FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VI

JEE (Main)-2025

TEST DATE: 20-01-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. B

Sol. for t < 1

$s_A = 2t$

$s_B = t^2$

separation at t = 1 sec

$\Delta\ell = 1$ ...(i)

For t > 1,

$\Delta s = |1 + t^2 - 2 - t|$

$\frac{d(\Delta s)}{dt} = 0$

T = 0.5 sec

So, $\Delta s_{max} = 0.25 + 1 = 1.25$ m

2. D

Sol. E = U + K.E. = 4J

At x = 6m, K.E. = 0

3. A

Sol. Total energy = $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\varepsilon_0\left(\frac{\varepsilon}{t}\right)^2 V$

$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\varepsilon_0\frac{B^2d^2v^2}{d^2}V$

$\frac{dE}{dt} = 0$

$\Rightarrow kxv + v\dfrac{dv}{dt}(m + \varepsilon_0 VB^2) = 0$

$\Rightarrow a = -\dfrac{kx}{(m + \varepsilon_0 vB^2)}$

4. C

Sol. till deformation, $v_A - v_B = v_0$

$\Delta K.E. = \dfrac{1}{2}mv^2 - \dfrac{1}{2}m\left(\dfrac{v}{3}\right)^2 = \dfrac{4}{9}mv^2$

5. D

Sol. Take component of gravitational acceleration along the rope and minimize the time taken.

6. C

Sol. $K.E_i = \dfrac{1}{2}mv^2 = k_0$

By conservation of angular momentum

$mvR = mv'\dfrac{R}{\eta}$

$\Rightarrow v' = \eta v$

$K.E_F = \eta^2 K$

$W = \Delta K.E. = (\eta^2 - 1)K_0$

7. D

Sol. Total energy = $-\dfrac{Gm^2}{r_0} + \dfrac{1}{2}m_t v_{com}^2$

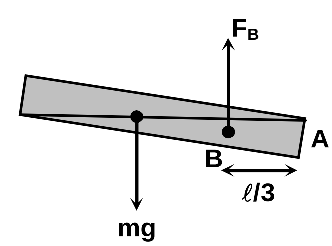
$v_{com} = \dfrac{v_0}{2}$, $m_t = 2m$

Putting total energy < 0

$v_0 < 2\sqrt{\dfrac{GM}{r_0}}$

8. C

Sol. B = centre of buoyancy force



9. B

Sol. Given $\dfrac{\ell_1}{v_1} = \dfrac{\ell_2}{v_2}$

$\Rightarrow \dfrac{\ell_1 A}{v_1} = \dfrac{\ell_2 A}{v_2}$

$\Rightarrow \frac{V_1}{v_2} = \frac{V_2}{v_2}$

$\Rightarrow \frac{V_1}{\sqrt{\frac{3RT_1}{M}}} = \frac{V_2}{\sqrt{\frac{3RT_2}{M}}}$

$\frac{V}{\sqrt{T}} = \text{constant}$

$\Rightarrow PV^{-1}$ = constant

$\Rightarrow C = C_V + \frac{R}{1-x}$

$\Rightarrow$ C = 2R

10. A

Sol. Let ρ_0 = density at 0°C
U_1, U_2 = volumes at 0°C
$m_1 = U_1\rho_0$
$m_2 = U_2\rho_0$
Heat gain = Heat loss
$\Rightarrow U_1\rho_0(S)(\theta - \theta_1) = U_2\rho_0(S)(\theta_2 - \theta)$
$\Rightarrow U_1(\theta - \theta_1) = U_2(\theta_2 - \theta)$
$\Rightarrow (U_1 + U_2)\theta = U_1\theta + U_2\theta_2$ …(i)
Thermal expansion
$\Delta V = V - V_1 - V_2$
$= (U_1 + U_2)[1 + \gamma\theta] - [U_1(1 + \gamma\theta_1) + U_2(1 + \gamma\theta_2)]$
$\Delta V = (U_1 + U_2)\gamma\theta - U_1\gamma\theta_1 - U_2\gamma\theta_2$
from equation (i) $\Delta V = 0$

11. B

Sol. If collision is inelastic at both walls then amplitude will change and time period does not depend on amplitude.

12. D

Sol. distance of images from B (taking rightward positive)
$\therefore \ell = -(x_0 - d)$
if ℓ = –ve, image is virtual
if ℓ = +ve, image is real

13. D

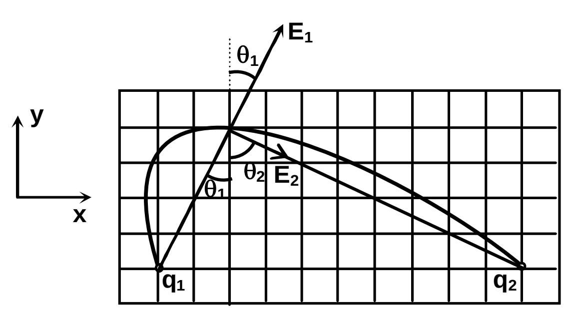
Sol. E_y at P = 0

$\frac{kq_1\cos\theta_1}{r_1^2} + \frac{kq_2\cos\theta_2}{r_2^2} = 0$

$\frac{q_1}{q_2} = -\left(\frac{r_1}{r_2}\right)^3$

$\Rightarrow q_2 = -q_1\left(\frac{r_2}{r_1}\right)^3 = -1\mu\mathbf{C}\left(\sqrt{\frac{80}{20}}\right)^3$

