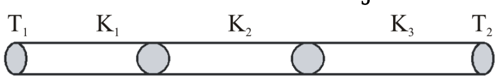
PART-1 : PHYSICS

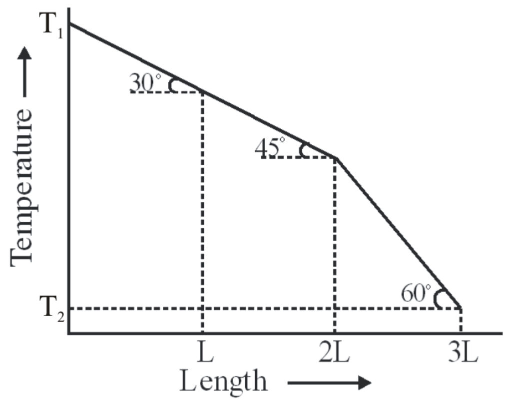
SECTION-I

1) Assume that you are facing south direction at a time when the sun is just setting. The scattered light that comes to you is polarized such that electric field is oscillating in a plane which is

(A) Horizontal with electric field along vertical direction.
(B) Vertical with electric field along east west direction.
(C) Vertical with electric field along vertical direction.
(D) Horizontal with electric field along east west direction.

2) Three rods of the same length and the same area of cross - section are joined. The temperature of two ends are T_1 and T_2 as shown in the figure. 

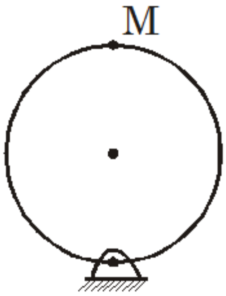
As we move along the rod the variation in temperature is as shown in the following graph [Rods are insulated from the surrounding except at the end faces]



(A) $\frac{K_1}{\sqrt{3}} = \frac{K_2}{3} = \frac{K_3}{1}$
(B) $\frac{K_1}{3} = \frac{K_2}{3} = \frac{K_3}{\sqrt{3}}$
(C) $\frac{K_1}{3} = \frac{K_2}{\sqrt{3}} = \frac{K_3}{1}$
(D) $\frac{K_1}{\sqrt{3}} = \frac{K_2}{1} = \frac{K_3}{3}$

3) A uniform disc of mass M and radius R is supported vertically by a pivot at its periphery as shown. A particle of mass M is fixed to the rim and raised to the highest point above the centre. The system is released from rest and it can rotate about pivot freely. The axis of rotation is horizontal and perpendicular to the plane of paper. The angular speed of the system when the attached object is

directly beneath the pivot, is



(A) $\sqrt{\frac{24g}{11R}}$

(B) $\sqrt{\frac{8g}{11R}}$

(C) $\sqrt{\frac{8g}{3R}}$

(D) $\sqrt{\frac{14g}{3R}}$

4) 1 mole of a monoatomic gas undergoes the process PT = constant. Then the molar heat capacity of the gas during the process will be equal to :

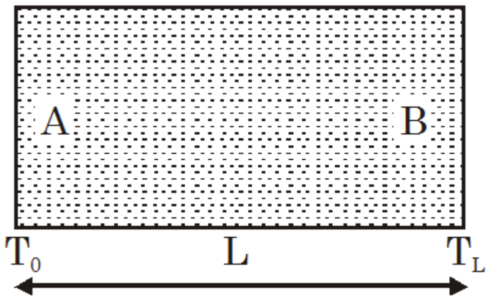
(A) 4R

(B) 2.5R

(C) 3.5 R

(D) 8R/3

5) The temperature of a mono-atomic gas in an uniform container of length 'L' varies linearly from T_0 to T_L as shown in the figure. If the molecular weight of the gas is M, then the time taken by a wave pulse in traveling from end A to end B is



(A) $\frac{2L}{\sqrt{T_L}+\sqrt{T_0}}\sqrt{\frac{3M}{5R}}$

(B) $\sqrt{\frac{3(T_L-T_0)}{5RML}}$

(C) $\frac{2L}{\sqrt{T_L}-\sqrt{T_0}}\sqrt{\frac{3M}{5R}}$

(D) $L\sqrt{\frac{M}{2R(T_L-T_0)}}$

6) Two sources of sound placed close to each other, are emitting progressive waves given by

$y_1 = 4\sin 600\pi t$ and $y_2 = 5\sin 608\pi t$. An observer located near these two sources of sound will hear

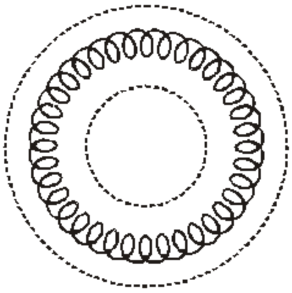
(A) 4 beats per second with intensity ratio 25 : 16 between waxing and waning.

(B) 8 beats per second with intensity ratio 25 : 16 between waxing and waning.

(C) 8 beats per second with intensity ratio 81 : 1 between waxing and waning.

(D) 4 beats per second with intensity ratio 81 : 1 between waxing and waning.

7) A spring of mass M and force constant K in form of a circular loop performing oscillations moving radially in and out in free space keeping its center at rest. Time period of oscillation is given by (Assume cross sectional radius of spring is very small than radius of spring loop)



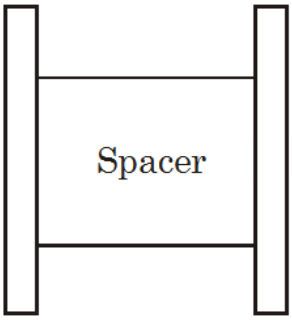
(A) $\sqrt{\frac{M}{K}}$

(B) $2\pi\sqrt{\frac{M}{K}}$

(C) $2\pi\sqrt{\frac{M}{3K}}$

(D) $2\pi\sqrt{\frac{2M}{K}}$

8) A capacitor is to be designed to operate, with constant capacitance, in an environment of fluctuating temperature (less than 5%). As shown in the figure, the capacitor is a parallel plate capacitor with 'spacer' to change the distance for compensation of temperature effect. If α_1 be the co-efficient of linear expansion of plates and α_2 that of spacer, the condition for no change in capacitance with change of temperature is (The capacitance of the capacitor is equal to C and spacer have insulated ends). The dielectric constant of spacer is 1.



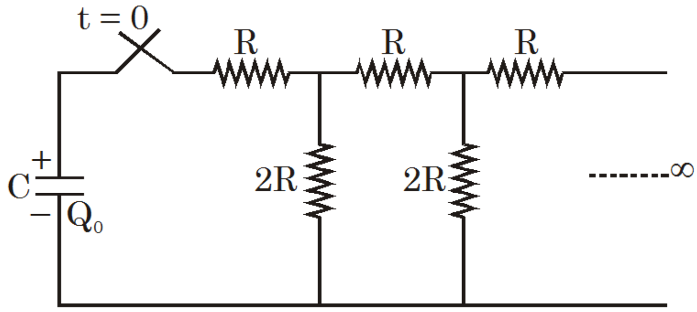
(A) $\alpha_1 = \alpha_2$

(B) $\alpha_1 = 2\alpha_2$

(C) $2\alpha_1 = \alpha_2$

(D) $2\alpha_1 = 3\alpha_2$

9) A capacitor is being discharged by infinite ladder of resistors, starting from Q_0 charge. Total heat dissipated in horizontal resistors is



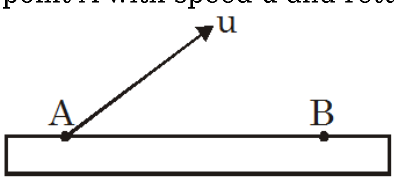
(A) $\frac{Q_0^2}{4C}$

(B) $\frac{Q_0^2}{6C}$

(C) $\frac{Q_0^2}{3C}$

(D) $\frac{Q_0^2}{5C}$

10) A large flat metal surface has a uniform charge density ($+\sigma$). An electron of mass m and charge e leaves the surface at the point A with speed u and returns to it at point B. Disregarding gravity, the maximum value of AB is



(A) $\frac{u^2 m \varepsilon_0}{\sigma e}$

(B) $\frac{u^2 e \varepsilon_0}{m\sigma}$

(C) $\frac{2\varepsilon_0 m u^2}{e\sigma}$

(D) $\frac{u^2 \sigma e}{\varepsilon_0 . m}$

11) Let A_n be the area enclosed by the n^{th} orbit in a hydrogen atom. The graph of □n (A_n / A_1) against □n (n)

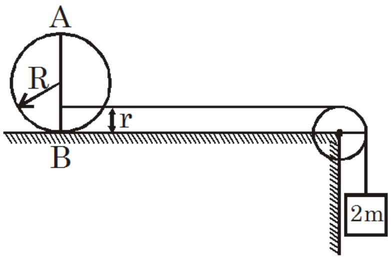
(A) Will not pass through the origin

(B) Will have certain points lying on a straight line with slope 4

(C) Will be a monotonically increasing non linear curve

(D) Will be a circle

12) An uniform ring of radius R, is fitted with a massless rod AB along its diameter. An ideal horizontal string (whose one end is attached with the rod at a height r) passes over a smooth pulley and other end of the string is attached with a block of mass double the mass of ring as shown. The co-efficient of friction between the ring and the surface is μ. When the system is released from rest,



the ring moves such that rod AB remains vertical. The value of r is

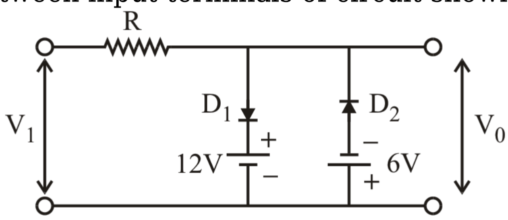
(A) $R\left(1-\frac{3\mu}{2(1+\mu)}\right)$

(B) $R\left(1-\frac{\mu}{2(1+\mu)}\right)$

(C) $R\left(2-\frac{3\mu}{2(1+\mu)}\right)$

(D) $R\left(1-\frac{3\mu}{(1+\mu)}\right)$

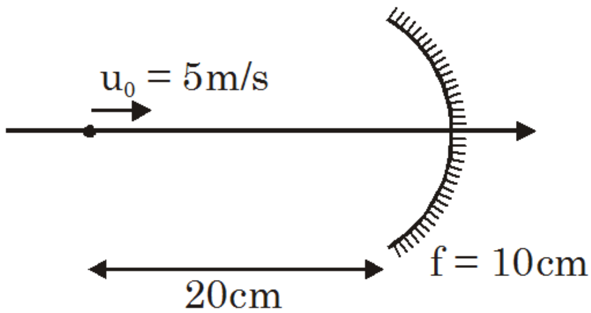
13) A sinusoidal peak voltage of 15V connected between input terminals of circuit shown in figure.



