# FIITJEE

# ALL INDIA TEST SERIES

## FULL TEST – III

## JEE (Main)-2025

## TEST DATE: 09-01-2025

## ANSWERS, HINTS & SOLUTIONS

## Physics

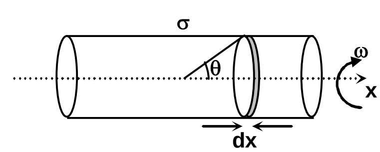
PART – A

SECTION – A

1. B

Sol. Conceptual

2. A




Sol. $dB = \dfrac{\mu_0 \dfrac{\omega}{2\pi}\sigma 2\pi R dx \cdot R^2}{2(R^2+x^2)^{3/2}} = \dfrac{\mu_0 \omega\sigma R^3}{2}\left\{\dfrac{dx}{(R^2+x^2)^{3/2}}\right\}$

$x = R\cot\theta \Rightarrow dx = -R\text{cosec}^2\theta\, d\theta$

$R^2 + x^2 = R^2\text{cosec}^2\theta$

$\Rightarrow dB = -\dfrac{\mu_0\omega\sigma R}{2}\left(\dfrac{d\theta}{\text{cosec}\theta}\right)$

$B = \left|\int dB\right| = \left|\dfrac{\mu_0\omega\sigma R}{2}\right|\left|\int_0^{\pi} -\sin\theta d\theta\right| = \mu_0\omega\sigma R$

3. C

Sol. Net force on the cylinder by the liquid is zero.

4. D

Sol. For outer sphere, $\dfrac{dQ}{dt} = \dfrac{IAe}{h\nu}$

Potential on inner sphere = 0

5. C

Sol. Let S be the dip

$$\Rightarrow \quad \tan\left[\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\right] = \frac{B\sin\delta}{B\cos\delta\cos 60^\circ}$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}} = 2\tan\delta$$

$$\Rightarrow \delta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

6. B

Sol. Magnetic field is in the direction of magnetic moment.

7. A

Sol. $B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{a+x} + \frac{1}{a-x}\right]$

8. C

Sol. Parallel to the reflecting surface, component of image's velocity is equal to the component of object's velocity.

9. A

Sol. $N \cdot \frac{\lambda D}{d} = \frac{(\mu - 1)\ell d}{d}$

$N = \frac{5}{3} = \left(1 + \frac{2}{3}\right)$

10. A

Sol. $M = \frac{\phi_B}{I} = \frac{BA}{I} = \left[ML^2T^{-2}A^{-2}\right]$

11. B

Sol. For very high $\omega$, $x_L \approx \infty$ and $x_C = 0$

12. B

Sol. Hint; friction provides both, centripetal as well as tangential acceleration.
when coin is about to slip on the disc,

$m\sqrt{\left(\omega^2 r\right)^2 + \alpha^2 r^2} = \mu mg$

On solving we get
T = 2s.

13. C

Sol. formula, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$\because \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = 2\sqrt{2}$

14. A

Sol. Fission of a nucleus is feasible only if the binding energy of daughter nuclei is more than the parent nucleus.
A = 55 will have more BE than 110.

A = 70 will have same BE as 110 but A = 40 will have more B.E.
A = 100 will have same BE as 110 but A = 10 will have lesser B.E.
A = 90 will have same BE as 110 but A = 20 will have lesser B.E.

15. D

Sol. formula, $\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_m} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\frac{1}{f} = \frac{1}{400\text{cm}}$

$\frac{1}{f_{eq}} = \frac{2}{400\text{cm}} + \frac{2}{25\text{cm}}$

$\Rightarrow f_{eq} = \frac{200\text{cm}}{17}$

Therefore, distance is $2f_{eq} = \frac{400\text{cm}}{17}$

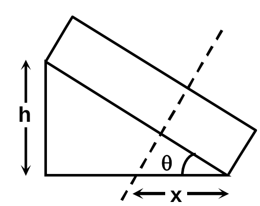
16. A

Sol. Number of emitted electrons $= \frac{9.6 \times 10^{-7} \times 2 \times 10^{-4} \times 25}{6 \times 1.6 \times 10^{-19} \times 10^{5}}$ = $5 \times 10^4$ electrons

q = + ne = $8 \times 10^{-15}$ C.

17. A

Sol. Consider an arbitrary moment when the wedge has travelled a distance x into region II.



