ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
UNIT TEST # 02
21-07-2024

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	B	D	A	C	C	C	A	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	B	A	C	B	C	D	C	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	10	10	2	17	2	5	580	60	100	30

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	C	C	B	A	D	A	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	A	B	C	A	D	B	A	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	3	15	15	2	9	1	6	25	0

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	D	A	C	B	C	C	C	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	D	C	B	D	C	D	B	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	0	4	740	30	3	120	13	3	7

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (A)

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{R^2}}\right), x << R$$

$$E_{net} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{h}{b} - 1 + \frac{h}{a}\right)$$

Put $b = 2a$

$$E_{net} = \frac{\sigma h}{4\epsilon_0 a}$$

$$= Ch$$

$$C = \frac{\sigma}{4\epsilon_0 a}$$

2. Ans (B)

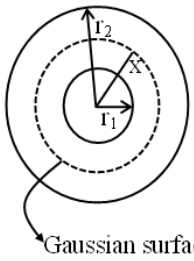
$$W_{2\to1} = q(V_1 - V_2)$$

$$= q\left[\left(\frac{KQ_1}{R} + \frac{KQ_2}{\sqrt{R^2 + R^2}}\right) - \left(\frac{KQ_2}{R} + \frac{KQ_1}{\sqrt{R^2 + R^2}}\right)\right]$$

$$\frac{Kq(Q_1 - Q_2)}{R}\left(1 - \frac{1}{\sqrt{2}}\right)$$

3. Ans (B)

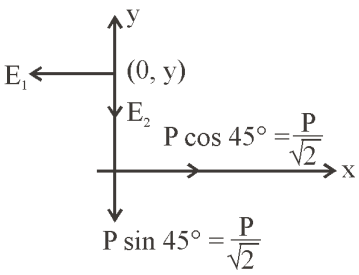
$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\epsilon_0}$$



$$E.4\pi x^2 = \frac{Q \times \frac{4}{3}\pi\left(x^3 - r_1^3\right)}{\frac{4}{3}\pi\left(r_2^3 - r_1^3\right)}/\epsilon_0$$

$$E = \frac{Q}{4\pi\epsilon_0 x^2}\left(\frac{x^3 - r_1^3}{r_2^3 - r_1^3}\right)$$

5. Ans (A)



$$E_P = \sqrt{E_1^2 + E_2^2}$$

$$E_1 = \frac{KP/\sqrt{2}}{y^3}$$

$$E_2 = \frac{2KP/\sqrt{2}}{y^3}$$

$$\therefore\ E_P = \frac{KP\sqrt{5}}{\sqrt{2}y^3}$$

6. Ans (C)

$$V = \frac{KQ_{total}}{r}$$

$$V \propto Q_{total}$$

8. Ans (C)

$$F \propto 10(90)$$

After contact, charge on each becomes

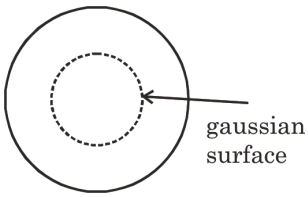
$$= \frac{10 + (-90)}{2} = -40\ \mu C$$

$$F' \propto (40)(40)$$

$$\frac{F'}{F} = \frac{40 \times 40}{10 \times 90} = \frac{16}{9}$$

$$F' = \frac{16F}{9}$$

9. Ans (A)



For spherical charge distribution we can apply

Gauss theorem

$$\int E \cdot ds = \frac{q_{in}}{\epsilon_0}$$

$$E4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = \int_0^{R/2} \rho dv$$

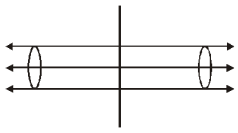
$$= \int_0^{R/2} \frac{Ar}{R} 4\pi r^2 dr$$

$$q_{in} = \frac{A4\pi}{R}\left[\frac{r^4}{4}\right]_0^{R/2} = \frac{A\pi R^3}{16}$$

$$\frac{E4\pi R^2}{4} = \frac{A\pi R^3}{16\epsilon_0}$$

$$\frac{AR}{16\epsilon_0} = \frac{2R}{\epsilon_0} \Rightarrow A = 2$$

10. Ans (D)



$\oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\epsilon_0}$

$2 \times (200)10^2 = \frac{q_{in}}{\epsilon_0}$

$q_{in} = 8.85 \times 10^{-12} \times 4 \times 10^4$

$= 3.540 \times 10^{-7}\ Nm^2/C$

11. Ans (C)

Velocity at ground (means zero height) is non-zero therefore one is incorrect and velocity versus height is non-linear therefore two is also incorrect.

