TEST CODE : 113103

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IIT-JEE

Batch – GROWTH (JUNE) | MAJOR Test-02 (PAPER – I)

Time: 3 Hours | **Test Date: 24th November 2024** | **Maximum Marks: 180**

Name of the Candidate: ____ Roll No.______________

Centre of Examination (in Capitals):____

Candidate's Signature:____________________________ Invigilator's Signature:___________________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space)
   on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 54 questions.
3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 18 questions each and every **PART** has three sections.
   - **(i)** **Section-I** contains **8** Numerical Value questions.
     Marking scheme: +3 for correct answer, 0 if not attempted and 0 in all other cases.
   - **(ii)** **Section-II** contains **6 Question** Multiple Choice Option with more than one correct answer.
     Marking scheme: (+4 for correct answer, 0, if not attempted and +1 partial marking —2 in all other cases.
   - **(iii)** **Section-III** contains **4** questions the answer to only 4 questions, is an **List Match Question.**
     Marking scheme: +3 for correct answer, 0 if not attempted and —1 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For integer-based questions, the answer should be in decimals only not in fraction.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

TEST SYLLABUS

Batch – GROWTH (June) | Major Test-02 (Paper – I)

24th November 2024

Mathematics	:	Compound Angle & Trigonometric Eq Quadratic Eq St. Line
Physics	:	NLM & Friction WEP Circular Motion, Centre of Mass, Momentum & Collision
Chemistry	:	Chemical Bonding Thermodynamics-1 Thermochemistry & Thermodynamics-2

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821\,Lit\ atm\ K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023\times10^{23}$
Planck's Constant	h	$= 6.626\times10^{-34}\,Js$
		$= 6.25\times10^{-27}\,erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66\times10^{-27}\,kg$
1 eV		$= 1.6\times10^{-19}\,J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I (Numerical Value)

1. The maximum value of $\frac{1}{\sin^6 x+\cos^6 x}, (x \in R)$ is________

Ans. (4)

Sol. $\frac{1}{(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)}=\frac{1}{1-\frac{3}{4}(\sin 2x)^2}$

$=\max.=\frac{1}{1-\frac{3}{4}(1)};\ \min=\frac{1}{1-\frac{3}{4}(0)}$

max = 4, min = 1

2. Total number of solution $\sin x.\tan 4x=\cos x$ in x $\in (0,\pi)$ is__________

Ans. (5)

Sol. $\sin x\cdot\tan 4x=\cos x$

$\sin x\cdot\sin 4x=\cos x\cdot\cos 4x$

$\cos 4x\cdot\cos x-\sin x\cdot\sin 4x=0$

$\cos(4x+x)=0$

$\cos 5x=0$

$5x=(2n-1)\frac{\pi}{2}\quad (n\in I)$

$x=(2n-1)\frac{\pi}{10}$

$\therefore\quad x=\frac{\pi}{10},\frac{3\pi}{10},\frac{5\pi}{10},\frac{7\pi}{10},\frac{9\pi}{10}$

Number of solutions = 5

3. If the difference of roots of the equation (a −2)x^2 – (a – 4)x – 2 = 0 is equal to 3 then the greatest value of a.

Ans. (3)

Sol.

$$(a-2)x^2-(a-4)x-2=0$$
$$|\alpha-\beta|=3$$
$$\sqrt{(\alpha+\beta)^2-4\alpha\beta}=3$$
$$(\alpha+\beta)^2-4\alpha\beta=9$$
$$\left(\frac{a-4}{a-2}\right)^2-4\frac{(-2)}{a-2}=9$$
$$(a-4)^2+8(a-2)=9(a-2)^2$$
$$a^2+16-8a+8a-16=9(a^2+4-4a)$$
$$a^2=9a^2+36-36a$$
$$8a^2-36a+36=0$$
$$2a^2-9a+9=0$$
$$2a^2-6a-3a+9=0$$
$$2a(a-3)-3(a-3)=0$$
$$(2a-3)(a-3)=0$$

$$a=\frac{3}{2} \text{ or } 3$$

4. If $\cos x+\cos^2 x=1$. Let $E=\sin^{12}x+3\sin^{10}x+3\sin^8 x+\sin^6 x+2$, then the value of $\log_{\tan\frac{\pi}{3}} E$ is:

Ans. (2)

Sol.

$\cos x+\cos^2 x=1 \Rightarrow \cos x=\sin^2 x$

Now, $E=\sin^{12}x+3\sin^{10}x+3\sin^8 x+\sin^6 x+2$

$=\cos^6 x+3\cos^5 x+3\cos^4 x+\cos^3 x+2=(\cos^2 x+\cos x)^3+2=1+2=3$

$\log_{\tan\frac{x}{3}} E=\log_{\sqrt{3}} 3=2$

5. If $3a+2b+6c=0$ the family of straight lines $ax+by+c=0$ passes through a fixed point. If the coordinates of fixed point is (p,q). Find 6(p+q).

Ans. (5)

Sol. Given, $3a+2b+6c=0$

or $\frac{a}{2}+\frac{b}{3}+c=0$ ....(i)

and family of straight lines is

$ax+by+c=0$ ....(ii)

Subtracting Eqs. (i) from (ii), then

$$a\left(x-\frac{1}{2}\right)+b\left(y-\frac{1}{3}\right)=0$$

which is equation of a line passing through the point of intersection of the lines

$$x-\frac{1}{2}=0 \quad \text{and} \quad y-\frac{1}{3}=0$$

$\therefore$ The coordinates of fixed point are $\left(\frac{1}{2},\frac{1}{3}\right)$

6. If the distance of the point (2,3) from the line 2x – 3y + 9 = 0 measured along the line 2x – 2y + 5 = 0 is d. Find d^2.

Ans. (32)

Sol. Since slope of the line 2x – 2y + 5 = 0 is 1, it makes an angle $\frac{\pi}{4}$ with positive direction of X – axis.

The equation of the line through (2,3) and making an angle $\frac{\pi}{4}$ in parametric form

$$\frac{x-2}{\cos\left(\frac{\pi}{4}\right)}=\frac{y-3}{\sin\left(\frac{\pi}{4}\right)}=r \text{ or } \frac{x-2}{\frac{1}{\sqrt{2}}}=\frac{y-3}{\frac{1}{\sqrt{2}}}=r$$

Coordinates of any point on this line are $\left(2+\frac{r}{\sqrt{2}},3+\frac{r}{\sqrt{2}}\right)$.

This point lies on the line 2x – 3y + 9 = 0

$$\Rightarrow \quad 2\left(2+\frac{r}{\sqrt{2}}\right)-3\left(3+\frac{r}{\sqrt{2}}\right)+9=0$$

$$\Rightarrow \quad -\frac{r}{\sqrt{2}}+4=0$$

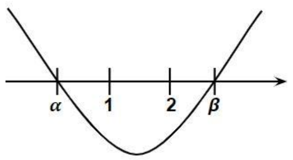
$$\therefore \quad r=4\sqrt{2}$$

$$\therefore \quad d^2=r^2=32$$

7. The number of integral values of a for which one root of equation $(a-5)x^2-2ax+a-4=0$ is smaller than 1 and the other greater than 2.

Ans. (18)

Sol.



$(a-5)x^2-2ax+a-4=0$

$(a \neq 5)$ as equation has two roots.

$\Rightarrow x^2-\left(\frac{2a}{a-5}\right)x+\left(\frac{a-4}{a-5}\right)=0$

Let $f(x)=x^2-\left(\frac{2a}{a-5}\right)x+\left(\frac{a-4}{a-5}\right)$

Required conditions are

(i) f(1)<0 and (ii) f(2)<0

(i) $f(1)<0$

$\Rightarrow \frac{(-9)}{a-5}<0$

$\Rightarrow a \in (5,\infty) \quad \ldots(1)$

(ii) $f(2)<0$

$\Rightarrow \frac{a-24}{a-5}<0$

$\Rightarrow 5<a<24$

$\Rightarrow a \in (5,24) \quad \ldots(2)$

From (1) and (2)

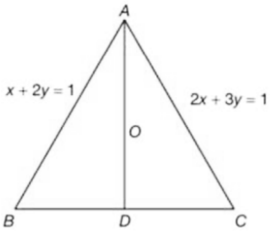
a∈(5,24)

Hence, the number of integral values of a is 18.

8. If the orthocentre of the triangle formed by $2x+3y-1=0, x+2y-1=0, ax+by-1=0$ is at the origin, then $\frac{b-a}{4}$ equals ____________.

Ans. (4)

Sol. Solving $2x+3y=1, x+2y=1, A=(-1,1)$



Orthocentre $=(0,0)$

$\Rightarrow$ Slope if altitude $AD=-1$

Equation of BC is $x-y=k$

Solving $x-y=k$ and $x+2y=1$, we get,

$$B=\left(\frac{1+2k}{3},\frac{1-k}{3}\right)$$

Slope of OB $=\frac{1-k}{1+2k}$, slope of AC $=\frac{-2}{3}$

$$\therefore \frac{1-k}{1+2k}=\frac{3}{2}\Rightarrow k=\frac{-1}{8}$$

Equation of BC is $x-y+\frac{1}{8}=0$

$\Rightarrow\ -8x+8y-1=0$

$\Rightarrow\ a=-8, b=8$

Hence, the correct answer is (4).

