ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Advanced)
PART TEST
05-01-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	A	B	A		
SECTION-I (ii)	Q.	5	6	7	8	9	10
	A.	A,C	A,B	A,B,C	A,B,C,D	C,D	A,B,C,D
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	12.00	120.00	4.33	0.75	632.80	3.40 to 3.41
SECTION-II (ii)	Q.	7	8	9			
	A.	1234	336	4800			

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	A	B	C		
SECTION-I (ii)	Q.	5	6	7	8	9	10
	A.	B,C,D	C,D	B,C	A,B	A,B	A,B,D
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	4.00	1.00	4.00	212.30	21.25	23.85
SECTION-II (ii)	Q.	7	8	9			
	A.	12	7	50			

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	B	D	D		
SECTION-I (ii)	Q.	5	6	7	8	9	10
	A.	A,C,D	A,C,D	B,C	B,C,D	B	B,C,D
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	1.00	103.00	23.00	32.00	2001.00	6.25
SECTION-II (ii)	Q.	7	8	9			
	A.	4	11	8			

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

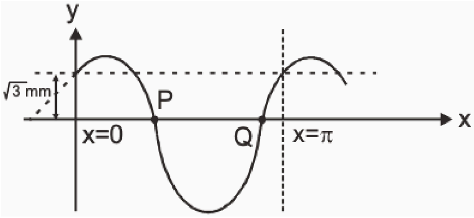
1. Ans (C)

$$\vec{E} = \frac{E_0}{2}[\cos(\omega t - ky) - \cos(\omega t + ky)]\hat{k}$$

$$\vec{B} = \frac{E_0}{2c}[\cos(wt - ky)\hat{i} + \cos(wt + ky)\hat{i}]$$

$$= \frac{E_0}{c}\cos ky \cos\omega t\,\hat{i}$$

2. **Ans (A)**



Wave is heading towards positive x-direction

Equation of y

$y = A\sin(\omega t + \frac{2\pi}{3})$

Wave equation $y = (2\times10^{-3})\sin(10t - 2x + \frac{2\pi}{3})$

Given

Relative velocity of P & Q = $2\omega A = 4$ cm/s

$\Rightarrow 2\omega(2\times10^{-3}) = 4\times10^{-2}$

$\omega = 10$ rad/s

From snapshot $\lambda = \pi m \Rightarrow k = \frac{2\pi}{\lambda} = 2m^{-1}$

Wave equation = $(2\times10^{-3})\sin(10t - 2x + \frac{2\pi}{3})$

4. **Ans (A)**

$$\frac{T}{A} = y\frac{dy}{dx}$$

$$\frac{T}{A} = 10^6 \times \frac{\pi}{2\sqrt{2}}$$

PART-1 : PHYSICS

SECTION-I (ii)

5. **Ans (A,C)**

$4a = 4g - T$

$4a = T$

$T = 20$ N

$a = 5$

$$50 = \frac{n}{2\times0.6}\sqrt{\frac{20}{\frac{1}{20}}}$$

$n = 3$

$0.2 = \frac{1}{2}5t^2$

$t_1 = \sqrt{0.08}$

$t_2 = \sqrt{0.16}$

9. **Ans (C,D)**

$y = a_0\sin(kx)\sin(\omega t + \phi)$

$\frac{kL}{3} = n\pi$ and $\frac{kL}{2} = \left(n + \frac{1}{2}\right)\pi$

$\Rightarrow n = 1, k = \frac{3\pi}{L}$

Also ; $a' = \left|a_0\sin\left(\frac{3\pi}{L}\times\frac{5L}{6}\right)\right| = a_0$

PART-1 : PHYSICS

SECTION-II (i)

1. **Ans (12.00)**

A and C parts are in open at one end whereas B part is closed at both and hence in fundamental mode

$$\frac{\ell V}{4\times5} = \frac{mV}{2\times80} = \frac{nV}{4\times15}$$

$$\frac{\ell}{1} = \frac{m}{8} = \frac{n}{3}$$

2. **Ans (120.00)**

Minimum frequency (fundamental)

Where $\ell = 1 \Rightarrow f = \frac{V}{0.2} = \frac{1}{0.2}\sqrt{\frac{1.6\times10^{11}}{2500}} = 40$K Hz

Next higher frequency where $\ell = 3$

$f = 3\times40 = 120$ K Hz.

