Test Pattern

JEE (Main)

FULL SYLLABUS

19-01-2025

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

JEE(Main + Advanced) : ENTHUSIAST & LEADER COURSE (SCORE-I)

ANSWER KEY

PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B	B	C	A	C	C	B	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	A	B	A	D	D	C	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	25	1	60	256	4					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	C	B	C	C	A	C	A	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	C	D	C	C	D	D	B	B
SECTION-II	Q.	1	2	3	4	5					
	A.	4	5	492	2	98					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	A	A	A	C	A	C	D	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	C	C	D	A	A	B	A	A	B
SECTION-II	Q.	1	2	3	4	5					
	A.	17	10	101	56	10					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (D)**

$dQ = dU + dW$

$TdS = dU + PdV$

$dU = C_V dT$

at constant volume $PdV = 0$

$\left(\frac{dT}{dS}\right)$ at constant volume $= \frac{T}{C_V}$

at constant pressure

$dQ = C_P dT = TdS$

$\frac{dT}{dS} = \frac{T}{C_P}$

$C_P > C_V$

so, slope at constant volume > slope at constant pressure

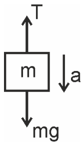
so, path b is isobaric and path a is isochoric.

In isochoric process work done is zero.

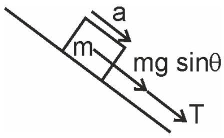
and here in isobaric process work done is positive.

Work done in path b > work done in path a.

2. Ans (B)



$mg - T = ma$ .... (1)



$mg\sin\theta + T = ma$ ...... (2)

adding (1) & (2)

$a = \frac{g}{2}(1+\sin\theta)$

$T = m(g-a)$

$= \frac{mg}{2}(1-\sin\theta)$

5. Ans (A)

$f_{app} = f\left[\frac{v + v_0}{v - v_s}\right]$

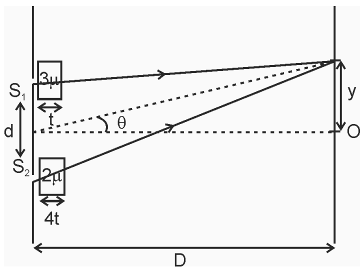
$= f\left[\frac{v + \frac{v}{3}}{v - \frac{v}{3}}\right] = f\frac{\frac{4v}{3}}{\frac{2v}{3}}$

$f_{app} = 2f$

$\%\text{ change} = \left(\frac{f_{app} - f}{f}\right) \times 100$

$= 100\%$

6. Ans (C)



Path difference

$\Delta x = (2\mu - 1)4t + d\sin\theta - (3\mu - 1)t = 0$

$5\mu t - 3t + \frac{yd}{D} = 0$

$y = \frac{tD(3 - 5\mu)}{d}$

7. Ans (C)

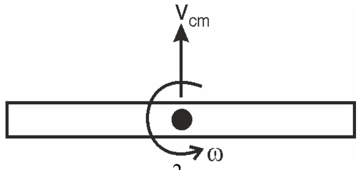
$\frac{1}{2}Kx^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}m_1(2v)^2 = \text{constant}$

$2kxv + 2m_2va + 8m_1va = 0$

$Kx + (m_2 + 4m_1)\,a = 0$

$a = \frac{-kx}{(m_2 + 4m_1)}$

8. Ans (B)



$\frac{J \times L}{2} = \frac{ML^2}{12} \times \omega;\ \omega = \frac{6J}{ML}$

$J = Mv_{cm}$

$v_{cm} = \frac{J}{M}$

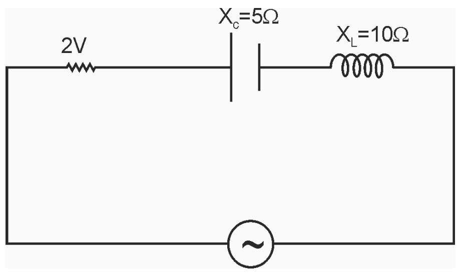
time taken in one rotation $= \frac{2\pi}{\omega}$

distance travelled by c.m in one rotation =

$V_{cm} \times \frac{2\pi}{\omega}$

$= \frac{J}{M} \times \frac{2\pi}{6J} \times ML = \frac{\pi L}{3}$

9. Ans (C)



$\tan\phi = \frac{V_L - V_C}{V_R};\ \frac{1}{\sqrt{3}} = \frac{V_L - V_C}{2}$

$V_L - V_C = \frac{2}{\sqrt{3}}$

$V_{Source} = \sqrt{(V_L - V_C)^2 + {V_R}^2}$

$= \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 2^2} = \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}}$

