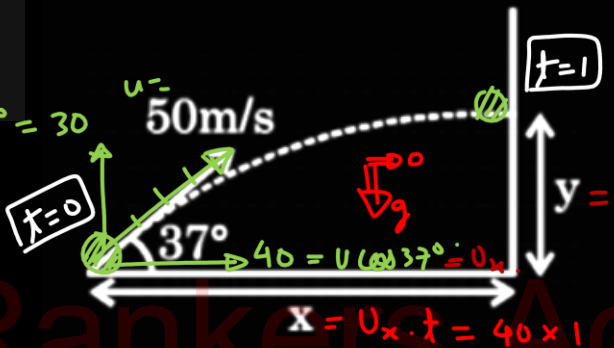

PHYSICS
Rankers Academy JEE

7

A particle is projected as shown in figure. If the particle strikes at the wall after 1 sec , then the ratio $\frac{x}{y}$ will be (approximately)



$U_y = U\sin 37° = 30$

$40 = U\cos 37° = U_x$

$x = U_x \cdot t = 40 \times 1$

$x = 40$

$y = U_y t + \frac{1}{2} a_y t^2 = 30(1) + \frac{1}{2}(-10)(1)^2 = 25$

$y = 25$

(A) 1 (B) 1.6

(C) 2.6 (D) 3

$\frac{x}{y} = \frac{40}{25} = \frac{8}{5} = 1 + \frac{3}{5}$

$\frac{x}{y} = 1.6$

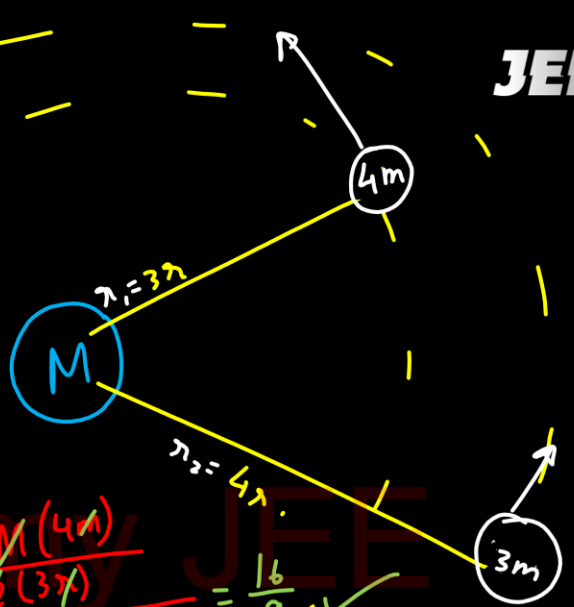
2

Two satellite A and B, having masses in the ratio of 4: 3, are revolving in circular orbit of radii 3r and 4r respectively around the earth the ratio of total mechanical energy of A to B is

(A) 9: 16 (B) 16: 9

(C) 1: 1 (D) 4: 3

$$TE = -\frac{GMm}{2r}$$



$$\frac{TE_1}{TE_2} = \frac{-\frac{GM(4m)}{2(3r)}}{-\frac{GM(3m)}{2(4r)}} = \frac{16}{9}$$

3

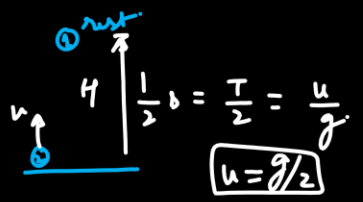
A juggler throws ball vertically upwards with same initial velocity in air. When the first ball reaches its highest positions, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is

(A) $g/3n^2$ (B) $2gn$

(C) g/n^2 (D) $g/2n^2$

$T = \frac{2u}{g}$ $H = \frac{u^2}{2g}$

M-1 $n = 2$ ☺ ✓ JEE 1



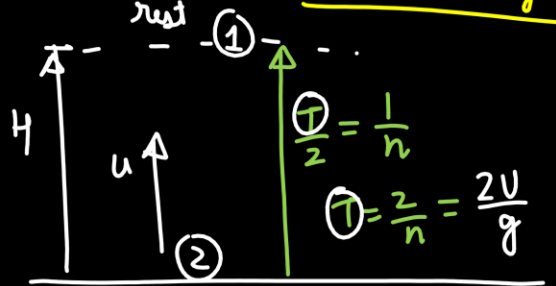
$H = \frac{u^2}{2g}$

$H = \frac{g^2/4}{2g} = g/8$

M-2

'n' balls in 1 sec

1 ball every '1/n' sec



$H = \frac{u^2}{2g} = \frac{g^2/n^2}{2g} = \frac{g}{2n^2}$

4

A nucleus with mass number 220 initially at rest emits an α-particle. If the Q-value of the reaction is 5.5 MeV, the kinetic energy of the α-particle is (in MeV)

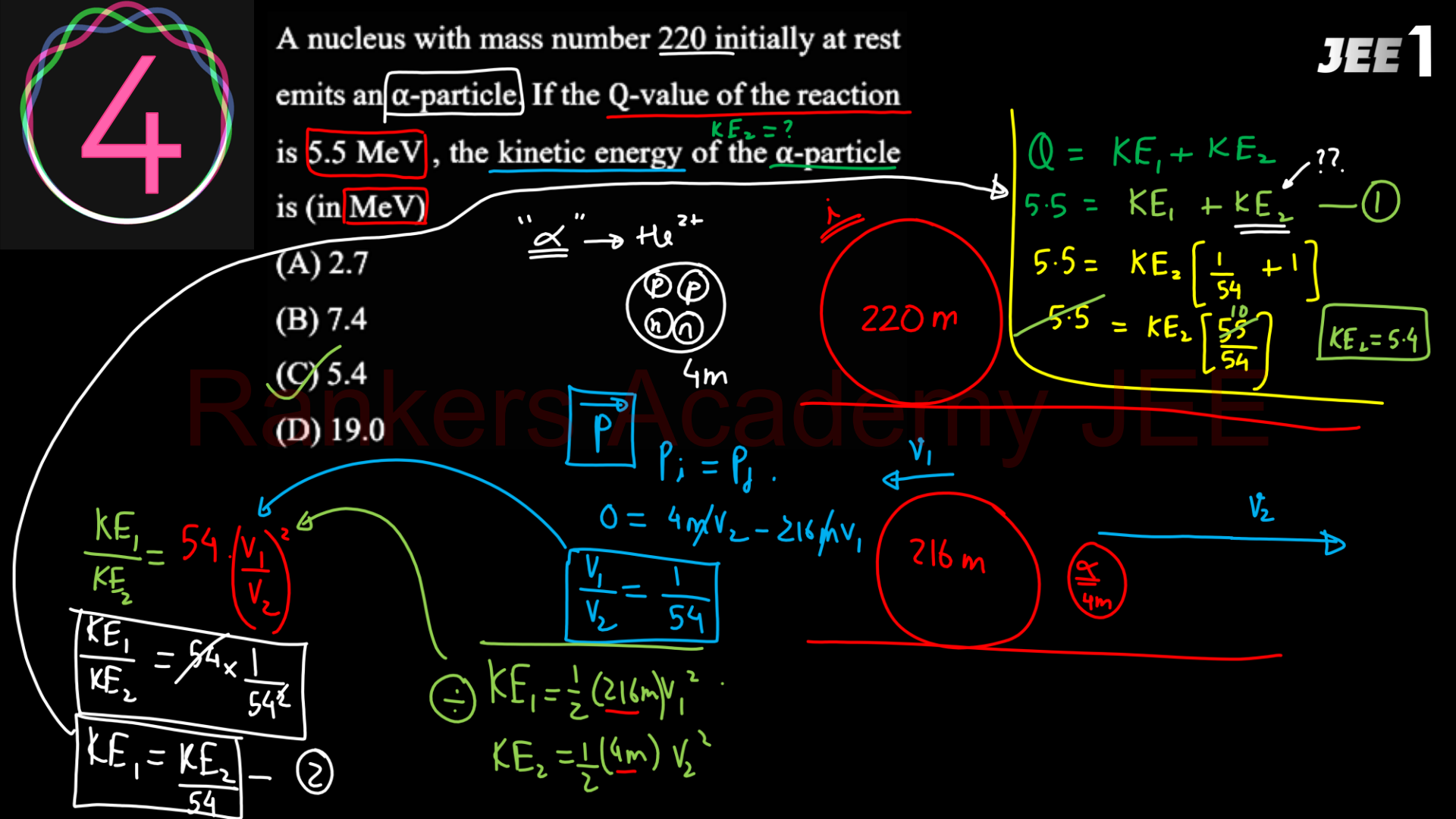
(A) 2.7

(B) 7.4

(C) 5.4

(D) 19.0

JEE 1



5

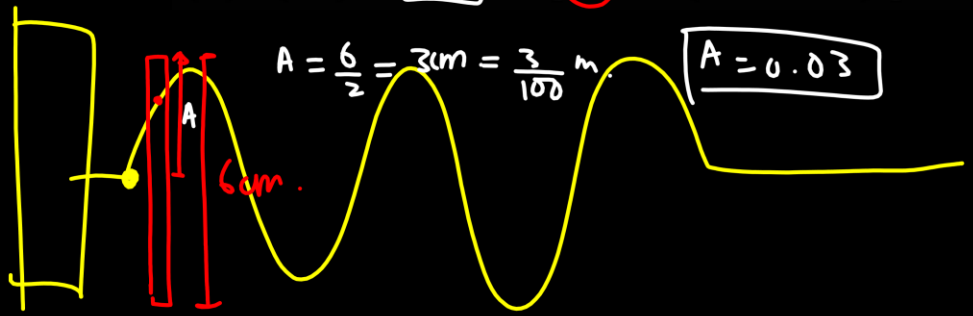
A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave?

(A) $Y(x,t) = 0.03 \sin [5.1x - (0.2 \times 10^3)t]$

(B) $Y(x,t) = 0.06 \sin [5.1x - (1.5 \times 10^3)t]$

(C) $Y(x,t) = 0.06 \sin [0.8x - (0.5 \times 10^3)t]$

(D) $Y(x,t) = 0.03 \sin [5.1x - (1.5 \times 10^3)t]$



$A = \frac{6}{2} = 3cm = \frac{3}{100} m.$

$A = 0.03$

$+x$

$y = A \sin(\omega t - kx)$ or $y = A \sin(kx - \omega t)$

$k = 2\pi$

$v = \nu \lambda$

$\frac{v}{\nu} = \lambda$

$\lambda = \frac{}{}$

$k = \frac{}{}$

$\omega = 2\pi \nu$

$\omega = 2(3.14)(245)$

$\omega = 490 \times 3.14 \simeq 1538.6$

$\omega \simeq 1.5 \times 10^3$

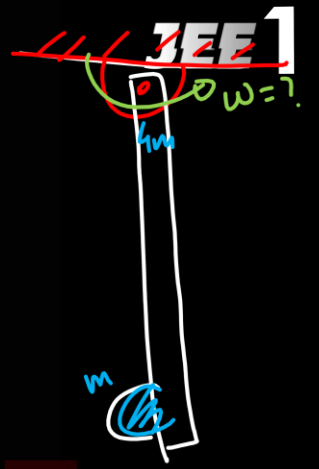
JEE 1

6

A rod of mass ~~20~~ 4m kg & length 10 m is hinged at A & hanging vertically. A bullet of mass ~~5~~ m kg moving with velocity 10 m/s strikes to one end of rod and stick to it. Find angular velocity of rod just after the collision.

(A) $\frac{7}{3}$ rad/sec (B) $\frac{3}{7}$ rad/sec

(C) $\frac{3}{10}$ rad/sec (D) 3rad/sec



$L_0 = 0$

L_0 → conserved.

$$mu\,L = \left[\frac{1}{3}(4m)L^2 + mL^2\right]\omega$$

$$\frac{u}{L} = \frac{7}{3}\omega$$

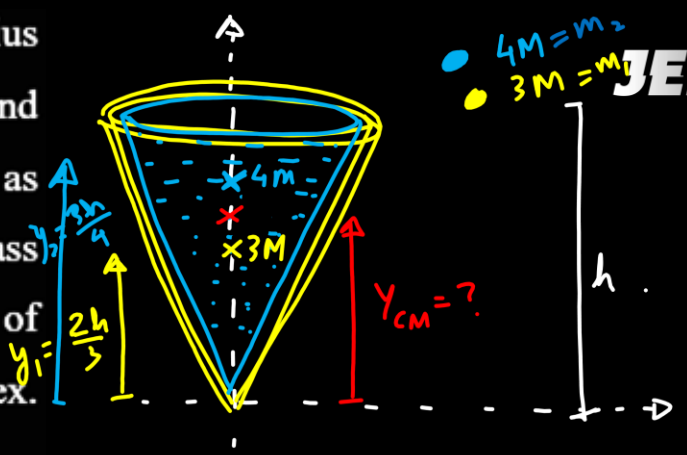
$$\omega = \frac{3u}{7L} = \frac{3}{7}\ \text{rad/sec.}$$

7

An ice cream cone of mass 3 M has base radius R and height h. Assume its wall to be thin and uniform. When ice cream is filled inside it (so as to occupy the complete conical space) its mass becomes 7 M. Find the distance of the centre of mass of the ice cream filled cone from its vertex.

(A) $\frac{3h}{4}$ (B) $\frac{2h}{3}$

(C) $\frac{5h}{7}$ (D) $\frac{6h}{7}$

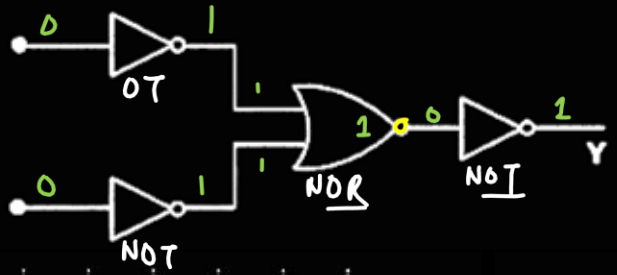


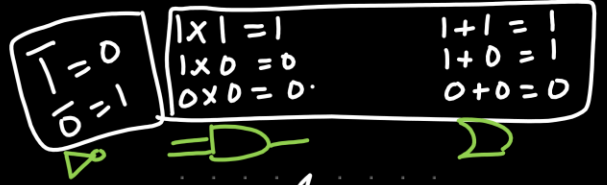
$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

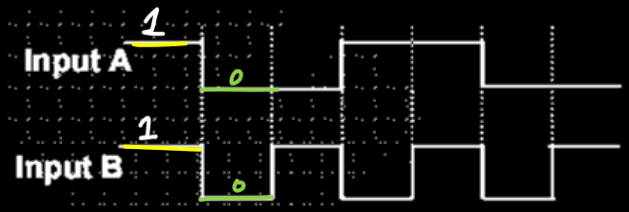
$$Y_{CM} = \frac{3M\left(\frac{2h}{3}\right) + 4M\left(\frac{3h}{4}\right)}{7M} = \frac{5h}{7}$$

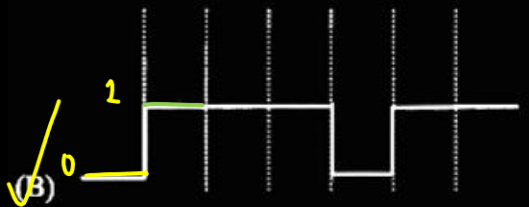
8

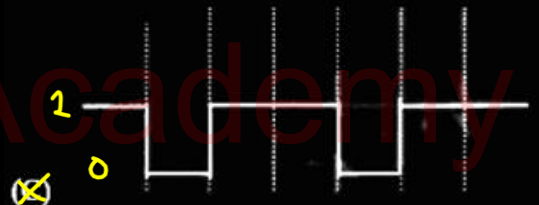
The logic circuit as shown has the input wave forms ' A ' and ' B ' as shown in the figure. Pick out the correct output waveform.

