DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Main)

UNIT TEST # 04

01-09-2024

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	B	B	C	A	C	A	B	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	B	A	D	B	D	D	B	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	5	2	30	200	10	172	10	2	25

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	D	A	C	B	B	C	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	C	A	D	C	C	B	D
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	4	5	4	7	8	2	4	3	1

PART-3 : MATHEMATICS

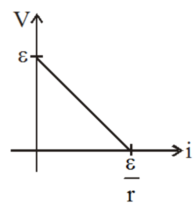
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	A	A	A	C	D	A	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	A	C	A	B	C	A	C	B	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	-2	-5	3	5	1	2	16	180	25	48

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (A)



$V = \varepsilon - ir$

$\varepsilon = 10V$

$\dfrac{\varepsilon}{r} = 2$

$\dfrac{10}{r} = 2$

$r = 5\Omega$

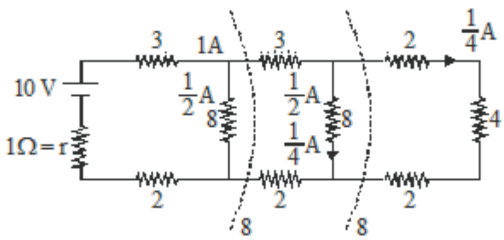
$i_{mix} = \dfrac{\varepsilon}{r} = \dfrac{10}{5} = 2A$

2. Ans (D)

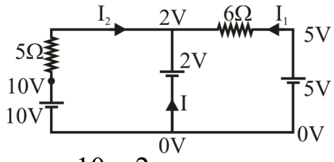
Equivalent resistance $R_{eq} = 10\ \Omega$ so current passing through battery and $3\ \Omega$ resistance is

$i = \dfrac{10}{10} = A$

and current passing through $4\ \Omega$ is 0.25 A



3. Ans (B)



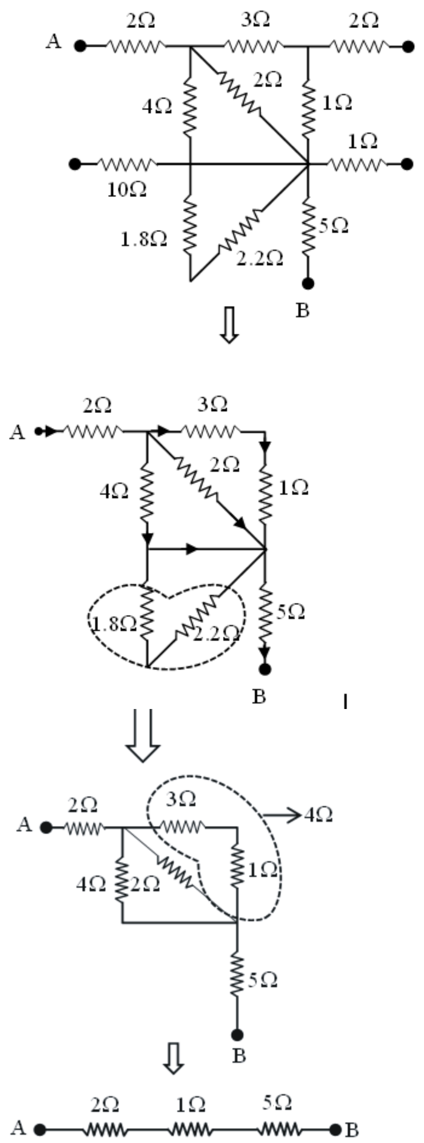
$$I_2 = \frac{10-2}{5} = 1.6\text{ A}$$

$$I_1 = \frac{5-2}{6} = 0.5\text{A}, \quad I_1 + I_2 + I = 0$$

$$I = -(I_1 + I_2)$$

$$= -2.1\text{ A}$$

4. Ans (B)



$R_{AB} = 8\Omega$

5. Ans (C)

When C is fused R↑, hence I↓ Thus brightness of A will decrease. Also as now entire current I will pass through B, hence brightness of B will increase.

7. Ans (C)

$$R = \frac{V}{i_g} - G$$

$$910 = \frac{V}{10 \times 10^{-3}} - 90$$

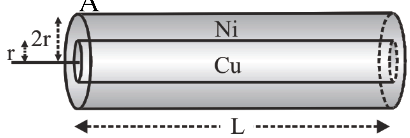
$$V = 10\text{ volt}$$

$$N(0.1) = 10$$

$$N = 100$$

8. Ans (A)

$$R = \rho\frac{\ell}{A}$$



Resistance of copper wire $R_{Cu} = \rho_c \dfrac{\ell}{\pi r^2}$

$(\because A = \pi r^2)$

Resistance of Nickle wire

$$\because A_{Ni} = \pi(2r)^2 - \pi r^2 = 3\pi r^2$$

$$\Rightarrow R_{Ni} = \rho_n \frac{\ell}{3\pi r^2}$$

Both wire are connected in parallel. So equivalent resistance

$$R = \frac{R_{Cu}R_{ni}}{R_{Cu}+R_{ni}} = \left(\frac{\rho_C \rho_n}{3\rho_C + \rho_n}\right)\frac{\ell}{\pi r^2}$$