$\cos\phi = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{\sqrt{3}}{2}$

on calculating $R = 5\sqrt{3}\Omega$

$\tan\phi = \frac{X_L - X_C}{R} = \frac{1}{\sqrt{3}}$

$X_L - X_C = \frac{R}{\sqrt{3}}$

$Z = \sqrt{\frac{R^2}{3} + R^2} = \sqrt{\frac{75}{3} + 75} = 10\Omega$

$i = \frac{V}{Z} = \frac{4}{\sqrt{3} \times 10} = \frac{2}{5\sqrt{3}}A$

10. Ans (A)

$mv_0 2R = mvR\ \text{Sin}\theta$

$2v_0 = v\sin\theta$

$$\frac{1}{2}mv_0^2 - \frac{GM_e m}{2R} = \frac{1}{2}mv^2 - \frac{GM_e m}{R}$$

$$v_0{}^2 - \frac{GM_e}{R} = v^2 - \frac{2GM_e}{R}$$

$$v^2 = v_0{}^2 + \frac{GM_e}{R}$$

$$v^2 = \frac{GM_e}{7R} + \frac{GM_e}{R} = \frac{8}{7}\frac{GM_e}{R}$$

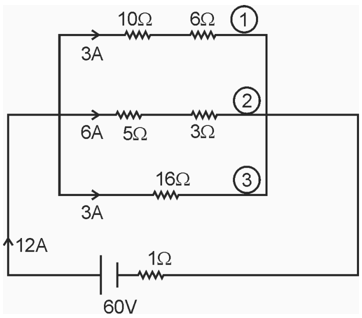
$$2\times\sqrt{\frac{GM_e}{7R}} = \sqrt{\frac{8}{7}\frac{GM_e}{R}}\sin\theta$$

$$2 = 2\sqrt{2}\sin\theta$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$\theta = 45^o$

11. Ans (A)



Given circuit is balanced wheat stone bridge. &

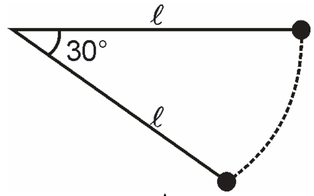
Circuit will look like above digaram

$R_{eq} = 5\Omega$

current through the battery $i = \frac{60}{5} = 12A$

current will divide in inverse ratio of resistance in branch (1), (2) & (3)

12. Ans (C)



$W_g + W_{F_E} = \Delta K.E.$

$mg\ell\ \sin 30 + mg\frac{\ell}{2}\sin 30^\circ - QE(\ell - \ell\cos 30^o) = \frac{1}{2}I\omega^2 - 0$

$$mg\frac{\ell}{2} + mg\frac{\ell}{4} - Q\frac{mg}{Q}\ell\left(1-\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}+m\ell^2\right)\omega^2$$

$$mg\ell\left(\frac{1}{2}+\frac{1}{4}-1+\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\times\frac{4}{3}m\ell^2\omega^2$$

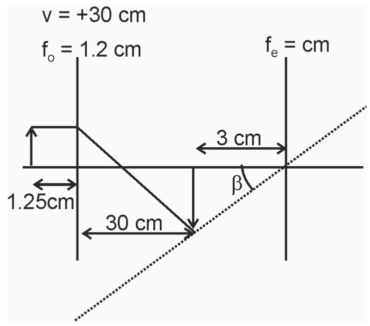
$$\left(\frac{\sqrt{3}}{2}-\frac{1}{4}\right)g = \frac{2}{3}\ell\omega^2$$

$$\omega = \sqrt{\frac{3g\left(2\sqrt{3}-1\right)}{8\ell}}$$

$v = \omega\ell$

$$v = \sqrt{\frac{3g\ell\left(2\sqrt{3}-1\right)}{8}}$$

13. Ans (A)



$f_0 = 1.2$ cm, $f_e = 3$ cm

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$\frac{1}{1.2} = \frac{1}{v_0} - \frac{1}{(-1.25)}$$

$$\Rightarrow \frac{1}{v_0} = \frac{5}{6} - \frac{4}{5} = \frac{1}{30}$$

Magnifying power for image at infinity

$$= \left(\frac{v_0}{u_0}\right)\left(\frac{D}{f_e}\right) = \left(\frac{30}{1.25}\right)\left(\frac{25}{3}\right) = 200$$

15. Ans (A)

When any input is 0 then current flows and $v_y = 0$

and when both input is 1

No current flows and $v_y = 1$

16. Ans (D)

$\mu_r = 1.6,\ \varepsilon_r = 6.4$

$B = 4.5 \times 10^{-2}$

$E = ?$

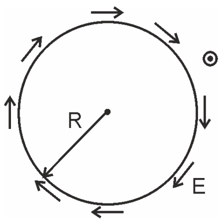
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}},\ v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\frac{c}{v} = \sqrt{\mu_r\varepsilon_r} = \sqrt{1.6 \times 6.4}$$

$$\frac{E}{B} = v = \frac{3 \times 10^8}{\sqrt{1.6 \times 6.4}} = 9.3 \times 10^7 \text{m/s}$$

$E = 4.5 \times 10^{-2} \times 9.3 \times 10^7 = 4.2 \times 10^6$

17. Ans (D)



Due to the time varying magnetic field, an induced electric field E is set up, F =qE

Ring starts rotating when the torque of force due to induced electric field becomes equal to the torque of maximum friction force.

