TEST CODE : 112004

IIT-JEE

Batch – Growth (May) | Minor Test-04

Time: 3 Hours | **Test Date: 04^{th} August 2024** | **Maximum Marks: 300**

Name of the Candidate: ____________________ Roll No.__________

Centre of Examination (in Capitals):____________________

Candidate's Signature:__________________ Invigilator's Signature:__________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

2. This Test Booklet consists of 90 questions.

3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 30 questions each and every **PART** has two sections.

(i) **Section-I** contains 20 multiple choice questions with only one correct option. Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

(ii) **Section-II** contains 10 questions the answer to only 5 questions, is an INTEGERAL VALUE.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.

5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.

7. For integer-based questions, the answer should be in decimals only not in fraction.

8. If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.

TEST CODE : 112004

TEST SYLLABUS

Batch – Growth (May) | Minor Test-04

04h August 2024

Mathematics: Sequence & Series

Physics: Kinematics-2D

Chemistry: Periodic Table & Periodic Properties

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821$ Lit atm $K^{-1}mol^{-1}$
		$= 1.987 \approx 2$ Cal $K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}$ Js
		$= 6.25 \times 10^{-27}$ erg.s
1 Faraday		= 96500 Coulomb
1 calorie		= 4.2 Joule
1 amu		$= 1.66 \times 10^{-27}$ kg
1 eV		$= 1.6 \times 10^{-19}$ J

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m / s^2$

PART-A: MATHEMATICS

SECTION-I

1. What is the n^{th} term of the sequence 25, –125, 625, –3125, ...?

(A) $(-5)^{2n-1}$

(B) $(-1)^{2n}\, 5^{n+1}$

(C) $(-1)^{2n-1}\, 5^{n+1}$

(D) $(-1)^{n-1}\, 5^{n+1}$

Ans. (D)

Sol. Given series $25, -125, 625, -3125 \ldots \ldots$ is geometric progression.

$$a = t_1 = 25, t_2 = -125$$
$$r = \frac{t_2}{t_1} = \frac{-125}{25} = -5$$
$$n^{th} \text{ term } (t_n) = \mathrm{ar}^{\mathrm{n}-1} = (25)(-5)^{\mathrm{n}-1}$$
$$= 5^2(-1)^{\mathrm{n}-1}(5)^{\mathrm{n}-1} = (-1)^{\mathrm{n}-1}(5)^{2+n-1}$$
$$= (-1)^{\mathrm{n}-1}5^{\mathrm{n}+1}$$

2. The sum of the series $3-1+\frac{1}{3}-\frac{1}{9}+\ldots.$ is equal to

(A) $\frac{20}{9}$

(B) $\frac{9}{20}$

(C) $\frac{9}{4}$

(D) $\frac{4}{9}$

Ans. (C)

Sol. The series is $\frac{3}{3^6}-\frac{3}{3^1}+\frac{3}{3^2}-\frac{3}{3^3}+\cdots.\infty$

This is a G.P. with first term $a = 3$

and common ratio $r = -\frac{1}{3}$

$\therefore$ Sum $S = \frac{3}{1-\left(-\frac{1}{3}\right)} = \frac{3\times 3}{3+1} = \frac{9}{4}$

3. If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P, then x equals

(A) $\log_3 4$

(B) $1 - \log_3 4$

(C) $1 - \log_4 3$

(D) $\log_4 3$

Ans. (B)

Sol.

Hint: $2\log_9(3^{1-x}+2)=\log_3(4.3^x-1)+1$

$\Rightarrow \log_3(3^{1-x}+2)=\log_3 3(4.3^x-1)$

$\Rightarrow 3^{1-x}+2=3(4.3^x-1)$

$\Rightarrow \frac{3}{3^x}+2=4.3^{x+1}-3$

$\Rightarrow \frac{3}{3^x}+2=12\cdot 3^x-3$

$\Rightarrow 12\cdot(3^x)^2-5(3^x)-3=0$

$\Rightarrow (4(3^x)-3)(3(3^x)+1)=0$

$\because 3^x>0 \ \therefore 4(3^x)-3=0$

$\therefore 3(3^x)+1\neq 0 \ \Rightarrow 3^x=\frac{3}{4}$

4. The sum of the infinite series

$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+.......$ is equal to:

(A) $\frac{9}{4}$

(B) $\frac{15}{4}$

(C) $\frac{11}{4}$

(D) $\frac{13}{4}$

Ans. (D)

Sol.

$$S=1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\cdots$$
$$\frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+\frac{12}{3^4}+\cdots$$
$$\frac{2S}{3}=1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+\frac{5}{3^4}+\cdots+ \text{ up to infinite terms}$$
$$\Rightarrow \frac{2S}{3}=\frac{4}{3}+\left[\frac{5}{9}\left(\frac{1}{1-1/3}\right)\right]$$
$$\Rightarrow \frac{2S}{3}=\frac{13}{6}$$
$$\Rightarrow S=\frac{13}{4}$$

5. If $x=\sum_{n=0}^{\infty}a^n,\ y=\sum_{n=0}^{\infty}b^n,\ z=\sum_{n=0}^{\infty}c^n$, where a, b, c are in A.P. and $|a|<1, |b|<1, |c|<1, abc\neq 0$, then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

(D) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-(a+b+c)$

Ans. (C)

Sol.

$$x = 1 + a + a^2 = \cdots$$
$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$
$$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$
$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

6. Let a, b, c be real numbers, each greater than 1, such that

$\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c = 3.$ If the value of b is 9, then the value of 'a' must be

(A) $\sqrt[3]{81}$

(B) $\frac{27}{2}$

(C) 18

(D) 27

Ans. (D)

Sol. $\frac{2\ln a}{3\ln b} + \frac{3\ln b}{5\ln c} + \frac{5\ln c}{2\ln a} = 3$

By A.M. = G.M.

$\frac{2\ln a}{3\ln b} = 1$

$\Rightarrow a^2 = b^3 \Rightarrow a = (3^6)^{1/2} = 3^3 \Rightarrow a = 27$

7. If m arithmetic (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to:

(A) 36

(B) 37

(C) 38

(D) 39

Ans. (D)

Sol. Let $3, A_1, A_2, A_3, \ldots\ldots A_m, 243$ are in arithmetic progression with m arithmetic means.

Common difference $\mathrm{d} = \frac{243-3}{m+1} = \frac{240}{m+1}$

Let $3, G_1, G_2, G_3, 243$ are in geometric progression with 3 geometric means.

Common ratio $r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{\frac{1}{4}} = 3$

Given $G_2 = A_4$

$$\Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$$
$$\Rightarrow 27 = 3 + \frac{960}{m+1}$$
$$\Rightarrow m + 1 = 40$$
$$\Rightarrow m = 39$$

8. If the arithmetic mean and the geometric mean of the p^{th} and q^{th} terms of the sequence −16, 8, −4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to ______.

(A) 4

(B) 5

(C) 6

(D) 7

Ans. (C)

Sol.

Let $t_p \& t_q$ is the p^{th} & q^{th} terms of the series $-16, 8, -4, 2, \ldots$

Here $a = -16, r = -\frac{1}{2}$

Then, $t_p = -16\left(-\frac{1}{2}\right)^{p-1}$

$$t_q = -16\left(-\frac{1}{2}\right)^{q-1}$$

The roots of the given quadratic equation $4x^2 - 9x + 5 = 0$ is $x = 1, \frac{5}{4}$.

$$AM = \frac{5}{4}, GM = 1(\because AM \geq GM)$$

Now, given $\frac{5}{4} = \frac{t_p + t_q}{2}, 1 = t_p t_q$

So, $1 = 256\left(-\frac{1}{2}\right)^{p+q-2} \Rightarrow 2^{p+q-2} = (-1)^{p+q-2} 2^8$

$$\Rightarrow 2^{p+q-10} = (-1)^{p+q-2}$$

It is equal when $2^{p+q-10} = 1 = (-1)^{p+q-2}$

Hence, $p + q - 10 = 0 \Rightarrow p + q = 10$

9. G.M. and H.M. of two numbers are 10 and 8 respectively. The numbers are :

(A) 5, 20

(B) 4, 25

(C) 2, 50

(D) 1, 100

Ans. (A)

Sol. Hints: $\sqrt{ab} = 10 \Rightarrow ab = 100$

$\frac{2ab}{a+b} = 8$

a + b = 25

So a = 5, b = 20

10. Consider two G.Ps 2, 2^2, 2^3,... and 4, 4^2, 4^3, ... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560
(B) 1540
(C) 1330
(D) 2600

Ans. (C)

Sol.

