Sri Chaitanya IIT Academy.,India.

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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.S60_Elite, Target & LIIT-BTs JEE-MAIN Date: 29-12-2024

Time: 09.00Am to 12.00Pm GTM-11/06 Max. Marks: 300

KEY SHEET

MATHEMATICS

1	**3**	2	**3**	3	**2**	4	**3**	5	**1**
6	**2**	7	**4**	8	**3**	9	**2**	10	**4**
11	**3**	12	**3**	13	**4**	14	**1**	15	**3**
16	**1**	17	**1**	18	**3**	19	**3**	20	**1**
21	**44**	22	**8**	23	**1**	24	**3**	25	**5**

PHYSICS

26	**3**	27	**4**	28	**1**	29	**3**	30	**4**
31	**2**	32	**2**	33	**4**	34	**4**	35	**1**
36	**3**	37	**4**	38	**1**	39	**2**	40	**4**
41	**4**	42	**1**	43	**4**	44	**1**	45	**3**
46	**4**	47	**3**	48	**4**	49	**25**	50	**900**

CHEMISTRY

51	**1**	52	**3**	53	**1**	54	**2**	55	**1**
56	**3**	57	**2**	58	**4**	59	**3**	60	**3**
61	**4**	62	**4**	63	**1**	64	**2**	65	**3**
66	**4**	67	**3**	68	**3**	69	**4**	70	**1**
71	**97**	72	**2**	73	**4**	74	**1**	75	**9**

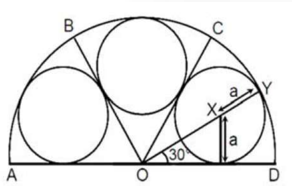
SOLUTION
MATHEMATICS

1. take *Let* $\sec x - \tan x = t$

$then\ (2\sec x\ \tan x)dx = \left(1 - \frac{1}{t^2}\right)dt$

$I = -\frac{1}{9t^9} + \frac{1}{11t^{11}} + c \Rightarrow \frac{1}{p} + \frac{1}{q} = 2$

2. From the diagram, $\angle AOB = \angle BOC = \angle COD = 60^0 \quad \Rightarrow \angle YOD = \frac{\pi}{6}$



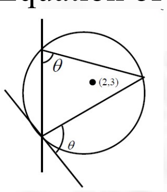
= Let X be the centre of right-hand circle, $OX \sin 30^0 = a$
Now $r = OY = 2a + a = a = r/3$

3. Given parabola is $(x-1)^2 + (y-3)^2 = \left(\frac{5x - 12y + 17}{13}\right)^2$

Focus $= (1,3)$, directrix is

$5x - 12y + 17 = 0$ $\therefore$ Length of latus rectum $= 2\left|\frac{5 - 36 + 17}{13}\right| = \frac{28}{13}$

4. Equation of tangent at origin is



$-2(x+0) - 3(y+0) = 0 \Rightarrow 2x + 3y = 0 \qquad \tan\theta = \frac{7}{4} \Rightarrow \left|\frac{m + \frac{2}{3}}{1 - \frac{2m}{3}}\right| = \frac{7}{4} \Rightarrow \frac{3m+2}{3-2m} = \frac{7}{4}$

$\Rightarrow 12m + 8 = 21 - 14m \Rightarrow 26m = 13 \Rightarrow m = \frac{1}{2} \qquad \therefore y = \frac{1}{2}x \Rightarrow x - 2y = 0$

5. $\Delta = 0, a = 5$

$\int_0^{-10} f(x)dx = \int_0^{-5} f(x)dx + \int_{-5}^{-10} f(x+5)dx + \int_{-85}^{-10} f(x)dxdx = 2\int_{-5}^{-10} dx = -10$

6. let $5^x = t; t > 0 \qquad A.m \geq G.m \qquad y^2 + 5y - (2+a) = 0; where\ y = t + \frac{1}{t} \geq 2$ Since $a \geq 12$

