PHYSICS

The period of oscillation of a simple pendulum is $\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$ Measured value of L is 20.0 cm known to 1 mm accuracy Measured value of L is 20.0 cm known to 1 mm accuracy and time for $\boxed{\text{100 oscillations}}$ of the pendulum is found to be 90 s using wrist watch of 1 s resolution. The accuracy in the determination of g is

(A) 3% (B) 1%

(C) 5% (D) 2%

$$\boxed{\left|\frac{\Delta g}{g}\right| \times 100 = ?}$$

$$\boxed{\begin{matrix} \Delta L = 0.1\text{cm.} & \Delta t = 1\text{ s} \\ L = 20\text{cm} & t = 90\text{ s} \end{matrix}}$$

$n = 100.$

$t = nT$

$$\boxed{t = 200\pi\sqrt{\frac{L}{g}}}$$

your exp.

$$g = 200^2\pi^2 L^{1} t^{-2}$$

$\boxed{\text{error}}$

$$\left|\frac{\Delta g}{g}\right|^{\times 100} = 1\left|\frac{\Delta L}{L}\right|^{\times 100} + 2\left|\frac{\Delta t}{t}\right|^{\times 100}$$

$$\%\text{ error } g = \frac{0.1 \times 100}{20}\% + \frac{2(1) \times 100}{90}\%$$

$$= 2.7\%$$

A car moves on a straight track from station A to the station B, with an acceleration $\boxed{a} = (b - cx)$, where b and c are constants and x is the distance from station A. The maximum velocity between the two stations is

(A) $b/\sqrt{c}$ (B) b/c

(C) $c/\sqrt{a}$ (D) $\sqrt{b}/c$

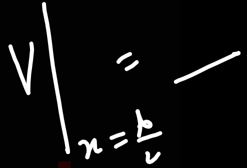
$\boxed{M-1}$

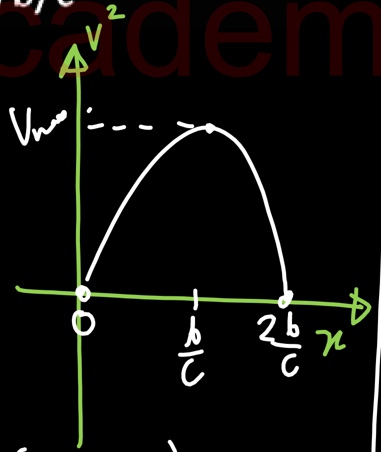
$a = -c\left(x - \frac{b}{c}\right)$

$v \cdot \frac{dv}{dx} = b - cx$

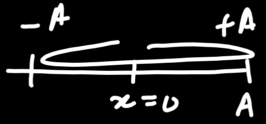
$\int_0^v v\,dv = \int_0^x (b - cx)\,dx$

max $\rightarrow \frac{v^2}{\cancel{2}} = 2bx - \frac{cx^2}{\cancel{2}} = x(2b - cx)$





SHM $\boxed{a = -\omega^2(x - 0)}$

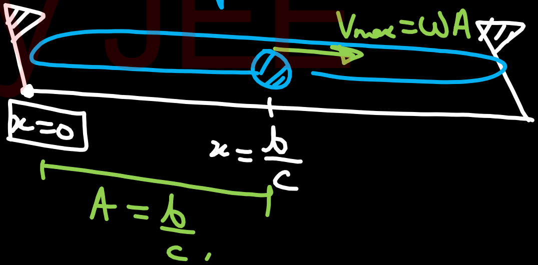


given

$a = -c\left(x - \frac{b}{c}\right)$

$a = -\omega^2(x - x_0)$

$\omega = \sqrt{c}$

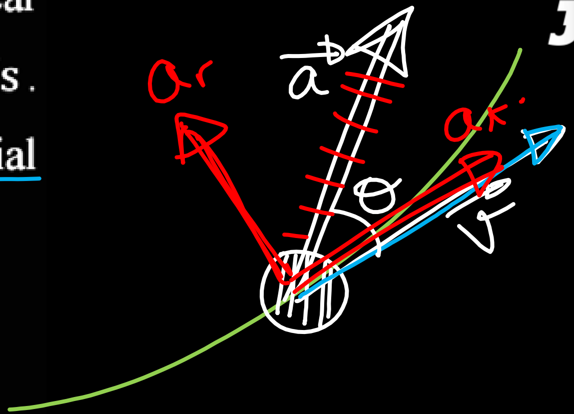


Ans $= \omega A = \sqrt{c} \cdot \frac{b}{c}$

Velocity of a particle moving in a curvilinear path varies with time as $\boxed{\vec{v} = (2t\hat{i} + t^2\hat{j})}$ m/s. Here, t is in second. At t = 1 s. The tangential acceleration of particle is

(A) $\frac{2}{\sqrt{5}}$ m/s^2 (B) $\frac{3}{\sqrt{5}}$ m/s^2

(C) $\frac{4}{\sqrt{5}}$ m/s^2 (D) $\frac{6}{\sqrt{5}}$ m/s^2



$a_t = a\cos\theta$

$a_t = \frac{\vec{a}\cdot\vec{v}}{v}$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = (2)\hat{i} + (2t)\hat{j}}$$

$\boxed{t=1}$

$$\boxed{\begin{aligned}\vec{v} &= 2\hat{i} + 1\hat{j}\\ \vec{a} &= 2\hat{i} + 2\hat{j}\end{aligned}}$$

$\boxed{\text{at } t=1}$

$$a_t = \frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}}$$

A simple harmonic motion is represented by

$$y = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]\,\text{cm}$$

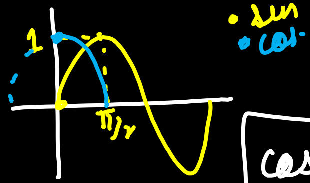
The amplitude and time period of the motion are

(A) 5 cm, $\frac{3}{2}$ S

(B) 10 cm, $\frac{2}{3}$ S

(C) 5 cm, $\frac{2}{3}$ S

(D) 10 cm, $\frac{3}{2}$ S

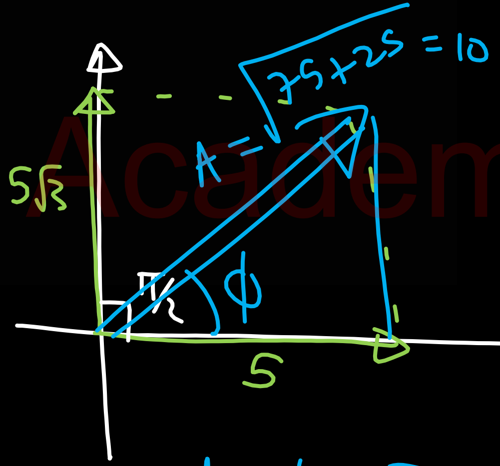


cos is ahead by π/2

$$y = 5\sin(\boxed{3\pi}t + 0) + 5\sqrt{3}\sin(3\pi t + \pi/2)$$

$$= \boxed{A}\sin(3\pi t + \phi)$$

$$y = 10\sin(\boxed{3\pi}t + \pi/3)$$



$\tan\phi = \sqrt{3}$

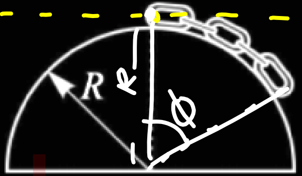
$\boxed{\omega, \nu, T}$

$\omega = 3\pi$

$T = \frac{2\pi}{\omega} = \frac{2}{3}$

5

A chain of length ℓ and mass m lies on the surface of a smooth hemisphere of radius R > l with one end tied to the top of the hemisphere. Gravitational potential energy of the chain with reference level of the top of the hemisphere is :



$l = R\phi$

$\boxed{\phi = \frac{l}{R}}$

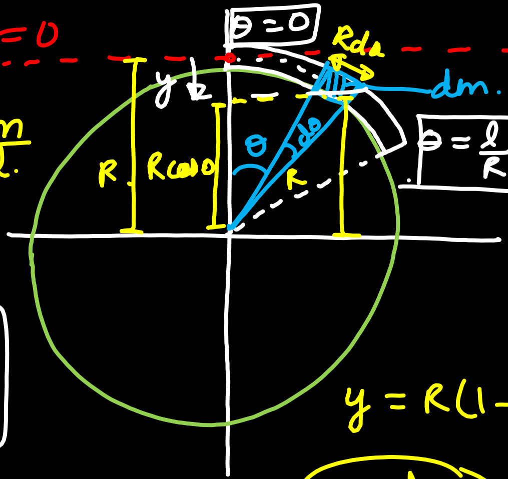
(A) ~~mgR~~

(B) ~~$\frac{mR^2g}{l}$~~

(C) ~~$mgR\sin\frac{l}{R}$~~

(D) $\underbrace{\frac{mR^2g}{\ell}\left[\sin\frac{\ell}{R} - \frac{\ell}{R}\right]}_{-ve}$

$U = -\frac{mgR^2}{\ell}(\phi - \sin\phi)$



$\mu = \frac{m}{\ell}$

$\boxed{\theta = \frac{\ell}{R} = \phi}$

$y = R(1-\cos\theta)$

$dm = \mu . Rd\theta = \frac{m}{\ell}Rd\theta.$

P.E. of element

$dU = -dm\, g\, y$

$\int dU = -\frac{mR}{\ell}gR\int_0^\phi (1-\cos\theta)d\theta.$

$U = -\frac{mgR^2}{\ell}\left[\theta - \sin\theta\right]_0^\phi$

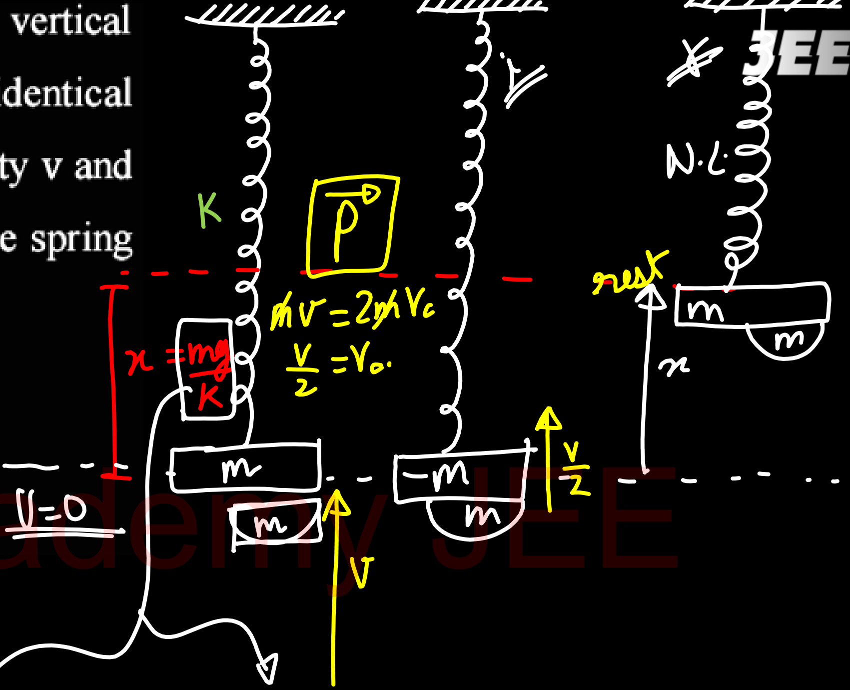
6

$e=0$

Block A of mass m is hanging from a vertical spring of force constant k. Another identical block B strikes the block A with velocity v and sticks to it. The value of v for which the spring just attains natural length is

(A) $\sqrt{\frac{6m}{k}}g$ (B) $\sqrt{\frac{8m}{k}}g$

(C) $\sqrt{\frac{24m}{k}}g$ (D) $\sqrt{\frac{12m}{k}}g$



$$\boxed{V=\sqrt{\frac{6m}{k}g^2}}$$

$$E_i = E_f$$

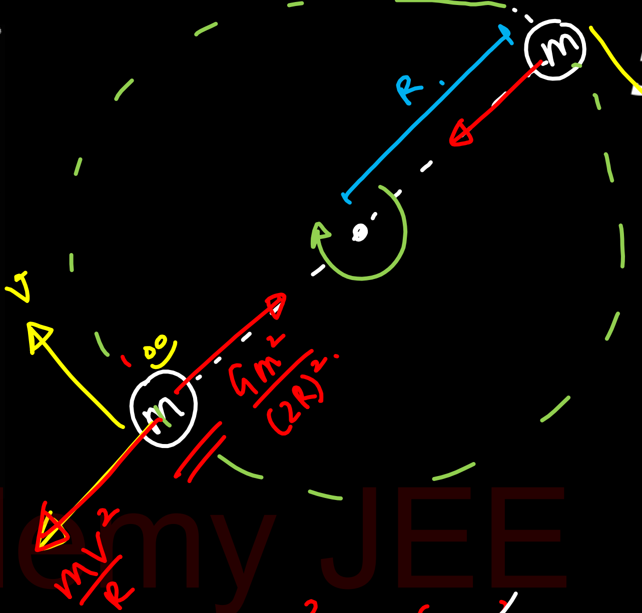
$$\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2 = 2mgx$$

$$\frac{mv^2}{4} + \frac{1}{2}\frac{m^2g^2}{k} = 2\frac{m^2g^2}{k}$$

Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (B) $v = \sqrt{\frac{Gm}{2R}}$

(C) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (D) $v = \sqrt{\frac{4Gm}{R}}$



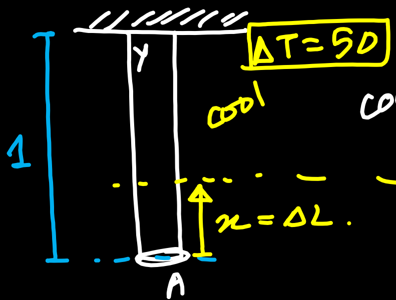
$$\frac{\cancel{m}V^2}{\cancel{R}} = \frac{Gm^2}{4R^2}$$