$\Rightarrow qER = \mu mgR$

$qE = \mu mg \quad .... (1)$

$$E = \frac{R}{2}\frac{dB}{dt}$$

$$E = \frac{R}{2} \times 16t = 8tR \ ....(2)$$

From eq. (1) & (2)

$$\mu = \frac{24qR}{mg}$$

At t = 3 sec.

18. Ans (C)

Least count

$$= \frac{\text{Pitch}}{\text{no. of divisions on circular scale}} = \frac{0.5}{100} = 0.005\text{mm}$$

positive zero error = $3 \times 0.005 = 0.015$ mm

Reading = $4.5 + 48 \times 0.005 - 0.015 = 4.725$ mm

19. Ans (D)

$Q = (B.E)_P - (B.E)_R$

$= (104 + 116)(6.4) - (220)(5.6) = 176$ MeV

20. Ans (B)

$$\frac{1}{u} + \frac{1}{29} = \frac{7}{20}$$

$$\frac{1}{u} = \frac{7}{120} - \frac{7}{29}$$

$$= \frac{29 \times 7 - 120}{120 \times 29}$$

u = 41.93 cm

PART-1 : PHYSICS

SECTION-II

1. Ans (25)

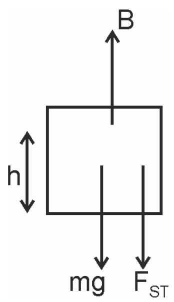
$$\text{Total Energy} = \frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2}\frac{T}{A}\left[\frac{\Delta l_{cu}}{l_{cu}} \times V_{cu} + \frac{\Delta l_{steel}}{l_{steel}} \times V_{steel}\right]$$

$$= \frac{1}{2}\frac{T}{A}\left[\frac{T}{Y_{cu}A}(Al_{cu}) + \frac{T}{Y_{steel}A}(Al_{steel})\right]$$

$$= \frac{1}{2}\frac{T^2}{A}\left[\frac{l_{cu}}{Y_{cu}} + \frac{l_{steel}}{Y_{steel}}\right] = 0.25\text{J}$$

2. Ans (1)



$mg + F_{ST} = B$

$mg + 4aS = a^2 h\rho_{w}g$

$$10 + 4 \times \frac{10}{4} = 10h$$

$\Rightarrow h = 2$

3. Ans (60)

$$Q = \frac{3}{4}mL$$

$2Q = mL + ms(50-20)$

$$\Rightarrow \frac{3}{2}mL = mL + 30ms$$

$$\Rightarrow \frac{L}{2} = 30s$$

$$\Rightarrow \frac{L}{s} = 60$$

4. **Ans (256)**

$P \propto T^4, T \propto \frac{1}{\lambda}$

$\therefore P \propto \frac{1}{\lambda_0^4} \Rightarrow P' = P\left(\frac{64\times 4}{81}\right)$

5. **Ans (4)**

We use $\frac{hC}{\lambda} = \phi + 3eV_0$ and $\frac{hC}{2\lambda} = \phi + eV_0$

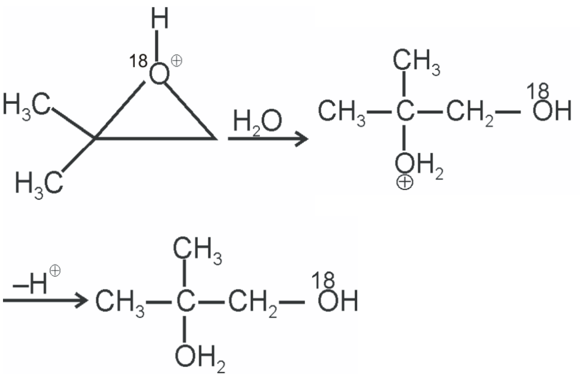
Solving both equations

$\frac{hC}{2\lambda} = 2\phi \Rightarrow \phi = \frac{hC}{4\lambda}$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (A)**



On protonated epoxide, nucleophilic attack occur at α-carbon Where more stable carbocation could be formed.

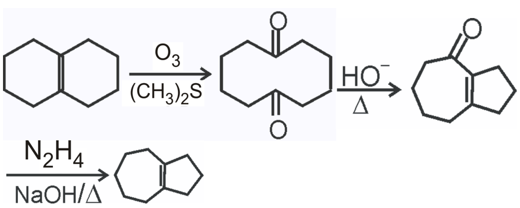
2. **Ans (D)**

(d) Acetal is formed by cyclisation.

