FIITJEE RBT-5 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 3 | **QP CODE: 100959** | **PAPER - 1**

Time Allotted: 3 Hours **Maximum Marks: 204**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains **Four (04)** multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A** (**05–12**) contains Eight (8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **Part-B (1 – 8)** contains Eight **(08)** Numerical based questions, the answer of which maybe positive or negative numbers or decimals **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**____________________

Enrolment Number :____

SECTION – I : PHYSICS

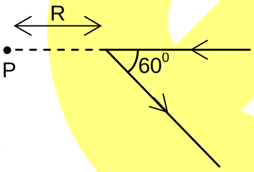
PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in

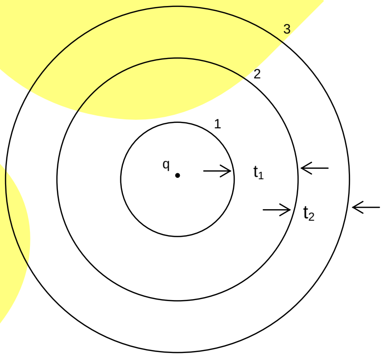
(A) X-ray region (B) ultra-violet region
(C) infra-red region (D) visible region

2. A long straight wire, carrying a current I is bent at its mid point to form an angle of 60^0 . AT a point P, distance R from the point of bending the magnetic field is



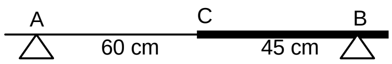
(A) $\frac{(\sqrt{2}-1)\mu_0 i}{4\pi R}$ (B) $\frac{(\sqrt{2}+1)\mu_0 i}{4\pi R}$
(C) $\frac{\mu_0 i}{4\sqrt{3}\pi R}$ (D) $\frac{\mu_0 i}{8R}$

3. Figure show three spherical equipotential surface 1, 2 and 3 round a point charge q. The potential difference $V_1 - V_2 = V_2 - V_3$. If t_1 and t_2 be the distance between them. Them



(A) $t_1 = t_2$ (B) $t_1 > t_2$
(C) $t_1 < t_2$ (D) $t_1 \leq t_2$

4. A steel wire of length 60 cm and area of cross section $10^{-6} m^2$ is joined with an aluminium wire of length 45 cm and area of cross section $3 \times 10^{-6} m^2$. The composite string is stretched by a tension of 80 N. Density of steel is 7800 kg m^{-3} and that of aluminium is 2600 kg m^{-3}. The minimum frequency of turning fork, which can produce standing wave in it with node at joint is



(A) 357.3 Hz (B) 375.3 Hz
(C) 337.5 Hz (D) 325.3 Hz

Space For Rough Work

PART – A (Maximum Marks: 32)

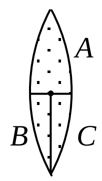
*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Which of the following are true?
(A) Isotopes of hydrogen can fuse to form helium.
(B) Helium isotopes may fuse to form Iron.
(C) Helium isotopes may fuse to form carbon .
(D) Helium isotopes cannot fuse because they are highly stable.

6. A particle moves such that its position from origin is given by $x = 2 + 3\sin^2 \omega t$. Choose correct statement about its motion.
(A) Particle performs S.H.M. of amplitude $\frac{3}{2}$m and frequency $\frac{\omega}{\pi}$.
(B) Particle performs oscillations but no S.H.M.
(C) Particles performs S.H.M. about mean position at $x = \frac{7}{2}$m
(D) Particle performs oscillations of amplitude 3m.

7. Suppose the pressure P and the density ρ of the air are related as $\frac{P}{\rho^n}$ = constant regardless of height (n is constant here and M is the molecular weight of air.)
(A) The corresponding temperature gradient is $\frac{Mg(1-n)}{nR}$
(B) The corresponding temperature gradient is $\frac{Mg(1-n^2)}{nR}$
(C) $\frac{dT}{d\rho}$ is proportional to ρ^{n-2}
(D) $\frac{dP}{d\rho}$ is proportional to ρ^{n-1}

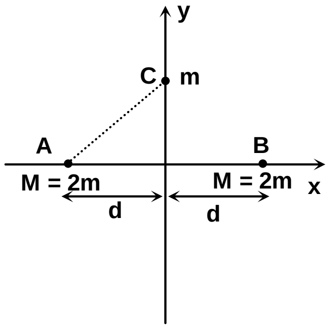
8. A thin, symmetric double-convex lens of power *P* is cut into three parts *A*, *B* and *C* as shown. The power of:

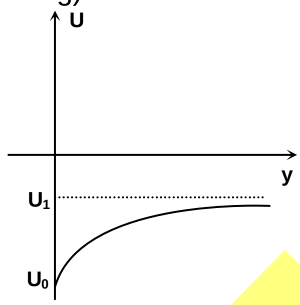


(A) *A* is *P*
(B) *A* is 2*P*
(C) *B* is $\frac{P}{2}$
(D) *B* is $\frac{P}{4}$

Space For Rough Work

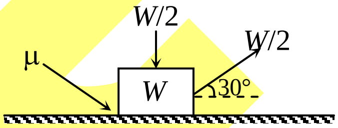
9. Two point masses, each of mass M are kept at rest at points A and B respectively. A third point m is released from infinity with a negligible speed, so that it can move along y-axis under the influence of mutual gravitational attraction on it due to point masses kept at A and B respectively as shown in the figure -1. Figure –2 represents the potential energy of system (includes m, M at A and M at B) with position of m at y-axis. (Neglect any other forces other than gravity) (given $Gm^2/d = 12$ Joule($m = 6$ kg). Choose the correct option(s)





(A) Point mass m will perform periodic motion (B) $U_1 = -24$ Joule
(C) Maximum speed of particle is 24 m/s (D) Maximum speed of particle is 4 m/s

10. A block of weight W is kept on a rough horizontal surface (friction co-efficient μ). Two forces $W/2$ each are applied as shown in the figure. Choose the correct statement.



(A) for $\mu < \frac{\sqrt{3}}{5}$ block will move.

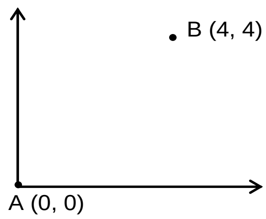
(B) for $\mu < \frac{\sqrt{3}}{5}$, work done by friction force is negative (in ground frame).

(C) $\mu > \frac{\sqrt{3}}{5}$, friction force will perform positive work (in ground frame).

(D) for $\mu < \frac{\sqrt{3}}{5}$ block will not move.

11. Two particles A and B located as shown starts moving simultaneously with velocities (constant)

$\vec{V}_A = +\frac{4}{\sqrt{3}}\hat{i}$ and $\vec{V}_B = -4\hat{j}$



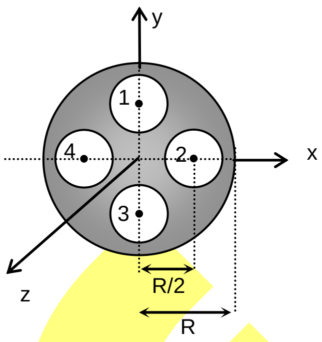
(A) the shortest distance between them is $2(\sqrt{3}-1)$M
(B) the distance between hem first increases then decreases.

(C) the distance between them first decreases then increases.

(D) the magnitude of relative velocity of A w.r.t. B is 4 m/s.

Space For Rough Work

12. Four identical sphere of radius R/4 are taken out from a uniform sphere of mass M and radius R as shown in the figure. The density of sphere is uniform. Choose the correct option(s).