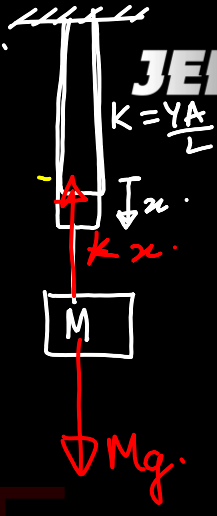
$$V = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

A thin rod having a length of 1 m and area of cross section $3 \times 10^{-6}\ m^2$ is suspended vertically from one end. The rod is cooled form 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are $2 \times 10^{11} Nm^{-2}$ and $2 \times 10^{-5} K^{-1}$, respectively. The value of M____ is kg (Take $g = 10 m^{-2}$)

(A) 30
(B) 60
(C) 90
(D) 15

$Y = \checkmark$
$A = \checkmark$
$L = \checkmark$
α





$\Delta L = L \alpha \Delta T$

$x = 1 \times 2 \times 10^{-5} \times 50$

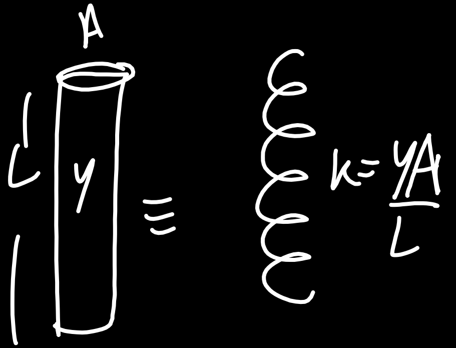
$\boxed{x = 10^{-3} m}$

$Kx = Mg$

$\frac{YA}{L} \cdot x = Mg$

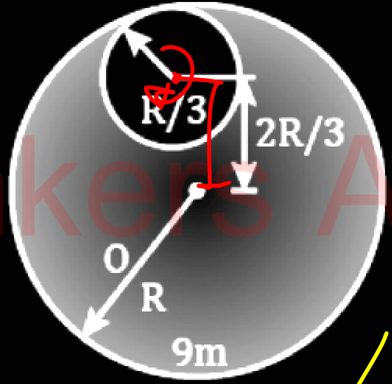
$\frac{2 \times 10^{11} \times 3 \times 10^{-6}}{1} \times 10^{-3} = M \times 10$

$\boxed{M = 60}$



9

A disc has mass $\boxed{9\ m}$ and radius R. A hole of radius R/3 is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre ' O ' of the disc and perpendicular to the plane of the disc is

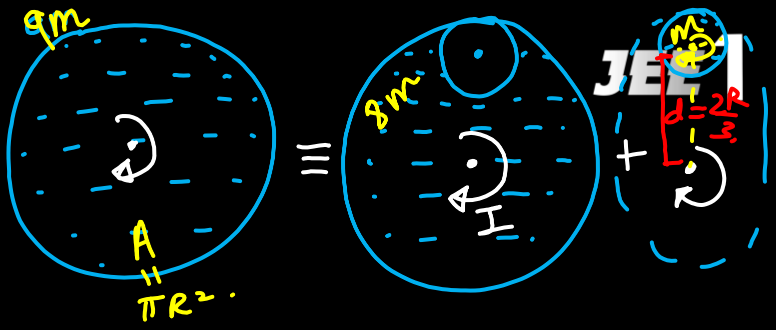


(A) ~~8 mR²~~

(B) $4\ mR^2$

(C) $\frac{40}{9}\ mR^2$

(D) $\frac{37}{9}\ mR^2$



Ans.

$$\frac{1}{2}(9m)r^2 = I + \left[I_{cm} + Md^2\right]$$

$$\frac{9}{2}mr^2 = I + \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + m\left(\frac{2R}{3}\right)^2\right]$$

$$\underline{I = 4mR^2}$$

10

Two coherent sources of light interfere. The intensity ratio of two sources is $\underset{I}{1}:\underset{4I}{4}$. For this interference pattern if the value of $\boxed{\dfrac{I_{max}+I_{min}}{I_{max}-I_{min}}}$ is equal to $\boxed{\dfrac{2\alpha+1}{\beta+3}}$, then $\boxed{\dfrac{\alpha}{\beta}}$ will be

(A) 1.5

(B) 2

(C) 0.5

(D) 1

$\boxed{\begin{array}{l} I_1 = I \\ I_2 = 4I \end{array}}$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$+1$

$I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = 9I$

$I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2 = I$

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{10\cancel{I}}{8\cancel{I}} = \boxed{\frac{5}{4} = \frac{2\alpha+1}{\beta+3}}$$

2

1

$$\boxed{\frac{\alpha}{\beta} = \frac{2}{1}}$$

11

A body of mass 2 kg moving with a speed of 4 ms^{-1} makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the centre of mass of two body system after the collision is $\frac{x}{10}$ ms^{-1}. Then the value of ' x ' is ____

(A) 5 (B) 25

(C) 10 (D) 15

2kg $\xrightarrow{4m/s}$ m rest.

$e = 1$

2kg $\xrightarrow{1m/s}$ m $= \frac{6}{5}$ $\longrightarrow$ $v = ?$ $v = 5$

$\longrightarrow +$

$\vec{P}$: $8 = 2 + mV$ — (1)

C: $e = \frac{V_{sep}}{V_{app}}$

$1 = \frac{V-1}{4}$

$\boxed{V = 5}$ — (2)

$6 = 5m$

$\boxed{m = \frac{6}{5}}$

$$\boxed{\vec{V}_{COM} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2}}$$

$$V_{COM} = \frac{8 + 0}{2 + \frac{6}{5}} = \boxed{2.5} = \frac{x}{10} \quad \boxed{x = 25}$$

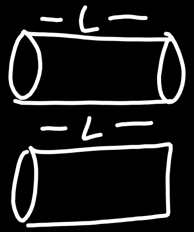
or $\frac{2(1) + \left(\frac{6}{5}\right)5}{2 + \frac{6}{5}}$.

12

A closed organ pipe and an open pipe of same length produce 6 beats per sec when they are set into vibrations simultaneously with their fundamental frequency. If the length of each pipe is doubled, then the number of beats produced is

(A) 4 (B) 3

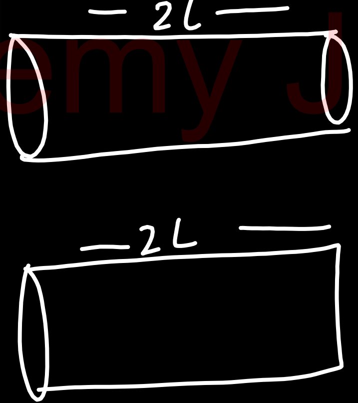
(C) 5 (D) 7



$\frac{V}{2L}$, $\frac{V}{4L}$ beat.

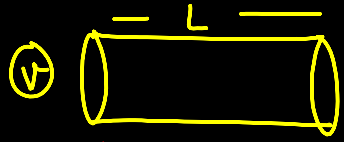
$$\frac{V}{2L} - \frac{V}{4L} = 6$$

$$\boxed{f_B = |f_1 - f_2|}$$



$\frac{V}{2(2L)}$, $\frac{V}{4(2L)}$

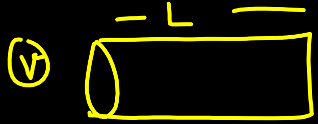
$$\frac{V}{4L} - \frac{V}{8L} = 3$$



$f_0 = \frac{V}{2L}$

$2f_0$

$3f_0$



$f_0 = \frac{V}{4L}$

$3f_0$

$5f_0$

13

An ideal gas is expanding such that $\boxed{PT^2 = \text{const}}$ constant. The coefficient of $\boxed{\text{volume expansion}}$ of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$

(C) $\frac{3}{T}$ ✓ (D) $\frac{4}{T}$

JEE 1

$PV = nRT$

$P = \frac{nRT}{V}$

$\left(\frac{nRT}{V}\right)T^2 = \text{const.}$

$V^{-1}T^3 = C$

$\frac{1}{V} = \frac{C}{T^3}$ — (1)

$V = \frac{T^3}{C}$

$\frac{dV}{dT} = \frac{3T^2}{C}$ — (2)

$\Delta V = V \boxed{\gamma} \Delta T$

$\gamma = \frac{\Delta V}{V \Delta T}$

$\boxed{\gamma = \frac{dV}{V\,dT}} = \left(\frac{1}{V}\right)\left(\frac{dV}{dT}\right) = \frac{\cancel{C}}{T^3} \cdot \frac{3T^2}{\cancel{C}} = \frac{3}{T}$

14

The rms speed of hydrogen molecules at a certain temperature is 300 m/s. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen, the rms speed will become

(A) 100 m/s (B) 300 m/s

(C) 600 m/s (D) $300\sqrt{2}$ m/s

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

H | H

$T \rightarrow 2T$

$M \rightarrow \frac{M}{2}$

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{\cancel{3R}T_1}{M_1}}}{\sqrt{\frac{\cancel{3R}T_2}{M_2}}}$$

$$\frac{300}{V_2} = \sqrt{\frac{\frac{T}{M}}{\frac{2T}{M/2}}} = \frac{1}{2}$$

$$\boxed{V_2 = 600}$$

15

Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50 $=n$. Further, it is found that the screw gauge has a zero error of -0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(A) 3.32 mm (B) 3.73 mm

(C) 3.67 mm (D) 3.38 mm

JEE 1

$$\text{Reading S.G.} = (MSR) + [CSR \times L.C] - \text{zero error}$$

$$= 3mm + \left[35 \times \left(\frac{P}{n}\right)\right] - (-0.03m)$$

$P = 0.5 mm.$

$n = 50$

$$= 3.38 mm$$

16

JEE 1

Two spheres of radii a and b respectively are charged and joined by a wire. The ratio of electric field of the spheres is

same pot.

(A) a/b
(B) b/a
(C) a^2/b^2
(D) b^2/a^2

$$V = \frac{kQ_1}{a} = \frac{kQ_2}{b}$$

$$\frac{Q_1}{Q_2} = \frac{a}{b}$$

$$\frac{E_1}{E_2} = \frac{\frac{kQ_1}{a^2}}{\frac{kQ_2}{b^2}} = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{b}{a}$$

$$= \frac{a}{b} \times \frac{b^2}{a^2} = \frac{b}{a} \;//$$

17

JEE 1

Two resistance R_1 and R_2 are made of different materials. The temperature coefficient of the material of R_1 is α and of the material of R_2 is $-\beta$. The resistance of the $\boxed{\text{series}}$ combination of R_1 and R_2 will not change with temperature, if R_1/R_2 equals

(A) $\frac{\alpha}{\beta}$

(B) $\frac{\alpha+\beta}{\alpha-\beta}$

(C) $\frac{\alpha^2+\beta^2}{\alpha\beta}$

(D) $\frac{\beta}{\alpha}$ ✓

$R_{eq} = R_1 + R_2$

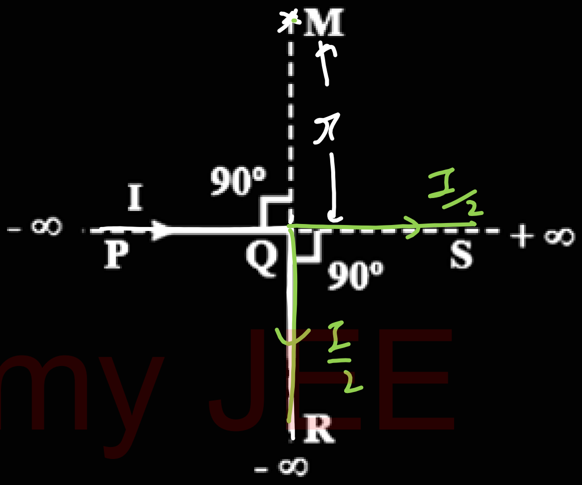
$(R_1+R_2)' = R_1(1+\alpha\Delta T) + R_2(1-\beta\Delta T)$

$= (R_1+R_2) + \underbrace{(R_1\alpha - R_2\beta)}_{0}\Delta T$

$R_1\alpha = R_2\beta \Rightarrow \frac{R_1}{R_2} = \frac{\beta}{\alpha}$

18

An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is B_1. Now another infinitely long straight conductor QS is connected at Q so that the current is I/2 in QR as well as in QS, The current in PQ remaining unchanged. The magnetic field at M is now B_2. The ratio B_1/B_2 is given by



(A) $\frac{1}{2}$

(B) 1

(C) $\frac{2}{3}$ ✓

(D) 2

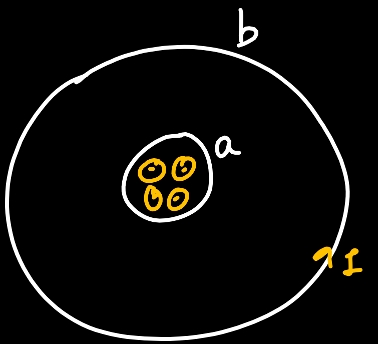
$B_1 = \frac{\mu_0 i}{4\pi r}$ — 1

$B_2 = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i/2}{4\pi r}$ — 2

$\frac{B_1}{B_2} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$

19

A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $\boxed{I = I_0 \cos(\omega t)}$. The emf induced in the smaller inner loop is nearly



(A) $\frac{\pi\mu_0 I_0}{2} \cdot \frac{a^2}{b} \omega \cos(\omega t)$ (B) $\frac{\pi\mu_0 I_0 b^2}{a} \omega \cos(\omega t)$

(C) $\frac{\pi\mu_0 I_0}{2} \cdot \frac{a^2}{b} \omega \sin(\omega t)$ (D) $\pi\mu_0 I_0 \frac{a^2}{b} \omega \sin(\omega t)$

$$\phi = B(\pi a^2)$$

$$= \frac{\mu_0 \pi a^2}{2b}(I_0 \cos \omega t) \quad \text{—(1)}$$

$$\varepsilon = -\frac{d\phi}{dt} = +\frac{\mu_0 \pi a^2}{2b} i_0 \omega (\sin \omega t)$$

20

When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case

(A) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(B) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$

(C) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(D) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$ ✓

$$3\times\left(eV=\frac{hc}{\lambda_1}-\varphi \quad (1)\right)$$

$$3eV=\frac{hc}{\lambda_2}-\varphi \quad (2)$$

$$\frac{3hc}{\lambda_1}-3\varphi=\frac{hc}{\lambda_2}-\varphi$$

$$\varphi=\frac{3}{2}\frac{hc}{\lambda_1}-\frac{hc}{2\lambda_2}$$

finally $$eV'=\frac{hc}{\lambda_3}-\varphi=hc\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$$

21

JEE 1

Calculate the work done in J when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are $10^5\ N/m^2$ and 6 litre respectively. The final volume of the gas is 2 litre. Molar specific heat of the gas at constant volume is $\frac{3R}{2}$.

