ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Main)

UNIT TEST # 07

27-10-2024

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C	A	B	B	B	A	D	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	A	A	B	C	D	D	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	32	6	175	16	5	4	6	20	800

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	D	C	A	B	D	D	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	A	A	B	C	B	B	D	D	A
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	6	9	5	8	10	3	6	3	4	3

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	A	B	B	B	A	D	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	C	B	A	B	A	B	B	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3375	64	4	4	4	10	8	6	2	4

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (A)

$$H = \frac{1}{2}Li_0^2 = \frac{1}{2}L\left(\frac{E}{R_1}\right)^2$$

2. Ans (C)

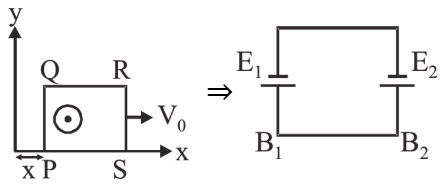
$$i = i_0 \sin\omega t$$

$$i_0 = i_0 \sin\omega t$$

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2}\sqrt{LC}$$

3. Ans (A)



$$\varepsilon_{QR} = \varepsilon_{SP} = 0$$

$$\varepsilon_1 = V_0 B_1 d$$

$$\varepsilon_2 = V_0 B_2 d$$

$$\varepsilon = \varepsilon_2 - \varepsilon_1 = V_0 d(B_2 - B_1)$$

$$= V_0 d\left[B_0\left(1 + \frac{x+d}{a}\right) - B_0\left(1 + \frac{x}{a}\right)\right] = \frac{V_0 d^2 B_0}{a}$$

4. **Ans (B)**

$\phi_i = NBA = NA\,\mu_0\,ni$

$\phi_f = 0$

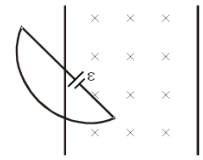
$q = \dfrac{\Delta\phi}{R} = \dfrac{NA\mu_0 ni}{R}$

$= \dfrac{100 \times \pi(0.01)^2 \times 4\pi \times 10^{-7} \times 2 \times 10^4 \times 4}{10\pi^2}$ coul.

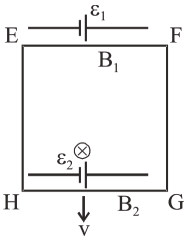
$= 32\mu C$

5. **Ans (B)**

$\boxed{\varepsilon = \dfrac{B\omega R^2}{2}}$



6. **Ans (B)**



$\varepsilon_{EH} = \varepsilon_{FG} = 0$

$\varepsilon = \varepsilon_2 - \varepsilon_1$

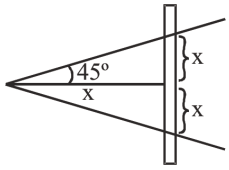
$= vB_2a - vB_1a$

$= va\left[\dfrac{B_0(y+a)}{a} - \dfrac{B_0 y}{a}\right] = vB_0a$

7. **Ans (A)**

$\varepsilon = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(BA)$

$= B\,\dfrac{d}{dt}\left(\dfrac{1}{2} \times 2x \times x\right) = B\,2x\,\dfrac{dx}{dt}$



$= 2B(Vt)\,V = 2\,BV^2\,t$

$= 2 \times 0.35\,(5.2)^2 \times 3 = 56.8$ V

8. **Ans (D)**

$i = \dfrac{\varepsilon}{R_{eq}} = \dfrac{V B\ell}{R + \dfrac{R_1R_2}{R_1 + R_2}}$

10. **Ans (A)**

$F = \dfrac{\mu_0 Ii \times L}{2\pi L/2} - \dfrac{\mu_0 Ii \times L}{2\pi 3L/2}$

12. **Ans (C)**

$T.R = I\,.\,\alpha$

$T.R = \dfrac{MR^2}{2}\,\dfrac{a}{R}$

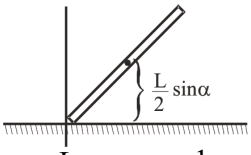
$2T = Ma$ ...(1)

$T - Mg = 0$ ...(2)

