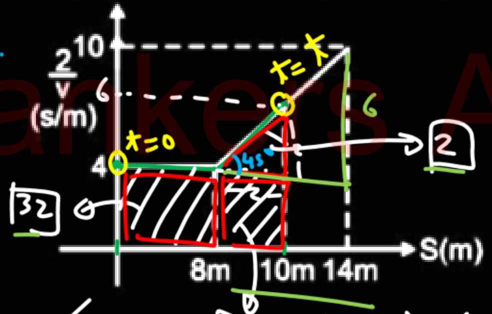
# PHYSICS

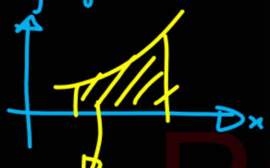
Figure shows $\frac{2}{v}$ v/s s curve for a particle of mass 2 kg moving in a straight line. If the time (in seconds) taken by the particle to achieve a displacement of 10 m from start is t. (v = velocity, s = displacement ), then find the value of (t − 20).

$\boxed{m = 2}$




(A) 1 s (B) 21 s
(C) 3 s (D) 42 s

$y = f(x)$



$$\boxed{\text{Area}_{y\text{-}x} = \int y \cdot dx}$$

$$\text{Area}_{\left(\frac{2}{v} - s\right)} = \int \left(\frac{2}{v}\right) \cdot ds$$

$$\text{Area} = 2\int \frac{ds}{\left(\frac{ds}{dt}\right)} = 2\int_0^t dt$$

$$= 2t$$

$32 + 8 + 2$

$\boxed{42}$

$\boxed{t = 21}$

$$\boxed{\text{Ans} = t - 20 = 1}$$

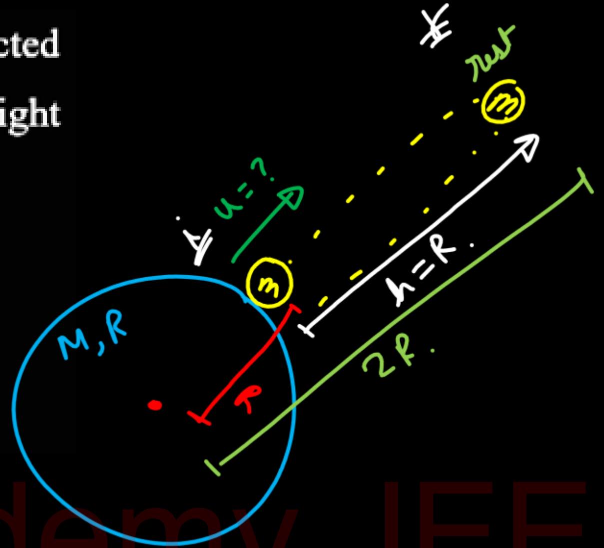
2

JEE1

With what velocity should a particle be projected from the surface of earth so that it's height becomes equal to radius of earth

(A) $\left(\frac{GM}{R}\right)^{\frac{1}{2}}$ (B) $\left(\frac{8GM}{R}\right)^{\frac{1}{2}}$

(C) $\left(\frac{2GM}{R}\right)^{\frac{1}{2}}$ (D) $\left(\frac{4GM}{R}\right)^{\frac{1}{2}}$






$$E_i = E_f$$

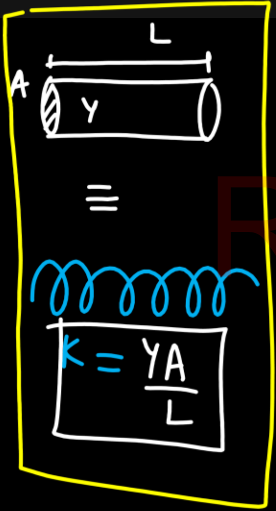
$$-\frac{GM\cancel{m}}{R} + \frac{1}{2}\cancel{m}u^2 = -\frac{GM\cancel{m}}{2R} + 0$$

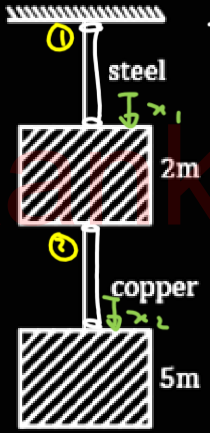
$$\frac{1}{\cancel{2}}u^2 = \frac{GM}{\cancel{2}R}$$

$$u = \sqrt{\frac{GM}{R}}$$

3

If the ratio of diameters, lengths and young's modulus of steel and copper wires shown in the figure are p, q and s respectively, then the corresponding ratio of increase in their lengths would be $\frac{Mq}{Nsp^2}$ then find value of M + N

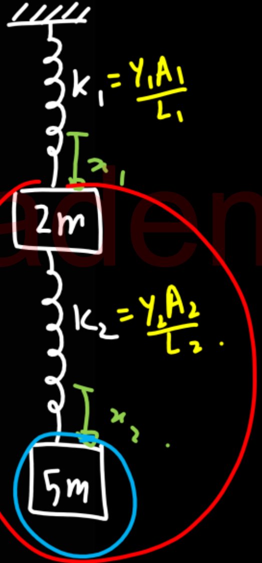



$$\frac{x_1}{x_2} = \frac{M\, q}{N\, sp^2}$$

$$\frac{d_1}{d_2} = p$$

$$\boxed{\frac{A_1}{A_2} = p^2} \qquad \boxed{\frac{L_1}{L_2} = q} \qquad \boxed{\frac{Y_1}{Y_2} = s}$$




(A) 8
(B) 12
(C) 7
(D) 5

$K_1 x_1 = 7Mg$

$K_2 x_2 = 5Mg$ $\quad \div$

$$\frac{x_1}{x_2} = \frac{7}{5}\cdot\frac{K_2}{K_1} = \frac{7}{5}\,\frac{\frac{Y_2A_2}{L_2}}{\frac{Y_1A_1}{L_1}}$$

$$\frac{x_1}{x_2} = \frac{7}{5}\left(\frac{Y_2}{Y_1}\right)\cdot\left(\frac{A_2}{A_1}\right)\cdot\left(\frac{L_1}{L_2}\right)$$

$$\frac{x_1}{x_2} = \frac{7}{5}\,\frac{q}{s\,p^2}$$

M = 7
N = 5

$\boxed{M + N = 12}$

km hr

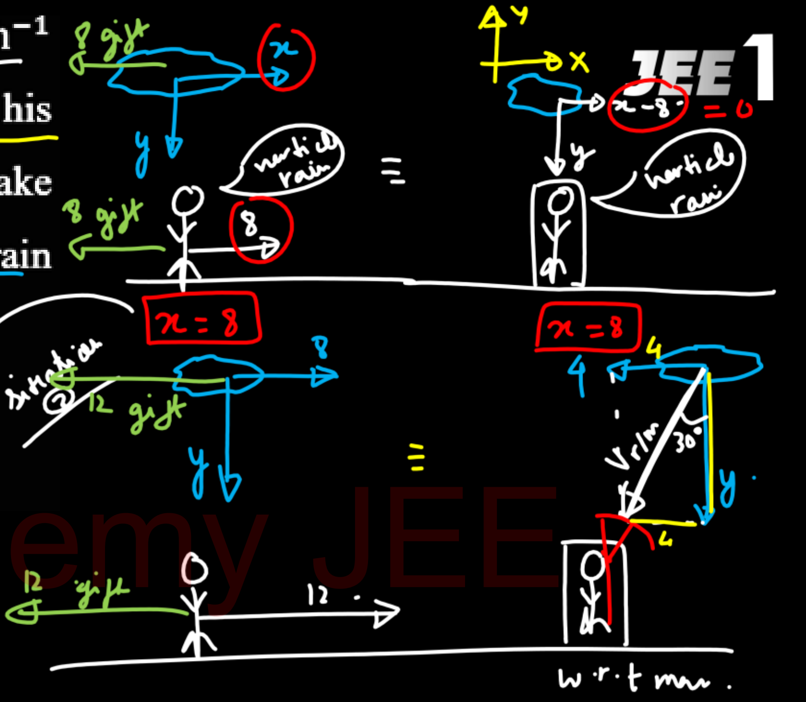
A man running on a horizontal road at 8 km h$^{-1}$ finds the rain falling vertically. He increases his speed to 12 km h$^{-1}$ and find that the drops make angle 30° with vertical. Find the speed of the rain with respect to the road.

(A) 5.5 km h$^{-1}$

(B) $4\sqrt{7}$ km h$^{-1}$

(C) 2.2 km h$^{-1}$

(D) 12 km h$^{-1}$






$\vec{V}_r = x\hat{i} - y\hat{j}$

$$\boxed{\text{Ans } V_r = \sqrt{x^2+y^2}}$$

$$\tan 30^\circ = \frac{4}{y}$$

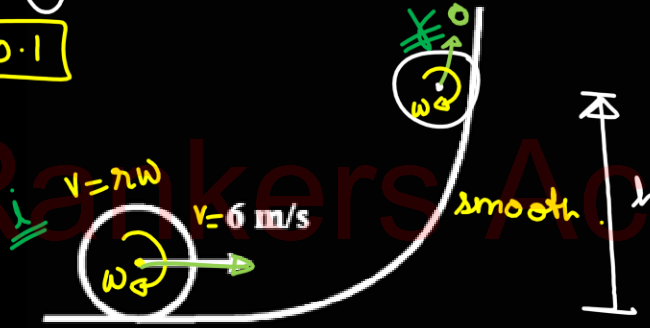
$$\boxed{y = 4\sqrt{3}}$$

$$V_r = \sqrt{(2\times4)^2+(4\sqrt{3})^2} = 4\sqrt{4+3} = \boxed{4\sqrt{7}}$$

5

A disc of radius 0.1 m rolls without sliding on a horizontal surface with a velocity of 6 m/s. It then ascend a smooth continuous track as shown in figure. The height upto which it will ascend is $\frac{x}{5}$ m. Value of x is:- [Take $g = 10\ m/s^2$]

$\boxed{r = 0.1}$




(A) 18
(B) 37
(C) 27
(D) 9 ✓

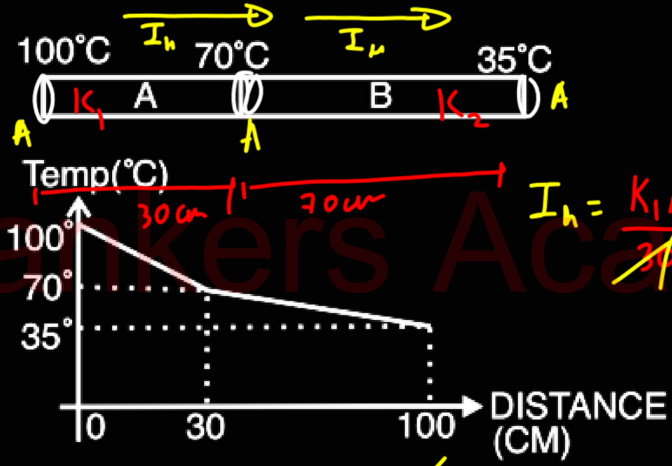
$E_i = E_f$

$\frac{1}{2}mv^2 + \cancel{(\ )} = mgh + \cancel{(\ )}$

$h = \frac{v^2}{2g} = \frac{36}{20} = 1.8 = \frac{18}{10} = \frac{9}{5} = \frac{x}{5}$

$\boxed{x = 9}$

Two different rods A and B are kept as shown in figure. The variation of temperature of different cross sections with distance is plotted in a graph shown in figure. The ratio of thermal conductivities of A and B is-



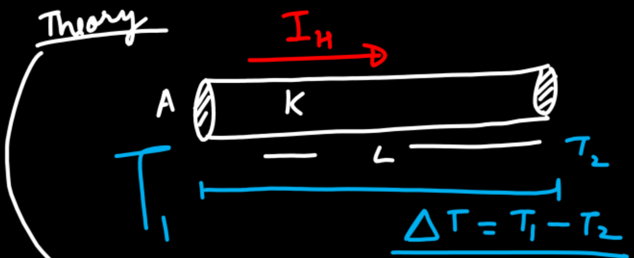

(A) 2

(B) 0.5

(C) 1

(D) 2/3

Theory




$\Delta T = T_1 - T_2$

$$\boxed{I_H = \frac{KA}{L}\cdot(\Delta T)}$$

$\frac{K_1}{K_2} = ?$

$$I_h = \frac{K_1 A \cdot (\cancel{30})}{\cancel{30}} = \frac{K_2 A (\cancel{35})}{\cancel{70}_{\,2}}$$

$$\boxed{\frac{K_1}{K_2} = \frac{1}{2}}$$

A nucleus rupture into two nuclear parts which have their velocity ratio equal to $\boxed{2:1}$. What will be the ratio of their nuclear size (nuclear radius)

(A) $2^{1/3}:1$

(B) $1:2^{1/3}$

(C) $3^{1/2}:1$

(D) $1:3^{1/2}$

Ans

$$\boxed{R = R_0 A^{1/3}}$$

$$\boxed{\frac{R_1}{R_2} = ?}$$


$$\boxed{P_i = P_f}$$

$$0 = 0$$

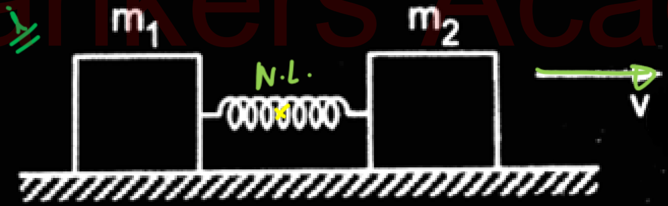
$$\frac{m_1}{m_2} = \frac{1}{2} = \frac{A_1}{A_2}$$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$$

8

Two blocks of mass $m_1$ and $m_2 (m_1 < m_2)$ are connected with an ideal spring on a smooth horizontal surface as shown in figure. At $t = 0$, $m_1$ is at rest and $m_2$ is given a velocity v towards right. At this moment, spring is in its natural length. Then choose the correct alternative:

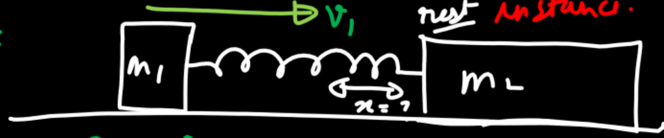



(A) Block of mass $m_2$ will be finally at rest after some time.
(B) Block of mass $m_2$ will never come to rest.
(C) Both the blocks will be finally at rest.
(D) None of these

$\vec{P}_{sys} \rightarrow$ conserved.

$\boxed{E} \rightarrow$ conserved.

Let's assume $m_2 \rightarrow$ rest at some instance.