$[(3)^{5/3} = 6.19]$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$PV^\gamma = \text{const}$$

$$10^5 \times (6\ \text{litr})^{5/3} = P_2 (2\ \text{litr})^{5/3}$$

$$P_2 = 3^{5/3} \times 10^5\ Pa$$

$$= 6.19 \times 10^5\ Pa \quad \text{——} \ (1)$$

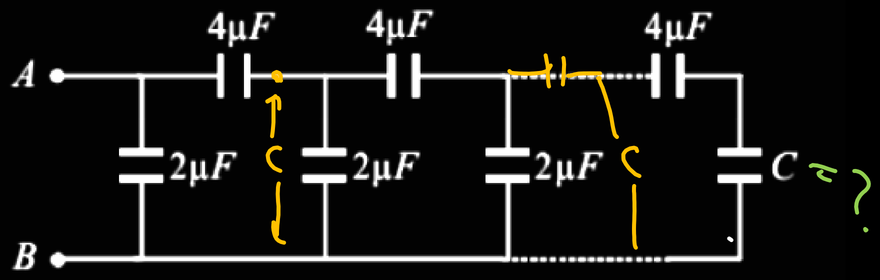
$$W = \frac{10^5 \times 10^{-3}\left[1 \times 6 - 6.19 \times 2\right]}{\frac{5}{3} - 1}$$

$$= -\frac{300}{2}\left[6.38\right] = -3 \times 319 = -\boxed{957}\ J$$

22

JEE 1

A finite ladder is constructed by connecting several sections of 2 μF, 4μF capacitor combinations as shown in figure. It is terminated by a capacitor of capacitance C. What value should be chosen for C. (in μF) such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between?



$$\frac{4C}{4+C}+2=C$$

$$\cancel{4C}+8+2C=\cancel{4C}+C^2$$

$$C^2-2C-8=0$$

$$C=\frac{2+\sqrt{4+32}}{2}$$

$$=1+3=\boxed{4}\,\mu F$$

23

JEE 1

When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25J_2$ then the value of R in Ω is

$J_1 = i_1^2 R$

$$J_1 = \left(\frac{2\varepsilon}{R+2r}\right)^2 R \quad \text{— (1)}$$

$$J_2 = \left(\frac{\varepsilon}{R+\frac{r}{2}}\right)^2 R \quad \text{— (2)}$$

$J_1 = 2.25\, J_2$

$$\left(\frac{2\varepsilon}{R+2r}\right)^2 R = 2.25\left(\frac{\varepsilon}{R+\frac{r}{2}}\right)^2 R$$

$$2\left(R+\frac{r}{2}\right) = 1.5\,(R+2r)$$

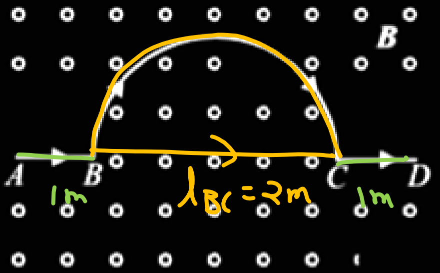
$$(2-1.5)R = 2r$$

$$R = 4r$$

$$= \boxed{4}\,\Omega$$

JEE 1

A wire ABCD is bent in the form shown here in the figure. Segments AB and CD are of length 1 m each while the semicircular loop is of radius 1 m. A current of 5 A flows from A towards the end D and the whole wire is placed in a magnetic field of 0.5 T directed out of the page. The force acting on the wire is ____ N.



$$l_{AD} = (1+2+1)$$
$$= 4m$$

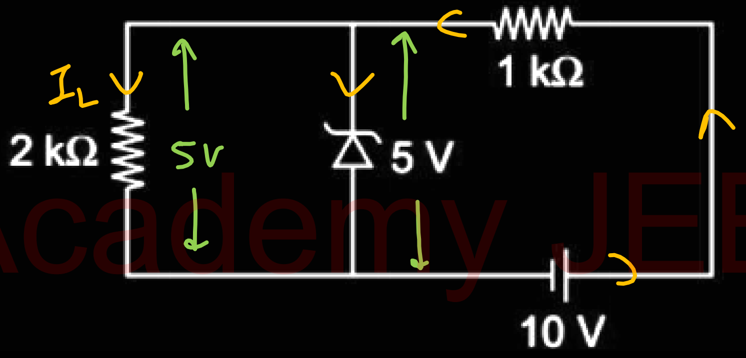
$$F = ilB$$
$$= 5(4)0.5$$
$$= \boxed{10}\ N$$

25

In connection with the circuit drawn below, the value of current flowing through 2kΩ resistor is

_____ × 10^{-4} A

JEE 1

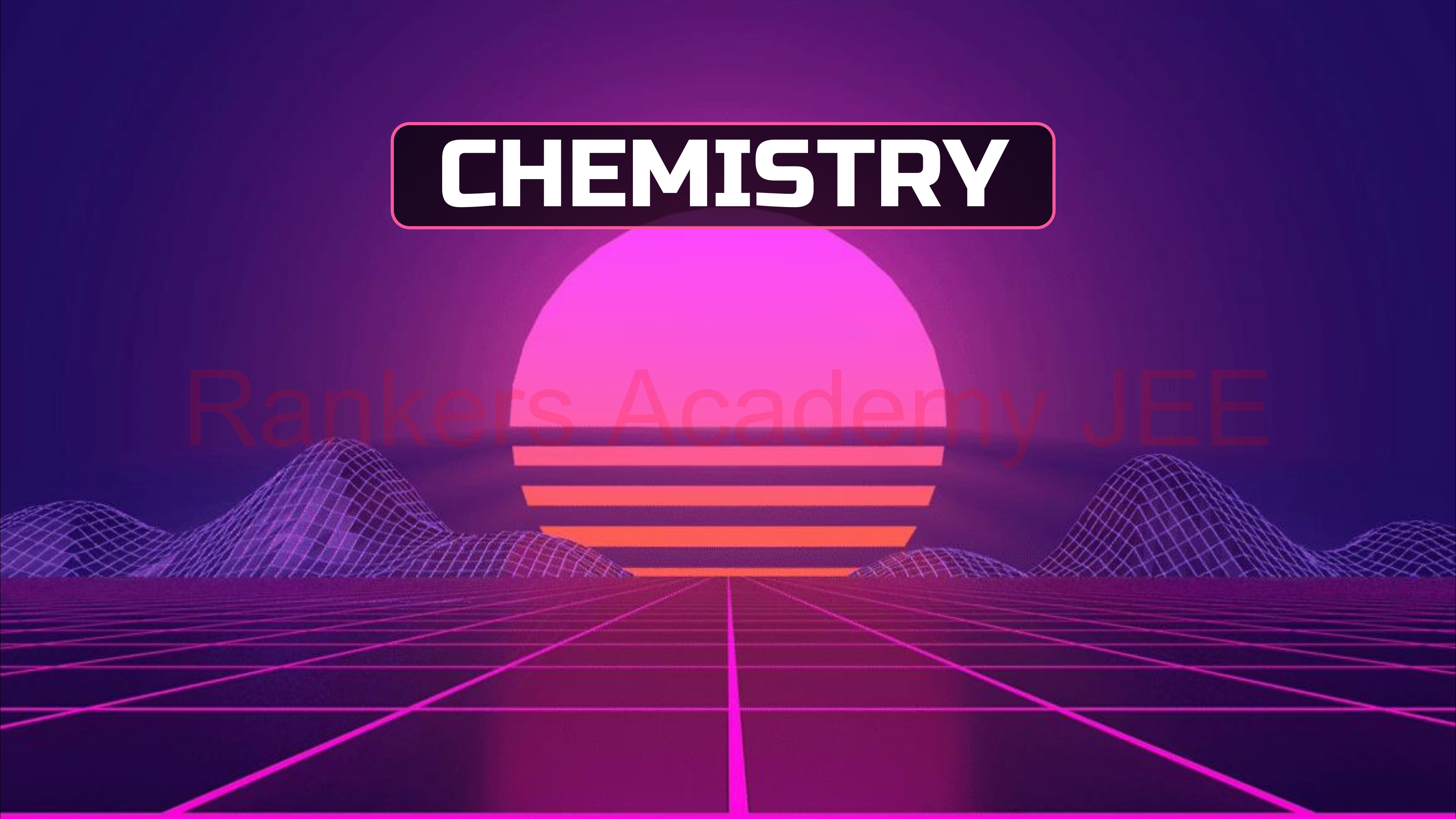


$I_L = \frac{V_2}{R_L}$

$= \frac{5V}{2k\Omega}$

$= 2.5\,mA$

$= \boxed{25} \times 10^{-4}$ A


CHEMISTRY

1

JEE 1

An organic amino compound reacts with aqueous nitrous acid at low temperature to produce an oily nitrosoamine. The compound is

(A) CH_3NH_2 (1° amine)

(B) $CH_3CH_2NH_2$ (1° amine)

(C) $CH_3CH_2NHCH_2CH_3$ (2° amine)

(D) $(CH_3CH_2)_3$ N (3° amine)

2

Given that (at T = 298 K)

$Cu(s)|Cu^{2+}(1.0M) \parallel Ag^{+}(1.0M)|Ag(s)$

$E^{\circ}_{cell} = 0.46\ V$

$Zn(s)|Zn^{2+}(1.0M) \parallel Cu^{2+}(1.0M)|Cu(s)$

$E^{\circ}_{cell} = 1.10\ V$

Then E_{cell} for

$Zn|Zn^{2+}(0.1M)$

$\parallel Ag^{+}(1.0M)|Ag$ at 298 K will be

(A) 1.59 V

(B) 1.53 V

(C) 2.53 V

(D) cannot be calculated due to insufficient data

$E^{0}_{Ag^{+}/Ag} - \cancel{E^{0}_{Cu^{+2}/Cu}} = 0.46V$ — 1

$\cancel{E^{0}_{Cu^{+2}/Cu}} - E^{0}_{Zn^{+2}/Zn} = 1.10$ — 2

$E^{0}_{Ag^{+}/Ag} - E^{0}_{Zn^{+2}/Zn} = 1.56V$

$\underset{}{Zn} + 2\underset{1m}{Ag^{+}} \longrightarrow \underset{0.1m}{Zn^{+2}} + 2Ag \qquad E^{0}_{cell}$

$E_{cell} = E^{0}_{cell} - \frac{0.059}{2}\log(0.1)$

$E_{cell} = E^{0}_{cell} + 0.0295$

$= (E^{0}_{Ag^{+}/Ag} - E^{0}_{Zn^{+2}/Zn}) + 0.0295$

$= 1.56 + 0.0295$

$= \underline{1.59V}$

3

What is the end product of following sequence of reaction

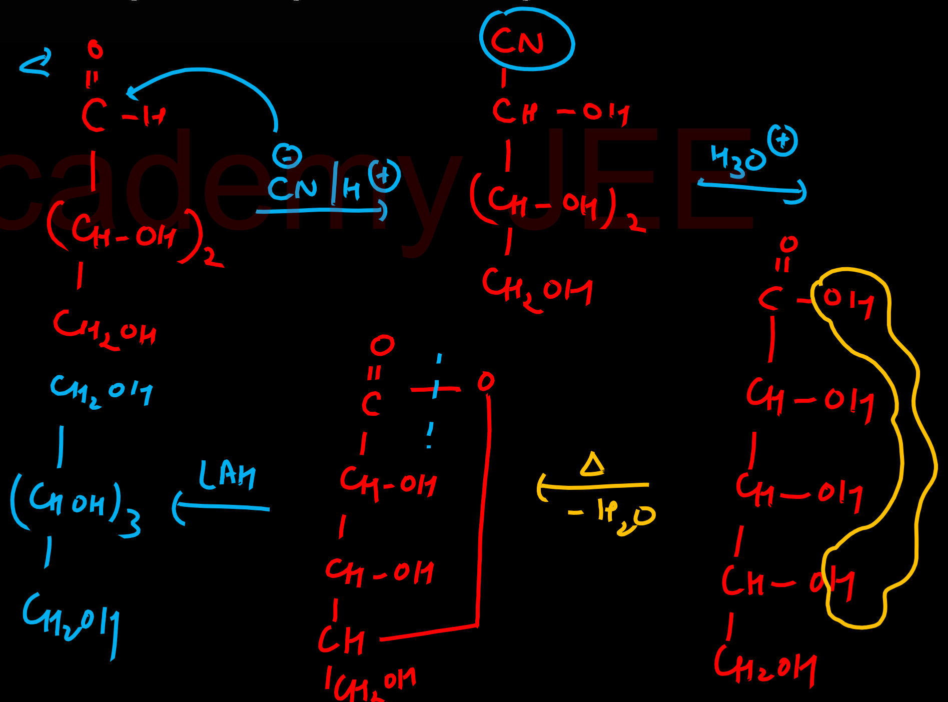
$$\underset{CH_2OH}{\overset{CHO}{(CH-OH)_2}} \xrightarrow{KCN/H^+} \xrightarrow{H_3O^+} \xrightarrow{\Delta} \xrightarrow{LiAlH_4}$$

