

FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – I
PAPER –1
TEST DATE: 26-12-2024

Time Allotted: 3 Hours

Maximum Marks: 180

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts**.
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections: Section-A & Section-B**.

Section – A (01 – 04, 18 – 21, 35 – 38): This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A (05 –07, 22 – 24, 39 – 41): This section contains **NINE (9)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A (08 – 11, 25 – 28, 42 – 45): This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B (12 – 17, 29 – 34, 46 – 51): This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

Physics

PART – I

SECTION – A

(One Options Correct Type)

This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

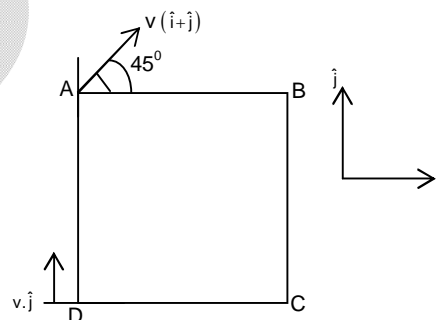
1. A ball is projected from a point in one of the two smooth parallel vertical walls against the other in a plane perpendicular to both. After being reflected at each wall impinge again on the second at a point in the same horizontal plane as it started from. If “e” is the coefficient of restitution, “a” is the distance between the walls and “R” is the free range on horizontal surface, then which of the following is correct?
- (A) $Re^3 = a(1 + e + e^2)$ (B) $Re = a(1 + e^2 + e^3)$
 (C) $Re^2 = a(1 + e + e^2)$ (D) $R = a(1 + e + e^2)$

2. A hemisphere rests in equilibrium on a rough ground and against an equally rough wall, if the equilibrium is limiting, then the inclination θ of the base to the horizontal is
- (A) $\theta = \sin^{-1} \left[\frac{3}{8} \left\{ \frac{\mu + \mu^2}{1 + \mu^2} \right\} \right]$ (B) $\theta = \sin^{-1} \left[\frac{8}{3} \left\{ \frac{\mu + \mu^2}{1 + \mu^2} \right\} \right]$
 (C) $\theta = \cos^{-1} \left[\frac{8}{3} \left\{ \frac{\mu + \mu^2}{1 + \mu^2} \right\} \right]$ (D) $\theta = \cos^{-1} \left[\frac{3}{8} \left\{ \frac{\mu + \mu^2}{1 + \mu^2} \right\} \right]$

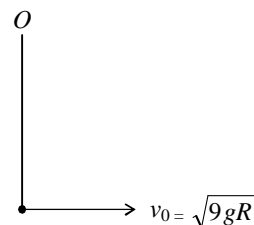
3. The rigid square plate ABCD moves in the xy plane.

If $V_A = v(\hat{i} + \hat{j})$ and $\bar{V}_D = v\hat{j}$, then V_C/V_B is

- (A) 1 (B) $\sqrt{2}$
 (C) $\frac{1}{\sqrt{2}}$ (D) 0



4. A ball is projected with horizontal velocity $v_0 = \sqrt{9gR}$ at the bottom most point attached with inextensible string of length R and fixed at O as shown. Tension in the string in horizontal position
- (A) 3 mg (B) 5 mg
 (C) 7 mg (D) 9 mg



SECTION – A

(One or More than one correct type)

This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

5. A long cable is constructed of a solid conductor of radius 'r' in a co-axial hollow cylindrical conductor of inner and outer radii R_1 and R_2 respectively. I_0 current flows in both the conductors in opposite directions. The magnetic field B at a distance x from the axis of the cable is

(A) $B = \frac{\mu_0 I_0}{2\pi x}$ when $r < x < R_1$

(B) $B = \frac{\mu_0 I_0}{2\pi r^2}$ when $x < r$

(C) $B = \frac{\mu_0 I_0 (R_2^2 - x^2)}{2\pi x (R_2^2 - R_1^2)}$ when $R_1 < x < R_2$

(D) $B = 0$, when $x > R_2$

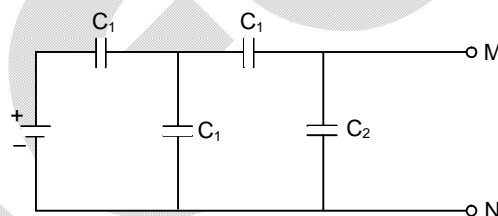
6. If the capacitance ratio $\frac{C_2}{C_1} = \eta$, then the potential difference V between points M and N in the given figure is

