PHYSICS

In a typical combustion engine the work done by a gas molecule is given by $\boxed{W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be:

(A) $[MLT^{-1}]$ (B) $[M^0LT^0]$ ✓

(C) $[M^2LT^{-2}]$ (D) $[MLT^{-2}]$

$W = \alpha^2 \beta \, \boxed{e^{-\boxed{\frac{\beta x^2}{kT}}}}$

same

$[\square] = M^0L^0T^0 = 1$

$[\alpha] = ?$

$\frac{[\beta][x]^2}{[kT]} = M^0L^0T^0 = 1$

$PV = nRT$

$PV = NkT$

Joule

$[x] = L$

$[kT] = ML^2T^{-2}$

$[PV] = ML^2T^{-2}$

$[W] = [\alpha]^2[\beta]$

$ML^2T^{-2} = [\alpha]^2[\beta]$

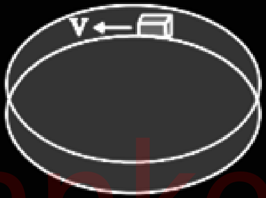
$\cancel{M}L^2\cancel{T^{-2}} = [\alpha]^2 \cancel{M}\cancel{T^{-2}}$

$[\alpha] = M^0L^1T^0$

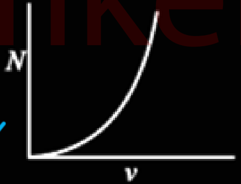
$[\beta]\cancel{L^2} = M\cancel{L^2}T^{-2}$

$[\beta] = MT^{-2}$

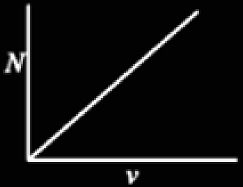
A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v) ?

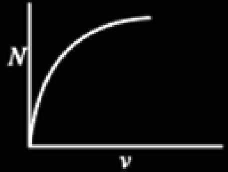


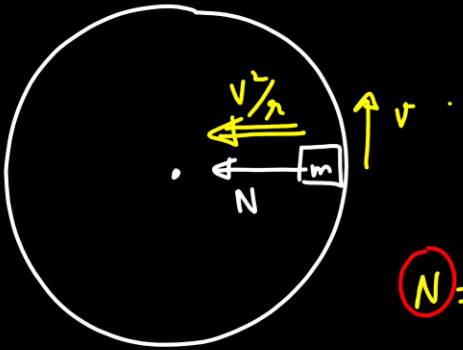
UCM

(A)


(C)

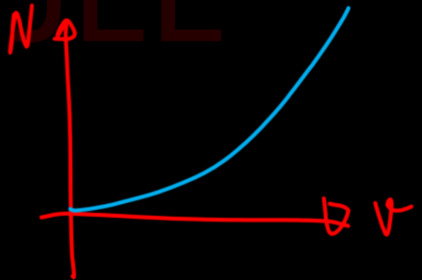

(B)


(D)




$$N = \frac{mv^2}{R}$$

$$\boxed{y = k\ x^2}$$



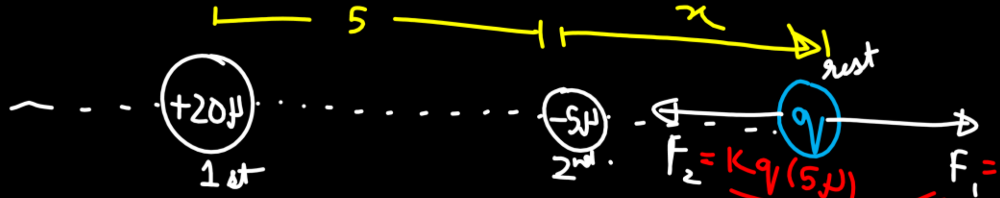
3

Two particles A and B having charges 20μC and −5μC respectively are held fixed with a separation of 5 cm . At what position a third charged particle should be placed so that it does not experience a net electric force?

$$\boxed{F = \frac{kq_1q_2}{r^2}}$$

$\boxed{\begin{matrix}\text{cm}\\ \mu\text{C}\end{matrix}}$

(A) At 5 cm from 20μC on the left side of system.
(B) At 1.25 cm from −5μC between two charges.
(C) At midpoint between two charges.
(D) At 5 cm from −5μC on the right side.



$F_2 = \frac{Kq(5\mu)}{x^2}$ = $F_1 = \frac{Kq(20\mu)}{(5+x)^2}$

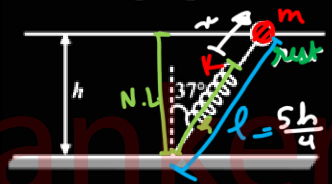
$$\frac{\cancel{K}\cancel{q}(5\mu)}{x^2} = \frac{\cancel{K}\cancel{q}(\overset{4}{\cancel{20\mu}})}{(5+x)^2}$$

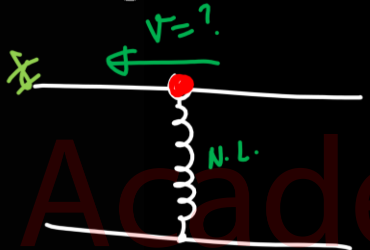
$$\frac{1}{x} = \frac{2}{5+x}$$

$$2x = 5 + x.$$

$$\boxed{x = 5\text{ cm}}$$

One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h. Initially the spring makes an angle of 37° with the vertical when the system is released from rest. The speed of the ring when the spring becomes vertical is





(A) $\sqrt{\frac{k}{m}}$ (B) $\frac{h}{2}\sqrt{\frac{k}{m}}$

(C) $\frac{h}{3}\sqrt{\frac{k}{m}}$ (D) $\frac{h}{4}\sqrt{\frac{k}{m}}$

$l\cos 37^\circ = h.$

$l = \frac{5h}{4}$

$x = l - h$

$\boxed{x = h/4}$

$E_i = E_f.$

$\cancel{\frac{1}{2}} kx^2 = \cancel{\frac{1}{2}} mv^2.$

$k\left(\frac{h}{4}\right)^2 = mv^2.$

$\boxed{\frac{h}{4}\sqrt{\frac{k}{m}} = v}$

5

A photoelectric surface is illuminated successively by monochromatic light of wavelength λ and $\lambda/2$ if the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface of the material is :

(h = Planck's constant, c = speed of light)

(A) $\frac{2hc}{\lambda}$ (B) $\frac{hc}{3\lambda}$

(C) $\frac{hc}{2\lambda}$ (D) $\frac{hc}{\lambda}$

Case-1 (λ)

$$\left[\boxed{\frac{hc}{\lambda} = \phi + K}\right] \times 3 \quad (1)$$

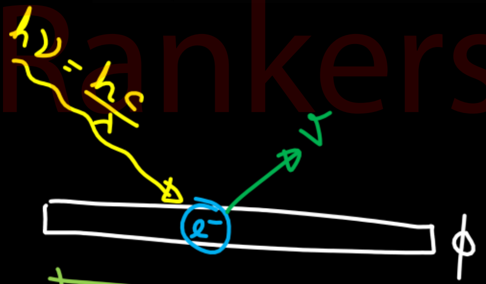
$\phi = ?$

Case-2 $(\lambda/2)$

$$\boxed{\frac{hc}{(\lambda/2)} = \phi + 3K} \quad (2)$$

$$\frac{hc}{\lambda} = 2\phi$$

$$\boxed{\phi = \frac{hc}{2\lambda}}$$



$$\boxed{\frac{hc}{\lambda} = \phi + K}$$

6

A rod of length L has non-uniform linear mass density given by $\mu = \cancel{\rho(x)} = a + b\left(\frac{x}{L}\right)^2$ where a and b are constants and $0 \le x \le L$. The value of x coordinate of the centre of mass of the rod

(A) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(B) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

(C) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

(D) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

μ	kg/m
σ	kg/m^2
ρ	kg/m^3

M-1 let $\boxed{b = 0}$

$\boxed{\mu = a = \text{constt}}$

uniform rod.

$x=0$ $x=L$

then COM $\boxed{x = \frac{L}{2}}$

$$\boxed{\mu = a + \frac{b}{L^2}x^2}$$

$\mu = a$ $\quad$ $\mu = a+b$

x $\quad$ dx $\quad$ COM

$\boxed{x=0}$ $\quad$ $dm = \mu\, dx$ $\quad$ $\boxed{x=L}$

$$\boxed{X_{com} = \frac{\int dm\, x^*}{\int dm}}$$

$$dm = \mu \cdot dx = \left(a + \frac{bx^2}{L^2}\right) \cdot dx.$$

$$x^* = x$$

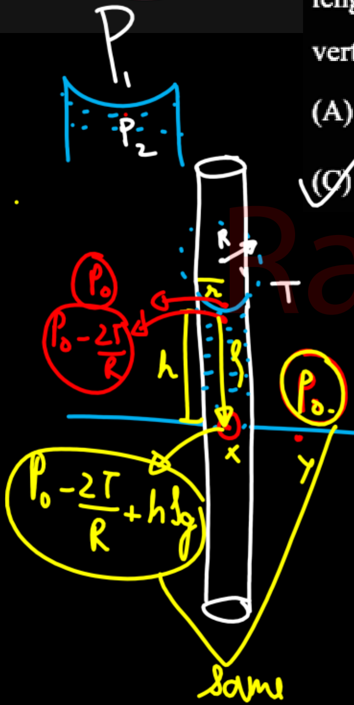
$$X_{com} = \frac{\int_{x=0}^{x=L}\left(ax + \frac{bx^3}{L^2}\right) \cdot dx}{\int_{x=0}^{x=L}\left(a + \frac{bx^2}{L^2}\right) \cdot dx} = \ldots$$

$$= \frac{3}{4}\frac{(2a+b)}{(3a+b)}L$$

When a capillary is dipped vertically inside a liquid, the liquid rises up to a height $\boxed{h}$. In the same capillary, the same liquid is filled in horizontal position and then kept in vertical position with both ends in air. The maximum length of liquid column which can be kept in vertical position is

(A) $\frac{h}{2}$ (B) h

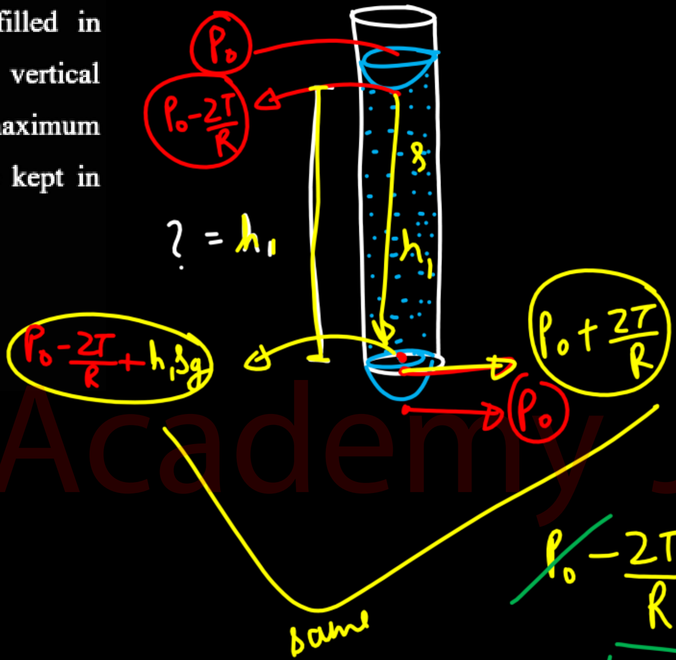
(C) $2h$ (D) $\frac{3h}{2}$



$\cancel{P_0} - \frac{2T}{R} + h\rho g = \cancel{P_0}$

$\boxed{h = \frac{2T}{R\rho g}}$

$\boxed{R = \frac{r}{\cos\theta}}$



$\cancel{P_0} - \frac{2T}{R} + h_1\rho g = \cancel{P_0} + \frac{2T}{R}$

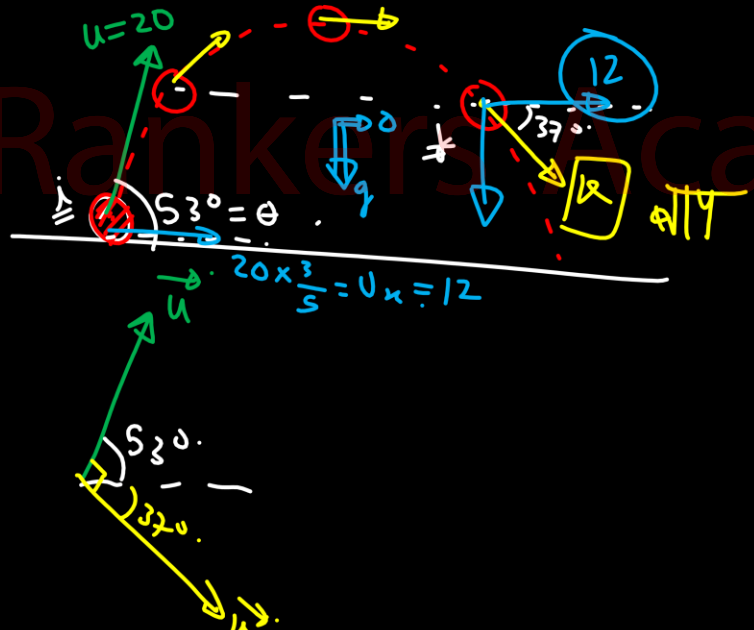
$\boxed{h_1 = \frac{4T}{R\rho g}}$

$h_1 = 2h$

8 A projectile is projected with velocity 20 m/s at an angle of 53° with horizontal. Speed of the projectile when its velocity is perpendicular to its initial velocity, is

(A) 16 m/s (B) 12 m/s

(C) 15 m/s (D) 18 m/s



$V \cos 37^\circ = 12$

$V \times \frac{4}{5} = 12$

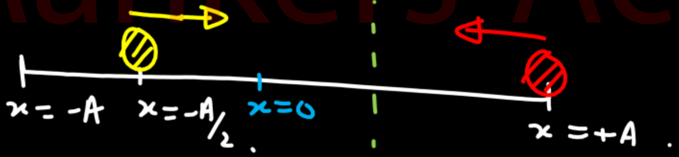
$\boxed{V = 15}$

9

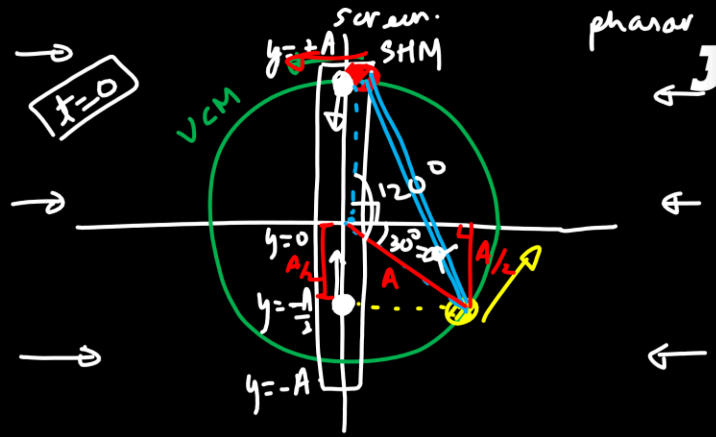
Two particles are in SHM in a straight line about same equilibrium position. Amplitude A and time period T of both the particles are equal. At time $t = 0$, one particle is at displacement $y_1 = +A$ and the other at $y_2 = -\frac{A}{2}$ and they are approaching towards each other. The time after which they cross-each other will be

~~(A) $\frac{T}{3}$~~ ~~(B) $\frac{T}{4}$~~

$\boxed{\text{(C) } \frac{T}{6}}$ ✓ (D) $\frac{T}{5}$

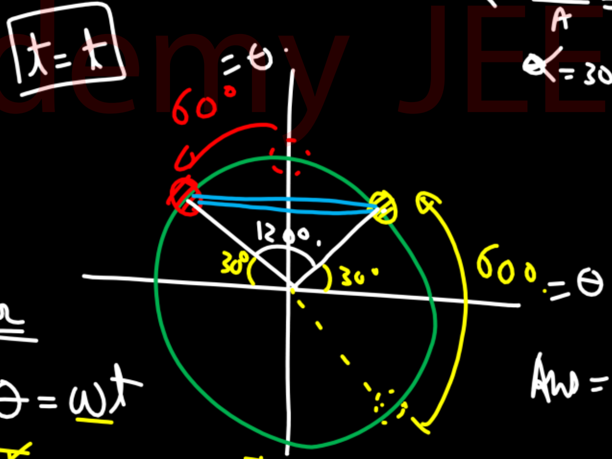


$\boxed{\text{Ans} < T/4}$



$\sin\alpha = \frac{A/2}{A} = \frac{1}{2}$

$\alpha = 30°$



or

$\theta = \omega t$

$60° \rightarrow \frac{\pi}{3} = \frac{2\pi}{T} \cdot t$ $\boxed{t = T/6}$

$\text{Ans} = t = \left(\frac{60°}{360°}\right) \cdot T$

$t = \frac{T}{6}$ Ans.