SECTION-II (One or More than One Correct)

9. The numerical value of $\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}$ is equal to :

(A) $-\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $-\frac{3}{2}$ (D) $\frac{1}{2}$

Ans. (D)

Sol. $\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}$

$$=\frac{\cos\left(\frac{\pi}{7}+\frac{3-1}{2}.\frac{2\pi}{7}\right)}{\sin\frac{\pi}{7}}.\sin\frac{3\pi}{7}$$

$$=\frac{\cos\left(\frac{3\pi}{7}\right).\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{7}}=\frac{\sin\frac{6\pi}{7}}{2\times\sin\frac{\pi}{7}}=\frac{\sin\frac{\pi}{7}}{2\times\sin\frac{\pi}{7}}=\frac{1}{2}$$

10. Angle between the lines $2x^2-7xy+3y^2=0$ are

(A) 45º (B) 135º (C) 60º (D) 120º

Ans. (A, B)

Sol. $\tan\theta=\frac{2\sqrt{(-7/2)^2-2\times 3}}{2+3}\Rightarrow$ $\tan\theta=1$

$\Rightarrow\ \theta=45^\circ$

Angle between the lines 45º and 135º

11. If x is real, the maximum and minimum values of expression $\frac{x^2+14x+9}{x^2+2x+3}$ will be

(A) 4, - 5
(B) 5, -4
(C) -4, 5
(D) -4, -5

Ans. (A)

Sol. $y=\frac{x^2+14x+9}{x^2+2x+3}$

$x^2y+2xy+3y-x^2-14x-9=0$

$(y-1)x^2+x(2y-14)+3y-9=0$

$D\geq 0$

$(2y-14)^2-4(y-1)(3y-9)\geq 0$

$4(y-7)^2-4(3y^2-12y+9)\geq 0$

$y^2+49-14y-3y^2+12y-9\geq 0$

$-2y^2-2y+40\geq 0$

$y^2+y-20\leq 0$

$(y+5)(y-4)\leq 0$

$y\in[-5,4]$

Maximum value = 4

Minimum value = -5

12. Which of the following is/are correct.

(A) $(\tan x)^{\ln(\sin x)}>(\cot x)^{\ln(\sin x)}, \forall x\in(0,\pi/4)$

(B) $4^{\ln \operatorname{cosec} x}<5^{\ln \operatorname{cosec} x}, \forall x\in(0,\pi/2)$

(C) $(1/2)^{\ln(\cos x)}<(1/3)^{\ln(\cos x)}, \forall x\in(0,\pi/2)$

(D) $2^{\ln(\tan x)}>2^{\ln(\sin x)}, \forall x\in(0,\pi/2)$

Ans. (A,B,C,D)

Sol.

(a) For $x\in\left(0,\frac{\pi}{4}\right)$, $\tan x<\cot x$

Also $\ln(\sin x)<0$

$\Rightarrow$ $(\tan x)^{\ln(\sin x)}>(\cot x)^{\ln(\sin x)}$

(b) For $x\in\left(0,\frac{\pi}{2}\right)$, $\operatorname{cosec} x\geq 1$

$\Rightarrow$ $\ln(\operatorname{cosec} x)\geq 0\Rightarrow 4^{\ln \operatorname{cosec} x}<5^{\ln(\operatorname{cosec} x)}$

(c) $x\in\left(0,\frac{\pi}{2}\right)$ $\Rightarrow$ $\cos x\in(0,1)$

$\Rightarrow \ln(\cos x)<0$ Also, $\frac{1}{2}>\frac{1}{3}$

$\Rightarrow$ $\left(\frac{1}{2}\right)^{\ln(\cos x)}<\left(\frac{1}{3}\right)^{\ln(\cos x)}$

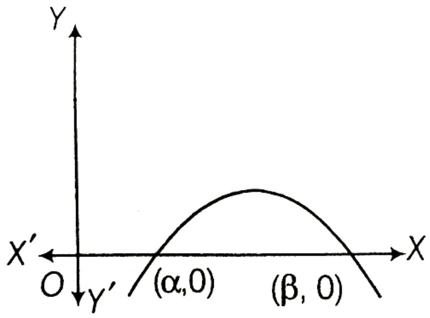
(d) For $x\in\left(0,\frac{\pi}{2}\right)$

Since, $\sin x<\tan x$, we get

$\ln(\sin x)<\ln(\tan x)$

$\Rightarrow$ $2^{\ln(\sin x)}<2^{\ln(\tan x)}$

13. The adjoining graph of $y = ax^2 + bx + c$ shows that



(A) a < 0
(B) $b^2 - 4ac > 0$
(C) c > 0
(D) a and b are of opposite signs.

Ans. (A,B,D)

Sol. I. $a < 0,$ graph open to downward.

II. $D > 0,$ (distinct real roots)

$b^2 - 4ac > 0$

III. c < 0, graph will intersect y – axis below the x – axis.

IV. Sum of roots > 0

$$\frac{-b}{a} > 0$$

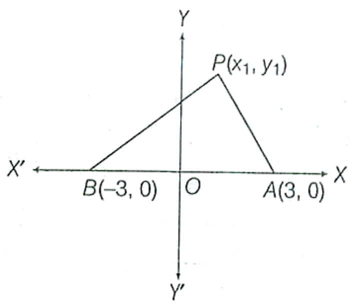
b and a are of opposite sign.

14. Find the equation of the locus of a point which moves so that the difference of its distances from the points (3,0) and (-3,0) is 4 units.

(A) $\frac{x^2}{4} + \frac{y^2}{5} = 1$
(B) $\frac{x^2}{5} + \frac{y^2}{4} = 1$
(C) $\frac{x^2}{4} - \frac{y^2}{5} = 1$
(D) $\frac{x^2}{5} - \frac{y^2}{4} = 1$

Ans. (C)

Sol. Let $P(x_1, y_1)$ be the moving point whose locus is required and A (3,0) and B(-3,0) be the given fixed points.



By hypothesis

$$|PB| - |PA| = 4 \text{ (assume } |PB| > |PA|)$$

$$\Rightarrow \sqrt{(x_1+3)^2 + (y_1-0)^2} - \sqrt{(x_1-3)^2 + (y_1-0)^2} = 4$$

$$\Rightarrow \sqrt{(x_1^2 + y_1^2 + 6x_1 + 9)} = 4 + \sqrt{(x_1^2 + y_1^2 - 6x_1 + 9)}$$

Squaring both sides then,

$$x_1^2 + y_1^2 + 6x_1 + 9 = 16 + x_1^2 + y_1^2 - 6x_1 + 9 + 8\sqrt{(x_1^2 + y_1^2 - 6x_1 + 9)}$$

or $$(12x_1 - 16) = 8\sqrt{(x_1^2 + y_1^2 - 6x_1 + 9)}$$

or $$(3x_1 - 4) = 2\sqrt{(x_1^2 + y_1^2 - 6x_1 + 9)}$$

Again, squaring both sides, then

$$9x_1^2 - 24x_1 + 16 = 4x_1^2 + 4y_1^2 - 24x_1 + 36$$

or $$5x_1^2 - 4y_1^2 = 20$$

$$\Rightarrow \frac{x_1^2}{4} - \frac{y_1^2}{5} = 1$$

Changing (x_1, y_1) by (x, y), then

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

which is the required locus of P.

SECTION-III (Match List Type)

15. Match the following:

	List-I		List-II
(I)	If $x^2 + px + q = 0$ is the quadratic equation whose roots are a – 2 and b – 2, where a and b are the roots of $x^2 - 3x + 1 = 0$, then p + q is	(P)	-1
(II)	If both the roots of $k(6x^2+3)+rx+2x^2-1=0$ and $2(6k+2)x^2+px+2(3k-1)=0$ are same, then 2r–p is equal to	(Q)	-3
(III)	If $x^2 + \lambda x + 1 = 0$ and $(b-c)x^2 + (c-a)x + (a-b) = 0$ have both the roots common, then $[\lambda - 1]$ is (where [.] denotes the greatest integer function)	(R)	0
(IV)	If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root $a \neq 0$, then $\frac{a^3+b^3+c^3}{abc}$ is equal to	(S)	3

(A) (I) → (R); (II) → (R); (III) → (Q); (IV) → (S)
(B) (I) → (P); (II) → (R); (III) → (S); (IV) → (Q)
(C) (I) → (S); (II) → (R); (III) → (Q); (IV) → (P)
(D) (I) → (Q); (II) → (S); (III) → (R); (IV) → (P)

Ans. (A)

Sol. (I) a+b=3,ab=1 and –p=a –2+b –2,q=(a –2)(b –2)

$\Rightarrow -p=a+b-4, q=ab-2(a+b)+4$

$\Rightarrow -p=3-4$ and $q=1-2(3)+4$

$\Rightarrow (p,q)=(1,-1)$

$\Rightarrow p+q=0$

(II)

Given equations can be written as

$k(6x^2+3)+rx+2x^2-1=0$ ...(i) and

$2(6k+2)x^2+px+2(3k-1)=0$ ...(ii)

Condition for common roots is that the corresponding coefficients must be proportional.

