ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Advanced)
FULL SYLLABUS
04-02-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE ALL STAR BATCH (SCORE-II)

ANSWER KEY — PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4				
	A.	B	D	D	B				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,B,D	A,C,D	A,C	A,C,D	A,B,C	A,B		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	2	5	5	4	5	1	8	6

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4				
	A.	C	C	C	B				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,B	A,B,C,D	B,C,D	C,D	A,B,D	A,B		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	3	9	3	8	9	7	7	6

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4				
	A.	A	C	A	A				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,C,D	A,C	A,C	A,C,D	B,C	C,D		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	9	2	6	1	9	2	2	3

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

2. Ans (D)

$$\oint \vec{B}.\vec{dl} = \mu_0(i_1 + i_3 + i_2 - i_3) = \mu_0(i_1 + i_2)$$

[Since for the given direction of circulation i_3 entering at PSTU is positive while i_3 at PQRS is negative].

Alternative solution

$$\oint_{ABCDA} \vec{B}.\vec{dl} = \oint_{ABCA} \vec{B}.\vec{dl} + \oint_{CDAC} \vec{B}.\vec{dl} = \mu_0 i_1 + \mu_0 i_2$$

$$= \mu_0(i_1 + i_2)$$

3. Ans (D)

$$\vec{g}_P = \vec{g}_{Sphere} + \vec{g}_{cavity}$$

$$|\vec{g}| = \left[\frac{GM_1}{r_1^2} + \left(-\frac{GM_2}{r_2^2}\right)\right]$$

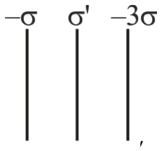
$$|\vec{g}| = \frac{GM_1}{r_1^2} - \frac{GM_2}{r_2^2}$$

$$M_1 = \rho\frac{4}{3}\pi R^3 \text{ and } M_2 = \rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

PART-1 : PHYSICS

SECTION-I (ii)

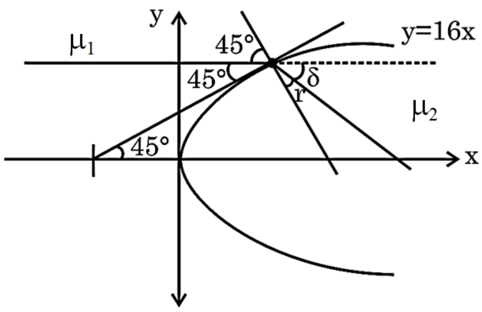
5. **Ans (A,B,D)**



$$-\frac{\sigma}{2\epsilon_0}-\frac{\sigma'}{2\epsilon_0}+\frac{3\sigma}{2\epsilon_0}=0$$

$\sigma' = +2\sigma$

7. **Ans (A,C)**



$y=\sqrt{kx}$

$y=\sqrt{k}(x)^{1/2}$

$\frac{dy}{dx}=\frac{\sqrt{k}}{2\sqrt{x}}$ for $y = 8$

$x = 4$

$\frac{dy}{dx}=\frac{\sqrt{k}}{2\sqrt{x}}=1$

$\tan\theta = 1,\ \theta=\frac{\pi}{4}$

$\mu_1 \sin 45° = \mu_2 \sin r$

$\sin r = \frac{\mu_1}{\sqrt{2}\mu_2};$

$\mu_1 = 1 \quad \mu_2 = \sqrt{2}$

$\sin r = \frac{1}{2} \rightarrow r = 30° = \frac{\pi}{6};\ \delta = \frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$

if $\mu_1 = \sqrt{2}$ and $\mu_2 = 1$

Then at $x = 4$ $y = 8$; $i = \theta_2 = \sin^{-1}\left(\frac{1}{\mu}\right) = 45$

so ray just grazes the surface and angle of deviation

at that point will be :

