

Competishun

52/6, Opposite Metro Mas Hospital, Shipra Path, Mansarovar

Date: 11/11/2024

Time: 3 hours

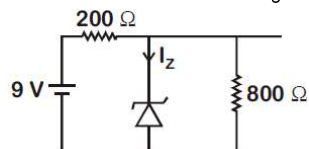
Max. Marks: 300

UTS-I_MT-1 (24-25)

Physics

Single Choice Question

Q1 The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



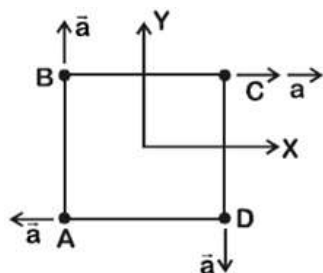
The current I_Z through the Zener is

- a) 15 mA b) 7 mA c) 10 mA d) 17 mA

Q2 The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s. The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

- a) 2.25 b) 4.25 c) 6.25 d) 8.25

Q3 Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



- a) Zero b) $a(\hat{i} + \hat{j})$ c) $\frac{a}{5}(\hat{i} + \hat{j})$ d) $\frac{a}{5}(\hat{i} - \hat{j})$

Q4 Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is:-

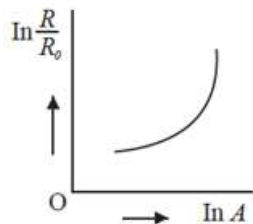
- a) $\left[2G \frac{(m_1 - m_2)}{r} \right]^{1/2}$ b) $\left[\frac{2G}{r} (m_1 + m_2) \right]^{1/2}$ c) $\left[\frac{r}{2G(m_1 m_2)} \right]^{1/2}$ d) $\left[\frac{2G}{r} m_1 m_2 \right]^{1/2}$

Q5 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3 \times 10^8 \text{ m/s}$) :

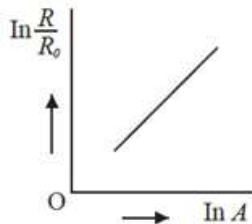
- a) $20 \times 10^{-8} \text{ N}$ b) $35 \times 10^{-8} \text{ N}$ c) $15 \times 10^{-8} \text{ N}$ d) $10 \times 10^{-8} \text{ N}$

- Q6** Which of the following figure represents the variation of $\ln \left(\frac{R}{R_0} \right)$ with $\ln A$ (If R = radius of a nucleus and A = its mass number)

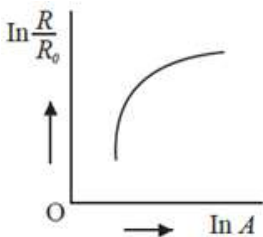
a)



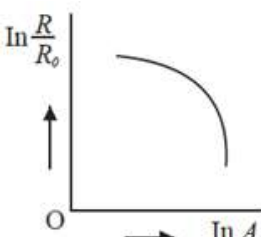
b)



c)



d)

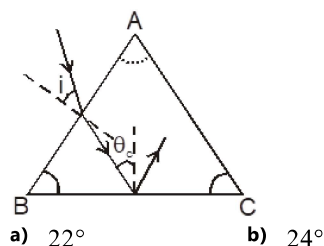


- Q7** The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v is in m/s and x is in m. Find its acceleration in m/s^2 when passing through the point $x = 2\text{m}$
- a) 0 b) 5 c) 11 d) 30

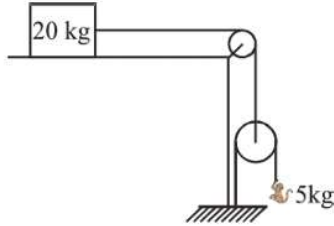
- Q8** A particle is projected from a point $P(2, 0, 0)\text{m}$ with a velocity 10 m/s making an angle 45° with the horizontal. The plane of projectile motion passes through a horizontal line PQ which makes an angle of 37° with positive x-axis, xy plane is horizontal. The coordinates of the point where the particle will strike the line PQ is: (Take $g = 10\text{ m/s}^2$)

