Test Pattern

JEE (Main)
FULL SYLLABUS
08-01-2025

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C	D	A	A	A	C	C	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	B	D	A	B	A	C	C	B	D
SECTION-II	Q.	1	2	3	4	5					
	A.	3	3	60	798	125					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	B	A	C	A	D	D	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	A	B	C	D	B	D	B	B	D
SECTION-II	Q.	1	2	3	4	5					
	A.	3	6	8	4	3					

PART-3 : MATHEMATICS

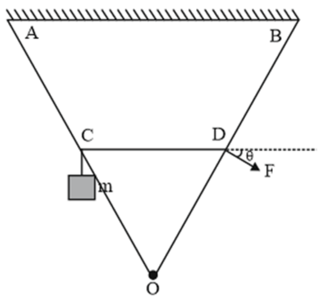
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	D	D	B	B	C	B	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	A	C	B	D	B	D	C
SECTION-II	Q.	1	2	3	4	5					
	A.	1	4	99	3	44					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (A)**



$T_0 = mg\ell\frac{1}{2} = F\,\ell\sin(60+\theta)$

$\frac{mg}{2} = F\sin(60+\theta)$

to get $F_{min}\, 60+\theta = 90^\circ \quad \Rightarrow \theta = 30^\circ$

2. **Ans (C)**

Chemical energy converted to mechanical energy

3. **Ans (D)**

COM can not shift and COM is at a distance of $\frac{L}{6}$ from 0.

$\Delta\ell = \frac{L}{6} \propto \Delta\theta$

4. **Ans (A)**

When diode in RB , current through the resistor is Zero. So out put is 10 volt

When diode in RB current through the resistor is (10-IR), max IR is 35 volt

5. Ans (A)

$i = \frac{12}{100}e^{-\frac{t}{T}}$

8. Ans (C)

$n\left(\frac{C_p + C_v}{2}\right) dT = nC_v dT + pdv$

$n\left(\frac{C_p - C_v}{2}\right) dT = \frac{nRT}{v}dv$

$\left(\frac{C_p - C_v}{2}\right)\frac{dT}{T} = dv$

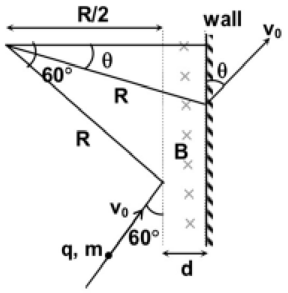
$\frac{C_v}{4R}\ell nT = \ell nCv \Rightarrow T^{1/2} = Cv$

$(Pv)^{\frac{1}{2}} = Cv \Rightarrow Pv^{-1} = \text{con s } k = -1 = 1$

10. Ans (A)

$\frac{IA}{c} = K(Amp)$

11. Ans (C)



$R\cos\theta - \frac{R}{2} = d = \frac{R\left(\sqrt{3}-1\right)}{2}$

$\cos\theta - \frac{1}{2} = \frac{\sqrt{3}}{2} - \frac{1}{2} \Rightarrow \theta = 30°$

So, total angle rotates inside the magnetic field

$= 30° + 30° = 60°$

Time period $\frac{m}{qB}\frac{\pi}{3} = \frac{\pi m}{3qB}$

12. Ans (B)

$F \propto \frac{x}{(x^2+a^2)^{3/2}}; a = \frac{F}{m}$

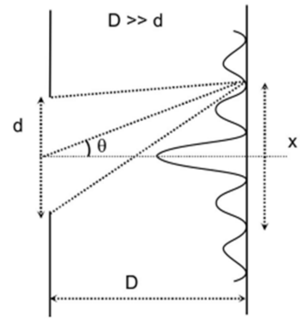
13. Ans (D)

$\tan\theta = \frac{x}{2D}$

For second minima

$\sin\theta = \frac{2\lambda}{d} \approx \tan\theta \Rightarrow \frac{x}{2D} = \frac{2\lambda}{d}$

$\Rightarrow x = \frac{4\lambda D}{d}$



14. Ans (A)

$V_e = \sqrt{2gR_e}$

$V_s = \frac{3}{8}\sqrt{2gR_e} = \sqrt{\frac{9gR_e}{32}} = \sqrt{\frac{R_e^2 g}{R_e + h}}$

$9R_e + 9h = 32R_e \Rightarrow h = \frac{23}{9}R_e$

Now, total energy at height 'h' = total energy at the surface of the earth

$0 - \frac{GM_e m}{R_e} = \frac{1}{2}mv^2 - \frac{GM_e m}{R_e}$

$\frac{1}{2}mv^2 = \frac{GM_e m}{R_e} - \frac{GM_e m}{R_e + \frac{23}{9}R_e}$

$\frac{1}{2}mv^2 = \frac{23}{32}\frac{GM_e m}{R_e}$

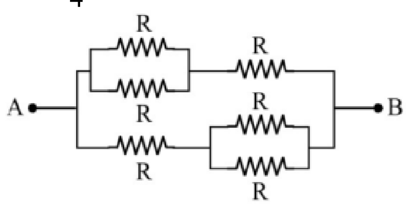
$v = \sqrt{\frac{23}{16}\frac{GM_e}{R_e}}, \sqrt{gR_e} = 8 \text{ Km/sec}$

