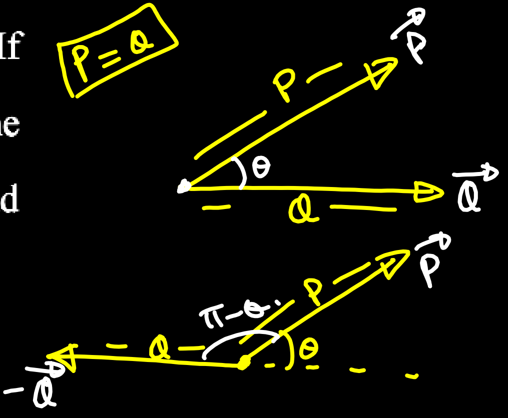
PHYSICS

Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $|\vec{P}+\vec{Q}|$ is n times the magnitude of $|\vec{P}-\vec{Q}|$, then angle between $\vec{P}$ and $\vec{Q}$ is

$\vec{P}+(-\vec{Q})$



(A) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$ (B) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(C) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (D) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

$$|\vec{P}+\vec{Q}| = n|\vec{P}-\vec{Q}|$$

$$\sqrt{P^2+Q^2+2PQ\cos\theta} = n\sqrt{P^2+Q^2+2PQ\cos(\pi-\theta)}$$

$$\cancel{P^2}+\cancel{P^2}+2\cancel{P^2}\boxed{\cos\theta} = n^2\left[\cancel{P^2}+\cancel{P^2}-2\cancel{P^2}\boxed{\cos\theta}\right]$$

$$\cos\theta = \frac{n^2-1}{n^2+1}$$

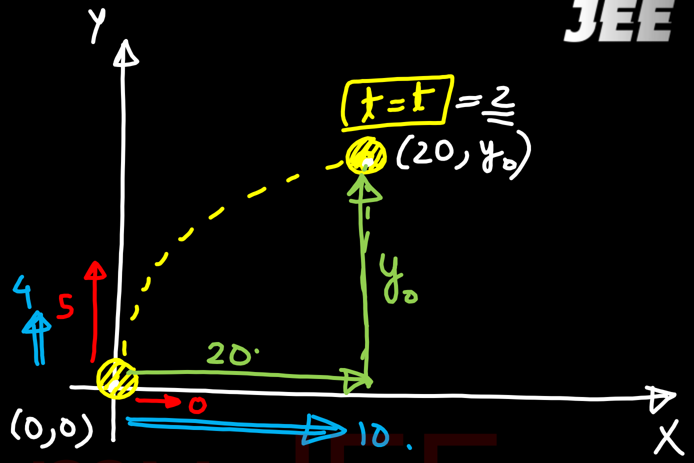
Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}\ \text{m s}^{-1}$, a particle moves in the $x - y$ plane with a constant acceleration of $(10\hat{i} + 4\hat{j})\text{ms}^{-2}$. At time t, its coordinates are $(20\text{ m}, y_0\text{ m})$. The values of t and y_0 are, respectively

(A) 2 s and 24 m

~~(B)~~ 4 s and 52 m

~~(C)~~ 5 m and 25 m

(D) 2 s and 18 m



$$\boxed{s = ut + \frac{1}{2}at^2}$$

X

$$20 = \frac{1}{2}(10)t^2$$

$$\boxed{t = 2}$$

Y

$$y_0 = 5(t) + \frac{1}{\cancel{2}}(4)t^2$$

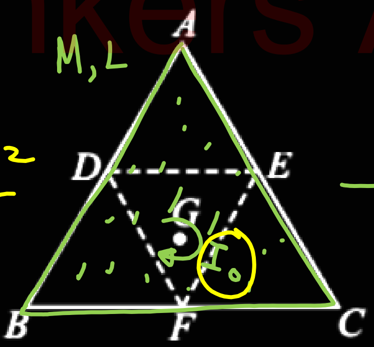
$$y_0 = 10 + 8 = 18$$

An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then

format

$I = \boxed{KML^2}$

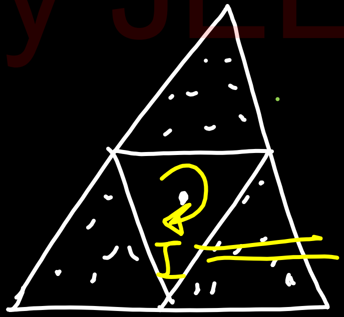
$I_0 = KML^2$



—

$\frac{M}{4}, \frac{L}{2}$

=



$I_0 - \frac{I_0}{16}$

$I_1 = K\frac{M}{4}\left(\frac{L}{2}\right)^2 = \frac{I_0}{16}$

$\boxed{\text{(A) } I = \frac{15}{16} I_0}$

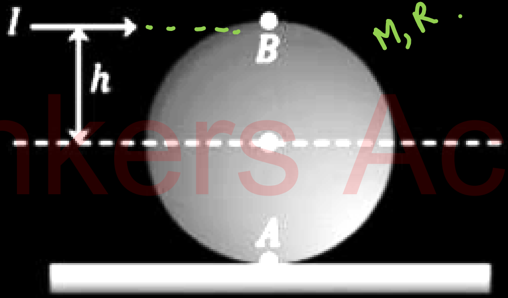
(B) $I = \frac{9}{16} I_0$

(C) $I = \frac{3}{4} I_0$

(D) $I = \frac{I_0}{4}$

4

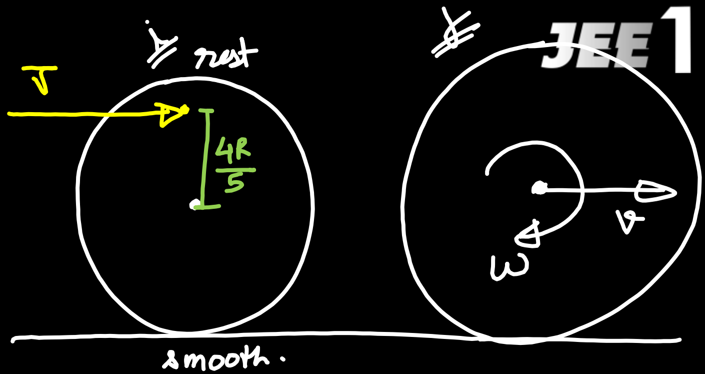
A uniform solid sphere is placed on a smooth horizontal surface. At a height $\boxed{h = \frac{4R}{5}}$ above the centre line, an impulse J is given horizontally to the sphere. Assume the mass and radius of the sphere being m and R respectively.



Find angular velocity ω of the sphere after the impulse.

(A) $\frac{4J}{mR}$ (B) $\frac{2J}{mR}$

(C) $\frac{J}{mR}$ (D) $\frac{J}{2mR}$



T_x: $\vec{P}_i + \vec{J} = \vec{P}_f$

$0 + J = Mv$ $\boxed{v = \frac{J}{M}}$

COM.

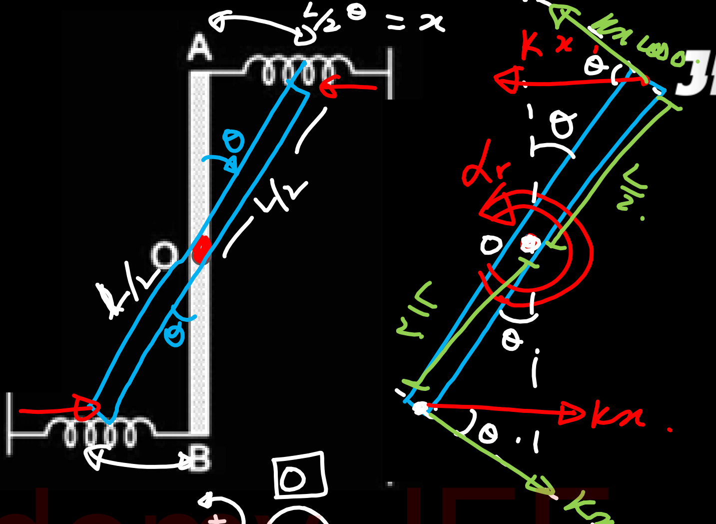
R_x: $\vec{L}_i + \vec{J}_R = \vec{L}_f$

$0 + J\frac{\cancel{4}^{2}\cancel{R}}{\cancel{5}} = \frac{\cancel{2}}{\cancel{5}}MR^{\cancel{2}}\omega$

$\frac{2J}{MR} = \omega$

5

Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre O and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is $\nu = \frac{1}{T}$



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

SHM-Tx

$\boxed{a_r = \omega^2 x}$

$\boxed{\alpha_r = \omega^2 \theta}$

SHM-Rx

B

$\theta \simeq$ small.

$\theta \simeq \sin\theta \simeq \tan\theta$

$\cos\theta \simeq 1$

Rx $\tau_R = I\alpha_r$

$2kx\cos\theta\left(\frac{L}{2}\right) = \frac{ML^2}{12}\alpha_r$

SHM

$2k\frac{L}{2}\theta\frac{L}{2} = \frac{ML^2}{12}\alpha_r$

$\alpha_r = \left(\frac{6k}{m}\right)\theta$

$\omega = \sqrt{\frac{6k}{m}}$

$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{6k}}}$

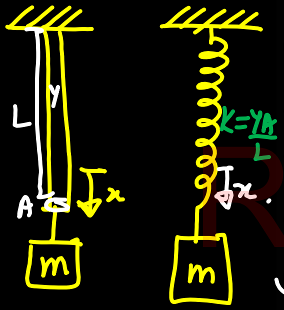
6

In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1 m (measured using a scale of least count= 1mm), a weight of mass 1 kg (measured using a scale of least count = 1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment?