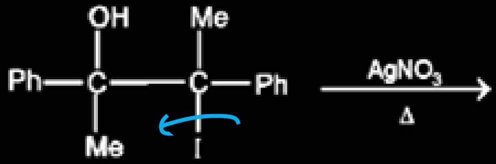
(A) $CH_2OH-(CH-OH)_3-CH_2OH$

(B) $CHO-(CH-OH)_3-CH_2OH$

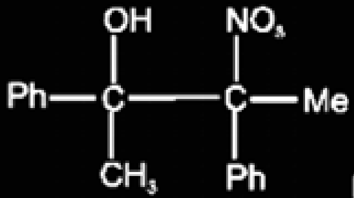
(C) $CH_2OH-\underset{|}{\overset{|}{C}}(H)(OH)-CH_2OH$

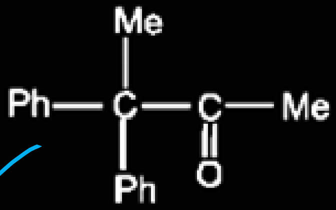
(D) $CH_2OH-CH-OH-CH-OH-CH_2OH$

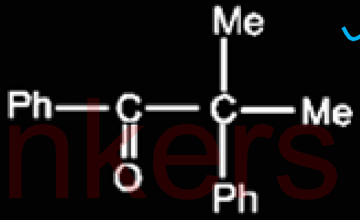




? Major product is

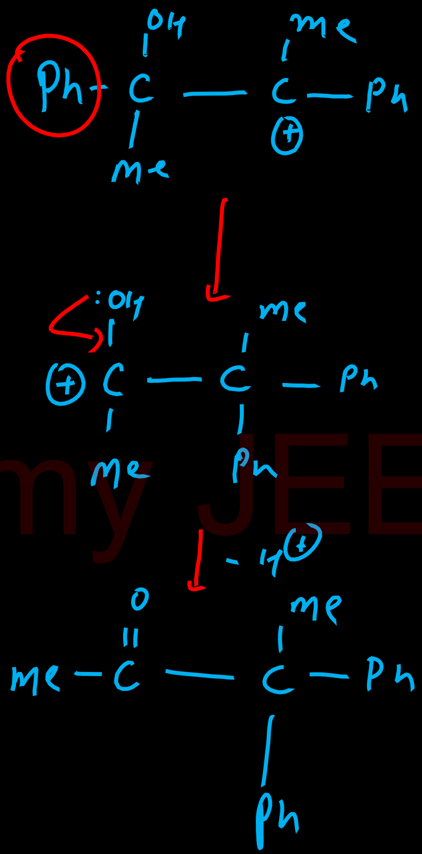
(A) 

(B) 

(C) 

(D) None of These

(given : Ph > me migratory aptitude)



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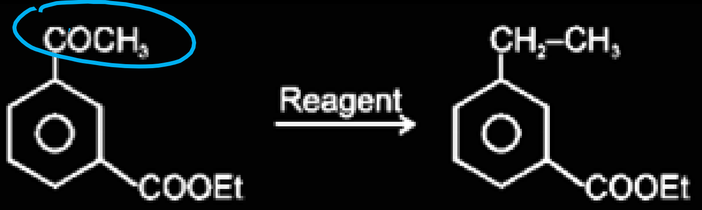
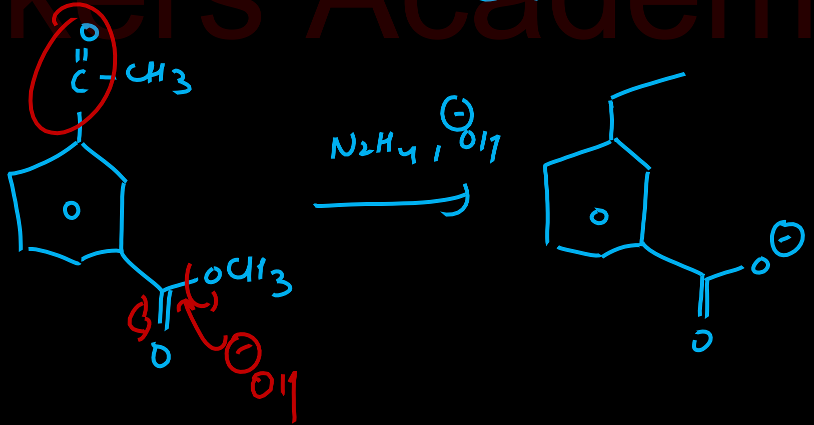


Which of the following reagent can be used for above conversion?

(A) $LiAlH_4$ (B) H_2, Pt

(C) Zn, Hg, Conc. HCl (D) N_2H_4, KOH



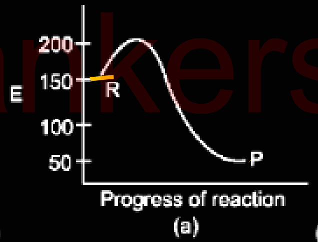
6

An exothermic chemical reaction proceeds by two stages. Reactants $\xrightarrow{\text{STAGE 1}}$ intermediate $\xrightarrow{\text{STAGE 2}}$ products. The activation energy of stage 1 is 50 kJ mol^{-1}. The overall enthalpy change of the reaction is -100 kJ mol^{-1}. Which diagram could represent the energy level diagram for reaction?

$\Delta H < 0$

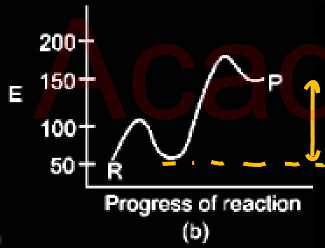
$\Delta H > 0$

(A)



(a)

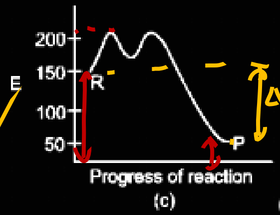
(B)



(b)

$\Delta H = E_P - E_R$

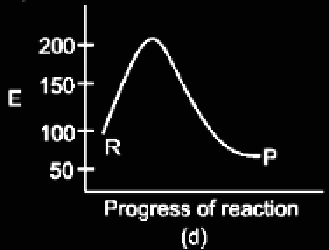
(C)



(c)

ΔH

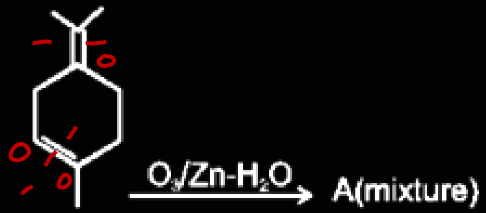
(D)

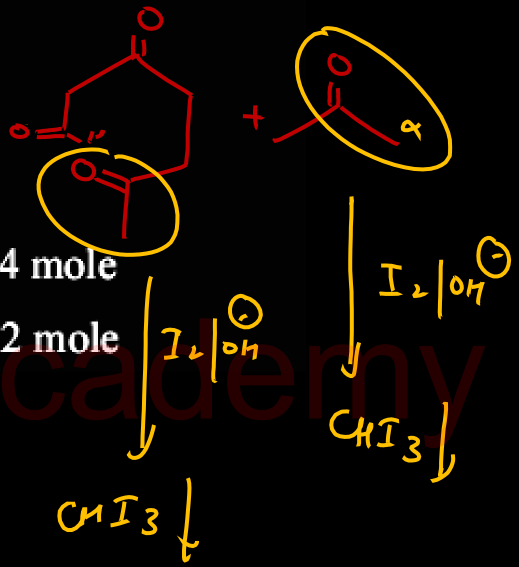


(d)

7

When mixture of ' A ' reacts with I_2/OH^-, than how many moles of CHI_3 will form?





(A) 3 mole

(B) 4 mole

(C) 1 mole

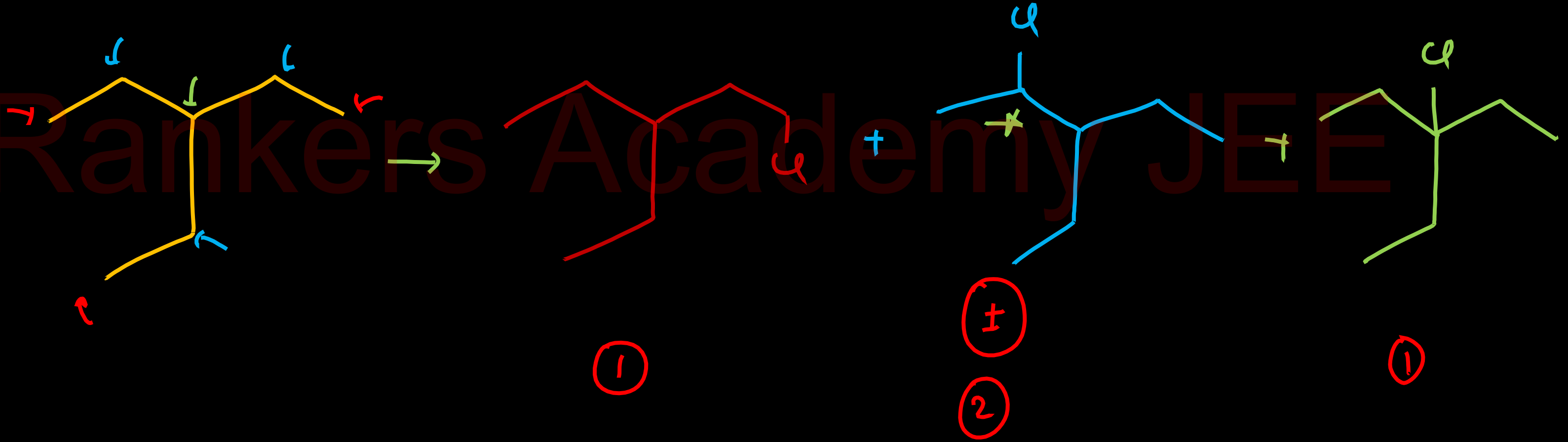
(D) 2 mole

8

The value of M and N are respectively:

(A) 3,2 (B) 3,3

(C) 4,3 (D) 3,4



9

One mole of an ideal diatomic gas ($C_v = 5$cal) was transformed from initial 25°C and 1 L to the state when temperature is 100°C and volume 10 L. The entropy change of the process can be expressed as (R = 2 calories/mol/K)

(A) $3\ln\frac{298}{373} + 2\ln 10$

(B) $5\ln\frac{373}{298} + 2\ln 10$

(C) $7\ln\frac{373}{298} + 2\ln\frac{1}{10}$

(D) $5\ln\frac{373}{298} + 2\ln\frac{1}{10}$

$T_1 = 298K \quad T_2 = 373K$

$V_1 = 1$ litre

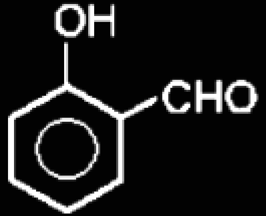
$V_2 = 10$ litre

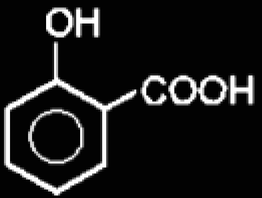
$$\Delta S = nC_{v,m}\ln\frac{T_2}{T_1} + nR\ln\frac{V_2}{V_1}$$

$$= 5\ln\frac{373}{298} + 2\ln\frac{10}{1}$$

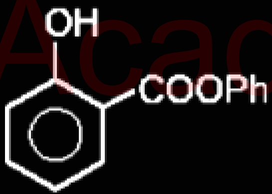
10

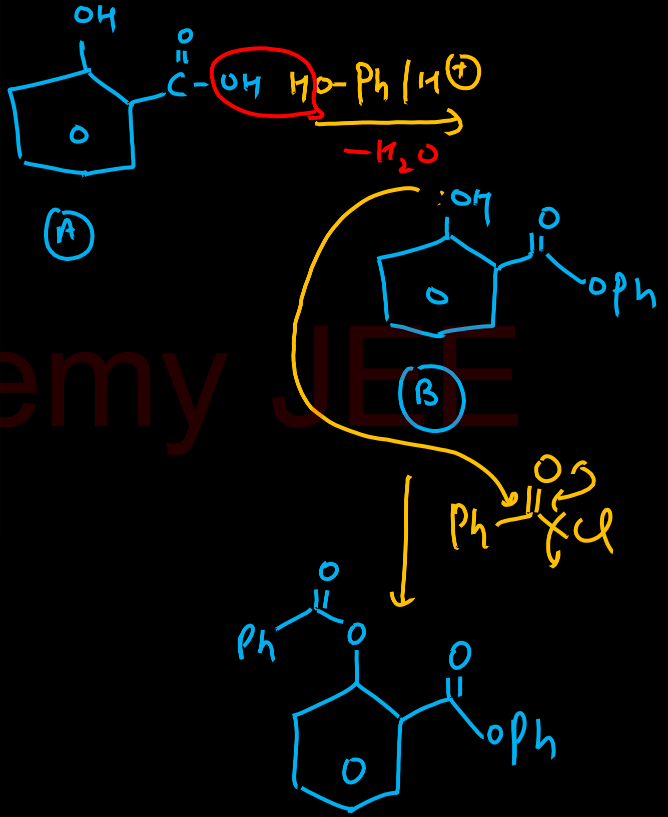
Give the product (P) of the above reaction sequence.

