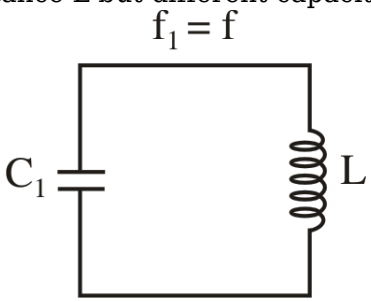
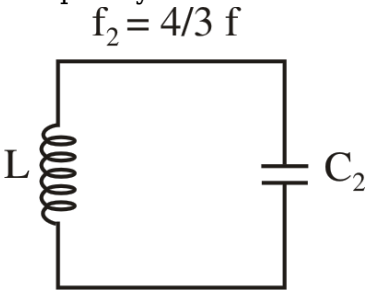
PART-1 : PHYSICS

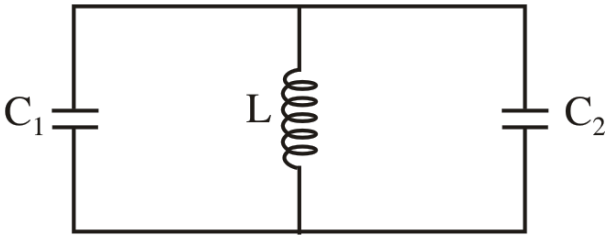
SECTION-I (i)

1) A circuit consisting of an ideal coil and an ideal capacitor is called a resonant circuit. The two electrical resonant circuits shown, with identical inductance L but different capacitances C, oscillate completely without resistance at the given frequencies.





What is the oscillation frequency f_{12} (natural frequency) of the following coupled system?



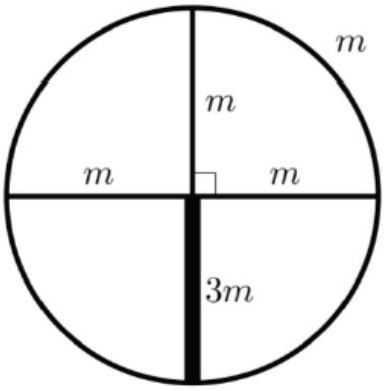
(A) $\frac{2}{3}f$

(B) $\frac{3}{4}f$

(C) $\frac{4}{5}f$

(D) $\frac{5}{4}f$

2) A wheel of radius R has a thin rim and four spokes, each of which have uniform density. The entire rim has mass m. three of the spokes each have mass m. and the fourth spoke has mass 3m. The wheel is suspended on a horizontal frictionless axle passing through its center. If the wheel is slightly rotated from its equilibrium position, what is the angular frequency of small oscillations?



(A) $\sqrt{\frac{g}{3R}}$

(B) $\sqrt{\frac{g}{2R}}$

(C) $\sqrt{\frac{2g}{3R}}$

(D) $\sqrt{\frac{g}{R}}$

3) An empty freight car on a level railroad track has a mass M. A chute above the freight car opens and grain falls down into the car at a rate of r, measured in kilograms per second. The grain falls a vertical distance h before hitting the bed of the freight car without bouncing up. How much normal force is exerted on the freight car from the rails at a time t after the grain begins to hit the bed of the car? Assume the grain starts from rest.

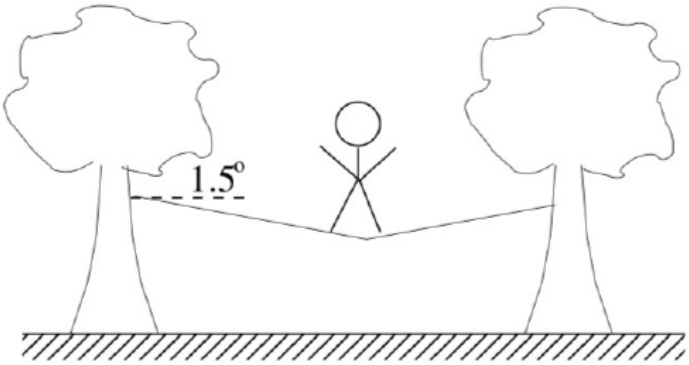
(A) Mg + rtg

(B) $Mg + r\sqrt{gh}$

(C) $Mg + r\sqrt{gh} + rtg$

(D) $Mg + r\sqrt{2gh} + rtg$

4) A massless cable of diameter 2.54 cm (1 inch) is tied horizontally between two trees 18 m apart. A tightrope walker stands at the center of the cable, giving it a tension of 7300 N. The cable stretches and makes an angle of 1.50° with the horizontal. The Young's modulus of the cable is:



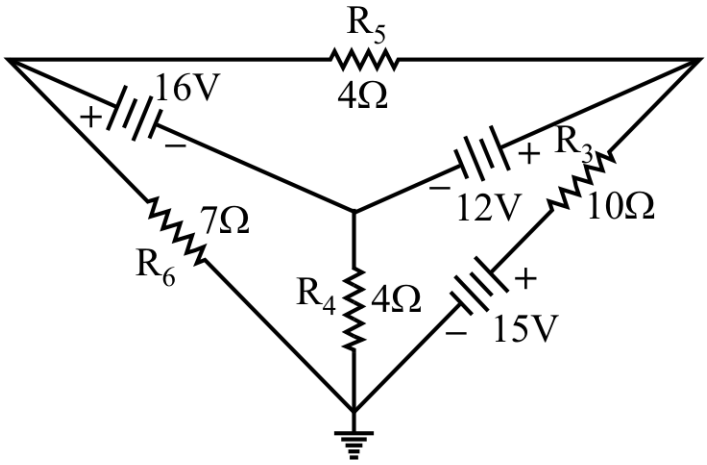
(A) 1.5×10^6 N/m^2

(B) 2.2×10^9 N/m^2

(C) 2.4×10^{10} N/m^2

(D) 4.2×10^{10} N/m^2

SECTION-I (ii)



1) For the circuit shown in the figure :

(A) Current through R_5 is approximately 4 A.

(B) Current through 12V battery is approximately 3.2 A.

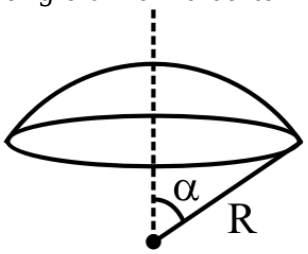
(C) Current through R_6 is approximately 1.7 A

(D) Current through R_3 is approximately 0.7 A.

2) An insulating sphere of radius R is coated with a uniform surface charge density σ on its exterior surface.

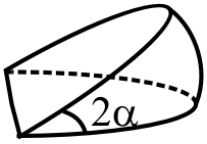
Case-I :-

Suppose that we cut off a spherical cap corresponding to a half-angle α from a certain axis (i.e. the cap has base radius R sinα) and remove the rest of the sphere.



Case-II :-

A spherical wedge of half-angle α (a slice of watermelon) is extracted from the sphere and the rest of the sphere is removed instead.



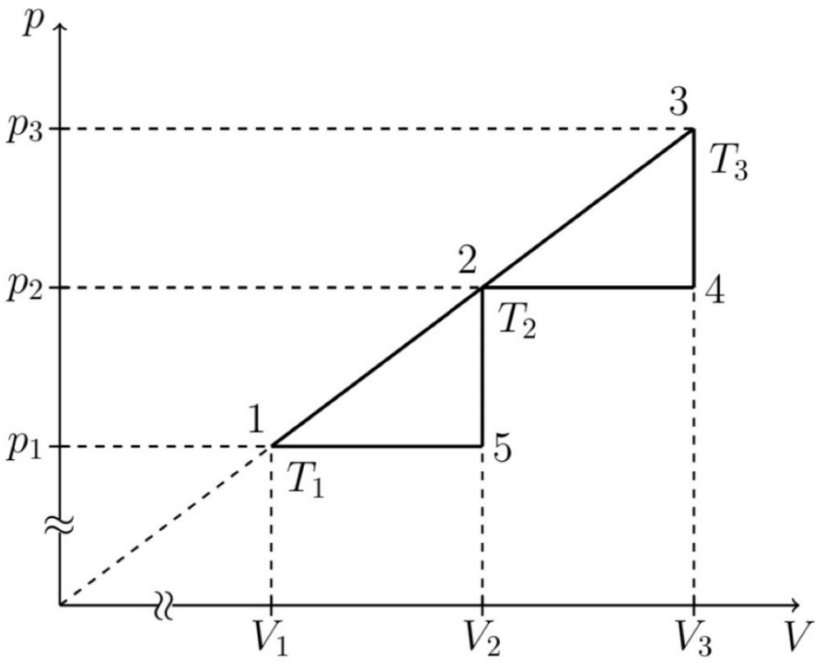
(A) The electric field at the center of original sphere in case-I is $\frac{\sigma \sin\alpha}{4\epsilon_0}$.

