

FIITJEE RBT-4 for (JEE-Advanced)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 4

QP CODE: 100958

PAPER - 2

Time Allotted: 3 Hours

Maximum Marks: 186

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

- Attempt ALL the questions. Answers have to be marked on the OMR sheets.
- This question paper contains **Three Sections**.
- Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
- Each **Section** is further divided into **Two Parts: Part-A & B** in the OMR.
- Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
- Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

- Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
- On the OMR sheet, darken the appropriate bubble with **Blue/Black Ball Point Pen** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
- OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Part.

- (i) **PART-A (01-08)** contains (8) Multiple Choice Questions which have **One or More Correct** answer.
Full Marks: +4 If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: +1 For darkening a bubble corresponding to **each correct option**, provided NO incorrect option is darkened.
Zero Marks: 0 If none of the bubbles is darkened.
Negative Marks: -2 In all other cases.
For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only (A) and (D) will result in **+2 marks**; and darkening (A) and (B) will result in **-2 marks**, as a wrong option is also darkened.
- (ii) **Part-A (09-12)** – This section contains Two (02) List-Match Sets, each List-Match set has Two (02) Multiple Choice Questions. Each List-Match set has two lists: List-I and List-II. FOUR options are given in each Multiple Choice Question based On List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question. Each question carries **+3 Marks** for correct combination chosen and **-1 marks** for wrong options chosen.
- (iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate : _____

Batch : _____ Date of Examination : _____

Enrolment Number : _____

BATCH – Two Year CRP325 batches

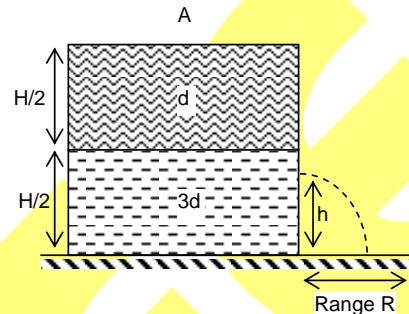
SECTION – I : PHYSICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE is correct**.

1. A container of large uniform cross sectional area A resting on a horizontal surface holds two immiscible non viscous and incompressible liquids of density d and $3d$, each of height $H/2$. The lower density liquid is open to the atmosphere having pressure P_0 . A tiny hole of area a ($a \ll A$) is punched to the vertical side of lower container at a height h ($0 < h < H/2$) for which range is maximum.



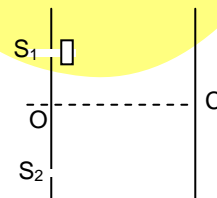
(A) $h = H/3$

(B) Range $R = \frac{2H}{3}$

(C) Range $R = \frac{3H}{2}$

(D) Velocity of efflux $v = \sqrt{\frac{2}{3}gH}$

2. A YDSE is performed in a medium of refractive index μ_1 . In front of one of the slits say S_1 as shown, thin glass slab of refractive index μ_2 ($< \mu_1$) is kept. If initially the central maxima was formed on the central line OC then



- (A) central maxima will shift upwards
 (B) central maxima will shift downwards
 (C) the waves reaching on the screen at C from S_1 will lead the waves reaching from S_2
 (D) the waves reaching C from S_1 will lag from the waves reaching from S_2

3. Two projectile are thrown at the same time from two different points. The projectile thrown from the origin has initial velocity $3\hat{i} + 3\hat{j}$ with respect to earth. The projectile has initial velocity $a\hat{i} + b\hat{j}$ with respect to earth thrown from the point (10, 5). (\hat{i} is a unit vector along horizontal, \hat{j} along vertical). If the projectile collides after two second, then the

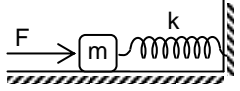
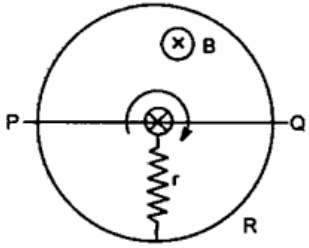
(A) value of a is -2

(B) value of a is $\frac{1}{2}$

(C) value of b is $\frac{1}{2}$

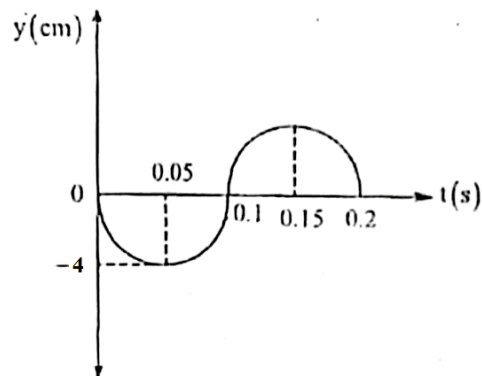
(D) value of b is -2

Space For Rough Work

4. A constant force F is applied on a spring block system as shown in figure. The mass of the block is m and spring constant is k . The block is placed over a smooth surface. Initially the spring was unstretched. Choose the correct alternative (s)
- 
- (A) the block will execute SHM (B) amplitude of oscillation is $F/2k$
- (C) time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$ (D) the maximum speed of block is $\sqrt{\frac{2Fx - kx^2}{m}}$
(here $x = F/k$)
5. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then
- (A) $W_{nc} - \Delta U = \Delta k$ (B) $W_c = -\Delta U$
- (C) $W_{nc} + W_c = \Delta k$ (D) $W_{nc} - \Delta U = -\Delta k$
6. A particle of charge q and mass m moves rectilinearly under the action of an electric field $E = \alpha - \beta x$. Here, α and β are positive constants and x is the distance from the point where the particle was initially at rest. Then,
- (A) the motion of the particle is oscillatory.
- (B) the amplitude of the particle is $\left(\frac{\alpha}{\beta}\right)$
- (C) the mean position of the particle is at $x = \left(\frac{\alpha}{\beta}\right)$
- (D) the maximum acceleration of the particle is $\frac{q\alpha}{m}$
7. In figure, R is a fixed conducting ring of negligible resistance and radius a . PQ is a uniform rod of resistance r . It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω . There is a uniform magnetic field of strength B pointing inward and r is a stationary resistance. Then:
- 
- (A) current through r is zero.
- (B) current through r is $\frac{2B\omega a^2}{5r}$
- (C) direction of current in external resistance r is from centre to circumference.
- (D) direction of current in external resistance r is from circumference to center.

Space For Rough Work

8. For a certain transverse standing wave on a long string, an antinode is formed at $x = 0$ and next to it, a node is formed at $x = 0.10$ meter, the displacement $y(t)$ of the string particle at $x = 0$ is shown in figure.

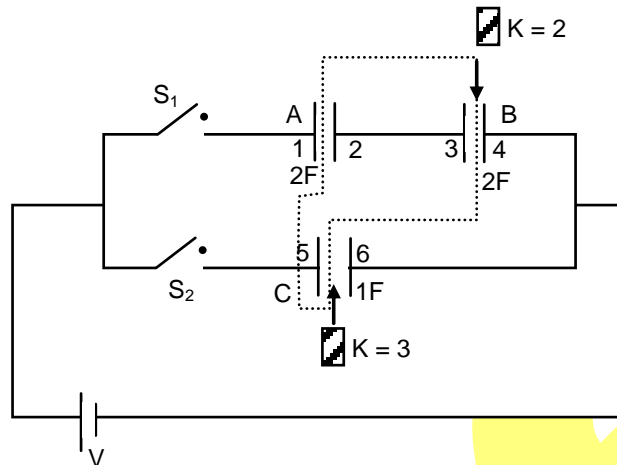


- (A) Transverse displacement of the particle at $x = 0.05$ m and $t = 0.05$ sec is $-2\sqrt{2}$ cm
 (B) Transverse displacement of the particle at $x = 0.04$ m and $t = 0.025$ sec is $-2\sqrt{2}$ cm
 (C) Speed of travelling wave that interfere to produce this standing wave is 2 m/s.
 (D) The transverse velocity of the string particle at $x = \frac{1}{15}$ m and $t = 0.1$ sec is 20π cm/sec.

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)



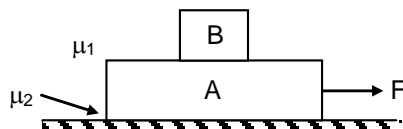
Change in quantities between initial and final situation is given in list-I and its numerical value in list-II.

