ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
PART TEST
05-01-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	B	C	C	A	B	D	A	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	B	D	B	C	C	D	A	D
SECTION-II	Q.	1	2	3	4	5					
	A.	4	1	2	2	4					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	B	A	B	B	C	B	C	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	B	B	B	A	C	C	B	B
SECTION-II	Q.	1	2	3	4	5					
	A.	100	20	1000	8	4					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B	A	D	D	C	D	C	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	B	C	D	B	B	A	B	A
SECTION-II	Q.	1	2	3	4	5					
	A.	18	3	0	0	4					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (C)

$I = \frac{I_0}{2}(\cos^2 37^\circ)^3$

$I_1 = \frac{I_0}{2}$

$I_2 = \frac{I_0}{2}\cos^2(\theta)$

$I_3 = \frac{I_0}{2}(\cos^4\theta)$

$I_4 = \frac{I_0}{2}\cos^6\theta$

$= \frac{I_0}{2} \times \left(\frac{4}{5}\right)^6$

$\frac{I}{I_0} \times 100 = \frac{1}{2}\left(\frac{4}{5}\right)^6 \times 100 = 13.1\%$

2. Ans (C)

For first minimum $a \sin\theta = \lambda$

So $\frac{\lambda}{a} = \sin 37^\circ = \frac{3}{5}$

Now for first secondary maximum $a \sin\theta = \frac{3\lambda}{2}$

Therefore $\sin\theta = \frac{3\lambda}{2a} = \frac{3}{2}\left(\frac{3}{5}\right) = \frac{9}{10}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{9}{10}\right)$

3. Ans (B)

Factual question

4. Ans (C)

Suppose the charge on the capacitor at time t is Q.

The electric field between the plates of the capacitor is $E = \frac{Q}{\varepsilon_0 A}$. The flux through the area considered is

$$\Phi_E = \frac{Q}{\varepsilon_0 A}.\frac{A}{4} = \frac{Q}{4\varepsilon_0}.$$

The displacemet current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0\left(\frac{1}{4\varepsilon_0}\right)\frac{dQ}{dt} = \frac{i}{4}.$$

5. Ans (C)

$$4 = \frac{I_{max} + I_{min}}{I_{max} - I_{min}} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{5}{3} = \left(\frac{A_{max}}{A_{min}}\right)^2$$

$$\therefore \frac{A_{max}}{A_{min}} = 1.3$$

6. Ans (A)

x is number of loops

$$\frac{450}{375} = \frac{x+1}{x} \Rightarrow x = 5$$

$$f_0 = \frac{375}{5}$$

$$f_0 = \frac{V}{2L}$$

$$L = \frac{V}{2f_0} = \frac{1}{2\times 75} \times \sqrt{\frac{360}{36\times 10^{-3}}}$$

$$= \frac{100}{2\times 75} = \frac{2}{3}m$$

$$m = 36\times 10^{-3} \times \frac{2}{3} = 24\times 10^{-3} kg$$

7. Ans (B)

$$20 = 10\log\left(\frac{r^2}{r_2^2}\right)$$

$$r_2 = 0.1r$$

shift = r − 0.1 r = 0.9 r. = 90m

8. Ans (D)

Amplitude of electric field

$E_0 = B_0 C = 60$ V/m

direction of $E = \hat{k} \times \hat{i}$

9. Ans (A)

path diff = 6d − 2d = 4d

$$4d = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{8}$$

10. Ans (A)

$$I = \frac{I_0}{4} + \frac{I_0}{4} + 2\sqrt{\frac{I_0}{4}\times\frac{I_0}{4}}\cos\theta$$

$$\cos\theta = \frac{-1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$$K\Delta x = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{3}$$

$$\Delta x = \frac{\lambda}{3}$$

$$d\sin\theta = \Delta x$$

$$\sin\theta = \frac{\Delta x}{d} = \frac{\lambda}{3d}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

11. Ans (B)

Second overtone of open pipe = $\frac{3V}{2\ell_1}$

second overtone of closed pipe = $\frac{5V}{4\ell_2}$

Since, these frequency are same

$$\therefore \frac{3V}{2\ell_1} = \frac{5V}{4\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{4\times 3}{2\times 5} = \frac{6}{5}$$

