PART-1 : PHYSICS

SECTION-I (i)

1) A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then (α, β, γ, δ) is given by

(A) (n, –n, –n, –2n)

(B) (2n, –n, –2n, –2n)

(C) (2n, –n, –n, –n)

(D) (n, –n, –2n, –n)

2) An infinitely long wire, located on the z-axis, carries a current I along the +z-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B}.\vec{dl}$ along a straight line from the point $(-\sqrt{3}a, a, 0)$ to $(\sqrt{3}a, a, 0)$ is given by : [μ_0 is the magnetic permeability of free space.]

(A) $7\mu_0 I/24$

(B) $7\mu_0 I/12$

(C) $\mu_0 I/3$

(D) $\mu_0 I/6$

3) Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [ε_0 is the permittivity of free space]

(A) $q^2/(8\pi\varepsilon_0 R^3 m)$

(B) $q^2/(16\pi\varepsilon_0 R^3 m)$

(C) $q^2/(32\pi\varepsilon_0 R^3 m)$

(D) $q^2/(4\pi\varepsilon_0 R^3 m)$

4) A block of mass 2.5 kg moves along the x-direction subject to the force $F = (-10x + 5)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are

(A) –0.5 m, 2.5 kg m/s

(B) 0.5 m, 0 kg m/s

(C) 0.5 m, –2.5 kg m/s

(D) –1 m, 5 kg m/s

SECTION-I (ii)

1) A particle of mass *m* is moving in a circular orbit under the influence of the central force F(*r*) = –*kr*, corresponding to the potential energy $V(r) = kr^2 / 2$, where *k* is a positive force constant and *r* is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, *h* is the Planck's constant, and *n* a positive integer. If *v* and *E* are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct ?

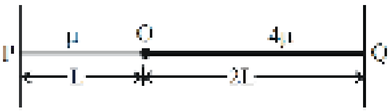
(A) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$

(B) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

(C) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(D) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$

2) Two uniform string of mass per unit length μ and 4μ, and length *L* and 2*L*, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct ?



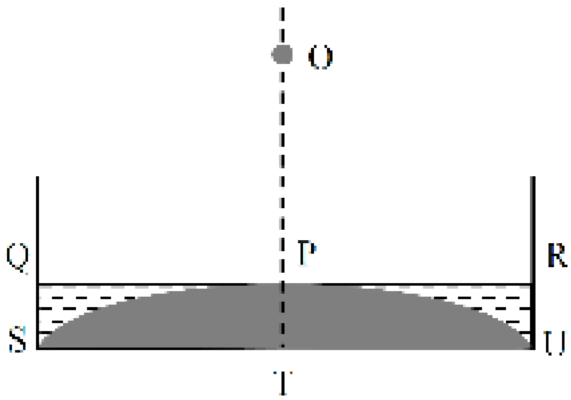
(A) No vibrational mode with an antinode at O is possible for the composite string.

(B) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.

(C) With a node at O, the minimum frequency of vibration of the composite string is v_0.

(D) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.

3) A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index *n* up to the level QPR. If the image of a point object O at a height of *h* (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct ?



(A) For $n = 1.42$, $h = 40$ cm.

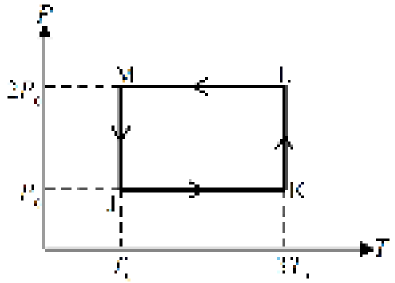
(B) For $n = 1.35$, $h = 52$ cm.

(C) For $n = 1.45$, $h = 60$ cm.

(D) For $n = 1.48$, $h = 75$ cm.

SECTION-I (iii)

1) One mole of a monatomic ideal gas undergoes the cyclic process J → K → L → M → J, as shown in the P-T diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [R is the gas constant.]

	List-I		List-II
(P)	Work done in the complete cyclic process	(1)	0
(Q)	Change in the internal energy of the gas in the process JK	(2)	$-3RT_0 \ln 2$
(R)	Heat given to the gas in the process KL	(3)	$3RT_0$
(S)	Change in the internal energy of the gas in the process MJ	(4)	$RT_0 - 4RT_0 \ln 2$
		(5)	$-2RT_0 \ln 2$

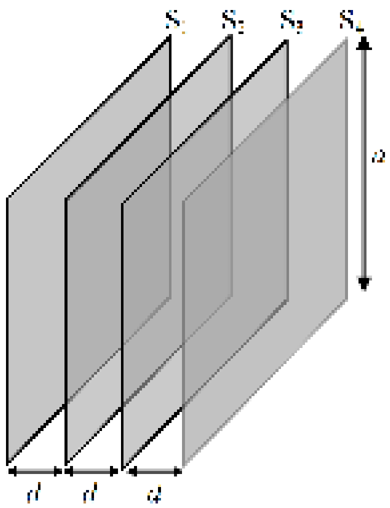
(A) P → 1;Q → 3;R → 5;S → 4

(B) P → 5;Q → 3;R → 2;S → 1

(C) P → 5;Q → 4;R → 1;S → 1

(D) P → 2;Q → 5;R → 3;S → 4

2) Four identical thin, square metal sheets, S_1, S_2, S_3 and S_4, each of side a are kept parallel to each other with equal distance d ($<< a$) between them, as shown in the figure. Let $C_0 = \varepsilon_0 a^2/d$, where ε_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The capacitance between S_1 and S_4, with S_2 shorted to S_3, is	(1)	$3C_0$
(Q)	The capacitance between S_1 and S_2, with S_3 shorted to S_1, and S_2 shorted to S_4, is	(2)	$C_0/2$
(R)	The capacitance between S_1 and S_4, with S_2 and S_3 not connected, is	(3)	$C_0/3$
(S)	The capacitance between S_1 and S_3, with S_2 shorted to S_4, is	(4)	$2C_0/3$
		(5)	$2C_0$

(A) P → 2;Q → 1;R → 3;S → 4

(B) P → 4;Q → 2;R → 3;S → 1

(C) P → 2;Q → 4;R → 1;S → 4

(D) P → 4;Q → 3;R → 2;S → 1

3) A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0. The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	If $n = 2$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(1)	60° and 0°
(Q)	If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(2)	150°
(R)	If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of ϕ_0 will be	(3)	0°
(S)	If $n = \sqrt{2}$ and $\theta_0 = 45°$, then all the possible values of α will be	(4)	30° and 0°
		(5)	45° and 0°

(A) P → 3;Q → 2;R → 4;S → 3

(B) P → 5;Q → 1;R → 2;S → 4

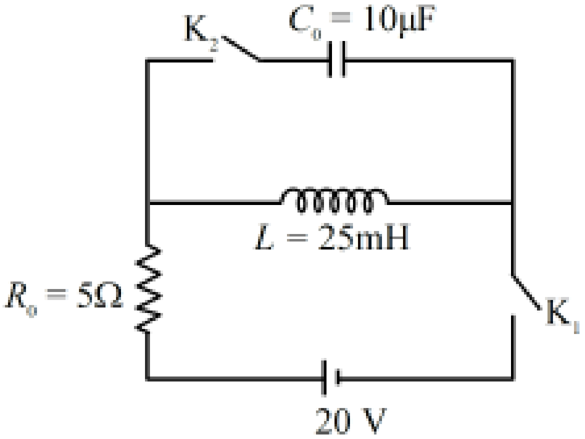
(C) P → 3;Q → 1;R → 4;S → 2

(D) P → 3;Q → 1;R → 2;S → 5

4) The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open and there is no

charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1. After a long time, the current attains a steady state value I_2. Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular

0



frequency ω . Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The value of ω_0 in kilo-radians/s is	(1)	0
(Q)	The value of V_0 in Volt is	(2)	2
(R)	The value of I_1 in Ampere is	(3)	4
(S)	The value of I_2 in Ampere is	(4)	20
		(5)	200

(A) P → 2;Q → 5;R → 1;S → 3

(B) P → 2;Q → 1;R → 4;S → 3

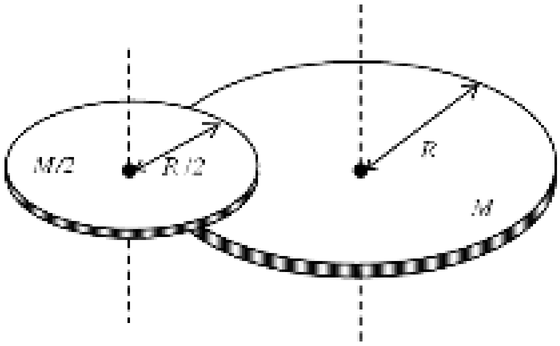
(C) P → 1;Q → 3;R → 2;S → 5

(D) P → 4;Q → 3;R → 2;S → 1

SECTION-II

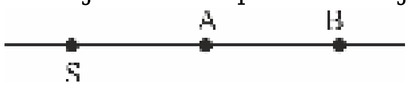
1) The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from –23°C to 27°C is nk, the value of $\frac{n}{50}$ is ______. [Given : 0 K = –273 °C.]

2) A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the mass $\frac{M}{2}$ and radius R / 2 is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω about its axis as seen from ground. If the angular speed at which the large disc rotates is ω / n, then

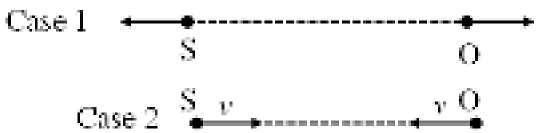


the value of n is ______.

3) A point source S emits unpolarized light uniformly in all directions. At two points A and B, on the same line as S the ratio $r = I_A / I_B$ of the intensities of light is 2. If a polaroids having 45° angle between their pass-axes is placed just before point A and just before point B respectively, then the new value of r will be _____.

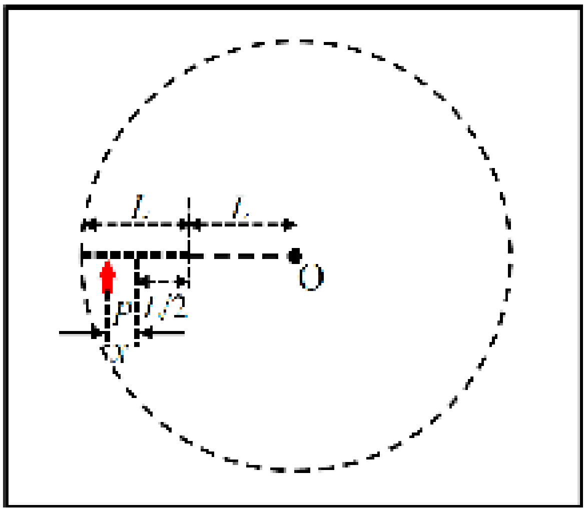


4) A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move away from each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 200 Hz. However, when the observer and the source move towards each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is _____.



5) Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio t_1/t_2 is _____.

6) A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse is imparted to the rod at a distance $x = L/n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is _____.