(B) The electric field at the center of original sphere in case–I is $\frac{\sigma \sin^2\alpha}{4\epsilon_0}$.

(C) The electric field at the center of original sphere in case–II is $\frac{\sigma \sin\alpha}{4\epsilon_0}$.

(D) The electric field at the center of original sphere in case II is $\frac{\sigma \sin^2\alpha}{4\epsilon_0}$.

3) A heat engine, with an ideal gas composed of diatomic molecules as the working substance, operates in a cycle 1 → 2 → 3 → 4 → 2 → 5 → 1, whose pV-diagram is shown in Figure. Points 1, 2, and 3 lie on a straight line passing through the origin, with point 2 being the midpoint of segment 1 → 3. The lowest temperature in the cycle is T_{min}, and the maximum temperature of the cycle is k-times higher. Take T_{min} = 300 K and k = 4.



(A) The value of T_2 is 675 K.

(B) The ratio $\frac{V_3}{V_1}$ is 2.

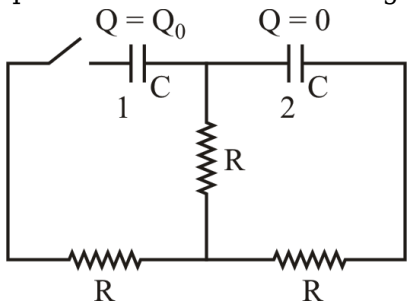
(C) The efficiency of cyclic process is $\frac{25}{9}\%$.

(D) The efficiency of cyclic process is $\frac{25}{18}\%$.

SECTION-II (i)

Common Content for Question No. 1 to 2

The capacitor 1 has an initial charge Q_0 and capacitor 2 is initially uncharged. The switch is closed at t = 0.

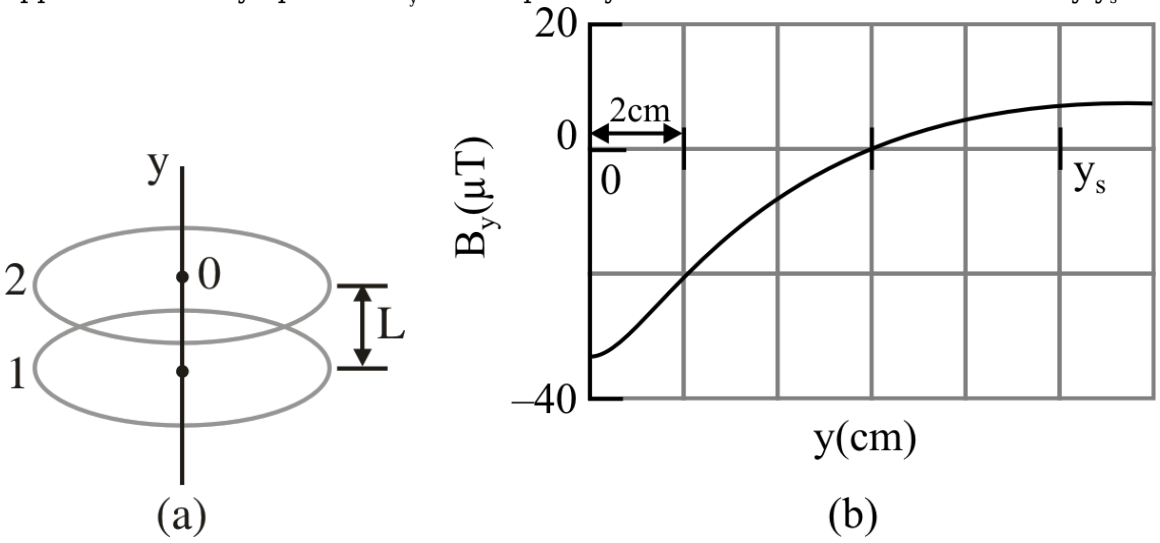


1) The maximum charge on capacitor 2 is $\frac{Q_0}{\sqrt{\alpha}}$. The value of α is.

2) The time at which the charge on capacitor 2 is maximum is αRC□nβ. The value of α + β is

Common Content for Question No. 3 to 4

In figure-(a), two circular loops, with different currents but the same radius of 4.0 cm, are centered on a y axis. They are initially separated by distance L = 3.0 cm, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with y component B_y. That component is to be measured as loop 2 is gradually moved in the positive direction of the y axis. Figure-(b) gives B_y as a function of the position y of loop 2. The curve approaches an asymptote of B_y = 7.20 μT as y → ∞. The horizontal scale is set by y_s = 10.0 cm.

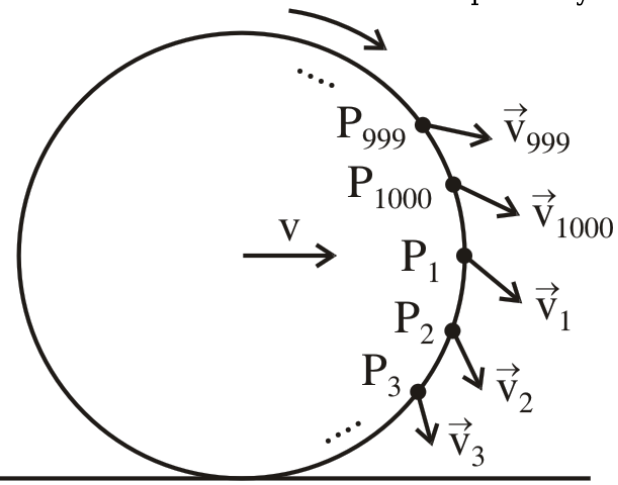


3) The current in loop-2 (in A) is :

4) The current in loop-1 (in A) is :

SECTION-II (ii)

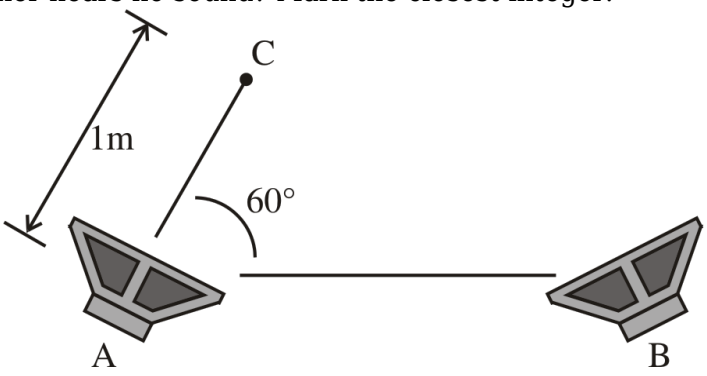
1) A rigid ring rolls without slipping along horizontal ground, with constant translational velocity v = 5.0 m s^{-1} towards the right. Consider 1000 points P_1, P_2, ... , -P_{1000} on the ring, evenly spaced across the ring's full circumference, with the first point P_1 taken to be the ring's rightmost point. Denote their velocities as $\vec{v}_1, \vec{v}_2, \ldots\ldots, \vec{v}_{1000}$ respectively. The value of $|\vec{v}_1 + \vec{v}_2 + \vec{v}_3 \ldots\ldots\ldots \vec{v}_{1000}|$ (in m/s)



is

2) The drawing shows a loudspeaker A and point C, where a listener is positioned. A second loudspeaker B is located some-where to the right of loudspeaker A. Both speakers vibrate in phase and are playing a 68.6-Hz tone. The speed of sound is 343 m/s. What is the closest (in m) to speaker

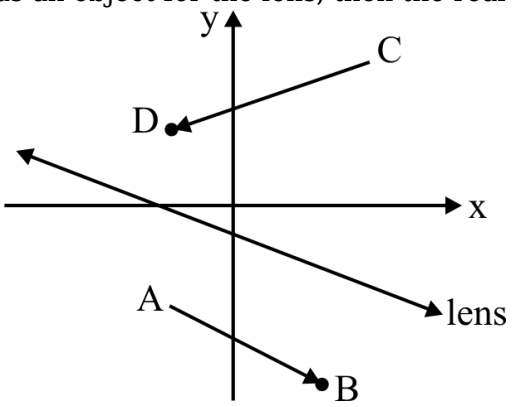
A that speaker B can be located, so that the listener hears no sound? Mark the closest integer.



(Assume both speakers behave as point sources)

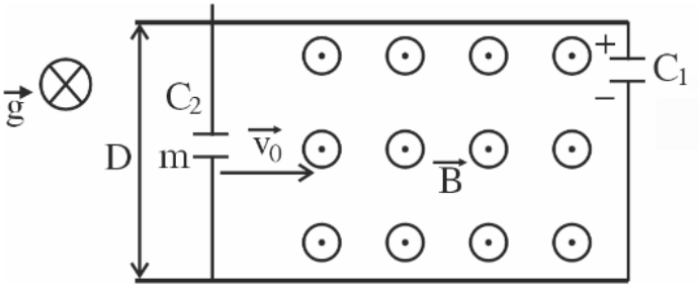
3) Three perfect linear polarizers are stacked normal to a central axis along which is incident a beam of natural light of intensity I_0. If the first and last polarizers are crossed and if the middle one rotates at a rate 2π rad/s about the axis. Find the frequency (in 1/sec) of oscillation of intensity of emerging light?