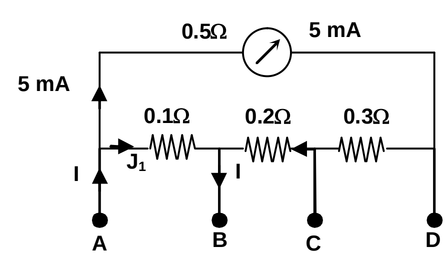

The area of the top surface inside the region II = ax sec θ
Force on it = ax sec θ ΔP = ax sec θ [ΔP =1]
Component of the force opposite velocity = ax sec θ. sin θ = ax tan θ.
If it further moves by dx then the work done = ax tanθ dx

$\therefore \quad \frac{1}{2}mv_0^2 = a\tan\theta \int_0^b x\,dx$

$\Rightarrow \quad v_0 = \sqrt{\frac{abh}{M}}$

18. B

Sol. $J_1 \times 0.1 = 5 \times I$
$J_1$ = 50 mA
I = 50 + 5 = 55 mA




19. C

Sol. No. of electrons hitting the target per second

$N = \frac{10 \times 10^{-3}}{e}$, $E = N \times 150 \times 10^3$ eV = 1500 J/s

Heat $= \frac{99}{100} \times \frac{1500}{402}$ cal/s

20. C

Sol. Effective length = $(r_2 + r_1)$

## SECTION – B

21. 13

Sol. $(Mg + T)\sin\theta = Ma$ …(i)

$T + ma\sin\theta - mg = 0$ …(ii)

From (i) and (ii)

$$a = \frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$$

22. 2

Sol. $\because \frac{E_0Z^2}{n^2} + \frac{hc}{\lambda} = 0$

$$\Rightarrow \frac{-13.6eV\times 4}{n^2} + \frac{1224eVnm}{90nm} = 0$$

$\Rightarrow n = 2$

23. 9

Sol. According to stefan's law, the power radiated by a black body at absolute temperature T is given by

$\theta = \sigma AT^4$ …(1)

According to wein's displacement law

$\lambda_m T = b$

$\Rightarrow T = \frac{b}{\lambda_m}$ …(2)

From (1) and (2)

$$\theta = \sigma A\left(\frac{b}{\lambda_m}\right)^4$$

$$= \frac{\sigma Ab^4}{\lambda_m{}^4}$$

For a sphere of radius r, $A = 4\pi r^2$

Hence $\theta = \frac{\sigma b^4 4\pi r^2}{\lambda_m^4} = K\frac{r^2}{\lambda_m^4}$

Where $K = 4\pi\sigma b^4$ is a constant.

Hence $\theta_1 = K\frac{4_1^2}{(\lambda_m^4)_1}$

$$\theta_2 = \frac{Kr_2^2}{(\lambda_m^4)_2}$$

$$\therefore \frac{\theta_1}{\theta_2} = \left(\frac{r_1}{r_2}\right)^2 \cdot (\lambda_m^4)_2$$

$$= \left(\frac{3}{5}\right)^2 \times \left(\frac{500}{300}\right)^4 = (5/3)^2$$

24. 11

Sol. formula, $\frac{dQ}{dt} = K\left(\overline{T} - T_S\right)$

$\because K\left(75 - T_S\right)t_1 = K\left(65 - T_S\right)t_2$

$\Rightarrow t_2 = 11\text{min}$

25. 98

Sol. formula, $f = \frac{n}{2L}V$

$\because \frac{f_0 + 5}{f_0 - 5} = \frac{1}{0.95}$

$\Rightarrow f_0 = 195\text{Hz}$

$\because f_0 = \frac{1}{2L}V$

$\Rightarrow L = \frac{V}{2f_0} = \frac{382.2}{2 \times 195} = 0.98\text{m}$

# Chemistry

## PART – B

### SECTION – A

26. B

Sol. $5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$
$5 \rightarrow 3 \rightarrow 1$
Total = 6

27. B

Sol. $6KMnO_4 + 10FeC_2O_4 + 24H_2SO_4 \longrightarrow 6MnSO_4 + 5Fe_2(SO_4)_3 + 20CO_2 + 3K_2SO_4 + 24H_2O$

Volume of $CO_2 = \frac{20}{6} \times 0.1 \times 22.4 = 7.47L$

28. C

Sol. Work done in free expansion is zero.

29. C

Sol. $W = \frac{108}{96500} \times 0.1 \times 965 = 0.0108$ g

30. B

Sol. $\pi = iCRT$
$= 2.4 \times 0.1 \times 0.0821 \times 300$
= 5.91 atm.

31. D

Sol. As per MOT $O_2$ has 2 unpaired electron in its antibonding $2p_\pi$ orbital.

32. A

Sol. Fructose, glucose and mannose give same osazone.

