PHYSICS

7

The value of x in the formula $Y = \frac{2mgl^x}{5bt^3e}$ where m is the mass, ' g ' is acceleration due to gravity, l is the length, ' b ' is the breadth, ' t ' is the thickness and e is the extension and Y is Young's Modulus, is

(A) 3 (B) 2

(C) 1 (D) 4

$[Y] = \frac{[m][g][l]^x}{[b][t]^3[e]}$

$ML^{-1}T^{-2} = \frac{MLT^{-2}L^x}{L^5}$

$L^3 = L^x$

$x = 3$

$[b] = L^1$

$[t] = L^1$

$[g] = LT^{-1}$

$[l] = L$

$[Y] = \frac{[F]/[A]}{[\Delta L]/[L]} = \frac{MLT^{-2}}{L^2}$

$[Y] = ML^{-1}T^{-2}$

2

A person swims at 135° to current of river, to meet target on reaching opposite point. The ratio of person's velocity to river water velocity is

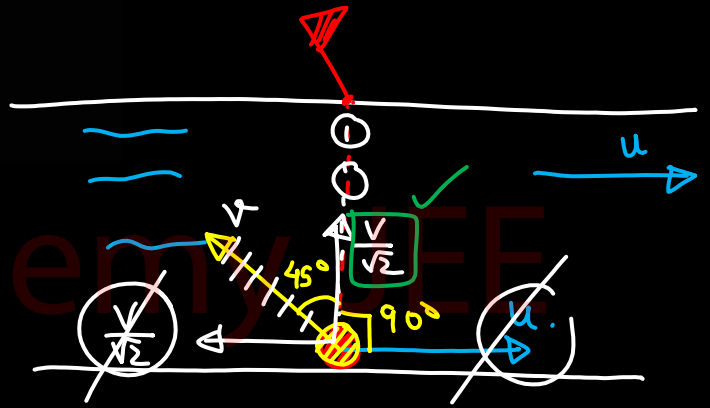
(A) $\sqrt{3}:1$

(B) $\sqrt{2}:1$

(C) $1:\sqrt{2}$

(D) $1:\sqrt{3}$

$\frac{v}{u} = ?$



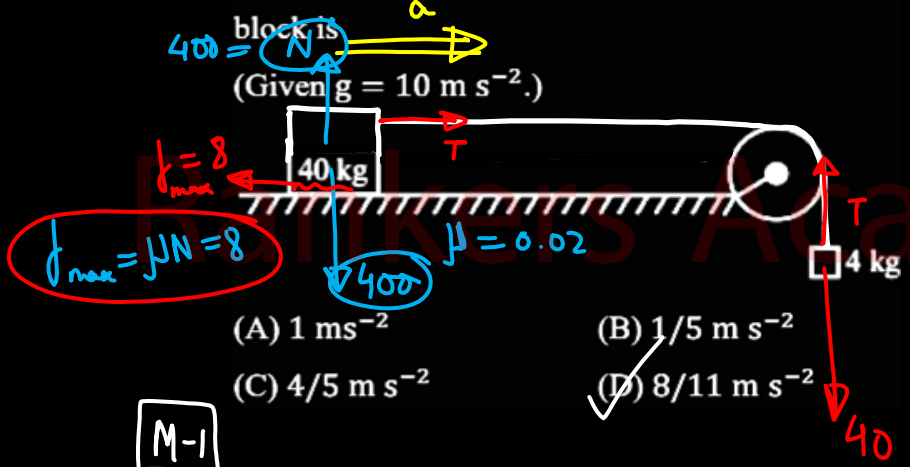
$u = \frac{v}{\sqrt{2}}$

$\frac{v}{u} = \sqrt{2}$

3

A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02 . The acceleration of block is

(Given g = 10 m s^{-2}.)



(A) 1 ms^{-2}

(B) 1/5 m s^{-2}

(C) 4/5 m s^{-2}

(D) 8/11 m s^{-2}

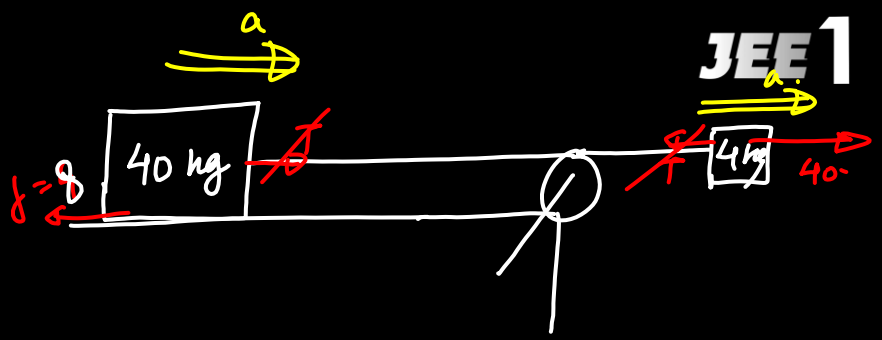
M-1

$40 - T = 4a$

$T - 8 = 40a$

$32 = 44a$

$a = \frac{8}{11}$



$32 = 44a$

$a = \frac{8}{11}$

A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(A) 850 J (B) 950 J

(C) 875 J (D) 900 J

$W_{0\to5} = \Delta KE_{0\to5}$

$W_{0\to5} = \frac{1}{2}(2)(30)^2 - 0 = 900$

JEE 1

diff. x

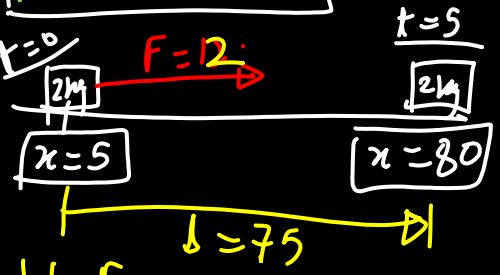
$v = 6t$

$u = 0 = v|_{t=0}$.

$v|_{t=5} = 30$

$a = \frac{dv}{dt} = 6$.

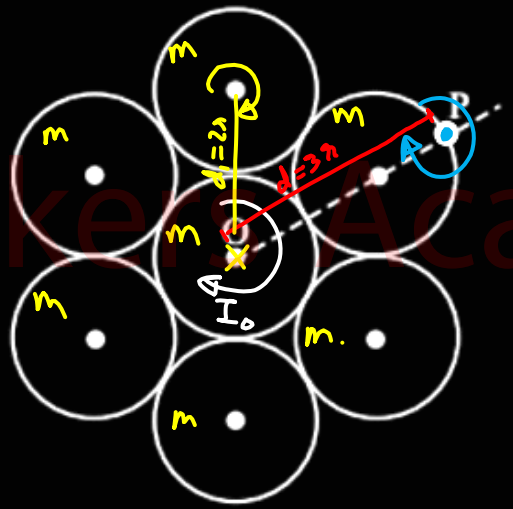
$F = ma = 12$



$W = Fd = 900$

5

Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(A) $\frac{19}{2}MR^2$

(B) $\frac{55}{2}MR^2$

(C) $\frac{73}{2}MR^2$

(D) $\frac{181}{2}MR^2$

Ans = $I_P = I_0 + (7m)\cdot d^2$, $R = 3r$

JEE 1

$I_0 = \frac{1}{2}mr^2 + 6\left[\frac{1}{2}mr^2 + m d_1^2\right]$, $d_1 = 2r$

$I_0 = mr^2\left[\frac{1}{2} + 3 + 24\right]$

$\boxed{I_0} = \frac{55}{2}mr^2$

$I_P = \boxed{I_0} + 7m(3r)^2$

$I_P = \frac{55}{2}mr^2 + 63mr^2$

$I_P = \left(\frac{55+126}{2}\right)mr^2 = \frac{181}{2}mr^2$.

6

A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube.

(Given $\alpha = 5.0 \times 10^{-4}\ °C^{-1}$)

(A) $2.4 \times 10^6\ cm^3$ (B) $1.2 \times 10^5\ cm^3$

(C) $6.0 \times 10^4\ cm^3$ (D) $4.8 \times 10^5\ cm^3$

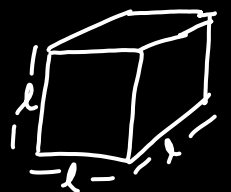
JEE 1

$6l^2 = 24$

$l^2 = 4$

$l = 2$ ☺

$V = l^3 = 8$



$\Delta T = 10$

$\alpha : \beta : \gamma$

$1 : 2 : 3$

$\frac{\alpha}{\gamma} = \frac{1}{3}$

$\gamma = 3\alpha$

$\gamma = 15 \times 10^{-4}$

$\Delta V = ?$

$\Delta V = V \gamma \Delta T$

$\Delta V = 8(15 \times 10^{-4})(10) = 0.12\ m^3$

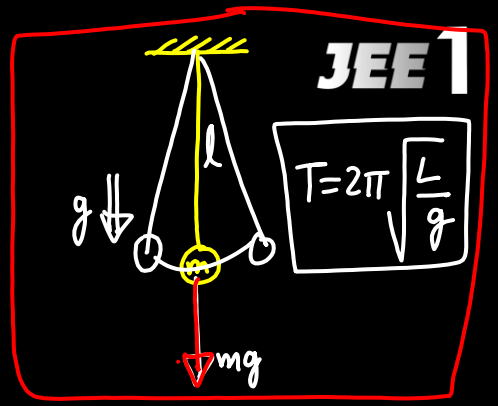
$1.2 \times 10^5\ (cm)^3$

7

A bob of mass m suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length times that of the bob and the length of the thread is increased by 1/3rd of the original length, then the time period of the simple harmonic oscillations will be

(A) $\frac{3}{4}T$ (B) $\frac{4}{3}T$

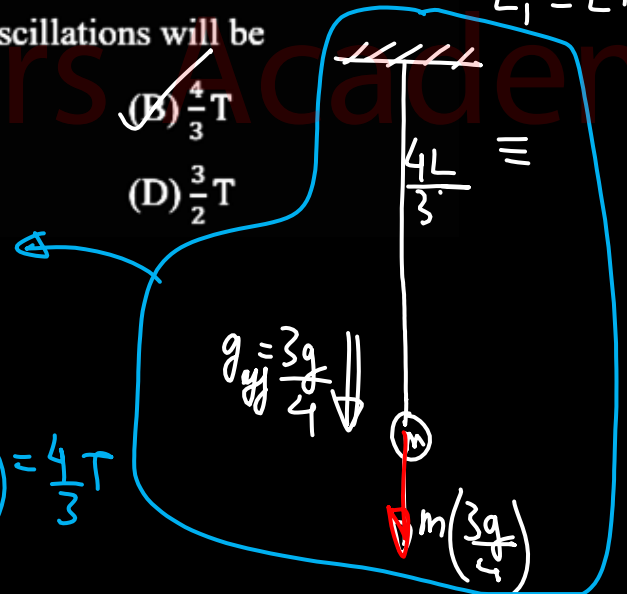
(C) T (D) $\frac{3}{2}T$



$L_1 = L + \frac{L}{3} = \frac{4L}{3}$

$T_1 = 2\pi\sqrt{\frac{(4L/3)}{(3g/4)}}$

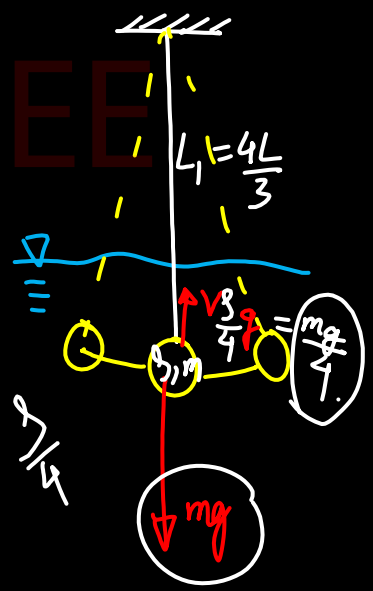
$T_1 = \frac{4}{3}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{4}{3}T$



$m = V\rho$

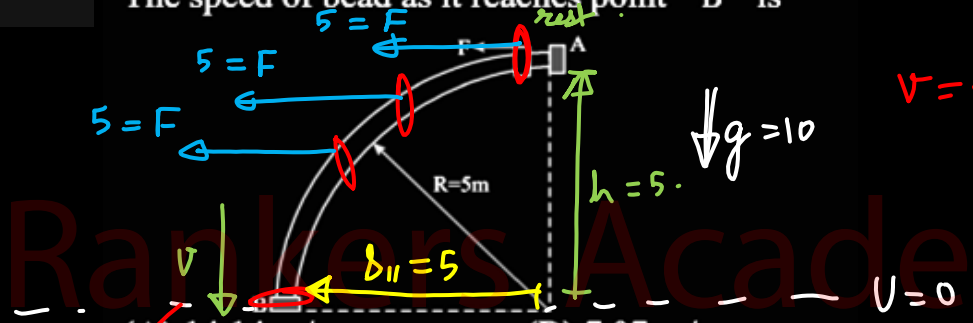
$g_{yy} = \frac{3g}{4}$

3/4



8

A bead of mass $\frac{1}{2}$ kg starts from rest from "A" to move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force F = 5 N as shown. The speed of bead as it reaches point " B " is



