
PHYSICS
Rankers Academy JEE

7

A ball is allowed to fall from a height 1.0 m. If the value of coefficient of restitution is 0.6, then after the impact ball will go upto :

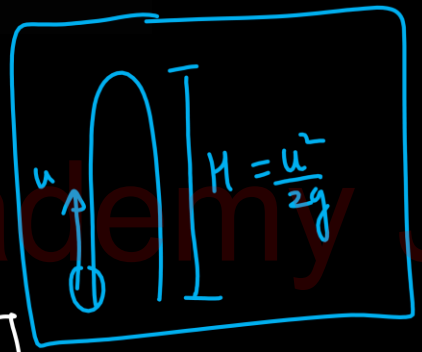
(A) 0.16 m (B) 0.36 m

(C) 0.40 m (D) 0.60 m

JEE 1

M-1

M-2



$$h_1 = \frac{(ev)^2}{2g} = \frac{e^2 \cdot 2gh}{2g} = e^2 h = (0.6)^2 \cdot 1 = 0.36$$

2

The potential energy of a body is given by :

$U = 40 + 6x^2 - 7xy + 8y^2 + 32z$

where U is in joule and x, y, z in metre. Deduce the x, y and z components of the force (in newton) on the body when it is in position $(-2, 0, +5)$

(A) $F_x = -32; F_y = -14; F_z = 24$

(B) $F_x = -14; F_y = -32; F_z = 24$

(C) $F_x = 24; F_y = -32; F_z = -14$

(D) $F_x = 24; F_y = -14; F_z = -32$

$x = -2$
$y = 0$
$z = 5$

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

JEE 1

$$\vec{F} = -(12x - 7y)\hat{i} - (-7x + 16y)\hat{j} - (32)\hat{k}$$

F_x F_y F_z

$$F_x\Big|_{\substack{x=-2\\y=0\\z=5}} = -(-24 - 0) = 24$$

$$F_y\Big|_{\substack{x=-2\\y=0\\z=5}} = -14$$

$$F_z = -32$$

3

306 J of heat is required to raise the temperature of 2 moles of an ideal gas at constant pressure from 25°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is

(A) 306 J (B) 153 J

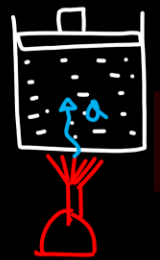
(C) 140 J (D) 80 J

$n = 2$

$\Delta T = 10$

JEE 1

Theary.

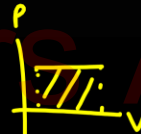


$Q = \Delta U + W$

$C_p - C_v = R$

$R = 8.314$

const. P



$Q = \Delta U + W$ ($W = P\Delta V = nR\Delta T$)

$Q = nC_v\Delta T + nR\Delta T$

$Q = n\Delta T(C_v + R)$

$Q = nC_p\Delta T$

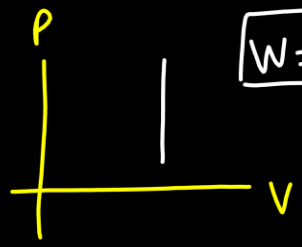
$306 = 2C_p(10)$

$C_p = 15.3$

$C_v = 15.3 - 8.314$

$C_v \simeq 7$

const. V



$Q_1 = \Delta U + W$ ($W = 0$)

$Q_1 = nC_v\Delta T$

$Q_1 = 2(7)(10)$

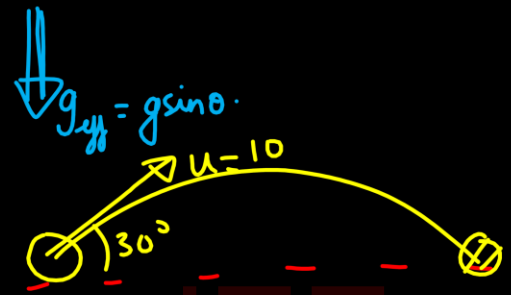
$Q_1 = 140$

4

A small ball is projected up a smooth inclined plane with an initial speed of 10 m/s along the direction at 30° to the bottom edge of the slope. It returns to the edge after 2 s . The ball is in contact with the inclined plane throughout the process. What is the inclination angle of the plane?

(A) 30°
(B) 37°
(C) 53°
(D) 60°

JEE 1



$$T = \frac{2u\sin 30°}{g_{yy}}$$

$$2 = \frac{2 \times 10 \times \frac{1}{2}}{g\sin\theta}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30°$$

5

If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is :-

(A) $2S^2Y$

(B) $\frac{S^2}{2Y}$

(C) $\frac{2Y}{S^2}$

(D) $\frac{S}{2Y}$

M-1

stress = S

Energy Density = $\frac{1}{2}$ (stress) × (strain)

$= \frac{1}{2} \cdot S \cdot \frac{S}{Y}$

$= \frac{S^2}{2Y}$

$Y = \frac{\text{stress}}{\text{strain}}$

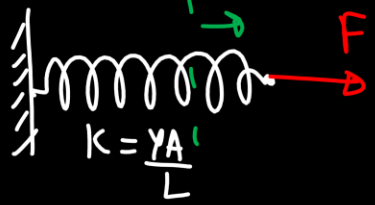
$Y = \frac{S}{\text{strain}}$

$\text{strain} = \frac{S}{Y}$

M-2

$S = F/A$

$\equiv$



$F = kx$

$x = \frac{F}{K} =$

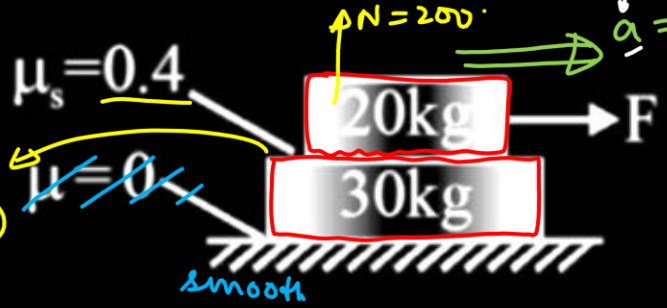
Energy Density = $\frac{U}{\text{Vol.}} = \frac{\frac{1}{2}kx^2}{A \cdot L} = \frac{1}{2} \frac{\frac{YA}{L} \cdot \frac{F^2}{K^2}}{A \cdot L} = \frac{1}{2} \frac{Y}{L} \cdot \frac{F^2}{\left(\frac{YA}{L}\right)^2}$

$= \frac{1}{2} \frac{Y F^2}{Y^2 A^2}$

E.D. = $\frac{1}{2} \frac{S^2}{Y}$

6

For the given diagram for what maximum value of force both block will move together?

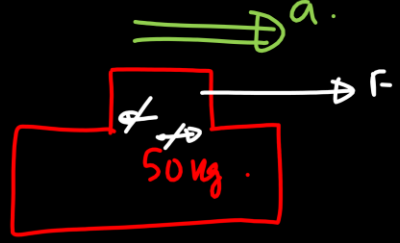


(A) 150 N (B) 200 N

(C) $\frac{400}{3}$ N (D) 80 N

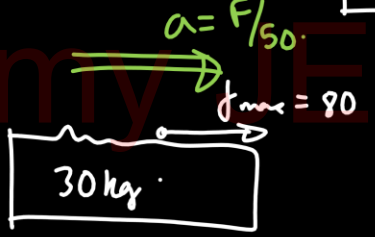
$f_{max} = \mu_s N = 80$

$a = \frac{F}{50}$



$F = 50a$

$a = \frac{F}{50}$



$80 = 30\left(\frac{F}{50}\right)$

$F = \frac{400}{3}$

JEE 1

7

In an A.C. circuit, voltage applied is $V = 220 \sin 100t$. If the impedance is 110Ω and phase difference between current and voltage is $60°$, the power consumption is equal to

(A) 55 W (B) 110 W

(C) 220 W (D) 330 W

JEE 1

$Z = 110\ \Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{220/\sqrt{2}}{110} = \frac{\sqrt{2}}{1}$

$\phi = \pi/3$

$V = V_0 \sin(\omega t)$

$I = I_0 \sin(\omega t + \pi/3)$

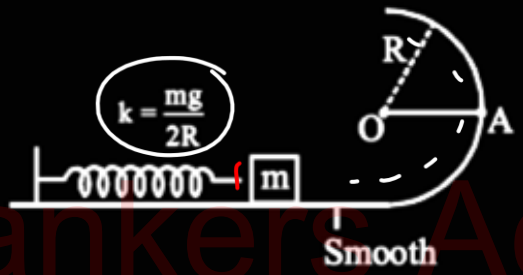
$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{220}{\sqrt{2}}$

$I_{rms} = \frac{I_0}{\sqrt{2}}$

$P = V_{rms} \cdot I_{rms} \cdot \cos\phi = \frac{220}{\sqrt{2}} \times \frac{\sqrt{2}}{1} \cdot \frac{1}{2} = 110$

8

Block is taken toward left so that spring is compressed. Now the block is released from rest. Block move on smooth horizontal surface after leaving spring, then on circular path R.



V_{CM}

$U_{min} = \sqrt{5gR}$

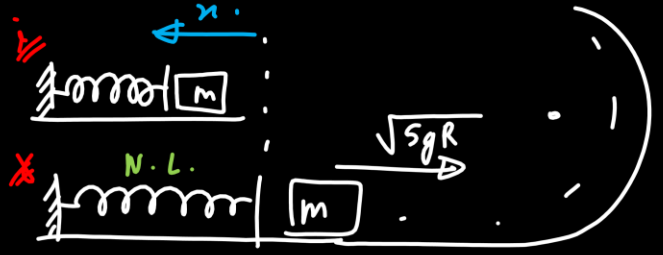
By how much distance spring should be compressed so that block can complete circular motion :

(A) $R\sqrt{5}$

(B) $R\sqrt{10}$

(C) $5R$

(D) $\frac{5}{2}R$



$\frac{1}{2}kx^2 = \frac{1}{2}m(\sqrt{5gR})^2$

$\frac{1}{2} \cdot \frac{mg}{2R} \cdot x^2 = \frac{1}{2} m \cdot 5gR$

$x^2 = 10R^2$

$x = \sqrt{10}\,R$

9

A string of length 1.5 m with its two ends clamped is vibrating in the fundamental mode. The amplitude at the centre of the string is 4 mm . The minimum distance between the two points having amplitude of 2 mm is

(A) 1 m (B) 75 m

(C) 60 m (D) 50 m

Ans = $d = 1.5 - 2x_1$

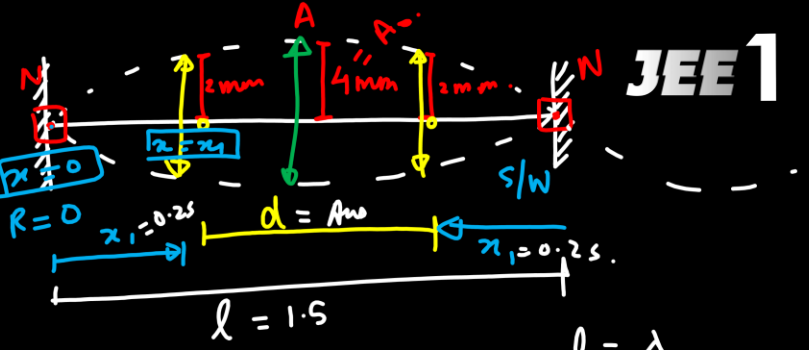
$A_0 = 2A$

$R = A_0 = 4 \times 10^{-3}$ m

$y_{s/w} = \boxed{A_0 \cdot \sin(kx)} \cdot \sin(\omega t + \phi)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3}$

$y_{s/w} = \underbrace{\left(\frac{4}{1000}\right) \cdot \sin\left(\frac{2\pi}{3} \cdot x\right)}_{R} \cdot \sin(\omega t + \phi)$



JEE 1

$l = \frac{\lambda}{2}$

$\lambda = 3$

(given)

$R\big|_{x=x_1} = \frac{2}{1000}$

$\left(\frac{4}{1000}\right) \cdot \sin\left(\frac{2\pi}{3} \cdot x_1\right) = \frac{2}{1000}$

$\frac{2\pi}{3} \cdot x_1 = \frac{\pi}{2}$

$x_1 = 0.25$

$\sin\left(\frac{2\pi}{3} \cdot x_1\right) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

Ans = $d = 1.5 - 2x_1$

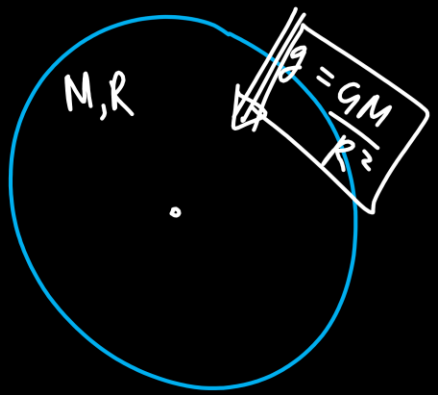
$= 1$

10

What will be the acceleration due to gravity on the surface of the moon if its radius is $1/4^{th}$ the radius of the earth and its mass is $1/80^{th}$ the mass of the earth.

(A) g/6 (B) g/5

(C) g/7 (D) g/8



$M_1 = \frac{M}{80}$

$R_1 = \frac{R}{4}$

$g_1 = \frac{GM_1}{R_1^2} = \frac{G\left(\frac{M}{80}\right)}{\left(\frac{R}{4}\right)^2} = \frac{16}{80} \cdot \frac{GM}{R^2} = \frac{16}{80} \cdot g = g/5$

JEE 1

11

An α - particle (He^{2+}) and a proton (p) are accelerated from rest through the same potential difference. V

The ratio of de-Broglie wavelength acquired by above two particals will be.

