FIITJEE RBT-4 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 3 | **QP CODE: 100957** | **PAPER - 1**

Time Allotted: 3 Hours **Maximum Marks: 204**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains **Four (04)** multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A** (**05–12)** contains Eight **(**8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **Part-B (1 – 8)** contains Eight **(08)** Numerical based questions, the answer of which maybe positive or negative numbers or decimals **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

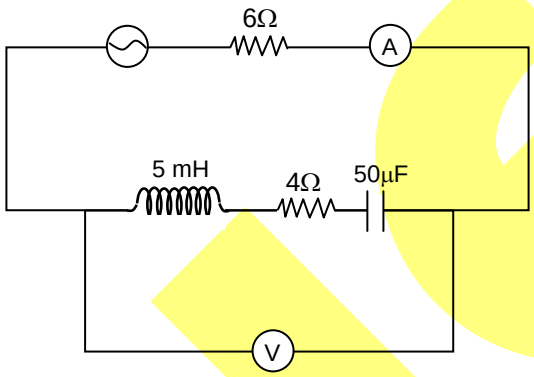
SECTION – I : PHYSICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A particle moves along the y-axes of a coordinate system, with a force component $F_y = (2\ N/m^3)y^3$ acting on it. As the particle moves from the origin to y = 3 m, how much work is is done on it by the force?
   (A) 0 J (B) 40.5 J
   (C) –40.5 J (D) 162 J

2. In the circuit shown in given below figure, the AC source gives a voltage V = 20 cos (2000 t). Neglecting source resistance, the voltmeter and ammeter reading will be: (approximately)



   (A) 0 V, 0.47 A (B) 1.68 V, 0.47 A
   (C) 0 V, 1.4 A (D) 5.6 V, 1.4 A

3. Two coils X and Y are placed in a circuit such that a current change of 3 A in coil X causes the change in magnetic flux by 1.2 Wb in coil Y. The value of mutual inductance of the coil is:
   (A) 0.2 H (B) 0.4 H
   (C) 0.6 H (D) 3.6 H

4. The temperature gradient in a rod of 0.5 m length is 80°C/m. If the temperature of hotter end of the rod is 30°C, then the temperature of the cooler end is:
   (A) 40°C (B) –10°C
   (C) 10°C (D) 0°C

Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having a mass m and charge –q, is kept at the middle point of line AB. It displaced through a distance x perpendicular to AB. Assume x << d. Then choose the correct option(s).
(A) Force experienced by C is proportional to x
(B) Force experienced by C is proportional to d
(C) Particle C may execute SHM with time period $\left(\frac{m\pi^3\varepsilon_0 d^3}{qQ}\right)^{1/2}$
(D) Particle C may execute SHM with time period $\left(\frac{m\pi^2\varepsilon_0 d^2}{qQ}\right)^{1/2}$

6. The equivalent resistance of a group of resistances is R. If another resistance is connected in parallel to the group, its new equivalent becomes R_1 and if it is connected in series to the group, its new equivalent becomes R_2 we have:
(A) $R_1 > R$ (B) $R_1 < R$
(C) $R_2 > R$ (D) $R_2 < R$

7. Radiations of monochromatic waves of wavelength 400 nm are made incident on the surface of metals Zn, Fe and Ni of work functions 3.4 eV, 4.8 eV and 5.9 eV respectively. Which of the following is (are) correct?
(A) Maximum KE associated with photoelectrons from the surface of any metal is 0.3 eV.
(B) No photoelectron are emitted from the surface of Ni.
(C) If the wavelength of source of radiation is doubled then KE of photoelectrons is also doubled.
(D) Photoelectrons will be emitted from the surface of all the three metals if the wavelength of incident radiations < 200 nm.

8. A diverging lens of focal length f_1 is placed in front of and coaxially with a concave mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement.
(A) The beam diameters of the incident and reflected beam must be the same.
(B) $d = 2|f_2| - |f_1|$
(C) $d = 2|f_1| - |f_2|$
(D) If the entire arrangement is immersed in water, the conditions will remain unaltered.

Space For Rough Work

9. A positive charge is passing through an electromagnetic field in which $\vec{E}$ & $\vec{B}$ are directed towards y-axis and z-axis respectively. If a charge particle passes through the region undeviated, then its velocity may be represented by (here a, b & c are constant)

(A) $\vec{v} = \frac{E}{B}\hat{i} + a\hat{j}$ (B) $\vec{v} = \frac{E}{B}\hat{i} + b\hat{k}$

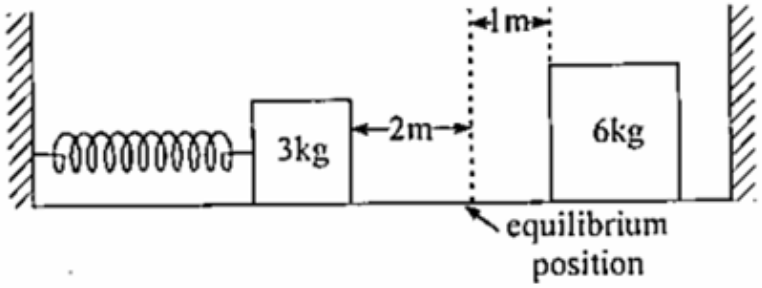
(C) $\vec{v} = \frac{E}{B}\hat{i} + c\hat{i}$ (D) $\vec{v} = \frac{E}{B}\hat{i}$

10. An ideal gas undergoes a process such that $P \propto \frac{1}{T}$. The molar heat capacity of this process is 33.24 J/mol K. For one mole, choose correct options. (R = 8.31)

(A) The work done by the gas is $2R\Delta T$ (B) Degree of freedom of the gas is 4.

(C) Degree of freedom of the gas is 3. (D) $\gamma = \left(\frac{C_P}{C_V}\right)$

11. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to a spring with a force constant k = 900 Nm^{-1} which is compressed 2 m from the equilibrium position as shown in given below figure. The 6 kg mass is at rest at 1 m from mean position. 3 kg mass strikes the 6 kg mass and the two stick together:



(A) Velocity of the combined masses immediately after the collision is 10 ms^{-1}

(B) Velocity of the combined masses immediately after the collision is 5 ms^{-1}

(C) Amplitude of the resulting oscillation is $\sqrt{2}$ m.

(D) Amplitude of the resulting oscillation is $\sqrt{5/2}$ m.

12. If x, v and a denote displacement, velocity and acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(A) $aT + 2\pi v$ (B) $\frac{aT}{v}$

(C) $a^2T^2 + 4\pi^2v^2$ (D) $\frac{aT}{x}$

Space For Rough Work

PART – B (Maximum Marks: 24)

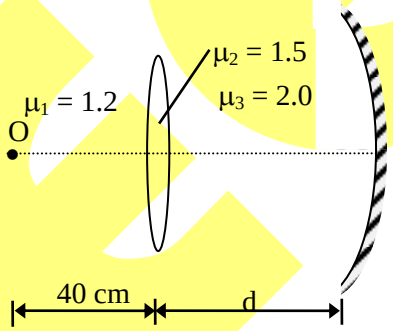
(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. A gas containing hydrogen like ions with atomic number Z, emits photons in transition $n + 2 \rightarrow n$, where n = Z. There photons fall on a metallic plate and eject electrons having minimum de-Broglie wavelength λ of 5000 Å. Find the value of 'Z' (nearest integer value) if the work function of metal is 4.2 eV.

