PART-1 : PHYSICS

SECTION-I

1) The expression for the angular momentum of a point mass about origin is given by
$\vec{L} = 2\alpha t \ell n\left(\frac{t}{t+1}\right)\hat{k}$ where α is a certain constant and 't' is time in seconds.

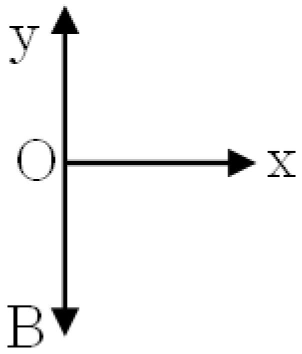
(A) The torque acting on the mass as a function of time is $2\alpha\left[\frac{1}{t+1} + \ell n\left(\frac{t}{t+1}\right)\right]\hat{k}$.

(B) The mass can pass through the point (0, 0, 4).

(C) If the position of the particle at t = 1s is (0, 2, 0), the x component of its momentum at t = 1 s is $p_x = \alpha \ell n(2)$.

(D) If the position of the particle at t = 1s is (0, 2, 0), the z component of its momentum at t = 1 s is $p_z = 0$.

2) A plane electromagnetic wave of the radiation field from a distant wire antenna travels toward the plane of the paper (which is in the -z direction). At time t = 0 s, the wave strikes the paper at normal incidence. The magnetic field vector at point O in the figure points in the -y direction and has a magnitude of 4.0×10^{-8} T at that moment. This magnitude is maximum. The frequency of the wave is 1.0×10^{16} Hz.



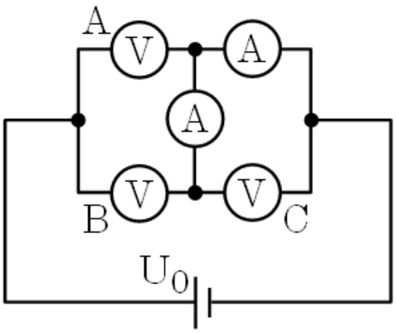
(A) The associated electric field E at time t = 0 s at the point O is $12\hat{i}$ V/m.

(B) The associated electric field E at time t = 2.5×10^{-17} s at z = 0.75×10^{-8} m is $6\sqrt{2}\hat{i}$ V/m.

(C) The associated electric field E at time t = 5×10^{-17} s at z = 1.5×10^{-8} m is $-6\sqrt{2}\hat{i}$ V/m.

(D) The associated electric field E at time t = 7.5×10^{-17} s at z = 2.25×10^{-8} m is $-12\hat{i}$ V/m.

3) To an ideal battery with EMF 1.3 V we connected a bridge circuit assembled from three identical voltmeters and two identical milliammeters (see figure). It is known that the reading of one of the milliammeters is three times the other. Resistance of voltmeters are more than milliammeters.



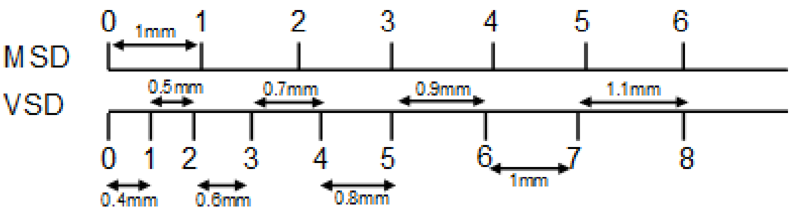
(A) Reading of Voltmeter A is 1.0 V.

(B) Reading of Voltmeter B is approx 0.62 V.

(C) Reading of Voltmeter C is 0.4 V.

(D) Reading of Voltmeter C is approx 0.68 V.

4) Consider a specially designed vernier calliper in which Vernier divisions are not equi spaced and main scale divisions are equispaced. 1 main scale division is equal to 1mm. Vernier scale has 8 divisions. When zero of the vernier scale is matched with zero of main scale, these 8 divisions of vernier scale coincide with 6 division of main scale division as shown.



When nothing is put between the jaws, zero of the main scale lies between zero and 1 division of Vernier scale and 4^{th} Vernier scale division exactly coincide with one of the main scale division. To measure the diameter of a cylindrical wire it is put between the jaws. Zero of vernier scale lies between 4^{th}& 5^{th} of main scale division and 6^{th} division of Vernier scale division coincide with one of the main scale division. Diameter of the wire is:

(A) Zero error is 0.2mm

(B) Zero error is –0.2mm

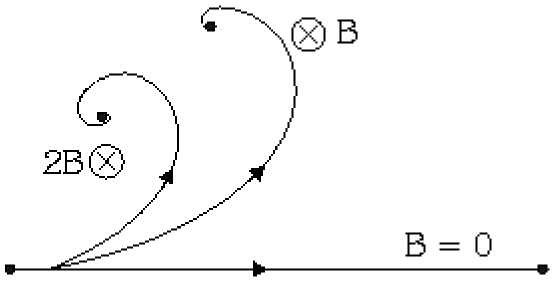
(C) Diameter is 4.3 mm

(D) Diameter is 4.1 mm

SECTION-II (i)

Common Content for Question No. 1 to 2

A charged particle enters a region in which there is a frictional force proportional to the particle's velocity $(-k\vec{v})$, and the particle stops 10 cm from its entry point. If the particle repeats its motion when a homogeneous magnetic field of magnitude B, perpendicular to the plane of its trajectory, is also present, then the particle comes to rest at a displacement of 6 cm from its entry point (see figure).
(velocity with which particle enters the region is to be taken same in all cases)

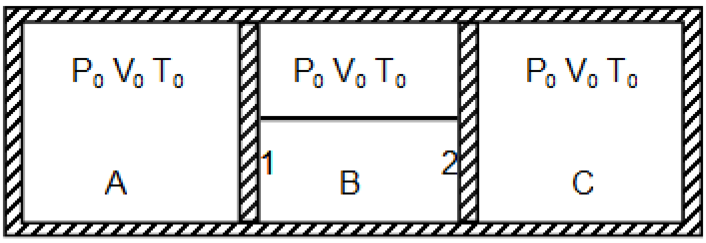


1) If the magnetic field were twice as large the particle would stop at a displacement of $\frac{30}{\sqrt{N}}$ cm from entry point, where N is:

2) The magnitude of the magnetic field for which the particle would stop at a displacement of $5\sqrt{2}$ cm from entry point is $\frac{nB}{4}$, where n is:

Common Content for Question No. 3 to 4

Consider an adiabatic chamber, divided in three equal parts as shown. Pistons are massless and can slide freely inside the chamber, without any leakage of gas. Area of cross section of each piston is A. Piston-1 is conducting and piston-2 is adiabatic. Piston 1 and 2 are connected by a rigid massless rod whose thermal coefficient of linear expansion and heat capacity is negligible. Each part contains same gas, each at temperature T_0, pressure P_0 and volume V_0. Adiabatic exponent of gas is $\gamma = 2$. Now using an electrical arrangement gas in part B is heated slowly so that its temperature becomes $2T_0$.

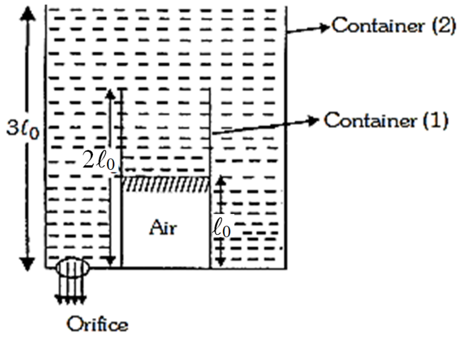


3) Final temperature of gas in chamber A is nT_0 where n is:

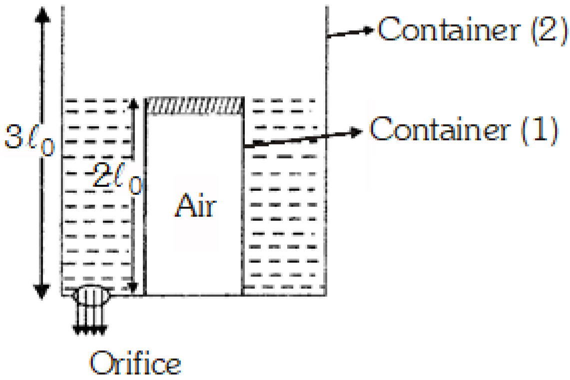
4) Final pressure of gas in chamber A is $P = \left(\frac{N+\sqrt{17}}{8}\right)P_0$, where N is :-

Common Content for Question No. 5 to 6

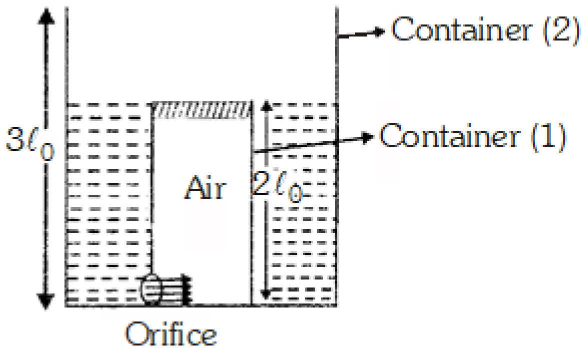
A small container (1) of height $2\ell_0$ and cross-section area 'A' is kept in a big container (2) of height $3\ell_0$ and cross-section area 2A. The bottom of container (1) is welded with the bottom of container (2). A massless piston is put on the container (1). The container (2) is completely filled with water and initially the length of air column in container (1) is ℓ_0. Now a small orifice is made at the bottom of the container (2). Assume the temperature of water and air remains constant in all the following processes.



