Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Main)
UNIT TEST # 03
04-08-2024

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	C	C	B	C	A	A	B	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	B	A	B	C	C	C	A	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	2	3	6	7	2	2	400	100	4

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	B	C	D	C	B	C	C	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	C	C	B	D	D	C	B	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	2	5	2	9	250	-1450	400	-374	4

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A	B	A	B	B	C	B	C	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	A	D	A	C	C	C	C	B	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	7	3	5	6	-1	-1	0	4	9	3

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (B)**

$$Mg - B = Mf$$

$$B - (M - CM)g = (M - CM)f$$

$$CMg = (2M - CM)f$$

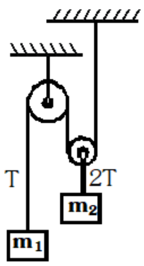
$$Cg + Cf = 2f$$

$$C = \frac{2f}{g+f}$$

2. **Ans (B)**

$$T = m_1 g$$

$$2T = M_2 g$$



$$\Rightarrow \frac{1}{2} = \frac{M_1}{M_2}$$

3. **Ans (C)**

$$F_{MIN} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

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4. **Ans (C)**

Blocks A and C both move due to friction. But less friction is available to A as compared to C because normal reaction between A and B is less.

Maximum friction between A and B can be:

$$f_{max} = \mu m_A g = \left(\frac{1}{2}\right) mg$$

∴ Maximum acceleration of A can be

$$a_{max} = \left(\frac{f_{max}}{2}\right) = \frac{g}{2}$$

further $a_{max} = \dfrac{m_D g}{3m + m_D}$

or $\dfrac{g}{2} = \dfrac{m_D g}{3m + m_D}$

∴ (C) is the right answer

5. **Ans (B)**

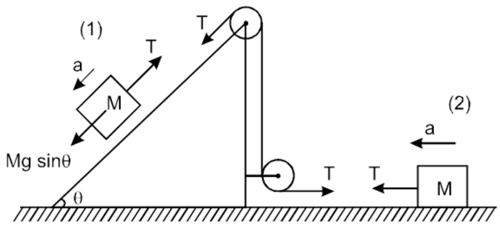
[NCERT pg # 113]

$$S_{relative} = \frac{1}{2} a_{relative}\, t^2$$

$$t = \sqrt{\frac{2 \times 5}{(3-2)}} = \sqrt{10} \text{ sec}$$

$$s_{trolley} = 0 + \frac{1}{2} \times 3 \times \left(\sqrt{10}\right)^2 = 15 \text{ m}$$

6. **Ans (C)**



$Mg\sin\theta - T = Ma$ [Newton's II law for block 1]

$T = Ma$ [Newton's II law for block 2]

By dividing both equations

$$2T = Mg\sin\theta \quad T = \frac{Mg\sin\theta}{2}$$

7. **Ans (A)**

When there is no friction, $a = g\sin\theta$

When there is friction, $a' = g(\sin\theta - \mu\cos\theta)$

If the length of the inclined plane is *d*, then

$$d = \frac{1}{2}at^2 = \frac{1}{2}a'(2t)^2$$

or $a = 4a'$

or $g\sin\theta = 4g(\sin\theta - \mu\cos\theta)$

$\sin\theta = 4\sin\theta - 4\mu\cos\theta$

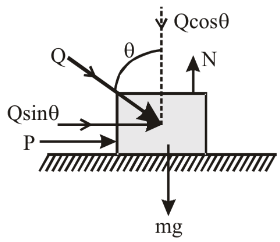
or $g\sin\theta - 4g(\sin\theta - \mu\cos\theta)$

$\sin\theta = 4\sin\theta - 4\mu\cos\theta$

$4\mu\cos\theta = 3\sin\theta$

$$\mu = \frac{3}{4}\tan\theta$$

8. **Ans (A)**



Applied force

$f_a = P + Q\sin\theta$

Normal reaction

$N = mg + Q\cot\theta$

$f_\ell = \mu N = \mu(mg + Q\cos\theta)$

Now, condition for no slipping

$f_a \le f_\ell$

$P + Q\sin q \le \mu(mg + Q\cos\theta)$

$$\mu \ge \frac{P + Q\sin\theta}{mg\ell\, Q\cos\theta}$$

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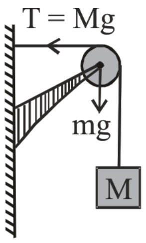
10. Ans (D)