(A) 14.14 m/s
(B) 7.07 m/s
(C) 5m/s
(D) 25m/s

$E_i + W_F = E_f$

JEE 1

$0 + mgh + F(5) = \frac{1}{2}mv^2 + 0$

$25 + 25 = \frac{1}{4}v^2$

$v = \sqrt{200} = 10\sqrt{2} = 14.14$ m/s

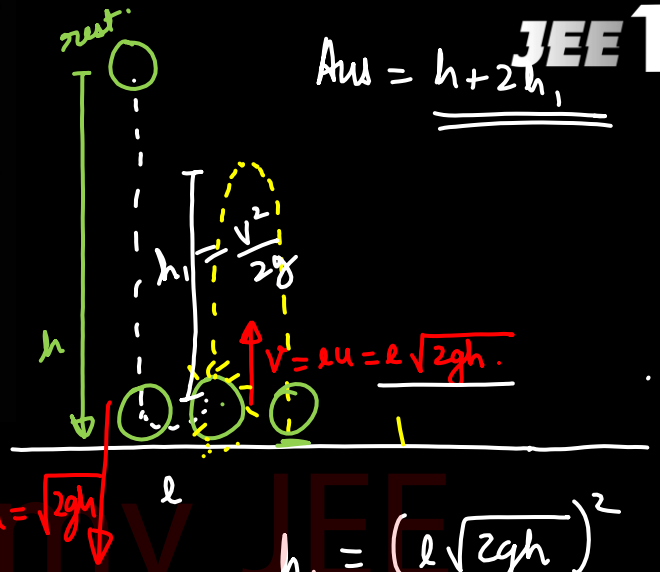
9

A ball is dropped from a height ' h ' on to a floor of coefficient of restitution ' e '. The total distance covered by the ball just before second hit is

(A) $h(1-2e^2)$ (B) $h(1+2e^2)$

(C) $h(1+e^2)$ (D) he^2

$mgh = \frac{1}{2}mv^2$



Ans = $h + 2h_1$

$h_1 = \frac{(e\sqrt{2gh})^2}{2g}$

$h_1 = e^2h$

Ans = dist. = $h + 2(e^2h)$

10

Two different artificial satellites orbiting with same time period around the earth having angular momenta 2:1. The ratio of masses of the satellites is

(A) 2: 1 (B) 1: 2

(C) 1: 1 (D) 1:3

$T^2 \propto r^3$

$T \propto r^{3/2}$

or M2

$\frac{dA_1}{dt} = \frac{dA_2}{dt}$

$\frac{L_1}{2m_1} = \frac{L_2}{2m_2}$

$\frac{L_1}{L_2} = \frac{m_1}{m_2} = \frac{2}{1}$

$T = \frac{2\pi r}{v_0}$

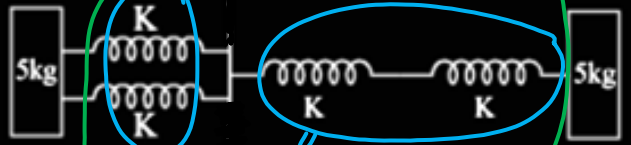
$v_0 = \sqrt{\frac{GM}{r}}$

$\frac{GMm}{r^2}$ $\quad \frac{mv_0^2}{r}$ $\quad v_0$ $\quad m$ $\quad L$ $\quad r$ $\quad M$

$2L$ $\quad v_0$ $\quad 2m$

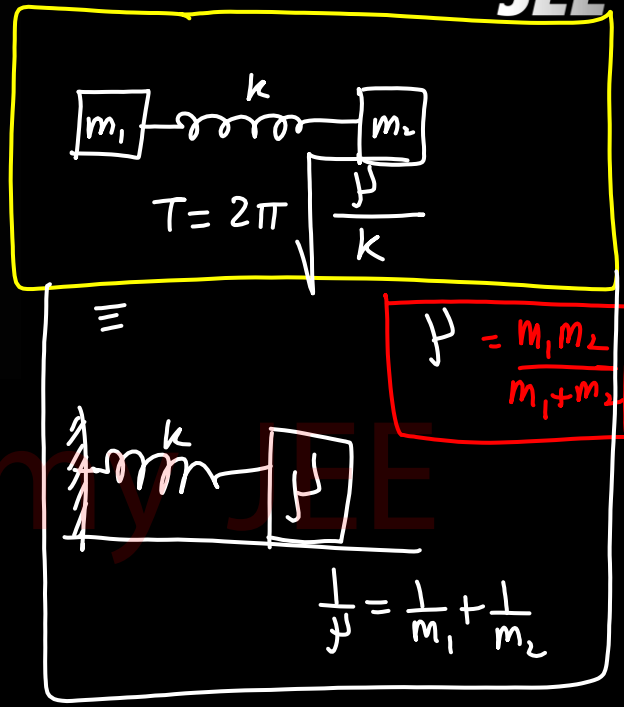
11

In the given spring block system if $k = 25\pi^2$ Nm^{-1}, find time period of oscillation.



(A) 1sec (B) 2sec
(C) 3sec (D) 4sec

JEE 1

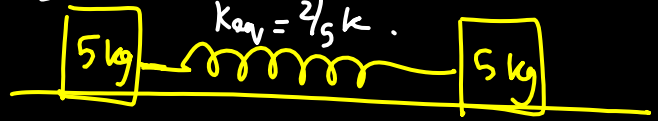


$2k$. $\frac{k}{2}$

$= \quad //$

$k_{eq} = \frac{(2k)\left(\frac{k}{2}\right)}{2k + \frac{k}{2}} = \frac{2}{5}k$

$\mu = \frac{5\times5}{5+5} = \frac{5}{2}$



$$T = 2\pi\sqrt{\frac{\mu}{k_{eq}}} = 2\pi\sqrt{\frac{5\times5}{2\times2k}} = \frac{2\pi\times5}{2}\sqrt{\frac{1}{25\pi^2}} = 1$$

12

Two containers of equal volume containing the same gas at pressure P_1 and P_2 and absolute temperature T_1 and T_2 respectively were connected with narrow capillary tube. The gas reaches a common pressure P and a common temperature T. The ratio P/T is equal to

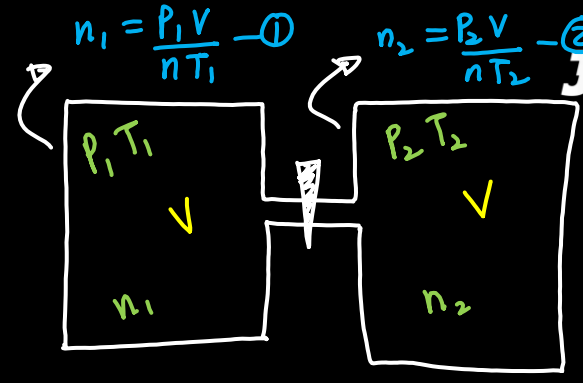
(A) $\frac{P_1}{T_1}+\frac{P_2}{T_2}$

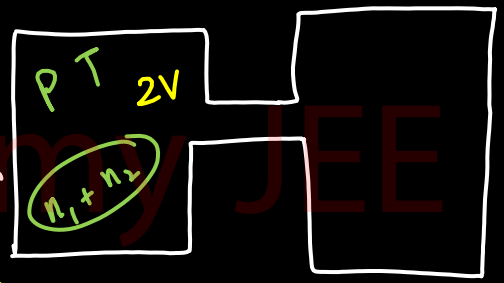
(B) $\frac{1}{2}\left(\frac{P_1}{T_1}+\frac{P_2}{T_2}\right)$

(C) $\frac{P_1T_2+P_2T_1}{T_1+T_2}$

(D) $\frac{P_1T_2-P_2T_1}{T_1-T_2}$

$n=\frac{PV}{RT}$





1&2

$n_1+n_2=\frac{P(2V)}{RT}$ -③.

$\frac{P_1V}{RT_1}+\frac{P_2V}{RT_2}=\frac{2PV}{RT}$

13

A sine wave is travelling in a medium. The minimum distance between the two particles, always having same speed, is

(A) $\frac{\lambda}{4}$

(B) $\frac{\lambda}{3}$

(C) $\frac{\lambda}{2}$

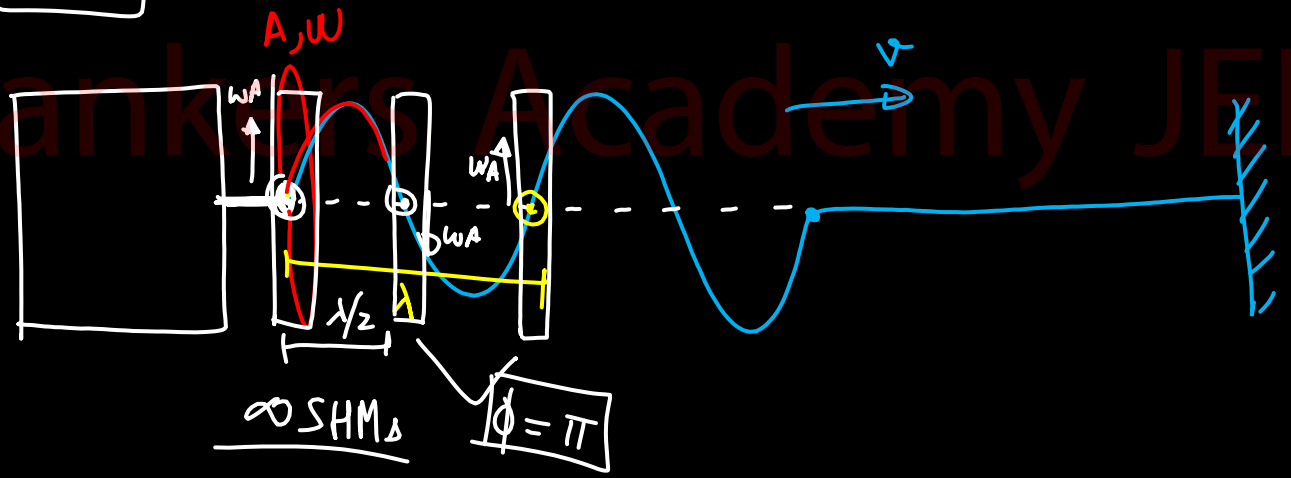
(D) λ ✗

JEE 1

$\frac{\phi}{\Delta} = \frac{2\pi}{\lambda}$ ×

$\phi = 2\pi$

$\phi = \pi$

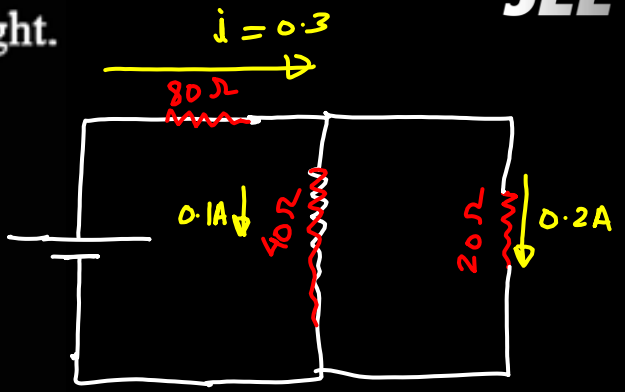


14

A current of 0.10 A flows through the 25Ω resistor represented in the diagram to the right. The current through the 80Ω resistor is:

(A) ~~0.10 A~~ (B) 0.20 A

(C) 0.30 A (D) 0.40 A



15

A ball of mass ' m ' moving with speed ' u ' undergoes a head-on elastic collision with a ball of mass ' nm ' initially at rest. Find the fraction of the incident energy transferred to the second ball.

