FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
PART TEST – I
PAPER –1
TEST DATE: 17-11-2024

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

Section – A *(01 – 04, 18 – 21, 35 – 38)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A *(05 –07, 22 – 24, 39 – 41):* This section contains **NINE (9)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A *(08 – 11, 25 – 28, 42 – 45):* This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B *(12 – 17, 29 – 34, 46 – 51):* This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

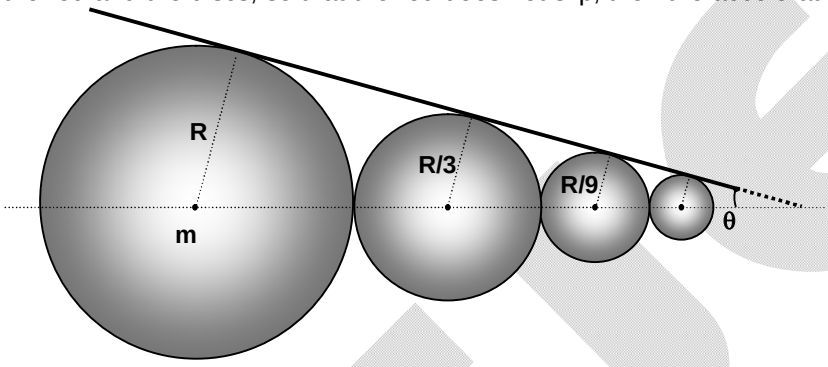
Physics

PART – I

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

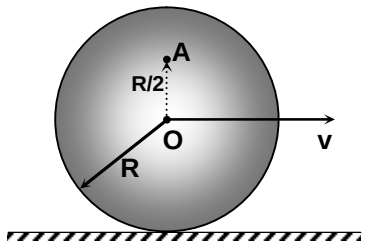
1. An infinite number of uniform discs having same thickness and made of same material are pivoted at their respective centres and arrange in such a manner that centres of all the discs are at the same horizontal level. Radius of biggest disc is R and successive discs have radii $\left(\frac{1}{3}\right)^{rd}$ of radius of disc to its left. A rod having same mass as the mass of largest disc is placed over the discs, touching all of them. There is no friction between the discs but friction is sufficient between the rod and the discs, so that the rod does not slip, then the acceleration of the rod is



(A) $\frac{8g}{25}$ (B) $\frac{3g}{25}$

(C) $\frac{g}{25}$ (D) $\frac{g}{50}$

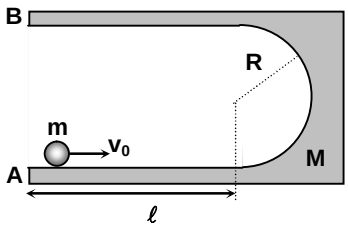
2. A sphere is rolling on the horizontal surface without any slipping at the contact point. Find the radius of curvature of the trajectory of the point A when it is at R/2 distance vertically above the centre of the sphere (as shown in the figure).



(A) $\frac{9R}{2}$ (B) $\frac{R}{2}$

(C) $\frac{3R}{2}$ (D) R

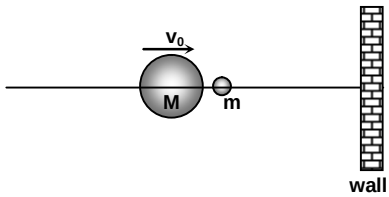
3. A particle of mass m moving with a velocity v_0 enters a stationary object of mass M (free to move) as shown in the figure. All surfaces are frictionless and the system shown in the figure is lying on a smooth horizontal plane. Time taken by the particle of mass m to move from end A to end B of the object of mass M is



(A) $\frac{2\ell+\pi R}{v_0}$ (B) $\ell n\left(\frac{\ell+R}{v_0}\right)$

(C) $\frac{\ell}{v_0}\ell n\left(\frac{\ell+\pi R}{\ell}\right)$ (D) $\frac{\pi R}{v_0}\ell n\left(\frac{\ell+\pi R}{\pi R}\right)$

4. Two beads of mass M and m can move without friction on a horizontal wire as shown in the figure. Initially M is moving with a speed v_0 and m is stationary. During subsequent motion speed of m will increase. If all collisions are perfectly elastic then maximum speed attained by m is (here M >> m)



(A) v_0 (B) $\sqrt{\frac{m}{M}}v_0$

(C) $\sqrt{\frac{M}{m}}v_0$ (D) ∞

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$, where $\vec{A}$ is a constant non zero vector and $\vec{L}$ is the angular momentum of the body about that point. Choose the correct option(s).