Net force on clamp



$$F_{net} = \sqrt{[(M+m)g]^2 + (Mg)^2}$$

$$= \sqrt{[(M+m)]^2 + M^2}\ g$$

13. Ans (B)

$$F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$$

Force applied by sphere with cavity = force applied by complete sphere – force applied by removed sphere.

$$F_2 = \frac{GMm}{(2R)^2} - \frac{G \times \frac{M}{8} m}{\left(\frac{3R}{2}\right)^2}$$

$$= \frac{7GMm}{36R^2} \quad = \frac{F_2}{F_1} = \frac{7}{9}$$

14. Ans (A)

$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2} = 1yr'$$

$$T' = \frac{2\pi}{\sqrt{G(2M)}} (2R)^{3/2} = 2yr$$

15. Ans (B)

Gravitational force provides the required centripetal force for orbiting the satellite

$$\frac{m\upsilon^2}{R} = \frac{K}{R} \quad \text{because } \left(F \propto \frac{1}{R}\right)$$

$\therefore u \propto R^\circ$

16. Ans (C)

Kinetic energy = Potential energy

$$m(k\upsilon_e)^2 = \frac{mgh}{1+\frac{h}{R}} \Rightarrow \frac{1}{2} mk^2\, 2gR =$$

$$\frac{mgh}{1+\frac{h}{R}} \Rightarrow h = \frac{Rk^2}{1-k^2}$$

Height of Projectile from the earth's surface = h

Height from the centre $r = R + h = R + \frac{Rk^2}{1-k^2}$

By solving $r = \frac{R}{1-k^2}$

17. Ans (C)

$$T \propto r^{3/2}$$

$$\left(\frac{T}{2}\right) \propto (r')^{3/2}$$

$$\left(\frac{r'}{r}\right)^{3/2} = \frac{1}{2}$$

$$\left(\frac{r'}{r}\right) = \frac{1}{2^{2/3}}$$

$$r' = \frac{r}{2^{2/3}} = \frac{r}{(4)^{1/3}}$$

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18. **Ans (C)**

$$V_0 = \sqrt{\frac{GM}{r}} \Rightarrow V_0 \propto \frac{1}{\sqrt{r}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{V}{V_2} = \sqrt{\frac{4R}{R}}, V_2 = \frac{V}{2}$$

20. **Ans (C)**

$T^2 = \frac{4\pi^2}{GM} r^3$. If G is variable then time period, angular velocity and orbital radius also changes accordingly.

PART-1 : PHYSICS

SECTION-II

1. **Ans (4)**

$$g_p = \frac{GM_p}{R_{p^2}} \Rightarrow g_p = \frac{4\ GM_e}{9\ R_{e^2}} \quad ..(1)$$

$$\left[g = \frac{GMe}{R_e^{\ 2}}\right] \quad ..(2)$$

by (1) & (2)

$$g_p = \frac{4}{9}g, \quad w_p = mg_p = \frac{4}{9}mg, \quad [mg = 9N]$$

$$w_p = \frac{4}{\cancel{9}} \times \cancel{9}N = 4N$$

2. **Ans (2)**

K.E. required for satellite to escape from earth's gravitational field

$$\frac{1}{2}mv_e^0 = \frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2 = \frac{GMm}{R}$$

K.E. required for satellite to move in circular orbit

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2 = \frac{GMm}{2R}$$

The ratio between these two energies = 2

4. **Ans (6)**

Gravitational force between the masses are same.