33. D

Sol. Molecule with formula [M(aa)bcde] type has 6 geometrical isomers and all are optically active.

34. C

Sol. $K_a = \frac{S_1^2 \times [H^+]_2}{S_2^2}$ Where, $S_1$ = solubility at pH = 7

$S_2$ = solubility at pH = 3

$[H^+]_2 = H^+$ ion concentration at pH = 3

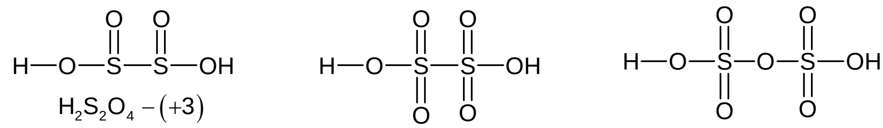
$K_a = \frac{10^{-8} \times 10^{-3}}{10^{-6}} = 10^{-5}$

$pK_a = 5$

35. D

Sol. Pairing should only start when all degenarate orbitals have at least 1 electron.

36. D

Sol. 

$H_2S_2O_4 - (+3)$ $H_2S_2O_6 - (+5)$ $H_2S_2O_7 - (+6)$

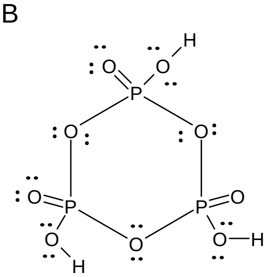
37. D

Sol. No conjugation with $-\ddot{N}H-$

38. A

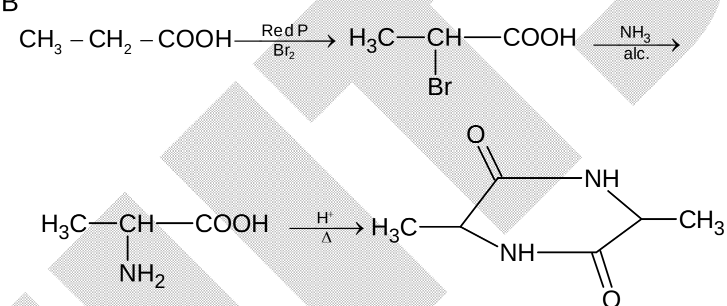
Sol. $$\log\frac{(K_{eq})_2}{(K_{eq})_1} = \frac{(E_f - E_b)}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

39. B

Sol. 

40. B

41. B

Sol. 

42. C

Sol. For hydrocarbons containing same carbon.

$$HOC \propto \frac{1}{\text{stability}}$$

More the number of $\alpha$-H, more will be stability.

43. D

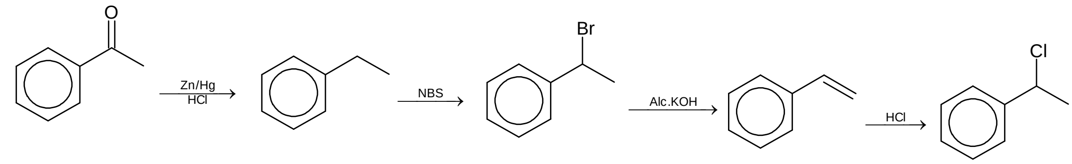
Sol. $MnO_2$ do allylic oxidation.

44. B

Sol. A, C and D will have plane of symmetry.

45. A

Sol.

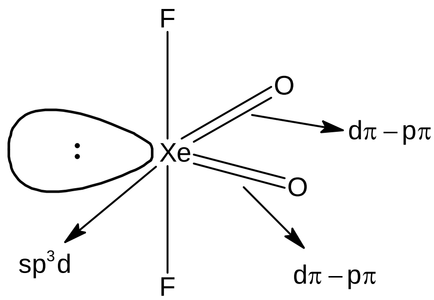



## SECTION – B

46. 33

Sol.

$XeO_2F_2 \equiv$




x = Number of d-orbitals used = 3

(one for hybridization + two for $\pi -$bonds)

$\therefore 11x = 33$

47. 894

Sol. $NH_3 + (NH_4)_2SO_4$ will form a basic buffer.

$[NH_3] = 0.1\,M$

$[NH_4^+] = 2 \times 0.1 = 0.2M$

$$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$$pOH = 4.76 + \log\frac{0.2}{0.1} = 5.06$$

$pH = 14 - 5.06 = 8.94$

Since, $Z = 8.94 \Rightarrow 100Z = 894$

48. 200

Sol.