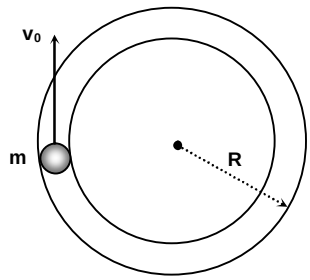
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instant of time.

(B) $\vec{L}$ does not change with time.

(C) The component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

(D) The magnitude of $\vec{L}$ does not change with time.

6. A small ball of mass m is placed in a circular tube of negligible width, mass M and radius R which is kept on a horizontal plane in a gravity free space. Friction is absent between the tube and the ball. Ball is given a velocity v_0 as shown. Then choose the correct option(s).



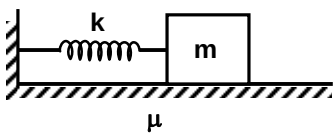
(A) The path of ball relative to centre of mass of the (ball + tube) system will be circular.

(B) The path of ball relative to centre of mass of the (ball + tube) system will be elliptical.

(C) Radius of curvature of the trajectory of the ball relative to ground at the time of projection of the ball is $\frac{MR}{m+M}$

(D) Normal force between the tube and the ball if M = 2m, at the time of projection of the ball is $\frac{2mv_0^2}{3R}$

7. A block of mass m is connected to a spring of spring constant k and kept on a rough horizontal surface having coefficient of friction μ as shown in the figure. Initially spring is compressed by a value d $\left(d > \frac{2\mu mg}{k}\right)$



(A) There can be more than one value of d for which the system can lose its entire mechanical energy.

(B) There can be only one value of d for which the system can lose its entire mechanical energy.

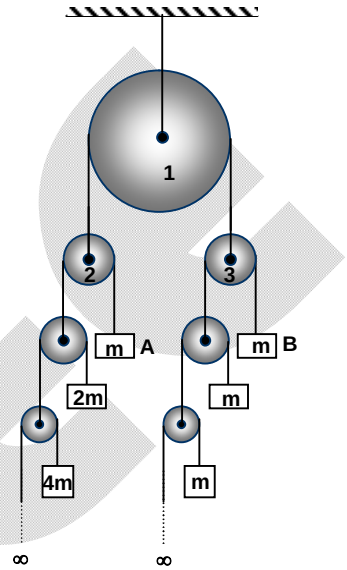
(C) The values of d for which the system can lose its entire mechanical energy are in A.P.

(D) The values of d for which the system can lose its entire mechanical energy are in G.P.

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

8. Consider the strings and pulleys system as shown in the figure. (There is no friction between the strings and pulleys)

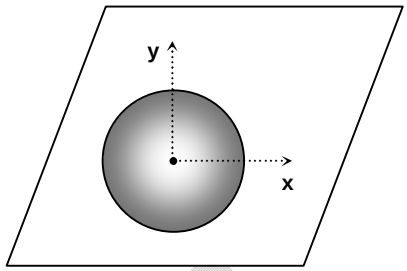


List –I		List –II	
(P)	Acceleration of block A	(1)	$\frac{g}{13}$
(Q)	Acceleration of block B	(2)	$\frac{8g}{13}$
(R)	Acceleration of pulley 2	(3)	$\frac{42mg}{13}$
(S)	Tension in the string between pulleys 1 and 2	(4)	$\frac{21mg}{13}$
		(5)	$\frac{7g}{13}$

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (2) (S) → (4)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (2) (R) → (1) (S) → (3)
(D) (P) → (2) (Q) → (5) (R) → (1) (S) → (3)

9. A uniform circular disc placed on a frictionless horizontal plank is rotating with a constant angular velocity $\vec{\omega}$ about its vertical axis. Now the plank is made to move with a constant acceleration $\vec{a}$ on a straight path. Initially the centre of the disc is at the origin of xyz space which is fixed with the plank and xy plane is on the plank. List-I gives some possible values of $\vec{a}$ and $\vec{\omega}$ and List-II gives the trajectory of instantaneous axis of rotation (ICR) of the disc relative to the plank.