9. Ans (B)

$$V_1 = \frac{15E}{5+15} = \frac{3E}{4} = 0.75E$$

$$V_2 = \frac{30E}{5+35} = \frac{6E}{7} = 0.85E$$

$$V_3 = \frac{10E}{10+5} = \frac{2E}{3} = 0.67E$$

$$V_2 > V_1 > V_3$$

11. Ans (A)

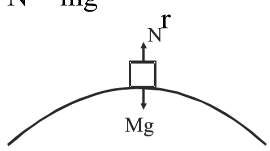
$$V_{max} = \sqrt{\frac{(\mu + \tan\theta)}{(1 - \mu\tan\theta)}Rg}$$

$$= \sqrt{\frac{1+0.5}{1-0.5} \times 1000 \times 10} = 172\text{m/s}$$

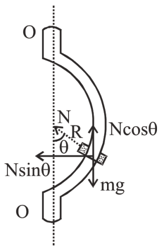
12. Ans (D)

$$Mg - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r}$$



14. Ans (A)

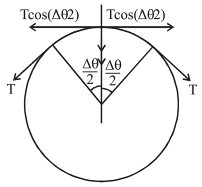


$N\sin\theta = m(R\sin\theta)w^2$

$N\cos\theta = mg$

$$\boxed{\cos\theta = \frac{g}{R\omega^2}}$$

15. Ans (D)



Net force towards centre

$= 2T\sin\dfrac{\Delta\theta}{2} = (\Delta m)r\omega^2$

$2T\dfrac{\Delta\theta}{2} = (\Delta m)r\omega^2$

When $\dfrac{\Delta\theta}{2}$ is small

$T = \left[\dfrac{\Delta m}{\Delta\theta}\right] r\omega^2$

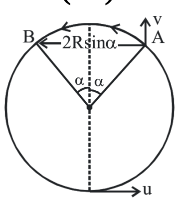
$T = \dfrac{0.4}{2\pi \times 1000} \times \dfrac{0.628}{2 \times 3.14} \times \omega^2$

$T = \dfrac{0.4}{2\pi \times 100} \times (2\pi \times 60)^2$

T = 90 Newton approx

$T \approx 9.2$ Kgf

16. Ans (B)



By solving projectile motion from A to B

$2R\sin\alpha = \dfrac{v^2 \sin 2\alpha}{g}$

$v = \sqrt{\dfrac{gR}{\cos\alpha}}$

by solving vertical circular motion from lowest point to A

$KE_L + PE_L + KE_A + PE_A$

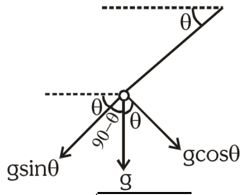
$\dfrac{1}{2}mu^2 + 0 = \dfrac{1}{2}m\dfrac{gR}{\cos\alpha} + mg(R + R\cos\alpha)$