5. **Ans (632.80)**

The mirror moves $\Delta L_2 = 3.164$ mm $= 3.164\times10^{-3}$ m.

We can use Equation 22.33 to find

$\lambda = \frac{2\Delta L_2}{\Delta m} = 6.328\times10^{-7}$ m = 632.8 m

A measurement of ΔL_2 accurate to four significant figures allowed us to determine λ to four significant figures. This happens to be the neon wavelength that is emitted as the laser beam in a helium-neon laser.

PART-1 : PHYSICS

SECTION-II (ii)

8. **Ans (336)**

Suppose level of liquid rises in left tube by ℓ.

$\pi r_1^2 \ell = \pi r_2^2 (0.1)$

$\ell = 0.4$

$\ell = \frac{\lambda}{2} = 0.4$

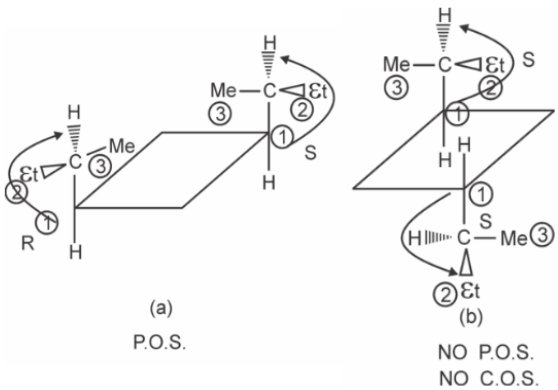
$\lambda = 0.8$

V = 336 m/s.

PART-2 : CHEMISTRY

SECTION-I (i)

2. **Ans (A)**



3. **Ans (B)**

$C_xH_{12} + (x+3)O_2 \longrightarrow xCO_2 + 6H_2O$

At 400 K H_2O is gaseous

Let $P_{C_xH_{12}} = P$

$$\frac{XP + 6P}{P + (X+3)P} = \frac{1.2}{1}$$

X = 6

4. **Ans (C)**

(A) It is an example of homogeneous catalysis.

(B) It fails at high pressure.

(D) Pumice stone is colloid of solid sol category.

PART-2 : CHEMISTRY

SECTION-I (ii)

7. **Ans (B,C)**

(A) TiO_2 is positively charged sol.

(D) Lamp black is emulsifying agent for w/o emulsion.

8. **Ans (A,B)**

1 litre solution has 1 mol MCl and x mol $MgCl_2$

(Formula mass of MCl = a)

$z = \frac{10^3 x}{1240 - a - 95x}$ & $5 = 1 + 2x, x = 2$

a = 50

so molar mass of M = 50 – 35.5 = 14.5 g/mol.

$[Mg^{2+}] = \frac{x}{1} = 2\text{mol/L}$

$[M^+] = 1$ mol/L

9. **Ans (A,B)**

(A) $MnO_4^- + I^- \rightarrow MnO_2 + IO_3^-$

$3 \times 0.2 \times 500 = n_{I^-}(6)1000$

$n_{I^-} = 0.05$

(B) I_2 disproportionates in strongly basic solution.

Strong acid can breakdown the starch.

(C) Fact

(D) $3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$

PART-2 : CHEMISTRY

SECTION-II (i)