$q_2 = -8\mu C$



14. B

Sol. $\frac{k\theta}{r_A^2} = 3, \frac{k\theta}{r_A} = 7$

$\frac{3k\theta}{r_B^2} = 3 \Rightarrow r_B = \sqrt{3}\, r_A$

So now, $v_B = \frac{2kQ}{r_B} = \sqrt{3} \times 7$

15. B

Sol. Before earthing the charge on the outer surface of the shell is 2q and after earthing, $V_S = 0$

$\frac{kQ}{r} + \frac{k(2q)}{2r} = 0$

Q = –q

$\therefore$ charge flown to the earth, $\Delta q = 2q - (-q) = 3q$

16. C

Sol. Finally K.E. = P.E. = E_0

At closest approach K.E. = $\frac{E_0}{2}$

(parallel component of velocity will be constant)

$KE_i + PE_i = KE_f + PE_f$

$E_0 + E_0 = \frac{E_0}{2} + PE_f$

$PE_F = \frac{3E_0}{2}$

17. D

Sol. $\frac{1}{2} \times \frac{4m}{5} v^2 = \Delta E = 40.8 \times 1.6 \times 10^{-19}$

$v = 9.89 \times 10^4$ m/s

18. A

Sol. For z > 0 motion $R_x = \frac{mV}{qB_y}$

For z < motion $R_y = \frac{mV}{qB_x}$

(x, y) coordinate at 1st time = $(-2R_x, 0)$

at 2nd time = $(-2R_x, -2R_y)$

at 3rd time = $(-4R_x, -2R_y)$

19. A

Sol. $R = \frac{mV}{qB}$

$\frac{R}{R_0} = \tan\theta$

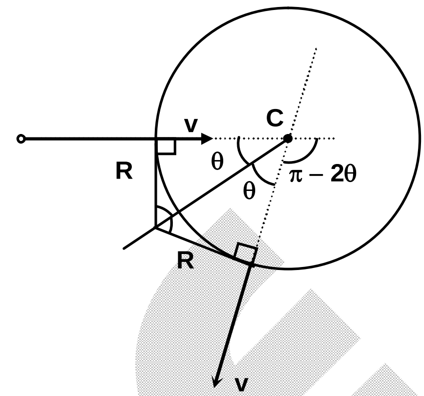
$R = R_0 \tan\theta$

$\frac{mV}{qB} = R_0 \tan\theta$

$V \propto \tan\theta$ …(i)

Also, $t = \frac{m}{qB}(\pi - 2\theta)$

For more V, more will be θ and lesser will be t.



20. C

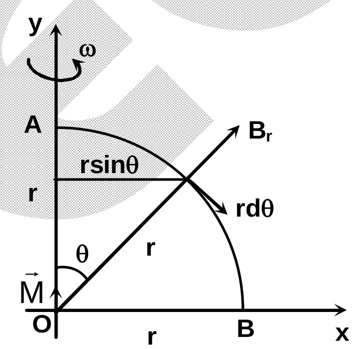
Sol. $B_r = \frac{2\mu_0 M\cos\theta}{4\pi r^3}$

$d\varepsilon = B_r \omega r\sin\theta r d\theta$

$d\varepsilon = \frac{2\mu_0 M\cos\theta}{4\pi r^3}\omega r^2 \sin\theta d\theta$

$\varepsilon = \frac{\mu_0 M\omega}{4\pi r}\int_0^{\pi/2} \sin 2\theta d\theta$

$\varepsilon = \frac{\mu_0 M\omega}{4\pi r}$



SECTION – B

21. 16

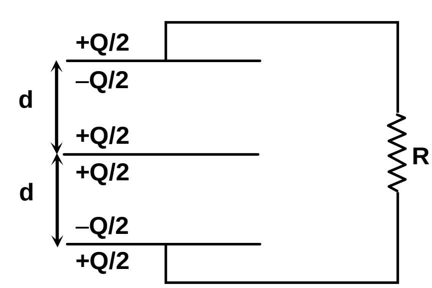
Sol. $U_i = \frac{Q^2}{4C}$ and $U_f = \frac{Q^2}{2\left(2C + \frac{2C}{3}\right)} = \frac{3Q^2}{16C}$, $\left(C = \frac{\varepsilon_0 A}{d}\right)$

Heat dissipated in the resistor,

$H = U_i - U_f$

$H = \frac{Q^2}{4C} - \frac{3Q^2}{16C}$

$H = \frac{Q^2 d}{16\varepsilon_0 A}$



22. 5

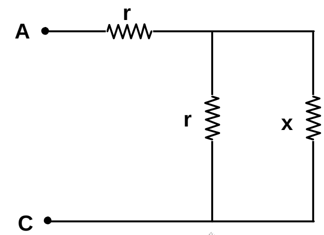
Sol. Using flux conservation,

$5 \times 1 = 1 \times I$

$\Rightarrow I = 5$ amp

23. 6

Sol. Let $R_{AC} = x$



$$\left(\frac{rx}{r+x}\right)+r = x$$

$$x^2 - rx - r^2 = 0$$

$$x = \frac{r(\sqrt{5}+1)}{2}$$

So, $R_{AB} = 2x = (\sqrt{5}+1)r$

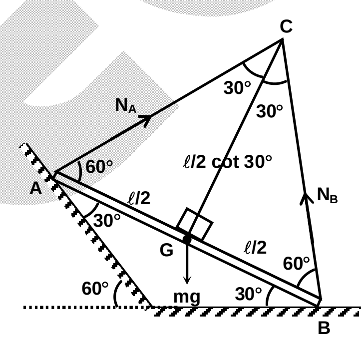
24. 5

Sol. $$\int_0^2 (mg - T)dt = M\Delta V$$

ΔV = 5 m/s

25. 6

Sol. 'C' is the point through which the instantaneous axis of rotation passes and G is the centre of mass of the rod.



