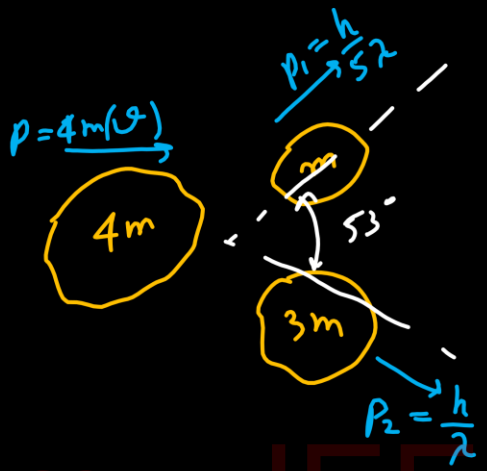

PHYSICS
Rankers Academy JEE

7

Find fraction of increase in K.E when an object in motion undergoes sudden explosion and breaks into two parts such that their de-broglie wave-lengths are in ratio 5: 1. The direction of motion of parts make an angle of 53° with each other and mass of the part with lower wave-length is thrice that of one with higher wavelength

(A) $\frac{1}{6}$ (B) $\frac{5}{3}$

(C) $\frac{2}{3}$ (D) $\frac{7}{6}$



$P_{initial} = P_{final}$

$(4m)v = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 53°}$

$\Rightarrow 4mv = \frac{h}{\lambda}\sqrt{\frac{1}{5^2} + \frac{1}{1^2} + 2\frac{1}{5}\times\frac{3}{5}}$

$\Rightarrow 4mv = \frac{h}{5\lambda}\sqrt{32}\sqrt{\lambda}$ — (1)

7

$$K_i = \frac{1}{2}(4m)v^2 = 2mv^2 \quad \text{—②}$$

$$K_f = \frac{p_1^2}{2m} + \frac{p_2^2}{2(3m)}$$

$$= \frac{\left(\frac{h}{5\lambda}\right)^2}{2m} + \frac{\left(\frac{h}{\lambda}\right)^2}{6m}$$

$$K_f = \frac{(3+25)}{150}\frac{h^2}{m\lambda^2}$$

$$= \frac{14h^2}{75m\lambda^2} \quad \text{—③}$$

$$\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i}$$

$$= \frac{K_f}{K_i} - 1$$

$$= \frac{\frac{14h^2}{75m\lambda^2}}{2mv^2} - 1$$

$$= \frac{7}{75}\left(\frac{h^2}{m^2v^2\lambda^2}\right) - 1$$

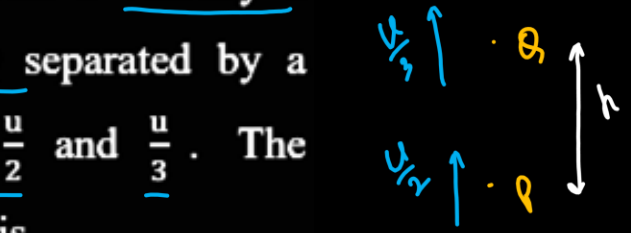
$$= \frac{7}{\cancel{75}_3}\left(\frac{\cancel{25}}{2}\right) - 1 = \frac{7}{6} - 1 = \frac{1}{6}$$

2

A stone projected upwards with a velocity u reaches two points P and Q separated by a distance 'h' with velocities $\frac{u}{2}$ and $\frac{u}{3}$. The maximum height reached by it is

(A) $\frac{9h}{5}$ (B) $\frac{18h}{5}$

(C) $\frac{36h}{5}$ (D) $\frac{72h}{5}$



P to Q $v^2 = u^2 + 2as$

$\left(\frac{u}{3}\right)^2 = \left(\frac{u}{2}\right)^2 - 2gh$

$\frac{u^2}{9} = \frac{u^2}{4} - 2gh$

$2gh = \frac{5u^2}{36}$

$h = \frac{5}{36}\left(\frac{u^2}{2g}\right)$

$= \frac{5}{36} h_{max}$

$h_{max} = \frac{36h}{5}$

$\frac{u^2}{2g} = h_{max} = ?$

3

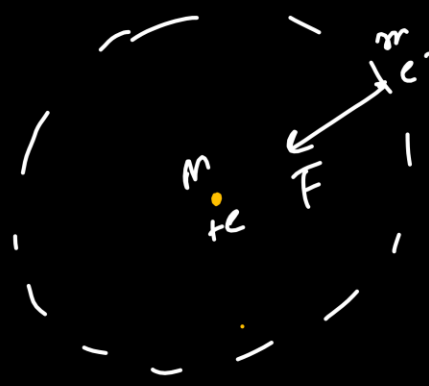
If we assume only gravitational attraction between proton (mass M) and electron (mass m) in a hydrogen atom and also assume Bohr's quantization condition, then the expression for the n^{th} orbit energy of the H -atom will be

(A) $-\frac{(GMm)^2}{n^2h^2}$

(B) $-\frac{2\pi^2\, m(GMm)^2}{n^2\, h^2}$

(C) $-\frac{2\pi^2(GMm)^2}{n^2\, h^2}$

(D) none of the above



JEE 1

$F = \frac{mv^2}{r}$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$\frac{GMm}{r} = mv^2$ — (2)

$\frac{GMm}{2r} = \frac{mv^2}{2}$ — (1)

$E = U + K$

$= -\frac{GMm}{r} + \frac{GMm}{2r}$

$E = -\frac{GMm}{2r} = -\frac{GMm}{2\left(\frac{n^2h^2}{4\pi^2}\frac{GM}{m^2}\right)}$

$= -2\pi^2 m\frac{(GMm)^2}{(nh)^2}$

$mvr = \frac{nh}{2\pi}$ — (1)

$r = \frac{n^2h^2}{4\pi^2 GMm^2}$

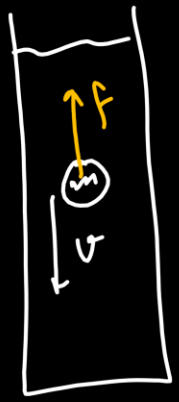
$m\sqrt{\frac{GM}{r}}\, r = \frac{nh}{2\pi}$

4

A solid ball of volume V experiences a viscous force F when falling with a speed v in a liquid. If another ball of volume 8 V with the same velocity v is allowed to fall in the same liquid, it experiences a force

(A) F (B) 16 F

(C) 4 F (D) 2 F



$$V = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$F = 6\pi\eta r v$$

$$F = 6\pi\eta \sqrt[3]{\frac{3V}{4\pi}}\, v \quad \text{— (1)}$$

$$F' = 6\pi\eta \sqrt[3]{\frac{3(8V)}{4\pi}}\, v$$

$$F' = \sqrt[3]{8}\, F = 2F$$

5

Calculate the magnetic moment associated with the loop carrying current I_0 as shown in figure.

$\vec{m} = Ni\vec{A}$

$m = iA$

$= I_0\left(\frac{\sqrt{3}l^2}{4}\right)$

$= I_0\frac{\sqrt{3}}{4}(\sqrt{2}a)^2$

$= \frac{\sqrt{3}I_0a^2}{2}$

(A) $\frac{3\sqrt{3}}{2}I_0a^2$

(B) $\frac{2\sqrt{3}}{2}I_0a^2$

(C) $\frac{2}{3}I_0a^2$

(D) $\frac{\sqrt{3}}{2}I_0a^2$

6

A square plate of edge $\frac{a}{2}$ is cut out from a uniform square plate of edge a, as shown in the figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through O (centre of the square of side a) and perpendicular to plane of the plate is

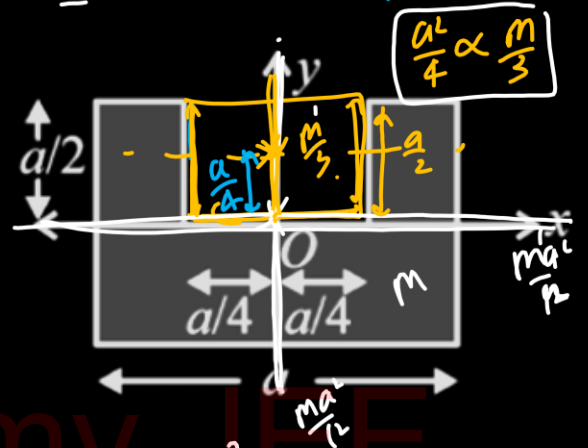
(A) $\frac{9}{64}Ma^2$ (B) $\frac{3}{16}Ma^2$

(C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

$m \propto \left(a^2 - \left(\frac{a}{2}\right)^2\right) = \frac{3a^2}{4}$

JEE 1

$\frac{a^2}{4} \propto \frac{m}{3}$



$I_{total} = \frac{m'a^2}{6} = \left(m + \frac{m}{3}\right)\frac{a^2}{6}$

$= \frac{4ma^2}{18}$ — ①

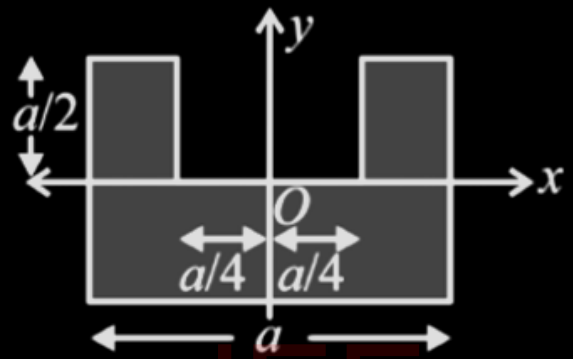
$I_{Remain} = I_{total} - I_{cavity}$ *

$I_{cavity} = \frac{\frac{m}{3}\left(\frac{a}{2}\right)^2}{6} + \frac{m}{3}\left(\frac{a}{4}\right)^2 = \frac{ma^2}{12}\left[\frac{1}{6} + \frac{1}{4}\right]$

$= \frac{5ma^2}{144}$ — ①

6

A square plate of edge $\frac{a}{2}$ is cut out from a uniform square plate of edge a, as shown in the figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through O (centre of the square of side a) and perpendicular to plane of the plate is



(A) $\frac{9}{64}Ma^2$

(B) $\frac{3}{16}Ma^2$

(C) $\frac{5}{12}Ma^2$

(D) $\frac{Ma^2}{6}$

$I_{xmaij} = I_{total} - I_{cavity}$

$= \frac{4ma^2}{18} - \frac{5ma^2}{144}$

$= \left[\frac{2\times 18 - 5}{144}\right] ma^2 = \frac{3}{16}ma^2$

7

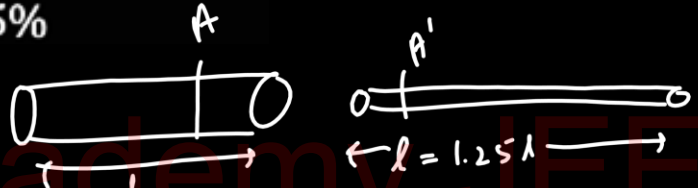
A wire of 1Ω has length of 1 m . It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is

(A) 76%
(B) 56%
(C) 12.5%
(D) 25%

R_1

$\frac{\Delta l}{l}$

% change $= 2\frac{\Delta l}{l}$ ✗ if $\Delta l \ll l$



$V = Al = A'l' = A'(1.25l)$

$A' = \frac{A}{1.25}$ — ①

$R' = \frac{\rho l'}{A'} = \frac{\rho (1.25l)}{\left(\frac{A}{1.25}\right)}$

$R' = (1.25)^2 R$

$\frac{\Delta R}{R} = (1.25)^2 - 1 = 0.5625 \times 100 = 56.25\%$

8

In the determination of refractive index of material of a parallel sided slab using a travelling microscope the following observations are made. Given least count of microscope is 0.001 cm . Then the value of refractive index of material of slab is

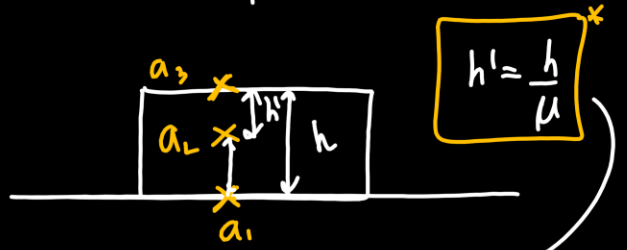
Reading of the microscope when focused on								
Mark Made on paper			Mark on paper Through the slab			Particles on top of the glass surface		
M.S.R. M (cm)	V.S.R. N (cm)	a_1=M+N×L.C. (cm)	M.S.R. M (cm)	V.S.R. N (cm)	a_2=M+N×L.C. (cm)	M.S.R. M (cm)	V.S.R. N (cm)	a_3=M+N×L.C. (cm)
3.10	19	3.119 cm	3.20	15	3.215 cm	3.30	10	3.310 cm

(A) 2.01

(B) 1.50

(C) 1.25

(D) 2.25



$h' = \frac{h}{\mu}$

$\mu = \frac{h}{h'}$

$\mu = \frac{a_3 - a_1}{a_3 - a_2}$

$= \frac{3.310 - 3.119}{3.310 - 3.215}$

$= \frac{191}{95} \approx 2.01$

9

The root mean square speed of the molecules of a diatomic gas is v. When the temperature is doubled, the molecules dissociate into two atoms. The new root mean square speed of the individual atom is

(A) $\sqrt{2}v$ (B) v

(C) 2v (D) 4v

$v = \sqrt{\frac{3RT}{m_0}}$ → $2T$, m_0 → $\left(\frac{m_0}{2}\right)$ Dissociates

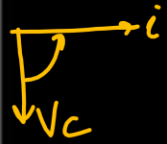
$v' = \sqrt{\frac{3R(2T)}{\frac{m_0}{2}}} = 2v$

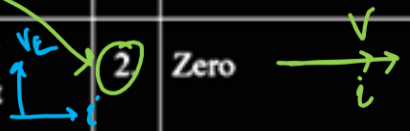
10

JEE 1

Match the List-I with the List-II.