5) When the height of water level in container (2) becomes $2\ell_0$, the piston reaches the topmost portion in container (1). The atmospheric pressure is $N\rho_w g\ell_0$, where N is: (density of water is ρ_w)



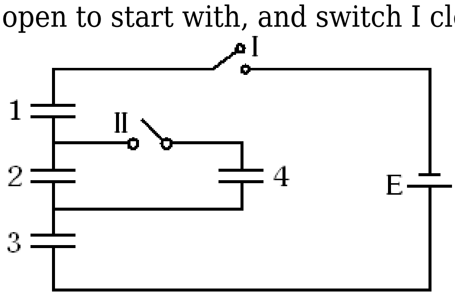
6) Now the piston is held (fixed) at that position. The orifice is closed and another orifice is made at the lowest position of container (1). The height up to which the water will rise in the container (1) at



equilibrium is $(N-\sqrt{M})\ell_0$ then (N + M) is :

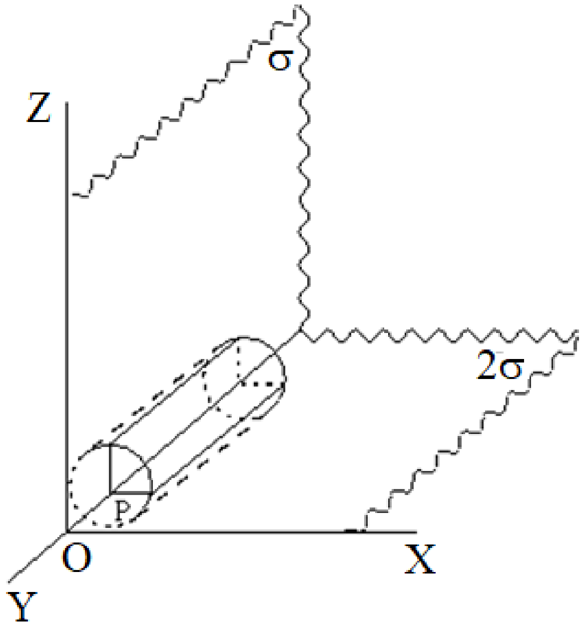
SECTION-II (ii)

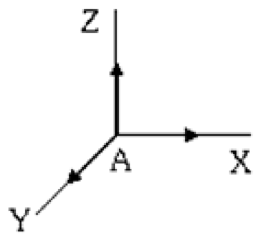
1) Four identical condensers are connected as shown in figure, and joined to a battery E. Switch II is open to start with, and switch I closed. Switch I is then opened and switch II is closed.



List-I		List-II	
(P)	ratio of energy stored in condenser ‘1’ before and after closing switch II	(1)	1 : 1
(Q)	ratio of energy stored in condenser ‘2’ before and after closing switch II	(2)	1 : 4
(R)	ratio of charge on condenser ‘2’ before and after closing switch II	(3)	4 : 1
(S)	ratio of potential difference across condenser ‘3’ before and after closing switch II	(4)	2 : 1

2) Two infinite perpendicular planes were charged uniformly with surface densities of the charges being σ and 2σ respectively. A long hollow cylinder of radius R, whose axis coincides with the line of intersection of the planes, is given a surface charge –3σ, as shown in figure.





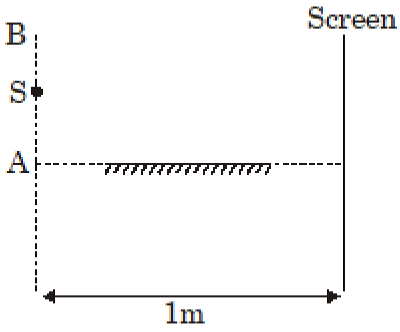
List-I		List-II	
(P)	x-component of electric field at a point (x,y,z) inside cylinder	(1)	$\frac{\sigma}{2\varepsilon_0}\left[1-\frac{6Rx}{(x^2+z^2)}\right]$
(Q)	z-component of electric field at a point (x,y,z) inside cylinder	(2)	$\frac{\sigma}{2\varepsilon_0}$
(R)	x-component of electric field at a point (x,y,z) outside cylinder	(3)	$\frac{\sigma}{2\varepsilon_0}\left[2-\frac{6Rz}{(x^2+z^2)}\right]$
(S)	z-component of electric field at a point (x,y,z) outside cylinder	(4)	$\frac{\sigma}{\varepsilon_0}$

SECTION-III

1) Very thin non-conducting rod of length L is uniformly charged along the length with total charge of Q. A small conducting ring of radius R is made of very thin wire, its center coincides with one of the ends of the rod, and the plane of the ring is perpendicular to the rod. Ring has uniform charge q. With what net force F does the rod act on the ring? If the tension in ring due to charge on rod is T,

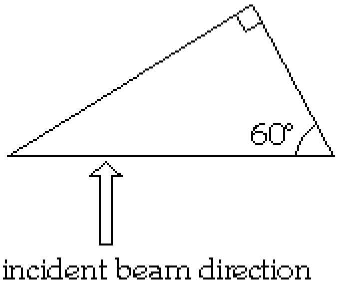
find F/T. Round off to nearest integer. (Take $L = \sqrt{3}R$)

2) In an interference experiment as shown in the figure, the source plane and screen are separated by a distance 1m. At a certain position of source, fringe width is $\frac{1}{4}$mm and by moving the source away from mirror along the line AB by 0.6 mm, the fringe width changed to $\frac{1}{6}$mm, then wavelength



of light used is (N × 1000) Å, where N is:

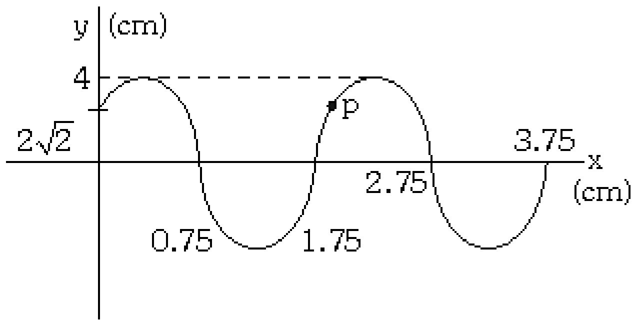
3) A narrow beam of light is incident on a 30° – 60° – 90° prism kept in air as shown figure. The refractive index of the prism is μ = 2.1. The total clockwise deviation of the beam is $\frac{2\pi}{N}$ radians,



where N (≠ 2) is:

4) A snap shot, of a vibrating string at t = 0, is shown in the figure. A particle at point P is observed moving up with velocity $\left(20\sqrt{2}\pi\right)$ cm/s and the tangent at the point P makes an angle $\theta = \tan^{-1}\left(2\sqrt{2}\pi\right)$ with the positive x-axis.

The equation of the wave is $y = (4\text{cm})\sin\left[\left(2N\pi\,s^{-1}\right)t - \left(\frac{M\pi}{2}\text{cm}^{-1}\right)x + \frac{L\pi}{4}\right]$

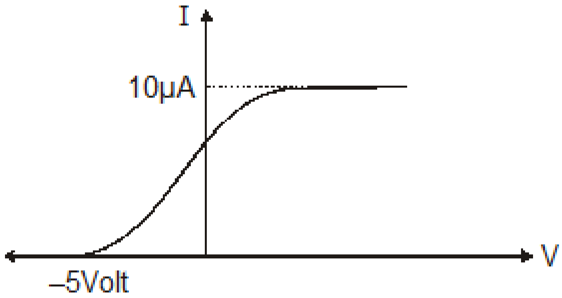


(L → smallest positive integer). Find (N + M + L).

5) At low temperatures, the molar heat capacity of solids at constant pressure depends on the absolute temperature T according to the law $C_T = C_{T_0}\left(\frac{T}{T_0}\right)^3$. One mole (40 g) of solid argon at

temperature 9 K is brought into thermal contact with 600 gm of solid argon at a temperature of 1 K in an insulated chamber. What will be the temperature (in K) of the system when thermal equilibrium is established? The heat capacity of chamber is negligible. Round off to the nearest integer if necessary

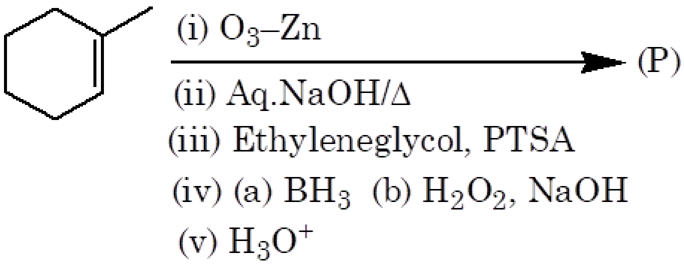
6) In the photoelectric experiment, if we use a monochromatic light, the I-V curve is as shown. The work function of the metal is 2eV (Assume percentage efficiency of photo emission = 10^{-3} %, i.e. number of photoelectrons emitted are (10^{-3}) % of number of photons incident on metal.) The power of light used in Watts is:



PART-2 : CHEMISTRY

SECTION-I

1) In the following chemical reaction sequence which is/are correct.



(A) ‘P’ gives yellow ppt. with I_2/OH^-.

(B) ‘P’ on treatment with $NaBH_4$ / EtOH will form a diol.

(C) If 1 mole of ‘P’ is treated with Sodium metal then 22.4 L of H_2(g) is released at 0°C, 1 atm.

(D) ‘P’ is an aldehyde.

