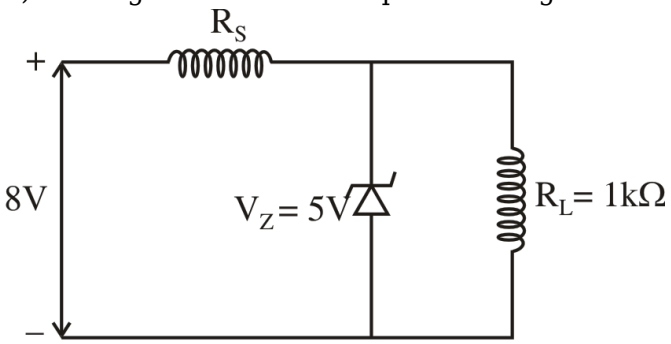
PART-1 : PHYSICS

SECTION-I

1) In the given circuit if the power rating of zener diode is 10 mW, the value of R_s is:



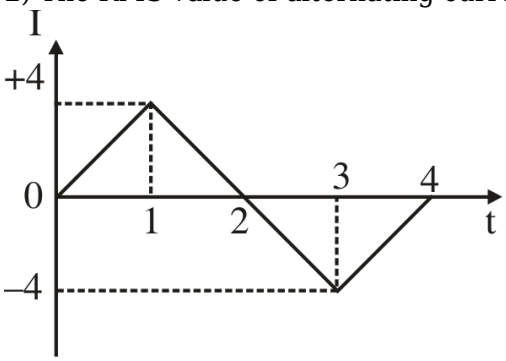
(A) $\frac{3}{4}k\Omega$

(B) $\frac{3}{7}k\Omega$

(C) $\frac{4}{3}k\Omega$

(D) $\frac{4}{7}k\Omega$

2) The RMS value of alternating current shown in the diagram below is :-



(A) 0

(B) $\frac{4}{\sqrt{3}}$

(C) $\frac{8}{\sqrt{3}}$

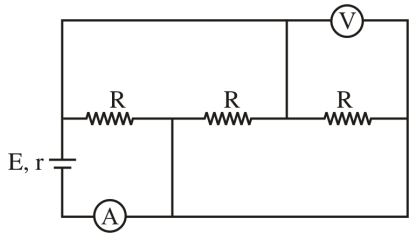
(D) $\frac{2}{\sqrt{3}}$

3) **Statement-I :** The hysteresis curve for transformer cores must be narrow.

Statement-II : The material for transformer cores must have high retentivity.

(A) Statement-I is True, Statement-II is False.
(B) Statement-I is False, Statement-II is True.
(C) Both Statement-I and Statement-II are False.
(D) Both Statement-I and Statement-II are True.

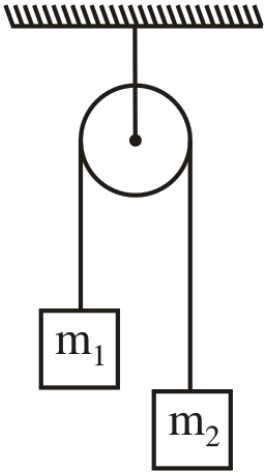
4) In the circuit shown in figure, ammeter and voltmeter are ideal. If E = 4V, R = 9 Ω and r = 1Ω,



then readings of ammeter and voltmeter are :-

(A) 1A, 3V
(B) 2A, 3V
(C) 1A, 4V
(D) 2A, 4V

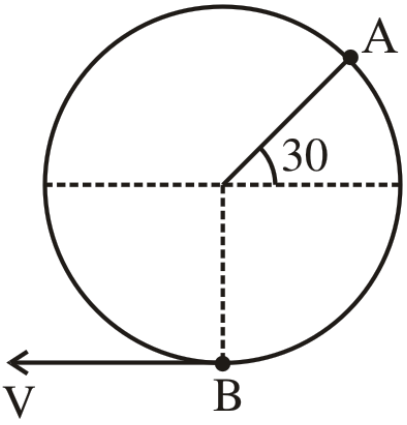
5) A light string passing over a smooth light fixed pulley connects two blocks of mass m_1 & m_2. If the



acceleration of the system is $\frac{g}{4}$ then the ratio of masses is :-

(A) $\frac{2}{3}$
(B) $\frac{5}{3}$
(C) $\frac{7}{3}$
(D) $\frac{8}{3}$

6) A particle of mass 4kg attached to a massless string is released from A. Find the speed of the



particle when it reaches point B. Length of the string is 20 m.

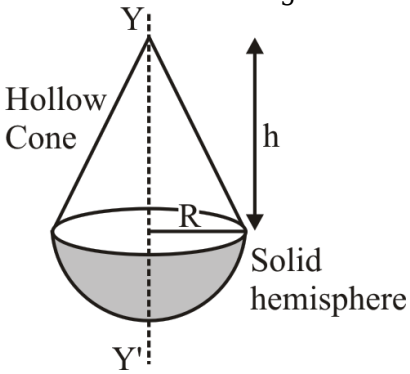
(A) $10\sqrt{3}$ m/s

(B) $10\sqrt{5}$ m/s

(C) $3\sqrt{10}$ m/s

(D) $6\sqrt{10}$ m/s

7) Find moment of inertia of the system shown in the diagram about YY'. Both the cone & the



hemisphere have the same mass M.

(A) 0.7 MR^2

(B) 0.9 MR^2

(C) 7/6 MR^2

(D) 5/6 MR^2

8) A metal rod of mass 50 kg is having its one end on the rough ground and the other end on the shoulder of a man. The rod makes an angle 37° with the horizontal. Assuming no friction at the surface of the shoulder, the force experienced by the man is :-

(A) 100 N

(B) 200 N

(C) 150 N

(D) 50 N

9) Two metallic wires P & Q have same volume and are made up of same material. If their area of cross sections are in the ratio 2 : 1 and to produce same extension in both wires, the applied forces are F_1 & F_2 respectively. Then $\frac{F_1}{F_2}$ is :-

(A) 4

(B) 8

(C) 16

(D) 32

10) If the rms speed of hydrogen molecule at a given temperature & pressure is 4 km/s the rms speed of oxygen molecule at the same condition is :-

(A) 1 km/s

(B) 1.5 km/s

(C) 2 km/s

(D) 2.5 km/s

11) One main scale division of a vernier caliper is equal to 2mm. If 9^{th} division of main scale coincides with 10^{th} division of vernier scale the least count of vernier caliper is :-

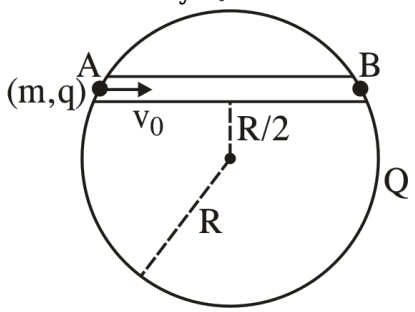
(A) 0.1 mm

(B) 0.05 mm

(C) 0.3 mm

(D) 0.2 mm

12) A uniformly charged non-conducting solid sphere has a narrow tunnel inside it. A charge q of mass m is projected as shown with velocity v_0. Find minimum value of v_0 so that particle reaches B.



Take charge on sphere as Q.

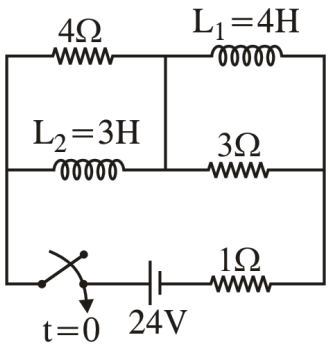
(A) $v_0 = \sqrt{\frac{2KQq}{4mR}}$

(B) $v_0 = 0$

(C) $v_0 = \sqrt{\frac{6KQq}{4mR}}$

(D) $v_0 = \sqrt{\frac{3KQq}{4mR}}$

13) Find the current through battery just after closing the switch and long time after closing switch.



Switch is closed at t = 0.