10

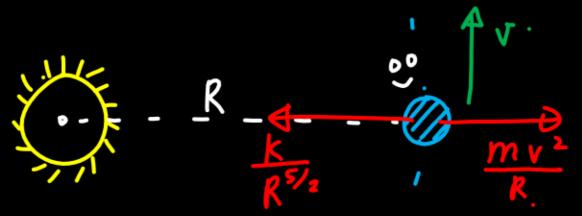
Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is inversely proportional to $R^{5/2}$, then the time period of revolution is proportional to $R^{k/4}$. Find the value of k.

(A) 3 (B) 5

(C) 7 (D) 9

$F = \frac{GMm}{R^2}$

$\boxed{F = \frac{K}{R^{5/2}}}$



$$\frac{K}{\cancel{R^{5/2}}_{R^{3/2}}} = \frac{mv^2}{\cancel{R}}$$

$$v = \sqrt{\frac{K}{m}}\, R^{-3/4}$$

$$T = \frac{2\pi R^1}{v}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} \frac{R^1}{R^{-3/4}} = \ldots R^{\frac{7}{4}}$$

$$T \propto R^{k/4}$$

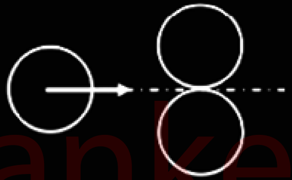
$\boxed{k = 7}$

JEE 1

11

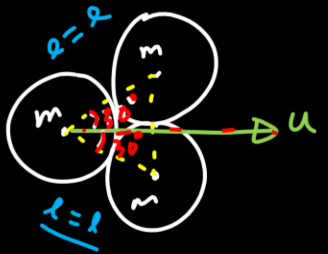
Two smooth identical stationary spheres are kept touching each other on a smooth horizontal floor as shown. A third identical sphere moving horizontally with a constant speed hits both stationary spheres symmetrically. If after collision the third sphere moves in same direction with one fourth of its initial speed, the coefficient of restitution will be

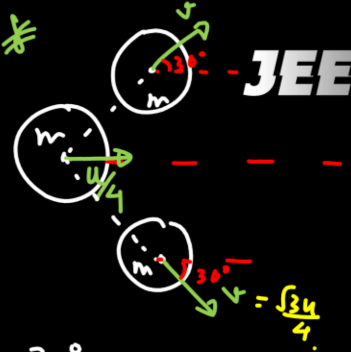
$e = ?$



(A) 2/3 (B) 1/3

(C) ¼ (D) 1/6



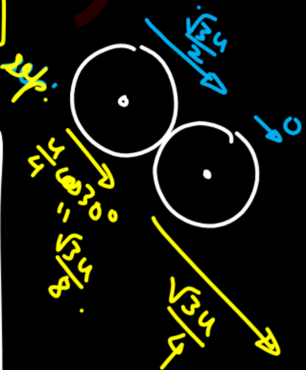


P $\rightarrow$ x

$\cancel{m}u = \cancel{m}\frac{u}{4} + 2\cancel{m}v\cos 30°$

$\frac{3u}{4} = \sqrt{3}v$

$\boxed{v = \frac{\sqrt{3}u}{4}}$



C

$e = \frac{V_{sep.}}{V_{app}}$

$e = \frac{\frac{\cancel{\sqrt{3}u}}{4} - \frac{\cancel{\sqrt{3}u}}{8}}{\frac{\cancel{\sqrt{3}u}}{2}}$

$\boxed{e = \frac{1}{4}}$

12

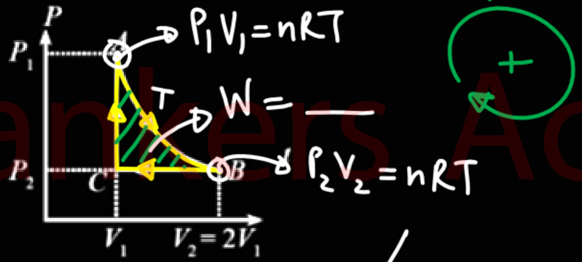
n moles of perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following process.

$A \rightarrow B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and Pressure changes from P_1 to P_2.

$B \rightarrow C$: Isobaric compression at pressure P_2 to initial volume V_1.

$C \rightarrow A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total work done in the complete cycle $\boxed{\text{ABCA}}$ is:



(A) $nRT\left(\ln 2 + \frac{1}{2}\right)$

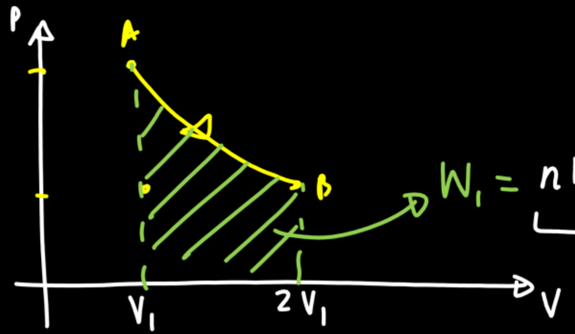
(B) $nRT\left(\ln 2 - \frac{1}{2}\right)$ ✓

(C) 0

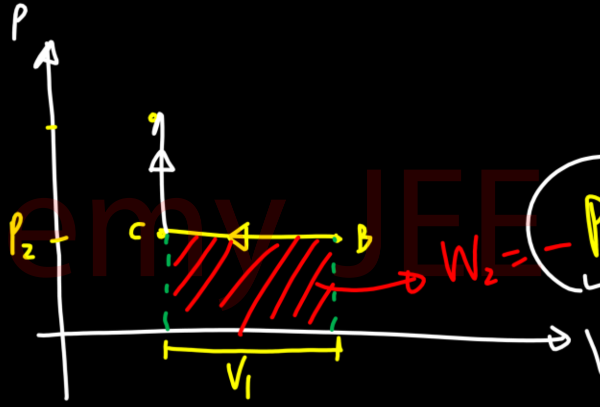
(D) $nRT\ln 2$

$$P_1V_1 = P_2V_2 = nRT$$

$$V_1 = \frac{V_2}{2}$$



$$W_1 = \underbrace{nRT\ln\left(\frac{2V_1}{V_1}\right)}$$



$$W_2 = -P_2V_1 = -\frac{P_2V_2}{2} = -\frac{nRT}{2}$$

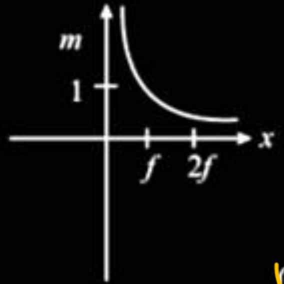
$$W = W_1 + W_2$$

$$W = \underbrace{nRT\ln(2)} + \underbrace{\left(-\frac{nRT}{2}\right)}$$

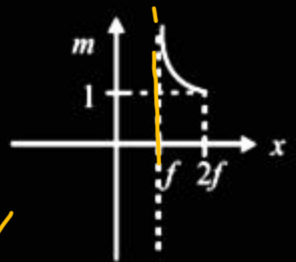
An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by (Graphs are drawn schematically and are not to scale)

(A)

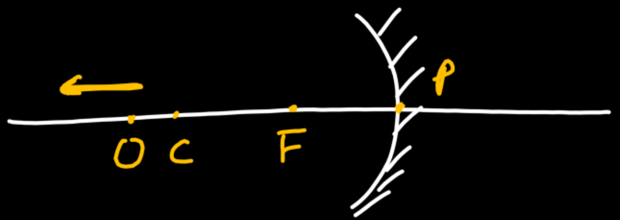
(B)



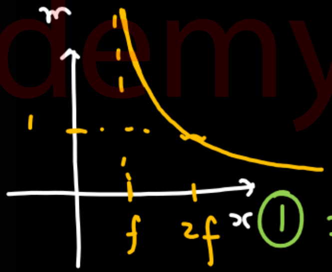
(C)



(D)



$$\boxed{m = -\frac{v}{u} = \frac{f}{f-u}} = \frac{f}{f+x}$$



$$m = \frac{f}{f+x}$$

① $x = -f$ (at F) $m \to \infty$

② $x \to -\infty$ (at ∞) $m = 0$ (point image)

③ $x = -2f$ $m = -1$ (at C)

14

JEE 1

An electron of a hydrogen like atom, having $\boxed{Z = 4}$, jumps from 4^{th} energy state to 2^{nd} energy state, The energy released in this process, will be:

$n = 4$

$n = 2$

(Given $Rch = 13.6eV$)

Where R = Rydberg constant c = Speed of light in vacuum h = Planck's constant

(A) 13.6 eV
(B) 10.5 eV
(C) 3.4 eV
(D) 40.8 eV ✓

$$\Delta E = 13.6\, Z^2 \left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= 13.6 \times 4^2 \left[\frac{4-1}{4^2}\right]$$

$$= 13.6 \times 3$$

$$= 40.8\, eV$$

15

Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

(A) In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

(B) The energy in electromagnetic wave is divided equally between electric and magnetic fields.

(C) Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

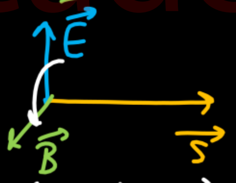
(D) The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

(E) The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

$$\boxed{U_E = U_B}$$

$$\frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0} \Rightarrow B_0 = \sqrt{\mu_0 \varepsilon_0}\, E_0$$

$$B_0 = \frac{E_0}{c} \Rightarrow \boxed{E_0 = B_0 c}$$



$$(\vec{E} \times \vec{B}) \parallel \vec{S}$$

$$\frac{B}{E} \neq c$$

Choose the most appropriate answer from the options given below:

(A) (D) only

(B) (B) and (D) only

(C) (B), (D) and (E) only

(D) (A), (B) and (E) only

16

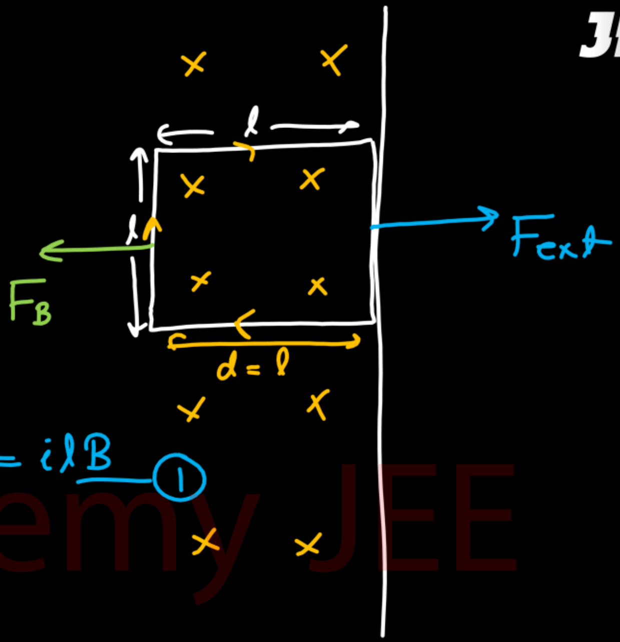
A square loop of area 25 cm^2 has a resistance of 10Ω. The square loop is placed at the edge of a uniform magnetic field of magnitude 40.0 T. The plane of the loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field 'slowly' and uniformly in 1.0 sec, will be

(A) 2.5×10^{-3} J

(B) 1.0×10^{-3} J

(C) 1.0×10^{-4} J

(D) 5×10^{-3} J



$F_{ext} = F_B = ilB$ (1)

$W = F_{ext}\, l$

$= (ilB)\, l$

$= \left(\frac{Bvl}{R}\right) l^2 B$

$= \frac{B^2 l^3}{R}\left(\frac{l}{t}\right) = \frac{B^2 l^4}{Rt} = \frac{40^2 \times (5\times10^{-2})^4}{10 \times 1} = 160 \times 625 \times 10^{-8}$

$= 10^{-3}$ J

17

JEE 1

In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen 2 m from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is

(A) 1.5 mm (B) 3 mm

(C) 4.5 mm (D) 6 mm

$y = n_1 \beta_1 = n_2 \beta_2$

$\frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d}$

$\boxed{n_1 \lambda_1 = n_2 \lambda_2}$ *

$n_1 \times 750 = n_2 \times 900$

$n_2 = 5$

$y = \frac{n_2 \lambda_2 D}{d} = \frac{5 \times 900 \times 10^{-9} \times 2}{2 \times 10^{-3}}$

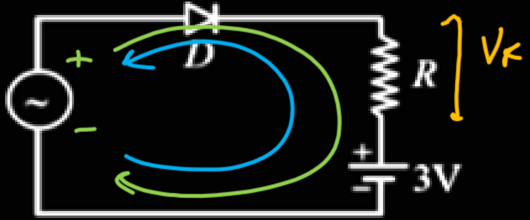
$= 45 \times 10^{-4}$

$= 4.5 \times 10^{-3}$ m

18

Choose the correct waveform that can represents the voltage across R of the following circuit, assuming the diode is ideal one:

$\boxed{V_i = 10\sin\omega t}$



at $t>0$ $V_i > 3V$

Diode F.B.

