FIITJEE

ALL INDIA TEST SERIES

FULL TEST – II

JEE (Main)-2025

TEST DATE: 05-01-2025

Physics **PART – A**

SECTION – A

1. B

Sol. $v_y = u_y - gt$ ...(i)

$x = u_x t$ ...(ii)

$$v_y = u_y - g\frac{x}{u_x}$$

$$\Rightarrow \frac{g}{u_x} = \tan 45^\circ \Rightarrow u_x = 10 \text{ m/s}$$

Also, Range $R = \frac{2u_x u_y}{g}$

$$40 = \frac{2 \times 10 \times u_y}{g} \Rightarrow u_y = 20 \text{ m/s}$$

$$\tan\theta = \frac{u_y}{u_x} = 2$$

$$\Rightarrow \theta = \tan^{-1}(2)$$

2. B

Sol. Maximum loss in kinetic energy

$$\Delta K.E_{max} = \frac{1}{2}\frac{Mm}{(M+m)}u^2$$

$$\Delta K.E_{max} \le \frac{3}{4}\left(\frac{1}{2}mu^2\right)$$

$$\frac{1}{2}\frac{Mm}{(M+m)}u^2 \le \frac{3}{4}\left(\frac{1}{2}mu^2\right)$$

$$\Rightarrow \frac{M}{m} \le 3$$

3. D

Sol. $-\frac{GMm}{R}+\frac{1}{2}mv^2=0$

$\frac{GMm}{R}=\frac{1}{2}mv^2$

$v=\sqrt{\frac{2GM}{R}}$

$\Delta v=(\sqrt{2}-1)\sqrt{\frac{GM}{R}}$

4. B

Sol. $R=\frac{\sqrt{2mk}}{Bq}$

$m \,\alpha\, R^2q^2$

$\frac{m_1}{m_2}=\left(\frac{R_1q_1}{R_2q_2}\right)^2$

$\left(\frac{6}{5}\times\frac{1}{2}\right)^2=\frac{9}{25}$

5. C

Sol. $V_A=\frac{KP\cos 37°}{\left(\frac{5R}{3}\right)^2}=\frac{9KP}{25R^2}\times\frac{4}{5}$

$V_A=\frac{36KP}{125R^2}$

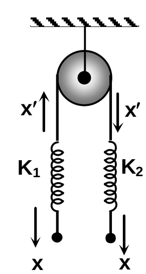
6. B

Sol. $F_R=2K_2(x-x')$

$F_R=\frac{4K_1K_2x}{(K_1+K_2)}$

$K_2(x-x')=k_1(x+x')$

$x'=\frac{(K_2-K_1)x}{(K_1+K_2)}$



7. B

Sol. $\phi_{AFC}=\phi_{cube}=\frac{\lambda a}{4\varepsilon_0}$

8. C

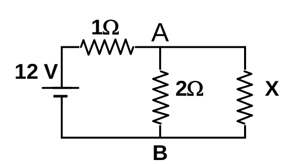
Sol. Let equivalent resistance across battery be x

$\frac{2x}{2+x}+1=x$

$\Rightarrow$ x = 2 Ω

$V_{AB}=6V$

$I_{AB}=3A$



9. B

Sol. $T2\pi r\cos\theta=Mg$

$\Rightarrow M\propto r$

$$\frac{M_1}{M_2}=\frac{r_1}{r_2}=\frac{r}{4r}$$
$$M_2=4M_1$$
$$M_2=4M$$

10. D
Sol. $U_i+W_B=U_f+\text{Heat}$
$$\frac{CV^2}{4}+\frac{CV^2}{2}=\frac{CV^2}{2}+\text{Heat}$$
$$\text{Heat}=\frac{CV^2}{4}$$

11. D
Sol. $I_D=\varepsilon_0\frac{d\phi_E}{dt}=\varepsilon_0(\text{slope})$

12. B
Sol. $\vec{v}_{OG}=9\hat{i}+12\hat{j}$, $\vec{v}_{MG}=-2\hat{i}$
For x-axis : $\vec{v}_{IM}=-m^2\vec{v}_{OM}$
For y-axis: $\vec{v}_{I_y}=-m\vec{v}_{O_y}$
$$\vec{v}_{IG}=-46\hat{i}-24\hat{j}\ \text{m/s}$$

13. A
Sol. $e=\int\frac{\mu_0 Idxv}{2\pi x}=\frac{\mu_0 Iv}{2\pi}\ell n2$
$q_{max}=Ce$

14. C
Sol. $a=\frac{g\sin\theta}{\left(1+\frac{I_{cm}}{MR^2}\right)}$
$$\frac{\sin\theta_D}{\left(1+\frac{1}{2}\right)}=\frac{\sin\theta_H}{\left(1+\frac{2}{3}\right)}$$
$$\frac{2\sin\theta_D}{3}=\frac{3\sin\theta_H}{5}$$
$$\frac{\sin\theta_D}{\sin\theta_H}=\frac{9}{10}$$

