CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
FULL SYLLABUS
15-01-2025

JEE(Main + Advanced) : ENTHUSIAST & LEADER COURSE (SCORE-I)

ANSWER KEY PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C	D	C	A	C	D	A	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	B	C	A	C	A	D	C	A
SECTION-II	Q.	1	2	3	4	5					
	A.	72	10	225	30	10					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	B	B	B	D	A	B	C	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	A	D	B	B	A	C	C	B
SECTION-II	Q.	1	2	3	4	5					
	A.	18	6	6	6	3					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	D	D	B	D	C	A	A	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	B	A	A	C	B	A	C	A
SECTION-II	Q.	1	2	3	4	5					
	A.	1	15	25	15	7					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (A)**

Let V_0 = peak voltage

$$V = \frac{\int_0^{\pi/\omega} V_0 \sin \omega t\, dt}{T} = \frac{V_0}{\pi}$$

For full wave rectifier, $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{\pi V}{\sqrt{2}}$

2. **Ans (C)**

For decay $X^{230} \to Y^{226} + He^4 + Q$

$$K_\alpha = \left(\frac{A-4}{A}\right) Q \Rightarrow Q = \frac{6.8 \times 230}{226} = 6.92\,\text{MeV}$$

For $X^{230} \to {*Y}^{226} + He^4$

$$\sqrt{2m_y K_y} = \sqrt{2m_\alpha K_\alpha} \Rightarrow K_y = \frac{4 \times 5.2}{226} = 0.09\,\text{MeV}$$

$$E_\gamma = Q - K_\alpha - K_y = 6.92 - (5.2 + 0.09)$$

$$= 1.63\ \text{MeV}$$

3. Ans (D)

$$\frac{V}{R}\left(1-e^{-\frac{tR}{L}}\right)+\frac{V}{R'}e^{\frac{-t}{R'C}}=\text{constant}$$

$$\Rightarrow \frac{V}{R'}e^{\frac{-t}{R'C}}=\frac{V}{R}e^{-\frac{Rt}{L}}$$

$$\therefore R=R' \ \& \ \frac{1}{R'C}=\frac{R}{L}\Rightarrow R=\sqrt{\frac{L}{C}}$$

4. Ans (C)

Focal length of lenses change on dipping in water, whereas that of mirrors do not change. Diverging lens and convex mirror cannot form real image mirror cannot form real image for real object. Refer to ray diagram.

5. Ans (A)

KE = 100 + 50 = 150eV

v = 150 volt

$$\lambda=\sqrt{\frac{150}{V}}$$

$\lambda = 1Å$

6. Ans (C)

Energy density $=\frac{1}{2}\epsilon_0 E^2$

Avg. energy density with electric field

$$=\frac{1}{2}\epsilon_0 E_{rms}^2$$

$$=\frac{1}{2}\epsilon_0\frac{E_0^2}{2}$$

$$=\frac{1}{4}\times 8.8\times 10^{-12}\times 4^2$$

$$=35.2\times 10^{-12}\,\text{J/m}^3$$

7. Ans (D)

$$\frac{4}{3}\pi r^3(n-n_0)g=6\pi\mu r V_T.$$

$$V_T=\frac{2r^2[n-n_0]}{9\mu}g$$

$$P_g=\frac{4}{3}\pi r^3.n.g.\frac{2r^2(n-n_0)g}{9\mu}$$

$$=\frac{8\pi r^5 n}{27\mu}[n-n_0]g^2$$

8. Ans (A)

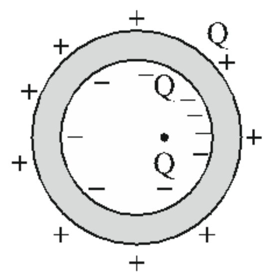
$T^2 \alpha\, r^3$

$$\left(\frac{T_E}{T_P}\right)^2=\left(\frac{r_E}{r_P}\right)^3$$

$$\frac{1}{27}=\left(\frac{1.5\times 10^8}{r_p}\right)^3$$

$r_p = 4.5\times 10^8$ km

9. Ans (B)



Negative induced charge density on right surface will be move.

10. Ans (C)

For polytropic process PV^m = constant

$$C=C_v+\left(\frac{R}{1-m}\right)\ \left(\text{Here } m=\frac{3}{2} \text{ and } C_v=\frac{3R}{2}\right)$$

$$\Rightarrow C=\frac{3R}{2}+\frac{R}{1-\frac{3}{2}}=\frac{3R}{2}-2R=-0.5R$$

$$Q=nC\Delta T=1(-0.5R)(-26)=+13R$$

11. Ans (B)

For 3rd overtone, wavelength

$$\lambda=\frac{2\ell}{4}=\frac{\ell}{2}$$

$$P=P_0\sin kx=P_0\sin\left[\frac{2\pi}{\lambda}\right]x$$

$$=P_0\sin\left[2\pi\times\frac{2}{\ell}\times\frac{\ell}{16}\right]$$

$$=P_0\sin\left[\frac{\pi}{4}\right]$$

$$=\frac{P_0}{\sqrt{2}}$$

12. Ans (B)

As sound takes finite time to travel so sound received at t = 5 sec, should have been emitted earlier.