(A) If $E=110V$ and $\eta = 2$ then $V= 10$ volt

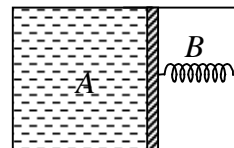
(B) If $E=285V$ and $\eta = 3$ then $V= 15$ volt

(C) If $E=110V$ and $\eta = 1/2$ then $V= 10$ volt

(D) If $E=380V$ and $\eta = 3$ then $V= 20$ volt



7. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S into two equal parts A and B. Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant k is connected with piston and the wall of the container as shown. Initially spring is unstretched. Gas in chamber A is allowed to expand. Let in equilibrium spring is compressed by x_0 . Then



(A) final pressure of the gas is $\frac{kx_0}{S}$

(B) work done by the gas is $\frac{1}{2} kx_0^2$

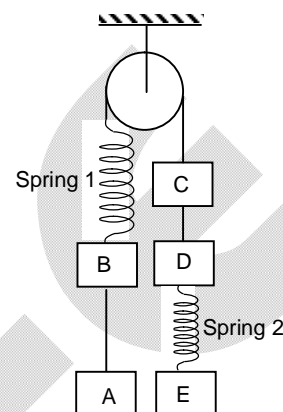
(C) change in internal energy of the gas is $-\frac{1}{2} kx_0^2$

(D) temperature of the gas is decreased

SECTION – A (Matching List Type)

This section contains **FOUR (04)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

8. The system shown below is initially in equilibrium
 $m_A = m_B = 3\text{kg}$
 $m_C = m_D = m_E = 2\text{kg}$

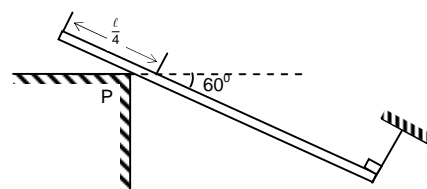


List-I		List-II	
(P)	Just after the spring 2 is cut the block D	(1)	Accelerates up
(Q)	Just after the spring 2 is cut the block B	(2)	Accelerates down
(R)	Just after the spring 1 is cut, the block A	(3)	Momentarily at rest
(S)	Just after the spring 1 is cut, the block D	(4)	Has acceleration g
		(5)	Constant velocity

The correct option is:

- (A) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 4$; $S \rightarrow 2,3$ (B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2,4$; $S \rightarrow 2$
 (C) $P \rightarrow 1$; $Q \rightarrow 3,4$; $R \rightarrow 2$; $S \rightarrow 4$ (D) $P \rightarrow 2$; $Q \rightarrow 3$; $R \rightarrow 4$; $S \rightarrow 2,3$

9. A uniform rod of mass m and length ℓ is in equilibrium under the action of contact forces, gravity and tension in string as shown in the figure.

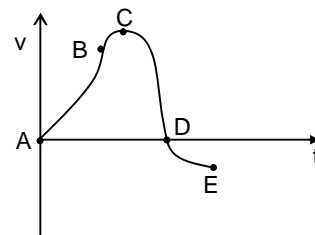


List-I		List-II	
(P)	The tension in the string is	(1)	$\frac{2mg}{7}$
(Q)	The frictional force acting on the rod is	(2)	$\frac{mg}{3}$
(R)	If the string is cut, then just after it normal reaction on the rod is	(3)	$\frac{mg}{6}$
(S)	Just before the string is cut, the normal reaction on the rod is	(4)	$\frac{\sqrt{3} mg}{2}$
		(5)	Zero

The correct option is:

- (A) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$ (B) $P \rightarrow 1$; $Q \rightarrow 2$; $R \rightarrow 3$; $S \rightarrow 4$
 (C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$ (D) $P \rightarrow 2$; $Q \rightarrow 1$; $R \rightarrow 4$; $S \rightarrow 3$

10. The velocity time graph of a particle moving along x-axis is shown in figure. Match the entries of List-I with entries of List-II.



List-I		List-II	
(P)	For AB particle is	(1)	Moving in positive x-direction with increasing speed.
(Q)	For BC particle is	(2)	Moving in positive x-direction with decreasing speed
(R)	For CD particle is	(3)	Moving in negative x-direction with increasing speed
(S)	For DE particle is	(4)	Moving in negative x-direction with decreasing acceleration.
		(5)	Moving in positive x-direction with increasing acceleration.

