PART-1 : PHYSICS

SECTION-I

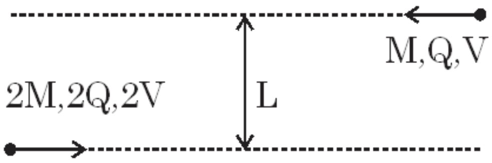
1) A boy throws several balls out of the window of his house at different angles to the horizontal. All balls are thrown at speed $u = \sqrt{10}$ m/s and it was found that all of them hit the ground making an angle of 60° or larger than that with the horizontal. Find the height (in meter) of the window above the ground. [g = 10 m/s^2]

(A) 1.75
(B) 1.5
(C) 1.85
(D) 2

2) A ball of mass (m) is rolling without sliding down an inclined plane. Choose the correct statement.

(A) Net force applied by ball on inclined plane is less than it’s own weight(mg)
(B) Net force applied by ball on inclined plane is larger than it’s own weight(mg)
(C) Net force applied by ball on inclined plane is equal to it’s own weight(mg)
(D) We cannot determine the relation between net force by ball on inclined plane and weight(mg)

3) Two particles approach each other from very large distance as shown, (consider gravitational force between the particle is negligible). If the least separation between the particles is b, what is the speed of M in center of mass frame when particles are closest ?



(A) LV/b
(B) 2LV/b
(C) 3LV/b
(D) 4LV/b

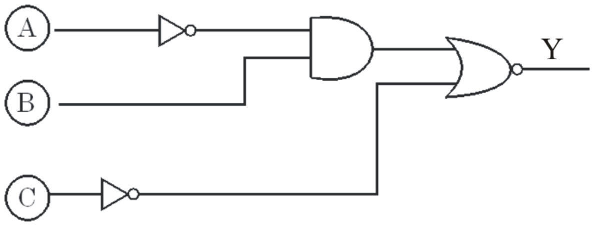
4) An ice cream cone of mass 3M has base radius R and height h. Assume its wall to be thin and uniform. When ice cream is filled inside it (so as to occupy the complete conical space) its mass becomes 7M. Find the distance of the centre of mass of the ice cream filled cone from its vertex.

(A) $\frac{3h}{4}$
(B) $\frac{2h}{3}$

(C) $\frac{5h}{7}$

(D) $\frac{6h}{7}$

5) For the logic in circuit shown in figure, choose the correct output from following options.



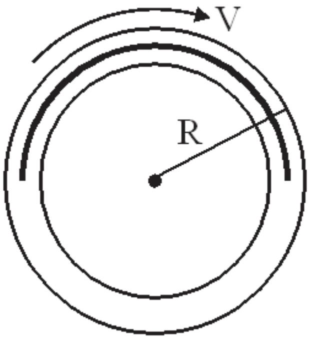
(A) $Y = \bar{A}B + \bar{C}$

(B) $Y = \overline{\bar{A}B}\,.\,C$

(C) $Y = A\bar{B} + \bar{C}$

(D) $Y = \overline{A\bar{B} + C}$

6) A metal wire having mass M is bent in the shape of a semicircle of radius R and is sliding inside a smooth circular groove of radius R present in a horizontal table. The wire just fits into the groove and is moving at a constant speed V. Find the magnitude of angular speed of COM of wire.



(A) $\frac{V}{R}$

(B) $\frac{3\pi V}{2R}$

(C) $\frac{\pi V}{2R}$

(D) None of these

7) A total charge Q is spread onto a non-conducting, flat circular annular disc of inner radius a and outer radius b. The charge distributed is such that the charge density is given by $\sigma = \frac{k}{r^3}$, when r is the distance from the center. Find the potential at the center of disc.

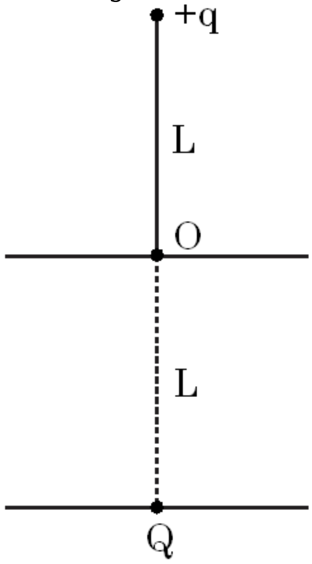
(A) $\frac{1}{8\pi\varepsilon_0}Q\left(\frac{a+b}{ab}\right)$

(B) $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{a+b}{ab}\right)$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$

8) A particle of mass m and charge +q is attached to a light insulating thread of length L. The other end of the thread is secured at point O. Exactly below point O, there is a small ball having charge +Q fixed on an insulating horizontal surface. The particle remains in equilibrium vertically above the ball with tension in string equal to 2mg. Distance of the ball from point O is L. Find time period of



SHM if it is slightly displaced

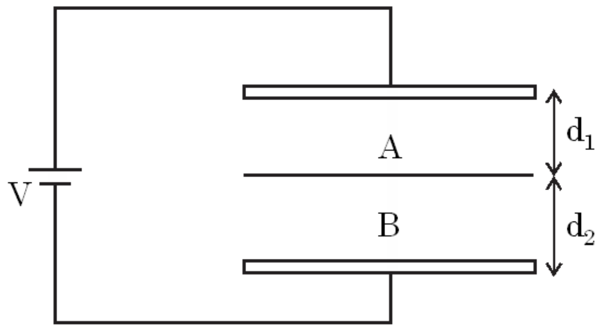
(A) $2\pi\sqrt{\frac{2L}{g}}$

(B) $2\pi\sqrt{\frac{4L}{g}}$

(C) $2\pi\sqrt{\frac{5L}{g}}$

(D) $2\pi\sqrt{\frac{6L}{g}}$

9) The figure shows a parallel plate capacitor filled with two different layers of materials. The upper layer 'A' is a material of thickness d_1 and conductivity σ_1 and the lower layer 'B' is a material of thickness d_2 and conductivity σ_2 . The capacitor is connected across an ideal battery of emf 'V'



volt. The electric field in the material A is :

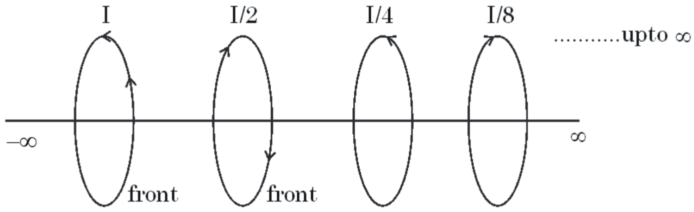
(A) $\dfrac{V\sigma_1}{d_1\sigma_1 + d_2\sigma_2}$

(B) $\dfrac{V\sigma_2}{d_1\sigma_1 + d_2\sigma_2}$

(C) $\dfrac{V\sigma_1}{d_1\sigma_2 + d_2\sigma_1}$

(D) $\dfrac{V\sigma_2}{d_1\sigma_2 + d_2\sigma_1}$

10) Infinite number of circular current carrying loops are coaxially placed with their planes parallel as shown in the diagram. The current in the first loop is I, in second loop is I/2, in third loop is I/4 and so on. In the first loop the current is upwards in the portion close to the observer. The value of the line integral of magnetic field over the axis from $-\infty$ to $+\infty$ i.e., $\int_{-\infty}^{+\infty} \vec{B}.\,\vec{dx}$ is :



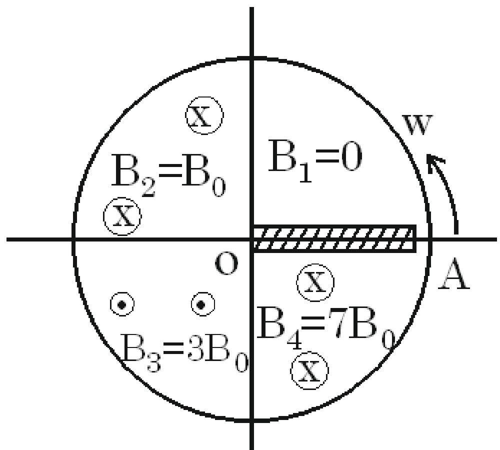
(A) $\dfrac{4\mu_0 I}{3}$

(B) $\dfrac{2\mu_0 I}{3}$

(C) $\dfrac{-2\mu_0 I}{3}$

(D) $\dfrac{-4\mu_0 I}{3}$

11) A conducting rod OA has one of its ends at the centre O and the other end at the periphery of a circular region of radius R which contains four different uniform magnetic fields in the four



0 0 0

quadrants as shown $B_1 = 0$, $B_2 = B$, $B_3 = 3B$; $B_4 = 7B$. B_3 is out of the plane while B_2 and B_4 are into the plane. The average emf induced in one cycle is

(A) $\dfrac{11}{4}B_0\omega R^2$

(B) $\frac{11}{2}B_0\omega R^2$

(C) $\frac{5}{8}B_0\omega R^2$

(D) $\frac{8}{2}B_0\omega R^2$

12) A cylinder is closed at both ends and has insulating walls. It is divided into two compartments by an adiabatic partition that is perpendicular to the axis of the cylinder. Each compartment contains 1.00mol of oxygen that behaves as an ideal gas with $\gamma = \frac{7}{5}$. Initially, the two compartments have equal volumes and their temperatures are 550K and 250K. The partition is then allowed to move slowly until the pressures on its two sides are equal. Find the final temperature in the compartment which has initial temperature 550K. (Take : $\left(\frac{550}{250}\right)^{\frac{5}{7}} = 1.756$).

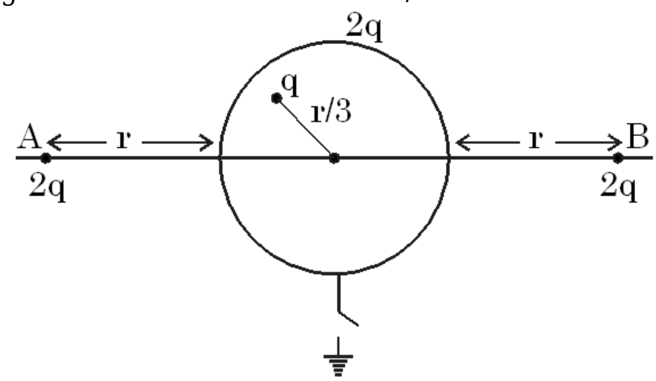
(A) 510K

(B) 590K

(C) 210K

(D) 290K

13) The figure shows a conducting shell of radius r having charge 2q on it. Two point charges, of magnitude 2q each are placed at distance 2r from the centre of the shell at A and B respectively. A charge q is residing inside the shell at distance r/3 from its centre. The charge flow to earth if the switch is closed is



(A) 2q

(B) 3q

(C) 5q

(D) None of these

14) A combination of 0.250kg of water at 20.0°C, 0.400kg of aluminium at 26.0°C and 0.100kg of copper at 100°C is mixed in an insulated container and allowed to come to thermal equilibrium. Ignore any energy transfer to or from the container and determine the final temperature of the mixture. (s_{Water} = 4186 J kg^{-1} $°C^{-1}$, $s_{Aluminium}$ = 900 J kg^{-1} $°C^{-1}$, s_{Copper} = 387 J kg^{-1} $°C^{-1}$)

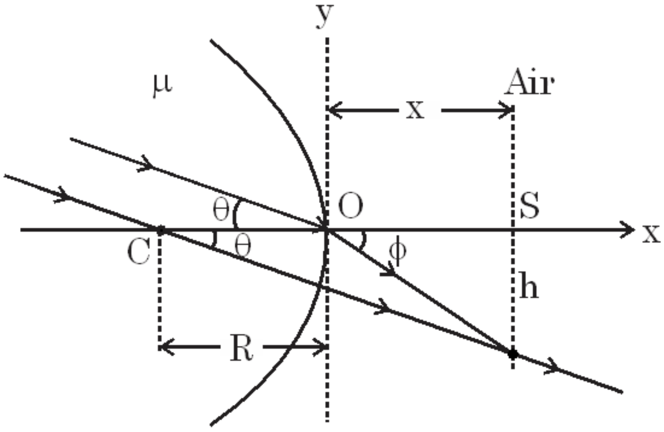
(A) 23.6°C

(B) 40°C

(C) 20°C

(D) 26°C

15) A spherical surface of radius R separates air from a medium of refractive index μ. Parallel beam of light is incident, from medium side, making a small angle θ with the principal axis of the spherical surface. Choose the correct relation between R, x and μ.