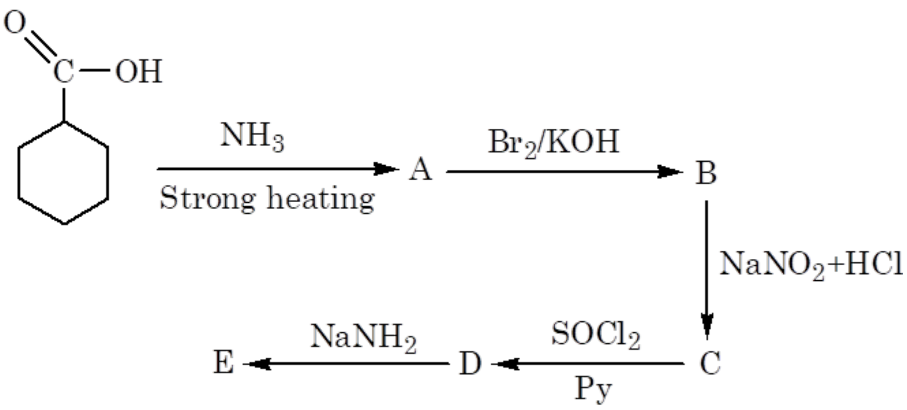
2) During the preparation of $K_2Cr_2O_7$, Chromite ore is fused with Na_2CO_3 in free excess of air. Then which is are correct?

(A) Chromite ore has iron in +2 oxidation state.

(B) Oxidation number of Cr in fusion product is +6.

(C) In the fusion product all ‘Cr–O’ bond length are equal.

(D) In the product Cr has no unpaired electron but still it is coloured.



3)
Which is/are correct about above reaction sequence?

(A) 'C' is a 2° alcohol.

(B) 'E' can't decolorize $Br_2 + H_2O$.

(C) 'C' to 'D' conversion is nucleophilic substitution reaction.

(D) Formation of 'B' from 'A' is dehydration reaction.

4) $\left(\Delta_f^0 G\right)_{Cr_2O_3(s)} = -527\ kJmol^{-1}$

$\left(\Delta_f^0 G\right)_{Al_2O_3(s)} = -827\ kJmol^{-1}$

$\left(\Delta_f H^0\right)_{Cr_2O_3(s)} = -1100\ kJ$

$\left(\Delta_f H^0\right)_{Al_2O_3(s)} = -1600\ kJ$

Which is/are correct?

(A) $Cr_2O_3(s)$ is reduced by Al(s) in given standard condition.

(B) $Al_2O_3(s)$ is reduced by Cr(s) in given standard condition.

(C) $\Delta_r H^\circ$ for the reduction of Cr_2O_3 by Al is –500 kJ.

(D) Reduction of Cr_2O_3 by Al is spontaneous at all temperature.

SECTION-II (i)

Common Content for Question No. 1 to 2

Sulphate salt of an element 'A' of 3-d series is (X). On heating 'X' produces mixture of two gases. Both the gaseous product contains S–atom. Addition of few drops of HNO_3 to the solution of 'X', followed by addition of KSCN produces blood red colour. Red colour disappears upon addition of excess of sodium oxalate, due to the formation of (Y). Then answer the following questions.

1) How many statement is/are true?

(i) Element 'A' on treatment with dilute HCl forms salt in +3 oxidation state and releases H_2(g).

(ii) Decomposition of Sulphate salt (X) produces an oxide in +3 oxidation state of element.

(iii) Gases are SO_2 and SO_3.

(iv) Element is diamagnetic.

(v) 'Y' is paramagnetic complex.

2) Find P + Q + R for compound (Y)

(P) Number of unpaired electron(s)
(Q) Number of stereoisomers
(R) Number of oxygen atoms

Common Content for Question No. 3 to 4

The vapour pressure of two miscible liquids A and B are 300 and 500 mm Hg respectively. In a flask, 10 mole of A is mixed with 12 mole of B. However, as soon as 'B' is added, 'A' starts polymerizing into a completely insoluble solid. The polymerisation follows first order kinetics. After 100 min, 1 mole of a non–volatile solute is dissolved, which arrests the polymerisation completely. The final vapour pressure of the solution is 400 mm of Hg.
Assume negligible volume change on mixing and polymerisation and ideal behaviour for the final solution. [log2 = 0.3, $\log_e 10 = 2.3$]

3) If rate constant for the first order is 'K' min^{-1}, then find the value of $\left(\frac{10^4 K}{2.3}\right)$ is:

4) If initially mixed A and B has non–ideal behaviour with negative deviation from Raoult's law and there is no polymerisation, then how many of the following are true:
(i) Attraction between A and B is more than individual attraction of A.....A and B.....B.
(ii) Potential energy of the system decreases on mixing.
(iii) $\Delta S_{System} > 0$
(iv) $\Delta S_{Surrounding} < 0$
(v) $\Delta G_{Mixing} < 0$

Common Content for Question No. 5 to 6
Amino acids are connected by peptide bonds. A decapeptide (Mol. wt. 796) on complete hydrolysis gives glycine (Mol. wt. 75), alanine and phenylalanine. Glycine contributes 39.14% to the total weight of the hydrolysed products. Then answer the following questions.

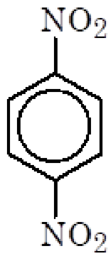
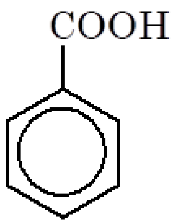
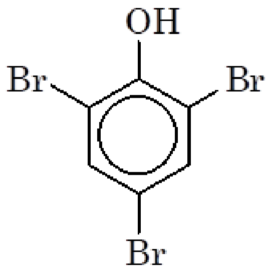
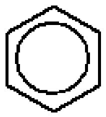
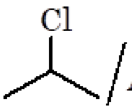
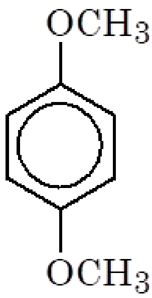
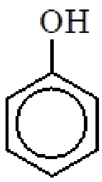
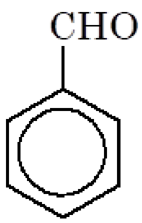
5) Find the total number of Glycine units present in the decapeptide.

6) This decapeptide consists of an octapeptide part with the following properties -
(i) All the glycine units are used in this octapeptide part & it is single specific sequence
(ii) This octapeptide chain is fixed with an ordered linkage
(iii) The remaining two amino acids apart from this octapeptide are alanine and phenylalanine.
Then how many such decapeptides are possible

SECTION-II (ii)

1) Match the Lists.

	List-I		**List-II**

(P)	 (i) Sn+HCl; (ii) $NaNO_2$+HCl; (iii) H_2O(warm); (iv) NaOH; (v) CH_3–I(excess)	(1)	
(Q)	Propyne (i) Red–hot, Fe-tube; (ii) $KMnO_4/H^+,\Delta$; (iii) NaOH/CaO,Δ; (iv) CO/HCl/$AlCl_3$	(2)	
(R)	 (i) /$AlCl_3$; (ii) O_2/H_3O^+; (iii) Br_2/H_2O	(3)	
(S)	 (i) Zn-dust/Δ; (ii) CH_3–C(=O)–Cl/$AlCl_3$; (iii) I_2/OH^-; (iv) H^+	(4)	

2) Match the Lists.

	List-I		List-II (pH)
(P)	10 ml of 0.1 M Na_2HPO_4 + 10 ml of 0.1 M HCl	(1)	$pH = pKa_1 + \log\frac{[salt]}{[acid]}$
(Q)	10 ml of 0.1 M Na_3PO_4 + 15 ml 0.2 M HCl	(2)	$pH = pKa_2 + \log\frac{[salt]}{[acid]}$
(R)	10 ml of 0.1 M H_3PO_4 + 15 ml 0.1 M NaOH	(3)	H_3PO_4 is the product
(S)	15 ml of 0.1 M Na_3PO_4 + 40 ml of 0.1 M HCl	(4)	$pH = \frac{1}{2}(pKa_2 + pKa_1)$

SECTION-III

1) To form a complete monolayer of acetic acid on 1g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unabsorbed. To neutralize the unabsorbed acetic acid, 40 mL

of 1 M NaOH solution was required. If each molecule of acetic acid occupies **P** × 10^{-21} m^2 surface area on charcoal, the value of $\left(\frac{P}{5}\right)$ is ______.
[**Use given data:** Surface area of charcoal = 1.5 × 10^2 m^2g^{-1}; Avogadro's number (N_A) = 6.0 × 10^{23} mol^{-1}]

2) Given:
$\lambda^0_{H^+} = 350\ Scm^2mol^{-1}$
$\lambda^0_{A^-} = 50\ Scm^2mol^{-1}$
Molar conductivity of 0.038 M 'HA' solution is 20 Scm^2mol^{-1} at 298 K. What is the pH of 0.01 M NaA salt solution. **(Express your answer to nearest integer)**.

3) An organic compound (A) having molecular formula $C_5H_{10}O$ responds +ve to Baeyers reagent as well as Lucas reagent. It shows geometrical isomerism as well as optical isomerism. (A) on heating with MnO_2 followed by treatment with Cl_2/NaOH forms (B) (organic product containing one carbon less than (A)). (B) when heated with NaOH/CaO forms organic compound (C). What is number of H–atoms in (C).

4) Find the number of combination(s) of reagents in which pink coloured solution or precipitate or product(s) is/are formed.
(i) $MnCl_2 + H_2S(g) + HCl$
(ii) $Mn^{2+} + S_2O_8^{2-}$
(iii) $Mn^{2+} + NaBiO_3 + HNO_3$
(iv) $Mn^{2+} + PbO_2 + HNO_3$
(v) $K_2MnO_4 + Cl_2$
(vi) $ZnCl_2 + H_2S + NH_4OH(aq.)$

5) Suppose de–Broglie equation is valid in Bohr model. In H–like sample, all electrons are in that orbit which has path length of the orbit equal to 3 times of de-Broglie wavelength of electrons in that orbit. This orbit of H–like sample has total energy equal to the ground state energy of H–atom. Now we have same H-like sample having all electrons in the ground state. If photons of energy 126.566 eV are given to this ground state sample and de-Broglie wavelength of the emitted electron is (P × 10^{-10}) metres then find nearest integer value of (P) ?

6) Consider the following reaction sequence.
$$[PtCl_4]^{2-} \xrightarrow[-2Cl^-]{+2Py} P \xrightarrow[-Cl^-]{+NH_3} Q \xrightarrow[-Py]{+NH_3} R$$
Find the sum of number of stereoisomers of complexes P, Q and R.

