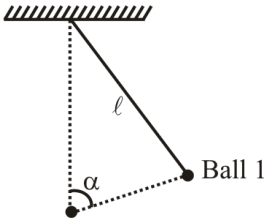
PART-1 : PHYSICS

SECTION-I

1) A small charged ball 'q' of mass 'm' is suspended on an insulating string of length □. Another identical charged ball is slowly moved by a student towards the first one from a large distance. Eventually, the second ball is placed at the original location of the first one as shown in figure. At that moment, the first ball is elevated a small distance h above its original position and remains at rest. Then



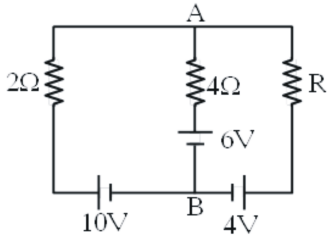
(A) $\cos\alpha = \frac{kq^2}{mg\ell h}$

(B) $\cos\alpha = \frac{kq^2}{2mgh\ell}$

(C) $\cos\alpha = \frac{kq^2}{4mgh\ell}$

(D) $\cos\alpha = \frac{kq^2}{8mgh\ell}$

2) For what value of R in the circuit as shown, current passing through 4Ω resistance will be zero:-



(A) 1Ω

(B) 2Ω

(C) 3Ω

(D) 4Ω

3) The length of a solenoid is 0.3 m and the number of turns is 2000. The area of cross-section of the solenoid is $1.2 \times 10^{-3} m^2$. Another coil of 300 turns is wrapped over the solonoid. A current of 2A is passed through the solenoid and its direction is changed in 0.25 sec. then the induced emf in coil

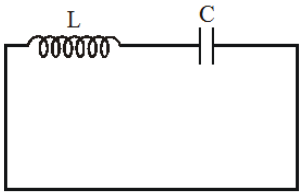
(A) 4.8×10^{-2} V

(B) 4.8×10^{-3} V

(C) 3.2×10^{-4} V

(D) 3.2×10^{-2} V

4) In an LC circuit the capacitor has maximum charge q^0. The value of $\left(\frac{dI}{dt}\right)_{max}$ is :-



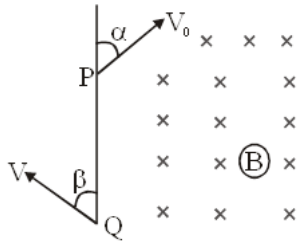
(A) $\frac{q_0}{LC}$

(B) $\frac{q_0}{\sqrt{LC}}$

(C) $\frac{q_0}{2LC}$

(D) $\frac{2q_0}{LC}$

5) A particle of charge –q & mass 'm' enters a uniform magnetic field 'B' (Perpendicular to paper inwards) at P with a speed V_0 at an angle α & leaves the field at Q with speed V at angle β as shown



then :-

(A) α = β

(B) $PQ = \frac{2mV_0 \sin\alpha}{qB}$

(C) Particle remains in field for time $t = \frac{2(\pi - \alpha)m}{Bq}$

(D) All of above

6) Match list-I with list-II :

List-I		List-II	
(P)	Ultravoilet rays	(i)	Study crystal structure
(Q)	Microwaves	(ii)	Green house effect
(R)	Infrared waves	(iii)	Sterilizing surgical instrument

(S)	X-rays	(iv)	Radar system

(A) P→iii; Q→iv; R→ii; S→i

(B) P→iii; Q→i; R→ii; S→iv

(C) P→iv; Q→i; R→ii; S→iii

(D) P→ii; Q→iii; R→i; S→iv

7) The temperature of 5 mole gas changed from 100°C to 120°C. Change in internal energy is 80 J. The total heat capacity of the gas at constant volume will be :-

(A) 8 JK^{-1}

(B) 0.8 JK^{-1}

(C) 4 JK^{-1}

(D) 0.4 JK^{-1}

8) A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass
2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?

(A) 0.06 m

(B) 0.03 m

(C) 0.24 m

(D) 0.12 m

9) A cylindrical tube (L = 120 cm.) is resonant with a tuning fork of frequency 330 Hz. If it is filling by water then to get resonance again, minimum length of water column is (v_{air} = 330 m/s)

(A) 45 cm

(B) 60 cm

(C) 25 cm

(D) 20 cm

10) A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :-

(A) convergent lens of focal length 3.5R

(B) convergent lens of focal length 3.0 R

(C) divergent lens of focal length 3.5 R

(D) divergent lens of focal length 3.0 R

11) A monochromatic source of light operating at 200W emits 4×10^{20} photons per second then (λ) wavelength of light is →

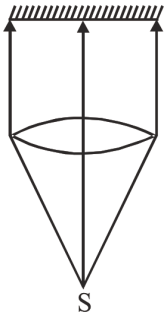
(A) 200 nm

(B) 400 nm

(C) 100 nm

(D) 1800 nm

12) A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in figure. The mass of mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Then the power of the source needed to support the weight of the mirror. ($g = 10\ m/s^2$) :-



(A) 100 MW

(B) 200 MW

(C) 50 MW

(D) 10 MW

13) 200 MeV of energy may be obtained per fission of U^{235}. A reactor is generating 1000 kW of power. The rate of nuclear fission in the reactor is :-

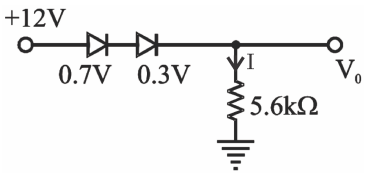
(A) 1000

(B) 2×10^8

(C) 3.125×10^{16}

(D) 931

14) In the given network the value of current I is :-



(A) 1.96 mA

(B) 2.14 mA

(C) 0

(D) None of the above

15) Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (–1, 5), (2, –3), (–1, –4). Find the coordinates of their centre of mass.

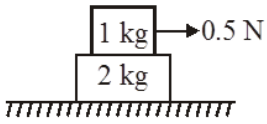
(A) $\left(\frac{1}{7}, \frac{23}{14}\right)$

(B) $\left(\frac{1}{7}, \frac{19}{14}\right)$

(C) $\left(\frac{3}{14}, \frac{23}{14}\right)$

(D) None

16) A force of 0.5 N is applied on the upper block as shown in figure. The coefficient of static friction between the two blocks is 0.1 and that between the lower block and the surface is zero. The work done by the lower block on the upper block for a displacement of 3 m of the upper block is :-



(A) 1 J

(B) –1 J

(C) 2 J

(D) –2 J

17) A stone is tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is :-

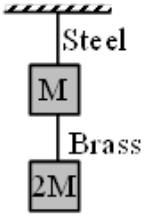
(A) $\sqrt{u^2 - 2gL}$

(B) $\sqrt{2gL}$

(C) $\sqrt{u^2 - gL}$

(D) $\sqrt{2(u^2 - gL)}$

18) If the ratio of lengths, radii and Young's modulus of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths would be :-



(A) $\frac{2a^2c}{b}$

(B) $\frac{3a}{2b^2c}$

(C) $\frac{2ac2ac}{b^2\ b^2}$

(D) $\frac{3c}{2ab^2}$

19) Surface tension of soap solution is 2×10^{-2} N/m. The work done in producing a soap bubble of radius 2 cm is :-