$u = \sqrt{gR\sec\alpha + 2gR(1 + \cos\alpha)}$

17. Ans (D)

by COME

$v = \sqrt{2gl\sin\theta}$



$a_T = \sqrt{a_{cp}^2 + a_{t^2}}$

$a_{cp} = \dfrac{v^2}{r} = \dfrac{2g\ell\sin\theta}{\ell}$

$= 2g\sin\theta$

and $a_t = g\cos\theta$

$a_{net} = \sqrt{(g\cos\theta)^2 + (2g\sin\theta)^2}$

$= g\sqrt{1 + 3\sin^2\theta}$

18. Ans (D)

$\dfrac{dy}{dx} = \tan\theta = x$

20. Ans (A)

By energy conservation

$TE_A = TE_B$

$\dfrac{1}{2}mu^2 = \dfrac{1}{2}mv_B^2 + mgR$

PART-1 : PHYSICS

SECTION-II

2. Ans (5)

$i = \dfrac{6-2}{1+3} = 1\text{ A}$

$V_2 = \varepsilon + ir = 2 + 1 \times 3 = 5V$

3. Ans (2)

$\dfrac{P}{Q} = \dfrac{S}{625}$

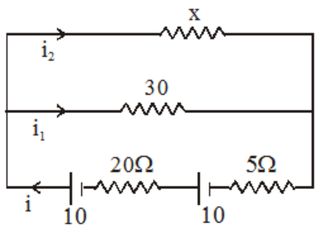
$\dfrac{Q}{P} = \dfrac{S}{676}$

$\therefore \dfrac{S}{625} = \dfrac{676}{S}$

S $= 25 \times 26$

$= 650\ \Omega$

4. Ans (30)



$E_1 = E - ir$ $\quad\quad$ $E_2 = E - ir$

$= 10 - i20 = 0$ $\quad$ $= 10 - 0.5 \times 5$

$i = 0.5\text{ A}$ $\quad\quad$ $= 7.5\text{ V}$

$E_{net} = E_1 + E_2 = 7.5\text{ V}$

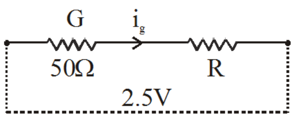
$i = i_1 + i_2$

$$0.5 = \frac{7.5}{x} + \frac{7.5}{30}$$

$x = 30\ \Omega$

5. Ans (200)

$I_g = 4 \times 10^{-4} \times 25 = 10^{-2}\text{ A}$



$2.5 = (50 + R)\ 10^{-2}$

$\therefore\ R = 200\ \Omega$

6. Ans (10)

$$r = \frac{100}{\sqrt{19}} m$$

at 2 sec, $v = 2 \times 2^2 + 2 = 10$ m/s

$v = 2t^2 + t$

$$a_{cp} = \frac{v^2}{r} = \frac{100}{100/\sqrt{19}} = \sqrt{19}\ \text{m/s}^2$$

$$a_t = \frac{dv}{dt} = ut + 1$$

at t = 2 sec

$a_t = 9\ \text{m/s}^2$

then, $a_{net} = \sqrt{a_{cp}^2 + a_t^2}$

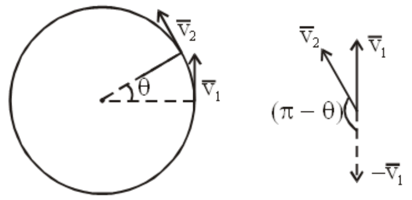
$$a_{net} = \sqrt{(\sqrt{19})^2 + 9^2} = 10\ \text{m/s}^2$$

7. Ans (172)

$$V_{max} = \sqrt{\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right) Rg}$$

$$= \sqrt{\left(\frac{\tan 45 + 0.5}{1 - 0.5\tan 45}\right) \times 1000 \times 9.8} = 172\text{ m/s}$$

8. Ans (10)

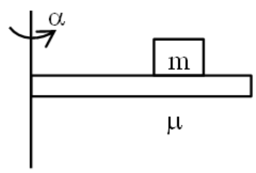


$$|\Delta\bar{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi - \theta)}$$

$$= 2v\sin\frac{\theta}{2} \text{ since } [|\bar{v}_1| = |\bar{v}_2|]$$

$= (2 \times 10) \times \sin(30°) = 10$ m/s

9. Ans (2)



$\mu mg = m\sqrt{a_t^2 + a_n^2}$

$r\omega^2 = (\alpha t)^2 = x^2$

$(0.5 \times 10)^2 = x^4 + 9$

$x^2 = 16$

$x = 2$

10. Ans (25)

$\omega = \omega_0 + \alpha t$

$20 = 0 + \alpha(5)$

$\alpha = 4\text{ rad/s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(20)^2 = 0 + 2(4)\ \Delta\theta$

$$\Delta\theta = \frac{400}{8} = 50\text{ rad}$$

$$n = \frac{\Delta\theta}{2\pi} = \frac{25}{\pi}\text{rev.}$$

PART-2 : CHEMISTRY

SECTION-I

2. **Ans (C)**

$K_3[\overset{+1}{\underset{3d^{10}}{Cu}}(CN)_4]$ Hybridization sp^3, tetrahedral, diamagnetic

6. **Ans (B)**

In $[Cr(NH_3)_6]^{3+}$

EAN of Cr = 24 – 3 + 12 = 33

8. **Ans (C)**

$[Cr(NH_3)_4Cl_2]$ Cl

11. **Ans (C)**

n-factor for $FeSO_4 \Rightarrow$ Fe (+2 to + 3) × 1 = 1

n -factor for $FeC_2O_4 \Rightarrow$

Fe(+2 to + 3) × 1 = $1e^-$ loss

C(+3 to + 4) × 2 = 2e– loss = 3

Equivalent of FeSO4 = Equivalent of $KMnO_4$

$\Rightarrow$ x mol × 1 = y mol × 5 .... (i)

Similarly,

Equivalent of FeC_2O_4 = Equivalent of $KMnO_4$

$\Rightarrow$ x mol × 3 = (mol of $KMnO_4$ = ?) × 5 ...(ii)

$\therefore$ Mol of KMnO4 required = 3y

12. **Ans (C)**

Compounds where oxidation state of central atom $\Rightarrow$ O.S. between min. & max.

$\Downarrow$

Act as both

Oxidant, Reductant

13. **Ans (A)**

Caro's acid = H_2SO_5

= Peroxomono-sulphuric acid

Marshell's acid = $H_2S_2O_8$

= Peroxodi-sulphuric acid

The oxidation state in both the acids is +6 each because oxidation state cannot be greater than the number of valuence electrons.

