FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Main)-2025

TEST DATE: 16-11-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. B

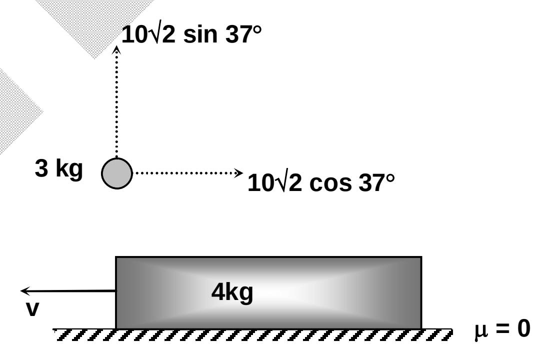
Sol. From conservation of momentum

$4v = 3(10\sqrt{2})\cos 37^\circ$

Minimum length required is

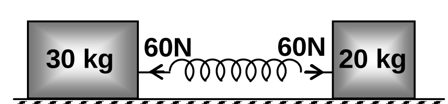
$\ell = (10\sqrt{2}\cos 37^\circ + v)\left(\frac{20\sqrt{2}\sin 37^\circ}{10}\right)$

$\ell = 33.6$ m



2. A

Sol. Initially spring force is $(200\ \text{N/m})\left(\frac{30\text{m}}{100}\right) = 60\text{N}$



Here only 20 kg block will move.
From work energy theorem, when spring attains its natural length

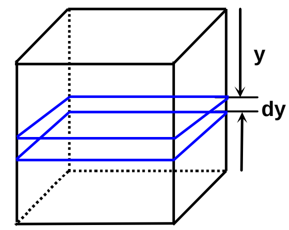
$W_{sp} + W_{fr} = \Delta k$

$+\frac{1}{2}kx^2 - \mu mgx = \Delta k$

Substituting value we get

$\Delta k = 0$

3. C



Sol. $I = \int_0^{\ell}\left(\frac{M}{\ell}dy\right)\left(\frac{\ell^2}{12}+y^2\right) = \frac{5M\ell^2}{12}$

4. C

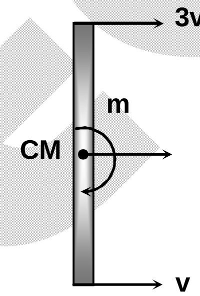
Sol. Speed of particle after 1st collision with wall is

$v_1 = (4+3)+3 = 10$ m/s towards left

Speed of particle after 2nd collision with wall is

$v_2 = (10+6) = 22$ m/s towards right

5. D



Sol. $\omega = \frac{3v - v}{2\ell} = \frac{v}{\ell}$

$v_{CM} + \frac{v}{\ell}\ell = 3v$

$v_{CM} = 2v$

$K.E. = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$

$= \frac{1}{2}m(2v)^2 + \frac{1}{2}m\frac{(2\ell)^2}{12}\left(\frac{v^2}{\ell^2}\right)$

$= 2mv^2 + \frac{mv^2}{6} = \frac{13}{6}mv^2$

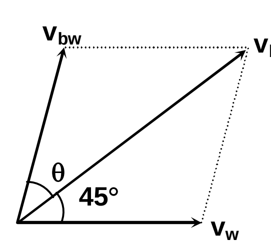
6. D

Sol. $x_{cm} = \frac{-\pi\left(\frac{R}{4}\right)^2\left(\frac{-3R}{4}\right)}{\pi R^2 - 2\pi\left(\frac{R}{4}\right)^2} = \frac{+\frac{3\pi R^3}{64}}{\frac{7\pi R^2}{8}} = \frac{3R}{56}$

$y_{cm} = \frac{-\pi\left(\frac{R}{4}\right)^2\left(\frac{3R}{4}\right)}{\pi R^2 - 2\pi\left(\frac{R}{4}\right)^2} = \frac{-3R}{56}$

$\vec{r} = \frac{3R}{56}(\hat{i} - \hat{j})$

7. C



Sol. $v_{bw}\sin\theta = v_w \sin 45°$

$15\sin\theta = \frac{5}{\sqrt{2}}$

$\Rightarrow \sin\theta = \frac{1}{3\sqrt{2}}$

Time $T = \dfrac{5\text{ km}}{v_w \cos 45° + v_{bw}\cos\theta}$

$$T = \frac{5}{\frac{5}{\sqrt{2}} + \frac{15\sqrt{7}}{3\sqrt{2}}} = \frac{\sqrt{2}}{1+\sqrt{7}}$$

$$T = \left(\frac{\sqrt{34}-\sqrt{2}}{16}\right)\text{hr}$$

8. A

Sol. Since net applied force on the block is zero, so frictional force is zero.

