FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – IX
PAPER –1
TEST DATE: 04-05-2025

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A

Sol. $\frac{dm}{dx} = kx$

$\Rightarrow \int_0^m dm = k\int_0^{\ell} xdx \Rightarrow k = \frac{2m}{\ell^2}$

$dI = x^2 dm = x^2 \frac{2m}{\ell^2} xdx$

$I = \frac{2m}{\ell^2}\int_0^{\ell} x^3 dx = \frac{m\ell^2}{2}$

$X_{cm} = \int xdm = \frac{2\ell}{3}$

$-(mg\sin 30°)\frac{2\ell}{3}\theta = \frac{m\ell^2}{2}\alpha \Rightarrow \alpha = -\frac{2g}{3\ell}\theta$

$T = 2\pi\sqrt{\frac{3\ell}{2g}}$

2. C

Sol. $V = \frac{\pi D^2 L}{4}$

$\frac{\Delta V}{V} = 2\frac{\Delta D}{D} + \frac{\Delta L}{L} = 2\times\frac{0.1}{4} + \frac{0.1}{5}$

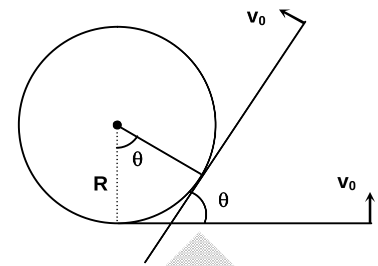
% change in volume = $\frac{\Delta V}{V}\times 100 = 7\%$

3. B

Sol. Wire 1, 3, 6 and 7 are inside the loop.

4. C

Sol. T and v are perpendicular so speed is constant.



$$\theta = \frac{\ell - x}{R}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{dx}{Rdt}$$

$$\Rightarrow \frac{v_0}{x} = -\frac{dx}{Rdt}$$

$$\Rightarrow xdx = -Rv_0 dt$$

$$\Rightarrow t = \frac{\ell^2}{2Rv_0}$$

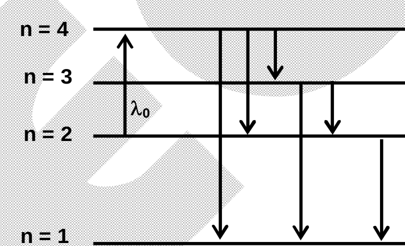
Now check the options.

5. A, B, D

Sol. Let n be principle quantum number of final excited state

$$\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$$

The principle quantum number of initial state is n = 2 as shown in figure.



6. B, C, D

Sol. $f_H = \left(\frac{v}{v - v_c}\right)f$

$$\lambda_H = \frac{v}{f_H} = \frac{v - v_c}{f}$$

$$f_D = \left(\frac{v + v_0}{v + v_c}\right)f$$

$$f_R = \left(\frac{v + v_0}{v}\right)\left(\frac{v}{v - v_c}\right)f = \left(\frac{v + v_0}{v - v_c}\right)f$$

beat frequency = $f_R - f_D = \frac{2v_c(v + v_0)f}{v^2 - v_c^2}$.

7. A, C

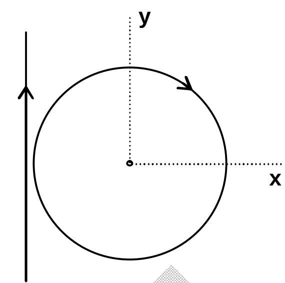
Sol. Conceptual

8. B

Sol. (P) $B_P = \frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{4\pi a} = \frac{\mu_0 I}{2\pi a}$

$$B_Q = \frac{\mu_0 I}{4\pi a}(\sin 90^\circ - \sin 45^\circ) + \frac{\mu_0 I}{4\pi a}(\sin 90^\circ + \sin 45^\circ) = \frac{\mu_0 I}{2\pi a}$$

$$\frac{B_P}{B_Q} = 1$$



(Q) $$B_P = \frac{\mu_0 \pi I}{2\pi r} + \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{r}$$

$$B_{QZ} = \frac{\mu_0 \pi I}{2\pi\sqrt{2}r}\cos 45kd + \frac{\mu_0 I r^2}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 I}{4r}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$B_{Qx} = \frac{\mu_0 I \pi}{2\pi\sqrt{2}r}\sin 45^\circ = \frac{\mu_0 I}{4r}$$

$$B_Q = \sqrt{B_{Qx}^2 + B_{Qz}^2} = \frac{\mu_0 I}{4r}\sqrt{\frac{(5+2\sqrt{2})}{2}}$$

$$\frac{B_P}{B_Q} = \frac{4\sqrt{2}}{\sqrt{5+2\sqrt{2}}}$$

(R) $$B_P = 2\times\frac{\mu_0 I}{4\pi r}(\sin 90^\circ + \sin 45^\circ) = \frac{\mu_0 I}{2\pi r}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$B_Q = 2\times\frac{\mu_0 I}{4\pi r}(\sin 90^\circ - \sin 45^\circ) = \frac{\mu_0 I}{2\pi r}\left(1-\frac{1}{\sqrt{2}}\right)$$

Here r is perpendicular distance of pints P and Q from any one straight current carrying wire

$$\frac{B_P}{B_Q} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = 3+2\sqrt{2}$$