(A) 

(B) 

(C) 

(D) 



11

JEE 1

The brown gas prepared by the action of concentrated nitric acid on copper is an equilibrium mixture of dinitrogen tetraoxide and nitrogen dioxide

$$1\ N_2O_4(g) \overset{\text{Endothermic}}{\rightleftharpoons} \underset{\text{Brown}}{2NO_2(g)}$$

Which one of the following change would result in a $\underbrace{\text{darkening of the colour}}$?

(1) Increase in pressure ✗

(2) Increase in temperature ✓

(3) Addition of a catalyst ✗

(4) Removal of dinitrogen tetraoxide by liquefaction ✗

12

JEE 1

A complex of certain metal has the magnetic moment of $\underbrace{4.91 BM}$ whereas another complex of the same metal with same oxidation state has $\underbrace{\text{zero magnetic moment}}$. The metal ion could be

(A) Co^{2+}

(B) Mn^{2+}

(C) Fe^{2+}

(D) Fe^{3+}

$n = 4$

$n = 0$

$Fe : 3d^6 4s^2$

$\boxed{Fe^{+2} : 3d^6 4s^0}$

Case I d^4 $\boxed{\uparrow \mid \uparrow \mid \uparrow \mid \uparrow \mid \;}$ $n = 4$

$\boxed{\uparrow\downarrow \mid \uparrow \mid \uparrow \mid \; \mid \;}$ $n = 2$

Case II d^6 $\boxed{\uparrow\downarrow \mid \uparrow \mid \uparrow \mid \uparrow \mid \uparrow}$ $n = 4$

$\boxed{\uparrow\downarrow \mid \uparrow\downarrow \mid \uparrow\downarrow \mid \; \mid \;}$ $n = 0$

13

Which of the following species/molecule have same shape and same hybridisation of its central atom

(A) XeF_2, CO_2

(B) $I_3^-, SnCl_2$

(C) Cationic part of PCl_5 (s) and PBr_5 (s)

(D) BF_4^-, BF_2^+

JEE 1

$\boxed{A}$ ×

XeF_2
- sp^3d
- linear

CO_2
- sp
- linear

$\boxed{B}$ ×

I_3^-
- sp^3d
- linear

$SnCl_2$
- sp^2
- Bent

$\boxed{C}$ ✓

PCl_4^+
- sp^3
- Td

PBr_4^+
- sp^3
- Td

$\boxed{D}$

BF_4^-
- sp^3
- Td

BF_2^+
- sp
- linear

14

The electron of a hydrogen atom is in its nth Bohr orbit having de-Broglie wavelength of 13.4Å. The value of n is

(A) 2 (B) 4

(C) 6 (D) 5

$$\lambda_{Debroglie} = \sqrt{\frac{150}{V}} = 13.4\,Å$$

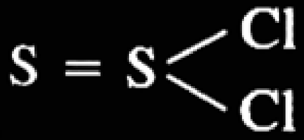
$V = 0.85$ volts

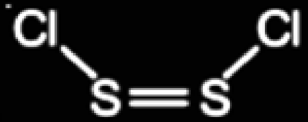
$KE = 0.85\,eV$

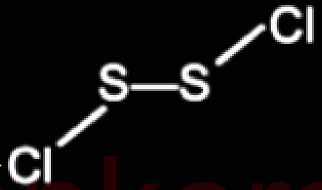
Total $E = -0.85\,eV$

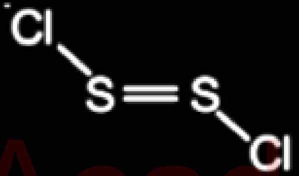
15

Which of the following is correct structure of S_2Cl_2 ?

(A) 

(B) 

(C) 

(D) 

16

The maximum amount of CH_3Cl that can be prepared from 20 g of CH_4 and 10 g of Cl_2 by the following reaction, is :

$CH_4 + Cl_2 \longrightarrow CH_3Cl + HCl$, (presume that no other reaction is taking place)

(A) 3.625 mole
(B) 0.141 mole
(C) 1.41 mole
(D) 0.365 mole

$$\underset{\frac{20}{16}}{CH_4} + \underset{\frac{10}{71}}{\overset{L.R}{Cl_2}} \longrightarrow CH_3Cl + HCl$$

moles of L.R $= \frac{10}{71}$

moles of Product formed $= \frac{10}{71} = 0.141$ mole

17

JEE 1

In order to precipitate a metal ion M^{2+} in the form of metal sulphide (MS) from saturated solution of H_2S, the best way is to

(A) Add an acid (B) Increase H_2S in solution

(C) Raise the pH (D) Heat the solution

$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

$$M^{2+} + S^{2-}\uparrow \longrightarrow MS$$

pH ↑ → H^+ decreases → Eq^m shifts forward → Precipitation occurs

18

JEE 1

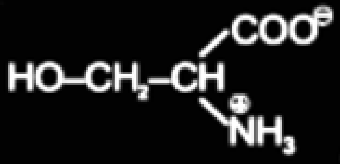
The boiling point of C_6H_6, CH_3OH, $C_6H_5NH_2$ and $C_6H_5NO_2$ are 80°C, 65°C, 184°C and 212°C respectively. Which will show highest vapour pressure at room temperature :-

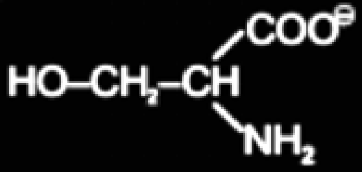
(A) C_6H_6
(B) CH_3OH
(C) $C_6H_5NH_2$
(D) $C_6H_5NO_2$

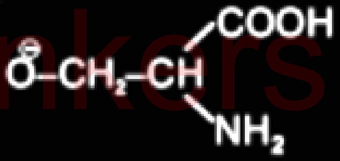
Vapour Pressure $\propto \dfrac{1}{B.P}$

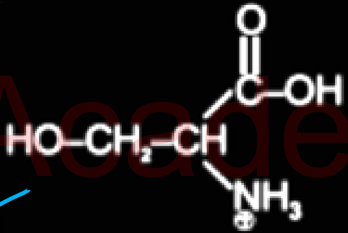
19

Which of the following amino acid structure is favoured at pH = 1 ?

(A) 

(B) 

(C) 

(D) 

When PH = 1, solution is highly acidic, amino acid exists in cationic form.

20

JEE 1

A reaction system in equilibrium according to reaction $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$ in one litre vessel at a given temperature was found to be 0.12 mole each of SO_2 and SO_3 and 5 mole of O_2. In another vessel of one litre contains 32 g of SO_2 at the same temperature. What mass of O_2 must be added to this vessel in order that at equilibrium 20% of SO_2 is oxidized to SO_3 ?

(A) 0.4125 g

(B) 11.6 g

(C) 1.6 g

(D) 5.55 g

$2SO_2 + O_2 \rightleftharpoons 2SO_3$

0.12   5 mol   0.12

$$K_c = \frac{\cancel{(0.12)^2}}{\cancel{(0.12)^2}(5)} = \frac{1}{5} = 0.2$$

$2SO_2 + O_2 \rightleftharpoons 2SO_3$

t = 0   0.5 mol. x 0

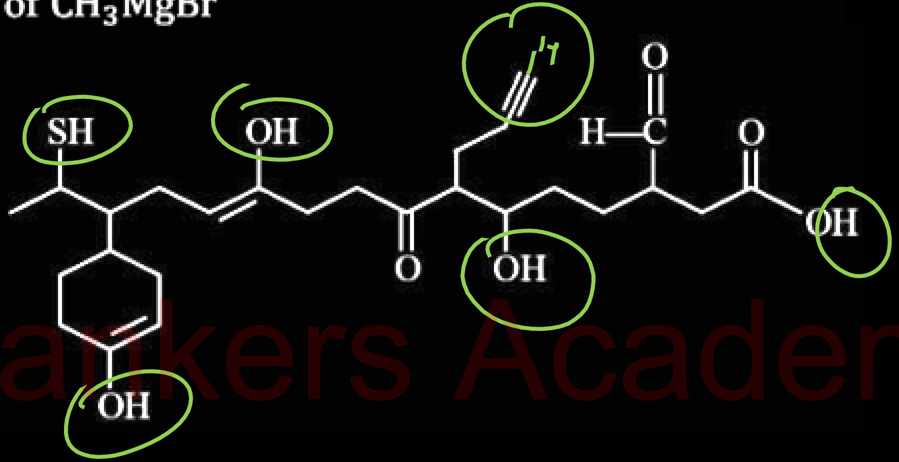
t = 0   0.4 mol $x - 0.05$ 0.1 mol.

$$K_c = \frac{(0.1)^2}{(0.4)^2(x-0.05)} = 0.2$$

$x = 0.3625$ mol.

21

Find number of moles of CH_4 obtained when 1 mole of the given compound reacts with excess of CH_3MgBr



6

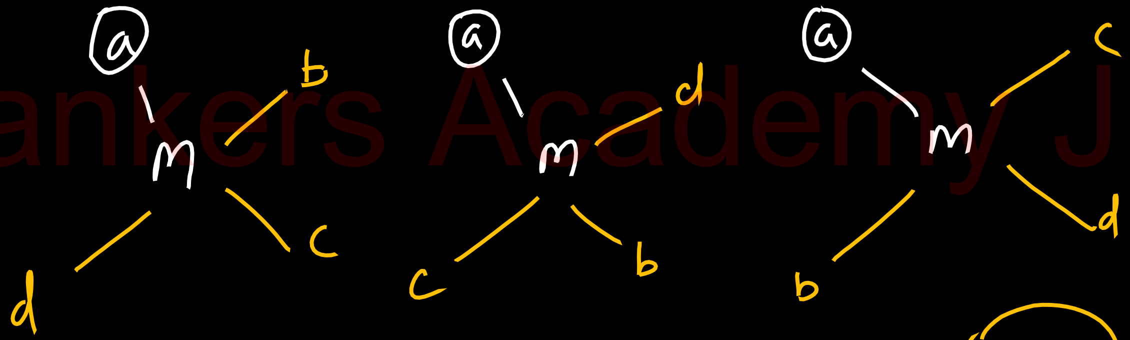
$\overset{\ominus}{CH_3} + H^{\oplus} \longrightarrow CH_4(g)$

Base Acid

22

Total number of geometrical isomers for the complex $[RhBr(H_2O)(PPh_3)(NH_3)]$ is :

(Mabcd)



Ans 3

23

JEE 1

On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 75kPa and 50kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1}) in Kpa ____ is (nearest integer)

$$P_{solution} = p^{\circ}_{H} X_H + p^{\circ}_{O} X_O$$

$$= 75 X_H + 50(1 - X_H)$$

$$= 25 X_H + 50 = 61$$

$$X_H = \frac{n_H}{n_H + n_O} = \frac{0.25}{0.25 + \frac{35}{114}}$$

$$X_H = \frac{0.25}{0.557}$$

24

JEE 1

A first order reaction takes 40 min for 30% decomposition. Calculate $t_{1/2}$. In min (Given $\log 7 = 0.845$) (nearest integer)

$K = \frac{1}{t} \times \ln \frac{[A]_0}{[A]_t}$

$K = \frac{1}{40} \ln \frac{100}{70}$

$K = \frac{1}{40} \times 2.303 (1 - 0.845)$

$K = \frac{2.303}{40} \times 0.155$

$t_{1/2} = \frac{\ln 2}{K} = \frac{2.303 \log 2}{K}$

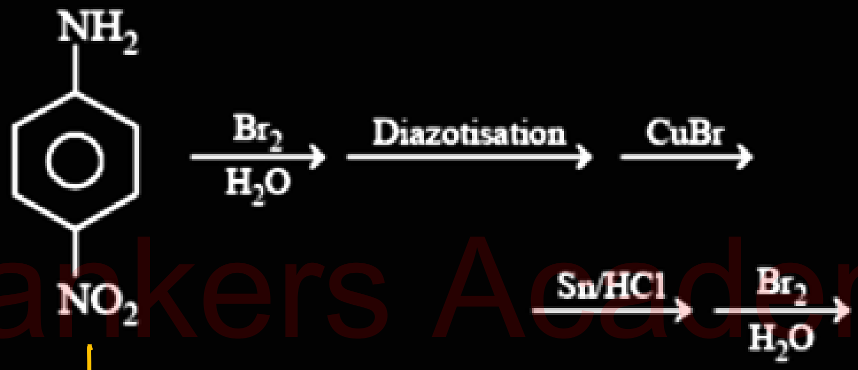
$t_{1/2} = \frac{\cancel{2.303} \times 0.301}{\cancel{2.303} \times 0.155} \times 40$

