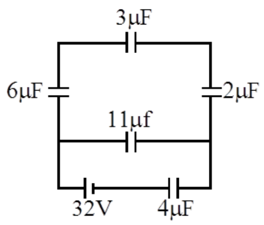
PART-1 : PHYSICS

SECTION-I



1) Choose correct option regarding given circuit.

(A) Potential difference across 4μF is 20V

(B) Charge at 3μF is 8μC

(C) Energy stored in 2μF is 32μJ

(D) Ratio of charges on 3μF & 11μF are 3 : 11

2) A solid cylinderical wire of radius 'R' carries a current 'I'. The magnetic field is 5μT at a point, which is '2R' distance away from the axis of wire. Magnetic field at a point which is R/3 distance inside from the surface of the wire is :-

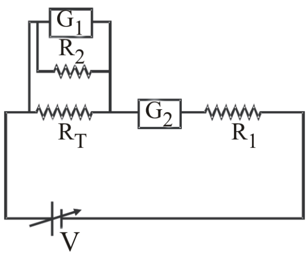
(A) $\frac{10}{3}\mu T$

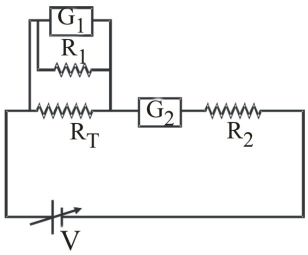
(B) $\frac{20}{3}\mu T$

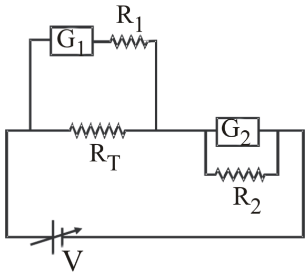
(C) $\frac{5}{3}\mu T$

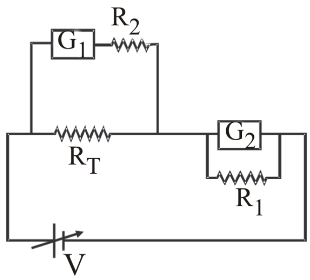
(D) $\frac{40}{3}\mu T$

3) To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :-

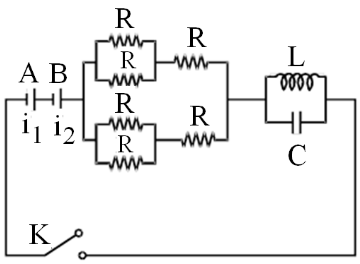
(A) 

(B) 

(C) 

(D) 

4) In the circuit shown A and B are two cells of same emf E but different internal resistances r_1 and r_2 ($r_1 > r_2$) respectively. Find the value of R such that the potential difference across the terminals of cell A is zero after long time after the key K is closed :-



(A) $R = \frac{4}{3}(r_1 - r_2)$

(B) $R = (r_1 - r_2)$

(C) $R = r_1 + r_2$

(D) $R = \frac{r_1 r_2}{r_1 + r_2}$

5) A fully charged capacitor C with initial charge Q_0 is connected to a coil of self inductance L at t = 0. The time at which are energy is stored equally between the electric and the magnetic field is :-

(A) $\frac{\pi}{4}\sqrt{LC}$

(B) $2\pi\sqrt{LC}$

(C) $\sqrt{LC}$

(D) $\pi\sqrt{LC}$

6) The magnetic field of an electromagnetic wave is given by

$\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right) \frac{Wb}{m^2}$

The associated electric field will be :

(A) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(\hat{i} - 2\hat{j}\right)\frac{V}{m}$

(B) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right)\frac{V}{m}$

(C) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z - 6 \times 10^{15} t\right)\left(-2\hat{j} + \hat{i}\right)\frac{V}{m}$

(D) $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(-\hat{i} + 2\hat{j}\right)\frac{V}{m}$

7) A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now :

(A) f

(B) $\frac{f}{2}$

(C) $\frac{3f}{4}$

(D) 2f

8) The maximum intensity in Young's double-slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen ?
(Given that screen is at a distance D = 10 d)

(A) I_0

(B) $\frac{I_0}{2}$

(C) $\frac{3}{4}I_0$

(D) $\frac{I_0}{4}$

9) Focal length of a biconvex lens in air is 40 cm. The focal length of the lens in water is 120 cm. If refractive index of water is $\frac{4}{3}$, then what is refractive index of the material of the lens

(A) 1.4

(B) 1.5

(C) 1.6

(D) 1.7

10) A particle of mass 'm' is projected from ground with velocity 'u' making angle θ with the vertical. The de-Broglie wavelength of the particle at the highest point is -

(A) ∞

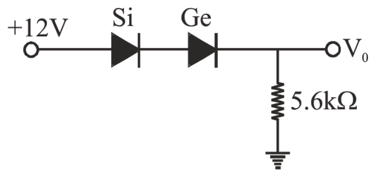
(B) h/mu sinθ

(C) h/mu cosθ

(D) h/mu

11) What is the number of α and β- particle emitted in the following radioactive decay ?
$_{90}X^{200} \rightarrow {}_{80}Y^{168}$

(A) 6 and 6
(B) 8 and 8
(C) 8 and 6
(D) 6 and 8

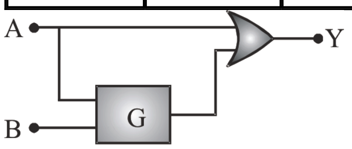


12) In the network shown in figure determine V_0 :-

(A) 11V
(B) 12V
(C) 9.5 V
(D) 13V

13) Identify the logic gate G in the combination of gates shown in figure. The truth table is shown here

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

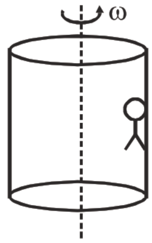


(A) OR gate
(B) AND gate
(C) NOR gate
(D) XOR gate

14) A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :
[Take R = 8.3 J/ K mole]

(A) 10 kJ
(B) 0.9 kJ
(C) 6 kJ
(D) 14 kJ

15) In a rotor a hollow vertical cylindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If radius of rotor is 2m and μ_s = 0.2 what will be the minimum angular velocity at which floor may be removed :-



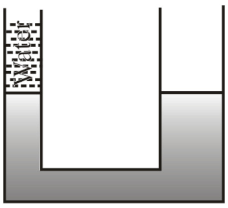
(A) 2 rad/s

(B) 5 rad/s

(C) 10 rad/s

(D) None of these

16) A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water :-



(A) 1.2 cm

(B) 2.35 cm

(C) 0.56 cm

(D) 0.8 cm

17) A ball of mass m is fired vertically upwards from the surface of the earth with velocity nv_e, where v_e is the escape velocity and n < 1. Neglecting air resistance, to what height will the ball rise? (Take radius of the earth as R)

