Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)
PART TEST
29-12-2024

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	B,C,D	B,C	A,B,C	A,C	B,C,D	A,B,C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	B	A	D	A				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	2.40	7.10	3.42 to 3.43	20.00	2.42	8.00	9.00	1.37 to 1.38

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,C,D	A,C,D	A,C	A,B,C,D	B,C,D	A,B,C		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	A	C	B	D				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	24.00	0.25	8.00	9.00	4.00	5.00	6.00	87.00

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	B,C	A,C,D	B,C	A,C	B,C	B,C		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	C	D	B	D				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	3.00	1.00	4975.00	7.00	967.00	5049.00	6.00	10.00

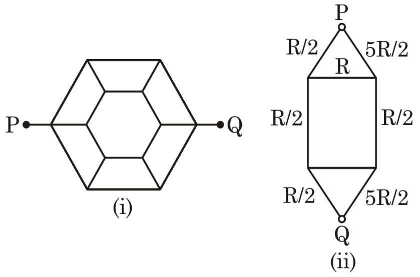
HINT – SHEET

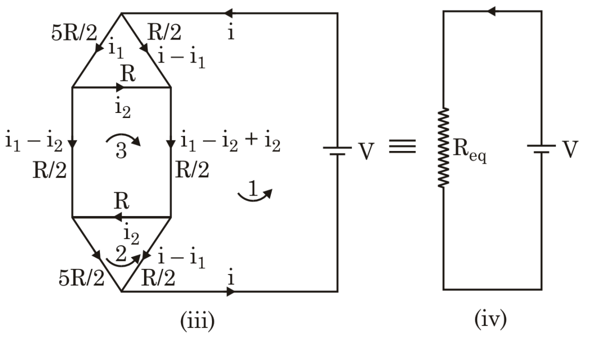
PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (B,C,D)**

Compressing the hexagonal box and folding about the line PQ, the circuit is reduced as shown in the figure (ii).





Now, let a battery is connected between point P and Q as shown in the figure (iii) and its equivalent is shown in the figure (iv).

Applying KVL in loop 1

$$(i-i_1)\frac{R}{2}(i-i_1+i_2)\frac{R}{2}+(i-i_1)\frac{R}{2}=V \qquad ....(i)$$

Applying KVL in loop 2

$$i_1\frac{5R}{2}+i_2R-(i-i_1)\frac{R}{2}=0 \qquad ....(ii)$$

Applying KVL in loop 3

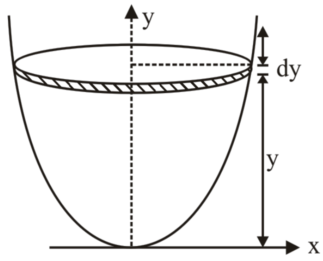
$$i_2R+(i-i_1+i_2)\frac{R}{2}+i_2R=(i_1-i_2)\frac{R}{2} \qquad ....(ii)$$

From (i), (ii) and (iii)

$$i=\frac{20V}{23r}=\frac{V}{R_{eq}} \Rightarrow R_{eq}=\frac{23R}{20}$$

2. **Ans (B,C)**

$x^2 = 2y$



Area of circular shape $= \pi x^2 = 2\pi y$

Volume of liquid flowing out in dt sec. is

$= \sqrt{2gy}\,(10\times 10^{-4})m^3$

Let level of liquid decrease by dy in dt sec.

So $-2\pi y\,dy = 10^{-3} = \sqrt{2gy}\,dt$

$$\int_2^0 -2\pi\times 10^3\, y^{1/2}dy = \int_0^t \sqrt{2g}\,dt$$

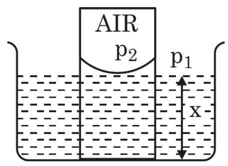
$$2\pi\times 10^3\,\frac{(2)^{3/2}}{3/2}=\sqrt{2}\sqrt{g}t$$

$$\frac{8\pi}{3}\times 10^3=\sqrt{g}t$$

$$t=\frac{8}{3}\times 10^3\text{ sec}=\frac{8}{3}\times\frac{1000}{60\times 60}=\frac{40}{9\times 6}=\frac{20}{27}\text{hr}$$

3. **Ans (A,B,C)**

Let ℓ be the length of the capillary tube. Let x be the length of the capillary tube dipped in the liquid at which the liquid level inside and outside the tube is the same.



$\therefore$ Initial pressure $P_1 = p_a$ (atmospheric)

Final pressure $p_2 = p_a + \frac{2\sigma}{r}$, where σ is surface tension and r is radius of the tube.

Initial volume of air $V_1 = \ell a$, where a is area of cross-section.

Final volume of air $V_2 = (\ell - x)a$.

By Boyle's law, $p_1V_1 = p_2V_2$

$p_a \times \ell a = (p_a + p)(\ell - x)a,$

where $p = \frac{2\sigma}{r}$ is the excess pressure over and above the atmosphere.