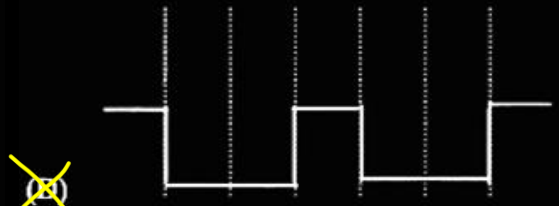






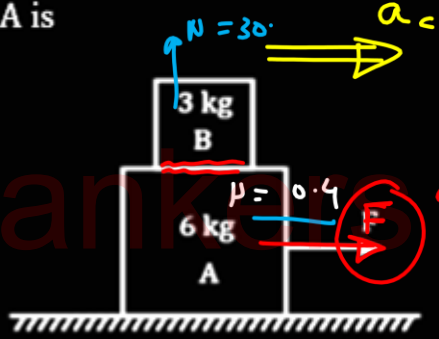






9

Two blocks A and B of masses 6 kg and 3 kg rest on a smooth horizontal surface as shown in adjoining Fig. If coefficient of friction between A and B is 0.4 , the maximum horizontal force which can make them move without B sliding on A is



(A) 72 N (B) 40 N

(C) 36 N (D) 20 N

μN.

$f_{max} = 12$

a_c

3kg

$f_{max} = 12$

$12 = 3\,a_c$

$a_c = 4$ ← max.

$a_c = 4$

$f_{max} = 12$

6kg.

F.

$F - 12 = 6 \times 4$

$F = 36$

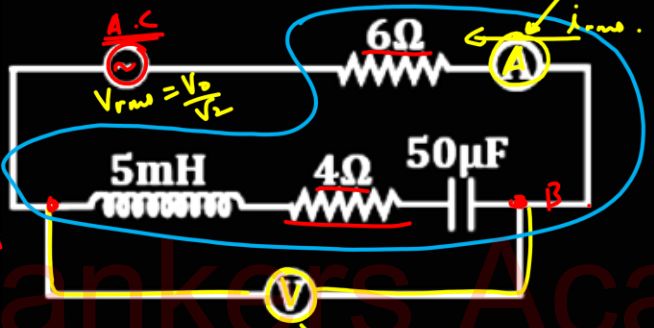
4

9kg

$F = 36$

10

In the circuit shown in the figure, the ac source gives a voltage $V = 20\cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter reading will be ($\sqrt{2} \approx 1.4$)



(A) 0 V, 0.47 A

(B) 1.68 V, 0.47 A

(C) 0 V, 1.4 A

(D) 5.6 V, 1.4 A

LCR ckt.

$V = 20\cos(\omega t)$, $V_0 = 20$

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$\omega = 2000$

reading i_{rms}

$R = 10$

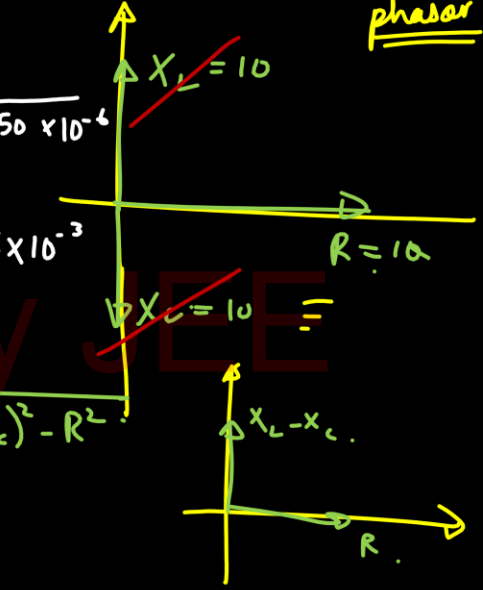
$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}}$

$X_C = 10$

$X_L = \omega L = 2000 \times 5 \times 10^{-3}$

$X_L = 10$

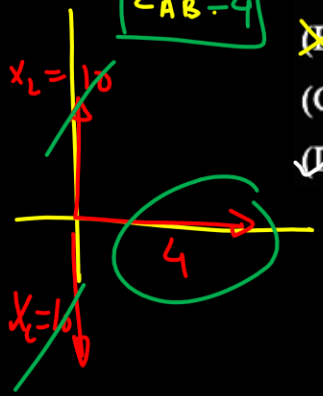
phasor (2)



$Z = \sqrt{(X_L - X_C)^2 - R^2}$

$Z = 10$

$Z_{AB} = 4$



reading of B

$V_{AB} = i_{rms} \cdot Z_{AB} = 1.4 \times 4 = 5.6$ V

reading (A)

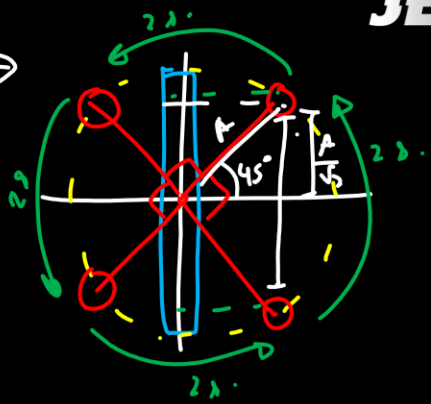
$i_{rms} = \frac{V_{rms}}{Z} = \frac{20}{\sqrt{2} \times 10} = \sqrt{2} = 1.4$ A

11

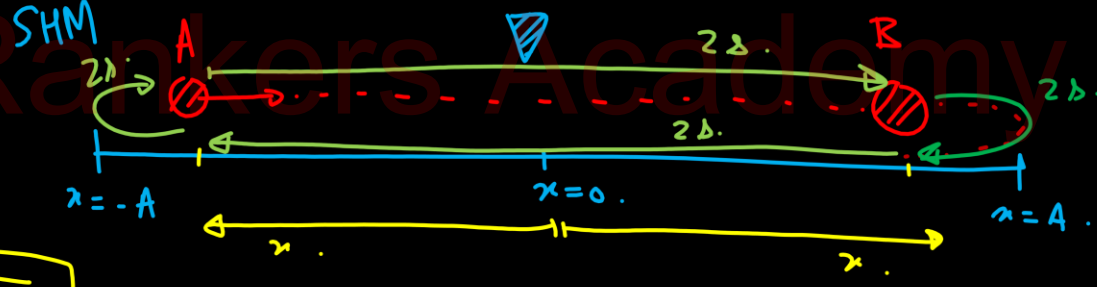
The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2s. After another 2s it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8

(C) 6 (D) 4



$T = 8s$



$V = \omega\sqrt{A^2 - x^2}$

12

If the ratio $C_p/C_v = \gamma$, the change in internal energy of the given gas when the volume changes from V to 2 V at constant pressure P is

(A) $\frac{R}{(\gamma-1)}$ (B) PV

(C) $\frac{PV}{(\gamma-1)}$ (D) $\frac{\gamma PV}{(\gamma-1)}$

$Q = \Delta U + W$

JEE 1

$\frac{C_p}{C_v} = \gamma$

$C_v = f\frac{R}{2} = \frac{R}{\gamma-1}$

$C_p = \frac{\gamma R}{\gamma-1}$

$\Delta U = nC_v\Delta T$

$\Delta U = \frac{nR\Delta T}{\gamma-1}$

$\Delta U = \frac{PV}{\gamma-1}$

$PV_i = nRT_i$ ⊖

$PV_f = nRT_f$

$P\Delta V = nR\Delta T$

$P(2V-V) = nR\Delta T$

13

Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50 . Further, it is found that the screw gauge has a zero error of -0.03 mm . While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35 . The diameter of the wire is (in mm)

(A) 1.38
(B) 2.38
(C) 3.38
(D) 2.54

pitch = $\frac{1\text{mm}}{2} = 0.5\text{mm}$ — (1)

L.C. = $\frac{\text{pitch}}{\text{No. of CSD}}$ = $\frac{0.5}{50}\text{mm} = 0.01\text{mm}$ — (2) *

Reading = MSR + CCSD × L.C. − zero error *

diameter = 3mm + 35 × 0.01mm − (−0.03mm)

= 3.38mm

JEE 1

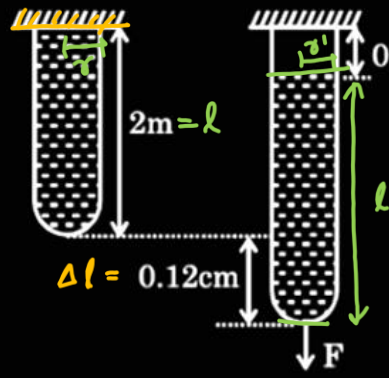
14

A uniform glass tube of length 2 m is closed at the lower end. It is completely filled with water and rigidly fixed at upper end. Now the tube is stretched downwards. Although the length of tube increases by 0.12 cm , the length of water column increases only by 0.08 cm .

(Assume water is incompressible). What is poisson's ratio σ for glass? Find $\frac{1}{\sigma}$.

(A) 1/3 (B) 3

(C) 1/2 (D) 2



JEE 1

$\ell' = 2m + 0.08cm$
$= 2.0008\ m$
$= 200.08\ cm$ — (1)

$V = \pi r^2 \ell = \pi r'^2 \ell'$

$\frac{r'}{r} = \sqrt{\frac{\ell}{\ell'}} = \sqrt{\frac{200\ cm}{200.08\ cm}}$

$\frac{r'}{r} - 1 = (1.0004)^{-\frac{1}{2}} - 1$

$\frac{\Delta r}{r} = 1 - \frac{1}{2} \times .0004 - 1$

$\frac{\Delta r}{r} = -0.0002$ — (2)

$\sigma = \frac{\varepsilon_{Lateral}}{\varepsilon_{Longitudinal}} = \frac{-\frac{\Delta r}{r}}{\frac{\Delta \ell}{\ell}}$ *

14

JEE 1

A uniform glass tube of length 2 m is closed at the lower end. It is completely filled with water and rigidly fixed at upper end. Now the tube is stretched downwards. Although the length of tube increases by 0.12 cm , the length of water column increases only by 0.08 cm .

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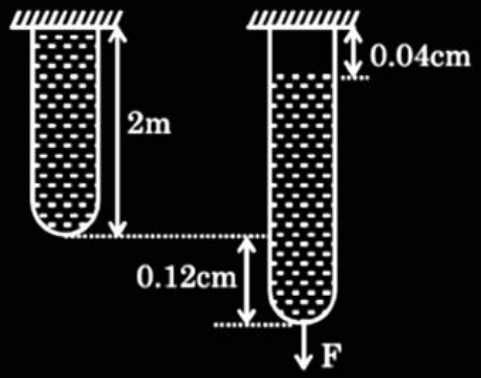


$\ell = 2m = 200cm$

$\Delta\ell = 0.12cm$

Hence $\frac{\Delta\ell}{\ell} = \frac{0.12}{200}$ (3)

$\sigma = \frac{-\Delta r/r}{\Delta\ell/\ell}$

$= \frac{+0.0002}{0.12/200}$

$\sigma = \frac{0.04}{0.12} = \frac{1}{3} \Rightarrow \frac{1}{6} = 3$

15

JEE 1

A plane Electromagnetic Wave travelling along the positive X-direction has a wavelength of 3 mm . The variation in the electric field occurs in the Y-direction with an amplitude 66Vm^{-1}. The equations for the electric and magnetic fields as a function of x and t are respectively

(A) $E_y = 66\cos\pi \times 10^{11}\left(t - \frac{x}{c}\right)$,

$B_z = 1.1 \times 10^{-7}\cos\pi \times 10^{11}\left(t - \frac{x}{c}\right)$

(B) $E_y = 11\cos 2\pi \times 10^{11}\left(t - \frac{x}{e}\right)$,

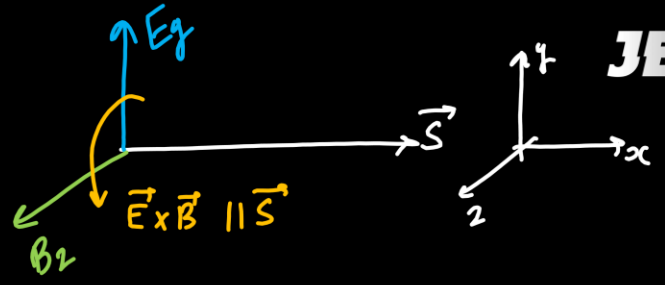
$B_y = 11 \times 10^{-7}\cos 2\pi \times 10^{11}\left(t - \frac{x}{c}\right)$

(C) $E_x = 66\cos\pi \times 10^{11}\left(t - \frac{x}{c}\right)$,

$B_x = 2.2 \times 10^{-7}\cos\pi \times 10^{11}\left(t - \frac{x}{c}\right)$

(D) $E_y = 66\cos 2\pi \times 10^{11}\left(t - \frac{x}{c}\right)$,

$B_z = 2.2 \times 10^{-7}\cos 2\pi \times 10^{11}\left(t - \frac{x}{c}\right)$



$\lambda = 3\times10^{-3}\,m$

$\omega = 2\pi f = 2\pi\left(\frac{c}{\lambda}\right) = 2\pi \times \frac{3\times10^{8}}{3\times10^{-3}} = 2\pi\times10^{11}$ —— (1)

$E_y = E_0\cos\omega\left(t - \frac{x}{c}\right) = 66\,V/m\,\cos 2\pi\times10^{11}\left(t - \frac{x}{c}\right)$

Similarly $B_z = B_0(\;\;\;\;)$

$= \frac{E_0}{c}(\;\;\;\;) = \frac{66}{3\times10^{8}}\cos 2\pi\times10^{11}(\;\;\;\;)$

$= 2.2\times10^{-7}$ ∥

16

JEE 1

A combination of 0.250 kg of water at 20.0°C, 0.400 kg of aluminium at 26.0°C and 0.100 kg of copper at 100°C is mixed in an insulated container and allowed to come to thermal equilibrium. Ignore any energy transfer to or from the container and determine the final temperature of the mixture.

($s_{Water} = 4186\ J\ kg^{-1}\ °C^{-1}$, $s_{Aluminium} = 900\ J\ kg^{-1}\ °C^{-1}$, $s_{Copper} = 387\ J\ kg^{-1}\ °C^{-1}$)

(A) 23.6°C (B) 40°C

(C) 20°C (D) 26°C

Calorimeter

$Q_{given} = Q_{accept}$

$Q_{Cu} = Q_{water} + Q_{Al}$

$m_{Cu} s_{Cu} (100-T) = m_w s_w (T-20) + m_{Al} s_{Al} (T-26)$

$\Rightarrow 0.1 \times 387 \times (100-T) = 0.25 \times 4186 (T-20) + 0.4 \times 900 (T-26)$

$1445.2\,T = 34180$

$T = 23.64\,°C$

17

JEE 1

The units of $\frac{CV}{\rho\varepsilon_0}$ is the same as that of

(C :capacitance; V: Voltage; ρ: Specific resistance; ε_0: Absolute permittivity)

(A) Charge

(B) Current

(C) Time

(D) Frequency

$Q = CV$

$\frac{Q}{\varepsilon_0} = EA = \phi$ (flux) — ①

$\rho = R \cdot \ell = \frac{V}{i} \cdot \ell$ — ②

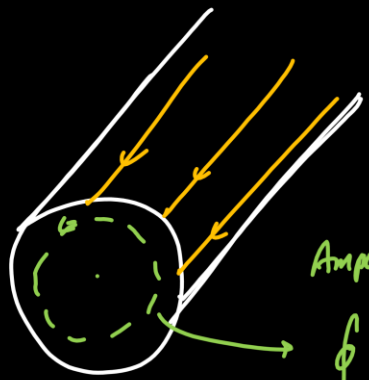
$\frac{CV}{\rho\varepsilon_0} = \frac{E \times A\cancel{\ell}}{\frac{V}{i} \times \cancel{\ell}} = i$ (current)

18

JEE 1

Assertion : A current I flows along the length of an infinitely long straight and thin walled pipe. Then, the magnetic field at any point inside the pipe is zero.