(A) $\frac{R}{n^2}$

(B) $\frac{R}{(1-n^2)}$

(C) $\frac{Rn^2}{(1-n^2)}$

(D) Rn^2

18) A particle travels such that $\frac{dV}{dt} = -a\sqrt{V}$ where a is constant. Its initial velocity is v_0. The time at which it stops is :-

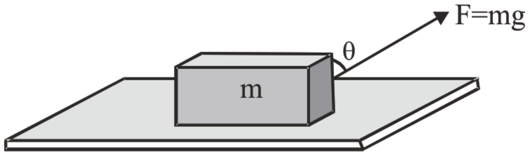
(A) $\frac{2\sqrt{V_0}}{a}$

(B) $\frac{\sqrt{V_0}}{a}$

(C) $2a\sqrt{V_0}$

(D) $a\sqrt{V_0}$

19) A force f is acting on a block of mass m. Coefficient of friction between block & surface is μ. The block can be pulled along the surface if :-



(A) $\tan\theta \geq \mu$

(B) $\cot\theta \geq \mu$

(C) $\tan\frac{\theta}{2} \geqslant \mu$

(D) $\cot\frac{\theta}{2} \geqslant \mu$

20) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

(A) $\frac{3}{2}MR^2$

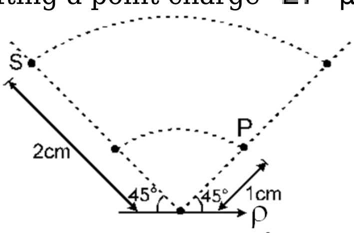
(B) $\frac{7}{5}MR^2$

(C) $\frac{2}{5}MR^2$

(D) $\frac{MR^2}{4}$

SECTION-II

1) Find out work done by electric field in shifting a point charge $\frac{4\sqrt{2}}{27}$ μC from point P to S which are shown in the figure is $\frac{100}{x}$ J. Then find x.



Dipole moment $P = 2 \times 10^{-6}$ C-m

2) A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index 5/3 is poured inside the bucket, then microscope have to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is (in cm) :

3) A rest helium atom emits a photon of wavelength 0.1Å. The recoil energy of the atom due to the

emission of photon will be (in eV)

4) A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in ºC, if this thermometer in the contact with the object reads $x_0/2$?

5) A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

PART-2 : CHEMISTRY

SECTION-I

1) At what ratio of total pressure of reaction
$X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$ have same value of K_p if degree of dissociation of X and Z are same :-

(A) 1 : 36

(B) 1 : 4

(C) 1 : 9

(D) 1 : 3

2) For reaction A → B, the rate constant
$k_1 = A_1e^{-Ea_1/(RT)}$ and for the reaction
X → Y, the rate constant $k_2 = A_2e^{-Ea_2/(RT)}$.
If $A_1 = 10^8$, $A_2 = 10^{10}$ and E_{a_1} = 600 cal/mol,
E_{a_2} = 1800 cal/mol, then the temperature at which $k_1 = k_2$ is :
(Given : R = 2 cal/K-mol)

(A) 1200 K

(B) 1200 × 4.606 K

(C) $\frac{1200}{4.606}$K

(D) $\frac{600}{4.606}$K

3) From the following E° values, of half cells :-
(i) $A + e \rightarrow A^-$, E° = -0.34
(ii) $B^- + e \rightarrow B^{2-}$; E° = + 1.30 V
(iii) $C^- + 2e \rightarrow C^{3-}$; E° = – 1.30
(iv) $D + 2e \rightarrow D^{2-}$; E° = + 0.78 What combination of two half cells would result in a cell with the largest potential ?

(A) ii and iii

(B) ii and iv

(C) i and iii

(D) i and iv

4) 1 Mole of liquid A and 2 moles of liquid B make a solution having observed vapour pressure of 42 torr. The vapour pressure of pure A and pure B are 45 torr and 36 torr respectively. The described solution :-

(A) is an ideal solution

(B) shows negative deviation

(C) is a minimum boiling azeotrope

(D) has volume less than the sum of individual volumes of both components.

5) 17.4% (W/V) solution of potassium sulphate
(mol. wt. = 174) is isotonic with 4% (W/V) aqueous solution of NaOH. If NaOH is 100% ionised, the degree of ionisation of potassium sulphate is :-

(A) 50%

(B) 75%

(C) 40%

(D) 60%

6) Suppose two elements X and Y combine to form two compounds XY_2 and X_2Y_3. If 0.05 mole of XY_2 weighs 5 g while 3.011×10^{23} molecules of X_2Y_3 weighs 85 g, then atomic masses of X and Y are respectively :

(A) 20, 30

(B) 30, 40

(C) 40, 30

(D) 80, 60

7) Find enthalpy of neutralisation of NH_4OH and HCN in aqueous solution if enthalpy of ionisation of NH_4OH and HCN are 7 kJ/mol and 8 kJ/mol. also enthalpy of ionisation of H_2O is 57.3 kJ/mole.

(A) –15 kJ/mol

(B) – 42.3 kJ/mol

(C) +1 kJ/mol

(D) 42.3 kJ/mol

8) $[Co(NH_3)_5NO_2]\ Cl_2$ and $[Co(NH_3)_5ONO]Cl_2$
are related to each other as :-

(A) Geometrical isomers

(B) Linkage isomers

(C) Coordination isomers

(D) ionisation isomers

9)

Which of the following show colour due to charge transfer ?

(A) MnO_4^-

(B) $Cr_2O_7^{-2}$

(C) CrO_4^{-2}

(D) All

10) Which element is having lowest melting and boiling point?

(A) Ti

(B) Cu

(C) Zn

(D) Mn

11) Which of the following pairs is expected to form colourless compound :

(A) La^{+3}, Pr^{+3}

(B) La^{+3}, Lu^{+3}

(C) Nd^{+3}, Pm^{+3}

(D) Eu^{+3}, Tb^{+3}

12) Which of the following reaction is not feasible :-

(A) $F_2 + 2Cl^- \rightarrow 2F^- + Cl_2$

(B) $Cl_2 + 2Br^- \rightarrow 2Cl^- + Br_2$

(C) $Br_2 + 2F^- \rightarrow 2Br^- + F_2$

(D) $Br_2 + 2I^- \rightarrow 2Br^- + I_2$

13) Most basic is :-

(A) KOH

(B) NaOH

(C) $Mg(OH)_2$

(D) ClOH

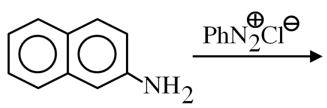
14) Which of the following cations is not precipitated by $NH_4Cl + NH_4OH$?