$\delta = \frac{\pi}{4}$

8. **Ans (A,C,D)**

Frequency of surface MN$(f') = \frac{10f}{7}$

$\lambda_A = \frac{7v}{f'} = \frac{49v}{10f}$

$\lambda_B = \frac{7v}{f'} = \frac{49v}{10f}$

$f_A\left(\frac{7v-\frac{v}{6}}{7v}\right)f' = \left(\frac{205}{147}\right)f$

$f_A\left(\frac{7v+\frac{v}{6}}{7v}\right)f' = \left(\frac{640}{441}\right)f$

10. **Ans (A,B)**

P_{output} of bulb = 10 W

$I = \frac{10}{4\pi(2)^2}$

Power incident on plate $= \frac{10}{4\pi(2)^2}\times\frac{2}{10^4}$

Energy of each photon $= \frac{12400}{1000}$ eV $= 12.4$ eV

Total no. of photons falling on plate per second

$$=\frac{\left[\frac{10\times 2}{4\pi(2)^2\times 10^4}\right]}{12.4\times 1.6\times 10^{-19}}=n$$

No. of e^- ejected per second $= \frac{n}{10^6}$

Saturation current $= \frac{n}{10^6}\times e$

$$=\frac{\left[\frac{10\times 2}{4\pi(2)^2\times 10^4}\right]}{12.4\times 10^6}=3.2\times 10^{-12}\ \text{A}$$

$KE_{max} = (12.4 - 10)$

$= 2.4$ eV

PART-1 : PHYSICS

SECTION-III

3. Ans (5)

$t = \frac{d}{v}$ and $v = \sqrt{\frac{2eV}{m}}$ (2')

When the electron enters the disk, the impulse is

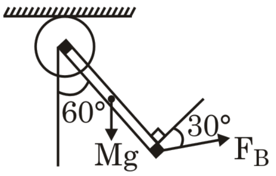
$m\Delta v = t(evB)$ (2')

$\Rightarrow \Delta v = \frac{evBt}{m} = \frac{eBd}{m}$

$\Rightarrow \frac{\Delta v}{v} = \frac{eBd}{mv} = \frac{r}{L}$ where $r \leqslant R$ (3')

$\Rightarrow B = \frac{r}{dL}\sqrt{\frac{2mV}{e}}$ (1')

4. Ans (4)



$F_B = m.B,\ B = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}}$

in equilibrium,

$F_B \cos 30°.\ell = Mg\frac{\ell}{2}.\sin 60°$

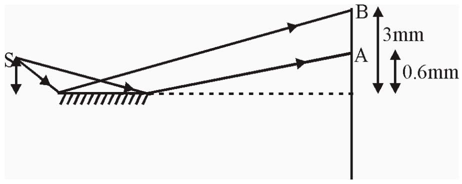
$m.\frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}} = \frac{Mg}{2}$

$m \times \frac{4\pi \times 10^{-7} \times 1000 \times \frac{100}{\pi} \times 10^{-2}}{2 \times 8}$

$= \frac{2 \times 10^{-6} \times 10}{2}$

$m = \frac{16}{40} = 0.4\ A - m$

6. Ans (1)



At A

$\frac{(0.6)(1.2) \times 10^{-6}}{6} = n_1\left(4 \times 10^{-7}\right)$

$\Rightarrow n_1 = 0.3$

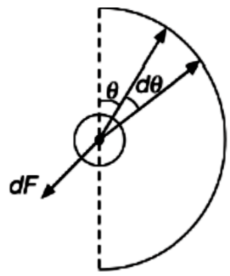
At B

$\frac{(3)(1.2) \times 10^{-6}}{6} = n_2\left(4 \times 10^{-7}\right) \Rightarrow n_2 = 1.5$

So, there will be two maximas in the pattern.

7. Ans (8)

Magnetic field due to small ring at distance R from the centre



$B = \frac{\mu_0}{4\pi}\frac{M}{R^3}$, where $M = I\pi a^2$

$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{I\pi a^2}{R^3} = \frac{\mu_0 Ia^2}{4R^3}$

$\Rightarrow dF = BI_0 d\ell = BI_0(Rd\theta) = I_0 Rd\theta\frac{\mu_0 Ia^2}{4R^3}$