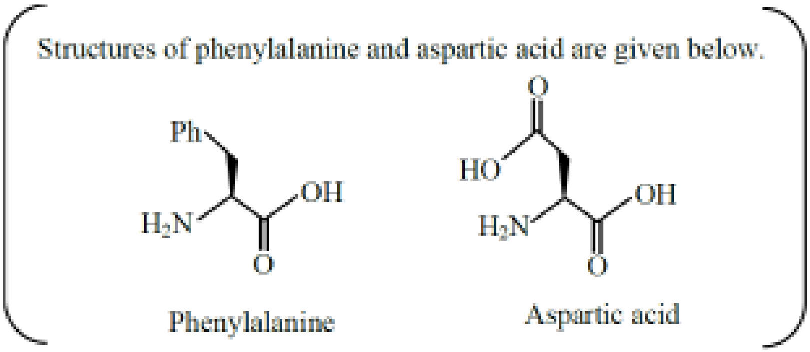
PART-2 : CHEMISTRY

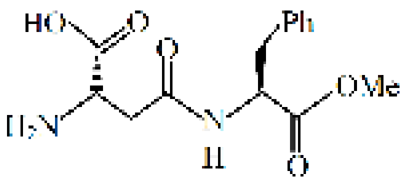
SECTION-I (i)

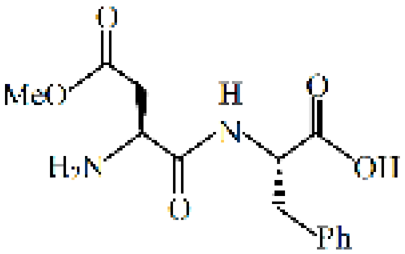
1) A closed vessel contains 10 g of an ideal gas **X** at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas **Y** is added to it and the pressure becomes 6 atm. The ratio of root mean square velocities of **Y** and **X** at 300 K is

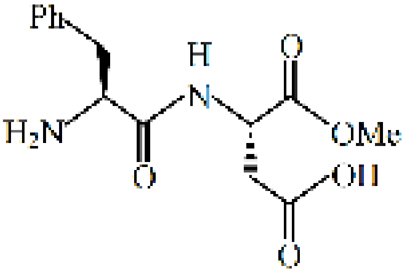
(A) $2\sqrt{2} : \sqrt{3}$
(B) $2\sqrt{2} : 1$
(C) 1 : 2
(D) 2 : 1

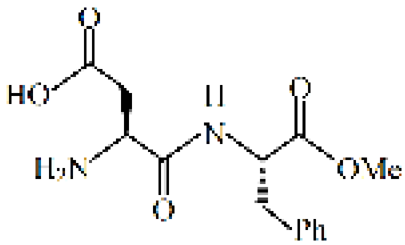
2) Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is



(A) 

(B) 

(C) 

(D) 

3) The species formed on fluorination of phosphorus pentachloride in a polar organic solvent are :

(A) $[PF_4]^+ [PF_6]^-$ and $[PCl_4]^+ [PF_6]^-$

(B) PF_3 and PCl_3

(C) $[PCl_4]^+ [PCl_4F_2]^-$ and $[PCl_4]^+ [PF_6]^-$

(D) PF_5 and PCl_3

4) The option in which at least three molecules follow Octet Rule is

(A) HCN, C_2H_2 , ClO_2 , H_3PO_4

(B) NO_2 , O_3 , NH_3 , NO

(C) BF_3 , ClO_3 , CH_4 , $HClO_4$

(D) XeF_2 , C_3H_4 , O_3 , C_6H_6

SECTION-I (ii)

1) Among the following, the correct statement(s) for electrons in an atom is(are)

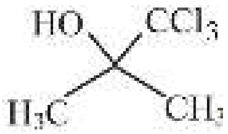
(A) According to Bohr's model, the magnitude of velocity of electrons increases with decrease in values of n.

(B) According to Bohr's model, the most negative energy value for an electron is given by n = 1, which corresponds to the most stable orbit.

(C) Uncertainty principle rules out the existence of definite paths for electrons.

(D) The energy of an electron in 2*s* orbital of an atom is higher than the energy of an electron that is infinitely far away from the nucleus.

2) Reaction of *iso*-propylbenzene with O_2 followed by the treatment with H_3O^+ forms phenol and a by-product **P**. Reaction of **P** with 3 equivalents of Cl_2 gives compound **Q**. Treatment of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.
The correct statement(s) regarding **P**, **Q**, **R** and **S** is(are)

(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives


(B) Reaction of **R** with O_2 in the presence of light gives phosgene gas

(C) **Q** react with CO_2 in presence of NaOH followed by $H_3O^{\oplus}$ gives salicylic acid.

(D) **S** on heating gives **P**

3) The compound (s) having peroxide linkage is(are) :

(A) H_3PO_5

(B) $H_2S_2O_8$

(C) $H_4P_2O_8$

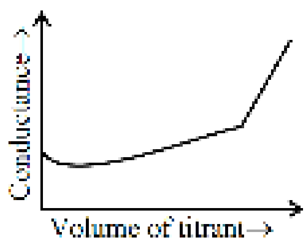
(D) $H_2S_2O_5$

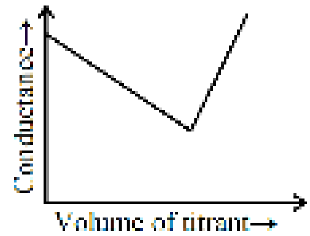
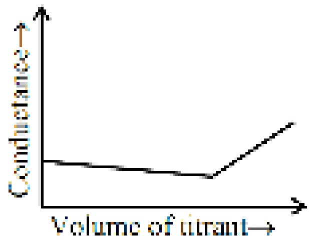
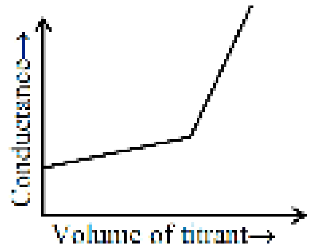
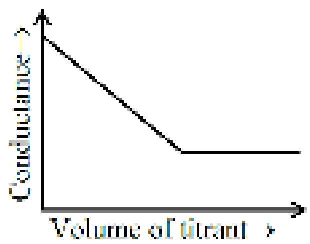
SECTION-I (iii)

1) In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.
The limiting ionic conductivity (Λ_0) values (in mS m^2 mol^{-1}) for different ions in aqueous solutions are given below :

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
Λ_0	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.
Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

List-I		**List-II**	
(P)	Titrate : $AgNO_3$ Titrant : KCl	(1)	

(Q)	Titrate : KCl Titrant : $AgNO_3$	(2)	
(R)	Titrate : NaOH Titrant : HCl	(3)	
(S)	Titrate : CH_3COOH Titrant : NaOH	(4)	
		(5)	

(A) P → 4;Q → 3;R → 2;S → 5

(B) P → 2;Q → 4;R → 3;S → 1

(C) P → 3;Q → 4;R → 2;S → 5

(D) P → 4;Q → 3;R → 2;S → 1

2) Based on **VSEPR** model, match the xenon compounds given in **List-I** with the corresponding geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

List-I		List-II	
(P)	XeO_3	(1)	Trigonal bipyramidal and two lone pair of electrons
(Q)	XeF_4	(2)	Tetrahedral and one lone pair of electrons
(R)	XeO_2F_2	(3)	Octahedral and two lone pair of electrons
(S)	$XeOF_4$	(4)	Trigonal bipyramidal and one lone pair of electrons
		(5)	Octahedral and one lone pair of electron

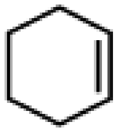
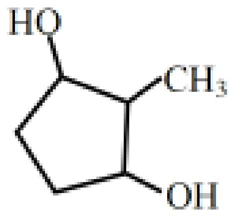
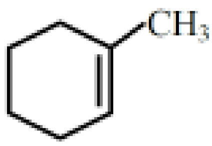
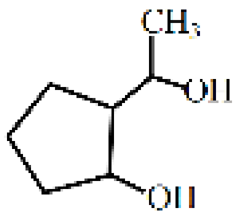
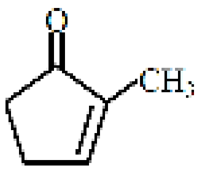
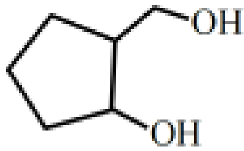
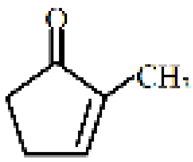
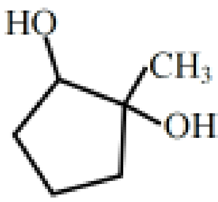
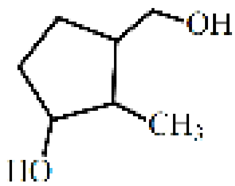
(A) P → 5;Q → 2;R → 3;S → 1

(B) P → 2;Q → 3;R → 4;S → 1

(C) P → 4;Q → 3;R → 2;S → 1

(D) P → 2;Q → 3;R → 4;S → 5

3) **List-I** contains various reaction sequences and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		List-II
(P)	 i) O_3, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH_3, b) H_2O_2, NaOH v) H_3O^+ vi) $NaBH_4$	(1)	
(Q)	 i) O_3, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH_3, b) H_2O_2, NaOH v) H_3O^+ vi) $NaBH_4$	(2)	
(R)	 i) ethylene glycol, PTSA ii) a) $Hg(OAc)_2$, H_2O, b) $NaBH_4$ iii) H_3O^+ iv) $NaBH_4$	(3)	
(S)	 i) ethylene glycol, PTSA ii) a) BH_3, b) H_2O_2, NaOH iii) H_3O^+ iv) $NaBH_4$	(4)	
		(5)	

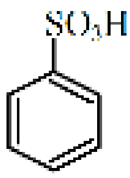
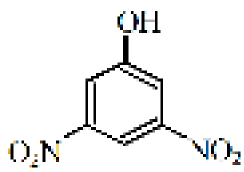
(A) P → 3;Q → 2;R → 1;S → 4

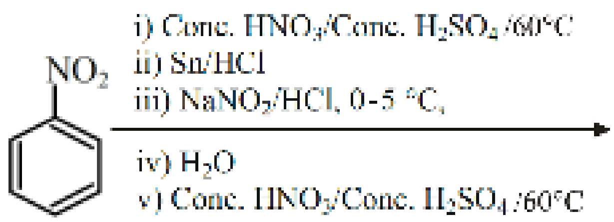
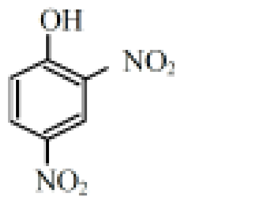
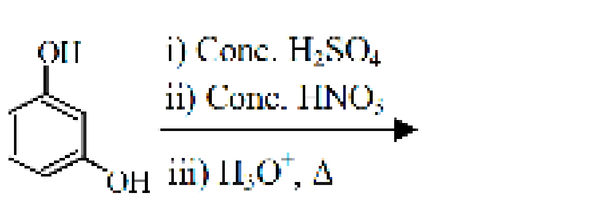
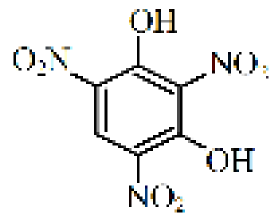
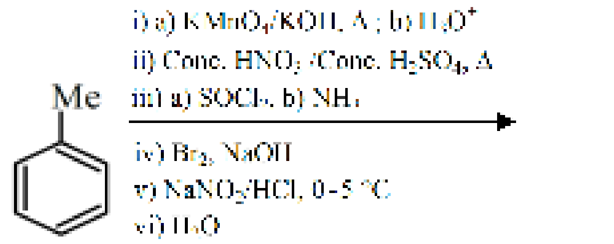
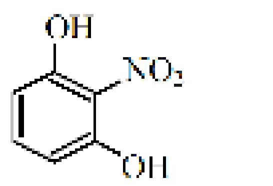
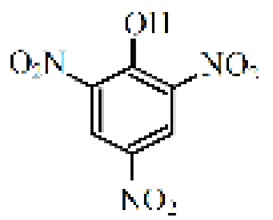
(B) P → 3;Q → 5;R → 4;S → 1

(C) P → 3;Q → 2;R → 4;S → 1

(D) P → 2;Q → 5;R → 4;S → 1

4) **List-I** contains various reaction sequences and **List-II** contains different phenolic compounds. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

	List-I		List-II
(P)	 i) molten NaOH, H_3O^+ ii) Conc. HNO_3	(1)	

(Q)		(2)	
(R)		(3)	
(S)		(4)	
		(5)	

(A) P → 2;Q → 5;R → 4;S → 3

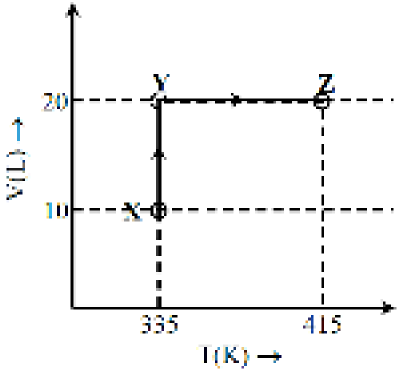
(B) P → 5;Q → 3;R → 4;S → 1

(C) P → 2;Q → 5;R → 3;S → 1

(D) P → 5;Q → 2;R → 3;S → 4

SECTION-II

1) Consider the following volume-temperature (V – T) diagram for the expansion of 5 moles of an



ideal monoatomic gas. Considering only P-V work is involved, the total change in internal energy (in Joule) for the transformation of state in the sequence **X → Y → Z** is ______ .

