Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Main)
FULL SYLLABUS
30-12-2024

JEE(Main) : Enthusiast Course (PHASE : I(A), I(B), I & II)

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	A	B	C	B	A	A	A	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	C	A	A	B	B	D	B	C
SECTION-II	Q.	1	2	3	4	5					
	A.	12	6	2	10	2					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	C	C	D	B	C	C	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	A	B	C	B	D	A	C	D
SECTION-II	Q.	1	2	3	4	5					
	A.	6	3	5	37	8					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	D	A	C	A	D	A	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	A	C	D	B	B	A	A	D
SECTION-II	Q.	1	2	3	4	5					
	A.	2022	0	0	1	2					

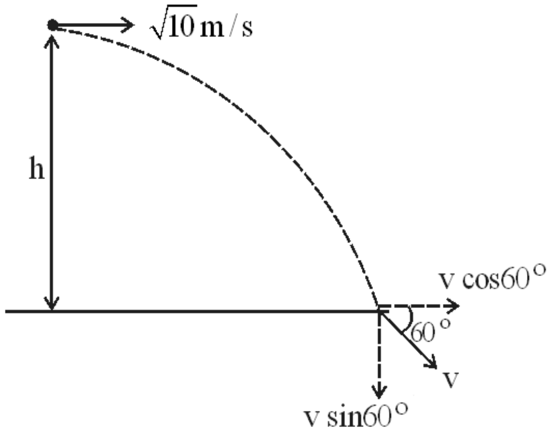
HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (B)**

Minimum angle of 60º is possible when projectile is thrown horizontally



$\therefore$ Horizontal velocity remains constant in projectile motion

$\therefore \sqrt{10} = V\cos 60°$

$\Rightarrow V = 2\sqrt{10}$

Also, vertical motion is free fall motion

$\therefore \sqrt{2gh} = V\sin 60$

$\Rightarrow \sqrt{20h} = 2\sqrt{10}\times\dfrac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{20h} = \sqrt{30}$

$\Rightarrow h = 1.5\text{ m}$

2. **Ans (A)**

For ball, $I = \dfrac{2}{5}mR^2$ (assuming solid sphere)

In case of pure rolling on incline,

$N = mg\cos\theta$

and $f = \dfrac{mg\sin\theta}{1+\frac{I}{mR^2}} = \dfrac{mg\sin\theta}{1+\frac{2}{5}} = \dfrac{5}{7}mg\sin\theta$

Net force applied on incline

$= \sqrt{N^2+f^2} = \sqrt{m^2g^2\cos^2\theta + \dfrac{25}{49}m^2g^2\sin^2\theta}$

$= mg\sqrt{\cos^2\theta+\sin^2\theta-\dfrac{24}{49}\sin^2\theta}$

$= mg\sqrt{1-\dfrac{24}{49}\sin^2\theta}$

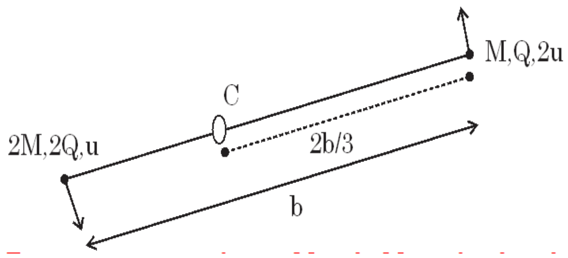
$\because$ Quantity inside square root is less than one.

$\therefore$ This net force is less than mg.

3. **Ans (B)**

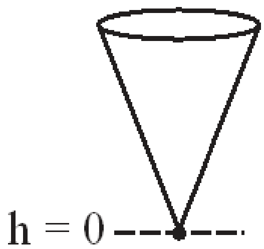
In CM frame

Use linear momentum conservation and angular momentum conservation about CM



4. **Ans (C)**

For hollow cone, COM is at $\dfrac{2h}{3}$ from vertex and for solid cone, COM is at $\dfrac{3h}{4}$ from vertex



$M_h = 3m$

$M_s = 7m - 3m = 4m$

$\therefore y_{COM} = \dfrac{M_hy_h + M_sy_s}{M_h+M_s}$

$= \dfrac{(3m)\left(\frac{2h}{3}\right)+(4m)\left(\frac{3h}{4}\right)}{7m}$

$= \dfrac{5Mh}{7m} = \dfrac{5h}{7}$

5. **Ans (B)**

$\overline{(\bar{A}B)+\bar{C}}$

$= \overline{\bar{A}\ B}.\bar{\bar{C}}$

$= \bar{A}\ \ B\ .\ C$

6. **Ans (A)**

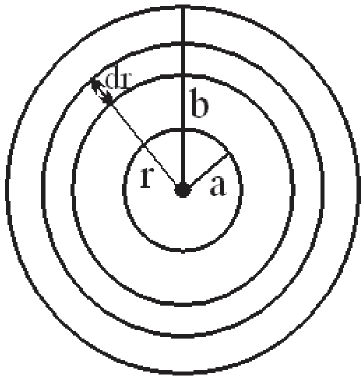
$\because$ Angular speed of entire wire and that of COM of wire will be same about centre of circular groove. Also, for any point of wire:

$v = R\omega \Rightarrow \omega = \dfrac{v}{R}$

$\therefore$ For COM, $\omega = \dfrac{v}{R}$

7. Ans (A)

$\int dQ = \int \sigma 2\pi r\, dr$



$Q = \int_a^b \frac{k}{r^3} 2\pi r\, dr$; $Q = 2\pi k \int_a^b \frac{dr}{r^2}$

$Q = 2\pi k \frac{(b-a)}{ab}$; $\int dV = \int \frac{kdQ}{r}$

$V = \int \frac{1}{4\pi\varepsilon_0} \frac{\sigma 2\pi r\, dr}{r} = \frac{k}{4\pi\varepsilon_0} 2\pi \int_a^b \frac{dr}{r^3}$

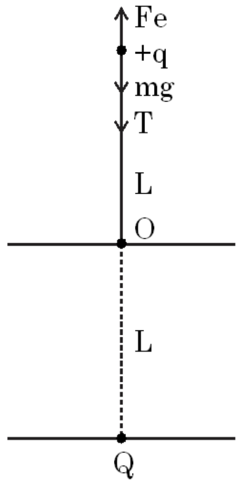
$= \frac{1}{4\pi\varepsilon_0} \frac{k2\pi}{2} \left(\frac{b^2-a^2}{a^2b^2}\right)$