List-I		List-II	
(I)	Change in amount of + charge on capacitor	(P)	0
(II)	Change in energy	(Q)	$\frac{1}{3}$
(III)	Change in potential difference	(R)	$\frac{1}{36}$
(IV)	Final charge inside closed surface inclosing plate 2, 3 and 5 of different capacitor.]	(S)	$\frac{1}{6}$
		(T)	$\frac{1}{2}$

9. S_1 is closed and S_2 open, and $V = 1$ volt. In initial situation capacity are given. When capacitors are fully charged then dielectric of dielectric constant shown in figure is inserted in capacitor B to fill space between plates completely for final situation. Then for capacitor B. Choose correct option
- (A) I \rightarrow Q, II \rightarrow R, III \rightarrow S, IV \rightarrow P (B) I \rightarrow P, II \rightarrow Q, III \rightarrow R, IV \rightarrow S
(C) I \rightarrow R, II \rightarrow S, III \rightarrow Q, IV \rightarrow P (D) I \rightarrow Q, II \rightarrow S, III \rightarrow P, IV \rightarrow R
10. Now switch S_2 is also closed and $V = 1/6$ volt. In initial situation, capacitor C is completely charged then in final situation, di electric is inserted completely as shown in figure. For capacitor C.
- (A) I \rightarrow Q, II \rightarrow R, III \rightarrow T, IV \rightarrow S (B) I \rightarrow P, II \rightarrow Q, III \rightarrow T, IV \rightarrow R
(C) I \rightarrow Q, II \rightarrow R, III \rightarrow P, IV \rightarrow T (D) I \rightarrow R, II \rightarrow S, III \rightarrow T, IV \rightarrow P

Space For Rough Work

List Match Set (11-12)



List-I		List-II	
(I)	Acceleration of A or B	(P)	$\frac{4}{3}$
(II)	Normal reaction between blocks	(Q)	$\frac{2}{3}$
(III)	Frictional force between blocks f_1	(R)	20
(IV)	Frictional force on surface f_2	(S)	0
		(T)	1

11. $F = 4$ Newton, $m_A = 4$, $m_B = 2$, $\mu_1 = 0.1$ and $\mu_2 = 0$, frictional force between blocks is f_1 and between surface and block A is f_2 ($g = 10 \text{ m/sec}^2$)
- (A) I \rightarrow Q, II \rightarrow P, III \rightarrow R, IV \rightarrow S (B) I \rightarrow Q, II \rightarrow S, III \rightarrow P, IV \rightarrow R
 (C) I \rightarrow S, II \rightarrow R, III \rightarrow Q, IV \rightarrow P (D) I \rightarrow Q, II \rightarrow R, III \rightarrow P, IV \rightarrow S
12. $F = 4$ Newton, $m_A = 4$, $m_B = 2$, $\mu_1 = 0$ and $\mu_2 = \frac{1}{45}$
- (A) I \rightarrow QS, II \rightarrow R, III \rightarrow S, IV \rightarrow P (B) I \rightarrow QS, II \rightarrow RS, III \rightarrow S, IV \rightarrow T
 (C) I \rightarrow QR, II \rightarrow P, III \rightarrow R, IV \rightarrow Q (D) I \rightarrow QS, II \rightarrow S, III \rightarrow P, IV \rightarrow R

(PART – B)

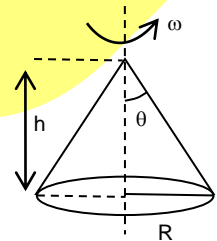
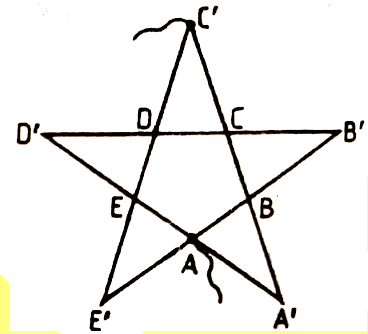
(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. Toluene liquid of volume 300 cm^3 at 0°C is contained in a beaker. Another quantity of toluene of volume 110 cm^3 at 100°C is in another beaker. (the combined volume is 410 cm^3). $\Delta V \text{ cm}^3$ is change in volume of the mixture of the toluene liquids when they are mixed together. Given the coefficient of volume expansion $\gamma = 0.001/\text{C}$ and all forms of heat losses can be ignored. Value of ΔV is

Space For Rough Work

2. Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. Velocity of tuning fork is small in comparison to velocity of sound. (velocity of sound in air is 340 m/s).
3. A five-pointed regular star has been formed from a uniform wire. Calculate the equivalent resistance between points A and C'. Take $\sin 18^\circ = 1/3$ and the resistance of the sections $AA' = A'B = BB' = \dots = r = \frac{1}{2} \Omega$.
4. Three concentric thin spherical shells are of radii a , b and c ($a < b < c$). The first and third are connected by a fine wire through a small hole in the second. The second is connected to earth. Find the capacity of the condenser formed in μF . ($a : b : c = 1 : 2 : 3$, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, $a = 90 \text{ m}$)
5. 'Q' charge is uniformly distributed over the curve surface of a right circular cone of semi-vertical angle θ and height h . The cone is uniformly rotated about its axis at angular velocity ω . Magnetic moment associated with the cone is found to be $nQ\omega h^2 \tan^2 \theta$ where value of 'n' is
6. In an experiment of photoelectric effect light of wavelength 400 nm is incident on a metal plate at the rate of 5W. the potential of the collector plate is made sufficiently positive with respect to emitter so that the current reaches the saturation value. Assuming that on the average one out of every 10^6 photons is able to eject a photoelectron, find the photocurrent in the circuit. (in micro ampere)



Space For Rough Work

SECTION – II : CHEMISTRY

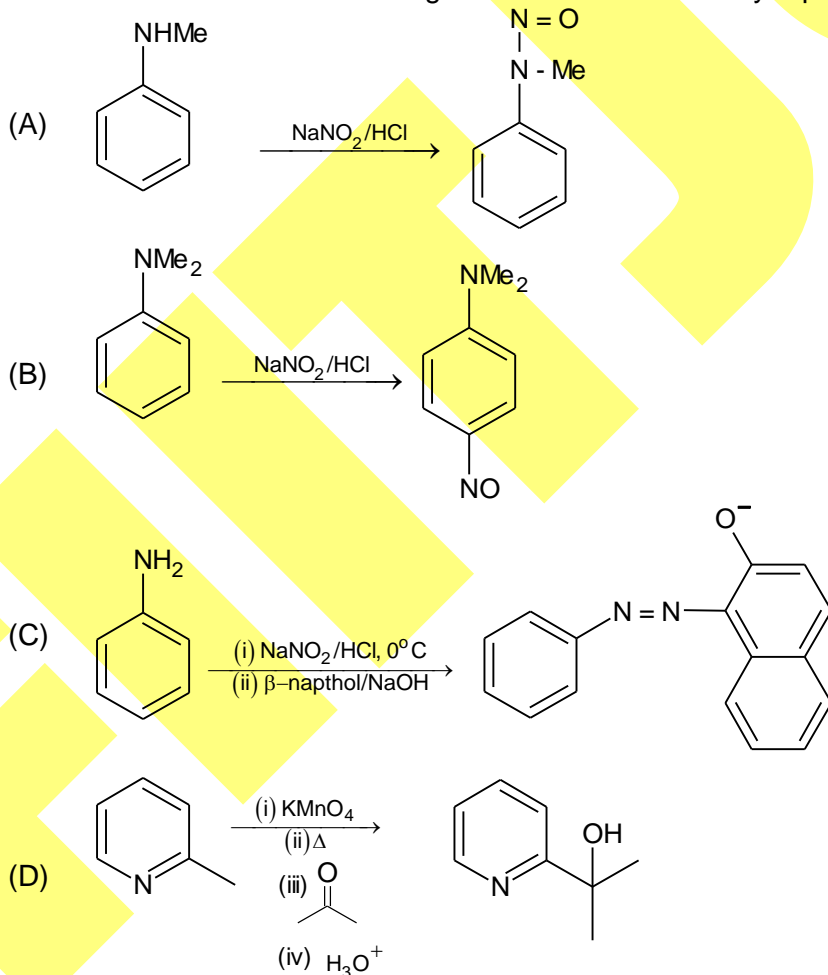
(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE is correct**.

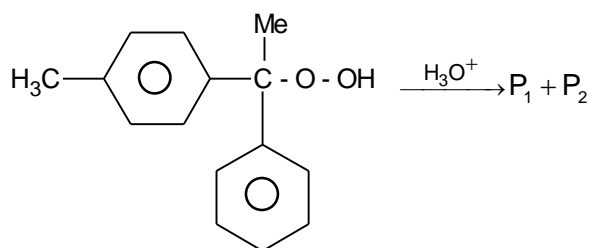
- Which of the following pairs produce same gas?
 (A) $(\text{Ca}_3\text{N}_2 + \text{dil. HCl})$ and $(\text{NH}_4\text{NO}_2 \text{ on heating})$
 (B) $[(\text{NH}_4)_2\text{Cr}_2\text{O}_7 \text{ on heating}]$ and $(\text{NH}_4\text{NO}_2 \text{ on heating})$
 (C) $(\text{NH}_4\text{NO}_3 \text{ on heating})$ and $[\text{Hg}(\text{NO}_3)_2 \text{ on heating}]$
 (D) $(\text{NH}_4\text{Cl on heating})$ and $[\text{NaNO}_3 + \text{Zn} + \text{NaOH}_{(\text{aq.})} \text{ on heating}]$
- A current is passed through 500 mL of an aqueous solution of CaI_2 . After sometime, it is observed that 50 millimoles of I_2 have been formed. Which of the following statements is(are) correct?
 (A) The number of faradays of charge passed through the solution is 0.10 F.
 (B) The volume of dry H_2 at STP that has been formed during electrolysis is 1120 mL.
 (C) The pH of the solution is nearly 0.7.
 (D) The mass of calcium produced is 2.0 g.