14. **Ans (C)**

$2BrO_3^- + 12H^+ 10Br^- \longrightarrow 6Br_2 + 6H_2O$

10 mole e^- required for formation of 6 moles of Br_2

n-factor of $Br_2 = \frac{10}{6} = \frac{5}{3}$

eq. wt. = $\frac{\text{mol. wt.}}{n} = \frac{m}{5/3} = \frac{3m}{5}$

15. **Ans (A)**

meq. $FeSO_4(NH_4)_2SO_4.6H_2O$ = meq of $KMnO_4$

(n = 1)

$\frac{W}{392} \times 1 \times 1000 = 0.1 \times 50$; W = 1.96 g

Hence, % purity of Mohr's salt

$= \frac{1.96}{2.5} \times 100 = 78.4\%$

16. **Ans (D)**

M_4O_5

$M_x(+1)\ M_y(+3)\ O_5$

x + y = 4

$$\frac{x + 3y = 10}{-2y = -6} \qquad \frac{x + 3y = 10}{-2y = -6}$$

y = 3 ; x = 1

17. **Ans (C)**

$M^{+3} \longrightarrow M^{+n}$ (Reduction)

n-factor = (3 – n) $^{e-}$ gain

$SO_3^{-2} \rightarrow SO_4^{-2}$ (oxidation)

n-factor = (+ 4 to + 6) = $2e^-$ loss

Total loss = Total gain

0.1 M × (3 – n) × 50 ml

= 0.1 M × (2) × 25 ml

3 – n = 1

n = 2

18. **Ans (C)**

$$\overset{1.5\ -2}{P_4S_3} \longrightarrow \underset{14e^-}{\overset{+5}{2P_2O_5}} + \underset{18e^-}{\overset{+4}{3SO_2}}$$

19. **Ans (B)**

In above reaction

valency factor of $N_2H_4 = 4$

$\therefore$ eq. wt. of N2H4 = $\frac{\text{mol. wt}}{\text{valency factor}} = \frac{32}{4} = 8$

and valency factor of $KIO_3 = 4$

$\therefore$ eq. wt. of $KIO_3 = \frac{214}{4} = 53.5$

20. **Ans (D)**

Balanced chemical reaction is

$2MnO_4^- + 3CN^- + H_2O$

$\longrightarrow 2MnO_2 + 3CNO^- + 2OH^-$

PART-2 : CHEMISTRY

SECTION-II

3. **Ans (5)**

	Ions in aq. solution
$[Cr(NH_3)_5Cl]Cl_2$	3
$[Cr(NH_3)_4Cl_2]Cl$	2

$3 + 2 = 5$ ions

5. **Ans (7)**

$[CoCl_6]^{-3}$, $Co^{+3} \Rightarrow 4s^2 3d^7 = d^6$

$\boxed{\uparrow}\boxed{\uparrow}$ $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$ (4)

$[Cr(NH_3)_6]^{+3}$, $Cr^{+3} \Rightarrow 4s^2 3d^5 d^3$

$\boxed{}\boxed{}$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ (3)

$[Zn(NH_3)_4]^{+2}$, $Zn^{+2} \Rightarrow 4s^2 3d^{10}$

unpaired $e^- = 0$

Sum $= 7e^-$

6. **Ans (8)**

Valency factor of $H_2S = 8$.

7. **Ans (2)**

eq. of MnO_4^- = eq. of A^{+x}

$1 \times 5 = 1.67 \times (5 - x)$

$x = 2$

8. **Ans (4)**

$\underbrace{Na_2C_2O_4 + H_2C_2O_4}_{448\ g}$

Let moles of each be x

$x \times 134 + x \times 90 = 448$

$x = 2$

Equivalents of NaOH required = Equivalent of $H_2C_2O_4$

(only for $H_2C_2O_4$) $= 2 \times 2 = 4$

9. **Ans (3)**

$P_2H_4 \xrightarrow{-10e^-} \underset{[\text{Compound Y}]}{(P_2H_4)^{+10}}$

in compound Y;

$2x + 4 = +10$

$2x = +6 \Rightarrow [x = +3]$

10. **Ans (1)**

$BrO_3^- \rightarrow Br^-$; n-factor = (+ 5 to – 1) = 6 e^- gain

$N_2H_4 \longrightarrow N_2$; n-factor = (– 2 to 0) × 2 = $4e^-$ loss

Total loss = Total gain

$n \times 4e^- = \frac{2}{3} \times 6e^-$

n = 1 mole

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (A)**

$\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx = g(x) + c$

Put $x = \cos 2\theta$

$dx = -2\sin 2\theta \cdot d\theta$

$= \int \frac{1}{\cos 2\theta} \tan\theta(-4\sin\theta \cdot \cos\theta)d\theta$

$= \int \frac{1}{\cos 2\theta}(-4\sin^2\theta)d\theta$

$= -2\int \frac{1-\cos 2\theta}{\cos 2\theta}d\theta$

$= -\frac{2}{2}\ln|\sec 2\theta + \tan 2\theta| + 2\theta + c$

$= \ln|\sec 2\theta - \tan 2\theta| + 2\theta + c$

$= \ln\left|\frac{1-\sin 2\theta}{\cos 2\theta}\right| + \cos^{-1}x + c$

$= \underbrace{\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x}_{g(x)} + c$

$\therefore g(1) = 0$

$g(x) = \ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x$

$g\left(\frac{1}{2}\right) = \ln\left|2-\sqrt{3}\right| + \frac{\pi}{3}$

$g\left(\frac{1}{2}\right) = \ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| + \frac{\pi}{3}$