7. $Applying\ R_1 \to R_1 - R_2 \qquad f(x) = \begin{vmatrix} \cos x - \tan x & 0 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} = (\cos x - \tan x)(x^2 - 2x^2)$

$= -x^2(\cos x - \tan x) \quad \therefore f'(x) = -2x(\cos x - \tan x)$

$-x^2\left(-\sin x-\sec^2 x\right) \quad \therefore \lim_{x\to\infty}\frac{f'(x)}{x}=\lim_{x\to\infty}\left[-2\left(\cos x-\tan x\right)\right] \quad +\lim_{x\to\infty}x\left(\sin x\sec^2 x\right)] \quad =-2\times 1=-2$

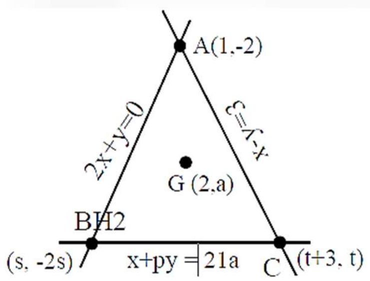
8. $$\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 2a & 2b & 2c\\ a^2 & b^2 & c^2\end{vmatrix}$$

9. $f^1(0)=\underset{h\to 0}{lt}\dfrac{f(0-h)-f(0)}{-h} \quad Rf^1(0)=\underset{h\to 0}{lt}\dfrac{f(0+h)-f(0)}{h}$

$=\underset{h\to 0}{lt}\left(\dfrac{h\left(3e^{1/h}+4\right)}{2-e^{1/h}}-0\right)\left(\dfrac{1}{h}\right) \quad \underset{n\to 0}{lt}\left(\dfrac{-h\left(3e^{-1/h}+4\right)}{2-e^{-1/h}}-0\right)\left(\dfrac{-1}{h}\right)=2 \quad =\underset{h\to 0}{lt}\left(\dfrac{3+4e^{-1/h}}{2e^{-1/h}-1}\right)=-3$

Since $Lf^1(0)\neq Rf^1(0)$ $\therefore f(x)$ is differentiable *at x =0*. But *f(x)* is continuous at *x=0*

10. 2x + y = 0 ------(1)
x – y = 0 –----(2)
x + py = 21a -------(3)
solving (1) & (2) $\Rightarrow$ A (1, -2)



centriod of triangle ABC is $\left(\dfrac{4+s+t}{3},\dfrac{-2-2s+t}{3}\right)=(2,a)$

$\Rightarrow s+t=2.......(4)\Rightarrow s=-a,t=2+a \;\; -2s+t=3a+2......(5)$

Solving (4) 795) we get

$B(-a,2a);C(a+5,a+2)\therefore$ Distance$(BC)^2=122$

11. Statement-1 General term$=\dfrac{10!}{\alpha!\beta!\gamma!}2^{\alpha/2}3^{\beta/3}5^{T/6}$ for rational terms

$\alpha=0,2,4,6,8,10, \quad \beta=0,3,6 \quad \gamma=0,6$

Hence possible sets= $(4,6,0),(4,0,6);(10,0,0)$

Hence, there are s rotational terms. $\therefore$ required $=\dfrac{10!}{4!6!}2^2 5\dfrac{10!}{10!}2^5=12632.$

Statement-3 $t_r+1,the(r+1)$ in the expansion of

$\left(5^{1/6}+2^{1/8}\right)^{10}$ *is* $t_r+1={}^{100}C_r\left(5^{1/6}\right)^{100-r}\left(2^{1/8}\right)^r$

As 5 and 2 are relatively prime, t_r+1 will be rational if $\frac{100-r}{6}$ and $\frac{r}{8}$ are both integers. i.e
If 100-r is a multiple of 6 and r is a multiple of 8. As $0 \le r \le 100$, multiple of 8 upto 100 and corresponding value of 100-r r=0,8,16,24,…….88,96
100-r=100,92,84, 76,…..12,4
Out of 100-r, multiple of 6 are 84,60,36,12 $\therefore$ There are just four rational terms
$\Rightarrow$ Number of irrational terms is 101-4=97