The correct option is:

- (A) $P \rightarrow 1,4$; $Q \rightarrow 3$; $R \rightarrow 1,4$; $S \rightarrow 2,4$ (B) $P \rightarrow 1,5$; $Q \rightarrow 1$; $R \rightarrow 2$; $S \rightarrow 3,4$
 (C) $P \rightarrow 1,4$; $Q \rightarrow 1$; $R \rightarrow 3$; $S \rightarrow 2,5$ (D) $P \rightarrow 1,4$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 1,5$

11. List-I shows the cylindrical region of radius r where a downward magnetic field \vec{B} exists, and \vec{B} is increasing at the rate of $\frac{dB}{dt}$. A rod PQ is placed in different situation as shown. Match the list-I with the correct statement in list-II regarding the induced e.m.f. in rod.

List-I		List-II	
(P)		(1)	Induced e.m.f. in rod PQ is $\frac{1}{2}r^2\theta \frac{dB}{dt}$
(Q)		(2)	Induced e.m.f. in rod PQ is less than $\frac{1}{2}r^2\theta \frac{dB}{dt}$
(R)		(3)	End P is positive with respect to point Q

(S)		(4)	End Q is positive with respect to point P
		(5)	End P and Q are at same potential

The correct option is:

- (A) $P \rightarrow 3,4$; $Q \rightarrow 5$; $R \rightarrow 5,1$; $S \rightarrow 1,4$ (B) $P \rightarrow 3,2$; $Q \rightarrow 4$; $R \rightarrow 1,5$; $S \rightarrow 2,1$
 (C) $P \rightarrow 2,4$; $Q \rightarrow 1,3$; $R \rightarrow 4,1$; $S \rightarrow 2,3$ (D) $P \rightarrow 2,3$; $Q \rightarrow 1,4$; $R \rightarrow 3,1$; $S \rightarrow 2,4$

SECTION – B

(Numerical Answer Type)

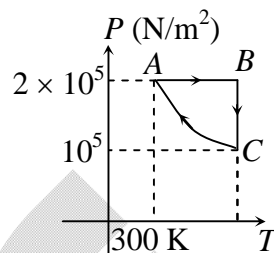
This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

12. An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at the both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is $\frac{3}{n}$ then $n = ?$
13. As shown in figure light string PQR is stretched by force $F = (3 - 10kt)$ N, where k is a constant and t is time in second. At time $t = 0$, a pulse is generated at the end P of the string. For the value of k (in N/s) if the value of force becomes zero as the pulse reaches point Q. Given mass per unit length of string is 0.03 kg/m.
-
14. In a thermally isolated system, two boxes filled with an ideal gas are connected by a valve. When the valve is in closed position, states of the box 1 and 2, respectively, are (1 atm, V , T) and (0.5 atm, $4V$, T). When the valve is opened, what is approximately the final pressure (in atm) of the system is P , then find the value of $10P$.
15. Two long parallel rails separated by a distance ' ℓ ' are connected by a movable connector of mass ' m '. Terminals of the rails are connected across a resistor ' R ' and a capacitor C as shown in figure. A uniform magnetic field B perpendicular to the plane of rails is switched on and the connector is dragged by a constant force F . The terminal velocity (in m/s) of the connector is $\frac{1}{v}$, then find v . (Neglect the friction between rails & connector and take $B\ell = 2\sqrt{FR}$).
-

16. Find the mass ratio M_D/M_H of deuterium and hydrogen (in near integer value) if their $H\alpha$ lines have wavelengths of 6561.01 \AA and 6562 \AA respectively.

17. Two moles of a monatomic ideal gas is taken through a cyclic process shown on pressure(P)-temperature(T) diagram in figure. Process CA is represented as $PT = \text{Constant}$. If efficiency of given cyclic process is

$$1 - \frac{3x}{12\ln 2 + 15}, \text{ then find the value of 'x'.$$



Chemistry

PART – II

SECTION – A

(One Options Correct Type)