2. Potential energy of a particle moving along x-axis is given by $U = \frac{x^3}{3} - \frac{9x^2}{2} + 20x$. Find out position (value of x) of stable equilibrium state.

3. The figure shows an arrangement of a equi-convex lens and a concave mirror. A point object O is placed on the principal axis at a distance 40 cm from the lens such that the final image is also formed at the position of the object. If the radius of curvature of the concave mirror is 80 cm. The distance d comes out to be 10 n cm. Find n. The focal length of the lens in air is 20 cm.



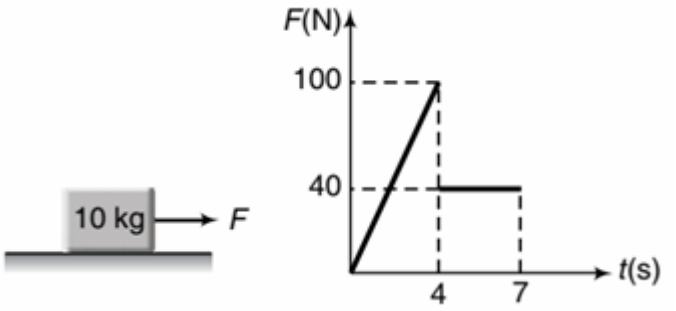
4. A ring of mass 3.15 kg is rolling without slipping with linear velocity 1 m/sec on a smooth horizontal surface. A rod of same mass is fitted along its one diameter. Find total kinetic energy of the system (in J).

5. A source of sound is travelling towards a stationary observer. The frequency of sound heard by the observer is three times of the original frequency. The velocity of sound v = 332.25 m/sec.

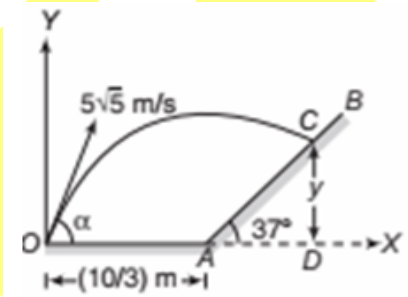
6. A light inextensible thread is wound round a solid cylindrical reel of mass m = 1.5 kg and radius a = 10 cm. The end of the string is held fixed and the reel is allowed to fall so that the thread unwinds. If the axis of the reel remains horizontal and tension is 20 n, value of 'n' is ($g = 10$ m/sec^2)

Space For Rough Work

7. The 10 kg block is resting on the horizontal surface when the force F is applied to it for 7s. The variation of F with time is shown. Calculate the maximum velocity reached by the block during which the block is in motion. The coefficients of static and kinetic friction are both 0.50. ($g = 9.8\ m/s^2$)



8. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}\ m/s$ at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of 37° with horizontal as shown in figure. If the particle does not rebound, calculate maximum height from the ground to which the particle rises. ($g = 10\ m/s^2$).



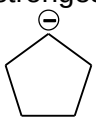
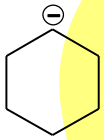
Space For Rough Work

SECTION – II : CHEMISTRY

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. Under which condition, the internal energy of a substance may be zero?
   (A) -273°C, 0.01 atm
   (B) 273°C, 1 atm
   (C) 298 K, 10^{-4} atm
   (D) None of these

2. The strongest base out of the following is
   (A) 
   (B) 
   (C) 
   (D) 

3. At constant volume, one mole of an ideal monoatomic gas was heated from 1400 to 1420 K.
   $\left[C_v = \frac{3R}{2}, R = 8.3\,JK^{-1}mol^{-1}\right]$
   Choose the incorrect statement
   (A) ΔU or ΔE = 249 J mol^{-1}
   (B) Q = 249 J mol^{-1}
   (C) Change in kinetic energy = 249 J mol^{-1}
   (D) Entropy change = 249 J K^{-1} mol^{-1}

4. The product of which reaction can form a conjugated diene when it is treated with alcoholic KOH?
   (A) $CH_3CH_2CH=CH \xrightarrow{Cl_2/CCl_4}$
   (B) $CH_3CH=CHCH_3 \xrightarrow{HCl}$
   (C) $CH_3CH_2CH=CHCH_3 \xrightarrow{Cl_2/CCl_4}$
   (D) $CH_3CH_2CH=CHCH_3 \xrightarrow{HCl}$

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Which property(s) of NH_3 and NF_3 is/are given in correct order?
   (A) Dipole moment: $NH_3 > NF_3$
   (B) Bond angle: $NH_3 > NF_3$
   (C) Lewis basicity: $NH_3 > NF_3$
   (D) Number of hydrogen bonds formed with H_2O: $NH_3 > NF_3$

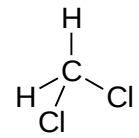
Space For Rough Work

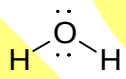
6. Crystal field theory is not applied on complex(es) like
(A) $[Ni(CO)_4]$ (B) $[Fe(CO)_5]$
(C) $[Zn(NH_3)_4]CO_3$ (D) $K_4[Fe(CN)_6]$

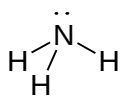
7. The boiling point of water in which aqueous solution(s) is/are higher than 100°C?
(A) 0.2 M $Al_2(SO_4)_3$ (B) 1 M CH_3OH in water
(C) 0.1 M NaCl (D) 1 M $NaCONH_2$

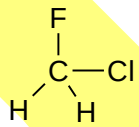
8. The first ionization enthalpies of two elements P and Q respectively are 8.1 and 10.2 eV. The electron gain enthalpies of P and Q are respectively -3.1 and -6.6 eV respectively. The percentage ionic character of the covalent bond P – Q is given as
% ionic character = $16(END) + 3.5(END)^2$
Where END is the electronegativity difference **in Mulliken scale** between P and Q.
Choose correct statements
(A) The electronegativity of P is 1.05
(B) Q is more electronegative than P
(C) percentage covalent character of P – Q bond is 80.5
(D) P – Q bond is purely covalent with 100% covalent character

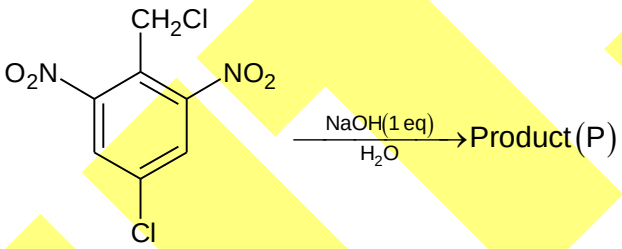
9. In which of the following molecule(s) the bond containing H and the central atom contains more than 25% s-orbital character?

(A) 

(B) 

(C) 

(D) 

10. 