(A) 3A, 24A

(B) 24A, 3A

(C) 3A, 3A

(D) Incomplete information

14) For hydrogen spectrum (Take 1/R = 912Å)

	List-I		**List-II**
(P)	Smallest wavelength of Lyman series.	(1)	6566 Å
(Q)	Wavelength of 1st line of Lyman series.	(2)	912 Å
(R)	Largest wavelength of Lyman series.	(3)	1216 Å
(S)	Smallest wavelength of Balmer series.	(4)	3648 Å

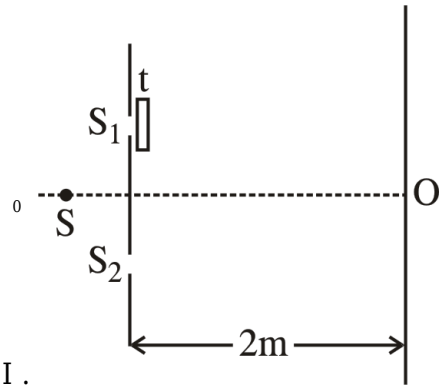
(A) P→2, Q→3, R→3, S→4

(B) P→2, Q→3, R→1, S→4

(C) P→3, Q→2, R→4, S→1

(D) P→3, Q→3, R→2, S→1

15) In a YDSE setup light of wavelength 500 nm is used. Distance between slits is 1mm and screen is placed 2m away from slits. A thin slab of thickness t = 3 μm and refractive index μ = 3/2 is placed in front of one slit as shown. What is the intensity at centre of screen O? Given intensity due to single slit is I .



(A) $3I_0$

(B) $4I_0$

(C) $I_0/2$

(D) $2I_0$

16) The electric field of an electromagnetic wave is given by :

$\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)(-\hat{i} + 2\hat{j})$ V/m

The associated magnetic field will be :-

(A) $\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)(-2\hat{i} - \hat{j}) \frac{Wb}{m^2}$

(B) $\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)(-\hat{i} + 2\hat{j}) \frac{Wb}{m^2}$

(C) $\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)(2\hat{i} + \hat{j}) \frac{Wb}{m^2}$

(D) $\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)(2\hat{i} - \hat{j}) \frac{Wb}{m^2}$

17) Find the height from the surface of the earth at which the acceleration due to gravity changes by 1% :- (Take : radius of the earth = 6400 km)

(A) 64 km

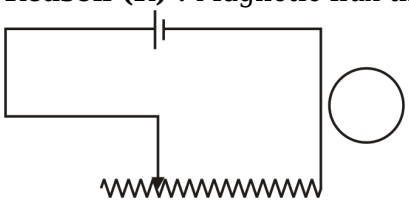
(B) 16 km

(C) 128 km

(D) 32 km

18) **Assertion (A) :** A conducting ring is placed close to the circuit in the plane of the circuit as shown in the diagram. When resistance of rheostat is decreased, clockwise current is induced in the ring.

Reason (R) : Magnetic flux through the ring is into the page and increasing.



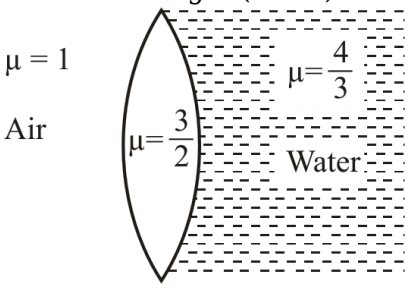
(A) (A) is true, (R) is true, (R) is the correct explanation of (A).

(B) (A) is true, (R) is true, (R) is NOT the correct explanation of (A).

(C) (A) is true, (R) is false.

(D) (A) is false, (R) is true.

19) An equiconvex lens of radius of curvature 20 cm and refractive index μ = 3/2 is placed as shown. Find focal length (in cm) of the lens if rays are entering the lens from air.



(A) 30

(B) 40

(C) 60

(D) 80

20) In photoelectric effect metal X is used in two different experimental setups. In first setup λ_1 = 3100 Å is used while in second λ_2 = 6200 Å is used and it is observed that the ratio of maximum speed of electrons in both cases is $\sqrt{5} : 1$. The work function of X is :-

(A) 1.5 eV

(B) 6 eV

(C) 3 eV

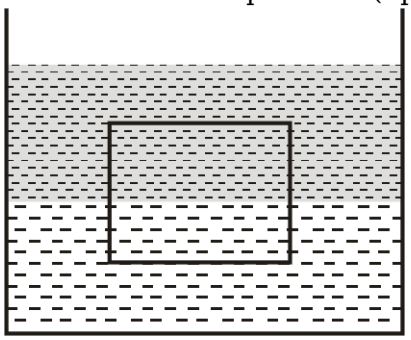
(D) 9 eV

SECTION-II

1) A parallel plate capacitor is charged by a battery connected between its plates. The battery is now disconnected and a dielectric slab (ε_r = 2) of same area as plates but having thickness half the distance between plates, is immersed. Find the % change in its potential energy after inserting the slab.

2) The pressure & volume of an ideal gas are related as $PV^{5/3}$ = (Constant). The work done when the gas is taken form state A(P_1, V_1) to state B(P_2, V_2) is $\frac{x}{2}(P_1V_1 - P_2V_2)$ then the value of x is:

3) A cube of ice floats partly in water and partly in liquid x. The ratio of volume of ice immersed in water to that in liquid X is: (Specific gravity of liquid X = 0.4, Specific gravity of Ice = 0.9)



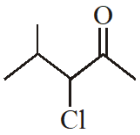
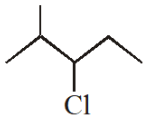
4) In a cyclotron α-particles (which are doubly ionized helium atoms) are accelerated. The radius of the greatest circular path is 50 cm. The magnitude of magnetic flux density is 1 Tesla. To what greatest energy (in MeV) can these particles be accelerated in this cyclotron? If the answer is x fill the value of 10x. Take mass of helium = 6.4×10^{-27} kg.

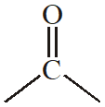
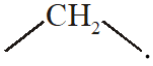
5) Neon 23 undergoes β decay ${}_{10}Ne^{23} \rightarrow {}_{11}Na^{23} + {}_{-1}^{0}\beta + \bar{\nu}$. Atomic mass of ^{23}Ne and ^{23}Na are 22.9945 u and 22.9898u respectively. Mass of electron is 0.51 $\frac{MeV}{c^2}$. The maximum kinetic energy of β^- particle is to the nearest integer (in MeV). (Take 1u = 931.5 MeV/c^2)

PART-2 : CHEMISTRY

SECTION-I

1) Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) :  can be subjected to Wolff-Kishner reduction to give  .

Reason (R) : Wolff-Kishner reduction is used to convert  into  .

In the light of the above statements, choose the correct answer from the options given below :

(A) Both A and R are true but R is NOT the correct explanation of A.

(B) A is true but R is false.

(C) A is false but R is true.

(D) Both A and R are true and R is the correct explanation of A.

2)

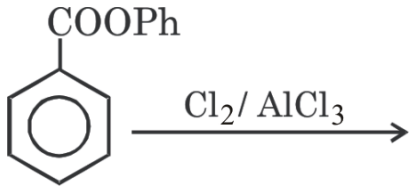
Which of the following is not reducing disaccharide ?

(A) Cellobiose

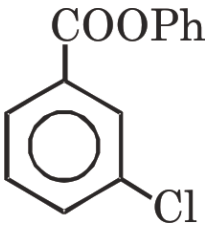
(B) Lactose

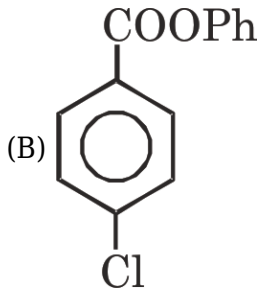
(C) Maltose

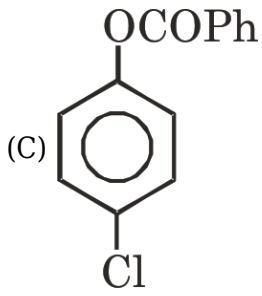
(D) Sucrose

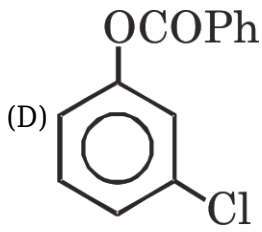
3) 

Major product of above reaction is :

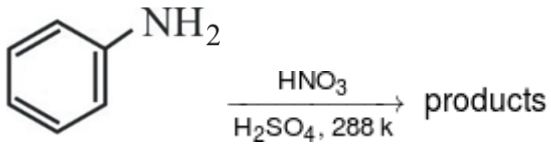
(A) 

(B) 

(C) 

(D) 

4) With respect to the following reaction, consider the given statements :



(i) o-Nitroaniline and p-nitroaniline are the predominant products
(ii) p-Nitroaniline and m-nitroaniline are the predominant products
(iii) HNO_3 acts as an acid
(iv) H_2SO_4 acts as an acid

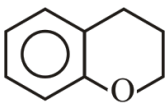
(A) (i) and (iii) are correct statements.