(S) $$B_P = 2\times\frac{\mu_0 I}{4\pi\ell}(\sin 45^\circ + \sin 45^\circ) = \frac{2\sqrt{2}\mu_0 I}{\pi\ell}$$

$$B_Q = 4\times\frac{\mu_0 I}{4\pi\frac{\ell}{\sqrt{2}}}(\sin\theta+\sin\theta)\sin 45^\circ = \frac{\sqrt{2}\mu_0 I}{\pi\ell}\left(\frac{\ell/2}{\frac{\sqrt{3}\,\ell}{2}} + \frac{\ell/2}{\frac{\sqrt{3}\,\ell}{2}}\right)\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{3}}\frac{\mu_0 I}{\pi\ell}$$

$$\frac{B_P}{B_Q} = \sqrt{6}$$

9. A

Sol. (P) frequency of sound receive by the reflector

$$f_r = \frac{v-v_r}{v}f$$

The frequency of reflected sound

$$f' = \frac{v}{v+v_r}f_r = \frac{v-v_r}{v+v_r}f = \frac{330-33}{330+33}\times 660$$

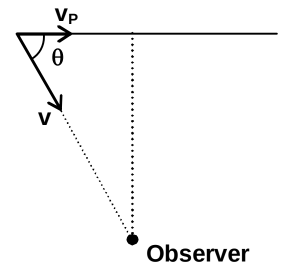
(Q) frequency of sound reaching the hill

$$f_h = \frac{v}{v-v_b}f$$

The frequency of reflected sound

$$f' = \frac{v+v_b}{v}f_h = \frac{v+v_b}{v-v_b}f = \frac{330+16.5}{330-16.5}\times 627 = 693\text{ Hz}$$

(R) $f' = \frac{v}{v - v_P\cos\theta} f = \frac{v}{\left(v - v_P\frac{v_P}{v}\right)} f = \frac{v^2}{v^2 - v_P^2} f$

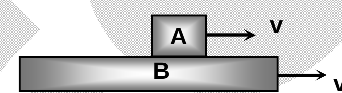


$\frac{v^2}{v^2 - \left(\frac{v}{2}\right)^2} f = \frac{4}{3} \times 450 = 600$ Hz

(S) $f' = \sqrt{\frac{c-v}{c+v}} f = \sqrt{\frac{c - \frac{c}{2}}{c + \frac{c}{2}}} \times 500\sqrt{3} = 500$ GHz

10. C

Sol. using conservation of momentum



$10 \times 8 + 12 \times 20 = 20v$

$v = 16$ m/s

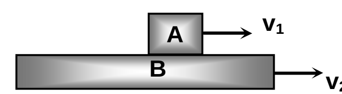
work done by friction on A

$w_f = \frac{1}{2}(8)\left[(16)^2 - (10)^2\right] = 624$ J

work done by friction on B

$w_f = \frac{1}{2}(12)\left[(16)^2 - (20)^2\right] = -864$ J

Velocity of block A when the loss of energy becomes half of its maximum value



$\frac{1}{2}(8)(10)^2 + \frac{1}{2}(12)(20)^2 = \frac{1}{2}(8)v_1^2 + \frac{1}{2}(12)v_2^2 + 120$ ...(i)

By using of conservation of linear momentum

$10 \times 8 + 12 \times 20 = 8v_1 + 12v_2$ ...(ii)

From (i) and (ii)

$v_1 = (16 - 3\sqrt{2})$ m/s

11. A

Sol. Information Based

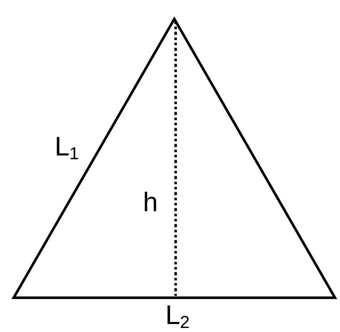
SECTION – B

12. 1

Sol. $kx = P_0A$

13. 4

Sol. $h^2 = L_1^2 - \frac{L_2^2}{4}$



Differentiating and using $dL = L\alpha\,\Delta t$

$0 = 2L_1\alpha_1L_1\Delta t - \frac{1}{4}2L_2\alpha_2L_2\Delta t$

$L_1L_1\alpha_1\Delta t = \frac{1}{4}L_2L_2\alpha_2\Delta t$

$\Rightarrow \frac{L_2}{L_1} = 2\sqrt{\frac{\alpha_1}{\alpha_2}} = 2\sqrt{\frac{4\times10^{-6}}{1\times10^{-6}}} = 4$

14. 2

Sol. $\vec{v} = \frac{v_0}{2}\left(\sqrt{3}\hat{i} + \hat{j}\right)$

For crossing origin,

$k\frac{2\pi m}{qB} = \frac{2(v_0/2)}{qE/m}$; k is an integer

$\Rightarrow B = \left(\frac{2\pi E}{v_0}\right)k$

For minimum value; k = 1

$\Rightarrow B_{min} = \frac{2\pi E}{v_0}$

15. 2

Sol. Let displacement be x.