$v = \sqrt{\frac{23}{16}gR_e} = \sqrt{\frac{23}{2}} \text{ Km/sec}$

15. Ans (B)

The given circuit can be simplified as

$R_{eq} = \frac{3}{4}R$



16. Ans (A)

$$L_H = M_2 v\frac{L}{2} = \frac{M_1L^2}{3}\omega$$

$$\omega = \frac{3M_2v}{2M_1L} \quad \ldots(1)$$

$$\text{for } e = 1,\ v_{CM} = v = \frac{\omega L}{2} \quad \ldots(2)$$

$$\Rightarrow \frac{2v}{L} = \frac{3M_2v}{2M_1L}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{3}{4}$$

17. Ans (C)

$$E = L\frac{di}{dt}$$

$$di = \frac{E}{L}dt$$

$$i = \frac{E}{L}t$$

$$i = \frac{2}{4}\times t$$

$$i = 0.5t$$

$$5 = 0.5t$$

$$t = 10\,\text{sec}$$

18. Ans (C)

When one electron is removed, the remaining atom is hydrogen like atom whose energy in first orbit is

$$E_1 = -(2)^2(13.6\text{ eV}) = -54.4\text{ eV}$$

$\therefore$ to remove both electrons energy required is

$$(24.6 + 54.4)\text{eV} = 79\text{eV}$$

19. Ans (B)

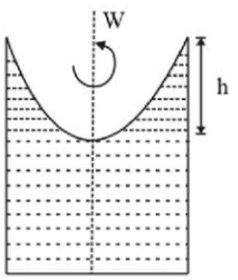
$$V_{source} = \sqrt{V_R^2 + V_C^2}$$

$$\therefore\ V_C = \sqrt{V_{source}^2 - V_R^2}$$

$$= \sqrt{(20)^2 - (12)^2}$$

$$= 16\text{ V}$$

20. Ans (D)

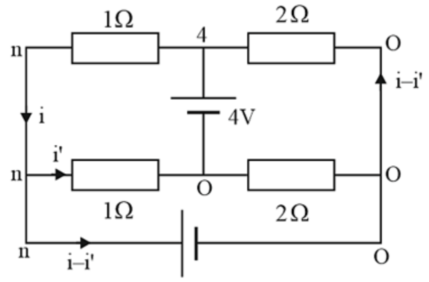


$$h = \frac{r^2\omega^2}{2g} = \frac{(0.05)^2(2\times 2\pi)^2}{2\times g} = 0.02\text{m}$$

PART-1 : PHYSICS

SECTION-II

1. Ans (3)



$$(4 - n) - n = 0 - 2,\ n = 3$$

2. Ans (3)

I_0 is intensity of unpolarized incident light.

$$I = \frac{I_0}{2}\cos^2 30^\circ \cos^2 60^\circ = 3\,\text{wattm}^{-2}.$$

3. Ans (60)

$$v_2 = \frac{60}{100}v_1 = \frac{3}{5}v_1$$

$$\frac{A_t}{A_i} = \frac{2v_2}{v_1 + v_2} = \frac{2\left(\frac{v_2}{v_1}\right)}{1 + \left(\frac{v_2}{v_1}\right)}$$

$$= \frac{2\left(\frac{3}{5}\right)}{1 + \left(\frac{3}{5}\right)} = \frac{3}{4}$$

4. Ans (798)

The magnetising current I_M is the additional current that needs M to be passed through the windings of the solenoid in the absence of the core which would give a B value as in the presence of the core. Thus

$B = \mu_r n\ (I + I_M)$.

Using I = 2A, B = 1 T. we get I_M = 794 A.

5. **Ans (125)**

The lead slab is fixed and the force is applied parallel to the narrow face as shown in Fig. The area of the face parallel to which this force is applied is

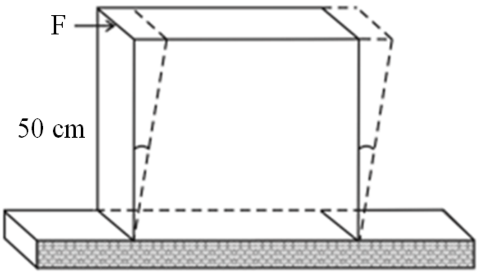
A = 50 cm x 10 cm

= 0.5 m x 0.1 m = 0.05 m^2

Therefore, the stress applied is

= (9.4×10^4 N/0.05 m²)

= 1.80×10^6 N.m^2



We know that shearing strain = (Δx/L) = Stress/G.