$T = Mg$ and $a = 2g$

$\alpha = \dfrac{2g}{R}$

13. **Ans (A)**

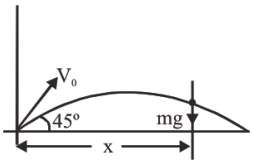


$Mg\,\dfrac{L}{2}\sin\alpha = \dfrac{1}{2}\,I\,\omega^2$

$Mg\,\dfrac{L}{2}\sin\alpha = \dfrac{1}{2}\,\dfrac{ML^2}{3}\,\omega^2$

$\therefore w = \sqrt{\dfrac{3g\sin\alpha}{L}}$

14. **Ans (A)**



$t = mgx$

$\ell = mg(V_0\cos 45t)$

$\dfrac{dJ}{dt} = \dfrac{mgV_0}{\sqrt{2}}t$

$\int_0^J dJ = \int_0^t \dfrac{mgV_0}{\sqrt{2}}t\,dt$

$J = \dfrac{mgV_0}{\sqrt{2}}\,\dfrac{t^2}{2}$

$J = \dfrac{mgV_0^3}{2\sqrt{2}g^2}$

$J = \dfrac{1}{2\sqrt{2}}\,\dfrac{mV_0^3}{g}$

15. Ans (A)

$$V = \sqrt{\frac{2gh}{1+K^2/R^2}} = \sqrt{\frac{2gh}{1+2/5}} = \sqrt{\frac{10}{7}gh}$$

16. Ans (B)

$W_1(y\cot 30°) = W_2\, y\cot 60°$

$$\frac{W_1}{W_2} = \frac{\cot 60^o}{\cot 30^o} = 1:3$$

17. Ans (C)

According to problem disc is melted and recasted into a solid sphere so their volume will be same

$$V_{Disc} = V_{Sphere} \Rightarrow \pi R^2_{Disc}\, t = \frac{4}{3}\pi R^3_{Sphere}$$

$$\Rightarrow \pi R^2_{Disc}\left(\frac{R_{Disc}}{6}\right) = \pi R^3_{Sphere}\left[t = \frac{R_{Disc}}{6}, \text{given}\right]$$

$$\Rightarrow \pi R^3_{Disc} = 8R^3_{Sphere} \Rightarrow R_{Sphere} = \frac{R_{Disc}}{2}$$

Moment of inertia of disc $I_{Disc} = \frac{1}{2}MR^2_{Disc} = I$

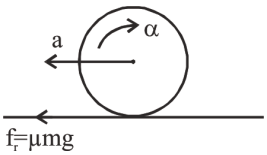
Given $M(R_{Disc})^2 = 2I$

Moment of inertia of sphere

$$I_{sphere} = \frac{2}{5}NR^2_{Sphere}$$

$$= \frac{2}{5}M\left(\frac{R_{Disc}}{2}\right)^2 = \frac{M}{10}(R_{Disc})^2 = \frac{2I}{10} = \frac{I}{5}$$

18. Ans (D)



$a = \mu g$

time at which V become zero

$$0 = V - \mu g t \Rightarrow t = \frac{V}{\mu g}$$

by $\tau = I\alpha \Rightarrow \mu mgR$

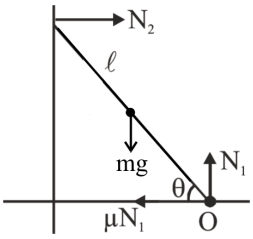
$$= \frac{2}{3}mR^2\alpha \Rightarrow \alpha = \frac{3\mu g}{2R}$$

$$\omega = \frac{3V}{R} - \frac{3\mu g}{2R}\cdot\frac{V}{\mu g} = \frac{3V}{2R}$$

19. Ans (D)

For equilibrium

$mg = N_1$



$\mu N_1 = N_2$

Taking moments about 0

$$N_2\, \ell \sin\theta = mg\frac{\ell}{2}\cos\theta$$

20. Ans (C)

$$\mu_{min} = \frac{\tan\alpha}{1+\frac{mR^2}{I}} = \frac{\tan\alpha}{1+2} = \frac{1}{3}\tan\alpha$$

