FIITJEE RBT-8 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 1

QP Code: 100965

PAPER - 1

Time Allotted: 3 Hours **Maximum Marks: 234**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-07) –** Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(i) **Part-A (08-13) –** Contains six (06) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **Part-B (01-08)** contains eight (08) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

Name of the Candidate :____________________________________

Batch :________________ **Date of Examination :**________________

Enrolment Number :____________________________________

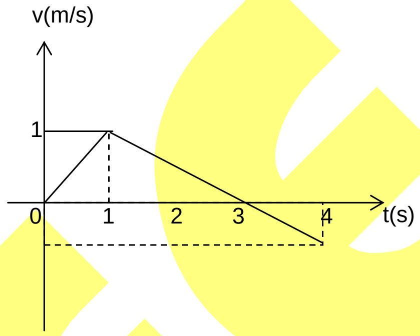
SECTION-1 : PHYSICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. The velocity time graph of a particle is shown for time interval t = 0 to t = 4 sec. Choose correct options.



(A) the displacement of the particle is equal to $\frac{5}{4}$m

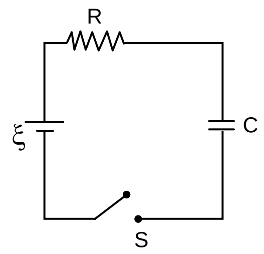
(B) the distance covered by the particle is equal to $\frac{7}{4}$m

(C) the average velocity of the particle is $\frac{1}{2}$m/s over 4 s

(D) the motion is uniformly accelerated for 4s

2. Using screw gauge the radius of the wire is found to be 2.50 mm. The length of wire is measured by using a scale and is found to be 50.0 cm. If mass of wire is measured as 25 g, then mark the correct statement(s) [Take π = 3.14]
(A) The density has to be computed upto 2 significant digit.
(B) The least count of scale used to measure length of wire is 1 mm.
(C) The density of wire is 2.5 g/cm^3.
(D) The least count of screw gauge is 0.01 mm.

3. As situation shown in figure the maximum value of rate of energy stored in the capacitor after the switch is closed



(A) $\frac{\xi^2}{2R}$

(B) $\frac{\xi^2}{4R}$

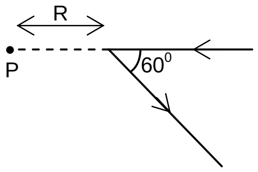
(C) $\frac{\xi^2}{8R}$

(D) none of these

4. A metal cylinder of mass 0.5 kg is heated electrically by a 12 W heater in a room at 15°C. The cylinder temperature rises uniformly to 25°C in 5 min and finally becomes constant at 45°C. Assuming that the rate of heat loss is proportional to the excess temperature over the surroundings,
(A) the rate of loss of heat of the cylinder to surrounding at 20°C is 2 W
(B) the rate of loss of heat of the cylinder to surrounding at 45°C is 12 W
(C) specific heat capacity of metal is $\frac{240}{\ln(3/2)}$J/kg°C
(D) none of the above

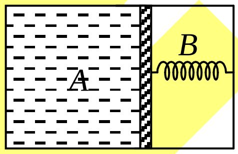
Space For Rough Work

5. A long straight wire, carrying a current I is bent at its mid point to form an angle of 60^0 . AT a point P, distance R from the point of bending the magnetic field is



(A) $\frac{(\sqrt{2}-1)\mu_0 i}{4\pi R}$ (B) $\frac{(\sqrt{2}+1)\mu_0 i}{4\pi R}$ (C) $\frac{\mu_0 i}{4\sqrt{3}\pi R}$ (D) $\frac{\mu_0 i}{8R}$

6. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S into two equal parts A and B. Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant k is connected with piston and the wall of the container as shown. Initially spring is unstretched. Gas in chamber A is allowed to expand. Let in equilibrium spring is compressed by x_0. Then



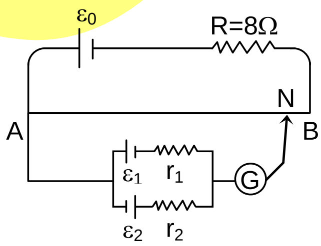
(A) final pressure of the gas is $\frac{kx_0}{S}$

(B) work done by the gas is $\frac{1}{2}kx_0^2$

(C) change in internal energy of the gas is $\frac{1}{2}kx_0^2$

(D) temperature of the gas is decreased.

7. A battery of emf $\varepsilon_0 = 12V$ is connected across a 4m long uniform wire having resistance 4Ω/m. The cells of small emfs $\varepsilon_1 = 2V$ and $\varepsilon_2 = 4V$ having internal resistance 2Ω and 6Ω respectively are connected as shown in the figure. If galvanometer shows no deflection at the point N, the distance of point N from the point A is equal to :



(A) $\frac{1}{6}$m (B) $\frac{1}{3}$m

(C) 25 cm (D) 50 cm

Space For Rough Work

(Single Correct Choice Type)

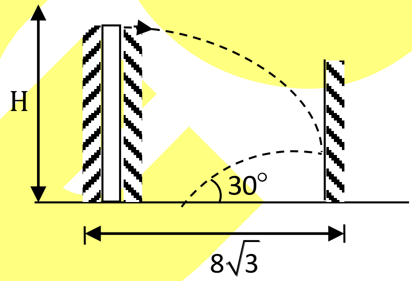
This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. An object of mass m is thrown vertically upwards in air with an initial speed v_0. The air applies a drag force which is proportional to the instantaneous velocity of the particle in the opposite direction of the motion of ball. Then the time it takes to reach maximum height above ground is [$V_T = mg/c$] $\left[\vec{F} = -c(\vec{v})\right]$

(A) $\frac{V_T}{g}\ln\left[1-\frac{v_0}{g}\right]$ (B) $\frac{V_T}{g}\ln\left[1+\frac{v_0}{v_T}\right]$

(C) $\frac{V_T}{2g}\ln\left[1+\frac{2v_0}{v_T}\right]$ (D) $\frac{V_T}{g}\ln\left[1+\frac{2v_0}{V_T}\right]$

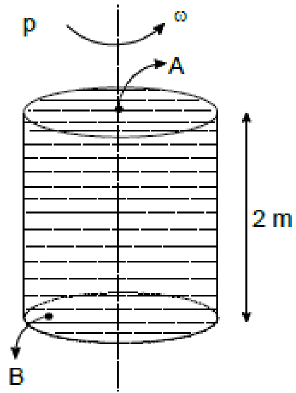
9. A ball is thrown horizontally from the top of a tower of unknown height. Ball strikes a vertical wall whom plane is normal to the plane of motion of ball. Collision is elastic and ball falls on ground at mid-point between tower and wall. Ball strikes the ground at angle of 30° with horizontal. The height of tower is



(A) 2 m (B) 4 m (C) 6 m (D) 8 m

10. A 50 Hz 20V source is connected to a resistance of 100 Ω an inductance of 1 H and a capacitance of 10μF all in series. Calculate approximate time (in sec) which the resistance (thermal capacity 2J/^{0}C) will get heated by 10^{0}C.
(A) 4 (B) 5 (C) 6 (D) 7