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

| | $PCl_5(g)$ | $PCl_3(g)$ | $Cl_2(g)$ |
|---|---|---|---|
| Ini. | 3 | – | – |
| Equi. | $\frac{3}{4}(1-0.4)$ | $\frac{3\times0.4}{4}$ | $\frac{3\times0.4}{4}$ |

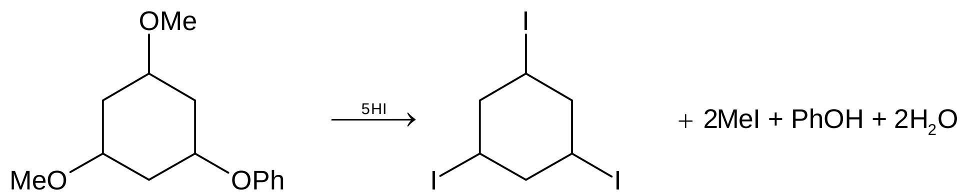
$$K_C = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{\frac{3\times0.4}{4}\times\frac{3\times0.4}{4}}{\frac{3}{4}\times(1-0.4)}$$

$$K_C = \frac{0.4\times3\times0.1}{0.6} = 0.2$$

$\therefore 1000K_C = 1000 \times 0.2 = 200$

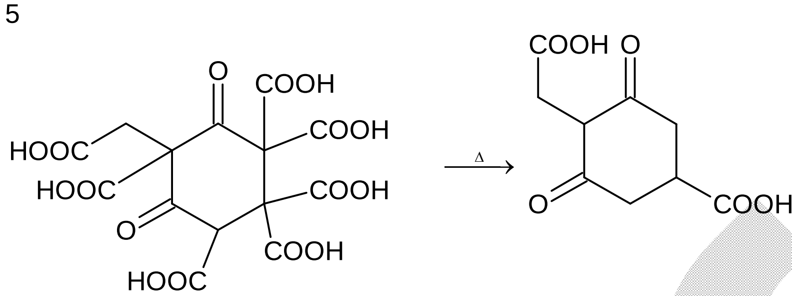
49. 5

Sol.




50. 5

Sol.



# Mathematics

## PART – C

### SECTION – A

51. C

Sol. 4cos18° – 3sec18° – 2tan18°

$$= \frac{2(1+\cos 36^\circ - \sin 18^\circ) - 3}{\cos 18^\circ} = 0$$

52. B

Sol. $$\frac{\sqrt{5}-1}{4} \times \frac{1}{\sin x} + \frac{\sqrt{10+2\sqrt{5}}}{4} \times \frac{1}{\cos x} = 2$$

$$\frac{\sqrt{5}-1}{4}\cos x + \frac{\sqrt{10+2\sqrt{5}}}{4}\sin x = 2\sin x \cos x$$

$$\sin\left(x + \frac{\pi}{10}\right) = \sin 2x = \sin(\pi - 2x)$$

$$x = \frac{\pi}{10}, \frac{3\pi}{10}$$

53. C

Sol. $$\sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{1+(r^2-5r+6)}\right)$$

$$= \sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{1+(r-2)(r-3)}\right)$$

$$= \sum_{r=2}^{\infty} \tan^{-1}\left(\frac{(r-2)-(r-3)}{1+(r-2)(r-3)}\right)$$

$$= \sum_{r=2}^{\infty} \tan^{-1}(r-2) - \tan^{-1}(r-3)$$

$$= \left(\tan^{-1}0 - \tan^{-1}(-1)\right) + \left(\tan^{-1}1 - \tan^{-1}0\right) + \left(\tan^{-1}(2) - \tan^{-1}1\right) + \tan^{-1}(n-2) - \tan^{-1}(4-3)$$

$$= \tan^{-1}(n-2) - \tan^{-1} - 1 = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

54. C

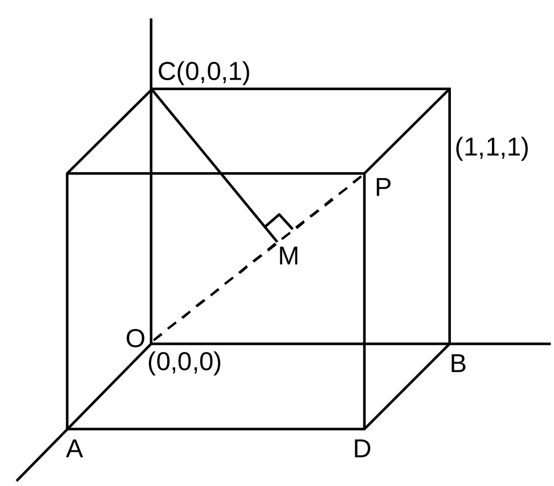
Sol. $\overrightarrow{OP} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{OC} = k$

$$OM = \frac{\overrightarrow{OP} \cdot \overrightarrow{OC}}{|\overrightarrow{OP}|}$$

$$OM = \frac{1}{\sqrt{3}}$$

$$CM^2 = OC^2 - OM^2 = 1 - \frac{1}{3} = \frac{2}{3}, \; CM = \sqrt{\frac{2}{3}}$$