$= \frac{1}{4\pi\varepsilon_0} \frac{2\pi k}{2} \frac{(b-a)}{ab} \frac{(b+a)}{ab}$

$= \frac{1}{8\pi\varepsilon_0} Q \left(\frac{a+b}{ab}\right)$

8. Ans (A)

In initial equilibrium position



$T + mg = Fe$

$2mg + mg = Fe$

$Fe = 3mg$ ....(1)

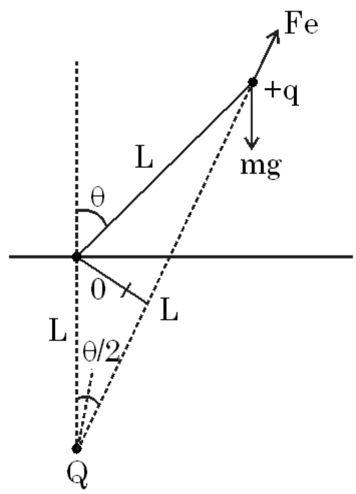
When slightly displaced

We will neglects slight change in Fe as distance b/w charges will charge.

$i_0 = I\alpha$

$mg.\, L \sin\theta - FeL \sin\frac{\theta}{2} = mL^2\alpha$

From (1) $Fe = 3mg$



$mgL\theta - 3mgL\frac{\theta}{2} = mL^2\alpha$

$-\frac{mg}{2} L\theta = mL^2\alpha$

$\alpha = -\left(\frac{g}{2L}\right)$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2L}{g}}$

9. Ans (D)

$R_1 = \dfrac{d_1}{\sigma_1 A},\ R_2 = \dfrac{d_2}{\sigma_2 A}$

$\therefore i = \dfrac{V}{\dfrac{d_1}{\sigma_1 A} + \dfrac{d_2}{\sigma_2 A}}$

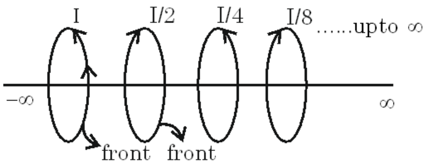
$\therefore\ j = \dfrac{i}{A} = \dfrac{V}{\dfrac{d_1}{\sigma_1} + \dfrac{d_2}{\sigma_2}}$

$j = \sigma E \quad \therefore \quad E = \dfrac{j}{\sigma}$

$\therefore\ E_A = \dfrac{j}{\sigma_1} = \dfrac{V}{d_1 + \dfrac{\sigma_1 d_2}{\sigma_2}}$

$= \dfrac{V\,\sigma_2}{\sigma_1 d_2 + \sigma_2 d_1}$

10. Ans (C)



Lets take a square loop of infinite length with one of the sides being the axis of the circular loops itself.

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left[-I + \dfrac{I}{2} - \dfrac{I}{4} + \dfrac{I}{8} - \ldots\right]$

But apart from the axial side, $\int \vec{B} \cdot d\vec{\ell}$ for all other sides will be zero as magnetic field along those sides is negligible.

$\therefore \displaystyle\int_{-\infty}^{\infty} \vec{B} \cdot d\vec{x} + 0 + 0 + 0 = -\mu_0 I \left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\right]$

$= -\mu_0 I \left[\dfrac{1}{1 - \left(\frac{-1}{2}\right)}\right] = \dfrac{-2\mu_0 I}{3}$

11. Ans (C)

Emf induced in the first quadrant equal to zero.

Emf induced in the second quadrant

$= \displaystyle\int_{x=0}^{x=R} B_0\,(dx)\,\omega x = \frac{B_0 \omega R^2}{2}$

Emf induced in the third quadrant $= \dfrac{-3B_0\omega R^2}{2}$

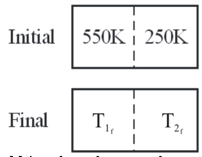
Emf induced in the fourth quadrant

$= \dfrac{7B_0\omega R^2}{2}$

$\therefore$ Avg. $= \dfrac{\frac{B_0\omega R^2}{2} - \frac{3B_0\omega R^2}{2} + \frac{7B_0\omega R^2}{2}}{4}$

$= \dfrac{5B_0\omega R^2}{8}$

12. Ans (A)



Work done by adiabatically expanding gas is equal and opposite to work done by adiabatically compressing gas.

$\therefore\ \dfrac{nR}{1-\gamma}\left(T_{1_f} - 550\right) = -\dfrac{nR}{1-\gamma}\left(T_{2_f} - 250\right)$

$\Rightarrow\ T_{i_f} + T_{2_f} = 800$ ....(i)

$\because\ PV^{\gamma} =$ constant for both sides

$\therefore\ P_{1_i}V_{1_i}^{\gamma} = P_{1_f}V_{1_f}^{\gamma}$ and $P_{2_i}V_{2_i}^{\gamma} = P_{2_f}V_{2_f}^{\gamma}$

Dividing above equations:

$\dfrac{P_{1_i}V_{1_i}^{\gamma}}{P_{2_i}V_{2_i}^{\gamma}} = \dfrac{P_{1_f}V_{1_f}^{\gamma}}{P_{2_f}V_{2_f}^{\gamma}} \Rightarrow \dfrac{P_{i_i}}{P_{2_i}} = \left(\dfrac{V_{i_f}}{V_{2_f}}\right)^{\gamma}$

$(\because\ V_{1_i} = V_{2_i}\ \&\ P_{1_f} = P_{2_f})$

$\Rightarrow\ \dfrac{nR\,T_{1_i}/V_{1_i}}{nR\,T_{2_i}/V_{2_i}} = \left(\dfrac{nRT_{1_f}/P_{1_f}}{nRT_{2_f}/P_{2_f}}\right)^{\gamma}$

$\Rightarrow\ \dfrac{T_{1_i}}{T_{2_i}} = \left(\dfrac{T_{1_f}}{T_{2_f}}\right)^{\gamma}$

$(\because\ V_{1_i} = V_{2_i}\ \&\ P_{1_f} = P_{2_f})$

$\Rightarrow\ \dfrac{T_{1_f}}{T_{2_f}} = \left(\dfrac{550}{250}\right)^{\frac{1}{(7/5)}}$ ...(ii)

Solving (i) & (ii),

$T_{1_f} = 510K$

13. Ans (C)

When switch is open, charge on inner surface of shell = −q.

Charge on outer surface = 2q − (−(q)

= 3q. (as total charge on shell = 2q)

When switch is closed, potential of the shell becomes zero.

