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A right Choice for the Real Aspirant

ICON Central Office - Madhapur - Hyderabad

SEC: Sr.S60_Elite, Target & LIIT-BTs

JEE-MAIN(Theory Based Test)

GTM-10/05

Date: 27-12-2024 Max. Marks: 300

KEY SHEET

MATHEMATICS

Time: 09.00Am to 12.00Pm

1	2	2	2	3	2	4	4	5	4
6	1	7	4	8	4	9	2	10	4
11	1	12	2	13	2	14	1	15	3
16	2	17	4	18	3	19	3	20	1
21	66	22	6	23	6	24	6	25	6

PHYSICS

26	1	27	4	28	3	29	1	30	3
31	2	32	2	33	1	34	2	35	3
36	4	37	4	38	4	39	1	40	3
41	4	42	1	43	3	44	1	45	2
46	1234	47	134	48	123	49	23	50	124

CHEMISTRY

51	2	52	2	53	1	54	4	55	3
56	2	57	1	58	3	59	3	60	3
61	1	62	4	63	4	64	2	65	1
66	2	67	2	68	2	69	3	70	1
71	6	72	4	73	3	74	9	75	4

SOLUTIONS MATHEMATICS

1. Statement-I: $f(x) = x^3$ Not satisfies

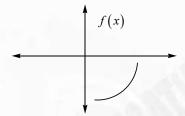
Statement-II: For odd if $\lim_{x\to 0}$ exists it is 0

Statement-III: from graph, it is correct

Statement-IV:
$$f(x) = \begin{cases} \cos(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$
 Not satisfies

2. Statement-II: $f(x) = -x^3$ at x=0 contradiction

Statement-III: $(f^2(x))^1 = 2f(x)f'(x) = -ve$



Statement-IV:

contradiction

3. $S_1 \rightarrow Z$ can be zero

 $S_2, S_3, S_4 \rightarrow$ Use properties of complex numbers

4. Let $f: R \to R$ is defined by $f(x) = (x^2 + \sin x)(x-1)$

Then,
$$f(1^+) = f(1^-) = f(1) = 0$$

Let $(fg): R \to R$ is defined by (fg)(x) = f(x).g(x)

Let
$$(fg) = h(x) = f(x).g(x)$$
 then $h: R \to R$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

If g is differentiable at x=1

$$h'(1) = f'(1)g(1) + 0, \quad [\because f(1) = 0]$$

If g(x) is differentiable then h(x) I also differentiable (true)

If g(x) is differentiable at x=1, then fg is also differentiable at x=1

If g(c) is continuous at x=1, then $g(1^+) = g(1^-) = g(1)$

$$h'(1^+) = \lim_{h \to 0^+} \frac{h(1+h) - h(1)}{h}$$

$$h'(1^+) = \lim_{h \to 0^+} \frac{f(1+h)g(1+h) - 0}{h} = f'(1)g(1)$$

$$h'(1^{-}) = \lim_{h \to 0^{+}} \frac{f(1-h)g(1-h)-0}{-h} = f'(1)g(1)$$

 $\Rightarrow h(x) = f(x).g(x)$ is differentiable at x=1(True)

So, If g is continuous at x=1, THEN fg is differentiable at x=1.

Option (b) (d)
$$h'(1^+) = \lim_{h \to 0^+} \frac{h(1+h) - h(1)}{-h}$$

$$h'(1^+) = \lim_{h \to 0^+} \frac{f(1+h)g(1+h)}{h} = f'(1)g(1^+)$$

$$h'(1^{-}) = \lim_{h \to 0^{+}} \frac{f(1-h)g(1-h)}{-h} = f'(1)g(1^{-})$$

$$\Rightarrow g(1^+) = g(1^-)$$

So, it does not mean that if fg is differentiable at x=1, then fg is continuous or differentiable at x=1

5. Statement-I: Use by parts 2 times

Statement-II: Indefinite integral +c must be kept

Statement-IV: $F(x) = \int \sin^2 x dx = g(x) + c$

 $F(x+\pi) = F(x)$ Might not satisfy for every 'c', (also Integral contains linear expression in x)

6. Given,
$$f(x) = \int_0^{x \tan^{-1} x} \frac{e^{t - \cos t}}{1 + t^{2023}} dt$$

$$\mathbf{f}'(\mathbf{x}) = \frac{e^{x \tan^{-1} x - \cos(x \tan^{-1} x)}}{1 + (x \tan^{-1} x)^{2023}} \cdot \left(\frac{x}{1 + x^2} + \tan^{-1} x\right)$$

For
$$x < 0$$
, $\tan^{-1} x \in \left(-\frac{\pi}{2}, 0\right)$

For
$$x \ge 0$$
, $\tan^{-1} x \in \left[0, \frac{\pi}{2}\right]$ $\Rightarrow x \tan^{-1} \forall x \in R$

And
$$\frac{x}{1+x^2} + \tan^{-1} x = \begin{cases} > 0 & \text{For } x > 0 \\ < 0 & \text{For } x < 0 \\ 0 & \text{For } x = 0 \end{cases}$$



i.e Point of minima

So, f(x) is minimum at x=0

Here minimum value is $f(0) = \int_0^0 = 0$.