$V_R = V_i - 3V$

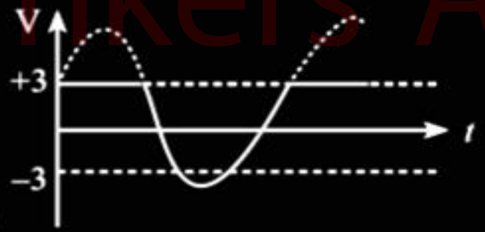
$\boxed{V_R = 10\sin\omega t - 3}$

$V_i \le 3V$

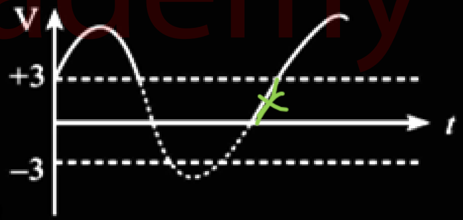
Diode R.B. $i=0$

$\Rightarrow V_R = iR$

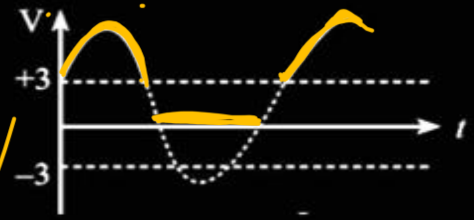
$\boxed{V_R = 0}$



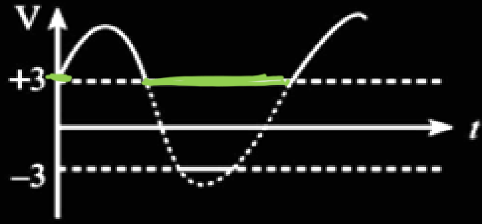
(A)



(C)



(B)



(D)

19

JEE 1

The relation between internal energy U, pressure P and volume V of a gas in an adiabatic process is $\boxed{U = a + bPV.}$ Where a and b are constant. What is the effective value of adiabatic constant γ ?

(A) $\frac{a}{b}$

(B) $\frac{b+1}{b}$

(C) $\frac{a+1}{a}$

(D) $\frac{b}{a}$

$\gamma = \frac{C_p}{C_v} = ?$

$U = a + b(nRT)$

$\Delta U = bnR\Delta T$

$nC_v\Delta T = bnR\Delta T$

$C_v = bR$ — (1)

$C_p - C_v = R$

$C_p = (b+1)R$ — (2)

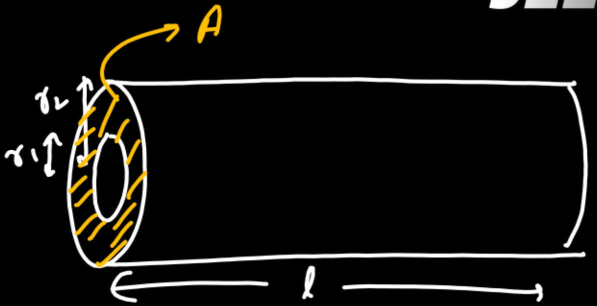
$\gamma = \frac{C_p}{C_v} = \frac{b+1}{b}$

20

A hollow cylindrical conductor has length of 3.14 m , While its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor in $n \times 10^{-3} \Omega$. If the resistivity of the material is $2.4 \times 10^{-8} \Omega$ m. The value of the n is

(A) 2 (B) 4

(C) 6 (D) 8



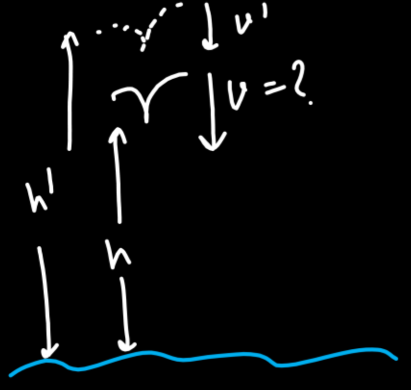
$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi (r_2^2 - r_1^2)}$$

$$= \frac{2.4 \times 10^{-8} \times \cancel{3.14}\,m}{\cancel{\pi} \times (4^2 - 2^2) \times 10^{-6}\,m^2}$$

$$= \frac{2.4 \times 10^{-2}}{12} = \boxed{2} \times 10^{-3}\,\Omega$$

21

A fish rising vertically upward with a uniform velocity of 8 ms^{-1}, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms^{-1}. If the refractive index of water is 4/3, then the actual velocity of the diving bird to pick the fish, will be ____ ms^{-1}.



$$h' = \mu h$$

$$\frac{dh'}{dt} = \mu \frac{dh}{dt}$$

$$\Rightarrow \boxed{v' = \mu v} \quad (1)$$

$$v_{rel} = v_0 + v'$$

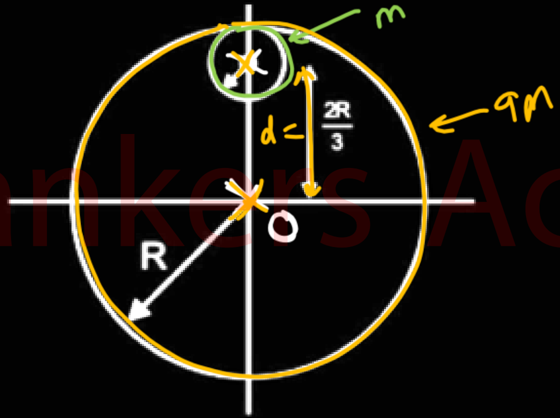
$$12 = v_0 + \mu v$$

$$12 = 8 + \frac{4}{3} v$$

$$v = 3 \, m/s$$

22

From a circular disc of radius R and mass 9M, a small disc of radius R/3 is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is _____ MR^2



$9m \equiv \pi R^2$

$m \equiv \pi\left(\frac{R}{3}\right)^2$

$$\boxed{I_{rem} = I_{total} - I_{cavity}}$$

Parallel axis Th

$I_0 + md^2$

$$= \frac{(9m)R^2}{2} - \left[\overbrace{\frac{m\left(\frac{R}{3}\right)^2}{2} + m\left(\frac{2R}{3}\right)^2}\right]$$

$$= mR^2\left[\frac{9}{2} - \frac{1}{18} - \frac{4}{9}\right]$$

$$= mR^2\left(\frac{81 - 1 - 8}{18}\right) = \frac{72\,mR^2}{18}$$

$$= \boxed{4}\,mR^2$$

23

In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, '4 linear scale divisions' are clearly visible while 46th division the circular scale coincide with the reference line. The diameter of the wire is ______ $\times 10^{-2}$ mm.

MSR

$$\boxed{L.C. = \frac{\text{pitch}}{NCSD}} = \frac{0.5\,mm}{100} \quad (1)$$

$$\text{zero error} = +6 \times L.C. \quad (2)$$

$$\boxed{\text{Reading} = MSR + CCSD \times L.C. - \text{zero error}}$$

$$= 4\,MSD + 46 \times L.C. - 6 \times L.C.$$

$$= 4(0.5\,mm) + \frac{40 \times 0.5}{100}$$

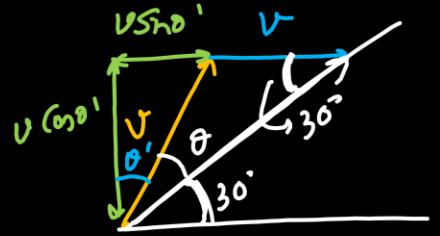
$$= 2 + 0.2$$

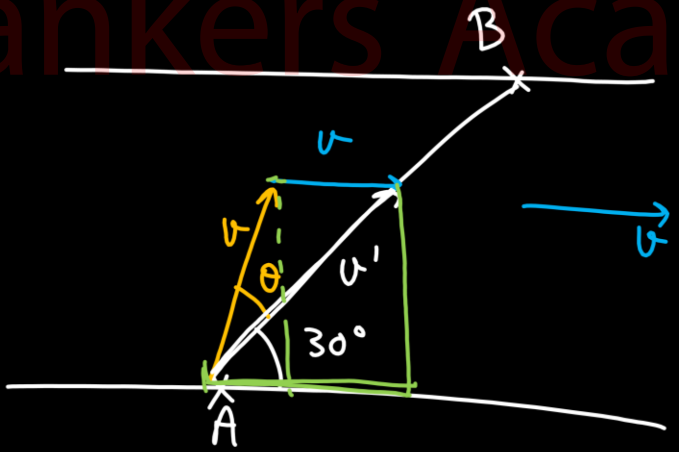
$$= 2.2\,mm$$

$$= \boxed{220} \times 10^{-2}\,mm$$

24

A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer in still water is same as that of the river. The angle θ with the line AB should be ____ °, so that the swimmer reaches point B.





$$\tan 30^\circ = \frac{v\cos\theta'}{v\sin\theta' + v}$$

$$1 + \sin\theta' = \sqrt{3}\cos\theta'$$

$$1 + \sin^2\theta' + 2\sin\theta' = 3(1 - \sin^2\theta')$$

$$\sin\theta' = \frac{1}{2} \qquad \theta' = 30^\circ$$

$$\theta = 90^\circ - \theta' - 30^\circ$$

$$= 30^\circ$$

25

JEE 1

In the figure shown below, the charge on the left plate of the 10μF capacitor is $-30\mu C$. The charge (in μC) on the right plate of the 6μF capacitor is:

$-Q_1 \; +Q_1 = ?$

In Parallel $V_1 = V_2$

$30\mu C = Q_1 + Q_2$ — (1)

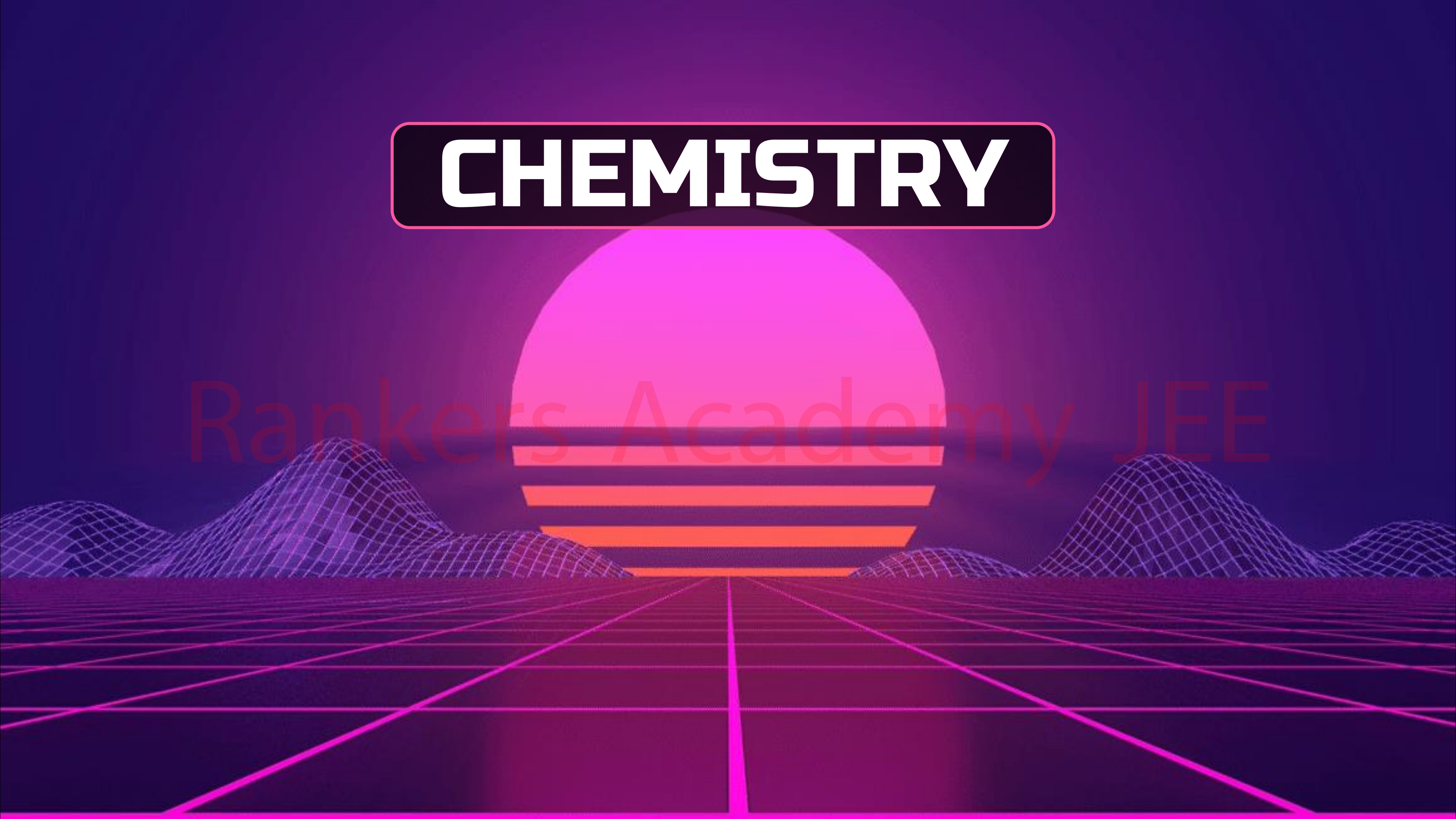
$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{6\mu F}{4\mu F} = \frac{3}{2}$ — (2)

from (1) & (2)

$Q_1 = \frac{3}{3+2} \times 30\mu C$

$= \boxed{18}\mu C$


CHEMISTRY

Match List - I with List - II:

	List – I		List – II
(A)	$[Mn(H_2O)_6]^{2+}$	(I)	Colourless
(B)	$[Cr(H_2O)_6]^{3+}$	(II)	Green
(C)	$[Fe(H_2O)_6]^{2+}$	(III)	Purple
(D)	$[Sc(H_2O)_6]^{3+}$ $3d^0$	(IV)	Pink

Choose the correct answer from the options given below:

(A) A - III, B - IV, C - I, D - III

(B) A - IV, B - II, C - III, D - I

(C) A - IV, B - III, C - I, D - II

(D) A - IV, B - III, C - II, D - I

JEE 1

Configuration	Example	Colour
$3d^0$	Sc^{3+}	colourless
$3d^0$	Ti^{4+}	colourless
$3d^1$	Ti^{3+}	purple
$3d^1$	V^{4+}	blue
$3d^2$	V^{3+}	green
$3d^3$	V^{2+}	violet
$3d^3$	Cr^{3+}	violet
$3d^4$	Mn^{3+}	violet
$3d^4$	Cr^{2+}	blue
$3d^5$	Mn^{2+}	pink
$3d^5$	Fe^{3+}	yellow
$3d^6$	Fe^{2+}	green
$3d^6 3d^7$	$Co^{3+} Co^{2+}$	bluepink
$3d^8$	Ni^{2+}	green
$3d^9$	Cu^{2+}	blue
$3d^{10}$	Zn^{2+}	colourless

JEE 1

Transition metals in M^{+2} state

V^{2+}	d^3	violet	→	V	→	very
Cr^{2+}	d^4	Blue	→	B	→	Beautiful and
Mn^{2+}	d^5	Pink	→	P	→	Pretty
Fe^{2+}	d^6	green	→	G	→	Girl
Co^{2+}	d^7	Pink	→	P	→	PRIYA
Ni^{2+}	d^8	green	→	G	→	Going to
Cu^{2+}	d^9	Blue	→	B	→	Bank to
Zn^{2+}	d^{10}	Colourless	→	C	→	Collect money