V_{shell} = Work done against electric field in moving a unit charge from infinity to surface

= Work done against the field of 2q (at A), 2q (at B) and q' (on outer surface of the shell) from infinity to surface

Note that the external field will have no contribution from the charges inside the shell due to shielding effect.

$\therefore V_{shell}$ = work done against the field of 2q, 2q, & q' from infinity to centre of the sheel.

(as field inside shell due to these charges = 0)

= potential at centre due to 2q, 2q & q'.

$\Rightarrow\ 0 = \dfrac{k2q}{2r} + \dfrac{k2q}{2r} + \dfrac{kq'}{r}$

$\Rightarrow\ 0 = q + q + q' \quad \therefore\ q' = -2q$

$\therefore$Charge flow $= 3q - (q')$

$= 3q - (-2q) = 5q$

14. Ans (A)

$$T_{mixture} = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$$

Substituting the values

$T_{mixture} = 23.6C$

15. Ans (A)

Snell's Law at O

$\mu \sin\theta = 1 \sin\phi$

for small angle

$\sin\theta \simeq \tan\theta$

$\sin\phi \simeq \tan\phi$

$\mu \tan\theta = \tan\phi$

$$\mu\frac{h}{R+x} = \frac{h}{x}$$

$\mu x = R + x$

$$x = \frac{R}{\mu - 1}$$

16. Ans (B)

$${}^{A}_{Z}Y \rightarrow {}^{A}_{Z-1}X + {}^{0}_{+1}\beta + \nu$$

$Q = [(m_Y - zm_e) - (m_x - (z-1)m_e) - m_e]c^2$

$\Rightarrow Q = (m_Y - m_x - 2m_e)c^2$

For reaction to be feasible $Q > 0$

$\Rightarrow m_Y - m_x - 2m_e > 0$

$\Rightarrow m_Y - m_x > 2m_e$

17. Ans (B)

$N = N_s + N_u$ (1)

$$\frac{N}{3} = N_s + \frac{N_u}{x} \quad (2)$$

Divide equation (1) by equation (2)

$$3 = \frac{N_s + N_u}{N_s + \frac{N_u}{x}};\ 3 = \frac{N_s/N_u + 1}{N_s/N_u + \frac{1}{x}}$$

$$\text{Let } \frac{N_s}{N_u} = t;\ 3 = \frac{t+1}{t+\frac{1}{x}}$$

$$3t + \frac{3}{x} = t + 1$$

$$2t = 1 - \frac{3}{x} \Rightarrow \boxed{t = \frac{x-3}{2x}}$$

18. Ans (D)

In sample x no impurity level seen, so it is undoped. In sample y impurity energy level lies closer to the conduction bond so it is doped with fifth group impurity.

In sample z, impurity energy level lies closer to the valence band so it is doped with third group impurity.

19. Ans (B)

Susceptibility χ for a paramagnatic substance varies with absolute temperature as $\chi_1 = \frac{c}{T}$

$$\therefore \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$$

$$\therefore \chi_2 = \frac{300}{350} \times 2.5 \times 10^{-5}$$

$= 2.14 \times 10^{-5}$

$\therefore$Magnetisation at 350 k is

$I = \chi H = 2.14 \times 10^{-5} \times 2000\ Am^{-1}$

$= 4.28 \times 10^{-2} Am^{-1}$

20. Ans (C)

Using energy conservation

$$mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{5L}} = \sqrt{\frac{30}{5 \times 1.5}} = 2 \text{ rad/s}$$

Now, at final position, consider an elemental mass dm having width dx at a distance x from end A.

Linear momentum of this element:

$dP = (dm)\ v$

$$= \left(\frac{M}{L}dx\right)(x\omega) = \frac{M}{L}\omega\ xdx$$

$$\therefore P = \int dP = \frac{M}{L}\omega\int_0^L x\,dx$$

$$= \frac{M\omega}{L}\left(\frac{L^2}{2}\right) = \frac{M\omega L}{2}$$

$$= \frac{5 \times 2 \times 1.5}{2} = 7.5 \text{ kg m/s}$$

PART-1 : PHYSICS

SECTION-II

1. **Ans (12)**

$i = \epsilon_0 A \dfrac{dE}{dt}$

2. **Ans (6)**

Using Bernoulli's theorem at surface of water-gas boundary and just outside nozzle:

$P_{gas} = P_{atm} + \dfrac{1}{2}\rho v^2$

[Neglecting pressure due to height of water & also neglecting speed of water-gas boundary]

$\Rightarrow\ 4 \times 10^6 = 1 \times 10^6 + \dfrac{1}{2} \times 10^3 v^2$

$\Rightarrow\ v^2 = 6 \times 10^3$

$\therefore\ n = 6$

3. **Ans (2)**

$\tan 30^\circ = \dfrac{B_V}{B_H + B_R} = \dfrac{1}{\sqrt{3}}$

$\tan 60^\circ = \dfrac{B_V}{B_H - B_R} = \sqrt{3}$

Dividing, we get

$\dfrac{B_H - B_R}{B_H + B_R} = \dfrac{1}{3} \Rightarrow 3B_H - 3B_R = B_H + B_R$

$\Rightarrow\ 2B_H = 4B_R \quad \therefore\ B_R = \dfrac{B_H}{2}$

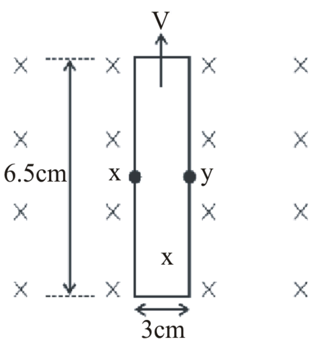
$50\dfrac{\mu_0 i}{2R} = \dfrac{B_H}{2}$

$\therefore\ R = \dfrac{\mu_0 i \times 50}{B_H} = \dfrac{(4\pi \times 10^{-7}) \times 3 \times 50}{4 \times 3 \times 10^{-5}}$