(A) $x = \dfrac{R}{\mu - 1}$

(B) $x = \dfrac{R}{\mu}$

(C) $x = \dfrac{R}{\mu + 1}$

(D) $x = \dfrac{R(\mu + 1)}{\mu}$

16) For the β^+ decay process a reaction is done as follows ${}^{A}_{Z}Y \to {}^{A}_{Z-1}X + {}^{0}_{+1}\beta + \upsilon$. If mass of neutrino particle is negligible and atomic mass of Y is m_Y and atomic mass of X is m_X then choose correct option for β^+ decay reaction to be feasible, m_e is the mass of β^+ particle.

(A) $m_X - m_Y > 2m_e$

(B) $m_Y - m_X > 2m_e$

(C) $m_X + m_Y > 2m_e$

(D) $m_Y - m_X > m_e$

17) In a radioactive process a sample contains two isotopes, one stable and the other unstable. Number of stable nuclei is equal to N_s and the number of unstable nuclei is equal to N_u. After certain time the activity of sample decreased to $\dfrac{1}{x}$ times initial activity. After this certain time total number of nuclei (excluding decayed nuclei) became one third of intial nuclei, then find the ratio of N_s/N_u.

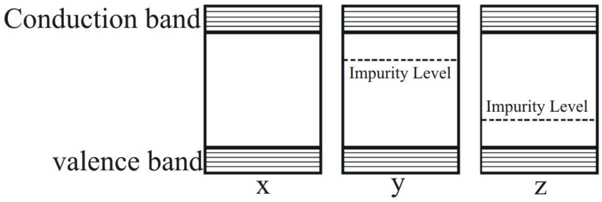
(A) $\dfrac{x+3}{2}$

(B) $\dfrac{x-3}{2x}$

(C) $\dfrac{x-2}{x}$

(D) $\dfrac{x-1}{x}$

18) The energy band diagrams for three semiconductor samples of silicon are as shown. We can then choose the correct statement



(A) Sample X is undoped while samples Y and Z have been doped with a trivalent and a pentavalent impurity respectively

(B) Sample X is undoped while both samples Y and Z have been doped with a pentavalent impurity

(C) Sample X has been doped with equal amounts of trivalent and pentavalent impurities while samples Y and Z are undoped

(D) Sample X is undoped while samples Y and Z have been doped with a pentavalent and a trivalent impurity respectively

19) The magnetic susceptibility of aluminium at 300 K is 2.5×10^{-5}. The magnetic intensity H inside a solenoid with aluminium core is 2000 Am^{-1}. If the temperature of the core is raised to 350 K what will be magnetisation at 350 K?

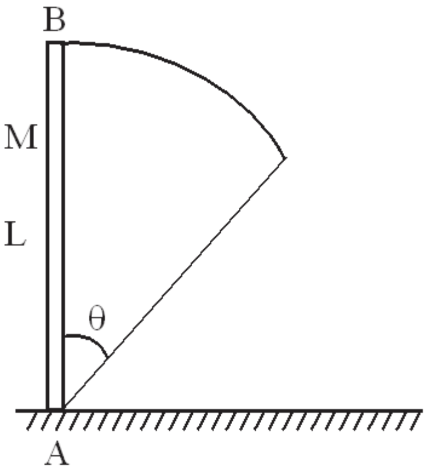
(A) 2.14×10^{-4} Am^{-1}

(B) 4.28×10^{-2} Am^{-1}

(C) 4.28×10^{-4} Am^{-1}

(D) 2.14×10^{-2} Am^{-1}

20) A rod of mass M = 5kg and length L = 1.5m is held vertical on a table as shown. A gentle push is given to it and it starts falling. Friction is large enough to prevent end A from slipping on the table. Find the sum of linear momentum of all the particles of the rod when it rotates through an angle $\theta = 37°$.



(A) 15 kg m/s

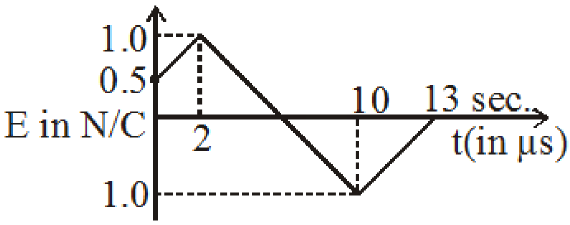
(B) 5 kg m/s

(C) 7.5 kg m/s

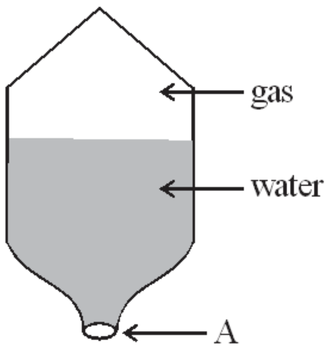
(D) 3 kg m/s

SECTION-II

1) The electric field through an area of $2m^2$ varies with time as shown in the graph. The time (in sec) at which greatest displacement current through the area is observed, is

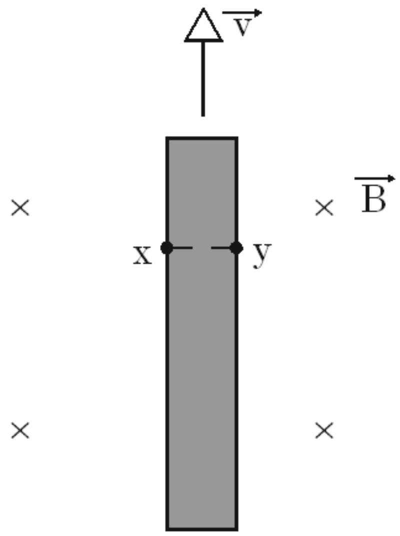


2) A container having pressurized gas in its upper part and water in its lower part. Pressure of the gas is 4.0 MPa. Mass of empty container is 1.0kg and mass of its content is also 1.0kg. The nozzle at the bottom is opened to impart a vertical acceleration to the container. The square of velocity at the exit of the nozzle is $n \times 10^3$ (in m^2/s^2). Then fill the value of n. Neglect the pressure due to height of water in the container and take atmosphere pressure to be 1.0 MPa. ($g = 10\ ms^{-2}$)

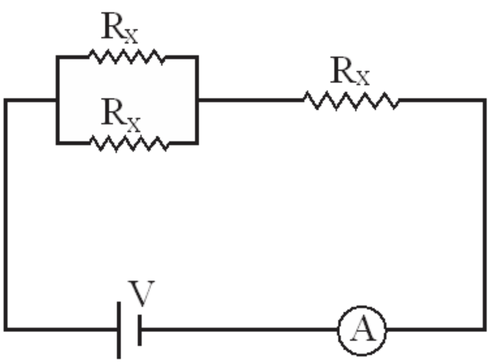


3) A current carrying coil, having current $\frac{3}{4}A$ and number of turns 50 is placed such that its plane is vertical and axis is along magnetic meridian. At the centre of the coil, a dip needle is placed. Axis of rotation is horizontal and is in the plane of the coil. Initially the angle of dip is found to be 30° when the given current is flowing through the coil. The angle of dip becomes 60° when this current is reversed. Assuming that the magnetic field due to coil is smaller than the horizontal component of earth's magnetic field, $B_H = 3 \times 10^{-5}$ T if radius of the coil in metre is $\frac{k\pi}{4}$ find the value of k.

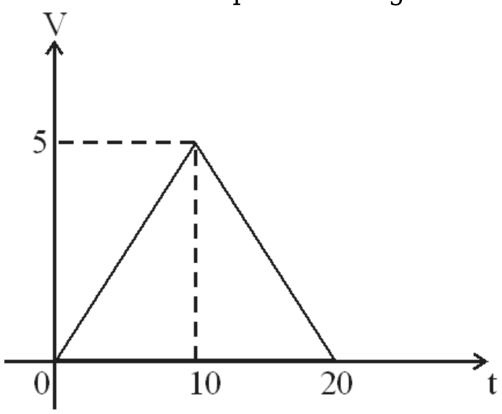
4) A metal strip 6.5 cm long, 3 cm wide, and 0.76 mm thick moves with constant speed v through a magnetic field B = 1.2 mT perpendicular to the strip, as shown in figure. A potential difference of 3.6 μV is measured between the points x and y across the strip. Calculate the speed v in (cm/sec)



5) The emf(V) of source changes with time after switch is closed at t = 0. The variation of V with time t is shown in figure below. Each resistance has resistance R_x which depends on voltage V_x across it as:
$v_x \leq 2$; $R_x = 1\Omega$; $v_x > 2$; $R_x = 1\Omega$



Find current in Amperes through ideal Ammeter at t = 10s.



PART-2 : CHEMISTRY

SECTION-I

1) Which of the following options is incorrect :-

(A) Order of electronegativity of carbon atom - $C_2H_6 < C_2H_4 < C_2H_2$

(B) Order of electron affinity (magnitude only) - Cl > Br > F > I

(C) Order of stability of oxidation state - $Ge^{+2} < Sn^{+2} < Pb^{+2}$

(D) Order of second ionisation energy - O > F > N > C

2) The incorrect statement among the following is :-

(A) Hydration energy - $Al^{+3} > Mg^{+2}$

(B) magnetic moment - $O_2 > N_2^{\ominus}$

(C) Dipole moment - H–F < H_3C–F

(D) Thermal stability - $Li_2CO_3 < BeCO_3$

3) Regarding borax which of the following statement is incorrect :-

(A) It contains two six membered heterocyclic rings

(B) Its aqueous solution can act as buffer

(C) It give blue bead with copper salt in reducing flame

(D) In its structure, two four-coordinated boron centers and two three-coordinated boron centers are present.

4) In which of the following reaction H_3PO_3, will form as a product

(A) $\underset{\text{(Red)}}{P_4} \xrightarrow{NaOH}$

(B) $H_3PO_2 \xrightarrow{\Delta}$

(C) $PCl_3 \xrightarrow{EtOH}$

(D) $PCl_5 \xrightarrow{EtOH}$

$$\begin{tikzcd}
\text{Limestone} \arrow[r, "\Delta"] & A + B \arrow[d, "\text{Carbon, }\Delta"] \\
E + F & C + D \arrow[l, "N_2"', "\Delta"]
\end{tikzcd}$$

5)
Compound 'D' reacts with iron to form (X). Choose the correct statement about (X)

(A) It is a paramagnetic complex

(B) It is a high spin complex

(C) It has dsp^3 hybridisation and square pyramidal geometry

(D) It obeys the EAN rule

6) Atoms of small non metallic atoms get trapped inside the crystal lattice of iron to form a compound [X]. Which of the following statements is not true for [X] ?

(A) [X] has higher melting point than iron

(B) [X] has more metallic conductivity than iron

(C) [X] is harder than iron

(D) [X] is chemically inert

7) Which of the following statements is true?

(A) NH_3 is an electron precise hydride

(B) In ice, each oxygen atom is surrounded in a regular tetrahedral by four hydrogen atoms

(C) 10 volume solution of H_2O_2 contains 3% H_2O_2 by weight.