List-I		List-II	
A.	Phase difference between current and voltage in a purely resistive AC circuit	1.	$\frac{\pi}{2}$; current leads voltage
B.	Phase difference between current and voltage in a pure inductive AC circuit	2.	Zero
C.	Phase difference between current and voltage in a pure capacitive AC circuit	3.	$\frac{\pi}{2}$; current lags voltage
D.	Phase difference between current and voltage in an LCR series circuit	4.	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$





Choose the most appropriate answer from the options given below:

(A) A-2, B-3, C-1, D-4

(B) A-1, B-3, C-4, D-2

(C) A-2, B-3, C-4, D-1

(D) A-2, B-4, C-3, D-1

11

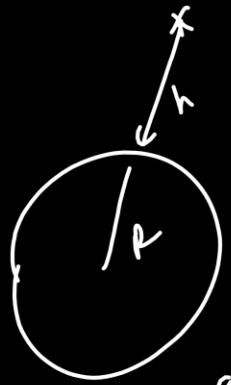
The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is

(A) 2R

(B) $\frac{R}{\sqrt{2}}$

(C) R/2

(D) $\sqrt{2}R$



JEE 1

$g' = \frac{GM}{(R+h)^2}$

$g' = \frac{GM}{R\left(1+\frac{h}{R}\right)^2}$

$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\frac{g}{9} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$3 = 1+\frac{h}{R} \Rightarrow \boxed{h = 2R}$

12

A 'beat frequency' of 17 Hz is observed when the fundamental mode of two hollow tubes is excited. The tubes differ in length but have similar construction with an open end and one closed end. The shorter tube length is 80 cm . What is the length of the longer tube?

(v = 340 m/s)

(A) 85 cm

(B) 90 cm

(C) 95 cm

(D) 100 cm

JEE 1

Closed organ pipe

$l = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow f = \frac{v}{4l}$ *

$\Delta f = f_1 - f_2$ beat

$17 = \frac{v}{4L_1} - \frac{v}{4L_2}$

$17 = \frac{v}{4}\left[\frac{1}{0.8} - \frac{1}{L_2}\right]$

$17 = \frac{340}{4}\left[\frac{10}{4} - \frac{1}{L_2}\right]$

$\frac{1}{5} = \frac{5}{2} - \frac{1}{L_2}$

$\frac{1}{L_2} = \frac{25-4}{20}$

$L_2 = \frac{20}{21} = 0.95\ m$

13

In Young's double slit experiment using monochromatic light, the fringe patterns shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima now is the same as the observed fringe shift upon the introduction of the mica sheet. The wavelength of light is

(A) 5762Å (B) 5825Å

(C) 5892Å (D) 6500Å

$\Delta = \frac{(\mu-1)tD}{d}$ — (1)

$\beta' = \frac{\lambda(2D)}{d}$ — (2)

$\beta' = \Delta$

$\frac{2\lambda D}{d} = \frac{(\mu-1)tD}{d}$

$\lambda = \frac{(\mu-1)t}{2}$

$= \frac{0.6 \times 1.964 \times 10^{-6}}{2}$

$= 1964 \times 3 \times 10^{-10}$

$= 5892\ \text{Å}$

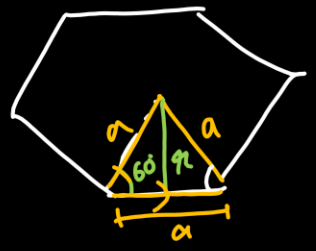
14

Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (ampere) in units of $\frac{\mu_0 I}{\pi}$ is:

(A) $250\sqrt{3}$ (B) $50\sqrt{3}$

(C) $500\sqrt{3}$ (D) $5\sqrt{3}$

JEE 1



$B_{hexa} = N6\, B_{side}$

$= N6\left[\frac{\mu_0 i}{4\pi r}(\cos\alpha + \cos\beta)\right]$

$= N^2 6 \frac{\mu_0 i}{4\pi\left(\frac{\sqrt{3}a}{2}\right)} \times (2\cos 60°)$

$= \frac{50\sqrt{3}}{a}\left(\frac{\mu_0 i}{\pi}\right)$

$= \frac{50\sqrt{3}}{0.1m}\left(\frac{\mu_0 i}{\pi}\right) = 500\sqrt{3}$

15

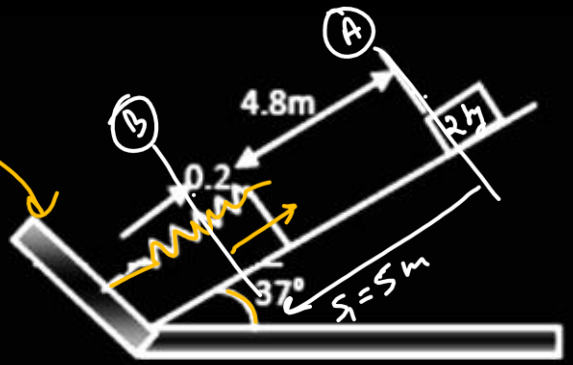
Figure shows a massless spring fixed at the bottom end of an inclined of inclination $37°(\tan 37° = 3/4)$. A small block of mass 2 kg start slipping down the incline from a point 4.8 m away from free end of spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance 1 m up the inclined, then
($g = 10\ m/s^2$)

(A) coefficient of friction between block and inclined is 0.5.

(B) coefficient of friction between block and inclined is 0.75.

(C) value of spring constant is 4000 N/m.

(D) value of spring constant is 2000 N/m.



JEE 1

$W_{all} = \Delta K$ b/w A & B

$$mg\,s_1\sin\theta - \mu mg\cos\theta\, s_1 - \frac{1}{2}kx^2 = 0 - 0$$

$$mg\,s_1(\sin\theta - \mu\cos\theta) = \frac{1}{2}kx^2$$

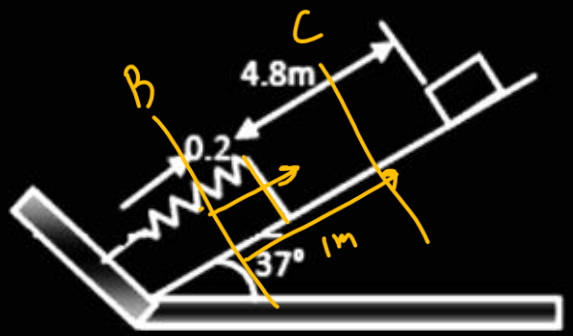
$$20\times 5\left(\frac{3}{5} - \frac{4\mu}{5}\right) = \frac{1}{2}kx^2 \quad \text{— (1)}$$

15

B to C

$\Delta W = \Delta KE$

$-mgS\sin\theta_2 - \mu mgS\cos\theta_2 + \frac{1}{2}kx^2 = 0$



$20\times 1\left(\frac{3}{5}+\mu\left(\frac{4}{5}\right)\right) = \frac{1}{2}kx^2$ — (2)

(1) = (2)

$20\left(\frac{3}{5}+\frac{4\mu}{5}\right) = \frac{1}{2}kx^2 = 20\times 5\left[\frac{3-4\mu}{5}\right]$

$20\left[\frac{3}{5}+\frac{4}{5}\times\frac{1}{2}\right] = \frac{1}{2}k\times(0.2)^2$

$\frac{40\times 100}{0.2\times 0.2} \quad k = 1000$

$12+16\mu = 60-80\mu$

$96\mu = 48$

$\mu = 0.5$

16

JEE 1

An ideal gas undergoes an isothermal process. The pressure (P) of the gas is plotted against the mean free path λ of the molecules. Select the correct graph.

$T = \text{const}$

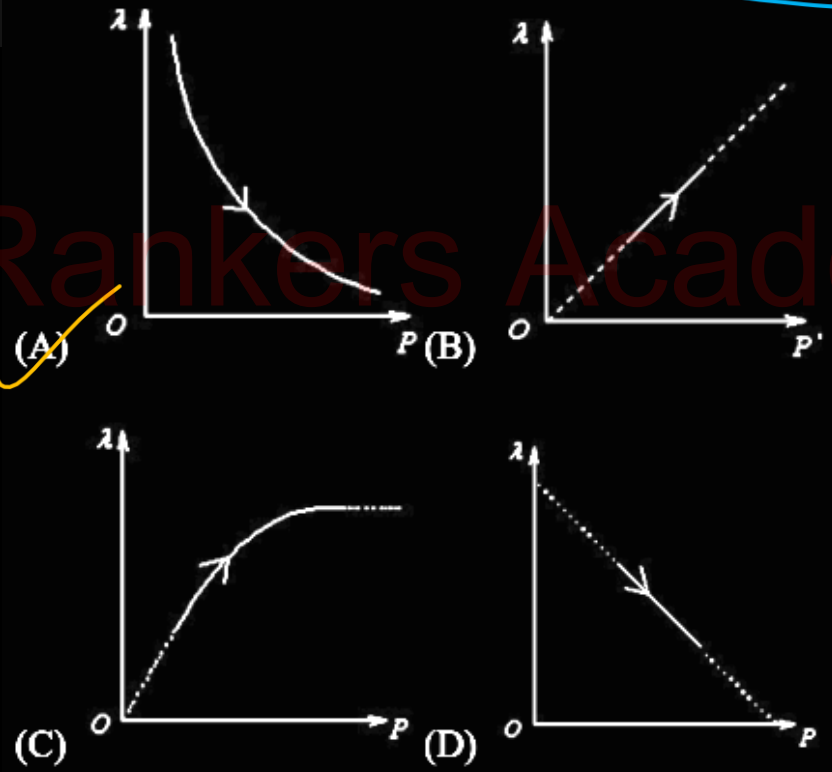
$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{kT}{\sqrt{2}\pi P d^2} \quad *$$

$\lambda P \propto T$

$\lambda P = \text{const}$

$$\lambda \propto \frac{1}{P}$$

Rat hyperbola



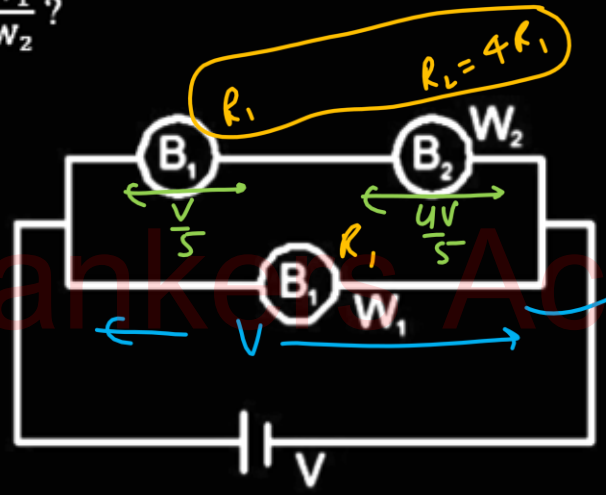
17

JEE 1

There are two bulbs $B_1(P, V), B_2(P, 2V)$ their rated power and voltages are mentioned with them. Calculate the ratio of consumed power $\frac{W_1}{W_2}$?