(A) 1.4% (B) 0.14%

(C) 9% (D) 0.9%

JEE 1



$r = 0.2$ cm, $\Delta r = 0.001$ cm

$l = 1$ m, $\Delta l = 10^{-3}$ m

$m = 1$ kg, $\Delta m = 10^{-3}$ kg

$x = 0.5$ cm. $\Delta x = 0.001$ cm

$\left|\frac{\Delta Y}{Y}\right| \times 100 = ?$

$Kx = mg.$

$\boxed{\frac{YA}{L} x = mg}$

$\to Y = \frac{mgL}{Ax}.$

$\boxed{Y = \frac{g}{\pi} m l r^{-2} x^{-1}}$

const

error

$\left|\frac{\Delta Y}{Y}\right| \times 100 = \left|\frac{\Delta m}{m}\right| \times 100 + \left|\frac{\Delta l}{l}\right| \times 100 + 2\left|\frac{\Delta r}{r}\right| \times 100 + \left|\frac{\Delta x}{x}\right| \times 100$

% error Y $= 0.1\% + 0.1\% + 1\% + 0.2\%$

$= 1.4\%$

7

If the potential energy between two molecules is given by $\boxed{U = -\frac{A}{r^6} + \frac{B}{r^{12}}}$, then at equilibrium, separation between molecules, and the potential energy are

$F = 0$

(A) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$

(B) $\left(\frac{B}{A}\right)^{1/6}, 0$

(C) $\boxed{\left(\frac{2B}{A}\right)^{1/6}}, \boxed{-\frac{A^2}{4B}}$

(D) $\boxed{\left(\frac{2B}{A}\right)^{1/6}}, -\frac{A^2}{2B}$

JEE 1

$\boxed{F = -\frac{dU}{dr}.}$

$U = -A r^{-6} + B r^{-12}$

$-\frac{dU}{dr} = -\left[6 A r^{-7} - 12 B r^{-13}\right] = 0$

$\cancel{6} A r^{-7} = \overset{2}{\cancel{12}} B r^{-13}$

$r^6 = \frac{2B}{A}$

$\boxed{r = \left(\frac{2B}{A}\right)^{1/6}}$

2nd part

$U\Big|_{r=\left(\frac{2B}{A}\right)^{1/6}} = -\frac{A}{\left(\frac{2B}{A}\right)} + \frac{B}{\left(\frac{2B}{A}\right)^2} = -\frac{A^2}{4B}$

8

JEE 1

In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be

(A) $[M^2LT^{-2}]$ (B) $[MLT^{-2}]$

(C) $[MLT^{-1}]$ (D) $[M^0LT^0]$ ✓

$W = \alpha^2\beta \, e^{-\frac{\beta x^2}{kT}}$

same unit

$\frac{[\beta][x]^2}{[K][T]} = M^0L^0T^0 = 1$

$[PV] = ML^2T^{-2}$

PV
nRT
NkT
kT

$PV = nRT$

$PV = \frac{N}{N_A} \cdot RT$

$PV = NkT$

$[W] = [\alpha]^2[\beta]$

$\cancel{M}L^2\cancel{T^{-2}} = [\alpha]^2[\cancel{MT^{-2}}]$

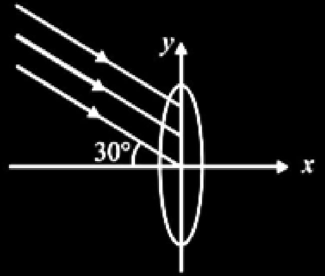
$[\alpha] = L$

$\frac{[\beta]\,\cancel{L^2}}{M\cancel{L^2}T^{-2}} = 1$

$[\beta] = MT^{-2}$

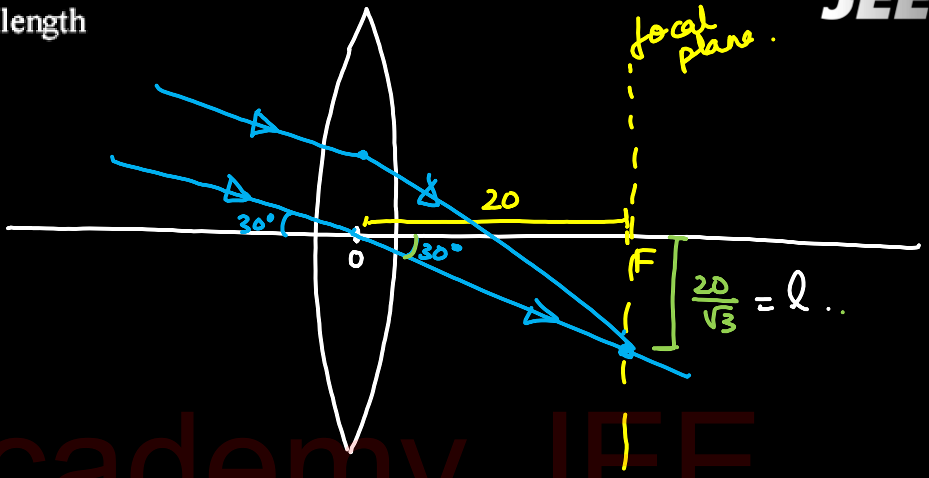
paraxial

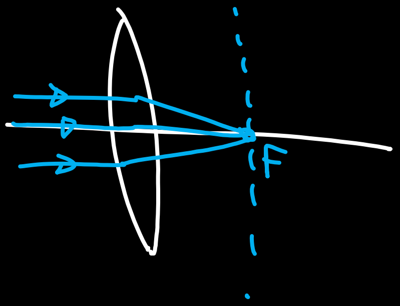
Parallel rays are focussed by the convex lens (lens is placed along y-axis) of focal length 20 cm at the point



~~(A) (20,0)~~ (B) (20, −20)

(C) (20, −10) (D) $\left(20, -\frac{20}{\sqrt{3}}\right)$ ✓





$\tan 30° = \frac{l}{20}$

$\boxed{l = \frac{20}{\sqrt{3}}}$

$\left(20, -\frac{20}{\sqrt{3}}\right)$

A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body

~~(A)~~ escapes from the planet's gravitational field

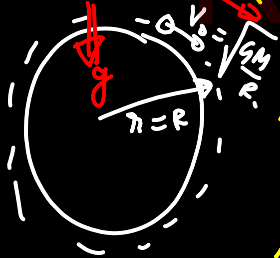
(B) starts moving in an elliptical orbit around the planet

~~(C)~~ falls vertically downwards towards the planet

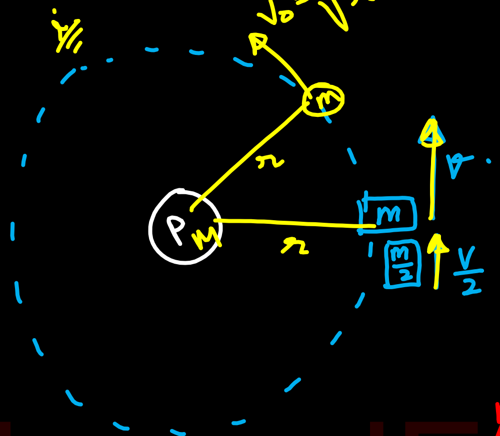
~~(D)~~ continues to move in a circular orbit.

$\boxed{V_0} \to \boxed{\sqrt{2}V_0}$ escape.

$V_0 = \sqrt{\frac{GM}{R}}$



$V_0 = \sqrt{\frac{GM}{r}} = v$

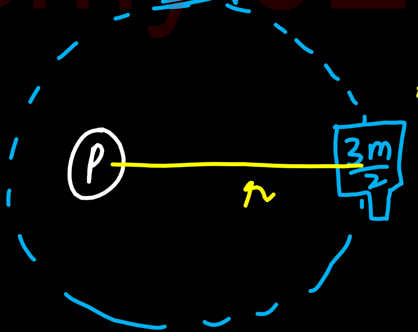


$\boxed{l = 0}$

$mv + \frac{m}{2}\frac{v}{2} = \left(\frac{3m}{2}\right)v_c$

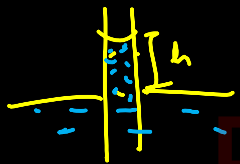
$\frac{5v}{4} = \frac{3v_c}{2}$

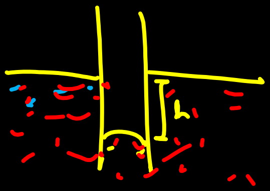
ellipse



$v_c = \frac{5}{6}v$

The ratio of surface tensions of mercury ① and water ② is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1 while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to. approx





(A) $\boxed{\frac{2}{5} = 0.4}$

(B) $\frac{4}{5} = 0.8$

(C) $\frac{3}{5} = 0.6$

(D) $\frac{2}{3} = 0.66$

$\boxed{\frac{T_1}{T_2} = \frac{15}{2}}$

$\boxed{\frac{\rho_1}{\rho_2} = \frac{13.6}{1}}$

$\boxed{\frac{h_1}{h_2} = \frac{1}{1}}$

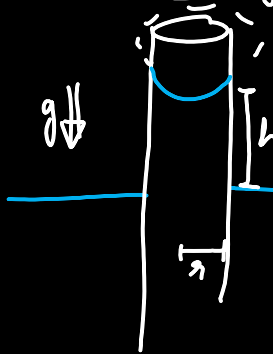
$\boxed{\frac{r_1}{r_2} = ?}$

$\theta_1 = 135°$.

$\theta_2 = 0°$

$\boxed{\frac{\cos\theta_1}{\cos\theta_2} = \frac{-\frac{1}{\sqrt{2}}}{1} = \left|-\frac{1}{\sqrt{2}}\right|}$

Theory



$h = \frac{2T\cos\theta}{r\rho g}$

ρ, T, θ

$\frac{h_1}{h_2} = \frac{\frac{\cancel{2}T_1\cos\theta_1}{r_1\rho_1\cancel{g}}}{\frac{\cancel{2}T_2\cos\theta_2}{r_2\rho_2\cancel{g}}}$

$1 = \frac{\left(\frac{T_1}{T_2}\right)\left|\left(\frac{\cos\theta_1}{\cos\theta_2}\right)\right|}{\left(\frac{r_1}{r_2}\right)\left(\frac{\rho_1}{\rho_2}\right)}$

$\left(\frac{r_1}{r_2}\right) = \frac{15}{2} \times \frac{1}{\sqrt{2}} \cdot \frac{1}{13.6} = \frac{15}{13.6 \times 2.82}$

$\frac{15}{40}$

12

JEE 1

A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. ' m ' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of ' m ' is close to (Latent heat of water = 540cal^{-1}, specific heat of water = 1cal g $^{-1}$°C^{-1})

g °C cal

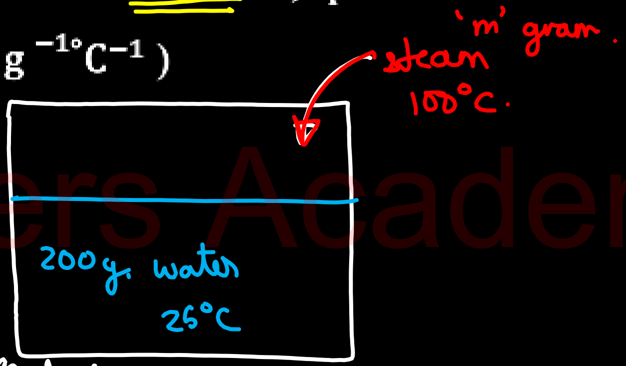
~~(A)~~ $\boxed{2}$

(B) 4

(C) 3.2

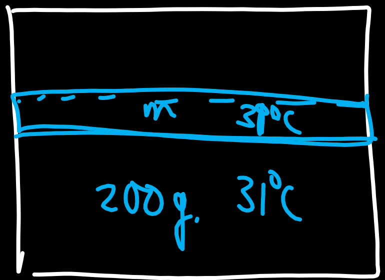
(D) 2.6

i



good calirometer.

f



Q = mL

Q = msΔT

Heat Lost = Heat gained

$mL + m\cancel{s}^{1}(100-31) = 200\cancel{s}^{1}(31-25)$

$m(540) + 69m = 1200$

$\boxed{m = \frac{1200}{609} \simeq 2}$

13

JEE 1

A particle starts from rest and moves with an acceleration of $\boxed{a = 2 + |t-2|}$ m/s^2, the velocity of the particle at $t = 4$sec is