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2$ Ans $= \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2}$

(A) $\frac{n}{n+1}$ (B) $\frac{n}{(n+1)^2}$

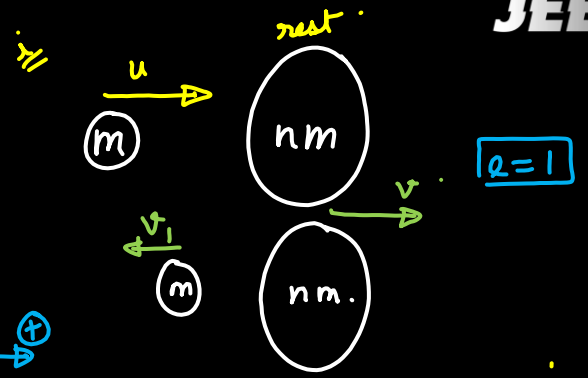
(C) $\frac{2n}{(1+n)^2}$ (D) $\frac{4n}{(1+n)^2}$

$v = u - (nv - u)$

$\frac{v(n+1)}{2} = u$

$\frac{1}{2}m v^2 = \frac{1}{2}\left(\frac{2}{n+1}\right)^2 u^2$

$\frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{4}{(n+1)^2}$



$e = 1$

P

C

$mu = nmv - mv_1$

$v_1 = nv - u$ — (1)

$e = \frac{V_{sep}}{V_{app}}$

$1 = \frac{v + v_1}{u}$

$v = u - v_1$ — (2)

16

A proton, a deuteron and an α-particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is _____ and their speed is _____, in the ratio.

(A) 4: 2: 1 and 2: 1: 1

(B) 1: 2: 4 and 1: 1: 2

(C) 2: 1: 1 and 4: 2: 1

(D) 1: 2: 4 and 2: 1: 1

$F = qvB$

$F = \frac{q}{m} pB \propto \frac{q}{m}$

$p = (m)v_1 = (2m)v_2 = (4m)v_3$

$v \propto \frac{1}{m}$

$v_1 : v_2 : v_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{4}$

$V_1 : V_2 : V_3 = 4 : 2 : 1$

$F_1 : F_2 : F_3 = \frac{e}{m} : \frac{e}{2m} : \frac{2e}{4m}$

$= 1 : \frac{1}{2} : \frac{1}{2}$

$\therefore 2 : 1 : 1$

17

JEE 1

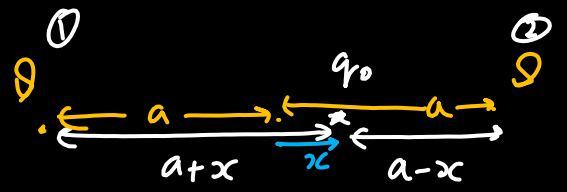
Two identical positive charge Q each are fixed at a distance of ' 2a ' apart from each other. Another point charge q_0 with mass ' m ' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of change q_0 will be

(A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

(B) $\sqrt{\frac{q_0 Q}{4\pi^3\varepsilon_0 ma^3}}$

(C) $\sqrt{\frac{2\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

(D) $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0 Q}}$



$$F_{net} = F_2 - F_1$$

$$= \frac{kQq_0}{(a-x)^2} - \frac{kQq_0}{(a+x)^2}$$

$$= \frac{kQq_0}{(a^2-x^2)^2}\,4ax$$

$x \ll a$

$$F_{net} = \frac{4kQq_0}{a^3}\,x$$

$F = (m\omega^2)x$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ma^3}{4kQq_0}}$$

18

JEE 1

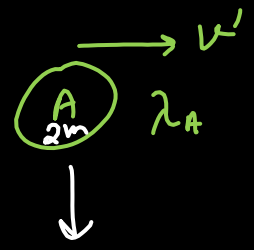
A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively

(A) $\lambda_A, \frac{\lambda_A}{2}$

(B) $2\lambda_A, \lambda_A$

(C) $\lambda_A, 2\lambda_A$

(D) $\frac{\lambda_A}{2}, \lambda_A$



$2mv' = mv - \frac{mv}{2}$

$v' = \frac{v}{4}$

$\lambda_A = \frac{h}{2m\left(\frac{v}{4}\right)}$ — ①

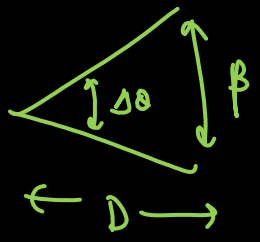
$\lambda_B = \frac{h}{mv}$ $\lambda_C = \frac{h}{\frac{mv}{2}}$

$\lambda_B = \frac{\lambda_A}{2}$ $\lambda_C = \lambda_A$

19

In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to

(A) 0.17°
(B) 0.57°
(C) 1.7°
(D) 0.07°



$\Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d} \Rightarrow \Delta\theta = \frac{500 \times 10^{-9}}{5 \times 10^{-5}}$

$\Delta\theta = \frac{1}{100} = 0.01\ \text{rad}$

$0.01 \times \frac{180^\circ}{\pi}$

$\frac{1.8}{3.14} \approx 0.6^\circ$

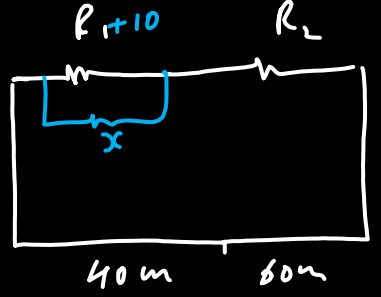
20

JEE 1

In the experimental set up of metre bridge, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is

(A) 40Ω (B) 20Ω

(C) 60Ω (D) 30Ω



40cm

+10cm

$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$ — (1)

$\frac{R_1+10}{R_2} = \frac{50}{50}$ — (2)

(2) ÷ (1)

$\frac{R_1+10}{R_1} = \frac{3}{2}$

$R_1 = 20\Omega$

$\frac{\frac{(R_1+10)x}{R_1+10+x}}{R_2} = \frac{2}{3}$

$\frac{30 \times x}{(30+x)30} = \frac{2}{3}$

$x = 60$

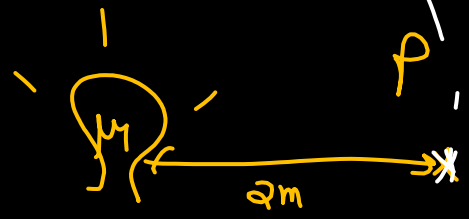
21

JEE 1

A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is ____

(Rounded off to the nearest integer)

[Take $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$, $c = 3 \times 10^8$ ms^{-1}]



$I = \frac{\eta P}{4\pi r^2}$

$\left(\frac{1}{2}\varepsilon_0 E_0^2\right) c = \frac{\eta P}{4\pi r^2}$

$E_0 = ?$

$E_0 = \sqrt{\frac{2\eta P}{4\pi\varepsilon_0 r^2 c}}$

$= \sqrt{\frac{2 \times (1.25 \times 10^{-2} \times 1000)}{\frac{1}{9\times10^9} \times 4 \times 3 \times 10^8}}$

$= \sqrt{\frac{5}{4} \times \frac{3}{2} \times 100}$

$= 10\sqrt{\frac{15}{8}} \approx 10 \times 1.369$

≈ 13.7

$= \frac{137}{10}$

22

The Vernier constant of Vernier callipers is 0.1 mm and it has zero error of −0.05 cm. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be _____ × 10^{-2} cm.

L.C.

JEE 1

L.C. = 0.1 mm = 0.01 cm

Reading = MSR + L.C. × CVSD − Zero error

= 1.7 cm + (0.01 cm) × 5 − (−0.05)

= 1.7 + 0.1 = 1.8 cm = 180 × 10^{-2} cm

23

In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304Å. The corresponding difference for the Paschan series (in Å) is____

JEE 1

$\frac{1}{\lambda_1} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$

$\lambda_1' = \frac{16 \times 9}{7R}$

$\frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$ $\lambda_2' = \frac{9}{R}$

$\frac{1}{\lambda_1} = R\left[1 - \frac{1}{4}\right] \Rightarrow \lambda_1 = \frac{4}{3R}$

$\frac{1}{\lambda_2} = R\left[1 - \frac{1}{\infty^2}\right] \Rightarrow \lambda_2 = \frac{1}{R}$

$\lambda_1 - \lambda_2 = 304\ \text{Å}$

$\frac{1}{3R} = 304\ \text{Å}$ — (1)

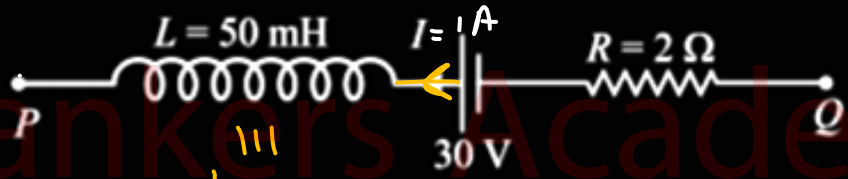
$\lambda_1' - \lambda_2' = \frac{9}{R}\left(\frac{16}{7} - 1\right)$

$\Delta\lambda = 912 \times \frac{9 \times 9}{7}$

$= \boxed{10553}\ \text{Å}$

24

A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is_____



JEE 1

MH②

$V_P - V_B = -L\frac{di}{dt} + \mathcal{E} - iR$

$= -0.05 \times (-100) + 30 - 2$

$= 5 + 30 - 2$

$= 33$

MH①

$V_P - V_B = \left|L\frac{di}{dt}\right| + \mathcal{E} - iR$

$= 0.05 \times 100 + 30 - 2$

$= 33\ V$

25

A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 m V/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is _____

$j = \sigma E$

$i = (\sigma E) A$

$= 5 \times 10^7 \times 10^{-2} \times \pi \times \frac{1}{4} \times 10^{-6}$

$= \frac{5\pi}{4} \times 10^{-1} = \frac{500}{4}\pi \times 10^{-3} = \frac{125}{4}\pi \times 10^{-3}$

$x = 5$

$= (5^3)\ \pi$ mA

CHEMISTRY

1

In which of the following cases upon mixing, the resulting solution will be a buffer solution :

I. 25ml of 0.1 M HNO_3 and 25ml of 0.1 M $NaNO_3$

II. 25ml of 0.1 M NH_4OH and 50ml of 0.1 M HCl

III. 50ml of 0.1 M NaOH and 10ml of 0.1M H_3PO_3

IV. 25ml of 0.1 M HCOOH and 25ml of 0.1 M $Ba(OH)_2$

(A) I and III

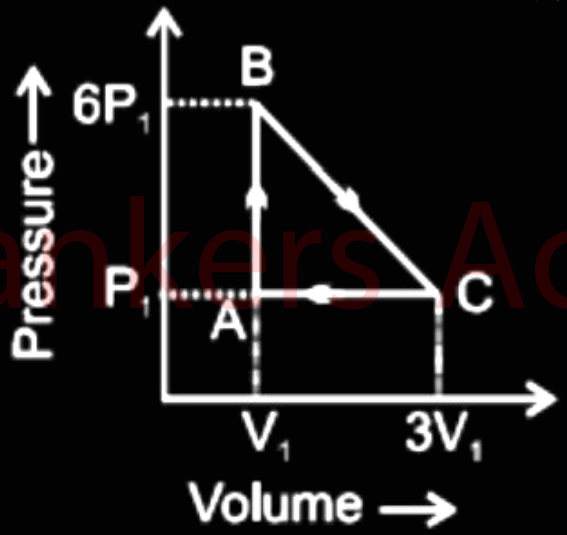
(B) I and II

(C) II & IV

(D) None of these

2

An ideal gas is taken around the cycle ABCA as shown in P-V diagram. The value of q for the cycle is equal to :



(A) $7P_1V_1$

(B) $-5P_1V_1$

(C) $5P_1V_1$

(D) $-7P_1V_1A$

$W = \frac{1}{2} \times \text{Base} \times \text{height}$

$= \frac{1}{2} \times 2V_1 \times 5P_1$

$= 5P_1V_1$

For clockwise, W will have − sign

$W = -5P_1V_1$

$q = 5P_1V_1$

Accor to 1st law of thermo.