- a) $(10, 6, 0)\text{m}$ b) $(8, 6, 0)\text{m}$ c) $(10, 8, 0)\text{m}$ d) $(6, 10, 0)\text{m}$

- Q9** A light is incident on face AB of an equilateral glass prism ABC . After refraction at AB , the ray is incident on face BC at the angle slightly greater than critical angle so that it gets reflected from face BC and finally emerges out from face AC . Net deviation angle of the ray is 112° anticlockwise. The angle of incidence ' i ' has value :

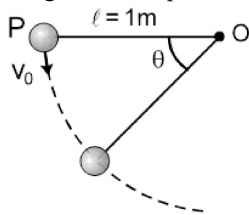


- Q10** The monkey climbs up the rope with acceleration of 2 m/s^2 , with respect to string (which monkey is holding). If all surfaces are smooth, string and pulley are light, what will be the acceleration of 20 kg blocks?



- a) 2 m/s^2 b) 3 m/s^2 c) 4 m/s^2 d) 0

- Q11** The sphere at P is given a downward velocity v_0 and swings in a vertical plane at the end of a rope of $\ell = 1\text{ m}$ attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can withstand a maximum tension equal to three times the weight of the sphere. Then the value of v_0 will be : ($g = \pi^2\text{ m/s}^2$)

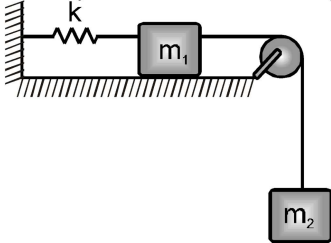


- a) $\frac{g}{2}ms$ b) $\frac{2g}{3}ms$ c) $\sqrt{\frac{3g}{2}}ms$ d) $\frac{g}{3}ms$

- Q12** A uniform smooth rod is placed on a smooth horizontal floor is hit by a particle moving on the floor, at a distance $\frac{\ell}{4}$ from one end. Then the distance travelled by the centre of the rod after the collision when it has completed three revolution will be: [$e \neq 0$ & ' ℓ ' is the length of the rod]

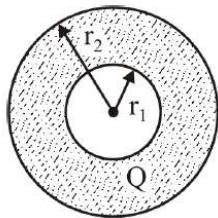
- a) $2\pi\ell$ b) can't be determined
c) $\pi\ell$ d) none of these

- Q13** m_1 & m_2 are connected with a light inextensible string with m_1 lying on smooth table and m_2 hanging as shown in figure. m_1 is also connected to a light spring which is initially unstretched and the system is released from rest.



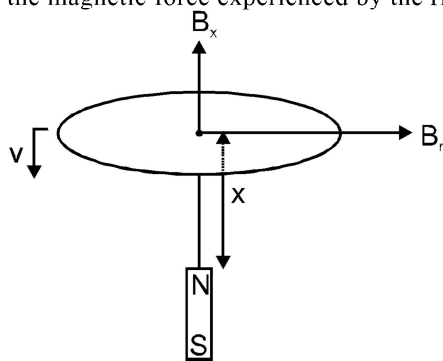
- a) system performs SHM with angular frequency given by $\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$
b) system performs SHM with angular frequency given by $\sqrt{\frac{k}{m_1 + m_2}}$
c) tension in string will be zero when the system is released
d) maximum displacement of m_1 will be $\frac{m_2 g}{k}$

- Q14** A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 (See figure). The electric field at distance x from centre for $r_1 < x < r_2$ will be :-



- a) $\frac{Q(x^3 - r_1^3)}{4\pi\epsilon_0 x^2 r_2^3}$ b) $\frac{Q(x^3 - r_1^3)}{4\pi\epsilon_0 x^2 (r_2^3 - r_1^3)}$ c) $\frac{Qx}{4\pi\epsilon_0 (r_2^3 - r_1^3)}$ d) $\frac{Q(x^3 - r_1^3)}{4\pi\epsilon_0 r_1^2 (r_2^2 - r_1^2)}$