Assume diodes are ideal. In the output wave form,

(A) Positive peaks of input clipped at +12V and negative peaks at -6V
(B) Positive peaks of input clipped at +6V and negative peaks at –12 V
(C) Positive peaks of input clipped at +12V and negative peaks at –12 V
(D) Positive peaks of input clipped at +6V and negative peaks at -6 V

14) An object moves with a uniform velocity u_0 along the axis of a concave spherical mirror. Consider the instant shown in diagram, object is moving with u_0 = 5 m/s and f = –10 cm. If object is at the centre of curvature at this instant, then the magnitude of acceleration of image at this instant is a



m/s^2. Find a.

(A) 5
(B) 10
(C) 25
(D) None of these

15) Only thin film shows interference because

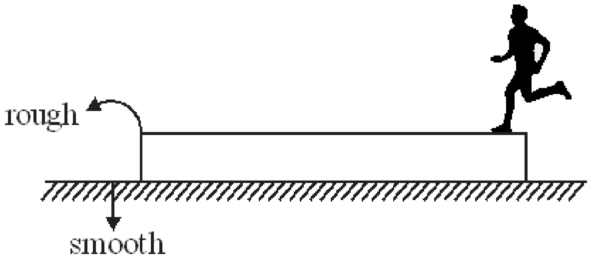
(A) In thick film, light is absorbed in large amount.

(B) In thick film, condition of coherence is not met.

(C) In thick film, interference is not for visible wavelengths.

(D) Light get scattered by thick film.

16) A plank is on a smooth horizontal ground. A man is standing at one end of plank as shown in figure. Whole system is initially at rest. When man starts walking without slipping :



Consider the following statements:

i. Friction between man and plank is static.
ii. Friction between man and plank is kinetic.
iii. Net work done by friction on system (man + plank) is 0.
iv. Work done by friction on man is positive
v. Work done by friction on man is negative
vi. Work done by friction on plank is positive
vii. Work done by friction on plank is negative

Which of the above statements are correct?

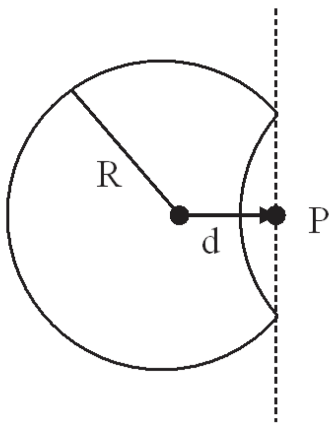
(A) i, iii, iv, vii

(B) i, iv, vi

(C) i, iii, v, vi

(D) ii, v, vi

17) A curved portion of radius of curvature R is cut from a uniformly charged non–conducting sphere of radius R having charge density ρ as shown in figure. Find electric field at point P lying



symmetrically w.r.t the ends.

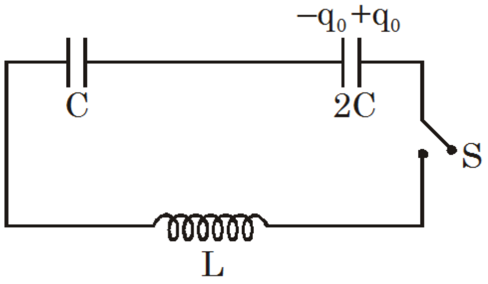
(A) $\frac{2\rho d}{3\epsilon_0}$

(B) $\frac{\rho d}{3\epsilon_0}$

(C) $\frac{\rho d}{6\epsilon_0}$

(D) $\frac{\rho d}{2\epsilon_0}$

18) The switch in the circuit below is open. The capacitor of capacitance 2C carries electric charge q_0. The capacitor of capacitance C is uncharged and there is no electric current in the inductor. Find the value of the maximum current through the switch after it is closed.



(A) $\frac{q_0}{\sqrt{6LC}}$

(B) $\frac{q_0}{\sqrt{8LC}}$

(C) $\frac{\sqrt{2}q_0}{\sqrt{3LC}}$

(D) $\frac{q_0}{\sqrt{LC}}$

19) A thin elastic loop of natural length ▯ is kept slightly on a water film in gravity free space and inner film of loop is bursted keeping outer film intact. If force constant of loop is k and loop follow hooke's law, then radius of loop in equilibrium is (Surface Tension of water is S)

(A) $\frac{k\ell}{\pi k - S}$

(B) $\frac{k\ell}{2\pi k - S}$

(C) $\frac{2k\ell}{\pi - S}$

(D) $\frac{k\ell}{2\pi k - 2S}$

20) $F = \frac{\alpha}{\beta} e^{\left(-\frac{x}{\alpha kt}\right)}$, F = Force, k = Boltzmann constant, t = temperature, x = distance. The dimensions of β is

(A) $\left[MLT^{-4}\right]$

(B) $\left[M^{-2}L^{-2}T^{4}\right]$

(C) $\left[MLT^{-2}\right]$

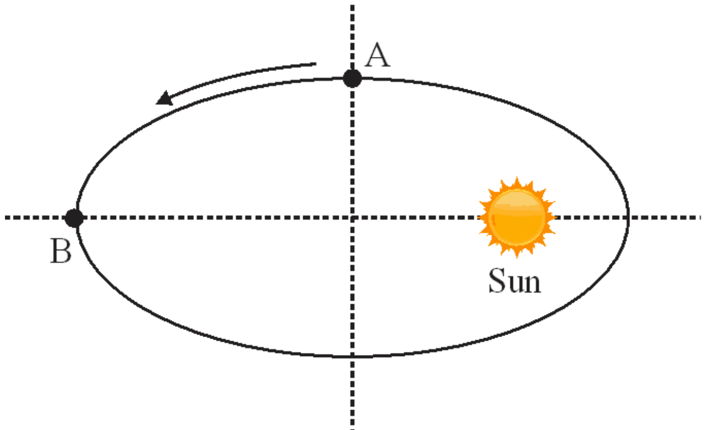
(D) $\left[M^{2}LT^{-1}\right]$

SECTION-II

1) In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm and each rotation corresponds to one main scale division. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, what is the thickness (in 10^{-5} m) of the wire?

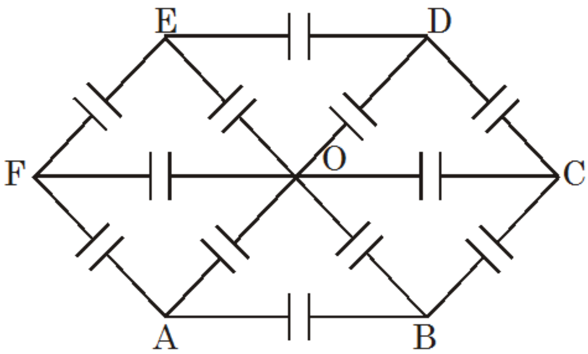
2) A paramagnetic sample shows a magnetization of 8 Am^{-1} when placed in an external magnetic field of 0.6T at a temperature of 4K. When the same sample is placed in an external magnetic field of 1.2T at a temperature of 16K, the magnetization (in Am^{-1}) is

3) A planet is revolving around the sun in an elliptical orbit having eccentricity $e\left(=\frac{\pi}{4}\right)$. If the time period of revolution is 16 months then find the time taken by the planet in going from A to B in



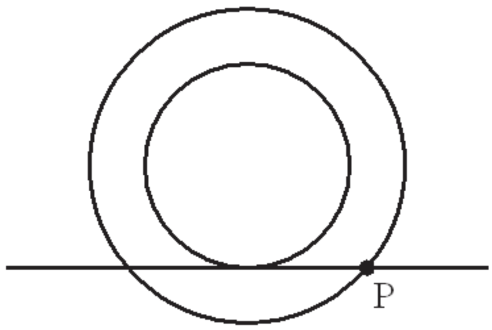
months.

4) All the capacitors in the given figure has a capacitance C_0. The equivalent capacitance between



the points A and O is x μF. Find x, if C_0 = 9 μF.

5) A spool is rolling on horizontal surface with its inner radius in contact with surface. The outer surface of spool performs trochoidal motion going above and below the horizontal level. Any point P on outer circumference has velocity direction at 30N degrees angle with horizontal, when it is just



crossing horizontal level. Then find the value of N.

PART-2 : CHEMISTRY

SECTION-I

1) For the reaction $2NO(g) + H_2(g) \rightarrow N_2O(g) + H_2O(g)$ at 900K. following data are observed.

Initial pressure of NO(atm)	Initial pressure of H_2(atm)	Rate
0.15	0.40	0.02
0.075	0.40	0.005
0.150	0.2	0.01

Find out the rate law and order of reaction-

(A) $k\left[P_{NO_2}\right]^2\left[P_{H_2}\right]^1, 3$

(B) $k\left[P_{NO_2}\right]^2\left[P_{H_2}\right]^1, 2$

(C) $k\left[P_{NO_2}\right]^2\left[P_{H_2}\right]^1, 1$

(D) $k\left[P_{NO_2}\right]^2\left[P_{H_2}\right]^1, 0$

2) **Statement-1 :** The voltage of mercury cell remains constant for longer period of time.
Statement-2 : It is because net cell reaction does not involve ions.

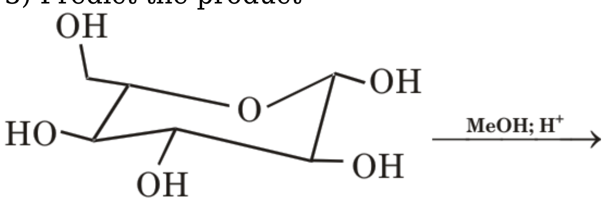
(A) If both the statements are true and statement-2 is the correct explanation of statement-1

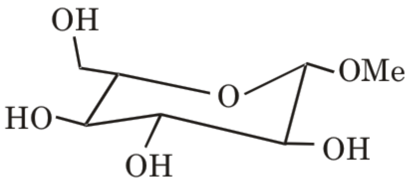
(B) If both the statements are true but statement-2 is not the correct explanation of statement-1

(C) If statement-1 is true and statement-2 is false

(D) If statement-1 is false and statement-2 is true

3) Predict the product



(A)


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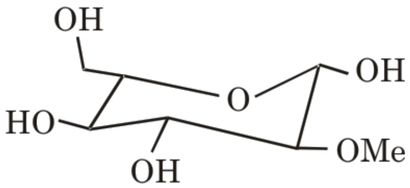
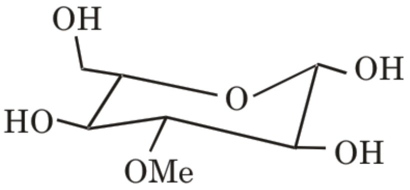
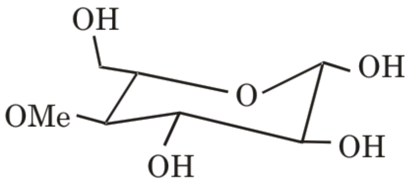
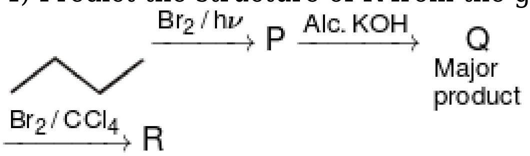
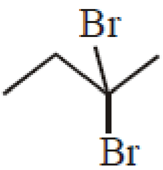