$\Rightarrow \frac{12k+4}{6k+2}=\frac{p}{r}$

$\Rightarrow \frac{6k-2}{3k-1}=2$

$\Rightarrow 2r-p=0$

(III) $\because$ $x=1$ satisfies

$(b-c)x^2+(c-a)x+(a-b)=0$

$\therefore$ $x=1$ satisfies $x^2+\lambda x+1=0$

Then $1+\lambda+1=0 \Rightarrow \lambda=-2$

$\Rightarrow [\lambda-1]=[-2-1]=-3$

(IV) $ax^2+bx+c=0$ and $bx^2+cx+a=0$ has a common root, then

$\frac{x^2}{ab-c^2}=\frac{x}{cb-a^2}=\frac{1}{ac-b^2}$

$\frac{x^2}{ab-c^2}=\frac{1}{ac-b^2}$

$\Rightarrow x^2=\frac{ab-c^2}{ac-b^2}$.......(1)

$\frac{x}{cb-a^2}=\frac{1}{ac-b^2}$

$\Rightarrow x=\frac{cb-a^2}{ac-b^2}$

Using (1)

$\frac{ab-c^2}{ac-b^2}=\left(\frac{cb-a^2}{ac-b^2}\right)^2$

$\Rightarrow (cb-a^2)^2=(ab-c^2)(ac-b^2)$

$\Rightarrow b^2c^2+a^4-2a^2bc=a^2bc-ab^3-ac^3+c^2b^2$

$\Rightarrow a^3+b^3+c^3=3abc$

$$\Rightarrow \frac{a^3+b^3+c^3}{abc}=3$$

16. Let $\cos(\theta+70°)=\frac{-1}{3}$ where $\theta \in (0°,110°)$

	Column-I		Column-II
(P)	$\tan(\theta+70°)=$	(1)	$2\sqrt{2}$
(Q)	$\cos(160°+\theta)=$	(2)	$\frac{9+4\sqrt{2}}{7}$
(R)	$\sin(20°-\theta)$	(3)	$-2\sqrt{2}$
(S)	$\tan(25°+\theta)=$	(4)	$\frac{-2\sqrt{2}}{3}$
		(5)	$\frac{-1}{3}$

(A) P-5 ; Q-3 ; R-4 ; S-1

(B) P-3 ; Q-4 ; R-5 ; S-2

(C) P-3 ; Q-5 ; R-2 ; S-4

(D) P-2 ; Q-4 ; R-5 ; S-3

Ans. (B)

Sol. $\cos\left(\theta+70^\circ\right)=\frac{-1}{3}$ where $\theta\in\left(0^\circ,110^\circ\right)$

(P) $\theta+70^\circ\in\left(70^\circ,180^\circ\right)$

$\therefore\ \tan\left(\theta+70^\circ\right)=-2\sqrt{2}$

(Q) $\cos\left(160^\circ+\theta\right)=\cos\left(90^\circ+\theta+70^\circ\right)=-\sin\left(\theta+70^\circ\right)=\frac{-2\sqrt{2}}{3}$

(R) $\sin\left(20^\circ-\theta\right)=\sin\left(90^\circ-\left(\theta+70^\circ\right)\right)=\cos\left(\theta+70^\circ\right)=\frac{-1}{3}$

(S) $\tan\left(25^\circ+\theta\right)=\tan\left(70^\circ+\theta-45^\circ\right)=\frac{\tan\left(70^\circ+\theta\right)-1}{1+\tan\left(70^\circ+\theta\right)}=\frac{-2\sqrt{2}-1}{1-2\sqrt{2}}=\frac{9+4\sqrt{2}}{7}$

17. Match the following:

	List-I		List-II
(I)	The points on the line $x+y=2$, which are at a unit distance from the line $4x+3y=12$ are	(P)	(1, 1)
(II)	At what point the origin be shifted such that the equation $x^2+xy-3x-y+2=0$ does not contain any 1st degree term and constant term	(Q)	(11, -9)
(III)	If the algebraic sum of the perpendicular distances from the points $(2,0),(0,2)$ and $(1,1)$ to a variable line is zero, then the line passes through the fixed point, which is	(R)	(9, -7)
(IV)	If O is the origin and P is a variable point on $x^2=2y$, then point which lies on the locus of the midpoint of OP is	(S)	(4, 2)
		(T)	(2, 4)

(A) (I) $\rightarrow$ (P,Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (P)
(B) (I) $\rightarrow$ (P,Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (T)
(C) (I) $\rightarrow$ (P); (II) $\rightarrow$ (S); (III) $\rightarrow$ (R); (IV) $\rightarrow$ (Q)
(D) (I) $\rightarrow$ (S); (II) $\rightarrow$ (R,Q); (III) $\rightarrow$ (P); (IV) $\rightarrow$ (Q)

Ans. (B)

Sol. $I\rightarrow(P,Q);II\rightarrow(P);III\rightarrow(P);IV\rightarrow(T)$

(I) $(t,2-t)\Rightarrow\left|4(t)+3(2-t)-12\right|=5\Rightarrow t=11,1$

(II) Let shifted to $(a,b)\Rightarrow(x+a)^2+(x+a)(y+b)\ -3(x+a)-(y+b)+2=0$ for given is true it $a=1$ and $b=1$

(III) Line passed through the centroid

(IV) $P(h,k) \Rightarrow h^2 = 2k$. Let, (α,β) be the locus the point then $h = 2\alpha, k = 2\beta \therefore x^2 = y$

18. Match the following:

List-I		List-II	
(I)	Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$	(P)	4
(II)	Show that $\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ$	(Q)	$2\cos\theta$
(III)	Find the value of $\dfrac{1-\tan^2\left(\dfrac{\pi}{4}-A\right)}{1+\tan^2\left(\dfrac{\pi}{4}-A\right)}$	(R)	$\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right)^2$
(IV)	Find the value of $\dfrac{1+\sin 2\theta}{1-\sin 2\theta}$	(S)	$\sin 2A$

(A) (I) → (P); (II) → (Q); (III) → (R); (IV) → (S)
(B) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)
(C) (I) → (Q); (II) → (P); (III) → (S); (IV) → (R)
(D) (I) → (S); (II) → (Q); (III) → (R); (IV) → (S)

Ans. (C)

Sol. (I) We have, $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8}}}$

$\Rightarrow \quad \text{LHS} = \sqrt{2+\sqrt{2+\sqrt{2(2\cos^2 4\theta)}}}$

$\left[\because 1+\cos 8\theta = 2\cos^2\dfrac{8\theta}{2}\right]$

$\Rightarrow \quad \text{LHS} = \sqrt{2+\sqrt{2+\sqrt{(4\cos^2 4\theta)}}}$

$\Rightarrow \quad \text{LHS} = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$\Rightarrow \quad \text{LHS} = \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$\Rightarrow \quad \text{LHS} = \sqrt{2+\sqrt{2(2\cos^2 2\theta)}} \quad [\because 1+\cos 4\theta = 2\cos^2 2\theta]$

$\Rightarrow \quad \text{LHS} = \sqrt{2+2\cos 2\theta} = \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2(2\cos^2\theta)} = 2\cos\theta = \text{RHS}$

(II) We have,

$$\text{LHS} = \sqrt{3}\ \text{cosec}\ 20^\circ - \sec 20^\circ$$

$$\Rightarrow \quad \text{LHS} = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$\Rightarrow \quad \text{LHS} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$\Rightarrow \quad \text{LHS} = \frac{2\left\{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right\}}{\sin 20^\circ \cos 20^\circ.}$$

$$\Rightarrow \quad \text{LHS} = \frac{2(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sin 20^\circ \cos 20^\circ}$$

$$\Rightarrow \quad \text{LHS} = \frac{2\sin(60^\circ - 20^\circ)}{\sin 20^\circ \cos 20^\circ} \Rightarrow \text{LHS} = \frac{2\sin 40^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{4\sin 40^\circ}{2\sin 20^\circ \cos 20^\circ} = \frac{4\sin 40^\circ}{\sin 40^\circ} = 4 = \text{RHS}$$

(III) $$\frac{1-\tan^2\left(\frac{\pi}{4}-A\right)}{1+\tan^2\left(\frac{\pi}{4}-A\right)} = \frac{1-\tan^2\theta}{1+\tan^2\theta} \quad \left(\text{where } \frac{\pi}{4}-A=\theta\right)$$

$$= = \cos 2\theta = \cos\left(\frac{\pi}{2}-2A\right) = \sin 2A$$

(IV) L.H.S. = $$\frac{1+\sin 2\theta}{1-\sin 2\theta} = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}$$

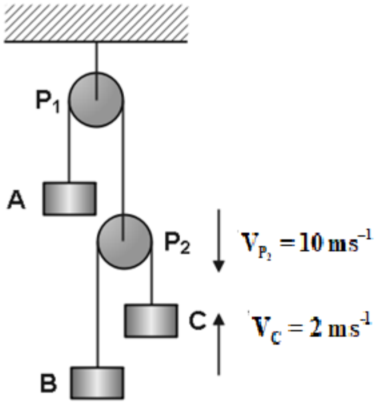
$$= \left(\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}\right)^2 = \left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2$$

[dividing numerator and denominator by $\cos\theta$]

PART-B: PHYSICS

SECTION-I (Numerical Value)

19. The three blocks shown in Figure, move with constant velocities. Find the ratio of magnitudes of velocities of blocks B and A $\left(=\frac{V_B}{V_A}\right)$. Given $V_{P_2} = 10\text{ ms}^{-1}\downarrow, V_C = 2\text{ ms}^{-1}\uparrow$.