- Q15** A conducting ring of radius r and resistance R is moving downward with a constant velocity (V). Due to a fixed magnet, axial magnetic field varies only with the axial distance (x) from north pole as $B_x = \frac{1}{\pi x}$ Tesla and the radial magnetic field varies as $B_r = \frac{2}{\pi x}$ Tesla. When the ring is at a height x from the north pole of the magnet, the magnetic force experienced by the ring is :



- a) $\frac{2r^3 v}{Rx^3}$ upwards b) $\frac{2r^3 v}{Rx^3}$ downwards c) $\frac{4r^3 v}{Rx^3}$ upwards d) $\frac{4r^3 v}{Rx^3}$ downwards

- Q16** The primary winding of the transformer to power the radio receiver has 1200 turns with input voltage of 120 V. What should be the number of turns of the transformer in secondary winding to power the lamp filament, if it requires 3.6 V voltage and current of 1A? Secondary has a resistance of 0.4 ohms. The loss in the primary winding may be neglected.

- a) 40 b) 36 c) 48 d) 18

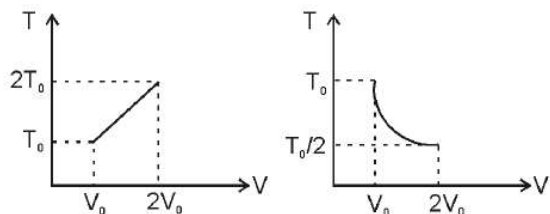
- Q17** A string under a tension of 90 N produces 10 beats/second when it is vibrated in its fundamental mode along with a tuning fork. When the tension is the string is increased to 160N. It sounds in resonance in its fundamental mode with same tuning fork. Calculate fundamental frequency of tuning fork.

- a) 100 Hz b) 40 Hz c) 150 Hz d) 200 Hz

- Q18** In Young's double slit experiment, we get 60 fringes in the field of view of monochromatic light of wavelength 4000 \AA . If we use monochromatic light of wavelength 6000 \AA , then the number of fringes that would be obtained in the same field of view is :

- a) 60 b) 90 c) 40 d) 1.5

- Q19** For two thermodynamic process temperature and volume diagram are given. In first process, it is a straight line having initial and final coordinates as (V_0, T_0) and $(2V_0, 2T_0)$, where as in second process it is a rectangular hyperbola having initial and final coordinates (V_0, T_0) and $(2V_0, T_0/2)$. Then ratio of work done in the two processes must be



- a) 1 : 2 b) 2 : 1 c) 1 : 1 d) None of these

- Q20** Two spheres of radii r_1 and r_2 have densities ρ_1 and ρ_2 and specific heats s_1 and s_2 respectively. If they are heated to the same temperature, then the ratio of their rates of cooling initially in the same surrounding will be: (assume that both surface has same emissivity)

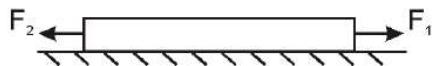
- a) $\frac{r_2 \rho_2 s_2}{r_1 \rho_1 s_1}$ b) $\frac{r_2 \rho_2 s_1}{r_1 \rho_1 s_2}$ c) $\frac{r_1 \rho_1 s_1}{r_2 \rho_2 s_2}$ d) $\frac{r_2 \rho_1 s_1}{r_1 \rho_2 s_1}$

Numerical

- Q21** An unknown quantity x is measured using an experiment by measuring a length ℓ (in cm) from scale having least count of 1 cm. Formula used is $x = R \frac{\ell}{100 - \ell}$. R is known accurately. Find the percentage error in measurement of ' x ' for $\ell = 50$ cm.

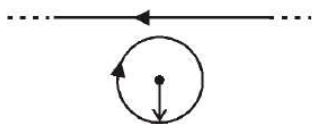
- Q22** A particle is displaced from point $A(0,0,0)$ to $D(2,0,2)$ via point $B(3,4,6)$ and $C(6,9,4)$. If two constant forces $\vec{F}_1 = (\hat{i} + \hat{j})n$ and $\vec{F}_2 = (3\hat{i} + 4\hat{k})N$ apart from other forces were acting on the particle during the whole journey then find the total work (in Joule) done by these two forces on the particle.