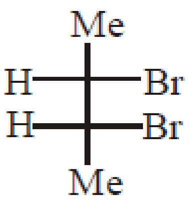
(C) 

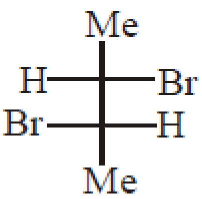
(D) 

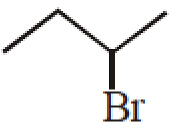
4) Predict the structure of R from the given sequence of reaction :



(A) 

(B) 

(C) 

(D) 

5) Which of the following statement is correct for BrF_5 :

(A) Total number of lone pairs present is 15

(B) Total number of maximum number of atoms in one plane = 5

(C) Molecule is polar

(D) Bromine has its maximum covalency in this molecule

6) Match List I with List II:

List - I (Complexes)	**List - II (Hybridisation)**
A $[Ni(CO)_4]$	I sp^3

B $[Co(CN)_4]^{3-}$ II dsp^2

C $[Fe(CN)_6]^{2+}$ III sp^3d^2

D $[Fe(H_2O)_6]^{2+}$ IV d^2sp^3

(A) A - II, B – I, C- III, D - IV

(B) A - I, B – II, C- III, D - IV

(C) A - II, B – I, C- IV, D - III

(D) A - I, B – II, C- IV, D - III

7) In a constant pressure calorimeter 224 ml of 0.1M KOH (aq.) solution is reacted with 50 ml of 0.1M H_2SO_4 (aq.) solution then increase in temperature of solution will be (assume heat capacity of calorimeter is negligible) - Given :
Specific heat of solution = 1 cal/gm–K
Density of solution = 1 gm/ml HCl(aq.) + NaOH(aq.) → NaCl (aq.) + H_2O(l) $\Delta H = -13.7$ kcal/eq.

(A) 0.5 K

(B) 1 K

(C) 2K

(D) 2.4 K

8) Ratio of solubilities of gases N_2 & O_2 in water (as mole of gas in large volume of water) from air at 25º & 1 atm will be, if air is 20 % by mole of O_2 & 80% by mole of N_2

Given : $K_H (N_2) = 2 \times 10^4$ atm

$K_H (O_2) = 10^4$ atm

(A) 8 : 1

(B) 1 : 8

(C) 2 : 1

(D) 1 : 2

9) The correct order of atomic radii trends in decreasing order for lanthanoids-
Given, Ce(Z = 58)
Eu(Z = 63)
Gd(Z = 64)
Lu(Z = 71)

(A) Eu > Lu > Ce > Gd

(B) Eu > Ce > Lu > Gd

(C) Eu > Ce > Gd > Lu

(D) Ce > Eu > Gd > Lu

10) When CO_2 gas is passed through the aqueous solution of sodium chromate

(A) ppt. of $Cr(OH)_3$ is formed

(B) Orange colour solution of $Na_2Cr_2O_7$ is formed

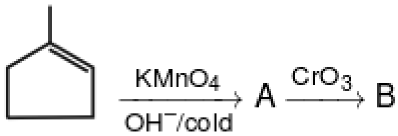
(C) Green ppt. of $Cr_2(CO_3)_3$ is formed

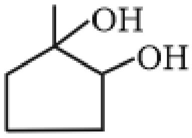
(D) Cr is oxidise to +7 oxidation state

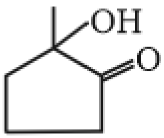
11) $CH_3CH_2COOH \xrightarrow{NH_3} \xrightarrow{\Delta} \xrightarrow{Br_2/KOH} (A)$ Which is not correct statement for (A)

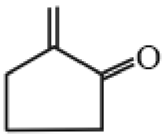
(A) It gives carbyl amine reaction.
(B) It gives mustard oil reaction.
(C) Forms alcohol with $NaNO_2$/ HCl
(D) Forms yellow oily liquid with $NaNO_2$/ HCl

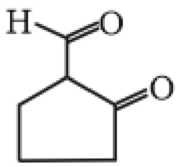
12) Identify B in the following scheme ?



(A) 

(B) 

(C) 

(D) 

13) Which of the following statement is correct for paper chromatography ?

(A) Water present in the mobile phase gets absorbed by the paper which then forms the stationary phase.
(B) Water present in the pores of the paper forms the stationary phase.
(C) Paper sheet forms the stationary phase.
(D) Paper and water present in its pores together form the stationary phase.

14) Given below are two statements:
Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga. **Statement II :** The d orbitals in Ga are completely filled.
In the light of the above statements, choose the most appropriate answer from the options given below

(A) Statement I is incorrect but statement II is correct.

(B) Both the statements I and II are correct

(C) Statement I is correct but statement II is incorrect

(D) Both the statements I and II are incorrect

15) Phosphine, ammonia and acetylene can be formed by treating water with :-

(A) Mg_3P_2, Li_3N, Al_4C_3

(B) Ca_3P_2, Mg_3N_2, CaC_2

(C) Ca_3P_2, $CaCN_2$, Be_2C

(D) Ca_3P_2, NH_4NO_3, Mg_2C_3

16) Concentrated nitric acid, upon long standing, turns yellow-brown due to the formation of -

(A) NO

(B) NO_2

(C) N_2O

(D) N_2O_4

17) Which of the following bonds has the highest bond energy ?

(A) Se—Se

(B) Te—Te

(C) S—S

(D) O—O

18) $C_6H_5CH_3 \xrightarrow{CrO_2Cl_2} A \xrightarrow{H_2O} B$

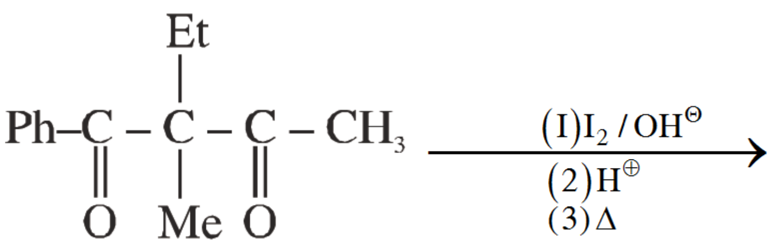
The functional group present in B and name of the reaction would be–

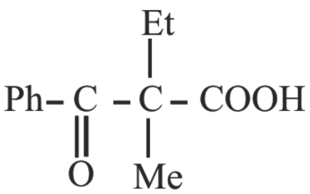
(A) –CHO, Gattermann aldehyde synthesis

(B) –CHO, Etard reaction

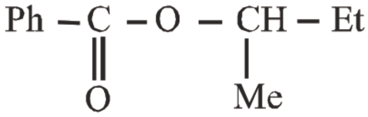
(C) $-COCH_3$, Friedel Crafts reaction

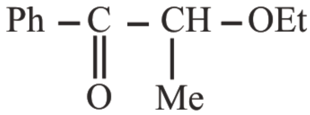
(D) –CHO, Oxo reaction

19)  product is

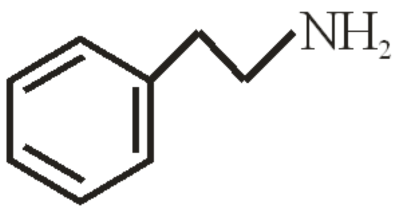
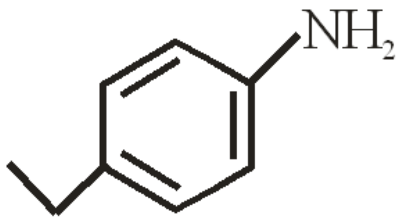
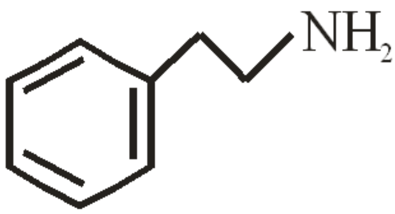
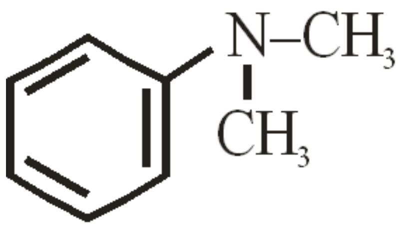
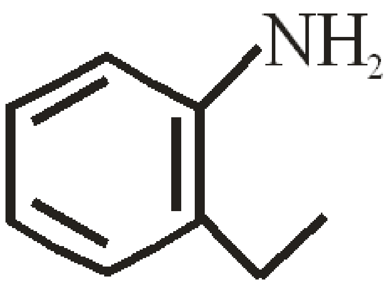
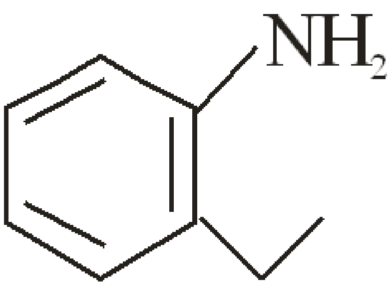
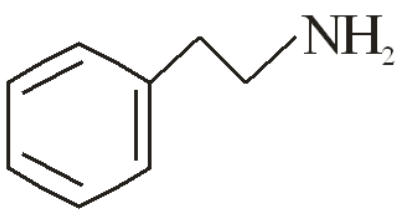
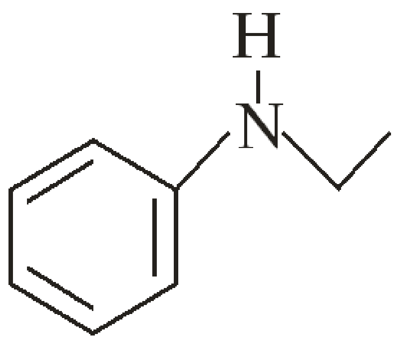
(A) 

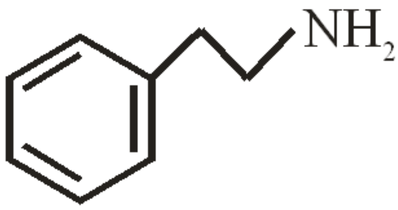
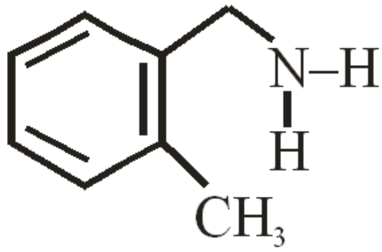
(B) 

(C) 

(D) 