Correct statement(s) regarding product(P) is
(A) it is an alcohol
(B) it is a phenol
(C) the longer C – Cl bond breaks in the reaction
(D) the NO_2 groups favours substitution of benzylic chloride

Space For Rough Work

11. Which of the following reactions produce(s) a ketone?

(A) $CH_3COOH \xrightarrow[2.Heat]{1.\ Ca(OH)_2}$

(B) $CH_3CH_2\overset{\overset{Cl}{|}}{\underset{\underset{Cl}{|}}{C}}CH_3 \xrightarrow{H_2O/OH^-}$

(C) $CH_3-\underset{Cl}{\underset{|}{CH}}-\underset{Cl}{\underset{|}{CH}}-CH_3 \xrightarrow[2.\ H_2SO_4/Hg^{2+}]{1.NaNH_2}$

(D) $CH_3-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_3 \xrightarrow{Cu/300^oC}$

12. $CH_3CH_2OH \xrightarrow{Cu/300^oC} (X) \xrightarrow[\Delta]{dil.NaOH} P$

$(X) \xrightarrow[(2)\ H^+]{(1)\ I_2/NaOH} Q + .PPt.$

$P \xrightarrow{NH_2OH/H^+} R$

$P \xrightarrow[2.H_3O^+]{1.CH_3MgBr} S$

Correct statement(s) regarding products of above reaction(s) is/are

(A) P, Q and R contain pi-bonds
(B) P and R exhibit geometrical isomerism
(C) R and S show optical isomerism
(D) The ppt. is yellow colour

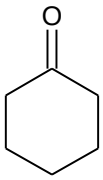
PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

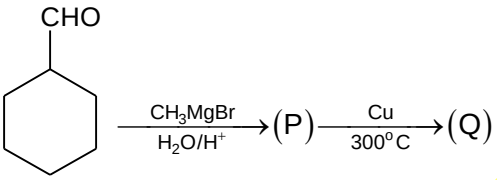
1. A dicarboxylic acid(P) forms a white precipitate when reacts with aqueous $Ca(OH)_2$ solution. If (P) is the simplest dicarboxylic acid. How many moles of gases are evolved when one mole of (P) is heated at 85°C.

Space For Rough Work

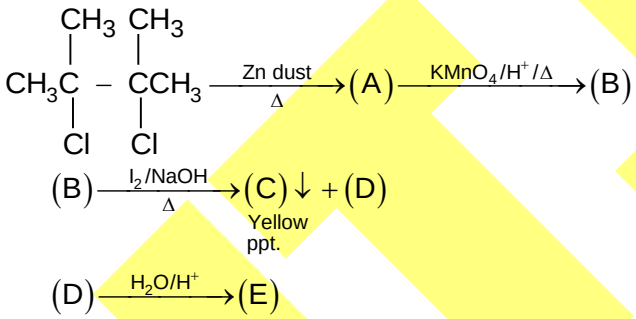
2. Two moles of  reacts with dilute NaOH to produce one mole of product(P). Heating of (P) forms (Q). Reaction of (Q) with Zn-Hg/Conc.HCl(excess) forms product(R)
If x = Number of C – C sigma bonds present in (R)
y = Number of C = C bonds present in (R)
z = Number of 3°-carbon atoms present in (R), then the value of (x + y + z) is

3. A vessel contains equal number of moles of CH_4 and SO_2 gases at a particular temperature. If the ratio of rates of effusion of CH_4 to SO_2 through an orifice made on the vessel, is expressed as x : y, the value of (x + y) is:

4. The p^{K_a} of the α-COOH, α-NH_2 and side chain COOH groups of an α-amino acid are respectively 2.1, 9.82 and 3.86. What is the pI of the amino acid?

5. 

If the number of hydrogen atom(s) is/are present in a molecule of (Q) is X. What is $\frac{X}{10}$?

6. 

If the number of oxygen atom(s) are present in (E) is y, what is the value of $\frac{y}{10}$?

7. The depression in freezing point of an aqueous solution of NaCl is 18.6. How many moles of NaCl is present in 200 g of solvent(water)?
[K_f of water = 1.86 K kg mol^{-1}]
[Assume complete dissociation of NaCl]

8. A metal crystallized in b.c.c unit cell. What will be the diameter of the metal atoms in pm unit if the edge-length of the unit cell is $\sqrt{3}$ pm?

Space For Rough Work

SECTION – Iii : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. If $f(x) = x^4 \tan(x^3) - x \ln(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is

   (A) 0 (B) 6
   (C) 12 (D) 24

2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = 0$, then incorrect statement is

   (A) $\vec{b}$ is in the plane of $\vec{a}$ and $\vec{c}$ (B) $\vec{b}$ is parallel to $\vec{a} \times \vec{c}$
   (C) angle between $\vec{a}$ and $\vec{c}$ is 30^o (D) angle between $\vec{a}$ and $\vec{c}$ is 150^o

3. Let a, b, c are distinct real numbers, not equal to one. If $ax + y + z = 0$, $x + by + z = 0$ and $x + y + cz = 0$ has a non – trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to:

   (A) –1 (B) 1
   (C) zero (D) none of these

4. The volume enclosed by the planes $|3x - 4| + |2y - 3| + |z + 4| = 3$ is

   (A) 4 (B) 6
   (C) 8 (D) None of these

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is

   (A) $\frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$ (B) $\frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$
   (C) $\frac{-\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$ (D) none of these

Space For Rough Work

6. If A and B are two events such that $P(A)=3/4$ and $P(B)=5/8$, then

(A) $P(A\cup B)\geq 3/4$ (B) $P(A'\cap B)\leq 1/4$

(C) $3/8\leq P(A\cap B)\leq 5/8$ (D) $3/8\leq P(A\cap B)\leq 5/8$

7. If $f(x)=2x-\sin x$ and $g(x)=\sqrt[3]{x}$, then

(A) range of gof is R (B) gof is one – one

(C) both f and g are one – one (D) both f and g are onto

8. $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$ equals

(A) $\lim\limits_{x\to 0}\dfrac{\ln(1+x)-x}{x^2}$ (B) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

(C) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}$ (D) $\lim\limits_{x\to 0}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x^2+2x}}}$

9. The solution/s of the equation $9\cos^{12}x+\cos^2 2x+1=6\cos^6 x\cos 2x+6\cos^6 x-2\cos 2x$ is/are

(A) $x=n\pi+\dfrac{\pi}{2}, n\in I$ (B) $x=n\pi\pm\cos^{-1}\sqrt[4]{\dfrac{2}{3}}, n\in I$

(C) $x=n\pi\pm\cos^{-1}\sqrt{\dfrac{2}{3}}, n\in I$ (D) $x=n\pi, n\in I$

10. If a straight line through a point $P(\alpha,2), \alpha\neq 0$, meets the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ at A and D and x – and y–axes at B and C respectively such that $PA\times PD=PB\times PC$, then 'α' can lie in the interval

(A) $[7,\infty)$ (B) $(-12,-8)$

(C) $(-5,0)$ (D) $(10,\infty)$

11. Tangent drawn at point P(1, 3) of a parabola intersects its tangent at vertex at M(–1, 5) and cuts the axis of parabola at T. If R(–5, 5) is a point on SP; where S is focus of the parabola, then

(A) slope of axis is –3

(B) radius of circumcircle of ΔSMP is $\sqrt{\dfrac{5}{2}}$ units

(C) $(ST)^2-(SM)^2=(PM)^2$

(D) tangent cuts the axis of parabola at T (–3, 7)