Let t' $\longrightarrow$ time at which source emits sound which is detected at t = 5 sec.

So $\left(505 - \frac{1}{2}gt'^2\right) - \left(50t - \frac{1}{2}gt^2\right) = V\,(t - t')$

Here t = 5 sec, v = 300 m/s

By solving t' = 4 sec and $v_s = gt' = 40$ m/s

So $f_{ap} = f_{act}\left[\frac{v}{v - v_s}\right] = 1300\left[\frac{300}{300-40}\right] = 1500$ Hz.

13. Ans (B)

Speed of 1st bob just before collision = $\sqrt{g\ell_1}$

Speed of 2nd bob to complete vertical loop

$= \sqrt{4g\ell_2}$ [due to light rod]

As identical mass exchange velocity on head on elastic collision,

so $\sqrt{g\ell_1} = \sqrt{4g\ell_2} \Rightarrow \ell_1 = 2\ell_2$

PART-1 : PHYSICS

SECTION-II

1. Ans (72)

Index error in u = 1 cm

u = 8 cm

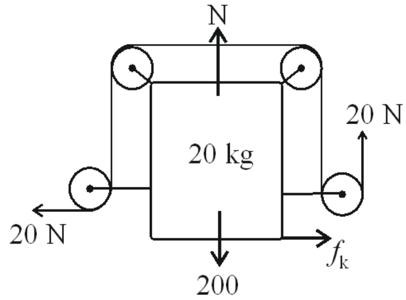
Index error in v = –1 cm

v = 17 + 1 = 18

$\frac{1}{f} = \frac{1}{18} + \frac{1}{8}$

$f = \frac{72}{13} = \frac{9 \times 8}{13}$

2. Ans (10)



N + 20 = 200

N = 180

Kinetic friction

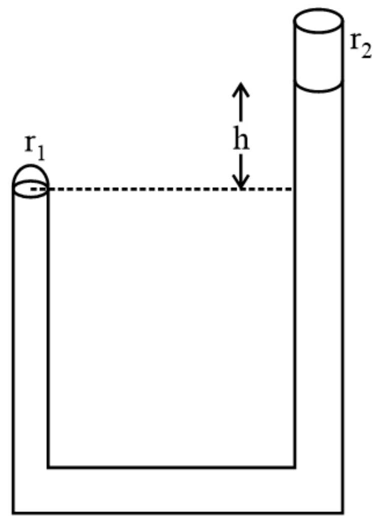
$f_k = \mu N = 0.1 \times 180 = 18$ N

$\therefore$ 20 kg will slide

$a = \frac{20-18}{20} = 0.1\ \text{m/s}^2 = 10\ \text{cm/s}^2$

3. Ans (225)

When water is about to flow out of the left tube, the surface gets convex with radius of curvature r_1.



$P_0 + \frac{2T}{r_1} = P_0 - \frac{2T}{r_2} + \rho gh$

$\Rightarrow 2T\frac{(r_1 + r_2)}{r_1 r_2} = \rho gh \Rightarrow h = \frac{2T(r_1 + r_2)}{r_1 r_2 \rho g}$

$= \frac{2 \times 0.075 \times 0.3 \times 10^{-3}}{0.1 \times 10^{-3} \times 0.2 \times 10^{-3} \times 10^{3} \times 10}$

$= 0.75 \times 0.3 = 0.225$ m = 225 mm

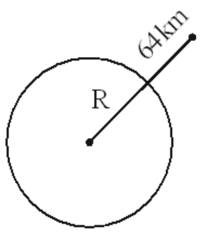
4. Ans (30)

$T + mg_{eff} = m\omega^2 (R + 64)$

$T = m\omega^2 (R + 64) - mg_{eff}$

$$\omega^2 = \frac{GM}{R^3} \Rightarrow T = \frac{mGM}{R^3}(R+64) - mg\left[1 - \frac{2h}{R}\right]$$

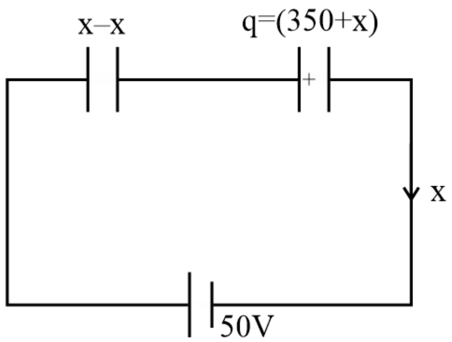
$$T = \frac{GMm}{R^2} + \frac{64\,GMm}{R^3} - mg + \frac{2mg(64)}{R}$$



$$T = \frac{64\,mg}{R} + \frac{128mg}{R}$$

$= 30$ N

5. Ans (10)



Apply K.V.L.