 $\psi(x)$ and $\phi(x)$ can be negative functions

- 7. Use definition of Area. $S_1(F), S_2(F)$
- 8. When $x \in [0,1]$, then $\frac{dy}{dx} + 2y = 1 \Rightarrow y = \frac{1}{2} + C_1 e^{-2x}$ $\therefore y(0) = 0 \Rightarrow y(x) = \frac{1}{2} \frac{1}{2} e^{-2x}$

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Here,
$$y(1) = \frac{1}{2} - \frac{1}{2}e^{-2x} = \frac{e^2 - 1}{2e^2}$$

When
$$x \notin [0,1]$$
 then $\frac{dy}{dx} + 2y = 0 \Rightarrow C_2 = \frac{e^2 - 1}{2}$

:
$$y(1) = \frac{e^2 - 1}{2} \Rightarrow \frac{e^2 - 1}{2} = c^2 e^{-2} \Rightarrow C_2 = \frac{e^2 - 1}{2}$$

$$y(x) = \left(\frac{e^2 - 1}{2}\right)e^{-2x} \Rightarrow y\left(\frac{3}{2}\right) = \frac{e^2 - 1}{2e^3}$$

9.
$$S_1 \rightarrow V(ax+b) = a^2V(x)$$

$$S_2 \rightarrow \text{mean}(ax \pm b) = a \text{ mean}(X) \pm b$$

Mode
$$(x_1, x_2, x_n) = K$$

Mode of
$$(ax_1 \pm \alpha, ax_2 \pm \alpha, \dots ax_n \pm \alpha) = ak \pm \alpha$$

central tendencies depend on shift of origin & scaling

Variance depends on scaling but not shift of origin

$$f - g = x$$
 x is rational

10.
$$=-x$$
 x is irrational

11. Use graphs of
$$\sin^{-1}(\sin x)$$
 and $\cos^{-1}(\cos x)$

14.
$$P(X_i > Y_i) + P(X_i < Y_i) + P(X_i = Y_i) = 1$$

And
$$P(X_i > Y_i) = P(X_i < Y_i) = p$$

For
$$i = 2$$
 $P(X_2 = Y_2) = P(5,5) + P(4,4) = \frac{5}{12} \times \frac{5}{12} \times 2 + \frac{1}{6} \times \frac{1}{6}$

$$= \frac{25}{72} + \frac{1}{36} = \frac{27}{72} = \frac{3}{8} \quad P(X_2 > Y_2) = P(10,0) = \frac{5}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{1}{6} \times 2 = \frac{5}{16}$$

For
$$i = 3$$
 $P(X_3 = Y_3) = P(6,6) + P(7,7) = \frac{1}{6 \times 6 \times 6} + \frac{5}{12} \times \frac{1}{6} \times \frac{5}{12} \times 6 = \frac{77}{432}$

$$P(X_3 > Y_3) = \frac{1}{2} \left(1 - \frac{77}{432} \right) = \frac{355}{864}$$

$$III \rightarrow T, IV \rightarrow S$$

16.
$$S_1$$
: $a = b = c = 0$ Then it is not linear

S2 is obviously True

17. Since, given
$$|a+b+c| = |a+b-c|$$
 $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}+\vec{b}-\vec{c}|^2$

$$\Rightarrow 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \Rightarrow 4\vec{a} \cdot \vec{c} = 0$$

So, (B) is incorrect

Now,
$$|\vec{a} + \lambda \vec{c}|^2 \ge |\vec{a}|^2$$
 True $\forall \lambda \in R$ (A) is correct.