- Products of which of the following reactions is/are correctly represented.



Space For Rough Work

4.



$\text{P}_2 + \text{FeCl}_3 \rightarrow \text{Violet Colour}$

$\text{P}_1 + \text{NaOI} \rightarrow \text{P}_3 \downarrow + \text{P}_4$
yellow

The correct statement/s for the above sequence is/are

- (A) P_3 on reaction with Ag gives acetylene
 (B) P_4 on reaction with sodalime gives toluene
 (C) P_4 on reaction with sodalime gives benzene
 (D) P_1 on reaction with LiAlH_4 gives a compound which gives blue colour in Victor Meyer test

5.

During the production of iron and steel.

(A) The oxide ore is primarily reduced to iron by solid coke according to the reaction.



(B) The oxide ore is reduced by the carbon monoxide according to the reaction



(C) Major silica impurities are removed as calcium silicate slag by addition of a fluxing agent lime stone.

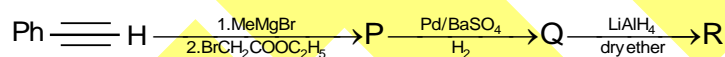
(D) The converter slag containing phosphorus is used as a fertilizer.

6.

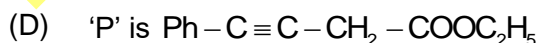
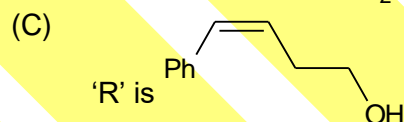
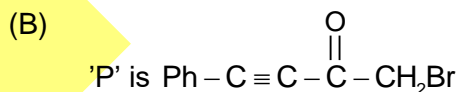
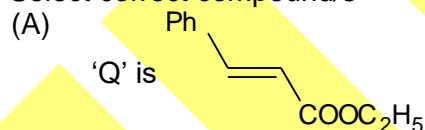
Select the correct statement(s)

- (A) the value of rate constant cannot exceed the value of Arrhenius factor
 (B) molecularity of multiple step reaction can be obtained from mechanism
 (C) half life of a fourth order reaction is linearly dependent on initial concentration of reactant
 (D) reactions with order ≥ 1 cannot get complete infinite time interval

7.



Select correct compound/s



Space For Rough Work

8. Which of the following reactions will have α -hydroxy acid as a product?

- (A) $\text{CH}_3\text{-CHO} \xrightarrow[\text{(ii) H}_2\text{O/H}^+/\Delta]{\text{(i) NaCN/HCl}}$
- (B) $\text{CH}_3\text{-CHO} + \text{Br-CH}_2\text{-COOC}_2\text{H}_5 \xrightarrow[\text{(iii) HOH/H}^+/\Delta]{\text{(i) Zn, (ii) NH}_4\text{Cl/HOH}}$
- (C) $\begin{array}{c} \text{X} \\ | \\ \text{CH}_2\text{-CH}_2\text{-COOH} \end{array} \xrightarrow{\text{NaOH/HOH}}$
- (D) $\text{CH}_3\text{CH}_2\text{Br} \xrightarrow[\text{(iii) NaOH}]{\text{(i) NaCN followed by hydrolysis, (ii) Red P/Br}_2}$

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Match the lists and answer the following question.

List – I		List – II	
Alcohols subjected to Lucas reagent		Observation	
(I)	$\begin{array}{c} \text{CH}_3 \\ \\ \text{H}_3\text{C}-\text{C}-\text{OH} \\ \\ \text{CH}_3 \end{array}$	(P)	White turbidity after boiling only
(II)	PhCH_2OH	(Q)	Turbidity instantaneously
(III)	$\begin{array}{c} \text{H}_3\text{C} \\ \\ \text{H}_3\text{C}-\text{CH}-\text{OH} \end{array}$	(R)	Appearance of turbidity after 5-10 minutes of addition of reactants
(IV)	$\text{H}_3\text{C}-\text{CH}_2\text{CH}_2\text{CH}_2\text{OH}$	(S)	No visible change observed
		(T)	Red colour is obtained

9. Which combination is correct according to the above list?

- (A) I \rightarrow R (B) II \rightarrow Q
(C) III \rightarrow S (D) IV \rightarrow T

Space For Rough Work

Match the lists and answer the following question.

List – I		List – II	
Alcohols subjected to Lucas reagent		Observation	
(I)	$\begin{array}{c} \text{CH}_3 \\ \\ \text{H}_3\text{C}-\text{C}-\text{OH} \\ \\ \text{CH}_3 \end{array}$	(P)	White turbidity after boiling only
(II)	PhCH_2OH	(Q)	Turbidity instantaneously
(III)	$\begin{array}{c} \text{H}_3\text{C} \\ \\ \text{H}_3\text{C}-\text{C}-\text{OH} \end{array}$	(R)	Appearance of turbidity after 5-10 minutes of addition of reactants
(IV)	$\text{H}_3\text{C}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{OH}$	(S)	No visible change observed
		(T)	Red colour is obtained

10. Which combination is correct according to the above list?

- (A) I \rightarrow R
(C) III \rightarrow Q

- (B) II \rightarrow S
(D) IV \rightarrow P

List Match Set (11-12)

Match the Lists and answer the following question.

List – I		List – II	
(I)	Neso Silicate	(P)	$(\text{SiO}_3)_n^{2n-}$
(II)	Soro Silicate	(Q)	$(\text{Si}_2\text{O}_5)_n^{2n-}$
(III)	Cyclic Silicate	(R)	$(\text{Si}_2\text{O}_7)^{6-}$
(IV)	Phyllo Silicate	(S)	None of the O atom is shared
		(T)	All oxygen atoms are shared

11. Which combination is correct according to the above list?

- (A) I \rightarrow Q
(C) III \rightarrow P

- (B) II \rightarrow S
(D) IV \rightarrow P

Space For Rough Work

Match the Lists and answer the following question.

List- I		List – II	
(I)	Neso Silicate	(P)	$(\text{SiO}_3)_n^{2n-}$
(II)	Soro Silicate	(Q)	$(\text{Si}_2\text{O}_5)_n^{2n-}$
(III)	Cyclic Silicate	(R)	$(\text{Si}_2\text{O}_7)^{6-}$
(IV)	Phyllo Silicate	(S)	None of the O atom is shared
		(T)	All oxygen atoms are shared

12. Which combination is correct according to the above list?
 (A) I \rightarrow S (B) II \rightarrow P
 (C) III \rightarrow Q (D) IV \rightarrow T

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimal Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

- 3.64 g of silver salt of an organic dibasic acid yields, on strong heating, 2.16 g of silver. If the weight percentage of carbon in acid 8 times the weight percentage of hydrogen and one-half the weight percentage of oxygen, if the molecular formula of the acid is written as $\text{C}_x\text{H}_y\text{O}_z$. Then find $x + y + z = ?$
 [Atomic weight of Ag = 108]
- How many of the following statements are incorrect?
 (a) Spontaneous adsorption of gases on solid surface is an endothermic process as entropy decreases during adsorption.
 (b) Formation of micelles takes place when temperature is below Kraft Temperature (T_k) and concentration is above critical micelle concentration (CMC).
 (c) A colloid of $\text{Fe}(\text{OH})_3$ is prepared by adding a little excess [required to completely precipitate Fe^{3+} ions as $\text{Fe}(\text{OH})_3$] of NaOH in FeCl_3 solution the particles of this sol will move towards anode during electrophoresis.
 (d) According to Hardy-Schulze rule for the coagulation of As_2S_3 sol flocculation value of Fe^{3+} ion will be more than Ba^{2+} or Na^+ .
 (e) In Micelle formation of ionic soap, entropy decreases
- How many of following is/are paramagnetic in nature?
 $[\text{NiCl}_4]^{2-}$, $[\text{Ni}(\text{CO})_4]$, $[\text{PtCl}_4]^{2-}$, $[\text{Cu}(\text{NH}_3)_4]^+$, $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$, $\text{K}_2[\text{NiF}_6]$, $[\text{Ni}(\text{NH}_3)_6]^{2+}$, $[\text{Co}(\text{CN})_6]^{3-}$, $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$, $[\text{Cr}(\text{CN})_6]^{4-}$, $[\text{Fe}(\text{NH}_3)_6]^{2+}$, $[\text{PtCl}_6]^{2-}$

Space For Rough Work

4. Na_2HPO_4 and NaH_2PO_4 on heating at high temperature produce a chain sodium pentaphosphate quantitatively of ideal molar ratio of Na_2HPO_4 to NaH_2PO_4 is $x : y$ & total charge on pentaphosphate anion is z then calculate value of $\frac{x}{y} \times z$.
5. 10 g Neon initially at a pressure of 506.625 KPa and temperature of 473 K expand adiabatically to a pressure of 202.65 KPa. Calculate entropy change (in JK^{-1}) of universe of expansion is a free expansion.
6. The value of rate constant for the gas phase reaction $2\text{NO}_2 + \text{F}_2 \longrightarrow 2\text{NO}_2\text{F}$ is $38 \text{ dm}^3 \text{ mol}^{-1} \text{ s}^{-1}$ at 300 K. the order of reaction is _____?
Write "1000" if the order cannot be predicated with given information
-

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE is correct**.