9. B

Sol. For block

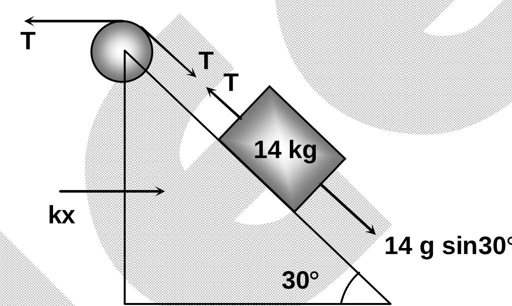
$T = 14g \sin 30°$

$T = 7g$

Taking wedge + block

$T = kx$

$\therefore x = 7$ cm



10. C

Sol. $x = \dfrac{1}{2}a(n-1)^2$ …(i)

$y = \dfrac{1}{2}an^2$ …(ii)

$x + y = \dfrac{1}{2}at^2$

$\Rightarrow \dfrac{a(n-1)^2}{2} + \dfrac{an^2}{2} = \dfrac{at^2}{2}$

$\Rightarrow t^2 = (n-1)^2 + n^2$

$\Rightarrow t^2 = n^2 + 1 - 2n + n^2$

$\Rightarrow t^2 = 2n^2 - 2n + 1$

$\therefore t = \sqrt{2n^2 - 2n + 1}$

11. B

Sol. $\vec{u}_1 = u\hat{i} - \sqrt{2gh}\hat{j}$

$\vec{v}_1 = -v\hat{i} - \sqrt{2gh}\hat{j}$

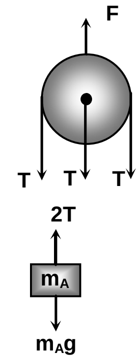
$\vec{u}_1 \perp \vec{v}_1 \Rightarrow \vec{u}_1 \cdot \vec{v}_1 = 0$

$\Rightarrow -uv + 2gh = 0$

$h = \dfrac{uv}{2g}$

12. C

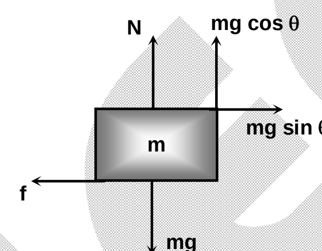
Sol. $F = 3T$
$2T = m_A g$
$T = 20\ N$



From (i) $F = 60\ N$
$\therefore\ 20\,t = 60$
$\Rightarrow t = 3$ sec

13. A

Sol. $mg\sin\theta \ge f$
$\Rightarrow mg\sin\theta \ge \mu N$
$\Rightarrow mg\sin\theta \ge \mu mg(1 - \cos\theta)$
$$\Rightarrow \frac{\sin\theta}{1-\cos\theta} \ge \mu$$
$$\Rightarrow \cot\left(\frac{\theta}{2}\right) \ge \mu$$



14. D

Sol. $$P = Fv = mav = m\left(v\frac{dv}{dx}\right)v$$
$$\Rightarrow \int_0^x P\,dx = \int_x^v mv^2 dv$$
$$\Rightarrow Px = \frac{m}{3}\left(v^3 - u^3\right)$$
$$\Rightarrow v^3 = u^3 + \frac{3Px}{m}$$
$$\Rightarrow v = \left(u^3 + \frac{3Px}{m}\right)^{1/3}$$