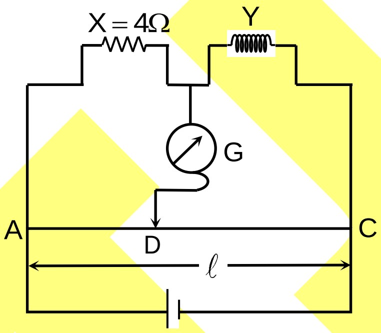
(A) Gravitational field at the origin is zero.
(B) Gravitational potential at the centre of the spherical cavity is $-\frac{GM}{64R}(81-2\sqrt{2})$
(C) The work done by gravitational field in moving a point mass from centre of third cavity to centre of second cavity is zero.
(D) Gravitational potential at the centre of second cavity is $-\frac{GM}{64R}(75-2\sqrt{2})$

PART – B (Maximum Marks: 24)

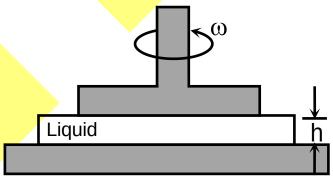
(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. Figure shows a meter bridge wire AC has a uniform cross-section. The length of wire AC is 100 cm. X is a standard resistor of 4 Ω and Y is a coil. When Y is immersed in melting ice the null point is at 40 cm from point A. When the coil Y is heated to 100°C, a 12 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point.

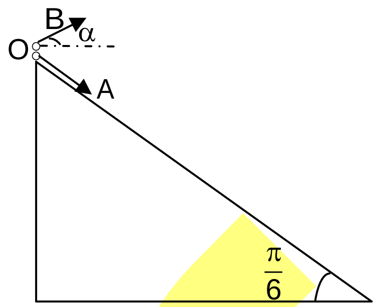


2. A circular disc of a diameter d is slowly rotated in a liquid of large viscosity μ at a small distance h from a fixed surface. An expression for torque T necessary to maintain an angular velocity ω is $\frac{\mu\pi d^4\omega}{Xh}$. Find the value of X.



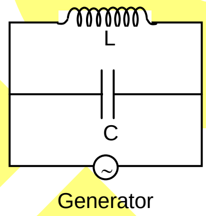
Space For Rough Work

3. Particle A is released from the point on a smooth inclined plane as shown in the diagram. At same instant particle B is projected from the same point with a speed u. making a angle α with a horizontal. If they collide on the inclined plane find $\frac{\alpha^{\circ}}{30}$.



4. A cubical body floats in a mercury bath with half of its volume submerged. What fraction of the body will be inside mercury if a layer of water is poured on the mercury covers the body completely? (Sp. gr. of mercury = 13.6)

5. For the circuit shown in the figure, the current through the inductor is 0.6 A, while the current through the capacitor is 0.4 A. Find the current (in A) drawn from the generator



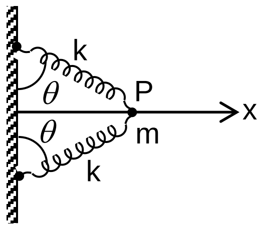
6. Two rods of different materials having coefficient of thermal expansion α_1, α_2 and Young's modulus Y_1, Y_2 are fixed between two massive walls. The rods are heated such that they undergo the same increase in temperature. If $\alpha_1 : \alpha_2 = 2 : 3$, then thermal stresses in rods are equal, then the ratio $Y_1 : Y_2$ is _____ .

7. A student determines a dimensionless quantity,

$$B = \frac{e^n}{Z\varepsilon_0 hc}.$$

Where, e = electric charge,
ε_0 = permittivity of vacuum,
h = Plank's constant
and c = speed of light.
Find the value of n.

8. Two identical springs each of stiffness k are welded at a point P. If a particle of mass m is welded at P. The period of oscillation of the particle in the direction of x is $\frac{\pi}{\sin\theta}\sqrt{\frac{Ym}{k}}$. Find Y



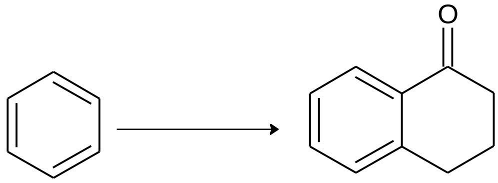
Space For Rough Work

SECTION – II : CHEMISTRY

PART – A (Maximum Marks: 12)

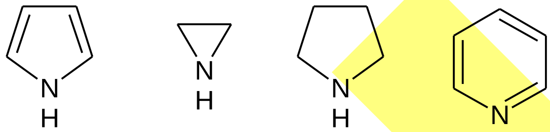
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. The appropriate reagents required for carrying out the following transformation are



(A) (i) Succinic anhydride, $AlCl_3$ (ii) Zn/Hg, Conc.HCl (iii) Polyphosphoric acid
(B) (i) Maleic anhydride, $AlCl_3$ (ii) H_2N-NH_2, KOH (iii) H_2SO_4
(C) (i) Succinic anhydride, $FeCl_3$ (ii) $LiAlH_4$ (iii) H_2SO_4
(D) (i) Pthalic anhydride, $F_3B.OEt_2$ (ii) $HS(CH_2)_2SH$, H^+ (iii) Raney Ni (iv) Polyphosphoric acid

2. The correct order of the pK_a values of the conjugate acids of hetereocyclic compounds given below:



(I) (II) (III) (IV)

(A) II > III > I > IV
(B) IV > II > III > I
(C) III > II > IV > I
(D) III > IV > II > I

3. 2 moles of a non-volatile solute was dissolved in 8 moles of a solvent. If P^o is the vapour pressure of the pure solvent and P_S is the vapour pressure of the solution, then the correct relationship is

(A) $P^o - P_S = \frac{8}{10}$
(B) $\frac{P^o}{P_S} = \frac{5}{4}$
(C) $P^o + P_S = \frac{4}{5}$
(D) $P^o \times P_S = \frac{10}{8}$

4. Which of the following transition element cannot form complex by using the 3d orbital?

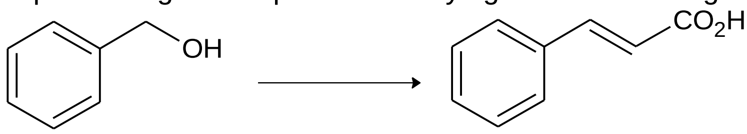
(A) Co
(B) Mn
(C) Zn
(D) Fe

Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. The appropriate reagents required for carrying out the following transformation are



(A) (i) PCC, CH_2Cl_2 (ii) $Ph_3P = CHCOOEt$ (iii) aq. NaOH, Heat then acidify
(B) (i) CrO_3, H_2SO_4, aq acetone (ii) Ac_2O, NaOAc
(C) (i) MnO_2 (ii) $CH_2(CO_2H)_2$, piperidine, pyridine
(D) (i) PCC, CH_2Cl_2 (ii) $BrCH_2CO_2C(CH_3)_3$, Zn (iii) H_3O^+ + hot

6. The ion(s) that exhibit only charge transfer band in the absorption spectra (UV-Visible region) is/are
(A) $[Cr(C_2O_4)_3]^{3-}$
(B) $[CrO_4]^{2-}$
(C) $[ReO_4]^{-}$
(D) $[NiO_2]^{2-}$

7. The type(s) of interaction(s) that hold layers of graphite together is(are)
(A) π-π stacking
(B) van der Waal's
(C) hydrogen bonding
(D) columbic

8. True statement(s) about Langmuir adsorption isotherm is(are)
(A) valid for monolayers coverage
(B) all adsorption sites are equivalent
(C) there is dynamic equilibrium between force gas and adsorbed gas
(D) adsorption probability is independent of occupancy at the neighbouring sites

9. The diatomic molecule(s) that has(have) two π-type bond is(are)
(A) B_2
(B) C_2
(C) N_2
(D) O_2

10. For which of the following electrochemical cells $E_{Cell} = E^{o}_{Cell}$?
(A) $Zn(s) \mid Zn^{2+}(1\ M) \parallel Ag^{+}(1\ M) \mid Ag(s)$
(B) $Fe(s) \mid Fe^{2+}(0.01\ M) \parallel Cu^{2+}(0.01\ M) \mid Cu(s)$
(C) $Sn(s) \mid Sn^{2+}(0.01\ M) \parallel Ag^{+}(0.01\ M) \mid Ag(s)$
(D) $Pt, H_2(1\ atm) \mid H^{+}(1\ M) \parallel Au^{+}(1\ M) \mid Au(s)$

Space For Rough Work

11. $Cl_2\,(\text{excess}) + NH_3 \longrightarrow P + Q$

$Cl_2 + NH_3\,(\text{excess}) \longrightarrow R + S$

Which of the following is/are the product(s) of above reactions?