1. **Ans (4.00)**

P = 2

Q = 6

R = 4

P + Q – R

= 2 + 6 – 4 = 4

2. **Ans (1.00)**

P = 7, Q = 5, R = 8, S = 3

P + Q – R – S

7 + 5 – 8 – 3 = 1

3. **Ans (4.00)**

$$-0.12-\frac{0.06}{2}\log\left(\frac{1}{x}\right)=-0.24$$

$$\log\frac{1}{x}=\frac{0.12\times2}{0.06}=4$$

$$X=10^{-4}$$

4. **Ans (212.30)**

$Zn|Zn^{2+}||Ag^{+}|Ag$

For standard state

$[Zn^{2+}]=1M$

$[OH^{-}]=1M$

$\therefore [Ag^{+}]=K_{SP}$

$$E=1.562-\frac{0.06}{2}\log\frac{1}{(2\times10^{-8})^2}$$

$E=1.1$

$\Delta G^\circ=-nFE^0=-212300\ J$

5. **Ans (21.25)**

$Ag^{+}\quad+\quad I^{-}\longrightarrow AgI\downarrow$

x milimol $\quad$ 50ml

M mol/ℓ

$5I^{-}\quad+\quad IO_3^{-}\longrightarrow I_2$

50 m – x $\quad$ 0.1M

50 ml

$5I^{-}\quad+\quad IO_3^{-}\longrightarrow I_2$

20 ml $\quad$ 0.1M

M $\quad$ 30 ml

$(20M)=0.1\times30\times5$

$$M=\frac{3}{4}$$

$$50\left(\frac{3}{4}\right)-x=0.1\times50\times5$$

$x=12.5$

$$\%\ W/W\ of\ AgNO_3=\frac{12.5\times170\times10^{-3}}{10}\times100=21.25$$

6. **Ans (23.85)**

$Na_2CO_3\xrightarrow{\Delta}$ No reaction

$Na_2SO_4\xrightarrow{\Delta}$ No reaction

$2NaHCO_3\xrightarrow{\Delta}Na_2CO_3+H_2O+CO_2$

$$\text{Mole of } CO_2=\frac{2.5\times3}{\frac{1}{12}\times300}=0.3$$

Mole of $NaHCO_3=0.6$

$0.6\times1+(n\times2)=1.5\times1$

$n=0.45$

$$\%\ of\ Na_2CO_3=\frac{0.45\times106}{200}\times100=23.85\%$$

PART-2 : CHEMISTRY

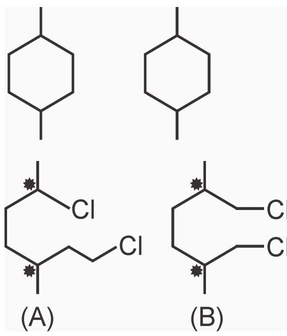
SECTION-II (ii)

7. **Ans (12)**

x = 4

y = 8

8. **Ans (7)**



PART-3 : MATHEMATICS

SECTION-I (i)

1. Ans (C)

for $[x-y]=0,\ [x+y]=5$

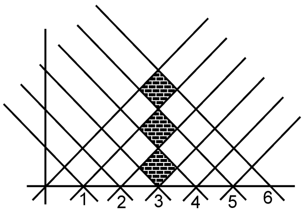
$\Rightarrow 0\le x-y<1,\ 5\le x+y<6$

similarly for $1\le x-y<12,\ 4\le x+y<5$ and so an

Required area = region marled by section lies

= are of rectangles (ABDC + DEGF + GHJI)

$= 3\times\frac{1}{2}.\ 1. = \frac{3}{2}$



2. Ans (B)

$y' = \sec^2(x+y)\,[1+y']$

$\Rightarrow y' = (1+y^2)(1+y')$

$\Rightarrow y^2y' + y^2 + 1 = 0$

$\Rightarrow y' = -1 - \frac{1}{y^2}$

$\Rightarrow y'' = \frac{2}{y^3}y' = \frac{2}{y^3}\left(-1-\frac{1}{y^2}\right) = -\frac{2}{y^3} - \frac{2}{y^5}$

$\Rightarrow y''' = \frac{6}{y^4}y' + \frac{10}{y^6}y'$

$= \left(\frac{6}{y^4}+\frac{10}{y^6}\right)\left(-1-\frac{1}{y^2}\right)$

$= \frac{-(6y^2+10)(1+y^2)}{y^8} = -\frac{6y^4+16y^2+10}{y^8}$

$\Rightarrow n = 3$

3. Ans (D)

$$I = \int \frac{2\sin x\cos^2 x + 2\sin x}{\cos^7 x + 6\cos^3 x + 4\cos x}dx$$

Put $\cos x = t$

$$I = -2\int\frac{t^2+1}{t^7+6t^3+4t}dt = -2\int\frac{\frac{1}{t^5}+\frac{1}{t^7}}{1+\frac{6}{t^4}+\frac{4}{t^6}}dt$$

