Test Pattern
JEE (Main)
PART TEST
29-12-2024

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY | PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	A	C	A	A	C	C	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	B	C	C	B	B	D	B	D	A
SECTION-II	Q.	1	2	3	4	5					
	A.	8	2	12	6	3					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	A	D	C	A	C	C	B	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	C	D	D	B	C	A	D	C	C
SECTION-II	Q.	1	2	3	4	5					
	A.	5	3	4	3168	3					

PART-3 : MATHEMATICS

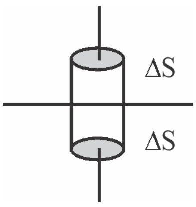
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	A	A	C	B	A	C	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	A	A	D	D	A	A	C	A	C
SECTION-II	Q.	1	2	3	4	5					
	A.	6	13	2	4	10					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (D)**



$$E_1 = \frac{\sigma}{K_1\varepsilon_0}$$

$$E_2 = \frac{\sigma}{K_2\varepsilon_0}$$

$$E_2\Delta S - E_1\Delta S = \frac{\sigma'\Delta S}{\varepsilon_0}$$

$$\frac{\sigma}{K_2\varepsilon_0} - \frac{\sigma}{K_1\varepsilon_0} = \frac{\sigma'}{\varepsilon_0}$$

$$\sigma\left[\frac{1}{K_2} - \frac{1}{K_1}\right] = \sigma'$$

$$\sigma\left[\frac{1}{6} - \frac{1}{2}\right] = \sigma'$$

$$\sigma\left[\frac{2-6}{12}\right] = \sigma'$$

$$-\frac{\sigma}{3} = \sigma'$$

2. Ans (D)

$$\frac{Q^2}{2A\epsilon_0}x = \frac{1}{2}kx^2$$

$$Q = \sqrt{k\epsilon_0 Ax}$$

$$x < d$$

$$\Rightarrow Q = \sqrt{k\epsilon_0 Ad}$$

4. Ans (C)

$$\vec{J}_P < \vec{J}_Q$$

$$\vec{J} = \sigma\vec{E}$$

5. Ans (A)

$E_{eq} = 8 \times 10 = 80$ V

$r_{eq} = 8 \times 0.2 = 1.6$ V

$$I = \frac{E_{eq}}{r_q} = \frac{80}{1.6} = 50A$$

P.d. across the battery

$V = E - Ir = 10 - 50 \times 0.2 = 0$

6. Ans (A)

$$A\ell\sigma g - A\rho xg = A\ell\sigma a$$

$$g = \frac{\rho gx}{\sigma\ell} + a$$

$$g = \frac{\rho gx}{\sigma\ell} + v.\frac{dv}{dx}$$

$$\int_0^x \left(g - \frac{\rho gx}{\sigma\ell}\right)dx = \int_0^v vdv \Rightarrow \frac{V^2}{2} = gx - \frac{\rho gx^2}{\sigma\ell 2}$$

At maximum displacement $v = 0$

$$x = \frac{2\sigma\ell}{3} = 2 \times \frac{1}{3} \times 6 = 4m$$

7. Ans (C)

$$m.\frac{dv}{dt} = -6\pi\eta rv$$

$$\frac{dv}{dt} = -\frac{v}{2}$$

$$\frac{dv}{v} = \frac{-dt}{2}$$

$$-\int_2^1 \frac{dv}{v} = \frac{1}{2}\int_{t_1}^{t_2} dt$$

$$-[\ln v]_2^1 = \frac{1}{2}\Delta t$$

$$\ln 2 = \frac{\Delta t}{2}$$

$$\boxed{2\ln 2 = \Delta t}$$

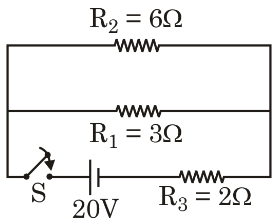
8. Ans (C)

$$C_{eq} = \frac{\varepsilon_0 l}{d}[1 - x + kx]$$

$$I = \frac{V\,v\varepsilon_0 l}{d}[k-1] = 4 \times 4 = 16\mu A$$

9. Ans (A)

$t = 0$;