(A) $1:\sqrt{2}$ (B) $1:2\sqrt{2}$

(C) $1:4\sqrt{2}$ (D) $1:8$

$(K.E.) = qV$

JEE 1

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(K.E.)}} = \frac{h}{\sqrt{2mqV}}$$

		mass	charge
α	(p p n n)	4m	2e
proton	(p)	m	e

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\frac{h}{\sqrt{2(4m)(2e)V}}}{\frac{h}{\sqrt{2(m)(e)V}}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

12

The resistance of a wire of ~~100 cm~~ length is 10Ω. Now, it is cut into 10 equal parts and all of them are twisted to form a single bundle. Its resistance is

(A) 1Ω (B) 0.5Ω

(C) 5Ω (D) 0.1Ω ✓

JEE 1

$\frac{1}{R_{eq}} = \left[\frac{1}{R} + \frac{1}{R} + \cdots\right]$

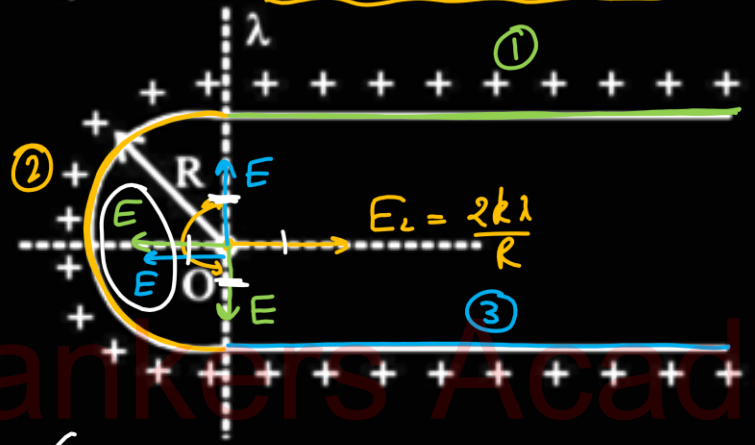
$R_{eq} = R/n$

$R_{eq} = \frac{1}{10} = 0.1\Omega$.

13

JEE 1

In the figure shown, if the linear charge density is λ, then the net electric field at O will be



(A) Zero

(B) $\frac{k\lambda}{R}$

(C) $\frac{2k\lambda}{R}$

(D) $\frac{\sqrt{2}k\lambda}{R}$

$$E_\perp = \frac{k\lambda}{r}(\sin\alpha + \sin\beta)$$

$$E = \frac{k\lambda}{R} \quad —①$$

$$E_\parallel = \frac{k\lambda}{r}[\cos\beta - \cos\alpha]$$

$$E = \frac{k\lambda}{R} \quad —②$$

14

JEE 1

Given below are two statements χ

Statement I: Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement II: Diamagnetic is a result of orbital motions of electrons developing magnetic moment opposite to the applied magnetic field.

Choose the correct answer from the options given below

(A) Both statements I and statement II are true

(B) Both statement I and statements II are false

(C) Statement I is true but statement II is false

(D) Statement I is false but statement II is true

$\chi_{pm} \propto \frac{1}{T}$ * Curie's Law

$\chi_{fem} \propto \frac{1}{(T-T_c)}$

15

Two concentric and coplanar coils have radii a and b(>> a) as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then the total charge circulating the inner coil is

(A) $\frac{\mu_0 \pi i a^2}{2Rb}$

(B) $\frac{\mu_0 iab}{2R}$

(C) $\frac{\mu_0 iab}{2a}\frac{\pi b^2}{R}$

(D) $\frac{\mu_0 ib}{2\pi R}$

$\frac{dq}{dt} = i = \frac{\mathcal{E}}{R} = \frac{-\left(\frac{d\phi}{dt}\right)}{R}$

$q = \frac{\int -d\phi}{R} = -\frac{\Delta\phi}{R}$ — (1)

$\phi = BA$

$= \left(\frac{\mu_0 i}{2b}\right) \times (\pi a^2)$ — (2)

$q = \frac{\mu_0 i \pi a^2}{2bR}$

Ans

16

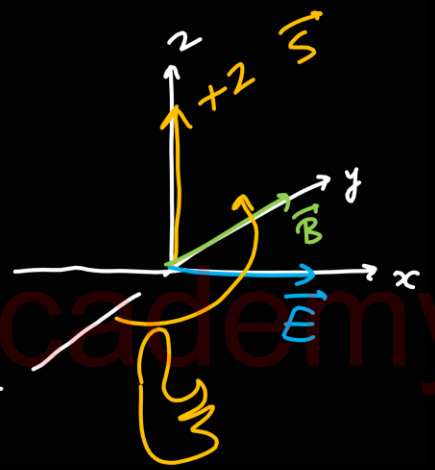
The electric and magnetic field, associated with an E.M wave propagating along the +z axis, can be represented by

(A) $[\vec{E} = E_0\hat{j}, \vec{B} = B_0\hat{k}]$

(B) $[\vec{E} = E_0\hat{i}, \vec{B} = B_0\hat{j}]$

(C) $[\vec{E} = E_0\hat{k}, \vec{B} = B_0\hat{i}]$

(D) $[\vec{E} = E_0\hat{j}, \vec{B} = B_0\hat{i}]$



$\vec{S} \parallel (\vec{E} \times \vec{B})$ *

$\hat{i} \times \hat{j} = \hat{k}$

Alternately

$\vec{E}\,(\hat{j} \times -\hat{i}) = \hat{k}$

17

A gate has the following truth table

P	1	1	0	0
Q	1	0	1	0
R	1	0	0	0

The gate is

(A) NOR

(B) OR

(C) NAND

(D) AND

18

The ratio of the density of oxygen nucleus $^{16}_{8}O$ and helium nucleus $^{4}_{2}He$ is

(A) 4: 1

(B) 8: 1

(C) 1: 1

(D) 2: 1

JEE 1

Nuclear Physics

$R = R_0 A^{1/3}$ *

$$\rho = \frac{m}{\text{Volume}} = \frac{(mA)}{\frac{4}{3}\pi(R_0 A^{1/3})^3} =$$

independent of A

it is same for All Nuclei

19

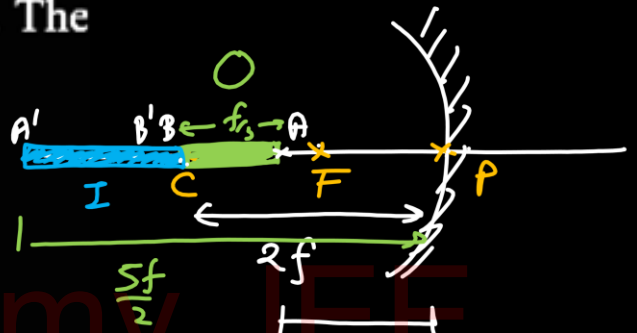
A thin rod of length f/3 lies along the axis of a concave mirror of focal length f. One end of its magnified image touches an end of the rod. The length of the image is

(A) f

(B) $\frac{1}{2}$f

(C) 2 f

(D) $\frac{1}{4}$f



for A'

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}+\frac{1}{\left(-\frac{5f}{3}\right)}=\frac{1}{-f}$$

$$\frac{1}{u}=\frac{1}{\frac{5f}{2}}-\frac{1}{f}$$

$$v=\ominus\frac{5f}{2}$$

→ same side as O

$$A'B'=\frac{5f}{2}-2f=\frac{f}{2}$$

Ans

20

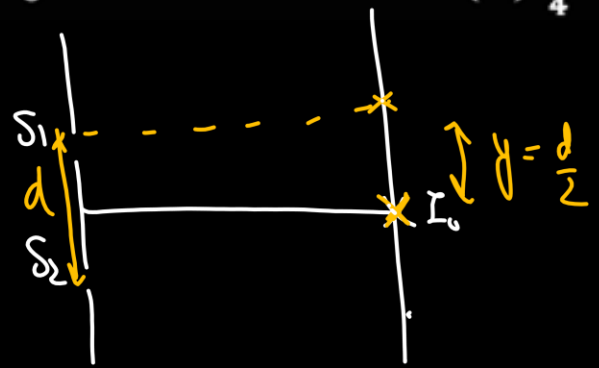
The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10\,d$

(A) $\frac{I_0}{2}$ ✓

(B) $\frac{3}{4}I_0$

(C) I_0

(D) $\frac{I_0}{4}$



JEE 1

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta\phi = \frac{2\pi \times \lambda}{\lambda 4} = \frac{\pi}{2} \quad \text{— (2)}$$

$$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$= I_0 \cos^2\left(\frac{\pi/2}{2}\right)$$

$$= I_0 \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{I_0}{2}$$ Ans

$$\Delta x = d \sin\theta$$

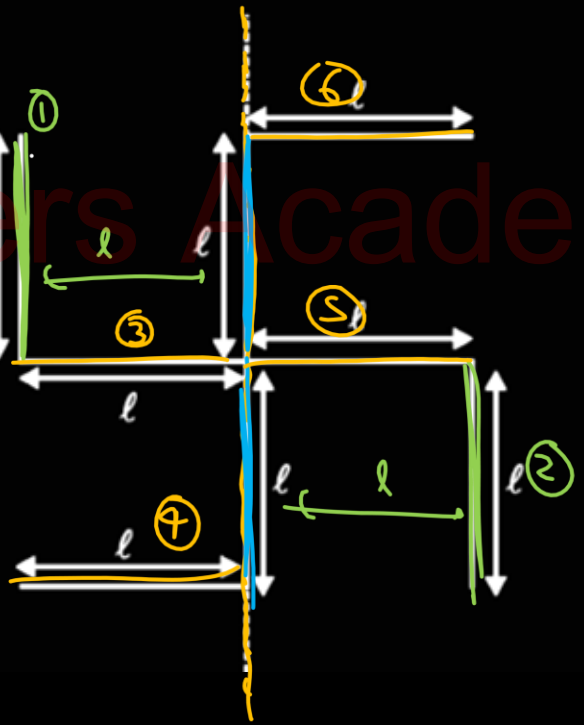
$$\Delta x = d\left(\frac{y}{D}\right)$$

$$= \frac{d\left(\frac{d}{2}\right)}{10\,d}$$

$$= \frac{d}{20} = \frac{5\lambda}{20} = \frac{\lambda}{4}$$ — (1)

21

In the shown figure the SWASTIK is made of uniform rods has linear mass density of λ and dimension are shown in figure. The moment of inertia about the dashed axis is $x\lambda\ell^3$, where value 3x is.



$$I = 2m\ell^2 + 4\left(\frac{m\ell^2}{3}\right)$$

$$= \frac{10}{3}(m\ell^2)$$

$$= \frac{10}{3}(2\ell)\ell^2$$

$$= \frac{10}{3}2\ell^3$$

$$3x = \left(\frac{10}{3}\right)3 = 10 \text{ Ans}$$

22

The difference between threshold wavelength for two metal surfaces A and B having work function $\phi_1 = 6eV$ and $\phi_2 = 4.5eV$ in nm is _____. (Given, hc = 1242 eV nm)

$\phi = \frac{hc}{\lambda_0} \Rightarrow \boxed{\lambda_0 = \frac{hc}{\phi}}$ *

$\Delta\lambda = \lambda_2 - \lambda_1$

$= hc\left[\frac{1}{\phi_2} - \frac{1}{\phi_1}\right]$

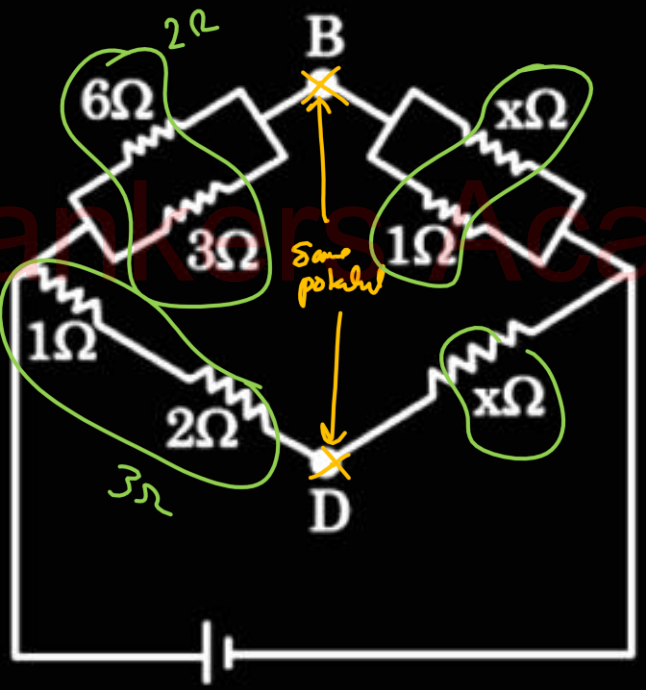
$= 1242\ eV\text{-}nm\left[\frac{1}{4.5eV} - \frac{1}{6eV}\right]$

$= \frac{1242(4-3)}{18}\ nm = 69\ nm$

23

If the potential difference between B and D is zero and the value of X is $\frac{1}{n}\Omega$. The value of n is

____.



Balanced wheatstone Bridge

$$\frac{2\Omega}{3\Omega} = \frac{x}{x+1}$$

$$x = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\frac{1}{n} = \frac{1}{2}$$

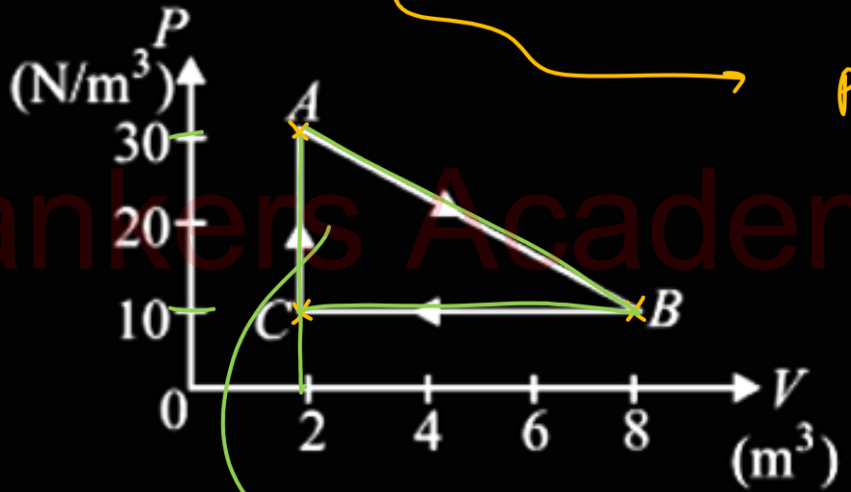
$\boxed{n=2}$ Ans

JEE 1

24

JEE 1

A gas undergoes the cyclic process shown in figure. The cycle is repeated 100 times per minute. The power generated is _____ W.



