FIITJEE RBT-9 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

QP Code: 100967 **Paper-1**

Time Allotted: 3 Hours **Maximum Marks: 180**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

(i) **PART – A *(01 – 04)*:** This section contains Four **(04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer. Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

PART – A *(05 – 10)*: This section contains **Six (06)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(ii) **PART – B *(1 – 3)*:** This section contains **Three (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.** Each question carries **+4 marks** for correct answer and **there will be no negative marking.**

(iii) **PART – B *(4 – 9)*:** This section contains Six **(06)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places. Each question carries **+2 marks** for correct answer and **there will be no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**___________________

Enrolment Number :____

SECTION – I : PHYSICS

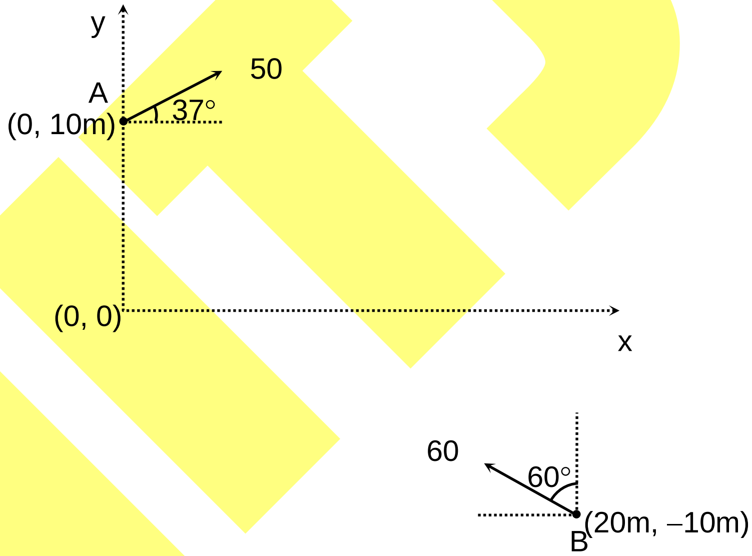
PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A circular disc of radius R caries surface charge density $\sigma(r) = \sigma_0\left(1-\frac{r^2}{R^2}\right)$, where σ_0 is a constant and r is the distance from the centre of the disc. Assume that disc is in xz-plane and its axis is y-axis. We have an imaginary hollow sphere of radius R and centre at $\left(0, \frac{3}{5}R, 0\right)$. What is the flux of electric field through this imaginary sphere?

(A) $\frac{408}{625}\left(\frac{\sigma_0\pi R^2}{\varepsilon_0}\right)$
(B) $\frac{272}{625}\left(\frac{\sigma_0\pi R^2}{\varepsilon_0}\right)$
(C) $\frac{544}{625}\left(\frac{\sigma_0\pi R^2}{\varepsilon_0}\right)$
(D) $\frac{136}{625}\left(\frac{\sigma_0\pi R^2}{\varepsilon_0}\right)$

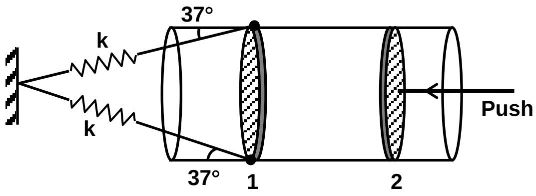
2. Two particles A and B are projected in the vertical plane with velocity 50 m/s at an angle of 37° with horizontal and with velocity 60 m/s at an angle of 60° with vertical respectively as shown in the figure. Find the minimum distance between the particles A and B during the motion? (Take g = 10 m/s² and $\tan 37° = \frac{3}{4}$)



(A) 10 m
(B) 15 m
(C) 20 m
(D) 8 m

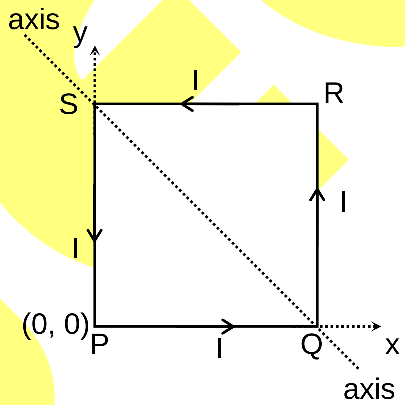
Space For Rough Work

3. A long container has air enclosed inside at room temperature and atmospheric pressure 10^5 N/m^2 between two movable pistons. It has a volume 20,000 cc. The area of cross section is 100 cm^2 and force constant of springs is k = 781.25 N/m^2. Now push the right piston isothermally and slowly till it reaches the original position of the left piston-1 which is movable. What is the final length of air column. Assume that spring is initially relaxed and length of spring is very large as compare to separation between two piston.



(A) 100 cm
(B) 75 cm
(C) 50 cm
(D) 200 cm

4. A uniform rigid, square loop of mass 'm' and side 'ℓ' is free to rotate about an axis in xy plane passing through two corner as shown in figure. A constant current I is flowing in the loop. In space magnetic field $\vec{B}(x) = B_0\left(1+\frac{x}{\ell}\right)(\hat{i}+\hat{k})$ is present. Initially loop is held at rest in xy-plane. On releasing its instantaneous angular acceleration will be (space is gravity free space)



(A) $\frac{9IB}{\sqrt{2}m}$
(B) $\frac{5IB}{2\sqrt{2}m}$
(C) $\frac{9IB}{2\sqrt{2}m}$
(D) $\frac{5\sqrt{2}IB}{2m}$

Space For Rough Work

PART – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A gaseous mixture at 300 K and 2×10^5 N/m^2 pressure contains 6g of H_2 and 8g of He. The mixture is expanded four times its initial volume, through an isobaric heating process. Then it is **isochoricaly** cooled unit its temperature again becomes 300 K. After that the gas mixture is isothermally compressed to its original volume. Choose the correct option(s). (take $\ell n2 = 0.695$)

(A) Ratio of molar specific heat (γ) of the mixture is $\frac{31}{21}$

(B) Ratio of heat absorb in isobaric expansion to that of heat rejected in isochoric cooling is $\frac{31}{21}$

(C) Heat rejected in isothermal compression is more than the heat rejected in isochoric cooling.

(D) Efficiency of the complete cycle is $\frac{161}{930}$.

6. An electron, initially at rest, is released far away from a proton (fixed in space). The de-broglie wavelength of the electron when it is at distance r from proton is λ. The ratio of its Kinetic energy at this distance with the kinetic energy of the electron in ground state of hydrogen atom (Bohr model) with the radius of its orbit being r, is μ. Pick the correct option(s).

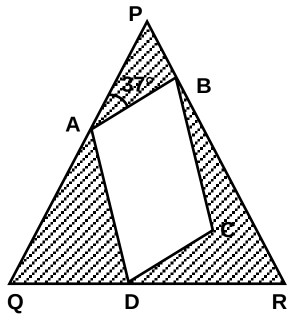
(A) $\lambda \propto \sqrt{r}$

(B) $\lambda \propto \frac{1}{r}$

(C) $\mu = \frac{1}{2}$

(D) $\mu = 2$

7. A rectangular cavity ABCD is carved inside an equilateral prism PQR of refractive index $\left(\frac{8}{5}\right)$ as shown in figure. If δ_{min} is the deviation in a ray entering at face PQ and emerging at face PR, without passing through AB or CD, then which of the following is/are correct.



(A) There is no effect of cavity of the value of δ_{min}.

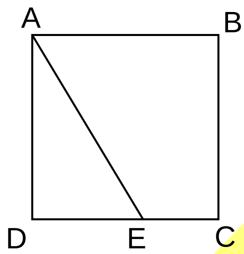
(B) The minimum deviation $\delta_{min} = 46°$

(C) If the cavity is filled with water ($\mu_W = 4/3$), the $\delta_{min} = 46°$

(D) If the cavity is filled with a liquid of refractive index $\mu = 2$, then $\delta_{min} = 23°$

Space For Rough Work

8. ABCD is square as shown in figure, resistance of side AB, BC, CD and DA are 8Ω, 2Ω, 14Ω and 22Ω respectively. A point E lies on the CD such that a uniform wire of resistance 32Ω is connected across AE and constant potential difference is applied across A and C then B and E are equipotential. Choose the correct option(s).