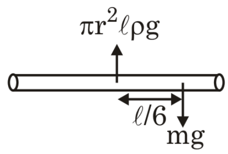
$$i = \frac{20}{4} = 5A$$

$t \longrightarrow \infty$

$$\frac{20}{8} = \frac{5}{2}A$$

$q_{max} = (1\ \mu F) \times 5V = 5\ \mu C$

10. Ans (B)



$$\text{Torque} = (\pi r^2\ell\rho g)\frac{\ell}{6} = \frac{\pi r^2\ell\rho g}{6}$$

$$I\alpha = \frac{\pi r^2\ell^2\rho g}{6}$$

$$\alpha = \frac{\pi r^2\ell^2\rho g}{6I}$$

12. Ans (B)

$R = R_0(1 + \alpha\ \Delta T)$

$R = 5(1 + 0.005 \times 20)$

$R = 5(1 + 0.1)$

$R = 5.5$

$$I = \frac{10}{5.5} = \frac{20}{11}$$

13. Ans (C)

$W = \Delta U - W_B$;

$$-\frac{1}{2}CV^2$$

19. **Ans (D)**

h does not depend on inclination.

20. **Ans (A)**

$20 = 5(R + 0.2)$

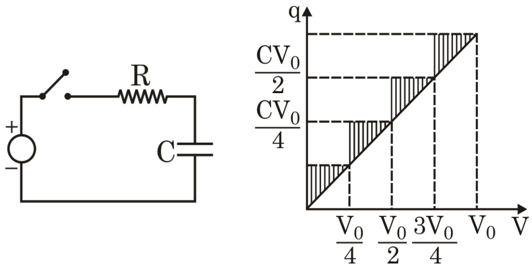
$4 = R + 0.2$

$R = 3.8\ \Omega$

PART-1 : PHYSICS

SECTION-II

1. **Ans (8)**



$$\text{Heat lost} = \frac{1}{2}C\left(\frac{V_0}{4}\right)^2 + \frac{1}{2}\left(\frac{V_0}{4}\right) \times C\frac{V_0}{4}$$

$$+\frac{1}{2}C\left(\frac{V_0}{4}\right)^2 + \frac{1}{2}\left(\frac{V_0}{4}\right)^2$$

$$= 4 \times \frac{1}{2}C\frac{V_0^2}{16} = \frac{1}{4}CV_0^2 \times \frac{1}{2}$$

2. **Ans (2)**

$10\ VSD = 8\ MSD$

$1 VSD = \dfrac{8 \times 1mm}{10}$

$L.C. = 1\ MSD - 1\ VSD = 0.2\ mm$

3. **Ans (12)**

$E2\pi r^2 = \dfrac{q_1}{K\varepsilon_0}$

$E2\pi r^2 = \dfrac{q_2}{2K\varepsilon_0}$

$[2K\varepsilon_0 2\pi r^2 + K\varepsilon_0 2\pi r^2]E = q$

$E = \dfrac{q}{3K\varepsilon_0 2\pi r^2}$

$$V_A - V_B = \int E,.dr = \frac{q}{6\pi\varepsilon_0 K}\int_R^{2R}\frac{1}{r^2}dr$$

$$= \frac{q}{6\pi\varepsilon_0 K}\left[\frac{1}{R} - \frac{1}{2R}\right]$$

4. **Ans (6)**

$I = \dfrac{M}{2}(R_1^2 + R_2^2)$

$dI = \dfrac{M}{2}[2R_1dR_1 + 2R_2dR_2] + \dfrac{dM}{2}[R_1^2 + R_2^2]$

$\dfrac{dI}{I} \times 100 = 5.8\%$

PART-2 : CHEMISTRY

SECTION-I

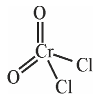
1. **Ans (C)**

As electro positivity of the metal decreases from Ti to Fe

6. **Ans (A)**

$K_2Cr_2O_{7(s)} + 4NaCl_{(s)} + \text{conc. } 6H_2SO_4 \rightarrow$

$2KHSO_4 + \underset{\text{Chromyl Chloride}}{2CrO_2Cl_2} + 4NaHSO_4$



7. **Ans (C)**

X = black, MnO_2, Y= green, K_2MnO_4, Z=$KMnO_4$

$\underset{(X)}{2MnO_2} + 2K_2CO_3 + \underset{\text{air}}{O_2} \xrightarrow{\Delta} \underset{\substack{\text{green}\\(Y)}}{2K_2MnO_4} + 2CO_2(g)$

$\underset{(X)}{2K_2MnO_4} + Cl_2 \rightarrow \underset{\text{(Z) Purple}}{2KMnO_4} + 2KCl$

8. **Ans (C)**

Gd^{3+} has 7 unpaired electrons.

