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IIT-JEE (Mains)
Batch: Growth (June) | Minor Test - 9

Time: 3:00 | **Test Date: 15th December 2024** | **Maximum Marks: 300**

Name of the Candidate: ______________________________ Roll No.____________

Centre of Examination (in Capitals): ______________________________

Candidate's Signature: ____________________ Invigilator's Signature: ____________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

2. This Test Booklet consists of 75 questions.

3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS and PART C - CHEMISTRY** having 25 questions each and every **PART** has two sections.

(i) **Section-I** contains 20 multiple choice questions with only one correct option.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

(ii) **Section-II** contains 5 questions, is an NUMERICAL VALUE.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.

5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.

7. **For the numerical based questions in Section-II of Mathematics, Physics, Chemistry, the answer should be in whole number only.**

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TEST SYLLABUS

Batch – Growth (June) | Minor Test- 09

15^{th} December 2024

Mathematics: **Binomial Theorem**

Physics: **Gravitation**

Chemistry: **Ionic Eq.**

Useful Data Chemistry:

Gas Constant	R	$= 8.314\ JK^{-1}mol^{-1}$
		$= 0.0821\ Lit\ atm\ K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}\ Js$
		$= 6.25 \times 10^{-27}\ erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66 \times 10^{-27}\ kg$
1 eV		$= 1.6 \times 10^{-19}\ J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

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PART-A-MATHEMATICS

SECTION: I

1. Let the term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ has the value p and q be the sum of the coefficients of its middle terms, then(p-q) equals

(A) 0

(B) $2.{}^9C_4$

(C) ${}^9C_5$

(D) ${}^9C_3$

Ans. **(D)**

Sol. T_{r+1} in$\left(x^2-\frac{1}{x}\right)^9is{}^9C_r\ x^{2(9-r)}\left(-\frac{1}{x}\right)^r={}^9\ C_r\cdot x^{18-3r}\cdot(-1)^r$

for term independent of x, 18 - 3r=0

$\Rightarrow r = 6$

$\therefore$ the 7^{th} term is independent of x and equals ${}^9C_6={}^9\ C_3=84$

Also, there are 10 terms, hence 5^{th}term and 6th are the two middle terms.

$T_5 = {}^9C_4\cdot x^6$, $T_6 = {}^9C_5\cdot(-x)^3$

$\therefore$ q = coefficient of 5^{th} + coefficient of 6^{th} term $= {}^9C_4-{}^9C_5=0$

hence p = 84; q = 0

$\therefore p - q = {}^9C_3$.

2. The greatest terms of the expansion $(2x+5y)^{13}$ when x = 10, y = 2 is:

(A) ${}^{13}C_5.20^8.10^5$

(B) ${}^{13}C_6.20^7.10^4$

(C) ${}^{13}C_4.20^9.10^4$

(D) None of these

Ans. **(C)**

Sol. $\frac{-T_{r+1}}{T_r}=\frac{14-r}{r}\cdot\frac{5y}{2n}=\frac{14-r}{r}\cdot\frac{1}{2}>1$

$\Rightarrow r<4\frac{2}{3}$

$\Rightarrow 5^{th}$ term is greatest $\Rightarrow\ T_5={}^{13}\ C_4(2x)^9(5y)^4$

3. Sum of the coefficient of all the integral powers of 'x' in the expansion of $\left(1+2\sqrt{x}\right)^{40}$, is

(A) $\frac{3^{40}-1}{2}$

(B) $\frac{3^{40}+1}{2}$

(C) $\frac{3^{40}}{2}$

(D) $\frac{3^{40}-2}{2}$

Ans. **(B)**

Sol. Coefficient of the integral power of 'x' are

$S = {}^{40}C_0 + {}^{40}C_2 \cdot 2^2 + {}^{40}C_4 \cdot 2^4 + \text{.........} + {}^{40}C_{40} \cdot 2^{40}$

Now $(1 + 2)^{40} = {}^{40}C_0 + {}^{40}C_1 \cdot 2 + \text{......} + {}^{40}C_{40} \cdot 2^{40}$

$(1 - 2)^{40} = {}^{40}C_0 - {}^{40}C_1 \cdot 2 + \text{......} + {}^{40}C_{40} \cdot 2^{40}$

Add

$3^{40} + 1 = 2\,[{}^{40}C_0 + {}^{40}C_2 \cdot 2^2 + {}^{40}C_4 \cdot 2^4 + \text{.........}] = 2S$

$\therefore S = \dfrac{3^{40}+1}{2}$

4. If $\sum_{k=10}^{2006} {}^{k}C_{10}$ simplifies to ${}^{n}C_p$ where p is prime then (n + p) has the value equal to

(A) 2017

(B) 2018

(C) 2019

(D) 2020

Ans. **(B)**

Sol. $S = {}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + {}^{13}C_{10} + \text{..........} + {}^{2006}C_{10}$

$= \underline{{}^{11}C_0 + {}^{12}C_1} + {}^{13}C_2 + {}^{14}C_3 + \text{..........} + {}^{2006}C_{1996}$

$= {}^{12}C_1 + {}^{12}C_2 + \text{..........} + {}^{13}C_2$ and so on

$\therefore S = {}^{2007}C_{1996} = {}^{2007}C_{11} = {}^{n}C_p$

$\therefore n + p = 2018$

5. The expansion of $(1 + x)^n$ has 3 consecutive terms with coefficients in the ratio 1 : 2 : 3 and can be written in the form ${}^{n}C_k : {}^{n}C_{k+1} : {}^{n}C_{k+2}$. The sum of all possible values of (n + k) is:

(A) 18

(B) 21

(C) 28

(D) 32

Ans. **(A)**

Sol. $\dfrac{{}^{n}C_k}{{}^{n}C_{k+1}} = \dfrac{1}{2} \Rightarrow \dfrac{n!}{k!(n-k)!}\,\dfrac{(k+1)!\,(n-k-1)!}{n!} = \dfrac{1}{2}$

or $\dfrac{k+1}{n-k} = \dfrac{1}{2}$

$2k + 2 = n - k$

$n - 3k = 2$ .... (1)

$$\frac{^nC_{k+1}}{^nC_{k+2}} = \frac{2}{3}$$

$$\frac{n!}{(k+1)!\,(n-k-1)} \cdot \frac{(k+2)!\,(n-k-2)!}{n!} = \frac{2}{3}$$

$$\frac{k+2}{n-k-1} = \frac{2}{3}$$

$3k + 6 = 2n - 2k - 2$
$2n - 5k = 8$ .... (2)
From (1) and (2) $n = 14$ and $k = 4$
$\therefore n + k = 18$

6. If $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$ then $\sum_{r=0}^{n} \frac{r}{^nC_r}$ equals

(A) $(n-1)\, a_n$

(B) $n\, a_n$

(C) $n\, a_n/2$

(D) cannot be determined

Ans. **(C)**

Sol.

$$\mathbf{a_n} = \frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + \cdots \ldots.$$

$$S = \frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + \frac{3}{^nC_3} + \ldots\ldots\ldots + \frac{n}{^nC_n}$$

$$S = \frac{n}{^nC_0} + \frac{n-1}{^nC_n} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$2S = n\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + \ldots\ldots\right] = \frac{na_n}{2}$$

7. Let C_r's denotes the combinational coefficients in the expansion of the binomial $(1+x)^{14}$, then the value of the sum $\sum_{r=1}^{15} r \cdot C_{r-1}$ is

(A) 2^{15}

(B) 2^{16}

(C) 2^{17}

(D) 2^{18}

Ans. **(C)**

Sol. $S = 1 \cdot C_0 + 2 \cdot C_1 + 3 \cdot C_2 + \cdots \ldots + 15 \cdot C_{14}$

Re-writing the sum as reverse order

$S = 15 \cdot \bar{C}_0 + 14\ C_1 + 13\ C_2 + \ldots\ldots + C_{14}.$

$2S = 16[C_0 + C_1 + C_2 + \cdots \ldots + C_{14}] = 16 \cdot 2^{14}$
$S = 2^3 \cdot 2^{14} = 2^{17} \Rightarrow (B)$

Alternatively:

Consider expansion of $(1 + x)^{14}$. Multiply both sides by x. Differentiate w.r.t. x and put x=1 to get the answer.