55. B

Sol. Line is parallel to the plane as $\left(\hat{i}-\hat{j}+4\hat{k}\right)\cdot\left(\hat{i}+5\hat{j}+\hat{k}\right)=1-5+4=0$ from the plane

x + 5y + z – 5 = 0 is

$= \left|\frac{2-10+3-5}{\sqrt{1+25+1}}\right| = \frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$

56. D

Sol. $|\vec{c}-\vec{a}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c}$

$|\vec{c}\times\vec{a}| = |\vec{b}|$

$|\vec{c}||\vec{a}|\sin\theta = |\vec{b}|$

$|\vec{c}|2\sin\theta = 3$

$|\vec{c}| = \frac{3}{2\sin\theta} = \frac{3}{2}\text{cosec}\theta$

$|\vec{c}-\vec{a}|^2 = \frac{9}{4}\text{cosec}^2\theta + 4 - 4\frac{3}{2\sin\theta}\cos\theta$

$= \frac{9}{4}\text{cosec}^2\theta + 4 - 6\cot\theta$

$= \frac{9}{4}(1 + \cot^2\theta) + 4 - 6\cot\theta$

$= \frac{9}{4} + 4 + \frac{9}{4}\cot^2\theta - 6\cot\theta$

$= \frac{25}{4} + \frac{1}{4}(9\cot^2\theta - 24\cot\theta)$

$= \frac{25}{4} + \frac{9}{4}\left(\cot^2\theta - 2\frac{4}{3}\cot\theta + \frac{16}{9} - \frac{16}{9}\right)$

$= \frac{25}{4} - 4 + \frac{9}{4}\left(\cot\theta - \frac{4}{3}\right)^2$

$|\vec{c}-\vec{a}|^2 = \frac{9}{4} + \frac{9}{4}\left(\cot\theta - \frac{4}{3}\right)^2$

$|\vec{c}-\vec{a}| \ge \frac{3}{2}$

57. C

Sol. $T_n = \sum_{r=1}^{n} T_r - \sum_{r=1}^{n-1} T_r = \frac{n(n+1)(n+2)}{3}$

$\frac{1}{T_n} = \frac{3}{n(n+1)(n+2)} = 3\left(\frac{n+2-n}{n(n+1)(n+2)}\right)$

$\frac{1}{T_n} = \frac{3}{2}\left(\frac{1}{n(n+1)} = \frac{1}{(n+1)(n+2)}\right)$

$\frac{1}{T_n} = \frac{3}{2}(v_{n-1} - v_n)$ where $v_n = \frac{1}{(n+1)(n+2)}$

$\sum\frac{1}{T_n} = \frac{3}{2}(v_0 - v_n) = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$

$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{T_n} = \frac{3}{4}$

58. B

Sol. $x = \log_\lambda a = \log_a b = \log_b \sqrt{c}$

$x^3 = \log_\lambda a \,.\, \log_a b \,.\, \log_b \sqrt{c}$

$x^3 = \frac{\log a}{\log \lambda} \times \frac{\log b}{\log a} \times \frac{\log \sqrt{c}}{\log b} = \frac{1}{2}\frac{\log c}{\log \lambda}$

$x^3 = \frac{1}{2}\log_\lambda c \Rightarrow \log_\lambda c = 2x^3$

Now, $2x^3 = n(x)^{n+1}$ so $n = 2$

59. C

Sol. $10! = 2^8.3^4.5^2.7^1$

so sum of odd divisors is $\left(\frac{3^5-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)\left(\frac{7^2-1}{7-1}\right) = 30008$

60. C

Sol. $\left(1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)^{10} = \frac{\left(1+x+x^3+x^4\right)^{10}}{x^{30}}$

coefficient of $x^{30}$ in $(1 + x + x^3 + x^4)^{10}$

$= {}^{10}C_{10}\ {}^{10}C_0 + {}^{10}C_9\ {}^{10}C_3 + {}^{10}C_8\ {}^{10}C_6 + {}^{10}C_7\ {}^{10}C_9$

$= 1 + 1200 + 9450 + 1200$

$= 11851$

last digit of $(11853)^{11851} = 7$.

