
PHYSICS
Rankers Academy JEE

7

max. error

The following measurements are obtained on a single phase load V = 220 V ± 1%, I = 5.0 A ± 1% and ~~W~~ P = 555 W ± 2%. If the power factor is calculated using these measurements, the worst case error in the calculated power factor in percent is _____ (Given answer up to one decimal place).

(A) 2 (B) 0

(C) 4 (D) 3

power factor

$P = VI\cos\phi$

$\cos\phi = P^{(1)} V^{(-1)} I^{(-1)}$

$\left|\frac{\Delta\gamma}{\gamma}\right| \times 100 = \left|\frac{\Delta P}{P}\right| \times 100 + \left|\frac{\Delta V}{V}\right| \times 100 + \left|\frac{\Delta I}{I}\right| \times 100$

% error = 2% + 1% + 1%

= 4%

JEE 1

2

The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option:

(A) $R_1 > R_2$ and $H_1 = H_2$

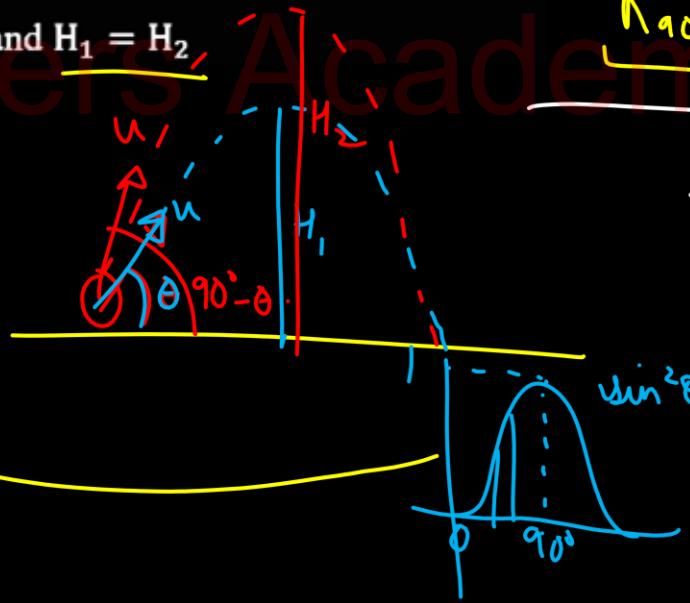
(B) $R_1 = R_2$ and $H_1 < H_2$

(C) $R_1 < R_2$ and $H_1 < H_2$

(D) $R_1 = R_2$ and $H_1 = H_2$

$\theta = 42°$, $90° - \theta = 48°$

$H_1 < H_2$



$$R_\theta = \frac{u^2}{g} \cdot \sin(2\theta)$$

$$R_{90°-\theta} = \frac{u^2}{g} \sin(2(90° - \theta))$$

$$R_{90°-\theta} = \frac{u^2}{g} \sin(2\theta)$$

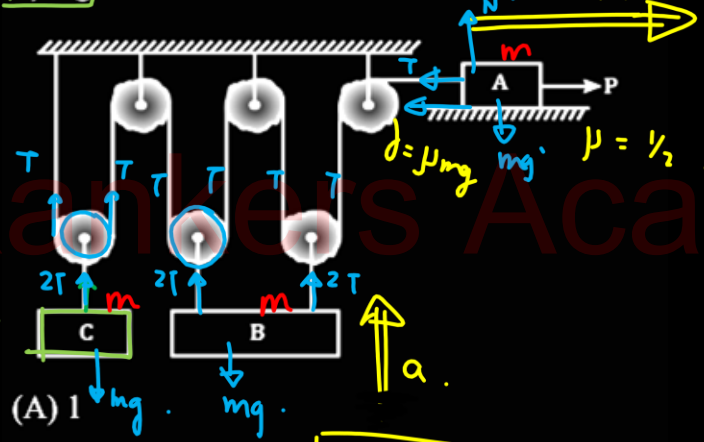
$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H_1 = \frac{u^2}{2g} \cdot \sin^2 42°$$

$$H_2 = \frac{u^2}{2g} \cdot \sin^2 48°$$

3

Three blocks A, B and C of mass m each are arranged in pulley mass system as shown. Coefficient of friction between block A and horizontal surface is equal to 0.5 and a force P acts on ' A ' in the direction shown. The value of P/mg so that block ' C ' doesn't move is –



(A) 1

(B) 2

(C) 3

(D) 5

$P/mg = ?$

$4T - mg = ma$ — ①

$P - T - \mu mg = m(4a)$ — ②

$P - \frac{mg}{2} - \frac{mg}{2} = 4ma$ = Q

$\mu mg = ma$

$a = g$

use in Q

rest

$2T = mg$

$T = \frac{mg}{2}$ ☺

$\sum \vec{T} \cdot \vec{a} = 0$

$-Ta_1 + 4Ta_2 = 0$

$a_1 = 4a_2$

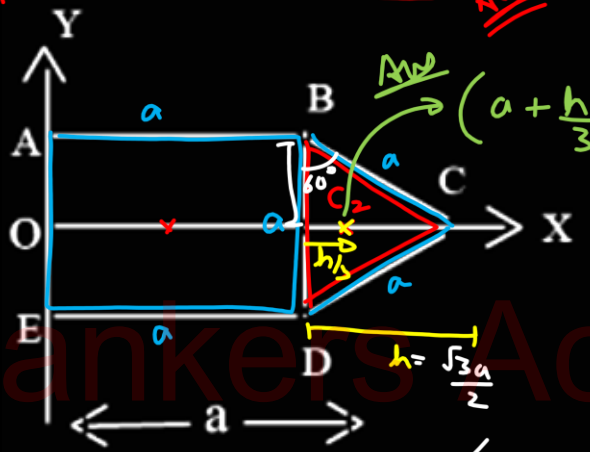
$P - mg = 4mg$

$P = 5mg$

$\frac{P}{mg} = 5$

4

A uniform lamina ABCDE is made from a square ABDE and an equilateral triangle BCD. Find the center of mass of the triangle.



Ans $C_2 \equiv \left(a + \frac{1}{3}\frac{\sqrt{3}a}{2}, 0\right) \equiv \left(a + \frac{a}{2\sqrt{3}}, 0\right)$

Ans $\left(a + \frac{h}{3}, 0\right)$.

(A) $(a, 0)$

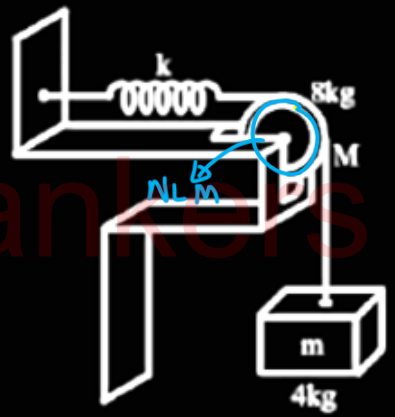
(B) $\left(a + \frac{a}{2\sqrt{3}}, 0\right)$ ✓

(C) $\left(a + \frac{a}{\sqrt{3}}, 0\right)$

(D) $\left(a - \frac{a}{\sqrt{3}}, 0\right)$ ✗

5

A block of mass m = 4 kg is attached to a spring of spring constant ($k = 32 Nm^{-1}$) by a rope that hangs over a frictionless pulley. If the system starts from rest with the spring unstretched, find the speed of the block after it falls 1 m.



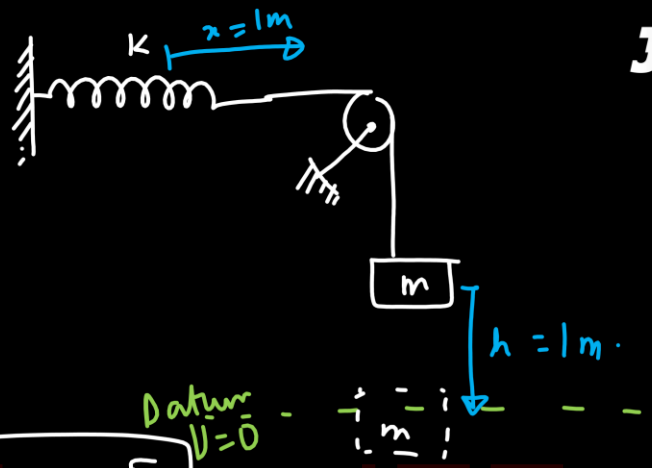
$\sqrt{9} = 3$ $\quad$ $\sqrt{16} = 4$

(A) 1.4 m/s

(B) 3.46 m/s

(C) 2.4 m/s

(D) 0.4 m/s



$E_i = E_f$

$mg(1) = \frac{1}{2}k(1)^2 + \frac{1}{2}mV^2$

$40 = 16 + 2V^2$

$V = \sqrt{12} = 3.46$

6

A block (3m) moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. (m, 2m) If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

JEE 1

$P_i = P_f$

$120m = 60m + 2mv$

$v = 30$

$K_i = \frac{1}{2}(3m)(40)^2$

$K_i = 2400m$

$K_f = \frac{1}{2}m(60)^2 + \frac{1}{2}(2m)30^2$

$K_f = 1800m + 900m$

$K_f = 2700m$

$\Delta W = \frac{K_f - K_i}{K_i} = \frac{300m}{2400m} = \frac{1}{8}$

7

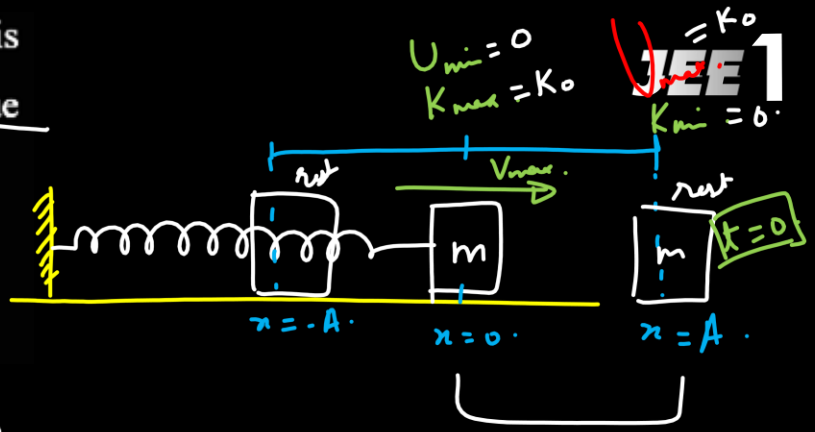
For a particle executing S.H.M. The K.E 'K' is given by $K = K_0 \sin^2 \omega t$. The maximum value of P.E is

(A) K_0

(B) Zero

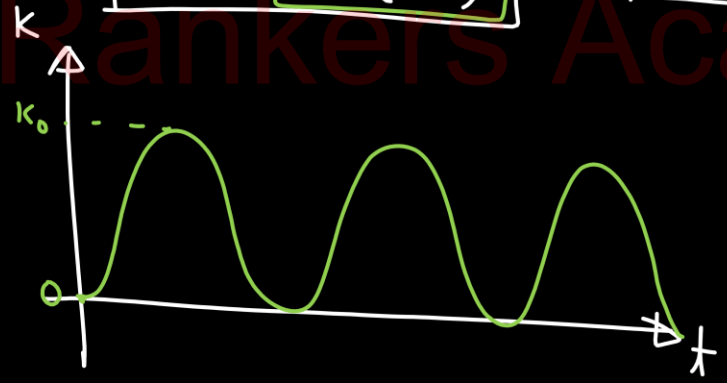
(C) $\frac{K_0}{2}$

(D) $\frac{K_0}{4}$



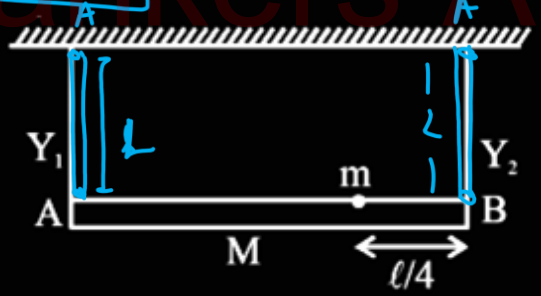
$K = K_0 \sin^2(\omega t)$

$K_{max} = K_0$



8

A rigid rod AB of mass M and length ℓ is suspended horizontally from two vertical wires having same length and same area of crosssection. When a mass ' m ' is placed at a distance $\frac{\ell}{4}$ from end B, rod remains horizontal. If the ratio of Young modulus of two wires $\frac{Y_1}{Y_2}$ is $\left(\frac{xM+m}{2M+ym}\right)$ then find the value of $(x+y)$.