8. Number of rational terms in the expansion of $\left(2^{\frac{1}{7}}+3^{\frac{1}{4}}\right)^{198}$ is equal to

(A) 7

(B) 8

(C) 9

(D) 10

Ans. **(A)**

Sol. T_{r+1}= $198C_r(2)^{\frac{198-r}{7}}(3)^{\frac{r}{4}}$r= 0, 1, 2 . . . ., 198

∴For r= 16, 44, 72, 100, 128, 156, 184 the terms in the expansion are rational.

9. The value of expression $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}$, is equal to

(A) n + 2

(B) $\frac{n}{2}$

(C) n

(D) n + 1

Ans. **(B)**

Sol.

$$\sum_{r=0}^{n-1} \frac{1}{1+\frac{{}^nC_{r+1}}{{}^nC_r}} = \sum_{r=0}^{n-1} \frac{r+1}{n+1} = \frac{1}{n+1}\sum_{r=0}^{n-1}(r+1)$$

$= \frac{n}{2}$

10. In the expansion of $\left(2x+\frac{1}{x}\right)^6$,if the coefficient of $x^2$is k times the coefficient of x^{-2} then k equals

(A) 4

(B) 8

(C) 16

(D) 32

Ans. **(C)**

Sol.

$$T_{r+1} = 6C_r(2x)^{6-r} \cdot \frac{1}{(2x)^r} = 6C_r \cdot 2^{6-2r} \cdot x^{6-2r}$$

if $\quad 6-2r=2 \quad \Rightarrow \quad r=2$

if $\quad 6-2r=-2 \quad \Rightarrow \quad r=4$

coefficient of $x^2 = {}^6C_2 \cdot 2^2 = 60$

coefficientof$x^2 = {}^6C_4 \cdot \frac{1}{2^2} = \frac{15}{4}$

$\Rightarrow k = 16$

11. The remainder when $\left(\sum_{r=1}^{5} {}^{20}C_{2r-1}\right)^6$ is divided by 11, is

(A) 1

(B) 2

(C) 3

(D) 4

Ans. **(C)**

Sol.

$$\left(\sum_{r=1}^{5} {}^{20}C_{2r-1}\right)^6$$

$$\sum_{r=1}^{5} {}^{20}C_{2r-1} = 20C_1 + 20C_3 + 20C_5 + 20C_7 + 20C_9$$

We know

${}^{20}C_1 + {}^{20}C_3 + \ldots\ldots + {}^{20}C_{19} = 2^{19}$

$2({}^{20}C_1 + {}^{20}C_3 + \ldots\ldots + {}^{20}C_9) = 2^{19}$

${}^{20}C_1 + {}^{20}C_3 + \ldots\ldots + {}^{20}C_{18} = 2^{18}$

$(2^{18})^6 = 2^{108}$ when divided by 11.

$= 8(2^5)^{21}$

$= 8(33 - 1)^{21}$

$= 8((-1)^{21} + {}^{21}C_1(3) + \ldots\ldots\ldots\ldots)$ divisible by 11

remainder $= -8$

12. If 5^{97} is divided by 52, then the remainder obtained is

(A) 3

(B) 5

(C) 4

(D) 0

Ans. **(B)**

Sol. We know that, $5^4 = 625 = 52 \times 12 + 1$

$\Rightarrow 5^4 = 52\lambda + 1$, where λ is a positive integer.

$$\Rightarrow (5^4)^{24} = (52\lambda + 1)^{24}$$
$$= {}^{24}C_0(52\lambda)^{24} + {}^{24}C_1(52\lambda)^{23} + {}^{24}C_2(52\lambda)^{22} + \cdots + {}^{24}C_{23}(52\lambda) + {}^{24}C_{24} \text{ (by binomial theorem)}$$
$$\Rightarrow 5^{96} = 52[\,{}^{24}C_0 52^{23}\lambda^{24} + {}^{24}C_1 52^{23}\lambda^{22} + \cdots + {}^{24}C_{23}\lambda] + 1$$
$$= (a \text{ multiple of } 52) + 1$$

On multiplying both sides by 5 , we get

$5^{97} = 5^{96} \cdot 5 = 5$ (a multiple of 52) $+ 5$

Hence, the required remainder is 5 .

13. The coefficient of the term independent of x in $\left[\sqrt{\left(\frac{x}{3}\right)}+\frac{\sqrt{3}}{x^2}\right]^{10}$ is

(A) $\frac{5}{3}$

(B) $\frac{4}{5}$

(C) 6

(D) $\frac{1}{2}$

Ans. **(A)**

Sol. Given, $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{x^2}\right]^{10}$

General term, $T_{r+1}={}^{10}C_r\left(\frac{x}{3}\right)^{\frac{1}{2}(10-r)}\left(\frac{\sqrt{3}}{x^2}\right)^r$

$\Rightarrow T_{r+1}={}^{10}C_r\left(\frac{1}{3}\right)^{\frac{10-r}{2}}(\sqrt{3})^r x^{\frac{1}{2}(10-r)-2r}$

For the term independent of x, put

$\frac{1}{2}(10-r)-2r=0$

$\Rightarrow\quad r=2$

$\therefore\quad T_{2+1}=T_3={}^{10}C_2\left(\frac{1}{3}\right)^{\frac{8}{2}}(\sqrt{3})^2$

$=\quad 45\times\frac{1\times 3}{81}=\frac{5}{3}$

14. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

(A) $ab=1$

(B) $\frac{a}{b}=1$

(C) $a+b=1$

(D) $a-b=1$

Ans. **(A)**

Sol. In the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$

$$T_{r+1}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$$

$$={}^{11}C_r\frac{a^{11-r}}{b^r}\cdot x^{22-3r}$$

For the coefficient of x^7, put $22-3r=7$

$\Rightarrow r=5$

$\therefore\ T_6={}^{11}C_5\frac{a^5}{b^5}\cdot x^7$

[redacted]

$\therefore$ Coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ is ${}^{11}C_5\frac{a^6}{b^5}$

Similarly, the coefficient of x^{-7} in the expansion $\left(ax-\frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6\frac{a^5}{b^6}$

Now, ${}^{11}C_3\frac{a^6}{b^5}={}^{11}C_6\frac{a^5}{b^6}$

$\Rightarrow ab=1$

15. Let $C_1, C_2, C_3 \ldots$ are the usual binomial coefficients where $C_r={}^nC_r$. Let $S=C_1+2C_2+3C_3+\cdots+nC_n$, then S is equal to

(A) $n2^n$

(B) 2^{n-1}

(C) $n2^{n-1}$

(D) 2^{n+1}

Ans. **(C)**

Sol. Let $S=C_1+2C_2+3C_3+\cdots+nC_n=\sum_{r=1}^{n} r\cdot {}^nC_r$

$$=\sum_{r=1}^{n} r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}\left[\because {}^nC_r=\frac{n}{r}\,{}^{n-1}C_{r-1}\right]$$

$$=n\sum_{r=1}^{n}{}^{n-1}C_{r-1}$$

$$=n\left[{}^{n-1}C_0+{}^{n-1}C_1+{}^{n-1}C_2+\cdots+{}^{n-1}C_{n-1}\right]$$

$$=n2^{n-1}$$

16. The first integral term in the expansion of $(\sqrt{3}+\sqrt[3]{2})^9$, is the

(A) 2^{nd} term

(B) 3^{rd} term

(C) 4^{th} term

(D) 5^{th} term

Ans. **(C)**

Sol. 4^{th} term

$$(\sqrt{3}+\sqrt[3]{2})^9=\left(3^{\frac{1}{2}}+2^{\frac{1}{3}}\right)^9$$

$$\therefore T_{r+1}={}^9C_r\left(3^{\frac{1}{2}}\right)^{9-r}\left(2^{\frac{1}{3}}\right)^r$$

For first integral term,

$r=3$

$\Rightarrow T_{3+1}={}^9C_3 3^3\cdot 2^1$

i.e., $T_{3+1}=T_4$

17. If $(5+2\sqrt{6})^n=I+f, n$ and $I\in N, 0\le f<1$ then I will be

(A) Even number

(B) Odd number

(C) Negative integer

(D) Can't predict

Ans. **(B)**

Sol. $I + f = (5+2\sqrt{6})^n$ ...(i)

$f' = (5-2\sqrt{6})^n$ ...(ii)

adding (i) and (ii) we get the answer.

18. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is

(A) $\frac{4}{9}$

(B) $\frac{1}{9}$

(C) $\frac{1}{4}$

(D) $\frac{9}{4}$

Ans. **(D)**

Sol.

$$\left(\frac{x^{\frac{1}{3}}}{3}+\frac{x^{\frac{-2}{3}}}{2}\right)^{18}$$

$$t_7 = {}^{18}C_6\left(\frac{x^{\frac{1}{3}}}{3}\right)^{12}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{6} = {}^{18}C_6\frac{1}{(3)^{12}}\cdot\frac{1}{2^6}$$

$$t_{13} = {}^{18}C_{12}\left(\frac{x^{\frac{1}{3}}}{3}\right)^{6}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12} = {}^{18}C_{12}\frac{1}{(3)^{6}}\cdot\frac{1}{2^{12}}\cdot x^{-6}$$

$$\mathbf{m} = {}^{18}\mathbf{C}_6\cdot 3^{-12}\cdot 2^{-6};\ \mathbf{n} = {}^{18}\mathbf{C}_{12}\cdot 2^{-12}\cdot 3^{-6}$$

$$\left(\frac{n}{m}\right)^{\frac{1}{3}} = \left(\frac{2^{-12}\cdot 3^{-6}}{3^{-12}\cdot 2^{-6}}\right)^{3} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$$

19. ${}^{n-1}C_r = (k^2-8)\ {}^nC_{r+1}$ if and only if:

(A) $2\sqrt{2} < k \le 3$

(B) $2\sqrt{3} < k \le 3\sqrt{2}$

(C) $2\sqrt{3} < k < 3\sqrt{3}$

(D) $2\sqrt{2} < k < 2\sqrt{3}$

Ans. **(A)**

Sol. ${}^{n-1}C_r = (k^2-8){}^nC_{r+1}$

$r+1 \ge 0, r \ge 0$

$r \ge 0$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = k^2-8$$

$\frac{r+1}{n} = k^2 - 8$

$\Rightarrow k^2 - 8 > 0 \ (k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$

$\mathbf{k} \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty) \ldots (\mathrm{I})$

$\therefore \mathbf{n} \geq \mathbf{r} + 1, \frac{\mathbf{r+1}}{\mathbf{n}} \leq 1$

$\Rightarrow \mathbf{k}^2 - 8 \leq 1$

$\mathrm{k}^2 - 9 \leq 0$

$-3 \leq \mathbf{k} \leq 3 \ldots (\Pi)$

rom equation (I) and (II) we get $k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$

20. If for some m, n ${}^6C_m$ + 2 (${}^6C_{m+1}$) + ${}^6C_{m+2}$ > ${}^8C_3$ and ${}^{n-1}P_3$: ${}^nP_4$ 1:8, then ${}^nP_{m+1}$ + ${}^{n+1}C_m$ is equal to

(A) 380

(B) 376

(C) 384

(D) 372

Ans. **(D)**

Sol. ${}^6\mathrm{C}_\mathrm{m} + 2({}^6\mathrm{C}_{\mathrm{m}+1}) + {}^6\mathrm{C}_{\mathrm{m}+2} >^8 \mathrm{C}_3 \ {}^7\mathrm{C}_{\mathrm{m}+1} + {}^7\mathrm{C}_{\mathrm{m}+2} >^8 \mathrm{C}_3 \ {}^8\mathrm{C}_{\mathrm{m}+2} > \ {}^8\mathrm{C}_3$

$\therefore \mathrm{m} = 2$

And ${}^{\mathrm{n}-1}\mathrm{P}_3 \colon {}^{\mathrm{n}}\mathrm{P}_4 = 1:8$

$\frac{(\mathrm{n}-1)(\mathrm{n}-2)(\mathrm{n}-3)}{\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2)(\mathrm{n}-3)} = \frac{1}{8}$

$\therefore \mathbf{n} = 8$

$\therefore {}^\mathrm{n} \mathrm{P}_{\mathrm{m}+1} + {}^{\mathrm{n}+1}\mathrm{C}_\mathrm{m} = {}^8 \mathrm{P}_3 + {}^9\mathrm{C}_2$

$= 8 \times 7 \times 6 + \frac{9 \times 8}{2}$

= 372

SECTION: II

21. For $\mathrm{n} \in \mathrm{N}$, in the expansion of $\left(\sqrt[4]{\mathrm{x}^{-3}} + a\sqrt[4]{\mathrm{x}^5}\right)^\mathrm{n}$, the sum of all the binomial coefficients lie between 200 and 400 . Also, the term independent of x is 448 , then the value of a is

Ans. **(8)**

Sol.

[redacted]

$200 < {}^nC_0 + {}^nC_1 + \cdots + {}^nC_n < 400$
$\Rightarrow 200 < 2^n < 400$
$\Rightarrow n = 8$
$T_{r+1} = 8^8C_r\left(\sqrt[4]{x^{-3}}\right)^{8-r}\left(a\sqrt[4]{x^5}\right)^r$
$\Rightarrow\ T_{r+1} = {}^8C_r a^{rx^{2r-6}}$

For this term to be independent of **x**,

$2r - 6 = 0 \Rightarrow r = 3$
$T_4 = {}^8C_3 a^3 \Rightarrow 448 = 56a^3 \Rightarrow a^3 = 8$
$\Rightarrow a = 2$

22. If the 6^{th} term in the expansion of $\left(\frac{1}{x^{\frac{8}{3}}} + x^2\log_{10} x\right)^8$ is 5600 , then the value of x is

Ans. **(10)**

Sol.

$T_6 = {}^8C_5\left(\frac{1}{x^{\frac{8}{3}}}\right)^{8-5}(x^2\log_{10} x)^5$
$= 56\left(\frac{1}{x^8}\right)(x^2\log_{10} x)^5$
$= 56x^2(\log_{10} x)^5$
$T_6 = 5600$
$\Rightarrow 56x^2(\log_{10} x)^5 = 5600$
$\Rightarrow x^2(\log_{10} x)^5 = 100$
$\Rightarrow x^2(\log_{10} x)^5 = 10^2$
$\Rightarrow x = 10$

23. Let α and β be non-zero complex numbers. If the coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{\beta x}\right)$ is equal to the coefficient of x^{-7} in $\left(\alpha x - \frac{1}{\beta x^2}\right)^{11}$ then find the value of $(\alpha\beta)$.