$u = 0$
$v = ?$

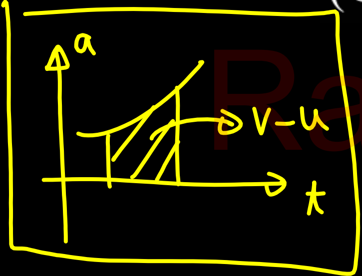
(A) 2 m/s

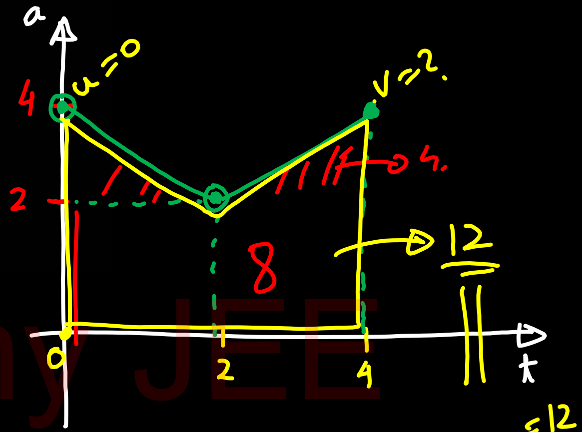
(B) 4 m/s

(C) zero

(D) 12 m/s

Theory





$$a = \begin{cases} 2-(t-2) & 0 \le t < 2 \\ \cancel{2}+(t-\cancel{2}) & t \geq 2 \end{cases}$$

$$\boxed{a = \begin{cases} 4-t & t \in [0,2) \\ t & t \in [2,\infty) \end{cases}}$$

$v-u = 12$

$\boxed{V = 12}$

14

n moles of an ideal gas with constant volume heat capacity C_V $= C_V'$ undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is

$n=1$

$P\uparrow V\uparrow = nRT\uparrow$

$P = \text{const.}$

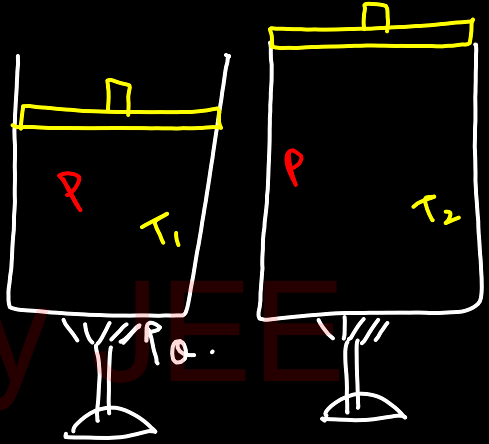
$P\Delta V = nR\Delta T$

(A) $\dfrac{nR}{C_V' - nR}$ $\quad \dfrac{R}{C_v - R}$

(B) $\dfrac{4nR}{C_V' - nR}$ $\quad \dfrac{4R}{C_v - R}$

(C) $\dfrac{nR}{C_V' + nR}$ $\quad \dfrac{R}{C_v + R}$ ✓

(D) $\dfrac{4nR}{C_V' + nR}$ $\quad \dfrac{4R}{C_v + R}$



$Q = \Delta U + W$

$$\boxed{Q = nC_v\Delta T + nR\Delta T}$$

$$\text{Ans} = \frac{W}{Q} = \frac{\cancel{n}R\cancel{\Delta T}}{\cancel{n}C_v\cancel{\Delta T} + \cancel{n}R\cancel{\Delta T}} = \frac{R}{C_v + R}$$

$$= \frac{nR}{\frac{C_v'}{n} + nR}$$

Ans

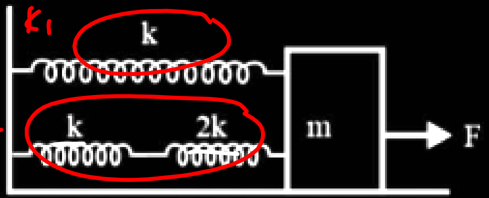
$C_v \rightarrow 1\text{ mole}$

$C_v' = \boxed{n}\, C_v$

$\dfrac{C_v'}{n} = C_v$

15

The springs shown are all unstretched. Find the time period of the displaced body.



$\frac{2k}{3} = \frac{k(2k)}{3k} = K_2$

(A) $T = 2\pi\sqrt{\frac{6\,m}{5k}}$

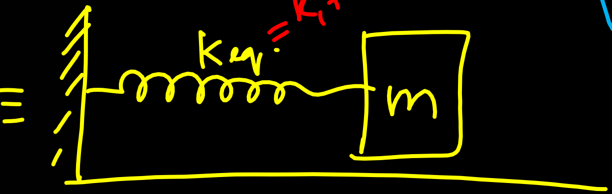
(B) $T = 2\pi\sqrt{\frac{m}{k}}$

$\boxed{\text{(C) } T = 2\pi\sqrt{\frac{3\,m}{5k}}}$

(D) $T = 2\pi\sqrt{\frac{3\,m}{2k}}$

$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$=$



$K_{eq} = k + \frac{2k}{3} = \frac{5k}{3}$

$T = 2\pi\sqrt{\frac{3m}{5k}}$

16

JEE1

A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $= 1.6 \times 10^{-19}$C)

(A) 2.0×10^{-24} kg (B) 1.6×10^{-19} kg

(C) 1.6×10^{-27} kg (D) 9.1×10^{-31} kg

$$R = \frac{mv}{qB} \quad — (1)$$

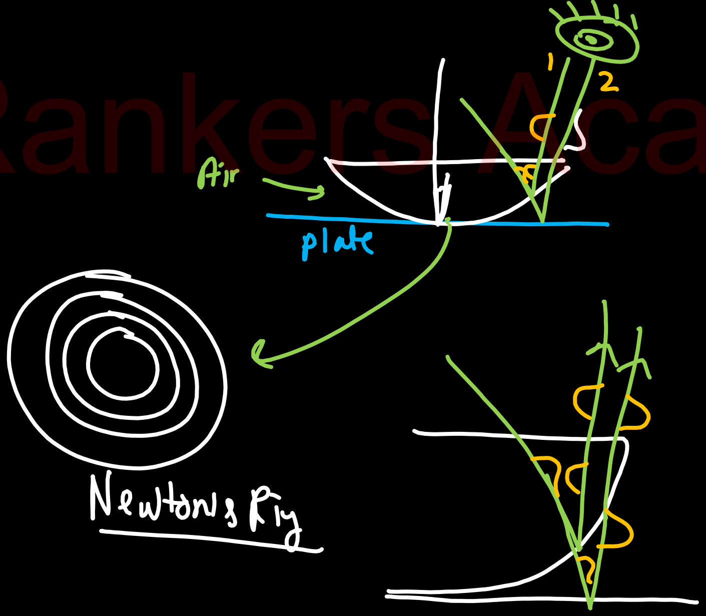
$$qvB = qE \Rightarrow E/B = v \quad — (2)$$

$$R = \frac{m}{qB}\left(\frac{E}{B}\right)$$

$$m = \frac{qB^2R}{E} = \frac{1.6\times10^{-19} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2}\times10^{-2}}{100} = 2\times10^{-24}\ \text{kg}$$

17

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.



Statement-1: When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2: The centre of the interference pattern is dark.

(A) Statement-1 is true, Statement-2 is false.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(C) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

(D) Statement-1 is false, Statement-2 is true.

18

JEE1

A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is

(A) 2.0%

(B) 1.0%

(C) 0.5%

(D) 2.5%

$$R = \frac{\rho l}{A} = \frac{\rho l}{(Vol/l)} = \frac{\rho l^2}{V}$$

$$R \propto l^2 \Rightarrow \frac{\Delta R}{R} = 2\frac{\Delta l}{l} \quad \left\{ \text{for small of } \frac{\Delta l}{l} \right\}$$

$$= 2 \times 0.5\% = 1\%$$

19

JEE 1

A circuit connected to ac source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\pi/4$ between the emf e and current i. Which of the following circuits will exhibit this?

(A) RC circuit with $R = 1k\Omega$ and $C = 10\mu F$

(B) RL circuit with $R = 1k\Omega$ and $L = 10mH$

(C) RC circuit with $R = 1k\Omega$ and $C = 1\mu F$

(D) RL circuit with $R = 1k\Omega$ and $L = 1mH$

$\omega = 100$

$\boxed{X_L \text{ or } X_C = R}$

$X_C = \frac{1}{\omega C} = R$

$\Rightarrow RC = \frac{1}{\omega} = \frac{1}{100}$

Opt A $1000 \times 10 \times 10^{-6}$

$= \frac{1}{100}$

20

JEE 1

When a certain photosensitive surface is illuminated with monochromatic light of frequency υ, the stopping potential for the photocurrent is $\frac{-V_0}{2}$. When the surface is illuminated by monochromatic light of frequency $\frac{\upsilon}{2}$ the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

(A) $\frac{4\upsilon}{3}$

(B) 2υ

(C) $\frac{5\upsilon}{3}$

(D) $\frac{3\upsilon}{2}$

$K_{max} = eV_0 = h\upsilon - \varphi$

$2\left(e\frac{V_0}{2} = h\upsilon - \varphi\right)$ ①

$eV_0 = \frac{h\upsilon}{2} - \varphi$ ②

$2h\upsilon - 2\varphi = \frac{h\upsilon}{2} - \varphi$

$\frac{3}{2}h\upsilon = \varphi$

$\frac{3h\upsilon}{2} = h\upsilon_0$

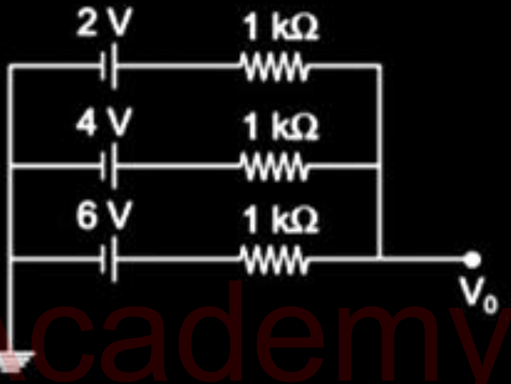
$\upsilon_0 = \frac{3\upsilon}{2}$

21

Integer

JEE 1

In the given figure, the value of V_0 will be ____ V.



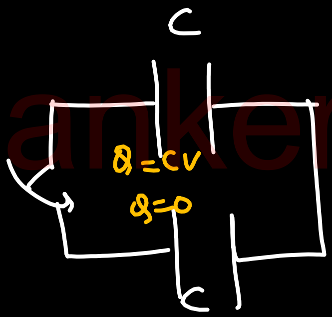
$$E_{eq} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}}$$

$$= \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$= \frac{\frac{2}{1} + \frac{4}{1} + \frac{6}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}} = \frac{12}{3} = \boxed{4}\,V$$

22

A capacitor of capacitance 50pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is ____ nJ.



$$U_i = \frac{1}{2}CV^2 \quad \Big| \quad U_f = 2\left[\frac{\frac{1}{2}\left(\frac{CV}{2}\right)^2}{C}\right] = \frac{1}{4}CV^2$$

$$\text{Loss} = \frac{1}{4}CV^2$$

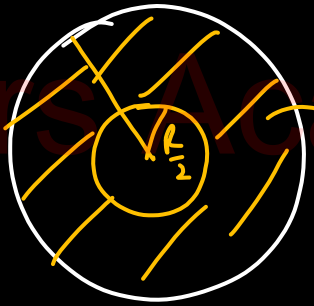
$$= \frac{1}{4} \times 50 \times 10^{-12} \times (100)^2$$

$$= 12.5 \times 10^{-12+4}$$

$$= \boxed{125} \times 10^{-9}$$

23

The current density in a cylindrical wire of radius 4 mm is $4 \times 10^6 \text{Am}^{-2}$. The current through the outer portion of the wire between radial distances $\frac{R}{2}$ and R is _____ πA



$i = jA$

$= j\left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]$

$i = j\pi \frac{3}{4} R^2$

$= \cancel{4} \times 10^6 \times \frac{3}{\cancel{4}} \times (4 \times 10^{-3})^2 \pi$

$= \boxed{48}\pi$

24

JEE 1

A coil of inductance 2H having negligible resistance is connected to a source of supply whose voltage is given by $\boxed{V = 3t}$ volt (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4 s is ______ J.