Given two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively, Also given the geometric mean of all the $60 \div \mathrm{n}$ terms is $(2)^{\frac{23}{8}}$,

So, $\left((2^1 2^2 \ldots .2^{60})(4^1 \cdot 4^2 \ldots \ldots .4^n)\right)^{\frac{1}{60+n}} = 2^{\frac{23s}{8}}$

$$\Rightarrow \left(2^{30\times 61} 4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}} = 2^{\frac{23s}{8}}$$

$$\Rightarrow 2^{1830+n^2+n} = 2^{\frac{(225)(60+n)}{8}}$$

On comparing both side we get,

$$\Rightarrow 1830 + \mathrm{n}^2 + \mathrm{n} = \frac{225(60+\mathrm{n})}{8}$$

$$\Rightarrow 8\mathrm{n}^2 - 217\mathrm{n} + 1140 = 0$$

$$\Rightarrow \mathrm{n} = 20, \frac{57}{8}$$

Now $\sum_{k=1}^{n} k(n-k) = \sum_{k=1}^{20} nk - k^2 = \frac{n^2(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$$= \frac{20^2 \times 21}{2} - \frac{20 \times 21 \times 41}{6} = 1330$$

11. Find the sum $\sum\sum_{0\le i<j\le 20} 1$.

(A) 180
(B) 190
(C) 200
(D) 210

Ans. (B)

Sol.

$$\sum_{0\le i<j\le n}\sum 1 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}1-\sum_{i=j}\sum 1}{2}$$

$$=\frac{\left(\sum_{j=1}^{n}1\right)\left(\sum_{j=1}^{n}1\right)-\sum_{j=1}^{n}1}{2}$$

$$=\frac{n^2-n}{2}$$

$$=\frac{n(n-1)}{2}$$

12. Let x and y be two positive real numbers such that x + y = 1. Then the minimum value of $\frac{1}{x}+\frac{1}{y}$ is

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(A) 2

(B) $\frac{5}{2}$

(C) 3

(D) 4

Ans. (D)

Sol. x + y = 1 and x, y > 0 $\frac{x+y}{2}\geqslant\frac{2}{\frac{1}{x}+\frac{1}{y}}$

Apply AM ≥ HM $\frac{1}{x}+\frac{1}{y}\geqslant 4$

13. The sum of the series

$\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+....$ up to 10 terms is

(A) $\frac{45}{109}$

(B) $-\frac{45}{109}$

(C) $\frac{55}{109}$

(D) $-\frac{55}{109}$

Ans. (D)

Sol. General term of the sequence,

$$T_r = \frac{r}{1 - 3r^2 + r^4}$$
$$T_r = \frac{r}{r^4 - 2r^2 + 1 - r^2}$$
$$T_r = \frac{r}{(r^2 - 1)^2 - r^2}$$
$$T_r = \frac{r}{(r^2 - r - 1)(r^2 + r - 1)}$$
$$T_r = \frac{\frac{1}{2}[(r^2 + r - 1) - (r^2 - r - 1)]}{(r^2 - r - 1)(r^2 + r - 1)}$$
$$= \frac{1}{2}\left[\frac{1}{r^2 - r - 1} - \frac{1}{r^2 + r - 1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2}\left[\frac{1}{-1} - \frac{1}{109}\right] = \frac{-55}{109}$$

14. The greatest value of x^2y^3 for positive x and y and 3x + 4y = 5 is $\frac{p}{q}$ (in reduced form). Find p + q.

(A) 19
(B) 20
(C) 21
(D) 22

Ans. (A)

Sol. Given that $3x + 4y = 5$

Since we have expression x^2y^3, we consider

$$2\left(\frac{3x}{2}\right) + 3\left(\frac{4y}{3}\right) = 3x + 4y = 5$$

Using A.M. ≥ G.M. for weighted means, we get

$$\frac{2\left(\frac{3x}{2}\right) + 3\left(\frac{4y}{3}\right)}{2 + 3} \geq \left[\left(\frac{3x}{2}\right)^2 \left(\frac{4y}{3}\right)^3\right]^{\frac{1}{5}}$$

$$\Rightarrow \quad \left(\frac{3x + 4y}{5}\right)^5 \geq \left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^3 x^2y^3$$

$$\Rightarrow \quad x^2y^3 \leq \left(\frac{2}{3}\right)^2 \left(\frac{3}{4}\right)^3$$

15. If an infinite GP has the first term x and the sum 5, then which of the following is correct?

(A) x < – 10
(B) –10 < x < 0
(C) 0 < x < 10
(D) x > 10

Ans. (C)

Sol. Sum of an infinite G.P. with first term x and common ratio r(< 1) is

$$s = \frac{x}{1 - r}$$

From question s = 5

then,

$$5 = \frac{x}{1-r}$$
$$1 - r = \frac{x}{5}$$
$$r = 1 - \frac{x}{5}$$
For, $-1 < 1 - \frac{x}{5} < 1$
$$-2 < -\frac{x}{5} < 0$$
$$0 < x < 10$$

16. The sum to infinite term of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.....$ is

(A) 3
(B) 4
(C) 6
(D) 2

Ans. (A)

Sol. We have

$$S = 1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......\infty \qquad ....(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+.....\infty \qquad ....(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\cdots\dots\dots\infty$$
$$\Rightarrow \frac{2}{3}S = \frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\cdots\dots\dots\infty$$
$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3}\times\frac{3}{2} \Rightarrow S = 3$$

17. If the $(2p)^{th}$ term of a H.P. is q and the $(2q)^{th}$ term is p, then the $2(p+q)^{th}$ term is –

(A) $\frac{pq}{2(p+q)}$

(B) $\frac{2pq}{p+q}$

(C) $\frac{pq}{p+q}$

(D) $\frac{p+q}{pq}$

Ans. (D)

Sol. If a is the first term and d is the common difference of the associated A.P.

$$\frac{1}{q}=\frac{1}{a}+(2p-1)d,\ \frac{1}{p}=\frac{1}{a}+(2q-1)d$$

$$\Rightarrow d=\frac{1}{2pq}$$

$$\frac{1}{p}-(2q-1)\frac{1}{2pq}=\frac{1}{a}$$

$$\frac{2q-2q+1}{2pq}=\frac{1}{a}$$

$$\left(\frac{1}{a}=\frac{1}{2pq}\right)$$

$$T_{p+q}=\frac{1}{a}+(p+q-1)d$$

$$=\frac{1}{2pq}+\frac{(p+q-1)}{2pq}$$

$$T_{p+q}=\frac{p+q}{2pq}$$

$$2T_{p+q}=\frac{p+q}{pq}$$

18. If sum of the first 21 terms of the series $\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+$ ..... where x > 0 is 504, then x is equal to

(A) 243

(B) 9

(C) 7

(D) 81

Ans. (D)

Sol. Let

$$S=\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+\cdots+\log_{9^{1/22}} x$$
$$\Rightarrow S=2\log_9 x+3\log_9 x+\cdots\ldots+22\log_9 x$$
$$\Rightarrow S=\log_9 x(2+3+\cdots\ldots+22)$$
$$\Rightarrow S=\log_9 x\left\{\frac{21}{2}(2+22)\right\}$$
$$\Rightarrow S=252\log_9 x$$

Given,

$$S=504$$
$$\Rightarrow 252\log_9 x=504$$

19. Let $a_1, a_2, \ldots \ldots a_{40}$ be in AP and $h_1, h_2, \ldots . h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then a_4h_7 is

(A) 2

(B) 3

(C) 5
(D) 6

Ans. (D)

Sol. Let d be the common difference of the AP. Then,

$$a_{10} = 3 \Rightarrow a_1 + 9d = 3$$
$$\Rightarrow 2 + 9d = 3 \Rightarrow d = \frac{1}{9}$$
$$\therefore a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$$

Let D be the common difference of $\frac{1}{h_1}, \frac{1}{h_2}, \ldots . \frac{1}{h_{10}}$.