61. D

Sol. Since f(x) is odd function

so $f''(x)$ is also odd function

so $f(x) + f''(x)$ is odd function

so $\int_{-\pi/2}^{\pi/2} (1+x^4)(f(x)f''(x))dx = 0$

$2\lambda + 3 = 0,\ \lambda = -\frac{3}{2}$

62. A

Sol. $|A| = 1$

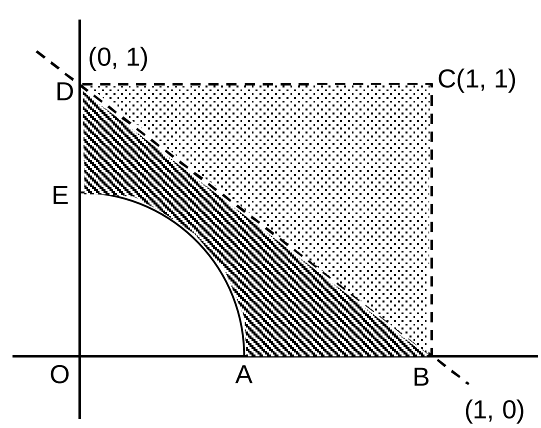
$= \det((|A| + 2)I) = (|A| + 2)^3 = (1 + 2)^3 = 27$

63. A

Sol. Prob $= \frac{\text{Area of region ABDE}}{\text{Area of region ABCDE}}$

$= \frac{\frac{1}{2}\times 1\times 1-\frac{1}{4}\pi\frac{1}{4}}{1\times 1-\frac{1}{4}\pi\frac{1}{4}} = \frac{8-\pi}{16-\pi}$

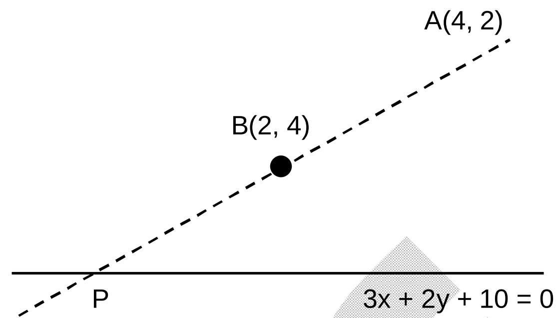
Prob. $= \frac{8-\pi}{16-\pi}$



64. A

Sol. It will be point P if A, B, P are collinear.

Coordinate of P(–22, 28)




65. B

Sol. Radical centre will be the orthocentre of $\Delta$ABC. Since sides 4x – 7y + 10 = 0 and 7x + 4y = 15 are perpendicular. So intersection will be orthocentre (radical centre). So radical centre is (1, 2).

66. B

Sol. PM = $p_1$

TZ = $p_2$

QN = $p_3$

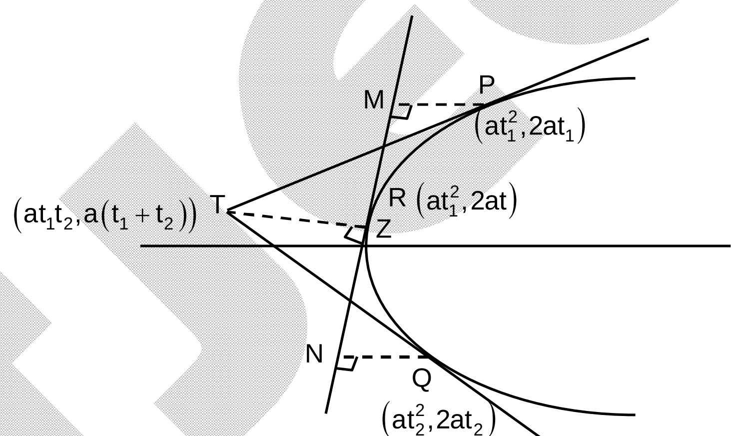
Equation of tangent MN is

ty = x + $at^2$, x – ty + $at^2$ = 0

$$PM = p_1 = \frac{at_1^2 - 2att_1 + at^2}{\sqrt{1+t^2}}$$

$$TZ = p_2 = \frac{at_1t_2 - t(t_1+t_2)a + at^2}{\sqrt{1+t^2}}$$

$$QN = p_3 = \frac{at_2^2 - 2att_2 + at^2}{\sqrt{1+t^2}}$$




$$p_1p_3 = \frac{(at_1^2 - 2att_1 + at^2)(at_2^2 - 2att_2 + at^2)}{1+t^2}$$

$$p_2^2 = p_1p_3$$

67. D

Sol. $$1\int_0^x f(t)\,dt + xf(x) = (x+1)xf(x) + \int_0^x tf(t)\,dt$$

f(x) + f(x) + xf′(x) = $(x^2 + x)$ f′(x) + (2x + 1) f(x) + xf(x)