11. A closed tank of water is rotating about a vertical axis as shown in figure and at same time the entire tank is accelerated upward at 4 m/s^2. If the rate of rotation is 10 rad/sec. The difference in pressure between points A and B is equal to (10 × a.01) kPa. There is a point B at the bottom of the tank at a radial distance of 0.5 m from the axis of rotation and point A is at the top on the axis of rotation. Find the value of a. (take g = 9.8 m/s^2)
(A) 1 (B) 2
(C) 3 (D) 4



Space For Rough Work

12. If $\bar{A} \times \bar{B} = \bar{C} + \bar{D}$, then the correct alternative is
(A) $\bar{B}$ is parallel to $(\bar{C} + \bar{D})$
(B) $\bar{A}$ is perpendicular to $\bar{C}$
(C) component of $\bar{C}$ along $\bar{A}$ = component of $\bar{D}$ along $\bar{A}$
(D) component of $\bar{C}$ along $\bar{A}$ = - component of $\bar{D}$ along $\bar{A}$

13. If θ is the angle with horizontal with which a projectile must be fired to escape from earth's gravitational pull then
(A) $0^o \le \theta < 45^o$ (B) $0^o \le \theta \le 180^o$ (C) $\theta = 90^o$ (D) $\theta = 45^o$

PART – B
(Numerical based)

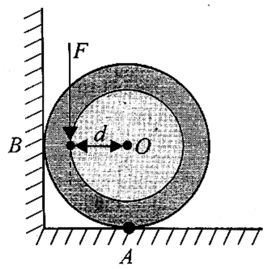
This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. In a thermally isolated system, two boxes filled with an ideal gas are connected by a valve. When the valve is in closed position, states of the box 1 and 2, respectively, are (1 atm, V, T) and (0.5 atm, 4V, T). When the valve is opened, what is approximately the final pressure (in atm) of the system?

2. Three simple harmonic motions of equal amplitude 2.25m and equal time periods in the same direction combine. The phase of the second motion is 60^0 ahead of the first and the phase of the third motion is 60^0 ahead of the second. Find the amplitude of the resultant motion (in m).

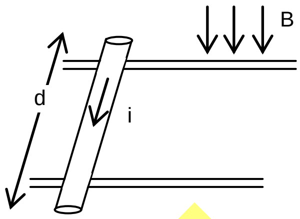
3. On a horizontal frictionless frozen lake, a girl (36 kg) and a box (9 kg) are connected to each other by means of a rope. Initially they are 18 m apart. The girl exerts a horizontal force on the box, pulling it towards her. How far has the girl travelled (in m) when she meets the box?

4. A solid cylinder with r = 0.1 m and mass M = 2 kg is placed such that it is in contact with the vertical and a horizontal surface as shown in figure. The coefficient of static friction is μ = (1/3) for both the surfaces. Find the distance d (in cm) from the centre of the cylinder at which a force F = 40 N should be applied vertically so that the cylinder just starts rotating in anticlockwise direction.



Space For Rough Work

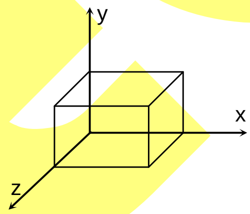
5. A cylindrical uniform rod of mass 0.72 kg and radius 6 cm rests on two parallel rails, that are d = 50 cm apart. The rod caries a current I = 48A (In the direction shown) and rolls along the rails without slipping. If it starts from rest, uniform magnetic field of magnitudes 0.25 T is directed perpendicular to the rod and the rail, then the friction force(In N) between rod and rails is



6. A wire of length L and 3 identical cells of negligible internal resistance are connected in series, when the temperature of the wire is raised by (ΔT) in time t due to the current. The same temperature rise is observed in the same time when N similar cells are connected in series with a wire of length 2L but of same material and cross-section. Find the value of N.

7 A thin wire of area of crossection $A = 10^{-2} m^2$ is used to make a ring of radius $r = 10^{-1} m$. This ring is placed on a smooth horizontal floor & is given angular velocity ω = 20 rad/s about its centre. Find out stress in the ring (mass per unit length of wire λ = 1 kg/m)

8. Electric field $\vec{E} = x\hat{i} + y\hat{j}$ exist in the region. The flux linked with the surface of cube of side ‘a’ as shown in the figure is ka^3 then ‘k’ is



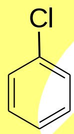
Space For Rough Work

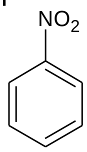
SECTION-2 : CHEMISTRY

PART – A

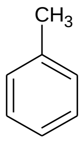
(Multi Correct Choice Type)

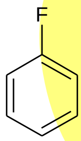
This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. Which of the following compound(s) is/are more reactive than chlorobenzene  towards electrophilic attack?

(A) 

(B) 

(C) 

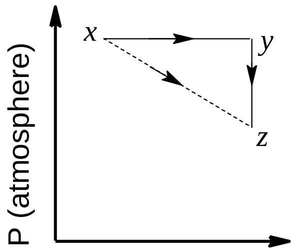
(D) 

2. For a reaction A $\rightarrow$ B, E_a for the forward reaction $(E_a)_f$ and backward reaction $(E_a)_b$ is 19 kJ/mole and 9 kJ/mole respectively. Potential energy of A is 12 kJ/mole
Find out which of the following options are correct?
(A) Heat of reaction is 10 kJ/mol
(B) It is an exothermic reaction
(C) The threshold energy of reaction is 31 kJ/mol
(D) The potential energy of product is 22 kJ/mol

3. Which of the following hydrogen halides react(s) with $AgNO_3(aq)$ to give a precipitate that dissolves in $Na_2S_2O_3(aq)$?
(A) HCl (B) HF
(C) HBr (D) HI

Space For Rough Work

4. For an ideal gas, consider only $P-V$ work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct? [Take ΔS as change in entropy and w as work done]

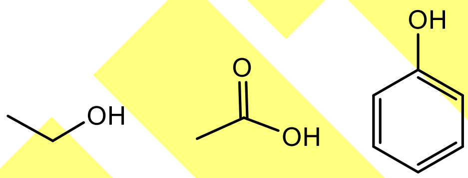


(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$
(B) $W_{x\to z} = W_{x\to y} + W_{y\to z}$
(C) $W_{x\to y\to z} = W_{x\to y}$
(D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

5. A salt heated with dil. H_2SO_4 and subsequently treated with few drops of dil. $KMnO_4$. The solution looses the colour of $KMnO_4$ indicates the presence of:
(a) Nitrite ion
(b) nitrate ion
(c) Oxalate ion
(d) sulfite ion

6. A solution of 1 mole benzene $\left(p^0_{\text{benzene}} = 42\text{mm of Hg}\right)$ and 2 mole toluene $\left(p^0_{\text{toluene}} = 36\text{mm of Hg}\right)$ at normal temperature & pressure, will have
(A) total vapour pressure 38 mm of Hg
(B) mole fraction of vapours of benzene above liquid mixture is 7/19
(C) positive deviation from Raoult's law
(D) negative deviation from Raoult's law

7. Correct acidity order of the following substrate in H_2O solvent.



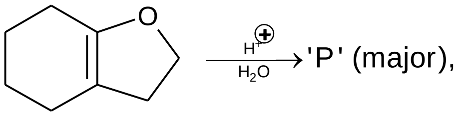
(a) (b) (c)

(A) b < c < a
(B) b >a
(C) b >c >a
(D)c >b

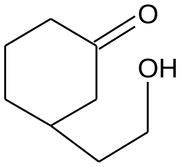
Space For Rough Work

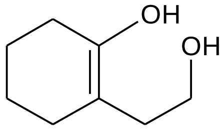
(Single Correct Choice Type)

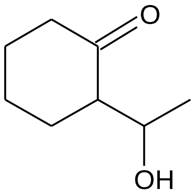
This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

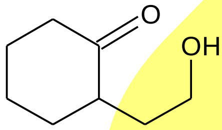
8. 