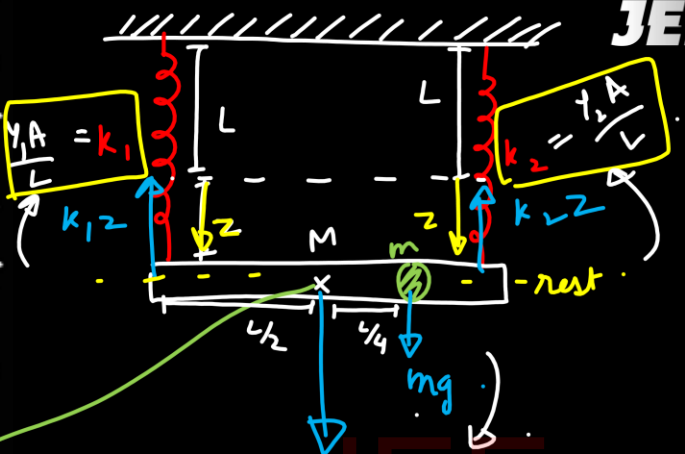


(A) 2 (B) 3

(C) 5 (D) 7

$K = \frac{YA}{L}$

$\frac{Y_1}{Y_2} = \frac{K_1}{K_2}$



$k_1z + k_2z = (M+m)g$ — (1)

com. $\tau = \tau$

$k_1z\left(\frac{L}{2}\right) + mg\left(\frac{L}{4}\right) = k_2z\left(\frac{L}{2}\right)$

$k_2z - k_1z = \frac{mg}{2}$ — (2)

① + ②

$$2K_2 z = Mg + 3\frac{mg}{2}$$

① − ②

$$2K_2 z = Mg + \frac{mg}{2}$$

÷

$$\frac{K_2}{K_1} = \frac{\left(M + \frac{3}{2}m\right) \times 2}{\left[M + \frac{m}{2}\right] \times 2}$$

$$\frac{Y_2}{Y_1} = \frac{K_2}{K_1} = \frac{2M + 3m}{2M + m}$$

$$\frac{Y_1}{Y_2} = \frac{2M + 1m}{2M + 3m}$$

$x = 2$

$y = 3$

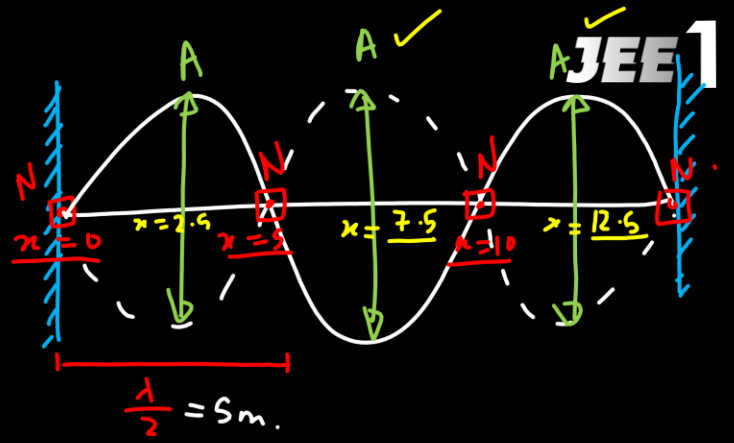
$x + y = 5$

9

The equation of a stationary wave is $y = 4\sin\left(\frac{\pi x}{5}\right)\cos(100\pi t)$. The wave is formed using a string of length 20 cm . The second and 3rd antinodes are located at positions (in cm)

(A) 7.5,12.5 Draw (B) ~~2.5,7.5~~

(C) 12.5,17.5 ~~(D) 5,10~~



JEE 1

$y_{s/w} = \underbrace{2A\sin(kx)}_{R} \cdot \cos(\omega t)$

$k = \frac{\pi}{5} = \frac{2\pi}{\lambda}$

$\lambda = 10$

NODE

$R|_{x=0} = 0$

Nodes

$\sin\left(\frac{\pi}{5}x\right) = 0$

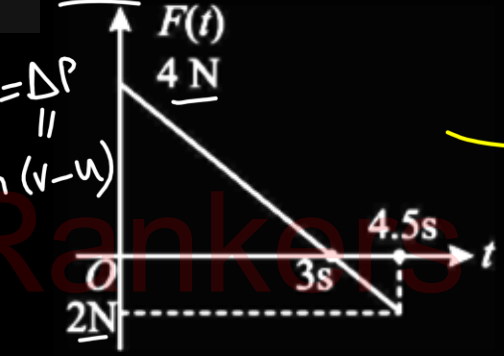
$= \sin(0)$

$= \sin(\pi)$

$= \sin(2\pi)$

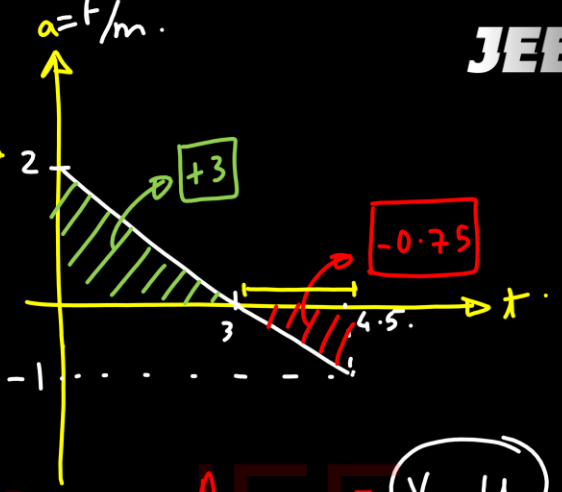
10

A block of mass 2 kg is free to move along the x -axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force F(t) in the x -direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is



(A) 4.50 J
(C) 5.06 J
(B) 7.50 J
(D) 14.06 J

$Area_{F-t} = J = \Delta P$
$= m(v-u)$



$Area_{a-t} = v - u$

$2.25 = v - 0$

$v = 2.25$

$K_f = \frac{1}{2}mv^2 = \frac{1}{2}(1)(2.25)^2$

11

Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be

$m = 1$
$R = 1$

(A) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$

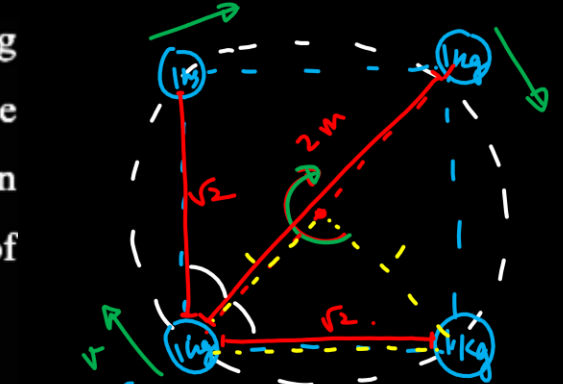
(B) $\sqrt{G(1+2\sqrt{2})}$

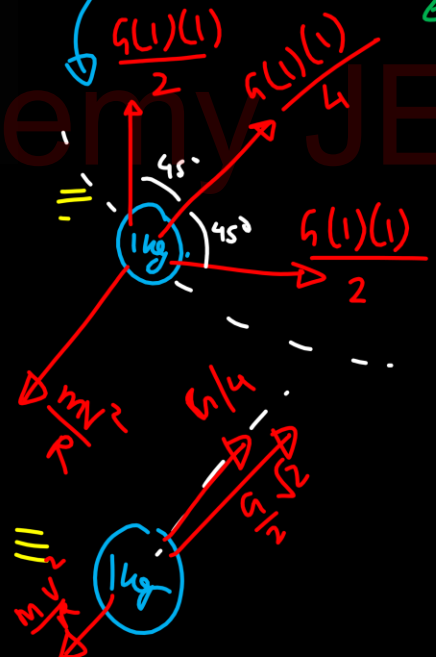
(C) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

(D) $\sqrt{\frac{G}{4}(1+2\sqrt{2})}$

$$\frac{mv^2}{R} = \frac{G}{4} + \frac{G}{2}\sqrt{2}$$

$$v = \sqrt{\frac{G}{4}(1+2\sqrt{2})}$$





12

A particle of mass m is moving horizontally at speed v perpendicular to a uniform rod of length d and mass M = 6 m. The rod is hinged at centre O and can freely rotate in horizontal plane about a fixed vertical axis passing through its centre 0 . The hinge is frictionless. The particle strikes and sticks to the end of the rod. The angular speed of the system just after the collision:

(A) 2v/3 d

(B) 3v/2 d

(C) v/3 d

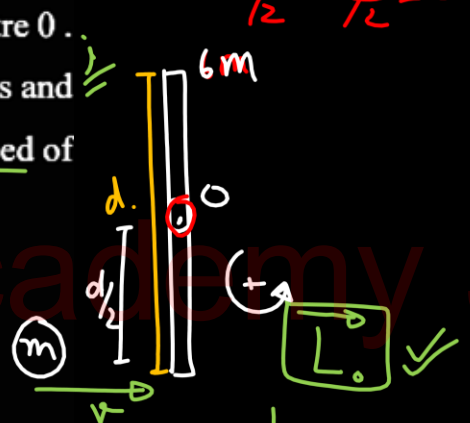
(D) 2v/d

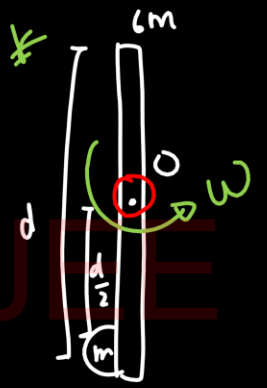
about O - hinge.

$\tau = 0$

L_0 → const.

$L_0 = \frac{Jd}{2} - \frac{Jd}{2} = 0$





$L_i = L_f$

$mv\frac{d}{2} = \left[\frac{(6m)d^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega$

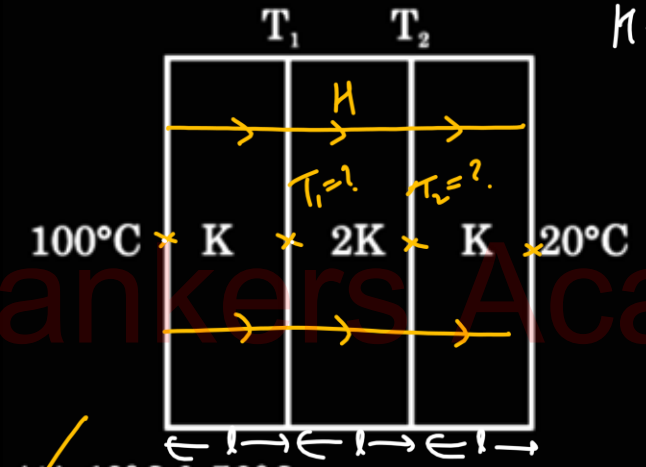
$\frac{mvd}{2} = \frac{3}{4}md^2\omega$

$\omega = \frac{2v}{3d}$

13

JEE 1

Three slabs of equal area and thickness are arranged as shown in the figure. Find the value of T_1 and T_2 in steady state :-



(A) 68°C & 52°C

(B) 62°C & 58°C

(C) 60°C & 50°C

(D) 50°C & 30°C

$$H = \frac{dQ}{dt} = \frac{KA}{\ell}(100 - T_1) = \frac{2KA}{\ell}(T_1 - T_2) = \frac{KA}{\ell}(T_2 - 20)$$

$$3T_1 - 2T_2 = 100 \quad (1)$$

$$2(T_1 + T_2 = 120) \quad (2)$$

$$5T_1 = 340$$

$$T_1 = 68^\circ C$$

$$T_2 = 180 - T_1 = 120 - 68 = 52$$

14

JEE 1

Magnetic field associated with electromagnetic wave whose electric field is given by

$\vec{E} = 2.1\sin(3 \times 10^8 t - 1.8Z)\hat{i}$ N/C, is

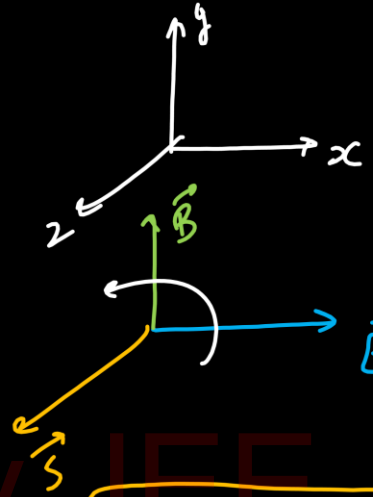
(A) $\vec{B} = 1.26 \times 10^{-8}\sin(3 \times 10^8 t - 1.8Z)\hat{j}$ T