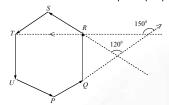
$$\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) = \overrightarrow{PQ} \times \overrightarrow{RT}$$
 (using triangle law)

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$$= |\overrightarrow{PQ}| \times |\overrightarrow{RT}| \sin 150 \, \hat{n} \neq 0 \Rightarrow Statement - 1 \, is \, true.$$

Also,
$$\overrightarrow{PQ} \times \overrightarrow{RS} = \left| \overrightarrow{PQ} \right| \times \left| \overrightarrow{RS} \right| \sin 120^{\circ} \times \hat{n}_{1} \neq 0$$

And
$$\overrightarrow{PQ} \times \overrightarrow{ST} = |\overrightarrow{PQ}| \times |\overrightarrow{ST}| \sin 180^{\circ} \times \hat{n}_{2} \neq 0$$
 : Statement – 2 is false

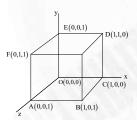


- 18. $AB = 0 |B| \neq 0 \quad AB = 0 \Rightarrow A = 0$
- 19. Draw graphs
- 20. Refer P & C synapses Integrated and objective
- 21. Since area of $\triangle OCP$ $\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}; PC = \sqrt{7}$



$$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2 = 2(5+7) = 24$$

22.



Equation of face diagonal OD line is $I_1: \vec{r} = \lambda(\hat{i} + \hat{j})$

Equation of main diagonal BE is $I_2: \vec{r} = \hat{j} + \mu(\hat{i} - \hat{j} + \hat{k})$

In other case S.D is zero.

23. Statement-I: Refer to chord properties of conics

Statement -II: $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\} \Rightarrow \sin \theta = (\sqrt{2} + 1) \cos \theta \Rightarrow \tan \theta = \sqrt{2} + 1$

$$Now, Q = \left\{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\right\} \implies \cos \theta = \left(\sqrt{2} - 1\right) \sin \theta \implies \tan \theta = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$\Rightarrow \tan \theta = \sqrt{2} + 1 \qquad \therefore P = Q \qquad \text{Statement -III: } (a - b)^2 + (b - c)^2 + (c - b)^2 = 0$$

24. Statement-I: Use addition theorem of probability



Statement-II: Total number of relation both symmetric and reflexive = $2^{\left(\frac{n^2-n}{2}\right)}$

And total number of symmetric relation = $2^{\left(\frac{n^2+n}{2}\right)}$

Then number of symmetric relation which are not reflexive =

$$2^{\frac{n(n+1)}{2}} - 2^{\left(\frac{n(n-1)}{2}\right)} = 2^{10} - 2^6 = 1024 - 64 = 960$$

25. Apply De morgan's law $(B \cup C)' = B' \cap C$

Complement of set B containing subsets of A which do not contains 2 but element 1.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of (3, 4, 5, 6, 7) whose sum of elements plus 1 is composite.

Number of 5 elements subset = 1

Number of 4 elements subset which does not include 3 or 7=3

Number of 3 elements subset = 6 (except selecting $\{3, 4, 5\}, (3, 6, 7), \{4, 5, 7\} \text{ or } \{5, 6, 7\}$)

Number of 2 elements subset =7 (except selecting $(3, 7), \{4, 6\}, \{5, 7\}$)

Number of 1 elements subset -3 (except selecting {4} or {6})

Number of 0 elements subset = 1

Here, =1+3+6+7+3+1=21

Therefore, $n(B \cup C) = 27 - 21 = 107$

Statement-II:
$$x = (8\sqrt{3} + 13)^{13} = {}^{13}C_0(8\sqrt{3})^{13} + {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + \dots (i)$$

 $x' = (8\sqrt{3} - 13)^{13} = {}^{13}C_0(8\sqrt{3})^{13} - {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + \dots (ii)$
By (i)-(ii)
 $x - x' = 2 \Big[{}^{13}C_0(8\sqrt{3})^{13}(13)^1 + {}^{13}C_3(8\sqrt{3})^{10}(13)^3 \dots \Big]$

Therefore, x - x' is even integer, hence [x] is even

Now,
$$y = (7\sqrt{2} + 9)^9 = {}^9C_0(7\sqrt{2})^9 + {}^9C_1(7\sqrt{2})^8(9)^1 + {}^9C_2(7\sqrt{2})^7(9)^2$$
(iv)

By (iii) - (iv)

$$y - y' = 2 \left[{}^{9}C_{1} \left(7\sqrt{2} \right)^{8} \left(9 \right)^{1} + {}^{9}C_{3} \left(7\sqrt{3} \right)^{6} \left(9 \right)^{3} + \dots \right]$$

y - y' = Even integer hence [y] is even

Statement –IV: ns is quadratic without constant term

Statement –V:

Given expression:
$$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$$

We know that using binomial theorem,

$$(x+a)^n + (x-a)^n = 2 \lceil {}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \dots \rceil$$

