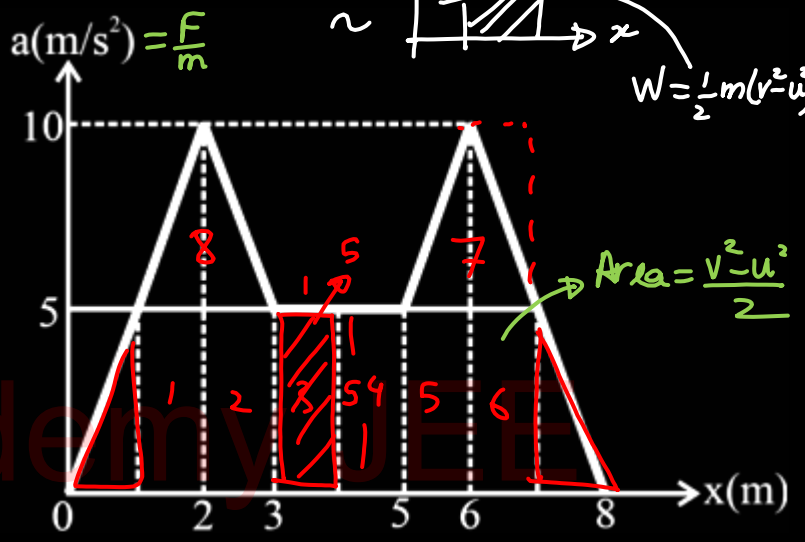
7

A car is moving on a horizontal road whose acceleration versus position graph is drawn below. At the initial moment of time when particle was at $x = 0$, its speed was $\sqrt{10}$ m/s. Find the speed of the particle when it reaches at $x = 8$ m.

(A) $\sqrt{84}$ m/s (B) 10 m/s

(C) $\sqrt{110}$ m/s (D) 11 m/s



$u = \sqrt{10}$

$\text{Area} = 9(5) = \frac{v^2 - u^2}{2}$

$90 = v^2 - 10$

$\boxed{v = 10}$

2

A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement $\vec{d} = 4\hat{i}$. If the particle has a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

(A) 9 J
(B) 12 J
(C) 10 J
(D) 15 J

JEE 1

$W = \vec{F} \cdot \vec{d} = 12$

$KE_i = 3$

KE_f

$W = \Delta KE$

$12 = KE_f - KE_i$

$KE_f = 15$

3

Match List-I with List-II.

List-I	List-II
(A) Torque	(i) MLT^{-1}
(B) Impulse	(ii) MT^{-2}
(C) Tension	(iii) ML^2T^{-2}
(D) Surface tension	(iv) MLT^{-2}

ML^2T^{-2}

$J = \Delta P$

MLT^{-2}

$\frac{F}{L}$ =

$[J] = MLT^{-1}$

~~(A) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)~~

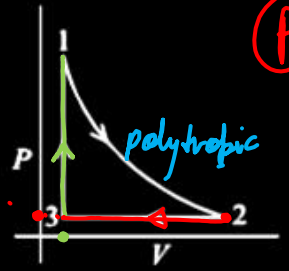
(B) (A)-(iii); (B)-(i); (C)-(iv); (D)-(ii)

~~(C) (A)-(i); (B)-(iii); (C)-(iv); (D)-(ii)~~

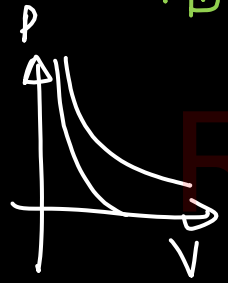
~~(D) (A)-(ii); (B)-(i); (C)-(iv); (D)-(iii)~~

4

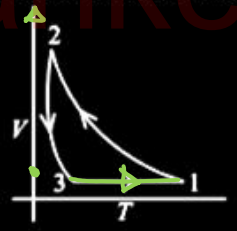
Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure?



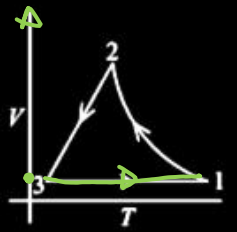
where, $1 \to 2$ is adiabatic. (Graphs are schematic and are not to scale)



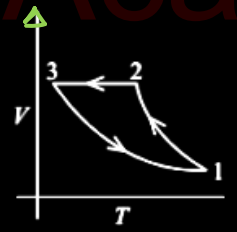
(A)



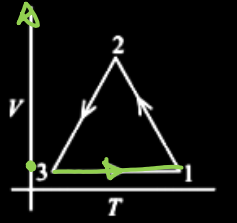
(B)



(C)



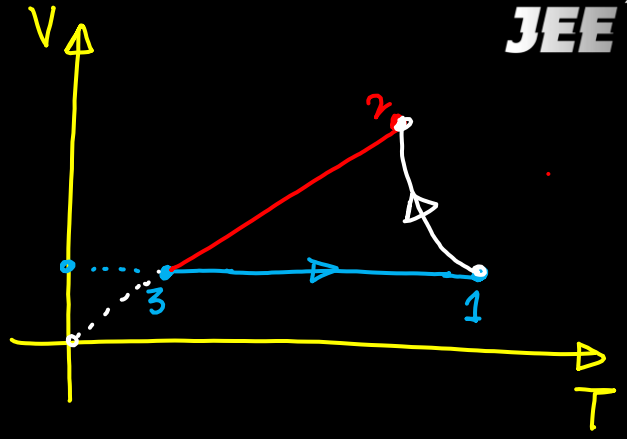
(D)



$PV = nRT$

$PV = nRT$ constt.

$V = \frac{nR}{P} T$



$1 \to 2$ $x = 2$ eg.

PV^x = constt.

PV^2 = constt = k.

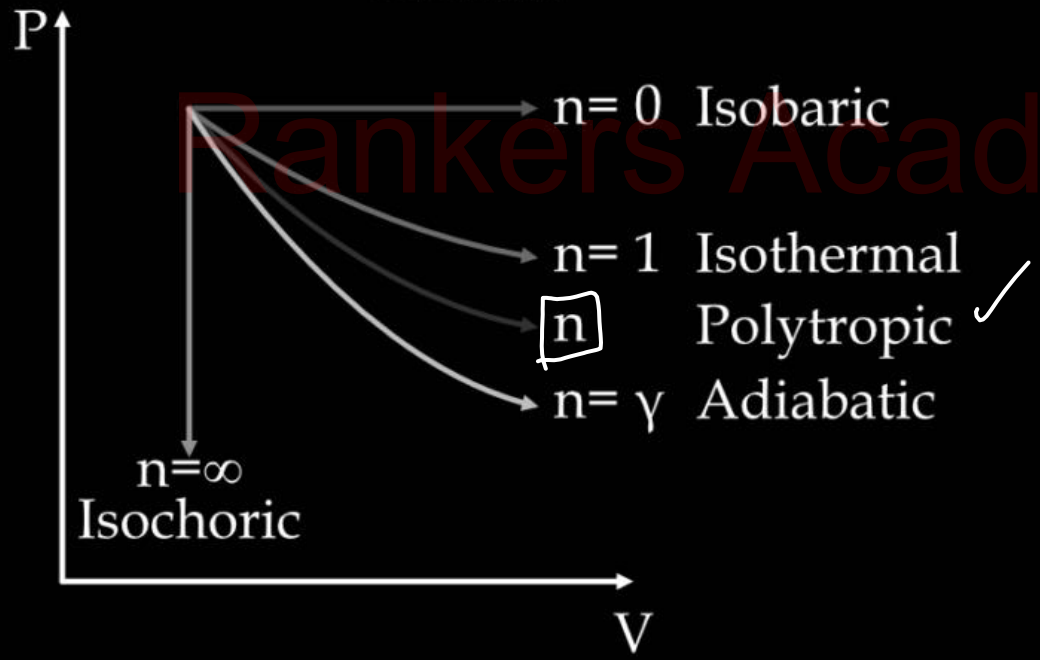
$PV^2 = nRTV$ PV

constt $= \frac{K}{nR} = VT$

4

Thermodynamic Process

$PV^n = \text{constt.}$



5

A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance ' h ' the square of angular velocity of wheel will be

$\omega = ?$

(A) $2gh$

(B) $\frac{2gh}{I+mr^2}$

(C) $\frac{2mgh}{I+mr^2}$

(D) $\frac{2mgh}{I+2mr^2}$

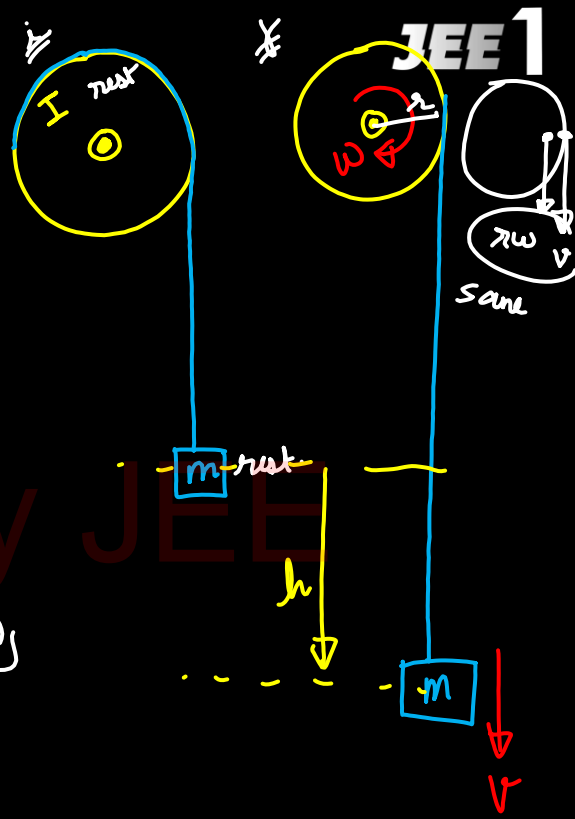
$V = r\omega$

Energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgh = mR^2\omega^2 + I\omega^2$$

$$\omega^2 = \frac{2mgh}{I+mR^2}$$



6

Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R. Assertion A: Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and respectively, then $R = 4\sqrt{h_1 h_2}$

Reason R: Product of said heights.

$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right) \cdot \left(\frac{u^2 \cos^2\theta}{2g}\right) = \frac{u^4}{g^2} \frac{4\sin^2\theta \cdot \cos^2\theta}{16} = \frac{R^2}{16}$

Choose the correct answer:

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

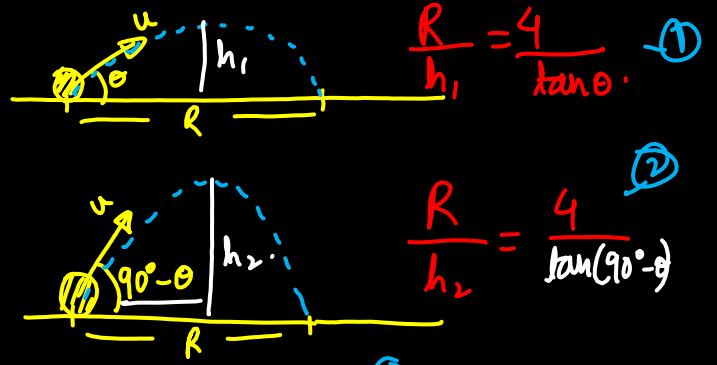
~~(D) A is false but R is true.~~

$R_\theta = \frac{u^2}{g}\sin(2\theta)$

$R_{90^\circ-\theta} = \frac{u^2}{g}\sin(2\theta)$

$H = \frac{u^2 \sin^2\theta}{2g}$

$\frac{R}{H} = \frac{4}{\tan\theta}$



$\frac{R}{h_1} = \frac{4}{\tan\theta}$ ①

$\frac{R}{h_2} = \frac{4}{\tan(90^\circ-\theta)}$ ②

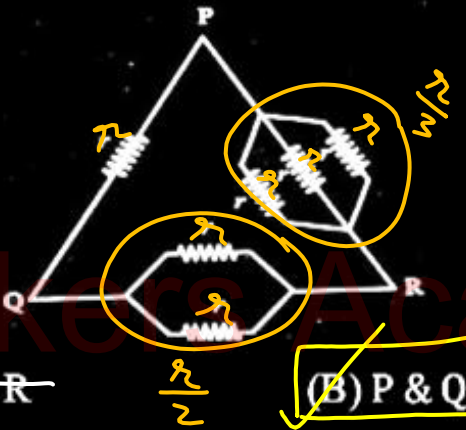
① × ②

$\frac{R^2}{h_1 h_2} = 16$

$R = 4\sqrt{h_1 h_2}$

7

Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between:

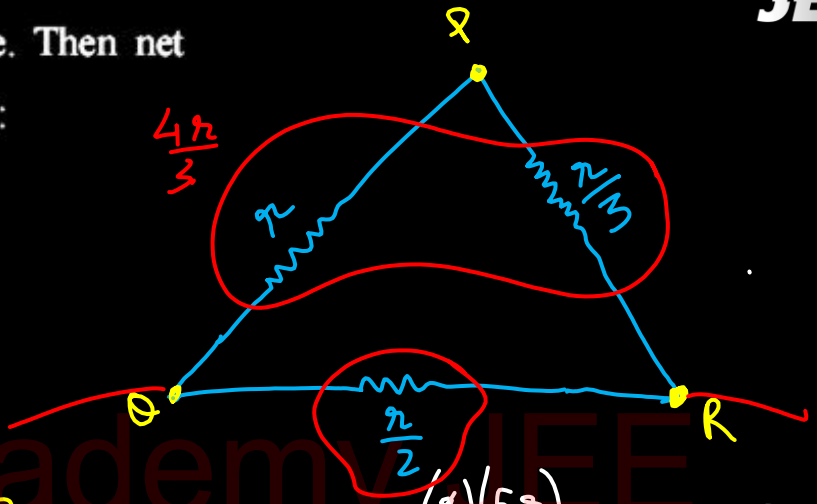


~~(A) P & R~~

(B) P & Q

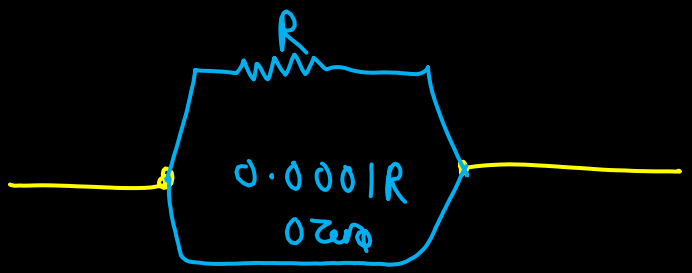
~~(C) Q & R~~

(D) any two points



$$R_{eq_{PQ}} = \frac{\left(r\right)\left(\frac{5r}{6}\right)}{\frac{11r}{6}} = \frac{5r}{11}$$

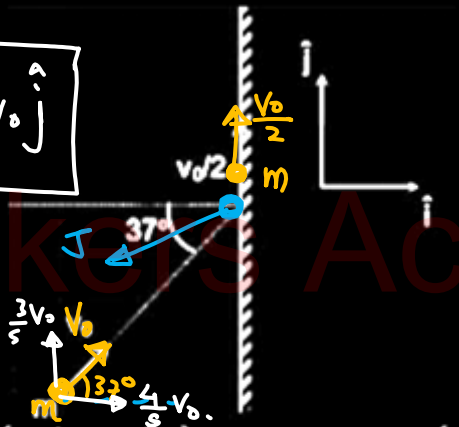
$$R_{eq_{PR}} = \frac{\left(\frac{2r}{2}\right)\left(\frac{r}{2}\right)}{\left(\frac{3r}{2}+\frac{r}{3}\right)} = \frac{\frac{r}{2}}{\frac{9+2}{6}} = \frac{3r}{11}$$



$$R_{eq_{QR}} = \cdots = \frac{4r}{11}$$

8

A ball collides with a wall and after collision moves parallel to the wall as shown in the figure. The impulse acting on the ball during the collision is



$\vec{P_i} = \frac{4}{5}mv_0\hat{i} + \frac{3}{5}mv_0\hat{j}$

$\vec{P_f} = \frac{mV_0}{2}\hat{j}$

$\vec{P_i} + \vec{J} = \vec{P_f}$

$\vec{J} = \vec{P_f} - \vec{P_i}$

$\vec{J} = -0.8mV_0\hat{i} - 0.1mV_0\hat{j}$

(A) $mv_0\left(-\frac{4}{5}\hat{i} - \frac{1}{10}\hat{j}\right)$

(B) $mv_0\left(-\frac{4}{5}\hat{i} - \frac{1}{5}\hat{j}\right)$

(C) $mv_0\left(-\frac{3}{5}\hat{i} - \frac{1}{10}\hat{j}\right)$

(D) $mv_0\left(-\frac{3}{5}\hat{i} - \frac{1}{5}\hat{j}\right)$

9

A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is

(A) 2.6 m from the mirror, real

(B) 1 m from the mirror, virtual

(C) 1 m from the mirror, real

(D) 2.6 m from the mirror, virtual.

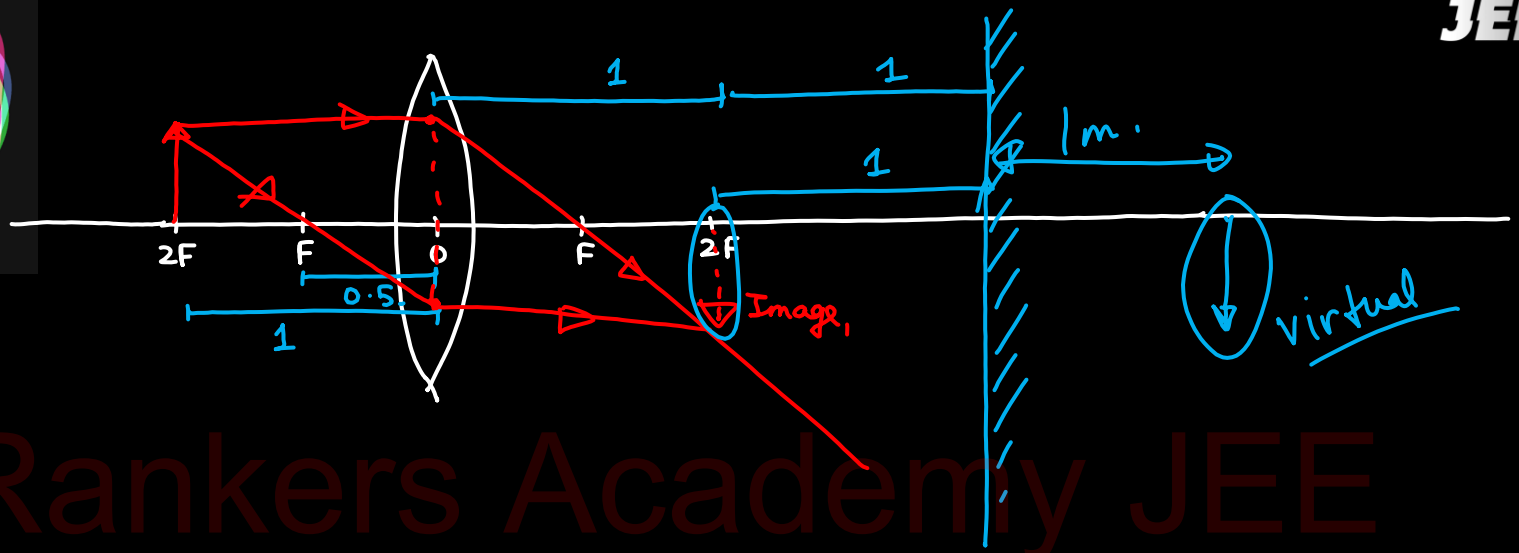
$u = -1$
$f = +\frac{1}{2}$.

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$2 = \frac{1}{v} - \frac{1}{(-1)}$

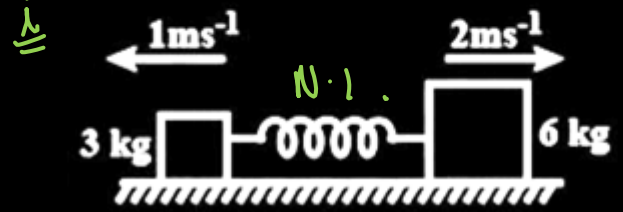
$v = 1$

9



10

Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface they are connected by a light spring of force constant $k = 200 Nm^{-1}$. Initially the spring is unstretched and velocities of $1\ ms^{-1}$ and $2\ ms^{-1}$ are imparted in opposite directions to the respective blocks as shown in figure. The maximum extension of the spring will be



(A)15 cm

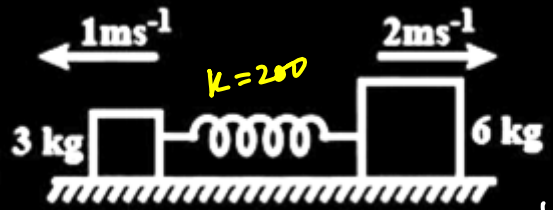
(B) 20 cm

(C)25 cm

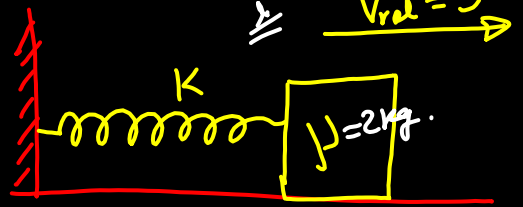
(D) 30 cm

JEE 1

10



$\equiv$



$$\frac{1}{\mu}=\frac{1}{m_1}+\frac{1}{m_2}$$

$$\mu=\frac{m_1 m_2}{m_1+m_2}=\frac{3\times 6}{9}=2$$

$$\frac{1}{2}\mu V_{rel}^2=\frac{1}{2}kx^2$$

$$\frac{1}{2}(2)(3)^2=(10)^2x^2$$

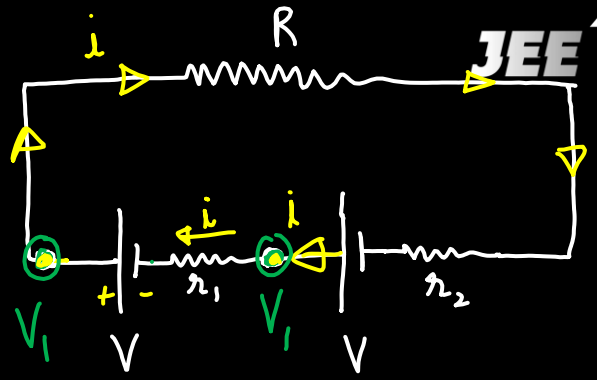
$$x=0.3\ m$$

11

Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistance of the two sources are r_1 and $r_2 (r_1 > r_2)$. If the potential difference across the source of internal resistance r_1 is zero, then the value of R will be

(A) $r_1 - r_2 = R$ (B) $\frac{r_1 r_2}{r_1 + r_2} = R$

(C) $\frac{r_1 + r_2}{2} = R$. (D) $r_2 - r_1 = R$



$i = \frac{2V}{R + r_1 + r_2}$ — ①

$V_1 - i r_1 + V = V_1$

$V = i r_1$ — ②

R = ?

① & ②

$i = \frac{2 i r_1}{R + r_1 + r_2}$

$R + r_1 + r_2 = 2 r_1$

$R = r_1 - r_2$

12

JEE 1

Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions.

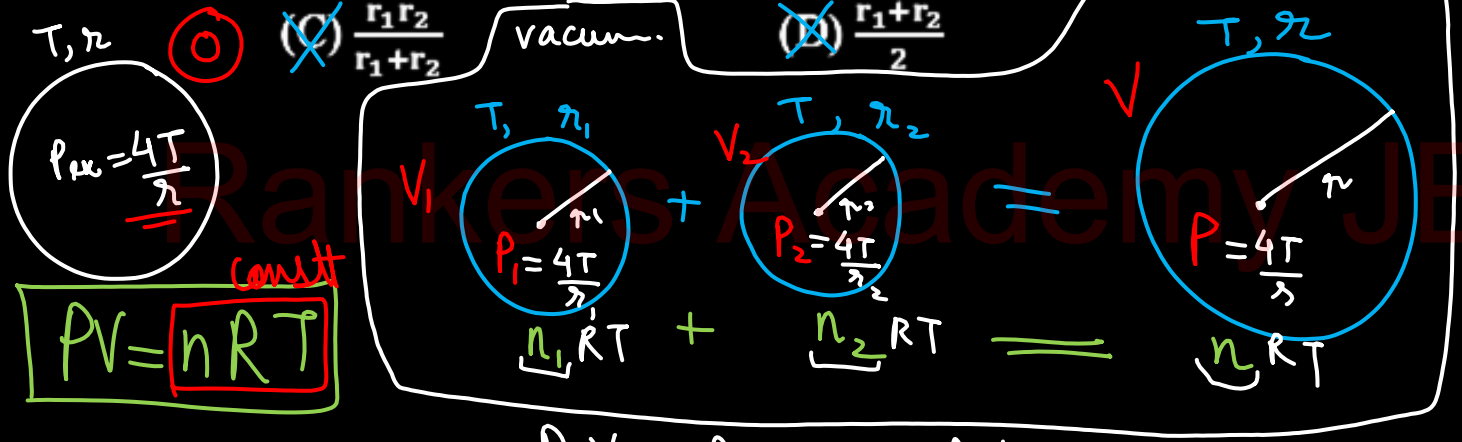
The resulting bubble has a radius equal to

(A) $\sqrt{r_1 r_2}$ (B) $\sqrt{r_1^2 + r_2^2}$

(C) $\frac{r_1 r_2}{r_1 + r_2}$ (D) $\frac{r_1 + r_2}{2}$

$r = ?$

Vacuum isothermal.