Identify the product ‘P’

(A) 

(B) 

(C) 

(D) 

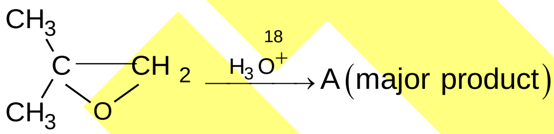
9. Which of the following ‘gem’ diols is stable?

(A) $H_5C_2-C(OH)_2-H$

(B) $Br_3C-C(OH)_2-H$

(C) $CF_3-C(OH)_2-H$

(D) $C_6H_5CH_2-C(OH)_2-CH_2C_6H_5$

10. 

The product (A) has the structure

(A) $(CH_3)_2\overset{OH}{O}-CH_2\overset{18}{O}H$

(B) $(CH_3)_2\overset{^{18}OH}{C}-CH_2OH$

(C) $CH_3\overset{^{18}OH}{CH}-\overset{OH}{CH}-CH_3$

(D) $(CH_3)_2\overset{OH}{C}-CH_2\overset{18}{O}H$

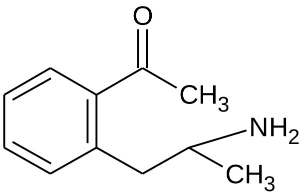
Space For Rough Work

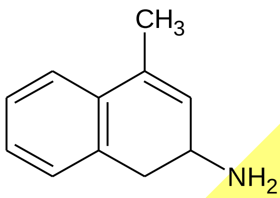
11. The flocculating power of the given ions for the specified colloidal sols will be such that:

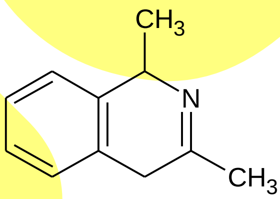
Arsenic sulphide sol.	**Ferric hydroxide sol**
(A) $\left[Fe(CN)_6\right]^{4-} > PO_4^{3-} > SO_4^{2-} > Cl^-$	$Al^{3+} > Ba^{2+} > Na^+$
(B) $Al^{3+} > Ba^{2+} > Na^+$	$\left[Fe(CN)_6\right]^{4-} > PO_4^{3-} > SO_4^{2-} > Cl^-$
(C) $Na^+ > Ba^{2+} > Al^{3+}$	$Cl^- > SO_4^{2-} > PO_4^{3-} > \left[Fe(CN)_6\right]^{4-}$
(D) $Cl^- > SO_4^{2-} > PO_4^{3-} > \left[Fe(CN)_6\right]^{4-}$	$Na^+ > Ba^{2+} > Al^{3+}$

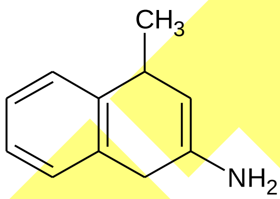
12. In an amino acid the carboxylic acid group has $K_a = 10^{-4}$ and amino group has $K_b = 10^{-5}$ at 298 K. The isoelectric point of that amino acid is
(A) 4
(B) 5
(C) 6.5
(D) 4.5

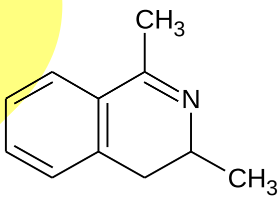
13. The compound shown below undergoes a spontaneous cyclization on long standing, the product of this intermolecular reaction is:



(A)


(B)


(C)


(D)


Space For Rough Work

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

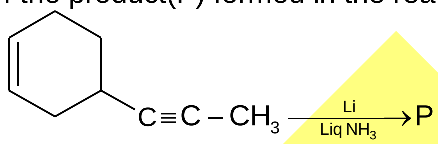
1. The successive ionization energies of an s-block element are
8.6, 12.9, 1630.2, 2406.2 eV, etc
If atomic mass of the elements is 39.8 g mol^{-1}, what will be the molar mass of it's normal oxide?

2. 5 mole of a mixture of $Na_2C_2O_4$ and Na_2CO_3 required 400 mL of 0.5 M acidified $KMnO_4$ for complete oxidation. What is the mole fraction of $Na_2C_2O_4$ in the mixture?

3 If the wavelength(λ) of the electronic transition n = 4 $\rightarrow$ n = 1 in a hydrogen atom is expressed as $\left(\frac{x}{y}\times 10^{-7}\right)$m. What is the value of $\left(\frac{x+y}{10}\right)$?
[Rydberg constant = 10^7 m^{-1}]

4. $CH_4 \xrightarrow[h\nu]{Cl_2(excess)} A + B + C + D$
If x is the molar mass of the most acidic product and y is the molar mass of the heaviest product, then what is the value of (x + y) in g mol^{-1} unit?

5. If the product(P) formed in the reaction



Has molar mass M_0 then $\frac{M_0}{4}$ is

6. pH of solution obtained by mixing 20 mL of 0.2 M NaOH with 50 mL of 0.2 M acetic acid ($K_a = 1.8 \times 10^{-5}$) is (log2 = 0.30, log3 = 0.47)

7. 100 mL of hard water is passed through a column of the ion exchange resin RH_2. The water coming off the column requires 15.17 mL of 0.0265 M NaOH for its titration. The hardness of water as ppm of Ca^{2+} is

8. $\text{Phenol} \xrightarrow[\text{(ii)}CO_2/140°C]{\text{(i)}NaOH} A \xrightarrow{H^+/H_2O} B \xrightarrow[CH_3COOH,\Delta]{Al_2O_3} C$
Molecular weight of C is m. What is the value of m/18

Space For Rough Work

SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. The equation $\sin^4 x + \cos^4 x = a$ has a real solution for

(A) all values of a
(B) $a = \frac{1}{2}$
(C) $a = \frac{7}{10}$
(D) a = 1

2. Let PQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true?

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$
(B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$
(C) $|\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 48\sqrt{3}$
(D) $\vec{a}.\vec{b} = -72$

3. If $\alpha = x_1x_2x_3$ and $\beta = y_1y_2y_3$ are two 3 – digit numbers, then the number of pairs of α and β can be formed so that α can be subtracted from β without borrowing, is

(A) $2!\times10!\times10!$
(B) $(45)(55)^2$
(C) $3^2.5^3.11^2$
(D) 136125

4. If all the roots of the equation $z^4 + az^3 + bz^2 + cz + d = 0$ $(a,b,c,d \in R)$ are of unit modulus, then

(A) $|a| \le 4$
(B) $|b| \le 4$
(C) $|c| \le 4$
(D) $|b| \le 6$

Space For Rough Work

5. Which of the following statement(s) is (are) correct?

(A) The coefficient of x^2 in the expansion of $\sum_{r=0}^{100} {}^{100}C_r (x-4)^{100-r}(5)^r$ is equal to 4950.

(B) If the sixth term in the expansion of $\left(x^{\frac{-8}{3}} + x^2 \log_{10} x\right)^8$ is 5600, then x is equal to 1000.