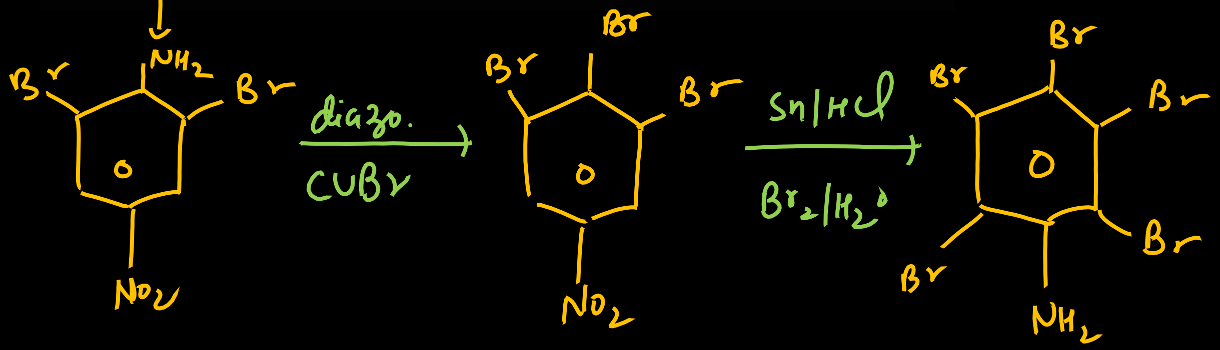
$t_{1/2} = \frac{301}{155} \times 40$

$= \underline{78}$

25

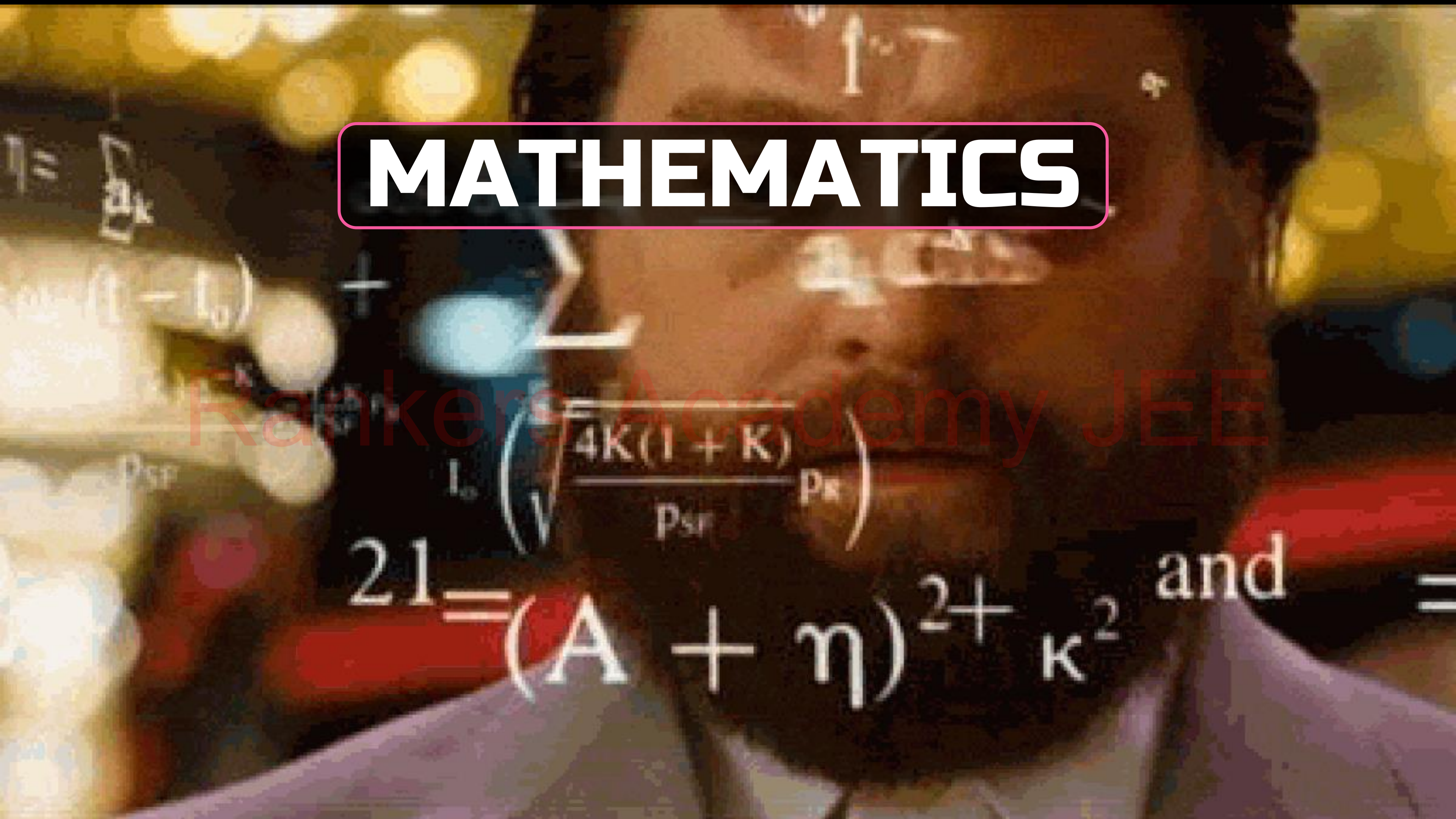
Find the total number of bromine atoms present in the end product of following scheme of reactions.





5

MATHEMATICS



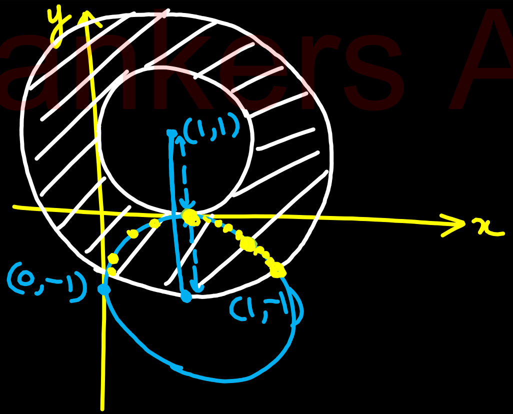
Let $A = \{z \in \mathbb{C} : 1 \le |z - (1 + i)| \le 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B :

(A) is an empty set

(B) contains exactly two elements

(C) contains exactly three elements

(D) is an infinite set



2

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18 (B) 24

(C) 36 (D) 96

JEE1

$3x^2 + \lambda x - 1 = 0 < {}^{\alpha}_{\beta}$

$\alpha + \beta = -\frac{\lambda}{3}$; $\alpha\beta = -\frac{1}{3}$

$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$

$\Rightarrow \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = 15$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 15(\alpha\beta)^2$

$\Rightarrow \left(-\frac{\lambda}{3}\right)^2 - 2\left(-\frac{1}{3}\right) = 15\left(-\frac{1}{3}\right)^2 = \frac{5}{3}$

$\Rightarrow \frac{\lambda^2}{9} = 1 \Rightarrow \lambda = 3, -3$

$\boxed{\begin{array}{l} \alpha + \beta = -1 \text{ or } 1 \\ \alpha\beta = -1/3 \end{array}}$

$$6\left(\alpha^3+\beta^3\right)^2$$

$$=6\left[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\right]^2$$

$$=6\left[(-1)^3-3\left(-\frac{1}{3}\right)(-1)\right]^2$$

$$=6\left[-1-1\right]^2$$

$$=24$$

$$6\left[(1)^3+3\left(+\frac{1}{3}\right)(1)\right]^2$$

$$=24$$

Let $S = \{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda$, then λ is equal to :

(A) 218

(B) 221 ✓

(C) 663

(D) 1717

$n = 1, 3, 5, 7, 9, 11 \ldots\ldots 49$

$S = \sqrt{n} = \{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{9}, \sqrt{11} \ldots, \sqrt{49}\}$

$a =$

$|\operatorname{adj} A| = |A|^{n-1} = |A|^{3-1} = |A|^2$

$|A| = 1 + a^2$

Ans : $100\lambda = \sum |A|^2 = \sum (1+a^2)^2$

$\cancel{100}\lambda = \dfrac{\cancel{1} \times \cancel{25} \times \overset{13}{\cancel{26}} \times \overset{17}{\cancel{51}}}{\underset{3}{\cancel{6}}}$

$\lambda = 13 \times 17$

$= 221.$

a	a^2	$1+a^2$	$(1+a^2)^2$
$\sqrt{1}$	1	2	2^2
$\sqrt{3}$	3	4	4^2
$\sqrt{5}$	5	6	6^2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sqrt{49}$	49	50	50^2

$$\sum(1+a^2)^2 = 2^2+4^2+6^2+\cdots+50^2$$

$$= 2^2(1^2+2^2+3^2+\cdots+25^2) = 4\cdot\frac{25\cdot 26\cdot 51}{6}$$

JEE 1

4

The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0,1]$ is :

(A) $3 + \frac{\sin(1)\cos^2(1/2)}{2}$

(B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$

(D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

$$f(x) = |(2x-1)(x+2)| + \frac{\sin 2x}{2}$$

$$f(x) = \begin{cases} -(2x^2+3x-2) + \frac{\sin 2x}{2}; & x \in (0, 1/2) \\ (2x^2+3x-2) + \frac{\sin 2x}{2}; & x \in (1/2, 1) \end{cases}$$

$+ \quad - \quad +$

$-2 \quad 0 \quad \frac{1}{2} \quad 1$

$$f'(x) = \begin{cases} -(4x+3) + \cos 2x; & x \in (0, 1/2) \\ \in(-5,-3) + \in\left(\frac{1}{2}, 1\right) = -ve & 4x \in (0,2),\ 4x+3 \in (3,5) \\ (4x+3) + \cos 2x; & x \in (1/2, 1) \\ \in(5,7) \quad \in(-1,1) = +ve & 4x \in (2,4),\ 4x+3 \in (5,7) \end{cases}$$

$- \quad +$

$0 \quad \frac{1}{2} \quad 1$

f':

Min
$x = 1/2$

$f(0) = 2$

$f(1) = 3 + \sin 1 \cos 1 = 3 + \frac{\sin 2}{2} \longrightarrow \text{Max}$

$f\left(\frac{1}{2}\right) = \frac{\sin 1}{2} \longrightarrow \text{Min}$

Ans: $(3 + \sin 1 \cos 1) + \frac{\sin 1}{2}$

$= 3 + \sin 1 \left(\cos 1 + \frac{1}{2}\right)$

$= 3 + \frac{\sin 1 (2\cos 1 + 1)}{2}$

5

JEE 1

Let $\hat{a}, \hat{b}$ and be unit vectors. If $\vec{c}$ be vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to :

$36|c|^2$

Ans : $36|\vec{c}|^2$.

(A) $6(3-\sqrt{3})$

(B) $3+\sqrt{3}$

(C) $6(3+\sqrt{3})$

(D) $6(\sqrt{3}+1)$

$$|\hat{b}|^2 = |\vec{c} + 2(\vec{c} \times \hat{a})|^2$$

$$1 = |\vec{c}|^2 + 4|\vec{c} \times \hat{a}|^2 + 2.2\underbrace{\vec{c}\cdot(\vec{c}\times\hat{a})}_{=0}$$

$$1 = |\vec{c}|^2 + 4|\vec{c}|^2|1|\sin^2 15^\circ$$

$$1 = |\vec{c}|^2\left(1 + 4\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2\right)$$

$$1 = |\vec{c}|^2\left(1 + \cancel{4}\frac{(\cancel{4}^{2} - 2\sqrt{3})}{\cancel{8}_{2}}\right) \Rightarrow 1 = |c|^2(3-\sqrt{3})$$

$$36|c|^2 = \frac{36}{3-\sqrt{3}}$$

$$= \frac{36\,(3+\sqrt{3})}{3^2-(\sqrt{3})^2}$$

$$= \frac{\overset{6}{\cancel{36}}\,(3+\sqrt{3})}{\cancel{9-3}}$$

$$= 6(3+\sqrt{3})$$

6

JEE 1

If $|x| < 1$ and $|y| < 1$, then the sum to infinity of the series $(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots\ldots\ldots$ upto ∞ is

(A) $\frac{(x+y-xy)}{(1-x)(1-y)}$ ✓

(B) $\frac{(x-y+xy)}{(1-x)(1-y)}$

(C) $\frac{(x+y-xy)}{(1+x)(1+y)}$

(D) infinite

$= (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + - - -$

$\Rightarrow \frac{x^2-y^2}{x-y} \qquad \frac{x^3-y^3}{x-y} \qquad \frac{x^4-y^4}{x-y} + - -$

$\Rightarrow \left(\frac{1}{x-y}\right)\left[x^2-y^2+x^3-y^3+x^4-y^4+ - -\right]$

$\Rightarrow \frac{1}{x-y}\left[(x^2+x^3+x^4+ -)-(y^2+y^3+y^4+ -)\right]$

JEE 1

6

$$= \frac{1}{x-y}\left[\frac{x^2}{1-x} - \frac{y^2}{1-y}\right]$$

$$= \frac{1}{x-y}\left[\frac{x^2 - x^2y - y^2 + xy^2}{(1-x)(1-y)}\right]$$

$$= \frac{(x^2-y^2) + xy(y-x)}{(x-y)(1-x)(1-y)}$$

$$= \frac{\cancel{(x-y)}\left[x+y-xy\right]}{\cancel{(x-y)}(1-x)(1-y)}$$

Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1,2,3, ... ,18 and are arranged in the increasing order $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

$\underbrace{x_1}_{{}^6C_1 = 6}\ \boxed{7}\ \underbrace{x_3}_{{}^3C_1 = 3}\ \boxed{11}\ \underbrace{x_5}_{{}^7C_1 = 7}$

$N^r = 6 \times 3 \times 7$

$D^r : {}^{18}C_5 = \frac{18!}{13!\,5!}$

$= \frac{14 \times \cancel{15} \times \cancel{16}^{2} \times 17 \times 18}{\cancel{120}\,\cancel{8}}$

$= 14 \times 2 \times 17 \times 18$

$\text{Prob} = \frac{\cancel{6} \times \cancel{3} \times \cancel{7}}{\cancel{14}^{2} \times 2 \times 17 \times \cancel{18}}$

$= \frac{1}{68}.$

8

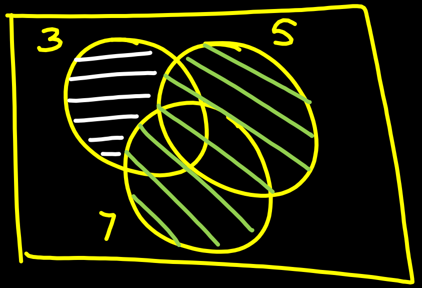
If the probability that a number selected from the set $\{1,2,3, \dots\dots, 1000\}$ is divisible by 3 but neither divisible by 5 nor by 7 is $\frac{m}{n}$, $= \frac{229}{1000}$