Space For Rough Work

12. Suppose that f(x) is a differentiable invertible function, $f(x) \neq 0$ and $h(x) = \int_1^x f(t)dt$. Given that $f(1) = f'(1) = 1$ and g(x) is inverse of f(x).,
Let $G(x) = x^2 g(x) - xh(g(x)) \ \forall x \in R$. Which of the following are correct:
(A) G'(1) = 2 (B) G'(1) = 3
(C) G"(1) = 2 (D) G"(1) = 3

PART – B (Maximum Marks: 24)

(Numerical **Type)**

This section contains ***Eight (08)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. Let $A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and $x = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$. If $P = AXB, Q = BX^T A$ and $Tr.\left((PQ)^{10}\right) = a^{10} + b^{10}$, where $a < b$, then find the value of $b - 2a$ _____
(Tr (M) denotes trace of Matrix M and M^T denotes transpose of matrix M)

2. Let $\beta = \int_0^1 \frac{2 + 3x + 4x^2}{2\sqrt{x^2 + x + 1}}$, then find the value of β^2.

3. Suppose that the circle $x^2 + y^2 - 4ax - 2ay + 4a^2 = 0 (a > 0)$ touches the lines $y = mx$ at A and B such that AB = 3. Then the value of $\frac{4a}{\sqrt{5}}$ is

4. If |z – 3| = Re z, |w – 3| = Re w and $\arg(z - w) = \frac{\pi}{4}$, then find the value of $Im(z + w)$.

5. The smallest value of 'a' for which the inequality $(a-1)x^2 - (a+1)x + (a-1) \geq 0$ holds for all $x \geq 2$ is k then the value of $\frac{3k}{14}$ is

6. If roots of $x^3 + ax^2 + bx - \frac{1}{9} = 0$ are in A.P. the $\frac{121}{16}(2a^3 - 9ab + 1)$ is _____.

7. If $\sum_{k=0}^{100} \left(\frac{k}{k+1}\right)^{100} C_k = \frac{a(2^{100}) + b}{c}$ where $a, b, c \in N$, then find the least value of $\frac{(a+b+c)}{670}$.

8. A straight line cuts the x-axis at point $A(1,0)$, and y-axis at point B, such that $\angle OAB = \alpha \left(\alpha > \frac{\pi}{4}\right)$. C is middle point of AB and B' is a mirror image of point B with respect to line OC and C' is a mirror image of point C with respect to line BB', then the ratio of the areas of triangles ABB' and BB'C' is equal to

Space For Rough Work

Q. P. Code: 100957

Answers

SECTION – I : PHYSICS

PART – A

1.	B	2.	D	3.	B	4.	B
5.	AC	6.	BC	7.	BD	8.	AB
9.	BD	10.	ABD	11.	AC	12.	CD

PART – B

1.	2	2.	5	3.	3	4.	5.25
5.	221.50	6.	0.25	7.	5.2	8.	4.45

SECTION – II : CHEMISTRY

PART – A

1.	D	2.	D	3.	D	4.	C
5.	ABCD	6.	AB	7.	ACD	8.	BC
9.	AD	10.	ACD	11.	ABC	12.	ABD

PART – B

1.	2	2.	15	3.	3	4.	2.98
5.	1.4	6.	0.2	7.	1	8.	1.5

SECTION – III : MATHEMATICS

PART – A

1.	A	2.	B	3.	B	4.	B
5.	A	6.	ABCD	7.	ABCD	8.	BC
9.	AB	10.	ABD	11.	ABCD	12.	AD

PART – B

1.	3	2.	3	3.	3	4.	6
5.	0.50	6.	30.25	7.	0.30	8.	2.00

Answers & Solutions

SECTION – I : PHYSICS

PART – A

1. **B**

Sol. $W = \int_0^3 F dy = 2\int_0^3 y^3 dy = \frac{1}{2}\left[y^4\right]_0^3 = \frac{81}{2} = 40.5$

2. **D**

Sol. Circuit impedance for series RLC circuit is given as

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$R = 10\Omega$, $X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\ \Omega$.

$$X_C = \frac{1}{\omega_C} = \frac{10^6}{2000 \times 5}$$

As $X_L = X_C$ so circuit is in resonance and current in circuit is given as

$$i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2A$$

$$\Rightarrow\ i_{rms} = \frac{2}{\sqrt{2}} = 1.41\,A$$

Voltmeter reading is equal to the voltage across the resistance as at resonance phasor voltage of capacitor and inductor gets cancelled out

$V_{rms} = i_{rms} R = 4 \times 1.41 = 5.64$ V

3. **B**

Sol. Flux change in coil Y is related to current change in coil X as

$\phi = Mi$

$\Rightarrow\ d\phi = M\, di$

$\Rightarrow\ M = \frac{d\phi}{di}$

$\Rightarrow\ M = \frac{1.2}{3} = 0.4$ H

4. **B**

Sol. Temperature gradient

$$\frac{dT}{dx} = 80°C/m$$

$$\Rightarrow\ \int_{30}^{T} dT = -80\int_0^{0.5} dx$$

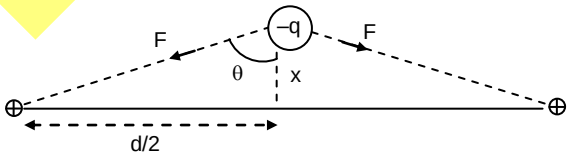
$\Rightarrow\ T - 30 = -80(0.5)$

$\Rightarrow\ T - 30 = -40$

$\Rightarrow\ T = -10°C$

5. **AC**

Sol.



$$F_{net} = 2F\cos\theta = 2\frac{k\,Qqx}{\left(\frac{d^2}{4}+x^2\right)^{\frac{3}{2}}}$$

If x << d

Acceleration $\bar{a} = -\frac{2\,kQq}{m\left(\frac{d}{2}\right)^3}\bar{x}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md^3}{16}\frac{4\pi\varepsilon_0}{Q\,q}} = \left(\frac{m\pi^3\varepsilon_0 d^3}{q\,Q}\right)^{\frac{1}{2}}$$

6. BC

Sol. Suppose resistance connected is r.

R2 = R + r > R

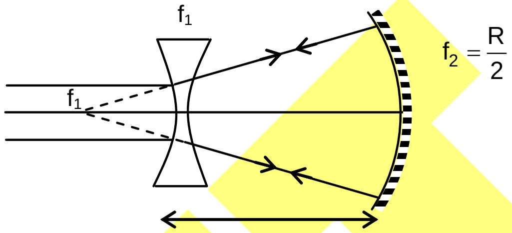
7. **BD**

Sol. $\Delta E = \frac{hc}{\lambda} = \frac{12375}{4000}$ = 3.09375 ≃ 3.1 eV.

For photoelectric effect, ΔE should be greater than work function.

8. **AB**

Sol.



