

Rankers Academy JEE

(1001CJA101021240026)

Test Pattern



CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)

PART TEST

08-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

| | | | | | | | | | | | |
|------------|----|----|----|----|----|----|----|----|----|----|----|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | B | C | A | D | A | C | B | B | C | C |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | D | C | D | D | D | B | A | C | B | B |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | | | | | |
| | A. | 4 | 3 | 9 | 3 | 4 | | | | | |

PART-2 : CHEMISTRY

| | | | | | | | | | | | |
|------------|----|------|------|----|----|----|----|----|----|----|----|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | B | B | D | A | D | B | A | C | B | C |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | D | C | D | D | D | B | D | B | D | C |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | | | | | |
| | A. | 9480 | 1263 | 3 | 10 | 4 | | | | | |

PART-3 : MATHEMATICS

| | | | | | | | | | | | |
|------------|----|----|----|----|----|----|----|----|----|----|----|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | C | D | D | A | B | C | A | A | D | C |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | C | B | B | C | B | A | D | C | C | B |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | | | | | |
| | A. | 0 | 16 | 2 | 50 | 8 | | | | | |

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (B)

$$W_{\text{net}} = W_g + W_{\text{spring}}$$

$$= mg(h + d) + \frac{1}{2}k(0^2 - d^2)$$

$$W_{\text{net}} = mg(h + d) - \frac{1}{2}kd^2$$

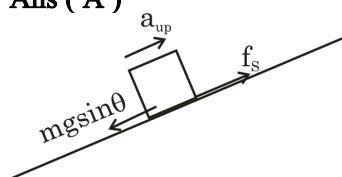
2. Ans (C)

$$\frac{1}{2}mv_0^2 + 0 = 4 + 0$$

$$\frac{1}{2}(0.5)v_0^2 = 4$$

$$v_0 = 4 \text{ m/s}$$

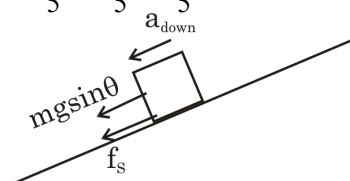
5. Ans (A)



$$a_{\text{up}} = \frac{f_s - mg \sin 37^\circ}{m}$$

$$(a_{\text{up}})_{\text{max}} = \frac{\mu mg \cos 37^\circ - mg \sin 37^\circ}{m}$$

$$= \frac{4g}{5} - \frac{3g}{5} = \frac{g}{5} = 2 \text{ m/s}^2$$



$$a_{\text{down}} = \frac{mg \sin \theta + f_s}{m}$$

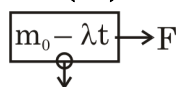
$$= g \sin \theta + \mu g \cos \theta$$

$$= \frac{3g}{5} + \frac{4g}{5} = \frac{7g}{5} = 14 \text{ m/s}^2$$

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HS-1/8

For More Material Join: @JEEAdvanced_2025

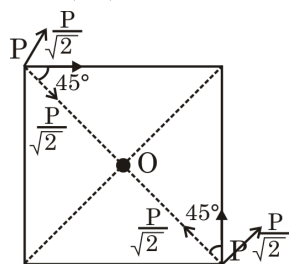
6. **Ans (C)**


$$\vec{F} = (m_0 - \lambda t) \frac{d\vec{v}}{dt}$$

 7. **Ans (B)**

$$v_f = 0, I = m(\sqrt{2g(2)} - 0) = 1\sqrt{40}\text{N} - S$$

$$I = \sqrt{40} \text{ N} - S$$

 8. **Ans (B)**


$$\text{for O :- } \frac{\frac{kp}{\sqrt{2}}}{\left(\frac{a}{\sqrt{2}}\right)^3} \times 2 = \frac{\frac{kp}{\sqrt{2}} \cdot 2}{\frac{a^3}{2\sqrt{2}}} = \frac{4kp}{a^3}$$