12. Let N be $(3\lambda+6, 2\lambda+7, 2\lambda+7)$ such PN is perpendicular to the line
Then $\lambda=-1 \quad \therefore N=(3,5,9) \qquad \therefore PN=7$

13. Since each has equally 9 different possible results for A and B to draw a ball from the packet independently, the total number of possible events is $9^2=81$. *From* $a-2b+10>0$ we get $2b<a+10$. We find that when $b=1,2,3,4,5$ a can take any value in 1,2,3,.....,9 to make the inequality hold. Then we have $9\times 5=45$ admissible events

When b = 6, a can be 3, 4,…., 9 and there are 7 admissible events
When b = 7, a can be 5, 6, 7, 8, 9 and there are 5 admissible events
When b = 8, a can be 7, 8, 9 and there are 3 admissible events
When b = 9, a can be 9 and there are 1 admissible events

So, the required probability is $\frac{45+7+5+3+1}{81}=\frac{61}{81}$

14. $I.F=e^{\int\left(\frac{3x^2}{1+x^3}\right)dx}=1+x^3$

$y\left(1+x^3\right)=\int\frac{1-\cos(2x)}{2}dx \qquad y\left(1+x^3\right)=\frac{x}{2}-\frac{1}{4}\sin 2x+C$

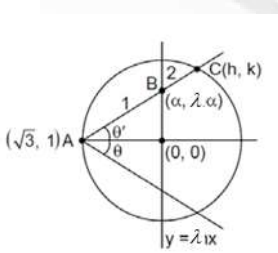
15. $(a,a)\notin R$

If $(a,b)\in R \Rightarrow (b,a)\in R$

If $(a,b)\in R,(b,c)\in R \Rightarrow (a,c)\notin R$

16. The line can be written as
$y=\lambda x$ *and curve as* $x^2+y^2=4$

Let C(h,k) be a point on the circles and $A\left(\sqrt{3},1\right)$ be given point, then $\frac{h+2\sqrt{3}}{3}=a$



$\Rightarrow h=3\alpha-2\sqrt{3} \qquad \frac{k+2}{3}=\lambda\alpha \Rightarrow k=3\lambda\alpha-2$

Now, this point (h, k) lies on the circle $\qquad \Rightarrow\left(3\alpha-2\sqrt{3}\right)^2+\left(3\lambda\alpha-2\right)^2=4$

$9\alpha^2+12-12\sqrt{3}\alpha+9\lambda^2\alpha^2+4-12\lambda\alpha=4 \quad \Rightarrow 9(1+\lambda^2)\alpha^2-12\alpha(\sqrt{3}+\lambda)+12=0$

$3(1+\lambda^2)\alpha^2-4\alpha(\sqrt{3}+\lambda)+4=0 \quad 16(\sqrt{3}+\lambda)^2-4\times3(1+\lambda^2)(4)>0$

$(\sqrt{3}+\lambda)^2-3(1+\lambda^2)>0 \quad 2\sqrt{3}\lambda-2\lambda^2>0 \quad 2\sqrt{3}\lambda-2\lambda^2>0$

$2\lambda^2-2\sqrt{3}\lambda>0 \quad \lambda\in(0,\sqrt{3})$

17. Given, $\sqrt{1+\cos 2x}=\sqrt{2}\cos^{-1}(\cos x) \quad \therefore \sqrt{2}|\cos x|=\sqrt{2}\,x$

For all $x\in\left[\frac{\pi}{2},\pi\right],-\cos x=x \quad \Rightarrow$ No Solution

18. Ortho-centres of triangles formed by three tangents and corresponding normal to a parabola are equidistant from axis of parabola

