PHYSICS

1

If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $\boxed{P = EL^2M^{-5}G^{-2}}$ are

(A) $[M^1\ L^1\ T^{-2}]$

(B) $[M^0\ L^1\ T^0]$

(C) $[M^{-1}\ L^1\ T^2]$

(D) $[M^0\ L^0\ T^0]$ ✓

JEE 1

$E = mc^2$

$L = mvr_\perp$

$[E] = ML^2T^{-2}$

$[L] = ML^2T^{-1}$

$[M] = M^1$

$[G] = \left[\dfrac{Fr^2}{m_1 m_2}\right] = \dfrac{MLT^{-2}L^2}{M^2}$

$[G] = M^{-1}L^3T^{-2}$

$[P] = M^1L^2T^{-2}(ML^2T^{-1})^2 M^{-5}(M^{-1}L^3T^{-2})^{-2}$

$[P] = M^{1+2-5+2}\ L^{2+4-6}\ T^{-2-2+4}$

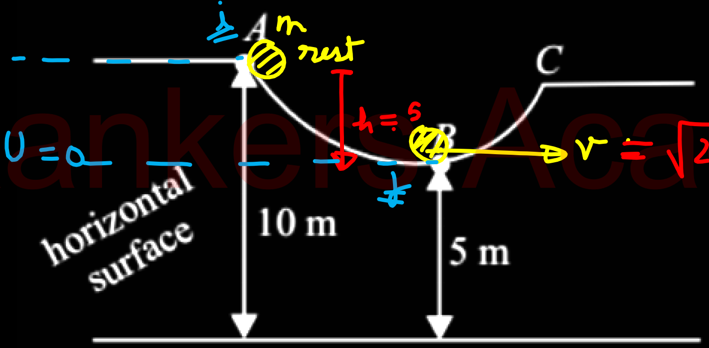
$[P] = M^0L^0T^0 = 1$

2

As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take $g = 10\ \text{m/s}^2$) Find x

$E_i = E_f.$

$mgh = \frac{1}{2} mv^2$



$v = \sqrt{2gh} = \sqrt{100} = 10\ \text{m/s}.$

(A) $\sqrt{10}$ m/s

(B) 10 m/s ✓

(C) $\sqrt{20}$ m/s

(D) 20 m/s

3

The trajectory of a projectile near the surface of the earth is given as $\boxed{y = 2x - 9x^2}$. If it were launched at an angle $\cancel{\theta_0}$ θ with speed $\cancel{v_0}$ u then ($g = 10\ m\ s^2$)

(A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(B) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

(C) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

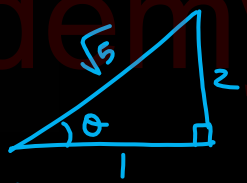
(D) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$

$$\boxed{y = x\tan\theta\left(1 - \frac{x}{R}\right)}$$

$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

$\tan\theta = 2$ (= 2), $\frac{g}{2u^2\cos^2\theta} = 9$

$\tan\theta = 2$



$$\boxed{\sin\theta = \frac{2}{\sqrt{5}}}$$

$$\boxed{\cos\theta = \frac{1}{\sqrt{5}}}$$

$$\frac{\cancel{10}^{\,5}}{\cancel{2}u^2\left(\frac{1}{5}\right)} = 9$$

$$\boxed{u = \frac{5}{3}}$$

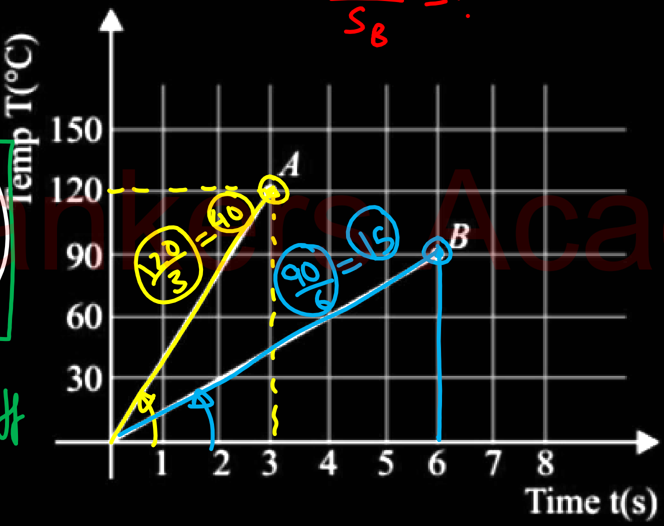
4

Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is

$\frac{S_A}{S_B} = ?$

$\boxed{Q = mS\Delta T}$

$$\boxed{\underbrace{\left(\frac{Q}{t}\right)}_{\text{same}} = \underbrace{m}_{\text{same}}\,\underbrace{S}_{\text{diff}}\,\underbrace{\left(\frac{\Delta T}{t}\right)}_{\text{diff}}}$$



(A) $\frac{3}{4}$ (B) $\frac{8}{3}$

(C) $\frac{3}{8}$ (D) $\frac{4}{3}$

slope$_A$ = 40.

$$\left(\frac{Q}{t}\right) = m S_A \left(\frac{\Delta T}{t}\right)_A \quad \text{— (1)}$$

slope$_B$ = 15

$$\left(\frac{Q}{t}\right) = m S_B \left(\frac{\Delta T}{t}\right)_B \quad \text{— (2)}$$

$\div$

$$1 = \frac{S_A}{S_B}\left(\frac{40}{15}\right)$$

Ans $= \frac{S_A}{S_B} = \frac{\cancel{15}\,3}{\cancel{40}\,8}$

5

JEE 1

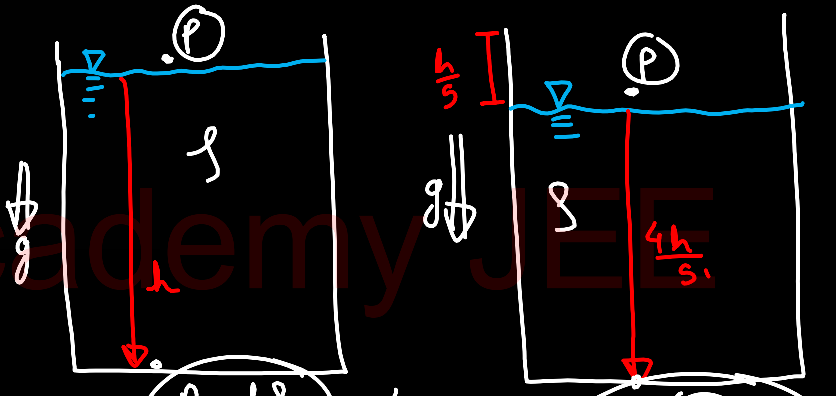
The pressure at the bottom of a tank of water is 4P where P is the atmosphere pressure. If the water is drawn out till level of water is lowered by onefifth, the pressure at the bottom of the tank will now be

(A) 2P

(B) $\frac{13}{5}$P

(C) $\frac{17}{5}$P

~~(D) $\frac{4}{5}$P~~



$$P + h\rho g = 4P$$

$$h\rho g = \underline{\underline{3P}}$$

$$\text{Ans} = P + \left(\frac{4}{5}h\right)\rho g$$

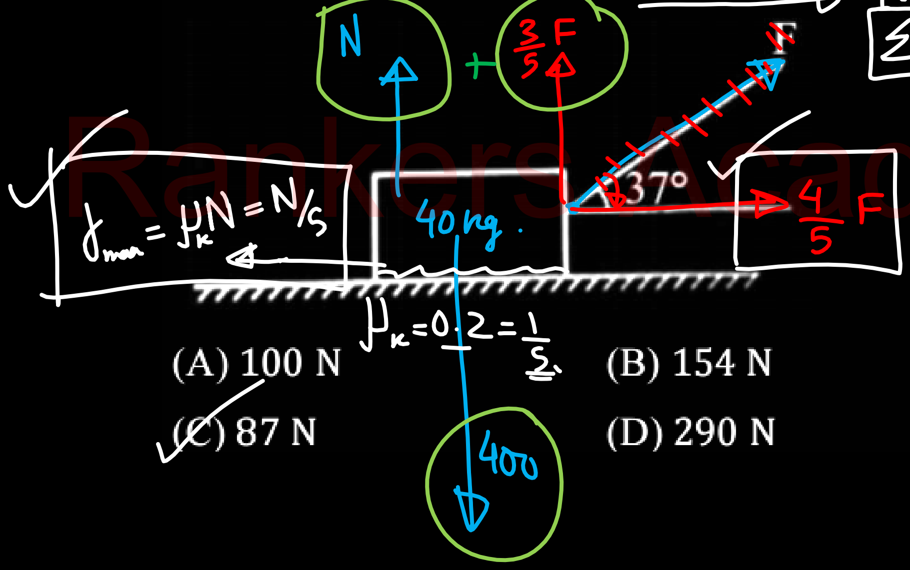
$$= P + \frac{4}{5}(3P)$$

$$\underline{\underline{\text{Ans}}} = \frac{17P}{5}$$

6

A 400 N block is dragged along a horizontal surface by an applied force $\vec{F}$ as shown. The coefficient of kinetic friction is $\boxed{\mu_k = 0.2}$ and the $\boxed{\text{block moves at constant velocity}}$. The magnitude of $\vec{F}$ is : (close to)

V = const. $\boxed{a = 0}$ $\boxed{\Sigma F = 0}$



$\mu_k = 0.2 = \frac{1}{5}$

(A) 100 N (B) 154 N

(C) 87 N (D) 290 N

$N + \frac{3F}{5} = 400$ — ①

$\frac{4F}{\cancel{5}} = \frac{N}{\cancel{5}}$ — ②

$4F + \frac{3F}{5} = 400$

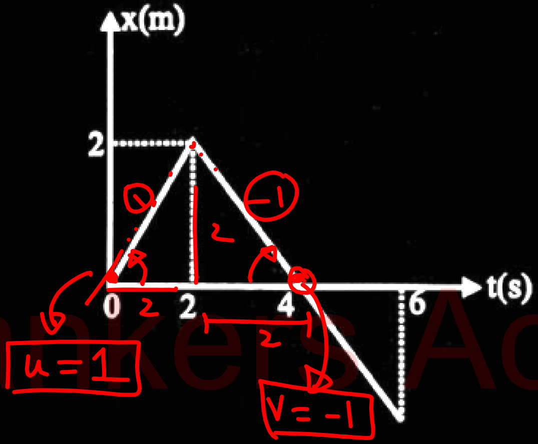
$23F = 200$

$F = \frac{2000}{23}$

$\boxed{F = 87}$

7

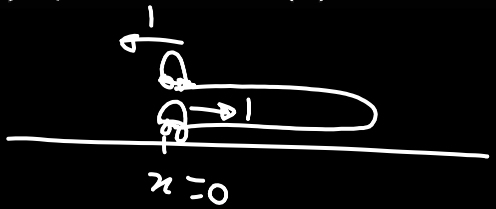
Position-time graph of particle of mass 2 kg is shown in figure. Total work done on the particle from t = 0 to t = 4 s is



$v = \frac{dx}{dt}$

(A) 8 J (B) 4 J

(C) 0 J (D) can't be determined



$W_{0\to4} = \Delta KE_{0\to4}$

$W_{0\to4} = \frac{1}{2}m(v^2 - u^2)$

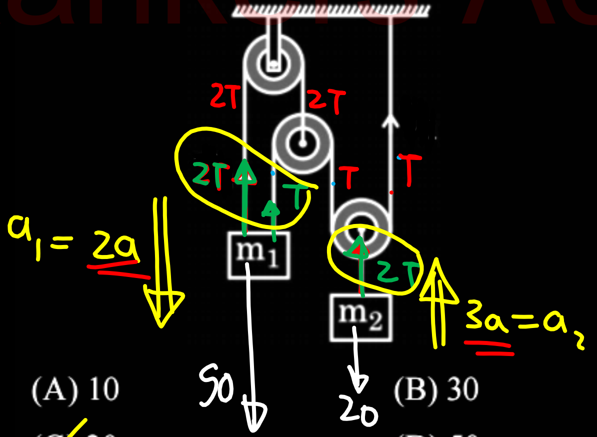
$= \frac{1}{2}m(1^2 - 1^2)$

$= 0$

8

Two blocks of masses $m_1 = 5$ kg and $m_2 = 2$ kg are connected by threads which pass over the pulleys as shown in the figure. The threads are massless and the pulleys are massless and smooth. The blocks can move only along the vertical direction. If the acceleration of m_1, if expressed in simplest form, is equal to $\boxed{\left(\frac{2n}{n-1}\right)}$ m/sec^2, calculate ' n '.