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m}$ = 12 J K^{-1} mol^{-1} and gas constant, R = 8.3 J K^{-1} mol^{-1}]

2) Consider the following reaction,

$$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$$

which follows the mechanism given below:

$2NO(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} N_2O_2(g)$ (fast equilibrium)

$N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g)$ (slow reaction)

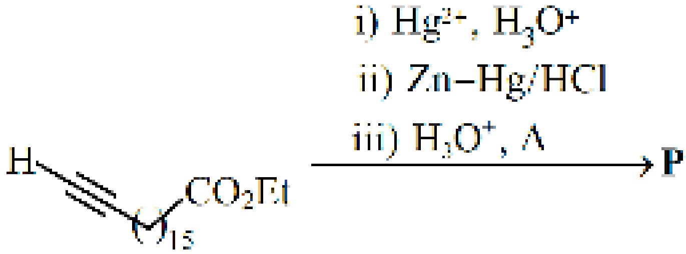
$N_2O(g) + H_2(g) \xrightarrow{k_3} N_2(g) + H_2O(g)$ (fast reaction)

The order with respect to H_2 is.

3) Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollens' test, whereas **Q** on acidification gives positive Tollens' test. Treatment of **P** with excess cyclohexanone in the presence of catalytic amount of *p*-toluenesulfonic acid (PTSA) gives product **R**.
Total number of methylene groups ($-CH_2-$) in **R** is ______.

4) Among $[V(CO)_6]^-$, $Cr(CO)_5$, $Cu(CO)_3$, $[Mn(CO)_6]^+$, $Fe(CO)_5$, $[Co(CO)_3]^{2-}$, $[Cr(CO)_4]^{4-}$, $[Ti(CO)_6]^{-2}$, $[Co(CO)_4]^{-1}$ and $Ir(CO)_3$, the total number of species isoelectronic with $Ni(CO)_4$ is_______.
[Given, atomic number : V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

5) In the following reaction sequence, the major product **P** is formed.



1 mole of P react with $\frac{1}{3}$ mole of glycerol in presence of an acid as catalyst gives product Q. Reaction of Q with excess NaOH followed by the treatment with $CaCl_2$ yields R, quantitatively.

Starting with $\frac{1}{3}$ mole of Q, the amount of R produced in gm is ________
[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

6) Among the following complexes, the total number of diamagnetic species is_______.
$[Mn(NH_3)_6]^{3+}$, $[MnCl_6]^{3-}$, $[Ni(CO)_4]$, $[CoF_6]^{3-}$, $[Fe(NH_3)_6]^{3+}$, O_3, BaO_2, $[Pt(NH_3)_2Br_2]$, $[Co(en)_3]Cl_2$ and $[Fe(CN)_6]^{4-}$
[Given, atomic number : Mn = 25, Fe = 26, Co = 27;
en = $H_2NCH_2CH_2NH_2$]

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let f(x) is a continuous and differentiable on $(0, \infty)$ such that f(1) = 2 and $\lim_{t \to x} \frac{t^3 f(x) - x^3 f(t)}{t^2 - x^2} = 2$ for each x > 0. Then for all x > 0, f(x) is equal to

(A) $f(x) = x^3 + \frac{1}{x}$

(B) $f(x) = x^3 + \frac{1}{x^2}$

(C) $f(x) = 3x^3 - \frac{1}{x}$

(D) $f(x) = 3x^3 - \frac{1}{x^2}$

2) Let $\frac{\pi}{2} < x < \pi$ and $\tan x = \frac{-24}{7}$, then value of $\sin\frac{13x}{2}(\sin 7x - \cos 7x) + \cos\frac{13x}{2}(\sin 7x + \cos 7x)$ is equal to

(A) $-\frac{7}{5}$

(B) $\frac{7}{5}$

(C) $\frac{4}{3}$

(D) $-\frac{4}{3}$

3) A student appears for a quiz consisting of only, objective type questions, each question having 4 options of which exactly one option is correct and he answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{4}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is :

(A) $\frac{4}{9}$

(B) $\frac{5}{12}$

(C) $\frac{5}{7}$

(D) $\frac{5}{9}$

4) Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let S(p, q) be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{9}{5}$ then which of the following options is correct ?

(A) p = 3, q = 2

(B) p = 6, q = 2

(C) p = 3, q = 1

(D) p = 6, q = 1

SECTION-I (ii)

1) Let S = $\{a+b\sqrt{3} : a, b \in Z\}$, $T_1 = \left\{\left(2-\sqrt{3}\right)^n : n \in N\right\}$ and $T_2 = \left\{\left(2+\sqrt{3}\right)^n : n \in N\right\}$, then which of the following is (are) true ?

(A) $Z \cup T_1 \cup T_2 \subset S$

(B) $T_2 \cap \left(0, \frac{1}{2025}\right) = \phi$
(where ϕ denotes null set)

(C) $T_1 \cap \left(0, \frac{1}{100}\right) \neq \phi$

(D) For any a, b ∈ Z, $\cos\left(\pi\left(a+b\sqrt{3}\right)\right) + i\sin\left(\pi\left(a+b\sqrt{3}\right)\right) \in Z$ if and only if b = 0, where $i = \sqrt{-1}$

2) Let R^2 denote R × R. Let S = {(a, b, c) : a, b, c ∈ R and $ax^2 + 2bxy + cy^2 < 0$, ∀ (x, y) ∈ R^2 – {(0, 0)}} then which of the following statements is(are) True ?

(A) (–2, 3, –4) ∈ S

(B) If (–1, 2, c) ∈ S, then c < –4

(C) For some given (a, b, c) ∈ S, the system of equation $\begin{matrix} ax+by=1 \\ bx+2cy=-1 \end{matrix}$, may be inconsistent

(D) For some given (a, b, c) ∈ S, the system of equation $\begin{matrix} ax+by=1 \\ bx+cy=-1 \end{matrix}$, has a unique solution

3) Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points P = (3, 4, 5) & Q(6, 4, 9). Let dist (X, Y) denote distance between two point X and Y in R^3.
Let S = {X ∈ $\mathbb{R}^3$: $(\text{dist }(X, P))^2 - (\text{dist }(X, Q))^2 = 100$} and T = {Y ∈ $\mathbb{R}^3$: $(\text{dist }(Y, Q))^2 - (\text{dist }(Y, P))^2 = 100$}, then which of the following is True ?

(A) There is a triangle whose area is 2 sq. unit & all vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

SECTION-I (iii)

1) Let α, β and γ be the distinct roots of the equation $x^3 - 7x + 6 = 0$. Consider the set T = {α, β, γ} for a 3 × 3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i = 1, 2, 3 and j = 1, 2, 3.
Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for i > j. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

(A) P → 2;Q → 4;R → 3;S → 5

(B) P → 3;Q → 2;R → 4;S → 2

(C) P → 2;Q → 2;R → 5;S → 2

(D) P → 3;Q → 4;R → 3;S → 1

2) Let α ∈ R be such that lines :

$L_1 : \frac{x-1}{2} = \frac{y-1}{3} = \frac{z-4}{\alpha}$ and

$L_2 : \frac{x}{3} = \frac{y}{4} = \frac{z-2}{4}$

intersect let R_1 be the point of intersection of L_1 & L_2. Let O = (0, 0, 0) and $\hat{n}$ be a unit vector normal to plane containing lines L_1 & L_2 :

List-I		List-II	
(P)	α =	(1)	$\frac{2}{\sqrt{21}}$
(Q)	A possible choice for $\hat{n}$ =	(2)	4
(R)	$\overrightarrow{OR_1}$ =	(3)	$\left(\frac{4\hat{i}}{\sqrt{21}} - \frac{2\hat{j}}{\sqrt{21}} - \frac{\hat{k}}{\sqrt{21}}\right)$
(S)	$\lvert\overrightarrow{OR_1}.\hat{n}\rvert$	(4)	2
		(5)	$3\hat{i} + 4\hat{j} + 6\hat{k}$

(A) P → 2;Q → 3;R → 4;S → 2

(B) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 5; S \rightarrow 3$

(C) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 1$

(D) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 5; S \rightarrow 1$

3) Let line $2y = x + 3$ touch a circle with centre $(\alpha, 0)$, $(\alpha > 0)$ and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 2\sqrt{5}$ + . Match List-I to List-II.

List-I		List-II	
(P)	$\alpha =$	(1)	(1, 2)
(Q)	r =	(2)	$\sqrt{5}$
(R)	$A_1 =$	(3)	2
(S)	$B_1 =$	(4)	(3, -2)
		(5)	1

(A) $P \rightarrow 3; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 1$

(B) $P \rightarrow 5; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 2$

(C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

(D) $P \rightarrow 5; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 4$

4) Let $f : R \rightarrow R$ & $g : R \rightarrow R$, be functions defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ & $g(x) = \begin{cases} 1 - x, & 0 \leqslant x < 1 \\ 0, & \text{otherwise} \end{cases}$ where a, b, c, d ∈ R. Let h(x) be a function, $h : R \rightarrow R$, $h(x) = af(x) + b(g(x) + g(1 - x)) + c(x - g(x)) + dg(x)$, $x \in R$

Match List-I & List-II

List-I		List-II	
(P)	If a = 0, b = 1, c = 0 & d = 0	(1)	h is one-one
(Q)	If a = 1, b = 0, c = 0, d = 0	(2)	h is onto
(R)	If a = 0, b = 0, c = 1, d = 0	(3)	h is differentiable
(S)	If a = 0, b = 0, c = 0 & d = 1	(4)	Range of h is [0, 1]
		(5)	Range of h is {0, 1}

(A) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 4$

(B) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 5$

(C) $P \rightarrow 5; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 4$

(D) $P \rightarrow 5; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 5$

SECTION-II

1) Let a = $2\sqrt{3}$ & b = $\sqrt[3]{3\sqrt{2}}$

$2x + 3y = \log_a 144 + \log_b \sqrt{18}$

$2x - y = 2\log_6 ab^3$

then value of 3x + 4y is equal to

2) Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that f(1) = –7 suppose that $\sqrt{2}i$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If α_1, α_2, α_3 and α_4 are all the roots of the equation f(x) = 0, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to

3) A group of 12 students $S_1, S_2, S_3 \ldots, S_{12}$ are to be divided into three teams X, Y, Z of sizes 3, 4 & 5 respectively. If S_1 & S_2 can not go to team X & S_3 can not go to team Y, then number of ways to form such teams is 'N', then $\frac{N}{2}$ is equal to

4) Let $\overrightarrow{OP} = (\alpha - 1)\hat{i} + \hat{j} + 2\hat{k}$, $\overrightarrow{OQ} = (\beta - 2)\hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \hat{k}$ be three vectors, where a, b ∈ $\mathbb{R}$ – {0} and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right).\overrightarrow{OR} = 0$ and the point (α, β, 2) lies on the plane 3x + y – z – □ = 0, then the value of □ is

5) Let S = $\left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & 2a & d \\ 1 & 2b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-2, 2\} \right\}$, where |A| denotes the determinant of A. Then the number of elements is S is

6) Let X be a random variable and let P(X = x) denote the probability that X takes the value x. Suppose that the points (x, P(X = x)), x = 1, 2, 3, 4, 5 lie on a fixed straight line in the xy-plane and P(X = x) = 0 for all x ∈ $\mathbb{R}$ – {1, 2, 3, 4, 5}. If the mean of X is $\frac{10}{3}$ and the variance of X is α, then the value of 9α is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	**C**	**C**	**C**	**C**

SECTION-I (ii)

Q.	5	6	7
A.	**A,C,D**	**A,B,C**	**C,D**

SECTION-I (iii)

Q.	8	9	10	11
A.	**B**	**A**	**C**	**A**

SECTION-II

Q.	12	13	14	15	16	17
A.	**275**	**16**	**4**	**288**	**3**	**18**

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	18	19	20	21
A.	**C**	**D**	**C**	**D**

SECTION-I (ii)

Q.	22	23	24
A.	**A,B,C**	**A,B,C,D**	**A,B,C**

SECTION-I (iii)

Q.	25	26	27	28
A.	**D**	**D**	**C**	**B**

SECTION-II

Q.	29	30	31	32	33	34
A.	**4800**	**1**	**14**	**6**	**303**	**5**

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	35	36	37	38
A.	**A**	**B**	**D**	**B**

SECTION-I (ii)

Q.	39	40	41
A.	**A,B,C,D**	**B,D**	**A,B,C**

SECTION-I (iii)