Then, $\mathrm{h}_{10} = 3$

$$\Rightarrow \quad \frac{1}{h_{10}} = \frac{1}{3} \Rightarrow \frac{1}{2} + 9D = \frac{1}{3}$$
$$\Rightarrow \quad 9D = -\frac{1}{6} \Rightarrow D = -\frac{1}{54}$$
$$\therefore \quad \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}$$
$$\Rightarrow \quad h_7 = \frac{18}{7}$$
$$\therefore \quad a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

20. Let a_1, a_2, a_3, ... be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to
(A) 60
(B) 61
(C) 62
(D) 63

Ans. (A)

Sol. Given,
$a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers,
And the product of fourth and sixth terms is 9
So, $a_4 \cdot a_6 = 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3$
Also given the sum of fifth and seventh terms is 24 ,

So, $a_5 + a_7 = 24$
$\Rightarrow a_5 + a_5 r^2 = 24$
$\Rightarrow (1 + r^2) = 8 \Rightarrow r = \sqrt{7}$
Now using $a_5 = 3 \Rightarrow a_1 r^4 = 3 \Rightarrow a_1 = \frac{3}{r^4} = \frac{3}{49}$
Then, $a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7$

$$= \frac{3}{49} \times \frac{3}{49} \times (\sqrt{7})^8 + \frac{3}{49} \times \sqrt{7} \times \frac{3}{49} \times (\sqrt{7})^3 \times \frac{3}{49} \times (\sqrt{7})^8 + \frac{3}{49} \times (\sqrt{7})^4 + \frac{3}{49} \times (\sqrt{7})^6$$
$$= 9 + 27 + 3 + 21 = 60$$

SECTION-II

21. The arithmetic mean of numbers a, b, c, d, e is M. What is the value of (a – M) + (b – M) + (c – M) + (d – M) + (e – M) + 1 ?

Ans. 1

Sol. Given M = $\frac{a+b+c+d+e}{5}$

$\Rightarrow$ a + b + c + d + e = 5M

$\Rightarrow$ a + b + c + d + e – 5m = 0

22. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 =

Ans. 5

Sol.

Clearly, $1+2+3+\cdots\ldots+n-2 \leq 1224 \leq 3+4+\cdots\ldots..n$

$\Rightarrow \frac{(n-2)(n-1)}{2} \leq 1224 \leq \frac{(n-2)}{2}(3+n)$

$\Rightarrow n^2-3n-2446 \leq 0$ and $n^2+n-2454 \geq 0$

$\Rightarrow 49 < n < 51 \Rightarrow n = 50$

$\therefore \frac{n(n+1)}{2}-(2k+1) = 1224 \Rightarrow k = 25 \Rightarrow k-20 = 5$

23. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then find a + 4r.

Ans. 4

Sol. Since, sum = 4 and second term $= \frac{3}{4} \Rightarrow \frac{a}{1-r} = 4$, and ar $= \frac{3}{4} \Rightarrow \frac{a}{1-\frac{3}{43}} = 4$

$\Rightarrow (\mathrm{a}-1)(\mathrm{a}-3) = 0 \Rightarrow \mathrm{a} = 1$ or $\mathrm{a} = 3$

$\Rightarrow \frac{\mathrm{a}}{1-\frac{3}{4a}} = 4$

$\Rightarrow (\mathrm{a}-1)(\mathrm{a}-3) = 0$

$\Rightarrow \mathrm{a} = 1$ or $\mathrm{a} = 3$

24. The sum of the series 1 + 2.2 + 3.2^2 + 4.2^3 +....+ 100.2^{99} is a × 2^{100} + 1. Find a.

Ans. 99

Sol. Let $S = 1+2.2+3.2^2+4.2^3+\cdots.+100.2^{99}$

It is an arithmetico - geometric series. On multiplying Eq.

(i) by 2 and then subtracting it from Eq. (i), we get

$$S = 1 + 2.2 + 3.2^2 + 4.2^3 + \cdots + 100.2^{99}$$
$$2\ S = 1.2 + 2.2^2 + 3.2^3 + \cdots + 99.2^{99} + 100.2^{100}$$
$$-S = 1 + 2 + 2^2 + 2^3 \ldots + 2^{99} - 100.2^{100}$$
$$\Rightarrow -S = \frac{1(2^{100} - 1)}{2-1} - 100.2^{100}$$
$$\Rightarrow -S = 2^{100} - 1 - 100.2^{100}$$
$$\Rightarrow -S = -1 - 99.2^{100}$$
$$\Rightarrow S = 99.2^{100} + 1$$

25. Let m be the minimum possible value of $\log_3 3^{y_1} + 3^{y_2} + 3^{y_3}$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of ($\log_3 x_1 + \log_3 x_2 + \log_3 x_3$), where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2 (m^3) + \log_3(M^2)$ is

Ans. 8

Sol. Applying A.M-G.M inequality,

$$\left(\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3}\right) \geq (3^{y_1} \cdot 3^{y_2} \cdot 3^{y_3})^{\frac{1}{3}} = (3^{y_1+y_2+y_3})^{\frac{1}{3}}$$
$$\Rightarrow 3^{y_1} + 3^{y_2} + 3^{y_3} \geq 81$$

So, $m = \log_3 (81) = 4$

Now, $\log_3\ x_1 + \log_3\ x_2 + \log_3\ x_3 = \log_3 (x_1 \cdot x_2 \cdot x_3)$

$\because \frac{x_1+x_2+x_3}{3} \geq (x_1 \cdot x_2 \cdot x_3)^{\frac{1}{3}} \Rightarrow \mathbf{x_1 \cdot x_2 \cdot x_3 \leq 27}$ (applying A.M-G.M inequality)

So, $\mathrm{M} = \log_3\ 27 = 3$

Thus, $\log_2 (\mathrm{m}^3) + \log_3 (\mathrm{M}^2) = 8$

26. If S = $\frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then 160 S is equal to ______

Ans. 305

Sol. Given, if $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \cdots \ldots \infty \ldots$ (i)

Dividing by 5 on both sides of series

$$\frac{S}{5} = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \frac{19}{5^5} + \cdots \ldots \infty \ldots$$

Subtracting equations (i)&(ii)

$$S - \frac{S}{5} = \left[\frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \cdots \ldots \infty\right] - \left[\frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \frac{19}{5^5} + \cdots \ldots \infty\right]$$
$$\frac{4\ S}{5} = \left[\frac{7}{5} + \left(\frac{9}{5^2} - \frac{7}{5^2}\right) + \left(\frac{13}{5^3} - \frac{9}{5^3}\right) + \left(\frac{19}{5^4} - \frac{13}{5^4}\right) + \cdots \ldots \infty\right]$$
$$\frac{4\ S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \cdots \ldots \infty$$
$$\left(\frac{4\ S}{5} - \frac{7}{5}\right) = k = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \cdots \ldots \infty \ldots \text{ (iii)}$$

Let $\left(\frac{4\ S}{5} - \frac{7}{5}\right) = k$

$$\frac{k}{5} = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + \frac{8}{5^6} + \cdots \ldots \infty$$

Subtracting equation (iii) & (iv)

$$\frac{4k}{5} = \frac{2}{5^2} + \frac{2}{5^3} + \frac{2}{5^4} + \frac{2}{5^5} \ldots \ldots \infty$$

Common ratio

$$r=\frac{\frac{2}{3^3}}{\frac{2}{5^2}}=\frac{1}{5},\ r=\frac{\frac{2}{s^4}}{\frac{2}{5^3}}=\frac{1}{5}$$

$$\frac{4k}{5}=\frac{2}{25}\left\{\frac{1}{1-\frac{1}{5}}\right\}=\frac{1}{10}$$

$$k=\frac{1}{8}$$

Now $\frac{4\,S}{5}-\frac{7}{5}=\frac{1}{8}$

$$\frac{4\,S}{5}=\frac{7}{5}+\frac{1}{8}$$

$$S=\frac{61}{32}$$

$$160\ S=160\times\frac{61}{32}$$

$$=305$$

27. The sum of the series $\sum_{r=1}^{99}\left(\frac{1}{r\sqrt{r+1}+(r+1)\sqrt{r}}\right)$ is $\frac{p}{q}$ (in reduced form). Find p + q.