$\Rightarrow dF_x = dF \sin\theta$

$\Rightarrow dF_x = \frac{\mu_0 II_0 a^2 \sin\theta d\theta}{4R^2}$

$\Rightarrow F_x = \frac{\mu_0 II_0 a^2}{4R^2}\int_0^{\pi} \sin\theta\ d\theta$

$\Rightarrow F_x = \left(\frac{\mu_0 II_0 a^2}{4R^2}\right)2$

$\Rightarrow F_x = \frac{\mu_0 II_0 a^2}{2R^2}$

and $F_y = 0$

$\Rightarrow F_{net} = F_x = \frac{\mu_0 II_0 a^2}{2R^2} = 8\ N$

PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (C)**

Consider following equations

$H_2SO_4 \rightarrow H^+ + HSO_4^-$

$HSO_4^- \rightleftharpoons H^+ + SO_4^{2-}$

$Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$

$PbSO_4(s) = Pb^{2+}_{(aq)} + SO_4^{2-}(aq)$

$Ka_2 = 1.2 \times 10^{-4} = \dfrac{[H^+][SO_4^{2-}]}{[HSO_4^-]}$

$K_{sp} = 1.6 \times 10^{-8} = [Pb^{2+}][SO_4^{2-}]$

$\dfrac{1.2 \times 10^{-4}}{1.6 \times 10^{-8}} = \dfrac{[H^+]}{[HSO_4^-][Pb^{2+}]}$

$[Pb^{2+}] = \dfrac{1.6 \times 10^{-8}}{1.2 \times 10^{-4}} \times \dfrac{[HSO_4^-]}{[H^+]}$

As $[H^+] \approx [HSO_4^-]$

$[Pb^{2+}] = 1.33 \times 10^{-4}$

PART-2 : CHEMISTRY

SECTION-I (ii)

6. **Ans (A,B,C,D)**

$P_T = P^\circ_A X_A + P^\circ_B X_B$ (According to Raoult's law)

$480 = 300\ (X) + 600\ (1 - X)$

$X = 0.4$

Now Y is the mole fraction of A in vapour mixture

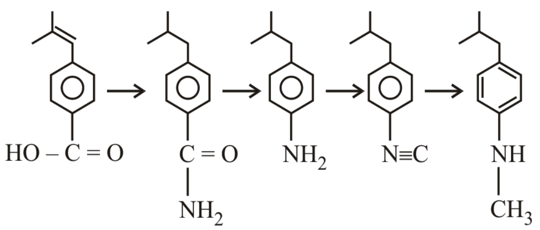
$P_T \cdot Y_A = P^\circ_A \cdot X_A$

$Y_A = 0.25$

$\dfrac{1}{P} = \dfrac{Y_A}{P^\circ_A} + \dfrac{Y_B}{P^\circ_B}$ (when $Y_A = 0.4$)

$P = \dfrac{3000}{7}$

10. **Ans (A,B)**



PART-2 : CHEMISTRY

SECTION-III

1. **Ans (3)**

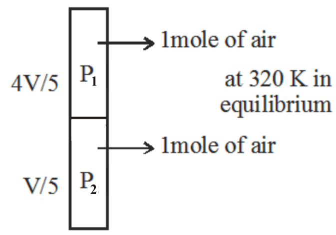
$1.109 = 1.100 - \dfrac{0.06}{2} \log \dfrac{0.1}{[Cu^{2+}]}$

$[Cu^{2+}] = 0.2$ M

$\Delta n = 0.2 \times 0.1 - 0.1 \times 0.1 = 0.01$

$w = 0.01 \times 135 = 1.35$ gm

2. **Ans (9)**



$P_1 + P_w = P_2$

Assuing total volume = V

$\left[\begin{array}{l} P_w = \text{ Pressure exerted due to} \\ \text{weight of things on below} \\ \text{portion by above portion} \end{array}\right]$

$P_1 \times \dfrac{4V}{5} = P_2 \times \dfrac{V}{5}$

$\left[\begin{array}{l} \text{Initial ratio of} \\ \text{volume } = 4:1 \end{array}\right]$

$4P_1 = P_2$

$P_w = 3P_1$

When temperature is changed to T^1

$P_1' \times \dfrac{3V}{4} = nRT^{-1}$

$P_2' \times \dfrac{V}{4} = nRT^{-1}$

$P_2' = 3P_1'$

$\left[\begin{array}{l} \text{Initial ratio of} \\ \text{volume } = 4:1 \end{array}\right]$