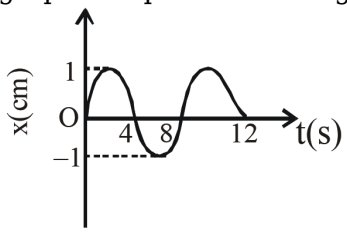
(A) $64\pi \times 10^{-6}$ J

(B) $32\pi \times 10^{-6}$ J

(C) $16\pi \times 10^{-6}$ J

(D) $8\pi \times 10^{-6}$ J

20) The x-t graph of a particle undergoing SHM is shown below. The acceleration of the particle at t = 8/3 s is :-



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

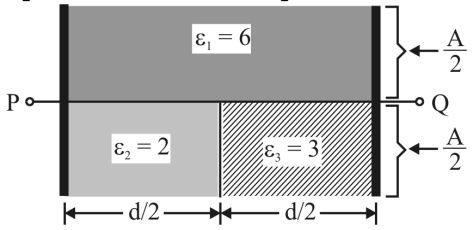
(B) $-\frac{\pi^2}{32}$ cm/s^2

(C) $\frac{\pi^2}{32}$ cm/s^2

(D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

SECTION-II

1) Three dielectric of relative primitivities $\varepsilon r_1 = 6$, $\varepsilon r_2 = 2$ and $\varepsilon r_3 = 3$ are introduced in a parallel plate capacitor of plate area A and separation d. The effective capacitance between P and Q is $\frac{x\varepsilon_0 A}{d}$. Then $\frac{5}{7}x$ will be.



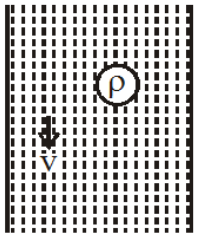
2) Two ideal monoatomic & diatomic gases are mixed with one another to form an ideal gas mixture. The equation of the adiabatic process of the mixture is PV^γ = constant, where $\gamma = 11/7$. If n_1 & n_2 are the number of moles of the monoatomic & diatomic gases in the mixture respectively, find the ratio n_1/n_2 :-

3) On placing a thin sheet of mica of thickness 12×10^{-7} m in the path of one of interferring beams in a Young's experiment, it is found that the central bright band shifts a distance equal to the width of a bright fringe. If the wavelength of light used is 12×10^{-7} m. then find refractive index of mica.

4) In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides $(A \approx 118)$, having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ____ MeV.

5) A liquid of density $\rho = 1000$ kg/m^3 and coefficient of viscosity $\eta = 0.1$ Ns/m^2 is flowing down in vertical pipe of large cross section. A small ball of density $\rho_0 = 100$ kg/m^3 and r = 5 cm will be at rest

in flowing liquid, if velocity of flowing liquid is 10 k m/s. Then find the value of k.



PART-2 : CHEMISTRY

SECTION-I

1) For the equilibrium,
$SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$,

What is the temperature at which $\frac{K_P(atm)}{K_C(M)} = 3$?

(A) 0.027 K

(B) 0.36 K

(C) 36.54 K

(D) 273 K

2) The following data is given for reaction between A and B :-

S.No.	[A]	[B]	Rate
	$mol.1^{-1}$	$mol.1^{-1}$	$mol.1^{-1}sec^{-1}$
I	1×10^{-2}	2×10^{-2}	2×10^{-4}
II	2×10^{-2}	2×10^{-2}	4×10^{-4}
III	2×10^{-2}	4×10^{-2}	8×10^{-4}

Which of the following are correct statements -
(a) Rate constant of the reaction 10^{-4}
(b) Rate law of the reaction is k[A][B]
(c) Rate of reaction increases four times on doubling the concentration of both the reactant.

(A) a, b and c

(B) a and b

(C) b and c

(D) c alone

3) If standard reduction potentials of Ni^{2+}/Ni and Au^{3+}/Au couples are –0.25 V and 1.50 V respectively, then E_{cell} for
$Ni\ |Ni^{2+}\ (0.01M)||Au^{3+}\ (0.1M)|\ Au$ will be :-

(A) $> E°_{cell}$

(B) $= E°_{cell}$

(C) $< E^\circ_{cell}$

(D) unpredictable

4) How many gm of solid KOH must be added to 100 mL of a buffer solution ? Which is 0.1 M each w.r.t. acid HA and salt KA to make the pH of solution 6.0. [Given : pK_a (HA) = 5] :-

(A) 0.458

(B) 0.327

(C) 5.19

(D) 0.925

5) In a compound Carbon=52.2%, Hydrogen = 13%
Oxygen = 34.8% by mass are present and vapour density of the compound is 46. The molecular formula of the compound is :-

(A) $C_3H_6O_2$

(B) $C_4H_{10}O_2$

(C) $C_4H_{12}O_2$

(D) $C_2H_6O_3$

6) Hydrazine reacts with KIO_3 in presence of HCl as;

$$N_2H_4 + IO_3^- + 2H^+ + Cl^- \rightarrow ICl + N_2 + 3H_2O$$

The equivalent masses of N_2H_4 and KIO_3 respectively are : (K = 39, I = 127) :-

(A) 8,87

(B) 8,35.6

(C) 16,53.5

(D) 8,53.5

7) For the reactions, $2A + B \rightarrow C$,
$\Delta H = 400$ kJ mol^{-1} and $\Delta S = 2$ kJ $K^{-1}mol^{-1}$
Calculate temperature above which reaction become spontaneous ? (considering ΔH and ΔS to be constant over the temperature range) :-

(A) 22

(B) 11

(C) 44

(D) 10

8) Which of the following has an optical isomer ?

(A) $[Co(en)(NH_3)_2]^{2+}$

(B) $[Co(H_2O)_4(en)]^{3+}$

(C) $[Co(en)_2(NH_3)_2]^{3+}$

(D) $[Co(NH_3)_3Cl]^+$

9) Type of hybridization of boron in diborane is :-

(A) sp

(B) sp^2

(C) sp^3

(D) sp^3d^2

10) Which of the following statements is not correct?

(A) Copper liberates hydrogen from acids.

(B) In its higher oxidation states, manganese forms stable compounds with oxygen and fluorine.

(C) Mn^{3+} and Co^{3+} are oxidising agents in aqueous solution.

(D) Ti^{2+} and Cr^{2+} are reducing agents in aqueous solution.

11) Which of the following statement is correct when a mixture of NaCl and $K_2Cr_2O_7$ is gently warmed with conc. H_2SO_4 ?
(A) A deep red vapor is evalved
(B) The vapour when passed into NaOH solution gives a yellow solution of Na_2CrO_4
(C) Chlorine gas is evolved
(D) Chromyl chloride is formed

(A) A,B,D

(B) A,B,C

(C) B,C,D

(D) All are correct

12) Which of the following set of elements have almost same radii :-

(A) Y & La

(B) Ti & Zr

(C) Mo & W

(D) V & Nb

13) Which of the following ion has maximum complex forming tendency ?