$$= -2\frac{-1}{24}\int\frac{dz}{z} = \frac{1}{12}\log|z| + C$$

$$= \frac{1}{12}\log\left(1+\frac{6}{t^4}+\frac{4}{t^6}\right)+C$$

$$= \frac{1}{12}\log\left(1+\frac{6}{\cos^4 x}+\frac{4}{\cos^6 x}\right)+C$$

$$= \frac{1}{12}\log\left(\frac{\cos^6 x + 6\cos^2 x + 4}{\cos^6 x}\right)+C$$

4. Ans (D)

$f(x) = \int_0^1 \frac{dt}{f(xt)}$ $\quad xt = z\ \ dt = \frac{dz}{x}$

$f(x) = \frac{1}{x}\int_0^x\frac{dz}{f(z)} \Rightarrow xf(x) = \int_0^x \frac{dz}{f(z)}$

$xf'(x) + f(x) = \frac{1}{f(x)}$

$xf'(x) = \frac{1-(f(x))^2}{f(x)}$

$\frac{2f(x)f'(x)}{1-(f(x))^2} = \frac{2}{x}$

Integrate $-\log(1-f(x)^2) = 2\log x + c$

$f(1) = 0 \qquad c = 0$

$\log(1-f(x)^2) = \log\frac{1}{x^2}$

$f(x) = \sqrt{1-\frac{1}{x^2}}$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. Ans (A,C,D)

(A) $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0 \Rightarrow F(x) = \int \frac{e^{\sin x}dx}{x}$

$$I = \int_1^4 \frac{3e^{\sin x^3}}{x}dx = \int_1^4 \frac{3x^2 e^{\sin x^3}}{x^3}dx$$

$$= \int_1^{64} \frac{e^{\sin t}}{t}dt = [F(x)]_1^{64} = F(64) - F(1)$$

$\therefore F(64) - F(1) = F(k) - F(1)$

$\therefore k = 64$

(B) $I = \int_{\sin\theta}^{\text{cosec}\theta} f(x)\,dx = \int_{\text{cosec}\theta}^{\sin\theta} f\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)dt$

$$= \int_{\text{cosec}\theta}^{\sin\theta} f(t)\;dt = -I$$

$\therefore I = 0$

(C) $I = \int_{\pi}^{\pi} \frac{2x(1+\sin^2 x)}{1+\cos^2 x}dx \quad I = 0\ (\because f(-x) = -f(x))$

(D) $\sin x - \cos x = t \Rightarrow 1 - 2\sin x\cos x = t^2$

$$\Rightarrow \frac{1-t^2}{2} = \sin x\cos x\ (\sin x + \cos x)\,dx = dt$$

$$I = \int_{-1}^{1}\left(\frac{1-t^2}{2}\right)^2 dt = \frac{1}{2}\int_0^1 (1 - 2t^2 + t^4)\,dt$$

$$= \frac{1}{2}\left(t - \frac{2t^3}{3} + \frac{t^5}{5}\right)_0^1 = \frac{4}{15}$$

6. Ans (A,C,D)

$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow 0 < \sin x < 1$

$\Rightarrow -\alpha < -\alpha\sin x < 0$

$\Rightarrow e^{-\alpha} < e^{-\alpha\sin x} < e^0 = 1$

$$\Rightarrow \int_0^{\pi/2} e^{-\alpha}\,dx < \int_0^{\pi/2} e^{-\alpha\sin x}\,dx < \int_0^{\pi/2} 1\,dx$$

$$\Rightarrow \frac{\pi}{2}e^{-\alpha} < I < \frac{\pi}{2}$$

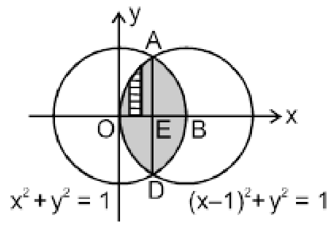
7. Ans (B,C)

Solving the given equation of circle, we get

$$A \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right); D \equiv \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