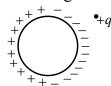
:. The given expression =
$$2 \left[{}^{5}C_{0}x^{5} + {}^{5}C_{2}x^{3}(x^{3} - 1) + {}^{5}C_{4}x(x^{3} - 1)^{2} \right]$$



PHYSICS

26.
$$V_d = \mu E = \frac{\mu V}{d} \Rightarrow v_d \propto V$$
. If V (potential difference)

27. Net charge = 0



Net charge = 0

28. Acceleration of charged particle $\vec{a} = \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B})$

Released from rest $\Rightarrow \vec{a} = \frac{q}{m}\vec{E} = a_0 (west)$ $\Rightarrow \vec{E} = \frac{ma_0}{e} (west)$

When it is projected towards north, acceleration due to magnetic force = $2a_0$

Therefore magnetic field = $\frac{2ma_0}{ev_0}(down)$, Direction of magnetic field $\overrightarrow{F}_m \rightarrow (\overrightarrow{v} \times \overrightarrow{B})$

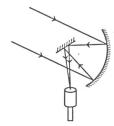
 $-\hat{i} \rightarrow (\hat{j} \times -\hat{k})$, The direction of magnetic force is towards west, for that the direction of magnetic field is vertically downward $(-\hat{k})$

- 29. Here $\vec{\tau} = \vec{M} \times \vec{B} = \vec{O}(:\vec{M} \uparrow \uparrow \vec{B})$, Also parallel currents attract each other.
- 30. As seen from graph, $\Delta \ell_A = \Delta \ell_B \Rightarrow \frac{F_A L_A}{\pi r_A^2 Y_A} = \frac{F_B L_B}{\pi r_R^2 Y_B} \Rightarrow \frac{10 \times L}{\pi r_A^2 \times Y} = \frac{40 \times L}{\pi r_B^2 \times Y} \Rightarrow \frac{r_A}{r_B} = \frac{1}{2}$
- 31. Balance A read less than 2kg & balance B read more than 5kg due to buoyancy
- 32. $m = \frac{4}{3}\pi r^3 \rho m$, Keeping m constant, if r is halved, ρ will increased by a factor of 8.

Now, $V_0 \propto r^2 \rho$ $V_0' \propto \frac{r^2}{4} (8\rho) \text{ or } V'O \propto 2r^2 \rho$, Dividing, $\frac{V_0'}{V_0} = 2 \text{ or } V_0' = 2V_0$ or $V_0' = 2V_0$

- 33. Saturation current is different so intensity is different but stopping potential is same and frequency is same so cathode material also same.
- 34. Slope of position-time graph gives velocity
- 35. Frictional force between two surfaces in contact always opposes relative motion between the surfaces in contact
- 36. $\mu = \frac{\text{Real thickness}(x)}{Apparent thickness}(y) \Rightarrow y = \frac{x}{\mu}, \frac{y_v}{y_R} = \frac{x}{\mu_v} \times \frac{\mu_R}{x} = \frac{\mu_R}{\mu_v}, \text{Since } \mu_v > \mu_R \qquad \therefore y_R > y_v$

Red colour has maximum apparent thickness. Hence red colour is least raised.



37.



In a reflecting telescope secondary mirror turns light towards eyepiece which is outside the telescope tube

38. Due to scattering of light by water droplets

39.
$$\frac{Rotational \ K.E}{Total \ K.E} = \frac{2}{5} \Rightarrow \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} = \frac{2}{5} \Rightarrow \frac{\frac{1}{2}mK^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{2}mK^2\omega^2} = \frac{2}{5}$$
$$\Rightarrow \frac{K^2}{K^2 + R^2} = \frac{2}{5} \Rightarrow K^2 = \frac{2}{3}R^2 \text{ or } K = \sqrt{\frac{2}{3}}R, \text{ So it will be hollow sphere}$$

- 41. $P = \overrightarrow{F} \cdot \overrightarrow{v}$, Tension is always perpendicular to velocity
- 42. $\left(x_m \propto \frac{1}{T}\right)$ for paramagnetic material
- 43. Since conducting shells come in contact they have same potential hence will flow to outer shell.
- 44. $f \propto V$ & $V \propto \sqrt{T}$ (where T is Temperature)