$\therefore\ p_a \times \ell = p_a \times \ell + p\ell - p_a x - px$

$$\therefore\ x = \frac{p\ell}{(p_a + p)} = \frac{\left(\frac{2\sigma}{r}\right)\ell}{p_a + \frac{2\sigma}{r}} = \frac{\ell}{1 + \frac{p_a}{\left(\frac{2\sigma}{r}\right)}}$$

$$= \frac{0.11}{1 + \frac{10^5}{5 \times 10^3}} = \frac{0.11}{1+20} = \frac{0.11}{21} = 5.23 \times 10^{-3} m$$

$$\text{Excess pressure} = \frac{(2 \times 5.06 \times 10^{-2})}{(2 \times 10^{-5})} = 5.06 \times 10^3\ N/m^2$$

$\therefore$ Excess pressure is 5 kN/m^2 (approximately)

4. **Ans (A,C)**

$$\text{Least count} = \frac{0.5}{100} = 0.005\ mm$$

Zero error = 0 + 0.005 × 2 = 0.01 mm

So, true diameter = 0.5 × 8 + 0.005 × 83 – 0.01 = 4.405 mm

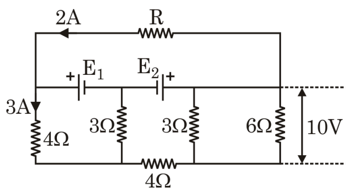
5. **Ans (B,C,D)**

Let potential of point A is x and potential of point B is zero. Consider charge flown through 3V battery is q_0.

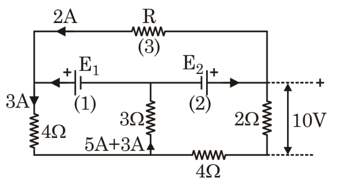
$2(3 - x) + q_0 + (0 - x)2 = 0$ ...(1)

$-q_0 - (x - 3) \times 1 + (2 - x + 3)2 = 0$ ...(2)

6. **Ans (A,B,C,D)**



After redrawing the circuit.



(a) $I_4 = 5A$

(b) From loop (1) to (1)

$-8(3) + E_1 - 4(e) = 0 \quad \Rightarrow \quad E_1 = 36$ volt

From loop (2) to (2)

$+ 4(5) + 5(2) - E_2 + 8(3) = 0$

$E_2 = 54$ volt

(c) From loop (3) to (3)

$-2R - E_1 + E_2 = 0$

$$R = \frac{E_2 - E_1}{2} = \frac{54}{2} - 36 = 9\,\Omega$$

Ans. (a) 5.00 A **(b)** 36.0 V, 54.0 V **(c)** 9.00 Ω

PART-1 : PHYSICS

SECTION-I (ii)

7. **Ans (B)**

(A) - (q); (B) - (p,q,r,s,t); (C) - (p,q,r,s,t); (D) - (s)

A generalised circuit for all the circuit can be given by

Applying KVL in mesh - I

$4 - i_2 \times 2 - (i_1 - i_2) \times 4 - 6 = 0$

$\Rightarrow\ 2i_1 + 4i_1 - 4i_2 + 2 = 0$

$\Rightarrow\ 3i_1 - 2i_2 = -1 \qquad \ldots(i)$

Applying KVL in mesh - II

$6 - (i_2 - i_1) \times 4 - i_2 \times R - E = 0$

$4i_1 - i_2\,(4 + R) = E - 6 \quad \ldots(ii)$

Now

(I) $i_1 - i_2 = 0 \Rightarrow\ i_1 = i_2$

Putting in (i) and (ii) we get

$4 + R = E - 2\ \Rightarrow\ R = E - 6$

& R should be positive and non-zero.

(II) $i_2 - i_1 > 0\ \Rightarrow\ i_2 > i_1$

Now from (i)

$2(i_1 - i_2) + i_1 = -1 \Rightarrow\ 2(i_1 - i_2) = -1 - i_1$

Now as $i_2 > i_1$

$i_1 - i_2 < 0$

$\therefore\ -1 - i_1 < 0 \Rightarrow\ i_1 + 1 > 0$

$i_1 > -1$ & $i_2 > i_1$

$\therefore\ i_1 > -1$

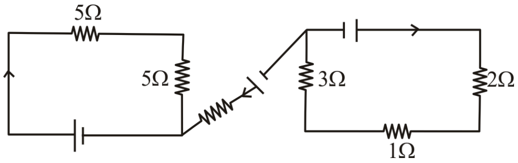
Now from (ii)

$4(i_1 - i_2) - i_2 R = E - 6$

$\Rightarrow\ 4(i_1 - i_2) = E - 6 + i_2 R$

$\Rightarrow\ 4(i_1 - i_2) < 0$

8. Ans (A)

(P) 

As branch CD is not a part of any closed loop, I = 0.

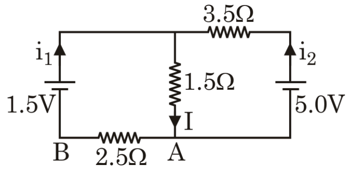
Current through BC : $i_1 = \frac{1}{10} = 0.1$ A

Current through DA : $i_2 = \frac{3}{6} = 0.5$ A

$\therefore\ V_A - 1 \times 0.5 - 3 \times 0.5 + 0.5 - 4 \times 0 + 5 \times 0.1 = V_B$

$V_A + 2 - 3 = V_B$ or $V_A - V_B = 1$ V

$I = 0, V_{AB} = 1V$

(Q) 