(C) Let $A_n = {}^nC_0\,{}^nC_1 + {}^nC_1\,{}^nC_2 + \ldots + {}^nC_{n-1}\,{}^nC_n$ and $\frac{A_{n+1}}{A_n} = \frac{15}{4}$, then the sum of possible values of n is equal to 6.

(D) If $A_k = \frac{{}^nC_k}{{}^nC_k + {}^nC_{k+1}}$ and $\sqrt[3]{\sum_{k=0}^{n-1} A_k} = 4$, then n is equal to 128.

6. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be respectively given by $f(x)=|x|+1$ and $g(x)=x^2+1$. Defined $h:\mathbb{R}\to\mathbb{R}$ by

$$h(x)=\begin{cases}\max\{f(x), g(x)\} & \text{if } x\le 0\\ \min\{f(x), g(x)\} & \text{if } x>0\end{cases}$$

The number of points at which h(x) is not differentiable is

(A) more than 2
(B) more than 2 but less than 4
(C) 3
(D) less than 5

7. Identify correct statement(s) about conic $\sqrt{(x-5)^2+(y-7)^2}+\sqrt{(x+1)^2+(y+1)^2}=12$.

(A) centre is (2, 3)
(B) hyperbola with foci (5, 7) and (–1, –1)
(C) ellipse with major axis $4x-3y+1=0$
(D) eccentricity is $\frac{5}{7}$

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. If $\tan\theta_1$, $\tan\theta_2$, $\tan\theta_3$ and $\tan\theta_4$ are the roots of the equation $x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0$ then $\tan(\theta_1+\theta_2+\theta_3+\theta_4)$ is equal to

(A) $\sin\beta$
(B) $\cos\beta$
(C) $\tan\beta$
(D) $\cot\beta$

Space For Rough Work

9. If a vertex of triangle is (3, 3) and the mid points of two sides through this vertex are $\left(2, \frac{2}{3}\right)$ and $\left(4, \frac{3}{2}\right)$ then the centroid of the triangle is given by

(A) (1, 3) (B) (3, 0)
(C) (3, 4/9) (D) (0, 3)

10. Three normals are drawn from the point (14, 7) to the parabola $y^2 - 16x - 8y = 0$. The coordinates of the feet of the normals are

(A) (0, 0), (8, –16), (3, –4) (B) (0, 0), (8, 16), (3, –4)
(C) (0, 0), (–8, 16), (3, –4) (D) None of these

11. For two 3×3 matrices A and B, let $A + B = 2B'$ and $3A + 2B = I_3$, where B's is the transpose of B and I_3 is 3×3 identity matrix. Then

(A) $10A + 5B = 3I_3$ (B) $5A + 10B = 2I_3$
(C) $3A + 6B = 2I_3$ (D) $B + 2A + I_3$

12. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to:

(A) (4, 5) (B) (–4, –5)
(C) (–4, 3) (D) (–4, 5)

13. Let $f : R \to R$ be a continuous onto function satisfying $f(x) + f(-x) = 0, \forall x \in R$. If $f(-3) = 2$ and $f(5) = 4$ in $[-5, 5]$, then the equation $f(x) = 0$ has:

(A) exactly three real roots (B) exactly two real roots
(C) at least five real roots (D) at least three real roots

Space For Rough Work

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. Through a given point P (5, 2),m secants are drawn to cut the circle $x^2+y^2=25$ at points $A_1(B_1), A_2(B_2), A_3(B_3), A_4(B_4)$ and $A_5(B_5)$ such that $PA_1+PB_1=5$, $PA_2+PB_2=6$, $PA_3+PB_3=7$, $PA_4+PB_4=8$ and $PA_5+PB_5=9$.
Find the value of $\dfrac{\left(\sum_{i=1}^{5} PA_i^2+\sum_{i=1}^{5} PB_i^2\right)}{2}$.
[Note : $A_r(B_r)$ denotes that the line passing through P (5, 2) meets the circle $x^2+y^2=25$ at two points A_r and B_r]

2. The quadratic polynomial $P(x)=ax^2+bx+c$ has two different zeroes including –2. The quadratic polynomial $Q(x)=ax^2+cx+b$ has two different zeroes including 3. If α and β be the other zeroes of P(x) and Q (x) respectively, then find the value of $\dfrac{3\alpha}{5\beta}$.

3. Let L denotes the number of surjective functions $f: A \to B$, where set A contains 4 elements and set B contains 3 elements.
M denotes number of elements in the range of the function.
$f(x)=\sec^{-1}(\operatorname{sgn} x)+\operatorname{cosec}^{-1}(\operatorname{sgn} x)$ where $\operatorname{sgn} x$ denotes signum function of x.
And N denotes coefficient of t^5 in $(1+t^2)^5(1+t^3)^8$. Find the value of $1.285(LM+N)$.

4. Let $0 \le \beta_r \le 1$ and $\sum_{r=1}^{k}\cos^{-1}\beta_r=\dfrac{k\pi}{2}$ for any $k \ge 1$ and $A=\sum_{r=1}^{k}(\beta_r)^r$, then $\lim_{x\to A}\dfrac{(1+x^2)^{1/3}-(1-2x)^{1/4}}{x+x^2}$ is equal to λ, then $6.258\lambda=$

Space For Rough Work

5. Given a function g continuous on R such that $\int_0^1 g(t)dt = 2$ and $g(1) = 5$. If $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)dt$, then the value of $1.85\left(f'''(1) - f''(1)\right)$ is equal to:

6. If $F(x) = \int \frac{(1+x)\left[\left(1-x+x^2\right)\left(1+x+x^2\right)+x^2\right]}{1+2x+3x^2+4x^3+3x^4+2x^5+x^6}dx$ then find the value of $6.76\left[F(99)-F(3)\right]$.
[Note: [k] denotes greatest integer less than or equal to k.]

7. If $6+f''(x) = x^2 + f^2(x)$ is the differential equation of a curve and let P be the point of minima of this curve then the number of tangents which can be drawn from P to the circle $x^2+y^2=4$ is k then 8.71k equals to

8. If a, b, c, d are distinct integers form an increasing AP such that $d = a^2+b^2+c^2$, then find the value of $8.69(a+b+c+d)$.

Space For Rough Work

QP Code: 100965

Answers

SECTION-1 : PHYSICS

PART – A

1.	**AB**	2.	**ABCD**	3.	**B**	4.	**ABC**
5.	**C**	6.	**ABCD**	7.	**C**	8.	**B**
9.	**C**	10.	**B**	11.	**D**	12.	**D**
13.	**B**						

PART – B

1.	**0.60**	2.	**4.50**	3.	**3.60**	4.	**6**
5.	**2**	6.	**6.00**	7.	**400**	8.	**2.00**

SECTION-2 : CHEMISTRY

PART – A

1.	BCD	2.	ACD	3.	ACD	4.	AC
5.	ACD	6.	AB	7.	B,C	8.	D
9.	C	10.	B	11.	B	12.	C
13.	D						

PART – B

1.	55.8	2.	0.1	3.	3.1	4.	273.5
5.	30.5	6.	4.50(Range 4.50 to 4.60)				
7.	80.40(Range 80.00 to 80.10)			8.	10		

SECTION-3 : MATHEMATICS

PART – A

1.	BCD	2.	ACD	3.	BCD	4.	ACD
5.	ACD	6.	ABCD	7.	AC	8.	D
9.	C	10.	B	11.	A	12.	D
13.	D						

PART – B

1.	107.50	2.	6.60	3.	97.66	4.	3.13
5.	5.55	6.	20.28	7.	17.42	8.	17.38

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **AB**

Sol. Displacement = Area under v-t graph.