15. C

Sol. Applying work energy theorem
$$W_{gr} + W_N + W_{fr} = k_f - k_i$$
$$\Rightarrow (mg\sin\theta)x + 0 - \frac{\mu_0 mg\cos\theta x^2}{2} = 0$$
$$\therefore\ x = \frac{2\tan\theta}{\mu_0}$$

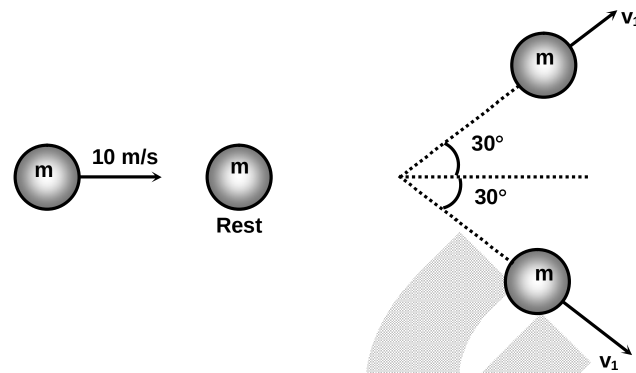
16. A

Sol. Area under power versus position graph is $\frac{mv^3}{3} - \frac{mu^3}{3}$
$$2 \times 8 \times 13 = \frac{24}{3}(v^3 - 1)$$
$\Rightarrow v = 3$ m/s

17. C

Sol. $0 = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ$

$\Rightarrow v_1 = v_2$



18. D

Sol. $0.4 = \frac{9.8}{2} t_1^2 \Rightarrow t_1 = \sqrt{\frac{0.8}{9.8}}$

$0.9 = \frac{9.8}{2} t_2^2 \Rightarrow t_2 = \sqrt{\frac{1.8}{9.8}}$

$\therefore$ required time

$t = t_2 - t_1 = 1/7$ sec

19. D

Sol. $g_{eff} = g + \frac{g}{10} = \frac{11}{10} g$

$T = \frac{2(3)(1.5)}{(4.5)} \left(\frac{11g}{10}\right) = \frac{11g}{5}$

$\therefore$ reading of spring balance is

$\frac{2T}{g} = 4.4$ kg

20. D

Sol. $a_B = \frac{F - kx_0}{M}$, $a_A = \frac{kx_0}{M}$

$a_{BA} = a_B - a_A$

$\therefore a_{BA} = \frac{F - 2kx_0}{M}$

SECTION – B

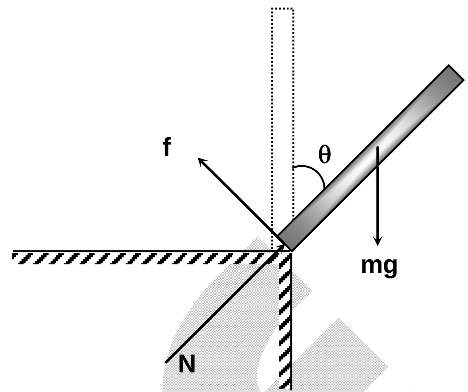
21. 5

Sol. When the block breaks of the surface

$\cos\theta = \frac{2}{3}$

$\therefore \tau = MgR \sin\theta$

$\Rightarrow \tau = \frac{\sqrt{5}}{3} MgR$



22. 4

Sol. From COME

$$\frac{1}{2}\frac{m\ell^2}{3}\omega^2 = mg\frac{\ell}{2}(1-\cos\theta) \quad \ldots(i)$$

From equation of motion

$$mg\cos\theta - N = \frac{m\omega^2\ell}{2} \quad \ldots(ii)$$

From (i) and (ii)

$$N = \frac{mg}{2}(5\cos\theta - 3)$$

N becomes zero when $\cos\theta = \frac{3}{5}$

$\therefore\ \sin\theta = \frac{4}{5}$

23. 1

Sol. $a_A = g,\ a_B = g$

$\therefore\ \frac{a_B}{a_A} = \frac{g}{g} = 1$

24. 7

Sol. x = –3

U = 107 J

$\therefore\ 5x^2 - 20x + 2 = 107$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow$ x = 7m, –3m