Reason : $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$



Ampere's Law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$

$\Rightarrow = \mu_0 (0)$

$B_{in} = 0$

Read the assertion and reason carefully to mark the correct option out of the options given below:

(A) Both Assertion and Reason are true and the reason is the correct explanation of the assertion.

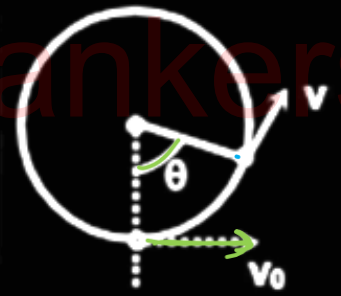
(B) Both Assertion and Reason are true but reason is not the correct explanation of the assertion.

(C) Assertion is true but Reason is false.

(D) Assertion and Reason both are false.

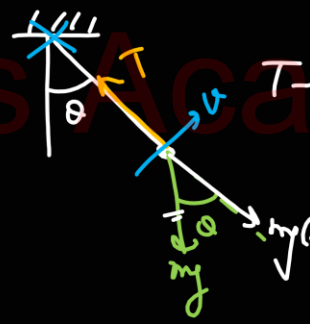
19

A pendulum of length $\ell = 1$ m having a bob of mass $m = 1$ kg is hanging from a rigid support. If the bob is projected horizontally with a velocity $v_0 = \sqrt{35}$ m/s. The tension in the string is 6 k Newton when angle made by the string is 60° from vertical as shown. Find the value of k.



(A) 2 (B) 5

(C) 3 (D) 6



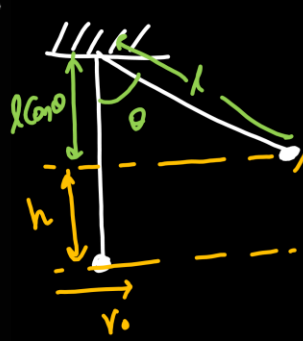
$F = \frac{mv^2}{r}$

$T - mg\cos\theta = \frac{mv^2}{\ell}$

$T = \frac{mv^2}{\ell} + mg\cos\theta$

$= \frac{1\times 25}{1} + 1\times 10\times\frac{1}{2}$

$= 25 + 5 = 30 \Rightarrow 6k = 30 \Rightarrow k = 5$



$\frac{mv^2}{2} = \frac{mv_0^2}{2} - mgh$

$v^2 = v_0^2 - 2gh$

$v^2 = v_0^2 - 2g\ell(1-\cos\theta)$

$= 35 - 2\times 10\times 1\left(1-\frac{1}{2}\right)$

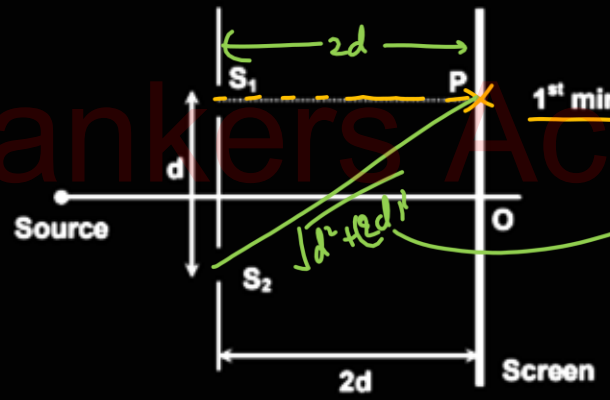
$= 35 - 10 = 25$

$v^2 = 25$ — (1)

20

JEE 1

Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit S_1 ?



$\Delta x = \frac{\lambda}{2}$ (1st minima)

$S_2P - S_1P = \frac{\lambda}{2}$

$\Rightarrow \sqrt{5}d - 2d = \frac{\lambda}{2}$

$d = \frac{\lambda}{2(\sqrt{5}-2)}$

(A) $\frac{\lambda}{2(\sqrt{5}-2)}$

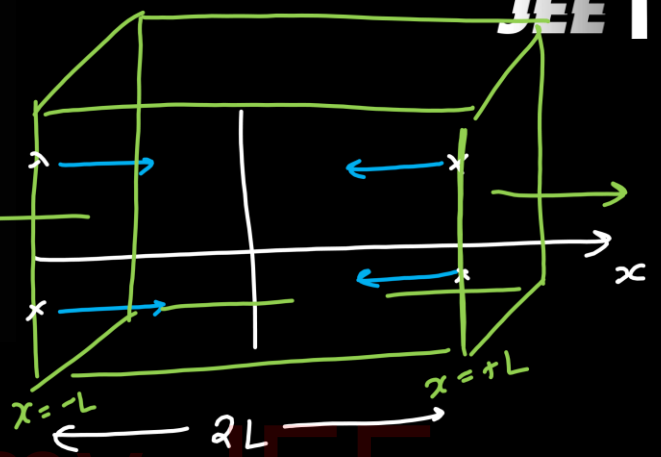
(B) $\frac{\lambda}{(\sqrt{5}-2)}$

(C) $\frac{\lambda}{2(5-\sqrt{2})}$

(D) $\frac{\lambda}{(5-\sqrt{2})}$

21

The electric potential in a medium of dielectric constant 'unity' is $\phi(x, y, z) = ax^2$ where ' a ' is a constant of suitable dimensions. The total charge contained in a cube of dimensions $-L \leq x, y, z \leq L$ is $-na\varepsilon_0 L^3$. Find n.



$V = ax^2$

$E_x = -\frac{\partial V}{\partial x}$

$E_x = -2ax$ — ①

at $x = L$

$E_x = -2aL$ — ②

$\phi = q_{in}/\varepsilon_0$

$-2E(2L)^2 = \frac{q_{in}}{\varepsilon_0}$

$\Rightarrow q_{in} = -2E\varepsilon_0(2L)^2$

$= -2(2aL)\varepsilon_0 4L^2 = -16a\varepsilon_0 L^3$

$n = 16$ Ans

22

A particle is moving on X-axis and has potential energy $U = 2 - 20x + 5x^2$ joule, where x is position. The particle is released at $x = -3$. If the mass of the particle is 0.1 kg, then the maximum velocity (in m/s) of the particle is 25β. Find the value of β.

$a = 0 \quad \frac{dv}{dt} = 0$

Def^n of P.E.

$dU = -dw = -F dx$

$$F = -\frac{dU}{dx} = -(-20 + 10x)$$

$$F = 20 - 10x$$

$$m\left(v\frac{dv}{dx}\right) = 20 - 10x$$

At a_{max} $\quad 0 \Rightarrow x = 2$

$$m\int_0^{v_{max}} v\,dv = \int_{-3}^{2}(20 - 10x)\,dx \qquad (v = 0)$$

$$0.1 \times \frac{v^2}{2} = 20(5) - \frac{10}{2}\left[2^2 - 3^2\right]$$

$$\frac{v^2}{20} = 100 + 25$$

$$v^2 = 2500$$

$$v = 50$$

$$25\beta = 50 \Rightarrow \boxed{\beta = 2}$$

23

A compound microscope has a magnification of 30. The focal length of its eyepiece is 5 cm. Assuming the final image to be formed at least distance of distinct vision (25 cm). The magnification produced by the objective is.

$m_o = ?$

$m = 30$

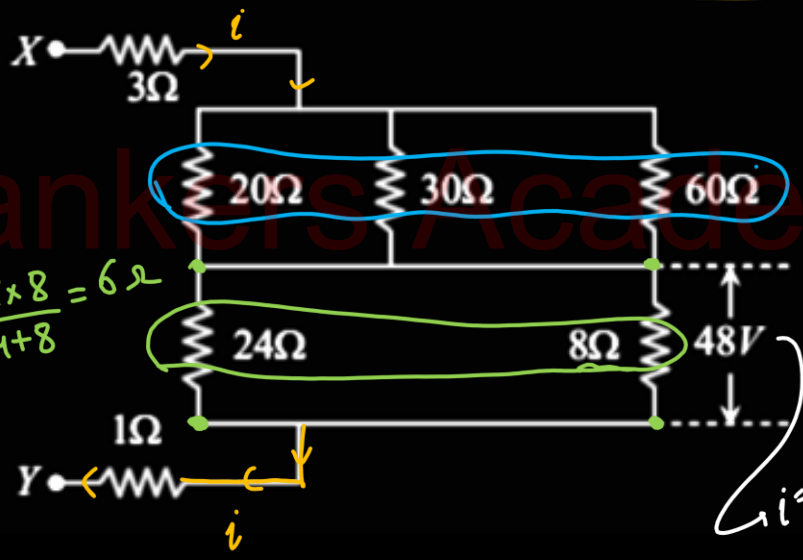
$\Rightarrow m_o \times m_e = 30$

$\Rightarrow m_o \times \left[1 + \frac{D}{f_e}\right] = 30$

$\Rightarrow m_o \times \left[1 + \frac{25}{5}\right] = 30 \Rightarrow m_o = 5$ Ans

24

The potential difference across 8 ohm resistance is 48 volt as shown in the figure. The value of potential difference across X and Y points will be _____ (in Volts)



$V = i R_{eq}$

$= 8 \times (3 + 10 + 6 + 1)$

$= 8 \times 20$

$= 160\ V$

10Ω

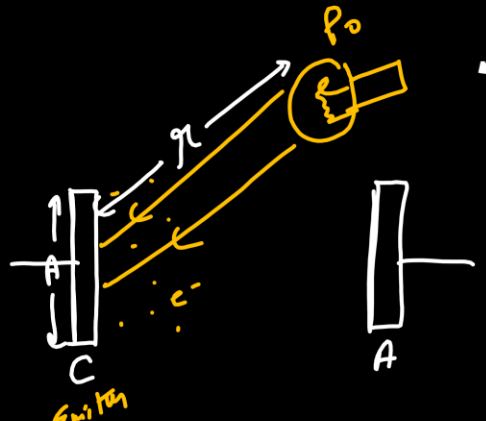
$\frac{24 \times 8}{24+8} = 6\Omega$

$i = \frac{V}{R} = \frac{48}{6} = 8$

JEE 1

25

When a monochromatic point source of light is at a distance 0.2 m from a photoelectric cell, the saturation current and cut-off voltage are 12.0 mA and 0.5 V . If the same source is placed at 0.4 m away from the same photoelectric cell, then the saturation current, now, will be _____ (in mA)



JEE 1

$i = \frac{dq}{dt} = \left(\frac{dn}{dt}\right)e = \eta\left(\frac{dn}{dt}\right)e$

$= \eta \frac{P}{\left(\frac{hc}{\lambda}\right)} e = \frac{\eta e (IA)}{\left(\frac{hc}{\lambda}\right)}$

$i = \frac{\eta e}{hc}(P_0 A) \propto \frac{1}{r^2}$

$i = \left(\frac{dn}{dt}\right) \times e \propto$ no. of photons $\propto \frac{1}{r^2}$

$i \propto \frac{1}{r^2}$ *

$i_1 r_1^2 = i_2 r_2^2$

$12 \times 0.2^2 = i_2 \times 0.4^2$

$i_2 = \frac{12}{4} = 3\,mA$

CHEMISTRY

7

JEE 1

Match List-I with List-II

	List-I (Hybridization)		List-II (Complex)
(a)	$[Ni(CO)_4]$	I.	sp^3
(b)	$[Ni(CN)_4]^{2-}$	II.	sp^3d^2
(c)	$[Co(CN)_6]^{3-}$	III.	d^2sp^3
(d)	$[CoF_6]^{3-}$	IV.	dsp^2

Choose the correct answer from the options given below:

(A) a-IV, b-I, c-III, d-II

(B) a-I, b-IV, c-III, d-II

(C) a-I, b-IV, c-II, d-III

(D) a-IV, b-I, c-II, d-III

2

In the Finkelstein reaction, the solvent medium, reactant and product are respectively.

(A) acetone, alkyl chloride, alkyl iodide

(B) water, alkyl chloride, alkyl iodide

(C) acetone, alkyl iodide, alkyl chloride

(D) water, alkyl iodide, alkyl chloride

$R-X + NaI \xrightarrow{\text{dry acetone}} R-I + NaX$

3

Identify the INCORRECT order with the properties

(A) $F^- > Cl^- > Br^- > I^-$ (Enthalpy of hydration) $\propto \frac{1}{size}$

(B) $H_2O > H_2S < H_2Se < H_2Te$ (Melting point)

(C) $H_2O > H_2Te > H_2Se > H_2S$ (Boiling point)

(D) $PH_3 < AsH_3 < SbH_3 < NH_3$ (Boiling point)

Table 7.7: Properties of Hydrides of Group 16 Elements

Property	H_2O	H_2S	H_2Se	H_2Te
m.p/K	273	188	208	222
b.p/K	373	213	232	269
H–E distance/pm	96	134	146	169
HEH angle (°)	104	92	91	90
$\Delta_f H$/kJ mol^{-1}	–286	–20	73	100
$\Delta_{diss} H$ (H–E)/kJ mol^{-1}	463	347	276	238
Dissociation constant[a]	1.8×10^{-16}	1.3×10^{-7}	1.3×10^{-4}	2.3×10^{-3}

(i) *Reactivity towards hydrogen*: All the elements of Group 15 form hydrides of the type EH_3 where E = N, P, As, Sb or Bi. Some of the properties of these hydrides are shown in Table 7.2. The hydrides show regular gradation in their properties. The stability of hydrides decreases from NH_3 to BiH_3 which can be observed from their bond dissociation enthalpy. Consequently, the reducing character of the hydrides increases. Ammonia is only a mild reducing agent while BiH_3 is the strongest reducing agent amongst all the hydrides. Basicity also decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3 \geq BiH_3$. Due to high electronegativity and small size of nitrogen, NH_3 exhibits hydrogen bonding in solid as well as liquid state. Because of this, it has higher melting and boiling points than that of PH_3.