(B) (i) and (iv) are correct statements.

(C) (ii) and (iv) are correct statements.

(D) (ii) and (iii) are correct statements.

5) Consider the reaction of HI with the following:

(I)  (II) 

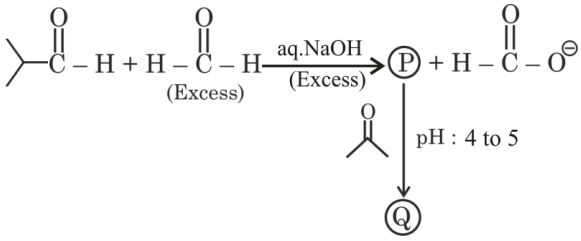
Which forms di-iodide on reaction with HI (excess)?

(A) I and II both

(B) II only

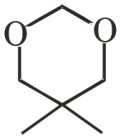
(C) I only

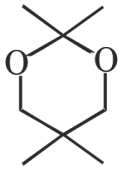
(D) none

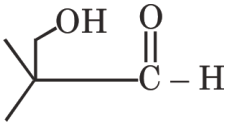


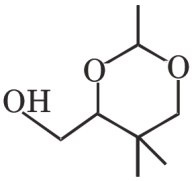
6)

Q is :

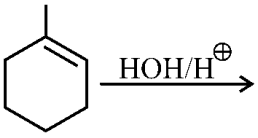
(A) 

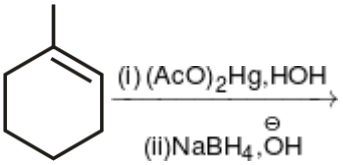
(B) 

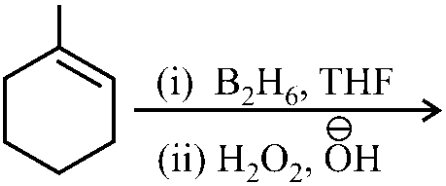
(C) 

(D) 

7) In which of the following reaction, 2º alcohol is obtained as major product.

(A) 

(B) 

(C) 

(D) None of these

8) Which of the following is incorrect regarding GRISS-I□osvay test of Nitrite?

(A) Diazotised acid couples with 1-Napthylamine to form Red-Azo dye.

(B) Acidification with acetic acid ensures the formation of HNO_2 which thereby undergoes Diazotisation with Sulphanilic acid.

(C) The formation of Red-Azo dye will always take place irrespective of the concentration of test solution.

(D) Red-Azo dye has total three nitrogen atom in one single molecule.

9) The pair in which both the ions show PINK color of their aquated ions.

(A) (Mn^{2+} , Fe^{2+})

(B) (Mn^{2+} , Co^{2+})

(C) (Ni^{2+} , Co^{3+})

(D) (Cr^{3+} , Mn^{3+})

10) The compound that is sure to have a bridging oxo linkage among the following.

(A) C_3O_2

(B) N_2O_4

(C) N_2O_5

(D) $Na_2S_2O_3$

11) The pair of complexes in which both complexes show same type of structural isomerism as well as same number of geometrical isomers.

(A) $[Co(NH_3)_4Cl_2]Br$ & $[Ni(en)_2(NH_3)_2]Cl_2$

(B) $[Pt(NH_3)_4Cl_2]Br_2$ & $[Fe(gly)_3]$

(C) $[Pt(NH_3)_4Br_2]Cl_2$ & $Na_2[CoBr_2Cl_2]$

(D) $[Co(NO_2)_2(NH_3)_4]Cl$ & $[Fe(NH_3)_4(CN)_2]Br$

12) **Statement-I :** In the periodicity of the graph IE(ionization energy) v/s Z(atomic number) (only Z = 1 to Z = 60), the maxima of the curve corresponds to noble gases and minima of the curve corresponds to Alkali metals.
Statement-II : Generally, across a period, the increase in nuclear charge outweighs the shielding effect. Consequently the outer electrons are held more tightly.

(A) Statement I is correct but Statement II is incorrect.

(B) Both statements are correct

(C) Statement I is incorrect but Statement II is correct

(D) Both Statements are incorrect

13) Which of the following statement is correct?

(A) Ionic solids are unstable because sum of ionization enthalpy & electron gain enthalpy is positive

(B) The Van der Waal's radius represents the overall size of the atom which includes its valence shell in a bonded situation.

(C) Enthalpy needed to break the two O-H bonds is identical in H_2O

(D) According to VSEPR theory, two or three electron pairs of a multiple bond are treated as a single super pair.

14) In preparation of a very important chemical "X" used as an oxidant for preparation of many compounds, fusion of chromite ore with K_2CO_3 in presence of air takes place to obtain a yellow solution "Y". Acidification of yellow solution "Y" gives orange solution of "X".
The oxidation state on metal atom in anion of "X" & "Y" respectively are.

(A) +6, +3

(B) +6, +6

(C) +3, +3

(D) +3, +6

15) Some atomic orbitals of hydrogen like atom is given in List-I and the number of radial nodes(a) and angular nodes (b) present in that orbital is given in List-II as (a, b).

List-I		**List-II**	
(P)	2p	(1)	(0, 2)
(Q)	3d	(2)	(2, 1)
(R)	4p	(3)	(0, 1)
(S)	4d	(4)	(1, 2)

(A) P-1, Q-2, R-3, S-4

(B) P-3, Q-1, R-4, S-2

(C) P-3, Q-1, R-2, S-4

(D) P-1, Q-3, R-2, S-1

16) How many millimoles of acidified $KMnO_4$ solution will be required to just oxidise one litre saturated solution of CaC_2O_4 : (K_{sp} of $CaC_2O_4 = 2.5 \times 10^{-7}$)

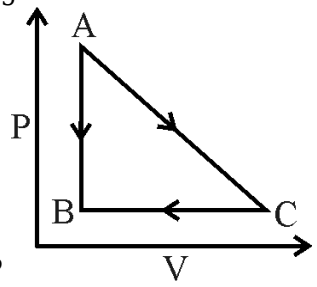
(A) 0.1

(B) 0.2

(C) 1

(D) 2

17) Consider the modes of transformation of a gas from state A to state B as show in the following P-V diagram. Which one of the following is true ?



(A) $\Delta H = q$ along path $A \rightarrow C$

(B) ΔS is same along both paths, $A \rightarrow B$ and $A \rightarrow C \rightarrow B$

(C) W is same along both paths, $A \rightarrow B \rightarrow C$ and $A \rightarrow C \rightarrow B$

(D) $W > 0$ along both paths, $A \rightarrow B$ and $A \rightarrow C$

18) pOH of 0.1 molar aqueous NaCN solution found to be 2. Then value of dissociation constant of HCN in its aqueous solution will be

(A) 1.11×10^{-3}

(B) 10^{-2}

(C) 10^{-11}

(D) 9×10^{-12}

19) A (aq) → B (aq) + C (aq) is a first order reaction :

Time	t	∞
Moles of reagent 'R'	n_1	n_2

Reaction progress is measure with the help of titration of reagent 'R'. If all A, B and C reacted with the reagent and have 'n' factors in the ratio of 1 : 2 : 3 with the reagent, the rate constant (k) in terms of t, n_1 and n_2 is
('n' factor of reagent is same for reaction with A, B as well as C)

(A) $K = \frac{1}{t}\ell n\left(\frac{n_2}{n_2 - n_1}\right)$

(B) $K = \frac{1}{t}\ell n\left(\frac{2n_2}{n_2 - n_1}\right)$

(C) $K = \frac{1}{t}\ell n\left(\frac{4n_2}{n_2 - n_1}\right)$

(D) $K = \frac{1}{t}\ell n\left(\frac{4n_2}{5(n_2 - n_1)}\right)$

20) 3.0 molal NaOH solution has a density of 1.12 g/mL. The % w/v of the solution is-
[Atomic mass : Na = 23, H = 1, O = 16]

(A) 2.97

(B) 3

(C) 12

(D) 3.5

SECTION-II

1) The difference in number of SINGLY OCCUPIED MOLECULAR ORBITALS (S.O.M.O.) in B_2 & O_2^{2-} is,_____

2) The number of black colored precipitates (in aqueous solution) among
NiS , Sb_2S_3 , As_2S_3 , $NaHCO_3(s)$, MnS , ZnS , $HgCO_3.3HgO$, CuS , $Cr(OH)_3$, $A\square(OH)_3$ is_____.