2f(x) + xf′(x) = $x^2$f′(x) + xf′(x) + (3x + 1) f(x)

$x^2$f′(x) + (3x + 1) f(x) = 2f(x)

$x^2$f′(x) = (2 – 3x – 1) f(x)

$x^2$f′(x) = (1 – 3x) f(x) …(1)

Let y = f(x), $\frac{dy}{dx}$ = f′(x)

$$x^2\frac{dy}{dx} = (1-3x)y$$

$$\frac{dy}{y} = \left(\frac{1-3x}{x^2}\right)dx = \frac{1}{x^2}dx - \frac{3dx}{x}$$

$$\int\frac{dy}{y} = \int\frac{1}{x^2}dx - 3\int\frac{dx}{x} + \log_e c$$

$$\log_e y = -\frac{1}{x} - 3\log_e x + \log_e c$$

$$\log_e y + 3\log_e x - \log_e c = -\frac{1}{x}$$

$$\log_e\left(\frac{yx^3}{c}\right) = -\frac{1}{x}, \frac{yx^3}{c} = e^{-1/x}$$

$$y = \frac{c}{x^3 e^{1/x}} \Rightarrow f(x) = \frac{c}{x^3 e^{1/x}}$$

$$x = 1, f(1) = \frac{1}{e} \Rightarrow c = 1$$

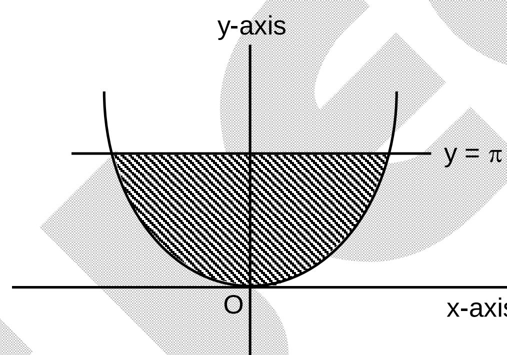
$$f(x) = \frac{1}{x^3 e^{1/x}}, f(-1) = -\frac{1}{e^{-1}} = -e$$

68. A

Sol. $= 2\int_0^{\pi} \sqrt{y}\,dy$

$= \frac{2\times 2}{3}\left|y^{3/2}\right|_0^{\pi}$

$= \frac{4}{3}\left(\pi\sqrt{\pi}\right) = \frac{4\pi\sqrt{\pi}}{3}$




69. C

Sol. Let $x_n = \tan(\theta_n)$

$$\tan(\theta_{n+1}) = \frac{\tan(\theta_n)}{1+\sqrt{1+\tan^2\theta_n}} = \frac{\tan\theta_n}{1+\sec\theta_n} = \frac{\sin\theta_n}{1+\cos\theta_n} = \tan\left(\frac{\theta_n}{2}\right)$$

$$\Rightarrow \quad \theta_{n+1} = \frac{\theta_n}{2} \quad \ldots(1)$$

$$n = 1, x_1 = \tan(\theta_1) = \sqrt{3}, \theta_1 = \frac{\pi}{3}$$

$$\theta_2 = \frac{\theta_1}{2} = \frac{\pi}{2.3}, \theta_3 = \frac{\theta_2}{2} = \frac{\pi}{2^2.3}, \theta_4 = \frac{\theta_3}{2} = \frac{\pi}{2^3.3}$$

so on $\theta_n = \frac{\pi}{3.2^{n-1}}, \tan\theta_n = \tan\left(\frac{\pi}{3.2^{n-1}}\right)$

$$\lim_{n\to\infty} 2^n \frac{\tan\left(\frac{\pi}{3.2^{n-1}}\right)}{\left(\frac{\pi}{3.2^{n-1}}\right)} \times \frac{\pi}{3.2^{n-1}} = \frac{2^n \times \pi^2}{3.2^n} = \frac{2\pi}{3}$$

70. B

Sol. Mean = 8, let missing observation are x, y then

$x + y = 14$ ...(1)

variance = 16,

$x^2 + y^2 = 100$ ...(2)

