FIITJEE

ALL INDIA TEST SERIES

FULL TEST – V

JEE (Main)-2025

TEST DATE: 15-01-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. A

Sol. $R = \frac{V}{I} = \frac{8}{2} = 4\Omega$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.5}{8} + \frac{0.2}{2}$

$\frac{\Delta R}{R} \times 100 = \left[\frac{0.5}{8} + \frac{0.2}{2}\right] \times 100 = \frac{50}{8} + \frac{20}{2}$

= 6.25 + 10 = 16.25%

2. B

Sol. $\frac{1}{2}mv^2 = hf - \phi = hf - hf_{th}$

$f_{th} = f - \frac{\frac{1}{2}mv^2}{h}$

$= 7.21 \times 10^{14} - \frac{\frac{1}{2} \times 9.1 \times 10^{-31} \times (6 \times 10^5)^2}{6.02 \times 10^{-34}} = 4.49 \times 10^{14} \text{Hz}$

So, N = 4.49

3. D

Sol. $\beta = \frac{\lambda D}{d}$, $y = \frac{\beta}{3}$

$\Delta X = \frac{dy}{D} = \frac{d}{D}\left(\frac{\lambda D}{3d}\right) = \frac{\lambda}{3}$

$$\frac{\phi}{2\pi}=\frac{\Delta x}{\lambda},\ \phi=\frac{2\pi}{3}$$

$$I_P=2I_0\left(1+\cos\frac{2\pi}{3}\right)=I_0$$

$$I_{max}=4I_0$$

$$\text{Ratio}=\frac{4I_0}{I_0}=4$$

4. D
Sol. Theoretical

5. A
Sol. Total resistance of wire = 20 × 2 ×0.5 = 20Ω
Town gets power at 4000 V
Power given to town 1200 kW

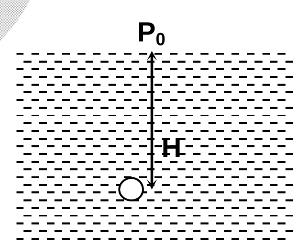
$$P = VI,\ I=\frac{1200\times10^3}{4000}=300\text{ A}$$

Power loss = $I^2R = (300)^2 \times 20 = 1800$ kW

6. B
Sol. Lenz's law

7. C
Sol. For lens A
U = –30, f = +10,v = +15
Image of A must be as focus of B
So, separation is 5 cm

8. A
Sol.



$$P_{outside}=P_0+\rho gH$$

$$P_{inside}-P_{outside}=\frac{2T}{r}$$

Pressure greater than atmospheric

$$\rho gH+\frac{2T}{r}=10^3\times9.8\times0.1+\frac{2\times0.075}{2\times10^{-3}}$$

$= 980 + 75 = 1055\ Nm^{-2}$

9. B
Sol. g_1(gravity due to complete sphere at P)

$$=\frac{G8M}{R^3}\left(\frac{R}{2}\right)=\frac{4GM}{R^2}$$

Mass of cavity = M

g_2 (gravity due to removed portion at P) = $\frac{GM}{R^2}$

$$g_P=\frac{4GM}{R^2}-\frac{GM}{R^2}=\frac{3GM}{R^2}$$

10. C
Sol. Equal to $\frac{1}{10}$th of its volume

11. A
Sol. $dA = 2\pi r dr$

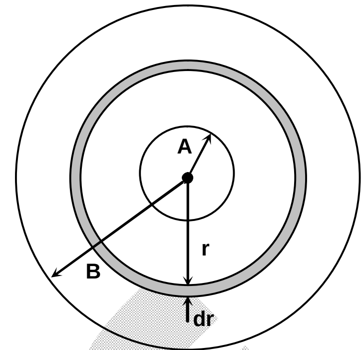
$dM = \sigma dA = \frac{\sigma_0}{r} 2\pi r dr = 2\pi\sigma_0 dr$

$dI = (dM)r^2$

$dI = 2\pi\sigma_0 r^2 dr$

$I = 2\pi\sigma_0 \int_A^B r^2 dr = \frac{2\pi\sigma_0}{3}\left[B^3 - A^3\right]$

Moment of inertia about diameter is $\frac{I}{2} = \frac{\pi\sigma_0}{6}\left[B^3 - A^3\right]$



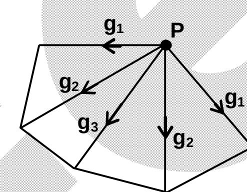
12. D
Sol. Gravitational force on star at P

$= 2\vec{g}_1 + 2\vec{g}_2 + g_3$

$= 2(\vec{g}_1 + \vec{g}_2) + g_3$

$= 2\vec{g}_3 + \vec{g}_3 = 3g_3$

$g_3 = \frac{GM^2}{(2\ell)^2} = \frac{GM}{4\ell^2}$



13. B
Sol. $\Delta\ell = \ell \propto \Delta T = 2 \times 10^{-5} \times 200$

$= 4 \times 10^{-3}$ m

Strain = $\frac{4\times10^{-3}}{2} = 2\times10^{-3}$

$Y = \frac{\text{stress}}{\text{strain}}$

Stress = Y × strain

$= 2 \times 10^{11} \times 2 \times 10^{-3}$

$= 4 \times 10^{8}$

Tension = Stress × area

$= 4 \times 10^{8} \times 10^{-4}\ m^2 = 4 \times 10^{4}$ N

14. B
Sol. Total energy remain constant

$= \frac{1}{2}kA^2 = 0.25$

$\frac{1}{2}k(0.1)^2 = 0.25$

$\Rightarrow$ k = 50 N/m

15. C
Sol. Equation of trajectory is given by

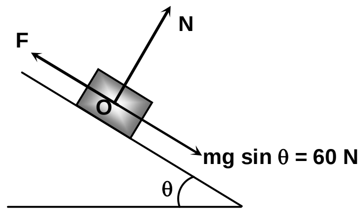
$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\alpha = \tan\theta,\ \beta = \frac{g}{2u^2\cos^2\theta}$