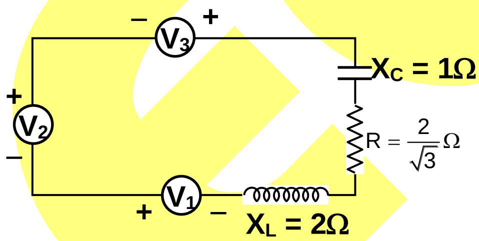
(A) Resistance of DE is 10Ω

(B) Resistance of EC is 10Ω

(C) Equivalent resistance across AC is $\frac{20}{3}\Omega$

(D) Equivalent resistance across AC is 10Ω

9. Three alternating voltage sources $V_1 = 3\sin\omega t$, $V_2 = 5\sin(\omega t + 30°)$ and $V_3 = 5\sin(\omega t - 127°)$ volt connected across a resistor, inductor and capacitor as shown in the figure, then choose the correct option(s)



(A) The maximum current in the resistors is 3 ampere.

(B) The maximum current in the resistor is 2 ampere.

(C) Effective impedance of the circuit is $\sqrt{\frac{7}{3}}\Omega$

(D) Maximum potential of the three AC source is $\sqrt{21}$ volts.

10. Two vernier calipers A and B both have M.S.D. of 1 mm. 10 vernier scale division have same length as 7 main scale division of vernier A and 8 vernier scale division have same length as 3 main scale division for vernier B.

(A) The least count of vernier A is more than the vernier B.

(B) The least count of vernier A is less than the vernier B.

(C) The least count of vernier B is 0.125 mm.

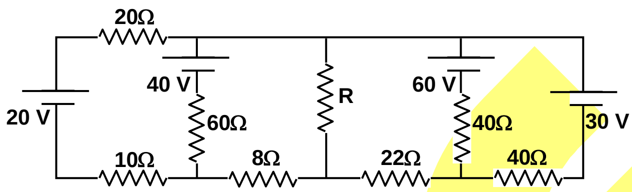
(D) The magnitude of difference in least count of vernier A and vernier B is 0.025 mm.

Space For Rough Work

PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. In the given circuit, the value of R so that thermal power generated in R will be maximum is 4.2mΩ. Find the value of m.



2. Velocity of an object in rectilinear motion is given as function of time by $v = 4t - 3t^2$, where v is in m/s and t is in seconds. Its average speed over the time interval from t = 0 second to t = 2 seconds, is $\frac{8K}{27}$ m/s. Find the value of K.

3. An open pipe 68.48 cm long and a closed pipe 58.4 cm long, both having same diameter, are producing their second overtone, and these are in unison. Determine the end correction in centimeter of these pipes.

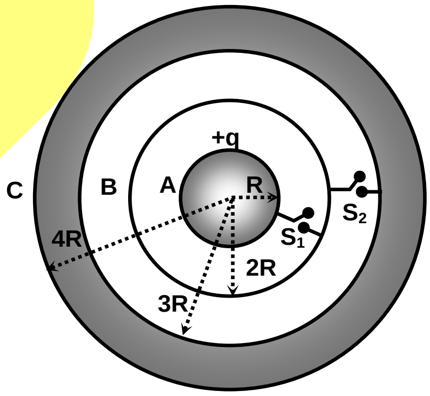
PART – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 4 and 5

Question Stem

Three conducting bodies A(solid sphere), B (spherical shell) and C(hollow sphere with thickness R) are arranged as shown in the figure. A charge q is given to the inner sphere. When S_1, S_2 both are open capacity of the system is C_1 and when S_1 is closed but S_2 is open capacity is C_2, then $(C_2 - C_1) = \frac{X\pi\varepsilon_0 R}{55}$. When S_2 is closed but S_1 is opened capacity is C_3 and When both S_1 and S_2 are closed capacity is C_4 then $(C_4 - C_3) = \frac{Y\pi\varepsilon_0 R}{3}$.



4. The value of X is ....

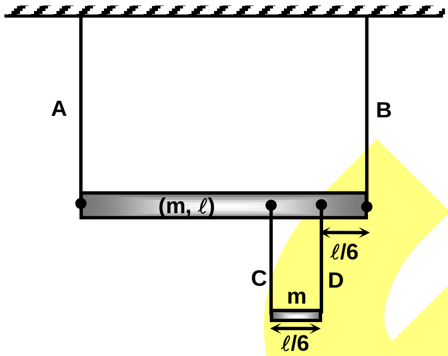
5. The value of Y is ....

Space For Rough Work

Question Stem for Question Nos. 6 and 7

Question Stem

A uniform rod of mass ‘m=10kg’ and length ‘ℓ=20m’ is held horizontally by two vertical strings of negligible mass and another rod of mass ‘m’ is also held by two strings with the rod as shown in the figure. The tension in the string ‘A’ immediately after the string B and D is cut is X newton and the tension in the string ‘A’ immediately after the string B and C is cut is Y newton. (Take g = 10 m/s^2)



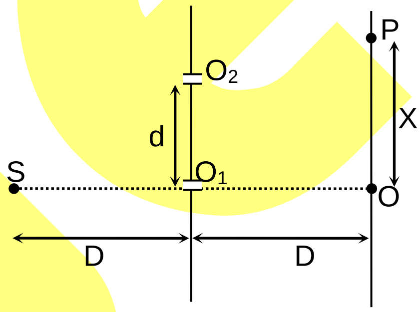
6. The value of X is ………..

7. The value of Y is ………..

Question Stem for Question Nos. 8 and 9

Question Stem

A monochromatic light source is kept at S having wavelength λ in a modified YDSE setup. The minimum value of d so that there is a dark fringe at O is $d_{min} = \sqrt{\frac{D\lambda}{M}}$. For the value of d_{min}, the distance of the 2nd nearest bright fringe from O is X(as shown) = Kd_{min}.



8. The value of M is………………

9. The value of K is………………

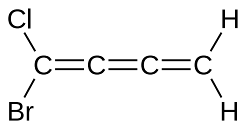
Space For Rough Work

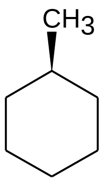
SECTION – II : CHEMISTRY

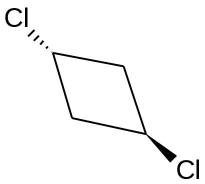
PART – A (Maximum Marks: 12)

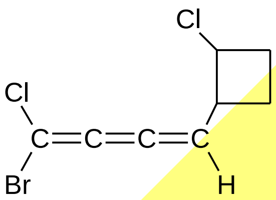
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

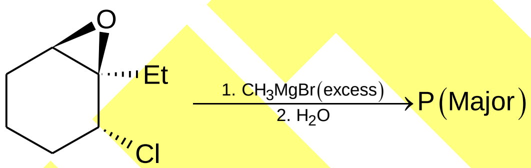
1. Optically active is

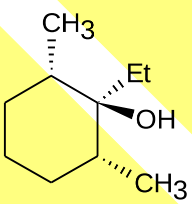
(A) 

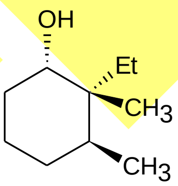
(B) 

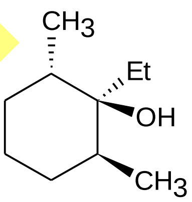
(C) 

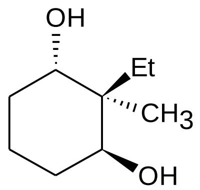
(D) 

2. 

(A) 

(B) 

(C) 

(D) 

3. 4 mole of a mixture O_2 and O_3 is reacted with excess of acidified solution of KI. The liberated iodine require 1 L of 2 M hypo solution for complete reaction. The mole percent of O_3 in the initial sample is

(A) 25
(B) 30
(C) 75
(D) 50

4. Square pyramidal shape is

(A) XeF_2
(B) ICl_4^-
(C) XeF_5^+
(D) XeF_5^-

Space For Rough Work

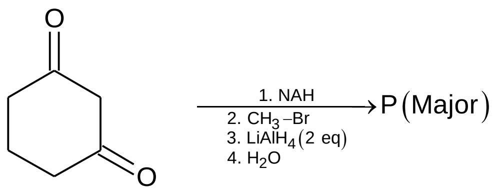
PART – A (Maximum Marks: 24)

This section contains ***SIX (06)*** *questions. Each question has* ***FOUR*** *options (A), (B), (C) and (D).* ***ONE OR MOER THAN ONE*** *of these four option(s) is (are) correct answer(s).*

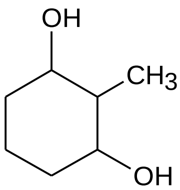
5. High spin complex is/are

(A) $[Fe(H_2O)_6]^{2+}$ (B) $[FeF_6]^{3-}$

(C) $[Fe(CN)_6]^{3-}$ (D) $[Co(H_2O)_6]^{3+}$

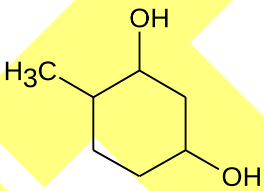
6. 