(A) Al^{3+}

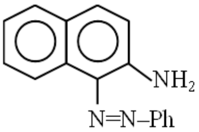
(B) Cr^{3+}

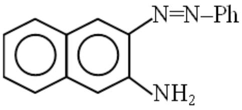
(C) Fe^{2+}

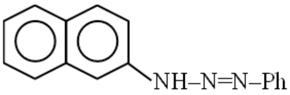
(D) Fe^{3+}

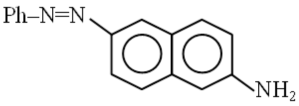
15) 

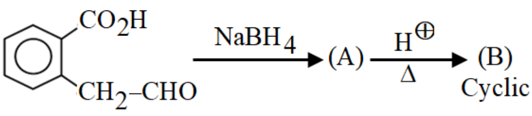
Major product is :

(A) 

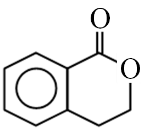
(B) 

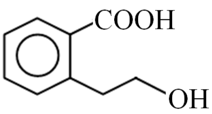
(C) 

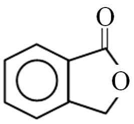
(D) 

16) 

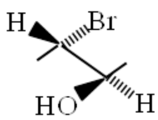
Compound (B) is :-

(A) 

(B) 

(C) 

(D) (A) and (C) both

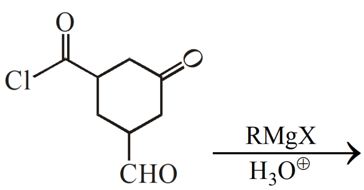
17) The correct IUPAC name of the compound :- 

(A) (2R, 3R)-3-bromo-2-butanol

(B) (2R, 3S)-3-bromo-2-hydroxy butane

(C) (2R, 3S)-3-bromo-2-butanol

(D) (2S, 3R)-2-bromo-3-butanol

18) 

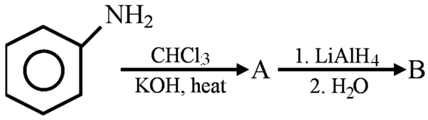
How many molecules of RMgX are consumed in the above given reaction ?

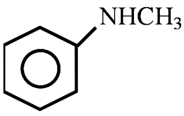
(A) 2

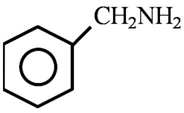
(B) 4

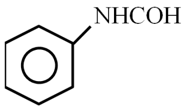
(C) 5

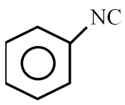
(D) 6

19) The end product (B) of the reaction sequence :- 

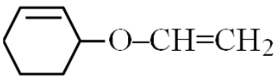
(A) 

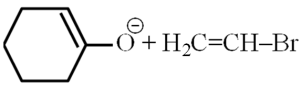
(B) 

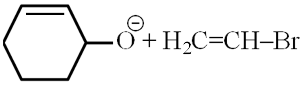
(C) 

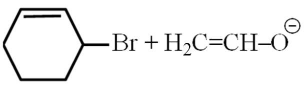
(D) 

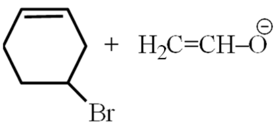
20) The best method for synthesis of given ether by Williamson's ether synthesis is :



(A) 

(B) 

(C) 

(D) 

SECTION-II

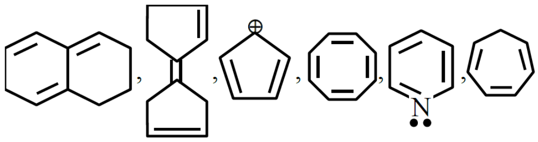
1) A weak acid 0.1M HA is 1% ionised, pH of the solution will be –

2)

How many of the following on heating will produce an oxide of nitrogen ?
$(NH_4)_2Cr_2O_7$, NH_4NO_3, NH_4NO_2, KNO_3, $Pb(NO_3)_2$

3) Out of B_2, C_2, N_2, N_2^+, O_2^-, how many species are paramagnetic?

4) Total number of aromatic compounds among the following compounds is _____.



5) X g of ethylamine is subjected to reaction with $NaNO_2/HCl$ followed by water; evolved dinitrogen gas which occupied 2.24 L volume at STP. X is ____ $\times 10^{-1}$ g

PART-3 : MATHEMATICS

SECTION-I

1) Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9 (b + c) is equal to :

(A) 9
(B) 15
(C) 7
(D) 11

2) Let $A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ and
$B = \{y_1, y_2, y_3\}$ The total number of onto function from f : A → B. If exactly three element x in set A such that $f(x) = y_1$ is equal to -

(A) $12 \times {}^7C_3$
(B) $15 \times {}^7C_3$
(C) $16 \times {}^7C_3$
(D) $14 \times {}^7C_3$

3) The number 'a' is randomly selected from the set {0, 1, 2, 3, ... 98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is :

(A) $\frac{1}{16}$
(B) $\frac{2}{16}$
(C) $\frac{4}{16}$
(D) $\frac{3}{16}$

4) If matrix $A = [a_{ij}]_{3\times3}$, $B = [b_{ij}]_{3\times3}$, where

$a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0 \ \forall$ i, j, then A^4B^3 is :-

(A) singular

(B) zero matrix

(C) symmetric

(D) skew symmetric

5) Let $M = \begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$ and $N = \frac{M^2}{2}$.
If $(a-b)^2 + (d-e)^2 = 36$, $(b-c)^2 + (e-f)^2 = 64$,
$(a-c)^2 + (d-f)^2 = 100$, then value of $|N|$ is equal to :

(A) 1152

(B) 48

(C) 144

(D) 288

6) If w is an Imaginary fifth root unity, then

$\log_{\sqrt{2}}\left|1 + w + w^2 + w^3 - \frac{1}{w}\right| = ?$

(A) 2

(B) 1

(C) 0

(D) None

7) If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1 - \frac{x}{3}, & 0 \le x \le 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,
then range of (fog(x)) is :