$i = ?$

$V = L\frac{di}{dt}$

$3\int_0^4 t\,dt = L\int_0^i di$

$\frac{3t^2}{2L} = i$

$i = \frac{3\times 4^2}{4} = 12\,A$

$U = \frac{1}{2}Li^2$

$= \frac{1}{2}\times 2\times 12^2$

$= \boxed{144}$

JEE 1

25

The electric field in an electromagnetic wave is given by $E = (50\ NC^{-1})\sin\omega(t - x/c)$. The energy contained in a cylinder of volume V is 5.5×10^{-12} J. The value of V is cm^3.

(Given: $\varepsilon_0 = 8.8 \times 10^{-12} C^2 N^{-1} m^{-2}$)

$$u = \frac{U}{V} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$V = \frac{U}{\frac{1}{2}\varepsilon_0 E_0^2} = \frac{5.5\times10^{-12}}{\frac{1}{2}\times 8.8\times10^{-12}\times 50\times50}\ m^3$$

$$= \frac{\cancel{2}\times\cancel{5}}{\cancel{8}\times\cancel{2500}}\times(10^6\ cm^3)$$

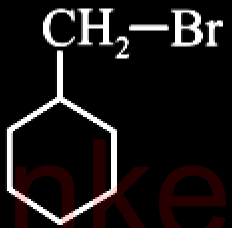
$$4 \quad 50 \quad = \frac{1000\cancel{000}}{2\cancel{000}} = \boxed{500}$$

CHEMISTRY

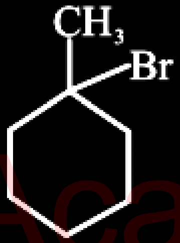
1

The product C of the following sequence of reaction

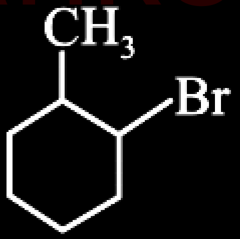
(A)



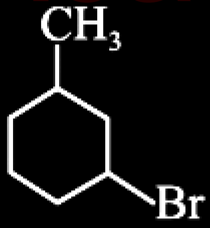
(B)

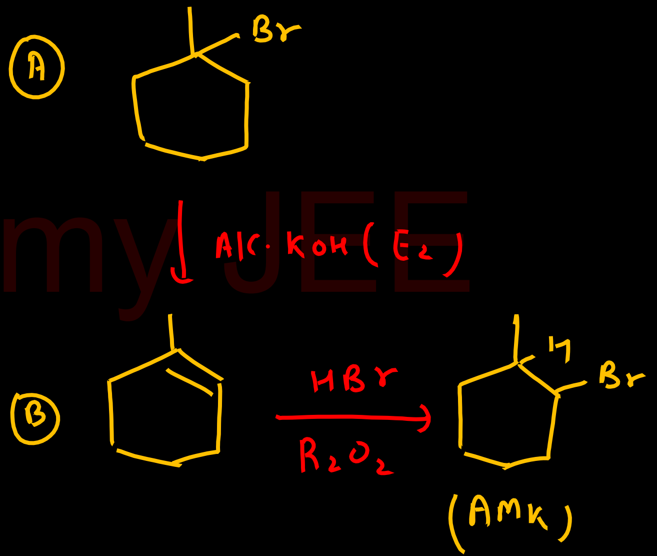


(C)



(D)

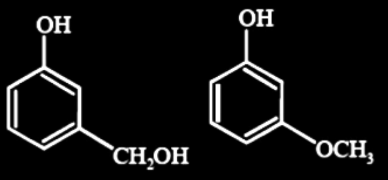




2

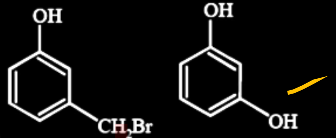
JEE 1

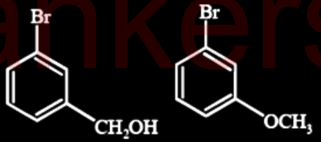
Two isomeric compounds (I) and (II) are heated with HBr –

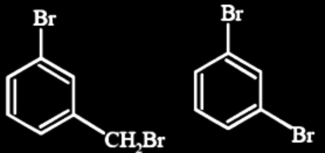


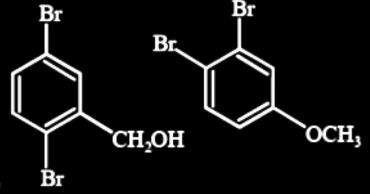
(I) (II)

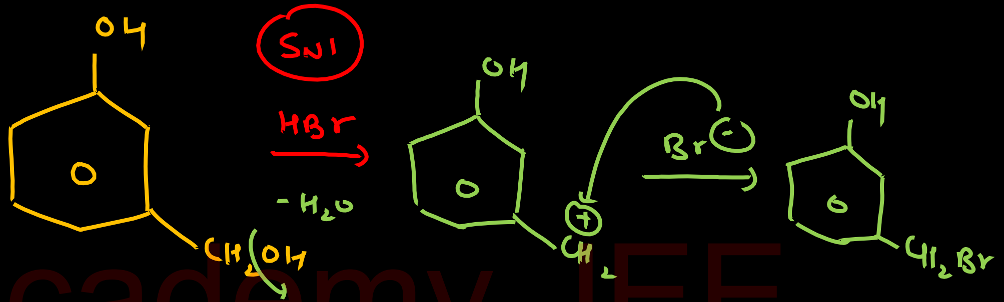
The products obtained are –

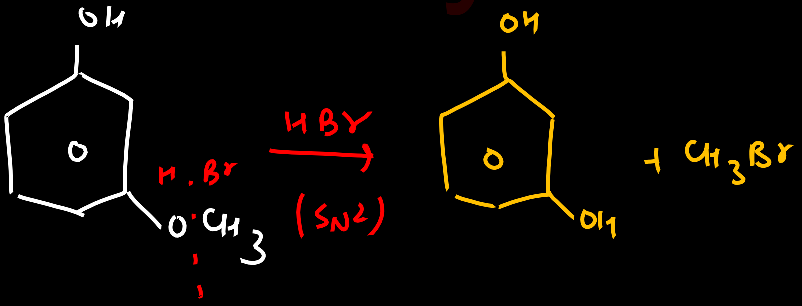
(A)


(B)


(C)


(D)






3

JEE 1

Vapour pressure of solution containing 6 g of a non-volatile solute in 180 g water is 20.0 torr. If 1 mol water is further added vapour pressure increases by 0.02 torr. Calculate vapour pressure of water.

(A) 20.22 torr (B) 21.22 torr

(C) 20.04 torr (D) 22.22 torr

$\frac{p^0 - p_s}{p_s} = \frac{n}{N}$ ∴ $\frac{p^0}{p_s} = \frac{n+N}{N}$

$p_s = 20$ torr

$n = \frac{6}{M}$, $N = \frac{180}{18} = 10$

$p_s = 20.02$ torr

$N = 10 + 1 = 11$

$n = \frac{6}{M}$

$\frac{p^0}{20} = \frac{n+10}{10}$ ∴ $p^0 = 2n + 20$ — (1)

$\frac{p^0}{20.02} = \frac{n+11}{11}$ — (2)

n ✓

$p^0 = 22.22$ torr

4

The E° in the given figure is about :

(A) 0.5 V ~~(B)~~ 0.6 V

(C) 0.7 V (D) 0.8 V

$$\frac{2.16 + 0.45 + 1.07}{6} = 0.6$$

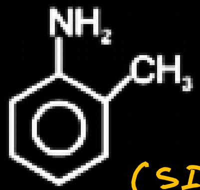
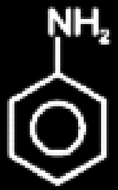
$$-n_4 F E_4^\circ = -n_1 F E_1^\circ - n_2 F E_2^\circ - n_3 F E_3^\circ$$

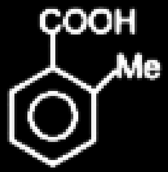
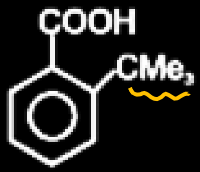
$$E_4^\circ = \frac{n_1 E_1^\circ + n_2 E_2^\circ + 1\, E_3^\circ}{n_4} = \frac{4(0.54) + (1 \times 0.45) + (1 \times 1.07)}{6}$$

$$= \underline{0.6V}$$

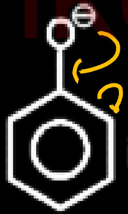
5

Select correct statement from the following:

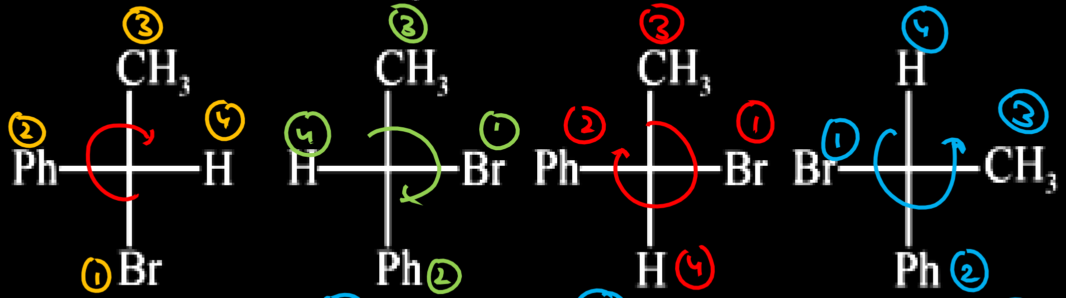
(A)  (SIP) is more basic than 

(B)  is more acidic than 

(C) $HC\equiv CH$ is more acidic than NH_3

(D)  is more stable than $Me-\overset{\overset{O}{\|}}{C}-O^{\ominus}$

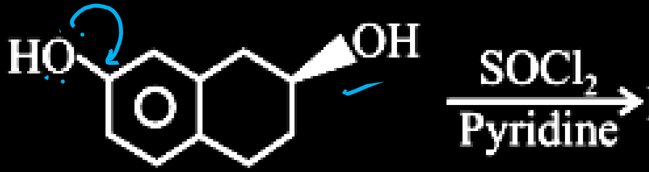
The pair of enantiomers among the followings for 1-Bromo-1-Phenyl ethane are :



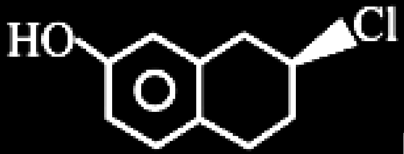
I II III IV

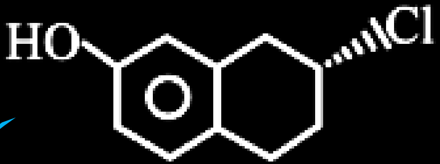
(A) I and IV (B) II and IV

(C) II and III (D) I and II

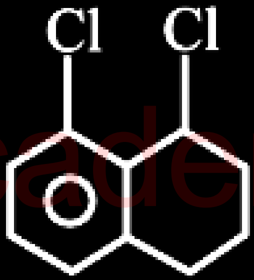


Product is :