Ans. **(1)**

Sol. The coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{\beta x}\right)^{11} = 11C_5(\alpha)^6(\beta)^{-5}$

Also, the coefficient of x^{-7} in $\left(\alpha \mathbf{x} - \frac{1}{\beta \mathbf{x}^2}\right)^{11} = 11C_6(\alpha)^5\ (\beta)^{-6}$

Now according to question, we get

$11_{C_5} \cdot \frac{(\alpha)^6}{(\beta)^5} = \frac{{}^{11}C_6 \cdot (\alpha)^5}{(\beta)6} \quad \Rightarrow \quad \alpha\beta$

=1

24. Let P be the 7^{th} term from the beginning and Q be the 7^{th} term from the end in the expansion of $\left(\sqrt[3]{3} + \frac{1}{\sqrt[3]{4}}\right)^n$ where n ∈ N. If 12 P = Q, then find the value of n.

Ans. **(9)**

Sol.

[redacted]

$P = {}^{n}C_6 \left(3^{\frac{1}{3}}\right)^{n-6} \cdot \left(4^{\frac{-1}{3}}\right)^{6}$

$Q = {}^{n}C_{n-6} \left(3^{\frac{1}{3}}\right)^{6} \cdot \left(4^{\frac{-1}{3}}\right)^{n-6}$

$\therefore \frac{Q}{P} = 12 \Rightarrow (12)^{\frac{n-6}{3}} = (12)^1 \Rightarrow \frac{n-6}{3}$

$= 1 \Longrightarrow n = 9$ Ans.

25. Number of integral terms in the expansion of $\left((7)^{1/2} + (11)^{1/6}\right)^{824}$ is equal to ________

Ans. **(60)**

Sol. General term in expansion of $\left((7)^{1/2} + (11)^{1/6}\right)^{824}$ is $t_{r+1} = {}^{824}C_r (7)^{\frac{824-r}{2}} (11)^{r/6}$

For integral term, r must be multiple of 6.

Hence $r = 0, 6, 12, \ldots\ldots\ldots\ldots 822$

PART-B-PHYSICS

SECTION: I

26. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is

(A) $\frac{R}{\sqrt{2}}$

(B) $\frac{R}{2}$

(C) $\sqrt{2}R$

(D) $2R$

Ans. **(D)**

Sol.

$$\text{As, } g(h) = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g}{9} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \left(1+\frac{h}{R}\right) = 3$$

$$\Rightarrow \frac{h}{R} = 2 \Rightarrow h = 2R$$

27. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is

(A) $\sqrt{\frac{Gm}{2R}}$

(B) $\sqrt{\frac{Gm}{R}}$

(C) $\sqrt{\frac{Gm}{4R}}$

(D) $\sqrt{\frac{Gm}{3R}}$

Ans. **(C)**

Sol.

$$\frac{mv^2}{R} = \frac{Gm^2}{(2R)^2}$$

$$v = \sqrt{\frac{Gm}{4R}}$$

28. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(A) $\frac{5GmM}{6R}$

(B) $\frac{2GmM}{3R}$

(C) $\frac{GmM}{2R}$

(D) $\frac{GmM}{3R}$

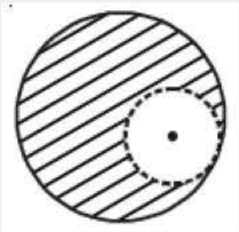
Ans. **(A)**

Sol. At surface, $E = -\frac{GMm}{R}$

In orbit, $E = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$

$\Rightarrow$ Required energy $= \frac{5GMm}{6R}$

29. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is



(G = gravitational constant)

(A) $\frac{-GM}{2R}$

(B) $\frac{-GM}{R}$

(C) $\frac{-2GM}{3R}$

(D) $\frac{-2GM}{R}$

Ans. **(B)**

Sol. $V = V_1 - V_2$

$$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$V_2 = -\frac{3G\left(\frac{M}{8}\right)}{2\left(\frac{R}{2}\right)}$$

$$\Rightarrow V = \frac{-GM}{R}$$

30. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is

(A) $\frac{L}{m}$

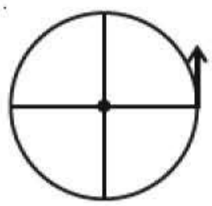
(B) $\frac{4L}{m}$

(C) $\frac{L}{2m}$

(D) $\frac{2L}{m}$

Ans. **(C)**

Sol.



$\therefore mv_0R = L$

$\therefore v_0 = \dfrac{L}{mR}$

$\therefore T = \dfrac{2\pi R}{v_0}$

Areal velocity $= \dfrac{\pi R^2}{T}$

$\Rightarrow \dfrac{dA}{dt} = \dfrac{\pi R^2 v_0}{2\pi R} = \dfrac{Rv_0}{2} = \dfrac{L}{2m}$

31. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be

(A) $\frac{2}{\sqrt{3}}$s

(B) $\frac{3}{2}$s

(C) $\frac{\sqrt{3}}{2}$s

(D) $2\sqrt{3}$s

Ans. **(D)**

Sol.

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$T_2 = 2\sqrt{\frac{g_1}{g_2}}$

$g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}$

$T_2 = 2\sqrt{3}$ s

32. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling radially towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be

(A) Such that it escapes to infinity

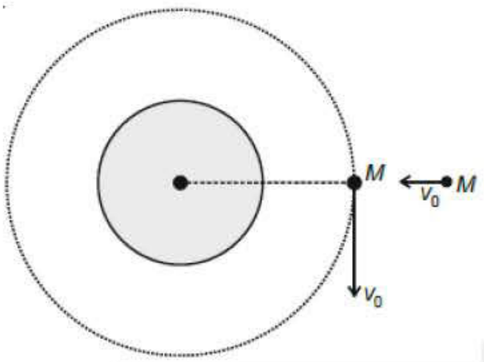
(B) In a circular orbit of a different radius

(C) In an elliptical orbit

(D) In the same circular orbit of radius R

Ans. **(C)**

Sol.



$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision

$$mv_0(-\hat{j}) + mv_0(-\hat{\imath}) = 2m\vec{v}$$
$$\vec{v} = -\frac{v_0}{2}\hat{\imath} - \frac{v_0}{2}\hat{j}$$
$$|\vec{v}| = \frac{v_0}{\sqrt{2}} = 0.7v_0$$
$$\because v < v_0$$

$\therefore$ The path will be elliptical.

33. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of inner radius a and outer radius $2a$ and mass $2M$. The gravitational field at distance $3a$ from the centre will be

(A) $\frac{GM}{9a^2}$

(B) $\frac{2GM}{9a^2}$

(C) $\frac{GM}{3a^2}$

(D) $\frac{2GM}{3a^2}$

Ans. **(C)**

Sol. $E = \frac{GM}{(3a)^2} + \frac{2GM}{(3a)^2}$

$E = \frac{GM}{3a^2}$

34. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is

(A) 1

(B) 4

(C) 3

(D) 2

Ans. **(B)**

Sol.

$$v=\sqrt{\frac{2GM}{R}}$$

$$\frac{V_1}{V_2}=\sqrt{\frac{M_1R_2}{M_2R_1}}$$

35. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance ' r ' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as

(A) $T^2 \propto \frac{1}{R^3}$

(B) $T^2 \propto R$

(C) $T \propto R$

(D) $T^2 \propto R^3$

Ans. **(B)**

Sol. $m\omega^2 R=\frac{GMm}{R^2}$

$M=\int_0^R 4\pi r^2 dr\left(\frac{K}{r}\right)=2\pi K R^2$

$\omega^2 \mathrm{R}=2 \mathrm{G}\pi \mathrm{~K}$

$\mathrm{T}^2 \propto \mathrm{R}$

36. Two planets have masses M and $16M$ and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is

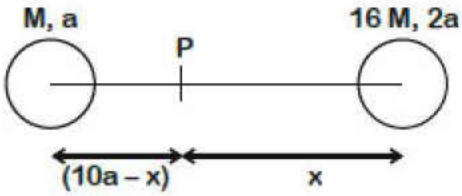
(A) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(B) $\sqrt{\frac{GM^2}{ma}}$

(C) $2\sqrt{\frac{GM}{a}}$

(D) $4\sqrt{\frac{GM}{a}}$

Ans. **(A)**



Sol.