Table 7.2: Properties of Hydrides of Group 15 Elements

Property	NH_3	PH_3	AsH_3	SbH_3	BiH_3
Melting point/K	195.2	139.5	156.7	185	–
Boiling point/K	238.5	185.5	210.6	254.6	290
(E–H) Distance/pm	101.7	141.9	151.9	170.7	–
HEH angle (°)	107.8	93.6	91.8	91.3	–
$\Delta_f H^\ominus$/kJ mol^{-1}	–46.1	13.4	66.4	145.1	278
$\Delta_{diss} H^\ominus$(E–H)/kJ mol^{-1}	389	322	297	255	–

4

JEE 1

Electronic configuration $[Xe]4f^{7}5\,d^{1}6s^{2}$ is of

(A) Gd

(B) Tb

(C) Dy

(D) Eu

64 :

Gd

58 59 60 61 62 63 64

$4f^{7}5d^{1}6s^{2}$

Table 8.9: Electronic Configurations and Radii of Lanthanum and Lanthanoids

Atomic Number	Name	Symbol	Electronic configurations*			Radii/pm		
			Ln	Ln^{2+}	Ln^{3+}	Ln^{4+}	Ln	Ln^{3+}
57	Lanthanum	La	$5d^1 6s^2$	$5d^1$	$4f^0$		187	106
58	Cerium	Ce	$4f^1 5d^1 6s^2$	$4f^2$	$4f^1$	$4f^0$	183	103
59	Praseodymium	Pr	$4f^3 6s^2$	$4f^3$	$4f^2$	$4f^1$	182	101
60	Neodymium	Nd	$4f^4 6s^2$	$4f^4$	$4f^3$	$4f^2$	181	99
61	Promethium	Pm	$4f^5 6s^2$	$4f^5$	$4f^4$		181	98
62	Samarium	Sm	$4f^6 6s^2$	$4f^6$	$4f^5$		180	96
63	Europium	Eu	$4f^7 6s^2$	$4f^7$	$4f^6$		199	95
64	Gadolinium	Gd	$4f^7 5d^1 6s^2$	$4f^7 5d^1$	$4f^7$		180	94
65	Terbium	Tb	$4f^9 6s^2$	$4f^9$	$4f^8$	$4f^7$	178	92
66	Dysprosium	Dy	$4f^{10} 6s^2$	$4f^{10}$	$4f^9$	$4f^8$	177	91
67	Holmium	Ho	$4f^{11} 6s^2$	$4f^{11}$	$4f^{10}$		176	89
68	Erbium	Er	$4f^{12} 6s^2$	$4f^{12}$	$4f^{11}$		175	88
69	Thulium	Tm	$4f^{13} 6s^2$	$4f^{13}$	$4f^{12}$		174	87
70	Ytterbium	Yb	$4f^{14} 6s^2$	$4f^{14}$	$4f^{13}$		173	86
71	Lutetium	Lu	$4f^{14} 5d^1 6s^2$	$4f^{14} 5d^1$	$4f^{14}$	–	–	–

* *Only electrons outside [Xe] core are indicated*

JEE 1

10 mole of Zn metal reacts with 0.25 M NaOH , produced H_2 gas & Na_2ZnO_2. H_2 gas reacts with 28 litre of O_2 at N.T.P. completely. Find volume of NaOH required for this reaction

(A) 20 lit (B) 10 lit

(C) 25 lit (D) 5 lit

$M = \frac{n}{V}$

$0.25 = \frac{5}{V}$ or $V = \frac{5}{0.25} = 20$ lit

0.25M

$Zn + 2NaOH \longrightarrow H_2 + Na_2ZnO_2$

5 mol 2.5 mol

$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$

2.5 mol 28 lit = 1.25 mol

6

Statement I: All the elements of group- 13 reacts with all the Halogens to form trihalides with +3 oxidation number of central atom.

Statement II: General stoichiometry of reaction is $2E + 3X_2 \rightarrow 2EX_3$

(where E = group 13 elements)

(A) Both Statement I and Statement II are correct.

(B) Statement I is correct but Statement II is incorrect.

(C) Both Statement I and Statement II are incorrect

(D) Statement I is incorrect but Statement II is correct.

(iii) Reactivity towards halogens

These elements react with halogens to form trihalides (except TlI_3).

$$2E(s) + 3\,X_2\,(g) \rightarrow 2EX_3\,(s) \qquad (X = F, Cl, Br, I)$$

7

JEE 1

45 gm of ethylene glycol ($C_2H_6O_2$) is mixed with 600 gm of water. What is the difference between elevation in boiling point and depression in freezing point of the solution? (for water $K_f = 1.86$ K kg/mole and $K_b = 0.52$ K kg/mole)

(A) 1.608 K

(B) -2.232 K

(C) +0.624 K

(D) -1.608 K

Find m,

$$\frac{45 \times 1000}{62 \times 600} = 1.20$$

$\Delta T_b = m K_b$

$\Delta T_f = m K_f$

so $m K_b - m K_f = m(K_b - K_f) = 1.2\,(0.52 - 1.86)$

$= -1.608$

8

Which of the following statement is true for $IO_2F_2^-$ according to VSEPR theory?

(A) The lone pair and two I – O double bonds occupy the equatorial positions of trigonal bipyramid

(B) It has sp^3d hybridization and is T -shaped.

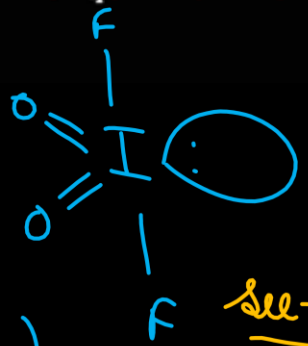
(C) Its structure is analogous to SF_4

(D) (A) and (C) both

$IO_2F_2^-$

8

4 B.P + 1 L.P = 5 (sp^3d)



See-Saw

9

JEE 1

What will be the pH of 1%(w/w) aqueous solution of salt NaA ? SBWA

[Given: $K_a(HA) = 10^{-6}$,

density of solution = 1.2gm/ml,

Molecular mass of HA = 98, so $A^- = 97$

Atomic mass of Na = 23].

(A) 4.5 (B) 9

(C) 10.5 (D) 9.5

$pH = \frac{1}{2}(pK_w + pK_a + \log c)$

$= \frac{1}{2}(14 + 6 + (-1))$

$= 9.5$

1% solution with density 1.2 gm/ml

$\frac{12g \times 1000}{120 \times 1000 ml} = 0.1\ M$

10

JEE 1

Cellulose is a polysaccharide composed of

(A) β-D-glucose

(B) α – D - glucose

(C) β-D-galactose

(D) Both (A) and (B)

Starch cellulose Glycogen

α-glucose β-glucose α-glucose

11

JEE 1

Identify CORRECT statement

(A) For ions $Co^{+2}, Ni^{+2}, Mn^{+2}$ group reagent is $(NH_4)_2CO_3$ in the presence of NH_4OH

(B) Colour of PbI_2 is black yellow

(C) When $FeCl_3$ reacts with $K_4[Fe(CN)_6]$ then prussian blue colour $Fe_4[Fe(CN)_6]_3$ is formed

(D) When $MnCl_2$ reacts with NaOH , pink colour $Mn(OH)_2$ is formed

$MnCl_2 + NaOH \rightarrow Mn(OH)_2$

yellow

Cations – Pb^{2+}, Cu^{2+}, Al^{3+}, Fe^{3+}, Zn^{2+}, Ni^{2+}, Ca^{2+}, Ba^{2+}, Mg^{2+}, NH_4^+

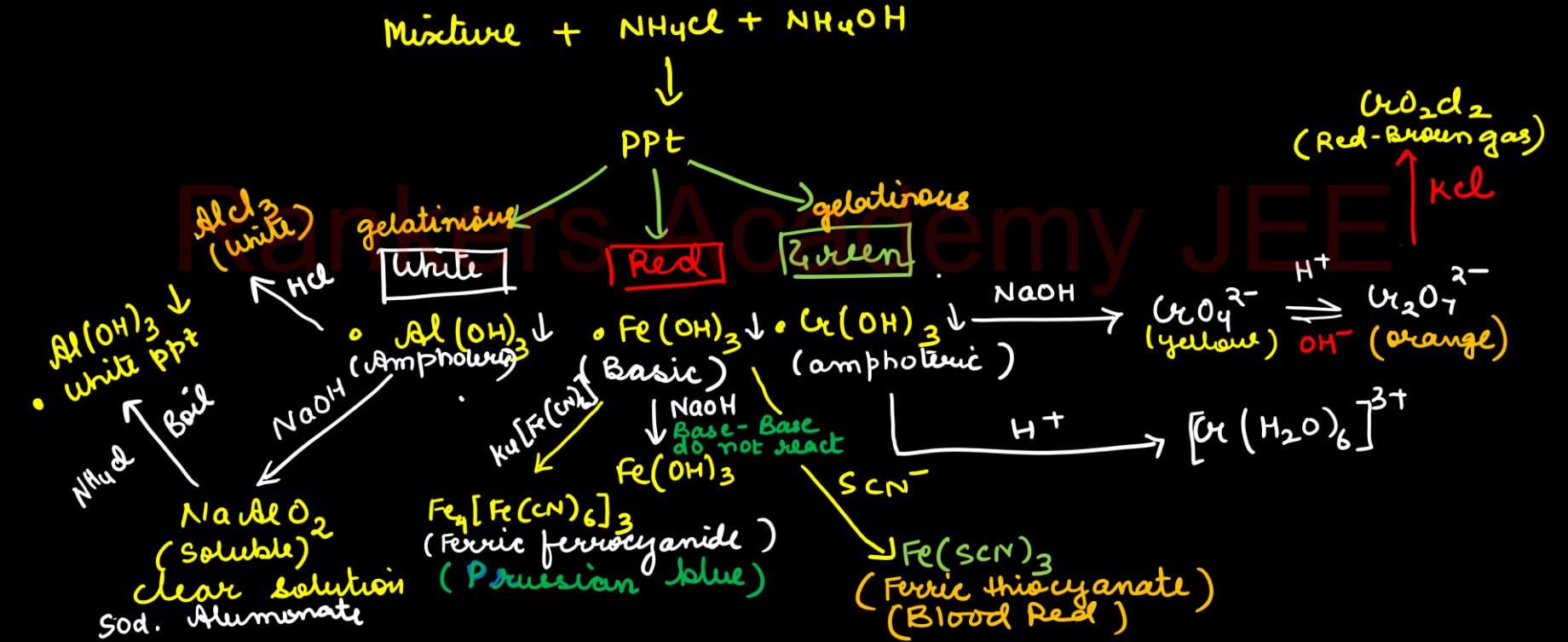
Group	Group Reagent		Basic Radical	Composition and Colour of the ppt.	
Zero	NaOH		NH_4^+ ✓	NH_3 gas is evolved.	
I	dil Hcl	आज	Ag^+	AgCl	White
		होगी	Hg_2^{2+}	Hg_2Cl_2	White
		Problem	Pb^{2+} ✓ → (I & II दोनो में Test देता है)	$PbCl_2$	White
II	H_2S in presence of dil Hcl	क्योंकी	Cu^{2+} ✓	CuS	Black
		होंगी	Hg^{2+}	HgS	Black
		Punjabi	Pb^{2+}	PbS	Black
		कुड़िया	Cd^{2+}	CdS	Yellow
		BIMAAR	Bi^{2+}	Bi_2S_3	Black Brown
		असली बात	As^{3+}	As_2S_3	Yellow
		सबू	Sb^{3+}	Sb_2S_3	orange
		सुनो	Sn^{2+}	SnS	Brown
			Sn^{4+}	SnS_2	Yellow

III	NH_4OH in presence of NH_4Cl	अलस	Al^{3+} ✓	$Al(OH)_3$	White gelatin ppt
		फैक	Fe^{3+} ✓	$Fe(OH)_3$	Red
		कर	Cr^{3+}	$Cr(OH)_3$	Dirty green
IV	H_2S in presence of NH_4OH	Nikita	Ni^{2+} ✓	NiS	Black
		Zeenat	Zn^{2+} ✓	ZnS	Bluish white
		को	Co^{2+}	CoS	Black
		मनाओ	Mn^{2+}	MnS	Buff coloured (flesh)
V	$(NH_4)_2CO_3$ in presence of NH_4OH	Use फ्रूट बीमारी			
		सर	Sr^{2+}	$SrCO_3$	White
		के	Ca^{2+} ✓	$CaCO_3$	White
		बाल	Ba^{2+} ✓	$BaCO_3$	White
VI	Na_2HPO_4 No group reagent	मांगेगी	Mg^{2+} ✓	$Mg(NH_4)PO_4$	White
	No group reagent	नाक	Na^+		
		बहेगी	K^+		

Group - 3 (Al^{3+}, Fe^{3+}, Cr^{+3})

आलस फेक कर

Grp Reagent : NH_4OH in presence of NH_4Cl



12

What is the correct order for the properties mentioned in bracket

(A) Sc < V > Ni < Zn (1^{st} ionization enthalpy)

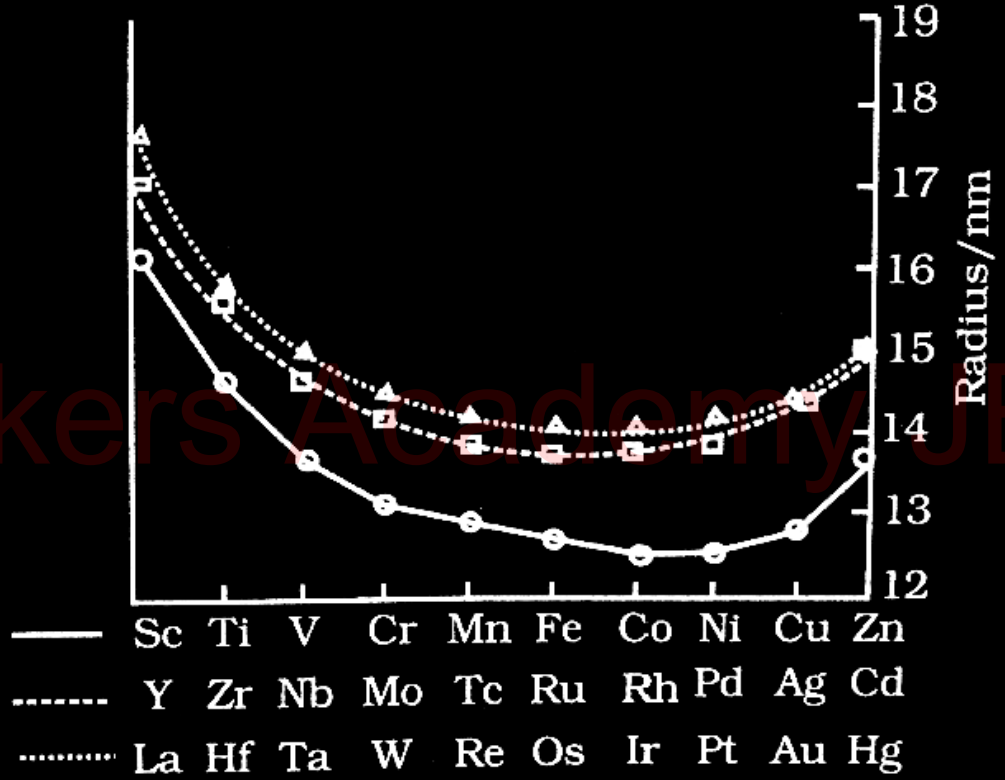
(B) Sc > V > Co < Zn (Atomic radius)

(C) Sc < V < Ni < Zn (Density (gm/cm^3))

(D) Sc < V > Mn > Zn (Enthalpy of atomization)

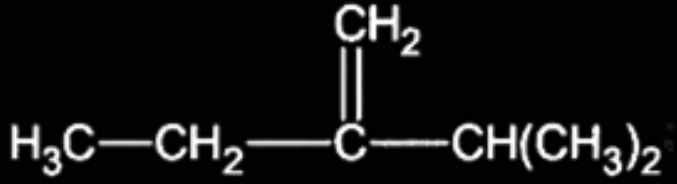
Table 8.2: Electronic Configurations and some other Properties of the First Series of Transition Elements

