

Sol. $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$

$\angle AOB = \frac{\pi}{3}$



$\therefore \left[\vec{a} \ \ \vec{b} \ \ \vec{c}\right]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$= \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4 \therefore \left[\vec{a} \ \ \vec{b} \ \ \vec{c}\right] = 2$

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 1 \Rightarrow \left(\vec{b} - \vec{c}\right).\vec{a} = 0 \Rightarrow \vec{b} - \vec{c} \perp \vec{a}$

$\left|\left(\vec{b} - \vec{c}\right) \times \vec{a}\right| = \sqrt{2} \times \sqrt{2} = 2$, since $\left|\vec{b} - \vec{c}\right| = BC = \sqrt{2}$

The equation of OA and BC are given by

$\vec{r} = s\vec{a}, \vec{r} = \vec{b} + t\left(\vec{b} - \vec{c}\right)$

The shortest distance is

$$\frac{\left|\vec{b}.\left(\left(\vec{b} - \vec{c}\right) \times \vec{a}\right)\right|}{\left|\left(\vec{b} - \vec{c}\right) \times \vec{a}\right|} = \frac{\left|\left[\vec{a} \ \ \vec{b} \ \ \vec{c}\right]\right|}{2} = \frac{2}{2} = 1$$

51. 1

Sol. Length of latus rectum

$= 2a\left(1 - e^2\right) = \frac{32}{5} \Rightarrow a\left(1 - \frac{9}{25}\right) = \frac{16}{5}$

$\therefore a = 5, b = a\sqrt{1 - e^2} = 5\sqrt{1 - \frac{9}{25}} = 4$

The ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

The normal at $(5\cos\theta, 4\sin\theta)$ is $\frac{5x}{\cos\theta} - \frac{4y}{\sin\theta} = 5^2 - 4^2 = 9$

The distance of the centre from it is $d = \frac{9}{\sqrt{\frac{25}{\cos^2\theta} + \frac{16}{\sin^2\theta}}}$

$= \frac{9}{\sqrt{25\sec^2\theta + 16\operatorname{cosec}^2\theta}} = \frac{9}{\sqrt{41 + 25\tan^2\theta + 16\cot^2\theta}}$

But $25\tan^2\theta + 16\cot^2\theta \ge 2.5.4 = 40$

by A.M. – G.M. inequality.

$\therefore d \le \frac{9}{\sqrt{41 + 40}} = 1$