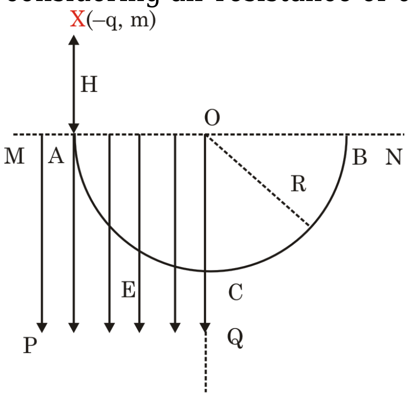
PART-1 : PHYSICS

SECTION-I (i)

1) As shown in the figure, MPQO represents a uniform electric field directed downward (the electric field exists only within this boundary), with an electric field strength of $E = \frac{mg}{q}$. ACB is a fixed smooth semicircular track with a radius R, and A, B are the endpoints of the track's horizontal diameter. AC is $\frac{1}{4}$ of a circle. A small sphere with mass m and charge –q is released from rest at point X, at a height H = R, sliding along the tangent line into the semicircular track without considering air resistance or any energy loss. The correct statements are:



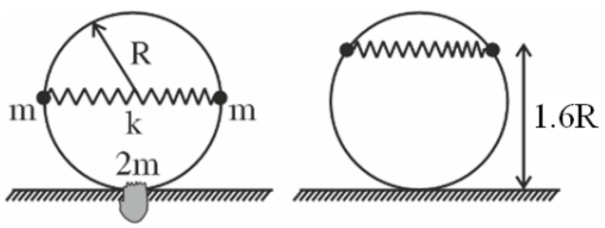
(A) The normal force on the sphere from the track at point just before C is 2mg.

(B) When the sphere moves along the AC portion, its acceleration remains constant.

(C) Increasing E appropriately could result in the sphere's speed at point C becoming zero.

(D) If $E = \frac{2mg}{q}$, for the sphere to just reach point C along the track, the height H should be adjusted to $\frac{R}{2}$.

2) Along a thin ring of radius R and mass 2m, two beads, each of mass m, can slide. The beads are connected by a spring with stiffness k. Initially, the ring is held against a vertical wall on a horizontal surface, and the beads are in unstable equilibrium along the ring's diameter. The diagram shows a top view. After a small push, the beads start moving away from the wall. At the moment when the distance from the beads to the wall reaches a maximum value of 1.6R, the ring is released. (Neglect gravity)



(A) The length ℓ of the undeformed spring is 1.8 R.

(B) The maximum speed v of the beads as they move away from the wall is $0.2R\sqrt{\frac{k}{2m}}$.

(C) The distance of the beads from the wall when they return to the diameter is 1.3 R.

(D) The distance of the beads from the wall when they return to the diameter is 1.4 R.

3) Three satellites orbit the Earth. The first moves in a circular orbit with a radius R_1, the second in a circular orbit with a radius R_2, where $R_2 > R_1$. The third satellite follows an elliptical path, touching the smaller circular orbit at its perigee in point P and the larger circular orbit at its apogee in point A.

- v_{k_1} as the instantaneous speed on the circular trajectory with radius R_1,
- v_{k_2} as the instantaneous speed on the circular trajectory with radius R_2,
- v_p as the instantaneous speed at the perigee of the elliptical trajectory,
- v_a as the instantaneous speed at the apogee of the elliptical trajectory.

T_1 is the orbital time period of satellite in radius R_1.
T_2 is the orbital time period of satellite in radius R_2.
T_3 is the orbital time period of the satellite in elliptical orbit.

(A) $v_a < v_{k_2} < v_{k_1} < v_p$

(B) $v_a < v_{k_1} < v_p < v_{k_2}$

(C) $T_1 = 2\pi\sqrt{\frac{R_1^3}{GM}}$

(D) $T_3 = \pi\sqrt{\frac{(R_1 + R_2)^3}{2GM}}$

4) A 1.5-meter-long steel rod is clamped at both ends and set into longitudinal vibrations, creating standing waves along its length. The speed of longitudinal waves in steel is v = 5000 m/s. The rod vibrates in its third harmonic mode.

(A) The fundamental frequency of longitudnal standing waves is $\frac{5000}{3}$Hz.

(B) The equation for displacement along the rod can be $S = A \sin(2\pi x) \cos(10000\,\pi t)$.

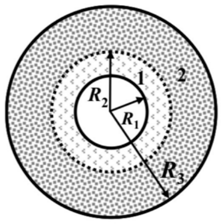
(C) The positions of nodes is x = 0, 0.5 m, 1m, 1.5m.

(D) The position having maximum stress along the rod x = 0, 0.5 m, 1 m and 1.5 m.

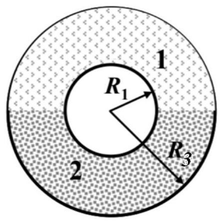
5) The space between two concentric conductor spherical shells of radii R_1 and R_3, is filled with two types of media. The dielectric constant and the conductivity of medium-1 and medium-2 are ε_1, σ_1 and ε_2, σ_2, respectively. The voltage between the two shells is V.

In case-A, the media form two concentric shells with the conductor shells, and the radius of the boundary between the two media is R_2.

In Case-B, medium-1 fills the upper hemisphere and medium-2 fills the other half.



Case-A



Case-B

(A) The current from inner shell to outer shell in Case A is $4\pi K$ where $\frac{1}{K} = \frac{1}{V}\left[\frac{1}{\sigma_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{1}{\sigma_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)\right]$.

(B) The total charge on the boundary of two medium in case A is $4\pi K\left(\frac{\varepsilon_2}{\sigma_2} - \frac{\varepsilon_1}{\sigma_1}\right)$ where $\frac{1}{K} = \frac{1}{V}\left[\frac{1}{\sigma_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{1}{\sigma_2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)\right]$

(C) The current from inner shell to outer shell in case B is $2\pi K(\sigma_1 + \sigma_2)$ where $K = V\frac{R_1R_3}{R_3 - R_1}$.

(D) The total free charge on the upper half in case B of the shell is $2\pi K\varepsilon^{0}\varepsilon_1$ where $K = V\frac{R_1R_3}{R_3 - R_1}$.

6) A standing electromagnetic wave is formed by the superposition of two plane waves with equal amplitude and frequency, travelling in opposite directions along the z-axis. The electric field of the standing wave is given by $\vec{E}(z,t) = E_0\cos(kz)\cos(\omega t)\hat{i}$

(A) The electric field is linearly polarized in the x-direction.

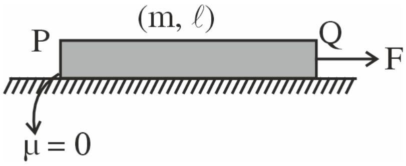
(B) The magnetic field vector of the standing wave oscillates in the y-direction.

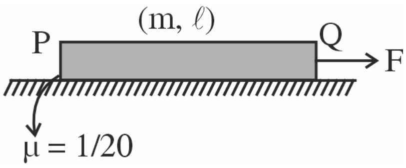
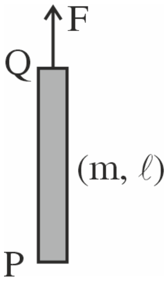
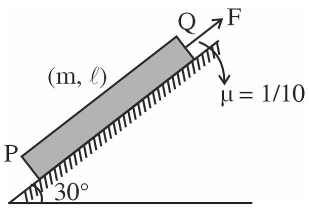
(C) The nodes of the electric field occur at positions $z = \frac{n\pi}{k}$, where n is an integer.

(D) The nodes of magnetic field occur at positions $z = \frac{n\pi}{k}$, where n is an integer.

SECTION-I (ii)

1) In List -I a constant force F is applied on the rod of mass m and length 'ℓ' such that in each case rod moves. A transverse pulse is created at the end point P in each case. The time to move the pulse from P to Q is given in List -II. Then match the following.