This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

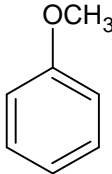
18. P_4O_{10} can react with
 (A) HCl (B) HNO_3
 (C) HBr (D) HI
19. In photoelectric experiment, the emission of the number of photoelectrons depends on
 (A) the frequency of incident radiation (B) the intensity of incident radiation
 (C) the wavelength of incident radiation (D) the wave number of incident radiation
20. The most thermally unstable complex is
 (A) $[Cr(NH_3)_6][Co(CN)_6]$ (B) $[Cr(NH_3)_2(CN)_4][Co(NH_3)_4(CN)_2]$
 (C) $[Cr(NH_3)_3(CN)_3][Co(NH_3)_3(CN)_3]$ (D) $[Cr(NH_3)(CN)_5][Co(NH_3)_5(CN)]$
21. If Λ_m = molar conductivity of the solution of an electrolyte.
 Λ_m^0 = limiting molar conductivity of the solution
 α = degree of dissociation of the electrolyte
 (A) $\frac{\Lambda_m^0}{\Lambda_m} = \alpha$ (B) $\frac{\Lambda_m}{\Lambda_m^0} = \alpha$
 (C) $\Lambda_m^0 \times \Lambda_m = \alpha$ (D) $\Lambda_m^0 - \Lambda_m = \alpha$

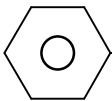
SECTION – A

(One or More than one correct type)

This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

22. The correct statement(s) regarding CH_4 is
 (A) it is a tetrahedral molecule
 (B) it undergoes substitution reaction
 (C) it can be formed by the reaction between CH_3MgBr and phenol
 (D) it diffuses at a faster rate than dioxygen
23. Which of the following substances on heating produces PH_3 ?
 (A) PH_4Cl (B) H_3PO_3
 (C) H_3PO_4 (D) Ca_3P_2

24. The correct statement(s) regarding anisole  is/are

- (A) it can be formed by treating phenol with hot CH_2N_2 .
 (B) it can undergo halogenation reaction(both electrophilic and free radical mechanism).
 (C) it is oxidized to benzoic acid.
 (D) it can be produced by treating  with CH_3Br .

SECTION – A
(Matching List Type)

This section contains **FOUR (04)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

25. List-I contains ionic solids and list-II states regarding their crystal structure.

	List-I		List-II
(P)	NaCl	(1)	Cations occupy half of the tetrahedral voids of FCC unit cell
(Q)	CsCl	(2)	Cations occupy octahedral voids of FCC unit cells
(R)	ZnS	(3)	Coordination number of the cation is eight
(S)	CaF_2	(4)	Cations and anions have different coordination numbers
		(5)	Coordination number of both cations and anions is four

- (A) $P \rightarrow 3; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 5$ (B) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$
 (C) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3$ (D) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 5; S \rightarrow 1$

26. List-I contains complexes and their behaviour is mentioned in list-II.

	List-I		List-II
(P)	$[\text{Cu}(\text{CN})_4]^{2-}$	(1)	Colourless complex
(Q)	$[\text{FeF}_6]^{3-}$	(2)	Back donation from metal to ligand takes place
(R)	$[\text{Cr}(\text{en})_3]^{3+}$	(3)	Coordination number is six
(S)	$[\text{Ni}(\text{CO})_4]$	(4)	Square planar complex
		(5)	Chelate complex

- (A) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 2$ (B) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 1$
 (C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$ (D) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 3; S \rightarrow 2$

27. Match the lists.

	List-I (Reactions)		List-II (Properties of products)
(P)		(1)	The product upon ozonolysis forms only glyoxal(OHC – CHO)
(Q)		(2)	The product is an alcohol but does not undergo oxidation with acidified KMnO_4 solution
(R)		(3)	Product contains two benzene nuclei
(S)		(4)	The intermediate shows aromaticity
		(5)	The product undergoes $\text{S}_{\text{N}}2$ reaction

 (A) $\text{P} \rightarrow 3; \text{Q} \rightarrow 2; \text{R} \rightarrow 1; \text{S} \rightarrow 4$

 (B) $\text{P} \rightarrow 4; \text{Q} \rightarrow 2; \text{R} \rightarrow 5; \text{S} \rightarrow 3$

 (C) $\text{P} \rightarrow 2; \text{Q} \rightarrow 1; \text{R} \rightarrow 3; \text{S} \rightarrow 4$

 (D) $\text{P} \rightarrow 1; \text{Q} \rightarrow 3; \text{R} \rightarrow 4; \text{S} \rightarrow 2$

28. Match the lists.

	List-I (Species of iron)		List-II (Characteristics)
(P)	Fe^{3+}	(1)	Undergoes dsp^3 hybridization to form a mononuclear carbonyl
(Q)	Fe^{2+}	(2)	Forms blood-red colouration with KSCN
(R)	Fe^{4+}	(3)	The spin-magnetic moment is identical to that of Fe^{2+} ion
(S)	Fe	(4)	The aqueous solution of its sulphate is green coloured
		(5)	Oxide is colourless