CG = $\frac{\ell}{2}\cot 30° = \frac{\ell\sqrt{3}}{2}$

The moment of inertia about the instantaneous axis of rotation is

$$I = \frac{m\ell^2}{12} + m\left(\frac{\ell\sqrt{3}}{2}\right)^2$$

$$I = \frac{m\ell^2}{12} + \frac{3m\ell^2}{4}$$

$$I = \frac{5m\ell^2}{6}$$

Now, $mg\frac{\ell\sqrt{3}}{4} = \left(\frac{5m\ell^2}{6}\right)\alpha$

$$\alpha = \frac{3\sqrt{3}g}{10\ell}$$

Hence k = 6

Chemistry

PART – B

SECTION – A

26. C

Sol.

	A	B
At. wt.	40	80
Given wt.	x gm	2x gm
No. of moles	$\frac{x}{40}$	$\frac{2x}{80}$
No. of atom	$\frac{x}{40}\times N_A$	$\frac{2x}{80}\times N_A$

But according to question $=\frac{x}{40}\times N_A=Y$

27. C

Sol. Meq. of H_2O_2 = Meq. of I_2

$$N\times V=\frac{W}{E}\times 1000$$

$$\frac{V_S}{5.6}\times 5=\frac{0.508}{127}\times 1000$$

$$V_S=4.48$$

28. B

Sol. $5f^{14}6d^{3}=17e^{-}$

29. C

Sol. $\Delta S=nC_P\ell n\frac{T_2}{T_1}$

$$=2\times\frac{5}{2}R\ell n\frac{600}{300}=5R\ell n2$$

30. B

Sol. $K_P=P_{CO_2}$ ... (1)

$K_P'=\frac{[P_{CO}]^2}{P_{CO_2}}$ ... (2)

Multiply Eq. (1) and (2), we get

$K_P.K_P'=[P_{CO}]^2$

$P_{CO}=\sqrt{K_P.K_P'}=\sqrt{4\times 10^{-2}\times 2.0}==2\sqrt{2}\times 10^{-1}=0.28$

31. B

Sol. $M_{LiCl}=42.5,\quad \Delta T_f=0.343^o$

$$\Delta T_f=K_f\times m=1.86\times\frac{4.13}{42.5}$$

$$i=\frac{\Delta T_f(\text{obs.})}{\Delta T_f(\text{Theor.})}=\frac{0.343}{1.86\times(4.13/42.5)}=\frac{0.343\times 42.5}{1.86\times 4.13}$$

$$=1.898\simeq 1.9$$

32. B

Sol. $k = \dfrac{2.303}{t}\log\dfrac{P_i}{P_f}$

$$\begin{array}{lcccc} & CH_3-N=N-CH_3 & \longrightarrow & C_2H_6 & + \; N_2 \\ t=0 & 200 & & & \\ t=t & 200-x & & x & x \end{array}$$

$P_t = 200 - x + x + x = 200 + x$

$Pt = 200 + x = 350$

$x = 350 - 200 = 150$

$K = \dfrac{2.303}{40\times 60}\log\dfrac{200}{200-150}$

$K = \dfrac{2.303}{40\times 60}\log\dfrac{200}{50} = \dfrac{2.303}{40\times 60}\times\log 4$

$K = \dfrac{2.303}{2400}\log 4$

$= \dfrac{2.303}{2400}\times 0.602 = 5.77\times 10^{-4}$

33. C

Sol. Facts

34. A

Sol. Facts

35. D

Sol. $10KI + 2KMnO_4 + 8H_2SO_4 \longrightarrow 5I_2 + 2K_2SO_4 + 2MnSO_4 + 8H_2O$

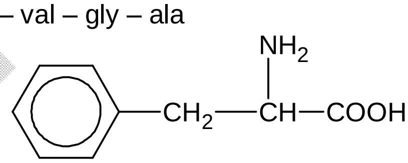
$2KMnO_4 + KI + H_2O \longrightarrow 2MnO_2 + 2KOH + KIO_3$

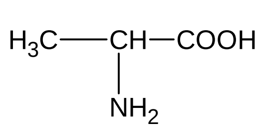
36. D

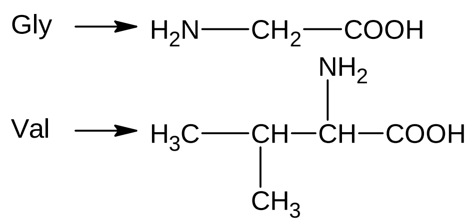
Sol. Condition of John Teller Theorem →
- (1) Non-linear
- (2) Electronic degeneracy