Velocity should be given so as to reach a point where field is zero.

$$\frac{16M}{x^2}=\frac{M}{(10a-x)^2}$$
$$x=8a$$

By CoE

$$-\frac{GMm}{8a}-\frac{16GMm}{2a}+\frac{1}{2}mv^2=-\frac{16GMm}{8a}-\frac{GMm}{2a}$$

$$v=\frac{3}{2}\sqrt{\frac{5GM}{a}}$$

37. Consider two satellites S_1 and S_2 with periods of revolution 1 hr and 8 hr respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is:

(A) 1:4

(B) 1:8

(C) 2:1

(D) 8:1

Ans. **(D)**

Sol.

$$\frac{\omega_1}{\omega_2}=\left(\frac{2\pi}{T_1}\right)\times\left(\frac{T_2}{2\pi}\right)=\frac{T_2}{T_1}=\frac{8}{1}$$

38. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be

(A) $\frac{2}{9}F$

(B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$

(D) F

Ans. **(C)**

Sol.

Let the masses are m and distance between them is l, then $F=\frac{Gm^2}{l^2}$.

When 1/3rd mass is transferred to the other then masses will be $\frac{4m}{3}$ and $\frac{2m}{3}$. So new force will be

$$F' = \frac{G\frac{4m}{3}\times\frac{2m}{3}}{l^2} = \frac{8}{9}\frac{Gm^2}{l^2} = \frac{8}{9}F$$

39. Two satellites A and B, having masses in the ratio $4:3$, are revolving in circular orbits of radii $3r$ and $4r$ respectively around the earth. The ratio of total mechanical energy of A to B is

(A) 9:16

(B) 16:9

(C) 1:1

(D) 4:3

Ans. **(B)**

Sol.

$$U = -\frac{GM_e m}{2r}$$

So, $\frac{U_A}{U_B} = \frac{m_A}{m_B}\times\frac{r_B}{r_A}$

$$= \frac{4}{3}\times\frac{4}{3} = \frac{16}{9}$$

40. If the radius of the earth were to shrink by 1%, its mass remaining the same, the value of g on the earth's surface would

(A) increase by 0.5%

(B) increase by 2%

(C) decrease by 0.5%

(D) decrease by 2%.

Ans. **(B)**

Sol. $g = \frac{GM}{R^2} \Rightarrow \frac{dg}{g} = -2\frac{dR}{R}$

$\frac{dR}{R} = -1\% \Rightarrow \frac{dg}{g} = 2\%$

41. Two thin rings each of radius R are coaxially placed at a distance R. The rings have a uniform mass distribution and have mass m_1 and m_2 respectively. Then the work done in moving a mass m from centre of one ring to that of the other is

(A) zero

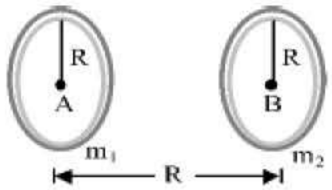
(B) $\frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

(C) $\frac{\text{Gm}(\sqrt{2})(m_1+m_2)}{R}$

(D) $\frac{\text{Gmm}_1(\sqrt{2}+1)}{\text{m}_2\text{R}}$

Ans. **(B)**

Sol.



$$V_A = \begin{pmatrix} \text{Potential at} \\ \text{Adueto A} \end{pmatrix} + \begin{pmatrix} \text{Potential at} \\ \text{Adueto B} \end{pmatrix}$$

$$\Rightarrow V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and } V_B = \begin{pmatrix} \text{Potential at} \\ \text{B due to A} \end{pmatrix} + \begin{pmatrix} \text{Potential at} \\ \text{B due to B} \end{pmatrix}$$

$$\Rightarrow V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A\to B} = m(V_B - V_A)$

$$\Rightarrow W_{A\to B} = \frac{Gm(m_1 - m_2)(\sqrt{2} - 1)}{\sqrt{2R}}$$

42. Three identical point masses, each of mass 1 kg lie in the $x - y$ plane at points $(0,0), (0,0.2 \text{ m})$ and $(0.2 \text{ m}, 0)$. The net gravitational force on the mass at the origin is

(A) $1.67 \times 10^{-9}(\hat{\imath} + \hat{\jmath})\text{N}$

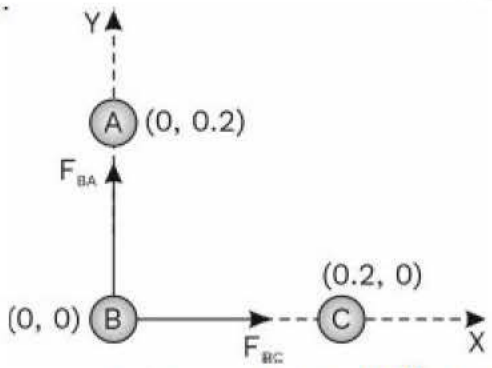
(B) $3.34 \times 10^{-10}(\hat{\imath} + \hat{\jmath})\text{N}$

(C) $1.67 \times 10^{-9}(\hat{\mathbf{i}} - \hat{\mathbf{j}})\text{N}$

(D) $3.34 \times 10^{-10}(\hat{\imath} - \hat{\jmath})\text{N}$

Ans. **(A)**

Sol. Let particle A lies at origin, particles B and C on y and X -axis respectively



$$\overrightarrow{F_{BC}} = \frac{Gm_C m_B}{r_{BC}^2}\hat{i} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.2)^2}\hat{i} = 1.67 \times 10^{-9}\hat{i}\text{ N}$$

Similarly, $\overrightarrow{F_{BA}} = 1.67 \times 10^{-9}\hat{\jmath}\text{ N}$

$\therefore$ Net force on particle A

$$\vec{F} = \vec{F}_{BC} + \vec{F}_{BA} = 1.67 \times 10^{-9}(\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{N}$$

43. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(A) zero

[redacted]

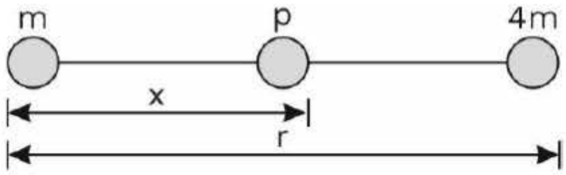
(B) $-\frac{4Gm}{r}$

(C) $-\frac{6Gm}{r}$

(D) $-\frac{9Gm}{r}$

Ans. **(D)**

Sol. Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \frac{Gm}{x^2} = \frac{G(4\ m)}{(r-x)^2} \text{ or } \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4} \text{ or } x = \frac{r}{3} \ldots \text{ (i)}$$

Gravitational potential at a point P is

$$v = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$v = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r-\frac{r}{3}\right)} = -9\frac{Gm}{r} \qquad \text{Using (i)}$$

44. In a certain region of space, the gravitational field is given by $-k/r$, where r is the distance and k is a constant. If the gravitational potential at $r = r_0$ be V_0, then what is the expression for the gravitational potential (V):

(A) $V_0 + k\ln(r/r_0)$

(B) $V_0 + k\ln(r_0/r)$

(C) $V_0 + k\log(r/r_0)$

(D) $V_0 + k\log(r_0/r)$

Ans. **(A)**

Sol. Gravitational field = -gravitational potential gradient

$$= -\frac{dV}{dr} = -\frac{k}{r}$$

Integrating, we get;

$$\int_{V_0}^{V} dV = \int_{r_0}^{r} \frac{k}{r} dr \Rightarrow [V]_{V_0}^{V} = k[\ln r]_{r_0}^{r}$$

$$\Rightarrow V - V_0 = k[\ln r - \ln r_0]$$

$$\Rightarrow V = V_0 + k\ln(r/r_0)$$

45. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is:

(A) $(3/2)\mathrm{mv}^2$

(B) mv^2

(C) $2mv^2$

(D) $(1/2)\mathrm{mv}^2$

Ans. **(B)**

Sol. At height r from center of earth. orbital velocity $=\sqrt{\frac{GM}{r}}$

By energy conservation

$$\text{KE of m}+\left(-\frac{\mathrm{GMm}}{\mathrm{r}}\right)=0+0$$

(At infinity, $\mathrm{PE}=\mathrm{KE}=0$)

$$\Rightarrow \text{ KE of ' m}' = \frac{\mathrm{GMm}}{\mathrm{r}}=\left(\sqrt{\frac{\mathrm{GM}}{\mathrm{r}}}\right)^2 \mathrm{m}=\mathrm{mv}^2$$

SECTION: II

46. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i=v_e\sqrt{\frac{10}{x}}$. The value of x will be

Ans. **(11)**

Sol.

$$-\frac{GM_e m}{R}+\frac{1}{2}mv^2=-\frac{Gm_e m}{11R}$$

$$v=\sqrt{\frac{20}{11}\frac{\mathrm{GM}_e}{R}}$$

$$v_e=\sqrt{\frac{2\mathrm{GM}_e}{R}}$$

$$v=\sqrt{\frac{10}{11}}\cdot v_e$$

47. The radius in kilometer to which the present radius of earth (R = 6400kms) to be compressed so that the escape velocity is increased 10 times is

Ans. **(64)**

Sol.

$$v_e = \sqrt{\frac{2GM}{R}} \qquad 10v_e = \sqrt{\frac{2GM}{R'}}$$

$$\Rightarrow R' = \frac{R}{100} = 64 \text{ km}$$

48. A planet revolves about the sun in elliptical orbit. The areal velocity $\left(\frac{dA}{dt}\right)$ of the planet is $4.0 \times 10^{16}\ m^2/s$. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is

Ans. **(40)**

Sol. $\frac{L}{2m} = \frac{dA}{dt}$ (L = angular momentum)

$$\frac{mv_{max}r_{min}}{2\ m} = \frac{dA}{dt}; v_{max} = \frac{2dA/dt}{r_{min}} = 40$$

49. A satellite is seen after each 8 hours over equator at a place on the earth when its sense of rotation is opposite to the earth. The time interval after which it can be seen at the same place when the sense of rotation of earth and satellite is same will be:

Ans. **(24)**

Sol. Given $8 = \frac{2\pi}{\omega_1+\omega_2} = \frac{2\pi}{\frac{2\pi}{T_1}+\frac{2\pi}{T_2}}, T_1 = 24$ hours for earth.

$\Rightarrow T_2 = 12$ hours (T_2 being the time period of satellite, it will remain same as the distance from the centre of the earth remains constant).

$$\Rightarrow T = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{\frac{2\pi}{T_2} - \frac{2\pi}{T_1}} = 24 \text{ hours}$$

50. A uniform solid sphere of mass M and radius R is surrounded symmetrically by a uniform thin spherical shell of equal mass and radius $2R$. The value of gravitational potential at a distance $(3/2)R$ from the centre is $-\frac{(A)}{6}\frac{GM}{R}$. Then find the value of A.

Ans. **(7)**

Sol. Due to solid sphere, Gravitational potential,

$$V_1 = -\frac{GM}{3R/2} = -\frac{2GM}{3R}$$

Due to spherical shell,

Gravitational potential, $V_2 = -\frac{GM}{2R}$

Net Gravitational potential

$$= -\frac{2GM}{3R} - \frac{GM}{2R} = -\frac{7GM}{6R}$$

[redacted]

PART-C-CHEMISTRY

SECTION: I

51. Nicotinic acid ($K_a = 1.4 \times 10^{-5}$) is represented by the formula HNiC. Calculate its per cent dissociation in a solution which contains 0.10 mole of nicotinic acid per 2.0 litre of solution.

(A) 0.1 %

(B) 1 %

(C) 10 %

(D) 2 %

Ans. **(A)**

Sol. Given, $C = (0.1/2) = 5 \times 10^{-2}$ mol litre^{-1}; $K_a = 1.4 \times 10^{-5}$

HNiC	$\Rightarrow$	H^+	+	NiC^-
C		0		0
C (1- α)		$C\alpha$		$C\alpha$

For Nicotinic acid:

$\therefore K_a = \frac{C\alpha^2}{(1-\alpha)} = C\alpha^2$ ($\because \alpha$ is small $\therefore 1 - \alpha = 1$)

$\therefore 1.4 \times 10^{-5} = 5 \times 10^{-2} \times \alpha^2$

$\therefore \alpha = 1.67 \times 10^{-2}$ or 1.67%

52. Calculate the conc. of fluoroacetic acid when $[H^+] = 1.50 \times 10^{-3}$M. K_a of acid $= 2.6 \times 10^{-3}$.

(A) 2.37×10^{-3} M

(B) 3.6×10^{-3} M

(C) 2.37×10^{-5} M

(D) 3.6×10^{-5} M

Ans. **(A)**

Sol.

$CH_2FCOOH \rightleftharpoons$	CH_2FCOO^-	+	H^+
C	0		0
C(1- α)	$C\alpha$		$C\alpha$

Given, $[H^+] = C\alpha = 1.5 \times 10^{-3}$; $K_a = 2.6 \times 10^{-3}$

Also, $\because K_a = \frac{C\alpha^2}{(1-\alpha)}$

$$\therefore \quad 2.6 \times 10^{-3} = \frac{1.5 \times 10^{-3} \cdot \alpha}{(1-\alpha)}$$

$$\therefore \quad \alpha = 0.634$$

$$\therefore \quad C\alpha = 1.5 \times 10^{-3}$$

$$C = \frac{1.5 \times 10^{-3}}{0.634}$$

$$= 2.37 \times 10^{-3} \text{M}$$

53. 0.16 g of N_2H_4 are dissolved in water and the total volume made upto 500 mL . Calculate the percentage of N_2H_4 that has reacted with water in this solution. The K_b for N_2H_4 is 4.0×10^{-6}M.

(A) $\alpha = 3 \times 10^{-2}$

(B) $\alpha = 2 \times 10^{-3}$

(C) $\alpha = 1 \times 10^{-2}$

(D) $\alpha = 2 \times 10^{-2}$

Ans. **(D)**

Sol.

	$N_2H_4 + H_2O \rightleftharpoons$	$N_2H_5^+ +$	OH^-
Before dissociation	C	0	0
After dissociation	$C(1-\alpha)$	$C\alpha$	$C\alpha$

Also, $K_b = \frac{C\alpha^2}{(1-\alpha)}$

Assuming $1 - \alpha \simeq 1$

$K_b = C\alpha^2$

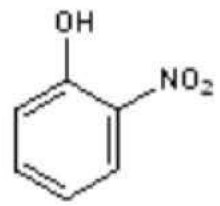
$[N_2H_4] \quad = C = \frac{0.16 \times 1000}{32 \times 500} = 0.01$

Given, $K_b = 4 \times 10^{-6}$M

$$\alpha^2 \quad = \frac{4 \times 10^{-6}}{0.01} = 4 \times 10^{-4}$$

$\alpha = 2 \times 10^{-2}$ or 0.02 or 2%

54. A saturated solution of o-nitrophenol, has a pH equal to 4.53. What is the solubility of o-nitrophenol in water? pK_a for o-nitrophenol is 7.23. (log 1.5 = 0.17)



(o-nitrophenol)