Now area = 2[OBAO] = 2[area OEAO + EBAE] = 2

$$\left[\int_0^{x_E}\sqrt{[1-(x-1)^2]}\,dx + \int_{x_E}^{x_B}\sqrt{1-x^2}\,dx\right]$$



$$= 2\left[\int_0^{1/2}\sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1}\sqrt{1-x^2}\,dx\right]$$

$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \text{ square units}$$

8. Ans (B,C,D)

(A) Order of the differential equation is 2

(B) $\dfrac{xdy-ydx}{\sqrt{x^2+y^2}}=dx \Rightarrow \dfrac{\frac{xdy-ydx}{x^2}}{\sqrt{\left(1+\frac{y^2}{x^2}\right)}}=\dfrac{dx}{x}$

$\therefore \ell n\left|\dfrac{y}{x}+\sqrt{1+\dfrac{y^2}{x^2}}\right|=\ell n|cx|$

$\therefore \dfrac{y}{x}+\dfrac{\sqrt{x^2+y^2}}{x}=cx$

i.e. $y+\sqrt{x^2+y^2}=cx^2$

(C) $y=e^x(A\cos x+B\sin x)$

$\dfrac{dy}{dx}=e^x(A\cos x+B\sin x)+e^x(-A\sin x+B\cos x)$

$=y+e^x(-A\sin x+B\cos x)$

$\Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}+e^x(-A\sin x+B\cos x)+e^x$

$(-A\cos x-B\sin x)$

$\Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}+e^x(-A\sin x+B\cos x)-y$

$\Rightarrow \dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}+\dfrac{dy}{dx}-y-y=2\left(\dfrac{dy}{dx}-y\right)$

(D) $(1+y^2)\dfrac{dx}{dy}+x=2e^{\tan^{-1}y}$

$\Rightarrow \dfrac{dx}{dy}+\dfrac{1}{1+y^2}x=2\dfrac{e^{\tan^{-1}y}}{1+y^2}$

$\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

$\Rightarrow e^{\tan^{-1}y}=2\displaystyle\int e^{\tan^{-1}y}.\dfrac{e^{\tan^{-1}}}{1+y^2}dy$

$\Rightarrow e^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

9. Ans (B)

$\dfrac{\left[\frac{ydx-xdy}{y^2}\right]}{\left(\frac{x}{y}+1\right)\sqrt{\frac{x}{y}}}+d\left(\dfrac{x^2+y^2}{2}\right)=0$

$\Rightarrow \dfrac{x^2+y^2}{2}+2\tan^{-1}\sqrt{\dfrac{x}{y}}=\lambda$

10. Ans (B,C,D)

(A) $f(x)=f(2-x)\Rightarrow f'(x)=-f'(2-x)$ ……(i)

put $x=\dfrac{1}{2},\dfrac{1}{4}$ we get

$f'\left(\dfrac{1}{2}\right)=-f'\left(\dfrac{3}{2}\right), f'\left(\dfrac{1}{4}\right)=-f'\left(\dfrac{7}{4}\right)$

we get $f'\left(\dfrac{3}{2}\right)=f'\left(\dfrac{7}{4}\right)=0$

putting x = 1 in equation (1) we get $f'(1)=-f'(1)=0$

$\Rightarrow f'(1)=0$ $f'(x)=0$ will have atleast 5 roots in [0, 2]

$f''(x)=0$ will have atleast 4 roots in [0, 2]

(B) $f'(x)=-f'(2-x)$

$f'(1+x)=-f'(1-x)$

$I=\displaystyle\int_{-1}^{1}f'(1+x)x^2e^{x^2}dx$

$I=\displaystyle\int_{-1}^{1}f'(1-x)x^2e^{x^2}dx$

on adding we get $2I=0\Rightarrow I=0$

(C) $I=\displaystyle\int_0^1 f(1-t)e^{-\cos\pi t}dt-\int_1^2 f(2-t)e^{\cos\pi t}dt$

