PHYSICS

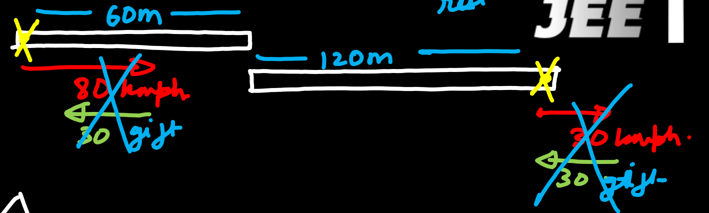
factor $= \frac{5}{18}$

A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the opposite direction is

(A) $\frac{25}{11}$ (B) $\frac{3}{2}$

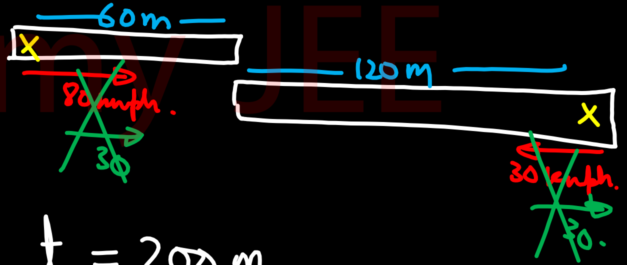
(C) $\frac{5}{2}$ (D) $\frac{11}{5}$ ✓

i) $V_{rel} = 50 \text{ kmph} \times \left(\frac{5}{18}\right)$ m/s.



$t_1 = \frac{200 \text{ m}}{250/18 \text{ m/s}}$

ii) $110 \text{ kmph} \times \frac{5}{18} = \frac{550}{18}$ m/s



$t_2 = \frac{200 \text{ m}}{550/18}$

$\frac{t_1}{t_2} = \frac{\cancel{55}^{11}}{\cancel{25}5} = \frac{11}{5}.$

2

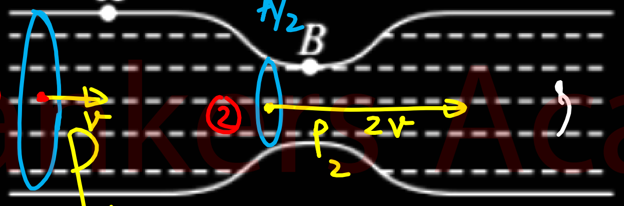
S.I.

Water flows in a horizontal tube (see figure). The pressure of water changes by $\boxed{700\ \mathrm{N\,m^{-2}}}$ $= P_1 - P_2$ between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water $= 1000\ \mathrm{kg\,m^{-3}}$)

$Q = Av$

$Av = Q = Av$

$A = 40\,cm^2 = (40 \times 10^{-4})\,m^2$



(A) 3020 cm^3/s

(B) 2420 cm^3/s

(C) 2720 cm^3/s

(D) 1810 cm^3/s

B.T ① → ②

$P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho(2v)^2$

$P_1 - P_2 = \frac{3}{2}\rho v^2$

$7\cancel{00} = \frac{3}{\cancel{2}} \times \cancel{1000}\,5\; v^2$

$$\boxed{v = \sqrt{\frac{7}{15}}\ m/s}$$

$Q = Av = \frac{40}{10000}\sqrt{\frac{7}{15}} \simeq 2.836 \times 10^{-3}\ m^3/s$

N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?

(A) 0

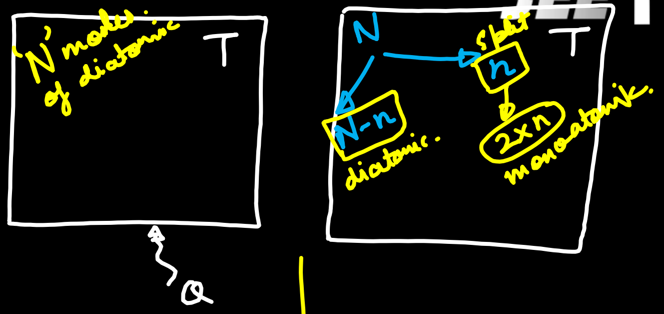
(B) $\frac{5}{2}$nRT

(C) $\frac{1}{2}$nRT

(D) $\frac{3}{2}$nRT

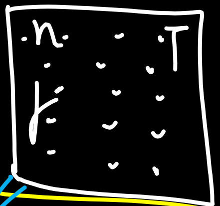
$\gamma_{di} = 5$

$\gamma_{mono} = 3$



$KE_i = \frac{5}{2}NRT$

$KE_f = \frac{5}{2}(N-n)RT + \frac{3}{\cancel{2}}(\cancel{2}n)RT$



$\boxed{KE_{gas} = \frac{f}{2}nRT}$

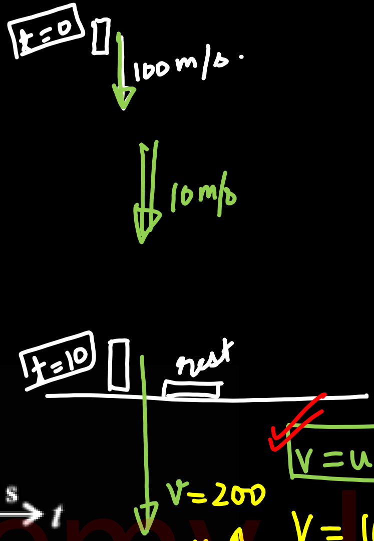
$\Delta KE = KE_f - KE_i$

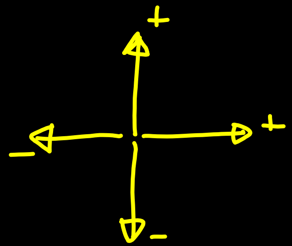
$= \frac{5}{2}(\cancel{N}-n)RT + 3nRT - \frac{5}{2}\cancel{N}RT$

$\boxed{\Delta KE = \frac{1}{2}nRT}$

4

When a bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10 second the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time t = 20 s will be (Take $g = 10\ m/s^2$).





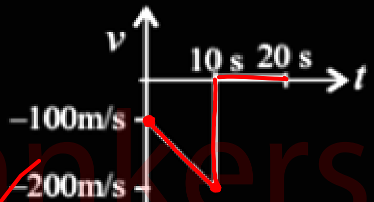
$\boxed{v = u + at}$ ↓ ⊕

$v = 200$

$V = 100 + 10(10) = 200$

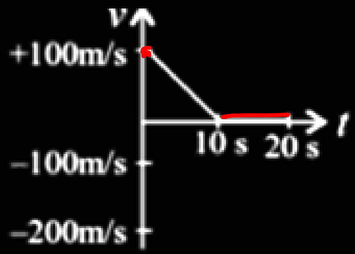
(10 m/s)/s

(A)

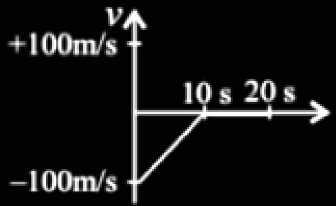


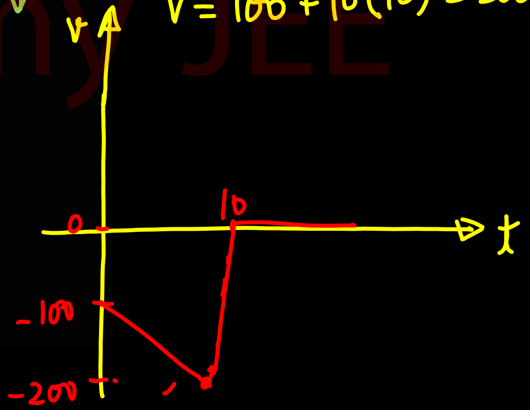
(C)

(B)



(D)





JEE 1

Rankers Academy JEE

5

JEE 1

The force is given in terms of time t and displacement x by the equation $F = A\cos Bx + C\sin Dt$. The dimensional formula of $\frac{AD}{B}$ is

(A) $[M^2 L^2 T^{-3}]$ (B) $[M^1 L^1 T^{-2}]$

(C) $[M^0 L T^{-1}]$ (D) $[ML^2 T^{-3}]$ ✓

$[F] = MLT^{-2}$

$[C] = \boxed{[A] = MLT^{-2}}$

$[x] = L$

$[t] = T$

$[B][x] = 1 = M^0L^0T^0$

$\boxed{[B] = L^{-1}}$

$[D][t] = 1$

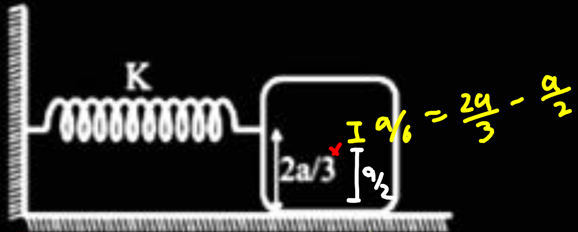
$\boxed{[D] = T^{-1}}$

$Ans = \frac{[A][D]}{[B]}$

$= \frac{MLT^{-2} \cdot T^{-1}}{L^{-1}}$

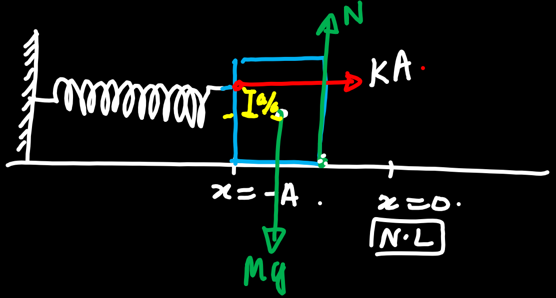
$= ML^2T^{-3}$

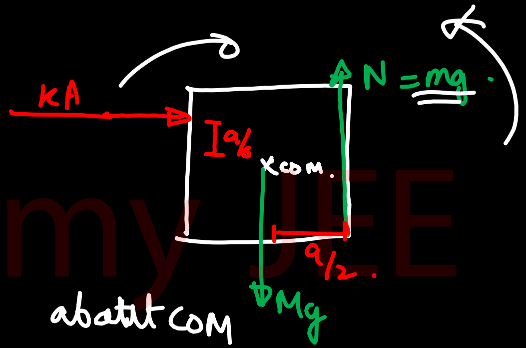
Find maximum amplitude for safe SHM (block does not topple during SHM) of a cubical block of side 'a' on a smooth horizontal floor as shown in figure (spring is massless)



(A) $\frac{mg}{3K}$ (B) $\frac{3mg}{K}$

(C) $\frac{2mg}{3K}$ (D) None





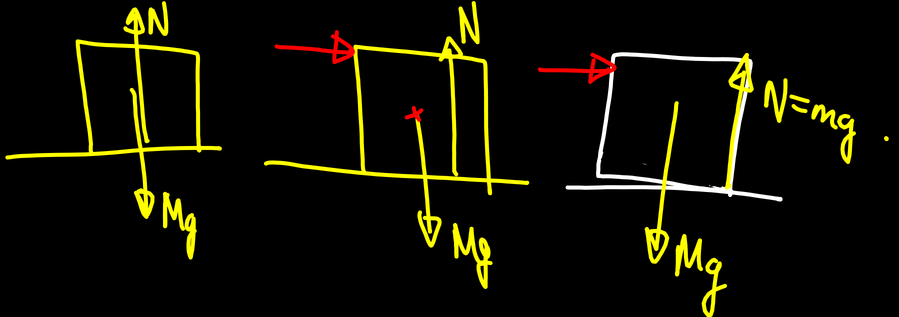
about COM

$$\tau_+ = \tau$$

$$KA\frac{a}{6} = Mg\frac{a}{2}$$

N

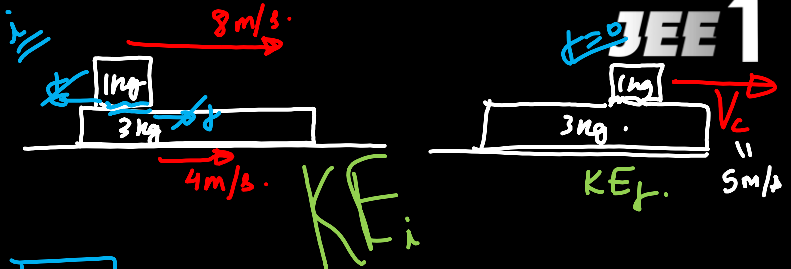
$$\boxed{A = \frac{3Mg}{k}}$$



7

At an instant t = 0, Block A of mass 1 kg is moving with speed 8 m/s towards right on rough surface of block B of mass 3 kg. Block B, which is placed on smooth horizontal surface is moving with speed 4 m/s towards right at same instant ($t = 0$). The net work done by the frictional force in long time is :- (block A will not fall from block B)

(A) –2 J
(B) –6 J
(C) –4 J
(D) –8 J



$\boxed{\vec{P}}$

$P_i = P_f$

$8 + 12 = 4V_c$

$\boxed{V_c = 5}$

$E_i + W_f = E_f$

$\Rightarrow W_f = KE_f - KE_i$

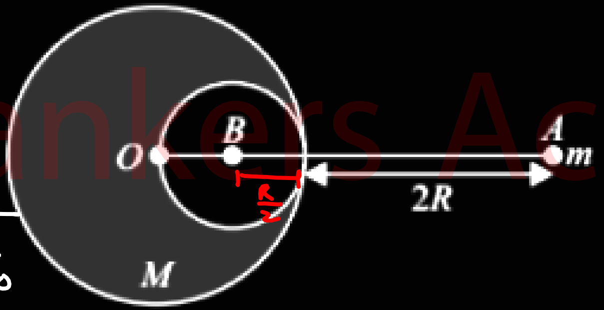
$W_f = \frac{1}{2}(4)(5)^2 - \left[\frac{1}{2}(1)8^2 + \frac{1}{2}(3)4^2\right]$