For distance, consider magnitude of area only.

2. **ABCD**

Sol. Based on fact of errors and significant figures.

3. **B**

Sol. $q = q_0(1 - e^{-t/RC})$

$$U = \frac{q^2}{2C} = \frac{q_0^2(1 - e^{-t/RC})^2}{2C}$$

Rate = $\frac{dU}{dt}$

Rate will be maximum when $\frac{d^2U}{dt^2} = 0$.

4. **ABC**

Sol. For cylinder:

$$\left(\frac{dq}{dt}\right)_{net} = ms\frac{dT}{dt}$$

= Rate of heat inflow – Rate of heat loss to surroundings.

5. **C**

Sol. $B = \frac{\mu_0 I}{4\pi d}(\text{Sin}\,\theta_1 + \text{Sin}\,\theta_2)$

Use this formula for both straight sections.

6. **ABCD**

Sol. Final gas pressure $P = \frac{F}{A} = \frac{kx}{S}$

Work done by gas = Spring energy = $\frac{1}{2}kx_0^2$.

Temperature of gas will decrease.

7. **C**

Sol. Due to auxiliary battery E_0, find current in potentiometer wire. Then find $V_A - V_N$. This must be equal to potential difference produced by E_1 and E_2 combinely.

8. **B**

Sol. $m\frac{dV}{dt} = -mg - cv$

$$m\frac{dv}{mg + cv} = -dt$$

$$m\int_{V_0}^{0}\frac{dV}{mg + cv} = -\int_0^t dt$$

9. **C**

Sol. Assuming wall to be absent, it can be treated as standard projectile case having range

$= 8\sqrt{3} + 4\sqrt{3}$

$= 12\sqrt{3}$ m.

10. **B**

Sol. $I_{rms} = \dfrac{V_{rms}}{Z}$

$= \dfrac{20}{\sqrt{(X_L - X_C)^2 + R^2}}$

$\Delta q = C\Delta T = (I_{rms})^2 Rt$

11. **D**

Sol. Pressure difference along vertical = $\rho g_{eff} h$

$= \rho(g + a)h$

Pressure difference along horizontal = $\rho\left(\dfrac{r}{2}\omega^2\right)(r)$

12. **D**

Sol. $\vec{A} \times \vec{B} = \vec{C} + \vec{D}$

$\vec{A} \cdot \left(\vec{A} \times \vec{B}\right) = \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{D}$

$O = \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{D}$

$\vec{A} \cdot \vec{C} = -\vec{A} \cdot \vec{D}$

$\dfrac{\vec{A} \cdot \vec{C}}{A} = \dfrac{-\vec{A} \cdot \vec{D}}{A}$

$\Rightarrow$ option (D).

13. **B**

Sol. Escape speed is independent of angle of projection.

PART – B

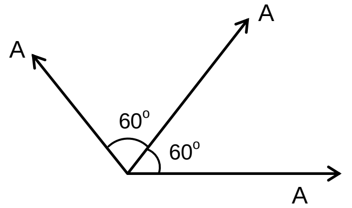
1. **0.60**

Sol. Number of moles of system remains conserved.

$\dfrac{(1)\,V}{RT} + \dfrac{(0.5)\,(4V)}{RT} = \dfrac{PV}{RT} + \dfrac{P(4V)}{RT}$

2. **4.50**

Sol. Use vector resultant method to find resultant amplitude.



3. **3.60**

Sol. As $F_{ext} = 0$, girl and box will meet at their centre of mass.

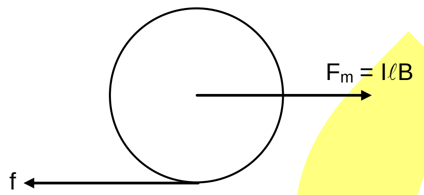
4. **6**

Sol. At the verge of rotation:

For translation:

$F_{net} = 0$ (Along horizontal and vertical)
$\tau_{net} = 0$ (About centre of cylinder)
Use: $f = \mu \times N$ ($N \to$ Normal reaction from adjoining surface)

5. **2**
Sol. For translation:



$I\ell B - f = ma$ ...(1)

For rotation about centre:

$$f(R) = \left(\frac{mR^2}{2}\right)(\alpha) \quad ...(2)$$

Use $\alpha = \frac{a}{R}$

6. **6.00**
Sol. Only wire will contribute to heating as cells are ideal.

$$I = \frac{\text{Net E.M.F.}}{\text{Resistance}}$$

$I^2Rt = mS\Delta T$

7. **400**
Sol. $Td\theta = \lambda r\, d\theta \omega^2 r$
$T = \lambda r^2 \omega^2$

$$\text{Stress} = \frac{T}{A} = \frac{\lambda r^2 \omega^2}{A}$$
$$= 4$$

8. **2.00**
Sol. Flux = $2a^3$.

SECTION-2 : CHEMISTRY
PART – A

1. BCD
Sol. Fluorine is more activating than chlorine due to greater +R effect.

2. ACD
Sol. Heat of reaction = 19 – 9 = 10(It is not a exothermic reaction)

3. ACD
Sol. $HX + AgNO_3 \rightarrow AgX \downarrow + HNO_3\left(X = Cl, Br, I\right)$

$AgX + 2Na_2S_2O_3 \rightarrow Na_3\left[Ag\left(S_2O_3\right)_2\right] + NaX$

4. AC
Sol. $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}\left[\text{entropy}\left(\text{S}\right)\text{is a state function, hence additive}\right]$

$W_{x\to y\to z} = W_{x\to y}\left[\text{work done in Y} \rightarrow \text{Z is zero as it is an isochoric process}\right]$

5. ACD
Sol. Nitrate ion is not oxidized by $KMnO_4$.

6. AB
Sol. $p = p_A^0 x_A + p_B^0 x_B A \rightarrow$ Benzene, B$\rightarrow$Toluene

$= 42 \times \frac{1}{3} + 36 \times \frac{2}{3}$

$= \frac{114}{3} = 38\text{mm}$ of Hg

Mole fraction of benzene in vapour = $\frac{p_{\text{benzene}}}{p_{\text{total}}} = \frac{42/3}{38}$ = 7/19

7. B, C
Sol. $b \rightarrow pK_a = 4.76$

$c \rightarrow pK_a = 9.95$

$a \rightarrow pK_a \approx 16 - 17$

8. D
Sol. Cleavage of ether takes place.

9. C
Sol. gem diols are stabilised by electron withdrawing groups.

10. B
Sol. Cleavage of ether takes place in acidic medium .

11. B
Sol. 1. Negatively charged arsenic sulphide solution is coagulated by higher valence of active cation.
2. Positive sol. $\left[Fe(OH)_3\right]$ is coagulated by higher valency of active anion.