$P = \frac{W}{\text{time}} = \frac{100 \times 60}{60 \text{ sec}}$

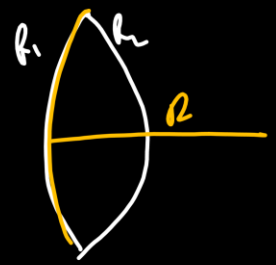
$= 100$ Watt.

$W = \frac{1}{2} \times \text{Base} \times \text{Ht}$

$= \frac{1}{2} \times (8-2)\,m^3 \times (30-10)\,\frac{N}{m^2} = 60$ J. ①

25

A double convex lens of focal length 6 cm is made of glass of refractive index 1.5. The radius of curvature of one surface is double that of other surface. The value of larger radius of curvature is ____ cm .



JEE 1

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \times \longrightarrow \frac{1}{6\text{cm}} = (1.5 - 1)\left[\frac{1}{R} - \frac{1}{-2R}\right]$$

$$\frac{1}{6} = \frac{1}{2}\left[\frac{3}{2R}\right]$$

$$R = \frac{9}{2}\text{cm} \Rightarrow 2R = 9\text{cm}$$

Ans

CHEMISTRY

7

JEE 1

Give the correct order of initials **T** (true) or **F** (false) for following statements.

(I) Top positions of L other-Mayer's atomic volume curve are occupied by Alkali metals. T

(II) Number of elements presents in the fifth period of the periodic table are 32. : 5S 4d 5P = 9 orbitals, 18 elements

(III) 2nd ionisation potential of Mg is less than the 2nd ionisation potential of Na.

Mg: $1s^2 2s^2 2p^6 3s^1$

Na^+: $1s^2 2s^2 2p^6$

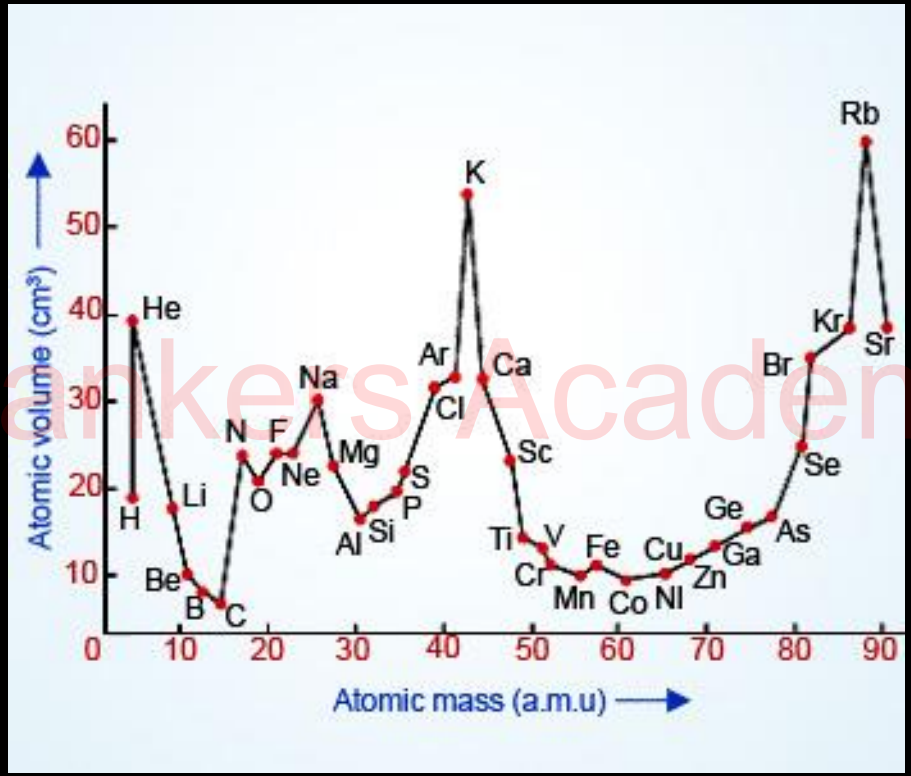
(IV) A p-orbital can take maximum of six electrons. 1↓

(A) TFTF

(B) TFFT

(C) FFTF

(D) TTFF



2

$TeF_5^-, XeF_2, I_3^+, XeF_4, PCl_3$

Which of the following shape does not describe to any of the above species?

(A) Square pyramidal (B) Square planar

(C) Trigonal planar (D) Linear

1. TeF_5^-
sp^3d^2 : Sq. Pyramidal

2. XeF_2 : Linear
sp^3d

3. I_3^+ : $\underline{I}I_2^{\oplus}$: Bent shape

4. XeF_4
sp^3d^2 : Sq. planer

5. $\underline{P}Cl_3$
sp^3 : Trigular Pyramidal

3

Calculate the volume of Cl_2 gas (in ml) liberated at 1 atm & 273 K when 1.74gm MnO_2 reacts with 2.19 gm HCl according to the following reaction with % yield 40 . (Volume of 1 mole at 1 atm, 273 K is 22.4 litre)

$$MnO_2 + HCl \longrightarrow MnCl_2 + Cl_2 + H_2O$$

Take [Mn = 55, Cl = 35.5, O = 16]

(A) 336 ml (B) 112 ml

(C) 134.4ml (D) 44.8 ml

E.R L.R

$MnO_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$

$= \frac{1.74g}{87} \quad \frac{2.19g}{36.5}$

$= 0.02 \quad 0.06$

If 4 moles of HCl gives 1 mol Cl_2

0.06 moles of HCl gives 0.015 mol of Cl_2

Since yield is 40%

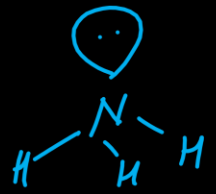
so 40% of 0.015 moles of Cl_2 = 0.006 mol

Volume of Cl_2 Produced = $0.006 \times 22.4 = 134.4$ ml

4

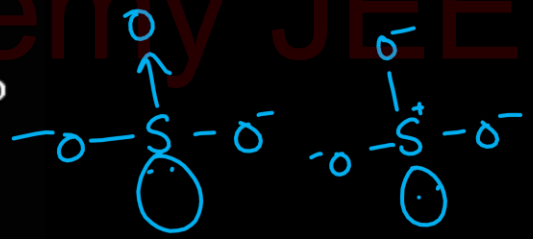
Choose the correct statement:

(A) NH_3 is having bond angle of 109°28′.



(B) The direction of the dipole moment of NF_3 is as shown in the diagram

(C) In the Lewis structure of SO_3^{2-}, there is no π-bond.



(D) sp^2 hybrid orbital is consisting of 33.33% 'p' character.

33.3% s-character

66.6% p-character

5

JEE 1

An electron is moving in 3rd orbit of Li^{+2} and its separation energy is y. The separation energy of an electron moving in 2nd orbit of He^{+} is.

(A) 4y/9 (B) y/9

(C) −y/9 (D) y

$E_n = -13.6 \frac{Z^2}{n^2}$

Separation Energy $= -\left(-13.6 \frac{Z^2}{n^2}\right) = 13.6 \frac{Z^2}{n^2}$

For Li^{+3} : $E = 13.6 \frac{3^2}{3^2} = 13.6\,eV = y$

For He^{+} : $E = 13.6 \frac{2^2}{2^2} = 13.6\,eV = y$

6

Give the correct order of initials True (T) or False (F) for following statements.

(I) Number of electrons having '$l = 0$' is 10 in Pd .

(II) The value of Z_{eff} for 3d electron of Cr & 3d electron of Mn is same as number of electron in 'd' subshell of Cr & Mn are same.

(III) Multiplicity of Fe is equal to Ni^{+2}.

(IV) Value of $\left(\frac{l}{n}\right)$ for last electron of element having atomic number 57 is 0.4

(A) T T T T (B) F T T T

(C) T F T F (D) F F F T

I : Pd (46)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^0\ 3d^{10}$

only 8 e^- with $l = 0$

s-subshell

$Z_{eff} = Z - \sigma$

IV La (57)

$5d^1\ 6s^2$

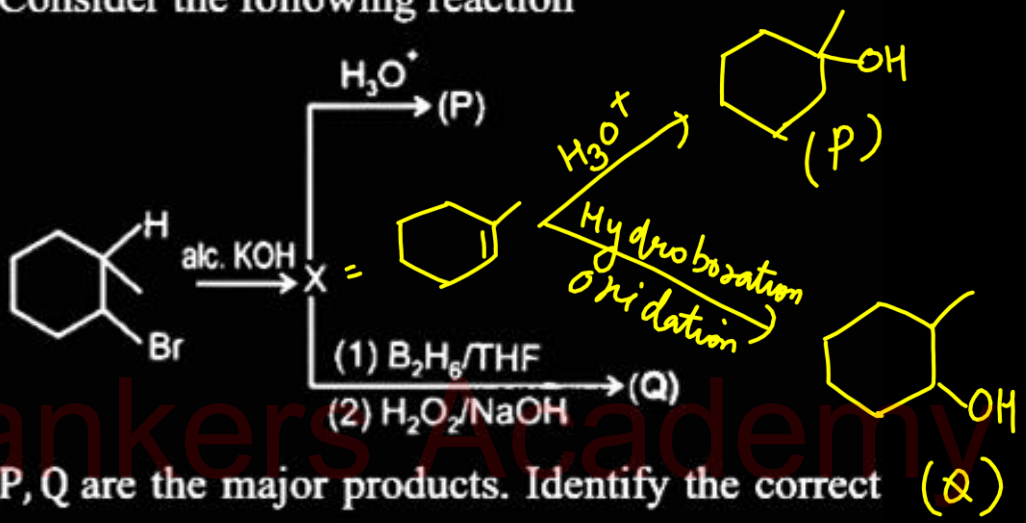
$n = 5 \quad l = 2$

$\frac{l}{n} = \frac{2}{5} = 0.4$

7

JEE 1

Consider the following reaction



P, Q are the major products. Identify the correct statement.

(A) P and Q are identical

(B) P and Q are positional isomers

(C) P and Q are functional isomers

(D) P and Q are metamers

8

JEE 1

For the reversible system,

$X(g) \rightleftharpoons Y(g) + Z(g)$

a certain quantity of X was heated at pressure P at a certain temperature.

The equilibrium partial pressure of X was found to be $\frac{P}{7}$. What is the value of K_p at given temperature

(A) $\frac{6P}{7}$

(B) $\frac{9P}{7}$

(C) $\frac{36P}{7}$

(D) 6 P

$X \rightleftharpoons Y + Z$

$t=0$ P 0 0

$t=eq$ $P-\frac{6P}{7}$ $\frac{6P}{7}$ $\frac{6P}{7}$

$K_p = \frac{P_Y \; P_Z}{P_X}$

$= \frac{\frac{6P}{7} \times \frac{6P}{7}}{\frac{P}{7}} = \frac{36P}{7}$

9

JEE 1

Calculate $[SO_4^{2-}]$ ions, in a solution obtained by mixing 100ml, 0.2M $BaCl_2$ and 100 ml 0.6 M Na_2SO_4. $[K_{sp}(BaSO_4) = 10^{-10}]$

(A) 10^{-9}M

(B) 0.3 M

(C) 5×10^{-10}M

(D) 0.2 M

$BaCl_2 + Na_2SO_4 \rightarrow BaSO_4 + 2NaCl$

20 mmol	60 mmol	0	0
0	40 mmol	20 mmol	40 mmol

SO_4^{2-} ions contributers in solution

$Na_2SO_4 = \frac{40 \text{ mmol}}{200 \text{ ml}} = 0.2 \text{ M}$

$BaSO_4 = s$

Since $BaSO_4$ K_{sp} is very less it contributes negligible SO_4^{2-}

Total $SO_4^{2-} = 0.2 + s^{\to 0}$

$= 0.2$M

10

JEE 1

Which of the following is/are incorrect regarding nitrogen family.

(A) Nitrogen is restricted to a maximum covalency of 4 as only four orbitals are available for bonding. ✓

$1s^2 2s^2 2p^3$

(B) The single N – N bond is weaker than the single P – P bond. ✓ due to small size, l.p-l.p repulsion will be more.