(A) La^{+3}

(B) Ce^{+3}

(C) Eu^{+3}

(D) Lu^{+3}

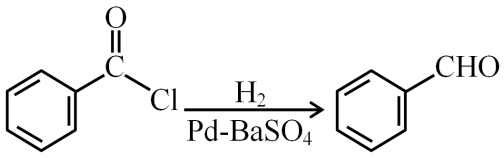
14) The option with only amphoteric oxides is :-

(A) Cr_2O_3, NO, SnO, PbO

(B) NO, B_2O_3, PbO, SnO_2

(C) Cr_2O_3, BaO, SnO, SnO_2

(D) ZnO, A$\square_2O_3$,PbO, PbO_2

15) 

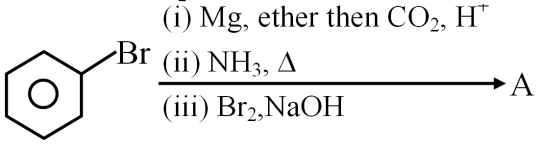
This reduction reaction is known as:

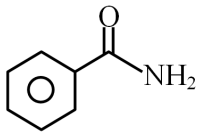
(A) Rosenmund reduction
(B) Wolff-Kishner reduction
(C) Stephen reduction
(D) Etard reduction

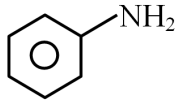
16) Sugar which does not give reddish brown precipitate with Fehling's reagent is:

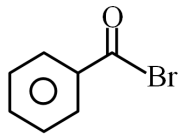
(A) Sucrose
(B) Lactose
(C) Glucose
(D) Maltose

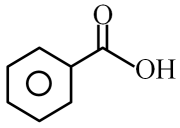
17) The final product A, formed in the following multistep reaction sequence is:



(A) 

(B) 

(C) 

(D) 

18) Match List - I with List - II.

	List-I (Reactants)		**List-II (Products)**
(A)	Phenol, Zn/Δ	(I)	Salicylaldehyde
(B)	Phenol, $CHCl_3$, NaOH, HCl	(II)	Salicylic acid
(C)	Phenol, CO_2, NaOH, HCl	(III)	Benzene

(D)	Phenol, Conc. HNO_3	(IV)	Picric acid

Choose the correct answer from the options given below.

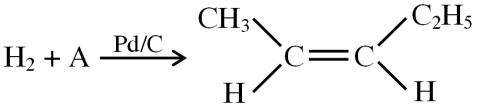
(A) (A)-(IV), (B), (II), (C)-(I), (D)-(III)

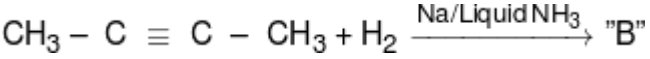
(B) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(C) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

19) In the given reactions identify A and B.





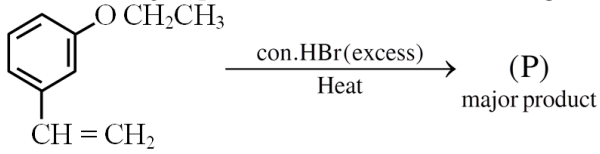
(A) A : 2 – Pentyne B : trans – 2 – butene

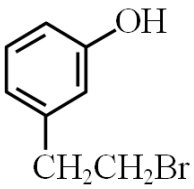
(B) A : n – Pentane B : trans – 2 – butene

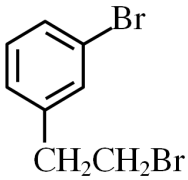
(C) A : 2 – Pentyne B : Cis – 2 – butene

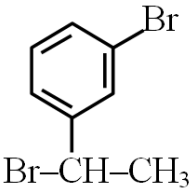
(D) A : n – Pentane B : Cis – 2 – butene

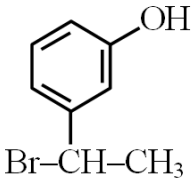
20) The major product(P) in the following reaction is



(A) 

(B) 

(C) 

(D) 

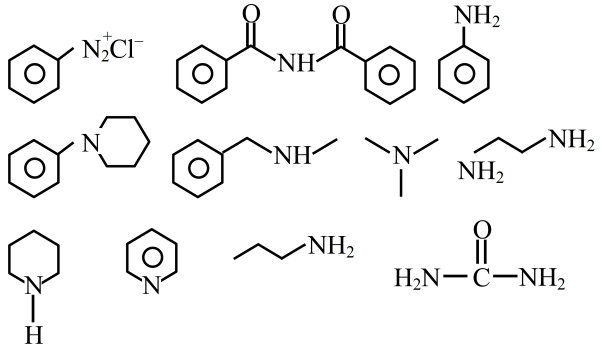
SECTION-II

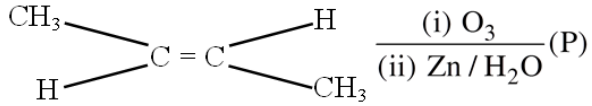
1) A solution of isopropyl alcohol and propyl alcohol has a vapour pressure 200 mm of Hg if it has 25% mole of isopropyl alcohol. Another solution of same components containing 25% mole propyl alcohol has vapour pressure 300 mm of Hg.Then vapor pressure of isopropyl alcohol in mm of Hg is :

2) Calculate enthalpy change for the reaction in Kcal/mol $H_2(g) + C_2H_4(g) \rightarrow C_2H_6(g)$
The bond energies are, H–H ⇒ 110, C–H ⇒ 100,
C–C ⇒ 80 & C=C ⇒ 150 Kcal mol^{-1}

3) Find sum of number of unpaired electron in $[CoCl_6]^{-3}$, $[Cr(NH_3)_6]^{+3}$ and $[Zn(NH_3)_4]^{+2}$:-

4) Number of compounds which give reaction with Hinsberg's reagent is_____.



5) 

Consider the given reaction. The total number of oxygen atoms present per molecule of the product (P) is____.

PART-3 : MATHEMATICS

SECTION-I

1) If $R = \int_0^1 x^{50}(2-x)^{50}dx, K = \int_0^1 x^{50}(1-x)^{50}dx$ which of the following is true :-

(A) $R = 2^{50}\ K$

(B) $R = 2^{-50}\ K$

(C) $R = 2^{100}\ K$

(D) $R = 2^{-100}\ K$

2) If $f(x) = \int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$ and

f(0) = 0, then f(1) is equal to

(A) $\log\left(1+\sqrt{2}\right)$

(B) $\log\left(1+\sqrt{2}\right)-\frac{\pi}{4}$

(C) $\log\left(1+\sqrt{2}\right)+\frac{\pi}{4}$

(D) None of these

3) $\frac{dy}{dx} = (x^3 - 2x\sin^{-1}y)\sqrt{1-y^2}$. General solution will be :

(A) $2\sin^{-1}y = (x^2 - 1) + Ce^{-x^2}$

(B) $2\cos^{-1}y = (x^4 + 1) + C$

(C) $e^{x^2}\sin^{-1}y = (x^2 - x) + C$

(D) $2\cos^{-1}y = (x^2 - 1)\,e^{-x^2} + C$

4) Let $f(x) = x^2 - 3x + 2$ be a function, $\forall\, x \in R$
the area bounded by the curve $|f(|x|)|$ between
$1 \le |x| \le 2$ and x-axis is :