37. C

Sol.
(i) Val – gly – phe – ala
(ii) Phe – gly – val – ala
(iii) Val – phe – gly – ala
(iv) Phe – val – gly – ala

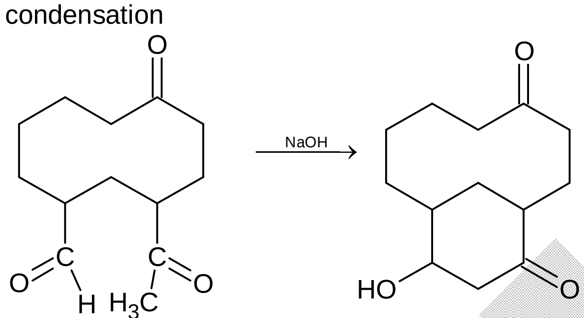
Phe →


Ala →


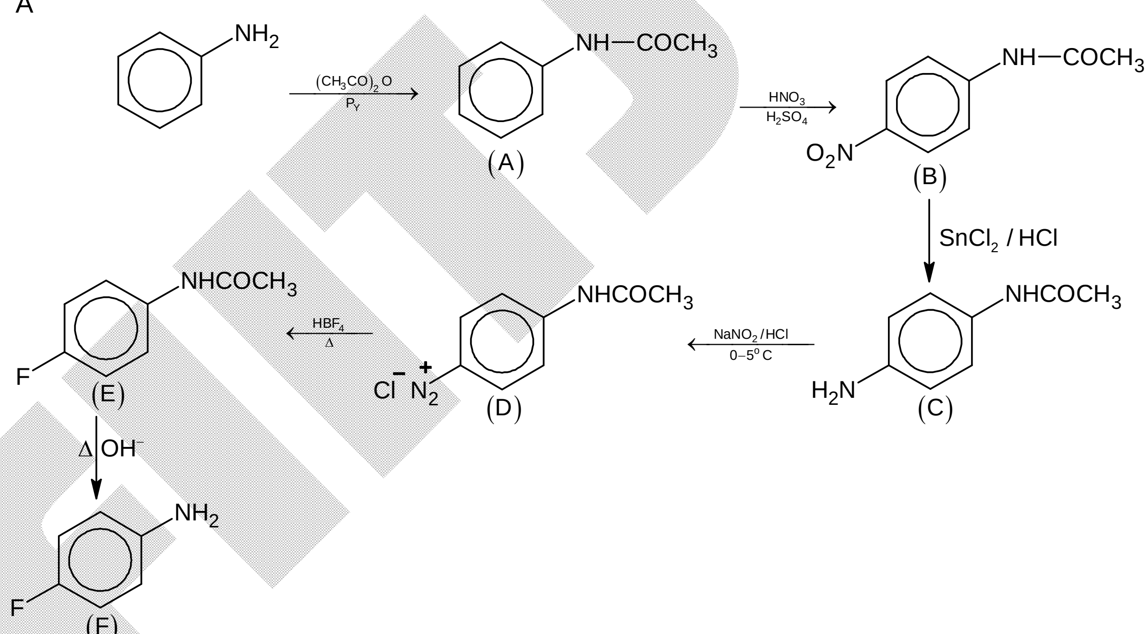


38. D
Sol. Facts

39. D
Sol. Aldehyde is more reactive then ketone towards necleophilic addition, which is the 1[st] step of Aldol condensation



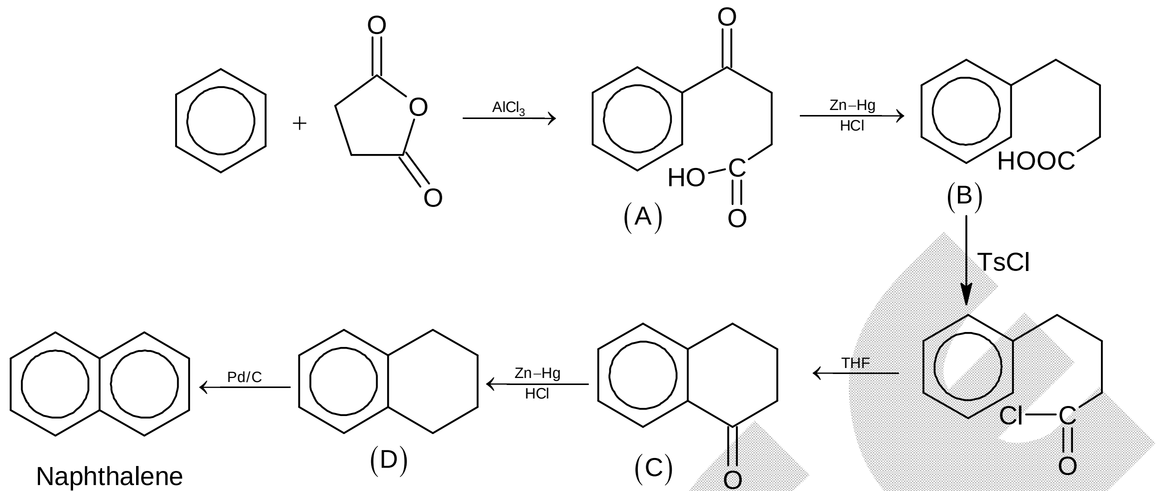
40. A
Sol.