(A) (0, 1]

(B) [0, 3)

(C) [0, 1]

(D) [0, 1)

8) $\lim_{x \to \frac{\pi}{2}} \left(\frac{1}{(x - \frac{\pi}{2})^2} \int_{x^3}^{(\frac{\pi}{2})^3} \cos\left(\frac{1}{t^3}\right) dt \right)$
is equal to

(A) $\frac{3\pi}{8}$

(B) $\frac{3\pi^2}{4}$

(C) $\frac{3\pi^2}{8}$

(D) $\frac{3\pi}{4}$

9) Suppose

$$f(x) = \frac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$$

Then the value of f '(0) is equal to

(A) π

(B) 0

(C) $\sqrt{\pi}$

(D) $\frac{\pi}{2}$

10) For 0 < a < 1, the value of the integral

$$\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}$$ is :

(A) $\frac{\pi^2}{\pi + a^2}$

(B) $\frac{\pi^2}{\pi - a^2}$

(C) $\frac{\pi}{1 - a^2}$

(D) $\frac{\pi}{1 + a^2}$

11) The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$
is equal to :

(A) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

(B) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/2} + C$

(C) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right) + C$

(D) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{3} + C$

12) If y = y(x) is the solution curve of the differential equation $(x^2 - 4)\,dy - (y^2 - 3y)dx = 0$, $x > 2$, $y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :

(A) $\dfrac{3}{1+(8)^{1/4}}$

(B) $\dfrac{3}{1+2\sqrt{2}}$

(C) $\dfrac{3}{1-2\sqrt{2}}$

(D) $\dfrac{3}{1-(8)^{1/4}}$

13) For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if
$y(x) = \int \dfrac{\operatorname{cosec}x + \sin x}{\operatorname{cosec}x \sec x + \tan x \sin^2 x} dx$
$\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to :

(A) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(B) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(D) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

14) The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$ is :

(A) 12

(B) 14

(C) 18

(D) 21

15) If the shortest distance of the parabola
$y^2 = 4x$ from the centre of the circle
$x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

(A) 16

(B) 24

(C) 20

(D) 36

16) Let $a_1, a_2, \ldots\ldots a_{10}$ be 10 observations such that

$\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(A) 5
(B) $\sqrt{5}$
(C) 10
(D) $\sqrt{115}$

17) Let R be the interior region between the lines 3x – y + 1 = 0 and x + 2y – 5 = 0 containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is :

(A) $(-3,-1) \cup \left(-\frac{1}{3}, 1\right)$
(B) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$
(C) $(-3, 0) \cup \left(\frac{2}{3}, 1\right)$
(D) $(-3,-1) \cup \left(\frac{1}{3}, 1\right)$

18) Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b, which passes through the foci of the hyperbola. If $e_1e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through (0, 2) is :

(A) $4\sqrt{5}$
(B) $\frac{8\sqrt{5}}{3}$
(C) $\frac{10\sqrt{5}}{3}$
(D) $3\sqrt{5}$

19) If α, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of
4 cosθ + 5 sinθ = 0, then the value of tan α is :

(A) $\frac{10-\sqrt{10}}{6}$
(B) $\frac{10-\sqrt{10}}{12}$
(C) $\frac{\sqrt{10}-10}{12}$
(D) $\frac{\sqrt{10}-10}{6}$

20) The sum of the solutions $x \in \mathbb{R}$ of the equation $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is :

(A) 0

(B) 1

(C) –1

(D) 3

SECTION-II

1) For the series

$$s = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 \ldots\ldots,$$

if the of the first 10 terms is K, then $\frac{4K}{101}$ equal to

2) Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersects the x-axis, then the value of m + n is :

3) Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right) dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to.....

4) The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is :

5) The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	**B**	**B**	**C**	**A**	**A**	**D**	**A**	**B**	**C**	**B**	**C**	**A**	**B**	**A**	**B**	**C**	**C**	**A**	**D**	**A**

SECTION-II

Q.	21	22	23	24	25
A.	**3**	**75**	**2**	**25**	**3**

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	**B**	**D**	**A**	**C**	**A**	**C**	**B**	**B**	**D**	**C**	**B**	**C**	**A**	**C**	**A**	**A**	**C**	**B**	**A**	**C**

SECTION-II

Q.	46	47	48	49	50
A.	**3**	**2**	**3**	**1**	**45**

PART-3 : MATHEMATICS

SECTION-I

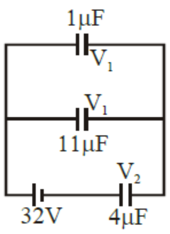
Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	**C**	**D**	**D**	**A**	**D**	**A**	**D**	**C**	**C**	**C**	**A**	**A**	**D**	**B**	**C**	**B**	**B**	**C**	**C**	**C**

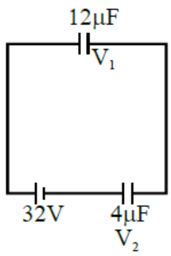
SECTION-II

Q.	71	72	73	74	75
A.	**5**	**119**	**2**	**5**	**2521**

SOLUTIONS

PART-1 : PHYSICS

1)




$V_2 = \frac{12}{12+4} \times 32$ ∴ $V_1 = 8V = 24$ V

∴ Charge at 3μf = CV = 1μF × 8V = 8μc

2) r = 2R, $5\mu T = \frac{\mu_0 I}{2\pi(2R)} \Rightarrow \frac{\mu_0 I}{\pi R} = 20\mu T$

At distance $\frac{R}{3}$ from suface means $\frac{2R}{3}$ from centre.

$$r = \frac{R}{3}, B = \frac{\mu_0 Ir}{2\pi R^2} = \frac{\mu_0 I\left(\frac{2R}{3}\right)}{2\pi R^2} = \frac{\mu_0 I}{3\pi R}$$

$$= \frac{1}{3} \times 20\mu T = \frac{20}{3}\mu T$$

3) Voltmeter is a galvanometer with high resistance in series. Ammeter is a galvanometer with small resistance in parallel