Now, the ratio of fundamental frequencies :

$$\frac{f_1}{f_2} = \frac{\frac{V}{2\ell_1}}{\frac{V}{4\ell_2}} \Rightarrow \frac{2\ell_2}{\ell_1}$$

= 10 : 6 = 5 : 3

13. Ans (B)

$$\frac{T}{A} = y.\frac{\Delta\ell}{\ell} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}}$$

$$\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{y.\frac{\Delta\ell}{\ell}}{\rho}}$$

$$\Rightarrow f_1 = \frac{1}{2\times 2}\sqrt{\frac{2.2\times 10^{11}\times 0.01}{7.7\times 10^3}}$$

$$f_1 = \frac{1000}{4}\times\sqrt{\frac{2}{7}} = 132.5\,Hz$$

$$f_2 = 2f_1 = 265\,Hz \Rightarrow \Delta f = 132.5\,Hz$$

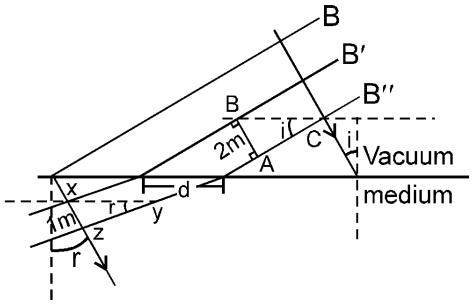
15. Ans (B)

$$f = \frac{330}{330-10}\times 512 = \frac{33}{32}\times 512$$

19. Ans (A)

In Δ ABC ; $\sin(i) = \frac{2}{d}$ In Δ xyz ; $\sin(r) = \frac{1}{d}$

$$\Rightarrow \frac{\sin i}{\sin r} = 2 = \mu$$



PART-1 : PHYSICS

SECTION-II

1. Ans (4)

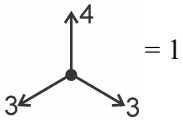
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \frac{1}{2}$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{4}$$

$$\frac{\rho_1}{\rho_2} = 4$$

2. Ans (1)



$= 1$

3. Ans (2)

$$\text{time} = \frac{\text{distance}}{\text{relative velocity}}$$

$$\text{Speed of } y_1 = \frac{1}{2}\,m/sec.$$

$$\text{Speed of } y_2 = \frac{1}{2}\,m/sec.$$

$$\text{time} = \frac{2}{1} = 2\ sec.$$

4. Ans (2)

$$f = \frac{nV}{2\ell} = \frac{V}{2\ell} = \sqrt{\frac{\frac{T}{\mu}}{2\ell}} = C\sqrt{T}$$

$$\ell n(f) = \ell n(c) + \frac{1}{2}\ell n(T)$$

$$\frac{1}{f}df = \frac{1}{2}\frac{1}{T}dT$$

$$\frac{dT}{T} = 2\left(\frac{df}{f}\right)$$

$$= 2\times\frac{6}{600}$$

$$\frac{dT}{T}\times 100 = 2\%$$

5. Ans (4)

$$f \propto \frac{1}{L}$$

$$f_A = \frac{K}{30}\ \&\ f_B = \frac{K}{25}$$

$$\frac{f_A}{f_B} = \frac{25}{30} = \frac{5}{6}$$

$$f_A - f_B = 4$$

$$\Rightarrow f_A = 20\ \&\ f_B = 24$$

$$f_A + f_B = 44$$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (C)

Only oxidising agent $PO_4^{3\Theta}$, $SO_4^{2\Theta}$

ON $\Rightarrow$ $PO_4^{3\Theta}$, $SO_4^{2\Theta}$

+5 +6

sum = 11

2. Ans (A)

Reverse of disproportionation reaction is called comproportionation reaction.