$\Delta U = q + W$

$\Delta U = 0$ for a cyclic process

So $q = -W$

JEE 1

3

The electrons identified by quantum numbers n and ℓ :

(a) $n = 5, \ell = 1$

(b) $n = 5, \ell = 0$

(c) $n = 4, \ell = 2$

(d) $n = 4, \ell = 1$

can be placed in order of increasing energy as :

(A) (c) < (d) < (b) < (a)

(B) (d) < (b) < (c) < (a)

(C) (b) < (d) < (a) < (c)

(D) (a) < (c) < (b) < (d)

$n + \ell$ rule

i) $E \propto n + \ell$

ii) $E \propto n$ (when $n+\ell$ value is same)

a > c > b > d

4

$A(s) \rightleftharpoons 2B(g) + C(g)$

The above equilibrium was established by initially taking A(s) only. At equilibrium, B is removed so that its partial pressure at new equilibrium becomes $1/3^{rd}$ of original total pressure. Ratio of total pressure at new equilibrium and at initial equilibrium will be :

(A) 2/3 (B) 14/13

(C) 5/3 (D) 17/19

$A(s) \rightleftharpoons 2B(g) + C(g)$

eq-1 $2P \quad P$

eq-1 $P \quad P'$

$P_{T_1} = 3P$

$P_{T_2} = P + P'$

$K_P = (2P)^2(P) = 4P^3$

$K_P = P^2 \times P'$

$4P^3 = P' \times P^2$

$P' = 4P$

$P_{T_2} = P + 4P = 5P$

$\frac{P_{T_2}}{P_{T_1}} = \frac{5P}{3P} = \frac{5}{3}$

5

A magnetic moment of 1.73BM will be shown by one amongst the following :

(A) $[Ni(CN)_4]^{2-}$ $n=0$

(B) $\overset{IV}{TiCl_4}$ $Ti^{+4} = 3d^0 4s^0$, $n=0$

(C) $[CoCl_6]^{4-}$ $Co^{+2} = 3d^7$, W.L $\boxed{1\downarrow\ |1\downarrow\ |1\ |1\ |1}$, $n=3$

~~(D)~~ $[Cu(NH_3)_4]^{2+}$ $Cu^{2+} = 3d^9$, $n=1$

$\mu = 1.73$ BM

$\boxed{n=1}$

6

0.1 molar aqueous solution of NaCl and water are separated by semi-permeable membrane at 27°C. If external pressure of 3 atm is applied on solution side then $\left(R = \frac{1}{12}\ L\ atm\ mol^{-1}\ K^{-1}\right)$:

(A) Osmosis will stop.

(B) Osmosis will continue.

(C) Reverse osmosis will occur.

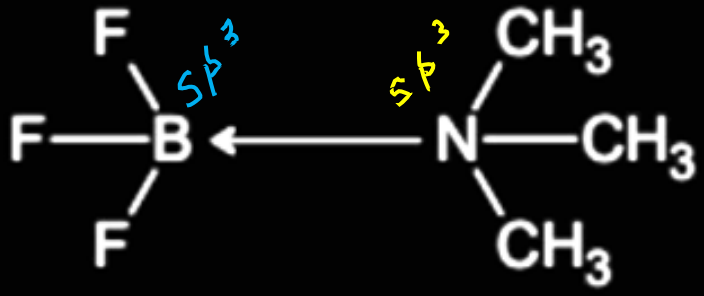
(D) Solute will move from solution to solvent.

$\pi = iCRT$

$= 2 \times 0.1 \times \frac{1}{12} \times 300 = 5\ atm$

7

Choose the correct statement regarding the following molecule



(A) ∠FBF bond angle is 120°

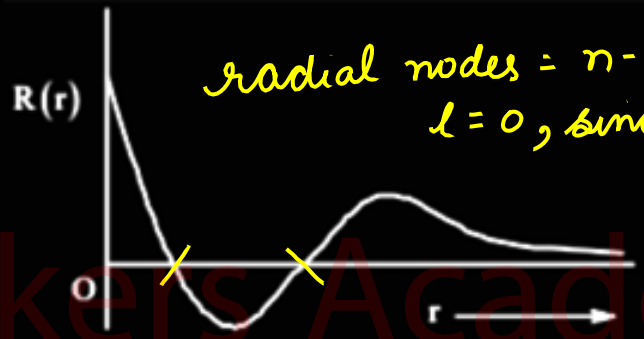
(B) Nitrogen undergoes sp^2 hybridization

(C) $p\pi - p\pi$ back bond between B & F in the molecule is more dominant than that in isolated BF_3 molecule

(D) All central atoms have the same type of hybridization

8

The radial part of wave function of an orbital is plotted against distance from nucleus. Which orbital represent below graph?



radial nodes = $n - l - 1 = 2$

$l = 0$, since graph does not start from origin

So orbital = 3S

(A) 1 s

(B) 2 s

(C) 3 s

(D) 2p

9

Heat of the following reaction in bomb calorimeter is -1415 kJ. ΔU

$$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$$

What is the heat released if 1.4 g C_2H_4 is combusted in open atmosphere at 27°C ? $\rightarrow \Delta H$

($R = 8.3 JK^{-1} mol^{-1}$)

(A) -71.0 kJ (B) -1415 kJ

(C) -710 kJ (D) -1420 kJ

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta n_g = 2 - 4 = -2$

$\Delta H = -1415\,kJ + \underbrace{(-2)(8.314\times10^{-3})(300)}_{5\,kJ}$

$= -1415\,kJ - 5\,kJ$

$= -1420\,kJ$

$28\,g \text{ —— } -1420$

$1\,g \text{ —— } \frac{-1420}{28}$

$1.4\,g \text{ —— } \frac{-1420}{28}\times 1.4$

$= -71\,kJ$

10

Peptide bond between two α-amino acids is actually:

(A) ether bond

(B) ester bond

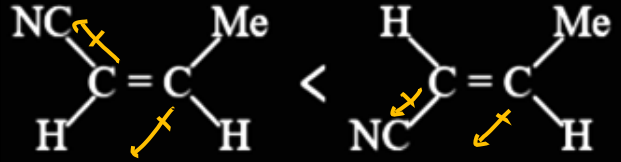
(C) amide bond

(D) glycoside bond

11

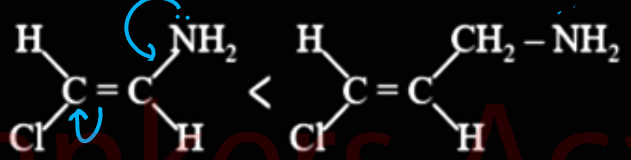
Which option is incorrect?

(A) Dipole moment

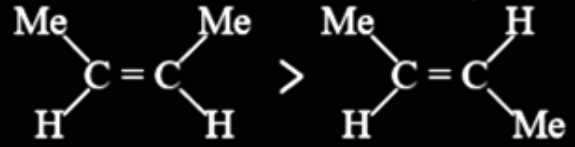


$\vec{\mu} \propto \frac{1}{\theta}$

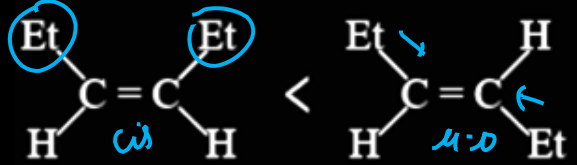
(B) Basicity



(C) Melting point $\propto \frac{1}{\text{Stability}}$



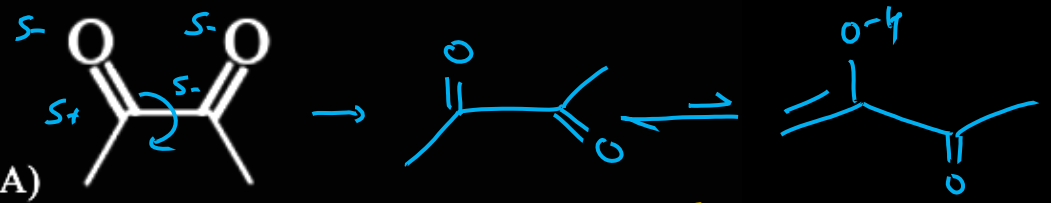
(D) Stability



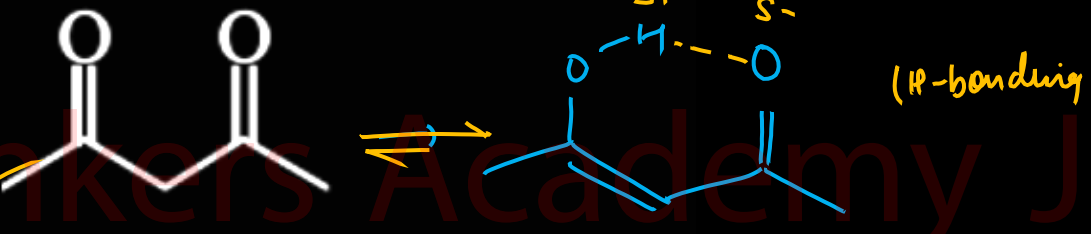
12

The compound which predominantly exist in its enol form is

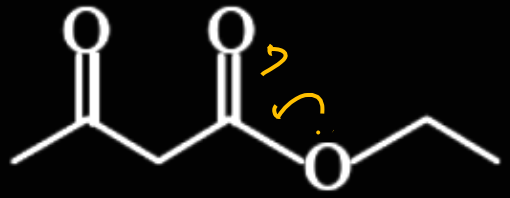
(A)



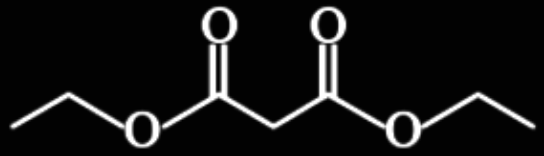
(B)



(C)



(D)

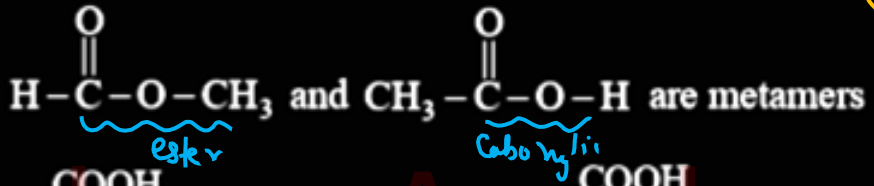


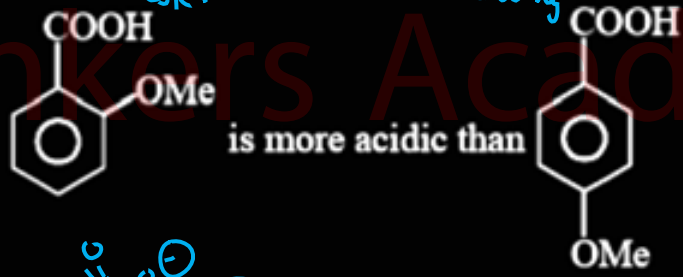
13

Which statement is correct:

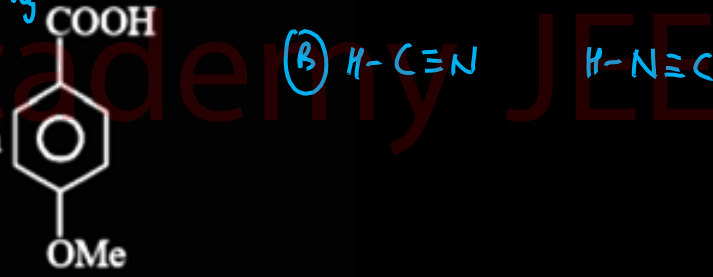
(A) Me_3C-OH reacts faster than $MeCH_2-OH$ with Na-metal

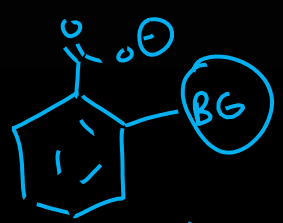
(B) HCN and HNC are functional isomers

(C)  and are metamers

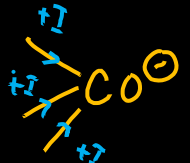


is more acidic than



(D) 

(SIR)

(A) $Me_3COH + Na \rightarrow$ 

(Stability)

(B) H-C≡N H-N≡C

14

JEE 1

Reaction-(P) :  Baeyer's Reagent → Product

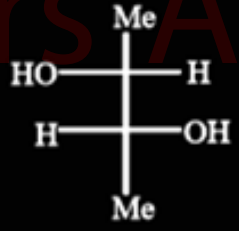
Reaction-(Q) :  (i) mCPBA, (ii) $H_3O^{\oplus}$ → Product

Reaction-(R) :  Baeyer's Reagent → Product

Reaction-(S) :  (i) mCPBA, (ii) $H_3O^{\oplus}$ → Product

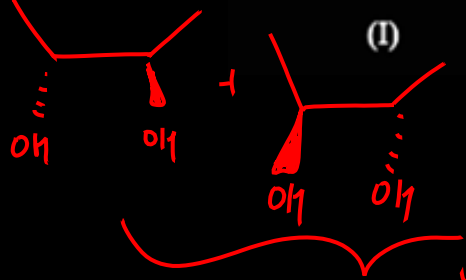
These four reactions give three products.