PART-1 : PHYSICS

SECTION-II

1. Ans (2)

$$e = B\ell v = \frac{8}{10}\times 3\times 5 = 12 \text{ Volt}$$

$$\therefore q = CE(1 - e^{-t/t})$$

$$\therefore 24 = 6\times 12(1 - e^{-t/t}) \Rightarrow e^{-t/t} = \frac{2}{3}$$

$$\therefore i = \frac{E}{R}e^{-t/\tau} = \frac{12}{4}\times\frac{2}{3} = 2$$

3. Ans (6)

$$B = \frac{\mu_0 I}{\ell}$$

from coil

$\phi = BA\cos 0°$

$$= \frac{\mu_0 I}{\ell}\pi r^2$$

$$= \frac{\mu_0 \pi r^2 I_0 \cos 300t}{\ell}$$

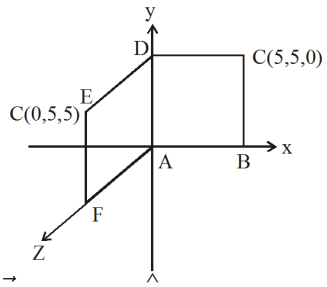
$$\varepsilon = -\frac{d\phi}{dt}$$

induced current $i = \frac{\varepsilon}{R}$

Magnetic moment of coil is

$M = i\pi\pi^2$

4. Ans (175)



$\vec{A}_{ABCD} = 25\hat{k}$

$\vec{A}_{ADEF} = 25\hat{i}$

$\vec{A}_{net} = 25\hat{i} + 25\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{k}$

$\phi = \vec{B}.\vec{A} = 25 \times 3 + 25 \times 4$

$\phi = 175$ Wb

6. Ans (5)

Rod rotates about its one end in a horizontal plane

$$\therefore t = I\alpha \Rightarrow \frac{Mg}{2} \times \frac{5L}{6} = \frac{ML^2}{3} \times \alpha$$

$$\Rightarrow \alpha = \frac{5g}{4L}$$

10. Ans (800)

$$E = \frac{L^2}{2I} \quad \therefore E \propto L^2 \Rightarrow \frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$\frac{E_2}{E_1} = \left[\frac{L_1 + 200\% \text{ of } L_1}{L_1}\right]^2 = \left[\frac{L_1 + 2L_2}{L_1}\right]^2$$

$= (3)^2 \Rightarrow E_2 = 9E_1$

Increment in kinetic energy

$\Delta E = E_2 - E_1 = 9E_1 - E_1$

$\Delta E = 8E_1 \therefore \frac{\Delta E}{E_1} = 8$ or percentage increase

$= 800\%$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (C)

$r_1 \quad r_2 \quad r_3 \quad r_4$

$\underbrace{r_1 \quad 4r_1}_{1:4} \quad \underbrace{9r_1 \quad 16r_1}_{1:4}$

$n = 1, 2 \quad \Delta E = 10.2$ eV

$n = 2, 4 \quad \Delta E = 2.55$ eV

2. Ans (A)

Larger the value of $E°_{RP}$ larger is tendency for reduction and consequently stronger will be the oxidant. Similarly, smaller the value of $E°_{RP}$ larger the tendency for oxidation and consequently stronger will be the reductent.

3. Ans (D)

$$\frac{\Delta E_{2\to1}}{\Delta E_{3\to2}} = \frac{\left(\frac{-13.6ev.z^2}{2^2}\right) - \left(\frac{-13.6ev.z^2}{12}\right)}{\left(\frac{-13.6ev.z^2}{3^2}\right) - \left(\frac{-13.6ev.z^2}{2^2}\right)}$$

$$= \frac{-\frac{1}{4} + 1}{-\frac{1}{9} + \frac{1}{4}} = \frac{+3/4}{+5/9 \times 4} = \frac{27}{5}$$

6. Ans (B)

$$\Lambda_m = \frac{\left(\frac{1}{96} \times 1.29 \times 10^{-2} \times 85\right) \times 1000}{0.052}$$

7. Ans (D)

$$\Delta E = 13.6 \times (5)^2 \left(\frac{1}{(3)^2} - \frac{1}{(4)^2}\right) \approx 16.53 \text{ eV}$$

8. Ans (D)

Theory based.