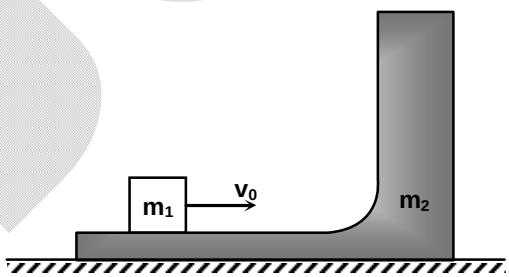


List –I		List –II	
(P)	$\vec{a} = -2\hat{j}$ m/s^2, $\vec{\omega} = 4\hat{k}$ rad/s	(1)	$y = 4$
(Q)	$\vec{a} = -2\hat{j}$ m/s^2, $\vec{\omega} = -4\hat{k}$ rad/s	(2)	$y = 4x^2$ $(x \le 0)$
(R)	$\vec{a} = -2\hat{i}$ m/s^2, $\vec{\omega} = 4\hat{k}$ rad/s	(3)	$y = 4x^2$ $(x \ge 0)$
(S)	$\vec{a} = -2\hat{i}$ m/s^2, $\vec{\omega} = -4\hat{k}$ rad/s	(4)	$x = 4y^2$ $(y \ge 0)$
		(5)	$x = 4y^2$ $(y \le 0)$

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (2) (S) → (4)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
(D) (P) → (2) (Q) → (3) (R) → (4) (S) → (5)

10. A block of mass m_1 is projected on a smooth wedge of mass m_2 with an initial horizontal velocity v_0 as shown in the figure. All the surfaces are smooth and the velocity v_0 is sufficient so that the block m_1 does not leave contact with the wedge m_2. Choose the correct statement(s).

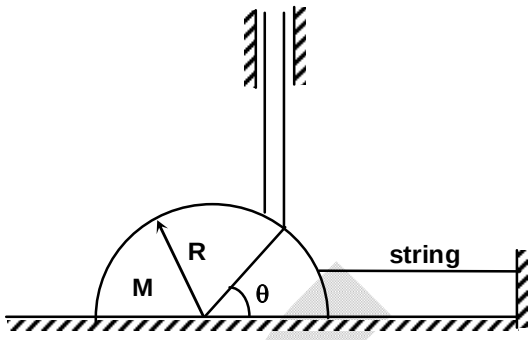


List –I		List –II	
(P)	The maximum height achieved by the block m_1 from the horizontal surface of projection is	(1)	$\frac{(m_2 - m_1)v_0}{m_1 + m_2}$
(Q)	The velocity of the block m_1 when it again comes back on the horizontal surface of the wedge m_2 is	(2)	$\frac{2m_1v_0}{m_1 + m_2}$
(R)	The velocity of the wedge m_2 when the block m_1 again comes back on the horizontal surface of the wedge m_2 is	(3)	$\frac{m_1v_0}{m_1 + m_2}$
(S)	When the block m_1 is at maximum height, its velocity is	(4)	$\frac{m_2v_0^2}{2g(m_1 + m_2)}$
		(5)	$\frac{m_1v_0^2}{2g(m_1 + m_2)}$

The correct option is:

(A) (P) → (4) (Q) → (1) (R) → (2) (S) → (3)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (4) (Q) → (1) (R) → (3) (S) → (2)
(D) (P) → (4) (Q) → (1) (R) → (1) (S) → (3)

11. A smooth hemisphere of mass M and radius R rests on a smooth horizontal ground. A vertical rod of mass 'm' is held between two smooth guide walls supported on the hemisphere as shown in the figure. A horizontal string tied to the hemisphere keeps the system at rest.



List –I		List –II	
(P)	Tension in the string just before the string cut is	(1)	$\frac{mg}{M\tan\theta + m\cot\theta}$
(Q)	Normal reaction between the rod and the hemisphere just after the string cut is	(2)	$\frac{mg\cot\theta}{M\tan\theta + m\cot\theta}$
(R)	Acceleration of the hemisphere just after the string cut is	(3)	$\frac{mMg}{\cos\theta(M\tan\theta + m\cot\theta)}$
(S)	Acceleration of the rod just after the string cut is	(4)	$mg\cot\theta$
		(5)	$Mg\cot\theta$

The correct option is:

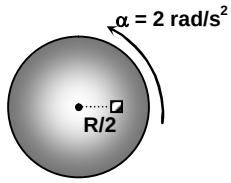
(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)
(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)
(C) (P) → (4) (Q) → (3) (R) → (2) (S) → (5)
(D) (P) → (3) (Q) → (1) (R) → (1) (S) → (3)