$R_1 = \frac{V^2}{P}$ — (1)

$R_2 = \frac{(2V)^2}{P} = 4R_1$ — (2)



$W_1 = \frac{V^2}{R_1} = \frac{V^2}{\frac{V^2}{P}} = P$

$W_2 = \frac{\left(\frac{4V}{5}\right)^2}{R_2} = \frac{\frac{16V^2}{25}}{\frac{4V^2}{P}} = \frac{4P}{25}$

$\frac{W_1}{W_2} = \frac{25}{4}$

(A) $\frac{25}{4}$

(B) $\frac{4}{25}$

(C) $\frac{10}{4}$

(D) $\frac{4}{10}$

18

A heavy nucleus X having mass number 200 gets disintegrated into two small fragments Y and Z of mass numbers 80 and 120 respectively. If binding energy per nucleon for the parent atom X is 6.5 MeV and for daughter nuclei Y and Z are 7 MeV and 8 MeV respectively. Energy released in the decay will be

(A) 200 MeV

(B) 240 MeV

(C) 220 MeV

(D) 180 MeV

JEE 1

$^{200}X \longrightarrow {}^{80}Y + {}^{120}Z$

$Q = B.E._P - B.E_R$ *

$= BE_Y + BE_Z - BE_X$

$= (80 \times 7) + (120 \times 8) - (200 \times 6.5)$

$= 560 + 960 - 1300$

$= 220$ MeV

19

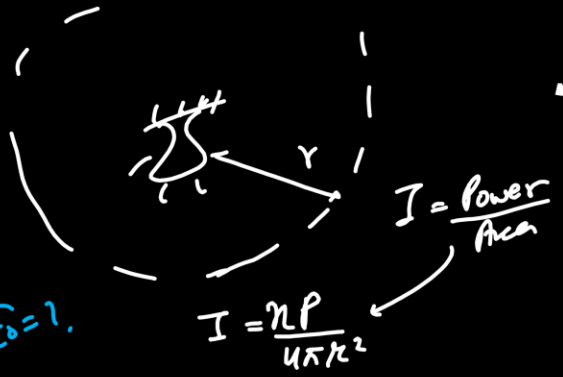
A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 10 m from the lamp will be

(A) 1.34 V/m

(B) 2.68 V/m

(C) 5.36 V/m

(D) 9.37 V/m



$E_0 = ?$

$$\left(\frac{1}{2}\varepsilon_0 E_0^2 c\right) = \frac{\eta P}{4\pi r^2}$$

$$E_0 = \sqrt{\frac{\eta P}{(2\pi\varepsilon_0)\, r^2 c}}$$

$$= \sqrt{\frac{0.03 \times 100\ \text{W} \times (9\times10^9 \times 2)}{10^2 \times 3\times10^8}}$$

$$= \frac{3}{\sqrt{5}} = \frac{3}{2.23} \simeq 1.34$$

20

STATEMENT-1 : Inductance plays same role in the electrical circuits as mass plays in the mechanical circuits.

STATEMENT-2 : Greater the value of inductance, harder it is to change the current in the circuit.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement- 1 is False, Statement- 2 is True

$U = \frac{1}{2}Li^2$ $\quad L \equiv m$

$K = \frac{1}{2}mv^2$ $\quad v \equiv i$

$\varepsilon = -L\frac{di}{dt}$ $\quad \frac{di}{dt} = \frac{\varepsilon}{L}$

$L \uparrow \Rightarrow \frac{di}{dt} \downarrow$

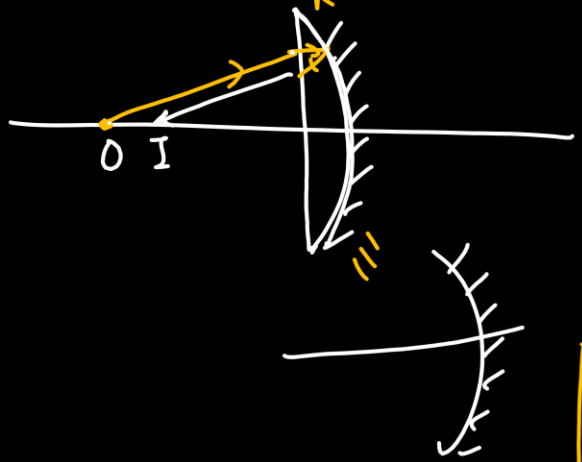
$F = -m\frac{dv}{dt}$ $\quad m \equiv L$ $\quad i \equiv v$

21

A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance in cm from this lens an object is placed in order to have a real image of the size of the object?

placed at C: inverted & real of same size

JEE 1



$P_{eq} = P_L + P_m + P_L$

$P_{eq} = 2P_L + P_m$

$\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$

$\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$ *

$\frac{1}{f_{eq}} = \frac{1}{\left(\frac{-30}{1.5-1}\right)} - \frac{2}{(1.5-1)}\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$\frac{1}{f_{eq}} = \frac{1}{-15} - \frac{1}{30}$

$\frac{1}{f_{eq}} = \frac{2+1}{-30}$

$f_{eq} = -10$ → concave mirror

$R_{eq} = -20\,cm$

C from P = 20 cm

22

JEE 1

A wire of length $l = 6 \pm 0.06$ cm and radius of cross-section $r = 0.5 \pm 0.005$ cm and mass $m = 0.3 \pm 0.003$gm . Maximum percentage error in density is _____ %.

$$\rho = \frac{m}{\text{Volume}} = \frac{m}{(\pi r^2 l)}$$

$$\frac{d\rho}{\rho} = \frac{dm}{m} + \frac{2dr}{r} + \frac{dl}{l}$$

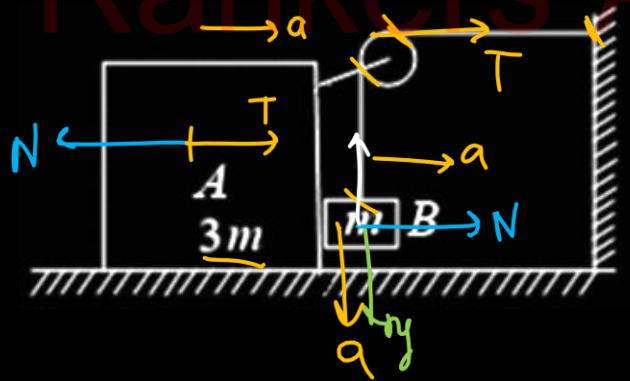
$$\frac{\Delta\rho}{\rho} \times 100\% = \left(\frac{\Delta m}{m} + 2\times\frac{\Delta r}{r} + \frac{\Delta l}{l}\right)\times 100$$

$$= \left(\frac{0.003}{0.3} + 2\times\frac{0.005}{0.5} + \frac{0.01}{6}\right)\times 100 = (0.9 + 0.02 + 0.9)\times 100$$

$$= 4\%.$$

23

In the system shown all the surfaces are frictionless while pulley and string are massless. Mass of block A is 3m and that of block B is m. If the acceleration of block B with respect to ground after system is released from rest is $\sqrt{n}$, m/s^2 then the value of ' n ' is (take $g = 10\ m/s^2$)



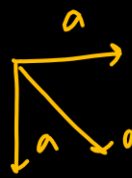
(B) $mg - T = ma$ — (1)

$N = ma$ — (2)

(A) $T - N = (3m)a$ — (3)

$mg = 5ma \Rightarrow a = \frac{g}{5}$

$a_{BA} = a\sqrt{2}$



$= \frac{g}{5}\sqrt{2}$

$= \frac{10}{5}\sqrt{2}$

$= 2\sqrt{2}$

$= \sqrt{8}$ Ans

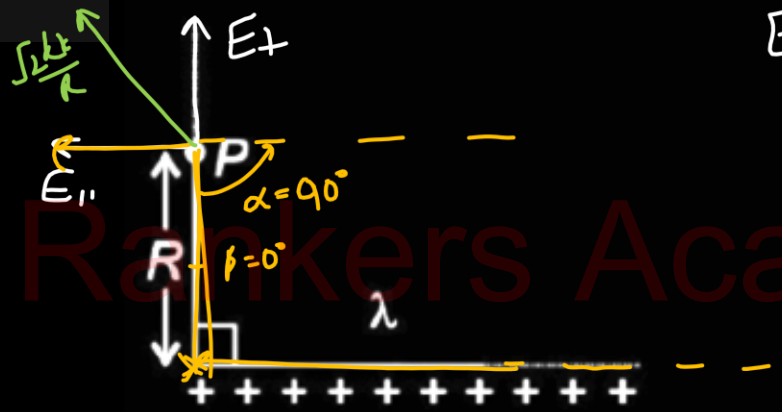
JEE 1

24

JEE 1

Electric field at point ' P ' due to long rod having uniform charge density, λ as shown is $\frac{\lambda}{\sqrt{n}\pi\varepsilon_0 R}$,

Find n.



$$E_\perp = \frac{k\lambda}{R}\left[\sin\alpha + \sin\beta\right] = \frac{k\lambda}{R}$$

$$E_{||} = \frac{k\lambda}{R}\left[\cos\beta - \cos\alpha\right] = \frac{k\lambda}{R}$$

$$E_{net} = \frac{\lambda}{\sqrt{n}\pi\varepsilon_0 R} = \frac{\sqrt{2}k\lambda}{R}$$

$$\sqrt{n} = 2\sqrt{2}$$

$$n = 8$$

25

When photon of energy 4.0 eV (E) strikes the surface of a metal A , the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wave length λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$eV. If the de-Broglie wave length of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is _____ eV.

ϕ_B

$K_{max} = E - \phi$

$T_A = 4 - \phi_A$ — (1)

$T_B = 4.5 - \phi_B$ — (2)

$T_B = T_A - 1.5$ — (3)

(4) (3)

$T_B = 0.5$ eV

$T_A = 2$ eV

$\phi_B = 4.5 - T_B$

$= 4$ eV

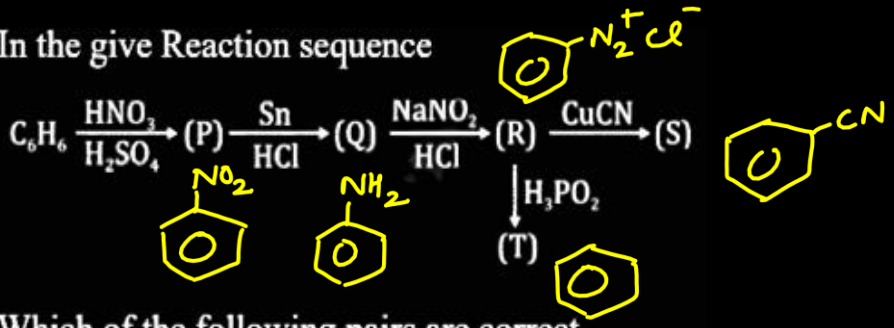
$\frac{h}{\sqrt{2mT_B}} = \frac{2h}{\sqrt{2mT_A}}$

$T_A = 4T_B$ — (4)

CHEMISTRY

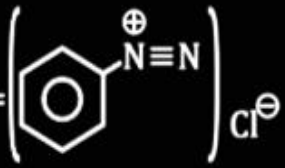
7

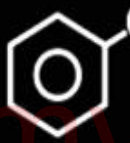
In the give Reaction sequence

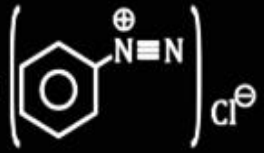


Which of the following pairs are correct

(A) (P) =  ; (S) = 

(B) (S) =  ; (R) = 

(C) (S) =  ; (T) =

(D) (T) =  ; (S) = 

2

$[Co(NH_3)_4(NO_2)_2]Cl$ exhibits

$-NO_2$
$-ONO$

(a) linkage isomerism, geometrical isomerism and optical isomerism

(b) linkage isomerism, ionization isomerism and optical isomerism

(c) linkage isomerism, ionization isomerism and geometrical isomerism

(d) ionization isomerism, geometrical isomerism and optical isomerism

3

Assume that the decomposition of HNO_3 can be represented by the following equation

$$4HNO_3(g) \rightleftharpoons 4NO_2(g) + 2H_2O(g) + O_2(g)$$

and that at a given temperature 400 K and pressure 30 atm the reaction approaches equilibrium. At equilibrium partial pressure of HNO_3 is 2 atm . Calculate K_C in $(mol/L)^3$ at 400 K. (Use R = 0.08 atm – L/mol. K)

(A) 28 (B) 35

(C) 30 (D) 32

JEE 1

$4HNO_3 \rightleftharpoons 4NO_2 + 2H_2O + O_2$

P_i 0 0 0

$P_i - 4x$ $4x$ $2x$ x

2 atm 16 8 atm 4 atm

$4x + 2x + x = 28$

$7x = 28$

$x = 4$

$K_P = \frac{[16]^4[8]^2[4]}{2^4} = \frac{(2)^{16}(2)^6(2)^2}{2^4}$

$K_P = 2^{20}$

$K_P = K_C (RT)^{\Delta n}$

$K_C = \frac{2^{20}}{(0.08 \times 400)^3} = \frac{2^{20}}{2^{15}} = 2^5 = 32$

4

Consider the following conversions :

(i) $O_{(g)} + e^- \rightarrow O^-_{(g)}, \Delta H_1$ exo

(ii) $F_{(g)} + e^- \rightarrow F^-_{(g)}, \Delta H_2$ exo

(iii) $Cl_{(g)} + e^- \rightarrow Cl^-_{(g)}, \Delta H_3$ exo

(iv) $O^-_{(g)} + e^- \rightarrow O^{2-}_{(g)}, \Delta H_4$ endo

That according to given information the incorrect statement is:

(A) ΔH_3 is more negative than ΔH_1 and ΔH_2

(B) ΔH_1 is less negative than ΔH_2

(C) $\Delta H_1, \Delta H_2$ and ΔH_3 are negative whereas ΔH_4 is positive

(D) ΔH_1 and ΔH_3 are negative whereas ΔH_2 and ΔH_4 are positive

5

Identify P and Q products in given reaction

$KI + MnO_4^- \xrightarrow{H^+} P + Mn^{2+}$ I_2

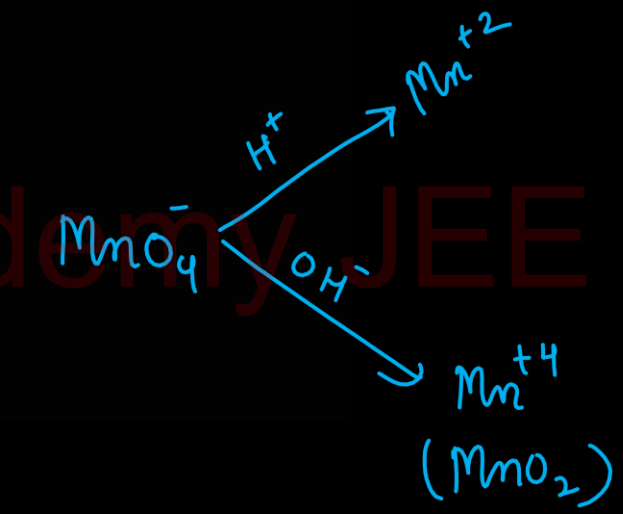
$KI + MnO_4^- \xrightarrow{OH^-} IO_3^- + Q$ = MnO_2

(A) IO_3^-, MnO_2 respectively

(B) I_2, Mn^{2+} respectively

(C) IO_3^-, Mn^{2+} respectively

(D) I_2, MnO_2 respectively



JEE 1

6

Given below are two statements:

Statement I: Vapour pressure of o-nitrophenol is low due to intra molecular hydrogen bonding.