2. **Ans (B)**

$$\int\left(\frac{x^2+1}{(x+1)^2}\right)e^x.dx$$

$$=\int\left(\frac{x^2-1+2}{(x+1)^2}\right)e^x\,dx$$

$$=\int\left(\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right)e^x\,dx$$

$$=\int\left(f(x)+f'(x)\right)e^x\,dx$$

$$=f(x)\,e^x+c$$

Where $f(x)=\dfrac{x-1}{x+1}$

$$f'(x)=\frac{2}{(x+1)^2}$$

$$f''(x)=\frac{-4}{(x+1)^3}$$

$$=\frac{12}{(x+1)^4}$$

$$f''(1)=\frac{12}{16}=\frac{3}{4}$$

3. **Ans (A)**

$$I=\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$$

$$\frac{\sqrt{3}}{2}\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)}dx$$

$$\int\frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^\circ+\sin 2x}dx$$

$$\int\frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\int\frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\frac{1}{2}\left(\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}-\int\frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$$

$$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$$

4. **Ans (A)**

$$I(x)=\int\underset{\text{II}}{\sec^2 x}.\underset{\text{I}}{\sin^{-2022}x}\,dx-2022\int\sin^{-2022}x\,dx$$

$$=\tan x.(\sin x)^{-2022}+\int(2022)\tan x.(\sin x)^{-2023}\cos x\,dx$$

$$-2022\int(\sin x)^{-2022}dx$$

$$I(x)=(\tan x)\,(\sin x)^{-2022}+C$$

At $X=\pi/4$, $2^{1011}=\left(\dfrac{1}{\sqrt{2}}\right)^{-2022}+C \therefore C=0$

Hence $I(x)=\dfrac{\tan x}{(\sin x)^{2022}}$

$$I(\pi/6)=\frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}}=\frac{2^{2022}}{\sqrt{3}}$$

$$I(\pi/3)=\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}=\frac{2^{2022}}{\left(\sqrt{3}\right)^{2021}}$$

$$=\frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$$

$$3^{1010}I(\pi/3)=I(\pi/6)$$

5. **Ans (A)**

$$\int\frac{2x^3-1}{x^4+x}dx$$

$$\int\frac{2x-\frac{1}{x^2}}{x^2+\frac{1}{x}}dx$$

$$x^2+\frac{1}{x}=t$$

$$\left(2x-\frac{1}{x^2}\right)dx=dt$$

$$\int\frac{dt}{t}=\ell n(t)+C$$

$$=\ell n\left(x^2+\frac{1}{x}\right)+C$$

6. **Ans (C)**

$$\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx=\int\frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$$

Let $x-\alpha=t$

$$\Rightarrow\int\frac{\sin(t+2\alpha)}{\sin t}dt=\int\cos 2\alpha dt+\int\cot(t)\sin 2\alpha dt$$

$$=t.\cos 2\alpha+\ell n|\sin t|.\sin 2\alpha+C$$

$$=(x-\alpha)\cos 2\alpha+\ln|\sin(x-a)|.\sin 2\alpha+C$$

7. Ans (D)

$$\int e^{\sec x}(\sec x \tan x\ f(x) + (\sec x\ \tan x\ + \sec^2 x)\ dx$$

$$= e^{\sec x}\ f(x) + C$$

Diff. both sides w.r.t. 'x'

$$e^{\sec\ x}(\sec\ x\ \tan x\ f(x) + (\sec\ x\ \tan\ x + \sec^2 x))$$

$$= e^{\sec\ x} \times \sec x \tan x\ f(x) + e^{\sec x}\ f\,\text{¢}(x)$$

$$f'(x) = \sec^2 x + \tan x \sec x$$

$$\Rightarrow f(x) = \tan x + \sec x + c$$

8. Ans (A)

$$\int \frac{\cos x\, dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}$$

$$= \frac{-6}{-6}\int \frac{\cos x\, dx}{\sin^7 x\left(\frac{1}{\sin^6 x}+1\right)^{2/3}}$$

$$= -\frac{1}{6} \times 3\left(\frac{1}{\sin^6 x}+1\right)^{\frac{1}{3}} + c$$

$$= -\frac{1}{2}\frac{\left(1+\sin^6 x\right)^{\frac{1}{3}}}{\sin^2 x} + c$$

Hence, $l = 3$ and $f(x) = -\dfrac{1}{2\sin^2 x}$

so, $\lambda f\left(\dfrac{\pi}{3}\right) = -2$

Remark : Technically, this question should be marked as bonus. Because f(x) and l cannot be found uniquely. For example, another such f(x) and l can be $-\dfrac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