Choose correct option is/are

(A) P is 

(B) Number stereo isomers in P is 4

(C) Number of stereo isomer in P is 3

(D) P is 

7. The correct statement for $2p_z$ orbital is/are

(A) $\psi_{n,\ell,m} \propto \left(\frac{Z}{a}\right)^{3/2}\left(\frac{Zr}{a}\right)e^{-2r/2a}\cos(\theta)$

(B) $\psi_{n,\ell,m} \propto \left(\frac{Z}{a}\right)^{3/2}\left(2-\frac{Zr}{a}\right)e^{-2r/2a}$

(C) xy plane is nodal plane

(D) $2p_z$ ungerade orbital

Space For Rough Work

8. Which of the following complexes is/are shows the co-ordination isomerism?

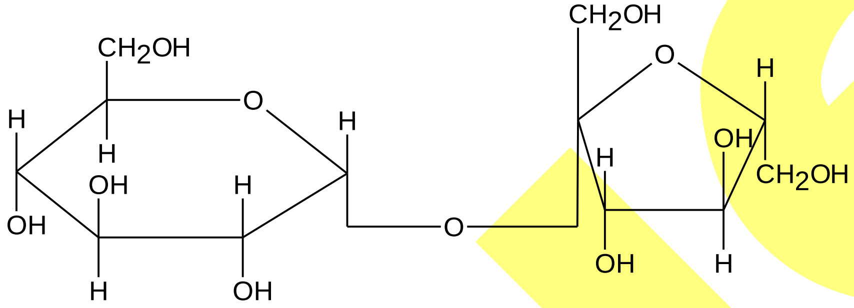
(A) $\left[Pt\left(NH_3\right)_4\right]\left[PtCl_4\right]$ (B) $\left[CO\left(en\right)_3\right]\left[Cr\left(CN\right)_6\right]$

(C) $\left[Ag\left(NH_3\right)_2\right]\left[AgCl_2\right]$ (D) $\left[CO\left(NH_3\right)_6\right]\left[CO\left(NO_2\right)_6\right]$

9. Choose homopolymer:

(A) Butadiene-styrene (Buna-S)
(B) Polythene
(C) Polypropene
(D) Nylon 6, 6

10. Which statement are correct about following

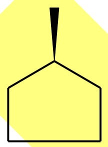


(A) Due to presence of Hemiacetal linkage it is non reducing sugar
(B) It is sucrose
(C) Due to absence of Hemiacetal group it can't give Tollen's test
(D) Left side is a $\alpha-D-$Glucose and right side unit is $\beta-D-$fructose

PART – B (Maximum Marks: 12)

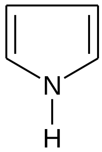
*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

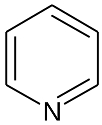
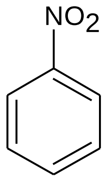
1. Number of monohalogen derivative (excluding stereo isomers) for the following is



Space For Rough Work

2. Total number of correct statement among the following is

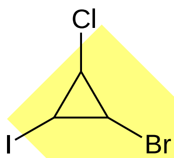
(A)  is aromatic

(B)  is more reactive than  for EAS reaction

(C) In $H_3C-\left(\overset{O}{\overset{\|}{C}}\right)-\dot{C}H_2$ the presence of $-\overset{O}{\overset{\|}{C}}-$ group stabilise the radical

(D) Adiabatic irreversible expansion for ideal gas, $[\Delta S]_{system} = 0$

(E) In $F-\overset{\bullet}{\underset{F}{\underset{|}{C}}}-F$ the state of hybridization of radical is sp^3

3. Total number of diastereomeric pair for  is x then value of $\frac{x}{6}$ is

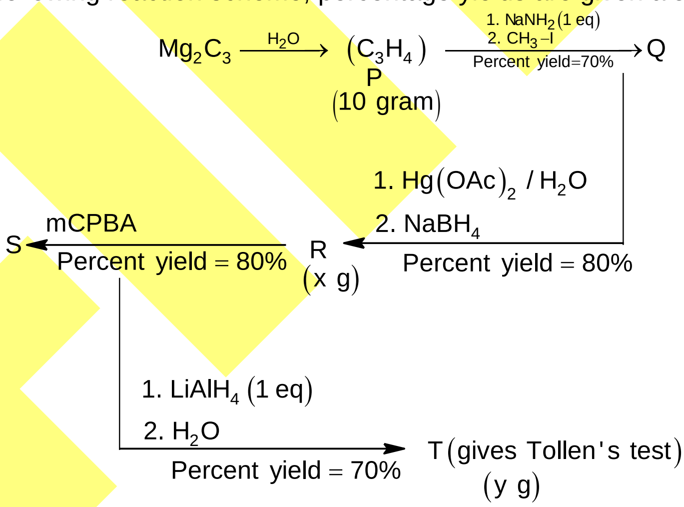
PART – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 4 and 5

Question Stem

For the following reaction scheme, percentage yields are given along the arrow



If mass of R and T are x and y g respectively then

4. The value of x is_______

5. The value of y is_______

Question Stem for Question Nos. 6 and 7

Question Stem

When 2 mole of P(g) is introduced in a closed rigid 1 litre vessel maintained at constant temperature. Following equilibrium are established

$P(g) \rightleftharpoons Q(g) + R(g), \quad K_{C_1} = W \text{ mole / litre}$

$R(g) \rightleftharpoons S(g) + Q(g), \quad K_{C_2} = Z \text{ mole / litre}$

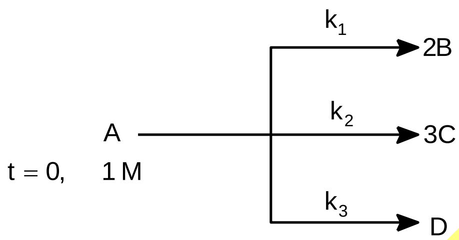
If pressure at equilibrium is twice the initial pressure and $\frac{[R]_{eq}}{[Q]_{eq}} = \frac{1}{5}$

6. The value of W is_________mol/L

7. The value of Z is_________mol/L

Question Stem for Question Nos. 8 and 9

Question Stem



$k_1 = 0.1 \text{ min}^{-1}$

$k_2 = k_3 = 0.2 \text{ min}^{-1}$

If C_t, D_t at $t = 4.606 \text{ min}^{-1}$ are W and Z mol/L

8. The value of W is __________

9. The value of Z is __________

Space For Rough Work

SECTION – III : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. $\int_0^2 \left(\sqrt{1+x^3} + \sqrt[3]{x^2+2x}\right)$ is equal to
   (A) 4
   (B) 5
   (C) 6
   (D) 7

2. Assume that P = {(x, y) : $x^2 + 2y^2 = 10$} and Q = {(x, y) : y = mx + c}. If $P \cap Q = \phi$ for all $m \in [-1, 1]$, then minimum positive integral value of c is
   (A) 3
   (B) 4
   (C) 5
   (D) 10

3. The locus of the centre of the circle which touches the circle $(x - 1)^2 + y^2 = 9$ and the line x = 6 is a curve whose directrix is
   (A) x = 1
   (B) y = 9
   (C) y = –4
   (D) x = 9

4. If f(x) is a 4 degree polynomial satisfying $f(x) = \frac{x}{x+1}$ for x = 0, 1, 2, 3, 4, then f(6) is
   (A) $\frac{12}{7}$
   (B) 0
   (C) 1
   (D) –6

Space For Rough Work

PART – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. The value of definite integral $\int_{-\infty}^{k}\frac{\left(\sin^{-1}e^{x}+\sec^{-1}e^{-x}\right)}{\left(\tan^{-1}e^{k}+\tan^{-1}e^{x}\right)\left(e^{x}+e^{-x}\right)}dx$ (where $k \in R$) is