3. **Ans (C)**

According to Le Chateliar's principle addition of inert gas doesn't effect equilibrium at constant volume.

4. **Ans (B)**



6. **Ans (C)**

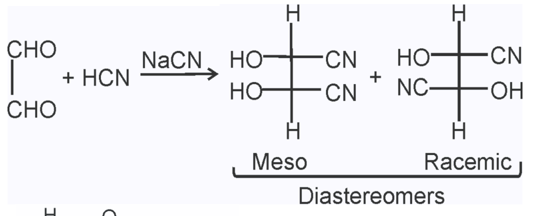
at constant pressure

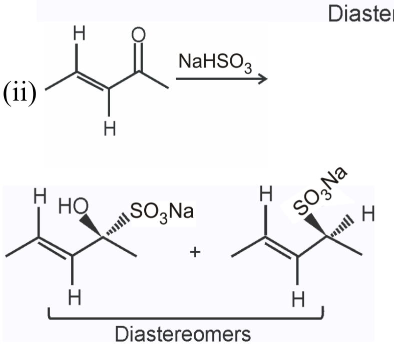
$\Delta S = nC_P \ell n\frac{T_2}{T_1}$

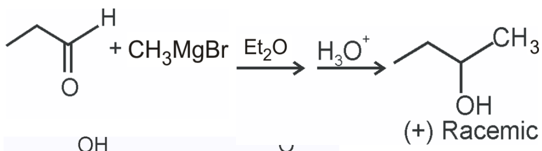
8. **Ans (C)**

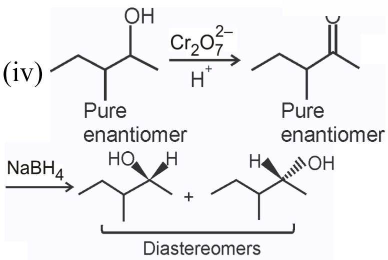
Electronic configuration of O_2^- according MOT.

9. **Ans (A)**

(i) 

(ii) 

(iii) 

(iv) 

10. **Ans (A)**

Z (atomic number)

$a \longrightarrow F$ 9

$b \longrightarrow Q$ 17

$c \longrightarrow Br$ 35

$d \longrightarrow I$ 53

17. **Ans (D)**

According to K_H data from NCERT.

18. **Ans (D)**

NCERT statements.

20. **Ans (B)**

NCERT

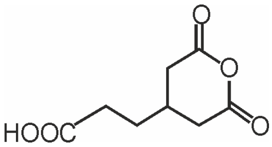
PART-2 : CHEMISTRY

SECTION-II

1. **Ans (4)**

$$Ph-\overset{\overset{\large O}{\|}}{C}-CH_3 + 4KOH + 3I_2 \rightarrow$$
$$Ph-COOK + CHI_3 + 3KI + 3H_2O$$

2. **Ans (5)**



3. **Ans (492)**

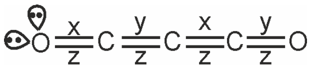
$MgCO_3.\ CaCO_3 \longrightarrow CaO + 2CO_2 + MgO$

$\frac{1.84}{184} \times 10^3$

${}^{n}CO_2 = 2 \times 10 = 20$

${}^{V}CO_2 = 20 \times 24.6 = 492$

4. **Ans (2)**



5. **Ans (98)**

$Pb(s) + PbO_2(s) + 2H_2SO_4\,(aq) \longrightarrow 2PbSO_4 + 2H_2O$

2 electrons are involved for 2 molecules of sulphuric acid.

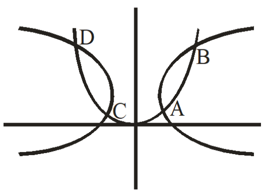
Hence the n-factor for sulphuric acid is 1.

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (B)**

Family of curves passing through the intersection of the parabola and hyperbola is



$x^2 - y^2 - a^2 + \lambda\,(x^2 - y) = 0.$

i.e.$(1 + \lambda)x^2 - y^2 - \lambda y - a^2 = 0$

For this equation to represent a circle

$1 + \lambda = -1 \Rightarrow$

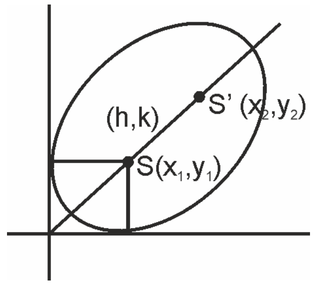
$\therefore x^2 + y^2 - 2y + a^2 = 0$

More over this equation represent a real circle, if $g^2 + f^2 - c > 0$.