	List-I		List-II
(I)	 (m = 4kg, ℓ = 20 m, F = 5N)	(P)	12 sec

(II)	 (m = 9kg, ℓ = 32 m, F = 8N)	(Q)	20 sec
(III)	 (m = 7kg, ℓ = 14 m, F = 2N)	(R)	8 sec
(IV)	 (m = 10kg, ℓ = 20 m, F = 2N)	(S)	14 sec
		(T)	7 sec

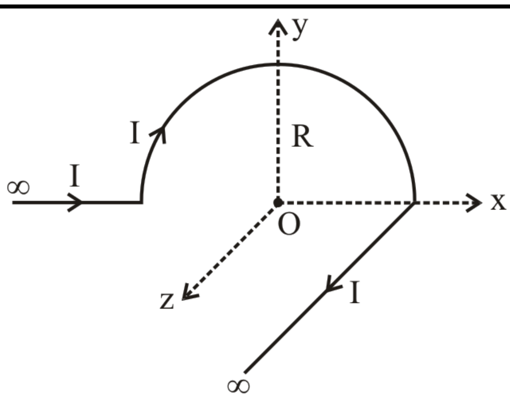
(A) I → R;II → P;III → S;IV → T

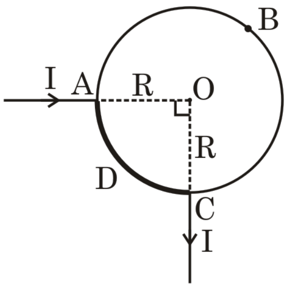
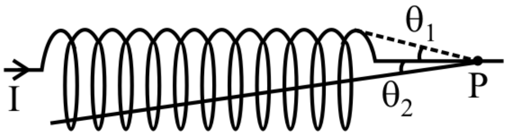
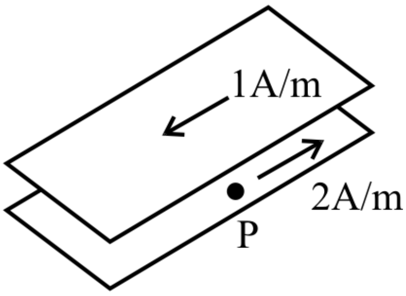
(B) I → R;II → R;III → Q;IV → T

(C) I → R;II → P;III → S;IV → Q

(D) I → T;II → R;III → Q;IV → Q

2) Match the magnetic field (in T) in list-II to the circuit in list-I. (Take : $\pi^2 = 10$)

	List-I		List-II
(I)	 Magnetic field at O. Take R = 1m, $I = \sqrt{\frac{10}{11}}A$.	(P)	$\frac{\mu_0}{2}$

(II)	 Thickness of ABC is double the thickness of ADC. Take $R = \frac{9}{56}$m, I = 1A. Magnetic field at O.	(Q)	1.5 μ_0
(III)	 $\theta_1 = 60°$, $\theta_2 = 30°$ n = $1 m^{-1}$, $I = (\sqrt{3}+1)A$ Magnetic field at P on the axis of the solenoid.	(R)	$\frac{\mu_0}{4}$
(IV)	 Magnetic field at P between two infinite current carrying sheets.	(S)	μ_0
		(T)	$\frac{\mu_0}{3}$

(A) I → R;II → Q;III → P;IV → S

(B) I → R;II → S;III → P;IV → Q

(C) I → Q;II → P;III → R;IV → S

(D) I → R;II → S;III → P;IV → P

3) Suppose we have ice at 0°C, water at 50°C and steam at 100°C. Some amount of each is taken and mixed as described in List-I. For each situation, find the final temperature of the mixture out of possible values in List-II. Assume that there is no heat loss to surroundings. Latent heat of fusion of ice = 80 cal/gm, latent heat of vapourisation of water = 540 cal/gm, specific heat of water = 1 cal/gm°C.

	List-I		**List-II**
(I)	20 gm ice + 40 gm water	(P)	0°C
(II)	50 gm ice + 10 gm steam	(Q)	6.67°C
(III)	100 gm water + 10 gm steam	(R)	40°C
(IV)	60 gm ice + 16 gm water + 5 gm steam	(S)	50°C
		(T)	100°C

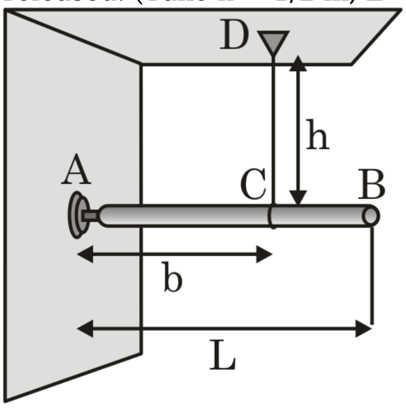
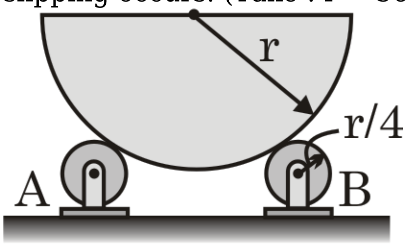
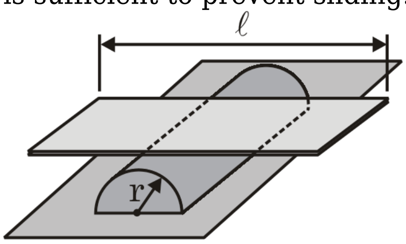
(A) I → P;II → R;III → T;IV → P

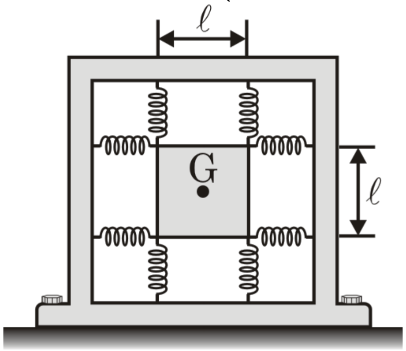
(B) I → Q;II → R;III → T;IV → T

(C) I → Q;II → R;III → T;IV → P

(D) I → P;II → T;III → T;IV → P

4) Match the angular frequency (in rad/s) in list-II with the system in list-I.

	List-I		**List-II**
(I)	A uniform rod of length L hinged at A and by a vertical wire CD. End B is given a small horizontal displacement and then released. (Take h = 1/2 m, L = 2m, b = 5/3 m)  	(P)	1
(II)	A half section of uniform cylinder of radius r and mass m rests on two casters A and B, each of which is a uniform cylinder of radius r/4 and mass m/8. The half cylinder is rotated through a small angle and released and that no slipping occurs. (Take : r = 56/33 m)  	(Q)	4
(III)	A thin plate of length ℓ rests on a half cylinder of radius r. The plate is displaced by a small angle and released. Friction is sufficient to prevent sliding. (Take r = 1/10 m, $\ell = \sqrt{12}$m)  	(R)	2

(IV)	A square plate of mass m is held by eight springs, each of constant k. The plate is rotated through a small angle about G and released. (Take $k = \frac{1}{12}$ N-m, m = 1kg) 	(S)	5
		(T)	3

(A) I → S;II → R;III → P;IV → P

(B) I → S;II → P;III → R;IV → P

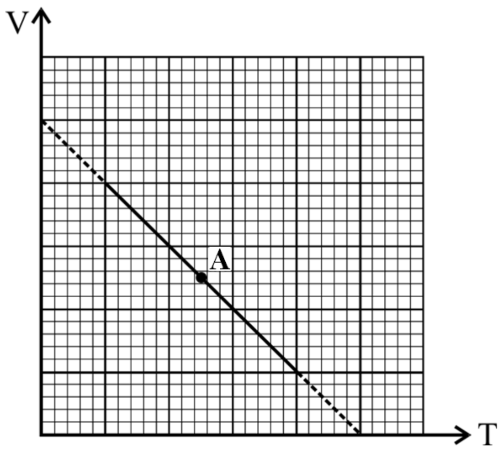
(C) I → S;II → Q;III → P;IV → P

(D) I → R;II → S;III → P;IV → P

SECTION-II

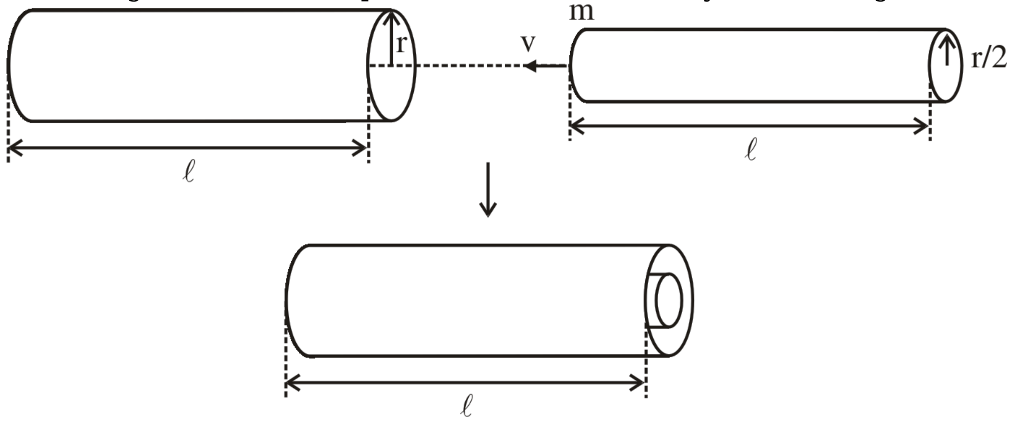
1) A charged particle is in a magnetic field within a three-dimensional space described by the standard Cartesian axes x, y, z. The magnetic field strength and direction varies with time and space. Initially, the particle has velocity $v_x = 200$ m s^{-1} and $v_y = 210$ m s^{-1}. After travelling in the magnetic field for some time, the particle has travelled a distance of d = 500 m and has $v_z = 290$ m s^{-1}. Find $|\langle \vec{a} \rangle|$ (in m/s^2), the magnitude of the average acceleration of the particle. Assume there is no gravity.

