PART-1 : PHYSICS

SECTION-I

1)

A monochromatic source of light operating at 200W emits 4×10^{20} photons per second then (λ) wavelength of light is →

(A) 200 nm
(B) 400 nm
(C) 100 nm
(D) 1800 nm

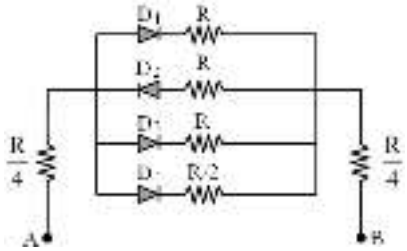
2) Light with an energy flux of 25×10^4 Wm^{-2} falls on a perfectly reflecting surface at normal incidence. If the surface area is 15 cm^2, the average force exerted on the surface is :-

(A) 1.25×10^{-6} N
(B) 2.50×10^{-6} N
(C) 1.20×10^{-6} N
(D) 3.0×10^{-6} N

3) The density 'd' of nuclear matter varies with nucleon number A is :-

(A) $d \propto A^3$
(B) $d \propto A^2$
(C) $d \propto A$
(D) $d \propto A°$

4) In the following circuits P–N junction diodes D_1, D_2 and D_3 are ideal for the following potentials of A and B. The correct increasing order of resistance between A and B will be



(i) –10V, –5V (ii) –5V, –10V
(iii) –4V, –12V

(A) (i) < (ii) < (iii)
(B) (iii) < (ii) < (i)
(C) (ii) = (iii) < (i)

(D) (i) = (iii) < (ii)

5) The potential energy of a particle varies with distance x as $U = \frac{A\sqrt{x}}{x + B}$. The dimensions of AB are :-

(A) $ML^{5/2}T^{-2}$

(B) $M^1L^2T^{-2}$

(C) $M^{3/2}L^{3/2}T^{-2}$

(D) $M^1L^{7/2}T^{-2}$

6) The velocity time relation of an electron starting from rest is given by, v = Kt, where K = 2 m/s^2. The distance (in m) traversed in 3 sec is:

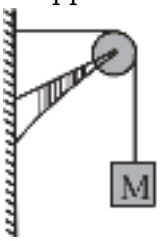
(A) 9

(B) 10

(C) 12

(D) 15

7) A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given :-



(A) $\sqrt{2}$ Mg

(B) $\sqrt{2}$ mg

(C) $\sqrt{(M+m)^2 + m^2}$ g

(D) $\sqrt{(M+m)^2 + M^2}$ g

8) An elastic string of unstretched length □ and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is :-

(A) $\frac{1}{2}kx^2$

(B) $\frac{1}{2}k(x^2 + y^2)$

(C) $\frac{1}{2}ky(2x + y)$

(D) $\frac{1}{2}kx(x + 2y)$

9) The height at which the weight of a body becomes $1/9^{th}$ its weight on the surface of earth (radius of earth is R) :-

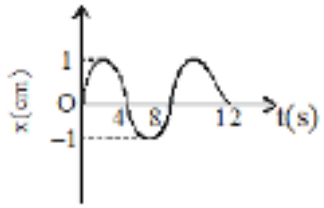
(A) h = 3R

(B) h = R

(C) $h = \frac{R}{2}$

(D) h = 2R

10) The x-t graph of a particle undergoing SHM is shown below. The acceleration of the particle at t = 8/3 s is :-



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

(B) $-\frac{\pi^2}{32}$ cm/s^2

(C) $\frac{\pi^2}{32}$ cm/s^2

(D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

11) An air bubble in a glass slab with refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness (in cm) of the slab is :-

(A) 12

(B) 16

(C) 8

(D) 10

12) In a Young's double slit experiment, light has a frequency of 6×10^{14} Hz. The distance between the centres of adjacent bright fringes is 0.75 mm. If the screen is 1.5 m away then find the distance between the slits :

(A) 1 mm

(B) 1 μm

(C) 1 m

(D) 1 cm

13) Two vibrating strings of the same materials but lengths L and 2L have radii 2r and r, respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental notes, the one of length L with frequency v_1 and the other with frequency v_2. The ratio v_1/v_2 is given by :

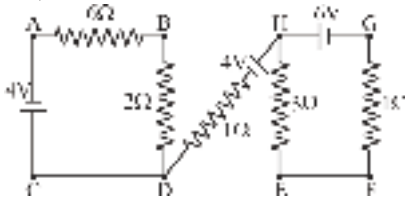
(A) 1

(B) 2

(C) 3

(D) 4

14) In the network shown in the figure, choose the incorrect statement :-



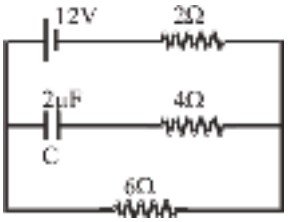
(A) Potential difference between points H and B is 5 V

(B) Potential difference between points D and B is 1 V

(C) Potential difference between points D and E is 1 V

(D) Potential difference between points D and G is 2 V

15) Find the charge on the capacitor C in the following circuit :-



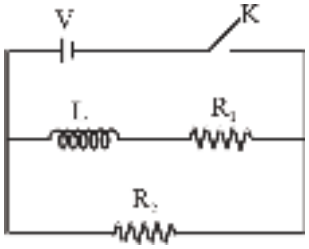
(A) 12 μC

(B) 14 μC

(C) 20 μC

(D) 18 μC

16) In the circuit shown below, the key K is closed at t = 0. The current through the battery is :-



(A) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(B) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

(C) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞

(D) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

17) Power dissipated in pure inductance will be :-

(A) $\frac{1}{2}Li^2$

(B) $2Li^2$

(C) $\frac{Li^2}{4}$

(D) Zero

18) Two small conducting spheres of equal radius have charges +10μC and –20μC respectively and placed at a distance R from each other experience force F_1. If they are brought in contact and separated to the same distance, they experience force F_2. The ratio of F_1 to F_2 is:-

(A) 1 : 8

(B) – 8 : 1

(C) 1 : 2

(D) – 2 : 1

19) The magnetic moment of a magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg/m^3 then intensity of magnetisation will be :-

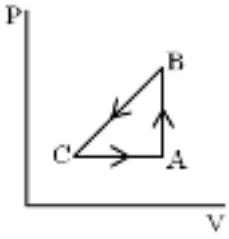
(A) 0.9 A/m

(B) 0.09 A/m

(C) 9 A/m

(D) 90 A/m

20) A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60 J of heat during CA. A work 50 J is done on the gas during the part BC. The internal energy of the gas at A is 1560 J. The work done by the gas during the part CA is :



(A) 20 J

(B) 30 J

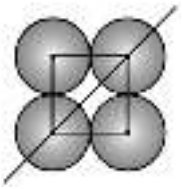
(C) –30 J

(D) –60 J

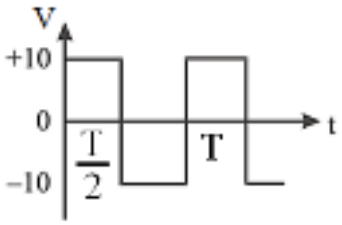
SECTION-II

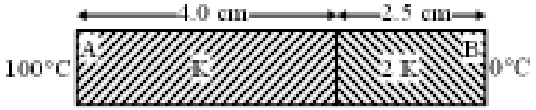
1) The energy band gap of semiconducting material to produce violet (wavelength = 4000 Å) LED is _____ eV. (Round off to the nearest integer).