10. Ans (C)

$HCl\,(aq.) \longrightarrow H^+\,(aq.) + Cl^-\,(aq.)$

$0.1\ M \qquad 0.1\ M$

$CH_3COOH + H_2O \rightleftharpoons CH_3COO^- + H_3O^+$

Initial $\quad 0.01$

final $\quad (0.01 - x) \quad x \quad 0.1+x$

$$K_a = \frac{(x + 0.1)(x)}{(0.01 - x)}$$

assuming that $0.1 + x \approx 0.1$ and $x = 0.01\ \alpha$

$$K_a = \frac{(0.1)(0.01\alpha)}{0.01(1 - \alpha)}$$

$$\alpha = \frac{K_a}{0.1} = 1.6 \times 10^{-4}$$

11. Ans (D)

Number of moles of HCl

$= \frac{MV}{1000} = \frac{0.1 \times 40}{1000} = 0.004$

Number of moles of NaOH

$= \frac{MV}{1000} = \frac{0.45 \times 10}{1000} = 0.0045$

Remaining moles of NaOH after neutralization

$= 0.0005$

Molarity of OH^- $= \frac{0.0005}{50} \times 1000 = 0.01$ M

pOH = 2 $\quad \therefore$ pH = 14 – 2 = 12

13. Ans (A)

Basic strength $\propto$ +M, +I $\propto \frac{1}{-M, -I}$

+M ($-NH_2$) > + H($-CH_3$)

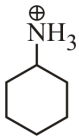
14. Ans (B)

Acid strength $\propto -1 \propto \frac{1}{+1}$

17. Ans (B)

Due to ortho effect. Acidic nature increase.

18. Ans (D)



– does not accept proton

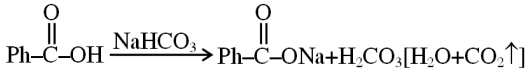
– octet of N atom is compete

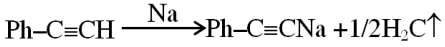
19. Ans (D)

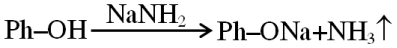
Acidic strength $\propto$ –I power $\propto K_a$

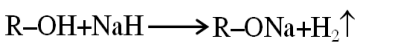
Also –I power decreased as distance increase.

20. Ans (A)









$A + B + C + D = 44 + \frac{2}{2} + 18 + 2 = 65$

PART-2 : CHEMISTRY

SECTION-II

4. Ans (8)

$K_h = \frac{K_w}{K_a} = \frac{10^{-14}}{10^{-4}} = 10^{-10}$

$h = \sqrt{\frac{K_h}{C_o}} = \sqrt{\frac{10^{-10}}{10^{-2}}} = 10^{-4}$

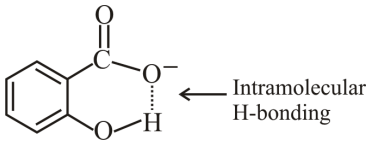
$pH = \frac{1}{2}[pK_w + pK_a + \log C_o]$

$pH = \frac{1}{2}[14 + 4 + \log 10^{-2}]$

$pH = \frac{1}{2}[16] = 8$

7. Ans (6)

Acids, I, III, IV, VII, VIII and IX are all stronger than benzoic acid. I is stronger because of stabilisation of conjugate base by intramolecular H-bonding. III is stronger because from *meta* position, –OH exert only –I effect, its electron donating resonance effect has no role on acidic strength.



IV is stronger acid due to loss of planarity of –COOH with phenyl ring, hence absence of electron donating resonance effect as phenyl rings on –COOH increases acidic strength. VII is stronger because a sulphonic acid is stronger than a carboxylic acid.

VIII is stronger because electron withdrawing inductive effect of one –COOH over other increases acidic strength.

IX is stronger due to only –I effect of methoxy group operate from *meta* position but not its electron donating resonance effect.

8. Ans (3)

Acidic strength $\propto -1, -M \propto \frac{1}{+I, +M}$

9. Ans (4)

Compounds which are more acidic than H_2CO_3 soluble in $NaHCO_3$ c, d, f and g soluble in $NaHCO_3$

PART-3 : MATHEMATICS

SECTION-I

1. Ans (C)

$P^4 = (I - P)(I - P)$

$= I - 2P + P^2 = 2I - 3P$

$P^6 = (2I - 3P)(I - P) = 5I - 8P$

$\Rightarrow \boxed{n = 6}$

3. Ans (A)

$AB + A + B + I = I$

$\Rightarrow A(B+I)+I(B + I) = I \Rightarrow (A+I)(B+I) = I$

so $(B + I)$ & $(A + I)$ are inverse of each other

$(B+I)(A+I) = (A+I)(B + I) \Rightarrow AB = BA$

so $(A+B)^2 = A^2 + 2AB + B^2$

4. Ans (B)

$$A = \begin{bmatrix} \alpha & \beta & 0 \\ \alpha^2 & \beta^2 & 0 \\ \alpha^3 & \beta^3 & 0 \end{bmatrix} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ x & y & z \end{bmatrix}$$