5. **Ans (7)**

$V_e = \sqrt{\frac{2GM}{R}}$ Given the velocity projection of the body

$$= v = \frac{3}{4}v_e = \frac{3}{4}\sqrt{\frac{2GM}{R}}$$

Total energy on earth = Total energy at maximum height h

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0 + \left(-\frac{GMm}{R+h}\right)$$

$$\frac{1}{2}m\frac{9}{16}\cdot\frac{2GM}{R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{9}{16} - 1 = -\frac{R}{R+h} \text{ or } -\frac{R}{R+h} = \frac{-7}{16}$$

$$7R + 7h = 16R$$

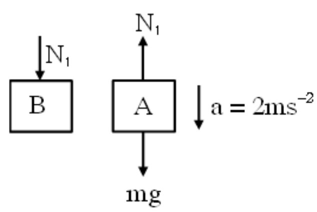
$$7h = 9R \Rightarrow h = \frac{9R}{7}$$

6. **Ans (2)**

Let A applies force N_1 on B

Then B also applies an opposite force N_1 on A

As shown



For A $\quad mg - N_1 = ma$

$$N_1 = m(g - a) = 0.5\ (10 - 2)$$

$$N_1 = 4$$

$$N_1 = 2x \Rightarrow x = 2$$

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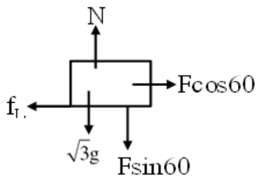
7. **Ans (2)**



$N = F\sin 60 + \sqrt{3}g$

$f_L = \mu(F\sin 60 + \sqrt{3}g)$

$f_L = \frac{F}{4} + \frac{g}{2}$

$F\cos 60 = f_L$

$\frac{F}{2} = \frac{F}{4} + 5$

$\frac{F}{4} = 5 \Rightarrow F = 20N$

9. **Ans (100)**

$f_s = \mu\, mg$

$= 0.4 \times 30 \times 10$

$= 120$ N

$F < f_s$

So $f = 100$ N

10. **Ans (4)**

$\frac{dp}{dt} = F = -3t$

$\int_{3'}^{1} dp = \int_0^T -3t\,dt$

$(P_3^1) = -\left(\frac{3t^2}{2}\right)_0^T$

$1 - 3 = -\frac{3T^2}{2}$

$-\frac{4}{3} = -T^2$

$T = \frac{2}{\sqrt{3}}$ sec

PART-2 : CHEMISTRY

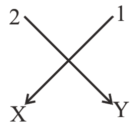
SECTION-I

1. **Ans (C)**

Species	ℓp on central atom	Hybridization
SF_4, XeO_2F_2	1	sp^3d
PCl_4^+	0	sp^3
SOF_4	0	sp^3d

2. **Ans (A)**

Valency of element X is 2(2 electrons in the outermost shell) while that of element Y is 1(1 electron in the outermost shell). So the formula of the compound between X and Y is XY_2



3. **Ans (B)**

Bond length depends on size of atoms order of bond length :

Si – Si > P – P > Cl – Cl

Bond energy $\propto \frac{1}{\text{Bond length}}$

Hence, correct order of bond energy is :

Si – Si < P – P < Cl – Cl

4. **Ans (C)**

In diamond, a network of covalent bond is present. In melting of diamond, breaking of covalent bonds takes place.

5. **Ans (D)**

O-Nitrophenol has intramolecular H-bond. Due to intramolecular H-bond, intermolecular bond becomes weaker hence, volatility of O-nitrophenol is higher than p-nitrophenol.

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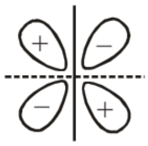
6. Ans (C)

Structure of π^*_{2py} is,



It has two nodal planes.

7. Ans (B)

$2p_x + 2p_x$ will form s-bond

$2p_y + 2p_y$ and $2p_x + 3d_{xy}$ will form π-bond but

$2p\pi$-$2p\pi$ bond is stronger than $2p\pi$-$3d\pi$

8. Ans (C)

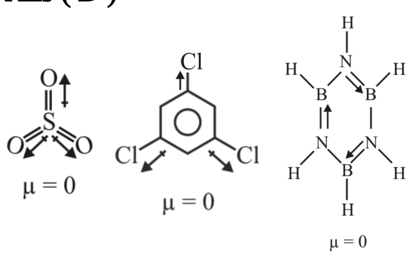
'N' can form NCl_3, N_2O_5 and Ca_3N_2 but can not form NCl_5. Due to absence of d-orbital's, 'N' can not expand its valency to 5.

9. Ans (C)

$H_3Te^+ < H_3Se^+ < H_3S^+ < H_3O^+$

They all have pyramidal shape. As electro negativity of central atom decreases, bond angle decreases.