1. If $f(x) = \begin{vmatrix} \sec^2 x & 1 & 1 \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cot^2 x \end{vmatrix}$ then

(A) $\int_{-\pi/4}^{\pi/4} f(x) dx = \frac{1}{16}(3\pi + 8)$

(B) $f'\left(\frac{\pi}{2}\right) = 0$

(C) Maximum value of $f(x)$ is 1

(D) Minimum value of $f(x)$ is 0.

2. If $A(\alpha, \beta) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & e^\beta \end{bmatrix}$, then

(A) $A(\alpha, \beta)' = A(-\alpha, \beta)$

(B) $A(\alpha, \beta)^{-1} = A(-\alpha, -\beta)$

(C) $\operatorname{Adj}(A(\alpha, \beta)) = e^\beta A(-\alpha, -\beta)$

(D) $A(\alpha, \beta)' = A(\alpha, -\beta)$

3. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

(A) $\frac{1}{6}(n-2)(n-3)(n-4)$

(B) ${}^{n-2}C_3$

(C) ${}^{n-3}C_3 + {}^{n-3}C_2$

(D) None of these

4. Let $\alpha(n) = 1 - \frac{1}{2} + \frac{1}{3} - \dots + (-1)^{n-1} \frac{1}{n}$

(A) $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} = \alpha(2n)$

(B) $\alpha(2n) < 1 \forall n$

(C) $\alpha(2n) \geq 0.5 \forall n$

(D) $0.5 < \alpha(n) < 1 \forall n$

5. If $\int_0^\alpha \frac{dx}{1 - \cos \alpha \cos x} = \frac{A}{\sin \alpha} + B (a \neq 0)$. Then possible values of A and B are

(A) $A = \frac{\pi}{2}, B = 0$

(B) $A = \frac{\pi}{4}, B = \frac{\pi}{4 \sin \alpha}$

(C) $A = \frac{\pi}{6}, B = \frac{\pi}{\sin \alpha}$

(D) $A = \pi, B = \frac{\pi}{\sin \alpha}$

Space For Rough Work

6. The solution of $\frac{dy}{dx} = \sqrt{y-x}$ is given by
 (A) $x + C = 2\sqrt{y-x} + 2\log(\sqrt{y-x} - 1)$ (B) $x^2 + C = 2\sqrt{y-x} + \log(\sqrt{y-x} - 1)$
 (C) $x + C = (y-x)^2 + \log(y-x-1)$ (D) $\sqrt{y-x} - 1 = C' e^{x/2 - \sqrt{y-x}}$
7. If sines of the angle A and B of a triangle ABC satisfy the equation $c^2x^2 - c(a+b)x + ab = 0$, then the triangle
 (A) is acute angled (B) is right angled
 (C) is obtuse angled (D) satisfies $\sin A + \cos A = \frac{(a+b)}{c}$
8. In the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n$, $n \in \mathbb{N}$,
 (A) number of terms is $2n+1$ (B) coefficient of constant term is $2^n - 1$
 (C) coefficient of x^{2n-2} is n (D) coefficient of x^2 is n

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-12)

9. Match the following

List – I		List – II	
(I)	The radical axis of two circles	(P)	Subtends a right angle at a point of intersection.
(II)	The common tangent to two intersecting circles of equal radii	(Q)	Is perpendicular to the line joining the centres.
(III)	The common chord of two intersecting circles	(R)	Is parallel to the line joining the centres.
(IV)	The line joining the centres of two circles intersecting orthogonally	(S)	Is bisected by the line joining the centres.
		(T)	Is parallel to common normals

Which is correct option?

- (A) I \rightarrow Q
 (C) III \rightarrow R

- (B) II \rightarrow P
 (D) IV \rightarrow S

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10. Match the following

List - I		List - II	
(I)	$f(x) = x \operatorname{sgn}(x-1)$	(P)	$\lim_{x \rightarrow 1} f(x)$ does not exist
(II)	$f(x) = \frac{\sin\left(\sin\left(\tan\left(\frac{x^2}{2}\right)\right)\right)}{\log \cos 3x}$	(Q)	$\lim_{x \rightarrow 0} f(x)$ doesn't exist
(III)	$f(x) = \frac{\sqrt[3]{1+\tan^{-1} 3x} - \sqrt[3]{1-\sin^{-1} 3x}}{\sqrt{1-\sin^{-1} 2x} - \sqrt{1+\tan^{-1} 2x}}$	(R)	$\lim_{x \rightarrow 0} f(x) = \frac{-1}{9}$
(IV)	$f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$	(S)	$\lim_{x \rightarrow 0} f(x) = -1$
		(T)	$\lim_{x \rightarrow 0} f(x) = 2$

Which is correct option?

(A) I \rightarrow S(B) II \rightarrow R(C) III \rightarrow Q(D) IV \rightarrow P11. Match the sequence a_1, a_2, a_3, \dots whose n th term is given on the left with properties of the sequence on the right.

List - I		List - II	
(I)	$a_n = \int_0^{\pi/2} \frac{1 - \cos 2nx}{1 - \cos 2x} dx$	(P)	a_1, a_2, a_3, \dots are in A.P.
(II)	$I_n = \int_0^{\pi/4} \tan^n x dx$ and $a_n = \frac{1}{I_{n+1} + I_{n+3}}$	(Q)	a_1, a_2, a_3, \dots are in G.P.
(III)	$I_n = \int_0^{\pi/4} \frac{\sin^2 nx}{\sin x} dx \quad \forall n \geq 0$ and $a_n = I_n - I_{n-1} \quad \forall n \geq 1$	(R)	a_1, a_2, a_3, \dots are in H.P.
(IV)	$b_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx, \quad a_n = b_n - b_{n-1} \quad \forall n \geq 1$	(S)	a_1, a_2, a_3, \dots is constant sequence.
		(T)	$a_{n+1} = a_n + a_n^2$

Which is correct option?

(A) I \rightarrow Q(B) II \rightarrow T(C) III \rightarrow R(D) IV \rightarrow S

Space For Rough Work

12. Lines $y^2 - xy = 0$ make Intercepts on curve given on R.H.S of Matrix.

List – I		List – II	
(I)	2	(P)	$\frac{x^2}{3} + y^2 = 1$
(II)	4	(Q)	$x^2 + \frac{y^2}{3} = 1$
(III)	$\sqrt{6}$	(R)	$x^2 + y^2 = 4$
(IV)	$2\sqrt{3}$	(S)	$y^2 = 2\sqrt{2}x$
		(T)	$x^2 - y^2 = 2$

Which is correct option?

(A) I \rightarrow Q

(B) II \rightarrow P

(C) III \rightarrow S

(D) IV \rightarrow R

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

- If $f : [0, 1] \rightarrow \mathbb{R}$ is a continuous function satisfying $\int_0^1 f(x) dx = \frac{1}{3} + \int_0^1 (f(x^2))^2 dx$, then the reciprocal of $\frac{1}{8} f\left(\frac{1}{4}\right)$ is _____
- Any chord of conic $x^2 + y^2 + xy = 1$ passing through origin is bisected at point (p, q), then $\frac{(p+q+1)}{8}$ equals _____
- Compute the square of the value of the expression $\frac{4 + \sec 20^\circ}{\operatorname{cosec} 20^\circ}$
- Let $f(x) = x - \frac{1}{x}$. Then find the number of different solutions to the equation $f(f(f(x))) = 1$.
- If the total number of selections that can be made from n different books each having 5 copies taking at least 2 copies of each book is 256, then n =
- The positive integral value of n such that $1.2^1 + 2.2^2 + 3.2^3 + 4.2^4 + \dots + n.2^n = 2 + 2^{n+5}$, is

Space For Rough Work

Q. P. Code: 100958
Answers
SECTION – I : PHYSICS

(PART – A)

- | | | | |
|--------|---------|-------|--------|
| 1. ABD | 2. BC | 3. AC | 4. ACD |
| 5. ABC | 6. ABCD | 7. BD | 8. ACD |
| 9. A | 10. C | 11. D | 12. A |

(PART – B)

- | | | | |
|---------|------------------------------|---------|---------|
| 1. 0.75 | 2. 1.50 | 3. 0.50 | 4. 0.11 |
| 5. 0.25 | 6. 1.6 (Range: 1.60 to 1.62) | | |

SECTION – II : CHEMISTRY

(PART – A)

- | | | | |
|--------|-------|---------|--------|
| 1. BD | 2. AB | 3. ABCD | 4. ACD |
| 5. BCD | 6. AD | 7. CD | 8. AD |
| 9. B | 10. D | 11. C | 12. A |

(PART – B)

- | | | | |
|-------------------------------|---|------|--|
| 1. 16 | 2. 4 | 3. 6 | |
| 4. -4.66 (Range -4.6 to -4.7) | 5. 3.81 JK ⁻¹ (Range 3.7 to 3.9) | | |
| 6. 2 | | | |

SECTION – III : MATHEMATICS

(PART – A)

- | | | | |
|---------|--------|--------|--------|
| 1. ABCD | 2. ABC | 3. ABC | 4. ABC |
| 5. AB | 6. AD | 7. BD | 8. AC |
| 9. A | 10. A | 11. C | 12. A |

(PART – B)

- | | | | |
|---------|----------|---------|---------|
| 1. 0.25 | 2. 0.125 | 3. 3.00 | 4. 1.00 |
| 5. 4.00 | 6. 17.00 | | |

Answers & Solutions

SECTION – I : PHYSICS

(PART – A)

1. **ABD**

Sol. Bernoulli's Theorem for an orifice at depth 'x' in liquid '3d'.

$$P_o + \left(dg \frac{H}{2} + 3dgx \right) = P_o + \frac{1}{2} \times 3d \times v^2 \quad \dots(1)$$

$$\frac{H}{2} - x = \frac{1}{2}gt^2 \quad \dots(2)$$

$$R = Vt \quad \dots(3)$$

Solve for R and apply maxima/minima.