3. Ans (B)

$K_2Cr_2O_7 + Fe_{0.8}O \longrightarrow Cr^{3+} + Fe^{3+}$

gm equivalent of $K_2Cr_2O_7$ = gm equivalent of $Fe_{0.8}O$

$\Rightarrow 0.2 \times 10 \times 5 = x \times 0.4$

x = 30 mole

4. Ans (A)

$$\text{No. of atoms} = \frac{\text{wt. of substance} \times N_A \times \text{atomicity of "O"}}{\text{m.w.}}$$

$$y = x\left[\frac{N_A \times \text{Atomicity of "O"}}{\text{m.w.}}\right]$$

$$4 \times 10^{22} = \frac{6 \times 10^{23} \times \text{Atomicity}}{60}$$

Atomicity = 4

5. Ans (B)

	A	B
%mole	50%	50%
atomic mass	m	(m+1)

$$M_{Avg.} = \frac{\text{Total mass}}{\text{Total mole}}$$

$$10 = \frac{50 \times M + 50(m+1)}{100}$$

M = 9.5

6. Ans (B)

Organic compound $\longrightarrow NH_3$

POAC : N

$$n_{N\times1} = \frac{51}{17} \times 1$$

n_N = 3 mole

$$\%N \text{ in organic compound} = \frac{3 \times 14}{140} \times 100$$

= 30%

7. Ans (C)

Metal having lower SRP value than hydrogen, produce hydrogen gas in dilute acidic solution.

8. Ans (B)

A : $H_2(g) \longrightarrow 2H^{\oplus}(aq.)$

1 atm 10^{-2}M

C : $2H^{\oplus}(aq.) \longrightarrow H_2(g)$

10^{-3}M 4 atm

Cell Rx : $H_2(g) + 2H^+(aq.) \longrightarrow 2H^+(aq.) + H_2(g)$

1 atm 10^{-3} M 10^{-2} M 4 atm

$$E_{cell} = 0 - \frac{0.06}{2}\log\frac{[10^{-2}]^2[4]}{(10^{-3})^2(1)}$$

$= -0.03 \times 2.6 = -0.078$ V

9. Ans (C)

For electrolysis of (aq.) $CuSO_4$

Anode : $2H_2O(\ell) \longrightarrow 4e^{\Theta} + 4H^{\oplus} + O_2$

Cathode : $Cu^{2+}(aq.) + 2e^{\Theta} \longrightarrow Cu(5)$

10. Ans (D)

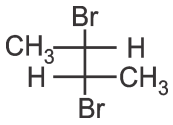
$$\lambda_E = \frac{\lambda m}{n - \text{factor}}$$

$$\frac{\lambda_E}{\lambda m} = \frac{1}{n - \text{factor}}$$

$$= \frac{1}{2}$$

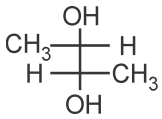
11. Ans (D)

Product mixure 1:



(Recemic mixure, Threo)

Product mixure 2:

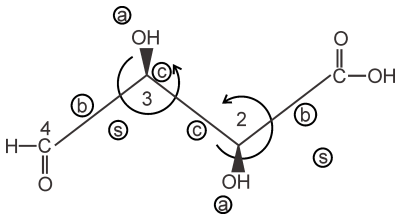


(Recemic mixure, Threo)

12. Ans (B)

its fact

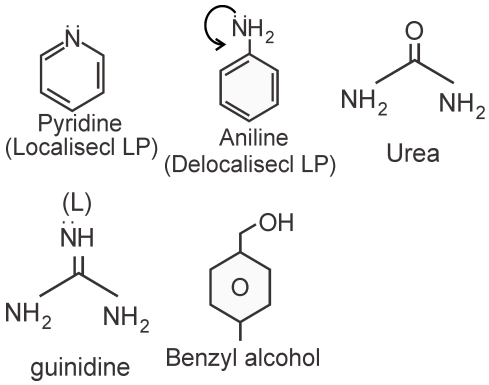
13. Ans (B)



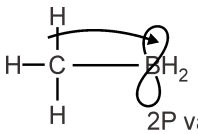
14. Ans (B)

Ortho Effect

15. Ans (B)

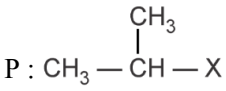


16. Ans (A)

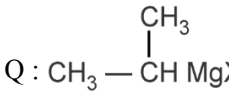


as C = 0 is stable ; it doesn't participate in hyperconjugation

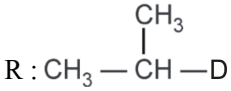
17. Ans (C)