3) $CH_3CH_2CH_2\text{–}COOH \xrightarrow[\text{(ii) } Cr_2O_3,\ 780K,\ 20atm]{\text{(i) NaOH, Electrolysis}} \underset{\text{(Product)}}{P}$
Number of sp^2 carbon atoms present in product "P" is :

4) The photoelectric current from Na (work function, w_0 = 2.3 eV) is stopped by the output voltage of the cell

$Pt(s)|H_2(g, 1bar)|HCl(aq., pH = 1)|AgCl(s)|Ag(s)$
Then the energy of incident photon on Na is _____ $\times 10^{-2}$ eV.
[Given, $2.303\frac{RT}{F} = 0.06V; E^0_{Cl^-|AgCl|Ag} = 0.22V$]

5) How many gm urea (NH_2CONH_2) should be dissolved in 88.2 gm water in order to decrease the vapour pressure by 2% ?

PART-3 : MATHEMATICS

SECTION-I

1) If the function $f(x) = cxe^{-x} - \frac{x^2}{2} + x$ is decreasing for every $x \in (-\infty, 0]$, then the least value of c^2 is equal to

(A) 1
(B) 2
(C) 3
(D) 4

2) The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is

(A) $xe^{\frac{x}{y}} = e$
(B) $xe^{\frac{y}{x}} = e$
(C) $ye^{\frac{y}{x}} = e$
(D) $ye^{\frac{x}{y}} = e$

3) **Assertion (A) :** If R is a relation defined on set of natural numbers N such that $R = \{(x, y) : x, y \in N \text{ and } x + y = 24\}$ then R is an equivalence relation. **Reason (R) :** A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

(A) A is True, R is True; R is a correct explanation for A.
(B) A is True, R is True; R is not a correct explanation for A.
(C) A is True, R is False
(D) A is False, R is True.

4) Let $f(x) = 12\left(\frac{e^{3x} - 3e^x}{e^{2x} - 1}\right)$ be defined for $x > 0$ and g(x) be the inverse of f(x). If $\int_8^{27} g(x)dx = a\ell n3 - b\ell n2 - c$ then the value of (a – (b + c)) is

(A) 7

(B) 6

(C) 5

(D) 71

5) Let f(x) be a non constant twice differentiable function on R such that f(2 + x) = f(2 – x) and $f'\left(\frac{1}{2}\right) = f'(1) = 0$. Then minimum number of root(s) of equation f ''(x) = 0 in (0, 4) is/are

(A) 2

(B) 4

(C) 5

(D) 6

6) A circle of radius 2 units having centre in fourth quadrant passes through the vertex and focus of parabola $y^2 = 4x$ and touches the parabola $y = -\left(x - \frac{1}{2}\right)^2 - \alpha$, $\alpha > 0$ then the value of $(2\alpha - 4)^2$ is

(A) 15

(B) 18

(C) 22

(D) 25

7) Value of $\int_0^{\frac{\pi}{2}} \frac{\sin 8x}{\sin x} dx$ is

(A) $\frac{152}{105}$

(B) $\frac{52}{105}$

(C) $\frac{52}{35}$

(D) $\frac{152}{35}$

8) If the line x – 1 = 0 divides the area bounded by the curves $2x + 1 = \sqrt{4y + 1}$, y = x and y = 2 in two regions of area A_1 and A_2 ($A_1 < A_2$) then $\left(A_1^{-2} - A_2^{-2}\right)$ is equal to

(A) 4

(B) 5

(C) 6

(D) 8

9)

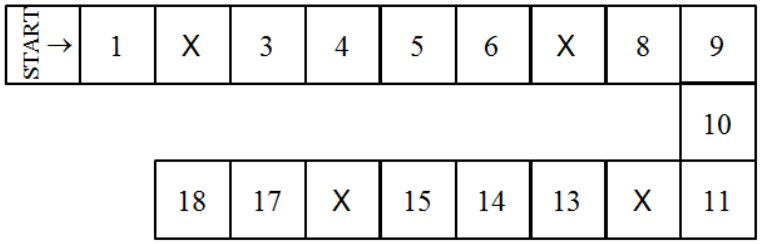
List-I		List-II	
(A)	Let a_1, a_2, a_3 ..... be AP. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$ then $4a_2$ is equal to	(I)	16
(B)	$(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + ... 20(21)^{19} = K(20)^{19}$, then $\frac{K}{100}$ is equal to	(II)	4
(C)	If number of integral solution to the equation $x + y + z = 21$ where $x \geq 1, y \geq 3, z \geq 4$ is K then $\frac{K+7}{7}$ is	(III)	15
(D)	Let $\frac{1}{16}$, a and b be in GP and $\frac{1}{a}, \frac{1}{b}$, 6 be in AP where a, b > 0 then 72 (a + b) is	(IV)	14

(A) (A) – II, (B) – IV, (C) – IV, (D) – I

(B) (A) – I, (B) – II, (C) – I, (D) – IV

(C) (A) – II, (B) – I, (C) – IV, (D) – I

(D) (A) – IV, (B) – II, (C) – I, (D) – II



10) A game board is shown in diagram.
Player take turns to roll an ordinary die, then move their counter forward from **'START'** a number of squares equal to the number rolled with the die. If a player's counter ends its move on a cross marked square, then it is moved back to **START**. Let α denotes the probability that player's counter is on **START** after rolling the die twice and let β denotes the probability that after rolling the die thrice, a player's counter is on square numbered 17, then the value of $\frac{\alpha}{\beta}$ is

(A) 24

(B) 21

(C) 20

(D) 18

11) If $(\sin^{-1}a)^2 + (\cos^{-1}b)^2 + (\sec^{-1}c)^2 + (\text{cosec}^{-1}d)^2 = \frac{5\pi^2}{2}$ then the value of $(\sin^{-1}a)^2 - (\cos^{-1}b)^2 + (\sec^{-1}c)^2 - (\text{cosec}^{-1}d)^2$ is

(A) $-\pi^2$

(B) $-\frac{\pi^2}{2}$

(C) 0

(D) $\frac{\pi^2}{2}$

12) Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ and ϕ to the x-axis. If $\tan\theta \tan\phi = 2$ then value of $2\alpha^2 - \beta^2$ is

(A) 7

(B) –7

(C) 1

(D) –1

13) Let $\frac{\cot 3^\circ}{\cot^2 3^\circ - 3} + \frac{3\cot 9^\circ}{\cot^2 9^\circ - 3} + \frac{9\cot 27^\circ}{\cot^2 27^\circ - 3} + \frac{27\cot 81^\circ}{\cot^2 81^\circ - 3} = x\cot 27^\circ + y\cot 87^\circ$. Then value of 4(x + y) is

(A) 40

(B) 42

(C) 38

(D) 84

14) Let P(x) be polynomial $x^3 + ax^2 + bx + c$ where a, b, c ∈ R. If P(–3) = P(2) = 0 and P'(–3) < 0. Which of the following is a possible value of c.

(A) –27

(B) –18

(C) –6

(D) –3

15) **Statement-I :** Let B be matrix of order 3 × 3 and adj B = A. If M and N are matrices of order 3 × 3 such that det (M) = 1 = det (N) then $\text{adj}(N^{-1}BM^{-1}) = MAN$. (where det(X) denotes determinant of matrix X ; adj (Y) denotes adjoint of matrix Y) **Statement-II :** If P is Non-singular square matrix of order 3 × 3 then $\text{adj}(P^{-1}) = (\text{adj } P)^{-1}$

(A) Statement-I is true, Statement-II is false.

(B) Statement-I is false, Statement-II is true.

(C) Statement-I is true, Statement-II is true.

(D) Statement-I is false, Statement-II is false.

16) Let $f(x) = (x^2 - 9)|x^3 - 6x^2 + 11x - 6| + \frac{x}{1 + |x|}$ and

$g : (-2, 2) \to R$, $g(x) = [x]\,|x^2 - 1| + \sin\left(\frac{\pi}{[x] + 3}\right) - [x + 1]$

If **'m'** denotes number of points where f(x) is not differentiable and **'n'** denotes the number of points where g(x) is discontinuous then **(m + n)** is (where [.] denotes the Greatest Integer Function)

(A) 3

(B) 2

(C) 4

(D) 6

17) Equation of plane which passes through the point of intersection of the lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is ax + by + cz + 50 = 0 then |a + b + c| is

(A) 12

(B) 7

(C) 15

(D) 5

18) Let f(x) be a real valued function such that $f(x)=\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos x+|u| f(u))\,du$. If M and m are the maximum and minimum values of the function f(x) respectively then $\frac{M}{m}$ is

(A) $\frac{2-\pi}{6+\pi}$

(B) $-\frac{(\pi+1)}{(\pi+6)}$

(C) $-\frac{(\pi+1)}{(\pi+3)}$

(D) $\frac{(2\pi)}{(3-\pi)}$

19) An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of all 50 candidates, then $\mu+\sigma^2$ is equal to _____.