2) Four disc each of mass m and radius R are placed as shown in figure then what will be the moment of inertia of the system about an axis passing through centre of diagonally opposite disc and

in the same plane as shown in figure is NmR^2. Then N is :- 

3) A 2.0m long string with a linear mass density of 5.2×10^{-3} kg m^{-1} and tension 52N has both of its ends fixed. It vibrates in a standing wave pattern with four antinodes. Frequency of the vibraion (in Hz) is :-

4) The r.m.s. voltage of the wave form shown is : 

5) As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)$ K, then the value of α will be _____. 

PART-2 : CHEMISTRY

SECTION-I

1) Wrong match is :-

(A) B < C < N < O (I^{st} IP)

(B) $Al^{+3} < Mg^{+2} < Na^{+} < F^{-}$ (Ionic radius)

(C) Li < Na < K (Metallic radius)

(D) I < Br < F < Cl (EA)

2) What is the proper order of decreasing F–X–F angle in the compounds below (where X is central atom) ?
(a) BF_3 (b) BeF_2 (c) CF_4

(A) a > b > c

(B) b > c > a

(C) c > b > a

(D) b > a > c

3) Hybridisation of which complex in correctly matched :-

(A) $AuCl_4^-$ – sp^3

(B) $[Co(OX)_3]^{3-}$– sp^3 d^2

(C) $RhCl(PPh_3)_3$–dsp^2

(D) $[Fe(NH_3)_6]^{2+}$ – d^2sp^3

4) Which of the following complex is optically active ?

(A) trans-$[Co(NH_3)_4Cl_2]^+$

(B) $[Cr(H_2O)_6]^{3+}$

(C) cis-$[Co(NH_3)_2(en)_2]^{3+}$

(D) trans-$[Co(NH_3)_2(en)_2]^{3+}$

5) When acidified solution of $K_2Cr_2O_7$ is shaken with aqueous solution of $FeSO_4$, then :

(A) $Cr_2O_7^{2-}$ ion is reduced to Cr^{3+} ions

(B) $Cr_2O_7^{2-}$ ion is converted to CrO_4^{2-} ions

(C) $Cr_2O_7^{2-}$ ion is reduced to Cr

(D) $Cr_2O_7^{2-}$ ion is converted to CrO_3

6) Which of the following statement is not correct ?

(A) Lu^{+3} has the strongest tendency toward complex formation among trivalent lanthanoid ions

(B) Ce has maximum composition in misch metal

(C) f-block elements can have electrons from f^0 to f^{14}

(D) Nd, Np and Nb all are f-block elements

7)

What is the hybridization on the centre atom of SiO_2.

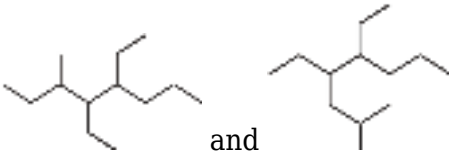
(A) sp

(B) sp^2

(C) sp^3

(D) sp^3d

8) Given compound shows which type of isomerism?

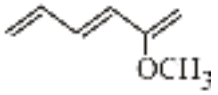
 and

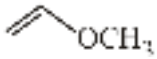
(A) Chain isomerism

(B) Positional isomerism

(C) Functional group isomerism

(D) Metamerism

(I)  (II) 

(III) 

9) For the given compound the correct order of resonance energy is :-

(A) III > I > II

(B) II > I > III

(C) I > II > III

(D) III > II > I

10) In following conversion

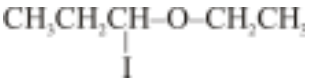
$CH_3CH{=}CH{-}O{-}CH_2CH_3 \xrightarrow[\text{heat}]{\text{Conc. HI}}$

major product is :

(A) $CH_3CH{=}CHI$ and CH_3CH_2I

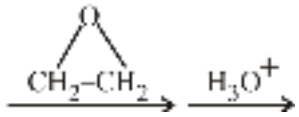
(B) $CH_3CH = CHI$ and CH_3CH_2OH

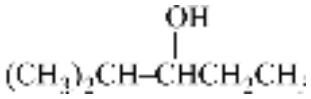
(C) CH_3CH_2CHO and CH_3CH_2I

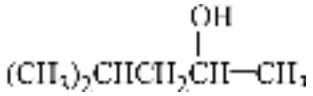
(D) 

11) What is the major organic product of the following sequence of reaction?

$(CH_3)_2CHCH_2{-}OH \xrightarrow{PBr_3} \xrightarrow{Mg}$



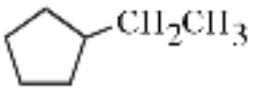
(A) 

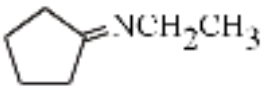
(B) 

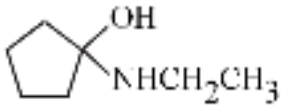
(C) $(CH_3)_2CHCH_2CH_2{-}OH$

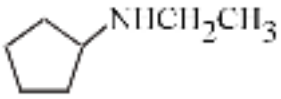
(D) $(CH_3)_2CHCH_2CH_2CH_2OH$

12) What is the major product of the following reaction ?  $+ CH_3CH_2NH_2 \xrightarrow[Pt]{H_2}$?