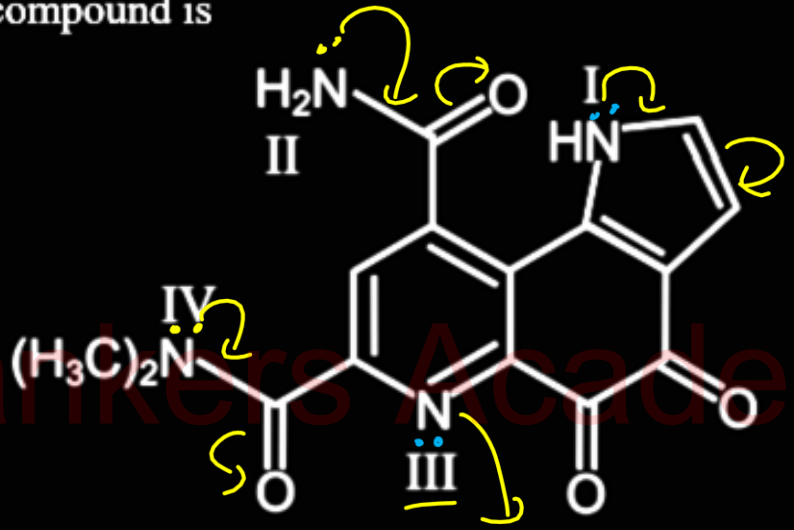
(C) The catenation tendency is weaker in nitrogen as compared to phosphorous. ✓

(D) Nitrogen forms pπ – pπ bond as well as pπ – dπ bonds. no vacant d so no pπ-dπ

11

JEE 1

The most basic nitrogen in the following compound is



(A) I

(B) II

(C) III

(D) IV

l.p is not participating in resonance
So maximum availability

12

JEE 1

When $K_4[Fe(CN)_6]$ is added to $FeCl_3$, the complex compound formed is :

(A) $Fe_3[Fe(CN)_6]_4$

(B) $Fe_4[Fe(CN)_6]_3$

(C) $K_2Fe[Fe(CN)_6]$

(D) $K_2Fe_3[Fe(CN)_6]_2$

$[Fe(CN)_6]^{4-} + Fe^{3+} \longrightarrow Fe_4[Fe(CN)_6]_3$

Group-3 (Al^{3+}, Fe^{3+}, Cr^{+3})

आलस फेक कर

JEE 1

Grp Reagent : NH_4OH in presence of NH_4Cl

Sequence
1. NaOH
2. H^+ (जो भी है intensity बढ़ेगी)

Mixture + NH_4Cl + NH_4OH

↓

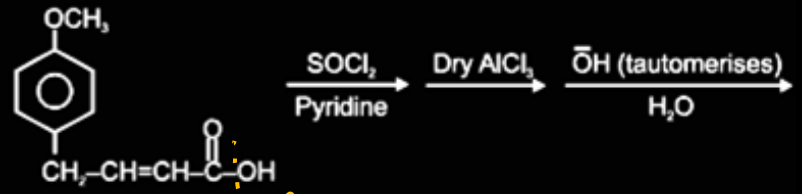
PPt

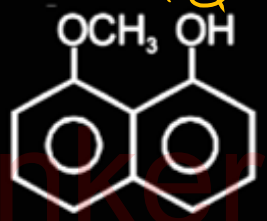
- $AlCl_3$ (white) ← HCl ← gelatinous White
- Red
- gelatinous Green

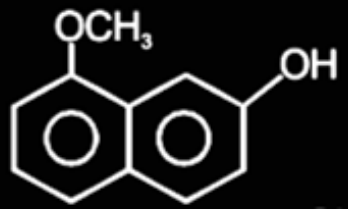
- $Al(OH)_3$ ↓ (Amphoteric) + NaOH → $NaAlO_2$ (Soluble) Clear solution Sod. Aluminate
- $NaAlO_2$ + NH_4Cl → Boil → $Al(OH)_3$ ↓ White ppt
- $Fe(OH)_3$ ↓ (Basic) + $K_4[Fe(CN)_6]$ → $Fe_4[Fe(CN)_6]_3$ (Ferric ferrocyanide) (Prussian blue)
- $Fe(OH)_3$ + NaOH → Base-Base do not react
- $Fe(OH)_3$ + SCN^- → $Fe(SCN)_3$ (Ferric thiocyanate) (Blood Red)
- $Cr(OH)_3$ ↓ (amphoteric) + NaOH → CrO_4^{2-} (yellow) ⇌ (H^+ / OH^-) $Cr_2O_7^{2-}$ (orange)
- $Cr(OH)_3$ + H^+ → $[Cr(H_2O)_6]^{3+}$
- CrO_2Cl_2 (Red-Brown gas) ← KCl

13

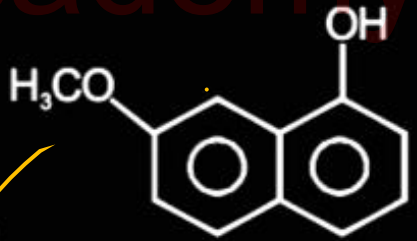
The end product of the following reaction sequence is

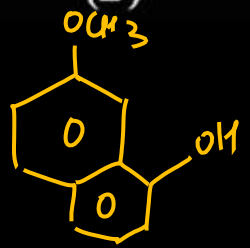


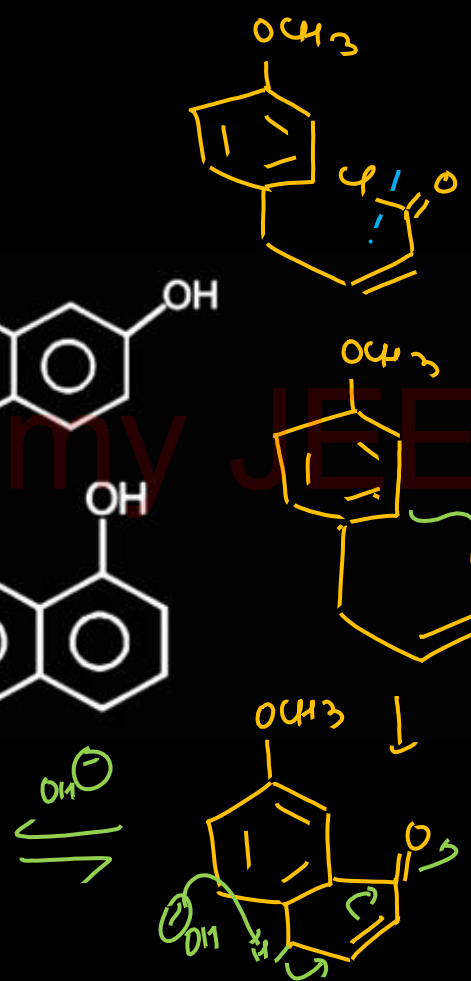
(A) 

(B) 

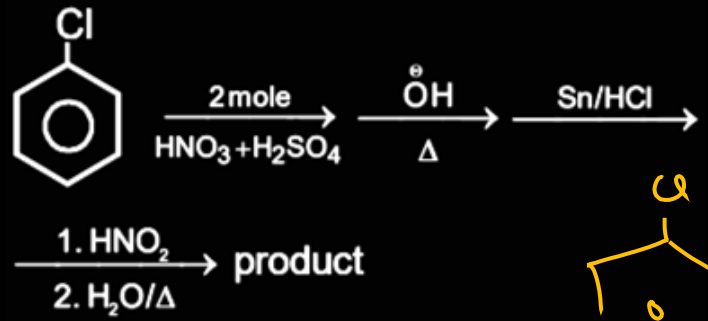
(C) 

(D) 

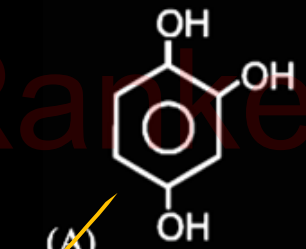




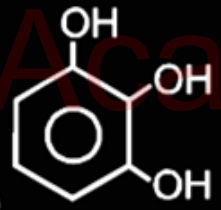
14



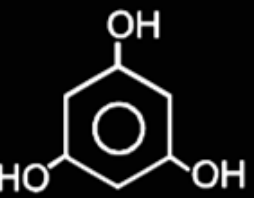
Final product is



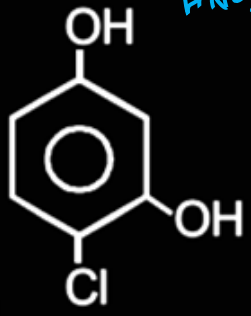
(A)



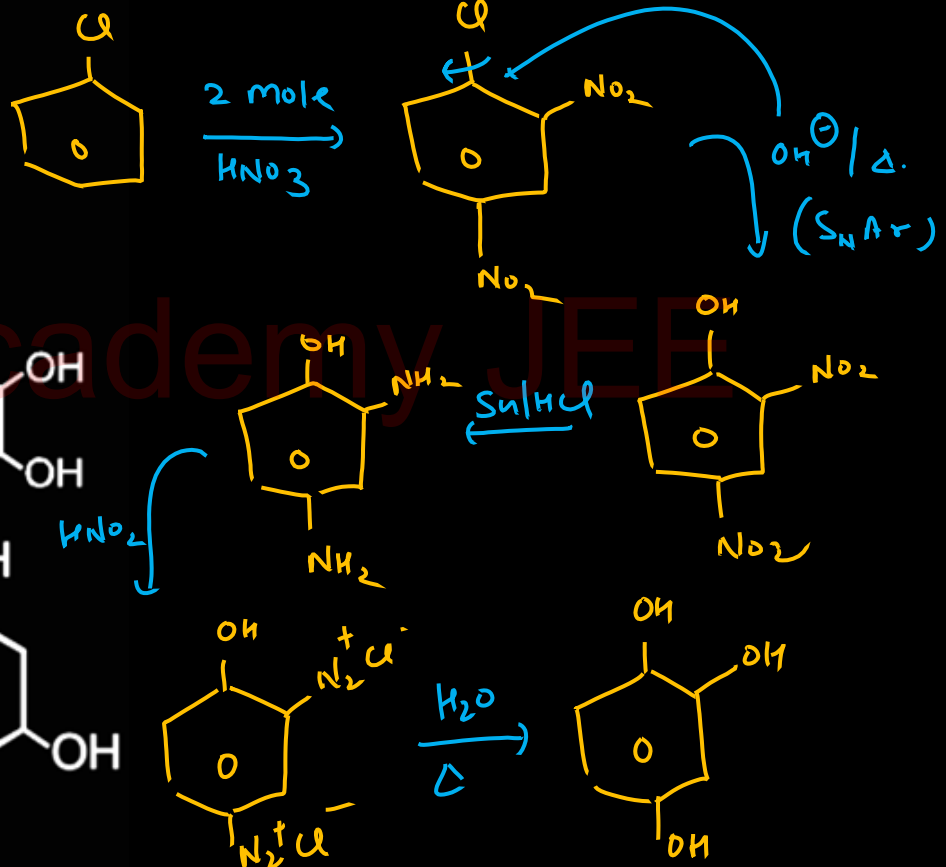
(B)



(C)

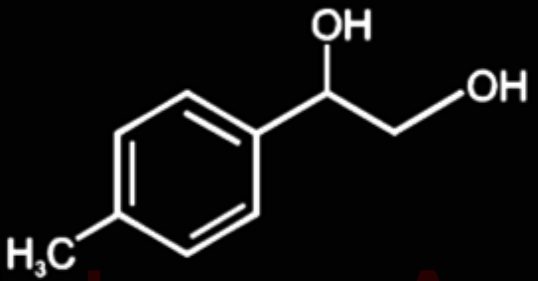


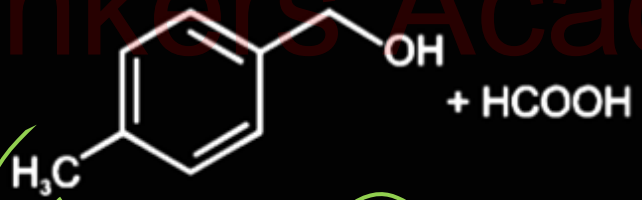
(D)

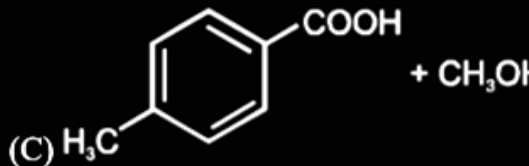


15

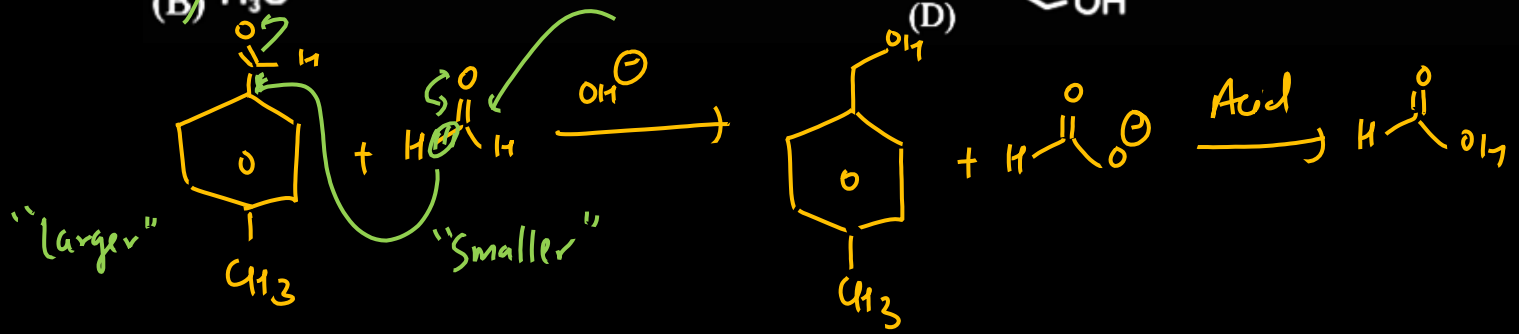
The reaction of 50% aq KOH on an equimolar mixture of 4-methylbenzaldehyde and formaldehyde followed by acidification gives –