$P_2' = P_1' + P_w$

$2P_1' = P_w - 3P_1$

So new temperature

$P_1 \dfrac{4V}{5} = nR \times 320$

$P_1' = \dfrac{3B}{4} = nR \times T^1$

$\dfrac{P_1}{P_1'} = \dfrac{4 \times 4}{3 \times 5} = \dfrac{320}{T^1}$

$\dfrac{2}{3} \times \dfrac{4 \times 4}{3 \times 5} = \dfrac{320}{T^1}$ $\quad T^1 = 450$ K

3. Ans (3)

$g = 9.8\ m/s^2$

$m = 50\ kg$

$\Rightarrow$ Now 27 g Al = 1 mol Al

From equation -

$Al_2O_3 + 3C \longrightarrow 2Al + 3CO$

$\because$ To produce 2 moles of Al, required standard free energy = 588 kJ

$\therefore$ To produce 1 mol of Al, energy required = $\frac{588}{2}$ kJ

$\Rightarrow$ Energy produced when mass is dropped from 2m once = mgh J = $50 \times 9.8 \times 2$ J = 980 J

$\therefore$ No. of times the athlete will have to lift the mass

$$= \frac{\text{Energy required}}{980\ \text{J}} = \frac{588 \times 1000}{2 \times 980} = 300$$

6. Ans (7)

i, ii, iii, v, vi, viii, ix

7. Ans (7)

Molecular weight of glycine $75 \times 2 = 150$

Molecular weight of asparagine $132 \times 2 = 264$

Total molecular weight of all 4 amino acids

$150 + 264 = 414$

Molecular weight of tetrapeptide

$= 414 - 54$ (sum of $3H_2O$) $= 360$

% of N in tetrapeptide

$Z = \frac{84}{360} \times 100$

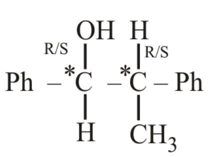
$X = 0.3\ Z$

$X = \frac{3}{10} \times \frac{84}{360} \times 100$

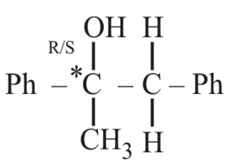
$X = 7$

8. Ans (6)

$4 + 2 = 6$



2° Alcohol (4)



3° Alcohol (2)

PART-3 : MATHEMATICS

SECTION-I (i)

1. Ans (A)

$$\int \frac{(x^2-1)\ dx}{\frac{(x^2+1)}{x^2} \cdot x^3 \sqrt{\left(x+\frac{1}{x}-1\right)\left(x+\frac{1}{x}+1\right)}}$$

$$\int \frac{\left(1-\frac{1}{x^2}\right)\ dx}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-1}},$$

put $x + \frac{1}{x} = t$

$$\int \frac{dt}{t\sqrt{t^2-1}} = \sec^{-1}t + c$$

2. Ans (C)

$$P = \frac{P\ (S_1 \cap (E_1 = E_3))}{P\ (E_1 = E_3)} = \frac{P\ (B_{1,3})}{P\ (B)}$$

$P(B) = P(B_{1,3}) + P(B_{1,4}) + P(B_{3,4})$

$\uparrow$	$\uparrow$	$\uparrow$
If 1,3 chosen at start	If 1,4 chosen at start	If 3,4 chosen at start

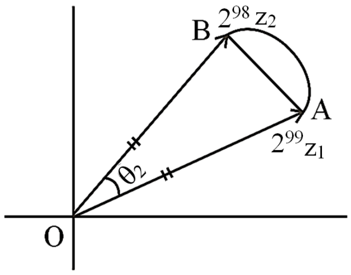
$$P\ (B_{1,3}) = \frac{1}{3} \times \frac{1 \times {}^3C_1}{\underbrace{{}^4C_2}_{\text{1 is definitely chosen from } F_2}} \times \frac{1}{\underbrace{{}^5C_2}_{\text{1,3 chosen from } G_2}}$$

$$P\ (B_{1,4}) = \frac{1}{3} \times \frac{1 \times {}^2C_1}{\underbrace{{}^3C_2}_{\text{1 is definitely chosen from } F_2}} \times \frac{1}{\underbrace{{}^5C_2}_{\text{1,3 chosen from } G_2}}$$