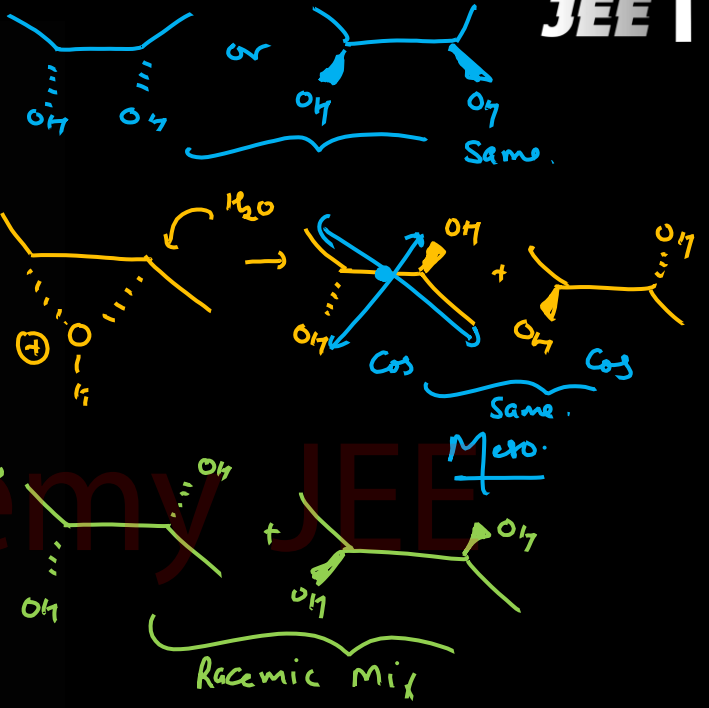
(I)



(II)

(III)





Select the correct statement :

(A) In reaction-P; III product is obtained.

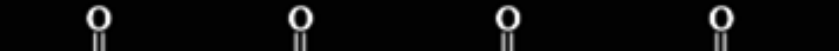
(B) In reaction-Q; I, II product is obtained.

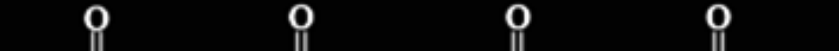
(C) In reaction-R; III product is obtained.

(D) In reaction-S; III product is obtained.

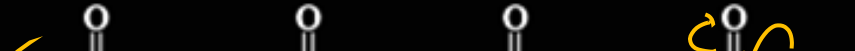
15

Correct order of rate of reaction of MeMgBr will be

(A) 

(B) 

(C) 

(D) 

16

Sodium salt of which acid can produce ethene on electrolysis.

(A) Maleic acid

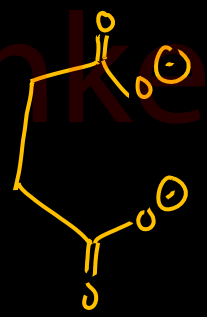
(B) Fumaric acid

(C) Succinic acid

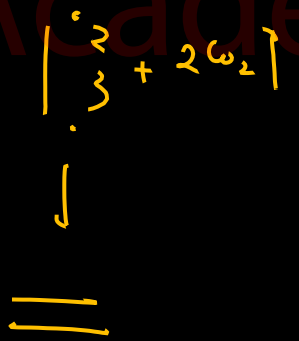
(D) Glutaric acid

JEE 1

n=0 O
n=1 M
n=2 S
G
A
P

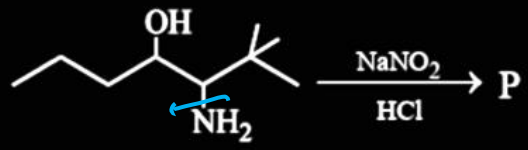


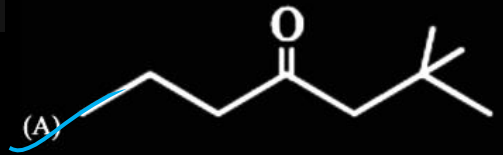
Electrolysis

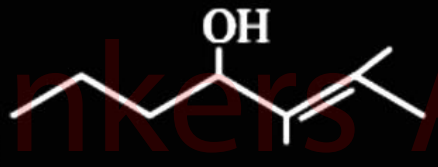


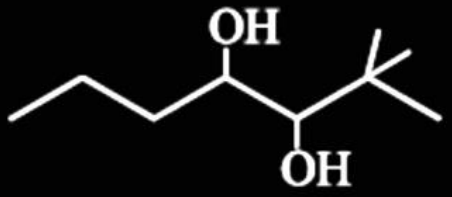
17

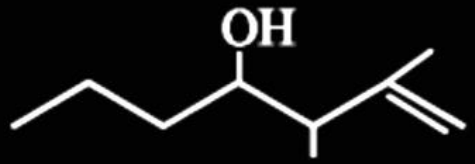
Predict the major product P in following reaction:

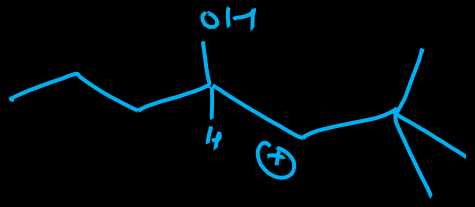


(A) 

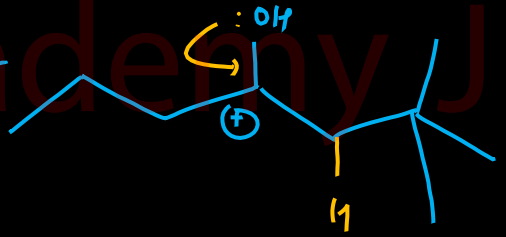
(B) 

(C) 

(D) 

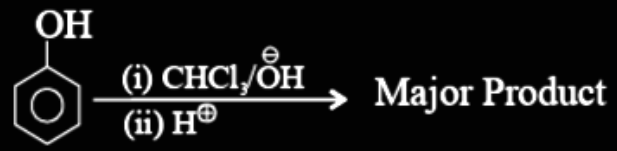


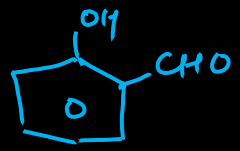
Hydride migration



18

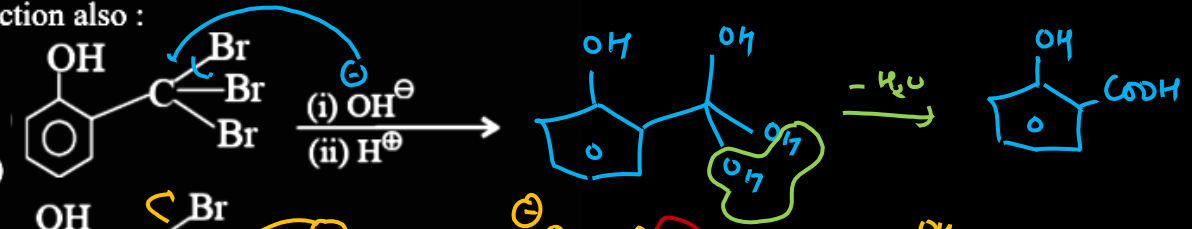
JEE 1



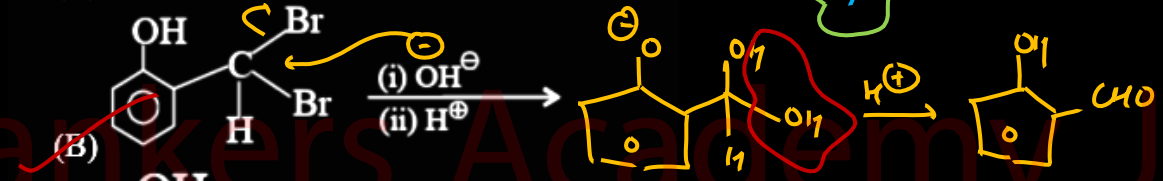


Major product can be prepared by following reaction also :

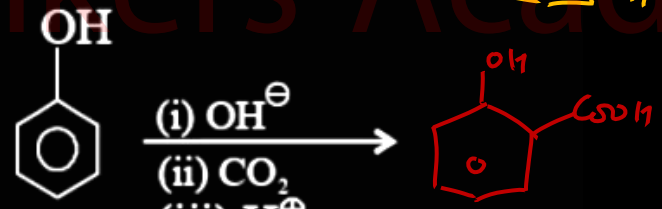
(A)



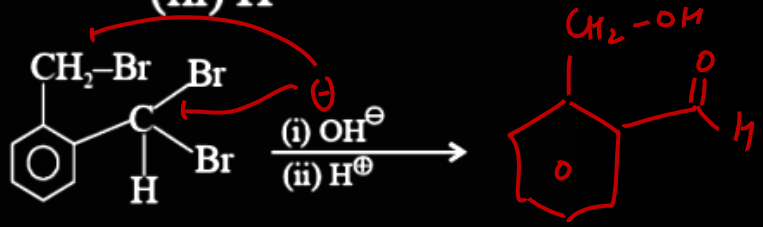
(B)



(C)



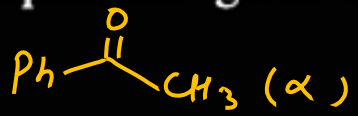
(D)



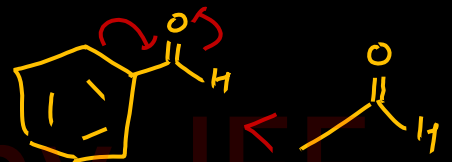
19

Identify the correct combination of true (T) and false (F) of the given three statements.

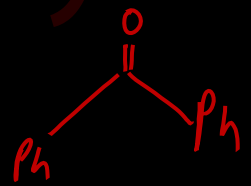
STATEMENT-1 : Acetophenone gives aldol condensation.



STATEMENT-2 : Benzaldehyde is more reactive than acetaldehyde towards nucleophilic addition.



STATEMENT-3 : Benzophenone has α - Hydrogens.



(no α H)

(A) TTT

(B) FFF

(C) TFF

(D) TFT

20

The equivalent conductances of CH_3COONa, HCl and NaCl at infinite dilution are 91,426 and 126 S cm^2eq^{-1} respectively at 25°C . The equivalent conductance of 1MCH_3COOH solution is 19.55 S cm^2eq^{-1} . The pH of solution is (Take log 5 = 0.7)

(A) 5.3 (B) 4.3

(C) 2.3 (D) 1.3

$CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$

$(C - C\alpha)$ $C\alpha$ $C\alpha$

$[H^+] = C\alpha = 1\alpha = \alpha$

$\alpha = \frac{\Lambda_{eq}^c}{\Lambda_{eq}^\infty} = \frac{19.55}{(91+426)-126}$

$= 0.05$

$pH = -\log[H^+] = -\log 5\times10^{-2}$

$= 2 - \log 5$

$= 2 - 0.7$

$= 1.3$

21

100 mL, 0.2 M CH_3COOH is mixed with 100 mL, 0.1MNaOH find out pH of resulting solution.

(Given : $K_{a(CH_3COOH)} = 2 \times 10^{-5}$)(log 2 = 0.3, log 3 = 0.48) Give your answer to the nearest single digit integer.

Ans 5

$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

	CH_3COOH	$NaOH$	CH_3COONa	H_2O
$t=0$	20 mmol	10 mmol	0	
	10 mmol	0	10 mmol	

$$PH = PK_a + \log \frac{Salt}{acid}$$

$$\because [Salt] = [Acid]$$

$$\Rightarrow PH = PK_a = -\log (2 \times 10^{-5})$$

$$= 5 - \log = 4.7 \approx 5$$

22

One mole of glycol is added to 380 g water and the solution was cooled to −5°C. How much gram of ice is formed in this process? [K_f of $H_2O = 1.86$ KKgmol^{-1}]

Ans 8

$\Delta T_f = m K_f$

$= \frac{1 \times 1000}{380} \times 1.86$

$= 4.89°C$

New F P = −4.89°C

We cooled it to −5

$5 = \frac{1}{x} \times 1000 \times 1.86$

$x = \frac{1000 \times 1.86}{5}$

$= 372$ g

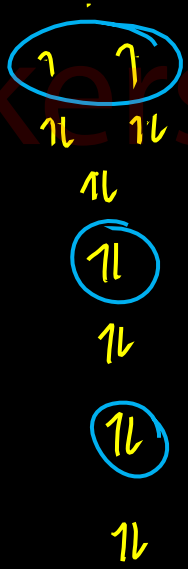
So ice $380 - 372 = 8$ g

23

Ratio of number of antibonding molecular orbital electrons to the number of bonding molecular orbital electrons of O_2 is x/10. Fill x

JEE 1

$$\text{In } O_2 \quad = \frac{n_a}{n_b} = \frac{6}{10} = \frac{x}{10}$$



Ans 6

24

The mass of $Ca(NO_3)_2$ in mg that is present in 50ml of solution with 9.50ppm of Ca is.....

(Molar mass of $Ca(NO_3)_2 = 164\ g\ mol^{-1}$)

Round off the ans to nearest integer.