4) A convergent ideal thin lens is placed such that the plane of the lens is perpendicular to the plane of the paper, and the optical axis lies within the plane of the paper. Define points A(–4.0, –4.2), B(3.0, –9.0), C(10.0, 10.5) and D(–1.5, 4.5) within the plane of the paper. All the coordinates given are in cm. If the vector $\overrightarrow{AB}$ is used as an object for the lens, then the real image $\overrightarrow{CD}$ is formed. Find



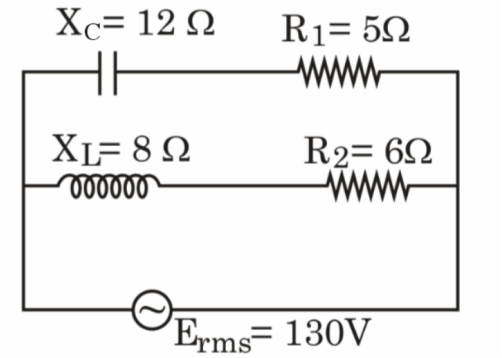
the focal length of the lens (in cm).

5) Two sufficiently long horizontal rails with zero resistance are connected to a charged capacitor with capacitance C_1. The distance between the rails is D. The rails are placed in a vertical constant magnetic field with an induction of B. Initially, the rails are connected to an uncharged capacitor with capacitance C_2, and this capacitor is simultaneously given an initial velocity v_0 along the rails. The terminals of the capacitor C_2 can slide along the rails without friction or loss of contact. The mass of the capacitor C_2 and its terminals is m. Find the initial charge Q (in μC) the capacitor C_1, assuming that capacitor C_2 comes to rest during its motion. (Take m = 1kg, v_0 = 2 m/s, D = 1m, C_1 =



1μF, C_2 = 2 μF and B = 1T.

6) The amount of power delivered by the ac source in the resistance R_1 of the circuit given below is X watt, then X is :-



PART-2 : CHEMISTRY

SECTION-I (i)

1) When diazoaminobenzene is treated with a small quantity of aniline hydrochloride in the presence of anilene, then

(A) Deep-Red coloured product is formed

(B) Azobenzene is formed

(C) Sodium fusion extract of dye formed in the above reaction, fails to produce prussian blue/ green colour with $FeSO_{4(aq)}$ and Conc. H_2SO_4

(D) Orange dye is formed, when product formed in the above reaction is treated with
(i) $NaNO_2$ + HC□, 0°C (ii) H_2O, Δ

2) When equal volumes of two pure liquids phenol and aniline are mixed together, then

(A) Resultant Free energy of the liquid mixture is greater than its free energy in pure states

(B) Solution shows negative deviation from Raoult's law

(C) For this process, 'ΔS' of system is less than zero due to Exothermic nature of this process.

(D) For this process, $\Delta S_{surrounding}$ is less than zero due to exothermic nature of this process.

3) Which one of the following statement regarding potash alum is **INCORRECT** ?

(A) During its preparation, dilute H_2SO_4 is used to prevent the hydrolysis of mixture

(B) Aqueous solution of potash alum gives white precipitate with perchlorate aqueous solution

(C) It is used as Fixing agent in dye industry

(D) This salt shrinks on heating initially.

4) A H-like species emitted a photon corresponding to the first line of Lyman series. The photon liberated a photoelectron from He^+ in ground state. The de-Broglie wavelength of the photoelectron is 2 Å. Then, the atomic number of H-like species is
Given : Energy of electron in ground state of H-atom = – 13.6 eV/atom
h = 6.626 × 10^{-34} J-sec, mass of electron = 9.1 × 10^{-31} kg

(A) 3

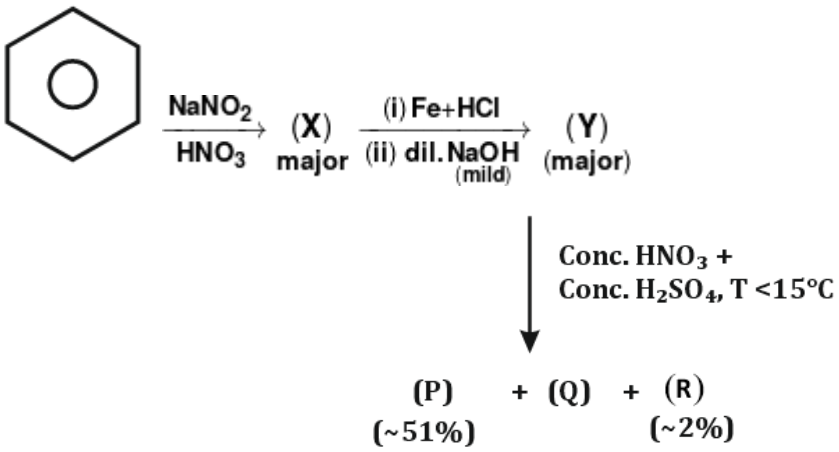
(B) 2

(C) 4

(D) 6

SECTION-I (ii)

1)



(P), (Q) and (R) are isomers

Which of the following statement(s) is / are **CORRECT** ?

(A) Boiling point order : (P) > (Q) > (R)

(B) Basicity order : (R) > (Q) > (P)

(C) Melting point order : (P) > (Q) > (R)

(D) 'X' gives grey precipitate or silver mirror, when it reacts with (i) $Zn/NH_4C\square$ and (ii) $[Ag(NH_3)_2]$ OH, Δ

2) At 25°C, the dissociation constant (K_a) of weak mono basic acid (HA) is numerically equal to the dissociation constant (K_b) of its conjugate base (A^-). Then, identify the **CORRECT** option(s)

(A) $K_a(HA) = 10^{-7}$

(B) pH of 0.1M HA aqueous solution ≈ 3.0

(C) pH of 0.1 M aqueous solution of A^- ≈ 10.0

(D) pH of aqueous solution containing 0.1 M HA and 0.01 M HC□ ≈ 2.0

3) A complex salt is represented as $CoC\square_3 . xNH_3$ and it's 0.1 molal solution in water has 'ΔT_f' of 0.372°C.

Given :

Cryoscopic and Ebullioscopic constants for H_2O are 1.86 K-kg mol^{-1} and 0.52 K-kg mol^{-1} respectively.

Assume complex salt undergoes complete ionisation in H_2O

Which of the following statement(s) is/are **CORRECT** regarding this complex salt ?

(A) IUPAC name of this salt is Pentaamminechloridocobalt(III) chloride

(B) Diamagnetic in nature

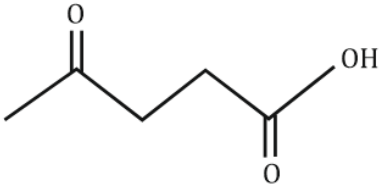
(C) Total stereomers possible for this salt = 2

(D) Given solution will boil between 100°C to 101°C

SECTION-II (i)

Common Content for Question No. 1 to 2

Levulinic acid is a versatile chemical that can be used to make pharmaceuticals, plastics and fuels. It



(Levulinic acid)

can be made from fructose. Furanose form of Fructose can be converted to levulimic acid as follows :

Furanose form of D(–) - Fructose $\xrightarrow{-H_2O}$ (P) (contains aldehyde) + other tri substituted alkenes

(P) $\xrightarrow{-2H_2O}$ (Q) (major) $\xrightarrow{\text{Finally}}$ Levulinic acid + (R)

Given :

'Q' has diatropic nature

(R) gives '+' ve $NaHCO_3$ test and Tollen's test

Atomic mass : C–12, H–1, O–16, Ag–108, N–14

1) (Q) + α-naphthol $\xrightarrow[H_2SO_4]{\text{Conc}}$ Coloured product (S)

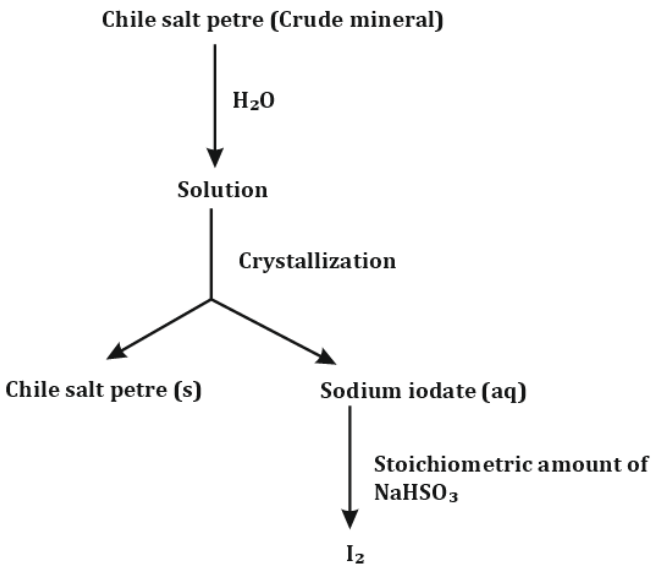
Total number of hetero atoms present in one Cationic unit of 'S' is .....