4) After long time L → Conductor

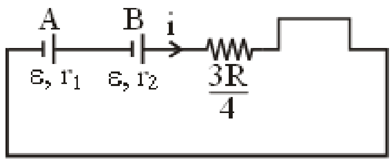
C → Blocks current

$V_A = 0$

$\varepsilon - ir_1 = 0$

$\varepsilon = ir_1$

$$\varepsilon = \frac{2\varepsilon r_1}{r_1 + r_2 + \frac{3R}{4}}$$



$r_1 + r_2 + \frac{3R}{4} = 2r_1$

$\frac{3R}{4} = r_1 - r_2$

$R = \frac{4}{3}(r_1 - r_2)$

5) As $\omega^2 = \dfrac{1}{LC}$ or $\omega = \dfrac{1}{\sqrt{LC}}$

Maximum energy stored in capacitor = $\dfrac{1}{2}\dfrac{Q_0^2}{C}$

Let at any instant t, the energy be stored equally between electric and magnetic field. The energy stored in electric field at instant t is $\dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}\left[\dfrac{1}{2}\dfrac{Q_0^2}{C}\right]$

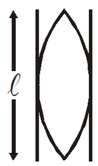
or $Q^2 = \dfrac{Q_0^2}{2}$ or $Q = \dfrac{Q_0}{\sqrt{2}}$

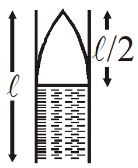
$\Rightarrow Q^0 \cos\omega t = \dfrac{Q_0}{\sqrt{2}}$

or $\omega t = \dfrac{\pi}{4}$

or $t = \dfrac{\pi}{4\omega} = \dfrac{\pi}{4 \times (1/\sqrt{LC}} = \dfrac{\pi\sqrt{LC}}{4}$

6) If we use that direction of light propagation will be along . Then (4) option is correct.
Detailed solution is as following.
Magnitude of E = CB
$E = 3 \times 10^8 \times 1.6 \times 10^{-6} \times \sqrt{5}$
$E = 4.8 \times 10^2 \sqrt{5}$
$\vec{E}$ and $\vec{B}$ are perpendicular to each other $\Rightarrow \vec{E}.\vec{B} = 0$
$\Rightarrow$ either direction of $\vec{E}$ is $\hat{i} - 2\hat{j}$ or $-\hat{i} + 2\hat{j}$
From given option
Also wave propagation direction is parallel to $\vec{E} \times \vec{B}$ which is $-\hat{k}$ $\Rightarrow \vec{E}$ is along $\left(-\hat{i} + 2\hat{j}\right)$

7) 



$\dfrac{\lambda}{2} = \ell$ $\qquad$ $\dfrac{\lambda}{4} = \dfrac{\lambda\ell}{2}$

$\boxed{\lambda = 2\ell}$ $\qquad$ $\boxed{\lambda = 2\ell}$

v = fλ $\qquad$ v = f'λ

$\boxed{f = \dfrac{v}{\lambda} = \dfrac{v}{2\ell}}$ $\qquad$ $\boxed{f' = \dfrac{v}{\lambda} = \dfrac{v}{2\ell} = f}$

$\boxed{f' = f}$

8) Path difference, $\Delta x = \dfrac{yd}{D}$

Here, $y = \dfrac{5\lambda}{2}$

and $D = 10d = 50\lambda$ (as $d = 5\lambda$)

So, $\Delta x = \left(\dfrac{5\lambda}{2}\right)\left(\dfrac{5\lambda}{50\lambda}\right) = \dfrac{\lambda}{4}$

Corresponding phase difference will be

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

or $\frac{\phi}{2} = \frac{\pi}{2}$

$$\therefore \quad I = I_0\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

9) $\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1\right) - \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

μ_o = R.I. of surrounding

$$\frac{f_W}{f_a} = \frac{\mu - 1}{\frac{\mu}{\mu_W} - 1} \; ; \; 3 = \frac{(\mu - 1)}{\left(\frac{3\mu}{4} - 1\right)}$$

$$\frac{9\mu}{4} - 3 = \mu - 1 \; ; \; \frac{5\mu}{4} = 2$$

$\mu = 1.6$

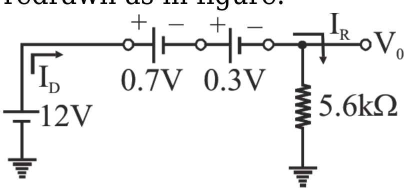
10)

Velocity at higher point = u $\sin\theta$

$$\lambda_D = \frac{h}{mu\sin\theta}$$

11) ${}_{90}X^{200} \rightarrow {}_{80}Y^{168} + a \times {}_{2}\alpha^{4} + b \times {}_{-1}\beta^{0}$

12) Direction of current ID is in the same direction as arrow in the diode symbol. Both the diodes are in "on" state; hence voltage drops across their terminal V_{T_1} and V_{T_2} are 0.7 V and 0.3 V respectively. The network function because E = 12 V > (0.7 V + 0.3 V). The circuit is redrawn as in figure.



$V_0 = E - V_{T1} - V_{T2} = 12 - 0.7 - 0.3 = 11V$

14) $Q = nC_v\Delta T$ as gas in closed vessel

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

Q = 10000 J = 10 kJ

17) By conservation of energy

$$\frac{-GMem}{Re} + \frac{1}{L}m(nVe)^2 = \frac{-GMem}{(Re+h)}$$

$$\frac{-\cancel{G}\cancel{M}e\,\cancel{m}}{Re} + \frac{1}{L}\cancel{m}n^2\frac{2GMe}{Re} = \frac{-\cancel{G}\cancel{M}e\,\cancel{m}}{Re+h}$$

$$\frac{1}{Re}\left[n^2-1\right]=\frac{-1}{Re+h}$$

$$Re + h = \frac{-Re}{n^2-1}$$

$$h = \frac{Re}{1-n^2} - Re$$

$$h = Re\left[\frac{n^2}{1-n^2}\right]$$

18) $\frac{dV}{dt} = -a\sqrt{v}$

$$\frac{dV}{v^{1/2}} = -adt$$

$$\int v^{-1/2}dv = \int -adt$$

$$2\sqrt{v} = -at + c$$

at $t = 0$ $v = V0$

$$2\sqrt{V_0} = c$$

 $2\sqrt{v} = -at + 2\sqrt{V_0}$

put $v = 0$

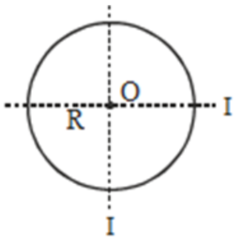
$$t = \frac{2\sqrt{V_0}}{a}$$

19) $F \sin\theta \geq f$ ...(1)

$f = \mu(mg - F\cos\theta)$ ...(2)

$mg\sin\theta \geq \mu mg(1-\cos\theta)$

$\cot\left(\frac{\theta}{2}\right) \geq \mu$



20) Given, $I = \frac{MR^2}{4}$ So, MI of disc about an axis passing through centre and perpendicular to the disc, is given by

$\rightarrow I_C = 2I = \frac{MR^2}{2}$ (from ⊥r axis theorem)

Now, to calculate MI about an axis normal to the disc and passing through a point on the edges, we use parallel axis theorem.