Therefore the displacement Δx = (Stress x L)/G

= (1.8×10^6 N m^{-2} x 0.5m)/(7.2×10^6 N m^{-2})

= 125 mm

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (B)**

Due to d-d transition of Cr^{3+} ion in Al_2O_3 lattice

2. **Ans (B)**

The ΔH_{eg} of oxygen is lowest among group 16 elements.

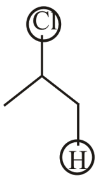
3. **Ans (B)**

Pb^{+4} is a stronger oxidising agent and I^- is a stronger reducing agent but Cl^- is a milder reducing agent and hence $PbCl_4$ exists at room temperature.

4. **Ans (A)**

$$pH = pK_a + \log \frac{[CH_3COO^-]}{CH_3COOH}$$

5. **Ans (C)**



Dehydrohalogenation is concertated and require anti periplanner orientation and show primary kinetic isotopic effect of H/D.

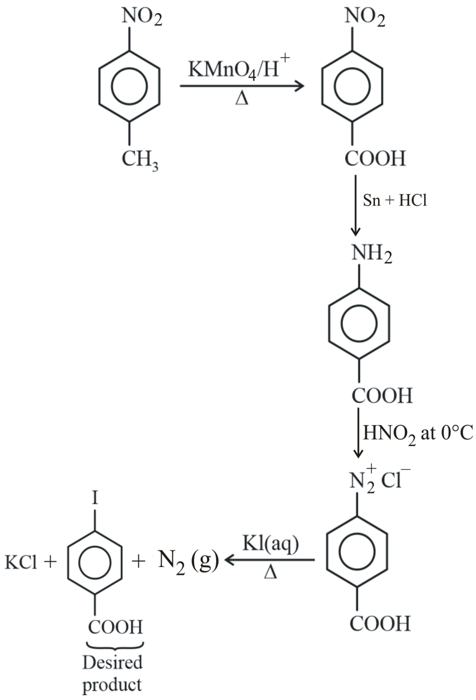
6. **Ans (A)**

Factual

7. **Ans (D)**

The magnetic properties of the actinoids are more complex than the lanthanoids.

8. **Ans (D)**



9. **Ans (B)**

Vitamin B_1 – Beri-Beri

Rest are correct

10. **Ans (C)**

Total heat absorbed by Aℓ = nC_PdT

$$= \frac{54}{27} \times 24 \times 20$$

= 48 × 20J

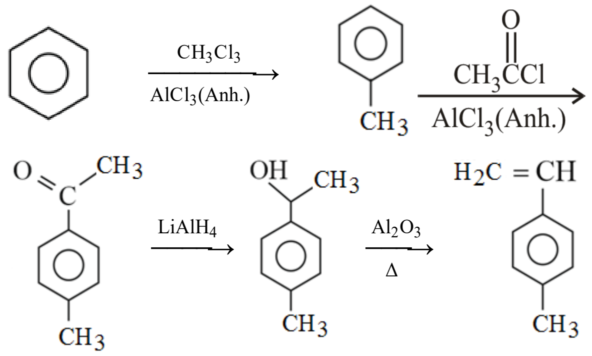
= 960 J

393.5 KJ energy released by 1 mol of $C_{(S)}$

Therefore, 960 J energy released by $\dfrac{960 \times 1}{393.5 \times 10^3}$ mol of $C_{(S)}$

= 2.44×10^{-3}mol of $C_{(S)}$

12. **Ans (A)**



13. **Ans (B)**

$\lambda_m^0\,(CH_3COONa) = \dfrac{K \times 1000}{C}$

$= \dfrac{80 \times 10^{-5} \times 1000}{0.1/110} = \dfrac{80 \times 10^{-2} \times 110}{0.1}$

$= 880\text{mho cm}^2\ \text{mol}^{-1}$

As the solution is fairly diluted

So,

$\lambda_m^0\,(CH_3COONa) = \lambda_m^0\left(CH_3COO^-\right) + \lambda_m^0\left(Na^+\right)$

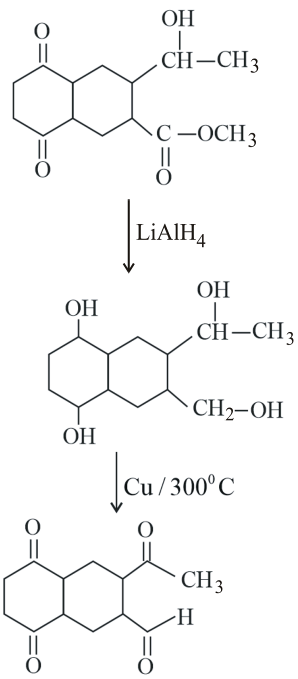
$\Rightarrow 880 = \lambda_m^0\left(CH_3COO^-\right) + 500$

$\Rightarrow \lambda_m^0\left(CH_3COO^-\right) = 380\text{mho cm}^2\ \text{mol}^{-1}$

14. **Ans (C)**

Theory based

15. **Ans (D)**



(Z)