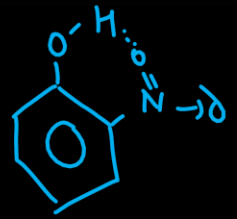
Statement II: O-nitrophenol is steam volatile.

(A) Statement I is true but Statement II is false

(B) Both Statement I and Statement II are true

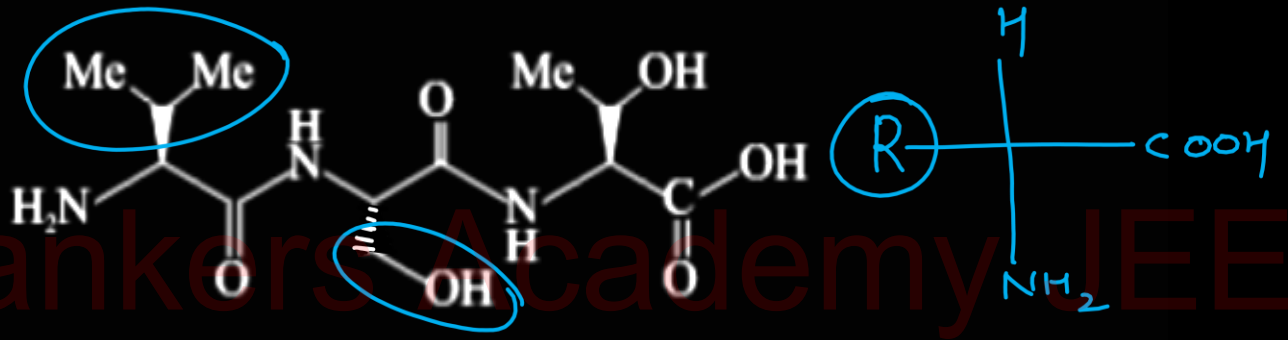
(C) Both Statement I and Statement II are false

(D) Statement I is false but Statement II is true



7

The correct sequence of amino acids present in the tripeptide given below is

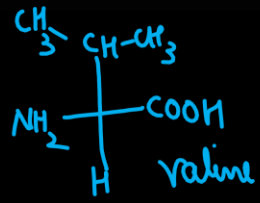


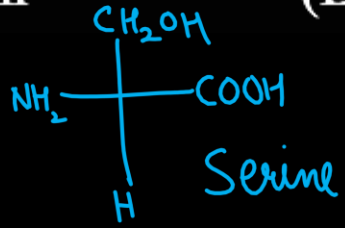
(A) Thr-Ser-Leu

(B) Leu-Ser-Thr

(C) Val-Ser-Thr

(D) Thr-Ser-Val





8

In a polythionic series $(H_2S_nO_6)$ if $[n = 4]$.

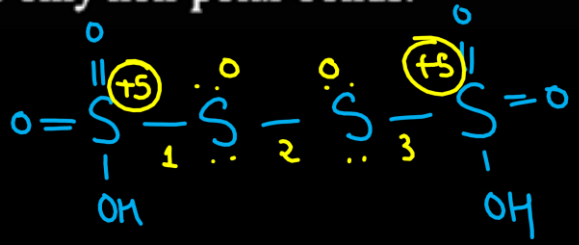
Which of the following option is incorrect?

(A) Absolute oxidation state of S is +5 as well as zero.

(B) Number of S-S linkage are three

(C) It has sp^3 sulphur atom.

(D) It has only non-polar bonds.



JEE 1

9

JEE 1

Which of the following statements about p-block elements are incorrect?

(I) Salts of hypohalous acids are alkaline in nature due to hydrolysis.

(II) BiF_3 is covalent while NF_3 and PF_3 are ionic in nature

Polar covalent

(III) Boiling point of water is higher that that of hydrogen fluoride

100°C

19.5°C

(IV) Stability of hydrides of carbon family increases down the group from C to Pb

(A) (II) and (IV) Only

(B) (I) and (III) Only

(C) (III) and (IV) Only

(D) All of these

10

If 20.0 g of $CaCO_3$ is treated with 20.0 g of HCl, how many grams of CO_2 can be produced according to the reaction:

$$CaCO_3(s) + 2HCl(aq) \longrightarrow CaCl_2(aq) + H_2O(l) + CO_2(g)$$

(A) 8.80 g (B) 7.70 g

(C) 8.00 g (D) 7.20 g

JEE 1

L.R E.R

$CaCO_3 + 2HCl \longrightarrow CaCl_2 + H_2O + CO_2$

20 g 20 g

0.2 mol 0.2 mol

0.2 mol of CO_2 = 0.2 × 44 g = 8.8 g of CO_2 will be formed

11

If last line of Lyman series of H -atom has wavelength λ_1 Å and 2nd line of Balmer series has wavelength λ_2 Å then :

(A) $\frac{16}{\lambda_1} = \frac{9}{\lambda_2}$

(B) $\frac{16}{\lambda_2} = \frac{3}{\lambda_1}$

(C) $\frac{4}{\lambda_1} = \frac{1}{\lambda_2}$

(D) $\frac{16}{\lambda_1} = \frac{3}{\lambda_2}$

JEE 1

Last line of lyman series

$\frac{1}{\lambda_1} = R$

2nd line of Balmer series

$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{16}\right] = \left(\frac{1}{\lambda_1}\right)\left(\frac{3}{16}\right) \Rightarrow \frac{16}{\lambda_2} = \frac{3}{\lambda_1}$

12

Find the solubility product of copper (II) iodate, $Cu(IO_3)_2$, by iodometric titration in an acidic solution. 26 ml of 0.1 M sodium thiosulphate solution is needed to titrate 20 ml of saturated aq. Solution $Cu(IO_3)_2$

(A) 4.0×10^{-6} (B) 1.0×10^{-6}

(C) 8.8×10^{-6} (D) 1.0×10^{-4}

$Cu(IO_3)_2 \rightleftharpoons Cu^{2+} + 2IO_3^-$

$IO_3^- + Na_2S_2O_3 \rightleftharpoons I_2 + Na_2S_4O_6$

Applying the law of equivalence

$(m_{eq})_{IO_3^-} = (m_{eq})_{Na_2S_2O_3}$

$5 \times 20 \times M = 26ml \times 0.1M \times 1$

$M = 2.6 \times 10^{-2} M$

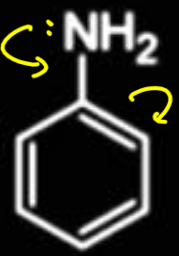
$K_{sp} = (Cu^{2+})(IO_3^-)^2$

$= (1.3 \times 10^{-2})(2.6 \times 10^{-2})^2$

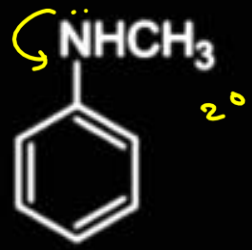
$= 8.78 \times 10^{-6}$

13

Which of the following is the strongest base ?

(A) 

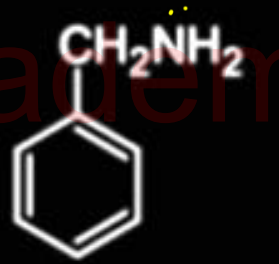
1°

(B) 

2°

(C) 

SIP effect

(D) 

(D) > (B) > (A) > (C)

14

For a hypothetical reaction, $A + B \rightarrow C$, the following data were obtained. The overall order of reaction is

Experiment	[A](mol L^{-1})	[B](mol L^{-1})	Rate of reaction (mol $L^{-1}s^{-1}$)
I	0.25	0.25	3.0×10^{-3}
II	0.50	0.25	6.0×10^{-3}
III	0.50	0.50	1.20×10^{-2}

(A) 0 (B) 1

(C) 2 (D) 3

$r = k[A]^x[B]^y$

$(2)^x \propto 2$

$x = 1$

$(2)^y \propto 2$

$y = 1$

$x + y = 2$ Ans.

JEE 1

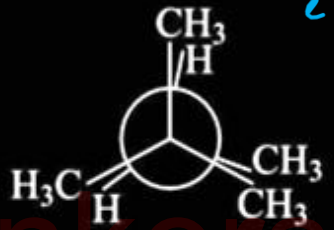
15

In the given four conformational isomers, which of the following has minimum torsional strain and minimum van der Waals strain?

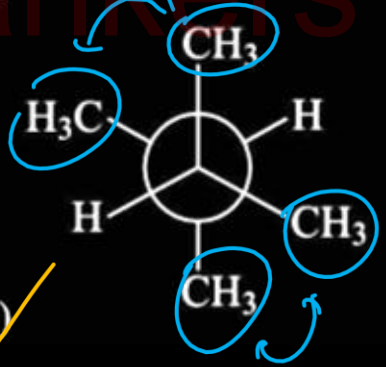
↳ B.P repulsion

e⁻ cloud

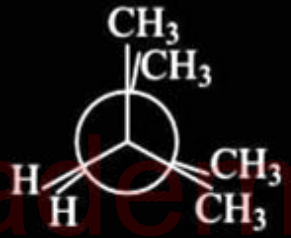
(A)



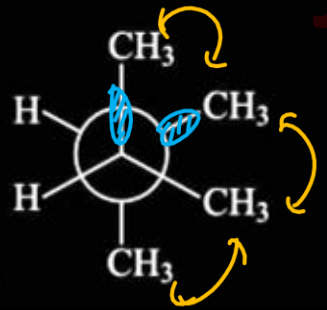
(B)



(C)



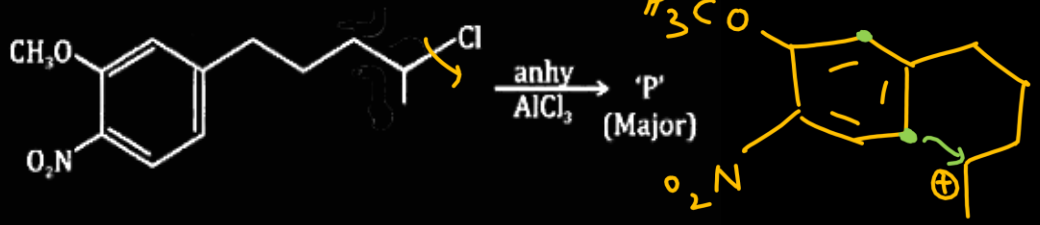
(D)



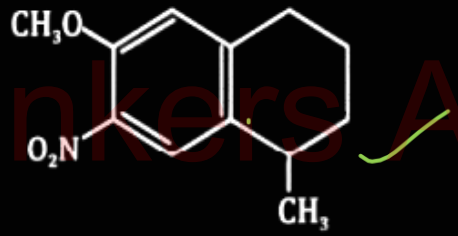
16

JEE 1

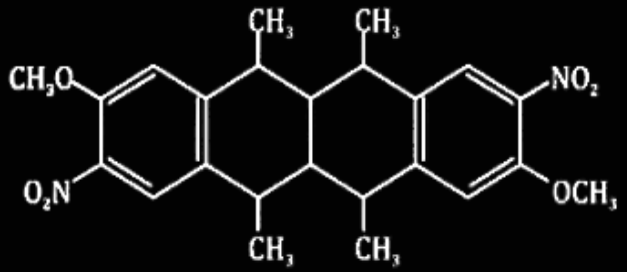
The major product ' P ' formed in the given reaction is



(A)

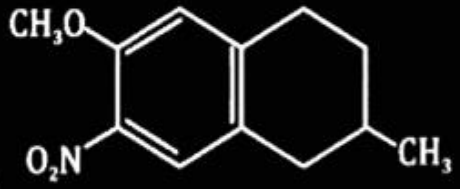


(B)



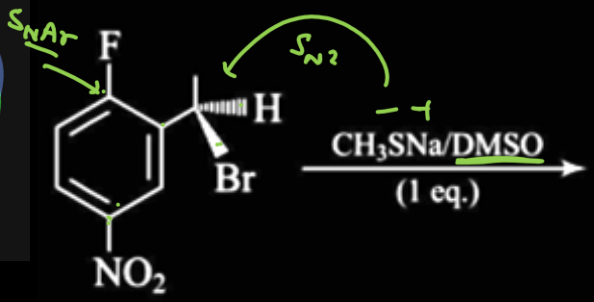
(C)

(D)

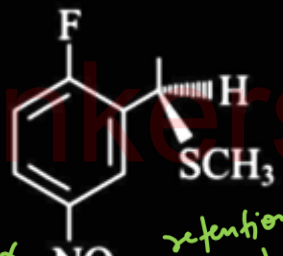


17

Identify the product in the following reaction?