(A) $\frac{\pi}{2}\ln 2\left(2\tan^{-1}e^{k}\right)$

(B) independent of k

(C) dependent on k

(D) $\frac{\pi}{2}\ln 2$

6. Let g(x) be a cubic polynomial with leading coefficient unity such that the roots of g(x) = 0 are the squares of the roots of $x^3 + x + 1 = 0$, then

(A) g(3) is a prime number

(B) last two digits of $(g(1))^{50}$ are 49

(C) when $(g(2))^{2022}$ is divided by 5, remainder is 2

(D) g(x) = 0 has all 3 roots real

7. Let P and Q be two square matrix of order 3, then which of the following statement is/are always CORRECT?

(A) PQP^T is symmetric matrix

(B) PQ – QP is skew symmetric matrix

(C) If $Q = |P|P^{-1}$, $|P| \neq 0$, then $adj(P^T) - Q$ is skew symmetric matrix

(D) If $Q + P^T = 0$ and P is skew symmetric matrix then Q^{15} is also skew symmetric matrix

Space For Rough Work

8. A tangent L_1 is drawn to the curve $x^2 - 4y^2 = 16$ at point A in first quadrant whose abscissa is 5. Another tangent L_2 parallel to L_1 meets the curve at B. L_3 and L_4 are normal to the curve at A and B lines L_1, L_2, L_3, L_4 forms a rectangle, then

(A) equation of normal at B is $12x + 10y + 75 = 0$

(B) area of rectangle L_1 L_2 L_3 L_4 is $\frac{2400}{61}$

(C) radius of largest circle inscribed in the rectangle is $\frac{32}{\sqrt{61}}$

(D) radius of the circle circumscribing the rectangle is $\frac{\sqrt{109}}{2}$

9. The integral point satisfying $|z - 1| + |z + 1| \le 6$ in the argand plane for which $||z + \omega^2| - |z + \omega||$ is minimum (where ω is complex cube root of unity) is/are

(A) (2, 0)

(B) (1, 2)

(C) (–3, 0)

(D) (0, 2)

10. Let for $x \in [1, \infty)$, $\int_1^x f(x)dx = \left(\frac{x^2+1}{2x}\right)\left(f(x) - x + \frac{2x}{x^2+1}\right) = \ln 2$, then for $x \in [1, \infty)$ which of the following is/are TRUE?

(A) $f(1) = \ln 2$

(B) Range of $g(x) = \frac{xf'(x) - f(x)}{x}$ is [0, 2]

(C) $\lim_{x\to 3}\left[\frac{f(x)}{x}\right] = 2$

(D) $h(x) = \frac{f(x)}{x}$ is an odd function

(where [.] denotes greatest integer function)

Space For Rough Work

PART – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. If P is the number of ways in which a person can walk up a stairway which has 11 steps if he can take 1 or 2 steps up the stairs at a time, then $\frac{P}{16}$ is equal to

2. In a triangle ABC, side AC = 4 units and sin A sin B + sin B sin C + sin C sin A = $\frac{9}{4}$. If A is the area of the triangle ABC, then $\frac{A}{\sqrt{3}}$ is equal to

3. If p is the least positive integer which satisfies the equation $|x-2| + |x^2 - 9x + 20| = |x^2 - 8x + 18|$ and q is the minimum value of $\sin^2 x + \sin x + 1$ and α is the only positive root of $\left((2021)^{2022} - 1\right)x^2 + \left(p - (2021)^{2022}\right)x = 1$, then the value of $\left(\alpha^{2022} - 1\right)p^2 + \left(p - 1 - \alpha^{2017}\right)pq + 8q$ is

PART – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 4 and 5

Question Stem

Let $A_n = [A_{ij}]$ be a $n \times n$ matrix such that $A_{ij} = \begin{cases} k & ; \quad i = j \\ 1 & ; \quad |i-j| = 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$. Let B_n denote the determinant of matrix A_n

4. If k = 2, then B_{2022} is equal to

5. If k = 1, then $\sum_{p=1}^{2022} |B_p|$ is equal to (where |.| denotes absolute value function)

Space For Rough Work

Question Stem for Question Nos. 6 and 7

Question Stem

Let $A_1, A_2, A_3, \ldots\ldots A_{12}$ be a regular polygon of 12 sides whose centre is origin. Let the complex numbers representing vertices $A_1, A_2, A_3, \ldots\ldots A_n$ be $z_1, z_2, z_3, \ldots\ldots z_{12}$ respectively. Let $OA_1 = OA_2 = OA_3 = \ldots\ldots OA_{12} = 1$ (where O is origin)

6. The value of $|A_1 A_2| \times |A_1 A_3| \times |A_1 A_4| \times \ldots\ldots |A_1 A_{12}|$ is equal to

7. The value of $|A_1 A_2|^2 + |A_1 A_3|^2 + \ldots\ldots |A_1 A_n|^2$ is

Question Stem for Question Nos. 8 and 9

Question Stem

Suppose 12 students of a class are asked a question successively in a random order and exactly 4 of them know the answer. Assume that none of them are aware of the answers given by other students, answer the following

8. If the probability that 6^{th} student to be asked the question is the first one to know the answer is k, then 99k is equal to

9. If the probability that 1^{st} and 12^{th} student knows the answer is k, then 99k is equal to

Space For Rough Work

Q. P. Code: 100967

ANSWERS

SECTION-1 : PHYSICS

PART – A

1.	B	2.	C	3.	A	4.	A
5.	A, B, D	6.	A, D	7.	A, B, C	8.	A, C
9.	A, C, D	10.	B, C, D				

PART – B

1.	4	2.	4	3.	2	4.	288.00
5.	32.00	6.	25.00	7.	25.00	8.	02.00
9.	03.00						

SECTION – 2 : CHEMISTRY

PART – A

1.	D	2.	A	3.	A	4.	C
5.	A, B	6.	A, B	7.	A, C, D	8.	A, B, D
9.	B, C	10.	B, C, D				

PART – B

1.	4	2.	3	3.	4	4.	10.08
5.	03.44	6.	01.00	7.	04.00	8.	01.08
9.	00.36						

SECTION – 3 : MATHEMATICS

PART – A

1.	C	2.	B	3.	D	4.	B
5.	B, D	6.	A, B	7.	C, D	8.	A, B, C, D
9.	A, C	10.	A, B, C				

PART – B

1.	9	2.	4	3.	6	4.	2023.00
5.	1348.00	6.	12.00	7.	24.00	8.	04.00
9.	09.00						

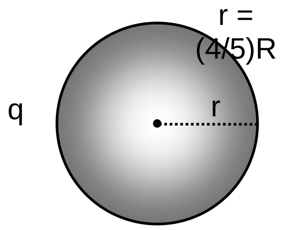
Answers & Solutions

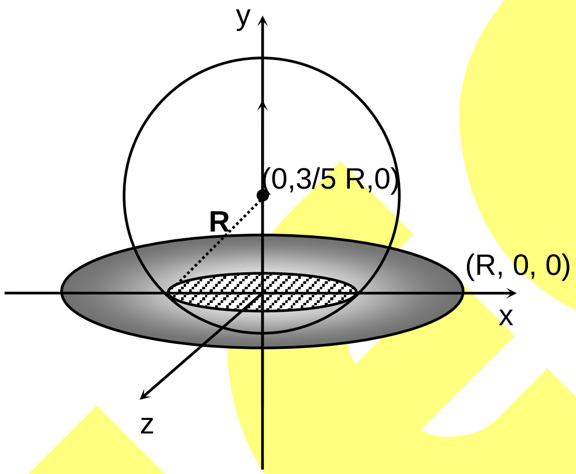
SECTION-1 : PHYSICS

PART – A

1. B

Sol.





Charge on circular part

$$q = \int \sigma(r) \cdot 2\pi r \cdot dr$$

$$= \int_0^{(4/5)R} 2\pi\sigma_0 \left(r - \frac{r^3}{R^2} \right) dr = \frac{272}{625}\left(\sigma_0 \pi R^2\right)$$

$$\phi = \frac{q}{\varepsilon_0} = = \frac{272}{625}\left(\frac{\sigma_0 \pi R^2}{\varepsilon_0}\right)$$

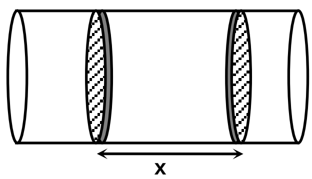
2. C

Sol. The path of projectile motion as observed by the other projectile motion is a straight line. The vertical component 30 m/s will get cancelled. Hence B will travel horizontal with respect to A. Hence minimum separation = 20 m.