$\Rightarrow n = 7$

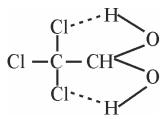
$\mu_s = \sqrt{7(7+2)} = \sqrt{63} = 7.9$

10. **Ans (B)**

Ionic radii inversly proportional to Zeff

12. **Ans (C)**

Due to intramolecular H-bonding



13. **Ans (D)**

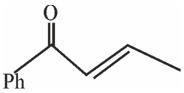
Semicarbazide react with both aldehyde & ketone and forms semicarbazone.

15. **Ans (B)**

Birch reduction results into ANTI ADDITION - Tarns product is major.

16. **Ans (C)**

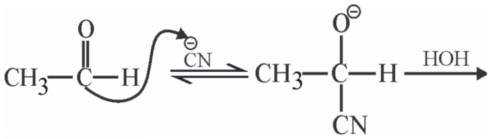
(a) Due to bulky base hoffmann product is preferred.

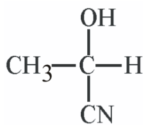
(b) reaction follows εl_{CB} Mech hence carbanion is stable anc'  is the major product.

(c) ε_1 Mechanism involve rearrangement of C^+.

(d) Pyrolysis of ester is an example of syn elimination

17. **Ans (A)**

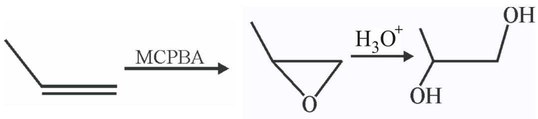


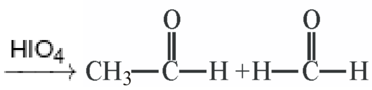


18. **Ans (D)**

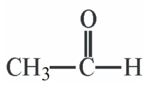
SN^2 reactions are very sensitive to steric factor.

19. **Ans (C)**





can be distinguished by Haloform Test only

 give +ve Haloform Test.

20. **Ans (C)**

Ion	Observed magnetic moment (BM)
Fe^{2+}	5.3 – 5.5
Mn^{2+}	5.96
Co^{2+}	4.4 – 5.2
Ni^{2+}	2.9 – 3.4

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (5)**

Eka - Aluminium (Z)=31

Eka-Boron (Z)=21

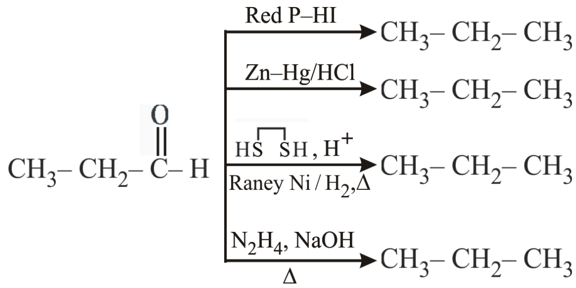
2n = 31 - 21 =10

n = 5

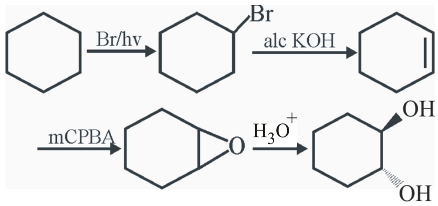
2. **Ans (3)**

Li, B and P

3. **Ans (4)**



4. **Ans (3168)**



5. **Ans (3)**

	V	Cr	Mn	Fe	Co	Ni	Cu
$E^{\ominus}/V(M^{2+}/M)$	-1.18	-0.91	-1.18	-0.44	-0.28	-0.25	+0.34