(take g = 10 m/s^2)



(A) 10 (B) 30

(C) 20 (D) 50

$$\boxed{\sum \vec{T} \cdot \vec{a} = 0}$$

$$-3\cancel{T} a_1 + 2\cancel{T} a_2 = 0$$

$$\frac{a_1}{a_2} = \frac{2a}{3a}$$

$$50 - 3T = 10a \quad \times 2 \text{ — } (1)$$

$$2T - 20 = 6a \quad \times 3 \text{ — } (2)$$

$$100 - \cancel{6T} = 20a$$

$$\cancel{6T} - 60 = 18a$$

$$40 = 38a$$

$$\boxed{a = \frac{20}{19}}$$

$$\text{Ans} = a_1 = 2a = \frac{40}{19} = \frac{2n}{n-1}$$

$$\boxed{n = 20}$$

$$\text{Ans} = a_1 = \boxed{2a} = \frac{2n}{n-1}$$

9

A body of mass 3 kg collides elastically with another body at rest and then continues to move in the the original direction with one-half of its original speed. What is the mass of the target body (in kg)?

$m = ?$

(A) 2

~~(B) 3~~

(C) 1

(D) 6

$\boxed{C}$

$e = \frac{V_{sep.}}{V_{app.}}$

$\boxed{e = 1}$

u

i

3kg.

m

$u/2$

v

$\oplus$

f

$\boxed{\vec{P}}$

3kg.

m

$\cancel{J}$ J

$3u = 3\frac{u}{2} + mv$ — ①

$1 = \frac{v - u/2}{u}$

$v = \frac{3u}{2}$ — ②

①&②

$3\frac{\cancel{u}}{2} = m\left(\frac{3\cancel{u}}{2}\right)$

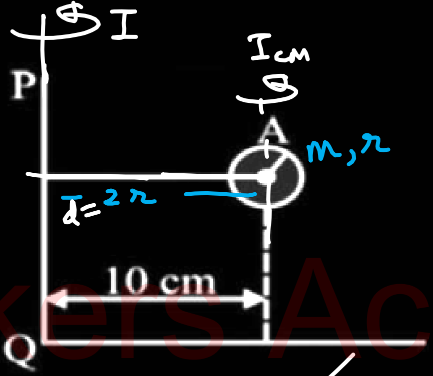
$\boxed{m = 1}$

JEE 1

CGS

Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is

$\boxed{r=5}$



(A) 55
(B) 110
(C) 11
(D) 5

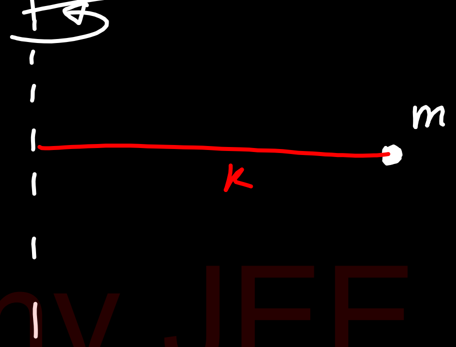
$$I = \underline{I_{cm}} + md^2$$

$$I = \frac{2}{5}mr^2 + m(2r)^2$$

$$\boxed{I = \frac{22}{5}mr^2} \quad -①$$

$$\boxed{I = mk^2} \quad -②$$

$\equiv$



$$\frac{22}{5}\cancel{m}r^2 = \cancel{m}k^2$$

$$\frac{(22)(25)}{5} = k^2$$

$$\boxed{k = \sqrt{110}}\ \text{cm}$$

Given below are two statements

Statement I: A car moving towards East takes a turn and moves towards North the speed remains unchanged. The acceleration of the car is not zero. True..

Statement II: A car and a truck moving with the same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.

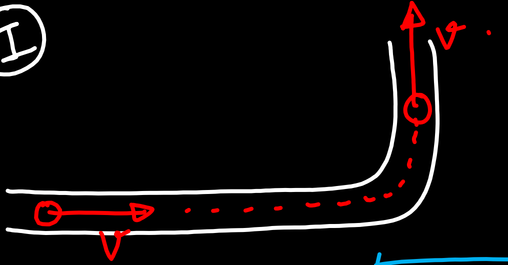
(A) Statement I is incorrect but Statement II is correct

(B) Statement I is correct but Statement II is incorrect.

(C) Both Statement I and Statement II are correct.

(D) Both Statement I and Statement II are incorrect.

I

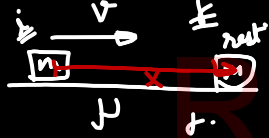


$\vec{v} \to$ changing

$\vec{v} \neq$ const

$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$

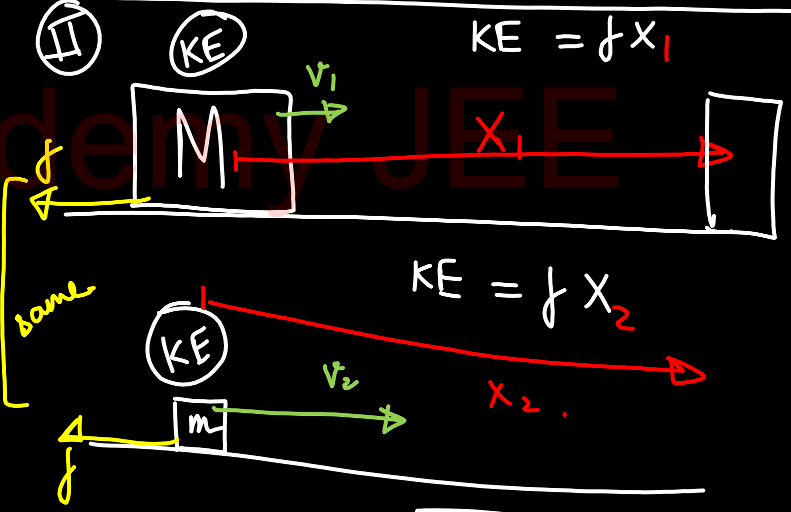
$\vec{a} \neq 0$



$\frac{1}{2}mv^2 = \mu mg X + 0$

$\boxed{X = \frac{KE}{f}}$

II



$KE = f X_1$

$KE = f X_2$

same

$\boxed{X_1 = X_2}$

12

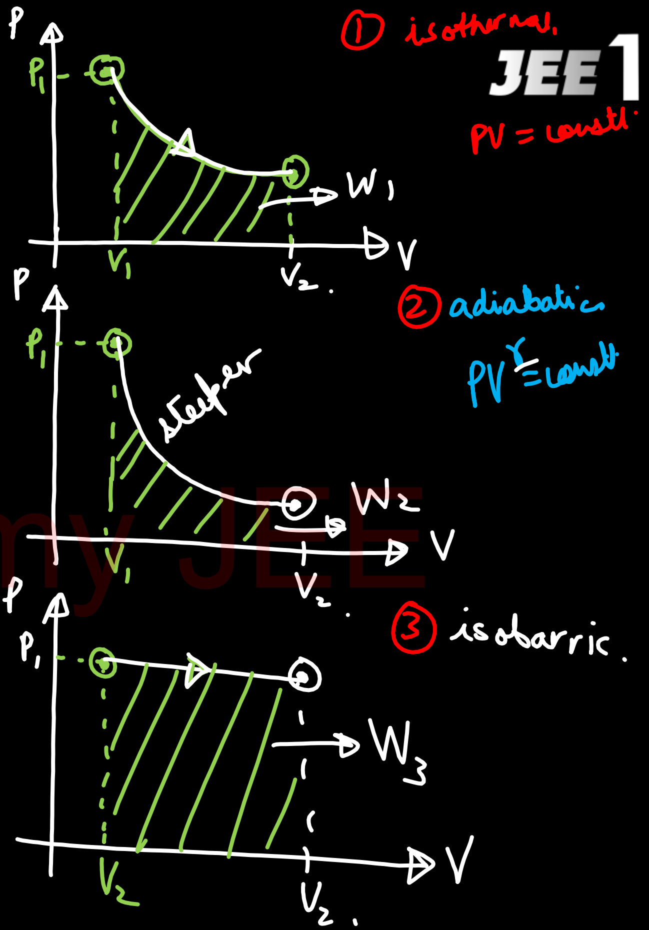
Starting with $\boxed{\text{same initial conditions,}}$ an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1, if the process is purely isothermal. W_2, if the process is purely adiabatic and W_3, if the process is purely isobaric. Then, choose the correct option

PVT

(A) $W_1 < W_2 < W_3$ (B) $W_2 < W_3 < W_1$

(C) $W_3 < W_1 < W_2$ (D) $W_2 < W_1 < W_3$

$$W_2 < W_1 < W_3$$



13

Three simple harmonic motions of equal amplitude $\boxed{2.25\text{ m}}$ $= A$ and equal time periods in the same direction combine. The phase of the second motion is 60° ahead of the first and the phase of the third motion is 60° ahead of the second. Find the amplitude of the resultant motion (in m).

(A) 9
(B) 9/2
(C) 3
(D) 6

T, ω, ν

$\boxed{\omega = 2\pi\nu = \frac{2\pi}{T}}$

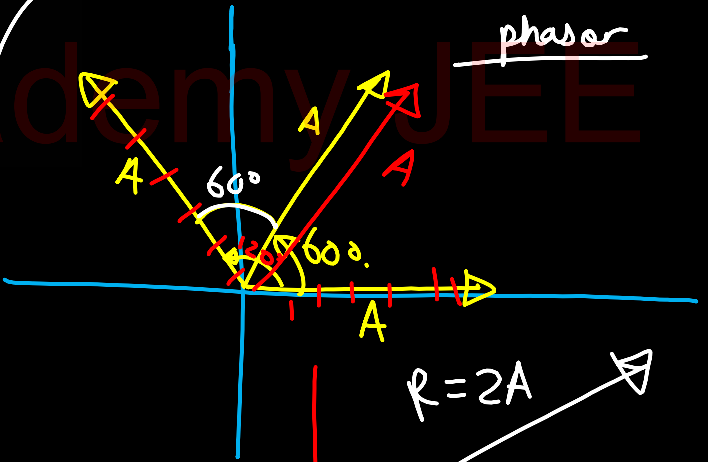
$x_1 = A \sin(\omega t + 0)$

$x_2 = A \sin(\omega t + 60^\circ)$

$x_3 = A \sin(\omega t + 120^\circ)$

$x = x_1 + x_2 + x_3 = $ ______

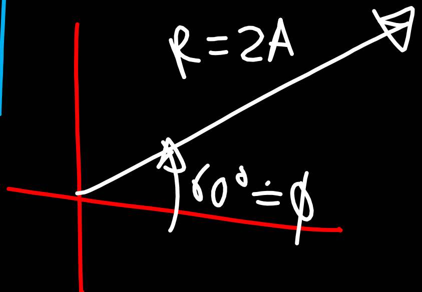
phasor



$x = \boxed{R} \sin(\omega t + \phi)$

$x = \boxed{2A} \sin(\omega t + 60^\circ)$

Ans $= 2A = 4.5$



14

JEE 1

Two vibrating strings of the same material but of lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and other with frequency f_2. Find the ration $\frac{f_1}{f_2}$?