 (A) $\text{P} \rightarrow 2; \text{Q} \rightarrow 1; \text{R} \rightarrow 3; \text{S} \rightarrow 4$

 (B) $\text{P} \rightarrow 4; \text{Q} \rightarrow 3; \text{R} \rightarrow 1; \text{S} \rightarrow 2$

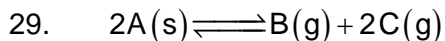
 (C) $\text{P} \rightarrow 2; \text{Q} \rightarrow 4; \text{R} \rightarrow 3; \text{S} \rightarrow 1$

 (D) $\text{P} \rightarrow 4; \text{Q} \rightarrow 2; \text{R} \rightarrow 1; \text{S} \rightarrow 3$

SECTION – B

(Numerical Answer Type)

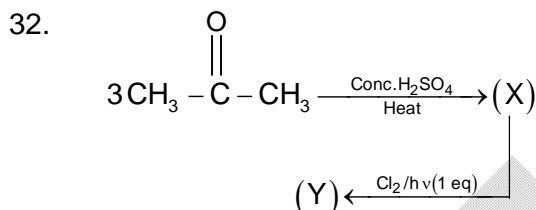
This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.



Above reaction attains equilibrium at 240°C and 18 atm pressure. What is the equilibrium constant K_p of the reaction?

30. 'X' is an aliphatic monocarboxylic acid. It is the simplest acid which can form only one α -chloro product(Y), upon treatment with Cl_2 in presence of red phosphorus(H.V.Z reaction). What is the molar mass of X in g mol^{-1} unit?

31. AB is a polar molecule. Its bond length is 5 \AA . If the maximum dipole moment of the molecule is $b \times 10^{-29}$ coulomb meter, what is the value of '100b'?
[Assume charge of electron = 1.602×10^{-19} coulomb]



What is the molar mass of compound (X) in g mol^{-1} unit?

33. Salt(A) produces golden yellow flame on flame test. Aqueous solution of (A) produces a white precipitate with barium chloride solution. The precipitate is insoluble in dilute mineral acids. On thermal decomposition in presence of SiO_2 , (A) forms a salt (B) and gas(C) when treated with an acid(D), salt(B) forms NaCl , SiO_2 and H_2O . What is the molar mass of (C)?

34. 418.52 mL of x M acidified potassium permanganate solution can completely oxidize 523.15×10^{-3} moles of Sn^{2+} ions to Sn^{4+} ions. What is the value of 10x?



Mathematics

PART – III

SECTION – A

(One Options Correct Type)

This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

35. Let $f(x) = 2 \operatorname{cosec}(2x) + \sec(x) + \operatorname{cosec}(x)$ then the minimum value of $f(x)$ for $x \in \left(0, \frac{\pi}{2}\right)$ is
- (A) $\frac{1}{\sqrt{2}-1}$ (B) $\frac{2}{\sqrt{2}-1}$
 (C) $\frac{1}{\sqrt{2}+1}$ (D) $\frac{2}{\sqrt{2}+1}$
36. Let $f(x) = \lim_{n \rightarrow \infty} \tan^{-1} \left(4n^2 \left(1 - \cos \frac{x}{n} \right) \right)$ and $g(x) = \lim_{n \rightarrow \infty} \frac{n^2}{2} \ln \cos \left(\frac{2x}{n} \right)$. Then $\lim_{x \rightarrow 0} \left(\frac{e^{-2g(x)} - e^{f(x)}}{x^6} \right)$ is equal to
- (A) $-\frac{5}{3}$ (B) $\frac{5}{3}$
 (C) $-\frac{8}{3}$ (D) $\frac{8}{3}$
37. Number of values of x in $[-4, 4]$ where $f(x) = [3x + 14] + |4x^2 - 1|(2x^2 + 3x - 2) + \sin\left(\frac{\pi x}{2}\right)$ is non – derivable, is equal to
- [Note: $[k]$ denotes the largest integer less than or equal to k .]
 (A) 23 (B) 24
 (C) 25 (D) 26
38. Let b and c be real numbers and define the polynomial $P(x) = x^2 + bx + c$. Suppose that $P(P(1)) = P(P(2)) = 0$, and that $P(1) \neq P(2)$. Find $P(0)$.
- (A) $-\frac{1}{2}$ (B) 0
 (C) $-\frac{3}{2}$ (D) $\frac{1}{2}$