(A) 25

(B) 20

(C) 15

(D) 30

20) Let α be the root of $x^2 + x + 1 = 0$. For some values of 'n', if $(1-\alpha+\alpha^2)(1-\alpha^2+\alpha^4)(1-\alpha^3+\alpha^6)(1-\alpha^4+\alpha^8)\ldots(1-\alpha^n+\alpha^{2n})=a^b$ where (a, b) is orthocentre of triangle with (2, 12), $(5, 12+\sqrt{3})$ and $(3, 12-\sqrt{3})$ as co-ordinates of its vertices. Then sum of possible values of n is

(A) 33

(B) 15

(C) 35

(D) 23

SECTION-II

1) Let **X** and **Y** be the set of words which can be formed using all the letters of the words SHREYANSH and SANIDHYA respectively. A set is randomly chosen and a word is selected at random from it. If the probability that it contains atleast one pair of alike letters together is $\frac{p}{q}$ (where p and q are coprime) then the value of (q – 3p) is

2) Let $\vec{a}, \vec{b}, \vec{c}$ be the vectors representing three coterminous edges of tetrahedron such that $\hat{} = \vec{b} \hat{} \vec{c} = \vec{c} \hat{} \vec{a} = \frac{\pi}{3}$ and $4\vec{a}\cdot\vec{a} + 3\vec{b}\cdot\vec{b} + 2\vec{c}\cdot\vec{c} = 144$. If V is volume of the tetrahedron, then the maximum value of V is (where $\vec{a} \hat{} \vec{b}$ represent angle between $\vec{a}$ and $\vec{b}$)

3) If the complete set of values of 'a' for which the function f : R → R defined by $f(x) = 2\sin 2x - 3\cos^2 x - (a^2 + a - 7)x + 5$, $a \in R$ is strictly increasing is [p, q] (where p, q are integers) then |p + q| is

4) Let $7\overbrace{55.....5}^{r}7$ denotes (r + 2) digit number where the first and last digit are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + ... + 7\overbrace{5.....5}^{99\text{ times}}7$. If $S = \frac{68.10^{\lambda} + 11020}{81}$, then λ

5) A point P(x, y) moves in xy plane in such a way that $\sqrt{2} \le |x + y| + |x - y| \le 3\sqrt{2}$. Area of region representing all possible of point P is equal to

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	A	B	B	B	B	A	A	D	D	A	A	B	C	D	D	B	A

SECTION-II

Q.	21	22	23	24	25
A.	25	3	5	125	4

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	C	D	C	C	C	B	C	C	B	C	D	B	D	B	C	B	B	D	D	C

SECTION-II

Q.	46	47	48	49	50
A.	2	2	6	264	6

PART-3 : MATHEMATICS

SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	A	D	D	A	B	A	A	B	B	A	C	A	A	A	C	C	A	A	A	C

SECTION-II

Q.	71	72	73	74	75
A.	3	8	1	101	16

SOLUTIONS

PART-1 : PHYSICS

1)

P = VI
$10 = 5\ I_1 \Rightarrow I_1 = 2$ mA
Across load resistance
$5 = I_2 1 \Rightarrow I_2 = 5$ mA
$I_s = 7$ mA
Vs = 3V $\Rightarrow R_S = \frac{3}{7}k\Omega$

2)

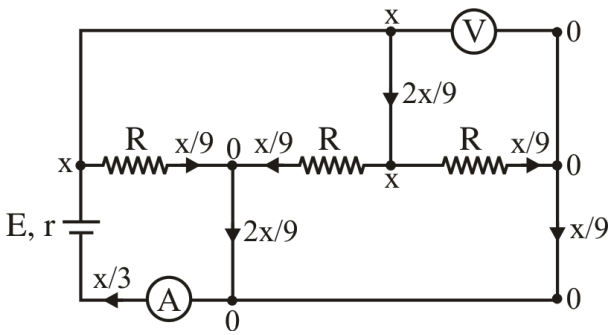
$I = 4t = 0 \le t \le 1$

$$I_{rms}^2 = \frac{\int_0^1 (4t)^2 dt}{\int_0^1 dt} = \frac{16}{3}$$

$$I_{rms} = \frac{4}{\sqrt{3}}$$

3)

Fact



4)

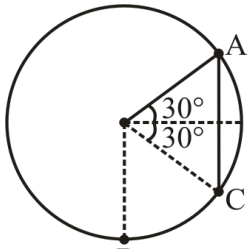
$$x = 4 - \frac{x}{3} \times 1$$

x = 3

5) $$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{g}{4}$$

$\Rightarrow 4m_1 - 4m_2 = m_1 + m_2$
$\Rightarrow 3m_1 = 5m_2$

$$\Rightarrow \frac{m_1}{m_2} = \frac{5}{3}$$



6)

Velocity just before,

$C = \sqrt{2g\ell} = 20 m/s$

Velocity just after,

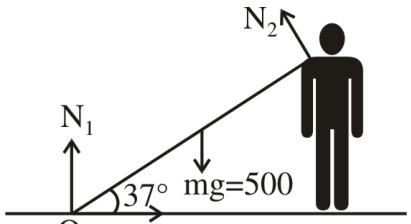
$C = 20 \cos 30° = 10\sqrt{3}$ m/s

Velocity at B $= \sqrt{(10\sqrt{3})^2 + 2g\frac{\ell}{2}} = 10\sqrt{5}$ m/s

7)

$I = I_{cone} + I_{hemisphere}$

$= \frac{1}{2}MR^2 + \frac{2}{5}MR^2 = 0.9\ MR^2$



8)

Torque about O = 0

$500\left(\frac{\ell}{2}\cos 37^o\right) - N_2 \times \ell = 0$

$\Rightarrow N_2 = 500 \times \frac{1}{2} \times \frac{4}{5}$

$\Rightarrow N_2 = 200$ N

9) $F = YA\frac{\Delta\ell}{\ell} = Y.\Delta\ell.\frac{A.A}{A.\ell} = Y.\Delta\ell.\frac{A^2}{V}$

$F \propto A^2$

$\Rightarrow \frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = 4:1$

10) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{V_1}{V_2} = \sqrt{\frac{32}{2}}$$

$$\Rightarrow \frac{4}{V_2} = 4$$

$\Rightarrow V_2 = 1$ km/sec

11) 9 MSD = 10 VSD

$$1\text{VSD} = \frac{9}{10}\text{ MSD}$$

1VSD = 0.9 × 2 mm = 1.8 mm

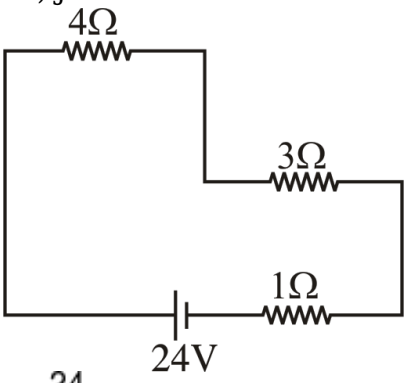
LC = 1 MSD – 1 VSD

= 2 mm – 1.8 mm = 0.2 mm

12) $$\frac{1}{2}mv^2 + \frac{KQq}{R} = \frac{KQq}{2R}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$$

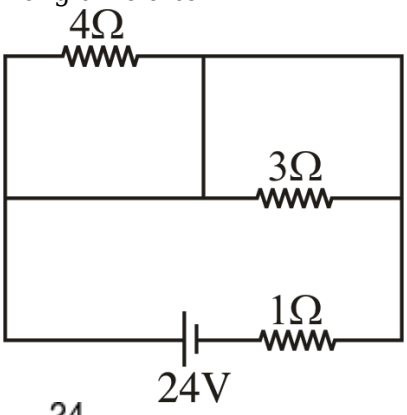
$$\Rightarrow v = \sqrt{\frac{3KQq}{4mR}}$$

13) Just after



$$I = \frac{24}{8} = 3A$$

Long time after



$$I = \frac{24}{1} = 24A$$

14)

Lyman : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$;
First Lyman line means n = 2, shortest Lyman line means n → ∞
Balmer : $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$
First Balmer line means n = 3, shortest Balmer line means n → ∞

15) $\Delta x_0 = (\mu - 1)t = \left(\frac{3}{2} - 1\right) 3\mu m = 1.5\mu m = 3\lambda$
⇒ There is constructive interference at O.
⇒ $4I_0$

16) $\hat{E} = \frac{-\hat{i} + 2\hat{j}}{\sqrt{5}}$
$\hat{V} = -\hat{K}$
$\hat{B} = \hat{V} \times \hat{E} = \frac{\hat{j} + 2\hat{i}}{\sqrt{5}}$

17) $g\left(1 - \frac{2h}{R}\right) = 0.99\,g$
⇒ h = 0.005 R = 32 km

18)

As magnetic flux is into the plane and increasing, induced current will be anti-clockwise, thereby creating out of the plane magnetic flux.