SECTION – B

(Numerical Answer Type)

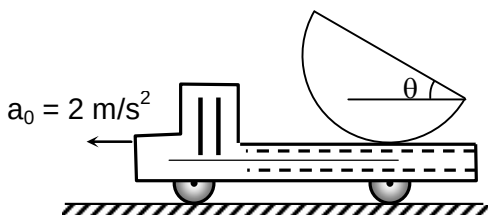
*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

12. A horizontal circular disc of radius R = 2m starts rotating with a constant angular acceleration α = 2 rad/s^2 about its vertical axis. A block of mass m = 2 kg is kept at a distance R/2 from the centre of the disc. The coefficient of friction between the disc and the block is μ_S = 0.4, μ_k = 0.3. If just after the block starts slipping, the accelerations of the block with respect to ground and with respect to disc are α m/s^2 and β m/s^2 respectively. Then find the value of $\left(\frac{\alpha}{\beta}\right)$. (g = 10 m/s^2)

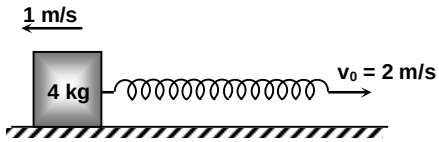


13. A uniform hollow hemisphere of radius R = 1m rests on the rough horizontal surface of a truck as shown in the figure. The truck starts moving with a constant horizontal acceleration a_0 = 2m/s^2. The friction is sufficient between the hemisphere and the horizontal surface of the truck to prevent slipping. The system is released from rest when θ = 37°. If initial angular acceleration of the hollow hemisphere is $\left(\frac{a}{b}\right)$ rad/s^2.

Find the value of (a + b).

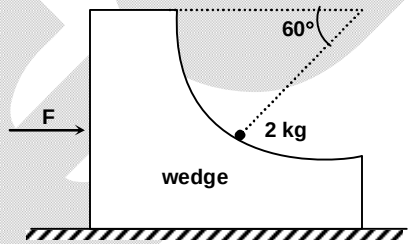


14. The spring block system lies on a smooth horizontal surface. The free end of the light spring is being pulled towards right with a constant velocity v_0 = 2 m/s. At t = 0, the spring of spring constant k = 100 N/m is unstretched and the block has a velocity 1 m/s towards left. If the maximum elongation in the spring is x cm. Find the value of x.



15. A smooth massive vertical wall is approaching a stationary man on the ground with a constant horizontal velocity u. When it is at a horizontal distance of 10 m from the man, the man throws a ball with a velocity 10 m/s at an angle 37° from horizontal which after completely elastic rebound from the wall it reaches back directly into his hands. If the velocity u of the wall is $\left(\frac{a}{b}\right)$ m/s, then find the value of (a + b).

16. A wedge is placed on a smooth horizontal surface. Wedge contains a circular quadrant of radius 25 cm as shown in the figure. An insect of mass 50 gm crawls on the circular path with a constant speed $\frac{1}{2}$ m/s relative to wedge. A horizontal force F is applied on the wedge, so that it does not move. Find the value of F in millinewton when the radial line of position of the insect makes an angle 60° with the horizontal.



17. A point moves in the xy plane according to the law x = 4 sin6t and y = 4(1 – cos6t), (where x and y are in metres and t in sec). Find the distance (in m) covered by the particle in first 4 seconds.

Chemistry

PART – II

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

18. The time period of revolution of electron in the 2^{nd} orbit of He^+ ion is x sec. The time period of revolution of electron in the 3^{rd} orbit of Li^{2+} ion is

(A) $\frac{9x}{16}$ sec (B) $\frac{8x}{27}$ sec

(C) $\frac{9x}{4}$ sec (D) $\frac{3x}{2}$ sec

19. For the reaction $2NO(g)+Cl_2(g) \rightleftharpoons 2NOCl(g)$; NO(g) and Cl_2(g) are initially taken in mole ratio 1 : 1 in an evacuated vessel of fixed volume. The total pressure at equilibrium is found to be 1 atm. The moles of NOCl(g) are one half of that of Cl_2(g) at equilibrium, the value of K_P for the reaction is:

(A) $1\,atm^{-1}$ (B) $9\ atm^{-1}$

(C) $4\ atm^{-1}$ (D) $16\ atm^{-1}$

20. Among the following, the correct statement is
(A) Sodium reacts with dinitrogen to form sodium nitride.
(B) Sodium is the least powerful reducing agent in aqueous solution among all alkali metals
(C) Potassium superoxide is a diamagnetic substance
(D) Dilute solution of sodium metal in liquid ammonia is pale yellow in colour

21. The decomposition of N_2O_5 in CCl_4 at certain temperature follows first order kinetics. If 8.4 ml of O_2 gas is evolved 10 minutes after the start of the experiment and 16.8 ml of O_2 gas is evolved after a very long time, then the volume of the O_2 gas that would evolve 20 minutes after the start will be: [All volumes are measured under similar conditions]
(A) 9.6 ml
(B) 10.4 ml
(C) 14.2 ml
(D) 12.6 ml

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

22. Among the following, the **INCORRECT** statement(s) for photoelectric effect is/are
(A) The number of photoelectrons ejected is directly proportional to the frequency of incident light.
(B) The kinetic energy of the photoelectrons is directly proportional to the wavelength of incident light
(C) There is no time lag between the striking of light beam and the ejection of electrons from the metal surface
(D) The work function of a metal is directly proportional to the frequency of incident light on metal surface

23. Among the following, the **CORRECT** statement(s) is/are
(A) 1st ionization energy of Boron is less than the 1st ionization energy of Beryllium.
(B) Electron gain enthalpy of Helium is greater than the electron gain enthalpy of Neon.
(C) The covalency of Aluminium in $\left[AlCl(H_2O)_5\right]^{2+}$ is 3.
(D) The ionic radius of F^- is bigger than the ionic radius of Na^+.

24. Among the following, the molecule/ion having square pyramidal structure is/are
(A) XeF_5^+ (B) PF_5
(C) XeF_5^- (D) BrF_5

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

25. Based on VSEPR model, match the molecule given in List-I with corresponding structure and the number of lone pair on the central atom given in List-II and choose the correct option

List – I		List – II	
(P)	$XeOF_4$	(1)	See-saw and one lone pair of electrons
(Q)	ClF_3	(2)	Pyramidal and one lone pair of electrons
(R)	SF_2Cl_2	(3)	Linear and three lone pairs of electrons
(S)	XeF_2	(4)	Square pyramidal and one lone pair of electrons
		(5)	T-shaped and two lone pairs of electrons

(A) P → 4; Q → 2; R → 4; S → 3
(B) P → 4; Q → 5; R → 1; S → 3
(C) P → 5; Q → 3; R → 4; S → 1
(D) P → 4; Q → 1; R → 2; S → 3

26. List-I contains molecules and List-II contains statements related to those molecules. Match the molecules in List-I with corresponding statement in List-II and choose the correct option.

List – I		List – II	
(P)	NF_3, SO_2, H_2S	(1)	All molecules are non polar
(Q)	$BeCl_2$, BCl_3, BeH_2	(2)	All molecules have same geometry
(R)	BF_3, CS_2, PCl_5	(3)	All molecules have same hybridization of central atom
(S)	NH_3, CH_4, XeO_4	(4)	All molecules are polar
		(5)	All molecules have incomplete octet of the central atom

(A) P → 5; Q → 2; R → 4; S → 1
(B) P → 2; Q → 4; R → 5; S → 1
(C) P → 4; Q → 5; R → 1; S → 3
(D) P → 4; Q → 5; R → 2; S → 3

27. If the shortest wavelength of spectral line of hydrogen atom in Lyman series is $x\ A^o$, then match the wavelength given in List-I with their corresponding value given (in A^o) in List-II and choose the correct option:

List – I		List – II	
(P)	Shortest wavelength in Balmer series of Li^{2+}	(1)	$\frac{9x}{5}$
(Q)	Longest wavelength in Balmer series of He^{+}	(2)	$\frac{x}{9}$
(R)	Longest wavelength in Lyman series of He^{+}	(3)	$\frac{4x}{9}$
(S)	Shortest wavelength in Lyman series of Li^{2+}	(4)	$\frac{4x}{27}$
		(5)	$\frac{x}{3}$

(A) P → 3; Q → 5; R → 1; S → 4
(B) P → 4; Q → 5; R → 2; S → 3
(C) P → 4; Q → 3; R → 5; S → 1
(D) P → 3; Q → 1; R → 5; S → 2

28. Match the order of elements given in List-I with the properties of elements given in List-II and choose the correct option