$PV = nRT$ (const)

$r^2 = r_1^2 + r_2^2$

$r = \sqrt{r_1^2 + r_2^2}$

$P_1 V_1 + P_2 V_2 = PV$

$$\left(\frac{4T}{r_1}\right)\left[\frac{4}{3}\pi r_1^3\right] + \left(\frac{4T}{r_2}\right)\left[\frac{4}{3}\pi r_2^3\right] = \left(\frac{4T}{r}\right)\left[\frac{4}{3}\pi r^3\right]$$

13

A solid metallic cube having total surface area $24\ m^2$ is uniformly heated. If its temperature is increased by $10°C$, calculate the increase in volume of the cube. (Given $\alpha = 5.0 \times 10^{-4}\ °C^{-1}$)

(A) $2.4 \times 10^6\ cm^3$ (B) $6.0 \times 10^4\ cm^3$

(C) $1.2 \times 10^5\ cm^3$ (D) $4.8 \times 10^5\ cm^3$

α	β	γ
1	2	3

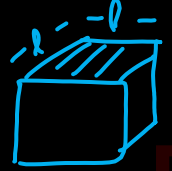
JEE 1

$\Delta L = L\alpha(\Delta T)$

$\Delta A = A\beta(\Delta T)$

$\Delta V = V\gamma\Delta T$

$\gamma = 3\alpha = 15 \times 10^{-4}$



$6l^2 = 24$

$l^2 = 4$

$l = 2$

$V = l^3 = 8$

8 3α 10

ΔV = V γ ΔT

$\Delta V = 8\left(15 \times 10^{-4}\right) 10\ m^3$

14

A nucleus with mass number 184 initially at rest emits an α particle. If the Q value of the reaction is 5.5MeV, calculate the kinetic energy of the α-particle.

(A) 5.38MeV

(B) 0.12MeV

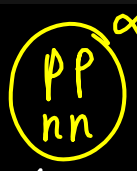
~~(C) 5.0MeV~~

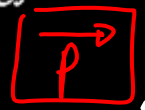
~~(D) 5.5MeV~~

Ans = $KE_\alpha = \frac{1}{2}(4m)v^2$

Ans $= 2mv^2$

JEE 1





$0 = 4mv - 180mv_1$

$180mv_1 = 4mv$

$v_1 = \frac{v}{45}$

Energy

$\frac{1}{2}(180m)\left(\frac{v}{45}\right)^2 + \frac{1}{2}4mv^2 = 5.5\,MeV$

$\frac{90}{45\times45}mv^2 + 2mv^2 = 5.5\,MeV$

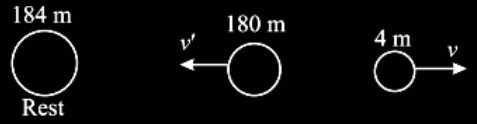
$2mv^2\left[1+\frac{1}{45}\right] = 5.5MeV$

$2mv^2 = 5.5\,MeV \times \frac{45}{46}$

14

JEE 1

(*a*): Mass number, A = 184; Q_{value} = 5.5 MeV Let the velocity of α-particle is *v* and for $180m$, it is *v*′.



Use conservation of momentum,

$184m \times 0 = 180\ mv' - 4mv$

$$v' = \frac{4v}{180}$$

Now using conservation of energy,

$$\frac{1}{2}(4m)v^2 + \frac{1}{2}(180m)v'^2 = 5.5\text{MeV}$$

$$\frac{1}{2}\cdot 4mv^2\left[1+45\times\left(\frac{4}{180}\right)^2\right] = 5.5\text{MeV (Using (i))}$$

Here,

$$\text{K}\cdot\text{E}_\alpha = \frac{1}{2}\left(4mv^2\right), \text{K}\cdot\text{E}_\alpha\left(1+45\times\left(\frac{4}{180}\right)^2\right) = 5.5\text{MeV}$$

$$K\cdot E_\alpha = \frac{5.5}{1+45\left(\frac{4}{180}\right)^2} = 5.38\text{MeV}$$

15

The magnetic field of a plane electromagnetic wave is $\vec{B} = 3 \times 10^{-8} \sin [200\pi(y + ct)]\hat{i}$ T, where $c = 3 \times 10^{8}$ m s $^{-1}$ is the speed of light. The corresponding electric field is

(A) $\vec{E} = 9\sin [200\pi(y + ct)]\hat{k}$ V/m

(B) $\vec{E} = -10^{-6}\sin [200\pi(y + ct)]\hat{k}$ V/m

(C) $\vec{E} = 3 \times 10^{-8}\sin [200\pi(y + ct)]\hat{k}$ V/m

(D) $\vec{E} = -9\sin [200\pi(y + ct)]\hat{k}$ V/m.

$E = BC$

JEE 1

em wave dir. $\vec{E} \times \vec{B}$

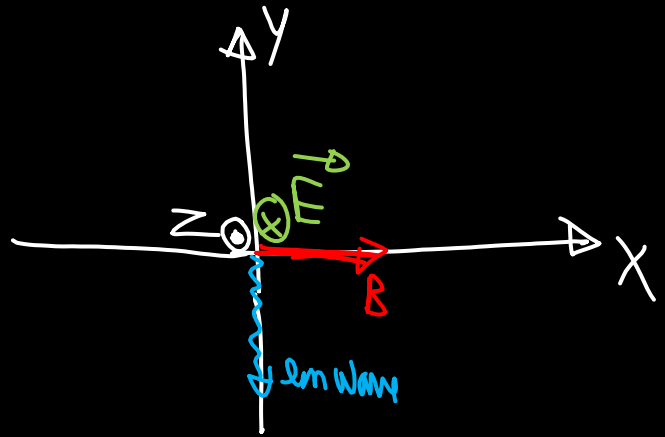
wave

$\vec{E}$ $\vec{B}$

$y = A\sin(\omega t - kx)$ → $+x$

$y = A\sin(\omega t + kx)$ ← $-x$

$\vec{E} = 9 \cdot \sin[200\pi(y + ct)](-\hat{k})$



15

JEE 1

The magnetic field of a plane electromagnetic wave is $\vec{B} = 3 \times 10^{-8} \sin [200\pi(y + ct)]\hat{i}\ T$, where $c = 3 \times 10^{8}\ m\ s^{-1}$ is the speed of light.

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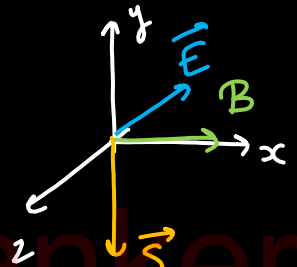
(A) $\vec{E} = 9\sin [200\pi(y + ct)]\hat{k}\ V/m$

(B) $\vec{E} = -10^{-6}\sin [200\pi(y + ct)]\hat{k}\ V/m$

(C) $\vec{E} = 3 \times 10^{-8}\sin [200\pi(y + ct)]\hat{k}\ V/m$

(D) $\vec{E} = -9\sin [200\pi(y + ct)]\hat{k}\ V/m.$

[-ve-y axis]



$\vec{E} \times \vec{B} \parallel \vec{S}$

$E_0 = B_0 c$

$= 3 \times 10^{-8} \times 3 \times 10^{8} = 9\ V/m$

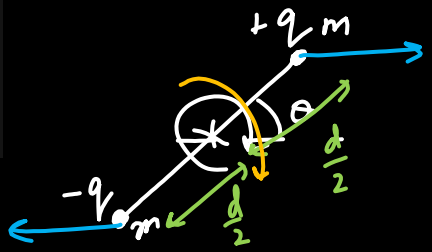
16

JEE 1

An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is

(A) $\sqrt{\frac{2qE}{md}}$ (B) $\sqrt{\frac{qE}{2md}}$

(C) $2\sqrt{\frac{qE}{md}}$ (D) $\sqrt{\frac{qE}{md}}$



$\tau = pE\sin\theta$

$\tau = I\alpha$

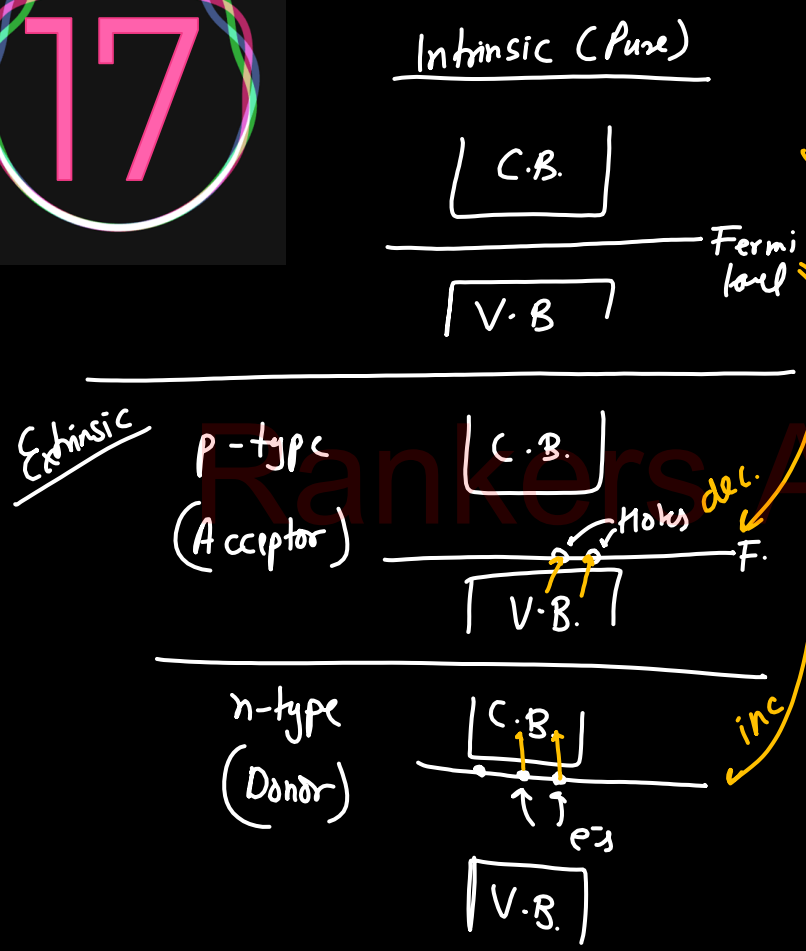
$\Rightarrow pE\sin\theta = \left[2m\left(\frac{d}{2}\right)^2\right]\alpha$

$\Rightarrow (qd)E(\theta) = \left(\frac{md^2}{2}\right)\alpha \quad \Rightarrow \alpha = \left(\frac{2qE}{md}\right)\theta$

$\alpha = \omega^2\theta$

ω^2

17



For extrinsic semiconductors, when doping level is increased

(A) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.

(B) Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward.

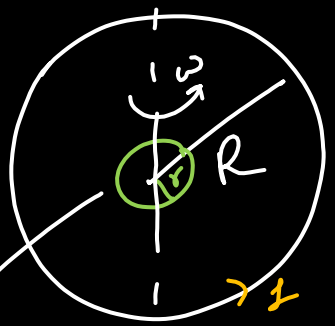
(C) Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F K$ and downward for $T < T_F K$, where T_F is Fermi temperature.

(D) Fermi-level of p and n-type semiconductors will not be affected.