Lucas test is used to detect alcoholic functional group

16. **Ans (B)**

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$\dfrac{0.1}{2}M \qquad\qquad \dfrac{1}{2}M$

$\dfrac{0.1}{2} - x \qquad x \qquad \dfrac{1}{2} + x$

$\simeq \dfrac{0.1}{2} \qquad x \qquad \dfrac{1}{2}$

$K_a = 10^{-5} = \dfrac{x \times \frac{1}{2}}{\frac{0.1}{2}}$

$x = 10^{-6}$

17. **Ans (D)**

As per Henry's law

$x_{Ar} = \dfrac{P_{Ar}}{K_H} = \dfrac{0.403}{40.3 \times 10^3} = 1 \times 10^{-5}$

$\Rightarrow \dfrac{n_{Ar}}{n_{Ar} + n_{H_2O}} = 1 \times 10^{-5}$ [1 litre of H_2O= 55.56 mol]

$\Rightarrow \dfrac{n_{Ar}}{55.56} = 1 \times 10^{-5}$

$\Rightarrow n_{Ar} = 55.56 \times 10^{-5}$ mol

18. **Ans (B)**

Total work = $W_{BC} + W_{DE}$

$= -(10 \times 5) - (5 \times 15)$

$= -125$ L atm

$\Delta U = \Delta Q + W$

$\Rightarrow nC_V\,dT = \Delta Q + W$

$\Rightarrow 0 = \Delta Q + W \quad (dT = 0,\ T_A = T_E)$

isothermal path ACE

$\Rightarrow \Delta Q = -W$

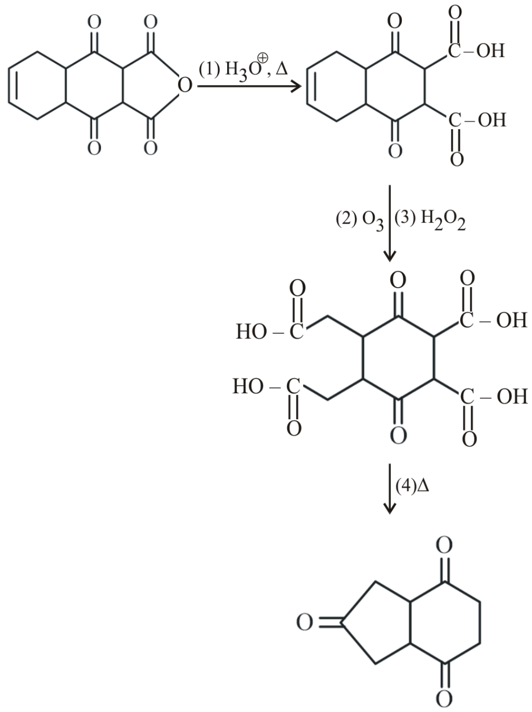
19. **Ans (B)**

$2KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (3)**



Number of keto groups in product = 3

The number of carbon atoms in product = 9

2. **Ans (6)**

$$pI = \left(\frac{pK_{a_1} + pK_{a_3}}{2}\right) = \frac{2.0 + 10.0}{2} = 6$$

3. **Ans (8)**

$n_{eq}\ Fe_2(C_2O_4)_3 + n_{eq}\ FeC_2O_4 = n_{eq}\ KMnO_4$

or, $4.5 \times 6 + 4.5 \times 3 = n \times 5$

$\therefore n = 8.1$

4. **Ans (4)**

ZnS - White, NiS - black

5. **Ans (3)**

Area = $\pi r^2 = 549$

$$\Rightarrow \frac{22}{7} \times \left[\frac{0.529 \times n^2}{1}\right]^2 = 549 \text{ .. (1)}$$

Circumference, $2\pi r = 83.05$

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{0.529 \times n^2}{1} = 83.05 \text{ ... (2)}$$

eq. (1) / (2)

$$\frac{\frac{22}{7} \times (0.529 \times n^2)^2}{2 \times \frac{22}{7} \times (0.529 \times n^2)} = \frac{549}{83.05}$$

$$\frac{0.529 \times n^2}{2} = \frac{549}{83.05}$$

$$\Rightarrow n^2 = \frac{2 \times 549}{83.05 \times 0.259}$$

$\Rightarrow n = 5$

Therefore, Max no. of possible line = $5 \longrightarrow 4$, $4 \longrightarrow 3$, $3 \longrightarrow 2$ of a H - atom

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (D)**

Let the equation of the circle be $x^2 + y^2 = r^2$. As the chords are bisected by the x-axis, their y-coordinates are 5, 4 and 2 respectively. Let their x-coordinates be a, a +d and a + 2d respectively

Then the coordinates of the extremities of the chords are (a, 5), (a+d, 4) and (a+2d, 2) respectively.