JEE 1

Ans 2

$$9.50 = \frac{W_{Ca}}{50} \times 10^6$$

$$\frac{W_{Ca}}{40} = \text{moles of } Ca = \text{moles of } Ca(NO_3)_2$$

$$\text{mass of } Ca(NO_3)_2 \text{ in mg} = \frac{W_{Ca}}{40} \times 164 \times 10^3$$

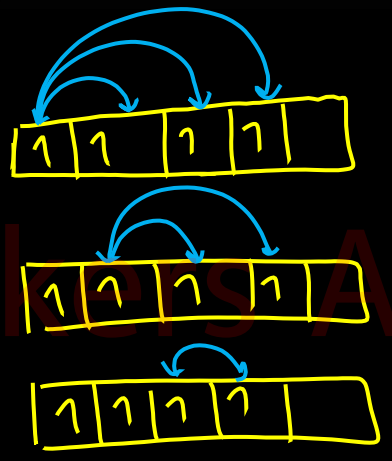
$$= 1.943 \approx 2$$

25

If x is the number of exchanges possible for $3d^4$ electronic configuration, then the value of x is _____ .

JEE 1

Ans 6

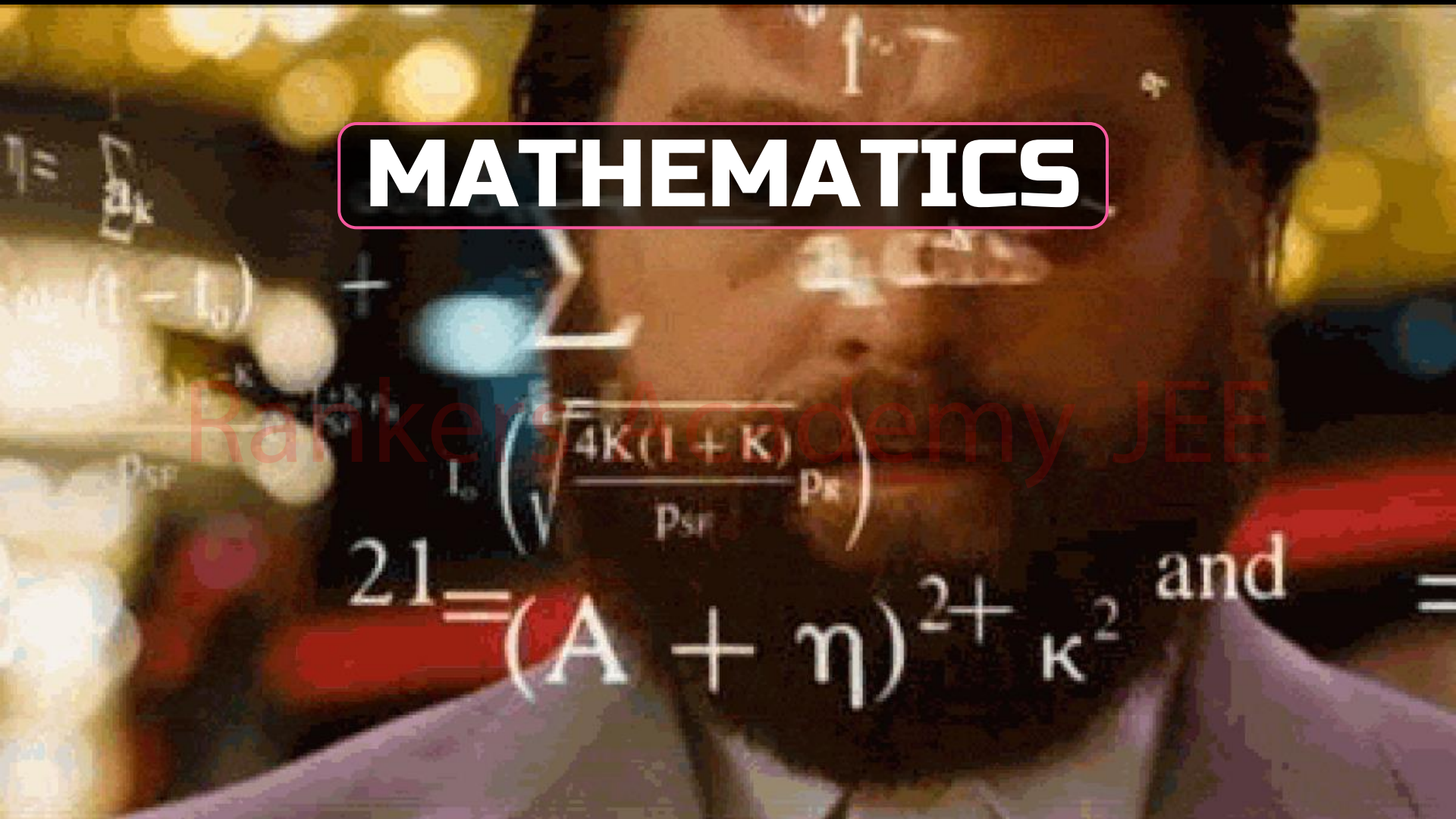


3

2

1

6


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

Acute angle between the lines $L_1: x + y = 0 = y + z$ and $L_2: 2x + y + z = 0, x + 2y + z = 0$ is -

$z = -3x$

(A) $\cos^{-1}\frac{3}{11}$

(B) $\cos^{-1}\sqrt{\frac{3}{11}}$

(C) $\sin^{-1}\sqrt{\frac{2}{11}}$

(D) $\sin^{-1}\frac{2}{11}$

$\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

$L_1: \quad x = -y = z$

$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$

$\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

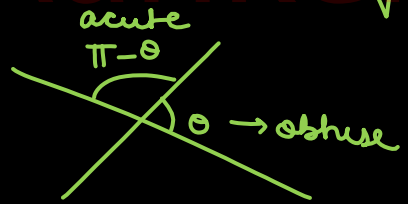
$L_2: \quad x = y = \frac{z}{-3}$

$\frac{x}{1} = \frac{y}{1} = \frac{z}{-3}$

$\vec{b_2} = \hat{i} + \hat{j} - 3\hat{k}$

$$\cos\theta = \frac{1 - 1 - 3}{\sqrt{3}\ \sqrt{11}}$$

$$\cos\theta = -\sqrt{\frac{3}{11}}$$



$\sin^{-1}\sqrt{\frac{8}{11}}$

2

If $y = e^{\log_e \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)} + \frac{1}{2}\tan^{-1} x$, where $x \in (0,1]$, then $\frac{dy}{dx}$ is equal to -

(A) $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ (B) $\frac{1}{1+x^2}$

(C) $\frac{1}{2(1+x^2)}$ (D) $\frac{1}{\sqrt{1+x^2}}$

JEE 1

$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) + \frac{1}{2}\tan^{-1} x$

Let $x = \tan\theta$

$x \in [0,1]$

$\theta = \tan^{-1} x \in (0, \frac{\pi}{4}]$

$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) + \frac{1}{2}\theta$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) + \frac{\theta}{2}$

$y = \tan^{-1}\left(\frac{2\sin^2\theta/2}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) + \frac{\theta}{2}$

$= \tan^{-1}(\tan\theta/2) + \theta/2$

$= \frac{\theta}{2} + \frac{\theta}{2} = \theta$ ✓

$y = \theta = \tan^{-1} x$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

3

Value of $\lim_{x\to 0} \dfrac{(\ln(1+\tan x))\cdot(\sin x - x)\cdot x^3}{\tan^2(2\pi\sec x)\cdot x^3}$ is -

(A) $\dfrac{\pi^2}{2}$

(B) $-\dfrac{\pi^2}{2}$

(C) $-\dfrac{1}{6\pi^2}$ ✓

(D) $\dfrac{1}{6\pi^2}$

JEE 1

$\ln(1+\tan x) \longleftrightarrow \tan x$

$\lim_{x\to 0} \dfrac{\ln(1+x)}{x} = 1$

$\lim_{x\to 0} \dfrac{x-\sin x}{x^3} = \dfrac{1}{6}$

$\lim_{x\to 0} \dfrac{\tan x}{x} = 1$

$\tan^2(2\pi\sec x) \to 0$

$= \lim_{x\to 0} \dfrac{\tan x}{x} \cdot x \cdot \left(-\dfrac{1}{6}x^3\right)$ ($\dfrac{\tan x}{x} = 1$)

$= \lim_{x\to 0} \dfrac{-\frac{1}{6}x^4}{\tan^2(2\pi\sec x)}$ → M2 LH.

JEE 1

M1

$$\lim_{x\to 0} \frac{\frac{-1}{6}x^4}{\tan^2(2\pi - 2\pi\sec x)}$$

$$= \lim_{x\to 0} \frac{\frac{-1}{6}x^4}{(2\pi - 2\pi\sec x)^2}$$

$$= \frac{-1}{6}\cdot\frac{1}{4\pi^2} \lim_{x\to 0} \frac{x^4}{(1-\sec x)^2}$$

$$= \frac{-1}{24\pi^2} \lim_{x\to 0} \frac{x^4\cos^2 x}{(1-\cos x)^2}$$

$$= \frac{-1}{24\pi^2} \lim_{x\to 0} \left(\frac{x^2}{1-\cos x}\right)^2 (1)$$

$$= \frac{-1}{24\pi^2}\left(\frac{1}{(1/2)}\right)^2$$

$$= \frac{-1}{24\pi^2}(4)$$

$$= \frac{-1}{6\pi^2}$$

4

$\int \text{cosec}\left(\tan^{-1}\sqrt{x^2-1}\right)dx, x > 1$ is equal to -

(A) $\frac{1}{\sqrt{x^2-1}} + C$

(B) $x\sqrt{x^2-1} + C$

(C) $\frac{x}{\sqrt{x^2-1}} + C$

(D) $\sqrt{x^2-1} + C$

Let $x = \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$= \int \text{cosec}\left(\tan^{-1}\sqrt{\sec^2\theta - 1}\right)\sec\theta\tan\theta\, d\theta$

$\sqrt{\sec^2\theta - 1} = \tan\theta$

$\theta = \tan^{-1}(\tan\theta)$

$= \int \text{cosec}\,\theta\sec\theta\tan\theta\, d\theta = \int \sec^2\theta\, d\theta = \tan\theta + C = \sqrt{x^2-1} + C$

5

Let $f(n) = \tan^{-1}\left(\frac{4}{1+3.4}\right) + \tan^{-1}\left(\frac{6}{1+8.9}\right) + \tan^{-1}\left(\frac{8}{1+15.16}\right) + \cdots$ n terms, then $\lim_{n\to\infty} f(n)$ is -

(A) less than $\frac{\pi}{8}$ but greater than $\frac{\pi}{12}$

(B) less than $\frac{\pi}{12}$

(C) greater than $\frac{\pi}{8}$ $(> 22.5^\circ)$

(D) equal to $\frac{\pi}{8}$

$\frac{\pi}{8} \to 22.5^\circ$ | $\frac{\pi}{12} = 15^\circ$

$\tan 22.5 = \sqrt{2} - 1 \approx 1.4 - 1 \approx 0.4$ | $\tan 15 = 2 - \sqrt{3} \approx 0.3$

$T_n = \tan^{-1}\left(\frac{2(n+1)}{1 + ((n+1)^2 - 1)(n+1)^2}\right)$

$= \tan^{-1}\left(\frac{2(n+1)}{1 + (n^2+2n)(n+1)^2}\right)$

$= \tan^{-1}\left(\frac{2(n+1)}{1 + n(n+2)(n+1)^2}\right)$

$= \tan^{-1}\left(\frac{2(n+1)}{1 + (n(n+1))((n+1)(n+2))}\right)$

$= \tan^{-1}\left(\frac{(n+1)(n+2) - (n+1)n}{1 + (n(n+1))(n+1)(n+2)}\right)$

5

JEE 1

$T_n = \tan^{-1}(n+1)(n+2) - \tan^{-1} n(n+1)$

$S_n = \sum_{n=1}^{\infty} T_n = \tan^{-1}(2\times3) - \tan^{-1} 1\times2$

$+ \tan^{-1}(3\times4) - \tan^{-1}(2\times3)$

$= \tan^{-1}(n+1)(n+2) -$

$\Rightarrow \tan^{-1}(n+1)(n+2) - \tan^{-1} 2$

$S_\infty = \tan^{-1}\infty - \tan^{-1} 2$

$= \frac{\pi}{2} - \tan^{-1} 2$

$= \cot^{-1} 2$

$= \tan^{-1}\frac{1}{2}.$

$= \tan^{-1}(0.5)$

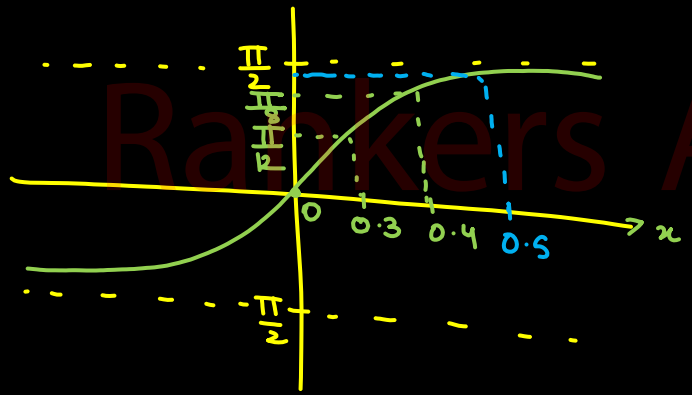
$> \tan^{-1}(\sqrt{2}-1)$

$> \tan^{-1}(2-\sqrt{3})$

5

JEE 1

$y = \tan^{-1} x$



$\tan^{-1} x$ ↑

6

A real valued function f satisfying the equation $f(x) + 2f(-x) = x^2 - x$, then sum of real roots of equation $f(f(x)) = 0$ is -