$v^2 = 2gh$

$v\frac{dv}{dh} = 2g = \text{const.}$

$\frac{dv}{dh} = 2g = \text{const.}$

$\frac{dv}{dh} = \frac{\text{cons tan t}}{v}$

Here we can see slope is very high when velocity is low therefore at Maximum height the slope should be very large which is in option C and as velocity increases slope must decrease there for option C is correct.

12. Ans (A)

For $0 \le x \le 200$

$v = mx + C$

$v = \frac{1}{5}x + 10$

$a = \frac{vdv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right)$

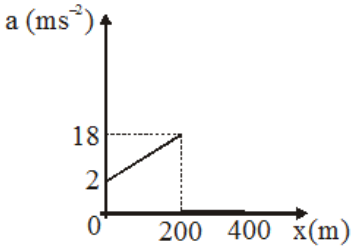
$a = \frac{x}{25} + 2 \Rightarrow$ Straight line till $x = 200$

for $x > 200$

$v = \text{constant}$

$\Rightarrow a = 0$

$\Rightarrow$



Hence most approriate option will be (1), otherwise it would be BONUS.

13. Ans (B)

$u = v_A$

$\theta = 60^\circ$

$h = \frac{v_A^2}{2g} \times \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3v_B^2}{40}$

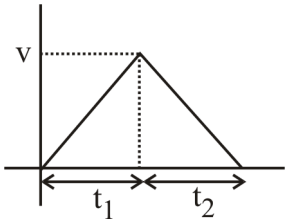
$h = \frac{v_B^2}{10}$

$\frac{3v^2}{40} = \frac{v_B^2}{10}$

14. Ans (A)

$$1500 = \frac{1}{2} \times v \times t_1 + \frac{1}{2} \times v \times t_2$$

$$\frac{3000}{v} = t_1 + t_2 \quad \text{.....(i)}$$

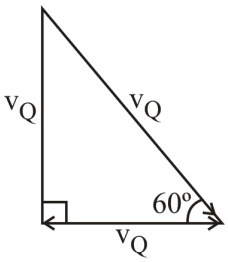


$$t_1 = \frac{v}{5} \qquad t_1 + t_2 = \frac{3v}{10}$$

$$t_2 = \frac{v}{10} \quad \text{from equation (i) and (ii)}$$

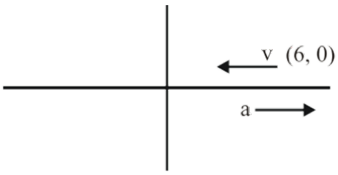
$$t_1 + t_2 = 30 \text{ sec}$$

15. Ans (C)



$$v_B = \frac{v_Q}{\cos 60^o} = 2v_Q = 40 \text{ km/hr}$$

16. Ans (B)



$0 = -24 + a \times t$

$t = 4$ sec, $a = 6 m/s^2$

$$S = -24 \times 4 + \frac{1}{2} \times 6 \times 16$$

$\Rightarrow -96 + 48 = -48$ (left)

$\Rightarrow$ coordinate = (−42, 0)

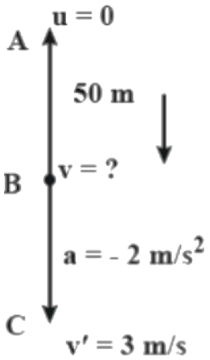
$x_{axis} = 6 - 48 = (-42)$

17. Ans (C)

A ⟶ B

$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times 9.8 \times 50$

$v^2 = \sqrt{980} m/s$



From B ⟶ C

$V^2 = u^2 + 2as$

$(3)^2 = 980 - 2 \times 2 \times s$

$s = 242..75$

Total height = 242.75 + 50

= 292.75 = 293 m

18. Ans (D)

The initial velocity travelled is 625m

The acceleration is $\alpha = 10 m/s^2$

The time is t = 5s

Apply the equation of motion

$v = u + at$

The velocity is v = 0 + 10.5 = 50m/s.