(A) 1: 2
(B) 2: 1
(C) 1: 4
(D) 1: 1

$v = \sqrt{\frac{T}{\mu}}$

T, μ

L

$\boxed{f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}}$

μ

ρ S

$\boxed{\mu = \rho S}$

1m

mass $= \mu$ S

mass $= \rho \times$ vol

$\mu = \rho (S)$

$T, \mu_1 = \rho S_1 = \rho \pi (2r)^2$

$L_1 = L$

$f_1 = \frac{1}{2L_1}\sqrt{\frac{T}{\mu_1}}$ — ①

$T, \mu_2 = \rho S_1 = \rho \pi r^2$

$L_2 = 2L$

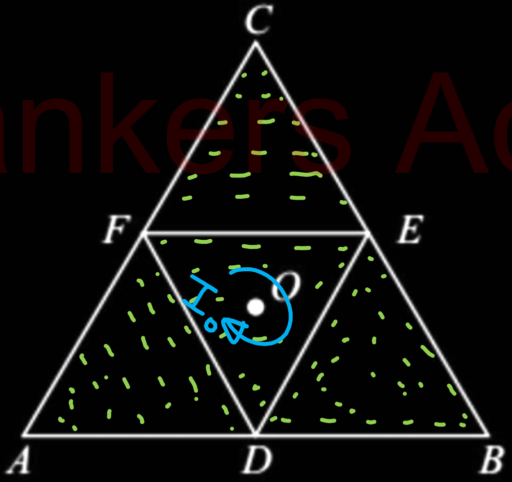
$f_2 = \frac{1}{2L_2}\sqrt{\frac{T}{\mu_2}}$ — ②

① ÷ ②

$$\frac{f_1}{f_2} = \frac{\frac{1}{\cancel{2L}}\sqrt{\frac{\cancel{T}}{\cancel{\rho}\cancel{\pi}(4r^2)}}}{\frac{1}{\cancel{2}\,4L}\sqrt{\frac{\cancel{T}}{\cancel{\rho}\cancel{\pi}\cancel{r^2}}}} = \frac{1}{1}$$

15

Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is $\boxed{I_0}$ as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is

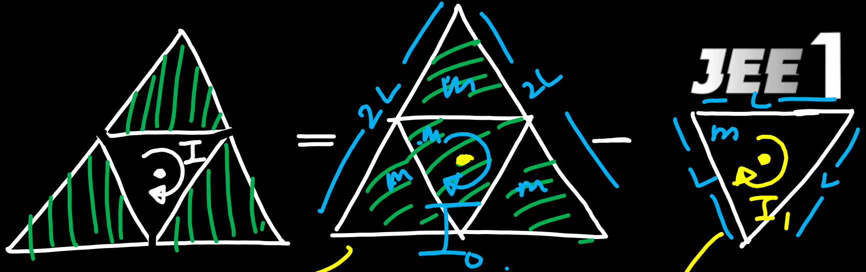


(A) $\frac{7I_0}{8}$ (B) $\frac{31I_0}{32}$

(C) $\frac{3I_0}{4}$ (D) $\boxed{\frac{15I_0}{16}}$

$I = k m l^2$

$\boxed{I=?}$



$I = I_0 - I_1$

$I_0 = k(4m)(2L)^2$

$\frac{I_0}{16} = kmL^2$

$I_1 = k m L^2$

I_0

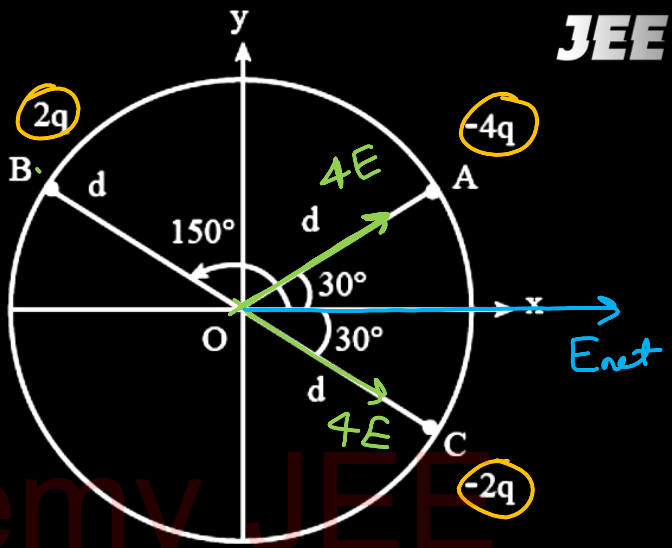
$I = I_0 - \frac{I_0}{16} = \frac{15 I_0}{16}$

16

Three charged particles A, B and C with charges $-4q, 2q$ and $-2q$ are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is.

(A) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (B) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(C) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (D) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$



$$E = \frac{kq}{d^2}$$

$$E_{net} = 2(4E)\cos 30^\circ$$

$$= 4\sqrt{3}E$$

$$= 4\sqrt{3}\frac{kq}{d^2} = \frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$$

17

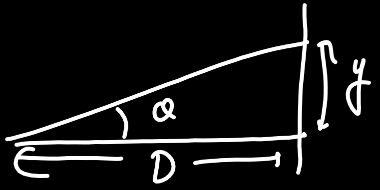
In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that D ≫ d ≫ λ. If the fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is

(A) $\frac{\beta}{2}$

(B) $\frac{\beta}{4}$

(C) $\frac{\beta}{3}$

(D) $\frac{\beta}{6}$



$$I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

$$2I_0 = 4I_0\cos^2\frac{\phi}{2}$$

$$\cos^2\frac{\phi}{2} = \frac{1}{2}$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{2}$$

$$\boxed{\frac{\Delta x}{\lambda} = \frac{\Delta\phi}{2\pi}}$$

$$\Delta x = \frac{\lambda}{4}$$

$$\boxed{\Delta x = d\sin\alpha}$$

$$d\sin\theta = \frac{\lambda}{4}$$

$$d\left(\frac{y}{D}\right) = \frac{\lambda}{4}$$

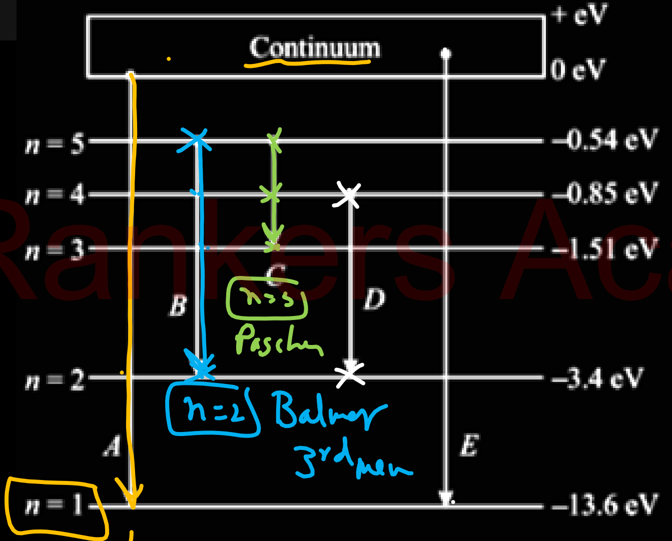
$$y = \frac{\lambda D}{4d}$$

$$= \frac{\beta}{4}$$

JEE 1

18

In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent



$\frac{1}{\lambda_{min}} \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

Series limit

(A) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

(B) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

(C) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.

(D) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

19

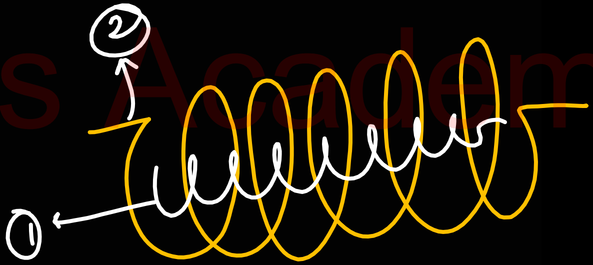
There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The ratio of mutual inductance to the self inductance of the inner-coil is

(A) $\frac{n_2}{n_1}$

(B) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$

(C) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$

(D) $\frac{n_1}{n_2}$



$$m = \frac{\phi}{i} = \frac{B_2 A_1}{i}$$

$$= \frac{(\mu_0 n_2 i)(\pi r_1^2)(n_1 l)}{i} \Rightarrow m = \mu_0 n_1 n_2 \pi r_1^2 l \quad \text{— (1)}$$

$$L = \mu_0 n_1^2 \pi r_1^2 l \quad \text{— (2)}$$

(1) ÷ (2)

$$\frac{m}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

20

JEE 1

Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

(A) In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

(B) The energy in electromagnetic wave is divided equally between electric and magnetic fields. $U_B = U_E$

(C) Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

(D) The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

(E) The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

$$\frac{B_0}{E_0} = \frac{1}{c}$$

Choose the most appropriate answer from the options given below.

(A) (D) only

(B) (B) and (D) only

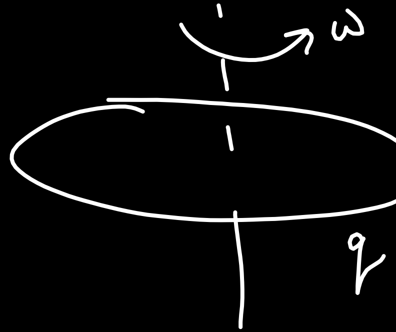
(C) (B), (C) and (E) only

(D) (A), (B) and (E) only

21

A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rads^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to

_____ $\times 10^{-5}$ C ($\mu_0 = 4\pi \times 10^{-7}$ n/A^2)



$i = \frac{q}{T} = \frac{q}{\left(\frac{2\pi}{\omega}\right)}$

$i = \frac{q\omega}{2\pi}$ — (1)

$B = \frac{\mu_0 i}{2R}$

$B = \frac{\mu_0 q \omega}{2R(2\pi)}$

$q = \frac{4\pi B R}{\mu_0 \omega}$

$= \frac{\cancel{4\pi} \times 3.8 \times 10^{-9} \times 0.1\text{m}}{\cancel{4\pi} \times 10^{-7} \times 40\pi}$

$= \frac{38}{4\pi} \times 10^{-5}$

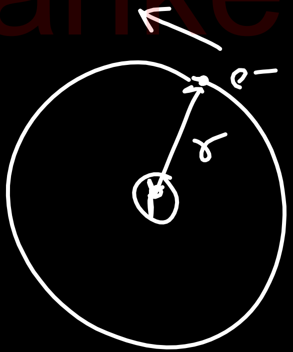
$= \frac{38}{12.} \times 10^{-5} = \boxed{3} \times 10^{-5}$

22

Experimentally it is found that 12.8eV energy is required to separate a hydrogen atom into a proton and an electron. So the orbital radius of the electron in a hydrogen atom is $\frac{9}{x} \times 10^{-10}$ m.

The value of the x is:

$\left(1\text{eV} = 1.6 \times 10^{-19}\text{ J}, \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm/C}^2\right.$ and electronic charge $= 1.6 \times 10^{-19}\text{C})$



$$E = \frac{k(e \times -e)}{2r}$$

Total Energy

$$B.E. = -E = \frac{ke^2}{2r}$$

$$r = \frac{ke^2}{2(B.E.)}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^{\cancel{2}}}{2 \times 12.8\,\cancel{e}V}$$

$$= \frac{9 \times \cancel{1.6}}{2 \times \cancel{12.8}} \times 10^{-10}$$

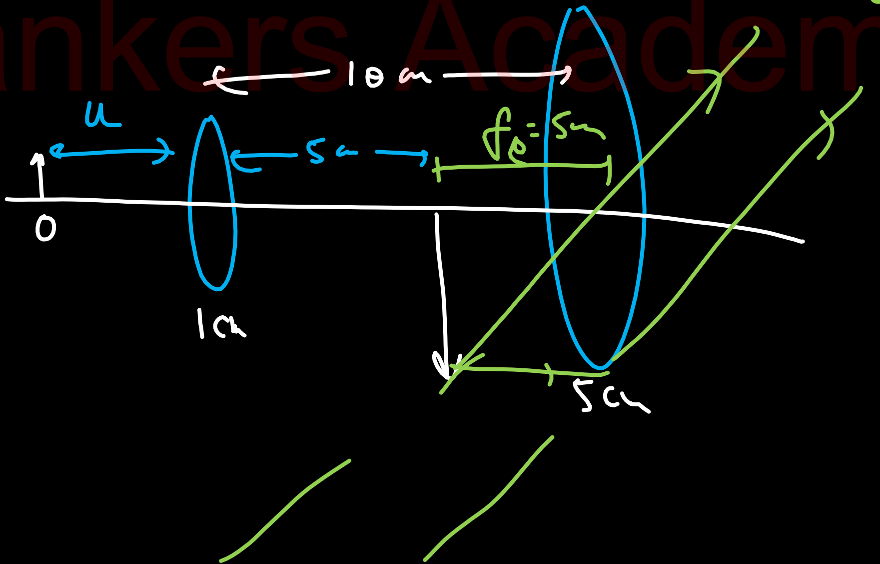
8

$$= \frac{9}{\boxed{16}} \times 10^{-10}$$

23

A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is____

Normal Adjustment



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o}$$

$$\frac{1}{+5\,\text{cm}} - \frac{1}{u} = \frac{1}{+1\,\text{cm}}$$

$$\frac{1}{u} = \frac{1}{5} - 1$$

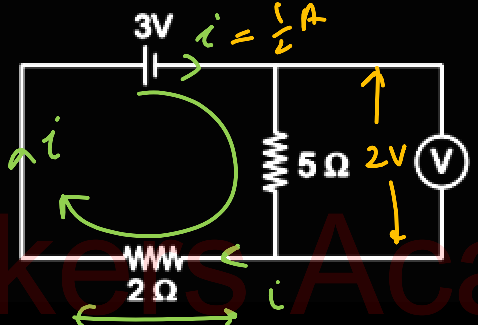
$$u = -\frac{5}{4}$$

$$u = -\frac{50}{40}$$

$$\text{distance} = |u| = \frac{\boxed{50}}{40}$$

24

As shown in the figure the voltmeter reads 2V across 5 Ω resistor. The resistance of the voltmeter is ______ Ω.