(B) $\vec{B} = 1.8 \times 10^{-8}\sin(3 \times 10^8 t - 1.8Z)\hat{j}$ T

(C) $\vec{B} = 0.7 \times 10^{-8}\sin(3 \times 10^8 t - 1.8Z)\hat{j}$ T

(D) $\vec{B} = -1.26 \times 10^{-8}\sin(3 \times 10^8 t - 1.8Z)\hat{j}$ T

+z-direction



$\vec{E} \times \vec{B} \parallel \vec{S}$ *

$\omega = 3 \times 10^8$

$k = 1.8$

$v = \frac{\omega}{k} = \frac{3 \times 10^8}{1.8}$ — (1) *

$E = Bv \Rightarrow B = \frac{E}{v} = \frac{2.1}{3 \times 10^8} \times 1.8 = 1.26 \times 10^{-8}$ — (2) *

15

JEE 1

In a hydrogen atom, electron jumps from 4th excited state to 2nd excited state. Wavelength of photon emitted is [R : Rydberg constant]

$n = 5$

$n = 3$

(A) $\frac{225}{16R}$

(B) $\frac{225}{4R}$

(C) $\frac{100}{21R}$

(D) $\frac{100}{4R}$

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{5^2}\right]$$

$$\frac{1}{\lambda} = R\left(\frac{25-9}{9\times 25}\right)$$

$$\lambda = \frac{225}{16R}$$

16

In a Vernier calipers, one main scale division is 1 mm and 9 main scale divisions are equal to 10 vernier scale divisions. When nothing is put between jaws of the calipers, zero of the Vernier scale lies to the right side of zero of the main scale and the 2nd division of the Vernier scale coincides with a main scale division. While measuring inner diameter of a hollow cylinder the zero of Vernier scale lies between 1.7 cm and 1.8 cm of the main scale. Also, 8th division of Vernier scale coincides with a main scale division, inner diameter of the cylinder is

(A) 17.2 mm
(B) 17.4 mm
(C) 17.6 mm
(D) 17.8 mm

1 MSD = 1mm

9 MSD = 10 VSD

L.C. = MSD − VSD

$= MSD - \frac{9}{10} MSD$

$= \frac{1}{10} MSD = \frac{1}{10} \times 1mm = 0.1mm$ — (1)

zero = +2 × L.C. — (2)

Reading = MSR + CVSD × L.C. − zero

= 1.7 cm + 8 × L.C. − 2 × L.C.

= 1.7 cm + 6 × L.C.

= 1.76 cm = 17.6 mm

17

JEE 1

Two charged spherical conductors of radius R_1 and R_2 are connected by a wire. Then the ratio of surface charge densities of the spheres (σ_1/σ_2) is :

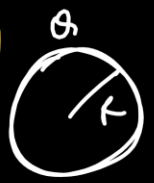
(A) $\frac{R_1}{R_2}$

(B) $\frac{R_2}{R_1}$

(C) $\sqrt{\left(\frac{R_1}{R_2}\right)}$

(D) $\frac{R_1^2}{R_2^2}$

$V = \frac{kQ}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{R}$



$V = \frac{\sigma R}{\varepsilon_0}$

$V_1 = V_2$ *

$\sigma = \frac{Q}{4\pi R^2}$

$\frac{\sigma_1 R_1}{\varepsilon_0} = \frac{\sigma_2 R_2}{\varepsilon_0}$

$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

18

Column - I gives certain physical terms associated with flow of current through a metallic conductor. Column - II gives some mathematical relations involving electrical quantities. Match Column - I and Column - II with appropriate relations.

Column-I	Column-II
(A) Drift Velocity	(P) $\frac{m}{ne^2\rho}$
(B) Electrical Resistivity	(Q) nev_d
(C) Relaxation Period	(R) $\frac{eE}{m}\tau$
(D) Current Density	(S) E/J

(A) (A) – (R), (B) – (S), (C) – (P), (D) – (Q)

(B) (A) – (R), (B) – (S), (C) – (Q), (D) – (P)

(C) (A) – (R), (B) – (P), (C) – (S), (D) – (Q)

(D) (A) – (R), (B) – (Q), (C) – (S), (D) – (P)

$v = u + at$

$v_d = 0 + \left(\frac{eE}{m}\right)\tau$

$v_d = \frac{eE\tau}{m}$ A → R

$j = nev_d$ D → Q

v_d

$j = ne\left(\frac{eE\tau}{m}\right)$

$j = \sigma E$ $\sigma = \frac{ne^2\tau}{m}$

$j = \frac{E}{\rho}$ $\rho = \frac{m}{ne^2\tau}$

$\rho = E/j$

B – S

$\tau = \frac{m}{he^2\rho}$ C → P

JEE 1

19

JEE 1

According to kinetic theory of gases the average kinetic energy of a monoatomic gas molecule is

(A) $3k_BT$

(B) $\frac{3}{2}k_BT$

(C) $2k_BT$

(D) k_BT

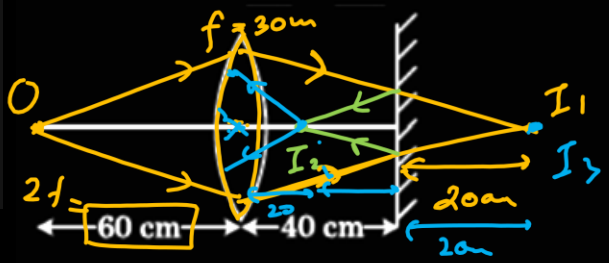
$E_{molecule} = \frac{1}{2}kT$ per degree

$K_{molecule} = \frac{f}{2}kT$ *

$K_{monoatom} = \frac{3}{2}kT$

20

A point object is placed at a distance of 60 cm from a convex lens of focal length 30 cm .



If a plane mirror were put perpendicular to the principal axis of the lens and at a distance of 40 cm from it, the final image would be formed at a distance of:

(A) 20 cm from the lens, it would be a real image

(B) 30 cm from the lens, it would be a real image.

(C) 30 cm from the plane mirror, it would be a virtual image.

(D) 20 cm from the plane mirror, it would be a virtual image.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-60} = \frac{1}{+30}$

$\Rightarrow v_1 = +60$

I_2 = 20 cm from M Real Image

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v_2} - \frac{1}{-20} = \frac{1}{+30}$

$\frac{1}{v_2} = \frac{1}{30} - \frac{1}{20}$

$v_2 = -60$

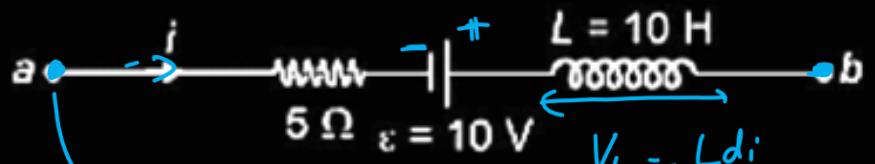
60 cm from lens behind (right side) Virtual.

20cm from M.

JEE 1

21

If at an instant the value of i is 1 A and it is increasing at the rate of 1 A/s then $V_a - V_b$ at that instant will be _____ volt



$V_L = L\frac{di}{dt}$

$V_a - V_b = +iR - \varepsilon + L\frac{di}{dt}$

$= (+1 \times 5) - 10 + 10\left(+1\,A/s\right)$

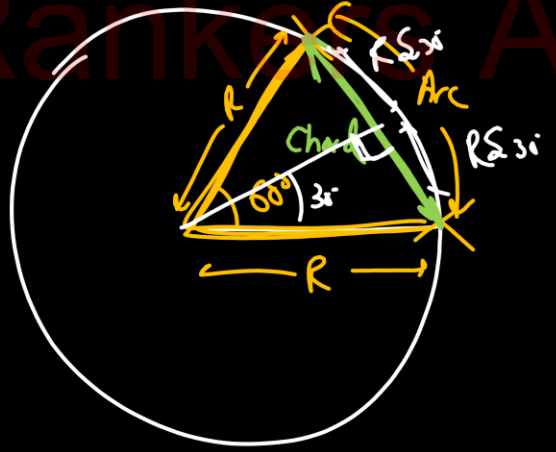
$= 5 - 10 + 10$

$= +5\ A$

22

JEE 1

A particle is moving in a circle of radius R with constant speed. Time period of the particle is T = 2.30 s . In time $t = T/6$ if the difference between average speed and average velocity of the particle is 2 ms^{-1}, find the radius R of the circle to the (nearest) integer (in m).



$$\text{speed} = \frac{\text{Arc}}{t} = \frac{\frac{\pi}{3}R}{\frac{T}{6}} = \frac{2\pi R}{T} \quad (1)$$

$$\text{velocity} = \frac{\text{chord}}{t} = \frac{2R\sin 30^\circ}{T/6} = \frac{6R}{T} \quad (2)$$

$$\frac{2\pi R - 6R}{T} = 2 \text{ m/s}$$

$$R(2\pi - 6) = 2 \times 2.3$$

$$R = \frac{4.6}{6.28 - 6}$$

$$= \frac{460}{28} = \frac{115}{7}$$

$$= 16.\ldots$$

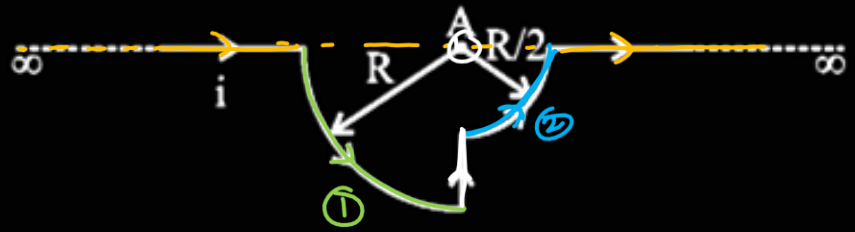
$$= 16$$

23

JEE 1

Find the magnetic field (in μT) at point A.

(Given : $i = \frac{20}{\pi}$ Amp, R = 1 m)



$B_1 = \frac{\mu_0 i}{8R}$

$B_2 = \frac{\mu_0 i}{8(\frac{R}{2})} = \frac{\mu_0 i}{4R}$

$B_{net} = B_1 + B_2 = 3\frac{\mu_0 i}{8R} = \frac{3 \times 4\pi \times 10^{-7}}{8 \times 1} \times \frac{20}{\pi} = 3 \times 10^{-6} = 3\mu T$

24

Two lighter nuclei combined to from a comparatively heavier nucleus by the relation given below:

$${}_{1}^{2}X + {}_{1}^{2}X = {}_{2}^{4}Y$$

The binding energies per nucleon for ${}_{1}^{2}X$ and ${}_{2}^{4}Y$ are 1.1 MeV and 7.6 MeV respectively. The energy released in the process is ____ MeV

$$Q = B.E._p - B.E._R$$ *

$$= B.E._y - 2(B.E._x)$$

$$= (4 \times 7.6) - 2(2 \times 1.1)$$

$$= 30.4 - 4.4 = 26 \text{ MeV}$$

25

In the two separate set-ups of the Young's double slit experiment, fringes of equal width are observed when lights of wavelengths in the ratio 1: 2 are used. If the ratio of the slit separation in the two cases is 2: 1, the ratio of the distances between the plane of the slits and the screen, in the two set ups is n: 1. Find 'n'.

$\beta_1 = \beta_2$

$\frac{\lambda_1 D_1}{d_1} = \frac{\lambda_2 D_2}{d_2}$

$\frac{D_1}{D_2} = \frac{\lambda_2/\lambda_1}{d_2/d_1}$

$= \frac{2:1}{1:2}$

$= \boxed{4:1}$

CHEMISTRY

7

How much AgCl will be formed by adding 1.70 g of $AgNO_3$ in 200 mL of 5 N HCl solution (Ag = 108, N = 14, O = 16)

(A) 1.70 g (B) 17 g

(C) 1.435 g (D) 14.35 g

L.R

$AgNO_3 + HCl \longrightarrow AgCl + HNO_3$

$\frac{1.70}{170}$ = 0.01 moles

200 × 5 N = 1000 meq = 1 eq

0.01 moles

$W_g = 0.01 \times 143.5$

= 1.435 g

2

Optical isomerism is not shown by the complex:

(A) $[Cr(ox)_3]^{3-}$ (B) Cis$[Co(en)_2Cl_2]^+$

(C) Trans $[Co(en)_2Cl_2]^+$ (D) $[Cr(en)_3]^{3+}$

JEE 1

3

Chlorobenzene on treatment with sodium in dryether gives diphenyl. The name of the reaction is

(A) Fittig reaction

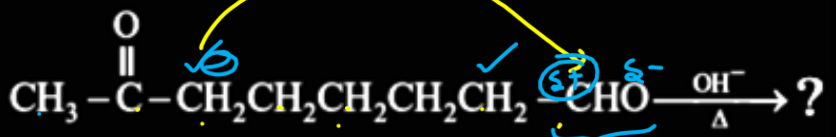
(B) Wurtz Fittig reaction

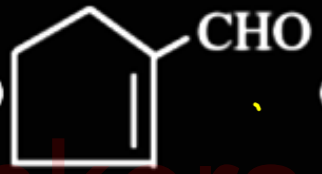
(C) Gattermann reaction

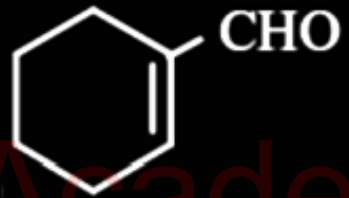
(D) Wurtz reaction

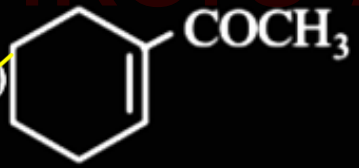
4

In the following reaction, the final product is

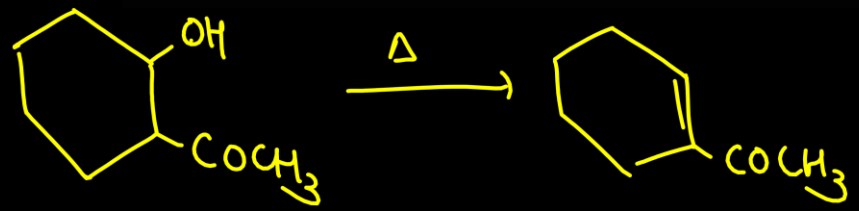


(A) 

(B) 

(C) 

(D) 



5

JEE 1

Which of the following species has the shortest bond length?