$\Rightarrow 0 + 1 - a^2 > 0.$

$\Rightarrow a \in (-1, 1)$

2. **Ans (B)**



$SS' = 2ae$

$\Rightarrow (x_1 - x_2)^2 + (y_1 - y_2)^2 = 4a^2e^2$

$\Rightarrow (x_1 + x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2$

$= 4(a^2 - b^2)$

$\Rightarrow (2h)^2 + (2k)^2 - 4(x_1x_2 + y_1y_2) = 4(a^2 - b^2)$

Perpendiculars from focii upon any tangent $= b^2$

Also $x_1x_2 = y_1y_2 = b^2$

$\Rightarrow\ h^2 + k^2 = a^2 + b^2$

$\Rightarrow x^2 + y^2 = 16 + 4 = 20.$

Which is circle.

3. **Ans (A)**

$P_n = P_{n-1}\,P(T) + P_{n-2}\,P(T)\,P(H)$

$P_n = \frac{P_{n-1}}{2} + \frac{P_{n-2}}{4};$

$P_1 = 1,\ P_2 = \frac{3}{4};\ P_3 = \frac{3}{8} + \frac{1}{4}$

$P_3 = \frac{5}{8}\ ;\ P_4 = \frac{P_3}{2} + \frac{P_2}{4}; = \frac{8}{16}$

Alternatively: Clearly $p_1 = 1$ and p_2

$= 1 - P(H\,H) = 1 - \frac{1}{4} = \frac{3}{4}$

Now for $n \geq 3$,

Compute $P_3 = \frac{5}{8};\ P_4 = \frac{1}{2}$

Hence $P_2 = \frac{12}{16};\ P_3 = \frac{10}{16};\ P_4 = \frac{8}{16}$

$\Rightarrow\ P_2, P_3, P_4$ are in A.P.

$P_n = \underbrace{(1-p)}_{T}\,p_{n-1} + \underbrace{p}_{H}\,\underbrace{(1-p)}_{T}\,p_{n-2}$

4. Ans (A)

$$z = \frac{1}{1-\cos\theta + 2i\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2} - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$$\mathrm{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$$

$$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$$

$$3\cos^2\frac{\theta}{2} = \frac{3}{2}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

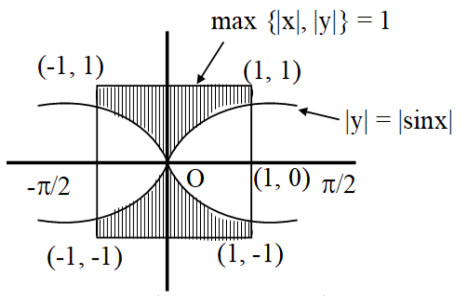
$$\theta \in (0, \pi)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta - [-\cos\theta]_0^{\frac{\pi}{2}}$$

$$= -(0-1) = 1$$

5. Ans (A)



$$\therefore \text{Area} = 4\left(1 - \int_0^1 \sin x\, dx\right) = 4\cos(1)$$

6. Ans (C)

Given D.E. is

$$2xy\,dy = (x^2+1)\,dx + y^2\,dx$$

$$\Rightarrow \frac{2xy\,dy - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx$$

$$\Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \frac{y^2}{x} = x - \frac{1}{x} + c$$

$$\Rightarrow y^2 + 1 = x^2 + cx$$

$$\Rightarrow x^2 - y^2 + cx - 1 = 0$$

$$\Rightarrow \frac{(x+(c/2))^2}{1+c^2/4} - \frac{(y-0)^2}{1+c^2/4} = 1$$

Which represents a family of rectangular hyperbola with center on x-axis.

7. Ans (A)

$C_1 : x^2 + y^2 = r^2$

$\therefore C_2 : x^2 + y^2 = 2r^2$

Let C_3 is $x^2 + y^2 + 2gx + 2fy = 0$

common chord of C_3 and C_2 is

$gx + fy + r^2 = 0$ and this is a tangent to C_1

$$\Rightarrow \left|\frac{r^2}{\sqrt{g^2+f^2}}\right| = r$$

$$\Rightarrow g^2 + f^2 = r^2$$

$\Rightarrow$ Locus of $(-g, -f)$ is C_1

8. Ans (C)

Consider F(x) = cot

$$\left(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|)\right)$$

But $|\sin x| + |\cos x| \Rightarrow [1, \sqrt{2}],\ \forall\, x \in R$

$\therefore F(x) = \cot(\cos^{-1}(1) + \sin^{-1}(-1))$

$$= \cot\left(0 - \frac{\pi}{2}\right) = 0 = g(3)$$

(As $F(x) = 0, \forall\, x \in D_F$)]