- Q23** Two opposite forces $F_1 = 120\text{N}$ and $F_2 = 80\text{N}$ act on an heavy elastic plank of modulus of elasticity $y = 2 \times 10^{11} \text{ N/m}^2$ and length $L = 1\text{m}$ placed over a smooth horizontal surface. The cross-sectional area of plank is $A = 0.5\text{m}^2$. If the change in the length of plank is $x \times 10^{-9} \text{ m}$, then find x ?



- Q24** When an air bubble rises from the bottom to the surface of a lake, its radius becomes double. The depth of the lake is d and the atmospheric pressure is equal to the pressure due to column of water 10m high. Assume constant temperature and neglect the effect of surface tension and viscosity. Find the value of $\frac{d}{10}$.

- Q25** The radius of a coil of wire with N turns is 0.22 m, and 3.5 A current flows clockwise in the coil as shown. A long straight wire carrying a current 54A toward the left is located 0.05 m from the edge of the coil. The magnetic field at the centre of the coil is zero tesla. The number of turns N in the coil are:



Chemistry

Single Choice Question

Q26 A compound 'X' on treatment with Br_2/NaOH , provided $\text{C}_3\text{H}_9\text{N}$, which gives positive carbylamine test. Compound 'X' is :

- a) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CONH}_2$ b) $\text{CH}_3\text{COCH}_2\text{NHCH}_3$
 c) $\text{CH}_3\text{CH}_2\text{COCH}_2\text{NH}_2$ d) $\text{CH}_3\text{CON}(\text{CH}_3)_2$

Q27 The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by $\Delta G^\circ = A - BT$

Where A and B are non-zero constants. Which of the following is true about this reaction?

- a) Exothermic if $B < 0$ b) Endothermic if $A > 0$
 c) Endothermic if $A < 0$ and $B > 0$ d) Exothermic if $A > 0$ and $B < 0$

Q28 SO_2Cl_2 (sulphuryl chloride) reacts with water to give a mixture of H_2SO_4 and HCl . What volume of $0.2\text{M Ba}(\text{OH})_2$ is needed to completely neutralize 25 mL of $0.2\text{M SO}_2\text{Cl}_2$ solution:

- a) 25 mL b) 50 mL c) 100 mL d) 200 mL

Q29 A dye absorbs a photon of wavelength λ and re-emits the same energy into two photons of wavelength λ_1 and λ_2 respectively. The wavelength λ is related with λ_1 and λ_2 as-

- a) $\lambda = \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2}$ b) $\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
 c) $\lambda = \frac{\lambda_1^2 \lambda_2^2}{\lambda_1 + \lambda_2}$ d) $\lambda = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)^2}$

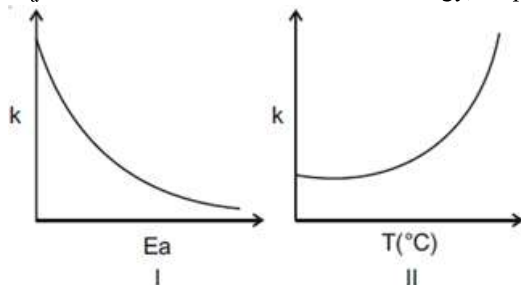
Q30 In the cell

$\text{Pt(s)}|\text{H}_2(\text{g}, 1\text{bar})|\text{HCl(aq)}|\text{AgCl(s)}|\text{Ag(s)}|\text{Pt(s)}$ the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of $(\text{AgCl}/\text{Ag}, \text{Cl}^-)$ electrode is:

$$\left\{ \text{Given, } \frac{2.303RT}{F} = 0.06 \text{ V at } 298 \text{ K} \right\}$$

- a) 0.20 V b) 0.40 V c) 0.76 V d) 0.94 V

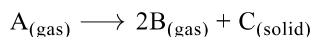
- Q31** Consider the given plots for a reaction obeying Arrhenius equation ($0^\circ\text{C} < T < 300^\circ\text{C}$) : (k and E_a are rate constant and activation energy, respectively)



Choose the correct option:

- a) I is wrong but II is right b) Both I and II are correct
c) Both I and II are wrong d) I is right but II is wrong
- Q32** Wilkinson catalyst is ($\text{Et} = \text{C}_2\text{H}_5$)
a) $[(\text{Ph}_3\text{P})_3\text{IrCl}]$ b) $[(\text{Ph}_3\text{P})_3\text{RhCl}]$ c) $[(\text{Et}_3\text{P})_3\text{IrCl}]$ d) $[(\text{Et}_3\text{P})_3\text{RhCl}]$

- Q33** For a first order reaction, the value of rate constant for the reaction



(If P_0 is initial pressure and P_t is pressure of mixture at time t)

- a) $\frac{1}{t} \ln \left(\frac{P_0}{P_0 - P_t} \right)$ b) $\frac{1}{t} \ln \left(\frac{P_0}{2P_0 - P_t} \right)$ c) $\frac{1}{t} \ln \left(\frac{2P_0}{3P_0 - P_t} \right)$ d) $\frac{1}{t} \ln \left(\frac{2P_0}{2P_0 - P_t} \right)$

- Q34** Correct name is written against which of the following chemical formulae ?

- a) Mg_3N_2 Magnesium nitrite b) $\text{Ni}(\text{HSO}_3)_2$ Nickel (II) sulphite
c) $\text{Sr}(\text{PO}_3)_2$ Stronsium phosphate d) CsOBr Cesium hypobromite

- Q35** Given below are two statements:

Statement I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

- a) Statement I is true but statement II is false
b) Both statement I and statement II are true
c) Statement I is false but statement II is true
d) Both statement I and statement II are false

- Q36** The thermal stability of the hydrides of group 15 follows the order:

- a) $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 < \text{BiH}_3$ b) $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{BiH}_3$
c) $\text{PH}_3 > \text{NH}_3 > \text{AsH}_3 > \text{SbH}_3 < \text{BiH}_3$ d) $\text{AsH}_3 < \text{PH}_3 > \text{SbH}_3 > \text{BiH}_3 > \text{NH}_3$

- Q37** S^{2-} and SO_3^{2-} ions can be distinguished by using—

- a) $(\text{CH}_3\text{COO})_2\text{Pb}$ b) $\text{Na}_2[\text{Fe}(\text{CN})_5\text{NO}]$
c) Both (1) and (2) d) none of these

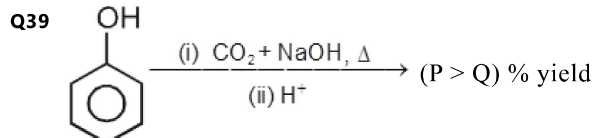
Q38 Given below are two statements:

Statement I: Tropolone is an aromatic compound and has 6π electrons in ring.

Statement II: π electrons of $>C=O$ group in tropolone is involved in aromaticity.

In the light of the above statements, choose the correct answer from the options given below:

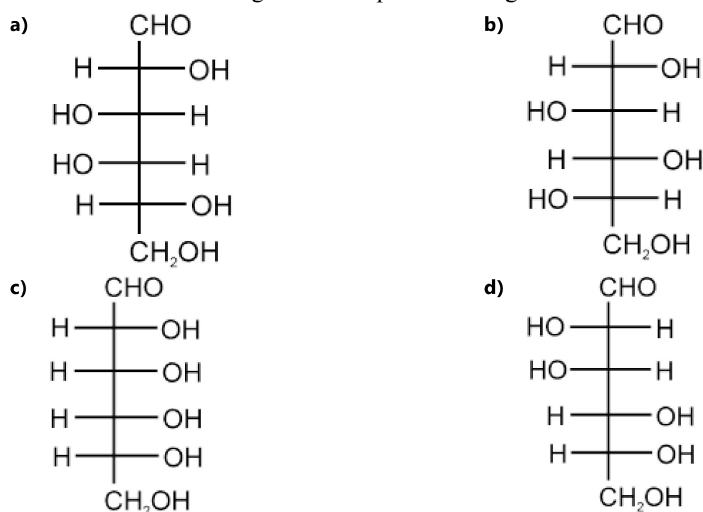
- a) Both Statement I and Statement II are true
- b) Statement I is true but Statement II is false
- c) Statement I is false but Statement II is true
- d) Both Statement I and Statement II are false



Select the correct option :

- a) Boiling point : (P > Q)
- b) Melting point : (Q > P)
- c) Water solubility : (P > Q)
- d) Acid Strength : (Q > P)

Q40 Which of the following is a C-4 epimer of D-glucose.