3. A

Sol. $P_1V_1 = P_2V_2$

$$(10^5)(20{,}000 \times 10^{-6}) = \left\{10^5 + \frac{(2k\cos^2 37°)x}{100 \times 10^{-4}}\right\}(100 \times 10^{-4})x$$



$x^2 + x - 2 = 0$

$x = 1$ m = 100 cm

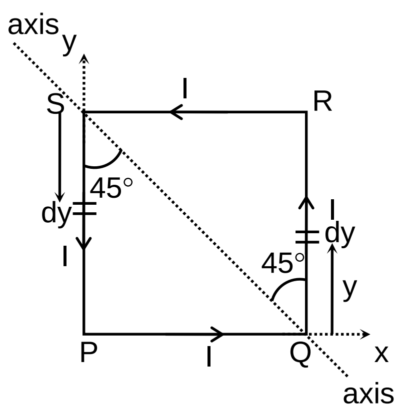
4. A

Sol. Torque on the loop due to magnetic field

$$\tau = \int_0^{\ell} (I2B_0 dy)\frac{y}{\sqrt{2}} + \int_0^{\ell} (IB_0 dy)\frac{y}{\sqrt{2}}$$

$$I\alpha = \frac{3IB_0\ell^2}{2\sqrt{2}}$$

$$\frac{1}{6}m\ell^2\alpha = \frac{3IB_0\ell^2}{2\sqrt{2}} \Rightarrow \alpha = \left(\frac{9IB_0}{\sqrt{2}m}\right)$$



5. A, B, D

Sol. $f = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{3\times5+2\times3}{3+2} = \frac{21}{5}$

$\gamma = 1 + \frac{2}{f} = \frac{31}{21}$

Let initial volume and pressure are V_0 and P_0 respectively

Heat absorb in isobaric expansion

$Q_1 = \frac{\gamma}{\gamma-1}(3P_0V_0) = 9.3P_0V_0$

Heat rejected in isochoric cooling

$Q_2 = \frac{1}{\gamma-1}(3P_0V_0) = 6.3P_0V_0$

Heat rejected in isothermal compression $Q_3 = nRT_0\ell n4 = P_0V_0\ell n4$

Efficiency = $\frac{Q_1 - Q_2 - Q_3}{Q_1} = \left(\frac{161}{930}\right)$

6. A, D

Sol. $\frac{1}{2}mu^2 - \frac{ke^2}{r} = 0$

$\Rightarrow u = \sqrt{\frac{2k}{mr}}e$

$\lambda = \frac{h}{mu} \propto \frac{1}{u} \propto \sqrt{r}$

$KE_1 = \frac{1}{2}mu^2 = \frac{ke^2}{r}$

For Bohr model, $\frac{1}{2}mv^2 = \frac{ke^2}{2r} = KE_2$

$\Rightarrow \mu = \frac{KE_1}{KE_2} = 2$

7. A, B, C

Sol. The deviation produced by ABCD is zero. Hence the cavity will not have any effect on the deviation.

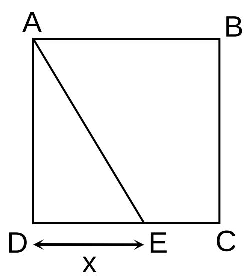
$\mu = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{8}{5}$

$\sin\left(\frac{\delta_{min} + A}{2}\right) = \left(\frac{8}{5}\right)\sin\left(\frac{60^\circ}{2}\right)$

$\delta_{min} = 46^\circ$

8. A, C

Sol. Equivalent resistance between A and E



$y = \frac{x+22}{x+22+32}$

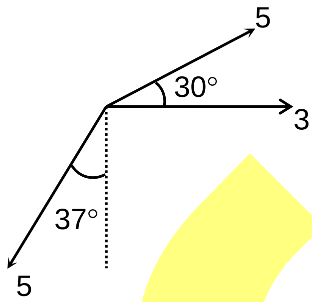
For B and E to be equipotential

$\frac{R_{AE}}{R_{AR}} = \frac{R_{EC}}{R_{RC}}$

$\Rightarrow \frac{(22+x)\times32}{(54+x)\times(14-x)} = \frac{8}{2} = 4$

Solve to get : $x = 10\ \Omega$

9. A, C, D



Sol. $z = \sqrt{(2-1)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} = \sqrt{\frac{7}{3}}\Omega$

$V_1 = 3\sin\omega t,\ V_2 = 5\sin(\omega t + 30°),\ V_3 = 5\sin(\omega t - 127°)$

$V_{max} = \sqrt{21}$ volts

$I_{max} = \frac{V_{max}}{z} = \frac{\sqrt{21}}{\sqrt{\frac{7}{3}}} = 3$ amp

10. B, C, D

Sol. For A, $1V = \frac{7S}{10} = 0.7$ mm

Least count of A = 5 – 0.7 × 7 = 0.1 mm

For B, 1V = $\frac{3S}{8}$ = 0.375 mm

Least count of B = 2 – 0.375 × 5 = 0.125

Difference = 0.125 – 0.1 = 0.025 mm

PART – B

1. 4

Sol. $R = \frac{28 \times 42}{70} = 16.8\Omega$

2. 4

Sol. v = 4t – $3t^2$

S = $2t^2 - t^3$ + C

When v = 0, ⇒ t = 0 and t = 4/3

S(t = 0) = C, $S\left(t = \frac{4}{3}\right) = \frac{32}{27} + C$, S(t = 2) = 0

Distance traveled = $\frac{32}{27} + \frac{32}{27} = \frac{64}{27}$

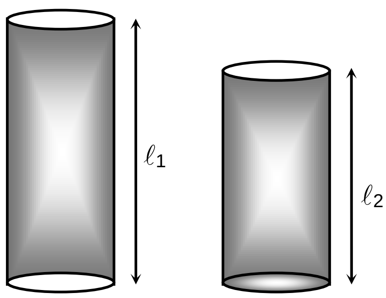
Average speed = $\frac{64}{27 \times 2} = \frac{32}{27}$ m/s

3. 2

Sol. $\frac{3v}{2\ell_1} = \frac{5v}{4\ell_2}$

$\frac{3v}{2(\ell_1 + 2e)} = \frac{5v}{4(\ell_2 + e)}$

$e = \frac{12\ell_2 - 10\ell_1}{8} = 2\,\text{cm}$



4. 288.00

5. 32.00

Sol. **(for Q.4-5):**

When S_1, S_2 both are open capacity of the system is C_1. Then C_1.

$$C_1 = \frac{\frac{4\pi\varepsilon_0(3R)(R)}{(3R-R)}(4\pi\varepsilon_0 4R)}{\frac{4\pi\varepsilon_0(3R)(R)}{(3R-R)} + 4\pi\varepsilon_0 4R}$$

$$C_1 = \frac{(4\pi\varepsilon_0 R)\left(\frac{3}{2}\right)(4)}{\left(\frac{3}{2}+4\right)} = \frac{(4\pi\varepsilon_0 R)(3.4)}{11}$$

$$C_1 = \frac{48\pi\varepsilon_0 R}{11}$$

When S_1 is closed but S_2 is open capacity is C_2. Find C_2.

$$C_2 = \frac{\frac{4\pi\varepsilon_0(3R)(2R)}{(3R-2R)}(4\pi\varepsilon_0 .4R)}{\frac{4\pi\varepsilon_0(3R)(2R)}{(3R-2R)} + 4\pi\varepsilon_0 4R} = \frac{(4\pi\varepsilon_0 R)(6)(4)}{(6+4)} = \frac{48\pi\varepsilon_0 R}{5}$$

When S_2 is closed but S_1 is opened capacity is C_3. Find C_3

$$C_3 = \frac{\frac{4\pi\varepsilon_0(2R)(R)}{(2R-R)}(4\pi\varepsilon_0 4R)}{\frac{4\pi\varepsilon_0(2R)(R)}{(2R-R)} + 4\pi\varepsilon_0 4R} = \frac{4\pi\varepsilon_0 R\cdot 2\cdot 4}{6} = \frac{16\pi\varepsilon_0 R}{3}$$