The distance travelled in the first 5s is

$$s = ut + \frac{1}{2}at^2$$

$$= 0.5 + \frac{1}{2} \times 10 \times (5)^2$$

= 125 m

The distance travelled in the following 10s is

$S_1 = ut_1 = 50.10 = 500$ m

The total distance travelled is

$d = s + S_1 = 125 + 500 = 625$ m

19. **Ans (C)**

$s = ut - \frac{1}{2}gt^2$

$-78.4 = 0 - \frac{1}{2} \times 9.8 \times t^2$

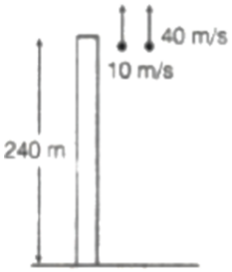
$\Rightarrow t = 4$ sec.

Horizontal distance $= 150 \times 4 = 600$ m

20. **Ans (A)**

Concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.

Consider the stones thrown up simultaneously as shown in the diagram below.



Considering motion of the second particle with respect to the first we have relative acceleration

$|a_{21}| = |a_2 - a_1| = g - g = 0$

Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$-240 = 10t - 1/2 \times 10t^2$

$t^2 - 2t - 48 = 0$

$t^2 - 8t + 6t - 48 = 0$

$t = 8 - 6$ (not possible)

Thus, distance covered by second particle with respect to first particle in 8 s is $S_{12} = (v_{21})t = (40 - 10)(8s)$

$= 30 \times 8 = 240$m

Similarly , time taken b second particle to strike the ground is given by

$-240 = 40t - \frac{1}{2} \times 10 \times t^2$

$-240 = 40t - 5t^2$

$5t^2 - 40t - 240 = 0$

$t^2 - 8t - 48 = 0$

$t^2 - 12t + 4t - 48 = 0$

$t(t - 12) + 4(t - 12) = 0$

$t = 12, -4$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase up to 12 s when the second particle strikes the ground.

PART-1 : PHYSICS

SECTION-II

2. **Ans (10)**

$\phi = \vec{E} \cdot \vec{S} = \left(5\hat{i} + 2\hat{j}\right) \cdot 2\hat{i}$

$= 10$

4. **Ans (17)**

Electric field intensity at a point on the exis of a uniformly charged ring is given by

$$E = \frac{qr}{4\pi\varepsilon_0(R^2 + r^2)^{3/2}}$$

where r is the distance of the point from the centre of the ring (radius R).

For this intensity to be maximum, $\frac{dE}{dr} = 0$

Now, $\frac{dE}{dr} = \frac{q}{4\pi\varepsilon_0}\left[(R^2 + r^2)^{-3/2} + r\left(-\frac{3}{2}\right)(R^2 + r^2)^{-3/2}(2r)\right] = 0$

$\Rightarrow \frac{q}{4\pi\varepsilon_0}(R^2 + r^2)^{-5/2}[(R^2 + r^2) - 3r^2] = 0$

$(R^2 + r^2) - 3r^2 = 0$

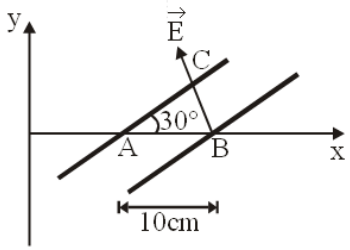
or $r = \frac{R}{\sqrt{2}} = \frac{0.24}{1.414} = 0.17$ m

5. **Ans (2)**

Equipotential surfaces shown in the figure are equidistant, parallel and have equal potential difference between two consecutive surfaces. This information is clear indication of uniform electric field which is given as

$\left|\vec{E}\right| = -\frac{\Delta V}{\Delta r} = \frac{20 - 10}{BC} = \frac{10}{10\sin 30^\circ}$

$\therefore E = \frac{10}{5} = 2$ V/cm



6. **Ans (5)**

Distance travelled in last second during upward journey = distance travelled in 1st second during downward journey.

so $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 1^2 = 5m$

7. **Ans (580)**

$x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now $\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$

$\therefore v = 580\ m^2/s^2$

8. **Ans (60)**

For max. height h = 5 m

initial velocity

$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10\ m/s$

So time taken for max-height

$t = \frac{u}{g} = \frac{10}{10} = 1sec$

So balls are thrown at interval of 1 sec.