$$\text{for A :- } \sqrt{2} \frac{kp}{a^3}$$

$$\frac{E_0}{E_A} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

 10. **Ans (C)**

$$v_{\text{disk}} = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + r^2} - r)$$

$$v_{\text{quarter disk}} = \frac{\sigma}{8\epsilon_0} (\sqrt{R^2 + r^2} - r)$$

 11. **Ans (D)**

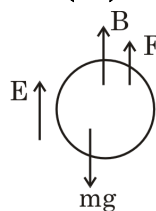
$$I = \int_0^{T_0} F dt = \int_0^{T_0} (600 - 2 \times 10^5 t) dt$$

$$\text{for } T_0, F = 0 \Rightarrow 600 = 2 \times 10^5 T_0 \Rightarrow T_0 = 3 \times 10^{-3} \text{ sec}$$

$$I = 600 T_0 - 2 \times 10^5 \frac{T_0^2}{2}$$

$$= 600 \times 3 \times 10^{-3} - 10^5 \times 9 \times 10^{-6}$$

$$= 9 \times 10^{-1} = 0.9 \text{ N-S}$$

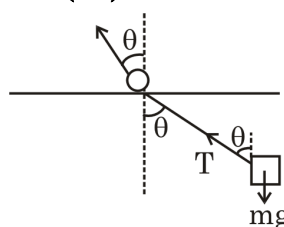
 12. **Ans (C)**


$$B + F = mg$$

$$\rho_0 \frac{4}{3} \pi R^3 g + qE = \rho \cdot \frac{4}{3} \pi R^3 g$$

$$qE = (\rho - \rho_0) \frac{4}{3} \pi R^3$$

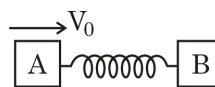
$$N = \frac{(\rho - \rho_0) \frac{4}{3} \pi R^3}{eE}$$

 13. **Ans (D)**


$$T = mg \cos \theta$$

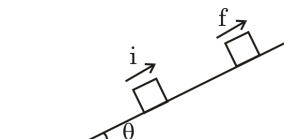
$$\tan \theta = \mu$$

$$T = mg \frac{1}{\sqrt{1 + \mu^2}}$$

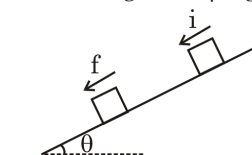
 14. **Ans (D)**


$$\frac{1}{2} \frac{2m}{3} \cdot V_0^2 = \frac{1}{2} kx_0^2$$

$$k = \frac{2mV_0^2}{3x_0^2}$$

 15. **Ans (D)**


$$\Delta KE = -mgsin\theta - \mu mg \cos\theta$$



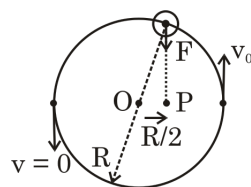
$$\Delta KE = mgsin\theta - \mu mg \cos\theta$$

16. **Ans (B)**

$$F = -\frac{dU}{dr} = -\frac{km}{r^2}$$

$$\int dv = km \int \frac{dr}{r^2}$$

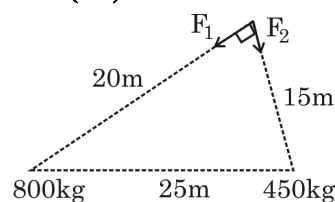
$$U = -\frac{km}{r} + C$$



$$-\frac{km}{3R/2} + 0 = \frac{1}{2}mv_0^2 - \frac{km}{R/2}$$

$$\frac{1}{2}mv_0^2 = 2\frac{km}{R} - \frac{2km}{3R} = \frac{4km}{3R}$$

$$v_0^2 = \frac{8k}{3R} \Rightarrow v_0 = \sqrt{\frac{8k}{3R}}$$

 19. **Ans (B)**


$$E_1 = -\frac{G(800)}{20^2} = -2G$$

$$E_2 = \frac{-G(450)}{15^2} = -2G$$

 20. **Ans (B)**

$$V = \frac{kQ}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

$$0 + \frac{kQq_1}{2R} \left(3 - \frac{1}{4} \right) = \frac{1}{2}mv^2 + \frac{kQq_1}{(2R)}$$