Since these points lie on the circle $x^2 + y^2 = r^2$,

$a^2 + 25 = r^2$ .... (1)

$(a + d)^2 + 16 = r^2$ ... (2)

$(a + 2d)^2 + 4 = r^2$ ... (3)

(2) - (1) $\Rightarrow 2ad + d^2 = 9$

(3) - (2) $\Rightarrow 2ad + 3d^2 = 12$

$\therefore 2d^2 = 3 \Rightarrow d = \sqrt{\frac{3}{2}}$

$\Rightarrow 2a\sqrt{\frac{3}{2}} = 9 - \frac{3}{2} = \frac{15}{2}$

$\therefore a = \frac{15}{4} \times \sqrt{\frac{2}{3}} = \frac{15}{2\sqrt{6}}$

$\therefore r^2 = a^2 + 25 = \frac{15^2}{4 \times 6} + 25$

$= 25\left[\frac{3}{8} + 1\right]$

$= \frac{25 \times 11}{8}$

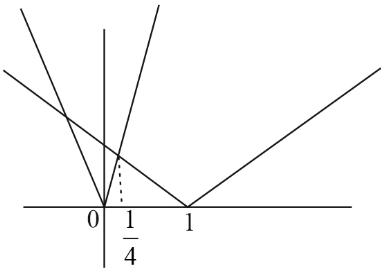
$\Rightarrow r^2 = \frac{275}{8}$

2. Ans (A)

f(x) is of the form

$a + b - |a - b| = 2\min\{a, b\}$

$= 2\min\{3|x|, |x-1|\}$



f(x) has a local maximum at $x = \frac{1}{4}$

and local minimum at $x = 0$ and $x = 1$

3. Ans (D)

R is true. So

$\lim\limits_{x \to \frac{\pi}{2} - 0} f(x) = \lim\limits_{\substack{h \to 0 \\ h > 0}} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)}$

$= \lim\limits_{h \to 0} \frac{1 - \cos^3 h}{3\sin^2 h}$

$= \lim\limits_{h \to 0} \frac{(1 - \cos h)\left(1 + \cos h + \cos^2 h\right)}{3(1 - \cos h)(1 + \cos h)}$

$= \lim\limits_{h \to 0} \frac{1 + \cos h + \cos^2 h}{3(1 + \cos h)}$

$= \frac{3}{3(2)} = \frac{1}{2} = f\left(\frac{\pi}{2}\right) = a$

Therefore a = 1/2. Also

$\lim\limits_{x \to 0+0} f(x) = \lim\limits_{\substack{h \to 0 \\ h > 0}} \left[\frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)}\right]$

$= \lim\limits_{h \to 0} \frac{b(1 - \cos h)}{(-2h)^2}$

$= \lim\limits_{h \to 0} \frac{2b\sin^2 \frac{h}{2}}{4h^2}$

$= \lim\limits_{h \to 0} \frac{b\sin^2 \frac{h}{2}}{2h^2}$

$= \lim\limits_{h \to 0} \frac{b}{8}\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 = \frac{b}{8}$

Now $\frac{b}{8} = a = \frac{1}{2} \Rightarrow b = 4$

Therefore $a = \frac{1}{2}$ and $b = 4$

Both **A** and **R** are correct and **R** is the correct explanation of **A**

4. Ans (D)

$\left(2 + \sqrt{3}\right)^n + \left(2 - \sqrt{3}\right)^n$ = even integer $\forall\, n \in N$.

$I_n + f_n + f_n^1$ = Integer, where $\left(2 - \sqrt{3}\right)^n = f_n^1$

$\Rightarrow\ fn + f_n^1$ = Integer

$$\boxed{\begin{array}{c} 0 < fn < 1 \\ 0 < f_n^1 < 1 \\ \hline 0 < f_n + f_n^1 < 2 \end{array}}$$

$\Rightarrow\ \boxed{f_n + f_n^1 = 1}$

$\Rightarrow\ (1 - f_n)\, T_n$

$= f_n^1\ T_n = \left(2 - \sqrt{2}\right)^n \times \left(2 + \sqrt{3}\right)^n$

$= 1\ \forall\, n \in N$.

5. Ans (B)

$f_n(0) = 0$ and $f_n^1(x) = x^n \sin x$ (even if n is odd)

$\Rightarrow\ f_n(x)$ is odd f^n. So

$\int\limits_{-1}^{1} f_{2025}(x) = 0,\ f_n^1(\pi) = 0$ and $f_0(\pi/2) = 1$.