$= \underline{-6J}$

8

$\vec{F} = m\vec{g}$

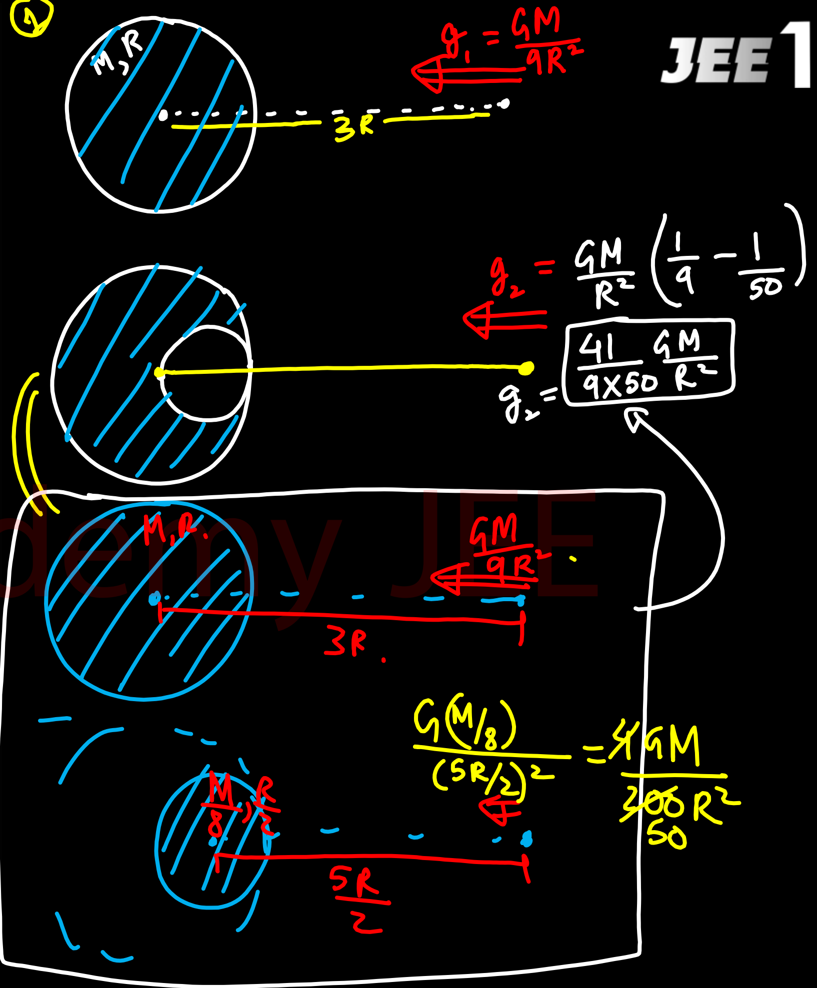
A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force $\boxed{F_1}$ $= mg_1$. Now a spherical cavity of radius (R/2) is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is



$$\frac{F_1}{F_2} = \frac{\cancel{m}g_1}{\cancel{m}g_2} = \frac{\frac{1}{\cancel{9}}}{\frac{41}{\cancel{9}\times 50}} = \frac{50}{41}$$

~~(A) 41: 50~~ ~~(B) 25: 36~~

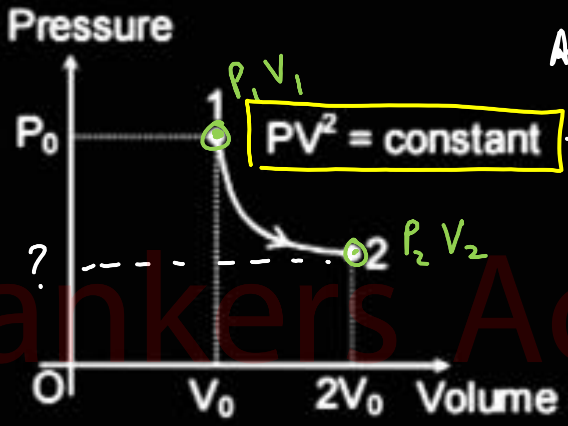
(C) 36: 25 (D) 50: 41 ✓



9

JEE 1

An ideal monoatomic gas undergoes a thermodynamics process form 1 → 2 as shown in the figure. The change in internal energy is



(A) $\frac{3}{4}P_0V_0$

(B) $-\frac{3}{4}P_0V_0$

(C) $\frac{3}{2}P_0V_0$

(D) $-\frac{3}{2}P_0V_0$

$\boxed{f = 3}$

$C_v = \frac{fR}{2}$

$\boxed{C_v = \frac{3R}{2}}$

$P_1 = P_0$

$V_1 = V_0$

$P_2 = ?$

$V_2 = 2V_0$

$P_0 \cancel{V_0^2} = P_2(2\cancel{V_0})^2$

$\boxed{P_2 = P_0/4}$

$P_1V_1 = nRT_1$

$P_2V_2 = nRT_2$

$(P_2V_2 - P_1V_1) = nR\Delta T$

Ans = $\Delta U = nC_v\Delta T = ?$

$\Delta U = n\frac{3R}{2}\Delta T = \frac{3}{2}\boxed{nR\Delta T}$

$\Delta U = \frac{3}{2}[P_2V_2 - P_1V_1]$

$\Delta U = \frac{3}{2}\left[\left(\frac{P_0}{4}\right)(2V_0) - P_0V_0\right]$

$= \frac{3}{2}\left[-\frac{1}{2}P_0V_0\right]$

$= -\frac{3}{4}P_0V_0$

10

JEE 1

A wire of length $\ell = (6 \pm 0.06)$ cm and radius $r = (0.5 + 0.005)$ cm has mass $m = (0.3 \pm 0.003)$ gm maximum percentage error in density is :-

1% error · 1% error · 1% error ·

(A) 4 (B) 2

(C) 1 (D) 6.8

$\rho = \frac{\text{mass}}{\text{vol.}} = \frac{m}{\pi r^2 \ell}$

$\boxed{\rho = \left(\frac{1}{\pi}\right) m^{1} r^{-2} \ell^{-1}}$

const.

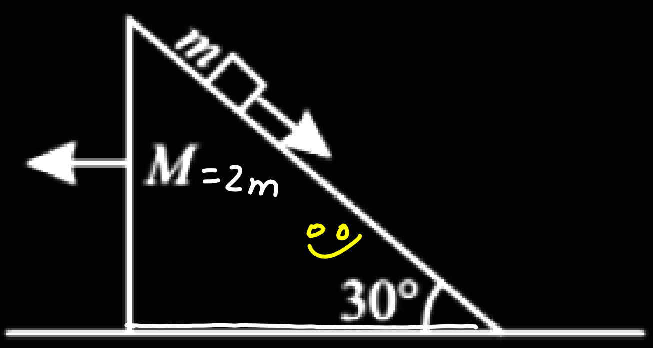
error $\left|\frac{\Delta\rho}{\rho}\right| \times 100 = 1\left|\frac{\Delta m}{m}\right| \times 100 + 2\left|\frac{\Delta r}{r}\right| \times 100 + 1\left|\frac{\Delta \ell}{\ell}\right| \times 100$

$1(1\%) + 2(1\%) + 1(1\%) = 4\%$

$\%\rho$ error $= 1(1\%) + 2(1\%) + 1(1\%) = 4\%$

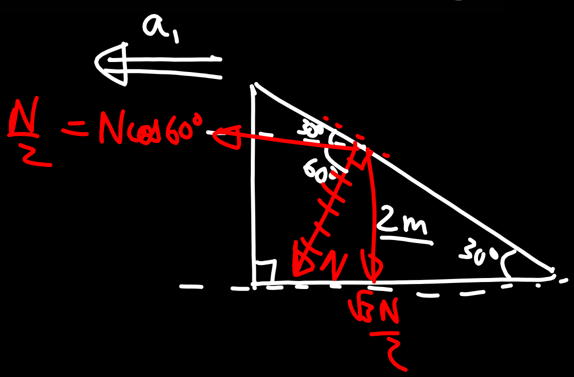
11

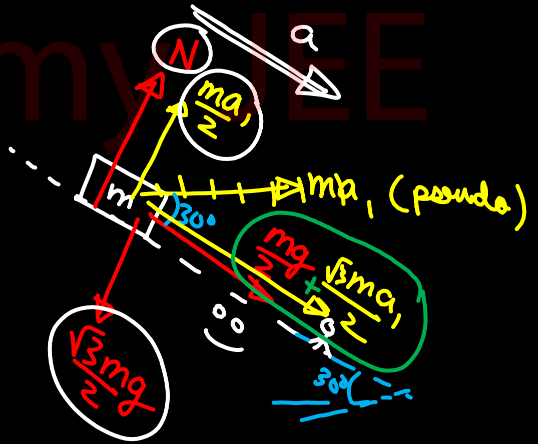
A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is ______ Given m = 8 kg, M = 16 kg Assume all the surfaces shown in the figure to be frictionless.



(A) $\frac{3}{5}g$ (B) $\frac{4}{3}g$

(C) $\frac{6}{5}g$ (D) $\frac{2}{3}g$





11

$\frac{N}{2} = 2ma_1$

$N = 4ma_1$ — ①

$N = mg\frac{\sqrt{3}}{2} - \frac{ma_1}{2}$ — ②

$\frac{\cancel{m}g}{2} + \frac{\cancel{m}a_1\sqrt{3}}{2} = \cancel{m}a$ — ③

① in ②

$4ma_1 = mg\frac{\sqrt{3}}{2} - \frac{ma_1}{2}$

$\frac{9}{\cancel{2}}\cancel{m}a_1 = \cancel{m}g\frac{\sqrt{3}}{\cancel{2}}$

$a_1 = \frac{g\sqrt{3}}{9}$

$\frac{g}{2} + \frac{\cancel{\sqrt{3}}}{2}\cdot\frac{g\cancel{\sqrt{3}}}{\cancel{9}\,3} = a$

$a = \frac{g \times 3}{2 \times 3} + \frac{g}{6} = \frac{4g}{6} = \frac{2g}{3}$

12

Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is

(A) $\frac{2}{5}\frac{ML^2}{\pi^2}$

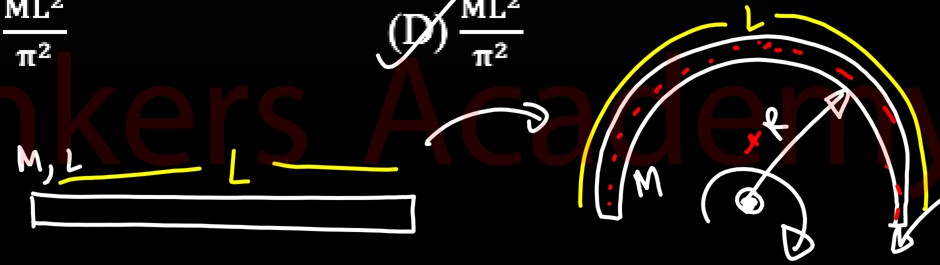
(B) $\frac{1}{2}\frac{ML^2}{\pi^2}$

(C) $\frac{1}{4}\frac{ML^2}{\pi^2}$

(D) $\frac{ML^2}{\pi^2}$

$\pi R = L$

$R = \frac{L}{\pi}$



$I = MR^2 = M\left(\frac{L}{\pi}\right)^2$

13

JEE 1

A convex lens of focal length 20 cm produces images of the same magnification $\boxed{2}$ when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is

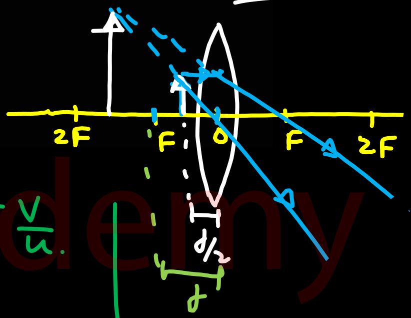
(A) 5: 3 (B) 3: 1

(C) 2: 1 (D) 4: 3

$\boxed{\frac{x_1}{x_2} > 1}$

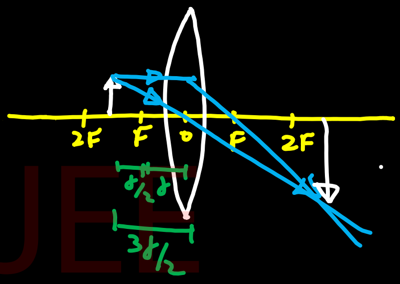
virtual image 2x.

$\boxed{m = 2}$



real image 2x.