So, $I = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

21) Work done by electric field :

$w = PE_i - PE_f = q\,[V_i - V_f]$

$$= q\left[\frac{KP\cos 45^o}{(1\times10^{-2})^2} - \frac{KP\cos 135^o}{(2\times10^{-2})^2}\right]$$

work done = $\frac{100}{3}$ J

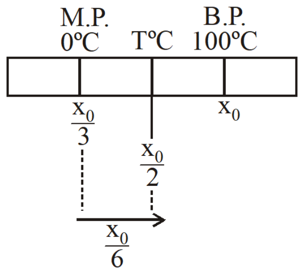
22) $d\left(1-\frac{1}{\mu}\right) = 30$

$d = \frac{30\times5}{2} = 75cm$

23)

$P_{atom} = P_{photon} = \frac{h}{\lambda}$

KE = $\frac{P_{atm}^2}{2m} = \frac{h^2}{2\lambda^2 m}$



24)

$\Rightarrow$ $TC = \frac{x_0}{6}$ & $\left(x_0 - \frac{x_0}{3}\right) = (100 - 0C)$

$x_0 = \frac{300}{2} \Rightarrow TC = \frac{150}{6} = 25C$

25) k = $\frac{1}{2}m\omega^2A^2\cos^2\omega t$

U = $\frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$

Hence ratio is 3 (most appropriate)

PART-2 : CHEMISTRY

26) $X \rightleftharpoons 2Y \qquad Z \rightleftharpoons P + Q$

$1 - \alpha_1 \quad 2\alpha_1 \quad 1 - \alpha_2 \quad \alpha_2 \quad \alpha_2$

$K_{P_1} = \frac{(2\alpha_1)^2}{(1-\alpha_1)} \times \left(\frac{P_1}{1+\alpha_1}\right)'$

$K_{P_2} = \frac{\alpha_2^2}{(1-\alpha_2)} \times \left(\frac{P_2}{1+\alpha_2}\right)'$

$$\frac{K_{P_1}}{K_{P_2}} = 1 = \frac{4\alpha_1^2}{(1-\alpha_1^2)}P_1 \Big/ \left(\frac{\alpha_2^2 P_2}{1-\alpha_2^2}\right)$$

$$1 = \frac{4P_1}{P_2}$$ ($\alpha_1 = \alpha_2$) Given

$$\frac{P_1}{P_2} = \frac{1}{4}$$

27) $A_1.e^{-E_{a_1}/RT} = A_2.e^{-E_{a_2}/RT}$

$$\frac{A_2}{A_1} = e^{\left(E_{a_2}-E_{a_1}\right)/RT}$$

$$10^2 = e^{\frac{1200}{RT}}$$

$$2\ln 10 = \frac{1200}{RT}$$

$$2 \times 2.303 = \frac{1200}{2 \times T}; \quad T = \frac{600}{4.606}K$$

29) vapour pressure expected by Raoult's law,

$P_s = P_A^o X_A + P_B^o X_B = 45 \times \frac{1}{3} + 36 \times \frac{2}{3}$ = 39 torr.

Thus observed vapour pressure (42 torr) is greater than expected value. Hence this solution shows positive deviation and therefore form minimum boiling azeotrope.

31) 0.05 mol XY_2 = 5 gm

1 mol XY_2 = $\frac{5}{0.05}$ = 100

X + 2Y = 100 ...(i)

3.01×10^{23} molecule X_2Y_3 = 85 gm

1 mol X_2Y_3 = N_A molecule = 170 gm

2X + 3Y = 170 ...(ii)

On solving,

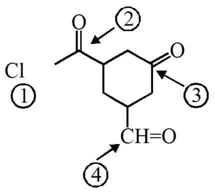
X = 40, Y = 30

35) Zn has no unpaired e^-

36) La^{+3} has f^0 & Lu^{+3} has f^{14} configuration so no possibility of f–f transition.

37) $Br_2 + 2F^- \rightarrow 2Br^- + F_2$

(this reaction is not possible)

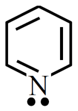
43) 

Consumed mole of R.MgX = 4

46) $[H^+] = C\alpha = 0.1 \times \frac{1}{100} = 10^{-3}$ M
$pH = -\log[H^+] = -\log 10^{-3} = 3$
Ions in aq. solution

47)

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$
$NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$
$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$
$KNO_3 \xrightarrow{\Delta} KNO_2 + O_2$
$Pb(NO_2)_2 \xrightarrow{\Delta} PbO + 2NO_2 + O_2$

49) 

50) $\underset{\text{Mol.wt.45g}}{CH_3CH_2NH_2} \xrightarrow{NaNO_2+HCl} \xrightarrow{H_2O} CH_3CH_2\text{–}OH + \underset{14g}{N_2}$
given : N_2 evolved is 2.24 L i.e. 0.1 mole.
i.e. $CH_3CH_2NH_2$ (ethyl amine) will be 4.5 g
(= 0.1 mole)
Hence the answer = 45×10^{-1} g

PART-3 : MATHEMATICS

52) ${}^7C_3 \times 1 \times (2^4 - 2)$

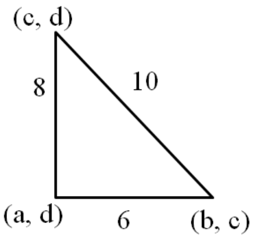
53)

3^a	1	3	7	9
7^b	7	3	1	9

Case-1 : $3^a \rightarrow$ end with 1 and
$7^b \rightarrow$ end with 7
Case-2 : $3^a \rightarrow$ end with 7 and
$7^b \rightarrow$ end with 1
Case-3 : $3^a \rightarrow$ end with 9 and
$7^b \rightarrow$ end with 9
$3 \times \left(\frac{25}{100} \times \frac{25}{100}\right) = \frac{3}{16}$

54)

Hence $a_{ij} = -a_{ij} \Rightarrow A^T = -A$ and $B^T = B$ and A, B are 3×3 matrices,
Hence $|A| = 0 \Rightarrow |A^4B^3| = 0 \Rightarrow A^4B^3$ is singular.