List – I		List – II	
(P)	Br < F < Cl	(1)	Electronegativity
(Q)	Na < K < Li	(2)	Atomic radius
(R)	Li < B < Be	(3)	Standard reduction potential
(S)	Al < Ga < B	(4)	Electron affinity
		(5)	1^{st} ionization energy

(A) P → 4; Q → 3; R → 5; S → 1
(B) P → 2; Q → 1; R → 5; S → 3
(C) P → 4; Q → 1; R → 3; S → 5
(D) P → 1; Q → 4; R → 2; S → 3

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

29. Among H_2, Li_2, C_2, O_2, N_2, B_2, F_2
The total number of diamagnetic species is_____

30. For the redox reaction
$$xS_2O_3^{2-}(aq) + yBr_2(\ell) + zH_2O(\ell) \longrightarrow aSO_4^{2-}(aq) + bBr^-(aq) + cH^+(aq)$$
If x + y + z = 8, then the value of x + b is_____

31. At certain temperature when $SO_3(g)$ is added to a evacuated closed vessel at a pressure of 2 atm, it undergoes the reaction and attained the following equilibrium
$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$$
At equilibrium 80% of $SO_3(g)$ has reacted. Volume remains constant. The value of K_P for the equilibrium at this temperature is x. the value of 10x is_____

32. 48 ml of 0.5 M benzoic acid solution was mixed with **x** ml of 1.2M sodium benzoate solution and the pH of the resulting solution was found to be 4.5. The value of x is_____
(pK_a of benzoic acid is 4.2, log2 = 0.3)

33. For a zero order reaction $A \longrightarrow P$, the half-life is 50 sec. The time (in sec.) required for $\frac{3}{4}$th of the initial amount of reactant A to get reacted is_____

34. In a closed evacuated vessel at 450 K solid $(NH_4)_2S$ was taken which decomposes as
$(NH_4)_2S(s) \rightleftharpoons 2NH_3(g) + H_2S(g)$
At equilibrium the total pressure in the flask was found to be 9 atm. K_P for the above equilibrium at 450 K is_____

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

35. Assume that $M=\{(x,y)\,|\,3x^2+y^2=4\}$ and $N=\{(x,y)\,|\,y=mx+c\}$. If $M\cap N\neq\phi$, for all $m\in\mathbb{R}$. Then c takes values from

(A) $\left[-\frac{\sqrt{55}}{2},\frac{\sqrt{55}}{2}\right]$ (B) $\left[-\frac{\sqrt{65}}{2},\frac{\sqrt{65}}{2}\right]$

(C) $\left[-\frac{2\sqrt{85}}{3},\frac{2\sqrt{85}}{3}\right]$ (D) $\left[-\frac{2\sqrt{5}}{3},\frac{2\sqrt{5}}{3}\right]$

36. If the area of region $\mathbb{R}=\{(x,y)\in\mathbb{R}\times\mathbb{R}: 0\le x\le 2;\ 0\le y\le 1\}$ and $\frac{1}{3}\le xy\le 1$ is λ square unit.

Then λ is equal to

(A) $\frac{2}{3}+\frac{2}{3}\ln 2-\frac{1}{3}\ln 3$ (B) $\frac{3}{2}+\frac{3}{2}\ln 2-3\ln 3$

(C) $\frac{2}{3}-\frac{2}{3}\ln 2+\frac{1}{3}\ln 3$ (D) none of these.

37. For non–negative integers n, define the function $h_n(x)$ as $h_n(x)=\sum_{k=1}^{n}\frac{\sin kx}{k+\cos kx}$, where $x\in\mathbb{R}-(2m+1)\pi$, $m\in\mathbb{Z}$. Let $h_n'(x)$ denote the derivative of $h_n(x)$. Which of the following statements is **INCORRECT**?

(A) There exists $\alpha\in(0,2\pi)$ such that $h'_n(\alpha)=0$, for all $n\ge 1$

(B) $h_3\left(\frac{\pi}{2}\right)=\frac{2}{3}$

(C) for all n, $h_n(x)$ is an odd function

(D) $h_n(x)$ periodic with period $\frac{2\pi}{n+1}$.