SECTION – A

(One or More than one correct type)

This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

39. Consider $L = \lim_{x \rightarrow 0} \frac{\sin(x^2) + 2\cos(bx) - ax^4 - 2}{e^{ax} - 1 - ax - 2x^2 - \frac{a^3 x^3}{6}}$, $a, b \in \mathbb{I}$. Identify the correct statement(s)?
- (A) If $L = \frac{25}{8}$ then $a + b = 3$ (B) If $L = \frac{25}{8}$ then $a + b = -3$
- (C) If $L = \frac{-23}{8}$ then $a + b = -1$ (D) If $L = \frac{-23}{8}$ then $a + b = 1$
40. Let $f(x) = x^2 \sin \frac{1}{x}$ for $0 < x \leq 1$ and $f(0) = 0$. If $g(x) = x^2$ for $x \in [0, 1]$ then which of the following statement(s) is/are correct?
- (A) $f(x)$ is differentiable in $[0, 1]$ (B) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ does not exist
- (C) $\lim_{x \rightarrow 0} f'(x)g'(x)$ does not exist (D) $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ does not exist
41. If f is differentiable on $[a, b]$ and if $f(a) = f(b) = 0$, then for some $x \in (a, b)$
- (A) $xf(x) + f'(x) = 0$ (B) $xf(x) - f'(x) = 0$
- (C) $50f(x) + f'(x) = 0$ (D) $f'(x) + (\ln 50)f(x) = 0$

SECTION – A

(Matching List Type)

This section contains **FOUR (04)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

42. Match the following

	List - I		List - II
(P)	If A and B are two matrices of order 3 such that $AB = 0$, $A^2 + B = I$, then $\text{tr}(A^2 + B^2)$ is equal to [Note: $\text{tr}(P)$ denotes the trace of matrix P.]	(1)	0
(Q)	Let $y = f(x)$ satisfy the differential equation, $y \ln y + xy' = 0$ where $y(1) = e$ and $\lim_{x \rightarrow \infty} x(f(x) - 1) = e^\lambda$, then λ equals	(2)	1
(R)	If $\int_1^e 2^{\ln x} dx + \int_{1+e}^{2+e} e^{\log_2(x-e)} dx = \lambda$ then $[\lambda]$ is equal to [Note: $[k]$ denotes greatest integer less than or equal to k.]	(3)	2

(S)	If $P(\alpha + 1, \alpha - 3)$ be any point in xy plane, then the number of integral values of α for which the point P lies between the lines $x + 2y = 1$ and $2x + 4y = 14$, is	(4)	3
		(5)	4

The correct option is

- (A) $P \rightarrow (1) \quad Q \rightarrow (4) \quad R \rightarrow (5) \quad S \rightarrow (2)$ (B) $P \rightarrow (4) \quad Q \rightarrow (1) \quad R \rightarrow (5) \quad S \rightarrow (2)$
 (C) $P \rightarrow (4) \quad Q \rightarrow (1) \quad R \rightarrow (2) \quad S \rightarrow (5)$ (D) $P \rightarrow (2) \quad Q \rightarrow (5) \quad R \rightarrow (1) \quad S \rightarrow (4)$

43. Match List - I with List - II:

List - I		List - II	
(P)	The least value of $8 z - 7 + 6 z - 5 $; $z \in \mathbb{C}$	(1)	12
(Q)	The least value of $\left \frac{z - 7 + 11i}{\sqrt{2}} \right + \left \frac{z + 5 - i}{\sqrt{2}} \right + \left \frac{z + 4}{\sqrt{2}} \right $; $z \in \mathbb{C}$ is	(2)	13
(R)	Find the least value of $ z - 3 - 4i + z - i + z + z - 1 $; $z \in \mathbb{C}$	(3)	24
(S)	The least value of $ z - 3 ^2 + z - 5 + 2i ^2 + z - 1 + i ^2$	(4)	10
		(5)	$5 + \sqrt{2}$

The correct option is

- (A) $P \rightarrow (3) \quad Q \rightarrow (5) \quad R \rightarrow (4) \quad S \rightarrow (2)$ (B) $P \rightarrow (5) \quad Q \rightarrow (4) \quad R \rightarrow (3) \quad S \rightarrow (1)$
 (C) $P \rightarrow (5) \quad Q \rightarrow (3) \quad R \rightarrow (1) \quad S \rightarrow (2)$ (D) $P \rightarrow (3) \quad Q \rightarrow (2) \quad R \rightarrow (1) \quad S \rightarrow (4)$