$P_i = P_f$

$m_2 v = m_1 v_1$

$\boxed{v_1 = \left(\frac{m_2}{m_1}\right) \cdot v}$

$\boxed{K_i = \frac{1}{2} m_2 v^2}$

$K_f = \frac{1}{2} m_1 v_1^2$

$= \frac{1}{2} \cancel{m_1} \cdot \frac{m_2^2}{m_1^{\cancel{2}}} \cdot v^2$

$\boxed{K_f = \left(\frac{m_2}{m_1}\right) \cdot \frac{1}{2} m_2 v^2}$

$K_i < K_f$

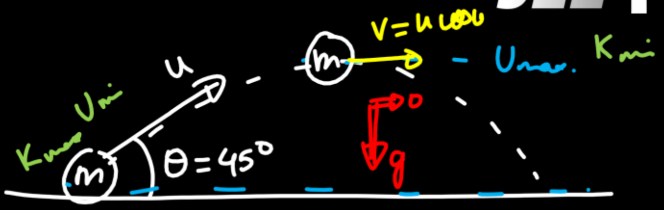
N.P.

9

An object of mass m is projected with momentum P at such an angle that is its maximum height (H) is 1/4th of its horizontal range ( R ). Its minimum kinetic energy in its path will be

(A) $\frac{p^2}{8m}$ (B) $\frac{p^2}{4m}$

(C) $\frac{3p^2}{4m}$ (D) $\frac{p^2}{m}$




$\boxed{p = mu}$ — ①

$\boxed{H = \frac{1}{4}R}$ (given)

formula → $\left(\frac{R}{H}\right) = \frac{4}{\tan\theta} = 4$

$\tan\theta = 1$

$\boxed{\theta = 45^\circ}$

$$\frac{\frac{u^2(2\sin\theta\cos\theta)}{g}}{\frac{u^2\sin^2\theta}{2g}} = \frac{4}{t}$$

$u = \frac{p}{m}$

$K_{min} = \frac{1}{2}mv^2$

$= \frac{1}{2}mu^2\cos^2\theta$ (45°)

$= \frac{1}{2}mu^2 \cdot \frac{1}{2}$

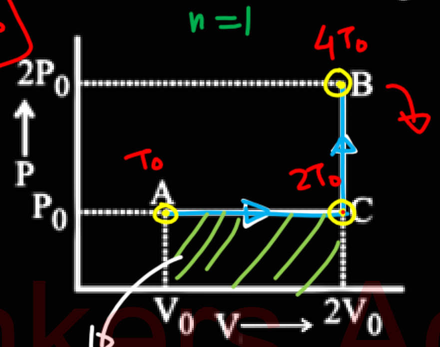
$= \frac{1}{4}mu^2$

$= \frac{1}{4}m \cdot \frac{p^2}{m^2} = \frac{p^2}{4m}$

One mole of monoatomic gas is brought from state A to state B such that temperature at A is $T_0$. Find the heat absorbed along the path ACB

$f = 3$ $\quad$ $\boxed{C_v = \frac{fR}{2}} = \frac{3R}{2}$

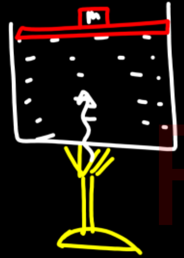
$\boxed{P_0V_0 = nRT_0}$




Area $= P_0V_0$.

(A) $\frac{11}{2}RT_0$ $\quad$ (B) $\frac{9}{2}RT_0$

(C) $\frac{7}{2}RT_0$ $\quad$ (D) $\frac{5}{2}RT_0$



$\boxed{Q = \underbrace{\Delta U}_{nC_v\Delta T} + \underbrace{W}_{\text{area}_{P\text{-}V} \text{ or } \int P\cdot dV}}$

for $\boxed{A \to C \to B}$

$\boxed{Q = \Delta U + W}$

$Q = nC_v\Delta T + \text{Area of PV}$

$Q = 1\left(\frac{3R}{2}\right)(3T_0) + P_0V_0$

$Q = \frac{9}{2}RT_0 + nRT_0$

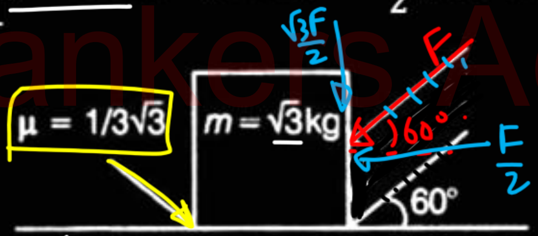
$\boxed{Q = 5.5\,RT_0}$

11

JEE 1

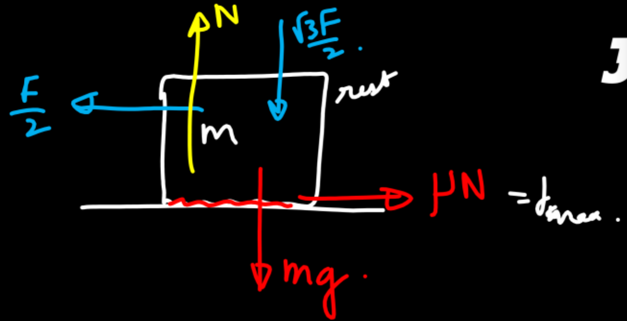
As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $1/3\sqrt{3}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be

$$\left[g = 10\ \text{ms}^{-2}; \sin 60° = \frac{\sqrt{3}}{2}; \cos 60° = \frac{1}{2}\right]$$




(A) 10 ✓ (B) 13

(C) 16 (D) 19




$$\frac{F}{2} = \mu N \quad —(1)$$

$$N = \frac{\sqrt{3}F}{2} + mg \quad —(2)$$

$$\frac{F}{2} = \mu\left[\frac{\sqrt{3}F}{2} + mg\right]$$

$$\frac{F}{2} = \frac{1}{3\cancel{\sqrt{3}}}\left[\frac{\cancel{\sqrt{3}}F}{2} + 10\cancel{\sqrt{3}}\right]$$

$$\frac{F}{2} - \frac{F}{6} = \frac{10}{3}$$

$$3F - F = 20$$

$$\boxed{F = 10}$$

12

What are the dimensions of (A/B) in the relation $F = A\sqrt{x} + Bt^2$, where F is the force, x is the distance and t is time?

(A) $ML^2T^{-2}$
(B) $L^{-1/2}T^2$
(C) $L^{-1/2}T^{-1}$
(D) $LT^{-2}$

$[x] = L$
$[t] = T$

$[F] = MLT^{-2}$

$[A][x]^{1/2} = MLT^{-2}$
$[A]\, L^{1/2} = MLT^{-2}$
$[A] = ML^{1/2}T^{-2}$

$[B][t]^2 = MLT^{-2}$
$[B]\, T^2 = MLT^{-2}$
$[B] = MLT^{-4}$

$\left[\frac{A}{B}\right] = \frac{[A]}{[B]} = L^{-1/2}T^2$

13

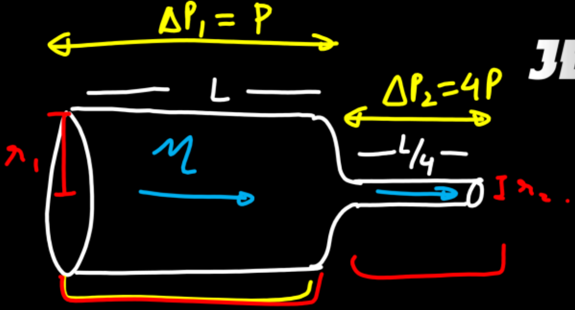
JEE 1

Two tubes of radii $r_1$ and $r_2$, and lengths $l_1$ and $l_2$, respectively, are connected in series and a liquid flows through each of them in streamline conditions. $P_1$ and $P_2$ are pressure differences across the two tubes. If $P_2$ is $4P_1$ and $\boxed{l_2 \text{ is } \frac{l_1}{4}}$, then the radius $\boxed{r_2}$ will be equal to:

(A) $r_1$ (B) $2r_1$

(C) $4r_1$ (D) $\frac{r_1}{2}$

viscous flow



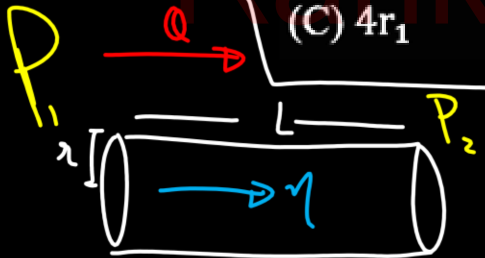

$Q_1 = Q_2$

$$\frac{\cancel{\pi} r_1^4 (\cancel{P})}{\cancel{8\eta} \cancel{L}} = \frac{\cancel{\pi} r_2^4 (4\cancel{P})}{\cancel{8\eta}\left(\frac{\cancel{L}}{4}\right)}$$

$$\frac{\Delta P_1}{\Delta P_2} = \boxed{\frac{P_1}{P_2} = \frac{1}{4}}$$

$$\left(\frac{r_1}{r_2}\right)^4 = 16 = 2^4$$

$$\frac{r_1}{r_2} = 2 \qquad \boxed{\frac{r_1}{2} = r_2}$$



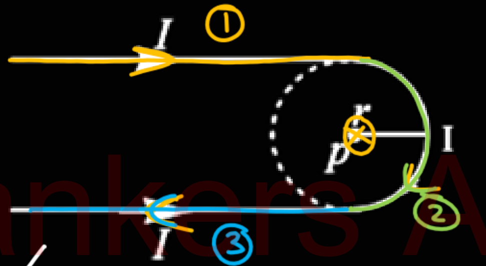

$$\boxed{\Delta P = P_1 - P_2}$$

$$\boxed{Q = \frac{\pi r^4 \cdot \Delta P}{8\eta L}}$$

14

JEE 1

A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle?




(A) $\frac{\mu_0 I}{4\pi r}(2+\pi)$ (B) $\frac{\mu_0 I}{4\pi r}(2-\pi)$

(C) $\frac{\mu_0 I}{2\pi r}(2-\pi)$ (D) $\frac{\mu_0 I}{2\pi r}(2+\pi)$

$$\vec{B}_3 = \vec{B}_1 = \frac{\mu_0 i}{4\pi r} \otimes$$

$$\vec{B}_2 = \frac{\mu_0 i}{4r} \otimes$$

$$B_{net} = B_1 + B_2 + B_3$$

$$= \frac{\mu_0 i}{4r}\left(1 + \frac{2}{\pi}\right)$$

$$= \frac{\mu_0 i}{4\pi r}(2+\pi)$$

15

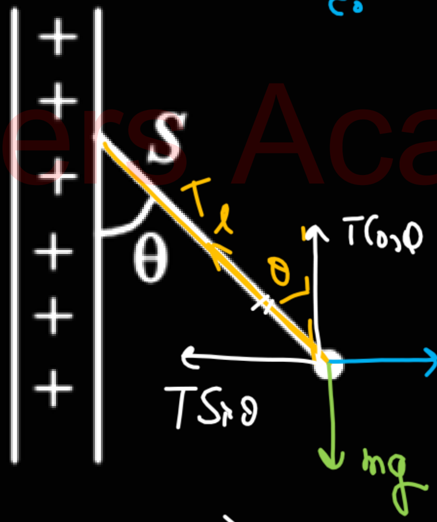
A charged ball hangs from a silk thread of length l. It makes an angle θ with a large charged conducting sheet P as shown in the figure. The surface charge density σ of the sheet is proportional to

$E = \frac{\sigma}{\varepsilon_0}$ — (1)

$T\sin\theta = \frac{q\sigma}{\varepsilon_0}$

$T\cos\theta = mg$

$\boxed{\sigma \propto \tan\theta}$




$F = qE = q\frac{\sigma}{\varepsilon_0}$ — (1)

(A) cos θ  ~~(B)~~ cot θ

~~(C)~~ sin θ  (D) tan θ

16

JEE 1

A 0.5 m wire of mass 5 g in its second overtone (one end fixed and one end free) is in resonance with organ pipe of 1 m in its $\boxed{\text{fundamental mode.}}$ If tension in wire (in Newton) is T and if speed of sound in air is 320 m/s, then find the value of $\left(\frac{T}{2.56}\right)$.

$T = ?,\ \mu,\ v' = \sqrt{\frac{T}{\mu}}$ speed


(A) 2
(B) 4 ✓
(C) 1
(D) 1.6

$\mu = \frac{m}{l} = \frac{5\times10^{-3}\,kg}{0.5\,m}$

$= 10^{-2}\ kg/m$ — (1)

first overtone

$f = \frac{v}{4L} = \frac{320}{4\times1} = 80\ Hz$ — (2)

$l = \frac{5\lambda}{4} = \frac{5}{4}\frac{v'}{f}$

$\Rightarrow v' = \frac{4fl}{5} = \frac{4\times80\times0.5}{5\times1}$

$= 32\ m/s$

$v' = \sqrt{\frac{T}{\mu}} \Rightarrow T = v'^2\mu = 32^2\times10^{-2}$

$T = 10.24$

$T/2.56 = \frac{10.24}{2.56} = 4$

17

JEE 1

What is the ratio of the circumference of the first Bohr orbit for the electron in the hydrogen atom to the de-Broglie wavelength of electrons having the same velocity as the electron in the $\boxed{\text{first Bohr}}$ orbit of the hydrogen atom?