(A) 

(B) 

(C) 

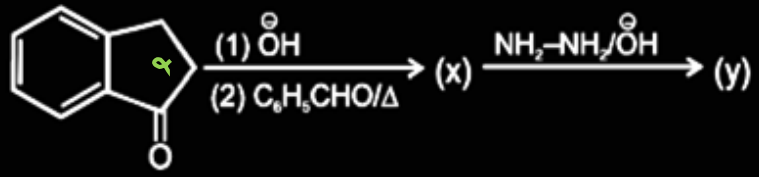
(D) 



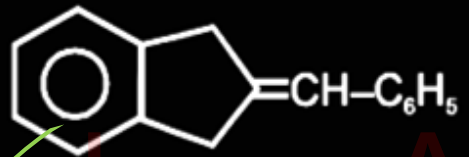
16

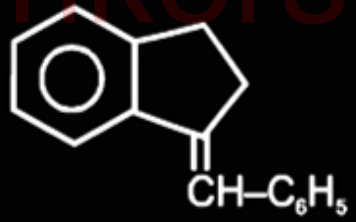
JEE 1

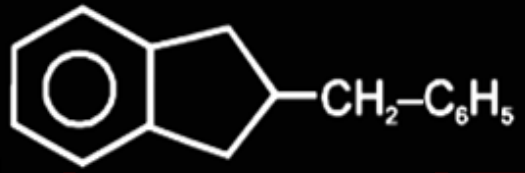
In the given reaction sequence

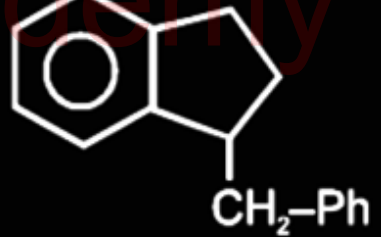


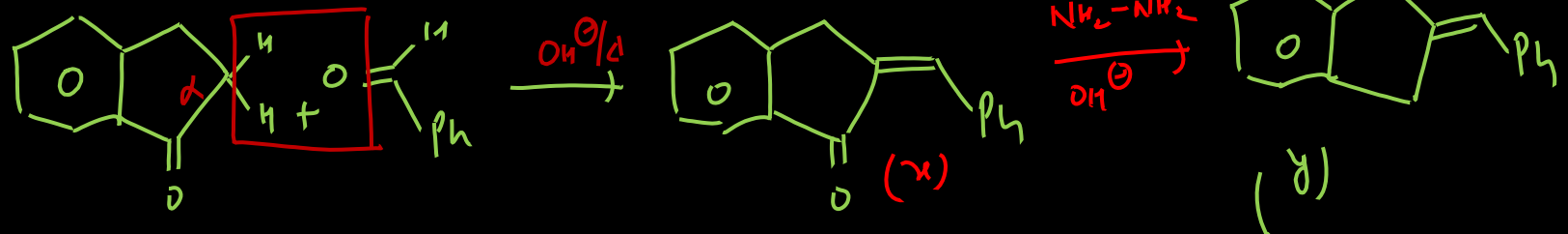
Product (y) will be :

(A) 

(B) 

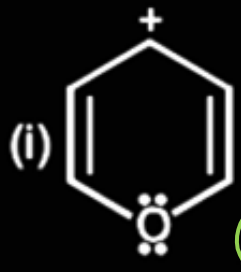
(C) 

(D) 



17

Which of the following compound(s) is/are an aromatic compounds?

(i) 

$6\pi e^-$

(A)

(ii) 

Non aromatic

Non Planar

sp^3

(iii) 

Peripheral Resonance

$14\pi e^-$ "Aromatic"

(iv) 

$10\pi e^-$

Aromatic

(A) I, II and III

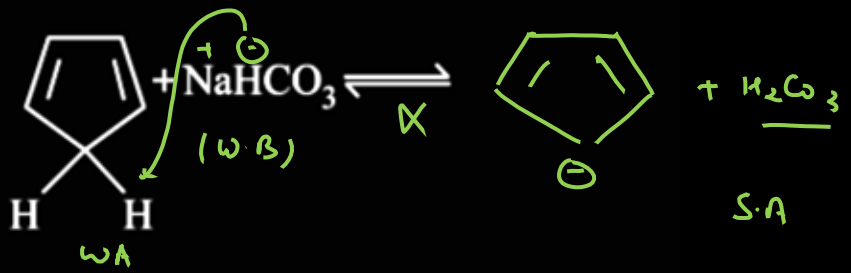
(B) III and IV only

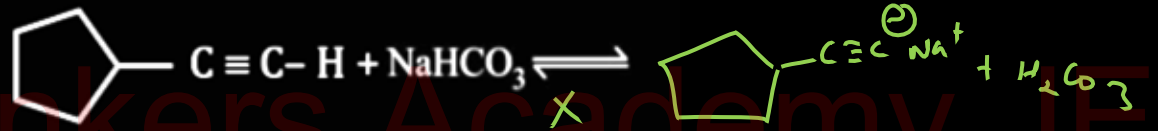
(C) IV only

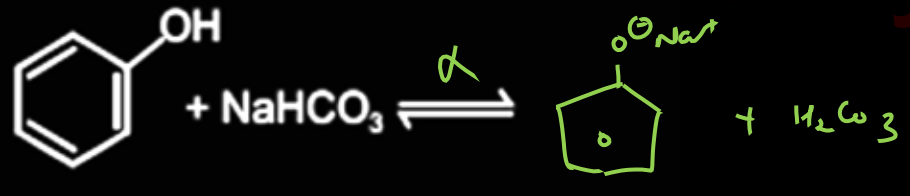
(D) I, III and IV

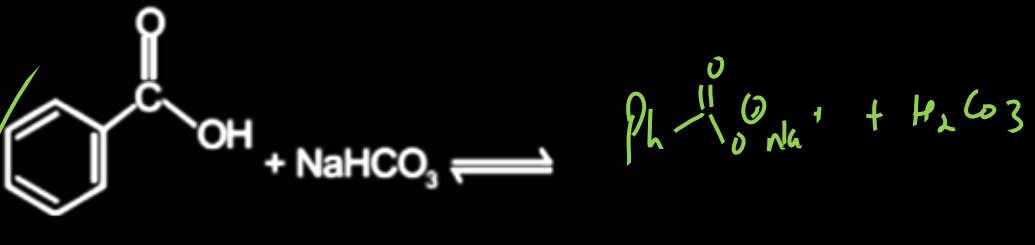
18

In which of the following reactions equilibrium will shift toward right?

(A) 

(B) 

(C) 

(D) 

19

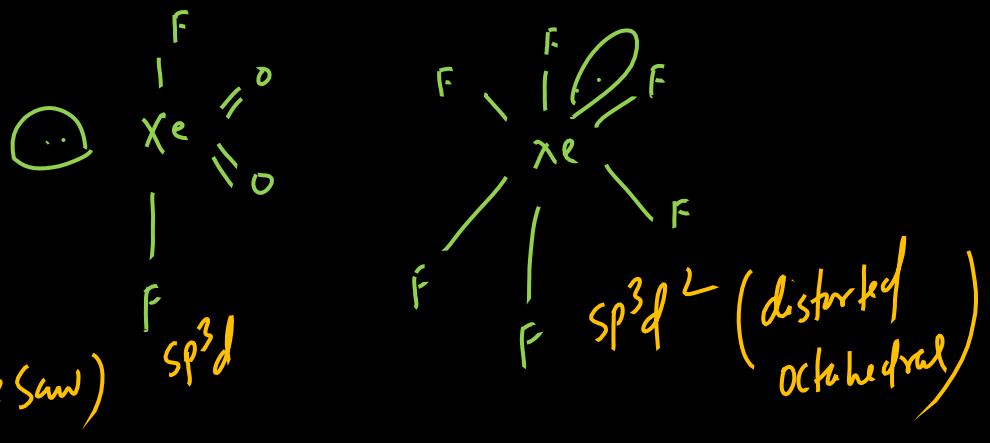
In case of XeO_2F_2 and XeF_6, Xe is with

(A) Same hybridization but with different geometry

(B) Different hybridization with same geometry

(C) Different hybridization and different geometry

(D) Same geometry and same hybridization



20

P(g) dissociates in a closed container into Q(g) according to the following reaction

P(g) → 3Q(g) If order of reaction is one and after 23 minute of starting of reaction total pressure of all gases is 2.6 times of initial pressure. The approximate value of rate constant is. (Assume temperature remains constant in the reaction and initially only P(g) is present in the container) [Use log5 = 0.698]

(A) 0.0016 minute^{-1} (B) 0.002 minute^{-1}

(C) 0.07 minute^{-1} (D) 0.008 minute^{-1}

$P(g) \rightleftharpoons 3Q(g)$

$t=0$	P_0	–
$t=23$	(P_0-x)	$3x$

$(P_0-x)+3x = 2.6P_0$

$P_0+2x = 2.6P_0$

$x = 0.8P_0$

$K = \frac{1}{t} \times 2.303 \log \frac{P_0}{P_0-x}$

$= \frac{1}{23} \times 2.303 \log \frac{P_0}{P_0-0.8P_0}$ → 5

21

A list of species having the formula XZ_4 is given below.

$XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, [Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

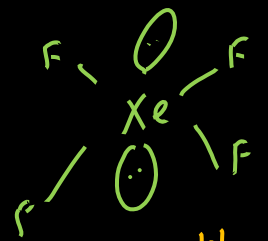
JEE 1

4d & 5d series

Zero, always back pair.

sp3

XeF_4



Square planar

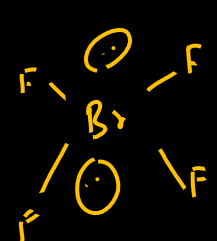
SF_4

See Saw

SiF_4

Tetrahedral

BrF_4^-

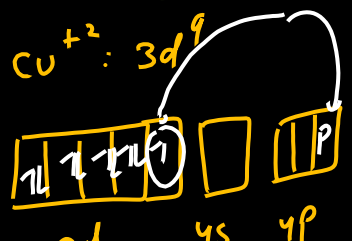


Square planar

$[Cu(NH_3)_4]^{+2}$

4

$Cu^{+2}: 3d^9$



dsp^2

22

Elevation in boiling point of an aqueous glucose solution is 0.6 . The value of K_b for water is 0.52 $Kmolal^{-1}$. The mole fraction of glucose in the solution is x. find 100x.

$\Delta T_b = i K_b m$

$0.6 = 1 \times 0.52\, m$

molality $= \frac{60}{59} = \frac{15}{13}$

$(15/13)$ mole Solute in 1000g H_2O.

$\chi_{glucose} = \frac{n_{glucose}}{n_{glucose} + n_{H_2O}}$

$= \frac{15/13}{15/13 + 1000/18}$

$= 0.0201$

$\therefore$ $100x = 2$

23

JEE 1

For the following electrochemical cell at 298 k ,

$Pt(s)|H_2(g, 1bar)|H^+(aq, 1M)||M^{+3}(aq), M^{+1}(aq)|Pt(s)$

$E_{cell} = 0.086\ V$ when $\frac{[M^{+1}(aq)]}{[M^{+3}(aq)]} = 10^x$

Given $E^\circ_{M^{+3}/M^{+1}} = 0.145\ V$,

$2.303\frac{RT}{F} = 0.059\ V$

The value of x is ?