(A) $\frac{2}{3}$ sq. unit

(B) $\frac{1}{6}$ sq. unit

(C) $\frac{1}{3}$ sq. unit

(D) $\frac{1}{2}$ sq. unit

5) If $n \in N$, $n \in [1, 100]$, then sum of all n for which. H.C.F of (15, n) = 1, will be :-

(A) 2418

(B) 2632

(C) 2733

(D) None

6) If the system of equation $x - ky + z = 0$;
$kx + 3y - kz = 0$; $3x + y - z = 0$ have non trival solution then all value of k will be:-

(A) {2, -3}

(B) R-{2,-3}

(C) R-{2}

(D) None of these

7) How many 6 digit palindrom number can be made by using digits 0, 1, 2, 6, 7, 8 which are even :-

(A) 108

(B) 144

(C) 72

(D) 36

8) If matrix $A = [a_{ij}]_{3 \times 3}$ and $a_{ij} + a_{ji} = 0$ and element $a_{ij} \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7\}$, then number of matrix A is equal to :-

(A) 3375

(B) 2744

(C) 6750

(D) 5488

9) If two distinct numbers a and b are chosen at random from the set {1, 2, 3, ... 50} then the probability that $4^a + 4^b + 3$ is divisible by 5 is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{24}{49}$

(D) $\frac{12}{49}$

10) Area of region enclosed by locus of z given by $\text{Arg}(z + i) - \text{Arg}(z - i) = \frac{2\pi}{3}$ and imaginary axis is -

(A) $\frac{2\pi}{9} - \frac{1}{\sqrt{3}}$

(B) $\frac{4\pi}{9} - \frac{1}{\sqrt{3}}$

(C) $\frac{2\pi}{9} - \frac{2}{\sqrt{3}}$

(D) $\frac{4\pi}{9} - \frac{2}{\sqrt{3}}$

11) The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is :

(A) $\frac{\sqrt{17} + 3}{2}$

(B) $\frac{\sqrt{17} + 5}{2}$

(C) 5

(D) $\frac{9-\sqrt{17}}{2}$

12) Let f (x) = min {1, 1 + x sin x}, $0 \le x \le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0)

(B) (1, 0)

(C) (1, 1)

(D) (2, 1)

13) If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha} = \frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is :

(A) $4\sqrt{(1-x^2)}\left(1-2x^2\right)$

(B) $4x\sqrt{(1-x^2)}\left(1-2x^2\right)$

(C) $2x\sqrt{(1-x^2)}\left(1-4x^2\right)$

(D) $4\sqrt{(1-x^2)}\left(1-4x^2\right)$

14) If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5

(B) 10.5

(C) 12

(D) 11

15) $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$
is equal to

(A) $\frac{3}{16}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{9}{32}$

16) The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$

is equal to :

(A) –1

(B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{4}$

17)

Let the line L intersect the lines
$x - 2 = -y = z - 1$, $2(x + 1) = 2(y - 1) = z + 1$
and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$
Then which of the following points lies on L ?

(A) $\left(-\frac{1}{3}, 1, 1\right)$

(B) $\left(-\frac{1}{3}, 1, -1\right)$

(C) $\left(-\frac{1}{3}, -1, -1\right)$

(D) $\left(-\frac{1}{3}, -1, 1\right)$

18) Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$ where α and β are integers and αβ = –6. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2 =$

(A) 17

(B) 24

(C) 21

(D) 19

19) Let the circles $C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2$ and $C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1, then $(\alpha + \beta) + 4\left(r_1^2 + r_2^2\right)$ equals :

(A) 110

(B) 130

(C) 125

(D) 145

20) Let H : $\frac{-x^2}{a^2}+\frac{y^2}{b^2}=1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to :

(A) 170

(B) 171

(C) 169

(D) 172

SECTION-II

1) The value of integral $\int_0^4 \min\{|x-1|, |x-2|, |x-3|\}\,dx = \frac{a}{b}$ where a and b are co-prime, then the value of (a + b) is :-

2) Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A + I)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then the value of $a + b + c + d$ is :-

3) The number of one-one function
$f : \{a, b, c, d\} \to \{0, 1, 2, \ldots, 10\}$ such that
$2f(a) - f(b) + 3f(c) + f(d) = 0$ is _____ .

4) The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is :

5) A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	C	A	A	A	D	A	C	D	A	A	B	A	C	A	A	B	D	B	A	D

SECTION-II

Q.	21	22	23	24	25
A.	3	3	2	236	5

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	C	C	A	A	C	D	A	C	C	A	A	C	D	D	A	A	B	C	A	D

SECTION-II

Q.	46	47	48	49	50
A.	350	-20	7	5	1

PART-3 : MATHEMATICS

SECTION-I

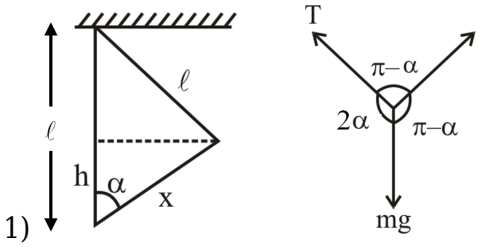
Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	C	B	A	C	B	A	A	A	D	B	A	B	B	D	B	B	B	D	B	B

SECTION-II

Q.	71	72	73	74	75
A.	5	2	31	96	30

SOLUTIONS

PART-1 : PHYSICS



1)

By lami's theorem

$$\left(\frac{Kq_1q_2}{x^2}\right)\frac{1}{\sin 2\alpha} = \frac{mg}{\sin(\pi-\alpha)}$$

x = 2cosa

$$x = \frac{h}{\cos\alpha}$$

$x^2 = 2h$

$$\frac{Kq_1q_2}{2\ell h(2\sin\alpha\cos\alpha)} = \frac{mg}{\sin\alpha}$$

$$\cos\alpha = \frac{Kq_1q_2}{4\ell hmg}$$

2)

When current through resistance is zero then

$V_A - V_B = 6V$

so potential difference across 2Ω resistance would be 10 – 6 = 4V

 Current through 2Ω resistance is 2A

Now $2 = \frac{\text{net emf}}{\text{net resistance}} = \frac{10-4}{2+R}$

R = 1Ω

3) $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

$$E = \frac{M\Delta i}{\Delta E}$$

$$E = \left(\frac{\mu_0 N_1 N_2 A}{\ell}\right)(2i)$$

4)

Let $q = q_0 \sin\omega t$

$$I = \frac{dq}{dt} = q_a\omega\cos\omega t$$

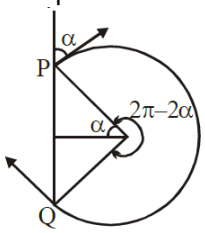
$$\frac{dI}{dt} = \frac{d^2q}{dt^2} = -q_0\omega^2\sin\omega t$$

$\frac{dI}{dt}$ maximum at sin ωt = -1

$\left(\frac{dI}{dt}\right)_{max} = q_0\omega^2 \quad \omega = \frac{1}{\sqrt{LC}}$

5)

α = β



$t = T\left\{\frac{2\pi - 2\alpha}{2\pi}\right\}$

PQ = 2R sin α

7) dU = mCvdT

$\Rightarrow mCv = \frac{dU}{dT} = \frac{80}{20} = 4$ J/K

8) $v = n \cdot \lambda = \sqrt{\frac{T}{m}}$

$v \propto \lambda \propto \sqrt{T}$

$\frac{0.06}{\lambda} = \sqrt{\frac{2 \times g}{(2+6) \times g}}$

λ = 0.12 m

9) In resonance ,

$n_{fork} = n_{wp}$

$330 = \frac{x \times 330}{4\ell}$

$\Rightarrow \ell = \frac{x}{4}$

$= \frac{1}{4}, \frac{3}{4}, \frac{5}{4}\ldots\ldots$

= 25 cm, 75 cm, 125 cm....