When both S_1 and S_2 are closed. Capacity is C_4 is . Then C_4 is equal to

$C_4 = 4\ \pi\varepsilon_0 4R$

$C_4 = 16\ \pi\varepsilon_0 R$

6. 25.00

7. 25.00

Sol. **(for Q.6-7):**

When B and D is cut

mg – T = ma

taking torque about mid point of rod

$$T\cdot\frac{\ell}{2} = \frac{m\ell^2}{12}\times\frac{a}{\ell/2}$$

T = mg/4

8. 02.00

9. 03.00

Sol. **(for Q.8-9):**

There is a dark fringe at O if path difference = $\lambda/2$

$$\Delta x = SO_2O - SO_1O = \frac{\lambda}{2}$$

$$2\sqrt{D^2+d^2} - 2D = \frac{\lambda}{2}$$

$$2D\left(1+\frac{d^2}{2D^2}\right) - 2D = \left(\frac{\lambda}{2}\right)$$

$$d_{min} = \sqrt{\frac{D\lambda}{2}}$$

The bright fringe is formed at P if the path differnece = λ

$\Delta x = SO_1P - SO_2P = \lambda$

$$\left\{\left(D+\sqrt{D^2+X^2}\right) - \left(\sqrt{D^2+d^2}+\sqrt{D^2+(x-d)^2}\right)\right\} = \lambda$$

$$D + D\left(1+\frac{x^2}{2D^2}\right) - D\left(1+\frac{d^2}{2D^2}\right) - D\left(1+\frac{(x-d)^2}{2D^2}\right) = \lambda$$

$$\frac{x^2}{2D} - \frac{d^2}{2D} - \frac{(x-d)^2}{2D} = \lambda$$

$$x = 2\sqrt{\frac{\lambda D}{2}} = 3\,d_{min}$$

SECTION – 2 : CHEMISTRY

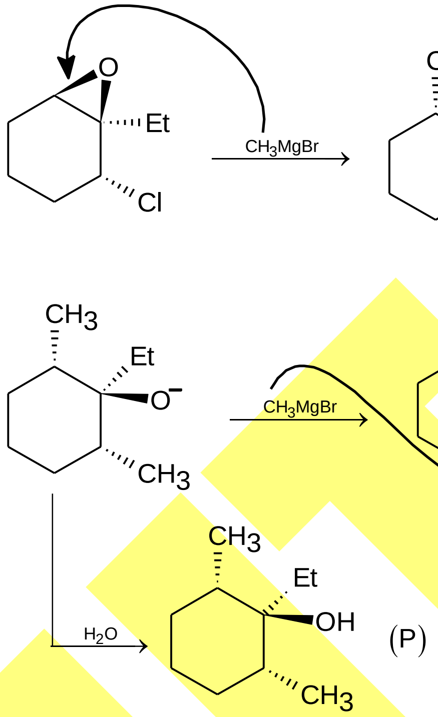
PART – A

1. D

Sol. Due to absence of plane of symmetry. Option (D) is optically active.

2. A

Sol.



3. A

Sol. $KI + O_3 \longrightarrow KOH + I_2 + O_2$

$I_2 + Na_2S_2O_3 \longrightarrow NaI + Na_2S_4O_6$

Law of chemical equivalence,

$(ne)_{I_2} = (ne)_{Na_2S_2O_3} = (ne)_{O_3}$

$\therefore (ne)_{Na_2S_2O_3} = (ne)_{O_3}$

$\Rightarrow 2\times1 = 2\times[\text{mole}]_{O_3}$

$[\text{mole}]O_3 = 1$

Mole of % $O_3 = \frac{1}{4}\times100 = 25\%$

4. C

Sol. $XeF_2 \longrightarrow$ linear

$ICl_4^- \longrightarrow$ square planar

$XeF_5^+ \longrightarrow$ square pyramidal

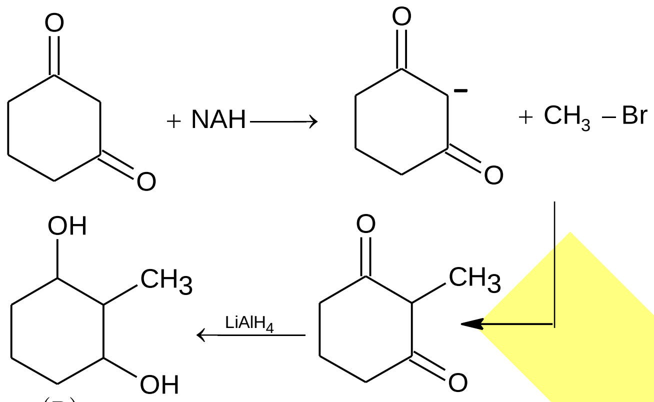
$XeF_5^- \longrightarrow$ pentagonal planar

5. A, B

Sol. A and B are high spin complex [NO pairing] C and D are low spin complex.

6. A, B

Sol.



(P)

Number of stereo isomers in P is 4.

7. A, C, D

Sol. $\psi_{n,\ell,m} \propto \left(\frac{Z}{a}\right)^{3/2}\left(\frac{Zr}{a}\right)e^{-2r/2a}\cos(\theta)$ and xy plane is nodal plane

Number of nodal plane = 1

$2p_z$ is ungerade orbital.

8. A, B, D

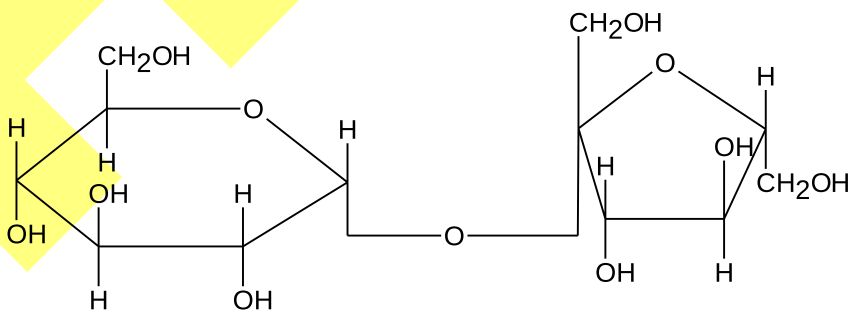
Sol. $\left[Ag(NH_3)_2\right]\left[AgCl_2\right]$ will not show the co-ordination isomerism.

9. B, C

Sol. Polythene and polypropene are homopolymer.

10. B, C, D

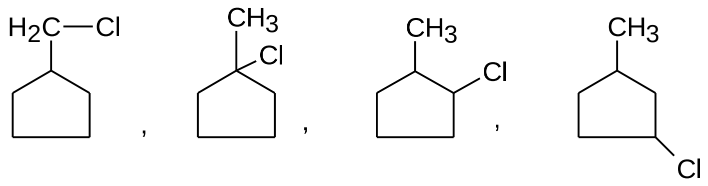
Sol.



Structure is of sucrose which is non reducing sugar due to absence of Hemiacetal group.

PART – B

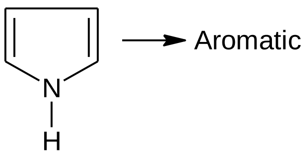
1. 4

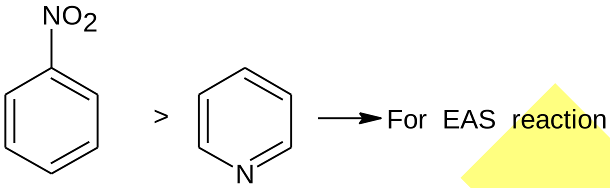
Sol. 

2. 3

Sol. $F-\dot{C}(F)-F \rightarrow$ hybridization sp^3.

$-\overset{O}{\overset{||}{C}}-$ group stabilize the radical.





$[\Delta S]_{system} > 0$ for adiabatic irreversible process.