15. B
Sol. $\frac{nR\Delta T}{n\left(R+\frac{C_V}{n}\right)\Delta T}$
$$\Rightarrow\frac{nR}{C_V+nR}$$

16. A
Sol. Conceptual

17. B

Sol. $2I_0 = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$\frac{1}{\sqrt{2}} = \cos\left(\frac{\phi}{2}\right)$

$\frac{\phi}{2} = \frac{\pi}{4}$

$\phi = \frac{\pi}{2}$

$\Rightarrow \Delta x = \frac{\lambda}{4}$

$\frac{yd}{D} = \frac{\lambda}{4}$

$y = \frac{\lambda D}{4d}$

18. B

Sol. $\Delta E = \frac{hC}{\lambda}$

$\frac{E}{2} = \frac{hC}{\lambda_2}$ ...(i)

$2E = \frac{hC}{\lambda_1}$ ...(ii)

$\frac{1}{4} = \frac{\lambda_1}{\lambda_2}$

19. B

Sol. Conceptual

20. A

Sol. $-\int_0^x mg\cos\theta kx^2 dx + \int_0^x mg\sin\theta dx = \frac{1}{2}mv^2$

$-mg\cos\theta\frac{kx^3}{3} + mg\sin\theta x = K.E.$

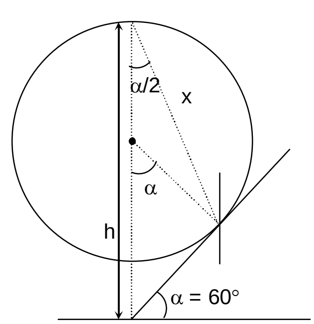
For K.E. to be maximum

$\frac{dK.E.}{dx} = -mg\cos\theta kx^2 + mg\sin\theta = 0$

SECTION – B

21. 2

Sol. For time 't' all beads lie on a circle



$x = \frac{h}{\sqrt{3}}$

$x = \frac{1}{2}g\cos\left(\frac{\alpha}{2}\right)t^2$, $\frac{h}{\sqrt{3}} = \frac{1}{2}g\cos 30°t^2$

$\frac{h}{\sqrt{3}} = \frac{1}{2}g\frac{\sqrt{3}}{2}t^2$

t = 2 sec

22. 2

Sol. Magnetic field due to upper loop $(B_1) = \frac{\mu_0(i)}{4\pi r_1}\left(\frac{2\pi}{3}\right)(-\hat{z})$

Magnetic field due to lower loop (B_2) = $(B_2) = \frac{\mu_0(i)}{4\pi r_2}\pi(+\hat{z})$

23. 3

Sol. $q\left[v_i - v_f\right] = k_f - k_i$

$10^{-6}\left[4\times10^5 - v_f\right] = \frac{1}{2}\times2\times10^{-3}[200-100]$

$4\times10^5 - v_f = 10^5$

$v_f = 3\times10^5$ volts

n = 3

24. 3

Sol. For instantaneous values

Let $I = I_0\sin(\omega t - \phi)$

$V_R = IR = I_0 R\sin(\omega t - \phi)$

$\Rightarrow I_0\sin(\omega t-\phi) = 2$

$\frac{E_0}{z}\sin(\omega t - \phi) = 2$

$\cos\phi = \frac{R}{z} = \frac{2z}{E_0}$

$\Rightarrow x_L = \sqrt{3}\Omega$

$V_L + V_R = E = 7$

$\Rightarrow E = 7V$

$E = E_0\sin(\omega t)$

$7 = 7\sin(\omega t)$

$\sin\omega t = 1$

$\omega t = \frac{\pi}{2}$

25. 2

Sol. $\frac{2\pi}{\lambda} = \frac{\pi}{4}$

λ = 8 cm

I^{st} Node: $\frac{\lambda}{4} = 2$ cm

Chemistry

PART – B

SECTION – A

26. C

Sol. $Cl_2O_7(g) \longrightarrow 2ClO_2(g) + \frac{3}{2}O_2(g)$

Rate $= -\frac{d[Cl_2O_7]}{dt} = \frac{1}{2}\frac{d[ClO_2]}{dt} = \frac{3}{2}\frac{d[O_2]}{dt}$

$\frac{d[O_2]}{dt} = 50 \times \frac{3}{2} = 75$ mm Hg

Rate $k[Cl_2O_7]^o = k = 50$

$t_{1/2} = \frac{P_o}{2k} = \frac{600}{2 \times 50} = 6$ sec

27. D

Sol.
1. $BrO_3^- + 3H_2O + 6e^- \longrightarrow Br^- + 6OH^-$ $\Delta G_1^o = ?$
2. $BrO_3^- + 2H_2O + 4e^- \longrightarrow BrO^- + 4OH^-$ $\Delta G_2^o = -4F(0.54)$
3. $BrO^- + H_2O + 2e^- \longrightarrow Br^- + 2OH^-$ $\Delta G_3^o = -2F(0.17)$

Eq. (2) + (3) = 1, $= \Delta G_1^o = \Delta G_2^o + \Delta G_3^o = -F(4 \times 0.54 + 2 \times 0.17) = -2.5$ VF

$E = \frac{\Delta G_1^o}{nF} = -\frac{(2.5)VF}{6F} = 0.41$ V

28. C

Sol. Rate is faster when the substituent activates the ring (+I or/and +R, o/p) and the rate is slower when the substituent deactivates the ring (– I, R; m). Halogen deactivates the ring (– I, +R, – I > +R) but the orientation is o/p.