20) $\frac{KE_1}{KE_2} = \frac{5}{1} = \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi}$
$5\left(\frac{hc}{\lambda_2} - \phi\right) = \frac{hc}{\lambda_1} - \phi$
$\frac{5hc}{\lambda_2} - \frac{hc}{\lambda_1} = 4\phi$
$\phi = \frac{1}{4}\left[\frac{5hc}{\lambda_2} - \frac{hc}{\lambda_1}\right]$
$\phi = \frac{1}{4}\left[\frac{5 \times 12400}{6200} - \frac{12400}{3100}\right]$
$\phi = \frac{1}{4} \times [10 - 4] = \frac{6}{4} = \frac{3}{2} = 1.5$

21) $\frac{\frac{Q^2}{2C_i} - \frac{Q^2}{2C_F}}{\frac{Q^2}{2C_i}} \times 100$

$$\frac{\frac{d}{\varepsilon_0 A}-\frac{3d}{4\varepsilon_0 A}}{\frac{d}{\varepsilon_0 A}-\frac{1}{4}} \times \frac{\frac{1}{C_i}-\frac{1}{C_F}}{\frac{1}{C_i}} \times 100 = \frac{\frac{1}{C_0}-\frac{3}{4C_0}}{\frac{1}{C_0}} \times 100$$

$$\frac{1}{4} \times 100 = 25\%$$

22) PV^x = constant

$$W = \frac{nR\Delta T}{1-x}$$

$$W = \frac{P_2V_2 - P_1V_1}{1-\frac{5}{3}}$$

$$= \frac{P_2V_2 - P_1V_1}{-\frac{2}{3}}$$

$$= \frac{3}{2}(P_1V_1 - P_2V_2)$$

23) $F_B = F_g$

$\Rightarrow V_1\rho_w g + V_1\rho_x g = (V_1 + V_2)\, P_I g$

$\Rightarrow V_1 + V_2\frac{\rho_x}{\rho_w} = (V_1 + V_2)\frac{\rho_I}{\rho_w}$

$\Rightarrow V_1 + V_2 \times 0.4 = (V_1 + V_2)\, 0.9$

$\Rightarrow 0.1\, V_1 = 0.5V_2$

$= \Rightarrow \frac{V_1}{V_2} = \frac{5}{1}$

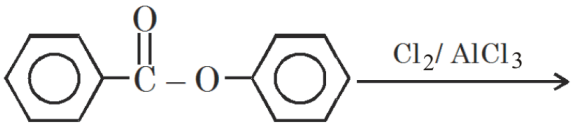
25) $Q = (M_{Ne} - M_{Na})c^2$

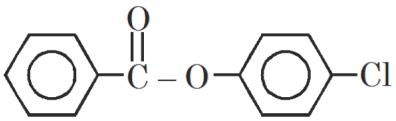
$= 0.0047 \times 931.5$ MeV $= 4.378$ MeV

PART-2 : CHEMISTRY

26) Wolff-Kishner reduction is not suitable for base sensitive group.

28)





40) Number of radoa; mpdes = (n – □ – 1)

Number of angular nodes = □

41) Solubility of $CaC_2O_4 = \sqrt{K_{sp}} = 5 \times 10^{-4}$ M
gm equivalent of $CaC_2O_4 \equiv KMnO_4$

$5 \times 10^{-4} \times 2 = \frac{n}{1000} \times 5$

n = 0.2

42) S is a state function so ΔS will be same is path
A → B and A → C → B

43) $CN^- + H_2O \rightleftharpoons HCN + OH^-$

0.1–x x x

$\frac{10^{-14}}{K_a} = \frac{x^2}{0.1-x}$ $x = 10^{-2}$

$K_a = 9 \times 10^{-12}$

45)

m = 3 ⇒ If $w_{solvent}$ = 1000 gm
Then moles of NaOH = 3

$\% \text{ (w/v)} = \frac{\frac{3 \times 40}{(1000+120)}}{1.12} \times 100 = 12$

48)

Product P is benzene

49) $\frac{1}{2}H_2 \rightarrow H^+ + e^{\ominus}$

$e^{\ominus} + AgCl_{(s)} \rightarrow Ag_{(s)} + Cl^{\ominus}$

$\frac{1}{2}H_2 + AgCl_{(s)} \rightarrow H^+_{(aq)} + Ag_{(s)} + Cl^{\ominus}_{(aq)}$

$E = \varepsilon^0 - \frac{.06}{1} \log \frac{[H^+]\,[Cl^{\ominus}]}{P_{H_2}^{\frac{1}{2}}}$

$E = 0.22 - .06 \log \frac{\left(10^{-1}\right)\left(10^{-1}\right)}{1^{\frac{1}{2}}}$

E = 0.22 + .12 = .34 volt
⇒ total energy of photon will be (for Na)
= 2.3 + 0.34 = 2.64 eV

50) $\frac{p^o - p_s}{p_s} = \frac{n}{N}$

or, $\frac{2}{98} = \frac{\frac{w}{60}}{\frac{88.2}{18}} \Rightarrow w = 6$

PART-3 : MATHEMATICS

51) $f'(x) = c[e^{-x} - xe^{-x}] - x + 1 \quad = ce^{-x}(1 - x) + (1 - x)$
$f'(x) = (1 - x)(ce^{-x} + 1) \le 0$
$ce^{-x} + 1 \le 0$
$c \le -e^{x} \; \forall \; x \le 0$
$c \in (-\infty, -1]$
Least value of $c^2 = 1$

52) $Y - y = m(X - x)$ For x-intercept $y = 0$ $X = x - \frac{y}{m}$ $\Rightarrow x - \frac{y}{m} = y$
$\frac{dy}{dx} = \frac{y}{x-y}$
$xdy - ydy = ydx \Rightarrow \frac{-ydy}{y^2} = \frac{ydx - xdy}{y^2}$
$-\frac{dy}{y} = d\left(\frac{x}{y}\right)$
$-\ell ny = \frac{x}{y} + C$
$x = 1, y = 1, C = -1$
$-\ell ny = \frac{x}{y} - 1$ or $\ell ny = \frac{-x}{y} + 1$
$y = e \cdot e^{-\frac{x}{y}}$

53) Assertion is False but Reason is True.