55) 

$\Delta = \frac{1}{2} \times 6 \times 8 = 1\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{vmatrix}$

$\frac{1}{2} \times 6 \times 8 = \frac{1}{2}|M|$; $|M| = 48$

$|N| = \left|\frac{M^2}{2}\right|$

$|N| = \frac{1}{23}|M|^2 = \frac{48 \times 48}{-8} = 288$

56) Here $w^5 = 1$; $\frac{1}{w} = w^4$

& also $1 + w + w^2 + w^4 = 0$

Now $\log_{\sqrt{2}}\left|1 + w + w^2 + w^3 - \frac{1}{w}\right|$

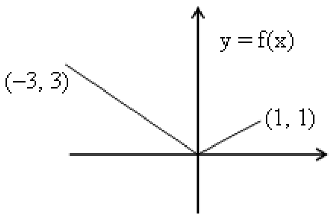
$= \log_{\sqrt{2}}\left|1 + w + w^2 + w^3 - w^4\right|$

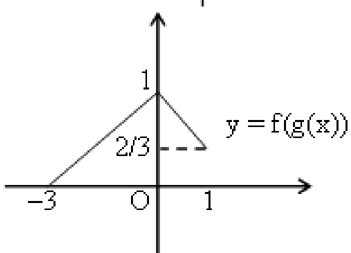
$= \log_{\sqrt{2}}\left|-2w^4\right| = \log_{\sqrt{2}}2 = \log_{\sqrt{2}}\left(\sqrt{2}\right)^2 = 2$

57) $f(g(x)) = \begin{cases} 2 + 2g(x) & , \quad -1 \le g(x) < 0 \quad .....(1) \\ 1 - \frac{g(x)}{3} & , \quad 0 \le g(x) \le 3 \quad .....(2) \end{cases}$

By (1) $x \in \phi$

And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$





Range of $f(g(x))$ is [0, 1]

58) Using L' hopital rule

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{0 - \cos x \times 3x^2}{2\left(x - \frac{\pi}{2}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\sin\left(x - \frac{\pi}{2}\right)}{2\left(x - \frac{\pi}{2}\right)} \times \frac{3\pi^2}{4} = \frac{3\pi^2}{8}$$

59) $f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$

$$= \lim_{h \to 0} \frac{(2^h + 2^{-h})\tan h \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{(7h^2 + 3h + 1)^3 h} = \sqrt{\pi}$$

60) $I = \int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2};\ 0 < a < 1$

$$I = \int_0^{\pi} \frac{dx}{1 + 2a\cos x + a^2}$$

$$2I = 2\int_0^{\pi/2} \frac{2(1 + a^2)}{(1 + a^2)^2 - 4a^2\cos^2 x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1 + a^2).\sec^2 x}{(1 + a^2)^2.\sec^2 x - 4a^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2.(1 + a^2).\sec^2 x}{(1 + a^2)^2.\tan^2 x + (1 - a^2)^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\frac{2.\sec^2 x}{1+a^2}.dx}{\tan^2 x + \left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$\Rightarrow I = \frac{2}{(1 - a^2)}\left[\frac{\pi}{2} - 0\right];\ I = \frac{\pi}{1 - a^2}$$

61) $I = \int \frac{x^8 - x^2}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$

Let $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$

$$\Rightarrow \frac{1}{1 + \left(x^3 + \frac{1}{x^3}\right)^2} . \left(3x^2 - \frac{3}{x^4}\right) dx = dt$$

$$\Rightarrow \frac{x^6}{x^{12} + 3x^6 + 1} . \frac{3x^6 - 3}{x^4} dx = dt$$

$$I = \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\ln|t| + C$$

$$I = \frac{1}{3}\ln\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right| + C$$

$$I = \ln\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|^{1/3} + C$$

Hence option (A) is correct

62) $(x^2 - 4)dy - (y^2 - 3y)dx = 0$

$$\Rightarrow \int \frac{dy}{y^2 - 3y} = \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3}\int \frac{y - (y-3)}{y(y-3)}dy = \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3}(\ln|y-3| - \ln|y|) = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

$$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

At $x = 4,\ y = \frac{3}{2}$; $\therefore C = \frac{1}{4}\ln 3$

$$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + \frac{1}{4}\ln(3)$$

At x = 10 ; $\frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{2}{3}\right| + \frac{1}{4}\ln(3)$

$\ln\left|\frac{y-3}{y}\right| = \ln 2^{3/4}$, $\forall x > 2, \frac{dy}{dx} < 0$

as $y(4) = \frac{3}{2} \Rightarrow y \in (0, 3)$ $-y + 3 = 8^{1/4}.y$

$$y = \frac{3}{1 + 8^{1/4}}$$

63) $y(x) = \int \frac{(1 + \sin^2 x)\cos x}{1 + \sin^4 x}dx$

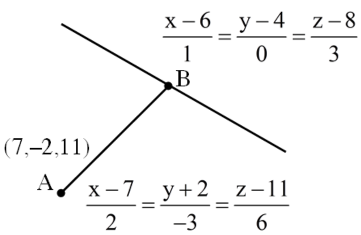
Put sinx = t

$$\int \frac{1 + t^2}{t^4 + 1}dt = \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(t - \frac{1}{t}\right)}{\sqrt{2}} + C$$

$x = \frac{\pi}{2}, t = 1$ $\therefore C = 0$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$$

64) B = (2λ +7, -3λ - 2, 6λ + 11)



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$$

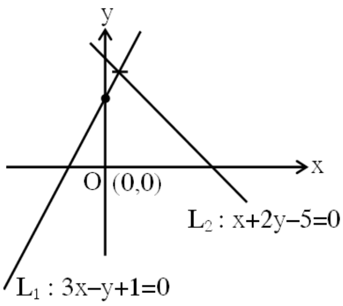
$-3\lambda - 6 = 0$

$\lambda = -2$
$B \Rightarrow (3, 4, -1)$
$$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$$
$$= \sqrt{16+36+144} = \sqrt{196} = 14$$

65) $y^2 = 4x$
$a = 1$
Point $p(t^2, 2t)$
centre of circle (25)
$z = d^2 = (t^2 - 2)^2 + (2t - 8)^2$
$\frac{dz}{dt} = 2(t^2 - 2).2t + 2(2t - 8).2$
After solving
$t = z$
$P(4, 4)$, centre $(2, 8)$
$d^2 = (4 - 2)^2 + (4 - 8)^2$
$= 4 + 16 = 20$