Rate of $C_6H_6 = C_6D_6$, since no kinetic isotope effect is observed when H is replaced by D.

Hence, the order is as given in option (C).

29. D

Sol. Hydrolysis of $SbCl_3$

$SbCl_3 + H_2O \rightleftharpoons SbO^+Cl^- + HCl \longrightarrow SbO^+ + 3Cl^-$

NF_3 does not undergo hydrolysis, due to high N – F bond strength.

Pentahalides are thermally less stable than the corresponding trihalides, e.g. thermal stability of PCl_3 is less than PCl_5

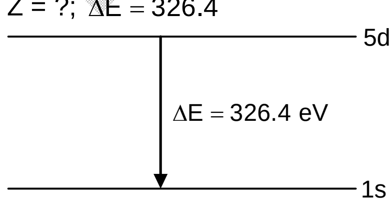
$PCl_5 \xrightarrow{\Delta} PCl_3 + Cl_2$

30. A

Sol. The sols obtained in the two cases will be oppositely charged so coagulate each other.

31. D

Sol. Z = ?; $\Delta E = 326.4$

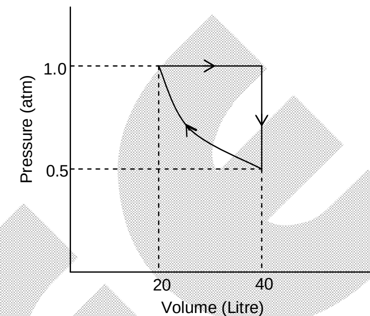


$$= 13.6\ Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\text{eV}$$

$$\Rightarrow 326.4 = 13.6\times Z^2\left[\frac{1}{(1)^2}-\frac{1}{(5)^2}\right]\Rightarrow Z = 5$$

32. B

Sol. $W_{total} = W_{AB} + W_{BC} + W_{CA}$



$= 1(40-20)+0+\left(-nRT\ell n\frac{20}{40}\right)$

$= -20$ litre atm + $2\times 2.303\times 0.0821\log 2\times 300$

$= -20$ litre atm + 34.15 litre atm

$= 14.15$ litre atm

$= 14.15\times 100$ J

= 1415 J

33. C

Sol. meq. of $KMnO_4$ = meq. of $K_xH(C_2O_4)_y$

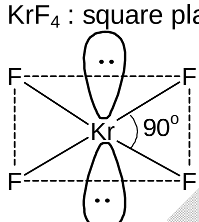
$0.2 \times 5 \times 8 = 10 \times 0.2 \times$ n-factor

n-factor = 4 $\Rightarrow$ x = 3

$K_3H(C_2O_4)_2$

34. A

Sol. KrF_4 : square planar.



35. D

Sol. With bulky base, Hofmann's elimination occur to give the major elimination product.

36. C

Sol. $3Br_2 + 6CO_3^{--} + 3H_2O \longrightarrow 5Br^- + BrO_3^- + 6HCO_3^-$

$$EW(Br_2) = \frac{M}{2}+\frac{M}{10} = \frac{3M}{5}$$

37. C

Sol.

$$A(g) + B(g) \underset{K_b=1.6\times10^{-4}}{\overset{K_f=6.4\times10^{-4}}{\rightleftharpoons}} 2C(g)$$

2	2	0	initially
$2-x/2$	$2-x/2$	$2x/2$	equilibrium

$$K_c = \frac{[C]^2}{[A][B]} = \frac{4x^2\times 4}{4(2-x)^2} = \frac{6.4\times10^{-4}}{1.6\times10^{-4}}$$