(i) 

(ii) 

(iii) 

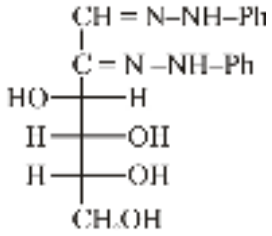
(iv) 

(A) (i)

(B) (ii)

(C) (iii)

(D) (iv)

13) 

The given osazone can be obtained by :

(A) D-glucose

(B) D-mannose

(C) D-Idose

(D) Both (A) & (B)

14) For the reaction
$2A(g) + 3B(g) \rightleftharpoons 3A(g) + 2B(g)$; ΔH = –ve.
An increase in pressure shows that :

(A) forward reaction is favoured

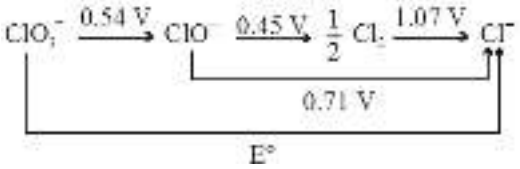
(B) no effect

(C) backward reaction is favoured

(D) all of the above

15)

The Eº in the given diagram is :-



(A) 0.60

(B) 0.3

(C) 0.9

(D) 0.1

16) Two solutions A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution B is_____.
(In nearest integer)

(A) 22

(B) 11

(C) 33

(D) 5.5

17) For $[CrCl_3.xNH_3]$, elevation in boiling point of one molal solution is triple of one molal aqueous solution of urea. Assuming 100% ionisation of complex molecule and coordination number as six, calculate the value of x.

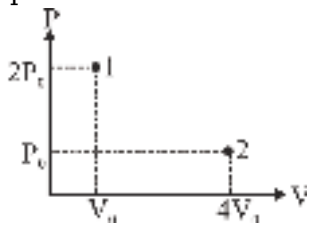
(A) 10

(B) 1.5

(C) 5.00

(D) 2.5

18) A liquid is subjected to an adiabatic process from state 1 to state 2 as shown in the graph. What is the enthalpy change for the process?



(A) $\Delta H = \frac{2\gamma P_0 V_0}{\gamma - 1}$

(B) $\Delta H = -P_0V_0$

(C) $\Delta H = \frac{3\gamma P_0 V_0}{\gamma - 1}$

(D) Zero

19) The following reaction is at equilibrium at 298K.
$2NO(0.00001\ bar) + Cl_2(g, 0.01\ bar) \rightleftharpoons 2NOCl\ (g, 0.01\ bar)$
ΔG° for the reaction is :-

(A) –45.65 kJ

(B) –28.53 kJ

(C) –22.83 kJ

(D) –57.06 kJ

20) For the reaction,
$CaO + 2HCl \rightarrow CaCl_2 + H_2O$
1.23 g of CaO is reacted with excess of hydro-chloric acid and 1.85 g of $CaCl_2$ is formed. What is the percent yield ?

(A) 76.1

(B) 86.3

(C) 95.1

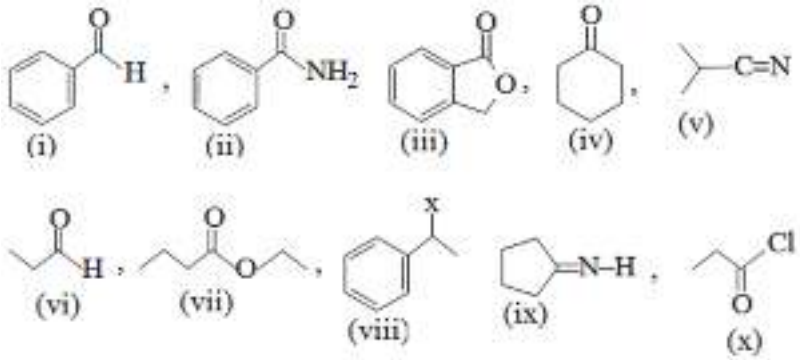
(D) None of these

SECTION-II

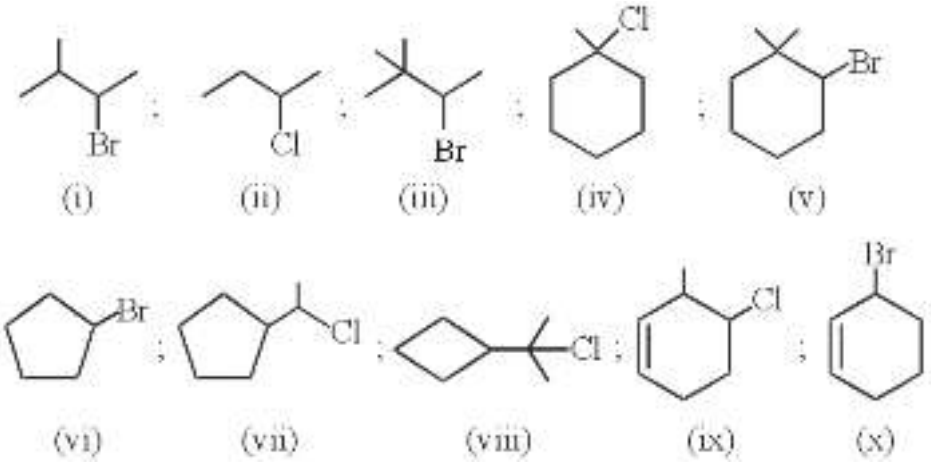
1) How many species are paramagnetic ?
(i) Na
(ii) Ca
(iii) K
(iv) Mg
(v) Cr
(vi) Cu

2) How many of these elements have greater IE_1 than both adjacent elements along a period?
B, S, O, Li, Mg, P, Na, K

3) Identify number of compounds which are reduced by $NaBH_4$.



4) How many substrates will show rearrangement during S_N1 reaction.



5) 4 mole of a mixture of Mohr's salt and $Fe_2(SO_4)_3$
requires 500 mL of 1 M $K_2Cr_2O_7$ for complete oxidation in acidic medium. The mole % of the Mohr's salt in the mixture is :-

PART-3 : MATHEMATICS

SECTION-I

1) A man saves Rs. 100 in the first month of his service. In each of the subsequent months his saving increases by twice of the saving of immediately previous month. His total saving from the start of service will be Rs. 409500 after :-

(A) 10 months

(B) 14 months

(C) 12 months

(D) 19 months

2) If $\alpha, \beta, \gamma, \delta$ are the roots of
$x^4 - 100x^3 + 2x^2 + 4x + 10 = 0$ then
$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ is equal to :-

(A) $\frac{2}{5}$

(B) $\frac{1}{10}$

(C) 4

(D) $\frac{-2}{5}$

3) If matrix $A = [a_{ij}]_{3 \times 3}$ and $a_{ij} + a_{ji} = 0$ and
element $a_{ij} \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7\}$,
then number of matrix A is equal to :-

(A) 3375

(B) 2744

(C) 6750

(D) 5488

4) $ax + y + z = 0$, $x - by + 3z = 0$ and $x - y - z = 0$ has a non-trivial solution, then :-

(A) a = 1 or b = 5

(B) a = –1 or b = –3

(C) a = –1 or b = 3

(D) a = 1 or b = 3

5) A person goes office by car or scooter or bus or train, probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}, \frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train are