Ans. 19

Sol.

$$T_r=\frac{1}{\sqrt{r}\sqrt{r+1}[\sqrt{r}+\sqrt{r+1}]}=\frac{\sqrt{r+1}-\sqrt{r}}{\sqrt{r}\sqrt{r+1}}$$

$$=\frac{1}{\sqrt{r}}-\frac{1}{\sqrt{r+1}}$$

$=V(r)-V(r+1)$, where $V(r)=\frac{1}{\sqrt{r}}$

Required sum, $\sum_{r=1}^{99}(V(r)-V(r+1))=V(1)-V(100)$

$$=1-\frac{1}{\sqrt{100}}$$

$$=1-\frac{1}{10}=\frac{9}{10}$$

28. Let a_1, a_2, a_3, ... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. Find the least positive integer n for which $a_n < 0$.

Ans. 25

Sol.

$\because a_1, a_2, a_3, \ldots\ldots$ are in H.P.

$\therefore \frac{1}{a_1},\frac{1}{a_2},\frac{1}{a_3}\ldots\ldots$ are in A.P.

$\therefore\ \frac{1}{a_1}=\frac{1}{5}$ and $\frac{1}{a_{20}}=\frac{1}{25}$

$$\frac{1}{a_1}+19d=\frac{1}{a_{20}}\Rightarrow\frac{1}{5}+19\ d=\frac{1}{25}\Rightarrow d=\frac{-4}{475}$$

Now $\frac{1}{a_n}=\frac{1}{5}+(n-1)\left(\frac{-4}{475}\right)$

Clearly $a_n<0$, if $\frac{1}{a_n}<0\Rightarrow\frac{1}{5}-\frac{4n}{475}+\frac{4}{475}<0$

$\Rightarrow\ -4n<-99$ or $n>\frac{99}{4}=24\frac{3}{4}\ \therefore n\geq 25$

29. The sum to 10 terms of the series 3 + 15 + 35 + 63 + ... is S. Find $\frac{S}{10}$.

Ans. 153

Sol. $S_n = 3 + 15 + 35 + 63 + ... + T_{n-1} + T_n$

$$S_n = 3 + 15 + 35 + 63 + ... + T_{n-1} + T_n \quad (1)$$
$$S_n = \quad 3 + 15 + 35 + 63 + ... + T_{n-1} + T_n \quad (2)$$
$$0 = 3 + [12 + 20 + 28 + ... + (n-1) \text{ terms}] - T_n$$

[Subtracting (2) from (1)]

$$\Rightarrow T_n = 3 + \frac{(n-1)}{2}[2 \times 12 + (n-1-1) \times 8]$$

$$= 3 + (n-1)(12 + 4n - 8)$$

$$= 3 + (n-1)(4n+4)$$

$$= 4n^2 - 1$$

$$\Rightarrow S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}(4k^2 - 1) = 4\sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} 1$$

$$= 4\left\{\frac{n(n+1)(2n+1)}{6}\right\} - n$$

$$= \frac{n}{3}(4n^2 + 6n - 1)$$

Put (n = 10) for S and then $\frac{S}{10}$

30. The minimum value of $f(x) = x + \frac{1}{x} + \frac{1}{x + \frac{1}{x}}$ for x > 0 is $\frac{p}{q}$ (in reduced form). Find p + q.

Ans. 7

Sol. First, consider the function

$$g(x) = x + \frac{1}{x}.$$

If $1 \le x < y$, then

$$\begin{aligned} g(y) - g(x) &= y + \frac{1}{y} - x - \frac{1}{x} \\ &= y - x + \frac{1}{y} - \frac{1}{x} \\ &= y - x + \frac{x - y}{xy} \\ &= (y - x)\left(1 - \frac{1}{xy}\right) \\ &= \frac{(y - x)(xy - 1)}{xy} \\ &> 0. \end{aligned}$$

Thus, $g(x)$ is increasing on the interval $[1, \infty)$.

By AM-GM (and what we just proved above),

$$x+\frac{1}{x} \geq 2$$

So,

$$g\left(x+\frac{1}{x}\right) \geq 2+\frac{1}{2}=\frac{5}{2}.$$

Equality occurs when $x=1$, to the minimum value of $f(x)$ for $x>0$ is $\frac{5}{2}$.

In particular, we cannot use the following argument: By AM-GM,

$$x+\frac{1}{x}+\frac{1}{x+\frac{1}{x}} \geq 2 \sqrt{\left(x+\frac{1}{x}\right) \cdot \frac{1}{x+\frac{1}{x}}}=2.$$

However, we cannot conclude that the minimum is 2 , because equality can occur only when $x+\frac{1}{x}=1$, and this is not possible.

PART-B: PHYSICS

SECTION-I

31. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is (g = 10 m/s^2)

(A) $\sqrt{3} : 2$

(B) $2 : \sqrt{3}$

(C) 1 : 1

(D) 4 : 9

Ans. (D)

Sol. The formula of range of projectile,

$$\mathrm{R}=\frac{\mathrm{u}^2 \sin 2 \theta}{\mathrm{g}}$$

$$\mathrm{R}_1=\frac{\mathrm{u}_1^2 \sin 2 \theta_1}{\mathrm{g}} ; \mathrm{R}_2=\frac{\mathrm{u}_2^2 \sin 2 \theta_2}{\mathrm{g}}$$

$$\frac{\mathrm{R}_1}{\mathrm{R}_2}=\frac{\mathrm{u}_1^2 \sin 2 \theta_1}{u_2^2 \sin 2 \theta_2}=\frac{40^2 \sin \left(2 \times 30^{\circ}\right)}{60^2 \sin \left(2 \times 60^{\circ}\right)}=\frac{4}{9}$$

32. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be :

(A) 50 m

(B) $50\sqrt{2}$ m

(C) 100 m

(D) $100\sqrt{2}$ m

Ans. (C)

Sol. Range of projectile

$$R = \frac{v^2 \sin 2\theta}{g} (\because R \propto \sin(2\theta))$$

$$\frac{R_1}{R_2} = \frac{\sin(2\theta_1)}{\sin(2\theta_2)} = \frac{\sin(2 \times 15)}{\sin(2 \times 45)} = \frac{\sin 30^\circ}{\sin 90^\circ}$$

$$\Rightarrow \frac{50}{R_2} = \frac{1}{2} \Rightarrow R_2 = 100 \text{ m}$$

33. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be.

(A) 5 m

(B) $10\sqrt{2}$ m

(C) 200 m

(D) 10 m

Ans. (A)

Sol. The trajectory of projectile projected. From the ground is given by

$$y = x - \frac{x^2}{20}$$

For maximum height,

$$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{2x}{20} = 0 \ \therefore x = 10$$

Hence, the maximum height attained by the projectile,

$$y_{max} = 10 - \frac{100}{20} = 5 \text{ m}$$

34. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at the point of projection to its kinetic energy at the highest point of flight will be :

(A) 1 : 2

(B) 1 : 4

(C) 4 : 1

(D) 4 : 3

Ans. (D)

Sol. Let u be the speed of projectile at initial point.

$$\therefore \text{ KE} = \frac{1}{2} mu^2$$

At maximum height speed $= u\cos 30^\circ$

$$\therefore \text{ KE} = \frac{1}{2} m(u\cos 30^\circ)^2$$

$$\frac{KE_{initial}}{KE_{top}} = \frac{\frac{1}{2} m(u)^2}{\frac{1}{2} m(u\cos 30^\circ)^2} = \frac{4}{3}$$

35. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be : (Given g = 10 m/s^2)

(A) $20\sqrt{3}\ ms^{-1}$

(B) $40\sqrt{3}\ ms^{-1}$

(C) 20 ms^{-1}

(D) Zero

Ans. (A)

Sol. Given,

Initial velocity of projectile, $u = 40\ m/s$

Angle, $\theta = 30°$

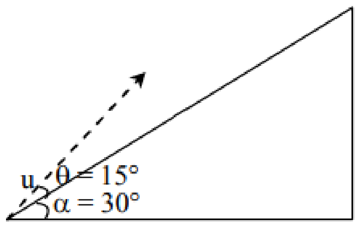
Time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 40 \times 1}{10 \times 2} = 4s\ (\because g = 10\ m/s^2)$$

It means projectile is at maximum height at t = 2 s. At maximum height vertical component of velocity is zero.