$i = \frac{1}{2}A$

$(3-2) = 1V$

$2V$

$i = \frac{V}{R} = \frac{1V}{2\Omega} = \frac{1}{2}A$

$V = i R_{eq}$

$2V = \frac{1}{2}\left(\frac{5V}{5+V}\right)$

$20 + 5V = 5V$

$\boxed{V = 20\,\Omega}$

25

If the charge on a capacitor is increased by 2C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C) ____

$Q \to Q + 2$

JEE 1

$U \to 1.44\,U$

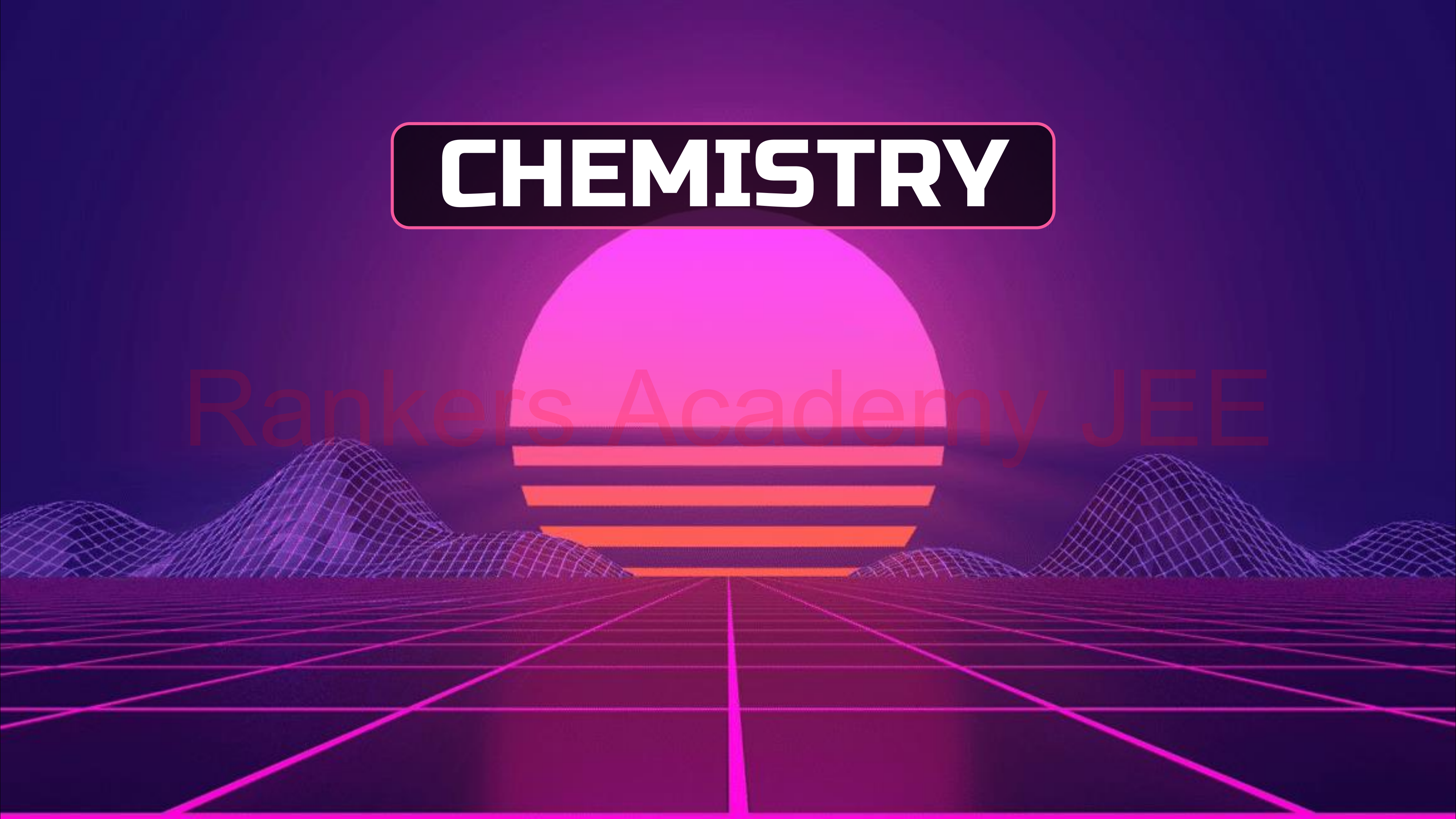
$$\frac{(Q+2)^2}{2C} = 1.44\left(\frac{Q^2}{2C}\right)$$

$$(Q+2) = 1.2\,Q$$

$$2 = 0.2\,Q$$

$$\boxed{Q = 10\ C}$$

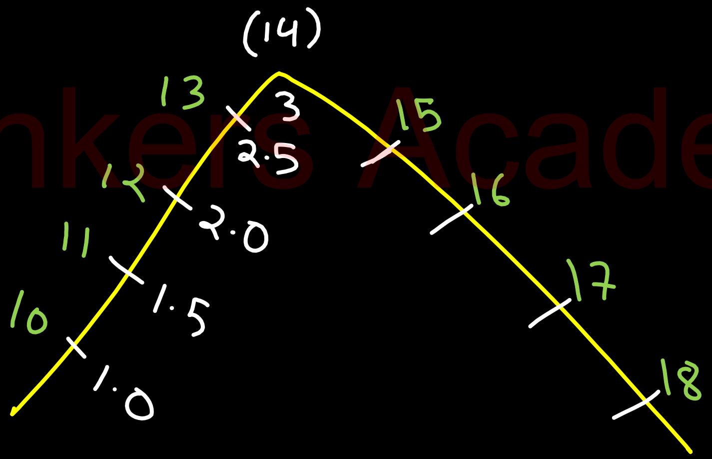
CHEMISTRY



The diatomic molecule(s) that has (have) two π-bonds is (are) (according to MO theory):

(A) B_2 = 10 (1.0) (B) C_2 = 12 (2.0)

(C) N_2 = 14 (3.0) (D) O_2 = 16 (2.0)



2

JEE 1

In a closed flask at 400 K solid $(NH_4)_2 S$ was taken. At equilibrium a constant pressure of 3 atm was found in the flask. What is the K_P of the reaction

$$(NH_4)_2 S(s) \rightleftharpoons 2NH_3(g) + H_2 S(g)$$

at 400 K is

(A) 1 (B) 2

(C) 3 (D) 4

So $P = 1$ atm

$K_P = (2P)^2 (P)$

$= 4P^3$

$= 4(1)^3 = 4$

$$(NH_4)_2 S(s) \rightleftharpoons \underset{2P}{2NH_3(g)} + \underset{P}{H_2S(g)}$$

Pressure at Eq^m = $P + 2P = 3P = 3$ atm

3

de-Broglie wavelength of an electron in the first Bohr orbit of H-atom is

(A) 1.7Å (B) 3.3Å

(C) 6.8Å (D) 5.5Å

$$n\lambda = 2\pi r$$

$$\lambda = 2\pi r$$

$$= 2 \times 3.4 \times 0.529 \frac{(1)^2}{1} \text{Å}$$

$$= 3.3\text{Å}$$

4

JEE 1

Which set of Quantum numbers is consistent with the theory?

(A) $n = 2, \ell = 0, m = -1, s = +\frac{1}{2}$

(B) $n = 2, \ell = 0, m = 0, s = -\frac{1}{2}$

(C) $n = 1, \ell = -1, m = 0, s = -\frac{1}{2}$

(D) $n = 3, \ell = 3, m = -3, s = +\frac{1}{2}$

$m = -l$ to $+l$

for s subshell $l = 0$

5

JEE 1

If one mole of a ideal gas $\left(C_P = \frac{5R}{2}\right)$ is expanded isothermally at 300 K until its volume is tripled and $\underbrace{P_{ext} = 0}$ then which one of the following option is correct?

(A) $W \neq 0 \; \Delta S = \text{infinity}$

(B) $W = 0 \; \Delta S = R\ell n3$

(C) $\Delta E = 0 \; \Delta S = 0$

(D) $\Delta E = 0 \; \Delta S = \text{infinity}$

$\Delta S = nR \ln \frac{V_2}{V_1}$

$\Delta S = R \ln 3$

$\Delta U = q + W$

$W = -P\Delta V$

When $P_{ext} = 0$, $W = 0$

6

JEE 1

Which of the following ions gives blood red colour with ammonium thiocyanate?

(A) Fe^{+3} (B) Cu^{2+}

(C) Cd^{2+} (D) Sn^{2+}

7

Which of the following is not a group IV basic radical?

(A) Co^{2+}

(B) Ni^{2+}

(C) Cu^{2+}

(D) Zn^{2+}

8

Which of the following is an inner orbital complex?

(A) $[Ni(H_2O)_6]^{2+}$ (B) $[Co(NH_3)_6]^{+3}$

(C) $[CoF_6]^{3-}$ (D) $[Mn(H_2O)_6]^{2+}$

9

The correct order of ionic radii is

(A) $La^{3+} < Ce^{3+} < Pm^{3+} < Yb^{3+}$

(B) $La^{3+} < Pm^{3+} < Ce^{3+} < Yb^{3+}$

(C) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$

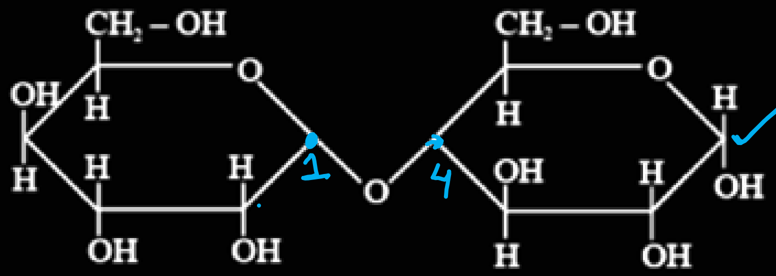
(D) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$

Due to Lanthenoid contraction

10

JEE 1

Incorrect statement about given carbohydrate is



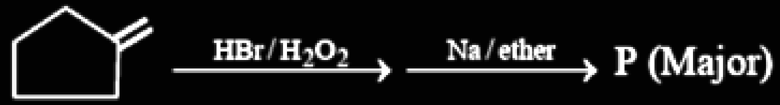
Lactose

(A) Above compound is a reducing sugar

(B) Above compound undergo mutarotation

(C) Above compound is a non-reducing sugar

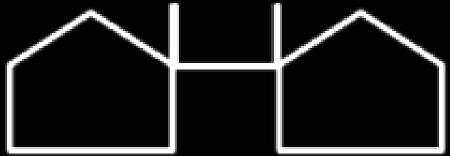
(D) Above compound has a glycosidic linkage

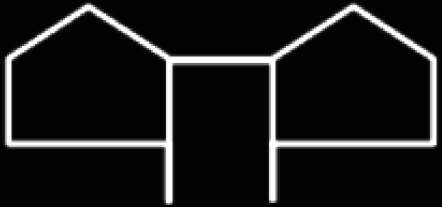


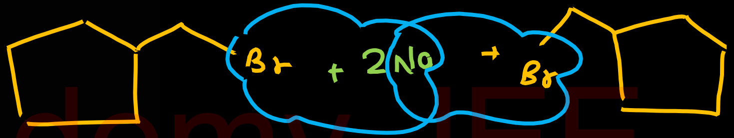
P is

(A) 

(B) 

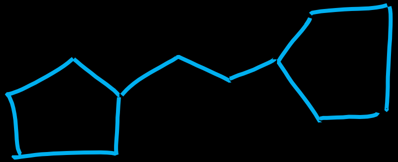
(C) 

(D) 



(AMK)

wurtz reaction



12

Which oxidising agent can not be used to convert 1° alcohol to corresponding carboxylic acid.