$\boxed{m = -2}$



$m = 2 = \frac{v}{u}$

$v = 2u$

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

$$\frac{1}{f} = \frac{1}{2u} - \frac{1}{u}.$$

$$\frac{1}{f} = \frac{-1}{2u}$$

$$\boxed{u = -f/2} = x_1$$

$m = -2 = \frac{v}{u}.$

$v = -2u$

$$\frac{1}{f} = \frac{1}{-2u} - \frac{1}{u}.$$

$$-\frac{1}{f} = \frac{3}{2u}$$

$$\boxed{u = -\frac{3f}{2}} = x_2$$

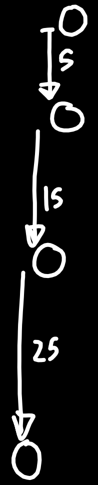
$$\boxed{\frac{x_2}{x_1} = \frac{3}{1}}$$

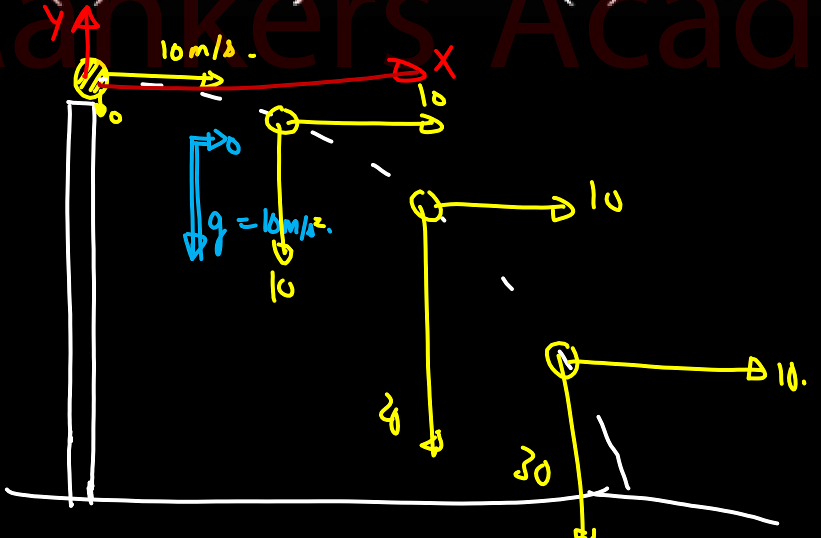
14

A body is projected horizontally from a height of $\boxed{78.4 \text{ m}}$ with a velocity 10 ms^{-1}. Its velocity after 3 seconds is $(g = 10 \text{ms}^{-2})$ (Take direction of projection as $\hat{i}$ and vertically upward direction as $\hat{j}$)

(A) $10\hat{i} - 30\hat{j}$ (B) $10\hat{i} + 30\hat{j}$

(C) $20\hat{i} - 30\hat{j}$ (D) $10\hat{i} + 10\sqrt{3}\hat{j}$



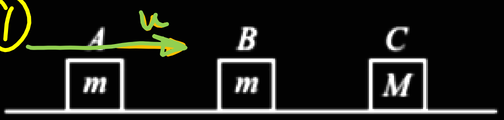


X	Y
$U_x = 10$	$U_y = 0$
$a_x = 0$	$a_y = -10$
$V_x = U_x + a_x t$	$V_y = U_y + a_y t$
$\boxed{V_x = 10}$	$\boxed{V_y = 0 - 10t}$

$$\boxed{\vec{V} = 10\hat{i} + (-10t)\hat{j}}$$

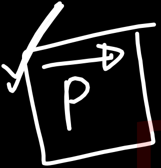
15

Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed ~~v~~ u towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$ of the initial kinetic energy is lost in whole process. What is value of M/m ?



(A) 5 (B) 2

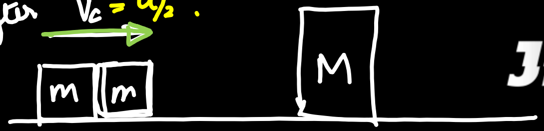
(C) 3 (D) 4



$e=0$

$KE_i = \frac{1}{2}mu^2$

2 after $V_c = u/2$

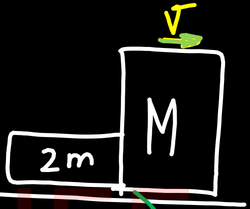


$\vec{P_1} = \vec{P_2}$

$\cancel{m}u = (2\cancel{m})V_c$

$V_c = u/2$.

3



$P_1 = P_2 = P_3$

$mu = (2m+M)v$

$$\boxed{v = \frac{mu}{2m+M}}$$

$KE_f = \frac{1}{6}KE_i$

$\frac{1}{\cancel{2}}(2m+M)v^2 = \frac{1}{\cancel{2}\,6}mu^2$

$$\frac{\cancel{(2m+M)}\,m^2\cancel{u^2}}{(2m+M)^{\cancel{2}}} = \frac{\cancel{m}\cancel{u^2}}{6}$$

$6m = 2m+M$

$$\boxed{M = 4m}$$

16

A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is

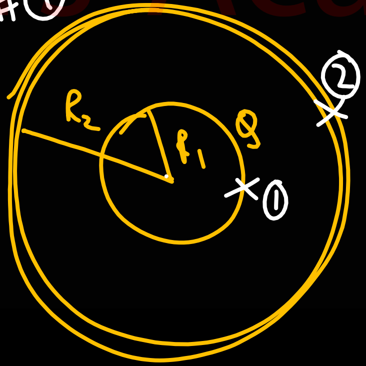
(A) 4 V

(B) V

(C) 2 V

(D) -2 V

M#1

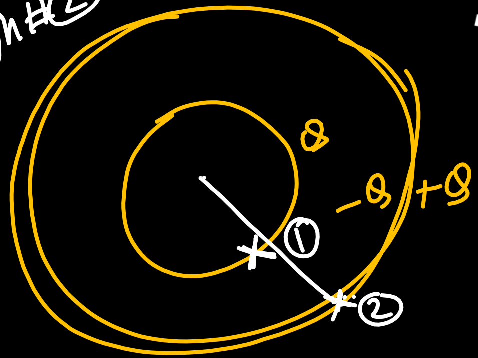


$$V_1 = \frac{kQ}{R_1}$$

$$V_2 = \frac{kQ}{R_2}$$

$$V = V_1 - V_2 = \frac{kQ}{R_1} - \frac{kQ}{R_2} \quad \text{—(1)}$$

M#2



$$V_2 = \frac{kQ}{R_2}$$

$$V_1 = \frac{kQ}{R_1} + \cancel{\frac{k(-Q)}{R_2}} + \cancel{\frac{kQ}{R_2}}$$

$$V = V_1 - V_2 = \frac{kQ}{R_1} - \frac{kQ}{R_2} \quad \text{—(1)}$$

16

JEE 1

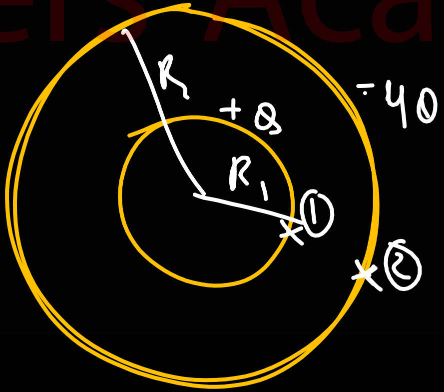
A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is

(A) 4 V

(B) V

(C) 2 V

(D) -2 V



$$V_1' = \frac{kQ}{R_1} + \frac{k(-4Q)}{R_2}$$

$$V_2' = \frac{kQ}{R_2} + \frac{k(-4Q)}{R_2}$$

$$V' = V_1' - V_2'$$

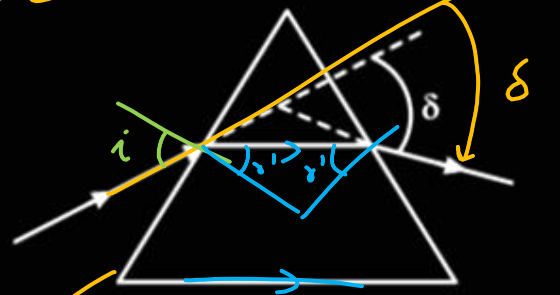
$$= \frac{kQ}{R_1} - \frac{kQ}{R_2}$$

Same as Eq^n 1

17

JEE1

The angle of deviation through a prism is minimum when



(A) incident ray and emergent ray are symmetric to the prism $i = i'$

(B) the refracted ray inside the prism becomes parallel to its base $r = r'$

(C) angle of incidence is equal to that of the angle of emergence $i = i'$

(D) ~~when angle of emergence is double the angle of incidence.~~

(A) Only statements (A) and (B) are true.

(B) Only Statements (B) and (C) are true.

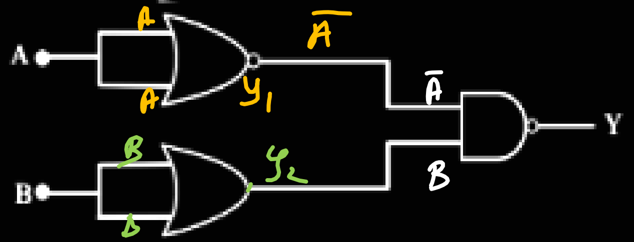
(C) Statements (A), (B) and (C) are true.

(D) Only statement (D) is true.

18

JEE 1

For the following circuit and given inputs A and B, choose the correct option for output ' Y '



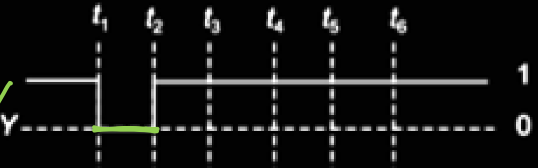
$$y_1 = \overline{A + A}$$
$$= \bar{A} \cdot \bar{A}$$
$$= \bar{A} \quad (1)$$

$$y_2 = B + B$$
$$= B \quad (2)$$

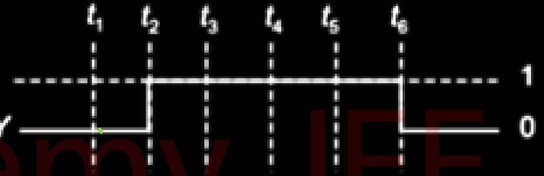
$$y = \overline{\bar{A} \cdot B} = \bar{\bar{A}} + \bar{B}$$

$$\boxed{y = A + \bar{B}}$$

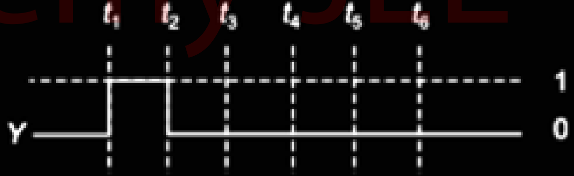
(A)



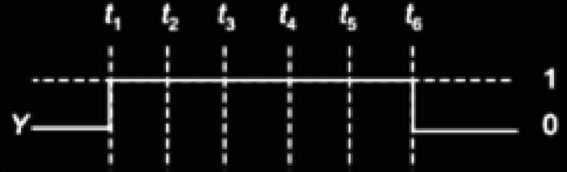
(B)



(C)



(D)



19

JEE 1

A particle moving in the xy plane experiences a velocity dependent force $\boxed{\vec{F} = k(v_y\hat{i} + v_x\hat{j})}$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If a is the acceleration of the particle, then which of the following statements is true for the particle?

(A) Quantity $\vec{v} \times \vec{a}$ is constant in time.

(B) $\vec{F}$ arises due to magnetic field.

(C) Kinetic energy of particle is constant in time.

(D) Quantity $\vec{v} \cdot \vec{a}$ is constant in time.

Sol^n

$$\vec{a} = \frac{\vec{F}}{m} = \frac{k}{m}(v_y\hat{i} + v_x\hat{j}) \quad \text{— (1)}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad \text{— (2)}$$

$$a_x = \frac{k}{m}v_y \quad \text{— (3)} \qquad a_y = \frac{k}{m}v_x \quad \text{— (4)}$$

$$(\vec{v} \times \vec{a}) = \frac{k}{m}(v_y^2 - v_x^2)\hat{k}$$

$$\frac{d(\vec{v} \times \vec{a})}{dt} = \frac{k}{m}\left[2v_y a_y - 2v_x a_x\right]\hat{k}$$

(4) (3)

$$= \frac{k^2}{m^2}\left[2v_x v_y - 2v_x v_y\right]\hat{k}$$

$$\frac{d(\vec{v} \times \vec{a})}{dt} = 0$$

$$\Rightarrow \boxed{(\vec{v} \times \vec{a}) = \text{const}}$$

19

JEE 1

A particle moving in the xy plane experiences a velocity dependent force $\vec{F} = k(v_y\hat{i} + v_x\hat{j})$, where v_x and v_y are the x and y components of its velocity $\vec{v}$. If a is the acceleration of the particle, then which of the following statements is true for the particle?

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~~(B)~~ $\vec{F}$ arises due to magnetic field.

~~(C)~~ Kinetic energy of particle is constant in time.

~~(D)~~ Quantity $\vec{v} \cdot \vec{a}$ is constant in time.

due to $\vec{B}$ $\quad \vec{F} \cdot \vec{v} = 0$

$\vec{a} \cdot \vec{v} = 0$

Eqⁿ ① & ② $\quad \vec{v} \cdot \vec{a} = \frac{k}{m}(2v_x v_y) \neq 0 \Rightarrow$ option B & D are incorrect.