(D) Calgon is a zeolite used to remove hardness of water

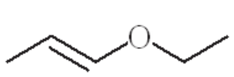
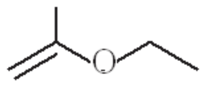
8) $\underset{(A)}{C_5H_{10}O} \xrightarrow[\text{dry HCl}]{CH_3OH} \underset{(B)}{C_6H_{14}O_2}$

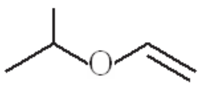
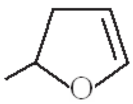
$B \xrightarrow{H_3O^+} X + Y + CH_3OH$

$X + Y \xrightarrow[NaOH]{I_2}$ Yellow precipitate

$X + Y \xrightarrow[\text{reagent}]{\text{Tollen's}}$ Silver mirror

A can be

(i)  (ii) 

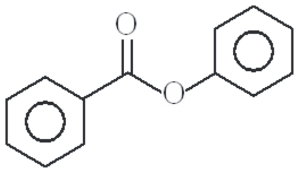
(iii)  (iv) 

(A) All of these

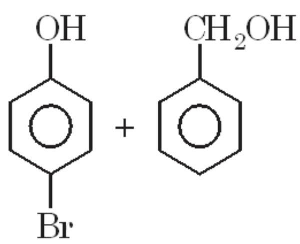
(B) (i), (ii) and (iii) only

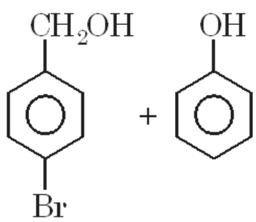
(C) (i) and (iii) only

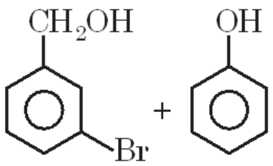
(D) (i), (ii) and (iv) only

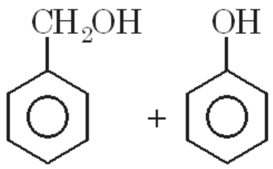
9)  $\xrightarrow[\text{(1eq)}]{Br_2/Fe}$ A $\xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } LiAlH_4}$ Products

The products are

(A) 

(B) 

(C) 

(D) 

10) $CH_3COPh \xrightarrow[\Delta]{\text{dil. NaOH}} A \xrightarrow{HBr} B \xrightarrow[\text{tbuOH}]{\text{tbuOK}/\Delta} A+C$

$C \xrightarrow{\text{Ozonolysis}} D + CO_2$

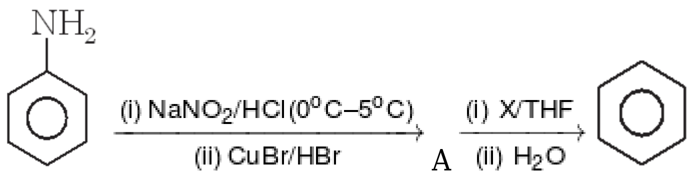
Which of the following statements is incorrect about D.

(A) It gives  on reaction with PhMgBr

(B) It gives +ve iodoform test

(C) It gives +ve test with Tollen's Reagent

(D) It decolorizes Bromine water solution

11) 

Which statement is incorrect about X ?

(A) It reacts with water to give H_2

(B) It reacts with N_2 to give nitrate salt

(C) It combines with O_2 to form oxides

(D) It can displace Cu from an aqueous solution of $CuSO_4$.

12) $PhCH_2-\overset{\overset{O}{\parallel}}{C}-OC_2H_5 \xrightarrow[\text{(ii) } H_3O^+ \text{ (iii) } \Delta]{\text{(i) } C_2H_5ONa} ?$

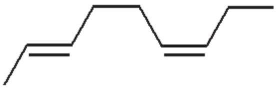
(A) $Ph-CH_2-\overset{\overset{O}{\parallel}}{C}-\underset{\underset{Ph}{|}}{CH}-\overset{\overset{O}{\parallel}}{C}-OC_2H_5$

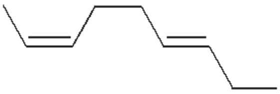
(B) $PhCH_2-\overset{\overset{O}{\parallel}}{C}-\underset{\underset{Ph}{|}}{CH}-\overset{\overset{O}{\parallel}}{C}-OH$

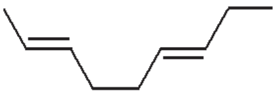
(C) $Ph-CH_2-\overset{O}{\overset{\|}{C}}-CH_2-Ph$

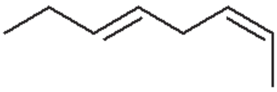
(D) $Ph–CH_2CH_2CH_2Ph$

13) $CH \equiv C-CH_2CH_2C \equiv C-CH_3 \xrightarrow[\text{(ii) Na/Liq } NH_3 \\ \text{(iii) } CH_3CH_2Br \\ \text{(iv) } H_2/Pd/BaSO_4]{\text{(i) } CH_3Li}$

(A) 

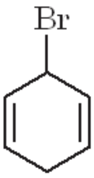
(B) 

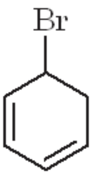
(C) 

(D) 

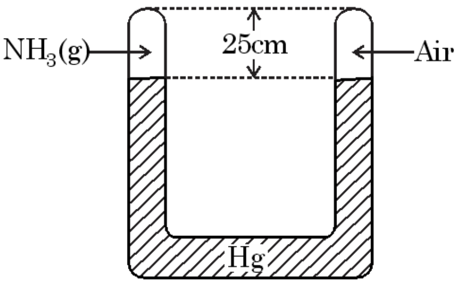
14) Which of the following alkyl bromides undergo fastest SN_1 reaction in water ?

(A) CH_3CH_2Br

(B) 

(C) 

(D) CD_3CD_2Br

15) NH_3(g) is sealed in a U-tube manometer of uniform area of cross-section at 80 cm of Hg, as shown 

After sometime, due to the dissociation of NH_3, it was observed that the air column has shrunk by 5cm. Assuming that temperature remains constant, what is the degree of dissociation of NH_3 ?

(A) 0.375

(B) 0.5

(C) 0.65

(D) 0.357

16) To the dissociation reaction $2N_2O_5(g) \rightleftharpoons 4NO_2(g) + O_2(g)$, $O_2(g)$ was added, maintaining its partial pressure constant.

(A) K_P increases and equilibrium shifts backwards

(B) K_P remains constant and the equilibrium shifts forwards

(C) K_P remains constant and the equilibrium remains undisturbed

(D) K_P remains constant and the equilibrium shifts backwards

17) 1 mole of an ideal monoatomic gas occupying a volume of 20L at 1 atm undergoes single step adiabatic compression against a constant external pressure of 10 atm. The final volume of gas is -

(A) 20 L

(B) 2 L

(C) 10.6 L

(D) 9.2 L

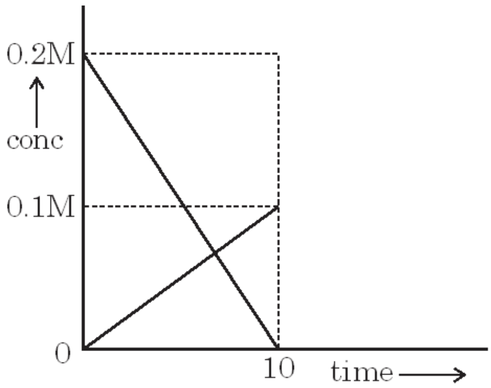
18) What is the solubility of $CaCO_3$ in 0.4M H_2CO_3 solution ? Assume H_2CO_3 to be stable in aqueous solution for the duration of the reaction and $Ca(HCO_3)_2$ to be completely soluble. K_{sp} of $CaCO_3$ = 10^{-9}M K_1, K_2 of H_2CO_3 = 10^{-7} and 10^{-11} respectively

(A) 1 g per L

(B) 1 mol per L

(C) 0.1 g per L

(D) 0.1 mol per L



19)

For the reaction $xA(g) \rightarrow yB(g)$, which of the following statements are correct ?

(i) Reaction is a zero order reaction

(ii) Rate constant for the reaction is 0.02 s^{-1}

(iii) If concentration of A is changed, then rate constant changes, but rate of reaction remains constant.

(iv) The reaction is a complex reaction

(A) (i), (ii) and (iv) are correct

(B) (i), (iii) and (iv) are correct

(C) Only (i) and (iv) are correct

(D) All statements are correct

20) Electrons in H atom get excited from the ground state to an orbit in which the wavelength of the electron is 10 times the wavelength of an electron in the ground state of He^{+}. How many spectral lines are observed in the visible range when these electrons are de-excited to the ground state ?

(A) 10

(B) 15

(C) 4

(D) 3

SECTION-II

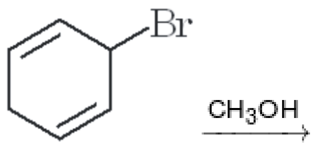
1) $U(s) + 3ClF_3(\ell) \rightarrow X(g) + Y(g)$

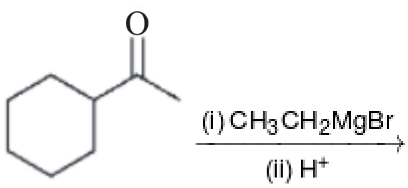
X is uranium containing compound. Oxidation state of uranium in X is :-

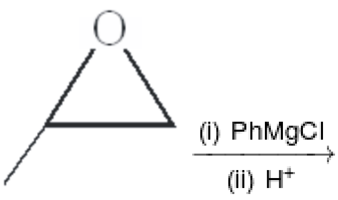
2) How many of the following give a racemic mixture as the major product ? All chiral reactants used are optically pure.

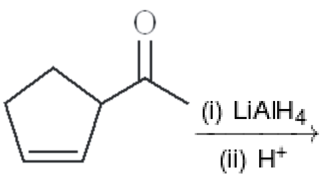
(i) $(CH_3)_3C-Br \xrightarrow{H_2O}$

(ii) $Ph-\underset{\substack{|\\ CH_3}}{CH}-I \xrightarrow[\text{Acetone}]{\text{NaI}}$

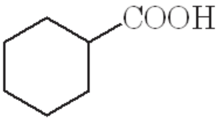
(iii) 

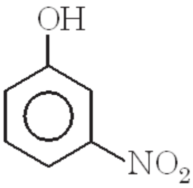
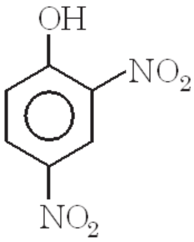
(iv) 

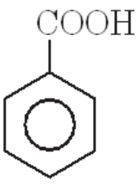
(v) 

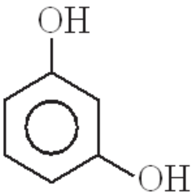
(vi) 

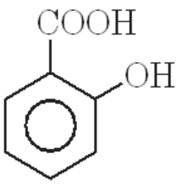
3) Number of compounds more acidic than  are

(i) CH_3CH_2OH (ii) 

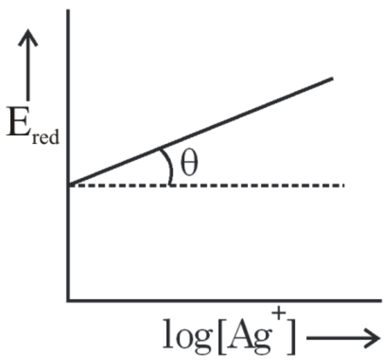
(iii)  (iv) 

(v)  (vi) CH_3SO_3H

(vii)  (viii) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$

(ix) $CH\equiv CH$ (x) 

4) The plot of Reduction Potential (E_{red}) vs $[Ag^+]$ for $Ag^+|Ag$ half cell is given as



Given :

$E^0_{Ag^+|Ag} = 0.8V$ (independent of temperature) ; $\dfrac{2.303RT}{F} = 0.06$ at 300K; $\tan\theta = 0.062$

What is the temperature of the solution in °C

5) The radius of A^+ = 1.4Å and B^- = 2.6 Å. What is the edge length of the cubic unit cell of AB in Å ?