$\frac{\alpha}{\beta} = \frac{2u^2\cos^2\theta\tan^2\theta}{g} = 2\frac{(20)^2\cos^2 45^\circ\tan 45^\circ}{10} = 40$

16. B

Sol.



17. C

Sol. $v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{6^2 - 4^2} = \omega\sqrt{20}$

Speed of per particle is tripled = $3\omega\sqrt{20} = \omega 3\sqrt{20}$

New amplitude

$3\omega\sqrt{20} = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - 4^2}$

$9 \times 20 = A^2 - 16$

$A^2 = 180 + 16 = 196$ cm

$A = \sqrt{196} = 14$ cm

18. D

Sol. $\frac{dT}{dt} = -k(T - T_S)$

$\frac{40-60}{7} = -k\left[\frac{40+60}{2} - 10\right]$

$\frac{T-40}{7} = -k\left[\frac{T+40}{2} - 10\right]$

$\frac{-\frac{20}{7}}{\frac{T-40}{7}} = \frac{(50-10)}{\left[\frac{T+40}{2} - 10\right]}$

T = 28°C

19. D

Sol. $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A \rho} = \frac{kT}{\sqrt{2}\pi d^2 \rho} = 102\mu m$

20. A

Sol. $y = m\frac{\lambda_1 D}{d} = \frac{P\lambda_2 D}{d}$

$\frac{m}{P} = \frac{5200}{6500} = \frac{4}{5}$

SECTION – B

21. 8

Sol. $y_2 = 3\left[\sin 400\pi t + \sqrt{3}\cos 400\pi t\right]$ cm

$= 6\sin\left(400\pi + \frac{\pi}{3}\right)$

A = 6 + 2 = 8 cm

22. 4

Sol. $x = 0,\ v_{max} = \frac{10}{4}$

$v = 0$, x = amplitude = 10

$\frac{\text{Amplitude}}{v_{max}} = 4$

23. 5

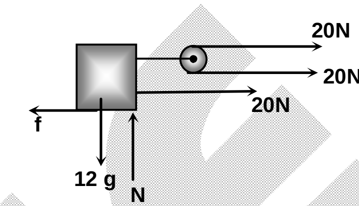
Sol. N = 120

f = 60

$\mu N = f$

$\mu(120) = 60$

$\mu = 0.5 = \frac{5}{10}$



24. 2

Sol. $\vec{v} = 2t\hat{i} + t^2\hat{j}$

at t = 1, $\vec{v} = 2\hat{i} + \hat{j}$

$|\vec{v}| = \sqrt{5}$ m/s

$\vec{a} = \frac{dv}{dt} = 2\hat{i} + 2t\hat{j}$

at t= 1 $|\vec{a}| = |2\hat{i} + 2\hat{j}| = \sqrt{8}$ m/s^2

$|\vec{v}| = \sqrt{4t^2 + t^4}$

$a_T = \frac{d|v|}{dt} = \frac{1}{2}\frac{[8t + 4t^3]}{\sqrt{4t^2 + t^4}}$

at t = 1, $a_T = \frac{8 + 4}{2\sqrt{4+1}} = \frac{6}{\sqrt{5}}$

$a^2 = a_T^2 + a_r^2 \Rightarrow 8 = \frac{36}{5} + a_r^2$

$a_r = \frac{2}{\sqrt{5}}$

$R = \frac{v^2}{a_r} = \frac{5}{2/\sqrt{5}} = \frac{5\sqrt{5}}{2} = \frac{a\sqrt{b}}{2}$

a =5, b = 5 $\Rightarrow \frac{5+4}{5} = \frac{9}{5} = 1.8 \approx 2.0$

25. 2

Sol. 19 MSD = 20 VSD

$1\,\text{VSD} = \frac{19}{20}\text{MSD}$

L.C. = MSD – VSD = $\frac{\text{MSD}}{20} = 0.1\text{mm}$

MSD = 2mm

N = 2

Chemistry

PART – B

SECTION – A

26. C
Sol. Conceptual

27. C
Sol. $Fe^{2+} + 6CN^- \rightleftharpoons \left[Fe(CN)_6\right]^{4-};\ K_f = 10^{35};$

$\Delta G_1^o = -2.303RT\log K_f = -199704.69\ J$

$Fe^{3+} + e^- \rightleftharpoons Fe^{2+};\ E^o = 0.77\ V;$

$\Delta G_2^o = -96500 \times 0.77 = -74305\ J$

$\left[Fe(CN)_6\right]^{4-} \rightleftharpoons \left[Fe(CN)_6\right]^{3-} + e^-;\ E^o = -0.36\ V;$