(A) NO^{2-} (B) NO^{+}

(C) NO (D) NO^{-}

Bond length $\propto \frac{1}{B.O}$

	no of e^-	B.O
NO^{2-}	17	1.5
NO^{+}	14	3
NO	15	2.5
NO^{-}	16	2.0

6

JEE 1

Which of the following does not represent the correct sequence of property indicated?

(A) $Sc^{3+} > Cr^{3+} > Fe^{3+} > Mn^{3+}$ – Stability of +3 ions

(B) Sc < Ti < Cr < Mn – number of oxidation states

+2 to +7

n=5 n=2 n=3 n=4

(C) $Mn^{2+} < Ni^{2+} < CO^{2+} < Fe^{2+}$ – Magnetic moment

depends upon no. of unpaired e^-

(D) FeO > CoO > NiO > CuO – Basic nature

$Fe^{+2}: 3d^6$

n=4

$Mn^{+2}: 3d^5$

n=5

7

JEE 1

Which of the following reaction is non-redox?

(A) $2\overset{+5}{\underset{}{N}}a\overset{-2}{\underset{}{N}}O_3 \longrightarrow 2\overset{+3}{\underset{}{N}}a\overset{-2}{\underset{}{N}}O_2 + \overset{0}{O_2}$ Redox

(B) $\overset{+}{K}\overset{-}{Cl} + \overset{+}{N}a\overset{-}{N}O_3 \longrightarrow \overset{+}{K}\overset{-}{N}O_3 + \overset{+}{N}a\overset{-}{Cl}$ (No change in O.S)

(C) $\overset{0}{Fe} + \overset{+1}{H_2}\overset{+6}{S}O_4 \longrightarrow \overset{+2}{Fe}SO_4 + \overset{0}{H_2}$

(D) $4\overset{0}{Ag} + 8CN^- + \overset{0}{O_2} + H_2O \longrightarrow$

$4[\overset{+1}{Ag}(CN)_2]^- + 4OH^-$

$-2+1=-1$

8

JEE 1

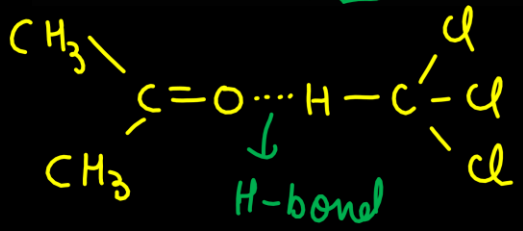
Which of the following combinations are correct for a binary solution, in which the solute and solvent are liquid?

(A) C_6H_6 and $C_6H_5CH_3$; $\Delta H_{sol} > \Delta V_{sol} = 0$

(B) $H_3C-\overset{\overset{O}{\|}}{C}-CH_3$ and $CHCl_3$; $\Delta H_{sol} < 0; \Delta V_{sol} < 0$

(C) H_2O and HCl; $\Delta H_{sol} > 0; \Delta V_{sol} < 0$

(D) H_2O and C_2H_5OH; $\Delta H_{sol} < 0; \Delta V_{sol} > 0$



ΔV = -ve
ΔH = -ve

9

JEE 1

Which of the following properties is not shown by NO?

NO, CO, N_2O, H_2O are neutral oxides.

(A) It is a neutral oxide

(B) It combines with oxygen to form nitrogen dioxide

(C) Its bond order is 2.5

(D) It is diamagnetic in gaseous state

NO = 15 = 2.5

odd e^- species are para

10

Which of the following reactions of glucose can be explained only by its Cyclic structure?

(A) Glucose forms penta acetate

(B) Glucose reacts with hydroxylamine to form an oxime

(C) Penta acetate of glucose does not react with hydroxylamine

(D) Glucose is oxidized by nitric acid to gluconic acid

JEE 1

11

JEE 1

An aqueous solution of colourless metal sulphate M , gives a white ppt., with NH_4OH. This was soluble in excess of NH_4OH. On passing H_2S through this solution a white ppt. Is formed. The metal M in the salt is:

(A) Ca (B) Ba
(C) Al (D) Zn

$CaSO_4$: white
$BaSO_4$: white
$Al_2(SO_4)_3$: white
$ZnSO_4$: colourless

$Zn^{2+} + H_2O + NH_3 \rightarrow Zn(OH)_2$

$Zn(OH)_2 + 4NH_3 \rightarrow [Zn(NH_3)_4]^{2+}$

$Zn^{2+} + H_2S \rightarrow ZnS$ white ppt

12

JEE 1

. Which one of the following pairs of isomers and types of isomerism are correctly matched-

(P) $[Co(NH_3)_5(NO_2)]Cl_2$ and $[Co(NH_3)_5(ONO)]Cl_2$ ... linkage

(Q) $[Cu(NH_3)_4](PtCl_4)$ and $[Pt(NH_3)_4][CuCl_4]$ .... co-ordination

(R) $[Pt(NH_3)_4Cl_2]Br_2$ and $[Pt(NH_3)_4Br_2]Cl_2$ .....ionization

Select the correct answer using the codes given below:

(A) Q and R only

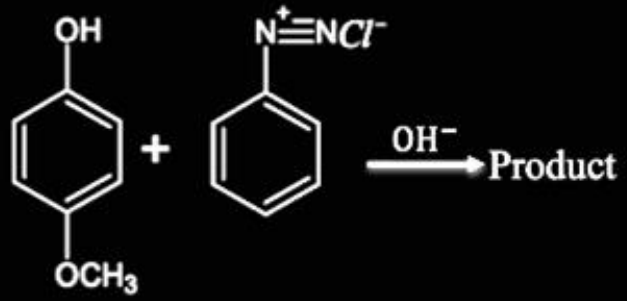
(B) P, Q and R

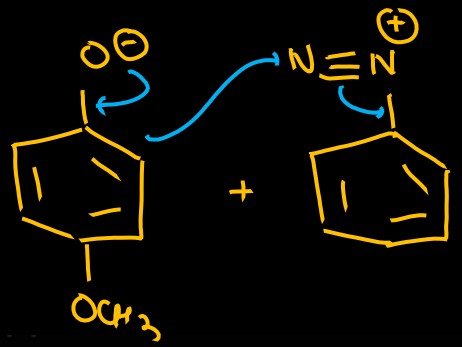
(C) P and R only

(D) P and Q only

13

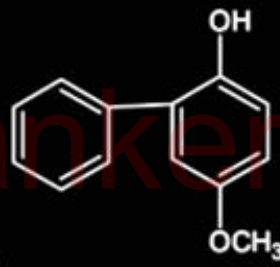
Product of reaction is:



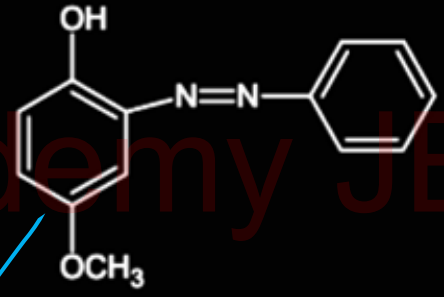


JEE 1

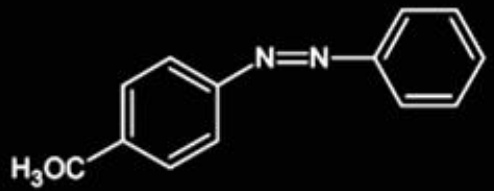
(A)



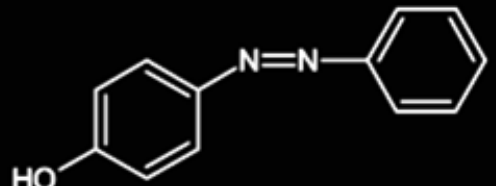
(C)



(B)

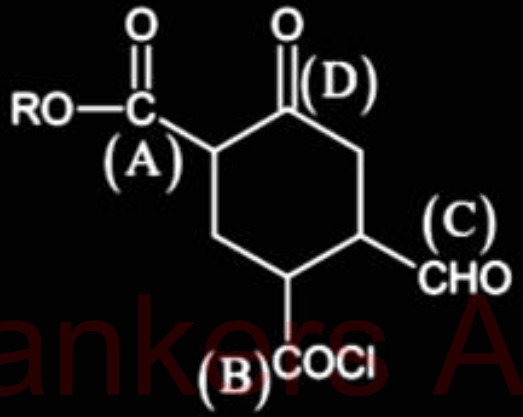


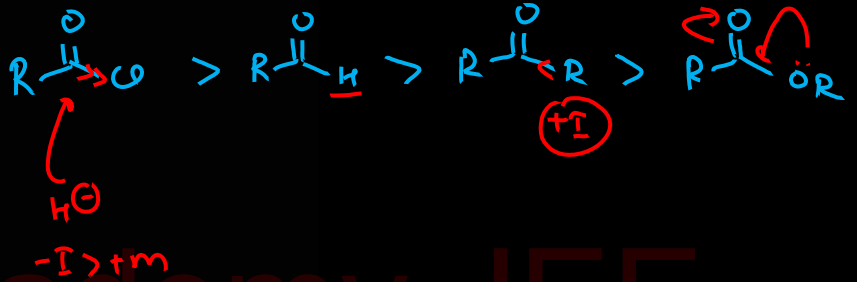
(D)



14

Find the correct order of reactivity of carbonyl groups with $LiAlH_4$:





(A) A > B > C > D

(B) B > C > D > A

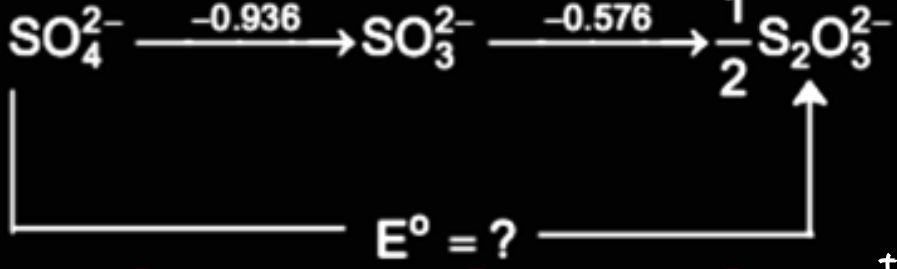
(C) D > C > B > A

(D) B > D > C > A

15

JEE 1

Consider the standard reduction potentials (in volts) as shown below. Find E°.