$\therefore$ x = 7 m is maximum x-coordinate

25. 60

Sol. From conservation of momentum

(1 kg) (9 m/s) = (1 + 3)v

$\therefore\ v = \frac{9}{4}$ m/s

From conservation of energy

$$mg\ell(1 - \cos\theta_{max}) = \frac{1}{2}(1\text{kg})(9)^2 - \frac{1}{2}(4\text{kg})\left(\frac{9}{4}\right)^2$$

$\therefore\ \theta_{max} = 60°$

Chemistry

PART – B

SECTION – A

26. B

Sol. $\left|\Delta_{eg}H^{o}\right|$ of Noble gases

$He = 48\ kJ\ mol^{-1}$

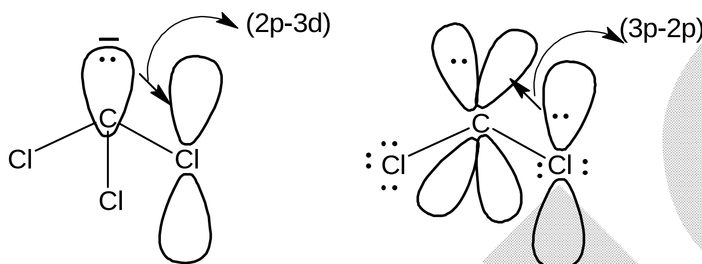
$Ne = 116\ kJ\ mol^{-1}$

$Xe = 77\ kJ\ mol^{-1}$

$Rn = 68\ kJ\ mol^{-1}$

27. C

Sol.

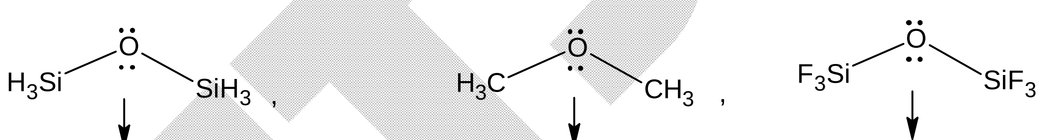


$\overset{\cdot\cdot}{C}Cl_3^-$ ⟶ Electron rich, i.e. Nucleophile.

$:CCl_2$ ⟶ Electron deficient, i.e. Electrophile.

28. A

Sol.



Backbonding decreases basicity. | Lone pair freely available | Strong electron withdrawing group so decreases basicity

29. C

Sol. CN^- and N_2 are isoelectronic species and bond order = 3.

CN and N_2^+ are isoelectronic species and bond order = 2.5.

$\therefore$ Stability of CN^- > CN and curve shows more decrease in PE.

30. C

Sol. Due to very small size and high inter electronic repulsions the formation of $O^{--}(g)$ from $O^-(g)$ becomes endothermic in nature.

Most of the compounds of oxygen are formed by $O^{--}(g)$ and it can be explained on the basis of high lattice enthalpy.

31. B

Sol. $FeC_2O_4 \longrightarrow Fe^{++}(aq) + \begin{array}{l} COO^- \\ | \\ COO^- \end{array}(aq)$

$$\left.\begin{array}{l} Fe^{++}(aq) \longrightarrow Fe^{+++}(aq) + e^- \\ \begin{array}{l} COO^- \\ | \quad (aq) \\ COO^- \end{array} \longrightarrow 2CO_2(g) + 2e^- \end{array}\right\}$$

$$Fe^{++}(aq) + \begin{array}{l} COO^- \\ | \quad (aq) \\ COO^- \end{array} \longrightarrow Fe^{+++}(aq) + 2CO_2(g) + 3e^-$$

In acidic medium

$$MnO_4^-(aq) + 8H^+(aq) + 5e^- \longrightarrow Mn^{++}(aq) + 4H_2O(\ell)$$

$$\therefore 5FeC_2O_4 + 3KMnO_4 + 12H_2SO_4 \longrightarrow \frac{5}{2}Fe_2(SO_4)_3 + 3MnSO_4 + \frac{3}{2}K_2SO_4 + 12H_2O + 10CO_2$$

OR

$$5FeC_2O_4(s) + 3MnO_4^-(aq) + 24H^+(aq) \longrightarrow 5Fe^{+++}(aq) + 3Mn^{++}(aq) + 10CO_2(g) + 12H_2O(\ell)$$

Since 5 mols of FeC_2O_4 requires 3 mols of $KMnO_4$.