45.
$$E = -y\hat{i} - x\hat{j}$$
 $\tan \theta = \frac{E_y}{E_x} = \frac{x}{y}$

- 46. Since only 6 different wavelengths are excited, therefore highest excited stat is n-4. Two wavelengths are shorter than λ_0 , Initially atoms were in excited state n=2 Corresponding transitions are $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$, $2 \rightarrow 1$.
- 47. After completion of every one oscillation particle returns to initial point with same velocity which it has initially at that point
- 48. For (A): Orbital speed $v_0 = \sqrt{\frac{GM}{r}}$, For (B): Time period of revolution $T^2 \propto r^3$ For (C/D): Total energy $= -\frac{GMm}{2r}$
- 49. No. of collisions per unit area $\infty \frac{1}{time\ between\ two\ collisions \times area}$ $n/A \propto \frac{V_{rms}}{\text{distance\ b/w\ walls} \times A} \quad V_1\ rms \propto \sqrt{T}\ , \qquad \text{Distance\ b/w\ walls} \times A\ \text{volume}$ So $\frac{n}{A} \propto \frac{T^{1/2}}{v}$, If both T and V are halved, $\frac{n}{A}$ increases.
- 50. According to the standard Model, free protons are stable particles because their spontaneous decay has never been observed.

 Free neutrons are unstable and decay into a proton, an electron, and an antineutrino through a process called beta decay.

CHEMISTRY

51. Key-2

Sol: Diastereomer of each other

52. Key: 2

Sol: (A) is more stable radical and undergoes Markovnikov addition to form (B)

- 53. I,III,IV are statements are Correct
- 54. KEY-4

55.
$$\Lambda_m = \frac{k}{M} \times 1000; (x+y) = \frac{k}{M} \times 1000$$

$$\Rightarrow M = \frac{1000 \times k}{\left(x + y\right)}$$

Solubility
$$\left(in \ gL^{-1}\right) = \frac{k \times 1000 \times 188}{x + y}$$

- 56. H₃PO₂ reduces diazonium salt into benzene derivative.
- 57. KEY-1

SOL.
$$A_x B_y \rightarrow xA^{y+} + yB^{x-}$$

$$1-\alpha$$
 $x\alpha$ $y\alpha$

$$i = 1 - \alpha + x\alpha + y\alpha$$

$$\alpha = \frac{i-1}{(x+y-1)}$$

58. KEY-3

SOL. : Cannizaro reaction

- O_2 and Xe have comparable ionization energies
- 60. KEY-3

SOL.
$$\log k = \log A - \frac{E_a}{2.303RT} (y = c + mx)$$

Slope =
$$\frac{-E_a}{2.303R} = \frac{1}{2.303} (given) \left(\tan \theta = \frac{1}{2.303} \right)$$

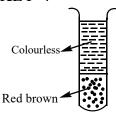
$$-E_a = 2.303R \times slope$$

$$=2.303 \times \frac{R}{2.303} = R = 2cal$$

61. $Na_2Cr_2O_7$ is hygroscopic and that is why, it is not used as a primary standard in volumetric analysis.

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62. KEY-4



SOL.

$$Cl_2 + 2NaBr \rightarrow Br_2 + 2NaCl$$

 Cl_2 oxidises Br^- to Br_2 in trichloro ethane is reddish brown in colour. Two layers are observed as trichloro ethane is immisible with water.

63. KEY-4

- SOL. Solubility is directly proportional to K_{Sp} . MnS has highest K_{Sp} anong the given substances and hence has highest solubility
- 64. KEY-2

SOL. Conceptual

OH
$$CH_3$$
 CH_3 CH_3 CH_3 CH_3 CH_3 CH_3

- 65. KEY-1
- 66. Key-2
- sol. C1⁻ is weak ligand so no pairing of electrons take palce. so [NiCl₄]⁻² is tetrahedral. pph₃ group is bulkier one so it favours tetrahedral geometry, through pph₃ is strong field ligand.
- 67. KEY-2

SOL: (b) Bond dissociation energy orders

$$Cl_2 > Br_2 > F_2 > I_2$$

242.6 192.8 158.8 151.1 kj/mol

68.
$$PV^{\gamma} = K \left(K \text{ is constant} \right) \qquad dPV^{\gamma} + P\gamma V^{\gamma-1} dV = 0$$

$$\frac{dP}{P} - \frac{\gamma V^{\gamma-1}}{V^{\gamma}} dV \qquad \frac{dP}{P} = -\gamma \frac{dV}{V}$$

- 69. KEY-3
- 70. KEY-1
- 71. KEY-6

SOL: CONCEPTUAL

- 72. pair of diastereomers is formed (m), therefore, two fractions are obtained
- 73. P= 1-Bromo 3-chloro Benezen
- 74. KEY-9

SOL.
$$B_2H_6 - 6 = x B_3N_3H_6 - 12 = y$$

$$\frac{x+y}{2} = \frac{18}{2} = 9$$

75. n-factor = 4