$= \pi \times 10^{-2} \times 50$ m

$k = 2$

4. **Ans (10)**

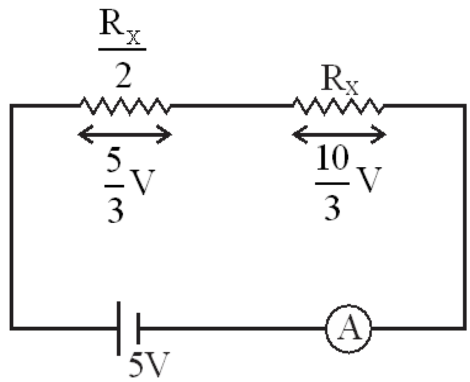


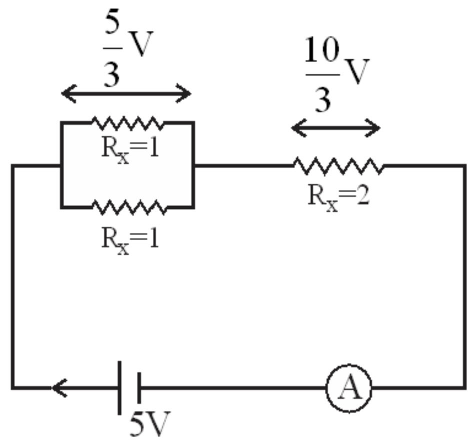
emf = Blv

$\Rightarrow 3.6 \times 10^{-6} = (1.2 \times 10^{-3})(3 \times 10^{-2})\, v$

$\Rightarrow v = 10$ cm/s

5. **Ans (2)**





$i = \dfrac{5}{2 + \frac{1}{2}} = 2A$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (B)**

Correct order of EA = Cl > F > Br > I

2. **Ans (D)**

Due to high polarising power of Be^{+2} , $BeCO_3$ is thermally less stable.

3. **Ans (C)**

4. **Ans (C)**

$PCl_3 + 3C_2H_5OH \xrightarrow{\Delta} 3C_2H_5Cl + H_3PO_3$

5. **Ans (D)**

$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

$CaO \xrightarrow{C,\ D} CaC_2 + CO$

$Fe + 5CO \rightarrow Fe(CO)_5$

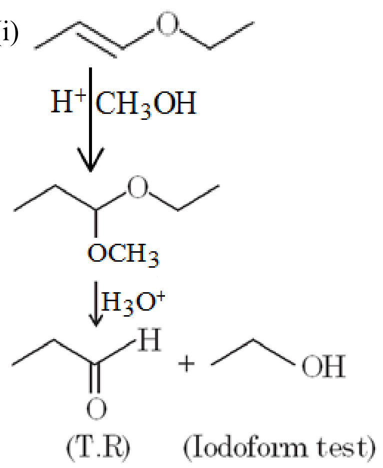
7. Ans (C)

Molarity of 10V H_2O_2 = 0.8 gm

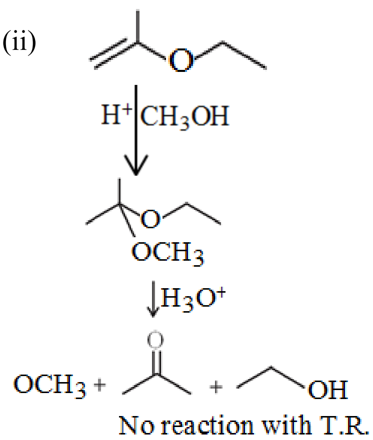
$\% \frac{w}{w} = \frac{0.89 \times 34}{1000} \times 100 = 3\%$

8. Ans (C)

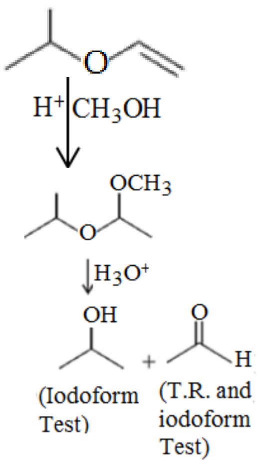
(i)

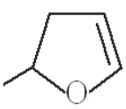


(ii)

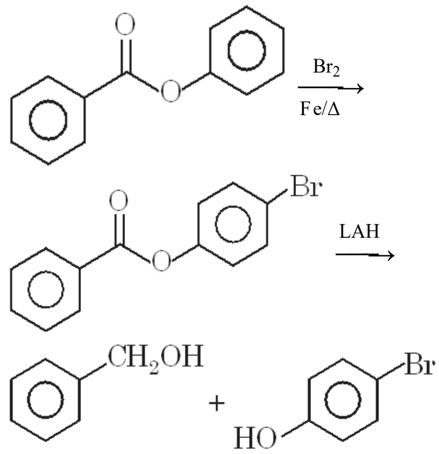


(iii)

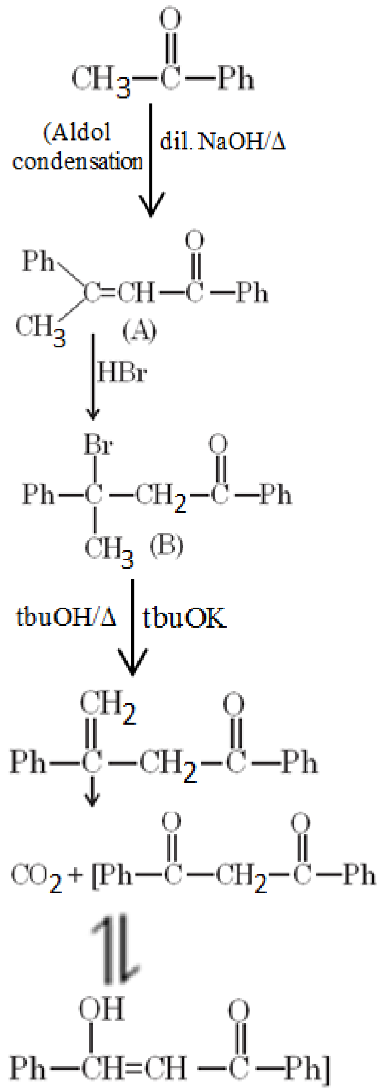


(iv)  has the formula C_5H_8O

9. Ans (A)



10. Ans (C)

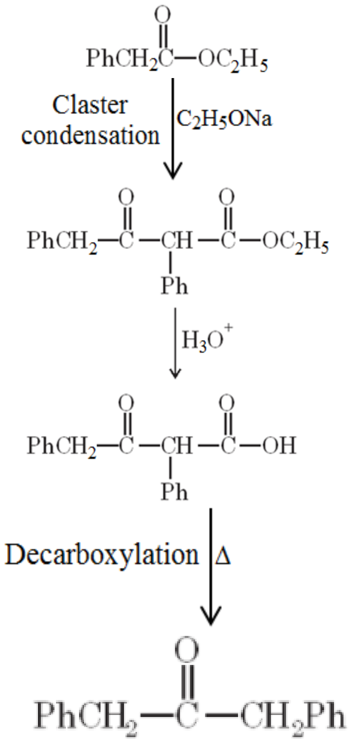


11. **Ans (B)**

$$PhNH_2 \xrightarrow[(0^0-5^0C)]{NaNO_2/HCl} PhN_2^+ \xrightarrow[HBr]{CuBr} PhBr$$