$|A| = 0 \Rightarrow$ Singular matrix

5. Ans (B)

$y^2 - 2y = -6x - 13$

$(y - 1)^2 = -6x - 12$

$(y - 1)^2 = -6(x + 2)$

$y^2 = -6x$

vertex $(-2, 1)$

Focus $x = a, y = 0,$

$x + 2 = -\frac{3}{2}, y - 1 = 0$

$x = -2 - \frac{3}{2} = -\frac{7}{2}, y = 1,$

Focus $\left(-\frac{7}{2}, 1\right)$

Ends of L.R. $x = a, y = \pm 2a$

$x = -\frac{7}{2}, \quad y - 1 = \pm 2 \times \frac{6}{4}$

$y = \pm 3 + 1$

$y = -2, y = 4$

$\left(-\frac{7}{2}, -2\right)$ or $\left(-\frac{7}{2}, 4\right)$

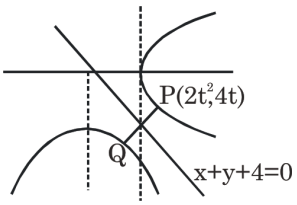
Foot of directrix

$x = -a \quad x + 2 = 3/2 \quad x = -1/2$

$\left(-\frac{1}{2}, 1\right)$

A–S, B–R, C–Q, D–P

9. Ans (A)



for minimum distance $\left.\frac{dy}{dx}\right|_P = -1$

$\Rightarrow t = -1$

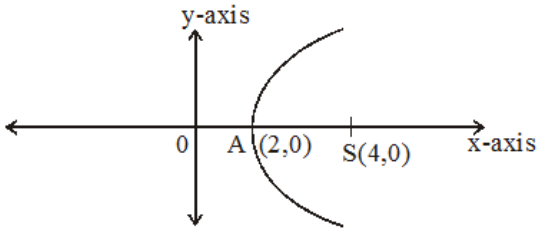
$\Rightarrow$ min distance = PQ = $2\sqrt{2}$

10. Ans (C)

$5y = 4x + 8$. Point of intersection of chord of contact with parabola $y^2 = 8x$ are $\left(\frac{1}{2}, 2\right), (8, 8),$

so that length $\frac{3}{2}\sqrt{41}$.

11. Ans (C)



equation of parabola is $y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

12. Ans (D)

Vertex is $(a^2,0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.(a^2) = 250$

$\Rightarrow a^3 = 125$ or $a = 5$

13. Ans (C)

After reflector ray must pass through focus.

S (0,2)

14. Ans (B)

Homogenising,

$x^2 - 6y\left(\frac{x + 2y}{-t}\right) = 0$

$\Rightarrow tx^2 + 6xy + 12y^2 = 0$

Coeff. of x^2 + coeff. of $y^2 = 0 \Rightarrow t = -12$

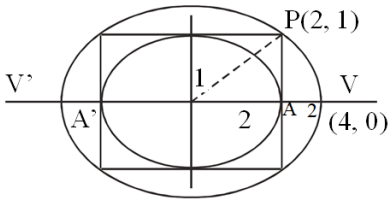
15. Ans (A)

$x^2 + 4y^2 = 4 \quad \Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{1} = 1$

$\Rightarrow \quad a = 2, b = 1 \quad \Rightarrow \quad P = (2, 1)$

Required Ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \quad \frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$

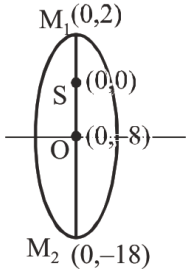


$\Rightarrow \quad \frac{4}{16} + \frac{1}{b^2} = 1$

$\Rightarrow \quad \frac{1}{b^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow b^2 = \frac{4}{3}$

$\therefore \quad \frac{x^2}{16} + \frac{y^2}{(4/3)} = 1 \Rightarrow x^2 + 12y^2 = 16$

16. Ans (B)



$b = 10$ & $be = 8$

$\Rightarrow e = \frac{4}{5}$ and $a = 6$

Let ellipse is $\frac{x^2}{a^2} + \frac{(y+8)^2}{b^2} = 100$

where $b = 10$ & $a = 6$

17. Ans (A)

Here and $2ae = 8m$; $e = \frac{4}{5}, a = 5m$

$\therefore \quad b^2 = a^2(1 - e^2) = 9 \quad \Rightarrow \quad b = 3$

Thus, required area = $\pi ab = 15\pi$ sq.metre.