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (B)**

Let $P(x) = ax^2 + bx + c$

$P'(x) = 2ax + b$

$\therefore P(x) - P'(x) = x^2 + 2x + 1$

$\Rightarrow ax^2 + (b - 2a)x + c - b = x^2 + 2x + 1$

$\Rightarrow a = 1, b = 4, c= 5$

$\therefore P(x) = x^2 + 4x + 5$

$$f(x) = \begin{cases} \left(\dfrac{x^2+4x+5}{10}\right)^{\frac{1}{\tan(x^2-1)}} & ; \quad x \neq 1 \\ e^{\frac{3x}{3(a^2-2a)+13}} & ; \quad x = 1 \end{cases}$$

$$e^{\lim\limits_{x\to1} \frac{x^2+4x-5}{10}\times\frac{1}{\tan(x^2-1)}} = e^{\frac{3}{3a^2-6a+13}}$$

$\Rightarrow 3a - 6a + 3 = 0$

$\Rightarrow a = 1$

2. Ans (C)

Let $y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$

$\ln y = \sin^2 x \cdot \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)$

$\frac{1}{y}y' = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) 2\sin x\cos x + \sin^2 x$

$\frac{2\sin x}{\sqrt{3e}} \quad \frac{\sqrt{3e}}{2} \quad (-\text{cosec}\, x \cot x)$

$\frac{dy}{dx} - 0 \Rightarrow \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) 2\sin x\cos x - \sin x\cos x = 0$

$\Rightarrow \sin x\cos x\left[2\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) - 1\right] = 0$

$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right) = 1 \Rightarrow \frac{3e}{4\sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}\left(\text{as } x \in \left(0, \frac{\pi}{2}\right)\right)$

$\Rightarrow \text{local max value} = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8} = \frac{k}{e}$

$\Rightarrow k^8 = e^{11} \Rightarrow \left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$

$= e^3 + e^6 + e^{11}$

3. Ans (A)

Given $f(x) = \ln|x| + bx^2 + ax$

$\therefore f'(x) = \frac{1}{x} + 2bx + a$

At $x = -1$, $f'(x) = -1 - 2b + a = 0$

$\Rightarrow a - 2b = 1$ ..... (i)

At $x = 2$, $f'(x) = \frac{1}{2} + 4b + a = 0$

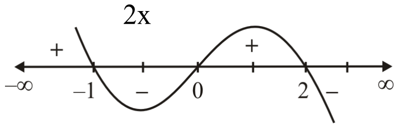
$\Rightarrow a + 4b = -\frac{1}{2}$ ..... (ii)

On solving (i) and (ii) we get $a = \frac{1}{2}, b = -\frac{1}{4}$

Thus, $f'(x) = \frac{1}{x} - \frac{x}{2} + \frac{1}{2} = \frac{2 - x^2 + x}{2x}$

$= \frac{-x^2 + x + 2}{2x} = \frac{-(x^2 - x - 2)}{2x}$

$= \frac{-(x+1)(x-2)}{2x}$



So maxima at $x = -1, 2$

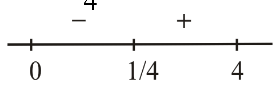
Hence both Assertion (A) and Reason (R) both are true and R is correct explanation of A.

4. Ans (A)

$f'(x) = 16x - \frac{1}{x}$

$f'(x) = 0 \Rightarrow x = \frac{1}{4}$

$\therefore a = \frac{1}{4}$



Let $P(x_1, y_1)$ be any point on parabola $y^2 = 2x$ diff.

w.r.t. x

$y' = \frac{1}{y_1} = m_T$

$\therefore \frac{1}{y_1} = \frac{3 - y_1}{4 - x_1}$

$\Rightarrow \frac{1}{y_1} = \frac{3 - y_1}{4 - \frac{y_1^2}{2}}$

$\Rightarrow 8 - y_1^2 = 6y_1 - 2y_1^2$

$\Rightarrow y_1^2 - 6y_1 + 8 = 0$

$\Rightarrow y_1 = 2, 4$

Point is P (2, 2) or (8, 4)

Tangent at (2, 2) is

$y - 2 = \frac{1}{2}(x - 2)$

As (–2, 0) satisfies $\Rightarrow$ Reject

$\therefore$ P (8, 4)