2) (R) $\xrightarrow[\Delta]{[Ag(NH_3)_2]OH}$ Coloured product (X) + other products

The maximum amount of (R) in gram required to consume completely one mole of $[Ag(NH_3)_2]OH$ in the formation of 'X' is .....

Common Content for Question No. 3 to 4

Caliche (Sodium iodate) is the main source for iodine, which is found in nature with chile salt petre



as crude chile salt petre **Reactions involved in the above process :**

$$IO_3^- + HSO_3^- \rightarrow I^- + SO_4^{2-} + H^+$$
$$I^- + IO_3^- + H^+ \rightarrow I_2 + H_2O$$

[Given : Atomic mass : Na = 23, S = 32, O = 16, H =1, I = 127]

3) Determine equivalent mass of I_2 (in gm $eq.^{-1}$) produced when iodide salt is treated with iodate salt in acidic medium

4) One litre of aqueous solution contains 5 gram of $NaIO_3$ is treated with stoichiometric quantity of $NaHSO_3$. Now an additional amount of same solution ($NaIO_{3(aq)}$) is added to resultant mixture to convert iodide into Iodine. What volume of solution **(in mL)** containing $NaIO_3$ must be added in step-II to bring in complete conversion of I^- to I_2

SECTION-II (ii)

1) (i) $PCl_3 + LiH \rightarrow (X)\uparrow + Y$

(ii) $(X) \xrightarrow[H_2SO_4]{HCHO\ (excess)} Salt\ \ (Z)$

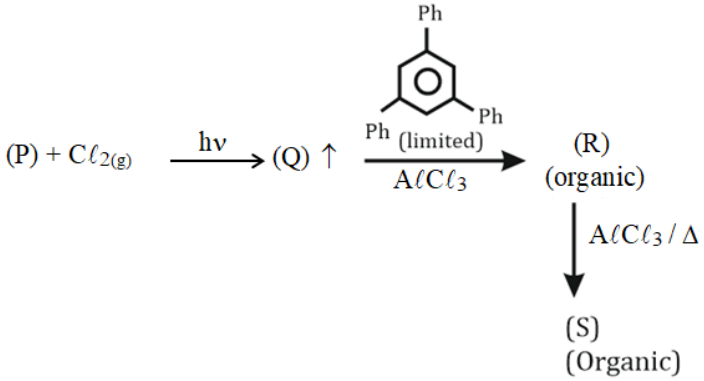
white solid which is used for making fire-proof cotton fabrics

The maximum amount (in gram) of **(Z)** formed when 13.75 gr of PC□$_3$ is used in the above reaction sequence is **(Nearest Integer)**

Given : % yield of reaction (i), (ii) are 100 and 50 respectively

Atomic mass : P-31, C□-35.5, Li-7, H-1 , C-12, O-16, S-32

2) $K_4[Fe(CN)_6] + \underset{(conc)}{H_2SO_4} + H_2O \rightarrow (P)\uparrow + \text{other products}$



Then, D.U of product (S) is

Given : (P) and (Q) are poisonous gases

Consider major products in all above reactions

3)

$K_2Cr_2O_7$ is a good oxidising agent in acidic medium. By how much **(in volt)** oxidising power of $K_2Cr_2O_7$ decreases when pH changes from 0 to 14 at 298 K. Assume that activities of all other components are constant at unity. **(If the answer is X volt then find the value of 100X)**

Given : $\frac{2.303RT}{F} = 0.06\,\text{Volt}$

4) One litre of 1M H_2O_2 solution was left open, causing first order decomposition of H_2O_2 with rate constant equal to 6.93×10^{-2} sec^{-1}. Determine amount of energy released **(in kJ)**, if this solution is reacted with excess of $I^-_{(aq)}$ in acidic medium after 20 sec from the start.

Given : Assume no change in volume of H_2O_2 solution

$\Delta H_f^0\left[H_2O_{2(aq)}\right] = -200\,\text{kJ/mole}$

$\Delta H_f^0\left[I^-_{(aq)}\right] = -60\,\text{kJ/mole}$

$\Delta H_f^0\left[I_{2(aq)}\right] = 60\text{kJ/mole}$

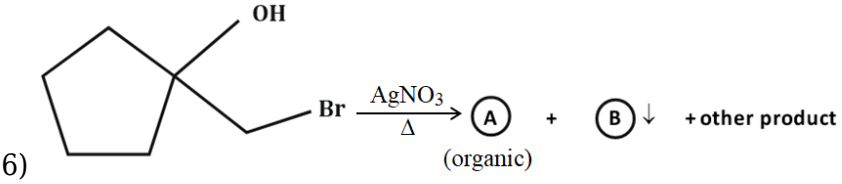
$\Delta H_f^0\left[H_2O_{(\ell)}\right] = -290\,\text{kJ/mole}$

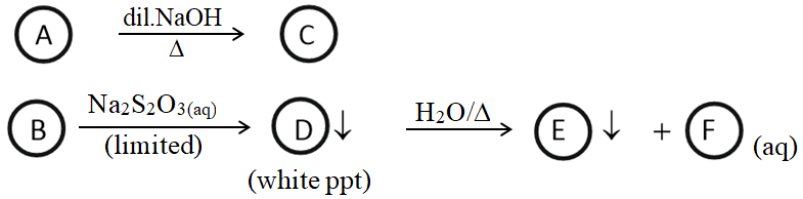
$\Delta H_f^0\left[H^+_{(aq)}\right] = 0$

5) Among the following the total number of species which is / are paramagnetic

$C_2, O_2, S_2, O_2^+, N_2, N_2^-, CN^-, B_2, NO_2, NO^+, C\ell O_2, O_3$

6)





If D.U of product (C) = 'X'

Spin only magnetic moment of product (E) = 'Y' B.M

Then, the value of [100 Y + X] is

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let $f(x) = x^2 - 3x - 7$. Let g(x) and h(x) be two quadratic polynomials also, both with the coefficient of x^2 equal to 1. Rohan computes each of the three sums f(x) + g(x) , g(x) + h(x) , h(x) + f(x) , and is surprised to find that each pair of these sums has a common root, and these three common roots are distinct. If g(0) = 2, then h(0) is

(A) $\frac{33}{52}$

(B) $\frac{19}{33}$

(C) $\frac{52}{19}$

(D) $\frac{71}{19}$

2) The volume enclosed by the planes $|5x-6| + |10y-7| + |z+4| = 10$ is

(A) $\frac{10}{3}$

(B) $\frac{40}{3}$

(C) $\frac{80}{3}$

(D) $\frac{20}{3}$

3) The area of the region defined as and $y \le e^{||x| \ln |x||}$, $x^2 + y^2 - 2(|x| + |y|) + 1 \ge 0$ and $y \ge 0$, where $|x| \le 1$, if α is the x-coordinate of the point of intersection of curves $y = e^{||x| \ln |x||}$ & $x^2 + y^2 - 2(|x| + |y|) + 1 = 0$ in 1st quadrant, is

(A) $4\left[-\int_0^{\alpha} e^x \ln x\,dx + \int_{\alpha}^{1} \left(1 - \sqrt{1-(x-1)^2}\right) dx\right]$

(B) $4\left[\int_0^{\alpha} e^x \ln x\,dx - \int_1^{\alpha} \left(1 - \sqrt{1-(x-1)^2}\right) dx\right]$

(C) $2\left[-\int_0^{\alpha} e^x \ln x\,dx + \int_{\alpha}^{1} \left(1 - \sqrt{1-(x-1)^2}\right) dx\right]$

(D) $2\left[\int_0^{\alpha} ex\ln x\ dx + \int_{\alpha}^{1}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$

4) If $a + b + c = 35$, then maximum value of $\sqrt{3a+13}+\sqrt{3b+13}+\sqrt{3c+13}$ is (where $a, b, c \in R^+$)

(A) 12

(B) $12\sqrt{3}$

(C) 6

(D) $6\sqrt{3}$

SECTION-I (ii)

1) Let $A=\begin{bmatrix}\alpha & \beta & 0\\ \beta & \alpha & 0\\ 0 & 0 & \alpha+\beta\end{bmatrix}$ and $|2A^3| = 2^{12}$ where $\alpha,\beta \in I$, then possible values of tr(A) are-

(A) 2

(B) –2

(C) 6

(D) –4

2) Let f(x) be a function defined by $f(x)=\int_0^{3\pi/4}\left|x\cos t-3\sin t+\sqrt{2}x\right|\left(\frac{1+\sqrt{2}\cos t}{(\cos t+\sqrt{2})^3}\right)dt$ where x is any real number, then which of the following options is/are true ?