18. Ans (B)

Given ellipse is = 1 whose area is π ab. The auxiliary circle to the given ellipse is

$x^2 + y^2 = a^2$ whose area is πa^2.

Given that, $\pi a^2 = 2\pi ab \Rightarrow a = 2b$

Now , eccentricity of ellipse

$= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}} = \frac{\sqrt{3}}{2}$

19. Ans (B)

Any point on the given hyperbola is $P(\sqrt{2}\sec\theta, \tan\theta)$

Asymptotes are $x - \sqrt{2}y = 0, \quad x + \sqrt{2}y = 0$

Product of perpendiculars from P on these asymptotes

$= \frac{(\sqrt{2}\sec\theta - \sqrt{2}\tan\theta)(\sqrt{2}\sec\theta + \sqrt{2}\tan\theta)}{1+2}$

$= \frac{2\sec^2\theta - 2\tan^2\theta}{3} = \frac{2}{3}$

20. Ans (B)

$a_1e_1 = a_2e_2$ or $a_1^2e_1^2 = a_2^2e_2^2$

or $\quad a_1^2\left(1 - \frac{b_1^2}{a_1^2}\right) = a_2^2\left(1 + \frac{b_2^2}{a_2^2}\right)$

or $\quad a_1^2 - b_1^2 = a_2^2 + b_2^2$

$16 - a = \left(\frac{12}{5}\right)^2 + \left(\frac{9}{5}\right)^2 = \frac{225}{25} = 9$

$\therefore \quad a = 7$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (3375)

$1 \times 15 \times 15 \times 15 = 3375$

2. Ans (64)

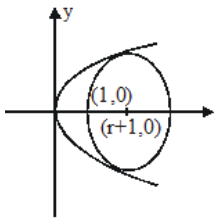
$A^{-1} + B^{-1} = 2I$

$\Rightarrow I + AB^{-1} = 2A$

$\Rightarrow B + A = 2AB$ or $2AB = I$

4. Ans (4)

Equation of circle is



$(x - r - 1)^2 + y^2 = r^2$

$(x - r - 1)^2 + 4x = r^2$

$D = 0$

$r = 4$

5. Ans (4)

$C \equiv (0, 3), S \equiv (4e, 0)$

$\equiv (4\sqrt{1 - \frac{9}{16}}, 0) \equiv (\sqrt{7}, 0)$

$r = |CS| = \sqrt{(\sqrt{7} - 0)^2 + (0 - 3)^2}$

$= \sqrt{7 + 9} = 4$

7. Ans (8)

$(x^2 - 4x + 4) - 3(y^2 + 2y + 1) = 11 + 4 - 3$

$(x - 2)^2 - 3(y + 1)^2 = 12$

$\frac{(x-2)^2}{12} - \frac{(y+1)^2}{4} = 1$

$a = 2\sqrt{3}, \ b = 2$

$e = \sqrt{1 + \frac{4}{12}} = \frac{2}{\sqrt{3}}$

focus, $x = \pm ae,$

$x - 2 = \pm 2\sqrt{3}.\ \frac{2}{\sqrt{3}}$

$x = 6, -2$

$y = 0$

$y + 1 = 0$

$y = -1$

$s\ (6, -1), s'\ (-2, -1)$

$ss^1 = 8$

8. Ans (6)

Here $2b = \frac{1}{3}(2ae) \Rightarrow b = \frac{ae}{3}$

$\Rightarrow b^2 = \frac{a^2e^2}{9} \Rightarrow a^2(e^2 - 1) = \frac{a^2e^2}{9}$

$\Rightarrow e = \frac{3}{2\sqrt{2}} = \frac{a}{b\sqrt{b}} \Rightarrow ab = 3 \times 2 = 6$

9. Ans (2)

Here equations of directrices are $x = \frac{21}{5}$ and

$x = -\frac{11}{5} \Rightarrow a + b = \frac{21}{5} + \left(-\frac{11}{5}\right) = \frac{10}{5} = 2$

10. Ans (4)

$|S_1P - S_2P| = 2a = 2\left(\alpha + \frac{1}{\alpha}\right) = 4.00$