6. Ans (B)

$(\sin^2 x)(2y\,dy) + (2\sin x\cos x\,dx)\,y^2 = 2x\,dx$

$\Rightarrow\ d\left(\sin^2 x - y^2\right) = 2x\,dx$ (upon integrating)

$\boxed{\Rightarrow (\sin^2 x)\,y^2 = x^2 + C}$

7. Ans (C)

$$S_3 = \underbrace{\frac{\alpha^3+1}{\alpha^5-\alpha^4-\alpha^3+\alpha^2}}_{\downarrow} + \frac{\beta^3+1}{\beta^5-\beta^4-\beta^3+\beta^2}$$

$$\frac{\alpha(\alpha+3)+1}{\alpha^2(\alpha-1)(\alpha^2-1)} = \frac{4\alpha+4}{(\alpha+3)(\alpha-1)(\alpha+2)}$$

$$= \frac{4\alpha+4}{3\alpha(\alpha+2)}$$

$$= \frac{4\alpha+4}{4\alpha+9} = \boxed{\frac{4}{9}}$$

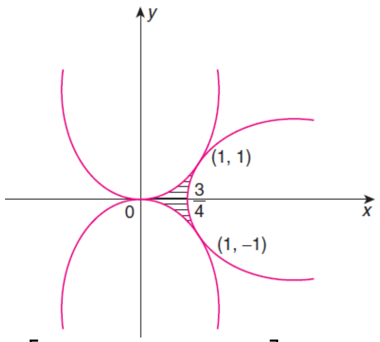
8. Ans (B)

Use result $\left|Z_1+\sqrt{Z_1^2-Z_2^2}\right| + \left|Z_1-\sqrt{Z_1^2-Z_2^2}\right|$

$= |Z_1+Z_2| + |Z_1-Z_2|$

9. Ans (A)

(A) The curves $y = x^2$, $y = -x^2$ are touched by $y^2 = 4x - 3$ at the points (1,1) and (1,–1), respectively (seeg. Fig). The curve $y^2 = 4x - 3$ cuts the x-axis at (3/4,0). The required area (shaded portion) is given by



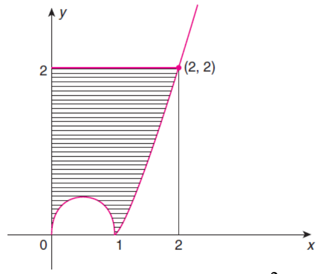
$$2\left[\int_0^1 x^2 dx - \int_{3/4}^1 \sqrt{4x-3}\,dx\right] = \frac{2}{3} - \frac{2}{4}\times\frac{2}{3}\left[(4x-3)^{3/2}\right]_{3/4}^{1}$$

$$= \frac{2}{3} - \frac{1}{3}(1-0) = \frac{1}{3}$$

(B) The curve $y = x(x-1)^2$ meets x-axis at (0,0) and (1,0) and $y \longrightarrow +\infty$ as $y \longrightarrow -\infty$. Therefore the required are (dotted portion fig) is equal to

$$(2\times 2)\ (= \text{area of the square}) - \int_0^2 (x-1)^2 x\,dx$$

9. Ans (A)



$$= 4 - \left[\frac{x^4}{4} - \frac{2}{3}x^3 + \frac{x^2}{2}\right]_0^2$$

$$= 4 - \left[\frac{16}{3} - \frac{16}{3} + \frac{4}{2}\right]$$

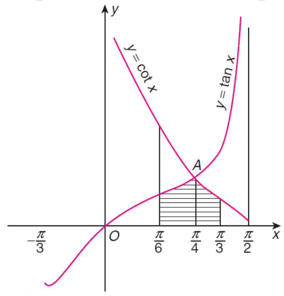
$$= 4 - \left[\frac{48-64+24}{12}\right]$$

$$= 4 - \frac{8}{12}$$

$$= 4 - \frac{2}{3}$$

$$= \frac{10}{3}$$

(C) $y = \tan x$ and $y = \cot x$ intersect in $A(\pi/4, 1)$. The required area (shadded part Fig) is



$$\int_{\pi/6}^{\pi/4} \tan x\,dx + \int_{\pi/4}^{\pi/3} \cot x\,dx = [\log_e \sec x]_{\pi/6}^{\pi/4} + [\log_e \sin x]_{\pi/4}^{\pi/3}$$

$$= \left(\log_e\sqrt{2} - \log_e\frac{2}{\sqrt{3}}\right) + \left(\log_e\frac{\sqrt{3}}{2} - \log_e\frac{1}{\sqrt{2}}\right)$$

$$= 2\log_e\sqrt{2} + 2\log_e\frac{\sqrt{3}}{2}$$

$$= \log_e 2 + \log_e\frac{3}{4} = \log\frac{3}{2}$$

(D) We have

(i) $3e^{-x} = 3$ if $x = 0$

(ii) $3e^{-x} = 2$ if $x = \log_e\frac{3}{2}$

(iii) $3e^{-x} = 1$ if $x = \log_e 3$

(iv) $[3e^{-x}] = 0$ if $x > \log_e 3$

$$\text{Therefore} \int_0^{\log_e 3} [3e^{-x}]\,dx = \int_0^{\log_e 3/2} 2dx + \int_{\log_e 3/2}^{\log_e 3} 1dx$$

$$= 2\left(\log_e\frac{3}{2}\right) + \left(\log_e 3 - \log_e\frac{3}{2}\right)$$