Ans. (2.20)

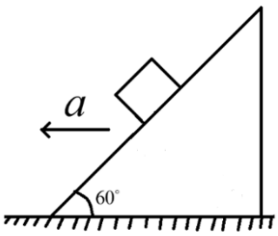
Sol. $V_A = V_{P_2} = 10\text{ ms}^{-1}$

For pulley P_2

$$V_{P_2} = \frac{V_B + (-V_C)}{2}$$

$$10 = \frac{V_B - 2}{2} = V_B = 22\text{ ms}^{-1}$$

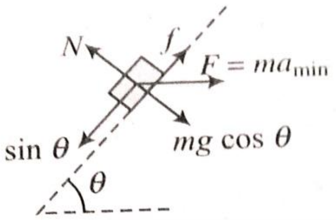
20. For the arrangement shown in Fig, find the minimum value of the horizontal acceleration a (in ms^{-2}) of the system so that the block does not slide on the wedge. Coefficient of limiting friction between block and wedge is $\frac{1}{\sqrt{3}}$. Angle of inclination of wedge is 60^0.



Ans. (5.77)

Sol. Here angle of inclination of the plane > angle of repose $\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ\right]$

Hence, the block has a tendency to slide down the plane.
Free body diagram of the block as seen from the frame of reference of the wedge.
For $a_{\min}$, limiting friction



will act in upward direction

Considering the equilibrium of M in the plane of wedge.

$$mg\sin\theta = f_1 + ma_{\min}\cos\theta$$

$$\Rightarrow f_1 = mg\sin\theta - ma_{\min}\cos\theta \qquad \text{(i)}$$

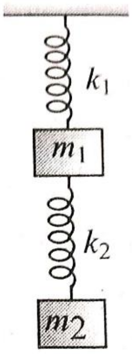
$$f_1 = \mu N$$

$$\Rightarrow mg\sin\theta - ma_{\min}\cos\theta = \mu\left(ma_{\min}\sin\theta + mg\cos\theta\right)$$

$$a_{\min} = g\left[\frac{\sin\theta - \mu\cos\theta}{\mu\sin\theta + \cos\theta}\right]$$

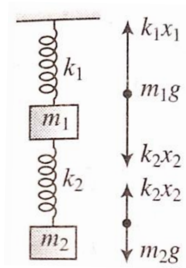
$$= g\left[\frac{tan\,\theta - \mu}{\mu\,tan\,\theta + 1}\right] = 10\left[\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{\sqrt{3}\cdot\frac{1}{\sqrt{3}}+1}\right] = \frac{10}{\sqrt{3}}\text{ms}^{-2} = 5.77\text{ms}^{-2}$$

21. Given $k_1 = 1500\ \text{Nm}^{-1}, k_2 = 500\ \text{Nm}^{-1}, m_1 = 2kg, m_2 = 1kg$. Find work done (in J) in slowly pulling down m_2 by 8 cm.



Ans. (1.20)

Sol. Let the initial extension in the springs of force constants k_1 and k_2, at equilibrium position, be x_1 and x_2. Then



$$x_2 = \frac{m_2 g}{k_2}, \qquad x_1 = \frac{(m_1+m_2)g}{k_1}$$

Let Δx_1 and Δx_2 be additional elongations caused by pulling m_2 by $\ell = 8cm$. Additional forces on m_1 are equal and in opposite directions.

$\Rightarrow k_1\Delta x_1 = k_2\Delta x_2$ ......(i)

Also, $\Delta x_1 + \Delta x_2 = \ell$ ....(ii)

Δx_1 and Δx_2 can be found from Eqs. (i) and (ii)

From work – energy theorem,

$$w_g + w_p + w_s = 0$$

(where w_p is the work done by the pulling force)

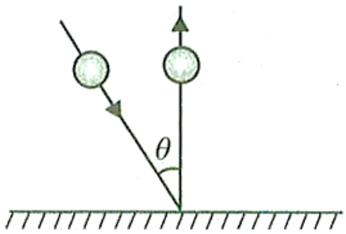
$$\Rightarrow w_p = -w_s - w_g = (U_2 - U_1) - \left[m_1 g\Delta x_1 + m_2 g(\Delta x_1 + \Delta x_2)\right]$$

where $U_2 = \frac{1}{2}k_1(x_1+\Delta x_1)^2 + \frac{1}{2}k_2(x_2+\Delta x_2)^2$

Putting the values, we get

$\Rightarrow w_p = 1.2\ J$

22. A ball of mass 1 kg moving with a velocity of 5 m/s collides elastically with rough ground at an angle θ with the vertical as shown in Fig. What can be the minimum coefficient of friction if ball rebounds vertically after collision? (given $\tan\theta = 2$)



Ans. (1.00)

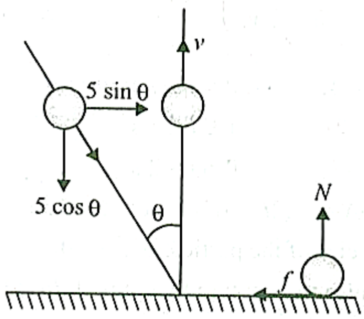
Sol. From impulse – momentum theorem,

$$\int N\,dt = m(v + 5\cos\theta) \quad \text{(i)}$$

$$\int f\,dt = m5\sin\theta$$

$$\mu\int N\,dt = m5\sin\theta \quad \text{(ii)}$$

$$\Rightarrow \quad \mu m(v + 5\cos\theta) = m5\sin\theta$$

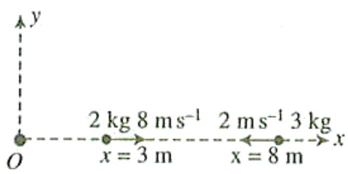


According to Newton's law of restitution,

$v = e\,5\cos\theta$ (e=1)

Solve to get $\mu = 1$

23. Figure shows position and velocities of two particles moving under mutual gravitational attraction in space at time t = 0. The position of centre of mass (in m) after one second is 'x'. Find 'x'.



Ans. (8.00)

Sol. As

$$\vec{F}_{ext} = 0; \vec{a}_{cm} = 0$$

$$\vec{V}_{cm} = \frac{2 \times 8\hat{i} - 3 \times 2\hat{i}}{2+3} = 2\hat{i}$$

$$x_{cm} = \frac{2 \times 3 + 3 \times 8}{2+3} = 6\,\text{m}$$

As $\vec{V}_{cm}$ is constant

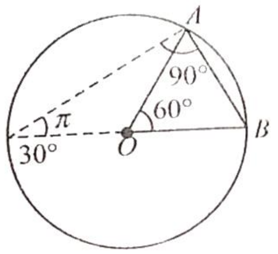
Therefore, CM will move 2 m in 1s.

$\therefore$ 6 + 2 = 8

24. A particle is moving in a circle of radius R with constant speed. Time period of the particle is T= 2.30s. In time t = T/6, if the difference between average speed and average velocity of the particle is 2 ms^{-1}, find the radius R of the circle (in m).

Ans. (16.10)

Sol. At t = T/6, particle will travel only 1/6 of the circle.



Average speed

$$= \frac{\text{Arc AB}}{\text{Time}} = \frac{(2\pi R)/6}{T/6} = \frac{2\pi R}{T}$$

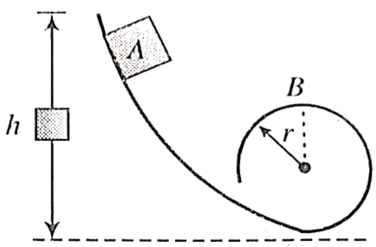
Average velocity

$$= \frac{\text{Chord AB}}{\text{Time}} = \frac{2R\sin 30^\circ}{T/6} = \frac{6R}{T}$$

Distance = $\frac{R}{T}\left[\frac{44}{7} - 6\right] = \frac{2R}{7T} = 2\text{ms}^{-1}(\text{given})$

R = 7T=7(2.3) =16.10m

25. A mass *m* starting from A reaches B of a frictionless track. On reaching B, it pushes the track with a force equal to *x* times its weight. If r = 4m and h = 25 m. Find *x*.



Ans. (7.50)

Sol. KE of blocks at B = PE at A – PE at B

$$\frac{1}{2}mv^2 = mgh - mg2r = mg(h-2r)$$

$$v^2 = 2g(h-2r) \text{.....(i)}$$

Also, $\dfrac{mv^2}{r} = xmg + mg$

or $v^2 = (x+1)rg$ ......(ii)

Equating Eqs. (i) and (ii), we get $2g(h-2r) = (x+1)gr$

or $2gh = (x+1)gr + 4gr = (x+5)gr$

$$h = \left(\frac{x+5}{2}\right)r$$

26. The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is $U = 6x - 8y$. If the initial velocity of the particle (in SI units) is $\vec{u} = -1.5\hat{i} + 2\hat{j}$, then find the total distance travelled (in m) by the particle in the first 2.5 seconds.