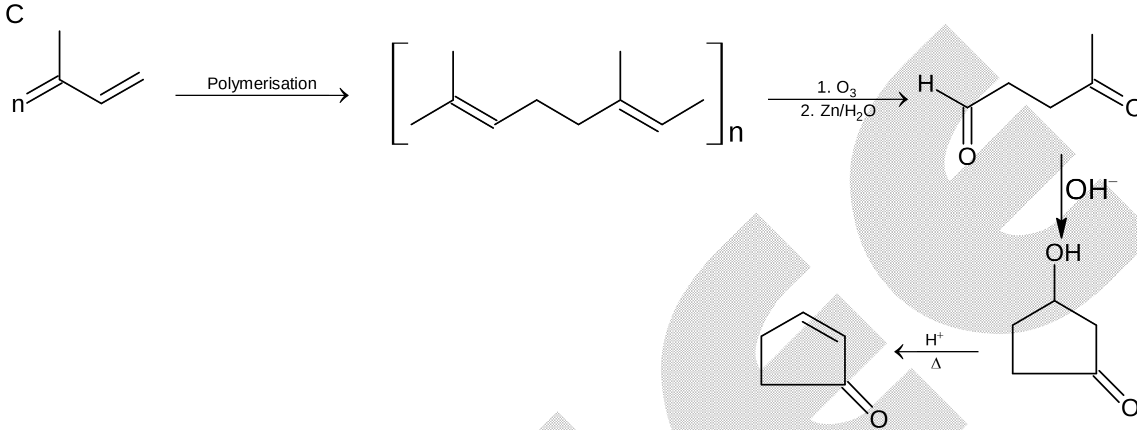
or $\dfrac{4x}{2-x}=4$ or $4x=8-4x$

or $8x=8$, $x=1$

Hence, $\dfrac{2x}{2}=\dfrac{2\times1}{2}=1$

38. C

Sol.



39. C

Sol.

$2A \rightleftharpoons A_2$

$1-\alpha \qquad \alpha/2$

$i=1-\dfrac{\alpha}{2}$ or $\alpha=2(1-i)$

$$K_{eq}=\frac{m\left(\frac{\alpha}{2}\right)}{m^2(1-\alpha)^2}=\frac{1-i}{m(2i-1)^2} \text{ where } i=\frac{\Delta T_b}{(\Delta T_b)}=\frac{\Delta T_b}{K_b m}$$

$$K_{eq}=\frac{1-\Delta T_b/K_b m}{m\left[(2\Delta T_b/K_b m)-1\right]^2}=\frac{K_b(K_b m-\Delta T_b)}{(2\Delta T_b-K_b m)^2}$$

40. D

Sol. $N_2H_5^+ + H_2O \rightleftharpoons N_2H_4 + H_3O^+$

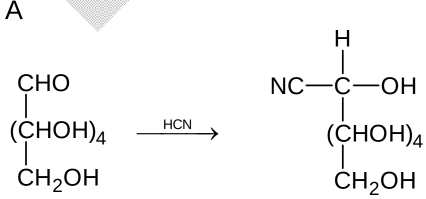
$K_h=\dfrac{[N_2H_4][H_3O^+]}{[N_2H_5^+]}=\dfrac{k_w}{k_b}=\dfrac{10^{-14}}{9.6\times10^{-7}}=\dfrac{x^2}{0.1}=1.04\times10^{-8}$

$\therefore x=3.2\times10^{-5}$

% hydrolysis = $\dfrac{3.2\times10^{-5}}{0.1}\times100=0.032$ %

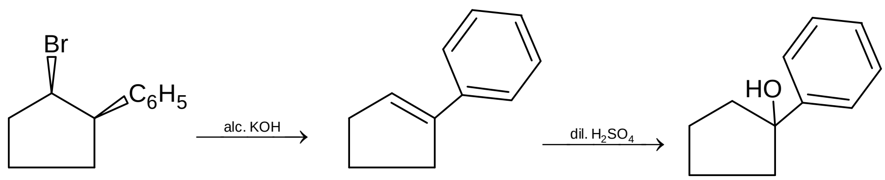
41. A

Sol.



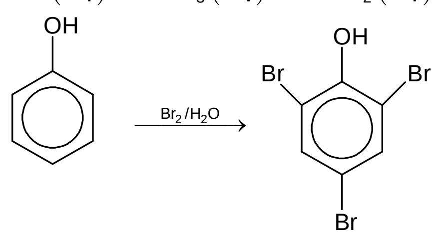
5 chiral carbons

42. A
Sol. (A) is incorrect.

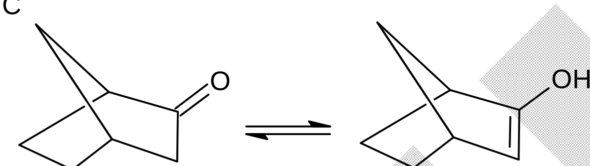


Cis-2-phenyl-1-bromocyclopentane

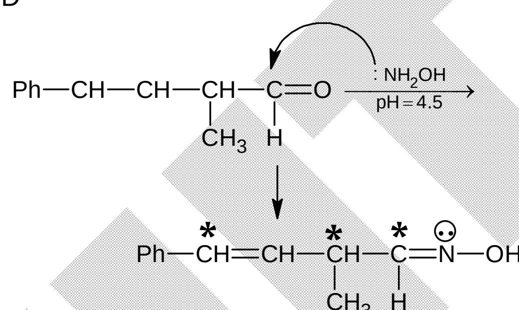
43. D
Sol. $KBr(aq.) + KBrO_3(aq.) \longrightarrow Br_2(aq.)$



44. C
Sol.



45. D
Sol.



3 stereocentres, hence no. of isomers = $2^3 = 8$

SECTION – B

46. 2
Sol. $Fe^{++} \equiv [Ar]_{18}\, 3d^6 \Rightarrow n = 4$

$Mn^{++} \equiv [Ar]_{18}\, 3d^5 \Rightarrow n = 5$

$Cr^{++} \equiv [Ar]_{18}\, 3d^4 \Rightarrow n = 4$

$Ni^{++} \equiv [Ar]_{18}\, 3d^8 \Rightarrow n = 2$

Since, $\mu_s = \sqrt{n(n+2)}$ BM

Fe^{++} and Cr^{++} will have same μ_s.