$$P\ (B_{3,4}) = \frac{1}{3} \times \left[\frac{{}^3C_2 \times 1}{\underbrace{{}^4C_2}_{\text{If 1 is not chosen from } F_2}} \times \frac{1}{{}^4C_2} + \frac{1 \times {}^3C_1}{\underbrace{{}^4C_2}_{\text{If 1 is chosen from } F_2}} \times \frac{1}{{}^5C_2}\right]$$

$$\frac{P\ (B_{1,3})}{P\ (B)} = \frac{1}{5}$$

3. **Ans (A)**



$P : |z_2 - z_1| + |z_3 - z_2| + \ldots |z_{100} - z_{99}| + |z_1 - z_{100}|$

$\geq ||z_2| - |z_1|| + ||z_3| - |z_2|| + \ldots$

$||z_{100}| - |z_{99}|| + ||z_{100}| - |z_1||$

$= (2-1) + (2^2 - 2^1) + (2^3 - 2^2) +$

$\ldots (2^{100} - 2^{99}) + (2^{100} - 1)$

$= 2^{100} - 1 + 2^{100} - 1 = 2^{101} - 2$

$\text{Arc} \geq |2^{98}z_2 - 2^{99}z_1|$

$\theta_2 \times OA \geq |2^{98}z_2 - 2^{99}z_1|$

$\Sigma\, \theta_2 \times 2^{99} \geq \Sigma\, |2^{98}z_2 - 2^{99}z_1|$

$\Rightarrow 2\pi \times 2^{99} \geq \Sigma\, |2^{98}z_2 - 2^{99}z_1|$

4. **Ans (A)**

$P_1 : x + y + z = a$

$P_2 : x + 5y - z = b$

$P_3 : x - y + 2z = c + 2$

for consistent

$P_3 = 3P_1 - 2P_2 \Rightarrow b = 3a - 2c - 4$

$\Rightarrow 3a - b - 2c - 4 = 0$

so $P : 3x - y - 2z - 4 = 0$

$D = \dfrac{4}{\sqrt{14}}$

$$M = \begin{vmatrix} a & -2 & b \\ 2 & 1 & c \\ 1 & 0 & 3 \end{vmatrix}$$

$= 3a - 2(c-6) + b(-1)$

$= 3a - b - 2c + 12$

$= 4 + 12 = 16$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. **Ans (A,C,D)**

$xf'(x) + (x\tan x + 1)f(x) = \sec x$

$$\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$$

Use LDE

$$y\,|x\sec x| = \int \frac{\sec x}{x}.\,|x\sec x|dx$$

$x \in \left(0, \dfrac{\pi}{2}\right) \quad \Rightarrow yx\sec x = \tan x + c$

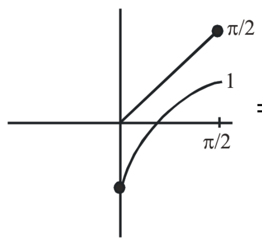
$f\left(\dfrac{\pi}{4}\right) = 0 \Rightarrow c = -1$

$$y = \frac{\tan x - 1}{x\sec x} = \frac{\sin x - \cos x}{x}$$

$$f\left(\frac{\pi}{3}\right) = \frac{3\left(\sqrt{3}-1\right)}{2\pi} \text{ (A)}$$

$f(x) = 1 \Rightarrow \sin x - \cos x = x$

$$\Rightarrow \sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = x$$

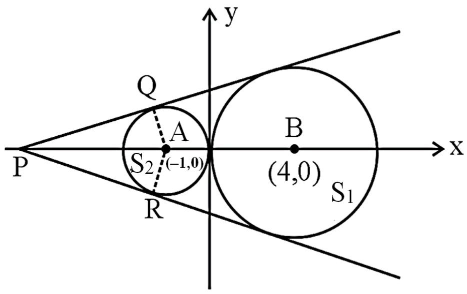


$\Rightarrow$ No solution

$$\int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx = \sqrt{2} - 1$$

Range of $x f(x)$ is finite.