(where m and n are relatively prime numbers)

then $(n - m)$ is $:= 1000 - 229 =$

(A) 667 (B) 717

(C) 771 (D) 766



Ans:
$n(3 \cup 5 \cup 7) - n(5 \cup 7)$

$n(3) = \left[\frac{1000}{3}\right] = 333$

$n(5) = \left[\frac{1000}{5}\right] = 200$

$n(7) = \left[\frac{1000}{7}\right] = 142$

$n(3\cap 5) = \left[\frac{1000}{15}\right] = \left[\frac{200}{3}\right] = 66$

$n(5\cap 7) = \left[\frac{1000}{35}\right] = \left[\frac{200}{7}\right] = 28$

$n(3\cap 7) = \left[\frac{1000}{21}\right] = 47$

$n(3\cap 5\cap 7) = \left[\frac{1000}{105}\right] = 9$

$n(3\cup 5\cup 7) = (333 + 2\cancel{0}0 + 1\cancel{4}2) - 66 - \cancel{28} - 47 + 9$

$=$

$n(5\cup 7) = 2\cancel{0}0 + 1\cancel{42} - \cancel{28}$

Ans: $333 - 66 - 47 + 9$

$333 - 104 = 229$

JEE 1

9

Let $f:[0,\infty)\to[0,\infty)$, $g:[0,\infty)\to[0,\infty)$ be two onto functions and $f(x)$ is an increasing function and $g(x)$ is a decreasing function. Also, $h(x) = f(g(x))$, $h(0) = 0$ and $p(x) = h(x^3 - 2x^2 + 2x) - h(4)$ then for every $x \in (0,2]$

(A) $p(x) \in [0, -h(4)]$

(B) $p(x) \in [-h(4), 0]$

(C) $p(x) \in (-h(4), h(4))$

(D) $p(x) \in (h(4), -h(4)]$

$f'(x) > 0$

$g'(x) < 0$

$h'(x) = \underbrace{f'(g(x))}_{+ve} \cdot \underbrace{g'(x)}_{-ve}$

$= -ve$

$h'(x) < 0$

$p(x) = h(x^3 - 2x^2 + 2x) - h(4)$

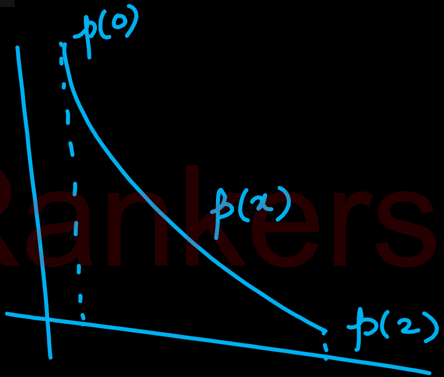
$p'(x) = \underbrace{h'(x^3 - 2x^2 + 2x)}_{-ve} \cdot \underbrace{(3x^2 - 4x + 2)}_{\text{quad} > 0} - 0$

$a > 0$

$D < 0$

$p'(x) < 0$

p is $\downarrow$ f^n.



$$p(x) \in [p(2), p(0)]$$
$$\in [0, -h(4)]$$

$$p(0) = \cancel{h(0)} - h(4) = -h(4)$$

$$p(2) = h(4) - h(4) = 0$$

10

JEE 1

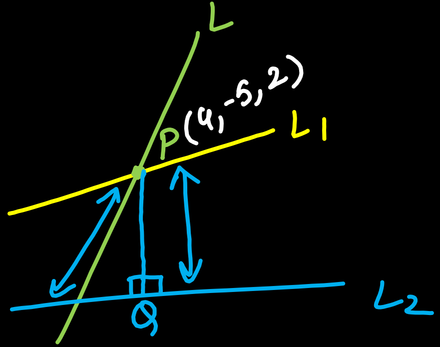
Let a line l pass through the origin and be perpendicular to the lines

$l_1: \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in R$ and $l_2: \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in R.$

If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $(\alpha + \beta + \gamma)$ is

(A) $\boxed{\frac{5}{9}}$ (B) $\frac{9}{5}$

(C) $\frac{19}{5}$ (D) $\frac{5}{3}$



$(-1+2\mu, 2\mu, 1+\mu)$

$(-1+\frac{2}{9}, \frac{2}{9}, 1+\frac{1}{9})$

$(-\frac{7}{9}, \frac{2}{9}, \frac{10}{9})$

(α, β, γ)

$L \perp L_1, L_2$

$L \parallel L_1 \times L_2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix} = -4\hat{i} + 5\hat{j} - 2\hat{k}$$

$$L: \frac{x-0}{-4} = \frac{y-0}{5} = \frac{z-0}{-2} = k$$

10

JEE 1

Solve for P

$$1+\lambda = -4k$$

$$-11+2\lambda = 5k$$

$$-7+3\lambda = -2k$$

$$-14+6\lambda = -4k$$

$$1+\lambda = -14+6\lambda$$

$$\boxed{\lambda = 3}$$

$$P:(4,-5,2)$$

$$\vec{PQ}\cdot L_2 = 0$$

$$2(-1+2\mu-4)+2(2\mu+5)+1(1+\mu-2)=0$$

$$2(-5+2\mu)+2(2\mu+5)+(\mu-1)=0$$

$$9\mu-1=0$$

$$\boxed{\mu = 1/9}$$

11

JEE 1

If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to

(A) 1 (B) 0

(C) 2 (D) 11

$$\boxed{r = \frac{np - m}{p + q}}$$

$$= \frac{(11)(3) - 9}{3 + 1}$$

$$= 6$$

$$T_7 = {}^{11}C_6 (\alpha x^3)^5 \left(\frac{1}{\beta x}\right)^6 = \boxed{{}^{11}C_6 \frac{\alpha^5}{\beta^6}} x^9$$

$$r = \frac{11(1) - (-9)}{1 + 3} = 5.$$

$$T_6 = {}^{11}C_5 (\alpha x)^6 \left(-\frac{1}{\beta x^3}\right)^5$$

$$= \boxed{{}^{11}C_5 \left(-\frac{\alpha^6}{\beta^5}\right)} x^{-9}$$

$$\cancel{{}^{11}C_5}\,\frac{\cancel{\alpha^5}}{\beta^{\cancel{6}}} = -\cancel{{}^{11}C_6}\,\frac{\alpha^6}{\cancel{\beta^5}}$$

$$\boxed{\alpha\beta = -1}$$

$$(\alpha\beta)^2 = 1$$

12

Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xydy = 0, y(1) = 1$. Then $6y^2(e)$ is equal to

(A) $3e^2$ (B) e^2

(C) $2e^2$ (D) $\frac{3e^2}{2}$

JEE 1

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy}$$

Homo Type $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{3v^2x^2 - x^2}{3vx^2} = \frac{3v^2 - 1}{3v} = v - \frac{1}{3v}$$

$$x\frac{dv}{dx} = -\frac{1}{3v}$$

$$3\int v\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{3v^2}{2} = -\ln|x| + c$$

$$\Rightarrow \frac{3}{2}\frac{y^2}{x^2} = -\ln|x| + c$$

$$\Rightarrow \boxed{c = 3/2}$$

JEE 1

$$\frac{3}{2}\frac{y^2}{x^2} = -\ln|x| + \frac{3}{2}$$

$x=e$ $$\frac{3}{2}\frac{y^2}{e^2} = -1 + \frac{3}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{3y^2}{\cancel{2}e^2} = \frac{1}{\cancel{2}}$$

$$\Rightarrow 3y^2 = e^2$$

$$\Rightarrow 6y^2 = 2e^2$$

13

JEE 1

If $a_1, a_2, \ldots\ldots, a_{4001}$ are in arithmetic progression and $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\cdots..+\frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$. Find the value of $|a_1 - a_{4001}|$.

(A) 20

(B) 32

(C) 60

(D) 30

$$\frac{1}{a_1a_2} = \frac{d}{d\,a_1a_2} = \frac{1}{d}\left(\frac{a_2-a_1}{a_1a_2}\right) = \frac{1}{d}\left(\frac{1}{a_1} - \cancel{\frac{1}{a_2}}\right)$$

$$\frac{1}{a_2a_3} = \frac{d}{d\,a_2a_3} = \frac{1}{d}\left(\frac{a_3-a_2}{a_2a_3}\right) = \frac{1}{d}\left(\cancel{\frac{1}{a_2}} - \cancel{\frac{1}{a_3}}\right)$$

$\vdots$

$$\frac{1}{a_{4000}a_{4001}} = \qquad = \frac{1}{d}\left(\cancel{\frac{1}{a_{4000}}} - \frac{1}{a_{4001}}\right)$$

13

JEE 1

$$10 = \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{4001}}\right)$$

$$\Rightarrow 10 = \frac{1}{d}\left(\frac{a_{4001} - a_1}{a_1 \, a_{4001}}\right)$$

$$\Rightarrow 1\cancel{0} = \frac{1}{\cancel{d}}\left(\frac{\cancel{a} + 4000\cancel{d} - \cancel{a}}{a(a_{4001})}\right)$$

$$\Rightarrow a_1 \cdot a_{4001} = 400 \text{ --- } (1)$$

$$a_2 + a_{4000} = 50$$

$$a_1 + a_{4001} = 50 \text{ --- } (2)$$

$$x^2 - Sx + P$$

$$x^2 - 50x + 400 \begin{cases} a_1 = 10 \\ a_{4001} = 40 \end{cases}$$

diff $= 30$. $\begin{cases} a_1 = 10 \text{ or } 40 \\ a_{4001} = 40 \text{ or } 10 \end{cases}$

14

JEE 1

If $\cos 1 + \cos 2 + \cos 3 + \cdots \ldots \ldots + \cos n = S_n$. That $\lim_{n\to\infty} \frac{S_n}{n} =$

(A) 1 (B) 2

(C) does not exists (D) 0

$\cos 1 + \cos 2 + \ldots + \cos n$

$$\lim_{n\to\infty} \frac{S_n}{n} = \frac{\sin n\left(\frac{1}{2}\right) \cos\left(1 + (n-1)\left(\frac{1}{2}\right)\right)}{n \sin\left(\frac{1}{2}\right)}$$

$$= \frac{\sin(n/2) \cos\left(\frac{n+1}{2}\right)}{n \boxed{\sin \frac{1}{2}}} \quad [-1,1] \qquad \boxed{\sin\frac{1}{2}} \to [-1,1]$$

$$= \frac{\text{Const}}{n \to \infty} \to 0$$

15

JEE 1

The sum of the infinite series $1+\left(1+\frac{1}{2}\right)\left(\frac{1}{3}\right)+\left(1+\frac{1}{2}+\frac{1}{2^2}\right)\left(\frac{1}{3^2}\right)+\cdots.\infty$ is :

(A) $\frac{12}{5}$

(B) $\frac{9}{5}$ ✓

(C) $\frac{8}{5}$

(D) $\frac{5}{3}$

$S = 1+\left(1+\frac{1}{2}\right)\left(\frac{1}{3}\right)+\left(1+\frac{1}{2}+\frac{1}{2^2}\right)\left(\frac{1}{3^2}\right)+ - - - \infty$

$\frac{S}{3} = \qquad 1\cdot\frac{1}{3} + \left(1+\frac{1}{2}\right)\left(\frac{1}{3^2}\right) + - - - \infty$

$\frac{2S}{3} = 1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2^2}\left(\frac{1}{3^2}\right) + - -$

$= 1+\frac{1}{6}+\frac{1}{6^2}+ - - \infty$

$$\frac{2S}{3} = \frac{1}{1-1/6} = \frac{1}{5/6} = \frac{6}{5}$$

$$\frac{\cancel{2}S}{3} = \frac{\cancel{6}\,3}{5}$$

$$S = \frac{9}{5}.$$

16

Let $a > 0, b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of $77a + 44b$ is equal to

(A) 100 (B) 110

(C) 120 (D) 130 ✓

$H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\begin{cases} e = \sqrt{1 + \frac{b^2}{a^2}} \\ \ell = \frac{2b^2}{a} \end{cases}$

$H_2 : -\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\begin{cases} e' = \sqrt{1 + \frac{a^2}{b^2}} \\ \ell' = 2a^2/b \end{cases}$

$$\begin{cases} e^2 = \frac{11}{14}\,l \\ 1 + \frac{b^2}{a^2} = \frac{11}{14}\left(\frac{2b^2}{a}\right) \end{cases}$$

$$e'^2 = \frac{11}{8}\,l'$$

$$1 + \frac{a^2}{b^2} = \frac{11}{8}\left(\frac{2a^2}{b}\right)$$

$$\begin{cases} a^2 + b^2 = \frac{11}{7}\,ab^2 \\ a^2 + b^2 = \frac{11}{4}\,a^2 b \end{cases}$$

$$\frac{\cancel{11}}{7}\,ab^2 = \frac{\cancel{11}}{4}\,a^2 b$$

$$\frac{b}{7} = \frac{a}{4}$$

$$\boxed{4b = 7a}$$

$$\overset{1}{\cancel{a^2}} + \frac{49\cancel{a^2}}{16} = \frac{11}{\cancel{7}}\,a\left(\frac{\overset{7}{\cancel{49}}\cancel{a^2}}{16}\right)$$

$$16 + 49 = 77a$$

$$\boxed{77a = 65}$$

$$4b = \cancel{7}\left(\frac{65}{\underset{11}{\cancel{77}}}\right)$$

$$\boxed{44b = 65}$$

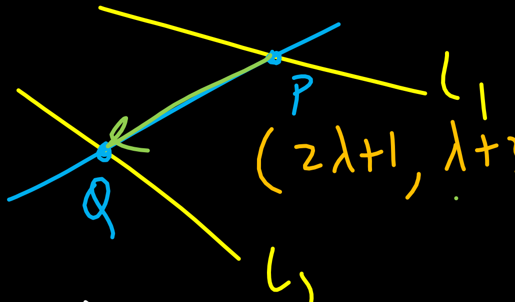
17

JEE 1

Consider the lines L_1 and L_2 given by $L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2} = \lambda$ and $L_2: \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3} = \mu$ A line having direction ratios $1, -1, -2$, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