PART-3 : MATHEMATICS

SECTION-I

1) There are several tea cups in the kitchen, some with handles and other without handles. The number of ways of selecting one cup without a handle and one cup with a handle is exactly 36. Then the minimum possible numbers of cups in the kitchen is equal to

(A) 20

(B) 12

(C) 10

(D) 28

2) The sum of values of $a\ (a \in I)$ such that the equation (x + 10) (x – a) + 5 = 0 has integral roots

(A) –4

(B) –16

(C) –20

(D) 0

3) If $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, for x, y, z > 0 then the least value of (x – 1)(y – 1)(z – 1) is

(A) 3

(B) 6

(C) 0

(D) 8

4) The number of points in square |x| + |y| = 2 which lie on the curve y = x + cosx and at which the tangent to the curve is parallel to the x-axis is

(A) 0

(B) 2

(C) 4

(D) 8

5) Let f(x) = max{|x|, x – [x]}, where [.] denotes greatest integer function, then $\int_{-1}^{1} f(x)\,dx$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{5}{4}$

(D) $\frac{7}{4}$

6) Length of the line segment joining the feet of the perpendicular drawn from the point (3, 4) on the pair of lines $x^2 - 4xy + y^2 = 0$ is

(A) $\frac{5\sqrt{3}}{2}$

(B) $5\sqrt{3}$

(C) $2\sqrt{3}$

(D) $\sqrt{3}$

7) Let z_1, z_2, z_3 be the complex numbers such that $|z_1 - 1| = |z_2 - 1| = |z_3 - 1|$ and $\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \frac{\pi}{6}$ then $z_2(z_2 - 1) - z_3(z_2 + 1) + (z_3 + 1)(z_3 - 1) =$

(A) 0

(B) 1

(C) 2

(D) –2

8) $\theta \in [0, \pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1 + |\sin\theta|) z_1 + (|\cos\theta| - 1) z_2 - 2z_3 = 0$. If complex numbers z_1, z_2, z_3 are non-zero then number of possible values of θ is :-

(A) 0

(B) 1

(C) 2

(D) More than 2

9) The curve satisfying the differential equation $(2x^2y - 2y^4)dx + (2x^3 + 3xy^3)dy = 0$ and passing through (1, 1) is given by $2\ln(xy) + \frac{y^m}{x^n} = 1$, $(m, n \in N)$, then the value of m + n is

(A) 5

(B) 4

(C) 3

(D) 6

10) The real no. x and y satisfying $(x - 4)^2 + (y - 5)^2 = 7^2$ then the minimum value of $\sqrt{x^2 + y^2 - 2x - 2y + 2}$ is

(A) 1

(B) 2

(C) 3

(D) 4

11) Two tangents on a parabola are x – y = 0 and x + y = 0. If (2, 3) is focus of the parabola then the equation of tangent at vertex is

(A) $4x - 6y + 5 = 0$

(B) $4x - 6y + 3 = 0$

(C) $4x - 6y + 1 = 0$

(D) $4x - 6y + \frac{3}{2} = 0$

12) If (6, 13) and (25, 8) are the foci of a hyperbola passing through the point (1, 1) then the eccentricity of the hyperbola is

(A) $\frac{\sqrt{386}}{38}$

(B) $\frac{\sqrt{386}}{25}$

(C) $\frac{\sqrt{386}}{13}$

(D) $\frac{\sqrt{386}}{12}$

13) The number of points of non-differentiability of function $f(x) = \max\{\text{sgn}x, 2^{-x}, x^2\}$,

$$\text{sgn}\,x = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

(A) 4

(B) 3

(C) 2

(D) 1

14) If f is a differentiable function satisfying $2f(x) = f(xy) + f\left(\frac{x}{y}\right), \forall x, y \in R^+$ f(1) = 0 and $f'(1) = \frac{1}{\ln 6}$, then the value of f(7776), is

(A) 3

(B) 4

(C) 5

(D) 6

15) If $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = 3\vec{c}$ then the value of $[\vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a} \;\; \vec{a} \times \vec{b}]$ ([.] denotes scalar triple product) is

(A) 2

(B) 3

(C) 7

(D) None of these

16) The standard deviation of 35 numbers is 50. If each number is increased by 10, then new standard deviation will be :-

(A) 40

(B) 50

(C) 60

(D) 500

17) The number of skew symmetric matrix of order at least 2 with distinct elements except principal diagonal formed by the elements $0, \pm1, \pm2, \pm3$ taking any number of elements at a time is

(A) 48

(B) 54

(C) 106

(D) 102

18) If $\Delta_r = \begin{vmatrix} r.r! & ({}^mC_r)^2 & 2r-1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \\ (m+1)!-1 & {}^{2m}C_m & m^2-1 \end{vmatrix}$, then $\sum_{r=0}^{m} \Delta_r$ is

(A) 0

(B) m

(C) m^2

(D) $m^2 + \sin^2 m$

19) The number of integral values of 'b' for which origin and the point (1,1,1) lie on the opposite side of the plane $2x + a^2y + abz - 1 = 0$, for all $a \in R - \{0\}$ is

(A) 3

(B) 2

(C) 1

(D) 0

20) Let x, y > 0. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30

(B) 32

(C) 36

(D) 40

SECTION-II

1) Let p be the product of the non-real roots of the equation $x^4 - 4x^3 + 6x^2 - 4x = 2021$ then find $(p - 1)^2$

2) How many tangents to the circle $x^2 + y^2 = 3$ are there which are normal to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

3) Find the no. of values of 'x' satisfying $\sin x . \sin\left(\frac{1}{x}\right) - 1 = 0, x \in R - \{0\}$

4) Let an ordered pair A be defined as A(x, y) where $x \in$ prime number, such that x < 10 and $y \in$ natural no. and $y \leqslant 10$. If the probability that the ordered pair A satisfies the relation $x^2 - 3y^2 = 1$ is P then 20P equals.

5) $\lim_{k \to \infty} \int_0^{k[x]} (kt - [kt])^k dt; k \in N$ is $\left[\frac{\lambda x}{2}\right]$ where [.] denotes greatest integer function then find the value of λ.

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	**B**	**A**	**B**	**C**	**B**	**A**	**A**	**A**	**D**	**C**	**C**	**A**	**C**	**A**	**A**	**B**	**B**	**D**	**B**	**C**

SECTION-II

Q.	21	22	23	24	25
A.	**12**	**6**	**2**	**10**	**2**

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	**B**	**D**	**C**	**C**	**D**	**B**	**C**	**C**	**A**	**C**	**B**	**C**	**A**	**B**	**C**	**B**	**D**	**A**	**C**	**D**

SECTION-II

Q.	46	47	48	49	50
A.	**6**	**3**	**5**	**37**	**8**

PART-3 : MATHEMATICS

SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	**B**	**C**	**D**	**A**	**C**	**A**	**D**	**A**	**A**	**B**	**A**	**D**	**A**	**C**	**D**	**B**	**B**	**A**	**A**	**D**

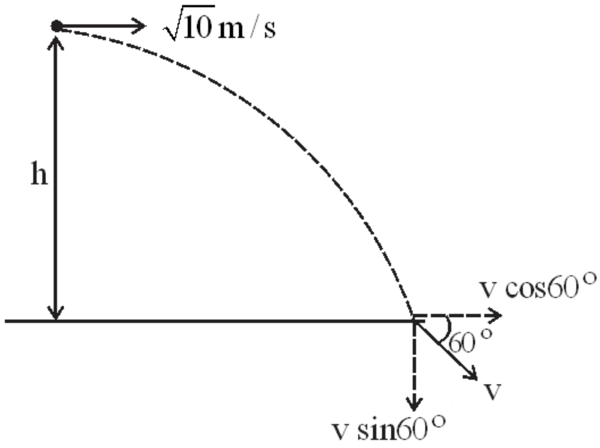
SECTION-II

Q.	71	72	73	74	75
A.	**2022**	**0**	**0**	**1**	**2**

SOLUTIONS

PART-1 : PHYSICS

1) Minimum angle of 60º is possible when projectile is thrown horizontally



 Horizontal velocity remains constant in projectile motion

 $\sqrt{10}$ = V cos 60°

$\Rightarrow$ $V = 2\sqrt{10}$

Also, vertical motion is free fall motion

$\therefore$ $\sqrt{2gh} = V \sin 60$

$\Rightarrow$ $\sqrt{20h} = 2\sqrt{10} \times \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\sqrt{20h} = \sqrt{30}$

$\Rightarrow$ $h = 1.5\,m$

2) For ball, $I = \frac{2}{5}mR^2$ (assuming solid sphere)

In case of pure rolling on incline,

$N = mg\cos\theta$

and $f = \dfrac{mg\sin\theta}{1 + \frac{I}{mR^2}} = \dfrac{mg\sin\theta}{1 + \frac{2}{5}} = \dfrac{5}{7}mg\sin\theta$

Net force applied on incline

$= \sqrt{N^2 + f^2} = \sqrt{m^2g^2\cos^2\theta + \dfrac{25}{49}m^2g^2\sin^2\theta}$

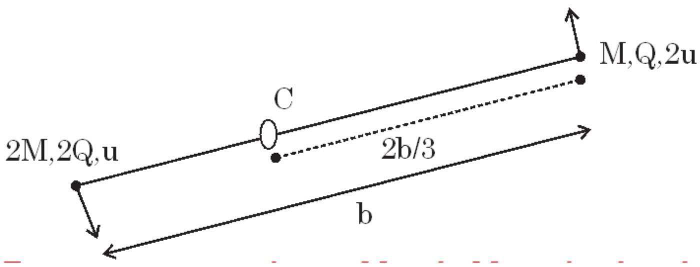
$= mg\sqrt{\cos^2\theta + \sin^2\theta - \dfrac{24}{49}\sin^2\theta} = mg\sqrt{1 - \dfrac{24}{49}\sin^2\theta}$

 Quantity inside square root is less than one.

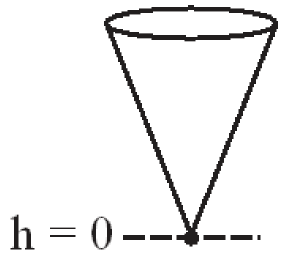
 This net force is less than mg.