6. **Ans (A,C)**



$r_2 = 1, r_1 = 4$

$\frac{PA}{PB} = \frac{1}{4} \quad \therefore P\left(\frac{1\times 4 - 4\times -1}{1-4}, 0\right)$

$\therefore P\left(-\frac{8}{3}, 0\right)$

Circum circle of ΔPQR

$\left(x+\frac{8}{3}\right)(x+1)+y^2 = 0$

$x^2+y^2+\frac{11}{3}x+\frac{8}{3} = 0$

7. **Ans (A,C)**

(A) $g(x) = x^{10} - f(2x)$, $g(0)\, g(1) < 0$ (A is true)

(B) $g'(x) = 2x - f(x) = 0 \;\; g(0) = 0 \;\; g(1) > 0$

(B is true)

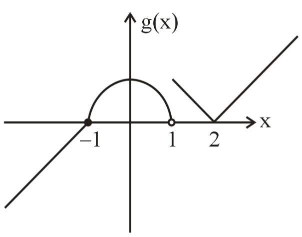
(C) $g(x) = \int_0^{\frac{\pi}{2}-x} f(3t)\sin 6t\, dt - x$, $g(0), g(1) < 0$

(C is true)

(D) $g(0) = 0, g(1) > 0$

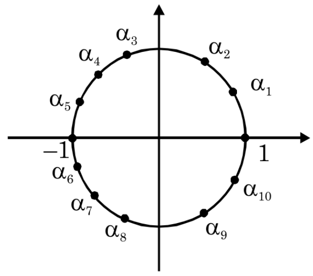
$g'(x) > 0$ so $g(x) \neq 0$ for $x \in (0, 1)$

8. **Ans (A,C,D)**



g(x) is discontinuous at x = 1 and non-differentiable at x = –1, 1, 2

9. **Ans (B,C)**



$|z| = 1$

$\alpha_k = e^{i\left(\frac{2\pi K}{11}\right)}$

$\forall k = \{0, 1, \ldots .10\}$

(A) for $\alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$, $\cos(\arg z) < 0$, from figure shown

(B) using Pythagoras theorem $|1-\alpha_i|^2 + |1+\alpha_i|^2 = 4$

(C) $|z+1|$ = distance of z from (–1), so it contains 6 distinct value

(D) $\arg(-1) + \arg\left(\frac{z+1}{z-1}\right) = \pi + \frac{\pi}{2}$, because angle in semi circle is right angle

10. **Ans (C,D)**

(A) $(ABA^T)^T = AB^TA^T$ which is not symmetric

(B) $(AB-BA)^T = (AB)^T - (BA)^T = B^TA^T - A^TB^T$

$\therefore AB - BA$ is not symmetric

(C) $B = |A|\frac{\text{adj. } A}{|A|} = \text{adj. } A$

$\text{adj } A^T - B = \text{adj } A^T - \text{adj. }(A)$

$(\because \text{adj } A^T = (\text{adj } A)^T)$

$((\text{adj } A)^T - \text{adj } A)^T = \text{adj. } A - (\text{adj. } A)^T$

$\therefore \text{adj.}(A^T) - B$ is skew symmetric

(D) $B + A^T = O$ and $A^T = -A$

$\Rightarrow B = A$

$B^{15} = A^{15}$, A is skew symmetric

$\therefore A^{15}$ is also skew symmetric

PART-3 : MATHEMATICS

SECTION-III

1. **Ans (9)**

$xy = 2^3 3^4 5^6 (x + y)$

or $xy - Sx - Sy + S^2 = S^2$ (where $S = 2^3.3^4.5^6$)

$\Rightarrow (x - S)(y - S) = 2^6 3^8 5^{12}$

So, number of positive integral solution

$= (6 + 1).(8 + 1)(12 + 1)$

$= 7 \times 9 \times 13 = 819$

2. **Ans (2)**

There are 4 even numbers & 5 odd numbers in $\{1,2,......9\}$ and required probability

$$= \frac{P(e_1e_2e_3) + P(o_1o_2e_1)}{P(e_1e_2e_3) + P(e_1e_2o_1) + P(o_1o_2o_3) + P(o_1o_2e_1)}$$