$\frac{2}{9}, \frac{1}{9}, \frac{4}{9}, \frac{1}{9}$ respectively. Given that he reaches office in time, then the probability that he travelled by a car is given by :-

(A) $\frac{1}{7}$

(B) $\frac{2}{7}$

(C) $\frac{3}{7}$

(D) None

6) If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 9$, then value of $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$ is equal to:-

(A) 216

(B) 18

(C) 64

(D) None

7) The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y where $x < y$ are 6, and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

(A) 162

(B) 320

(C) 674

(D) 420

8) The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to :

(A) – 1

(B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{4}$

9) $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to

(A) $\frac{3}{16}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{9}{32}$

10) Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

(A) 5

(B) 2

(C) 0

(D) 3

11) If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(A) 140

(B) 175

(C) 150

(D) 125

12) Let $f(x) = \begin{cases} x-1, & x \text{ is even}, \\ 2x, & x \text{ is odd}, \end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a))) = 21$, then $\lim\limits_{x \to a^-} \left\{ \frac{|x|^3}{a} - \left[\frac{x}{a}\right] \right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(A) 121

(B) 144

(C) 169

(D) 225

13) The differential equation of the family of circles passing the origin and having center at the line y = x is :

(A) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$

(B) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(C) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

(D) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

14) The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

(A) $\frac{8}{3}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{4}{3}$

15) Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0. If $\int_0^1 (1-x^{10})^{20}dx = a \times \beta(b, c)$, then 100(a + b + c) equals ____.

(A) 1021

(B) 1120

(C) 2012

(D) 2120

16) Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If I(0) = 3, then $I\left(\frac{\pi}{12}\right)$ is equal to :

(A) $\sqrt{3}$

(B) $3\sqrt{3}$

(C) $6\sqrt{3}$

(D) $2\sqrt{3}$

17) The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to :

(A) $\frac{\sqrt{1691}}{5}$

(B) $\frac{\sqrt{2009}}{5}$

(C) $\frac{\sqrt{1741}}{5}$

(D) $\frac{\sqrt{1541}}{5}$

18) Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $a \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals ____ .

(A) 1638

(B) 1575

(C) 1500

(D) None of these

19) Let $\alpha, \beta, \gamma, \delta \in Z$ and let A (α, β), B (1, 0),C (γ,δ) and D (1, 2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line 3y = 2x + 1, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

(A) 10

(B) 5

(C) 12

(D) 8

20) If the mirror image of the point P(3,4,9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ), then $14(\alpha + \beta + \gamma)$ is :

(A) 102

(B) 138

(C) 108

(D) 132

SECTION-II

1) If $1 + 2^2 + 2^3 + ...+ 2^{99}$ is divided by 5, then remainder is 'a', then the value of 3a + 5 is :-

2) Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

3) If

$$y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$$

then $96y'\left(\frac{\pi}{6}\right)$ is equal to :

4) $\int_{-4}^{0} (|x+1| + [x+3])\,dx$ (where [.] denotes greatest integer function) is equal to

5) Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$

subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{\ell}{m}(1+\sqrt{n})$, where $\square$ and m are co-prime numbers, then $\square^2 + m^2 + n^2$ is equal to

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	B	B	D	C	D	A	D	C	D	D	A	A	A	C	D	B	D	B	B	B

SECTION-II

Q.	21	22	23	24	25
A.	3	5	100	10	21

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	A	D	C	C	A	D	C	B	C	C	D	D	D	B	A	B	C	B	A	A

SECTION-II

Q.	46	47	48	49	50
A.	4	2	6	6	75

PART-3 : MATHEMATICS

SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	C	D	A	B	A	A	B	B	B	D	C	B	C	D	D	B	A	B	D	C

SECTION-II

Q.	71	72	73	74	75
A.	5	0	105	7	182

SOLUTIONS

PART-1 : PHYSICS

1) $n = \frac{P\lambda}{hc}$

2) Average force $F_{av} = \frac{\Delta p}{\Delta t} = \frac{2IA}{c}$

$= \frac{2 \times 25 \times 10^4 \times 15 \times 10^{-4}}{3 \times 10^8} = 2.50 \times 10^{-6}$ N

3)

Density of nuclear matter is independent of mass number.

4) **Case-I :** $V_A < V_B$ [(–10V) < (–5V)]

D_1 & D_3 are RB and D_2 is FB

$R_1 = \frac{R}{4} + R + \frac{R}{4} = \frac{3R}{2}$

Case-II : $V_A > V_B$

$\Rightarrow D_1$ & D_2 are FB and D_3 is RB

$R_2 = \frac{R}{4} + \frac{\frac{R}{2} \times \frac{R}{2}}{\left(\frac{R}{2} + \frac{R}{2}\right)} + \frac{R}{4} = \frac{3R}{4}$

Case-III : $V_A > V_B$

$\Rightarrow$ same as case-II

$R_3 = R$

5)

$V = \frac{A\sqrt{x}}{x + B}$

$B = L \quad A = \frac{ML^2T^{-2}L}{\sqrt{L}}$

$A = ML^{5/2}T^{-2}$

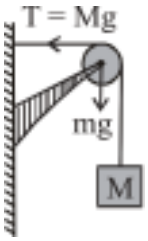
$AB = ML^{7/2}T^{-2}$

6) $a = \frac{dv}{dt} = k = 2$ m/s^2

$S = 0 + \frac{1}{2}(2)(3)^2 = 9$ m

7)

Net force on clamp



$F_{net} = \sqrt{[(M+m)g]^2 + (Mg)^2} = \sqrt{[(M+m)]^2 + M^2}\, g$

8) $W.D = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$

$= \frac{1}{2}k\left(x^2 - (x+y)^2\right)$

$= \frac{1}{2}k\left(x^2 - (x^2 + y^2 - 2xy)\right)$

$= -\frac{1}{2}k\left(y^2 + 2xy\right) = \frac{-1}{2}ky(2x+y)$

W.D on spring $= \frac{1}{2}ky\,(2x+y)$

9) $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$, $g' = \frac{g}{9}$

$\frac{g}{9} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$1 + \frac{h}{R} = 3$; $h = 2R$

10) according to graph

$x = A\sin\omega t$

$A = 1$ cm

and $T = 8$ s

so acceleration $= -A\omega^2\sin\omega t$

$= -(1)\left(\frac{2\pi}{8}\right)^2 \sin\left(\frac{2\pi}{8} \times \frac{8}{3}\right)$

$= -\frac{4\pi}{64} \times \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}\pi^2}{32}\text{cm/s}^2$