The strains are

$\varepsilon_1 = \frac{3\alpha\ell\Delta T - x}{\ell}$ and $\varepsilon_2 = \frac{2\alpha\ell\Delta T + x}{2\ell}$

For equilibrium, forces are balanced at the junction

$\Rightarrow Y\varepsilon_1 = 2Y\varepsilon_2$

$\Rightarrow x = \frac{\alpha\ell\Delta T}{2}$

16. 40

Sol. Applying snell's law

$1\sin 53° = \frac{4}{3}\sin r_1$

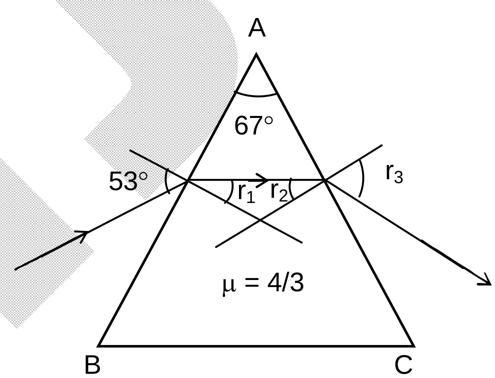
$r_1 = 37°$

So, $r_2 = 67° - 37° = 30°$

$\frac{4}{3}\sin 30° = \sin r_3$

$r_3 = 42°$

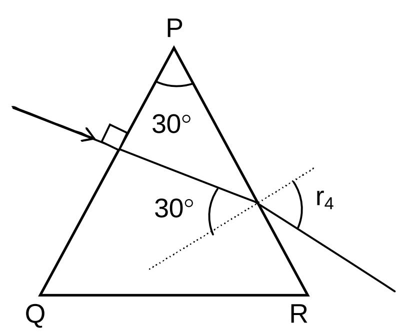
So, $\delta_1 = (53° - 37°) + (42° - 30°) = 28°$



$\frac{4}{3}\sin 30° = 1\sin r_4 \Rightarrow r_4 = 42°$

$\delta_2 = 42° - 30° = 12°$

So, net deviation $\delta = \delta_1 + \delta_2 = 40°$



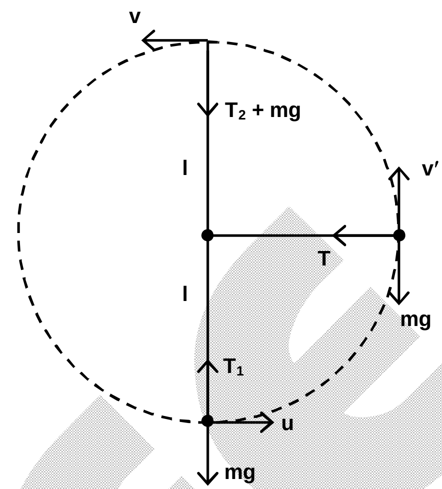
17. 9

Sol. $T_{max} = T_1$

$T_{min} = T_2$

$$T_1 - mg = \frac{mu^2}{l} \qquad \ldots(1)$$

$$T_2 + mg = \frac{mv^2}{l} \qquad \ldots(2)$$



From COME (of pendulum + earth system)

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2l)$$

$\therefore \quad u^2 = v^2 + 4gl \qquad \ldots(3)$

also $\frac{T_1}{T_2} = 2 \qquad \ldots(4)$

from (1), (2) (3) and (4)

$u = \sqrt{11gl}, \quad v = \sqrt{7gl}$

$\therefore \quad v' = \sqrt{9gl}$,

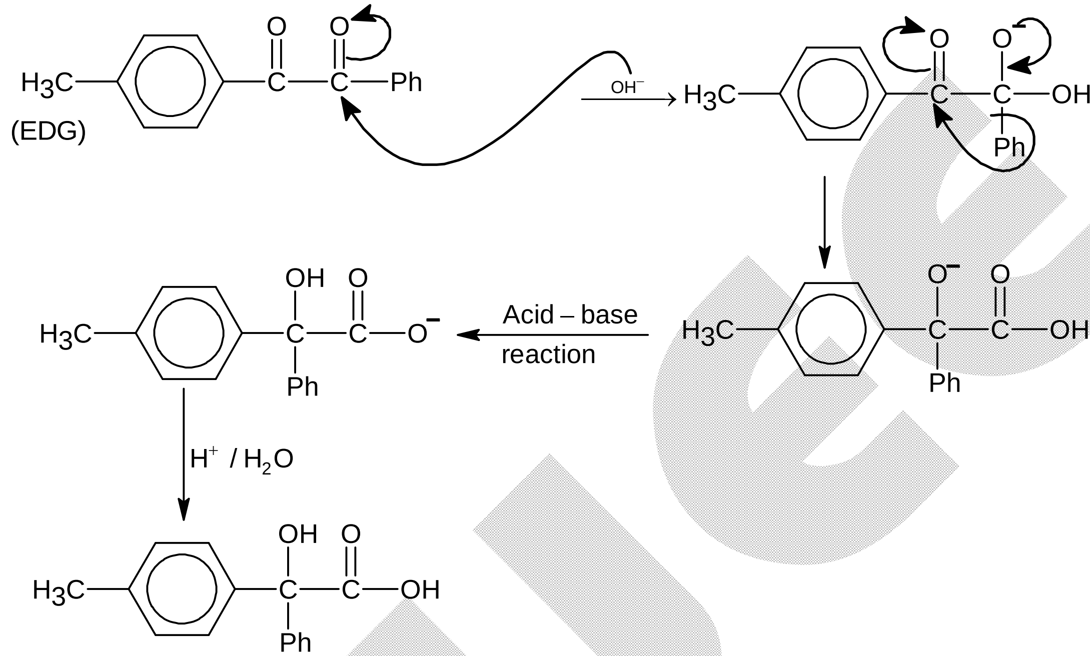
$$T = \frac{mv'^2}{l} = 9mg \qquad K = 9$$