(A) 1: 1

(B) 1: 2

(C) 1: 4

(D) 2: 1

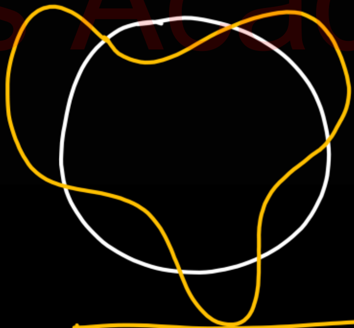
$l = \frac{nh}{2\pi}$

Concept

$mvr = \frac{h}{2\pi}$

$2\pi r = \frac{h}{mv}$

$\boxed{\text{Circumference} = \lambda}$



$\boxed{\text{Circumference} = 2\pi r = n\lambda}$

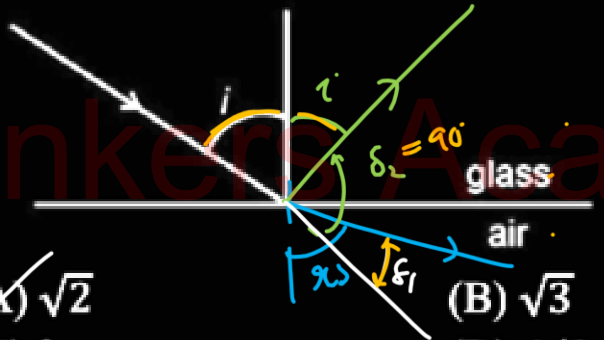
18

JEE 1

A ray of light travelling from glass to air is incident at angle i. Maximum $\boxed{\text{angle of deviation}}$ suffered for 'any angle' of incidence is $\frac{\pi}{2}$, then refractive index of glass is

Refraction $\delta_1 = r - i$ — ① ✗

TIR $\delta_2 = 180^\circ - 2i$ — ② ✓




Since $\delta = 90^\circ$

$180^\circ - 2i = 90^\circ$

$i = 45^\circ$

(A) $\sqrt{2}$ (B) $\sqrt{3}$
(C) 2 (D) 4/3

$\mu = \frac{1}{\sin i} \Rightarrow \mu = \frac{1}{\sin 45}$

$\boxed{\mu = \sqrt{2}}$

19

**Assertion:** - A p – n junction with reverse bias can be used as a photo diode to measure light intensity

**Reason:** In a reverse bias condition the current is small but it is more sensitive to changes in incident light intensity

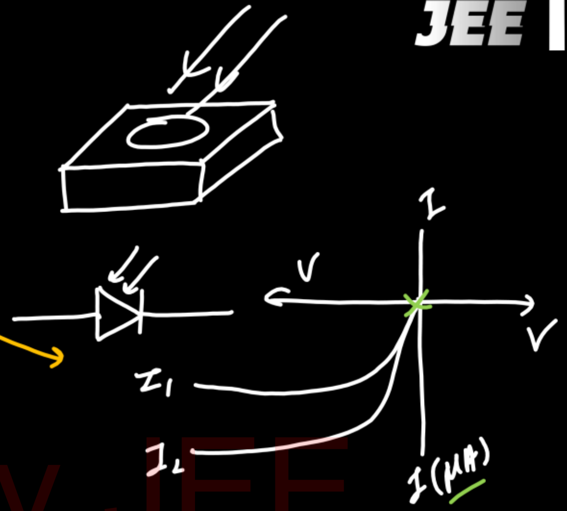
μA

(A) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion

(B) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion

(C) Assertion is correct but Reason is in correct

(D) Both Assertion and Reason are incorrect



20

JEE 1

In Young's double slit experiment using monochromatic light, the fringe patterns shifts by a certain distance ($\Delta$) on the screen when a mica sheet of refractive index 1.6 ($\mu$) and thickness 1.964 ($t$) micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled ($D \to 2D$). It is found that the distance between successive maxima now ($\beta'$) is the same as the observed fringe shift ($\Delta$) upon the introduction of the mica sheet. The wavelength of light is

(A) 5762Å (B) 5825Å

(C) 5892Å (D) 6500Å

$$\Delta = \frac{(\mu-1)tD}{d} \quad \text{—— (1)}$$

$$\beta' = \Delta$$

$$\frac{\lambda(2\cancel{D})}{\cancel{d}} = \frac{(\mu-1)t\cancel{D}}{\cancel{d}}$$

$$2\lambda = (\mu-1)t$$

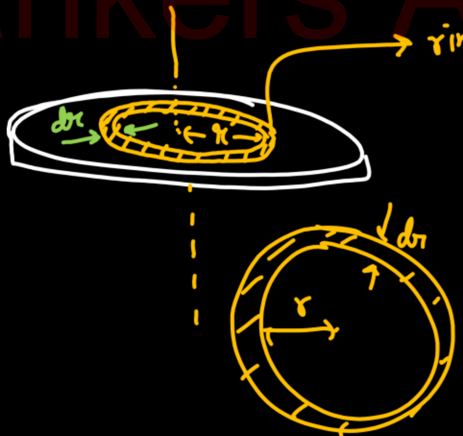
$$\lambda = \frac{(\mu-1)t}{2}$$

$$= \frac{0.6 \times 1.964 \times 10^{-6}\ m}{2}$$

$$= 3 \times 1964 \times 10^{-10} = 5892\ \text{Å}$$

21

Mass density of a disc is given by $\boxed{\sigma = \sigma_0 \frac{r}{R}}$, where $\sigma_0$ is constant, r is distance from centre and R is radius of disc. Moment of inertia of disc about an axis passing through centre and perpendicular to plane of disc is $\frac{n\pi\sigma_0 R^4}{m}$. Find the value of $n \times m$ is




ring element

$dI = (dm) r^2$

$= (\sigma\, dA) r^2$

$= \left(\sigma_0 \frac{r}{R}\right)(2\pi r\, dr) r^2$

$dA = \text{Circumference} \times \text{thickness}$
$= 2\pi r \times dr$ — (1)

$$\int_0^I dI = \frac{\sigma_0}{R} 2\pi \int_0^R r^4\, dr$$

$$I = \frac{\sigma_0}{R} \frac{2\pi R^5}{5}$$

$$I = \frac{2\pi\sigma_0 R^4}{5}$$

$n = 2 \quad m = 5$

$n \times m = 2 \times 5 = 10$

Ans

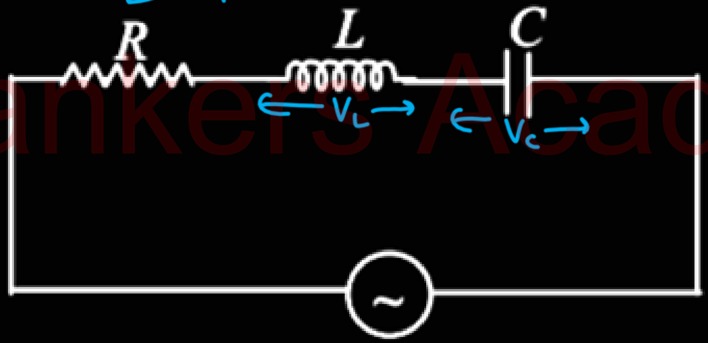
22

JEE 1

In the circuit shown, the value of L is 5 henry and the power factor of the circuit is 0.8 . It is also given that the voltage drop across capacitor is $\frac{2}{5}$ times the voltage drop across the inductor. Find impendence (in ohm) of the circuit.

$\cos\phi$

$Z = ?$




220 sin 314 $t$

$\omega$

$X_L = \omega L = 314 \times 5$ — (1)

$\cos\phi = \frac{R}{Z} = 0.8$

$\Rightarrow R = 0.8Z$ — (2)

$V_C = \frac{2}{5} V_L \Rightarrow X_C = 0.4 X_L$ — (3)

$Z = \sqrt{R^2 + (X_L - X_C)^2}$  Eq (3)  (2)

$\Rightarrow Z^2 = (0.8Z)^2 + (X_L - 0.4X_L)^2$

$\Rightarrow (1 - 0.64)Z^2 = (0.6X_L)^2$

$Z = X_L = \omega L = 314 \times 5 = 1570\,\Omega$

23

If the radius of the earth becomes half of its present value, with its mass remaining the same, the duration of one day will become ____ (in hrs)


Cons. of Angular momentum

$$L_1 = L_2$$

$$I\omega = I'\omega'$$

$$\frac{2}{5}mR^2\omega = \frac{2}{5}m\left(\frac{R}{2}\right)^2\omega'$$

$$\omega' = 4\omega$$

$$\frac{2\pi}{T'} = 4\,\frac{2\pi}{T}$$

$$T' = \frac{T}{4} = \frac{24\text{ hrs}}{4} = 6\text{ hrs}$$

24

JEE 1

When photon of energy 4.0 eV strikes the surface of a metal A , the ejected photoelectrons have maximum kinetic energy $T_A$ eV and de-Broglie wave length $\lambda_A$. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $\boxed{T_B = (T_A - 1.5)\text{eV}.}$ ④ If the de-Broglie wave length of these photoelectrons $\boxed{\lambda_B = 2\lambda_A}$, then the work function of metal B is _____ eV .

$\phi_B = ?$

$$\boxed{K_{max} = E - \phi}$$

$T_A = 4.0\,eV - \phi_A$ — ①

$\lambda_A = \frac{h}{p_A} = \frac{h}{\sqrt{2mT_A}}$ — ②

$T_B = 4.5 - \phi_B$ — ③

$\frac{h}{\sqrt{2mT_B}} = 2\frac{h}{\sqrt{2mT_A}}$

$\Rightarrow T_A = 4T_B$ — ⑤

from ④ & ⑤

$T_B = 0.5\,eV$ & $T_A = 2\,eV$

$\phi_B = 4.5 - T_B$

$= 4.5 - 0.5$

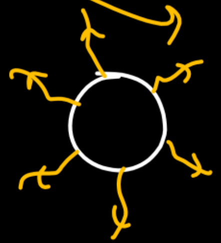
$= 4\,eV$ Ans

25

JEE 1

A solid sphere of diameter 0.1 m is at 427°C and is kept in an enclosure at 27°C. Take Stefan's constant $= \frac{20}{3} \times 10^{-8}\ W/m^2 K^4$, emissivity of the surface 0.84, specific heat 0.1kcal/kgK, density $= 9280\ kg/m^3$, $\boxed{J = 4200\ J/k\ cal}$. If $\boxed{\text{rate of decrease of temperature of the sphere}}$ is $N \times 10^{-3}$ °C/s. Then find $\left(\frac{N}{40}\right)$.

d, T, $T_0$, e



$$\frac{d\theta}{dt} = ms\frac{dT}{dt} = u = e\sigma A(T^4 - T_0^4)$$

Calorimetry

Stefan-Boltzmann's Law

$$\frac{dT}{dt} = \frac{e\sigma A}{ms}(T^4 - T_0^4)$$

$$\frac{dT}{dt} = \frac{e\sigma \cancel{4\pi R^2}}{\rho(\frac{4}{3}\pi R^3)}(T^4 - T_0^4)$$

$$= \frac{e\sigma 3}{\rho R}(T^4 - T_0^4)$$

$$= \frac{0.84 \times \left(\frac{20}{\cancel{3}} \times 10^{-8} \times \cancel{3}\right)\left[700^4 - 300^4\right]}{9280 \times \left(\frac{0.1m}{2}\right) \times 0.1\ \cancel{kcal}/kgK \times 4200\ J/\cancel{kcal}}$$

$$= \frac{8.4 \times \cancel{2} \times (\cancel{2401 - 81})}{\cancel{9280} \times 420 \times (0.05m)}$$

4 2

$$= \frac{1}{5} = 0.2 = N \times 10^{-3}$$

$$N = 200 \Rightarrow \frac{N}{40} = 5 \text{ Ans}$$

Rankers Academy JEE

# CHEMISTRY

State whether the given statements are true (T) or false( F).

T (P) $CrO_3$ in anhydrous medium oxidizes primary alcohols to aldehydes True

→ Mild oxidising agent

F (Q) Etard - reaction is used to prepare phenol Toluene → Benzaldehyde

F (R) Ortho - nitro phenol is more acidic than Para - nitro phenol H is involved in intramolecular H-bonding

T (S) Methane cannot be prepared by Kolbey's electrolysis

$R-COONa$ → $R$
$R-COONa$    $R$

(A) TTTT

(B) TFTF

(C) TTFT

(D) TFFT

2

JEE 1

Oxidation states of iron in the complexes $Na_2[Fe(CN)_5NO]$ and $[Fe(NO)(H_2O)_5]SO_4$ respectively are

(A) +2 and +1 (B) +1 and +2

(C) +2 and +2 (D) +3 and +3



$Na_2[Fe(CN)_5NO]$

$x-5+1+2=0$

$\boxed{x=+2}$

$[Fe(NO)(H_2O)_5]SO_4$

$x+1-2=0$

$x-1=0$

$\boxed{x=+1}$

3

JEE1

Two hexoses form the same osazone. Find the correct statement about these hexoses.

(A) Both of them must be aldoses

(B) They are epimers at C-3

(C) The carbon atoms 1 and 2 in both have the same configuration

(D) The carbon atoms 3, 4 and 5 in both have the same configuration

$$\begin{array}{l} CHO \\ | \\ CHOH \\ | \\ (CHOH)_3 \\ | \\ CH_2OH \end{array} \xrightarrow[-H_2O]{C_6H_5NHNH_2} \begin{array}{l} CH=N\bullet NH\ C_6H_5 \\ | \\ CHOH \\ | \\ COHOH_3 \\ | \\ CH_2OH \end{array}$$

Glucose
Phenyl hydrozone

$$\xrightarrow[\substack{-C_6H_5\,NH_2 \\ -H_2O}]{C_6H_5NHNH_2} \begin{array}{l} CH=N\bullet NH\ C_6H_5 \\ | \\ C=O \\ | \\ (CHOH)_3 \\ | \\ CH_2OH \end{array} \xrightarrow[-\,H_2O]{C_6H_6NHNH_2}$$



$$\begin{array}{l} CH=N\bullet NH\ C_6H_5 \\ | \\ C=N\bullet NH\ C_6H_5 \\ | \\ (CHOH)_3 \\ | \\ CH_2OH \end{array}$$

Glucosazone

Fructose also forms same osazone on reaction with phenyl hydrazine.

4

JEE 1

80 ml of $KMnO_4$ solution reacts with 3.4 gm of $Na_2C_2O_4 \cdot 2H_2O$ in acidic medium. Molarity of the $KMnO_4$ is

(A) 0.5 M

(B) 0.1 M

(C) 5 M

(D) 1 M



Law of equivalence

$(meq)_{KMnO_4} = (meq)_{\text{oxalic acid}}$

$M \times 80\,ml \times 5 = \frac{3.4}{126} \times 2$

$M = \frac{3.4 \times 2}{126 \times 400} = 0.13\,M$

$M \times V = \text{no of moles}$

$\text{Equivalence} = M \times n \times V$

$= \text{no. of moles} \times n$

5

JEE1

The correct order of the ionic radii of

$O^{2-}, N^{3-}, F^{-}, Mg^{2+}, Na^{+}$ and $Al^{3+}$ is:

(A) $N^{3-} < F^{-} < O^{2-} < Mg^{2+} < Na^{+} < Al^{3+}$

(B) $Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

13 12 11 9 8 7

(C) $Al^{3+} < Na^{+} < Mg^{2+} < O^{2-} < N^{3-}$

(D) $N^{3-} < O^{2-} < F^{-} < Na^{+} < Mg^{2+} < Al^{3+}$



All of them are isoelectronic species

For isoelectronic species,

$$\text{Size} \propto \frac{1}{\text{At. no}}$$

6

JEE 1

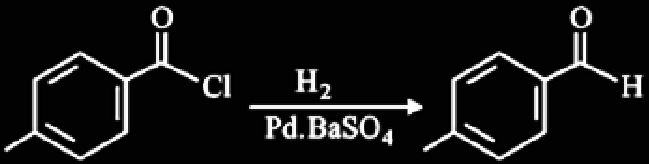
Match List-I with List - II and select the correct answer using the codes given below

| List-I | List-II |
|---|---|
| (a) $H_2O < H_2S < H_2Se < H_2Te$ B.D.E ↓ | 1. Bond Angle |
| (b) $H_2O > H_2S > H_2Se > H_2Te$ | 2. Melting and boiling point |
| (c) $H_2O \gg H_2S < H_2Se < H_2Te$ | 3. Acidic strength |

~~(A)~~ a – 1, b – 3, c – 2 (B) a – 3, b – 2, c – 1

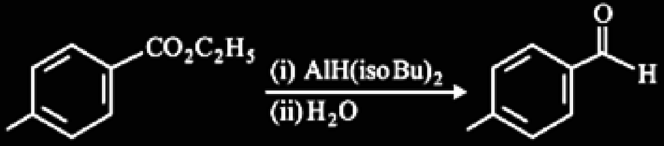
(C) a – 3, b – 1, c – 2 ~~(D)~~ a – 2, b – 3, c – 1

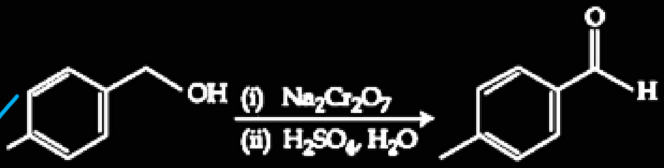
Which one of the following reactions does not represent correct combination of substrate and product under the given conditions?