(A) −0.326 V (B) −0.425 V

(C) −0.756 V (D) −0.512

$n_1 = 2$

$+6 \quad -2$
$SO_4^{-2} \longrightarrow SO_3^{-2}$ $\quad E_1^\circ = -0.936\ V$

$n_2 = 2$

$+4 \quad -2 \qquad +2 \quad -2$
$SO_3^{-2} \longrightarrow \frac{1}{2}S_2O_3^{-2}$ $\quad E_2^\circ = -0.576\ V$

$+6 \quad -2 \qquad +2 \quad -2$
$SO_4^{-2} \longrightarrow \frac{1}{2}S_2O_3^{-2}$ $\quad E_3^\circ = ?$

$n_3 = 4$

$E_3^\circ = \frac{n_1E_1^\circ + n_2E_2^\circ}{n_3}$

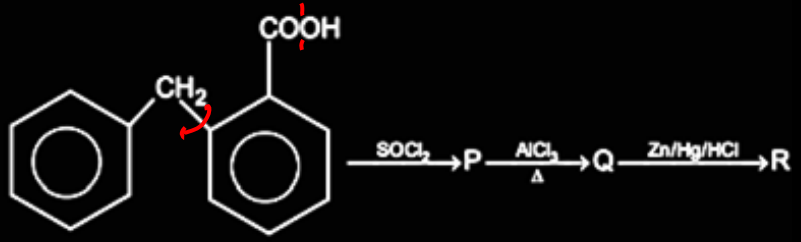
$-0.756\ V = \frac{2(-0.936) + 2(-0.576)}{4}$

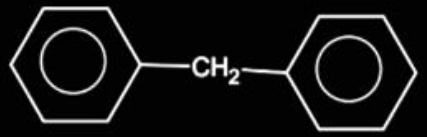
$\Delta^\circ G_3 = \Delta^\circ G_1 + \Delta^\circ G_2$

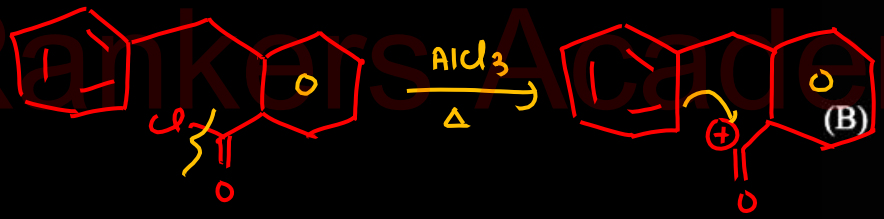
$-n_3FE_3^\circ = -n_1FE_1^\circ - n_2FE_2^\circ$

16

What is R in the following set of reactions?



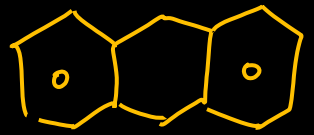
(A) 

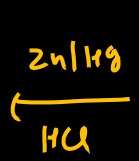


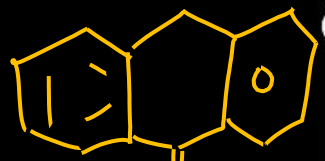
(B) 

(P)

(C) 







(R) (Q)

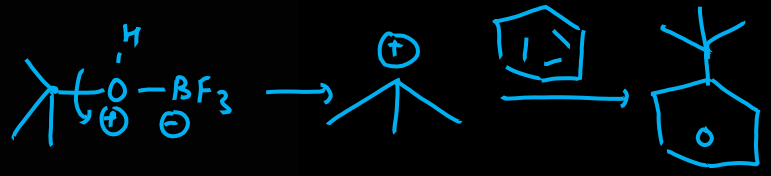
(D) 

17

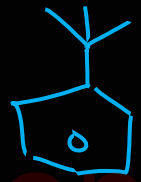
In which of the following reactions tertiary butyl benzene is formed?

JEE 1

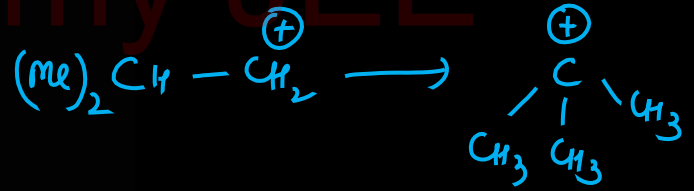
(A)  $+Me_3C\text{-}OH+BF_3$



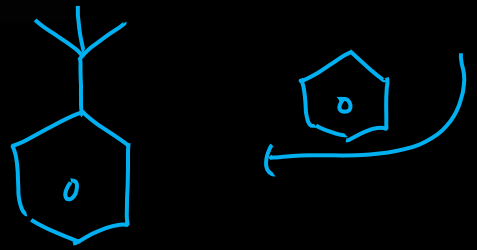
(B)  $+Me_2C{=}CH_2+H_2SO_4$



(C)  $+Me_2CH\text{-}CH_2Cl+AlCl_3$



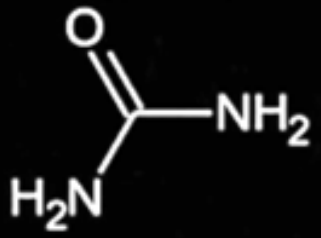
(D) All of these



18

Kjeldahl's method cannot be used to estimate nitrogen for which of the following compounds?

(A) 

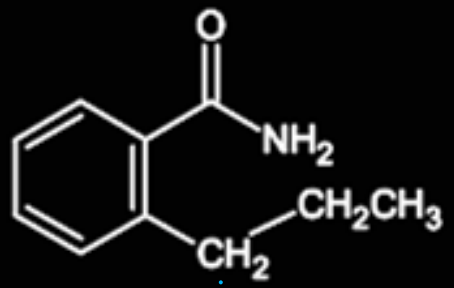
(B) 

(C) 

(D) 

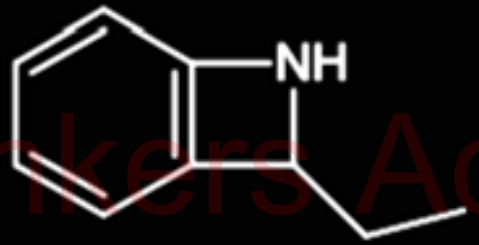
19

JEE 1

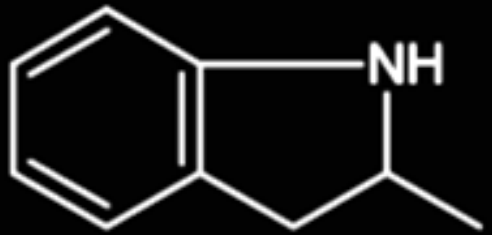


$\xrightarrow[\text{(ii) KOH}]{\text{(i) } Br_2/h\nu}$ The major product is :

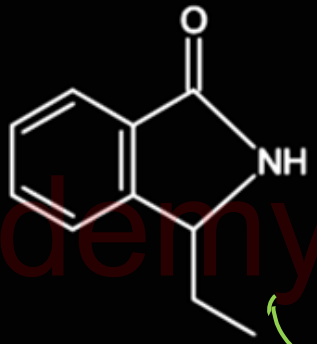
(A)



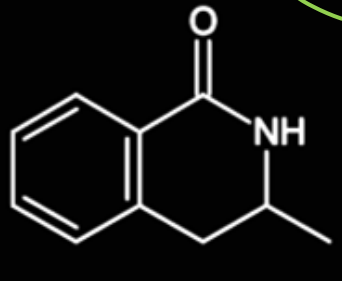
(B)

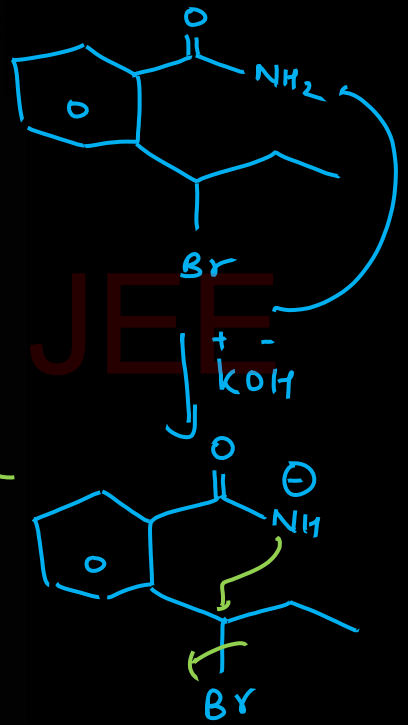


(C)



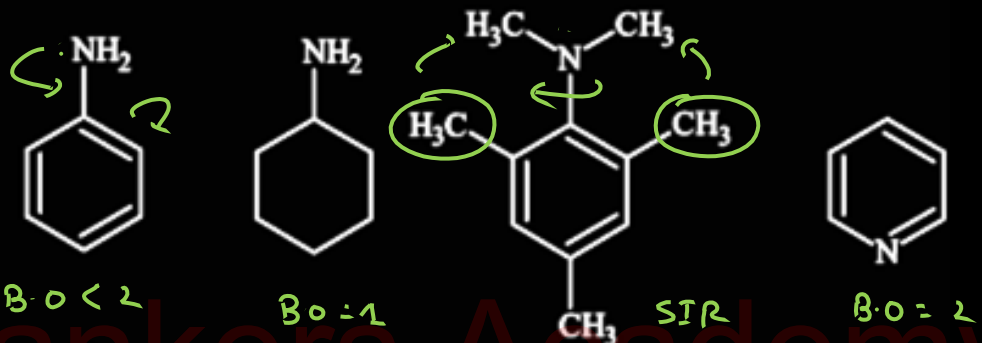
(D)





20

The correct order of C – N bond length in the following molecules is



(I) (II) (III) (IV)

(A) II > III > I > IV

(B) II > III = I > IV

(C) III > IV > I > II

(D) IV > III > IV > I

21

JEE 1

Consider the following first order competing reactions:

$X \xrightarrow{k_1} A + B$ and $Y \xrightarrow{k_2} C + D$

If 50% of the reaction of X was completed when 96% of the reaction of Y was completed, the ratio of their rate constants (k_2/k_1) is: (nearest integer)

(take log 2 = 0.30, log 5 = 0.70)

$(t_{50\%})_X = (t_{96\%})_Y$

$\frac{\ln 2}{k_1} = \left[\frac{1}{k_2} \ln \frac{100}{4}\right]$

$\frac{k_2}{k_1} = \frac{\ln 25}{\ln 2} = \frac{2 \ln 5}{\ln 2}$

$\frac{k_2}{k_1} = 2 \frac{\log 5}{\log 2}$

$= 2\left(\frac{0.70}{0.30}\right)$

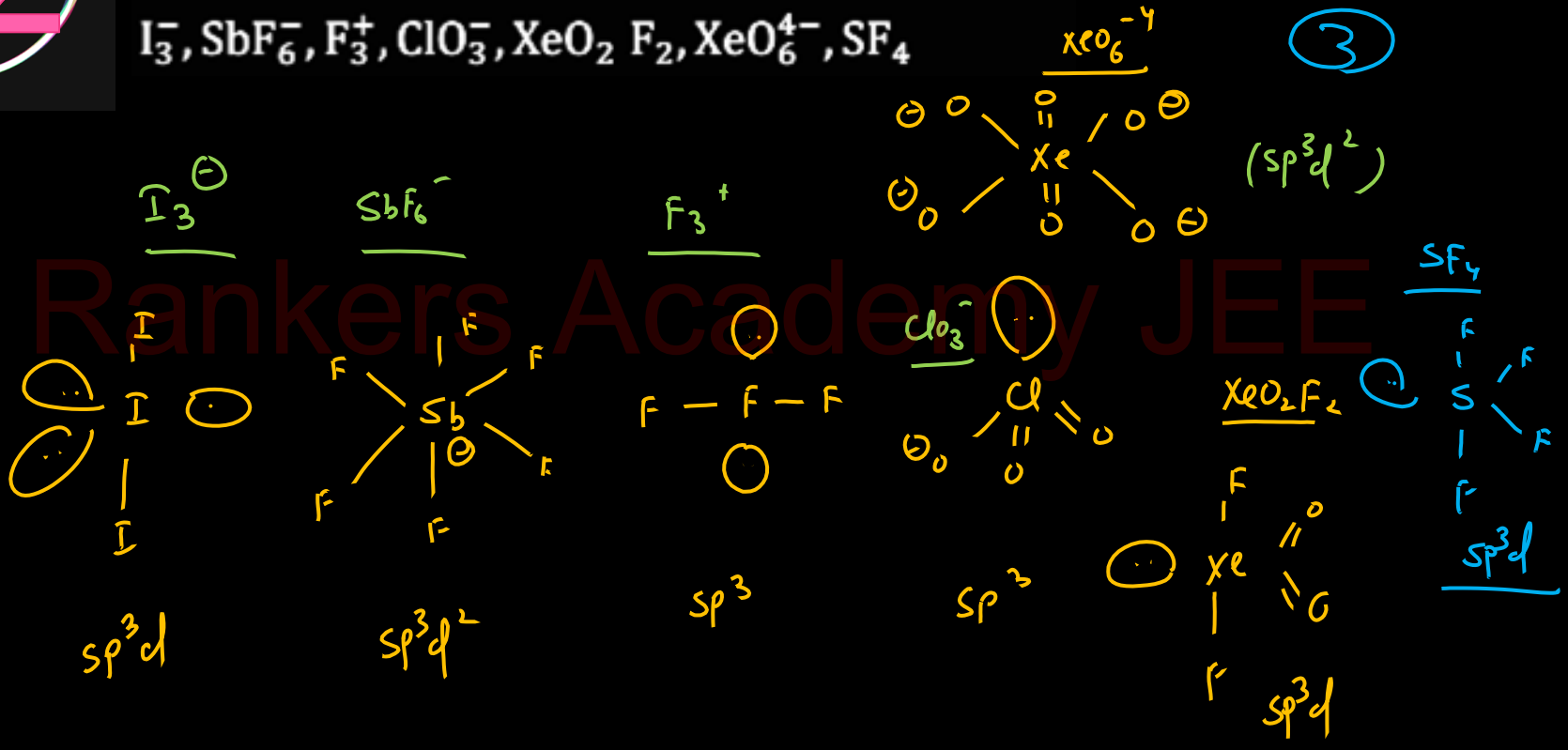
$\left(\frac{k_2}{k_1}\right) = \frac{14}{3} = 4.66 \approx 5$

22

How many of the following molecules or ions have sp^3d hybridisation of their central atom ?