19. Let $x_i-5=d_i\sigma_x^2=\sigma_d^2=\frac{\sum d_i^2}{n}-\left(\frac{\sum d_i}{n}\right)^2=\frac{125}{10}-\left(\frac{5}{10}\right)^2=\frac{25}{2}-\frac{1}{4}=\frac{49}{4}$

20. $(P)\ {}^{28}C_3=2600$

$(Q)\ {}^{26}C_3-{}^{20}C_3-{}^{21}C_3+{}^{15}C_3=585$

$(R)\ {}^{17}C_3=680$

$(S)\ {}^{24}C_2+{}^{19}C_2+{}^{14}C_2+{}^{9}C_2+{}^{4}C_2=580$

21. From fig it clear that $f(x)=\begin{cases}(1-x)^2 & 0\le x\le\frac{1}{3}\\ 2x(1-x) & \frac{1}{3}<x\le\frac{2}{3}\\ x^2 & \frac{2}{3}<x\le1\end{cases}$

The required area $A=\int_0^1 f(x) \quad =\int_0^{\frac{1}{3}}(1-x)^2\,dx+\int_{\frac{1}{3}}^{\frac{2}{3}}2x(1-x)\,dx+\int_{\frac{2}{3}}^{1}x^2dx$

$=\left[-\frac{1}{3}(1-x)^3\right]_0^{\frac{1}{3}}+\left[\left(x^2-\frac{2x^3}{3}\right)\right]_{\frac{1}{3}}^{\frac{2}{3}}+\left[\frac{x^3}{3}\right]_{\frac{2}{3}}^{1} \quad =\frac{17}{27}$ So, $\frac{p}{q}=\frac{17}{27}$ Hence p+q=17+27=44

22. $2^N<N!$ Which is true when $N\ge4$

$N=1(Not\ \ possible) \quad N=2\ i.e.,(1,1)(Not\ \ possible)$

$\therefore$ required probability $=\frac{36-3}{36}=\frac{33}{36}=\frac{11}{12} \quad \therefore m=11\,and\ \ n=12$

$Now, 4m-3n=4(11)-3)12=44-36-8$

23. Let $\vec{p}=2\hat{i}+3\hat{j}+5\hat{k};\vec{q}=\sin\alpha\sin\beta\hat{i}+\cos\beta+\cos\alpha\sin\beta\hat{k}$

$|\vec{q}|=\sqrt{\sin^2\alpha\sin^2\beta+\cos^2\beta+\cos^2\alpha\sin^2\beta}=1$

$\Rightarrow\sin\alpha\sin\beta=2\lambda;\cos\beta=3\lambda;\cos\alpha\sin\beta=5\lambda$

$1=38\lambda^2;\lambda=\frac{1}{\sqrt{38}} \quad \det A=\left[\frac{\sin\alpha\sin\beta}{\cos\beta}+\frac{1}{3}\right]=1$

24. $g'(2\pi)=3/7, g''(2\pi)=0$

25. $Let\ \vec{c}=\lambda\vec{a}+\mu\vec{b}$

Taking dot by $\vec{b}$

$0=\lambda(\vec{a}.\vec{b})+(\vec{b})^2=-\lambda+5\mu \quad \Rightarrow \lambda-5\mu=0.......(1)$

Again $\vec{a}.\vec{c}=7\Rightarrow \lambda\vec{a}^2+\mu(\vec{a}.\vec{b})=7 \quad \Rightarrow 3\lambda-\mu=7........(2)$

Solving (1) and (2) $\lambda=\frac{5}{2}, \mu=\frac{1}{2} \quad \Rightarrow \frac{2}{7}|\vec{c}|^2=\frac{1}{7}\times 35=5$

PHYSICS

26. In the first case the mechanical energy is completely converted into heat because of friction *i.e,* Decrease in mechanical energy $=\frac{1}{2}mv^2$

While is second case, a part of mechanical energy is converted into heat due to friction but another part of mechanical energy is retained in the from if potential energy of the block *i.e.,*

Decrease in mechanical energy $=\frac{1}{2}mv^2-mgh$

Therfore statement 1 is correct

Statement-2 is wrong. The coefficient of friction between the block and the surface does not depend on the angle of inclination.