Ans. (21.88)

Sol. $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j} = -6\hat{i} + 8\hat{j}$

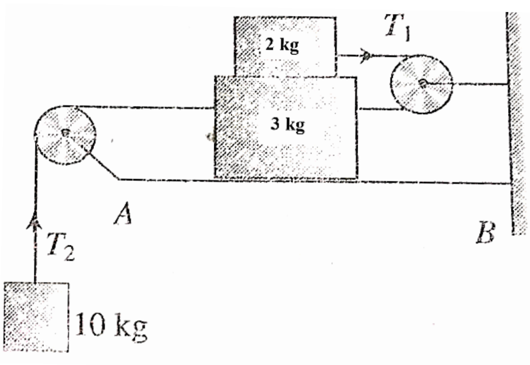
$\vec{a} = \frac{\vec{F}}{m} = -3\hat{\imath} + 4\hat{\jmath}$ has same direction as that of $\vec{u} = -1.5\hat{i} + 2\hat{j}$

Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line, so the distance covered

$$S = \tfrac{5}{2} \times 2.5 + \tfrac{1}{2} \times 5 \times 2.5^2 = 21.88m$$

SECTION – II (One or More than One Correct)

27. The coefficient of friction between the two blocks is 0.3, whereas the surface *AB* is smooth.



(A) Acceleration of the system of masses is (88/15) ms^{-2}.
(B) Net force acting on 3 kg mass is greater than that on 2 kg mass.
(C) Tension $T_2 > T_1$
(D) Since 10 kg mass is accelerating downwards, so the net force acting on it should be greater than any of the two blocks

Ans. (A,B,C)

Sol. (A) Let the acceleration of each block be *a*.

$$10g - T_2 = 10a, \quad T_2 - T_1 - f = 3a$$

$$T_1 - f = 2a, \quad \text{where } f = 0.3 \times 2g = 6N$$

From above equations

$$10g - 2f = 15a \Rightarrow 10 \times 10 - 2 \times 6 = 15a$$

$\Rightarrow$ a = 88/15 ms^{-2}

T_2 = 10g – 10a $= 10 \times 10 - 10 \times \dfrac{88}{15} = 41.3N$

$$T_1 = f + 2a = 6 + 2\times\frac{88}{15} = 17.7N$$

(B) Clearly $T_2 > T_1$

(C) This is correct because of greater mass of 3kg since acceleration is same for both.

(D) This is incorrect, because net force acting on 10 kg mass is greater due to its larger mass, not due to its acceleration downward.

28. A rocket, with an initial fuel mass of 1000 kg, is launched vertically upwards from rest under gravity (let gravity is constant throughout). The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 m/s relative to the rocket. Given ln2.5=0.916.

(A) The acceleration of rocket after 60 second from start is 40 ms^{-2}.

(B) The acceleration becomes constant after 100s

(C) The acceleration never becomes constant.

(D) The velocity of rocket after 60 second from blast is 1232.6 ms^{-1}

Ans. (A,B,D)

Sol. $a = \frac{u}{m}\frac{dm}{dt} - g$

$= \frac{2000}{400} \times 10 - 10 = 40\, ms^{-2}$

Since fuel is consumed after 100s, so acceleration is zero after 100s.

The velocity equation is given by

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

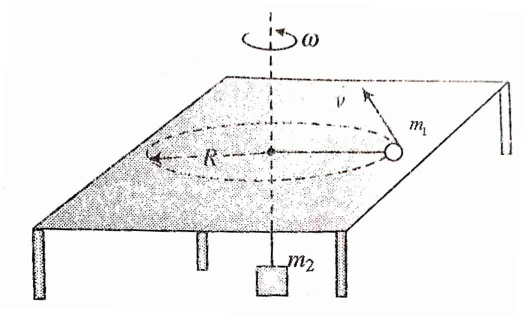
Here u = 0, t = 60 s, g = 10 m/s², $v_r = 2000 m/s, m_0 = 1000\, kg$ and m = 1000 – 10 × 60 = 400 kg

Putting these values, we get

$v = 0 - 600 + 2000\, ln\left(\frac{1000}{400}\right)$

or = 2000 ln 2.5 – 600 = 1232.6 m/s

29. A particle of mass m_1 moves in a circular path of radius R on a rotating table. A string connecting the particles m_1 and m_2 passes over a smooth hole made on the table as shown in fig. If mass m_1 does not slide relative to the rotating table, mark the correct options as applicable.



(A) The maximum friction force acting on the particle m_1 is $\left(m_1\omega^2 R - m_2 g\right)$ along radial direction of rotation.

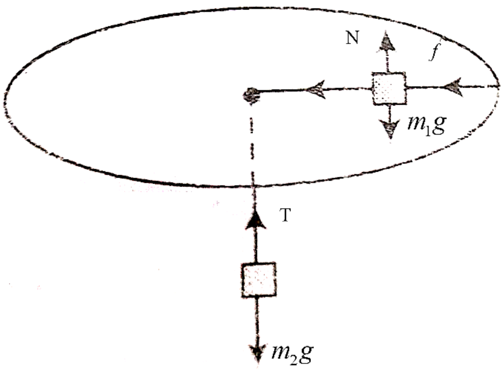
(B) The friction force acting on the block m_1 is $m_1\omega^2 R - m_2 g$ along tangent direction in the direction opposite to $\vec{v}$.

(C) The maximum angular velocity of the particle is $\sqrt{\left(\frac{m_2 + \mu m_1}{m_1}\right)\frac{g}{R}}$.

(D) The minimum angular velocity of the particle is $\sqrt{\left(\frac{m_2-\mu m_1}{m_1}\right)\frac{g}{R}}$.

Ans. (A,C,D)

Sol. **FBD:** Let the friction be directed radially inward. The forces on m_1 are $f\leftarrow$ (assumed). $T\leftarrow, N\uparrow, m_1g\downarrow$. The forces on m_2 are $m_2g\downarrow$ and $T\uparrow$ as shown in figure.



Force equation:
For m_1:
$N = m_1g$ ..... (i)
f + T = m_1a_r ..... (ii)
For m_2:
$T = m_2g$ .... (iii)
Law of static friction:
$f \le \mu N$ ....(iv)
$a_r = \frac{v^2}{R}$.....(v)
Solve the above equations, we have

$$f = \frac{m_1v^2}{R} - m_2g$$

Now substituting the above value of f and N in Eq. (iv)

we have $\left(\frac{m_1v^2}{R} - m_2g\right) \le \mu m_1 g$

This gives $v_{\max} = \sqrt{\left(\frac{m_2+\mu m_1}{m_1}\right)gR}$

30. A particle strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. If the coefficient of restitution between the particle and the floor is e, then

(A) the impulse delivered by the floor to the body is $mu(1+e)\sin\theta$

(B) $\tan\phi = e\tan\theta$

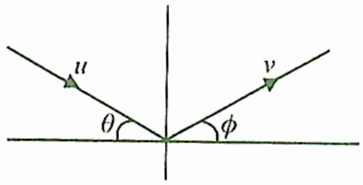
(C) $v = u\sqrt{1-(1-e)^2\sin^2\theta}$

(D) the ratio of final kinetic energy to the initial kinetic energy is $\left(\cos^2\theta + e^2\sin^2\theta\right)$

Ans. (A,B,D)

Sol.

$v \sin \varphi = eu \sin \theta, \quad v \cos \varphi = u \cos \theta$



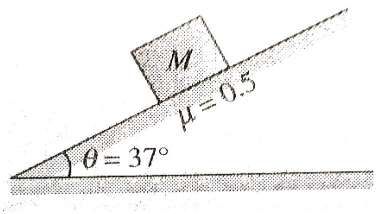
$$v = u\sqrt{\cos^2 \theta + e^2 \sin^2 \theta} = u\sqrt{1 - \sin^2 \theta + e^2 \sin^2 \theta}$$

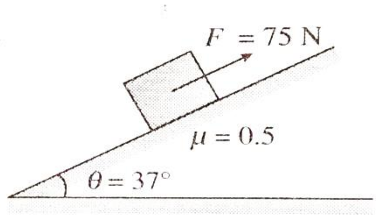
$$= u\sqrt{1 - (1 - e^2) \sin^2 \theta}$$

$$I = m(v \sin \phi + u \sin \theta) = mu \sin \theta (1 + e)$$

$$\text{Ratio of KE} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \cos^2 \theta + e^2 \sin^2 \theta$$

31. A block of mass M = 10 kg is placed on an inclined plane, inclined at angle $\theta = 37^\circ$ with horizontal. The coefficient of friction between the block and inclined is $\mu = 0.5$





(A) The acceleration of the block when it is released is 2 ms^{-2}

(B) Now a force F = 75 N is applied on block as shown. The acceleration of the block, if the block is initially at rest is 1 ms^{-2}.

(C) The force that should be added to 75 N force so that block the starts to move up the incline is 25 N.

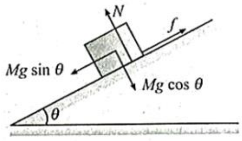
(D) The minimum force by which 75 N force should be replaced with so that the block does not move is 20 N.