Q.	42	43	44	45
A.	**A**	**D**	**C**	**C**

SECTION-II

Q.	46	47	48	49	50	51
A.	**10**	**16**	**5208**	**8**	**4**	**17**

PART-1 : PHYSICS

1) For the quantity to be dimensionless

$e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{d} = M^0L^0T^0A^0$

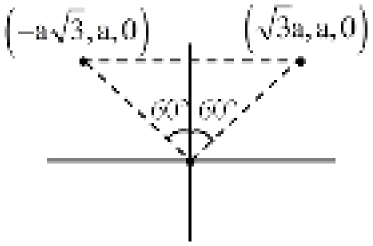
$\Rightarrow (AT)^{\alpha}\left(M^{-1}L^{-3}T^4A^2\right)^{\beta}\left(ML^2T^{-1}\right)^{\gamma}\left(LT^{-1}\right)^{\delta} = A^0M^0L^0T^0$

$\therefore \alpha + 2\beta = 0,\ \alpha + 4\beta - \gamma - \delta = 0,\ -\beta + \gamma = 0\ \&\ -3\beta + 2\gamma + \delta = 0$

$\therefore \alpha = -2\beta,\ \beta = \gamma\ \&\ \gamma = \delta$

 Option (C) satisfies the given condition

2) [θ_1 is anticlockwise hence taken negative]



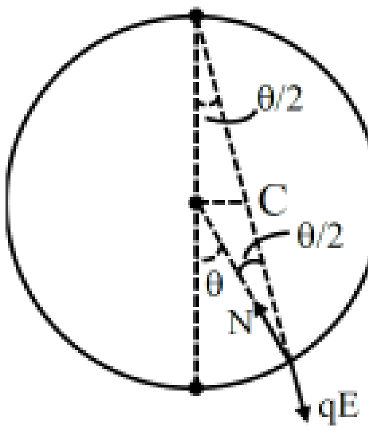
$\Rightarrow \tan\theta_1 = \frac{a\sqrt{3}}{a} = \sqrt{3}$

$\theta_1 = \frac{\pi}{3}$

$\Rightarrow \tan\theta_2 = \frac{a}{a} = \sqrt{3}$

$\theta_2 = \frac{\pi}{3}$

$\Rightarrow \int B.d\ell = \frac{\mu_0 I}{2\pi}\left[\frac{\pi}{3} + \frac{\pi}{3}\right] = \frac{\mu_0 I}{3}$



3)

Restoring force = $qE\sin\left(\frac{\theta}{2}\right)$

$\therefore \tau = qE\sin\left(\frac{\theta}{2}\right)R = I\alpha$

$E = \frac{Kq}{\left(2R\cos\frac{\theta}{2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{4R^2\cos^2\left(\frac{\theta}{2}\right)}$

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{qR}{4R^2\cos^2\left(\frac{\theta}{2}\right)}\sin\left(\frac{\theta}{2}\right)q = mR^2\alpha$$

For θ very small,

$$\frac{-q^2}{32\pi\varepsilon_0 R^3 m}\theta = \alpha$$

$$\therefore \omega^2 = \frac{q^2}{32\pi\varepsilon_0 mR^3}$$

Hence option (C)

4) $F = -10\left(x - \frac{1}{2}\right) = -10X \qquad \left(X = x - \frac{1}{2}\right)$

 Particle will perform SHM about $x = \frac{1}{2}$ with

ω = 2 rad/sec ⇒ T = π sec.

 Phase covered in $t = \frac{\pi}{4}$ second = 90°.

Given particle is at rest at x = 1m ⇒ x = 1 is extreme position.

 In $\frac{\pi}{4}$ sec, it will be at equilibrium

 x = 0.5 m and momentum = mωA = 2.5 × 2 × 0.5 = 2.5 kg m/s

Direction will be towards –ve x.

Hence option (C)

5) The central force will provide necessary centripetal force

$$\Rightarrow kr = \frac{mv^2}{r}$$

or, $kr^2 = mv^2$ .... (1)

By quantisation rule

$n\hbar = mvr$

or, $\frac{n\hbar}{r} = mv$ ... (2)

$$\frac{(1)}{(2)^2} \Rightarrow \frac{kr^2}{\frac{n^2\hbar^2}{r^2}} = \frac{mv^2}{m^2v^2}$$

$$\Rightarrow \frac{k}{n^2\hbar^2}r^4 = \frac{1}{m}$$

$$\Rightarrow r = \left(\frac{n^2\hbar^2}{km}\right)^{\frac{1}{4}} \Rightarrow r^2 = \frac{n\hbar}{\sqrt{mk}}$$

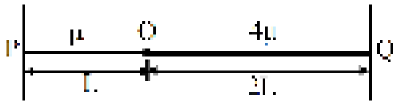
(A) $\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ from (1)

(B) $E = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 = \frac{n\hbar}{2}\sqrt{\frac{k}{m}} + \frac{1}{2}k\frac{n\hbar}{\sqrt{mk}}$

$$E = n\hbar\sqrt{\frac{k}{m}}$$

(C) Using (1), $K \cdot \frac{n\hbar}{\sqrt{mk}} = mv^2$

$\Rightarrow v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

6) 

$C_1 = \sqrt{\frac{T}{\mu}}, C_2 = \sqrt{\frac{T}{4\mu}} = \frac{C_1}{2}$

For node at O :

$L = \frac{n\lambda_1}{2}, 2L = \frac{m\lambda_2}{2}$ (n, m are integers)

$\lambda_1 = \frac{2L}{n}, \lambda_2 = \frac{4L}{m}$

$\frac{C_1}{\lambda_1} = \frac{C_2}{\lambda_2}$

$\Rightarrow \frac{C_1}{\frac{2L}{n}} = \frac{\frac{C_1}{2}}{\frac{4L}{m}}$

$\Rightarrow 4n = m$

For minimum frequency, n = 1, m = 4

$\therefore \nu_{min} = \frac{C_1 \times 1}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \nu_0$

The string will look like

Total no. of nodes = 6 including the end nodes

For antinode at O :

$L = (2n+1)\frac{\lambda_1}{4}$; $2L = (2n+1)\frac{\lambda_2}{4}$ (n, m are integers)

$\lambda_1 = \frac{4L}{(2n+1)}$; $\lambda_2 = \frac{8L}{(2m+1)}$

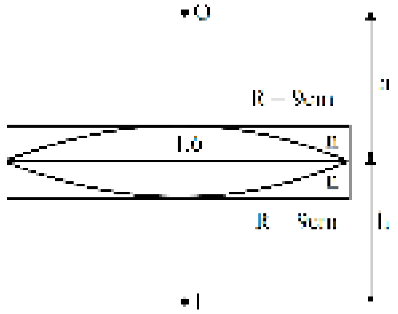
$\frac{C_1}{\lambda_1} = \frac{C_2}{\lambda_2}$

$\frac{C_1}{C_2} = \frac{\lambda_1}{\lambda_2}$

$2 = \frac{\frac{4L}{(2n+1)}}{\frac{8L}{(2m+1)}}$

$4 = \frac{(2m+1)}{(2n+1)} \Rightarrow \text{even} = \frac{\text{odd}}{\text{odd}} \Rightarrow$ This node is not possible

7) Since STU is a plane mirror, we can take mirror image of the whole situation about it and final image can be assumed to be at a distance h below the base.



Since object and image are at same distance from equivalent lens, hence $h = 2F_{eq}$

$$\frac{1}{F_{eq}} = \left(\frac{1.6-1}{1}\right)\left(\frac{2}{9}\right) + \frac{(n-1)}{1}\left(\frac{-2}{9}\right)$$

$$\frac{1}{\frac{h}{2}} = \frac{1.2}{9} + \frac{2(1-n)}{9}$$

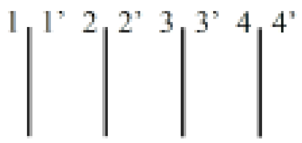
$$\frac{2}{h} = \frac{3.2-2n}{9}$$

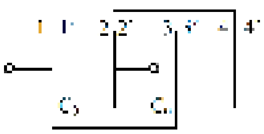
$$h = \frac{9}{1.6-n}\text{cm}$$

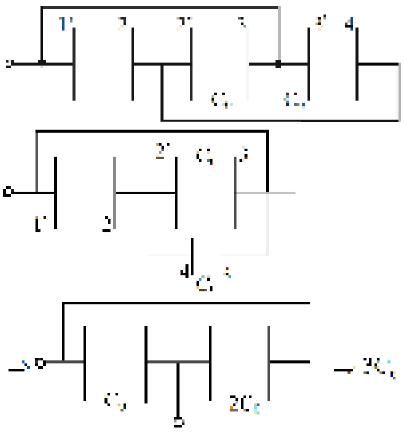
(A) for n = 1.42, h = 50 cm
(B) for n = 1.35, h = 36 cm
(C) for n = 1.45, h = 60 cm
(D) for n = 1.48, h = 75 cm

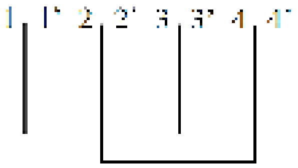
8)

J (P_0, V_0, T_0)
K $(P_0, 3V_0, 3T_0)$
M $(2P^0, , \frac{V_0}{2}, T^0)$
L $(2P^0, , \frac{3V_0}{2}\ 3T^0)$
$P_0V_0 = nRT_0$
JK → isobaric ⇒ $W = P_0(2V_0) = 2nRT_0$
$\Delta U = \frac{3}{2}nR(2T_0) = 3nRT_0$
KL → isothermal → $W = nR(3T)\ \ell n\left(\frac{1}{2}\right) = -3nRT^0 \ln 2$
$\Delta U = 0 \Rightarrow Q = -3nRT_0 \ell n2$
LM → isobaric $= 2P_0(-V_0) = -2nRT_0$
MJ → isothermal ⇒ $nRT_0 \ell n2$; $\Delta U = 0$
$WD_{net} = -2nRT_0 \ell n2$
P → 5, Q → 3, R → 2, S → 1

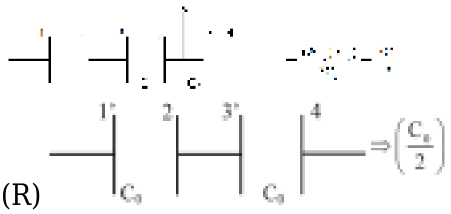
9) 

(P) 

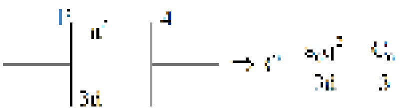




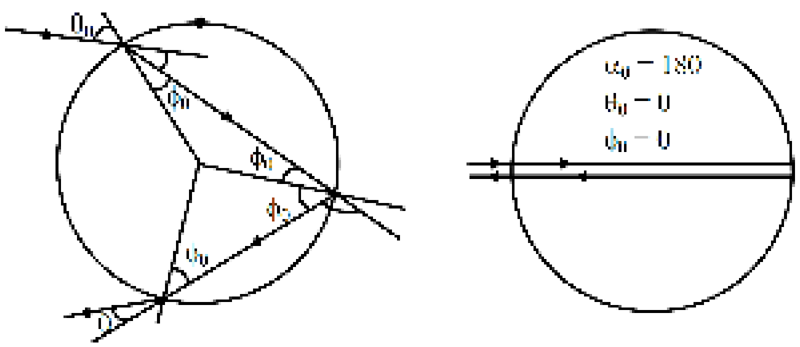
(Q)



(R)



(S)



10)

$\alpha = (\theta_0 - \phi_0) + (180 - 2\phi_0) + (\theta_0 - 2\phi_0)$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

(P) $\alpha = 180 + 2\theta_0 - 4\phi_0$

$180 = 180 + 2\theta_0 - 4\phi_0 \Rightarrow \theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = 2\sin\phi_0$ ....(ii)

From (i) & (ii)

$\sin\theta^0 = 2\sin(\theta^0/2) \Rightarrow \cos\left(\frac{\theta_0}{2}\right) = 1$

$\frac{\theta_0}{2} = 0$

$\Rightarrow \theta_0 = 0$

(Q) $\theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = \sqrt{3}\sin\phi_0$ ....(ii)

From (i) & (ii)

$$\sin\theta_0 = \sqrt{3}\sin\left(\frac{\theta_0}{2}\right)$$

$$\Rightarrow \cos\left(\frac{\theta_0}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\theta_0}{2} = 30, 150$$

$\theta_0 = 60, 300$ (Rejected)