27. Pseudo force is applied on a body only when the body is seen from an accelerated Observer

28. $av=$constant

29. conceptual

30. conceptual

31. $D=\frac{\mu_0 NI}{2\pi R}$

32. $f\alpha q_1q_2$

33. Net heat absorbed by one mole of diatomic gas in going from A $\rightarrow$B (isochoric process) and B $\rightarrow$ C (isobaric process) is $\Delta Q=C_V\Delta T+C_P\Delta T=\frac{5}{2}RT_0+\frac{7}{2}RT_0\Delta Q=6RT_0$

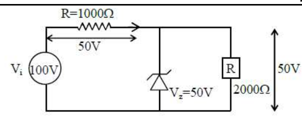
34. The gravitational force vanishes at the midway point between the planets, so the rocket only needs to have enough energy to get there. The initial and final gravitational potential energies are $U_i=-\frac{GMm}{R}-\frac{GMm}{3R}=-\frac{4GMm}{3R}\ and\ U_f=-\frac{2GMm}{2R}=-\frac{GMm}{R}$

35. $C_{AB}=\frac{24\times 8}{24+8}=6\mu F$

36. $P_0+\rho_1gh-\rho_2gh+\frac{2T}{r}=P_0=T=\frac{r}{2}(\rho_2-\rho_1)gh$

37. Conceptual

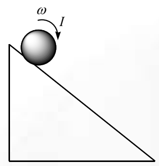
38. $I=\frac{50}{1000}=50mA \qquad R=1000\Omega$



$$I=\frac{50}{2000}=25mA, I_z=I_{1000}-I_{2000}=50-25=25\ mA$$

39. Use the basic concept of interference of light waves.

40. By conservation of energy, we have $\frac{1}{2}mv_c^2+\frac{1}{2}I_c\omega^2=mgh$



Moment of inertia of solid and hollow cylinders are given as $I_{solid}=\frac{MR^2}{2}$ $I_{hollow}=MR^2$

For pure rolling, we have $\omega=v_c/R$, As $I_{solid}<I_{hollow}\Rightarrow v_{solid}>v_{hollow}$

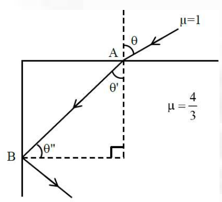
Hence solid cylinder will reach the bottom first.

41. $K_i=\frac{1}{2}m\left(u^2+\frac{u^2}{2}\right)=\frac{3}{4}mu^2$

$K_f=\frac{1}{2}(2m)\frac{u^2}{16}\times 10=\frac{5}{8}mu^2$

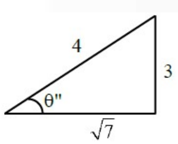
Loss in kinetic energy $\frac{3}{4}mu^2-\frac{5}{8}mu^2=\frac{1}{8}mu^2$

42.



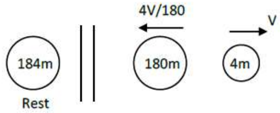
At maximum angle θ ray at point B goes in gazing emergence, at all less values of θ, TIR occurs. At point B $\frac{4}{3}\times\sin\theta''=1\times\sin 90^0$ $\theta''=\sin^{-1}\left(\frac{3}{4}\right)$ $\theta'=\left(\frac{\pi}{2}-\theta''\right)$

At point A $1\times\sin\theta=\frac{4}{3}\times\sin\theta'$ $\sin\theta=\frac{4}{3}\times\sin\left(\frac{\pi}{2}-\theta''\right)$ $\sin\theta=\frac{4}{3}\cos\left[\cos^{-1}\frac{\sqrt{7}}{4}\right]$



$\sin\theta=\frac{4}{3}\times\frac{\sqrt{7}}{4}$ $\theta=\sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$

43.