12. C
Sol. Isoelectronic point = $\frac{4+9}{2} = 6.5$

13. D
Sol. Cyclisation takes place by reaction between NH_2 group with the ketone part of the molecule.

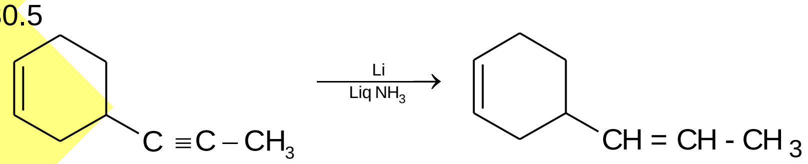
PART – B

1. 55.8
Sol. There is a large jump from $I.E_3$ which means the atom contains two valence electron.
$\therefore$ It's valency is 2
Let the atom is M
So formula of it's oxide = Mo
Molar mass = 39.8 + 16 = 55.8 g mol^{-1}

2. 0.1
Sol. Na_2CO_3 does not react with $KMnO_4/H^+$
M_{eq} of $KMnO_4/H^+ = 400 \times 0.5 \times 5 = 1000$
$\therefore M_{eq}$ of $Na_2C_2O_4 = 1000$
or $\frac{W}{E} \times 1000 = 1000$
$\therefore \frac{W}{E} = 1$
n-factor of $Na_2C_2O_4$ is 2 as it is converted to CO_2
$\therefore$ Moles of $Na_2C_2O_4 = = \frac{\text{Equivalent}}{\text{n} - \text{factor}} = \frac{W/E}{2} = \frac{1}{2}$
$\therefore$ Mole fraction $= \frac{1/2}{5} = 0.1$

3. 3.1
Sol. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
or, $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = R\left[1 - \frac{1}{16}\right] = \frac{15R}{16}$
or, $\lambda = \frac{16}{15R} = \frac{16}{15} \times 10^{-7} = \left(\frac{x}{y} \times 10^{-7}\right)$
$\therefore \frac{x+y}{10} = \frac{16+15}{10} = 3.1$

4. 273.5
Sol. x = $CHCl_3$, y is CCl_4
Molar mass of $CHCl_3$ = 119.5
Molar mass of y = 154
x + y = 119.5 + 154 = 273.5

5. 30.5
Sol.

$M_o = 122$, $\frac{M_o}{4} = 30.5$

6. 3.82
Sol. $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$
Initial 50×0.2 20×0.2

10 m moles   4 m moles

After Neutralization 6   –

$$pH = p^{K_a} + \log\frac{Salt}{Acid} = 4.76 + \log\frac{4}{6} = 3.82$$

7. 80.40

Sol. M_{eq} of Ca^{2+} = 15.17 × 0.0265 × 1 = 0.402

Hardness of water = $\frac{0.402 \times 20}{100 \times 1000} \times 10 = 80.4$

8. 10

SECTION-3 : MATHEMATICS
PART – A

1. BCD

Sol. We have $\sin^4 x + \cos^4 x \le \sin^2 x + \cos^2 x$, as $|\sin x| \le 1$ and $|\cos x| \le 1$

$\Rightarrow a \le 1$ (1)

Next, $\sin^4 x + \cos^4 x = a$

$\Rightarrow \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x = a$

$\Rightarrow \frac{1}{2}\sin^2 2x = 1 - a$

$\Rightarrow 1 - a \le \frac{1}{2}$ $\left[\because \sin^2 2x \le 1\right]$

$\Rightarrow \frac{1}{2} \le a$ (2)

From (1) and (2) we get $\frac{1}{2} \le a \le 1$. Note that $a = \frac{1}{2}$ for $x = \frac{\pi}{4}$ and $a = 1$ for $x = \frac{\pi}{2}$.

2. ACD

Sol. As $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow a^2 = b^2 + c^2 + 2\vec{b}.\vec{c}$

$\Rightarrow 144 = 48 + c^2 + 48 \Rightarrow c^2 = 48 \Rightarrow c = 4\sqrt{3}$

Again, $c^2 = a^2 + b^2 + 2\vec{a}.\vec{b}$

$\Rightarrow \vec{a}.\vec{b} = \frac{48 - 144 - 48}{2} = -72$

$\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = \left|\vec{a} \times \vec{b} + \vec{a} \times \vec{b}\right| = 2\left|\vec{a} \times \vec{b}\right|$

$= 2\sqrt{a^2b^2 - \left(\vec{a}.\vec{b}\right)^2} = 2\sqrt{12^2.48 - (-72)^2}$

$= 2.12\sqrt{48 - 36} = 2.12.\ 2\sqrt{3} = 48\sqrt{3}$

3. BCD

Sol. Since, α can be subtracted from β without borrowing, if $y_i \ge x_i$, for $i = 1, 2, 3$

Let $x_i = \lambda$

If $i = 1$, then $\lambda = 1, 2, 3, ...9$ and if $i = 2$ and 3, then $\lambda = 0, 1, 2, 3, ..., 9$

Hence, total number of ways of choosing the pairs α, β

$$= \left(\sum_{\lambda=1}^{9}(10 - \lambda)\right)\left(\sum_{\lambda=0}^{9}(10 - \lambda)\right)^2 = (45)(55)^2$$

4. ACD

Sol. $z^4 + az^3 + bz^2 + cz + d = 0 \left(a, b, c, d \in R\right)$

Let roots are z_1, z_2, z_3, z_4

$|z_1| = |z_2| = |z_3| = |z_4| = 1$

$z_1 + z_2 + z_3 + z_4 = -a$

$z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4 = b$

$z_1z_2z_3 + z_1z_2z_4 + z_1z_3z_4 + z_2z_3z_4 = -c$

$z_1z_2z_3z_4 = d$

$|z_1+z_2+z_3+z_4| \le |z_1|+|z_2|+|z_3|+|z_4|$

$|a| \le 4;\ \ |b| \le 6;\ \ |c| \le 4;\ \ |d| = 1$

5. ACD

Sol.

(a) Clearly, $E = ((x-4)+5)^{100} = (1+x)^{100}$

$\therefore$ Coefficient of x^2 in $(1+x)^{100} = {}^{100}C_2 = 4950$

(b) Clearly, ${}^8C_5\left(x^{-8/3}\right)^3\left(x^2\log_{10}x\right)^5 = 5600$

$\Rightarrow x^2(\log_{10}x)^5 = 100$

$\Rightarrow x = 10$

(c) Clearly

$A_n = {}^nC_0\,{}^nC_{n-1} + {}^nC_1\,{}^nC_{n-2} + \ldots + {}^nC_{n-1}\,{}^nC_0$

$= {}^{2n}C_{n-1}$

(We can also explain by using theory of permutation).

Now, $\dfrac{A_{n+1}}{A_n} = \dfrac{(2n+2)!}{(n)!(n+2)!} \times \dfrac{(n-1)!(n+1)!}{(2n)!}$

$= \dfrac{(2n+2)(2n+1)}{n(n+2)} = \dfrac{15}{4}$

$\Rightarrow n = 2, 4$

Hence, Sum $= 2+4 = 6$

(d) As $A_k = \dfrac{{}^nC_k}{{}^{n+1}C_{k+1}} = \dfrac{k+1}{n+1}$

$\Rightarrow \sum_{k=0}^{n-1} A_k = \dfrac{n}{2}$

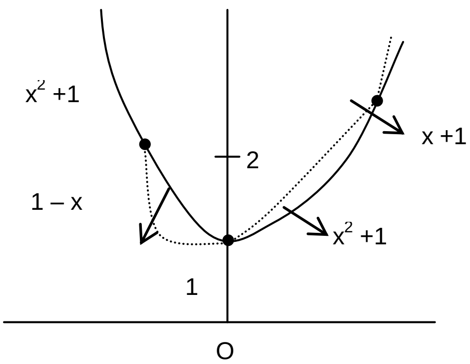
Now, $\sum_{k=0}^{n-1} A_k = 64$

$\Rightarrow \dfrac{n}{2} = 64 \Rightarrow n = 128$

6. ABCD

Sol.

$$h(x) = \begin{cases} x^2+1 & x \le -1 \\ |x|+1 & -1 < x < 0 \\ x^2+1 & 0 \le x < 1 \\ |x|+1 & x > 1 \end{cases}$$



Clearly 3 points of non – differentiability.