20) Isomeric amines with molecular formula $C_8H_{11}N$ give the following tests
Isomer (P) ⇒ Can be prepared by Gabriel phthalimide synthesis
Isomer (Q) ⇒ Reacts with Hinsberg's reagent to give solid insoluble in NaOH
Isomer (R) ⇒ Reacts with HONO followed by
β-naphthol in NaOH to give red dye.
Isomers (P), (Q) and (R) respectively are

	P	Q	R
(1)			
(2)			
(3)			

(4)			

(A) 1

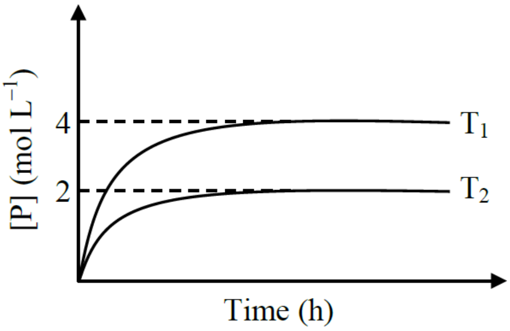
(B) 2

(C) 3

(D) 4

SECTION-II

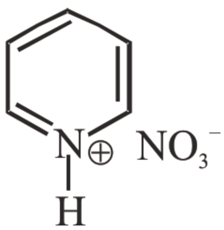
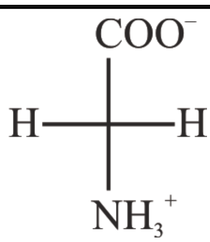
1) In a one-litre flask, 6 moles of A undergoes the reaction $A(g) \rightleftharpoons P(g)$. The progress of product formation at two temperatures (in Kelvin), T_1 and T_2, is shown in the figure:

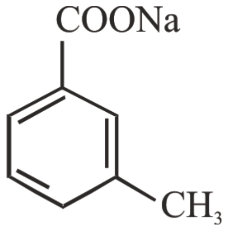


If $T_1 = 2T_2$ and $(\Delta G_2^\ominus - \Delta G_1^\ominus) = RT_2 \ln x$, then the value of x is ______.

[$\Delta G_1^\ominus$ and $\Delta G_2^\ominus$ are standard Gibb's free energy change for the reaction at temperatures T_1 and T_2, respectively.]

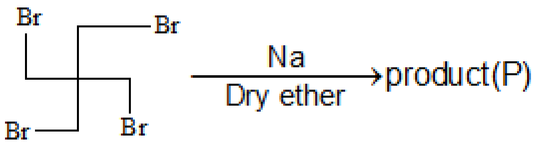
2)

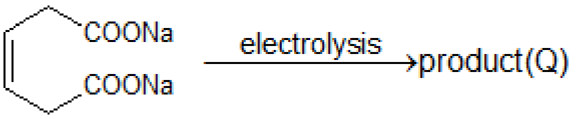
	List-I (Components)		**List-II (Nature)**
(P)		(1)	$7 + \frac{1}{2}(pKa + \log c)$
(Q)		(2)	$7 - \frac{1}{2}(pK_b + \log c)$

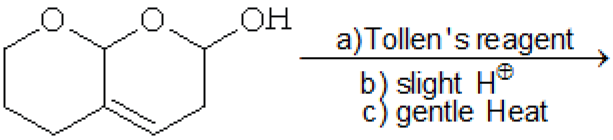
(R)		(3)	$7+\frac{1}{2}(pK_a-pK_b)$
(S)	NH_4CN	(4)	$\frac{1}{2}(pKa_1+pKa_2)$

3) Determine the number of total possible isomers of complex $[PtCl(NO_3)(NO_2)(SCN)]$

4) Consider the following test for a group-IV cation.
$M^{2+} + H_2S \rightarrow$ A (Black precipitate) + byproduct
A + aqua regia $\rightarrow$ B + NOCl + S + H_2O
B + KNO_2 + CH_3COOH $\rightarrow$ C + byproduct
The spin only magnetic moment value of the metal complex C is ______BM. (Nearest integer)

5) (i) 

(ii) 

(iii) 

X = Double bond equivalent of (P)
Y = Double bond equivalent of (Q)
Z = Number of moles of CO_2 evolved in reaction (iii) when one mole of the compound undergoes specified reaction sequence.
Find the value of (x + y + z) ?

PART-3 : MATHEMATICS

SECTION-I

1) The value of $\int \frac{(x^4+3x^2+5)dx}{x^3\sqrt{x}\sqrt{x^5\left(1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}\right)}}$ (where C is constant of integration)

(A) $2x^2\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}+C$

(B) $2x\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}+C$

(C) $2\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}+C$

(D) $x^3\sqrt{x}\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}+C$

2) A person is driving the car and the probability that he sees a red signal is $\frac{2}{3}$ and green signal is $\frac{1}{3}$. The probability that he meet an accident if he crosses the red signal is $\frac{1}{5}$ and the probability that he meet an accident if he crosses the green signal is $\frac{1}{8}$. Then, the probability that he sees a red signal if he does not meet an accident is

(A) $\frac{64}{99}$

(B) $\frac{32}{99}$

(C) $\frac{8}{99}$

(D) $\frac{35}{99}$

3) If α, β (β > α) are the roots of $g(x) = px^2 + qx + r = 0$ and f(x) is an even function, then $\int_{\alpha}^{\beta} \frac{e^{f\left(\frac{g(x)}{x-\alpha}\right)} \cdot dx}{e^{f\left(\frac{g(x)}{x-\alpha}\right)}+e^{f\left(\frac{g(x)}{x-\beta}\right)}}$ is equal to

(A) $\left|\frac{q}{p}\right|$

(B) $\left|\frac{q}{2p}\right|$

(C) $\frac{\sqrt{q^2-4pr}}{2|p|}$

(D) $\left|\frac{p}{q}\right|$

4) $\sum_{r=1}^{15} \frac{r \cdot 3^r}{(r+3)!} =$

(A) $\frac{1}{3!}-\frac{3^{16}}{18!}$

(B) $\frac{1}{2!}-\frac{3^{16}}{18!}$

(C) $\frac{1}{3!}-\frac{3^{17}}{18!}$

(D) $\frac{1}{2!}-\frac{3^{17}}{18!}$

5) Let $I=\int_0^1 \frac{\sin x}{\sqrt[4]{x}}\,dx$ and $J=\int_0^1 \frac{\cos x}{\sqrt[4]{x}}\,dx$

Which of the following is true ?

(A) $I<\frac{4}{7}$ and $J>\frac{4}{3}$

(B) $I<\frac{4}{7}$ and $J<\frac{4}{3}$

(C) $I>\frac{4}{7}$ and $J>\frac{4}{3}$

(D) $I>\frac{4}{7}$ and $J<\frac{4}{3}$

6) Non-zero vectors $\vec{p},\vec{q},\vec{r}$ are mutually perpendicular and of equal magnitude. If $\vec{x}$ is a vector which satisfies $\vec{p}\times[(\vec{x}-\vec{q})\times\vec{p}]+\vec{q}\times[(\vec{x}-\vec{r})\times\vec{q}]+\vec{r}\times[(\vec{x}-\vec{p})\times\vec{r}]=\vec{0}$, then $\vec{x}$ equals

(A) $\vec{p}+\vec{q}+\vec{r}$

(B) $\frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$

(C) $2(\vec{p}+\vec{q}+\vec{r})$

(D) $\frac{1}{3}(\vec{p}+\vec{q}+\vec{r})$

7) The value of $\lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)^{\frac{2}{n^2}}\left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}}\left(1+\frac{3^2}{n^2}\right)^{\frac{6}{n^2}}\cdots\left(1+\frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}$ is equal to

(A) $\frac{4}{e}$

(B) $\frac{3}{e}$

(C) $\frac{2}{e}$

(D) $\frac{1}{e}$

8) Let $f : R \to R$ be a function defined by
$f(x) = [x + 2] + [x]^2 - 8$ (where [.] denotes GIF)

(A) One-one onto

(B) Many-one onto

(C) One-one into

(D) Many-one into

9) In ΔABC, if $c^2 + a^2 = 2b^2$, then value of $\frac{\cot B}{\cot C + \cot A} =$
(Using usual notation)

(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{3}{2}$
(D) $\frac{5}{2}$

10) If the line $\frac{x-2}{3} = \frac{y+1}{5} = \frac{z-2}{+2}$ lies on the plane $\alpha x + 2y + z - \beta = 0$, then (α, β) equals

(A) (4, 8)
(B) (–8, 4)
(C) (–4, –8)
(D) (8, – 4)

11) Equation of common tangents with positive slope to the circle $x^2 + y^2 = 5$ and hyperbola $4x^2 - 9y^2 = 36$ is

(A) $2x - 3y \pm \sqrt{65} = 0$
(B) $3x - 2y \pm \sqrt{65} = 0$
(C) $2x - 3y \pm 5 = 0$
(D) $3x - 2y \pm 5 = 0$

12) Let z = x + iy, x, y ∈ R such that z ≠ 1 and |z – 1| ≠ 1. If $\frac{z-1}{e^{\frac{i\pi}{3}}} + \frac{e^{\frac{i\pi}{3}}}{z-1}$ is real, then locus of z is a straight line whose slope is (where $i = \sqrt{-1}$)

(A) $\frac{1}{\sqrt{3}}$
(B) $\sqrt{3}$
(C) 1
(D) $\frac{1}{2}$

13) If the function f : [0, 27] → R is diffrentiable and 0 < α < 1 < β < 2 < γ < 3, then $\int_0^{27} f(t)dt$ is equal to

(A) $3[\alpha f(\alpha) + \beta f(\beta) + \gamma f(\gamma)]$ for atleast one such triplet (α, β, γ)

(B) $3[\alpha^3 f(\alpha^3) + \beta^3 f(\beta^3) + \gamma^3 f(\gamma^3)]$ for atleast one such triplet (α, β, γ)

(C) $3[\alpha^2 f(\alpha) + \beta^2 f(\beta) + \gamma^2 f(\gamma)]$ for atleast one such triplet (α, β, γ)

(D) $3[\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3) + \gamma^2 f(\gamma^3)]$ for atleast one such triplet (α, β, γ)

14) Let ${}^{p}C_q$ be the number of ways in which 5 tickets are selected from 20 tickets from 1 to 20 so that no two consecutive numbered tickets are selected, then p – q may be

(A) 20

(B) 13

(C) 12

(D) 11

15) Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions f : A → A such that f(m·n) = f(m) · f(n) for every m, n ∈ A with m·n ∈ A is equal to _____.