JEE 1

18

JEE 1



At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

(A) $\frac{\mu_0 I}{4R}\omega\pi r^2 \sin\omega t$

(B) $\frac{\mu_0 I}{4R}\omega r^2 \sin\omega t$

(C) $\frac{\mu_0 I}{2R}\omega r^2 \sin\omega t$

(D) $\frac{\mu_0 I}{2R}\omega\pi r^2 \sin\omega t$

$\phi = B(\pi r^2)\cos\theta$

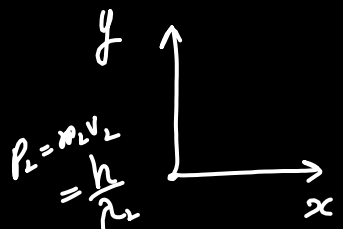
$\phi = \left(\frac{\mu_0 I}{2R}\right)\pi r^2 \cos\theta$

$\varepsilon_{\text{ind}} = -\frac{d\phi}{dt} = -\frac{\mu_0 I \pi r^2}{2R}\frac{d(\cos\theta)}{dt}$

$= +\frac{\mu_0 I}{2R}\pi r^2[\omega \sin\omega t]$

19

JEE 1



$p_1 = m_1 v_1 = \frac{h}{\lambda_1}$

Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ, of the final particle, is given by

(A) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(B) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(C) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

(D) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

$\vec{p}_f = \vec{p}_1 + \vec{p}_2$

$\vec{p}_f = \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$

$p_f = \sqrt{p_1^2 + p_2^2}$

$\left(\frac{h}{\lambda}\right)^2 = \frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}$

20

JEE 1

The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4^{th} VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division)

(A) 3.2 cm
(B) 3.21 cm
(C) 3.07 cm
(D) 2.99 cm

$MSD = 1\,mm$ — (1)

$10\,VSD = 9\,MSD$

$1\,VSD = \frac{9}{10}\,MSD$ — (2)

$L.C. = MSD - VSD$

$= MSD - \frac{9}{10}\,MSD$

$= \frac{1}{10}\,MSD = \frac{1}{10}\,mm = 0.1\,mm$

Zero error = +7 L.C.

Reading = MSR + (C.V.S.D × L.C.) − Zero

$= 3.1\,cm + 4 \times L.C. - 7 \times L.C.$

$= 3.10\,cm + (-3 \times 0.01\,cm)$

$= 3.07\,cm$

21

JEE 1

A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 \text{rad/s}$. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _____ Ω.

$Z_{min} = R$

$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{120 \times 120}{16}$$

$$= 30 \times 30$$

$$= 900\ \Omega$$

22

JEE 1

The escape velocity of a body on a planet 'A' is $12 kms^{-1}$. The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', is ____ kms^{-1}

$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 + 0$

$v = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}$

$v = \sqrt{\frac{8\pi}{3}G(\rho R^2)}$

$\rightarrow \sqrt{(4\rho)\left(\frac{R}{2}\right)^2}$

$v_e' = v_e = 12 km/s$

23

JEE 1

A monoatomic gas performs a work of $\frac{Q}{4}$, where Q is the heat supplied to it. The molar heat capacity of the gas will be ____R, during this transformation. Where R is the gas constant.

$$\Delta U = Q - W$$

$$\Delta U = +Q - \frac{Q}{4}$$

$$\Rightarrow nC_v\Delta T = \frac{3}{4}Q = \frac{3}{4}(nC\Delta T)$$

$$C_v = \frac{3}{4}C \Rightarrow C = \frac{4}{3} \times \left(\frac{3R}{2}\right) = \boxed{2}R$$

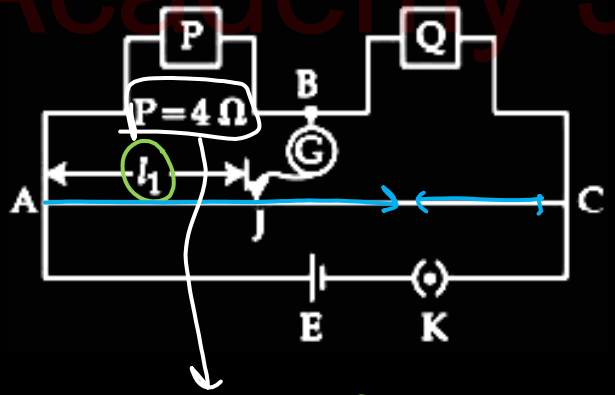
24

JEE 1

Resistance are connected in a meter bridge circuit. The balancing length l_1, is 40 cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80 cm measured from the same end. Then the value of x will be ____ Ω.

$$\frac{P}{Q} = \frac{40}{60} \quad \text{— (1)}$$

$$\frac{P+x}{Q} = \frac{80}{20} \quad \text{— (2)}$$



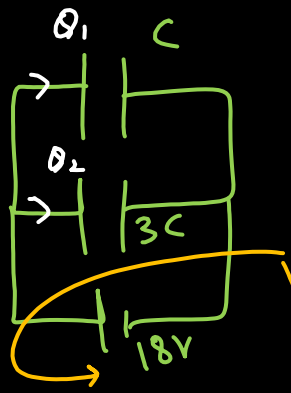
$$\frac{P+x}{P} = \frac{4}{2/3} = 6$$

$$1 + \frac{x}{P} = 6 \Rightarrow x = 5P = 5 \times 4 = 20\ \Omega$$

25

JEE 1

Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18 V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be ____V.



9C

$Q = Q_1 + Q_2 = C_1V + C_2V$

$Q = C_{eq}V$

$= (C + 3C)V$

$= 4CV$ — (1)

$Q_1' + Q_2' = Q = Q_1 + Q_2$ (cons. of charge)

$Q = C_{eq}'V'$

$= (9C + 3C)V' = 4CV$ (Eq. (1))

$V' = \frac{4CV}{12C} = \frac{V}{3} = \frac{18}{3} = 6V$

CHEMISTRY

1

The resistance of $1M - CH_3COOH$ solution is 250 Ω, when measured in a cell of cell constant 125 m^{-1}. The molar conductivity, in $\Omega^{-1} m^2 mol^{-1}$ is

(A) 5.0×10^{-4}

(B) 500

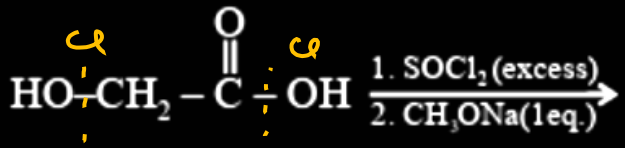
(C) 2×10^{-3}

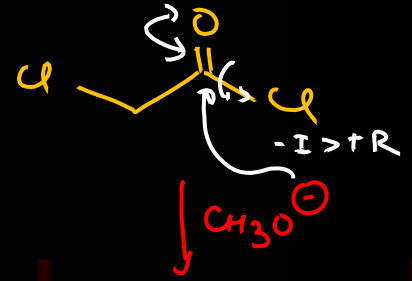
(D) 200

$\Lambda_m = \frac{K}{1000\,M} = \frac{GG^*}{1000\,M} = \left(\frac{1}{250}\right)^2 \frac{125}{1000 \times 1} = 5 \times 10^{-4}$

2

What is the major product of the following reaction sequence?





(A) $Cl–CH_2 – C(=O) – CH_3$

(B) $ClCH_2 – C(=O) – O – CH_3$

(C) $CH_3 – O – CH_2 – C(=O) – CH_3$

(D) $CH_3 – O – CH_2 – C(=O) – Cl$

3

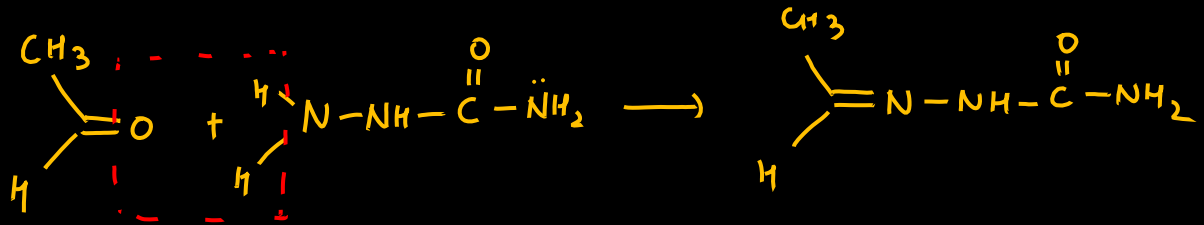
Acetaldehyde reacts with semicarbazide and forms semicarbazone. Its structure is

(A) $CH_3CH = NNHCON = CHCH_3$

(B) $CH_3CH = NNHCONH_2$

(C) $CH_3CH = N - \underset{\displaystyle OH}{\underset{|}{N}} - CONH_2$

(D) $CH_3CH = N{-}CONHNH_2$



4

JEE 1

Select the rate law that corresponds to the data shown for the following reaction

Exp.	$[A_0]$(M)	$[B_0]$(M)	Initial rate ($M\ s^{-1}$)
1	0.012	0.035	0.1
2	0.024	0.070	0.8
3	0.024	0.035	0.1
4	0.012	0.070	0.8

2 times, Const, 2 times, 8 times

(A) $r = K[B]^3$

(B) $r = K[B]^4$

(C) $r = K[A][B]^3$

(D) $r = K[A]^2[B]^3$

$R = K[A]^x[B]^y$

order w.r.t A

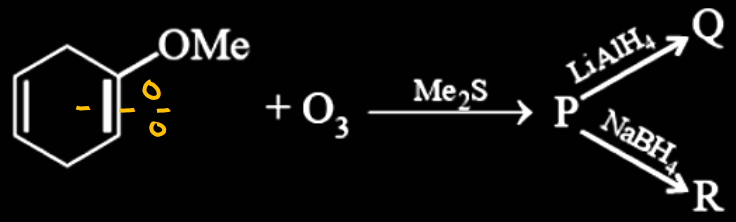
$x = 0$

order w.r.t B

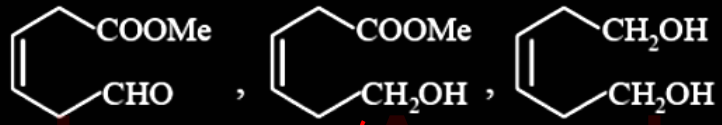
$(2)^y \propto 8$

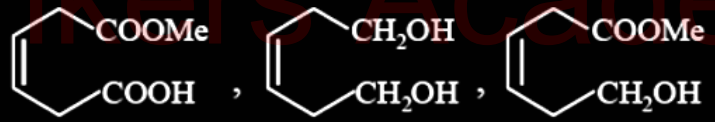
$y = 3$

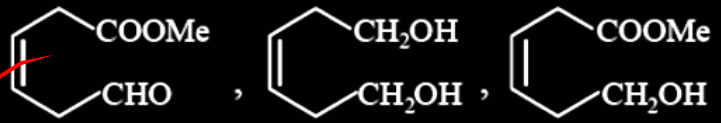
5

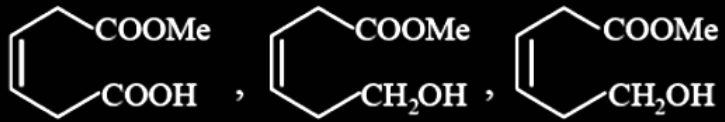


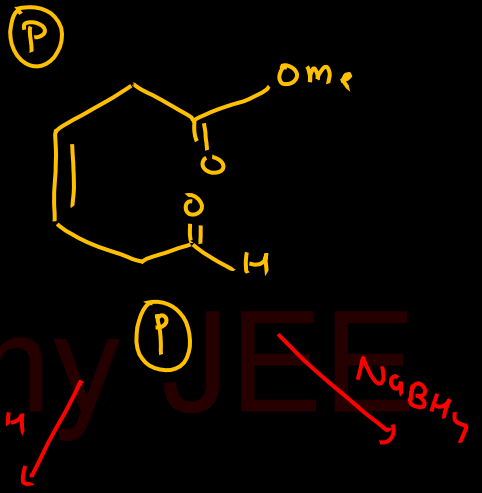
Product P, Q, R will be respectively:

(A) 

(B) 

(C) 

(D) 



6

JEE 1

Which order of basicity is correct for given groups : (aqueous phase)

(I) NH_3

(II) $C_2H_5NH_2$ 1^0

(III) $(C_2H_5)_2NH$ 2^0

(IV) $(C_2H_5)_3N$ 3^0

(A) IV > III > II > I

(B) III > IV > II > I

(C) III > II > IV > I

(D) I > II > III > IV

$R \rightarrow CH_3$ 2 1 3

$R \rightarrow C_2H_5$ 2 3 1

7

JEE 1

An amount of 2 moles of $KClO_3$ is decomposed completely to produce O_2 gas. How many moles of butene C_4H_8 can be burnt completely by the O_2 gas produced?