X must be Li or Mg

12. **Ans (C)**

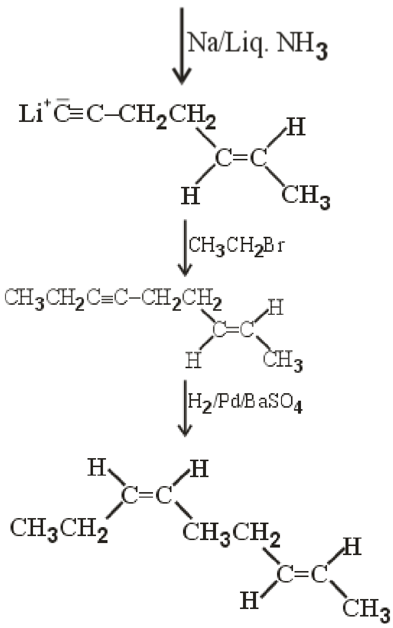


13. **Ans (A)**

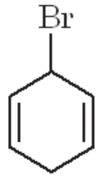
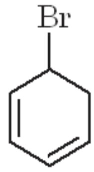
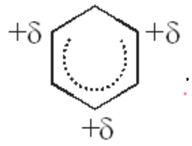
$CH\equiv C–CH_2CH_2C\equiv C–CH_3 + CH_3Li$

↓

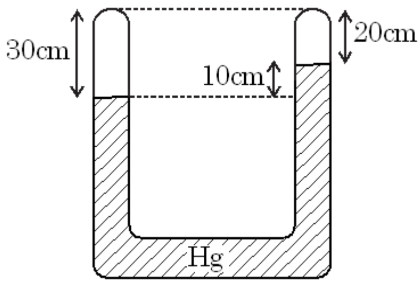
$Li^+\bar{C} \equiv C - CH_2CH_2C \equiv C - CH_3$



14. **Ans (B)**

The most stable intermediate carbocation is formed from both  and  that is . However (B) is less stable, and hence more reactive.

15. **Ans (C)**



For air : $P_1V_1 = P_2V_2$

$\Rightarrow 80 \times 25 = P_2 \times 20$

$\Rightarrow P_2 = 100$cm for Hg

For the left column :-

$P_{left} = P_{right} + 10$cm of Hg

$\Rightarrow P_{left} = 110$ cm of Hg

Also, as T is constant

$$\frac{P_1V_1}{n_1R} = \frac{P_1V_2}{n_2R}$$

Where,

n_1 = moles before dissociation

n_2 = moles after dissociation

$$\Rightarrow \frac{80 \times 25}{n_1} = \frac{110 \times 30}{n_2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{33}{20} = 1.65 (1)$$

For $NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2$

the n_1

eqn $n_1(1-\alpha)$ $\frac{n_1\alpha}{2}$ $\frac{3n_1\alpha}{2}$

$\therefore n_2 = n_1(1-\alpha) + \frac{n_1\alpha}{2} + \frac{3n_1\alpha}{2}$

$\Rightarrow n_2 = n_1(1+\alpha)$

$\Rightarrow \frac{n_2}{n_1} = 1 + \alpha = 1.65$

$\Rightarrow \alpha = 0.65$

16. **Ans (B)**

Since O_2 is added at constant partial pressure, the total pressure decreases and the total volume increases.

$2N_2O_5(g) \rightleftharpoons 4NO_2(g) + O_2(g)$

Let partial pressure at initial eql be $x \quad y \quad z$

$\therefore K_P = \frac{y^4 z}{x^2}$ ...(1)

Pressure after adding O_2, assuming that volume increases to 'a' times the initial volume $x/a \quad y/a \quad z$

$\therefore Q_P = \frac{(y/a)^4 z}{(x/a)^2} = \frac{y^4 z}{x^2} \times \frac{1}{a^2}$ ...(2)

$\therefore Q_P < K_P$ equilibrium shift fowards.

17. **Ans (D)**

$\Delta U = W$

$\Rightarrow \frac{P_2V_2 - P_1V_1}{\gamma - 1} = -P_{ext}\Delta V$

$\Rightarrow \frac{P_{ext}V_2 - 20}{5/3 - 1} = -P_{ext}(V_2 - 20)$

$\Rightarrow \frac{P_{ext}V_2 - 20}{2/3} = -P_{ext}V_2 + 20\,P_{ext}$

$\Rightarrow 3P_{ext}V_2 - 60 = -2P_{ext}V_2 + 40P_{ext}$

$\Rightarrow V_2 = \frac{60 + 40\,P_{ext}}{5\,P_{ext}} = \frac{12}{P_{ext}} + 8$

$V_{2_{min}} = 8L$ (when P_{ext} is max^m)

18. **Ans (A)**

$CaCO_3(s) \rightleftharpoons Ca^{2+}(aq.) + CO_3^{2-}(aq.)$

$K_{sp} = 10^{-9}$

$CO_3^{2-}(aq.) + H_2CO_3(aq.) \rightleftharpoons 2HCO_3^-(aq.)$

$K_c = \frac{K_1}{K_2} = 10^4$

$CaCO_3(s) + H_2CO_3\,(aq.) \rightleftharpoons$

$Ca^{2+}(aq.) + 2HCO_3^-\,(aq.)\ K = 10^{-5}$

in	0.4	–	–
eq^l	(0.4–x)	x	2x

$K = \frac{[Ca^{2+}][HCO_3^-]^2}{[H_2CO_3]}$

$\Rightarrow 10^{-5} = \frac{x.(2x)^2}{(0.4 - x)} \simeq \frac{4x^3}{0.4}$

$\Rightarrow x^3 = 10^{-6}$

$\Rightarrow x = 10^{-2}$ mol L^{-1}

$= 1$ g $CaCO_3$ per L of sol^n.

19. **Ans (C)**

(i) $\therefore -\frac{d[A]}{dt} = k$ (from the graph), it is a zero order reaction.

(ii) unit of rate constant for a zero order reaction is mol $L^{-1}s^{-1}$

(iii) Rate constant does not depend on concentration of the reaction

(iv) Zero order reaction is always a complex reaction.