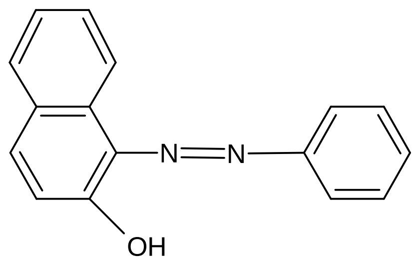
47. 5

Sol. The correct order in 4[th] period is

K < Ga < Ca < Ge < Se < As < Br < Kr

48. 9

Sol. Product 'P' is



49. 80

Sol. Let fraction of (+) isomer is x

32 = 40 × x + (1 – x) (– 40)

32 = 80x – 40

x = 0.9

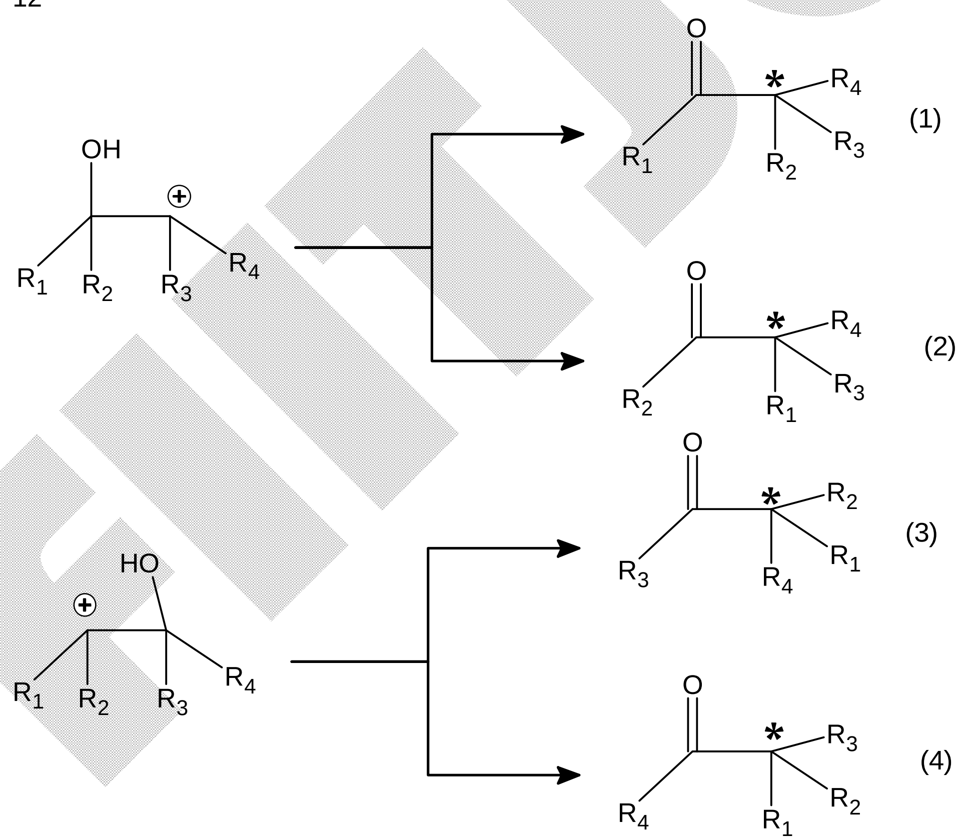
+ isomer is 90%

– isomer is 10%

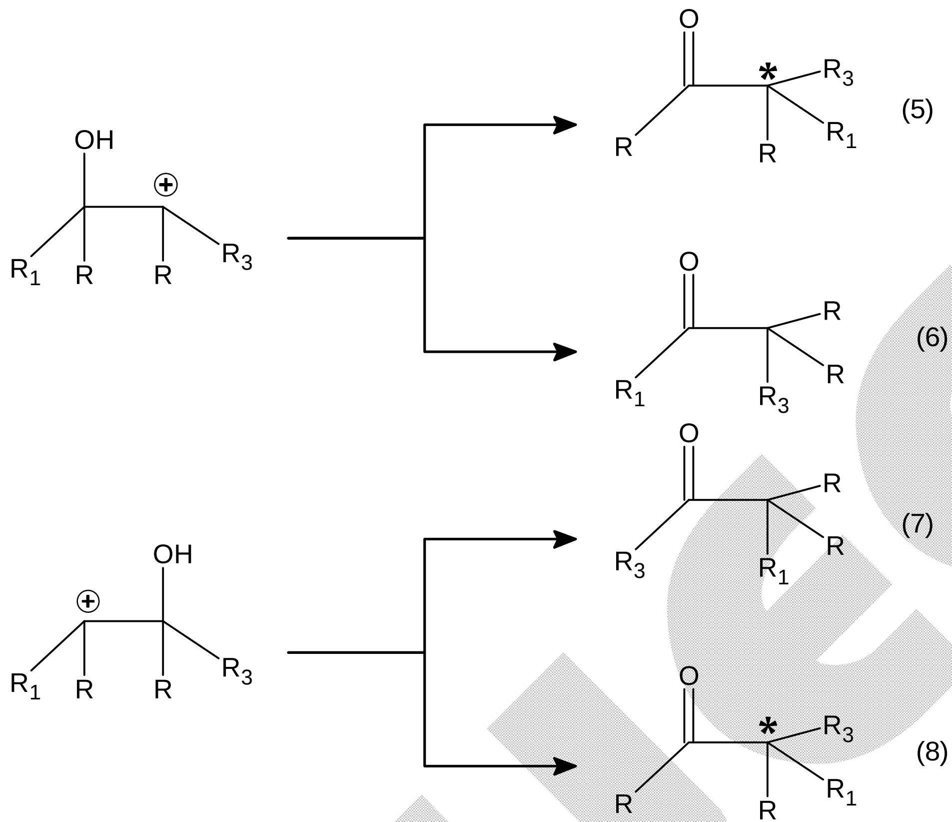
Optical purity = 90 – 10 = 80%

50. 12

Sol.



If $R_2 = R_4 \neq R_1 \neq R_3$



Structure (5) and (8) are identical.

Mathematics

PART – C

SECTION – A

51. C

Sol. $\because\ M^2 = M.M = MNM = MN = M$

sim $N^2 = N$

$\therefore\ M = M^2 = M^3 = \ldots\ldots$ and $N = N^2 = \ldots\ldots$

$(M^{2024} + N^{2024})^{2025} = (M + N)^{2025}$

Now, $(M + N)^2 = M^2 + N^2 + MN + NM = 2(M + N)$

$(M + N)^3 = 2(M + N)\,(M + N) = 2^2(M + N)$

$(M + N)^{2025} = \ldots\ldots = 2^{2024}(M + N)$

52. C

Sol. $\Rightarrow (x - 1)^{60} = a_0x^{60} + a_1x^{59} + \ldots\ldots\ a_{30}x^{30} + a_{31}x^{29} + \ldots\ldots\ a_{60}$