So per min. thrown balls are = 60

9. **Ans (100)**

Along x-axis

$100 = v \times t$

$t = \frac{100}{v}$ ...(1)

to along y-axis

$0.1 = \frac{1}{2} \times 9.8 \times t^2$ ...(2)

$0.1 = \frac{1}{2} \times 9.8 \times \left(\frac{100}{v}\right)^2$

from (1) & (2) v = 100 m/s

10. **Ans (30)**

$v = v_x\hat{i} + v_y\hat{j} = 6\hat{i} + 2\hat{j}$

We know, in case of projectile motion, horizontal component of the velocity remains constant.

$\therefore v_x = u_x = 6$

For y-component, using third equation of motion

$v_y^2 = u_y^2 - 2(10)(0,4)$

$\Rightarrow u_y^2 = 12$

$u_y = \sqrt{12}$

$= 2\sqrt{3}$

The angle of projection θ is,

$\tan\theta = \frac{u_y}{u_x}$

$\tan\theta = \frac{2\sqrt{3}}{6}$

$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (D)**

Reaction-I $\Rightarrow$ order = 0, $k_1 \Rightarrow M\ sec^{-1}$

Reaction-II $\Rightarrow$ order = 1, $k_2 \Rightarrow sec^{-1}$

2. **Ans (D)**

$K_1 = K_2$ (given)

$10^{15}e^{-\left(\frac{2000}{T}\right)} = 10^{14} \times e^{-\left(\frac{1000}{T}\right)}$

or $10 = e^{\left(\frac{1000}{T}\right)}$

on taking log of both side

$\log_e 10 = \log_e{}^{\{e^{1000/T}\}}$

$\log_e 10 = \frac{1000}{T} \times \log_e e$

$2.303 \times \log_{10} 10 = \frac{1000}{T} \times 1$

$T = \frac{1000}{2.303} = 434.2$ kelvin

4. Ans (C)

$r = K[A]^x [B]^y$

By I, II

A = doubled, B = same, r = doubled

$\therefore x = 1$

By II, III

A = same, B = doubled, r = doubled

$\therefore y = 1$

$r = K [A] [B]$

By I, $2 \times 10^{-4} = K \times (1 \times 10^{-2}) \times (2 \times 10^{-2})$

$K = 1$

5. Ans (B)

$$\text{At } t_{1/8} \Rightarrow \quad x = \frac{[A]_0}{8} \qquad x = Kt$$

$$\frac{[A]_0}{8} = Kt_{1/8} \qquad ...(1)$$

$$\text{At } t_{\frac{15}{16}} \Rightarrow x = [A]_0 \left(\frac{15}{16}\right) \frac{15}{16}[A]_0 = K \times t_{\frac{15}{16}} \qquad ...(2)$$

$$\frac{\text{eq. (1)}}{\text{eq. (2)}} = \frac{t_{1/8}}{t_{\frac{15}{16}}} = \frac{\frac{A_0}{8}}{\frac{15}{16}A_0} = \frac{16}{15} \times \frac{1}{8} = 2:15$$

6. Ans (A)

$$e^{-E_a/RT} = \left[\frac{e^{-E_{a_1}/RT} \times e^{-E_{a_2}/RT}}{e^{-E_{a_3}/RT}}\right]^{2/3}$$

$$E_a = \frac{2}{3}[E_{a_1} + E_{a_2} - E_{a_3}]$$

$$= \frac{2}{3}[180 + 80 - 50] = 140 \text{ kJ/mol.}$$

9. Ans (B)

$r = K[A]^x[B]^y$ .....(i)

$r' = K[9A]^x[B]^y \Rightarrow 3r = K(9)^x(A)^x(B)^y$

$\Rightarrow (3)^1 = (3)^{2x} \Rightarrow x = 1/2$

$x'' = K[A]^x[2B]^y$

$\Rightarrow 2r = (2)^y K[A]^x[B]^y$

$\Rightarrow 2r = (2)yr$

$\Rightarrow (2)^1 = (2)^y$

$\Rightarrow y = 1$

$\therefore$ Order $= 1 + \frac{1}{2} = \frac{3}{2}$

11. Ans (A)

SnO_2 (amphoteric)

CO_2, SiO_2 (acidic)

CaO (basic)

12. Ans (D)

C N O F

P S Cl

Br

I

In covalent hydrides, acidic strength increase in both directions.