$$\frac{kQq_1}{2R} \left(\frac{11}{4} \right) = \frac{1}{2}mv^2 + \frac{kQq_1}{2R}$$

$$\frac{kQq_1}{R} \left(\frac{11}{8} - \frac{1}{2} \right) = \frac{1}{2}mv^2$$

$$\frac{7}{8} \cdot \frac{kQq_1}{R} = \frac{1}{2}mv^2$$

$$v = \frac{\sqrt{7}}{2} \sqrt{\frac{kQq_1}{mR}}$$

PART-1 : PHYSICS

SECTION-II

 1. **Ans (4)**

$$v = \sqrt{\frac{2GM}{R}} \Rightarrow v \propto \frac{1}{\sqrt{R}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}} \Rightarrow \frac{v}{10v} = \sqrt{\frac{R_2}{6400}}$$

$$R_2 = 64 \text{ km}$$

 2. **Ans (3)**

$$N_1 = mg + ma = 80.5g$$

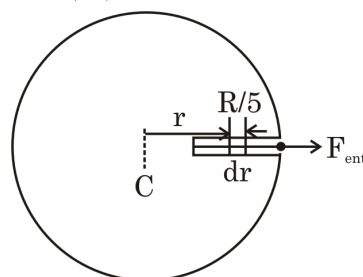
$$N_2 = mg - ma = 59.5g$$

$$-----$$

$$2mg = 140g \Rightarrow m = 70 \text{ kg}$$

$$70g + 70a = 80.5g \Rightarrow 70a = 10g$$

$$a = 1.5$$

 3. **Ans (9)**


$$dF = \frac{\rho r}{3} (4\pi G) \lambda dr$$

$$= \frac{\rho \lambda}{3} 4\pi G r dr$$

$$F_{\text{net}} = \int dF$$

$$= \frac{\rho \lambda}{3} 4\pi G \int_{4R/5}^R r dr$$

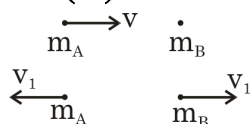
$$= \frac{\rho \lambda}{3} 4\pi G \cdot \frac{\left(R^2 - \frac{16R^2}{25} \right)}{2}$$

$$F_{\text{net}} = \frac{\lambda}{3} 4\pi G \frac{GR^2}{50} \cdot \frac{M}{\frac{4}{3}\pi R^3}$$

$$= \frac{9}{50} \left(\frac{GM}{R} \right) \lambda = \frac{9}{50} \left(\frac{GM}{R^2} \right) R \lambda$$

$$= \frac{9}{50} \times 1 \times 6 \times 10^5 \times 10^{-3} = 108 \text{ N}$$

4. Ans (3)



$$m_A v = m_B v_1 - m_A v_1$$

$$m_A v = (m_B - m_A) v_1$$

$$e = 1 = \frac{v_1 + v_1}{v} = \frac{2v_1}{v} \Rightarrow \frac{v_1}{v} = \frac{1}{2}$$

$$\frac{m_A}{m_B - m_A} = \frac{1}{2} \Rightarrow 2m_A = m_B - m_A$$

$$m_A = \frac{m_B}{3}$$

$$\frac{m_B}{m_A} = \frac{3}{1}$$

5. Ans (4)

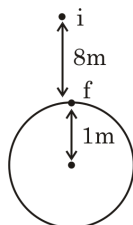
$$mg(8) + \frac{kq^2}{9} = \frac{kq^2}{1} + 0$$

