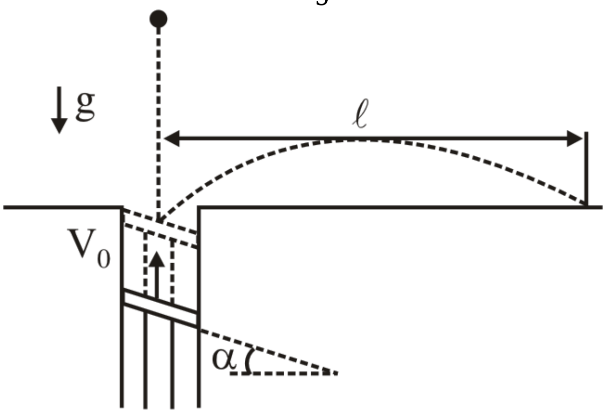
PART-1 : PHYSICS

SECTION-I (i)

1) In the horizontal surface of a large table, there is a small round hole into which a vertical cylindrical tube is inserted. Inside the tube, a piston moves upward with a constant velocity $V_0 = 6$ m/s, whose upper surface is tilted at an angle $\alpha = 15°$ to the horizontal. From a certain height above, a small ball falls (see the figure). At the moment immediately before an elastic collision of the ball with the piston, the velocity of the ball is directed downward and is equal to V_0. The point at which the collision occurs is exactly at the level of the table surface. After the collision with the piston, the ball falls onto the surface of the table at a distance ℓ from the hole in the table. Air resistance can be neglected. Let the angle of inclination of the top surface of the piston to the horizontal be adjustable. At what value of the angle α will the maximum possible flight distance ℓ_{max} of the ball be achieved?



(A) $\frac{1}{2}\cos^{-1}\left(\frac{-1+\sqrt{33}}{8}\right)$

(B) $\cos^{-1}\left(\frac{-2+\sqrt{33}}{8}\right)$

(C) $\cos^{-1}\left(\frac{-1+\sqrt{33}}{8}\right)$

(D) $\frac{1}{2}\cos^{-1}\left(\frac{-2+\sqrt{33}}{8}\right)$

2) You are in an airplane at an altitude of 10,000 m. The pupil of your eye is about 3.0 mm in diameter and the wavelength of light $\lambda = 550$ nm. If you look down at the ground, the minimum separation s between objects that you could distinguish is :

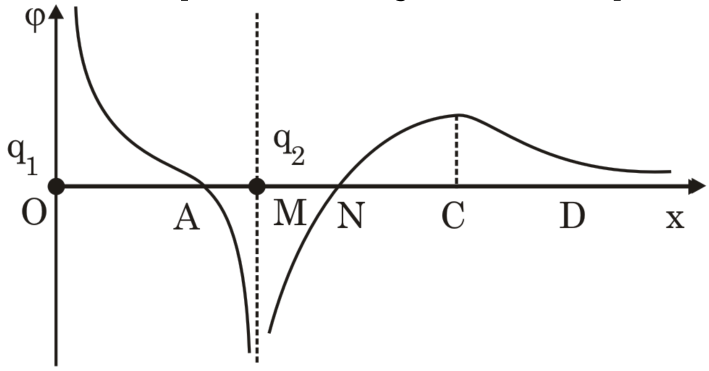
(A) 1.24 m

(B) 2.24 m

(C) 9.92 m

(D) 4.48 m

3) Two charges q_1 and q_2 are located at points O and M on the x-axis, respectively. The relationship between the potential φ along the x-axis and position is shown in the figure. Then :



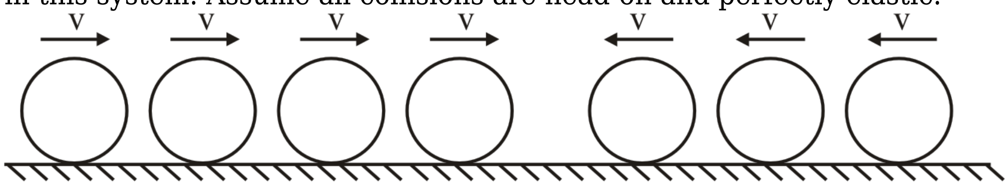
(A) The electric field strength at point N is zero.

(B) $|q_1| < |q_2|$

(C) The electric field direction between N and C is along the positive x-axis.

(D) If a negative charge is moved from point N to point D, the electric force first does positive work till C, then negative work.

4) Seven identical balls, each of mass m, slide on a smooth horizontal surface toward each other with the same speed v, as shown in the diagram. Determine the total number of collisions that will occur in this system. Assume all collisions are head-on and perfectly elastic.



(A) 9

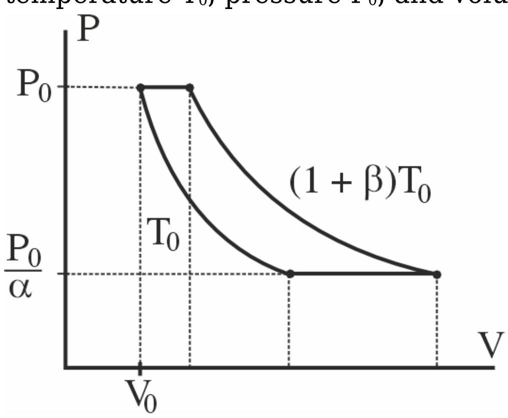
(B) 12

(C) 15

(D) 19

SECTION-I (ii)

1) Consider the following heat engine involving one mole of ideal monatomic gas. The gas begins at temperature T_0, pressure P_0, and volume V_0, and undergoes four reversible steps.



1. The gas is expanded at constant pressure until its temperature rises to $(1 + \beta)T_0$.

2. The gas is expanded at constant temperature until its pressure falls to $\frac{P_0}{\alpha}$.

3. The gas is contracted at constant pressure until its temperature falls back to T_0.
4. The gas is contracted at constant temperature until its pressure rises back to P_0.

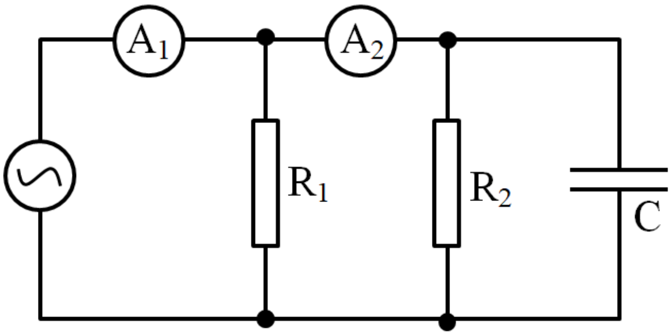
(A) The total heat supplied to the gas is $\frac{5}{2}\beta P_0 V_0$.

(B) The total heat rejected by the gas is $\beta P_0 V_0$.

(C) Total work done by the gas is $P_0 V_0 \beta \ell n\alpha$.

(D) Efficiency of the process is $\frac{\beta \ln \alpha}{(1+\beta)\ln \alpha + \frac{5\beta}{2}}$.

2) A harmonic AC voltage source with an effective value of 36 V and a frequency of 50 Hz has two identical resistors ($R_1 = R_2 = R$) and a capacitor connected in parallel (figure). Additionally, two ammeters A_1, A_2 with negligible resistance are included in the circuit. The measured currents are



$I_1 = 5A$, $I_2 = 4A$.

(A) The current through R_1 is 3A.

(B) The current through C is $\sqrt{13}A$.

(C) The phase angle between current through A_1 and source is $\tan^{-1}\left(\frac{\sqrt{13}}{\sqrt{12}}\right)$.

(D) The phase angle between current through A_2 and source is $\tan^{-1}\left(\frac{\sqrt{13}}{3}\right)$

3) A microscope with a numerical aperture (NA) of 1.25 is used to observe small biological samples with light of wavelength 500 nm. The resolving power of the microscope is influenced by factors such as the wavelength, numerical aperture, and immersion medium. Which of the following statements are correct about the resolving power of the microscope?

(A) Decreasing the wavelength of light used will increase the resolving power, allowing the microscope to distinguish smaller structures.

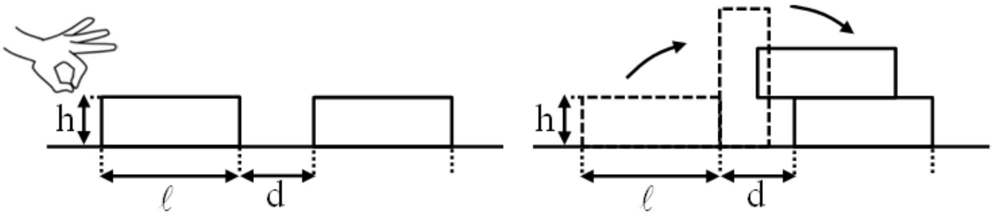
(B) Increasing the numerical aperture of the microscope's objective lens will improve the resolving power, reducing the minimum resolvable distance between two points.