$I=\displaystyle\int_0^1 f(t)e^{\cos\pi t}dt+\int_1^0 f(t)e^{\cos\pi t}dt=0$

$I=0$ and

$I=\displaystyle\int_0^2 f'(t)e^{\cos\pi t}dt$

$I=\displaystyle\int_0^2 f'(2-t)e^{\cos\pi t}dt=-\int_0^2 f'(t)e^{\cos\pi t}dt=-I$

$2I=0\Rightarrow I=0$

PART-3 : MATHEMATICS

SECTION-II (i)

1. **Ans (1.00)**

Differentiating both sides

$$\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}} = (Ax^2+Bx+C)\frac{(x+2)}{\sqrt{x^2+4x+3}}$$

$$+ (2Ax + B)\sqrt{x^2+4x+3} + \frac{\lambda}{\sqrt{x^2+4x+3}}$$

$x^3 - 6x^2 + 11x - 6 = (Ax^2 + Bx + C)(x + 2) +$

$(2Ax + B)(x^2 + 4x + 3) + \lambda$

compairng coefficients of like powers of x

$x^3 : 1 = A + 2A \Rightarrow A = 1/3]$

$x^2 : -6 = 2A + B + 8A + B$

$2B = -6 - 10.\frac{1}{3} \Rightarrow B = -\frac{14}{3}$

$x : 11 = 2B + C + 6A + 4B$

$C = 11 - 6\left(-\frac{14}{3}\right) - 6.\frac{1}{3} = 11 + 28 - 2 = 37$

Constant terms : $-6 = 2C + 3B + \lambda$

$\lambda = -6 - 2.37 - 3.\left(-\frac{14}{3}\right)$

$= -6 - 74 + 14 = -66.$

2. **Ans (103.00)**

Differentiating both sides

$$\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}} = (Ax^2+Bx+C)\frac{(x+2)}{\sqrt{x^2+4x+3}}$$

$$+ (2Ax + B)\sqrt{x^2+4x+3} + \frac{\lambda}{\sqrt{x^2+4x+3}}$$

$x^3 - 6x^2 + 11x - 6 = (Ax^2 + Bx + C)(x + 2) +$

$(2Ax + B)(x^2 + 4x + 3) + \lambda$

compairng coefficients of like powers of x

$x^3 : 1 = A + 2A \Rightarrow A = 1/3]$

$x^2 : -6 = 2A + B + 8A + B$

$2B = -6 - 10.\frac{1}{3} \Rightarrow B = -\frac{14}{3}$

$x : 11 = 2B + C + 6A + 4B$

$C = 11 - 6\left(-\frac{14}{3}\right) - 6.\frac{1}{3} = 11 + 28 - 2 = 37$

Constant terms : $-6 = 2C + 3B + \lambda$

$\lambda = -6 - 2.37 - 3.\left(-\frac{14}{3}\right)$

$= -6 - 74 + 14 = -66.$

3. **Ans (23.00)**

$$= \begin{bmatrix} 1+x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots & x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots \\ x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!} & 1+x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} e^{2x}+1 & e^{2x}-1 \\ e^{2x}-1 & e^{2x}+1 \end{bmatrix}$$

$\therefore f(x) = e^{2x} + 1 \ \&\ g(x) = e^{2x} - 1$

$$\int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

$$\int \frac{e^{2x}+1}{\sqrt{e^{2x}-1}}\,dx = \int \frac{e^{2x}}{\sqrt{e^{2x}-1}}dx + \int \frac{e^x}{e^x\sqrt{e^{2x}-1}}dx$$

$$= \sqrt{e^{2x}-1} + \sec^{-1}(e^x) + C$$

4. Ans (32.00)

$$= \begin{bmatrix} 1+x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots & x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots \\ x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!} & 1+x+\frac{2x^2}{2!}+\frac{2^2x^3}{3!}+\ldots \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} e^{2x}+1 & e^{2x}-1 \\ e^{2x}-1 & e^{2x}+1 \end{bmatrix}$$

$$\therefore f(x) = e^{2x}+1 \ \& \ g(x) = e^{2x}-1$$

$$\int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dx$$

$$\int \frac{e^{2x}+1}{\sqrt{e^{2x}-1}}\,dx = \int \frac{e^{2x}}{\sqrt{e^{2x}-1}}dx + \int \frac{e^{x}}{e^{x}\sqrt{e^{2x}-1}}dx$$