10. Ans (D)



All are symmetrical molecules so resultant d. M = 0

11. Ans (A)

$$V_1 = \frac{2 \times R \times 320}{4}$$

$$V_2 = \frac{2 \times R \times 640}{8}$$

$\Delta V = 0$

so w = 0

13. Ans (C)

$$\frac{500}{18} \times 75.6 \times (20-0) = n \times 6000 \times \frac{9}{18}$$

n = 14

15. Ans (B)

$\Delta_r G^o = \Delta_r H^o - T \times \Delta_r S^o$

$\Delta_r S^o = 2 \times 81 - 4 \times 24 - 3 \times 205$ J/mol

$\therefore \ \Delta_r H^o = -2258.1$ kJ/mol

$\Delta_r H^o = 2 \times \Delta_f H^o (Cr_2O_3, s)$

$$\therefore \ \Delta_r H^o (Cr_2O_3, s) = -\frac{2258.1}{2}$$

$= -1129.05$ kJ/mol

17. Ans (D)

$$\Delta S = 2.303 \times n \times R \times \log_{10}\left(\frac{P_1}{P_2}\right)$$

$$= 2.303 \times \left(\frac{64}{32}\right) \times 2 \times \log_{10}\left(\frac{1}{0.25}\right)$$

$= 5.52$ cal mol^{-1} K^{-1}

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19. **Ans (B)**

$HA \longrightarrow H^+ + A^- \; ; \; \Delta_r H = 1.4 \text{ kJ/mol}$

$\Delta H_{neutralization} = \Delta H_{ionization} + \Delta_r H$

$(H^+ + OH^- \longrightarrow H_2O) \; -55.95 = \Delta H_{ionization} - 57.3$

$\Delta H_{ionization}$ for 1 M HA = 1.35 kJ/mol

% heat utilized by 1 M acid for ionization

$= \dfrac{1.35}{1.4} \times 100 = 96.43\%$

so, acid is 100 – 96.43 = 3.57% ionized

20. **Ans (A)**

For adiabatic process $P \propto T^3$

$$\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}$$

$$(P)^{\frac{\gamma-1}{\gamma}} \propto T$$

$$P \propto T^{\left(\frac{\gamma}{\gamma-1}\right)}$$

According the question pressure is proportional

to the cube of its absolute temperature hence

$$\frac{\gamma}{(\gamma-1)} = 3 \Rightarrow \gamma = 3\gamma - 3$$

$$2\gamma = 3 \Rightarrow \gamma = \frac{C_P}{C_V} = \frac{3}{2}$$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (4)**

x = 6 ; y = 12 ; z = 2

y – x – z = 12 – 6 – 2 = 4

2. **Ans (2)**

O_2^+ (1) (unpaired)

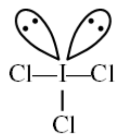
O_2^- (1)

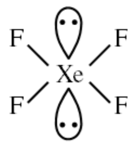
3. **Ans (5)**

5.0

$SO_3, PCl_5, CCl_4, PCl_3F_2$ and XeF_4 are non-polar

4. **Ans (2)**





$$\frac{x+y}{2} = \frac{2+2}{2} = 2$$

5. **Ans (9)**

Lattice energy depends on $\dfrac{q_1 q_2}{r}$

6. **Ans (250)**

$nC_v(T_2 - T_1) = -75$ cal

$0.1 \times \dfrac{3R}{2}(T_2 - T_1) = -75$

$T_2 - T_1 = \dfrac{-75}{0.3}$

$T_2 - 500 = -250$

$T_2 = 250$ K.

7. **Ans (-1450)**

$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$

$\Delta H = 2(-400 - 300) - 50$

= – 1450 kJ/mole

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8. Ans (400)

$$T = \frac{\Delta H_{vap}}{\Delta S_{vap}} = \frac{30 \times 10^3}{75} = 400K$$

10. Ans (4)

$$\int dw = \int pdv$$

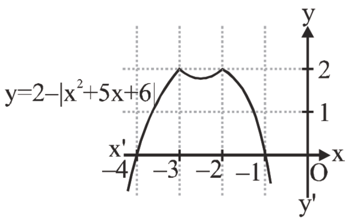
PART-3 : MATHEMATICS

SECTION-I

1. Ans (A)

f(x) will have maxima at $x = -2$ only if

$a^2 + 1 \geq 2$ or or $|a| \geq 1$.