2. **BC**

Sol. Introduction of sheet decrease optical path length.

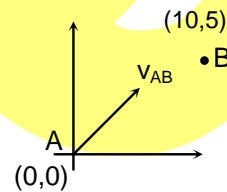
3. **AC**

$$\vec{v}_{AB} = (3-a)\hat{i} + (3-b)\hat{j}$$

$$\vec{a}_{AB} = \vec{0}$$

$$(3-a) \times 2 = 10 \text{ and } (3-b) \times 2 = 5$$

$$a = -2 \text{ and } b = \frac{1}{2}$$

4. **ACD**

$$\text{Sol. } Fx - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \sqrt{\frac{2Fx - kx^2}{m}} = v$$

$$\text{Where } x = \frac{F}{k}$$

5. **ABC**Sol. Using work energy theorem $W_c + W_{nc} = \Delta K$ Where $W_k = -\Delta U$.6. **ABCD**

$$\text{Sol. } a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x) \quad \dots(1)$$

$$a = 0 \text{ at } x = \frac{\alpha}{\beta}$$

i.e., force on the particle is zero at $x = \frac{\alpha}{\beta}$ So, mean position of the particle is at $x = \frac{\alpha}{\beta}$ Equation (1) can be written as $v \frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$

$$\therefore \int_0^v v dv = \frac{q}{m} \int_0^x (\alpha - \beta x) dx$$

$$\therefore v = \sqrt{\frac{2qx}{m} \left(\alpha - \frac{\beta}{2}x \right)}$$

$$v = 0 \text{ at } x = 0 \text{ and } x = \frac{2\alpha}{\beta}$$

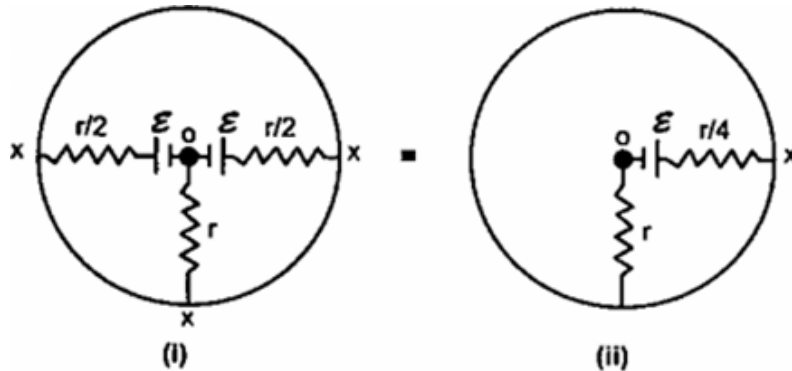
So, the particle oscillates between $x = 0$ and $x = \frac{2\alpha}{\beta}$

with mean position at $x = \frac{\alpha}{\beta}$.

Maximum acceleration of the particle is at extreme positions (at $x = 0$ or $x = \frac{2\alpha}{\beta}$) and $a_{\max} = q \alpha / m$ [from equation (1)].

7. **BD**

Sol. Equivalent circuit:



$$\text{Induced e.m.f. } \epsilon = \left(\frac{B\omega a^2}{2} \right) \quad (\because \text{radius} = a)$$

By nodal equation:

$$4 \left(\frac{x - \epsilon}{r} \right) + \left(\frac{x - 0}{r} \right) = 0$$

$$5x = 4\epsilon$$

$$\Rightarrow x = \frac{4\epsilon}{5} = \frac{2B\omega a^2}{5}$$

$$\text{and } I = \frac{x}{r} = \frac{2B\omega a^2}{5r}$$

Also direction of current in (i) will be toward negative terminal, i.e., from rim to origin alternating, by equivalent of cells figure.

$$I = \frac{\epsilon}{r + \frac{r}{4}} = \frac{4\epsilon}{5r} = \frac{2B\omega a^2}{5r}$$

8. **ACD**

Sol. $y(x, t) = -\cos kx \sin \omega t$

$$\begin{aligned} Y(0.05, 0.05) &= -4 \cos \left(\frac{2\pi}{0.4} 0.05 \right) \sin \left(\frac{2\pi}{0.2} 0.05 \right) \\ &= -4 \cos \left(\frac{\pi}{4} \right) \sin \left(\frac{\pi}{2} \right) = -2\sqrt{2} \text{ cm} \end{aligned}$$

$$v = f\lambda = \frac{0.4}{0.2} = 2 \text{ m/s}$$

$$\frac{\partial y}{\partial t} = -A\omega \cos(kx) \cos(\omega t)$$

$$= \sim 4 \left(\frac{2\pi}{0.2} \right) \cos \left(\frac{2\pi}{0.4} \frac{1}{15} \right) \cos \left(\frac{2\pi}{0.2} 0.1 \right)$$

$$\frac{\partial y}{\partial t} = 20\pi \text{ cm/sec} = \text{particle velocity.}$$

9. **A**

Sol. $C_{eq} = 1 \text{ F}$ of capacitors A and B before inserting dielectric. Final C_{eq} of A and B = $\frac{4}{3} \text{ F}$.

$$\therefore \Delta Q = (\Delta C)V = \left(\frac{4}{3} - 1 \right) \times 1 = \left(\frac{1}{3} \right)$$

$$\Delta U = \left| \frac{\left(\frac{4}{3} \right)^2}{2 \times 4} - \frac{(1)}{2 \times 2} \right| = \left| \frac{2}{9} - \frac{1}{4} \right| = \frac{1}{36}$$

$$\Delta V = \left| \frac{q_2}{C_2} - \frac{q_1}{C_1} \right| = \left| \frac{\left(\frac{4}{3} \right)}{4} - \frac{1}{2} \right| = \left| \frac{1}{3} - \frac{1}{2} \right| = \frac{1}{6}$$

10. **C**

Sol. $C_{\text{initial}} = 1 \text{ F}$

$C_{\text{final}} = 3 \text{ F}$

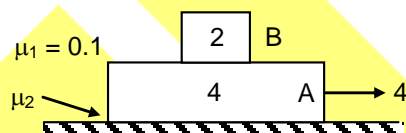
$$\Delta Q = (C_f - C_i)V = 2 \times \frac{1}{6} = \frac{1}{3}$$

$$\Delta U = \frac{1}{2} C_f V^2 - \frac{1}{2} C_i V^2 = \frac{1}{2} \times 2 \times \frac{1}{36} = \frac{1}{36}$$

$$Q_{\text{final}} = C_{\text{final}} V = 3 \times \frac{1}{6} = \frac{1}{2}$$

11. **D**

Sol.



If both block have together, their common acceleration

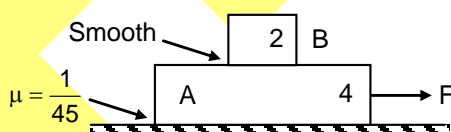
$$a = \frac{4}{6} = \left(\frac{2}{3} \right)$$

Maximum possible acceleration of block B = $\mu g = 1$ which is greater than common acceleration. Therefore both block will move together.

$$f_1 = ma = 2 \times \frac{2}{3} = \frac{4}{3}$$

12. **A**

Sol.



$$a = \frac{F - f}{4} = \frac{4 - \left(\frac{1}{45} \right) 60}{4} = \frac{2}{3}$$

B will be at rest

$$f = \mu(2 + 6)g = \frac{60}{45} = \frac{4}{3}$$

(PART – B)