2) One mole of an ideal monatomic gas performs a process whose chart in VT coordinates completely lies on a straight line. The heat capacity of the gas at the point A, equidistant from the points of intersection of the process line with the coordinate axes, is : (Take R = 8.3 J/mol - K)



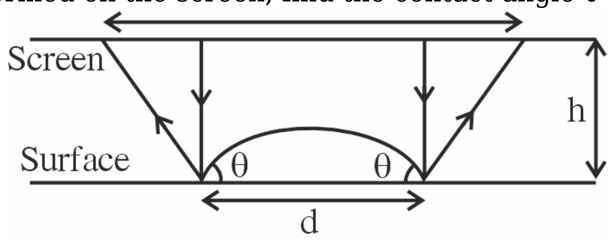
3) A fixed superconducting cylindrical shell A of radius r = 0.05 m and length ℓ = 100 m initially has uniform current flowing in its azimuthal direction (like a solenoid) with total magnitude I = 500 A. A

second superconducting cylindrical shell B of mass m = 2.5×10^{-6} kg, radius $\frac{r}{2}$ and length ℓ initially has no current flowing through it. It is positioned infinitely far away from shell A, and is allowed to move. Shell B is launched towards shell A with initial velocity v. The axes of symmetry of the two cylinders are aligned throughout the subsequent motion of shell B. Determine the velocity v such that shell B will fill the length of the blank space within shell A exactly, after a long time. The value



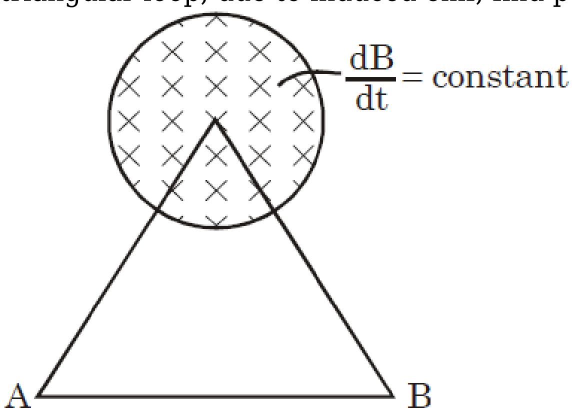
of v^2 (in m^2/s^2) is:

4) A drop of liquid of diameter d = 5.0 mm rests on a horizontal, flat solid surface. In order to find the contact angle θ between the drop and the surface, a collimated laser beam of identical diameter is shone vertically onto the liquid drop, and a translucent screen is set up horizontally at a height h = 12 cm above the drop. Assume that θ < 90°. The diagram is not drawn to scale. Given that a reflected image of diameter D = 5.10 cm is formed on the screen, find the contact angle θ (in



degree) between the liquid and solid surface.

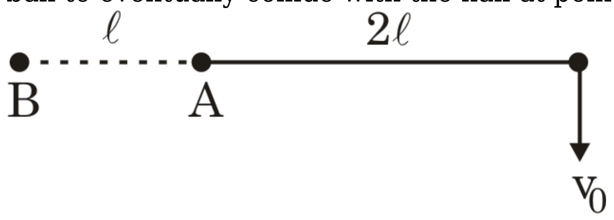
5) A uniform magnetic field is present in a cylindrical region as shown. This field is increasing uniformly with time. An equilateral loop is placed in such a way that its vertex coincide with centre of cylindrical region. Resistances of sides BC and CA are negligible whereas that of AB is 2Ω. If a current of magnitude 2A ampere flows in the triangular loop, due to induced emf, find potential



difference between points A and B (in volt).

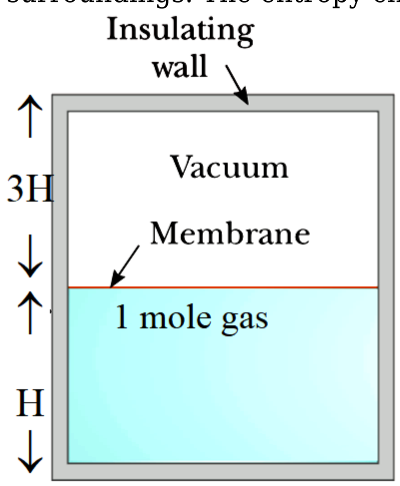
6) As shown in figure, a non-stretchable string of length 2ℓ has one end fixed to a nail at point A, and a small ball is tied to the other end. The small ball can move in the vertical plane. Initially, the ball is on the same horizontal line with A, and is launched with an initial velocity v_0 straight downward.

To the left of point A, at a distance ℓ from A, there is another nail at point B. When the string passes over point B, the ball will move upward. The minimum value of the initial speed v_0 required for the ball to eventually collide with the nail at point A is $\sqrt{\alpha g \ell}$. The value of α is :



7) A thin pole of length 1m is made to stand vertically on ground. One end of a massless thread of length 0.6 m is attached to top end of pole while at the other end a bob of mass 0.5 kg is fixed. Bob is released from rest from a position where string is taut & horizontal. An external torque τ_{ext} is needed about lower end of pole to keep it upright. Maximum value of τ_{ext} (in Nm) will be ________.

8) In the given figure, when the membrane separating the gas from the evacuated region is ruptured, the gas expands freely and irreversibly. The container is thermally insulated from its surroundings. The entropy change of the gas (in J/K) is. (Take : $R = \frac{25}{3}$ J/K-mol, $\ell n2 = 0.7$)



PART-2 : CHEMISTRY

SECTION-I (i)

1) In the process of N_2^- to N_2^+,

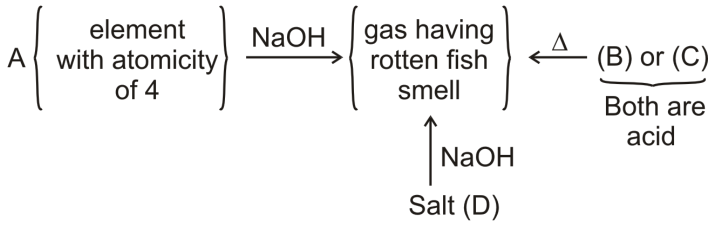
(A) I^{st} electron is removed from gerade BMO

(B) 2^{nd} electron is removed from ungerade BMO

(C) I^{st} electron is removed from gerade ABMO

(D) 2^{nd} electron is removed from gerade NBMO

2)



A, B, C, D are respectively

(A) Red 'P' , H_3PO_2 , H_3PO_3 , PH_4Cl

(B) White ''P' , H_3PO_3 , H_3PO_4 , PH_4I

(C) White ''P' , H_3PO_2 , H_3PO_3 , PH_4Br

(D) White ''P' , H_3PO_3 , H_3PO_2 , PH_4I

3) A solution of 0.1 M weak base (BOH) is titrated with 0.1 M of a strong acid (HA). The pH at half equivalence point of BOH is 11. Then select the correct statement(s) :

(A) K_b of base (BOH) is 10^{-3}

(B) Salt (BA) formed at equivalence point is 10^{-2}% hydrolysed.

(C) pH of solution at equivalence point is 6.15.

(D) During titration in an insulated container, temperature of solution increases upto equivalence point and then decreases.

4) Select correct statement(s) for FCC arrangement :

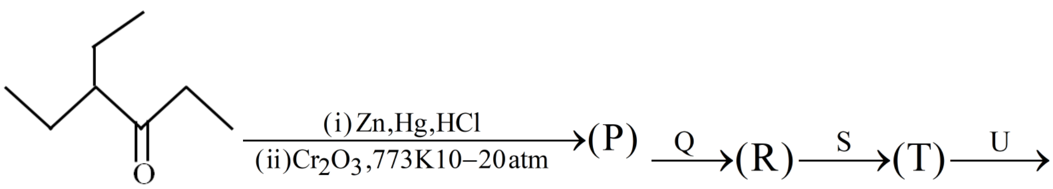
(A) Number of octahedral voids surrounding one octahedral voids is 12.

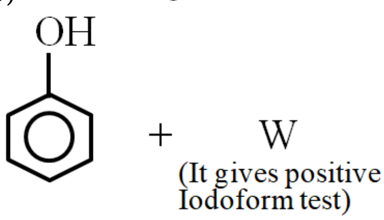
(B) Number of tetrahedral voids surrounding one tetrahedral void is 6.

(C) Number of octahedral void surrounding one tetrahedral void is 4.

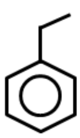
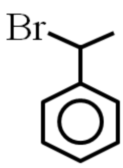
(D) Number of tetrahedral voids surrounding one octahedral void is 8.

5)

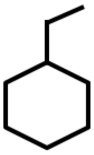


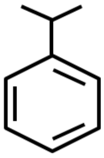


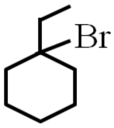
The correct option(s) is/are :

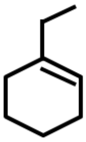
(A) P =  , R = 

, U = (i) O_2, hν (ii) $H^{\oplus}$, Δ

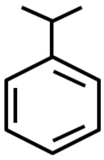
(B) P =  , Q = Br_2 (1 eq.) hν   T =



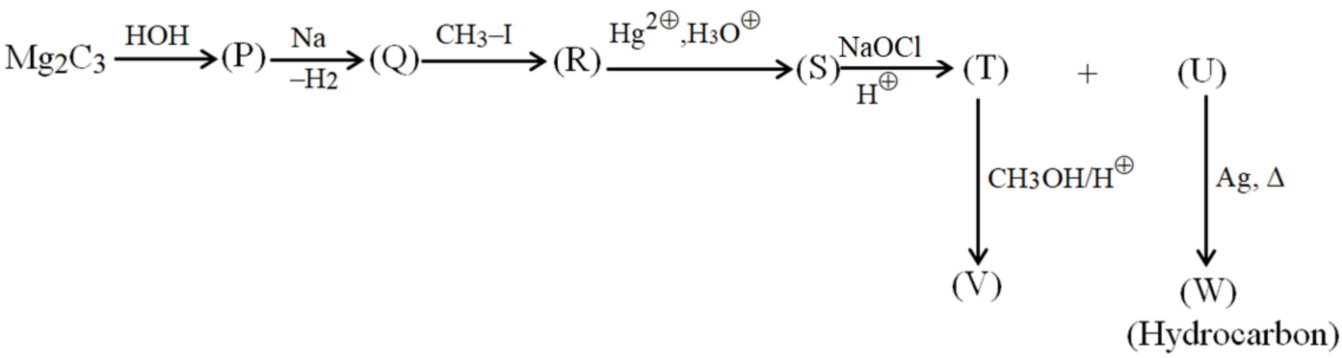
(C) R =  , S = Alc. KOH Δ , T =



(D)

Q = Br_2 (1 eq.), hν   S = CH_3MgBr T = 

6)



Which of the following statement(s) is/are correct about reaction sequence and products.