(A) 4 g/litre

(B) 1.085 g/litre

(C) 2.085 g/litre

(D) 3.085 g/litre

Ans. **(C)**

Sol. $\therefore\ [H^+] = C\alpha = C \times \sqrt{\frac{K_a}{C}} = \sqrt{K_a \cdot C}$

or $pH = -\log [H^+] = \frac{1}{2} pK_a - \frac{1}{2} \log C$

or $4.53 \ = \frac{1}{2} \times 7.23 - \frac{1}{2} \log C$

$C = 0.015\text{M}$

∴ Solubility in g/litre = 0.015 × molar mass of compound

= 0.015 × 139 = 2.085 g/litre

55. The K_a for formic acid and acetic acid are 2.1×10^{-4} and 1.1×10^{-5} respectively. Calculate relative strength of acids. (assuming same concentration of both acids)

(A) 1:1

(B) 2.36:1

(C) 4.36:1

(D) 1:4.36

Ans. **(C)**

Sol. Relative strength of weak acids $= \sqrt{\left(\frac{K_{a_1}}{K_{a_2}} \times \frac{C_1}{C_2}\right)}$

Given $C_1 = C_2$

Relative strength $= \sqrt{\frac{K_{a_1}}{K_{a_2}}}$

$= \sqrt{\frac{2.1\times10^{-4}}{1.1\times10^{-5}}} = 4.36$

Relative strength of $\text{HCOOH}:\text{CH}_3\text{COOH} = 4.36{:}1$

56. Calculate the pH of 10^{-1} M HCl.

(A) pH = 5

(B) pH = 8

(C) pH = 6

(D) pH = 1

Ans. **(D)**

Sol.

	$\text{HCl} \longrightarrow$	H^+ +	Cl^-
Before ionization	10^{-1}M	0	0
After ionization	0	10^{-1}	10^{-1}

$[\text{H}^+] = 10^{-1}$

∴pH = 1

57. Calculate the pH of solution obtained by mixing 10 mL of 0.1 M HCl and 40 mL 0.2M H_2SO_4 (log 3.4 = 0.53)

(A) pH = 0.3421

(B) pH = 1.4685

(C) pH = 1.3421

(D) pH = 0.47

Ans. **(D)**

Sol. meq. of $\text{H}^+$from $\text{HCl} = 10 \times 0.1 = 1$

meq. of $\text{H}^+$from $\text{H}_2\text{SO}_4 = 40 \times 0.2 \times 2 = 16$

∴ Total meq. of H^+in solution $= 1 + 16 = 17$

$\therefore\ [H^+] = \frac{17}{50} = 3.4 \times 10^{-1}$ $\left(\because [H^+] = \frac{Meq}{V \text{ in}}\right)$

$\therefore pH = -\log [H^+] = -\log [3.4 \times 10^{-1}] =$ **0.47**

58. Calculate the pH of resulting obtained by mixing 50 mL of 0.6 N HCl and 50 mL of 0.3 N NaOH (log 1.5 = 0.176)

(A) pH = 0.6239

(B) pH = 0.3239

(C) pH = 0.8239

(D) pH = 0.4239

Ans. **(C)**

Sol. meq. before reaction

$$\begin{array}{llll} HCl & + \quad NaOH & \longrightarrow NaCl & + H_2O \\ 50 \times 0.6 & 50 \times 0.3 & & \\ = 30 & 15 & & \\ = 15 & 0 & 15 & \end{array}$$

meq. after reaction
For monovalent electrolytes

$$\text{Molarity} = \text{Normality} = \frac{\text{milli equivalent}}{\text{Total volume}}$$

$$[H^+] = \frac{15}{100} = 0.15 \text{ M}$$

$$pH = -\log [H^+] = -\log 0.15 = 0.8239$$

59. What will be the resultant pH when 200 mL of an aqueous solution of HCl (pH = 2.0) is mixed with 300 mL of an aqueous solution of NaOH (pH = 12.0)? (log 2 = 0.30)

(A) pH = 9.3

(B) pH = 11.3

(C) pH = 2.69

(D) pH = 3.69

Ans. **(B)**

Sol.

$$\begin{bmatrix} \text{pH of HCl} = 2, \text{ pH of NaOH} = 12 \\ \therefore [HCl] = 10^{-2}M \therefore [NaOH] = 10^{-2}M \end{bmatrix}$$

$$\therefore\ [OH^-] = \frac{100 \times 10^{-2}}{500} = 2 \times 10^{-3}$$

$\therefore$ pOH = 2.6989

$\therefore$ pH = 11.3010

60. 100 mL solution is prepared by mixing 0.01 mol each of $H_2CO_3, NaHCO_3$, Na_2CO_3 and NaOH in water. Find out the pH of the resulting solution.

[Given : pK_{a_1} and pK_{a_2} of H_2CO_3 are 6.37 and 10.32 , respectively; $\log\ 2 = 0.30$]

(A) pH = 10.02

(B) pH = 12.02

(C) pH = 2.08

(D) pH = 9.02

Ans. **(A)**

Sol.

	$H_2CO_3 +$	$NaOH \longrightarrow$	$NaHCO_3 +$	H_2O
millimoles before reaction	10	10	10	
millimoles after reaction	0	0	10 + 10 = 20	

Final mixture acts as acidic buffer as it has 20 millimoles $NaHCO_3$ and 10 millimolo Na_2CO_3 (conjugate base)

$$\mathrm{pH} \quad = \mathrm{p}K_{a_2} + \log\left(\frac{\text{Conjugate base}}{\text{Acid}}\right)$$

$$\mathrm{pH} \quad = \mathrm{p}K_{a_2} + \log\left(\frac{1}{2}\right) = \mathrm{p}K_{a_2} - \log\frac{2}{1}$$

$$= 10.32 - \log\ 2 = 10.02$$

61. The solubility product of SrF_2 in water is 8×10^{-10}. Calculate its solubility in 0.1 M NaF aqueous solution.

(A) S = 3 x 10^{-8}

(B) S = 4 x 10^{-8}

(C) S = 8 x 10^{-8}

(D) S = 8 x 10^{-9}

Ans. **(C)**

Sol. $K_{sp} = [Sr^{2+}][F^-]^2$

Let solubility of SrF_2 be S mol litre $^{-1}$.

$$8 \times 10^{-10} = S[2S + 0.1]^2$$

$$\therefore\ S = \frac{8 \times 10^{-10}}{(0.1)^2} = 8 \times 10^{-8}\mathrm{M}$$

62. Calculate pH of 0.2 M solution of NH_4Cl. Hint K_b for NH_4OH is 1.8×10^{-5}. (log 1.054 = 0.022)

(A) pH = 3.271

(B) pH = 2.712

(C) pH = 4.9771

(D) pH = 5.712

Ans. **(C)**

Sol. $[\mathrm{H}^+] = C \cdot h = C\sqrt{\frac{K_\mathrm{H}}{C}} = \sqrt{K_\mathrm{H} \cdot C} = \sqrt{\frac{K_w}{K_b} \times C} = \sqrt{\frac{10^{-14} \times 0.2}{1.8 \times 10^{-5}}} = 1.054 \times 10^{-5}$

pH = -log[H^+] – log [1.054 x 10^{-5}] = 4.9771

63. Calculate pH of 0.1 M aqueous solution of $HCOONH_4$. Given pKa(HCOOH) = 3.8, pKb(NH_3) = 4.8.

(A) pH = 7.5

(B) pH = 6.5

(C) pH = 8.5

(D) pH = 2.5

Ans. **(B)**

Sol. The pH of salt $HCOONH_4$ (a salt of weak acid + weak base) is given by

$$\mathrm{HCOONH_4 + H_2O \rightleftharpoons HCOOH + NH_4OH}$$
$$\mathrm{pH} = 1/2[\log K_b - \log K_a - \log K_w]$$
$$\mathrm{pH} = 1/2[\mathrm{p}K_a + \mathrm{p}K_w - \mathrm{p}K_b]$$
$$\mathrm{pH} = 1/2[3.8 + 14 - 4.8] = 6.5$$