2. Ans (A)

$$\left(A + \frac{1}{A} + 1\right)\left(B + \frac{1}{B} + 1\right)\left(C + \frac{1}{C} + 1\right)\left(D + \frac{1}{D} + 1\right)$$

$= 3.3.3.3$

$= 3^4$

3. Ans (B)

A.M. $\geq$ G.M.

$$\Rightarrow \frac{1 + x + x^2 + \ldots\ldots + x^{100}}{101} \geq (1.x.x^2.x^3 \ldots\ldots\ldots x^{100})^{1/101}$$

$$\Rightarrow \left(\frac{1 + x + x^2 + \ldots\ldots + x^{100}}{101}\right) \geq \left(x^{\frac{100 \times 101}{2}}\right)^{\frac{1}{101}}$$

$$\Rightarrow \left(\frac{1 + x + x^2 + \ldots\ldots + x^{100}}{101}\right) \geq x^{50}$$

$$\Rightarrow \frac{1}{101} \geqslant \left(\frac{x^{50}}{1 + x + x^2 + \ldots\ldots + x^{100}}\right)$$

$$\therefore \text{exp.} \leq \frac{1}{101}$$

Greatest value $= \frac{1}{101}$.

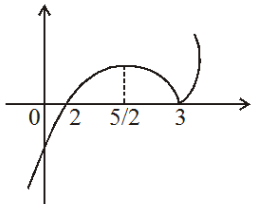
4. Ans (A)

For $f(x) = (x - 2)(x - 3) = x^2 - 5x + 6$,

$f(x) = 2x - 5 = 0$

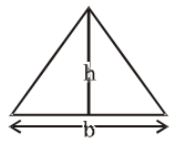
$\Rightarrow x = 5/2$

Now, the graph of $f(x) = (x - 2)|x - 3|$ is as follows:



Clearly, from the graph, f(x) increases in $(-\infty, 5/2) \cup (3, \infty)$

5. Ans (B)



$$A = \frac{1}{2}bh$$

$$\frac{dA}{dt} = \frac{1}{2}\left[\frac{db}{dt}h + b\frac{dh}{dt}\right]$$

$$2 = \frac{1}{2}[x.(10) + 20 \times 1]$$

$\Rightarrow x = -1.6$ cm/min.

6. Ans (B)

Let $g(x) = 4x^3 - 12x^2 + 11x - 3$

$\therefore\ g'(x) = 12x^2 - 24x + 11$

$= 12(x - 1)^2 - 1$

> 0 for x Î [2, 3]

Thus, g(x) is increasing in [2, 3].

$f(x)_{max} = f(3) = \log_{10}(4.27 - 12.9 + 11.3 - 3)$

$= \log_{10}(30) = 1 + \log_{10}3$

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7. Ans (C)

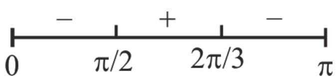
$f'(x) = -3\sin 2x\,(2\cos x + 1)(\cos x + 2)$

$f'(x) = 0 \Rightarrow \sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi$

or $(2\cos x + 1) = 0 \Rightarrow x = \frac{2\pi}{3}$

as $\cos x + 2 \neq 0$

sign scheme of $f'(x)$ is as follows.



8. Ans (B)

$$f'(x) = \frac{1}{x} - \frac{(2+x).2 - 2x.1}{(2+x)^2}$$

$$= \frac{1}{x} - \frac{4 + 2x - 2x}{(2+x)^2} = \frac{(2+x)^2 - 4x}{x(2+x)^2}$$

$$= \frac{4 + x^2 + 4x - 4x}{x(2+x)^2} = \frac{x^2 + 4}{x(2+x)^2}$$

$$= \frac{(x^2+4)x}{x^2(2+x)^2} > 0 \ \forall\ x > 0$$

9. Ans (C)

It is a fundamental property.