$$50 - \frac{x}{5} - \frac{(350+x)}{10} = 0$$

$$50 = \frac{x}{5} + \frac{350+x}{10}$$

$500 = 2x + 350 + x$

$$\frac{150}{3} = x$$

$x = 50\mu C$

Potential difference $5\mu F$ capacitor

$$\Delta v = \frac{q}{C} = \frac{50}{5} = 10 \text{ volts}$$

Potential difference across $10\mu F$ capacitor

$$\Delta V = \frac{350+x}{C} = \frac{350+50}{10} = 40\text{ V}$$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (D)

Solution	Molar. Conc.	Vant Hoff Factor	Osmotic Pressure
A	0.1	1	0.1 RT
B	0.1	1	0.1 RT
C	0.1	2	0.2 RT
D	0.1	3	0.3 RT

Solution D has maximum Osmotic pressure

2. Ans (D)

$$\lambda^o_{e_{Fe(OH)_3}} = (1.4 \times 10^{-2} + 2.2 \times 10^{-2} - 1.1 \times 10^{-2})Sm^2mol^{-1}$$

$= 2.5 \times 10^{-2}\ Sm^2\ mol^{-1}$

$$\lambda^o_{m_{Fe(OH)_3}} = 7.5 \times 10^{-2} Sm^2mol^{-1}$$

$$\text{Also, } \lambda^o_m = \frac{K}{1000s}$$

$$\Rightarrow S = \frac{7.5 \times 10^{-4}}{10^3 \times 7.5 \times 10^{-2}} = 10^{-5}\text{ mol L}^{-1}$$

$K_{sp} = 27s^4 = 27 \times 10^{-20} = 2.7 \times 10^{-19} M^4$

Solubility in 0.5 M NaOH

$[OH^-] = 0.5$ M

$[Fe^{3+}]\,[OH^-]^3 = 2.7 \times 10^{-19}$

$$\Rightarrow [Fe^{3+}] = \frac{2.7 \times 10^{-19}}{125 \times 10^{-3}} = 2.16 \times 10^{-18}$$

4. Ans (B)

	N_2	+	$3H_2$	=	$2NH_3$
Initial :	2 mole		excess		–
Final :	0				2×2=4 mole

Mass of NH_3 formed = (4 × 17)g = 68 g

5. **Ans (B)**

Option (A) : $E_{4\to2} + E_{2\to1} = E_{4\to1}$

Option (C) : $E_{4\to3} + E_{3\to2} = E_{2\to1} = E_{4\to1}$

Option (D) : $E_{4\to3} + E_{3\to1} = E_{4\to1}$

7. **Ans (A)**

$$\begin{array}{ccccccc} HCl & + & NaOH & \longrightarrow & NaCl & + & H_2O \\ 2\times0.2 & & 0.5 & & - & & - \\ - & & 0.1 & & 0.4 & & \end{array}$$

$$\begin{array}{ccccccc} HCN & + & NaOH & \longrightarrow & NaCN & + & H_2O \\ 0.5 & & 0.1 & & - & & - \\ 0.4 & & - & & 0.1 & & \end{array}$$

$pH = pK_a + \log\dfrac{0.1}{0.4}$

8. **Ans (B)**

EAN of $K_3[Fe(CN)_6] = 35$

$[Co(en)_3]Cl_3$ can show optical isomerism.

9. **Ans (C)**

$Be_{(g)} + e^- \to Be^-_{(g)}$ is an endothermic process

So $Be^-_{(g)}$ is unstable.

10. **Ans (C)**

Lesser value of $E^o_{Ni^{+2}/Ni}$ is due to high hydration energy.

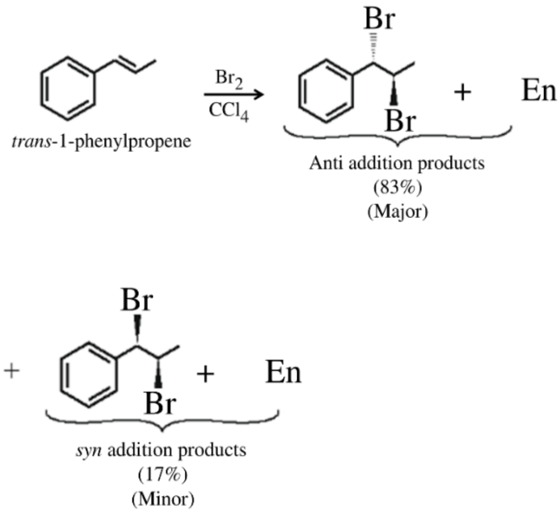
11. **Ans (D)**

Actionids on reaction with boiling water gives oxide and hydride.