P : $CH_3-CH(CH_3)-X$

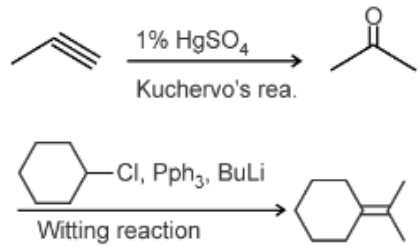


Q : $CH_3-CH(CH_3)-MgX$ (grignard reagent)



R : $CH_3-CH(CH_3)-D$

18. Ans (C)



19. Ans (B)

Stereo-isomers that are not mirror image of each other.

20. Ans (B)

PART-2 : CHEMISTRY

SECTION-II

1. Ans (100)

$$\lambda_m^{\infty} = \frac{K \times 1000}{5}$$

$$1000 = \frac{K \times 1000}{10^{-2}}$$

$$\Rightarrow K = 10^{-2}$$

$$\rho = 100$$

K_{sp} of $K_4[M(CN)_6] \Rightarrow 256.5^5 = 256 \times 10^{-10}$

$$S = 10^{-2}$$

2. **Ans (20)**

$NaOH + HCl \longrightarrow NaC\ell + H_2O$

10 mole

$Na_2CO_3 + 2HCl \longrightarrow H_2CO_3 + 2NaCl$

10 mole

$NaHCO_3 + HCl \longrightarrow H_2CO_3 + NaCl$

10 mole

gm eq. of NaOH + gm eq. of Na_2CO_3 + gm eq. of $NaHCO_3$ = gm eq. of HCl

$1 \times 10 + 2 \times 10 + 1 \times 10 = 2 \times V(L)$

$V = 20\ L$

3. **Ans (1000)**

$CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaCl$

0.1 mole $\frac{10.6}{10.6}$

$= 0.1$ mole

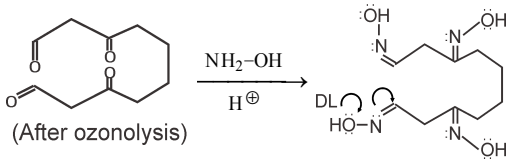
gm eq. of $CaCl_2$ = gm eq. of $CaCO_3$

$0.1 \times 2 = \frac{w}{100} \times 2$

$w = 10$ gm

Degree of hardness $= \frac{10}{10000} \times 10^6 = 1000$pm

4. **Ans (8)**



5. **Ans (4)**

(ii, iv , v, vi)

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (D)**

$$\int_0^{\pi/4} \frac{|\tan^{-1}\tan 2x| - |\sin^{-1}\sin 2x|}{|\tan^{-1}\tan 2x| + |\sin^{-1}\sin 2x|}dx$$

$$+ \int_{\pi/4}^{\pi/2} \frac{-(2x-\pi)-(\pi-2x)}{-(2x-\pi)+(\pi-2x)}dx$$

$$= \int_0^{\pi/4} \frac{2x-2x}{2x+2x}dx + \int_{\pi/4}^{\pi/2} 0\,dx = 0$$

2. **Ans (B)**

$$\frac{dy}{dx} - \frac{y}{1+x} = 2x(1+x)\sin x$$

$$\text{I. F.} = e^{-\int \frac{1}{1+x}dx} = e^{-\ell n(|1+x|)} = \frac{1}{|1+x|}$$

$$\frac{y}{|1+x|} = \int 2x(1+x)\frac{\sin x}{|1+x|}dx \quad (x > -1)$$

$$\frac{y}{|1+x|} = 2\int x\sin x dx = 2(-x\cos x + \sin x) + c$$

$$y(0) = 1 \Rightarrow \frac{y}{|1+x|} = 2(-x\cos x + \sin x) + 1$$

3. **Ans (A)**

$$\int \frac{6\sin x}{8+\sin 2x}dx = 3\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{8+\sin 2x}dx$$