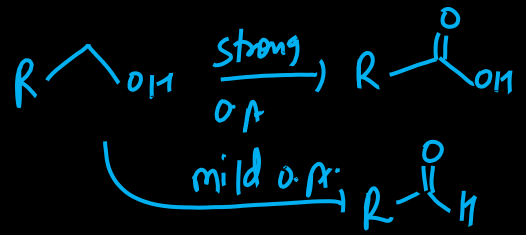
(A) $H^+/KMnO_4$

(B) $H^+/K_2Cr_2O_7$

(C) Jone's reagent CrO_3 / aq. H_2SO_4 / acetone

(D) PCC (Pyridinium chloro chromate)

1° Alcohol ⟶ Carboxylic



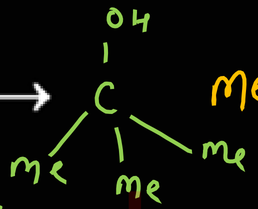
13

Best method to prepare

$$Me-\overset{\overset{\displaystyle O}{\|}}{C}-Me \text{ from } Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl.$$

(A) $Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl + \underset{\text{G.R}}{Me^{\ominus}MgBr} \longrightarrow$

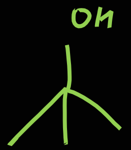
Highly Reactive



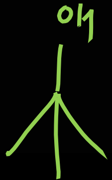
$$Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \xrightarrow[\text{or } Me_2CuLi]{(CH_3)_2Cd} Me-\overset{\overset{\displaystyle O}{\|}}{C}-Me$$

(B) $Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl + Me_2CuLi \longrightarrow$

(C) $Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl + MeLi \longrightarrow$

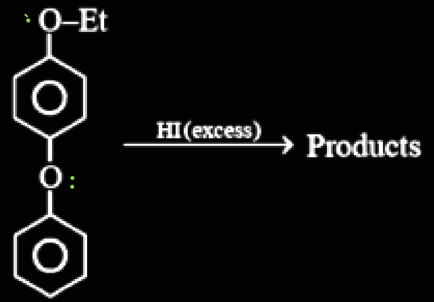


(D) $Me-\overset{\overset{\displaystyle O}{\|}}{C}-Cl + \underset{\text{(excess)}}{MeMgBr} \xrightarrow{H_3O^{\oplus}}$



14

JEE 1

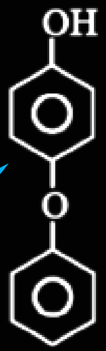


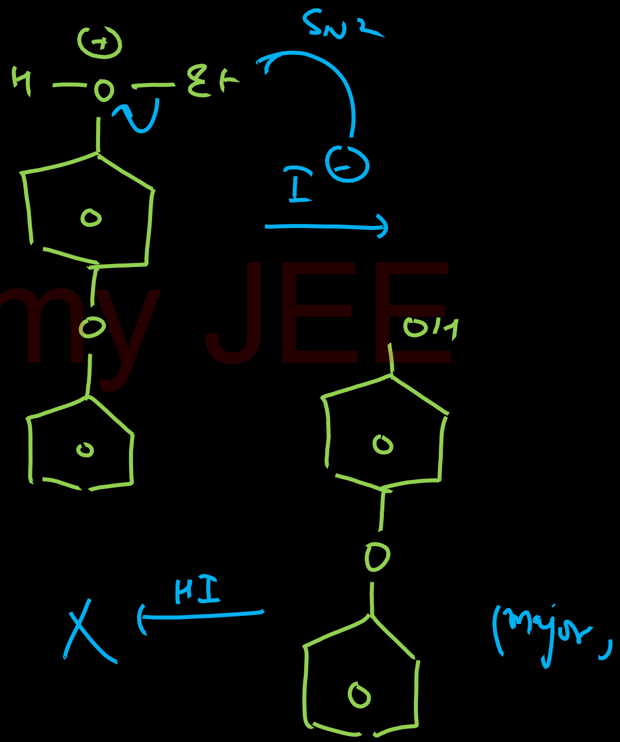
Which product forms during reaction as major product.

(A)


(B)


(C) EtOH

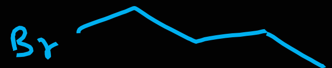
(D)




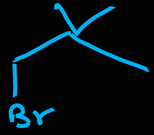
15

Arrange the compounds of given set in order of reactivity towards S_N2 displacement:

(I) 1-bromobutane

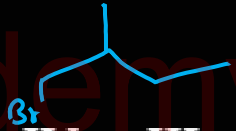


(II) 1-bromo-2, 2-dimethyl propane



(III) 1-bromo-2-methyl butane

(IV) 1-bromo-3-methyl butane

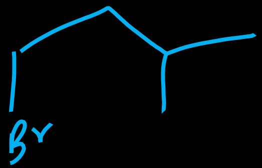


(A) I > II > III > IV

(B) I > IV > III > II

(C) I > III > IV > II

(D) IV > III > II > I



16

What is $\Delta G°$ for the following reaction?

$$Cu^{+2}(aq) + 2Ag(s) \longrightarrow Cu(s) + 2Ag^{+}$$

$E°_{Cu^{+2}/Cu} = 0.34\ V$, $E°_{Ag^{+}/Ag} = 0.8\ V$

(A) −44.5 kJ (B) 44.5 kJ

(C) −89 kJ (D) 89 kJ

$$\Delta G° = -nFE°_{cell}$$
$$= -2 \times 96500\, E°_{cell}$$
$$= -2 \times 96500\,(-0.46)\ \text{Joule}$$
$$= 0.46 \times 2 \times 96.5\ kJ$$
$$= \underline{89\ kJ}$$

$$E°_{cell} = E°_{Cu^{+2}/Cu} - E°_{Ag^{+}/Ag}$$
$$= 0.34 - 0.80$$
$$= \underline{-0.46\ V}$$

17

JEE1

The E_{cell} of the given cell is

$$(Pt)\ H_2 \left| \underset{1M}{HCl(aq)} \parallel \underset{10^{-4}M}{NaOH} \right| H_2(Pt)$$

(A) 0.59 V

(B) −0.59 V

(C) −0.236 V

(D) 0.236 V

$[H^+][OH^-] = 10^{-14}$

$[H^+] = \frac{10^{-14}}{10^{-4}} = 10^{-10}\,M$

$H_2 \longrightarrow \underset{1m}{2H^+} + 2e^-$

$2e^- + \underset{10^{-10}m}{2H^+} \longrightarrow H_2(g)$

$\underset{(10^{-10}m)}{2H^+} \longrightarrow \underset{1m}{2H^+} \quad E^\circ = 0$

$E_{cell} = \cancel{E^\circ_{cell}} - \frac{0.059}{2} \log \frac{(1)^2}{(10^{-10})^2}$

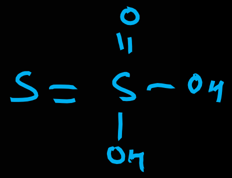
$E_{cell} = -0.59\,V$

18

JEE 1

Which of the following is not correctly matched?

(A) Thiosulphuric acid - $H_2 S_2O_3$

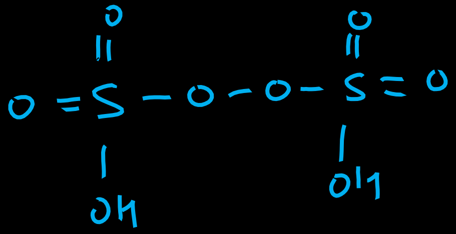


(B) Sulphurous acid - H_2SO_3

(C) Caro's acid - H_2SO_5

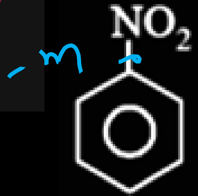
(D) Marshall's acid – $H_2 S_2O_6$

$H_2S_2O_8$

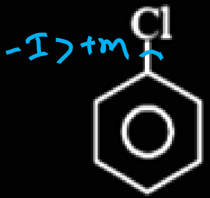


19

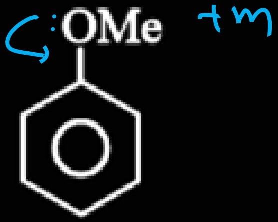
Arrange the following in the order of reactivity towards an electrophilic attack.



(I)



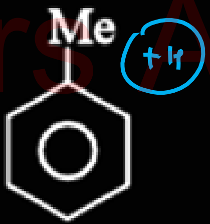
(II)



(III)

(IV)

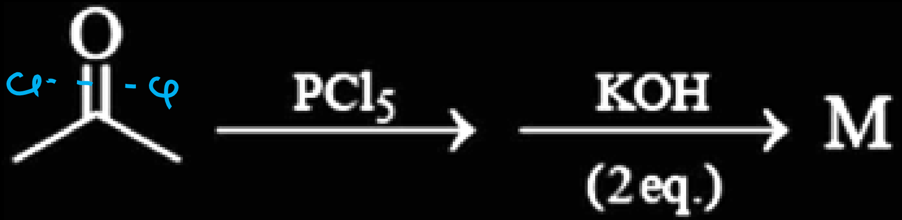
(IV)



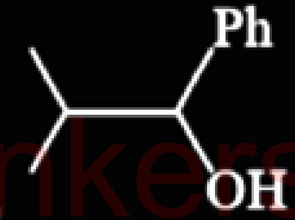
(V)

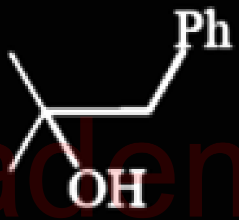
(A) V > IV > III > II > I
(B) III > V > IV > II > I
(C) III > IV > V > II > I
(D) V > IV > III > I > II

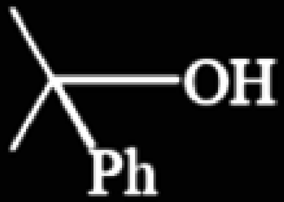
20

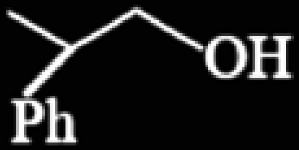


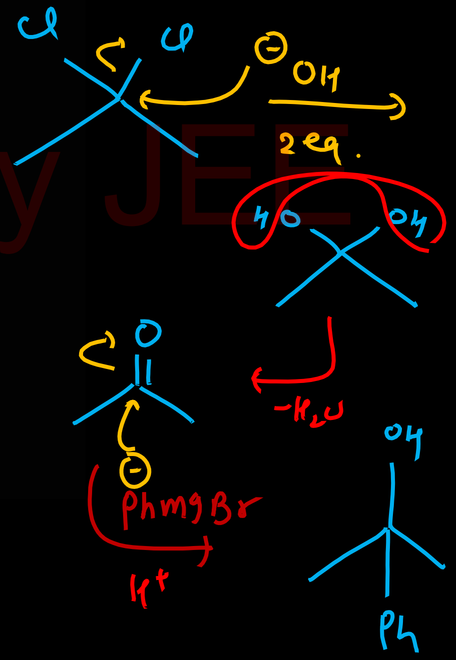
M on reaction with PhMgBr, followed by acidification gives

(A) 

(B) 

(C) 

(D) 



21

The value of abnormal and normal molecular mass of silver nitrate are 90.00 and 170.0 respectively. The percentage of dissociation of silver nitrate is______ (Rounded off to nearest integer)

$\boxed{89 \text{ Ans}}$

$AgNO_3 \longrightarrow Ag^+ + NO_3^-$; $n = 2$

$i = \frac{170}{90} = 1.88$

$\alpha = \frac{i-1}{n-1}$; $\alpha = i - 1$

$\alpha = 1.88 - 1 = 0.888$

% diss $= \alpha \times 100$

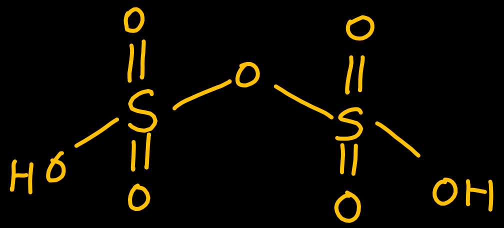
$= 0.888 \times 100$

$= 88.88 \approx 89$

22

The sum of $S = O$ and $S - S$ linkage in $H_2 S_2 O_7$ is ______

4 Ans



no of $S=O$ bonds $= 4$

$S-S = 0$

$\overline{\quad 4 \quad}$

23

How many of the following are paramagnetic in nature?

$[Fe(CN)_6]^{4-}$, $[Fe(CN)_6]^{3-}$, $[Cu(NH_3)_4]^{2+}$, $Ni(CO)_4$, $[Cr(H_2O)_6]^{3+}$, $[Ni(NH_3)_6]^{2+}$, $Fe(CO)_5$, $[Fe(H_2O)_6]^{3+}$, $[Co(en)_3]^{3+}$

di, di, Di, di

5

$Cr^{+3}: 3d^3$ ↑ | ↑ | ↑ | |

$Ni^{+2}: 3d^8$ ↑↓ | ↑↓ | ↑↓ | ↑ | ↑

$Co^{+3}: 3d^6$ ↑↓ | ↑↓ | ↑↓ | xx | xx | xx | xx | xx | xx

Fe^{+2} $3d^6$ ↑↓ | ↑↓ | ↑↓ | xx | xx

Fe^{+3} $3d^5$ ↑↓ | ↑↓ | ↑ | xx | xx

Cu^{+2} $3d^9$

$Ni^0 : 3d^8 4s^2$ ↑↓ | ↑↓ | ↑↓ | ↑↓ | ↑↓ | xx | xx | xx | xx

sp3

24

320ml of $\frac{M}{10}$ $KMnO_4$ solution in acidic medium when titrated required 112ml of H_2O_2 solution. The volume strength of H_2O_2 solution is ______ vol.