44. The number of 10 letter permutations comprising 4 a's, 3 b's and 3 c's such that

List - I		List - II	
(P)	a's are separated and all b's are together is equal to	(1)	18
(Q)	a's are separated and exactly two b's are together is equal to	(2)	20
(R)	no two adjacent letters are identical is equal to	(3)	150
(S)	no two b's are together, no two c's are together and all 'a' s are	(4)	180
		(5)	248

The correct option is

- (A) $P \rightarrow (2) \quad Q \rightarrow (4) \quad R \rightarrow (5) \quad S \rightarrow (1)$ (B) $P \rightarrow (1) \quad Q \rightarrow (2) \quad R \rightarrow (3) \quad S \rightarrow (4)$
 (C) $P \rightarrow (5) \quad Q \rightarrow (4) \quad R \rightarrow (2) \quad S \rightarrow (3)$ (D) $P \rightarrow (1) \quad Q \rightarrow (5) \quad R \rightarrow (3) \quad S \rightarrow (4)$

45. Match List - I with List - II:

List - I		List - II	
(P)	If the fourth term in the expansion of $\left(\frac{x}{a} + \frac{1}{x} \right)^n$ is $\frac{5}{2}$ then a is	(1)	2
(Q)	$\sum_{p=1}^4 \sum_{r=p}^4 {}^4C_r \cdot {}^rC_p = k$ then $\frac{k}{13}$ is	(2)	4
(R)	The coefficient of x^{13} in $(1-x)^5 (1+x+x^2+x^3)^4$	(3)	5
(S)	$\sum_{r=0}^4 {}^4C_r (r-2)^2 = k$ then $\frac{k}{2}$ is	(4)	8
		(5)	13

The correct option is

- (A) $P \rightarrow (1) \quad Q \rightarrow (3) \quad R \rightarrow (5) \quad S \rightarrow (4)$ (B) $P \rightarrow (1) \quad Q \rightarrow (3) \quad R \rightarrow (2) \quad S \rightarrow (5)$
 (C) $P \rightarrow (3) \quad Q \rightarrow (2) \quad R \rightarrow (4) \quad S \rightarrow (5)$ (D) $P \rightarrow (1) \quad Q \rightarrow (3) \quad R \rightarrow (2) \quad S \rightarrow (4)$

SECTION – B

(Numerical Answer Type)

This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

46. If $S_n = \sum_{r=1}^n \tan^{-1} \left(\frac{2(2r-1)}{4+r^2(r^2-2r+1)} \right)$ then find the value of $\lim_{n \rightarrow \infty} \sum_{n=2}^n (\cot(S_{n-1}) - \cot(S_n))$
47. For $p, n \in \mathbb{N}$, let $f(x) = 1 - x^p$ and $g_n(x) = \frac{n}{\frac{1}{f(x)} + \frac{1}{f(2x)} + \dots + \frac{1}{f(nx)}}$. Find the value of $\lim_{x \rightarrow 0} \frac{1 - g_n(x)}{x^p}$ at $n = 5$ and $p = 3$.
48. Let f and g be two real valued differentiable function on \mathbb{R} such that $\frac{f'(x)}{g'(x)} = \frac{g^2(x)}{f^2(x)} \forall x \in \mathbb{R}$. If $f(10) = 5$, $g(10) = 2$ and $g(2) = 1$, then find the value of $(f(2))^3$.
49. Let P be the point on the curve $4x^2 + \alpha^2 y^2 = 4\alpha^2, 0 < \alpha^2 < 8$ whose distance from $Q(0, -2)$ is greatest. If R is the reflection of P in the x -axis then find the least distance of R from the line $3x - 4y + 7 = 0$
50. If $\lim_{n \rightarrow \infty} \left(\prod_{r=1}^n \sin \left(\frac{r\pi}{4n} \right) \cos \left(\frac{r\pi}{4n} \right) \right)^{\frac{1}{n}} = \frac{a}{b}$, $a, b \in \mathbb{N}$ then find the least value of $(a+b)$.
51. The sum of all real value of k for which the equation $|x^2 - (7+k^2)x + 7k^2| + \sqrt{(x-3)(x-3k+2)} = 0$ has atleast one real solution.