$\theta_0 = 60$ & 0

(R) $\theta_0 = 2\phi_0$

$$\sin\theta_0 = \sqrt{3}\sin\phi_0$$

$$\sin 2\theta_0 = \sqrt{3}\sin\phi_0$$

$$\cos\phi_0 = \frac{\sqrt{3}}{2}$$

$\phi_0 = 30, 150$ (Rejected)

$\phi_0 = 30$ & 0 ....(i)

(S) $\sin 45 = \sqrt{2}\cos\phi_0$

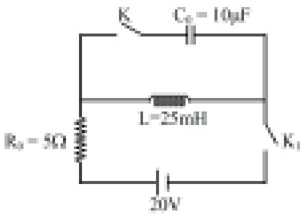
$\cos\phi_0 = 1/2$

$\phi_0 = 60$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

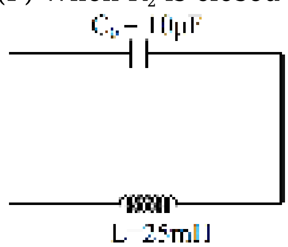
$\alpha = 180 + 90 - 120$ ....(iv)

$= 180 - 30$; $\alpha = 150°$



11)

(P) When K_2 is closed and K_1 open



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

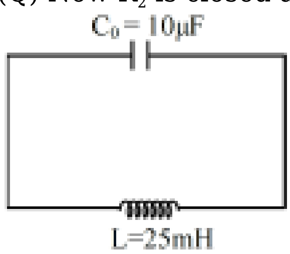
$$\omega_0 = \frac{1}{\sqrt{25\times10^{-3}\times10\times10^{-6}}} = \frac{1}{5\times10^{-4}}$$

$\omega_0 = 2 \times 10^3$ rad/s

$\omega_0 = 2$ kilo-radian/s

R → (2)

(Q) Now K_2 is closed and K_1 open



$\frac{1}{2}LI_2^2 = \frac{1}{2}CV_0^2$

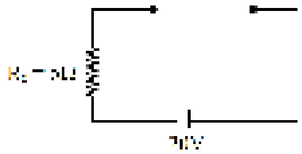
$25 \times 10^{-3} \times (4)^2 = 10 \times 10^{-6} \times V_0^2$

$V_0^2 = 2500 \times 16$

$V_0 = 50 \times 4 = 200$ V

S → (5)

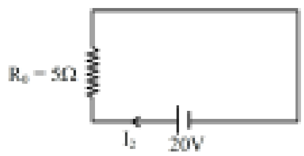
(R) When K_1 is closed current in R_0 is I_1 At t = 0; circuit will be



$I_1 = 0$

P → (1)

(S) After long time inductor behave as a wire so I_2



$I_2 = \frac{20}{5} = 4A$

Q → (3)

12) $T_i = -23°C = 250$ K

$T_f = 27°C = 300$ K

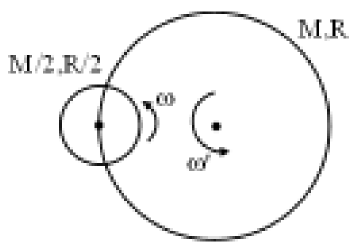
$Q = \int msdT$

$= \int 1 \cdot kT\, dT$

$= \int kT\, dT = K\int_{250}^{300} TdT$

$= \frac{K}{2}\left[T^2\right]_{250}^{300} = \frac{K}{2}\left[300^2 - 250^2\right]$

Q = 275 × 50



13)

On applying conservation of angular momentum about axis of larger disc.

$\frac{1}{2}.\frac{M}{2}\left(\frac{R}{2}\right)^2 . \omega - \frac{M}{2}(\omega'R).R - \frac{MR^2}{2}\omega' = 0$

$\Rightarrow \omega' = \frac{\omega}{16}$

Hence, n = 16

14) New intensity at B

$$I'_B = \left(\frac{I_B}{2}\right)\cos^2 45^\circ = \frac{I_B}{4}$$

New value of $\alpha = \dfrac{\frac{I_A}{2}}{\frac{I_B}{4}} = \dfrac{2I_A}{I_B}$

$= 2 \times 2;\ \alpha = 4$

15) Frequency received by observer

$f_0 = \left(\dfrac{C \pm V_0}{C \pm V_S}\right) f_S$, C is speed of sound

Case-1:

$$f_2 = \left(\frac{C - V}{C + V}\right) f_S$$

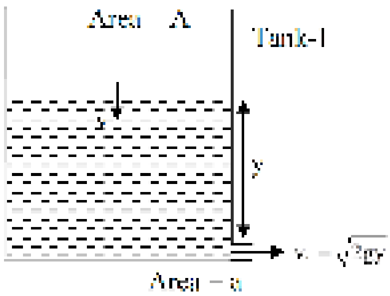
$$n = \left(\frac{C - V}{C + V}\right) 240$$

Case-2:

$$f_1 = \left(\frac{C + V}{C - V}\right) f_S$$

$$288 = \left(\frac{C + V}{C - V}\right) 240$$

16)

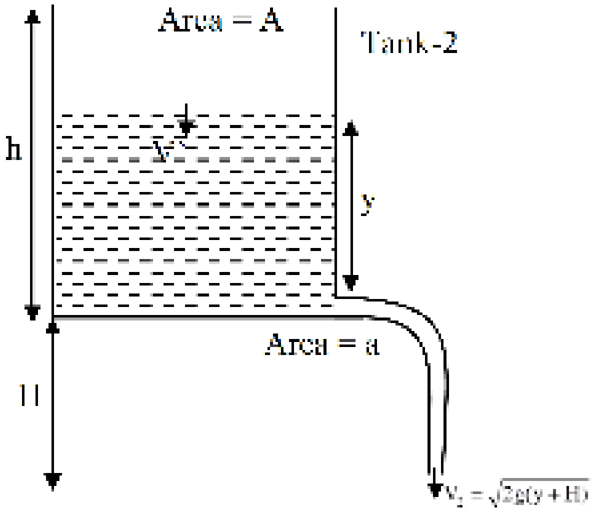


$Av = av_1$

$A\left(-\dfrac{dy}{dt}\right) = a\sqrt{2gy}$; $dt = \dfrac{A}{a\sqrt{2g}} \cdot \dfrac{-dy}{\sqrt{y}}$

$$\int_0^{t_1} dt = \frac{A}{a\sqrt{2g}} \int_h^0 -\frac{dy}{\sqrt{y}}$$

$t_1 = \dfrac{A}{a\sqrt{2g}} 2\sqrt{h}$; $t_1 = \dfrac{A}{a}\sqrt{\dfrac{2h}{g}}$



$Av' = av_2$

$$A\left(-\frac{dy}{dt}\right) = a\sqrt{2g(H+y)}$$

$$dt = -\frac{A}{a\sqrt{2g}}\frac{dy}{\sqrt{H+y}}$$

$$\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_H^0 \frac{dy}{\sqrt{H+y}}$$

$$t_2 = \frac{A}{a\sqrt{2g}}(2)(\sqrt{H+h}-\sqrt{H})$$ & $H = \frac{16h}{9}$

$$= \frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{5}{3}-\frac{4}{3}\right)$$

$$t_2 = \frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{1}{3}\right)$$

ratio $\frac{t_1}{t_2} = 3$

17) Linear impulse $\int Fdt$ = Δ momentum

= m (V_{cm} - 0)

P = m (ω r_{cm})

$$= m\omega\left(L+\frac{L}{2}\right)$$

$$P = m\omega\left(\frac{3L}{2}\right)$$ ....(i)

Angular impulse $\int \tau dt$ = Δ angular momentum

$$\int r \times Fdt = \Delta L$$

$r \times \int Fdt = I(\omega - 0)$, and I is moment of inertia about axis of rotation.

$\left(L+\frac{L}{2}+x\right) \times P$ = (I_{cm} + md^2)ω

$$= \left(\frac{mL^2}{12} + m\left(L + \frac{L}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right) P = mL^2 \left(\frac{1}{12} + \left(\frac{3}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right) P = mL^2 \left(\frac{7}{3}\right)\omega \quad \text{....(ii)}$$

Divide eq.-(i) & (ii)

$$\left(\frac{3L}{2} + x\right) = \frac{L\left(\frac{7}{3}\right)}{\left(\frac{3}{2}\right)}$$

$$\frac{3L}{2} + x = L\left(\frac{14}{9}\right)$$

$$x = \frac{L}{18}$$

PART-2 : CHEMISTRY

18) For Ideal Gas PV = nRT

 n ∝ P at constant T & V.

 mole = $\frac{\text{Mass}}{\text{Molar mass}}$

For gas X : $\frac{10}{M_X} \propto 2$ atm .... (1)

For gas X & Y : $\frac{10}{M_X} + \frac{80}{M_Y} \propto 6$ atm .... (2)

From (2) – (1)

$\frac{80}{M_y} \propto 4$ .... (3)

On dividing (1) by (3)

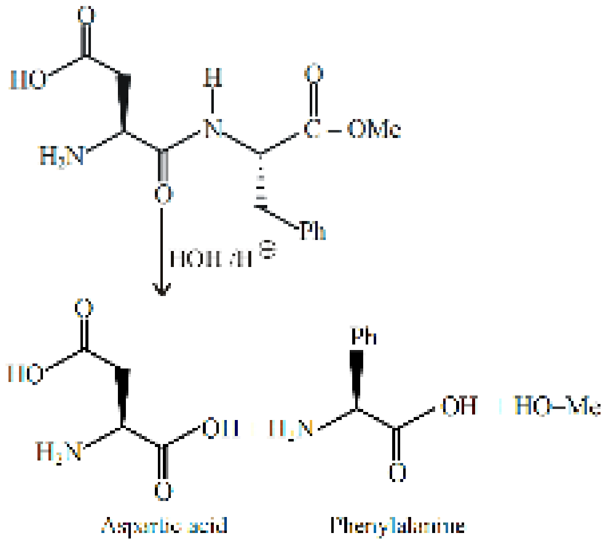
$$\frac{M_Y}{8M_X} = \frac{1}{2}$$

 $\frac{M_Y}{M_X} = 4$ .... (4)

 $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{rms} \propto \frac{1}{\sqrt{M}}$

 $\frac{(V_{rms})_X}{(V_{rms})_Y} = \sqrt{\frac{M_Y}{M_X}} = \sqrt{\frac{4}{1}} = \frac{2}{1}$

19) Aspartame structure is a dipeptide consisting aspartic acid and methyl ester of phenylalanine

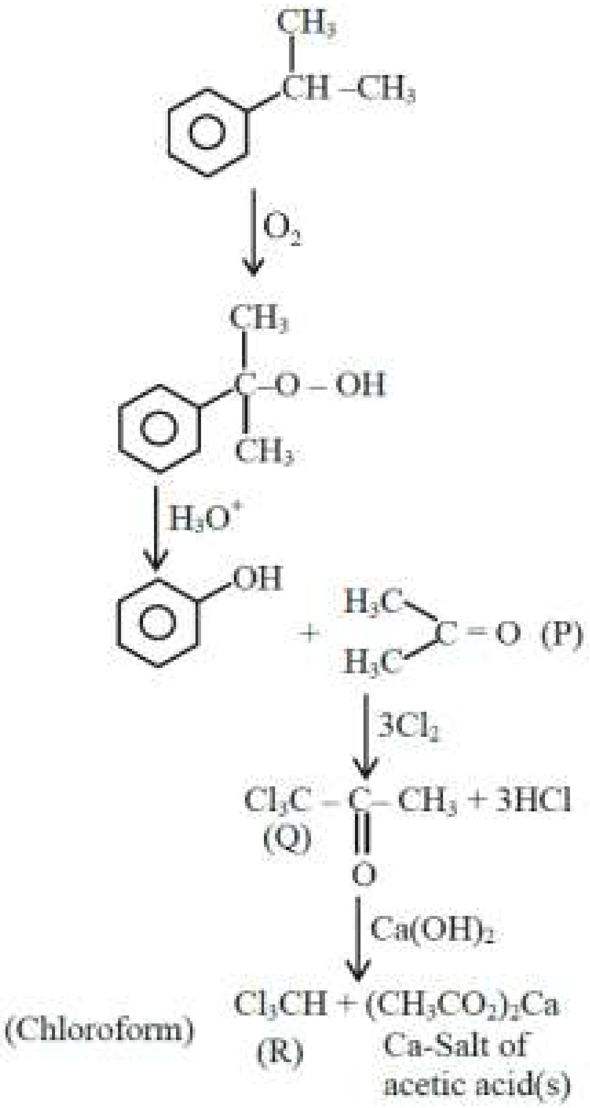


22) (A) Velocity of electron $V_n = 2.19 \times 10^6 \times \frac{Z}{n}$ m/sec.