13. **Ans (A)**

By definition

15. **Ans (B)**



17. **Ans (A)**

Under acidic condition $-NH_2$ group is stronger activating group than $-OH$

Naphthalene normally undergoes coupling at α-position.

More rapidly than β-positions.

18. **Ans (C)**

At P^I, aminoacid has net charge zero (Zwitter ionic form)

19. **Ans (C)**

X = 3 (a, d, f are 1° amines give positive carbyl amine test)

Y = 3 (b, e, g are 2° amines give oily yellow coloured nitroso compounds on reaction with aq.HNO_2)

Z = 3 (a, d, f are 1° amines react with Hinsberg reagent ($PhSO_2Cl$) and the product is soluble in aq. KOH

X + Y + Z = 9

20. **Ans (B)**

(i), (iv), (vii) and (viii) can be differentiated by both $K_2Cr_2O_7/H^+$ and NaOI(aq).

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (18)**

3, 7-dimethylocta –2, 6–dienal

$A + B + C + D = 3 + 7 + 2 + 6 = 18$

2. **Ans (6)**

$[OH^-]$ added $= \dfrac{36}{40} = 0.9 \text{ mol L}^{-1}$

$[H^+]$ after this $1 - 0.9 = 0.1$ M

$\Delta pH = 0.0591\ \Delta pH = 0.0591 \simeq 0.06$ V

4. **Ans (6)**

(i) $Ag_2O \xrightarrow{\Delta} Ag + O_2 \uparrow$

(ii) $KClO_3 \xrightarrow[\Delta]{MnO_2} KCl + O_2 \uparrow$

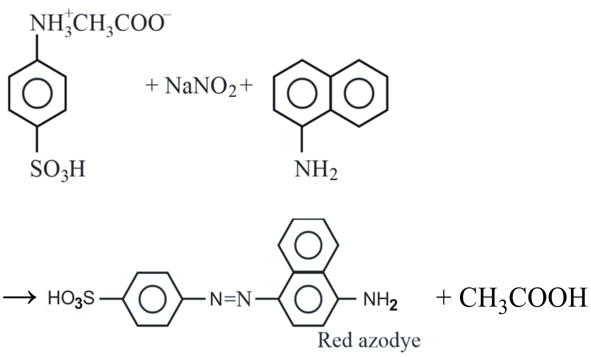
(iii) $Pb_3O_4 \xrightarrow{\Delta} PbO + O_2 \uparrow$

(iv) $Pt + HNO_3 + HCl \rightarrow PtCl_6^{-2} + NO + H_2O$

(v) $KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$

(vi) $NaNO_{2(s)} + \text{dil.} H_2SO_4 \rightarrow Na_2SO_4 + NO_2 \uparrow$

5. **Ans (3)**



PART-3 : MATHEMATICS

SECTION-I

1. **Ans (C)**

$\dfrac{x}{73} + \dfrac{y}{37} = 1$

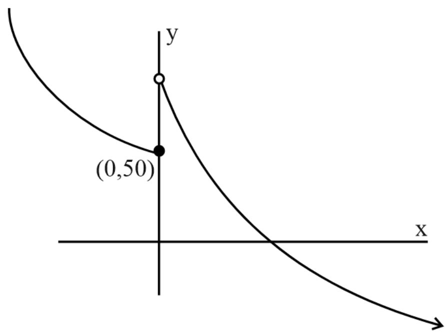
Let co-ordinate of the vertices of other diagonal are (x_1, y_1) & (x_2, y_2) and diagonals of the square bisects each other.

$\Rightarrow \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) \equiv \left(\dfrac{73}{2}, \dfrac{37}{2}\right)$