9. **Ans (D)**

(A) Equation of circle is $x^2 + \left(y - \frac{10}{3}\right)^2 = 1$

Consider a general point $(2t^2, 2t^3)$ on the curve $x^3 = 2y^2$, its distance from centre is

$$\ell^2 = 4t^4 + \left(2t^3 - \frac{10}{3}\right)^2 = 4\left[t^4 + \left(t^3 - \frac{5}{3}\right)^2\right]$$

$$\frac{d(\ell^2)}{dt} = 8t^2(t-1)(3t^2+3t+5)$$

$\Rightarrow t = 1$ is a point of local minima

$\Rightarrow \ell_{min} = \frac{2}{3}\sqrt{13}$

(B) Given curve is parabola of the form $(x-1)^2 = 4(y-2)$

Let AB is normal chord $A(2t_1+1, t_1^2+2), B(2t_2+1, t_2^2+2)$

$(AB)^2 = (t_1 - t_2)^2(4 + (t_1+t_2)^2)$ also $t_2 = -t_1 - \frac{2}{t_1}$

$AB^2 = 16\left(\frac{(1+t_1^2)^3}{t_1^4}\right)$; AB^2 is maximum at $t_1^2 = 2$

(C) Let $t = -x - \frac{\pi}{6}, t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$\Rightarrow \tan\left(x + \frac{2\pi}{3}\right) - \tan\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$

$\Rightarrow \cot t + \tan t + \cos t$

$\Rightarrow \left(\frac{2}{\sin 2t} + \cos t\right)$ of decreasing in $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$\Rightarrow$ Greatest value occur at $t = \frac{\pi}{6} \Rightarrow \frac{11\sqrt{3}}{6}$

(D) $(x-2)^2 + (y-2)^2 = 4$ .... (1)

Let equation of line is $y = mx$ put in equation (1)

$(x-2)^2 + (mx-2)^2 = 4$

$\Rightarrow x^2(1+m^2) - 4x(1+m) + 4 = 0$

$\Rightarrow$ Let M and N are (x_1, y_1) and (x_2, y_2)

$\Rightarrow (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \frac{32m}{(1+m^2)^2}$

10. **Ans (A)**

f(x) has no critical points in (a, b)

$\Rightarrow$ f is monotonic

$\Rightarrow \max\{f(x)\} = \max\{f(a), f(b)\}$

$\Rightarrow \min\{f(x)\} = \min\{f(a), f(b)\}$

11. **Ans (D)**

For idempotent matrix, $A^2 = A$

$A^{-1}A^2 = A^{-1}A$

{∵ A is non-singular}

$A = I$

$\Rightarrow$ Non-singular idempotent matrix is always a unit matrix

$\therefore \ell^2 - 3 = 1 \Rightarrow \ell = \pm 2$

$m^2 - 8 = 1 \Rightarrow m = \pm 3$

$S : \{-4, -3, -2, 0, 2, 3, 4\}$

$n^2 - 15 = 1 \Rightarrow n = \pm 4$

$p = q = r = 0$

$(-3 - 2 - 4 + 0 + 4 + 2 + 3)^2$

$= 2(3^2 + 2^2 + 1^2) + 2T$

$\Rightarrow T = -29$

$\therefore$ Absolute value of T is 29.

12. **Ans (C)**

$$I = \int\left(\frac{2\cos x(\cos x + \sin x)}{1 + \sin 2x} - \frac{2x}{1+\sin 2x}\right)dx$$

$$I = \int \frac{2\cos x}{\sin x + \cos x}dx - \int \frac{2x}{(\sin x + \cos x)^2}dx$$

After using integration by parts in first part,

$$I = \frac{2x\cos x}{\sin x + \cos x} + c$$

13. **Ans (C)**

$$h'(0) = \left(\lim_{h\to 0}\frac{h(0+h) - h(0)}{h}\right)$$

$$\left(e^{g(0)}.f^2(0)g(0) + e^{g(0)}f^2(0)\right) + 2e^{g(0)}f(0)f'(0)g(0)$$

For h'(0) to be exist either f(0) = 0 or g(0) = –1

14. **Ans (D)**

Let $A = \{a_1, a_2, a_3, ..., a_n\}$

A general element of A must satisfy one of the following possibilities

[here, general element by $a_i (1 \le i \le n)$]

(i) $a_i \in P, a_i \in Q$

(ii) $a_i \in P, a_i \notin Q$

(iii) $a_i \notin P, a_i \in Q$

(iv) $a_i \notin P, a_i \notin Q$

Therefore, for one element a_i of A, we have four choices (i), (ii), (iii) and (iv)

Total number of cases for all element $= 4^n$

And for one element a_i of A, such that $a_i \in P \cup Q$, we have there choices (i), (ii) and (iii)