$I = i_1 + i_2$

$1.5 = (i_1 + i_2) \times 1.5 + 2.5\, i_1$

(i.e.) $4\, i_1 + 1.5\, i_2 = 1.5 \Rightarrow 8i_1 + 3i_2 = 3$ ...(1)

and $5 = 3.5\, i_1 + 1.5\,(i_1 + i_2) = 1.5\, i_1 + 5\, i_2$

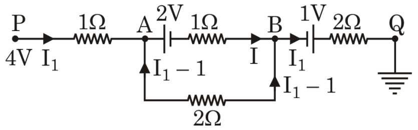
$\therefore\ 3\, i_1 + 10\, i_2 = 10$

$(1) \times 10 - (2) \times 3;$

$(80 - 9)\, i_1 = 0 \Rightarrow i_1 = 0,\ i_2 = 1A$

$\therefore\ I = i_1 + i_2 = 1A;\ V_A - V_B = 0 = V_{AB}$

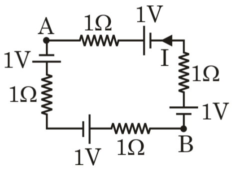
Hence, $I = 1A, V_{AB} = 0.$

(R) 

$4 - I_1 - 2(I_1 - I) + 1 - 2I_1 = 0$

(i.e.) $5 - 5I_2 + 2I = 0 \Rightarrow 5I_1 - 2I = 5$

$4 - I_1 - 2 - 3 - 3I - I = 0$

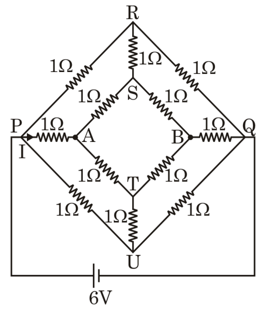
(S) 

$4 = 4I$ or $I = 1A$

$V_A + 1 - 1 \times 1 + 1 - 1 \times 1 = V_B$

$V_A - V_B = 0 = V_{AB}$

Hence, $I = 1A, V_{AB} = 0$

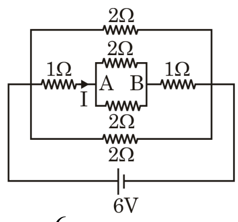
(T) 

PQ – symmetry line

Perpendicular to PQ will be at same potential.

$\therefore\ V_R = V_S$ and $V_T = V_U$

No current in RS and UT.



$I = \frac{6}{3} = 2A$

$V_A - V_B = V_{AB} = 2V$

Thus, $I = 2A, V_{AB} = 2V$

9. Ans (D)

(I) From free body diagram of the liquid above the sphere, $F_x = P_0\pi R^2 + \frac{1}{3}\pi R^2 \rho g$

Force of buoyancy on the sphere $= \frac{4}{3}\pi R^3 \rho g$

So, $F_y = P_0\pi R^2 + \frac{5}{3}\pi R^3 \rho g$

(II) Force of buoyancy on the disc $F_x = \frac{1}{3}\pi R^2 \rho g$

$F_y = P_0\pi R^2 + \frac{5}{3}\pi R^2 \rho g$

(III) From the free body diagram of the liquid in the container F_x and F_y are different with option (p) and (q)

(IV) $F_x = \left(P_0 + \rho g \frac{R}{3}\right) 4\pi R^2 = 4\pi P_0 R^2 + \frac{4}{3}\rho g \pi R^3$

$F_y = \left(P_0 + \rho g \frac{5R}{3}\right) \pi R^2 = P_0 \pi R^2 + \frac{5}{3}\rho g \pi R^3$

Force on the part open to atmosphere

$= P_0 3\pi R^2$

So, $F_B = \frac{1}{3}\rho g \pi R^3 = 4\pi R^2 \frac{4R}{3}\rho g - N$

(N = normal reaction)

$\therefore\ \sigma \geqslant \frac{\rho}{16}$

PART-1 : PHYSICS

SECTION-II

1. **Ans (2.40)**

$(\pi r^2)\sqrt{2gy} = \pi x^2\left(-\frac{dy}{dt}\right)$

$\Rightarrow\ (r^2)\sqrt{2gy} = x^2\lambda$

$\Rightarrow\ y\alpha x^4$

$\Rightarrow\ n = 4$

2. **Ans (7.10)**

1 MSD = 1 mm

1 VSD = 0.9 mm

L.C. = 0.1 mm

–ve error = 4 × 0.1 mm = 0.4 mm

Reading = 6 mm + 7 × 0.1 mm = 6.7 mm

Diameter = 6.7 + 0.4 = 7.1 mm

3. **Ans (3.42 to 3.43)**

$C = \frac{K\in_0 A}{d}$ = a constant.

For A to be minimum, d must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $\frac{V}{d_{min}} = E_{air}$

[E_{air} = Breakdown field for air]

$\therefore\ d_{min} = \frac{V}{E_{air}}$

Now $\frac{\in_0 A_{min}}{d_{min}} = C$

$\Rightarrow\ A_{min} = \frac{C}{\in_0}\frac{V}{E_{air}}$

$\therefore\ A_1 = \frac{CV}{\in_0 E_{air}}$

With dielectric, similar calculation gives

$A_2 = \frac{CV}{K\in_0 E_{dielect}}$

$\therefore\ \frac{A_1}{A_2} = \frac{KE_{dielec}}{E_{air}} = 3 \times 8 = 24$

4. **Ans (20.00)**

$\frac{q_1}{C_1} = \frac{q_2}{C_2};\ q_1 + q_2 = 2Q_0$

$C_1 = \frac{\varepsilon_0 A}{d_0 + vt};\quad C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$

$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$

$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0;\ q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$

$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt);\quad I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 20\text{ amp}$

$\therefore\ n = 5$

5. **Ans (2.42)**

Least count of screw gauge $= \frac{1}{100}$ mm = 0.01 mm

Diameter of the wire = (1 + 25 × 0.01) mm = 0.125 cm

Since $Y = \frac{4T\ell}{\pi d^2 \delta\ell}$

$\therefore\ \frac{\Delta Y}{Y} = \frac{\Delta\ell}{\ell} + \frac{2\Delta d}{d} + \frac{\Delta(\delta\ell)}{\delta\ell}$