13. Ans (A)

Correct order of electron affinity is

S > Se > Te > O

15. Ans (C)

Ununnilium ⟶ 110 ⟶ d-block

18. Ans (B)

According to (n + l)rule

PART-2 : CHEMISTRY

SECTION-II

3. Ans (15)

Check by condition.

4. Ans (15)

Apply formula

$$\text{Remaining conc}^n = \text{initial conc}^n \left(\frac{1}{2}\right)^{\text{No. of half lives}}$$

from given condition $[A]\left[\frac{1}{2}\right]^{t/5} = \frac{[A]}{4}\left[\frac{1}{2}\right]^{t/15}$

on solving for t, t = 15 min

5. Ans (2)

For 1st order reaction

$$t_{50\%} = \frac{\ln 2}{k}$$

$$\frac{1}{k} \cdot \ln \frac{100}{100 - 75} = t_{75\%} = 2\frac{\ln 2}{k}$$

$$\frac{t_{75\%}}{t_{50\%}} = 2$$

6. Ans (9)

Oxygen has second highest electronegative element after F.

7. Ans (1)

Due to inert pair effect the most stable oxidation state of Tℓ is +1.

9. Ans (25)

According to (n + *l*) rule

n = 5

5s ⟶ 1 (orbital)

5p ⟶ 3

5d ⟶ 5

5f ⟶ 7

5g ⟶ 9

⇒ 5H not possible

10. Ans (0)

Cu : $[Ar]4s^1 3d^{10}$

(Z = 29)

Cu^+ : $[Ar]3d^{10}$

⇒ 0 unpaired electrons.

PART-3 : MATHEMATICS

SECTION-I

1. Ans (A)

$3^{\log_{10} x} = 3^{\frac{\log_3 x}{\log_3 10}} = x^{\log_{10} 3}$

∴ Equations is

$x^{\log_{10}3} = 54 - x^{\log_{10}3}$

$2x^{\log_{10}3} = 54$

$x^{\log_{10}3} = 27$

$x = (27)^{\frac{1}{\log_{10}3}} = 27^{\log_3 10} = 3^{\log_3 10} = 1000$

x = 1000

2. Ans (B)

Give series is $\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$. A.P.

$\Rightarrow S_{24} = \frac{24}{2}\left[2\sqrt{2} + (24-1)\sqrt{2}\right] = 300\sqrt{2}$

3. Ans (D)

$$\sqrt{\underbrace{111.....1}_{200\text{ digits}} - \underbrace{222.....2}_{100\text{ digits}}}$$

$$\sqrt{\frac{(10^{200}-1)}{10-1} - 2\left[\frac{10^{100}-1}{10-1}\right]}$$

$$\sqrt{\frac{10^{200} - 2\times 10^{100} + 1}{9}} = \sqrt{\left(\frac{10^{100}-1}{3}\right)^2}$$

$$\Rightarrow 3.\left[\frac{10^{100}-1}{9}\right] = \underbrace{333.....3}_{\leftarrow 100\text{ digit}\rightarrow}$$

4. Ans (A)

$\sum_{n=1}^{100} a_{2n} = \alpha$

$a_2 + a_4 + \ldots\ldots\ldots + a_{2\omega} = \alpha$

{common ratio = r^2}

$\Rightarrow ar\left[\frac{1-r^{2\omega}}{1-r^2}\right] = \alpha$ .....(i)

and $\sum_{n=1}^{100} a_{2n-1} = \beta$

$\Rightarrow a_1 + a_3 + a_5 + \ldots\ldots + a_{199} = \beta$

$a\left[\frac{1-r^{2\omega}}{1-r^2}\right] = \beta$ ...(ii)

eq^n (i)/(ii)

$\Rightarrow \boxed{r = \frac{\alpha}{\beta}}$

5. Ans (C)

$(15a)^2 + (3b)^2 + (5c)^2 - (15a)$

$(15c) - (15a)(3b) - (3b)(5c) = 0$

$= \frac{1}{2}[(15a-3b)^2 + (3b-5c)^2 + (5c-15a)^2] = 0$

it is possible when $15a = 3b = 5c$

$\therefore b = \frac{5c}{3}, a = \frac{c}{3}$

$a + b = 2c$ b, c, a in A.P.