1. **0.75**

Sol. $m_1 = \rho_0 \times 300$

$m_2 = \rho_{100} \times 110$

Suppose final temp = T

$$m_1 S(T - 0) = m_2 S(100 - T)$$

$$\rho_0 \times 300 T = \rho_{100} \times 110(100 - T)$$

$$300 \rho_0 T = \frac{\rho_0 \times 110}{(1 + \gamma \times 100)} \times (100 - T)$$

$$300 T = \frac{110}{1.1} \times (100 - T)$$

$$3T = 100 - T$$

$$\Rightarrow T = 25^\circ$$

$$V_{\text{final}} = 300(1 + 0.001 \times 25) + 110(1 - 0.001 \times 75) \\ = 410 + 7.5 - 8.25 = 409.25 \text{ cm}^3$$

$$\text{Change} = 0.75 \text{ cm}^3.$$

2. **1.50**

Sol. Given: $f_1 - f_2 = 3$

$$\text{or} \quad \left(\frac{v}{v - v_s} \right) f - \left(\frac{v}{v + v_s} \right) f = 3$$

$$\text{or} \quad \left[\frac{1}{\left(1 - \frac{v_s}{v} \right)} - \frac{1}{\left(1 + \frac{v_s}{v} \right)} \right] f = 3$$

$$\text{or} \quad \left[\left(1 - \frac{v_s}{v} \right)^{-1} - \left(1 + \frac{v_s}{v} \right)^{-1} \right] f = 3$$

$$\text{or} \quad \left[\left(1 + \frac{v_s}{v} \right) - \left(1 - \frac{v_s}{v} \right) \right] f = 3$$

$$\text{or} \quad \frac{2v_s f}{v} = 3$$

$$\text{or} \quad \text{Speed of tuning fork } v_s = \frac{3v}{2f}$$

Substituting the values, we get

$$v_s = \frac{(3)(340)}{(2)(340)} = 1.5 \text{ m/s}$$

3. **0.50**

Sol. Each of angle A' , B' , C' , ... is 36° .

$$\therefore \text{Resistance of AB} = 2 \times r \sin 18^\circ = 2r \times \frac{1}{3} = \frac{2}{3} r$$

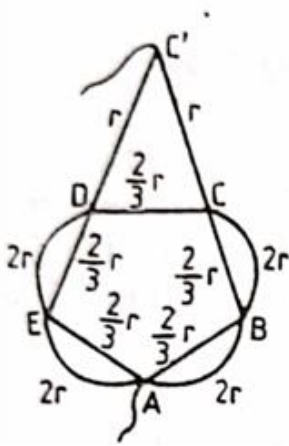


Fig. (a)

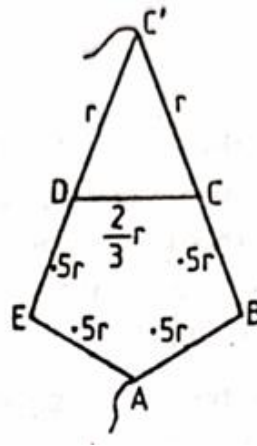


Fig. (b)

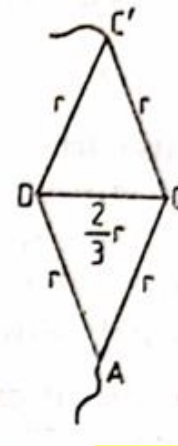


Fig. (c)

AA' and A'B are in series and so they sum up to $2r$. Similarly BB' and B'C, DD' and D'E, and EE' and E'A sum up to $2r$. This is shown in figure (a). In each side $2r$ and $\frac{2r}{3}$ in parallel sum up to $0.5r$, this is shown in figure b. The next reduction is shown in figure (c). D and C are equipotential points and hence DC' and CC' are in parallel and they sum up to $\frac{r}{2}$; AD and AC are in parallel and they sum up to $\frac{r}{2}$. So the required resistance is r .

4. 0.11

Sol. If a charge q is given to the outermost sphere C, some of the charge will go to the outer surface of sphere A. Some will go to the inner surface of C and the rest will go to the outer surface of C. The charge q_1 on sphere A will induce a charge $-q_1$ on the inner surface of sphere B. Charge q_2 on the inner surface of sphere C will induce a charge $-q_2$ on the outer surface of sphere B.

In this way three capacitors are formed. One between the outer surface of A and inner surface of B. Second between the outer surface of B and inner surface of C and third by the outer surface of C.

Their capacities are

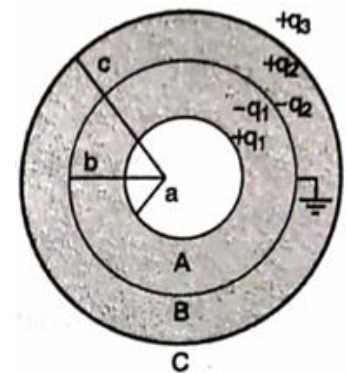
$$C_1 = 4\pi\epsilon_0 \frac{ab}{b-a};$$

$$C_2 = 4\pi\epsilon_0 \frac{bc}{c-b} \quad \text{and} \quad C_3 = 4\pi\epsilon_0 c$$

Hence the combined capacity is

$$C = C_1 + C_2 + C_3$$

$$C = 4\pi\epsilon_0 \left[\frac{ab}{b-a} + \frac{bc}{c-b} + c \right] = 4\pi\epsilon_0 \left[\frac{ab}{b-a} + \frac{c^2}{c-b} \right]$$



5. 0.25

Sol.

$$\text{Charge on ring} = q' = \frac{Q}{\pi RL} 2\pi x d\ell$$

$$\text{Magnetic moment due to ring} = \frac{q'\omega}{2\pi} \pi x^2$$

$$= \left(\frac{Q 2x d\ell}{\pi RL} \right) \frac{\omega x^2}{2}$$

$$= \frac{Q \omega d\ell}{RL} \left(\frac{R}{L} \ell \right)^3$$

$$= \frac{Q \omega R^2}{L^4} \ell^3 d\ell$$

$$\text{Total magnetic moment} = \frac{Q \omega R^2}{L^4} \int_0^L \ell^3 d\ell = \frac{Q \omega R^2}{4}$$

$$= \frac{Q \omega h^2 \tan^2 \theta}{4}$$

$$= 0.25 Q \omega h^2 \tan^2 \theta$$

6. **1.6 (Range: 1.60 to 1.62)**

$$\text{Sol. } E = \frac{12375}{4000} = 3.1 \text{ eV}$$

Number of photoelectrons emitted per second,

$$n = \left(\frac{1}{10^6} \right) \left(\frac{5}{3.1 \times 1.6 \times 10^{-19}} \right)$$

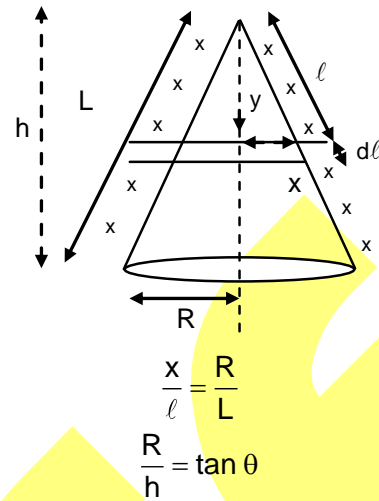
$$= 1.0 \times 10^{13} \text{ per second}$$

$$\therefore i = ne$$

$$= 1.0 \times 10^{13} \times 1.6 \times 10^{-19}$$

$$= 1.6 \times 10^{-6} \text{ A}$$

$$= 1.6 \mu\text{A}$$



SECTION – II : CHEMISTRY

(PART – A)

1. BD

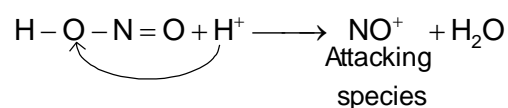
Sol. The gases formed in the options are

- (A) NH_3 and N_2
 (B) N_2 and N_2
 (C) N_2O and O_2
 (D) NH_3 and NH_3

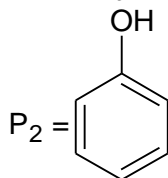
2. AB

Sol. $2\text{I}^- \longrightarrow \text{I}_2 + 2\text{e}^-$ \therefore One mole I_2 needs $2F$ of electricity.

3. ABCD

Sol. $\text{NaNO}_2 + \text{HCl} \longrightarrow \text{NaCl} + \text{HNO}_2$ 

4. ACD

Sol. $\text{P}_1 = \text{CH}_3\text{COCH}_3$  $\text{P}_4 = \text{CH}_3\text{COONa}$

5. BCD

Sol. The correct reaction in (A) is $\text{Fe}_2\text{O}_3 + \text{C} \longrightarrow 2\text{FeO} + \text{CO}$

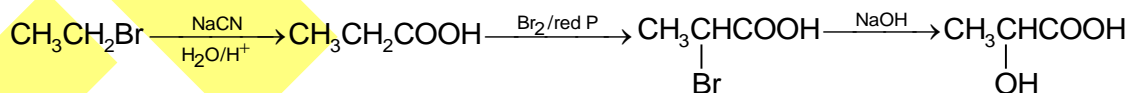
6. AD

Sol. All zero order reactions complete in two half lives.

7. CD

Sol. Q is $\text{Ph}-\text{CH}=\text{CHCH}_2\text{COOC}_2\text{H}_5$ (Cis-)

8. AD

Sol. $\text{CH}_3\text{CHO} \xrightarrow[\text{(ii) H}_2\text{O/H}^+/\Delta]{\text{(i) NaCN/HCl}} \text{CH}_3\text{CH}(\text{OH})\text{COOH}$ 

9. B

Sol. PhCH_2OH forms benzyl carbocation Ph^+CH_2 which is more stable and gives instantaneous result.