(A) 

(B) 

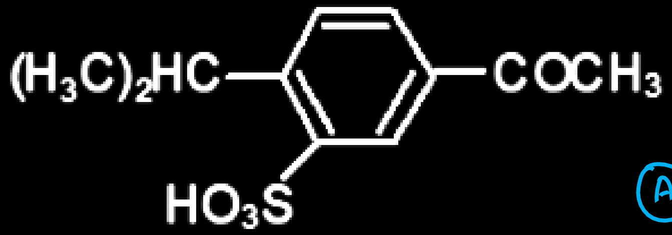
(C) 

(D) 

$S_N2 \longrightarrow$ inversion

8

The best sequence of reactions for preparation of the following compound from benzene is



(A) (i) $CH_3COCl/AlCl_3$ (ii) Oleum

(iii) $(CH_3)_2CH-Cl$ (1 mole) $/AlCl_3$

(B) (i) $(CH_3)_2CH-Cl$(1 mole)$/AlCl_3$

(ii) $CH_3COCl/AlCl_3$ (iii) Oleum

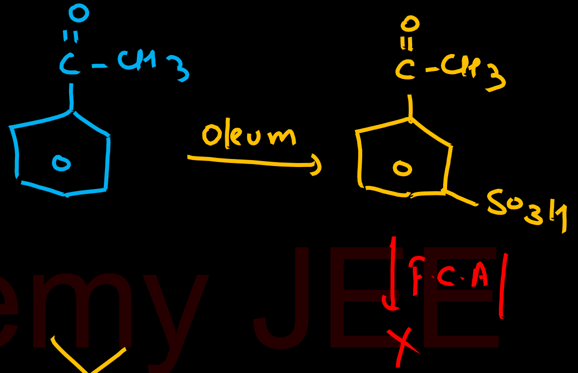
(C) (i) Oleum (ii) $CH_3COCl, AlCl_3$

(iii) $(CH_3)_2CH-Cl$ (1 mole) $/AlCl_3$

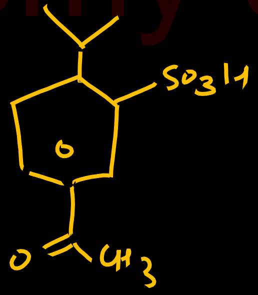
(D) (i) $(CH_3)_2CH-Cl$(1 mole)$/AlCl_3$

(ii) Oleum (iii) $CH_3COCl/AlCl_3$

A



B



Compound $CH_3-CH_2-\underset{\underset{PH-CH-OH}{|}}{CH}-CH_3$ can be prepared by

(A) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CHO \xrightarrow[(2)\ H_3O^{\oplus}]{(1)\ PhMgBr}$ $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-\overset{\overset{OH}{|}}{CH}-Ph$

(B) $CH_3-\underset{\underset{MgBr}{|}}{CH}-CH_3 \xrightarrow[(2)\ H_3O^{\oplus}]{(1)\ PhCH_2CHO}$ $\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{CH}}-\overset{\overset{OH}{|}}{C}-CH_2-Ph$

(C) $Ph-COCH_3 \xrightarrow[(2)\ H_3O^{\oplus}]{(1)\ CH_3CH_2-\overset{\overset{CH_3}{|}}{CH}-MgBr}$ $Ph-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-\overset{\overset{CH_3}{|}}{CH}-CH_2CH_3$

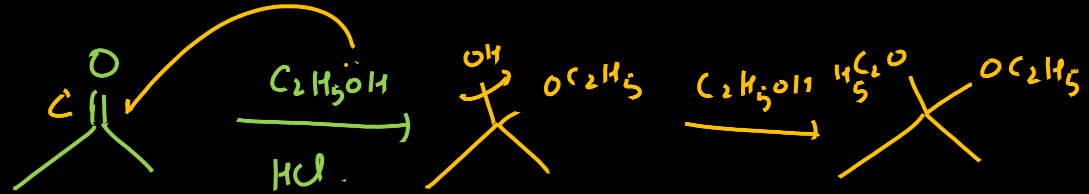
(D) $PhCHO \xrightarrow[(2)\ H_3O^{\oplus}]{(1)\ CH_3-CH_2-\overset{\overset{CH_3}{|}}{CH}-MgBr}$

10

Acetone is treated with excess of ethanol in the presence of hydrochloric acid. The product obtained is :

(A) $CH_3CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_3$ (B) $CH_3CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_2CH_2CH_3$

(C) $(CH_3)_2C\begin{matrix}\diagup OH \\ \diagdown OC_2H_5\end{matrix}$ (D) $(CH_3)_2C\begin{matrix}\diagup OC_2H_5 \\ \diagdown OC_2H_5\end{matrix}$



11

Which of the following chemical species have identical bond order?

i. $CN^{\ominus}$ ii. $O_2^{\ominus}$

iii. $NO^{\oplus}$ iv. $CN^{\oplus}$

(A) i, ii, iii (B) i, iv

(C) i, iii ✓ (D) i, iii, iv

$CN^{\ominus}$	$O_2^{\ominus}$	$NO^{\oplus}$	$CN^{\oplus}$
14	17	14	12
3	1.5	3	2

3

12

Match column-I with column-II

	Column-I		Column-II
(A)	Glycine	(p)	Optically inactive
(B)	Alanine	(q)	Aliphatic Amino acid
(C)	Lysine	(r)	Bear two –COOH
(D)	Glutamic acid	(s)	Bear two $-NH_2$

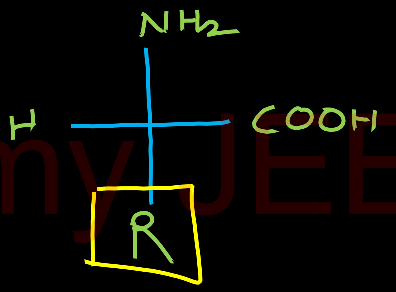
(A) A(q) B(p) C(r) D(s)

(B) A(p) B(q) C(s) D(r)

(C) A(q) B(p) C(s) D(r)

(D) A(s) B(q) C(p) D(r)

Backbone of amino acid



13

JEE 1

Which of following pair contains both coloured ions?

(A) Ti^{3+} and Ti^{4+}

(B) V^{4+} and Zn^{2+}

(C) V^{3+} and Zn^{2+}

(D) V^{4+} and V^{2+} ✓

Ti^{3+} : $3d^1 4s^0$ ⇒ colored

Ti^{4+} : $3d^0 4s^0$ ⇒ colourless

✓ V^{4+} : $3d^1 4s^0$ ⇒ coloured

Zn^{2+} : $3d^{10} 4s^0$ ⇒ colourless

V^{3+} : $3d^2 4s^0$ ⇒ coloured

✓ V^{2+} : $3d^3 4s^0$ ⇒ coloured

14

JEE 1

Select the $\boxed{\text{incorrect statement}}$ out of the following.

(A) $E^\circ Cu_{2+}/Cu$ has a positive value ✓

(B) Cu^{2+} cannot oxidise I^- to I_2 ✗ $Cu^{2+} + I^{-1} \longrightarrow Cu_2I_2 + I_2$ ($+1$) (0)

(C) Cu^{+} disproportionates in aqueous solutions to form Cu^{2+} and Cu $Cu^{+}(aq) \longrightarrow Cu^{2+} + Cu$ ✓

(D) $Cu^{2+}(aq)$ is more stable than $Cu^{+}(aq)$ as the hydration of Cu^{2+} is much more exothermic than that of Cu^{+} which compensates the second ionisation enthalpy of Cu ✓

15

A weak acid HA after treatment with 12ml of 0.1M strong base BOH has pH = 5. At the end point, the volume of same base required is 26.6ml. K_a of acid is :

[Given : $\log\left(\frac{1.2}{1.46}\right) = -0.085$]

(A) 8.2×10^{-6} (B) 8.2×10^{-5}

(C) 8.2×10^{-4} (D) 8.2×10^{-7}

$HA + BOH \rightleftharpoons AB + H_2O$

	HA	BOH	AB	H_2O
t=0	2.66 mmol	2.66 mmol	0	0
t = eq (consumed amt)	1.2 mmol	1.2 mmol	1.2 mmol	
(left amt)	2.66-1.2	2.66-1.2	1.2	

millimoles = M × V_{ml}

= 0.1M × 26.6 ml

At end point = 2.66 mmole

$pH = pK_a + \log\frac{Salt}{Acid}$

$5 = -\log K_a + \log\frac{1.2}{2.66-1.2}$

$\boxed{K_a = 8.2\times10^{-6}}$

16

JEE 1

In which of the following arrangements the order is $\boxed{\text{not according}}$ to the property indicated against it ?

(A) $Li < Na < K < Rb$: increasing metallic radius ✓

(B) $I < Br < F < Cl$: increasing electron gain enthalpy (with negative sign) ✓

(C) $B < C < \underset{}{\overset{2p^3}{N}} < \overset{2p^4}{O}$: increasing first ionisation enthalpy ✗

(D) $\underset{13}{Al^{3+}} < \underset{12}{Mg^{2+}} < \underset{11}{Na^{+}} < \underset{9}{F^{-}}$: increasing ionic size ✓

For isoelectronic sp

$$\text{size} \propto \frac{1}{\text{At. no}}$$

17

JEE 1

The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$(mol. wt. 329 g mol^{-1}) in 100 g of water (K_f = 1.86 K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2}

(C) -5.7×10^{-3} (D) -1.2×10^{-2}

$$K_3[Fe(CN)_6] \rightleftharpoons 3K^+ + [Fe(CN)_6]^{3-} \qquad \boxed{n = 4}$$

$$\boxed{\Delta T_f = i\, m\, K_f}$$

$$\Delta T_f = 4 \times \frac{0.1 \times 1\cancel{000}}{329 \times 1\cancel{00}} \times 1.86$$

$$= 4 \times \frac{1}{329} \times 1.86 = 2.3 \times 10^{-2}$$

So F.P $= -2.3 \times 10^{-2}$

18

A photon of 300 nm is absorbed by a gas which then re-emits two photons. One re-emitted photon has wavelength of 496 nm. Wavelength of other re-emitted photon is

(A) 300 nm (B) 496 nm

(C) 759 nm (D) 550 nm

$$\frac{\cancel{hc}}{300\,nm} = \frac{\cancel{hc}}{496\,nm} + \frac{\cancel{hc}}{\lambda}$$

$$\frac{1}{300\,nm} = \frac{1}{500} + \frac{1}{\lambda}$$ (rounding off)

$$\lambda = 750\,nm$$

19

The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of $10 dm^3$ to a volume of $100 dm^3$ at 27°C is