$\Delta G_3^o = +96500 \times 0.36 = 34740\ J$

$Fe^{3+} + 6CN^- \rightleftharpoons \left[Fe(CN)_6\right]^{3-};$

$\Delta G_4^o = \Delta G_1^o + \Delta G_2^o + \Delta G_3^o = -239269.69\ J$

$\Delta G_4^o = -2.303RT\log K_f'$

$\therefore K_f' = 8.59 \times 10^{41}$

28. A
Sol. $C_2H_5OH \longrightarrow V_1 = 20\ mL,\ d_1 = 0.7893\ g/mL$

$m_1 = 15.786\ g = w_B$

$H_2O \longrightarrow V_2 = 40\ mL,\ d_2 = 0.9971\ g/mL$

$m_2 == 39.884\ g = w_A$

Total mass = 55.67 g

$d_{sol.} = 58.16\ mL = \dfrac{\text{Total mass of solution}}{\text{density of solution}}$

$\%\ \text{change} = \dfrac{60 - 58.16}{60} \times 100 = 3.1\%$

29. D
Sol. Cationic part is ClO_2^+.

30. A
Sol. Conceptual

31. A
Sol. F^- is smallest size.

32. D
Sol. At equilibrium $\Delta G = 0$.

33. C
Sol. $n_{C_2H_4} = \dfrac{PV}{RT}$

$V_{C_2H_4} = \frac{2}{3} \times 3.67$ $\quad$ $V_{CH_4} = \frac{1}{3} \times 3.67$

$n_{C_2H_4} = \frac{1 \times 2 \times 3.67}{0.082 \times 3 \times 298}$ $\quad$ $n_{CH_4} = \frac{3.67}{0.082 \times 3 \times 298}$

Heat evolved $= \frac{2 \times 3.67}{3 \times 0.082 \times 298} \times (1400)$

Heat evolved $= \frac{3.67}{3 \times 0.082 \times 298} \times 900$

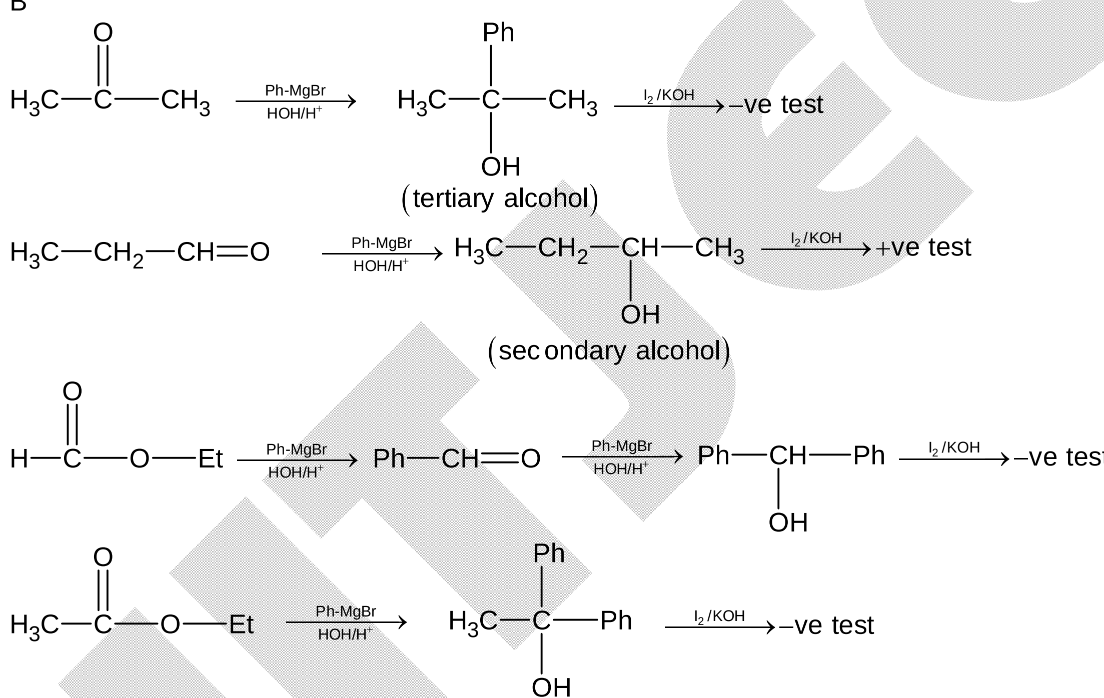
Total heat evolved from mixture = 140 + 45 = 185 kJ

34. A

Sol. $\alpha-$keratin is fibrous protein hence it is water insoluble.

35. B

Sol.