JEE 1

Transition metal ions in M^{+3} state

Sc^{3+}	d^0	Colourless	$\rightarrow$	C	$\rightarrow$	Cool	and
Ti^{3+}	d^1	Purple	$\rightarrow$	P	$\rightarrow$	Pretty	
V^{3+}	d^2	Green	$\rightarrow$	G	$\rightarrow$	Girl	
Cr^{3+}	d^3	violet	$\rightarrow$	V	$\rightarrow$	VINNY	is
Mn^{3+}	d^4	violet	$\rightarrow$	V	$\rightarrow$	very	
Fe^{3+}	d^5	Yellow	$\rightarrow$	Y	$\rightarrow$	Young	and
Co^{3+}	d^6	Blue	$\rightarrow$	B	$\rightarrow$	Beautiful	

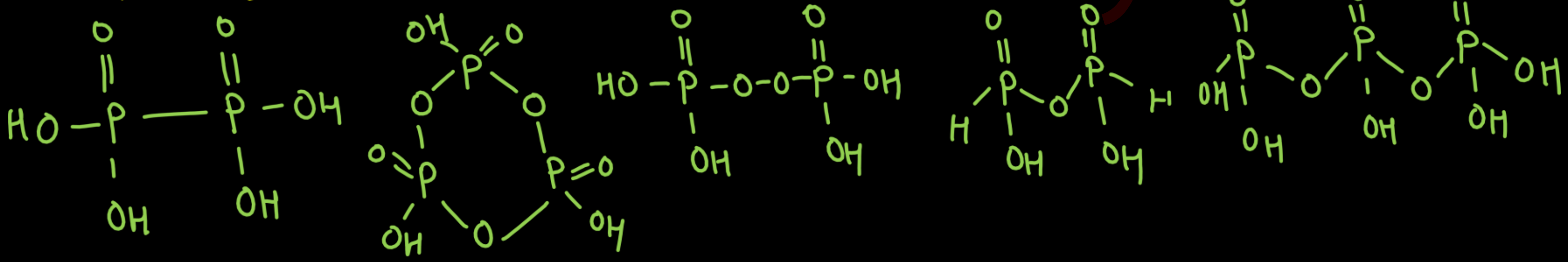
2

Consider the following phosphorus based oxoacids $\underbrace{H_4P_2O_6}$, $H_3P_3O_9$, $H_4P_2O_8$, $H_4P_2O_5$ and $H_5P_3O_{10}$ Amongst these oxoacids, the number of oxoacids with $P-O-P$ bond is ______

(A) 4 (B) 2

(C) 3 (D) 5

$H_4P_2O_6$ $H_3P_3O_9$ $H_4P_2O_8$ $H_4P_2O_5$ $H_5P_3O_{10}$



A_3B_4 is a sparingly soluble salt of molar mass is $x\ \text{g mol}^{-1}$ and solubility of this salt in water is $y\ \text{g L}^{-1}$, then the ratio of concentration of $\underline{A^{4+}(aq)}$ to $\underline{K_{sp}}$ of the salt is

(A) $\frac{1}{2304}\frac{x^6}{y^6}$ ✓

(B) $\frac{1}{256}\frac{x^6}{y^6}$

(C) $\frac{1}{768}\frac{y^6}{x^6}$

(D) $\frac{1}{1012}\frac{x^6}{y^6}$

$$A_3B_4 \rightleftharpoons \underset{3s}{3A^{4+}} + \underset{4s}{4B^{3-}}$$

$$K_{sp} = (3s)^3(4s)^4$$

$$\therefore s = \frac{y}{x}$$

$$K_{sp} = (3)^3(4)^4\left(\frac{y}{x}\right)^7$$

$$A^{4+} = 3s = 3\left(\frac{y}{x}\right)$$

The ratio

$$\Rightarrow \frac{[A^{4+}]}{K_{sp}} = \frac{\cancel{3}\cancel{\left(\frac{y}{x}\right)}}{(3)^{\cancel{3}\,2}(4)^4\left(\frac{y}{x}\right)^{\cancel{7}\,6}}$$

$$= \frac{1}{(3)^2(4)^4}\left(\frac{x}{y}\right)^6$$

$$= \frac{1}{2304}\left(\frac{x}{y}\right)^6$$

Compounds A and C in the following reaction are ______ .

$$CH_3CHO \xrightarrow[\text{(ii) } H_2O]{\text{(i) } CH_3MgBr} (A) \xrightarrow{H_2SO_4,\ \Delta} (B) \xrightarrow{\text{Hydroboration oxidation}} (C)$$

(i) identical

(ii) positional isomers

(iii) functional isomers

(iv) optical isomers

$$CH_3CHO \xrightarrow[H_2O]{CH_3MgBr} CH_3-\overset{OH}{\overset{|}{C}H}-CH_3 \xrightarrow{H_2SO_4,\Delta} CH_3-CH=CH_2 \xrightarrow{HBO} CH_3-CH_2-CH_2-OH$$

Propan-2-ol

Propan-1-ol

5

Given below are two statements:

Statement I: The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH by forming, a g~~re~~en- Red colored complex.

Statement II: The dimethyl glyoxime is a bidentate neutral anionic ligand. ✗

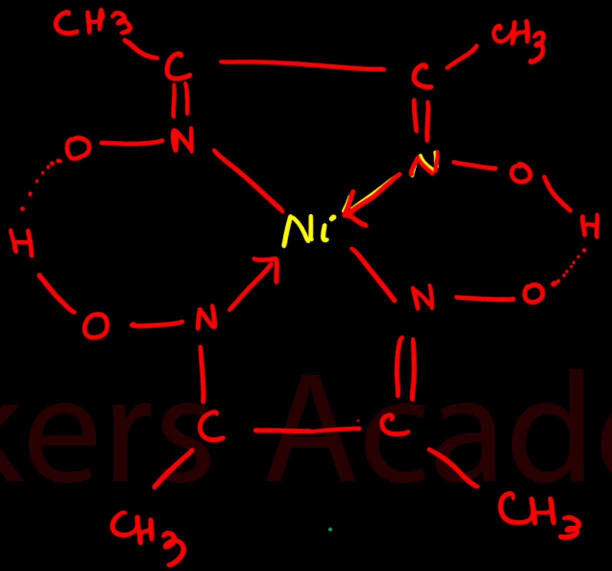
In the light of the above statements, choose the correct answer from the options given below:

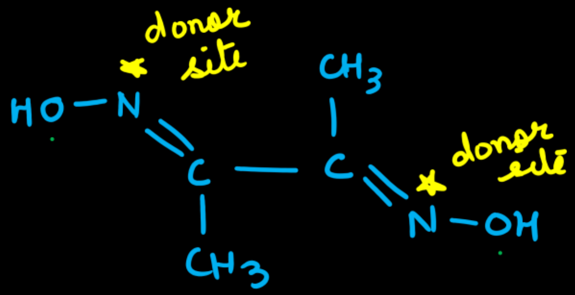
(A) Statement I is false but Statement II is true.

(B) Both Statement I and Statement II are false. ✓

(C) Statement I is true but Statement II is false.

(D) Both Statement I and Statement II are true.





Structure of dmg

Nickel complex with dimethyl glyoxime

Nickel bis(dimethyl glyoximate)

6

Which of the following is not essential amino acid

(A) Valine

(B) Lysine

(C) Histidine

(D) Glycine

GAGAGA → non-essential

CSTP → non-essential

7

Thermal decomposition of a Mn compound (X) at 513K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z, X, Y and Z respectively.

(A) K_2MnO_4, $KMnO_4$ and SO_2

(B) K_2MnO_4, $KMnO_4$ and Cl_2

(C) K_3MnO_4, K_2MnO_4 and Cl_2

(D) $KMnO_4$, K_2MnO_4 and Cl_2

$$\underset{(X)}{KMnO_4} \xrightarrow{513K} \underset{(Y)}{K_2MnO_4} + MnO_2 + O_2$$

$$MnO_2 + NaCl + H_2SO_4 \longrightarrow MnSO_4 + NaHSO_4 + \underset{(Z)}{Cl_2} + H_2O$$

JEE 1

8

Among the elements N, P, As, Sb and Bi the most stable oxidation state and nature of oxide of the element whose hydride has the highest Boiling point is

(A) +3 and acidic

(B) +3 and basic

(C) +5 and acidic

(D) +1 and amphoteric

+3 (More stable due to Inert pair effect)

+5

JEE1

9

$\frac{1}{2}x_2 + \frac{3}{2}y_2 \rightarrow xy_3; \Delta H_r = -30 \text{ kJ mol}^{-1}$

Entropies of x_2, y_2 and xy_3 are 60,40 and 50 J K^{-1} mol^{-1} respectively for the reaction. The temperature in kelvin at which the above reaction attains equilibrium.

(A) 750 (B) 650

(C) 550 (D) 880

$\Delta G = \Delta H - T\Delta S$

At eq^n, $\Delta G = 0$

$T = \frac{\Delta H^\circ}{\Delta S^\circ}$

$\Delta S^\circ = 50 - \left[\left(\frac{1}{\cancel{2}} \times \overset{30}{\cancel{60}}\right) + \left(\frac{3}{\cancel{2}} \times \overset{20}{\cancel{40}}\right)\right]$

$= 50 - 90 = -40$ J/K/mol

$T = \frac{-30 \times 10^3}{-40} = 750$ K

Which of the following will exhibit geometrical isomerism?

(i) 2,5-dichloro-3-hexene

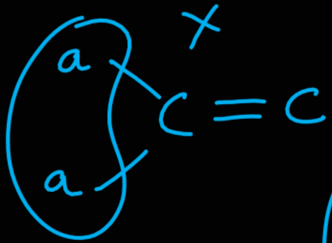
(ii) 1-phenyl-2-butene

(iii) 3-phenyl-1-butene

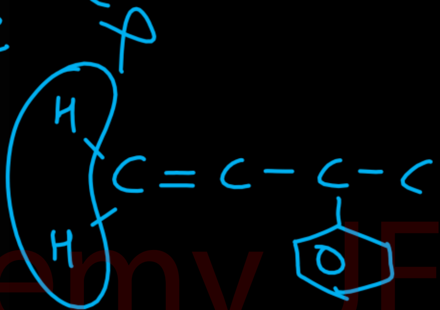
(iv) 1,4-dichloro-2-pentene

(A) (i) and (ii)　　(B) (iii) and (iv)

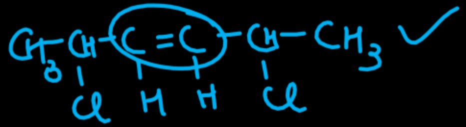
(C) (i), (ii) and (iv)　　(D) (ii) and (iv)



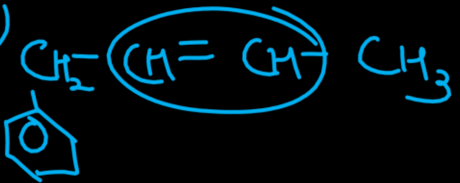
(iii)



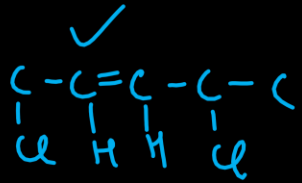
(i)



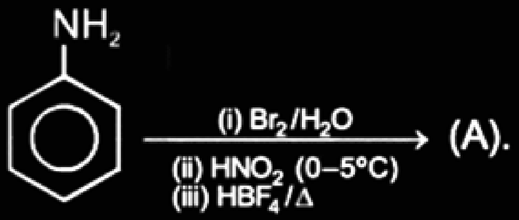
(ii)

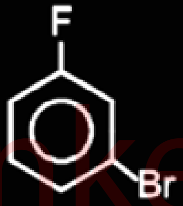


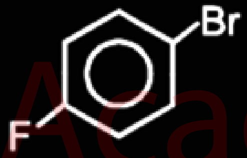
(iv)

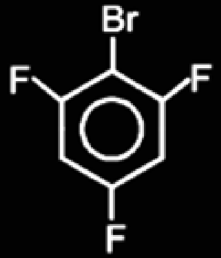


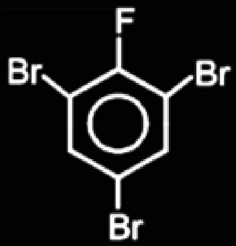
The major product (A) is

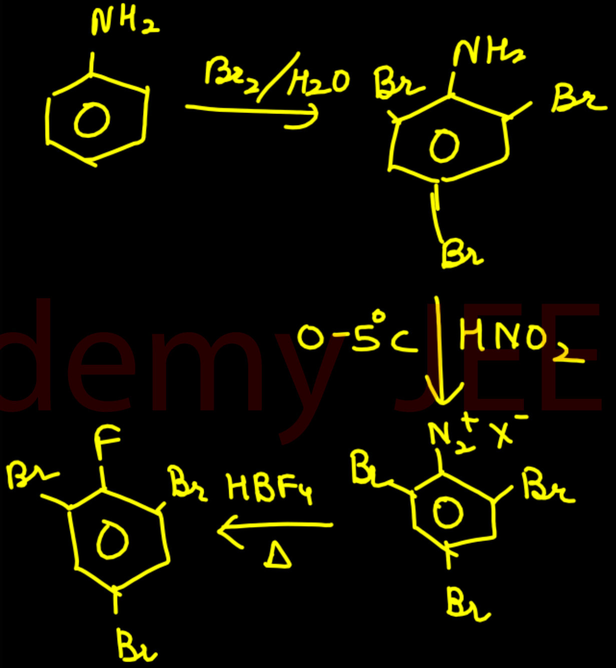


(A)


(B)


(C)


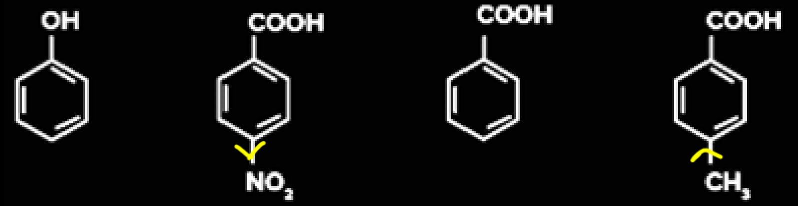
(D)




12

JEE1

The correct order of acidic strength character of the following compounds is



I II III IV

(A) IV > III > II > I (B) III > II > I > IV

(C) II > III > IV > I (D) I > II > III > IV

ewg acidity ↑

edg acidity ↓

13

Assertion (A) : Λ_m for weak electrolytes shows a sharp increase when the electrolytic solution is diluted.

Reason (R) : For weak electrolytes degree of dissociation increases with dilution of solution.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

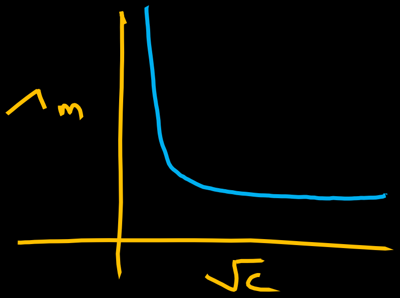
(C) A is true but R is false

(D) A is false but R is true

13

Assertion (A) : Λ_m for weak electrolytes shows a sharp increase when the electrolytic solution is diluted. $C = \frac{n}{V\uparrow}$, $C\downarrow$

Reason (R) : For weak electrolytes degree of dissociation increases with dilution of solution.