(A) $38.3\ J\ mol^{-1}\ K^{-1}$ (B) $35.8\ J\ mol^{-1}\ K^{-1}$

(C) $32.3\ J\ mol^{-1}\ K^{-1}$ (D) $42.3\ J\ mol^{-1}\ K^{-1}$

$$\Delta S = 2.303\, nR \log \frac{V_2}{V_1}$$

$$= 2.303 \times 2 \times 8.314 \log \frac{100}{10}$$

$$= 38.2\ J/mol/K$$

20

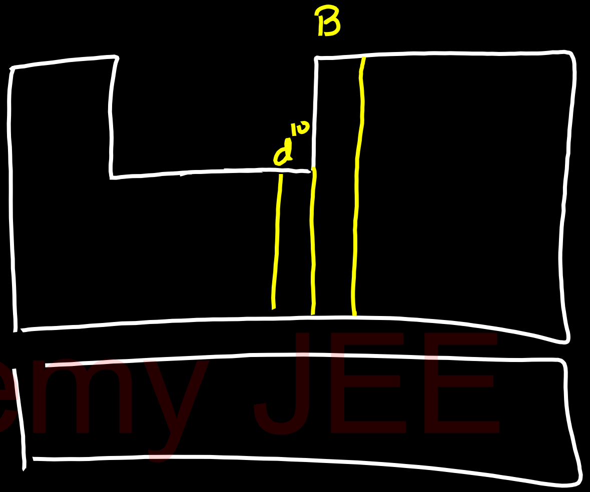
Following transition elements (IE) drops abruptly (Ga, In and TI). This is due to

(A) decrease in effective nuclear charge

(B) increase in atomic radius

(C) removal of an electron from the singly occupied np-orbitals of higher energy than the ns-orbitals of Zn, Cd and Hg

(D) None of the above is correct



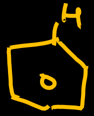
Zn, Cd, Hg : fully filled

B-family : np^1

JEE 1

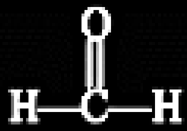
21

Total number of compounds which can produce benzene on reaction with PhMgBr

$Ph^{\ominus} + H^{\oplus} \rightarrow$ 

(Acid - base)

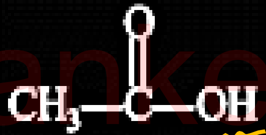
(I)



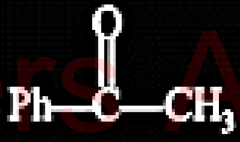
(II)

CH_3OH

(III)



(IV)



(V)

(VI)

(VII)

Ans. 4

22

JEE 1

The equivalent conductance of a N/10 NaCl solution at 25°C is 10^{-2} S $m^2 eq^{-1}$. Resistance of solution contained in the cell is 50Ω. Cell constant (in m^{-1}) is :

$$\Lambda_{eq} = \frac{K}{1000N} = \frac{GG^*}{1000N} = \frac{1}{R}\frac{G^*}{1000N}$$

$$10^{-2} = \frac{1}{50} \times \frac{G^*}{1000 \times 0.1}$$

$$G^* = 10^{-2} \times 5000 = \underline{50}$$

23

How much times faster would a reaction proceeds at 25°C than at 0°C if the activation energy is 65 kJ ?

JEE 1

$T_1 = 273K \quad , k_1$

$T_2 = 298K \quad k_2$

$$\log\left(\frac{k_2}{k_1}\right) = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\left(\frac{k_2}{k_1}\right) = \frac{65 \times 10^3}{2.303 \times 8.314}\left(\frac{1}{273} - \frac{1}{298}\right)$$

$$\therefore \frac{k_2}{k_1} = 11$$

24

10 mL of sulphuric acid solution (specific gravity = 1.84) contains 98% by weight of pure acid. Calculate the volume of 2 N NaOH solution required to just neutralize the acid. (nearest integer)

98% w/w

$W_{solute} = 98g$

$W_{solution} = 100g$

Volume of Solution (ml) $= \frac{100}{1.84}$ ml

m.eq of H_2SO_4 = m.eq of NaOH

$N_1V_1 = N_2V_2$

$(M_1 \times 2 \times 10) = 2V_2$

$\left[\frac{(98/98)}{\left(\frac{100}{1.84}\right)} \times 1000\right] \times 10 = V_2$

$\Rightarrow V_2(ml) = 184$

25

JEE 1

How many of the following reaction gives primary amine as product

(a) $CH_3CONH_2 \xrightarrow{Br_2/KOH}$ (b) $CH_3CN \xrightarrow{LiAlH_4}$

(c) $CH_3NC \xrightarrow{LiAlH_4}$ (d) $CH_3CONH_2 \xrightarrow{LiAlH_4}$

(a) CH_3-NH_2 1°

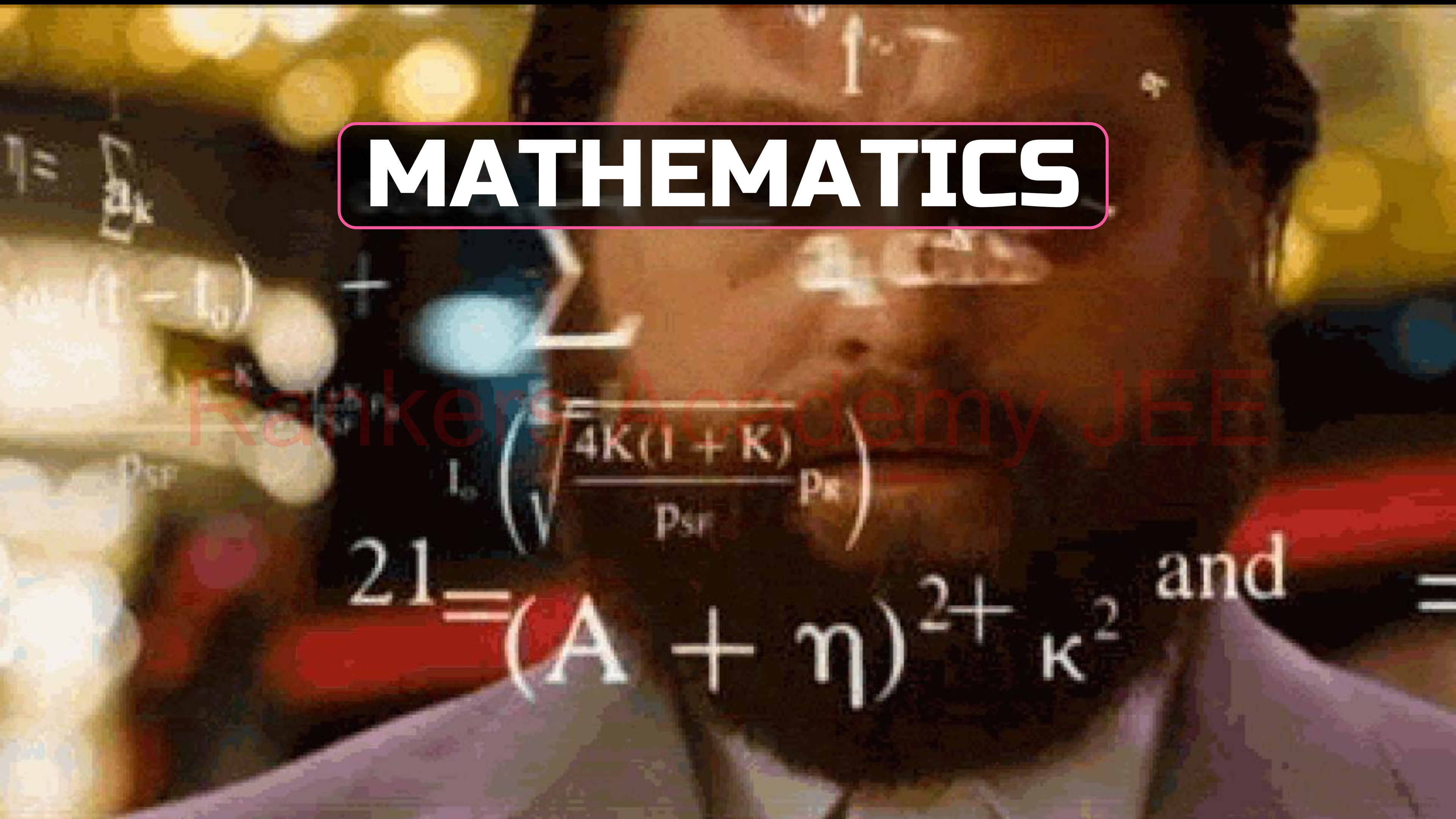
(b) $CH_3CH_2NH_2$ 1°

Ans. 3

(c) $CH_3-NH-CH_2$ 2°

(d) CH_3NH_2 1°

MATHEMATICS



If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are two values of λ for which the roots α, β are related by $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}$, is

(A) 254 (B) 200

(C) 350 (D) 154

$\lambda x^2 - \lambda x + x + 5 = 0$

$\lambda x^2 + (1-\lambda)x + 5 = 0$ α, β

$\alpha + \beta = \frac{\lambda - 1}{\lambda}$; $\alpha\beta = \frac{5}{\lambda}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{5}$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5}$

$\Rightarrow \frac{\left(\frac{\lambda-1}{\lambda}\right)^2 - 2\left(\frac{5}{\lambda}\right)}{5/\lambda} = \frac{4}{5}$

$\Rightarrow \frac{(\lambda-1)^2 - 10\lambda}{\cancel{5}\lambda} = \frac{4}{\cancel{5}}$

$\Rightarrow \lambda^2 + 1 - 12\lambda = 4\lambda$

$$\lambda^2 - 16\lambda + 1 = 0 \; \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

$$\lambda_1 + \lambda_2 = 16$$

$$\lambda_1 \lambda_2 = 1$$

$$\Rightarrow \frac{(\lambda_1+\lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2}$$

$$\Rightarrow \frac{16^2 - 2(1)}{1} = 254$$

2

If $x \in [0,2]$ and $g(x) = f(x) + f(2 - x)$. Also, $f''(x) < 0$, then $g(x)$

(A) increases in $[0,2]$

(B) decreases in $[0,2]$

(C) decreases in $[0,1)$ and increases in $(1,2]$

(D) increases in $[0,1)$ and decreases in $(1,2]$ ✓

$f'(x)$ decreasing f^n → $x_1 > x_2$, $f'(x_1) < f'(x_2)$

$g'(x) = f'(x) - f'(2-x)$

0 — 1 — 2

$x \in (0,1)$ | $x \in (1,2)$
$-x \in (-1,0)$ | $-x \in (-2,-1)$
$2-x \in (1,2)$ | $2-x \in (0,1)$
$x < 2-x$ | $x > 2-x$
$f'(x) > f'(2-x)$ | $f'(x) < f'(2-x)$
$g'(x) > 0$ | $g'(x) < 0$

3

If $a_1, a_2, \dots a_n$ are in arithmetic progression, where $a_1 > 0$ for all i.