So, 3 mols of FeC_2O_4 requires $3 \times \frac{3}{5}$ mols of $KMnO_4$ $\equiv 1.8$ mols.

32. A

Sol.

$$nX(g) \rightleftharpoons X_n(g)$$

Ini.	1	–
Equi.	$\frac{1-x}{V}$	$\frac{x}{n}\cdot\frac{1}{V}$

$$K_C = \frac{x}{n \times V} / \left(\frac{1-x}{V}\right)^n$$

$$K_C = \frac{xV^{n-1}}{(1-x)^n . n}$$

Since $x << 1 \Rightarrow (1-x)^n \cong 1$

$$K_C = \frac{xV^{n-1}}{n} \qquad \dots (1)$$

Total mols at equilibrium (n_{eq}) = $1 - x + \frac{x}{n} = 1 + \left(\frac{1-n}{n}\right)x$

$$n_{eq} = 1 - \left(\frac{n-1}{n}\right)x \qquad \dots (2)$$

Now, $PV = n_{eq.}RT$

$$\Rightarrow \frac{PV}{RT} = n_{eq}$$

Using (1) and (2)

$$\frac{PV}{RT} = 1 - \left(\frac{n-1}{n}\right)\left(\frac{nK_C}{V^{n-1}}\right)$$

$$\frac{PV}{RT} = 1 - \frac{(n-1)K_C}{V^{n-1}}$$

33. D

Sol. Since $E = \phi + KE_{max}$

$$4.25 = \phi_x + T_x, \lambda_x = \frac{h}{\sqrt{2m_e T_x}}$$

$$4.20 = \phi_y + T_y, \lambda_y = \frac{h}{\sqrt{2m_e T_y}}$$

Since, $T_x - T_y = 1.50$

$\lambda_y = 2\lambda_x$

$\Rightarrow \phi_x = 2.25\text{ eV}, T_x = 2.00\text{ eV}$

$\phi_y = 3.70\text{ eV}, \quad T_y = 0.50\text{ eV}$

$$\lambda_y = \frac{h}{\sqrt{2m_e T_y}}$$

$$\lambda_y = \frac{6.6\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times0.5\times1.6\times10^{-19}}}$$

$$\lambda_y = \frac{6.6}{3\times\sqrt{1.6}}\times10^{-9}$$

$$\lambda_y = \frac{6.6}{3\times1.26}\times10^{-9}$$

$\lambda_y = 1.746\times10^{-9}\text{ m} = 1746\text{ pm}$

34. B

Sol. Let order of the reactin is n.

$$\therefore -\frac{1}{3}\frac{d[X]}{dt} = K[X]^n$$

$$\Rightarrow -[X]^{-n}d[X] = 3kdt$$

$$\Rightarrow -\int[X]^{-n}d[X] = 3k\int dt \qquad \Rightarrow \frac{1}{(n-1)[X]^{n-1}} = 3kt + C$$

At $t = 0, [X] = [X_o] \Rightarrow C = \dfrac{1}{(n-1)[X_o]^{n-1}}$

$$\Rightarrow \frac{1}{(n-1)[X]^{n-1}} = 3kt + \frac{1}{(n-1)[X_o]^{n-1}}$$

$$\Rightarrow [X]^{1-n} = 3k(n-1)t + [X_o]^{1-n}$$

Comparing with graph

$1 - n = -3 \Rightarrow n = 4$

$\therefore 3k(n-1) = \tan 45^o \Rightarrow k = \dfrac{1}{9}$

Rate of the reaction $= -\dfrac{1}{3}\dfrac{d[X]}{dt} = k[X]^4$

Rate $= \dfrac{1}{9}\times(0.2)^4 = \dfrac{16\times10^{-4}}{9}\text{ M}\min^{-1}$

Rate $= \dfrac{16}{9}\times10^{-4}\text{ M min}^{-1}$

35. C

Sol.