(A) 432

(B) 400

(C) 452

(D) 462

16) A curve passing through (1, 2) and satisfying the differential equation $\int_0^x t^2 y(t)dt = x^3 y(x)\ (x > 0)$ is

(A) $x^2 y = 2$

(B) $x^2 + y^2 = 5$

(C) $\frac{x^2}{2} + \frac{y^2}{8} = 1$

(D) $y^2 = 4x$

17) If $a_1, a_2, a_3, \dots a_{2n}$ are in A.P. with common difference d, then $\tan^{-1}\left(\frac{2d}{1+a_1a_3}\right) + \tan^{-1}\left(\frac{2d}{1+a_3a_5}\right) + \tan^{-1}\left(\frac{2d}{1+a_5a_7}\right) + \dots + \tan^{-1}\left(\frac{2d}{1+a_{2n-3}.a_{2n-1}}\right)$ is equal to

(A) $\tan^{-1}(a_{2n-1}) - \tan^{-1}(a_3)$

(B) $\tan^{-1}(a_{2n}) - \tan^{-1}(a_1)$

(C) $\tan^{-1}(a_1) - \tan^{-1}(a_{2n-1})$

(D) $\tan^{-1}(a_{2n-1}) - \tan^{-1} a_1$

18) If $f : R \to R^+$ be a decreasing function and such that $\lim_{x\to\infty} \frac{f(x)}{f(\sqrt{3}x)} = 1$, then $\lim_{x\to\infty} \frac{f(x)}{f(\sqrt{2}x)}$ equals

(A) $\sqrt{\frac{2}{3}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{3}}$

(D) 1

19) Let $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$, then A^{48} is

(A) $\begin{bmatrix} 1 & 0 \\ \left(\frac{1}{3}\right)^{48} & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ \frac{3}{2}\left(1-\frac{1}{3^{48}}\right) & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 16 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 16 & 1 \end{bmatrix}$

20) If tanα, tanβ are the roots of the equation $x^2 + px + q = 0$, then the value of expression $\sin^2(\alpha + \beta) + p\sin(\alpha + \beta)\cos(\alpha + \beta) + q\cos^2(\alpha + \beta)$ is equal to

(A) $p + q$

(B) $\frac{p+q}{q}$

(C) q

(D) $\frac{p}{p+q}$

SECTION-II

1) If the quadratic equation $\alpha^2(x+1)^2 + \beta^2(2x^2 - x + 1) - 5x^2 - 3 = 0$ is satisfied for all $x \in R$, then number of ordered pair (α, β) which is possible is/are

2) If $\omega = e^{i\frac{2\pi}{3}}$, a, b, c, x, y, z are non-zero complex numbers such that $a + b + c = x$, $a + b\omega + c\omega^2 = y$ and $a + b\omega^2 + c\omega = z$, then $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is equal to (where $i = \sqrt{-1}$)

3) Let $I = \int_{0}^{2} \frac{15 + 4x - 2x^2}{e^{6(x-1)} + 1} dx$, then $\frac{6}{49} I =$

4) If A and B are coefficients of x^{11} in the expansion of $(1 + x)^{22}$ and $(1 + x)^{21}$ respectively, then $\frac{A}{B}$ is equal to

5) The number of points where the function $f(x) = \min\left\{e^x, \frac{3}{2}, 1 + e^{-x}\right\}$ is not differentiable are

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	**C**	**C**	**A**	**C**	**A**	**D**	**A**	**C**	**C**	**A**	**B**	**A**	**A**	**D**	**B**	**C**	**B**	**A**	**D**	**B**

SECTION-II

Q.	21	22	23	24	25
A.	**215**	**4**	**6**	**20**	**3**

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	**A**	**A**	**A**	**B**	**C**	**D**	**A**	**C**	**C**	**B**	**D**	**B**	**B**	**A**	**B**	**B**	**C**	**B**	**B**	**A**

SECTION-II

Q.	46	47	48	49	50
A.	**8**	**2313**	**12**	**0**	**6**

PART-3 : MATHEMATICS

SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	**C**	**A**	**C**	**B**	**B**	**B**	**A**	**D**	**B**	**C**	**B**	**B**	**D**	**D**	**A**	**A**	**D**	**D**	**C**	**C**

SECTION-II

Q.	71	72	73	74	75
A.	**4**	**3**	**2**	**2**	**2**

PART-1 : PHYSICS

2) $K_1 A \tan 30^\circ = K_2 A \tan 45^\circ = K_3 A \tan 60^\circ$

$\Rightarrow K_1 \tan 30^\circ = K_2 \tan 45^\circ = K_3 \tan 60^\circ$

$\Rightarrow \dfrac{K_1}{3\sqrt{3}} = \dfrac{K_2}{3} = \dfrac{K}{\sqrt{3}} \Rightarrow \dfrac{K_1}{3} = \dfrac{K_2}{\sqrt{3}} = \dfrac{K_3}{1}$

3) $Mg\,(2R) + Mg\,(4R) = \dfrac{1}{2}\left[\dfrac{3}{2}MR^2 + M(2R)^2\right]\omega^2$

$6MgR = \dfrac{11}{4}MR^2\omega^2 \;\therefore \omega = \sqrt{\dfrac{24g}{11}}$

4) PT = const $\quad PV = nRT,\; T = \left(\dfrac{PV}{nR}\right)$

$\dfrac{P^2V}{nR}$ = const $\quad C = \dfrac{3R}{2} + 2R = \dfrac{7R}{2}$

$P^2V = k'$ $\quad C = 3.5\,R$

$PV^{1/2} = k$

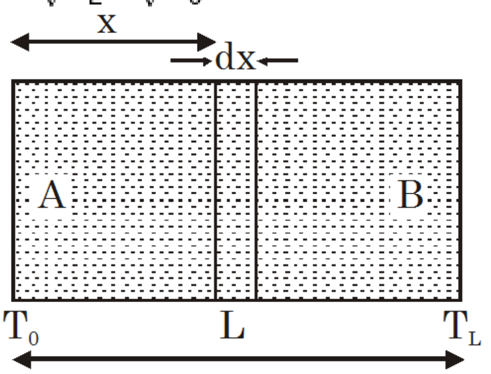
$C = C_v + \dfrac{R}{1-n}$

$C = \dfrac{3}{2}R + \dfrac{R}{1-\frac{1}{2}}$

5) $\nu = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{5RT}{3M}}$

$dx = C.dt = \sqrt{\dfrac{5R}{3M}\left[T_0 + \left(\dfrac{T_L - T_0}{L}\right)x\right]}\,dt,$

$t = \dfrac{2L}{\sqrt{T_L} + \sqrt{T_0}}\sqrt{\dfrac{3M}{5R}}$



6) Given $y_1 = 4\sin 600\pi t,\; y_2 = 5\sin 608\,\pi t$

$\therefore \omega_1 = 600\pi$ or $2\pi\upsilon_1 = 600\pi$ or $\upsilon_1 = 300$

$A_1 = 4$

and $\omega_2 = 608\pi$ or $2\pi\upsilon_2 = 608\pi$ or $\upsilon_2 = 304$

$A_2 = 5$

Number of beats heard per second

$= \upsilon_2 - \upsilon_1 = 304 - 300 = 4$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(4+5)^2}{(4-5)^2} = \frac{81}{1}$$

7) Let natural radius of loop is R and at any instant radius is r then mechanical energy of loop is

$$\frac{1}{2}K(2\pi r - 2\pi R)^2 + \frac{1}{2}Mv^2 = \text{costant}$$

$$K2\pi(r - R)2\pi + \frac{Mvdv}{dr} = 0$$

$$a = -4\pi^2\frac{K}{M}(r - R) \Rightarrow T = \sqrt{\frac{M}{K}}$$

8) $C = \frac{\varepsilon_0 A}{L}$ $\therefore \text{loc} C = \log\varepsilon_0 + \log A - \log L$

$$\frac{dC}{C} = \frac{dA}{A} - \frac{dL}{L} \Rightarrow \frac{dC}{C} = 2\alpha_1 dT - \alpha_2 dT = 0$$

$\therefore 2\alpha_1 = \alpha_2$

9) Total power in horizontal resistors at any time

$$= i^2R + \frac{i^2}{4}R + \frac{i^2}{16}R + \ldots\ldots\infty \quad = \frac{4}{3}i^2R$$

Total power in vertical resistors

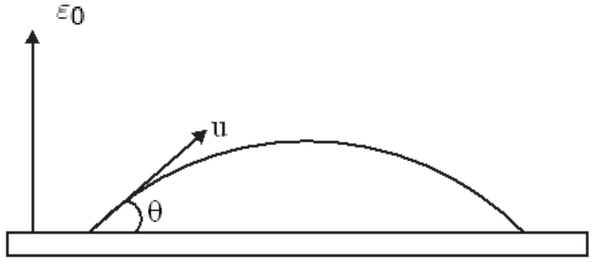
$$= \frac{i^2}{4}2R + \frac{i^2}{16}2R + \ldots\ldots\infty \quad = \frac{2}{3}i^2R$$

$$\frac{P_{Horizontal}}{P_{Vertical}} = 2$$

10)

Electric field due to the large metal plate

$$E = \frac{\sigma}{\varepsilon_0}$$



The projected electron's motion will be on a curved path like a projectile

 AB will be maximum when θ = 45°

Range $AB_{max} = \frac{u^2}{a}$

$$a = \frac{\rho E}{m} = \frac{\rho\sigma}{\varepsilon_0 m} = \frac{u^2}{\left(\frac{\rho\sigma}{\varepsilon_0 m}\right)} = \frac{\varepsilon_0 m u^2}{\rho\sigma}$$

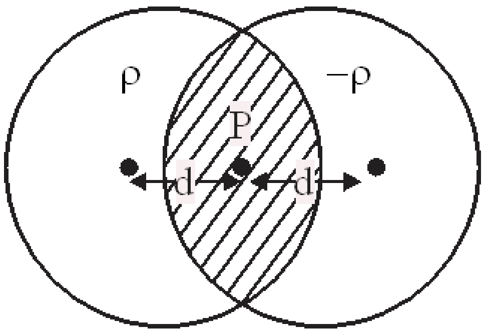
11) $A_n = \pi r^2$ $\quad \Rightarrow A_n = \pi\left(r_r n^2\right)^2$

$A_n = \pi r_0^2 n^4$ $\quad$ so, $\ell n \dfrac{A_n}{A_l} = 4\ell n\ n$

12) $T(R-r) = \mu mgR,\ 2mg - T = 2ma,$
$T - \mu mg = ma$
On solving, we get
$\therefore r = R\left(1 - \dfrac{3\mu}{2(1+\mu)}\right)$

13) Positive cycle : D_1 - forward, D_2 - reverse
Negative cycle D_1 - reverse, D_2 - forward

14) $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \rightarrow$ differentiating twice w.r.t. time
$|a_i| = \dfrac{2m^3 u_0^2}{f}$
$u_0 \rightarrow$ velocity of object
$m \rightarrow$ magnification
$f \rightarrow$ focal length.