(A) HCl (B) NH_4Cl

(C) N_2 (D) H_2

12. $POCl_3 + H_2O \longrightarrow \text{Products}$

The product(s) of above reaction is/are

(A) H_3PO_4 (B) HPO_3

(C) H_3PO_3 (D) H_3PO_2

PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. Hydrolysis of 15.45 g of benzonitrile produced 10.98 g of benzoic acid. The percentage yield of acid formed is __________

2. The total number of isomers possible for the molecule $[Co(NH_3)_4Cl(NO_2)]^+$ is ________

3. Specific rotation of the (R)– enantiomer of a chiral compound is 48. The specific rotation of a sample of this compound which contains 25% of (S)-enantiomer is

4. A carbonyl compound(P) reacts with conc.H_2SO_4 to form 1, 3, 5-trimethyl benzene. If the molar mass of 'P' is X g mol^{-1}. What is the value of $\frac{X}{4}$?

5. 300 mL of 0.5 M CH_3COOH was mixed with 200 mL of 0.75 M NaOH. What is the pH of the solution mixture after complete reaction?
[K_a of $CH_3COOH = 10^{-5}$, log 0.3 = -0.5]

6. The half-life of a zero order reaction is 2.4 minute. How much time in minute is required for 100% completion of the reaction?

Space For Rough Work

7. $$HOOC-CH_2-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$$

is an alpha-amino acid. The pK_a values of the functional groups present in it are

Functional group	pK_a
α - COOH	2.1
$\alpha-NH_3^+$	9.8
γ - COOH	3.9

If the isoelectric point of the amino acid is X, what is the value of $\frac{X}{2}$?

8. $$NO(g) \xrightarrow{Heat} A(g) + B(g)$$

$$A(g) \xrightarrow{Mg(s)} C(s) \xrightarrow{H_2O} D(soln) + E(g)$$

$$B(g) \xrightarrow{K(s)} F(s) \xrightarrow{H_2O} G(soln) + H(sol) + B(g)$$

If x = Number of atom(s) that constitute one molecule of C(s).

y = Number of unpaired electrons present in a molecule of F(s)

What is the value of $\left(\frac{x+y}{4}\right)$?

Space For Rough Work

SECTION – III : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. $\frac{(2x+1)}{(x^2+4x+1)^{3/2}}dx$

(A) $\frac{x^3}{(x^2+4x+1)^{1/2}}+C$ (B) $\frac{x}{(x^2+4x+1)^{1/2}}+C$

(C) $\frac{x^2}{(x^2+4x+1)^{1/2}}+C$ (D) $\frac{1}{(x^2+4x+1)^{1/2}}+C$

2. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is $\frac{1}{5}$. The probability that bus B will be late is $\frac{7}{25}$. The probability that bus B is late given that bus A is late is $\frac{9}{10}$. Then the probabilities:

(i) neither bus will be late on a particular day and
(ii) bus A is late given that bus B is later, are respectively

(A) $\frac{2}{25}$ and $\frac{12}{28}$ (B) $\frac{18}{25}$ and $\frac{22}{28}$

(C) $\frac{7}{10}$ and $\frac{18}{28}$ (D) $\frac{12}{25}$ and $\frac{2}{28}$

3. Number of solution of the equation $\frac{3\sin\theta-\sin3\theta}{1+\cos\theta}+\frac{3\cos\theta+\cos3\theta}{1-\sin\theta}=4\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)$ in the interval $(-10\pi, 8\pi]$ is equal to:

(A) 8 (B) 9
(C) 16 (D) 18

4. If a circle of radius 3 units is touching the lines $\sqrt{3}y^2-4xy+\sqrt{3}x^2=0$ in the first quadrant then the length of chord of contact to this circle, is:

(A) $\frac{\sqrt{3}+1}{2}$ (B) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(C) $3\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$ (D) $3\frac{\left(\sqrt{3}+1\right)}{2}$

Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A function f is defined $f(x)=\int_0^{\pi}\cos t\cos(x-t)dt,\ 0\le x\le 2\pi$ then which of the following hold(s) good?

(A) $f(x)$ is continuous but not differentiable in $(0, 2\pi)$

(B) Maximum value of f is π

(C) There exists atleast one $c\in(0, 2\pi)$ such that $f'(c)=0$

(D) Minimum value of f is $-\frac{\pi}{2}$

6. Let $f(x)$ is a real valued function defined by : $f(x)=x^2+x^2\int_{-1}^{1}t.f(t)dt+x^3\int_{-1}^{1}f(t)dt$ then which of the following hold(s) good?

(A) $\int_{-1}^{1}t.f(t)dt=\frac{10}{11}$

(B) $f(1)+f(-1)=\frac{30}{11}$

(C) $\int_{-1}^{1}t.f(t)dt>\int_{-1}^{1}f(t)dt$

(D) $f(1)-f(-1)=\frac{20}{11}$

7. A function $y=f(x)$ satisfying the differential equation $\frac{dy}{dx}.\sin x-y\cos x+\frac{\sin^2 x}{x^2}=0$ is such that, $y\to 0$ as $x\to\infty$ then the statement which is correct is:

(A) $\lim_{x\to 0}f(x)=1$

(B) $\int_0^{\pi/2}f(x)dx$ is less than $\frac{\pi}{2}$

(C) $\int_0^{\pi/2}f(x)dx$ is greater than unity

(D) $f(x)$ is an odd function

8. Let $f(x)$ be a differentiable function satisfying $\int_0^{2x}x f(t)dt+2\int_x^0 t f(2t)dt=2x^4-2x^3$, for all $x\in R$ then which of the following is/are correct?

(A) Minimum value of $f(x)$ is equal to $\frac{-3}{4}$.

(B) $f(|x|)$ is non – derivable at exactly one value of x.

(C) Area bounded by $y=f(x)$ and x – axis is equal to 2

(D) $\lim_{x\to 1}\frac{f(x)}{x-1}$ exists and is equal to 3.

Space For Rough Work

9. Given the matrices A and B as $A=\begin{bmatrix}1 & -1\\ 4 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 2 & -2\end{bmatrix}$. The two matrices X and Y are such that XA = B and AY = B then which of the following hold(s) true?

(A) $X=\frac{1}{3}\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}$
(B) $Y=\frac{1}{3}\begin{bmatrix}3 & 0\\ 4 & 0\end{bmatrix}$
(C) $\det.X=\det.Y$
(D) $3(X+Y)=\begin{bmatrix}4 & -1\\ 4 & 2\end{bmatrix}$

10. If $A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$ and $D(\vec{d})$ are four points such that
$\vec{a}=-2\hat{i}+4\hat{j}+3\hat{k};\ \vec{b}=2\hat{i}-8\hat{j};\ \vec{c}=\hat{i}-3\hat{j}+5\hat{k};\ \vec{d}=4\hat{i}+\hat{j}-7\hat{k}$
d is the shortest distance between the lines AB and CD, then which of the following is true?

(A) $d=0$, hence AB and CD intersect
(B) $d=\dfrac{\left[\overrightarrow{AB}\,\overrightarrow{CD}\,\overrightarrow{BD}\right]}{\left[\overrightarrow{AB}\times\overrightarrow{CD}\right]}$
(C) AB and CD are skew lines and $d=\dfrac{23}{13}$
(D) $d=\dfrac{\left[\overrightarrow{AB}\,\overrightarrow{CD}\,\overrightarrow{AC}\right]}{\left[\overrightarrow{AB}\times\overrightarrow{CD}\right]}$

11. TP and TQ are tangents to parabola $y^2=4x$ and normals at P and Q intersect at a point R on the curve The locus of the centre of the circle circumscribing ΔTPQ is a parabola whose:

(A) vertex is (1, 0)
(B) foot of directrix is $\left(\frac{7}{8},0\right)$
(C) length of latus – rectum is $\frac{1}{4}$
(D) focus is $\left(\frac{9}{8},0\right)$.