$0.086 = 0.145 - \frac{0.059}{2} \times x$

$\frac{0.059}{2} \times x = 0.059$

$x = 2$

$H_2(g) + M^{+3} \longrightarrow 2H^+ + M^{+1}$

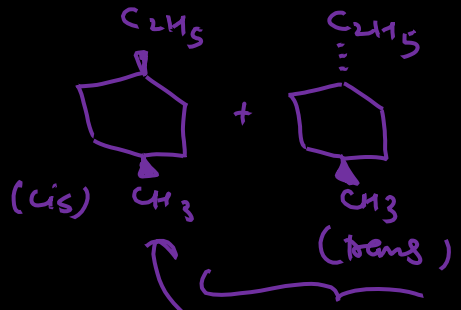
$E_{cell} = \left[E^\circ_{M^{+3}/M^{+1}} - E^\circ_{H^+/H_2}\right] - \frac{0.059}{2} \log \frac{[M^{+1}]}{[M^{+3}]} \frac{[H^+]^2}{P_{H_2}}$; $1M$; $1M$; 10^x

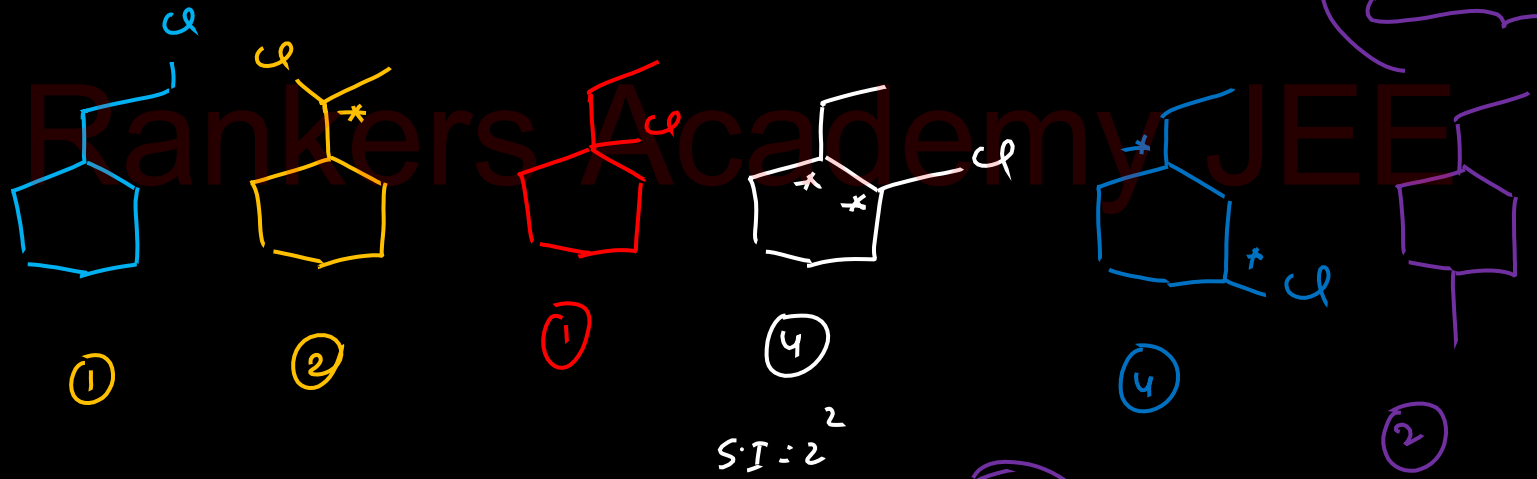
$0.086 = 0.145 - \frac{0.059}{2} \log 10^x$

24

(include stereoisomer)

JEE 1





Ans. 14

25

JEE 1

Calculate enthalpy change (in kcal) in adiabatic compression of 5 moles of ideal triatomic non-linear gas against a constant external pressure of 2 atm starting from initial pressure of 1 atm and initial temperature of 300 K . (Given : R = 2cal/mol. K and vibrational degrees of freedom are inactive).

$\gamma = 4/3$

$\frac{C_{p,m}}{C_{v,m}} = 4/3$

$C_{p,m} - C_{v,m} = R$

$C_{v,m} = 3R$

$C_{p,m} = 4R$

$\Delta U = q + w$ ($q = 0$)

$n C_{v,m} (T_2 - T_1) = -P_{ext}(V_2 - V_1)$

$n(3R)(T_2 - 300) = -2\left(\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1}\right)$

$q = 0$, $P_{ext} = 2$ atm $P_1 = 1$ atm

$T_1 = 300$ k

$3(T_2 - 300) = -2\left[\frac{T_2}{2} - \frac{300}{1}\right]$

$\Delta H = n C_{p,m} \Delta T$

$= 5(4R)(T_2 - T_1)$

$\Delta H = 20 \times 2 (T_2 - 300)$

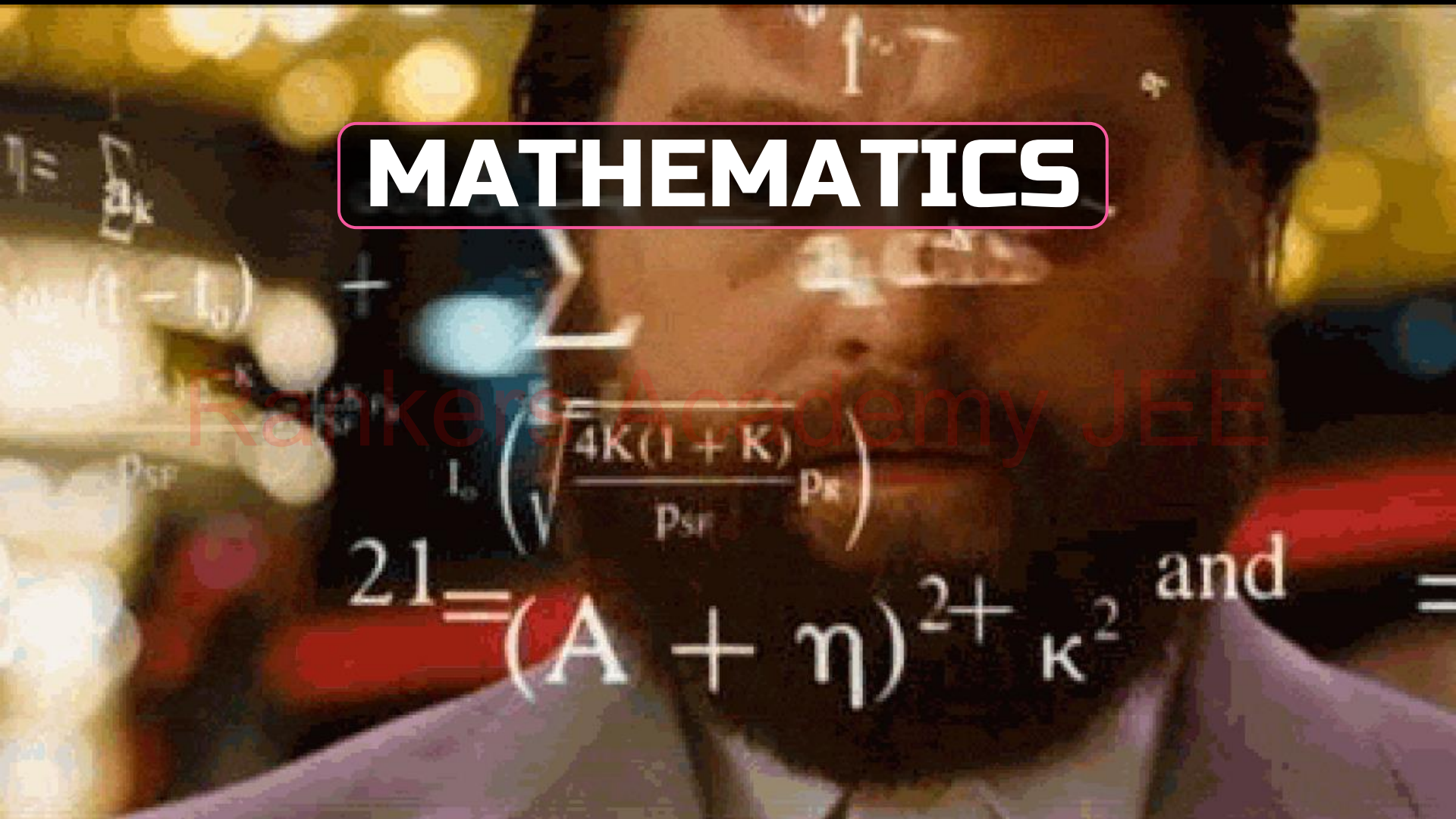
$= 40(T_2 - 300)$ cal $= 40(375 - 300)$

$= 40 \times 75 = 3000$ cal $\Rightarrow$ 3 kcal

$T_2 = 375$ k

$3T_2 - 900 = -T_2 + 600$

$T_2 = \frac{1500}{4} = 375$ k


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^{2+} \kappa^2$ and

If number of words which can be formed using all the letters from the word "EDUCATION" so that vowels are always in alphabetical order and A, N and E occupy 2^{nd}, 3^{rd} and 4^{th} positions only is 6λ, then value of λ is

(A) 30 (B) 5

(C) 6 (D) 10

U I O

D C T

_ A N E _ I O _ U

$\left(^{5}C_{3}\right)(3!) = (10)(6)$

2

$A = \{\alpha, \beta, \gamma, \eta\}$ then number of symmetric relations that can be defined on set A .

(A) 512 (B) 4

(C) 16 (D) 1024

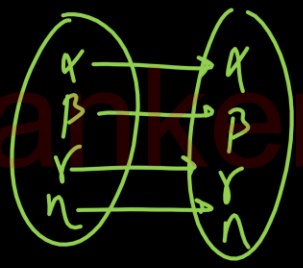
JEE 1

Formula:

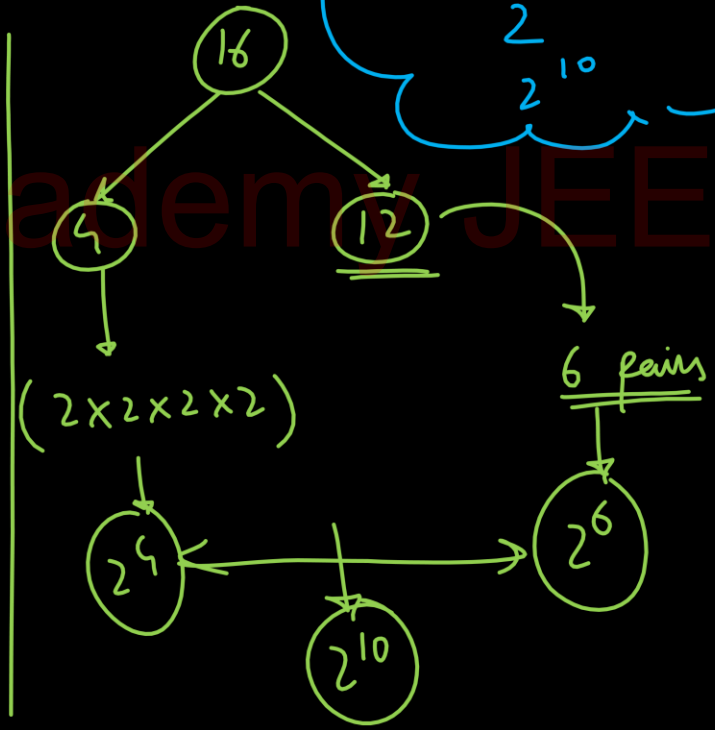
$2^{n(n+1)/2}$

$2^{4(5)/2}$

2^{10}



Total ordered pairs = 16



3

JEE 1

Consider the system of equations

$$\alpha x - y + z = \alpha$$
$$x - \alpha y + z = 1$$
$$x - y + \alpha z = 1$$

$$\Delta = \begin{vmatrix} \alpha & -1 & 1 \\ 1 & -\alpha & 1 \\ 1 & -1 & \alpha \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} (\alpha+2) & -(\alpha+2) & (\alpha+2) \\ 1 & -\alpha & 1 \\ 1 & -1 & \alpha \end{vmatrix}$$

If L, M and N denotes the number of integral values of α in interval $[-10,10]$ for which the system of the equations has unique solution, no solution and infinite solutions respectively, then the value of $(L - M + N)$ is _____ .

(A) 20 (B) 21

(C) 19 (D) 22

$$(\alpha+2)\begin{vmatrix} 1 & -1 & 1 \\ 1 & -\alpha & 1 \\ 1 & -1 & \alpha \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$(\alpha+2)\begin{vmatrix} 1 & -1 & 1 \\ 0 & -\alpha+1 & 0 \\ 0 & 0 & (\alpha-1) \end{vmatrix}$$

$$\Delta = (\alpha+2)\left(-(\alpha-1)^2\right)$$

unique

$\Delta \neq 0$

$\alpha \neq -2, 1$

$L = 19$

$\Delta = 0$

$\alpha = -2, 1$

$$\Delta_x = \begin{vmatrix} \alpha & -1 & 1 \\ 1 & -\alpha & 1 \\ 1 & -1 & \alpha \end{vmatrix} = -\Delta = 0$$

$$\Delta_y = \begin{vmatrix} \alpha & \alpha & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\Delta_z = \begin{vmatrix} \alpha & -1 & \alpha \\ 1 & -\alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \infty$ soln for both $\alpha = -2, 1$

$M = 0$ & $N = 2$

4

In an increasing geometric progression the sum of the first and the last term is 99 the product of the second and the last but one term is 288 and the sum of all the terms is 189. Then the number of terms in the progression is equal to

(A) 5 (B) 6

(C) 7 (D) 8

$a + ar^{n-1} = 99$

$(ar)(ar^{n-2}) = 288$

$(a)(ar^{n-1}) = 288$

$\begin{cases} a + l = 99 \\ al = 288 \end{cases}$

$a + \frac{288}{a} = 99$

$a^2 - 99a + 288 = 0$

$a^2 - 96a - 3a + 288 = 0$

$(a-96)(a-3) = 0$

$a = 96 \Rightarrow l = 3$ ✗

$a = 3 \Rightarrow l = 96$ ✓

$$\text{Sum} = \frac{a(1-r^n)}{(1-r)} = 189$$

$$\Rightarrow \frac{a - ar^n}{1-r} = 189 \quad (= l)$$

$$\Rightarrow \frac{a - (ar^{n-1})r}{1-r} = 189$$

$$\Rightarrow \frac{3 - (96)r}{1-r} = 189$$

$$3 - 96r = 189 - 189r$$

$$93r = 186$$

$$\boxed{r = 2} \checkmark$$

$\because l = ar^{n-1}$

$$96 = (3)(2)^{n-1}$$

$$32 = (2)^{n-1}$$

$$n-1 = 5 \Rightarrow \boxed{n = 6}$$

5

JEE 1

The coefficient of x^{20} in the expansion of

$(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ is

(A) ${}^{30}C_{10}$

(B) ${}^{30}C_{15}$

(C) ${}^{30}C_{25}$

(D) ${}^{30}C_{20}$

$(1+x^2)^{40}\left(\left(x+\frac{1}{x}\right)^2\right)^{-5}$

$(1+x^2)^{40}\left(\left(\frac{x^2+1}{x}\right)^{-10}\right)$

$(1+x^2)^{40}\,\frac{x^{10}}{(1+x^2)^{10}}$

$(1+x^2)^{30}\cdot x^{10}$

Coeff of x^{10}

$r=5$

$30C_5$

$(1+x^2)^{30}$

$T_{r+1} = {}^{30}C_r (1)^{30-r}(x^2)^r$

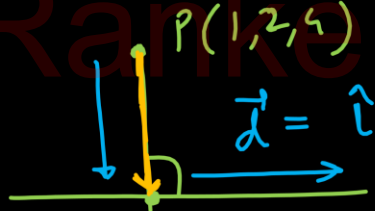
$30C_r\, x^{2r}$

6

The length of the perpendicular (in units) from the point (1,2,4) on the straight line $\frac{x-2}{1} = \frac{y-7}{2} = \frac{z-3}{-1} = \lambda$ lies in the interval