$e_1e_2e_3$ shows order that 1st numbers is even, 2^{nd} is even, 3^{rd} is even

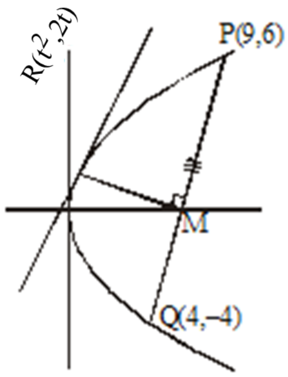
Hence required probability =

$$\frac{\left(\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\right)}{\left(\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{4}{9}\times\frac{3}{8}\times\frac{5}{7}\right)+\left(\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\right)+\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\right)}$$

$= \frac{13}{28} = \frac{p}{q} \Rightarrow q - 2p = 2$

3. **Ans (6)**

Let $R = (t^2, 2t)$



Equation of tangent

at $R(t^2, 2t)$

$ty = x + t^2$

$\Rightarrow$ Slope of tangent $= \frac{1}{t}$

slope of PQ = 2

For min. area of ΔRPQ,

$\frac{1}{t} = 2 \Rightarrow t = \frac{1}{2}$

Equation of PQ : $y + 4 = 2(x - 4)$

$\Rightarrow 2x - y - 12 = 0$

$\Rightarrow RM = \frac{\left|\frac{1}{2} - 1 - 12\right|}{\sqrt{5}} = \frac{25}{2\sqrt{5}}$, $PQ = \sqrt{125}$

$\therefore$ Area of $\Delta PRQ = 5\sqrt{5}.\frac{25}{2\sqrt{5}} = \frac{125}{4}$

$\therefore \left[\frac{\sqrt{5A}}{2}\right] = \left[\frac{\sqrt{\frac{625}{4}}}{2}\right] = \left[\frac{25}{4}\right] = 6$

4. Ans (1)

$$y=\lim_{n\to\infty}\left[\frac{(n+x)(n+2x)\ldots(n+2nx)}{n^{2n}}\right]^{\frac{1}{2n}}$$

$$.\left(\tan\frac{\pi}{2n}.\tan\frac{2\pi}{2n}\ldots\right)^{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{x}{n}\right)\left(1+\frac{2x}{n}\right)\ldots\left(1+\frac{2nx}{n}\right)\right]^{\frac{1}{2n}}$$

$$\times\prod_{r=1}^{n-1}\left[\tan\left(\frac{r\pi}{2n}\right)\right]^{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\prod_{r=1}^{2n}\left(1+\frac{rx}{n}\right)^{\frac{1}{2n}}\prod_{r=1}^{n-1}\left(\tan\frac{r\pi}{2n}\right)^{\frac{1}{n}}$$

$$\therefore\ \ell n\, y=$$

$$\lim_{n\to\infty}\left\{\frac{1}{2n}\sum_{r=1}^{2n}\ell n\left(1+\frac{rx}{n}\right)+\frac{1}{n}\sum_{r=1}^{n-1}\ell n\left(\tan\frac{r\pi}{2n}\right)\right\}$$

$$=\frac{1}{2}\int_0^2\ell n(1+tx)\,dt+\int_0^1\ell n\left(\tan\frac{t\pi}{2}\right)dt$$

$$=\frac{1}{2x}\int_0^{2x}\ell n(1+t)\,dt+\frac{2}{\pi}\int_0^{\pi/2}\ell n\tan t\,dt$$

$$=\frac{1}{2x}\left[(1+t)\,\ell n(1+t)-(1+t)\right]_0^{2x}$$

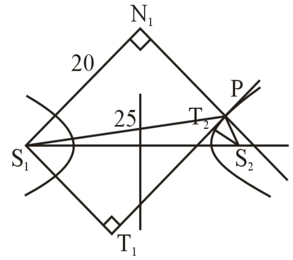
$$+\frac{2}{\pi}\int_0^{\frac{\pi}{2}}(\ell n\sin t-\ell n\cos t)\,dt$$

$$=\frac{1}{2x}\left[(1+2x)\,\ell n(1+2x)-2x\right]+\frac{2}{\pi}(0)$$

$$\Rightarrow\lim_{x\to0}\ell ny=\lim_{x\to0}\frac{(1+2x)\,\ell n(1+2x)}{2x}-1=0$$