(A) 0.5 (B) 1.0

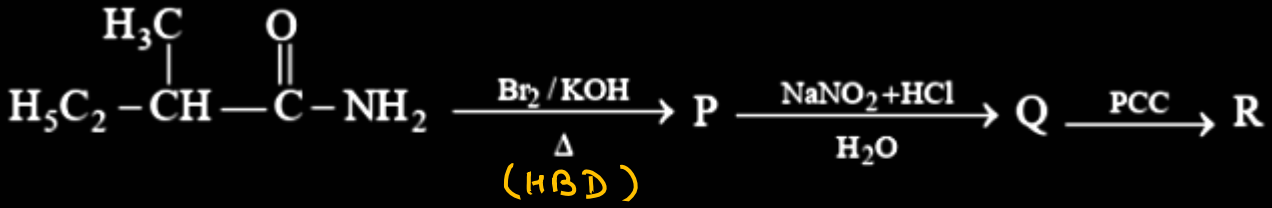
(C) 2.0 (D) 3.0

$2\,KClO_3 \longrightarrow 2\,KCl + 3\,O_2$

2 mole 3 mole

$C_4H_8 + 6O_2 \longrightarrow 4CO_2 + 4H_2O$

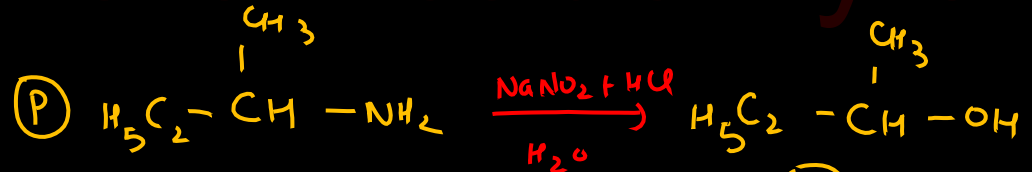
8

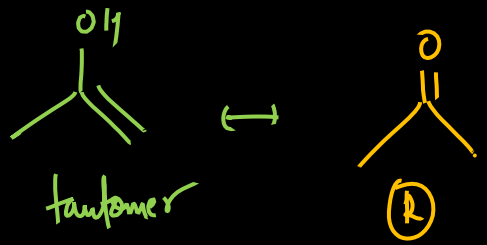


Final product R does not give

(A) Haloform reaction (B) Tautomerisation

(C) Tollen's Test (D) NaHSO$_3$ Test





9

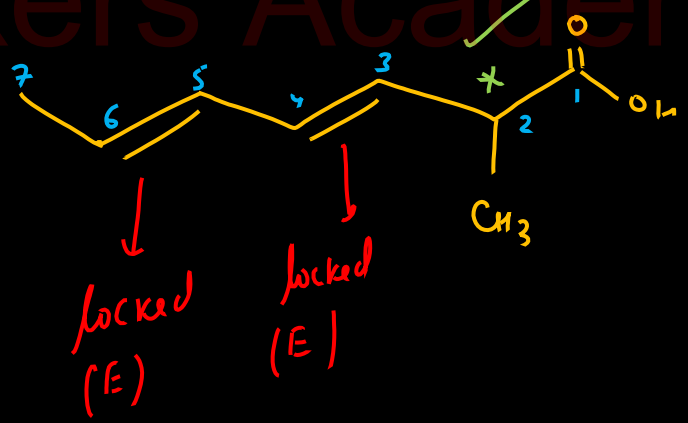
How many stereoisomers will be formed of 2-methyl hepta-3E, 5E-dienoic acid?

(A) 4

(B) 16

(C) 8

(D) 2



$SI = 2^n$

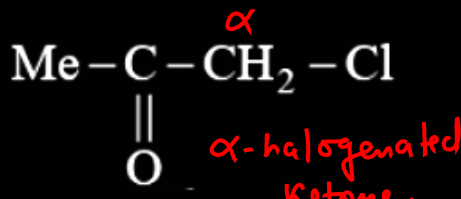
$= 2^1$

$= 2$

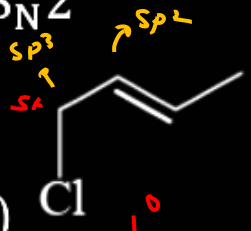
10

Find out correct order of reactivity of given compounds with respect to S_N2

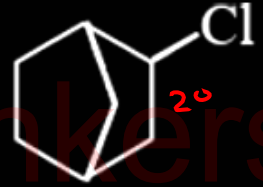
JEE 1



(P)



(Q)



(R)

(S) $Me - \ddot{O} \leftarrow CH_2 - Cl$

(A) p > s > q > r

(B) r > q > s > p

(C) s > p > r > q

(D) q > r > p > s

11

JEE 1

The magnitude of crystal field splitting energy Δ_t in tetrahedral complexes is less than that in octahedral fields. Consider the following statements.

(SI) : There are four ligands instead of six, so the ligand field is only two third the size and hence the ligand field splitting is also two third the size.

(SII) : The direction of orbitals does not coincide with the direction of the ligands. This reduces the crystal field splitting by roughly a further two thirds.

(A) SI is true, SII is false

(B) SI is false, SII is true

(C) SI and SII both are true

(D) SI and SII both are false

factor due to no of ligands $= \frac{4}{6} = \frac{2}{3}$

factor due to direction of approach $= \frac{2}{3}$

$$\Delta_t = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}\Delta_o$$

12

Which d-block element has highest melting point?

(A) Ru

(B) Os

(C) W

(D) Hf

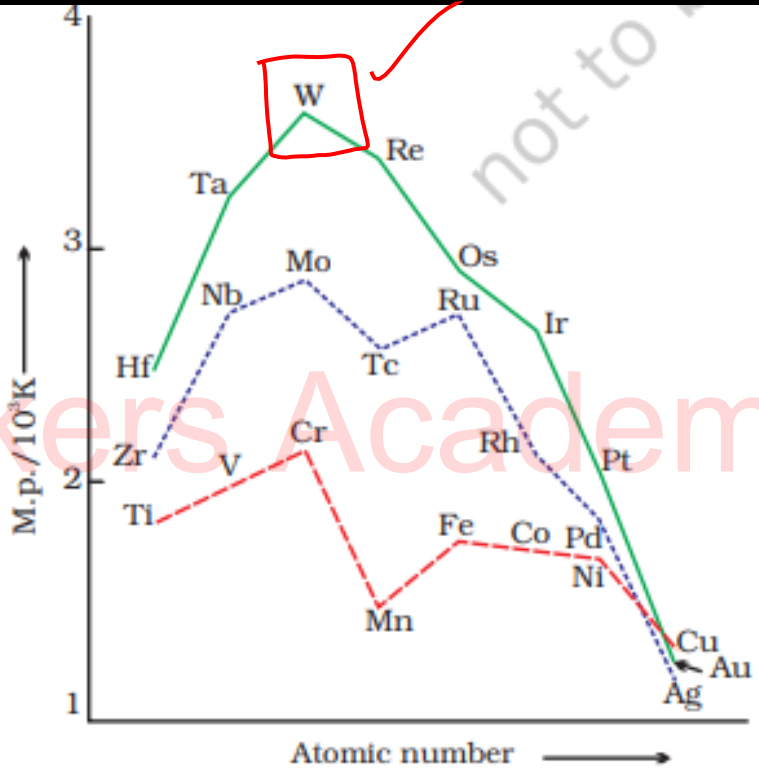


Fig. 8.1: Trends in melting points of transition elements

13

The radial component of Schrodinger wave equation for hydrogen atom is

$$\psi_{2s} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right)e^{\frac{-r}{a_0}},$$

where a_0 is Bohr's radius. If the radial node in 2s orbital is at r_0, then r_0 would be equal to

(A) $\frac{a_0}{2}$ (B) $2a_0$

(C) $\sqrt{2}a_0$ (D) $\frac{a_0}{\sqrt{2}}$

$$2 - \frac{r}{a_0} = 0$$

$$2 = \frac{r}{a}$$

$$r_0 = 2a_0$$

14

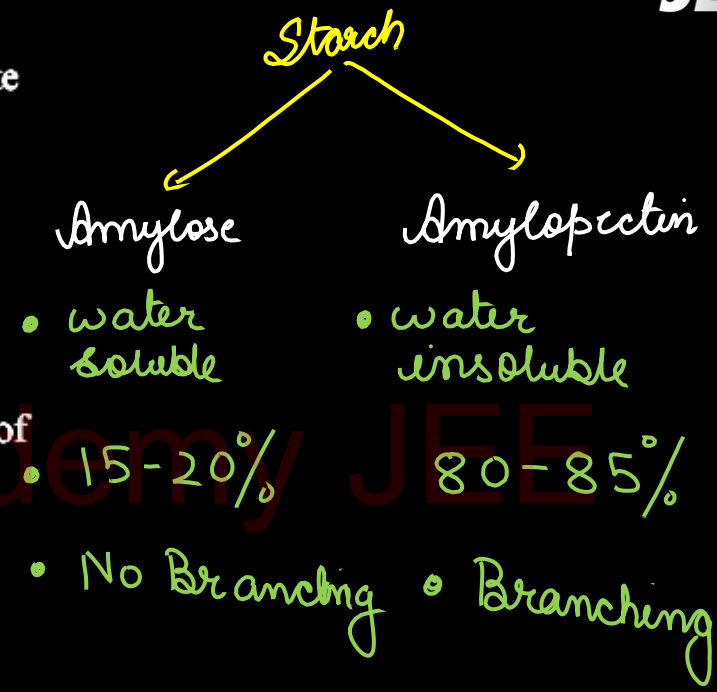
The incorrect statement is

(A) Amylose is water soluble which constitute 15 – 20% of starch

(B) Amylopectin is water soluble which constitute 80 – 85% of starch

(C) Amylopectin is water insoluble

(D) Amylopectin is branched chain polymer of α-D-glucose



15

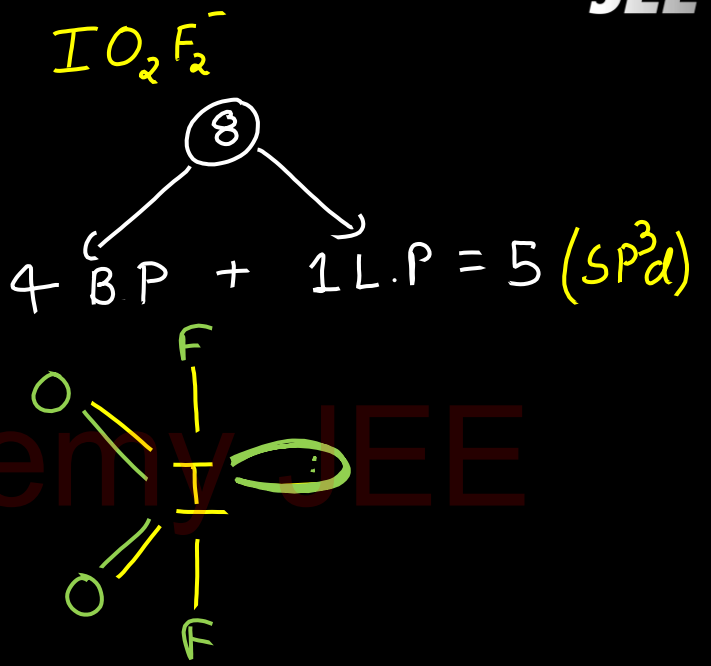
Which of the following statement is not true for $IO_2F_2^-$ according to VSEPR theory?

(A) The lone pair and two I-O double bonds occupy the equatorial positions of trigonal bipyramid

(B) It has sp^3d hybridization and is See-Saw shaped

(C) Its structure is analogous to SF_4

(D) One of the F occupy equatorial position



Shape: See-saw
Geo: TBP

16

A monoatomic ideal gas expands by doing q/2 work as a result of absorption of 'q' amount of heat. The molar heat capacity for the process is:

(A) R (B) 2R

(C) 3R (D) 4R

$q \propto \Delta T$

$q = nC\Delta T$

$\Delta U = nC_v\Delta T$

$C_v = f\frac{R}{2}$

For monoatomic gas, $f = 3$

$C_v = \frac{3R}{2}$

1st law of thermodynamics

$\Delta U = q + W$

$\Delta U = q - \frac{q}{2} = \frac{q}{2}$

$2(nC_v\Delta T) = nC\Delta T$

$\Rightarrow \boxed{C = 2C_v} \Rightarrow C = 2\left(\frac{3R}{2}\right) = 3R$

17

JEE 1

20ml of $\frac{M}{10}$ NH_4OH solution is mixed with 16ml of $\frac{M}{10}$ HCl solution. The pH of the resulting solution is: (pK_b of $NH_4OH = 4.74$)

(A) 9.26 (B) 6.85

(C) 4.74 (D) 8.65

$NH_4OH + HCl \longrightarrow NH_4Cl + H_2O$

2 mmol	1.6 mmol	0	0
0.4 mmol	0	1.6 mmol	

to

teq.

no of millimoles = $M \times V_{ml}$

$POH = pK_b + \log \frac{Salt}{Base}$

$= 4.74 + \log \frac{1.6}{0.4}$

$POH = 5.34$

$PH = 14 - POH = 14 - 5.34$

$PH = 8.65$

18

JEE 1

A complex of iron and cyanide ions is 100% ionized at 1 molal. If its elevation in boiling point is 2.08°, then the complex is:

(Given $K_b = 0.52°C mol^{-1}$ kg)

(A) $Fe_4[Fe(CN)_6]_3$ $n = 7$

(B) $K_3[Fe(CN)_6]$ $n = 4$

(C) $K_4[Fe(CN)_6]$ $n = 5$

(D) $Fe_3[Fe(CN)_6]_2$ $n = 5$

For 100% dissociation, $n = i$

$$\Delta T_b = i\, m\, K_b$$

$$2.08 = i \times 1 \times 0.52$$

$\Rightarrow i = 4$ so $n = 4$

19

JEE 1

The atomic number of elements A, B, C and D are $Z-1$, Z, $Z+1$ and $Z+2$ respectively. If B is a noble gas, choose the correct option from the following options.