PART-3 : MATHEMATICS

SECTION-I

1) Which of the following is/are true :

(A) In any ΔABC, minimum value of $\sum \frac{\sqrt{\sin A}}{\sqrt{\sin B} + \sqrt{\sin C} - \sqrt{\sin A}}$ is 3

(B) If sec A tan B + tan A sec B = 30 then $\sec A \sec B + \tan A \tan B = \sqrt{901}$

(C) If sec A tan B + tan A sec B = 30 then $\sec A \sec B + \tan A \tan B = \sqrt{899}$

(D) Three lines y – z –1 = 0 = x, z +x +1 = 0 = y, x – z –1 = 0 =y intersect the xy plane at U, M, R then orthocenter of triangle UMR is (0, 1, 0)

2) If A and B are respectively a symmetric and skew symmetric matrix such that AB = BA then :

(A) $(A - B)^{-1}(A + B)$ is orthogonal matrix when A – B is non-singular

(B) $(A + B)^{-1}(A - B)$ is orthogonal matrix when A + B is non-singular

(C) $\det\left((A-B)^{-1}(A+B)\right) = 1$ and $\det\left((A+B)^{-1}(A-B)\right) = -1$

(D) $\det\left((A-B)^{-1}(A+B)\right) = -1$ and $\det\left((A+B)^{-1}(A-B)\right) = 1$

3) Let y = ƒ(x) be a differentiable function passing through M(1,–1) and has a property that the chord joining any two points $A(x_1,f(x_1))$ and $B(x_2, f(x_2))$ always intersects y-axis at $N(0,2x_1x_2)$. Then

(A) $\int_0^{1/2} f(x)dx = \frac{1}{12}$

(B) $\int_0^{1/2} f(x)dx = \frac{1}{24}$

(C) area bounded by ƒ(x) and x-axis is equal to $\frac{10}{3}$

(D) area bounded by ƒ(x) and x-axis is equal to $\frac{8}{3}$

4) If a,b,c,d ∈ R such that $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$, then equation $ax^3 + bx^2 + cx + d = 0$ has

(A) at least one root in (–1,0)

(B) at least one root in (–1,1)

(C) at least one root in (0,1)

(D) no root in (–1,1)

SECTION-II (i)

Common Content for Question No. 1 to 2

Let $f(x) = \lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x) = \lim_{n\to\infty}(1 - x + x\sqrt[n]{e})^n$ then consider the function y = h(x) and

h(x) = $\tan^{-1}(g^{-1}(f^{-1}(x)))$ then answer the following questions

1) $4\left|\lim_{x\to 0^+}\frac{\ell n(f(x))}{\ell n(g(x))}\right|$ is equal to :

2) Range of the function $y = h(x)$ is (a, b) then a + b is equal to :

Common Content for Question No. 3 to 4

Let $P_n(x)$ be a polynomial such that $P_n(x) = P_{n-1}(x-n),\ n \in N \cup \{0\}$ and $P_0(x) = x^{90} - x^{89} + x^{88} - x^{87} + ... + 1$

3) If $P_{20}(x) = P_0(x-\lambda)$, then λ equals

4) If the constant term in $P_{10}(x)$ is $\frac{1}{p}((55)^m - 1)$ where p and m ∈ N, then p + m is equal to :

Common Content for Question No. 5 to 6

If $I(m) = \int_0^{\pi}\ell n(1 - 2m\cos x + m^2)dx$, then

5) $\frac{I(81)}{I(3)}$ is -

6) I(3) – I(–3) is -

SECTION-II (ii)

1) Consider a triangle having vertices at the points $A(z_1) \equiv A\left(\frac{2}{\sqrt{3}}e^{i\pi/2}\right)$, $B(z_2) \equiv B\left(\frac{2}{\sqrt{3}}e^{-i\pi/6}\right)$ and $C(z_3) \equiv C\left(\frac{2}{\sqrt{3}}e^{-i(5\pi/6)}\right)$. A circle is inscribed in triangle ABC touching the side AB, BC and CA at $D(z_4)$, $E(z_5)$, $F(z_6)$ respectively, consider point P(z) on incircle of ΔABC then

	List-I		List -II
(P)	Value of $\left\|2\,\text{Re}(z_4\bar{z}_5 + z_5\bar{z}_6 + z_6\bar{z}_4)\right\|$ is	(1)	4
(Q)	$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta DEF}$ is equal to	(2)	1
(R)	$AP^2 + BP^2 + CP^2$ is equal to	(3)	5

(S)	$DP^2 + EP^2 + FP^2$ is equal to	(4)	2

2) Match the following List-I with List-II

$$A = \begin{bmatrix} -3 & -1 & 2 \\ 3 & 1 & -1 \\ 4 & 2 & 5 \end{bmatrix}, \ A\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ A\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

	List-I		**List-II**
(P)	Trace B is	(1)	256
(Q)	4 det(adj B) is	(2)	-10
(R)	Det(adj(adj (2B))) is	(3)	1
(S)	Sum of element of B is	(4)	-8

SECTION-III

1) Let E_1 and E_2 two ellipses. The area of ellipse E_2 is one-third the area of quadrilateral formed by tangents at the ends of latus rectum of ellipse $E_3(E_3 : 5x^2 + 9y^2 = 45)$. E_1, E_2 and E_3 are similar and concentric. E_1 is inscribed in E_2 in such a way that both E_1, E_2 touches each other and minor axis of E_2 coincides with major axis of E_1. If length of major axis of E_1 is equal to length of minor axis of E_2, then area of ellipse E_2 outside the ellipse E_1 is equal to :

2) f(x) is a fifth order polynomial in x with every root of f(x) = 0 is real and distinct. The number of real roots of $f''(x)\,f(x) - (f'(x))^2 = 0$ is :

3) In an organization number of women are μ times that of men. If α things are to be distributed among them then the probability that the number of things received by men are odd is $\left(\frac{1}{2} - \left(\frac{1}{2}\right)^{\alpha+1}\right)$. Then find the value of μ :

4) Let $\bar{a}$, $\bar{b}$, $\bar{c}$ be three vectors having magnitude 1, 2, 3 units respectively such that $\left|[\bar{a}\ \bar{b}\ \bar{c}]\right| = 6$. If $\bar{r}$ is unit vector coplanar with $\bar{b}$ and $\bar{c}$ and $\bar{r}.\bar{b} = 1$, then the value of $|(\bar{a} \times \bar{c}) \times \bar{r}|^2 + |(\bar{a} \times \bar{c}).\bar{r}|^2$ is equal to :

5) On the side AC of an acute angled triangle ABC, a point D is taken such that AD = 1, DC = 2 and BD is an altitude of ΔABC. A circle of radius 2 which passes through points A and D and touch a circle at the point D circumscribed about the ΔBDC. The area of ΔABC is $p\sqrt{q}$ (p → prime number

and q is natural number.) The value of $\frac{q-p}{2}$ is :

6) let $S = \sum_{r=1}^{99} \frac{2r^2 - 98r + 1}{(100-r)\,{}^{100}C_r}$ then the value of $\frac{[s]}{11}$ is (where [.] denotes greatest integer function) :

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4
A.	A,C,D	A,D	A,C	B,C

SECTION-II (i)

Q.	5	6	7	8	9	10
A.	73	3	2	9	2	4

SECTION-II (ii)

Q.	11	12
A.	1341	2413

SECTION-III

Q.	13	14	15	16	17	18
A.	4	6	6	4	5	7

PART-2 : CHEMISTRY

SECTION-I

Q.	19	20	21	22
A.	A,B	A,B,C,D	A,C	A,C

SECTION-II (i)

Q.	23	24	25	26	27	28
A.	3	19	10	4	5	6

SECTION-II (ii)

Q.	29	30
A.	3421	4321

SECTION-III

Q.	31	32	33	34	35	36
A.	5	8	6	4	6	6

PART-3 : MATHEMATICS

SECTION-I

Q.	37	38	39	40
A.	**A,B,D**	**A,B**	**B,C**	**B,C**

SECTION-II (i)

Q.	41	42	43	44	45	46
A.	**2**	**0**	**210**	**145**	**4**	**0**

SECTION-II (ii)

Q.	47	48
A.	**2134**	**4312**

SECTION-III

Q.	49	50	51	52	53	54
A.	**4**	**0**	**3**	**9**	**6**	**9**

SOLUTIONS

PART-1 : PHYSICS

1) $\vec{\tau} = \frac{d\vec{L}}{dt} = \left[2\alpha t\left(\frac{1}{t} - \frac{1}{t+1}\right) + 2\alpha \ell n\left(\frac{t}{t+1}\right)\right]\hat{k}$

$= 2\alpha\left[\frac{1}{t+1} + \ell n\left(\frac{t}{t+1}\right)\right]\hat{k}$

Particle can never be found at point (0, 0, 4) m

At t = 1s, $\vec{\tau} = 2\alpha\left[\frac{1}{2} - \ell n 2\right]\hat{k}$; $\vec{L} = -2\alpha\ell n(2)\hat{k}$

$\vec{r} \times \vec{p} = -2\alpha\,\ell n(2)\hat{k}$

$2\hat{j} \times (\vec{p}) = -2\alpha\ell n(2)\hat{k}$

$2\hat{j} \times (p_x\hat{i} + p_y\hat{j} + p_z\hat{k}) = -2\alpha\ell n(2)\hat{k}$

$p_z = 0,\ \ p_x = \alpha\ell n(2)$

2) $\vec{B} = -4 \times 10^{-8}\hat{j}\cos(kz + \omega t)$

$\vec{E} = c^2\left(\frac{\vec{B} \times \vec{k}}{\omega}\right)$

$= \frac{9 \times 10^{16}}{2\pi \times 10^{16}} \times \left(4 \times 10^{-8}\hat{j} \times \frac{2\pi}{3 \times 10^{-8}}\hat{k}\right)$