9. Ans (A)

$$I = \int \frac{d\theta}{\cos^2\theta\,(\tan 2\theta + \sec 2\theta)}$$

$$= \int \frac{\sec^2\theta\ d\theta}{\frac{2\tan\theta}{1-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}} = \int \frac{(1-\tan^2\theta)\sec^2\theta\ d\theta}{(1+\tan\theta)^2}$$

$$\tan\theta = t \Rightarrow \sec 2\theta\ d\theta = dt$$

$$I = \int \frac{1-t^2}{(1+t)^2}\,dt = \int \frac{(1-t)(1+t)}{(1+t)^2}\,dt$$

$$= \int \frac{1}{1+t} - \frac{t}{1+t}\ dt$$

$$= \ell n|1+t| - \int\left(\frac{1+t}{1+t} - \frac{1}{1+t}\right)\ dt$$

$$= \ell n|1+t| - t + \ell n|1+t|$$

$$= 2\ell n|1+t| - t + C]$$

$$= 2\ell n|1+\tan\theta| - \tan\theta + C$$

$$\lambda = -1,\ f(\theta) = 1 + \tan\theta$$

10. Ans (C)

Put $x = \tan 2\ \theta \Rightarrow dx = 2\tan\theta\ \sec 2\theta\ d\theta$

$$\underset{\text{I}}{\theta}.(\underset{\text{II}}{2\tan\theta\cdot\sec^2\theta})\,d\theta \quad \text{(By parts)}$$

$$\int \theta.(2\tan\theta\cdot\sec^2\theta)\,d\theta$$

↓ ↓

I II (By parts)

$$= \theta.\tan^2\theta - \int \tan^2\theta\ d\theta$$

$$= \theta.\tan^2\theta - \int (\sec^2\theta - 1)\ d\theta$$

$$= \theta(1 + \tan^2\theta) - \tan\theta + C$$

$$= \tan^{-1}\left(\sqrt{x}\right)\ (1+x) - \sqrt{x} + C$$

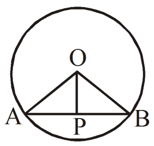
11. Ans (B)

Let the perimeter of the pentagon and decagon be 10x.

Then, each side of the pentagon is 2x and its area

$$\left(\text{using } \frac{1}{4}na^2\cot\frac{\pi}{n}\right) = 5x^2\cot\frac{\pi}{5} \qquad \text{... (i)}$$

$[\because n = 5 \text{ and } a = 2x]$



Also, as each side of decagon is x, so its area

$$= \frac{5}{2}x^2\cot\frac{\pi}{10}\ [\because n = 10 \text{ and } a = x]$$

$$\Rightarrow \frac{\text{Area of pentagon}}{\text{Area of decagon}} = \frac{2\cot 36^\circ}{\cot 18^\circ}$$

$$= \frac{2\cos 36^\circ\ \sin 18^\circ}{\sin 36^\circ\ \cos 18^\circ}$$

$$= \frac{2\cos 36^\circ\ \sin 18^\circ}{2\ \sin 18^\circ\ \cos 18^\circ\ \cos 18^\circ} = \frac{\cos 36^\circ}{\cos^2 18^\circ}$$

$$= \frac{2\cos 36^\circ}{2\cos^2 18^\circ} = \frac{2\cos 36^\circ}{1 + \cos 36^\circ}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{4\left\{1+\frac{\sqrt{5}+1}{4}\right\}} = \frac{2\left(\sqrt{5}+1\right)}{5+\sqrt{5}} = \frac{2}{\sqrt{5}}$$

12. Ans (A)

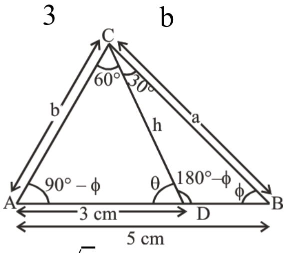
Let $\angle CDA = \theta \Rightarrow \angle CDB = 180^\circ - \theta$

and $\angle CBA = \phi \Rightarrow \angle CAB = 90^\circ - \phi$

And $AC = b$, $BC = a$ and $CD = h$

In ΔACD by sine rule

$$\frac{\sin 60}{3} = \frac{\sin \theta}{b} = \frac{\sin(90-\phi)}{h}$$



$$\Rightarrow \frac{\sqrt{3}}{6} = \frac{\sin\theta}{b} \quad ....(i)$$

In ΔCBD by sine rule,

$$\frac{\sin 30}{2} = \frac{\sin(180-\theta)}{a} = \frac{\sin\phi}{h}$$

$$\Rightarrow \frac{1}{4} = \frac{\sin\theta}{a} \quad ....(ii)$$

Dividing equation (i) by (ii), we get

$$\frac{\sqrt{3}}{6} \times 4 = \frac{a}{b} \Rightarrow a = \frac{2b}{\sqrt{3}} \quad ....(iii)$$

Now, in ΔABC by pythagoras theorem.

$$a^2 + b^2 = 5^2$$

$$\Rightarrow \left(\frac{2}{\sqrt{3}}b\right)^2 + b^2 = 25 \ \text{(using eq. (iii))}$$

$$\Rightarrow 4b^2 + 3b^2 = 75$$

$$\Rightarrow 7b^2 = 75 \Rightarrow b = 5\sqrt{\frac{3}{7}}\,\text{cm}$$

13. Ans (C)

Let r and R be the in radius and circumradius respectively of in circle and circumcircle of a regular polygon of side n. So,

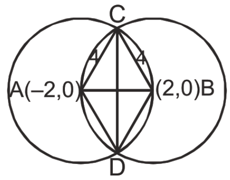
$$r + R = \frac{a}{2}\cot\frac{\pi}{n} + \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$= \frac{a}{2}\left(\frac{1+\cos\pi/n}{\sin\pi/n}\right) = \frac{a}{2}\,\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cdot\cos\frac{\pi}{2n}}$$

$$= \frac{a}{2}\cot\frac{\pi}{2n}$$

14. Ans (A)

The circles are with centers (2, 0) and (–2, 0) and radius 4.