$|R| = d + |f_1|$

$d = |2f_2| - |f_1|$

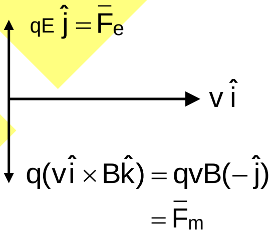
9. **BD**

Sol. $q\bar{E} + q(\bar{v}\times\bar{B}) = 0$

For, $\bar{E}, \bar{v}, \bar{B}$ mutually perpendicular.

qE = –qvB

$v = \frac{E}{B}$



$$q\bar{E}\,\hat{j} + q\left[(v_x\hat{i}+v_y\hat{j}+v_z\hat{k})\times B(\hat{k})\right] = 0$$

$$\Rightarrow \quad q(E - v_xB)\hat{j} + v_yBq\,\hat{i} = 0$$

$$\Rightarrow \quad v_x = \frac{E}{B} \text{ and } v_y = 0$$

$$\therefore \quad \bar{v} = \frac{E}{B}\hat{i} + b\,\hat{k}$$

10. **ABD**

Sol. Given PT = Constant

and $PV = nRT$

$\Rightarrow$ $P^2V = \text{constant}$

$\Rightarrow$ $PV^{1/2} = k$

From first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow$ $C = C_V + \left(\frac{R}{1-x}\right)$

$C = C_V + 2R$

$33.24 = \frac{R}{\gamma - 1} + 2R$

$\Rightarrow$ $\gamma = 1.5$

and $\gamma = 1 + \frac{2}{f}$

$\Rightarrow$ $f = 4.$

11. **AC**

Sol. Just before collision

$v = \omega\sqrt{A^2 - x^2}$

$v = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2} = \sqrt{\frac{900}{3}}\left[\sqrt{(4-1)}\right] = 30$

COM

$3 \times 30 = 9\ v'$

$\Rightarrow$ $v = 10$ (velocity just after collision)

When 3 kg mass is released the amplitude of its oscillation is 2 m and at a distance 1 m from the equilibrium position we can find the speed of it using the relation $v = [(k/m)(A^2 - x^2)]^{1/2}$ then by conservation of momentum we can find the resulting speed of the combined mass and the new amplitude using the above relation which gives options (A) and (C) are correct.

12. **CD**

Sol. $a = -\omega^2 x,\ v = \omega\sqrt{A^2 - x^2}$

$a^2T^2 + 4\pi^2v^2 = \omega^4x^2T^2 + 4\pi^2\omega^2(A^2 - x^2)$

$= \frac{4\pi^2}{T^2}\omega^2x^2T^2 + 4\pi^2\omega^2(A^2 - x^2)$

$= 4\pi^2\omega^2A^2 = \text{constant}$

$\frac{aT}{x} = -\frac{\omega^2xT}{x} = -\omega^2T = \text{constant.}$

PART – B

1. **2**

Sol. $\Delta E = 4.2 + \frac{12375}{5000} = 6.675$

$\because$ $\Delta E = 13.6\ Z^2\left(\frac{1}{n^2} - \frac{1}{(n+2)^2}\right) = 13.6\left[1 - \frac{1}{\left(1+\frac{2}{Z}\right)^2}\right]$

$$\Rightarrow \frac{1}{2} \simeq \left[1-\frac{1}{\left(1+\frac{2}{Z}\right)^2}\right] \Rightarrow \frac{1}{\left(1+\frac{2}{Z}\right)^2}=\frac{1}{2} \Rightarrow 1+\frac{2}{Z}=\sqrt{2}$$

$$\Rightarrow \frac{2}{Z}=0.41 \Rightarrow Z=\frac{2}{0.41} \simeq 5.$$

2. **5**

Sol. At stable equilibrium, U is minimum.

$$\frac{dU}{dx}=0 \text{ and } \frac{d^2U}{dx^2}>0$$

$$\Rightarrow \frac{1}{dx}\left(\frac{x^3}{3}-\frac{9x^2}{2}+20x\right)=0.$$

$\Rightarrow \quad x^2 - 9x + 20 = 0. \Rightarrow (x - 5)(x - 4) = 0.$

x = 5 and x = 4 are points of equilibrium.

And U minimum when $\frac{d^2U}{dx^2} > 0$. i.e. at x = 5.

3. **3**

Sol. $$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2}$$

$$\frac{2}{-(80-d)}-\frac{1.2}{-40}=\frac{1.5-1.2}{20}+\frac{2-1.5}{-20}$$

$$\Rightarrow \quad \frac{2}{-(80-d)}+\frac{1.2}{40}=\frac{0.3}{20}+\left(\frac{-0.5}{20}\right)$$

$$\frac{1.2}{40}+\frac{0.2}{20}=\frac{2}{(80-d)}$$

$$\frac{1.6}{40}=\frac{2}{(80-d)}$$

$$80 - d = 50$$

$$d = 30$$

$\Rightarrow n = 3.$

4. **5.25**

Sol. $$k=\frac{1}{2}mv_{cm}^2+\frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2+\frac{1}{2}\left\{\frac{m(2r)^2}{12}\right\}\left(\frac{v}{r}\right)^2+\frac{1}{2}mv_{rod}^2$$

$$=\frac{1}{2}mv_{cm}^2+\frac{1}{2}mv_{cm}^2+\frac{1}{2}mv_{cm}^2\times\frac{1}{3}+\frac{1}{2}mv_{cm}^2$$

$$=\frac{1}{2}\times 3.15\times 1\left[1+1+1+\frac{1}{3}\right]=\frac{3.15}{2}\times\frac{10}{3}=5.25$$

5. **221.50**

Sol. We use

$$n'=n\left(\frac{v}{v-v_s}\right)$$

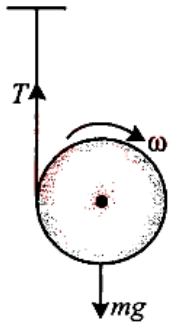
$$\Rightarrow \quad \frac{n'}{n}=\frac{v}{v-v_s}$$

$$\Rightarrow \quad \frac{v}{v - v_s} = 3$$

$$\Rightarrow \quad v_s = \frac{2v}{3}.$$

6. **0.25**

Sol. Let T be the tension in the thread and f, the linear acceleration of the reel as it falls.



For the downward translation,

$$(mg - T) = mf \qquad \ldots(1)$$

For the rotational motion of the reel, angular acceleration is

$$Ta = \frac{ma^2}{2}\alpha$$

$$T = \frac{ma\alpha}{2} = \frac{mf}{2} \qquad \ldots(2)$$

From equation (1) and (2)

$$\Rightarrow \quad 3T = mg$$

$$\therefore \quad T = \frac{mg}{3} = 1.5 \times \frac{10}{3} = 5 \text{ N}.$$

7. **5.2**

Sol. Block will start moving at, $F = \mu mg$

or $25t = (0.5)(10)(9.8) = 49$ N

$\therefore$ $t = 1.96$ s

Velocity is maximum at the end of 4 second.

$$\therefore \quad \frac{dv}{dt} = \frac{25t - 49}{10} = 2.5t - 4.9$$

$$\therefore \quad \int_0^{v_{max}} dv = \int_{1.96}^{4} (2.5\,t - 4.9)dt$$

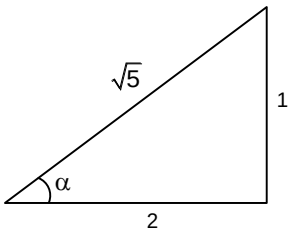
$$\therefore \quad v_{max} = 5.2 \text{ m/s}$$

8. **4.45**

Sol. Let, (x, y) be the coordinates of point C

$x = OD = OA + AD$

$$\therefore \quad x = \frac{10}{3} + y \cot 37^o = \frac{10 + 4y}{3} \qquad \ldots(i)$$



As point C lies on the trajectory of a parabola, we have

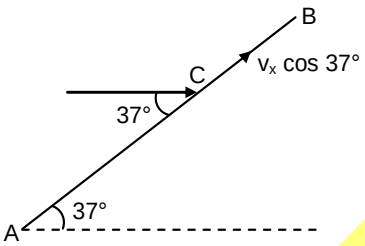
$$y = x \tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha) \qquad \ldots(ii)$$

Given that, $\tan\alpha = 0.5 = \frac{1}{2}$

Solving equations (i) and (ii), we get $x = 5$ m and $y = 1.25$ m.