$K = \frac{1}{2}m(v_x^2 + v_y^2)$

$\frac{dK}{dt} = \frac{m}{2}[2v_x a_x + 2v_y a_y]$

By Eqⁿ ③ & ④

$\frac{dK}{dt} = \frac{k}{2}[2v_x v_y + 2v_x v_y]$

$\frac{dK}{dt} = 2k v_x v_y \neq 0$

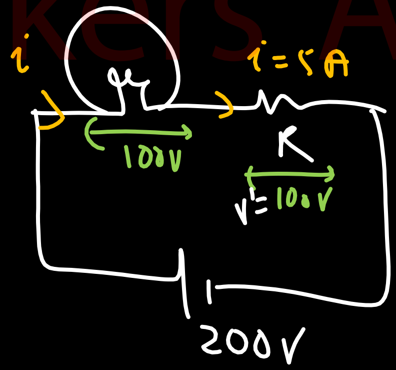
$K \neq$ const option C is incorrect

20

An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

(A) 30Ω (B) 5Ω

(C) 20Ω (D) 10Ω



$P = Vi$

$500W = 100\ i$

$i = 5A$

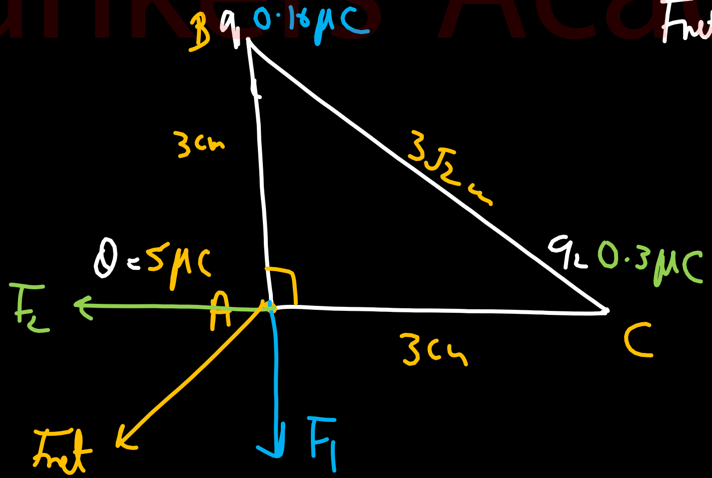
$V' = iR$

$100 = 5R$

$R = 20\Omega$

21

Three point charges of magnitude 5μC, 0.16μC and 0.3 μC are located at the vertices A, B, C of a right angled triangle whose sides are AB = 3 cm, BC = $3\sqrt{2}$ cm and CA = 3 cm and point A is the right angle corner. Charge at point A experiences ______ N of electrostatic force due to the other two charges.



$$F_{net} = \sqrt{F_1^2 + F_2^2}$$

$$= \frac{kQ}{r^2}\sqrt{q_1^2 + q_2^2}$$

$$= \frac{9\times10^{9}\times5\times10^{-6}}{9\times10^{-4}}\times10^{-8}\sqrt{16^2+30^2}$$

$$= 5\times10^{-1}\times\sqrt{1156}$$

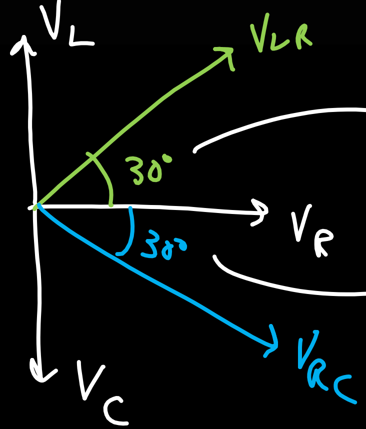
$$= 0.5\times34$$

$$= 17 \text{ Ans}$$

22

JEE 1

In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is ____ W



$\boxed{V_L = V_C}$

$\boxed{Z = R}$

$$P = \frac{V_{rms}^2}{R} = \frac{(220)^2}{200} = \frac{484}{2} = 242 \text{ Watt}$$

23

Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is _____ °.

$$\frac{I}{8} = \left(\frac{I}{2}\cos^2\theta\right)\cos^2(90° - \theta)$$

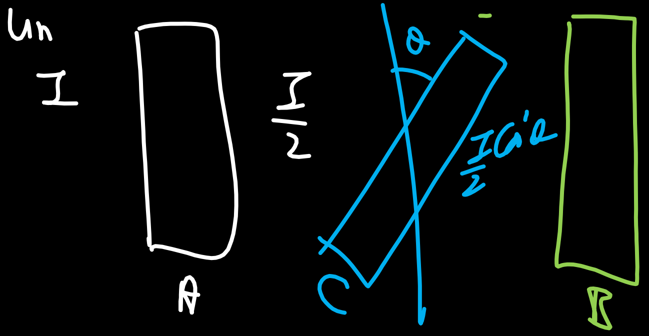
$$\frac{1}{4} = \sin^2\theta\cos^2\theta$$

$$\sin^2 2\theta = 1$$

$$\sin 2\theta = 1$$

$$2\theta = 90°$$

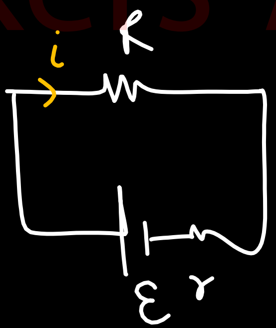
$$\boxed{\theta = 45°}$$



$I_B = \frac{I}{2}\cos^2 0° = \frac{I}{2}$

$\left(\frac{I}{2}\cos^2\theta\right)\sin^2\theta$

In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is_____



$$V = iR$$

$$\boxed{V = \frac{\varepsilon R}{(r+R)}}$$

$$1.25 = \frac{\varepsilon 5}{r+5} \quad \text{— (1)}$$

$$1 = \frac{\varepsilon 2}{r+2} \quad \text{— (2)}$$

$$\frac{5}{4} = \frac{5(r+2)}{2(r+5)}$$

$$r+5 = 2r+4$$

$$r = 1\Omega$$

$$\varepsilon = \frac{3}{2} = 1.5 = \boxed{\frac{15}{10}}$$

25

JEE 1

From the given data, the amount of energy required to break the nucleus of aluminium $^{27}_{13}Al$ is ______ $x \times 10^{-3}$ J.

Mass of neutron = 1.00866u

Mass of proton = 1.00726u

Mass of aluminium nucleus = 27.18846u

(Assume 1u corresponds to x J of energy)

(Round off to the nearest integer)

$$B.E. = (\Delta m)\,c^2$$

$$= \left(Z m_p + (A-Z) m_n - M_{Al}\right) c^2$$

$$= \left(13 \times 1.00726 + 14 \times 1.00866 - 27.18846\right)$$

$$(0.002716) \times c^2$$

$$u \rightarrow J$$

$$0.002716 \times x \text{ J}$$

$$\boxed{27}.16 \times 10^{-3} \times x \text{ J}$$

CHEMISTRY

JEE 1

1

The equilibrium concentrations of A, B and C for the reaction $A \rightleftharpoons B + C$ are 5, 2 and 2 mol/L respectively at 40°C. If 2.5 mol/L of A is removed and 1 mol/L of B is introduced into the reaction mixture, calculate the equilibrium concentration of C at the same temperature

(A) 2.8M
(B) 1.2M
(C) 1.8M
(D) 0.8M

	A	$\rightleftharpoons B$	$+ C$
$t = t_{eq}$	5 M	2 M	2 M
$t = t$	2.5 M	3 M	2 M
$t' = t_{eq}$	$(2.5+x)$ M	$(3-x)$ M	$(2-x)$ M

$K_{eq} : \frac{2 \times 2}{5} : \frac{4}{5}$

$Q = \frac{2 \times 3}{2.5} > K_{eq}$

backward

$K_{eq} = \frac{4}{5} = \frac{(2-x)(3-x)}{(2.5+x)}$

$\boxed{x = 0.8}$

$[C] = 2 - x = 2 - 0.8$

$= \underline{1.2\,M}$

Which of the following is a neutral oxide?

(A) NO

(B) NO_2

(C) N_2O_3

(D) N_2O_5

B, C, D $\longrightarrow$ Acidic

3

JEE 1

The correct IUPAC name of the compound $[Co(NH_3)_5NO_2]Cl_2$ is

(A) Pentaamminenitro cobalt (II) chloride

(B) Pentaamminenitro cobalt (III) chloride ✓

(C) Pentanitroammine cobalt (II) chloride

(D) Pentanitroammine cobalt (III) chloride

$x + 5(0) + 3(-1) = 0$

$\underline{x = +3}$

4

JEE1

Which of the following ions gives blood red colour with ammonium thiocyanate?

(A) Fe^{+3} (B) Cu^{2+}

(C) Cd^{2+} (D) Sn^{2+}

$Fe^{+3} + SCN^{-} \longrightarrow [Fe(SCN)]^{+2}$

blood red

5

JEE 1

Identify the colourless ion among the following

V^{2+}, Cr^{3+}, Zn^{2+} and Ti^{3+}

(A) V^{2+} (B) Cr^{3+}

(C) Zn^{2+} (D) Ti^{3+}

Unpaired e^- $\longrightarrow$ d-d transistion

V^{+2} : $3d^3$ $n : 3$

Cr^{+3} : $3d^3$ $n = 3$

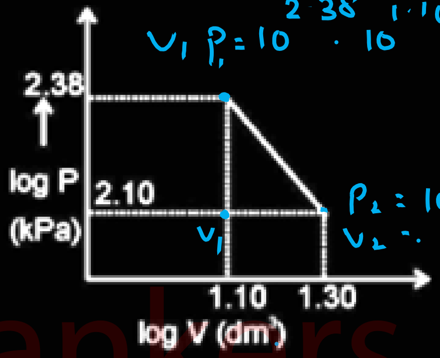
Zn^{+2} : $3d^{10}$ $\boxed{n : 0}$

Ti^{+3} : $3d^1$ $\boxed{n = 1}$

6

JEE 1

A graph between log P vs log V is plotted for an ideal gas. Which one of the following statements is true for the gas?



$V_1 P_1 = 10^{2.38} \cdot 10^{1.10} = 10^{3.48}$

$P_2 = 10^{2.10}$

$V_2 = 10^{1.30}$ ∴ $P_2 V_2 = 10^{3.30}$

(A) Monoatomic gas undergoing adiabatic change

(B) Monoatomic gas undergoing isothermal change

(C) Diatomic gas undergoing adiabatic change ✓

(D) Triatomic gas undergoing isothermal change

$PV = \text{Const}$ (Isothermal) ×

$PV^{\gamma} = \text{Const}$ (Adiabatic)

$\log P + \gamma \log V = \log k$

$2.38 + \gamma(1.10) = \log k$ — (1)

$2.10 + (1.30)\gamma = \log k$ — (2)

(1) − (2)

$\boxed{\gamma = \frac{7}{5}}$ → diatomic

7

Which of the following statement is correct?

(A) Gluconic acid is a dicarboxylic acid

(B) Gluconic acid is a partial oxidation product of glucose

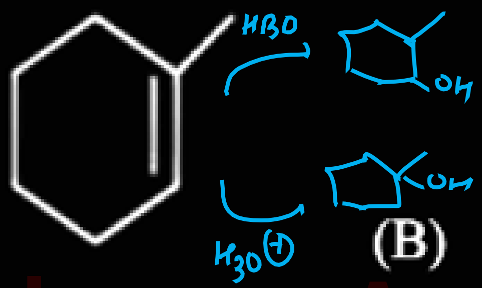
(C) Gluconic acid can form cyclic (acetal/hemiacetal) structure

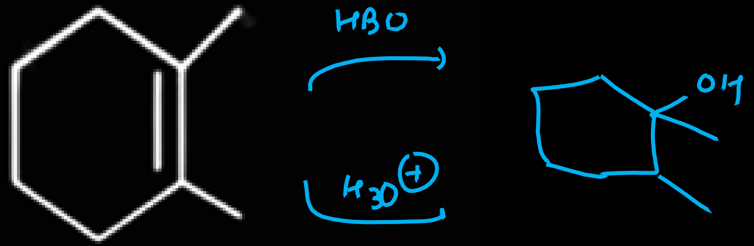
(D) Gluconic acid is obtained by oxidation of glucose with HNO_3

8

JEE 1

Hydroboration-oxidation and acidic hydration will not give the same products in case of

(A) 

(B) 

(C) $CH_2 = CH_2$ (D) $CH_3CH = CH - CH_3$

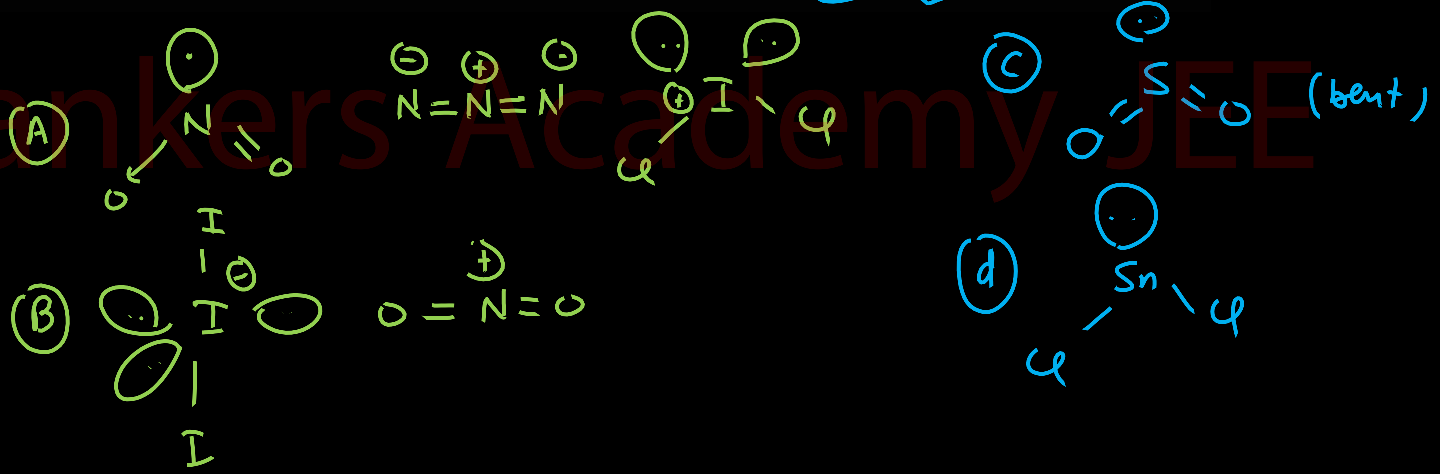
9 In which of the following groups all the members have linear shape?