Then, $\underbrace{\frac{1}{\sqrt{a_1}+\sqrt{a_2}}} + \underbrace{\frac{1}{\sqrt{a_2}+\sqrt{a_3}}} + \cdots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ is equal to

(A) $\frac{n^2(n+1)}{2}$

(B) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

(C) $\frac{n(n-1)}{2}$

(D) None of these

$$\Rightarrow \frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1}$$

$$\Rightarrow \frac{\cancel{\sqrt{a_2}}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\cancel{\sqrt{a_2}}}{d} + \dots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$$

$$\Rightarrow \frac{1}{d}\left[\sqrt{a_n}-\sqrt{a_1}\right]$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)}$$

$$= \frac{\cancel{a_1} + (n-1)\cancel{d} - \cancel{a_1}}{\cancel{d}\left(\sqrt{a_n} + \sqrt{a_1}\right)}$$

4

JEE 1

If $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$ is continuous

at $x = 0$, then the ordered pair (p, q) is

(A) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (B) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (D) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

LHL $\dfrac{(p+1)x + x}{x} = p+2$

$f(0) = q$

RHL $\dfrac{\cancel{\sqrt{x}}(\sqrt{1+x} - 1)}{x\cancel{\sqrt{x}}} = \dfrac{\cancel{1}+\cancel{x}-\cancel{1}}{\cancel{x}(\sqrt{1+x}+1)} = \dfrac{1}{2}$

$p + 2 = q = \dfrac{1}{2}$

5

JEE1

$$f(x) = \begin{cases} \frac{2}{\pi}\sin^{-1}\left(\sin\frac{\pi}{2}x\right), & 0 < x \le 1 \\ (a-1)^2 + (x-1)^2, & x > 1 \end{cases}$$

If $x = 1$ is point of local maximum of $f(x)$, then sum of all integral values of $|a|$ is

(A) 0 (B) 1

(C) 2 (D) 3

LHL < f(1) > RHL

$a = 1$

$|a| = 1$

Padosi Effect

$1 > (a-1)^2$

$-1 < a-1 < 1$

$0 < a < 2$

$\cancel{1} > a^2 - 2a + \cancel{1}$

$0 > a(a-2)$

$a \in (0,2)$

6

JEE 1

An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the left of S, then the side length of this triangle is

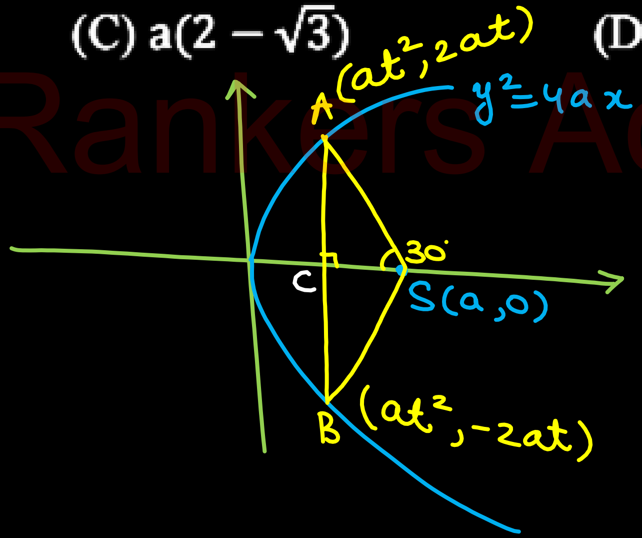
Ans : $4at = 4a(2-\sqrt{3})$

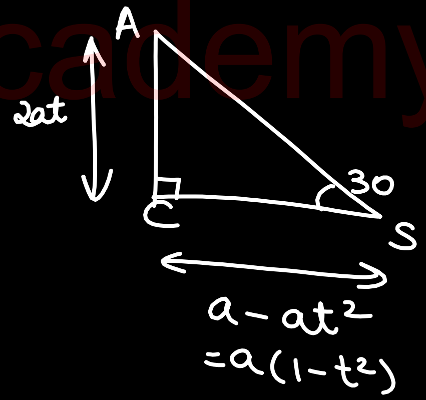
(A) $2a(2-\sqrt{3})$

(B) $4a(2-\sqrt{3})$

(C) $a(2-\sqrt{3})$

(D) $8a(2-\sqrt{3})$





$$\frac{1}{\sqrt{3}} = \frac{2\cancel{a}t}{\cancel{a}(1-t^2)}$$

$$\boxed{t^2 + 2\sqrt{3}t - 1 = 0}$$

$$t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$$

$$= \frac{-2\sqrt{3} \pm 4}{2}$$

$$= 2-\sqrt{3} \quad , \quad -2-\sqrt{3}$$

$\checkmark$ $\times$

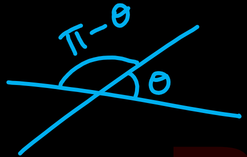
The acute angle between the lines $\frac{x-1}{a} = \frac{y+1}{b} = \frac{z}{c}$ and $\frac{x+1}{b} = \frac{y-3}{c} = \frac{z-1}{a}$ where $a > b > c$ and a, b, c are the roots of the equation $t^3 - t^2 - 4t + 4 = 0$ is

$\leftarrow a, b, c$

$abc = -4$

$a + b + c = 1$

$ab + bc + ca = -4$



(A) $\sin^{-1}\left(\frac{\sqrt{63}}{9}\right)$

(B) $\cos^{-1}\frac{4}{9}$

(C) $\tan^{-1}\left(\frac{2}{3}\right)$

(D) $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

Angle: $\theta, \pi - \theta$

$$\cos\theta = \frac{ab + bc + ca}{\sqrt{a^2+b^2+c^2}\sqrt{b^2+c^2+a^2}} = \frac{-4}{a^2+b^2+c^2}$$

$$= \frac{-4}{(a+b+c)^2 - 2(ab+bc+ca)}$$

$$= \frac{-4}{1-2(-4)} = -\frac{4}{9}.$$

8

JEE 1

The value of definite integral $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is:

(A) $\frac{\pi}{2}$

(B) π

(C) π^2 ✓

(D) $\frac{\pi^2}{2}$

$$I = \int_{-\pi}^{\pi} \cancel{\underbrace{\frac{2x}{1+\cos^2 x}}_{\text{odd}}} + \underbrace{\frac{2x\sin x}{1+\cos^2 x}}_{\text{even}} dx$$

Queen

$$= 2\int_{0}^{\pi} \frac{2x\sin x}{1+\cos^2 x} dx$$

$$I = 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$$

King

$x \to 0+\pi-x$

$$I = 4\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$

$$2I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$I = 2\pi\int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x}$$

Let $\cos x = t$

$-\sin x\,dx = dt$

$$I = 2\pi\int_1^{-1} \frac{-dt}{1+t^2}$$

$$= 2\pi\int_{-1}^{1} \frac{dt}{1+t^2}$$

$$= 2\pi \left.\tan^{-1}t\right|_{-1}^{1}$$

$$= 2\pi\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$$

$$= \pi^2$$

9

JEE1

Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(A) $\frac{1}{2}(\alpha-\beta)^2$

(B) $-\frac{a^2}{2}(\alpha-\beta)^2$

(C) 0

(D) $\frac{a^2}{2}(\alpha-\beta)^2$

$$\frac{2\left(\sin\left(\frac{ax^2+bx+c}{2}\right)\right)^2}{(x-\alpha)^2}$$

$$= \frac{2\left(\cancel{\sin}\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)\right)^2}{(x-\alpha)^2}$$

$$= 2\cdot\frac{a^2}{4}\,\frac{\cancel{(x-\alpha)^2}(x-\beta)^2}{\cancel{(x-\alpha)^2}}$$

$$= \frac{a^2}{2}(\alpha-\beta)^2$$

10

Let A be a matrix of order 3×3 such that $|A| = 3$. Let $B = 3\,A^{-1}$ and $C = \frac{\text{adj } A}{2}$, then the value of $|A^2 B^3 C^4|$ is

(A) $\frac{3^{16}}{2^{12}}$ ✓

(B) $\left(\frac{3}{2}\right)^{12}$

(C) $\frac{3^{10}}{2^{8}}$

(D) $\frac{3^{12}}{2^{14}}$

$|A^2 B^3 C^4| = |A|^2\,|B|^3\,|C|^4 = |A|^2 \cdot \frac{3^9}{|A|^3} \cdot \left(\frac{1}{8}\right)^4 |A|^8$

$|B| = |3A^{-1}| = 3^3|A^{-1}| = \frac{3^3}{|A|}$

$= \frac{3^9}{2^{12}} \cdot |A|^7$

$|C| = \left|\frac{1}{2}\text{adj}A\right| = \left(\frac{1}{2}\right)^3 |\text{adj}A|$

$= \left(\frac{1}{2}\right)^3 |A|^2$

$= \frac{3^9 \cdot 3^7}{2^{12}} = \frac{3^{16}}{2^{12}}$

11

JEE 1

If $\alpha^3 + \beta^6 = 2$ then the maximum value of the independent term of x in the expansion of $(\alpha x^{1/3} + \beta x^{-1/6})^9 (\alpha > 0, \beta > 0)$ is

(A) 42 (B) 68

(C) 84 (D) 148

$r = \frac{np - m}{p + q} = \frac{9\left(\frac{1}{3}\right) - 0}{\frac{1}{3} + \frac{1}{6}} = \frac{3}{3/6} = 6$

coeffi $= {}^9C_6 \underbrace{\alpha^3 \beta^6}_{=1}$

$= {}^9C_6 = \frac{7 \times \cancel{8}^{4} \times \cancel{9}^{3}}{\cancel{6}} = 28 \times 3 = 84$

$$\frac{\alpha^3+\beta^6}{2} \geq \sqrt{\alpha^3\beta^6}$$

$$\frac{2}{2}$$

$$1 \geq \alpha^3\beta^6$$

12

If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1-x^2)dy/dx$ equals-

(A) $x + y$ (B) $1 + xy$

(C) $1-xy$ (D) $xy - 2$

$$y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$y' = \frac{\cancel{\sqrt{1-x^2}} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}} + \frac{\sin^{-1}x}{\cancel{2}\sqrt{1-x^2}}(+\cancel{2}x)}{(1-x^2)}$$

$$(1-x^2)y' = \frac{1 + x\frac{\sin^{-1}x}{\sqrt{1-x^2}}}{\cancel{1+x^2}} = 1 + xy$$

13

If ' α ' is a complex number satisfying the relation $|\alpha - 1| = \alpha - 3(1+i)$. Then α equals to

(A) $-\frac{1}{4} + 3i$

(B) $\frac{1}{4} + 3i$

(C) $\frac{1}{4} - 3i$

(D) No such complex number exists

$\alpha = x + iy$

$|x + iy - 1| = x + iy - 3 - 3i$

purely real

$\sqrt{(x-1)^2 + y^2} = (x-3) + i\underbrace{(y-3)}_{=0}$

$\therefore y = 3$

$\Rightarrow \sqrt{(x-1)^2 + 9} = x - 3$

$\Rightarrow \cancel{x^2} - 2x + 10 = \cancel{x^2} - 6x + 9$

$\Rightarrow 4x + 1 = 0$

14

Number of 3×3 matrices M with entries from $\{0,1,2\}$ are there for which the sum of the diagonal entries of M^TM is 5, are k, then $\frac{k}{22}$ is

(A) 198 (B) 48

(C) 9 (D) 61

JEE 1

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Shortcut

$$tr(MM^T) = tr(M^TM) = a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2 = 5$$

$2^2, 1^2, 0, 0, 0, 0, 0, 0, 0 \longrightarrow {}^9C_2 \cdot 2 \cdot {}^7C_7 \cdot 1$

$= 72$

$1^2, 1^2, 1^2, 1^2, 1^2, 0^2, 0^2, 0^2, 0^2 \longrightarrow {}^9C_5 \cdot 1 \cdot {}^4C_4$

$126 = \frac{\cancel{6} \times 7 \times \cancel{8} \times 9}{\cancel{24}}$

15

JEE 1

If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then

(A) $4e^4 - 12e^2 - 27 = 0$

(B) $4e^4 + 8e^2 - 35 = 0$

(C) $4e^4 - 24e^2 + 27 = 0$

(D) $4e^4 - 24e^2 + 35 = 0$

$x = \frac{a}{e}$

$\frac{a}{e} = \frac{4}{\sqrt{5}}$

$\boxed{a^2 = \frac{16}{5}e^2}$

$e^2 = 1 + \frac{b^2}{a^2}$

$\boxed{b^2 = a^2(e^2-1)}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{16}{a^2} - \frac{12}{b^2} = 1$

$\Rightarrow \frac{16}{a^2} - \frac{12}{a^2(e^2-1)} = 1$

$\Rightarrow 16(e^2-1) - 12 = a^2(e^2-1)$

$\Rightarrow 16(e^2-1) - 12 = \frac{16}{5}e^2(e^2-1)$

$\Rightarrow 16e^2 - 28 = \frac{16e^4 - 16e^2}{5}$

$\Rightarrow 80e^2 - 140 = 16e^4 - 16e^2$

$\Rightarrow 16e^4 - 96e^2 + 140 = 0$

$4e^4 - 24e^2 + 35 = 0$

16

Consider the function $f(x) = \frac{1}{\sqrt{\cos^{-1}x - |\sin^{-1}x|}} + \ln\left(\frac{x}{2x-1}\right)$. Which of the following(s) is NOT the subset of domain of f(x) ?