Ans. (A,C,D)

Sol.

a. Here angle of repose $\alpha = \tan^{-1}(\mu_s)$

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.5$



The angle of inclination is greater than the angle of repose. The friction force on the block will act in upward direction.

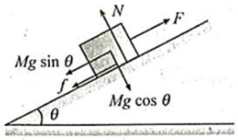
For the acceleration of block,

$$Mg \sin\theta - \mu N = Ma$$

$$\Rightarrow \quad a = g\sin\theta - \mu g\cos\theta$$

$$= 10(\sin 37° - 0.5\cos 37°)$$

b. If external force $F = 75$ N is applied on the block.



Let us find net driving force acting on block. Parallel to inclined two external forces are acting one in upward direction F and other is the component of weight in the direction downward the plane, $Mg \sin\theta$.

Net driving force $f_{\text{driving}} = F - Mg\sin\theta$

$\Rightarrow \quad F_{\text{driving}} = 75 - 10 \times 10 \times \sin 37° = 75 - 60 = 15\text{N}$

Maximum resisting force that oppose relative motion is maximum friction force (or f_{lim})

$$f_{\text{lim}} = \mu_s Mg\cos\theta$$

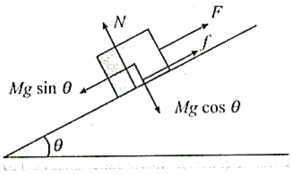
$$= 0.5 \times 10 \times 10 \times \cos 37° = 40 \text{ N}$$

Here $F_{\text{driving}} < f_{\text{resisting}}$. Hence, the block will not move and friction will be static and will act in the direction opposite to driving force, i.e., in downward direction.

c. To move the block, the least value of driving force should be 40 N. But in above case, driving force is 15 N (up). Hence, if we add $\Delta F = 25$ N in upward direction, the block will overcome maximum resistance force (or friction) and starts moving up.

$\therefore \quad 60 + 40 = 75 + \Delta F \quad \Rightarrow \Delta F = 25$ N

d. As resisting force which is maximum friction force is 40 N and the component of weight parallel to incline is 60 N and acting downward. If we remove F, then the driving forces will be the only component of the weight in the direction downward the incline plane.

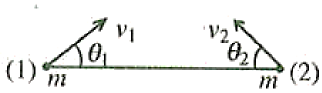


In this case, friction will act in upward direction. Hence, the required value of F to make block in equilibrium,

$$F + 40 = 60$$

or $\quad F = 20$ N

32. Two particles of equal mass m are projected from the ground with speeds v_1 and v_2 at angles θ_1, and θ_2 as shown in the figure. Given $\theta_2 > \theta_1$, and $v_1 \cos\theta_1 = v_2 \cos\theta_2$. Which statement/s is/are correct?



(A) Centre of mass of particles will move along a vertical line.
(B) Centre of mass of particles will move along a line inclined at some angle with vertical.
(C) Particle '1' will be above centre of mass level when both particles are in air.
(D) Particle '2' will be above centre of mass level when both particles are in air.

Ans. (A,D)

Sol.

$$v_{cm-x} = \frac{mv_1\cos\theta_1 + m(-v_2\cos\theta_2)}{m+m} = 0$$

So horizontal velocity of centre of mass is zero. Hence centre of mass will move in vertical direction.

Now $v_2 \cos\theta_2 = v_1 \cos\theta_1$

$\Rightarrow \quad \frac{v_2}{v_1} = \frac{\cos\theta_1}{\cos\theta_2} > 1 \qquad (\because\ \theta_2 > \theta_1)$

$\Rightarrow \quad v_2 > v_1$

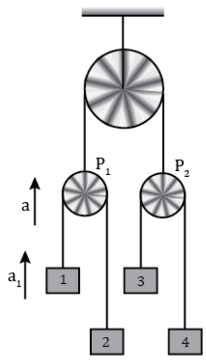
$\Rightarrow \quad v_2 \sin\theta_2 >> v_1 \sin\theta_1$

At any time, particle '2' will be at greater height than particle '1'. So particle '2' will be above centre of mass level.

SECTION – III (Match List Type)

33. In the system shown in Fig., masses of the blocks are such that when system is released, the acceleration of pulley P_1 is 'a' upwards and the acceleration of block 1 is a_1 upwards. It is found that the acceleration of block 3 is same as that of 1 both in magnitude and direction.

Given that $a_1 > a > a_1/2$. Match the following:



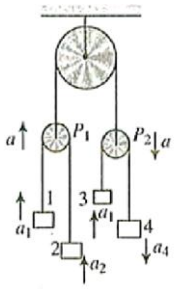
	Column I		Column II
1	Acceleration of 2	P	$2a + a_1$
2	Acceleration of 4	Q	$2a - a_1$
3	Acceleration of 2 w.r.t. 3	R	Upwards
4	Acceleration of 2 w.r.t. 4	S	Downwards
		T	$2(a - a_1)$

(A) $1 \to Q.,R.,2 \to P.,S.,3 \to S,T.,4 \to R.$ (B) $1 \to P.,R.,2 \to P.,S.,3 \to Q.,4 \to R,T$
(C) $1 \to R.,T.,2 \to P.,S.,3 \to S,Q.,4 \to R,S$ (D) $1 \to Q.,2 \to P.,R.,3 \to Q.,4 \to S,T$

Ans. (A)

Sol. $1 \to Q.,R.,2 \to P.,S.,3 \to S,T.,4 \to R$

Let the accelerations of various blocks are as shown in Fig. Pulley P_2 will have downward acceleration a.



Now $a = \dfrac{a_1 + a_2}{2} \Rightarrow a_2 = 2a - a_1 > 0$

So acceleration of 2 is upwards

and $a = \dfrac{-a_1 + a_4}{2} \Rightarrow a_4 = 2a + a_1 > 0$

So, acceleration of 4 is downwards.

Acceleration of 2 w.r.t. 3:

$$a_{2/3} = a_2 - a_3 = a_2 - a_1 = 2(a - a_1) < 0$$

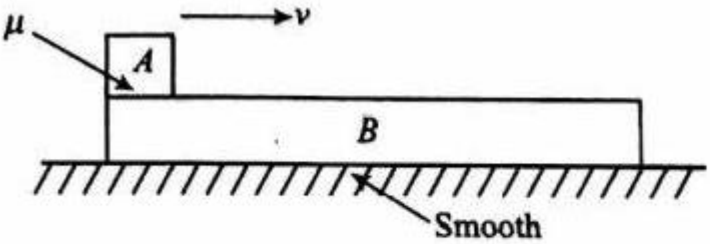
This is downwards,

Acceleration of 2 w.r.t. 4:

$$a_{2/4} = a_2 - (-a_4) = 4a > 0$$

This is upwards.

34. In Fig., block A is kept on a larger block B. Both are initially at rest. Friction exists between the blocks but there is no friction between B and floor. An impulse gives block A a velocity v as shown. For some displacement after this, match the entries of Column I with that of Column II.



	Column I		Column II
1	Work done by friction on B	P	positive
2	Work done by friction on A	Q	negative
3	Net Work done by friction on A and B	R	zero
4	Work done by friction on B in the frame of A.	S	positive, negative or zero.

(A) $1 \to P.,2 \to Q,R.,S.,3 \to Q,R.,4 \to R$ (B) $1 \to P.,2 \to Q.,3 \to Q.,4 \to Q$
(C) $1 \to Q.,2 \to R.,3 \to P.,4 \to R$ (D) $1 \to Q.,2 \to P.,3 \to R.,4 \to P$

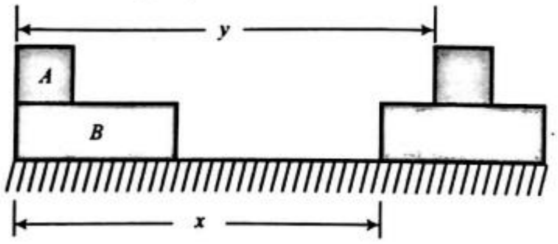
Ans. (B)

Sol. $1 \to P.,2 \to Q.,3 \to Q.,4 \to Q$

Friction forces on A and B will be acting as shown in figure.

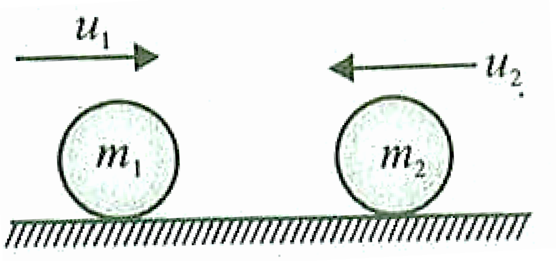
Both will be moving towards right after giving the impulse.

(i) Work done by friction on B should be positive.
(ii) Work done by friction on A should be negative.
(iii) Negative work done on A will be more than positive work on B by friction. This is because displacement of A will be more than that of B during any time.



(iv) In the frame of A,B will go towards left by a displacement of y−x. Hence work will be negative.

35. Two balls of masses m_1 and m_2 are moving towards each other with speeds u_1 and u_2, respectively. They collide head-on and their speeds are v_1 and v_2 after collision (m_1 = 8 kg, m_2= 2 kg, u_2 = 3 m/s).