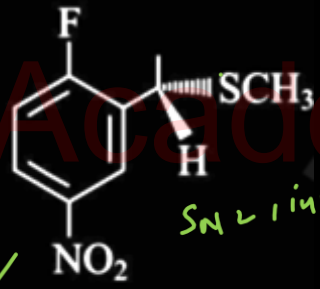


(A)



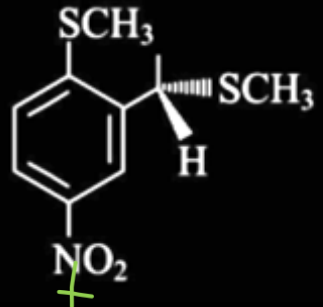
retention (SNAr)

(B)

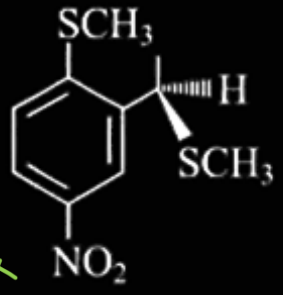


SN2 / inversion.

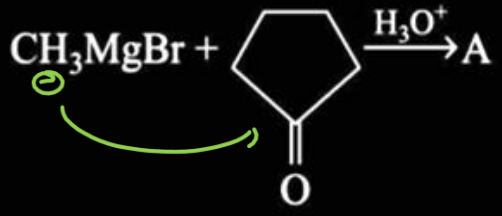
(C)

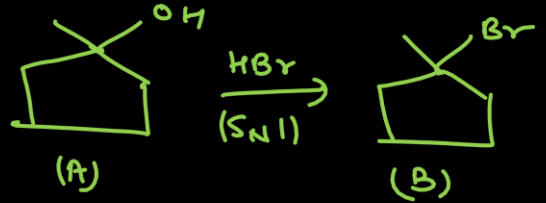


(D)

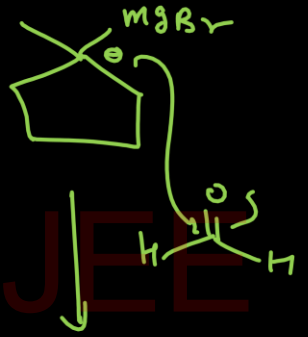


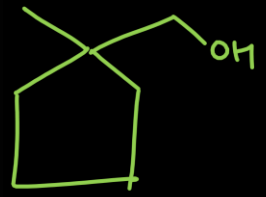
18





JEE 1

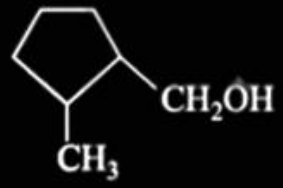


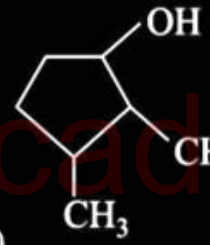


$\xrightarrow{HBr}$ B $\xrightarrow{Mg/ether}$ C $\xrightarrow[H_3O^+]{HCHO}$ D,

D is

(A) 

(B) 

(C) 

(D) 

19

When a certain conductivity cell was filled with 0.01 M solution of KCl , it had a resistance of 160Ω at 25°C, and when filled with 0.005 M NaOH , it had a resistance of 190Ω. If specific resistance of KCl solution is $700\Omega - cm$, specific conductance $(\Omega^{-1}\ cm^{-1})$ of NaOH solution is

(A) 0.00120
(B) 0.00170
(C) 0.00180
(D) 0.00190

G^* Same.

JEE 1

KCl
0.01M
$R_1 = 160\ \Omega$
$\rho_1 = 700$

NaOH
0.005M
$R_2 = 190$ ohm
$K = \frac{1}{\rho_2} = ?$

$K = G G^*$

$\frac{1}{\rho} = \frac{1}{R} G^*$

$\frac{1}{700} = \frac{1}{160} G^*$ — (1)

$\frac{1}{\rho_2} = \frac{1}{190} G^*$ — (2)

$\frac{1}{\rho_2} = \frac{1}{190} \times \frac{160}{700} = \frac{16}{19 \times 700}$

20

A solution has 1:4 mole ratio of pentane to hexane. The vapour pressure of pure hydrocarbons at 20°C are 440 mm Hg for pentane and 120 mm Hg for hexane. The mole fraction of pentane in vapour phase would be

(A) 0.786 (B) 0.549

(C) 0.478 (D) 0.200

$\frac{n_P}{n_H} = \frac{1}{4}$, $x_P = \frac{1}{5}$, $P_P^\circ = 440$ mm Hg

$x_H = \frac{4}{5}$ $P_H^\circ = 120$ mm Hg

$y_P = ?$

$$y_P = \frac{P_P}{P_{total}} = \frac{P_P^\circ x_P}{P_P^\circ x_P + P_H^\circ x_H}$$

$$= \frac{440 \times \frac{1}{5}}{\left(440 \times \frac{1}{5}\right) + \left(120 \times \frac{4}{5}\right)}$$

$$= \frac{440}{440 + 480} = \frac{44}{44+48}$$

$$= \frac{44}{92}$$

21

Out of

$I_3^-, ICl_2^-, BeCl_2, XeF_2, XeF_6, BrF_5, CH_2 = CH_2$

How many will have linear shape?

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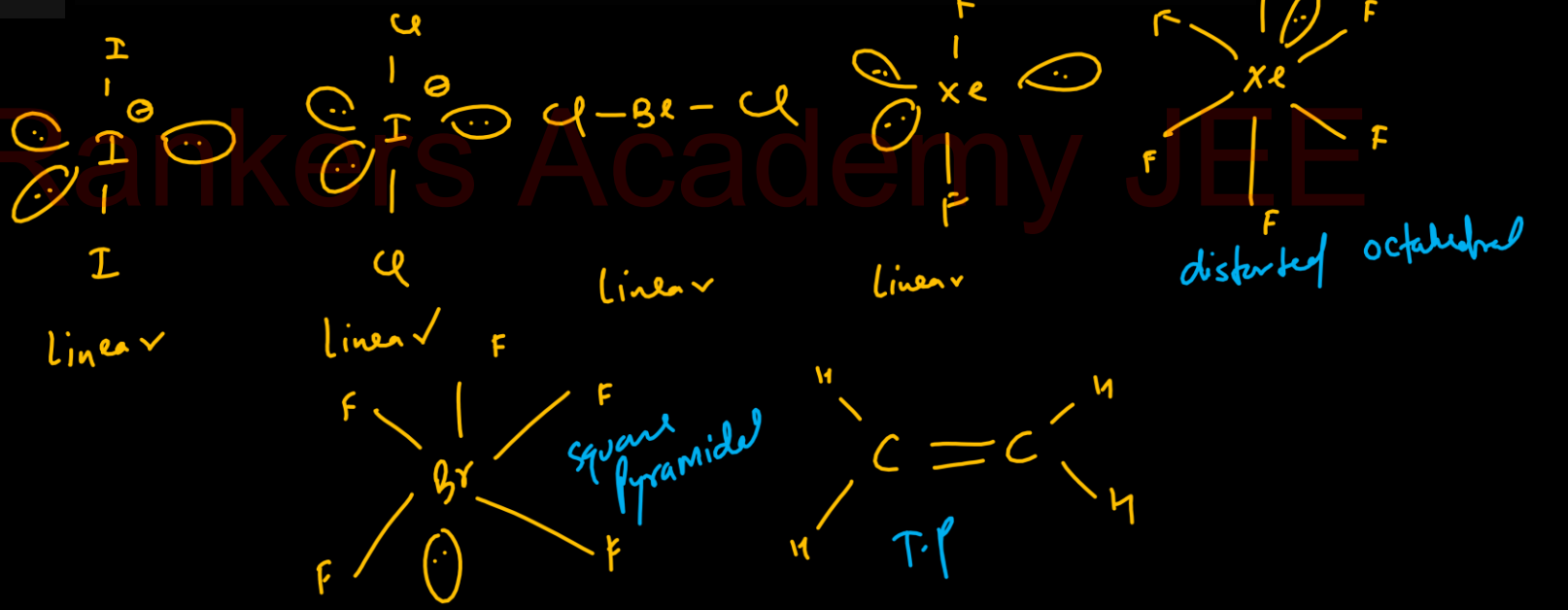
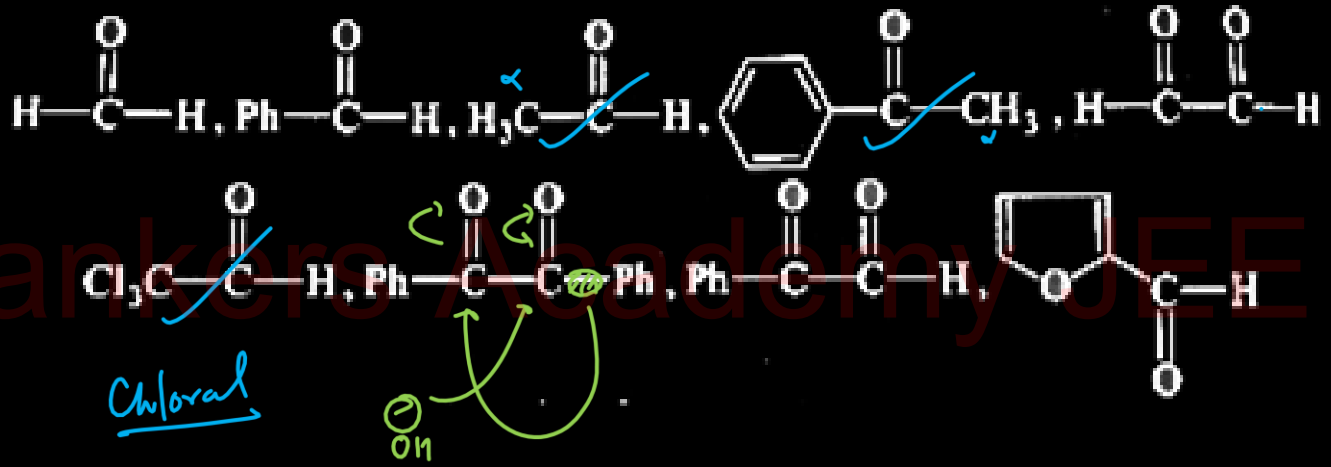


22

Find out number of substrates those cannot undergo Cannizzaro's reaction.



JEE 1

23

1 mole of an ideal gas is allowed to expand isothermally and reversibly at 27°C until its volume is tripled. Calculate ΔS_{sys} (Give answer as nearest integer in SI units).