54) $\int_{\ell n2}^{\ell n3} f(x)dx + \int_{8}^{27} g(y)dy = 27\ell n3 - 8\ell n2 \int_{\ell n2}^{\ell n3} f(x)dx = 12 - 12\ell n3 + 12\ell n2$ and
$\int_{8}^{27} g(y)dy = 39\ell n3 - 20\ell n2 - 12$
$a = 39 \;\; b = 20 \;\; c = 12$
$a - (b + c) = 7$

55) $f(2 + x) = f(2 - x)$ $f'(2 + x) = -f'(2 - x)$
Put $x = 0 \;\; f'(2) = 0$
$x = -1 \;\; f'(1) = -f'(3) = 0$
$x = -\frac{3}{2} \;\; f'\left(\frac{1}{2}\right) = -f'\left(\frac{7}{2}\right) = 0$
$f'\left(\frac{1}{2}\right) = 0 = f'(1) = f'(2) = f'(3) = f'\left(\frac{7}{2}\right)$
minimum roots of $f''(x) = 0$

56) $x(x-1)+y^2=0$

$S_1 : x^2+y^2-x=0$

Required circle

$x^2+y^2-x+\lambda y=0$

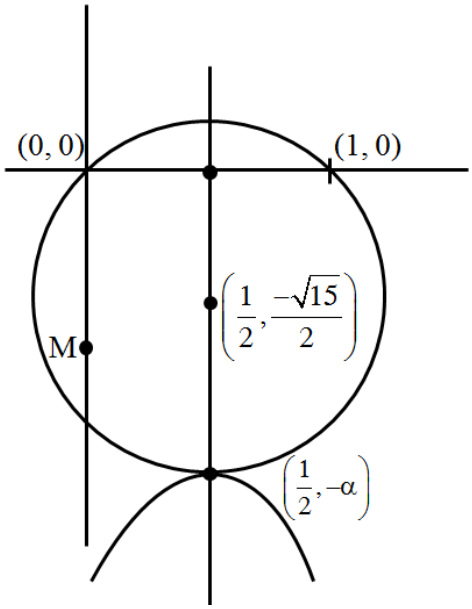
$g=-\frac{1}{2}$ $f=\frac{\lambda}{2}$, radius $=2$

$g^2+f^2-c=4$

$\frac{1}{4}+\frac{\lambda^2}{4}=4$

$1+\lambda^2=16 \quad \therefore \lambda^2=15 \quad \therefore \lambda=\sqrt{15}, -\sqrt{15}$

centre of circle $\left(\frac{1}{2},\frac{-\lambda}{2}\right)$



parabola $y+\alpha=-\left(x-\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2=-(y+\alpha)$

$-\frac{\sqrt{15}}{2}-2=-\alpha$

$\alpha=2+\frac{\sqrt{15}}{2}$

$2\alpha=4+\sqrt{15}$

$(2\alpha-4)^2=15$

57) $2\sin x\cos x=\sin 2x \quad 2\sin x\cos 3x=\sin 4x-\sin 2x$

$2\sin x\cos 5x=\sin 6x-\sin 4x$

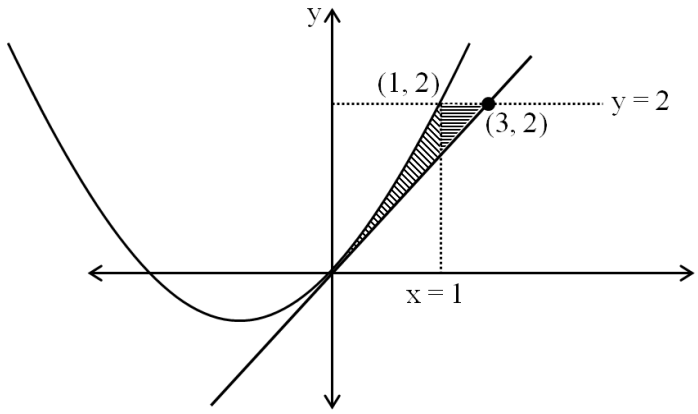
$2\sin x\cos 7x=\sin 8x-\sin 6x$

$2\sin x[\cos x+\cos 3x+\cos 5x+\cos 7x]=\sin 8x$

$$\int_0^{\frac{\pi}{2}}\frac{\sin 8x}{\sin x}=2.\int_0^{\frac{\pi}{2}}\cos x+\cos 3x+\cos 5x+\cos 7x$$

$$2\left[1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\right]$$

$$=2\left[\frac{105-35+21-15}{105}\right]=\frac{152}{105}$$



58)

$$A_1=\int_0^1\left(\frac{(2x+1)^2-1}{4}-x\right)dx=\frac{1}{3}$$

$A_2=\frac{1}{2}$

59) (A) $x = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + ...\infty$ $\quad \frac{1}{2}x = \frac{a_1}{2^2} + \frac{a_2}{2^3} + ...\infty$

$\frac{x}{2} = \frac{a_1}{2} + \left\{\frac{d}{2^2} + \frac{d}{2^3} + ..\infty\right\}$

$x = a_1 + d = 4$

$\Rightarrow a_2 = 4$

(C) $x + y + z = 21$

$(x + 1) + (y + 3) + (z + 4) = 13$

$x + y + z = 13 \quad \Rightarrow {}^{13+3-1}C_{3-1} = {}^{15}C_2 = 105$

(D) $\frac{1}{16}$, a, b in G. P. $\Rightarrow a^2 = \frac{b}{16}$

$\frac{1}{a}, \frac{1}{b}$, 6 in A. P. $\Rightarrow 2a = b + 6ab$

$16a^2 = \frac{2a}{1+6a}$

$(12a - 1)(4a + 1) = 0 \quad \Rightarrow a = \frac{1}{12}, \frac{-1}{4}$

$b = 16.\frac{1}{12}.\frac{1}{12} = \frac{1}{9}$

$72\left[\frac{1}{12} - \frac{1}{9}\right] = 6 + 8 = 14$

60)

START →	1	X	3	4	5	6	X	8	9
									10
	18	17	X	15	14	13	X	11	

To be on start after two throws

(1, 1), (2, 2), (1, 6), (6, 1), (5, 2), (3, 4), (4, 3), (6, 6)

$\alpha = 8 \cdot \left(\frac{1}{6} \times \frac{1}{6}\right) = \frac{2}{9}$

To be on square marked 17. He needs to throw sum of 17 in three throws

= (6, 5, 6), (5, 6, 6) = $2\left(\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}\right) = \frac{1}{108}$

Value of $\frac{\alpha}{\beta} = \frac{2}{9} \times \frac{108}{1} = 24$

61) $(\sin^{-1}a)^2 = \frac{\pi^2}{4}$, $(\cos^{-1}b)^2 = \pi^2$ $(\sec^{-1}c)^2 = \pi^2$, $(\operatorname{cosec}^{-1}d)^2 = \frac{\pi^2}{4}$

62) $\frac{x^2}{2} - \frac{y^2}{3} = 1$ $y = mx \pm \sqrt{2m^2 - 3}$

$(\beta - m\alpha)^2 = 2m^2 - 3$

$(\alpha^2 - 2)m^2 - 2\alpha\beta m + \beta^2 + 3 = 0$

$\tan\theta \tan\phi = 2$

$m_1m_2 = 2$

$\frac{\beta^2+3}{\alpha^2-2}=2 \quad \therefore\ \beta^2 = 2\alpha^2 - 7$

63) $\frac{\tan 3^\circ}{1-3\tan^2 3^\circ}+\frac{\tan 3^\circ}{8}-\frac{\tan 3^\circ}{8}$

$\Rightarrow \frac{3}{8}\tan 9^\circ - \frac{\tan 3^\circ}{8}$

Again $\frac{3}{8}\tan 9^\circ + \frac{3\tan 9^\circ}{1-3\tan^2 9^\circ} = \frac{9}{8}\tan 27^\circ$

and so on

$\frac{81\tan 243^\circ}{8} - \frac{\tan 3^\circ}{8}$

$\frac{81}{8}\cot 27^\circ - \frac{1}{8}\cot 87^\circ$

$x + y = \frac{80}{8}$

$4(x + y) = 40$

64) $P(x) = x^3 + ax^2 + bx + c$ $P(-3) = -27 + 9a - 3b + c = 0$ ...(i)

$P(2) = 8 + 4a + 2b + c = 0$ ...(ii)

$-35 + 5a - 5b = 0$

$a - b = 7$ ...(iii)

$P'(x) = 3x^2 + 2ax + b$

$P'(-3) = 27 - 6a + b < 0$

$27 - 6(a - b) - 5b < 0$

$27 - 6(7) - 5b < 0$

$-15 - 5b < 0$

$b + 3 > 0 \quad \square\ b > -3$

$a = 7 + b$

$a > 4$

$8 + (\geq 16) + 2\ (\geq -3) + c = 0$

$18 + c_{max} = 0 \quad c_{max} = -18$

$c < -18$

65) $\text{adj}(N^{-1}BM^{-1}) = (\text{adj } M^{-1})\ (\text{adj } B)\ (\text{adj } N^{-1})\ (\text{adj } M)^{-1}.\ A(\text{adj } N)^{-1}$

MAN $|M| = 1 = |N|$

Note: $P \rightarrow \text{adj } P^{-1} = |P^{-1}|\text{In}$

$\text{adj } P^{-1} = \frac{P}{|P|}$ ...(i)