(A) -3 (B) -6

(C) -9 (D) -10

$f(x) + 2f(-x) = x^2 - x$ — (1)

$x \to -x$

$f(-x) + 2f(x) = (-x)^2 - (-x)$

$2f(-x) + 4f(x) = 2x^2 + 2x$ — (2)

(2) − (1)

$3f(x) = x^2 + 3x$

$\boxed{f(x) = \frac{x^2}{3} + x}$

$f(f(x)) = \frac{(f(x))^2}{3} + f(x)$

$= \frac{\left(\frac{x^2}{3} + x\right)^2}{3} + \left(\frac{x^2}{3} + x\right)$

$0 = \left(\frac{x^2}{3} + x\right)\left(\frac{\frac{x^2}{3} + x}{3} + 1\right)$

$0 = x\left(\frac{x}{3} + 1\right)\left(\frac{x^2 + 3x + 9}{9}\right)$

$x = 0, -3,$ 2 complex roots

biquad $\begin{cases} 0 \\ -3 \end{cases}$ real

2 complex roots

7

If $I = \int_{\sqrt[3]{\ln 4}}^{\sqrt[3]{\ln 5}} \frac{x^2 \tan^{-1}(x^3)}{\tan^{-1}(x^3)+\tan^{-1}(\ln 20 - x^3)}\,dx$, then

$(4e^{6I})$ is

(A) 3 (B) 4

(C) 5 (D) 1

JEE 1

Let $x^3 = t$

$3x^2 dx = dt$

$$I = \frac{1}{3}\int_{\ln 4}^{\ln 5} \frac{\tan^{-1} t}{\tan^{-1} t + \tan^{-1}(\ln 20 - t)}\,dt \quad \text{—— (1)}$$

Using $t \to \ln 4 + \ln 5 - t$ or $\ln 20 - t$

$$I = \frac{1}{3}\int \frac{\tan^{-1}(\ln 20 - t)}{\tan^{-1}(\ln 20 - t) + \tan^{-1}(\ln 20 - (\ln 20 - t))}\,dt \quad \text{(2)}$$

(1) + (2)

$$2I = \frac{1}{3}\int_{\ln 4}^{\ln 5} \frac{\tan^{-1} t + \tan^{-1}(\ln 20 - t)}{\tan^{-1} t + \tan^{-1}(\ln 20 - t)}\,dt$$

$$2I = \frac{1}{3}(\ln 5 - \ln 4)$$

$$6I = \ln(5/4)$$

$$e^{6I} = 5/4$$

$$4e^{6I} = 5.$$

8

Total number of five digit numbers that can be formed having the property that every succeeding digit is greater than the preceding digit, is equal to -

(A) $^{10}C_5$

(B) $^{10}C_4$

(C) $^{9}C_4$

(D) $^{9}P_4$

$x_1 < x_2 < x_3 < x_4 < x_5$

$x_i \in \{1, \ldots 9\}$

$^{9}C_4 \times 1$

9

The line $4x - 3y + 15 = 0$ intersect the circle $x^2 + y^2 - 6x - 8y = 0$ at two points A and B. Maximum area of $\triangle ABC$, where C is a point on circumference of circle, will be -

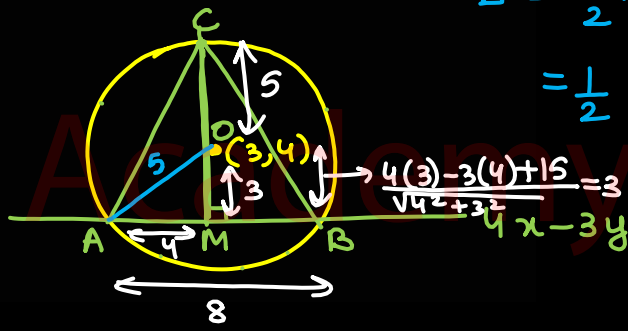
(A) 32 sq. units

(B) 16 sq. units

(C) 64 sq. units

(D) 8 sq. units

isos $\triangle$



$\Delta = \frac{1}{2} \times AB \times CM$

$= \frac{1}{2} (8)(8)$

$\frac{4(3)-3(4)+15}{\sqrt{4^2+3^2}} = 3$

$4x - 3y + 15 = 0$

JEE 1

10

The equation $x^2 + bx + c = 0$ has distinct roots. If 3 is subtracted from each root, the result are reciprocal of the original roots. Then the value of $(b^2 + 6c + bc)$, is

(A) 3 (B) 6

(C) 2 (D) 4

JEE 1

$x^2 + bx + c$ < α, β

$\alpha - 3 = \frac{1}{\alpha}$

$\beta - 3 = \frac{1}{\beta}$

$\alpha^2 - 3\alpha - 1 = 0$

$\beta^2 - 3\beta - 1 = 0$

$x^2 - 3x - 1 = 0$ < α, β

$b = -3$

$c = -1$

$b^2 + 6c + bc$

$9 - 6 + 3 = 6$

11

The value of $4\left(\lim_{x\to 0}\frac{\int_0^{\tan^{-1}x} t\tan^2 t\,dt}{\int_1^{\sec x}(t^2-1)dt}\right)$ is

(A) 4 ✓ (B) 8

(C) 2 (D) 1

JEE 1

$$\Rightarrow 4\lim_{x\to 0}\frac{(\tan^{-1}x)\left(\tan^2(\tan^{-1}x)\right)\cdot\frac{1}{1+x^2}}{(\sec^2 x-1)\sec x\tan x}$$

$$= 4\lim_{x\to 0}\frac{(\tan^{-1}x)(x^2)}{\underbrace{(1+x^2)}_{=1}(\tan^2 x)\underbrace{(\sec x}_{=1}\tan x)}$$

$$= 4\lim_{x\to 0}\frac{(\tan^{-1}x)(x^2)}{\tan^3 x}$$

$$= 4\cdot\frac{x\cdot x^2}{x^3} = 1$$

$$= 4.$$

12

JEE 1

Let R be a relation on $Z \times Z$ defined by $(a, b)R(c, d)$ if and only if $ad - bc$ is divisible by 5 . Then R is

(A) Reflexive and symmetric but not transitive

(B) Reflexive but neither symmetric not transitive

(C) Reflexive, symmetric and transitive

(D) Reflexive and transitive but not symmetric

Sym: $(a,b)\,R\,(c,d) \Rightarrow ad - bc = 5\lambda$

$(c,d)\,R\,(a,b) \Rightarrow cb - ad = -5\lambda = 5(-\lambda)$ ✓

Trans: (a,b) $(3,1)$ (c,d) $(5,10)$ $\Rightarrow (3)(10) - (1)(5) = 25$ ✓

(c,d) $(5,10)$ (e,f) $(1,3)$ $\Rightarrow (5)(3) - (10)(1) = 5$ ✓

$(a,b)\,R\,(e,f)$

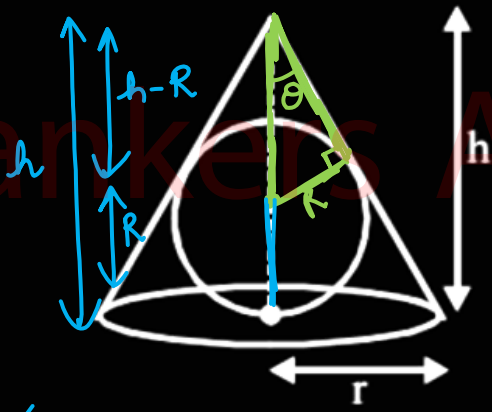
$af - be$

$3f - e$

$3(3) - 1 = 8$

13

A variable cone is such that ratio of radius of its base & height is $\frac{3}{4}$ at any instant. A variable sphere is inscribed in the cone in such a way that it always touches cone internally (as shown in figure). If at any instant the rate of change of height of cone is 8cm/sec, then the rate of change of radius of sphere at same instant is-



(A) 3 cm/sec

(B) 4 cm/sec

(C) $\frac{5}{2}$ cm/sec

(D) 6 cm/sec

$\frac{r}{h} = \frac{3}{4} = \tan\theta$

$\frac{dh}{dt} = 8$

$\frac{dR}{dt} = ?$

$\sin\theta = \frac{R}{h-R} = \frac{3}{5}$

$3h = 8R$

$R = \frac{3h}{8}$

$\frac{dR}{dt} = \frac{3}{8}\frac{dh}{dt} = \frac{3}{8}(8) = 3 \text{ cm/s}$

14

JEE 1

If A is a matrix of order 3 such that $|A| = 5$ and $B = \text{adj } A$, then the value of $||A^{-1}|.(AB)^T|$ is equal to (where $|A|$ denotes determinant of matrix A, A^T denotes transpose of matrix A, A^{-1} denotes inverse of matrix A, adjA denotes adjoint of matrix A)-

(A) 5 (B) 1

(C) 25 (D) $\frac{1}{25}$

order do, baahar ja.

formula

$|A^{-1}| = \frac{1}{|A|} = \frac{1}{5}$ ✓

$(AB)^T = B^T A^T$

$|AB| = |A||B|$

$|B| = |B^T|$

$|\text{adj}A| = |A|^{(n-1)}$

$\left|\frac{1}{5}(B^T A^T)\right| = \frac{1}{5^3}|B^T A^T|$

$= \frac{1}{5^3}|B^T||A^T|$

$= \frac{1}{5^3}|B||A|$

$= \frac{1}{5^3}\cdot|\text{adj}A|\cdot 5$

$= \frac{1}{5^2}|\text{adj}A|$

$= \frac{1}{5^2}|A|^{3-1}$

15

If $S = \{z \in C: |z - i| = |z + i| = |z - 1|\}$, then,

n(S) is:

$|z-(-i)|$

(A) 1 ✓

(B) 0

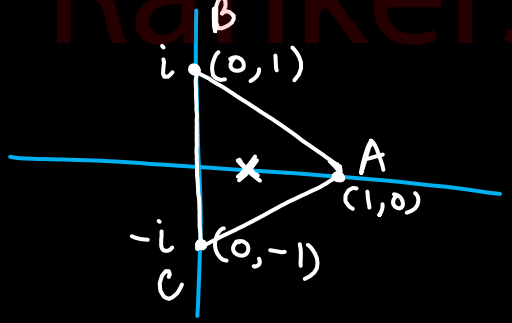
(C) 3

(D) 2

JEE 1

$|z - z_1| =$ dist b/w z & z_1

z_1 —— z



=

Circumcentre

=

16

The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(A) 406 (B) 130

(C) 142 (D) 136

$n = 15$, $r = 3$

$^{n+r-1}C_{r-1}$

$^{17}C_2$

$\frac{17 \times 16^8}{2} = 136$

17

The integral curve through origin satisfying the differential equation

$y^2(3x^9 + y)\frac{dy}{dx} = x^3(1 - 9x^5y^3)$ is

(A) $3(x^3y)^2 + y^3 - x^3 = 0$

(B) $4(x^3y)^3 + y^4 - x^4 = 0$

(C) $4(x^3y)^3 - y^4 + x^4 = 0$

(D) $3(x^3y)^2 - y^3 + x^3 = 0$

JEE 1

$x^9y^3 + \frac{y^4}{4} - \frac{x^4}{4} = C$

$C = 0$

$3x^9y^2dy + y^3dy = x^3dx - 9x^8y^3dx$

$(3x^9y^2dy + 9x^8y^3dx) + y^3dy - x^3dx = 0$

$\int d(x^9 \cdot y^3) + \int y^3dy - \int x^3dx = 0$

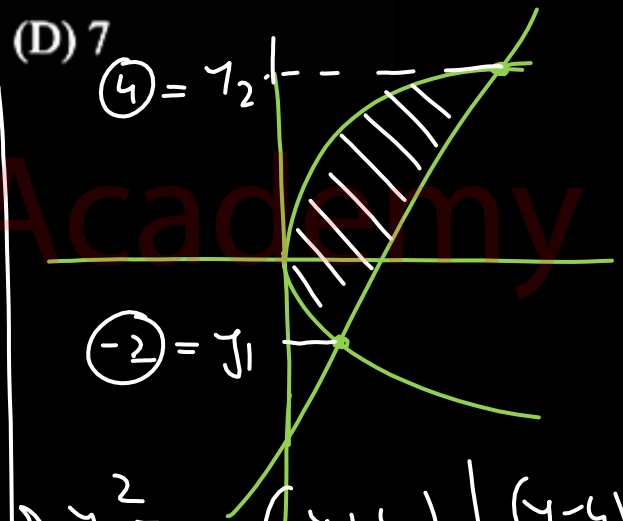
18

The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8 = 2x - 4 - 4 = 2(x-2) - 4$