I. A has highest electron affinity ✓

II. C exists in +2 oxidation state ✗

III. D is an alkaline earth metal ✓

(A) I and II

(B) II and III

(C) I and III

(D) I,II and III

Z = Noble gas ⇒ B

$Z-1$ = Halogen = A

$Z+1$ = Alkali metal = C

$Z+2$ = Alkaline earth = D

20

JEE 1

The increasing order of the first ionisation enthalpies of the elements B, P, S, and F is

(A) F < S < P < B

(B) P < S < B < F

(C) B < P < S < F

(D) B < S < P < F

F is a smallest halogen,

Among P & S

$P = ns^2 np^3$ → half-filled

$S = ns^2 np^4$

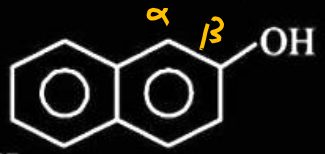
P > S

B < S < P < F

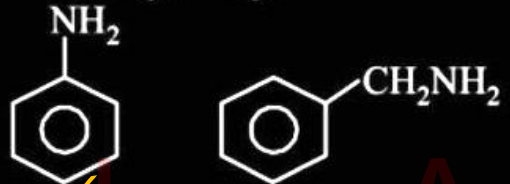
21

JEE 1

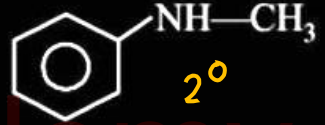
Number of amines which will form colourful dye with HNO_2 and



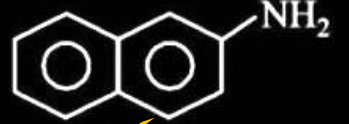
B P A
↓ ↓ ↓
R O Y



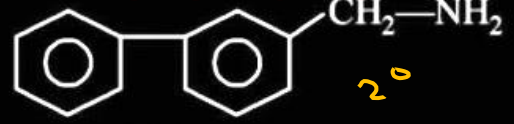
(I) (II) ✗



(III) ✗



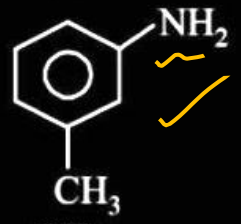
(IV)



(V) ✗

1° Aromatic Amines.

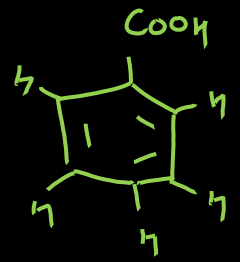
(VI) ✗



(VII)

22

A quantity of 2.0 g of benzoic acid dissolved in 26.0 g of benzene shows a depression in freezing point equal to 1.60 K. Molal depression constant of benzene is 4.88 $K - kg\ mol^{-1}$. The percentage association of the acid is (nearest integer)



JEE 1

$\Delta T_f = i K_f m$

$1.60 = i \times 4.88 \times \frac{2}{122} \times \frac{1000}{26}$ (u)

$\frac{26}{13}$

$i = \frac{1.60 \times 13}{40 \times 100} = 0.52$ (4)

$2B \longrightarrow B_2$

t : 0	1	-
t : t	$1-\alpha$	$\frac{\alpha}{2}$

$i = \left(1 - \alpha + \frac{\alpha}{2}\right) = 1 - \frac{\alpha}{2} = 0.52$

$\alpha = 0.96$

$i = 96.1$

23

JEE 1

The reduction potential at 25°C for $Fe^{3+} \mid Fe^{2+}$ electrode is +0.718 V. If $E^{\circ}_{Fe^{2+}|Fe} = -0.44$ V and $E^{\circ}_{Fe^{3+}|Fe} = -0.04$ V, the ratio of molar concentrations of Fe^{2+} to Fe^{3+} ions in solution is (2.303RT/F = 0.06, log 5 = 0.7)

$Fe^{+3} + e^- \longrightarrow Fe^{+2}$ $E_{Fe^{+3}/Fe^{+2}} = 0.718$ V

$0.718 = E^{\circ}_{Fe^{+3}/Fe^{+2}} - \frac{0.06}{1} \log \frac{[Fe^{+2}]}{[Fe^{+3}]}$

0.76

Ans 5

① $Fe^{+3} + 3e^- \longrightarrow Fe$ $E^{\circ}_1 = -0.04$V

② $Fe \longrightarrow Fe^{+2} + 2e^-$ $E^{\circ}_2 = +0.44$V

$e^- + Fe^{+3} \longrightarrow Fe^{+2}$ $E^{\circ}_{Fe^{+3}/Fe^{+2}}$ ③

$E^{\circ}_{Fe^{+3}/Fe^{+2}} = \frac{(3x - 0.04) + (0.44 \times 2)}{1}$

$= 0.76$V

24

A solution is made by mixing 200ml of 0.1 M $-$ $FeSO_4$, 200ml of 0.1M $-$ $KMnO_4$ and 600ml of 1M $-$ $HClO_4$. A reaction occurs in which Fe^{2+} and MnO_4^- convert to Fe^{3+} and Mn^{2+}. If the molarity of H^+ ion in the final solution is ' x ' M, then the value of 1000x is

$Fe^{+2} \longrightarrow Fe^{+3} + e^-$] x 5

$5e^- + 8H^+ + MnO_4^- \longrightarrow Mn^{+2} + 4H_2O$

$5Fe^{+2} + MnO_4^- + 8H^+ \longrightarrow 5Fe^{+3} + Mn^{+2} + 4H_2O$

24

JEE 1

LR

$5Fe^{+2} + MnO_4^- + 8H^+ \longrightarrow 5Fe^{+3} + Mn^{+2} + 4H_2O$

$t:0$	mole	$\frac{20}{1000}$	$\frac{20}{1000}$	$\frac{600}{1000}$
$t:t$	—		$x = [H^+] : \left[\frac{600}{1000} - \frac{8}{5} \times \frac{20}{1000}\right]$	

$5 \longrightarrow 8$

$\frac{20}{1000} \longrightarrow \frac{8}{5} \times \frac{20}{1000}$

$1000x = [600 - 32]$

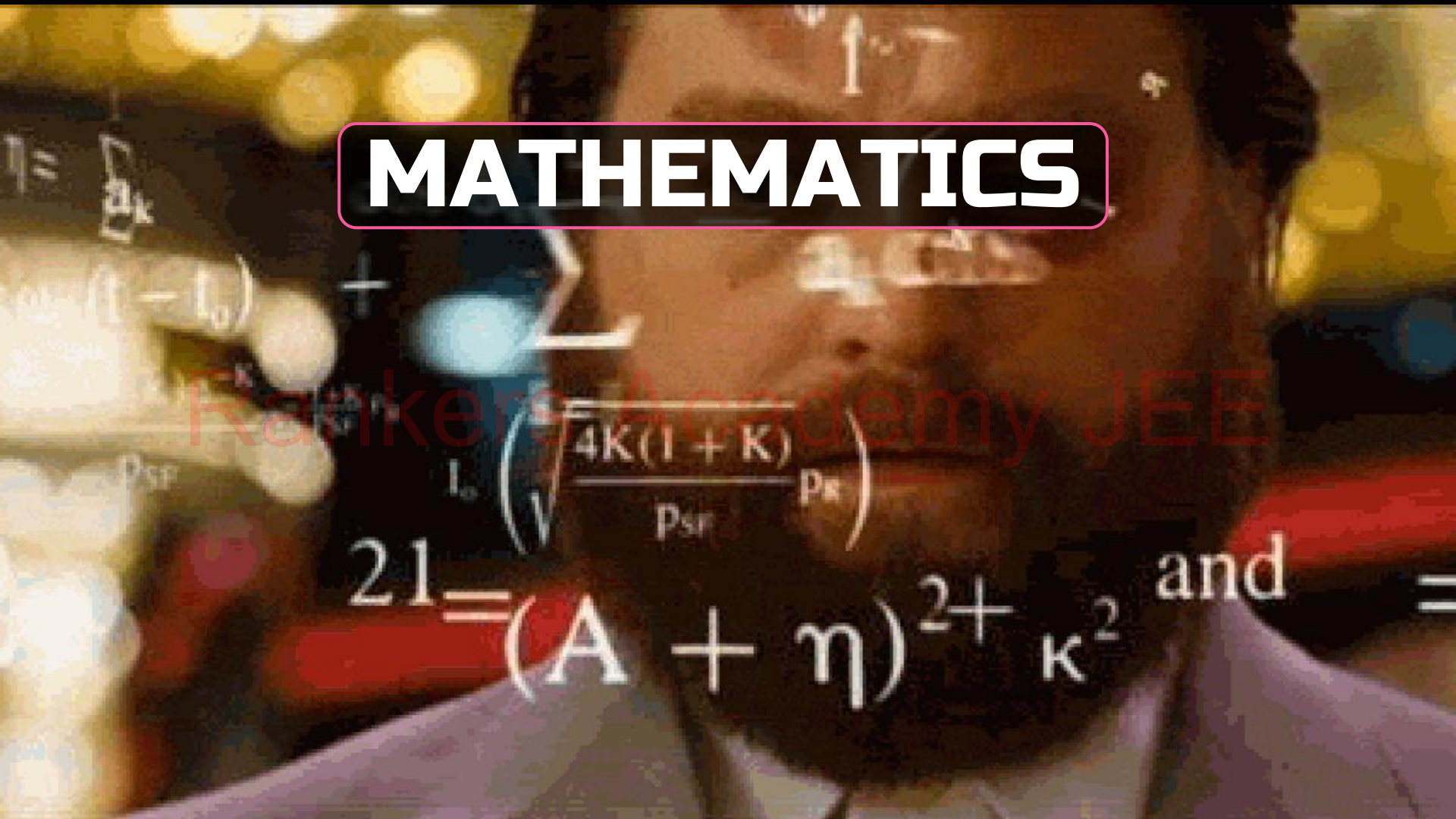
$= 568$

25

JEE 1

. Number of compounds which forms aldehyde and ketone both in the same molcules on ozonolysis among the given compounds are

Ans. 1

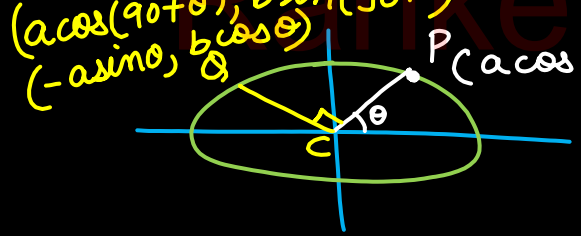

MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

P and Q are points on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose centre is C. The eccentric angle of P and Q differ by a right angle. If angle PCQ is minimum, then the eccentric angle of P can be

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}=\theta$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{12}$



JEE 1

$m_{PC}=m_1=\frac{b}{a}\tan\theta$

$m_{QC}=m_2=-\frac{b}{a}\cot\theta$

$\tan\alpha=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\alpha=\left|\frac{\frac{b}{a}(\tan\theta+\cot\theta)}{1-\frac{b^2}{a^2}}\right|$

$\frac{\tan\theta+\cot\theta}{2}\geq\sqrt{\tan\theta\cot\theta}$

2

A real valued function f defined as $f(x) = 2x^2 - \ln|x|$ has

(A) Two minima and one maxima

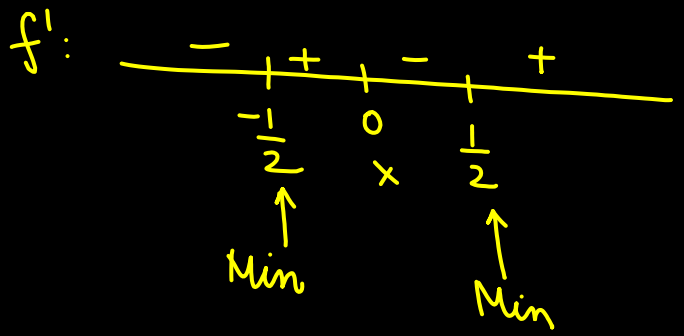
(B) Two maxima and one minima

(C) Only two minima

(D) Only two maxima

$$f(x) = \begin{cases} 2x^2 - \ln x & ; x > 0 \\ 2x^2 - \ln(-x) & ; x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 4x - \frac{1}{x} & ; x > 0 \\ \frac{4x^2 - 1}{x} & \\ 4x - \frac{(+1)}{+x} & ; x < 0 \end{cases}$$