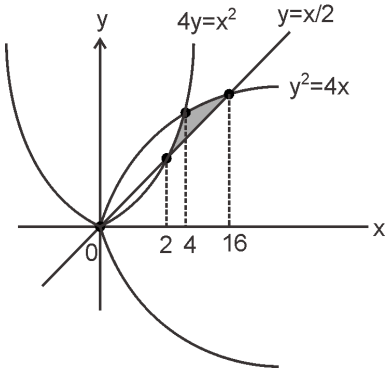
$$= 3\int \frac{\sin x+\cos x}{8+1-(\sin x-\cos x)^2}dx + 3\int \frac{\sin x-\cos x}{7+(\sin x+\cos x)^2}dx$$

$$= 3\int \frac{dt}{9-t^2} - 3\int \frac{dv}{7+v^2}$$

$$= \frac{-3}{6}\ell n\left|\frac{t-3}{t+3}\right| - \frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{v}{\sqrt{7}}\right) + c$$

$$= \frac{-1}{2}\ell n\left|\frac{\sin x-\cos x-3}{\sin x+\cos x+3}\right| \frac{-3}{\sqrt{7}}\tan^{-1}\left(\frac{\sin x+\cos x}{\sqrt{7}}\right)$$

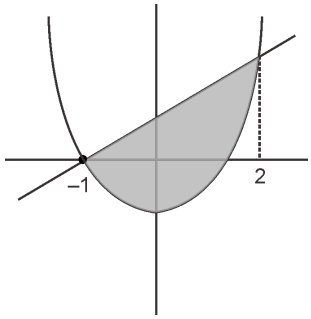
4. **Ans (D)**



$$A = \int_2^4 \frac{x^2}{4}dx + \int_4^{16} \sqrt{4x}dx - \int_2^{16} \frac{x}{2}dx$$

$$= \frac{49}{3}$$

5. Ans (D)



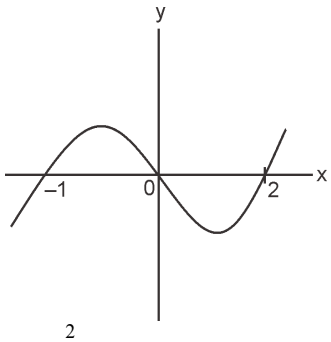
$$A = \int_{-1}^{2} \left((x+1) - \left(x^2 - 1\right)\right) dx$$

$$= \left(\frac{x^2}{2} + x - \left(\frac{x^3}{3} - x\right)\right)_{-1}^{2}$$

$$= \left(4 - \left(\frac{8}{3} - 2\right)\right) - \left(\frac{1}{2} - 1 - \left(\frac{-1}{3} + 1\right)\right)$$

$$= 4 - \frac{2}{3} - \left(\frac{-1}{2} - \frac{1}{3}\right) = 4 + \frac{1}{2} = \frac{9}{2}$$

6. Ans (C)



$$A = \int_{0}^{2} \left(x^3 - x^2 - 2x\right) dx$$

in (0, 2) I is −ve so it is minimum

7. Ans (D)

$$I = 6\int_{0}^{\pi} \left[\sqrt{2}\left|\sin\left(x - \frac{\pi}{4}\right)\right|\right] dx$$

$$6\left[\int_{0}^{\frac{\pi}{4}} (0)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (0)\,dx + \int_{\frac{\pi}{2}}^{\pi} 1\,dx\right]$$

$$= 6 \times \frac{\pi}{2} = 3\pi$$

8. Ans (C)

$$\int (\sin(2\ell nx) + 2\cos(2\ell nx))\,dx$$

$$\int \left(\sin(2\ell nx) + 2x\frac{\cos(2\ell nx)}{x}\right) dx$$

$$\int \left(f(x) + x \cdot f'(x)\right) dx = xf(x)$$

$$f(x) = x \cdot \sin(2\ell nx) + c,\ f(e^{\pi}) = 1$$

$$1 = e^{\pi}\sin(2\ell n(e^{\pi})) + c \Rightarrow c = 1$$

$$f(x) = x\sin(2\ell nx) + 1$$

$$f\left(e^{\frac{\pi}{8}}\right) = e^{\frac{\pi}{8}}\sin\left(2\ell n\left(e^{\frac{\pi}{8}}\right)\right) + 1$$