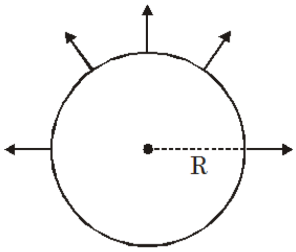


17)
$E_p = \dfrac{\rho d}{3\epsilon_0} + \dfrac{\rho d}{3\epsilon_0} = \dfrac{2\rho d}{3\epsilon_0}$
$\vec{E}$ due to shaded region will not be that as it is O net change region.
So, E_P is electric field due to remaining portion of each sphere. Hence, due to one sphere it is $\dfrac{\rho d}{3\epsilon_0}$.

18) $\dfrac{q_0^2}{2 \times 2c} = \dfrac{2(q_0/3)^2}{2 \times 2C} + \dfrac{(q_0/3)^2}{2 \times C} + \dfrac{1}{2}Li^2$
$\dfrac{q_0^2}{6C} = Li^2$

$i = \frac{q_0}{\sqrt{6LC}}$

19) In equilibrium of upper half of loop 2kx = 4RS



$x = \frac{2SR}{k} \Rightarrow 2\pi R = \ell + \frac{2SR}{k}$

$R = \frac{k\ell}{2\pi k - 2S}$

20) $F = \alpha\beta e^{\left(-\frac{x}{\alpha kt}\right)} \Rightarrow \frac{x}{\alpha} = \left[ML^2T^{-2}\right]$

$\alpha = \left[M^{-1}L^{-1}T^2\right] \Rightarrow \left[MLT^{-2}\right] = \left[M^{-1}L^{-1}T^2\right][\beta]^{-1}$

$\beta = \left[M^{-2}L^{-2}T^4\right]$

21) Least count $= \frac{0.25}{5 \times 100} cm = 5 \times 10^{-4} cm$

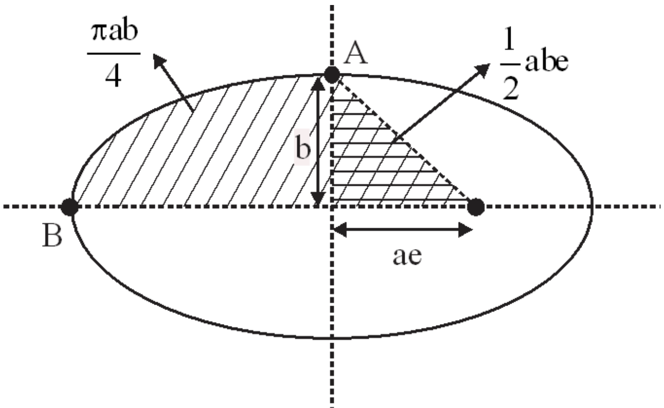
Reading $= 4 \times 0.05 cm + 30 \times 5 \times 10^{-4} cm$

$= (0.2 + 0.0150)\, cm = 2.15\, mm$

22) $I = xH$ and $x = \frac{C}{T}$ (according to curie law)

Magnetization (T)

$\propto \frac{\text{Magnetic Induction (B)}}{\text{Absolute temperature (T)}} \Rightarrow \frac{I_2}{I_1} = \frac{B_2T_1}{B_1T_2}$



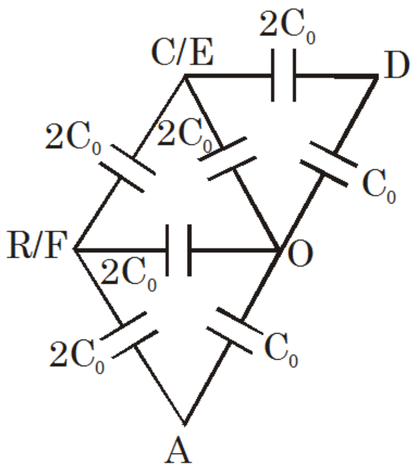
23)

$t = \frac{T}{\pi ab}\left(\frac{\pi ab}{4} + \frac{1}{2}abe\right) \Rightarrow t = T\left(\frac{1}{4} + \frac{e}{2\pi}\right)$

$t = 18\left(\frac{1}{4} + \frac{1}{8}\right) \Rightarrow t = 6$ months

24) Using the parallel axis symmetry it can be reduced to the following circuit.

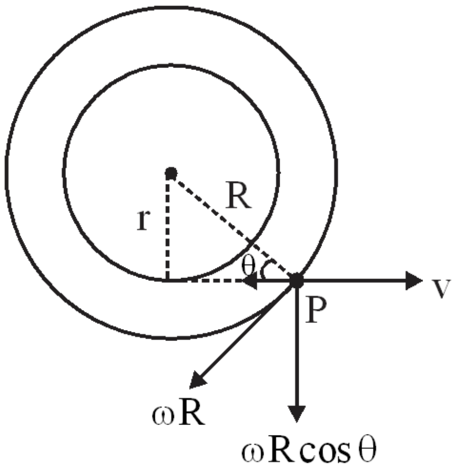
Solving this circuit will give us equivalent capacitance between A and O as $\left(\frac{20}{9}\right) C_0$



25) Horizontal velocity of point

$= v - \omega R \sin\theta \quad = \omega r - \omega R\left(\frac{r}{R}\right) = 0$

So velocity must be perpendicular.



PART-2 : CHEMISTRY

26) Rate law equation in terms of pressure is

$\text{Rate} = k\left[P_{NO_2}\right]^m\left[P_{H_2}\right]^n$

Substituting the given data in rate law equation we get,

(i) $0.02 = (0.15)^m[0.4]^n$

(ii) $0.005 = (0.075)^m[0.4]^n$

(iii) $0.010 = (0.15)^m\,[0.2]n$

Divide equation(i) by (ii)

$$\frac{0.02}{0.005} = \frac{k[0.15]^m[0.4]^n}{k[0.075]^m[0.4]^n}$$

or $4 = [2]^m \therefore m = 2$

Divide equation (i) by (iii)

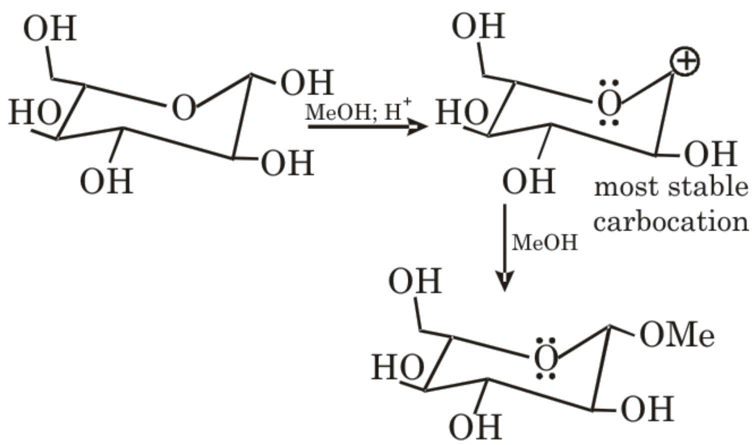
$$\frac{0.02}{0.01} = \frac{k[0.15]^m[0.4]^n}{k[0.15]^m[0.2]^n}$$

or $2 = (2)^n \therefore n = 1$

27)

Because of low oxidation potential of Ag it is get easily reduced. Means that it is having higher reduction potential

$Zn(s) + HgO(s) \rightarrow ZnO(s) + Hg(\ell)$



28)

30) BrF_5 is polar molecule which have sp^3d^2 hybridisation.

32) $H^+_{(aq)} + OH^-_{(aq)} \rightarrow H_2O(\ell)$ $\Delta H = -13.7 \frac{Kcal}{eq}$

no. of m_{eq} of OH^- = 224 × 0.1 × 1

= 22.4

no. of m_{eq} of H^+ = 50 × 0.1 × 2

= 10 (Limiting reagent)

A neutralisation 10 m_{eq} of acid will occur

Heat released = 13.7 × 0.01 × 1000 cal. = 137 cal.

$\because q = mS\Delta T$

$137 = (224 + 50) \times 1 \times \Delta T$

$\therefore \Delta T = 0.5$ K

33)

in air, $P_{N_2} = 0.8$ atm, $P_{O_2} = 0.2$ atm

$P = K_H X$ (Henry's law)

$0.8 = 2 \times 10^4 X_{N_2}$ ....(1)

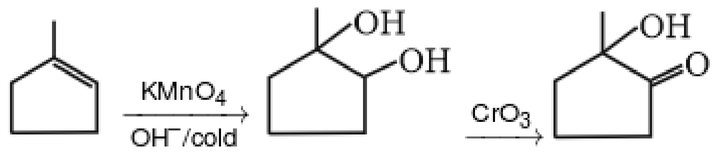
$0.2 = 10^4 X_{O_2}$ ....(2)

Ratio of solubility of N_2 & O_2 is 2 : 1

35) $$Na_2CrO_4 \xrightarrow[\substack{CH_2CO_3 \\ \text{(acidic)}}]{CO_2 + H_2O} \underset{\text{(orange)}}{Na_2CrO_7}$$

36) $$CH_3CH_2COOH \xrightarrow{NH_3} CH_3CH_2CO\overset{\ominus}{O}\overset{\oplus}{N}H_4 \xrightarrow{\Delta} CH_3CH_2CONH_2 \xrightarrow{Br_2/KOH} \underset{1^\circ\ \text{Amine}}{CH_3CH_2NH_2}$$

1° amine forms alcohol with $NaNO_2$ / HCl and not yellow oily liquid.

37) 

38) In paper chromatography, a special quality paper known as chromatography paper is used. Paper contains water trapped in it, which acts as the stationary phase.