(A) $[-1,0)$ (B) $\left(\frac{1}{3}, \frac{1}{\sqrt{2}}\right)$

(C) $\left(\frac{1}{2}, \frac{3}{5}\right)$ (D) $\left(-1, \frac{-1}{\sqrt{2}}\right)$

$\frac{x}{2x-1} > 0$

$+ \quad - \quad +$

$0 \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}}$

$x \in (-\infty, 0) \cup (\frac{1}{2}, \infty)$ — ①

$\cos^{-1}x > |\sin^{-1}x|$

$0 \le \sin^{-1}x < \frac{\pi}{2}$

$\frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x$

$\frac{\pi}{4} > \sin^{-1}x > 0$

$\frac{1}{\sqrt{2}} > x \ge 0$

$-\frac{\pi}{2} < \sin^{-1}x < 0$

$\frac{\pi}{2} - \sin^{-1}x > -\sin^{-1}x$

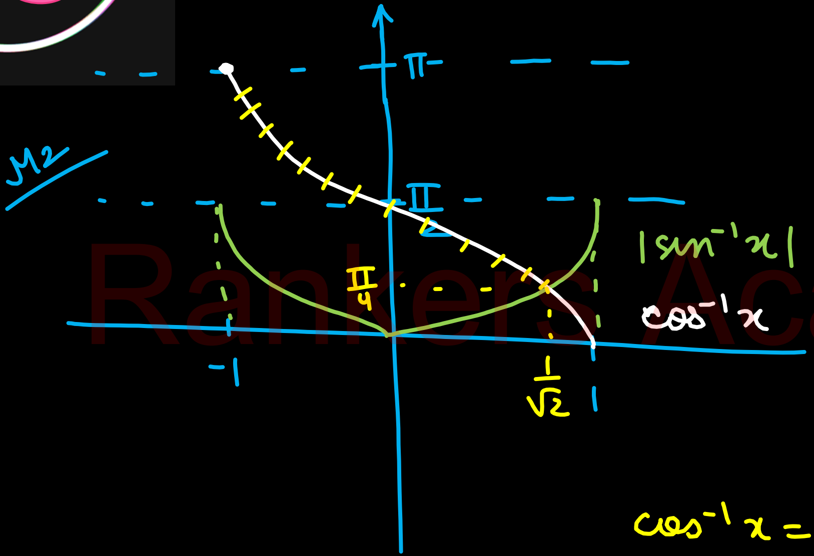
$\frac{\pi}{2} > 0$

$x \in (-1, 0)$

$\bigcup$

$x \in (-1, \frac{1}{\sqrt{2}})$

intersection $(-1, 0] \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$



$\cos^{-1}x = \sin^{-1}x = \pi/4$

17

Let $\alpha, \beta (\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in N$ then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to

(A) 15 (B) 16

(C) 17 (D) 18

$x^2 - x = 4$

$P_{16} - P_{15} = 4P_{14}$

$P_{15} - P_{14} = 4P_{13}$

$= \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$

$= \frac{(P_{15} - P_{14})}{P_{13}} \frac{(P_{16} - P_{15})}{P_{14}}$

$= 4 \cdot 4 = 16$

18

The number of integral solution of

$2x + 2y + z = 20 (x, y, z \geq 0)$ is

(A) 55 (B) 30

(C) 66 ✓ (D) 45

$2(x+y) + z = 20$

even + $\boxed{}$ = even

$\therefore z$ even

$z = 0, 2, 4 \ldots 20$

18

JEE 1

$2(x+y)+z=20$

$x+y$	z	Count
10	0	$10+2-1C_{2-1}=11$
9	2	$9+2-1C_{2-1}=10$
8	4	$=9$
7	6	$=8$
6	8	$=7$
5	10	$=6$
4	12	$=5$
3	14	$=4$
2	16	$=3$
1	18	$=2$
0	20	$=1$

Ans: $1+\dots+11$

$$=\frac{11(12)}{2}$$

$$=66$$

19

The solution of the differential equation $\frac{dy}{dt} = \frac{\tan y}{(1+t)} + (1+t)e^t \sec y$ is, where ' c ' is an arbitrary constant

(A) $\cos y = (e^t + c)(t+1)$

(B) $\cos y = (e^t - c)(x-1)$

(C) $\sin y = (e^t + c)(t-1)$

(D) $\sin y = (e^t + c)(t+1)$

$\Rightarrow \frac{dy}{dt} - \frac{\tan y}{1+t} = (1+t)e^t \sec y$

$\Rightarrow \cos y \frac{dy}{dt} - \frac{\sin y}{1+t} = (1+t)e^t$

$\Rightarrow$ Let $\sin y = z$

$\Rightarrow \frac{dz}{dt} - \frac{z}{t+1} = e^t(t+1)$

$$IF = e^{\int \frac{-1}{t+1} dt} = e^{-\ln|t+1|} = e^{\ln \frac{1}{t+1}} = \frac{1}{t+1}$$

$$\frac{z}{t+1} = \int \frac{1}{t+1} \cdot e^{t}(t+1)\, dt$$

$$\Rightarrow \frac{\sin y}{t+1} = e^{t} + c$$

20

JEE 1

The system of equations

px + (p + 1)y + (p − 1)z = 0

(p + 1)x + py + (p + 2)z = 0

(p − 1)x + (p + 2)y + pz = 0 has a non-trivial solutions for

$\Delta = 0$

(A) Exactly two real values of p.

(B) no real value of p.

(C) Exactly one real value of p.

(D) infinitely many real value of P.

$$\begin{vmatrix} p & p+1 & p-1 \\ p+1 & p & p+2 \\ p-1 & p+2 & p \end{vmatrix} = 0$$

$$0 = p\left(p^2-(p+2)^2\right) - (p+1)\left[p(p+1)-(p-1)(p+2)\right] + (p-1)\left[(p+1)(p+2)-p(p-1)\right]$$

$$0 = \cancel{p^3} - \cancel{p^3} - 4p^2 - 4p - (p+1)(2) + (p-1)(4p+2)$$

$$= -\cancel{4p^2} - 4p - 2p - 2 + \cancel{4p^2} - 2p - 2$$

$8p+4=0$

$p = -\frac{1}{2}$

21

A bag contains 3 red, 2 white and 2 black balls. Two balls are drawn at random and none of them is found to be a white ball. The probability that both balls are red is $\frac{a}{b}$ (where a, b are coprime) then b – a is equal to Ans: 7.

$\boxed{\begin{matrix} 3R \\ 2W \\ 2B \end{matrix}}$

$$\frac{\overset{2R}{\dfrac{{}^3C_2}{\cancel{{}^7C_2}}}}{\underset{2R}{\dfrac{{}^3C_2}{\cancel{{}^7C_2}}} + \underset{2B}{\dfrac{{}^2C_2}{\cancel{{}^7C_2}}} + \underset{1R\,1B}{\dfrac{{}^3C_1\,{}^2C_1}{\cancel{{}^7C_2}}}} = \frac{3}{3+1+6}$$

$$= \frac{3}{10} = \frac{a}{b}$$

22

JEE 1

If $|\bar{a}| = |\bar{b}| = 2, |\bar{c}| = 1, (\bar{a} - \bar{c}) \cdot (\bar{b} - \bar{c}) = 0$

Then the value of $|\bar{a} - \bar{b}|^2 + 2\bar{c} \cdot (\bar{a} + \bar{b})$ is equal to

$\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} + |c|^2 = 0$

Ans $= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} + 2\vec{b} \cdot \vec{c}$

$= 2^2 + 2^2 + 2(|c|^2)$

$= 4 + 4 + 2(1)$

$= 10$

23

JEE1

In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35. Then the first term of this geometric progression is Ans: 28

GP: a, ar, ar^2, ar^3, ar^4

Reci: $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \frac{1}{ar^3}, \frac{1}{ar^4}$

$$49 = \frac{a(1+r+r^2+r^3+r^4)}{\frac{1}{a}\left(1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}\right)}$$

$$49 = \frac{a^2\cancel{(1+r+r^2+r^3+r^4)}\,r^4}{\cancel{r^4+r^3+r^2+r+1}}$$

$$\boxed{7 = ar^2}$$

$a + ar^2 = 35$

$a + 7 = 35 \Rightarrow a = 28.$

24

If the mean and variance of eight numbers 3,7,9,12,13,20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to

JEE 1

$$\Rightarrow \frac{3+7+9+12+13+20+x+y}{8} = 10$$

$$\Rightarrow \frac{64+x+y}{8} = 10$$

$$\Rightarrow x+y = 16 \quad \text{— (1)}$$

$$x^2+y^2 = 148 \quad \text{— (2)}$$

$$\Rightarrow \frac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} - 10^2 = 25$$

$$\Rightarrow 9+49+81+\cancel{1}44+\cancel{1}69+\cancel{400}+x^2+y^2 = \cancel{1000}\ \cancel{600}\ 400$$

$$\Rightarrow \cancel{839} + \cancel{44+69} + x^2+y^2 = 200 - 52 = 148$$

$x+y=16$

$x^2+y^2+2xy=256$

$148+2xy=256$

$xy=\frac{256-148}{2}=\frac{108}{2}=54$

25

If N is the sum of all 3 digit natural numbers which on division with 7 leave a remainder of 3, then find sum of digits of N $\longrightarrow$ 18.

JEE 1

$7k+3$ ✓

$101, 108, \ldots\ldots 997$

$a = 101$

$d = 7$

$n = 129$

$S_n =$

$997 = 101 + (n-1)7$

$n = \frac{896}{7} + 1$

$n = 129$

$$S_n = \frac{129}{\cancel{2}}\left[\cancel{2}(101) + \overset{64}{\cancel{128}}(7)\right]$$

$$= 129\left[101 + 448\right]$$

$$= 129\left[549\right] = 70821$$

$$N = 70821$$

sum of digits $= 18$