3. 4

Sol. Number of diastereomeric pair 8_{C_2-4}

$= \dfrac{\lfloor 8}{\lfloor 2 \lfloor 6} - 4$

$= \dfrac{7 \times 8}{2} - 4$

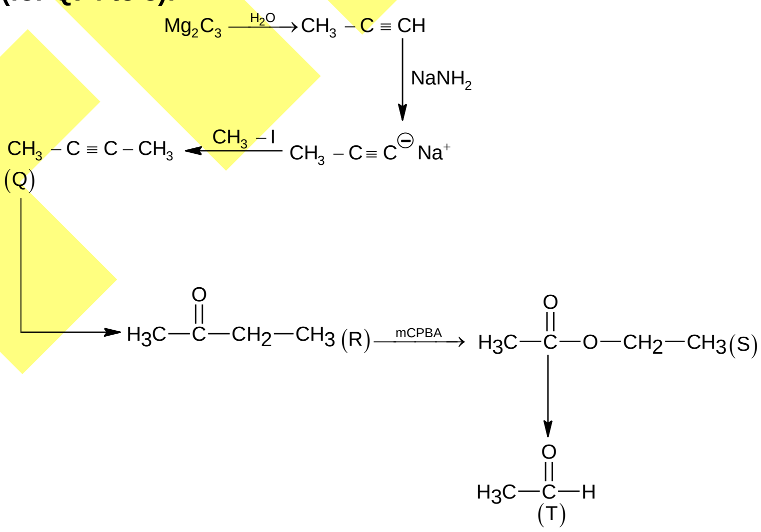
$x = 24$

$\therefore \dfrac{x}{6} = 4$

4. 10.08

5. 03.44

Sol. **(for Q. 4 to 5):**



$\because [\text{Mass}]_{CH_3-C\equiv C-H} = 10,\ [M]_{CH_3-C\equiv C-H} = 40$ gram/mole

$\therefore [\text{Mole}]_{CH_3-C\equiv CH} = \frac{10}{40} = \frac{1}{4}$

$\therefore \text{Mole of Q} = \frac{1}{4} \times \frac{70}{100} = \frac{7}{40}$

$\therefore \text{Mole of R} = \frac{7}{40} \times \frac{80}{100}$

$\therefore \text{Mass of R} = \frac{7}{40} \times \frac{80}{100} \times [72]$

$x = 10.08\,g$

$\text{Mass of T} = \frac{7}{40} \times \frac{80}{100} \times \frac{80}{100} \times \frac{70}{100} \times [44]$

6. 01.00

7. 04.00

Sol. **(for Q. 6 to 7):**

$P \rightleftharpoons$	$Q\ +$	R
2	0	0
$2-x$	$x+y$	$x-y$
$R \rightleftharpoons$	$S\ +$	Q
x	0	0
$x-y$	y	$y+x$

$2 - x + x + y + x - y + y = 4$

$x + y = 2$

$\frac{x-y}{x+y} = \frac{1}{5}; y = \frac{2}{3}x$

Hence, $x = 1.2$

$y = 0.8$

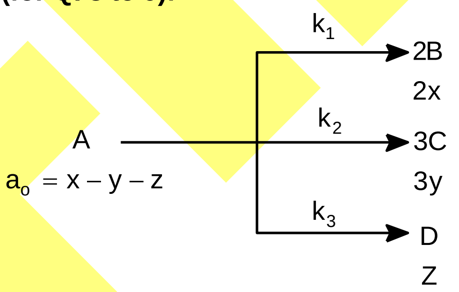
$K_{C_1} = W = \frac{(x-y)(x+y)}{2-x} = \frac{2 \times 0.4}{0.8} = 1$

$K_{C_2} = Z = \frac{(x+y)(y)}{x-y} = \frac{2 \times 0.8}{0.4} = 4$

8. 01.08

9. 00.36

Sol. **(for Q. 8 to 9):**



$A_t = a_o \times e^{-[k_1+k_2+k_3]t}$

$= 1 \times e^{-[0.5]4.606}$

$\therefore A_t = [10]^{-1} = \frac{1}{10} = 0.1\,M$

$\frac{k_1}{k_2} = \frac{x}{y} \Rightarrow \frac{1}{2} = \frac{x}{y}$ … (1)

$\frac{k_2}{k_3} = \frac{y}{z} \Rightarrow y = z$ … (2)

$A_t = a_o - [x + y + z]$

$0.1 = 1 - [x + y + z]$

$x + y + z = 1 - 0.1 = 0.9$ … (3)

$A_t = 0.1\,M,\ B_t = 2x = 0.36\,M$

$C_t = 3y = 1.08\ M,\ D_t = 0.36\ M$

SECTION – 3 : MATHEMATICS

PART – A

1. C

Sol. Let $(x^2 + 2x)^{\frac{1}{3}} = y \Rightarrow x^2 + 2x + 1 = y^3 + 1 \Rightarrow x = -1 + (y^3 + 1)^{\frac{1}{2}} \Rightarrow f^{-1}(x) = -1 + (x^3 + 1)^{\frac{1}{2}}$

$\int_0^2 (f^{-1}(x) + 1 + f(x))dx \Rightarrow \int_0^2 f^{-1}(x)dx$

Let $f^{-1}(x) = t \Rightarrow x = f(t) \Rightarrow \frac{dx}{dt} = f'(t)$; $\int tf'(t)dt = tf(t) - \int_0^2 f(t) = 2f(2) - \int_0^2 f(x)dx$

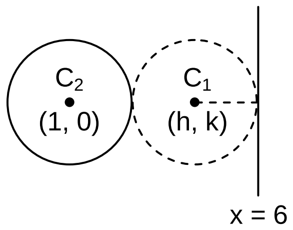
$\Rightarrow \int_0^2 (f^{-1}(x) + 1 + f(x))dx = 2f(2) - \int_0^2 f(x)dx + \int_0^2 1dx + \int_0^2 f(x)dx = 2f(2) + 2 = 2 \times 2 + 2 = 6$

2. B

Sol. $10\,m^2 + 5 - c^2 < 0$
Thus $c^2 > 15$

3. D

Sol. $C_1C_2 = r_1 + r_2 \Rightarrow \sqrt{(h-1)^2 + k^2} = r + 3$ and $h = 6 - r$
$\Rightarrow r = 6 - h$; $(h - 1)^2 + k^2 + r^2 + 9 + 6r$
$\Rightarrow h^2 + 1 - 2h + k^2 = (6 - h)^2 + 9 + 6(6 - h)$
$\Rightarrow h^2 + 1 - 2h + k^2 = 36 + h^2 - 12h + 9 + 36 - 6h$
$\Rightarrow k^2 = 16h + 80 \Rightarrow y^2 = -16(x - 5)$
Equation of directrix is $x - 5 = 4 \Rightarrow x = 9$



4. B

Sol. Let $g(x) = f(x)(x + 1) - x \Rightarrow g(0) = g(1) = g(2) = g(3) = g(4) = 0$
$\therefore g(x) = f(x)(x + 1) - x = kx(x - 1)(x - 2)(x - 3)(x - 4)$

Put $x = -1 \Rightarrow 1 = kx - 1x - 2x - 3x - 4x - 5 \Rightarrow k = -\frac{1}{120}$

$\therefore f(x)(x+1) - x = -\frac{x(x-1)(x-2)(x-3)(x-4)}{120}$

Put $x = 6 \Rightarrow f(6) \times (7 - 6) = -\frac{6 \times 5 \times 4 \times 3 \times 2}{120} \Rightarrow f(6) = 0$

5. B, D

Sol. $\int_{-\infty}^{k} \frac{(\sin^{-1} e^x + \cos^{-1} e^x)}{(\tan^{-1} e^k + \tan^{-1} e^x)} \frac{e^x}{e^{2x} + 1}$. Let $\tan^{-1} e^x = t$

$\frac{\pi}{2} \int_0^{\tan^{-1}(e^k)} \frac{1}{(t + \tan^{-1} e^k)} dt$

6. A, B

Sol. $\alpha^2 = x \Rightarrow \alpha = \sqrt{x}$ replace x by $\sqrt{x}$ in $x^3 + x + 1 = 0$. We get $g(x) = x^3 + 2x^2 + x - 1$
$\Rightarrow g(3) = 47 \Rightarrow g(1) = 3 \Rightarrow 3^{50} = (10-1)^{25} = {}^{25}C_0 10^{25} + \ldots\ldots {}^{25}C_{23}10^2 + {}^{25}C_{24}10 - 1$
$\Rightarrow g'(x) > 0\ \forall x \in R\ \therefore$ g(x) has only 1 real root