6. Ans (B)

$a + md, a + nd, a + rd \longrightarrow$ G.P

$\Rightarrow (a+nd)^2 = (a+md)(a+rd)$

$\Rightarrow 2an + n^2d = a(m+r) + mrd$

$\frac{d}{a} = \frac{(m+r-2n)}{(n^2-mr)}$

m, n, r are in H.P. $\Rightarrow n = \frac{2mr}{(m+r)}$

$\Rightarrow m + r - 2n = \frac{2(mr-n^2)}{n}$

$\frac{d}{a} = \frac{-2}{n}$

7. Ans (C)

$f(x) = \sqrt{\log_2(4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} + 1)}$ is

defined if

$\log_2(4\sin^2 x - 2(\sqrt{3}\sin x + \sin x) + (\sqrt{3}+1) \geq 0$

or $4\sin^2 x - 2\sin x(\sqrt{3}+1) + \sqrt{3} + 1 \geq 1$

or $\sin^2 x - \sin x\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) + \frac{\sqrt{3}}{4} \geq 0$

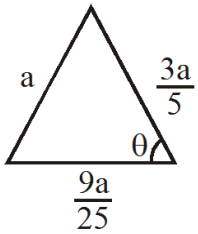
or $\left(\sin x - \frac{\sqrt{3}}{2}\right)\left(\sin x - \frac{1}{2}\right) \geq 0$

i.e., $-1 \leq \sin x \leq \frac{1}{2}$ or $\frac{\sqrt{3}}{2} \leq \sin x \leq 1$

or $x \in \left[-\pi, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

Number of integral solutions = 4 + 1 + 1 = 6

8. Ans (C)



$b^2 = ac$

$\frac{9b^2}{25c^2} = \frac{5c}{a} \times \frac{a}{3b}$

$3b = 5c$

$\cos\theta > 0$

$\theta < 90°$

9. Ans (C)

$S=10^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 \ldots 10(11)^9 \ldots(1)$

$\frac{11}{10} S = 11(10)^8 + 2(11)^2(10)^7 + \ldots 11^{10} \quad \ldots(2)$

Sub (1) – (2)

$-\frac{S}{10} = 10^9 + 11 . 10^8 + 11^2 . 10^7 + \ldots 11^9 - 11^{10}$

$-\frac{S}{10} = 10^9 + \frac{11 . 10^8 \left[1 - \left(\frac{11}{10}\right)^9\right]}{\left[1 - \frac{11}{10}\right]} - 11^{10}$

$\Rightarrow S = 10^{11} = K10^9$

$\Rightarrow K = 100$

11. Ans (C)

$\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x(1-\sin x)}{8\left(\frac{\pi}{2}-x\right)^3}$

$= \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)\left(1-\cos\left(\frac{\pi}{2}-x\right)\right)}{8\times\left(\frac{\pi}{2}-x\right)\times\left(\frac{\pi}{2}-x\right)^2}$

$= \frac{1}{8} \times 1 \times \frac{1}{2}$

$= \frac{1}{16}$

12. Ans (A)

(i) $\lim\limits_{x\to\infty} \sec^{-1}\left(\frac{x}{\sin x}\right) = \sec^{-1}\left(\frac{\infty}{\sin\infty}\right)$

$= \sec^{-1}\left(\frac{\infty}{\text{any value between } -1 \text{ to } 1}\right)$

$= \sec^{-1}(\pm\infty) = \frac{\pi}{2}$

(ii) $\lim\limits_{x\to\infty} \sec^{-1}\left(\frac{\sin x}{x}\right) = \sec^{-1}\left(\frac{\sin\infty}{\infty}\right)$

$= \sec^{-1}\left(\frac{\text{any value between } -1 \text{ to } 1}{\infty}\right)$

$= \sec^{-1} 0 =$ not defined

Hence, (i) exists but (ii) does not exist.

13. Ans (D)

By newton's Leibniz formula.