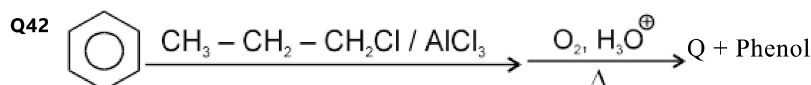


Q41 Given below are two statements :

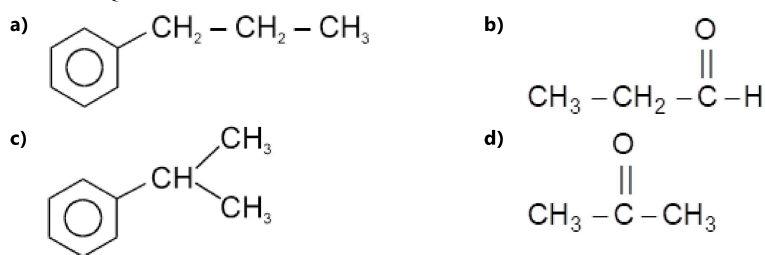
Statement I : The nucleophilic addition of sodium hydrogen sulphite to an aldehyde or a ketone involves proton transfer to form a stable ion.

Statement II : The nucleophilic addition of hydrogen cyanide to an aldehyde or a ketone yields amine as final product. In the light of the above statements, choose the most appropriate answer from the options given below :

- a) Both Statement I and Statement II are true.
- b) Statement I is true but Statement II is false.
- c) Statement I is false but Statement II is true
- d) Both Statement I and Statement II are false.



What is Q ?



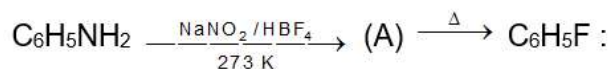
Q43 An halide $C_5H_{11}Br$ on treatment with a alc KOH given 2-pentene only the halide will be

- a) $CH_3-CH_2-CH_2-CH_2-CH_2-Br$
- b) $CH_3-CH_2-CH_2-\overset{\overset{Br}{|}}{CH}-CH_3$
- c) $CH_3-CH_2-\overset{\overset{Br}{|}}{CH}-CH_2-CH_3$
- d) $CH_3-\overset{\overset{CH_3}{|}}{CH}-\overset{\overset{Br}{|}}{CH}-CH_3$

Q44 Mixture of volatile components A and B has total vapour pressure (in Torr) $p = 254 - 119 x_A$ where x_A is mole fraction of A in mixture. Hence p_A^0 and p_B^0 are (in Torr)

- a) 254,119 b) 119,254 c) 135,254 d) 119,373

Q45 In the reaction,



The compound (A) is known as :

- a) m-nitro fluorobenzene b) a mixture of fluoroanilines
c) benzene diazonium fluoride d) benzene diazonium tetrafluoroborate

Numerical

Q46 In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is

Q47 The pH of $\frac{M}{100}$ aqueous of solution of monobasic organic acid is 4 at 300K. Find pK_b of corresponding conjugated base of organic acid at 300K.

Q48 The coordination number of Th in $K_4[Th(C_2O_4)_4(OH_2)_2]$ is ($C_2O_4^{2-} = Oxalato$)

Q49 How many of the following are paramagnetic: C_2 , B_2 , O_2^{-2} , BN , Cl_2^+ & NO^+ .