(A) f(x) is continuous and differentiable everywhere

(B) f(x) is continuous for all x but non-differentiable at two values of x

(C) Minimum value of f(x) is $\frac{3}{2}$ for $x \in R$

(D) Minimum value of f(x) is $\frac{3}{4}$ for $x \in R$

3) Let function y = f(x) satisfies the differential equation $x^2\frac{dy}{dx} = y^2e^{\frac{1}{x}}\ (x \neq 0)$ and $\lim_{x\to 0^-} f(x) = 1$. Identify the correct statement(s) :

(A) Range of f(x) is $(0,1)-\left\{\frac{1}{2}\right\}$

(B) f(x) is bounded

(C) $\lim_{x\to 0^+} f(x) = 1$

(D) $\int_0^e f(x)\,dx > \int_0^1 f(x)\,dx$

SECTION-II (i)

Common Content for Question No. 1 to 2

N dice are rolled once, then the probability of getting sum L is expressed as P_r (L, N), where dice is a r sided dice with faces marked 1, 2, 3, 4, ... r.

1) Value of $9.\sqrt{P_9(15,2)}$ is

2) Value of P_4 (9, 3) – P_4 (9, 4) is

Common Content for Question No. 3 to 4

$$A.\,adj\left(A^2\right) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

If A is 3 × 3 matrix with real entries such that

If B is inverse of matrix A such that $A = \lambda B^2 + \mu B + \gamma I$ where λ, μ, γ are scalars and I is identity matrix of order 3 × 3

3) The value of μ is :

4) The value of det(adj (adj A)) is :

SECTION-II (ii)

1) In ΔABC, let B(3,3,–9) and AB = AC and altitude AD is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{3}$. If co-ordinates of C are (α,β,γ), then |α| + |β| + |γ| is

2) Let ellipse $\frac{x^2}{25} + \frac{y^2}{p^2} = 1$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{q^2} = 1$ are confocal and meet at points A,B,C and D. If foci of ellipse subtend 90° at point A and e_1 and e_2 are eccentricities of ellipse and hyperbola then $\left(\frac{1}{e_1^2} + \frac{1}{e_2^2}\right)$ is

3) Consider the following lines :

$L_1 : x - \sqrt{3}y + 3 = 0,\ L_2 : x - 2y + 4 = 0$
$L_3 : x - 5y + 25 = 0,\ L_4 : x + y + 1 = 0$
$L_5 : x - y + 1 = 0,\ L_6 : x - 4y + 16 = 0$

Let H(L_m, L_n, L_k) denotes orthocenter of triangle formed by lines L_m, L_n, L_k and S(L_m, L_n, L_k) denotes circumcircle of triangle formed by lines L_m, L_n, L_k

If $H(L_1, L_3, L_5) \equiv (x_1, y_1)$ and $(x_3, 0) \in S(L_1, L_3, L_5) \cap S(L_2, L_4, L_6)$, where $x_3 > 0$. Then $x_1^2 + x_3^2$ is

4) Let $n = 2^{31} \cdot 3^{19}$, then number of factors of n^2 that are less than n and are not factor of n, are

5) Let the mean and standard deviation (non-zero) of marks of class A of 100 students be respectively 40 and α, and the mean and standard deviation (non-zero) of marks of class B of n students be respectively 55 and 30 – α. If the mean and variance of the marks of the combined class of 100 + n students are respectively 50 and 350, then the sum of variances of classes A and B is

6) If $\left[2\left(\vec{a} \times \vec{b}\right) \;\; 2\left(\vec{b} \times \vec{c}\right) \;\; 2\left(\vec{c} \times \vec{a}\right)\right] = 800$ and $\vec{b}.\left(\vec{a} \times \left(\vec{b} \times \left(\vec{c} \times \vec{b}\right)\right)\right) = -250$, then $\left|\vec{b}\right|$ is equal to $\left(\left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right] > 0\right)$

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	C	A	D	D

SECTION-I (ii)

Q.	5	6	7
A.	C,D	B,C	A,B,C

SECTION-II (i)

Q.	8	9	10	11
A.	27.00	4.50	2.60 to 2.70	0.86 to 0.90

SECTION-II (ii)

Q.	12	13	14	15	16	17
A.	5000	4	4	3	3	500

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	18	19	20	21
A.	D	B	D	A

SECTION-I (ii)

Q.	22	23	24
A.	A,C,D	A,C,D	B,C,D

SECTION-II (i)

Q.	25	26	27	28
A.	4.00	23.00	152.40	200

SECTION-II (ii)

Q.	29	30	31	32	33	34
A.	10	18	196	50	7	4

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	35	36	37	38
A.	C	C	C	B

SECTION-I (ii)

Q.	39	40	41
A.	B,C	A,D	A,B,D

SECTION-II (i)

Q.	42	43	44	45
A.	2.00	0.00	3.00	1.00

SECTION-II (ii)

Q.	46	47	48	49	50	51
A.	9	2	2	589	500	5

SOLUTIONS

PART-1 : PHYSICS

3) The (upward) normal force on the freight car must equal the total downward forces experienced by the freight ear. The freight car experiences three1 downward forces:
The freight car's own weight:
$F_{g,car} = Mg$
The weight of the grain accumulating in the freight car. The grain accumulates at a (mass) rate r, so after a time t the car contains a total grain mass rt. Thus the total weight of the grain after time t is
$F_{g,grain} + rtg$
The force from absorbing the momentum of the falling grain when the grain lands on the cart. Over some time interval Δt, an amount of grain with mass $\Delta m = r\Delta t$ falls onto the cart. The grain falls from a height h, so it lands on the cart with speed $v = \sqrt{2gh}$. Thus the grain carries a momentum
$\Delta p = \Delta m v = r\Delta t\sqrt{2gh}$
Thus the rate at which the momentum is delivered to the cart, or the force experienced by the cart, is

$$F_{collision} = \frac{\Delta p}{\Delta t} = \frac{r\Delta t\sqrt{2gh}}{\Delta t} = r\sqrt{2gh}$$

Thus the normal force, which equals the total downward force on the cart, is
$F_{g,car} + F_{g,grain} + F_{collision} = Mg + rtg + r\sqrt{2gh}$

4) The strain of the cable is

$$\frac{\Delta L}{L} = \frac{\frac{L_0}{\cos\theta} - L_0}{L_0} = \frac{1}{\cos\theta} - 1 \approx \frac{\theta^2}{2}$$

where L_0 is the initial length. Then the Young's modulus is

$$Y = \frac{(7300\ \text{N})}{\pi(0.0127\ \text{m})^2}\left(\frac{\left(\frac{1.50\pi}{180}\right)^2}{2}\right)^{-1} = 4.2 \times 10^{10}\ \text{N/m}^2$$

10) $$B_y = \frac{\mu_0 i_1 R^2}{2\pi(R^2 + z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2\pi(R^2 + z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ and $z_2^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum corresponding to By = 0 would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).
(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1. We obtain $i_1 = (45/16\pi)$ A ≈ 0.90 A.
(b) With loop 2 at y = 0.06 m we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2 + L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2 + z_2^2)^{3/2}}$$

We obtain

$$i_2 = \left(\frac{117\sqrt{13}}{50\pi}\right) A \approx 2.7A$$

14)

Intensity of light after passing through first polarizer, $I_0 = \frac{I_0}{2}$ intensity of light after passing through second polarizer, $I_2 = \frac{I_0}{2}\cos^2\omega t$ intensity of light after passing through third polarizer, $I_3 = \frac{I_0}{2}\cos^2\omega t.\sin^2\omega t$

$$I_3 = \frac{I_0}{16}(1-\cos 4\omega t)$$

$$2\pi f = 4\times 2\pi \Rightarrow f = 4/\text{sec}$$

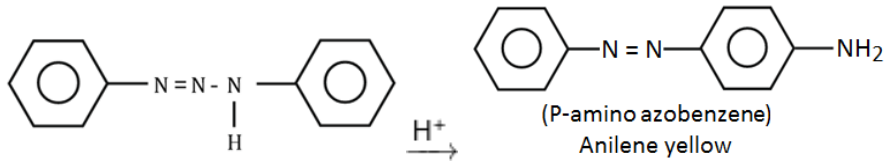
17) Power = $V_{rma} \times I_{rm} \cos\phi$

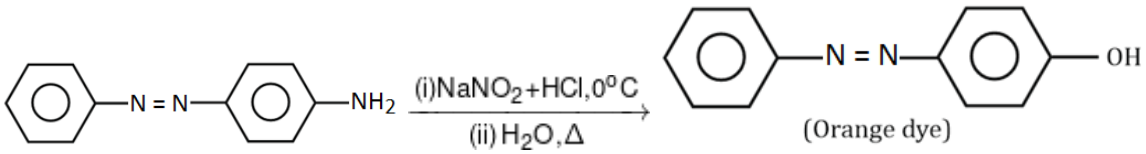
$= 130 \times \frac{130}{13}.\frac{5}{13} = 500\text{watt}$

PART-2 : CHEMISTRY

18)

In mild acidic condition, N–N coupling azo compound rearranges to C – N coupling compound.