JEE 1

8 Ans

Applying law of equivalence,

meq $KMnO_4$ = meq H_2O_2

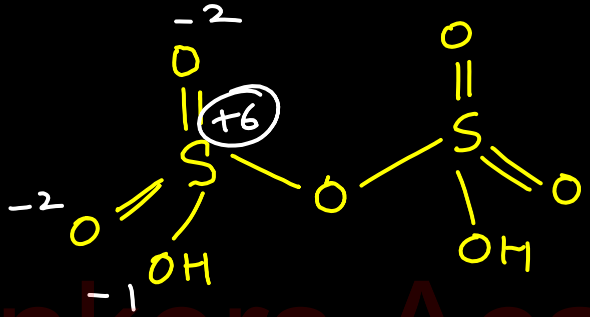
$$320 \times \frac{1}{10} \times 5 = 112 \times M \times 2$$

$$M = 0.714$$

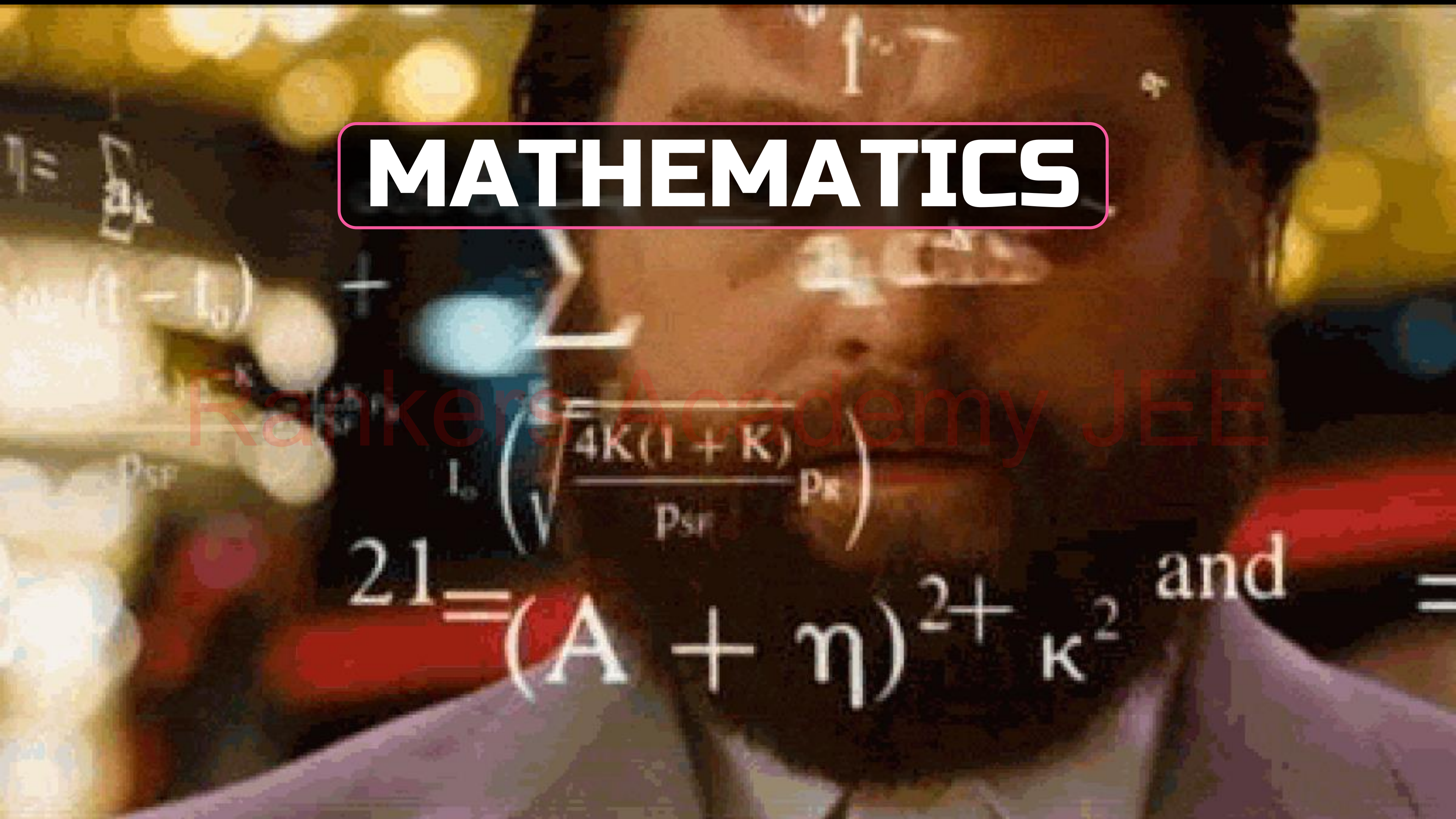
Volume strength $= 11.2 \times 0.714 = 7.99 \approx 8$

25

The oxidation state of sulphur atom in $H_2S_2O_7$ is <u>6</u>



MATHEMATICS



Let $A = \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}$ and P be any orthogonal matrix such that $Q = PAP^T$ and let $R = [r_{ij}]_{2\times 2} = P^T Q^8 P$ Then $r_{11} =$

(A) 81 (B) $81\sqrt{3}$

(C) 243 (D) $243\sqrt{3}$

$PP^T = P^T P = I$

$R = P^T Q^8 P$

$= P^T (PAP^T)(PAP^T) \ldots (PAP^T) P$

$R = A^8$

$A^2 = A \cdot A = \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -2\sqrt{3}-2 \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2 A = \begin{bmatrix} 3 & -2\sqrt{3}-2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3} & -2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3\sqrt{3} & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

$$A^n = \begin{bmatrix} 3^{n/2} & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

$$R = A^8 = \begin{bmatrix} 3^4 & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

$r_{11} = 81$ ✓

2

JEE 1

Let $\bar{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ and $\bar{b} = 2\hat{i} - \hat{j} + 5\hat{k}$ be two vectors such that $\bar{a} \times \bar{b} = 3\hat{i} - 4\hat{j} - 2\hat{k}$ then the projection of $\bar{b} + 3\bar{a}$ on $\bar{b} + \bar{a}$ is

(A) $\frac{14\sqrt{5}}{3}$ (B) $\frac{17}{\sqrt{5}}$

(C) $\frac{24\sqrt{5}}{7}$ (D) $\frac{37}{\sqrt{5}}$ ✓

$\vec{a} \times \vec{b} = 3\hat{i} - 4\hat{j} - 2\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & \beta \\ 2 & -1 & 5 \end{vmatrix}$

$3\hat{i} - 4\hat{j} - 2\hat{k} = (10+\beta)\hat{i} - (5\alpha - 2\beta)\hat{j} + (-\alpha - 4)\hat{k}$

$\beta = -7$ $\alpha = -2$

$\vec{a} = -2\hat{i} + 2\hat{j} - 7\hat{k}$ ✓

$3\vec{a} = -6\hat{i} + 6\hat{j} - 21\hat{k}$

Ans : $\frac{(\vec{b} + 3\vec{a}) \cdot (\vec{b} + \vec{a})}{|\vec{b} + \vec{a}|}$

$\vec{b} + 3\vec{a} = -4\hat{i} + 5\hat{j} - 16\hat{k}$

$\vec{b} + \vec{a} = \hat{j} - 2\hat{k}$

Ans : $\frac{5 + 32}{\sqrt{5}}$

A, B, C are three finite sets and U is Universal set such that $A \cup B \cup C = U$ Let $n(A) = 17$; $n(B) = 13$; $n(C) = 15$; $n(U) = 30$; $n(A \cap B) = x$, $n(B \cap C) = y$; and $n(A \cap B \cap C) = z$; If $x + y = 13$; $y + z = 7$ and $x + z = 12$ then $n(A \cup B \cap C')$ is

(A) 12 (B) 13

(C) 14 (D) 15

$x + y = 13$

$y + z = 7$

$x + z = 12$

$x = 9,\ y = 4,\ z = 3$

$13 + 8 - \alpha + \alpha + 11 - \alpha = 30$

$32 - \alpha = 30$

$\alpha = 2$

Ans : $\underbrace{8 - \alpha}_{6} + \underbrace{6 + 3}_{9}$

15 ✓

JEE 1

4

The number of solutions of the equation

$\sin^{-1}\left(\frac{x^2}{x^2+1}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \pi$ is

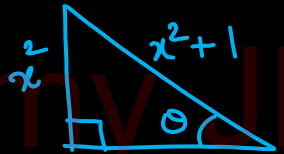
(A) 1 (B) 2

(C) 0 (D) 3

$\boxed{\cos^{-1}(-x) = \pi - \cos^{-1}x}$

$\sin^{-1}\left(\frac{x^2}{x^2+1}\right) = \pi - \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\underbrace{\sin^{-1}\left(\underbrace{\frac{x^2}{x^2+1}}_{\in[0,1)}\right)}_{\in[0,\pi/2)} = \underbrace{\cos^{-1}\left(\underbrace{\frac{1-x^2}{x^2+1}}_{\in[0,1]}\right)}_{\in[0,\pi/2]}$



$\sqrt{(x^2+1)^2-(x^2)^2}$

$= \sqrt{2x^2+1}$

$\theta = \sin^{-1}\frac{x^2}{x^2+1}$

$= \cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$

4

JEE 1

$$\cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right) = \cos^{-1}\left(\frac{1-x^2}{x^2+1}\right)$$

$$\Rightarrow \frac{\sqrt{2x^2+1}}{\cancel{x^2+1}} = \frac{1-x^2}{\cancel{x^2+1}}$$

$$\Rightarrow 2x^2+\cancel{1} = \cancel{1}+x^4-2x^2$$

$$\Rightarrow x^4 = 4x^2$$

$$\Rightarrow x = 0, \pm 2$$

$x=0$: $\sin^{-1}(0)+\cos^{-1}(-1) = 0+\pi = \pi$ ✓

$x=2$: $\sin^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

$53^\circ + 53^\circ \neq \pi$ ✗

$x=-2$: $\sin^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

$53^\circ + 53^\circ \neq \pi$ ✗

5

JEE 1

Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$, where $r, s \in \{1,2,3\}$ and $\underbrace{z = \frac{-1+i\sqrt{3}}{2}}$. $z=\omega$ If $P^2 = -I_2$, then total number of ordered pairs (r,s) is

(A) 0 (B) 1

(C) 2 (D) 3

$\omega^3 = 1$ ✓

$1+\omega+\omega^2 = 0$ ✓

$1+\omega^r+\omega^{2r} = 0$; $r \neq 3k$

$= 3$; $r = 3k$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & (\omega)^r \end{bmatrix} \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} \underline{(-\omega)^{2r} + \omega^{4s}} & \underbrace{(\omega^r + (-\omega)^r)}\omega^{2s} \\ (\omega^r + (-\omega)^r)\omega^{2s} & \omega^{4s} + \omega^{2r} \end{bmatrix} = \begin{bmatrix} \underline{-1} & \underline{0} \\ 0 & -1 \end{bmatrix}$$

$-1 = (-\omega)^{2r} + (\omega)^{4s}$

$0 = \underbrace{(\omega^r + (-\omega)^r)}\underbrace{\omega^{2s}}$

$\therefore \omega^r + (-\omega)^r = 0$

$r = 1, 3$

5

JEE 1

$r=1 \longrightarrow \quad -1 = (-\omega)^2 + \omega^{4s}$

$-1 = \omega^2 + \omega^{4s}$

$= \omega^2 + (\omega^4)^s$

$-1 = \omega^2 + \omega^s$

$(1,1)$

$s=1 ✓ \longrightarrow \quad \omega^2+\omega^1=-1 ✓$

$s=2 \longrightarrow \quad \omega^2+\omega^2=-1-i\sqrt{3}$ X

$s=3 \longrightarrow \quad \omega^2+\omega^3=\omega^2+1=-\omega$ X

$r=3 \longrightarrow \quad -1 = (-\omega)^6 + \omega^{4s}$

$-1 = 1 + \omega^{4s}$

$\omega^{4s} = -2$

$(\omega^4)^s = -2$

$\omega^s = -2$ X

6

Let $f: R \to R$ be a continuous onto function satisfying $f(x) + f(-x) = 0, \forall x \in R$. If $f(-3) = 2$ and $f(5) = 4$ in $[-5,5]$, then the equation $f(x) = 0$ has:

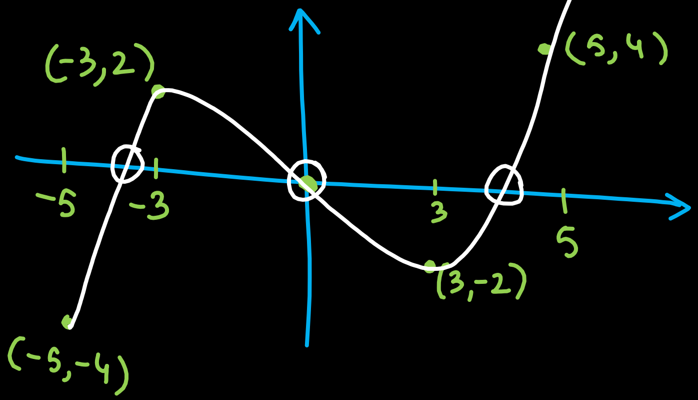
(A) exactly three real roots
(B) exactly two real roots
(C) at least five real roots
(D) at least three real roots

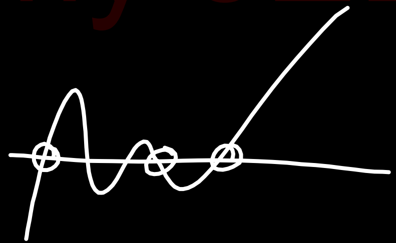
= range

$f(-x) = -f(x)$

odd f^n ✓

$f(0) = 0$ ✓





7

JEE 1

The function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as

$f(x) = \frac{x}{1+x^2}$ is

(~~A~~) Invertible

(~~B~~) Injective but not surjective

(C) Surjective but not injective

(D) neither injective nor surjective

$f'(x) = \frac{(1+x^2) - 2x^2}{(1+x^2)^2}$

$= \frac{1-x^2}{(1+x^2)^2}$

f' $-$ $+$ $-$; -1, 1

non-mono $\Rightarrow$ M-1

$y = \frac{x}{1+x^2}$

Case 1 quad ; $y \neq 0$

$\Rightarrow x^2y - x + y = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$y \neq 0 \longrightarrow 1 - 4y^2 \geq 0$

$y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Case 2 :

$y = 0 \longrightarrow x = 0$

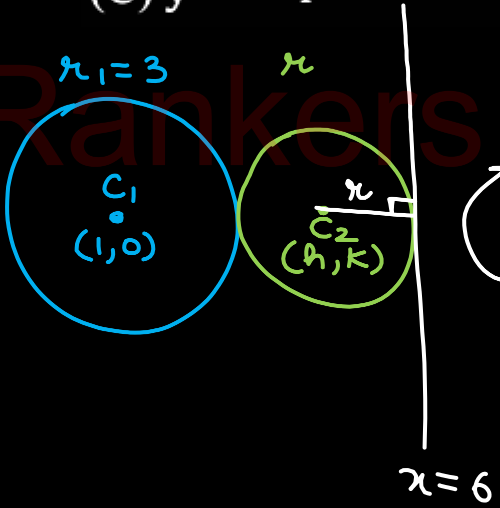
Range $\in \left[-\frac{1}{2}, \frac{1}{2}\right]$

The locus of the centre of the circle which touches the circle $(x-1)^2+y^2=9$ and the line $x=6$ is a curve whose directrix is

(A) $x = 1$ (B) $y = 9$

(C) $y = -4$ (D) $x = 9$

$c_1c_2 = r_1 + r_2$



$$\sqrt{(h-1)^2+k^2} = 3 + r \quad \text{—— (1)}$$

$$|6-h| = r$$

$h \in [1,6]$

$$\sqrt{(h-1)^2+k^2} = 3+|6-h|$$

$$= 3+6-h$$

$$\Rightarrow (h-1)^2+k^2 = 81+h^2-18h$$

$$\Rightarrow \cancel{h^2}-2h+1+k^2 = 81+\cancel{h^2}-18h$$

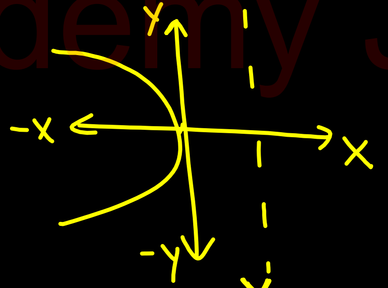
$$\Rightarrow k^2 = -16h+80$$

$$\Rightarrow k^2 = -16(h-5)$$

$$\Rightarrow y^2 = -16(x-5)$$

Let $x-5 = X$; $y = Y$

$$\Rightarrow Y^2 = -16X$$



$a = -4$

$X = -a$

$X = +4$

$x-5 = +4$

$x = 9$

9

JEE 1

If T_n and t_n represents n^{th} terms of 2 different arithmetic progressions respectively such that $\frac{T_n}{t_n} = \frac{n+1}{3n-2}$, then ratio of their sum of first 5 terms is equal to-

(A) $\frac{3}{4}$

(B) $\frac{4}{7}$ ✓

(C) $\frac{5}{8}$

(D) $\frac{6}{11}$

$AP_1 : a_1, d_1$

$AP_2 : a_2, d_2$

$$\frac{a_1 + (n-1)d_1}{a_2 + (n-1)d_2} = \frac{n+1}{3n-2} \quad \text{—— (1)}$$

$$\frac{S_n}{s_n} = \frac{\cancel{\frac{n}{2}}(2a_1 + (n-1)d_1)}{\cancel{\frac{n}{2}}(2a_2 + (n-1)d_2)} = \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} =$$

$$\frac{S_5}{s_5} = \frac{a_1 + 2d_1}{a_2 + 2d_2}$$

$\boxed{n-1 = 2}$ $\therefore n = 3$ ✓

$$\frac{a_1 + 2d_1}{a_2 + 2d_2} = \frac{4}{7}.$$

10

JEE 1

In the expansion of $(x+y+z)^n$ if coefficient of $x^2y^3z^\alpha$ is 210, then α is (where $n \in N$)-

(A) 0 (B) 1

(C) 2 (D) 3

$\frac{n!}{2!\,3!\,\alpha!}$ $x^2y^3z^\alpha$

$2+3+\alpha = n$

$n = \alpha + 5$.

$\Rightarrow 210 = \frac{(\alpha+5)!}{2!\,3!\,\alpha!}$

$\Rightarrow 210 = \frac{\boxed{(\alpha+1)}(\alpha+2)(\alpha+3)(\alpha+4)(\alpha+5)}{2\times 6}$

$\Rightarrow (\alpha+1)(\alpha+2)\ldots(\alpha+5) = 2\times 6\times 210$

$= 2\times 2\times 3\times 2\times 5\times 3\times 7$

$= 3, 4, 5, 6, 7$

$\alpha + 1 = 3$

$\alpha = 2$.

11

JEE 1

$\lim_{x\to 0} \dfrac{\sin\left(\sin\left(\tan\frac{x^2}{2}\right)\right)}{\log\cos 3x}$ is equal to

(A) -1

(B) $-\frac{1}{9}$

(C) $-\frac{2}{9}$

(D) does not exist

$$\lim_{x\to 0} \frac{\frac{x^2}{2}}{\ln(\cos 3x)}$$

M1

LH

$$\lim_{x\to 0} \frac{x}{-3\,\frac{\sin 3x}{\cos 3x}}$$

$$= -\frac{1}{3}\cdot\left(\frac{x}{\frac{\sin 3x}{3x}\cdot 3x}\right)$$

$$= -\frac{1}{3}\cdot\frac{1}{3} = -\frac{1}{9}$$

M2

$$\frac{\frac{x^2}{2}}{\ln(1+(\cos 3x-1))}$$

$$\lim_{x\to 0} \frac{\frac{x^2}{2}}{\cos 3x - 1} = \frac{x^2/2}{-2\sin^2 3x/2}$$

$$= \frac{x^2/2}{-2\left(\frac{3x}{2}\right)^2} = -\frac{1}{9}$$

12

Let $\int \frac{dx}{x^4(x^2+1)} = f(x) + C$ where $f(1) = \frac{\pi}{4} + \frac{2}{3}$

then $f(1) + f(-1)$ is-

(A) 0 (B) $\frac{\pi}{2} - 4$

(C) $\frac{\pi}{2}$ (D) -4

$$\int \frac{dx}{x^4(x^2+1)}$$

M2 $x = \tan\theta$ —— $x = 1 \longrightarrow \theta = \pi/4$

$f(x) = \pi/4 + 2/3$

$\Rightarrow \int \cot^4\theta \, d\theta$

$\Rightarrow \int \cot^2\theta(\operatorname{cosec}^2\theta - 1)d\theta$

$\Rightarrow \int \cot^2\theta \operatorname{cosec}^2\theta \, d\theta - \int \cot^2\theta \, d\theta$

$\Rightarrow -\frac{\cot^3\theta}{3} - \int \operatorname{cosec}^2\theta - 1 \, d\theta$

$f(\theta) = -\frac{\cot^3\theta}{3} + \cot\theta + \theta + C$

$\cancel{\frac{\pi}{4}} + \cancel{\frac{2}{3}} = \cancel{-\frac{1}{3} + \frac{1}{1}} + \cancel{\frac{\pi}{4}} + C$

$C = 0$ ✓

$\therefore f(\theta) = \text{odd } f^n$:

12

JEE 1

M1

$$\int \frac{dx}{x^4(x^2+1)}$$

$$\Rightarrow -\int \frac{(x^4-1)-x^4}{x^4(x^2+1)}\,dx$$

$$= -\int \frac{x^4-1}{x^4(x^2+1)} - \frac{\cancel{x^4}}{\cancel{x^4}(x^2+1)}\,dx$$

$$= -\int \frac{x^2-1}{x^4} - \frac{1}{x^2+1}\,dx$$

$$= -\int \frac{1}{x^2} - \frac{1}{x^4} - \frac{1}{x^2+1}\,dx$$

$$= \int \frac{1}{x^4} + \frac{1}{x^2+1} - \frac{1}{x^2}\,dx$$

$$f(x) \Rightarrow \frac{x^{-3}}{-3} + \tan^{-1}x + \frac{x^{-1}}{+1} + c$$

$x=1$

$$\frac{\pi}{4} + \frac{2}{3} = \frac{-1}{3} + \frac{\pi}{4} + 1 + c \Rightarrow c = 0$$

$f(x) =$ odd f^n : $f(-1) = -f(1)$

13

JEE 1

The slope of the graph of the function $f(x) = \frac{x^5}{20} - \frac{x^4}{12} + 5$ is increasing for x belonging to

(A) $(1, \infty)$ (B) $(0,1)$

(C) $(-\infty, 0)$ (D) $(-\infty, 0) \cup \left(\frac{4}{3}, \infty\right)$

Slope = f'

$f'' = ?$

$f'(x) = \frac{x^4}{4} - \frac{x^3}{3}$

$f''(x) = x^3 - x^2$

$f''(x) = x^2(x-1)$

$-$ $-$ $+$

0 1

POI Min

14

JEE 1

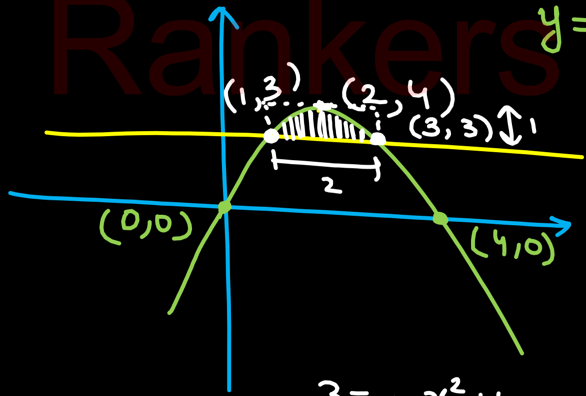
Area bounded by curves $y = -x^2 + 4x$ and $y = \left[\frac{x+12}{4}\right]$ for $x \in [0,4]$ is (where [.] denotes greatest integer function) –

(A) $\frac{4}{3}$ ✓ (B) $\frac{8}{3}$

(C) $\frac{16}{3}$ (D) $\frac{22}{3}$

$y = -(x^2-4x)$

$= -x(x-4)$



$3 = -x^2+4x$

$x = 1, 3$

$y = \left[3 + \frac{x}{4}\right]$

$= 3 + \left[\frac{x}{4}\right]$

$x \in [0,4]$

$\frac{x}{4} \in [0,1]$

$\left[\frac{x}{4}\right] = 0$

$y = 3$ ✓

JEE 1

14

M2

$$\text{Area} = \int_1^3 -x^2+4x-3\,dx$$

M1

$$\frac{2}{3}(2)(1)$$

$$\frac{4}{3}.$$

15

If x, y, z are in A.P, then the value of determinant $\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}$ is