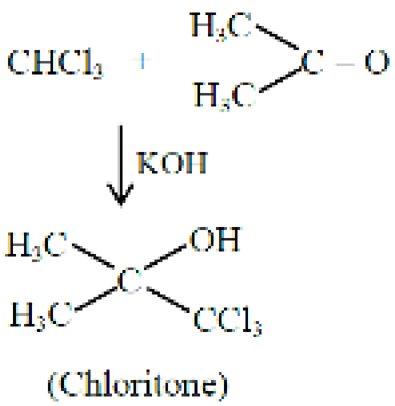
(B) Energy of electron $E_n = -13.6 \times \frac{Z^2}{n^2}$ eV/atom.

(C) Uncertainity principle talks about probability of finding electrons in different regions around the nucleus rather than definite paths.

(D) With increase in distance of electron from the nucleus, its energy increases.



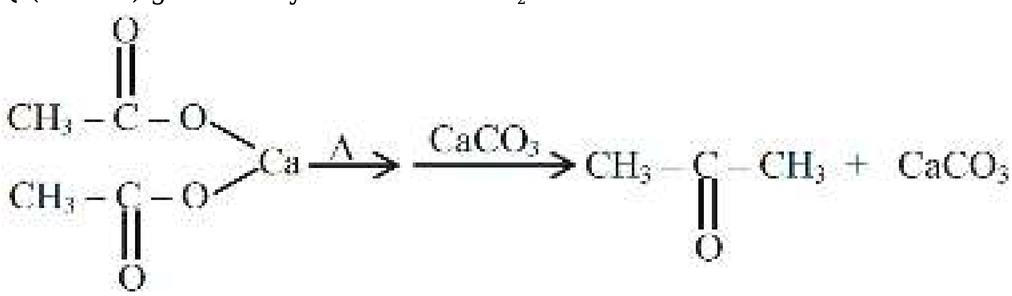
23)



(A)

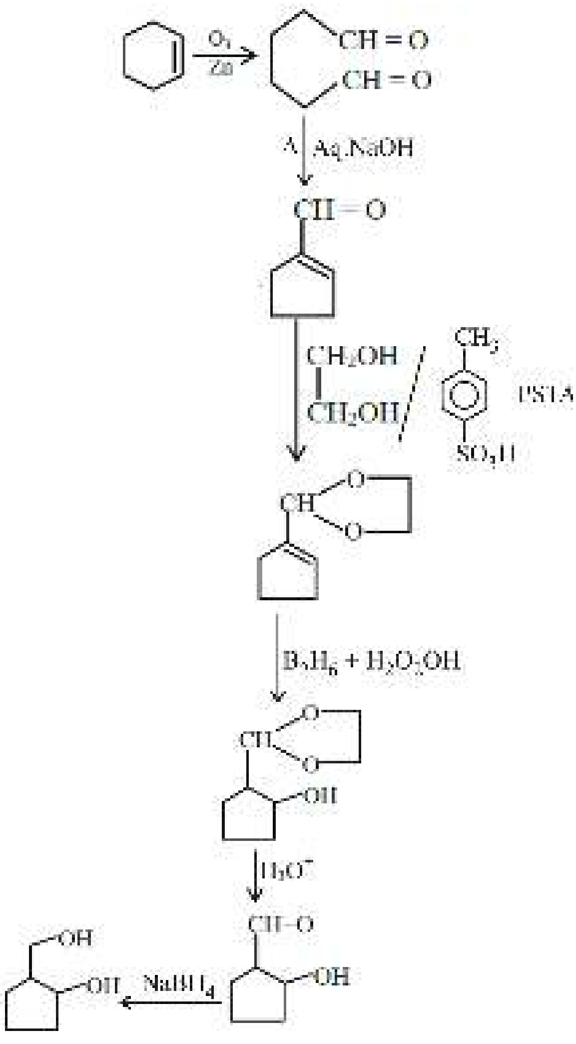
(B) $\underset{(R)}{CHCl_3} \xrightarrow{O_2} \underset{\text{Phosgene gas}}{COCl_2} + HCl$

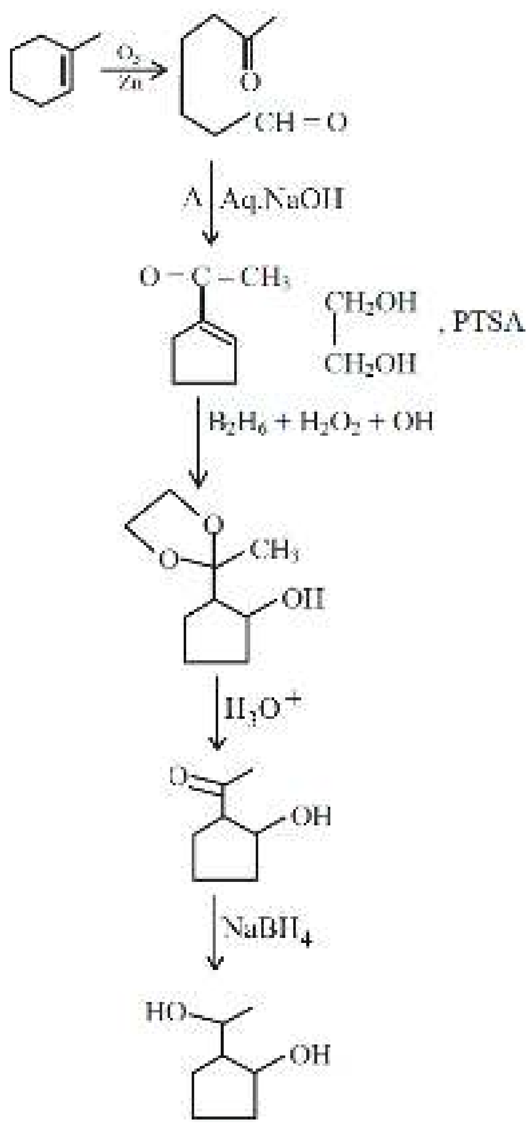
(C) Q (Phenol) gives salicylic acid with CO_2 and reaction is known as Kolbe schmidt reaction



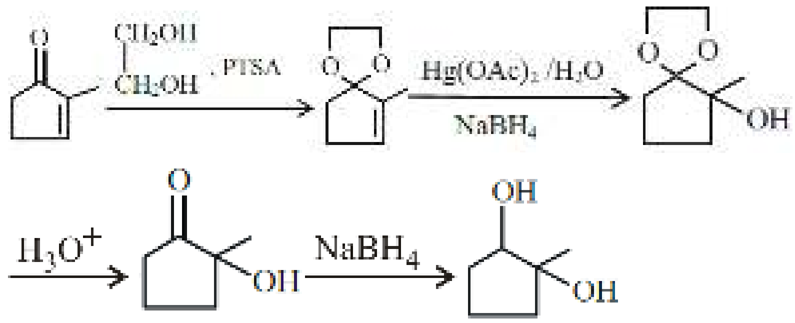
(D)

25) **Option (P) :**
On adding KCl solution to $AgNO_3$ solution precipitation of AgCl will occur due to which already present Ag^+ ions will be replaced by K^+ ions in solution. So conductance of solution will increase. After complete precipitation of AgCl further added KCl will increase the number of ions in resulting solution so conductance will increase further.
Option (Q) :
On adding $AgNO_3$ solution to KCl solution precipitation of AgCl will occur due to which Cl^- already present will be replaced by NO_3^- ions. So conductance of solution will decrease till equivalence point. After complete precipitation of AgCl, further added $AgNO_3$ will increase the number of ions in resulting solution so conductance will increase.
Option (R) :
On adding HCl solution to NaOH solution, OH^- will be replaced by Cl^- ions so conductance of solution decreases. After complete neutralisation further added HCl will increase number of ions in the solution. So conductance will increase futher.

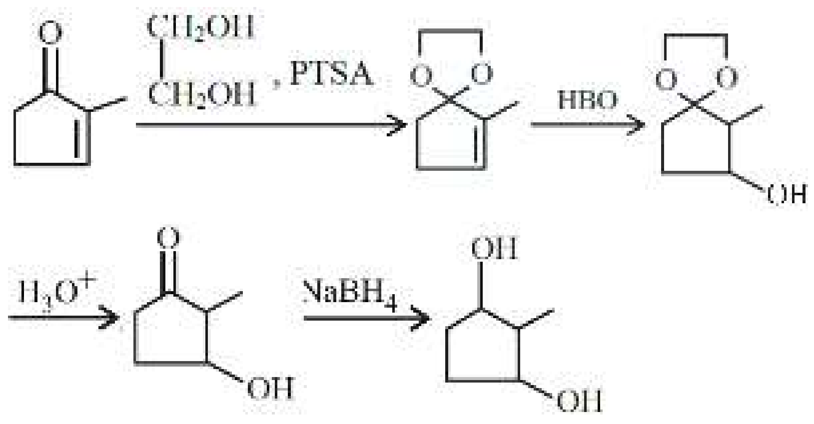
27) 



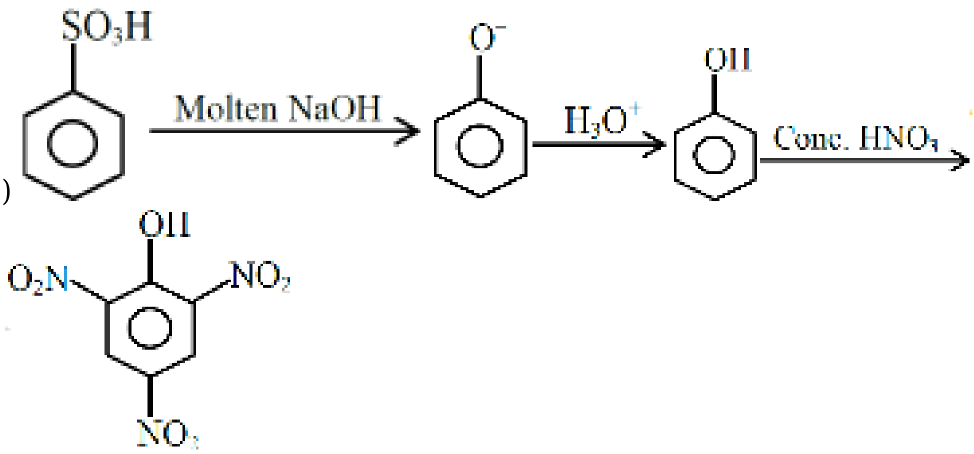
O₃/Zn
Aq.NaOH
CH₂OH
CH₂OH
, PTSA
B₂H₆ + H₂O₂ + OH
H₃O⁺
NaBH₄