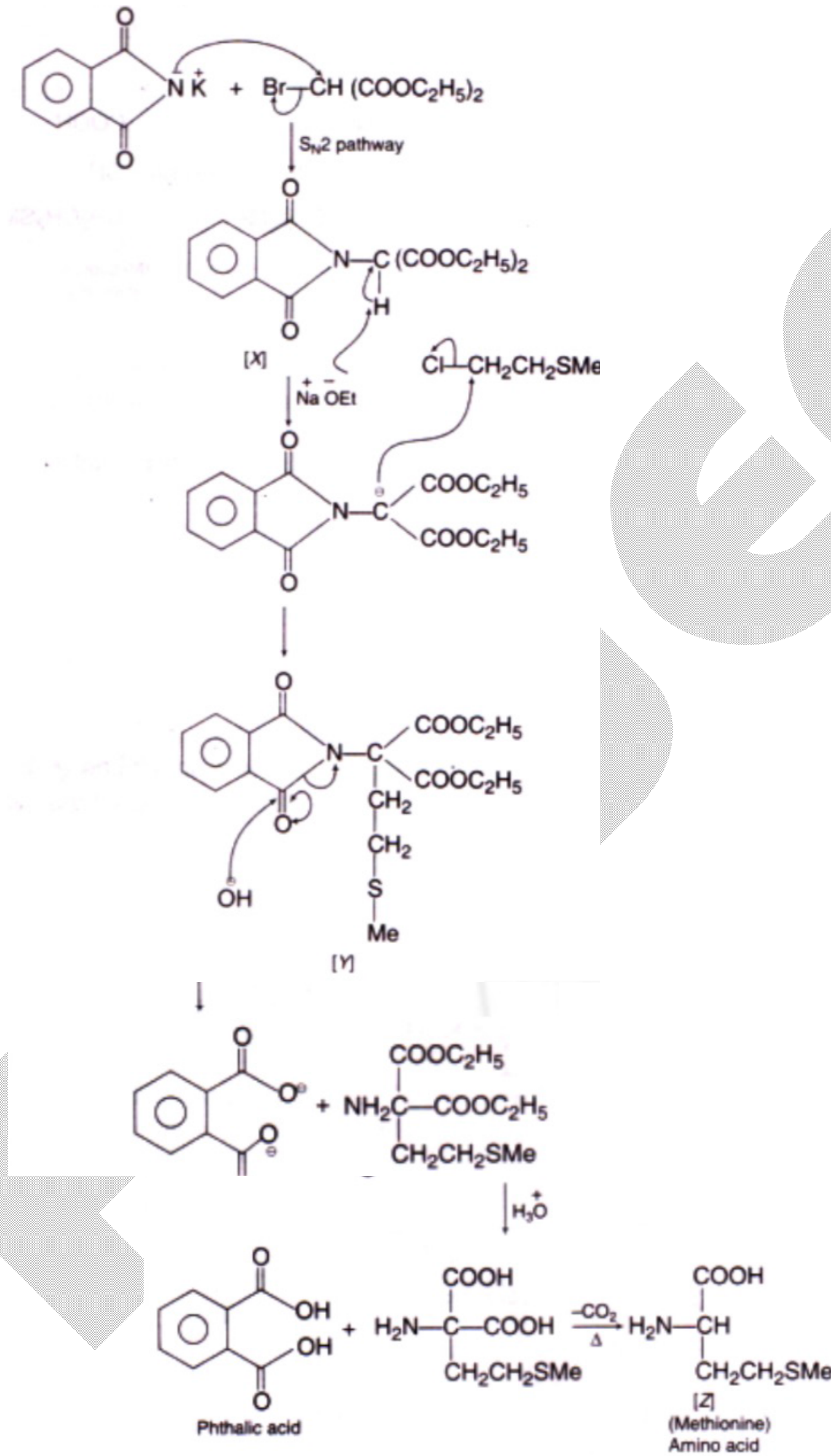


36. B

Sol. In Q compound plane of symmetry and centre of symmetry is not present.

37. B

Sol.



S$_N$2 pathway
[X]
Na OEt
Cl—CH$_2$CH$_2$SMe
[Y]
H$_3$O$^+$
Phthalic acid
$-CO_2$ / Δ
[Z]
(Methionine)
Amino acid

38. A

Sol. For Li^{2+}, n = 6 to n = 3. For He^{+}, the similar transition is 4 to 2. It means when electron of He^{+} absorbs energy at n = 2 it will go to n = 4. Energy of 4^{th} orbit of He^{+} = $-13.6\times\frac{2^2}{4^2} = -3.4\,eV$

39. C

Sol. $P = \frac{nRT}{V} = \frac{1}{4}\times 0.083\times 400 = 8.3$

$\therefore N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$8.3 - x \qquad 2x$

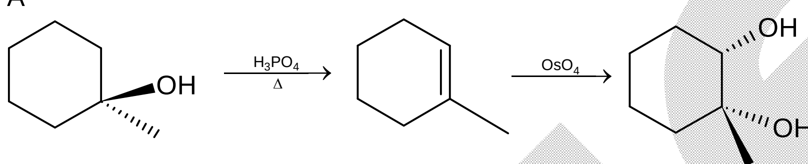
$8.3 + x = 11.6$

$\therefore x = 3.3$

$\therefore P_{NO_2} = 2x = 6.6$ bar

40. A

Sol.