9. Ans (A)

$$f'(x) = x + e^x \cdot \lambda$$

$$\Rightarrow \int f'(x)dx = \int (x + e^x \cdot \lambda)dx$$

$$\Rightarrow f(x) = \frac{x^2}{2} + \lambda e^x + c$$

$$f(0) = 2 \Rightarrow c = 2 - \lambda$$

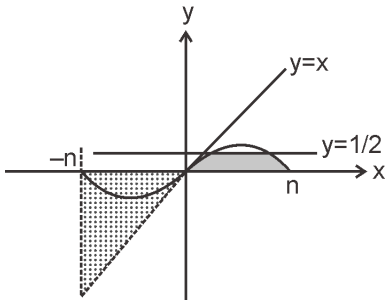
$$f(x) = \frac{x^2}{2} + \lambda e^x + 2 - \lambda \Rightarrow f(2) = 4 + \lambda e^2 - \lambda$$

$$\int_{0}^{1} \left(\frac{x^2}{2} + \lambda e^x + 2 - \lambda\right) dx = \lambda$$

$$\Rightarrow \frac{1}{6} + \lambda e - \lambda + 2 - \lambda = \lambda$$

$$\Rightarrow \lambda(3 - e) = \frac{13}{6}$$

10. Ans (C)



$$\text{Area} = \frac{1}{2}\pi \times \pi + \int_{0}^{\frac{\pi}{6}} \sin dx + \left(\frac{5\pi}{6} - \frac{\pi}{6}\right)\frac{1}{2} + \int_{\frac{5\pi}{6}}^{\pi} \sin dx$$

$$= \frac{\pi^2}{2} + \frac{2 - \sqrt{3}}{2} + \frac{\pi}{3} + \frac{2 - \sqrt{3}}{2}$$

11. Ans (D)

$x + y = t$

$\frac{dt}{dx} - 1 + 2xt = xt^2 - 1$

$\frac{dt}{dx} = x\left(t^2 - 2t\right)$

$\Rightarrow \int \frac{dt}{t(t-2)} = \int x dx$

$t \neq 0, 2$

$\frac{1}{2}\ell n\left|\frac{t-2}{t}\right| = \frac{x^2}{2} + c$

$\Rightarrow \frac{1}{2}\ell n\left|\frac{x+y-2}{x+y}\right| = \frac{x^2}{2} + c$

$c = 0, x = 0, y = 1$

$\frac{x+y-2}{x+y} = -e^{x^2}$

$\Rightarrow y = \frac{2-x}{1+e^{x^2}}$

or $x + y = 2$

or $y + x = 0$

12. Ans (B)

$\lim\limits_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1} = -1$

$\int \frac{\text{sgn}(-f(x))\ell n\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$

$\ell n\left(x+\sqrt{1+x^2}\right) = t$

$\frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx = dt$

$\int_0^{\ell n\left(\sqrt{2}+1\right)} tdt = \left(\frac{t^2}{2}\right)_0^{\ell n\left(\sqrt{2}+1\right)}$

13. Ans (B)

$\int \frac{\cot x}{\sqrt{\text{cosec } x - 1}} dx = \int \frac{\cot x \cdot \text{cosec } x}{\text{cosec } x\sqrt{\text{cosec } x - 1}} dx$

$\text{cosec } x - 1 = t^2 \Rightarrow -\cot x \cdot \text{cosec } x = 2t\, dt$

$\int \frac{-2t\, dt}{(t^2+1)\sqrt{t^2}} = -\int \frac{2dt}{t^2+1} = -2\tan^{-1}t + c$

14. Ans (C)

$\lim\limits_{n\to\infty} \sum_{r=1}^{6n} \frac{1}{n}\left(\frac{\frac{r}{n}}{1+\left(\frac{r}{n}\right)^2}\right) = \int_0^6 \frac{x}{1+x^2} dx$