Velocity at $t = 2s = V_x = u\cos\theta = 40\cos 30° = 20\sqrt{3}\ ms^{-1}$.

36. A plane is inclined at an angle α = 30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1}, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to: (Take g = 10 ms^{-2})



(A) 20 cm

(B) 18 cm

(C) 26 cm

(D) 14 cm

Ans. (A)

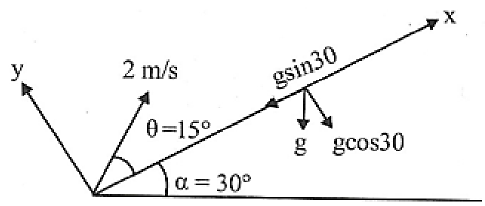
Sol. On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

Distance, $S = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$



$$= (2\cos 15°)\frac{4\ \sin 15°}{10\cos 30°} - \left(\frac{1}{2} \times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3} - 16}{60} \simeq 0.1952\ m \simeq 20\ cm$$

37. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Two identical balls A and B thrown with same velocity 'u' at two different angles with hroizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then R = $4\sqrt{h_1h_2}$

Reason R : Product of said heights.

$$h_1h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right).\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the CORRECT answer:

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Ans. (A)

Sol. We know that if range is same for two angles of projection, then these angle must be complementary. Let first angles of projection be ' θ ' then second will be $(90-\theta)$

$\therefore h_1 = \frac{u^2\sin^2\theta}{2g}$ and $h_2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

$\therefore h_1h_2 = \frac{u^2\sin^2\theta}{2g}\cdot\frac{u^2\cos^2\theta}{2g}$. So, reason is correct

$$\Rightarrow \sqrt{h_1h_2} = \frac{u^2\sin\theta\cos\theta}{2g} \Rightarrow 4\sqrt{h_1h_2} = \frac{4u^2\sin\theta\cos\theta}{2g}$$

$$\Rightarrow 4\sqrt{h_1h_2} = \frac{u^2(2\sin\theta\cos\theta)}{g} \Rightarrow 4\sqrt{h_1h_2} = \frac{u^2\sin^2\theta}{g} = R$$

So, assertion is correct.

38. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is

(A) 1

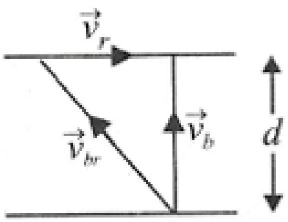
(B) 3

(C) 4

(D) $\sqrt{41}$

Ans. (B)

Sol. Shortest route corresponds to $\vec{v}_b$ perpendicular to river flow



$$\therefore t = \frac{d}{v_b} = \frac{d}{\sqrt{v_{br}^2 - v_r^2}}$$

or $t = \frac{d}{v_b} = \frac{1\text{ km}}{\frac{1}{\frac{1}{4}}}$

or $\frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}} \Rightarrow v_r = 3\text{ km/h}$

39. A fighter jet is flying horizontally at 200 m/s at height of 1 km from ground. If jet wants to destroy a terrorist base camp, how far from the camp should it drop the bomb ?

(A) $2\sqrt{3}$ km

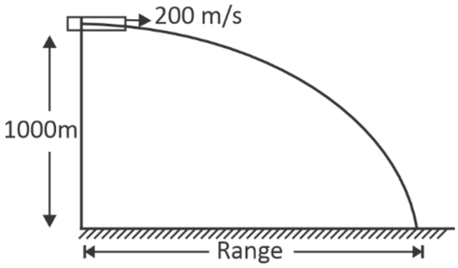
(B) $\sqrt{2}$ km

(C) $2\sqrt{2}$ km

(D) $2\sqrt{5}$ km

Ans. (C)

Sol.



$R = u\sqrt{\frac{2H}{g}}$

$R = 200\sqrt{\frac{2 \times 1000}{10}}$

$R = 2\sqrt{2}$ km

40. Ship A is sailing towards north-east with velocity $\bar{v} = 30\hat{i} + 50\hat{j}$ km/hr, where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distnace from B in :

(A) 4.2 hrs.

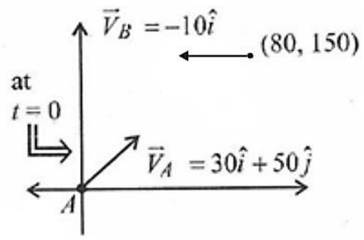
(B) 2.6 hrs.

(C) 3.2 hrs.

(D) 2.2 hrs.

Ans. (B)

Sol.



After time ' t '

$\vec{r}_A = 30t\hat{i} + 50t\hat{j}$

$\vec{r}_B = (80 - 10t)\hat{i} + 150\hat{j}$

$|\vec{r}| = |\vec{r}_B - \vec{r}_A|$

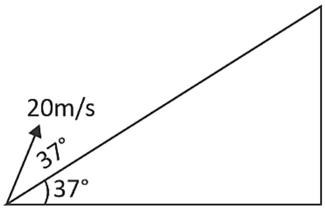
$=|(80-40t)\hat{i}+(150-50t)\hat{j}|$

For ' r ' min, $\frac{d}{dt}(|r|^2)=0$

$\Rightarrow 2(80-40t)\times -40+2(150-50t)\times -50=0$

Solving above we get $t=2.6\text{hr}$.

41. A particle projected on plane, then find time of flight.



(A) 2 sec
(B) 4 sec
(C) 8 sec
(D) 3 sec

Ans. (D)

Sol. $T=\dfrac{2u\sin\beta}{g\cos\alpha}$

$T=\dfrac{\cancel{2}\times 20\times 3}{\cancel{10}\times \frac{3}{\cancel{5}}5}=3\text{ sec}$

42. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$):

(A) $\theta_0=\sin^{-1}\frac{1}{\sqrt{5}}$ and $v_0=\frac{5}{3}ms^{-1}$

(B) $\theta_0=\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0=\frac{3}{5}ms^{-1}$

(C) $\theta_0=\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0=\frac{5}{3}ms^{-1}$

(D) $\theta_0=\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0=\frac{3}{5}ms^{-1}$

Ans. (C)

Sol. Given, $y=2x-9x^2$

On comparing with,

$y=x\tan\theta-\dfrac{gx^2}{2u^2\cos^2\theta}$,

We have,

$\tan\theta=2$ or $\cos\theta=\dfrac{1}{\sqrt{5}}$

and $\frac{g}{2u^2\cos^2\theta} = 9$ or $\frac{10}{2u^2(1/\sqrt{5})^2} = 9$

$\therefore u = 5/3 \text{ m/s}$

43. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

(A) $\sqrt{\frac{2ghv^2+1}{h^2}}$

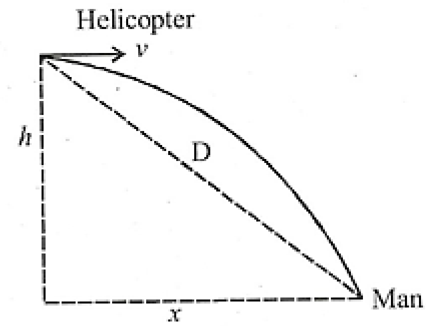
(B) $\sqrt{2ghv^2+h^2}$

(C) $\sqrt{\frac{2v^2h}{g}+h^2}$

(D) $\sqrt{\frac{2gh}{v^2}+h^2}$

Ans. (C)

Sol.