3) In CM frame

Use linear momentum conservation and angular momentum conservation about CM



4) For hollow cone, COM is at $\frac{2h}{3}$ from vertex and for solid cone, COM is at $\frac{3h}{4}$ from vertex



$M_h = 3m$

$M_s = 7m - 3m = 4m$

$\therefore y_{COM} = \dfrac{M_h y_h + M_s y_s}{M_h + M_s}$

$= \dfrac{(3m)\left(\frac{2h}{3}\right) + (4m)\left(\frac{3h}{4}\right)}{7m}$

$= \dfrac{5Mh}{7m} = \dfrac{5h}{7}$

5) $\overline{(\bar{A}B) + \bar{C}}$

$= \overline{\bar{A}\;B}.\bar{\bar{C}}$

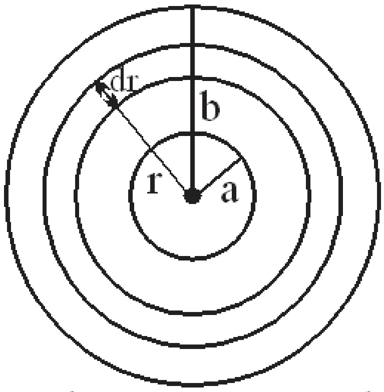
$= \bar{A}\;B.C$

6) □ Angular speed of entire wire and that of COM of wire will be same about centre of circular groove. Also, for any point of wire:

$v = R\omega \Rightarrow \omega = \dfrac{v}{R}$

□ For COM, $\omega = \dfrac{v}{R}$

7) $\int dQ = \int \sigma 2\pi r\, dr$



$$Q=\int_a^b \frac{k}{r^3}2\pi r\,dr \quad Q=2\pi k\int_a^b \frac{dr}{r^2}\ ;$$

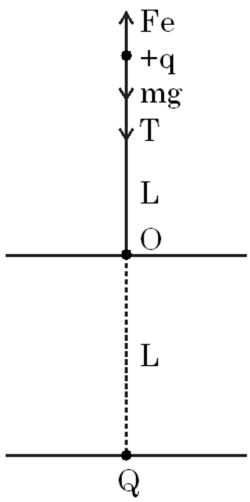
$$Q=2\pi k\frac{(b-a)}{ab}\ ;\ \int dV=\int \frac{kdQ}{r}$$

$$V=\int \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r\,dr}{r}=\frac{k}{4\pi\varepsilon_0}2\pi\int_a^b \frac{dr}{r^3}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{k2\pi}{2}\left(\frac{b^2-a^2}{a^2b^2}\right)=\frac{1}{4\pi\varepsilon_0}\frac{2\pi k}{2}\frac{(b-a)}{ab}\frac{(b+a)}{ab}$$

$$=\frac{1}{8\pi\varepsilon_0}Q\left(\frac{a+b}{ab}\right)$$

8) In initial equilibrium position



T + mg = Fe

2mg + mg = Fe

Fe = 3mg ....(1)

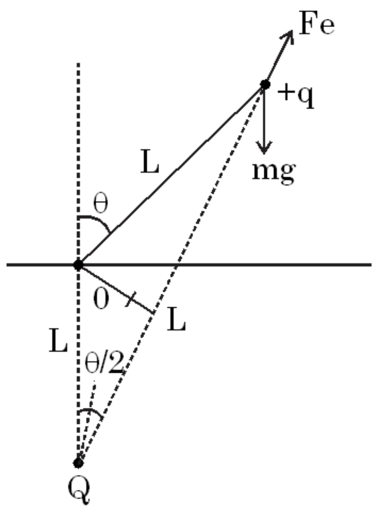
When slightly displaced

We will neglects slight change in Fe as distance b/w charges will charge.

$i_0 = I\alpha$

$$mg.L\sin\theta - FeL\sin\frac{\theta}{2} = mL^2\alpha$$

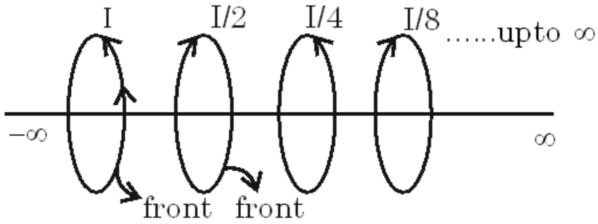
From (1) Fe = 3mg



$$mgL\theta - 3mgL\frac{\theta}{2} = mL^2\alpha$$
$$-\frac{mg}{2}L\theta = mL^2\alpha$$
$$\alpha = -\left(\frac{g}{2L}\right)$$
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2L}{g}}$$

9) $R_1 = \frac{d_1}{\sigma_1 A},\ R_2 = \frac{d_2}{\sigma_2 A}$

$$\therefore i = \frac{V}{\frac{d_1}{\sigma_1 A} + \frac{d_2}{\sigma_2 A}}$$
$$\therefore j = \frac{i}{A} = \frac{V}{\frac{d_1}{\sigma_1} + \frac{d_2}{\sigma_2}}$$
$$j = \sigma E \ \therefore\ E = \frac{j}{\sigma}$$
$$\therefore E_A = \frac{j}{\sigma_1} = \frac{V}{d_1 + \frac{\sigma_1 d_2}{\sigma_2}}$$
$$= \frac{V\sigma_2}{\sigma_1 d_2 + \sigma_2 d_1}$$



10)

Lets take a square loop of infinite length with one of the sides being the axis of the circular loops itself.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\left[-I + \frac{I}{2} - \frac{I}{4} + \frac{I}{8} - \dots\right]$$

But apart from the axial side, $\int \vec{B}\cdot d\vec{\ell}$ for all other sides will be zero as magnetic field along those sides is negligible.

$$\therefore \int_{-\infty}^{\infty} \vec{B} \cdot d\bar{x} + 0 + 0 + 0 = -\mu_0 I\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...\right]$$

$$= -\mu_0 I\left[\frac{1}{1-\left(\frac{-1}{2}\right)}\right] = \frac{-2\mu_0 I}{3}$$

11) Emf induced in the first quadrant equal to zero. Emf induced in the second quadrant

$$= \int_{x=0}^{x=R} B_0 (dx)\, \omega x = \frac{B_0\omega R^2}{2}$$

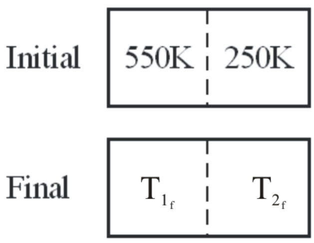
Emf induced in the third quadrant = $\dfrac{-3B_0\omega R^2}{2}$

Emf induced in the fourth quadrant

$$= \frac{7B_0\omega R^2}{2}$$

$$\therefore \text{Avg.} = \frac{\frac{B_0\omega R^2}{2} - \frac{3B_0\omega R^2}{2} + \frac{7B_0\omega R^2}{2}}{4}$$

$$= \frac{5B_0\omega R^2}{8}$$



12)

Work done by adiabatically expanding gas is equal and opposite to work done by adiabatically compressing gas.

 $\dfrac{nR}{1-\gamma}\left(T_{1_f} - 550\right) = -\dfrac{nR}{1-\gamma}\left(T_{2_f} - 250\right)$

$\Rightarrow T_{i_f} + T_{2_f} = 800$ ....(i)

$\because PV^{\gamma}$ = constant for both sides

 $P_{1_i}V_{1_i}^{\gamma} = P_{1_f}V_{1_f}^{\gamma}$ and $P_{2_i}V_{2_i}^{\gamma} = P_{2_f}V_{2_f}^{\gamma}$

Dividing above equations:

$$\frac{P_{1_i}V_{1_i}^{\gamma}}{P_{2_i}V_{2_i}^{\gamma}} = \frac{P_{1_f}V_{1_f}^{\gamma}}{P_{2_f}V_{2_f}^{\gamma}} \Rightarrow \frac{P_{i_i}}{P_{2_i}} = \left(\frac{V_{i_f}}{V_{2_f}}\right)^{\gamma}$$

$\left(\because V_{1_i} = V_{2_i} \;\&\; P_{1_f} = P_{2_f}\right)$

$$\Rightarrow \frac{nRT_{1_i}/V_{1_i}}{nRT_{2_i}/V_{2_i}} = \left(\frac{nRT_{1_f}/P_{1_f}}{nRT_{2_f}/P_{2_f}}\right)^{\gamma} \Rightarrow \frac{T_{1_i}}{T_{2_i}} = \left(\frac{T_{1_f}}{T_{2_f}}\right)^{\gamma}$$

$\left(\because V_{1_i} = V_{2_i} \;\&\; P_{1_f} = P_{2_f}\right)$

$\Rightarrow \dfrac{T_{1_f}}{T_{2_f}} = \left(\dfrac{550}{250}\right)^{\frac{1}{(7/5)}}$ ...(ii)

Solving (i) & (ii),

$T_{1_f} = 510K$

13) When switch is open, charge on inner surface of shell = –q.
Charge on outer surface = 2q – (–(q)
= 3q. (as total charge on shell = 2q)
When switch is closed, potential of the shell becomes zero.
V_{shell} = Work done against electric field in moving a unit charge from infinity to surface
= Work done against the field of 2q (at A), 2q (at B) and q' (on outer surface of the shell) from infinity to surface
Note that the external field will have no contribution from the charges inside the shell due to shielding effect.
∴ V_{shell} = work done against the field of 2q, 2q, & q' from infinity to centre of the sheel.
(as field inside shell due to these charges = 0)
= potential at centre due to 2q, 2q & q'.
$\Rightarrow \quad 0 = \frac{k2q}{2r} + \frac{k2q}{2r} + \frac{kq'}{r}$
$\Rightarrow \quad 0 = q + q + q' \therefore q' = -2q$
∴ Charge flow = $3q - (q')$
= 3q – (–2q) = 5q

14) $T_{mixture} = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$
Substituting the values
$T_{mixture} = 23.6C$

15) Snell's Law at O
$\mu \sin\theta = 1 \sin\phi$
for small angle
$\sin\theta \simeq \tan\theta$
$\sin\phi \simeq \tan\phi$
$\mu \tan\theta = \tan\phi$
$\mu \frac{h}{R+x} = \frac{h}{x}$
$\mu x = R + x$
$x = \frac{R}{\mu - 1}$

16) ${}^{A}_{Z}Y \rightarrow {}^{A}_{Z-1}X + {}^{0}_{+1}\beta + \nu$
$Q = [(m_Y - zm_e) - (m_X - (z-1)m_e) - m_e]c^2$
$\Rightarrow \quad Q = (m_Y - m_X - 2m_e)c^2$
For reaction to be feasible Q > 0
$\Rightarrow \quad m_Y - m_X - 2m_e > 0$
$\Rightarrow \quad m_Y - m_X > 2m_e$

17) $N = N_s + N_u$ (1)
$\frac{N}{3} = N_s + \frac{N_u}{x}$ (2)
Divide equation (1) by equation (2)

$3 = \frac{N_s + N_u}{N_s + \frac{N_u}{x}}$; $3 = \frac{N_s/N_u + 1}{N_s/N_u + \frac{1}{x}}$

Let $\frac{N_s}{N_u} = t$; $3 = \frac{t+1}{t+\frac{1}{x}}$

$3t + \frac{3}{x} = t + 1$

$2t = 1 - \frac{3}{x} \Rightarrow \boxed{t = \frac{x-3}{2x}}$

18) In sample x no impurity level seen, so it is undoped. In sample y impurity energy level lies closer to the conduction bond so it is doped with fifth group impurity.
In sample z, impurity energy level lies closer to the valence band so it is doped with third group impurity.

19) Susceptibility χ for a paramagnatic substance varies with absolute temperature as $\chi_1 = \frac{C}{T}$

$\therefore \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$

$\therefore \chi_2 = \frac{300}{350} \times 2.5 \times 10^{-5}$

$= 2.14 \times 10^{-5}$

$\therefore$ Magnetisation at 350 k is

$I = \chi H = 2.14 \times 10^{-5} \times 2000\ Am^{-1}$

$= 4.28 \times 10^{-2} Am^{-1}$

20) Using energy conservation

$mg\frac{L}{2}(1 - \cos\theta) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$

$\Rightarrow \omega = \sqrt{\frac{3g}{5L}} = \sqrt{\frac{30}{5 \times 1.5}} = 2\ rad/s$

Now, at final position, consider an elemental mass dm having width dx at a distance x from end A.
Linear momentum of this element:

$dP = (dm)\ v$

$= \left(\frac{M}{L}dx\right)(x\omega) = \frac{M}{L}\omega\ xdx$

$\therefore P = \int dP = \frac{M}{L}\omega \int_0^L x\,dx$

$= \frac{M\omega}{L}\left(\frac{L^2}{2}\right) = \frac{M\omega L}{2}$

$= \frac{5 \times 2 \times 1.5}{2} = 7.5\ kg\,m/s$

21) $i = \epsilon_0 A \frac{dE}{dt}$

22) Using Bernoulli's theorem at surface of water-gas boundary and just outside nozzle:

$P_{gas} = P_{atm} + \frac{1}{2}\rho v^2$

[Neglecting pressure due to height of water & also neglecting speed of water-gas boundary]