$\therefore$ Normal at P (8, 4) is

$y - 4 = -4(x - 8)$

$= -4x + 32$

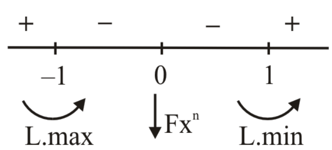
$\Rightarrow 4x + y = 36$

5. Ans (C)

$f(x) = x + 3x^{1/3}$

$f'(x) = 1 - x^{-2/3}$

$= \frac{x^{2/3} - 1}{x^{2/3}} = \frac{\left(x^{1/3} - 1\right)\left(x^{1/3} + 1\right)}{x^{2/3}}$



6. Ans (B)

$$f(x) = \begin{cases} x-1; & x = \text{Even} \\ 2x; & x = \text{odd} \end{cases}$$

$f(f(f(a))) = 1$

C-1 : If a = even

$f(a) = a - 1 = \text{odd}$

$f(f(a)) = 2(a-1) = \text{even}$

$f(f(f(a))) = 2a - 3 = 1 \Rightarrow a = 2$

C-2 : If a = odd

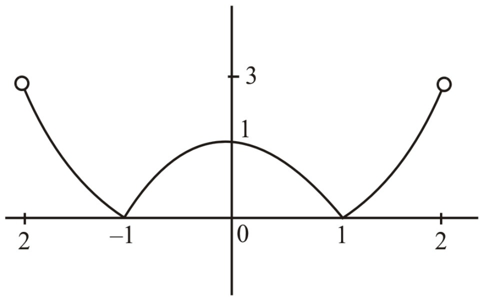
$f(a) = 2a = \text{even}$

$f(f(a)) = 2a - 1 = \text{odd}$

$f(f(f(a))) = 4a - 2 = 1$ (Not possible)

Hence a = 2

$\therefore g(x) = |x^2 - 1|\ ; x \in (-2, 2)$



$\Rightarrow$ continious everywhere not differentiable at x = –1, 1

$\therefore m = 0, n = 2$

hence $m + n = 2$

7. Ans (A)

$f(x) = 25 \Rightarrow x = 3$

$$f'(x) = 2(x + \log_3 x)\left[1 + \frac{1}{x\log_e 3}\right] + 2x$$

$$\left.\frac{d}{dx} f^{-1}(x)\right|_{x=25} = \left.\frac{1}{f'(x)}\right|_{x=3}$$

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{2(4)\left\{\frac{3\ln 3 + 1}{3\ln 3}\right\} + 6}$$

$$= \frac{3\ln 3}{8 + 42\ln 3}$$

8. Ans (C)

$$f(x) = \begin{cases} ax^3 + bx^2 & ; \quad x^2 < 1 & \Rightarrow -1 < x < 1 \\ \frac{1}{x} & ; \quad x^2 > 1 & \Rightarrow x < -1 \text{ or } x > 1 \\ \frac{a+b+1}{2} & ; \quad x = 1 & \\ \frac{b-a-1}{2} & ; \quad x = -1 & \end{cases}$$

$\because$ f(x) is continuous at x = 1 $\Rightarrow$ a + b = 1

and f(x) is continuous at x = –1 $\Rightarrow$ b – a = –1

$\therefore$ a = 1, b = 0

$\therefore$ A(–1, 3) and B(1, –1)

$\therefore g'(x) = \lambda(x-1)(x+1)$

$$g(x) = \lambda\left(\frac{x^3}{3} - x\right) + c$$

$$g(-1) = \frac{2\lambda}{3} + c = 3$$

$$g(1) = -\frac{2\lambda}{3} + c = -1$$

$c = 1$ and $\lambda = 3$

$\therefore g(x) = x^3 - 3x + 1$

$\therefore g(2) = 3$

9. Ans (D)

(A) $\displaystyle\lim_{x\to\infty} \frac{2}{\sqrt{x^2+1} + \sqrt{x^2-1}} = \frac{2}{\infty} = 0$

(B) $\displaystyle\frac{2}{25} \lim_{x\to 0} \frac{1-\cos x}{25x^2} \times \frac{25x^2}{3x^2}$