Time of fall of packet

$t = \sqrt{\frac{2h}{g}}, x = \sqrt{\frac{2h}{g}}v$

Horizontal range (x) = time × horizontal component of velocity (v)

∴ Required distance

$$D = \sqrt{x^2+h^2}$$

$$= \sqrt{\left(\sqrt{\frac{2h}{g}}v\right)^2 + h^2}$$

or, $D = \sqrt{\frac{2hv^2}{g}+h^2}$

44. Two projectiles are projected at 30° and 60° with the horizontal with the same speed. The ratio of the maximum heights attained by the two projectiles respectively is:

(A) $2:\sqrt{3}$

(B) $\sqrt{3}:1$

(C) 1 : 3

(D) $1:\sqrt{3}$

Ans. (C)

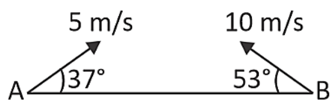
Sol. Maximum height in projectile motion

$$H_{max} = \frac{u^2 \sin^2 \theta}{2\,g}$$

For two projectiles $\theta_1 = 30°$ and $\theta_2 = 60°$

$$\therefore \frac{H_1}{H_2} = \frac{\frac{u^2 \sin^2 30°}{2\,g}}{\frac{u^2 \sin^2 60°}{2\,g}} = \frac{1}{3}$$

45. Find $\vec{v}_A - \vec{v}_B$



(A) $5\hat{i} + 10\hat{j}$

(B) $10\hat{i} - 5\hat{j}$

(C) $5\hat{i} - 10\hat{j}$

(D) $10\hat{i} + 5\hat{j}$

Ans. (C)

Sol. $\vec{v}_A - \vec{v}_B = \left(5\cos 37\hat{i} + 5\sin 37\hat{j}\right) - \left[10\cos 53(-\hat{i}) + 10\sin 53\hat{j}\right]$

$= 5 \times \frac{4}{5}\hat{i} + 5 \times \frac{3}{5}\hat{j} - \left[10 \times \frac{3}{5}(-\hat{i}) + 10 \times \frac{4}{5}\hat{j}\right]$

$= 4\hat{i} + 3\hat{j} - \left[6(-\hat{i}) + 8\hat{j}\right]$

$= 4\hat{i} + 3\hat{j} + 6\hat{i} - 8\hat{j}$

$\vec{v}_A - \vec{v}_B = 10\hat{i} - 5\hat{j}$

$\vec{a}_A - \vec{a}_B = 0$

46. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be_____ ms^{-1} (given, g = 10 ms^{-2}).

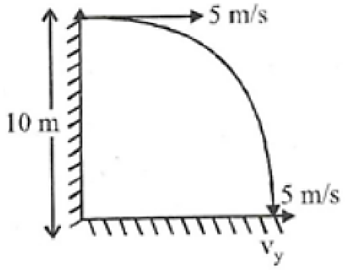
(A) 15

(B) 20

(C) 30

(D) 25

Ans. (A)

Sol. 

Given, $v_x = 5\ \text{m/s}$

$v_y = \sqrt{2gh} = \sqrt{200}$

$v_{net} = \sqrt{v_x^2 + v_y^2} = \sqrt{(5)^2 + (\sqrt{200})^2}$

$= \sqrt{25 + 200} = 15\ \text{m/s}$

47. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}$u. The time of flight of the projectile is :

(A) $\frac{u}{2g}$

(B) $\frac{u}{g}$

(C) $\frac{2u}{g}$

(D) $\frac{\sqrt{3}u}{g}$

Ans. (B)

Sol. At highest point, only horizontal component of velocity exists

So, $u\cos\theta = \frac{\sqrt{3}}{2}u \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

$T = \frac{2u\sin\theta}{g} = \frac{2 \times u \times \sin 30^\circ}{g} = \frac{u}{g}$

48. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is

(A) 192 m

(B) 136 m

(C) 272 m

(D) 68 m

Ans. (C)

Sol.

As $v^2 = u^2 + 2as$

$\Rightarrow 0 = u^2 - 2gH \Rightarrow u^2 = 2gH \Rightarrow u = \sqrt{2 \times 10 \times 136}$

$R_{max} = \frac{u^2}{g} = \frac{2 \times 10 \times 136}{10} = 272\ \text{m}$

49. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is :

(A) $y = x - 5x^2$

(B) $y = 2x - 5x^2$

(C) $4y = 2x - 5x^2$

(D) $4y = 2x - 25x^2$

Ans. (B)

Sol. From equation, $\vec{v} = \hat{i} + 2\hat{j}$

Þ $x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

50. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h in still water. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

(A) 90°

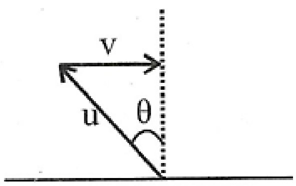
(B) 150°

(C) 120°

(D) 60°

Ans. (C)

Sol.



$\sin\theta = \frac{u}{v} = \frac{2}{4} = \frac{1}{2}$

Or $\theta = 30^\circ$

with respect to flow,

$= 90^\circ + 30^\circ = 120^\circ$

SECTION-II

51. A particle is projected at an angle of 45^o from a point lying 2 m from the foot of a wall. It just touches the top of the Wall and falls on the ground 4m from it. If height of the wall is x/3 m. Find x (g = 10 ms^{-2})

Ans. 4

Sol. $R = 6 = \frac{4^2 \sin 90}{g}$

$u = \sqrt{60}$ ...(1)

$$y = x\tan\theta\left(1-\frac{x}{R}\right)$$

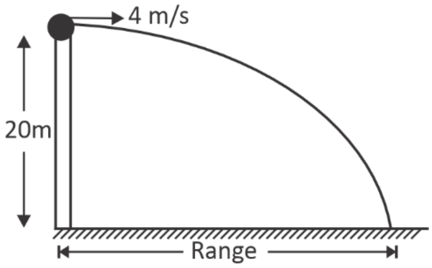
$$y = 2\times 1\left(1-\frac{2}{6}\right)$$

$$= \frac{4}{3}$$

52. A stone horizontally projected from a tower of height 20m, if velocity of projection of stone is 4 m/s. Then find range in meter value travelled by stone?

Ans. 8

Sol.



$$\text{Range} = u\sqrt{\frac{2H}{g}}$$

$$R = 4\sqrt{\frac{2\times \cancel{20}}{\cancel{10}}}$$

R = 8m

53. A stone projected from ground with velocity 10 m/s at an angle of 30° from horizontal axis, then calculate its time of flight in S value?

Ans. 1

Sol. $T = \dfrac{2u\sin\theta}{g}$

$$T = \frac{2\times 10}{10}\times\frac{1}{2}\quad\left[\sin 30^\circ = \frac{1}{2}\right]$$

T = 1 sec

54. A particle projected from ground with velocity $\vec{u} = (3\hat{i}+5\hat{j})$ m/s. Then calculate its range in meter value?

Ans. 3

Sol. $R = \dfrac{2u_x\times u_y}{g} \Rightarrow \dfrac{2\times 3\times 5}{10} = 3\text{m}$

55. Given trajectory equation is $y = \left(\sqrt{3}x - \frac{x^2}{\sqrt{3}}\right)$m. Then write its range in meter value?

Ans. 3

Sol. As we know $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$$y = \sqrt{3}\,x\left[1 - \frac{x}{3}\right]$$

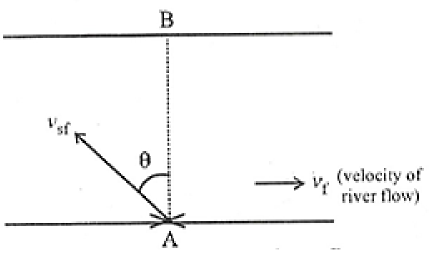
on comparing

R = 3m

56. A person is swimming with a speed of 10 m/s with respect to river at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is ____.

Ans. 5

Sol.



Velocity of swimmer w.r.t. river flow $= v_{sf} = 10$ m/s

Person is swimming at an angle of 120°.