3

The value of $\lim_{x\to 0} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is

(A) Zero (B) $\frac{1}{2}$

(C) 2 (D) Non-existent

JEE 1

$\sin x \longleftrightarrow x$

Shortcut

$$\lim_{x\to 0} \frac{x+2(x)}{\sqrt{x^2+2x+1}-\sqrt{x^2-x+1}}$$

$$\lim_{x\to 0} \frac{3x}{(x^2+2x+1)-(x^2-x+1)} \cdot (\sqrt{x^2+2x+1}+\sqrt{x^2-x+1})$$

$$= \sqrt{0+0+1}+\sqrt{0-0+1}$$

$$= 2$$

4

The straight line L is perpendicular to the line $4x - y = 1$. The area of the triangle formed by the line L and the coordinate axis is 8. Then the equation of the straight line with positive intercept on y-axis, is

$m = \frac{-4}{-1} = 4$

$c > 0$

(A) $4x + y = 8$

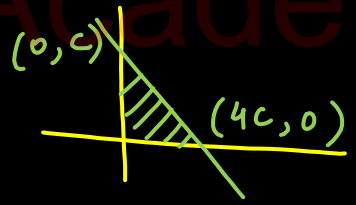
(B) $x + 4y = 4$

(C) $x + 4y = 8$

(D) $4x + y = 4$

$(4)(m_L) = -1$

$m_L = -\frac{1}{4}$



Area $= \frac{1}{2}(4c)(c)$

$= 2c^2 = 8$

$c = 2$

$L : y = -\frac{1}{4}x + c$

$y = -\frac{x}{4} + 2$

$\Rightarrow 4y + x = 8$

5

A square matrix P satisfies $P^2 = I - P$, where I is identity matrix. If $P^n = 13I - 21P$, then n is

(A) 6 (B) 8

(C) 9 (D) 10

JEE 1

Shortcut

$P^2 = I - P$

$P^4 = I^2 + P^2 - 2IP$

$= I + (I - P) - 2P$

$P^4 = 2I - 3P$

$P^8 = 4I + 9(I - P) - 12P$

$= 13I - 21P$

$P^6 = P^4 P^2 = (2I - 3P)(I - P) = 2I - 2P - 3P + 3(I - P)$

$= 5I - 8P$

6

In an experiment with 8 observations $x_i, i = 1, 2, 3, \ldots 8$, the following results were available: $\sum_{i=1}^{8} x_i^2 = 510, \sum_{i=1}^{8} x_i = 58$ If at the time of calculation the observation 21 was wrongly taken as 12 , then the corrected variance is

(A) $\frac{1967}{16}$ (B) 100

(C) $\frac{1967}{8}$ (D) $\frac{1967}{64}$

$x_1 \ldots x_7, 12$

$\rightarrow (x_1 + \ldots + x_7) + 12 = 58$

$\rightarrow (x_1 + \ldots + x_7) = 58 - 12 = 46$

$x_1 \ldots x_7, 21$

$\Rightarrow (x_1 + \ldots + x_7) + 21$

$\Rightarrow 46 + 21 = 67$

$\sum x_i = 67 \quad \bar{x} = \frac{67}{8}$ correct Mean

$x_1^2 + \ldots + x_7^2 + 12^2 = 510$

$(x_1^2 + \ldots + x_7^2) + 21^2 =$

$= 510 - 12^2 + 21^2$

$= 510 + 9(33)$

$= 510 + 297$

$= 807$

6

JEE 1

$$\text{var} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{807}{8} - \left(\frac{67}{8}\right)^2$$

$$= \frac{807 \times 8 - 67^2}{64}$$

7

The number of complex numbers z, satisfying the equations $|z+5|+|z-5|=10$ and $|z+1|=2$, is

(A) Zero (B) 1

(C) 2 (D) Infinite

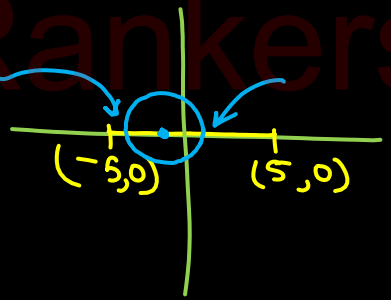
JEE 1

$|z-z_1|=r$

⊙ centre z_1 radius r

$z_1 \equiv (-1,0)$ $r=2$

$|z-k_1|+|z-k_2|=|k_1-k_2|$ ∴ $z, k_1, k_2 \rightarrow$ collinear



$k_1 \equiv (-5,0)$

$k_2 \equiv (5,0)$

∴ z lies on x-axis

z purely real

8

A man throws a fair coin 9 numbers of time and gets 2 points for each head he throws and 1 point for each tail he throws. The probability that he gets exactly 12 points is

(A) $\frac{7}{256}$ (B) $\frac{21}{256}$

(C) $\frac{7}{128}$ (D) $\frac{21}{128}$

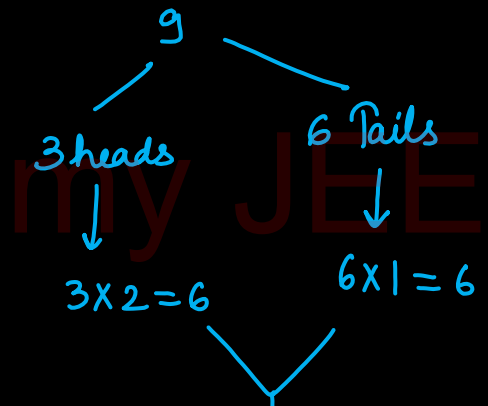
H	T	
9	0	18
8	1	17

6 3 15

4 5 13

3 6 12

2 7 11



$$\binom{9}{3}\left(\frac{1}{2}\right)^3 \binom{6}{6}\left(\frac{1}{2}\right)^6 = \frac{{}^9C_3}{2^9}$$

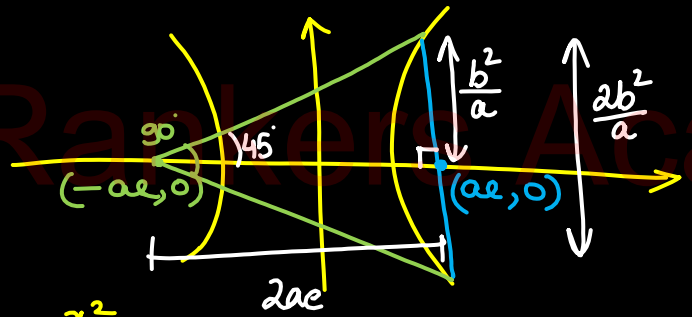
H

9

In given hyperbola the latus rectum subtends right angle at the other focus of the hyperbola, then its eccentricity is

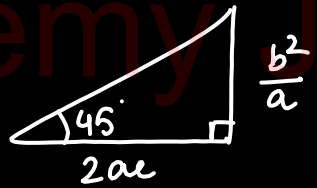
(A) $\sqrt{2}+1$ (B) $2+\sqrt{2}$

(C) $\sqrt{2}$ (D) $2+\sqrt{3}$



$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$L.R. = \frac{2b^2}{a}$



$\therefore \frac{b^2}{a} = 2ae$

$\frac{b^2}{a^2} = 2e$

9

JEE 1

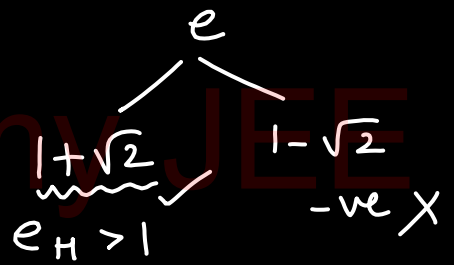
$$e^2 = 1 + \frac{b^2}{a^2}$$

$$e^2 = 1 + (2e)$$

$$\Rightarrow e^2 - 2e - 1 = 0$$

$$\Rightarrow e = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$



10

Given that $(\vec{\beta}-\vec{\alpha})\cdot(\vec{\beta}+\vec{\alpha})=8$ and $\vec{\alpha}\cdot\vec{\beta}=2$.

Also, $|\vec{\alpha}|=1$, then angle between $(\vec{\beta}-\vec{\alpha})$ and $(\vec{\beta}+\vec{\alpha})$ is

(A) $\cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$ (B) $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$

(C) $\cos^{-1}\left(\frac{5}{\sqrt{21}}\right)$ (D) $\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$

JEE 1

$|\beta|^2-|\alpha|^2=8$

$|\beta|=3$

$\vec{\alpha}\cdot\vec{\beta}=(1)(3)\cos\theta=2$

$\cos\theta=\frac{2}{3}$

$$\cos^{-1}\left(\frac{(\vec{\beta}-\vec{\alpha})\cdot(\vec{\beta}+\vec{\alpha})}{|\vec{\beta}-\vec{\alpha}|\,|\vec{\beta}+\vec{\alpha}|}\right)$$

$$=\cos^{-1}\left(\frac{8}{\sqrt{6}\sqrt{14}}\right)$$

$$=\cos^{-1}\left(\frac{8}{2\sqrt{3}\sqrt{7}}\right)$$

10

JEE 1

$$|\vec{\beta}+\vec{\alpha}|^2 = |\vec{\beta}|^2 + |\vec{\alpha}|^2 + 2\vec{\alpha}\cdot\vec{\beta}$$

$$= 9 + 1 + 2(2)$$

$$= 14$$

$$|\vec{\beta}-\vec{\alpha}|^2 = |\vec{\beta}|^2 + |\vec{\alpha}|^2 - 2\vec{\alpha}\cdot\vec{\beta}$$

$$= 9 + 1 - 2(2)$$

$$= 6$$

11

If a set A has 10 elements, then the ratio of total number of reflexive relations to total number of symmetric relations is

(A) 2^{35} (B) 2^{45}

(C) 2^{42} (D) 2^{47}

JEE 1

Elements : a, b - - - j

100

Ref : (a, a), (b, b), ⋮, (j, j) — 10 combi — Must come

90 combi (2 ways) — 2^{90}

$1 \times 2^{90} = 2^{90}$ ways $= 2^{n^2-n}$

Sym (a, b) ⟶ (b, a)

$2^{10} \cdot 2^{45}$

$= 2^{n} \, 2^{\frac{(n^2-n)}{2}}$

$= 2^{\frac{n^2+n}{2}}$

$= 2^{\frac{100+10}{2}} = 2^{55}$

12

If $a_n = n^2 + 5n + 6$ and $\sum_{r=1}^{668} \frac{1}{a_r} = \frac{668}{\lambda}$, then λ equals

(A) 671 (B) 2014

(C) 2013 (D) 668

JEE 1

$$\sum_{n=1}^{668} \frac{1}{(n+2)(n+3)} \qquad 1 = (n+3) - (n+2)$$

$$\Rightarrow \sum \frac{1}{n+2} - \frac{1}{n+3}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} \cdots - \frac{1}{671}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{671} = \frac{668}{3 \cdot 671} = \frac{668}{\lambda}$$

$$\lambda = (3)(671)$$

13

Let $f_k(x) = \frac{\sin^k x + \cos^k x}{k}$ then $f_4(x) - f_6(x)$ is equal to

(A) $\frac{1}{4} - \frac{1}{6}\cos^2 2x$

(B) $\frac{1}{12} + \frac{1}{4}\sin^2 2x$

(C) $\frac{1}{3} - \frac{1}{4}\cos^2 x$

(D) $\frac{1}{12}$

JEE 1

$$\Rightarrow \frac{\sin^4 x + \cos^4 x}{4} - \frac{\sin^6 x + \cos^6 x}{6}$$

$$\Rightarrow \frac{\sin^4 x + \cos^4 x}{4} - \frac{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)}{6}$$

$$\Rightarrow \frac{6(s^4 + c^4) - 4(s^4 + c^4 - s^2c^2)}{24}$$

$$\Rightarrow \frac{2(s^4 + c^4 + 2s^2c^2)}{24} = \frac{(s^2 + c^2)^2}{12} = \frac{1}{12}$$

14

JEE 1

$I =$ The value of $\int_{1/3}^{3} \frac{\tan\left(x^2-\frac{1}{x^2}\right)}{\sin\left(x+\frac{1}{x}\right)} \frac{dx}{x}$ is equal to