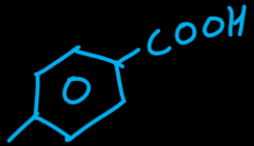
(A) 


Rosenmund Red

(B) 


(C) 


(D) 

Given below are two statements:

Statement I: The identification of $Ni^{2+}$ is carried out by dimethyl glyoxime in the presence of $NH_4OH$.

$Ni^{+2} + 2DMg^{-} \rightarrow [Ni(DMg)_2]$

Statement II: The dimethyl glyoxime is a bidentate neutral ligand.

negative



In the light of the above statements, choose the correct answer from the options given below:

(A) Statement I is false but Statement II is true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Both Statement I and Statement II are true.

Group-4 ($Ni^{2+}$, $Zn^{2+}$, $Co^{2+}$, $Mn^{2+}$)

Grp reagent : $H_2S$ in $NH_4OH$

Mixture + $H_2S$ $\xrightarrow{\text{Base}}$ ppt

ppt → $\boxed{NiS}$, $\boxed{ZnS}$, $\boxed{MnS}$, $\boxed{CoS}$

sequence :
1. $CH_3COOH$
2. dil HCl
3. $HNO_3$
4. dmg

$\boxed{NiS}$ Black ppt $\xrightarrow{HNO_3}$ soluble clear sol$^n$ $\xrightarrow{\text{dmg}}$ $Ni(dmg)_2$ (Rosy Red ppt)

$\boxed{ZnS}$ (white ppt) $\xrightarrow{\text{dil HCl}}$ Soluble (clear solution)

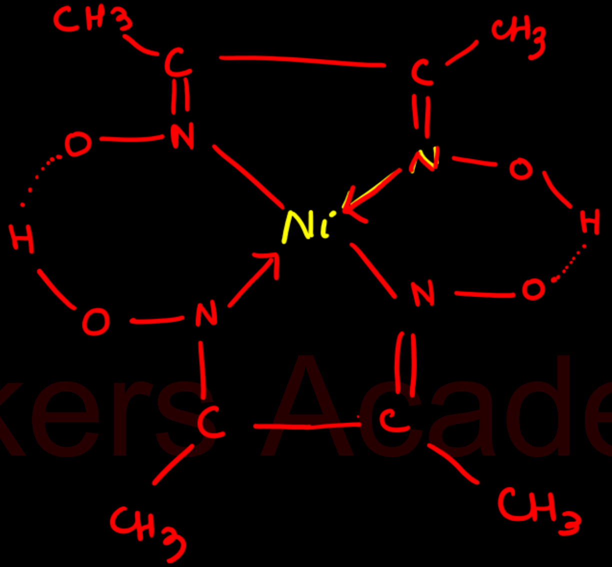
$\boxed{ZnS}$ $\xrightarrow{NaOH}$ $Zn(OH)_2$ $\xrightarrow{\text{excess } NH_3}$ $[Zn(NH_3)_4]^{2+}$ (clear solution)

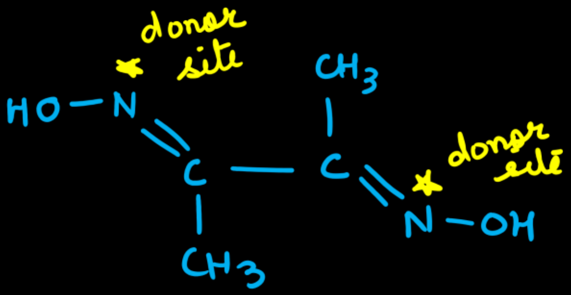
$Zn(OH)_2$:
- white ppt
- Amphoteric

$Zn(OH)_2$ $\xrightarrow{\text{excess NaOH}}$ $Na_2Zn(OH)_4$

$\boxed{MnS}$ $\xrightarrow{CH_3COOH}$ Pink ppt

$\boxed{CoS}$ $\xrightarrow{HNO3}$ Soluble (clear sol$^n$) $\xrightarrow{\text{DMG}}$ $Co(dmg)_2$ Red solution





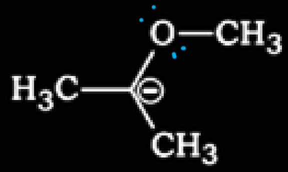

Structure of dmg

Nickel complex with dimethyl glyoxime

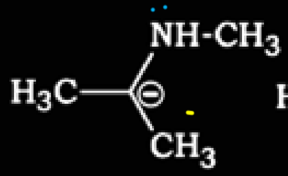
Nickel bis(dimethyl glyoximate)

JEE 1

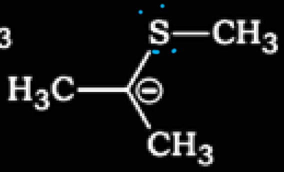
The correct order of stability of the following carbanions is



P



Q



R

S has vacant-d orb.

(A) R > Q > P

(B) R > P > Q

(C) Q > P > R

(D) P > Q > R

10

JEE 1

When NaCl is heated with $K_2Cr_2O_7$ in the presence of $H_2SO_4$, a red gas (A) is evolved. The gas when passed through aqueous NaOH . solution turns it yellow(B), which gives yellow ppt.(C) with $(CH_3COO)_2$ Pb. Then which of the following is correct?



(A) C is $PbO_2$

(B) A is $CrO_2Cl_2$

(C) B is $Cr_2O_3$

(D) C is $PbCrO_2$

$NaCl + K_2Cr_2O_7 + H_2SO_4 \longrightarrow \underset{A}{CrO_2Cl_2}$ (Red)

$CrO_2Cl_2 + NaOH \longrightarrow \underset{B}{Na_2CrO_4}$ (yellow)

$Na_2CrO_4 + (CH_3COO)_2Pb \longrightarrow \underset{C}{PbCrO_4}$ (yellow ppt)

The correct bond order for NO and $NO^+$ are respectively

(A) 3,5/2 (B) 3,2

(C) 3,7/2 (D) 5/2,3

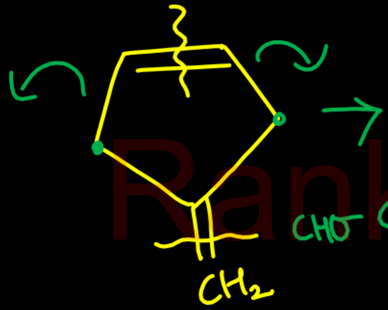
Bond order

NO : 15 2.5

$NO^+$ : 14 3

The structure of X will be :

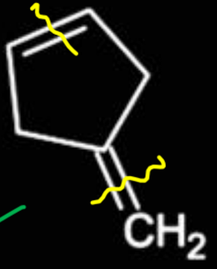
$$\mathrm{X} \xrightarrow{O_3/Zn,\ H_2O} H-\overset{\overset{\large O}{\|}}{C}-CH_2-\overset{\overset{\large O}{\|}}{C}-CH_2-\overset{\overset{\large O}{\|}}{C}-H + HCHO$$



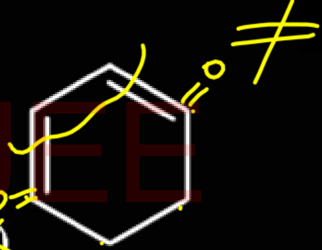
$$CHO-CH_2-\underset{\underset{O}{\|}}{C}-CH_2-CHO$$

$HCHO$

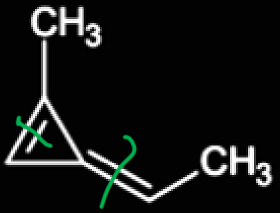
(A)



(B)



(C)



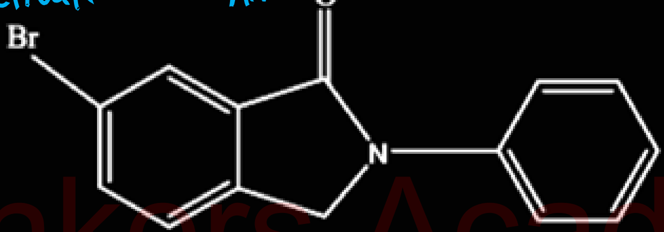
(D)

13

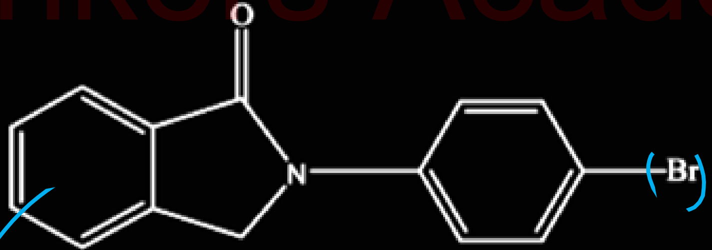
JEE 1


The Major Product formed is

deactivate Activate

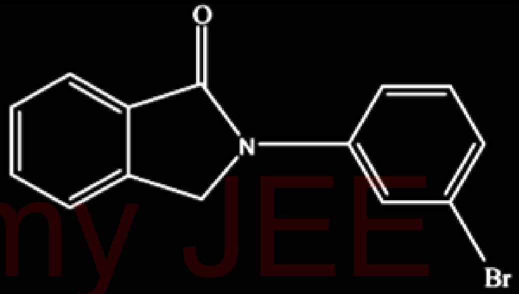



(A)

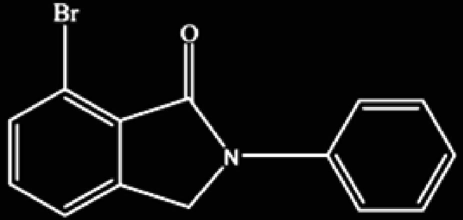



(B)

major



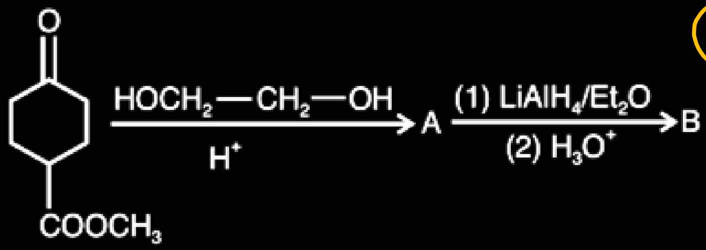

(C)

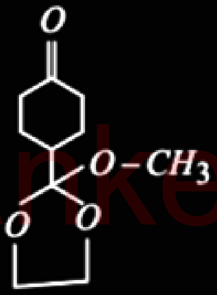
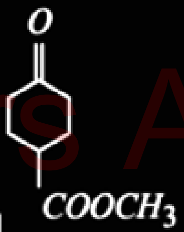


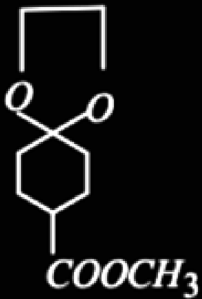
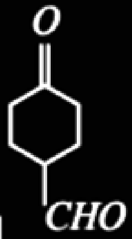

(D)

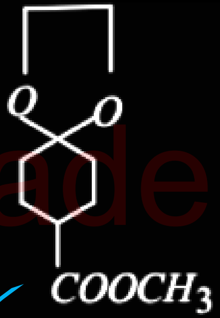
14

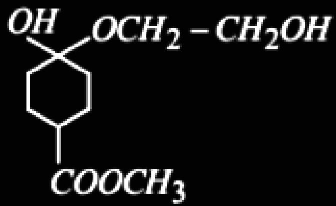
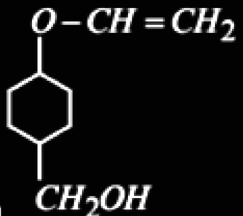
The product (A) and (B) respectively

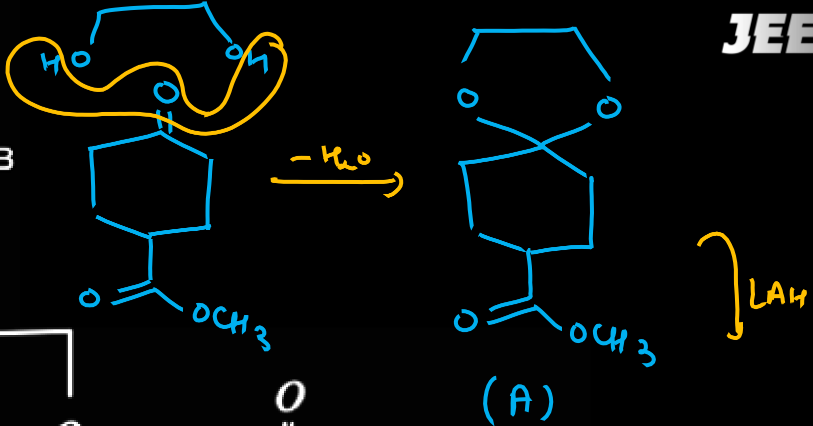


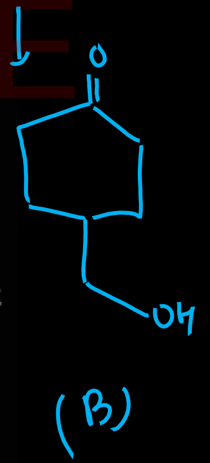

(A)  and 


(B)  and 


(C)  and 


(D)  and 

15

Let $v_1$ be the frequency of the series limit line of the Lyman series, $v_2$ be the frequency of the first line of the Lyman series, and $v_3$ be the frequency of the series limit line of the Balmer series. Then

(A) $v_1 - v_2 = v_3$

(B) $v_2 - v_1 = v_3$

(C) $v_2 = \frac{1}{2}(v_1 - v_3)$

(D) $v_1 + v_2 = v_3$

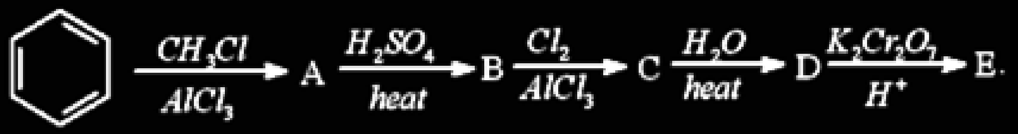
$\nu \propto \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\nu_1 = RZ^2\left(\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right) = RZ^2$

$\nu_2 = RZ^2\left(\frac{1}{(1)^2} - \frac{1}{(2)^2}\right) = \frac{3}{4}RZ^2$

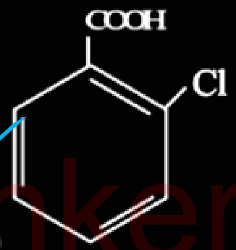
$\nu_3 = RZ^2\left(\frac{1}{(2)^2} - \frac{1}{(\infty)^2}\right) = \frac{1}{4}RZ^2$

$\therefore \boxed{\nu_1 - \nu_2 = \nu_3}$

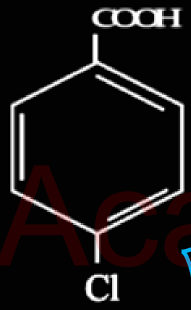


CH3Cl
AlCl3
A
H2SO4
heat
B
Cl2
AlCl3
C
H2O
heat
D
K2Cr2O7
H+
E.