$\therefore\ a_0 = {}^{60}C_0,\ a_1 = -\,{}^{60}C_1,\ a_2 = {}^{60}C_2,\ \ldots\ldots$

$\therefore\ k = a_0 + a_1 + \ldots..\ a_{60} = 0$

$k - a_{30} = -\,{}^{60}C_{30}$

53. B

Sol. We have

In S_1 unit place can have 1 or 3 or 5

In S_2 it is ${}^5C_3 \times 3 \times \dfrac{4!}{2!} + {}^5C_2(9 \times 2 + 6) = 600$

54. C

Sol. Centre of circle is (2, 0) lie on line $z(1 - i) + \bar{z}(1 + i) = 4$

Hence intersection points is two.

55. A

Sol. $T_{(\theta)} = \dfrac{1}{2}(1+\cos(60-2\theta))+\left(\sin^2\theta-\dfrac{3}{4}\right)+\dfrac{1}{2}(1+\cos(60^\circ+2\theta^\circ))$

$= 1+\dfrac{1}{2}(2\cos{}^\circ\cos 2\theta^\circ)+\dfrac{1}{4}(-1-2\cos\theta^\circ)=\dfrac{3}{4}$

$4\sum\theta T_{(\theta)} = 4\sum_{\theta=1}^{30}\dfrac{3\theta}{4} = 3\sum\theta = 1395$

56. B

Sol. Let the G.P. of three unequal numbers be given by a, ar, ar^2, $r \neq 1$ hence $r > 1$ for increasing G.P.

$ar^2 \leq 100 \ \Rightarrow a \leq \dfrac{100}{r^2}$

The number of geometric progression = $\sum_{r=2}^{100}\left[\dfrac{100}{r^2}\right]$ where [.] is G.I.F.