10. Ans (C)

$$f(x) = \int_0^4 e^{|x-t|}dt = \int_0^x e^{(x-t)}dt + \int_x^4 e^{(t-x)}dt$$

$$= e^x + e^{4-x} - 2 \geqslant 2e^2 - 2$$

11. Ans (D)

$\therefore$ Let $P\left(t, \frac{t^2}{2}\right)$ be a point on $x^2 = 2y$ and A be $(0, 5)$

If $AP = d$

$$\Rightarrow z = d^2 = t^2 + \left(\frac{t^2}{2} - 5\right)^2$$

$$\therefore \frac{dz}{dt} = 2t + 2\left(\frac{t^2}{2} - 5\right).t$$

$$= t^3 - 8t$$

$$= t(t^2 - 8)$$

$$\Rightarrow \frac{d^2z}{dt^2} = 3t^2 - 8$$

$$= \frac{dz}{dt} = 0 \Rightarrow t = 0 \text{ or } t = \pm 2\sqrt{2}$$

at $t = 0$, $\frac{d^2z}{dt^2}$ is – ive

At $t = \pm 2\sqrt{2}$, $\frac{d^2z}{dt^2}$ is + ive

Hence, the closest point is $(2\sqrt{2}, 4)$

12. Ans (A)

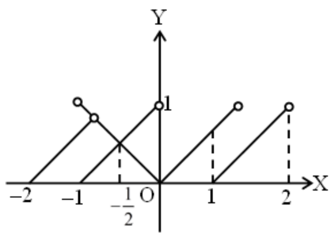
$\because f'(x) = |x| - \{x\}$

$\because f(x)$ is decreasing

$\therefore f'(x) < 0$

$\Rightarrow |x| - \{x\} < 0$

$\Rightarrow |x| < \{x\}$



It is clear from the figure $x \in \left(-\frac{1}{2}, 0\right)$

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13. **Ans (D)**

$$f(x)=\sqrt{((\sqrt{x-1})-2)^2}+\sqrt{((\sqrt{x-1})-3)^2}$$

$$=\left|\sqrt{x-1}-2\right|+\left|\sqrt{x-1}-3\right|$$

$$\therefore\quad f'(2)=-\frac{1}{\sqrt{x-1}}\ \&\ \text{put } x=2$$

14. **Ans (A)**

$$\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)$$

$$=\frac{g(g(x)).g'(x)-g(x)g'(g(x)).g'(x)}{(g(g(x)))^2}$$

$$\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)_{atx=4}$$

$$=\frac{g(g(4)).g'(4)-g(4).g'(g(4)).g'(4)}{(g(g(4))^2}\quad ...(1)$$

Now, f & g are inverse to each other

$\therefore\ g(f(x))=x$

$$g'(f(x))=\frac{1}{f'(x)}$$

$$g'(4)=\frac{1}{f'(0)}=\frac{1}{3}$$

and $g(4)=0$

$$\therefore\ \text{Deri}=\frac{g(0)\times\frac{1}{3}-0}{(g(0))^2}$$

And $g(0)=-1$

$$\therefore\ \text{Deri}=\frac{-1\times\frac{1}{3}}{1}=-\frac{1}{3}$$

15. **Ans (C)**

$$y=(\cos x)^{(\cos x)^{(\cos x)\ldots\ldots\infty}}$$

$$\Rightarrow y=(\cos x)^y$$

$$\log y=y\log\cos x$$

$$\frac{1}{y}\frac{dy}{dx}=y\frac{1(-\sin x)}{\cos x}+\frac{dy}{dx}\log\cos x$$

$$\left(\frac{1}{y}-\log\cos x\right)\frac{dy}{dx}=-y\tan x$$

$$\frac{dy}{dx}=\frac{-y^2\tan x}{1-y\log\cos x}$$

$$\frac{dy}{dx}=\frac{y^2\tan x}{y\log\cos x-1}$$

16. **Ans (C)**

$$y=e^{\ln(\sin-1x)}\ ;\ 0<x<1$$

$$\Rightarrow y=\sin^{-1}x$$

$$\Rightarrow \sin y=x$$

$$\cos y\frac{dy}{dx}=1,\ \frac{dy}{dx}=\frac{1}{\cos y}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\sin^2 y}}\quad \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