11) $\mu = \frac{hac}{happ}$

$1.5 = \frac{has}{5+3}$

hac= 12cm

12) Fringe width $\beta = \frac{\lambda D}{d} = \frac{c\,D}{\nu\,d}$

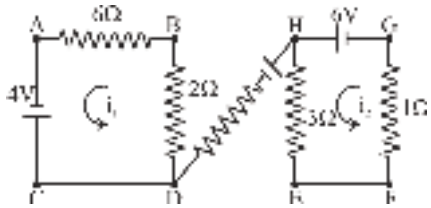
$$\Rightarrow d = \frac{c\,D}{\nu\,\beta} = \frac{3 \times 10^8 \times 1.5}{6 \times 10^{14} \times 75 \times 10^{-5}}$$

$= 10^{-3}$ m = 1 mm

13) $v = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{M}} = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 \rho}}$

Thus, $v \propto \frac{1}{Lr} = \frac{v_1}{v_2} = \frac{L_2 r_2}{L_1 r_1} = \frac{2L}{L} \cdot \frac{r}{2r} = 1$

14)



$i_1 = \frac{4}{8} = \frac{1}{2}A$

$i_2 = \frac{6}{4} = \frac{3}{2}A$

$V_H - V_D = 4V$

$V_D - V_B = 1V$

$V_H - V_B = 5V$

$V_H - V_D = 4V$ ....(i)

$V_H - V_G = 6V$ ....(ii)

Equation (ii) – (i)

$V_D - V_G = 2V$

15) $V_C = V_{6\Omega} = i \times 6 = \left(\frac{12}{2+6}\right) \times 6 = 9V$

$q = CV = 2 \times 9 = 18\ \mu C$

16) At t = 0, current does not flow through inductor

$\Rightarrow i = \frac{V}{R_2}$ At $t = \infty \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$\Rightarrow i = \frac{V(R_1 + R_2)}{R_1 R_2}$

17)

$P_{av} = t_{rms} \times i_{rms} \times \cos\phi$

for pure inductance, $\phi = 90°$, $\cos 90° = 0$

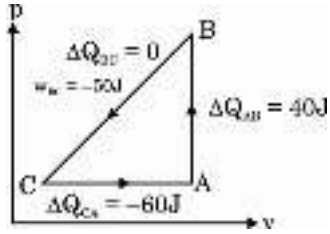
$P_{av} = 0$

18) $F \propto Q_1 Q_2 \Rightarrow \frac{F_1}{F_2} = \frac{Q_1 Q_2}{Q_1 Q_2} = \frac{10 \times -20}{-5 \times -5} = -\frac{8}{1}$

19) $d = \frac{m}{V} \Rightarrow V = \frac{m}{d}$

$V = \frac{75 \times 10^{-3}}{7.5 \times 10^{3}} = 10^{-5}\ m^3$

$I = \frac{M}{V} = \frac{9 \times 10^{-7}}{10^{-5}} = .09\ A/m$

20) 

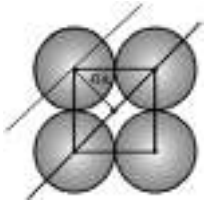
$\Delta Q_{cycle} = 40 - 60 = \Delta W$

$\Rightarrow \Delta W = -20J = W_{BC} + W_{CA}$

$\Rightarrow W_{CA} = -20J - W_{BC} = -20 - (-50) = 30\ J$

21) $E_g = \frac{hc}{\lambda} = \frac{1242}{\lambda(nm)} = \frac{1242}{400} = 3.105$

Answer rounded to 3 eV

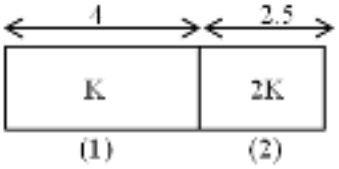
22) 

$I = 2\left(\frac{MR^2}{4}\right) + 2\left[\frac{mR^2}{4} + m\left(\sqrt{2}R\right)^2\right] = 5mR^2$

23) $f = \frac{4v}{2\ell} = \frac{2\sqrt{\frac{T}{\mu}}}{\ell} = \frac{2\sqrt{\frac{52}{5.2 \times 10^{-3}}}}{2}$

f = 100 Hz

24) $V_{rms} = \sqrt{\frac{10^2 + 10^2}{2}} = 10V$



25)

$$\frac{\Delta Q}{\Delta t} = \left(\frac{1}{R}\right)\Delta T$$

R : Thermal resistivity

$$\therefore R_1 = \frac{L_1}{K_1 A} = \frac{L_1}{K(120)}$$

$L_1 = 4$ cm

$A = 120\ cm^2$

$$R_2 = \frac{2.5}{(2K)(120)}$$

Now, R_{eq} of this series combination

$R_{eq} = R_1 + R_2$

where $L_{eq} = 4 + 2.5 = 6.5$

$$\frac{L_{eq}}{K_{eq}(A)} = \frac{4}{K(120)} + \frac{5}{\frac{2}{2K(120)}}$$

$$\frac{6.5}{K_{eq}(120)} = \frac{4}{K(120)} + \frac{5}{4K(120)}$$

$$\frac{6.5}{K_{eq}} = \frac{21}{4K}$$

$$K_{eq} = \frac{26}{21}K = \left(1 + \frac{5}{21}\right)K$$

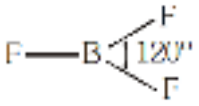
$\therefore$ a = 21

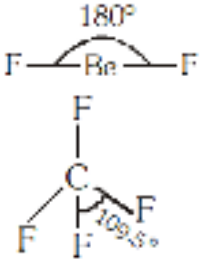
PART-2 : CHEMISTRY

26)

I.P. B < C < O < N

27)





28) (A) $AuCl_4^-$ – dsp^2

(B) $[Co(OX)_3]^{3-}$ – d^2sp^3

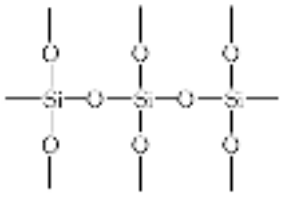
(C) $RhCl(PPh_3)_3$ – dsp^2

(D) $[Fe(NH_3)_6]^{2+}$ – sp^3d^2

30) $6Fe^{2+} + Cr2O_7^{2-} + 14H^+ \rightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$.

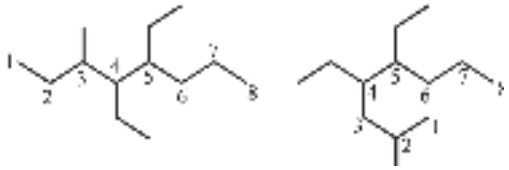
31) Nb belongs to d - block.