20. **Ans (D)**

For an e^- in a Bohr's orbit,

$2\pi r = n\lambda$

$\Rightarrow 2\pi r_0 \frac{n^2}{z} = n\lambda$

$\Rightarrow \lambda = 2\pi r_0 \frac{n}{z}$

$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{n_2 z_1}{n_1 z_2}$

$\Rightarrow \frac{10}{1} = \frac{n_2 \times 2}{1 \times 1}$

$\Rightarrow n_2 = 5$

Upon de-excitation visible lines in Balmer series corresponds to the transitions

$5 \rightarrow 2$

$4 \rightarrow 2$

$3 \rightarrow 2$

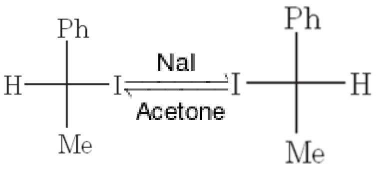
PART-2 : CHEMISTRY

SECTION-II

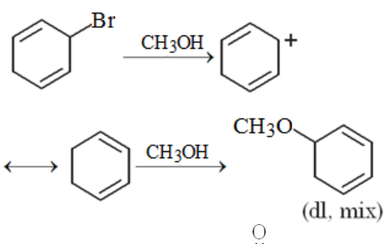
2. Ans (3)

(i) $(CH_3)_3C-Br \xrightarrow{H_2O} (CH_3)_3C-OH \rightarrow$ Achiral

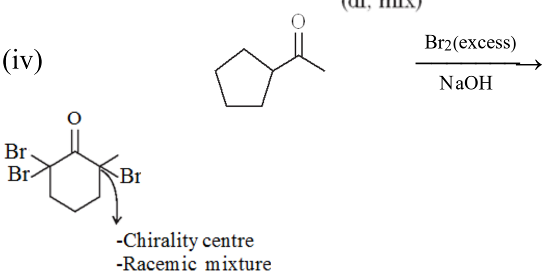
(ii)



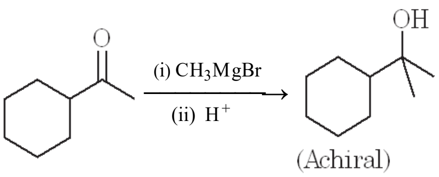
(iii)



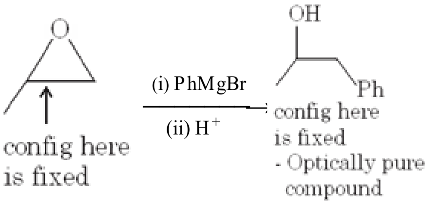
(iv)



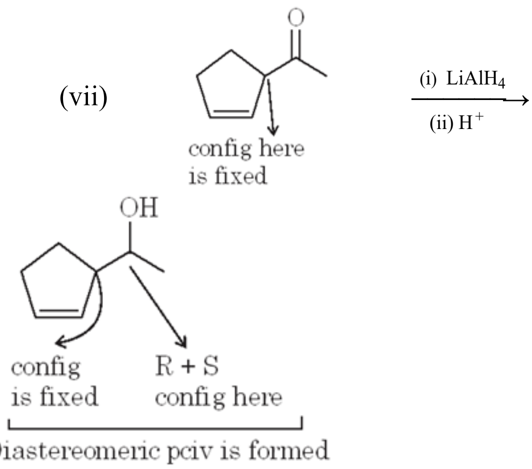
(v)



(vi)



(vii)



3. Ans (5)

(ii), (iv), (v), (vi) and (x)

4. Ans (37)

$$Ag^+(aq) + e^- \rightleftharpoons Ag(s)$$

$$E_{red} = E^0 - \frac{RT}{nF}\log\frac{1}{[Ag^+]}$$

$$\Rightarrow E_{red} = E^0 + \frac{RT}{F}\log[Ag^+]$$

$\therefore$ Slope of E_{red} vs $\log[Ag^+]$, $M = \dfrac{RT}{F}$

Given $\dfrac{RT}{F} = 0.06$ at 300K and 0.062 at the temperature of the solution.

$$\Rightarrow \frac{RT_1/F}{RT_2/F} = \frac{0.06}{0.062}$$

$$\Rightarrow T_2 = \frac{0.062}{0.06} \times T_1$$

$$= \frac{0.062}{0.06} \times 300 = 310K = 37^0C$$

5. Ans (8)

$$\frac{r_+}{r_-} = \frac{1.4}{2.6} = 0.538$$

$\Rightarrow$ C.N. = 6

The unit cell must be NaCl type

$\therefore$ Edge length $= 2(r_+ + r_-) = 8$ Å

PART-3 : MATHEMATICS

SECTION-I

1. Ans (B)

Let us assume that no. of cups with handle is n and without handle is m. No. of ways of selecting one cup each is ${}^mC_1 \, . \, {}^nC_1 = 36$, mn = 36

Now apply A.M. $\geqslant$ G.M.

$$\frac{m+n}{2} \geqslant \sqrt{mn}$$

$$\frac{m+n}{2} \geqslant 6$$

$$m + n \geqslant 12$$

So least value is '12'

2. Ans (C)

$(x + 10)(x - a) = -5$

Case (i) Either $x + 10 = 5,\ x - a = -1$

$x = 5,\ -5 - a = -1$

$a = -4$

or $x + 10 = -1,\ x - a = 5$

$x = -11,\ -11 - a = 5$

$a = -16$

Case (ii) $x + 10 = -5,\ x - a = 1$

or $x + 10 = 1,\ x - a = -5$ it gives same values of 'a'.

Sum of values of 'a' is $-16 - 4 = -20$

3. Ans (D)

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1;\ x, y, z > 0$

Let $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c} \Rightarrow a + b + c = 1$

Also $(x - 1)(y - 1)(z - 1)$

$= \left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right)$

$= \frac{(1-a)(1-b)(1-c)}{abc}$

$= \frac{(b+c)(a+c)(a+b)}{abc}$

Now by A.M. $\geqslant$ G.M.