$\therefore\ \theta = 120^\circ - 90^\circ = 30^\circ$

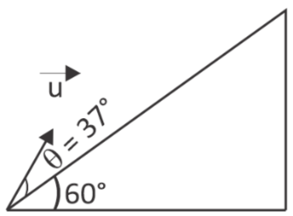
From figure, $\sin\theta = \frac{v_f}{v_{sf}}$

$$\Rightarrow \sin 30^\circ = \frac{x}{10} \Rightarrow \frac{1}{2} = \frac{x}{10} \Rightarrow x = \frac{10}{2} = 5 \text{ m/s}$$

57. A plane is inclined at angle (α = 60°) with respect to the horizontal. A particle is projected with speed u = 5 m/s from the base of plane, as shown in figure. The maximum height in meter from the plane upto which particle goes is $\frac{x}{10}\left(g = 10\text{ms}^{-2}\right)$. Then find x

Ans. 9

Sol.

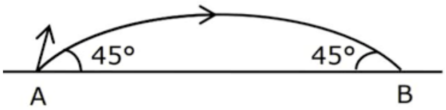


$$\text{Height} = \frac{u^2 \sin^2\beta}{2g\cos\alpha} \qquad \beta = \text{angle of projection}$$

$$H = \frac{(25)}{2\times 10\times\frac{1}{2}}\times\left(\frac{3}{2}\right)^2 \qquad \alpha = \text{angle of ineination}$$

H = 9/10

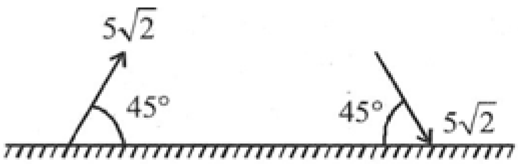
58. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is x × 10^{-2} $kgms^{-1}$. The value of x, to the nearest integer, is____

Ans. 5

Sol.



Change in momentum,

$$\Delta\vec{P} = \vec{P}_B - \vec{P}_A$$

$$\Delta\vec{P} = mV\cos\theta\hat{i} - mV\sin\theta\hat{j} - (mV\cos\theta\hat{i} + mV\sin\theta\hat{j})$$

$$= m\left(5\sqrt{2}\cos 45^\circ\hat{i} - 5\sqrt{2}\sin 45^\circ\hat{j}\right) - m\left(5\sqrt{2}\cos 45^\circ\hat{i} + 5\sqrt{2}\sin 45^\circ\hat{j}\right)$$

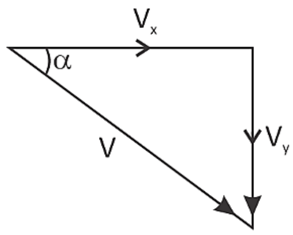
$$|\Delta\vec{P}| = 2\times\left(5\times 10^{-3}\right)\times(5) = 5\times 10^{-2}\,kgms^{-1}$$

$$\therefore \quad x = 5$$

59. A particle is projected horizontally with a speed 20 m/s from the top of a tower. After what time in S will the velocity be at 45° angle from the initial direction of projection? [g = 10 m/s^2]

Ans. 2

Sol.



Here V_x = U_x = 20 m/s, velocity in y-direction

$V_y = U_y + a_y t$

$V_y = 0 + gt$

$\therefore \tan\alpha = \frac{V_y}{V_x} = \frac{gt}{20}$

$\tan 45^\circ = \frac{gt}{20}$

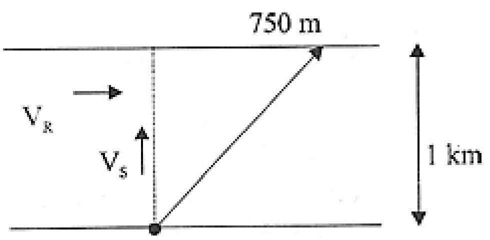
$20 = gt$

$t = 2$ sec

60. The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is ________ km h^{-1}.

Ans. 3

Sol.



We have

$1\text{km} = V_S \times t$

and, $750\text{m} = V_R \times t$ So, $\frac{1000\text{m}}{750\text{m}} = \frac{V_S}{V_R} \Rightarrow \frac{4}{3} = \frac{4}{V_R}$

$\Rightarrow V_R = 3\text{ km / hr}$

PART-C: CHEMISTRY
SECTION-I

61. In the long form of the periodic table, the valence shell electronic configuration of $5s^2 5p^4$ corresponds to the element present in :
(A) Group 16 and period 6
(B) Group 17 and period 6
(C) Group 16 and period 5
(D) Group 17 and period 5

Ans. (C)

Sol. Tellurium (Te) has $5s^2 5p^4$ valence shell configuration. It belongs to group 16 and present in period 5 of the periodic table.

62. The correct order of the atomic radii of C, Cs, Al, and S is :

(A) C < S < Al < Cs
(B) S < C < Cs < Al
(C) S < C < Al < Cs
(D) C < S < Cs < Al

Ans. (A)

Sol. On going down the group, size increases while going from left to right in a period, size decreases. So order is: C < S < Al < Cs.

63. The process requiring absorption of energy is :

(A) $N \longrightarrow N^-$

(B) $F \longrightarrow F^-$

(C) $Cl \longrightarrow Cl^-$

(D) $H \longrightarrow H^-$

Ans. (A)

Sol. Nitrogen has stable $2p^3$ configuration and also due to high e^- charge density at outermost orbital it requires energy to add one extra e^- in its outer most shell ie., its first electron gain enthalpy is positive.

64. In which of the following transformation least energy is required ?

(A) $F^-_{(g)} \rightarrow F_{(g)} + e^-$

(B) $P^-_{(g)} \rightarrow P_{(g)} + e^-$

(C) $S^-_{(g)} \rightarrow S_{(g)} + e^-$

(D) $Cl^-_{(g)} \rightarrow Cl_{(g)} + e^-$

Ans. (B)

Sol. If we consider the opposite process:

(a) $F_{(g)} + e^- \longrightarrow F^-_{(g)}$; ΔH_1

(b) $P_{(g)} + e^- \longrightarrow P^-_{(g)}$; ΔH_2

(c) $S_{(g)} + e^- \longrightarrow S^-_{(g)}$; ΔH_3

(d) $Cl_{(g)} + e^- \longrightarrow Cl^-_{(g)}$; ΔH_4

Order of energy released is : $\Delta H_4 > \Delta H_1 > \Delta H_3 > \Delta H_2$

So, $P^-_{(g)} \longrightarrow P_{(g)} + e^-$; Requires least energy.

65. The correct order of electronegativity for given elements is:
(A) C > P > At > Br
(B) Br > P > At > C
(C) P > Br > C > At
(D) Br > C > At > P

Ans. (D)

Sol.

Atom	E.N.
Br	2.86
C	2.5
At	2.2
P	2.1

66. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above 'E' in the group is

(A) $3s^2$, $3p^4$
(B) $3d^{10}$, $4s^2$, $4p^4$
(C) $4d^{10}$, $5s^2$, $5p^4$
(D) $2s^2$, $2p^4$

Ans. (A)

Sol. Period of the element just above E is 3 i.e., n = 3 group number = 16, so orbital is p and it contain $4e^-$ in last orbital.

E $\Rightarrow$ [Ar] $3d^{10}4s^24p^4$

Element above E $\Rightarrow$ [Ne] $3s^23p^4$

67. The correct order of atomic/ionic radii is:

(A) Sc > Ti > V > Cr
(B) Co > Ni > Cu > Zn
(C) $S^{2-} > Cl^- > O^{2-} > N^{3-}$
(D) None of these

Ans. (A)

Sol. (a) SC > Ti > V > Cr (size decrease initially in 3d-series)

(b) Correct order : Zn > Cu > CO $\approx$ Ni

(c) Correct order : $S^{2-} > Cl^- > N^{3-} > O^{2-}$

68. The first ionization enthalpies of Be, B, N and O follow the order

(A) O < N < B < Be
(B) Be < B < N < O
(C) B < Be < N < O
(D) B < Be < O < N

Ans. (D)

Sol. On moving form left to right period size of atom decrase, hence ionization energy increases.