32) SiO_2 is a giant molecule it means one Si atom with four oxygen atom and one oxygen atom combine with two Si atom



hybridisation of Si = sp^3

33)

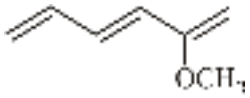


Locant position (3, 4, 5) & (2, 4, 5) differ along same parent chain.

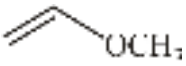
34) Resonance energy a stability due to resonance.

(I) 

Contineous conjugation

(II) 

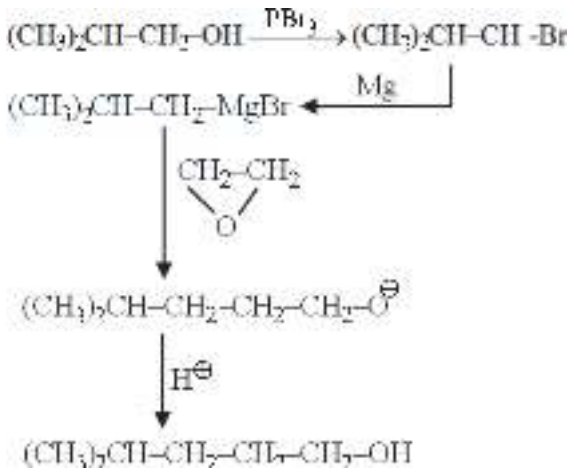
Cross conjugation

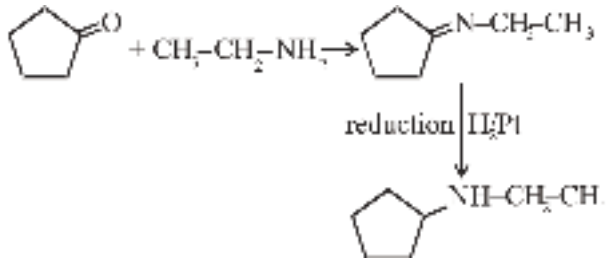
(III) 

Less conjugation thus.

I > II > III

35) CH_3–CH=CH– from unstable carb-cation, so it form alcohol and CH_2–CH_3 group from alkylhalide. We know that OH group is not stable on double bonded carbon it show tautomerism, that why CH_3–CH=CH– group form aldehyde.

36) 

37) 

38) D-Mannose is epimer of D-Glucose.

39) Since $\Delta n = 0$ and thus, increase in pressure has no effect.

40) $\overset{+5}{ClO_3} \rightarrow \overset{+1}{ClO}$ $n_1 = 4$ $E_1^o = 0.54$ ΔG_1^0

$\overset{+1}{ClO^-} \longrightarrow \frac{1}{2}\overset{0}{Cl_2}$ $n_2 = 1$ $E_2^o = 0.45$ ΔG_2^0

$\frac{1}{2}\overset{0}{Cl_2} \longrightarrow \overset{-1}{Cl^-}$ $n_3 = 1$ $E_3^o = 1.07$ ΔG_3^0

$\overset{+5}{ClO_3^-} \longrightarrow \overset{-1}{Cl^-}$ $n = 6$ $E^o = ?$ $\Delta G_4^0 = ?$

$\Delta G^o = \Delta G_1^0 + \Delta G_2^0 + \Delta G_3^0$

$nE^o = n_1E_1^0 + n_2E_2^0 + n_3E_3^0$ $E^o = \dfrac{4 \times 0.54 + 1 \times 0.45 + 1 \times 1.07}{6} \Rightarrow 0.6$ volt

41) 4 gm of NaOH in 100 L sol. $\Rightarrow 10^{-3}$ M sol.
9.8 gm of H_2SO_4 in 100 L sol. $\Rightarrow 10^{-3}$ M sol.
Mixture : 40L of 10^{-3} M NaOH and 10 L of 10^{-3} M H_2SO_4 sol.
Final Conc. of OH^-

$= \dfrac{10^{-3}(40 \times 1 - 10 \times 1 \times 2)}{40 + 10} = 6 \times 10^{-4}$ M

pOH = – log (6×10^{-4})
= 4 – log 6 = 4 – 0.60 = 3.40
pH = 14 – 3.40 = 10.60

42) ΔT_b (complex) = 3 × ΔT_b (area)
Thus, complex should furnish three ions. Therefore, complex is $[CrCl.xNH_3]\ Cl_2$
→ $[CrCl.xNH_3]^{2+} + 2Cl^-$
Also, co-ordination number of Cr is six, Thus, x = 5

43) Since liquid is expanding against external pressure P_0 hence work done.
$w = -P_0 (4V_0 - V_0) = -3P_0V_0$
$\Delta U = w = -3P_0V_0$
$\Rightarrow \Delta H = \Delta U + P_2V_2 - P_1V_1$
$= -3P_0V_0 + 4P_0V_0 - 2P_0V_0$

44) $\Delta G^\circ = -RT\ell nK$

$$= \frac{-8.314 \times 298 \times \ell n}{1000} \frac{(10^{-2})^2}{10^{-2}(10^{-5})^2}$$

$$= \frac{-8.314 \times 298 \times \ell n}{1000} (10^8)$$

$= -8.314 \times 298 \times 8 \times 2.3/1000 = -45.65$ kJ

45) The balanced equation is

$CaO + 2HCl \rightarrow$	$CaCl_2 +$	H_2O
1 mol	1 mol	
56 g	111 g	

56 g of CaO produces $CaCl_2$ = 111 g
1.23 g of CaO will produce $CaCl_2$

$$= \frac{111}{56} \times 1.23 = 2.43 \text{ g}$$

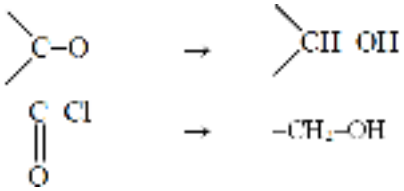
Thus, theoritical yield = 2.43 g
Actual yield = 1.85 g

$$\% \text{ yield} = \frac{1.85}{2.43} \times 100 = 76.1$$

46) (i), (iii), (v) & (vi) are paramagnetic.

47) Order of IE_1:-
Na < Mg > Al (In third period)
Si < P > S (In third period)

48) $NaBH_4$ can reduce only following group.
$-CH = O \rightarrow -CH_2OH$



$R-x \rightarrow R-H$
(2º/3º halide)

–C=NH → –CH_2–NH_2
Thus, (i), (iv), (vi), (viii), (ix), (x) are reduce by $NaBH_4$

49) S_N1 reaction given by those halides which directly produce 2°/3°/resonance stable carbocation.(i), (iii), (v), (vii), (viii), (ix) undergoes rearrangement of carbocation in order to convert into more stable carbocation.