39)

The first ionization energies (as in NCERT) are as follows:
B : 801 kJ/mol
Al : 577 kJ/mol
Ga : 579 kJ/mol
Ga : $[Ar]3d^{10}4s^24p^1$

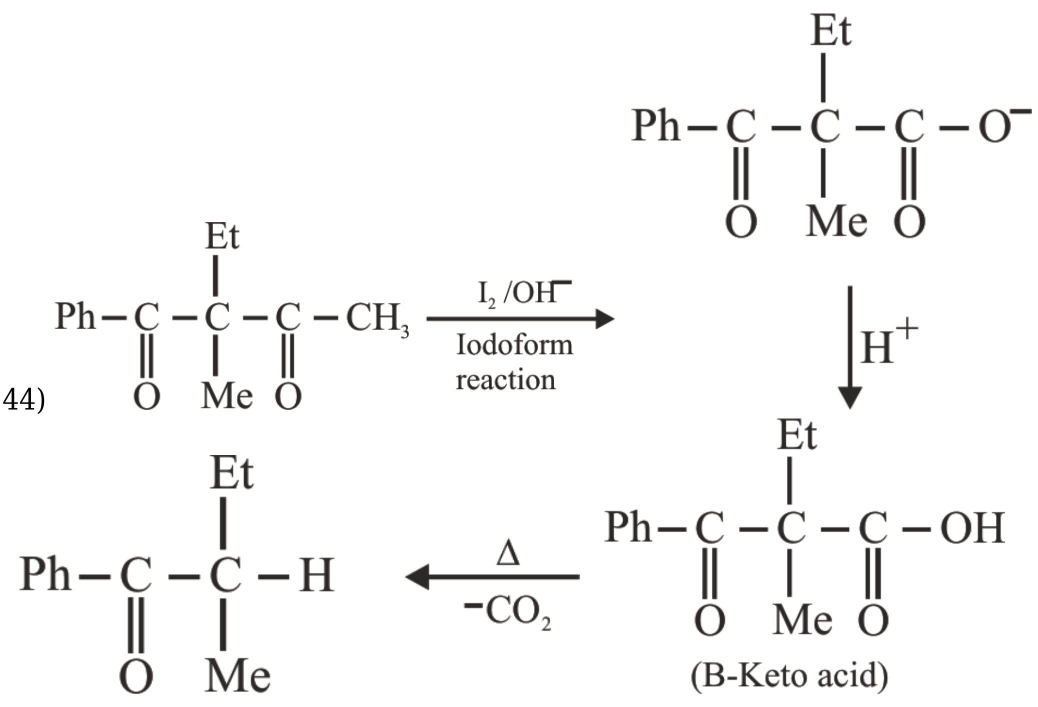
40) $Ca_3P_2 + H_2O \rightarrow Ca(OH)_2 + PH_3$
$Mg_3N_2 + H_2O \rightarrow Mg(OH)_2 + NH_3$
$CaC_2 + H_2O \rightarrow Ca(OH)_2 + C_2H_2$

41) Conc. HNO_3 decomposes partly on long standing in light as follows :

$2HNO_3 \rightarrow H_2O + 2NO_2 + \frac{1}{2}O_2$
This NO_2 gets dissolved in remaining HNO_3 and turns its colour yellow-brown which is due to the paramagnetic behaviour of NO_2.

42) Fact

43) The compound 'B' is benzaldehyde and the reaction.

44) 

45)

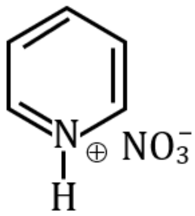
(P) Gabriel phthalimide synthesis is used for the preparation of aliphatic primary amines. Aromatic primary amines cannot be prepared by this method.
(Q) 2°-amines reacts with Hinsberg's reagent to give solid insoluble in NaOH
(R) Aromatic primary amine react with nitrous acid at low temperature (273 – 298 K) to form diazonium salts, which form Red dye with β-Naphthol

46) At T_1 K : $A(g) \rightleftharpoons P(g)$
t = 0   6
t = ∞   6 – x   x = 4 (from plot)

$\Rightarrow$ At T_1 K : $K_{P_1} = \frac{4}{2} = 2$
At T_2 K : $A(g) \rightleftharpoons P(g)$
t = 0   6
t = ∞   6 – y   y = 2 (from plot)

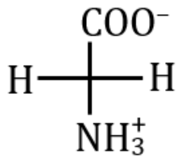
$\Rightarrow$ At T_2 K : $K_{P_2} = \frac{2}{4} = \frac{1}{2}$

Now, $\Delta G_2^o = -RT_2 ln K_{P_2} = -RT_2 ln \frac{1}{2}$
$\Rightarrow \Delta G_2^o = RT_2 ln 2$
$\Delta G_1^o = -RT_1 \ln K_{P_1} = -RT_1 ln 2 = -2RT_2 ln 2$
Given : $\Delta G_2^o - \Delta G_1^o = RT_2 ln 2 + 2RT_2 ln 2 = 3RT_2 ln 2 = RT_2 ln x$
$\Rightarrow x = 2^3 = 8$

(P) 

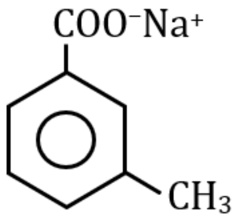
$\Rightarrow$ salt of weak base + strong acid

$$(2)\ pH = 7 - \frac{1}{2}(pk_b + \log c)$$

(Q) 

$\Rightarrow$ salt of w.A. + w.B.

$$(3)\ pH = 7 + \frac{1}{2}(pk_a + pk_b)$$

(R) 

$\Rightarrow$ salt of w.A. + w.B.

$$(1)\ pH = 7 + \frac{1}{2}(pk_a + \log c)$$

(S) $NH_4CN \Rightarrow$ salt of w.A. + w.B.

$$(3)\ pH = 7 + \frac{1}{2}(pk_a + pk_b)$$

47)

48) linkage isomer = (1 + 1)(1 + 1) = 4

M_{abcd} = 3 GI

Cl^- NO_3^- NO_2^- SCN^-

Cl^- NO_3^- NO_2^- NCS^-

Cl^- NO_3^- ONO^- SCN^-

Cl^- NO_3^- ONO^- NCS^-

49) $Co^{2+} + H_2S \rightarrow \underset{(A)}{CoS} \downarrow$ (Black)

$\underset{(A)}{CoS}$ + Aqua-regia $\rightarrow \underset{(B)}{Co^{2+}\ (aq)}$ + NOCl + S + H_2O

Co^{2+} (aq) + KNO_2 + CH_3COOH

$\downarrow$

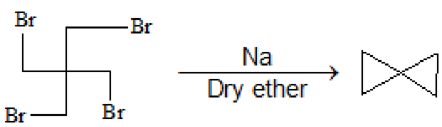
$K_3[Co(NO_2)_6]$ + NO + S + H_2O

In $K_3[Co(NO_2)_6]$, Co^{+3} : $3d^6\ 4s^0$

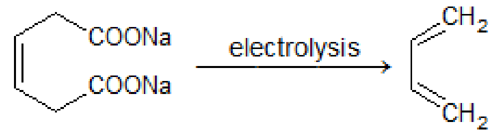
Co^{3+} : d^2sp^3 Hybridisation

Number of unpaired e^- = 0

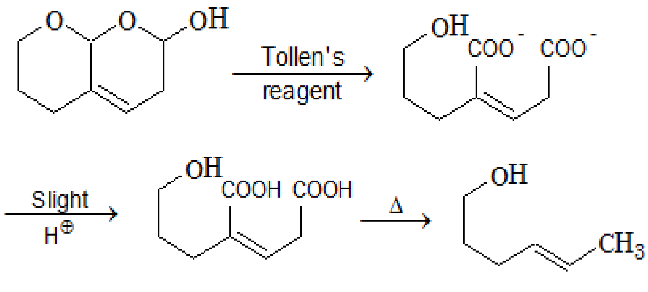
Magnetic moment = $\sqrt{n(n+2)}$ = 0 B.M

50) 

(P) Dou = 2, X = 2



(Q) Dou = 2, Y = 2



(R) + $2CO_2$, Z = 2

X + Y + Z = 6

PART-3 : MATHEMATICS

51) Divide by x^6 in numerator and denominator

$$\Rightarrow \int \frac{\left(\frac{1}{x^2}+\frac{3}{x^4}+\frac{5}{x^6}\right)dx}{\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}}$$

Now put $1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}=t^2$

$$\Rightarrow \int \frac{2t\,dt}{t} \Rightarrow 2\sqrt{1-\frac{1}{x}-\frac{1}{x^3}-\frac{1}{x^5}}+C$$

52) $P(R)=\frac{2}{3},\ P(G)=\frac{1}{3}$

$$P\left(\frac{\text{Accident}}{R}\right)=\frac{1}{5}, P\left(\frac{\text{Accident}}{G}\right)=\frac{1}{8}$$

$$P\left(\frac{R}{\bar{A}}\right)=\frac{P(R\cap\bar{A})}{P(\bar{A})}$$

$$=\frac{P(R)\cdot P\left(\frac{\bar{A}}{R}\right)}{P(R)P\left(\frac{\bar{A}}{R}\right)+P(G)\cdot P\left(\frac{\bar{A}}{G}\right)}$$

$$=\frac{\frac{2}{3}\cdot\frac{4}{5}}{\frac{2}{3}\cdot\frac{4}{5}+\frac{1}{3}\cdot\frac{7}{8}}=\frac{64}{99}$$

53) $g(x) = px^2 + qx + r = 0$ has roots α, β

$\Rightarrow g(x) = p(x - \alpha).(x - \beta)$

$$\text{Let } I=\int_{\alpha}^{\beta}\frac{e^{f\left(\frac{g(x)}{x-\alpha}\right)}}{e^{f\left(\frac{g(x)}{x-\alpha}\right)}+e^{f\left(\frac{g(x)}{x-\beta}\right)}}dx$$

$$=\int_{\alpha}^{\beta}\frac{e^{f(p(x-\beta))}}{e^{f(p(x-\beta))}+e^{f(p(x-\alpha))}}dx \quad ...(i)$$

Apply king's rule

$$I = \int_{\alpha}^{\beta} \frac{e^{f(p(\alpha-x))}}{e^{f(p(\alpha-x))} + e^{f(p(\beta-x))}} dx$$ ...(ii)

(replace x → a + b – x)

 f is even

$\Rightarrow \therefore f(p(\alpha - x)) = f(p(x - \alpha))$

& $f(p(\beta - x)) = f(p(x - \beta))$

Now add (i) and (ii)

$$\Rightarrow 2I = \int_{\alpha}^{\beta} dx \Rightarrow I = \frac{\beta - \alpha}{2}$$

$$\Rightarrow I = \frac{\sqrt{q^2 - 4pr}}{2|p|}$$

54) $\sum_{r=1}^{15} \frac{(r+3-3)\cdot 3^r}{(r+3)!} = \sum_{r=1}^{15} \frac{3^r}{(r+2)!} - \frac{3^{r+1}}{(r+3)!}$

$= \frac{3}{3!} - \frac{3^{16}}{18!}$

55) $I = \int_0^1 \frac{\sin x}{x^{1/4}} dx < \int_0^1 \frac{x \cdot dx}{x^{1/4}}$

$\Rightarrow I < \frac{4}{7} \quad (\because \sin x < x \forall x \in (0,1))$

$J = \int_0^1 \frac{\cos x}{x^{1/4}}.dx < \int_0^1 \frac{1}{x^{1/4}} dx$

$\Rightarrow J < \frac{4}{3} \quad (\because \cos x < 1 \forall x \in (0,1))$

56) $(\vec{p}.\vec{p})(\vec{x} - \vec{q}) - (\vec{p}.(\vec{x} - \vec{q}))\vec{p}$

$+(\vec{q}.\vec{q})(\vec{x} - \vec{r}) - (\vec{q}.(\vec{x} - \vec{r}))\vec{q}$

$+(\vec{r}.\vec{r})(\vec{x} - \vec{p}) - (\vec{r}.(\vec{x} - \vec{p})\vec{r} = 0$