Number of cases for all elements belong to $P \cup Q = 3^n$

Here, number of ways in which atleast one element of A does not belongs to

$P \cup Q = 4^n - 3^n$

15. **Ans (A)**

Clearly S_1 is true

$$D \equiv \frac{Z_2 + Z_3}{2}, E \equiv \frac{Z_3 + Z_1}{2}, F \equiv \frac{Z_1 + Z_3}{2}$$

$$\text{So, arg}\left(\frac{(Z_2 + Z_3)/2}{(Z_1 + Z_2)/2}\right) = \arg\left(\frac{(Z_3 + Z_1)/2}{(Z_2 + Z_3)/2}\right)$$

$$\Rightarrow \arg\left(\frac{(Z_2 + Z_3)^2}{(Z_1 + Z_3)(Z_1 + Z_2)}\right) = 0$$

$$\Rightarrow \frac{(Z_2 + Z_3)^2}{(Z_1 + Z_2)(Z_1 + Z_3)} = \text{purely real.}$$

16. **Ans (A)**

$$\alpha + \beta = -\frac{b}{a} \& \alpha\beta = \frac{c}{a}$$

$$\therefore A_{n+2} = \alpha^{n+2} + \beta^{n+2}$$

$$= (\alpha + \beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha\beta^{n+1} - \beta\alpha^{n+1}$$

$$= (\alpha + \beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha\beta(\alpha^n + \beta^n)$$

$$= \frac{-b}{a} A_{n+1} - \frac{c}{a} A_n$$

$$\Rightarrow aA_{n+2} + bA_{n+1} + cA_n = 0$$

17. **Ans (B)**

Clearly at 6^{th}, 7^{th} and 8^{th} toss there must be tail, head, head respectively and in first five tosses no any two consecutive heads obtained, for first five tosses

Case-I : all 5 "T" $\Rightarrow \frac{\underline{|5}}{\underline{|5}} = 1$ way

Case-II : 4 "T" $\Rightarrow \frac{\underline{|5}}{\underline{|4}\ \underline{|1}} = 5$ way

Case-III : 3 "T", 2"H" _T _T _T _

$\Rightarrow {}^4C_2 = 6$ ways using gap method

Case-IV 2 "T", 3"H" _T _T _

$\Rightarrow {}^3C_3 = 1$ way

$$\Rightarrow P(E) = \frac{1 + 5 + 6 + 1}{2^8} = \frac{13}{256}$$

18. **Ans (A)**

$$-8 \times {}^{18}C_{15} + 64 \times {}^{18}C_{16} - 2b\,{}^{18}C_{17} = 0$$

$$\therefore b = \frac{272}{3}$$

19. Ans (A)

$f(x) = x^2 + \lambda x + \mu\cos x$

Let α be the root of $f(x) = 0 \Rightarrow f(\alpha) = 0$

$\Rightarrow f(f(\alpha)) = f(0) = 0$ ($\because \alpha$ is root of $f(f(x) = 0$ also)

Now $f(0) = \mu = 0$

$f(x) = x^2 + \lambda x = 0$

$\Rightarrow x = 0, x = -\lambda$

$f(f(x)) = f(x^2 + \lambda x) = (x^2 + \lambda x)^2 + \lambda(x^2 + \lambda x)$

$= (x^2 + \lambda x)\ \{x^2 + \lambda x + \lambda\} = 0$

Will have same root $x = 0, x = -\lambda$

If $x^2 + \lambda x + \lambda = 0$ have no real roots

$\Rightarrow \lambda^2 - 4\lambda < 0$

$\Rightarrow 0 < \lambda < 4 \Rightarrow \lambda = 1,2,3$

But $\lambda = 0$ is also satisfy

(0,0), (1,0),(2,0),(3,0) are 4

or different (λ, μ) does exist.

20. Ans (B)

(2) We have,

$a \leqslant x_i \leqslant b\ ;\ i = 1, 2, \ldots\ldots, n$ ...(i)

$\Rightarrow \sum_{i=1}^{n} a \leqslant \sum_{i=1}^{n} x_i \leqslant \sum_{i=1}^{n} b$

$\Rightarrow na \leqslant \sum_{i=1}^{n} x_i \leqslant nb$

$\Rightarrow a \leqslant \frac{1}{n}\sum_{i=1}^{n} x_i \leqslant b$

$\Rightarrow a \leqslant \bar{X} \leqslant b$

$\Rightarrow -b \leqslant -\bar{X} \leqslant -a$ ...(ii)

From (i) and (ii) we get

$a - b \leqslant x_i - \bar{X} \leqslant b - a\ ;\ i = 1, 2, \ldots\ldots, n$

$\Rightarrow \left|x_i - \bar{X}\right| \leqslant b - a\ ;\ i = 1, 2, \ldots., n$

$\Rightarrow \left(x_i - \bar{X}\right)^2 \leqslant (b - a)^2\ ;\ i = 1, 2, \ldots., n$

$\Rightarrow \sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2 \leqslant n(b - a)^2$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2 \leqslant (b - a)^2$

$\Rightarrow \mathrm{Var}(X) \leqslant (b - a)^2$

PART-3 : MATHEMATICS

SECTION-II

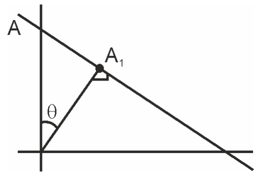
1. Ans (17)

$$\lim_{x\to\infty} \frac{\int_0^x e^{f(t)}dt}{\frac{e^{f(x)}}{g(x)}},\ \left(\frac{\infty}{\infty}\text{form}\right)$$

Applying L-Hopital Rule.