$$= \log_e\frac{9}{4} + \log_e 2$$

$$= \log_e\frac{9}{2}$$

10. Ans (C)

$A\left(2t_1, t_1^2\right)\ B\left(2t_2, t_2^2\right)$

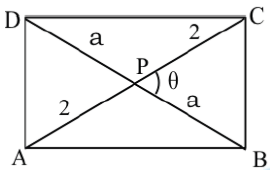
$t_1 t_2 = -4$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2t_1 & t_1^2 & 1 \\ 2t_2 & t_2^2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 20$$

solving $t_1 = \pm 4, \pm 1$

11. Ans (C)

P =(1, 1) by solving AC and BD



$\Delta PCB = 2,\ PC = 2$

Let PB = a

$\therefore \Delta PCB = \frac{1}{2} \times 2 \times a \times \sin\theta$

$2a = BD = \frac{20}{3}$ units

12. Ans (C)

We have $f_1(x) = 2\sum_{r=1}^{n} \frac{\sin\left(x + \frac{r\pi}{6}\right) - \left(x + (r-1)\frac{\pi}{6}\right)}{\cos\left(x + (r-1)\frac{\pi}{6}\right).\cos\left(x + \frac{r\pi}{6}\right)}$

$= 2\left[\left(\tan\left(x + \frac{\pi}{6}\right) - \tan x\right) + \left(\tan\left(x + \frac{2\pi}{6}\right) - \tan\left(x + \frac{\pi}{6}\right)\right)\right.$

$+ \left(\tan\left(x + \frac{3\pi}{6}\right) - \tan\left(x + \frac{2\pi}{6}\right)\right) + \ldots +$

$\left.\left(\tan\left(x + \frac{n\pi}{6}\right) - \tan\left(x + (n-1)\frac{\pi}{6}\right)\right)\right]$

$\Rightarrow f_1(x) = 2\left(\tan\left(x + \frac{n\pi}{6}\right) - \tan x\right)$

For $n = 3$, $f_1(x) = 2\left(\tan\left(\frac{\pi}{2} + x\right) - \tan x\right)$

$= 2(-\cot x - \tan x) = -2\frac{1}{\sin x \cos x}$

Now $\Rightarrow 2f_2(x) = f_1(x) - 2\tan\left(x + \frac{n\pi}{6}\right)$

$= 2\tan\left(x + \frac{n\pi}{6}\right) - 2\tan x - 2\tan\left(x + \frac{n\pi}{6}\right)$

$\therefore f_2(x) = -\tan x$

$\Rightarrow f_3(x) = -f_2(x)$, so $\Rightarrow f_3(x) = \tan x$

13. Ans (A)

$g(2-x) = g(2+x)$

$\&g(2+x)\sin x$ is an odd function. $\therefore I_1 = 0$

Now $g(2-(2-x)) = g(2+2-x) \Rightarrow g(x) = g9(4-x)$

$\Rightarrow g^1(x) = -g^1(4-x)$

So $I_2 = \int_0^4 \frac{1}{1 + e^{g^1(x)}}dx \longrightarrow (1)$

$= \int_0^4 \frac{dx}{1 + e^{g^1(4-x)}}$

$= \int_0^4 \frac{dx}{1 + e^{-g^1(x)}}$

$= \int_0^4 \frac{e^{g^1(x)}}{1 + e^{g^1(x)}}dx \longrightarrow (2)$

$(1) + (2) : 2I_2 = \int_0^4 1dx \Rightarrow I_2 = 2$

14. Ans (A)

$\frac{11!}{4!4!2!3!} \times 2!3! = 69300$

15. Ans (C)

Let z be a point such that

$\arg\left(\frac{z_1 - z}{z_2 - z}\right) = \frac{\pi}{4}$

Let $z = x + iy$. Then

$\arg\left(\frac{z_1 - z}{z_2 - z}\right) = \frac{\pi}{4} \Rightarrow (x - 9)(x - 3) + (y - 5)^2$

$= 6y - 30$

$\Rightarrow x^2 + y^2 - 12x - 16y + 82 = 0$

Now

$|z - 6 - 8i|^2 = (x - 6)^2 + (y - 8)^2$

$= x^2 + y^2 - 12x - 16y + 100$

$= (x^2 + y^2 - 12x - 16y + 82) + 18$

$= 0 + 18$

Therefore $|z - 6 - 8i| = 3\sqrt{2}$

16. **Ans (B)**

Consider the three-digit arrangement, aba. There are 10 choices for a and 10 choices for b (since it is possible for a = b), and so the probability of picking the palindrome is $\frac{10 \times 10}{10^3} = \frac{1}{10}$

Similarly, there is a $\frac{1}{26}$ probability of picking the three-letter palindrome