(C) If an immersion oil with a higher refractive index is used, the resolving power will improve.

(D) Using light of wavelength 1000 nm will improve the resolving power of the microscope.

4) A game of "Country Erasers" is played with two identical square-shaped erasers of length , thickness h and uniform mass m. They initially lie flat beside each other on a horizontal table, with their ends a distance d > h apart. In a winning move, a player flicks one eraser towards the other,

such that it rotates vertically and eventually lands resting flat on the other eraser, as shown below. Assume that neither of the erasers slip at any point of contact with the table or with each other throughout their motion.



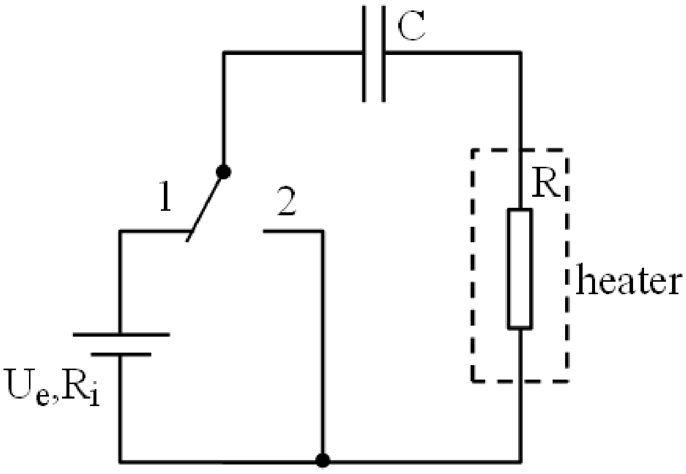
(A) The maximum value of d such that winning move is possible is $h+\sqrt{\frac{\ell^2}{4}-h^2}$.

(B) The minimum value of energy required by the flick for this to be possible is $\frac{mg(\ell-h)}{2}$.

(C) The minimum energy dissipated in the process is $\frac{mg(\ell-3h)}{2}$.

(D) This move is not possible.

5) The electric heater, whose schematic is shown in Fig., operates as follows :
The switch first moves to position 1, and the capacitor with capacitance C charges completely to the electromotive voltage of the source U_e. Then, the switch moves to position 2, and the capacitor fully discharges through the heating element of the heater, which has resistance R. This process repeats with frequency f. The source has an internal resistance R_i. Take C = 100 μF, R = 20 Ω, U_e = 90 V, R_i = 80Ω, f = 2s^{-1}.



(A) The heat generated in the heating element during the charging of capacitor is 81 mJ.

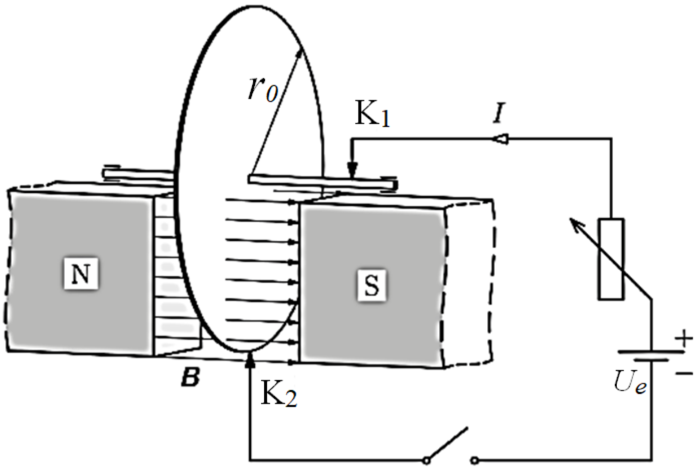
(B) The heat generated in the heating element during the discharging of capacitor is 405 mJ.

(C) The average power P of the heater is 4.86 W.

(D) The efficiency of the heater is $\frac{3}{5}$.

6) Barlow's wheel (Fig.) is a simple motor, whose construction is similar to Faraday's disk, but its operation is opposite. From a source of electromotive voltage U_e = 3.00 V, electric current is supplied to the metallic disk via a sliding contact K_1, which touches the axis of the disk, and a liquid contact K_2, made using a container of mercury, which touches the lower edge of the disk. The circuit resistance can be regulated using a rheostat in the range of R_1 = 3.00 Ω to R_2 = 6.00 Ω. The lower edge of the disk, together with contact K_2, is located in a transverse magnetic field between the poles of the magnet. For simplicity, assume that the magnetic induction is constant across the entire

area of the disk and has a value of B = 800 mT. The magnetic field will cause the disk to rotate. The disk has a radius r_0 = 120 mm and the moment of inertia about its axis is J = 5.00 × 10^{-4} kg.m^2.



Mechanical resistance is not considered.

(A) The direction of rotation of wheel when viewed from the source towards the north pole of the magnet is anti-clockwise.

(B) The maximum angular velocity of the wheel is independent of the resistance of the rheostat.

(C) The initial angular acceleration of the wheel just after the circuit is turned on, for R = R_1 is 11.52 rad/s^2.

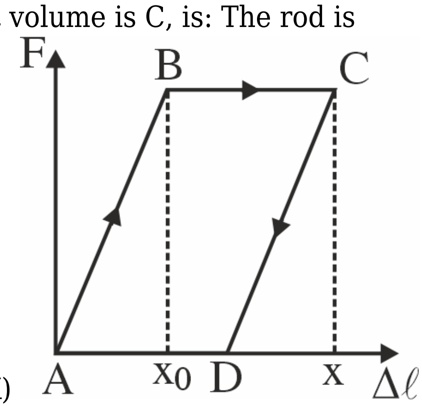
(D) The initial angular acceleration of the wheel just after the circuit is turned on, for R = R_2 is 5.76 rad/s^2.

SECTION-II

1) Residual deformation of an elastic rod can be roughly described within the framework of the following model. If the rod's extension $\Delta\ell < x_0$ (where x_0 is a given constant for this rod), the force required to cause the extension $\Delta\ell$ is determined by Hooke's law $F = k\Delta\ell$, where k is the stiffness of the rod. If $\Delta\ell > x_0$, the force no longer depends on the extension (the material of the rod begins to "flow"). If we now start removing the load, the extension of the rod will decrease along the path CD, which we simplify by assuming is straight and parallel to segment AB (see the figure). Thus, after the load is fully removed, the rod remains deformed (point D in the figure). The maximum temperature change ΔT (in K) of the rod if its heat capacity per unit volume is C, is: The rod is

0

thermally insulated. (Take : x = 2m, x = 1m, k = 20 N/m, C = 1 J-K)

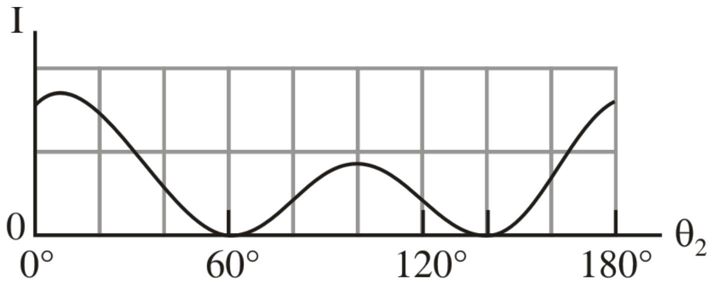


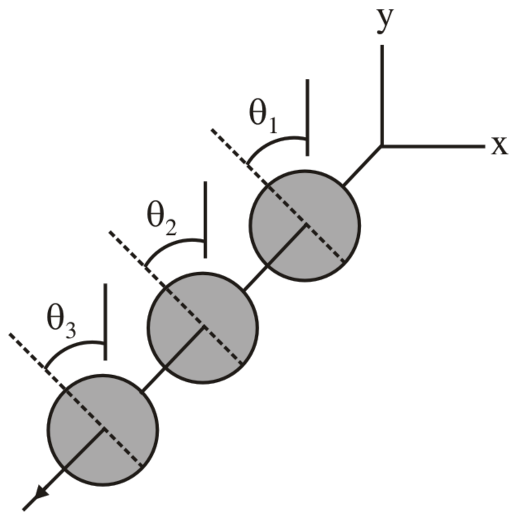
2) Room air conditioning can be described as a heat engine operating in reverse mode — it extracts heat Q_m from a room at temperature T_m and transfers heat $Q_v > Q_m$ to the air outside the house at temperature T_V. An electric compressor must supply work W to achieve this. Assume the device operates as a Carnot engine with maximum possible efficiency. Even though the room is insulated,

heat enters from outside at a rate $\frac{\Delta Q}{\Delta t} = k\Delta T$, where $\Delta T = T_v - T_m$ is the temperature difference between the outside environment and the room, and k is a constant. What is the minimum power P (in W) needed for the air conditioner to maintain a room temperature $T_m = 20°C$, given an outdoor temperature $T_v = 40°C$ and a typical value of the constant $k = 293\ W \cdot K^{-1}$?