41. B
Sol. It is undergo Cannizzaro is due to the absence of α – hydrogen in the compound.

42. B

Sol.

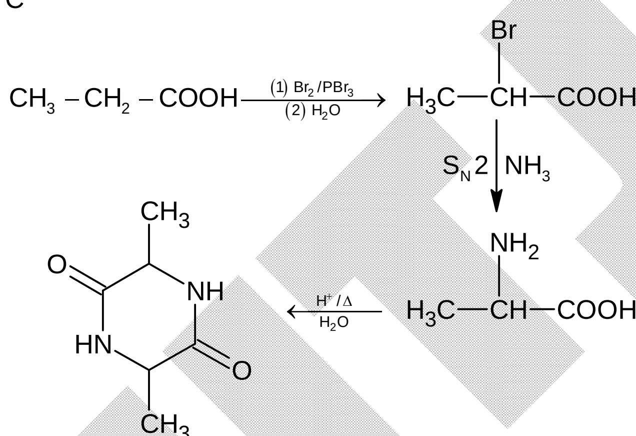


43. A

Sol. Finkelstein reaction is a halogen exchange reaction and this reaction is driven by difference of solubility of NaCl, NaBr with respect to NaI.

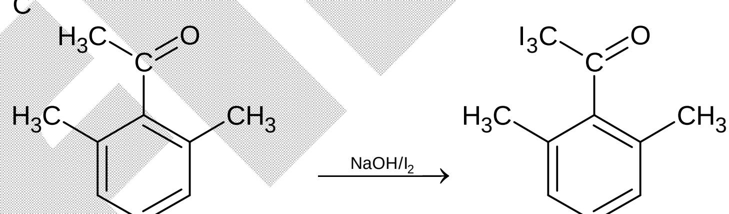
44. C

Sol.



45. C

Sol.



SECTION – B

46. 3

Sol. $Be_nAl_2Si_6O_{18}$

$Si_6O_{18}^{12-}$ is cyclic silicate

$2n + 6 - 12 = 0$

$n = 3$

47. 7

Sol. $W = -P_{ex}\Delta V$

$= -P_{ex}(V_2 - V_1)$

Hence $V_1 = 0$ (because of liquid)

$V_f = \dfrac{nRT}{P_{ext}}$

$W = -P_{ext}.\dfrac{nRT}{P_{ext}}$

$= -0.315 \times 0.083 \times 273$

$= -7.08$

= 7

48. 4

Sol. 2NO can replace 3CO.

49. 2

Sol. $M(OH)_x \rightleftharpoons M^{x+} + xOH^-$

$K_{SP} = S(xs)^x$

$4\times10^{-12} = x^x S^{x+1}$

$4\times10^{-12} = x^x\left(10^{-4}\right)^{x+1}$

$2^2\left(10^{-4}\right)^{2+1} = x^x\left(10^{-4}\right)^{x+1}$

x = 2

50. 3

Sol. 5 – 2
4 – 2
3 – 2

Mathematics

PART – C

SECTION – A

51. C

Sol. $\int_0^2 \left[\left|x^2-5x+4\right|\right]dx + \int_0^2 \left[\sin\frac{3\pi}{2}x\right]dx$

$I_1 = \int_0^1 \left[\left(x^2-5x+4\right)\right]dx + \int_1^2 \left[5x-x^2-4\right]dx$

$I_1 = \int_0^{\frac{5-\sqrt{21}}{2}} 3 + \int_{\frac{5-\sqrt{21}}{2}}^{\frac{5-\sqrt{17}}{2}} 2 + \int_{\frac{5-\sqrt{17}}{2}}^{\frac{5-\sqrt{13}}{2}} 1 + \int_{\frac{5-\sqrt{5}}{2}}^{2} 1dx$

$= \dfrac{14-\sqrt{17}-\sqrt{21}+\sqrt{5}-\sqrt{13}}{2}$

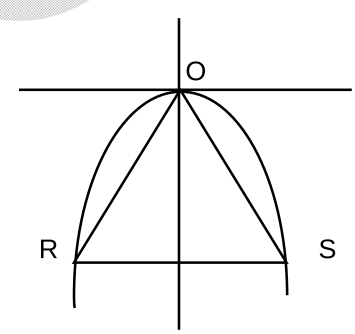
$I_2 = -\dfrac{2}{3}$.

52. A

Sol. Equation of normal to the parabola $x^2 = -4ay$ is

$x = my + 6m + 3m^3$.

Put $x = 0$, $y = -k \Rightarrow k = 6 + 3m^2 = 3(m^2 + 2)$



$x^2 = -12 \times \dfrac{-k}{506} = \dfrac{36\left(m^2+2\right)}{506}$

$x = \pm 6\sqrt{\dfrac{m^2+2}{506}}$

$\Rightarrow \quad \Delta ORS = \dfrac{1}{2} \times 12\sqrt{\dfrac{m^2+2}{506}} \times \dfrac{3\left(m^2+2\right)}{506} = 144$

$\Rightarrow \quad \left(\dfrac{m^2+2}{506}\right)^{3/2} = 8 = 4^{3/2}$

$\Rightarrow \quad m^2 + 2 = 2024$

$\Rightarrow \quad m^2 = 2022$.