$$= \sqrt{e^{2x}-1} + \sec^{-1}(e^{x}) + C$$

5. Ans (2001.00)

$$I_2 = \frac{1}{4}\int_{2001}^{4002} (\log_{2x}2)^2 dx \Rightarrow 4I_2 = \int_{2001}^{4002} (\log_{2x}2)^2\, dx$$

$$4I_2 + I_{4n} = \int_{2001}^{4002} \left\{ \frac{1+2\log_x 2}{(1+\log_x 2)^2} + \frac{1}{(1+\log_2 x)^2} \right\} dx$$

$$= \int_{2001}^{4002} \frac{(1+2\log_x 2+(\log_x 2)^2)}{(1+\log_x 2)^2}\, dx$$

$$= \int_{2001}^{4002} 1\, dx = 4002 - 2001 = 2001$$

$$4I_2 + I_{4n+2} = \int_{2001}^{4002} \left(\frac{1}{(1+\log_2 x)^2} + \frac{1-2\log_x 2}{(1+\log_x 2)^2} \right) dx$$

$$= \int_{2001}^{4002} \left\{ \frac{(\log_x 2)^2}{(1+\log_x 2)^2} + \frac{1-2\log_x 2}{(1+\log_x 2)^2} \right\} dx$$

$$\Rightarrow \int_{2001}^{4002} \frac{(1-\log_x 2)^2}{(1+\log_x 2)^2} dx = \int_{2001}^{4002} \left(\frac{\log_x \frac{x}{2}}{\log_x 2x} \right)^2 dx$$

$$= \int_{2001}^{4002} \left(\log_{2x}\frac{x}{2} \right)^2 dx \text{ put } 2x = t \Rightarrow \frac{dt}{dx} = 2$$

$$\Rightarrow \frac{1}{2}\int_{4002}^{8004} \left(\log_t \frac{t}{4} \right)^2 .\, Dt$$

6. Ans (6.25)

$$I_2 = \frac{1}{4}\int_{2001}^{4002} (\log_{2x}2)^2 dx \Rightarrow 4I_2 = \int_{2001}^{4002} (\log_{2x}2)^2\, dx$$

$$4I_2 + I_{4n} = \int_{2001}^{4002} \left\{ \frac{1+2\log_x 2}{(1+\log_x 2)^2} + \frac{1}{(1+\log_2 x)^2} \right\} dx$$

$$= \int_{2001}^{4002} \frac{(1+2\log_x 2+(\log_x 2)^2)}{(1+\log_x 2)^2}\, dx$$

$$= \int_{2001}^{4002} 1\, dx = 4002 - 2001 = 2001$$

$$4I_2 + I_{4n+2} = \int_{2001}^{4002} \left(\frac{1}{(1+\log_2 x)^2} + \frac{1-2\log_x 2}{(1+\log_x 2)^2} \right) dx$$

$$= \int_{2001}^{4002} \left\{ \frac{(\log_x 2)^2}{(1+\log_x 2)^2} + \frac{1-2\log_x 2}{(1+\log_x 2)^2} \right\} dx$$

$$\Rightarrow \int_{2001}^{4002} \frac{(1-\log_x 2)^2}{(1+\log_x 2)^2} dx = \int_{2001}^{4002} \left(\frac{\log_x \frac{x}{2}}{\log_x 2x} \right)^2 dx$$

$$= \int_{2001}^{4002} \left(\log_{2x}\frac{x}{2} \right)^2 dx \text{ put } 2x = t \Rightarrow \frac{dt}{dx} = 2$$

$$\Rightarrow \frac{1}{2}\int_{4002}^{8004} \left(\log_t \frac{t}{4} \right)^2 .\, dt$$

PART-3 : MATHEMATICS

SECTION-II (ii)

7. Ans (4)

Given the $f(x+y) = f(x) + f(y)$ .... (i)

Putting $x = 0$ and $y = 0$ in (i), we get

$f(0+0) = f(0) + f(0) \Rightarrow f(0) = 0$ .... (ii)

Now $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(x) + f(h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{f(h)}{h}$