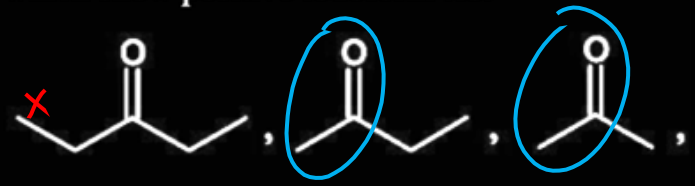
$I_3^-, SbF_6^-, F_3^+, ClO_3^-, XeO_2F_2, XeO_6^{4-}, SF_4$

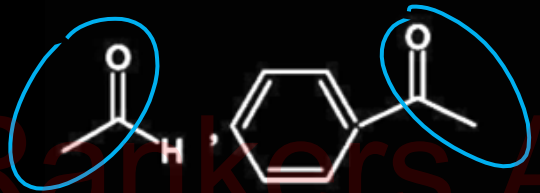
JEE 1

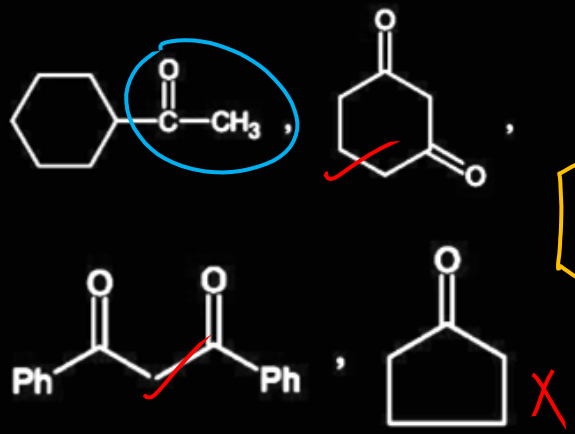


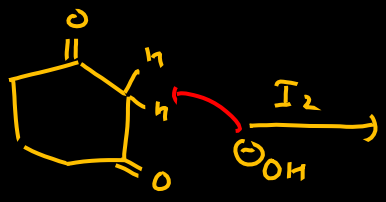
23

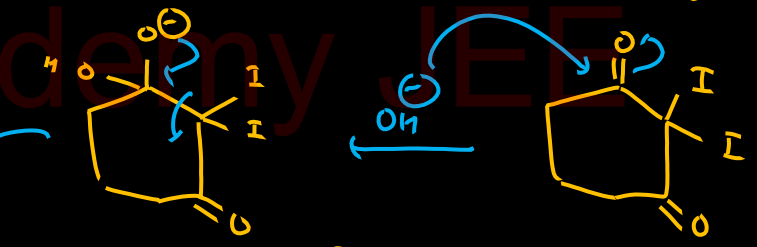
Examine the structural formulas of compounds given below and identify number of compounds which show positive iodoform test

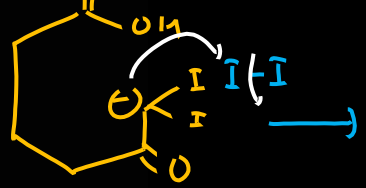


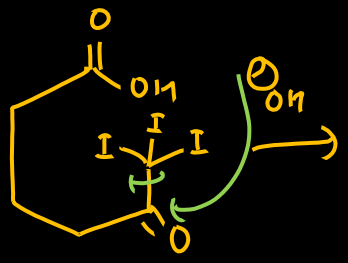


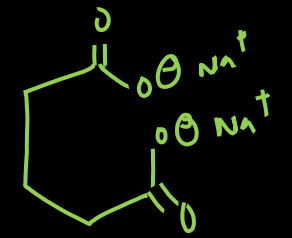


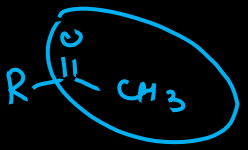












(7)

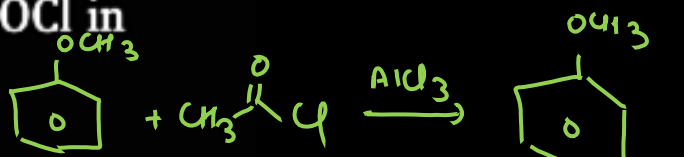
JEE 1

24

JEE 1

How many of the following statement is/are not true

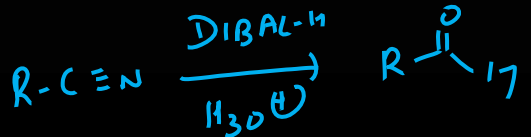
(P) Anisole does not react with CH_3COCl in presence of anhydrous $AlCl_3$.

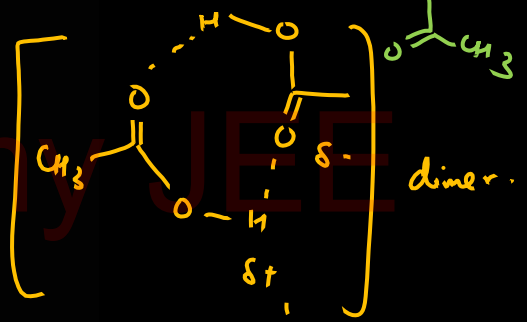


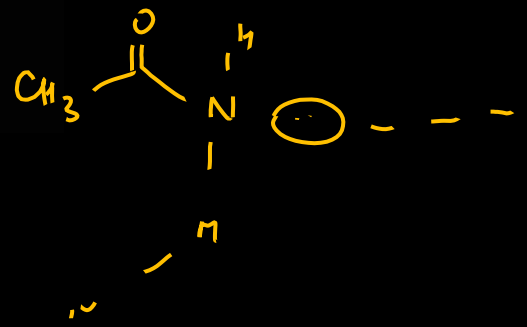
(Q) Acetamide has higher boiling point than acetic acid.

(R)Esters have fruity smell.

(S) DIBAL-H reduces unsaturated nitriles to unsaturated aldehydes (assume hydrolysis as needed).







25

100 mL of 0.2M CH_3COOH, 50 mL of 0.1 M HCl and 50 mL of 0.5 M NaOH are mixed together at 25°C If $K_a(CH_3COOH) = 10^{-5}$, then pH of the resulting solution will be; (nearest integer)

JEE 1

$HCl + NaOH \longrightarrow NaCl + H_2O$

5 mmole | 25 mmole | — | —

— | 20 mmole | 5 mmole | .

$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

20 mmole | 20 mmole | —

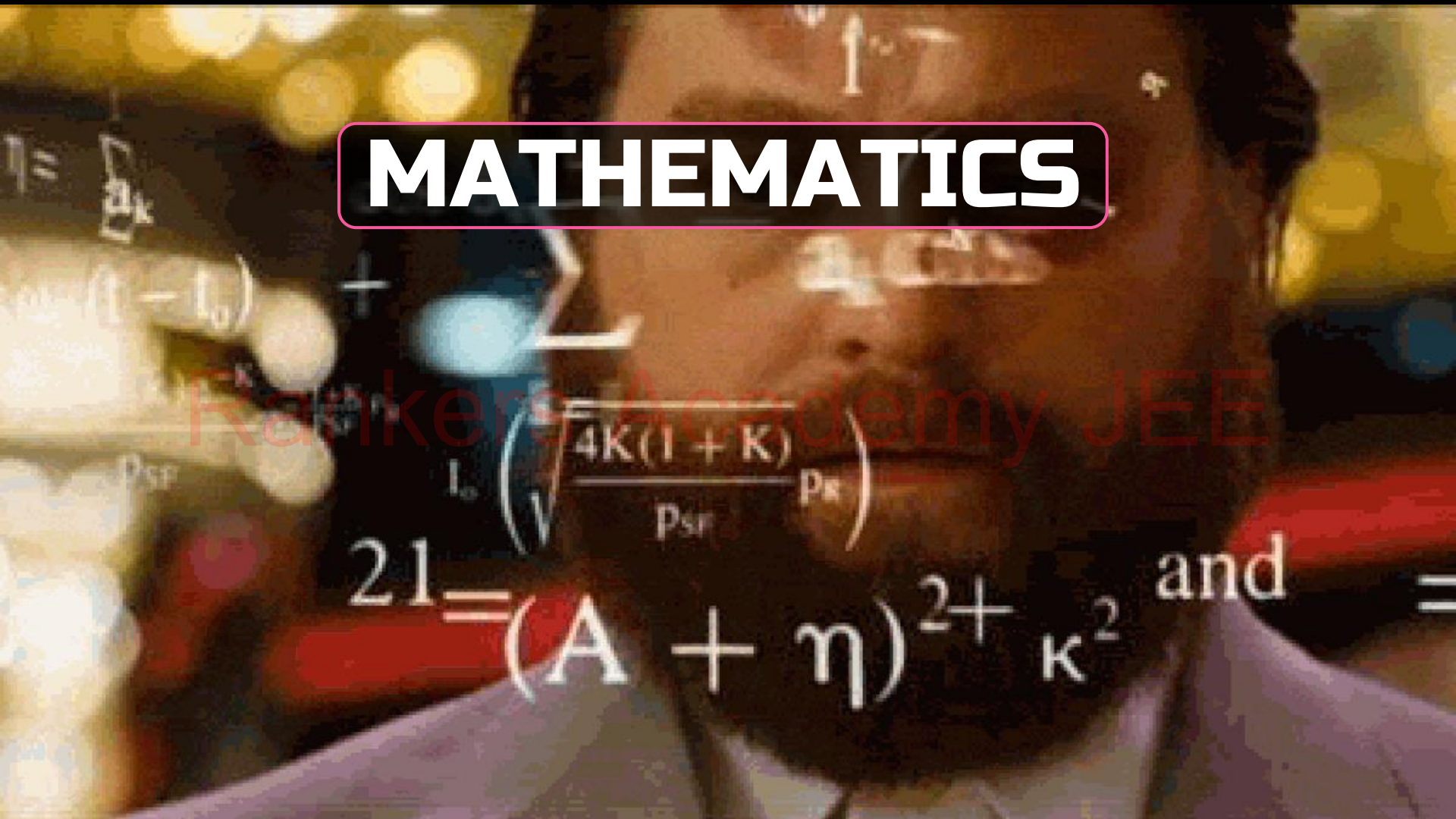
— | — | 20 mmole

$pH = 7 + \frac{1}{2}[pK_a + \log c]$

$[CH_3COONa] = \frac{20}{100+50+50} = 0.1$

$pH = 7 + \frac{1}{2}[-\log(10^{-5}) + \log 10^{-1}]$

$pH = 7 + 2 = 9$


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^{2+} \kappa^2$ and

7

If the value of $\sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{r^2-5r+7}\right)$ is equal to $\frac{a\pi}{b}$, where a and b are co-prime, the find the value of (a + b)

(A) 6 (B) 7

(C) 5 (D) 8

JEE 1

$$\sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{1+r^2-5r+6}\right)$$

$$\Rightarrow \sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{1+(r-2)(r-3)}\right)$$

$$\Rightarrow \sum_{r=2}^{\infty} \tan^{-1}\left(\frac{(r-2)-(r-3)}{1+(r-2)(r-3)}\right)$$

$$\Rightarrow \sum_{r=2}^{\infty} \tan^{-1}(r-2) - \tan^{-1}(r-3)$$

$$\Rightarrow \tan^{-1}0 - \tan^{-1}(-1) + \tan^{-1}1 - \tan^{-1}0 + \cdots + \tan^{-1}\infty$$

$$\Rightarrow \frac{\pi}{2} - \left(-\frac{\pi}{4}\right) = \frac{3\pi}{4}$$

$a = 3$

$b = 4$

2

The derivative of

$$f(x) = \frac{(1+\cos 2x)(1+\sin 2x)\ln(1+3x)}{(\sqrt{x^2+1}+\sqrt{x^2+4})(\sqrt[4]{x^2+1}+\sqrt[4]{x^2+16})} \text{ at } x = 0 \text{ is}$$

(A) 0 (B) 1/2

(C) 2/3 (D) Not defined

JEE 1

$$f(x) = \frac{(1+\cos 2x)(1+\sin 2x)}{(\sqrt{x^2+1}+\sqrt{x^2+4})(\sqrt[4]{x^2+1}+\sqrt[4]{x^2+16})} \cdot \ln(1+3x)$$

u v

$$f'(x) = u' \cdot v + u v'$$

$$= u' \cdot (\ln(1+3x)) + u \cdot \frac{1}{1+3x} \cdot 3$$

$$= u'(0) + \frac{2 \cdot 1}{3 \cdot 3} \cdot \frac{1}{1} \cdot 3 \Rightarrow \frac{2}{3}$$

$$= 0$$

3

A fair dice is thrown six times, it being known that each time a different digit is shown. The probability that a sum of 12 will be obtained in the first three throws is

(A) 5/24 (B) 25/216

(C) 3/20 (D) 1/12

6, 5, 1 → $\frac{1}{6} \cdot \frac{1}{5} \cdot \frac{1}{4} \times 3! \times 3 = \frac{3}{20}$

6, 4, 2

5, 4, 3

4

If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to

(A) 5 (B) 7

(C) 9 (D) 11

JEE 1

M1

$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$= {}^9C_r \cdot \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

$18 - 3r = 0 \Rightarrow r = 6$

$K = T_7 = {}^9C_6 \cdot \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6$

$= \frac{7}{18}$

coeff of x^m in $\left(ax^p + \frac{b}{x^q}\right)^n$

M2 Shortcut

$r = \frac{np - m}{p + q}$

$= \frac{9(2) - 0}{2 + 1}$

$= 6$ ✓

5

If the variance of the term in an increasing AP, $b_1, b_2, b_3, \ldots\ldots b_{11}$ is 90, then the common difference of this AP is.