7. C, D

Sol. $(PQP^T)^T = PQ^TP^T \Rightarrow PQP^T$ is not symmetric
$(PQ - QP)^T = Q^TP^T - P^TQ^T$
$Q = |P|\dfrac{\text{adj } P}{|P|} \Rightarrow Q = \text{adj } P$
$\Rightarrow (\text{adj }(P^T) - Q)^T = (\text{adj }(P^T) - \text{adj } P)^T = \text{adj } P - \text{adj } P^T$
and $Q = -P^T \Rightarrow Q = P \Rightarrow Q$ is also skew symmetric

8. A, B, C, D

Sol. Equation of L_1

$$5x - 4 \times y \times \frac{3}{2} = 16$$

$5x - 6y = 16$

$\therefore$ Equation of tangent of B

$5x - 6y = -16$

$\therefore$ Equation of normal at A

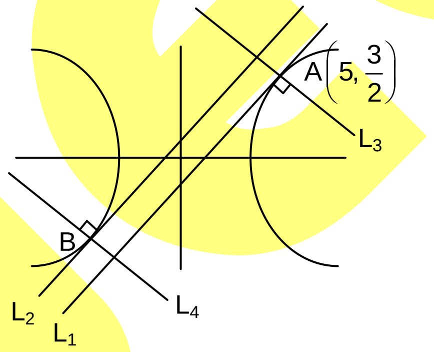
$6x + 5y = \dfrac{75}{2}$

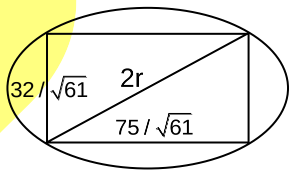
$\therefore$ Equation of normal at B is

$6x + 5y = -\dfrac{75}{2}$

Distance between tangents $\dfrac{32}{\sqrt{61}}$

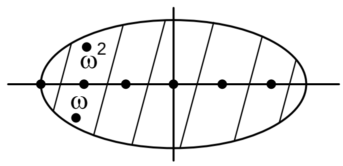
Distance between normals $\dfrac{75}{\sqrt{61}}$





9. A, C

Sol. $||z + \omega^2| - |z + \omega||$ is minimum when z lies on the major axis of the ellipse



$$\frac{x^2}{a} + \frac{y^2}{8} \le 1$$

10. A, B, C

Sol. $\displaystyle\int_1^x f(x)dx = \frac{f(x)}{2}\left(x + \frac{1}{x}\right) - \frac{x^2+1}{2} + 1$ using Leibnitz Rule

$$f(x) = \frac{f'(x)}{2}\left(x + \frac{1}{x}\right) + \frac{f(x)}{2}\left(1 - \frac{1}{x^2}\right) - x \Rightarrow \frac{xf'(x) - f(x)}{x^2} = \frac{2x}{x^2+1}$$

Integrating both sides, we get $\dfrac{f(x)}{x} = \ln(x^2+1)$

PART – B

1. 9

Sol. Let x_1 be the number of times he took 1 step up the staircase
Let x_2 be the number of times he took 2 step up the staircase
$x_1 + 2x_2 = 11$

			Number of ways
$x_1 = 1$	$x_2 = 6$	1, 2, 2, 2, 2, 2	$\frac{6!}{1!\,5!} = 6$
$x_1 = 3$	$x_2 = 4$	1, 1, 1, 2, 2, 2, 2	$\frac{7!}{3!\,4!} = 35$
$x_1 = 5$	$x_2 = 3$	1, 1, 1, 1, 1, 2, 2, 2	$\frac{8!}{3!\,4!} = 56$
$x_1 = 7$	$x_2 = 2$	1, 1, 1, 1, 1, 1, 1, 2, 2	$\frac{9!}{2!\,7!} = 36$
$x_1 = 9$	$x_2 = 1$		$\frac{10!}{9!\,1!} = 10$
$x_1 = 11$	$x_2 = 0$		1
			144

$\therefore$ P = 144

2. 4

Sol. 2 sin A sin B + 2 sin B sin C + 2 sin C sin A = $\frac{9}{2}$

cos(A – B) – cos(A + B) + cos(B – C) – cos(B + C) + cos(C – A) – cos(C + A) = $\frac{9}{2}$

$\Rightarrow$ cos(A – B) + cos(B – C) + cos(C – A) + cos A + cos B + cos C = $\frac{9}{2}$

$\Rightarrow$ cos(A + B) + cos(B – C) + cos(C – A) = $\frac{9}{2}$ – (cos A + cos B + cos C) $\ge \frac{9}{2} - \frac{3}{2} \ge 3$

$\Rightarrow$ cos(A – B) + cos(B – C) + cos(c – A) = 3

$\Rightarrow$ A = B = C $\therefore$ $A = \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3}$

3. 6

Sol. $(x - 2)(x^2 - 9x + 20) \ge 0 \Rightarrow x \in [2, 4] \cup [5, \infty)$

$\therefore$ p = 2

$\Rightarrow \left(\sin x + \frac{1}{2}\right)^2 + \frac{3}{4}$, least value is $\frac{3}{4}$ $\therefore$ $q = \frac{3}{4}$

$\Rightarrow$ x = 1 satisfies it $\therefore$ α = 1 and other root is $\frac{-1}{(2021)^{2022} - 1}$

$\therefore \alpha = 1,\ q = \frac{3}{4},\ p = 2$

4. 2023.00

5. 1348.00

Sol. **(for Q.4.-5):**

$A_1 = [k] \Rightarrow B_1 = k$; $A_2 = \begin{bmatrix} k & 1 \\ 1 & k \end{bmatrix} \Rightarrow B_2 = k^2 - 1$; $A_3 = \begin{bmatrix} k & 1 & 0 \\ 1 & k & 1 \\ 0 & 1 & k \end{bmatrix} \Rightarrow B_3 = KB_2 - 1B_1$

Similarly, $A_4 = \begin{bmatrix} k & 1 & 0 & 0 \\ 1 & k & 1 & 0 \\ 0 & 1 & k & 1 \\ 0 & 0 & 1 & k \end{bmatrix} \Rightarrow B_4 = kB_3 - B_2$

Thus, $B_n = KB_{n-1} - B_{n-2}$. If k = 2, $B_1 = 2$, $B_2 = 3$, $B_3 = 4$, $B_4 = 5$

Thus, $B_{2022} = 2023$
If k = 1, $B_n = B_{n-1} - B_{n-2}$; $B_1 = 1$, $B_2 = 0$, $B_3 = B_2 - B_1 = -1$, $B_4 = B_3 - B_2 = -1$
$B_5 = B_4 - B_3 = 0$; $B_6 = 1$ and it goes on
$B_{6n+1} = 1$; $B_{6n+2} = 0$; $B_{6n+3} = -1 \Rightarrow B_{6n-4} = -1$; $B_{6n+5} = 0$; $B_{6n} = 1$
Thus, $\sum_{k=1}^{2022} |B_p| = \frac{4}{6} \times 2022 = 1348$

6. 12.00

7. 24.00

Sol. **(for Q. 6.-7):**
$z_1, z_2, z_3, \ldots.. z$ are also the roots of $z^{12} = 1$ i.e. 12^{th} roots of unity
$z^{12} - 1 = (z - z_1)(z - z_2)(z - z_3) \ldots.. (z - z_{12})$
$\frac{z^{12}-1}{z-1} = (z - z_2)(z - z_3)(z - z_4).....(z - z + 2)$
$1 + z + z^2 + \ldots..z^{11} = (1 - z_1)(1 - z_3)(1 - z_4) \ldots.. (1 - z_{12})$
Put z = 1, $12 = |1 - z_2|\,|1 - z_3|\,|1 - z_4| \ldots.. |1 - z_{12}|$
$\Rightarrow |z_2 - z_1|^2 = |z_1|^2\,|(e^{i2\pi/12} - 1)|^2 = 2^2 \sin^2 \frac{\pi}{12} \Rightarrow |A_1 A_2|^2 + |A_1 A_3|^2 + \ldots.. |A_1 A_{12}|^2$
$= 4\left[\sin^2 \frac{\pi}{12} + \sin^2 \frac{2\pi}{12} + \sin \frac{3\pi}{12} + \ldots.. \sin^2 \frac{11\pi}{12}\right] = 2\left[11 - \left(\cos \frac{2\pi}{12} + \cos \frac{4\pi}{12} + \ldots.. \cos^2 \frac{2\pi}{12}\right)\right]$
$= 2(11 - (-1)) = 24$

8. 04.00

Sol. $\frac{4}{7} \times \frac{^8C_5}{^{12}C_5}$

9. 09.00

Sol. $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$