12. Let tangents at $A(z_1)$ and $B(z_2)$ are drawn to the circle $|z|=2$. Then which of the following is/are correct?

(A) The equation of tangent at A is given by $\frac{z}{z_1}+\frac{\bar{z}}{\bar{z}_1}=2$.
(B) If tangents at $A(z_1)$ and $B(z_2)$ intersect at $P(z_p)$, then $z_p=\dfrac{2z_1z_2}{z_1+z_2}$.
(C) Slope of tangent at $A(z_1)$ is $\frac{1}{i}\left(\dfrac{z_1+\bar{z}_1}{z_1+z_2}\right)$
(D) If points $A(z_1)$ and $B(z_2)$ on the circle $|z|=2$ are such that $z_1+z_2=0$, then tangents intersect at $\frac{\pi}{2}$.

Space For Rough Work

PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. One root of the cubic $2z^3-(5+6i)z^2+9iz+1-3i=0$ is real. If all the three roots of this cubic are plotted on the complex plane, then find the eight times area of the triangle formed by them.

2. Let $\{a_n\},\{b_n\},\{c_n\}$ be sequences such that
(I) $a_n+b_n+c_n=2n+1$;
(II) $a_nb_n+b_nc_n+c_na_n=2n-1$;
(III) $a_nb_nc_n=-1$
(IV) $a_n<b_n<c_n$
Then the value of $\left|\lim\limits_{n\to\infty}(n\alpha_n)\right|=\frac{m}{n}$ where a and b are co – prime then find the value of $(m+2n)$.

3. Let $f(x)=x^2+ax+3$ and $g(x)=x+b$, where $F(x)=\lim\limits_{n\to\infty}\frac{f(x)-x^{2n}g(x)}{1+x^{2n}}$. If $F(x)$ is continuous at $x=1$ and $x=-1$ then find the value of (a^2+b^2).

4. Let $f:R\to R^+$ be a differentiable function satisfying $f'(x)=2f(x)\forall x\in R$. Also $f(0)=1$ and $g(x)=f(x).\cos^2 x$. If n_1 represent number of points of local maxima of $g(x)$ in $[-\pi,\pi]$ and n_2 is the number of points of local minima of $g(x)$ in $[-\pi,\pi]$ and n_3 is the number of points in $[-\pi,\pi]$ where $g(x)$ attains its global minimum value, then find the value of $(n_1+n_2+n_3)$.

Space For Rough Work

5. $\int_1^2 \frac{(x^2-1)dx}{x^3.\sqrt{2x^4-2x^2+1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$.

6. Given a curve C. Let the tangent line at $P(x,y)$ on C is perpendicular to the line joining P and Q (1, 0). If the line $2x+3y-15=0$ is tangent to the curve C then the length of the tangent from the point (5, 0) to the curve C is $\sqrt{n}$ (where $n \in N$). Find the value of n.

7. If $\sin q \neq \cos q$ and x, y, z satisfy the equations

$$x\cos p - y\sin p + z = \cos q + 1$$
$$x\sin p + y\cos p + z = 1 - \sin q$$
$$x\cos(p+q) - y\sin(p+q) + z = 2$$

then find the value of $x^2+y^2+z^2$.

8. Find the number of solutions satisfying the equation $\cos\frac{x}{2} - \sin\frac{x}{2} = \sin x - 1$, where $|x-\pi| \le \frac{3\pi}{2}$.

Space For Rough Work

Q. P. Code: 100959
Answers

SECTION – I : PHYSICS

PART – A

1.	B	2.	C	3.	C	4.	C
5.	AC	6.	AC	7.	ACD	8.	AC
9.	ABD	10.	AB	11.	AC	12.	ABC

PART – B

1.	1	2.	4	3.	2	4.	0.46
5.	0.20	6.	1.50	7.	2.00	8.	2.00

SECTION – II : CHEMISTRY

PART – A

1.	A	2.	D	3.	B	4.	C
5.	ACD	6.	BC	7.	AB	8.	ABCD
9.	BC	10.	ABD	11.	ABC	12.	AB

PART – B

1.	60	2.	4	3.	24	4.	13.5
5.	9.25	6.	4.8	7.	1.5	8.	1.5

SECTION – III : MATHEMATICS

PART – A

1.	B	2.	C	3.	B	4.	C
5.	CD	6.	BD	7.	ABC	8.	ABD
9.	CD	10.	BCD	11.	ABD	12.	ABC

PART – B

1.	2	2.	5	3.	17	4.	8
5.	125	6.	3	7.	2	8.	4

Answers & Solutions

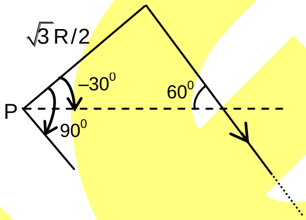
SECTION – I : PHYSICS

PART – A

1. **B**

Sol. $\lambda = \dfrac{1240\,\text{eV}\ \ \text{nm}}{11.2} \approx 1100\,\text{Å}$. Ultraviolet region

2. **C**



Sol. $B = \dfrac{\mu_0 I}{4\pi\sqrt{3}\,R/2}\left[\sin 90^o + \sin(-30^o)\right]$

$= \dfrac{\mu_0 I}{4\sqrt{3}\,R}$

3. **C**

Sol. $\because$ E is continuously decreasing along radially outward direction.

$\therefore \dfrac{V_1 - V_2}{t_1} > \dfrac{V_2 - V_3}{t_2}$; $\therefore$ $t_1 < t_2$

4. **C**

Sol. Mass per unit length, $\mu = \dfrac{m}{\ell} = \dfrac{\rho A\ell}{\ell} = \rho A$

$\mu_s = \mu_{A\ell} = 78 \times 10^{-4}$ kg/m

$\therefore$ Speed of wave is same in both wire

$V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{80 \times 10^4}{78}} = \dfrac{2 \times 10^2}{\sqrt{3.9}}$

$\nu_{min} = \dfrac{V}{\lambda_{max}} = \dfrac{200}{\sqrt{3.9} \times 0.3}$ $\left[\dfrac{\lambda_{max}}{2} = 15\text{ cm for C as a node}\right]$

$= 337.5$ Hz

5. **AC**

Sol. Iron is most stable nucleus.

6. **AC**

Sol. $x = 2 + \dfrac{3}{2}(1 - \cos 2\omega t)$

$x = 2 + \dfrac{3}{2} - \dfrac{3}{2}\cos 2\omega t$

$\therefore$ $x = \dfrac{7}{2}$m is mean position; Amplitude is $\dfrac{3}{2}$m.

7. **ACD**

Sol. The temperature gradient is given by

$\dfrac{dT}{dh} = \dfrac{dT}{d\rho}\cdot\dfrac{d\rho}{dP}\cdot\dfrac{dP}{dh}$ … (i)

From, gdh□dP =

$\dfrac{dP}{dh} = -\rho g$ … (ii)

Given that $\dfrac{p}{\rho^n}$ = c (constant)

$\therefore\ P = c\rho^n$

$$\frac{dP}{d\rho} = cn\rho^{n-1} \quad \text{… (iii)}$$

We know that

$$P = \rho\frac{R}{m}T$$

Or $c\rho^n = \rho\frac{R}{M}T$

$\therefore \quad T = \frac{M}{R}c\rho^{n-1}$

Or $\frac{dT}{d\rho} = \frac{M}{R}c(n-1)\rho^{n-2}$ … (iv)

From eps. (ii), (iii) and (iv) substituting these values in eq. (i) and solving, we get :

$$\frac{dT}{dh} = -\frac{Mg(n-1)}{nR}$$

8. **AC**

Sol. Let *f* be the focal length of lens.
Then focal length of part *A* is *f* and of part *B* and *C* is 2*f* each.
$\therefore$ Power of *A*, *B* and *C* is *P*, *P*/2 and *P*/2 respectively.

9. **ABD**

Sol. $U = \frac{-2GMm}{\sqrt{d^2+y^2}} + \frac{-GM^2}{2d}$

$= \frac{-2GMm}{\sqrt{d^2+y^2}} - \frac{2Gm^2}{d}$

$= -2Gm^2\left[\frac{2}{\sqrt{d^2+y^2}} + \frac{1}{d}\right]$

$= |\Delta U| = \frac{1}{2}mv^2$

10. **AB**

Sol. Show components of forces parallel to the surface (sliding force) and perpendicular to it.