We get our answer.

2. Ans (10)



Total distance AB = $4(1 + \cos\theta + \ldots\ldots\ \infty)$

$\frac{4}{1 - \cos\theta}$

3. Ans (101)

Given $\vec{B} \times \vec{C} = x\vec{A} + y\vec{B} + z\vec{C}$

dot with $\vec{B} \times \vec{C}$

$(\vec{B} \times \vec{C}) \cdot (\vec{B} \times \vec{C}) = x[\vec{A}\ \vec{B}\ \vec{C}]$

$x = \frac{(\vec{B} \times \vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]};\quad x = \frac{\vec{B}^2\vec{C}^2 - (\vec{B}\cdot\vec{C})^2}{[\vec{A}\ \vec{B}\ \vec{C}]}$

|||ly dot with $\vec{C} \times \vec{A}$ gives $\quad y = \frac{(\vec{B} \times \vec{C}) \cdot (\vec{C} \times \vec{A})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

and dot with $\vec{A} \times \vec{B}$ gives $\quad z = \frac{(\vec{B} \times \vec{C}) \cdot (\vec{A} \times \vec{B})}{[\vec{A}\ \vec{B}\ \vec{C}]}$

$[\vec{A}\ \vec{B}\ \vec{C}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 1(1 + 1) - 2(-2 - 3) = 12$

$\vec{B}^2\vec{C}^2 - (\vec{B}\cdot\vec{C})^2 = (6)(2) - (0) = 12$

$\therefore\quad x = \frac{12}{12} = 1$

$y = \frac{\begin{vmatrix} \vec{B}\cdot\vec{C} & \vec{B}\cdot\vec{A} \\ \vec{C}\cdot\vec{C} & \vec{C}\cdot\vec{A} \end{vmatrix}}{12} = \frac{\begin{vmatrix} 0 & -3 \\ 2 & 1 \end{vmatrix}}{12} = \frac{1}{2}\ ;$

$z = \frac{\begin{vmatrix} \vec{B}\cdot\vec{A} & \vec{B}\cdot\vec{B} \\ \vec{C}\cdot\vec{A} & \vec{C}\cdot\vec{B} \end{vmatrix}}{12} = \frac{1}{2}$

$100x + 10y + 8z$

$\Rightarrow\quad 100 + 5 - 4 = 109$

4. **Ans (56)**

$$I_2 = \int_0^1 \left(\frac{x}{7+x}\right)^{\frac{5}{2}} \left(\frac{1-x}{7+x}\right)^{\frac{7}{2}} \frac{dx}{(7+x)^2}$$

put $\frac{x}{7+x} = t$

$$I_2 = \frac{1}{7^{\frac{9}{2}}} \int_0^{\frac{1}{8}} (t)^{\frac{5}{2}} (1-8t)^{\frac{7}{2}} dt$$ and now substitute $8t = u$

after simplification $I_2 = \frac{I_1}{8^{\frac{7}{2}} \cdot 7^{\frac{9}{2}}}$

5. **Ans (10)**

Let $A(\alpha, \beta)$, the equation of the normal to the parabola at $(at^2, 2at)$ is $y + xt = 2at + at^3$

As it passes through (α,β) so $at^3 + (2a - \alpha)t - \beta = 0$ have three roots t_1, t_2, t_3

$\Rightarrow t_1 + t_2 + t_3 = 0$ ....(i)

Now $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ and $R(at_3^2, 2at_3)$

Equation of $C_1 \equiv (x - \alpha)(x - at_1^2) + (y - \beta)(y - 2at_1) = 0$

$C_2 \equiv (x - \alpha)\left(x - at_2^2\right) + (y - \beta)(y - 2at_2) = 0$

Common chord

$\equiv (t_1 + t_2)x + 2y - \alpha(t_1 + t_2) - 2\beta = 0$

From (i)

$\therefore m_1 = -\left(\frac{t_1 + t_2}{2}\right) = \frac{t_3}{2}$

Tangent to the parabola at R

$t_3 y = x + at_3^2 \Rightarrow m_2 = \frac{1}{t_3}$

So, $m_1 \times m_2 = \frac{1}{2}$