(A) 1 (B) 2

(C) 3 (D) 9

JEE 1

$\text{var} = 90 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$90 = a^2 + 10d^2 - a^2$

$d^2 = 9$ ✓

Terms: $a-5d, a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, a+4d, a+5d$

$\bar{x} = \frac{\sum x_i}{n} = \frac{(a-5d)+(a-4d)+\ldots+a+\ldots(a+4d)+(a+5d)}{11} = a$

$\frac{\sum x_i^2}{n} = \frac{(a-5d)^2+(a-4d)^2+\ldots+a^2+\ldots+(a+4d)^2+(a+5d)^2}{11}$

$= \frac{11a^2 + 2(1+4+9+16+25)d^2}{11} = \frac{11a^2 + 2 \cdot \frac{5 \cdot (5+1)(2 \cdot 5+1)}{6} d^2}{11} = a^2 + 10d^2$

6

The function $f(x) = |x^2 - 2x - 3| \cdot e^{9x^2-12x+4}$ is not different at exactly

cont.

(A) four points

(B) three points

(C) two points

(D) one point

$x^2 - 2x - 3 = 0$

$x = 3, -1$

non-diff

Number of subsets A of the set {1,2,3,4,5,6,7,8,9,10} having the property that no two elements of A sum to 11 ?

(A) 243 (B) 32

(C) 64 (D) 729

JEE 1

10, 1

9, 2

8, 3

7, 4

6, 5

10 ✓ 1 ×

10 × 1 ✓

10 × 1 ×

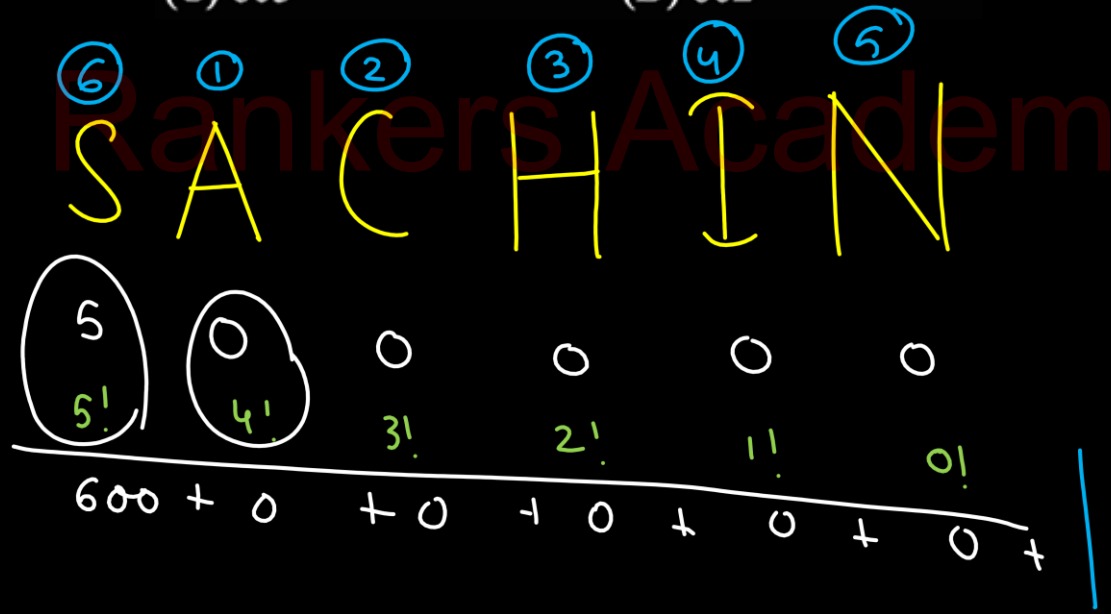
Ans : 3^5

8

If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number is

(A) 601 (B) 600

(C) 603 (D) 602



9

Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(A) $f(1) - 4f(-1) = 4$

(B) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.

(C) f is an odd function.

(D) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

$\lim_{x\to 0} 2 + \frac{f(x)}{x^3} = 4 \Rightarrow 2.$

JEE 1

$f(x) = ax^5 + bx^4 + cx^3$

$\lim_{x\to 0} \frac{ax^5 + bx^4 + cx^3}{x^3} = 2$

$\Rightarrow \lim_{x\to 0} ax^2 + bx + c = 2$

$\Rightarrow$ $\boxed{c = 2}$

$f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$f'(1) = 0 = 5a + 4b + 6$

$f'(-1) = 0 \Rightarrow 5a - 4b + 6$

$\boxed{b = 0}$ $\boxed{a = -6/5}$

$\boxed{f(x) = -\frac{6}{5}x^5 + 2x^3}$

$f(1) = 2 - \frac{6}{5} = \frac{4}{5}$

$f(-1) = \frac{6}{5} - 2 = -\frac{4}{5}$

$f'(x) = -6x^4 + 6x^2$

$= -6x^2(x+1)(x-1)$

10

Let f: (a, b) → R be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function g(x). If f(x) = 0 has exactly five distinct roots in (a, b), then g(x)g′(x) = 0 has at least:

(A) seven roots in (a, b)
(B) five roots in (a, b)
(C) three roots in (a, b)
(D) twelve roots in (a, b)

4 3

f(x) → 5 roots

g(x) = f′(x) → 4 roots

g′(x) → 3 roots

11

The number of solutions of the equation

$z^2 + \bar{z} = 0$ is

(A) 1 (B) 2

(C) 3 (D) 4

JEE 1

$z = x + iy$

$\bar{z} = x - iy$

$z^2 = x^2 - y^2 + 2xyi$

$z^2 + \bar{z} = 0$

$\Rightarrow x^2 - y^2 + 2xyi + x - iy = 0$

$\Rightarrow \underbrace{(x^2 - y^2 + x)}_{=0} + i\underbrace{(2xy - y)}_{=0} = 0 + 0i$

$2xy - y = 0$

$y(2x - 1) = 0$

$y = 0$

$x^2 + x = 0$

$x = 0, -1$

$\therefore 0 + 0i$

$-1 + 0i$

$x = \frac{1}{2}$

$y^2 = \frac{3}{4}$

$y = \pm \frac{\sqrt{3}}{2}$

$\therefore \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$\frac{1}{2} - \frac{\sqrt{3}}{2}i$

12

JEE 1

Let $\vec{\alpha} = 3\hat{i} + \hat{j}, \vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$ where $\vec{\beta_1} \parallel^{el} \vec{\alpha}$ and $\vec{\beta_2} \perp \vec{\alpha}$ and then $\vec{\beta_1} \times \vec{\beta_2} =$

(A) $-3\hat{i} + 9\hat{j} + 5\hat{k}$

(B) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(C) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

(D) $\frac{1}{2}(3\hat{i} - 9\hat{j} - 5\hat{k})$

$\vec{\beta_1} = \lambda\vec{\alpha}$

$= 3\lambda\hat{i} + \lambda\hat{j}$

$\vec{\beta_2} = \vec{\beta_1} - \vec{\beta}$

$= (3\lambda\hat{i} + \lambda\hat{j}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$= (3\lambda - 2)\hat{i} + (\lambda + 1)\hat{j} - 3\hat{k}$

$\vec{\alpha} \cdot \vec{\beta_2} = 0$

$3(3\lambda - 2) + 1(\lambda + 1) + 0(-3) = 0$

$10\lambda - 5 = 0 \Rightarrow \lambda = 1/2$

$$\vec{\beta_1} \times \vec{\beta_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$$

$$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$$

13

Let R be the relation defined in set of natural numbers as aRb $\Leftrightarrow$ ab is square of a natural number. Then relation R is

(A) Reflexive number

(B) Reflexive but not symmetric

(C) Equivalence

(D) Not transitive

① Ref: $(a,a) \rightarrow a^2$ ✓

② Symm: $(a,b) \rightarrow ab$ ✓

$(b,a) \rightarrow ba$ ✓

③ Trans:

$(a,b) \rightarrow ab = m^2$

$(b,c) \rightarrow bc = n^2$

$ab^2c = m^2n^2$

$ac = \left(\frac{mn}{b}\right)^2$

14

If $S_n = 1.n + 2.(n-1) + 3.(n-2) + \cdots \ldots + n.1$ and $S_{25} = 26\lambda$ then λ is

(A) 26 (B) 225

(C) 225/2 (D) 51/2

$$S_n = \sum_{r=1}^{n} r(n-r+1)$$

$$= \sum_{r=1}^{n} \left(r(n+1) - r^2 \right)$$

$$= (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$$

$$S_n = \frac{(n+1)(n)(n+1)}{2} - \frac{(n)(n+1)(2n+1)}{6}$$

$$S_{25} = \frac{26 \times 25 \times 26}{2} - \frac{25 \times 26 \times 51}{6}$$

$$= 26\left(\frac{225}{2}\right)$$

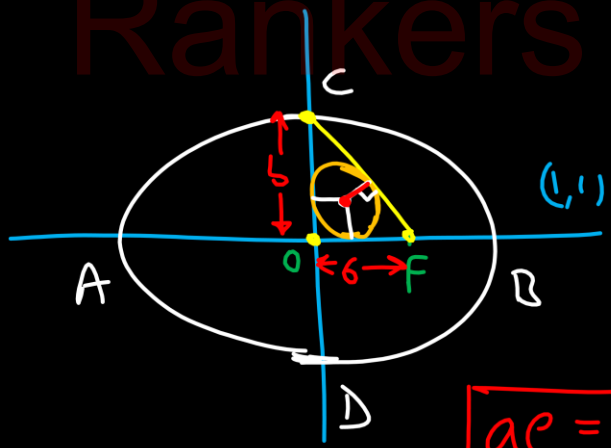
15

Point ' O ' is the centre of the ellipse with major axis AB and minor axis CD . Point F is one focus of the ellipse. If OF = 6 and the diameter of the inscribed circle of triangle OCF is 2 , then the product of (AB)(CD) is

(A) 65 (B) 62

(C) 78 (D) 91

$r = 1$



$\frac{x}{6} + \frac{y}{b} - 1 = 0$

(1,1) ∘

$\left| \frac{\frac{1}{6} + \frac{1}{b} - 1}{\sqrt{\frac{1}{36} + \frac{1}{b^2}}} \right| = 1$

$ae = 6$

$\left(\frac{1}{6} - \frac{5}{6}\right)^2 = \frac{1}{36} + \frac{1}{b^2}$

$\frac{1}{b^2} + \frac{25}{36} - \frac{5}{3b} = \frac{1}{36} + \frac{1}{b^2}$

$\frac{24}{36} = \frac{5}{3b}$

$b = 5/2$

$ae = 6$

& $b = \frac{5}{2}$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2 - (ae)^2$

$\frac{25}{4} = a^2 - 36$

$36 + \frac{25}{4} = a^2$

$\frac{169}{4} = a^2$

$a = \frac{13}{2}$

$AB = 13$

$CD = 5$

$(AB)(CD) = 65$

16

Suppose $0 < \alpha\beta < 1$. If $\alpha + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$ are roots of $3x^2 - 8x + 16 = 0$, then quadratic equation whose roots are α and β is

(A) $3x^2 - 2x + 1 = 0$ (B) $3x^2 - 4x + 6 = 0$

(C) $3x^2 - 2x + 6 = 0$ (D) $3x^2 - 4x + 8 = 0$

$$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = \frac{8}{3}$$

$$(\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{8}{3}$$

$$(\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{8}{3}$$

$$(\alpha + \beta)\left(1 + \frac{1}{\alpha\beta}\right) = \frac{8}{3} \quad \text{— (1)}$$