$= \frac{0.01}{50} + \frac{2 \times 0.001}{0.125} + \frac{0.001}{0.125} = 0.0242$

Percentage error $= \frac{\Delta Y}{Y} \times 100 = 2.42$

6. **Ans (8.00)**

$0 + \frac{\rho_1\omega^2}{2}\left(\frac{\ell}{2}\right) - \rho_2 g h_2 = P_0$

$\frac{\rho_1\omega^2}{8}\ell^2 = P_0 + \rho_2 g h_2$

$\omega = \sqrt{\frac{8(P_0 + \rho_2 g h_2)}{\rho_1\ell^2}}$

7. **Ans (9.00)**

$Q = C_{eq}V = \frac{a\varepsilon_0}{d + x}V$

$\frac{dQ}{dt} = -\frac{a\varepsilon_0}{(d+x)^2}V\frac{dx}{dt} = -\frac{a\varepsilon_0}{(d+x)^2}V\,v$

Rate of work done on the battery

$= -\left(\frac{dQ}{dt}\right)V = \frac{a\varepsilon_0 vV^2}{9d^2}$

8. **Ans (1.37 to 1.38)**

Initial charge (q_1) on the capacitor = 36 V × 250 mF = 9C.

Final charge (q_f) on the capacitor =

$$\left[\frac{4}{12+4}\times 12\right]\times\frac{250}{1000}C=\frac{3}{4}C$$

Time constant (τ) of the circuit =

$$\left(\frac{4\times 12}{4+12}+3\right)\times\frac{250}{1000}s=1.5s$$

Equation of charge on the capacitor

$$=q_f+(q_i-q_f)e^{-\frac{t}{\tau}}=\frac{3}{4}+\left(9-\frac{3}{4}\right)e^{\frac{-2t}{3}}=\frac{3}{4}+\frac{33}{4}e^{\frac{-2t}{3}}$$

Equation of current in the 3Ω resistor

$$i=\frac{-dq}{dt}=\frac{11}{2}e^{\frac{-2t}{3}}A.$$

Current at t = 3 ln 2s $i=\frac{11}{2}e^{-\frac{2}{3}\times 3\ln 2}=\frac{11}{8}A.$

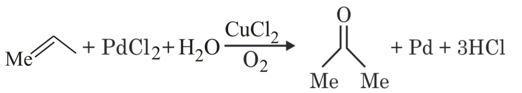
So x = 1.375

PART-2 : CHEMISTRY

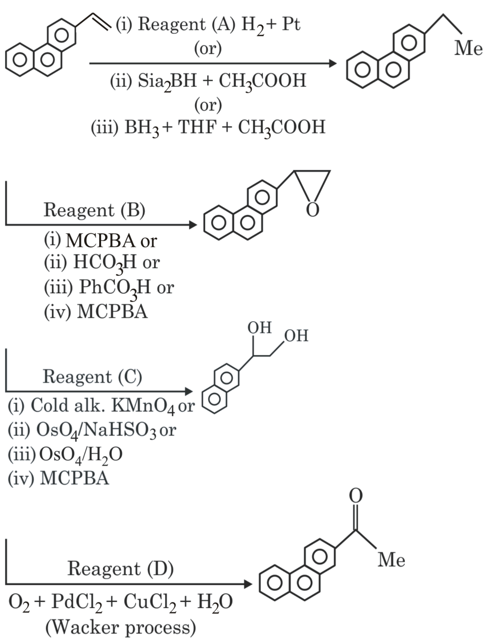
SECTION-I (i)

1. **Ans (A,C,D)**

Wacker process is used to convert alkene to carbonyl group.



$$CH_2=CH_2+PdCl_2+H_2O\xrightarrow[O_2]{CuCl_2}CH_3CHO+Pd+2HCl$$

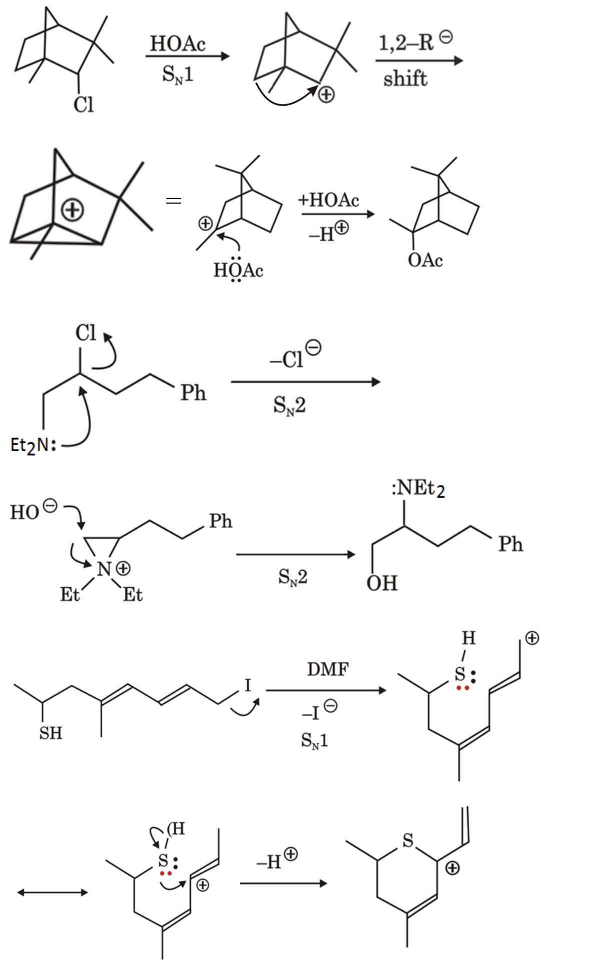


2. **Ans (A,C,D)**

Hg lies below H in ECS.