$$\Rightarrow\lim_{x\to0}y=\lim_{x\to0}e^{\ell ny}=e^0=1$$

5. Ans (9)



$$\frac{PS_1}{PS_2}=5 \qquad ....(1)$$

$$|PS_1-PS_2|=20 \qquad ....(2)$$

$$\Rightarrow PS_1=25\,,\ PS_2=5$$

Now $PN_1S_1T_1$ is a rectangle

Distance of S_1 from tangent at P is $\sqrt{(25)^2-(20)^2}$

$=15=S_1T_1$

Because $\Delta PS_1T_1\sim\Delta PS_2T_2$,

so $\Rightarrow\dfrac{PS_1}{S_1T_1}=\dfrac{PS_2}{S_2T_2}\Rightarrow\dfrac{PS_1}{PS_2}=\dfrac{S_1T_1}{S_2T_2}$

(Tangent is the internal angle bisector)

Distance of S_2 from tangent at P is $\dfrac{15}{5}=3$

$b^2=15\times3=45 \quad\Rightarrow 2b=6\sqrt{5}$

6. Ans (2)

Suppose

$$\begin{vmatrix}\frac{n(n+1)}{2} & n(n-1).2^{n-2}+n.2^{n-1}\\ n.2^{n-1} & 4^n\end{vmatrix}+\frac{n(n+1)}{2}.4^n=0$$

$$\frac{n(n+1)}{2}.4^n-n^2(n-1).2^{2n-3}-n^2 2^{2n-2}+n\frac{(n+1)}{2}4^n=0$$

$$n(n+1)-\frac{n^2(n-1)}{8}-\frac{n^2}{4}=0$$

$n=8$

$$\text{Now }\sum_{k=1}^{8}\frac{{}^8C_k}{k+1}=\sum_{k=1}^{8}\frac{k+1}{9}.\,{}^9C_{k+1}\frac{1}{k+1}$$

$$=\frac{1}{9}.\left[{}^9C_2+{}^9C_3+{}^9C_4+{}^9C_5+{}^9C_6+\ldots+{}^9C_9\right]$$

$$=\frac{1}{5}=\frac{1}{9}\left[2^9-10\right]=\frac{502}{9}=55.78$$

7. Ans (2)

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \left(\frac{\sqrt{85}}{3}\right)^2$$

$$\Rightarrow \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \frac{85}{9}$$

$$\Rightarrow \begin{vmatrix} 1 & \vec{a}.\vec{b} & 2 \\ \vec{a}.\vec{b} & 2 & 1 \\ 2 & 1 & 9 \end{vmatrix} = \frac{85}{9}$$

$$\Rightarrow \vec{a}.\vec{b} = \frac{2}{9}$$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = a^2 + b^2 + c^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right)$$

$$= 1 + 2 + 9 + 2\left(\frac{2}{9} + 1 + 2\right)$$

$$= 18 + \frac{4}{9}$$

8. Ans (3)

Let students have $x_1,\ x_2,\ x_3,\ x_4,\ x_5$ in increasing order of their weights

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = 40 \qquad \text{(Given)}$$

$$\frac{x_2 + x_3 + x_4 + x_5}{4} = 45 \qquad \text{(Given)}$$

$x_1 + x_2 + x_3 + x_4 + x_5 = 160 + x_5,$

(Sum of weights of all the students)

$x_2 + x_3 + x_4 + x_1 + x_5 = 180 + x_1,$

(Sum of weights of all the students)

$160 + x_5 = 180 + x_1 \qquad (1)$

Average maximum $\Rightarrow x_5$ is maximum

$x_5 - x_1 = 20\ ,\quad x_5 = x_1 + 20$

$(x_5)_{max} = (x_1)_{max} + 20 = 40 + 20 = 60$

Average maximum = 220/5 = 44

now, average is minimum when least weight is minimum

$x_1 = x_5 - 20$ min $\rightarrow$ Avg g heaviest (45)

$\therefore\ (\bar{x})\text{ min} = \frac{205}{5} = 41$

$\therefore\ (\bar{x})\text{ max} - (\bar{x})\text{ min} = 44 - 41 = 3$ kg