By the Principle of inclusion-exclusion, the total probability is

$\frac{1}{26} + \frac{1}{10} - \frac{1}{260} = \frac{35}{260} = \frac{7}{52}$

$\Rightarrow 7 + 52$

17. **Ans (D)**

Let diagonal elements are x_1, x_2, x_3, then $x_1 + x_2 + x_3 = 5$

Therefore, Coefficient of x^5 n $(x^0 + x^1 + x^2 + x^3 + x^4)^3$

Therefore 18

Now, total matrices are $18(5^6)$

$$\begin{bmatrix} x_1 & - & - \\ - & x_2 & - \\ - & - & x_3 \end{bmatrix}$$

18. **Ans (B)**

$$\sigma^2 = \frac{1}{n_1 + n_2}\left[n_1\sigma_1^2 + n_2\sigma_2^2 + \frac{n_1 n_2}{n_1 + n_2}(\bar{x}_1 - \bar{x}_2)^2\right]$$

$n_1 = 5 = n_2, \sigma_1^2 = 4, \sigma_2^2 = 5, \bar{x}_1 = 2, \bar{x}_2 = 4$

$$\sigma^2 = \frac{1}{10}\left[20 + 25 + \frac{25}{10} \times 4\right]$$

$$= \frac{1}{10}[45 + 10] = \frac{55}{10} = \frac{11}{2}$$

19. **Ans (D)**

Total sum $= 4^{37}$, $(4^{37} - 2^{37} - 2)$ = sum of coefficients divisible by 37

$$\Rightarrow \frac{a + b + c + 2}{2} = 5$$

20. **Ans (C)**

$n_1 = 2, n_2 = 2$

$$f^1(g(x)) \cdot g'(x) = \underbrace{\left(g^2(x) + g(x) + 1\right)}_{+ve} \cdot g^1(x)$$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (1)**

By applying

$R_1 \to aR_1, R_2 \to bR_2 \& R_3 \to cR_3$, we can write the given determinant as

$$\frac{1}{abc}\begin{vmatrix} -abc & a\left(b^2 + bc\right) & a\left(c^2 + bc\right) \\ b\left(a^2 + ac\right) & -abc & b\left(c^2 + ac\right) \\ c\left(a^2 + ab\right) & c\left(b^2 + ab\right) & -abc \end{vmatrix}$$

$$= \begin{vmatrix} -bc & a(b + c) & a(b + c) \\ b(a + c) & -ac & b(a + c) \\ c(a + b) & c(a + b) & -ab \end{vmatrix}$$

$$= \begin{vmatrix} -bc & ab + bc + ac & ab + bc + ac \\ ab + bc & -(ab + bc + ac) & 0 \\ bc + ac & 0 & -(ab + bc + ac) \end{vmatrix}$$

$C_2 \to C_2 - C_1 \;\; \& \;\; C_3 \to C_3 - C_1$

$$= (ab + bc + ac)^2\begin{vmatrix} -bc & 1 & 1 \\ ab + bc & -1 & 0 \\ bc + ac & 0 & -1 \end{vmatrix}$$

$= (ab + bc + ac)^3$

Now ab + bc + ac

$= \cot 50^\circ \cot 60^\circ + \cot 60^\circ \cot 70^\circ + \cot 50^\circ \cot 70^\circ = 1$

as $(50^\circ + 60^\circ + 70^\circ = 180^\circ)$

2. Ans (4)

$(a, a) \in R \ \forall \ a \in A$ (Reflexive)

$(a, b) \in R \Rightarrow (b, a) \in R$ (Symmetric)

If we pick subset {a, b} i,e. two element subset from set S it corresponds to adding both (a, b) and (b, a) in R (relation).

Total no. of 2-element subset $\binom{n}{2}$.

Now we want to pick collection of subsets of 2 elements.

So either select or reject.

Total Relation $= 2^{\binom{n}{2}}$

$\Rightarrow 2^{\binom{n}{2}} = 2^6 \ \binom{n}{2} = 6 \Rightarrow \boxed{n = 4}$

$\Rightarrow 2^{\binom{n}{2}} = 2^6 \Rightarrow \boxed{n = 4}$

3. Ans (99)

$f(n) = n\pi$

$$\sum_{n=2}^{10} n\pi + (n-1)\pi = \sum_{n=2}^{10} 2n\pi - \pi$$

$= 2\pi(54) - 9\pi = 99\pi$

4. Ans (3)

$$\overline{OG_1}.\overline{BG_2} = 0 \Rightarrow \frac{\bar{a}+\bar{b}+\bar{c}}{3} \cdot \frac{\bar{a}+\bar{c}-3\bar{b}}{3} = 0$$

$\Rightarrow a^2 + b^2 - 3b^2 + 2\bar{a}.\bar{c} - 2\bar{b}.\bar{c} - 2\bar{a}.\bar{b} = 0$

5. Ans (44)

Let $A = \sin 1^0 \sin 3^0 \ldots \sin 89^0$

$B = \sin 2° \sin 4° \sin 6° \ldots\ldots \sin 88°$

$\Rightarrow B = 2^{44} \sin 1^0 \cos 1^0 \sin 2^0 \cos 2^0 \ldots \sin 44^0 \cos 44^0$

$\Rightarrow B = 2^{44}. AB. \sqrt{2}$

$\Rightarrow A = 2^{\frac{89}{2}}$