Chemistry

PART – II

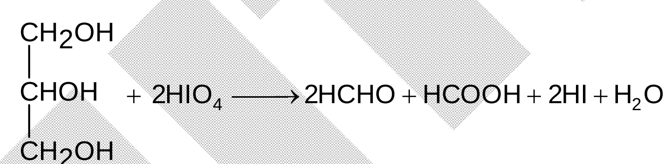
SECTION – A

18. A
Sol.

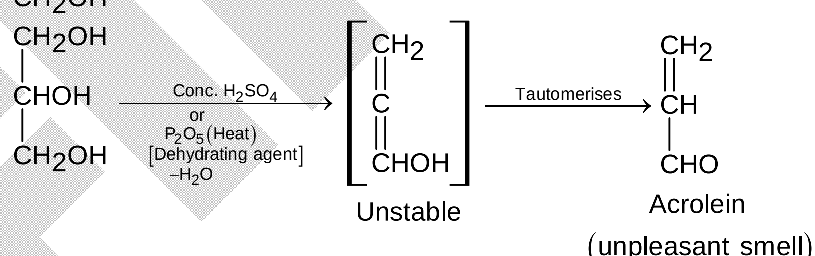


19. B
Sol. Beryl is cyclic silicate, two oxygen atoms per tetrahedron are shared to form closed ring.

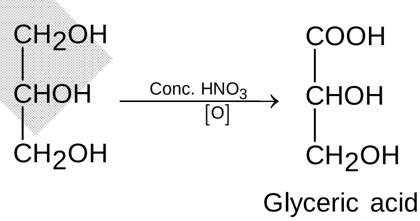
20. D
Sol. (A)



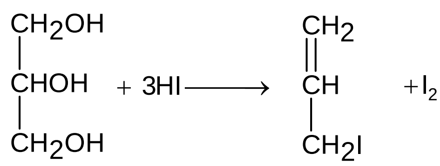
(B)



(C)

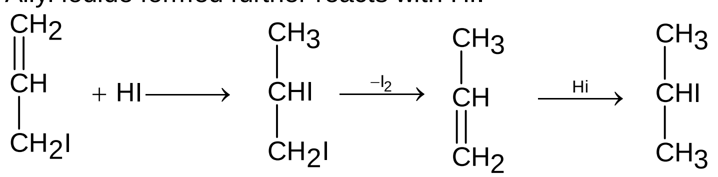


(D)



Allyl iodide

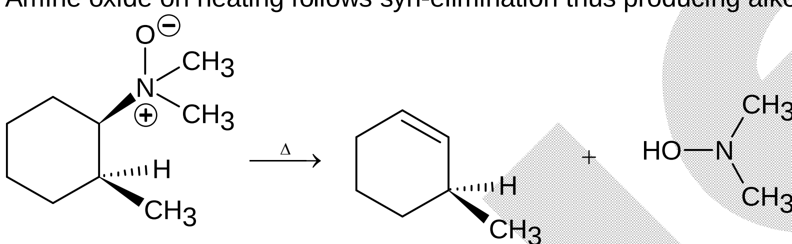
Allyl iodide formed further reacts with HI.



Unstable   Propene   Isopropyl iodide

21. B

Sol. Amine oxide on heating follows syn-elimination thus producing alkene



22. B, C, D

Sol. $E_{cell} = \frac{0.059}{2}\log\frac{\left[M^{2+}\right]_{right}}{\left[M^{2+}\right]_{left}}$

$0.236 = \frac{0.059}{2}\log\frac{[0.01]}{\left[M^{2+}\right]_{left}}$

$\left[M^{2+}\right]_{left} = 1\times10^{-10}\ M$

$\therefore$ Solubility of MX_2 in 0.2 M $X^- = 1\times10^{-10}$ M

K_{SP} of $MX_2 = \left[M^{2+}\right]\left[X^-\right]^2 = 1\times10^{-10}\times(0.2)^2 = 4\times10^{-12}\ M^3$

Solubility of MX_2 in water $= \left(\frac{K_{SP}}{4}\right)^{1/3} = \left(\frac{4.0\times10^{-12}}{4}\right)^{1/3} = 1.0\times10^{-4}$ M

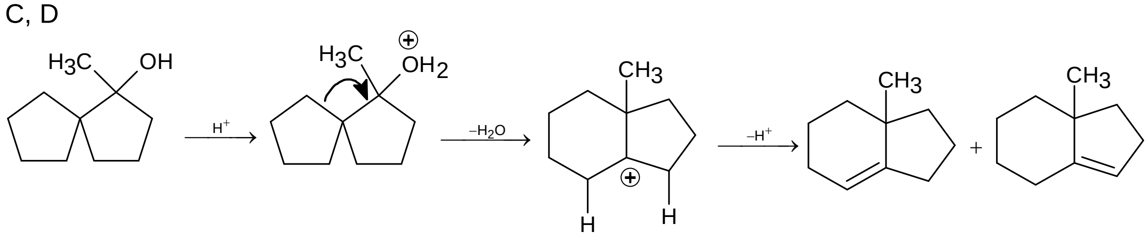
23. A, C

Sol. Haematite – Fe_2O_3 Bauxite – $Al_2O_3.2H_2O$
Magnetite – Fe_3O_4 Corundum – Al_2O_3
$\left.\begin{array}{l}\text{Bauxite} - Al_2O_3.2H_2O\\ \text{Corundum} - Al_2O_3\end{array}\right\}$ Oxide ores of same metal

Anglesite – $PbSO_4$ Cuprite – Cu_2O
Cerrusite – $PbCO_3$ $\underbrace{\text{Cassiterite} - SnO_2}_{\text{Oxide ores of different metals}}$

24. C, D

Sol.