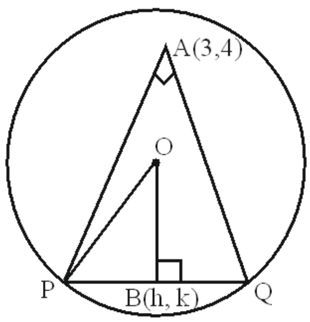
and $L_1 + L_2 = x_1 + x_2 = 73$

2. **Ans (C)**

$\Rightarrow$ for onto function $\lambda \geqslant 50$



3. **Ans (D)**



$\angle PAQ = 90^\circ$

$\Rightarrow$ Point A lies on the circle with diameter PQ

$\Rightarrow AB = PB = BQ \quad ...(1)$

Let mid point of PQ is B(h, k) $\Rightarrow OB = \sqrt{h^2 + k^2}$

in ΔPOB, $OP^2 = PB^2 + OB^2$

$PB^2 = 36 - h^2 - k^2$

Now, from eq. (1) $AB^2 = PB^2$

$(h - 3)^2 + (k - 4)^2 = 36 - h^2 - k^2$

$\Rightarrow h^2 + k^2 - 3h - 4k - \dfrac{11}{2} = 0$

$\Rightarrow$ center is $\left(\dfrac{3}{2}, 2\right)$

4. **Ans (D)**

Sum is obtained by putting $x = y = z = 1$

$\text{Sum} = (2 + 3 - 2)^n$

$\Rightarrow 3^n = 2187 \Rightarrow n = 7$

Coefficient of x^6 is

${}^6C_0 \times {}^8C_6 + 2 \times {}^6C_1\, {}^5C_3 + 4 \times {}^6C_2\, {}^2C_0 = 208$

5. **Ans (B)**

$$\frac{1}{2^n + 2^{1-n}} = \frac{2^{n-1}}{1 + 2^n.2^{n-1}} = \frac{(2-1).2^{n-1}}{1 + 2^n.2^{n-1}} = \frac{2^n - 2^{n-1}}{1 + 2^n.2^{n-1}}$$

$$\therefore \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{2^n + 2^{1-n}}\right)$$

$$= \sum_{n=1}^{\infty}\left\{\tan^{-1}(2^n) - \tan^{-1}(2^{n-1})\right\}$$

$$= \lim_{n\to\infty}\sum_{n=1}^{n}\left\{\tan^{-1}(2^n) - \tan^{-1}(2^{n-1})\right\}$$

$$= \lim_{n\to\infty}\left\{\tan^{-1}(2^n) - \tan^{-1}(1)\right\}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

$\sin^{-1}(\sin 3\lambda) = \sin^{-1}(\sin 3\pi/4) = \pi/4$

6. **Ans (D)**

Given $\vec{a} + 3\vec{b} = \lambda\vec{c}$

$2\vec{b} + 3\vec{c} = \mu\vec{a}$

$\Rightarrow 2\vec{b} + 3\vec{c} = \mu(\lambda\vec{c} - 3\vec{b})$

$\Rightarrow (2 + 3\mu)\vec{b} + (3 - \mu\lambda)\vec{c} = 0$

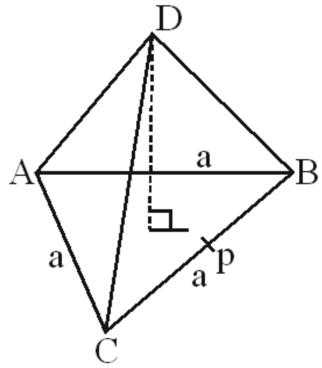
$\Rightarrow \mu = -\frac{2}{3}$ and $3 - \lambda\mu = 0$

Thus $2\vec{a} + 6\vec{b} + 9\vec{c} = \vec{0}$

$\Rightarrow 2\vec{a} + 9\vec{c} = -6\vec{b}$

$|2\vec{a} + 3\vec{b} + 9\vec{c}| = |-3\vec{b}| = 3$

7. **Ans (C)**



Let a point p lies on midpoint BC;

$P\,(1 + \lambda, 2 + 2\lambda, -\lambda)$

$\overrightarrow{AP} \cdot \overrightarrow{BC} = 0$

$\left((1+\lambda)\hat{i} + (2+2\lambda)\hat{j} - \lambda\hat{k}\right) \cdot \left(\hat{i} + 2\hat{j} - \hat{k}\right) = 0$

$\therefore\ \lambda = \frac{-5}{6}$

$\therefore\ p = \left(\frac{1}{6}, \frac{2}{6}, \frac{5}{6}\right)$

$\frac{\sqrt{3}}{2} \cdot a = AP = \frac{1}{6}\sqrt{30} \Rightarrow a = \frac{\sqrt{10}}{3}$

Normal vector of plane ABC is

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 1 & 2 & 0 \end{vmatrix} = 2\hat{i} - \hat{j}$$

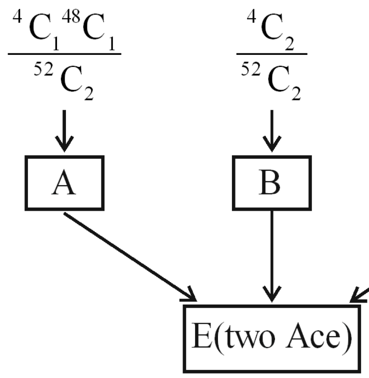
Plane ABC is $2x - y = 0$

$D = (1, 5, 0)$

$\therefore$ Height of tetrahedral $= \frac{3}{\sqrt{5}}$

$\therefore\ \text{Volume} = \frac{1}{3} \times \left(\frac{\sqrt{3}}{4} \cdot \frac{10}{9}\right)\frac{3}{\sqrt{5}} = \frac{1}{18}\sqrt{15}$