Nitrogen half half-filled p-orbital and Beryllium have half filled s-orbital so their ionization energy are greater than oxygen and Boron respectively. So the order is :

$$\underset{(2p^3)}{N} > \underset{(2p^4)}{O} > \underset{(2s^2)}{Be} > \underset{(2p^1)}{B}$$

69. The correct option with respect to the Pauling electronegativity values of the elements is:

(A) Te > Se

(B) Ga < Ge
(C) Si < Al
(D) P > S

Ans. (B)

Sol. Correct order of elecronegativity values of the element is

Si > Al; S > P; Se > Te; Ge > Ga.

70. Which of the following atom or ions has the smallest size?
(A) F
(B) F^-
(C) O
(D) N

Ans. (A)

Sol. F has smallest size due to effective nuclear charge.

71. The acidic, basic and amphoteric oxides, respectively, are:
(A) Na_2O, SO_3, Al_2O_3
(B) Cl_2O, CaO, P_4O_{10}
(C) N_2O_3, Li_2O, Al_2O_3
(D) MgO, Cl_2O, Al_2O_3

Ans. (C)

Sol. Generally, non-metal oxides are acidic in nature and metal oxides are basic in nature, Al_2O_3 is amphoteric.

72. The first ionisation potential of Na, Mg, Al and Si are in the order
(A) Na < Mg > Al < Si
(B) Na > Mg > Al > Si
(C) Na < Mg < Al < Si
(D) Na > Mg > Al < Si

Ans. (A)

Sol. First ionisation potential increases from left to right in a period.

IE_1 of Mg is higher than that of Na because of increased nuclear charge and also than that of Al because in Mg, a 3 s-electron has to be removed while in Al, it is the 3 p-electron. The IE_1 of Si is, however, higher than those of Mg and Al because of its increases nuclear charge. Thus, the overall order is : Na < Mg > Al < Si.

73. The correct order of increasing atomic radius of the following elements is :
(A) S < O < Se < C
(B) O < C < S < Se
(C) O < S < Se < C
(D) C < O < S < Se

Ans. (B)

Sol. Fact Based.

74. The elements with atomic numbers 101 and 104 belongs to, respectively :
(A) Group 11 and Group 4
(B) Actinoids and Group 6
(C) Actinoids and Group 4
(D) Group 6 and Actinoids

Ans. (C)

Sol. $_{90}Th \longrightarrow {}_{103}Lr$

Belongs to actinoids series and they all belongs to 3rd group. So atomic no. 101 element is actinoids and atomic number 104 element belongs to 4th group.

75. The incorrect statement is
(A) The first ionization enthalpy of K is less than that of Na and Li
(B) Xe does not have the lowest first ionization enthalpy in its group
(C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.
(D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Ans. (D)

Sol. $_{31}Ga$ is in group 13 with e^- configuration [Ar] $3d^{10}4s^24p^1$ whereas $_{30}Zn$ has e^- configuration as [Ar] $3d^{10}4s^2$. Removal of e^- from 4p orbital is easier than that from 4s orbital. Hence, first I.E. of Ga is lower than that of Zn.

76. The electronegativity of the following elements increases in the order
(A) C, N, Si, P
(B) N, Si, C, P
(C) Si, P, C, N
(D) P, Si, N, C

Ans. (C)

Sol. Electronegativity increases on moving from left to right in a period and decreases on moving from top to bottom in a group.

Si and P are placed in the 3rd period while C and N are placed in the 2nd period. Elements in 2nd period have higher electronegativities than those in the 3rd period. Since, N has smaller size and higher nuclear charge than C, its electronegativity is higher than that of C. Similarly, the electronegativity of P is higher than that of Si. Thus, the overall order is : Si, P, C, N.

77. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are :
(A) 16, 5 and 2
(B) 15, 5 and 3
(C) 16, 6 and 3
(C) 15, 6 and 2

Ans. (B)

Sol. Phosphorus has atomic number 15. Its group number is 15, number of valence electrons is 5 and valency is 3.

78. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively, are
(A) Chlorine, lithium and sodium
(B) argon, lithium and sodium
(C) argon, chlorine and sodium
(D) neon, sodium and chlorine

Ans. (C)

Sol. Ionization energy generally increases moving from left to right across the period.

So, the order is Na < Mg < ...... Cl < Ar.

79. Correct expression of "Allred and Rochow's" scale is :

(A) Electronegativity = $0.744\frac{Z_{eff.}}{r^2}+0.359$

(B) Electronegativity = $0.359\frac{r^2}{Z_{eff.}}+0.744$

(C) Electronegativity = $0.359\frac{Z_{eff.}}{r}+0.744$

(D) Electronegativity = $0.359\frac{Z_{eff.}}{r^2}+0.744$

Ans. (D)

Sol. According to Allred and Rochow scale

(C) $EN_{(AR)}=0.359\frac{Z_{eff.}}{r^2}+0.744$ (r : radius in Å)

80. Atomic radii of fluroine and neon in Angstrom units are respectively given by
(A) 0.72, 1.60
(B) 1.60, 1.60
(C) 0.72, 0.72
(D) None of these values

Ans. (A)

Sol. (i) Noble gases do not have covalent radii. They have only van der Waals radii.

(ii) Covalent radii is always smaller than corresponding van der Waals radii.

SECTION-II

81. How many following pairs have approximately the same atomic radii?
(A) Zr and Hf
(B) Al and Mg
(C) Al and Ga
(D) Na and Ne

Ans. 1

Sol. Due to poor shielding effect of d-electron. The screening effect decreases for the outer most electron.

Al > Ga, Zr and Hf (Lanthanide construction)

82. In how many pairs, all elements belong to same group in periodic table

(H, He) (Li, Be)

(Na, K) (F, Cl)

(S, Se) (Ga, Ge)

Ans. 3

Sol. Conceptual

83. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1} . The number of valence electrons in the element is :

Ans. 3

Sol. As difference in 3rd and 4th ionisation energies is high, so atom contains 3 valence electrons.

84. If the ionization enthalpy and electron gain enthalpy of an element are 310 and 86 kcal mol^{-1} respectively, then the electronegativity of the element on the Pauling scale is (Nearest integer)

Ans. 3

Sol. I.E. + E.A. = 310 + 86 = 396 kcal mol^{-1}

$= 396 \times 4.184 = 1655.28$ kJ/mol

$$\text{E.N.} = \frac{1655.28}{540} = 3.06$$

85. How many orders of radii are correct.

(A) $Pb > Pb^{2+} > Pb^{4+}$

(B) $In^{+} > Sn^{2+} > Sb^{3+} > Te^{4+}$

(C) Co > Ni > Cu > Zn

(D) $K^{+} > Li^{+} > Mg^{2+} > Al^{3+}$

Ans. 3

Sol. The correct order of radii for option C : Co ≈ Ni < Cu < Zn

86. How many correct order of electron affinity.

(A) S > O

(B) Al > B

(C) Mg > Na

(D) P > N

Ans. 3

Sol. Na has greater electron affinity than Mg due to $3s^1$ configuration.

87. How many of the following are smaller than $F^{-} \rightarrow Cl^{-}, Br^{-}, H^{-}, O^{-2}, S^{-2}$

Ans. 0

Sol. Conceptual

88. How many order of electronegativity are correct.

(A) Pauling scale (E.N. of F-atom) > Mulliken scale (E.N. of F-atom)

(B) Cl_2O (E.N. of Cl-atom) > Cl_2O_5 (E.N. of Cl-atom)

(C) Si < P > S

(D) Cu^{2+} (E.N.) > Cu^{+} (E.N.)

Ans. 1

Sol. Electronegativity of central atom is directly proportional to oxidation state.

Option A, B, C are incorrect.

89. How many pairs are, in which first species has lower ionisation energy than second species:

(i) N and O (ii) Br and K

(iii) Be and B (iv) F and O

(v) Li and Be (vi) O and S

(vii) Ba and Sr

Ans. 2

Sol. (v) Li < Be (vii) Ba < Sr

90. Total number of acidic oxides among

N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O and NO is ________.

Ans. 4

Sol. Acidic oxides are N_2O_3, NO_2, Cl_2O_7, SO_2

Basic oxides are CaO, Na_2O

Neutral Oxides are CO, NO, N_2O

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