25. C

Sol. (P) Non-superimposable mirror image; enantiomer.
(Q) Superimposable mirror image, identical.
(R) Configuration at one chiral carbon is different; Diasteromers
(S) Position of – OH is different; positional isomers.

26. D

Sol. (P) $pH = 7 + \frac{1}{2}\left(4.74 + \log\frac{25}{75}\right) = 9.1$

(Q) $pH = \frac{6.4 + 10.4}{2} = 8.4$

(R) $pOH = 4.74 + \log\frac{5}{5} = 4.74$

$pH = 14 - 4.74 = 9.26$

(S) $\alpha = \sqrt{\frac{2\times10^{-5}}{0.1}} = \sqrt{2}\times10^{-2}$

$\left[OH^-\right] = C\alpha = 0.1\times\sqrt{2}\times10^{-2} = \sqrt{2}\times10^{-3}$

$pOH = 3 - \log\sqrt{2} = 2.85$

$\Rightarrow pH = 11.15$

27. C

Sol. (P) Ti^{+3} is a oxidizing agent.
(Q) $(SiH_3)_3N$ has $p\pi - d\pi$ overlap between N and Si.
(R) Sn^{+2} is a reducing agent.
(S) $(CH_3)_3N$ is Lewis base.

28. A

Sol. (P) Markonikov's addition of H_2O : Oxymercuration-demercuration.
(Q) Anti- Markonikov's addition of H_2O : Hydroboration.
(R) Epoxidation at more electron rich alkene : mCPBA.
(S) Epoxidation at less electron rich alkene : H_2O_2 / OH^-

SECTION – B

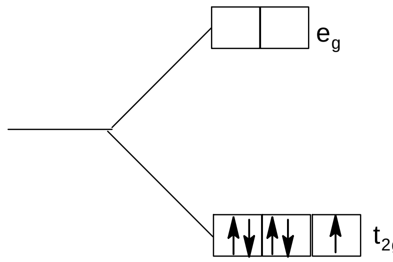
29. 7

Sol. All statements except (F) are true.
Insulin is a harmone and not co-enzyme.

30. 9

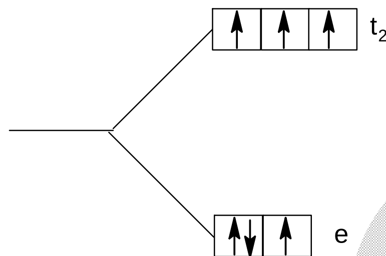
Sol.

$K_3[Fe(CN)_6]$: $Fe^{3+} = 3d^5$



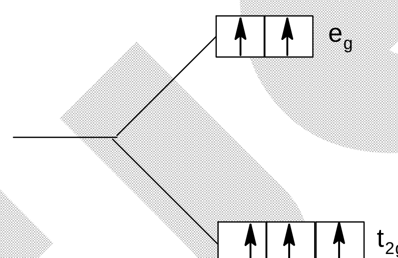
P = 1

$[FeCl_4]^{2-}$: $Fe^{2+} = 3d^6$



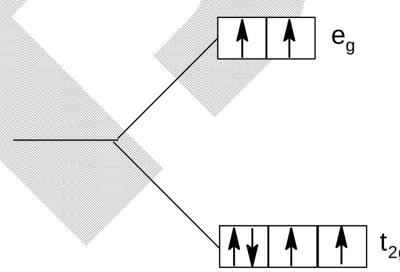
Q = 3

$[Fe(H_2O)_6]Cl_3$: $Fe^{3+} = 3d^5$



R = 3

$[FeF_6]^{4-}$: $Fe^{2+} = 3d^6$



S = 2

$\therefore P + Q + R + S = 1 + 3 + 3 + 2 = 9$

31. 17

Sol. the process involved are

$$\underset{(-10^\circ C)}{H_2O(s)} \xrightarrow{q_1} \underset{0^\circ C}{H_2O(s)} \xrightarrow{q_2} \underset{0^\circ C}{H_2O(\ell)} \xrightarrow{q_3} \underset{100^\circ C}{H_2O(\ell)} \xrightarrow{q_4} \underset{100^\circ C}{H_2O(g)} \xrightarrow{q_5} \underset{205.56^\circ C}{H_2O(g)}$$

$Q = q_1 + q_2 + q_3 + q_4 + q_5$

$= 5[2 \times 10 + 333 + 4.18 \times 100 + 2439 + 1.8 \times 105.56]$

= 17000 J

32. 50

Sol.

$$NaHCO_3 \longrightarrow \underset{1}{Na^+} + \underset{1}{HCO_3^-}$$

$$\underset{1-h}{HCO_3^-} + H_2O \longrightarrow \underset{h}{H_3O^+} + \underset{h}{CO_3^{-2}}$$