Element		Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
Atomic number		21	22	23	24	25	26	27	28	29	30
Electronic configuration											
	M	$3d^1 4s^2$	$3d^2 4s^2$	$3d^3 4s^2$	$3d^5 4s^1$	$3d^5 4s^2$	$3d^6 4s^2$	$3d^7 4s^2$	$3d^8 4s^2$	$3d^{10} 4s^1$	$3d^{10} 4s^2$
	M^+	$3d^1 4s^1$	$3d^2 4s^1$	$3d^3 4s^1$	$3d^5$	$3d^5 4s^1$	$3d^6 4s^1$	$3d^7 4s^1$	$3d^8 4s^1$	$3d^{10}$	$3d^{10} 4s^1$
	M^{2+}	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	$3d^8$	$3d^9$	$3d^{10}$
	M^{3+}	[Ar]	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	–	–
Enthalpy of atomisation, $\Delta_a H^\ominus$/kJ mol^{-1}		326	473	515	397	281	416	425	430	339	126
Ionisation enthalpy/$\Delta_i H^\ominus$/kJ mol^{-1}											
$\Delta_i H^\ominus$	I	631	656	650	653	717	762	758	736	745	906
$\Delta_i H^\ominus$	II	1235	1309	1414	1592	1509	1561	1644	1752	1958	1734
$\Delta_i H^\ominus$	III	2393	2657	2833	2990	3260	2962	3243	3402	3556	3837
Metallic/ionic radii/pm	M	164	147	135	129	137	126	125	125	128	137
	M^{2+}	–	–	79	82	82	77	74	70	73	75
	M^{3+}	73	67	64	62	65	65	61	60	–	–
Standard electrode potential $E^\ominus$/V	M^{2+}/M	–	−1.63	−1.18	−0.90	−1.18	−0.44	−0.28	−0.25	+0.34	−0.76
	M^{3+}/M^{2+}	–	−0.37	−0.26	−0.41	+1.57	+0.77	+1.97	–	–	–
Density/g cm^{-3}		3.43	4.1	6.07	7.19	7.21	7.8	8.7	8.9	8.9	7.1



13

The IUPAC name of the following compound is

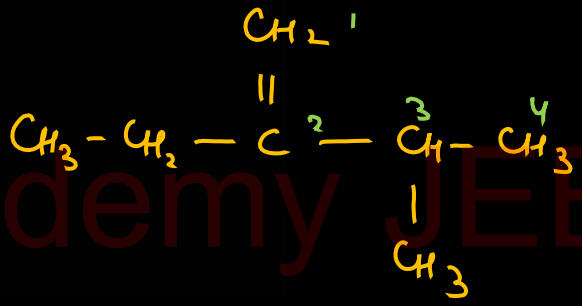


(A) ethylisopropyl ethane

(B) 2-isopropyl but-1-ene

(C) 2-methyl-3-ethyl but-3-ene

(D) 2-ethyl-3-methyl but-1-ene



14

JEE 1

λ_m° for NaCl, HCl and NaAc are 126.4, 425.9 and 91.0 SCm^2 $mole^{-1}$ respectively. If λ_m of 0.1 M CH_3COOH is $39.05 Scm^2$ $mole^{-1}$, find it's pH

(A) 1.5 (B) 1

(C) 2 (D) 2.5

$CH_3COOH \longrightarrow CH_3COO^- + H^+$

C — — (initial)

$(C - C\alpha)$ $C\alpha$ $C\alpha$ (equilibrium)

$[H^+] : C\alpha : 0.1\alpha = 10^{-1} \cdot 10^{-2} = 10^{-2}$

$pH = -\log[H^+] = -\log(10^{-2}) = 2$

$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{39.05}{390.5} = 10^{-1}$

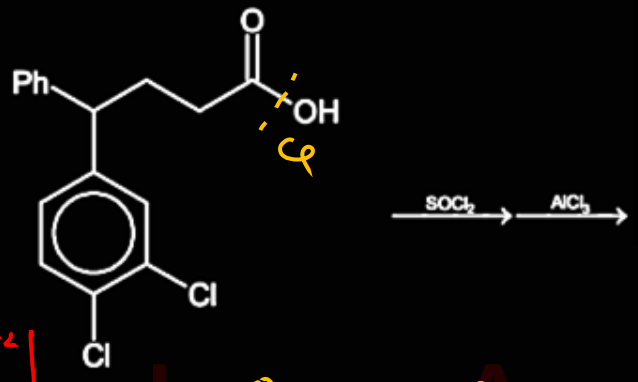
$\Lambda_m^\circ(CH_3COOH) = \Lambda_{NaAc}^\circ + \Lambda_{HCl}^\circ - \Lambda_{NaCl}^\circ$

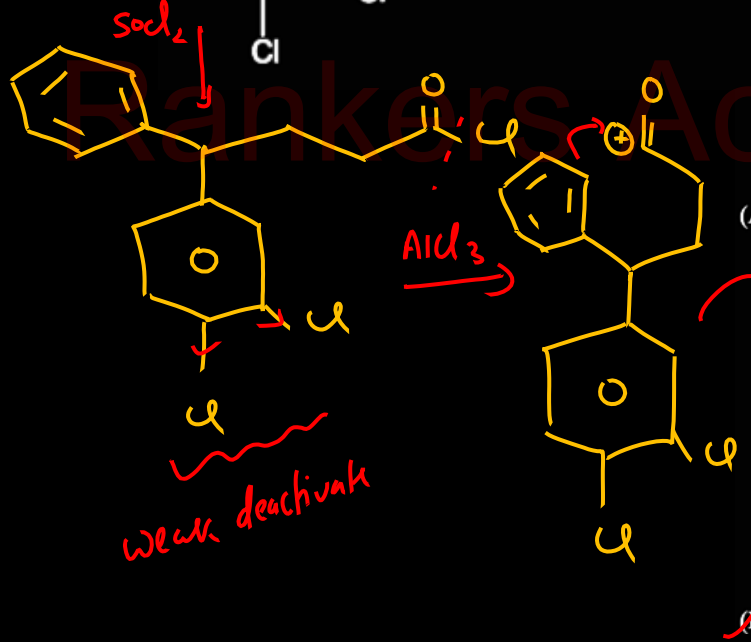
$= 91 + 425.9 - 126.4$

$= 390.5$

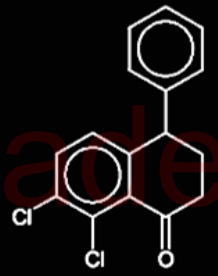
15

Major product obtained in the following synthesis is





(A)



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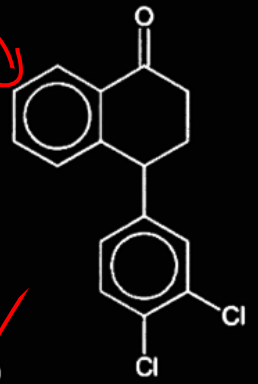
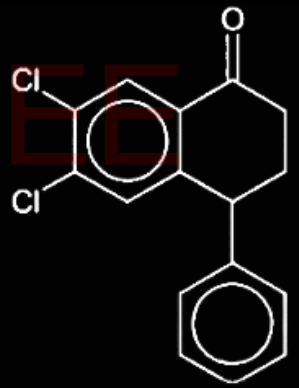
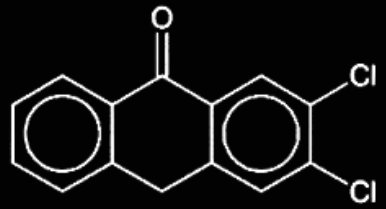


(C)

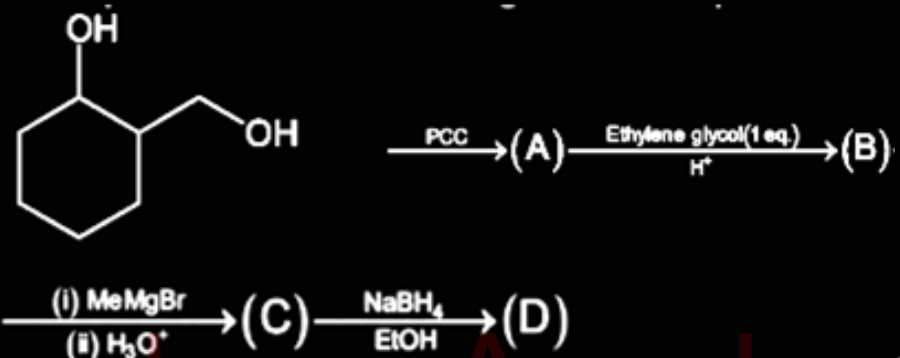


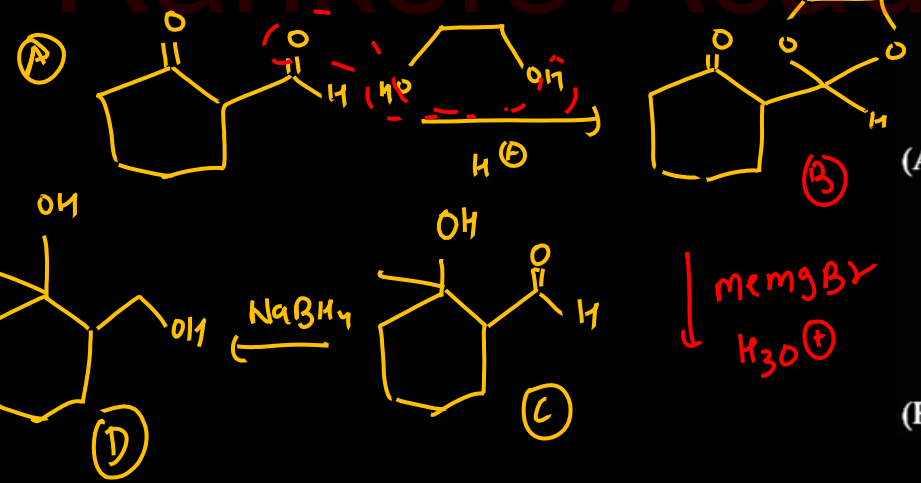
(D)



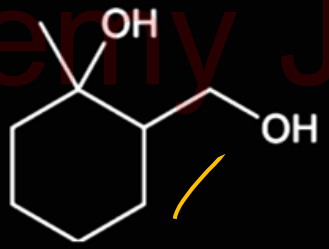
16

Final product ' D ' in the following reaction sequence is

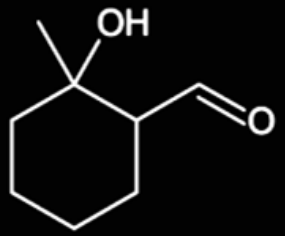




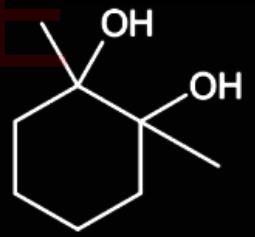
(A)



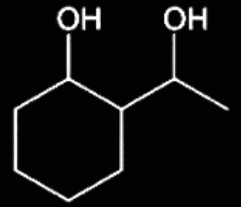
(B)



(C)



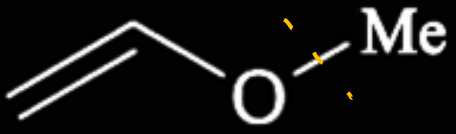
(D)

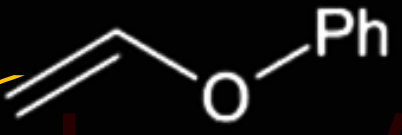


17

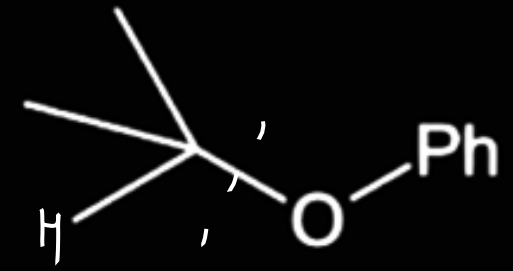
Which of the following ether can not be synthesized by Williamson synthesis?

JEE 1

(A) 

(B) 

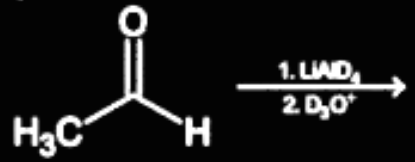
(C) 

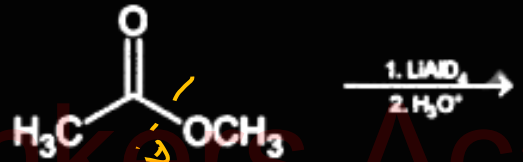
(D) 

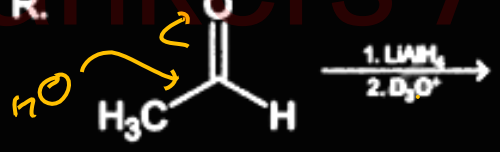
18

Match the reactant in group-I to product in group-II:

Group-I

P. 

Q. 

R. 

Group-II

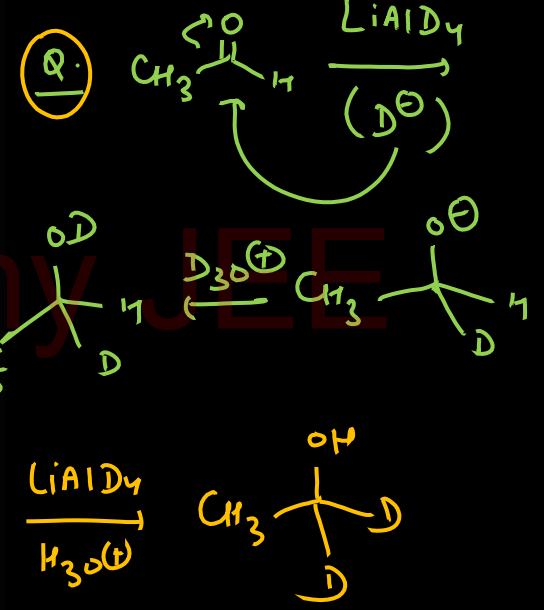
1. CH_3CHDOD
2. CH_3CH_2OD
3. CH_3CD_2OH

(A) P – 3, Q – 1, R – 2

(B) P – 1Q – 3, R – 2

(C) P – 1, Q – 2, R – 3

(D) P – 2, Q – 3, R – 1



19

Consider the following sequence of reaction

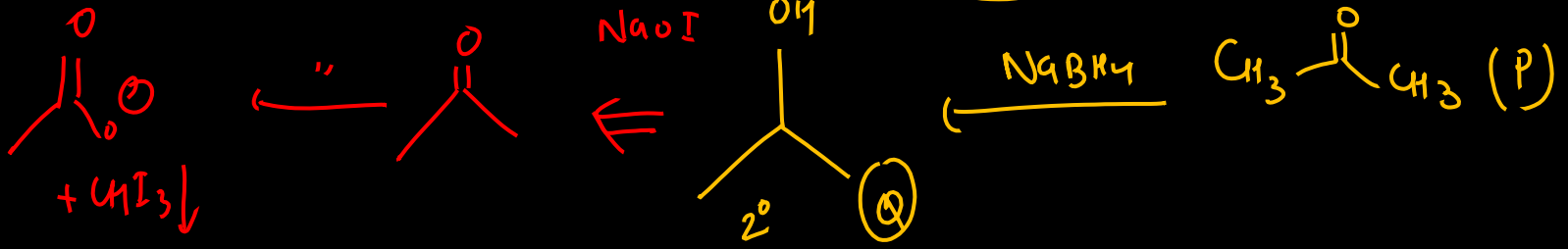
Product Q gives

(A) Instant turbidity with Lucas reagent

(B) Red colour in Victor-Mayer test (RBC)