$= 12\hat{i}\cos\left(\frac{2\pi}{3 \times 10^{-8}}z + 2\pi \times 10^{16}t\right)$

(B) $\vec{E} = 12\hat{i}\cos\left(\frac{2\pi}{3 \times 10^{-8}} \times \frac{3}{4} \times 10^{-8} + 2\pi \times 10^{16} \times \frac{5}{2} \times 10^{-17}\right)$

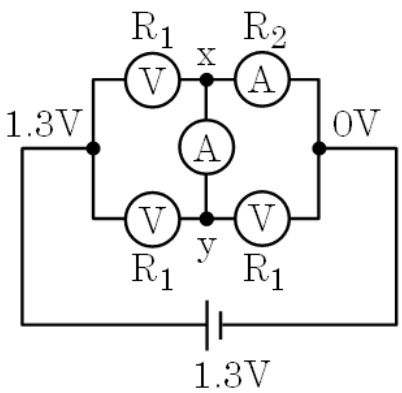
= 0

(C) $\vec{E} = 12\hat{i}\cos\left(\frac{2\pi}{3 \times 10^{-8}} \times \frac{3}{2} \times 10^{-8} + 2\pi \times 10^{16} \times 5 \times 10^{-17}\right)$

$= 12\hat{i}$

(D) $\vec{E} = 12\hat{i}\cos\left(\frac{2\pi}{3 \times 10^{-8}} \times \frac{9}{4} \times 10^{-8} + 2\pi \times 10^{16} \times 7.5 \times 10^{-17}\right)$

$= -12\hat{i}$



3)

y > x

$$\frac{y-x}{R_2}=\frac{x-0}{3R_2}$$

$$y=\frac{4x}{3}$$

$$\frac{x}{R_2}+\frac{x-1.3}{R_1}-\frac{x/3}{R_2}=0$$

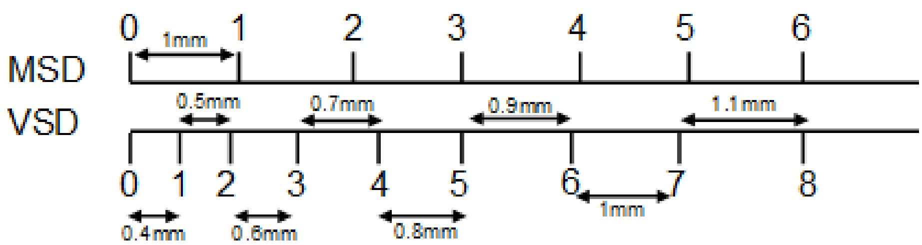
$$\frac{4x}{3R_1}+\frac{\frac{4x}{3}-1.3}{R_1}+\frac{x}{3R_2}=0$$

$$\Rightarrow \frac{2x}{3R_2}+\frac{x-1.3}{R_1}=0$$

$$-2\left(\frac{8x}{3R_1}-\frac{1.3}{R_1}+\frac{x}{3R_2}=0\right)$$

$$\Rightarrow \frac{1.3}{R_1}-\frac{13x}{3R_1}=0 \Rightarrow x=0.3$$

4)



1 VSD → 0.6 → 0.4

2 VSD → 0.1 → 0.9

3 VSD → 0.5 → 0.5

4 VSD → 0.8 → 0.2

5 VSD → 0 → 0

6 VSD → 0.1 → 0.9

7 VSD → 0.1 → 0.9

8 VSD → 0 → 0

Zero error = 0 – 0.2 mm = – 0.2 mm

Reading = 4 + 0.1 mm = + 4.1 mm

Diameter = (4.1 + 0.2) mm = 4.3 mm

5) $m\frac{d\vec{v}}{dt}=q(\vec{v}\times\vec{B})-k\vec{v}$

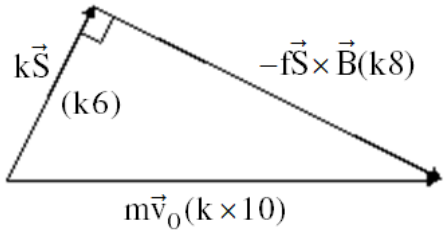
$$m\int_{\vec{v}_0}^{0} d\vec{v}=q\left(\int_{0}^{\vec{S}}(\vec{v}\,dt)\right)\times\vec{B}-k\int_{-0}^{\vec{S}}\vec{v}\,dt$$

$$m\vec{v}_0=-q\vec{S}\times\vec{B}+k\vec{S}$$

(a) when $\vec{B}=0 \rightarrow |\vec{S}|=10m \Rightarrow mv_0=k\times 10$

(b) when $|\vec{B}|=B \rightarrow |\vec{S}|=6m$

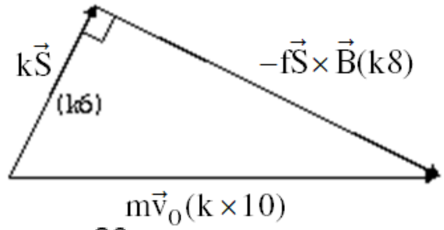
$qSB\sin 90^\circ=k\times 8$



$q\times6\times B = k\times8$

$qB = \frac{4}{3}k$

(c) $S^2 + \left(\frac{8}{3}S\right)^2 = 100$



$S = \frac{30}{\sqrt{73}}m$

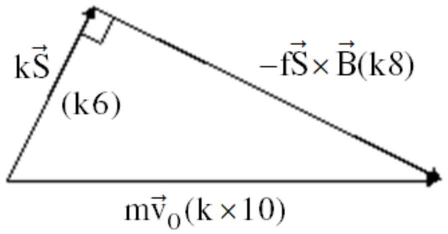
6) $m\frac{d\vec{v}}{dt} = q(\vec{v}\times\vec{B}) - k\vec{v}$ $\quad m\int_{\vec{v}_0}^{0} d\vec{v} = q\left(\int_{0}^{\vec{S}}(\vec{v}\,dt)\right)\times\vec{B} - k\int_{-0}^{\vec{S}}\vec{v}\,dt$

$m\vec{v}_0 = -q\vec{S}\times\vec{B} + k\vec{S}$

(a) when $\vec{B} = 0 \;\rightarrow\; |\vec{S}| = 10m \Rightarrow\; mv_0 = k\times10$

(b) when $|\vec{B}| = B \;\rightarrow\; |\vec{S}| = 6m$

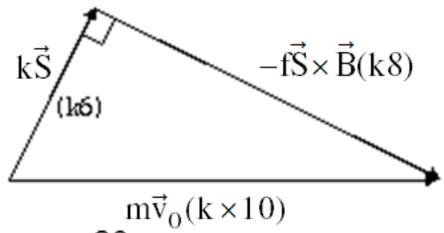
$qSB\sin90^\circ = k\times8$



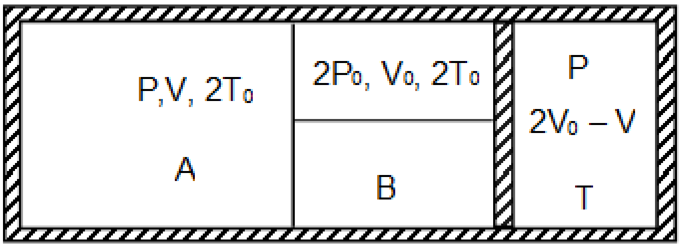
$q\times6\times B = k\times8$

$qB = \frac{4}{3}k$

(c) $S^2 + \left(\frac{8}{3}S\right)^2 = 100$



$S = \frac{30}{\sqrt{73}}m$

7) 

$\gamma = 2$

$\frac{1-\gamma}{\gamma} = \frac{-1}{2}$

$\frac{P_0V_0}{T_0} = \frac{PV}{2T_0} = \frac{2P_0V_0}{2T_0} = \frac{P(2V_0-V)}{T}$

$\frac{P_0V_0}{T_0} = \frac{2PV_0-PV}{T} = \frac{2V_0(P-P_0)}{T} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)}{T}$

In part C

$T = P^{\frac{1-\gamma}{\gamma}}$ = constant $\Rightarrow \frac{T}{\sqrt{P}}$ = constant $\Rightarrow \frac{T_0}{\sqrt{P_0}} = \frac{T}{\sqrt{P}}$

$T = \frac{T}{\sqrt{P_0}}\sqrt{P} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)}{\frac{T_0\sqrt{P}}{\sqrt{P_0}}} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)\sqrt{P_0}}{T_0\sqrt{P}}$

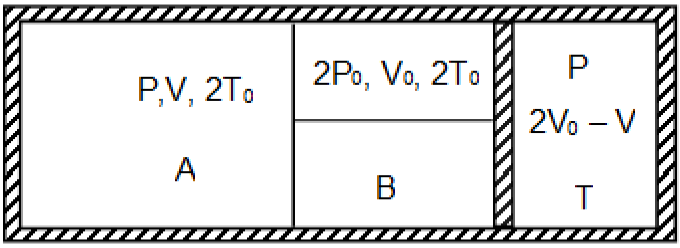
$\sqrt{P_0}\sqrt{P} = 2P - 2P_0$

Let $\sqrt{P} = x$

$2x^2 - \sqrt{P_0}x - 2P_0 = 0$

$\sqrt{P} = x = \left(\frac{\sqrt{P_0} \pm \sqrt{P_0 + 16P_0}}{4}\right) = \left(\frac{\sqrt{17}+1}{4}\right)\sqrt{P_0}$

$P = \frac{(\sqrt{17}+1)^2}{16}P_0 = \left(\frac{18+2\sqrt{17}}{16}\right)P_0 \Rightarrow P = \left(\frac{9+\sqrt{17}}{8}\right)P_0$

8) 