$i = 1 + h + h + 1 - h = 2 + h$
$0.372 = (2 + h) \times 0.08 \times 1.86$
$2 + h = 2.5$
$h = 50\%$

33. 20

Sol. Meq. of NaOH $+ \frac{1}{2}$Meq. of $Na_2CO_3 = 15 \times \frac{1}{20}$

Meq. of NaOH + Meq. of $Na_2CO_3 = 20 \times \frac{1}{20}$

Meq. of NaOH $= (30 - 20)\frac{1}{20}$

$\frac{W \times 1000}{40} = 0.5$

$W = 20 \times 10^{-3} = 20$ mg

34. 35

Sol. $5CO + I_2O_5 \longrightarrow I_2 + 5CO_2$

$I_2 \quad + \quad 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$

25 mm $\quad 100 \times \frac{1}{2} = 50$ mm

Millimoles of CO $= 125$

Wt. of CO $= 125 \times 10^{-3} \times 28 = 3.5$

Hence, % of CO $= \frac{3.5 \times 100\%}{10} = 35\%$

Mathematics
PART – III

SECTION – A

35. D

Sol. $4^4\left(\frac{\sin^{12}B+\sin^{12}C}{(\sin 2A+\sin 2B+\sin 2C)^4}\right) = \frac{\alpha^4+\beta^4}{\left(\frac{\alpha+\beta}{2}\right)^4}$ where α, β are distinct positive real number whose AM is a and two GM's are b, c

36. C

Sol. Let foot of perpendicular from (3, –1) on the directrix be (x_1, y_1)

So, $\frac{x_1-3}{2}=\frac{y_1+1}{1}=\frac{-(6-1-3)}{5}$

$\frac{x_1-3}{2}=\frac{y_1+1}{1}=\frac{-2}{5}$

$x_1=3-\frac{4}{5};\ y_1=-1-\frac{2}{5}$

$x_1=\frac{11}{5};\ y_1=-\frac{7}{5}$

Now image of (x_1, y_1) about tangent is focus, which is (x_2, y_2)

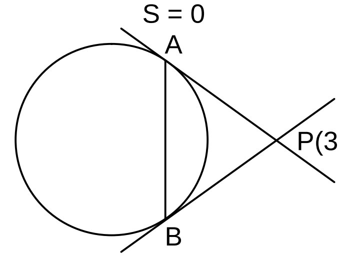
$\frac{x_2-\frac{11}{5}}{1}=\frac{y_2+\frac{7}{5}}{1}=\frac{-2\left(\frac{11}{5}-\frac{7}{5}-2\right)}{2}=\frac{6}{5}$

$x_2=\frac{17}{5};\ y_2=\frac{6}{5}-\frac{7}{5}=-\frac{1}{5}$

Focus is $\left(\frac{17}{5},-\frac{1}{5}\right)$

37. B

Sol. Let circle S = 0 is $x^2 + y^2 + 2gx + 2fy = 0$



Equation of AB as chord of contact

T = 0

$3x + 5y + g(x + 3) + f(y + 5) = 0$

$x(3 + g) + y(5 + f) + 3g + 5f = 0$ ..... (1)

Also, given equation of AB $x + 2y = 3$ ..... (2)

So, $\frac{3+g}{1}=\frac{5+f}{2}=\frac{3g+5f}{-3}$

$6 + 2g = 5 + f$ and $-9 - 3g = 3g + 5f$

$2g - f + 1 = 0$ and $6g + 5f + 9 = 0$

$g=-\frac{7}{8}$ and $f=-\frac{3}{4}$

38. B

Sol. $\frac{dy}{dx}-\frac{y}{x}=A$ when $A=\int_1^2 y\,dx$

I.F = $\frac{1}{x}$

So, $y \cdot \frac{1}{x} = \int A \cdot \frac{1}{x} dx + c = A \ln x + c$
$y = A\,x \ln x + cx$
Now $f(1) = 2 \Rightarrow 2 = c$
$y = A\,x \ln x + 2x$
Also, $A = \int_1^2 y dx = \int_1^2 (Ax\ln x + 2x) dx$

$$= A\left[\ln x \cdot \frac{x^2}{2} - \frac{x^2}{4}\right]_1^2 + \left[x^2\right]_1^2 = A\left[\ln 2 \cdot 2 - 1 - \left(0 - \frac{1}{4}\right)\right] + 4 - 1 = A\left[2\ln 2 - \frac{3}{4}\right] + 3$$

$$A\left[\frac{7}{4} - 2\ln 2\right] = 3;\ A = \frac{12}{7 - 8\ln 2}$$

39. A, C, D
Sol. (A) $a_{ij} = -a_{ji} \Rightarrow a_{ij} + a_{ji} = 0 \Rightarrow$ A is skew symmetric matrix
(C) $A^2 = 2A \Rightarrow A^3 = 2^2A \Rightarrow A^6 = 2^5A$
(D) A^6B^7 is a skew symmetric matrix of odd order