(A) Polymerisation of product V gives poly methyl acrylate.

(B) 3 moles of product W gives benzene as product when it is passed through red hot Cu tube.

(C) Dipole moment of product R is zero.

(D) Product P, R and W gives positive bromine water test.

SECTION-I (ii)

1)

List-I		List-II	
(P)	Cu^{2+} + $SnCl_2$ →	(1)	Black ppt.

(Q)	$Bi^{3+} + SnCl_2 \xrightarrow{OH^-}$	(2)	Green ppt.
(R)	$Cr^{3+} + (NH_4)_2S \rightarrow$	(3)	No ppt.
(S)	$Fe^{3+} + K_3[Fe(CN)_6] \rightarrow$	(4)	White ppt.
		(5)	Redox reaction

Choose the correct matching.

(A) P → 1,5;Q → 2;R → 3;S → 4,5

(B) P → 4,5;Q → 1;R → 2,5;S → 3

(C) P → 4;Q → 1,5;R → 2;S → 3

(D) P → 4,5;Q → 1,5;R → 2;S → 3,5

2) Consider the following galvanic cell at 25ºC

$$\underset{(1\ bar)}{Pt \mid H_2(g)} \mid \underset{(100\ ml,\ 0.2M)}{H_2CO_3(aq.)} + \underset{(100\ ml\ 0.2M)}{NaHCO_3(aq.)} \parallel \underset{(100\ ml\ 0.1M)}{CH_3COOH(aq.)} \mid \underset{(1\ bar)}{H_2(g) \mid Pt}$$

For H_2CO_3 : $Ka_1 = 10^{-7}$, $Ka_2 = 10^{-11}$

for CH_3COOH : $Ka = 10^{-5}$

Now some electrolyte is added in one of the compartment of the given cell in List-I and corresponding cell potential in List-II is measured. Select the correct option matched correctly :

Take $\frac{2.303RT}{F} = 0.06$ Volts

List-I		List-II	
(P)	If no electrolyte is added in either of the compartment of the cell, then	(1)	0.36 Volts
(Q)	If 100 ml 0.2 M NaOH is added in the anode compartment of the original cell	(2)	0.24 Volts
(R)	If 50 ml 0.1 M NaOH is added in the cathode compartment of the original cell then	(3)	0.3 Volts
(S)	If 100 mL 0.02 M HCl is added in the cathode compartment of the original cell then	(4)	0.12 Volts
		(5)	E.m.f. of the cell decreases after addition of electrolyte with respect to initial condition

(A) P → 4;Q → 5;R → 2;S → 3

(B) P → 2;Q → 1;R → 3;S → 5

(C) P → 2;Q → 1;R → 4,5;S → 3

(D) P → 4;Q → 1;R → 5;S → 4

3) Matching the List-I and List-II :

List-I		List-II	
(P)	Hair cream	(1)	Gel

(Q)	Foam rubber	(2)	Emulsion
(R)	Cheese	(3)	Sol
(S)	Paints	(4)	Foam
		(5)	Solid sol

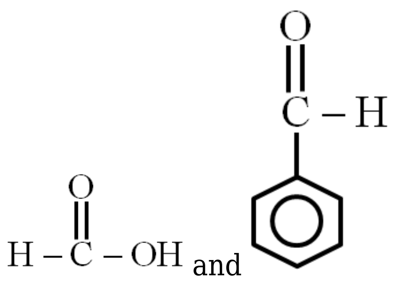
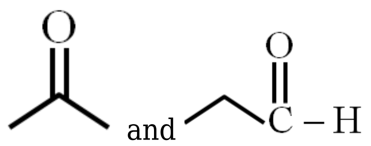
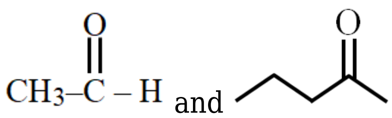
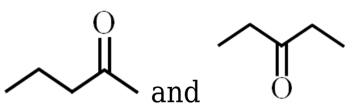
(A) P → 2;Q → 5;R → 1;S → 3

(B) P → 2;Q → 4;R → 1;S → 3

(C) P → 1;Q → 4;R → 2;S → 5

(D) P → 1;Q → 5;R → 3;S → 2

4)

List-I (Pair of compounds)		List-II (Differentiated by)	
(P)		(1)	Fehling solution
(Q)		(2)	Tollen's reagent
(R)		(3)	$NaHSO_3$
(S)		(4)	I_2, NaOH
		(5)	2,4-DNP

(A) P → 1,2,3;Q → 2,3,4;R → 1,2,5;S → 3,4

(B) P → 1,3,5;Q → 1,2,4;R → 1,2;S → 3,4

(C) P → 1,2,5;Q → 1,2,4,5;R → 1,2,3;S → 2,3,4

(D) P → 1,2,3;Q → 2,4;R → 1,2,4,5;S → 3,4,5

SECTION-II

1) $H_2O_2 + NH_2OH \rightarrow H_2O + (A)$

$Na_2O_2 + \text{Fe(II) salt} \xrightarrow{\text{fused}} (B) + Na^+$

$Ce(SO_4)_2 + H_2O_2 \rightarrow (C) + O_2\uparrow + H^+$

If x is the oxidation state of N-atom in 'A'

y is the oxidation state of Fe-atom in 'B' and z is the oxidation state of Ce-atom in 'C'.

Then find the value of $\dfrac{(x+y+z)}{\text{atomicity of compound 'A'}}$

2) $CaC_2 + N_2 \xrightarrow{1100^\circ C} C + \text{'M'}$

$M + H_2SO_4 \rightarrow$ white ppt. + 'N'

$N \xrightarrow{pH\ =\ 7\text{-}9} \text{Dimer of 'N'} \xrightarrow{\text{Pyrolysis}} \underset{(O)}{\text{Aromatic compound}}$

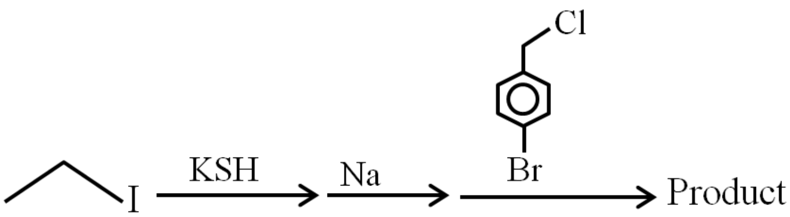
If the number of σ-bonds in 'O' = a

and number of lone pairs in 'O' = b

Then find the value of (a/b)

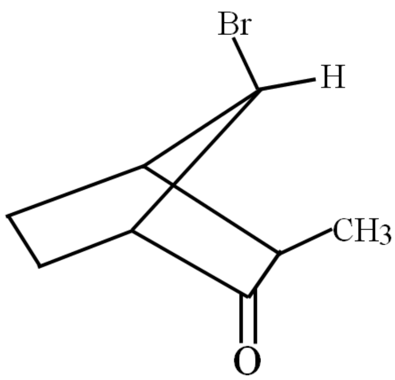
3) A silver coulometer is in series with a cell electrolyzing water. In a time of 1 minute at a constant current, 1.08 g silver got deposited on the cathode of the coulometer. What total volume (**in mL**) of the gases would have produced in other cell at 1atm & 273K if in this cell the anodic and cathodic efficiencies were 90% and 80% respectively. Assume gases collected are dry. [Atomic mass : Ag = 108]

4) For a fixed amount of real gas, when a graph of Z v/s P was plotted then at very high pressure slope was observed to be 0.01 atm^{-1}. At the same temperature if a graph is plotted between PV v/s P then for 2 moles of the gas 'Y' intercept is found to be 40 atm-liter. Calculate excluded volume in litres for 20 moles of the real gas.

5) 

% of sulphur by weight in product is

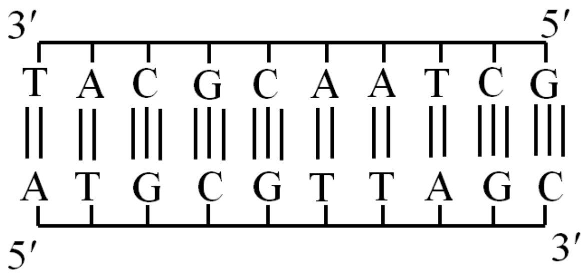
(Given atomic mass : H = 1, C = 12, Na = 23, S = 32, Cl = 35.5, Br = 80)

6) 

Compound (A)

Sum of total chiral centre(s) and total stereo isomers of compound (A) is :

7) A fragment of double helix of DNA is given as follows



Calculate total number of nitrogen atoms present in above fragment of DNA.