(A) NO_2, N_3^-, ICl_2^+

(B) N_3^-, I_3^-, NO_2^+

(C) XeF_2, C_2H_2, SO_2

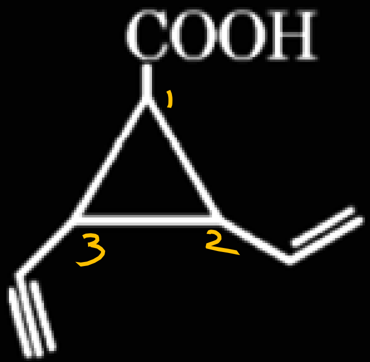
(D) CO_2, $BeCl_2$, $SnCl_2$



10

JEE 1

Correct IUPAC name of is



is

(A) 1-Ethenyl-2-ethynylcyclopropanoic acid

(B) 2-Ethenyl-3-ethynylcyclopropane carboxylic acid

(C) 2-Ethynyl-3-ethenylcyclopropane carboxylic acid

(D) 2-Ethenyl-3-ethynylcyclopropane-1-oic acid

11

The rate constant of a reaction at 300 K is half of the rate constant at 310 K. The activation energy of the reaction in $KJmol^{-1}$ is nearly [$\ln 2 = 0.7, R = 8.3 JK^{-1} mol^{-1}$]

(A) 35

(B) 66

(C) 98

(D) 54

$k'(300\,K) = \frac{1}{2} k(310\,K)$

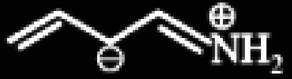
$\ln\left(\frac{k}{k'}\right) = \frac{E_a}{R}\left(\frac{1}{300} - \frac{1}{310}\right)$

$\ln 2 = \frac{E_a}{8.3}\left(\frac{1}{300 \times 31}\right)$

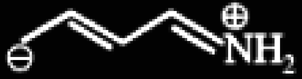
$\therefore E_a \simeq 54\,kJ\,mol^{-1}$

12

(I) neutral



(II)



(III)

(IV)

(V)

Stability order of given resonating structures of molecule is

(A) I > II > III > IV > V

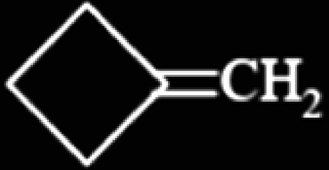
(B) I > IV > V > II > III

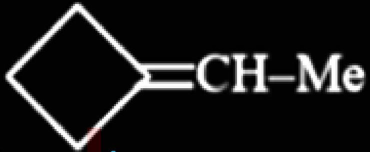
(C) I > II > III > V > IV

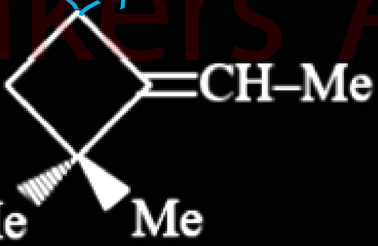
(D) V > IV > I > II > III

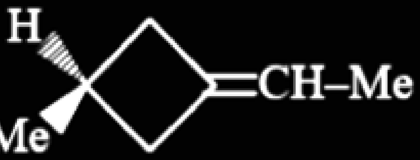
13

Which one of the following is capable of showing geometrical isomersim.

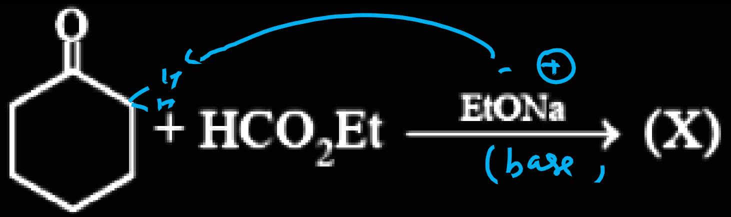
(A) 

(B) 

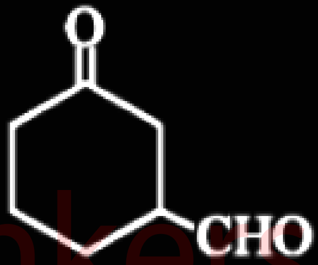
(C) 

(D) 

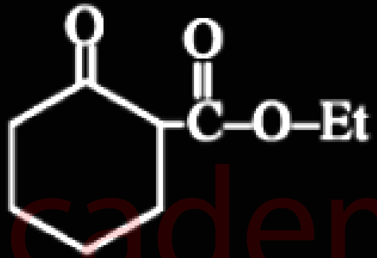
14



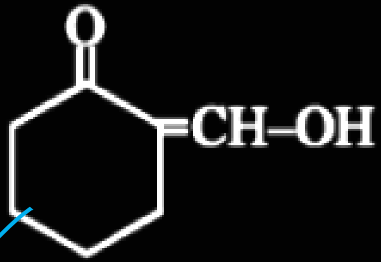
Identify unknown (X) in above reaction



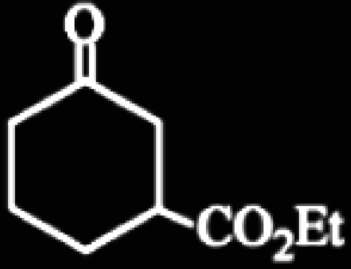
(A)



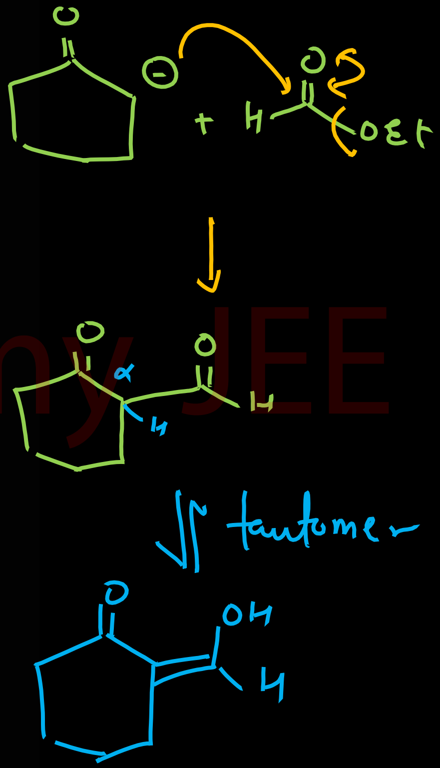
(B)



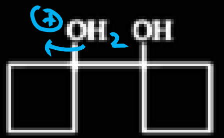
(C)



(D)

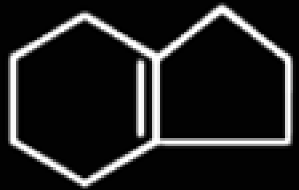


15



$\xrightarrow{H^+}$ (A) $\xrightarrow{NaBH_4}$ (B) $\xrightarrow[\Delta]{H^+}$ (C) (major)

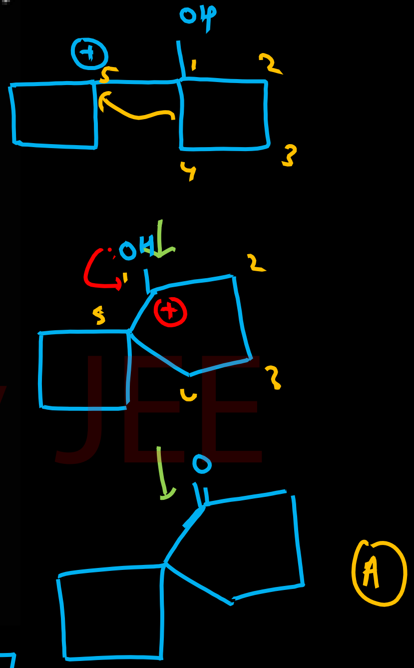
Product C is

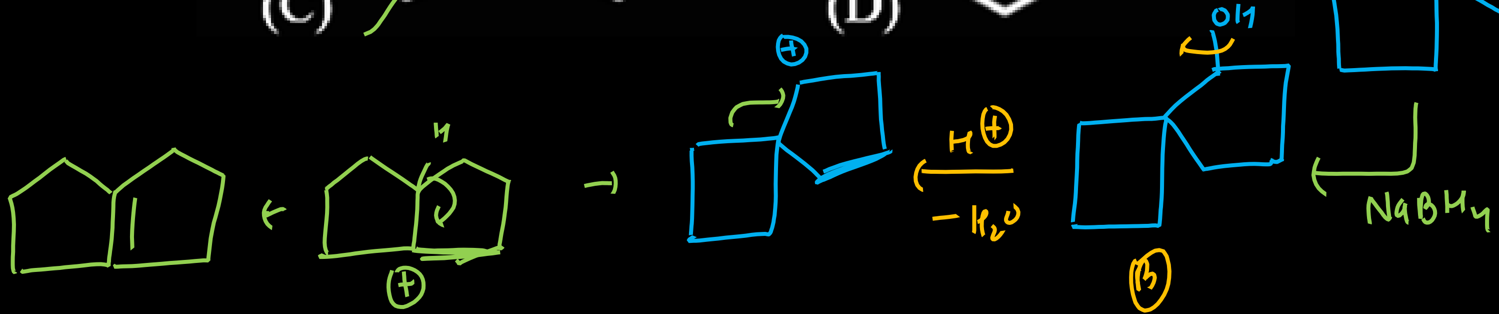
(A) 

(B) 

(C) 

(D) 





16

Two compounds I and II are eluted by column chromatography (adsorption of I > II). Which one of the following is a correct statement?

(A) II moves faster and has higher R_f value than I ✓

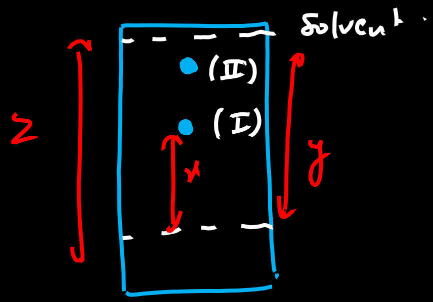
(B) I moves faster and has higher R_f value than II ×

(C) II moves slower and has higher R_f value than I

(D) I moves slower and has higher R_f value than II

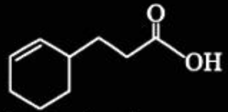
Mobility II > I

$$R_f : \frac{\text{distance travel by O.C}}{\text{distance travel by solvent}}$$



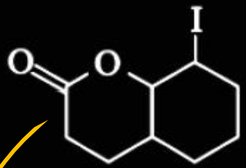
17

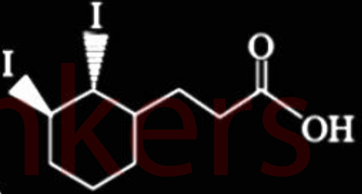
JEE 1

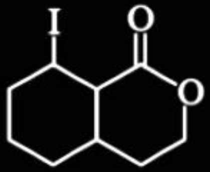


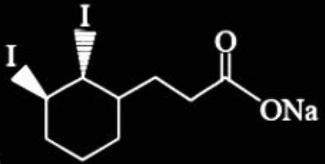
$\xrightarrow{I_2, NaHCO_3}$ Product.

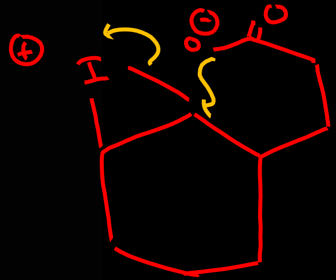
Major Product is

(A)


(B)


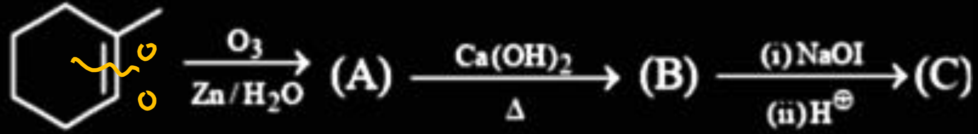
(C)


(D)




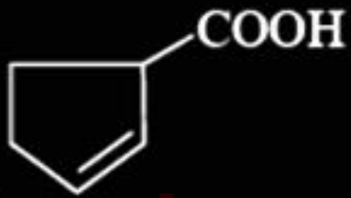
(Intramolecular)

18



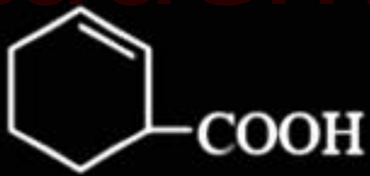
Identify product C

(A)

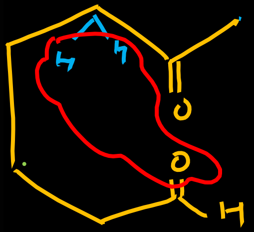


(B)

(C)



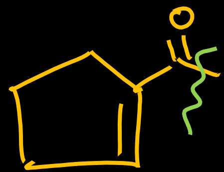
(D)



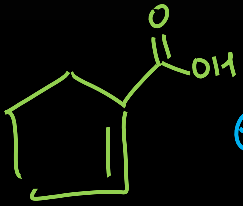
(A)

$Ca(OH)_2$ | Δ

(Aldol)



(B)



NaOI

$H^{\oplus}$

(Haloform)

19

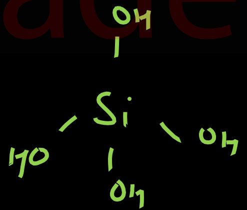
Which of the following is incorrect statement?

(A) PbF_4 is covalent in nature

(B) $SiCl_4$ is easily hydrolysed

(C) GeX_4 $(X = F, Cl, Br, I)$ is more stable than GeX_2

(D) SnF_4 is ionic in nature

(B) $SiCl_4 + H_2O \longrightarrow$ 

(C)

C
Si
Ge
Sn
Pb

+4 stability ↓

Which is the correct statement for the given acids?