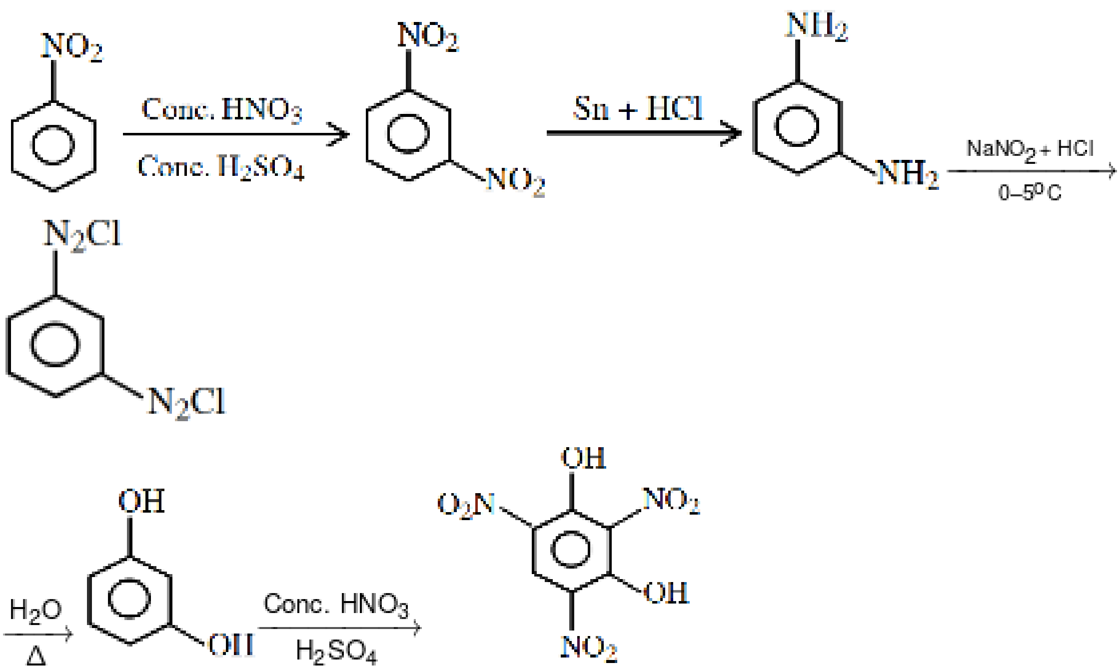


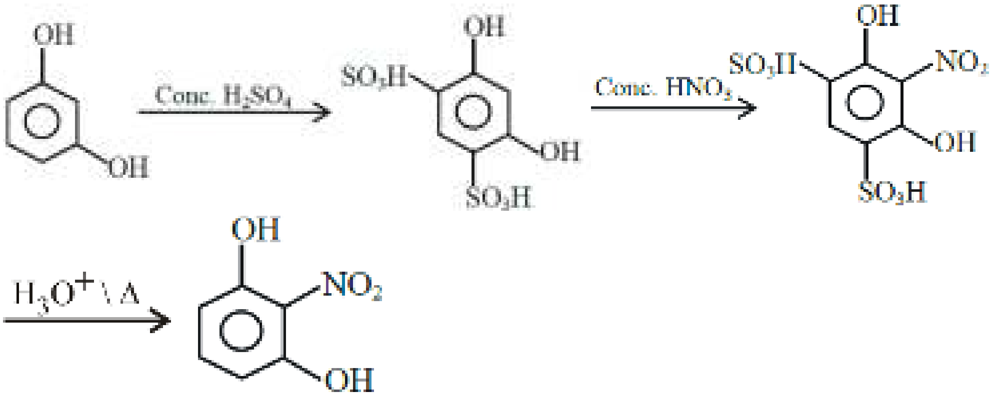
CH₂OH
CH₂OH
, PTSA
Hg(OAc)₂ /H₂O
NaBH₄
H₃O⁺
NaBH₄

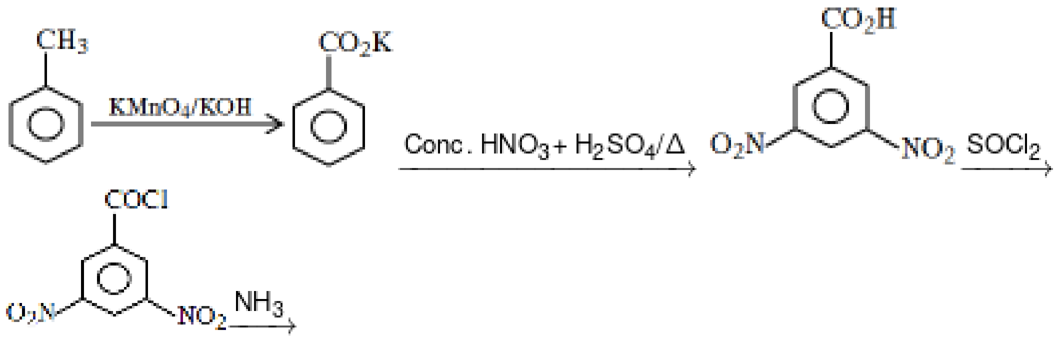


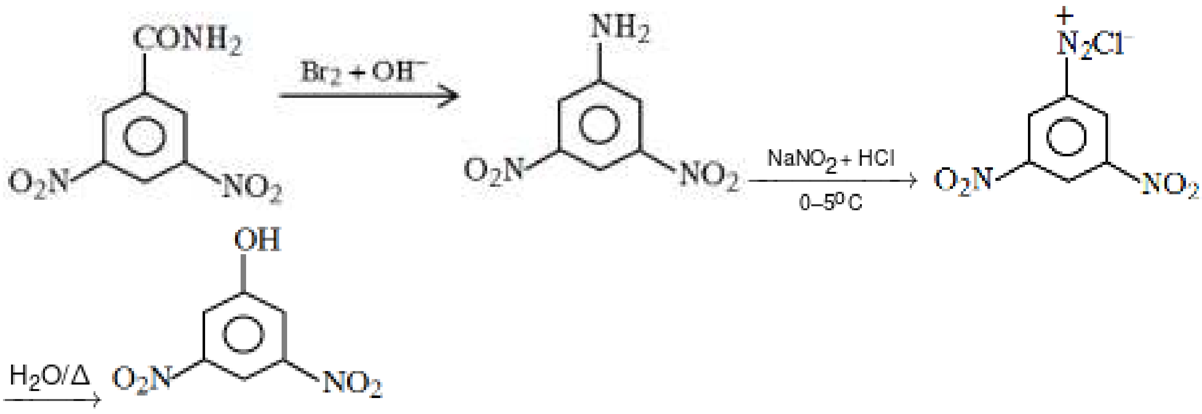
CH₂OH
CH₂OH
, PTSA
HBO
H₃O⁺
NaBH₄

28) 
Molten NaOH
H_3O^+
Conc. HNO_3


Conc. HNO_3
Conc. H_2SO_4
Sn + HCl
$NaNO_2$ + HCl
0–5^{0}C
H_2O
Δ
Conc. HNO_3
H_2SO_4


Conc. H_2SO_4
Conc. HNO_3
H_3O^+ \ Δ


$KMnO_4$/KOH
Conc. HNO_3 + H_2SO_4/Δ
$SOCl_2$
NH_3



29) $\Delta U = n\, C_{V,m}\, \Delta T$

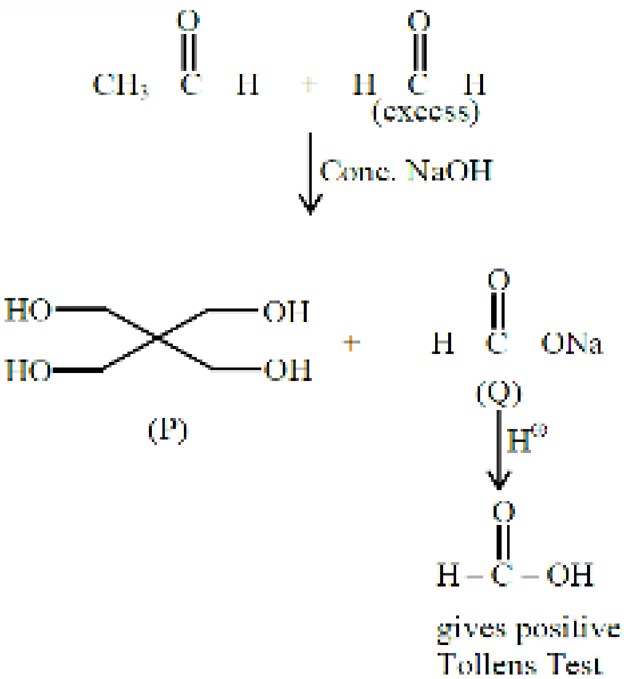
$= 5 \times 12 \times 80 = 4800$ Joules

30) Rate law $= k_2\,[N_2O_2]\,[H_2]$ [∵ slowest step of reaction is RDS]

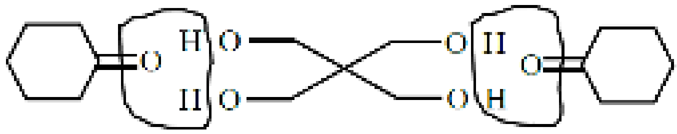
∴ $\dfrac{k_1}{k_{-1}} = \dfrac{[N_2O_2]}{[NO]^2}$

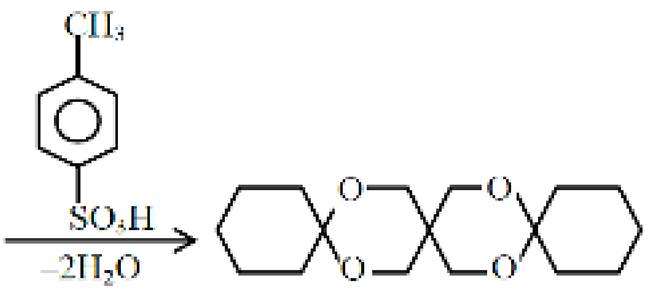
∴ $[N_2O_2] = \dfrac{k_1}{k_{-1}}\,[NO]^2$

∴ Rate $= k_2 \times \dfrac{k_1}{k_{-1}}\,[NO]^2\,[H_2]$



31)

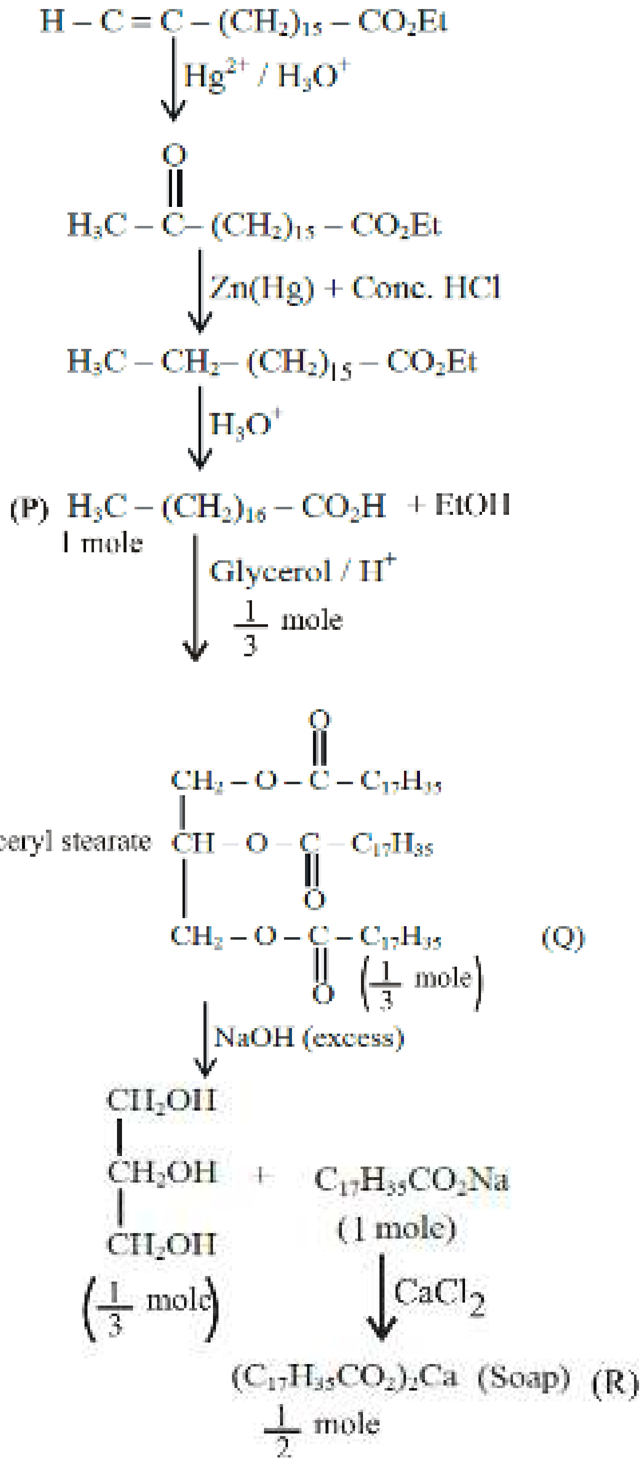




(R)

Total CH_2 in R = 14

33)



$\frac{1}{2}$ mole soap. = $\frac{1}{2}$ × 606 gm = 303 gm

PART-3 : MATHEMATICS

35) Apply L'hospital Rule

$$\lim_{t\to x}\frac{3t^2f(x)-x^3f'(t)}{2t}=2$$

$3x^2f(x) - x^3f'(x) = 4n$

$\Rightarrow x^3f'(x) - 3x^2 f(x) = -4x$

$\Rightarrow f'(x) - \frac{3}{x} f(x) = -\frac{4}{x^2}$

I.F. = $e^{-3\int \frac{1}{x}dx} = \frac{1}{n^3}$

$y\frac{1}{x^3} = \int -\frac{4}{x^5}dx$

$y\left(\frac{1}{x^3}\right) = \frac{1}{x^4} + C$

$f(x) = \frac{1}{x} + Cx^3 \Rightarrow C = 1$

36) $\left(\sin\frac{13x}{2}\sin 7x + \cos\frac{13x}{2}\cos 7x\right) + \left(\sin 7x\cos\frac{13x}{2} - \cos 7x\sin\frac{13x}{2}\right)$