8) When 0.2 mole of $K_4[Fe(CN)_6]$ crystal is treated with dilute H_2SO_4 until 33.33% of carbon atoms are lost in the form of HCN and then conc. H_2SO_4 is added in large excess. As a result the colourless, diamagnetic , diatomic gas comes out which is passed through x gm of Ni (solid bed) and gas is completely absorbed due to which 112 percent weight increase is observed for original Ni (solid) taken.
Find the value of x in gm.
[Atomic weight : Ni (58) , C (12) , O (16)]

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let P be a square matrix of order 4 such that, $16P^4 - 96P^3 + 216P^2 + 81I_4 = O$, and det(P) = 16 then which of following is/are true ? (Where det(P) represent determinant value of matrix P, O is null matrix and P^{-1} represents inverse of matrix P and adj(P) represents Adjoint of matrix P)

(A) $\det(\operatorname{adj}(2I - 3P^{-1})) = 3^6$

(B) $\operatorname{adj}(\operatorname{adj} P) = 2^8P$

(C) $\operatorname{adj}(\operatorname{adj}(\operatorname{adj} P)) = 2^{28}P^{-1}$

(D) $\det(\operatorname{adj}(2I - 3P^{-1})) = \pm 3^9$

2) Let a circle S = 0 with center C touches the line 2y = x, at a point P. A chord drawn through P at an angle 45° with the line 2y = x taken in anticlockwise direction intersects the circle at Q and also passes through R(10, 25). Let the co-ordinates of center C ≡ (0, 5).

(A) The length of the chord PQ is $2\sqrt{10}$

(B) Co-ordinates of point Q is (4, 7)

(C) The length of the chord PQ is $3\sqrt{10}$

(D) Co-ordinates of point Q is (4, 8)

3) Which of the following is/are true ?

(A) Let α = 5β and sin(α + β) + sin2β = 3 cos4β then the value of $\frac{\sin(\alpha - 3\beta)}{1 - \tan^2 2\beta} = \frac{3}{4}$ (where sin2β > 0)

(B) If $\alpha = 5\beta$ and $2\operatorname{cosec}\frac{\alpha-\beta}{4} = \frac{1}{\cos\alpha} + \frac{1}{\sin\beta}$, then number of values of β in $(0, \pi)$ is 5

(C) Let $\alpha = 5\beta$ and $\sin(\alpha + \beta) + \sin 2\beta = 3\cos 4\beta$ then the value of $\frac{\sin(\alpha-3\beta)}{1-\tan^2 2\beta} = \frac{4}{5}$ (where $\sin 2\beta > 0$)

(D) If $\alpha = 5\beta$ and $2\operatorname{cosec}\frac{\alpha-\beta}{4} = \frac{1}{\cos\alpha} + \frac{1}{\sin\beta}$, then number of values of β in $(0, \pi)$ is 4

4) Consider N = 15!. Which of the following is/are correct ?

(A) Product of all divisors of N is $(15!)^{2016}$

(B) Sum of all odd divisions of N = 15! is $(3^7 - 1)(5^4 - 1)(7^3 - 1)\frac{7}{2}$

(C) Product of all divisors of N = 15! which are not divisible of 5, is $(2^{11}.3^6.7^2.11.13)^{504}$

(D) Sum of all divisors of N = 15! which are perfect square of natural number is $\left(\frac{2^{12}-1}{3}\right)\left(\frac{3^8-1}{8}\right) \times 25$

5) If the n^{th} term of an arithmetic progression is $T_n = (a - 3)n^3 + (b - 4)n^2 + (a + b)n - 4$, $\forall\, n \in N$ then which of the following is/are true

(A) $a^2 + b^2 = 25$

(B) common difference of the AP is 7

(C) common difference of the AP is 9

(D) sum of first 10 terms of the AP is 345

6) Let $a_1 = 50$ and $a_1, a_2, \ldots, a_n$ be a sequence of number satisfying the rule $n(a_n - a_{n+1}) = n^3 + n^2 - a_n$, $n \in N$. Which of the following is/are true ?

(A) $\frac{a_{16}}{16} = -70$

(B) $a_{11} = -55$

(C) $a_{10} = 50$

(D) $\frac{a_{12}}{12} = -17$

SECTION-I (ii)

1) Match each entry in List-I to the correct entry in List-II

List-I		List-II	
(I)	Let $\tan(2\pi\lvert\sin\theta\rvert) = \cot(2\pi\lvert\cos\theta\rvert)$, where $\theta \in R$ and $f(x) = (\lvert\sin\theta\rvert + \lvert\cos\theta\rvert)^x$ then the value of $\lim_{x\to\infty}\left[\frac{2}{f(x)}\right] =$ (where [.] denotes the Greatest Integer Function)	(P)	0

(II)	If $\tan^2\alpha + \cot^2\alpha = \lim\limits_{x\to\infty}\left(\sqrt{x^2+8x+3} - \sqrt{x^2+4x+2}\right)$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then the possible value of $\frac{1}{2}(\tan\alpha + \cot\alpha)$	(Q)	1
(III)	If f(x) = [2 + 5|n| sinx] where n ∈ I, x ∈ [0,2π] has exactly 19 points of non differentiability then the possible value of 2|n| is (Where [.] denotes Greatest integer function)	(R)	4
(IV)	If n ∈ N and the set of equation $\cos^{-1}x + \left(\sin^{-1}y\right)^2 = \frac{n\pi^2}{4}$ and $\left(\sin^{-1}y\right)^2 - \left(\cos^{-1}x\right) = \frac{\pi^2}{16}$ is consistent then 'n' can be equal to	(S)	2
		(T)	7

(A) I → Q;II → R;III → Q;IV → P

(B) I → P;II → Q;III → S;IV → Q

(C) I → P;II → R;III → Q;IV → S

(D) I → Q;II → S;III → P;IV → R

2) Let f(x) = $2\sin^2\beta$ + 4 cos(x + β) sinx sin β + cos2(x + β) and $g(x) = \left(\lim\limits_{\alpha\to 0}(1+\alpha)^{1/\alpha}\right)^{\tan\left(\tan^{-1}x\right)}$ then

Match the entry in List-I to the correct entry in List-II

List-I		List-II	
(I)	Let $\lim\limits_{x\to 0}(f(x))^{\frac{1}{x^2}} = \ell$, then $2\square n\square^{-1}$ is equal to	(P)	$\frac{1}{2}$
(II)	$\lim\limits_{x\to 0}\frac{g(x)-f(x)}{2x}$ is equal to	(Q)	1
(III)	Let x ∈ (–∞,0], then number of integer(s) in the range of h(x) = 1 + g(x) is	(R)	2
(IV)	The number of solution(s) of the equation f(x) = $\frac{1}{2}$, where x ∈ [0,3] is	(S)	4
		(T)	3

(A) I → S;II → P;III → Q;IV → R

(B) I → P;II → S;III → R;IV → Q

(C) I → S;II → P;III → R;IV → Q

(D) I → R;II → S;III → Q;IV → P

3) Match the List-I and List-II

List-I	List-II

(I)	A straight line with negative slope passing through (1,4) meets the coordiantes axes at A and B. The minimum length of (OA + OB), where O being the origin, is	(P)	$5\sqrt{2}$
(II)	If the point P is symmetric to the point Q(4,–1) with respect to the bisector of the first quadrant, then the length of PQ is	(Q)	$3\sqrt{2}$
(III)	On the portion of the straight line x + y = 2 between the axis a square is constructed away from the origin, with this portion as one of its sides. If d dentoes the perpendicular distance of a side of this square from the origin then the maximum value of d is	(R)	5
(IV)	If the parametric equation of a line is given $x = 4 + \lambda\sqrt{2}$ and $y = -1 + \sqrt{2}\lambda$ where λ is a parameter, then the intercept made by the line on the x-axis is	(S)	9
		(T)	7

(A) I → R;II → P;III → Q;IV → S

(B) I → S;II → P;III → Q;IV → R

(C) I → S;II → R;III → Q;IV → P

(D) I → P;II → S;III → Q;IV → R

4) Let Π be the plane parallel to y-axis and containing the points (1,0,1) and (3,2,–1). A ≡ (4,0,0) and B ≡ (6,0,–2) are two points and P ≡ (x_0,y_0,z_0) is a variable point on the plane Π = 0.
Match List-I with List-II and select the correct answer using the code given below the list.