Hence, it does not liberate H_2 with HCl

4. **Ans (A,B,C,D)**



PART-2 : CHEMISTRY

SECTION-I (ii)

9. **Ans (B)**

$KO_2 + H_2O \longrightarrow KOH + H_2O_2 + O_2$

$Na_2O_2 + H_2O \longrightarrow NaOH + H_2O_2$

$NaCl \xrightarrow{\text{Electrolysis}} H_2$ (at cathode)

$NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$

10. **Ans (D)**

(P) $2KMnO_4+3HCOOK\xrightarrow[\text{solution}]{\text{alkaline}}2MnO_2+KHCO_3+2K_2CO_3+H_2O$

(Q) $2MnO_4^-+2OH^-\xrightarrow[\text{solution}]{\text{alkaline}}2MnO_4^{2-}+H_2O+O$

then

$2MnO_4^{2-}+2H_2O\rightarrow 2MnO_2+4OH^{\ominus}+2O$

(R) $2KMnO_4+H_2O+KI\rightarrow 2MnO_2+2KOH+KIO_3$

(S) $MnO_4^{\ominus}+H_2C_2O_4+H^{\oplus}\rightarrow Mn^{2+}+CO_2+H_2O$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (24.00)**

Product is $CH_2=CH-CH=CH_2$

value of $\mathbf{x} = 2$

value of $\mathbf{y} = 4$

value of $\mathbf{z} = 3$

$2 \times 4 \times 3 = 24$

2. **Ans (0.25)**

X = 4 (3, 4, 7, 9)

Y = 3 (3, 7, 9)

Z = 4 (5, 6, 7, 8)

Value of $\frac{X-Y}{Z}$ is

$= \frac{4-3}{4} = 0.25$

3. **Ans (8.00)**

1, 2, 3, 4, 7, 8, 9, 10 gives Tollen's test.

4. **Ans (9.00)**

1, 3, 5, 6, 8, 9, 10, 11, 12 will give diastereomeric pair.

6. **Ans (5.00)**

$AlCl_3$, $MgCl_2$, $FeCl_3$, BCl_3 , $BeCl_2$

7. **Ans (6.00)**

N_2, O, N, F, He, Ne

8. **Ans (87.00)**

$M_a = Zn$

$M_b = Cu$

$M_c = Ni$

$$2H_2O + 2MnO_4^- + 3Mn^{2+} \xrightarrow[OH^-]{ZnSO_4} 5MnO_2 + 4H^+$$

• $E^{\ominus}/V(Ni^{2+}/Ni) = -0.25$

• 'Silver' UK coins are a Cu/Ni alloy

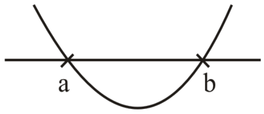
PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (B,C)**

$x^2 f''(x) + 4f'(x) + 2f(x) > 0$

$(x^2 f(x))'' > 0$



2. **Ans (A,C,D)**

$\lim_{x \to 0} \frac{f(x)}{x^2} = \pi.$

$f(0) = 0 = f'(0)$

$f''(0) = 2\pi;$ i.e. $f''(0) > 2\pi.$

$f'(x) = 0$ only for $x = 0$

i.e., $f'(x)$ changes sign at $x = 0$ only

3. **Ans (B,C)**

$f(x) = x^2 + x + 1 + \sin x$

$f'(x) = 2x + 1 + \cos x$

$f''(x) = 2 - \sin x > 0 \ \forall \ x \in R$

$\Rightarrow$ $f'(x)$ is monotonicaly increasing

Now $f'(-1) < 0$ & $f'(0) = 2 > 0$

$\therefore$ $f'(x)$ = has exactly one root in $(-1, 0)$ as $f'(x)$ is increasing function

4. **Ans (A,C)**

$$\lim_{n \to \infty}\left(n + 1 - \sum_{i=2}^{n}\sum_{k=2}^{i}\frac{1}{\underline{|k-1}} - \frac{1}{\underline{|k}}\right)$$

$$\lim_{n \to \infty}\left(1 + \sum_{i=2}^{n}\frac{1}{\underline{|i}}\right) = e$$

5. **Ans (B,C)**

Maxima at $x = 4k + 1 \Rightarrow 24$ points

Minima at $x = 4k + 2 \Rightarrow 25$ points

6. **Ans (B,C)**

For $x > 2$

$$f(x) = \int_0^1 (6-t)dt + \int_1^x (t+4)dt =$$

$$\left(6t - \frac{t^2}{2}\right)_0^1 + \left(\frac{t^2}{2} + 4t\right)_1^x =$$

$$\left(6 - \frac{1}{2}\right) + \left(\frac{x^2}{2} + 4x\right) - \left(\frac{1}{2} + 4\right)$$

$$= \frac{11}{2} + \frac{x^2}{2} + 4x - \frac{9}{2} = \frac{x^2}{2} + 4x - 1$$

$$f(x) = \begin{cases} 5x+1, & x \leqslant 2 \\ \frac{x^2}{2} + 4x - 1, & x > 2 \end{cases}$$

$$f'(x) = \begin{cases} 5, & x < 2 \\ x+4, & x > 2 \end{cases}$$

$f'(2^-) \neq f'(2^+)$ $\Rightarrow$ f is not differentiable

PART-3 : MATHEMATICS

SECTION-I (ii)