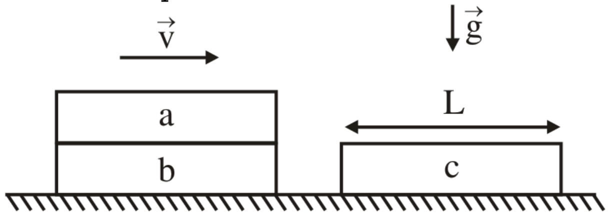
3) The electric field at a point P located at a height $\frac{\ell}{2\sqrt{6}}$ above the centroid of an equilateral triangle with side length ▯ and uniform surface charge density σ is $\frac{\sigma}{\alpha \epsilon_0}$. The value of α is

4) In figure, unpolarized light is sent into a system of three polarizing sheets. The angles θ_1, θ_2 and θ_3 of the polarizing directions are measured counterclockwise from the positive direction of the y axis. Angles θ_1 and θ_3 are fixed, but angle θ_2 can be varied. Figure gives the intensity of the light emerging from sheet 3 as a function of θ_2. What percentage of the light's initial intensity (approximately) is transmitted by the three-sheet system when $\theta_2 = 90°$? Take : $\sin 50° = 0.75$.





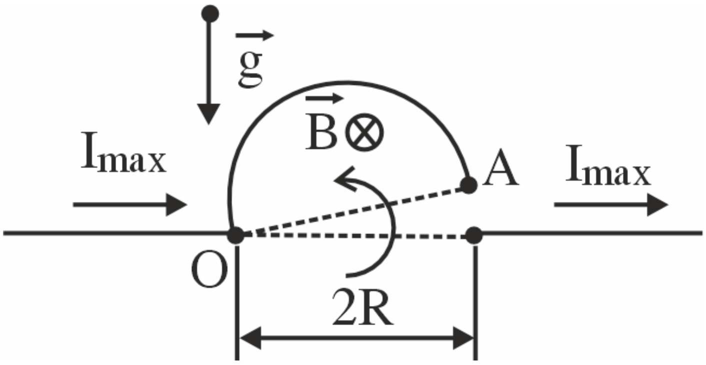
5) Board-(a) is lying on top of board-(b). Together, they form a single unit and move with some velocity v on a smooth horizontal surface. They collide with another identical board-(c). There is no friction between boards-(a) and (b), but there is friction between boards-(a) and (c). Upon collision, boards-(b) and (c) are firmly coupled and do not move relative to each other. The length of each board is L. Board -(a) stops moving relative to board-(b) and (c) at the moment it is entirely located on top of board-(c). Find the displacement Δx of board (a) from the moment of collision until the relative motion of the boards stops. If the value of Δx is $\frac{\alpha L}{21}$, the value of α is. Take : $\pi = \frac{22}{7}$. (all board are identical.)



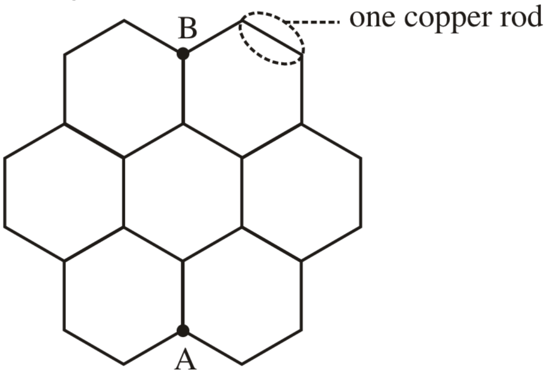
6) A beaker is fitted with a heating coil and stirrer and contains 40.0 cm^3 of liquid A. When the power dissipated in the heating coil is 4.80 W, the temperature of the contents rises from 15.0°C to 35.0°C in 400 s. The experiment is repeated using 20.0 cm^3 of liquid A mixed with 20.0 cm^3 of liquid

B. It is found that, with a heater power of 4.90 W, the temperature again rises from 15.0°C to 35.0°C in 400 s. The specific heat capacity, s of B (in $Jkg^{-1} K^{-1}$) is: (Density of A is 1.60×10^3 kg m^{-3}, Specific heat capacity of A is 8.60×10^2 J kg^{-1} K, Density of B is 2×10^3 kg m^{-3})

7) To break an electrical circuit when the current increases, a movable thin semicircular ring OA (figure) is used. The ring has a mass m = 3.0 g and a radius R = 1.0 cm, and it can freely (without friction) rotate in the vertical plane about point O. The system is located in a uniform horizontal magnetic field with an induction B = 1.5 T, perpendicular to the plane of the figure. Find the maximum current strength I_{max} (in A) at which the limiter disconnects the circuit.



8) Ram is bored, so he decides to use his collection of uniform thin copper rods, each of resistance R = 1.00 Ω, to create a rigid compound shape shown below. The copper rods form seven regular hexagons. The effective resistance R_{AB} between points A and B is $\frac{\alpha}{160}\Omega$. The value of α is:



PART-2 : CHEMISTRY

SECTION-I (i)

1) In Ellingham diagram, to indicate the boiling point of oxide of a metal and that of a metal, the nature of a curve will have turning point with

(A) Increasing slope and decreasing slope respectively.

(B) Decreasing slope and increasing slope respectively.

(C) Increasing and increasing slope respectively.

(D) Decreasing and decreasing slope respectively.

2)

A substance 'A' undergoes reactions by following three different parallel path :

$A \xrightarrow{K_1} B$; $K_1 = 5 \times 10^{-2}\ sec^{-1}$

$2A \xrightarrow{K_2} C$; $K_2 = 3 \times 10^{-2}\ sec^{-1}$

$3A \xrightarrow{K_3} D$; $K_3 = 5 \times 10^{-3}\ sec^{-1}$

Calculate average life time of substance (in sec.) [K_1, K_2, K_3 are rate constants for respective reactions]

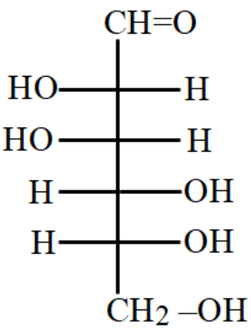
(A) $\frac{1000}{85}$

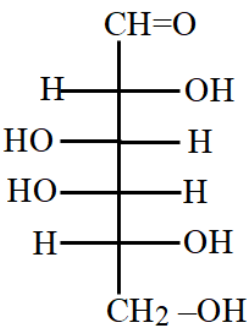
(B) $\frac{100}{85}$

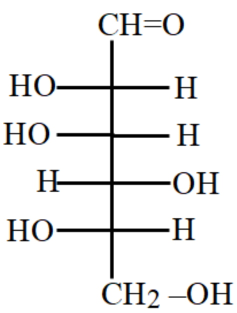
(C) 8

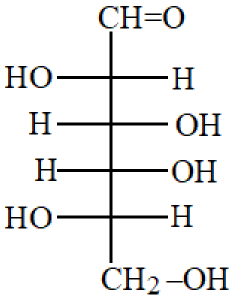
(D) 4

3) Which of the following stereoisomer of D-glucose gives same product with HNO_3 as obtained by oxidation of D-glucose with HNO_3.

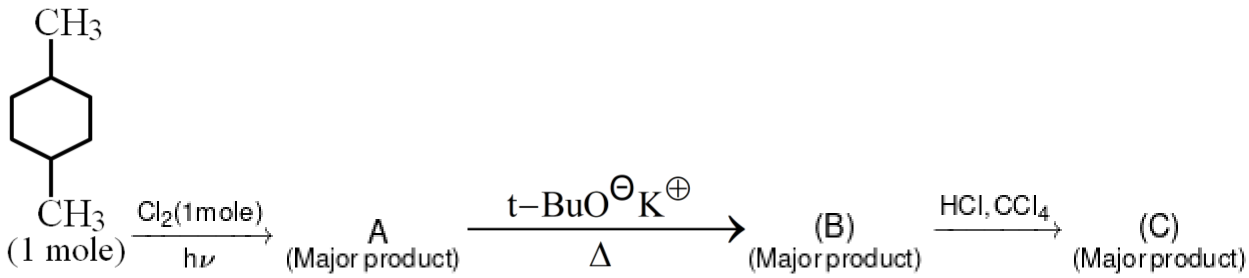
(A) 

(B) 

(C) 

(D) 

4) Consider the following reaction sequence, the correct statement is :



(A) Product A and C is same

(B) α-H in major product B is 2

(C) Product C is optically active.

(D) Major product B has one chiral centre.

SECTION-I (ii)

1) Choose correct match.

Substance		Maximum permissible limits in drinking water
(i)	Sulphate	< 500 ppm
(ii)	Fluoride	1 ppm
(iii)	Nitrate	50 ppm
(iv)	Lead	50 ppm

(A) (i)

(B) (ii)

(C) (iii)

(D) (iv)

2) $FeC_2O_4 \xrightarrow[\text{in absence of air}]{\Delta} \underset{\text{(Hot)}}{\text{Residue}} + \text{Gas} - A + \text{Gas} - B$

Gas-A is absorbed by Ethanolamine. Then which of the following option is/are correct?