(A) 24

(B) 1

(C) 0

(D) 4

$a = 0$

$x = 1,\ y = 2,\ z = 3$

$\begin{vmatrix} 2 & 3 & 2 \\ 3 & 4 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0$

16

JEE 1

If $\int \frac{\cos^3 x+\cos^5 x}{(1-\cos^2 x)(1+\sin^2 x)}dx = \sin x - \frac{A}{f(x)} - 6\tan^{-1}(f(x)) + C$ (where C is constant of integration and A is constant) and $g(x) = Af(x)$, then $g(x)$ is –

(A) $2\sin x$ (B) $2\cos x$

(C) $4\sin x$ (D) $4\cos x$

$$\int \frac{(1+\cancel{\cos^2 x})(\cancel{\cos^2 x})(\cos x)\,dx}{(\sin^2 x)(1+\sin^2 x)}$$

$1-\sin^2 x$ $(1-\sin^2 x)$

Let: $\sin x = t$

$\cos x\,dx = dt$

$$\int \frac{(2-t^2)(1-t^2)}{t^2(1+t^2)}dt$$

$$\int \left(\frac{t^4-3t^2+2}{t^4+t^2}\right)dt$$

$$\int \frac{(t^4+t^2)-4t^2+2}{(t^4+t^2)}dt$$

16

JEE 1

$$\int\left[1-\frac{4\cancel{t^2}}{\cancel{t^2}(t^2+1)}+\frac{2}{t^2(t^2+1)}\right]dt$$

$$t-4\tan^{-1}t+2\int\left(\frac{1}{t^2}-\frac{1}{t^2+1}\right)dt$$

$$t-4\tan^{-1}t+2\left(-\frac{1}{t}-\tan^{-1}t\right)+C$$

$$t-6\tan^{-1}t-\frac{2}{t}+C$$

17

JEE 1

Let there is a set of 12 natural numbers A = {481, 482, ... , 492}. The number of subset B of A which contain at least 3 elements such that difference of any two elements is at least 2 is –

(A) 66
(B) $2^{12} - 13$
(C) 364
(D) 309

max No of elements in subset.

$\{1, 3, 5, 7, 9, 11\}$

$A' = \{1, 2, 3, 4, \ldots, 11, 12\}$

$\boxed{n = 3}$:

$\xrightarrow{x} \square \xrightarrow{y} \square \xrightarrow{z} \square \xrightarrow{w}$

$\underbrace{1\ 2}\ 3\ \underbrace{4\ 5}\ 6\ \underbrace{7\ 8\ 9\ 10}\ 11\ \underbrace{12}$

$x + y + z + w = 9$

$x \geq 0$ | $w \geq 0$

$y \geq 1 \rightarrow y = 1 + t_1 ;\ t_1 \geq 0$

$z \geq 1 \rightarrow z = 1 + t_2 ;\ t_2 \geq 0$

$$x+y+z+w=9$$

$$x+(1+t_1)+(1+t_2)+w=9$$

$$\boxed{x+t_1+t_2+w=7}$$

$$\boxed{{}^{n+r-1}C_{r-1}}$$

$n=7$

$r=4$

$$=\boxed{{}^{10}C_3}$$

$\boxed{n=4}$:

$\xrightarrow{a} \square \xrightarrow{b} \square \xrightarrow{c} \square \xrightarrow{d} \square \xrightarrow{e}$

$\geq 0 \quad \geq 1 \quad \geq 1 \quad \geq 1 \quad \geq 0$

$$a+b+c+d+e=8$$

$$\left.\begin{matrix} n=5 \\ r=5 \end{matrix}\right\} \rightarrow {}^{5+5-1}C_{5-1} = {}^{9}C_4$$

$\boxed{n=5}$: ${}^{8}C_5$ & $\boxed{n=6}$: ${}^{7}C_6$

18

A square, of each side 2, lies above the x - axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x - axis, then the sum of the x - coordinates of the vertices of the square is

(A) $\sqrt{3}-2$ $x_0 + x_A$

(B) $2\sqrt{3}-1$

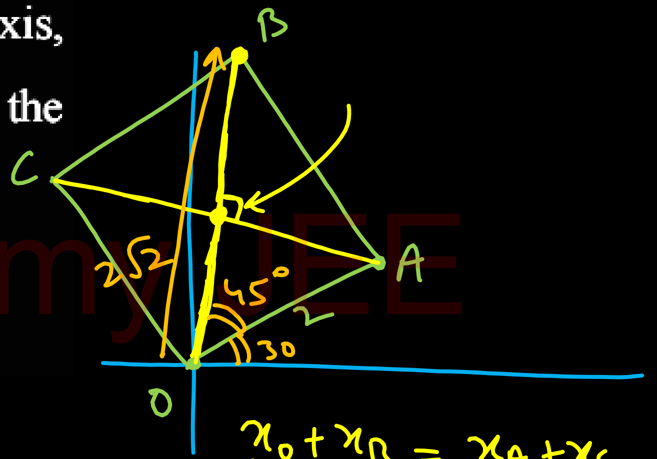
(C) $\sqrt{3}-1$ $x_B + x_C$

(D) $2\sqrt{3}-2$ ✓

$2(x_0 + x_B)$

$x_0 \rightarrow$ zero

✓ $\boxed{2x_B}$



$\frac{x_0 + x_B}{\cancel{2}} = \frac{x_A + x_C}{\cancel{2}}$

18

JEE 1

$$\frac{x-0}{\cos 75^\circ} = \frac{y-0}{\sin 75^\circ} = 2\sqrt{2}$$

$$x_B = 2\sqrt{2}\cos 75^\circ$$

$$= \cancel{2\sqrt{2}}\left(\frac{1}{\cancel{\sqrt{2}}} \times \frac{\sqrt{3}}{\cancel{2}} - \frac{1}{\cancel{\sqrt{2}}} \times \frac{1}{\cancel{2}}\right)$$

$$= (\sqrt{3}-1)$$

19

JEE 1

The sum of two natural numbers n_1 and n_2 is known to be equal to 100 . The probability that their product being greater than 1600 , is equal to:

(A) $\frac{20}{33}$ (B) $\frac{58}{99}$

(C) $\frac{13}{33}$ (D) $\frac{59}{99}$

$S = (n_1, n_2) : \{(1,99), (2,98), ---, (99,1)\}$

$n(S) = 99$

$(20,80), (21,79) ------ ,(79,21)$

20

JEE 1

A line L passing through the point P(1,4,3), is perpendicular to both the lines $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$.

If the position vector of point Q on L is (a_1, a_2, a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be:

(A) 16
(B) 15
(C) 2
(D) 12

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 4 \\ 3 & 2 & -2 \end{vmatrix} = \hat{i}(-10) - \hat{j}(-16) + \hat{k}(1)$$

$\langle -10, 16, 1 \rangle$

$L \equiv \frac{x-1}{-10} = \frac{y-4}{16} = \frac{z-3}{1} = \lambda$

$Q \equiv (-10\lambda+1, 16\lambda+4, \lambda+3)$

$P \equiv (1, 4, 3)$

$PQ = \sqrt{100\lambda^2 + 256\lambda^2 + \lambda^2}$

$(PQ)^2 = 357\lambda^2$

$\cancel{357} = \cancel{357}\lambda^2$

$\boxed{\lambda^2 = 1}$

$Q \equiv (-9, 20, 4)$

$\lambda = 1$

$a_1 + a_2 + a_3 = 15$

$\lambda = -1$

$Q \equiv (11, -12, 2)$

$a_1 + a_2 + a_3 = 1$

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JEE 1

If $a_1, a_2, a_3, \ldots\ldots, a_{21}$, are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$, then the value of $\sum_{i=1}^{21} a_i$ is ______ .

$$a_1 + a_{21} = (a_3 + a_{19}) = (a_5 + a_{17}) =$$

$$\& \quad a_{11} = \frac{a_1 + a_{21}}{2}$$

$$(a_1 + a_{21}) + (a_1 + a_{21}) + \frac{(a_1 + a_{21})}{2} = 10$$

$$\frac{5}{2}(a_1 + a_{21}) = 10 \Rightarrow \boxed{(a_1 + a_{21}) = 4}$$

$$\frac{n}{2}(a + l)$$

$$\frac{21}{2}(a_1 + a_{21})$$

$$\frac{21}{2} \times 4$$

$$42$$

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JEE 1

If $x = \frac{(t+2011)}{t^3}$ and $y = \frac{2011}{t} - \frac{2}{3}\ln t$, then $\frac{3\left(\frac{dy}{dx}\right)\left(3x\left(\frac{dy}{dx}\right)-1\right)^2-1}{20120}$ is

$x = \frac{1}{t^2} + \frac{2011}{t^3}$

$\frac{dy}{dt} = \left(-\frac{2011}{t^2} - \frac{2}{3t}\right)$

$\frac{dx}{dt} = \left(-\frac{2}{t^3} - \frac{6033}{t^4}\right)$

$\frac{dy}{dx} = \left(\frac{\cancel{-6033-2t}}{3t^2}\right)\left(\frac{t^4}{\cancel{-2t-6033}}\right)$

$\therefore \frac{dy}{dx} = \frac{t^2}{3}$

$\frac{\cancel{3}\left(\frac{t^2}{\cancel{3}}\right)\left[\cancel{3}x\left(\frac{t^2}{\cancel{3}}\right)-1\right]^2-1}{20120}$

$\frac{t^2\left(\cancel{1}+\frac{2011}{t}-\cancel{1}\right)^2-1}{20120}$

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JEE1

$$\frac{(2011)^2 - 1}{20120}$$

$$\frac{2010 \times 2012}{20120}$$

$$= \boxed{201}$$

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JEE 1

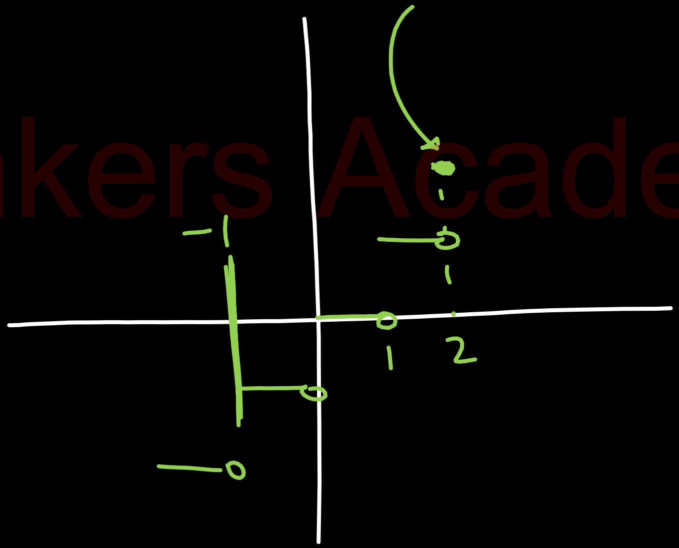
Number of points where f(x) = xsgn (x) + [2x] is discontinuous in x ∈ [−1,2] is (where [.] is greatest integer function and sgn (.) is signum function)

$$f(x) = \underbrace{x\,sgn(x)}_{} + [2x] = |x| + [2x]$$

$$\{-\tfrac{1}{2}, 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, 2\}$$

$$sgn(x) = \begin{cases} -1 ; & x < 0 \\ 0 ; & x = 0 \\ 1 ; & x > 0 \end{cases}$$

$$x\,sgn(x) = \begin{cases} -x ; & x < 0 \\ 0 ; & x = 0 \\ x ; & x > 0 \end{cases}$$


-1
1
2

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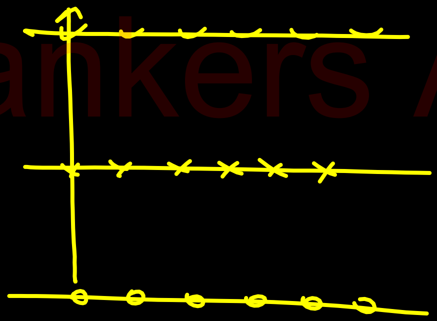
There are three parallel lines lying on the same plane. If 6 distinct points are taken on each of these lines, then maximum possible number of triangles with vertices at these points is-



$$^{18}C_3 - \left(^{6}C_3\right)3$$

$$= 756$$

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A square matrix M of order 3 satisfies $M^2 = I - M$, where I is an identity matrix of order 3 . If $M^n = 2I - 3M$, then n is equal to -

$n = 4$

$M^2 = I - M$

$M^3 = M(I - M)$

$= M - M^2$

$= M - (I - M)$

$M^3 = -I + 2M$

$M^4 = (I - M)^2$

$= I + M^2 - 2M$

$= I + I - M - 2M$

$M^4 = M(-I + 2M) = -M + 2M^2$

$= -M + 2(I - M)$