53. C

Sol. $S = 1 + \dfrac{x}{2\sqrt{5}} + \left(\dfrac{x}{\sqrt{5}}\right)^2 \dfrac{1}{6} + \left(\dfrac{x}{\sqrt{5}}\right)^3 \dfrac{1}{12} + \left(\dfrac{x}{\sqrt{5}}\right)^4 \dfrac{1}{20} + ...\infty$; $x = \sqrt{5}-\sqrt{3}$

$S = 1 + \dfrac{t}{2} + \dfrac{t^2}{6} + \dfrac{t^3}{12} + \dfrac{t^4}{20} + ...\infty$

$= 1 + t\left(\dfrac{1}{1}-\dfrac{1}{2}\right) + t^2\left(\dfrac{1}{2}-\dfrac{1}{3}\right) + t^3\left(\dfrac{1}{3}-\dfrac{1}{4}\right) + t^4\left(\dfrac{1}{4}-\dfrac{1}{5}\right) + ...\infty$

$= \left(1 + t + \dfrac{t^2}{2} + \dfrac{t^3}{3} + \dfrac{t^4}{4} + ...\right) - \dfrac{1}{t}\left(t + \dfrac{t^2}{2} + \dfrac{t^3}{3} + \dfrac{t^4}{4} + \dfrac{t^5}{5} + ...\infty\right) + 1$

$$= -\left(1-\frac{1}{t}\right)\log_e(1-t)+2$$
$$= 2+\left(\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)\log_e\left(1-\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}}\right)\right)$$
$$= 2+\frac{3+\sqrt{15}}{4}\log_e\frac{3}{5}.$$

54. B

Sol. $L_1 = \frac{x-\left(2\sqrt{\sin\theta}-3\right)}{4\sqrt{\sin\theta}} = \frac{y}{3\sqrt{3}} = \frac{z-\sqrt{\sin\theta}}{2\sqrt{\sin\theta}-3}$

$L_2 = \frac{x+2\sqrt{\cos\theta}-3}{-2\sqrt{\cos\theta}} = \frac{y}{\sqrt{3}} = \frac{z-\sqrt{\cos\theta}}{3-2\sqrt{\cos\theta}}$

If L_1 and L_2 are perpendicular then

$$-8\sqrt{\sin\theta}\sqrt{\cos\theta}+9-\left(2\sqrt{\sin\theta}-3\right)\left(2\sqrt{\cos\theta}-3\right)=0$$

$\Rightarrow \quad -12\sqrt{\sin\theta}\sqrt{\cos\theta}+6\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)=0$

$\Rightarrow \quad \left(\sqrt{\sin\theta}-\sqrt{\cos\theta}\right)^2=0$

$\Rightarrow \quad \tan\theta = 1$

$\theta = n\pi+\frac{\pi}{4}$; n is even

55. C

Sol. $P(A)=\frac{2}{5},\ P(B)=\frac{3}{5}$

$P(S)=\frac{1}{5}$

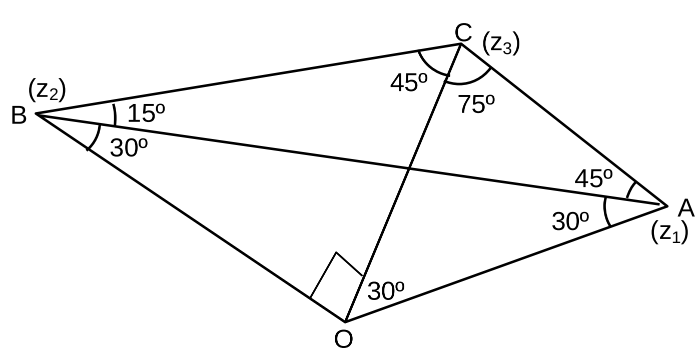
Also $5\times\frac{2}{5}x+\frac{3}{5}x=\frac{1}{5}\Rightarrow x=\frac{1}{13}$

$$\therefore \quad P\left(\frac{B}{D^C}\right)=\frac{P\left(B\cap D^C\right)}{P\left(D^C\right)}=\frac{\frac{3}{5}\times\frac{12}{13}}{\frac{80}{100}}=\frac{9}{13}.$$

56. C

Sol. $\angle ACB = \frac{2\pi}{3}$

Hence $\angle BDC = \frac{7\pi}{12}$.



57. C

Sol. $\frac{\pi}{2}-\cot^{-1}\cot\left(3-\frac{2}{x^2+|x|+1}\right)=\frac{\pi}{2}+\frac{1}{|x|+1}-3$

$\Rightarrow \quad \frac{2}{x^2+|x|+1}=\frac{1}{|x|+1}$

$\Rightarrow \quad x^2-|x|-1=0$

$\Rightarrow \quad |x|=\frac{1+\sqrt{5}}{2}\Rightarrow x=\pm\left(\frac{1+\sqrt{5}}{2}\right)$

$2\sin\theta=\frac{1+\sqrt{5}}{2}\Rightarrow \theta=\frac{3\pi}{10}.$

58. C

Sol. P(0, 6) lies on the line 3x + 4y – 24 = 0

Foot of perpendicular from P(0, 6) on the line

2x + y – 16 = 0 is Q(4, 8)