(A) Residue is black in colour

(B) Atomicity of gas-A is 3

(C) Gas-B is absorbed by aqueous suspension of CuCl.

(D) Gas-B can turn Baryta water milky

3) Select the correct statement(s) :

(A) According to collision theory, the rate of reaction is directly proportional to $\sqrt{T}\,.\,e^{-\frac{E_a}{RT}}$.

(B) A catalyst can not change equilibrium constant as well as equilibrium composition of a reaction mixture.

(C) A good solid catalyst for heterogeneous catalysis should have high magnitude of enthalpy of adsorption of the reactants on its surface.

(D) For reactions in liquid solutions, the solvent may strongly affect the rate constant.

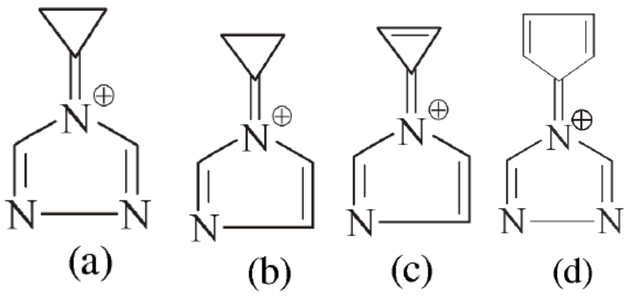
4) Select correct statement(s) :

(A) A liquid solution of two substances will always freeze entirely at one temperature.

(B) A liquid solution of two substances will never freeze entirely at one temperature.

(C) On increasing temperature, Henry constant (K_H) first increase and then decrease for most non-polar gas dissolved in water.

(D) For most non-polar gases, the value of K_H is less in benzene than in water.



5)
Which of the following statement(s) is/are correct about given compounds?

(A) Correct order of basic strength is : (c) > (b) > (a) > (d)

(B) Compound (c) has minimum rotational energy barrier .

(C) All compounds are bicyclo compounds

(D) Hybridisation state of all nitrogens in all compounds is sp^2

$$CH_3-CH_3 \xrightarrow{?} CH_3-CH_2-\overset{\overset{\large O}{||}}{C}-O-\overset{\overset{\large O}{||}}{C}-CH_2-CH_3$$

(Major product)

6)
Which of the following set of reagents can be applying for above interconversion.

(A) (i) Br_2 (1 eq.) hν, (ii) Mg/dry ether (iii) CO_2, $H_3O^{\oplus}$ (iv) Conc. H_2SO_4

(B) (i) Cl_2 (1 eq.) hν, (ii) KCN (iii) $H_3O^{\oplus}$ (iv) Conc. H_3PO_4

(C) (i) Br_2 (1 eq.) hν, (ii) AgCN (iii) $H_3O^{\oplus}$ (iv) Conc. H_2SO_4

(D) (i) Cl_2 (1 eq.), hν (ii) Mg/dry ether, (iii) HCHO, HOH (iv) $H^{\oplus}$, $KMnO_4$ (v) Conc. H_2SO_4

SECTION-II

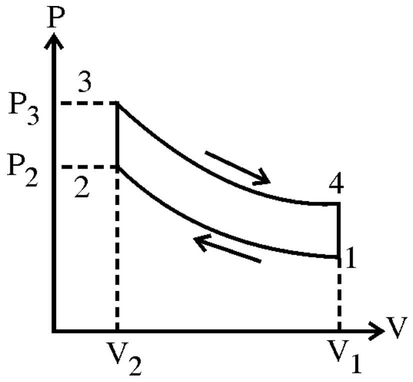
1) When one mole of B_2H_6 is reacted with D_2O (excess) then, number of moles of HD formed is_____.

2) For how many of the following elements, in the process of $M \rightarrow M^{3+}$, the number of f-electrons remains unchanged.
Pm (61) , Eu (63) , Gd (64) , Dy (66) , Ce (58) , Lu (71) , Th (90) , U (92) , Pu (94)

3) On the basis of the 18 electron rule, identify the first row transition metal for each of following and find the sum of the atomic number of these metals. (Considering all η^x-ligands are aromatic).
(a) $[M((CO)_7]^+$
(b) $[M(CO)_2(CS)(PPh_3)(Br)]$
(c) $[(\eta^3\text{-}C_3H_3)\ (\eta^5\text{-}C_5H_5)\ M(CO)]^-$
(d) $[(\eta^4\text{-}C_4H_4)\ (\eta^5\text{-}C_5H_5)\ M]^-$

4) A hypothetical particle 'Xeton" at rest with mass equivalent to a He nucleus absorbs 'n' photons of frequency, $\overset{0}{\nu}$. Its de-Broglie wavelength was later found to be $\frac{1}{8}\sqrt{\frac{h}{m\upsilon_0}}$, (where, m = mass of a proton), then value of n is

5) An otto cycle is an idealized thermodynamic cycle that describe the functioning of a typical spart ignition piston.
Assuming working substance to be ideal monoatomic gas. $3 \rightarrow 4$ and $1 \rightarrow 2$ are reversible adiabatic processes.
If $T_1 = 300$ K; $\frac{V_1}{V_2} = 8$; $\frac{P_3}{P_2} = \frac{4}{3}$
If value of $\frac{W_{234}}{W_{412}}$ is x, then calculate value of |3x|.
W_{412} = work done during process $4 \rightarrow 1 \rightarrow 2$



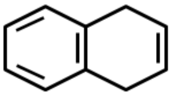
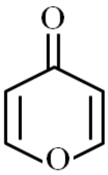
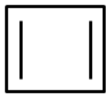
W_{234} = work done during process $2 \rightarrow 3 \rightarrow 4$

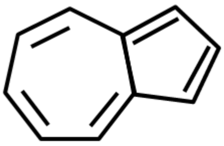
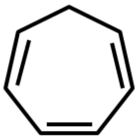
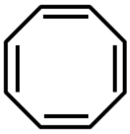
6) Dark Brown powder of PbO_2 is allowed to react with excess of KI and iodine liberated is reacted with N_2H_4 in another container. The volume of N_2 gas liberated from this second container at STP was measured out to be 1.135 litre. Find volume (in litre) of decimolar NaOH solution required to dissolve the same amount of PbO_2 present initially, completely and convert it to Na_2PbO_3.

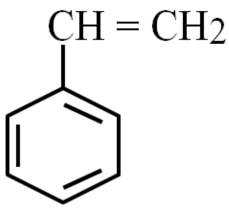
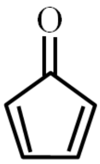
7) An optically pure aldehyde with molecular formula $C_5H_{10}O$ react with excess of formaldehyde gives product P and sodium formate in presence of conc. NaOH. Product P further react with cyclo

pentanone in mild acidic medium to gives product Q.
Calculate the value of x – y.
Whereas x is number of methylene $-\!\left(CH_2\right)\!-$ groups in product Q and Y is number of oxygen atoms in product Q.

8) Calculate total number of aromatic compounds/species :

(i)  (ii)  (iii) 

(iv)  (v)  (vi) 

(vii)  (viii)  (ix) 

PART-3 : MATHEMATICS

SECTION-I (i)

1) The number of solution of the equation $x^3 + x^2 + 3x + 2 \sin x = 0$ in $-2\pi \le x \le 4\pi$, is

(A) 4
(B) 3
(C) 2
(D) 1

2) Area enclosed by the graph of the function $y = \ell n^2 x - 1$ lying in the 4^{th} quadrant, is

(A) $\frac{2}{e}$
(B) $\frac{4}{e}$
(C) $2\left(e + \frac{1}{e}\right)$
(D) $4\left(e - \frac{1}{e}\right)$

3) The number of dissimilar terms in the expansion of $(1 + x + x^3)^{10}$ is

(A) 11
(B) 29
(C) 30
(D) 31

4) If the coefficient of x^{78} in the expansion of $(1 + x + 2x^2 + 4x^4)^{20}$ is $\lambda \cdot 2^{40}$ then λ is equal to

(A) 11
(B) 10
(C) 8
(D) 4

SECTION-I (ii)

1) Let z_1, z_2, z_3 are three non zero complex numbers denoting the vertices of an equilateral triangle A_1, A_2, A_3 respectively. Consider the statements
(1) $A_1A_2A_3$ is an equilateral triangle.
(2) $z_1 \bar{z}_2 = z_2 \bar{z}_3 = z_3 \bar{z}_1$
(3) $z_1^2 = z_2z_3$ and $z_2^2 = z_3z_1$
then which of the following is(are) correct.

(A) (2) ⇒ (1)
(B) (3) ⇒ (1)
(C) (2) ⇒ (3)
(D) (3) ⇒ (2)

2) Let α be a solution of the equation $2[x + 32] = 3[x - 64]$ (where [x] is the greatest integer less than or equal to x) and let $\beta = \prod_{r=1}^{9} \sin\left(\frac{2r-1}{18}\right)\pi$, then which of the following can be **True** ?