$V_2 = 3V_1$

$\log 3 = 0.477 \cdot R = 8.314\ J\ k^{-1}\ mol^{-1}$

$\Delta S_{sys} = n C_{v,m} \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$

$= nR \ln \frac{V_2}{V_1}$

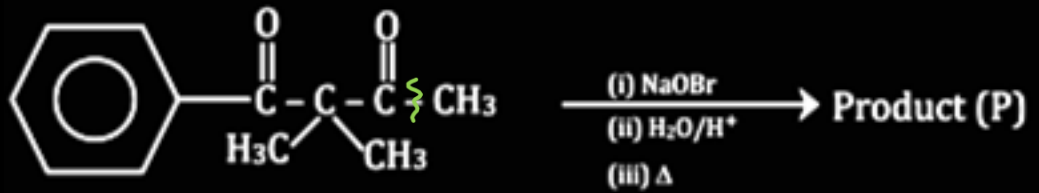
$= 1 \times 8.314 \times \ln 3$

$= 8.314 \times 2.303 \times 0.477 \approx 9$

24

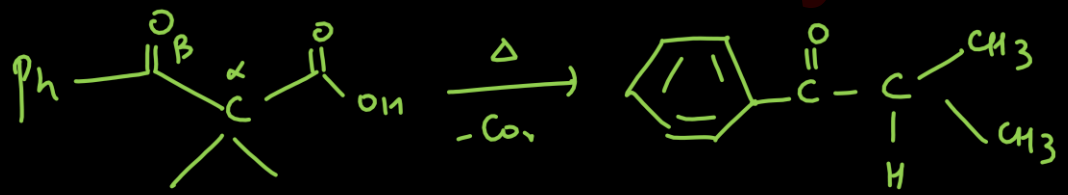
JEE 1

In the given reaction sequence



Total number of double bond equivalent present in product P is

NaOBr
H+

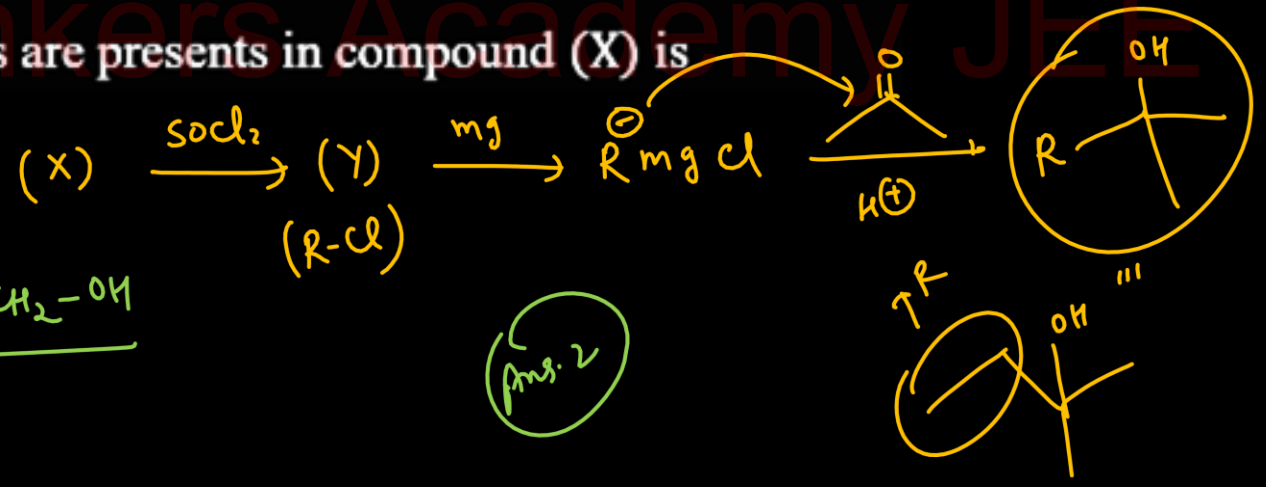


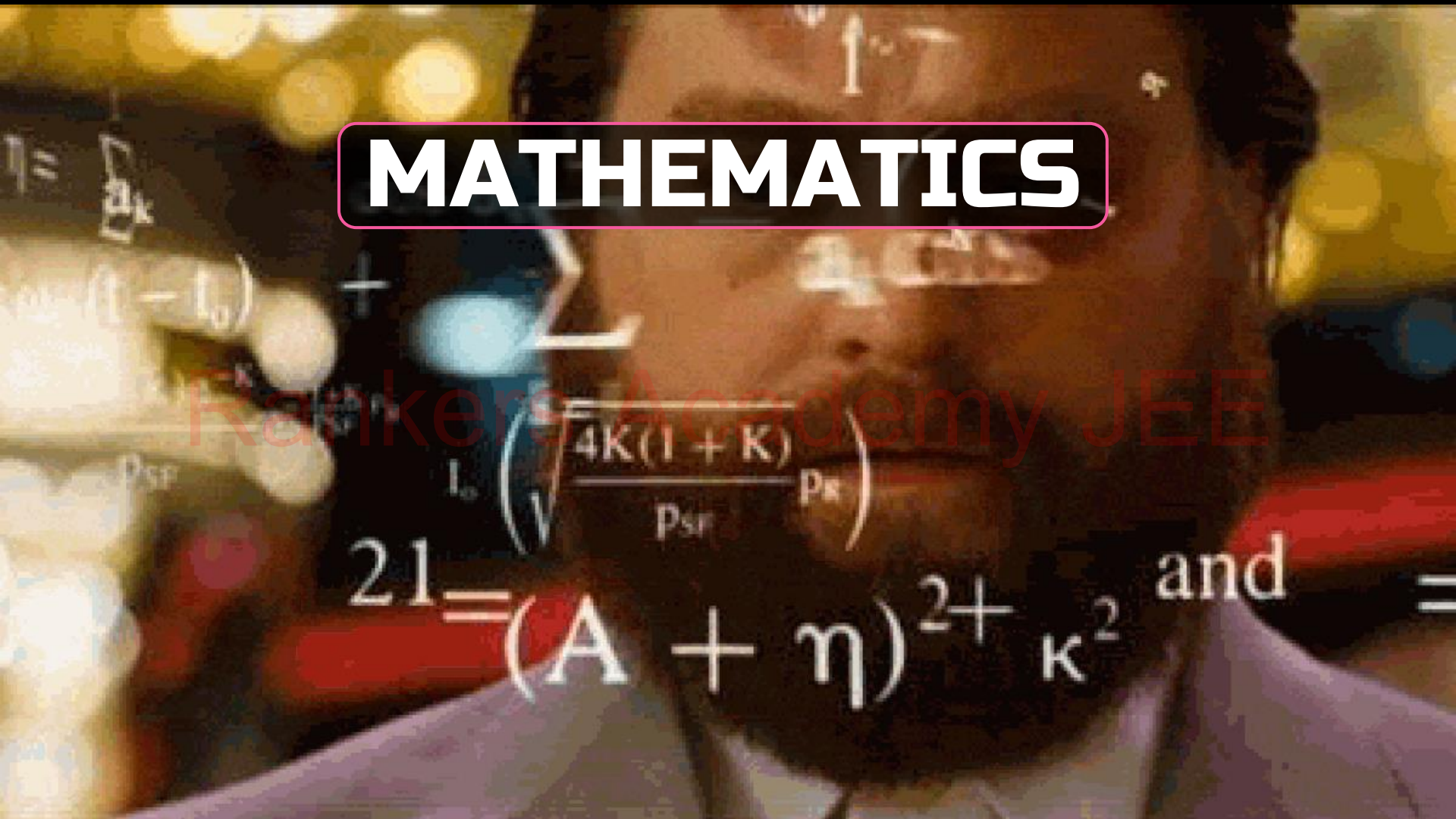
β-keto acid

(5)

25

A compound (X) reacts with thionyl chloride to give a compound (Y). (Y) reacts with Mg to form a Grignard reagent, which is treated with acetone and the product is hydrolysed to give 2-methyl-2-butanol. Total number of carbon atoms are presents in compound (X) is




MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^{2+} \kappa^2$ and

7

The value of the integral
$\int (x^2 + x)(x^{-8} + 2x^{-9})^{1/10} dx$ is

(A) $\frac{5}{11}(x^2 + 2x)^{11/10} + c$

(B) $\frac{5}{6}(x + 1)^{11/10} + c$

(C) $\frac{6}{7}(x + 1)^{11/10} + c$

(D) none of these

JEE 1

$\int (x+1)(x^2+2x)^{1/10} dx$

Let: $x^2 + 2x = t^{10}$

$(2x+2)dx = 10t^9 dt$

$(x+1)dx = 5t^9 dt$

$\int (t)\ 5t^9 dt$

$5\int t^{10} dt$

$5\frac{t^{11}}{11} + C$

$\frac{5}{11}(x^2+2x)^{11/10} + C$

JEE 1

If z_1 and z_2 are two non-zero complex number such that $|z_1 z_2| = 2$ and $\arg(z_1) - \arg(z_2) = \frac{\pi}{2}$, then the value of $3i\bar{z}_1 z_2$

(A) 2 (B) 2i

(C) 4 (D) 6

$$\begin{cases} |z_1||z_2| = 2 \\ \arg(z_1) = \frac{\pi}{2} + \arg(z_2) \end{cases}$$

$|z_1| \cdot |z_2| = 2$

$\arg(z_1) = \frac{\pi}{2} + \theta$

$\arg(z_2) = \theta$

$z_2 = 2e^{i\theta}$

$z_1 = (1)\, e^{i(\frac{\pi}{2}+\theta)}$

$= e^{i\pi/2} \cdot e^{i\theta}$

$= i \cdot e^{i\theta}$

$3i\bar{z}_1 z_2 = 3i(-i)(e^{-i\theta})(2e^{i\theta})$

3

The value of a for which all extremum of function $f(x) = x^3 - 3ax^2 + 3(a^2 - 1)x + 1$, lie, in the interval $(-2,4)$ is

(A) (3,4)

(B) $(-1,3)$

(C) $(-3,-1)$

(D) none of these

JEE 1

$f'(x) = 3x^2 - 6ax + 3(a^2 - 1)$

$= 3[x^2 - 2ax + a^2 - 1]$

$= 3[(x-a)^2 - 1^2]$

$= 3[(x-a+1)(x-a-1)]$

Roots: $x = a-1$, $x = a+1$

$a-1 > -2$

$a > -1$

$a+1 < 4$

$a < 3$

4

If $L = \lim_{x\to 0} \frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^8 \cdot x^4\left(e^{2x^4} - 1 - 2x^4\right)}$, then the value of $1/L$ is _____

(A) 3 (B) 1/3

(C) 6 (D) 1/6

$= \left[\frac{\quad}{x^8}\right] \to 2$

$$L = \lim_{x\to 0} \frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^{12}\,(2)}$$

$$L = \lim_{x\to 0} \frac{\left(x^4 - \frac{(x^4)^3}{3!}\right) - x^4\left(1 - \frac{(x^4)^2}{2!}\right) + x^{20}}{2x^{12}}$$

$$\left[\frac{2x^4}{e^{2x^4} - 1 - 2x^4}\right]$$

$$\left[\frac{\left(1 + 2x^4 + \frac{(2x^4)^2}{2!} + \frac{(2x^4)^3}{3!} + \cdots\right) - 1 - 2x^4}{x^8}\right]$$

$= 2$

$$L = \lim_{x \to 0} \frac{-\frac{x^{12}}{6} + \frac{x^{12}}{2} + x^{20}}{2x^{12}}$$

$$L = \lim_{x \to 0} \frac{1}{2}\left[-\frac{1}{6} + \frac{1}{2} + \cancel{x^8}\right]$$

$$L = \frac{1}{6}$$

5

Coefficient of x in the expansion of

$(1 - 2x^3 + 3x^5)\left(1 + \frac{1}{x}\right)^8$ is

(A) 154 (B) 164

(C) 146 (D) 156

$\left(1+\frac{1}{x}\right)^8 - 2x^3\left(1+\frac{1}{x}\right)^8 + 3x^5\left(1+\frac{1}{x}\right)^8$

x^1 x^{-2} x^{-4}

$X \quad -2\left({}^8C_2\right) + 3\left({}^8C_4\right)$

GT: $\left(1+\frac{1}{x}\right)^8$

${}^8C_r (1)^{8-r}\left(\frac{1}{x}\right)^r$

$\boxed{{}^8C_r (x^{-r})}$

6

Consider 2,5, a, 12, b be five observations. Now, 5 is added to each observations and the mean and variance of new observations are 14 and 27.6. Then ab is equal to

(A) 133 (B) 120

(C) 153 (D) 105

JEE 1

$a+b=26$ — (1)

$\bar{X}_{new} = \bar{X}_{old} + 5$

14

$\Rightarrow \bar{X}_{old} = 9$

$\sigma^2_{New} \Rightarrow \sigma^2_{old} = 27.6$

Now:

$\bar{X} = 9 = \frac{2+5+a+12+b}{5}$

$45 = 19 + a + b$

$$\sigma^2 = \frac{4+25+a^2+144+b^2}{5} - (9)^2$$

$\sigma^2 \rightarrow 27.6$

$$108.6 \times 5 = 29 + 144 + a^2 + b^2$$

$$\boxed{a^2+b^2 = 370} \quad -(2)$$

Now:

$$(a+b)^2 = (a^2+b^2)+(2ab)$$

$$(26)^2 = (370)+2(ab)$$

$$\boxed{ab = 153}$$ // Ans.