50) $FeSO_4 (NH_4)_2 SO_4 . 6H_2O + Fe_2 (SO_4)_3$
Mohr salt
x mol (4–x) mol
□ (Eq) $K_2Cr_2O_7$ = (Eq) Mohr solt
($Cr^{+6} \rightarrow Cr^{+3}$) ($Fe^{+2} \rightarrow Fe^{+3}$)
$N \times V_{(L)}$ = mol × n factor
1 × 6 × 0.5 = x × 1
x = 3 mol
$\therefore$ % of Mohr's salt in mixlure $= \frac{3}{4} \times 100 = 75\%$

PART-3 : MATHEMATICS

51) 100 + 200 + 400 + ... n terms (let) = 409500
⇒ $100.(2^n - 1) = 409500$
⇒ $2n = 4096 = 2^{12} \Rightarrow n = 12$

52) $x^4 - 100x^3 + 2x^2 + 4x + 10 = 0 \rightarrow \alpha, \beta, \gamma, \delta$
equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$ is
$10x^4 + 4x^3 + 2x2 - 100x + 1 = 0$
$\rightarrow \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$
S.O.R. $\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{-4}{10} = \frac{-2}{5}$

53) 1 × 15 × 15 × 15 = 3375

54) $\begin{vmatrix} a & 1 & 1 \\ 1 & -b & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$
a(b + 3) + 4 – 1 + b = 0
(ab + b) + 3a + 3 = 0
b = –3 or a = –1

55) $n = \left(\frac{1}{7} \times \frac{7}{9}\right) + \left(\frac{3}{7} \times \frac{8}{9}\right) + \left(\frac{2}{7} \times \frac{5}{9}\right) + \left(\frac{1}{7} \times \frac{8}{9}\right)$

$$m = \frac{1}{7} \times \frac{7}{9}$$

Required probability $= \frac{1}{7}$

56) $\left| z_1 z_2 z_3 \left(\frac{2|z_1|^2}{z_1} + \frac{3|z_2|^2}{z_2} + \frac{4|z_3|^2}{z_3} \right) \right|$

$|z_1||z_2||z_3|\ |2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3|$

$2 \times 3 \times 4 \times 9 = 216$

57) mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance $= \frac{1}{8}(16 + 25 + 36 + 36$

$+ 49 + 64 + x^2 + y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80 \ \square\ x = 4; y = 8$

$x^4 + y^2 = 256 + 64 = 320$

58) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$

$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{\pi}{7}\right)$

$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = -\frac{1}{2}$

59)

$$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$$

$$2\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{15}\right)$$

$$\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$$

$$2\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

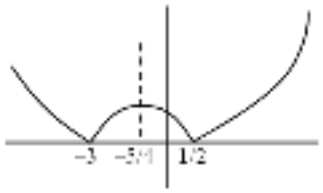
$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\pi-\frac{3\pi}{11}\right)\cos\left(\pi+\frac{5\pi}{11}\right)$$

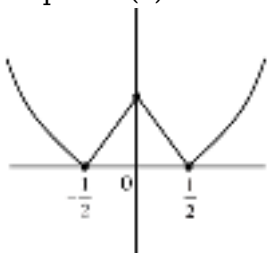
$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{8\pi}{11}\right)\cos\left(\frac{16\pi}{11}\right)$$

60) $f(x) = |2x^2 + 5|x| - 3|$

Graph of $y = |2x^2 + 5x - 3|$



Graph of f(x)



Number of points of discontinuity = 0 = m

Number of points of non-differentiability = 3 = n

61) $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$

Domain : $x^2 - 25 \geq 0$

$\Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

$x^2 + 2x - 15 > 0$

$\Rightarrow (x + 5)(x - 3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5 ; \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

62) $f(x) = \begin{cases} x - 1; & x = \text{even} \\ 2x; & x = \text{odd} \end{cases}$

$f(f(f(a))) = 21$

C-1: If a = even

$f(a) = a - 1 = \text{odd}$

$f(f(a)) = 2(a - 1) = \text{even}$

$f(f(f(a))) = 2a - 3 = 21 \Rightarrow a = 12$

C-2: If a = odd

$f(a) = 2a = \text{even}$

$f(f(a)) = 2a - 1 = \text{odd}$

$f(f(f(a))) = 4a - 2 = 21$ (Not possible)

Hence a = 12

Now $\lim_{x \to 12^-} \left(\frac{|x|^3}{12} - \left[\frac{x}{12} \right] \right)$

$= \lim_{x \to 12^-} \frac{|x|^3}{12} - \lim_{x \to 12^-} \left[\frac{x}{12} \right] = 144 - 0 = 144$

63) $C \equiv x^2 + y^2 + gx + gy = 0$ .....(1)

$2x + 2yy' + g + gy' = 0$

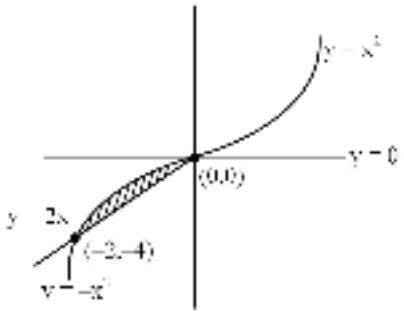
$$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$$

Put in (1)

$$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$$

$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$

64)



$$A = \int_{-2}^{0} -x^2 - 2x = \frac{4}{3}$$

65)

$$I = \int_0^1 1.\left(1 - x^{10}\right)^{20} dx$$

; $x^{10} = t$; $x = t^{1/10}$

$$dx = \frac{1}{10}(t)^{-9/10}dt$$

$$I = \int_0^1 (1-t)^{20}\frac{1}{10}(t)^{-9/10}dt$$

$$I = \frac{1}{10}\int_0^1 t^{-9/10}(1-t)^{20}dt$$

$a = \frac{1}{10}^{0}$ $b = \frac{1}{10}$ $c = 21$

66) $I(x) = \int \frac{6dx}{\sin^2 x(1-\cot x)^2} = \int \frac{6\,\mathrm{cosec}^2 x\,dx}{(1-\cot x)^2}$

Put $1 - \cot x = t$; $\mathrm{cosec}^2 x\,dx = dt$

$$I = \int \frac{6dt}{t^2} = \frac{-6}{t} + c \quad ; \quad I(x) = \frac{-6}{1-\cot x} + c,$$

$$I(0) = \frac{-6}{1-\cot 0} + c = 3 \Rightarrow c = 3$$

$$I(x) = 3 - \frac{6}{1-\cot x}, \quad I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1-(2+\sqrt{3})}$$

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{\sqrt{3}+1} = 3 + \frac{6(\sqrt{3}-1)}{2} = 3\sqrt{3}$$