$$mg(8) = kq^2 \cdot \frac{8}{9}$$

$$q^2 = \frac{9mg}{k}$$

$$q^2 = \frac{9 \times 80 \times 10^{-3} \times 9.8}{9 \times 10^9}$$

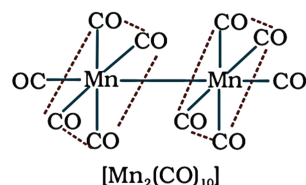
$$q = 28 \mu C$$



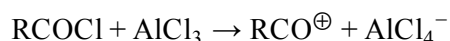
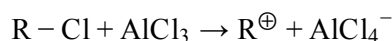
PART-2 : CHEMISTRY

SECTION-I

1. Ans (B)



2. Ans (B)



Here both R^{\oplus} and RCO^{\oplus} are acting as electrophile.

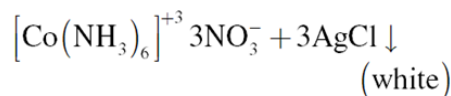
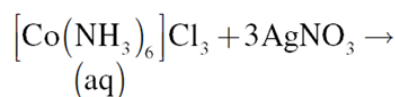
3. Ans (D)

Ti⁺³ configuration : 3d¹

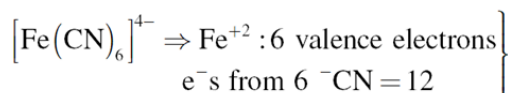
t_{2g} : ground state of complex, e_g : next higher state

∴ e⁻ excites from t_{2g} to e_g

8. Ans (C)



9. Ans (B)



Total 18 valence electrons

10. Ans (C)

set : 1 = d_{xy}, d_{yz}, d_{zx} : non - axial

set : 2 = d_{z²}, d_{x²-y²} : axial

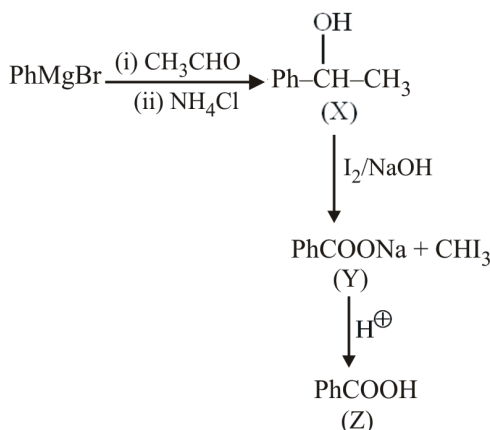
11. Ans (D)

The M-C π bond is formed by the donation of a pair electrons from a filled d orbital of metal into the vacant antibonding π* of carbon monoxide.

18. Ans (B)

C₂H₆ has neither lone pair nor π-bond.

19. Ans (D)



20. Ans (C)

X = Polar ⇒ cis - [Pt (gly)₂]

Y = Non-polar ⇒ trans - [Pt (gly)₂]

PART-2 : CHEMISTRY

SECTION-II

1. Ans (9480)

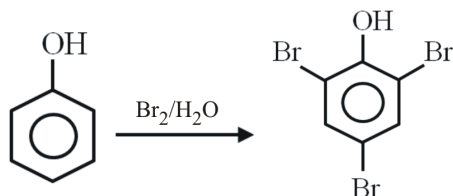
Δ₀ < P ⇒ so no forced pairing.

$$\text{CFSE} = 3 \times (-0.4\Delta_0) + 1 \times (+0.6\Delta_0)$$

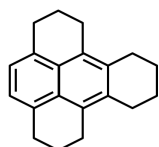
$$= -1.2\Delta_0 + 0.6\Delta_0 = -9480 \text{ cm}^{-1}$$

2. Ans (1263)

Moles of $\text{Cl}^- = 3 \times 0.0012 = 0.0036 = \text{moles of } \text{H}^+$
 moles of $\text{H}^+ = \text{moles of } ^-\text{OH}$
 $0.0036 = 28.5 \times 10^{-3} \times x$
 $x = 0.1263$

3. Ans (3)

4. Ans (10)

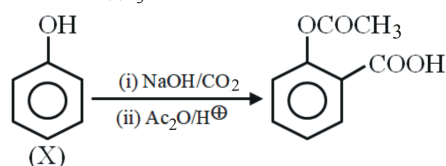
Formed product is



Hence its degree of unsaturation is 10.