(A) $[\alpha] = [\beta]$
(B) $\alpha = \frac{2051}{8}$
(C) $[\alpha]\left[\frac{1}{\beta}\right] = 1$
(D) $\left[\frac{1}{\alpha}\right] + \left[\frac{1}{\beta}\right] = 2^8$

3) Which of the following statement(s) is(are) correct ?

(A) If $5\sec^{-1}\alpha + 10\sin^{-1}\beta = 10\pi$ then the value of $\tan^{-1}\alpha + \cos^{-1}(\beta - 1)$ is $\frac{\pi}{4}$

(B) Minimum value of $\sin 2\pi x + \cos^2 \pi x$ is $\frac{\sqrt{5}+1}{2}$

(C) If $\sqrt{3}\tan 70^\circ = x + 4\sin 70^\circ$ then x is equal to 1

(D) If $f : (0, \pi) \to R$ be defined by $f(x) = \sum_{n=1}^{4}\left[1 + \sin\frac{x}{n}\right]$ (where [x] denotes largest integer less than or equal to x), then the sum of all the values in range of f(x) is equal to 10.

4) Let the function $f : (-\infty, \infty) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ be given by $f(x) = \sin^{-1}\left(\log_3\left(\frac{x^2-x+1}{x^2+x+1}\right)\right)$, then

(A) $f\left(\frac{1}{x}\right) = -f(-x)$

(B) f (x) is a strictly increasing function in $(-\infty, \infty)$

(C) f(x) is a surjective function

(D) f (x) is a injective function.

5) Let $A = [a_{ij}]$ be a square matrix of order 3 satisfying $A + \text{adj}(A) = \alpha A^T$. Identify which of the following statement(s) is(are) correct?

(A) If $\alpha = 1$ then $\det(A) = 0$.

(B) If $\alpha = 1$ and $\text{tr}(A) = 3$ then $\text{tr}(3A + \text{adj } A) = 9$.

(C) If $\alpha = -2$ then $\det(3A + \text{adj } A) = 0$.

(D) If $\alpha = -2$ and $\text{tr}(A) = 3$ then $\text{tr}(4A + \text{adj } A) = 6$

6) Number of ways in which 5 different toys can be distributed to 3 children if each child can get any number of toys is also equal to

(A) number of subsets A of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} such that no 2 elements of A have sum equal to 11.

(B) number of non-negative integral solution of the equation $xyz = 2310$.

(C) number of 6 digit number less than 200000 formed by using only the digits 1, 2 and 3.

(D) number of all possible selections of one or more questions from 5 given questions if each question having an alternative.

SECTION-II

1) Let $P_1, P_2, \ldots, P_n$ be the points on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $Q_1, Q_2, \ldots, Q_n$ are the corresponding points on the auxiliary circle of the ellipse. If the line joining C to Q_i (C is centre of ellipse) meets the normal at P_i with respect to the given ellipse at K_i and $\sum_{i=1}^{n} CK_i = 56$, then the value of n, is

2) A continuous function $f : R \to R$ satisfy the differential equation $f(x) = (1 + x^2)\left(1 + \int_0^x \frac{f^2(t)}{1+t^2}dt\right)$ then the value of $\frac{17}{15}f(-2)$, is

3) If $\lim_{n\to\infty} \int_{\frac{-1}{\sqrt[3]{n}}}^{\frac{1}{\sqrt[3]{n}}} x \ \ell n\,(1 + e^x + e^{2x} + e^{3x} + \ldots\ldots\ e^{2nx})\,dx = \prod_{k=2}^{m}\left(1 - \frac{1}{k^2}\right)$ then the value of m, is

4) Let W_1 and W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the smallest positive value of 'a' for which the line y = ax contains the centre of a circle that is externally tangent to W_2 and internally tangent to W_1. Given that $m^2 = \frac{p}{q}$ where p and q are relatively prime integers, then $\left[\frac{p+q}{26}\right]$ is (where [.] represent greatest integer function)

5) Let k_1 & k_2 be two values of k for which $f(x) = \begin{cases} x.\frac{\ell n(1+x)+\ell n(1-x)}{\sec x - \cos x}, & x \in (-1,0) \\ \left(k^2 - 3k - 1\right)\sin x + x^2, & x \in [0,\infty) \end{cases}$ is differentiable at x = 0, then the value of $\left(k_1^2 + k_2^2 - 3\right)$ is equal to

6) If the least integral value satisfying the equation $\log_3\sqrt{x^2 - 4x + 4} = 2^{\log_2\left(\log_3(|x|-2)\right)}$ is α, then the number of zeroes after decimal and before first significant digit in the number of $(\alpha)^{-4\alpha}$, is (where log2 = 0.3010)

7) Consider two matrices $A = \begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_2 & x_1 \\ 0 & 0 & x_3 \end{bmatrix}$ and $B = \begin{bmatrix} y_1 & 0 & 0 \\ y_3 & y_2 & 0 \\ y_2 & y_1 & y_3 \end{bmatrix}$, where each of $x_i, y_j \in \{-1, 0, 1\}\ \forall\ i, j = 1, 2, 3$, and if N is the number of possible ordered pair of matrices A and B for which det A = det B. Then the value of $\frac{N}{131}$, is

8) If $2x = \left(y^{\frac{1}{3}} + y^{\frac{-1}{3}}\right)$, then the value of $\frac{(x^2 - 1)}{y} \cdot \frac{d^2y}{dx^2} + \frac{x}{y} \cdot \frac{dy}{dx}$, is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	A	B	D	B

SECTION-I (ii)

Q.	5	6	7	8	9	10
A.	C,D	B,C	A,B,C	A,B,C	A,B,D	B,C,D

SECTION-II

Q.	11	12	13	14	15	16	17	18
A.	20	400	4	7	58	738	1	306

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	19	20	21	22
A.	B	C	C	B

SECTION-I (ii)

Q.	23	24	25	26	27	28
A.	A,B,C	A,B,C	A,B,D	C,D	A,B,D	A,B,D

SECTION-II

Q.	29	30	31	32	33	34	35	36
A.	6	5	101	8	4	2	5	5

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	37	38	39	40
A.	D	B	C	B

SECTION-I (ii)

Q.	41	42	43	44	45	46
A.	A,B,C,D	B,D	A,C	A,C	A,B,C	A,B,C

SECTION-II

Q.	47	48	49	50	51	52	53	54
A.	**8**	**1**	**3**	**6**	**8**	**9**	**3**	**9**

SOLUTIONS

PART-1 : PHYSICS

2) $\theta = \frac{1.22\lambda}{D} = \frac{s}{h}$

s = 2.24 m

3) The slope of the φ-x graph is equal to the negative value of the electric field intensity (–E), with the direction of the field being opposite to the slope direction. The field created by a positive charge has a positive potential, while the field created by a negative charge has a negative potential. If multiple field sources exist, the total potential is the algebraic sum of their potentials.
The slope of the φ-x graph is equal to the electric field intensity value –E. Statement C is incorrect because the slope at point N is not zero, meaning the electric field at point N is not zero, so statement A is incorrect. The slope of the tangent at point C is zero, indicating the field at C is zero. Since A and N are located to the right of point M, it can be deduced that $q_1 > q_2$, making statement B incorrect.
From point N to point D, the potential first rises and then falls. Thus, if a negative charge is moved from N to D, the electric force will first do positive work and then negative work, making statement D correct.

14) We note the points at which the curve is zero (θ_2 = 60° and 140°). We infer that sheet-2 is perpendicular to one of the other sheets at θ_2 = 60° and that it is perpendicular to the other of the other sheets when θ_2 = 140°. Without loss of generality, we choose θ_1 = 150°, θ_3 = 50°. Now, when θ_2 = 90°, it will be $|\Delta\theta|$ = 60° relative to sheet-1 and $|\Delta\theta'|$ = 40° relative to sheet-3. Therefore,

$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 7\%$

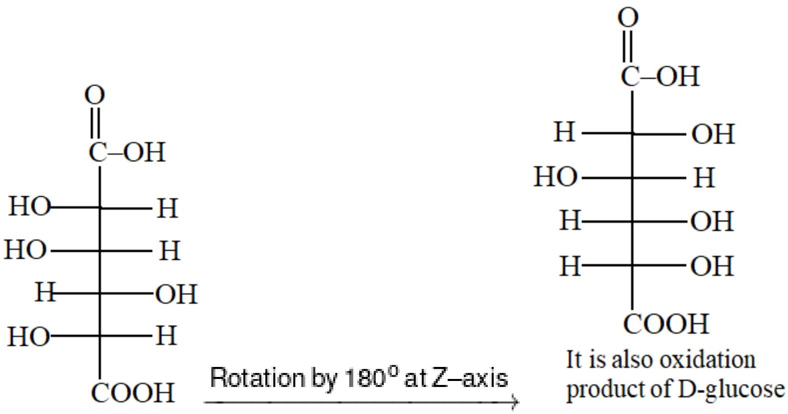
PART-2 : CHEMISTRY

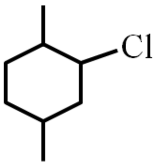
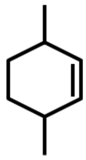
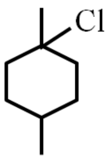
20) $K_{A_1} = K_1 ; K_{A_2} = 2K_2 ; K_{A_3} = 3K_3$