	Column I		Column II
1	The speed u_1 (in m/s) so that both balls move in same direction if coefficient of restitution is e = 0.5	P	$\frac{1}{14}$
2	The speed u_1 (in m/s) so that the maximum fraction of energy is transformed to m_2 (assume elastic collision)	Q	$\frac{1}{8}$
3	Coefficient of restitution if m_2 stops after collision and u_1 = 0.5 m/s	R	2
4	If collision is inelastic and u_1 = 3 m/s, the loss of kinetic energy (in J) after collision may be	S	4
		T	1

(A) $1 \to$ R,S.,$2 \to$ R.,$3 \to$ P.,$4 \to$ P, Q, R, S, T
(B) $1 \to$ R.,$2 \to$ S.,$3 \to$ P.,$4 \to Q, R, T$
(C) $1 \to$ Q,S.,$2 \to$ Q.,$3 \to$ P.,$4 \to$ P, Q, R
(D) $1 \to$ S.,$2 \to$ P,Q.,$3 \to$ R.,$4 \to$ P, Q

Ans. (A)

Sol. $1 \to R,S.,2 \to R.,3 \to P.,4 \to P,Q,R,S,T$

1 $8u_1 + 2(-3) = 8v_1 + 2v_2$

$$\frac{v_2 - v_1}{u_1 - (-3)} = e = 0.5$$

Solving, we get $v_1 = \dfrac{7u_1 - 9}{10}$, $v_2 = \dfrac{12u_1 + 6}{10}$

v_2 is always positive.

For $v_1 > 0$ $\quad u_1 > \dfrac{9}{7}$ m/s

ii. $8u_1 + 2(-3) = 8v_1 + 2v_2$

$$\frac{v_2 - v_1}{u_1 - (-3)} = e = 1$$

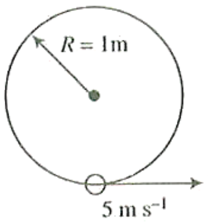
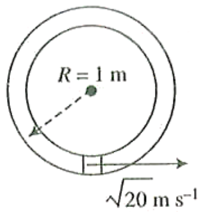
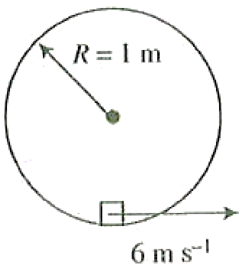
For maximum energy to transfer to m_2, $v_1 = 0$

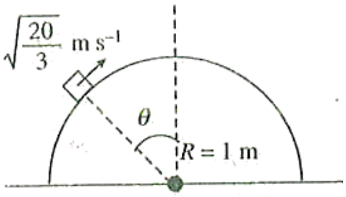
Solving, we get $u_1 = 2$ m/s.

iii. For this $v_2 = 0$

iv. Depends upon e.

36. In column I, a situation is depicted each of which is in vertical plane. The surfaces are frictionless. Match with appropriate entries in column II.

	Column I		**Column II**
p.	Bead is threaded on a circular fixed wire and is projected from the lowest point. 	a.	Normal force is zero at the top-most point of its trajectory.
q.	Block loosely fits inside the fixed small tube and is projected from lowest point 	b.	Velocity of the body is zero at the top-most point of its trajectory.
r.	Block is projected horizontally from lowest point of a smooth fixed cylinder. 	c.	Acceleration of the body is zero at the top-most point of its trajectory.

s.	Block is projected on a fixed hemisphere from angular position θ  $\cos\theta = 2/3$	d.	Normal force is radially outward at the top-most point of trajectory.
		e	Acceleration of body is tangential at the top most point of its trajectory.

(A) $p. \to b., q. \to a., d., r. \to c., s \to b., d.$

(B) $p. \to a., q. \to b., c., r. \to d., s \to b., a., e$

(C) $p. \to b., d., e., q. \to a., b., e, r. \to a., s \to b., c., d.$

(D) $p. \to a., q. \to b., r. \to c, s \to d$,e

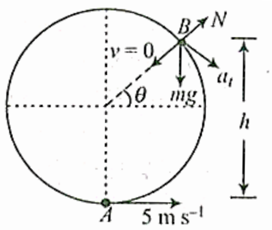
Ans. (C)

Sol. $p. \to b., d., e., q. \to a., b. e, r. \to a., s \to b., c., d.$

(A)

U = 5 m/s, let maximum height attained by bead is h, then

$$h = \frac{u^2}{2g} = \frac{5^2}{2 \times 10} = 1.25 \text{ m}$$



B is top most point of its trajectory where velocity is zero.

$$N = mg\sin\theta$$

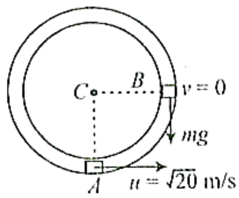
N is radially outward acceleration is not zero at B. It is along tangential direction as shown;

$$a_t = \frac{mg\cos\theta}{m} = g\cos\theta$$

(B)

Maximum height attained:

$$h = \frac{u^2}{2g} = \frac{(\sqrt{20})^2}{2 \times 10} = 1 \text{ m}$$



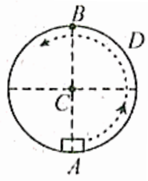
This is equal to radius.

B is the top most point of its trajectory. Here only force acting is mg, here acceleration is g downwards. Normal force is zero.

(C)

$u = 6\,\text{m s}^{-1}$, $\sqrt{2gR} < u < \sqrt{5gR}$

Block will leave contact at some point D and move in parabolic path afterwards. B is top most point of its trajectory where normal force is zero and acceleration is g downwards.



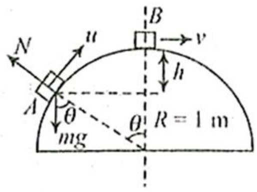
(D)

$\cos\theta = \frac{2}{3}$, $u = \sqrt{\frac{20}{3}}\,\text{m s}^{-1}$

$$mg\cos\theta - N = \frac{mu^2}{R}$$

$\Rightarrow \quad N = 0$

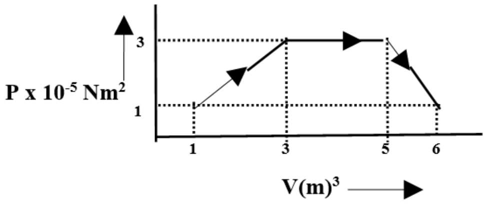
T(A), normal force is zero, but as it goes up normal force increase at B, v comes out to be zero. $N = mg$ at top.



PART-C: CHEMISTRY

SECTION-I (Numerical Value)

37. The net work done through a series of changes reported in figure for ideal gas is (-10^5x) Joule. Find x?



Ans. (12)

Sol. Net work done = (Area covered under PV curve)
$W = -12 \times 10^{-5}$J
$\Rightarrow x = 12$

38. 1 mole of a gas undergoes a change in state. 'H', 'S' & 'T' of the states are given as (25KJ, 72J/K, 700K) & (10KJ, 64J/K, 600K). What is the magnitude of ΔG in 'kJ' in this process?

Ans. (3)

Sol. $\Delta G = \Delta H - \Delta(TS)$

$= [(10000 - 25000) - (600 \times 64 - 700 \times 72)]$J

$= -3$kJ

39. What is the number of hybrid orbitals used by xenon when it forms the compound $XeOF_4$?

Ans. (6)

Sol. Xe is sp^3d^2 hybridised. Therefore, the number of hybrid orbitals = 6.

40. One mole of a non ideal gas undergoes a change of state (2.5atm, 4L, 90K) to (5atm,6L, 240K) when a change in internal energy is 40 Latm. What is the change in enthalpy of the process in L-atm?

Ans. (60)

Sol. $\Delta H = \Delta U + \Delta(PV)$

$\Rightarrow \Delta H = 40 + (P_2V_2 - P_1V_1) = 40 + (5 \times 6 - 2.5 \times 4) = 40 + 20 = 60$ L-atm

41. During a reversible adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The adiabatic index is $x/2$ find x ?

Ans. (3)

Sol. $T^{\gamma}P^{1-\gamma}$ = constant => $P \propto T^{(\gamma/\gamma-1)}$
Given that $P \propto T^3$

$\Rightarrow (\gamma/\gamma-1) = 3 \Rightarrow \gamma = 3/2 = x/2$

$\Rightarrow x = 3$

42. Calculate the maximum work done by the gas (in kJ) when 16 g of oxygen at 300 K expands from volume 5dm³ to 25 dm³ isothermaly. (Report answer in nearest integer)

Ans. (2)

Sol. Reversible work is maximum work

$$w = -2.303nRT\ \log_{10}\left(\frac{v_2}{v_1}\right) = 2.303\times\frac{16}{32}\times 8.314\times 300\times\log\frac{25}{5} = 2.01\times 10^3\text{ joule} = 2\text{kJ}$$

43. A sample of ideal gas ($\gamma = 1.4$) is heated at constant pressure if 98J of heat is supplied to gas, find ΔU in Joules.