5. Ans (4)

Cumene $\xrightarrow[\text{(ii) } \text{H}_3\text{O}^+]{\text{(i) } \text{O}_2(\text{Air})/\Delta}$



PART-3 : MATHEMATICS

SECTION-I

1. Ans (C)

Normal at (x_1, y_1) be $xx_1 - yy_1 = x_1^2 - y_1^2$

Pass's through $(3, 2)$

$$3x_1 - 2y_1 = x_1^2 - y_1^2 \quad \dots(i)$$

$$\text{Also } x_1y_1 = 4 \quad \dots(ii)$$

$$\text{So, } 3x_1 - \frac{8}{x_1} = x_1^2 - \frac{16}{x_1^2}$$

$$3x_1^3 - 8x_1 = x_1^4 - 16$$

$$\Rightarrow x_1^4 - 3x_1^3 + 8x_1 - 16 = 0$$

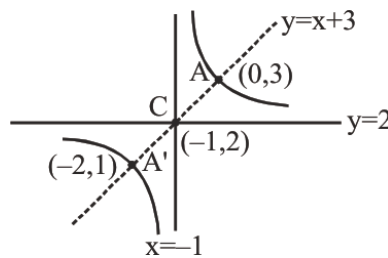
$$\sum x_i = 3 \quad \prod x_i = -16$$

$$\text{So, } \frac{\sum x_i}{\prod x_i} = \frac{-3}{16}$$

2. Ans (D)

$$y = 2 + \frac{1}{x+1}$$

$$\Rightarrow (y-2)(x+1) = 1$$



$$a = \sqrt{2}$$

$$\text{Length of L.R.} = 2\sqrt{2}$$

3. Ans (D)

$$\text{Let circle } S \equiv x^2 + y^2 - a^2 = 0$$

$$\text{and given hyperbola } xy = c^2$$

$$\text{Then, } x^2 + \frac{c^4}{x^2} = a^2 \quad \text{or} \quad x^4 - a^2x^2 + c^4 = 0$$

If four intersecting points are

$$\left(ct_1, \frac{c}{t_1}\right), \left(ct_2, \frac{c}{t_2}\right), \left(ct_3, \frac{c}{t_3}\right) \text{ and}$$

$$\left(ct_4, \frac{c}{t_4}\right), \text{ then}$$

$$(ct_1)(ct_2)(ct_3)(ct_4) = c^4$$

$$\therefore t_1 t_2 t_3 t_4 = 1$$

\therefore Reason is true.

For the point $(2, 2)$; $t_1 = 1$

For the point $(4, 1)$; $t_2 = 2$

For the point $(6, 2/3)$; $t_3 = 3$

For the point $\left(\frac{1}{4}, 16\right)$; $t_4 = \frac{1}{8}$

$$\text{Now, } t_1 t_2 t_3 t_4 = \frac{3}{4} \neq 1$$

\therefore Assertion is false.

4. Ans (A)

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{24}{49}} = \frac{5}{7}$$

$$\text{foci} = (\pm ae, 0) = (\pm 5, 0)$$

$$A \times \frac{5}{\sqrt{2}} = 5 \Rightarrow A = \sqrt{2} \text{ \& } B^2 = A^2(e^2 - 1) = 23$$

5. Ans (B)

Since each line of family of line passes through the focus of given parabola hence shortest intercept is L.R.