8. **Ans (A)**



A. One Ace in two lost cards

B. Both lost cards are Ace

C. Lost cards are not Ace

E. Two cards are drawn from remaining cards and both are Ace

Required probability $= 1 - P\left(\frac{C}{E}\right)$

$$= 1 - \frac{P(C)\cdot P\left(\frac{E}{C}\right)}{P(A)\cdot P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\cdot P\left(\frac{E}{C}\right)}$$

$$= 1 - \frac{\frac{{}^{48}C_2}{{}^{52}C_2}\cdot\frac{{}^{4}C_2}{{}^{50}C_2}}{\frac{{}^{4}C_1\,{}^{48}C_1}{{}^{52}C_2}\cdot\frac{{}^{3}C_2}{{}^{50}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2}\cdot\frac{{}^{2}C_2}{{}^{50}C_2} + \frac{{}^{48}C_2}{{}^{52}C_2}\cdot\frac{{}^{4}C_2}{{}^{50}C_2}}$$

$$= 1 - \frac{1128}{1225} = \frac{97}{1225}$$

9. **Ans (A)**

$$y\cos\left(a^2x^2\right)\left(a^2 2x\right) + \sin\left(a^2x^2\right)\frac{dy}{dx}$$

$$-x^b \sin y\frac{dy}{dx} + bx^{b-1}\cos y = 0$$

$$\int d\left(y\sin\left(a^2x^2\right)\right) + \int d\left(x^b\cos(y)\right) = \int 0\,dx$$

$$y\sin\left(a^2x^2\right) + x^b\cos y = c$$

10. **Ans (D)**

$R = \{(2,6), (2,10), (3,3), (3,6), (5,10)\}$

$\therefore$ 5 elements in R^{-1}

11. **Ans (C)**

Given sequence is an A.P.

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{3n^2}{2} - \frac{n}{2}$ on comparing,

$a = 1,\ d = 3$

Mean $= 19 = \frac{\Sigma x_i}{n} = \frac{S_n}{n}$

$\Rightarrow 2a + (n-1)d = 38 \Rightarrow \boxed{n = 13}$

Given sequence is 1, 4,.........,37

$\therefore x_n = 3n - 2.$

$$\text{Variance} = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 = \frac{\sum_{n=1}^{13}(3n-2)^2}{13} - 19^2 = 126$$

12. **Ans (D)**

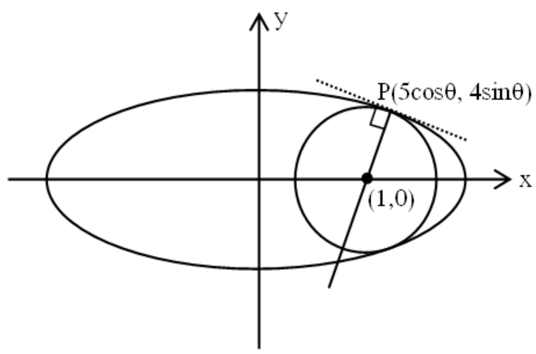
$$\Delta = \begin{vmatrix} 3\alpha^2 & \alpha^2+\alpha\beta+\beta^2 & \alpha^2+\alpha\gamma+\gamma^2 \\ \alpha^2+\alpha\beta+\beta^2 & 3\beta^2 & \beta^2+\beta\gamma+\gamma^2 \\ \alpha^2+\alpha\gamma+\gamma^2 & \beta^2+\beta\gamma+\gamma^2 & 3\gamma^2 \end{vmatrix}$$

$$= \begin{vmatrix} \alpha^2 & \alpha & 1 \\ \beta^2 & \beta & 1 \\ \gamma^2 & \gamma & 1 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}$$

$$\Delta = -[(\alpha-\beta)(\beta-\alpha)(\gamma-\alpha)]^2$$

$$\Rightarrow \Delta_{min} = -[(2-3)(3-5)(5-2)]^2 = -36$$

13. **Ans (B)**



Let $P(5\cos\theta, 4\sin\theta)$

Normal of ellipse at P;

$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$

$5x\sec\theta - 4y\,\mathrm{cosec}\,\theta = 9$

For largest circle this normal will pass through centre of given circle.