17. **Ans (C)**

$$y=\tan^{-1}\frac{4x}{1+5x^2}+\tan^{-1}\frac{2+3x}{3-2x}$$

$$=\tan^{-1}\frac{5x-x}{1+5x.x}+\tan^{-1}\frac{\frac{2}{3}+x}{1+\frac{2}{3}.x}$$

$$=\tan^{-1}5x-\tan^{-1}x+\tan^{-1}\frac{2}{3}+\tan^{-1}x$$

$$\Rightarrow\frac{dy}{dx}=\frac{5}{1+25x^2}$$

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18. **Ans (C)**

$$\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$$

$$=\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$$

$$+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$$

$$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$$

$$+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$$

$$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$$

$$+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$$

$$=\frac{1}{4}(3)+\frac{1}{4}(3)=\frac{3}{2}$$

19. **Ans (B)**

$$\frac{1}{\sin 1^\circ}\left[\frac{\sin 1^\circ}{\sin 1^\circ \sin 2^\circ}+\frac{\sin 1^\circ}{\sin 2^\circ \sin 3^\circ}+\ldots+\frac{\sin 1^\circ}{\sin 89^\circ \sin 90^\circ}\right]$$

$$\frac{1}{\sin 1^\circ}\left[\frac{\sin(2^\circ-1^\circ)}{\sin 1^\circ \sin 2^\circ}+\ldots+\frac{\sin(90^\circ-89^\circ)}{\sin 89^\circ \sin 90^\circ}\right]$$

$$\frac{1}{\sin 1^\circ}[(\cot 1^\circ-\cot 2^\circ)+\ldots\ldots+(\cot 89^\circ-\cot 90^\circ)]$$

$$\frac{\cot 1^\circ}{\sin 1^\circ}=\frac{\cos 1^\circ}{\sin^2 1^\circ}$$

20. **Ans (B)**

Given expression

$$=\frac{(1-\sin\alpha)-(1+\sin\alpha)}{\sqrt{1-\sin^2\alpha}}$$

$$=\frac{-2\sin\alpha}{|\cos\alpha|}=\frac{-2\sin\alpha}{-\cos\alpha}$$

$\left[\because \frac{\pi}{2}<\alpha<\pi \therefore \cos\alpha \text{ is } -\text{ve}\right]=2\tan\alpha$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (7)**

$f'(x)=3x^2-6(a-2)x+3a$

$f'(x)\geq 0\ \forall\ x\in(0,1]$

$f'(x)\leq 0\ \forall\ x\in[1,5)$

$\Rightarrow f'(x)=0$ at $x=1 \Rightarrow a=5$

$f(x)-14=(x-1)^2(x-7)$

$$\frac{f(x)-14}{(x-1)^2}=x-7$$

2. **Ans (3)**

$f'(x)=2xe^{-2x}-2x^2e^{-2x}$

$=2(1-x)x\,e^{-2x}.$

Now, $f'(x)=0 \Leftrightarrow x=1,0$

Also $f''(x)=2(1-x)e^{-2x}-2xe^{-2x}-4$

$(1-x)xe^{-2x}$, so $f''(1)$

$=-2e^{-2}<0$ and $f''(0)>0.$

Thus max $f(x)=f(1)=e^{-2}$

3. **Ans (5)**

$f(x)=\dfrac{x^2-x}{x^2+4x}$, to find $\dfrac{df^{-1}(x)}{dx}$ at $x=2$

First we have to find

$f^{-1}(x)=$

$$\Rightarrow x=\frac{y^2-y}{y^2+4y}=\frac{y(y-1)}{y(y+4)}$$

$$\Rightarrow y=\frac{-(1+4x)}{(x-1)}=\frac{(1+4x)}{(1-x)}=f^{-1}(x)$$

$$\frac{df^{-1}(x)}{dx}=\frac{(1-x)4-(1+4x)(-1)}{(1-x)^2}$$

$$=\frac{4-\cancel{4x}+1+\cancel{4x}}{(1-x)^2}$$

$$\left.\frac{df^{-1}(x)}{dx}\right]_{\text{at } x=2}=\frac{5}{(1-2)^2}=5$$

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4. Ans (6)