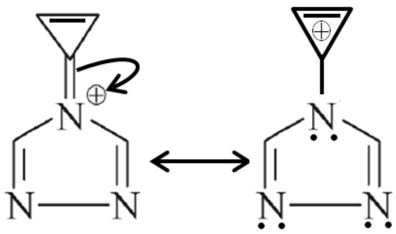
$T_{avg.} = \frac{1}{K_A} = \frac{1}{K_1 + 2K_2 + 3K_3} = 8$

21)

Oxidation product of C is



22) A =  B =  C = 

27) 

Both rings are aromatic
 minimum rotational barrier energy and due to easy breaking of π-bond it is most basic
* All compounds are bicyclic (Not bicyclo)

32) Energy of n photons = $nh\upsilon_0$
All this energy converts to KE of Xeton when Xteon absorbs these photons

$$KE = nh\upsilon_0 = \frac{1}{2}m_{zeton}V^2$$

$$nh\upsilon_0 = \frac{1}{2}(4m)\,V^2$$

Hence $v = \sqrt{\frac{nh\upsilon_0}{2m}}$

$$\lambda_{debroglie} = \frac{h}{(4m)\,v} = \frac{h}{4m\sqrt{\frac{nh\upsilon_0}{2m}}} = \frac{\sqrt{h}}{8\sqrt{m\upsilon_0}}$$

$$\Rightarrow \frac{\sqrt{h}}{4\sqrt{\frac{n}{2}}\sqrt{mv_0}} = \frac{1}{8}\sqrt{\frac{h}{mv_0}} \Rightarrow \frac{1}{8} = \frac{1}{4\sqrt{\frac{n}{2}}}$$

$$\Rightarrow 2\sqrt{\frac{n}{2}} \Rightarrow n = 8$$

33) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$T_2 = 300\,(8)^{2/3} = 1200$ K

$\frac{T_3}{T_2} = \frac{P_3}{P_2} = \frac{4}{3}$, $T_3 = 1600$ K

$\frac{T_4}{T_3} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{1600}{4} = 400\,\text{K}$

$|X| = \left|\frac{W_{234}}{W_{412}}\right| = \left|\frac{W_{34}}{W_{12}}\right| = \left|\frac{\Delta U_{34}}{\Delta U_{12}}\right| = \left|\frac{T_4 - T_3}{T_2 - T_1}\right|$

$|X| = \frac{1200}{900} = \frac{4}{3}$

$3|X| = 4$

34) $n_{N_2} = \frac{1.135}{22.7} = 0.05$ mol $= n_{N_2H_4}$ (n.f. = 4)

eq. N_2H_4 = eq. of I_2 = eq. of PbO_2

0.1 mole of PbO_2

$PbO_2 + 2NaOH \rightarrow Na_2PbO_3 + H_2O$

0.1 mole $\quad \frac{1}{10}$M

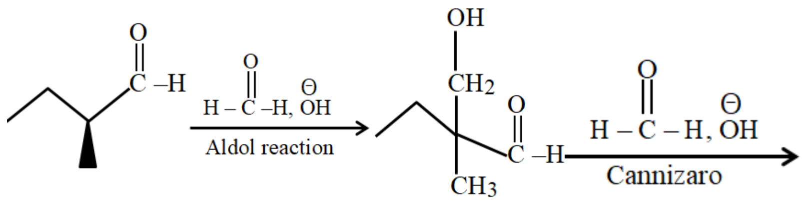
V = ?

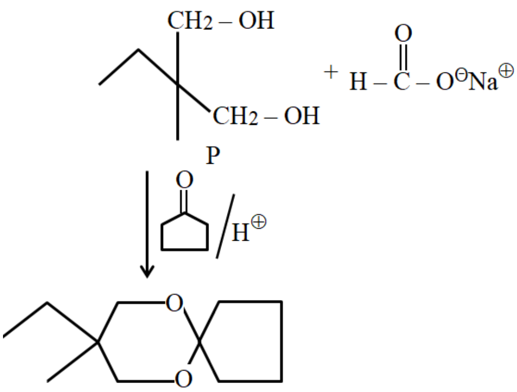
0.1 mol $PbO_2 \rightarrow$ 0.2 mole NaOH

$0.2 = \frac{1}{10} \times V$

V = 2 litre.

35)





Value of X = 7

Value of Y = 2

 X – Y

= 7 – 2 = 5

36)

i, ii, iv, vii, viii

PART-3 : MATHEMATICS

37) $f'(x) = 3x^2 + 2x + 3 + 2\cos x = \underbrace{3\left(x+\frac{1}{3}\right)^2}_{\geqslant 0} - \underbrace{\frac{1}{3} + 3 + 2\cos x}_{>0}$

f (x) is strictly increasing and f (0) = 0

38) $y = \ell n^2 x - 1$

$y' = \frac{2\ln x}{x} = 0 \Rightarrow x = 1$

x > 1, y ↑ and 0 < x < 1, y is ↓

$A = \left|\int_{1/e}^{e}(\ln^2 x - 1)dx\right|$

$= \left|\left[x\ln^2 x\right]_{1/e}^{e} - 2\int_{1/e}^{e}\left(\frac{\ln x}{x}\right)\cdot x\,dx - \left(e - \frac{1}{e}\right)\right|$

$= \left|\left(e - \frac{1}{e}\right) - 2\int_{1/e}^{e}\left(\frac{\ln x}{x}\right)\cdot x\,dx - \left(e - \frac{1}{e}\right)\right|$

$= \left|-2\left[\left[x\ln x\right]_{1/e}^{e} - \int_{1/e}^{e}dx\right]\right| = \left|-2\left[\left(e+\frac{1}{e}\right) - \left(e - \frac{1}{e}\right)\right]\right| = \left|\frac{4}{e}\right| = \frac{4}{e}$

39) $(1 + x + x^3)^{10} = {}^{10}C_0 + {}^{10}C_1(x + x^3) + {}^{10}C_2 x^2(1 + x^2)^2 + {}^{10}C_3 x^3(1 + x^2)^3 + \ldots\ldots$

$\ldots\ldots {}^{10}C^9 x^9(1 + x^2)^9 + {}^{10}C_{10} x^{10}(1 + x^2)^{10}$

$= a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{28}x^{28} + \underset{\times}{a_{29}x^{29}} + a_{30}x^{30}$

The expansion does not contain any term of x^{29}

∴ Number of different terms is 30

40) $(1 + x + 2x^2 + 4x^4)20$

$T_{r+1} = {}^{20}C_r \cdot (1 + x + 2x^2)^{20-r} \cdot (4x^4)^r$

For r = 19

$T_{19+1} = {}^{20}C_{19} \cdot (1 + x + 2x^2)^1 \cdot (4x^4)^{19}$

∴ Coefficient of $x^{78} = 2 \cdot {}^{20}C_{19} \cdot 4^{19} = 2 \cdot 20 \cdot 2^{38} = 10 \cdot 2^{40}$

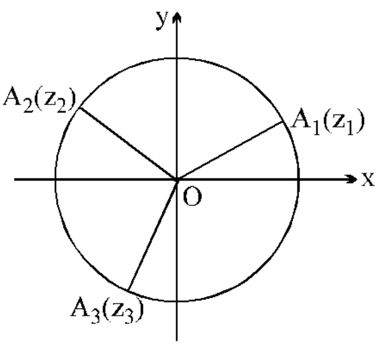
41) $z_1\bar{z}_2 = z_2\bar{z}_3 = z_3\bar{z}_1$ ....(2)

 $| z_1 | | z_2 | = | z_2 | | z_3 | = | z_3 | | z_1 |$

 $\frac{1}{|z_3|} = \frac{1}{|z_1|} = \frac{1}{|z_2|}$

 $| z_1 | = | z_2 | = | z_3 | = r$

$\Rightarrow$ z_1, z_2, z_3 lie on a circle of radius r.



$| z_1 |^2 = r^2 \quad \Rightarrow \bar{z}_1 = \frac{r^2}{z_1}$; |||ly $\bar{z}_2 = \frac{r^2}{z_2}$; $\bar{z}_3 = \frac{r^2}{z_3}$

from (2) $z_1 \frac{r^2}{z_2} = z_2 \frac{r^2}{z_3} = z_3 \frac{r^2}{z_1}$

$\frac{z_1}{z_2} = \frac{z_2}{z_3} = \frac{z_3}{z_1}$

hence amp z_1 – amp z_2 = amp z_2 – amp z_3 = amp z_3 – amp z_1

$\Rightarrow$ $A_1A_2A_3$ is equilateral triangle.