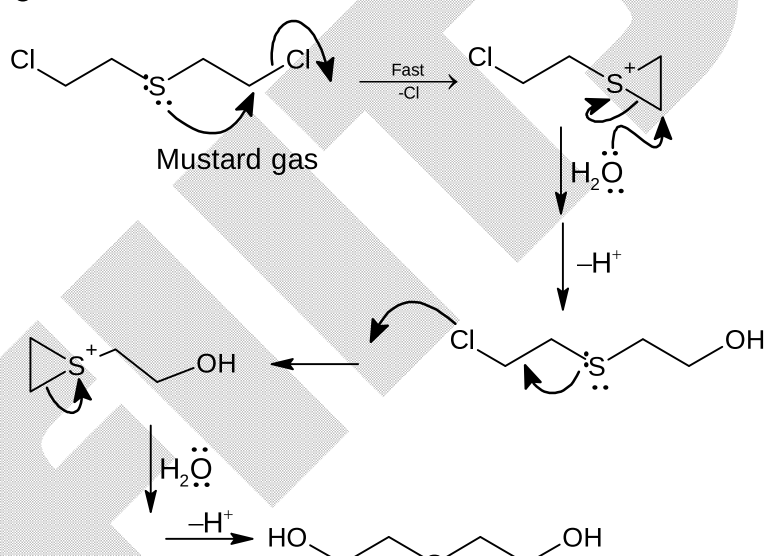


41. A

Sol. mole% of $Cl_2 = \frac{\alpha}{1+\alpha}\times 100$

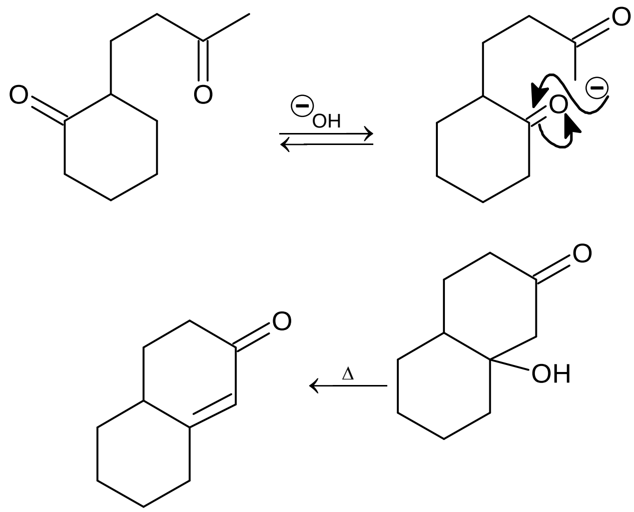
42. C

Sol.



43. B

Sol.



44. C

Sol. They have high melting point, are hard and chemically inert.

45. C

Sol. Equivalents of $K_2Cr_2O_7$ = Equivalents of Fe^{2+}

$M \times V \times nf = n \times nf$

$2 \times V \times 6 = n$

$12 \times V = n$

Equivalents of $KMnO_4$ = Equivalents of Fe^{2+}

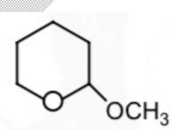
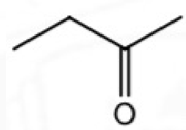
$M_1 \times V_1 \times nf_1 = n \times nf$

$2 \times V \times 5 = n \times 1$

$10\ V = n$

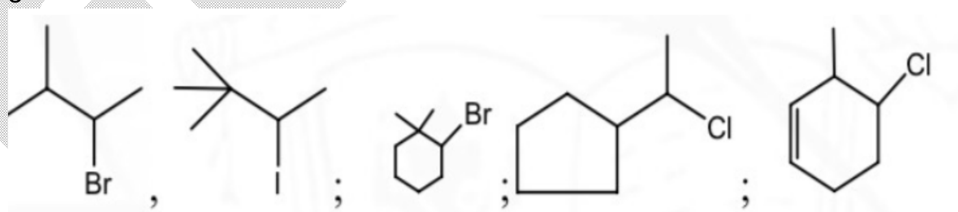
SECTION – B

46. 6

Sol. CH_3COOH, Methyl-β-D-fructofuranoside,  ;  are not oxidized.

47. 5

Sol.



48. 5

Sol. At $\frac{3}{4}$th of the equivalence point,

$pOH = pK_b + \log\frac{(3/4)}{(1/4)}$

$pOH = pK_b + \log(3)$

pH = 14 – pOH

$= 14 - pK_b - \log(3)$

$\Rightarrow 14 - pK_b = 9 \Rightarrow pK_b = 5$

$\therefore pK_b = 5$

$\therefore K_b = 10^{-5}$

$\therefore n = 5$

49. 0

Sol. $\left[CuCl_2Br_2\right]^{2-}$ is a tetrahedral complex.

50. 2

Sol. Rate of disappearance of 'A' $= -\frac{\Delta[A]}{\Delta t} = \frac{4\times10^{-2}}{40} = 10^{-3}$ mol L^{-1} s^{-1}

We know

$-\frac{1}{2}\frac{\Delta[A]}{\Delta t} = \frac{1}{2}\frac{\Delta[B]}{\Delta t} = \frac{1}{4}\frac{\Delta[C]}{\Delta t}$

$\therefore$ Rate of appearance of 'C'

$\Rightarrow \frac{\Delta[C]}{\Delta t} = -2\frac{\Delta[A]}{\Delta t}$

= 2 × 10^{-3} mol L^{-1} s^{-1}

Mathematics

PART – C

SECTION – A

51. A

Sol. $\dfrac{2\sin 40^\circ + \sin 20^\circ}{\cos 20^\circ \cdot \cos 30^\circ} = \dfrac{2\sin(60^\circ - 20^\circ) + \sin 20^\circ}{\cos 20^\circ \cdot \cos 30^\circ}$

52. D

Sol. $(5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix} = (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -(x+4) & 0 \\ 0 & 0 & -(x+4) \end{vmatrix}$ = (5x – 4) $(x + 4)^2$ = (A + Bx) $(x – A)^2$