$\frac{1}{2}(4m)v^2+\frac{1}{2}(180m)\left(\frac{4v}{180}\right)^2=5.5MeV \qquad \frac{1}{2}4mv^2\left[1+45\left(\frac{4}{180}\right)^2\right]=5.5MeV$

$\Rightarrow K.E_\alpha=\frac{5.5}{1+45.\left(\frac{4}{180}\right)^2}MeV \;\; K.E_\alpha=5.38MeV$

44. $i=\frac{|e|}{R}$

45. $1\times\frac{1}{2g}\left(\frac{p\sin\theta}{m}\right)^2=\frac{p\sin\theta}{mg}\times\frac{p\cos\theta}{m} \;\; \frac{1}{2}\sin^2\theta=\sin\theta\cos\theta\Rightarrow\tan\theta=2 \;\; \therefore\cos\theta=\frac{1}{\sqrt{5}}$

Minimum kinetic energy $=\left(\frac{p\cos\theta}{2m}\right)^2=\frac{p^2}{2m}\times\frac{1}{5}=\frac{p^2}{10m}$

46. $V=\frac{\omega}{k}=\frac{9\times10^8}{6}=1.5\times10^8 m/s$, Refractive index $\mu=\frac{C}{V}\Rightarrow\sqrt{K}=\frac{3\times10^8}{1.5\times10^8}$ $\therefore K=4$

47. $\tan\phi=\frac{x_C-x_L}{R} \qquad \tan 45=\frac{x_C-x_L}{R} \qquad x_C-x_L=R \qquad \frac{1}{\omega C}-\omega L=R$

$\frac{1}{\omega C}-300\times0.03=1 \qquad \frac{1}{\omega C}=10 \qquad C=\frac{1}{10\omega}=\frac{1}{10\times300} \qquad C=\frac{1}{3}\times10^{-3} \qquad x=3$

48. $KE=\frac{hc}{\lambda}-\phi.......(i)$

$e(3V_0)-\frac{hc}{\lambda_0}-\phi......(i) \qquad eV_0=\frac{hc}{2\lambda_0}-\phi........(ii)$

Using (i) & (ii) $\qquad \phi=\frac{hc}{4\lambda_0}=\frac{hc}{\lambda_t} \qquad \lambda_t=4\lambda_0$

49. $\therefore$ mean free path $\qquad \lambda=\frac{1}{\sqrt{2}\pi d^2n} \qquad \frac{\lambda_1}{\lambda_2}=\frac{d_2^2n^2}{d_1^2n_1} \qquad =\left(\frac{5}{10}\right)^2=0.25=25\times10^{-2}$

50. $\upsilon=\frac{2v}{2l}$

CHEMISTRY

51. $2\ C_3H_8 + 1C_4H_{10} + O_2 \rightarrow 10CO_2 + 13H_2O$

52. $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2}mv^2 \qquad \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$mv^2 = 2ch\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \qquad v^2 = \frac{2hc}{m}\left[\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right] \qquad v = \sqrt{\frac{2hc}{m}\left[\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right]}$

53. 2^{nd} electron affinity is positive

54. $p\pi - d\pi$ cannot formed by 2^{nd} period

55. $\frac{1}{2}H_2 + \frac{1}{2}X_2 \rightarrow HX \Delta H = -50$

$2a \qquad a \qquad 2a$

$\frac{2a}{2} + \frac{a}{2} - 2a = -50$

$3a - 4a = -100$

$a = 100$

56. $Ag_2CrO_4 \rightleftharpoons 2Ag^+ + CrO_4^{-2}$

$2.2\times10^{-4} 1.1\times10^{-4}$

$KSP = \left(2.2\times10^{-4}\right)^2\left(1.1\times10^{-4}\right) \qquad = 5.3\times10^{-12}$