64. What is conjugate base of ^-OH

(A) O_2

(B) H_2O

(C) O^-

(D) O^{2-}

Ans. **(D)**

Sol. -OH $\xrightarrow{-H^+}$ O^{2-}

65. If the pH of 0.1 M monobasic acid at 298 K is 5.0; the value of $\mathrm{p}K_a$ at the same temperature is:

(A) 5

(B) 9

(C) 8

(D) 6

Ans. **(B)**

Sol. For monobasic acid:

$$\begin{aligned} \mathrm{pH} &= -\frac{1}{2}\log CK_a \\ 5 &= -\frac{1}{2}\log(0.1K_a) \\ K_a &= 10^{-9} \\ \mathrm{p}K_a &= -\log_{10} K_a \\ &= -\log_{10}(10^{-9}) = 9] \end{aligned}$$

66. Which of the following is the correct quadratic form of the Ostwald's dilution law equation?

(A) $\alpha^2 C + \alpha K - K = 0$

(B) $\alpha^2 C - \alpha K + K = 0$

(C) $\alpha^2 C - \alpha K - K = 0$

(D) $\alpha^2 C + \alpha K + K = 0$

Ans. **(A)**

[redacted]

Sol. According to Ostwald's dilution law:

$$K = \frac{C\alpha^2}{1-\alpha}$$

or $C\alpha^2 + K\alpha - K = 0]$

67. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in $molL^{-1}$ at 298 K is:

(A) 1.0×10^{-15}

(B) 2.7×10^{-12}

(C) 3.2×10^{-10}

(D) 4.2×10^{-8}

Ans. **(A)**

Sol.

$Bi_2\ S_3 \rightleftharpoons 2Bi^{3+} + 3S^{2-}$

2s 3s

$K_{sp} = (2\ s)^2(3\ s)^3 = 108\ s^5$

$108\ s^5 = 1.08 \times 10^{-73}$

S = 1.0×10^{-15}

68. Which of the following is a buffer solution?

(A) HCl + NaCl

(B) H_2SO_4 + NaOH

(C) $NaHCO_3$ + Na_2CO_3

(D) HCl + NaOH

Ans. **(C)**

Sol. Weak acid + salt of weak acid with strong base is buffer.

69. For a concentrated solution of a weak electrolyte A_xB_y of concentration ' C ', the degree of dissociation ' α ' is given as:

(A) $\alpha = \sqrt{K_{eq.}/C(x+y)}$

(B) $\alpha = \sqrt{\frac{CK_{eq}}{xy}}$

(C) $\alpha = \left[\frac{K_{eq.}}{(C^{x+y-1}x^xy^y)}\right]^{1/x+y}$

(D) $\alpha = \frac{K_{eq.}}{Cxy}$

Ans. **(C)**

Sol. $A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$

t_0	C	0	0
$t_{teq.}$	$C(1-\alpha)$	$xC\alpha$	$yC\alpha$

[redacted]

$$K_{\text{eq}} = \frac{(xC\alpha)^x(yC\alpha)^y}{C(1-\alpha)}$$
$$\approx \frac{(xC\alpha)^x(yC\alpha)^y}{C} \qquad \ldots(1-\alpha)\approx 1$$
$$\alpha = \left[\frac{K_{\text{eq}}}{C^{x+y-1}x^x y^y}\right]^{1/x+y}$$

70. The ionization constant of ammonium hydroxide is 1.77×10^{-5} at 298 K. Hydrolysis constant of ammonium chloride is:

(A) 5.65×10^{-12}

(B) 5.65×10^{-10}

(C) 6.50×10^{-12}

(D) 5.65×10^{-13}

Ans. **(B)**

Sol. $K_h = \frac{K_w}{K_b} = \frac{10^{-14}}{1.77\times10^{-5}}$

$= 5.65 \times 10^{-10}]$

SECTION: II

71. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is ________. (Rounded off to the nearest integer)

[Given $\sqrt{2} = 1.41$]

Ans. **(141)**

Sol. Given : $[K_{sp}]_{PbI_2} = 8 \times 10^{-9}$

To calculate : solubility of PbI_2 in 0.1 M sol of $Pb(NO_3)_2$

(I) $Pb(NO_3)_2 \rightarrow Pb^{+2}_{(qq)} + 2NO_3^-(aq)$

$0.1M \quad - \quad -$

$- \quad 0.1M \quad 0.2M$

(II) $PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$

s

$= s + 0.1$

≈ 0.1

Now : $K_{sp} = 8 \times 10^{-9} = [Pb^{+2}][I^-]^2$

$\Rightarrow 8 \times 10^{-9} = 0.1 \times (2s)^2$

$\Rightarrow 8 \times 10^{-8} = 4s^2 \Rightarrow s = \sqrt{2} \times 10^{-4}$

$\Rightarrow S = 141 \times 10^{-6} M$

$\Rightarrow x = 141$

72. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is (nearest integer)

Ans. **(7)**

Sol. Since $(NH_4)_3PO_4$ is salt of weak acid (H_3PO_4) & weak base (NH_4OH).

$$\begin{aligned}\mathrm{pH} &= 7+\tfrac{1}{2}(\mathrm{pka}-\mathrm{pkb})\\ &= 7+\tfrac{1}{2}(5.23-4.75)\\ &= 7.24 \approx 7\end{aligned}$$

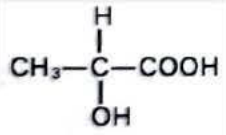
73. The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of $0.2\mathrm{M}\,CH_3COONa$ solution is mixed with 25 mL of $0.02\mathrm{M}\,CH_3COOH$ solution, the pH of the resultant solution is found to be equal to 5. The value of x is ________.

Ans. **(10)**

Sol. Buffer of HOAc and NaOAC

$$\mathrm{pH} = \mathrm{pKa} + \log\frac{0.1}{0.01}$$
$$5 = \mathrm{pKa} + 1$$
$$\mathrm{pKa} = 4$$
$$\mathrm{Ka} = 10^{-4}$$
$$\mathrm{x} = 10$$

74. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate sotution at 25°C is ________ $\times 10^{-1}$ (Nearest integer)

Lactic Acid 

Ans. **(85)**

Sol. Concentration of calcium lactate $= 0.005\mathrm{M}$,
Concentration of lactate ion $= (2 \times 0.005)\mathrm{M}$.
Calcium lactate is a salt of weak acid + strong base
∴ Salt hydrolysis will take place.

$$\begin{aligned}\mathrm{pH} &= 7+\tfrac{1}{2}(\mathrm{pKa}+\log\,\mathrm{C})\\ &= 7+\tfrac{1}{2}(5+\log\,(2\times 0.005))\\ &= 7+\tfrac{1}{2}[5-2\log\,10] = 7+\tfrac{1}{2}\times 3 = 8.5 = 85\times 10^{-1}\end{aligned}$$

75. 600 mL of 0.01 M HCl is mixed with 400 ml of $0.01\mathrm{M}\,H_2SO_4$. The pH of the mixture is ________ $\times 10^{-2}$. (Nearest integer)

[Given $\log\,2 = 0.30, \log\,3 = 0.48; \log\,5 = 0.69, \log\,7 = 0.84; \log\,11 = 1.04$]

Ans. **(186)**

Sol.

Total milimoles of $H^+ = (600 \times 0.01) + (400 \times 0.01 \times 2)$
$= 14$

$$[\mathrm{H}^+] = \frac{14}{1000} = 14 \times 10^{-3}$$
$$\begin{aligned}\mathrm{pH} &= 3 - \log\,14\\ &= 1.86\\ &= 186 \times 10^{-2}\end{aligned}$$

SPACE FOR ROUGH WORK

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