Hence, the coordinates of point C are (5 m, 1.25 m).

Let v_y be the vertical component of velocity of the particle just before collision at C.



Using $v_y = u_y + a_y t$, we have

$v_y = u \sin\alpha - g\,(x/u\cos\alpha)$ $(\because\ t = x/u\cos\alpha)$

$$= \frac{5\sqrt{5}}{\sqrt{5}} - \frac{10\times 5}{(5\sqrt{5}\times 2/\sqrt{5})} = 0$$

Thus, at C, the particle has only horizontal component of velocity

$$v_x = u\cos\alpha = 5\sqrt{5}\times\left(\frac{2}{\sqrt{5}}\right) = 10\ \text{m/s}$$

Given, that the particle does not rebound after collision. So, the normal component of velocity (normal to the plane AB) becomes zero. Now, the particle slides up the plane due to tangential component $v_x \cos 37^o = (10)\left(\frac{4}{5}\right) = 8\ \text{m/s}$

Let h be the further height raised by the particle. Then

$$mgh = \frac{1}{2}m(8)^2 \quad \text{or} \quad h = 3.2\ \text{m}$$

Height of the particle from the ground = y + h

$\therefore$ H = 1.25 + 3.2 = 4.45 m

SECTION – II : CHEMISTRY

PART – A

1. D

Sol. The internal energy never becomes zero. Because even at 0 K, the vibration motions of the ideal solids exist.

2. D

Sol. Larger the bond angle of the cycloalkanes, stronger is their basic strength. Because the p-orbital character of carbon atom increases, if the bond angle increases. So due to less s-orbital character, the carbon atom can't strongly attract the negative charge or electron cloud.

3. D

Sol. $\Delta V = 0,\ \therefore\ W = 0$

So, $\Delta U = q = nC_v\Delta T = 1\times\frac{3R}{2}\times(1420-1400) = 249\,J\ mol^{-1}$

$\Delta K.E = \frac{3}{2}nR(\Delta T) = \frac{3}{2}\times 1\times 8.3\times 20 = 249\ J\ mol^{-1}$

$\Delta S = nCv\ \ell n\frac{T_2}{T_1} = 0.176\ J\ K^{-1}\ mol^{-1}$

4. C

Sol. $CH_3CH_2CH=CHCH_3 \xrightarrow{Cl_2/CCl_4} CH_3CH_2\underset{Cl}{\overset{|}{C}}H\underset{Cl}{\overset{|}{C}}HCH_3 \xrightarrow{Alc.KOH} CH_3CH=CH-CH=CH_2$

5. ABCD

Sol. In NH_3, bond pair dipole-moment and lone-pair dipole moment are along same direction. In NF_3, they are in opposite direction.

6. AB

Sol. CFT based on electrostatic interaction between metal ions and ligands. In the given metal carbonyls, neither metal nor ligand carry charges. Though CO carries partial positive charge, but the metal carries no charge, hence no electrostatic attraction takes place between them.

7. ACD

Sol. The boiling point of water increases by adding non-volatile solutes.

8. BC

Sol. E.N of P = $\frac{8.1+3.1}{5.6} = 2$

E.N of Q = $\frac{10.2+6.6}{5.6} = 3$

% ionic character = $16(3-2) + 3.5(3-2)^2 = 19.5$

% covalent character = $100 - 19.5 = 80.5$

9. AD

Sol. $\cos\theta = \frac{S}{S-1}$

Where θ = bond angle containing hydrogen and central atom

S = % s-orbital character.

In H_2O, $\theta = 104.5^o$

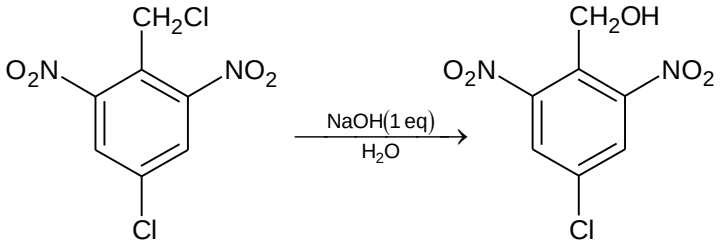
$\cos 104.5^o = -0.25 = \frac{S}{S-1}$

On solving S = 0.2, hence the O – H bond contain 20% s-orbital character and 80% p-orbital character.

$\therefore$ (B) is not correct similarly (C) will not be correct.

10. ACD

Sol.



11. ABC

Sol. $CH_3COOH + Ca(OH)_2 \longrightarrow Ca(CH_3COO)_2 \xrightarrow{\Delta} CH_3COCH_3 + CaCO_3$

$CH_3 - \underset{Cl}{CH} - \underset{Cl}{CH} - CH_3 \xrightarrow{NaNH_2} CH_3 - C \equiv C - CH_3 + CH_3CH_2C \equiv CH$

Both product will give $CH_3COCH_2CH_3$ upon hydrolysis with H_2SO_4/Hg^{2+}.

12. ABD

Sol. X = CH_3CHO

P = $CH_3 – CH = CH – CHO$

Q = HCOOH, PPt. = CHI_3

R = $CH_3 – CH = NOH$

S = CH_3COCH_3

PART – B

1. 2

Sol. (P) $\underset{COOH}{\overset{COOH}{|}} \xrightarrow[85^oC]{\Delta} CO(g) + CO_2(g) + H_2O(\ell)$

2. 15

Sol. x = 13, y = 0, z = 2

3. 3

Sol. When the moles are same

$$\frac{r_{CH_4}}{r_{SO_2}} = \sqrt{\frac{M_{SO_2}}{M_{CH_4}}} = \sqrt{\frac{64}{16}} = \sqrt{4} = 2$$

$\therefore$ x : y = 2 : 1 $\Rightarrow$ x + y = 3

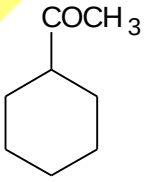
4. 2.98

Sol. $pI = \frac{1}{2}\left(p^{K_{a_1}} + p^{K_{a_2}}\right)$

$= \frac{1}{2}(2.1 + 3.86) = 2.98$

5. 1.4

Sol. Q is



6. 0.2

Sol. (A) is $(CH_3)_2C = C(CH_3)_2$

(B) is CH_3COCH_3
(C) = CHI_3
(D) = CH_3COONa
(E) = CH_3COOH

7. 1

Sol. $\Delta T_f = iK_f m = iK_f n \times \frac{1000}{W}$

or, $18.6 = 2 \times 1.86 \times n \times \frac{1000}{200}$

$\therefore$ n = 1

8. 1.5

Sol. $\sqrt{3}a = 4r = 2d$

or, $\left(\sqrt{3}\right)\left(\sqrt{3}\right) = 2d$

or, d = 3/2 = 1.5 pm

SECTION – III : MATHEMATICS

PART – A

1. A

Sol. As, f (x) = $x^4\tan(x^3) - x\ln(1 + x^2)$ is odd, $\frac{d^4f(x)}{dx^4} = 0$ at x = 0.