7. AC

Sol. $\sqrt{(x-5)^2+(y-7)^2} + \sqrt{(x+1)^2+(y+1)^2} = 12$

$SP + S'P = 2a \Rightarrow$ conic is ellipse

$$2ae = \sqrt{36+64}$$

$$2a = 12 \Rightarrow e = \frac{5}{6}$$

Major axis $y-7 = \frac{7+1}{5+1}(x-5)$

$\Rightarrow 4x-3y+1=0$

8. D

Sol. From the given equation we get

$S_1 = \tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \tan\theta_4 = \sin 2\beta$.

$S_2 = \sum \tan\theta_1 \tan\theta_2 = \cos 2\beta$

$S_3 = \sum \tan\theta_1 \tan\theta_2 \tan\theta_3 = \cos\beta$

and $S_4 = \tan\theta_1 \tan\theta_2 \tan\theta_3 \tan\theta_4 = -\sin\beta$

Now $\tan(\theta_1+\theta_2+\theta_3+\theta_4) = \frac{S_1 - S_3}{1 - S_2 + S_4}$.

$$= \frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta} = \frac{\cos\beta(2\sin\beta - 1)}{\sin\beta(2\sin\beta - 1)} = \cot\beta$$

9. C

Sol. Let $A \equiv (3, 3)$

Given that $F \equiv \left(2, \frac{2}{3}\right)$ and $E \equiv \left(4, \frac{3}{2}\right)$ are mid points of AB and AC.

Let $B \equiv (x_1, y_1)$ and $C \equiv (x_2, y_2)$

Using mid point formula.

Co – ordinate of $F \equiv \left(2, \frac{2}{3}\right) = \left(\frac{3+x_1}{2}, \frac{3+y_1}{2}\right)$

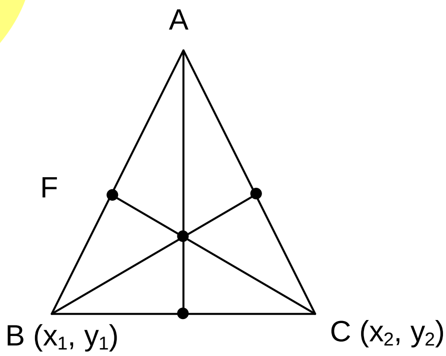
$\Rightarrow \quad (x_1, y_1) = \left(1, -\frac{5}{3}\right)$

Co – ordinate of $E = \left(4, \frac{3}{2}\right) = \left(\frac{3+x_2}{2}, \frac{3+y_2}{2}\right)$

$\Rightarrow (x_2, y_2) = (5, 0)$

$\therefore$ Centroid of triangle is

$$G(x,y) = \left(\frac{3+1+5}{3}, \frac{3-\frac{5}{3}+0}{3}\right) = \left(3, \frac{4}{9}\right)$$



10. B

Sol. Given parabola is $y^2 - 16x - 8y = 0$ ....(1)

Let the coordinates of the feet of the normal from (14, 7) be $P(\alpha, \beta)$

Now, equation of the tangent at $P(\alpha, \beta)$ to parabola (1) is $y\beta - 8(x+\alpha) - 4(y+\beta) = 0$

Or, $(\beta - 4)y = 8x + 8\alpha + 4\beta$ ....(2)

Its slope $= \frac{8}{\beta - 4}$

Equation of normal to parabola (1) at (a, b) is $\gamma-\beta=\frac{4-\beta}{8}(x-\alpha)$.

It passes through (14, 7),

$\therefore \quad 7-\beta=\frac{4-\beta}{8}(14-\alpha)$ or $\alpha=\frac{6\beta}{\beta-4}$ ……….(3)

Also, (α,β) lies on parabola (1), $\therefore \beta^2-16\alpha-8\beta=0$

Putting the value of a from (3) in (4), we get

$\beta^2-\frac{96\beta}{\beta-4}-8\beta=0 \quad \Rightarrow \quad \beta(\beta^2-12\beta-64)=0$

$\Rightarrow \beta(\beta-16)(\beta+4)=0$

$\therefore \quad \beta=0, 16, -4$

From (3), when $\beta=0$, $\alpha=0$

When $\beta=16$, $\alpha=8$ and when $\beta=-4, \alpha=3$

Hence, the feet of the normals are (0, 0), (8, 16) and (3, –4).

11. A

Sol. $A+B=2B'$

$\Rightarrow (A+B)'=(2B')'$

$\Rightarrow A'+B'=2B \quad (\because (A+B)'=A'+B')$

$\Rightarrow B=\frac{A'+B'}{2}$

Now, $A+\left(\frac{B'+A'}{2}\right)=2B' \quad [\because A+B=2B']$

$\Rightarrow 2A+B'+A'=4B' \Rightarrow 2A+A'=3B'$

$\Rightarrow A=\frac{3B'-A'}{2}$ ……..(1)

Also, $3A+2B=I_3$

$\Rightarrow 3\left(\frac{3B'-A'}{2}\right)+2\left(\frac{A'+B'}{2}\right)=I_3$

$\Rightarrow \left(\frac{9B'+2B'}{2}\right)+\left(\frac{2A'-3A'}{2}\right)=I_3$

$\Rightarrow (11B'-A')'=(2I_3)'$

$\Rightarrow 11B-A=2I_3$ …….(2)

Multiplying (2) by 3 and then adding (1) and (2), we get

$35B=7I_3 \Rightarrow B=\frac{I_3}{5}$

From (2), $11\frac{I_3}{5}-A=2I_3 \Rightarrow 11\frac{I_3}{5}-2I_3=A$

$\Rightarrow A=\frac{I_3}{5}$

$\therefore 5A=5B=I_3 \Rightarrow 10A+5B=3I_3$

12. D

Sol. $(5x-4)\begin{vmatrix}1 & 2x & 2x\\ 1 & x-4 & 2x\\ 1 & 2x & x-4\end{vmatrix}$

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -(x+4) & 0 \\ 0 & 0 & -(x+4) \end{vmatrix}$$

$$= (5x-4)(x+4)^2 = (A+Bx)(x-A)^2$$

$\Rightarrow \quad A = -4, B = 5$

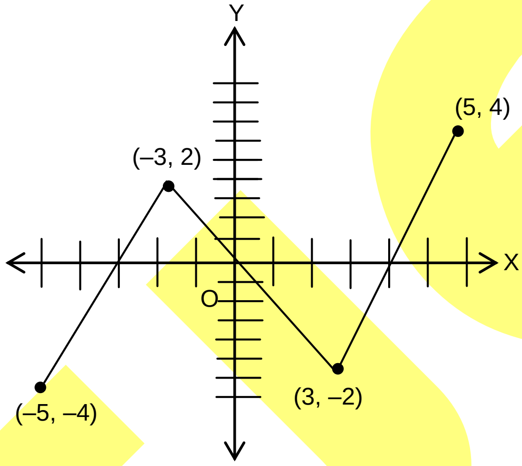
13. D

Sol. $f(x) + f(-x) = 0$

$\Rightarrow f(x)$ is an odd function. Since points (–3, 2) and (5, 4) lie on the curve.