$1 + x^2 = t \Rightarrow 2xdx = dt$

$-\int_1^{37} \frac{dt}{t} = \frac{1}{2}(\ell n(t))_1^{37}$

$= \frac{1}{2}\ell n37$

15. Ans (D)

$y + x\frac{dy}{dx} = 1$

$\Rightarrow y - x\frac{dx}{dy} = 1$

$\int ydy - \int xdx = \int dy$

$\Rightarrow \frac{y^2}{2} - \frac{x^2}{2} = y + c$

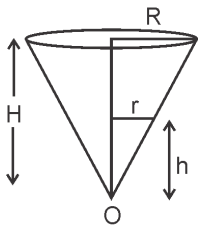
$\Rightarrow y^2 - 2y - x^2 = 2c$

16. Ans (B)

$y \cdot e^{\frac{x^2}{2}} = \int x \cdot e^{\frac{x^2}{2}} dx$

$ye^{\frac{x^2}{2}} = e^{\frac{x^2}{2}} + c$

17. Ans (B)



$$\frac{R}{H}=\frac{r}{h}=\frac{3}{9}$$

$R = 3m$, $H = 9m$

$$\frac{dv}{dt}=-4h^{\frac{1}{2}}$$

$$V=\frac{1}{3}\pi r^2 h$$

$$V=\frac{1}{3}\pi\times\frac{h^3}{9}$$

$$\frac{dv}{dt}=\frac{\pi h^2}{9}\frac{dh}{dt}$$

$$-4h^{\frac{1}{2}}=\frac{\pi h^2}{9}\frac{dh}{dt}$$

$$\Rightarrow -\int dt=\int\frac{\pi h^{\frac{3}{2}}}{36}dh$$

$$\Rightarrow t=\frac{-\pi h^{\frac{5}{2}}}{36}\times\frac{2}{5}+c$$

at $t = , h = 9 \Rightarrow c=\frac{3^5\times 2\pi}{5\times 36}$

$$t=\frac{-\pi h^{\frac{5}{2}}}{36}\times\frac{2}{5}+\frac{27\pi}{10}$$

18. Ans (A)

$f(x+y) = f(x) + f(y)$

then $f(x) = kx$

19. Ans (B)

$$I_\alpha=\left(\int_0^\pi \sin x.(0)dx+\int_\pi^{2\pi}\sin x\cdot(-1)dx\right)4\alpha$$

$$=4\alpha(\cos x)_\pi^{2\pi}=(2)(4\alpha)$$

$$\sum_{\alpha=1}^{20} I_\alpha = 8\,(1+2+\ldots+20)$$

$$= 1680$$

20. Ans (A)

(A) $\frac{xdy-ydx}{x^2}=\frac{xdx}{x^2}$

$$\Rightarrow\int d\left(\frac{y}{x}\right)=\int\frac{dx}{x}\Rightarrow\frac{y}{x}=\ell nx+c$$

(B) $x(xy-1)dy+y(xy+2)dx=0$

Put $xy = t$

$$y+x\frac{dy}{dx}=\frac{dt}{dx}$$

$$x(t-1)\left(\frac{dt}{dx}-\frac{t}{x}\right)\frac{1}{x}+\frac{1}{x}(t+2)=0$$

$$\Rightarrow(t-1)\frac{dt}{dx}-\frac{t(t-1)}{x}+\frac{t(t+2)}{x}=0$$

$$\Rightarrow(t-1)\frac{dt}{dx}+\frac{3t}{x}=0$$

$$\Rightarrow\int\frac{t-1}{t}dt=-3\int\frac{dx}{x}\Rightarrow t-\ell nt=-3\ell nx+c$$

$$\Rightarrow xy-\ell n(xy)=-3\ell nx+c$$

(C) $\frac{d}{dx}\left(\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\right)=0$

$$\Rightarrow\int\left(\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\right)dx=\int c_1dx$$

$$\Rightarrow\frac{1}{2}\left(\frac{dy}{dx}\right)^2=c_1x+c_2$$

$$\Rightarrow\frac{dy}{dx}=\sqrt{Ax+B}\Rightarrow dy=\int\sqrt{Ax+B}\,dx$$

$$\Rightarrow y=\frac{A}{2\sqrt{Ax+B}}+c$$

(D) $\frac{dy}{dx}=\frac{x+y}{x-2y}\Rightarrow y=xt\Rightarrow\frac{dy}{dx}=t+x\frac{dt}{dx}$