Q50 A vessel of 2.5 L was filled with 0.01 mole of Sb_2S_3 and 0.01 mole of H_2 to attain equilibrium at $440^\circ C$ as $Sb_2S_3(s) + 3H_2(g) \rightleftharpoons 2Sb(s) + 3H_2S(g)$
After equilibrium, the H_2S formed was analysed by dissolving it in water and treating with excess of Pb^{2+} to give 1.19 g of PbS as precipitate. The value of K_C of the reaction at $440^\circ C$ is ($Pb = 206$)

Mathematics

Single Choice Question

- Q51** The graph of the equation $x + y = x^3 + y^3$ is the union of
 a) line and an ellipse b) line and a parabola.
 c) line and hyperbola. d) pair of lines.
- Q52** The number of ordered quadruples (a_1, a_2, a_3, a_4) of positive odd integers that satisfy $a_1 + a_2 + a_3 + a_4 = 32$ is equal to
 a) 286 b) 4495 c) 680 d) 4040
- Q53** The coefficient of t^8 in the expansion of $(1 + 2t^2 - t^3)^9$, is
 a) 1680 b) 2140 c) 2520 d) 2730
- Q54** The complete set of values of the parameter α so that the point $P(\alpha, (1 + \alpha^2)^{-1})$ does not lie outside the triangle formed by the lines $L_1 : 15y = x + 1$, $L_2 : 78y = 118 - 23x$ and $L_3 : y + 2 = 0$, is
 a) $(0, 5)$ b) $[2, 5]$ c) $[1, 5]$ d) $[0, 2]$
- Q55** If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to
 a) 5 b) 2 c) 15 d) 10
- Q56** The area of the region bounded by the curve $C : y = \frac{x+1}{x^2+1}$ and the line $L : y = 1$, is
 a) $1 - \frac{1}{2}\ln 2 + \frac{\pi}{4}$ b) $\ln 2 - \frac{\pi}{4} + 1$ c) $\frac{1}{2}\ln 2 + \frac{\pi}{4} - 1$ d) $\ln 2 - \frac{\pi}{2} + 1$
- Q57** The solution set of inequality $(\cot^{-1}x)(\tan^{-1}x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1}x - 3\tan^{-1}x - 3\left(2 - \frac{\pi}{2}\right) > 0$, is
 a) $x \in (\tan 2, \tan 3)$ b) $x \in (\cot 3, \cot 2)$
 c) $x \in (-\infty, \tan 2) \cup (\tan 3, \infty)$ d) $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$
- Q58** **Statement-1:** The number of ordered pairs (x, y) satisfying the equation $x^2 + \frac{1}{x^2} = 2^{1-y^2}$ is 2.
Statement-2 : The range of $x^2 + \frac{1}{x^2}$ is $[2, \infty)$ and the range of 2^{1-y^2} is $(0, 2]$.
 a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 b) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
 c) Statement-1 is true, statement-2 is false.
 d) Statement-1 is false, statement-2 is true.
- Q59** The value of $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$ equals
 a) $-\ln 3$ b) 0 c) $-\ln 2$ d) $-\ln 5$

- Q60** An even polynomial function $f(x)$ satisfies a relation $f(2x) \left(1 - f\left(\frac{1}{2x}\right)\right) + f(16x^2y) = f(-2) - f(4xy)$, $x, y \in \mathbb{R} - \{0\}$ and $f(4) = -255$, $f(0) = 1$. Which of the following **are incorrect**?
- a) $f(x)$ has local maximum at $x = 0$.
- b) $f(x)f\left(\frac{1}{x}\right) \leq 0$
- c) Range of values of k for which $|f(x)| = k - 2$ has exactly four distinct solutions is $(2, 3)$.
- d) $\int_0^1 f(x) dx = \frac{3}{4}$.
- Q61** Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p , for which $P(\text{exactly one of } A, B \text{ occurs}) = \frac{5}{9}$, is :
- a) $\frac{1}{3}$ b) $\frac{2}{9}$ c) $\frac{4}{9}$ d) $\frac{5}{12}$
- Q62** $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :
- a) $\frac{3}{16}$ b) $\frac{1}{16}$ c) $\frac{1}{32}$ d) $\frac{9}{32}$
- Q63** If the function
- $$f(x) = f(x) = \begin{cases} \frac{1}{x} \log_e \left(\frac{1 + \frac{x}{a}}{1 - \frac{x}{b}} \right) & , x < 0 \\ k & , x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1} & , x > 0 \end{cases}$$
- is continuous at $x = 0$, then $\frac{1}{a} + \frac{1}{b} + \frac{4}{k}$ is equal to :
- a) -5 b) 5 c) -4 d) 4
- Q64** If the foot of the perpendicular from point $(4, 3, 8)$ on the line $L_1: \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is $(3, 5, 7)$, then the shortest distance between the line L_1 and line $L_2: \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :
- a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{6}}$ c) $\frac{\sqrt{2}}{\sqrt{3}}$ d) $\frac{1}{\sqrt{3}}$
- Q65** Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are co-prime, then $m+n$ is equal to
- a) 316 b) 314 c) 317 d) 315
- Q66** Let \vec{a} and let \vec{b} be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of \vec{b} on the vector $\vec{a} - \vec{b}$ is :
- a) $\frac{2}{\sqrt{21}}$ b) $2\sqrt{\frac{3}{7}}$ c) $\frac{2}{3}\sqrt{\frac{7}{3}}$ d) $\frac{2}{3}$