$\Rightarrow$ A = –4, B = 5

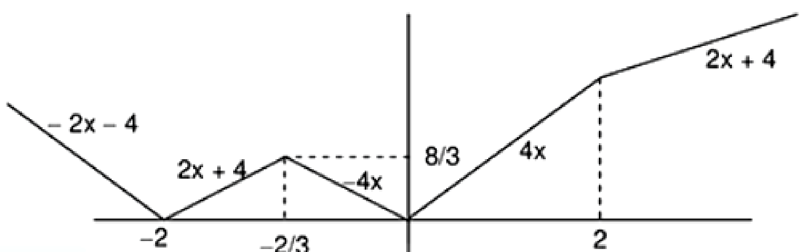
53. D

Sol. Since she hits the target successfully three times in exactly six attempts.
So 3rd hit must be the sixth time and in the first five attempts, there will be two hits. Now,

Required Probability $= {}^5C_2\left(\dfrac{1}{4}\right)^2\left(\dfrac{3}{4}\right)^3 \times \left(\dfrac{1}{4}\right)$

54. C

Sol.



55. B

Sol. $y_1 = \sin x + \cos x = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$

$y_2 = \sqrt{2}\left|\sin\left(\dfrac{\pi}{4} - x\right)\right|$

$\Rightarrow$ Area = $\displaystyle\int_0^{\pi/4}((\sin x + \cos x) - (\cos x - \sin x))\,dx + \int_{\pi/4}^{\pi/2}((\sin x + \cos x) - (\sin x - \cos x))\,dx$

$= 2\sqrt{2}\left(\sqrt{2} - 1\right)$

56. A

Sol. $\dfrac{dy}{dx} = \dfrac{y}{x} + \sec\dfrac{y}{x}$

Let y = vx

$\Rightarrow \dfrac{dv}{\sec v} = \dfrac{dx}{x}$

$\displaystyle\int \cos v\,dv = \int \dfrac{dx}{x}$

sinx = ln x + c

57. C

Sol. Given $(1 + x^n + x^{253})^{10} = \{(1 + x^{253}) + x^n\}^{10}$

Using the binomial expansion $(a + b)^n = {}^nC_0a^nb^0 + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + {}^nC_na^nb^n$,

$= {}^{10}C_0(1 + x^{253})^{10} (x^n)^0 + {}^{10}C_1(1 + x^{253})^9 (x^n)^1 + {}^{10}C_2(1 + x^{253})^8 (x^n)^2 + \dots + {}^{10}C_{10}(1 + x^{253})^0 (x^n)^{10}$

As $253 = 23 \times 11$ and $1012 = 253 \times 4$, also $n \le 22$

$\Rightarrow$ Coefficient of x^{1012} will come only from the first term, i.e. in

${}^{10}C_0(1 + x^{253})^{10} (x^n)^0 + = (1 + x^{253})^{10}$

The general term in the expansion of $(1 + a)^n$ is $T_{r+1} = {}^nC_ra^r$

Hence, the general term in the expansion of $(1 + x^{253})^{10}$ is $T_{r+1} = {}^{10}C_r(x^{253})^r = {}^{10}C_r(x^{253r})$

Since, $1012 = 253 \times 4$, hence $r = 4$

Thus, the required coefficient is $= {}^{10}C_r$.

58. B

Sol. $f(f(k)) = f(k + 3) = \frac{k+3}{2}$ and so, $f\left(\frac{k+3}{2}\right) = 27$.

If $\frac{k+3}{2}$ is odd, then $\frac{k+3}{2} + 3 = 27$ gives $k = 45$

Clearly $k = 45 \Rightarrow \frac{k+3}{2} = 24$ is even. So $\frac{k+3}{2}$ is even and $f\left(\frac{k+3}{2}\right) = \frac{k+3}{4} = 27$ gives $k = 105$

59. A

Sol. $\therefore T_n = \cot^{-1}\left[2a^{-1} + \frac{n(n+1)a}{2}\right] = \cot^{-1}\left(\frac{4+n(n+1)a^2}{2a}\right)$

$= \tan^{-1}\left[\frac{2a}{4+n(n+1)a^2}\right] = \tan^{-1}\left(\frac{a/2}{1+\frac{na}{2}\frac{(n+1)a}{2}}\right)$

$= \tan^{-1}\left((n+1)\frac{a}{2}\right) - \tan^{-1}\left(\frac{na}{2}\right)$

60. C

Sol. $\int \frac{\sec^2 x - 2010}{\sin^{2010} x} dx = \int \sec^2 x (\sin x)^{-2010} dx - 2010$

$\int \frac{1}{(\sin x)^{2010}} dx = I_1 - I_2$

Applying by parts on I_1, we get

$\frac{\tan x}{(\sin x)^{2010}} + 2010\int \frac{\tan x \cos x}{(\sin x)^{2011}} dx$

$= \frac{\tan x}{(\sin x)^{2010}} + 2010\int \frac{dx}{(\sin x)^{2010}}$

$\Rightarrow I_1 - I_2 = \frac{\tan x}{(\sin x)^{2010}} = \frac{P(x)}{(\sin x)^{2010}} + c \Rightarrow P(x) = \tan x$

$\therefore P\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

61. C

Sol. Taking three numbers.