 (2) $\Rightarrow$ (1) $\Rightarrow$ (A)

Also $\frac{z_1}{z_2} = \frac{z_2}{z_3} = \frac{z_3}{z_1}$

hence $z_2^2 = z_3z_1$ and $z_3^2 = z_1z_2$ & $z_1^2 = z_2z_3$

 (2) $\Rightarrow$ (3) $\Rightarrow$ (C)

42) We have

$2[x] + 64 = 3[x] - 192 \quad \Rightarrow [x] = 256 \quad \Rightarrow x \in [256, 257)$

Also, $\beta = \sin\frac{\pi}{18} \cdot \sin\frac{3\pi}{18} \ldots \sin\frac{9\pi}{18} \ldots \sin\frac{17\pi}{18}$

$= \sin^2\frac{\pi}{18} \cdot \sin^2\frac{3\pi}{18} \cdot \sin^2\frac{5\pi}{18} \cdot \sin^2\frac{7\pi}{18} = \sin^2 10° \cdot \sin^2 30° \cdot \sin^2 50° \cdot \sin^2 70°$

$\frac{1}{4}(\sin 10° \cdot \sin 50° \cdot \sin 70)^2 = \frac{1}{4}\left(\frac{\sin 30}{4}\right)^2$

$\therefore \beta = \frac{1}{4}\left(\frac{1}{8}\right)^2 = \frac{1}{256}$

43)

(A) $5 \sec^{-1}\alpha + 10 \sin^{-1} \beta = 10 \pi \;\; \sec^{-1} \alpha = \pi \;\&\; \sin^{-1} \beta = \pi/2$

$\Rightarrow \alpha = -1; \;\; \beta = 1$ So $\tan^{-1} (-1) + \cos^{-1} (0) = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

(B) $\sin 2\pi x + \cos 2\pi x = \sin 2\pi x + \frac{1}{2} + \frac{\cos 2\pi x}{2}$

Min. value = $\frac{-\left(\sqrt{5}+1\right)}{2}$

(C) We have $x = \frac{\sqrt{3}\cos 20^\circ - 4\sin 20^\circ.\cos 20^\circ}{\sin 20^\circ} = \frac{2\sin 60^\circ.\cos 20^\circ - 2\sin 40^\circ}{\sin 20^\circ}$

$= \frac{2\sin 60^\circ.\cos 20^\circ - 2\sin 40^\circ}{\sin 20^\circ} = 1$

(D) $f(x) = \sum_{n=1}^{4}\left[1+\sin\frac{x}{n}\right] = \sum_{n=1}^{4}\left(1+\left[\sin\frac{x}{n}\right]\right) = 4 + [\sin x] + \left[\sin\frac{x}{2}\right] + \left[\sin\frac{x}{3}\right] + \left[\sin\frac{x}{4}\right]$

$\left[\sin\frac{x}{3}\right]$ and $\left[\sin\frac{x}{4}\right]$ is zero in (0, p)

Hence $f(x) = 4 + [\sin x] + \left[\sin \frac{x}{2}\right]$

Now range of $f(x)$ is $\{4, 5\}$

Hence sum is $4 + 5 = 9 \Rightarrow$ (D) is incorrect.

44) (A) $f(x) = \sin^{-1}\left(\log_3\left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\right)$

$f(-x) = \sin^{-1}\left(\log_3\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)\right) = -f(x)$

$f\left(\frac{1}{x}\right) = f(x) = -f(-x)$

(B) $f'(x) = \frac{1}{\sqrt{1 - \left(\log_3\left(\frac{x^2-x+1}{x^2+x+1}\right)\right)^2}} \cdot \frac{\log_3 e}{\left(\frac{x^2-x+1}{x^2+x+1}\right)} \cdot \frac{2(x^2 - 1)}{(x^2 + x + 1)^2}$

+	–	+
	-1	1
	max. at $x = -1$	min at $x = 1$

Since $\frac{x^2 - x + 1}{x^2 + x + 1} \in \left[\frac{1}{3}, 3\right]$

∴ $f(x) = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

45) Given, $A + \text{adj}(A) = \alpha A^T$ ....(1)

(A) If $\alpha = 1$, $A + \text{adj}(A) = A^T \Rightarrow \text{adj}(A) = -(A - A^T)$

$\Rightarrow |\text{adj}(A)| = |-(A - A^T)| = 0$

∴ $A - A^T$ is skew symmetric.

(B) If $\alpha = 1$, $A + \text{adj}(A) = A^T \Rightarrow 3A + \text{adj}(A) = A^T + 2A$

So, $\text{tr}(3A + \text{adj}(A)) = \text{tr}(A^T + 2A) \equiv \text{tr}(3A) = 9$ (diagonal element are same in A^T and A).

(C) If $\alpha = -2$, $A + \text{adj}(A) = -2A^T$

$\Rightarrow 3A + \text{adj}(A) = -2A^T + 2A = 2(A - A^T) \Rightarrow |3A + \text{adj}(A)| = 0$

(D) If $\alpha = -2$, $A + \text{adj}(A) = -2A^T \Rightarrow 4A + \text{adj}(A) = 3A - 2A^T$

So, $\text{tr}(4A + \text{adj}(A)) = \text{tr}(3A - 2A^T) \equiv \text{tr}(A) = 3$.

46) (A) 3^5 ; There are 5 pair of numbers having sum equals to 11

(1, 10); (2, 9); (3, 8); (4, 7); (5, 6)

from every pair, one number can be taken in 3 ways

Hence number of subsets $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 35$

(B) 3^5 ; $xyz = 2310 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$

2 can be distributed to x, y, z in 3 ways.

3 can be distributed to x, y, z in 3 ways.

5 can be distributed to x, y, z in 3 ways.

7 can be distributed to x, y, z in 3 ways.

11 can be distributed to x, y, z in 3 ways.

Hence total ways is 3^5

(C) 3^5;

1					

1st place can be filled only in one way i.e. 1

remaining 5 places can be filled in 3^5 ways

Hence total number of ways $= 1 \cdot 3^5 = 3^5$ Ans.

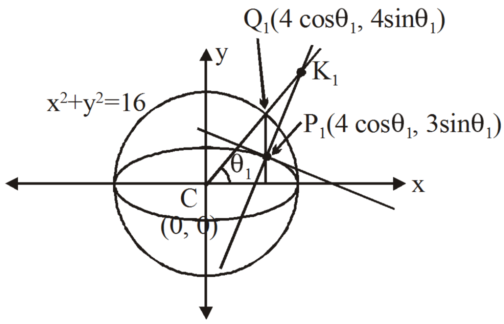
(D) $3^5 - 1$; Obvious.

47) Equation of normal at $P_1(4\cos\theta_1, 3\sin\theta_1)$ is

$$\frac{4x}{\cos\theta_1} - \frac{3y}{\sin\theta_1} = 7 \quad \dots (1)$$

Also, equation of CQ_1 is

$$y = \left(\frac{\sin\theta_1}{\cos\theta_1}\right)x \quad \dots (2)$$



∴ Solving (1) and (2), we get

$$\frac{4x}{\cos\theta_1} - \frac{3}{\sin\theta_1}\left(\frac{\sin\theta_1}{\cos\theta_1}\right)x = 7$$

$\Rightarrow \frac{x}{\cos\theta_1} = 7 \quad \Rightarrow \quad x = 7\cos\theta_1, \; y = 7\sin\theta_1$

So, $K_1 = (7\cos\theta_1, 7\sin\theta_1) \Rightarrow CK_1 = 7$

Similarly, $CK_2 = CK_3 = \dots = CK_n = 7$

∴ $\sum_{i=1}^{n} CK_i = 56 \quad \Rightarrow 7n = 56 \quad \Rightarrow n = \frac{56}{7} = 8$

48) $\frac{f(x)}{1+x^2} = 1 + \int_0^x \frac{f^2(t)\,dt}{1+t^2}$ [Note: $f(0) = 1$]

differentiate both sides w.r.t. x

$$\frac{(1+x^2)f'(x) - f(x)(2x)}{(1+x^2)^2} = \frac{f^2(x)}{(1+x^2)}$$

$$\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)y = y^2$$

$$\frac{1}{y^2}\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)\frac{1}{y} = 1$$

Let $\frac{-1}{y} = t$

$$\frac{dt}{dx} + \left(\frac{2x}{1+x^2}\right)t = 1$$

solving the above L.D.E,

$$f(x) = \frac{-3(1+x^2)}{x^3+3x-3}$$

$$f(-2) = \frac{-3(5)}{-8-6-3} = \frac{15}{17}$$

49) $I = \int_{\frac{-1}{\sqrt[3]{n}}}^{\frac{1}{\sqrt[3]{n}}} x \; \ell n\,(1 + e^x + e^{2x} + \dots\dots\dots e^{2nx})\,dx \quad \dots\dots\dots (1)$