6. Ans (C)

minimum value of E is the shortest distance between the parabola $y^2 = 4x$ and circle

$$(x+1)^2 + (y-14)^2 = 9.$$

Thus normal to the parabola $y^2 = 4x$, be

$y = mx - 2m - m^3$ passes through $(-1, 14)$ gives $m^3 + 3m + 14 = 0$, Thus only one real value $m = -2$

Hence corresponding point on the parabola is $(4, 4)$

Thus required minimum distance is

$$\sqrt{5^2 + 10^2} - 3 = 5\sqrt{5} - 3$$

7. Ans (A)

$$x^2 + y^2 + Ax + By + C = 0$$

is passing through $(0,0)$

$$C = 0$$

The tangent of the parabola $y = x^2$ at $(2, 4)$ is

$$4x - y - 4 = 0 \quad \dots(1)$$

The tangent of circle

$$x^2 + y^2 + Ax + By + C = 0 \text{ at } (2, 4) \text{ is}$$

$$(4 + A)x + (8 + B)y + 2A + 4B = 0 \quad \dots(2)$$

From Equation (1) and (2)

$$\frac{4 + A}{4} = \frac{8 + B}{-1} = \frac{2A + 4B}{-4}$$

$$A + 4B = -36 \quad \dots(3)$$

$$3A + 4B + 2C = -4 \quad \dots(4)$$

Circle passes through $(2, 4)$

$$A + 2B = -10$$

From (3), (4) and (5)

$$A = 16$$

$$B = -13$$

8. Ans (A)

$x = 1$ is a root of the equation

$$D = 0$$

$$\Rightarrow \frac{-b}{2a} = 1 \Rightarrow \frac{r - q}{2p - 2q} = 1$$

$$r - q = 2p - 2q$$

$$2p - q - r = 0$$

$$-2p + q + r = 0$$

$$px + qy + r = 0$$

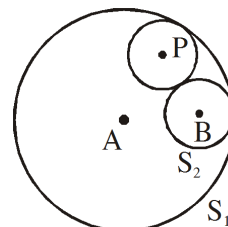
always passes through $(-2, 1)$

10. Ans (C)

$$S_1 : x^2 + y^2 = 9 \quad \begin{cases} r_1 = 3 \\ A(0, 0) \end{cases}$$

$$S_2 : (x - 2)^2 + y^2 = 1 \quad \begin{cases} r_2 = 1 \\ B(2, 0) \end{cases}$$

$$\therefore c_1 c_2 = r_1 - r_2$$



\therefore given circle are touching internally

Let a variable circle with centre P and radius r

$$\Rightarrow PA = r_1 - r \text{ and } PB = r_2 + r$$

$$\Rightarrow PA + PB = r_1 + r_2$$

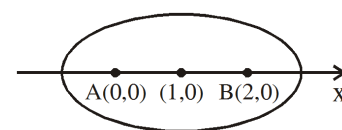
$$\Rightarrow PA + PB = 4 (> AB)$$

\Rightarrow Locus of P is an ellipse with foci at $A(0, 0)$ and

$B(2, 0)$ and length of major axis is $2a = 4$, $e = \frac{1}{2}$

$$\Rightarrow \text{centre is at } (1, 0) \text{ and } b^2 = a^2(1 - e^2) = 3$$

if x-ellipse



$$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

$$\text{which is satisfied by } \left(2, \pm \frac{3}{2}\right)$$

11. Ans (C)

$$\frac{3}{2a^2} + \frac{1}{b^2} = 1 \text{ and } 1 - \frac{b^2}{a^2} = \frac{1}{3}$$