(A) $2\sqrt{6}$ (B) $3\sqrt{2}$

(C) $4\sqrt{3}$ (D) 4



$P\,(2\lambda+1, \lambda+3, 2\lambda+2)$

$Q\,(\mu+2, 2\mu+2, 3\mu+3)$

$\vec{PQ} = (\mu-2\lambda+1)\hat{i} + (2\mu-\lambda-1)\hat{j} + (3\mu-2\lambda+1)\hat{k}$

$$\frac{\mu-2\lambda+1}{1} = \frac{2\mu-\lambda-1}{-1} = \frac{3\mu-2\lambda+1}{-2}$$

$$\boxed{\lambda = \mu = 3}$$

$\vec{PQ} = -2\hat{i} + 2\hat{j} + 4\hat{k}$

$|\vec{PQ}| = \sqrt{4+4+16} = \sqrt{24}$

JEE 1

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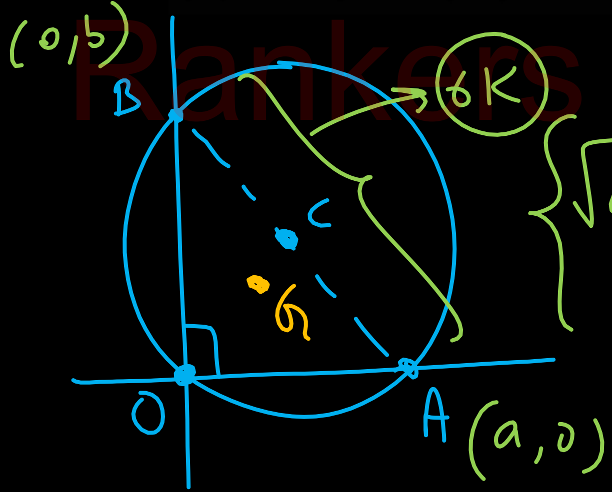
If a circle of constant radius 3k passes through the origin ' O ' and meets co-ordinate axes at A and B, then the locus of the centroid of the triangle OAB is always lies on :

(A) $x^2 + y^2 = (2k)^2$ (B) $x^2 + y^2 = (3k)^2$

(C) $x^2 + y^2 = (4k)^2$ (D) $x^2 + y^2 = (6k)^2$



$\sqrt{a^2+b^2} = 6K$

$a^2+b^2 = 36K^2$

$G \equiv$

$h = \dfrac{a+0+0}{3}$

$\Rightarrow a = 3h$

$K = \dfrac{0+0+b}{3}$

$b = 3K$

$\Rightarrow 9h^2 + 9k^2 = 36k^2$

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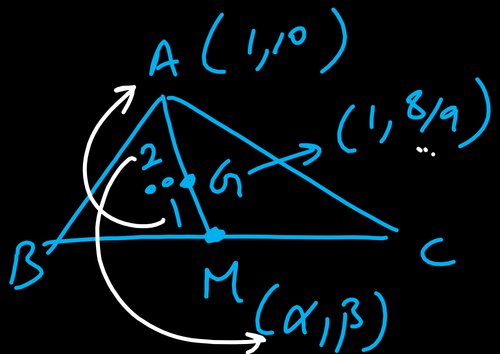
JEE 1

If in triangle ABC, $A \equiv (1,10)$, circumcentre $\equiv \left(-\frac{1}{3}, \frac{2}{3}\right)$ and orthocentre $\equiv \left(\frac{11}{3}, \frac{4}{3}\right)$, then the coordinates of mid-point of side opposite to A are :

(A) $\left(1, -\frac{11}{3}\right)$ (B) $(1,5)$

(C) $(1,-3)$ (D) $(1,6)$

ON G.C



$$G \equiv \left(\frac{-\frac{2}{3}+\frac{11}{3}}{3} \,;\, \frac{\frac{4}{3}+\frac{4}{3}}{3}\right)$$

$$\equiv \left(1, \frac{8}{9}\right)$$

$$\frac{8}{9} = \frac{2\beta + 10}{3}$$

$$4 = 3\beta + 15$$

$$3\beta = -11$$

$$\boxed{\beta = -11/3}$$

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JEE 1

If hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$ passes through the focus of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then eccentricity of hyperbola is :

(A) $\sqrt{2}$
(B) $\frac{2}{\sqrt{3}}$
(C) $\sqrt{3}$
(D) none of these

$(ae, 0)$

$(\sqrt{a^2-b^2}, 0)$

$b^2 = a^2(1-e^2)$

$b^2 = a^2 - a^2e^2$

$a^2e^2 = a^2 - b^2$

$ae = \sqrt{a^2-b^2}$

$\frac{a^2-b^2}{b^2} - 0 = 1$

$a^2 - b^2 = b^2$

$\boxed{a^2 = 2b^2}$

$e_H = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{3}$

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JEE 1

Let $f(\theta) = \sin\theta + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta + t\cos\theta) f(t)\,dt$.

Then the value of $\left|\int_0^{\frac{\pi}{2}} f(\theta)\,d\theta\right|$ is

$$f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} \sin\theta\, f(t)\,dt + \int_{-\pi/2}^{\pi/2} t\cos\theta\, f(t)\,dt$$

$$f(\theta) = \sin\theta + \sin\theta \underbrace{\int_{-\pi/2}^{\pi/2} f(t)\,dt}_{A} + \cos\theta \underbrace{\int_{-\pi/2}^{\pi/2} t\cdot f(t)\,dt}_{B}$$

$$f(\theta) = \sin\theta + A\sin\theta + B\cos\theta$$

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JEE 1

$$\underline{\underline{A}} = \int_{-\pi/2}^{\pi/2} f(t)\,dt$$

odd func

$$A = \int_{-\pi/2}^{\pi/2} (\sin t + A\sin t + B\cos t)\,dt$$

$$A = 2B\int_{0}^{\pi/2} \cos t\,dt \Rightarrow \boxed{A = 2B}$$

$$B = \int_{-\pi/2}^{\pi/2} t\cdot f(t)\,dt$$

odd func

$$B = \int_{-\pi/2}^{\pi/2} (t\sin t + At\sin t + Bt\cos t)\,dt$$

$$B = (1+A)\,2\int_{0}^{\pi/2} t\cdot\sin t\,dt$$

$$B = (2+2A)\left[\left(t(-\cos t)\right)_0^{\pi/2} - \int_0^{\pi/2} 1\cdot(-\cos t)\,dt\right]$$

$$\Rightarrow \boxed{B = 2+2A}$$

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JEE 1

$$\begin{cases} A = 2B \\ B = 2 + 2A \end{cases}$$

$$B = 2 + 4B$$

$$\begin{cases} B = -\frac{2}{3} \\ A = -\frac{4}{3} \end{cases}$$

$$f(\theta) = \sin\theta + A\sin\theta + B\cos\theta$$

$$= \sin\theta + \left(-\frac{4}{3}\right)\sin\theta + \left(-\frac{2}{3}\right)\cos\theta$$

$$= \left(-\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta\right)$$

$$\left|\int_0^{\pi/2}\left(-\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta\right)d\theta\right| = \left|-\frac{1}{3}(1) - \frac{2}{3}(1)\right|$$

$$= 1$$

22

JEE 1

If the shortest distance between the lines $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - a\hat{j})$ and $\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is $\sqrt{\frac{2}{3}}$, then the integral value of 'a' is equal to

$A \equiv (-1, 0, 3)$

$B \equiv (0, -1, 2)$

$\boxed{\vec{AB} = \hat{i} - \hat{j} - \hat{k}}$

$\vec{d_1} = \hat{i} - a\hat{j}$

$\vec{d_2} = \hat{i} - \hat{j} + \hat{k}$

$\vec{n} = \vec{d_1} \times \vec{d_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$\vec{n} = -a\hat{i} - \hat{j} + \hat{k}(a-1)$

$\frac{\vec{AB} \cdot \vec{n}}{|\vec{n}|} = \sqrt{\frac{2}{3}}$

$\frac{-a + 1 - (a-1)}{\sqrt{a^2 + 1 + (a-1)^2}} = \sqrt{\frac{2}{3}}$

$\boxed{a = 2}$

23

The coefficient of x^{98} in the expansion of $\frac{1+x}{1-x}$ if $|x| < 1$, is

JEE 1

$(1+x)(1-x)^{-1}$

$(1+x)(1+x+x^2+ \cdots \infty)$

$\boxed{\text{Ans: } 2}$

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JEE1

Let f: R → R be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x}+e}$.

Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+\cdots+f\left(\frac{99}{100}\right)$ is equal to $\boxed{11k}$, then k is 9

$$f(x) = \frac{2e^{2x}}{e^{2x}+e}$$

$$f(1-x) = \frac{2e^{2-2x}}{e^{2-2x}+e}$$

$$f(1-x) = \frac{2e^{2}}{e^{2}+e^{2x+1}}$$

$$= \frac{2e}{e+e^{2x}}$$

$$f(x)+f(1-x) = \frac{2e^{2x}+2e}{e+e^{2x}} = 2$$

1 → 99 : 2
2 → 98 : 2
3 → 97 : 2
⋮
49 → 51 : 2

$$f\left(\frac{50}{100}\right) \to f\left(\frac{1}{2}\right) \to 1$$

25

Let $A_1 = \{(x,y) : |x| \le y^2, |x| + 2y \le 8\}$ and $A_2 = \{(x,y) : |x| + |y| \le k\}$. If $27(\text{Area } A_1) = 5(\text{Area } A_2)$, then k is equal to:

A_1 : $|x| = y^2 \rightarrow y^2 = x$, $y^2 = -x$

$|x| + 2y = 8$

$x > 0$: $x + 2y = 8$

$$\boxed{\frac{x}{8} + \frac{y}{4} = 1}$$

$x < 0$: $-x + 2y = 8$

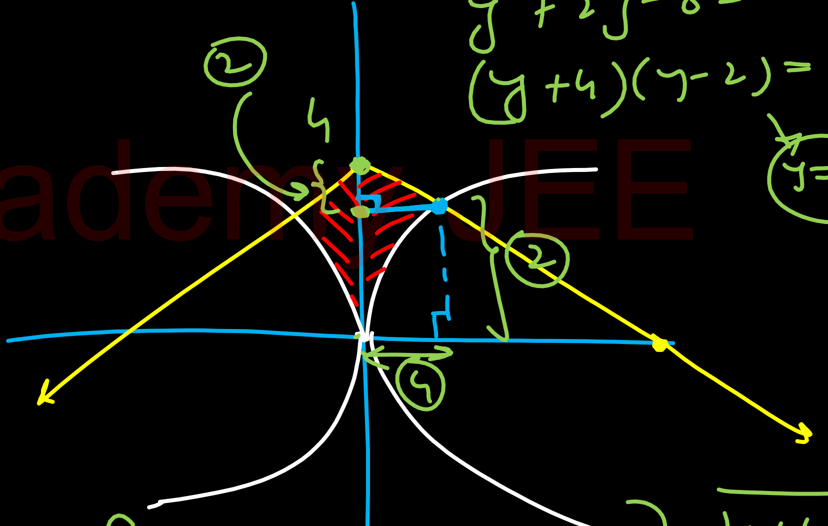
$y^2 = x$

$x + 2y = 8$

$y^2 + 2y - 8 = 0$

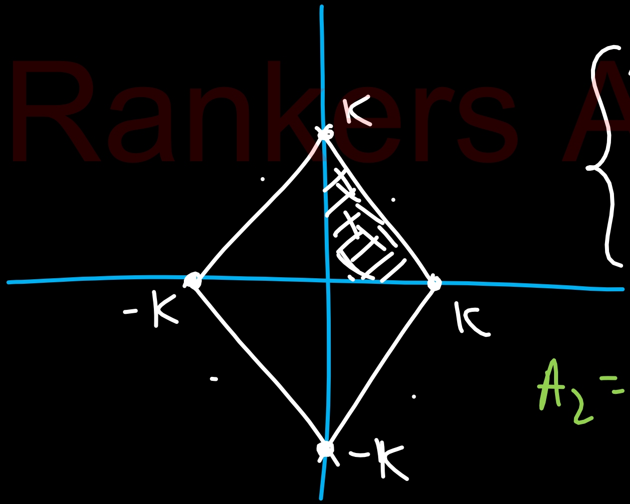
$(y+4)(y-2) = 0$

$y = 2$



$$A_1 = 2\left[\frac{1}{2}(4)(2) + \frac{1}{3}(4)(2)\right] = \boxed{16\left(\frac{5}{6}\right)}$$

$$|x|+|y| \le K$$



$$\begin{cases} x+y=K \\ \dfrac{x}{K}+\dfrac{y}{K}=1 \end{cases}$$

$$A_2 = 4\left(\frac{1}{2}\times K\times K\right)$$

$$= 2k^2$$

Now:

$$27(A_1) = 5(A_2)$$

$$\overset{9}{\cancel{27}}\left(\frac{\overset{4}{\cancel{16}}\times \cancel{5}}{\underset{\cancel{2}}{\cancel{6}}}\right) = \cancel{5}(\cancel{2}K^2)$$

$$K^2 = 36$$

$$\boxed{K=6}$$