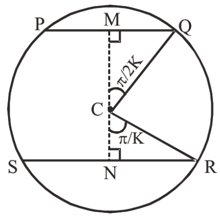


Therefore, the y-axis is their common chord.

ΔABC is equilateral. Hence

$$\text{Area of ADBC} = \frac{2\times\sqrt{3}}{4}(4)^2 = 8\sqrt{3}$$

15. Ans (B)



$$|MN| = |MC| + |CN| = \sqrt{3} + 1$$

$$\Rightarrow 2\cos\frac{\pi}{2K} + 2\cos\frac{\pi}{K} = \sqrt{3} + 1$$

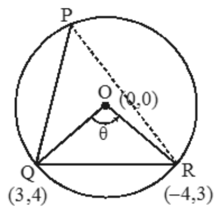
$$\Rightarrow \cos\frac{\pi}{2K} + \cos\frac{\pi}{K} = \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$\Rightarrow \frac{\pi}{K} = \frac{\pi}{3} \Rightarrow K = 3$$

16. Ans (C)

Let O (0, 0) be the centre of the given circle.

Clearly $\angle QPR = \frac{1}{2}\angle QOR = \frac{\theta}{2}$



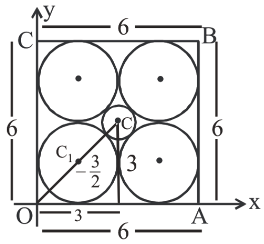
slope of QR $= \frac{3}{-4}$, slope of OQ $= \frac{4}{3}$

$\therefore$ OR and OQ are perpendicular

$\therefore \theta = \frac{\pi}{2}$

Hence $\angle QPR = \frac{\theta}{2} = \frac{\pi}{4}$

17. **Ans (A)**



$c\ (3, 3),\ OC = \sqrt{3^2 + 3^2} = 3\sqrt{2},\ c_1\left(\frac{3}{2}, \frac{3}{2}\right)$

$$c\,c_1 = \sqrt{\left(3 - \frac{3}{2}\right)^2 + \left(3 - \frac{3}{2}\right)^2} = \frac{3\sqrt{2}}{2}$$

Radius of smallest circle $= \frac{3\sqrt{2} - 3}{2}$

Equation of smallest circle

$$(x - 3)^2 + (y - 3)^2 = \left(\frac{3\sqrt{2} - 3}{2}\right)^2$$

18. **Ans (C)**

$$\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{\cos 2x\left(3 + \cos^2 2x\right)}{\cos 2x\left(1 - \sin^2 x\cos^2 x\right)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4\left(3 + \cos^2 2x\right)}{\left(4 - \sin^2 2x\right)} = x^3 - x^2 + 6$$

$\Rightarrow x^3 - x^2 + 2 = 0$

$\Rightarrow (x + 1)\ (x^2 - 2x + 2) = 0$

So, sum of real solutions $= -1$

19. **Ans (B)**

$3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$

$\Rightarrow\ 3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$

$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$

$\Rightarrow\ 5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$

$\Rightarrow\ \tan\alpha = -5\tan\beta \Rightarrow k = -5$

20. **Ans (A)**

$4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$;