40. C, D
Sol. $\cot 65^o + 2\tan 40^o = \frac{\sin 25^o}{\cos 25^o} + \frac{\sin 40^o}{\cos 40^o} + \tan 40^o$

$$= \frac{\sin 65^o}{\cos 25^o \cdot \cos 40^o} + \frac{\sin 40^o}{\cos 40^o} = \frac{1 + \sin 40^o}{\cos 40^o} = \frac{1 + \cos 50^o}{\sin 50^o} = \cot 25^o$$

and $\sqrt{3}\sin 80^o - 2\sin 50^o = 2\sin 60^o \cdot \sin 80^o - 2\sin 50^o$
$= \cos 20^o - \cos 40^o = \sin 10^o$

41. A, B, C
Sol. $\sum_{1 \le i < j \le 7} |a_i + a_j|^2 = \sum_{1 \le i < j \le 7} \left(|a_i|^2 + |a_j|^2 + a_i\bar{a}_j + \bar{a}_i a_j\right)$

$$= 2R^2 \cdot {}^7C_2 + \sum_{i \ne j} a_i\bar{a}_j = 42R^2 + \sum_{i=1}^{7}\sum_{j=1}^{7} a_i\bar{a}_j - \sum_{i=1}^{7} a_i\bar{a}_i$$

$$= 35R^2 + \left(\sum_{i=1}^{7} a_i\right)\left(\sum_{i=1}^{7} \bar{a}_i\right) - 7R^2$$

$$= 35R^2 + \left|\sum_{i=1}^{7} a_i\right|^2 \ge 35R^2$$

42. A
Sol. Given $\frac{abc}{6} = 32$, where A, B and C are respectively (a, 0, 0), (0, b, 0), (0, 0, c)
(P) Centroid of tetrahedron is $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$
$\Rightarrow a = 4\alpha,\ b = 4\beta,\ c = 4\gamma$
$\Rightarrow \alpha\beta\gamma = 3$
(Q) Equidistant point $(\alpha, \beta, \gamma) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
$\Rightarrow \alpha\beta\gamma = 24$

(R) The equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\therefore$ Foot of perpendicular from the origin is (α, β, γ) is $\left(\frac{\frac{1}{a}}{\sum\frac{1}{a^2}}, \frac{\frac{1}{b}}{\sum\frac{1}{a^2}}, \frac{\frac{1}{c}}{\sum\frac{1}{a^2}}\right)$

$\frac{1}{a\alpha}=\frac{1}{b\beta}=\frac{1}{c\gamma}=t$

$t=\sum\frac{1}{a^2} \Rightarrow t = (\alpha^2+\beta^2+\gamma^2)^{-1}$ and $a=\frac{\alpha^2+\beta^2+\gamma^2}{\alpha}$, $b=\frac{\alpha^2+\beta^2+\gamma^2}{\beta}$, $c=\frac{\alpha^2+\beta^2+\gamma^2}{\gamma}$

$abc = 6\times 32 \Rightarrow (\alpha^2+\beta^2+\gamma^2)^3 = 192\alpha\beta\gamma$

(S) Let P be (α, β, γ), then $PA \perp PB$

$\alpha(\alpha-a)+\beta(\beta-b)+\gamma\gamma = 0$, $PB \perp PC$

$\alpha\alpha+\beta(\beta-b)+\gamma(\gamma-c)=0$

$\Rightarrow \frac{a}{\frac{1}{\alpha}}=\frac{b}{\frac{1}{\beta}}=\frac{c}{\frac{1}{\gamma}}$

$(\alpha^2+\beta^2+\gamma^2)^3 = (192)\times(8\alpha\beta\gamma)$

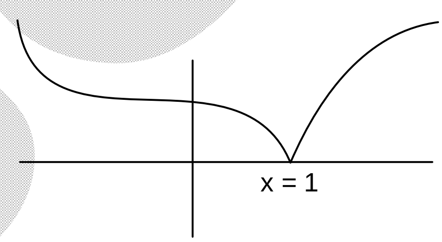
43. C

Sol. $f(x) = 2\max\{|x^3-1|, |x^2-1|\} = 2\left|x^3-1\right|$

$x = 1$ is point of non- differentiability

Area in 1st quadrant $= 2\int_0^1\left(1-x^3\right)dx = \frac{3}{2}$ sq. units

Required $\lim_{x\to 1} 2\left|\frac{1-x^3}{1-x^2}\right| = \lim_{x\to 1}\frac{2\left(1+x+x^2\right)}{1+x}=3$



44. D

Sol. (R) $\sqrt{2}\sin x = \sin t$

$\Rightarrow \sqrt{2}\cos x dx = \cos t dt$

$\int_0^{\pi/2}\left(1-\sin^2 t\right)^{9/2}\frac{1}{\sqrt{2}}\cos t dt = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\cos^{10} t dt$

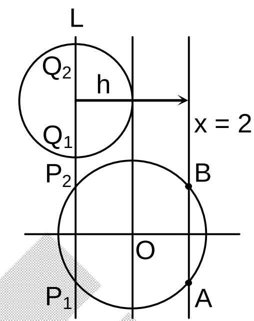
(S) $\lim_{x\to\infty} x\left\{\left(1+\frac{3}{x}\right)^{1/4}-\left(1+\frac{2}{x}\right)^{1/3}\right\}$

$\lim_{x\to\infty} x\left\{1+\frac{1}{4}\times\frac{3}{x}+.....-\left(1+\frac{1}{3}\times\frac{2}{x}+.....\right)\right\}$