(A) $\left(1, \frac{3}{2}\right)$ (B) (2,3)

(C) (0,2] (D) [4,5)



$\vec{PM} = (\lambda+1)\hat{i} + (2\lambda+5)\hat{j} + (-\lambda-1)\hat{k}$

$\vec{PM} \cdot \vec{d} = 0$

$(\lambda+1) + (4\lambda+10) + (\lambda+1) = 0$

$6\lambda = -12$

$\boxed{\lambda = -2}$

$\Rightarrow \vec{PM} = -\hat{i} + \hat{j} + \hat{k}$

$\Rightarrow |\vec{PM}| = \sqrt{3}$

JEE 1

7

The domain of

$f(x) = \log_{10}\{1 - \log_{10}(x^2 - 5x + 16)\}$ is

(A) (2,3)

(B) $(0, \infty)$

(C) [1,3]

(D) [2,3]

JEE 1

$1 - \log_{10}(x^2-5x+16) > 0$

$\log_{10}(x^2-5x+16) < 1$

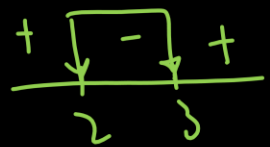
$0 < (x^2-5x+16) < (10)^1$

$x^2-5x+16 > 0$

always true ($\because D<0$)

$x^2-5x+6 < 0$

$(x-2)(x-3) < 0$



8

The standard deviation of n observation $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ is 2. If $\sum_{i=1}^{n} x_i = 20$ and $\sum_{i=1}^{n} x_i^2 = 100$, then n is :

(A) 10 or 20 (B) 5 or 10

(C) 5 or 20 (D) 5 or 15

$$\sigma^2 = 4 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

$$4 = \frac{100}{n} - \left(\frac{20}{n}\right)^2$$

$$4 = \frac{100}{n} - \frac{400}{n^2}$$

$$1 = \frac{25}{n} - \frac{100}{n^2}$$

$$n^2 = 25n - 100$$

$$n^2 - 25n + 100 = 0$$

$$(n-5)(n-20) = 0$$

9

The differential coefficient of $\sin^{-1}\frac{t}{\sqrt{1+t^2}}$ w.r.t. $\cos^{-1}\frac{1}{\sqrt{1+t^2}}$ is (given $t > 0$)

(A) 1

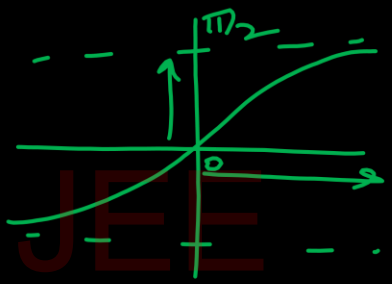
(B) -1

(C) 1/2

(D) None of these

$\theta = \tan^{-1} t \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

$t = \tan\theta$



$y = \sin^{-1}\frac{t}{\sqrt{1+t^2}}$

$y = \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)$

$= \sin^{-1}\sin\theta$

$= \theta$

$z = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

$z = \cos^{-1}\left(\frac{1}{\sec\theta}\right)$

$= \cos^{-1}\cos\theta = \theta$

10

JEE 1

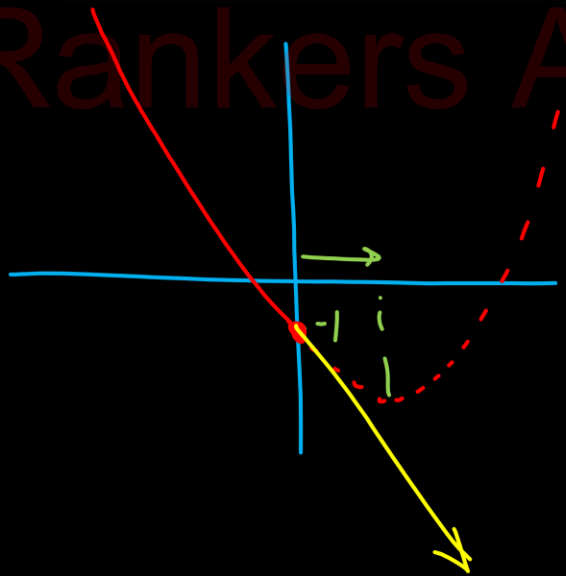
$f: R \to R$ is defined as

$$f(x) = \begin{cases} x^2 + 2mx - 1 & \text{for } x \le 0 \\ mx - 1 & \text{for } x > 0 \end{cases}$$. If $f(x)$ is one-one then m must lies in the interval

(A) $(-\infty, 0)$ (B) $(-\infty, 0]$

(C) $(0, \infty)$ (D) $[0, \infty)$

$V_n = \frac{-b}{2a}$



$\Rightarrow -\frac{2m}{2} > 0$

$m < 0$

If $f(x) = \begin{cases} x^2 \left|\cos \frac{\pi}{x}\right|, & x \neq 0, x \in R \\ 0, & x = 0 \end{cases}$, then ' f ' is

(A) differentiable both at $x = 0$ and $x = 2$

(B) differentiable at $x = 0$ but not differentiable at $x = 2$

(C) not differentiable at $x = 0$ but not differentiable at $x = 2$

(D) differentiable neither at $x = 0$ nor at $x = 2$

$x = 0$; $f(0) = 0$

$$RHD = \lim_{h \to 0} \frac{h^2 \left|\cos \frac{\pi}{h}\right| - 0}{h}$$

$$= 0$$

$$LHD = \lim_{h \to 0} \frac{h^2 \left|\cos\left(\frac{\pi}{-h}\right)\right| - 0}{-h}$$

$$= 0$$

$x = 2$

$f(2) = 0$

$$RHD = \lim_{h \to 0} \frac{(2+h)^2 \left|\cos\left(\frac{\pi}{2+h}\right)\right| - 0}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 \cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$= 4 \lim_{h \to 0} \frac{-\sin\left(\frac{\pi}{2+h}\right) \times \frac{-\pi}{(2+h)^2}}{1}$$

$$= \boxed{\pi}$$

$$LHD = \lim_{h \to 0} \frac{(2-h)^2 \left|\cos\left(\frac{\pi}{2-h}\right)\right| - 0}{(-h)}$$

$$4 \lim_{h \to 0} \frac{\left(+\cos\left(\frac{\pi}{2-h}\right)\right)}{(+h)}$$

$$4 \lim_{h \to 0} \frac{-\sin\left(\frac{\pi}{2-h}\right) \times \frac{-\pi}{(2-h)^2} \times (-1)}{1}$$

$$= \boxed{-\pi}$$

12

JEE 1

Let $A = [a_{ij}]_{3\times3}$ be a scalar matrix whose elements are the roots of the equation.

$x^9 - 6x^8 + 12x^7 - 8x^6 = 0$ and

$B = \text{diag}(b_{11}, b_{22}, b_{33})$ be a diagonal matrix where $b_{rr} = \sum_{j=1}^{3}\sum_{i=1}^{3} a_{ij}\ \forall r = 1,2,3.$

If $\frac{|\text{adj } B|}{|\text{adj (adj (adj } A))|} = \left(\frac{p}{q}\right)^6$ where p, q are relatively prime number, then $(q - 2p) =$

(A) 1 (B) 2

(C) 3 (D) 4

$x^6(x^3 - 6x^2 + 12x - 8) = 0$

$x^6(x-2)^3 = 0$

$\{0,0,0,0,0,0,2,2,2\}$

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$b_{rr} = 6 \Rightarrow B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \Rightarrow |B| = (6)^3$

$$= \frac{|\text{adj } B|}{|\text{adj adj adj } A|}$$

$$= \frac{|B|^2}{|A|^{2^3}}$$

$$= \frac{(6^3)^2}{(8)^8}$$

$$= \frac{6^6}{8^8}$$

$$= \frac{6^6}{2^{24}}$$

$$= \frac{6^6}{(2^4)^6}$$

$$= \left(\frac{6}{16}\right)^6$$

$$= \left(\frac{3}{8}\right)^6$$

$p = 3$

$q = 8$

$q - 2p = 8 - 6 = 2$

13

If $\lim_{x\to\infty}\left(\frac{x^3+x^2+x+1}{x+1}-ax^2-b\right)=4$ then

$|a|+|b|=$

(A) 1 (B) 3

(C) 4 (D) 7

JEE 1

$\lim_{x\to\infty}\left(\frac{(x+1)(x^2+1)}{(x+1)}-ax^2-b\right)=4$

$\lim_{x\to\infty}\left((1-a)x^2+(1-b)\right)=4$

$a=1$

$1-b=4$

$b=-3$

14

JEE 1

If $\sin^{-1}\left(\sin\frac{33\pi}{7}\right)+\cos^{-1}\left(\cos\frac{46\pi}{7}\right)+$

$\tan^{-1}\left(-\tan\frac{13\pi}{8}\right)+\cot^{-1}\left(\cot\left(\frac{-19\pi}{8}\right)\right)=\frac{a\pi}{b}$

then $a+b=$ _____

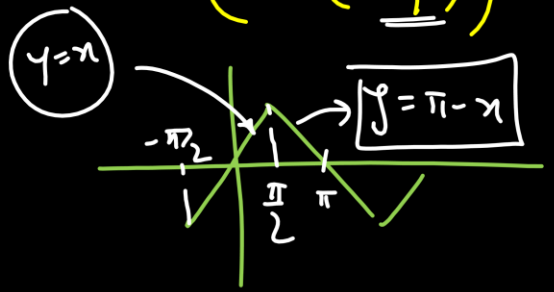
(A) 15 (B) 20

(C) 16 (D) 24

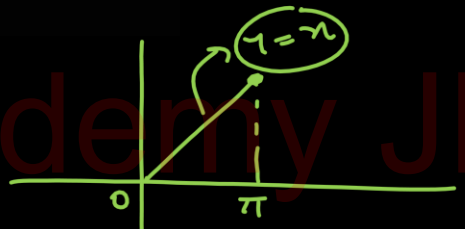
$\sin^{-1}\left(\sin\left(\frac{28\pi+5\pi}{7}\right)\right)$

$\sin^{-1}\left(\sin\left(\frac{5\pi}{7}\right)\right)=\pi-\frac{5\pi}{7}$

$=\frac{2\pi}{7}$



$\cos^{-1}\left(\cos\left(\frac{4\pi}{7}\right)\right)=\frac{4\pi}{7}$



$\tan^{-1}\left(-\tan\left(\frac{13\pi}{8}\right)\right)$

$\tan^{-1}\left(-\tan\left(2\pi-\frac{3\pi}{8}\right)\right)$

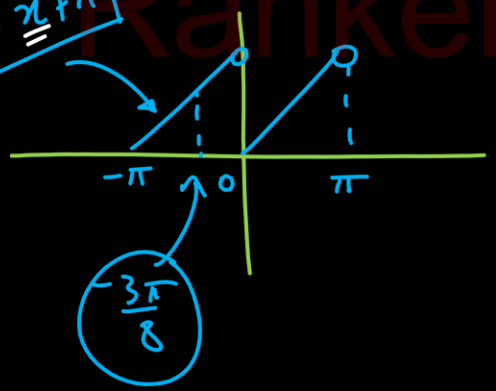
$\tan^{-1}\left(\tan 3\pi/8\right)=3\pi/8$

$\cot^{-1}\left(\cot\left(\frac{-19\pi}{8}\right)\right)$

$\cot^{-1}\left(\cot\left(-2\pi - \frac{3\pi}{8}\right)\right)$

$\cot^{-1}\left(\cot\left(\frac{-3\pi}{8}\right)\right) = \pi - \frac{3\pi}{8}$

$y = x + \pi$



Ans:

$\frac{2\pi}{7} + \frac{4\pi}{7} + \frac{3\pi}{8} + \pi - \frac{3\pi}{8}$

$\frac{6\pi}{7} + \pi$

$\frac{13\pi}{7}$

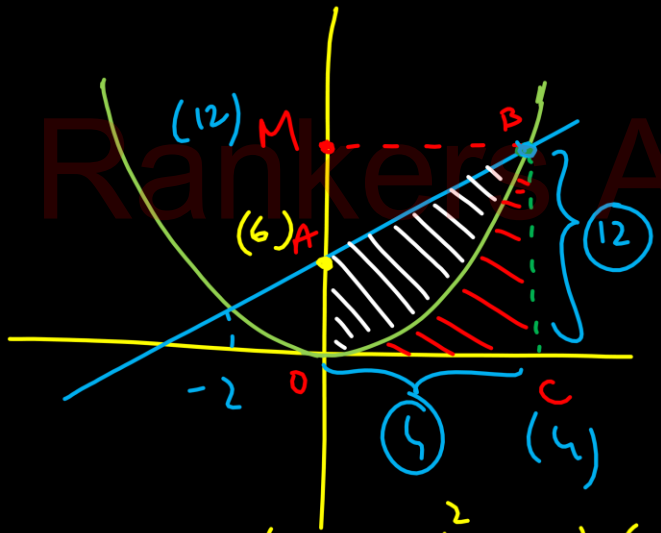
15

The area bounded by the parabola $4y = 3x^2$, the line $2y = 3x + 12$ and the y-axis is