7

If the solution of the differential equation $(2x + 3y - 1)dx + (4x + 6y - 5)dy = 0, y(0) = 2$ is $\alpha x + \beta y + 9(\ln|2x + 3y - \gamma|) = 12$, then $3\alpha + 2\beta + \gamma$ is

(A) 14 (B) 15

(C) 28 (D) 10

$$(4x+6y-5)dy = -(2x+3y-1)dx$$

$$\frac{dy}{dx} = \frac{-(2x+3y-1)}{(4x+6y-5)}$$

$$\frac{dy}{dx} = \left[\frac{-(2x+3y-1)}{2(2x+3y-1)-3}\right]$$

Let: $2x+3y-1 = t$

$$2 + 3\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dt}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dt}{dx}-2\right)=\left(\frac{-t}{2t-3}\right)$$

$$\frac{dt}{dx}=\left(\frac{-3t}{2t-3}+2\right)$$

$$\frac{dt}{dx}=\left(\frac{t-6}{2t-3}\right)$$

$$\int\left(\frac{2t-3}{t-6}\right)dt=\int dx$$

$$\Rightarrow \int\left[\frac{2t-12}{(t-6)}+\frac{9}{(t-6)}\right]dt=\int dx$$

$$\Rightarrow 2t+9\ln|t-6|=x+C$$

$$\Rightarrow 2(2x+3y-1)+9\ln|2x+3y-1-6|=x+C$$

$$\Rightarrow 3x+6y-2+9\ln|2x+3y-7|=C$$

$(0,2):\ 0+12-2+9\ln|-1|=C$

$\boxed{C=10}$

$3x+6y+9\ln|2x+3y-7|=12$

$\alpha=3,\ \beta=6,\ \gamma=7$

8

Box A contains four tickets numbered 1,2,3,4, box B contains 7 tickets numbered 1,2,3,4,5,6, 7. A ticket is drawn from each of the boxes. Let a_i be the number on the ticket drawn from Box A and let b_i be the number on the ticket drawn from box B. The probability that $a_i \times b_i$ is a multiple of 3 is

(A) $\frac{15}{28}$ (B) $\frac{13}{28}$

(C) $\frac{2}{3}$ (D) $\frac{11}{28}$

$1 - \left(\frac{3}{4} \times \frac{5}{7}\right)$

$1 - \frac{15}{28}$

9

If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$. Then $3\alpha + 10\beta$ equals to

(A) 97 (B) 95

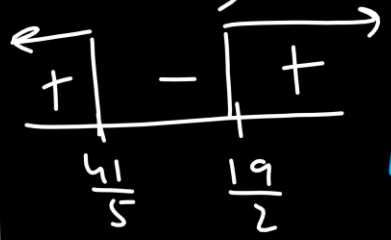
(C) 98 (D) 100

JEE 1

$-1 \le \left(\frac{3x-22}{2x-19}\right) \le 1$

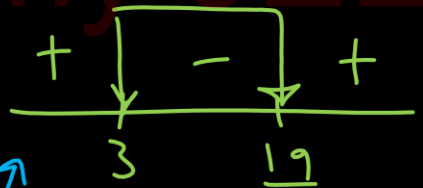
$\left(\frac{3x-22}{2x-19} + 1\right) \ge 0$

$\left(\frac{5x-41}{2x-19}\right) \ge 0$

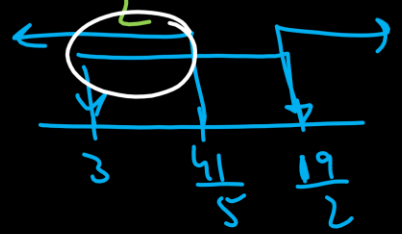


$\frac{3x-22}{2x-19} - 1 \le 0$

$\left(\frac{x-3}{2x-19}\right) \le 0$



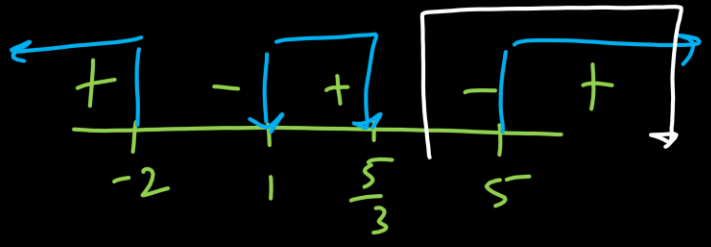
$D_1 \equiv$



$$\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)>0$$

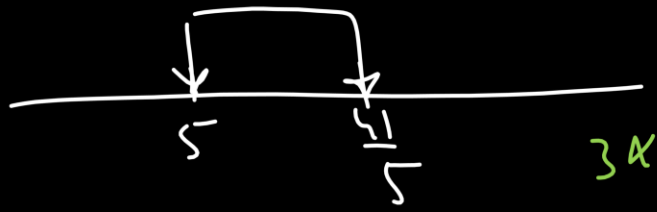
$$\left(\frac{3x^2-3x-5x+5}{x^2-5x+2x-10}\right)>0$$

$$\frac{(3x-5)(x-1)}{(x+2)(x-5)}>0$$



D_2

Now: $D_1 \cap D_2$



$\alpha = 5$

$\beta = \frac{41}{5}$

$3\alpha + 10\beta$

$15 + 82 = 97$

10

In an A.P of 99 terms, the sum of all the odd numbered terms is 2550. Then the sum of all 99 terms is

(A) 5039 (B) 5029

(C) 5019 (D) 5049

JEE 1

$a_1 + a_3 + a_5 + \cdots + a_{99} = 2550$

$\frac{50}{2}(a_1 + a_{99}) = 2550$

$a_1 + a_2 + a_3 + \cdots + a_{99}$

$\frac{99}{2}(a_1 + a_{99}) = S$

$\frac{50}{99} = \frac{2550}{S}$

$S = \frac{2550 \times 99}{50}$

11

Let [t] denote the greatestinteger less than or equal to t . Let $f:[0,\infty) \to R$ be a function defined by $f(x) = \left[\frac{x}{2} + 3\right] - [\sqrt{x}]$, Let S be the set of all points in the interval [0,8] at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.

(A) 8 (B) 9

(C) 11 (D) 17

$f(x) = \left[\frac{x}{2}\right] - [\sqrt{x}] + 3$

$\{0, 2, 4, 6, 8\}$ $\{0, 1, 4\}$

$S = \{1, 2, 6, 8\}$

$$= \begin{cases} 3 & ; x \in [0,1) \\ 2 & ; x \in [1,2) \\ 3 & ; x \in [2,4) \\ 3 & ; x \in [4,6) \\ 4 & ; x \in [6,8) \\ 5 & x = 8 \end{cases}$$

12

Let A = {1,2,3,4,5}, Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(A) 24 (B) 25

(C) 26 (D) 23

JEE 1

m = 16

n = 9

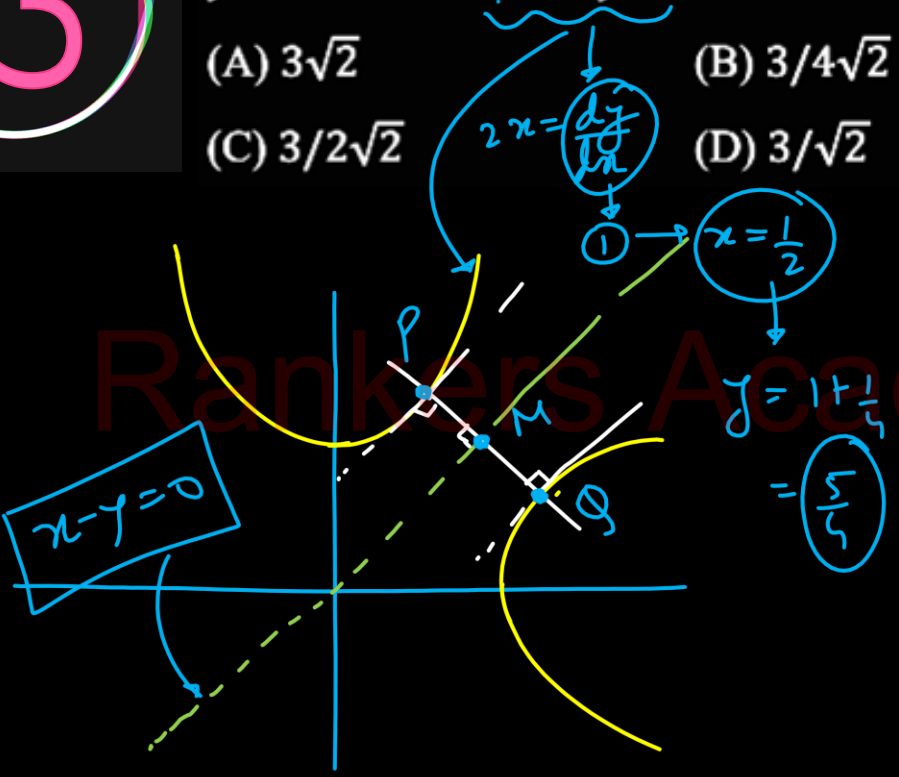
R = {(1,1), (1,2), (1,3), (1,4), (1,5)
(2,2), (2,3), (2,4), (2,5)
(3,3), (3,4), (3,5)
(4,4), (4,5), (5,4), (5,5)}

13

The shortest distance between the parabolas $y^2 = x - 1$ and $x^2 = y - 1$ is

(A) $3\sqrt{2}$ (B) $3/4\sqrt{2}$

(C) $3/2\sqrt{2}$ (D) $3/\sqrt{2}$



$PQ = 2(PM)$

$= 2\left(\left|\frac{\frac{1}{2} - \frac{5}{4}}{\sqrt{2}}\right|\right)$

$= \frac{3}{2\sqrt{2}}$

14

JEE 1

If $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2\left(\frac{x}{2}\right) \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$ and if

$\int_{-\pi/2}^{\pi/2} (1+x^4)(f(x)+f''(x))dx = 2\lambda + 3$, then the value of λ is

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $-\frac{1}{2}$ (D) $-\frac{3}{2}$

(Even)(odd)

$f(x) = \begin{vmatrix} E & E & O \\ E & E & O \\ 1 & 2 & O \end{vmatrix}$

$f(x) \rightarrow$ odd.

$f'(x) \rightarrow$ even

$f''(x) \rightarrow$ Odd.

$0 = 2\lambda + 3$

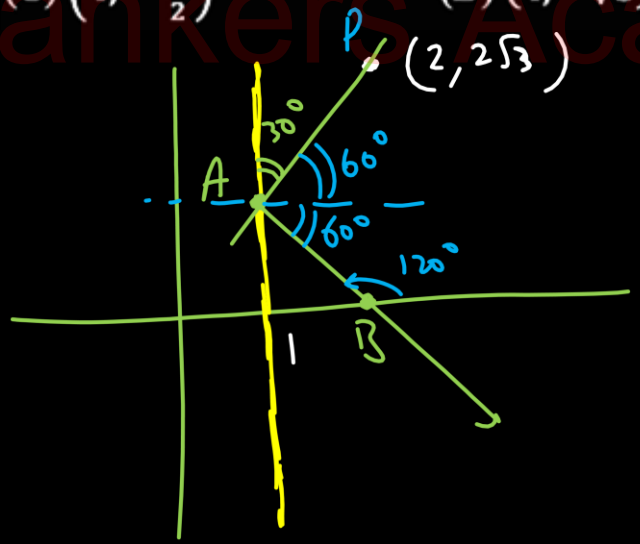
$\lambda = -3/2$

15

A ray of light coming from the point $(2,2\sqrt{3})$ is incident at an angle 30° on the line $x = 1$ at the point A. The ray gets reflected on the line $x = 1$ and meets x-axis at the point B. Then, the line AB passes through the point

(A) $\left(3,-\frac{1}{\sqrt{3}}\right)$ (B) $(3,-\sqrt{3})$

(C) $\left(4,-\frac{\sqrt{3}}{2}\right)$ (D) $(4,-\sqrt{3})$



JEE 1

PA :

$(y - 2\sqrt{3}) = \sqrt{3}(x - 2)$

$x = 1$: $y - 2\sqrt{3} = -\sqrt{3}$

$y = \sqrt{3}$

Eqn of AB :

$(y - \sqrt{3}) = (-\sqrt{3})(x - 1)$

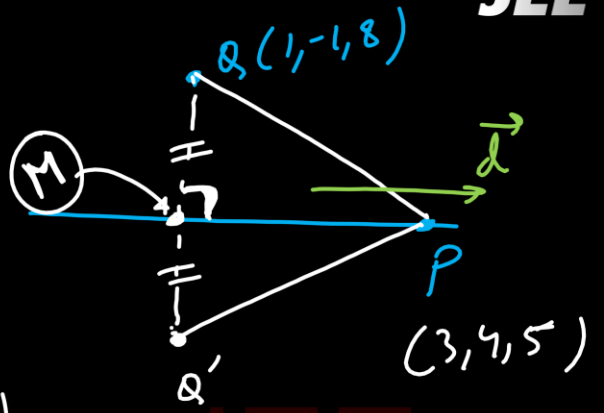
$y - \sqrt{3} = -\sqrt{3}x + \sqrt{3}$

16

Consider a line passing through the point P(3,4,5) and having direction ratios $(2,-2,1)$. If image of point Q(1,−1,8) w.r.t. given line is Q', then length of median of $\Delta PQQ'$ through vertex P is