$\frac{b+c}{2} \geqslant \sqrt{bc}$

$b + c \geqslant 2\sqrt{bc}$

$c + a \geqslant 2\sqrt{ac}$

$a + b \geqslant 2\sqrt{ab}$

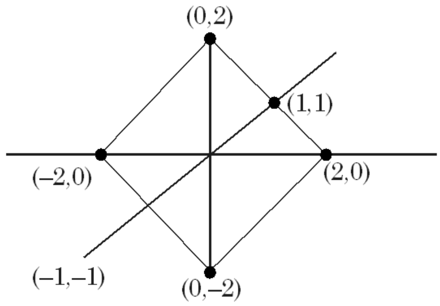
$(b+c)(c+a)(a+b) \geqslant 8abc$

$\frac{(b+c)(c+a)(a+b)}{abc} \geqslant 8$

4. Ans (A)

$y = x + \cos x$

$\frac{dy}{dx} = 1 - \sin x = 0 \Rightarrow \sin x = 1 \Rightarrow x = 2n\pi + \frac{\pi}{2}$



Putting value of x in given function

$y = x + \cos x$

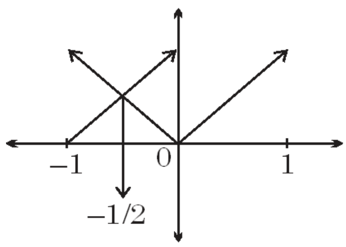
$y = 2n\pi + \frac{\pi}{2} + 0$

$y = 2n\pi + \frac{\pi}{2}$

Clearly point lie on the line $y = x$

Now x can be between -1 and 1 whereas least value of x is $\frac{\pi}{2}$, if $x > 0$ max value of x is $-\frac{\pi}{2}$ if $x < 0$. So no value of 'x'.

5. Ans (C)



Clearly, $f(x) = \begin{cases} -x, & -1 \leqslant x < -\frac{1}{2} \\ x + 1, & \frac{-1}{2} \leqslant x < 0 \\ x, & 0 \leqslant x \leqslant 1 \end{cases}$

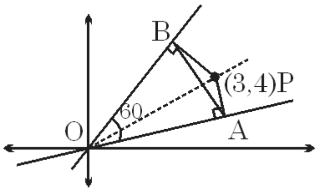
$\int_{-1}^{-1/2} -x.\,dx + \int_{-1/2}^{0} (x+1)\,dx + \int_{0}^{1} x.\,dx$

$-\frac{1}{2}[x^2]_{-1}^{-1/2} + \left[\frac{x^2}{2} + x\right]_{-1/2}^{0} + \left[\frac{x^2}{2}\right]_0^1$

$-\frac{1}{2}\left[\frac{1}{4} - 1\right] + \left[0 - \left(\frac{1}{8} - \frac{1}{2}\right)\right] + \frac{1}{2}$

$\frac{3}{8} + \left(\frac{3}{8}\right) + \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$

6. Ans (A)

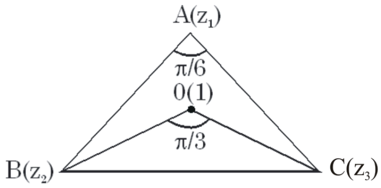


Apply sin rule in triangle OAB, $\frac{AB}{\sin 60^\circ} = OP$

$OP = \sqrt{3^2 + 4^2} = 5$

$AB = 5.\sin 60^\circ = \frac{5\sqrt{3}}{2}$

7. Ans (D)



z = 1 is circum center. So triangle OBC is equilateral

$z_2^2 + z_3^2 + 1 = z_2 + z_3 + z_2z_3$

Expression $z_2^2 - z_2 - z_2z_3 - z_3 + z_3^2 + 1 = 0$

$z_2(z_2 - 1) - z_3(z_2 + 1) + (z_3 + 1)(z_3 - 1) = -2$

So value is -2

8. Ans (A)

$1 + |\sin\theta| + |\cos\theta| - 1 - 2 = 0$

$\Rightarrow |\sin\theta| + |\cos\theta| = 2$ which is not possible.

9. Ans (A)

Divide throughout by x^3y, we get

$$2\frac{dx}{x} + 2\frac{dy}{y} - 2\frac{y^3}{x^3}dx + 3.\frac{y^2}{x^2}dy = 0$$

$$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + \frac{3xy^2dy - 2y^3dx}{x^3} = 0$$

$$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + \frac{3x^2y^2dy - 2xy^3dx}{x^4} = 0$$

$$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + d\left(\frac{y^3}{x^2}\right) = 0, \text{ integrate}$$

$$\Rightarrow 2(\ln x + \ln y) + \frac{y^3}{x^2} = C, \text{ Now put } x = y = 1$$

$$\Rightarrow 2\ln(1) + 1 = C \Rightarrow C = 1 \Rightarrow 2\ln(xy) + \frac{y^3}{x^2} = 1$$

$m = 3, n = 2 \Rightarrow m + n = 5$

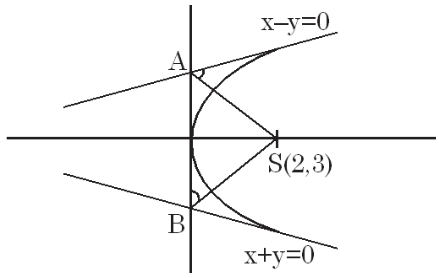
10. Ans (B)

Minimum value of $\sqrt{(x-1)^2 + (y-1)^2}$

Clearly minimum distance of circle from the point (1, 1)

$\sqrt{(4-1)^2 + (5-1)^2} = 5, 7 - 5 = 2$

11. Ans (A)



Equation of AS :

$(y - 3) = -1(x - 2)$

$y - 3 = -x + 2$

$x + y = 5$

For point 'A' solve

$x + y = 5$

$y = x$

$2x = 5$

$x = \frac{5}{2}, y = \frac{5}{2}$

$A\left(\frac{5}{2}, \frac{5}{2}\right)$

Equation of BS :

$(y - 3) = 1(x - 2)$

$y - 3 = x - 2$

$x - y = -1$

For point 'B' solve

$x - y = -1$

$x + y = 0$

$2x = -1$

$x = -\frac{1}{2}, y = \frac{1}{2}$

Now equation of line AB

$$y - \frac{1}{2} = \frac{\frac{5}{2} - \frac{1}{2}}{\frac{5}{2} + \frac{1}{2}}\left(x + \frac{1}{2}\right)$$

$$y - \frac{1}{2} = \frac{2}{3}\left(x + \frac{1}{2}\right)$$

$3y - \frac{3}{2} = 2x + 1$

$6y - 3 = 4x + 2$

$4x - 6y + 5 = 0$

Option 'A' is correct.

12. Ans (D)

S(6, 13) and S′(25, 8)

P(1, 1), S'P = 25, SP = 13

$S'P - SP = 12$

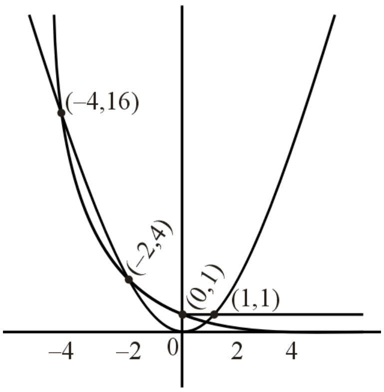
$2a = 12$

$S'S = \sqrt{(25-6)^2 + (8-13)^2}$

$= \sqrt{361 + 25} = \sqrt{386} = 2ae$

$e = \dfrac{\sqrt{386}}{12}$

13. Ans (A)



From graph clearly '4' points of non-differentiability.