(A) Phosphinic acid is a diprotic acid while phosphonic acid is a monoprotic acid

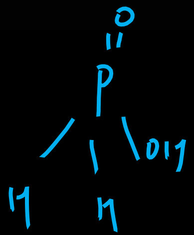
(B) Phosphinic acid is a monoprotic acid while phosphonic acid is a diprotic acid

(C) Both are triprotic acids

(D) Both are diprotic acids

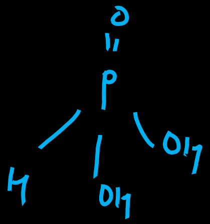
(hypophosphorous)
Phosphinic

H_3PO_2



Phosphonic (Phosphorous acid)

H_3PO_3



21

JEE 1

Two liquids P and Q form an ideal solution at 300 K, the vapour pressure of a solution of 1 mole of P and x mole of Q is 550 mm. If the vapour pressure of pure P and pure Q are 400 mm and 600 mm respectively, then x is ____

$n_P = 1$ mole $\quad X_P = \frac{1}{1+x}$

$n_Q = x$ mole $\quad X_Q = \frac{x}{1+x}$

$\therefore \boxed{x = 3}$

$$p_{total} = p_P^0 X_P + p_Q^0 X_Q$$

$$550 = 400\left(\frac{1}{1+x}\right) + 600\left(\frac{x}{1+x}\right)$$

22

JEE1

$Ca(OCl)_2 + KI \rightarrow aI_2 + bCaCl_2 + cH_2O + dKCl$

If stoichiometric coefficient of $Ca(OCl)_2$ is 1 , then the value of $(a + b + c + d)$ is (Assuming acidic medium is provided by HCl)

$Ca(OCl_2)_2 + 4KI + 4HCl \longrightarrow 2I_2 + CaCl_2 + 2H_2O + 4KCl$

23

JEE 1

Which orbit in Be^{3+} will have same radius as of 16^{th} orbit in H atom?

$$r_n \propto \frac{n^2}{Z}$$

$$\frac{n^2}{4} = \frac{(16)^2}{(1)}$$

$$n^2 = 4 \times (16)^2$$

$$n = 2 \times 16 = 32$$

24

JEE 1

On heating 4.9 g $KClO_3$, its mass is reduced by 0.384 g. Calculate the percentage of original $KClO_3$ sample, that has been decomposed.

sample

Take: K = 39, Cl = 35.5, O = 16

$$KClO_3(s) \xrightarrow{\Delta} KCl(s) + \frac{3}{2}O_2(g) \longrightarrow \text{escape.}$$

wt?

$$n_{O_2} = \frac{0.384}{32} = 1.2\times10^{-2} \text{ mole}$$

$$\frac{3}{2}\,O_2 \longrightarrow 1 \text{ mole } KClO_3$$

$$1.2\times10^{-2} \longrightarrow \frac{2}{3}\times1.2\times10^{-2} \text{ Mole } KClO_3$$

wt of pure $KClO_3$ $= \frac{2}{3}\times1.2\times10^{-2}\times122.5$

$= 0.98$ g

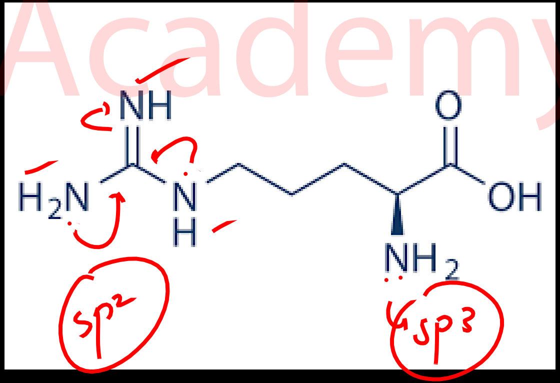
% wt $KClO_3$ $= \frac{0.98}{4.9}\times100$

$= \frac{98}{49}\times10 = 20\%$

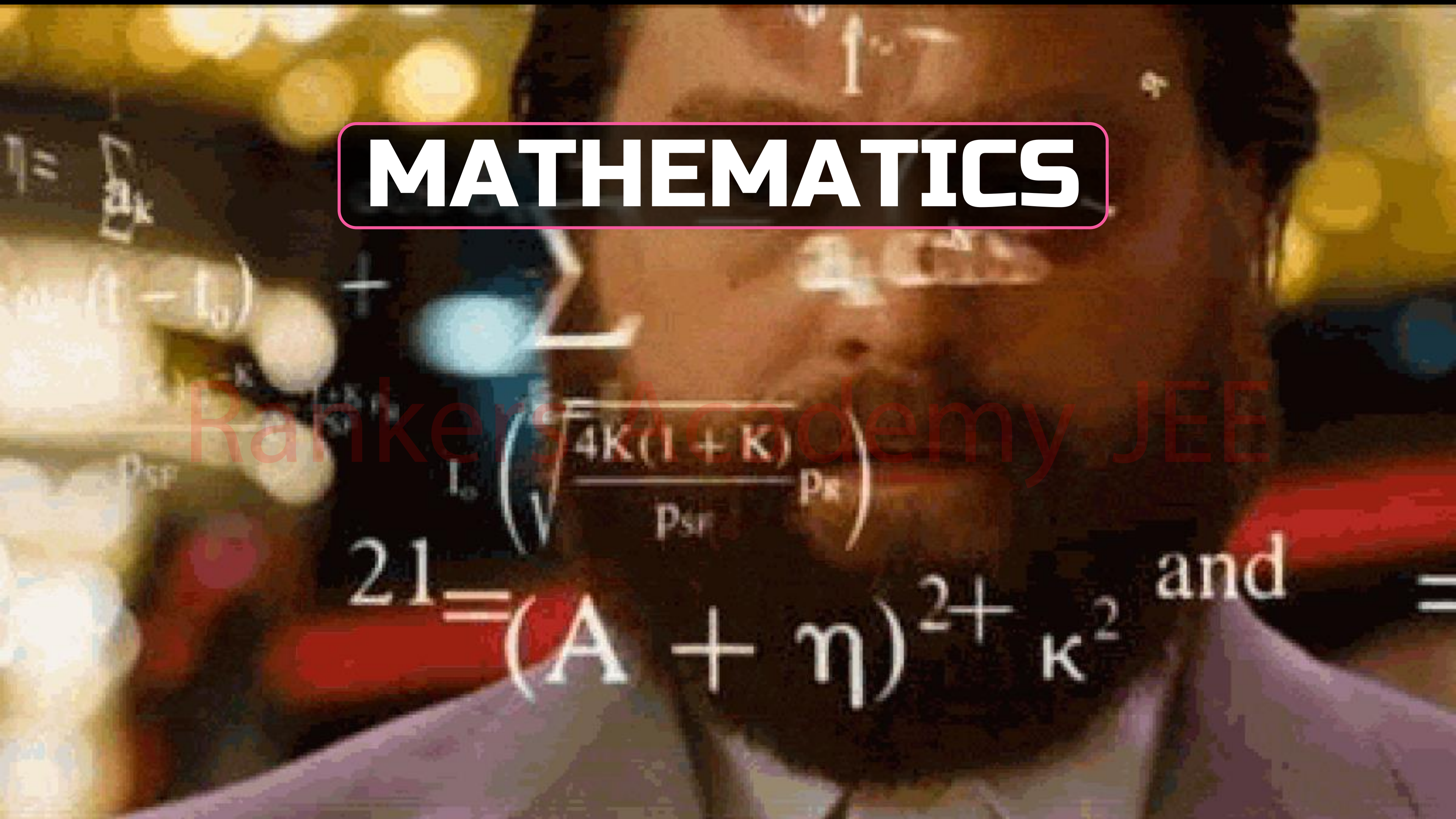
25

Arginine is nonessential amino acid which is present is human body. How many Nitrogen atoms are present in sp^2 hybrid state in Arginine.

3



MATHEMATICS



If $\lim_{x\to 0} \frac{Axe^x + B\log_e(1-x) + Cx^2e^{-x}}{x\tan^2(x/\sqrt{2})} = 2024$, then the value of $A + B + \frac{4C}{3}$ is

(A) 506
(B) 1012
(C) 2024
(D) 1518

$$\Rightarrow \frac{Ax\left(1+\frac{x}{1!}+\frac{x^2}{2!}\dots\right) + B\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\dots\right) + Cx^2\left(1-\frac{x}{1!}\dots\right)}{x\left(\frac{x}{\sqrt{2}}\right)^2} = 2024$$

$$\Rightarrow \cancel{2}\ \frac{(A-B)x + x^2\left(A-\frac{B}{2}+C\right) + x^3\left(\frac{A}{2}-\frac{B}{3}-C\right)}{x^3} = \cancel{2024}\ 1012$$

$A-B=0$

$A-\frac{B}{2}+C=0$

$\frac{A}{2}-\frac{B}{3}-C = 1012$

$A = B = 1518$

$C = -759.$

2

Let $J(x) = \int e^{\tan^2 x}\left(\frac{2\tan^3 x}{1+\tan^2 x}\right)dx$, $J(0) = 1$, then $J\left(\frac{\pi}{4}\right)$ is

(A) e

(B) $\frac{e}{2}$ ✓

(C) $\frac{e}{\sqrt{2}}$

(D) −e

JEE 1

$$J(x) = \int e^{\tan^2 x}\left(\frac{2\tan^3 x}{\sec^2 x}\right)dx$$

$$= \int e^{\underbrace{\tan^2 x}_{=t}}\left(\frac{\cancel{2}\sin^3 x}{\cos x}\right) dx \cdot 2\tan x\sec^2 x$$

$$\cancel{2}\tan x\sec^2 x$$

Let $\tan^2 x = t$

$2\tan x\sec^2 x\, dx = dt$

$$= \int e^t \frac{\sin^{\cancel{3}\,2} x}{\cancel{\cos x}}\, \frac{\cos^{\cancel{3}\,2} x}{\cancel{\sin x}}\, dt$$

$$= \frac{1}{4}\int e^t (\sin 2x)^2 dt$$

$$\left(\frac{2\tan x}{1+\tan^2 x}\right)^2$$

$$= \frac{1}{\cancel{4}}\int e^t \frac{\cancel{4}t}{(1+t)^2}\, dt$$

$$= \int e^t \frac{t}{(t+1)^2}\, dt \checkmark$$

$$= \int e^t \left(\frac{t+1-1}{(t+1)^2} \right) dt$$

$$= \int e^t \left(\frac{1}{t+1} - \frac{1}{(t+1)^2} \right) dt$$

$$= \frac{e^t}{t+1} + c$$

$$= \frac{e^{\tan^2 x}}{\tan^2 + 1} + c$$

$$J(x) = e^{\tan^2 x} \cos^2 x + c$$

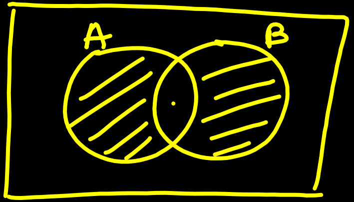
$$1 = e^0 (1) + c \Rightarrow c = 0$$

$$\boxed{J = \frac{e}{2}}$$

Let A be the set consisting of the first 2024 terms of arithmetic progression 1,5,9,13, ... and B be the set consisting of the first 2024 terms of the arithmetic progression 3,8,13,18, ... Then the number of elements which belongs to exactly one of the set A or B is/are

(A) 4048 (B) 3643
(C) 3238 (D) 3600

$\longrightarrow a_{2024} = 1 + (2023)(4)$
$= 1 + 8092$
$= 8093$



✓ $n(A) + n(B) - 2n(A \cap B)$

✓ $(A \cup B) - (A \cap B)$

A: $d_1 = 4$
B: $d_2 = 5$ $\Big\} \longrightarrow$ Common AP: $a = 13$

$d = \mathrm{LCM}(d_1, d_2) = 20$

$a_n \leq 8093$

$13 + (n-1)(20) \leq 8093$

$(n-1)\cancel{20} \leq \cancel{8080}\ 404$

$n \leq 405.$

$n(A) = 2024$

$n(B) = 2024$

Ans: $2024 + 2024 - 2(405)$
$= 4048 - 810$
$= 3238$

4

JEE 1

Let g be a differentiable function satisfying $\int_0^x (x-t+1)g(t)dt = x^4 + x^2$ for all $x > 0$.