$P^{-1} = \frac{\text{adj } P}{|P|} \Rightarrow P = |P|(\text{adj } P)^{-1}$

Also $M^{-1} = \frac{\text{adj } M}{|M|} \Rightarrow M^{-1} = \text{adj } M$

$(\text{adj } M)^{-1} = M$

Note $M^{-1}\ \text{adj } M^{-1} = \frac{\text{In}}{|M|}$

$\text{adj } M^{-1} = M$

66) $f(x) = (x-3)(x+3)\,|(x-1)(x-2)(x-3)| + \dfrac{x}{1+|x|}$

Not differentiable at x = 1, 2 ⇒ m = 2

For g(x)

$$\lim_{x\to -1^-} [x]\left|x^2-1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1]$$

$(-2)(0) + \sin\pi + 1 = 1$

$\lim\limits_{x\to -1^+} (-1)(0) + \sin\dfrac{\pi}{2} + 0 = 1$

Similarly $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to 1} g(x)$

⇒ Discontinuous at x = 1 and x = 0

⇒ n = 2

67) Point of intersection of lines is (4, 3, 5). For plane to be at maximum distance from origin normal to plane will be $4\hat{i}+3\hat{j}+5\hat{k}$ Equation of plane = 4(x – 4) + 3(y – 3) + 5(z – 5) = 0

4x + 3y + 5z = 50

–4x – 3y – 5z + 50 = 0

68)

$$f(x) = \cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,du + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |u|\,f(u)du$$

$$f(x) = \cos x + \pi\cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |u|\,f(u)du$$

$$f(x) = (1+\pi)\cos x + A\ ;\ A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |u|\,f(u)du$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ((1+\pi)\cos u + A)\cdot |u|\,du$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\pi)\,|u|\cos u\,du + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\,|u|\,du$$

$$A = 2\cdot\int_{0}^{\frac{\pi}{2}} (1+\pi)u\cos u + 2A\cdot\int_{0}^{\frac{\pi}{2}} u\,du$$

$$= 2\cdot(1+\pi)\left[\frac{\pi}{2}-1\right] + 2A\cdot\left[\frac{u^2}{2}\right]_0^{\frac{\pi}{2}}$$

$$A = 2(1+\pi)\left(\frac{\pi}{2}-1\right) + A\left[\frac{\pi^2}{4}-0\right]$$

$$A = \frac{-4(\pi+1)}{\pi+2}$$

$$f(x) = (1+\pi)\cos x - \frac{4(\pi+1)}{\pi+2}$$

$f_{max} = \frac{(\pi+1)}{\pi+2} \cdot (\pi-2)$; $f_{min} = -\left(\frac{\pi+1}{\pi+2}\right)[\pi+6]$

$\frac{M}{m} = \frac{2-\pi}{6+\pi}$

69) $\sigma_b^2 = 2$ (variance of boys) n_1 = no. of boys

$\bar{x}_b = 12$ n_2 = no. of girls

$\sigma_g^2 = 2$

$\bar{x}_g = \frac{50 \times 15 - 12 \times n_1}{30} = \frac{750 - 12 \times 20}{30} = 17 = \mu$

variance of combined series

$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1+n_2} + \frac{n_1 . n_2}{(n_1+n_2)^2}(\bar{x}_b - \bar{x}_g)^2$

$\sigma^2 = \frac{20 \times 2 + 30 \times 2}{20+30} + \frac{20 \times 30}{(20+30)^2}(12-17)^2$

$\sigma^2 = 8$

$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$

70) $1 + \alpha + \alpha^2 = 0$; $\alpha^3 = 1$

$1 + \alpha^2 = -\alpha$; $1 + \alpha = -\alpha^2$

considering first three consecutive terms

$\therefore (1 - \alpha + \alpha^2)(1 - \alpha^2 + \alpha^4)(1 - \alpha^3 + \alpha^6)$

$\Rightarrow (-2\alpha)(-2\alpha^2)(1) = 2^2$

$$\begin{cases} (2)^{2K} & , \ n = 3K \\ (2)^{2K+1}(-\alpha) & , \ n = 3K+1 \\ (2)^{2K+2} & , \ n = 3K+2 \end{cases}$$

since (2, 12) is orthocentre $\square$ $a^b = 2^{12}$

$2^{12} = 2^{2K}$ $\square$ K = 6 → n = 18

$2^{12} = 2^{2K+2}$ $\square$ K = 5 → n = 17

sum of possible values of n is 35

71) SHREYANSH Total words : $\frac{9!}{2!2!}$

for two alike letters together

n(A) : Two H together $= \frac{8!}{2!}$

n(B) : Two S together $= \frac{8!}{2!}$

n(A ∩ B) : H together, S together = 7!

n(A∪B) = $\frac{8!}{2!} + \frac{8!}{2!} - 7! \Rightarrow 8! - 7! = 7 \times 7!$

Required prob = $\frac{7 \times 7!}{9!} \times 2 \times 2 = \frac{7 \times 4}{9 \times 8} = \frac{7}{18}$

For word SANIDHYA

Total words = $\frac{8!}{2!}$

n(X) : two A together 7!

Required probability $= \frac{7! \times 2}{8!} = \frac{1}{4}$

Final probability $= \frac{1}{2} \times \frac{7}{18} + \frac{1}{2} \times \frac{1}{4} = \frac{14+9}{72} = \frac{23}{72}$

72)

Volume of tetrahedron $= \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} |\vec{a}|^2 & \frac{|\vec{a}||\vec{b}|}{2} & \frac{|\vec{a}||\vec{c}|}{2} \\ \frac{|\vec{a}||\vec{b}|}{2} & |\vec{b}|^2 & \frac{|\vec{b}||\vec{c}|}{2} \\ \frac{|\vec{c}||\vec{a}|}{2} & \frac{|\vec{c}||\vec{b}|}{2} & |\vec{c}|^2 \end{vmatrix}$$

$$= |\vec{a}|^2|\vec{b}|^2|\vec{c}|^2 \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix}$$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \frac{1}{2}|\vec{a}|\,|\vec{b}|\,|\vec{c}|^2$$

Now $\frac{4|\vec{a}|^2 + 3|\vec{b}|^2 + 2|\vec{c}|^2}{3} \geqslant \left(24|\vec{a}|^2|\vec{b}|^2|\vec{c}|^2\right)^2$

$$\frac{144}{3} \geqslant \left(24\left(|\vec{a}|\,|\vec{b}|\,|\vec{c}|\right)^2\right)^{\frac{1}{3}}$$

$$\frac{48 \times 48 \times 48}{24} \geqslant \left(|\vec{a}|\,|\vec{b}|\,|\vec{c}|\right)^2$$

$$|\vec{a}|\,|\vec{b}|\,|\vec{c}| \leqslant 48\sqrt{2}$$

$V_{max} = \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \frac{1}{6}(\leqslant 48) \leqslant 8$ ∴ $V_{max} = 8$

73) $f(x) = 2\sin 2x - 3\cos^2 x - (a^2 + a - 7)x + 5$ $f'(x) = 4\cos 2x + 3\sin 2x - (a^2 + a - 7) \geq 0$

$a^2 + a - 7 \leq 4\cos 2x + 3\sin 2x$

$a^2 + a - 7 \leq -5$

$a^2 + a - 2 \leq 0$

$(a + 2)(a - 1) \leq 0 \quad a \in [-2,1]$

$|p + q| = |-2 + 1| = 1$

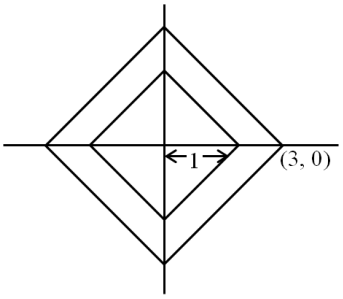
74) $T_r = 7 \times 10^r + \frac{50}{9}\left(10^{r-1} - 1\right) + 7$

$$\sum_{r=1}^{100} T_r = \frac{7\left(10\left(10^{100}-1\right)\right)}{9} + \frac{50}{9}\left(\frac{10^{100}-1-100}{9}\right) + 700$$

$$S = \frac{68.10^{101} + 11020}{81}$$

$\therefore \lambda = 101$

75) $1 \leqslant \left|\frac{x+y}{\sqrt{2}}\right| + \left|\frac{x-y}{\sqrt{2}}\right| \leqslant 3$ $\quad 1 \le |x| + |y| \le 3$



Area $4\left(\frac{1}{2}\times 3\times 3 - \frac{1}{2}\times 1\times 1\right)$

$4\left(\frac{9}{2}-\frac{1}{2}\right) = 16$