19)

(Phenol + Anilene) shows –ve deviation

$\therefore \Delta H_{mix} < 0$

$\Delta S_{sys} > 0$

$\Delta S_{sur} > 0$

$\Delta G_{sys} < 0$

20)

A. Potash alum - $K_2SO_4 . A\square_2(SO_4)_3 . 24H_2O$

B. Potash $alum_{(aq)} \xrightarrow{C\ell O_{4(aq)}^{-}} \underset{\text{Whiteppt}}{KC\ell O_4} \downarrow$

C. Alum forms $A\square(OH)_3$ on the cloth and dye sticks to this $A\square(OH)_3$

D. Alum swells on heating due to loss of water

21) $\lambda = \sqrt{\frac{150}{V}}\text{Å} \Rightarrow V = \frac{150}{4} = 37.5$

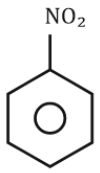
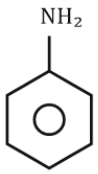
K.E of photo electron = 37.5 eV

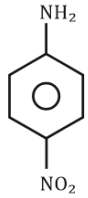
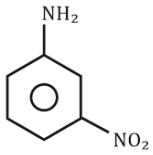
Energy required to remove electron $= \frac{13.6}{1} \times 4 = 54.4$ eV

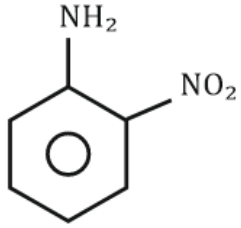
Total energy of photon = 91.9 eV

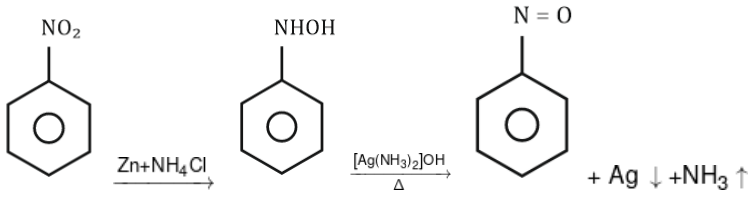
Now, $10.2\ Z^2 = 91.9$

$\Rightarrow Z = 3$

22) 'X' =  ; 'Y' =  ;

'P' =  ; 'Q' =  ;

'R' = 



23) (A) $K_a \cdot K_b = K_w \quad \Rightarrow K_a = K_b = 10^{-7}$ (B) $[H^+] = \sqrt{K_a.C} = \sqrt{10^{-8}} = 10^{-4}M$

pH = 4

(C) $A^- + H_2O \rightleftharpoons HA + OH^-$

$[OH^-] = \sqrt{K_b.C} = \sqrt{10^{-8}} = 10^{-4}M$

pOH = 4 ⇒ pH = 10.0

(D) $[H^+]_{Net} = [H^+]_{HA} + [H^+]_{HCl}$

$\approx [H^+]_{HC\ell} = 0.01M$

pH = 2.0

24) $\Delta T_f = i.\ K_f\ .$ molality

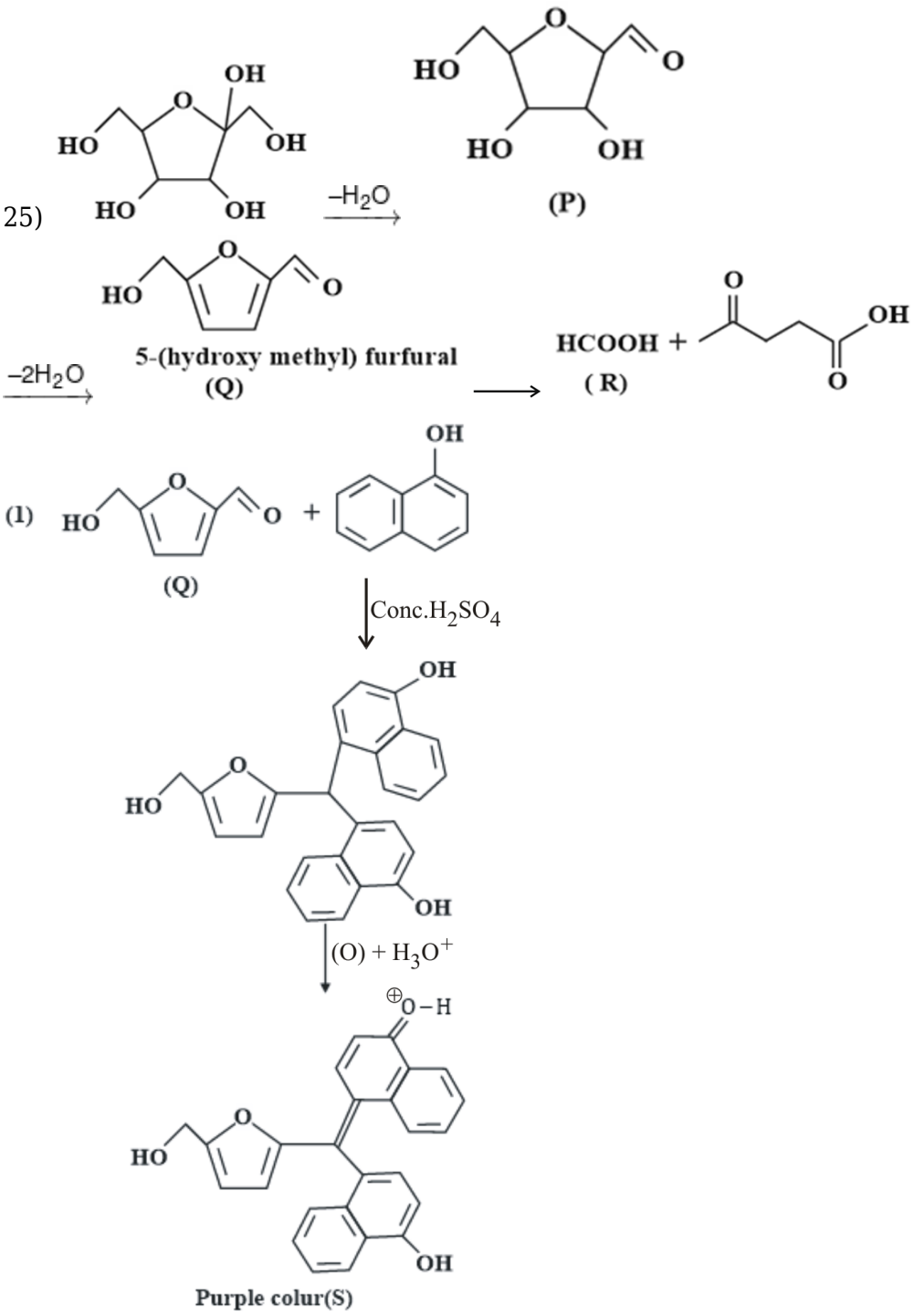
$0.372 = i \times 1.86 \times 0.1$

$i = 2$

Complex salt is $[Co(NH_3)_4C\square_2]\ C\square$

This salt has two G.I, but both isomers are optically inactive.