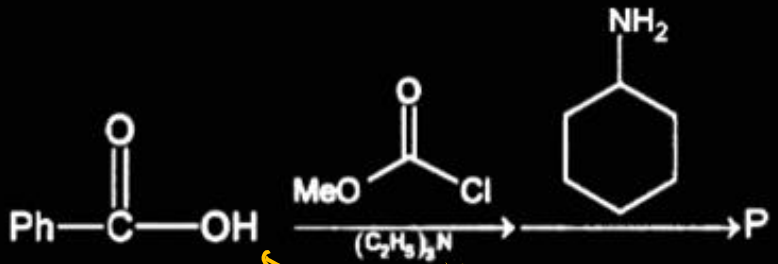
(C) Yellow precipitate in idodoform test

(D) Yellow precipitate with Brady's reagent (2,4 DNP)

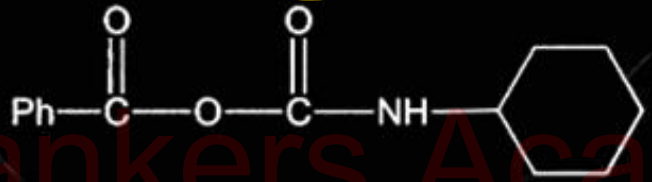


20

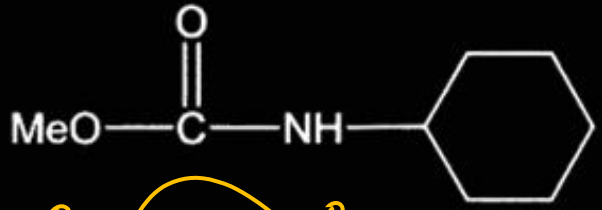
Major product P in the following reaction is



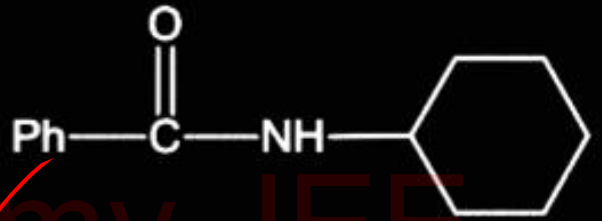
(A)



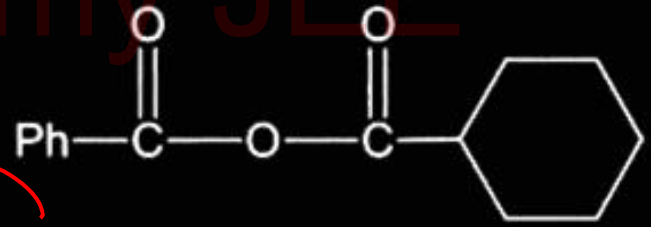
(B)

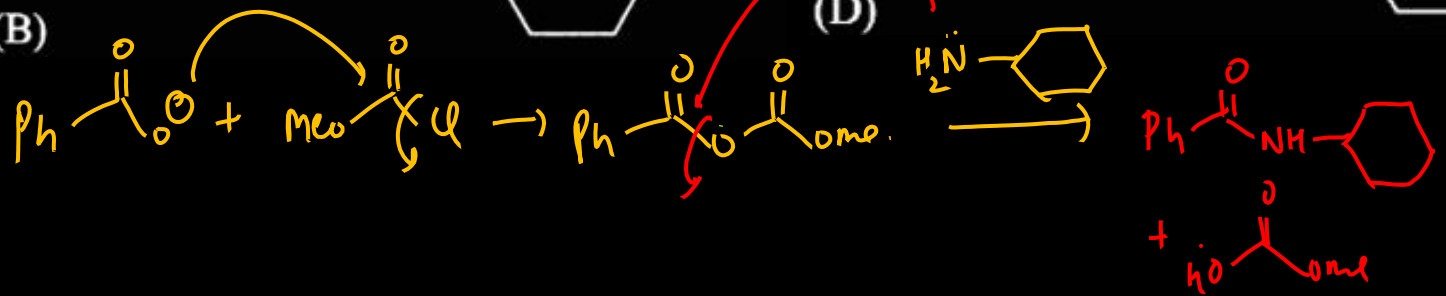


(C)



(D)





JEE 1

21

Density of an aqueous glucose solution is 1.18gm/mol. If it's concentration is 18% w/V then find out the difference between molarity and molality.

JEE 1

18% w/v → 18g glucose in 100 ml solution

$n_{glucose} = \frac{18}{180} = 0.1$

$M = \frac{n}{V} = \frac{0.1}{100} \times 1000 = 1\ M$

$m = \frac{n}{W_{solvent(g)}} \times 1000 = \frac{0.1 \times 1000}{100} = 1\ m$

$W_{sol} = d \times V$

$= 1.18 \times 100$

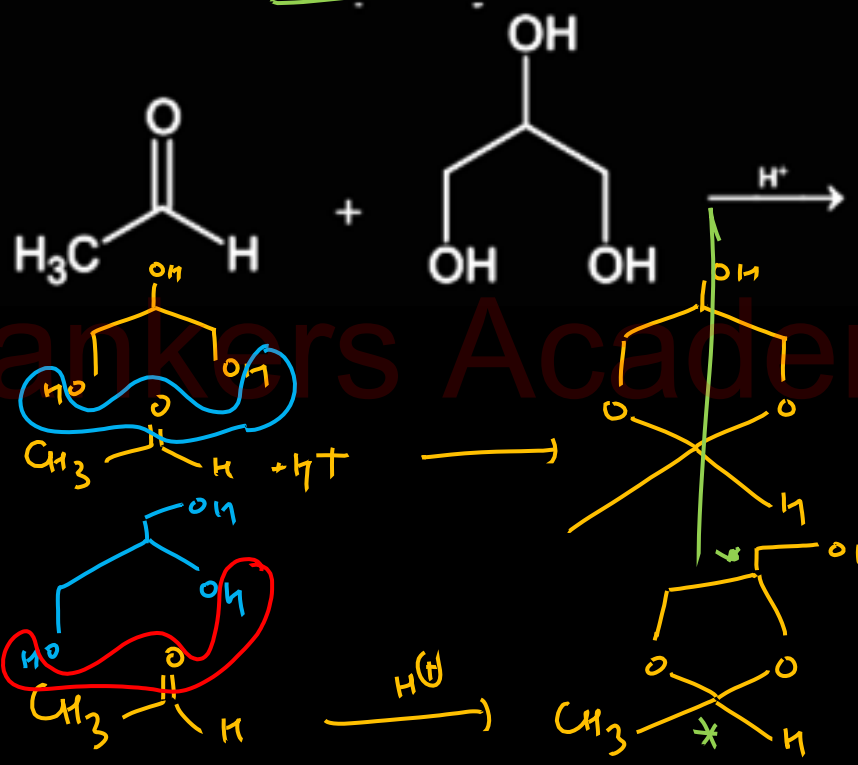
$= 118\ g$

$W_{solvent} = 118 - 18 = 100\ g$

Ans. 0

22

The number of optically active isomers formed in the following reaction is:



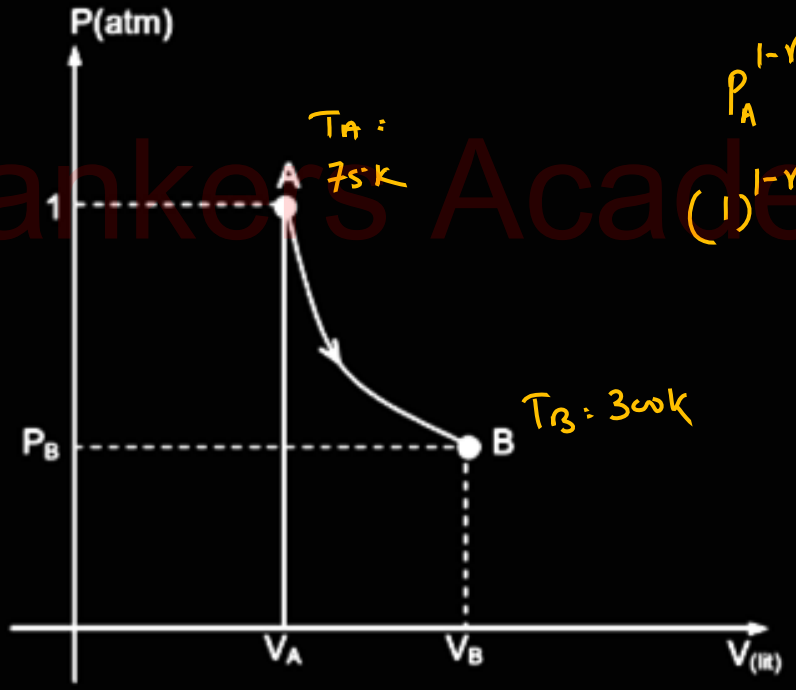
SI : 2^2 : 4

JEE 1

23

An adiabatic reversible process AB is performed for 1 mole monoatomic ideal gas. If temperature at point A and B are 75 K and 300 K respectively. What will be pressure (atm) at B.

JEE 1



$$P_A^{1-\gamma} T_A^{\gamma} = P_B^{1-\gamma} T_B^{\gamma}$$

$$(1)^{1-\gamma} (75)^{\gamma} = (P_B)^{1-\gamma} (300)^{\gamma}$$

$$\left(\frac{75}{300}\right)^{\gamma} = P_B^{1-\gamma}$$

$$\left(\frac{1}{4}\right)^{5/3} = P_B^{-2/3}$$

$$\therefore P_B = 2^5 = 32$$

24

How many of the following compounds is/are aromatic?

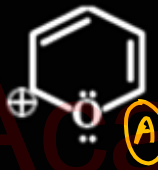
(i) 

(ii) 

(iii) 

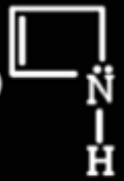
(iv) 

(v) 

(vi) 

(vii) 

(viii) 

(ix) 

(x) 

(xi) 

25

JEE 1

For a complex reaction $A \xrightarrow{K}$ products

$E_{a_1} = \frac{180\ kJ}{mol}; E_{a_2} = \frac{80\ kJ}{mol};$

$E_{a_3} = 50\ kJ/mol$

Overall rate constant k is related to individual rate constant by the equation $k = \left(\frac{k_1 \cdot k_2}{k_3}\right)^{2/3}$

Activation energy (kJ/mol) for the overall reaction is :- (nearest integer)

$K = Ae^{-Ea/RT}$:

$\left(\frac{A_1 e^{-Ea_1/RT} \cdot A_2 e^{-Ea_2/RT}}{A_3 e^{-Ea_3/RT}}\right)^{2/3}$

$e^{-Ea/RT} : \left(e^{-\frac{Ea_1 - Ea_2 + Ea_3}{RT}}\right)^{2/3}$

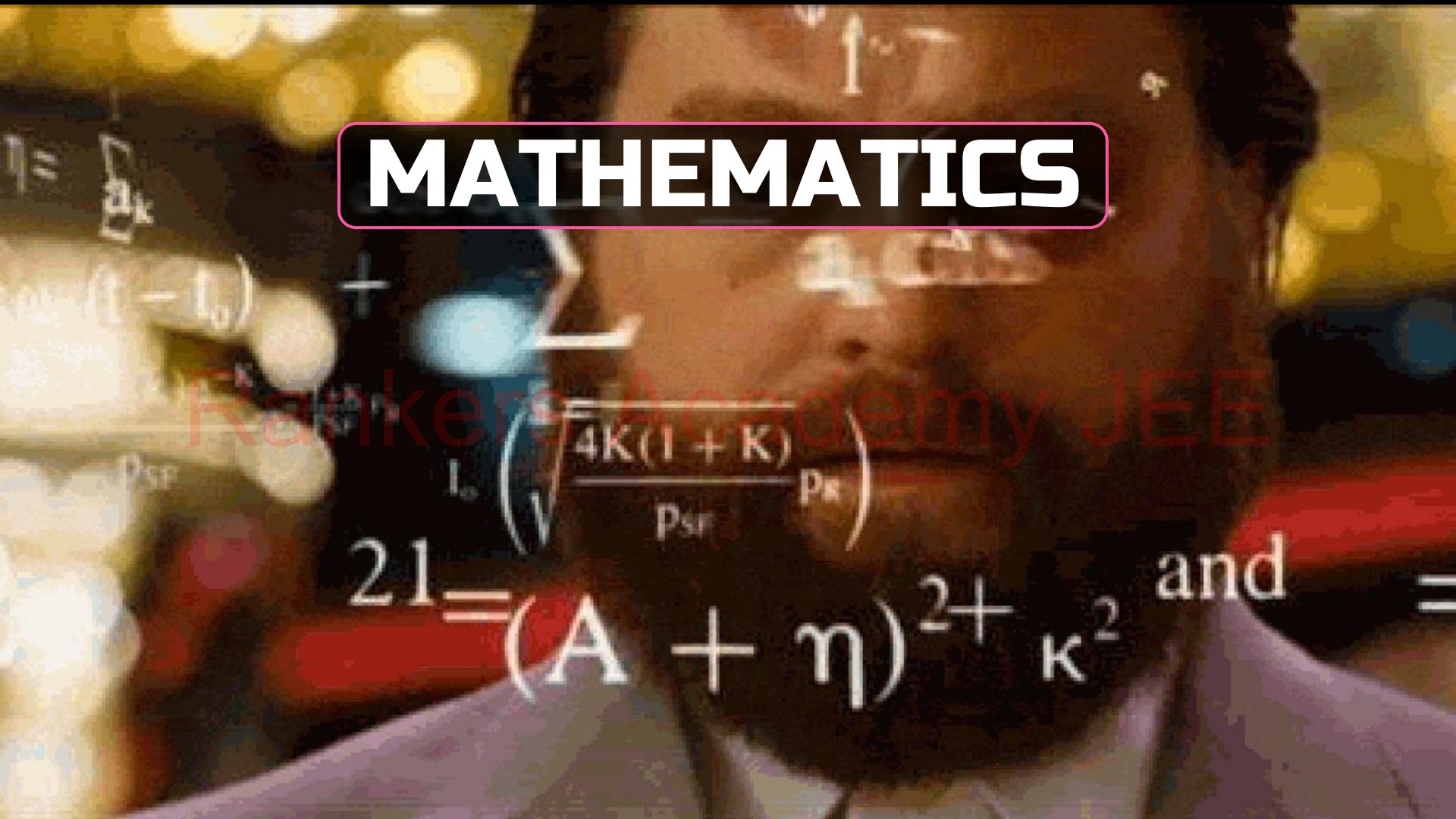
$-Ea : (-Ea_1 - Ea_2 + Ea_3)^{2/3}$

$Ea : [Ea_1 + Ea_2 - Ea_3]^{2/3}$

$\Rightarrow \left[180 + 80 - 50\right]^{\frac{2}{3}}$

$\in 210 \times \frac{2}{3}$

$\in$ 140


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^{2+} \kappa^2$ and

7

The integral $\int_0^{\pi/2} \frac{1}{3+2\sin x+\cos x}dx$ is equal to $\tan^{-1}(k) - \frac{\pi}{2k}$ then $k =$