Minimum water = 120–75

= 45 cm

10) $\frac{1}{f_a} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_m} = (_m\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

11) $n = \frac{P\lambda}{hc}$

13) Power = 1000 kW = 10^6 J/s

Rate of nuclear fission $= \frac{10^6}{200 \times 1.6 \times 10^{-13}}$

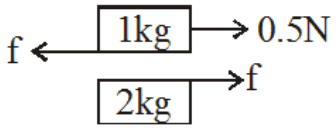
$= 3.125 \times 10^{16}$.

14) $I = \frac{12 - 0.7 - 0.3}{5.6 \times 10^3} = 1.96$ mA

15) $x_{com} = \frac{5 \times 1 + 3 \times -1 + 2 \times 2 + 4 \times -1}{14} = \frac{1}{7}$

$y_{com} = \frac{5 \times 6 + 3 \times 5 + 2 \times -3 + 4 \times -4}{14} = \frac{23}{14}$

16)



0.5 = 3a ⇒ a = 5/30

f = 2a = 2(5/30)= 1/3N

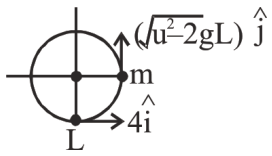
f_{max} = 0.1 × 10 = 10N

 f = 1/3 N will act

work done by lower block = –f(s)

= –1/3(3) = –1 Joule

17)



$|\vec{v_m} - \vec{u}|$

$= \left|\sqrt{u^2 - 2gL}\,\hat{j} - 4\hat{i}\right|$

$= \sqrt{\left(\sqrt{u^2 - 2gL}\right)^2 + 4^2}$

$= \sqrt{2\,(u^2 - gL)}$

18) $\Delta\ell = \frac{FL}{AY}$

$\frac{\Delta\ell_S}{\Delta\ell_B} = \frac{F_S}{F_B} \times \frac{L_S}{L_B} \times \frac{A_B}{A_S} \times \frac{Y_B}{Y_S}$

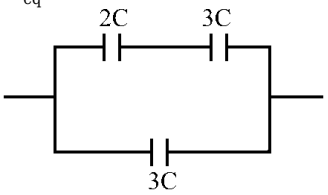
$$\frac{\Delta\ell_S}{\Delta\ell_B} = \left(\frac{3M}{2M}\right)(a)\left(\frac{1}{b^2}\right)\left(\frac{1}{c}\right)$$
$$\frac{\Delta\ell_S}{\Delta\ell_B} = \frac{3a}{2b^2c}$$

19) $W = 8\pi R^2T = 8 \times \pi \times (2 \times 10^{-2})2 \times 2 \times 10^{-2}$
$= 64\pi \times 10^{-6}$ J

20) according to graph
$x = A\sin\omega t$
A = 1 cm
and T = 8 s
so acceleration = $-A\omega^2\sin\omega t$
$$= -(1)\left(\frac{2\pi}{8}\right)^2 \sin\left(\frac{2\pi}{8} \times \frac{8}{3}\right)$$
$$= -\frac{4\pi}{64} \times \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}\pi^2}{32} \text{ cm/s}^2$$

21)

$$C_{eq} = \frac{21}{5} \times \frac{\in_0 A}{d}$$



22) $$\gamma_{mix} = \frac{n_1Cp_1 + n_2Cp_2}{n_1Cv_1 + n_2Cv_2}$$
$$\frac{11}{7} = \frac{n_1\left(\frac{5}{2}R\right) + n_2\left(\frac{7}{2}R\right)}{n_1\left(\frac{3}{2}R\right) + n_2\left(\frac{5}{2}R\right)} \quad \frac{n_1}{n_2} = 3$$

23)

$$\text{Shift} = \frac{D}{d}(\mu - 1)t = \frac{D\lambda}{d}$$
$$\mu - 1 = \frac{\lambda}{t} = \frac{12 \times 10^{-7}}{12 \times 10^{-7}} = 1$$
$\Rightarrow \mu = 2$

24)

$Q = BE_{Product} - BE_{Rectant}$

= 2(118) (8.6) – 236(7.6)
= 236 × 1 = 236 MeV

25) In fluid frame ball is moving with constant velocity (terminal) and resultant of all forces acting on it zero.
(A) Buoyant force
(B) viscous force
(C) mg

$v_1 = \frac{2}{9}\frac{(\rho - \rho_0)gr^2}{\eta} = \frac{2}{9} \times \frac{900 \times 10}{0.1}$
$\times 25 \times 10^{-4} = 50$ m/s

PART-2 : CHEMISTRY

26) $\frac{K_P}{K_C} = (RT)^{\Delta H}$
$3 = (0.08 \times T)^1$
$T = \frac{3}{0.08} = 36.54$

27) $r = K[A]^x [B]^y$
By I, II
A = doubled, B = same, r = doubled
$\Rightarrow x = 1$
By II, III
A = same, B = doubled, r = doubled
$\Rightarrow y = 1$
$r = K [A] [B]$
By I, $2 \times 10^{-4} = K \times (1 \times 10^{-2}) \times (2 \times 10^{-2})$
$K = 1$

28) $Q = \frac{\left[Ni^{+2}\right]^3}{\left[Au^{+3}\right]^2} = \frac{(0.01)^3}{(0.1)^2}$
$Q < 1$; $E > E°$

29) Let x mili-moles of NaOH is added
$6 = 5 + \log\left[\frac{s+x}{a-x}\right]; \frac{s+x}{a-x} = 10$
or $\frac{10+x}{10-x} = 10 \Rightarrow x = 8.18$
$\Rightarrow$ wt = $8.18 \times 10^{-3} \times 56 = 0.458$ gm

30) $\frac{C_{52.2}}{12}\frac{H_{13}}{1}\frac{O_{34.8}}{16}$

$C_{4.35}H_{13}O_{2.175}$
C_2H_6O = EF
Now VD × 2 = MW
⇒ 92 = MW
⇒ $C_4H_{12}O_2$

31) $\begin{bmatrix} N_2H_4 \rightarrow N_2\ ;\ n-\text{factor} = (-2 \text{ to } 0) \times 2 = 4 \\ IO_3^- \rightarrow ICl; n-\text{factor} = (+5 \text{ to } +1) \times 1 = 4 \end{bmatrix}$

$$E_{N_2H_4} = \frac{32}{4} = 8$$

$$E_{KIO_3} = \frac{214}{4} = 53.5$$

32) A reaction is spontaneous; ΔG = -ive
ΔH – TΔS < 0

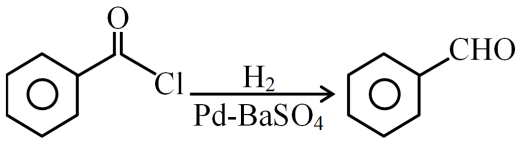
$$\Rightarrow T > \frac{\Delta H}{\Delta S}$$

$$\Rightarrow T > \frac{400}{2} \Rightarrow T > 200 \text{ K}$$

36) $K_2Cr_2O_7 + H_2SO_4(\text{conc}) + NaCl \rightarrow KHSO_4 + NaHSO_4 + H_2O + \underset{\text{Chromylchloride(red)}}{CrO_2Cl_2\uparrow}$
$CrO_2Cl_2 + NaOH \rightarrow + NaCl + H_2O$