$\because |\vec{p}| = |\vec{q}| = |\vec{r}| = \lambda(\text{let})$

$\Rightarrow \lambda^2(3\vec{x} - (\vec{p} + \vec{q} + \vec{r}))$

$= (\vec{p}.\vec{x})\vec{p} + (\vec{q}.\vec{x})\vec{q} + (\vec{r}.\vec{x})\vec{r}$

$\Rightarrow 3\vec{x} - (\vec{p} + \vec{q} + \vec{r}) = \vec{x}$

57) Let $L = \lim_{n\to\infty} \left(1 + \frac{1}{n^2}\right)^{\frac{2}{n^2}} \left(1 + \frac{2^2}{n^2}\right)^{\frac{4}{n^2}} \left(1 + \frac{3^2}{n^2}\right)^{\frac{6}{n^2}} \ldots \left(1 + \frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}$

$\Rightarrow \ell n L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2r}{n^2} \ell n \left(1 + \frac{r^2}{n^2}\right)$

$\Rightarrow \ell n L = \int_0^1 2x.dx.\ell n(1 + x^2)$

$$\Rightarrow \ell nL = \int_1^2 \ell nt.dt = (t\ \ell n\ t - t)_1^2$$

$$\Rightarrow \ell nL = 2\ell n2 - 1 \Rightarrow L = \frac{4}{e}$$

58) $f(x) = [x] + 2 + [x]^2 - 8$
$f(x) = ([x] + 3)([x] - 2)$
Clearly $f(x) = 0$ for $x \in [2, 3)$ many one
Also output of f(x) is always an integer.
$\Rightarrow$ f(x) is into

59) $\frac{\cot B}{\cot C + \cot A} = \frac{\frac{\cos B}{\sin B}}{\left(\frac{\sin(A+C)}{\sin A \sin C}\right)}$

$= \cos B.\frac{\sin A \sin C}{\sin^2 B}$

$= \frac{a^2 + c^2 - b^2}{2ac}.\frac{ac}{b^2} = \frac{1}{2}$

60) If line lies on plane
$\Rightarrow (3\hat{i} + 5\hat{j} + 2\hat{k}).(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 0$
$\Rightarrow 3\alpha + 10 + 2 = 0 \Rightarrow \alpha = -4$
Point (2, – 1, 2) on line must satisfy plane eqn.
$\Rightarrow 2\alpha - 2 + 2 - \beta = 0$
$\Rightarrow \beta = -8$

61) Eqn. of tangent of hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is
$y = mx \pm \sqrt{9m^2 - 4}$ ...(1)
Eqn. of tangent of circle
$y = mx \pm \sqrt{5m^2 + 5}$ ...(2)
Compare (1) and (2)
$\Rightarrow 9m^2 - 4 = 5m^2 + 5 \Rightarrow m = \frac{3}{2}$
required tangent
$3x - 2y \pm \sqrt{65} = 0$

62) Let $\frac{z-1}{e^{i\frac{\pi}{3}}} = t$

$\Rightarrow t + \frac{1}{t} = \text{purely real}$

$\Rightarrow t + \frac{1}{t} = \bar{t} + \frac{1}{\bar{t}} \Rightarrow (t - \bar{t})\left(1 - \frac{1}{t\bar{t}}\right) = 0$

$\Rightarrow t = \bar{t}$ (□ $|t| \neq 1$ □ $|z - 1| \neq 1$)
$\Rightarrow t = \text{purely real}$

$\Rightarrow \frac{x + y - 1}{e^{i\frac{\pi}{3}}} = \text{purely real}$

$\Rightarrow ((x-1)+iy)\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$

= purely real

$\Rightarrow -(x-1).\frac{\sqrt{3}}{2}+y.\frac{1}{2}=0$

$\Rightarrow -\sqrt{3}x+y+\sqrt{3}=0$

Slope of line = $\sqrt{3}$

63) Let $h(x)=\int_0^{x^3} f(t)dt$

$\Rightarrow h'(x)=3x^2.f(x^3)$

Apply LMVT on h(x)

for $(0,1)h'(\alpha)=\frac{h(1)-h(0)}{1-0}$ ...(1)

for $(1,2)h'(\beta)=\frac{h(2)-h(1)}{2-1}$ ...(2)

for $(2,3)h'(\gamma)=\frac{h(3)-h(2)}{3-2}$ ...(3)

Add (1), (2) and (3)

$\Rightarrow h(3)=h'(\alpha)+h'(\beta)+h'(\gamma)$

64) Required ways = ${}^{16}C_5$

 p – q = 16 – 5 or 16 – 11

= 11 or 5

65)

f(1) = 1; f(9) = f(3) × f(3)

i.e., f(3) = 1 or 3

Total function = 1 × 6 × 2 × 6 × 6 × 1 = 432

66) Differentiate both sides w.r.t. x

$x^2.y(x) = 3x^2.y(x) + x^3.y'(x)$

$\Rightarrow x\frac{dy}{dx}+2y=0$

$\Rightarrow \frac{dy}{y}+\frac{2dx}{x}=0$

$\Rightarrow \ell ny+2\ell nx=C$

$\Rightarrow x^2y=C'$

67) $\sum_{r=1}^{n-1}\tan^{-1}\left(\frac{2d}{1+a_{2r-1}.a_{2r+1}}\right)$

$\Rightarrow \sum_{r=1}^{n-1}\tan^{-1}\left(\frac{a_{2r+1}-a_{2r-1}}{1+a_{2r-1}.a_{2r+1}}\right)$

$\Rightarrow \sum_{r=1}^{n-1}\tan^{-1}(a_{2r+1})-\tan^{-1}(a_{2r-1})$

$\Rightarrow \tan^{-1}(a_{2n-1}) - \tan^{-1}(a_1)$

68) for x > 0

$x < \sqrt{2}x < \sqrt{3}x$

$\Rightarrow f(x) > f(\sqrt{2}x) > f(\sqrt{3}x)$

$\Rightarrow 1 > \frac{f\left(\sqrt{2}x\right)}{f(x)} > \frac{f\left(\sqrt{3}x\right)}{f(x)}$

Apply limit x → ∞

$\Rightarrow \lim_{x\to\infty} \frac{f\left(\sqrt{2}x\right)}{f(x)} = 1$

(by concept of sandwich theorem)

69) $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ \frac{1}{3}+\frac{1}{3} & 0+1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix}$

Now $A^3 = \begin{bmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ \frac{3}{3} & 1 \end{bmatrix} \Rightarrow A^{48} = \begin{bmatrix} 1 & 0 \\ \frac{48}{3} & 1 \end{bmatrix}$

70) $\tan\alpha + \tan\beta = -p$

and $\tan\alpha \cdot \tan\beta = q$

$\Rightarrow \tan(\alpha+\beta) = \frac{-p}{1-q}$

$\sin^2(\alpha+\beta) + p\sin(\alpha+\beta)$

$\cos(\alpha+\beta) + q\cos^2(\alpha+\beta)$

$= \cos^2(\alpha+\beta)$

$\left[\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\right]$

$= \frac{\frac{p^2}{(1-q)^2} - \frac{p\cdot p}{1-q} + q}{1 + \frac{p^2}{(1-q)^2}}$

$= \frac{p^2 - p^2(1-q) + q(1-q)^2}{(1-q)^2 + p^2} = q$

71) $(\alpha^2 + 2\beta^2 - 5)x^2 + x(2\alpha^2 - \beta^2) +$

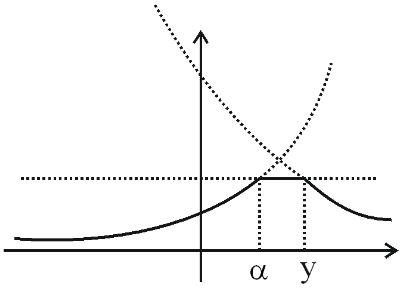
$(\alpha^2 + \beta^2 - 3) = 0$
$\forall x \in R$
$\therefore \alpha^2 + 2\beta^2 - 5 = 0$ & $2\alpha^2 - \beta^2 = 0$
& $\alpha^2 + \beta^2 = 3$
$\therefore \alpha^2 + 2.2\alpha^2 = 5$ & $\alpha^2 + 2\alpha^2 = 3$
$\Rightarrow \alpha^2 = 1$ & $\beta^2 = 2$
$\Rightarrow (\alpha, \beta) = (\pm 1, \pm\sqrt{2})$

72) ω is cube root of unity
$|x|^2 = (a + b + c)(\bar{a} + \bar{b} + \bar{c})$
$= |a|^2 + |b|^2 + |c|^2 + a\bar{b} + \bar{a}b$
$+ b\bar{c} + \bar{b}c + c\bar{a} + \bar{c}a$ ...(1)
$|y|^2 = y\bar{y} = (a + b\omega + c\omega^2)(\bar{a} + \bar{b}\omega^2 + \bar{c}\omega)$
$= |a|^2 + |b|^2 + |c|^2 + a\bar{b}\omega^2 + \bar{a}b\omega$
$+ b\bar{c}\omega^2 + \bar{b}c\omega + c\bar{a}\omega^2 + \bar{c}a\omega$ ...(2)
Similarly
$|z|^2 = z\bar{z} = (a + b\omega^2 + c\omega)(\bar{a} + \bar{b}\omega + \bar{c}\omega^2)$ ...(3)
Now add (1), (2), (3) and use $1 + \omega + \omega^2 = 0$
to get $|z|^2 + |y|^2 + |z|^2 = 3(|a|^2 + |b|^2 + |c|^2)$

73) $I = \int_0^2 \frac{17 - 2(x-1)^2}{e^{6(x-1)} + 1}$ ...(1)
Apply king's rule
$I = \int_0^2 \frac{17 - 2(1-x)^2}{e^{6(1-x)} + 1}$ ...(2)
Add (1) and (2)
$2I = \int_0^2 17 - 2(x-1)^2)dx$
$\Rightarrow 2I = \left(17x - \frac{2(x-1)^3}{3}\right)_0^2$
$\Rightarrow 2I = 34 - \frac{2}{3} - \frac{2}{3} \Rightarrow I = \frac{49}{3}$

74) $A = {}^{22}C_{11} = \frac{22}{11} \cdot {}^{21}C_{10}$
$B = {}^{21}C_{11}$
$\therefore \frac{A}{B} = 2$

75)



$e^{\alpha} = \frac{3}{2} \Rightarrow \alpha = \ell n\frac{3}{2}$

$e^{\beta} = 1 + e^{-\beta} \Rightarrow e^{\beta} = \frac{1+\sqrt{5}}{2}$

$\Rightarrow \beta = \ell n\left(\frac{1+\sqrt{5}}{2}\right)$

& $e^{-y} = \frac{1}{2} \Rightarrow y = \ell n2$

so at 2 points x = α, y curve has sharp corners.