$x \in [-2\pi, 2\pi]$

$\Rightarrow\ 4 - 4\cos^2 x - 4\cos^3 x + 9 - 4\cos x = 0$

$\Rightarrow\ 4\cos^3 x + 4\cos^2 x + 4\cos x = 13$

Since L.H.S. ≤ 12, equation has no roots

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (-2)**

$$I = \int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}}$$

$$I = \int \frac{dx}{\sqrt{\frac{\sin^3 x}{\cos^3 x}\cos^8 x}}$$

$$= \int \frac{\sec^4 x}{\sqrt{\tan^3 x}}dx$$

$$= \int \frac{(1 + \tan^2 x)\sec^2 x}{\sqrt{\tan^3 x}}\ dx$$

Let $\tan x = t \Rightarrow \sec^2 x\ dx = dt$

$$= \int \frac{(1 + t^2)dt}{\sqrt{t^3}}$$

$$= \int \left(t^{-3/2} + t^{1/2}\right) dt$$

$$= 2t^{-1/2} + \frac{2}{3}\ t^{3/2} + c$$

$$= -2\sqrt{\cot x} + \frac{2}{3}\sqrt{\tan^3 x} + c$$

2. **Ans (-5)**

$$I = \int \frac{x^9}{(4x^2 + 1)^6}dx$$

$$= \int \frac{dx}{x^3\left(4 + \frac{1}{x^2}\right)^6}$$

Let $4 + \frac{1}{x^2} = t$

$$-\frac{2}{x^3}dx = dt \quad \Rightarrow \frac{dx}{x^3} = \frac{-dt}{2}$$

$$= -\frac{1}{2}\int \frac{dt}{t^6}$$

$$= -\frac{1}{2}\ \frac{t^{-6+1}}{-6+1}$$

$$= \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + c$$

3. Ans (3)

$$\int \frac{\left(\frac{4}{x^5}+\frac{7}{x^8}\right)}{\left(1+\frac{1}{x^4}+\frac{1}{x^7}\right)^2}$$

Put $t = 1+\frac{1}{x^4}+\frac{1}{x^7}$

$$= \int \frac{-dt}{t^2} = \frac{x^7}{x^7+x^4+1}+C$$

4. Ans (5)

$$\int \frac{\sqrt{1+x^{1/3}}}{x^{2/3}}.dx = x$$

Let x1/3 = t

$$\frac{1}{3x^{2/3}}dx = dt$$

$$\Rightarrow \int 3\sqrt{t+1}dt \Rightarrow 3.\frac{(t+1)^{3/2}}{3/2}+c = 2t^{3/2}+c$$

$$\Rightarrow 2(1+x^{1/3})^{3/2}+C \Rightarrow k = 2, m =$$

$k + 2m = 2 + 3 = 5$

5. Ans (1)

$$\int x^5(1+x^3)^{2/3}dx$$

$1 + x^3 = t^2$ and $3x^2\,dx = 2t\,dt$

$$\because \int x^5(1+x^3)^{2/3}dx = \int x^3(1+x^3)^{2/3}x^2dx$$

$$= \int (t^2-1)(t^2)^{2/3}x^2dx$$

$$= \frac{2}{3}\int (t^2-1)\,t^{7/3}\,dt$$

$$= \frac{2}{3}\int \left(t^{13/3}-t^{7/3}\right)dt$$

$$= \frac{2}{3}\left[\frac{3}{16}t^{16/3}-\frac{3}{10}t^{10/3}\right]+c$$

$$= \frac{1}{8}(1+x^3)^{8/3}-\frac{1}{5}(1+x^3)^{5/3}+c$$

6. Ans (2)

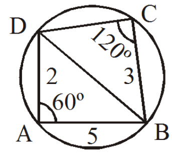
In ΔABD,

$$\cos 60° = \frac{2^2+5^2-BD^2}{2(5)}$$

$\Rightarrow BD^2 = 19$

Now, in ΔBCD

$$\cos 120° = \frac{CD^2+9-19}{2(3)(CD)}$$

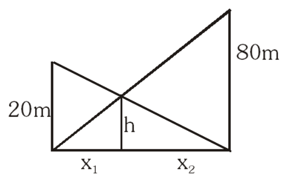


$\Rightarrow CD^2 + 3CD - 10 = 0$

$\Rightarrow CD = -5, 2$

$\Rightarrow CD = 2 \quad (\because CD \neq -5)$

7. Ans (16)



by similar triangle

$$\frac{h}{x_1} = \frac{80}{x_1+x_2} \qquad ...(1)$$

$$\text{by } \frac{h}{x_2} = \frac{20}{x_1+x_2} \qquad ..(2)$$

by (1) and (2)

$\frac{x_2}{x_1} = 4$ or $x^2 = 4x1$

$$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$$

or h = 16m

8. Ans (180)

Given lines are perpendicular

$(\because m_1m_2 = -1)$.

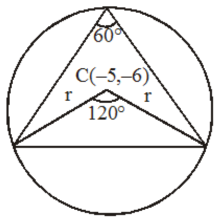
Hence P & Q are the end points of the diameter.

$\therefore$ Angle subtended by are PQ at its centre is 180°

9. Ans (25)

$$3\left(\frac{1}{2}r^2 . \sin 120^\circ\right) = 27\sqrt{3}$$

$$\frac{r^2}{2}\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$



$$r^2 = \frac{108}{3} = 36$$

$$\text{Radius} = \sqrt{25+36-C} = \sqrt{36}$$

$$\boxed{C = 25}$$

10. Ans (48)

$\cos 2x + a \sin x = 2a - 7$

$\Rightarrow\ 1 - 2\sin^2 x + a \sin x = 2a - 7$

$\Rightarrow\ 2\sin^2 x - a \sin x + 2a - 8 = 0$

$\Rightarrow\ \sin x = \frac{a-4}{2}, 2$ (Not possible)

Now, $-1 \leqslant \frac{a-4}{2} \leqslant 1$

$\Rightarrow\ 2 \leq a \leq 6$

$\therefore\ p = 2$ and $q = 6$

$r = \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$

$$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

$$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right] = 4$$

$\therefore\ pqr = 2 \times 6 \times 4 = 48$