$\Rightarrow 4 \times 10^6 = 1 \times 10^6 + \frac{1}{2} \times 10^3 v^2$

$\Rightarrow v^2 = 6 \times 10^3$

$\therefore n = 6$

23) $\tan 30^\circ = \frac{B_V}{B_H + B_R} = \frac{1}{\sqrt{3}}$

$\tan 60^\circ = \frac{B_V}{B_H - B_R} = \sqrt{3}$

Dividing, we get

$\frac{B_H - B_R}{B_H + B_R} = \frac{1}{3} \Rightarrow 3B_H - 3B_R = B_H + B_R$

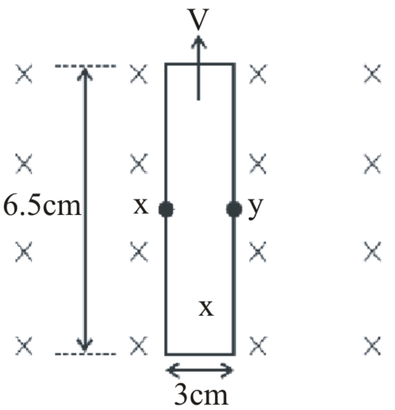
$\Rightarrow 2B_H = 4B_R \therefore B_R = \frac{B_H}{2}$

$50\frac{\mu_0 i}{2R} = \frac{B_H}{2}$

$\therefore R = \frac{\mu_0 i \times 50}{B_H} = \frac{(4\pi \times 10^{-7}) \times 3 \times 50}{4 \times 3 \times 10^{-5}}$

$= \pi \times 10^{-2} \times 50$ m

k = 2

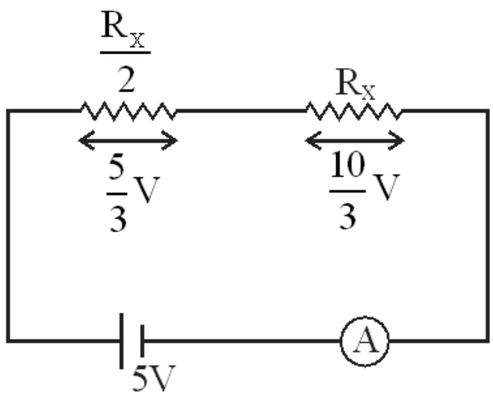


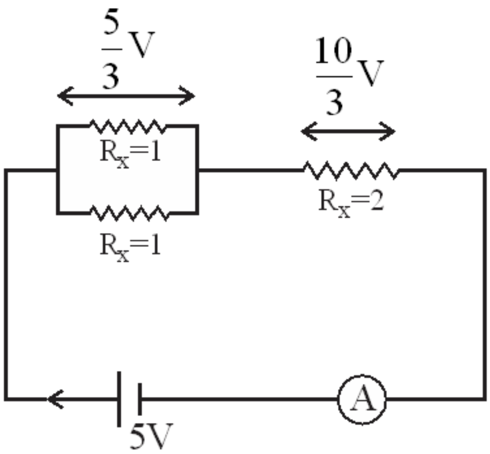
24)

emf = Blv

$\Rightarrow 3.6 \times 10^{-6} = (1.2 \times 10^{-3})(3 \times 10^{-2})v$

$\Rightarrow v = 10$cm/s

25) 



$$i = \frac{5}{2 + \frac{1}{2}} = 2A$$

PART-2 : CHEMISTRY

26)

Correct order of EA = Cl > F > Br > I

27)

Due to high polarising power of Be^{+2} , $BeCO_3$ is thermally less stable.

28)

29) $PCl_3 + 3C_2H_5OH \xrightarrow{\Delta} 3C_2H_5Cl + H_3PO_3$

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$$

$$CaO \xrightarrow{C,\ D} CaC_2 + CO$$

30)

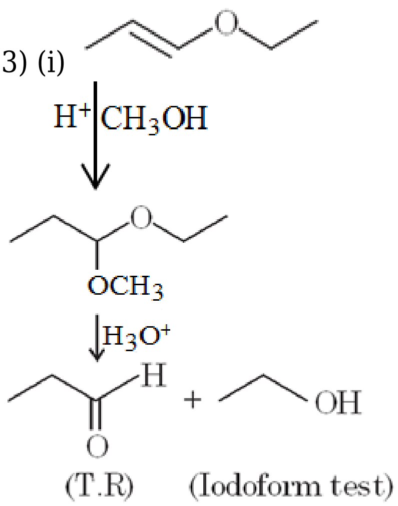
$Fe + 5CO \rightarrow Fe(CO)_5$

32)

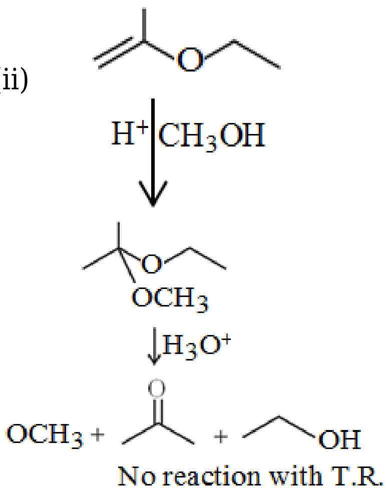
Molarity of 10V H_2O_2 = 0.8 gm

$$\%\frac{w}{w} = \frac{0.89 \times 34}{1000} \times 100 = 3\%$$

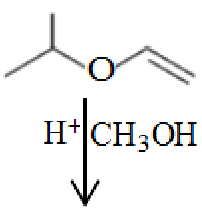
33) (i)

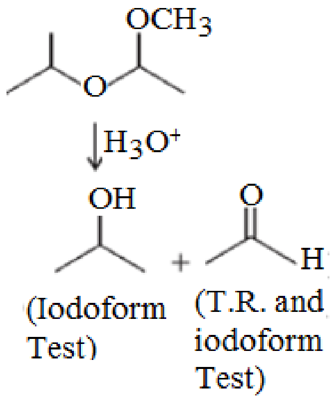


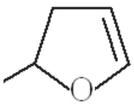
(ii)

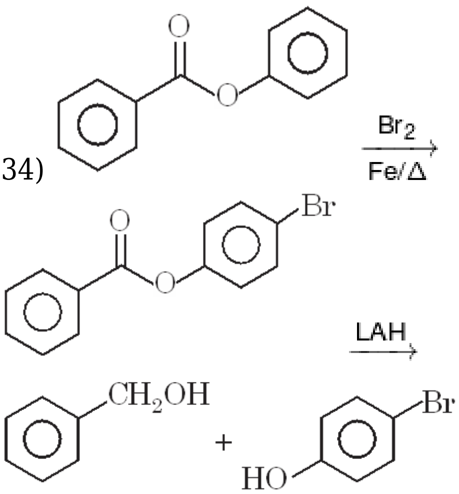


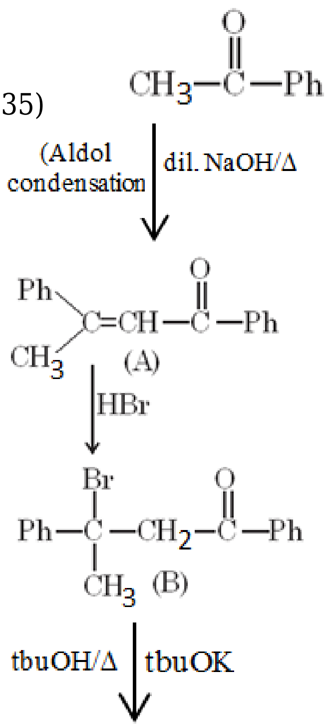
(iii)

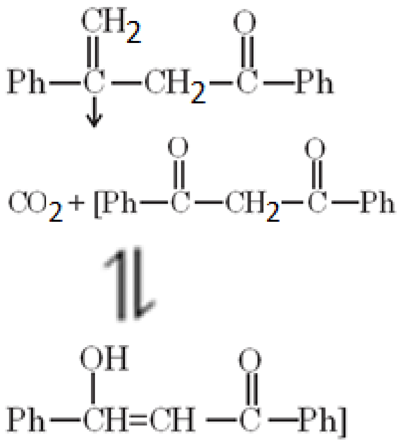




(iv)  has the formula C_5H_8O

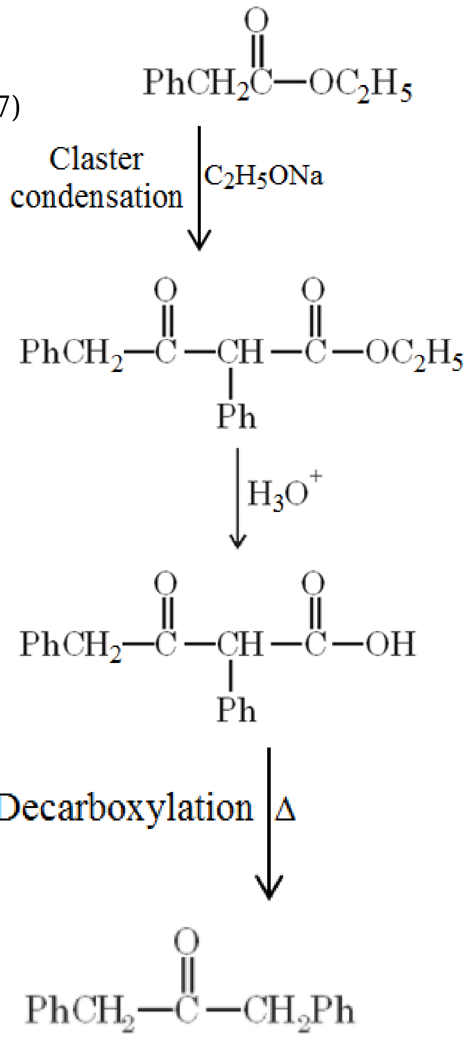
34)


35)




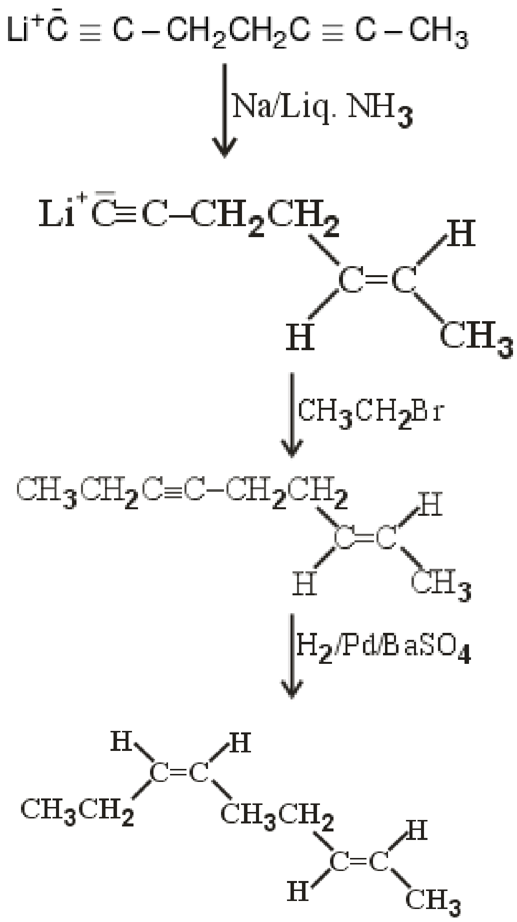
36) $PhNH_2 \xrightarrow[(0^0-5^0\,C)]{NaNO_2/HCl} PhN_2^+ \xrightarrow[HBr]{CuBr} PhBr$

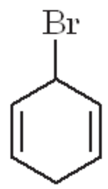
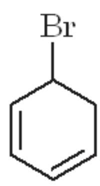
X must be Li or Mg

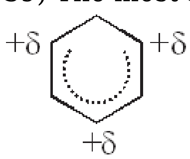
37) 

38) $CH\equiv C\text{-}CH_2CH_2C\equiv C\text{-}CH_3 + CH_3Li$

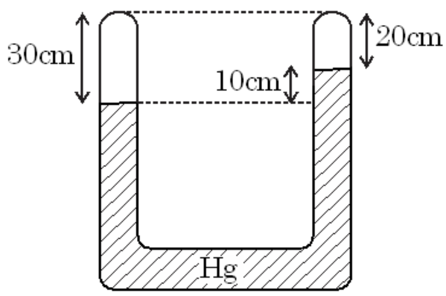
$\downarrow$



39) The most stable intermediate carbocation is formed from both  and  that is



. However (B) is less stable, and hence more reactive.