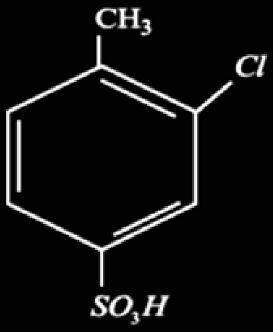

Find E.

(A)


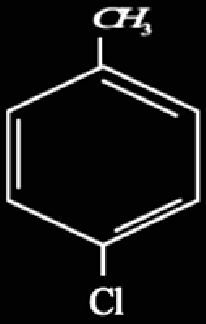
COOH
Cl


(B)


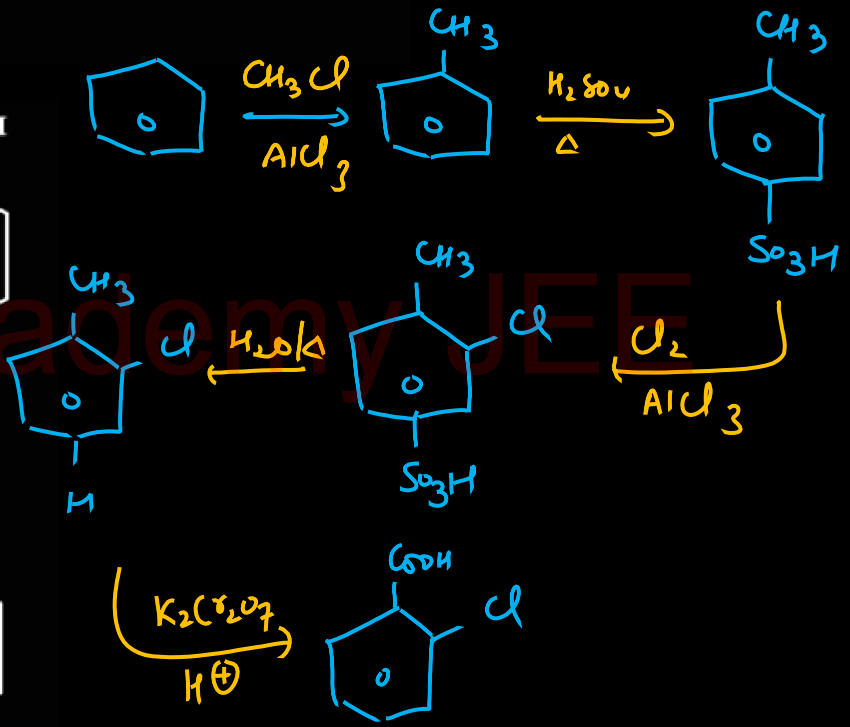
COOH
Cl


(C)


CH3
Cl
SO3H

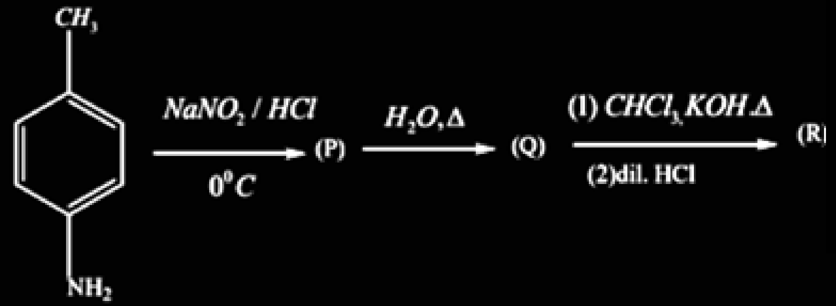

(D)


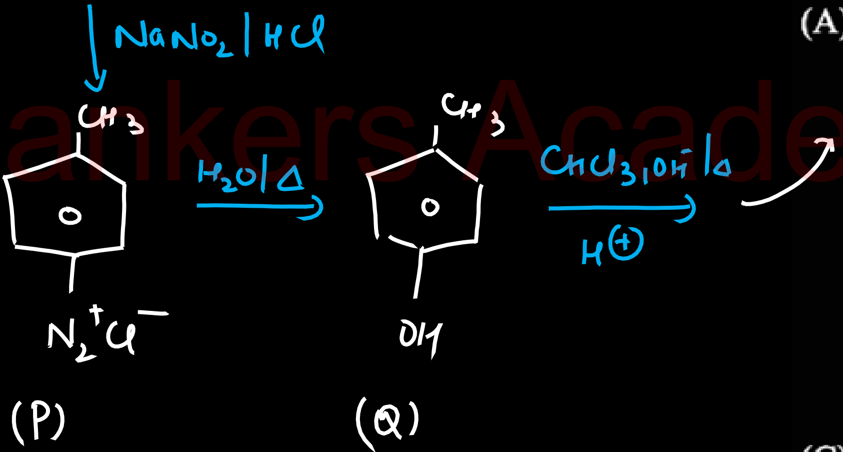
CH3
Cl

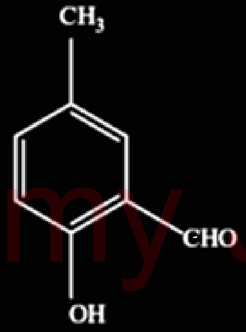



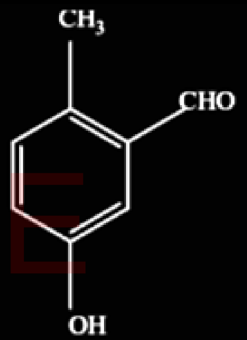
CH3Cl
AlCl3
CH3
H2SO4
Δ
CH3
SO3H
Cl2
AlCl3
CH3
Cl
SO3H
H2O/Δ
CH3
Cl
H
K2Cr2O7
H+
COOH
Cl

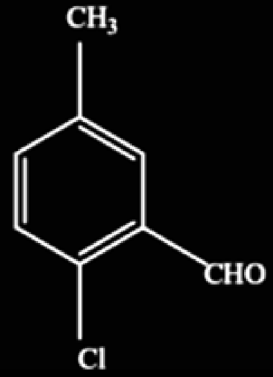
Identify R in the reaction

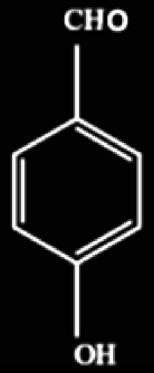



(A)



(B)



(C)



(D)


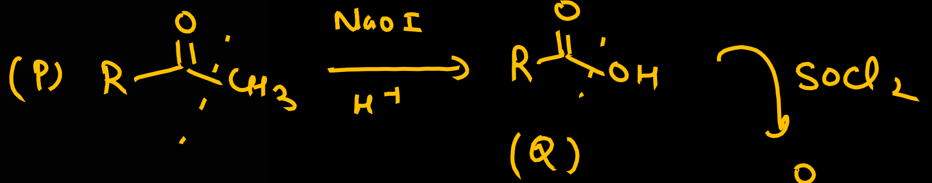
18

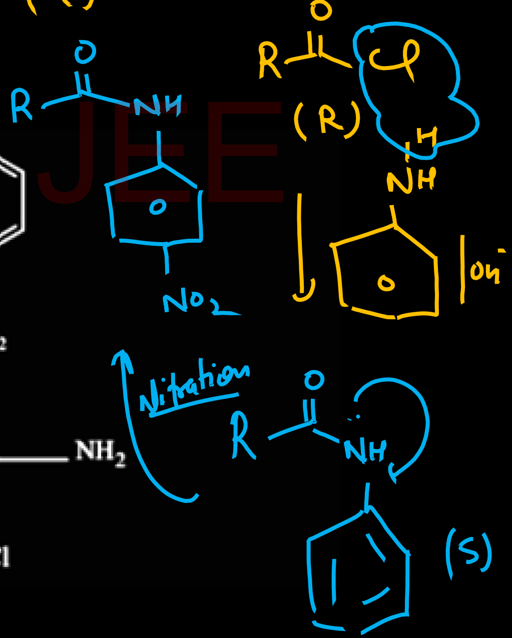
In the above reaction sequence (T) can be

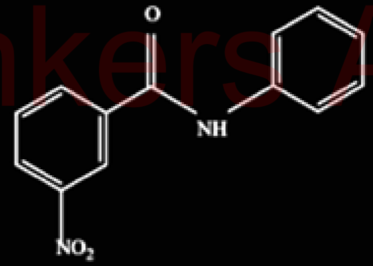
$$(P) \xrightarrow[(2)H^+]{(1)NaOI} (Q) \xrightarrow{SOCl_2} (R) \longrightarrow \xrightarrow[NaOH]{C_6H_5NH_2}$$

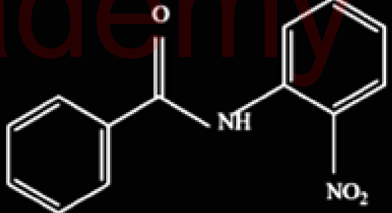
$$(S) \xrightarrow[conc.H_2SO_4(1\,equivalent)]{conc.HNO_3} (T)(major)$$

$\boxed{R = Ph}$

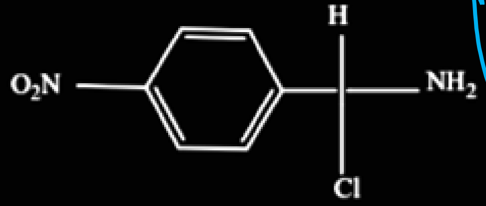



(A) 


(C) 


(B) 


(D) 


JEE 1

19

If 50% of $CO_2$ converts to CO at the following equilibrium:

$C(s) + CO_2(g) \rightleftharpoons 2CO(g)$

and the equilibrium pressure is 12 atm .

Calculate $K_p$

(A) 2 atm (B) 8 atm

(C) 16 atm (D) 1 atm

JEE 1



$$K_p = \frac{p_{CO}^2}{p_{CO_2}} = \frac{(8)^2}{4} = \frac{8 \times 8}{4} = 16 \text{ atm.}$$

$$C(s) + CO_2(g) \rightleftharpoons 2CO(g)$$

$t=0$   –   $p$   –

$t=t$   –   $(p-0.5p)$   $(2 \times 0.5p)$

$$[(p-0.5p) + p] = 12 \quad \therefore \frac{3}{2}p = 12 \quad \therefore \boxed{p = 8 \text{ atm}}$$

20

JEE 1

Half life of a first order reaction $A(aq.) \rightarrow 2B$ (aq.) is 40 min at 27°C and 20 min at 37°C. The activation energy of reaction is

$(\ln 2 = 0.7, R = 2.0 \text{cal mol}^{-1} K^{-1})$

(A) 19530cal/mol

(B) 13020cal/mol

(C) 26040cal/mol

(D) 6510cal/mol



$t_{1/2} = \frac{\ln 2}{k}$

$k_1 = \frac{\ln 2}{40}$ $T_1 = 300K$

$k_2 = \frac{\ln 2}{20}$ $T_2 = 310K$

$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$\ln 2 = \frac{E_a}{2}\left[\frac{1}{300} - \frac{1}{310}\right]$

$0.7 = \frac{E_a}{2}\left[\frac{10}{300 \times 310}\right]$ $\therefore E_a = 930 \times 2 \times 7$

$= 930 \times 14$

$=$

21

JEE 1

The conductivity of a weak acid HA of concentration 0.001 mol $L^{-1}$ is $2.0 \times 10^{-5}$ S $cm^{-1}$.
If $\Lambda_m^o(HA) = 190$ S $cm^2$ $mol^{-1}$, the ionization constant (Ka) of HA is equal to ______ $\times 10^{-6}$.
(Round off to the Nearest Integer)



$K = 2 \times 10^{-5}$ S $cm^{-1}$

$C = 10^{-3}$ M

$\Lambda_m = \frac{1000 K}{C} = \frac{1000 \times 2 \times 10^{-5}}{10^{-3}}$

$\Lambda_m = 20$

$\alpha = \frac{\Lambda_m}{\Lambda_m^o} = \frac{20}{190} = \frac{2}{19} >> 10\%$

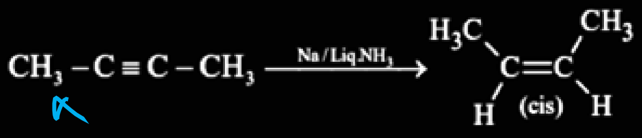
$Ka = \frac{C\alpha^2}{1-\alpha} = \frac{10^{-3}\left(\frac{2}{19}\right)^2}{\left(1 - \frac{2}{19}\right)} = \frac{4 \times 10^{-3}/19^2}{17/19}$

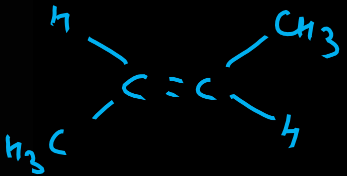
$\Leftarrow \frac{4 \times 10^{-3}}{19 \times 17}$

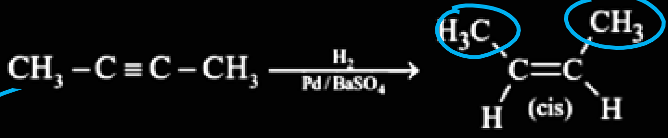
$\sim 12 \Leftarrow \frac{4000}{19 \times 17} \times 10^{-6}$

22

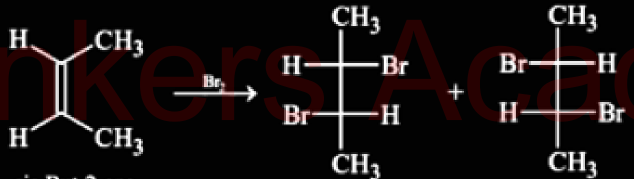
How Many of the reaction product is/are correct in the following

(A) 



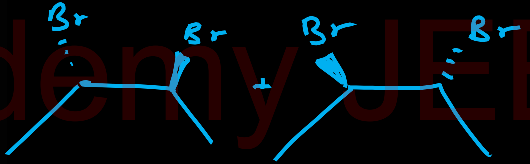

(B) 

3

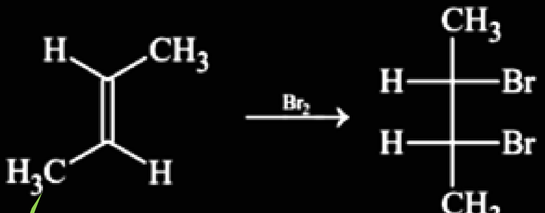
(C) 

cis-But-2-ene

(±)-2, 3-Dibromobutane

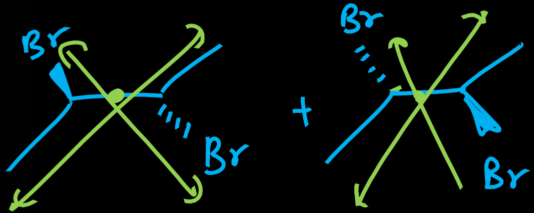



(Racemic)

(D) 

trans-But-2-ene

Meso -2, 3-dibromobutane




(identical)

cos

23

JEE 1

For the octahedral complexes of $Fe^{3+}$ in $SCN^-$( thiocyanato -S ) and in $CN^-$ ligand environments, the difference between the spin only magnetic moments in Bohr magnetons (when approximated to the nearest integer is

(Atomic number of Fe = 26)

4



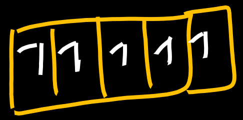
$\mu_1 = \sqrt{n(n+2)}$

$= \sqrt{35}$

$[Fe(SCN)_6]^{-3}$

$SCN \to WFL$

$Fe^{+3} : 3d^5$

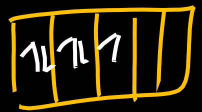


$n = 5$

$[Fe(CN)_6]^{-3}$

$CN \to SFL$

$Fe^{+3} : 3d^5$



$n = 1$

$\mu_2 = \sqrt{3}$

$\therefore |\mu_1 - \mu_2| = \sqrt{35} - \sqrt{3}$

$\simeq 5.92 - 1.73$

$\simeq 4$

24

JEE 1

If the freezing point of a 0.01 molal aqueous solution of a cobalt (III) chloride-ammonia complex(which have secondary valency of metal is 6 and it behaves as a strong electrolyte ) is $-0.0558°C$, the number of chloride(s) in the coordination sphere of the complex is ($K_f$ of water = $1.86\ K\ kg mol^{-1}$)

C.N

1

$\Delta T_f = i K_f m$

$0-(-0.0558) = i \times 1.86 \times 10^{-2}$

$\boxed{i = 3}$

$[Co(NH_3)_5Cl]Cl_2$

$\downarrow$

$[Co(NH_3)_5Cl]^{+2} + 2Cl^-$

25

JEE 1

The molar solubility of $Zn(OH)_2$ in 0.1 M NaOH solution is $x \times 10^{-18}$ M. The value of x is ______ (nearest integer)

(Given : the solubility product of $Zn(OH)_2$ is $2 \times 10^{-20}$)

Sol.