$x + 1, y + 1, z + 1$

AM $\ge$ GM.

$$\frac{(x+1)+(y+1)+(z+1)}{3} \geq \{(x+1)(y+1)(z+1)\}^{1/3}$$

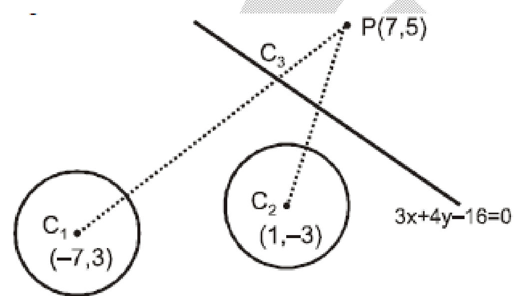
$$\left(\frac{13}{3}\right)^3 \geq xyz + xy + yz + zx + 11$$

$$\left(\frac{13}{3}\right)^3 - 11 \geq xyz + xy + yz + zx$$

equality holds when x = y = z but x + y + z = 10 and x, y, z are positive integers.
So maximum value will occur when any two of x, y, z are equal to 3 and third is equal to 4.

62. B

Sol. Image of the centre $C_2(1, -3)$ in the line $3x + 4y - 16 = 0$ is P (7, 5)
Now for $C_1C_2 + C_2C_3 + C_3C_1$ to be minimum C_1, C_3 and P should be on same line so $C_3 = (0, 4)$
Distance between C_3 and C_1
$= \sqrt{50} = 5\sqrt{2}$
radius of $C_1 = 3\sqrt{2}$
so radius of $C_3 = 2\sqrt{2}$
Equation of C_3 is $(x - 0)^2 + (y - 4)^2 = 8$
$x^2 + y^2 - 8y + 8 = 0$, a = 0, b = –8, c = 8



63. A

Sol. Let cups without handle equals to x & cups with handle equals to y
$\Rightarrow {}^xC_2 \times {}^yC_3 = 1200 = 2^4.3.5^2$

$$\frac{x(x-1)}{2} \times \frac{y(y-1)(y-2)}{6} = 2^4.3.5^2$$

x = 25, y = 4 and x = 16, y = 5
x + y is maximum when x = 25, y = 4
maximum possible cups is equal to 29

64. D

Sol. Given series,

$$\frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15} + \ldots.. + \frac{30^3 - 29^3 + 28^3 - 27^3 + \ldots. + 2^3 - 1^3}{15 \times 63}$$

Now finding $T_n = \dfrac{\sum_{k=1}^{n}\left[(2k)^3 - (2k-1)^3\right]}{n(4n+3)}$

$= \dfrac{\sum_{k=1}^{n} 4k^2 + (2k)^2 + 2k(2k-1)}{n(4n+3)}$

$= \dfrac{\sum_{k=1}^{n}(12k^2 - 6k + 1)}{n(4n+3)}$

$= \dfrac{2n(2n^2 + 3n + 1) - 3n^2 - 3n + n}{n(4n+3)}$

$= \dfrac{n^2(4n+3)}{n(4n+3)} = n$

So, $T_n = n$

Now finding $S_n = \sum_{n=1}^{n} T_n = \dfrac{15 \times 16}{2} = 120$

65. B

Sol. $2010^2 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 67^2$

Total divisors = $3 \times 3 \times 3 \times 3 = 3^4$ i.e. $(2 + 1)^4$ and $(1 + 1)^4$ of which are squares.

So, required probability = $\frac{26}{81}$

66. A

Sol. Three cases

Case I : $x > 0$

$f'(x) = 3\cos x - 2(x - \pi)\sin x - 1$

which is differentiable everywhere

Case II : $x < 0$

$f'(x) = \cos x - 2(x - \pi)\sin x + 1$

which is also differentiable everywhere

Case III : x = 0

LHD = RHD = 2

67. C

Sol. $\frac{dy}{dx} = |x| = 2 \quad \Rightarrow x = \pm 2$

at x = 2

$y = \int_0^2 |t|\,dt = \int_0^2 t\,dt = \left(\frac{t^2}{2}\right)_0^2 = 2$

Equation of tangent at (2, 2) is y – 2 = 2(x – 2)

At y = 0

–1 = x = 2

x = 1

at x = 2

$y = \int_0^{-2} |t|\,dt$

$y = -\int_{-2}^{0} -t\,dt$

$y = \frac{1}{2}\left(t^2\right)_{-2}^{0}$

$\therefore \; y = \frac{1}{2}(0 - (4)) = -2$

Equation of tangent at (–2, –2) is y + 2 = 2(x + 2)

At y = 0

1 = x + 2

x = –1

68. D

Sol. 'a' is real. So a = $\bar{a}$

$\Rightarrow \quad z^2 + z + 1 = \bar{z}^2 + \bar{z} + 1$

$\Rightarrow \quad (z - \bar{z})(z + \bar{z} + 1) = 0$

As z is imaginary

So $z - \bar{z} \neq 0$

$\Rightarrow \quad z + \bar{z} + 1 = 0$

$\Rightarrow \quad z + \bar{z} = -1 \;\; \forall\, z = x + iy$

$x = -\frac{1}{2}$

so $a = (x + iy)^2 + (x + iy) + 1$

$= (x^2 + x + 1 - y^2) + (2x + 1)\,yi \quad \forall\, x = -\frac{1}{2}$

$a = \frac{3}{4} - y^2$

so $a < \frac{3}{4} \quad \forall\, y^2 > 0$

so $a \neq \frac{3}{4}$

69. B

Sol. $\left|\vec{p}-\vec{q}\right|^2 + \left|\vec{q}-\vec{r}\right|^2 + \left|\vec{r}-\vec{p}\right|^2 = 9$