(A) 1 (B) 3

(C) 2 (D) 4

JEE 1

$$I = 2\int_0^1 \frac{dt}{3(1+t^2)+2(2t)+(1-t^2)} \quad \left(\because \tan\frac{x}{2} = t\right)$$

$$= \int_0^1 \frac{dt}{t^2+2t+2}$$

$$= \int_0^1 \frac{dt}{(t+1)^2+1} = \left[\tan^{-1}(t+1)\right]_0^1$$

$$= \tan^{-1}2 - \tan^{-1}1$$

$$= \tan^{-1}2 - \frac{\pi}{4}$$

$\therefore k = 2$

7

JEE 1

$$\int \frac{dx}{a\sin x + b\cos x + c}$$

✓ Let $\tan\frac{x}{2} = t$

$$2\int \frac{dt}{a(2t) + b(1-t^2) + c(1+t^2)}$$

2

If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to

(A) $\frac{27}{32}$ (B) $\frac{1}{24}$

(C) $\frac{3}{8}$ (D) $\frac{27}{16}$

JEE 1

$\alpha + \beta = 3/7$

$\alpha\beta = -2/7$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{7}\right)^2 - 2\left(-\frac{2}{7}\right)$

$= \frac{9}{49} + \frac{4}{7}$

$= \frac{9+28}{49} = \frac{37}{49}$

$\Rightarrow \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1 - (\alpha^2+\beta^2) + (\alpha\beta)^2}$

$\Rightarrow \frac{\frac{3}{7} + \left(+\frac{2}{7}\right)\left(\frac{3}{7}\right)}{1 - \frac{37}{49} + \frac{4}{49}} = \frac{\frac{3}{7}\left(1 + \frac{2}{7}\right)}{1 - \frac{33}{49}} = \frac{\frac{3}{7} \cdot \frac{9}{7}}{\frac{16}{49}} = \frac{27}{16}$

3

Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then the value of $\lim_{h\to 0} \frac{f(1-h)-f(1)}{h^3+3h}$ is equal to

(A) $\frac{-53}{3}$

(B) $\frac{22}{3}$

(C) $\frac{-22}{3}$

(D) $\frac{53}{3}$

JEE 1

$LHD = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$

Ans: $-\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h(h^2+3)}$

$= -\frac{f'(1)}{3} = -\left(\frac{+63}{3}\right)$

$f'(x) = 30x^9 - 56x^7 + 30x^5 - 63x^2 + 6x$

$f'(1) = 30 - 56 + 30 - 63 + 6 = -53$

4

If $A_1, A_2, A_3 \ldots\ldots\ldots$ are in A.P then

$\sum_{i=1}^{2n} (-1)^i \left(\frac{A_i + A_{i+1}}{A_i - A_{i+1}}\right)$ is equal to

(A) $2n-1$ $= 3$ (B) $n-1$ $= 1$

(C) $-2n$ $= -4$ (D) $n+2$ $= 3$

JEE 1

$n = 2$

$$\sum_{i=1}^{4} (-1)^i \frac{(A_i + A_{i+1})}{A_i - A_{i+1}}$$

$$= -\frac{(A_1 + A_2)}{A_1 - A_2} + \frac{A_2 + A_3}{A_2 - A_3} - \frac{(A_3 + A_4)}{A_3 - A_4} + \frac{A_4 + A_5}{A_4 - A_5}$$

$$= \frac{A_5 - A_1}{-d} = \frac{A_1 + 4d - A_1}{-d} = -4$$

5

JEE 1

The value of $\int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ is

(A) $\frac{1}{2\sqrt{11}} \ln \frac{(\sqrt{11}+1)^2}{10}$

(B) $\frac{1}{\sqrt{11}} \ln \frac{(\sqrt{11}+1)^2}{10}$

(C) $\frac{1}{2\sqrt{11}} \ln \frac{(\sqrt{11}-1)^2}{10}$

(D) $\frac{1}{\sqrt{11}} \ln \frac{(\sqrt{11}-1)^2}{10}$

$$I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})} \quad \text{—— (1)}$$

$x \to 0+1-x$

$$I = \int_0^1 \frac{dx}{(5+2(1-x)-2(1-x)^2)(1+e^{2-4(1-x)})}$$

$$I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{4x-2})} e^{-(2-4x)}$$

$$= \int_0^1 \frac{dx}{(5+2x-2x^2)\left(1+\frac{1}{e^{2-4x}}\right)}$$

$$= \int_0^1 \frac{e^{2-4x}\,dx}{(5+2x-2x^2)(e^{2-4x}+1)} \quad \text{—— (2)}$$

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = -\frac{1}{2}\int_0^1 \frac{dx}{x^2-x-\frac{5}{2}}$$

$$I = -\frac{1}{4}\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2-\frac{5}{4}-\frac{1}{4}} = -\frac{1}{4}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2-\left(\frac{\sqrt{11}}{2}\right)^2} = -\frac{1}{4}\cdot\frac{1}{2\left(\frac{\sqrt{11}}{2}\right)} \ln \left| \frac{x-\frac{1}{2}-\frac{\sqrt{11}}{2}}{x-\frac{1}{2}+\frac{\sqrt{11}}{2}} \right|_0^1$$

$$= \frac{-1}{2\sqrt{11}} \ln \left| \frac{\frac{1}{2} - \frac{\sqrt{11}}{2}}{\frac{1}{2} + \frac{\sqrt{11}}{2}} \right|$$

$$= \frac{-1}{2\sqrt{11}} \ln \left| \frac{\sqrt{11}-1}{\sqrt{11}+1} \right|$$

$$= \frac{-1}{2\sqrt{11}} \ln \left| \frac{(\sqrt{11}-1)^2}{10} \right|$$

$$= \frac{-1}{2\sqrt{11}} \ln \left| \frac{(\sqrt{11}-1)^2(\sqrt{11}+1)^2}{10(\sqrt{11}+1)^2} \right|$$

$$= \frac{-1}{2\sqrt{11}} \ln \left| \frac{10^2}{10(\sqrt{11}+1)^2} \right|$$

$$= \frac{1}{2\sqrt{11}} \ln \left| \frac{(\sqrt{11}+1)^2}{10} \right|$$

6

JEE 1

Let f be a function defined on the interval $[0,2\pi]$ such that

$\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t\,dt$ and $f(0) = 1$. Then the maximum value of $f(x)$ is _____.

(A) 3/4 (B) 9/4

(C) 9/8 (D) 3

$\rightarrow = -\int_0^x f(t)\tan t\,dt$

$\Rightarrow f'(x) - \sin 2x = -[f(x)\tan x]$

$\Rightarrow f'(x) + f(x)\tan x = \sin 2x$

$\Rightarrow \frac{dy}{dx} + (\tan x)\,y = \sin 2x$

$I.F. = e^{\int \tan x\,dx} \quad \ln|\sec x|$

$= e \qquad = |\sec x|$

$y(|\sec x|) = \int |\sec x| \cdot \sin 2x\,dx$

$y\sec x = 2\int \sin x\,dx = -2\cos x + c$

$x = 0,\ y = 1$

$1 = -2 + c \Rightarrow c = 3$

$y\sec x = -2\cos x + 3$

$y = f(x) = -2\cos^2 x + 3\cos x$

$= -2\left[\cos^2 x - \frac{3}{2}\cos x\right]$

$\frac{9}{8} - 2\left(\cos x - \frac{3}{4}\right)^2 = -2\left[\left(\cos x - \frac{3}{4}\right)^2 - \frac{9}{16}\right]$

$\frac{9}{8}$ -2 $\left(\frac{3}{4}\right)^2$ $\frac{9}{8}$ $-\frac{1}{2}$ $-\frac{9}{8}$

7

Function

$f(x) = \begin{cases} \text{sgn}([x]); & x \neq 1 \\ [\text{sgn}(x)]; & x = 1 \end{cases}$ is :

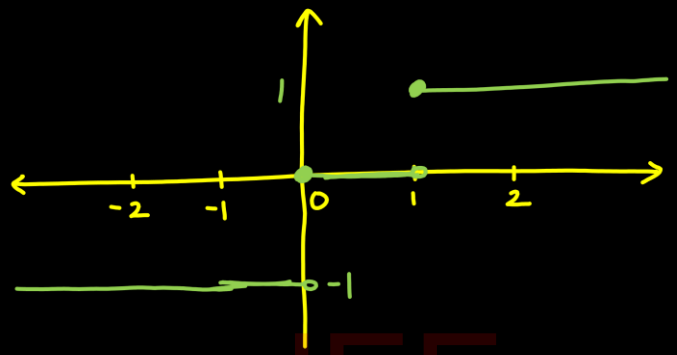
{where sgn (.) dentoes signum function and [.] denotes greatest integer function and I is the set of all integers}

(A) continuous but not differentiable at $x = 2$

(B) discontinuous at every integer

(C) non differentiable at $x = 0$ and 1 only

(D) non differentiable at every natural number



Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors satisfying $\vec{a} = \vec{b} \times \vec{c} + 2\vec{b}$ where $|\vec{b}| = |\vec{c}| = 2$ and $|\vec{a}| \leq$ 4. The sum of possible value(s) of $|2\vec{a} + \vec{b} + \vec{c}|$ is :

(A) 8 (B) 12

(C) 20 (D) 32

JEE 1

$\vec{a} = \vec{b} \times \vec{c} + 2\vec{b}$

$\vec{a} \cdot \vec{b} = \vec{b} \cdot (\vec{b} \times \vec{c}) + 2\vec{b} \cdot \vec{b}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = 2|\vec{b}|$

$\Rightarrow |\vec{a}| = 4\sec\theta \geq 4$

≥ 1

$\therefore |\vec{a}| = 4$

$\sec\theta = 1 \Rightarrow \theta = 0°$

$\vec{a} \parallel \vec{b}$

$|\vec{a}| = 4, |\vec{b}| = 2$

$\vec{a} = 2\vec{b}$

$\vec{a} = \vec{b} \times \vec{c} + 2\vec{b}$

$\vec{b} \times \vec{c} = 0$

$\vec{b} \parallel \vec{c}$

$\vec{b} = \vec{c}$ or $\vec{b} = -\vec{c}$

$|2\vec{a} + \vec{b} + \vec{c}|$

$|2(2\vec{b}) + \vec{b} + \vec{b}|$

$= 6|\vec{b}| = 6 \times 2 = 12$

$|2(2\vec{b}) + \vec{b} - \vec{b}|$

$= 4|\vec{b}| = 4 \times 2 = 8$

Sum $= 12 + 8 = 20$

9

If

$$\Delta_r = \begin{vmatrix} r.r! & ({}^mC_r)^2 & 2r-1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \\ (m+1)!-1 & {}^{2m}C_m & m^2-1 \end{vmatrix},$$

then $\sum_{r=0}^{m} \Delta_r$ is

(A) 0

(B) m

(C) m^2

(D) $m^2 + \sin^2 m$

JEE 1

$\sum r.r! = \sum (r+1-1)r!$

$= \sum (r+1)r! - r!$

$= \sum_{r=0}^{m} (r+1)! - r!$

$= (m+1)! - 0! + 0! - 1! = (m+1)! - 1$

$\sum {}^mC_r^2 = {}^mC_0^2 + {}^mC_1^2 + \cdots + {}^mC_m^2$

$= {}^{2m}C_m$

$\sum \Delta_r = \begin{vmatrix} \sum r.r! & \sum {}^mC_r^2 & \sum (2r-1) \\ = (m+1)!-1 & = {}^{2m}C_m & = m^2-1 \\ \sin^2 m^2 & \sin^2 m & \sin^2(m+1) \\ (m+1)!-1 & {}^{2m}C_m & m^2-1 \end{vmatrix}$

$= 0$

$\sum 2r-1 = \rightarrow 2\sum r - \sum 1$

$\frac{2m(m+1)}{2} - (m+1)$

$= m^2 - 1$

$-1 + (1 + 3 + \cdots + 2m-1)$

$-1 + (m^2)$

10

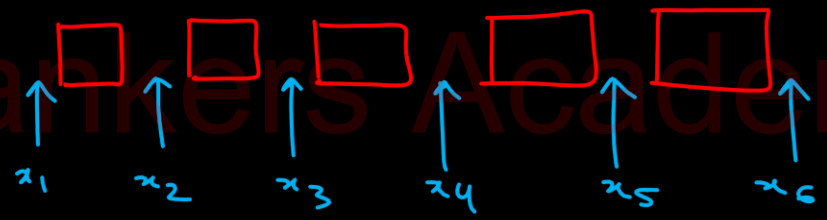
The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is

(A) 24 (B) 56

(C) 48 (D) 64

JEE 1

16 — 11 B, 5 R



$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_2, x_3, x_4, x_5 \geq 2$

$x_1, x_6 \geq 0$

$x_1 + x_2' + x_3' + x_4' + x_5' + x_6 = 3$

$= 3+6-1 C_{6-1}$

$= {}^{8}C_5 = \frac{8!}{5!3!} = 56$

11

If $z^2 + z + 1 = 0, z \in C$, then

$\left|\sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ is equal to

(A) 2 (B) 1

(C) ω (D) ω^2

JEE 1

Roots : $z = \omega, \omega^2$

$$= \left|\sum_{n=1}^{15} z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n \frac{z^n}{z^n} \frac{1}{z^n}\right| \Rightarrow \left|0 + 0 + 2(-1)\right| = 2$$

$$= \left|\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} \frac{1}{z^{2n}} + 2\sum_{n=1}^{15} (-1)^n\right|$$

$\omega^3 = 1$

$(\omega^2 + \omega^4 + \omega^6 + \ldots + \omega^{30})$

$= \omega^2\left(\frac{1-\omega^{30}}{1-\omega^2}\right)$

$\frac{1}{\omega^2} + \frac{1}{\omega^4} + \ldots + \frac{1}{\omega^{30}}$

$= \frac{1}{\omega^2}\left(\frac{1-\frac{1}{\omega^{30}}}{1-\frac{1}{\omega^2}}\right)$

$-1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 = -1$

12

JEE 1

The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is

(A) 270 (B) 269

(C) 2025 (D) 144

incorrect

correct

incorrect

$$\frac{x_1 + \cdots + x_8 + 45 + 50}{10} = 50$$

$$\frac{x_1 + \cdots + x_8 + 20 + 25}{10} = \text{correct mean}$$

$$\Rightarrow \frac{(45+50) - (20+25)}{10} = 50 - \text{correct mean}$$

$$\Rightarrow \text{correct mean} = 45$$

$$\frac{x_1^2 + \cdots + x_8^2 + 45^2 + 50^2}{10} - 50^2 = 144$$

$$\frac{x_1^2 + \cdots + x_8^2 + 20^2 + 25^2}{10} - 45^2 = V$$

$$\Rightarrow \frac{45^2 + 50^2 - 20^2 - 25^2}{10} - (50^2 - 45^2) = 144 - V$$

$$\Rightarrow \frac{65 \times 25 + 75 \times 25}{10} - 95 \times 5 = 144 - V$$

$$\Rightarrow 25 \times 14 - 95 \times 5 = 144 - V$$

13

If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, passes through the point $(6,1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is

(A) $\sqrt{11}$ (B) 4

(C) 3 (D) $2\sqrt{23}$

JEE 1

$(6,1) \rightarrow 36 + 1 - 12g + 6 - 19c = 0$

$12g + 19c = 43$ — (1)

$(+g, -3) \rightarrow g + 6c = 8$ — (2)

$c = 1$
$g = 2$

x-intercept $= 2\sqrt{(-g)^2 - (-19c)}$

$= 2\sqrt{4 + 19}$

$= 2\sqrt{23}$

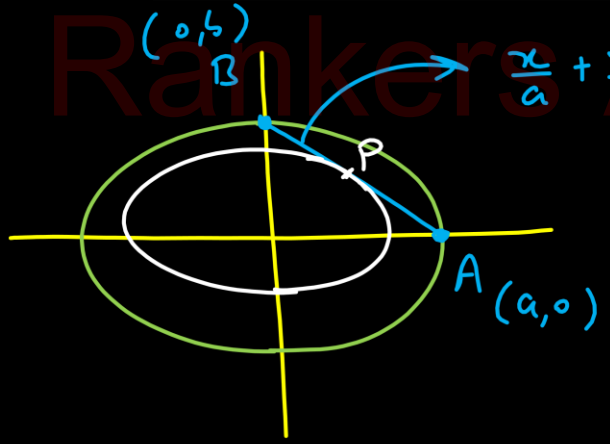
14

JEE 1

If $\alpha - \beta = \pi/2$, then the chord joining the points whose eccentric angles are α and β of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{k}$ where k is equal to _____ .