2. B

Sol. On expanding we get $(\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})(\vec{c}) = -\vec{b}$

$\Rightarrow \vec{c} - (\vec{a}.\vec{c})\vec{a} = \vec{b}$

$\Rightarrow$ (a) is correct and (b) is ruled out.

$\Rightarrow \left|\vec{c} - (\vec{a}.\vec{c})\vec{a}\right|^2 = \left|\vec{b}\right|^2$

If θ is the angle between $\vec{a}$ and $\vec{c}$ then

$4 + (2\cos\theta)^2 - 2(2\cos\theta)^2 = 1$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = 30^o; 150^o$

3. B

Sol. Since the system has non – trivial solution, therefore $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get $\Delta = \begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$

$\Rightarrow (a-1)(b-1)c - (1-c) + (1-b)(1-c) = 0$

$\Rightarrow (1-a)(1-b)c + (1-c) + (1-b)(1-c) = 0$

Dividing throughout by $(1-a)(1-b)(1-c)$, we get $\frac{c}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0$

$\Rightarrow -1 + \frac{1}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0$

$\Rightarrow \frac{1}{1-c} + \frac{1}{1-a} + \frac{1}{1-b} = 1.$

4. B

Sol. The volume is invariant under translation.

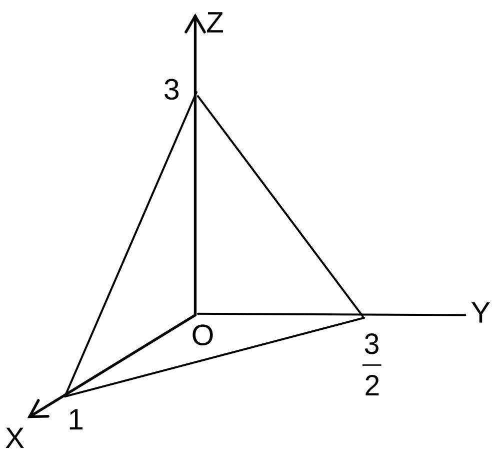
If $X = x - \frac{4}{3}, Y = y - \frac{3}{2}, Z = z + 4$, then the planes become $3|X| + 2|Y| + |Z| = 3$

The plane $3X + 2Y + Z = \pm3, -3X + 2Y + Z = \pm3.$

$3X + 2Y - Z = \pm3$, $3X + 2Y - Z = \pm3$ form an octahedron.

The plane $3X + 2Y + Z = 3$ forms a tetrahedron with coordinate planes in the positive octant of volume $\frac{1}{6}.1.\frac{3}{2}.3 = \frac{3}{4}$

Total volume $= 8 \times \frac{3}{4} = 6$



5. **A**

Sol. Integrating the given differential equation, we have $\frac{dy}{dx}=\frac{-\cos 3x}{3}+e^x+\frac{x^3}{3}+C_1$ but $y_1(0)=1$

so $1=\left(\frac{-1}{3}\right)+1+C_1\Rightarrow C_1=\frac{1}{3}$.

Again integrating, we get $y=\frac{-\sin 3x}{9}+e^x+\frac{x^4}{12}+\frac{1}{3}x+C_2$

but $y(0)=0$ so $0=0+1+C_2\Rightarrow C_2=-1$. Thus $y=\frac{-\sin 3x}{9}+e^x+\frac{x^4}{12}+\frac{1}{3}x-1$

6. ABCD

Sol. $A\subseteq A\cup B$

$\Rightarrow P(A)\le P(A\cup B)\Rightarrow P(A\cup B)\ge\frac{3}{4}$

Also,

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)$$
$$\ge P(A)+P(B)-1$$
$$=\frac{3}{4}+\frac{5}{8}-1=\frac{3}{8}$$

Now,

$$A\cap B\subseteq B$$

$\Rightarrow P(A\cap B)\le P(B)=\frac{5}{8}$

$\therefore \quad \frac{3}{8}\le P(A\cap B)=\frac{5}{8}$

and

$$P(A\cap B')=P(A)-P(A\cap B)$$

$\Rightarrow \frac{3}{4}-\frac{5}{8}\le P(A\cap B')\le\frac{3}{4}-\frac{3}{8}$

$\Rightarrow \frac{1}{8}\le P(A\cap B')\le\frac{3}{8}$

$\therefore P(A\cap B)=P(B)=P(A'\cap B)$ [Using Equation (1)]

$\Rightarrow \frac{3}{8}\le P(B)-P(A'\cap B)\le\frac{5}{8}$

$\Rightarrow 0\le P(A'\cap B)\le\frac{1}{4}$

7. ABCD

Sol. $(gof)(x)=(2x-\sin x)^{1/3}$

Range of (gof) (x) is R.

8. BC

Sol. $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$

$=\lim\limits_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\lim\limits_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{2}$

Option (A): $\lim\limits_{x\to0}\frac{\ln(1+x)-x}{x^2}=\lim\limits_{x\to0}\frac{\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots\right)-x}{x^2}=-\frac{1}{2}$

Option (B): $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}=\lim\limits_{x\to0}\dfrac{2\sin^2\dfrac{x}{2}}{4\times\left(\dfrac{x}{2}\right)^2}=\dfrac{1}{2}$

Option (C): $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-1}{x}=\lim\limits_{x\to0}\dfrac{(1+x)^{1/2}-(1)^{1/2}}{(1+x)-1}=\dfrac{1}{2}$

Option (D): $\lim\limits_{x\to0}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x^2+2x}}}=\lim\limits_{x\to0}\dfrac{1}{\sqrt{1+\sqrt{1+\dfrac{2}{x}}}}=\dfrac{1}{\sqrt{2}}$

9. AB

Sol. $9\cos^{12}x+\cos^2 2x+1-6\cos^6 x\cos 2x-6\cos^6 x+2\cos 2x=0$

$\Rightarrow \quad \left(3\cos^6 x-1-\cos 2x\right)^2=0$

$\Rightarrow \quad \cos^2 x\left(3\cos^4 x-2\right)=0$

$\Rightarrow \quad \cos x=0$

$x=n\pi+\dfrac{\pi}{2},\ n\in I$

or $\quad \cos^4 x=\dfrac{2}{3}$

$\cos x=\pm\sqrt[4]{\dfrac{2}{3}}$

$x=m\pi\pm\cos^{-1}\sqrt[4]{\dfrac{2}{3}},\ m\in I$

10. ABD

Sol. Let any point on the line through P be $x=\alpha+r\cos\theta,\ y=2+r\sin\theta$.

Put this in $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ and $xy=0$.