Q67 If the following system of linear equations

$$2x + y + z = 5$$

$$x - y + z = 3$$

$$x + y + az = b,$$

has no solution, then :

a) $a = -\frac{1}{3}, b \neq \frac{7}{3}$

b) $a \neq \frac{1}{3}, b = \frac{7}{3}$

c) $a \neq -\frac{1}{3}, b = \frac{7}{3}$

d) $a = \frac{1}{3}, b \neq \frac{7}{3}$

Q68 If $I(x) = \int e^{\sin^2 x} (\cos x \sin 2x - \sin x) dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to -

a) $-\frac{1}{2}e^{\frac{3}{4}}$

b) $e^{\frac{3}{4}}$

c) $\frac{1}{2}e^{\frac{3}{4}}$

d) $-e^{\frac{3}{4}}$

Q69 The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

a) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

b) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

c) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

d) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Q70 Let $g(x) = f(x) + f(1-x)$ and $f'(x) > 0, x \in (0,1)$.

If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then

$\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

a) $\frac{3\pi}{2}$

b) π

c) $\frac{5\pi}{4}$

d) $\frac{3\pi}{4}$

Numerical

Q71 Let $A = [a_{ij}]_{3 \times 3}$ be a matrix such that $AA^T = 4I$ and $2a_{ij} + c_{ij} = 0$ where c_{ij} is the cofactor of $a_{ij} \forall i$ and j and I is the unit matrix of order 3.

If $\begin{vmatrix} a_{11} + 4 & a_{12} & a_{13} \\ a_{21} & a_{22} + 4 & a_{23} \\ a_{31} & a_{32} & a_{33} + 4 \end{vmatrix} + 5\lambda \begin{vmatrix} a_{11} + 1 & a_{12} & a_{13} \\ a_{21} & a_{22} + 1 & a_{23} \\ a_{31} & a_{32} & a_{33} + 1 \end{vmatrix} = 0$, then

find the value of 100λ .

Q72 The number of solution(s) of the equation $z^2 = 4z + |z|^2 + \frac{16}{|z|^3}$ is (where $z = x + iy$, $x, y \in \mathbb{R}$, $i^2 = -1$ and $x \neq 2$)

Q73 For a positive constant t , let α, β be the roots of the quadratic equation $x^2 + t^2x - 2t = 0$. If the minimum value of $\int_{-1}^2 \left(\left(x + \frac{1}{\alpha^2}\right) \left(x + \frac{1}{\beta^2}\right) + \frac{1}{\alpha\beta} \right) dx$ is $\sqrt{\frac{a}{b}} + c$ where $a, b, c \in \mathbb{N}$, then find the least value of $(a + b + c)$.

Q74 $\lim_{x \rightarrow \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2} \sin 2x}$ is equal to

Q75 Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x-1) \frac{dy}{dx} + 2xy = \frac{1}{x-1}$

, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^\alpha + 1}{\beta e^\alpha}$. Then the value of $\alpha + \beta$ is equal to _____.