11. **AC**

Sol. To find least separation put the condition that $\vec{V}_A - \vec{V}_B$ should be along the line perpendicular to $\vec{r}_A - \vec{r}_B$.

12. **ABC**

Sol. Mass corresponding to removed sphere = $\frac{(4/3)\pi(R/4)^3}{(4/3)R^3}M = \frac{M}{64}$.

So, potential due to the sphere at any centre of cavity

$= -\frac{GM}{64R}\left(88-6-1-2\sqrt{2}\right) = \frac{-GM}{64R}\left(81-2\sqrt{2}\right)$

PART – B

1. **1**

Sol. According to given condition,

$$\frac{i\times 4}{i\times R_0} = \frac{AD}{DC} = \frac{40}{60} = \frac{2}{3}$$

$\Rightarrow \quad R = 12$

Now, $R = R_0(1+\alpha\Delta T)$

$12 = 6(1+\alpha 100)$

$\Rightarrow \quad \alpha = 1\times 10^{-2}$

So, $x = 1$

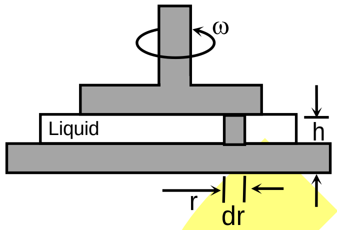
2. **4**

Sol. Consider an element of disc at a radius r and having a width dr.
Linearly velocity at this radius = ωr

Shear stress, $\tau = \mu \frac{du}{dy}$

Assuming, the gap h to be small, so that the velocity distribution may be assumed linear

$$\tau = \mu \times \frac{v}{h} = \mu \frac{\omega r}{h}$$



Viscous force, dF = τ × Area
= $\tau \times 2\pi r dr$

Torque dT on the element,

$$dT = dF \times r = \tau 2\pi r^2 dr$$

or $$dT = \frac{\mu\omega r}{h} \times 2\pi r^2 dr = \frac{2\pi\mu\omega r^2 dr}{h}$$

Total torque,

$$T = \int_0^{d/2} \frac{2\pi\mu\omega r^3 dr}{h} = \frac{\mu\pi d^4\omega}{4h}$$

Thus, x = 4

3. **2**

Sol. To collide A and B will have same displacement along the incline, in same time.

4. **0.46**

5. **0.20**

Sol. I_L and I_C will be in opposite phases.
$I_{net} = I_L - I_C$
$= 0.6 - 0.4 = 0.2$ A

6. **1.50**

Sol. Thermal stress = $Y\alpha\Delta\theta$
As, thermal stresses are equal

$$y_1\alpha_1\Delta\theta_1 = y_2\alpha_2\Delta\theta_2$$

$$\Rightarrow \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

7. **2.00**

Sol. Dimension of B = $[M^0L^0T^0]$
Dimension of e = [AT]
Dimension of $\varepsilon_0 = [A^2M^{-1}L^{-3}T^4]$
Dimension of h = $[ML^2T^{-1}]$
Dimension of c = $[LT^{-1}]$
Equating,

$$[M^0L^0T^0] = \frac{[AT]^n}{[A^2M^{-1}L^{-3}T^4][ML^2T^{-1}][LT^{-1}]}$$

$$\Rightarrow \quad [M^0L^0T^0] = [A^{n-2}L^0T^{n-2}]$$

$\therefore$ $n - 2 = 0$ or $n = 2$

8. **2.00**

Sol. $$\frac{\delta F}{\delta x} = k_{eff} \text{ (numerically)}$$

Let us find δx (static deformation of the system).
Since the particle is in equilibrium,

$\delta F = 2\delta F_s \sin\theta$

By Pythagoras theorem, $x^2 + y^2 = l^2$

Taking differentials of both sides, we have $2x\delta x + 2y\delta y = 2l\delta l$

Since y is constant, $\delta y = 0$

$\delta l = \frac{x}{l}\delta x$

$\frac{\delta F}{\delta x} = 2k\sin^2\theta$

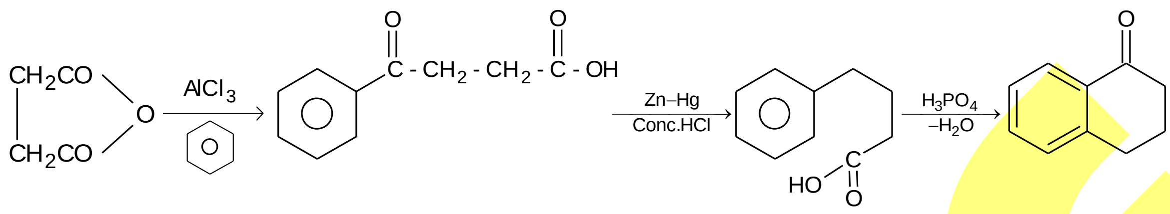
$k_{eff} = 2k\sin^2\theta$

Substituting k_{eff}, we have $T = \frac{\pi}{\sin\theta}\sqrt{\frac{2m}{k}}$

SECTION – II : CHEMISTRY

PART – A

1. A

Sol.


2. D

Sol. The order is due to the following order of basic strength: III > IV > II > I.

3. B

Sol. $\dfrac{p^o - p_s}{p^o} = \dfrac{2}{8+2} = \dfrac{2}{10}$

4. C

Sol. Zn has the full-filled $3d^{10}$ configuration.

5. ACD

Sol. Benzaldehyde is formed initially followed by nucleophilic addition at carbonyl carbon.

6. BC

Sol. Charge transfer band arises due to polarisation of either ligand or metal ion.

7. AB

Sol. π - π stacking makes an orderly arrangement of graphite layers which are held by van der Waal's forces.

8. ABCD

Sol. Langmuir adsorption is valid for chemical adsorption.

9. BC

Sol. Because the bond order of both N_2 and C_2 is three.

10. ABD

Sol. $E_{Cell} = E^o_{Cell} - \dfrac{0.0591}{n}\log\dfrac{C_1}{C_2}$

$E_{Cell} = E^o_{Cell}$ if $\dfrac{C_1}{C_2} = 1$

11. ABC

Sol. P = NCl_3, Q = HCl, R = NH_4Cl, S = N_2

12. AB

Sol. Hydrolysis followed by loss of water takes place from $POCl_3$. (Acts as a dehydrating agent)

PART – B

1. 60

Sol. $\%\text{yield} = \dfrac{\text{Experimental yield}}{\text{Theoretical yield}} \times 100 = \dfrac{10.98}{18.3} \times 100 = 60\%$

2. 4

Sol. Number of geometrical isomers = 2

Number of linkage isomer = 2

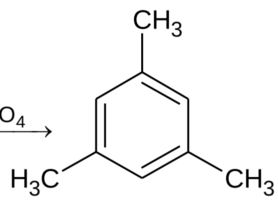
3. 24

Sol. S.P rotation of R- = +48°
S.P rotation of S- = -48°
Sample contains 75% R and 25% S
$\therefore$ S.P rotation of the sample

$$= \frac{75 \times (+48) + 25 \times (-48)}{100} = \frac{3600 - 1200}{100} = 24$$

4. 13.5

Sol. P is CH_3COCH_3

$3CH_3COCH_3 \xrightarrow{Conc.H_2SO_4}$ 

5. 9.25

Sol. $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

$$\left[CH_3COO^-\right] = \frac{150}{500} = 0.3$$

$$\therefore pH = \frac{1}{2}\left[p^{K_w} + p^{K_a} + \log C\right] = \frac{1}{2}[14 + 5 - 0.5] = 9.25$$

6. 4.8

Sol. $t_{Completion} = 2 \times t_{½}$ for a zero order reaction.