38. Suppose f is twice differentiable function satisfying $f(0)=-2$, $f'(0)=11$; $f''(0)=-8$, $f(2)=5$; $f'(2)=-3$; $f''(2)=7$ and also suppose that the function $g(x)=\frac{1}{x}\int_0^x f(t)dt$ has a critical point at $x=2$. Then

Statement – I : critical point of g(x) at x = 2 is local maxima,

Statement – II : $\int_0^2 f(t)dt=10$

(A) only statement – I is correct (B) only statement – II is correct

(C) both statements are correct (D) both statements are incorrect

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

39. For all real values continuous, differential and strictly increasing function f on the real line for all x satisfy the condition $f^2(x) = \int_0^x \left[(f(t))^2 + (f'(t))^2\right]dt + 2025$. Then the area bounded by curve $y = \frac{f(x)}{\sqrt{2025}}$ and lines x = 0 and y = e is/are

(A) $\frac{3}{\pi^3}\int_{-\pi}^{\pi} \frac{x^2}{1+\sin x+\sqrt{1+\sin^2 x}}dx$

(B) $e^{2025} - \int_0^1 e^x dx$

(C) $\int_1^{e^{2025}} \ln y\, dy$

(D) $\int_1^e \ln(e+1-y)dy$

40. If α, β ($\alpha < \beta$) are the roots of equations $1 - 8(\log_{10} x)^2 = \log_{10} x - 2(\log_{10} x)^2$. Then find **CORRECT** statements

(A) $\frac{1}{\alpha^4} + \beta^3 = 110$

(B) $\int_{-10^{1/2}\pi\alpha}^{\frac{\beta\pi}{10^{1/3}}} \frac{(2x)(1+\sin x)}{1+\cos^2 x}dx = \pi^2$

(C) $\alpha^2\beta^3 = 10$

(D) $\int_{-10^{1/2}\pi\alpha}^{\frac{\beta\pi}{10^{1/3}}} \frac{(2x)(1+\sin x)}{1+\cos^2 x}dx = 0$

41. Let $f:(0, 2\pi) \to \mathbb{R}$ be a differentiable function such that $\lim_{t\to x} \frac{f(x)\cos t - f(t)\cos x}{t-x} = \frac{\cos^2 x}{x^2}$; $\forall\ x(0, 2\pi)$. If $f(\pi) = -\frac{1}{\pi}$, then which of the following statement(s) is/are **TRUE**?

(A) f(x) has local minimum in the interval $\left(\frac{\pi}{2}, \pi\right)$

(B) f(x) is increasing for $x \in \left(0, \frac{\pi}{3}\right)$

(C) $\int_{1/2}^1 x^3 f'(x)\ dx < 0$

(D) $\frac{7}{30} \le \int_0^1 x^3 f(x)dx < \frac{1}{3}$

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

42. Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$ and $a_n I_{n+2} + b_n I_n = c_n \; \forall \; n \geq 1$. (where a_n & b_n are monic linear expression)

Match entry in List–I to the correct entry in List–II

List – I		List – II	
(P)	$a_1, a_2, a_3, \ldots$ are in	(1)	H.P.
(Q)	$a_1 - b_1, a_2 - b_2, a_3 - b_3, \ldots$ are in	(2)	A.G.P.
(R)	$\frac{\pi}{2} - c_1, \frac{\pi}{2} - c_2, \frac{\pi}{2} - c_3, \ldots$ are in	(3)	A.P.
(S)	$2^{a_1+b_1}, 2^{a_2+b_2}, 2^{a_3+b_3}, \ldots$ are in	(4)	G.P.
		(5)	A.P., G.P. & H.P.

The correct option is:

(A) (P) → (1); (Q) → (5); (R) → (2); (S) → (4)

(B) (P) → (3); (Q) → (1); (R) → (5); (S) → (4)

(C) (P) → (3); (Q) → (5); (R) → (1); (S) → (4)

(D) (P) → (1); (Q) → (5); (R) → (3); (S) → (2)

43. f , g, h : $\mathbb{R} \to \mathbb{R}$ be 3 differentiable functions such that $f(x) = x^3 + 3x + 1$, $g(f(x)) = x$ and $h(g(g(x))) = x$.