40) 

For air : $P_1V_1 = P_2V_2$

$\Rightarrow 80 \times 25 = P_2 \times 20$

$\Rightarrow P_2 = 100\text{cm for Hg}$

For the left column :-

$P_{left} = P_{right} + 10\text{cm of Hg}$

$\Rightarrow P_{left} = 110 \text{ cm of Hg}$

Also, as T is constant

$$\frac{P_1V_1}{n_1R} = \frac{P_1V_2}{n_2R}$$

Where,

n_1 = moles before dissociation

n_2 = moles after dissociation

$$\Rightarrow \frac{80 \times 25}{n_1} = \frac{110 \times 30}{n_2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{33}{20} = 1.65 \quad (1)$$

$$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2$$

For the n_1

eqn $n_1(1-\alpha) \quad \frac{n_1\alpha}{2} \quad \frac{3n_1\alpha}{2}$

$$\therefore n_2 = n_1(1-\alpha) + \frac{n_1\alpha}{2} + \frac{3n_1\alpha}{2}$$

$$\Rightarrow n_2 = n_1(1+\alpha)$$

$$\Rightarrow \frac{n_2}{n_1} = 1 + \alpha = 1.65$$

$$\Rightarrow \alpha = 0.65$$

41) Since O_2 is added at constant partial pressure, the total pressure decreases and the total volume increases.

$2N_2O_5(g) \rightleftharpoons 4NO_2(g) + O_2(g)$

Let partial pressure at initial eql be $\quad x \quad y \quad z$

$$\therefore K_P = \frac{y^4z}{x^2} \quad ...(1)$$

Pressure after adding O_2, assuming that volume increases to 'a' times the initial volume $\quad x/a \quad y/a \quad z$

$$\therefore Q_P = \frac{(y/a)^4z}{(x/a)^2} = \frac{y^4z}{x^2} \times \frac{1}{a^2} \quad ...(2)$$

$\therefore Q_P < K_P$ equilibrium shift fowards.

42) $\Delta U = W$

$$\Rightarrow \frac{P_2V_2 - P_1V_1}{\gamma - 1} = -P_{ext}\Delta V$$

$$\Rightarrow \frac{P_{ext}V_2 - 20}{5/3 - 1} = -P_{ext}(V_2 - 20)$$

$$\Rightarrow \frac{P_{ext}V_2 - 20}{2/3} = -P_{ext}V_2 + 20\,P_{ext}$$

$$\Rightarrow 3P_{ext}V_2 - 60 = -2P_{ext}V_2 + 40P_{ext}$$

$$\Rightarrow V_2 = \frac{60 + 40\,P_{ext}}{5\,P_{ext}} = \frac{12}{P_{ext}} + 8$$

$V_{2_{min}} = 8L$ (when P_{ext} is max^m)

43) $CaCO_3(s) \rightleftharpoons Ca^{2+}(aq.) + CO_3^{2-}(aq.)$

$K_{sp}=10^{-9}$

$CO_3^{2-}(aq.) + H_2CO_3(aq.) \rightleftharpoons 2HCO_3^{-}(aq.)$

$K_c = \frac{K_1}{K_2} = 10^4$

$CaCO_3(s) + H_2CO_3(aq.) \rightleftharpoons$

$Ca^{2+}(aq.) + 2HCO_3^{-}(aq.)$ $K = 10^{-5}$

in	0.4	–	–
eq[1]	(0.4–x)	x	2x

$$K = \frac{[Ca^{2+}][HCO_3^-]^2}{[H_2CO_3]}$$

$$\Rightarrow 10^{-5} = \frac{x.(2x)^2}{(0.4-x)} \simeq \frac{4x^3}{0.4}$$

$$\Rightarrow x^3 = 10^{-6}$$

$$\Rightarrow x = 10^{-2} \text{ mol L}^{-1}$$

= 1 g $CaCO_3$ per L of sol^n.

44) (i) $\therefore -\frac{d[A]}{dt} = k$ (from the graph), it is a zero order reaction.

(ii) unit of rate constant for a zero order reaction is mol $L^{-1}s^{-1}$

(iii) Rate constant does not depend on concentration of the reaction

(iv) Zero order reaction is always a complex reaction.

45) For an e^- in a Bohr's orbit,

$$2\pi r = n\lambda$$

$$\Rightarrow 2\pi r_0 \frac{n^2}{z} = n\lambda$$

$$\Rightarrow \lambda = 2\pi r_0 \frac{n}{z}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{n_2 z_1}{n_1 z_2}$$

$$\Rightarrow \frac{10}{1} = \frac{n_2 \times 2}{1 \times 1}$$

$$\Rightarrow n_2 = 5$$

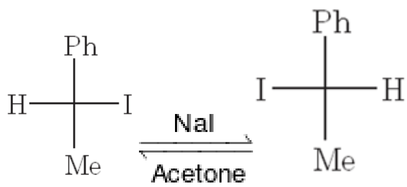
Upon de-excitation visible lines in Balmer series corresponds to the transitions

$5 \rightarrow 2$

$4 \rightarrow 2$

$3 \rightarrow 2$

47) (i) $(CH_3)_3C-Br \xrightarrow{H_2O} (CH_3)_3C-OH \rightarrow$ Achiral (ii)



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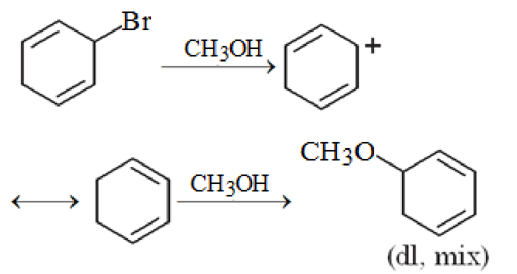
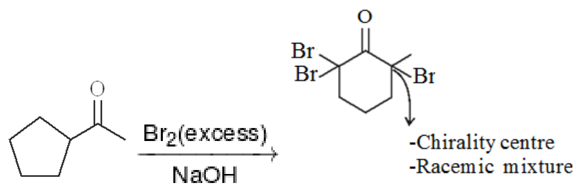
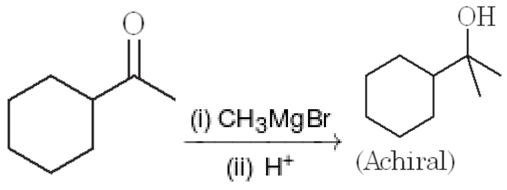
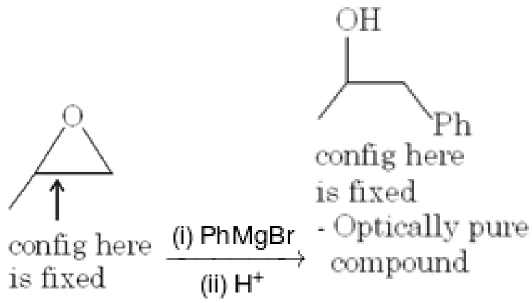
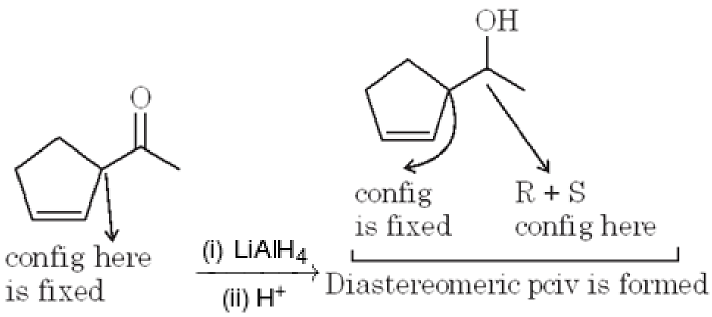


(iv) 

(v) 

(vi) 

(vii) 

48) (ii), (iv), (v), (vi) and (x)

49) $Ag^{+}(aq) + e^{-} \rightleftharpoons Ag(s)$

$$E_{red} = E^{0} - \frac{RT}{nF}\log\frac{1}{[Ag^{+}]}$$

$$\Rightarrow E_{red} = E^{0} + \frac{RT}{F}\log[Ag^{+}]$$

∴ Slope of E_{red} vs $\log[Ag^{+}]$, $M = \dfrac{RT}{F}$

Given $\dfrac{RT}{F} = 0.06$ at 300K and 0.062 at the temperature of the solution.

$$\Rightarrow \frac{RT_1/F}{RT_2/F} = \frac{0.06}{0.062}$$

$$\Rightarrow T_2 = \frac{0.062}{0.06} \times T_1$$

$$= \frac{0.062}{0.06} \times 300 = 310K = 37^0C$$

50) $\frac{r_+}{r_-} = \frac{1.4}{2.6} = 0.538$

$\Rightarrow$ C.N. = 6

The unit cell must be NaCl type

 Edge length = $2(r_+ + r_-)$ = 8 Å

PART-3 : MATHEMATICS

51) Let us assume that no. of cups with handle is n and without handle is m. No. of ways of selecting one cup each is ${}^mC_1 \cdot {}^nC_1 = 36$, mn = 36

Now apply A.M.$\geqslant$G.M.

$\frac{m+n}{2} \geqslant \sqrt{mn}$

$\frac{m+n}{2} \geqslant 6$

$m+n \geqslant 12$

So least value is '12'

52) (x + 10) (x – a) = –5

Case (i) Either x + 10 = 5, x – a = –1

x = 5, –5 – a = –1

a = –4

or x + 10 = –1, x – a = 5

x = – 11, –11 – a = 5

a = –16

Case (ii) x + 10 = –5, x – a = 1

or x + 10 = 1, x – a = –5 it gives same values of 'a'.

Sum of values of 'a' is –16 – 4 = –20

53) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1;\ x, y, z > 0$

Let $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c} \Rightarrow a + b + c = 1$

Also (x – 1) (y – 1) (z – 1)

$= \left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right)$

$= \frac{(1-a)(1-b)(1-c)}{abc}$

$= \frac{(b+c)\ (a+c)(a+b)}{abc}$

Now by A.M.$\geqslant$G.M.

$\frac{b+c}{2} \geqslant \sqrt{bc}$

$b + c \geqslant 2\sqrt{bc}$

$c + a \geqslant 2\sqrt{ac}$

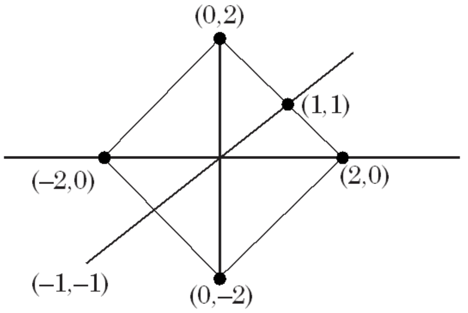
$a + b \geqslant 2\sqrt{ab}$

$(b+c)(c+a)(a+b) \geqslant 8abc$

$$\frac{(b+c)(c+a)(a+b)}{abc} \geqslant 8$$

54) y = x + cosx

$$\frac{dy}{dx} = 1 - \sin x = 0 \Rightarrow \sin x = 1 \Rightarrow x = 2n\pi + \frac{\pi}{2}$$



Putting value of x in given function

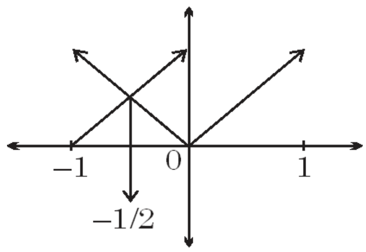
y = x + cosx

$y = 2n\pi + \frac{\pi}{2} + 0$

$y = 2n\pi + \frac{\pi}{2}$

Clearly point lie on the line y = x

Now x can be between –1 and 1 whereas least value of x is $\frac{\pi}{2}$, if x > 0 max value of x is $-\frac{\pi}{2}$ if x < 0. So no value of 'x'.