The value of $\int_0^1 \frac{12}{\underbrace{g'(x)+g(x)+10}} dx$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ ✓ (D) $\frac{\pi}{2}$

$$\Rightarrow \int_0^x x\, g(t)\,dt + \int_0^x (-t+1)g(t)\,dt = x^4 + x^2$$

$$\Rightarrow \underbrace{x \int_0^x g(t)dt}_{\text{Diff wrt } x} + \int_0^x (-t+1)g(t)\,dt = x^4 + x^2$$

$$\int_0^x g(t)dt + x\cancel{g(x)} + (\cancel{-x}+1)g(x) = 4x^3 + 2x$$

$$\Rightarrow \int_0^x g(t)\,dt + g(x) = 4x^3 + 2x$$

$$\Rightarrow \underbrace{g(x) + g'(x)} = 12x^2 + 2$$

$$\Rightarrow \int_0^1 \frac{12}{12x^2+2+10}\,dx$$

$$\Rightarrow \int_0^1 \frac{dx}{x^2+1} = \tan^{-1}x \Big|_0^1 = \pi/4.$$

5

JEE 1

If $\alpha + i\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)^{\frac{3n_1}{4}}(1-i)^{-2n_2}$, (where n_1 and n_2 are positive integer) then which of the following is FALSE (Given $i = \sqrt{-1}$)

T (A) $\alpha = 0$ if only one of n_1 and n_2 is odd

T (B) $\beta = 0$ if both n_1 and n_2 are odd

(C) $\alpha = 0$ if both n_1 and n_2 are even

T (D) $\beta = 0$ if both n_1 and n_2 are even

$$\alpha + i\beta = \left(e^{i\frac{2\pi}{3}}\right)^{\frac{3n_1}{4}}\left(\sqrt{2}\, e^{-i\pi/4}\right)^{-2n_2}$$

$$= e^{i\frac{\pi}{2}n_1} \cdot e^{i\frac{\pi n_2}{2}} (\sqrt{2})^{-2n_2}$$

$$= e^{i\frac{\pi}{2}(n_1+n_2)} \quad 2^{-n_2}$$

$$= 2^{-n_2}\left(\cos\left(\frac{\pi}{2}(n_1+n_2)\right) + i\sin\left(\frac{\pi}{2}(n_1+n_2)\right)\right)$$

(b) n_1, n_2 both odd

$n_1 = n_2 = 1$

$\alpha + i\beta = 2^{-1}(-1) =$ real

$\beta = 0.$

(a) $n_1 = 1$; $n_2 = 2$

$\alpha + i\beta = 2^{-2}(\quad + i(-1))$

$\alpha = 0.$

JEE 1

5

both even

Case 3 $\quad n_1 = n_2 = 2$

$$\alpha + i\beta = 2^{-2}\left(\cos\left(\frac{\pi}{2}\cdot 4\right) + i\sin\left(\frac{\pi}{2}\cdot 4\right)\right)$$

$$= 2^{-2}(1+0)$$

$$\boxed{\beta = 0}$$

6

If $A = \begin{bmatrix} x & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$; $\left(x \neq \frac{-11}{3}\right)$ and $\det(\operatorname{adj}(\operatorname{adj} A)) = (14)^4$. Then the sum of possible value(s) of x is

(A) 2

(B) -2

(C) 0

(D) $-\frac{22}{3}$ ✓

$|\operatorname{adj}(\operatorname{adj} A)| = 14^4$

$|A|^{(n-1)^2}$

$|A|^{(3-1)^2} = 14^4$

$\boxed{|A| = \pm 14}$

$|A| = 3x + 11$

$3x + 11 = 14$

$x = 1$

$3x + 11 = -14$

$x = -\frac{25}{3}$.

$-\frac{22}{3}$

7

JEE 1

$$\frac{2(\sin 1^\circ+\sin 2^\circ+\sin 3^\circ+\cdots+\sin 89^\circ)}{2(\cos 1^\circ+\cos 2^\circ+\cdots+\cos 44^\circ)+1} \text{ equals :-}$$

(A) $\sqrt{2}$ ✓
(B) $1/\sqrt{2}$
(C) ½
(D) 0

N^r: $2\left[(\sin 1^\circ+\sin 89^\circ)+(\sin 2^\circ+\sin 88^\circ)+(\sin 3^\circ+\sin 87^\circ)+\cdots\overset{(\sin 44^\circ+\sin 46^\circ)}{}+\underline{\underline{\sin 45^\circ}}\right]$

$=2\left[2\sin 45^\circ\cos 44^\circ+2\sin 45^\circ\cos 43^\circ+2\sin 45^\circ\cos 42^\circ+\cdots+2\sin 45\cos 1^\circ+\sin 45^\circ\right]$

$N^r=\underline{\underline{2\cdot\sin 45^\circ}}\left[2(\cos 44^\circ+\cos 43^\circ+\cdots+\cos 1^\circ)+1\right]$

$N^r=\sqrt{2}\ D^r$

$\frac{N^r}{D^r}=\sqrt{2}.$

8

If middle term in the expansion to $(x^{1/2}+x^k)^{16}$ is also independent term to x, then second term is :-

(A) $120x^6$ (B) $120x^5$

(C) $16x^7$ (D) $16x^5$

Middle term:

$T_9 = T_{8+1} = {}^{16}C_8 (x^{1/2})^8 (x^k)^8$

$= {}^{16}C_8 \; x^{4+8k} = 0$

$4+8k=0$

$k=-1/2$.

$(x^{1/2}+x^{-1/2})^{16}$

$T_2 = T_{1+1} = {}^{16}C_1 (x^{1/2})^{15} (x^{-1/2})^1$

$= 16 (x^7)$

9

The sum upto infinite terms of the series

$1^2+\frac{3^2}{2}+\frac{5^2}{2^2}+\frac{7^2}{2^3}+\cdots\infty$ is equal to

(A) 17

(B) 34

(C) 2

(D) 31

$$S = 1^2 + \frac{3^2}{2} + \frac{5^2}{2^2} + \frac{7^2}{2^3} + \frac{9^2}{2^4} - - -$$

$$\frac{S}{2} = \frac{1^2}{2} + \frac{3^2}{2^2} + \frac{5^2}{2^3} + \frac{7^2}{2^4} - -$$

$$\frac{S}{2} = 1^2 + \frac{3^2-1^2}{2} + \frac{5^2-3^2}{2^2} + \frac{7^2-5^2}{2^3} + \frac{9^2-7^2}{2^4} - -$$

$$\frac{S}{2} = 1 + \frac{\cancel{2}.4}{\cancel{2}} + \frac{8.\cancel{2}}{2^{\cancel{2}}} + \frac{12.\cancel{2}}{2^{\cancel{3}2}} + \frac{16}{2^3} -$$

JEE 1

9

$$\frac{S}{2} = 1 + \underbrace{\left[4 + \frac{8}{2} + \frac{12}{2^2} + \frac{16}{2^3} + \cdots\right]}_{S_1}$$

Ans: $\frac{S}{2} = 1 + S_1 \Rightarrow S = 2(1 + S_1)$ ✓

$$S_1 = 4 + \frac{8}{2} + \frac{12}{2^2} + \frac{16}{2^3} + \cdots$$

$$\frac{S_1}{2} = \frac{4}{2} + \frac{8}{2^2} + \frac{12}{2^3} + \cdots$$

$$\frac{S_1}{2} = 4 + \frac{4}{2} + \frac{4}{2^2} + \frac{4}{2^3} + \cdots = \frac{4}{1 - 1/2} = 8$$

$\frac{S_1}{2} = 8 \Rightarrow S_1 = 16$

Ans: $2(1 + S_1) = 34$

10

JEE 1

Given that $I_1 = \int_0^{\pi/2} \cos(\pi \sin x)dx$,

$I_2 = \int_0^{\pi/2} \cos(2\pi \sin^2 x)dx$

$I_3 = \int_0^{\pi/2} \cos(\pi \sin^2 x)dx$, then which of the following is $\boxed{\text{Incorrect}}$

(A) $I_1 \cdot I_3 \neq 0 \rightarrow = 0$

(B) $I_3 = 0$

(C) $I_1 + I_2 = 0$

(D) More than one option of above

$I_3 = \int_0^{\pi/2} \cos(\pi \sin^2 x)dx$

$x \to 0 + \pi/2 - x$

$I_3 = \int_0^{\pi/2} \cos(\pi \cos^2 x)dx = \int_0^{\pi/2} \cos(\pi(1-\sin^2 x))dx$

$= -\int \cos(\sin^2 x)dx = -I_3$

$\therefore I_3 = 0$

10

JEE 1

$$I_1 = \int_0^{\pi/2} \cos(\pi \sin x)\,dx$$

$$I_2 = \int_0^{\pi/2} \cos(2\pi \sin^2 x)\,dx$$

$$x \to \pi/2 - x$$

$$I_2 = \int_0^{\pi/2} \cos(2\pi \cos^2 x)\,dx$$

$$\cancel{2} I_2 = \cancel{2}\int_0^{\pi/2} \cos\left(\cancel{2}\pi\frac{(\sin^2 x + \cos^2 x)}{\cancel{2}}\right) \cdot \cos\left(\pi \cancel{2}\frac{(\sin^2 x - \cos^2 x)}{\cancel{2}}\right) dx$$

$$I_2 = -\int_0^{\pi/2} \cos(\pi \cos 2x)\,dx$$

$$I_2 = -\int_0^{\pi/2} \cos(\pi(\cos 2x))\,dx$$

Let $2x = t$

$$I_2 = -\frac{1}{2}\int_0^{\pi} \cos(\pi \cos t)\,dt$$

$$= -\frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \int_0^{\pi/2} \cos(\pi \cos t)\,dt$$

$$= -\int_0^{\pi/2} \cos(\pi \cos t)\,dt$$

$$= -I_1$$

11

JEE 1

If the system of equations $x - y - z = 0$, $x - y + \sin\theta z = 0$ and $x + \sin\theta y - z = 0$ have non-zero solution (x, y, z) then number of values of θ in $[0, 2023\pi]$ are

(A) 0 (B) 2022

(C) 1010 (D) 1011

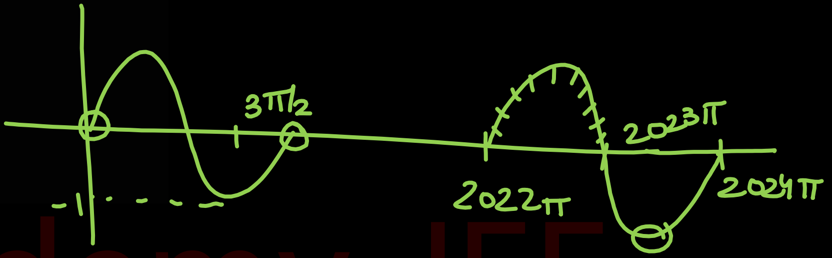
$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & -1 & \sin\theta \\ 1 & \sin\theta & -1 \end{vmatrix} = 0$$

$$\Rightarrow \cancel{1} - \sin^2\theta + (-\cancel{1} - \sin\theta) - (\sin\theta + 1) = 0$$

$$\Rightarrow -1 - \sin^2\theta - 2\sin\theta = 0$$

$$\Rightarrow (\sin\theta + 1)^2 = 0 \Rightarrow \sin\theta = -1$$



$[0, 2\pi] \rightarrow 1$ sol

$[0, 4\pi] \rightarrow 2$

$[0, 2022\pi] \rightarrow 1011$ sol.

$[2022\pi, 2023\pi] \rightarrow 0$ sol

$\rightarrow 1011$

12

If coefficient of t^8 in the expansion of $(2+9t^3+6t^4+t^5)^{10}$ is $5^a 2^b 3^c (a, b, c \in N)$, then value of $a+b+c$ is

(A) 15 (B) 14

(C) 20 (D) 13

JEE 1

$$\Rightarrow \left(2 + t^3(t^2+6t+9)\right)^{10}$$

$$\Rightarrow \left(2 + t^3(t+3)^2\right)^{10} \longrightarrow T_{r+1} = {}^{10}C_r\, 2^{10-r} \left(t^3(t+3)^2\right)^r$$

$r = 0, 1, 2 \ldots 10$

$$= {}^{10}C_r \cdot 2^{10-r} \left(t^{3r} (t+3)^{2r}\right)$$

$r=0 \longrightarrow \times$

$r=1 \longrightarrow (\quad) \, t^3(t+3)^2 \; \times$

$r=2 \longrightarrow {}^{10}C_2 \cdot 2^8 \; t^6 \; (t+3)^4$

${}^{10}C_2 \cdot 2^8 \; t^6$ (coeff. of t^2)

${}^{10}C_2 \cdot 2^8 \cdot t^6 \; ({}^4C_2 \; t^2 3^2)$

$^{10}C_2 \cdot 2^8 \cdot {}^4C_2 \cdot 3^2$

t_8

$5^a \cdot 2^b \cdot 3^c$

45

6

$3^2 \cdot \boxed{5} \cdot \boxed{2^8 \cdot 2} \, 3 \cdot 3^2$

$2^9 \cdot 3^5 \cdot 5^1$

13

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors satisfying $\vec{a} = \vec{b} \times \vec{c} + 3\vec{b}$ where $|\vec{b}| = |\vec{c}| = 2$ and $|\vec{a}| \leqslant 6$, then the sum of possible value(s) of $|2\vec{a} + \vec{b} + \vec{c}|$, is

(A) 28 (B) 26

(C) 30 (D) 24

JEE1

$\vec{a} = \vec{b} \times \vec{c} + 3\vec{b}$

dot with $\vec{b}$

$\vec{a}\cdot\vec{b} = 0 + 3|\vec{b}|^2 = 3(2)^2 = 12$

$|\vec{a}|(2)\cos\theta = 12$

$\cos\theta = \dfrac{6}{|a|} \geq 1$, $|a| \leq 6$

$\cos\theta = 1$; $|\vec{a}| = 6$

$\theta = 0$

$\vec{a} \parallel \vec{b}$

$\vec{a} = \lambda\vec{b}$

$|\vec{a}| = |\lambda||\vec{b}|$

$6 = |\lambda|(2)$

$|\lambda| = 3 \longrightarrow \lambda = \pm 3$

$\lambda = 3$ $\quad$ $\lambda = -3$

$\vec{a} = 3\vec{b}$

$\vec{b} \times \vec{c} = 0$

$\vec{b} = \vec{c}$

or

$\vec{b} = -\vec{c}$

or $\vec{a} = -3\vec{b}$

$-3\vec{b} = \vec{b} \times \vec{c} + 3\vec{b}$

$-6\vec{b} = \vec{b} \times \vec{c}$

$|-6||\cancel{\vec{b}}| = |\cancel{\vec{b}}||\vec{c}|\sin\theta$

$\sin\theta = 3$ X

Reject

$|2\vec{a} + \vec{b} + \vec{c}|$

$|6\vec{b} + \vec{b} + \vec{c}|$

$|7\vec{b} + \vec{c}|$

Case 1 $\quad \vec{b} = \vec{c}$

$|7\vec{b} + \vec{c}| = 8|\vec{b}| = 8(2)$
$= 16$

Case 2 $\quad \vec{b} = -\vec{c}$

$|7\vec{b} - \vec{b}| = 6|\vec{b}| = 6(2)$
$= 12$

Sum $= 16 + 12 = 28$.