Also PA = PB = 10

And Q is mid point of AB

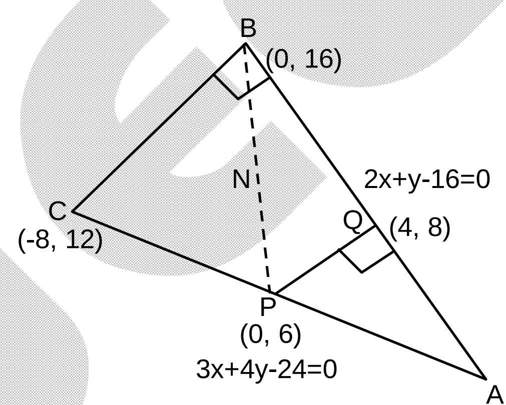
$\therefore$ BC parallel PQ (B is orthocentre)

$\therefore$ C is (–8, 12)

$r=\frac{\Delta}{S};\ \Delta=\frac{1}{2}\sqrt{80}\sqrt{320}=80$

$s=\frac{\sqrt{80}+\sqrt{320}+\sqrt{400}}{2}=\sqrt{80}\frac{\left(3+\sqrt{5}\right)}{2}.$

$r=6\sqrt{5}-10$



59. C

Sol. $f(x)=\frac{8x^5}{7\left(1+x^4\right)}$

$f(x)=\frac{4}{7}$ at $x=1$

Required area $A=\frac{4}{7}-\int_0^1\frac{8x^5}{7\left(1+x^4\right)}dx=\frac{\pi}{7}.$

60. D

Sol. Sum of 5 digits = 33 – (m + n)

(i) m + n = 3 (1, 2)
Number = 4!

(ii) m + n = 6(1, 5) number = 2 . 4!

(iii) m + n = 9(2, 7, & 3, 6) number 4! + 4!

(iv) m + n = 12(3, 9) and (5, 7) numbers = 4.4!

(v) m + n = 15(6, 9), number = 4!
Total numbers = 10 . 4!

61. C

Sol. $L_1=\lim_{x\to 0}\frac{\sin 2x}{\left(2\cos 4x\sin 3x+2\cos 4x\sin x-4\sin^2 2x\right)\times 1}$

$$= \lim_{x\to 0}\frac{\sin 2x}{2\cos 4x\cdot 2\sin 2x\cos x - 4\sin^2 2x} = \frac{1}{4}$$

$$L_2 = \lim_{x\to 0^+}\frac{e\left(e^{\sec\tan x^{1012n}-1}-1\right)}{\sec\left(\tan^{1012n}x\right)-1}\frac{\sec\left(\tan x^{1012n}\right)-1}{\sin x^{2025m}}$$

$$= e\lim_{x\to 0^+}\left(\frac{1-\cos\left(\tan x^{1012n}\right)}{\tan^2 x^{1012n}}\right)\left(\frac{\tan x^{1012n}}{x^{1012n}}\right)^2\frac{x^{2024n-2025m}}{\frac{\sin x^{2025m}}{x^{2025m}}}$$

$$= \frac{e}{2}\lim_{x\to 0^+}x^{2024n-2025m} = \frac{e}{2}$$

$\Rightarrow$ 2024n – 2025m = 0 $\Rightarrow \frac{n}{m} = \frac{2025}{2024}$

62. C

Sol. For common tangents

$25m^2 + 16 = r^2(1 + m^2)$

$\Rightarrow \quad r^2 = \frac{25m^2+16}{1+m^2}$

$$\Delta OPQ = \frac{1}{2}\left|\frac{r^2\left(1+m^2\right)}{m}\right| = \left|\frac{25m^2+16}{2m}\right| = \left|\frac{25m+\frac{16}{m}}{2}\right|$$

$$\left|25m+\frac{16}{m}\right| \ge 40$$

Minimum value occurs if $25m = \frac{16}{m} \Rightarrow m = \frac{4}{5}$

63. A

Sol. a = 119, b = 1728

$$I = \int_{\sqrt{119}}^{12}\frac{x\cos x^2}{\cos x^2+\cos\left(263-x^2\right)}dx = \frac{1}{2}\int_{119}^{144}\frac{\cos t\,dt}{\cos t+\cos(263-t)} = \frac{25}{4}$$

64. B

Sol. Equation of chord bisected at $(10t, 5t^2)$ of the ellipse is $\frac{x10t}{2000}+\frac{y5t^2}{500} = \frac{(10t)^2}{2000}+\frac{\left(5t^2\right)^2}{500}$

It passes through P(0, a)

$\Rightarrow \quad (a - 5) = 5t^2$; $t^2 = 0$

$a > 5$ and $\frac{(10t)^2}{2000}+\frac{\left(5t^2\right)^2}{500}-1<0$

$\Rightarrow \quad 0 < t^2 < 4$

$\therefore \quad 0 < \frac{a-5}{5} < 4 \Rightarrow 5 < a < 25$

$\Rightarrow \quad x + y = 20$

65. B

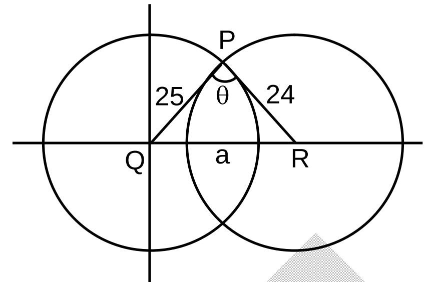
Sol. $\frac{1}{2}\times 25\times 24\times\frac{12}{25}=\frac{1}{2}a\frac{b}{2}$