25)



26) $HCOOH \xrightarrow[\Delta]{Ag(NH_3)_2OH} \underset{(X)}{Ag}\downarrow + NH_3\uparrow + CO_2\uparrow + H_2O$

$H-\overset{\overset{O}{\|}}{C}-OH \rightarrow CO_2 + 2e^-$

$Ag^+ + e^- \rightarrow Ag$

0.5 mole of HCOOH can reduce one mole of $[Ag(NH_3)_2]OH$

$W_{HCOOH} = 0.5 \times 46 = 23$

27) $5I^- + IO_3^- + 6H^+ \rightarrow 3I_2 + 3H_2O$

n-factor of H_2O = 5/3 = 1.67

28) (i) $IO_3^- + 3HSO_3^- \rightarrow I^- + 3SO_4^{2-} + 3H^+$

moles of $NaIO_3$ taken $= \frac{5}{M_{NaIO_3}}$ = moles of NaI formed

(ii) $5I^- + IO_3^- + 6H^+ \rightarrow I_2 + 3H_2O$

moles of $NaIO_3$ required

$= \frac{5}{5 \times M_{NaIO_3}} = \frac{1}{M_{NaIO_3}}$

Weight of $NaIO_3$ = 1

Volume of $NaIO_3$ required = 1000/5 = 200 mL

29) (i) $PC\ell_3 + 3LiH \rightarrow \underset{(P)}{PH_3} \uparrow + \underset{(Q)}{3LiC\ell}$

(ii) $PH_3 + \underset{(excess)}{HCHO} \xrightarrow{H_2SO_4} \underset{(R)}{[P(CH_2OH)_4]_2 SO_4}$

n_{PCl_3} taken $= \frac{13.75}{137.5} = 0.1$

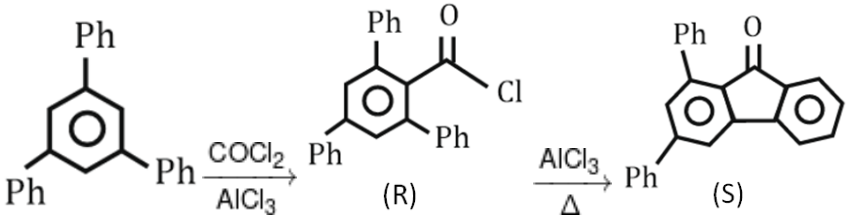
n_{PH_3} formed = 0.1

n_{salt} formed $= \frac{0.1}{2} \times \frac{50}{100} = 0.025$

$W_{salt} = 0.025 \times 406 = 10.15$ g

30) $K_4[Fe(CN)_6] + H_2SO_{4(Conc)} + H_2O \rightarrow K_2SO_4 + FeSO_4 + (NH_4)_2SO_4 + \underset{(P)}{CO} \uparrow$

$CO_{(g)} + C\ell_{2(g)} \xrightarrow{h\nu} \underset{(Q)}{COC\ell_{2(g)}}$



31) $Cr_2O_{7(aq)}^{2-} + 14H_{(aq)}^+ + 6e^- \rightarrow 2Cr_{(aq)}^{3+} + 7H_2O_{(\ell)}$

$$E = E^o - \frac{0.06}{6} \log \frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]^{14}}$$

$$E = E^o + 14(0.01) \log [H^+]$$

On changing pH from 0 to 14, $[H^+]$ changes from 1M to 10^{-14}M

$E_1 = E^oV$ and

$$E_2 = E^o - 14 \times 0.01 \times 14 = \left(E^0 - 1.96\right) V$$

$\therefore \Delta E = 1.96$ Volt

32) $2H_2O_2 \rightarrow 2H_2O + O_2$

Initial no.of moles of $H_2O_2 = 1$

$$t_{\frac{1}{2}} = \frac{0.693}{k} = 10 \text{ sec}$$

No.of moles of H_2O_2 after 20 sec = 1/4

$H_2O_2 + 2H^+ + 2I^- \rightarrow I_2 + 2H_2O$

$\Delta H^o_r = [60 + 2(-290)] - [-200 + 2(0) + 2(-60)]$

= – 520 + 320

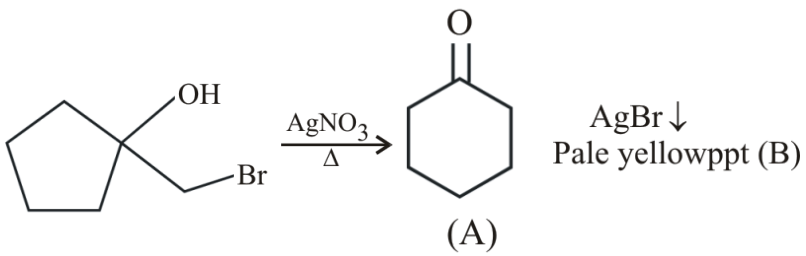
= – 200 kJ

Amount of heat released $= 200 \times \frac{1}{4} = 50\,kJ$

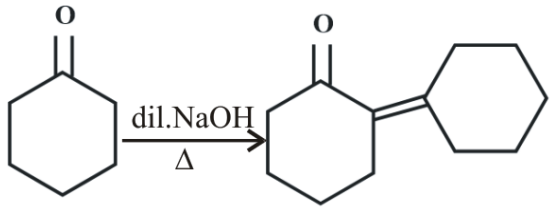
33) The following species are paramagnetic

$O_2, S_2, O_2^+, N_2^-, B_2, NO_2, C\ell O_2$

34)



(A)



(C)

$$AgBr + Na_2S_2O_3 \longrightarrow \underset{\text{(White ppt)}}{Ag_2S_2O_3\downarrow} + NaBr$$

$$\downarrow H_2O/\Delta$$

$$\underset{\text{(Black)}}{Ag_2S\downarrow} + H_2SO_4$$

(E) (F)

X = 4 ; Y = 0

PART-3 : MATHEMATICS

35) $g(x) = x^2 + ax + 2, h(x) = x^2 + bx + c$

$f + g = 2x^2 + (a-3)x - 5$ → r, t

$f + g = 2x^2 + (b-3)x + c - 7$ → r, s

$g + h = 2x^2 + (a+b)x + c + 2$ → s, t

$r + t = \frac{3-a}{2}$ ..... (i)

$r + g = \frac{3-b}{2}$ ..... (ii)

$s + t = -\frac{(a+b)}{2}$ ..... (iii)

(i) - (iii)

$r - s = \frac{3-b}{2}$

$r = \frac{3}{2}$ & $t = \frac{-5}{3}, rs = \frac{c-7}{2}$

$st = \frac{c+2}{2}$

$c = h(0) = \frac{52}{19}$

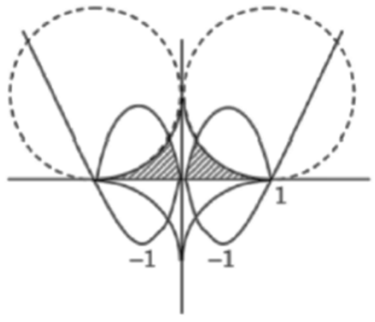
36) Volume will be independent of origin shifting $5|x| + 10|y| + |z| = 10$

v = 8[Volume of 5x + 10y + z = 10 in first octant]

$v = 8 \times \frac{1}{6}[1.2.10] = \frac{80}{3}$

37) Required area is

$$2\left[-\int_0^{\alpha} ex \ln x \, dx + \int_1^{\alpha} \left(1 - \sqrt{1-(x-1)^2}\right) dx\right]$$



38) $\sqrt{3a+13} = x$, $\sqrt{3b+13} = y$, $\sqrt{3x+13} = z$

$(x+y+z)^2 \leqslant 3\sum x^2$ (R.M.S $\geqslant$ A.M)

$(x+y+z)^2 \leqslant 3\left(3\sum a + 39\right)$

$x + y + z \leqslant 12\sqrt{3}$

39) $|2A^3| = 2^{12} \Rightarrow |A|^3 = 2^9 \Rightarrow |A| = 8$

$\Rightarrow (\alpha+\beta)^2(\alpha-\beta) = 8$

$(\alpha+\beta)^2 = 4 \ \& \ \alpha-\beta = 2$

$\Rightarrow \alpha+\beta = \pm 2$ and $\alpha-\beta = 2$

$\Rightarrow \alpha = 2, \beta = 0$ or $\alpha = 0, \beta = -2$

40) $$f(x) = \int_0^{3\pi/4} \left| x - \frac{3\sin t}{\cos t + \sqrt{2}} \right| \left(\frac{1+\sqrt{2}\cos t}{(\cos t+\sqrt{2})^2} \right) dt$$

$$f(x) = \int_0^1 |x-3u|\,du \Rightarrow f(x) = \begin{cases} \frac{3}{2}-x & ; \ x \leqslant 0 \\ \frac{x^2}{3}-x+\frac{3}{2} & ; \ 0 \leqslant x \leqslant 3 \\ x-\frac{3}{2} & ; \ x \geqslant 3 \end{cases}$$

41) $$x^2\frac{dy}{dx} = y^2 e^{\frac{1}{x}} \Rightarrow \frac{dy}{y^2} = \frac{e^{\frac{1}{x}}}{x^2}dx$$

Integrating both sides,

$$\frac{-1}{y} = \int \frac{e^{\frac{1}{x}}}{x^2}dx + C$$

$\Rightarrow$ Putting $\frac{1}{x} = t \Rightarrow \frac{-1}{x^2}dx = dt \Rightarrow -\frac{1}{y} - \int e^t dt + C \Rightarrow$

$$\frac{-1}{y} = e^{-} + C \Rightarrow \frac{-1}{y} = -e^{\frac{1}{x}} + C$$

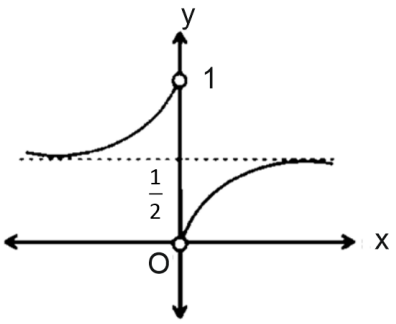
$\because \lim_{x\to 0^-} f(x) = 1 \Rightarrow -1 = 0 + C \Rightarrow C = -1$

$$\frac{-1}{y} = -e^{\frac{1}{x}} - 1 \Rightarrow y = \frac{1}{1+e^{\frac{1}{x}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(1+e^{\frac{1}{x}}\right)^2} e^{\frac{1}{x}} \left(\frac{-1}{x^2}\right) = \frac{e^{\frac{1}{x}}}{x^2\left(1+e^{\frac{1}{x}}\right)^2}$$

$\because \frac{dy}{dx} > 0 \ \forall x \in R - \{0\}$

$\lim_{x\to\pm\infty} \frac{1}{1+e^{\frac{1}{x}}} = \frac{1}{2}$ and $\lim_{x\to 0^+} \frac{1}{1+e^{\frac{1}{x}}} = 0$



Graph of the function is From the graph option (A), (B) and (D) are correct.