$= \left[\dfrac{100}{4}\right]+\left[\dfrac{100}{9}\right]+\left[\dfrac{100}{16}\right]+\left[\dfrac{100}{25}\right]+\left[\dfrac{100}{36}\right]+\left[\dfrac{100}{49}\right]+\left[\dfrac{100}{64}\right]+\left[\dfrac{100}{81}\right]+\left[\dfrac{100}{100}\right]$

$= 25 + 11 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 53$

57. A

Sol. $e^{\lim\limits_{x\to 0}\frac{2}{x}\left(\frac{a^x-1+b^x-1}{2}\right)} = e^{\ln ab} \Rightarrow ab = 6$

(a, b) = (1, 6), (6, 1), (2, 3), (3, 2)

$P(E) = \dfrac{4}{36} = \dfrac{1}{9}$

58. D

Sol. $f'(x) = ax(x-1) \Rightarrow f'(2) = 6 \Rightarrow a = 3$

$f'(x) = 3(x^2 - x) \Rightarrow f(x) = x^3 - \frac{3x^2}{2} + c$

$\because$ $f(2) = 2 \Rightarrow c = 0$

$\therefore$ $f(x) = x^2\left(x - \frac{3}{2}\right)$

59. D

Sol. $\log_a b = \log_a c \Rightarrow a = b^{1/b} = c^{1/c}$
from curve $y = x^{1/x}$, $x > 0$ we find $1 < a < e^{1/e}$.

60. B

Sol. $\left(2xydx + x^2dy\right) + x^2ydx + \left(\frac{y^3}{3}dx + y^2dy\right) = 0$

Put $x^2y = t$ and $\frac{y^3}{3} = u$

$(dt + tdx) + (udx + du) = 0$

$\int \frac{dt + du}{t + u} = -\int dx$

$\Rightarrow \ln(t + u) = -x + c$

$\Rightarrow x^2y + \frac{y^3}{3} = k'e^{-x}$ at $x = 1$, $y = 1$, $k' = \frac{4e}{3}$

Put $x = 0$, $\frac{y^3}{3} = \frac{4e}{3} \Rightarrow k = 4$

61. C

Sol. $\int_\alpha^\beta f(x)dt + \int_\alpha^\beta f^{-1}(x)dx = 13$

$\Rightarrow \beta^2 - \alpha^2 = 13 \Rightarrow \beta = 7, \alpha = 6$

62. A

Sol. Let point of intersection (h, k)

$\Rightarrow \frac{h}{a} + \frac{k}{b} = 1$ and $ah + kb = 1$, $\frac{a}{b} + \frac{b}{a} = 1$

$\left(\frac{h}{a} + \frac{k}{b}\right)(ah + kb) = 1$

$h^2 + k^2 + hk\left(\frac{b}{a} + \frac{a}{b}\right) = 1$

63. D

Sol. $e = \sqrt{\frac{2n+4}{n+1}}$

Put $n = 48$ then $e = \frac{10}{7}$ is a rational number

$\frac{x^2}{49} - \frac{y^2}{51} = 1 \quad \therefore \ \ell = \frac{2b^2}{a} = \frac{102}{7}$

64. C

Sol. PQ will be focal chord and its mid point will be circum centre.

$2h = a\left(t^2 + \frac{1}{t^2}\right), 2k = 2a\left(t - \frac{1}{t}\right)$, where P ≡ $(at^2, 2at)$

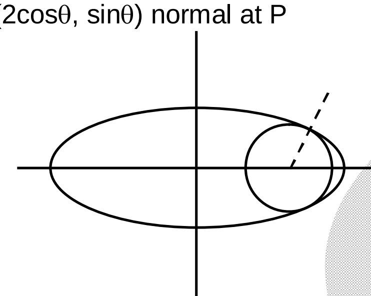
eliminate t, we get locus

locus is $y^2 = 2a(x - a)$, focus = $\left(\frac{3a}{2}, 0\right)$

65. C

Sol. Centre of circle should be on major axis.

If circle touches ellipse at P(2cosθ, sinθ) normal at P



$2x\sec\theta - y\,\mathrm{cosec}\theta = 3$ cuts major axis at $\left(\frac{3}{2}\cos\theta, 0\right)$ if r be the radius of circle then

$r^2 = \left(2\cos\theta - \frac{3}{2}\cos\theta\right)^2 + (\sin\theta - 0)^2 = 1 - \frac{3}{4}\cos^2\theta$

but $0 \le \cos^2\theta < 1$

$\frac{1}{2} \le r \le 1$

66. A

Sol. Let angle between $\bar{a}$ and $\bar{b}$ is α and $\bar{a}\times\bar{b}$ and $\bar{c}$ is β

$|(\bar{a}\times\bar{b})\cdot\bar{c}| = 6 \Rightarrow \sin\alpha.\cos\beta = 1$

so that $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular

$\therefore$ req. = $|\bar{a}\times\bar{c}|^2 |\bar{d}|^2 = 9$

67. C

Sol. $\bar{a} = \bar{b}\times\bar{c} + 2\bar{b}$

$\Rightarrow \bar{a}\cdot\bar{b} = 2|\bar{b}|^2 \Rightarrow |\bar{a}||\bar{b}|\cos\theta = 2|\bar{b}|^2$

$\cos\theta = \frac{4}{|\bar{a}|} \Rightarrow |\bar{a}| = 4 \Rightarrow \theta = 0°$