$f(x+y) = f(x).\ f(y)$

$\Rightarrow f'(x+y) = f'(x).f(y)$

Put $y = 5,\ x = 0$

$f'(5) = f'(0).f(5)$

$= 3 \times 2$

$= 6$

5. Ans (-1)

$$\ell ny = \frac{1}{2}\ell n(1+2x) + \frac{1}{2}\ell n(1+4x) - \frac{1}{3}\ell n(1+3x)$$

$$-\frac{1}{5}\ell n(1+5x) - \frac{1}{7}\ell n(1+7x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(1+2x)} + \frac{1}{(1+4x)}$$

$$-\frac{1}{(1+3x)} - \frac{1}{(1+5x)} - \frac{1}{(1+7x)}$$

$y'(0) = y(0)\cdot[1 + 1 - 1 - 1 - 1]$

$= -1\ (\because y(0) = 1)$

6. Ans (-1)

$$\text{Let } y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

$$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}(\tan x)$$

$$= \tan^{-1}\left(\frac{a}{b}\right) - x \left[\therefore\ -\frac{\pi}{2} < x < \frac{\pi}{2}\right]$$

$$\therefore \frac{dy}{dx} = 0 - 1 = -1$$

7. Ans (0)

$$y = \sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$$

$$\text{put } \frac{x+\sqrt{x}}{\sqrt{x}-1} = \sin\theta$$

$$\Rightarrow y = \sec^{-1}\left(\frac{1}{\sin\theta}\right) + \sin^{-1}(\sin\theta)$$

$$\Rightarrow y = \sec^{-1}(\operatorname{cosec}\theta) + \theta$$

$$\Rightarrow y = \sec^{-1}\left(\sec\left(\frac{\pi}{2} - \theta\right)\right) + \theta$$

$$y = \frac{\pi}{2} - \theta + \theta$$

$$\frac{dy}{dx} = 0$$

8. Ans (4)

Given that $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$

where $\alpha, \beta \in [-\pi, \pi]$

Now $\cos(\alpha - \beta) = 1 \Rightarrow \alpha - \beta = 0$ or $\alpha = \beta$

$\therefore \cos(\alpha + \beta) = 1/e \Rightarrow \cos 2\alpha = 1/e$

$\because 0 < 1/e < 1$ and $2\alpha \in [-2\pi, 2\pi]$

Therefore, there will be four values of α in $[-2\pi, 2\pi]$ and correspondingly four values of β.

Hence, there are four sets of (α, β)

9. Ans (9)

$$f(x) = 3\left(\cos x\cos\frac{5\pi}{6} - \sin x\sin\frac{5\pi}{6}\right) - 5\sin x + 2$$

$$f(x) = -\frac{13}{2}\sin x - \frac{3\sqrt{3}}{2}\cos x + 2$$

Range $\equiv [-7, 7] + 2$

$[-5, 9]$

Max value $= 9$

10. Ans (3)

$$\left(1 + \sqrt{3} + \tan 1^\circ\right)\left(1 + \sqrt{3} + \tan 2^\circ\right)$$

$$(\tan 1^\circ + \tan(60^\circ - 1^\circ))$$

$$\Rightarrow \frac{(\tan 1^\circ + \tan(60^\circ - 2^\circ))}{\left(1 + \tan^2 1^\circ\right)\left(1 + \tan^2 2^\circ\right)}$$

$$= (1 + \sqrt{3}\tan 1^\circ)(1 + \sqrt{3}\tan 2^\circ)$$

$$\frac{\left(\tan 1^\circ + \frac{\sqrt{3} - \tan 1^\circ}{1 + \sqrt{3}\tan 1^\circ}\right)\left(\tan 2^\circ + \frac{\sqrt{3} - \tan 2^\circ}{1 + \sqrt{3}\tan 2^\circ}\right)}{(1 + \tan^2 1^\circ)(1 + \tan^2 2^\circ)}$$

$$\left(1 + \sqrt{3}\tan 1^\circ\right)\left(1 + \sqrt{3}\tan 2^\circ\right)$$

$$= \frac{\left(\sqrt{3}\tan^2 1^\circ + \sqrt{3}\right)\left(\sqrt{3}\tan^2 2^\circ + \sqrt{3}\right)}{\left(1 + \tan^2 1^\circ\right)\left(1 + \tan^2 2^\circ\right)}$$

$$\left(1 + \sqrt{3}\tan 1^\circ\right)\left(1 + \sqrt{3}\tan 2^\circ\right)$$

$$\sqrt{3}\,.\,\sqrt{3} = 3$$

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