Using k ing

$$I = \int_{\frac{-1}{\sqrt[3]{n}}}^{\frac{1}{\sqrt[3]{n}}} -x \ln\left(1 + \frac{1}{e^x} + \frac{1}{e^{2x}} + \ldots\ldots\ldots + \frac{1}{e^{2nx}}\right) dx$$

$$= \int_{\frac{-1}{\sqrt[3]{n}}}^{+\frac{1}{\sqrt[3]{n}}} -x \ln\left(\frac{e^{2nx} + e^{(2n-1)x} + \ldots\ldots\ldots + e^x + 1}{e^{2nx}}\right) dx$$

$$I = -I + \int_{\frac{-1}{\sqrt[3]{n}}}^{+\frac{1}{\sqrt[3]{n}}} x \ln e^{2nx}\, dx$$

$$\therefore 2I = 2n\left(\frac{x^3}{3}\right)_{\frac{-1}{\sqrt[3]{n}}}^{\frac{1}{\sqrt[3]{n}}} = 2n\left(\frac{1}{3n} + \frac{1}{3n}\right) = \frac{4}{3}$$

$$\therefore I = \frac{2}{3}$$

50) W_1: $C_1 = (-5, 12)$ W_2: $C_2 = (5, 12)$

$r_1 = 16$ $r_2 = 4$

now, $CC_2 = r + 4$

$CC_1 = 16 - r$

let $C(h, k) = c(h, ah)$

$CC_1^{\,2} = (16 - r)^2$

$\Rightarrow (h + 5)^2 + (12 - ah)^2 = (16 - r)^2$

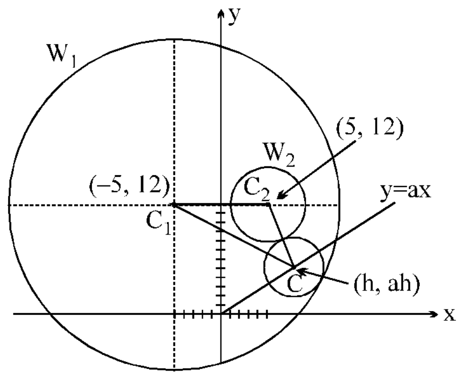
$CC_2^{\,2} = (4 + r)^2$

$\Rightarrow (h - 5)^2 + (12 - ah)^2 = (4 + r)^2$

By subtraction

$20h = 240 - 40r$

$\Rightarrow h = 12 - 2r \Rightarrow 12r = 72 - 6h$ ...(1)



By addition

$2[h^2 + 25 + a^2h^2 - 24ah + 144] = 272 - 24r + 2r^2$

$h^2(1 + a^2) - 24ah + 169 = 136 - 12r + r^2 = 136 + (6h - 72) + \left(\frac{12-h}{2}\right)^2$ [using (1)]

$\Rightarrow 4[h^2(1 + a^2) - 24ah + 169] = 4[64 + 6h] + (12 - h)^2 = 256 + 144 + h^2$

$\Rightarrow h^2(3 + 4a^2) - 96ah + 105 \cdot 4 - 36 \cdot 4 = 0$

$\Rightarrow h^2(3 + 4a2) - 96ah + 69 \cdot 4 = 0$; for 'h' to be real $D \geq 0$

$\Rightarrow (96a)^2 - 4 \cdot 4 \cdot 69 (3 + 4a^2) \geq 0$

$\Rightarrow 576a2 - 69.3 - 276a^2 \geq 0$

$300a^2 \geq 207 \Rightarrow a^2 \geq \frac{69}{100}$; hence m (smallest) $= \frac{13}{10}$

So, $m^2 = \frac{69}{100}$; $\therefore$ p + q = 169

51) $f(0^-) = \lim_{x\to 0} \dfrac{x\left[\dfrac{\ell n(1+x)}{x} - \dfrac{\ell n(1-x)}{-x}\right]x}{x^2 \dfrac{(1-\cos x)}{x^2}\left(\dfrac{1+\cos x}{\cos x}\right)} = 0$

& $f(0^+) = 0$

∴ function is always continuous.

$f(0^-) = \lim_{h\to 0} \dfrac{f(0-h) - f(0)}{-h}$

$= \lim_{h\to 0} -h\left(\dfrac{\ell n(1-h) + \ell n(1+h)}{(1-\cos^2 h)\cdot\frac{1}{\cosh}}\right) = 0$

& $f'(0^+) = k^2 - 3k - 1$

∴ function are differentiable at x = 0

$\Rightarrow k^2 - 3k - 1 = 0 \;\left\langle \begin{matrix} k_1 \\ k_2 \end{matrix}\right.$

$k_1 + k_2 = 3,\ k_1k_2 = -1$

∴ $k_1^2 + k_2^2 = (k_1 + k_2)^2 - 2k_1k_2 = 11$

52) $\log_3 |x - 2| = 2^{\log_2\left(\log_3(|x|-2)\right)}$

$\Rightarrow \log_3 |x - 2| = \log_3 (|x| - 2)$

$\Rightarrow |x - 2| = |x| - 2$

$\Rightarrow |x| - |x - 2| = 2$

Case-I : $x < 0$

$-x + x - 2 = 0$

$-2 = 2$ (no solution)

Case-II: $0 \le x < 2$

$x + (x - 2) = 2$

$\Rightarrow 2x = 4 \Rightarrow x = 2$ (no solution)

Case-III : $x \ge 2$

$x - (x - 2) = 2$

$\Rightarrow 2 = 2$

∴ $x \ge 2$

But $\log_3(|x| - 2) > 0 \Rightarrow |x| - 2 > 1 \Rightarrow |x| > 3$

∴ $x \in (3, \infty)$ is the solution set of the equation

Least integral value of x is 4.

Now, $(\alpha)^{-4\alpha} = (4)^{-16} = 2^{-32}$

$N = 2^{-32}$

$\log_{10}N = -32 \log_{10} 2 = -32 \times 0.301 = -9.632$

∴ Number of zeros after decimal and before first significant digit in 2^{-32} is 9

53) det. (A) = $x_1 \cdot x_2 \cdot x_3$

det. (B) = $y_1 \cdot y_2 \cdot y_3$

∴ $x_1 \cdot x_2 \cdot x_3 = y_1 \cdot y_2 \cdot y_3$

Both sides	Number of solutions.	
0	$({}^3C_1 \cdot 2 \cdot 2 + {}^3C_2 \cdot 2 + {}^3C_3)^2 = 19^2$	= 361
1	$({}^3C_2 \cdot 1 + {}^3C_3)^2 = 4^2$	= 16
– 1	$({}^3C_2 \cdot 1 + {}^3C_3)^2 = 4^2$	= 16
	N	= 393

$\therefore \frac{N}{131} = 3.$ Ans.]

54) $y^{2/3} - 2xy^{1/3} + 1 = 0$

$y^{1/3} = \left(x \pm \sqrt{x^2-1}\right) \Rightarrow \ln y = 3 \ln\left(x \pm \sqrt{x^2-1}\right)$

$\frac{y_1}{y} = \frac{\pm 3}{\sqrt{x^2-1}} \Rightarrow (x^2 - 1)\, y_1^2 = 9y^2$

$2x\, y_1^2 + (x^2 - 1)\, 2y_1y_2 = 18yy_1$

$xy_1 + (x^2 - 1)y_2 = 9y$ (As y_1 is not equal to 0, because y is not constant)

Dividing by y, we get

$\therefore \quad x\frac{y_1}{y} + (x^2 - 1)\frac{y_2}{y} = 9$ Ans.

Alternatively: Given $2x = \left(y^{\frac{1}{3}} + y^{\frac{-1}{3}}\right)$ .....(1)

Differentiate both side with respect to x, we get

$2 = \left(\frac{1}{3}y^{\frac{1}{3}-1} - \frac{1}{3}y^{\frac{-1}{3}-1}\right) y_1 \Rightarrow 2 = \frac{\left(y^{\frac{1}{3}} - y^{\frac{-1}{3}}\right) y_1}{3y}$

$\Rightarrow 6y = \left(y^{\frac{1}{3}} - y^{\frac{-1}{3}}\right) y_1$ (squaring)

$\Rightarrow 36y^2 = (4x^2 - 4)y_1^2$ {using equation (1) }

$\Rightarrow 9y^2 = (x^2 - 1)y_1^2$ ....(2)

Again differentiate both sides of above equation, we get

$18yy_1 = (x^2 - 1) \cdot 2\, y_1y_2 + 2xy_1^2$

$\Rightarrow \quad 9y = (x^2 - 1)y_2 + xy_1$ (As y is not constant so $y_1 \neq 0$)

$\Rightarrow \quad 9 = \frac{(x^2-1)}{y} \cdot y_2 + \frac{x}{y} \cdot y_1$