$\Rightarrow \bar{a} = 2\bar{b}$

Now $\bar{b}\times\bar{c} = 0 \Rightarrow \bar{b} = \bar{c}$ or $\bar{b} = -\bar{c}$

$\therefore |2\bar{a} + \bar{b} + \bar{c}|$ is either $|3\bar{a}|$ or $|2\bar{a}|$

$|2\bar{a} + \bar{b} + \bar{c}|$ is either 12 or 8

sum = 12 + 8 = 20

68. C

Sol. Normal vector of plane $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$

$\vec{n} = 2\left(\hat{i} + \hat{j} + 3\hat{k}\right)$

Equation of plane : x + y + 3z = 1

$\therefore$ a + b + c = 1 + 1 + 3 = 5

69. B

Sol. We write the elements A + A

1 + 1, 1 + a_1, 1 + a_2, …… 1 + a_{18}, 1 + 77, a_1 + 77, ….. a_{18} + 77, 77 + 77

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P. let common diff. be 'd'

77 = 1 + 19d $\therefore$ d = 4

$$\sum_{i=1}^{18} a_i = \frac{18}{2}\left[2a_1 + 17d\right] = 702$$

70. D

Sol.

x_i(observation)	0	2	2^2	2^n
f^i (frequency)	$^nC_0$	$^nC_1$	$^nC_2$	^nC_n

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 \times {}^nC_0 + 2\,{}^nC_1 + 2^2 \times {}^nC_2 + \ldots + 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n}$$

$$= \frac{3^n - 1}{2^n} = \frac{728}{2^n} \text{ (Given)}$$

$\Rightarrow$ n = 6.

SECTION – B

71. 30

Sol. $C_1 \equiv (1,1),\ r_1 = 1,\ C_2 \equiv (9,6),\ r_2 = 2$

$C_1M_1 \geq r_1,\ C_2M_2 \geq r_2$

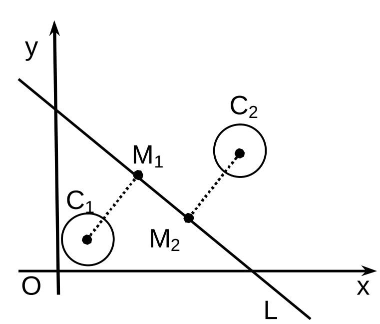
$|7 - k| \geq 5$

$k \geq 12$

and also $k \leq 41$

$k \in [12, 41]$

no of integral k = 30.



72. 41

Sol. Let $P(E_1) = x,\ P(E_2) = y,\ P(E_3) = z$

$\Rightarrow$ 3x(1 – y)(1 – z) = (1 – x) y(1 – z) = 9(1 – x)(1 – y)z = 3 (1 – x)(1 – y)(1 – z)

$$\frac{3x}{1-x} = \frac{y}{1-y} = \frac{9z}{1-z} = 3$$

$$\therefore\ x = \frac{1}{2},\ y = \frac{3}{4},\ z = \frac{1}{4}$$

Now $\begin{vmatrix} 1/2 & 3/4 & 1/4 \\ 3/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{vmatrix} = -\frac{9}{32}$

$\therefore \quad \frac{a}{b} = \frac{9}{32}$

$\therefore \quad a + b = 41$

73. 8

Sol. $(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow \quad 7\lambda - 2 - a = 0 \quad \ldots. \text{(i)}$

And $(5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

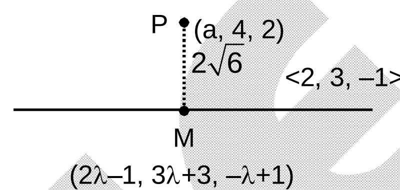
$\Rightarrow \quad \lambda = 1$

$\Rightarrow \quad a = 5$ from (i)

$(\alpha_1, \alpha_2, \alpha_3)$ is reflection of P

$\Rightarrow \quad \alpha_1 = -3, \alpha_2 = 8, \alpha_3 = -2$

$\therefore \quad a + \alpha_1 + \alpha_2 + \alpha_3 = 8.$



74. 9

Sol. Differentiate w.r.t. x

$f'(x) - \sin 2x = 0 - f(x)\tan x$

or $\quad y' + y\tan x = \sin 2x$, where $y = f(x)$

LDE, solution is $y\sec x = \int 2\sin x \, dx$

or $\quad y\sec x = -2\cos x + c$

$f(0) = 1 \Rightarrow c = 3 \therefore f(x) = 3\cos x - 2\cos^2 x$

$f(x) = -2\left(\cos x - \frac{3}{4}\right)^2 + \frac{9}{8}$

f(x) max = 9/8.

75. 5

Sol. eccentricity of ellipse = $\frac{1}{2}$

$\frac{de}{dt} = -0.1$ (given)

eccentricity of auxiliary circle = 0

$\int_{1/2}^{0} de = -0.1\int_{0}^{T} dt$

T is time at which it will becomes auxiliary circle

$-\frac{1}{2} = -0.1\,(T - 0)$

$\therefore$ T = 5 sec.