10. D

Sol. 1° -alcohols are less reactive than 2° - and 3° -alcohol.

11. C

Sol. Cyclic silicates contain $(\text{SiO}_3)_n^{2n-}$ units.

12. A

Sol. Meso silicates are not polymer. So they do not share any oxygen atom.

(PART – B)

1. 16

Sol. $x = 4, y = 6, z = 6$

2. 4

Sol. Statements a, c, d & e are incorrect.

3. 6

Sol. There are six paramagnetic complexes.

4. -4.66 (Range -4.6 to -4.7)

Sol. $2\text{Na}_2\text{HPO}_4 + 3\text{NaH}_2\text{PO}_4 \longrightarrow \text{Na}_7\text{P}_5\text{O}_{16} + 4\text{H}_2\text{O}$ $Z = -7, x = 2, y = 3$ 5. 3.81 JK^{-1} (Range 3.7 to 3.9)Sol. $nC_v(T_2 - T_1) = -Pdv$

6. 2

Sol. Since the unit of rate constant is $\text{dm}^3 \text{ mol}^{-1} \text{ s}^{-1} = \text{L mol}^{-1} \text{ s}^{-1}$
The reaction is second order.

SECTION – III : MATHEMATICS**(PART – A)**

1. ABCD

Sol. Applying $C_2 \rightarrow C_1 - \cos^2 x C_1$, we get $f(x) = \begin{vmatrix} \sec^2 x & 0 & 1 \\ \cos^2 x & \cos^2 x - \cos^4 x & \operatorname{cosec}^2 x \\ 1 & 0 & \cot^2 x \end{vmatrix}$

$$= (\cos^2 x - \cos^4 x)(\sec^2 x \cot^2 x - 1) \text{ [expanding along } C_2]$$

$$= \cos^2 x \sin^2 x (\operatorname{cosec}^2 x - 1)$$

$$= \cos^2 x \sin^2 x \cot^2 x = \cos^4 x$$

$$= \frac{1}{4}(3 + 4 \cot 2x + \cos 4x)$$

Thus $\int_{-\pi/4}^{\pi/4} f(x) dx$

$$= \frac{1}{4} \left(3x + 2 \sin 2x + \frac{1}{4} \sin 4x \right) \Big|_0^{\pi/4} = \frac{1}{16} (3\pi + 8)$$

It can be easily checked that each of (b), (c) and (d) also holds.

2. ABC

Sol. We have $A(\alpha, \beta)' = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & e^\beta \end{bmatrix}$

$$= \begin{bmatrix} \cos(-\alpha) & \sin(-\alpha) & 0 \\ -\sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & e^\beta \end{bmatrix} = A(-\alpha, \beta)$$

Also, $A(\alpha, \beta)A(-\alpha, -\beta)$

$$= \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & e^\beta \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & e^{-\beta} \end{bmatrix}$$

$$= I$$

$$\Rightarrow A(\alpha, \beta)^{-1} = A(-\alpha, -\beta)$$

Next, $\operatorname{Adj}(\alpha, \beta) = |A(\alpha, \beta)| A(\alpha, \beta)^{-1}$

$$= e^\beta A(-\alpha, -\beta)$$

3. ABC

Sol. Let x_0 be the number of objects to the left of the first object chosen, x_1 the number of objects between the first and the second, x_2 the number of objects between the second and the third and x_3 the number of objects to the right of the third object. We have

$$x_0, x_3 \geq 0, x_1, x_2 \geq 1 \text{ and } x_0 + x_1 + x_2 + x_3 = n - 1 \quad (1)$$

Put $x_1 = y_1 + 1$ and $x_2 = y_2 + 1$. Then (1) reads as $x_0 + y_1 + y_2 + x_3 = n - 5 \quad (2)$

where $x_0, y_1, y_2, x_3 \geq 0$

The number of non-negative integral solution of (2) is ${}^{n-5+3}C_3 = {}^{n-2}C_3$

We have ${}^{n-2}C_3 = \frac{1}{6}(n-2)(n-3)(n-4)$.

Also, ${}^{n-3}C_3 + {}^{n-3}C_2 = {}^{n-2}C_3$

4. ABC

Sol.
$$\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$$

$$= \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right)$$

$$= \left(1 + \frac{1}{3} + \dots + \frac{1}{2n-1}\right)$$

$$+ \frac{1}{2} \left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right) - \left(1 + \frac{1}{2} + \dots + \frac{1}{n}\right)$$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n}$$

$$\alpha(2n) < \underbrace{\frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n}}_{n \text{ times}} = 1 \text{ and } \alpha(2n) \geq \frac{1}{2n} \geq \frac{1}{2} \forall n$$

(D) is not correct as $\alpha(1) = 1$

5. AB

Sol.
$$I = \int_0^\alpha \frac{dx}{1 - \cos \alpha \cos x}$$

$$= \int_0^\alpha \frac{dx}{\left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}\right) - \cos \alpha \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)}$$

$$= \int_0^\alpha \frac{dx}{(1 - \cos \alpha) \cos^2 \frac{x}{2} + (1 + \cos \alpha) \sin^2 \frac{x}{2}}$$

$$= \int_0^\alpha \frac{dx}{2 \sin^2 \frac{\alpha}{2} \cos^2 \frac{x}{2} + 2 \cos^2 \frac{\alpha}{2} \sin^2 \frac{x}{2}}$$

$$= \frac{1}{2} \int_0^\alpha \frac{\left(\sec^2 \frac{\alpha}{2}\right) \sec^2 \frac{x}{2}}{\tan^2 \frac{\alpha}{2} + \tan^2 \frac{x}{2}} dx$$

Put $\tan \frac{x}{2} = t$, we have

$$I = \int_0^{\tan \frac{\alpha}{2}} \frac{\sec^2 \frac{\alpha}{2} dt}{t^2 + \tan^2 \frac{\alpha}{2}}$$

$$= \sec^2 \frac{\alpha}{2} \cot \frac{\alpha}{2} \left[\tan^{-1} \left(\frac{t}{\tan \frac{\alpha}{2}} \right) \right]_0^{\tan \frac{\alpha}{2}}$$

$$= \frac{2}{\sin \alpha} \cdot \frac{\pi}{4} = \frac{\pi}{2 \sin \alpha}$$

Thus $\frac{A}{\sin \alpha} + B = \frac{\pi}{2 \sin \alpha}$. $A = \frac{\pi}{2}$, $B = 0$ and $A = \frac{\pi}{4}$,

$B = \frac{\pi}{4 \sin \alpha}$ satisfy the last equation.

6. AD

Sol. Let $\sqrt{y-x} = t$

$$y - x = t^2$$

$$\frac{dy}{dx} = 2t \frac{dt}{dx} + 1$$

$$2t \frac{dt}{dx} + 1 = \sqrt{t}$$

$$\frac{2t}{\sqrt{t}-1} dt = dx \text{ which leads A and D as solutions}$$

7. BD

$$\text{Sol. } \sin A + \sin B = \frac{a+b}{c} = \frac{\sin A + \sin B}{\sin C} \Rightarrow \sin C = 1$$

$$\Rightarrow C = 90^\circ$$

$$A + B = 90^\circ \Rightarrow \cos A = \sin B$$

8. AC

$$\text{Sol. } \left(x^2 + 1 + \frac{1}{x^2}\right)^n$$

$$= {}^nC_0 + {}^nC_1 \left(x^2 + \frac{1}{x^2}\right) + {}^nC_2 \left(x^2 + \frac{1}{x^2}\right)^2 + \dots + {}^nC_n \left(x^2 + \frac{1}{x^2}\right)^n$$

This contains each of the term $x^0, x^2, x^4, \dots, x^{2n}, x^{-2}, x^{-4}, \dots, x^{-2n}$

Coefficient of constant term

$$= {}^nC_0 + ({}^nC_2)(2) + ({}^nC_4)({}^4C_2) + ({}^nC_6)({}^6C_3) + \dots$$

$$\neq 2^n - 1$$

Coefficient of x^{2n-2} in ${}^nC_{n-1} = n$

Coefficient of x^2 is ${}^nC_1 + ({}^nC_3)({}^3C_1) + ({}^nC_5)({}^5C_2) + \dots > n$

9. A

Sol. Let the equations of the circles be $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$.

(I) Equation of the radical axis is $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$

$$\text{Slope of the radical axis} = \frac{g_1 - g_2}{f_1 - f_2}$$

Slope of the line joining the centres $= \frac{f_1 - f_2}{g_1 - g_2}$ So, the radical axis is perpendicular to the line joining the centre

(II) Common tangent to the intersecting circles of equal radii is at the same distance from the centres of the two circles and hence is parallel to the line joining the centres.

(III) Since the line joining the centres of the circles to the mid – point of the common chord is perpendicular to the chord, it bisects the chord.

(IV) The line joining the centre of one to a point of intersection of tangent to the other circle. So by definition of orthogonality, they are perpendicular.