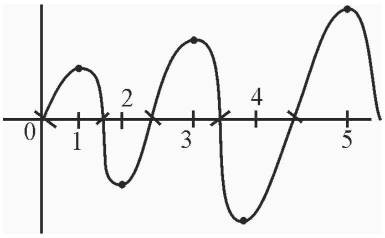
$$= \frac{2}{25} \times \frac{1}{2} \times \frac{25}{3} = \frac{1}{3}$$

(C) $\displaystyle\lim_{x\to\infty} 5\left[\left(\frac{4}{5}\right)^x + 1\right]^{\frac{1}{x}} = 5(0+1) = 5$

(D) $\displaystyle\lim_{x\to 0} \frac{x^2 - \sin^2 x}{\frac{\sin^2 x}{x^2} \times x^4}$

$$= \lim_{x\to 0} \frac{x^2 - \left[x - \frac{x^3}{3!} + \ldots\right]^2}{x^4} = \frac{1}{3}$$

10. Ans (C)



$(3f'f'' + f.f''')(x) = \left(\left(ff'' + (f')^2\right)(x)\right)'$

$\left(ff'' + (f')^2\right)(x) = \left((ff')(x)\right)'$

$\therefore \left(3f'f'' + f'''\right)(x) = \left(f(x).f'(x)\right)''$

Min. number of roots of f(x) = 5

Min. number of roots of f'(x) = 4

$\Rightarrow$ Min. number of roots of f(x).f'(x) = 5 + 4 = 9

$\therefore$ Min. number of roots of $\left(f(x).f'(x)\right)'' = 7$

11. Ans (B)

$f(x) = ||x-6| - |x-8|| - |x^2-4| + 3x - |x-7|^3$ is continuous $\forall x \in R$ and not differentiable at x = –2, 2, 6, 7 & 8

12. Ans (A)

Let f(x) = a (x – p) (x – q)

Then, g(x) = b(x – p) (x – q)

So, $h(x) = k(x-p)^2(x-q)^2$

no. of roots of h(x) . h'(x) is 3

So, roots of $\frac{d}{dx}\left(h(x)\cdot h'(x)\right) = 0$ are 4.

13. Ans (A)

$h(x) = (x-\alpha_1)(x-\alpha_2) ..... (x-\alpha_5)$ take log both side and differentiate

$$\frac{h'(x)}{h(x)} = \frac{1}{x-\alpha_1} + \frac{1}{x-\alpha_2} + ... + \frac{1}{x-\alpha_5}$$

again diff.

$$\frac{h\cdot h'' - (h')^2}{h^2} < 0 \ \forall x \in R$$

no real roots

14. Ans (D)

Let tangent y = mx + c

$x^4 - 2x^2 - x - (mx + c)$

$= (x^2 + ax + b)^2$

a = 0, b = – 1, m = – 1, c = – 1

x + y + 1 = 0 is tangent at (–1, 0) and (1, –2)

15. Ans (D)

$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{d^2y}{dx^2} = 2$

Now, $\frac{dx}{dy} = \frac{1}{2x}$

$\frac{d^2x}{dy^2} = \frac{-1}{2x^2} \times \frac{1}{2x} = \frac{-1}{4x^3}$

So, $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = \frac{-1}{2x^3}$

16. Ans (A)

$$\lim_{x\to0}\frac{\sin^2x}{e^{px}-qx-1} = \frac{1}{2}$$

$$\lim_{x\to0}\frac{2\cos 2x}{p^2e^{px}} = \frac{1}{2}$$

$N^r \longrightarrow 2, D^r \longrightarrow 4$

$p = q = \pm 2 \longrightarrow (p, q) = (2, 2). (-2, 2)$

17. Ans (A)

$$\lim_{x\to0}\frac{e^{x^3} - \left(1-x^3\right)^{\frac{1}{3}}}{x^3} + \lim_{x\to0}\frac{\left(1-x^2\right)^{\frac{1}{2}} - 1}{x^2}$$

$$= \lim_{x\to0}\frac{1 + x^3 - 1 + \frac{x^3}{3}}{x^3} + \frac{1 - \frac{x^2}{2} - 1}{x^2}$$

$$1 + \frac{1}{3} - \frac{1}{2} = \frac{5}{6}$$

18. **Ans (C)**

$$\sum_{n=0}^{\infty}\frac{(n+2020)^2}{n!}=?$$

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\Rightarrow x^{2020}e^x=\sum\frac{x^{n+2020}}{n!}$$