In the light of the above statements, choose the correct answer from the options given below:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

14

JEE 1

Arrange in order of decreasing reactivity towards electrophilic aromatic substitution reaction:

(I) Chlorobenzene

(II) Benzene

(III) Toluene

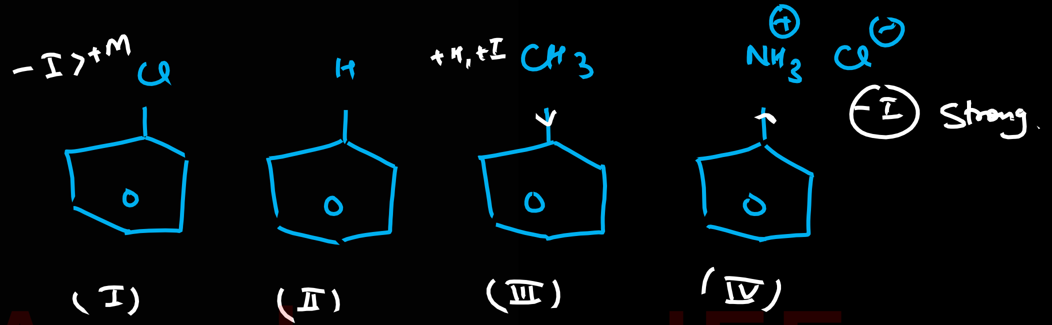
(IV) Anilinium chloride

(A) III > II > I > IV ✓

(B) IV > I > II > III

(C) II > I > IV > III

(D) I > II > IV > III



III > (II) > I > IV

Strong EDG   I:0   EWG

+M > +H > +I > I:0 > -I > -M

15

Iodination of benzene is not easily carried out. How can one prepare p-iodobenzoic acid from pnitro toluene?

(A) (i) $Br_2 + FeBr_3$ (ii) Mg in ether, CO_2 (iii) $3H_2/Pt$ catalyst (iv) HNO_2 at 0°C (v) KI solution

A

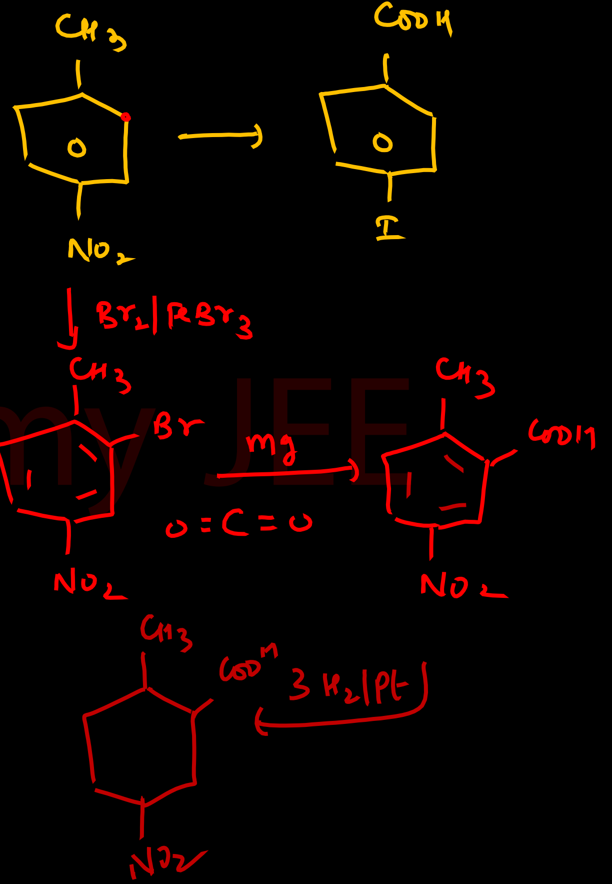
(B) (i) NBS in CCl_4 and Δ (ii) NaI in acetone

(Br)

(iii) $3H_2/Pt$ catalyst (iv) HNO_2 at 0°C (v) $H_3PO_2 + H_2O$

(C) (i) NBS in CCl_4 and Δ (ii) HNO_2 at 0°C (iii) $CuBr + HBr$ (iv) $KMnO_4/\Delta$ (v) KI solution

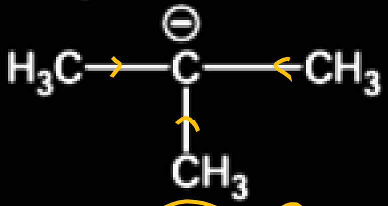
(D) (i) $KMnO_4/\Delta$ (ii) Sn + HCl (iii) HNO_2 at 0°C (iv) KI solution

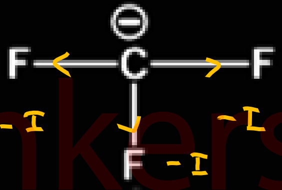


16

Which of the following carbanion is most stable?

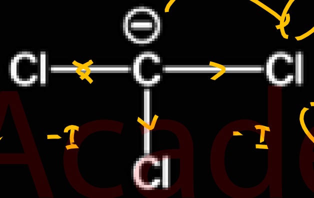
(A) 

(B) 

(C) 

-I -I

F -I

(D) 

vacant d

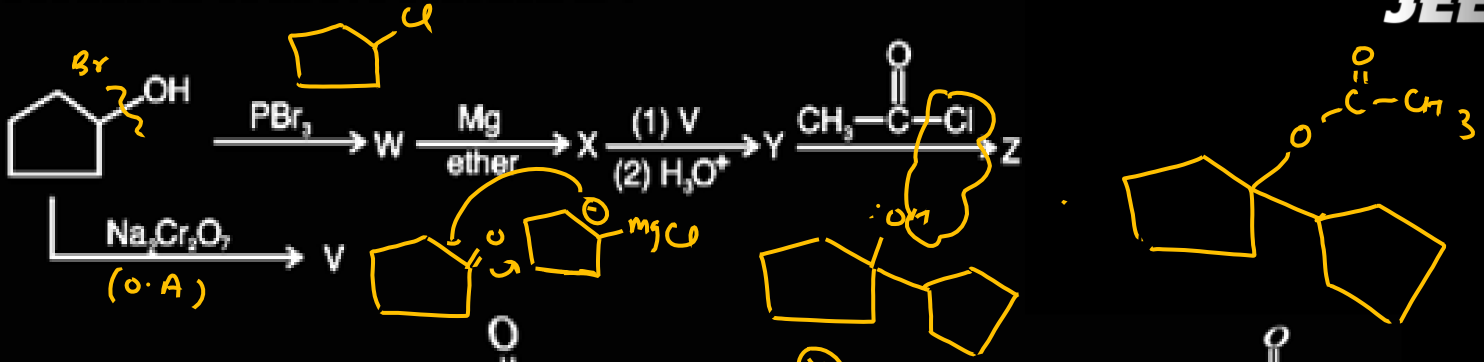
-I -I

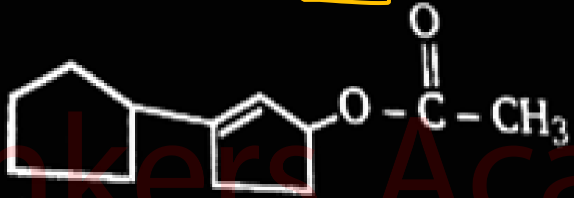
-I

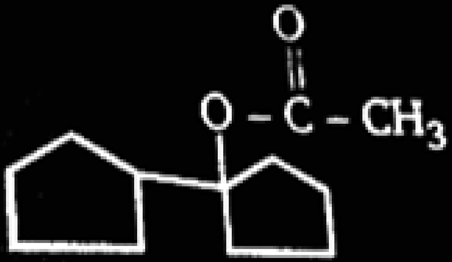
Only -I

-I , vacant 'd' Resonance

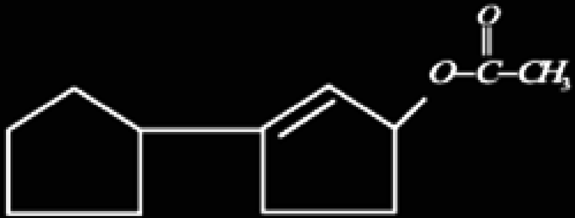
Product Z of above reaction is :



(A) 

(B) 

(C) 

(D) 

18

JEE 1

Assertion: Thin layer chromatography is an adsorption chromatography.

Reason: A thin layer of silica gel is spread over a glass plate of suitable size in thin layer chromatography which acts as an adsorbent.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

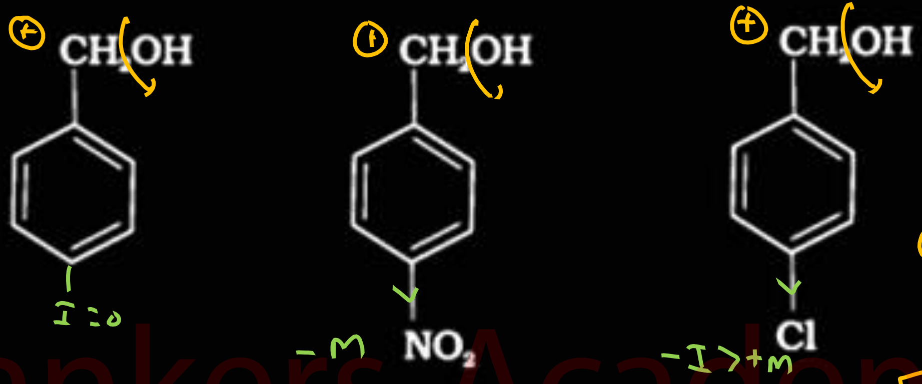
(C) A is true but R is false

(D) A is false but R is true

19

JEE 1

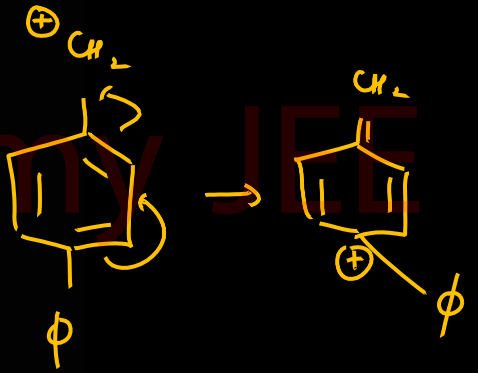
Mark the correct increasing order of reactivity of the following compounds with HBr/HCl



(a) (b) (c)

1° Alcohol → S_N2

2°, 3°, Benzylic, Allylic → S_N1



(stability)

(A) a < b < c

(B) b < a < c

(C) b < c < a

(D) c < b < a

20

JEE 1

The reaction $X(g) + 2Y(g) \longrightarrow Z(g) + W(g)$ is an elementary reaction. In an experiment partial pressures of X(g) and Y(g) are 0.60 and 0.80 atm respectively. Calculate the ratio of rate of reaction (when $P_Y = 0.20$ atm.) to the initial rate of reaction

(A) $\frac{1}{16}$ (B) $\frac{1}{24}$

(C) $\frac{1}{32}$ (D) $\frac{1}{48}$

$X(g) + 2Y(g) \longrightarrow Z(g) + W(g)$

$t=0$ 0.60 0.80 $-$ $-$

$t=t$ $(0.60-x)$ $(0.80-2x)$ x x

$=0.30$ atm. $=0.20$ atm

rate $= k[x][2y]$

$r_0 = k[0.60][0.80]^2$ — (1)

$0.80 - 2x = 0.20$

$\boxed{x = 0.30 \text{ atm}}$

$r_1 = k[0.30][0.20]^2$ — (2)

(2) ÷ (1)

$\frac{r_1}{r_0} = \frac{\cancel{k}[0.30]}{\cancel{k}[0.60]}\left[\frac{0.20}{0.80}\right]^2$

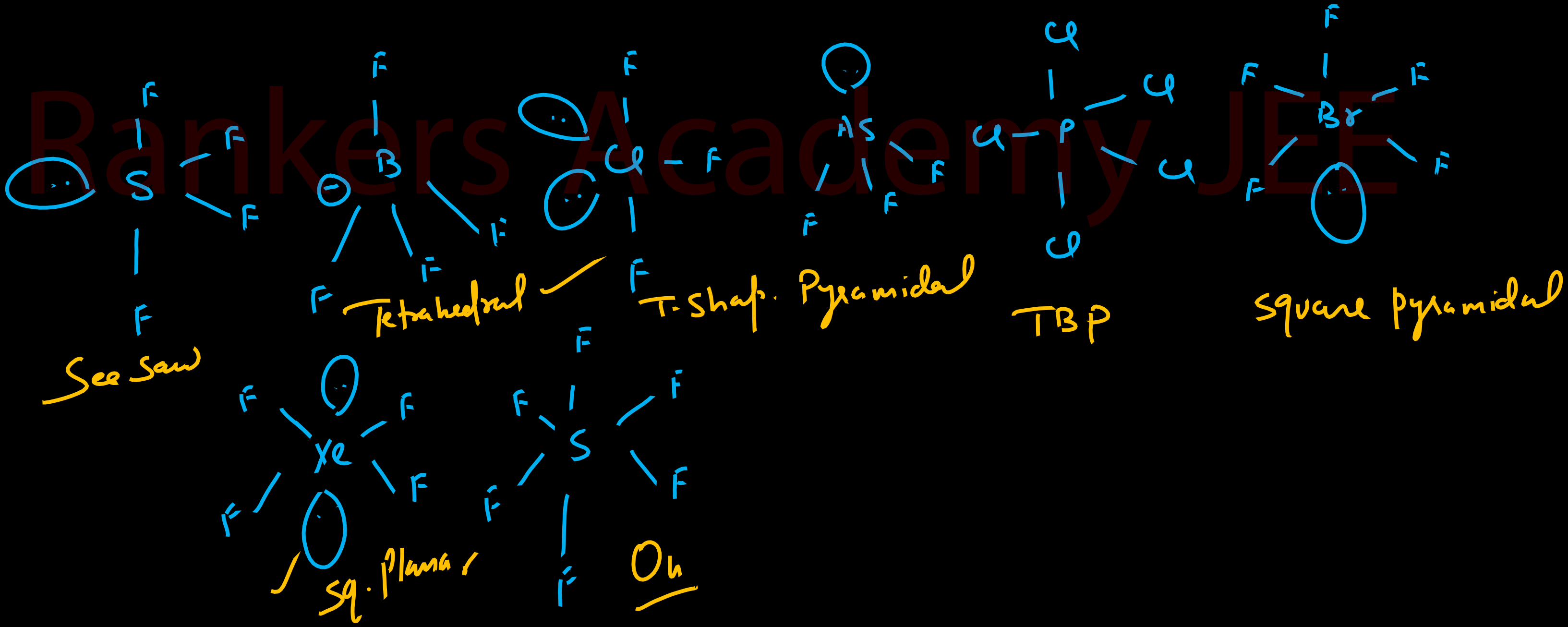
$\Leftarrow \frac{1}{2} \times \frac{1}{4^2}$

$= \frac{1}{32}$

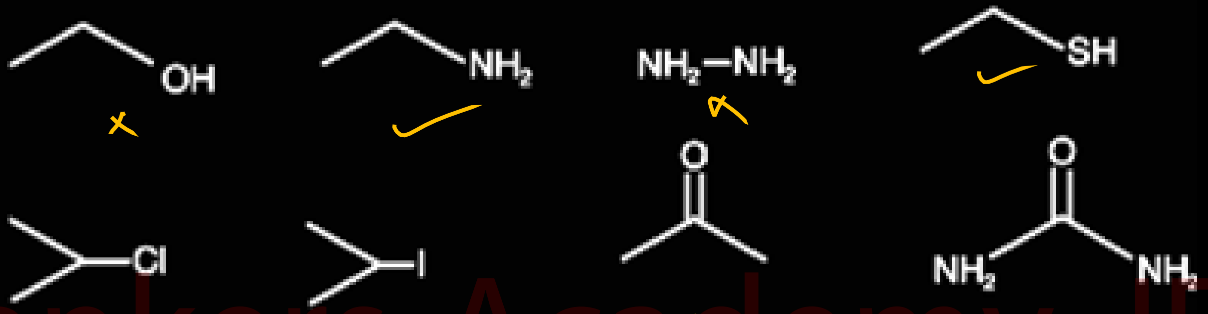
21

The number of species below that contain two lone pairs of electrons in their central atom is ______ . (Round off to the Nearest integer.)