Ans. (70)

Sol. γ = 1.4

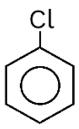
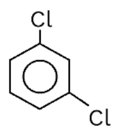
$$C_V = \frac{5}{2}R \quad C_P = \frac{7}{2}R$$

$$q_p = nC_{p,m}\Delta T$$

$$\Delta U = nC_{v,m}\Delta T$$

$$\frac{q_p}{\Delta U} = \frac{C_{p,m}}{C_{v,m}} = \gamma$$

$$\Delta U = \frac{q_p}{\gamma} = \frac{98}{1.4}$$

44. If the dipole moment of  is 2D, then calculate the dipole moment of  in Debye.

Ans. (2)

Sol. $\mu_{net} = \sqrt{\mu_1^2 + \mu_2^2 + 2\mu_1\mu_2\cos\theta}$

$$\theta = 120^\circ$$

$$\mu_{net} = 2$$

SECTION – II (One or More than One Correct)

45. Which of the following statements are **not** correct?

(A) All molecules with polar bonds have dipole moment.

(B) $SnCl_2$ is a nonlinear molecule.

(C) Intramolecular hydrogen bond is present in para-nitrophenol.

(D) Intermolecular hydrogen bond is present within the molecule.

Ans. (A), (C), (D)

Sol. (A) It is present within the molecule.
(B) It occurs between two different/identical molecules.

46. Which of the following statements are correct?

(A) Linear overlap of two atomic p-orbitals leads to a sigma bond.

(B) The bond angle H—N—H in NH_3 is greater than the bond angle H—As—H in AsH_3.

(C) Anhydrous HCl is a bad conductor of electricity but aqueous HCl is a good conductor.

(D) *o*-nitrophenol is steam volatile whereas *p*-nitrophenol is not.

Ans. (A), (B), (C), (D)

Sol. Conceptual

47. Which of the following pairs are isostructural?

(A) $\left[CO_3^{2-}, SO_3^{2-}\right]$ (B) $\left[XeF_2, IF_2^-\right]$

(C) $\left[NH_2^-, BeF_2\right]$ (D) $\left[SO_4^{2-}, BF_4^-\right]$

Ans. (B), (D)

Sol. CO_3^{2-} - Trigonal Planar

SO_3^{2-} - Pyramidal

XeF_2 - Linear

IF_2^- - Linear

NH_2^- - Bent

BeF_2 - Linear

SO_4^{2-} - Tetrahedral

BF_4^- - Tetrahedral

48. Which of the following statements are **not** correct?

(A) Electrovalent bond has directional characteristics.

(B) The crossed arrow of a dipole moment is directed from negative to positive centre.

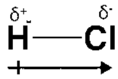
(C) The dipole moment of NH_3 is greater than that of NF_3.

(D) The two lone pair(s) on Cl in ClF_3 occupy opposite directions.

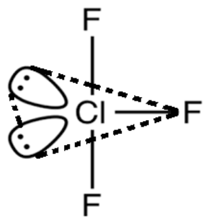
Ans. (A),(B), (D)

Sol.

- Ionic bond is non-directional.



- ClF_3



49. Which of the following statements are correct?

(A) For a reaction involving condensed phases ΔH is nearly equal to ΔU

(B) It is possible to calculate the value of ΔH for the reaction $H_2(g) + Br_2(l) \rightarrow 2HBr(g)$ from the bond - enthalpy data alone.

(C) The enthalpy of combustion of diamond and enthalpy of formation of carbon dioxide has the same value.

(D) The reaction $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$ is accompanied with decrease in entropy.

Ans. (A), (D)

Sol. (A) For a reaction involving condensed phases (solids or liquids), Δn_g is zero.

(B) Bond enthalpy data refers to gaseous species only.

(C) Enthalpy of combustion of graphite and enthalpy of formation of carbon dioxide refer to one and the same chemical equation.

(D) The reaction is accompanied with decrease in the gaseous species and, hence, decrease in entropy.

50. Which of the following statements are correct?

(A) First law of thermodynamics is not adequate in predicting the direction of the process.

(B) In an exothermic reaction, the total enthalpy of products is greater than that of reactants.

(C) In an endothermic reaction, the total enthalpy of products is greater than that of reactants.

(D) The standard enthalpy of formation of diamond is zero at 298 K and 1 atm pressure.

Ans. (A), (C)

Sol. (B) In an exothermic reaction, the total enthalpy of products is smaller than that of reactants.

(D) It is graphite which is more stable and thus has standard enthalpy of formation equal to zero.

SECTION-III (Match List Type)

51. Match each of the diatomic molecule in Column I with its property/properties in Column II.

	Column I		**Column I**
(a)	B_2	(p)	paramagnetic
(b)	N_2	(q)	Last electron in bonding M.O.
(c)	O_2^-	(r)	Last electron in anti bonding M.O.
(d)	O_2	(s)	Bond order $\geq$ 2
		(t)	Mixing of s and p orbitals

(A) (a) – (p), (r), (t); (b) – (r),(s), (t); (c) – (p), (q); (d) – (p), (q), (s)

(B) (a) – (r), (t); (b) – (q), (s), (t); (c) – (p), (r); (d) – (p), (r), (s)

(C) (a) – (p), (q),(t); (b) – (q),(s), (t); (c) – (p), (r); (d) – (p), (r), (s)

(D) (a) – (p), (q),(t); (b) – (q),(s), (t); (c) – (p), (q); (d) – (p), (r), (s)

Ans. (C)

Sol. (A)

Diboron, B_2 $\left[\sigma(2s)^2 \sigma(2s)^{*2} \pi(2p)^2\right]$

Dinitrogen, N_2 $\left[\sigma(2s)^2 \sigma(2s)^{*2} \pi(2p)^4 \sigma(2p)^2\right]$

Dioxygen, O_2 $\left[\sigma(2s)^2 \sigma(2s)^{*2} \sigma(2p)^2 \pi(2p)^4 \pi(2p)^{*2}\right]$

52. Column I lists some of hybridization schemes and Column II lists some of the compounds. Match each entry of Column I with the compounds mentioned in Column II.

	Column I		**Column I**
(a)	sp^3	(p)	$[Ni(CN)_4]^{2-}$
(b)	dsp^2	(q)	XeO_3
(c)	sp^3d	(r)	SO_3^{2-}
(d)	sp^3d^2	(s)	XeF_2
		(t)	XeF_4
		(u)	XeO_2F_2

(A) (a) – (r), (q); (b) – (p); (c) – (s), (t); (d) – (u)
(B) (a) – (s), (t); (b) – (q); (c) – (p), (u); (d) – (t)
(C) (a) – (q), (r); (b) – (p); (c) – (s), (u); (d) – (t)
(D) (a) – (t), (s); (b) – (u); (c) – (s), (t); (d) – (r)

Ans. (C)

Sol. XeO_3 – sp^3
XeF_2 – sp^3d
SO_3^{2-} – sp^3
XeF_4 - sp^3d^2
XeO_2F_2 – sp^3d
$[Ni(CN)_4]^{2-}$ - dsp^2

53. Column I lists a few physical processes and their expressions to compute these physical processes are listed in Column II. Match the correct choices.

	Column I		**Column I**
(a)	$\Delta_r G^\circ$	(p)	$\Delta_r U + (\Delta n_g)RT$
(b)	$\Delta_r H$	(q)	$RT\ ln(p_1/p_2)$ (isothermal process on an ideal gas)
(c)	ΔS_{sys}	(r)	q_{rev}/T
(d)	$w_{reversible}$	(s)	$-RT\ln K_{eq}$
		(t)	$nC_{p,m}\Delta T$

(A) (a) – (s); (b) – (t); (c) – (r); (d) – (q),(t)
(B) (a) – (t); (b) – (r), (p); (c) – (q); (d) – (r)
(C) (a) – (s); (b) – (p), (t); (c) – (r); (d) – (q)

(D) (a) – (r), (p); (b) – (t); (c) – (q); (d) – (s)

Ans. (C)

Sol.
$$\Delta G^{\circ} = -RT\ln K_{eq}$$
$$\Delta H = nC_{p,m}\Delta T$$
$$\Delta H = \Delta U + \Delta n_g RT$$
$$\Delta S = \frac{q_{rev}}{T}$$

54. Column I list expansion of an ideal gas under different conditions and the corresponding expressions to be used are listed in Column II. Identify the correct choice.

	Column I		**Column I**
(a)	Reversible isothermal expansion	(p)	$W = -p_{ext}(V_2 - V_1)$
(b)	Reversible adiabatic expansion	(q)	$W = -nRT\ ln\ (V_2/V_1)$
(c)	Irreversible isothermal expansion against a constant pressure	(r)	$W = C_V\Delta T$
(d)	Irreversible adiabatic expansion against a constant pressure	(s)	$W = -nRT\ ln\ (p_1/p_2)$
		(t)	pV^{γ} = constant

(A) (a) – (s), (q); (b) – (t), (r); (c) – (p); (d) – (q)
(B) (a) – (q), (s); (b) – (r), (t); (c) – (p); (d) – (p), (r)
(C) (a) – (q), (p); (b) – (p), (t); (c) – (r); (d) – (p)
(D) (a) – (r), (s); (b) – (s), (t); (c) – (p); (d) –(p), (r),(s)

Ans. (B)

Sol. Isothermal
$$\Delta U = w$$
$$w = C_v\Delta T$$

Isothermal Reversible : $w = -nRT\ln\frac{V_2}{V_1} = -nRT\ln\frac{P_1}{P_2}$

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