NaOH (0.1M)

$Zn(OH)_2 \rightleftharpoons Zn^{+2} + 2OH^-$

$s \quad (2s + 0.1)$

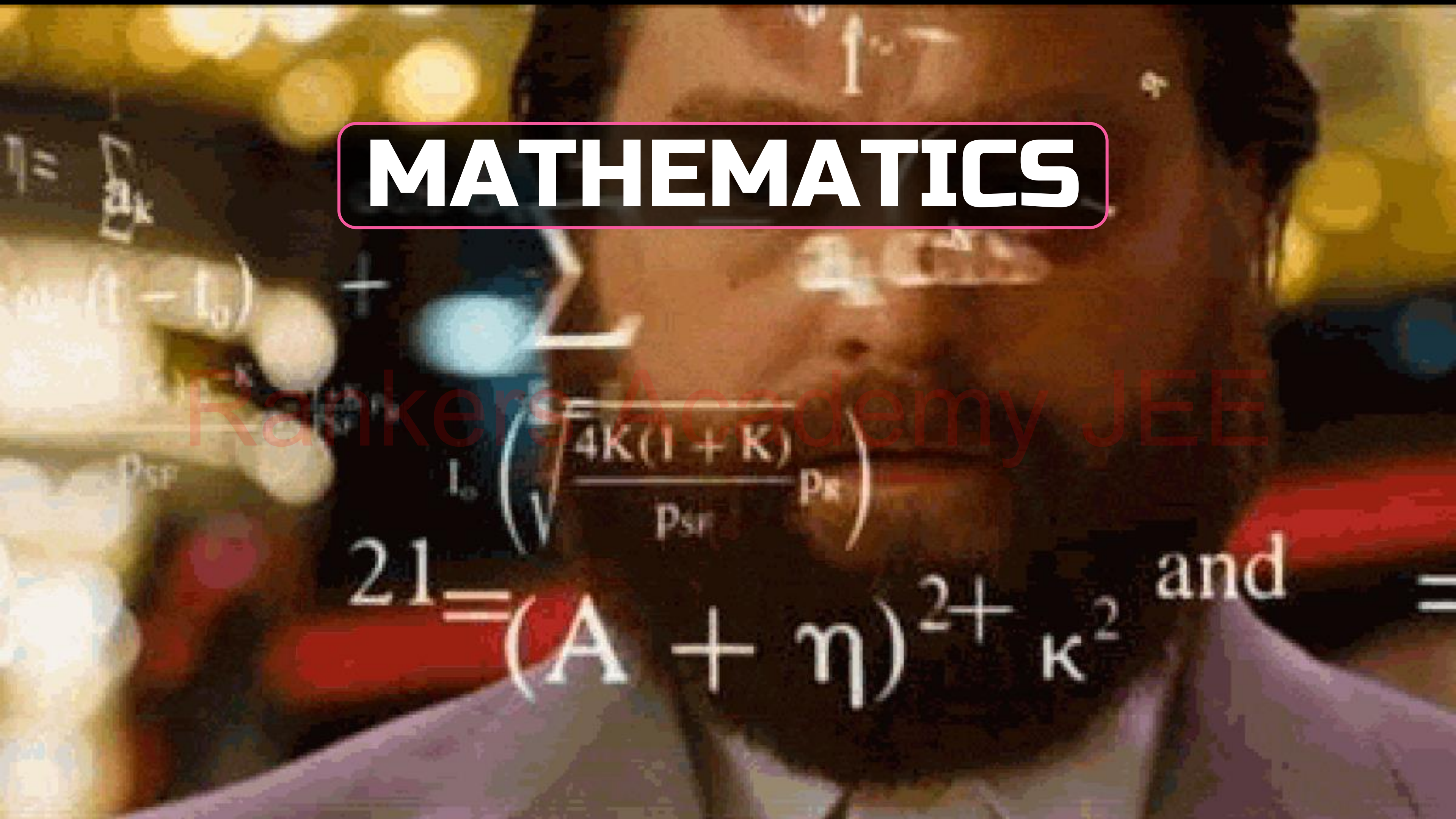
$K_{sp} = s(2s+0.1)^2$

$s << 0.1 \quad \therefore 2s + 0.1 \simeq 0.1$

$2 \times 10^{-20} = s(0.1)^2$

$\boxed{s = 2 \times 10^{-18}}$

$x = 2$


MATHEMATICS

If A, B, C are square matrix of third order and $|A| = 2, |B| = 3, |C| = 4$, $|A|$ means determinant value of A, then the value of $\left|\frac{A^3 \cdot B \cdot \text{Adj}C}{|C|}\right|$ equals to _____ .

(A) 8
(B) 6
(C) 4
(D) 12

$$\left|\frac{A^3 \cdot B \cdot \text{adj}\,C}{4}\right|$$

$$\Rightarrow \frac{1}{4^3}\left|A^3 \cdot B \cdot \text{adj}\,C\right|$$

$$\Rightarrow \frac{1}{4^3} \cdot |A^3|\,|B|\,|\text{adj}\,C|$$

$$\Rightarrow \frac{1}{4^3}\,|A|^3\,|B|\,|C|^{3-1}$$

$$\Rightarrow \frac{1}{4^{\cancel{3}}} \cdot 2^3 \cdot 3 \cdot \cancel{4}^2$$

$$\Rightarrow 6$$



JEE1

2

JEE 1

Three vectors a($|a| \neq 0$), b and c are such that $\boxed{a \times b = 3a \times c,}$ also $|a| = |b| = 1$ and $|c| = \frac{1}{3}$. If the angle between b and c is $60°$ and $|b - 3c| = \lambda|a|$, then the value of $\lambda^2$ is

(A) 1 ✓

(B) $\frac{1}{2}$

(C) 2

(D) 4

$\vec{a} \times \vec{b} - 3\vec{a} \times \vec{c} = 0$

$\vec{a} \times (\vec{b} - 3\vec{c}) = 0$

$\vec{a} \parallel \vec{b} - 3\vec{c}$

$\vec{b} - 3\vec{c} = \lambda\vec{a}$

Redundant

$\Rightarrow |\vec{b} - 3\vec{c}|^2 = \lambda^2|\vec{a}|^2$

$\Rightarrow |\vec{b}|^2 + 9|\vec{c}|^2 - 6|\vec{b}||\vec{c}|\cos 60° = \lambda^2|\vec{a}|^2$

$\Rightarrow 1 + 9\left(\frac{1}{9}\right) - 6(1)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \lambda^2(1)$

$\Rightarrow \lambda^2 = 1$

Let $f(x) = \cos 2x \cdot \cos 4x \cdot \cos 6x \cdot \cos 8x \cdot \cos 10x$

then $\lim_{x\to 0} \frac{1-(f(x))^3}{5\sin^2 x}$ is equal to

(A) 660 (B) 135

(C) 132 (D) 66 ✓

Replacement Trick

$x \to 0$

$\sin x \longleftrightarrow x$

$\tan x \longleftrightarrow x$

$\ln(1+x) \longleftrightarrow x$

$e^x - 1 \longleftrightarrow x$

$$\frac{1}{5} \lim_{x\to 0} \frac{(1-f(x))\,(1+(f(x))^2+f(x))}{x^2}$$

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$$

$$1-\cos\theta = \frac{\theta^2}{2}$$

$$\cos\theta = 1-\frac{\theta^2}{2}$$

$$= \frac{3}{5} \lim_{x\to 0} \frac{1-\cos 2x \cos 4x \cos 6x \cos 8x \cos 10x}{x^2}$$

$$= \frac{3}{5} \lim_{x\to 0} \frac{1-\left(1-\frac{4x^2}{2}\right)\left(1-\frac{16x^2}{2}\right)\left(1-\frac{36x^2}{2}\right)\left(1-\frac{64x^2}{2}\right)\left(1-\frac{100x^2}{2}\right)}{x^2}$$

$$= \frac{3}{5} \cdot \frac{1}{2}\left(4+16+36+64+100\right) = \frac{3}{10} \cdot 220 = 66$$

$=$

4

The remainder when

$$\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$$ is divided

by 5 is k then k equal to

(A) 8 (B) 6

(C) 4 (D) 12

$a_{11} = \frac{2014^{2014}}{5} = \frac{(2015-1)^{2014}}{5} = 1$

$a_{12} = 0$

$a_{13} = \frac{(2016)^{2016}}{5} = \frac{(2015+1)^{2016}}{5} = 1$

$a_{21} = \frac{2017^{2017}}{5} = \frac{(2015+2)^{2017}}{5} = \frac{4^{1008}\cdot 2^{1}}{5}$

$= \frac{(5-1)^{1008}\cdot 2}{5} = 2$

$a_{22} = \frac{2018^{2018}}{5}$

$= \frac{(2015+3)^{2018}}{5}$

$= \frac{9^{1009}}{5}$

$= \frac{(10-1)^{1009}}{5} = -1 = 4$

$a_{23} = \frac{2019^{2019}}{5} = \frac{(2020-1)^{2019}}{5} = -1 = 4$

$a_{31} = 0$

$a_{32} = (2020+1)^{2021} = 1$

$a_{33} = 2^{2022} = 4^{1011} = (5-1)^{1011} = -1$

$= 4$

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 4 & 4 \\ 0 & 1 & 4 \end{vmatrix}$$

$= 12 + 2 = 14$

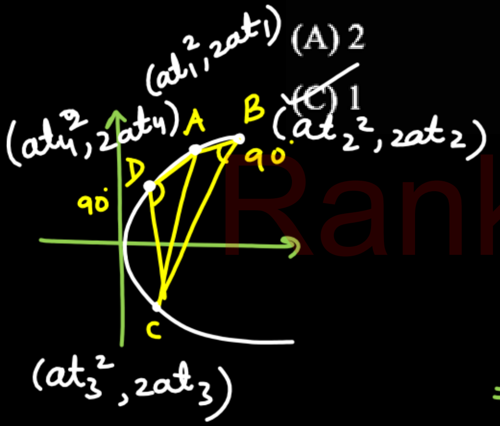
Rem = 4.

5

The chord AC of the parabola $y^2 = 4ax (a > 0)$ subtends an angle of 90° at points B and D on the parabola. If points A, B, C and D in order are represented by $(at_i^2, 2at_i), i = 1,2,3,4$ respectively then find the value of $\left|\frac{t_2+t_4}{t_1+t_3}\right|$. $= |-1| = 1$

(A) 2 (B) 3

(C) 1 (D) 6




$m_{AB} \cdot m_{BC} = -1$

$\frac{-2}{t_1+t_2} \cdot \frac{-2}{t_2+t_3} = -1$

$\Rightarrow (t_1+t_2)(t_2+t_3) = -4$ — 1

$m_{AD} \cdot m_{CD} = -1$

$\Rightarrow \frac{-2}{t_1+t_4} \cdot \frac{-2}{t_3+t_4} = -1$

$\Rightarrow (t_1+t_4)(t_3+t_4) = -4$ — 2

$\Rightarrow (t_1+t_2)(t_2+t_3) = (t_1+t_4)(t_3+t_4)$

$\Rightarrow t_1t_2 + \cancel{t_1t_3} + t_2^2 + t_2t_3$

$= \cancel{t_1t_3} + t_1t_4 + t_3t_4 + t_4^2$

$\Rightarrow t_2^2 - t_4^2 + t_1t_2 - t_1t_4 + t_2t_3 - t_3t_4 = 0$

$\Rightarrow (t_2-t_4)\underbrace{(t_2+t_4+t_1+t_3)}_{=0} = 0$



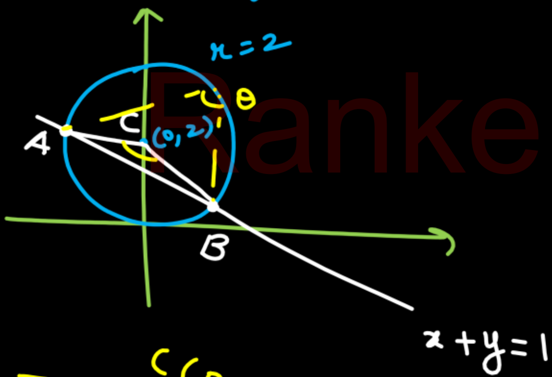
JEE 1

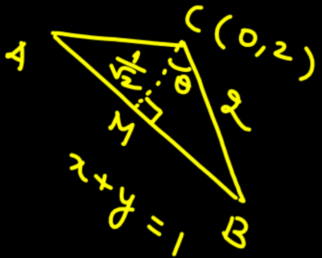
If the angle subtended by the chord of circle $x^2 + y^2 - 4y = 0$ along the line $x + y = 1$ at circumference of the larger segment is $\theta$, then $\sec^2 \theta =$ ______

$\angle ACB = 2\theta$

(A) 2 (B) 4

(C) 8 (D) 16




$CM = \left|\dfrac{0+2-1}{\sqrt{1^2+1^2}}\right| = \dfrac{1}{\sqrt{2}}$

$\cos\theta = \dfrac{1/\sqrt{2}}{2} = \dfrac{1}{2\sqrt{2}}$

$\sec\theta = 2\sqrt{2}$

$\sec^2\theta = 8$

7

Number of integral values of 'a' for which $f(x) = (b^2 + (a-1)b + 2)x + \int_0^x (\sin^4\theta + \cos^4\theta)dx$ is an increasing function $\forall x \in R$ and any $b \in R$, is ______ $-2, -1, 0, 1, 2, 3, 4$

(A) 2 (B) 5

(C) 10 (D) 7

JEE 1

Quad in $b \geq 0$.