Match entry in List–I to the correct entry in List–II

List – I		List – II	
(P)	$3g'(1)$	(1)	1
(Q)	$h(g(1))$	(2)	3
(R)	$\frac{1}{6} \cdot h'(0)$	(3)	5
(S)	$g(h(g(7)))$	(4)	7
		(5)	9

The correct option is:

(A) (P) → (3); (Q) → (1); (R) → (2); (S) → (4)

(B) (P) → (1); (Q) → (3); (R) → (2); (S) → (4)

(C) (P) → (3); (Q) → (1); (R) → (4); (S) → (2)

(D) (P) → (1); (Q) → (3); (R) → (4); (S) → (2)

44. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x)+f\left(1-\frac{1}{x}\right)=\tan^{-1}x$ for all $x \in \mathbb{R}-\{0\}$ (here $\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Match entry in List–I to the correct entry in List–II

List – I		List – II	
(P)	If $x \in (0, 1)$, then $f(x) + f(1 – x) =$	(1)	$\frac{3\pi}{8}$
(Q)	$4 \cdot \lim\limits_{x \to 1^-} f(x) =$	(2)	$\frac{\pi}{2}$
(R)	$\int_0^1 f(x)\,dx =$	(3)	$\frac{3\pi}{4}$
(S)	Let $g(x) = 2f(x) – \tan^{-1}x$, then $\lvert g(x) – g(1/x) \rvert =$ {given $x \in (0, 1)$}	(4)	π
		(5)	$\frac{3\pi}{2}$

The correct option is:
(A) (P) → (1); (Q) → (3); (R) → (2); (S) → (4)
(B) (P) → (1); (Q) → (5); (R) → (2); (S) → (4)
(C) (P) → (3); (Q) → (5); (R) → (4); (S) → (1)
(D) (P) → (3); (Q) → (5); (R) → (1); (S) → (4)

45. Let $f(x) = ax^2 + bx + c$ $(a \neq 0)$ and a, b, c $\in$ {–20, –19.....19, 20} and a is even. If f(f(1)) = f(f(2)) = f(f(3)) then. Match entry in List–I to the correct entry in List–II

List – I		List – II	
(P)	Maximum value of f(1) is	(1)	2
(Q)	Minimum value of f(1) is	(2)	16
(R)	Maximum value of b is	(3)	4
(S)	Minimum value of c is	(4)	0
		(5)	–12

The correct option is:
(A) (P) → (3); (Q) → (4); (R) → (2); (S) → (5)
(B) (P) → (4); (Q) → (3); (R) → (2); (S) → (1)
(C) (P) → (4); (Q) → (3); (R) → (1); (S) → (5)
(D) (P) → (3); (Q) → (4); (R) → (1); (S) → (5)

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

46. Let 'a' be a real number such that $\lim\limits_{n\to\infty} \frac{\sqrt[4]{1}+\sqrt[4]{2}+....+\sqrt[4]{n}}{n^{\frac{13}{4}}\left(\frac{1}{(an+1)^3}+\frac{1}{(an+2)^3}+...+\frac{1}{(an+n)^3}\right)} = 125$, then find the value of λ such that $\frac{4a^2(a+1)^2}{0.5+a} = \lambda$

47. Find the total number of real solutions of equation $e^{14x} - 2e^{7x} - e^{10x} - 6 \cdot e^{6x} + e^{3x} + 1 = 0$

48. If $\lim\limits_{n\to\infty} n^3 \int\limits_0^{1/n} x^{2025x+2}dx = L$, then find 333L

49. Find minimum value of the following expression $f(x) = \dfrac{\left(x+\frac{1}{x}\right)^4 - \left(x^4+\frac{1}{x^4}+6\right)}{\left(x+\frac{1}{x}\right)^2 - \left(x^2+\frac{1}{x^2}\right)}$. (when x > 0)

50. Let g(x) be a non–constant thrice differentiable function define on $\mathbb{R}$ such that
(i) g(x) = g(8 – x)
(ii) $g'(0) = 0 = g'(2) = g'(5) = g'(7)$
and if m is the minimum number of roots of the equation $(g''(x))^2 + g'(x)g'''(x) = 0$ in interval [0,8] then find m.

51. Let f : $\mathbb{R} \to \mathbb{R}$ and g : $\mathbb{R} \to \mathbb{R}$ be given $f(x) = |x^2 - 3x + 2| + 3$ and $g(x) = x^3 - 3x^2 + 3x + 1$ define h(x) as: $h(x) = \begin{cases} \max\{f(x), g(x)\}, & x \le 1 \\ \min\{f(x), g(x)\}, & x > 1 \end{cases}$. If ℓ = number of point(s) where h(x) is discontinuous and m = number of point(s) where h(x) is not differentiable.
Then find $\left|66\int\limits_{\ell}^{m} |x^2 - 3x + 2|\, dx\right|$