List-I		List-II	
(I)	If the equation of the plane Π = 0 is x + ay + bz = c, then |a + b + c| is	(P)	16
(II)	If (PA + PB) is minimum, then $\|4x_0 + y_0 + 2z_0\|$ is	(Q)	12
(III)	If (PA + PB) is minimum, then $\|x_0 + y_0 + z_0\|$ is	(R)	3
(IV)	If the reflection of the line AB in the plane Π = 0 is, $\frac{x-2}{1} = \frac{y-\alpha}{0} = \frac{z+\beta}{-1}$, then $(\alpha^4 + \beta^4)$	(S)	2
		(T)	10

(A) I → R;II → Q;III → P;IV → S

(B) I → Q;II → R;III → S;IV → P

(C) I → R;II → Q;III → S;IV → P

(D) I → R;II → S;III → Q;IV → P

SECTION-II

1) If $f(x)$ is defined on [-3, 3] by $f(x) = 4x^3 - 2x + 1$ and $g(x) = \frac{f(x) - f(-x)}{|x| + 2}$, then value of $\int_{-3}^{3} g(x)\,dx$ is equal to

2) Let $y = \frac{\left(e^{\sin x} - 1\right)^4 + (\cos x - 1)^3 e^{\sin x} + (e^x - e^{-x})^3 + x^3 + 2x^2 + 4}{e^x}$. The value of y"(0) is

3) Let $f(x) = e^{\{x\}}$, (where {.} denotes fractional part of 'x') then number of integers in the range of $f(x)$ is

4) Let AB : y = x be a chord of the parabola $y^2 = -4x + 4$ and P be any variable point on the ellipse $x^2 + 9(y - 4)^2 - 9 = 0$. Given that Δ_1 and Δ_2 be the minimum and maximum area of triangle ABP respectively. If $\lambda = \Delta_1\Delta_2$, then $\frac{\lambda}{4}$ equals

5) If the sum $\sum_{i=0}^{n-1} \arcsin\left(\frac{i(i+1) + n^2}{\sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}}\right) = (2n - 1)\frac{\pi}{\lambda}$, (where n ∈ N, n ≥ 2) then λ equals

6) If a, b, c ∈ N, the probability that $(a^2 + b^2 + c^2)$ is divisible by 13 is $\frac{m}{n}$, where m, n all relatively prime numbers, then (n – m) is

7) Let ABCD be a regular tetrahedron. Suppose point X,Y and Z lie on rays AB, AC and AD respectively such that XY = YZ = 7 and XZ = 5. The length of AX, AY and AZ are all distinct. If the volume of tetrahedron AXYZ is $\sqrt{\lambda}$, then λ equals

8)

Find the remainder when $43^{43^{43}}$ is divided by 40 :

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4	5	6
A.	**A,C**	**A,B,C**	**A,C,D**	**A,B,C,D**	**A,C,D**	**A,B,D**

SECTION-I (ii)

Q.	7	8	9	10
A.	**C**	**B**	**C**	**A**

SECTION-II

Q.	11	12	13	14	15	16	17	18
A.	**235.00 to 238.00**	**4.10 to 4.20**	**3.33**	**5.45 to 5.55**	**0.00**	**2.45 to 2.65**	**7.50**	**11.63 to 11.67**

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	19	20	21	22	23	24
A.	**C**	**C,D**	**A,C,D**	**A,B,C,D**	**A,D**	**B,C,D**

SECTION-I (ii)

Q.	25	26	27	28
A.	**C**	**C**	**A**	**B**

SECTION-II

Q.	29	30	31	32	33	34	35	36
A.	**2.80**	**2.50**	**140.00**	**4.00**	**13.85**	**12.00**	**75.00**	**20.00**

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	37	38	39	40	41	42
A.	**B,C,D**	**A,B**	**A,B**	**A,B,C**	**A,B,D**	**A,B,C**

SECTION-I (ii)

Q.	43	44	45	46
A.	**B**	**A**	**B**	**C**

SECTION-II

Q.	47	48	49	50	51	52	53	54
A.	0.00	8.00	2.00	12.00	4.00	12.00	122.00	27.00

SOLUTIONS

PART-1 : PHYSICS

9) (P) **6.67°C**

Heat needed to melt ice = 20 × 80 = 1600 cal

Heat that water can provide by coming to 0°C = 40(1)(50) = 2000 cal

So, all ice melts and final temperature is between 0°C and 50°C

This temperature can be found by conservation of heat.

(Q) 50 × 80 + 50 × 1(T – 0) = 10 × 540 + 10 × (100 –T)

60T = 2400 ⇒ T = 40°C

100 × 1 × (100 – 50) = m (540)

$m = \frac{5000}{540} = 9.25\text{gm}$

as m < 10 ⇒ T_f = 100°C

(S) 5 × 540 + 5 × 1 × (100 – 0) + 16 × 1(50 – 0) = m(80)

4000 = 80m ⇒ m = 50 gm

as m < 60 ⇒ T_f = 0°C

12) According to the definition the heat capacity is written as

$C = \frac{\delta Q}{\Delta T} = \frac{\Delta U + P\Delta V}{\Delta T} = C_V + P\frac{\Delta V}{\Delta T}$ .... (1)

Let the gas parameters at the point A be equal (P_0, V_0, T_0). Then, the equation of the process takes the following form

$\frac{V}{V_0} + \frac{T}{T_0} = 2$ ... (2)

For small deviations one derives

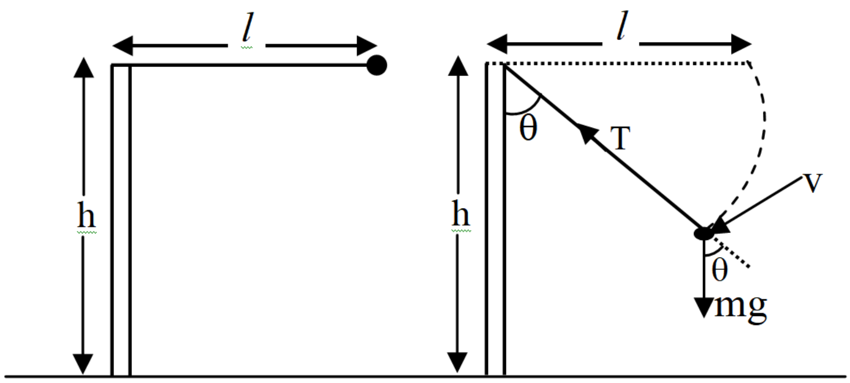
$\frac{\Delta V}{V_0} + \frac{\Delta T}{T_0} = 0$, ... (3)

$\frac{\Delta V}{\Delta T} = -\frac{V_0}{T_0}$ ... (4)

With the aid of the equation of state $P_0V_0 = RT_0$, one finally gets

$C = C_V + P\frac{\Delta V}{\Delta T} = C_V - P_0\frac{V_0}{T_0} = C_V - R = \frac{R}{2}$ ... (5)

17)



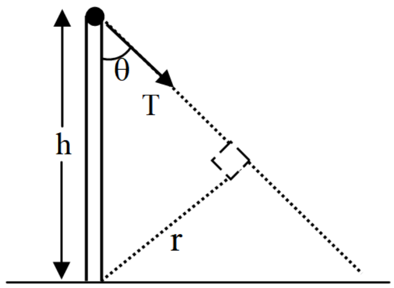
For bob conservation of mechanical energy : $mg\ell\cos\theta = \frac{1}{2}mv^2$

$v = \sqrt{2g\ell\cos\theta}$.....(1)

Centripetal force : $T - mg\cos\theta = \frac{mv^2}{\ell}$

$T = 3mg\cos\theta$ .....(2)

Torque of tension about lower point of pole



$\tau = Th\sin\theta$

$\tau = 3mgh\sin\theta\cos\theta$

$\tau = \frac{3}{2}mgh\sin 2\theta$

$\therefore\ \tau_{ext}$ should balance τ

$\tau_{ext} = \frac{3mgh\sin 2\theta}{2}$

$\theta = 45^o \Rightarrow \tau_{ext_{max}} = \frac{3mgh}{2}$

$\tau_{ext_{max}} = \frac{3}{2} \times 0.5 \times 10 \times 1$

$\tau_{ext_{max}} = 7.50$ Nm

PART-2 : CHEMISTRY

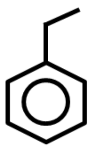
21) at half of equivalence point.

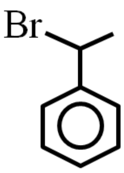
pH = 11

pOH = (14 – 11) = pK_b + log1

pK_b = 3

23)

P =  Q = Br_2 (1 eq.) hν

R =  S = CH_3MgBr

T = 

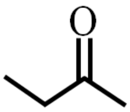
U = (i) O_2, hν (ii) $H^{\oplus}$, Δ

24)

P = CH_3–C≡CH

Q = $CH_3 - C \equiv \overset{\ominus}{C} Na^{\oplus}$

R = CH_3 - C ≡ C - CH_3

S = 

T = $CH_3 - CH_2 - \overset{\overset{\large O}{||}}{C} - OH$

U = CHI_3

V = $CH_3 - CH_2 - \overset{\overset{\large O}{||}}{C} - OCH_3$

W = CH ≡ CH

26)

(P) E_{cell} = 0.06 (P^Hanode – P^Hcathode)

pH Cathode = 3

for Anode $H_2CO_3 + Na_2CO_3 \rightarrow 2NaHCO_3$

	H_2CO_3	Na_2CO_3	$2NaHCO_3$
	20	10	0
	10	0	10

pH = PK_{a1} = 7

E_{cell} = 0.06 × (7 – 3) = 0.24

(Q) for Anode H_2CO_3 + NaOH → $NaHCO_3$

	H_2CO_3	NaOH	$NaHCO_3$
	20	20	20
	0	0	40

pH = 9

E_{cell} = 0.06 × (9 – 3) = 0.36

(R) for Anode CH_3COOH + NaOH → CH_3COOH

	CH_3COOH	NaOH	CH_3COOH
	10	5	0
	5	0	5

pH = 5

E_{cell} = 0.06 × (7 – 5) = 0.12

(S) P^H = 2

E_{cell} = 0.06 × (7 – 2) = 0.3

28)

Benzaldehyde do not give test with Fehling solution

* $NaHSO_3$ gives positive test with methyl ketone (not with Di ethyl ketone)

31) Charge passed = 0.01 Faraday

At the anode $\left(H_2O \rightarrow \frac{1}{2}O_2 + 2H^+ + 2e^-\right)$ with

90 % efficientcy 0.01 × 0.9 F have been used and will produce $\frac{1}{4}$ × 0.01 × 0.9 mole of O_2 i.e. 0.00225 mol O_2 .