$\therefore 5\sec\theta = 9 \Rightarrow \sec\theta = \frac{9}{5}$

$\therefore \cos\theta = \frac{5}{9}$ and $\sin\theta = \frac{\sqrt{56}}{9}$

Radius of C is $r = \sqrt{(5\cos\theta - 1)^2 + (4\sin\theta)^2}$

$$\therefore \text{Area} = \pi r^2 = \pi\left[\left(\frac{25}{9} - 1\right)^2 + 16.\frac{56}{81}\right] = \frac{128\pi}{9}$$

14. Ans (A)

Let $\int e^{x+x^2}(2x^2+3x+2)\,dx$

$= \int e^x\left\{e^{x^2}\cdot(ax+b)+e^{x^2}\cdot a+e^{x^2}\cdot 2x\,(ax+b)\right\}dx$

$\Rightarrow a=b=1$

$= e^x\cdot e^{x^2}(x+1)+C_1$

$= e^{x+x^2}(x+1)+C_1$

$f(x)=e^{x+x^2}(x+1)+C_1$

$f(0)=3 \Rightarrow C_1+1=3 \Rightarrow C_1=2$

$f(x)=e^{x+x^2}(x+1)+2 \therefore f(-1)=2$

15. Ans (A)

$L_R = 2\sqrt{2}C = 2\sqrt{2}, 2\sqrt{2} = 8$

Semi latus rectum $= \frac{8}{2} = \frac{2L_1L_2}{L_1+L_2}$

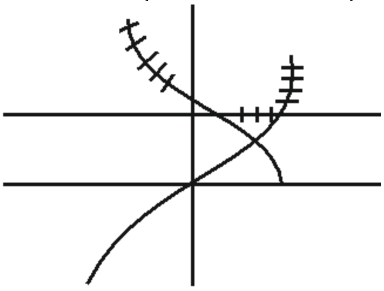
$2 = \frac{3L_2}{3+L_2} \Rightarrow 6 = L_2$

Length of focal chord $= L_1+L_2 = 3+6 = 9$

16. Ans (C)

The graph of the function

$f(x)=\max\left\{\sin^{-1}x, \frac{\pi}{3}, \cos^{-1}x\right\} - 1<x<1$



Clearly two points of non-differentiability so K is '2'.

Now the given quadratic equation

$mx^2+(m-1)x+1=0$

has the no '2' between the roots.

So, $m.\,f(2)<0$

$m\,(4m+(m-1)2+1)<0$

$m\,(6m-1)<0$

$0<m<\frac{1}{6}$

17. Ans (B)

If S is the sum of all possible products of first n natural numbers taking two at a time then

$(1+2+3+....+n)^2 = 1^2+2^2+3^2+........+n^2+2(s)$

So $S = \frac{n(n+1)(n-1)(3n+2)}{24} = f(n)$

Similarly

$g(n) = 1\cdot(n)+2\cdot(n-1)+3\cdot(n-2)+.....+n\cdot(1)$

General term of series is

$T_r = r(n-(r-1))$

$g(n) = \sum_{r=1}^{n} r(n-(r-1))$

$= \sum_{r=1}^{n}\left((n+1)r-r^2\right)$

$= (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$

$= (n+1)\frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)(n+2)}{6}$

$\lim_{n\to\infty} \frac{n(n+1)\ (n-1)\ (3n+2)}{24.\frac{n(n+1)(n+2)}{6}.n}$

$= \lim_{n\to\infty} \frac{(n-1)(3n-2)}{4n(n+2)} = \frac{3}{4}$

18. Ans (A)

$g'(x) = 4.f'\left(\frac{x^2}{4}\right)\cdot\frac{2x}{4} - f'(125-x^2).2x$

$= 2x\left(f'\left(\frac{x^2}{4}\right) - f'(125-x^2)\right)$

Now $f'\left(\frac{x^2}{4}\right) - f'(125-x^2) > 0$

when $\frac{x^2}{4} < 125-x^2$

$x^2 < 100$

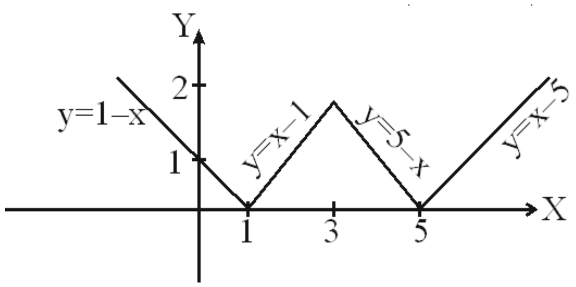
$\Rightarrow |x| < 10$

$\Rightarrow -10 < x < 10$

But for g(x) to be increasing either both x and $f'\left(\frac{x^2}{4}\right) - f'(125-x^2)$ should be positive or negative so set of values of 'x' is $(-\infty,-10)\cup(0,10)$.