(A) 8 (B) 9

(C) 6 (D) 7

$$\begin{cases} y^2 = 4(x-2) \\ y = 2(x-2) - 4 \end{cases}$$



$$\begin{cases} y^2 = 4X \\ y = 2X - 4 \end{cases} \longrightarrow y^2 = 2(y+4)$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0$$

$$y = 4, -2$$

18

$$\int_{-2}^{4} \left[\left(\frac{y+4}{2} \right) - \frac{y^2}{4} \right] dy$$

$$= 9$$

JEE 1

19

The line $L_1: 3x + 4y + 12 = 0$ is rotated through an angle of $\tan^{-1}\left(\frac{24}{7}\right)$ in anticlockwise about the point where it intersects the x-axis, line obtained in new position is denoted as $L_2 = 0$. If incentre of the triangle formed by $L_1 = 0, L_2 = 0$ and y-axis is $(4\lambda, 3k)$, then the value of $10k - 24\lambda$ is -

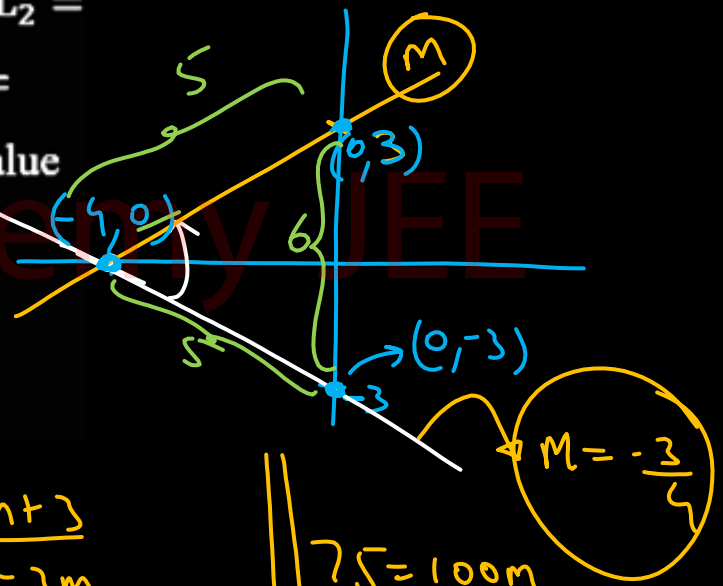
(A) -3

(B) 3

(C) 9

(D) -9

$I = \left(\frac{-24}{16}, 0\right)$



$\frac{24}{7} = \frac{m - (-3/4)}{1 + (m)(-3/4)}$

$\frac{24}{7} = \frac{4m+3}{4-3m}$

$96 - 72m = 28m + 21$

$75 = 100m$

$m = 3/4$

19

JEE 1

$$I \equiv \left(-\frac{3}{2}, 0\right) \equiv (4\lambda, 3K)$$

$$\begin{cases} \lambda = -\frac{3}{8} \\ K = 0 \end{cases}$$

$$10K - 24\lambda$$

$$0 - 24\left(\frac{-3}{8}\right) = 9$$

20

The number of point(s) whose perpendicular distances from yz, zx and xy planes are in A.P. and whose distances from x, y, z axes are $\sqrt{25}, \sqrt{20}, \sqrt{13}$ respectively, is-

(A) 8

(B) 16

(C) 4

(D) infinitely many points

JEE 1

$P \equiv (\alpha, \beta, \gamma)$

$|\alpha|, |\beta|, |\gamma| \rightarrow$ A.P.

$\sqrt{\beta^2+\gamma^2} = \sqrt{25}$

$\sqrt{\alpha^2+\gamma^2} = \sqrt{20}$

$\sqrt{\alpha^2+\beta^2} = \sqrt{13}$

$\beta^2+\gamma^2 = 25$

$\alpha^2+\gamma^2 = 20$

$\alpha^2+\beta^2 = 13$

$\rightarrow 2(\alpha^2+\beta^2+\gamma^2) = 58$

$\alpha^2+\beta^2+\gamma^2 = 29$

$\Rightarrow \alpha^2 = 4;\ \beta^2 = 9;\ \gamma^2 = 16$

$\alpha = \pm 2,\ \beta = \pm 3,\ \gamma = \pm 4$

21

If $f(x) = x^2 \ell n x$, where $f: [1, e] \to R$ and the maximum value of $f(x)$ is M, then $\ell n M$ is

$f'(x) = 2x \ln x + x$

$= x(2\ln x + 1)$

for : $x \in [1, e]$

$f'(x) > 0$

$\Rightarrow f(x) \uparrow ; x \in [1, e]$

$\therefore f(x)_{max} = e^2 \ln e$

$M = e^2$

$\ln M = 2$

22

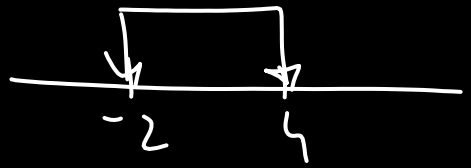
Let $A = \{x: |x-1| < 3\}$ and $B = \{x: x^2 - 2ax + a^2 - 4 \geq 0\}$ be two sets. If $A \cap B = \{x: -2 < x \leq 1\}$, then the smallest positive integral value of 'a' is

JEE 1

$A \equiv |x-1| < 3$

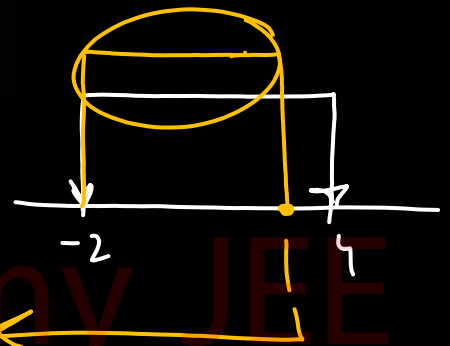
$-3 < (x-1) < 3$

$\boxed{-2 < x < 4}$



$A \cap B \equiv$

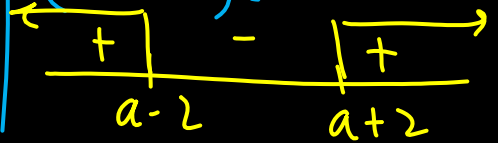
$-2 < x \leq 1$



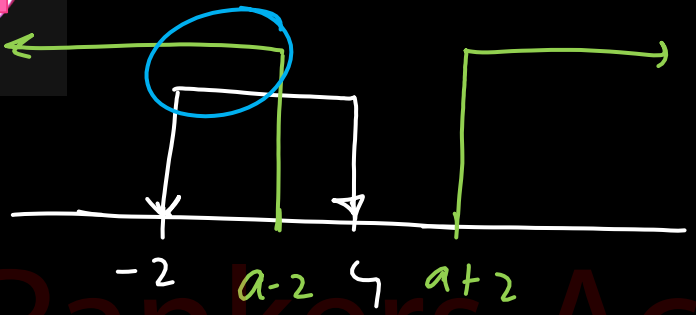
Now: $B \equiv$

$(x-a)^2 - 2^2 \geq 0$

$(x-a-2)(x-a+2) \geq 0$



22



$a-2=1$

$a=3$

23

If the coefficients of x^8 in $(x + \lambda x^2)^5$ and x^{11} in $(\mu x^3 - x^2)^4$ are equal, then the value of $\left|\frac{2\mu^3}{\lambda^3}\right|$ is → 5

JEE 1

$^5C_r\, x^{5-r} (\lambda x^2)^r$

$\left[{}^5C_r \lambda^r\right] x^{5+r}$ → 8

$r = 3$

$^5C_3 \lambda^3$

$10\lambda^3$

$^4C_r (\mu x^3)^{4-r} (-x^2)^r$

$\left[{}^4C_r \mu^{4-r} (-1)^r\right] x^{12-r}$ → 11

$r = 1$

$^4C_1 \mu^3 (-1)^1$

$-4\mu^3$

23

JEE 1

$$10\lambda^3 = -4\mu^3$$

$$5\lambda^3 = -2\mu^3$$

$$|s| = \left|\frac{-2\mu^3}{\lambda^3}\right|$$

✓

24

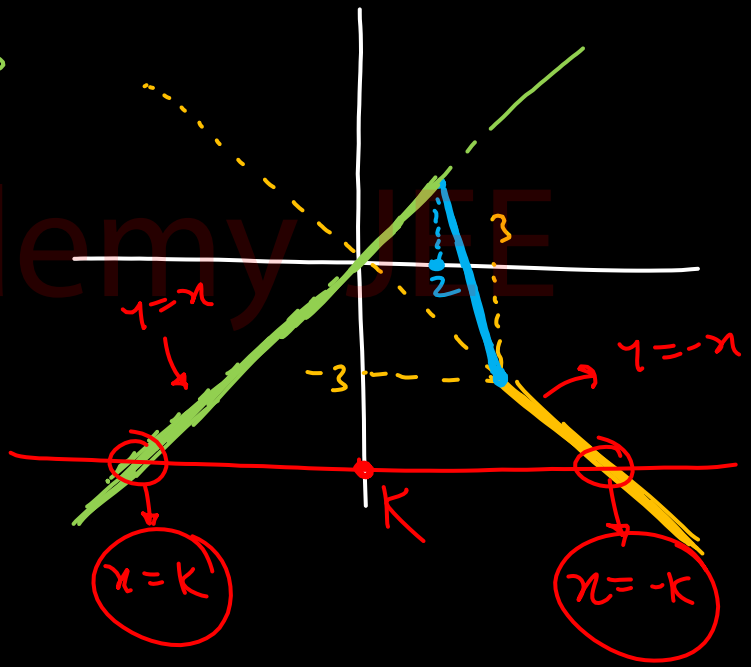
Sum of roots of $2|x-3|-3|x-2|=k$, where $k<-3$, is

JEE 1

$y = LHS$

$y = RHS$

	2	3	
	$-2x+6$	$-3x+6$	LHS
$-2x+6$			$= 2x-6-3x+6$
$3x-6$			
$= x$		$12-5x$	$= -x$



Sum = zero

25

$2^{10}\prod_{r=1}^{9}\cos\frac{r\pi}{19}$ is equal to

$$(2^{10})\left(\cos\frac{\pi}{19}\right)\left(\cos\frac{2\pi}{19}\right)\cdots\cdots\cos\left(\frac{9\pi}{19}\right)$$

$$2\left(2\cos\frac{\pi}{19}\right)\left(2\cos\frac{2\pi}{19}\right)\cdots\left(2\cos\frac{9\pi}{19}\right)$$

$$2\,\frac{\sin\frac{2\pi}{19}}{\sin\frac{\pi}{19}}\times\frac{\sin\frac{4\pi}{19}}{\sin\frac{2\pi}{19}}\times\frac{\sin\frac{6\pi}{19}}{\sin\frac{3\pi}{19}}\times\cdots\times\frac{\sin\frac{18\pi}{19}}{\sin\frac{9\pi}{19}}$$

25

JEE 1

$$2\,\frac{\sin\frac{10\pi}{19}\cdot\sin\frac{12\pi}{19}\cdot\sin\frac{14\pi}{19}\cdot\sin\frac{16\pi}{19}\cdot\sin\frac{18\pi}{19}}{\sin\frac{\pi}{19}\cdot\sin\frac{3\pi}{19}\cdot\sin\frac{5\pi}{19}\cdot\sin\frac{7\pi}{19}\cdot\sin\frac{9\pi}{19}}$$

$$= \boxed{2}$$