At the cathode $2H_2O \xrightarrow{+2e^-} H_2 + 2OH^-$

moles of H_2 produced = $\frac{0.01 \times 0.8}{2}$ mol

= 0.004 mol

Total moles produced of gases

= 0.004 + 0.00225 = 0.00625 mol

vol. at STP = 0.00625 × 22400 mL = 140 mL

32) $P(V - nb) = nRT$

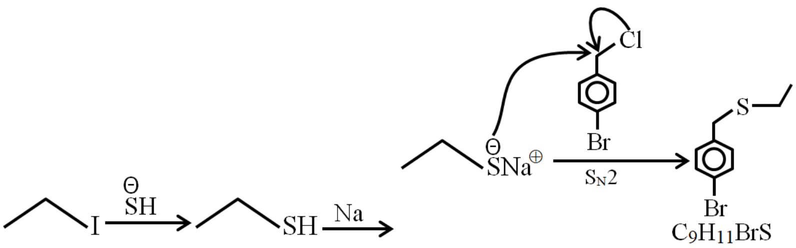
$\Rightarrow PV - nPb = nRT$

$\Rightarrow PV = nRT + nPb$ ..... (i)

$Z = \frac{PV}{nRT} = 1 + \frac{Pb}{RT}$ ..... (ii)

$\therefore \frac{b}{RT} = 0.01$ & $nRT = 40$

33)



% of sulphur in product

molecular weight of product = 231 ($C_9H_{11}BrS$)

% of S = $\frac{32}{231} \times 100$ = 13.85

34)

Total chiral centre = 4

Total stereo isomers = 8

∴ 4 + 8 = 12

35) Thymine contain 2 nitrogen atoms

Cytosine contain 3 nitrogen atoms

Guanine contain 5 nitrogen atoms

Adenine contain 5 nitrogen atoms

Nitrogen's in Thymine = 5 × 2 = 10

Nitrogen's in Cytosine = 5 × 3 = 15

Nitrogen's in Guanine = 5 × 5 = 25

Nitrogen's in Adenine = 5 × 5 = 25

Total nitrogen atoms = 75

36)

0.2 mole $K_4[Fe(CN)_6] \equiv$ 1.2 mole of C-atoms
33.33 % HCN is formed i.e. 0.4 mole of C-atoms are lost in the form of HCN.
So 0.8 mole CO is obtained.

$$\underset{0.8\text{ mole}}{4CO} + \underset{\substack{0.2\text{ mole}\\ =\ 11.6\text{ gm}}}{Ni} \rightarrow \underset{\substack{0.2\text{ mole}\\ =\ 34\text{ gm}}}{Ni(CO)_4}$$

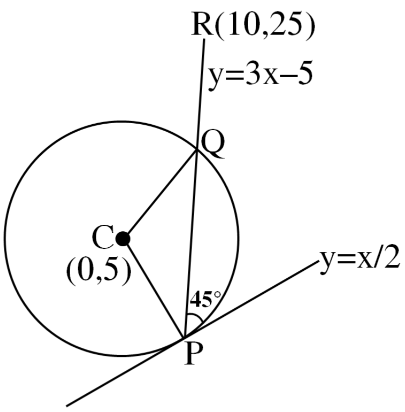
100 gm Ni → 212 gm final weight

$$x\text{ gm} \rightarrow \frac{212x}{100}$$

So, $(x - 11.6) + 34 = \dfrac{212x}{100}$
$x = 20.00$ gm

PART-3 : MATHEMATICS

37) $(2P - 3I)^4 = -216P$,
Take determinant on both sides we get
$|2P - 3I| = \pm 16 \times 27$
$\Rightarrow |2P - 3PP^{-1}| = \pm 16 \times 27$
$|P|\ |2I - 3P^{-1}| = \pm 16 \times 27$
$|2I - 3P^{-1}| = \pm 27$
Now verify the options



38)
Here let slope of chord PQ = m ⇒ m = 3
Also it passer through R(10, 25)
∴ Equation of chord PQ $y = 3x - 5$
⇒ Radius of circle

$$CP = \left|\frac{10}{\sqrt{5}}\right| = \sqrt{20}$$

∴ Distance of chord PQ from C,

$$d = \left|\frac{10}{\sqrt{10}}\right| = \sqrt{10}$$

∴ length of chord $PQ = 2\sqrt{CP^2 - d^2} = 2\sqrt{10}$
Here Point P lies on line $2y = x$ and $y = 3x - 5$
∴ $P \equiv (2, 1)$
Now $Q \equiv \left(2 + 2\sqrt{10}\cos\theta, 1 + 2\sqrt{10}\sin\theta\right)$

Here $\tan\theta = 3 \Rightarrow \cos\theta = \frac{1}{\sqrt{10}}, \sin\theta = \frac{3}{\sqrt{10}}$
 Q ≡ (4, 7)

39)

$\sin 6\beta + \sin 2\beta = 3\cos 4\beta$
$\Rightarrow 2\sin 4\beta.\cos 2\beta = 3\cos 4\beta$
$\Rightarrow \frac{\sin 4\beta . \cos 2\beta}{\cos 4\beta} = \frac{3}{2}$
$\Rightarrow \frac{\sin 2\beta \cos^2 2\beta}{\cos 4\beta} = \frac{3}{4}$
$\Rightarrow \frac{\sin(\alpha - 3\beta).\cos^2 2\beta}{\cos^2 2\beta - \sin^2 2\beta} = \frac{3}{4}$
$\Rightarrow \frac{2}{\sin\left(\frac{4\beta}{4}\right)} = \frac{1}{\cos\alpha} + \frac{1}{\sin\beta}$
$\Rightarrow \frac{1}{\sin\beta} = \frac{1}{\cos\alpha}$ $\sin\beta = \cos\alpha \Rightarrow \cos\left(\frac{\pi}{2} - \beta\right) = \cos(5\beta)$
$\Rightarrow 5\beta = 2n\pi \pm \left(\frac{\pi}{2} - \beta\right)$ $\beta = (4n-1)\frac{\pi}{8}$ or $\beta = (4n+1)\frac{\pi}{12}$

40)

$15! = 2^{11}.3^6.5^3.7^2.11.13 = N$
Sum of all odd divisors
$(2^0)(3^0 + 3^1 + \ldots 3^6)(5^0 + 5^1 + 5^2 + 5^3)(7^0 + 7^1 + 7^2)$
$(11^0 + 11^1)(13^0 + 13^1)$
Product of all divisors $= N^{\frac{4032}{2}} = N^{2016}$
Product of all divisors which are not divisible by $5 = \left(\frac{N}{5^3}\right)^{504}$

41)

$T_n = (a - 3)n^3 + (b - 4)n^2 + (a + b)n - 4$
for T_n of AP : a = 3, b = 4
$T_n = 7n - 4$
$d = T_n - T_{n-1}$
d = 7, first term = 3
$S_{10} = \frac{10}{2}(6 + 9 \times 7) = 345$

42) $\frac{a_n}{n} - \frac{a_{n+1}}{n+1} = n$
$\Rightarrow a_1 - \frac{a_{n+1}}{n+1} = \left(\frac{n(n+1)}{2}\right)$

$\Rightarrow 50 - n\left(\frac{n+1}{2}\right) = \frac{a_{n+1}}{n+1}$

Now check

43) **(P)** tan(2π|sinθ|) = $\tan\left(\frac{\pi}{2} - |\cos\theta|\, 2\pi\right)$

$\Rightarrow |\sin\theta| + |\cos\theta| = \frac{n}{2} + \frac{1}{4}$ ...(1)

⇒ sinθ $1 \le |\sin\theta| + |\cos\theta| \le \sqrt{2}$ then

$1 \leqslant \frac{n}{2} + \frac{1}{4} \leqslant \sqrt{2} \Rightarrow \frac{3}{2} \leqslant n \leqslant \frac{4\sqrt{2}-1}{2}$ then

n = 2 is only possible value.