7. **Ans (C)**

$$f(x) = \begin{cases} -6x & -1 < x < -\frac{2}{3} \\ 4 & -\frac{2}{3} \leqslant x \leqslant \frac{2}{3} \\ 6x & \frac{2}{3} < x < 1 \end{cases}$$

$g(x) = \{x\}$

$$\therefore f(g(x)) = \begin{cases} 4 & 0 \leqslant \{x\} \leqslant \frac{2}{3} \\ 6\{x\} & \frac{2}{3} < \{x\} < 1 \end{cases}$$

$\therefore$ c = 1 and d = 3

8. **Ans (D)**

$$f(x) = \frac{x^2+4x+3}{x^2+7x+14}$$

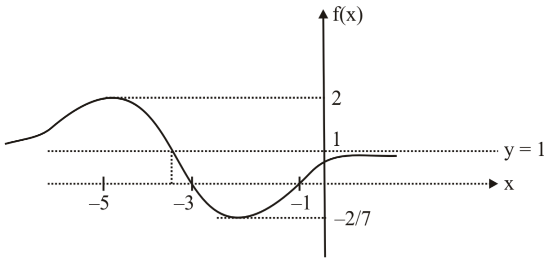
$$f'(x) = \frac{(3x+7)(x+5)}{x^2+7x+4}$$

$+$ -5 $-$ $-7/3$ $+$

$f(x)_{max} = 2$ at $x = -5$

$f(x)_{min} = \frac{-2}{7}$ at $x = \frac{-7}{3}$

Graph of $y = f(x)$



$$g(x) = \frac{x^2-5x+10}{x^2+5x+20}$$

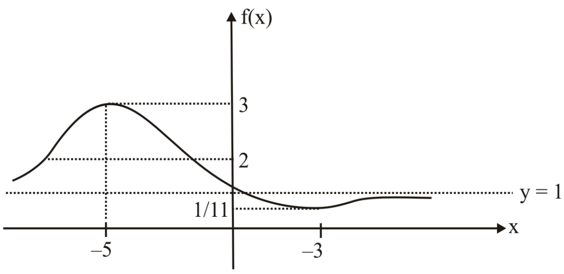
$$\frac{10(x+5)(x-3)}{(x^2+5x+20)^2}$$

$+$ -5 $-$ 3 $+$

$g(x)_{max.} = 3$ at $x = -5$

$g(x)_{min} = \frac{1}{11}$ at $x = 3$

Graph of $y = g(x)$



9. **Ans (B)**

(I) Equation of circle is $x^2 + \left(y - \frac{10}{3}\right)^2 = 1$

Consider a general point $(2t^2, 2t^3)$ on the curve $x^3 = 2y^2$, its distance from centre is

$$\ell^2 = 4t^4 + \left(2t^3 - \frac{10}{3}\right)^2 = 4\left[t^4 + \left(t^3 - \frac{5}{3}\right)^2\right]$$

$\frac{d(\ell)^2}{dt} = 8t^2 (t - 1)(3t^2 + 3t + 5) \Rightarrow t = 1$ is a point of local minima $\Rightarrow \ell_{min} = \frac{2}{3}\sqrt{13}$

(II) Let $t = -x - \frac{\pi}{6}, t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$$\Rightarrow \tan\left(x + \frac{2\pi}{3}\right) - \tan\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$$

$\Rightarrow \cot t + \tan t + \cos t \Rightarrow \frac{2}{\sin 2t} + \cos t$ of decreasing in $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$\Rightarrow$ Greatest value occur at $t = \frac{\pi}{6} \Rightarrow \frac{11\sqrt{3}}{6}$

(III) $I_1 = \lim\limits_{x\to\infty} -x^2\left(\left(1 + \frac{1}{x} + \frac{1}{x^3}\right)^{\frac{1}{3}} + \left(1 - \frac{1}{x} + \frac{1}{x^3}\right)^{\frac{1}{3}} - 2\right)$

$$= \lim_{x\to\infty} -x^2\left((1 + \alpha)^{\frac{1}{3}} + (1 - \beta)^{\frac{1}{3}} - 2\right)$$

$\left(\text{where, } \alpha = \frac{1}{x} + \frac{1}{x^3}, \beta = \frac{1}{x} - \frac{1}{x^3}\right)$

$$= \lim_{x\to\infty} -x^2\left(\frac{\alpha - \beta}{3} + \frac{1}{3}\left(\frac{1}{3} - 1\right)\frac{\alpha^2 + \beta^2}{2!} + \ldots\right)$$

$$= \lim_{x\to\infty} -x^2\left(\frac{2}{3x^3} - \frac{1}{9}\left(\frac{2}{x^2} + \frac{2}{x^6}\right) + \ldots\right) = \frac{2}{9}$$