Now diff

$$\Rightarrow\left(2020x^{2019}+x^{2020}\right)e^x=\sum(n+2020)\frac{x^{n+2019}}{n!}$$

Multiply by x

$$\left(2020x^{2020}+x^{2021}\right)e^x=\sum\frac{(n+2020)x^{n+2020}}{n!}$$

$$\left(2020^2x^{2019}+2021x^{2020}\right)e^x+e^x\left(2020x^{2020}+x^{2021}\right)$$

$$e^x\left(2020^2x^{2019}+4041x^{2020}+x^{2021}\right)=\sum\frac{(n+2020)^2x^{n+2019}}{n!}$$

$x=1$

$$\sum\frac{(n+2020)^2}{n!}=e\left((2020)^2+4041+1\right)$$

$$=e(4080400+4042)$$

$$=e(4084442)$$

19. **Ans (A)**

$$fog(x)=\begin{cases}\left|1-e^x+x\right|; & x<0\\ \left\{(x-1)^2-1\right\}; & 0<x<2\\ \left|1-(x-1)^2+1\right|; & x\geqslant 2\cup\{0\}\end{cases}$$

So, we will check at 0 & 2

Discontinuous at x = 0 and continuous at x = 2

20. **Ans (C)**

$f(x)=(\pi-4\tan^{-1}x)(\pi-2\sin^{-1}x)$

$$\lim_{x\to1^-}\frac{8\cdot\tan^{-1}\left(\frac{1-x}{1+x}\right)\cdot\sin^{-1}\sqrt{1-x^2}}{(1+x)\frac{(1-x)}{1+x}\cdot\sqrt{1-x^2}}$$

$=4$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (6)**

$$f(x)=\begin{cases}x^3+x^2 & 0<x<3\\ (x-3)^3+x^2 & 3\leqslant x<4\\ (x-3)^3+(x-4)^2 & 4\leqslant x<6\\ (x-6)^3+(x-4)^2 & 6\leqslant x<8\\ (x-6)^3+(x-8)^2 & 8\leqslant x<9\\ (x-9)^3+(x-8)^2 & 9\leqslant x<12\\ (x-12)^3+(x-12)^2 & 12\leqslant x<15\end{cases}$$

$f(3^-)=36$ $\quad f(3^+)=9$ $\quad$ D.C. at $x=3$

$f(4^-)=16$ $\quad f(4^+)=1$ $\quad$ D.C. at $x=4$

$f(6^-)=31$ $\quad f(6^+)=4$ $\quad$ D.C. at $x=6$

$f(8^-)=24$ $\quad f(8^+)=8$ $\quad$ D.C. at $x=8$

$f(9^-)=28$ $\quad f(9^+)=1$ $\quad$ D.C. at $x=9$

$f(12^-)=43$ $\quad f(12^+)=0$ $\quad$ D.C. at $x=12$

No. of points D.C. are 6

2. **Ans (13)**

Circles with centre (–6, 10) radius

$=\sqrt{36+100-120}=4$

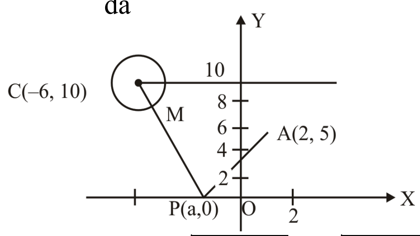
Now, let (a, 0) be a point on the X-axis. if y is the distance from A to P and P to M

$$y=\sqrt{(a-2)^2+25}+\sqrt{(a+6)^2+100}-4$$

$$\frac{dy}{dx}=\frac{2(a-2)}{2\sqrt{(a-2)^2+25}}+\frac{2(a+6)}{2\sqrt{(a+6)^2+100}}$$

$\frac{dy}{da}$ can be zero only if $a-2>0$ and $a+6<0$ not possible or $a-2<0$ and $a+6>0$, hence $a\in(-6,2)$.