SF_4, BF_4^-, $\cancel{ClF_3}$, AsF_3, PCl_5, BrF_5, $\cancel{XeF_4}$, SF_6 2



How many compounds will give positive Lassaigne's test?



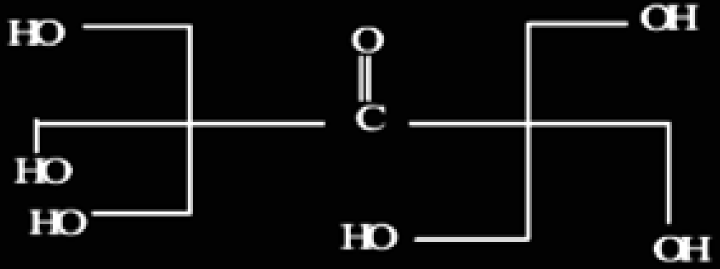
, NH_2OH

, diazonium salt

5

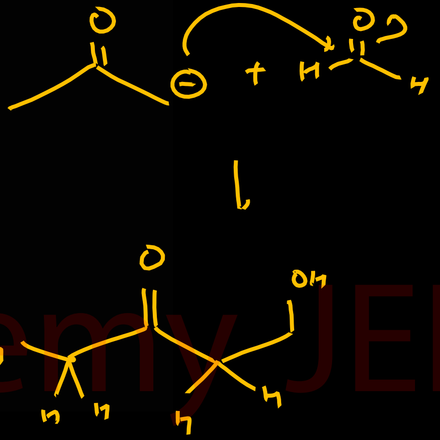
23

$$CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-CH_3 + xHCHO \xrightarrow{KOH}$$

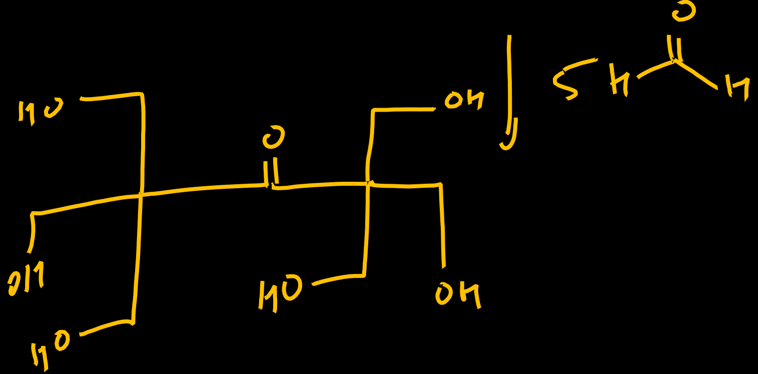


x = moles of HCHO consumed.

Value of (x) will be



6



24

In carius method of estimation of halogen's 250 mg of an organic compound gave 141 mg of AgBr . The percentage is x . Then x is (nearest integer)

(Take Atomic Weight Ag = 108, Br = 80)

$$O.C \longrightarrow AgBr$$

$w_1 : 250\,mg \qquad w_2 : 141\,mg$

$$wt\ Br : \frac{141\times10^{-3}}{188}\times 80$$

$$\%\ w\ of\ Br\ in\ O.C = \frac{141\times10^{-3}\times 80}{188\times 250\times10^{-3}}\times 100 = 24\%$$

25

JEE 1

How many grams of sucrose (mol. Wt. = 342) should be dissolved up to the nearest integer in 100 gm water in order to produce a solution with 105°C difference between the freezing point & boiling point temperature at 1 atm ?

(Unit : $k_f = 2K.kg\ mol^{-1}; k_b = 0.5\ K.kg\ mol^{-1}$)

$T_b^s - T_f^s = 105$ — ①

$\Delta T_f = \cancel{T_f^{\circ}} - T_f^s = i \times 2 \times m$

$-T_f^s = 2m$ — ②

$\Delta T_b = T_b^s - T_b^{\circ} = T_b^s - 100 = 0.5m$

$T_b^s = 100 + 0.5m$ — ③

② + ③ = ①

$2m + 100 + 0.5m = 105$

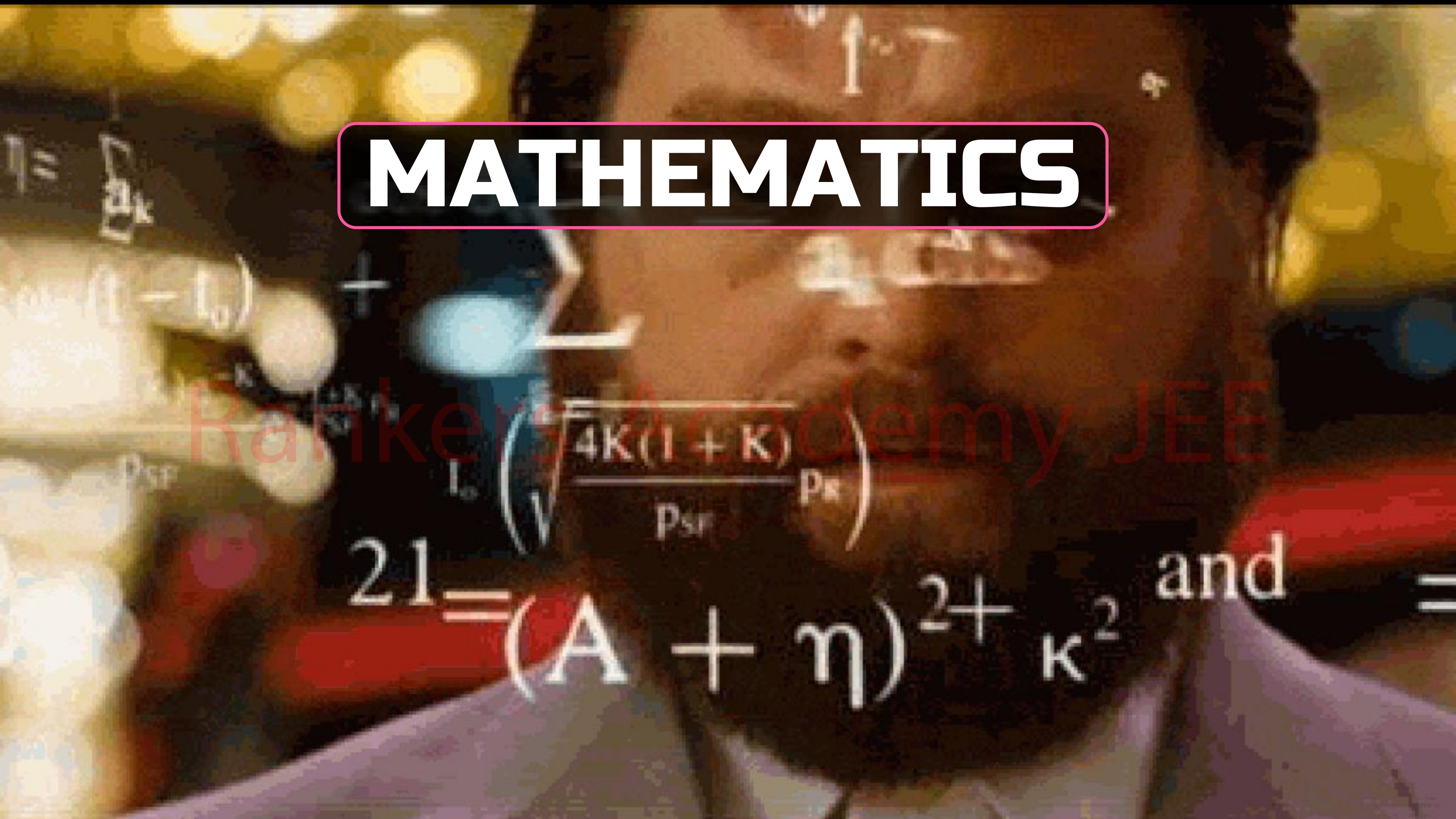
$\boxed{m = 2}$

$\frac{w}{342} \Big/ 100/1000 = 2$

$w = 34.2 \times 2$

$\boxed{w = 68.4g}$

MATHEMATICS



Let α and β be the roots of the equation $x^2 - 10x + 2 = 0$, then the value of $\frac{\alpha^{2028}+\beta^{2028}+8\alpha^{2022}+8\beta^{2022}}{\alpha^{2025}+\beta^{2025}}$ is

(A) 470 (B) 940

(C) 280 (D) 245

$$\Rightarrow \frac{(\alpha^{2028}+8\alpha^{2022})+(\beta^{2028}+8\beta^{2022})}{\alpha^{2025}+\beta^{2025}}$$

$$\Rightarrow \frac{\alpha^{2025}\left(\alpha^3+\frac{8}{\alpha^3}\right)+\beta^{2025}\left(\beta^3+\frac{8}{\beta^3}\right)}{\alpha^{2025}+\beta^{2025}}$$

$$\Rightarrow \frac{\alpha^{2025}(940)+\beta^{2025}(940)}{\alpha^{2025}+\beta^{2025}} = 940$$

$$\alpha^2 - 10\alpha + 2 = 0$$

$$\Rightarrow \alpha^2 + 2 = 10\alpha$$

$$\Rightarrow \alpha + \frac{2}{\alpha} = 10$$

$$\Rightarrow \alpha^3 + \frac{8}{\alpha^3} + 3\cdot\alpha\cdot\frac{2}{\alpha}\left(\alpha+\frac{2}{\alpha}\right) = 1000$$

$$\Rightarrow \alpha^3 + \frac{8}{\alpha^3} + 6(10) = 1000$$

$$\Rightarrow \alpha^3 + \frac{8}{\alpha^3} = 940$$

Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is :

(A) $\frac{4}{15}$ (B) $\frac{1}{3}$

(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

1: 6W 4B

2: 4W 6B

3: 5W 5B

$$P(B_2/W) = \frac{\frac{1}{3}\cdot\frac{4}{10}}{\frac{1}{3}\cdot\frac{6}{10}+\frac{1}{3}\cdot\frac{4}{10}+\frac{1}{3}\cdot\frac{5}{10}}$$

$$= \frac{4}{15}$$

3

Consider three sets $A = \{n \in N : n^2 \le n + 15000\}$, $B = \{5k + 4; k \in N\}$ and $C = \{3k; k \in N\}$. If the sum of all the elements of the set $A \cap (B - C)$ is equal to N, then number of ~~proper~~ divisors of N are

(A) 14 (B) 15

(C) 18 (D) 30

$1^2 \le 1 + 15000$

$2^2 \le 2 + 15000$

$\vdots$

$120^2 \le 120 + 15000$

$121^2 \le 121 + 15000$

$122^2 \le 122 + 15000$

$123^2 \ge 123 + 15000$

$A = \{1, 2, \ldots, 122\}$

$B = \{\cancel{9}, 14, 19, \cancel{24}, 29, 34, \cancel{39}, 44, 49, \cancel{54}, 59, 64, \cancel{69}, 74, 79, \cancel{84}, 89, 94, \cancel{99}, 104, 109, \cancel{114}, 119\}$

$C = \{3, 6, 9, 12 \ldots\}$

$B - C = \{14, 19, 29, 34, 44, 49, 59, 64, 74, 79, 89, 94, 104, 109, 119\}$

$= \frac{8}{2}(14 + 119) + \frac{7}{2}(19 + 109)$

$= 980$

$980 = 490 \times 2$

$= 2^2 \cdot 5^1 \cdot 7^2$

No. of divisors $= (2+1)(1+1)(2+1)$

$= 18$

4

Let the ellipse $E_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $E_2: \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1, A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals :

(A) $\frac{12\sqrt{6}}{5}$

(B) $6\sqrt{6}$

(C) $\frac{18\sqrt{6}}{5}$

(D) $\frac{24\sqrt{6}}{5}$ ✓

$e^2 = 1 - \frac{\text{chhota}}{\text{bada}}$

$\left(\frac{1}{\sqrt{3}}\right)^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{2}{3}$ — (1)

$\left(\frac{1}{\sqrt{3}}\right)^2 = 1 - \frac{A^2}{B^2} \Rightarrow \frac{A^2}{B^2} = \frac{2}{3}$ — (2)

$\frac{32}{\sqrt{3}} = \left(\frac{2b^2}{a^2}\right)\left(\frac{2A^2}{B^2}\right) \cdot aB$

$\frac{\cancel{32}}{\sqrt{3}} = \cancel{2} \cdot \frac{\cancel{2}}{3} \cdot \cancel{2} \cdot \frac{\cancel{2}}{3} \cdot aB$

$aB = 6\sqrt{3}$ — (3)

$2a\left(\frac{1}{\sqrt{3}}\right) = 4 \Rightarrow \boxed{a = 2\sqrt{3}}$

$\boxed{B = 3}$

$\frac{b^2}{12} = \frac{2}{3} \Rightarrow b^2 = 8 \Rightarrow \boxed{b = 2\sqrt{2}}$

$\frac{A^2}{3^{\cancel{2}}} = \frac{2}{\cancel{3}} \Rightarrow A^2 = 6 \Rightarrow \boxed{A = \sqrt{6}}$