$D \leq 0$

$\Rightarrow (a-1)^2 - 4(1)\left(3 - \frac{1}{2}\sin^2 2x\right) \leq 0$

$\Rightarrow (a-1)^2 \leq 4\left(3 - \frac{1}{2}\underbrace{\sin^2 2x}_{\in[0,1]}\right)$

$(a-1)^2 \leq [10, 12]$

$(a-1)^2 \leq 10$

$-\sqrt{10} \leq a-1 \leq \sqrt{10}$

$1-\sqrt{10} \leq a \leq 1+\sqrt{10}$

$1-3.1 \leq a \leq 1+3.1$

$f' \geq 0$

$\Rightarrow (b^2 + (a-1)b + 2) + \sin^4 x + \cos^4 x \geq 0$

$\Rightarrow b^2 + (a-1)b + 2 + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x \geq 0$

$\Rightarrow b^2 + (a-1)b + 2 + 1 - \frac{1}{2}\sin^2 2x \geq 0$

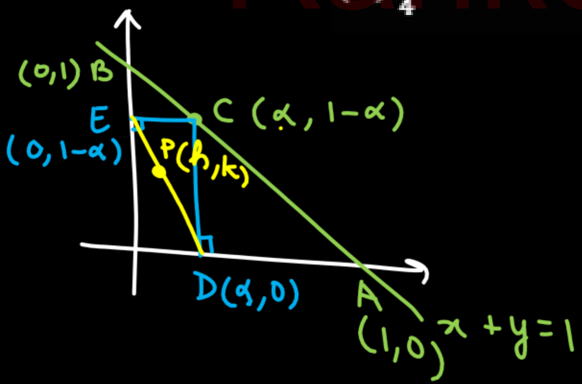
$\Rightarrow b^2 + (a-1)b + 3 - \frac{1}{2}\sin^2 2x \geq 0$

Let C be a variable point on the line $x + y = 1$ which intersect $x, y$ axes at A and B. D, E are feet of perpendicular from C on $x$ and $y$ axes respectively. Let P be a point on the line segment joining D and E, then area of the region traced by P is

(A) $\frac{1}{16}\log_e^2$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$



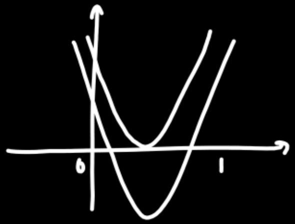



DE : $\frac{x}{\alpha} + \frac{y}{1-\alpha} = 1$

$\Rightarrow x - x\alpha + \alpha y = \alpha - \alpha^2$

$\Rightarrow \alpha^2 + \alpha(-x + y - 1) + x = 0$

$\Rightarrow \alpha \in (0,1)$




$D \geq 0$

$(-x+y-1)^2 \geq 4x$

$(-x+y-1) \geq 2\sqrt{x}$

or $\leq -2\sqrt{x}$

$0 < \frac{-(-x+y-1)}{2(1)} < 1$

$f(0) = x > 0$

$f(1) = 1 \cancel{-x} + y \cancel{-1} \cancel{+x} = y > 0$

$\int_0^1 x + 1 + 2\sqrt{x}\,dx = \frac{1}{2} + 1 + \frac{4}{3}$ ✗

or

$\int_0^1 x + 1 - 2\sqrt{x}\,dx$

$= \frac{1}{2} + 1 - \frac{4}{3}$ ✓

# 9

JEE 1

If different words are formed using all the letters from the word 'INDIANIDOL' in which 'a' denotes number of words which contains 'INDIA' but not 'INDIAN' and 'b' denotes number of words which contains 'INDIAN' but not 'IDOL' then:

~~(A)~~ $a + b = 6! + 2$ (B) $a + b = 6! - 2$ ✓

(C) $a - b = 4.5! + \cancel{2}\,4$ (D) $a - b = 4.5! - 2$



$a = \boxed{INDIA}\ N, I, D, O, L - \boxed{INDIAN}\ IDOL$

$= 6! - 5!$

$b = \boxed{INDIAN}\ IDOL - \boxed{INDIAN}\boxed{IDOL}$

$5! - 2!$

$a - b = 6! - 2.5! + 2!$

$= 720 - 240 + 2$

$= 482.$

10

JEE 1

Let $\alpha_n, \beta_n$ be the distinct roots of the equation $x^2 + (n+1)x + n^2 = 0$. If $\sum_{n=2}^{2021} \frac{1}{(\alpha_n+1)(\beta_n+1)}$ can be expressed in the form $\frac{a}{b}$, where a and b are positive integer, the value of (b – a), is:

(A) 1 ✓
(B) 3
(C) 6
(D) 9

$x^2 + (n+1)x + n^2 = 0 < \begin{matrix}\alpha_n \\ \beta_n\end{matrix}$

$x \to y - 1$

$(y-1)^2 + (n+1)(y-1) + n^2 = 0 < \begin{matrix}\alpha_n + 1 = y = x+1 \\ \beta_n + 1\end{matrix}$

$\Rightarrow y^2 - 2y + \cancel{1} + (n+1)y - n - \cancel{1} + n^2 = 0$

$\Rightarrow y^2 + (n-1)y + (n^2 - n) = 0 < \begin{matrix}\alpha_n + 1 \\ \beta_n + 1\end{matrix}$

$\sum_{n=2}^{2021} \frac{1}{(n^2 - n)} = \sum \frac{1}{n(n-1)}$

$= \sum \frac{1}{n-1} - \frac{1}{n}$

$= \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} \dots - \frac{1}{2021}$

$= 1 - \frac{1}{2021}$

$= \frac{2020}{2021}$

11



The general solution of the differential equation

$(x - y^2)dx + y(5x + y^2)dy = 0$ is

(A) $(y^2 + x)^4 = C|(y^2 + 2x)^3|$

(B) $(y^2 + 2x)^4 = C\,|\,(y)^2 + x)^3\,|$

(C) $|(y^2 + x)^3| = C(2y^2 + x)^4$

(D) $|(y^2 + 2x)^3| = C(2y^2 + x)^4$

$$2y\frac{dy}{dx} = \frac{2(y^2 - x)}{(y^2+5x)}$$

Let $y^2 = vx$

$$2y\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2(vx - x)}{vx+5x} = \frac{2(v-1)}{v+5}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2v-2}{v+5} - v = \frac{-(v^2+3v+2)}{v+5}$$

$$\Rightarrow \int\frac{(v+5)\,dv}{v^2+3v+2} = -\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{v+5}{(v+1)(v+2)}\,dv = -\int\frac{dx}{x}$$

$$= \int\frac{(v+2)+3}{(v+1)(v+2)}\,dv = -\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{dv}{v+1} + 3\int\frac{\overset{(v+2)-(v+1)}{dv}}{(v+1)(v+2)} = -\int\frac{dx}{x}$$

$$\Rightarrow \ln|v+1| + 3\ln\left|\frac{v+1}{v+2}\right| = -\ln|x| + \ln c$$

$$\Rightarrow \ln\left|\frac{(v+1)^4}{(v+2)^3}\right| = \ln\left|\frac{c}{x}\right| \Rightarrow \frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3} = \frac{c}{x}$$

12

JEE 1

If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to

(A) $52 \times 3^{24}$ (B) $56 \times 3^{25}$

(C) $28 \times 3^{25}$ (D) $56 \times 3^{24}$

$\boxed{x^2 + 3^{1/4}x + 3^{1/2} = 0}$ $\alpha$, $\beta$

$\Rightarrow (x^2 + 3^{1/2}) = -3^{1/4}x$

$\Rightarrow x^4 + 3 + 2\sqrt{3}x^2 = \sqrt{3}x^2$

$\Rightarrow \boxed{x^4 + 3 + \sqrt{3}x^2 = 0}$ $\alpha^2$, $\beta^2$

$\Rightarrow (x^4 + 3) = -\sqrt{3}x^2$

$\Rightarrow x^8 + 9 + 6x^4 = 3x^4$

$\Rightarrow \boxed{x^8 + 9 + 3x^4 = 0}$ $\alpha^4$, $\beta^4$

$\Rightarrow \alpha^4 \cdot \beta^4 = 9$

$3\alpha^8 + 27 + 9\alpha^4 = 0$

$\alpha^{12} = \alpha^8 \cdot \alpha^4$

$= -(3\alpha^4 + 9) \cdot \alpha^4$

$= -(3\alpha^8 + 9\alpha^4)$

$= -(-27) = 27$

$\beta^{12} = 27$

$\alpha^{96} = (\alpha^{12})^8 = 27^8$

$\beta^{96} = 27^8$

Ans:

$= 27^8(27 - 1) \times 2$

$= 3^{24} \times 52$

13

A sequence has (4n + 1) terms. The first (2n + 1) terms are in A.P with common difference 2 and the last (2n + 1) terms are in G.P with common ratio $\frac{1}{2}$. If the middle terms of A.P and G.P are equal, then the middle term of the sequence is _____. $= A+4n =$

(A) $(n+1)\cdot 2^{n+1}$ $= 2.2^2 = 8$

(B) $\frac{n\cdot 2^{n+1}}{2^{2n}-1}$ $= \frac{1.2^2}{2^2-1} = \frac{4}{3}$

(C) $n\cdot 2^n$ $= 1.2^1 = 2$

(D) $\frac{n\cdot 2^{n+1}}{2^n-1}$ $= \frac{1.2^2}{2-1} = 4$

M1

$\underbrace{A, A+2, \ldots, \boxed{A+4n}}_{\substack{(2n+1)\\ AP}}$, $\frac{A+4n}{2}, \frac{A+4n}{2^2}, \ldots \frac{A+4n}{2^{2n}}$

$\underbrace{\boxed{A+4n}, \frac{A+4n}{2}, \ldots}_{\substack{GP\\(2n+1)}}$

$d=2$

$r = 1/2$

Midterm $= (n+1)^{th}$ term

Midterm $= (n+1)^{th}$

M2 $n=1$

$A, A+2, \boxed{A+4}, \frac{A+4}{2}, \frac{A+4}{4}$

$A+2 = \frac{A+4}{2} \Rightarrow A=0$

$A+(n+1-1)(2) = (A+4n)\left(\frac{1}{2}\right)^{n+1-1}$

$\Rightarrow A+2n = \frac{A+4n}{2^n}$

$\Rightarrow A.2^n + 2.n.2^n = A+4n$

$\Rightarrow A = \frac{4n-2.n.2^n}{2^n-1}$

$\Rightarrow A+4n = \frac{4n-2.n^{n+1}}{2^n-1} + 4n$

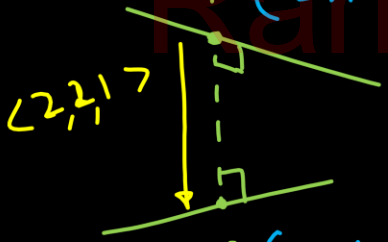
$\frac{n.2^{n+1}(-1+2)}{2^n-1} = \frac{\cancel{4n} - n.2^{n+1} + 2^{n+2}.n - \cancel{4n}}{2^n-1}$

The point on the line $\frac{x+2}{2} = \frac{y+6}{3} = \frac{z-34}{-10} = \lambda$ which is nearest to the line $\frac{x+6}{4} = \frac{y-7}{-3} = \frac{z-7}{-2} = \mu$ is $(a, b, c)$ then the value of $a + b + c =$

(A) 9 (B) 10

(C) 11 ✓ (D) 12

$P(2\lambda-2,\ 3\lambda-6,\ -10\lambda+34)$




$Q(4\mu-6,\ -3\mu+7,\ -2\mu+7)$

$$\vec{n} = \bar{d_1} \times \bar{d_2}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -10 \\ 4 & -3 & -2 \end{vmatrix}$$

$$= \hat{i}(-6-30) - \hat{j}(-4+40) + \hat{k}(-6-12)$$

$$= -36\hat{i} - 36\hat{j} - 18\hat{k}$$

$$= -18(2\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{PQ} = (4\mu - 2\lambda - 4)\hat{i} + (-3\mu - 3\lambda + 13)\hat{j} + (-2\mu + 10\lambda - 27)\hat{k}$$

$$\vec{n} \equiv \langle 2, 2, 1 \rangle$$

$$\frac{4\mu - 2\lambda - 4}{2} = \frac{-3\mu - 3\lambda + 13}{2} = \frac{-2\mu + 10\lambda - 27}{1}$$

$$\left\{\begin{matrix} \lambda = 3 \\ \mu = 2 \end{matrix}\right\} \to P \equiv (4, 3, 4)$$

$$\begin{matrix} 4 & 3 & 4 \\ \downarrow & \downarrow & \downarrow \\ a & b & c \end{matrix}$$

$$\boxed{a + b + c = 11}$$

15

If $Z_1$ and $Z_2$ are complex numbers such that

$Z_1 + Z_2 = Z_1^2 + Z_2^2 = \frac{2i}{\sqrt{3}}$. Then $2(\text{Re}(Z_1))^2 =$

(A) 1 (B) 2

(C) 1/2 (D) 1/4





$$z_1 + z_2 = \frac{2i}{\sqrt{3}}$$

$$z_1 = x + iy_1$$
$$z_2 = -x + iy_2$$

$$z_1 + z_2 = \cancel{i}(y_1 + y_2) = \frac{2\cancel{i}}{\sqrt{3}}$$

$$\boxed{y_1 + y_2 = 2/\sqrt{3}} \text{ — } 1$$

$$z_1^2 + z_2^2 = \frac{2i}{\sqrt{3}}$$

$$(x^2 - y_1^2 + 2ixy_1) + (x^2 - y_2^2 - 2ixy_2) = \frac{2i}{\sqrt{3}}$$

$$2x^2 - y_1^2 - y_2^2 = 0 \text{ — } 2 \Rightarrow 2x^2 = (y_1^2 + y_2^2)$$

$$\cancel{2}x(y_1 - y_2) = \cancel{1}/\sqrt{3} \text{ — } 3$$

$$\underbrace{y_1^2 + y_2^2}_{\downarrow} + 2y_1y_2 = \frac{4}{3}$$

$$\boxed{2x^2 + 2y_1y_2 = \frac{4}{3}}$$



**Now:**

$$x^2(y_1 - y_2)^2 = \frac{1}{3}$$

$$x^2\left(\underbrace{y_1^2 + y_2^2} - 2\underbrace{y_1y_2}\right) = \frac{1}{3}$$

$$x^2\left(2x^2 - \left(\frac{4}{3} - 2x^2\right)\right) = \frac{1}{3}$$

$$x^2 = \frac{1}{2}$$

$$\boxed{2x^2 = 1}$$

Ans

16

JEE 1

Let the function $f: R \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ be defined as $f(x) = \frac{\pi}{2} - 2\tan^{-1}(e^x)$, then f(x) is:

(A) odd function and strictly increasing in $(0, \infty)$

(B) odd function and strictly decreasing in $(-\infty, \infty)$ ✓

(C) even function and strictly decreasing in $(-\infty, \infty)$

(D) neither even nor odd and strictly increasing in $(-\infty, \infty)$

$f(x) \downarrow$

$$f(-x) = \frac{\pi}{2} - 2\tan^{-1}(e^{-x})$$

$$= \frac{\pi}{2} - 2\tan^{-1}\left(\frac{1}{e^x}\right)$$

$$= \frac{\pi}{2} - 2\left(\cot^{-1}(e^x)\right)$$

$$= \frac{\pi}{2} - 2\left(\frac{\pi}{2} - \tan^{-1}(e^x)\right)$$

$$= -\frac{\pi}{2} + 2\tan^{-1}(e^x)$$

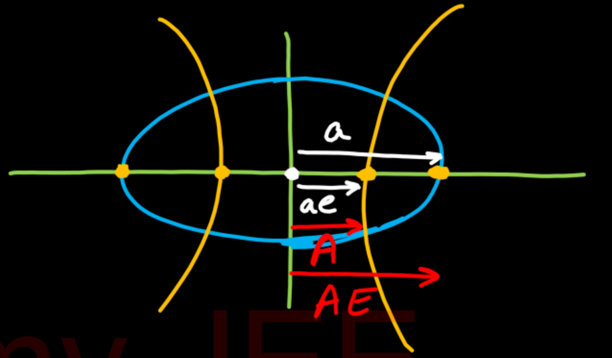
$$= -f(x)$$

17

Let the foci of the hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ be the vertices of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the foci of the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola be $e_E$ and $e_H$ respectively. If $e_E^2 = \frac{1}{2}$ and $(x, y)$ is point of intersection of ellipse and the hyperbola then $\frac{9x^2}{4y^2}$ is _____

(A) 9 (B) 1

(C) 4 (D) 6




$a = AE \;\&\; A = ae$

$a^2 = A^2E^2 \;\&\; A^2 = a^2e^2$

$A^2 = \frac{a^2}{2}$

$\cancel{a^2} = \frac{\cancel{a^2}}{2}E^2 \Rightarrow \boxed{E^2 = 2}$

JEE 1

$H:\quad B^2 = A^2(E^2-1)$

$B^2 = A^2(2-1)$

$\boxed{B^2 = A^2} \rightarrow B^2 = a^2/2$

$E:\quad b^2 = a^2(1-e^2)$

$b^2 = a^2\left(1-\frac{1}{2}\right)$

$b^2 = \frac{a^2}{2}$

$H:\quad \frac{x^2}{a^2/2} - \frac{y^2}{a^2/2} = 1$

$\frac{2x^2}{a^2} - \frac{2y^2}{a^2} = 1 \quad \text{---} (1)$

$E:\quad \frac{x^2}{a^2} + \frac{y^2}{a^2/2} = 1$

$\frac{x^2}{a^2} + \frac{2y^2}{a^2} = 1 \quad \text{---} (2)$

Add: $\frac{3x^2}{a^2} = 2 \Rightarrow x^2 = \frac{2a^2}{3} \Rightarrow y^2 = \frac{a^2}{6}$

$\therefore \frac{9x^2}{4y^2} = \frac{9}{\cancel{4}} \times \frac{\cancel{2}a^2}{\cancel{3}} \times \frac{\cancel{6}}{a^2} = \boxed{9}$

18

The marks obtained by 60 students in a certain subject out of 75 are given below:

| Marks | Number of Students | c.f. |
|---|---|---|
| 15 - 20 | 4 | 4 |
| 20 - 25 | 5 | 9 |
| 25 - 30 | 11 | 20 |
| 30 - 35 | 6 | 26 |
| 35 - 40 | 5 | 31 |
| 40 - 45 | 8 | |
| 45 - 50 | 9 | |
| 50 - 55 | 6 | |
| 55 - 60 | 4 | |
| 60 - 65 | 2 | |

What is the median?

(A) 35 (B) 38

(C) 39 (D) 40

$$\boxed{\frac{N}{2} = 30}$$

$$M = l + \frac{\left(\frac{N}{2} - c.f.\right)}{f} \times h$$

$$= 35 + \left(\frac{30 - 26}{\cancel{5}}\right) \times \cancel{5}$$

$$= 39$$

19

JEE 1

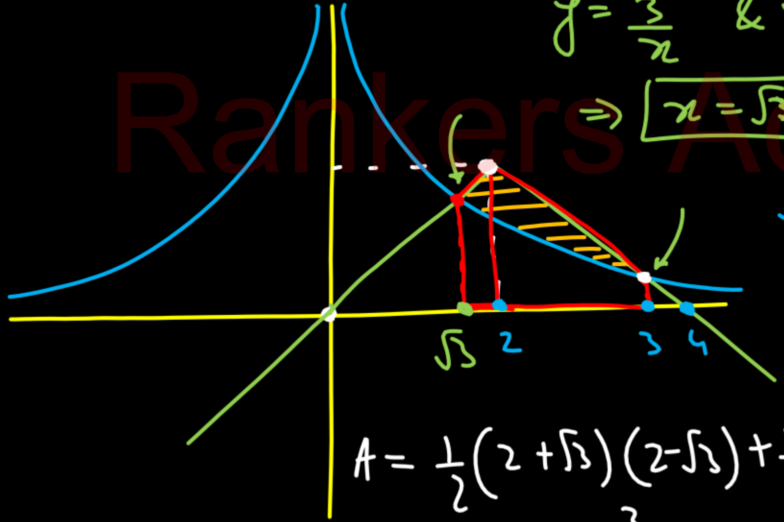
The area bounded by $y = 2 - |2 - x|; y = \frac{3}{|x|}$ is $\frac{k - 3\ln 3}{2}$, then k is equal to ______ .

(A) 2

(B) 4

(C) 3

(D) 6




$y = \frac{3}{x}$ & $y = x$

$\Rightarrow \boxed{x = \sqrt{3}}$

$y = \frac{3}{x}$ & $y = 4 - x$

$\frac{3}{x} = 4 - x$

$A = \frac{1}{2}(2+\sqrt{3})(2-\sqrt{3}) + \frac{1}{2}(2+1)(1)$

$- \int_{\sqrt{3}}^{3} \frac{3}{x}\,dx$

$y = 2 - |2 - x|$

$y = \begin{cases} 2 - (2 - x) & ; x < 2 \\ 2 + (2 - x) & ; x \geq 2 \end{cases}$

$y = \begin{cases} x & ; x < 2 \\ 4 - x & ; x \geq 2 \end{cases}$

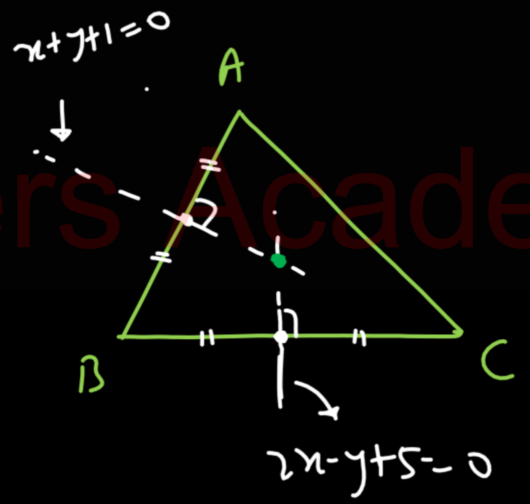
Consider a point $A(\alpha, \beta)$ in xy-plane image of A in the line $x + y + 1 = 0$ is B. Image of B in the line $2x - y + 5 = 0$ is C. Centroid of $\Delta^k ABC$ is G(3,2) then orthocentre of triangle ABC is

(A) $(-2,1)$

(B) $(8,3)$

(C) $(13,4)$

(D) $\left(\frac{1}{2}, \frac{3}{2}\right)$




2 : 1

O G C'

(x,y) (3,2) (-2,1)

$\frac{-4+x}{3} = 3 \Rightarrow x = 13$

$\frac{2+y}{3} = 2 \Rightarrow y = 4$

21

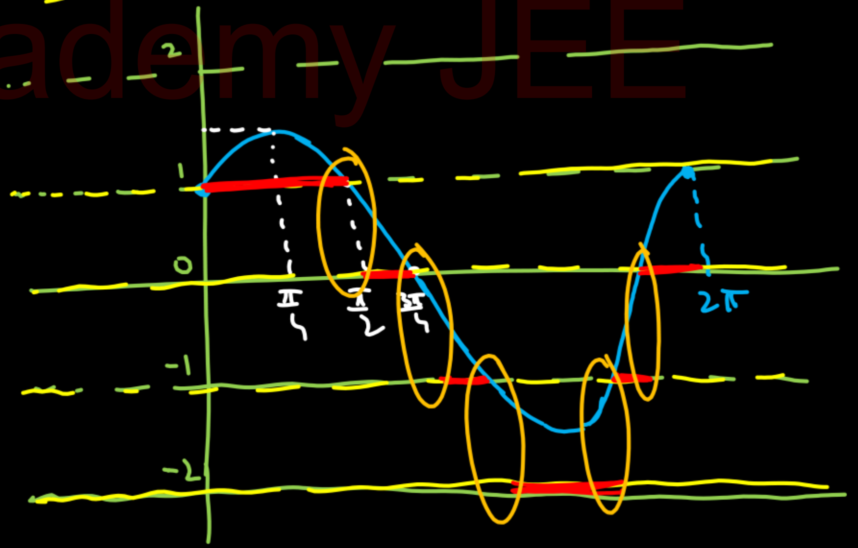
Find the number of points where f(x) = [sin x + cos x] (where [.] denotes greatest integral function), x ∈ (0,2π) is not continuous, is/are $\underline{5}$.

JEE 1

Now: $y = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$$f(x) = \left[\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)\right]$$

$$= \left[\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right]$$



22

JEE 1

If $\int \underbrace{(x^{2010}+x^{804}+x^{402})}(2x^{1608}+5x^{402}+10)^{\frac{1}{402}}dx = \frac{1}{10a}(2x^{2010}+5x^{804}+10x^{402})^{\frac{a}{402}}$,

Then (a – 400) is equal to 3.

$$\int \underline{(x^{2009}+x^{803}+x^{401})}\underbrace{(2x^{2010}+5x^{804}+10x^{402})}^{1/402}\,dx$$

Let: $2x^{2010}+5x^{804}+10x^{402}=t$

$$(4020x^{2009}+4020x^{803}+4020x^{401})\,dx = dt$$

$$= \int (t)^{1/402} \frac{dt}{4020}$$

$$= \frac{1}{4020} \times \frac{t^{\left(\frac{1}{402}+1\right)}}{\left(\frac{403}{402}\right)} + C$$

$$= \frac{1}{10 \times 403} (t)^{403/402} + C$$

$$\Rightarrow a = 403 \Rightarrow \boxed{a - 400 = 3}$$

23

JEE 1

If $\alpha$ and $\beta(\alpha < \beta)$ are the roots of the equation

$$\lim_{t\to\infty} \cos^{-1}\left[\sin\left(\tan^{-1}\left(\frac{\sqrt{tx}}{\sqrt{tx^2-3tx+t-1}-x}\right)\right)\right] = \frac{\pi}{6}$$

then the value of $(8^\alpha + 2^\beta - \alpha\beta)$ is

$$\lim_{t\to\infty}\left(\frac{\sqrt{tx}}{\sqrt{tx^2-3tx+t-1}-x}\right)=\sqrt{3}$$

$$\lim_{t\to\infty}\left(\frac{\sqrt{x}}{\sqrt{x^2-3x+1-\frac{1}{t}}-\frac{x}{t}}\right)=\sqrt{3}$$

$$\frac{\sqrt{x}}{\sqrt{x^2-3x+1}}=\sqrt{3}$$

$\boxed{x>0}$ :

$$x = 3(x^2-3x+1)$$

$$3x^2-9x+3=x$$

JEE 1

$3x^2 - 10x + 3 = 0$

$x = \frac{1}{3}$ $\qquad$ $x = 3$

$\alpha = \frac{1}{3}$ $\qquad$ $\beta = 3$

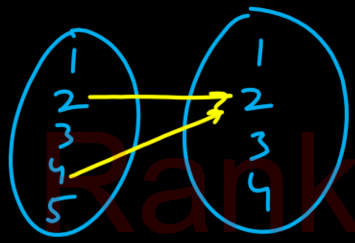


$8^{\alpha} + 2^{\beta} - \alpha\beta$

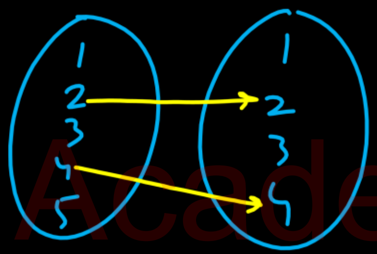
$2 + 8 - 1 = \boxed{9}$

24

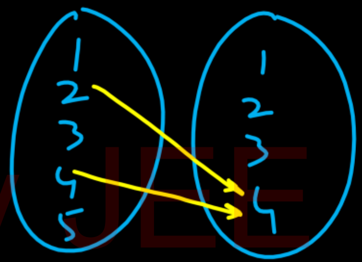
$f: \{1,2,3,4,5\} \rightarrow \{1,2,3,4\}$ such that $f(i) =$ even if 'i' is even and $f(i) \leq f(j) \forall i < j$. The units digit of number of such functions possible is




$2 \times 1 \times 3 = 6$




$2 \times 3 \times 1 = 6$




$4 \times 1 \times 1 = 4$

total = 16 ways

25

JEE 1

Sum of all possible values of x, satisfying the equation $\frac{[x]}{[x-2]} - \frac{8\{x\}+12}{[x-2][x]} = \frac{[x-2]}{[x]}$ is k , then [k] = ______ ([.] denotes G. I. F)

$$\frac{[x]}{([x]-2)} - \frac{8\{x\}+12}{([x]-2)([x])} = \frac{[x]-2}{[x]}$$

$$\frac{[x]^2 - 8\{x\} - 12}{([x]-2)\cancel{[x]}} = \frac{[x]-2}{\cancel{[x]}}$$

$$\cancel{[x]^2} - 8\{x\} - 12 = \cancel{[x]^2} - 4[x] + 4$$

$$-2\{x\} - 3 = -[x] + 1$$

$$\boxed{\frac{[x]-4}{2} = \{x\}}$$

$$0 \le \{x\} < 1$$

$$0 \le \frac{[x]-4}{2} < 1$$

$$0 \le [x]-4 < 2$$

$$\boxed{4 \le [x] < 6}$$

P-1: $[x] = 4 \rightarrow \{x\} = 0 \Rightarrow x = 4$

P-2: $[x] = 5 \rightarrow \{x\} = \frac{1}{2} \Rightarrow x = 5.5$

$\therefore [K] = [9.5] = 9$