$\Rightarrow$ ab = 576

Also $L^2 = a^2 - (r_1 - r_2)^2$

$\Rightarrow a^2 = 1296 \;\Rightarrow a = 36$

$\therefore\; b = \frac{576}{36} = 16$



66. A

Sol. $\because$ P AdjP = |P| I

$\therefore -2 + 2\lambda = 0 \Rightarrow \lambda = 1$

and $-7 - \mu + 6 = 0 \;\Rightarrow \mu = -1$

Also $(\text{adj } P)^{-1} + 14\text{adj}(P^{-1}) = 15\text{adj}(P^{-1})$

$= |P^{-1}|\,(P^{-1})^{-1} = 15\frac{P}{|P|} = -P$

67. B

Sol. $f(x) = \sec^2 x$

$g(x) = |\tan x| + |\cot x|$

m = 1, M = 2

$h(x) = x^2 - 3x + 2$

68. C

Sol. $I = \int\frac{(1-\log_e t)t^2}{(\log_e t)^4 - t^4}dt$; t = tanθ

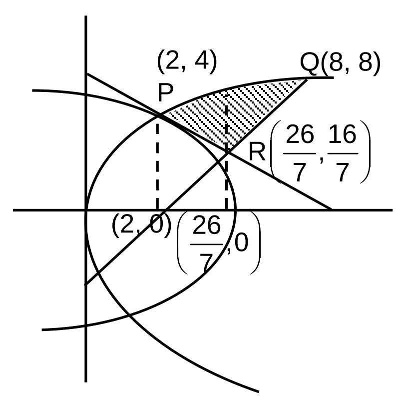
$= \int\frac{\frac{1-\log_e t}{t^2}}{\left(\frac{\log_e t}{t}\right)^4 - 1}dt$; $\frac{\ln t}{t} = u$

$= \int\frac{du}{(u^2-1)(u^2+1)} = \frac{1}{4}\ln\left|\left(\frac{\ln|\tan\theta| - \tan\theta}{\ln|\tan\theta| + \tan\theta}\right)\right| - \frac{1}{2}\tan^{-1}\left(\frac{\ln|\tan\theta|}{\tan\theta}\right) + c$

69. C

Sol. $A = \int_2^8 2\sqrt{2}\sqrt{x}\,dx - \left[\frac{1}{2}\left(4+\frac{16}{7}\right)\frac{12}{7} + \frac{1}{2}\left(8+\frac{16}{7}\right)\frac{30}{7}\right]$

$\frac{112}{3} - \frac{192}{7} = \frac{208}{21}$



70. C

Sol. n(A) = 122

$n((A \cap (B - C)) = 15$

sum of elements = 14 + 19 + 29 + 34 + 44 + 49 + ….. 104 + 109 + 119

= $\frac{8}{2}(14+119)+\frac{7}{2}(19+109)=980$

N = $2^2 . 5 . 7^2$

$\therefore$ Number of divisors = 18

SECTION – B

71. 1045

Sol. $h'(g(g(x)).g'(g(x)).g'(x) = 1$; $g(x) = f^{-1}(x)$

$h'(-1) \Rightarrow g(g(x)) = -1$

$\Rightarrow g(x) = f(-1) = 2$

$\Rightarrow x = f(2) = 53$

$\therefore h'(-1) = \frac{1}{g'(2)g'(53)}$

$g'(2)=\frac{1}{f'(-1)}$ and $g'(53)=\frac{1}{f'(2)}$

$\Rightarrow h'(-1) = 11 \times 95 = 1045$

72. 1

Sol. $XY = 66^{2025} = (62 + 4)^{2025} = 31\lambda + 4^{2025}$

$(4^3)^{675} = 31\mu + 2^{675}$

$2^{675} = (1 + 31)^{135} = 31k + 1$

73. 940

Sol. $\alpha + \frac{2}{\alpha} = 10$

$\Rightarrow \alpha^3 + \frac{8}{\alpha^3} = 940$

$$\frac{\alpha^{2025}\left(\alpha^3+\frac{8}{\alpha^3}\right)+\beta^{2025}\left(\beta^3+\frac{8}{\beta^3}\right)}{\alpha^{2025}+\beta^{2025}} = 940$$

74. 1

Sol. $\frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} = \frac{15c+25d+900}{24+c+d} = 31$ and $30+\frac{\frac{24+c+d}{2}-(c+d+3)}{11}\times 10 = \frac{340}{11}$

$\Rightarrow$ c = 6, d = 10.

75. 392

Sol. $(\vec{r}+\vec{a}-\vec{c})\times\vec{b}=0$

$\Rightarrow \vec{r}+\vec{a}-\vec{c}=\lambda\vec{b}$

$\because \vec{r}\cdot(\vec{a}+\vec{c})=0 \quad \therefore \lambda=\frac{|\vec{a}|^2-|\vec{c}|^2}{\vec{b}\cdot(\vec{a}+\vec{c})}=36$

$\therefore \vec{r}-36\vec{b}=\vec{c}-\vec{a}=2\hat{i}-8\hat{j}-18\hat{k}$