7. 1.5

Sol. $X = \frac{1}{2}[2.1 + 3.9] = \frac{1}{2}[6] = 3$

$$\frac{X}{2} = 1.5$$

8. 1.5

Sol. A = N_2, B = O_2, C = Mg_3N_2, D = $Mg(OH)_2$, E = NH_3, F = KO_2, G = KOH, H = H_2O_2

SECTION – III : MATHEMATICS

PART – A

1. B

Sol. $\dfrac{2x+1}{\left(x^2+4x+1\right)^{3/2}}dx=\int\dfrac{2x+1}{x^3\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}}dx$

$=\int\dfrac{2x^{-2}+x^{-3}}{\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}}dx$

Now put $\dfrac{1}{x^2}+\dfrac{4}{x}+1=t^2$

2. C

Sol.

(i) $P(A)=\dfrac{1}{5};P(B)=\dfrac{7}{25};P\left(\dfrac{B}{A}\right)=\dfrac{9}{10}$

$P\left(\overline{A}\cap\overline{B}\right)=1-P(A\cup B)$

$=1-\left[P(A)+P(B)-P(A\cap B)\right]$

$=1-\left[\dfrac{1}{5}+\dfrac{7}{25}-P(A).P\left(\dfrac{B}{A}\right)\right]$

$=1-\left[\dfrac{1}{5}+\dfrac{7}{25}-\dfrac{1}{5}.\dfrac{9}{10}\right]=\dfrac{7}{10}$

(ii) $P\left(\dfrac{A}{B}\right)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{P(A).P\left(\frac{B}{A}\right)}{P(B)}=\dfrac{\frac{1}{5}.\frac{9}{10}}{\frac{7}{25}}$

$=\dfrac{9}{50}\times\dfrac{25}{7}=\dfrac{9}{14}=\dfrac{18}{28}$

3. B

Sol. $\dfrac{3\sin\theta-\left(3\sin\theta-4\sin^3\theta\right)}{1+\cos\theta}+\dfrac{3\cos\theta+\left(4\cos^3\theta-3\cos\theta\right)}{1-\sin\theta}$

$=4\sqrt{2}\cos\left(\theta+\dfrac{\pi}{4}\right)$

$\Rightarrow 4\sin\theta(1-\cos\theta)+4\cos\theta(1+\sin\theta)$

$=4\sqrt{2}\cos\left(\theta+\dfrac{\pi}{4}\right)$

$\Rightarrow 4\sqrt{2}\sin\left(\theta+\dfrac{\pi}{4}\right)=4\sqrt{2}\cos\left(\theta+\dfrac{\pi}{4}\right)$

$\Rightarrow \tan\left(\theta+\dfrac{\pi}{4}\right)=1$

$\Rightarrow \theta+\dfrac{\pi}{4}=n\pi+\dfrac{\pi}{4},n\in I$

But $1+\cos\theta\neq 0$

$\therefore\ \theta=2n\pi\forall n\in I$

Hence number of solution are 9 i.e.

$\theta = -8\pi, -6\pi, -4\pi, -2\pi, 0, 2\pi, 4\pi, 6\pi, 8\pi$.

4. C

Sol. Given equation $\sqrt{3}y^2 - 4xy + \sqrt{3}x^2 = 0$

$\Rightarrow \quad \sqrt{3}y^2 - 3xy - xy + \sqrt{3}x^2 = 0$

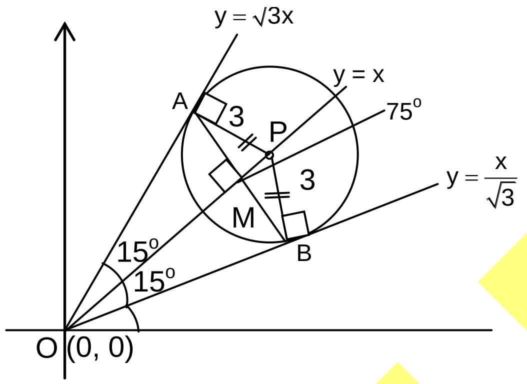
$\Rightarrow \quad \left(\sqrt{3}y - x\right)\left(y - \sqrt{3}x\right) = 0$

$\Rightarrow \quad y = \dfrac{x}{\sqrt{3}},\ y = \sqrt{3}\,x$

$\therefore \quad \angle APO = 75^o$

In ΔAMP, $\sin 75^o = \dfrac{AM}{3}$

$\Rightarrow \quad AM = 3\sin 75^o$



Now, length of chord of contact AB = 2AM

$= 2\left(3\sin 75^o\right) = 6\sin 75^o$

$= 6\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right) = 3\dfrac{\left(\sqrt{3}+1\right)}{\sqrt{2}}$

5. CD

Sol. $f(x) = \int_0^{\pi} \cos t \cos(x - t)\,dt$ ……(i)

$= \int_0^{\pi} -\cos t.\cos(x - \pi + t)\,dt$ (Using King)

$f(x) = \int_0^{\pi} \cos t.\cos(x + t)\,dt$ …..(ii)

Equation (i) + (ii) gives

$2f(x) = \int_0^{\pi} \cos t\left(2\cos x.\cos t\right)dt$

$\therefore f(x) = \cos x \int_0^{\pi} \cos^2 t\,dt = 2\cos x \int_0^{\pi/2} \cos^2 t\ dt$

$\Rightarrow f(x) = \dfrac{\pi\cos x}{2}$

Now verify. Only (a) and (b) are correct.

6. BD

Sol. We have $f(x) = x^2 + ax^2 + bx^3$

Where $a = \int_{-1}^{1} t.ft(dt)$ and $b = \int_{-1}^{1} f(t)\,dt$

Now $a=\int_{-1}^{1} t\left[(a+1)t^2+bt^3\right]dt$

$a=2b\int_{-1}^{1} t^4\,dt=\frac{2b}{5}$ ……(i)

Again $b=\int_{-1}^{1} f(t)dt=\int_{-1}^{1}\left[(a+1)t^2+bt^3\right]dt$

$2\int_{0}^{1}(a+1)t^2dt$

$b=\frac{2(a+1)}{3}$ …….(ii)

From equation (i) and (ii)

$\frac{5a}{2}=\frac{2(a+1)}{3}$

$\left(\frac{5}{2}-\frac{2}{3}\right)a=\frac{2}{3}$

$a=\frac{4}{11}$ and $b=\frac{10}{11}$

Hence $\int_{-1}^{1} t.f(t)dt=\frac{4}{11}$ and $\int_{-1}^{1} f(t)dt=\frac{10}{11}$

$\therefore$ $f(x)=(a+1)x^2+bx^3$

$\left.\begin{array}{l} f(1)=(a+1)+b \\ f(-1)=(a+1)-b \end{array}\right]$

$f(1)+f(-1)=2(a+1)=\frac{30}{11}$ and $f(1)-f(-1)=2b=\frac{20}{11}$

7. ABC

Sol. $f(x)=\frac{\sin x}{x}$

8. ABD

Sol. $x\int_{0}^{2x} f(t)dt+2\int_{x}^{0} t\,f(2t)dt=2x^4-2x^3$

Differentiate with respect to x

$x(2f(x))+\left(\int_{0}^{2x} f(t)dt\right)-2x\,f(x)=8x^3-6x^2$

Differentiate with respect to x

$2f(2x)=24x^2-12x$

$\Rightarrow$ $f(2x)=12x^2-6x$

Put $2x=y$

$\Rightarrow$ $f(y)=3y^2-3y$

$\therefore$ $f(x)=3x(x-1)$

Now, verify all the options.

9. CD

Sol. Not that A is non – singular but B is singular hence only A^{-1} exists

Now $XA=B$

$\Rightarrow$ $X=BA^{-1}$ …….(i)

and $AY=B$

$\Rightarrow \quad Y = A^{-1}B$ ....(ii)

Also $\quad A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 1 \\ -4 & 1 \end{bmatrix}$

Now verify.