$= \lim\limits_{h\to 0} f'(h) = f'(0)$

$\Rightarrow f(x) = \int f'(0)\,dx = x f'(0) + c$ .... (ii)

Putting $x = 0$ in (ii), we get

$f(0) = 0 + c \Rightarrow c = 0$ [$\therefore f(0) = 0$ from (ii)] .... (ii)

$\Rightarrow f(x) = xf'(0)$ .... (ii)

Thus $I_n = n\int_0^n f(x)\,dx = n\int_0^n xf'(0)\,dx$

$\Rightarrow I_n = \dfrac{n^3 \,.\, f'(0)}{2}$

therefore $I_1 + I_2 + I_3 + I_4 + I_5 = (1^3 + 2^3 + 3^3 + 4^3 + 5^3)$

$\Rightarrow 450 = \dfrac{f'(0)}{2} \,.\, \left\{\dfrac{5\,.\,(5+1)}{2}\right\}^2 \Rightarrow f'(0) = 4$

$\therefore f(x) = 4x$ (from equation (iii))

8. Ans (11)

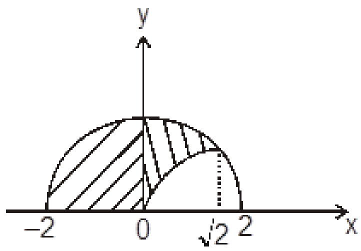
$y = \sqrt{4-x^2}, y = \sqrt{2}\sin\left(\dfrac{x\pi}{2\sqrt{2}}\right)$

intersect at $x = \sqrt{2}$

Area of the left of y-axis is π

Area to the right of y-axis

$= \displaystyle\int_0^{\sqrt{2}} \left(\sqrt{4-x^2} - \sqrt{2}\sin\dfrac{x\pi}{2\sqrt{2}}\right)dx$



$= \left(\dfrac{x\sqrt{4-x^2}}{2} + \dfrac{4}{2}\sin^{-1}\dfrac{x}{2}\right)_0^{\sqrt{2}} + \dfrac{4}{\pi}$

$\cos\dfrac{x\pi}{2\sqrt{2}}\Big|_0^{\sqrt{2}} = \left(1 + 2\,.\,\dfrac{\pi}{4}\right) + \dfrac{4}{\pi}(0-1)$

$= 1 + \dfrac{\pi}{2} - \dfrac{4}{\pi} = \dfrac{2\pi + \pi^2 - 8}{2\pi}$

$\therefore$ ratio $= \dfrac{2\pi^2}{2\pi + \pi^2 - 8}$

9. Ans (8)

for some $a \in \left(0, \frac{\pi}{2}\right)$

$kx = \cos x$

$ka = \cos a$

$k = \frac{\cos a}{a}$

$$I = \int_0^a (\cos x - kx)dx + \int_a^{\pi/2} (kx - \cos x)dx$$

$$= \sin x - \frac{kx^2}{2}\Big|_0^a + \frac{kx^2}{2} - \sin x\Big|_a^{\pi/2}$$

$$= \left(\sin a - \frac{ka^2}{2}\right) + \left(\frac{k\pi^2}{8} - 1\right) - \left(\frac{ka^2}{2} - \sin a\right)$$

$$I(a) = 2\sin a - ka^2 + \frac{k\pi^2}{8} - 1$$

putting $k = \frac{\cos a}{a}$,

$$I(a) = 2\sin a - a\cos a + \frac{\pi^2}{8}.\frac{\cos a}{a} - 1$$

$$\Rightarrow I'(a) = (\cos a + a\sin a)\left(1 - \frac{\pi^2}{8a^2}\right) = 0$$

$$\Rightarrow a = \frac{\pi}{2\sqrt{2}}$$

for $a < \frac{\pi}{2\sqrt{2}} \Rightarrow I'(a) < 0$ and $a > \frac{\pi}{2\sqrt{2}}$

$\Rightarrow I'(a) > 0 \Rightarrow$ minimum

now for $k = \frac{\cos a}{a}$; ; $k = \frac{2\sqrt{2}}{\pi} \cos\left(\frac{\pi}{2\sqrt{2}}\right)$,