42)

We can determine the value of $P_r(L,N)$ as $= \dfrac{\text{coefficient of } x^L \text{ in } \left(x^1+x^2+x^3+.....x^r\right)^N}{r^N}$

The value of $P_3(5,2) = P_3(5,3) = \dfrac{2}{9}$; $P_9(15,2) = \dfrac{4}{81}$

$P_6(14,3) = P_6(14,5) = \dfrac{5}{72}$; $P_3(10,4) = P_3(10,6) = \dfrac{10}{81}$

$P_4(9,3) = P_4(9,4) = \dfrac{5}{32}$

43) We can determine the value of $P_r(L,N)$ as $= \dfrac{\text{coefficient of } x^L \text{ in } \left(x^1+x^2+x^3+.....x^r\right)^N}{r^N}$

The value of $P_3(5,2) = P_3(5,3) = \dfrac{2}{9}$; $P_9(15,2) = \dfrac{4}{81}$

$P_6(14,3) = P_6(14,5) = \dfrac{5}{72}$; $P_3(10,4) = P_3(10,6) = \dfrac{10}{81}$

$P_4(9,3) = P_4(9,4) = \dfrac{5}{32}$

44) $\text{A. adj}\left(A^2\right) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\Rightarrow |A|^2 I = A \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ ..... (1)

$\therefore$ A. adj $\left(A^2\right) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ now determinant both sides

$|A| . \left|A^2\right|^2 = -1$

$\Rightarrow |A| = -1 \Rightarrow I = A \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\therefore B^2 - 3B^2 + B(1) + I(1) = 0$

$\Rightarrow B^{-1} = A = -B^2 + 3B - I$

45) $$A.\,adj\left(A^2\right)=\begin{bmatrix}1&0&2\\2&1&0\\1&0&1\end{bmatrix}$$

$$\Rightarrow |A|^2 I = A\begin{bmatrix}1&0&2\\2&1&0\\1&0&1\end{bmatrix} \quad \ldots\ldots\ldots (1)$$

$$\therefore A.\,adj\left(A^2\right)=\begin{bmatrix}1&0&2\\2&1&0\\1&0&1\end{bmatrix}$$ now determinant both sides

$$|A|.\left|A^2\right|^2=-1$$

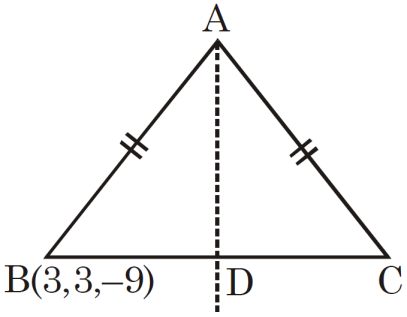
$$\Rightarrow |A|=-1 \Rightarrow I = A\begin{bmatrix}1&0&2\\2&1&0\\1&0&1\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}1&0&2\\2&1&0\\1&0&1\end{bmatrix}$$

$$\therefore B^2-3B^2+B(1)+I(1)=0$$

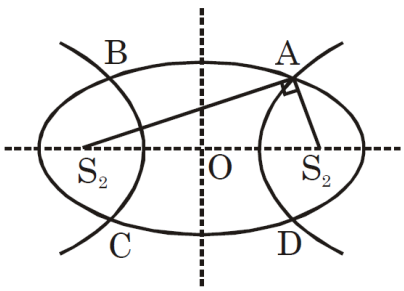
$$\Rightarrow B^{-1}=A=-B^2+3B-I$$

46) Let D is $(\lambda+1, 2\lambda+2, 3\lambda-3)$



$$\overrightarrow{BD}.(\hat{i}+2\hat{j}+3\hat{k})=0 \Rightarrow \lambda=-1$$

- D is mid point of BC
- C is (-3, -3, -3)



47)

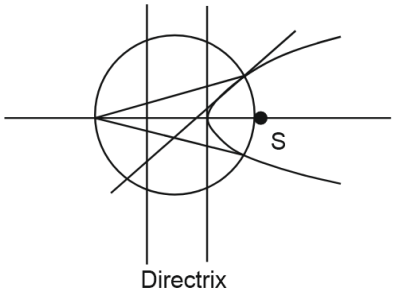
$AS_1 + AS_2 = 2a = 10$

$AS_2 - AS_1 = 2A = 6$

$AS_2 = 8$ & $AS_1 = 2$ & $S_1S_2 = \sqrt{68}$

for ellipse $2ae_1 = 2\sqrt{17} \Rightarrow e_1 = \frac{\sqrt{17}}{5}$

for hyperbola $2Ae_2 = 2\sqrt{17} \Rightarrow e_2 = \frac{\sqrt{17}}{3}$



48)

All lines are tangent to parabola $y^2 = 4x$

$x_1 = -1$ (Property of parabola)

$x_3 = 1$

49) Let $n^2 = 2^{62} \cdot 3^{38}$

Let F is a factor or $n^2 \Rightarrow 2^{\alpha}3^{\beta}$, $0 \leqslant \alpha < 62$; $0 \leqslant \beta \leqslant 38$

$\Rightarrow$ $n(F) = 6339 = 2457$

Let a, b are two factor, of n^2 such that a. b = n^2 when $a, b \neq n$

If a < n then b > n will exist in pairs

Number of factors less than n = 1228

Similarly if x and y are factors of n such that x. y = n and x and y are also factors of n^2

$\Rightarrow$ number of such factors = (32 × 20 –1) = 639

Req number of factors = 1228 – 639 = 589

A	B	A+B
$\bar{x}_1 = 40$	$\bar{x}_2 = 55$	$\bar{x} = 50$
$\sigma_1 = \alpha$	$\sigma_2 = 30 - \alpha$	$\sigma^2 = 350$
$n_1 = 100$	$n_2 = n$	$100 + n$

$$\bar{x} = \frac{100 \times 40 + 55n}{100 + n}$$

$$5000 + 50n = 4000 + 55n$$

$$1000 = 5n$$

50) $n = 200$

$$\sigma_1^2 = \frac{\sum x_i^2}{100} - 40^2$$

$$\sigma_2^2 = \frac{\sum x_i^2}{100} - 55^2$$

$$350 = \sigma^2 = \frac{\sum x_i^2 + \sum x_j^2}{300} - (\bar{x})^2$$

$$350 = \frac{\left(1600 + \alpha^2\right) \times 100 + \left[(30 - \alpha)^2 + 3025\right] \times 200}{300} - (50)^2$$

$2850 \times 3 = \alpha^2 + 2(30-\alpha)^2 + 1600 + 6050$
$8550 = \alpha^2 + 2(30-\alpha)^2 + 7650$
$= \alpha^2 + 2(30-\alpha)^2 = 900$
$= \alpha^2 - 40\alpha + 300 = 0$
$\alpha = 10, 30$
$\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$

51) $8\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right] = 800$
$\Rightarrow \quad [\vec{a}\,\vec{b}\,\vec{c}]^2 = 100$
$\vec{b}.\left(\vec{a}\times\left(\vec{b}\times\left(\vec{c}\times\vec{b}\right)\right)\right)$
$= \vec{b}.\left(\vec{a}\times\left(\left|\vec{b}\right|^2\vec{c} - -\left(\vec{b}.\vec{c}\right)\vec{b}\right)\right)$
$= \left|\vec{b}\right|^2\left[\vec{b}\,\vec{a}\,\vec{c}\right] - 0$
$\Rightarrow \quad \left|\vec{b}\right|^2 = 25$
$\left|\vec{b}\right| = 5$