(IV) Put $n = 1$; $f'(x) = f(x + 1) - f(x)$, $n = 2$;

$$f'(x) = \frac{f(x + 2) - f(x)}{2}$$

So, $f'(x) = \frac{f(x + 2) - f(x + 1) + f(x + 1) - f(x)}{2}$,

$$f'(x) = \frac{1}{2}f'(x + 1) + \frac{1}{2}f'(x)$$

$\Rightarrow f'(x) = f'(x + 1)\ \forall x \in R$

$\Rightarrow (f(x + 1) - f(x))' = 0\ \forall x \in R$

$\Rightarrow f(x + 1) - f(x) = c$ for a constant $c \in R$

$\Rightarrow f'(x) = c \Rightarrow f(x) = cx + d$

$f'(x) = c \therefore c = 2$

$f(0) = d = 3 \therefore f(x) = 2x + 3$

$$\frac{f(6)}{f(1)} = 3$$

10. **Ans (D)**

(I) $0 \quad \frac{1}{2} \quad 2 \quad 3 \quad \frac{7}{2}\ 4\ \frac{9}{2} \quad 5 \quad 6\ \frac{15}{2} \quad 8 \quad 9$

$$f\left(\frac{x + 13}{2}\right) = f\left(\frac{3 - x}{2}\right)$$

$f(x) = f(8 - x)$

$f'(x) = -f'(8 - x)$

$f'(2) = -f'(6) = 0$

$f'(3) = -f'(5) = 0$

$f'(4) = -f'(4) = 0$

$f'\left(\frac{9}{2}\right) = -f'\left(\frac{7}{2}\right) = 0$

$f'(0) = -f'(8)$; $h(x) = \frac{d}{dx}(f'(x) f''(x))$

Clearly: $h(x)$ has minimum 20 zeroes

(II) $x^4 - 7x^2 - 4x + 20 = (x^2 - 4)^2 + (x - 2)^2$

$x^4 + 9x^2 + 16 = (x^2 + 4)^2 + x^2$

Take the curve $y = x^2$. Both square roots can be interpreted as distances.

(III) $x = y = 1 \Rightarrow f^2(1) + f^2(2023) = 2 \times f(1)$

$\Rightarrow f(1) = 1$

$y = 1 \Rightarrow f(x) . f(1) + f(2023/x) f(2023) = 2f(x)$

$\Rightarrow f(x) = f(2023/x)\, f\left(\frac{2023}{x}\right)$

$y\text{by}\frac{2023}{x} \Rightarrow f(x) f(2023/x) = 1$

$\Rightarrow f(x) = 1, \forall x > 0$

(IV) $\lim\limits_{t\to\infty} \frac{\sqrt{tx}}{\sqrt{tx^2 - 3tx + t - 1} - x}$

$\tan\left[\sin\left(\cos\frac{\pi}{6}\right)\right]$

$$\frac{\sqrt{x}}{\sqrt{x^2 - 3x + 1}} = \frac{\sqrt{3}}{1}$$

$x = 3x^2 - 9x + 3$

$3x^2 - 10x + 3 = 0$

$\Rightarrow (3x - 1)(x - 3) = 0$

$\Rightarrow x = \frac{1}{3}, 3$

$\left(8^\alpha + 2^\beta - \alpha\beta\right) = 8^{\frac{1}{3}} + 2^3 - 1$

$\Rightarrow 2 + 8 - 1 = 9$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (3.00)**

Let $f(x) = x^4 + 4bx^3 + 12x^2 + 4x + 1$

$$f'(x) = 4x^3 + 12bx^2 + 24x + 4$$

$$f''(x) = 12x^2 + 24bx + 24$$

if $f(x)$ does not changes its concavity, then $f''(x)$ is always non negative $\Rightarrow D \le 0$

$$\Rightarrow 576b^2 - 4.12.24 \le 0$$

$$\Rightarrow b^2 - 2 \le 0 \Rightarrow b \in [-\sqrt{2}, \sqrt{2}]$$

hence number of integral values of b is 3.

2. **Ans (1.00)**

$$\text{Let } \frac{16r^2 + 16r + 6}{(2r+1)(2r+2)(2r+3)}$$

$$= \frac{A}{2r+1} + \frac{B}{2r+2} + \frac{C}{2r+3}$$

$$\Rightarrow A = 1, B = -6, C = 9$$

$$L = \lim_{n\to\infty}\sum_{r=0}^{n} 3^{2r+1}\left(\frac{1}{2r+1} - \frac{3}{2r+2} - \frac{3}{2r+2} + \frac{9}{2r+3}\right)$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n} \frac{3^{2r+1}}{2r+1} - \frac{3^{2r+2}}{2r+2} - \frac{3^{2r+2}}{2r+2} + \frac{3^{2r+3}}{2r+3}$$

$$= \ell n(1+3) + \ell n(1+3) - 3 = 2\,\ell n 4 - 3$$

$$\therefore [2\ell n4 - 3] = -1$$

3. **Ans (4975.00)**

$$y = [x] + \{x\}^2 \Rightarrow [y] = [x]$$

$$\text{Now, } y - [x] = \{x\}^2 \Rightarrow \{y\} = \{x\}^2$$

$$\therefore x = [x] + \{x\} = [y] + \sqrt{\{y\}}$$

$$\therefore g(x) = [x] + \sqrt{\{x\}}$$

Differentiable every where except integral values of x.