$$t+x\frac{dt}{dx}=\frac{x+xt}{x-2xt}=\frac{1+t}{1-2t}$$

$$\Rightarrow x\frac{dt}{dx}=\frac{1+t}{1-2t}-t=\frac{2t^2+1}{1-2t}$$

$$\Rightarrow\int\frac{1-2t}{1+2t^2}dt=\int\frac{dx}{x}\Rightarrow\int\frac{1}{1+2t^2}dt-\int\frac{2t}{1+2t^2}dt$$

$$=\ell nx+c$$

$$\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}t\right)-\frac{1}{2}\ell n\left(1+2t^2\right)=\ell nx+c$$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (18)

$$\int_0^{12} f(x)dx = \int_0^{6} f(x)dx + \int_6^{12} f(x)dx$$

$$= 9 + \int_6^{12} f(18-x)\,dx \quad \left(\begin{array}{c} f(18-x) = f(x-6) \\ \because\ f(12+x) = f(-x) = f(x) \\ f(6+x) = f(x-6) \end{array} \right)$$

$$= 9 + \int_6^{12} f(6+x)\,dx$$

$$= 9 + \int_0^{6} f(t)\,dt \ = 9 + 9 = 18$$

2. Ans (3)

$$ye^{y^2}\frac{dy}{dx} - \frac{e^{y^2}}{x} = \frac{1}{x^2}$$

$$e^{y^2} = t$$

$$2ye^{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - \frac{2t}{x} = \frac{2}{x^2}$$

$$\text{I. f.} = e^{-\int \frac{2}{x}dx} = e^{-2\ell nx} = \frac{1}{x^2}$$

$$t \cdot \frac{1}{x^2} = \int \frac{2}{x^4}dx = \frac{-2}{3x^3} + c$$

$$\frac{e^{y^2}}{x^2} = \frac{-2}{3x^3} + c \Rightarrow 3xe^{y^2} = -2 + 3cx^3$$

3. Ans (0)

$$f(x) = \left(1-x^5\right)^{\frac{1}{2}} \text{ then } f^{-1}(x) = \left(1-x^2\right)^{\frac{1}{5}}$$

$$f(0) = 1,\ f(1) = 0 \text{ so}$$

$$\int_0^{1} f(x)dx + \int_{f(0)}^{f(1)} f^{-1}(x)dx = 1\cdot f(1) - 0\cdot f(0)$$

$$\int_0^{1} \left(1-x^5\right)^{\frac{1}{2}} + \int_1^{0} \left(1-x^2\right)^{\frac{1}{5}}dx = 1\cdot f(1) - 0\cdot f(0)$$

$$= 0$$

4. Ans (0)

$$f'(x) = \ell nx + 1 - 1 - \frac{f(x)}{x\ell nx}$$

$$\frac{dy}{dx} - \frac{y}{x\ell nx} = \ell nx$$

$$y \cdot \ell nx = \int (\ell nx)^2 dx = x(\ell nx)^2 - 2\int x\ell nx\frac{1}{x}dx$$

$$y\ell nx = x(\ell nx)^2 - 2(x\ell nx - x) + c$$

$$f(1) = -1 \Rightarrow c = -2$$

$$y\ell nx = x(\ell nx)^2 - 2(x\ell nx - x) - 2$$

$$y = x(\ell nx) - 2x + \frac{2(x-1)}{\ell nx} \Rightarrow f(e) = e - 2$$

5. Ans (4)

$$\text{Put } x^4 = t \Rightarrow 4x^3dx = dt$$

$$\frac{1}{4}\int_0^{1} \frac{edt}{e^t(2-t)} \quad \underline{\text{king}} \quad = \frac{e}{4}\int_0^{1} \frac{dt}{e^{1-t}(1+t)}$$

$$= \frac{e}{4e}\int_0^{1} \frac{e^t dt}{1+t}$$