$\therefore \quad (3, -2)$ and $(-5, -4)$ will also lie on the curve

For minimum number of roots, graph of continuous function $f(x)$ is as follows.

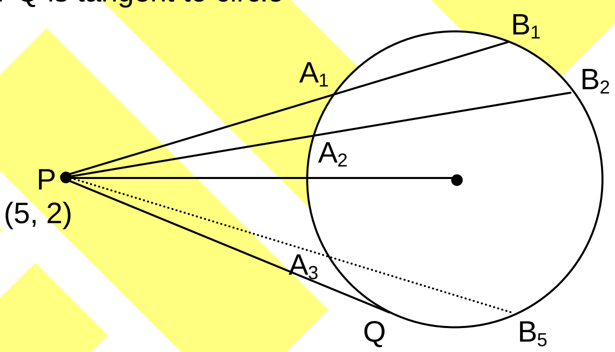


From the above graphs of $f(x)$ it is clear that equation $f(x) = 0$ has at least three real roots.

PART – B

1. 107.50

Sol. PQ is tangent to circle



$PA_1.PB_1 = PQ^2 = 4$

$PA_2.PB_2 = 4$

:

$PA_5.PB_5 = 4$

Given $PA_1 + PB_1 = 5$

:

$PA_5 + PB_5 = 9$

Square and Add

$\left(PA_1^2 + ...PA_5^2\right) + \left(PB_1^2 + PB_2^2 ... + PB_5^2\right)$

$$=\left[5^2+6^2+...+9^2\right]-2\left[4+4+4+4+4\right]$$
$$=255-40=215$$

2. 6.60

Sol. $P(x)=ax^2+bx+c=0\ (-2,\alpha)$

$Q(x)=ax^2+cx+b=0\ (3,\beta)$

$S_p+P_q=0$

$\left[S_p\to \text{sum of } P(x) \text{ and } P_q\to \text{Product of } P(x)\right]$

and $P_p+s_Q=0$

$\therefore\quad \alpha-2+3\beta=0$

$\Rightarrow\quad \alpha+3\beta=2$ ....(i)

and $-2\alpha+3+\beta=0$

$\Rightarrow\quad -2\alpha+\beta=-3$ .....(ii)

From equation (i) and (ii), we get

$\alpha=\dfrac{11}{7}$ and $\beta=\dfrac{1}{7}$

$\dfrac{\alpha}{\beta}=\dfrac{11}{7}\cdot\dfrac{7}{1}=11$

3. 97.66

Sol. $L:3^4-\left[{}^3C_1\left(2^4-2\right)+{}^3C_2\right]=36$

$M:$ If $x>0$, $\text{sgn}(x)=1$

$f(x)=0+\dfrac{\pi}{2}=\dfrac{\pi}{2}$

For $x=0\,f(x)$ is not defined

$\therefore$ For $x<0,\ f(x)=\pi-\dfrac{\pi}{2}=\dfrac{\pi}{2}$

$\therefore M=1$

N : Coefficient of $t^5=$ coefficient of t^2 in $\left(1+t^2\right)^5\times$ coefficient of t^3 in $\left(1+t^3\right)^8$

$=5\times 8=40$

Hence, L = 36; M = 1 and N = 40

$\Rightarrow LM+N=36+40=76$

4. 3.13

Sol. Since maximum value of $\cos^{-1}x$ in $[0,1]=\dfrac{\pi}{2},\sum_{r=1}^{k}\cos^{-1}\beta_r=\dfrac{k\pi}{2}$ is possible if and only if each $\cos^{-1}\beta_r=\dfrac{\pi}{2}\Leftrightarrow\beta_r=0$.

$A=\sum_{r=1}^{k}\left(\beta_r\right)^r=0$

Thus, $\lim_{x\to 0}\dfrac{(1+x)^{1/3}-(1-2x)^{1/4}}{x+x^2}$

$$= \lim_{x\to 0} \frac{\left(1+\frac{x^2}{3}+0(x^4)\right)-\left(1-\frac{x}{2}+0(x^2)\right)}{x(1+x)}$$

$$= \lim_{x\to 0} \frac{\frac{1}{2}+\frac{x}{3}+0(x^2)}{1+x} = \frac{1}{2}.$$

5. 5.55

Sol. $f(x) = \frac{1}{2}\left(x^2.\int_0^x g(t)dt - 2x.\int_0^x t.g(t)dt + \int_0^x t^2.g(t)dt\right)$

$\therefore$ Differentiate both sides with respect to x, we get

$f'(x) = x.\int_0^x g(t)dt - \int_0^x t.g(t)dt$ …….(i)

Again differentiate both sides with respect to x, we get

$f''(x) = \int_0^x g(t)dt$ ……(ii)

$\therefore$ $f''(1) = 2$

Also, $f'''(x) = g(x)$

$\Rightarrow$ $f'''(1) = g(1) = 5$

6. 20.28

Sol. $F(x) = \int \frac{(1+x)(1+x^2)^2}{(1+x)^2(1+x^2)^2}dx = \ln(1+x)+c$

$\therefore$ $[F(99)-F(3)] = [\ln 25] = 3$

7. 17.42

Sol. At P (x, y) for minima, $f'(x)=0,\ f''(x)>0$

$\Rightarrow x^2+f^2(x)-6>0 \Rightarrow x^2+y^2>6$

(i.e., P lies outside $x^2+y^2=6$)

$\therefore$ Number of tangents to $x^2+y^2=4$ is 2

8. 17.38

Sol. Here, sum of numbers i.e. $a+b+c+d$ is not given.

Let, $b=a+D,\ c=a+2D,\ d=a+3D, \forall D\in N$

According to hypothesis,

$a+3D = a^2+(a+D)^2+(a+2D)^2$

$\Rightarrow 5D^2+3(2a-1)D+3a^2-a=0$ ……..(i)

$$\therefore\ D = \frac{-3(2a-1)\pm\sqrt{9(2a-1)^2-20(3a^2-a)}}{10}$$

$$= \frac{-3(2a-1)\pm\sqrt{(-24a^2-16a+9)}}{10}$$

Now, $-24a^2-16a+9\ge 0$

$\Rightarrow$ $24a^2+16a-9\le 0$

$\Rightarrow$ $-\frac{1}{3}-\frac{\sqrt{70}}{3}\le a\le -\frac{1}{3}+\frac{\sqrt{70}}{12}$

$\Rightarrow$ $a=-1, 0$ $[\because a\in I]$

When $a = 0$ from Equation (i), $D = 0, \frac{3}{5}$ (not possible $\because D \in N$) and for $a = -1$

From equation (i), $D = 1, \frac{4}{5}$

$\therefore \quad D = 1 \quad [\because D \in N]$

$\therefore \quad a = -1, b = 0, c = 1, d = 2$

Then $a + b + c + d = -1 + 0 + 1 + 2 = 2$