38) Complex forming tendency ∝ polarising power (ϕ)

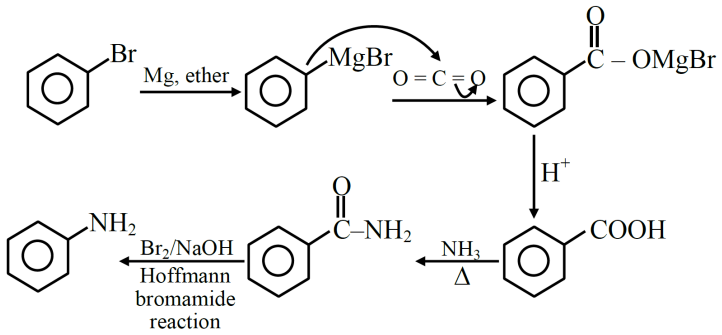
39) ZnO, Al_2O_3, PbO and PbO_2 are amphoteric oxides.

40) 

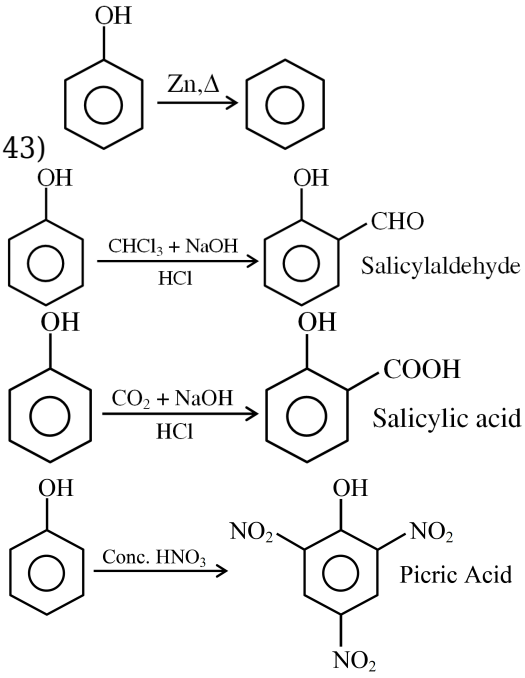
It is known as rosenmund reduction that is the partial reduction of acid chloride to aldehyde.

41) Sucrose do not contain hemiacetal group.
Hence it does not give test with Fehling solution.
While all other give positive test with Fehling solution.

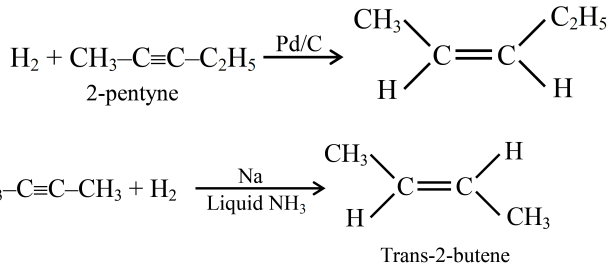
42)



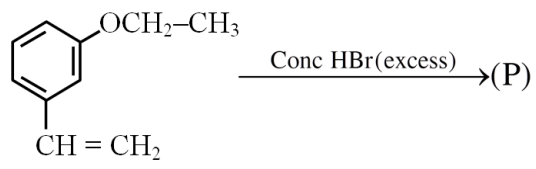
43)

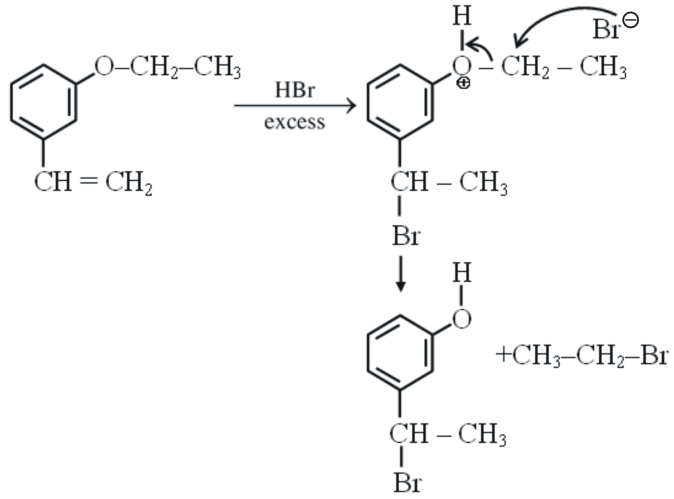


44)



45)



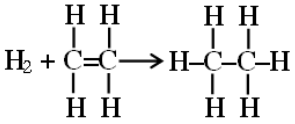


46) $\left(200 = \frac{P_i}{4} + \frac{P_P \times 3}{4}\right)$ ....(1)

$\left(300 = \frac{3P_i}{4} + \frac{P_P}{4}\right) \times 3$ ....(2)

$700 = \frac{9}{4}P_i - \frac{P_i}{4} = 2P_i$

$P_i = \frac{700}{2} = 350$ mm of Hg.

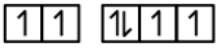


47)

$\Delta H = 110 + 150 - 80 - 2 \times 1$

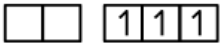
$= -20$ kcal/mol

48) $[CoCl_6]^{-3}$, $Co^{+3} \Rightarrow 4s^2 3d^7 = d^6$



(4)

$[Cr(NH_3)_6]^{+3}$, $Cr^{+3} \Rightarrow 4s^2 3d^5 d^3$



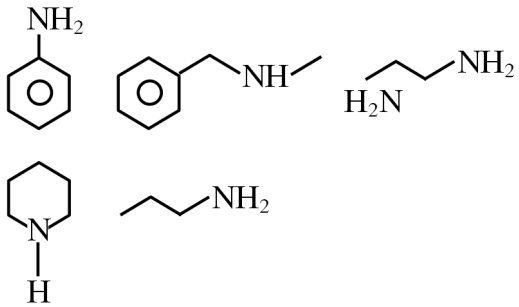
(3)

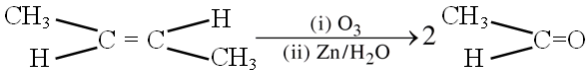
$[Zn(NH_3)_4]^{+2}$, $Zn^{+2} \Rightarrow 4s^2 3d^{10}$

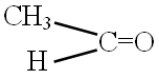
unpaired $e^- = 0$

Sum = $7e^-$

49)



50) 