$\gamma = 2$

$\frac{1-\gamma}{\gamma} = \frac{-1}{2}$

$\frac{P_0V_0}{T_0} = \frac{PV}{2T_0} = \frac{2P_0V_0}{2T_0} = \frac{P(2V_0-V)}{T}$

$\frac{P_0V_0}{T_0} = \frac{2PV_0-PV}{T} = \frac{2V_0(P-P_0)}{T} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)}{T}$

In part C

$T = P^{\frac{1-\gamma}{\gamma}}$ = constant $\Rightarrow \frac{T}{\sqrt{P}}$ = constant $\Rightarrow \frac{T_0}{\sqrt{P_0}} = \frac{T}{\sqrt{P}}$

$T = \frac{T}{\sqrt{P_0}}\sqrt{P} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)}{\frac{T_0\sqrt{P}}{\sqrt{P_0}}} \Rightarrow \frac{P_0}{T_0} = \frac{2(P-P_0)\sqrt{P_0}}{T_0\sqrt{P}}$

$\sqrt{P_0}\sqrt{P} = 2P - 2P_0$

Let $\sqrt{P} = x$

$$2x^2 - \sqrt{P_0}x - 2P_0 = 0$$
$$\sqrt{P} = x = \left(\frac{\sqrt{P_0} \pm \sqrt{P_0 + 16P_0}}{4}\right) = \left(\frac{\sqrt{17}+1}{4}\right)\sqrt{P_0}$$
$$P = \frac{(\sqrt{17}+1)^2}{16}P_0 = \left(\frac{18+2\sqrt{17}}{16}\right)P_0 \Rightarrow P = \left(\frac{9+\sqrt{17}}{8}\right)P_0$$

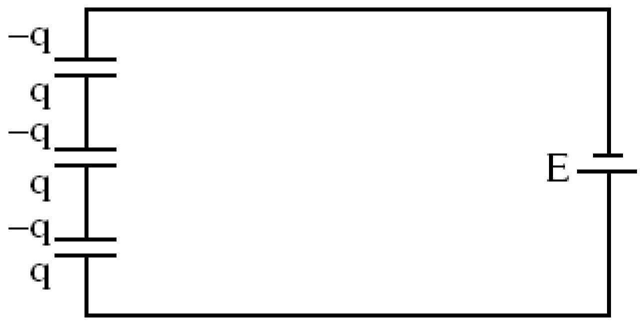
9)

$(p_0 + \rho g \times 2\ell_0) \times \ell_0 = p_0 \times 2\ell_0$
$p_0 = 2\rho g\ell_0$

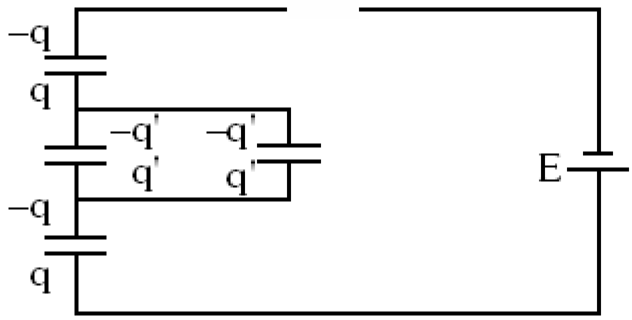
10)

$p_0 \times 2\ell_0 = (p_0 + \rho g(h - \ell))\,(2\ell_0 - \ell)$
$\ell = 2\ell_0 - h$
$h = 2\ell_0 - \ell$
$$4\rho g\ell_0^2 = (2\rho g\ell_0 + \rho g(2\ell_0 - 2\ell))\,(2\ell_0 - \ell)$$
$$4\ell_0^2 = 4\ell_0^2 - 2\ell\ell_0 + 4\ell_0^2 + 2\ell^2 - 4\ell\ell_0 - 2\ell\ell_0$$
$$2\ell^2 - 8\ell\ell_0 + 4\ell_0^2 = 0$$

$$q = \frac{CE}{3}$$



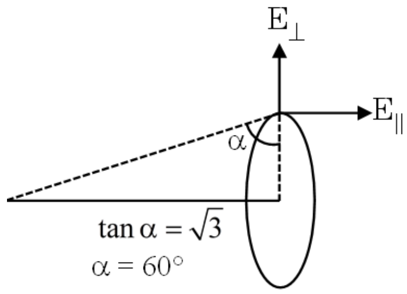
$$q = \frac{CE}{3}$$
$$q' = \frac{CE}{6}$$



11)

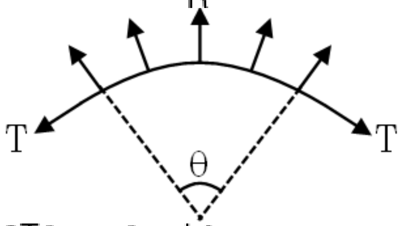
12)

Along the X-axis: $\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{6Rx}{(x^2+z^2)}\right]$
Along the Y-axis: 0.
Along the Z-axis: $\frac{\sigma}{2\varepsilon_0}\left[2 - \frac{6Rz}{(x^2+z^2)}\right]$



13)

$$F_{net} = \frac{qk\lambda(1-\cos\alpha)}{R}$$



$$\frac{2T\theta}{2} = \frac{q\theta}{2\pi} \times \frac{k\lambda}{R}\sin\alpha$$

$$\frac{F_{net}}{T} = \left(\frac{1-\cos\alpha}{\sin\alpha}\right) \times 2\pi$$

$$= \frac{1}{\sqrt{3}} \times 6.28 = 3.63$$

14)

Let AS = h

$$\frac{1}{4} = \frac{\lambda(1)}{2h} \qquad \text{...(i)}$$

$$\frac{1}{6} = \frac{\lambda(1)}{2(h+0.6)} \qquad \text{...(ii)}$$

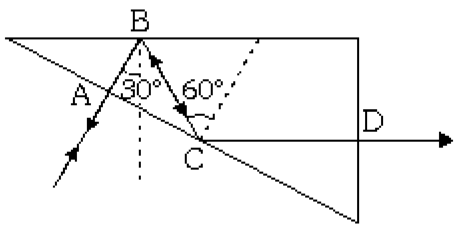
h = 1.2 mm

$\lambda = 0.6 \times 10^{-6}$ m = 6000 Å

15)

As seen from figure, for normal incidence at the bottom face the angle of incidence at B is 30°, and that at C is 60°, both of which are larger than the critical angle of the prism,

$$\theta_c = \sin^{-1}\frac{1}{n} = 28^\circ 26'$$



Hence the ray is totally reflected at B and C. Also, the ray is partially reflected back at the bottom and the right-hand faces. Therefore, the entire beam emerges either from the right-hand face, or back along the incident path.

16)

v_p = –slope × v_w

$20\sqrt{2}\pi cm/s = -2\sqrt{2}\pi\, v_\omega$

v_w = 10 cm/s in negative x-direction

λ = 2 cm;   f = 5 Hz

Y = (4 cm) sin (10πt + πx + π/4)

N = 5, M = -2, L = 1

17) nCdt + n'C'dT' = 0

$$1 \times \frac{C_0}{T_0^3}\int_9^T T^3 dT + \frac{15C_0}{T_0^3}\int_1^T T_0'^3 dT' = 0$$

$T^4 - 9^4 + 15(T^4 - 1^4) = 0$

$16T^4 = 6562$

$T^4 = 410.0625$

T = 4.5 K ≈ 5 K

18)

The energy of incident photons is given by

$h\nu = eV_s + f_0 = 2 + 5 = 7$ eV

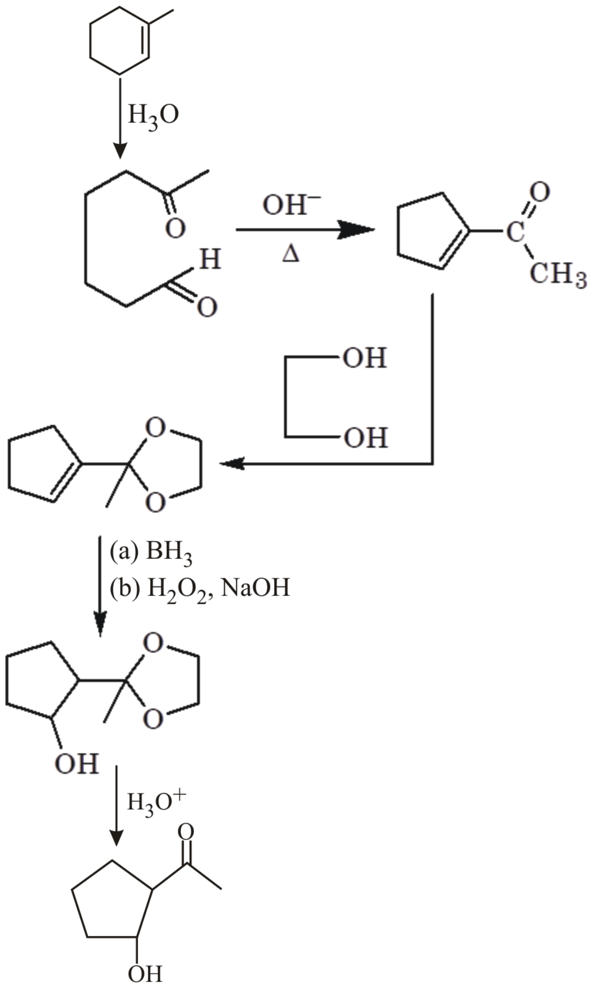
(Vs is stopping potential and ϕ_0 is work function)

Saturation current = 10^{-5} A = $\frac{\eta P}{h\nu}e = \frac{10^{-5}P}{7\times e}e$

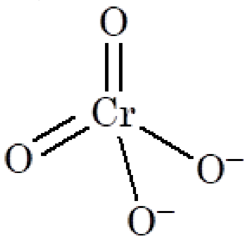
(h is photo emission efficiency)

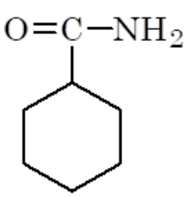
P = 7 W.