(A) 2 (B) 3

(C) 4 (D) 1



$$L \equiv \frac{x-3}{2} = \frac{y-4}{-2} = \frac{z-5}{1} = \lambda$$

$$M(\lambda) \equiv (2\lambda+3, -2\lambda+4, \lambda+5)$$

$$\vec{QM} = \left((2\lambda+2)\hat{i} + (-2\lambda+5)\hat{j} + (\lambda-3)\hat{k}\right)$$

$$\vec{QM} \cdot \vec{d} = 0$$

$$4\lambda+4+4\lambda-10+\lambda-3=0$$

$$9\lambda = 9$$

$$\lambda = 1$$

$\therefore M \equiv (5, 2, 6)$

$P \equiv (3, 4, 5)$

$PM = \sqrt{4 + 4 + 1}$

$= \sqrt{9}$

$= 3$

17

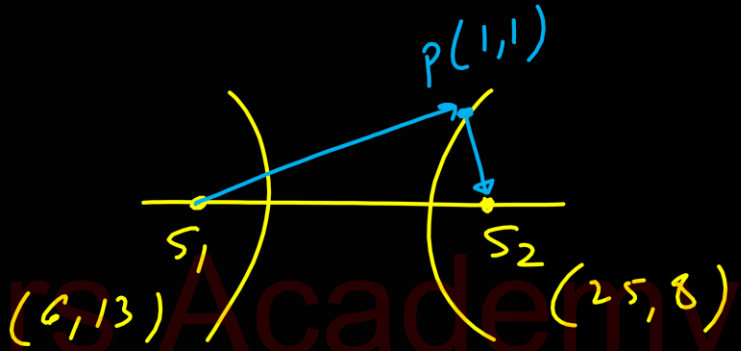
If (6,13) and (25,8) are the foci of a hyperbola passing through the point (1,1) then the eccentricity of the hyperbola is

(A) $\frac{\sqrt{386}}{38}$

(B) $\frac{\sqrt{386}}{25}$

(C) $\frac{\sqrt{386}}{13}$

(D) $\frac{\sqrt{386}}{12}$



$$\begin{cases} |PS_1 - PS_2| = 2a \\ S_1S_2 = 2ae \end{cases}$$

$$e = \frac{S_1S_2}{|PS_1 - PS_2|} = \frac{\sqrt{19^2 + 5^2}}{|13 - 25|} = \frac{\sqrt{386}}{12}$$

18

JEE 1

Let $a, b \in R$ consider the system of linear equations $2x + ay + 6z = 8, x + 2y + bz = 5$ and $x + y + 3z = 4$. Then which of the following statements is INCORRECT?

(A) if $a = 2$, then system of equations have infinitely many solution True

(B) if $a = 5$ and $b = 3$ then the system of equations have no solution True.

(C) if $a \neq 2$ and $b = 3$, then the system of equations have infinitely many solution

(D) if $a \neq 2$ and $b \neq 3$, then the system of equations have a unique solution True.

$$2x + ay + 6z = 8$$
$$x + 2y + bz = 5$$
$$x + y + 3z = 4$$

$$\Delta = \begin{vmatrix} 2 & a & 6 \\ 1 & 2 & b \\ 1 & 1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & a-2 & 0 \\ 1 & 1 & b-3 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= (a-2)(b-3)$$

$$\Delta_x = \begin{vmatrix} 8 & 5 & 6 \\ 5 & 2 & 3 \\ 4 & 1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 3 \end{vmatrix} \neq 0$$

19

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors and are such that

$\vec{a} \times \vec{b} = 3(\vec{a} \times \vec{c}), |\vec{a}| = |\vec{c}| = 2, |\vec{b}| = 5, |\vec{b} \times \vec{c}| = 5$

$,\vec{b} - 3\vec{c} = \lambda\vec{a}$, then a value of λ^2 is

(A) $\frac{61+30\sqrt{3}}{5}$ (B) $\frac{61-30\sqrt{3}}{4}$

(C) $\frac{1}{4}$ (D) $\frac{1}{9}$

$|\vec{b}||\vec{c}|\sin\theta = 5 \times 1$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

JEE 1

$\bar{a} \times \bar{b} - 3\bar{a} \times \bar{c} = 0$

$\bar{a} \times (\bar{b} - 3\bar{c}) = 0$

$\bar{a} \parallel (\bar{b} - 3\bar{c}) = 0$

$\bar{b} - 3\bar{c} = \lambda\bar{a}$

$|\bar{b} - 3\bar{c}|^2 = \lambda^2 |\bar{a}|^2$

$b^2 + 9c^2 - 6\,\bar{b} \cdot \bar{c} = \lambda^2 (a^2)$

$25 + 9(4) - 6\left(5 \times 2 \times \frac{\sqrt{3}}{2}\right)$

$= \lambda^2 (4)$

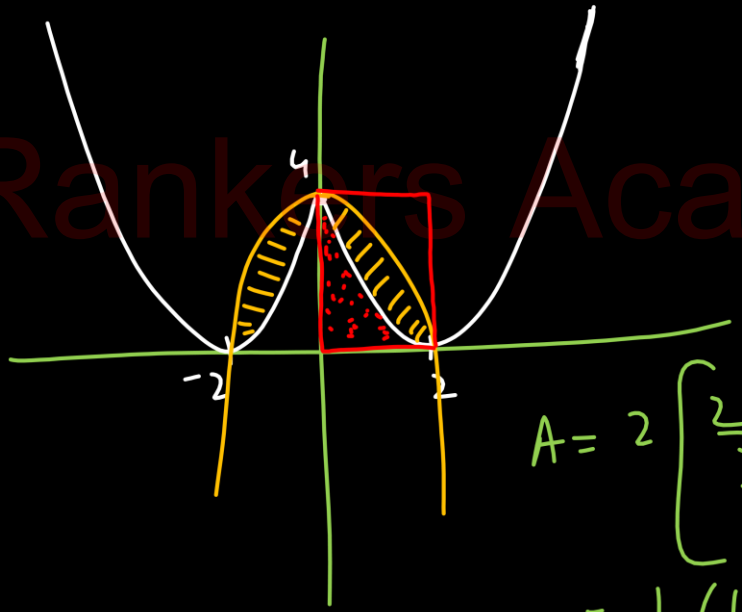
20

If area bounded by curves $y = (|x| - 2)^2$ and $y = 4 - x^2$ is ' A' (in eq. units), then 3A is :

(A) 3 (B) 16

(C) 48 (D) 6

JEE 1



$y = (|x| - 2)^2$

$= \begin{cases} (x-2)^2 ; & x \geq 0 \\ (-x-2)^2 ; & x < 0 \end{cases}$

$(-x-2)^2 \rightarrow (x+2)^2$

$A = 2\left[\frac{2}{3}(2 \times 4) - \frac{1}{3}(2 \times 4)\right]$

$= \frac{1}{3}(16)$

21

If A is a square matrix of order 3 such that $|A| = a, B = \text{adj}(A)$ and $|B| = b$, then $(ab^2 + a^2b + 1)\lambda =$ _____

(where $\frac{1}{2}\lambda = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \cdots$ upto ∞ and $a = 3$)

JEE 1

$|A| = a$

$|B| = |A|^2 = a^2$

$b = a^2$

$\therefore \boxed{a = 3, \quad b = 9}$

$\frac{\lambda}{2} = \frac{a/b}{\left(1 - \frac{a}{b^2}\right)}$

$\boxed{\lambda = \left(\frac{2ab}{b^2 - a}\right)}$

Ans.

$(3 \times 81 + 9 \times 9 + 1)\left(\frac{2 \times 3 \times 9}{81 - 3}\right)$

$= 225$

22

We have 21 identical balls, which needs to be distributed among 3 Boys A, B, C. Such that A always gets even number of balls. Number of possible ways of doing this is

a	b & c	$^{n+r-1}C_{r-1}$
0	21	22
2	19	20
4	17	18
6	15	16
8	13	⋮
⋮	⋮	⋮
20	1	2

→ Sum.

$2+4+6+\cdots+22$

$= 2(1+2+3+\cdots+11)$

$= 2\left(\frac{11\times 12}{2}\right)$

$= \boxed{132}$

$b+c=21$

$^{n+r-1}C_{r-1}$

$r=2$

↓

$^{n+1}C_1$

↓

$n+1$

23

Given that $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $A^n = A^{n-2} + A^3 - A \; \forall n \geq 4$, and $A^3 - A^2 - A + I = 0$.

$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ k & 1 & 0 \\ k & 0 & 1 \end{bmatrix}$. Then k is equal to

$A^n = A^{n-2} + A^3 - A$

$A^n - A^{n-2} = A^3 - A$

$n = 20 : A^{20} - A^{18} = (A^3 - A)$

$n = 18 : A^{18} - A^{16} = (A^3 - A)$

$n = 16 : A^{16} - A^{14} = (A^3 - A)$

$\vdots$

$n = 4 : A^4 - A^2 = (A^3 - A)$

$A^{20} - A^2 = 9(A^3 - A)$

$$A^{20} - A^2 = 9(A^2 + A - I) - 9A$$

$$A^{20} = 10A^2 - 9I$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{20} = \begin{bmatrix} 10 & 0 & 0 \\ 10 & 10 & 0 \\ 10 & 0 & 10 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 10 & 0 & 1 \end{bmatrix}$$

$K = 10$

24

Given that a, b are integers and the two real roots α, β of the equation $3x^2 + 3(a+b)x + 4ab = 0$ satisfy the relation $\alpha(\alpha+1) + \beta(\beta+1) = (\alpha+1)(\beta+1)$. The number of ordered pairs (a, b) is equal to

$\Rightarrow \alpha(\alpha+1) + \beta(\beta+1) = (\alpha+1)(\beta+1)$

$\Rightarrow \alpha^2 + \beta^2 + (\alpha+\beta) = \alpha\beta + (\alpha+\beta) + 1$

$+2\alpha\beta \qquad 2\alpha\beta$

$\Rightarrow (\alpha+\beta)^2 = 1 + 3(\alpha\beta)$

$\left(-\frac{3(a+b)}{3}\right)^2 = 1 + 3\left(\frac{4ab}{3}\right)$

$a^2 + b^2 + 2ab = 1 + 4ab$

$(a-b)^2 = 1$

$|a-b| = 1$ — (1)

Now:

$D \geq 0$

$3^{\not 4}(a+b)^2 - 4(1)(4ab) \geq 0$

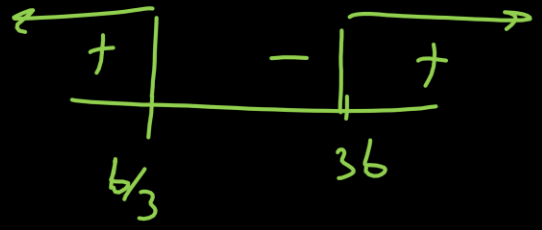
$3a^2 + 3b^2 + 6ab - 16ab \geq 0$

$3a^2 - 10ab + 3b^2 \geq 0$

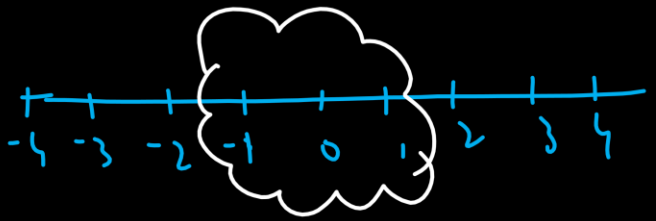
$3a^2 - ab - 9ab + 3b^2 \geq 0$

$a(3a-b) - 3b(3a-b) \geq 0$

$(3a-b)(a-3b) \geq 0$



$a \leq \frac{b}{3}$ (Or) $a \geq 3b$



$(0,1)$; $(-1,0)$, $(1,0)$, $(0,-1)$

25

If the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the points (5,5) is $(x-h)^2 + (y-k)^2 = 25$, then the value of $10(h^2 + k^2)$ must be _____ .

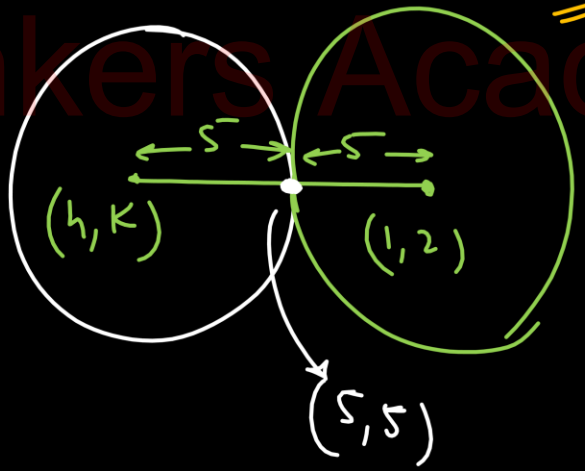
1450

JEE 1

$C \equiv (1,2)$

$r = \sqrt{1+4-(-20)}$

$= 5$



$\frac{h+1}{2} = 5 \Rightarrow h = 9$

$\frac{k+2}{2} = 5 \Rightarrow k = 8$