Hence total number of oxygen atom present per molecule  is 1

PART-3 : MATHEMATICS

51) In R, put x = 2t or dx = 2dt

$\therefore R = 2\int_0^{1/2} 2^{50} t^{50} 2^{50}(1-t)^{50}\,dt$ .....(i)

Now, $K = 2\int_0^{1/2} x^{50}(1-x)^{50}\,dt$ .....(ii)

From (i) & (ii),

$R = 2^{100} K$

52) Given, $f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Putting $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$\therefore f(x) = \int \frac{\tan^2\theta \cdot \sec^2\theta}{\sec^2\theta(1+\sec\theta)}\,d\theta$

$= \int \frac{\sec^2\theta - 1}{1+\sec\theta}\,d\theta = \int (\sec\theta - 1)\,d\theta$

$= \log(\sec\theta + \tan\theta) - \theta + C$

$f(x) = \log\left(\sqrt{1+x^2} + x\right) - \tan^{-1}x + C$

At $x = 0$, $f(0) = \log(1+0) - 0 + C \Rightarrow C = 0$

$\therefore f(x) = \log\left(\sqrt{1+x^2} + x\right) - \tan^{-1}x$

At $x = 1$, $f(1) = \log\left(1+\sqrt{2}\right) - \frac{\pi}{4}$

53) $\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = x^3 - 2x\sin^{-1}y$

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} + 2x\sin^{-1}y = x^3$

let $\sin^{-1} y = t$

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + 2xt = x^3$

I.F. $= e^{\int 2x dx} = e^{x^2}$

$\therefore$ solution is given by $t(e^{x^2}) = \int x^3 . e^{x^2} dx + c$ let $x^2 = t$

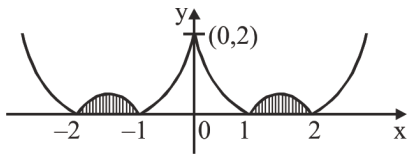
$\sin^{-1} y = e^{x^2} = \int \underset{I}{t}\ \underset{II}{e^t}.\frac{dt}{2} + c \qquad 2xdx = dt$

$\Rightarrow 2\sin^{-1} y\, e^{x^2} = \left(t.e^t - \int e^t dt\right) + 2C$

$\Rightarrow 2\sin^{-1} y\, e^{x^2} = (x^2 - 1)\, e^{x^2} + K$

$\Rightarrow 2\sin^{-1} y = (x^2 - 1) + K\, e^{-x^2}$

54)



$| f(|x|) | = | |x|^2 - 3|x| + 2 |$

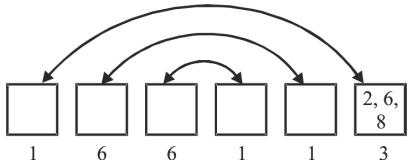
$$\text{Area} = 2\int_1^2 \left|x^2 - 3x + 2\right| dx$$

$$= -2\int_1^2 \left(x^2 - 3x + 2\right) dx$$

$$= -2\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2$$

$$= -2.\left[-\frac{1}{6}\right] = \frac{1}{3} \text{ Sq. unit}$$

57)



$= 108$

58) $1 \times 15 \times 15 \times 15 = 3375$

59) $4^a + 4^b + 3$

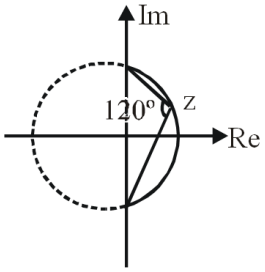
$= (5 - 1)^a + (5 - 1)^b + 3$

$= 5$ (integer) $+ (-1)^a + (-1)^b + 3$

$\Rightarrow$ a and b both must be even

$$\Rightarrow \text{prob} = \frac{{}^{25}C_2}{{}^{50}C_2} = \frac{25 \times 24}{50 \times 49} = \frac{12}{49}$$

60) Locus is are of circle with center $\left(-\frac{1}{\sqrt{3}}, 0\right)$ and radius $\frac{2}{\sqrt{3}}$



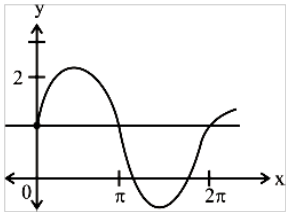
$$\text{Area} = \frac{1}{3} \times \pi \left(\frac{2}{\sqrt{3}}\right)^2 - \frac{1}{2} \times 2 \times \frac{1}{\sqrt{3}} = \frac{4\pi}{9} - \frac{1}{\sqrt{3}}$$

61)
$$f(x) = \begin{cases} x^2 - 4x - 2, & \forall x \in \left(-1, \frac{3-\sqrt{17}}{2}\right) \\ -x^2 + 2x + 2, & \forall x \in \left(\frac{3-\sqrt{17}}{2}, 2\right) \end{cases}$$

$x \in \left(-1, \frac{3-\sqrt{17}}{2}\right)$
f'(x) when
f'(x) = 2x – 4 = 0 ⇒ x = 2
f'(x) = 2(x – 2) ⇒ f'(x) is always ↓
f(2) = 2
f(–1) = 3
$f\left(\frac{3-\sqrt{17}}{2}\right) = \frac{\sqrt{17}-3}{2}$

$x \in \left(\frac{3-\sqrt{17}}{2}, 2\right)$
f'(x) when
f'(x) = –2x + 2
f'(x) = –2(x – 1)
f'(x) = 0 when x = 1
f(1) = 3

absolute minimum value = $\frac{\sqrt{17}-3}{2}$
absolute maximum value = 3
Sum = $\frac{\sqrt{17}-3}{2} + 3 = \frac{\sqrt{17}+3}{2}$

62) 

No. of non-differentiable points = 1 (m)
No. of not continuous points = 0 (n)
(m, n) = (1, 0)

63) $\frac{\sin^{-1}x}{\alpha} = \frac{\cos^{-1}x}{\beta} = k$

$\sin^{-1}x = k\alpha$

$\cos^{-1}x = k\beta$

$k = \frac{\pi}{2(\alpha+\beta)}$ ....(i)

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin(4\sin^{-1}x)$

$= 2\sin(2\sin^{-1}x)\cos(2\sin^{-1}x)$

$= 4x\sqrt{1-x^2}(1-2x^2)$

64) 3, 5, 7, 2k, 12, 16, 21, 24

Median = $\frac{2k+12}{2} = k+6$

M.D. = $\frac{\Sigma|x_i - M|}{8} = 6$

$= (k+3) + (k+1) + (k-1) + (6-k) + (6-k) +$
$(10-k) + (15-k) + (18-k) = 48$

$= 58 - 2k = 48$

$k = 5$

Median $= k + 6 = 11$

65) $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$

$2\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{15}\right)$

$\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$

$2\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$

$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)$

$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\pi-\frac{3\pi}{11}\right)\cos\left(\pi+\frac{5\pi}{11}\right)$

$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{8\pi}{11}\right)\cos\left(\frac{16\pi}{11}\right)$

$\frac{2.\sin\left(2^5\times\frac{\pi}{11}\right)}{2^5\sin\frac{\pi}{11}}$

$\frac{2.\sin\left(\frac{32\pi}{11}\right)}{32\sin\frac{\pi}{11}} = \frac{1}{16}$