(A) 10 sq. units (B) 20 sq. units

(C) 30 sq. units (D) 36 sq. units

JEE 1



$2(3x+12) = 3x^2$

$2(x+4) = x^2$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

$\text{Area} = \frac{1}{2}(6+12)(4) - \frac{1}{3}(4)(4^2) = 36 - 16 = 20$

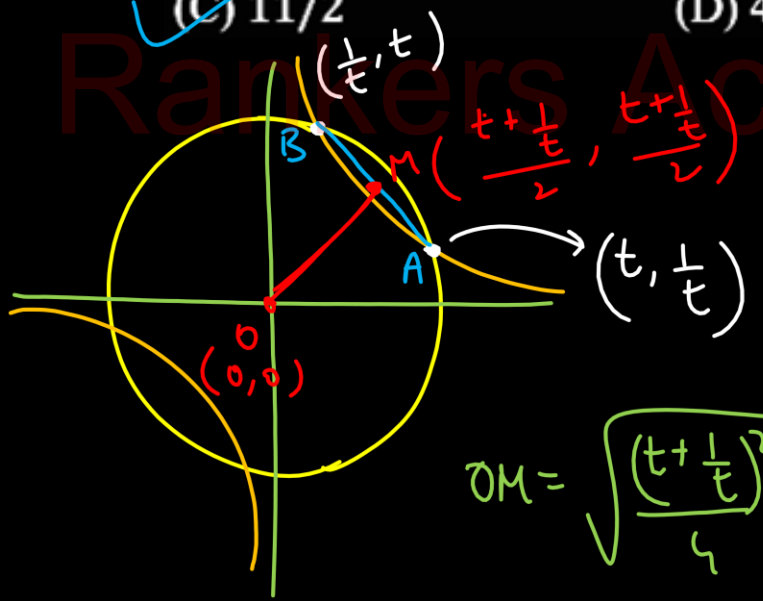
16

JEE 1

Let $x^2 + y^2 = r^2$ and $xy = 1$ intersect at A and B in the first quadrant. If $AB = \sqrt{14}$ units, then the square of the distance from the origin to mid point of AB is equal to

(A) 5 (B) 13/2

(C) 11/2 (D) 4



$$AB = \sqrt{\left(\frac{1}{t}-t\right)^2+\left(t-\frac{1}{t}\right)^2}$$

$$14 = \left(t-\frac{1}{t}\right)^2 \times 2$$

$$\left(t-\frac{1}{t}\right)^2 = 7$$

$$\left(t+\frac{1}{t}\right)^2 - 4 = 7$$

$$\left(t+\frac{1}{t}\right)^2 = 11$$

$$OM = \sqrt{\frac{\left(t+\frac{1}{t}\right)^2}{4}+\frac{\left(t+\frac{1}{t}\right)^2}{4}}$$

$$OM = \sqrt{\frac{11}{2}}$$

$$(OM)^2 = \frac{11}{2}$$

17

The projection of $2\hat{i} - 3\hat{j} + 4\hat{k}$ on the line whose equation is $\vec{r} = (3 + \lambda)\hat{i} + (3 - 2\lambda)\hat{j} + (5 + 6\lambda)\hat{k}$ where λ is a scalar parameter, is

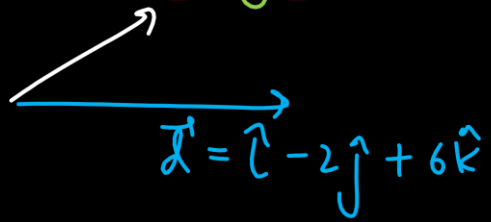
(A) $6\sqrt{41}$

(B) $\frac{32}{\sqrt{41}}$

(C) $\frac{16}{\sqrt{41}}$

(D) $\frac{7}{5}$

$\vec{p} = 2\hat{i} - 3\hat{j} + 4\hat{k}$



$\vec{d} = \hat{i} - 2\hat{j} + 6\hat{k}$

$$\Rightarrow \frac{\vec{p}\cdot\vec{d}}{|\vec{d}|} = \frac{2 + 6 + 24}{\sqrt{1 + 4 + 36}} = \frac{32}{\sqrt{41}}$$

18

The value of the integral

$I = \int_0^{\pi} \frac{x}{1+\tan^6 x} dx, x \neq \frac{\pi}{2}$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi^2}{4}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi^2}{2}$

JEE 1

$$I = \int_0^{\pi/2} \left(\frac{x}{1+\tan^6 x} + \frac{(\pi - x)}{1+\tan^6(\pi - x)} \right) dx$$

$$I = \pi \int_0^{\pi/2} \frac{1}{1+\tan^6 x} dx = \pi \int_0^{\pi/2} \frac{\cos^6 x}{\cos^6 x + \sin^6 x} dx = \pi \left(\frac{\pi}{4} \right)$$

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx = \frac{b-a}{2}$$

19

The solution of the differential equation $2\sqrt{x}e^{\sqrt{x}}dy + e^{\sqrt{x}}ydx = \sqrt{x}\sin x dx$ is (where, c is an arbitrary constant)

(A) $2ye^{\sqrt{x}} + \sin x = c$ (B) $y\sin x = e^{\sqrt{x}} + c$

(C) $ye^{\sqrt{x}} + \sin x = c$ (D) $2ye^{\sqrt{x}} + \cos x = c$

JEE 1

$$e^{\sqrt{x}}dy + \frac{e^{\sqrt{x}}}{2\sqrt{x}}y\,dx = \frac{\sin x}{2}dx$$

$$\int d(y \cdot e^{\sqrt{x}}) = \int \frac{\sin x}{2}dx$$

$$ye^{\sqrt{x}} = -\frac{1}{2}\cos x + C$$

20

For a complex number **Z**, if the argument of $3 + 3i$ and $(Z-2)(\bar{Z}-1)$ are equal, then the maximum distance of Z from the x -axis is equal to (where, $i^2 = -1$)

(A) $\left(\frac{1+\sqrt{2}}{2}\right)$ units

(B) 2 units

(C) $\frac{3}{2}$ units

(D) $\frac{(\sqrt{2}+2)}{2}$ units

$z = x+iy$

$\bar{z} = x-iy$

$\arg(3+3i) = \frac{\pi}{4}$

$\Rightarrow \arg(z\bar{z} - z - 2\bar{z} + 2) = \frac{\pi}{4}$

$\arg(x^2+y^2-x-iy-2x+2iy+2) = \frac{\pi}{4}$

$\arg((x^2+y^2-3x+2)+iy) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x^2+y^2-3x+2}\right) = \frac{\pi}{4}$

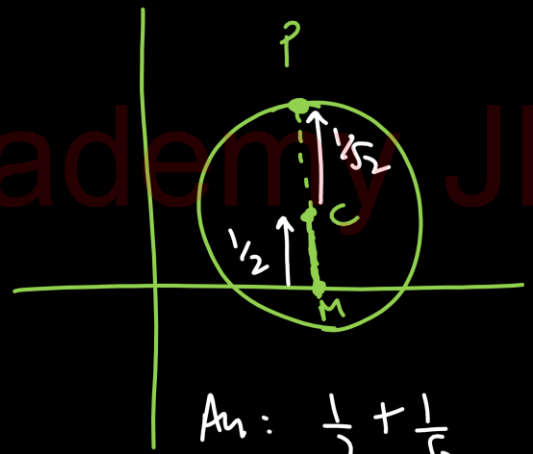
$\frac{y}{x^2+y^2-3x+2} = 1$

$$x^2 + y^2 - 3x - y + 2 = 0$$

$$C \equiv \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$r = \sqrt{\frac{9}{4} + \frac{1}{4} - 2}$$

$$= \frac{1}{\sqrt{2}}$$



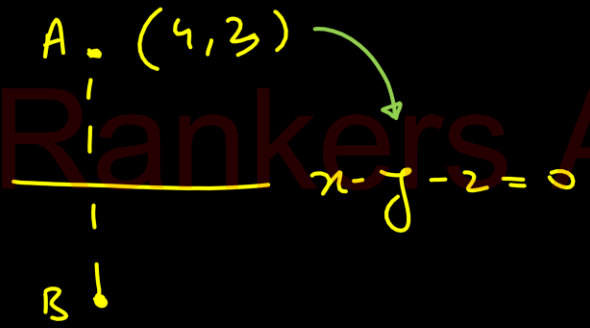
Ans: $\frac{1}{2} + \frac{1}{\sqrt{2}}$

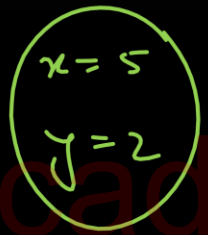
21

If the equation of reflection of the ellipse $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ with respect to the line $x-y-2=0$ is $16x^2+9y^2+k_1x-36y+k_2=0$ then $k_2-k_1=$

JEE 1

$L \equiv y = x-2$ & 45°





$$\frac{x-4}{1}=\frac{y-3}{-1}=-\frac{2(4-3-2)}{2}$$

$$=1$$

New ellipse:

$$\frac{(x-5)^2}{9}+\frac{(y-2)^2}{16}=1$$

$$16(x-5)^2+9(y-2)^2=144$$

$$16(x^2-10x+25)+9(y^2-4y+4)-144=0$$

$$16x^2+9y^2-160x-36y+400+36-144=0$$

$K_1=-160$

$K_2=436-144$

$K_2-K_1=436-144+160$

$=436+16$

$=452$

22

Let the value of $\frac{\cos 1^\circ+\cos 2^\circ+\cdots\ldots+\cos 44^\circ}{\sin 1^\circ+\sin 2^\circ+\cdots\ldots+\sin 44^\circ}$ be T, then the value of $(T-1)^2$ is

JEE 1

$$T=\frac{\cos(45^\circ-44^\circ)+\cos(45^\circ-43^\circ)+\cdots+\cos(45^\circ-1^\circ)}{\sin 1^\circ+\sin 2^\circ+\cdots+\sin 44^\circ}$$

$$T=\frac{\left(\frac{1}{\sqrt{2}}\cos 44^\circ+\frac{1}{\sqrt{2}}\sin 44^\circ\right)+\frac{1}{\sqrt{2}}\left(\cos 43^\circ+\sin 43^\circ\right)+\cdots+\frac{1}{\sqrt{2}}\left(\cos 1^\circ+\sin 1^\circ\right)}{\left(\sin 1^\circ+\sin 2^\circ+\cdots+\sin 44^\circ\right)}$$

$$T=\frac{1}{\sqrt{2}}\left[\frac{\left(\cos 44^\circ+\cos 43^\circ+\cdots+\cos 1^\circ\right)}{\left(\sin 1^\circ+\sin 2^\circ+\cdots+\sin 44^\circ\right)}+\frac{\left(\sin 44^\circ+\sin 43^\circ+\cdots+\sin 1^\circ\right)}{\left(\sin 1^\circ+\sin 2^\circ+\cdots+\sin 44^\circ\right)}\right]$$

$$T = \frac{1}{\sqrt{2}}\left[T+1\right]$$

$$\sqrt{2}T = T+1$$

$$(\sqrt{2}-1)T = 1$$

$$T = \frac{1}{\sqrt{2}-1}$$

$$\boxed{T = \sqrt{2}+1}$$

$$(T-1) = \sqrt{2}$$

$$(T-1)^2 = 2$$

23

The coefficients of a quadratic equation $ax^2 + bx + c = 0 (a \neq b \neq c)$ are chosen from first three prime numbers. If ' P ' be the probability that the equation has real roots, then the value of ' 18 P ' is

$ax^2+bx+c=0$; $\{2,3,5\}$

Total: $3 \times 2 \times 1 = 6$

For: $b^2 - 4ac \geq 0$

$b^2 \geq 4ac$

$b=2$ X

$b=3$ X

$b=5 \rightarrow (a,c) \equiv (2,3),(3,2)$

$\therefore P = \frac{2}{6}$

$P = \frac{1}{3}$

$18P = 6$

24

If the eccentricities of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ be e and e_1, then $\frac{1}{e^2}+\frac{1}{e_1^2}+10=$ (11)

JEE 1

$H: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{a^2+b^2}{a^2}}$

$H_1: -\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $e_1=\sqrt{1+\frac{a^2}{b^2}}=\sqrt{\frac{a^2+b^2}{b^2}}$

$\frac{1}{e^2}+\frac{1}{e_1^2}=1+10$

$+10$

25

JEE 1

Let $\alpha = \max_{x \in R}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$ and $\beta = \min_{x \in R}\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to

$8^{2\sin 3x} \cdot 4^{4\cos 3x}$

$2^{6\sin 3x} \cdot 2^{8\cos 3x}$

$2^{[6\sin 3x + 8\cos 3x]}$

$\alpha = 2^{10} \longrightarrow \alpha^{1/5} = 2^2 = 4$

$\beta = 2^{-10} \longrightarrow \beta^{1/5} = 2^{-2} = \frac{1}{4}$

$x^2 - \left(4 + \frac{1}{4}\right)x + 4 \times \frac{1}{4} = 0$

$x^2 - \frac{17}{4}x + 1 = 0$

$8x^2 - 34x + 8 = 0$

$b = -34;\ c = 8$

$c - b = 8 + 34 = 42$