$E_1: \frac{x^2}{12} + \frac{y^2}{8} = 1$

$E_2: \frac{x^2}{6} + \frac{y^2}{9} = 1$

$$\left(\frac{x^2}{12}+\frac{y^2}{8}=1\right)\times 2 \Rightarrow \frac{x^2}{6}+\frac{y^2}{4}=2$$

$$\frac{x^2}{6}+\frac{y^2}{9}=1$$

$$\Rightarrow \frac{y^2}{4}-\frac{y^2}{9}=1$$

$$\Rightarrow \frac{5y^2}{36}=1$$

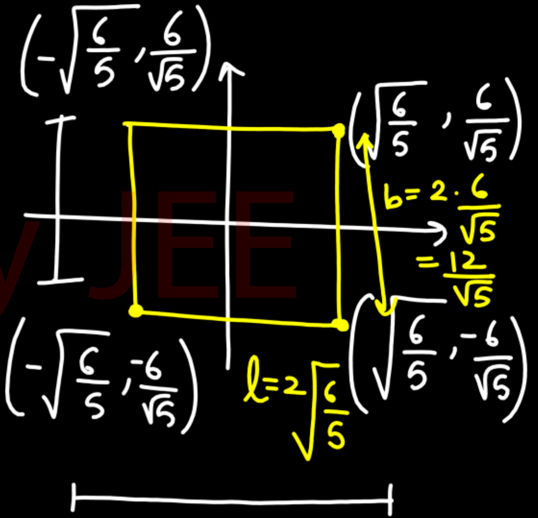
$$\Rightarrow \boxed{y=\pm\frac{6}{\sqrt{5}}}$$

$$\Rightarrow \frac{x^2}{6}+\frac{36/5}{9}=1$$

$$\Rightarrow \frac{x^2}{6}+\frac{4}{5}=1$$

$$\Rightarrow \frac{x^2}{6}=\frac{1}{5}$$

$$\Rightarrow \boxed{x=\pm\sqrt{\frac{6}{5}}}$$



$$\text{Area}=l\cdot b=2\sqrt{\frac{6}{5}}\cdot\frac{12}{\sqrt{5}}$$

$$=\frac{24}{5}\sqrt{6}.$$

5

JEE 1

Let $S = (-1, \infty)$ and $f : S \rightarrow R$ be defined as

$$f(x) = \int_{-1}^{x} (e^{t}-1)^{11}(2t-1)^{5}(t-2)^{7}(t-3)^{12}(2t-10)^{61}\,dt$$

Let p = Sum of squares of the values of x, where f(x) attains local maxima on S, and q = Sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is ______

(A) 17 (B) 27

(C) 25 (D) 20

Max : f' : $+ \longrightarrow -$

Min : f' : $- \longrightarrow +$

Max at $x = 0, 2$

Min at $x = \frac{1}{2}, 5$

$p = 0^2 + 2^2 = 4$

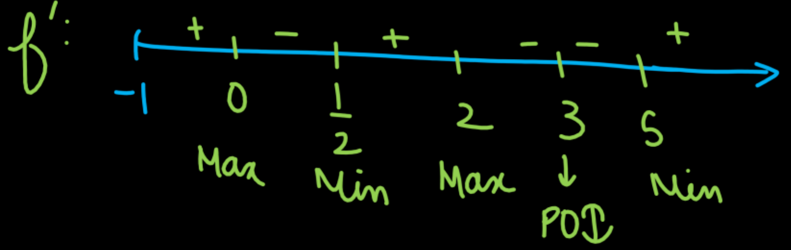
$q = \frac{1}{2} + 5 = \frac{11}{2}$

Ans: $p^2 + 2q$

$4^2 + 2\left(\frac{11}{2}\right) = 16 + 11$

$= 27$

$f'(x) = (e^{x}-1)^{11}(2x-1)^{5}(x-2)^{7}(x-3)^{12}(2x-10)^{61}$

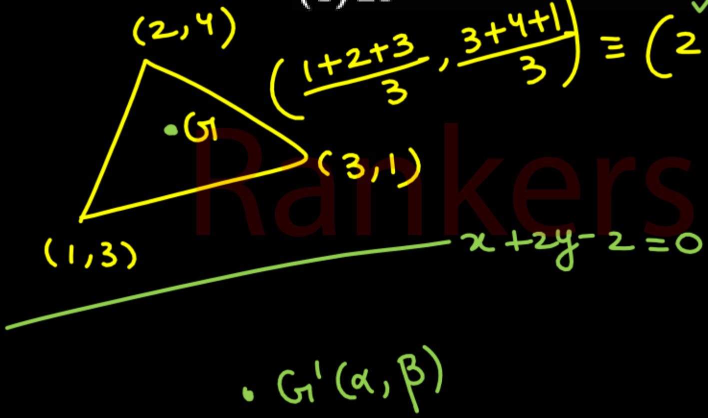


6

Let the triangle PQR be the image of the triangle with vertices (1,3), (3,1) and (2,4) in the line $x + 2y = 2$. If the centroid of Δ PQR is the point (α, β), then $15(\alpha - \beta)$ is equal to :

(A) 19 (B) 24

(C) 21 (D) 22 ✓



$$\left(\frac{1+2+3}{3}, \frac{3+4+1}{3}\right) \equiv \left(2, \frac{8}{3}\right)$$

$$\frac{\alpha - 2}{1} = \frac{\beta - 8/3}{2} = -2\left(\frac{\cancel{2} + 2\left(\frac{8}{3}\right) - \cancel{2}}{1^2 + 2^2}\right)$$

$$\frac{\alpha - 2}{1} = \frac{\beta - 8/3}{2} = -\frac{32}{15}$$

$$\alpha = -\frac{2}{15} \; ; \; \beta = -\frac{24}{15}$$

7

The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{7x+\frac{\pi}{2}}{2-\cos 2x} dx$ is:

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{4\sqrt{3}}$

(C) $\frac{\pi^2}{6\sqrt{3}}$ (D) $\frac{\pi^2}{3\sqrt{3}}$

JEE 1

$$I = \int_{-\pi/4}^{\pi/4} \frac{7x+\pi/2}{2-\cos 2x} dx$$

$$= \underbrace{\cancel{\frac{\pi}{4}\int_{-\pi/4}^{\pi/4} \frac{7x}{2-\cos 2x} dx}}_{\text{odd}} + \frac{\pi}{2}\underbrace{\int_{-\pi/4}^{\pi/4} \frac{dx}{2-\cos 2x}}_{\text{even}}$$

$$= \frac{\pi}{\cancel{2}} \cdot \cancel{2} \cdot \int_0^{\pi/4} \frac{dx}{2-\cos 2x}$$

$$I = \pi \int_0^{\pi/4} \frac{dx}{2-\cos 2x}$$

$$= \pi \int_0^{\pi/4} \frac{dx}{2-\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)}$$

$$= \pi \int_0^{\pi/4} \frac{\sec^2 x \, dx}{1+3\tan^2 x}$$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$$I = \pi \int_0^1 \frac{dt}{1+3t^2}$$

$$= \frac{\pi}{3} \int_0^1 \frac{dt}{t^2+\frac{1}{3}}$$

$$= \frac{\pi}{3} \cdot \sqrt{3} \cdot \left[\tan^{-1} t\sqrt{3}\right]_0^1$$

$$= \frac{\pi}{3}\sqrt{3}\left(\frac{\pi}{3}\right)$$

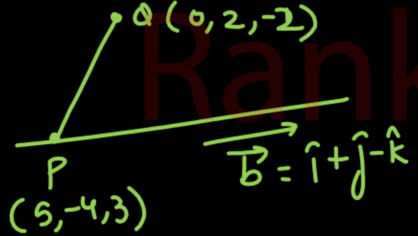
$$= \frac{\pi^2}{3\sqrt{3}}.$$

8

The distance of the point Q(0,2,−2) form the fine passing through the point P(5,−4,3) and perpendicular to the line $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in R$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in R$ is :

(A) $\sqrt{54}$ (B) $\sqrt{20}$

(C) $\sqrt{74}$ ✓ (D) $\sqrt{86}$



$\vec{b} \parallel \vec{b_1} \times \vec{b_2}$

$\parallel \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$

$\parallel (-9\hat{i} - 9\hat{j} + 9\hat{k})$

$\parallel (-9)(\hat{i} + \hat{j} - \hat{k})$

Ans : $\dfrac{|\vec{PQ} \times \vec{b}|}{|\vec{b}|}$

$= \dfrac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 5 \\ 1 & 1 & -1 \end{vmatrix}}{\sqrt{3}}$

$= \dfrac{|\hat{i} + 10\hat{j} + 11\hat{k}|}{\sqrt{3}} = \dfrac{\sqrt{1+100+121}}{\sqrt{3}}$

$= \sqrt{\dfrac{222}{3}} = \sqrt{74}$

9

JEE 1

Let a be the sum of all coefficients in the expansion of $(1-2x+2x^2)^{2023}(3-4x^2+2x^3)^{2024}$ and $b=\lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where $c, d, e \in R$, then d: c: e equals

both roots common

(A) 1: 2: 4
(B) 2: 1: 4
(C) 4: 1: 4
(D) 1: 1: 4

$x=1$

$a=(1-2+2)^{2023}(3-4+2)^{2024}=1$

$b=\lim_{x\to 0}\frac{\int_0^x \frac{\ln(1+t)}{t^{2024}+1}dt}{x^2}$ $\left(\frac{0}{0}\right)$

$=\lim_{x\to 0}\frac{\frac{\ln(1+x)}{x^{2024}+1}-0}{2x}=\frac{\overset{=1}{\frac{\ln(1+x)}{x}}}{2\,\boxed{=1}}=\frac{1}{2}$

$cx^2+dx+e=0$

$2bx^2+ax+4=0$

$x^2+x+4=0$

$D<0$

complex-conju

$$\boxed{\frac{c}{1}=\frac{d}{1}=\frac{e}{4}}$$

10

JEE 1

The sum of maximum and minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$ where gcd $(m, n) = 1$ then $m + n =$

(A) 217 (B) 182

(C) 201 (D) 195

$$y = \frac{2x^2-3x+8}{2x^2+3x+8}$$

$$\Rightarrow (2y)x^2 + 3xy + 8y = 2x^2 - 3x + 8$$

$$\Rightarrow 2(y-1)x^2 + 3x(y+1) + 8(y-1) = 0$$

$x \in R \Rightarrow D \geq 0$

$$\Rightarrow 9(y+1)^2 - 4 \cdot 8 \cdot 2 \cdot (y-1)^2 \geq 0$$

$$\Rightarrow (3(y+1))^2 - (8(y-1))^2 \geq 0$$

$$\Rightarrow (3y+3+8y-8)(3y+3-8y+8) \geq 0$$

$$\Rightarrow (11y-5)(11-5y) \geq 0$$

$$\Rightarrow (11y-5)(5y-11) \leq 0$$

$$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5}\right]$$

Min Max

$$\text{Sum} = \frac{5}{11} + \frac{11}{5} = \frac{121+25}{55} = \frac{146}{55} = \frac{m}{n}$$

$$m + n = 146 + 55 = 201$$

11

JEE 1

If the area of the region $\left\{(x, y), \frac{a}{x^2} \leq y \leq \frac{1}{x}, 1 \leq x \leq 2, 0 < a < 1\right\}$ is $(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to:

(A) 0 (B) -1

(C) 1 (D) 2

$$\Rightarrow \int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right) dx = \ln 2 - \frac{1}{7}$$

$$\Rightarrow \left[\ln x + \frac{a}{x}\right]_1^2$$

$$\Rightarrow \ln 2 + \left(\frac{a}{2} - \frac{a}{1}\right)$$

$$\Rightarrow \ln 2 - \frac{a}{2} = \ln 2 - \frac{1}{7}$$

$$\frac{a}{2} = \frac{1}{7}$$

$$\boxed{a = \frac{2}{7}}$$

$$7a - 3 = 2 - 3$$
$$= -1$$

12

JEE 1

For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace (A) = 3. If B = adj (adj (2A)), then the value of |B| + trace (B) equals :

(A) 56 (B) 132

(C) 174 (D) 280

$n=3$

$B = adj(adj(2A))$

$= |2A|^{n-2} \cdot (2A)$

$= |2A|^{3-2} (2A)$

$= |2A| \cdot (2A)$

$= 2^3 |A| \cdot (2A)$

$= 8 \cdot \frac{1}{2} \cdot 2A = 8A$

$\therefore \boxed{B = 8A}$

$|B| = |8A|$

$= 8^3 |A|$

$= 512 \cdot \frac{1}{2} = 256$

$tr(B) = tr(8A)$

$= 8\, tr(A)$

$= 8 \times 3 = 24$

Ans : $256 + 24 = 280$

13

JEE 1

Words from the letters of the word 'PROBABILITY' are formed by taking all at a time. The probability that both B's are together and both I's are together is :-

(A) $\frac{1}{55}$ (B) $\frac{2}{55}$

(C) $\frac{4}{165}$ (D) $\frac{7}{165}$

P R O $\boxed{B\ B}$ A $\boxed{I\ I}$ L T Y

$$\frac{9! \times 1 \times 1}{\left(\frac{11!}{2!\,2!}\right)} = \frac{\cancel{9!} \times 2! \times \cancel{2!}}{\cancel{11!}\ \cancel{10}_5 \times 11} = \frac{2}{55}$$

14

JEE 1

Let $A = \{3,4,6,7,8,9,10\}$ and R be the relation defined on A such that

$R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to

(A) 17 (B) 19

(C) 20 (D) 16

$A = \{10, 9, 8, 7, 6, 4, 3\}$

$R = \{(10,9), (10,8) - - - - - - - - , (4,3)\}$

x	y
10	9, 8, 7, 3
9	8, 7, 6, 4
8	7, 6, 3
7	6, 4
6	4, 3
4	3
3	—

15

JEE 1

Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}=\vec{a}+2\vec{b}+3(\vec{a}\times\vec{b})$, then $2|\vec{c}|$ is equal to:

(A) $\sqrt{55}$ (B) $\sqrt{37}$

(C) $\sqrt{51}$ (D) $\sqrt{43}$

$a^2+b^2+2\bar{a}\cdot\bar{b}=3$

$2\cos\theta=1$

$\boxed{\cos\theta=\frac{1}{2}}$

$|\bar{c}|^2=|\bar{a}+2\bar{b}+3(\bar{a}\times\bar{b})|^2$

$|\bar{c}|^2=1+4+9(\sin\theta)^2+4\cos\theta+0+0$

$|\bar{c}|^2=5+9\left(\frac{\sqrt{3}}{2}\right)^2+4\left(\frac{1}{2}\right)$

$|\bar{c}|^2=5+\frac{27}{4}+2$

$|\bar{c}|=\frac{\sqrt{55}}{2}$

16

JEE 1

Let S be the set of all solutions of the equation

$\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Then $\sum_{x\in S} 2\sin^{-1}(x^2-1)$ is equal to

(A) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (B) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(C) $\frac{-2\pi}{3}$ (D) 0

$\cos^{-1}(2x) = \pi + 2\cos^{-1}\left(\sqrt{1-x^2}\right)$

('cos' both sides)