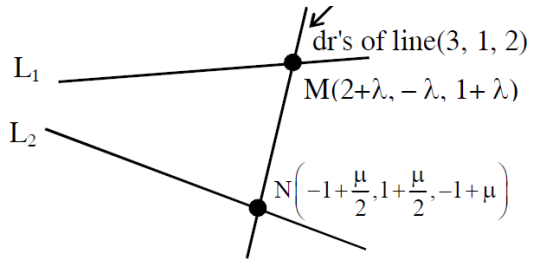
66) $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$

$$=\frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}}\times\cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$$

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}}\times\cos\left(\frac{4\pi}{7}\right)$$

$$=\frac{\sin\left(\frac{7\pi}{7}\right)+\sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$=\frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$



67)

$$L_1:\frac{x-2}{1}=\frac{y}{-1}=\frac{z-1}{1}=\lambda$$

$$L_2:\frac{x+1}{\frac{1}{2}}=\frac{y-1}{\frac{1}{2}}=\frac{z+1}{1}=\mu$$

dr of line MN will be

$<3+\lambda-\frac{\mu}{2},-1-\lambda-\frac{\mu}{2},2+\lambda-\mu>$ & it will be proportional to <3, 1, 2>

$$\therefore\frac{3+\lambda-\frac{\mu}{2}}{3}=\frac{-1-\lambda-\frac{\mu}{2}}{1}=\frac{2+\lambda-\mu}{2}$$

$4\lambda+\mu=-6$ $\qquad$ $4+3\lambda=0$

$$\Rightarrow\lambda=-\frac{4}{3}\&\mu=-\frac{2}{3}$$

 Coordinate of M will be $<\left(\frac{2}{3},\frac{4}{3},-\frac{1}{3}\right)$

and equation of required line will be.

$$\frac{x-\frac{2}{3}}{3}=\frac{y-\frac{4}{3}}{1}=\frac{z+\frac{1}{3}}{2}=k$$

So any point on this line will be

$$\left(\frac{2}{3}+3k,\frac{4}{3}+k,-\frac{1}{3}+2k\right)$$

$$\because\frac{2}{3}+3k=-\frac{1}{3}\Rightarrow k=-\frac{1}{3}$$

 Point lie on the line for

$k = -\frac{1}{3}$ is $\left(-\frac{1}{3}, 1, -1\right)$

68) Area of parallelogram $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$A = \frac{1}{2}\left|(\vec{a}+\vec{b}) \times (\vec{b}+\vec{c})\right| = \frac{\sqrt{21}}{2}$

so, $\vec{a}+\vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$

$\vec{b}+\vec{c} = -\hat{i} + \beta\hat{j}$

$$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 2 \\ -1 & \beta & 0 \end{vmatrix}$$

$= \hat{i}(-2\beta) - \hat{j}(2) + \hat{k}(\beta + \alpha)$

$\left|(\vec{a}+\vec{b}) \times (\vec{b}+\vec{c})\right| = \sqrt{4\beta^2 + 4 + (\alpha+\beta)^2} = \sqrt{21}$

$4\beta^2 + 4 + \alpha^2 + \beta^2 + 2\alpha\beta = 21$

$\alpha^2 + 5\beta^2 - 12 = 17$

$\alpha^2 + 5\beta^2 = 29$ and $\alpha\beta = -6$

and given α, β are integers

so, α = -3, β = 2

or

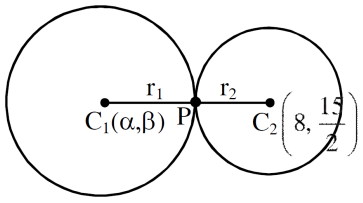
α = 3, β = -2

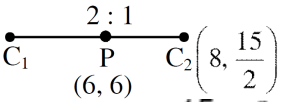
$(\alpha_1, \beta_1) = (-3, 2)$

$(\alpha_2, \beta_2) = (3, -2)$

$\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2 = 9 + 4 + 6 = 19$

69)





$\frac{16+\alpha}{3} = 6$ and $\frac{15+\beta}{3} = 6$

$\Rightarrow (\alpha, \beta) \equiv (2, 3)$

Also, $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_2 + r_2$

$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$

$\therefore (\alpha + \beta) + 4(r_1^2 + r_2^2)$

$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$

70) $H: \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, $e = \sqrt{3}$

$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \Rightarrow \frac{a^2}{b^2} = 2$

$a^2 = 2b^2$

length of L.R. $= \frac{2a^2}{b} = 4\sqrt{3}$

$a = \sqrt{6}$

P(α, 6) lie on $\frac{y^2}{3} - \frac{x^2}{6} = 1$

$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$

Foci = (0, ±be) = (0, 3) & (0, –3)

Let d_1 & d_2 be focal distances of P(α, 6)

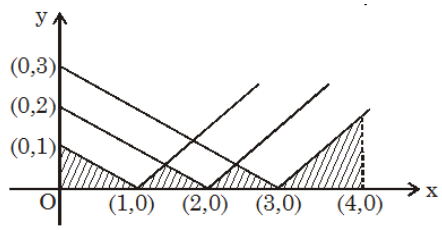
$d_1 = \sqrt{\alpha^2 + (6 + be)^2}$, $d_2 = \sqrt{\alpha^2 + (6 - be)^2}$

$d_1 = \sqrt{66 + 81}$, $d_2 = \sqrt{66 + 9}$

$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$

$\alpha^2 + \beta = 66 + 105 = 171$

71)



Required value

$= \frac{1}{2}(1)(1) + 2\left(\frac{1}{2} \times 1 \times \frac{1}{2}\right) + \frac{1}{2} \times 1 \times 1 = \frac{3}{2}$

73)

2f(a) + 3f(c) = f(d) – f(b)

Using fundamental principle of counting

Number of one-one function is 31

$\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

74) Let, mean = m = np

& variance = v = npq, p + q = 1

Sum = m + v = $\frac{165}{2}$

Product = mv = 1350

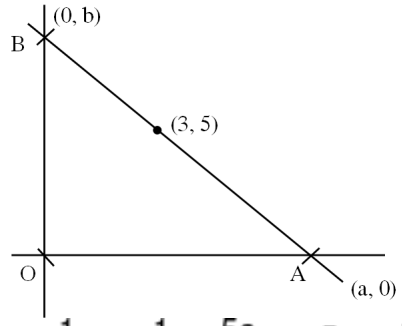
On solving,

m = np = 60

& v = npq = $\frac{45}{2}$ ⇒ q = $\frac{3}{8}$ ⇒ P = $\frac{5}{8}$

75) $\frac{x}{a}+\frac{y}{b}=1$

$\frac{3}{a}+\frac{5}{b}=1 \Rightarrow b=\frac{5a}{a-3}, a>3$



$A=\frac{1}{2}ab=\frac{1}{2}a\frac{5a}{(a-3)}=\frac{5}{2}\cdot\frac{a^2}{a-3}$

$=\frac{5}{2}\left(\frac{a^2-9+9}{a-3}\right)$

$=\frac{5}{2}\left(a+3+\frac{9}{a-3}\right)$

$=\frac{5}{2}\left(a-3+\frac{9}{a-3}+6\right)\geqslant 30$