$$\Rightarrow a^2 = 3, b^2 = 2$$

$$\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1 \quad \dots(i)$$

Its focus is $(1, 0)$

Now, eqn of circle is

$$(x-1)^2 + y^2 = \frac{4}{3} \quad \dots(ii)$$

Solving (i) and (ii) we get

$$y = \pm \frac{2}{\sqrt{3}}, x = 1$$

$$\Rightarrow PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

12. Ans (B)

equation of director circle

$$x^2 + y^2 = a^2 + b^2$$

$$x^2 + y^2 = 25 + 16$$

$$x^2 + y^2 = 41$$

13. Ans (B)

$$S_1 < 0$$

$$\frac{\left(7 - \frac{5h}{4}\right)^2}{25} + \frac{(-h)^2}{16} - 1 < 0$$

$$\Rightarrow 25h^2 - 140h + 192 < 0$$

$$\Rightarrow (5h - 12)(5h - 16) < 0$$

$$\Rightarrow \frac{12}{5} < h < \frac{16}{5}$$

∴ Integral value of h is 3

14. Ans (C)

$$\cos 5^\circ \cos 55^\circ \cos 65^\circ = \frac{1}{4} \cos 15^\circ$$

$$= \frac{1}{4} \cdot \frac{\sqrt{3} + 1}{2\sqrt{2}} = \frac{\sqrt{3} + 1}{8\sqrt{2}}$$

15. Ans (B)

BC

16. Ans (A)

$$\cot A + \cot B = p \quad \cot A \cot B = q$$

$$\frac{\sin(A+B)}{\sin A \sin B} = p \quad q - 1 = \frac{\cos(A+B)}{\sin A \sin B}$$

$$p^2 + (q-1)^2 = \frac{\sin^2(A+B)}{\sin^2 A \sin^2 B} + \frac{\cos^2(A+B)}{\sin^2 A \sin^2 B}$$

$$p^2 + (q-1)^2 = \frac{1}{\sin^2 A \sin^2 B}$$

$$(q-1)^2 = \frac{\cos^2(A+B)}{\sin^2 A \sin^2 B}$$

$$\cos^2(A+B) = \frac{(q-1)^2}{p^2 + (q-1)^2}$$

17. Ans (D)

$$S_k = \frac{k}{2} (2 \times 1 + (k-1) \times 1)$$

$$S_k = \frac{k}{2} (k+1)$$

$$\sum_{k=1}^{100} \frac{2}{k(k+1)} = \sum_{k=1}^{100} \left(\frac{2}{k} - \frac{2}{k+1} \right)$$

$$= 2 - \frac{2}{101} = \frac{200}{101}$$

$$S = \frac{200}{101} - 1$$

18. Ans (C)

$$\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \infty \right)$$

$$- \left[\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots \infty \right]$$

$$= \omega - \frac{1}{2^2} \left\{ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right\}$$

$$\omega - \frac{1}{4} \omega = \frac{3\omega}{4}$$

19. Ans (C)

$$2 \text{ (required sum)} = (1 - 1 + 2 - 2 + \dots + n - n)^2 -$$

$$(1^2 + 1^2 + 2^2 + 2^2 + \dots + n^2 + n^2)$$

20. Ans (B)

$$\text{We have, } 2b^2 - a^2 > a^2 + b^2 \Rightarrow b^2 > 2a^2$$

$$\Rightarrow \left| \frac{b}{a} \right| > \sqrt{2}$$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (0)

Intersection point of tangent at vertex and tangent is $(-1, 1)$

Now equation of line which is \perp from given tangent and passing through $(-1, 1)$ is

$$y - 1 = -\frac{1}{2}(x + 1) \Rightarrow 2y - 2 = -x - 1$$

$$x + 2y = 1$$

this line $x + 2y = 1$ and equation of axis satisfied the focus

$$\therefore x + 2y = 1 \text{ and } x = y \Rightarrow x = \frac{1}{3}, y = \frac{1}{3}$$