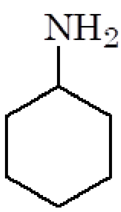
PART-2 : CHEMISTRY

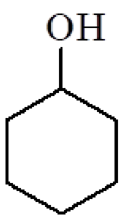
19) 

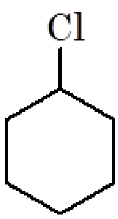
20) $\overset{+2}{Fe}O.\overset{+3}{Cr_2}O_3 \xrightarrow[\Delta]{Na_2CO_3 + O_2} Fe_2O_3 + Na_2CrO_4$

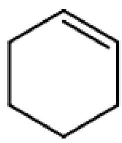


21) (A) 

(B) 

(C) 

(D) 

(E) 

22) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

$\Delta_rG^\circ = (-827 + 527) = -300$ kJ

$\Delta_rH^\circ = -1600 + 1100 = -500$ kJ

23) (ii), (iii) and (v) are correct statements.

24)

P = 5
Q = 2
R = 12

25)

Initial moles of A = 10
Let the number of moles of A when polymerization is arrested = n
Moles of B = 12
Moles of solute added = 1
Total moles = (n + 12 + 1) = (n + 13)

$$P = P_A^\circ X_A + P_B^\circ X_B$$

$$400 = 300 \times \frac{n}{n+13} + 500 \times \frac{12}{n+13}$$

400 n + 5200 = 300 n + 6000
100 n = 800
n = 8

For 1st order K = $\frac{2.303}{100} \log \frac{10}{8}$

$= \frac{2.303}{100} [1 - 3 \log 2]$

$= \frac{2.303 \times 0.1}{100}$

27) Total wt. of amino acid after addition of 9 mole of H2O = 796 + 9 × 18 = 958.
Now 39.14% is glycine, then total mass of glycine = 958 × 0.3914 = 375

Therefore number glycine units = $\frac{375}{75} = 5$

28) Octapeptide chain is fixed (8 amino acids involved in this can be taken as one unit). Since two more amino acids are present, hence 3! = 6 decapeptides are possible.

30) **[P-4, Q-3, R-2, S-1]**
(P) Final solution will be of NaH_2PO_4
1 mm of Na_2HPO_4 + 1 mm of HCl
 Amphoteric
Hence $pH = \frac{pKa_1 + pKa_2}{2}$

(R) 1 mm of H_3PO_4 & 1.5 mm of NaOH
 Final solution will have
NaH_2PO_4 + Na_2HPO_4
0.5 mm 0.5 mm
Handerson equation
Hence $pH = pK_2 + \log \frac{[salt]}{[acid]}$

(S) 1.5 mm Na_3PO_4 + 4 mm HCl

□ Final solution will have

NaH_2PO_4 + H_3PO_4

0.5 mm   1 mm

□ $pH = pK_1 + \log\frac{[salt]}{[acid]}$

31)

Millimole of acid taken = 100 × 0.5 = 50

Millimole of NaOH used = 40 × 1 = 40

Millimole of acid adsorbed = 50 – 40 = 10

Molecules of acid adsorbed = $10 \times 10^{-3} \times 6 \times 10^{23} = 6 \times 10^{21}$

Surface area occupied per molecule = $\frac{1.5 \times 10^2}{6 \times 10^{21}} = 0.25 \times 10^{-19} = 2500 \times 10^{-23}$

32) $\lambda^0_{HA} = 400$

$\lambda_{HA} = 20 \quad \alpha = \frac{1}{20}$

$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{C \times \frac{1}{400}}{1-\frac{1}{20}} = \frac{C}{19 \times 20}$

$= \frac{0.038}{19 \times 20} = \frac{38 \times 10^{-3}}{19 \times 20} = 10^{-4}$

$pH = \frac{1}{2}[14 + 4 - 2] = 8$

33) (A) $CH_3-CH=CH-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$ (B) $CH_3-CH=CH-\underset{\displaystyle O}{\underset{\|}{C}}-OH$ (C) $CH_3-CH=CH_2$

34) $MnCl_2 + H_2S \rightarrow$ No ppt.

Since MnS is obtained in basic medium.

35) $2\pi r = 3\lambda \Rightarrow \frac{2\pi r}{\lambda} = 3 = n$

T.E. in the orbit = –13.6 eV

$\Rightarrow -\frac{13.6Z^2}{3^2} = -13.6 \Rightarrow Z = 3$, Sample is Li^{2+}.

Ground state energy = –13.6 × 9 eV

Energy of ejected e^- = 126.566 – 122.4 = 4.166

$\lambda = \left(\frac{150}{4.16}\right)^{\frac{1}{2}} = 6Å$

36) P = $[Pt(Py)_2Cl_2]$ Q = $[Pt(Py)_2(NH_3)Cl]^+$

cis + trans   cis + trans

R = $[Pt(NH_3)_2(Py)Cl]^+$

cis + trans

PART-3 : MATHEMATICS

37) (A) $\sum \frac{\sqrt{\sin A}}{\sqrt{\sin B}+\sqrt{\sin C}-\sqrt{\sin A}}$ a = k sin A

$\sum \frac{\sqrt{a}}{\sqrt{b}+\sqrt{c}-\sqrt{a}}$ let $\sqrt{b}+\sqrt{c}-\sqrt{a}=x$

$\sqrt{c}+\sqrt{a}-\sqrt{b}=y$

$\sum \frac{y+z}{2x}$ $\sqrt{a}+\sqrt{b}-\sqrt{c}=z$

$\frac{1}{2}\sum \frac{y+z}{x}$ $\sqrt{a}+\sqrt{b}+\sqrt{c}=x+y+z$

$\frac{1}{2}\sum\left(\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{y}{z}+\frac{x}{z}\right)$

$\frac{1}{2}\left(\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{y}{z}+\frac{x}{z}\right)$

$\frac{1}{2}\left(\frac{y}{x}+\frac{x}{y}\right)+\left(\frac{x}{z}+\frac{z}{x}\right)+\left(\frac{y}{z}+\frac{z}{y}\right)$

$\frac{1}{2}(\geqslant(2+2+2) = \geqslant 3)$

(B,C) $(\sec A\sec B+\tan A\tan B)^2-(\sec A\tan B+\tan A\sec B)^2=1$

(D) U(0, 1, 0), M(–1, 0, 0), R(1, 0, 0) orthocenter is (0, 1, 0)

38) $A^T = A,\ B^T = B$

$\left((A-B)^{-1}(A+B)\right)\left((A-B)^{-1}(A+B)\right)^T$

$\left((A-B)^{-1}(A+B)\right)\left((A+B)^T(A-B)^{-1}\right)^T$

$(A^T+B^T)(A^T-B^T)^{-1}$

$(A-B)(A+B)^{-1}$

$(A-B)^{-1}(A-B)(A+B)(A+B)^{-1} = I$

 $(A-B)^{-1}(A+B)$ is orthogonal matrix

 $(A+B)^{-1}(A-B)$ is orthogonal

Det $(A-B)^{-1}(A+B)(A-B)^{-1}(A-B) = \det I = 1$

 Determinant of both will be 1 or –1

39) $f(x) = x - 2x^2$

$\therefore \int_0^{1/2} \left(x - 2x^2\right) dx = \frac{1}{24}$

and area = $\left| \int_0^{2} \left(x - 2x^2\right) dx \right| = \frac{10}{3}$

40) $\frac{a + 2c}{b + 3d} + \frac{4}{3} = 0$

$3a + 4b + 6c + 12d = 0$

$\int_0^1 (ax^3 + bx^2 + cx + d)dx = 0$

⇒ Option B and C are right

41) $= e^{\lim\limits_{n\to\infty} \left(\cos\sqrt{\frac{x}{n}} - 1\right) n}$

$= e^{\lim\limits_{n\to\infty} \left(-2\sin^2 \frac{1}{2}\sqrt{\frac{x}{n}}\right) n}$

$y = f(x) = e^{-x/2}$

$g(x) = e^{x \lim\limits_{n\to\infty} \left(\frac{e^{1/n} - 1}{1/n}\right)} = e^x$

$g^{-1}(x) = \ell n\, x, \quad f^{-1}(x) = 2\ell n \frac{1}{x}$

$g^{-1}(f^{-1}(x)) = \ell n\, f^{-1}(x) = \ell n \left(\ell n \frac{1}{x^2}\right)$

42) $= e^{\lim\limits_{n\to\infty} \left(\cos\sqrt{\frac{x}{n}} - 1\right) n} = e^{\lim\limits_{n\to\infty} \left(-2\sin^2 \frac{1}{2}\sqrt{\frac{x}{n}}\right) n}$

$y = f(x) = e^{-x/2}$

$g(x) = e^{x \lim\limits_{n\to\infty} \left(\frac{e^{1/n} - 1}{1/n}\right)} = e^x$

$g^{-1}(x) = \ell n\, x, \quad f^{-1}(x) = 2\ell n \frac{1}{x}$

$g^{-1}(f^{-1}(x)) = \ell n\, f^{-1}(x) = \ell n \left(\ell n \frac{1}{x^2}\right)$

43) $P_1(x) = P_0(x - 1)$

$P_2(x) = P_1(x - 2) = P_0(x - 3)$

$P_3(x) = P_2(x - 3) = P_0(x - 6)$

:

:

$P_{20}(x) = P_0(x - (1 + 2 + 3 \ldots + 20)$

$= P_0(x - 210)$

$P_{10}(x) = P_0(x - 55)$

For constant term put x = 0

$P_0(x) = (-55)^{90} - (-55)^{89} + \ldots\ldots + 1$

$= \frac{(55)^{91} - 1}{54}$, p+m = 145

44) $P_1(x) = P_0(x-1)$ $P_2(x) = P_1(x-2) = P_0(x-3)$

$P_3(x) = P_2(x-3) = P_0(x-6)$

:

:

$P_{20}(x) = P_0(x - (1 + 2 + 3 .... + 20)$

$= P_0(x - 210)$

$P_{10}(x) = P_0(x - 55)$

For constant term put x = 0

$P_0(x) = (-55)^{90} - (-55)^{89} + ... + 1$

$= \dfrac{(55)^{91} - 1}{54}$, p+m = 145

45) $I(m) = \int_0^{\pi} \ell n(1 - 2m\cos x + m^2)dx$ ...(1)

$I(-m) = \int_0^{\pi} \ell n(1 + 2m\cos x + m^2)dx$ ...(2)

$I(-m) = \int_0^{\pi} \ell n(1 + 2m\cos(\pi - x) + m^2)dx$

$I(-m) = \int_0^{\pi} \ell n(1 - 2m\cos x + m^2)dx$

I(m) = I(-m)

adding (1) and (2)

$I(m) + I(-m) = \int_0^{\pi} \ell n(1 - 2m^2\cos 2x + m^4)$

$I(m) + I(-m) = I(m^2)$ $I(9) = 2I_3$

$2I(m) = I(m^2)$ $I(81) = 2I_5$

$\dfrac{I(m^2)}{I(m)} = 2$

46) $I(m) = \int_0^{\pi} \ell n(1 - 2m\cos x + m^2)dx$ ...(1) $I(-m) = \int_0^{\pi} \ell n(1 + 2m\cos x + m^2)dx$ ...(2)

$I(-m) = \int_0^{\pi} \ell n(1 + 2m\cos(\pi - x) + m^2)dx$

$I(-m) = \int_0^{\pi} \ell n(1 - 2m\cos x + m^2)dx$

I(m) = I(-m)

adding (1) and (2)

$I(m) + I(-m) = \int_0^{\pi} \ell n(1 - 2m^2\cos 2x + m^4)$

$I(m) + I(-m) = I(m^2)$ $I(9) = 2I_3$

$2I(m) = I(m^2)$ $\qquad$ $I(81) = 2I_5$

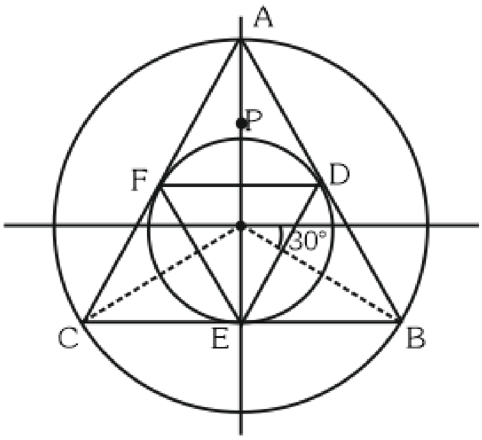
$\frac{I(m^2)}{I(m)} = 2$

47) Clearly point A, B, C lie on circle $|z| = \frac{2}{\sqrt{3}}$

ΔABC is equilateral Ar(ΔABC)

$= \frac{1}{2} \times \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \times 3 = \frac{1}{\sqrt{3}} \times 3 = \sqrt{3}$

Area $\Delta DEF = \frac{\sqrt{3}}{4}$



Let P(z) be any point on incircle

$AP^2 = |z - z_1|^2 = |z|^2 + |z_1|^2 - (\bar{z}z_1 + z_1\bar{z})$

$BP^2 = |z|^2 + |z_2|^2 - (\bar{z}z_2 + z_2\bar{z})$

$CP^2 = |z|^2 + |z_3|^2 - (\bar{z}z_3 + z_3\bar{z})$

$AP^2 + BP^2 + CP^2 = 5$

$DP^2 + EP^2 + FP^2$

$= |z - z_4|^2 + |z - z_5|^2 + |z - z_6|^2$

$= 3|z|^2 + |z_4|^2 + |z_5|^2 + |z_6|^2 - 2\,\mathrm{Re}(z(\overline{z_4 + z_5 + z_6})) = 2$

48)

AB = I

$B = A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 9 & -1 \\ -19 & -23 & 3 \\ 2 & 2 & 0 \end{bmatrix}$

$|\text{adj}(B)| = |B|^2 = \frac{1}{|A|^2} = \frac{1}{4}$

$|\text{adj}(\text{adj}(2B))| = |\text{adj}(2B)|^2 = \left((2B)^2\right)^2 = 2^{12}|B|^2 = 256$

49) $E_3 : \frac{x^2}{9} + \frac{y^2}{5} = 1 \Rightarrow e = \frac{2}{3}$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is 2x + 3y = 9
Area of quadrilateral = 27

Let $E_2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$E_1 : \frac{x^2}{b^2} + \frac{y^2}{c^2} = 1$

Required area = πb(a - c)

$b^2 = a^2(1 - e^2)$
$c^2 = b^2(1 - e^2)$
$c^2 = a^2(1 - e^2)^2$
$c = a(1 - e^2)$
$a - c = ae^2$

Area = $\pi abe^2 = 9 \times \frac{4}{9} = 4$

50) $f(x) = a(x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5)$

Take log and differentiable

$$\frac{f'}{f} = \frac{1}{x - x_1} + \frac{1}{x - x_2} + \frac{1}{x - x_3} + \frac{1}{x - x_4} + \frac{1}{x - x_5}$$

Differentiate again

$$\frac{ff'' - (f')^2}{(f)^2} = -\left(\frac{1}{(x - x_1)^2} + \frac{1}{(x - x_2)^2} + \ldots\ldots \frac{1}{(x - x_5)^2}\right) < 0$$

$\therefore ff'' - (f')^2 < 0$

51) Let p and q denote probability of things going to man and woman respectively Therefore

$p = \frac{1}{1 + \mu}$ and

$q = \frac{\mu}{1 + \mu}$ probability of men receiving r things is given by $P_r = {}^{\alpha}C_r \,.\, q^{\alpha - r} \,.p'$

So required probability is given by $P_1 + P_3 + P_5 + \ldots\ldots$

i.e. ${}^{\alpha}C_1 \,.\, q^{\alpha-1} p + {}^{\alpha}C_3 \,.\, q^{\alpha-3} p^3 + {}^{\alpha}C_5 \,.\, q^{\alpha-5} p^5 + \ldots\ldots$

$= \frac{1}{2}\left[(q + p)^{\alpha} - (q - p)^{\alpha}\right] = \frac{1}{2}\left[1 - \left(\frac{\mu - 1}{\mu + 1}\right)^{\alpha}\right]$

By comparison, we have $\left(\frac{\mu - 1}{\mu + 1}\right) = \frac{1}{2} \Rightarrow 2\mu - 2 = \mu + 1$ Thus μ = 3

52) $\left|[\bar{a}\,\bar{b}\,\bar{c}]\right| = 6$

$\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors $\bar{a} \,.\, \bar{r} = 0$

$$\left[\bar{b}\,\bar{c}\,\bar{r}\right] = 0 \Rightarrow \begin{vmatrix} \bar{b}.\bar{b} & \bar{b}.\bar{c} & \bar{b}.\bar{r} \\ \bar{c}.\bar{b} & \bar{c}.\bar{c} & \bar{c}.\bar{r} \\ \bar{r}.\bar{b} & \bar{r}.\bar{c} & \bar{r}.\bar{r} \end{vmatrix} = 0$$

$\bar{c} \,.\, \bar{r} = \frac{3\sqrt{3}}{2}$

$$[\vec{a}\,\vec{c}\,\vec{r}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{c} & \vec{a}.\vec{r} \\ \vec{c}.\vec{a} & \vec{c}.\vec{c} & \vec{c}.\vec{r} \\ \vec{r}.\vec{a} & \vec{r}.\vec{c} & \vec{r}.\vec{r} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 9 & \frac{3\sqrt{3}}{2} \\ 0 & \frac{3\sqrt{3}}{2} & 1 \end{vmatrix} = \frac{9}{4}$$

$$|(\bar{a} \times c) \times \bar{r}|^2 = ((\bar{a}.\bar{r})\bar{c} - (\bar{c}.\bar{r})\bar{a})^2$$

$$= \left|\frac{3\sqrt{3}}{2}\bar{a}\right|^2 = \frac{27}{4}$$

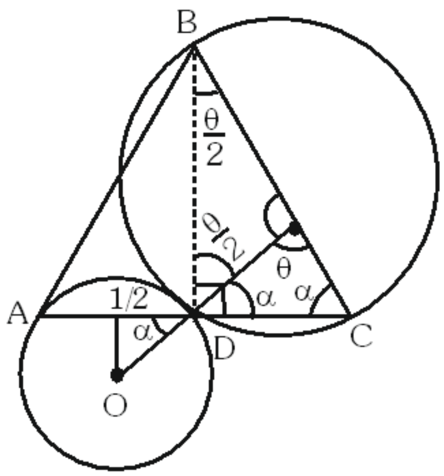
53) $\cos\alpha = \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$

$2\alpha + \theta = \pi$

BD = $2\cot\dfrac{\theta}{2}$

= $2\cot\left(\dfrac{\pi}{2} - \alpha\right)$

$= 2\tan\alpha = 2\sqrt{15}$



Area of ΔABC = $\dfrac{1}{2} \times 3 \times 2\sqrt{15} = 3\sqrt{15}$

P = 3, q = 15

54) $S = \sum \dfrac{(r+1)^2 - r(100-r)}{(100-r)\,{}^{100}C_r}$

$S = \sum \dfrac{r+1}{{}^{100}C_{r+1}} - \dfrac{r}{{}^{100}C_r}$

$= \dfrac{100}{{}^{100}C_{100}} - \dfrac{1}{{}^{100}C_1} = 100 - \dfrac{1}{100}$