$= \cos\frac{x}{2} + \sin\frac{x}{2} = \sqrt{1+\sin x} = \frac{7}{5}$

$\left(\text{as} \tan x = \frac{-24}{7}, \sin x = \frac{24}{25}\right)$

37) Let no. of questions guessed = x

No. of questions he knew = y

Prob. of a randomly selected

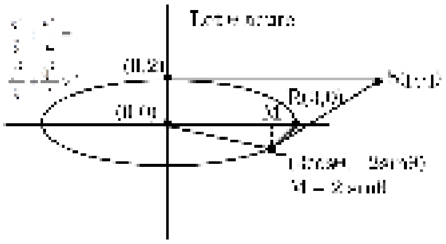
question being correct = P(c) = $\left(\frac{x}{x+y}\right)\frac{1}{4} + \frac{y}{(x+y)} \times 1$

$$P\left(\frac{G}{C}\right) = \frac{\left(\frac{x}{x+y}\right)\frac{1}{4}}{\left(\frac{x}{x+y}\right)\frac{1}{4} + \left(\frac{y}{x+y}\right)} = \frac{1}{6} \Rightarrow \frac{x}{x+4y} = \frac{1}{6}$$

$\Rightarrow 5x + 4y \Rightarrow x = \frac{4y}{5}$

Prob. that student knows the answer

of a randomly selected question = $\frac{y}{x+y} = \frac{y}{\frac{4y}{5}+y} = \frac{5}{9}$

38) 

Area of $\Delta ORP = \frac{9}{5}$

$\frac{1}{2} \times 3 \times |2\sin\theta| = \frac{9}{5}$

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $T\left(\frac{12}{5}, \frac{-6}{5}\right)$

Equation of tangent at (0, 2) : y = 2

Equation of tangent at T : $\frac{x}{9}\left(\frac{12}{5}\right) + \frac{y}{4}\left(\frac{-6}{5}\right) = 1$

q = 2, $P\left(\frac{4}{15}\right) - \frac{3}{5} = 1 \Rightarrow \frac{4P}{15} = \frac{8}{5}$

P = 6

39) $T_1 = \left(2-\sqrt{3}\right)^n = \left(p-q\sqrt{3}\right) < 1$, p, q ∈ z

$T_2 = \left(2+\sqrt{3}\right)^n = \left(p+q\sqrt{3}\right) > 1$, p, q ∈ z

$\cos\pi\left(2+b\sqrt{3}\right) + i\sin\pi\left(a+b\sqrt{3}\right) \in Z$ if

$\sin\pi\left(a+b\sqrt{3}\right) = 0 \Rightarrow b = 0$

40) $a\left(\frac{x}{y}\right)^2 + 2b\left(\frac{x}{y}\right) + c < 0, \forall x, y$

$4b^2 - 4ac < 0$

a < 0 ; D < 0 ⇒ $b^2 < ac$

(B) 4 + c < 0 ⇒ c < –4, True

(C) $\frac{a}{b} = \frac{b}{2c} \neq -1$ ⇒ $b^2 = 2ac$ but $b^2 < ac < 2ac$

so not possible, hence consistent always

(D) $\frac{a}{b} \neq \frac{b}{c}$ ⇒ $b^2 < ac$, so unique solution

41) S : $(x-3)^2 + (y-4)^2 + (z-5)^2 - (x-6)^2 - (y-4)^2 + (z-9)^2 = 100$

⇒ 6x + 8z + (9 + 16 + 25) – (36 + 16 + 81) = 100

S : 6x + 8z = 183

T : 6x + 8z = –17

Distance between planes = $\frac{200}{10} = 20$

42) $x^3 - 7x + 6 = 0$

$x^3 - x - 6x + 6 = 0$
$\Rightarrow (x - 1)(x^2 + x - 6) = 0$
$(x - 1)(x + 3)(x - 2) = 0$
Roots are $x = 1, 2, -3$, $\alpha + \beta + \gamma = 0$

(P) $\begin{bmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{bmatrix}$

first row can be arranged in 3! ways
& 2^{nd} row can be arranged in 2 ways
Req. ways = 12

(Q) $\begin{bmatrix} \alpha & \gamma & \beta \\ \gamma & \beta & \alpha \\ \beta & \alpha & \gamma \end{bmatrix}$

Diagonal entries in 3! ways
& all other entries in 1 way

(R) Let $M = (a_{ij})_{3 \times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in

$$\begin{bmatrix} 0 & p & q \\ -p & 0 & r \\ -q & -r & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ -r \end{bmatrix}$$

$\Delta = 0$, $\Delta_x = \Delta_y = \Delta_3 = 0$
Infinite solution

(S) $|M| = 0$, If $R_i = 0$

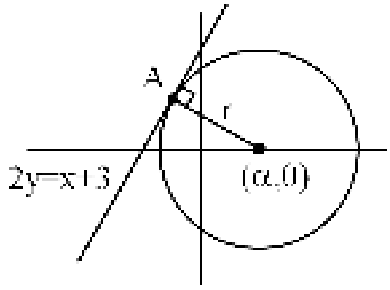
43) $L_1 : \frac{x-1}{2} = \frac{y-1}{3} = \frac{z-4}{\alpha}$ and
$L_2 : \frac{x}{3} = \frac{y}{4} = \frac{z-2}{4}$
$4x - 3y = 0$, $3x - 2y = 1$
$x = 3$, $y = 4$, $z = 6$, $\alpha = 2$
$R_1 = (3, 4, 6)$, $\overrightarrow{OR_1} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 3 & 4 & 4 \end{vmatrix} = \hat{i}(4) - \hat{j}(2) + \hat{k}(-1)$$

$= 4\hat{i} - 2\hat{j} - \hat{k}$

$$\left|\overrightarrow{OR_1}.\vec{n}\right| = \left|\frac{12-8-6}{\sqrt{21}}\right| = \frac{2}{\sqrt{21}}$$

44) $\left|\frac{\alpha+3}{\sqrt{5}}\right| = r$



$\alpha + 3 = \sqrt{5}r$
$\alpha + r = 2 + \sqrt{5}$
$\sqrt{5}r - 3 + r = 2 + \sqrt{5}$
$r\left(\sqrt{5}+1\right) = 5 + \sqrt{5}$
$r = \sqrt{5},\ \alpha = 2$
Equation of circle $(x - 2)^2 + y^2 = 5$
$x^2 + y^2 - 4x - 1 = 0$
Let A be (x_1, y_1)
$xx_1 + yy_1 - 2(x + x_1) - 1 = 0$
$\Rightarrow x(x_1 - 2) + yy_1 - 2x_1 - 1 = 0$
$x \qquad -2y \qquad +3 \qquad = 0$
$\frac{x_1-2}{1} = \frac{y_1}{-2} = \frac{-2x_1-1}{3}$
$x_1 = 1,\ y_1 = 2,\ A_1(1, 2)$
$B_1(3, -2)$ centre $(2, 0)$

45) $h(x) = af(x) + b(g(x) + g(1 - x)) + c(x - g(x)) + dg(x),\ x \in R$

$g(1-x) = \begin{cases} 1-(1-x) & , \quad 0 \leqslant 1-x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$; $g(x) = \begin{cases} 1-x & , \quad 0 \leqslant x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

$g(1-x) = \begin{cases} x & , \quad 0 \leqslant x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

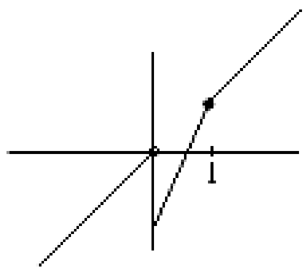
$g(x) + g(1-x) = \begin{cases} 1 & , \quad 0 \leqslant x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

(P) $b = 1,\ a = c = d = 0,\ h(x) = \begin{cases} 1 & , \quad 0 \leqslant x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

(Q) $a = 1,\ b = c = d = 0,\ h(x) = \begin{cases} x^2 \sin\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

(R) $c = 1,\ a = b = d = 0,$

$h(x) = \begin{cases} 2x-1 & , \quad 0 \leqslant x \leqslant 1 \\ x & , \quad \text{otherwise} \end{cases}$



(S) $d = 1,\ a = b = c = 0$

$h(x) = \begin{cases} 1-x & , \quad 0 \leqslant x \leqslant 1 \\ 0 & , \quad \text{otherwise} \end{cases}$

46) $\log_a 144 = \log_{\sqrt{12}}\left(\sqrt{12}\right)^4 = 4$
$\log_b \sqrt{18} = 3$
$ab^3 = 6\sqrt{6},\ 2\log_6 ab^3 = 3$
$\left.\begin{array}{l} 2x + 3y = 7 \\ 2x - y = 3 \end{array}\right\}\ x = 2,\ y = 1$

$3x + 4y = 10$

47) $f(x) = x^4 + ax^3 + bx^2 + c$,
$f(1) = 1 + a + b + c = -7$
$a + b + c = -8$
$4x^3 + 3ax^2 + 2bx = 0$ has roots $\sqrt{2}i, -\sqrt{2}i, 0$
$a = 0$, $4x^2 + 3ax + 2b = \frac{b}{2} = -2i^2$ $\Rightarrow b = 4$
$f(x) = x^4 + 4x^2 + c$, $c = -12$
$x^4 + 4x^2 - 12 = 0$
$(x^2 + 6)(x^2 - 2) = 0$
$x = x = \pm\sqrt{6}i, x^2 = \pm\sqrt{2}$
$\sum |\alpha_1|^2 = 6 + 6 + 2 + 2 = 16$

48) Case-I : If S_3 goes to term X $= {}^9C_2 \times \frac{9!}{4!5!} = 4536$
Case-II : If S_3 goes to team Z $= {}^9C_3 \times \frac{8!}{4!4!} \times \frac{1}{2!} \times 2! = 5880$
Total ways = 10416

49) $\begin{vmatrix} \alpha-1 & 1 & 2 \\ \beta-2 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 0$
$\Rightarrow (\alpha - 1)(3) - (\beta - 2)(-1) + 1(-5) = 0$
$3\alpha + \beta = 10$
$(\alpha, \beta, 2)$ lie on $3x + y - z - \ell = 0$
$3\alpha + \beta - 2 = \ell$
$\ell = 3\alpha + \beta - 2 = 8$

50) $|A| = \begin{vmatrix} 0 & 1 & c \\ 1 & 2a & d \\ 1 & 2b & e \end{vmatrix}$
$|-1(e-d) + c(2b-2a)| = 2$
$\left|\frac{(d-e)}{2} + c(b-a)\right| = 1$
$\frac{d-e}{2}$ must be integer so $\Rightarrow d = e$
so $d = e = 0$ or 1
$\Rightarrow |c(b - a)| = 1$
$c = 1$, $b - a = \pm 1$
$\Rightarrow$ No. of ways = 4

51) Let equation of line is $y = mx + c$

X	1	2	3	4	5
P(X)	m+c	2m+c	3m+c	4m+c	5m+c

$\Sigma P(x) = 1 \Rightarrow 15m + 5c = 1 \Rightarrow 3m + c = \frac{1}{5}$

Mean, $\mu = \Sigma P(x_i)\, x_i$

$= \sum_{r=1}^{5} r(mr + c) = 55m + 15c = \frac{10}{3}$

$\Rightarrow 11m + 3c = \frac{2}{3}$

$\Rightarrow m = \frac{1}{30}, c = \frac{1}{10}$

var.$(\alpha) = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2$

$\alpha = \sum r^2 (rm + c) - (\mu)^2$

$\alpha = m\sum r^3 + c\sum r^2 - (\mu)^2$

$= \frac{1}{30}(15)^2 + \frac{1}{10}(55) - \left(\frac{10}{3}\right)^2$

$\alpha = \frac{15}{2} + \frac{11}{2} - \frac{100}{9} \Rightarrow 9\alpha = 13 \times 9 - 100$

$9\alpha = 17$