$2x = -\cos\left(2\cos^{-1}\left(\sqrt{1-x^2}\right)\right)$

$2x = -\left(2\left(\cos\left(\cos^{-1}\sqrt{1-x^2}\right)\right)^2 - 1\right)$

$2x = -\left(2(1-x^2)-1\right)$

$2x = -2 + 2x^2 + 1$

$2x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{4+8}}{2\times 2}$

$\boxed{x = \frac{1\pm\sqrt{3}}{2}}$

JEE 1

$$S = \left\{ \frac{1-\sqrt{3}}{2} \right\}$$

$$2\sin^{-1}\left(\left(\frac{1-\sqrt{3}}{2}\right)^2 - 1\right)$$

$$2\sin^{-1}\left(\frac{1+3-2\sqrt{3}-4}{4}\right)$$

$$2\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$-\frac{2\pi}{3}$$

17

JEE 1

Let the system of linear equations

$$\left.\begin{aligned} x + y + kz &= 2 \\ 2x + 3y - z &= 1 \\ 3x + 4y + 2z &= k \end{aligned}\right\} \Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

have infinitely many solutions. Then the system

$$1(10) - 1(7) + k(-1) = 0$$

$$k = 3$$

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$ has :

$$\begin{cases} 4x + 5y = 7 \\ 7x + 8y = 10 \end{cases}$$

(A) infinitely many solutions ×

(B) unique solution satisfying $x - y = 1$

(C) no solution ×

(D) unique solution satisfying $x + y = 1$ ✓

$$3x + 3y = 3$$

$$\boxed{x + y = 1}$$

18

JEE 1

The curve satisfying the differential equation

$2xy(y^2\cos(x^2y) - 1) + x^2y'(y^2\cos(x^2y) + 1) = 0$

and passing through (0,1) is

(A) $\sin^2(x^2y) = x$

(B) $\sin(x^2y) = xy^2$

(C) $\sin(x^2y) = x^2y$

(D) $\sin(x^2y) = \frac{x^2}{y}$

$$2xy^3\cos(x^2y) - 2xy + x^2y^2\cos(x^2y)\frac{dy}{dx} + x^2\frac{dy}{dx} = 0$$

$$2xy^3\cos(x^2y)dx - 2xydx + x^2y^2\cos(x^2y)dy + x^2dy = 0$$

$$y^2\cos(x^2y)\left[2xydx + x^2dy\right] = 2xydx - x^2dy$$

$$\frac{\cancel{y}\cos(x^2y)\,d(x^2y)}{\cancel{y^2}} = \frac{2xy\,dx - x^2\,dy}{y^2}$$

$$\int \cos(x^2y)\,d(x^2y) = \int d\left(\frac{x^2}{y}\right)$$

$$\sin(x^2y) = \frac{x^2}{y} + C$$

$(0,1)$: $C = 0$

19

Let $A = \left\{z \in \mathbb{C} : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and

$B = \left\{z \in \mathbb{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$

Then $A \cap B$ is:

(A) a portion of a circle centered at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

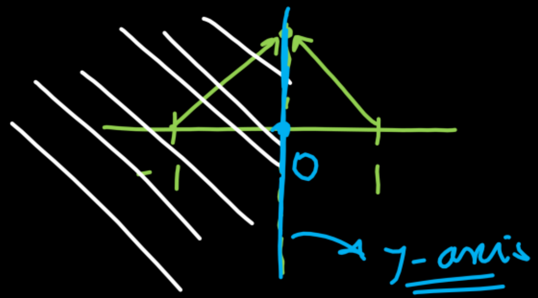
(B) a portion of a circle centered at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty set

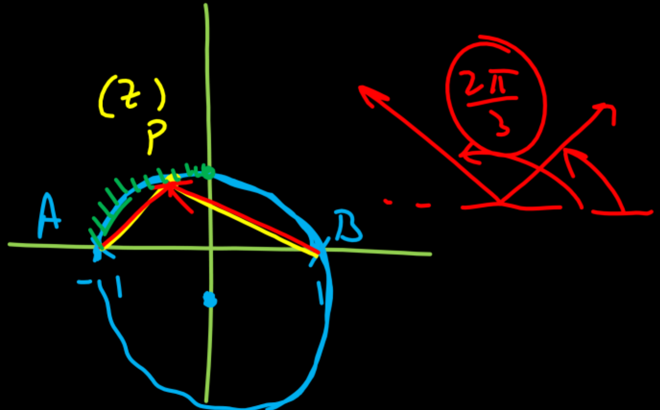
(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

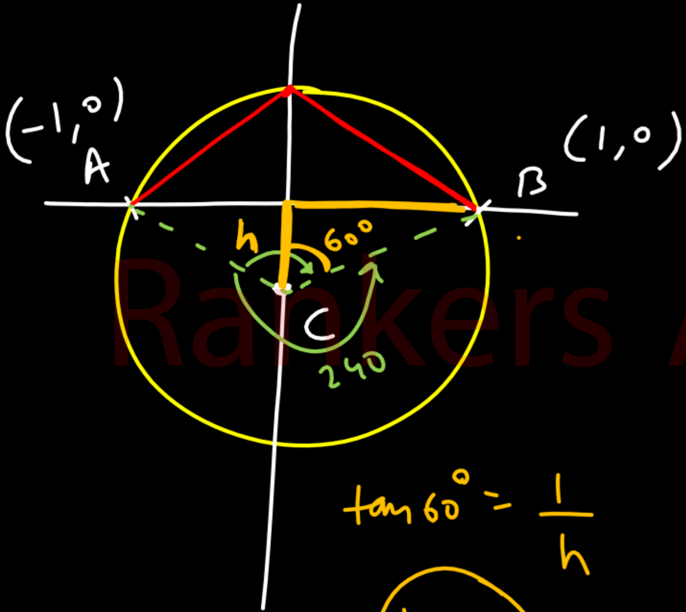
$|z_1 - z_2|$

$A = |z+1| < |z-1|$



$B = \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$

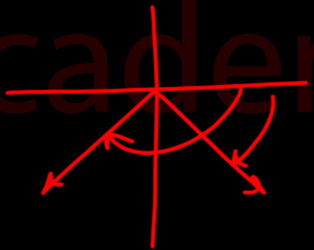


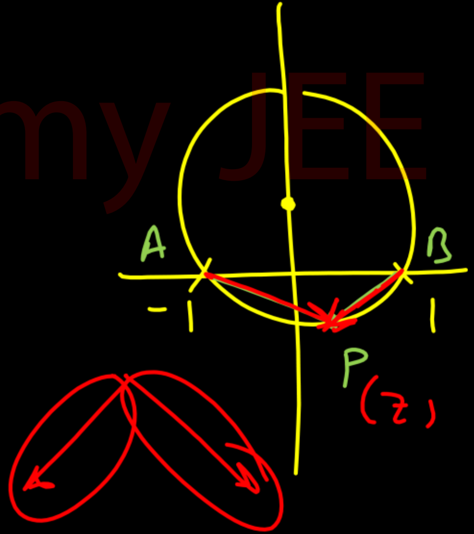


$$\tan 60^\circ = \frac{1}{h}$$

$$h = \frac{1}{\sqrt{3}}$$

$$\arg\left(\frac{z-1}{z+1}\right)$$





Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the are AB is $(1/6)^{th}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

(A) $3 + \sqrt{3}$ (B) 4

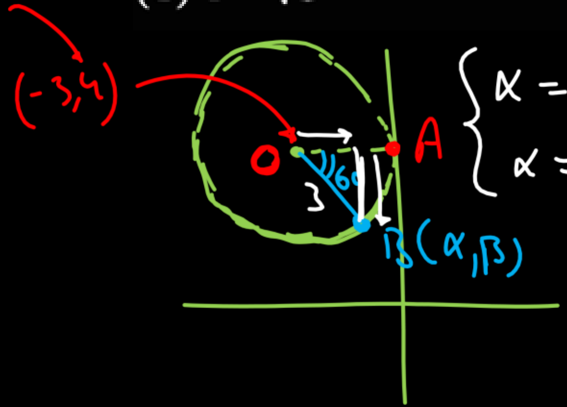
(C) $4 - \sqrt{3}$ (D) 3

$C_1 \equiv (1, -2)$

$r = \sqrt{1 + 4 + 4} = 3$

$\frac{x-1}{2} = \frac{y+2}{-3} = -2\frac{(2+6+5)}{(4+9)}$

$x = -3;\ y = 4$



$\alpha = -3 + 3\cos 60^\circ$

$\alpha = -\frac{3}{2}$

$\beta = 4 - 3\sin 60^\circ$

$\beta = 4 - \frac{3\sqrt{3}}{2}$

$\beta - \sqrt{3}\alpha \Rightarrow 4 - \cancel{\frac{3\sqrt{3}}{2}} - \sqrt{3}\cancel{\left(-\frac{3}{2}\right)}$

21

JEE 1

The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

If $\sum_{r=1}^{10} tr\,(A^{2r-1}) = \frac{2^m - 2}{n}$

Then m + n is equal to ______ .

(Here, Tr (A) means trace of matrix A.

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 32 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

JEE1

$$tr(A^1) + tr(A^3) + tr(A^5) + \cdots + tr(A^{19})$$

$$= 2^1 + 2^3 + 2^5 + \cdots \underline{\underline{10\text{-terms.}}}$$

$$= \frac{2\left((2^2)^{10} - 1\right)}{(2^2 - 1)}$$

$$= \frac{2(2^{20} - 1)}{3}$$

$$= \frac{2^{21} - 2}{3}$$

$$\left.\begin{array}{l} m = 21 \\ n = 3 \end{array}\right\} \rightarrow m + n = 24$$

22

Let f: R → R be a function defined as

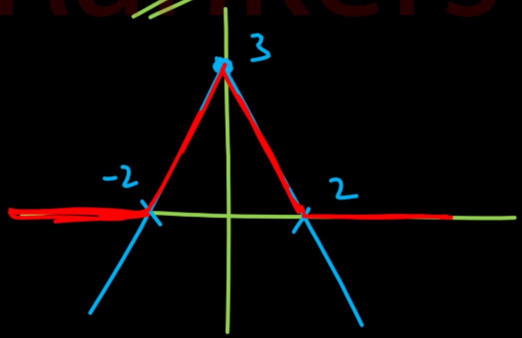
$$f(x) = \begin{cases} 3\left(1 - \dfrac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

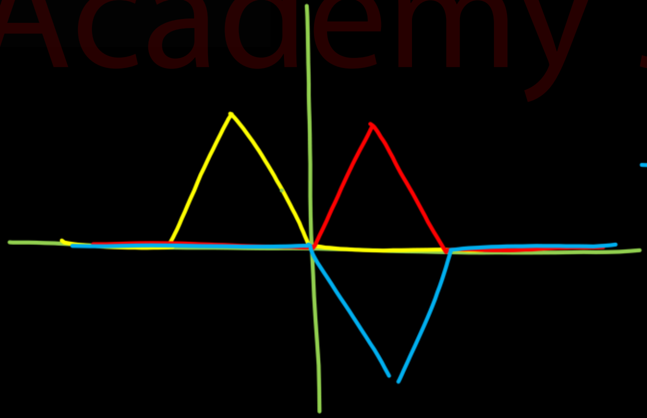
$3 - \frac{3}{2}|x|$

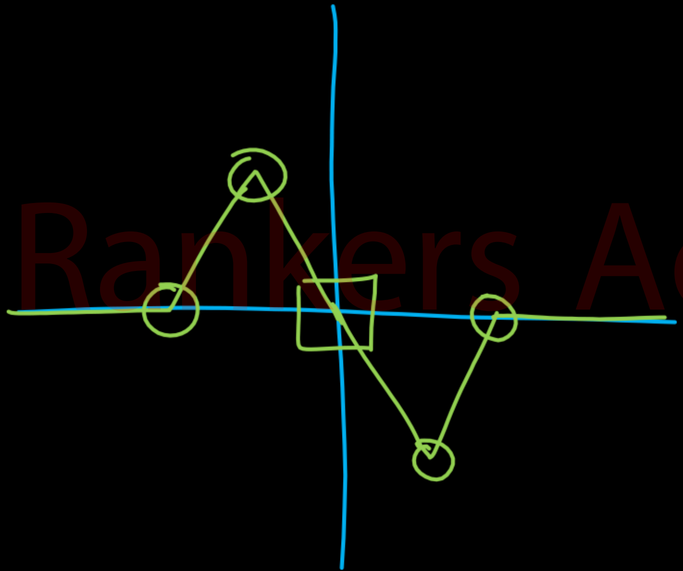
Let g: R → R be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in R where g is not continuous and not differentiable, respectively, then n + m is equal to 4

$3 + (-2)$

$3 - 2$







23

If $\sum_{r=1}^{20} \frac{r^2({}^{20}C_r)^2}{({}^{20}C_{r-1})} = \alpha(2^{18})$

Then number of divisors of α are ______ .

$$\sum_{r=1}^{20} r^{\cancel{2}}\left(\frac{20}{\cancel{r}} \times {}^{19}C_{r-1}\right)\left(\frac{{}^{20}C_r}{{}^{20}C_{r-1}}\right)$$

$$20\sum_{r=1}^{20} \cancel{r}\,{}^{19}C_{r-1}\left(\frac{20-r+1}{\cancel{r}}\right)$$

$$20\left[\sum_{r=1}^{20}(20)\,{}^{19}C_{r-1} - (r-1)\,{}^{19}C_{r-1}\right]$$

$$20\left[20\underbrace{\sum_{r=1}^{20}{}^{19}C_{r-1}}\right]$$

$$-20\left[\sum_{r=\cancel{1}\,2}^{20}\cancel{(r-1)}\,\frac{19}{\cancel{(r-1)}}\,{}^{18}C_{r-2}\right]$$

$$400(2^{19}) - 20(19)2^{18}$$

$$2^{18}(20)[40-19]$$

$$2^{18}(20)(21)$$

JEE1

$\alpha = 420$

$\alpha = 4 \times 105$

$\alpha = 4 \times 5 \times 21$

$\alpha = 2^2 \cdot 3^1 \cdot 5^1 \cdot 7^1$

$(2+1)(1+1)(1+1)(1+1)$

$3 \times 2 \times 2 \times 2$

24

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

24

JEE 1

Let S be the set of 6 - digit numbers $a_1a_2a_3a_4a_5a_6$ (all digits distinct) where $a_1 > a_2 > a_3 > a_4 < a_5 < a_6$. Then n(S) is equal to

$${}^{10}C_6 \cdot {}^{5}C_2$$

$$= 2100$$

$$\underbrace{a_1\, a_2\, a_3}\, a_4\, \underbrace{a_5\, a_6}$$

Eg: $\{3, \underbrace{5, 6, 7, 8, 9}\}$

25

JEE 1

Let $a_1, a_2 \dots \dots a_n$ be real numbers in A.P. Such that $a_1 = 15$ and a_2 is an integer. Given $\sum_{r=1}^{10} (a_r)^2 = 1185$. If $S(n) = \sum_{r=1}^{n} a_r$, then find the maximum value of n for which

$s(n) \geq s(n-1)$.

$$(15)^2 + (15+d)^2 + (15+2d)^2 + \dots + (15+9d)^2 = 1185$$

$$10(15)^2 + d^2\underbrace{(1^2+2^2+\dots+9^2)}_{\boxed{\frac{(9)(10)(19)}{6}}} + 2(15)(d)\underbrace{[1+2+3+\dots+9]}_{\frac{9\times 10}{2}} = 1185$$

$$\boxed{d=-1}$$

$$\overbrace{\underbrace{15, 14, 13, \dots, 2, 1}}, 0, \underline{\underline{-1}}$$

$$S(3) > S(2)$$

$$S(16) \geq S(15)$$

$$S(17) \overset{<}{\cancel{>}} S(16)$$