$\Rightarrow 2\left(\left|\vec{p}\right|^2 + \left|\vec{q}\right|^2 + \left|\vec{r}\right|^2\right) - 2\left(\vec{p}.\vec{q} + \vec{q}.\vec{r} + \vec{r}.\vec{p}\right) = 9$

$\Rightarrow 2\left(\left|\vec{p}\right|^2 + \left|\vec{q}\right|^2 + \left|\vec{r}\right|^2\right) + \left\{\left(\left|\vec{p}\right|^2 + \left|\vec{q}\right|^2 + \left|\vec{r}\right|^2\right) - \left|\vec{p}+\vec{q}+\vec{r}\right|^2\right\} = 9$

$\Rightarrow \left|\vec{p}+\vec{q}+\vec{r}\right| = 0 \quad \Rightarrow \vec{q}+\vec{r} = -\vec{p}$

70. A

Sol. $\frac{y}{x} = \frac{\cos 2^\circ \cos 6^\circ \cos 10^\circ ......\cos 86^\circ}{\cos 1^\circ \cos 2^\circ \cos 3^\circ ....\cos 89^\circ}$

$= 2^{44} \times \sqrt{2}\,\frac{\cos 2^\circ \cos 6^\circ \cos 10^\circ ....\cos 86^\circ}{\sin 2^\circ \sin 4^\circ ....\cos 88^\circ}$

$= \frac{2^{89/2}}{\cos 4^\circ \cos 8^\circ \cos 12^\circ ....\cos 88^\circ} = \frac{2^{89/2}}{\left(\frac{1}{2^{22}}\right)} = 2^{\frac{89}{2}+22} = 2^{\frac{133}{2}}$

$\frac{2}{7}\log_2(y/x) = \frac{2}{7}\log_2 2^{\frac{133}{2}} = \frac{2}{7} \times \frac{133}{2} = 19$

SECTION – B

71. 6

Sol. $(\alpha\beta)^3 = \alpha\beta$ and $\alpha^3 + \beta^3 = \alpha + \beta \;\Rightarrow\; \alpha\beta = 0, 1, -1$

If $\alpha\beta = 0 \;\Rightarrow (\alpha + \beta)^3 = \alpha + \beta \;\Rightarrow \alpha = \beta = 0, 1, -1$

Corresponding equation are $x^2 = 0$; $x^2 \pm x = 0$

If $\alpha\beta = 1 \;\Rightarrow (\alpha + \beta)^3 = 4(\alpha + \beta) \;\Rightarrow \alpha + \beta = 0, 2, -2$

Corresponding equation are $x^2 \pm 2x + 1 = 0$

If $\alpha\beta = -1 \;\Rightarrow (\alpha + \beta)^3 = 2(\alpha + \beta) = 0 \Rightarrow \alpha + \beta = 0$

Corresponding equation is $x^2 - 1 = 0$

72. 8

Sol. $\frac{x^2y^2 + 4 + 4xy + 2y^2 - 2xy^2 - 4y}{xy^2 + 2y} = \frac{(xy+2)^2 + 2y^2 - 2y(xy+2)}{y(xy+2)} = \frac{xy+2}{y} + \frac{2y}{xy+2} - 2 \geq 2\sqrt{2} - 2$

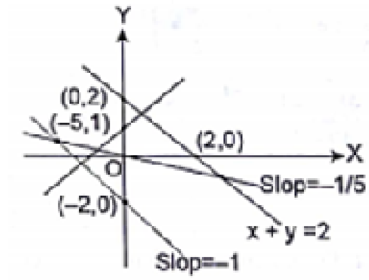
73. 6

Sol. $3y^2 + 2xy - 8y - x^2 + 4 = 0$ can be written as $(3y - x - 2)(y + x - 2) = 0$

The third line is passing through (–5, 1)

Let it be $p(x + 5) + q(y - 1) = 0$

Slope of third line $m = \frac{-p}{q}$



Now slope of the line joining (–5, 1) and (0, 0) is $-\frac{1}{5}$ and slopes of $x + y - 2 = 0$ is –1, these two are the extreme possibilities.

Hence O(0, 0) will be interior point of the triangle when slope of the third line $m \in \left(-1, -\frac{1}{5}\right)$

Comparing with interval (a, b), we get a = –1 and $b = -\frac{1}{5}$

$\Rightarrow a + \frac{1}{b^2} = -1 + 25 = 24$

74. 60

Sol. $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 5$

$6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 12\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

75. 32

Sol. Let (1, 1, 1), (–1, 1, 1), (1, –1, 1), (–1, –1, 1) be vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ rest of the vectors are $-\vec{a}, -\vec{b}, -\vec{c}, -\vec{d}$ and let us find the number of ways of selecting co-planar vectors.

Observe that out of any 3 coplanar vectors two will be collinear (anti parallel)

Number of ways of selecting the anti parallel pair = 4

Number of ways of selecting of third vector = 6

Total = 24

Number of non co-planar selections = $^8C_3 - 24 = 32$.