14

JEE1

Consider an infinitely decreasing GP, such that its sum is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ on the interval $[-4,3]$ and the difference between the first and second terms is 3 , if r is the common ratio find 27r.

$27\left(\frac{2}{3}\right) \leftarrow$ 27r.

(A) 2

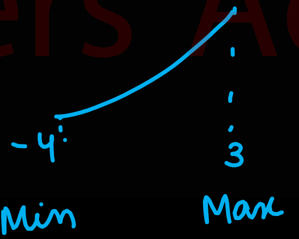
(B) 3/2

(C) 18 ✓

(D) 4

$f(x) = x^3 + 3x - 9$

$f'(x) = 3x^2 + 3 > 0$



$\text{Max} = f(3) = 3^3 + \cancel{3(3)} - \cancel{9}$

$= 27$

$S_\infty = \frac{a}{1-r}$; $|r| < 1$

$27 = \frac{a}{1-r}$ —— 1

$a - ar = 3$

$a(1-r) = 3$ —— 2

$\frac{27}{3} = \frac{\frac{a}{1-r}}{a(1-r)}$

$\rightarrow 9 = \frac{1}{(1-r)^2}$

$1 - r = \frac{1}{3}, -\frac{1}{3}$

✓ $r = 2/3$ $\quad$ $r = 4/3$ ✗

15

Greatest value of the function,

$f(x) = -1 + \frac{2}{2^{x^2}+1}$ is

(A) 1 (B) 3/2

(C) 2/3 (D) 0

$f'(x) = \frac{-2 \cdot 2^{x^2} \ln 2 \cdot 2x}{(\quad)^2}$

f' : + | −

0

Max

Ans: $f(0)$

16

JEE 1

The differential equation $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3\cos^2 y$ represents a family of curves given by the equation

(A) $x^6 + 6x^2 = C\tan y$

(B) $6x^2\tan y = x^6 + C$

(C) $\sin 2y = x^3\cos^2 y + C$

(D) None of these

$$\sec^2 y\frac{dy}{dx} + \frac{1}{x}\frac{(2\sin y\cos y)}{\cos^2 y} = \frac{x^3\cancel{\cos^2 y}}{\cancel{\cos^2 y}}$$

$$\tan y = z \Rightarrow \sec^2 y\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dz}{dx} + \frac{2}{x}(z) = x^3$$

$$I.F. = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

$$g(x^2) = \int x^3 \cdot x^2 \, dx$$

$$(\tan y)\, x^2 = \frac{x^6}{6} + C$$

17

How many six digit numbers are there in which no digit is repeated, even digits appear at even places, odd digits appear at odd places and the number is divisible by 4 ?

(A) 3600

(B) 2700

(C) 2160

(D) 1440

4 4 3 3 $\boxed{\_\ \ \_}$

O E O E O E

1 2, 16
3 2, 36
5 2, 56
7 2, 76
9 2, 96
→ 10 options

$\therefore 4 \times 4 \times 3 \times 3 \times 10$

18

If T_n denotes the nth term of the series

$2 + 3 + 6 + 11 + 18 + \cdots$, then T_{50} is

(A) $49^2 - 1$

(B) 49^2

(C) $50^2 + 1$

(D) $49^2 + 2$ ✓

JEE 1

2 3 6 11 18

1 3 5 7

2 2 2

$$2 + 1\frac{(n-1)}{1!} + \cancel{2}\frac{(n-1)(n-2)}{\cancel{2}!}$$

$$2 + (n-1) + n^2 - 3n + 2$$

$$\boxed{n^2 - 2n + 3} \longrightarrow \boxed{(n-1)^2 + 2}$$

19

JEE 1

For $x, y \in R$, let $\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1} y \in (0, \pi)$, then the sum of all the solutions of the equation

$\tan^{-1}\left(\frac{10x}{25-x^2}\right) + \cot^{-1}\left(\frac{25-x^2}{10x}\right) = \frac{\pi}{3}; 0 < |x| < 5$ is equal to

(A) $10 - 15\sqrt{3} - \frac{5}{\sqrt{3}}$

(B) $7\sqrt{10} - 15\sqrt{3} + \frac{5}{\sqrt{3}}$

(C) $10 - 5\sqrt{3} - \frac{5}{\sqrt{3}}$

(D) $10\sqrt{7} + 15\sqrt{3} - \frac{5}{\sqrt{3}}$

$\boxed{\cot^{-1} x = \tan^{-1}\frac{1}{x}}$

$\boxed{x > 0}$

$2\tan^{-1}\left(\frac{10x}{25-x^2}\right) = \frac{\pi}{3}$

$\tan^{-1}\left(\frac{10x}{25-x^2}\right) = \frac{\pi}{6}$

$\frac{10x}{25-x^2} = \frac{1}{\sqrt{3}}$

$10\sqrt{3}x = 25 - x^2$

$\boxed{x^2 + 10\sqrt{3}x - 25 = 0}$

$x = \frac{-10\sqrt{3} \pm \sqrt{300+100}}{2}$

$x = -5\sqrt{3} \pm 10$

$\therefore$ only one value

$\boxed{x = -5\sqrt{3} + 10}$

19

JEE 1

$$\boxed{x < 0}$$

$$\cot^{-1} x = \pi + \tan^{-1}\frac{1}{x}$$

$$\tan^{-1}\left(\frac{10x}{25-x^2}\right) + \pi + \tan^{-1}\left(\frac{10x}{25-x^2}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\left(\frac{10x}{25-x^2}\right) = -\frac{\pi}{3}$$

$$\frac{10x}{25-x^2} = -\sqrt{3}$$

$$10x = -25\sqrt{3} + \sqrt{3}x^2$$

$$\sqrt{3}x^2 - 10x - 25\sqrt{3} = 0$$

$$x = \frac{10 \pm \sqrt{100+300}}{2\sqrt{3}}$$

$$x = \frac{10 \pm 20}{2\sqrt{3}} \longrightarrow \boxed{x = -\frac{5}{\sqrt{3}}}$$

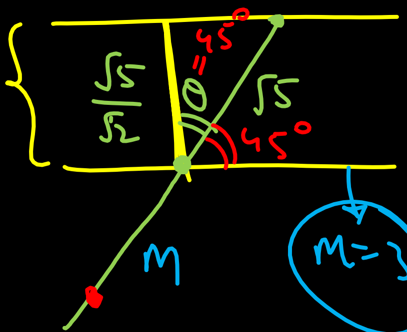
20

Let y = mx + c be a line passing through the point (3,4) and making an intercept of length $\sqrt{5}$ units between the lines y = 3x + 10 and y = 3x + 5. The sum of all possible values of m is

(A) $\frac{1}{2}$ (B) -2

(C) $\frac{5}{2}$ (D) $-\frac{3}{2}$

$$\frac{5}{\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{2}}$$



$m = -3$

$$\cos\theta = \frac{\sqrt{5}/\sqrt{2}}{\sqrt{5}} = 1/\sqrt{2}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$1 = \left|\frac{3 - m}{1 + 3m}\right|$$

(+): $1 + 3m = 3 - m$

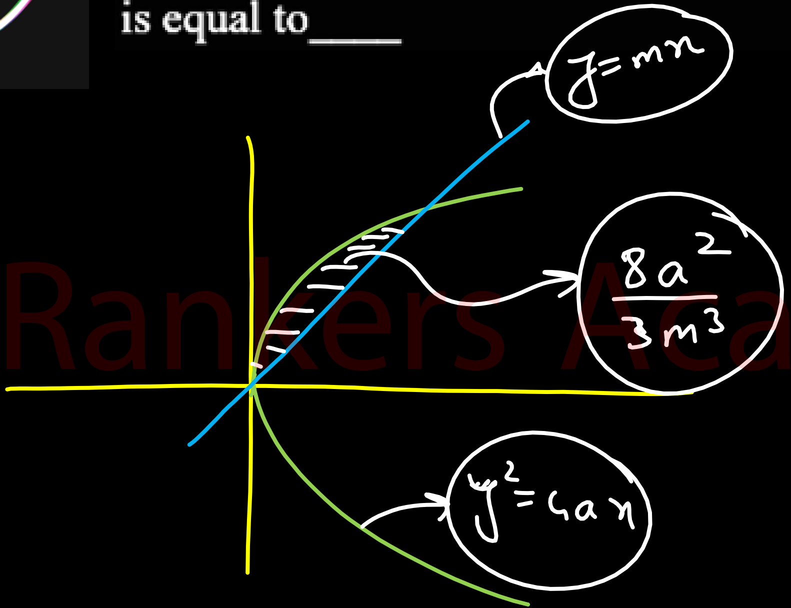
$4m = 2 \Rightarrow m = \frac{1}{2}$

(−): $-1 - 3m = 3 - m$

$-4 = 2m \Rightarrow m = -2$

21

If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x, \lambda > 0$, is $\frac{1}{9}$ then λ is equal to_____



$$\frac{8\cancel{\lambda^2}}{\cancel{3}\,\lambda^{\cancel{3}}} = \frac{1}{\cancel{9}_{3}}$$

$$\lambda = 24$$

22

If $y = \sec^{-1}\frac{\sqrt{x}-1}{x+\sqrt{x}} + \sin^{-1}\frac{x+\sqrt{x}}{\sqrt{x}-1}$ then $\frac{dy}{dx}$ is equal to :-

JEE 1

$$\underbrace{\underbrace{\cos^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)}\ +\ \underbrace{\sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)}}$$

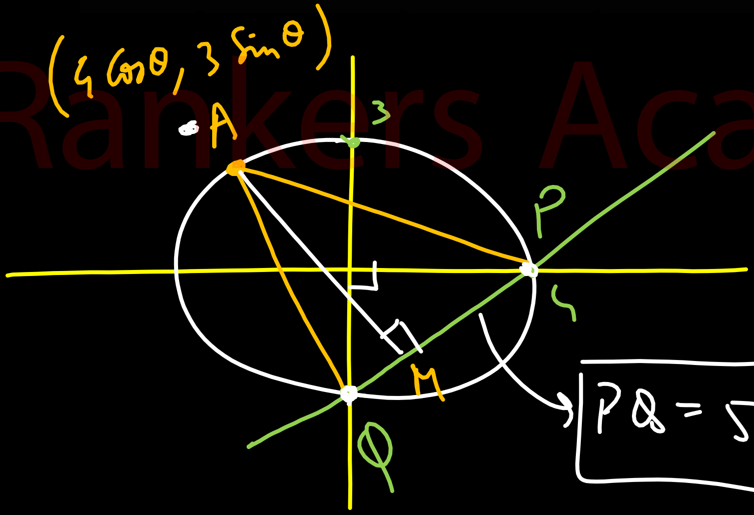
$$y = \frac{\pi}{2}$$

$$\boxed{y' = 0}$$

23

JEE 1

The straight line $\frac{x}{4} - \frac{y}{3} = 1$ intersects the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at two points P and Q. The number of points A on the ellipse such that area of Δ APQ is 15 sq units is ~~zero~~



$$\boxed{PQ = 5}$$

$$ar(\Delta APQ) = \frac{1}{2}(PQ)(h) = 15$$

$$\frac{\cancel{5}}{2}(h) = \cancel{15}\ 3$$

$$\boxed{h = 6}$$

$PQ \equiv 3x - 4y - 12 = 0$

$A \equiv (4\cos\theta, 3\sin\theta)$

$$AM = \left|\frac{12\cos\theta - 12\sin\theta - 12}{5}\right|$$

$$= \frac{12}{5}\left|\underbrace{\cos\theta - \sin\theta}_{(-\sqrt{2},\, \sqrt{2})} - 1\right|$$

$$\frac{12}{5}\left|-\sqrt{2} - 1\right|$$

$$\boxed{\frac{12(\sqrt{2}+1)}{5}}$$

max dist

24

$$\cos\left(2\tan^{-1}\frac{1}{7}\right) - \sin\left(4\tan^{-1}\frac{1}{3}\right) = \text{zero}$$

$\tan^{-1}\frac{1}{7} = \theta$

$\boxed{\tan\theta = \frac{1}{7}}$

$\cos 2\theta$

$$\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \frac{1-1/49}{1+1/49} = \boxed{\frac{48}{50}}$$

$$\sin\left(2\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)\right)$$

$$\sin\left(2\tan^{-1}\left(\frac{2/3}{1-1/9}\right)\right)$$

$$\sin\left(2\underbrace{\tan^{-1}\left(\frac{3}{4}\right)}_{A}\right)$$

$$\frac{2\tan A}{1+\tan^2 A} = \frac{2(3/4)}{1+9/16} = \frac{3}{2}\times\frac{16}{25} = \frac{48}{50}$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\frac{2}{3}\times\frac{9}{8}\,\frac{3}{4}$$

Let the mean of observations 1,3,4,7, x, y be 6 and their variance is 14 . If the mean deviation about the mean is k then the value of 60k is____

$$\overset{20}{\cancel{60}} \times \frac{10}{\cancel{3}} = \boxed{200}$$

$$6 = \frac{15 + x + y}{6}$$

$$\boxed{21 = x + y}$$

$$14 = \frac{1 + 9 + 16 + 49 + x^2 + y^2}{6} - 36$$

$$300 = 75 + x^2 + y^2$$

$$\boxed{x^2 + y^2 = 225}$$

144
81
225

$x = 12$
$y = 9$

$$MD = \frac{\sum |x_i - \bar{x}|}{6}$$

$$= \frac{(5 + 3 + 2 + 1 + 6 + 3)}{6}$$

$$= \frac{10}{3}$$

25

JEE 1