19. **Ans (C)**

Let us draw the graph of $f(x) = |2 - |x - 3||$



It is clear from the above graph that f(x) is not differentiable at x = 1, 3 and 5

$\Rightarrow S = \{1, 3, 5\}$

$\Rightarrow \sum_{x \in s} f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1 = 3$

20. **Ans (A)**

Let

$$x = 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{2^2} + 3^2 \cdot \frac{1}{2^3} + 4^2 \cdot \frac{1}{2^4} + \text{.................to}\infty$$

$$\frac{1}{2}x = 1^2 \cdot \frac{1}{2^2} + 2^2 \cdot \frac{1}{2^3} + 3^2 \cdot \frac{1}{2^4} + \text{.................to}\infty$$

$$\frac{1}{2}x = 1 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + 5 \cdot \frac{1}{2^3} + 7 \cdot \frac{1}{2^4} + \text{.................to}\infty$$

$$\frac{1}{4}x = 1 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + 5 \cdot \frac{1}{2^4} + \text{.................to}\infty$$

$$\frac{1}{4}x = \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \text{.................to}\infty$$

$$\Rightarrow \frac{1}{4}x = \frac{1}{2} + \frac{\frac{1}{2}}{1 - \frac{1}{2}}$$

$$\Rightarrow \frac{1}{4}x = \frac{1}{2} + 1$$

$$\Rightarrow x = 4 \cdot \frac{3}{2} = 6$$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (1)**

$M^2 = (ABA^{-1})(ABA^{-1}) = AB^2A^{-1}$

$\Rightarrow M^{10} = AB^{10}A^{-1}$

$$B^{10} = \begin{bmatrix} 1 & 0 \\ 0 & 2024^{10} \end{bmatrix}$$

Then

$$M^{10} = \frac{-1}{16}\begin{bmatrix} -1 & 5 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2024^{10} \end{bmatrix}\begin{bmatrix} 1 & -5 \\ -3 & -1 \end{bmatrix}$$

$$M^{10} = \frac{-1}{16}\begin{bmatrix} -1-15(2024)^{10} & 5-5(2024)^{10} \\ 3-3\times(2024)^{10} & -15-(2024)^{10} \end{bmatrix}$$

$$a = \frac{1+15(2024)^{10}}{16},\ b = \frac{-5+5(2024)^{10}}{16}$$

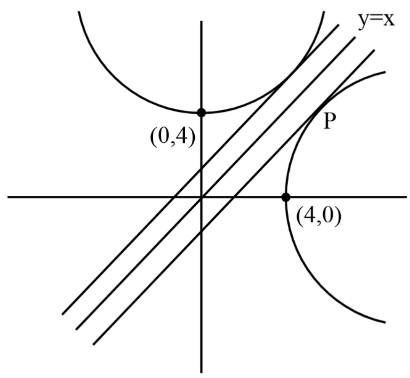
$a - 3b = 1$

2. **Ans (15)**

$y^2 = x - 4,\ x^2 = y - 4$

Both curve symmetric about $y = x$

Now, $y^2 = x - 4$



$$2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y} = 1\ ;\ y = \frac{1}{2}$$

So, $x = y^2 + 4 = \frac{1}{4} + 4 = \frac{17}{4}$

Now point P $\left(\frac{17}{4}, \frac{1}{2}\right)$

Shortest distance $= 2\frac{\left|\frac{17}{4} - \frac{1}{2}\right|}{\sqrt{2}} = \frac{15}{2\sqrt{2}}$

$d = \frac{15}{2\sqrt{2}}$

3. Ans (25)

$f(x+y) = 2^x f(y) + 4^y.f(x)$ ....(1)

Interchanging $x \longrightarrow y$

$f(x+y) = 2^y f(x) + 4^x.f(y)$ ....(2)

(1) – (2)

$f(y)[2^x - 4^x] + f(x)[4y - 2y] = 0$

$$\frac{f(x)}{4^x - 2^x} = \frac{f(y)}{4^y - 2^y} = K$$

$f(x) = K(4^x - 2^x)$

$f(1)=2K$

$2=2K$

$K = 1$

$f(x) = 4^x - 2^x$

$= (2^x)^2 - 2^x$

$$= \left(2^x - \frac{1}{2}\right)^2 - \frac{1}{4}$$

Least value of f(x) is $-\frac{1}{4}$

$K = -\frac{1}{4}$

$|K| = \frac{1}{4} = 0.25$

4. Ans (15)

$A = \{(1, 4)\ (2, 3)\ (3, 2)\ (4, 1)\}$

No. of non empty subsets $= 2^4 - 1 = 16 - 1 = 15$

5. Ans (7)

$5\cos 2x\,(1 - \tan x) - 5\tan x + 7 = 0$

$1 - \cos 2x - 5\tan x\,(1 + \cos 2x) + 6 + 6\cos 2x = 0$

$\tan^2 x - 5\tan x + 6 = 0$

$(\tan x - 2)(\tan x - 3) = 0$

Let $\tan\theta_1 = 2$ and $\tan\theta_2 = 3$

$\therefore$ Sum of solution $= \theta_1 + \theta_2 + \pi + \theta_1 + \pi + \theta_2$

$$= 2\pi + 2\left(\frac{3\pi}{4}\right) = \frac{7\pi}{2}$$