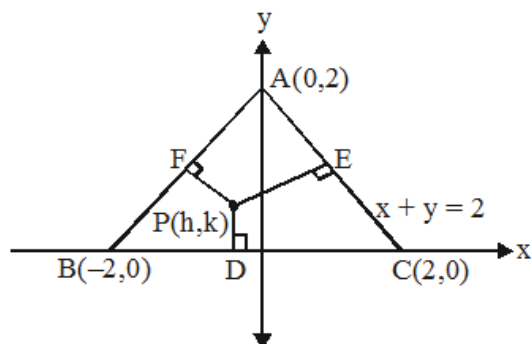
Now distance between focus and tangent at vertex is

$$a = \left| \frac{\frac{1}{3} + \frac{1}{3}}{\sqrt{2}} \right| \Rightarrow 4a = \frac{8}{3\sqrt{2}}$$

$$\Rightarrow 2a = \frac{4}{3\sqrt{2}} = L$$

$$= L^2 = \frac{8}{9}$$

2. Ans (16)



$$k^2 = \frac{1}{2} \cdot \left| \frac{h+k-2}{\sqrt{2}} \right| \cdot \left| \frac{h-k+2}{\sqrt{2}} \right|$$

$$4k^2 = -(h+k-2)(h-k+2)$$

$$4k^2 = (h+k-2)(-h+k-2)$$

$$4k^2 = (k-2)^2 - h^2$$

$$h^2 = (k-2)^2 - 4k^2$$

$$h^2 = -3k^2 + 4 - 4k$$

$$h^2 + 3k^2 + 4k = 4$$

$$h^2 + 3 \left(k^2 + \frac{4}{3}k + \frac{4}{9} \right) - 3 \cdot \frac{4}{9} = 4$$

$$h^2 + 3 \left(k + \frac{2}{3} \right)^2 = \frac{16}{3}$$

$$\frac{h^2}{\frac{16}{3}} + \frac{\left(k + \frac{2}{3} \right)^2}{\frac{16}{9}} = 1$$

$$e^2 = 1 - \frac{16/9}{16/3} = \frac{2}{3}$$

3. Ans (2)

Any point on hyperbola $(2\sec\theta, 3\tan\theta)$

chord of contact : $2\sec\theta x + 3\tan\theta y = 4$... (1)

equation of chord of circle, whose midpoint is (h,k)

$$hx + ky = h^2 + k^2 \quad \dots (2)$$

compare eq(1) and eq(2)

$$\frac{2\sec\theta}{h} = \frac{3\tan\theta}{k} = \frac{4}{h^2 + k^2}$$

$$\sec\theta = \frac{2h}{h^2 + k^2}, \tan\theta = \frac{4k}{3(h^2 + k^2)}$$

$$\frac{4h^2}{(h^2 + k^2)^2} - \frac{16k^2}{9(h^2 + k^2)^2} = 1$$

$$\frac{(36x^2 - 16y^2)}{9} = (x^2 + y^2)^2$$

$$\frac{9}{4}(4x^2 - 9y^2) = (x^2 + y^2) \text{ locus of mid point}$$

$$b = \frac{36}{9}, c = \frac{16}{9}$$

4. Ans (50)

$$(2\sec^2 A - \sec^4 A) - (2\operatorname{cosec}^2 A - \operatorname{cosec}^4 A)$$

$$= \frac{15}{4}$$

$$\sec^2 A(2 - \sec^2 A) - \operatorname{cosec}^2 A(2 - \operatorname{cosec}^2 A)$$

$$= \frac{15}{4}$$

$$(1 - \tan^4 A) - (1 - \cot^4 A) = \frac{15}{4}$$

$$\text{Put } \tan^4 A = t \quad -t + \frac{1}{t} = \frac{15}{4}$$

$$\frac{1-t^2}{t} = \frac{15}{4} \Rightarrow 4t^2 + 15t - 4 = 0$$

$$4t^2 + 16t - t - 4 = 0$$

$$t = -4, \frac{1}{4}$$

$$\tan^4 A = \frac{1}{4} \Rightarrow \tan^2 A = \frac{1}{2}$$

5. Ans (8)

$$\text{Length of tangent} = \sqrt{a\beta} = \sqrt{64} = 8$$