14. Ans (C)

For x = 1 $0 = f(y) + f\left(\dfrac{1}{y}\right)$, interchanging x and y, we have $2f(y) = f(xy) + f\left(\dfrac{y}{x}\right)$ adding

$f(xy) = f(x) + f(y)$

Partial differentiation with respect to 'x'

$f'(xy).\,y = f'(x)$..................(1)

Partial differentiation with respect to 'y'

$f'(xy).\,x = f'(y)$................(2)

$\dfrac{y}{x} = \dfrac{f'(x)}{f'(1)}$ put y = 1

$\dfrac{1}{x} = \dfrac{f'(x)}{f'(1)}$

$\dfrac{1}{\ln 6} \times \dfrac{1}{x} = f'(x)$

Integrate with respect to 'x'

$f(x) = \dfrac{1}{\ln 6}.\ln x + C$ put x = 1

$f(1) = 0 + C = 0$

$f(x) = \dfrac{\ln x}{\ln 6}$

$f(7776) = \dfrac{\ln(7776)}{\ln 6} = 5$

15. Ans (D)

$((\vec{b} \times \vec{c}).\vec{a})\vec{c} - ((\vec{b} \times \vec{c}).\vec{c})\vec{a} = 3\vec{c}$

$[\vec{a}\ \vec{b}\ \vec{c}]\vec{c} = 3\vec{c}$

$[\vec{a}\ \vec{b}\ \vec{c}] = 3$

We know that $[\vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}\ \ \vec{a} \times \vec{b}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 = 9$

16. Ans (B)

There will be no effect due to addition of constant in variables.

17. Ans (B)

$\begin{bmatrix} 0 & \otimes \\ -\otimes & O \end{bmatrix}$ If matrix is of order 2 × 2, then $\otimes$ this place can be filled in 6 ways so 6 matrices can be formed. If matrix is of order 3 × 3, then

$\begin{bmatrix} 0 & \otimes & \ominus \\ -\otimes & 0 & \oplus \\ -\ominus & -\oplus & 0 \end{bmatrix}$

$\otimes$ this place can be filled in 6 ways

$\ominus$ this place can be filled in 4 ways

$\oplus$ this place can be filled in 2 ways

So total matrices 6 × 4 × 2 = 48

Now total matrices 48 + 6 = 54

18. Ans (A)

$\sum_{r=0}^{m} r.r! = ((r+1)-1)r! = (r+1)! - r!$

$\sum_{r=0}^{m} (r+1)! - r! = (m+1)! - 1$

$({}^mC_0)^2 + ({}^mC_1)^2 + ({}^mC_2)^2 + \ldots\ldots + ({}^mC_m)^2 = {}^{2m}C_m$

$\sum_{r=0}^{m} (2r-1) = m^2 - 1$

So Row 1 and Row 3 is same so value of determinant is zero.

19. Ans (A)

Putting (0, 0, 0) expression is – 1 and putting (1, 1, 1) expression is $2 + a^2 + ab - 1$

Both should have opposite sign so

$2 + a^2 + ab - 1 > 0$

$a^2 + ab + 1 > 0$

its discriminant is negative

$D = b^2 - 4 < 0 = -2 < b < 2$

No. of integral value of 'b' is $\{-1, 0, 1\}$

20. Ans (D)

Using AM $\geqslant$ GM

$$\frac{x+x+x+y+y}{5} \geqslant (x^3 . y^2)^{1/5}$$

$$\frac{3x+2y}{5} \geqslant (2^{15})^{1/5}$$

$(3x+2y)_{min} = 40$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (2022)

$x^4 - 4x^3 + 6x^2 - 4x + 1 = 2022$

$(x-1)^4 = 2022$

$(x-1)^2 = -\sqrt{2022}$ because only non real roots required

$x^2 + 1 - 2x + \sqrt{2022} = 0$

$x^2 - 2x + 1 + \sqrt{2022} = 0$

p = product of roots $= 1 + \sqrt{2022}$

2. Ans (0)

Equation of normal at $P(3\cos\theta, 2\sin\theta)$ is

$3x\sec\theta - 2y\,\mathrm{cosec}\theta = 5$

Now it is tangent to circle so

$$\frac{5}{\sqrt{9\sec^2\theta + 4\,\mathrm{cosec}^2\theta}} = \sqrt{3}$$

But $\{9\sec^2\theta + 4\,\mathrm{cosec}^2\theta\} \geqslant (3+2)^2 = 25$

No such θ exist

3. Ans (0)

$$\sin x . \sin\left(\frac{1}{x}\right) = 1$$

$$\sin\left(\frac{1}{x}\right) = \mathrm{cosec}\,x$$

Clearly $\sin\left(\frac{1}{x}\right) = \mathrm{cosec}\,x = 1$ or

$\sin\left(\frac{1}{x}\right) = \mathrm{cosec}\,x = -1$ which is not possible for same value of x.

4. Ans (1)

$x \in$ prime and $x < 10 \Rightarrow x = 2, 3, 5, 7$

Total no. of A(x, y) pair $n(A) = 4 \times 10 = 40$ and

$x^2 - 3y^2 = 1 \Rightarrow x^2 = 3y^2 + 1$.

For above condition to be satisfied only two such pairs (2, 1) and (7, 4) are possible

$$\Rightarrow P(A) = \frac{2}{40} = \frac{1}{20} = P$$

5. Ans (2)

$$\lim_{k\to\infty}\int_0^{k[x]} (\{kt\})^k = \lim_{k\to\infty}\int_0^{k^2[x].1/k} (\{kt\})^k$$

$$= \lim_{k\to\infty} k^2[x]\int_0^{1/k} (kt)^k . dt$$

$$= \lim_{k\to\infty} k^2[x] . k^k . \left[\frac{t^{k+1}}{k+1}\right]_0^{1/k}$$

$$= \lim_{k\to\infty} k^2[x] . k^k . \frac{1}{k^{k+1}(k+1)}$$

$$= [x]\lim_{k\to\infty}\frac{k}{k+1} = [x] \text{ or } \lambda = 2$$