10. A

$$\text{Sol. For function } f \text{ in (a) } f(x) = \begin{cases} x, & x > 1 \\ 0, & x = 1 \\ -x, & x < 1 \end{cases} \text{ so } \lim_{x \rightarrow 1} f(x) \text{ does not exist.}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{\sin\left(\sin\left(\tan\left(\frac{x^2}{2}\right)\right)\right)}{\log \cos 3x} \\
 &= \lim_{x \rightarrow 0} \frac{\sin\left(\sin\left(\tan\left(\frac{x^2}{2}\right)\right)\right)}{\sin\left(\frac{\tan x^2}{2}\right)} \times \frac{\sin\left(\sin\left(\tan\left(\frac{x^2}{2}\right)\right)\right)}{\sin\left(\frac{\tan x^2}{2}\right)} \times \frac{\sin\left(\tan \frac{x^2}{2}\right)}{\tan \frac{x^2}{2}} \times \frac{\tan\left(\frac{x^2}{2}\right)}{\frac{x^2}{2}} \times \frac{\frac{x^2}{2}}{\log \cos 3x} \\
 &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{2x}{-3 \tan 3x} = -\frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + \tan^{-1} 3x} - \sqrt[3]{1 + \sin^{-1} 3x}}{\sqrt{1 - \sin^{-1} 2x} - \sqrt{1 + \tan^{-1} 2x}} \\
 &= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{1}{3} \tan^{-1} 3x + \dots\right) - \left(1 - \frac{1}{3} \sin^{-1} 3x + \dots\right)}{\left(1 - \frac{1}{2} \sin^{-1} 2x - \dots\right) - \left(1 + \frac{1}{2} \tan^{-1} 2x + \dots\right)} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{1}{3} \frac{\tan^{-1} 3x}{x} + \frac{1}{3} \frac{\sin^{-1} 3x}{x} + \frac{1}{x} (\text{highest power of } \sin^{-1} 3x \text{ and } \tan^{-1} 3x)}{-\frac{1}{2} \frac{\sin^{-1} 2x}{x} - \frac{1}{2} \frac{\tan^{-1} 2x}{x} + \frac{1}{x} (\text{highest power of } \sin^{-1} x \text{ and } \tan^{-1} 2x)} \\
 &= \frac{1+1}{-1-1} = -1 \\
 & \lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1} = -1 \text{ and } \lim_{x \rightarrow 0^+} \frac{e^{1/x} - 1}{e^{1/x} + 1} = 1
 \end{aligned}$$

11. C

Sol.

$$(I) \quad a_1 = \int_0^{\pi/2} dx = \frac{\pi}{2}, a_2 = \pi$$

Also, $a_{n+2} + a_n - 2a_{n+1}$

$$= \int_0^{\pi/2} \frac{2 \cos(2n+2)x - \cos(2n+4)x - \cos(2nx)}{1 - \cos 2x} dx$$

$$= \int_0^{\pi/2} \frac{2 \cos(2n+2)x - 2 \cos(2n+2)x \cos 2x}{1 - \cos 2x} dx$$

$$= \int_0^{\pi/2} 2 \cos(2n+2)x dx$$

$$= \frac{2}{2n+2} \sin(2n+2)x \Big|_0^{\pi/2} = 0$$

 $\therefore a_1, a_2, a_3, \dots$ are in A.P.

$$(II) \quad I_{n+1} + I_{n+3}$$

$$= \int_0^{\pi/4} \tan^{n+1} x \sec^2 x dx$$

$$= \frac{1}{n+2} \tan^{n+2} x \Big|_0^{\pi/4} = \frac{1}{n+2}$$

 $\therefore a_n = n+2$ $\Rightarrow a_1, a_2, \dots$ are in A.P.

$$\begin{aligned}
 \text{(III)} \quad I_n - I_{n-1} &= \int_0^{\pi/2} \frac{\sin^2(nx) - \sin^2(n-1)x}{\sin x} dx \\
 &= \int_0^{\pi/2} \frac{\sin(2n-1)x \sin x}{\sin x} dx \\
 &= -\frac{1}{2n-1} \cos(2n-1)x \Big|_0^{\pi/2} = \frac{1}{2n-1} \\
 &\Rightarrow a_1, a_2, a_3, \dots \text{ are in H.P.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(IV)} \quad b_n &= \int_0^{\pi/2} \frac{1 - \cos 2nx}{1 - \cos 2x} dx = \frac{n\pi}{2} [\text{see(a)}] \\
 a_n &= \frac{\pi}{2} \forall n.
 \end{aligned}$$

$\therefore a_1, a_2, a_3, \dots$ are in A.P., G.P. and H.P.

Also a_1, a_2, a_3, \dots is a constant sequence.

12. A

Sol.

(I) The given lines are $y = x$ and $y = 0$. The line $y = x$ meets $\frac{x^2}{3} + y^2 = 1$ at the points

$$x = \pm \frac{\sqrt{3}}{2}, y = \pm \frac{\sqrt{3}}{2}. \text{ So the required intercept is } \sqrt{2 \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right)^2} = \sqrt{6}, \text{ intercept on } y = 0 \text{ is } 2\sqrt{3}$$

(II) Required intercepts are $\sqrt{6}$ and 2.

(III) $y = x$ and $y = 0$ are diameters of the circle and hence the required intercepts is equal to the diameter of the circle which is 4.

(IV) $y = x$ meets the parabola $y^2 = 2\sqrt{2}x$ at points where $x = 0, 2\sqrt{2}$.

So that the points of intersection of the line and the parabola are $(0, 0)$ and $(2\sqrt{2}, 2\sqrt{2})$ and the required intercept is 4. Intercept on $y = 0$ is 0.

For T, $x = \pm\sqrt{2}$, and $0 = 2$ which is not possible hence no intercept or intercept $= 2\sqrt{2}$

(PART - B)

1. 0.25

Sol. Given $\int_0^1 f(x) dx = \frac{1}{3} + \int_0^1 (f(x^2))^2 dx$

Consider, $I_1 = \int_0^1 (f(x^2))^2 dx$

Put $x^2 = t \Rightarrow 2x dx = dt$

$$I_1 = \frac{1}{2} \int_0^1 \frac{(f(t))^2}{\sqrt{t}} dt$$

\therefore Equation (1) becomes

$$\int_0^1 \frac{t}{2\sqrt{t}} dt + \int_0^1 \frac{(f(t))^2 - 2\sqrt{t}f(t)}{2\sqrt{t}} dt \Rightarrow \int_0^1 (f(t) - \sqrt{t})^2 dt = 0$$

$$\therefore f(t) = \sqrt{t}$$

So, $f\left(\frac{1}{4}\right) = \frac{1}{2}$

So, reciprocal of $f\left(\frac{1}{4}\right) = 2$

2. **0.125**

Sol. Given curve is ellipse having centre (0, 0). All chords of ellipse through origin are bisected at origin
 $\Rightarrow p = q = 0$

3. 3.00

Sol. We have $\frac{4 + \sec 20^\circ}{\operatorname{cosec} 20^\circ} = \frac{\sin 20^\circ}{\cos 20^\circ} (4 \cos 20^\circ + 1)$

$$= \frac{2 \sin 40^\circ + \sin 20^\circ}{\cos 20^\circ}$$

$$= \frac{\sin 40^\circ + (\sin 40^\circ + \sin 20^\circ)}{\cos 20^\circ}$$

$$= \frac{\sin 40^\circ + 2 \sin 30^\circ \cos 10^\circ}{\cos 20^\circ}$$

$$= \frac{2 \sin 60^\circ \cos 20^\circ}{\cos 20^\circ} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

Hence square of the value of expression = 3

4. 1.00

Sol. Note that for the x interval $(-\infty, 0)$, $f(x)$ is strictly increasing and has range $(-\infty, \infty)$. For the x interval $(0, \infty)$, again, $f(x)$ is strictly increasing and has range $(-\infty, \infty)$.

Hence for all a, there is exactly one positive and one negative solution to $f(x) = a$.

Thus, there are 2^n solutions to $f^n(x) = a$, and the answer is $2^3 = 8$

5. 4.00

Sol. 2 or 3 or 4 or 5 copies of each book out of 5 identical copies can be selected in 4 ways. Since number of books is n, therefore, total number of selections = 4^n

Given $4^n = 256 \Rightarrow n = 4$

6. 17.00

Sol. Let $S_n = 1.2^1 + 2.2^2 + 3.2^3 + 4.2^4 + \dots + n.2^n$ (1)

$\Rightarrow 2.S_n = 1.2^2 + 2.2^3 + \dots + (n-1)2^n + n.2^{n+1}$ (2)

\therefore On Subtracting, we get

$\Rightarrow -S_n = (2^1 + 2^2 + 2^3 + \dots + 2^n) - n.2^{n+1}$

$$= 2(2^n - 1) - n.2^{n+1}$$

$\therefore S_n = n.2^{n+1} - 2(2^n - 1) = n.2^{n+1} - 2^{n+1} + 2 = 2 + (n-1)2^{n+1}$

But $2 + (n-1)2^{n+1} = 2 + 2^{n+5}$ (given)

$\Rightarrow n-1 = 2^4$

$\therefore n = 17$