Solving $\frac{dy}{da}=0$, gives $a=10$ (rejected) or $a=-\frac{2}{3}$



$$\text{Hence, } y_{min}=\sqrt{\frac{64}{9}+25}+\sqrt{\frac{256}{9}+100}-4$$

$$=\frac{17}{3}+\frac{\sqrt{1156}}{3}-4=\frac{17}{3}+\frac{34}{3}-4$$

$=17-4=13$

3. Ans (2)

Let a < b and $f(x) = |x - a| + |x - b|$, $\forall\ x \in R$

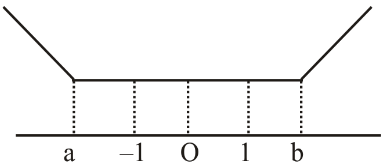
So, f(x) is decreasing in $(-\infty, a]$ constant in [a, b]

and increasing in $[b, \infty)$, we have

$f(0) = f(1) = f(-1)$

a & b $\in$ {-1, 0, 1}

$\therefore |a-b|_{min} = 2$

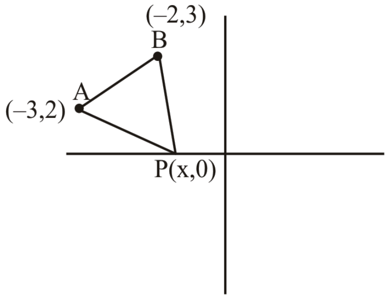


4. Ans (4)

$$S = \sqrt{(x+3)^2+4} - \sqrt{(x+2)^2+9}$$

$$S = \sqrt{(x+3)^2+(2)^2} - \sqrt{(x+2)^2+(3)^2}$$

$$S = \sqrt{(x-(-3))^2+(0-2)^2} - \sqrt{(x-(-2))^2+(0-3)^2}$$

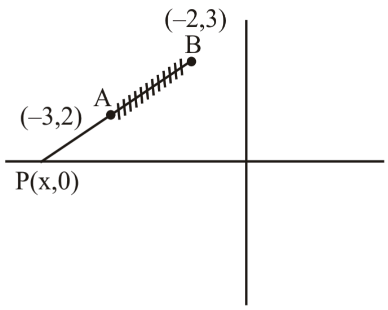


For $|PA - PB|_{max}$

$|PA - PB| \leqslant AB$

$|PA - PB|_{max} = AB$

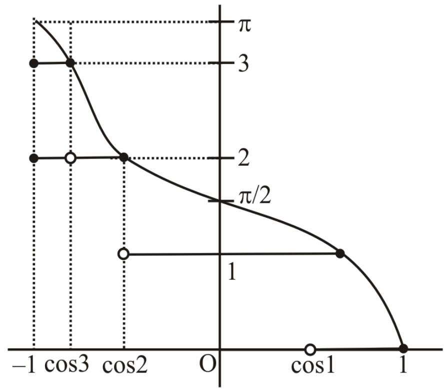
$\Rightarrow$ P, A, B are collinear



$AB = \sqrt{1+1} = \sqrt{2}$

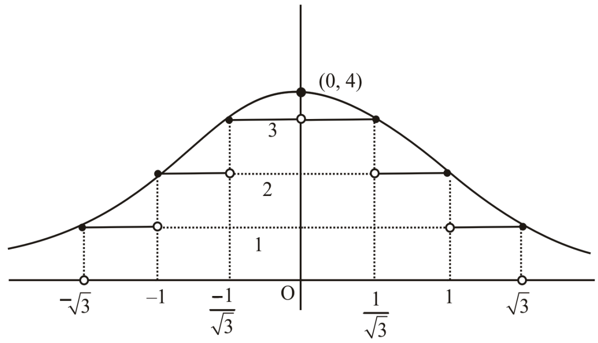
$S_{max} = \sqrt{2}$ $(S_{max})^4 = \left(\sqrt{2}\right)^4 = 4$

5. Ans (10)



D.C. at 3 points $\boxed{a = 3}$

$g(x) = \left[\dfrac{4}{1+x^2}\right]$ b = 7



a + b = 10