66) $\sum_{k=1}^{10} a_k = 50$
$a_1 + a_2 + \ldots + a_{10} = 50$ ....(i)
$\sum_{\forall k<j} a_k a_j = 1100$ ....(ii)
If $a_1 + a_2 + \ldots + a_{10} = 50.$
$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$
$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$
$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$
$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation 'σ'
$$= \sqrt{\frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$$
$$= \sqrt{30 - 25} = \sqrt{5}$$

67) $P(a^2, a + 1)$
$L_1 = 3x - y + 1 = 0$
Origin and P lies same side w.r.t. L_1
$\Rightarrow L_1(0) . L_1(P) > 0$
$\therefore 3(a^2) - (a + 1) + 1 > 0$



$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$ ....(1)

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0).L_2(P) > 0$

$\Rightarrow a^2 + 2(a+1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a+3)(a-1) < 0$

$\therefore a \in (-3, 1)$ ......(2)

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

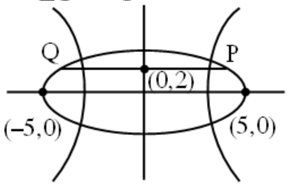
68) $H : \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad e_1 = \frac{5}{4}$

$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$

Also, ellipse is passing through $(\pm 5, 0)$

$\therefore$ a = 5 and b = 3

$E : \frac{x^2}{25} + \frac{y^2}{9} = 1$



End point of chord are $\left(\pm\frac{5\sqrt{5}}{3}, 2\right)$

 $L_{PQ} = \frac{10\sqrt{5}}{3}$

69) $4 + 5\tan\theta = \sec\theta$

Squaring : $24\tan^2\theta + 40\tan\theta + 15 = 0$

$\tan\theta = \frac{-10 \pm \sqrt{10}}{12}$

and $\tan\theta = -\left(\frac{10+\sqrt{10}}{12}\right)$ is Rejected.

70) $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

$\Rightarrow \dfrac{\cos 2x\,(3 + \cos^2 2x)}{\cos 2x\,(1 - \sin^2 x\cos^2 x)} = x^3 - x^2 + 6$

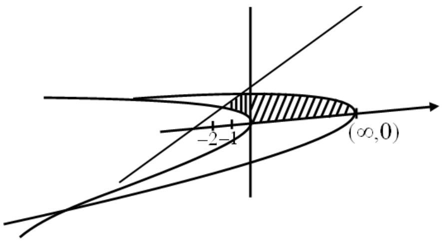
$\Rightarrow \dfrac{4(3 + \cos^2 2x)}{(4 - \sin^2 2x)} = x^3 - x^2 + 6$

$\Rightarrow \dfrac{4(3 + \cos^2 2x)}{(3 + \cos^2 2x)} = x^3 - x^2 + 6$

$x^3 - x^2 + 2 = 0 \Rightarrow (x + 1)(x^2 - 2x + 2) = 0$

so, sum of real solutions = –1

72)



$$A = \int_0^1 \left[\left(8 - 4y^2\right) - \left(-2y^2\right)\right] dy +$$

$$\int_1^{3/2} \left[\left(8 - 4y^2\right) - (2y - 4)\right] dy$$

$$= \left[8y - \frac{2y^3}{3}\right]_0^1 + \left[12y - y^2 - \frac{4y^3}{3}\right]_1^{3/2}$$

$= \dfrac{107}{12} = \dfrac{m}{n}$; □ m + n = 119

73) $f(x) = \displaystyle\int_0^x g(t)\ \ln\left(\frac{1 - t}{1 + t}\right) dt$

$$f(-x) = \int_0^{-x} g(t)\ln\left(\frac{1 - t}{1 + t}\right) dt$$

$$f(-x) = -\int_0^x g(-y)\ln\left(\frac{1 + y}{1 - y}\right) dy$$

$= -\displaystyle\int_0^x g(y)\ln\left(\frac{1 - y}{1 + y}\right) dy$ (g is odd)

$f(-x) = -f(x) \Rightarrow f$ is also odd

Now, $I = \displaystyle\int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2\cos x}{1 + e^x}\right) dx$ .....(1)

$$I = \int_{-\pi/2}^{\pi/2} \left(f(-x) + \frac{x^2 e^x \cos x}{1+e^x} \right) dx$$ .....(2)

$$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx = 2\int_0^{\pi/2} x^2 \cos x \, dx$$

$$I = \left(x^2 \sin x \right)_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x dx$$

$$= \frac{\pi^2}{4} - 2\left(-x\cos x + \int \cos x dx \right)_0^{\pi/2}$$

$$= \frac{\pi^2}{4} - 2(0+1) = \frac{\pi^2}{4} - 2 \Rightarrow \left(\frac{\pi}{2}\right)^2 - 2$$; $\therefore$ $\alpha = 2$

74) $$\cos\theta = \frac{\left(\alpha\hat{i} - 2\hat{j} + 2\hat{k}\right) \cdot \left(\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}\right)}{\sqrt{\alpha^2+4+4}\,\sqrt{\alpha^2+4\alpha^2+4}}$$

$$\cos\theta = \frac{\alpha^2 - 4\alpha - 4}{\sqrt{\alpha^2+8}\,\sqrt{5\alpha^2+4}}$$

$\Rightarrow \alpha^2 - 4\alpha - 4 > 0$

$\Rightarrow \alpha^2 - 4\alpha + 4 > 8 \quad \Rightarrow (\alpha - 2)^2 > 8$

$\Rightarrow \alpha - 2 > 2\sqrt{2}$ or $\alpha - 2 < -2\sqrt{2}$

$\alpha > 2 + 2\sqrt{2}$ or $\alpha < 2 - 2\sqrt{2}$

$\alpha \in (-\infty, -0.82) \cup (4.82, \infty)$

Least positive integral value of a $\Rightarrow 5$

75) Let the incorrect mean be μ' and standard deviation be σ'

We have $m' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$

As per given information correct

$\Sigma x_i = 180 - 10 + 12$

$\Rightarrow \mu$ (correct mean) $= \frac{182}{15}$

Also

$$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$$

Correct $\Sigma x_i^2 = 2295 - 100 + 144 = 2339$

σ^2 (correct variance) $= \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}$

Required value $= 15(\mu + \mu^2 + \sigma^2)$

$$= 15\left(\frac{182}{15} + \frac{182 \times 182}{15 \times 15} + \frac{2339}{15} - \frac{182 \times 182}{15 \times 15} \right)$$

$$= 15\left(\frac{182}{15} + \frac{2339}{15} \right) = 2521$$