$PA\times PD=PB\times PC$

$\Rightarrow 2\alpha\sin 2\theta+5\cos 2\theta=13$

$\Rightarrow 13\le\sqrt{4\alpha^2+25}$

$\Rightarrow |\alpha|\ge 6$

11. ABCD

Sol. Equation of tangent at P is x+y=4

Clearly mirror image of R(–5,5) lies on line PQ.

Now mirror image R′ or R

$\Rightarrow\dfrac{\alpha'+5}{1}=\dfrac{\beta'-5}{1}=\dfrac{-2(-5+5-4)}{2}=4$

$\Rightarrow (\alpha',\beta')\equiv(-1,9)$

Let PM cuts the axis at T; As M is midpoint of PT

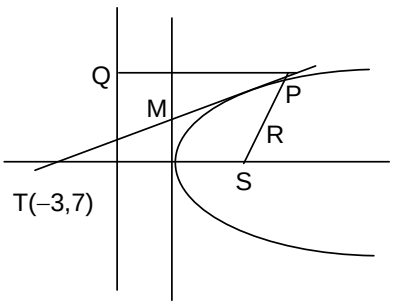
$\Rightarrow$ T is (–3, 7)

We know that SP = ST and $\angle SMP=\dfrac{\pi}{2}$

Equation SP $\equiv y-3=-\dfrac{1}{3}(x-1)$

$\Rightarrow$ x + 3y–10 = 0

Let S $\equiv$ (10 – 3β, β)



Again TS || PQ $\Rightarrow \frac{\beta-7}{13-3\beta}=\frac{9-3}{-1-1}=-3$

$\therefore$ focus is (–2, 4)

12. AD

Sol. $h'(x) = f(x) \Rightarrow h''(x) = f'(x)$
$h(1) = 0, f(1) = f'(1) = h'(1) = h''(1) = g(1)$
$f(g(x)) = x$
$f'(g(x)).g'(x) = 1$
$f'(g(1)).g'(1) = 1 \Rightarrow g'(1) = 1$
$G(x) = x^2.g(x) - x.h(g(x))$
$G'(x) = x^2 g'(x) + 2x.g(x) - h(g(x)) - x.h'(g(x)).g'(x)$
$= 2x.g(x) - h(g(x))$
$G''(x) = 2.g(x) + x.g'(x)$
$G'(1) = 2g(1) - h(g(1)) = 2g(1) - h(1) = 2$
$G''(1) = 2.g(1) + g'(1) = 3$

PART – B

1. 3

Sol. $XX^T = I$; Find PQ then $(PQ)^{10}$

2. 3

Sol. $\beta=\int_0^1 \frac{2x+3x^2+4x^3}{2\sqrt{x^4+x^3+x^2}}dx$

Put $\left(x^2+x^3+x^4\right)=t \Rightarrow \left(2x+3x^2+4x^3\right)dx=dt$

$\beta=\int_0^3 \frac{dt}{2\sqrt{t}}$

$\beta=\left[\sqrt{t}\right]_0^3=\sqrt{3} \Rightarrow \beta^2=3$

3. 3

Sol. $y=mx$ touches circle

$\Rightarrow D=0$ for $\left(m^2+1\right)x^2-2a(m+2)x+4a^2=0$

$\Rightarrow 4a^2(m+2)^2=4\left(m^2+1\right)\left(4a^2\right)$

$\Rightarrow m=0,\frac{4}{3}$

Let $A:(x_1,m_1x_1), B:(x_2,m_2x_2)$

$AB^2=9 \Rightarrow (x_1-x_2)^2+(m_1x_1-m_2x_2)^2=9$ where

$x_1=\frac{(m_1+2)}{m_1^2+1}a,\ x_2=\frac{(m_2+2)}{m_2^2+1}a,\ m_1=0,\ m_2=\frac{4}{3}$

get $a=\frac{3\sqrt{5}}{4}$

4. 6

Sol. Let $z=x_1+iy_1, w=x_2+iy_2$

$\therefore |z-3|=\text{Re}\,z \Rightarrow (x_1-3)^2+y_1^2=x_1^2$ ....(i)

$|w-3|=\text{Re}\,w \Rightarrow (x_2-3)^2+y_2^2=x_2^2$ ...(ii)

$\arg(z-w)=\frac{\pi}{4} \Rightarrow \frac{y_1-y_2}{x_1-x_2}=\tan\frac{\pi}{4}=1$ ...(iii)

By solving (i), (ii) and (iii) we get

$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{6}{y_1 + y_2}$

From (iii) $y_1 + y_2 = 6$

5. 0.50

Sol. $a\left(x^2 - x + 1\right) - \left(x^2 + x + 1\right) \geq 0$

$\Rightarrow a \geq \frac{x^2 + x + 1}{x^2 - x + 1}$ or $a \geq \frac{\left(x^2 - x + 1\right) + 2x}{x^2 - x + 1}$

$\Rightarrow a \geq 1 + \frac{2x}{x^2 - x + 1}$ or $a \geq 1 + \frac{2}{\left(x + \frac{1}{x}\right) - 1}$

Now, for $x \geq 2$, the minimum value of $x + \frac{1}{x}$ is $\frac{5}{2}$.

$\Rightarrow a \geq \frac{7}{3}$

6. 30.25

Sol. Let $\alpha - d, \alpha, \alpha + d$ are roots

$\therefore 3\alpha = -a$ (1)

$3\alpha^2 - d^2 = b$ (2)

$\alpha\left(\alpha^2 - d^2\right) = \frac{1}{9}$ (3)

From (1) and (2), $d^2 = \frac{a^2}{3} - b$

Put d^2 in (3) to get

$2a^3 - 9ab + 1 = 4$

7. 0.30

Sol. Let $S = \sum_{k=0}^{100} \left(\frac{k}{k+1}\right) {}^{100}C_k$

$= \sum_{k=0}^{100} \frac{\left((k+1) - 1\right)}{(k+1)} {}^{100}C_k = \left(\sum_{k=0}^{100} {}^{100}C_k\right) - \sum_{k=0}^{100} \frac{{}^{100}C_k}{k+1}$

$= 2^{100} - \frac{1}{101} \sum_{k=0}^{100} \left(\frac{101}{k+1} {}^{100}C_k\right)$

$= 2^{100} - \frac{1}{101} \sum_{k=0}^{100} {}^{101}C_{k+1}$

$2^{100} - \left(\frac{2^{101} - 1}{101}\right) = \frac{(101)2^{100} - 2^{101} + 1}{101} = \frac{99\left(2^{100}\right) + 1}{101}$

$= \frac{a\left(2^{100}\right) + b}{c}$ (Given)

So, $a = 99, b = 1, c = 101$

Hence, $(a + b + c)_{least} = 99 + 1 + 101 = 201.$

8. 2.00

Sol. $A \equiv (1,0)$, $B = \equiv (0, \tan\alpha)$, $C \equiv \left(\frac{1}{2}, \frac{\tan\alpha}{2}\right)$, $B' = \left(\frac{2\tan^2\alpha}{1+\tan^2\alpha}, \frac{\tan^3\alpha - \tan\alpha}{1+\tan^2\alpha}\right)$

$$\frac{\Delta ABB'}{\Delta BB'C} = 2$$