57. $M = \frac{25.3}{106}\times\frac{1000}{250} \qquad = 0.9547 = 0.955M$

$\left[Na^+\right] = 1.910\left[CO_3\right]^{-2} = 0.955$

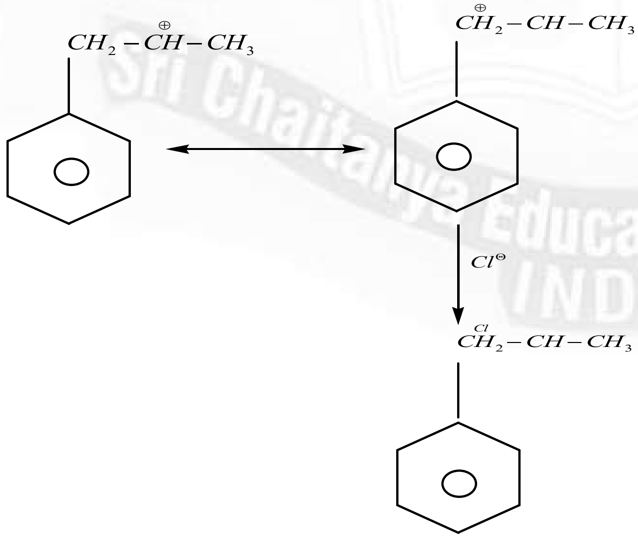
58.

$$CH_3 - CH_2 - CH_2 - Cl \xrightarrow{AlCl_3} CH_3 - CH_2 - CH_2^{+}$$

$$CH_3 - CH_2 - \overset{\oplus}{C}H_2 \xrightarrow[shift]{1,2-H^{\ominus}} CH_3 - CH^{\oplus} - CH_3$$

59. inductive effect

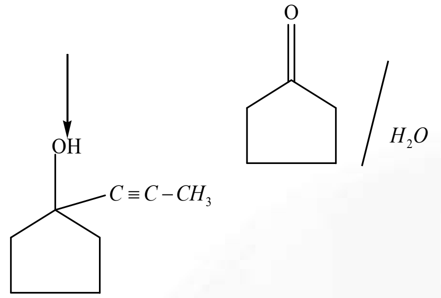
60.



61. $CH_2 - C \equiv CH \xrightarrow{CH_3CH_2MgCl}$

$CH_3-C\equiv C-MgCl$

$CH_2-C\equiv CH \xrightarrow{CH_3CH_2MgCl}$



62. Named reactions and uses

63. $\begin{array}{cccccccc} A & T & G & C & T & T & G & A \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ T & A & C & G & A & A & C & T \end{array}$

64. No SP carbons in benzyne

65. Optical Isomerism

66. B.P $PH_3 < ASH_3 < NH_3 < SbH_3$

67. H_3PO_4 is Tribasic

68. I.E $B > Tl > Ga > Al > In$

69. periodic property

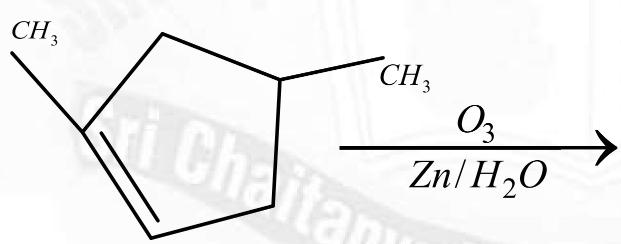
70. Sandmeyer and gatterman reactions

71. $\Delta G = \Delta G_p - \Delta G_r$

72. $\frac{t_1}{t_2} = \left(\frac{P_2}{P_1}\right)^{n-1}$

73. $CN^-, SCN^{-1}, NO_2^-, CNO^-$

74.



$CHO-CH_2-\underset{|\;CH}{CH}-CH-\overset{O\;\|}{C}-CH_3$

75. $:\ddot{\underset{..}{I}}-\ddot{\underset{..}{I}}^{\ominus}-\ddot{\underset{..}{I}}:$