67) Equation of chord with given middle point. $T = S_1$

$$\frac{x}{25}+\frac{y}{40}=\frac{1}{25}+\frac{1}{100}$$

$$\frac{8x+5y}{200}=\frac{8+2}{200};\ y=\frac{10-8x}{5} \quad ...(i)$$

$$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1$$ (put in original equation)

$$\frac{16x^2+100+64x^2-160x}{400}=1$$

$4x^2 - 8x - 15 = 0$

$$x=\frac{8\pm\sqrt{304}}{8};\ x_1=\frac{8+\sqrt{304}}{8};\ x_2=\frac{8-\sqrt{304}}{8}$$

Similarly, $y=\frac{10-18\pm\sqrt{304}}{5}=\frac{2\pm\sqrt{304}}{5}$

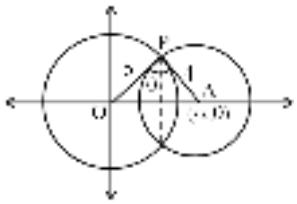
$$y_1=\frac{2-\sqrt{304}}{5};\ y_2=\frac{2+\sqrt{304}}{5}$$

Distance $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}=\frac{\sqrt{1691}}{5}$$

68) $C_1 : x^2+y^2=25,\ C_2 : (x-\alpha)^2+y^2=16$

$5 < a < 9$



$$\theta=\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$$

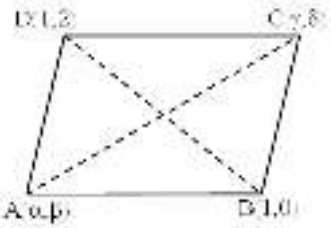
$$\sin\theta=\frac{\sqrt{63}}{8}$$

Area of $\Delta OAP = \frac{1}{2}\times\alpha\left(\frac{\beta}{2}\right)=\frac{1}{2}\times5\times4\sin\theta$

$$\Rightarrow \alpha\beta=40\times\frac{\sqrt{63}}{8}$$

$$\alpha\beta=5\times\sqrt{63}$$

$$(\alpha\beta)^2=25\times63=1575$$

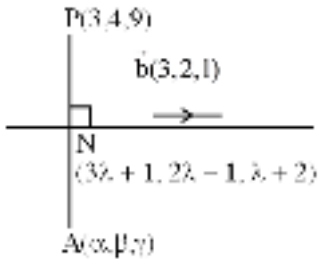


69)

Let E is mid point of diagonals

$$\frac{\alpha+\gamma}{2}=\frac{1+1}{2}\ \&\ \frac{\beta+\delta}{2}=\frac{2+0}{2}$$

$\alpha+\gamma=2 \qquad \beta+\delta=2$
$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$



70)
$\overrightarrow{PN}.\vec{b}=0$
$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$
$14\lambda=23 \Rightarrow \lambda=\frac{23}{14}$
$N\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$
$\therefore \frac{\alpha+3}{2}=\frac{83}{14} \Rightarrow \alpha=\frac{62}{7}$
$\frac{\beta+4}{2}=\frac{32}{14} \Rightarrow \beta=\frac{4}{7}$
$\frac{\gamma+9}{2}=\frac{51}{14} \Rightarrow \gamma=\frac{-12}{7}$
Ans. $14(\alpha+\beta+\gamma)=108$

71) $1+2+2^2+2^3+\ldots+2^{99}\ \frac{1(2^{100}-1)}{2-1}=2^{100}-1$
$=(5-1)^{50}-1=(1-5)^{50}-1=1-{}^{50}C_1.5+{}^{50}C_2\,5^2-{}^{50}C_3\,5^3+..+{}^{50}C_{50}\,550-1$
Which is divisible by 5 ∴ a = 0 ⇒ 3a + 5 = 5

72) $20=\frac{\sum_{i=1}^{7}|x_i-62|^2}{7}$
$\Rightarrow |x_1-62|^2+|x_2-62|^2+\ldots+|x_7-62|^2=140$
If $x_1=49$; $|49-62|^2=169$ then,
$|x_2-62|^2+\ldots..+|x_7-62|^2$ = Negative number which is not possible, therefore, no student can fail.

73) $y=\frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2x-5)\cos^3x$

$y=\frac{(\sqrt{x}+1)(\sqrt{x})\left((\sqrt{x})^3-1\right)}{(\sqrt{x})\left((\sqrt{x})^2+(\sqrt{x})+1\right)}+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

$y=(\sqrt{x}+1)(\sqrt{x}-1)+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

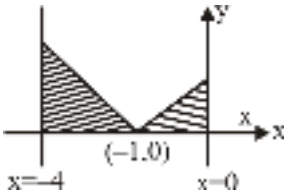
$y' = 1 - \cos^4 x \cdot (\sin x) + \cos^2 x (\sin x)$

$$y'\left(\frac{\pi}{6}\right) = 1 - \frac{9}{16} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{32 - 9 + 12}{32} = \frac{35}{32} = 96y'\left(\frac{\pi}{6}\right) = 105$$

74) $\int_{-4}^{0} (|x+1| + [x+3])\,dx$

$= A_1 + A_2$



Where A_1 = shaded area shown above

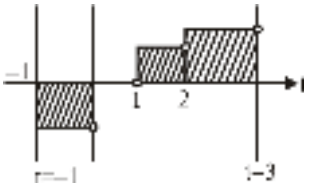
$$= \frac{1}{2} \times 3 \times 3 + \frac{1}{2} = 5$$

$$A_2 = \int_{-4}^{0} [x+3]\,dx$$

Let $x + 3 = t$

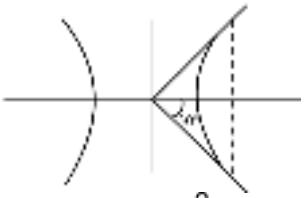
$$A_2 = \int_{-1}^{3} [t]dt$$

A_2 = Shaded area



$= -1 + 1 + 2 = 2 \Rightarrow I = 5 + 2 = 7$

75) LR subtends 60º at centre



$$\Rightarrow \tan 30^\circ = \frac{b^2/a}{ae} = \frac{b^2}{a^2 e} = \frac{1}{\sqrt{3}} \Rightarrow e = \frac{\sqrt{3}b^2}{9}$$

Also, $e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$

$\Rightarrow b^4 = 3b^2 + 27 \Rightarrow b^4 - 3b^2 - 27 = 0$

$$\Rightarrow b^2 = \frac{3}{2}(1 + \sqrt{13}) \Rightarrow \ell = 3,\ m = 2,\ n = 13$$

$\Rightarrow \square^2 + m^2 + n^2 = 182$