(A) ZERO

(B) $\frac{3}{2}$

(C) $\frac{1}{2}$

(D) $\frac{4}{3}$

Let $x = \frac{1}{t} \Rightarrow dx = \frac{-1}{t^2} dt$

$$I = \int_{3}^{\frac{1}{3}} \frac{\tan\left(\frac{1}{t^2}-t^2\right)}{\sin\left(\frac{1}{t}+t\right)} \cdot t \cdot \left(\frac{-1}{t^2}\right) dt$$

$$= -\int_{\frac{1}{3}}^{3} \frac{+\tan\left(t^2-\frac{1}{t^2}\right)}{\sin\left(t+\frac{1}{t}\right)} \left(+\frac{1}{t}\right) dt$$

$$= -I$$

$I = -I$

$2I = 0$

$I = 0$

15

The domain of the function

$$f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right) + \frac{1}{\log\{2-x\}}$$

(where (.) denotes the fractional part function) is

(A) $\{x: x \in I \text{ and } x \geq 3\}$

(B) $\{x: x \in I \text{ and } x \leq 4\}$

(C) $(-\infty, 0) \cup (0,1)$

(D) ϕ (empty set)

$\frac{1}{2}\left(x + \frac{1}{x}\right) \geq \frac{2}{2}$; $x > 0$

≥ 1

$\leq \frac{-2}{2}$; $x < 0$

≤ -1

$x = 1, -1$

$\{\cdot\} \in [0, 1)$

$\{2-x\} \neq 0$

$2 - x \neq \text{int}$

$x \neq \text{int}$

16

A curve passing through (1,2) and satisfying $\int_0^x t^2 y(t)dt = x^3 y(x)(x > 0)$ is

(A) $x^2y = 2$

(B) $x^2 + y^2 = 5$

(C) $\frac{x^2}{2} + \frac{y^2}{8} = 1$

(D) $y^2 = 4x$

JEE 1

Diff wrt x

$\Rightarrow x^2 y = 3x^2 y + x^3 y'$

$\Rightarrow -2x^2 y = x^3 y'$

$\Rightarrow \frac{-2}{x} = \frac{y'}{y}$

$\Rightarrow \int \frac{-2}{x} dx = \int \frac{dy}{y}$

$\Rightarrow -2\ln|x| = \ln|y| + \ln c$

$\Rightarrow \ln \frac{1}{x^2} = \ln|cy|$

$\Rightarrow |cy| = \frac{1}{x^2}$

$x = 1 \quad y = 2$

$|2c| = 1 \Rightarrow |c| = \frac{1}{2}$

$\left|\frac{y}{2}\right| = \frac{1}{x^2}$

17

The term which is independent of x in the binomial expansion of $\left(\frac{1-x}{x^{1/2}-x}+\frac{x-1}{x^{2/3}+x^{1/3}+1}\right)^{10}$ is T_{r+1}, then T_{r+1} is equal to

(A) ${}^{10}C_3$ (B) ${}^{10}C_4$

(C) ${}^{10}C_5$ (D) ${}^{10}C_2$

$$\frac{1-x}{\sqrt{x}-x}+\frac{(x^{1/3})^3-1^3}{x^{2/3}+x^{1/3}+1}$$

$$\frac{(1+\sqrt{x})(1-\sqrt{x})}{\sqrt{x}(1-\sqrt{x})}+(x^{1/3}-1)$$

$$=\left(1+\frac{1}{\sqrt{x}}\right)+(x^{1/3}-1)$$

17

JEE 1

$\left(x^{1/3} + \frac{1}{x^{1/2}}\right)^{10}$ ⟶ Term inde of x

coeffi of x^0

$r = \frac{np - m}{p + q} = \frac{10\left(\frac{1}{3}\right) - 0}{\frac{1}{3} + \frac{1}{2}} = \frac{10}{3} \cdot 6 = 4$

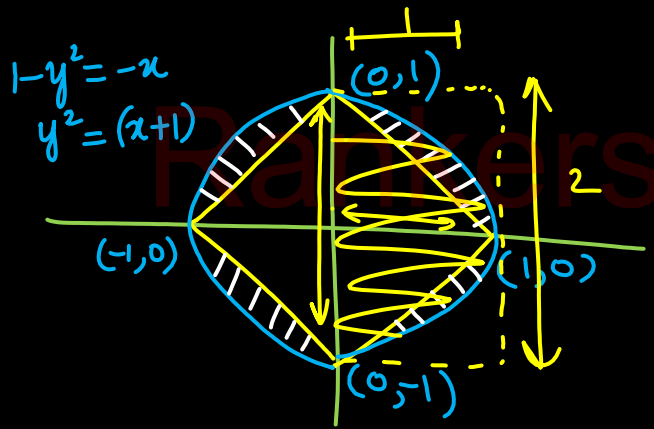
$T_5 = T_{4+1} = (10C_4)(x^0)$

18

The area of the region bounded by the curves $1 - y^2 = |x|$ and $|x| + |y| = 1$ is A, then 12A equals

(A) 2/3 (B) 8

(C) 4 (D) 14/3



$1 - y^2 = x$

$y^2 = 1 - x$

$y^2 = -(x-1)$

$= 2\left(\frac{2}{3}(1)(2) - \frac{1}{2}(2)(1)\right) = 2\left(\frac{4}{3} - 1\right) = \frac{2}{3}.$

$A = 2/3$

$12A = 12\left(\frac{2}{3}\right) = 8.$

19

Sum of the infinite terms of the series

$\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+$

$\cot^{-1}\left(3^2+\frac{3}{4}\right)+\cdots\infty$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\tan^{-1}2$ (D) $\tan^{-1}3$

$\tan^{-1}\left(\frac{4}{3+4\cdot1^2}\right)+\tan^{-1}\left(\frac{4}{3+4\cdot2^2}\right)+\cdots$

$T_n=\tan^{-1}\left(\frac{4}{3+4n^2}\right)=\tan^{-1}\left(\frac{1}{n^2+\frac{3}{4}}\right)$

$=\tan^{-1}\left(\frac{1}{1+n^2+\frac{3}{4}-1}\right)$

$=\tan^{-1}\left(\frac{1}{1+n^2-\frac{1}{4}}\right)$

19

JEE 1

$$= \tan^{-1}\left(\frac{(n+\frac{1}{2})-(n-\frac{1}{2})}{1+(n+\frac{1}{2})(n-\frac{1}{2})}\right)$$

$$T_n = \tan^{-1}\left(n+\tfrac{1}{2}\right) - \tan^{-1}\left(n-\tfrac{1}{2}\right)$$

$$S_n = \tan^{-1}\tfrac{3}{2} - \tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{5}{2} - \tan^{-1}\tfrac{3}{2}$$

$$\tan^{-1}\left(n+\tfrac{1}{2}\right) -$$

$$= \tan^{-1}\left(n+\tfrac{1}{2}\right) - \tan^{-1}\tfrac{1}{2}$$

$$= \tan^{-1}\infty - \tan^{-1}\tfrac{1}{2}$$

$$= \frac{\pi}{2} - \tan^{-1}\tfrac{1}{2} = \cot^{-1}\tfrac{1}{2}$$

$$= \tan^{-1} 2$$

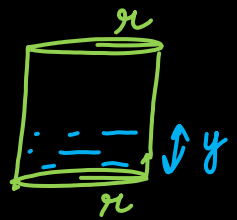
20

JEE 1

Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. It is measured in minutes and $k = \frac{1}{15}$ then the time to drain the tank if the water is 4 meter deep to start with is

(A) 30 min (B) 45 min

(C) 60 min (D) 80 min



$\frac{dy}{dt} \propto -\sqrt{y}$

$\frac{dy}{dt} = -k\sqrt{y}$

$\Rightarrow \frac{dy}{dt} = \frac{-\sqrt{y}}{15}$

$t = 0 \longrightarrow t = T$

$y = 4 \longrightarrow y = 0$

$\Rightarrow \int_4^0 \frac{dy}{\sqrt{y}} = -\int_0^T \frac{dt}{15}$

$\Rightarrow \frac{y^{1/2}}{1/2}\Big|_4^0 = -\frac{t}{15}\Big|_0^T$

$\Rightarrow 2(0-2) = -\frac{T}{15}$.

21

If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$, then $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$ ky, where k is equal to 25.

JEE 1

$2x = y^{1/5} + \frac{1}{y^{1/5}}$

$2x = t + \frac{1}{t}$

$t^2 - 2xt + 1 = 0$

$t = \frac{2x \pm \sqrt{4x^2-4}}{2}$

$y^{1/5} = x \pm \sqrt{x^2-1}$

$y = (x \pm \sqrt{x^2-1})^5$

$y = (x+\sqrt{x^2-1})^5$

$\frac{dy}{dx} = 5(x+\sqrt{x^2-1})^4\left(1+\frac{2x}{2\sqrt{x^2-1}}\right)$

$\frac{dy}{dx} = \frac{5(x+\sqrt{x^2-1})^5}{\sqrt{x^2-1}} = \frac{5y}{\sqrt{x^2-1}}$

$\frac{d^2y}{dx^2} = 5\left(\frac{\sqrt{x^2-1}\,\frac{dy}{dx} - y\,\frac{2x}{2\sqrt{x^2-1}}}{x^2-1}\right)$

$\frac{d^2y}{dx^2} = 5\left(\frac{(x^2-1)\frac{dy}{dx} - xy}{(x^2-1)^{3/2}}\right)$

21

JEE 1

$$5(x^2-1)\frac{\left((x^2-1)\frac{dy}{dx}-xy\right)}{(x^2-1)^{3/2}}+x\left(\frac{5y}{\sqrt{x^2-1}}\right)=ky$$

$$\Rightarrow 5(x^2-1)\frac{\frac{dy}{dx}}{(x^2-1)^{1/2}}=ky$$

$$\Rightarrow 5\left(\frac{dy}{dx}\right)\left(\sqrt{x^2-1}\right)=ky$$

$$\Rightarrow 5\left(\frac{5y}{\sqrt{x^2-1}}\right)\left(\sqrt{x^2-1}\right)=ky$$

$$\Rightarrow k=25$$

22

Given the relation $R = \{(2,3), (3,4)\}$ on the set $\{2,3,4\}$. The number of minimum number of ordered pairs to be added to R so that R is reflexive and symmetric [5]

JEE 1

(2,3) , (3,4)

{ (2,2), (3,3), (4,4)
(3,2) , (4,3)

(2,2) ~~(2,3)~~ (2,4)

(3,2) (3,3) ~~(3,4)~~

(4,2) (4,3) (4,4)

23

Value of n if the sum of $(n-1)$ terms of the series $\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\cdots$ is equal to $2+\sqrt{3}$ is

angles AP

$a=\pi/n$

$(n-1)$ terms

$d=\frac{\pi}{n}$

$$= \frac{\sin\left((n-1)\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)} \cdot \sin\left(\frac{\pi}{n}+(n-1-1)\frac{\pi}{2n}\right)$$

$$= \frac{\sin\left(\frac{\pi}{2}-\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)} \sin\left(\frac{\pi}{n}+\frac{n\pi}{2n}-\frac{\pi}{n}\right)$$

23

JEE 1

$$\frac{\cos\frac{\pi}{2n}}{\sin\frac{\pi}{2n}} = \cot\frac{\pi}{2n} = 2+\sqrt{3} = \cot 15^\circ$$

$$= \cot\frac{\pi}{12}$$

$$n = 6$$

24

Six numbers are in A.P. such that their sum is 3 and the first number is four times the third number. The sum of positive terms in the A. P is

Ans : 15.

JEE 1

$a, a+d, a+2d, a+3d, a+4d, a+5d$

$\Rightarrow \frac{6}{2}[2a+(6-1)d] = 3$

$\Rightarrow 3(2a+5d) = 3$

$\Rightarrow 2a+5d = 1$ —— (1)

$a = 4(a+2d) \Rightarrow 3a+8d=0$ —— (2)

$2a+5\left(-\frac{3a}{8}\right) = 1$

$a = 8$

$d = -3$

25

Let a real valued function f satisfy $f(x+y) = f(x) \cdot f(y), \forall x, y \in N$, where $f(1) = 3$, and $\sum_{x=1}^{n} f(x) = 120$. Then the value of n is 4 Ans.

JEE 1

$f(x+y) = f(x) \cdot f(y)$

$f(x) = a^x$

$f(1) = a^1 = a = 3$

$f(x) = 3^x$

$120 = 3^1 + 3^2 + \dots + 3^n$

$120 = 3\left(\frac{3^n - 1}{3 - 1}\right)$

$3^n = 81$

$n = 4$ ✓