Now:

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \frac{16}{3}$$

$$\alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = \frac{16}{3}$$

$$\Rightarrow \alpha\beta + \frac{1}{\alpha\beta} = \frac{10}{3}$$

$$\boxed{\alpha\beta = \frac{1}{3}} \quad \text{— (2)}$$

Now:

$$(\alpha+\beta)\left(1+\frac{1}{\alpha\beta}\right)=\frac{8}{3}$$

$$\boxed{\alpha+\beta=\frac{2}{3}}$$

$\therefore$ Ans: $x^2-\frac{2}{3}x+\frac{1}{3}=0$

$$\boxed{3x^2-2x+1=0}$$

17

JEE 1

Area bounded by $y = \{x\}^{[x]} + [x]$, x -axis between $x = 0$ and $x = 3$ (where [.] denotes greatest integer function and {.} denotes fractional part function) is

(A) 3 (B) $\frac{9}{2}$

(C) $\frac{29}{6}$ (D) $\frac{25}{6}$

$y = \{x\}^{[x]} + [x]$

$y = (x - [x])^{[x]} + [x]$

$$y = \begin{cases} (x-0)^0 + 0 & ; x \in [0,1) \\ (x-1)^1 + 1 & ; x \in [1,2) \\ (x-2)^2 + 2 & ; x \in [2,3) \end{cases}$$

$$y = \begin{cases} 1 & ; x \in (0,1) \\ x & ; x \in [1,2) \\ x^2 - 4x + 6 & ; x \in [2,3) \end{cases}$$

$$\int_0^1 1\,dx + \int_1^2 x\,dx + \int_2^3 (x^2-4x+6)\,dx$$

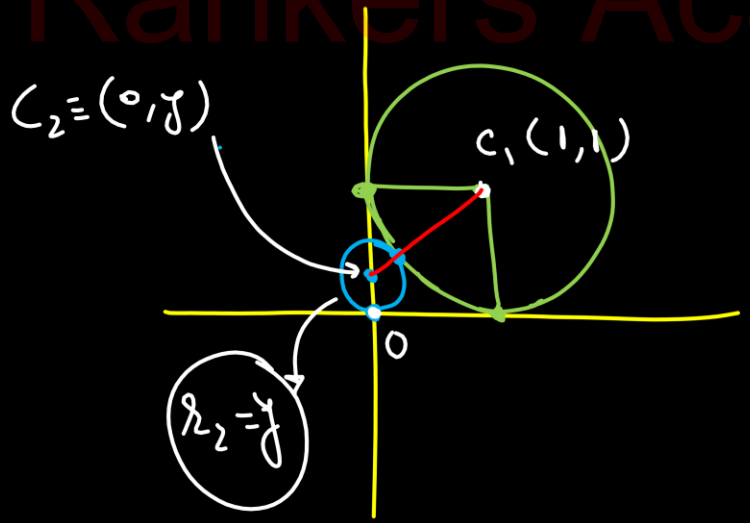
$$= \frac{29}{6}$$

18

Let C be the circle with centre at (1,1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to

(A) $\frac{\sqrt{3}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$



$\overline{C_1C_2} = r_1 + r_2$

$\sqrt{1 + (y-1)^2} = (1 + y)$

$1 + y^2 - 2y + x = 1 + y^2 + 2y$

$1 = 4y \Rightarrow y = \frac{1}{4}$

19

JEE 1

Solution of differential equation $\sin y\frac{dy}{dx} + \frac{1}{x}\cos y = x^4\cos^2 y$ is (Where C is the constant of integration)

(A) $x\sec y = x^6 + C$

(B) $6x\sec y = x + C$

(C) $6x\sec y = x^6 + C$

(D) $6x\sec y = 6x^6 + C$

$$\sin y\frac{dy}{dx} + \frac{1}{x}\cos y = x^4\cos^2 y$$

$$\tan y\sec y\frac{dy}{dx} + \frac{1}{x}\sec y = x^4$$

Let:

$$\begin{cases} \sec y = z \\ \sec y\tan y\frac{dy}{dx} = \frac{dz}{dx} \end{cases}$$

$$\frac{dz}{dx} + \frac{1}{x}z = x^4$$

$$I.f = e^{\int\frac{1}{x}dx} = e^{\ln x} = x$$

$$g(x) = \int x^4 \cdot x \, dx$$

$$(\sec y)\, x = \frac{x^6}{6} + C$$

$$6x \cdot \sec y = x^6 + K$$

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If a hyperbola centred at origin, has one of its directrix as $x = 2$. Also ordinate of one of the end point of a latus rectum is 12 . Then find its eccentricity.

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$

(C) 2 (D) $\frac{8}{3}$

JEE 1

$x = \frac{a}{e} = 2 \rightarrow a = 2e$

$\frac{b^2}{a} = 12$

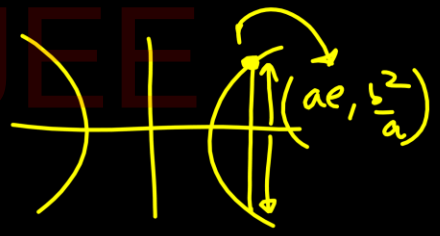
$b^2 = 12a \rightarrow b^2 = 24e$

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$e = \sqrt{1 + \frac{24e}{4e^2}}$

$e = \sqrt{1 + \frac{6}{e}}$

$\ell(LR) = \frac{2b^2}{a}$



$e^2 = 1 + \frac{6}{e}$

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If $A \cdot \text{adj}(A^2) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then absolute value of sum of elements of adj A is

(where adj (X) denotes adjoint of matrix X)

$A \cdot (\text{adj } A)^2 = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A \cdot \text{adj } A \cdot \text{adj } A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$|A| \text{ adj } A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ — (1)

$|A \cdot \text{adj } A^2| = 1 + 2(-1)$

$|A|\,|\text{adj } A^2| = -1$

$|A|\,|A^2|^2 = -1$

$|A|^5 = -1$

$\boxed{|A| = -1}$

$$\Rightarrow \text{adj } A = \begin{bmatrix} -1 & 0 & -2 \\ -2 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$= -8$$

$\therefore$ absolute value $= 8$ //

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JEE 1

Let f be a differentiable function satisfying $\int_0^{f(x)} f^{-1}(t)dt - \int_0^{x} (\cos t - f(t))dt = 0$ and $f(0) = 1$ The number of solutions of the equation $\begin{vmatrix} f(2x) & f(x) \\ \sin x & 2 \end{vmatrix} = 0$ in $(0, 2\pi)$ is

$x \cdot f'(x) - (\cos x - f(x)) = 0$

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$x\frac{dy}{dx} - \cos x + y = 0$

$x \cdot \frac{dy}{dx} + y = \cos x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{\cos x}{x}$

$I.F. = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$y(x) = \int \frac{\cos x}{x} \cdot x \, dx$

$y \cdot x = \sin x + C$

$(0,1) \Rightarrow C = 0$

$$y = \frac{\sin x}{x} = f(x)$$

Now:

$$2f(2x) - (\sin x) f(x) = 0$$

$$\not{2}\,\frac{\sin 2x}{\not{2}x} - \sin x \times \frac{\sin x}{x} = 0$$

$$\frac{2\sin x \cos x}{x} - \frac{\sin^2 x}{x} = 0$$

$$\frac{\sin x}{x}\left(2\cos x - \sin x\right) = 0$$

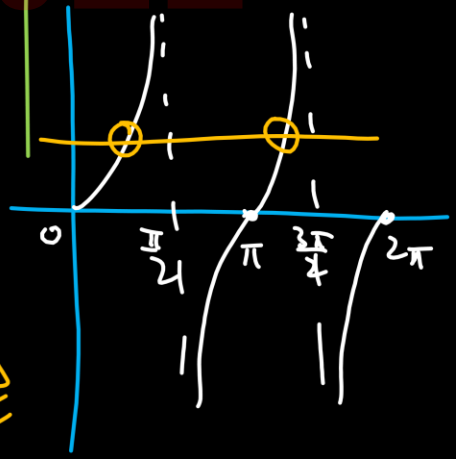
$x \neq 0$

$\sin x = 0$

$x = \pi$

$2\cos x - \sin x = 0$

$\tan x = 2$



Total: 1 + 2

= 3 Sol^n^s

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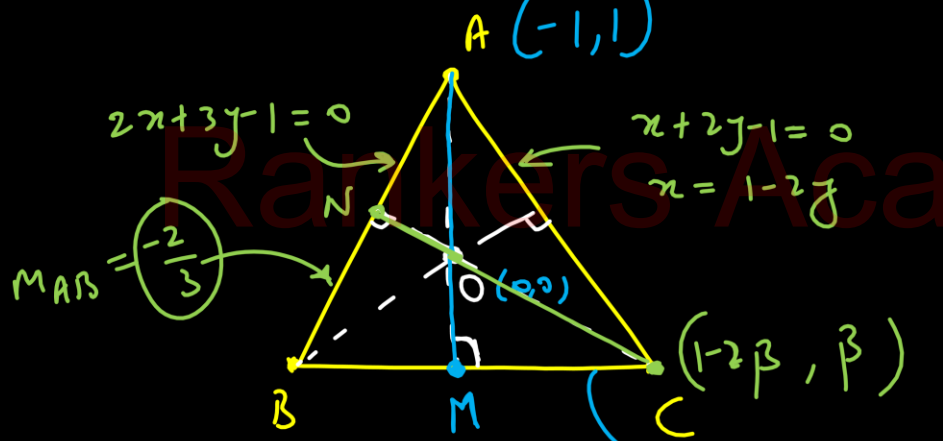
If the orthocentre of the triangle formed by $2x + 3y - 1 = 0, x + 2y - 1 = 0, ax + by - 1 = 0$ is at the origin, then $\frac{b-a}{4}$ equals _____ .

JEE 1



$M_{AB} = \frac{-2}{3}$

$M_{AO} = \frac{-1}{1} = -1$

$M_{BC} = 1$

$M_{CO} = \frac{\beta}{1-2\beta}$

$\Rightarrow \left(\frac{\beta}{1-2\beta}\right)\left(-\frac{2}{3}\right) = -1$

$2\beta = 3 - 6\beta$

$8\beta = 3$

$\beta = 3/8$

$\therefore C \equiv \left(1 - 2\left(\frac{3}{8}\right), \frac{3}{8}\right)$

$\equiv \left(\frac{1}{4}, \frac{3}{8}\right)$

Eq^n of BC:

$\left(y - \frac{3}{8}\right) = 1\left(x - \frac{1}{4}\right)$

$8y - 3 = 8x - 2$

$-8x + 8y - 1 = 0$

$ax + by - 1 = 0$

$a = -8 ; b = 8$

$\frac{b-a}{4} = \frac{8-(-8)}{4}$

$= \boxed{4}$ Ans

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JEE 1

The value of f(0) so that the function $f(x) = \frac{7\tan^7 x+\sin^6 x+2\sin^5 x}{(\sin x-\tan x)^2+(1-\cos 2x)^4+x^5}$ is continuous at x = 0, is 2.

$$\lim_{x\to 0} \frac{7\frac{\tan^7 x}{x^5} + \frac{\sin^6 x}{x^5} + 2\frac{\sin^5 x}{x^5}}{x\cdot\frac{(\sin x-\tan x)^2}{x^5\times x} + x^3\cdot\frac{(1-\cos 2x)^4}{x^5\times x^3} + 1}$$

$$\lim_{x\to 0} \frac{\sin x-\tan x}{x^3} = \text{finite}$$

$$\lim_{x\to 0}\left(\frac{1-\cos\theta}{\theta^2}\right)^4 = \frac{1}{2}$$

$$\frac{(1-\cos\theta)^4}{\theta^8} = \frac{1}{2}$$

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JEE 1

Let $0 \le a, b, c, d \le \pi$, where b and c are not complementary, such that $2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$ and such that $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$, then the value of $3\frac{\cos(a+d)}{\cos(b+c)}$ is _____.

$$\begin{cases} 2\cos a + 6\cos b + 7\cos c + 9\cos d = 0 \\ 2\sin a - 6\sin b + 7\sin c - 9\sin d = 0 \end{cases}$$

$$\begin{cases} (2\cos a + 9\cos d)^2 = (-6\cos b - 7\cos c)^2 \\ (2\sin a - 9\sin d)^2 = (6\sin b - 7\sin c)^2 \end{cases}$$

$4 + 81 + 36\cos a\cos d - 36\sin a\sin d$

$= 36 + 49 + 84\cos b\cos c - 84\sin b\sin c$

$\Rightarrow 36\cos(a+d) = 84\cos(b+c)$

$\Rightarrow 3\frac{\cos(a+d)}{\cos(b+c)} = \frac{84}{12} = 7$