$\lim_{x\to\infty} x\left\{\frac{\left(\frac{3}{4}-\frac{2}{3}\right)}{x}\right\}=\frac{9-8}{12}=\frac{1}{12}$

45. B

Sol. Area of ΔPAB is maximum or minimum as h in the figure is maximum or minimum



SECTION – B

46. 1

Sol. If x > 2 then $x^3 - 3x > 4x - x = x > \sqrt{x+2}$

$\Rightarrow |x| \le 2$ take x = 2 cos θ for some $\theta \in [0, \pi]$

$\Rightarrow 2\cos 3\theta = \sqrt{2(1+\cos\theta)}$

$\Rightarrow 2\sin\frac{7\theta}{4}\cdot\sin\frac{5\theta}{4} = 0$

$\Rightarrow \theta = 0, \frac{4\pi}{7}, \frac{4\pi}{5}$

$\therefore x = 2,\ 2\cos\frac{4\pi}{7}, -\frac{1}{2}\left(1+\sqrt{5}\right)$

47. 7

Sol. $\lim\limits_{x\to\infty}\frac{f(x)-f(\alpha x)}{x} = 0$

$\Rightarrow$ For any $\in$ > 0 there is δ > 0 such that |x| < δ and |f(x) – f(αx)| < $\in$ |x| using triangle inequality

$|f(x) - f(\alpha^n x) \le |f(x) - f(\alpha x)| + |f(\alpha x) - f(\alpha^2 x)| + ... + |f(\alpha^{n-1} x) - f(\alpha^n x)|$

$< \in |x|(1 + \alpha + \alpha^2 + ... + \alpha^{n-1}) = \in\frac{1-\alpha^n}{1-\alpha}|x| \le \frac{\in|x|}{1-\alpha}$

As, $n \to \infty$

$|f(x)| \le \frac{\in}{1-\alpha}|x|$

$\Rightarrow \lim\limits_{x\to\infty}\frac{f(x)}{x} = 0$

48. 7

Sol. For f({1, 2, 3}) and f(f{1, 2, 3}) to be disjoint, we must have f(1), f(2), f(3) $\notin$ {1, 2, 3}

If f(1) = f(2) = f(3) = 4 or 5, then one of f(4) and f(5) will have 4 and other have 5 choice. Number of possible function = 2 $\times$ 4 $\times$ 5 = 40

If two of f(1) and f(2) and f(3) are equal and rest is different then f(4), f(5) $\in$ {1, 2, 3}

$\Rightarrow$ Total number of possible function = 3 $\times$ 2 $\times$ 3 $\times$ 3 = 54

$\therefore$ N = 94

49. 4

Sol. Exactly four girls are together – Exactly four girls are together and B_1 B_2 are together

$p = \frac{5!\times 8!\times 58}{13!}$

50. 3

Sol. $\frac{1}{\sqrt{k}} = \frac{2}{\sqrt{k}+\sqrt{k}} < \frac{2}{\sqrt{k}+\sqrt{k-1}} = 2\left(\sqrt{k}-\sqrt{k-1}\right)$

Putting k = 2, 3, ..., 10,000 and adding $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+.....+\frac{1}{\sqrt{10,000}} < 2\left(\sqrt{10,000}-\sqrt{1}\right)$

$\Rightarrow A < 2 \times 99 \Rightarrow A < 198$

Similarly $\frac{1}{\sqrt{k}} = \frac{2}{\sqrt{k}+\sqrt{k}} > \frac{2}{\sqrt{k+1}+\sqrt{k}} = 2\left(\sqrt{k+1}-\sqrt{k}\right)$

$\Rightarrow\ A > 2\left\{\sqrt{10,000}-\sqrt{2}\right\} > 197$

$\therefore$ [A] = 197

$\therefore\ \left[\frac{[A]}{50}\right] = 3$

51. 5

Sol. $\alpha+\beta = \frac{a_2}{a_1},\ \alpha\beta = \frac{a_3}{a_1}$, $\alpha, \beta \in (1, 2)$

$\Rightarrow \alpha - 1, \beta - 1, 2 - \alpha, 2 - \beta \in (0, 1)$

A.M. $\geq$ G.M. $\Rightarrow \frac{(\alpha-1)+(2-\alpha)}{2} \geq \sqrt{(\alpha-1)(2-\alpha)}$

$\Rightarrow\ (\alpha-1)(2-\alpha) \leq \frac{1}{4}$ similarly $(\beta - 1)(2 - \beta) \leq \frac{1}{4}$

$0 < (\alpha-1)(2-\alpha)(\beta-1)(2-\beta) < \frac{1}{16}$ (Both can't equal to $\frac{1}{4}$ simultaneously $(\alpha \neq \beta)$

$0 < \left(\frac{a_3}{a_1}-\frac{a_2}{a_1}+1\right)\left(4-\frac{2a_2}{a_1}+\frac{a_3}{a_1}\right) < \frac{1}{16}$

$0 < \frac{(a_1-a_2+a_3)(4a_1-2a_2+a_3)}{a_1^2} < \frac{1}{16}$

$\frac{a_1^2}{16} > (a_1-a_2+a_3)(4a_1-2a_2+a_3)$

$\frac{a_1^2}{16} > f(1)\cdot f(2)\ \Rightarrow\ \frac{a^2}{16} > 1$

$\Rightarrow$ Least integral value of a is 5