$= \lim\limits_{x\to\pi/4} \frac{f\left(\sqrt{2}\sec x\right) . \sqrt{2}\sec x . \tan x - 0}{2x}$

$= \frac{f(2) . \sqrt{2} . \sqrt{2} . 1}{2 . \pi/4} = \frac{4}{\pi} f(2)$

16. Ans (D)

LHL = RHL

$P = -\frac{1}{2}$

17. Ans (C)

g(x) is continuous at x = 3

$\Rightarrow \quad 2k = 3m + 2 \quad .....(i)$

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}}, & 0 \leqslant x < 3 \\ m, & 3 < x \leqslant 5 \end{cases}$$

$g'(3^-) = g(g^+)$

$\Rightarrow \quad \frac{k}{4} = m$

$\Rightarrow \quad k = 4m \quad .....(ii)$

From (i) and (ii)

$m = \frac{2}{5}, \; k = \frac{8}{5}$

$\therefore \quad k + m = 2$

18. Ans (D)

$f(0^-) = \lim\limits_{h\to 0} -h+\{-h\} + (-h)\sin\{-h\}$

$= \lim\limits_{h\to 0} -h + (1 - h) + (-h)\sin(1-h) = 1$

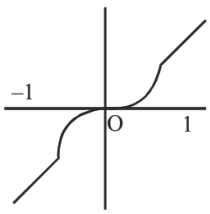
$f(0^+) = \lim\limits_{h\to 0} h+\{h\} + h\sin\{h\}$

$= 0$

Not continuous at x = 0.

19. Ans (B)

f(x) is clearly continuous for $x \in R$.



Thus, f(x) is non-differentiable at x = 1, –1.

20. Ans (C)

Divide above and below by x^m, then

$$K = \lim_{x\to\infty} \left(\frac{\sum\limits_{K=1}^{1000} \left(1 + \frac{k}{x}\right)^m}{1 + \frac{10^{1000}}{x^m}} \right)$$

$$= \frac{1 + 1 + 1 + \ldots\ldots \text{upto } 1000 \text{ times}}{1 + 0} = 1000$$

$= 10^3$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (4)

$$\log_{10}\left(\frac{x(x+2)}{5x+4}\right) = 0$$

$$\Rightarrow \frac{x(x+2)}{5x+4} = 1 \quad \Rightarrow x^2 - 3x - 4 = 0$$

$\Rightarrow (x - 4)(x + 1) = 0 \quad \Rightarrow x = 4, -1$

but x cannot be –1

2. Ans (0)

x can't be negative

4. Ans (740)

$T_3 = 7$

$T_7 = 2 + 3T_3$

$S_{20} = 740$

5. **Ans (30)**

$A.M = \frac{75}{4} \Rightarrow \frac{a+b}{2} = \frac{75}{4}$ .....(1)

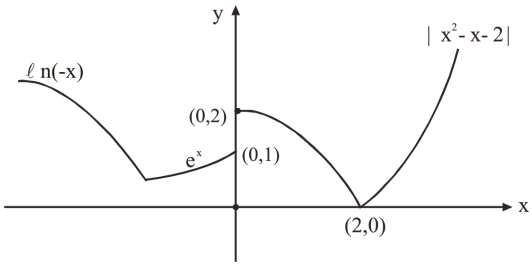
$G.M = 15 \Rightarrow \sqrt{ab} = 15$ .....(2)

from (1) and (2)

$a = 30$

6. **Ans (3)**

By graph it is clear that function is discontinuous at x=0 & non differentiable at x=2 & at the point where e^x & $\ell n\,(-x)$ intersect each other.



7. **Ans (120)**

$\lim\limits_{x\to0} \frac{3f(x) - 4f(3x) + f(9x)}{x^2} \left(\frac{0}{0}\text{form}\right)$

$\lim\limits_{x\to0} \frac{3f'(x) - 12f'(3x) + 9f'(9x)}{2x} \left(\frac{0}{0}\text{form}\right)$

$\lim\limits_{x\to0} \frac{3f''(x) - 36f''(3x) + 81f''(9x)}{2}$

$\frac{3f''(0) - 36f''(0) + 81f''(0)}{2}$

$= 24f''(0) = 24(5) = 120$

10. **Ans (7)**

$\lim\limits_{h\to0} \frac{f(h)}{h} = \lim\limits_{h\to0} \frac{kh}{h} = k$

And $f(x) = kx$

$f'(x) = k = 7$