$$\text{In } x \in (0, 1) \Rightarrow g(x) = \sqrt{x} \Rightarrow g'(x) = \frac{1}{2\sqrt{x}}$$

$$\text{Now, } g'(x) = 1 = \frac{1}{2\sqrt{x}} \Rightarrow x = \frac{1}{4}$$

$$\therefore \text{sum} = \sum_{r=0}^{99} r + \frac{1}{4} = 4950 + 25 = 4975$$

4. **Ans (7.00)**

$$f'(x) = x\ln x - e$$

$$\Rightarrow f'(x)\begin{cases} < 0 & ; \quad x \in (0, e) \\ > 0 & ; \quad x \in (e, \infty) \end{cases}$$

$$f(x)_{min} = f(e) = k - \frac{3}{4}e^2$$

$$\Rightarrow k - \frac{3}{4}e^2 \geqslant 0$$

$$k \geqslant \frac{3}{4}e^2 \Rightarrow (a+b) = 7$$

5. **Ans (967.00)**

Let 1, 3, 5, 7 and α are roots of f(x)

$$\Rightarrow f(x) = A(x-1)(x-3)(x-5)(x-7)(x-\alpha)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \left[\frac{1}{x-1} + \frac{1}{x-3} + \frac{1}{x-5} + \frac{1}{x-7} + \frac{1}{x-\alpha}\right]$$

Now, $f'(11) = 0$

$$o = \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{11-\alpha}$$

$$\Rightarrow 77\alpha = 967$$

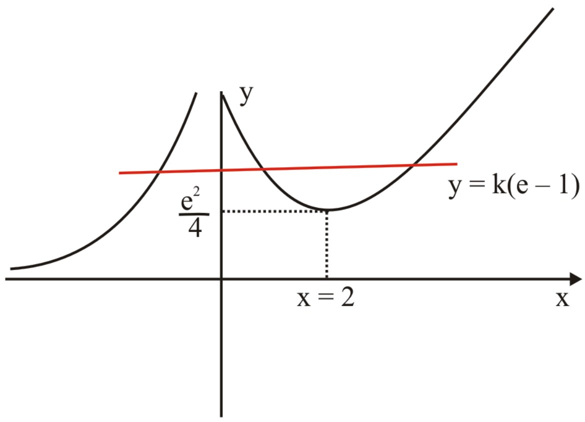
6. **Ans (5049.00)**

$$f(x) = e^x \lim_{n\to\infty} \sum_{r=1}^{n} \frac{(e^{r+1} - e^r)}{(e^r - 1)(e^{r+1} - 1)}$$

$$f(x) = e^x \lim_{n\to\infty}\left(\frac{1}{e-1} - \frac{1}{e^2-1} + \frac{1}{e^2-1} - \frac{1}{e^3-1} + \ldots\ldots - \frac{1}{e^{n+1}-1}\right)$$

$$f(x) = \frac{e^x}{e-1}$$

Now, $\frac{e^x}{x^2} = k(e-1)$



$$y = \frac{e^x}{x^2}$$

$$\frac{dy}{dx} = \frac{e^x \cdot (x-2)}{x^3}$$

for three solution

$$k(e-1) > \frac{e^2}{4}$$

$\because k \in I \quad k > \frac{e^2}{4(e-1)}$

Now, k = 2, 3, 4, ..., 99, 100

$$S = \sum_{k=2}^{100} k = \frac{100 \times 101}{2} - 1$$

= 5050 – 1 = 5049

7. **Ans (6.00)**

$$\lim_{x\to 0}\left(3 - \frac{P(x)}{x}\right) = 27$$

$\Rightarrow$ P(x) has no constant term

let $P(x) = ax^4 + bx^3 + cx^2 + dx$

$\Rightarrow 3 - d = 27 \Rightarrow d = -24$

$P(x) = ax^4 + bx^3 + cx^2 - 24x$

$P'(2) = 0,\ p(1) = -9,\ p'''(2) = 0$

$P'(x) = 4ax^3 + 3bx^2 + 2cx - 24$

$P''(x) = 12ax^2 + 6bx + 2c$

$P'''(x) = 24ax + 6b$

$a + b + c - 24 = -9$

$\Rightarrow a + b + c = 15$ ... (1)

$P'(2) = 0$

$\Rightarrow 4a(8) + 3b(4) + 2c(2) - 24 = 0$

$\Rightarrow 8a + 3b + c = 6$ ... (2)

$P'''(2) = 0$

$\Rightarrow 24a(2) + 6(b) = 0$

$\Rightarrow 8a + b = 0$ ... (3)

Solving (1), (2) & (3)

$a = 1,\ b = -8,\ c = 22$

$\Rightarrow P(x) = x^4 - 8x^3 + 22x^2 - 24x$

$P'(x) = 4x^3 - 24x^2 + 44x - 24$

$= 4[x^3 - 6x^2 + 11x - 6]$

$P'(x) = 4[(x-1)(x-2)(x-3)]$

$P''(x) = 4[3x^2 - 12x + 11] > 0\ \forall\ x \in [3, 4]$

$\Rightarrow P'(x) = 4[(4-1)(4-2)(4-3)]$

$= 4[(3)(2)(1)]$

$= 24 = 4M$

$\Rightarrow M = 6$

8. **Ans (10.00)**

Let $g(x) = f(x+3) - f(x)$ and $g(0) = k\ (k > 0)$

$g(3) = -k,\ g(6) = k,\ g(9) = -k,\ g(12) = k,\ g(15) = -k,\ g(18) = k$

$\Rightarrow g(x) = 0$ has at least 6 solutions in $x \in (0, 18)$

$\Rightarrow g'(x) = 0$ has at least 5 solutions in $x \in (0, 18)$

Let

$\Rightarrow h(x) = g(x) \cdot g'(x)$

$\Rightarrow h(x) = 0$ has at least 11 solutions in $x \in (0, 18)$

$\Rightarrow h'(x) = 0$ has at leat 10 solutions in $x \in (0, 18)$