55)

$$f(x) = \begin{cases} -x, & -1 \leqslant x < -\frac{1}{2} \\ x+1, & \frac{-1}{2} \leqslant x < 0 \\ x, & 0 \leqslant x \leqslant 1 \end{cases}$$

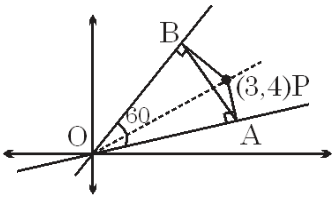
Clearly,

$$\int_{-1}^{-1/2} -x.dx + \int_{-1/2}^{0} (x+1)\,dx + \int_{0}^{1} x.dx$$

$$-\frac{1}{2}[x^2]_{-1}^{-1/2} + \left[\frac{x^2}{2} + x\right]_{-1/2}^{0} + \left[\frac{x^2}{2}\right]_0^1$$

$$-\frac{1}{2}\left[\frac{1}{4} - 1\right] + \left[0 - \left(\frac{1}{8} - \frac{1}{2}\right)\right] + \frac{1}{2}$$

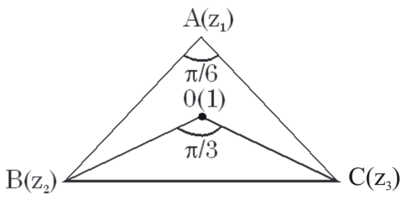
$$\frac{3}{8} + \left(\frac{3}{8}\right) + \frac{1}{2} = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$$

56) 

Apply sin rule in triangle OAB, $\frac{AB}{\sin 60^o} = OP$

$OP = \sqrt{3^2 + 4^2} = 5$

$AB = 5.\sin 60^o = \frac{5\sqrt{3}}{2}$

57) 

z = 1 is circum center. So triangle OBC is equilateral

$z_2^2 + z_3^2 + 1 = z_2 + z_3 + z_2 z_3$

Expression $z_2^2 - z_2 - z_2 z_3 - z_3 + z_3^2 + 1 = 0$

$z_2(z_2 - 1) - z_3(z_2 + 1) + (z_3 + 1)(z_3 - 1) = -2$

So value is –2

58) $1 + |\sin\theta| + |\cos\theta| - 1 - 2 = 0$

$\Rightarrow |\sin\theta| + |\cos\theta| = 2$ which is not possible.

59) Divide throughout by x^3y, we get

$2\frac{dx}{x} + 2\frac{dy}{y} - 2\frac{y^3}{x^3}dx + 3.\frac{y^2}{x^2}dy = 0$

$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + \frac{3xy^2dy - 2y^3dx}{x^3} = 0$

$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + \frac{3x^2y^2dy - 2xy^3dx}{x^4} = 0$

$\Rightarrow 2\frac{dx}{x} + 2\frac{dy}{y} + d\left(\frac{y^3}{x^2}\right) = 0$, integrate

$\Rightarrow 2(\ln x + \ln y) + \frac{y^3}{x^2} = C$, Now put x = y = 1

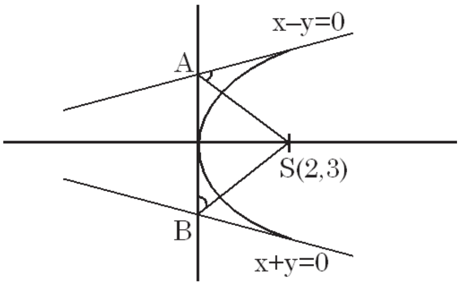
$\Rightarrow 2\ln(1) + 1 = C \Rightarrow C = 1 \Rightarrow 2\ln(xy) + \frac{y^3}{x^2} = 1$

$m = 3, n = 2 \Rightarrow m + n = 5$

60) Minimum value of $\sqrt{(x-1)^2 + (y-1)^2}$

Clearly minimum distance of circle from the point (1, 1)

$\sqrt{(4-1)^2 + (5-1)^2} = 5, 7 - 5 = 2$

61) 

Equation of AS :

(y – 3) = –1(x – 2)

y – 3 = -x + 2

x + y = 5

For point 'A' solve

x + y = 5

y = x

2x = 5

$x = \frac{5}{2}, y = \frac{5}{2}$

$A\left(\frac{5}{2}, \frac{5}{2}\right)$

Equation of BS :

(y – 3) = 1(x – 2)

y – 3 = x – 2

x – y = –1

For point 'B' solve

x – y = –1

x + y = 0

2x = -1

$x = -\frac{1}{2}, y = \frac{1}{2}$

Now equation of line AB

$y - \frac{1}{2} = \frac{\frac{5}{2} - \frac{1}{2}}{\frac{5}{2} + \frac{1}{2}}\left(x + \frac{1}{2}\right)$

$y - \frac{1}{2} = \frac{2}{3}\left(x + \frac{1}{2}\right)$

$3y - \frac{3}{2} = 2x + 1$

6y - 3 = 4x + 2

4x - 6y + 5 = 0

Option 'A' is correct.

62) S(6, 13) and $S'(25, 8)$

P(1, 1), S'P = 25, SP = 13

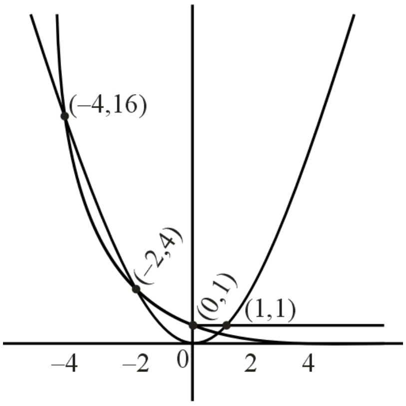
$S'P - SP = 12$

2a = 12

$S'S = \sqrt{(25-6)^2 + (8-13)^2}$

$= \sqrt{361 + 25} = \sqrt{386}$ = 2ae

$e = \frac{\sqrt{386}}{12}$



63)
From graph clearly '4' points of non-differentiability.

64) For x = 1 $0 = f(y) + f\left(\frac{1}{y}\right)$, interchanging x and y, we have $2f(y) = f(xy) + f\left(\frac{y}{x}\right)$ adding f(xy) = f(x) + f(y)
Partial differentiation with respect to 'x'
$f'(xy).y = f'(x)$ ....(1)
Partial differentiation with respect to 'y'
$f'(xy).x = f'(y)$ ....(2)
$\frac{y}{x} = \frac{f'(x)}{f'(1)}$ put y = 1
$\frac{1}{x} = \frac{f'(x)}{f'(1)}$
$\frac{1}{\ln 6} \times \frac{1}{x} = f'(x)$
Integrate with respect to 'x'
$f(x) = \frac{1}{\ln 6}.\ln x + C$ put x = 1
f(1) = 0 + C = 0
$f(x) = \frac{\ln x}{\ln 6}$
$f(7776) = \frac{\ln(7776)}{\ln 6} = 5$

65) $((\vec{b} \times \vec{c}).\vec{a})\vec{c} - ((\vec{b} \times \vec{c}).\vec{c})\vec{a} = 3\vec{c}$
$[\vec{a}\ \vec{b}\ \vec{c}]\vec{c} = 3\vec{c}$
$[\vec{a}\ \vec{b}\ \vec{c}] = 3$
We know that $[\vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}\ \ \vec{a} \times \vec{b}] = [\vec{a}\,\vec{b}\,\vec{c}]^2 = 9$

66) There will be no effect due to addition of constant in variables.

67) $\begin{bmatrix} 0 & \otimes \\ -\otimes & O \end{bmatrix}$ If matrix is of order 2 × 2, then $\otimes$ this place can be filled in 6 ways so 6 matrices can be formed. If matrix is of order 3 × 3, then

$$\begin{bmatrix} 0 & \otimes & \ominus \\ -\otimes & 0 & \oplus \\ -\ominus & -\oplus & 0 \end{bmatrix}$$

$\otimes$ this place can be filled in 6 ways

$\ominus$ this place can be filled in 4 ways

$\oplus$ this place can be filled in 2 ways

So total matrices 6 × 4 × 2 = 48

Now total matrices 48 + 6 = 54

68) $\sum_{r=0}^{m} r.r! = ((r+1)-1)r! = (r+1)! - r!$

$\sum_{r=0}^{m} (r+1)! - r! = (m+1)! - 1$

$({}^mC_0)^2 + ({}^mC_1)^2 + ({}^mC_2)^2 + \ldots\ldots + ({}^mC_m)^2 = {}^{2m}C_m$

$\sum_{r=0}^{m} (2r-1) = m^2 - 1$

So Row 1 and Row 3 is same so value of determinant is zero.

69) Putting (0, 0, 0) expression is – 1 and putting (1, 1, 1) expression is $2 + a^2 + ab - 1$

Both should have opposite sign so

$2 + a^2 + ab - 1 > 0$

$a^2 + ab + 1 > 0$

its discriminant is negative

$D = b^2 - 4 < 0 = -2 < b < 2$

No. of integral value of 'b' is {–1, 0, 1}

70) Using $AM \geqslant GM$

$\frac{x+x+x+y+y}{5} \geqslant (x^3.y^2)^{1/5}$

$\frac{3x+2y}{5} \geqslant (2^{15})^{1/5}$

$(3x+2y)_{min} = 40$

71) $x^4 - 4x^3 + 6x^2 - 4x + 1 = 2022$

$(x-1)^4 = 2022$

$(x-1)^2 = -\sqrt{2022}$ because only non real roots required

$x^2 + 1 - 2x + \sqrt{2022} = 0$

$x^2 - 2x + 1 + \sqrt{2022} = 0$

p = product of roots = $1 + \sqrt{2022}$

72) Equation of normal at $P(3\cos\theta, 2\sin\theta)$ is

$3x\sec\theta - 2y\,\mathrm{cosec}\theta = 5$

Now it is tangent to circle so

$\frac{5}{\sqrt{9\sec^2\theta + 4\,\mathrm{cosec}^2\theta}} = \sqrt{3}$

But $\{9\sec^2\theta + 4\operatorname{cosec}^2\theta\} \geqslant (3+2)^2 = 25$
No such θ exist

73) $\sin x . \sin\left(\frac{1}{x}\right) = 1$

$\sin\left(\frac{1}{x}\right) = \operatorname{cosec} x$

Clearly $\sin\left(\frac{1}{x}\right) = \operatorname{cosec} x = 1$ or $\sin\left(\frac{1}{x}\right) = \operatorname{cosec} x = -1$ which is not possible for same value of x.

74) $x \in$ prime and x < 10 $\Rightarrow x = 2, 3, 5, 7$
Total no. of A(x, y) pair n(A) = 4 × 10 = 40 and $x^2 - 3y^2 = 1 \Rightarrow x^2 = 3y^2 + 1$.
For above condition to be satisfied only two such pairs (2, 1) and (7, 4) are possible

$\Rightarrow P(A) = \frac{2}{40} = \frac{1}{20} = P$

75) $\lim_{k\to\infty} \int_0^{k[x]} (\{kt\})^k = \lim_{k\to\infty} \int_0^{k^2[x].1/k} (\{kt\})^k$

$= \lim_{k\to\infty} k^2[x] \int_0^{1/k} (kt)^k.dt$

$= \lim_{k\to\infty} k^2[x].k^k.\left[\frac{t^{k+1}}{k+1}\right]_0^{1/k}$

$= \lim_{k\to\infty} k^2[x].k^k.\frac{1}{k^{k+1}(k+1)}$

$= [x] \lim_{k\to\infty} \frac{k}{k+1} = [x]$ or $\lambda = 2$