$g(x) = \lim_{n\to\infty}\left[2\left(\frac{4}{5}\right)^x\right] = 0$

(Q) $\tan^2\alpha + \cot^2\alpha = \lim_{x\to\infty}\left(\sqrt{x^2+8x+3} - \sqrt{x^2+4x+2}\right) = 2$

given $\tan^2\alpha + \frac{1}{\tan^2\alpha} = 2 \Rightarrow \tan^2\alpha = 1 \Rightarrow \alpha = \pm\frac{\pi}{4}$

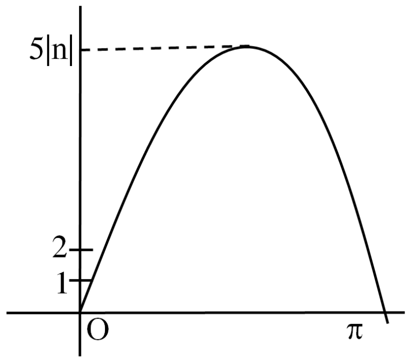
then $\frac{1}{2}(\tan\alpha + \cot\alpha) = 1, -1$

(R) [2 + 5|n| |sinx|] = 2 + [5|n|sinx]

Number of points of non derivability

= 4(5|n| – 1) + 3 = 20|n| – 1 = 19

⇒ |n| = 1



(S) Adding equation (1) and (2)

$\left(\sin^{-1}y\right)^2 = \frac{4n+1}{32}\pi^2$

$\Rightarrow 0 \leqslant \frac{4n+1}{32}\pi^2 + \frac{\pi^2}{4} \Rightarrow \frac{-1}{4} \leqslant n \leqslant \frac{7}{4}$ ...(3)

Also $\cos^{-1}x = \frac{4n-1}{32}\pi^2 \Rightarrow \frac{1}{4} \leqslant n \leqslant \frac{8}{\pi} + 1$ ...(4)

From equations (3) and (4) n = 1

44)

f(x) = cos2x and g(x) = e^x.

(I) $\lim_{x\to 0}(f(x))^{\frac{1}{x^2}} = e^{-2}$

(II) $\lim_{x\to 0}\frac{g(x) - f(x)}{2x} = \frac{1}{2}$

(III) h(x) = e^x + 1

f(x) x ∈ (-∞, 0], h(x) ∈ (1, 2]

(IV) cos2x = $\frac{1}{2}$ in [0, 3] has 2 solutions

45) **(I)** OA = 1 + 4cotθ

OB = 4 + tanθ

OA + OB = 5 + 4cosθ + tanθ ≥ 5 + $2\sqrt{4\cot\theta\tan\theta}$

(II) The reflection of P(4, –1) on y = x is Q(–1, 4)

Hence, PQ = $\sqrt{(4+1)^2+(-1-4)^2} = \sqrt{50} = 5\sqrt{2}$.

(III) $AB = 2\sqrt{2}$

$OC = \sqrt{2}$

The maximum value of d is

OF = $\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

(IV) The given line is x = y + 5

Hence, the intercept made by the x-axis is 5.

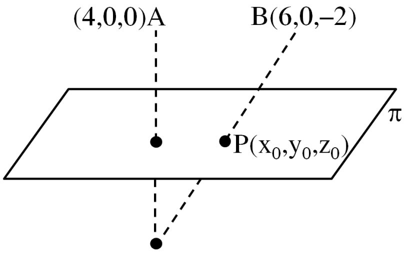
46)

Let the plane be αx + βz + 1 = 0

Pass through (1, 0, 1); (3, 2, –1)

∴ $\alpha = \frac{1}{2}; \beta = -\frac{1}{2}$

(I) π : x + z = 2



(II)

Both A and B are on same side of π.

Reflection of A in plane is A'(2, 0, –2)

Equation of line $A'B = \vec{r} - = 6\hat{i} - 2\hat{k} + \lambda(4\hat{i})$

For P : 6 + 4λ + 0 – 2 = 2 ⇒ $\lambda = \frac{-1}{2}$

∴ P(4, 2, –2)

∴ $|4x_0 + y_0 + 2z_0| = 12$

(IV) Also, A' will lie on $\frac{x-2}{1} = \frac{y-\alpha}{0} = \frac{z+\beta}{-1}$.

⇒ $\frac{2-2}{1} = \frac{0-\alpha}{0} = -\frac{2+\beta}{-1}$

⇒ α = 0, β = 2

∴ $\alpha^4 + \beta^4 = 16$

47)

g(x) = $\frac{f(x)-f(-x)}{|x|+2} = \frac{8x^3-4x}{|x|+2}$

∴ g(x) = g(–x) ∴ g(x) is odd.

48)

At x = 0, y = 4

$e^x y = x^3 + 2x^2 + 4$

$\Rightarrow y''(0) = 8$

For $y''(0)$, $(e^{\sin x} - 1)^4 + (\cos x - 1)^3 e^{\sin x} + (e^x - e^{-x})^3$ is always gives zero at x = 0

49)

$0 \le \{x\} < 1$

$1 \le e^{\{x\}} < e$

50) $\Delta = \frac{1}{2}.8.P = 4P$

$P = \left|\frac{3\cos\theta - 4 - \sin\theta}{\sqrt{2}}\right|$

$2\sqrt{2}\left(4 - \sqrt{10}\right) = \Delta_1$

$2\sqrt{2}\left(4 + \sqrt{10}\right) = \Delta_2$

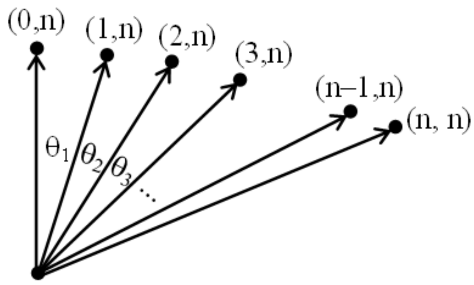
51)

For i = 0, 1, ..., n, define vectors $v_i = (i, n)$

Then, letting θ_{i+1} denote the angle between v_i and v_{i+1}, we have

$i(i + 1) + n^2 = v_i \cdot v_{i+1} = \sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}\cos\theta_{i+1}$

so $\theta_{i+1} = \text{arc}\cos\left(\frac{i(i+1) + n^2}{\sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}}\right)$



Also, the sum $\theta_1 + \theta_2 + ... + \theta_n$ of all these angles is the angle between the y-axis and the line y = x, so $\sum_{i=0}^{n-1}\theta_{i+1} = \frac{\pi}{4}$. Finally,

$\text{arc}\sin\left(\frac{i(i+1) + n^2}{\sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}}\right) = \frac{\pi}{2} - \text{arc}\cos\left(\frac{i(i+1) + n^2}{\sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}}\right)$

Therefore,

$\sum_{i=0}^{n-1}\text{arc}\sin\left(\frac{i(i+1) + n^2}{\sqrt{i^2 + n^2}\sqrt{(i+1)^2 + n^2}}\right) = \sum_{i=0}^{n-1}\left(\frac{\pi}{2} - \theta_{i+1}\right)$

$= \frac{n\pi}{2} - \frac{\pi}{4} = \frac{(2n-1)\pi}{4}$

52)

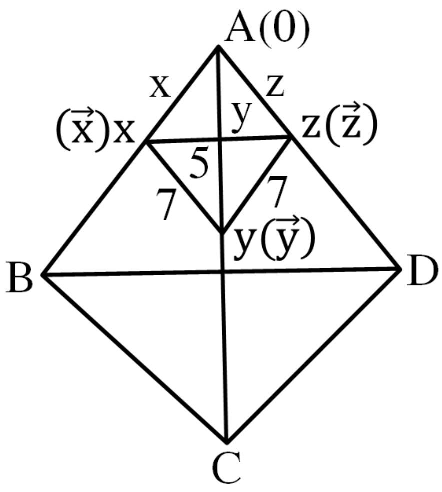
$a^2 = 13\lambda + r_1$ $\quad r_1 \in \{0, \pm1, \pm2, ..., \pm6\}$

$b^2 = 13\lambda + r_2$

$c^2 = 13\lambda + r_3$

$P = \frac{\text{fav}}{\text{Total}} = \frac{1 + 4 \times 3! + 4 \times 3! + 8 \times 3! + 4 \times 3! + 8 + 3!}{13^3}$

$P = \dfrac{169}{13^3} = \dfrac{1}{13}$



53)

$|\vec{x}| = x$; $|\vec{y}| = y$; $|\vec{z}| = z$

Volume of tetrahedron AXYZ = $\dfrac{xyz}{6\sqrt{2}}$

$x^2 + y^2 - xy = 7^2$ ...(1)

$y^2 + z^2 - yz = 7^2$ ...(2)

$x^2 + z^2 - xz = 5^2$ ...(3)

On solving 3 equations we get

xz = 12, $x + z = \sqrt{61}$, y = x + z

Volume of tetrahedron = $\dfrac{1}{6\sqrt{2}} = \dfrac{xz(x+z)}{6\sqrt{2}} = \sqrt{122}$

54)

$43^{43} = (40 + 3)^{43} = 40k + 3^{43}$ ....(1)

$3^{43} = (3^2)^{21}.3 = (8 + 1)^{31}.3$

$= 4n + 3$ ....(2)

∴ $43^{43^{43}} = (40 + 3)^{4n+3} = 40k + 3^{4n+3}$

$= 3^{4n+3}$

$= (81)^n \times 27$

$= (80 + 1)^n \times 27$

$= 80k + 27$

∴ Remainder = 27