(A) 1 (B) 2

(C) 1/2 (D) 3



$\frac{x}{a}+\frac{y}{b}=1$ — solve $D=0$

$\frac{x^2}{a^2}+\frac{1}{k}\left(k\left(1-\frac{x}{a}\right)\right)^2=\frac{1}{k}$

$\frac{x^2}{a^2}+\frac{(a-x)^2}{a^2}=\frac{1}{k}$

$2x^2+a^2-2ax=\frac{a^2}{k}$

$2x^2-2ax+a^2-\frac{a^2}{k}=0$

$D = 0$

$1 \cdot a^2 - k(2)a^2\left(1 - \frac{1}{k}\right) = 0$

$1 - 2 + \frac{2}{k} = 0$

$k = 2$

15

Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(A) $\alpha = 1$ (B) $\alpha = 4$

(C) $\beta = 8$ (D) $\beta = -8$

$(A^2)^2 = (3A + \alpha I)^2$

$21A + \beta I = 9A^2 + \alpha^2 I + 6\alpha A$

$= 9(3A + \alpha I) + \alpha^2 I + 6\alpha A$

$= (27 + 6\alpha)A + (9\alpha + \alpha^2)I$

$\alpha = -1$

$\beta = -8$

16

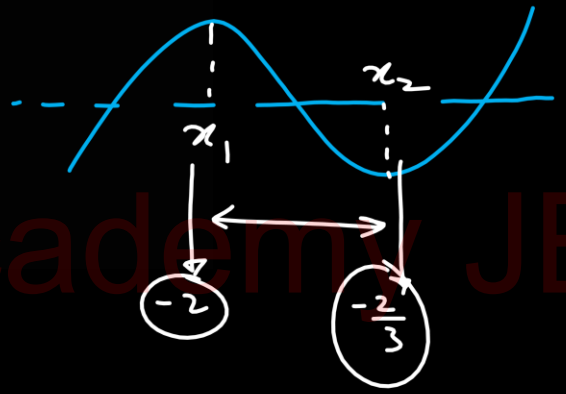
If $f(x) = x^3 + 4x^2 + \lambda x + 1$ is a monotonically decreasing function of x in the largest possible interval $\left(-2, \frac{-2}{3}\right)$ Then :-

(A) $\lambda = 4$

(B) $\lambda = 2$

(C) $\lambda = -1$

(D) λ has no real value



$f'(x) = 3x^2 + 8x + \lambda = 0$

Prod of roots $= \frac{\lambda}{3} = \frac{4}{3}$

$\lambda = 4$

17

If $A_\lambda = \begin{bmatrix} \lambda & \lambda-1 \\ \lambda-1 & \lambda \end{bmatrix}; \lambda \in N$ then

$|A_1| + |A_2| + \cdots . |A_{300}|$ is equal to

(A) $(299)^2$ (B) $(300)^2$

(C) $(301)^2$ (D) None of these

$|A_\lambda| = \lambda^2 - (\lambda-1)^2$

$= (2\lambda - 1)$

$1 + 3 + 5 + \cdots + \underbrace{\qquad\qquad}_{300\text{-term.}}$

18

JEE 1

Let $S_1 = \left\{x \in R - \{1,2\}: \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$

and $S_2 = \{x \in R: 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

Then, $S_1 \cup S_2$ is equal to

(A) $(-\infty, -2] \cup (1,2)$ (B) $(-\infty, -2] \cup [1,2]$

(C) $(-2,1] \cup (2, \infty)$. (D) $(-\infty, 2]$

$$\frac{(x+2)}{(x^2-3x+2)} \leq 0$$

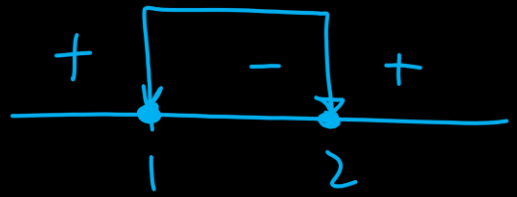
$$\frac{(x+2)}{(x-1)(x-2)} \leq 0$$

$x \in (-\infty, -2] \cup (1,2)$

$$(3^x)^2 - 3 \cdot 3^x - 9 \cdot 3^x + 27 \leq 0$$

$$3^x(3^x - 3) - 9(3^x - 3) \leq 0$$

$$(3^x - 3)(3^x - 9) \leq 0$$



19

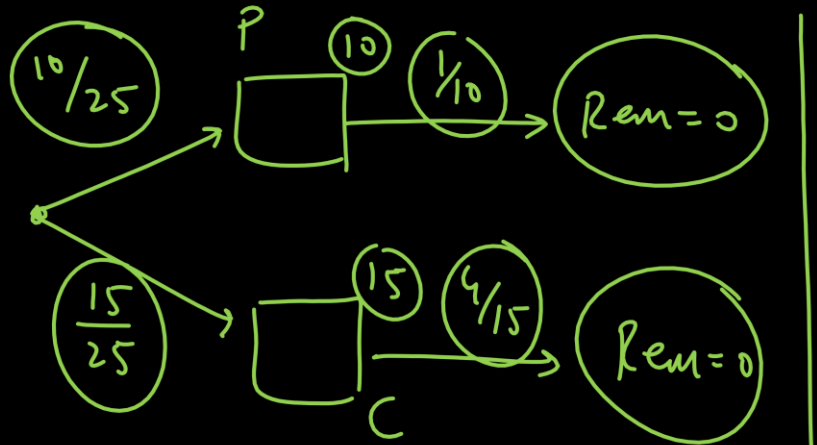
A number is selected at random from the first 25 natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, then it is divided by 2. The probability that there will be no remainder in the division is :

A. 11/30
B. 0.4
C. 0.2
D. None of these

$P = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$n(P) = 10$

$n(C) = 25 - 10 = 15$



$\frac{10}{25} \times \frac{1}{10} + \frac{15}{25} \times \frac{4}{15}$

$\frac{5}{25} = \frac{1}{5} = 0.2$

20

Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}, y(1) = 3$ be $y = y(x)$.

Then $y(2)$ is equal to:

(A) 15 (B) 11

(C) 13 (D) 17

$$\frac{dy}{dx} - \left(\frac{y}{x}\right) = \sqrt{\left(\frac{y}{x}\right)^2 + 16}$$

Let: $y = vx$

$$\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$$

$$v + x\frac{dv}{dx} - v = \sqrt{v^2 + 16}$$

$$\int \frac{dv}{\sqrt{v^2+16}} = \int \frac{dx}{x}$$

$$\ln\left|v + \sqrt{v^2+16}\right| = \ln cx$$

$$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} + 16} = cx$$

$$y + \sqrt{y^2 + 16x^2} = Cx^2$$

$(1, 3)$

$$3 + \sqrt{9 + 16} = C$$

$$\boxed{C = 8}$$

$x = 2$

$$y + \sqrt{y^2 + 64} = 8 \times 4$$

$$y^2 + 64 = (32 - y)^2$$

$$y^2 + 64 = 32 \times 32 - 64y + y^2$$

$$2 = 32 - 2y$$

$$2y = 30$$

$$\boxed{y = 15}$$

21

JEE 1

The probability distribution of X is:

X	0	1	2	3
P(X)	$\frac{1-d}{4}$	$\frac{1+2d}{4}$	$\frac{1-4d}{4}$	$\frac{1+3d}{4}$

For the minimum possible value of d, sixty times the mean of X is equal to $\Rightarrow \frac{5}{4}(60) = 75$

$\Sigma p_i x_i$

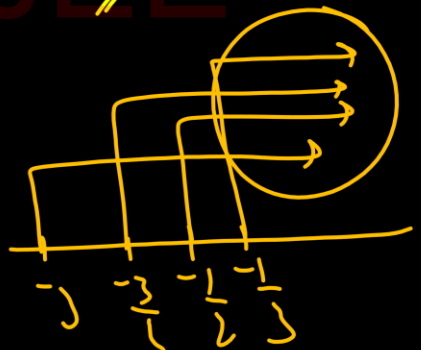
$0 + \frac{1+2d}{4} + \frac{2-8d}{4} + \frac{3+9d}{4}$

$= \boxed{\frac{6+3d}{4}}$

$\Sigma p_i = 1$

$0 \le \frac{1-d}{4} \le 1$ | $0 \le \frac{1+2d}{4} \le 1$ | $0 \le \frac{1-4d}{4} \le 1$ | $0 \le \frac{1+3d}{4} \le 1$

$d \le 1$; $d \ge -3$ | $d \ge -\frac{1}{2}$ | $1-4d \le 4$, $d \ge -\frac{3}{4}$ | $d \ge -\frac{1}{3}$

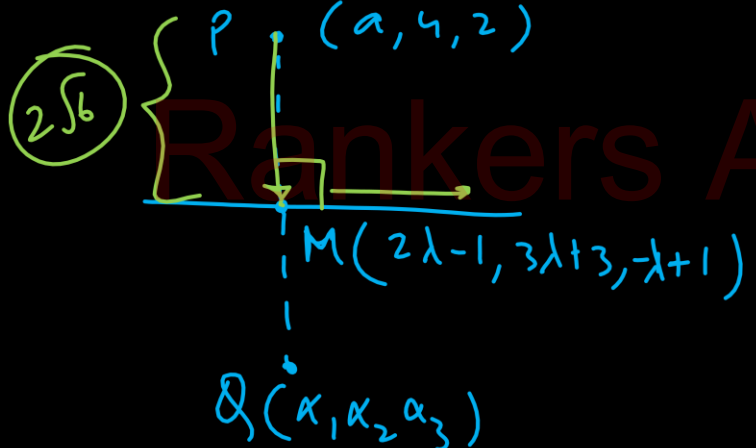


$\therefore d_{min} = -\frac{1}{3}$

22

JEE 1

If the length of the perpendicular drawn from the point $(a, 4, 2)$, $a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P on this line, then $a + \sum_{i=1}^{3} \alpha_i$ is equal to



$\vec{PM} = (2\lambda - 1 - a)\hat{i} + (3\lambda - 1)\hat{j} + (-\lambda - 1)\hat{k}$

$\vec{PM} \cdot \vec{d} = 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$14\lambda - 2a = 4$

$7\lambda - a = 2$

$\boxed{a = (7\lambda - 2)}$

$2\lambda - 1 - 7\lambda + 2$

$1 - 5\lambda$

$$|\vec{PM}| = \sqrt{(1-5\lambda)^2 + (3\lambda-1)^2 + (-\lambda-1)^2}$$

square

$$24 = \begin{pmatrix} 1+25\lambda^2-10\lambda \\ 9\lambda^2+1-6\lambda \\ \lambda^2+1+2\lambda \end{pmatrix}$$

$$24 = 35\lambda^2 + 3 - 14\lambda$$

$$35\lambda^2 - 14\lambda - 21 = 0$$

$\alpha = 1 \Rightarrow \alpha\beta = \frac{-21}{35}$

accept

$\beta = -\frac{21}{35}$

ignore $\because a > 0$

$\lambda = 1 \Rightarrow a = 5$

P (5, 4, 2)

M (1, 6, 0)

Q (−3, 8, −2)

Ans: $5 + (-3+8+(-2))$

(8)

Coefficient of in x^6 in $(1+x^4)^4(1+x^5)^5(1+x^6)^6$

$[(1+x)(1+x^2)^2(1+x^3)^3 \ldots (1+x^n)^n]$ is :-

$(1+x)(1+2x^2+x^4)(1+3x^3+3x^6+x^9)$

$(1+4x^4)(1+5x^5)(1+6x^6)(1+7x^7)(1+8x^8)\ldots\ldots$

$= 6+5+8+3+6$

$= 12+16$

$= 28$

24

JEE 1

If the sum of all the solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$$

$-1<x<1$, $x\neq 0$ is a $-\frac{4}{\sqrt{3}}$ then a is equal to

$x>0$: $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$

$2\left(2\tan^{-1}(x)\right)=\frac{\pi}{3}$

$\tan^{-1}x=\frac{\pi}{12}$

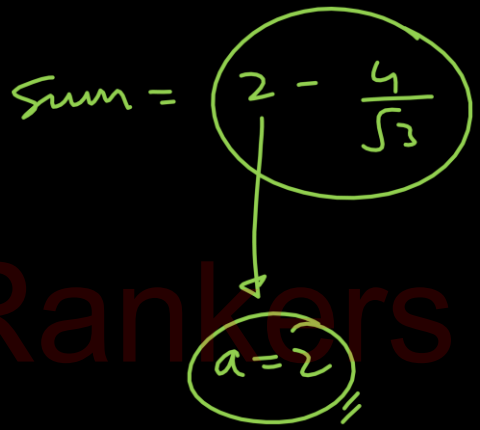
$x=\tan\left(\frac{\pi}{12}\right)=2-\sqrt{3}$

$x<0$: $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$

$4\tan^{-1}x=-\frac{2\pi}{3}$

$\tan^{-1}x=-\frac{\pi}{6}$

$x=-\frac{1}{\sqrt{3}}$

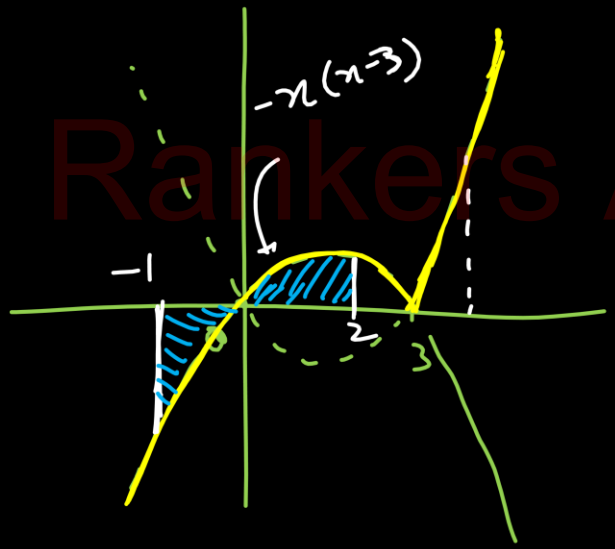


25

JEE 1

Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to

$$y = \begin{cases} x(x-3) ; & x \geq 3 \\ -x(x-3) ; & x < 3 \end{cases}$$



$$A = -\int_{-1}^{0} (-x^2+3x)\,dx + \int_{0}^{2} (-x^2+3x)\,dx$$

$$\Rightarrow 12A = 62$$