10. BCD

Sol. $\overrightarrow{AB} = \vec{b} - \vec{a} = 4\hat{i} - 12\hat{j} - 3\hat{k}$

$\overrightarrow{CD} = \vec{d} - \vec{c} = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\overrightarrow{AC} = \vec{c} - \vec{a} = 3\hat{i} - 7\hat{j} + 2\hat{k}$

$\overrightarrow{BD} = \vec{d} - \vec{b} = 2\hat{i} + 9\hat{j} - 7\hat{k}$

By definition $d = \frac{\left(\overrightarrow{AB} \times \overrightarrow{CD}\right).\overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ ....(i)

$= \frac{\left(\overrightarrow{AB} \times \overrightarrow{CD}\right).\overrightarrow{BD}}{\left|\overrightarrow{AB} \times \overrightarrow{CD}\right|}$ ......(ii)

$\overrightarrow{AB} \times \overrightarrow{CD} = 13\left(12\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$\therefore \quad \left|\overrightarrow{AB} \times \overrightarrow{CD}\right| = 169$

$\therefore \quad d = \frac{13\left(12\hat{i} + 3\hat{j} + 4\hat{k}\right)}{169}.\left(3\hat{i} - 7\hat{j} + 2\hat{k}\right)$

$= \frac{23}{13}$ [Using equation (i)]

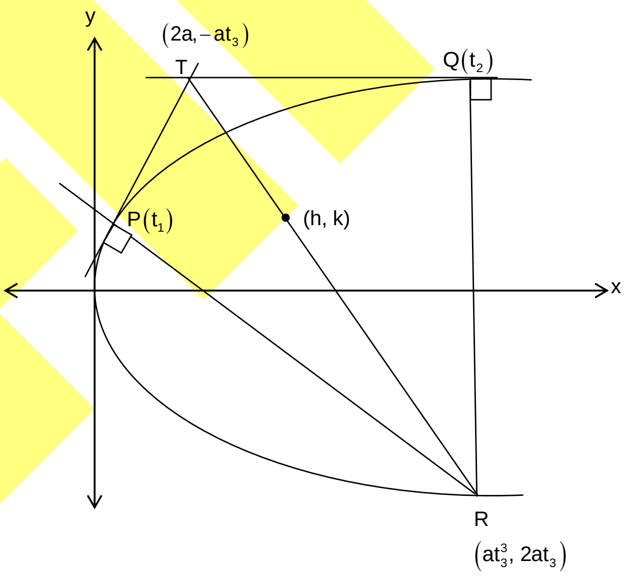
Also $d = \frac{13\left(12\hat{i} + 3\hat{j} + 4\hat{k}\right)}{169}.\left(2\hat{i} - 9\hat{j} + 7\hat{k}\right)$

$= \frac{23}{13}$ [Using equation (ii)]

11. ABD

Sol. We have $2h = t_3^2 + 2$ ....(i)

$2k = t_3$ ....(ii)



Here $a = 1$

$t_1 t_1 = 2$

$t_1 + t_2 + t_3 = 0$

$\therefore \quad 2h = 4k^2 + 2$

$\therefore \quad 2y^2 = x - 1$

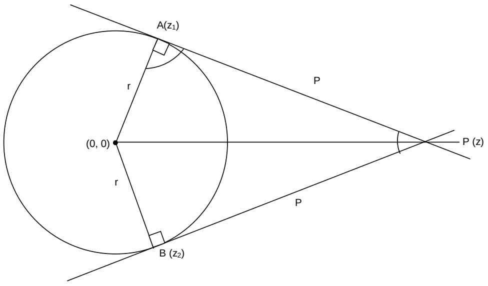
$$y^2 = \frac{1}{2}(x-1) \qquad \text{(Parabola)}$$

Now interpret.

12. ABC

Sol.

(A) $\arg\left(\frac{z - z_1}{0 - z_1}\right) = \frac{\pi}{2}$



$\Rightarrow \frac{z_1 - z}{z_1}$ is purely imaginary.

$$\frac{z_1 - z}{z_1} + \frac{\bar{z}_1 - \bar{z}}{\bar{z}_1} = 0$$

$$\Rightarrow \frac{z}{z_1} + \frac{\bar{z}}{\bar{z}_1} = 2$$

(B) By applying rotation, we get

$$\frac{z - z_1}{0 - z_1} = \frac{p}{r} e^{\frac{i\pi}{2}} \qquad \text{..........(i)}$$

Also $\frac{0 - z_2}{z - z_2} = \frac{r}{p} e^{\frac{i\pi}{2}}$ ....(ii)

$\therefore$ On multiplying equation (i) and (ii), we get

$$z_p = \frac{2z_1 z_2}{z_1 - z_2}$$

(C) As equation of tangent at $A(x_1 y_1)$ is $xx_1 + yy_1 = 4$

$\therefore$ Slope of tangent $\frac{-x_1}{y_1} = \frac{-2x_1}{2y_1}$

$$= -i\left(\frac{z_1 + \bar{z}_1}{z_1 - \bar{z}_1}\right) = \frac{1}{i}\left(\frac{z_1 + \bar{z}_1}{z_1 + \bar{z}_1}\right)$$

(D) Clearly, tangents are parallel lines.

(As $A(z_1)$ and $B(z_2)$ are ends of diameter of circle.)

PART – B

1. 2

Sol. Let α be the real root

$$2\alpha^3 - (5 + 6i)\alpha^2 + 9i\alpha + 1 - 3i = 0$$

Equating real and imaginary parts

$$2\alpha^3 - 5\alpha^2 + 1 = 0 \qquad \text{......(i)}$$

and $-6\alpha^2 + 9\alpha - 3 = 0$

$\Rightarrow$ $2\alpha^2 - 3\alpha + 1 = 0$

$\Rightarrow$ $2\alpha^2 - 2\alpha - \alpha + 1 = 0$

$2\alpha(\alpha-1)-1(\alpha-1)=0$

$\Rightarrow \qquad \alpha=1 \text{ or } \alpha=\frac{1}{2}$

But $\alpha=\frac{1}{2}$ only satisfy equation (i)

Hence, $z=\frac{1}{2}$ is the real root, say $z_3=\frac{1}{2}$

Now, $z_1+z_2+z_3=\frac{5+6i}{2}$;

$z_1+z_2=\frac{5+6i}{2}-\frac{1}{2}=\frac{4+6i}{2}$;

Hence, $z_1+z_2=(2+3i)$

Also, $z_1z_2z_3=-\left(\frac{1-3i}{2}\right)$;

$z_1z_2=-(1-3i)$

Hence, equation whose roots are z_1 and z_2

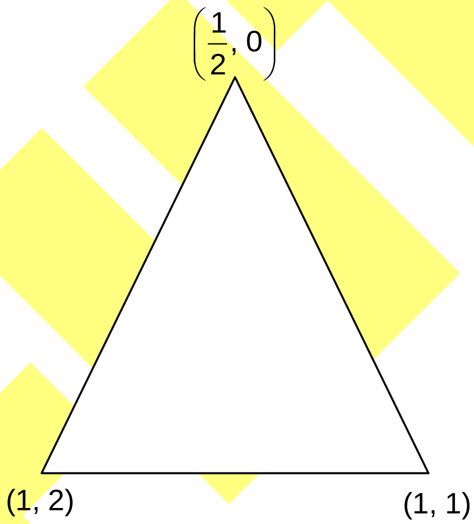
$z^2-(2+3i)z-(1-3i)=0$

$\Rightarrow\left[z-(1+2i)\right]\left[z-(1+i)\right]=0$

$z=1+2i$ or $1+i$

$$A=\frac{1}{2}\begin{vmatrix}\frac{1}{2} & 0 & 1\\ 1 & 2 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[\frac{1}{2}(2-1)+1(1-2)\right]$$



$$=\frac{1}{2}\left[\frac{1}{2}-1\right]=\left|-\frac{1}{4}\right|=\frac{1}{4} \text{ square units.}$$