On solving x = 6, y = 8 or x = 8, y = 6

## SECTION – B

71. 2

Sol. Let $x = t^3$, $dx = 3t^2\,dt$

$= \int t\left(t^2+t+1\right)\left(2t^2+3t+6\right)^3 3t^2 dt$

$= \int 3t^3\left(t^2+t+1\right)\left(\dfrac{2t^3+3t^2+6t}{t}\right)^3 dt$

$= \int 3t^3\left(t^2+t+1\right)\dfrac{\left(2t^3+3t^2+6t\right)^3}{t^3} dt$

$= 3\int\left(t^2+t+1\right)\left(2t^3+3t^2+6t\right)^3 dt$

Let $z = 2t^3 + 3t^2 + 6t$

$dz = (6t^2 + 6t + 6)\,dt = 6(t^2 + t + 1)dt$

$= \dfrac{3}{6}\int z^3 dz = \dfrac{1}{2}\cdot\dfrac{z^4}{4} = \dfrac{z^4}{8}$

$= \dfrac{1}{8}\left(2t^3+3t^2+6t\right)^4$

$f(x) = \dfrac{1}{8}\left(2x+3x^{2/3}+6x^{1/3}\right)^4$

$f(x) = \dfrac{1}{8}\left(2x+3x^{2/3}+6x^{1/3}\right)$

$f(1) = \dfrac{11}{8} \Rightarrow \dfrac{16}{11}f(1) = \dfrac{16}{11}\times\dfrac{11}{8} = 2$

72. 3

Sol. Given $z^2 = \bar{z}(2)^{1-|z|}$, where $z = x + iy$

$|z|^2 = |z|(2)^{1-|z|}$

$|z|(|z| - (2)^{1-|z|}) = 0$

$|z| = 0,\ x^2 + y^2 = 0$ ...(1)

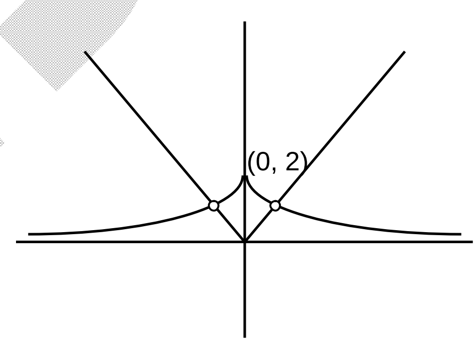
$\text{Arg}(z^2) = \text{Arg}(\bar{z})$

$2\text{Arg}z = -\text{Arg}z,\ \text{Arg}\,z = 0 \Rightarrow y = 0,\ z = x$

so x = 0 are soln (0, 0)

and $|z| = (2)^{1-|z|}$

$|x| = (z)^{1-|x|}$




73. 3

Sol. $\sin x = \dfrac{2 \pm \sqrt{4+4n^2(2n+1)}}{2n^2} = \dfrac{2 \pm 2\sqrt{1+2n^3+n^2}}{2n^2}$

$\sin x = \dfrac{1 \pm \sqrt{1+2n^3+n^2}}{n^2}$

$0 \le \dfrac{1+\sqrt{1+2n^3+n^2}}{n^2} \le 1$

$1 + \sqrt{2n^3+n^2+1} \le n^2$

$\sqrt{2n^3+n^2+1} \le n^2 - 1$

$2n^3 + n^2 + 1 \le n^4 + 1 - 2n^2$

$n^4 - 2n^2 - 2n^3 - n^2 \ge 0$

$n^2 - 2n - 3 \geq 0$
$n^2 - 3n + n - 3 \geq 0$
$n(n - 3) + (n - 3) \geq 0$
$(n - 3)(n + 1) \geq 0$
$n \in (-\infty, -1] \cup [3, \infty)$
Minimum positive integer value of n = 3

74. 4
Sol. $2b = a + c$
given $(a + 2b - c)(2b + c - a)(c + a - b) = \lambda abc$
$(a + a + c - c)(a + c + c - a)(2b - b) = \lambda abc$
$2a.2c.b = \lambda abc$
$4abc = \lambda abc$
$\lambda = 4$

75. 4
Sol. $n_1n_2 = 2n_1 - n_2$
$n_1n_2 + n_2 = 2n_1$
$n_2(n_1 + 1) = 2n_1$
$n_2 = \frac{2n_1}{1+n_1}, n_2 = 2 - \frac{2}{n_1+1}$ ...(1)
$n_2$ to be integer $\frac{2}{n_1+1}$ must be integer
so $n_1 + 1 = \pm 2$, $n_1 + 1 = \pm 1$
$n_1 = \pm 2 - 1$, $n_1 = -1 \pm 1$
$n_1 = 1, -3$, $n_1 = 0, -2$
$n_1 = 0, -2, -3, -1$
if $n_1 = 0$, $n_2 = 0$
$n_1 = 1$, $n_2 = 1$
$n_1 = -3$, $n_2 = 3$
$n_1 = -2$, $n_2 = 4$