$\therefore 100A=25$

2. 5

Sol. a_n, b_n, c_n are obviously the roots of the equation

$t^3-(2n+1)t^2+(2n-1)t+1=0$

$(t^3-t^2)-2n(t^2-t)-(t-1)=0$

$(t-1)\left[\ell^2-2nt-1\right]=0$

Hence, $t = 1$ or $t = \dfrac{2n \pm \sqrt{4n^2 + 4}}{2}$

$\therefore\ t = 1$ or $t = n + \sqrt{n^2 + 1}$ or $t = n - \sqrt{n^2 + 1}$

Since $a_n < b_n < c_n$

Hence, $n \to \infty$, $\underbrace{n - \sqrt{n^2 + 1}}_{a_n} < \underbrace{1}_{b_n} < \underbrace{n + \sqrt{n^2 + 1}}_{c_n}$

$\Rightarrow \left|\lim\limits_{n\to\infty} na_n\right| = \left|\lim\limits_{n\to\infty} n\left(n - \sqrt{n^2 + 1}\right)\right|$

$= \left|\lim\limits_{n\to\infty} n\left(\dfrac{-1}{n + \sqrt{n^2 + 1}}\right)\right| = \dfrac{1}{2}$

Hence, $m + 2n = 5$

3. 17

Sol. $F(x) = \begin{cases} f(x), & -1 < x < 1 \\ g(x), & x \in (-\infty, -1) \cup (1, \infty) \\ \dfrac{f(1) + g(1)}{2}, & x = 1 \\ \dfrac{f(-1) + g(-1)}{2}, & x = -1 \end{cases}$

Continuous at $x = 1$

$\Rightarrow F(1^-) = F(1^+) = F(1)$

$\Rightarrow f(1^-) = g(1^+) = \dfrac{f(1) + g(1)}{2}$

$\Rightarrow 4 + a = 1 + b = \dfrac{(4 + a) + (1 + b)}{2} = \dfrac{5 + a + b}{2}$.

$\therefore\ b - a = 3$ ....(i)

Continuous at $x = -1$

$\Rightarrow F(-1^-) = F(-1^+) = F(-1)$

$\Rightarrow b - 1 = 4 - a = \dfrac{(4 - a) + (b - 1)}{2}$

$\Rightarrow b - 1 = 4 - a = \dfrac{3 + b - a}{2}$

$\therefore\ a + b = 5$

From equation (i) and (ii)

$\Rightarrow a = 1, b = 4 \Rightarrow a^2 + b^2 = 17$

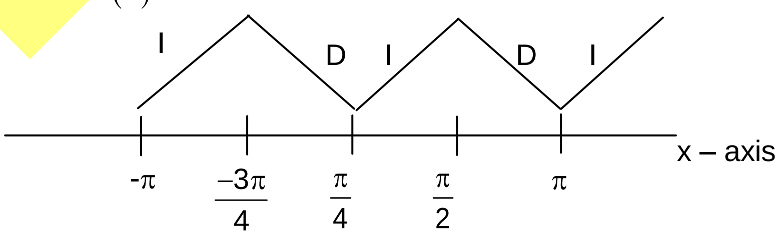
4. 8

Sol. Given $f'(x) = 2f(x)$

$\therefore\quad \dfrac{f'}{f} = 2 \Rightarrow f(x) = Ae^{2x}$

$f(0) = 1 = A$

$\therefore\quad f(x) = e^{2x}$



Now, $g(x) = e^{2x}.\cos^2 x$

$$g'(x) = e^{2x}\left(-2\cos x \sin x + 2\cos^2 x\right)$$

$$g'(x) = 2\cos x\, e^{2x}(\cos x - \sin x)$$

$$g'(x) = 0 \Rightarrow x = \frac{-3\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{-\pi}{2}$$

$\therefore$ Points of maxima are $\frac{-3\pi}{4}, \frac{\pi}{4}$ and π. Points of minima are $-\pi, \frac{-\pi}{2}, \frac{\pi}{2}$ and global minimum value occurs at $\frac{\pm\pi}{2}$ which is zero.

Hence, $n_1 = 3, n_2 = 3, n_3 = 2$

$\Rightarrow \quad n_1 + n_2 + n_3 = 8$

5. 125

Sol. $I = \int_1^2 \frac{(x^2 - 1)dx}{x^5.\sqrt{2 - 2x^{-2} + x^{-4}}}$ (Taking our from the radical sign)

$$= \int_1^2 \frac{(x^{-3} - x^{-5})dx}{\sqrt{2 - 2x^{-2} + x^{-4}}}.$$

Put $2 - 2x^{-2} + x^{-4} = t^2$; When $x = 2, t \to \frac{5}{4}$

$$+4(x^{-3} - x^{-5})dx = 2t\ dt$$

$$(x^{-3} - x^{-5})dx = \frac{t\ dt}{2}$$

$$I = \frac{1}{2}\int_1^{5/4} \frac{t\ dt}{t} = \frac{1}{2}[t]_1^{5/4} = \left(\frac{5}{4} - 1\right).\frac{1}{2} = \frac{1}{8} = \frac{u}{v};$$

Hence, $\frac{(1000).1}{8} = 125$

6. 3

Sol. $Y - y = m(X - x)$

Now, $\left(\frac{y - 0}{x - 1}\right)m = -1$

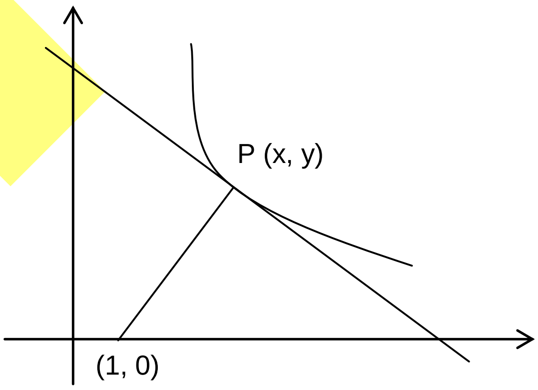
$$y\frac{dy}{dx} = 1 - x$$

Integrating $\frac{y^2}{2} = x - \frac{x^2}{2} + C$

$x^2 + y^2 - 2x = C$ ....(i)

This is the equation of a circle with centre (1, 0)

$\therefore 2x + 3y - 15 = 0$ is tangent at (1)



$\therefore$ Perpendicular from (1, 0) on the line = r

$$\left|\frac{2-15}{\sqrt{13}}\right| = \sqrt{1+C}$$

$$C+1=13$$

$\Rightarrow$ $C = 12$

Hence the curve is $x^2+y^2-2x=12$

Length of tangent is $\sqrt{S_1}=\sqrt{3}\equiv\sqrt{n}$

$\therefore$ $n=3$

7. 2

Sol. $$D=\begin{vmatrix}\cos p & -\sin p & 1\\ \sin p & \cos p & 1\\ \sin p & -\sin(p+q) & 1\end{vmatrix}$$

Applying $R_3 \to R_3-\cos qR_1+\sin qR_2$

$$=\begin{vmatrix}\cos p & -\sin p & 1\\ \sin p & \cos p & 1\\ 0 & 0 & 1+\sin q-\cos q\end{vmatrix}$$

$\Rightarrow$ $D=1+\sin q-\cos q$

Now $$D_x=\begin{vmatrix}\cos q+1 & -\sin p & 1\\ -\sin q+1 & \cos p & 1\\ 2 & -\sin(p+q) & 1\end{vmatrix}$$

Apply $C_1 \to C_1-C_3$ and then $R_3 \to R_3-\cos qR_1+\sin qR_2$ and expand

$D_x=(1+\sin q-\cos q)\cos(p+q)$

Similarly $D_y=-(1+\sin q-\cos q)\sin(p+q)$

and $D_z=1+\sin q-\cos q$

$\therefore$ $x=\cos(p+q); y=-\sin(p+q); z=1$

$\Rightarrow$ $x^2+y^2+z^2=2$

8. 4

Sol. On squaring, we get

$(1-\sin x)=(1-\sin x)^2$

$\Rightarrow (1-\sin x)(1-(1-\sin x))=0$

$\therefore$ Either $\sin x=0$ or $\sin x=1$

$\Rightarrow x=n\pi$ or $x=(4m+1)\frac{\pi}{2}$, where $n,m\in I$

But only $x=\frac{\pi}{2},\pi,2\pi,\frac{5\pi}{2}$ satisfy the condition $|x-\pi|\le\frac{3\pi}{2}$