	CH_3COOH	+ $NaOH$	$\rightleftharpoons$	CH_3COONa + H_2O
Ini.	$600 \times 2 = 1200$ milimoles	x milimoles		– –
Final	$(1200 - x)$ milimoles	–		x milimoles x milimoles

For acidic buffer

$$pH = pK_a + \log\frac{[Salt]}{[Acid]}$$

and for maximum buffer capacity [Salt] = [Acid]

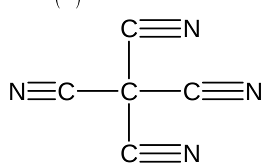
$\Rightarrow x = 1200 - x$

$\Rightarrow x = 600$ milimoles

$$\therefore W_{NaOH} = \frac{600}{1000} \times 40 = 24\,g$$

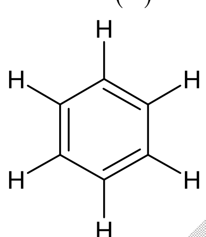
36. A

Sol. $\underset{(X)}{O=C=O} \Rightarrow \frac{\sigma}{\pi} = \frac{2}{2} = 1$



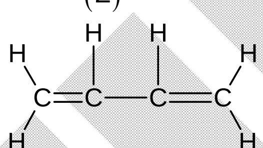
$\Rightarrow \frac{\sigma}{\pi} = \frac{8}{8} = 1$

(Y)



$\Rightarrow \frac{\sigma}{\pi} = \frac{12}{3} = 4$

(Z)



$\Rightarrow \frac{\sigma}{\pi} = \frac{9}{2} = 4.5$

(W)

$\therefore (X) = (Y) < (Z) < (W)$

37. C

Sol. $N_2 \equiv \sigma(1s)^2\,\sigma^*(1s)^2\,\sigma(2s)^2\,\sigma^*(2s)^2\,\pi(2p_x)^2 = \pi(2p_y)^2\,\sigma(2p_z)^2$

For N_2^+ = Bond order $= \frac{1}{2}(9-4) = 2.5$ and paramagnetic.

For N_2^- = Bond order $= \frac{1}{2}(10-5) = 2.5$ and paramagnetic.

So, $BO(N_2^+) = BO(N_2^-)$

But stability of N_2^+ > Stability of N_2^-

(Due to more number of electron(s) in ABMO in case of N_2^-)

38. C

Sol. $X \rightleftharpoons Y \Rightarrow K_1 = \frac{[Y]}{[X]} \Rightarrow [Y] = K_1[X]$

$X \rightleftharpoons Z \Rightarrow K_2 = \frac{[Z]}{[X]} \Rightarrow [Z] = K_2[X]$

Now, mole fraction of 'X' at equilibrium $= \frac{[X]_{eq}}{[X]_{eq} + [Y]_{eq} + [Z]_{eq}}$

$= \frac{[X]_{eq}}{[X]_{eq} + K_1[X]_{eq} + K_2[X]_{eq}} = \frac{1}{1 + K_1 + K_2}$

39. B

Sol.

	$2XY(g)$	$\rightleftharpoons$	$2X(g)$	$+\ Y_2(g)$
Ini.	P_o		–	–
Equi.	$P_o - 2x$		$2x$	x

$P_o - 2x + 2x + x = P$

$P_o + \frac{P}{9} = P \Rightarrow P_o = \frac{8P}{9}$ as $x = \frac{P}{9}$

$\therefore K_P = \frac{[X(g)]^2 [Y_2(g)]^1}{[XY(g)]^2}$

$K_P = \frac{(2x)^2 \times x}{(P_o - 2x)^2} = \frac{\left(\frac{2P}{9}\right)^2 \times \left(\frac{P}{9}\right)}{\left(\frac{8P}{9} - \frac{2P}{9}\right)^2}$

$K_P = \frac{4P^2 \times P}{9^2 \times 9} / \frac{36P^2}{9^2} = \frac{P}{81}$

$\Rightarrow \frac{K_P}{P} = \frac{1}{81}$

40. C

Sol. $T_n \propto \frac{n^3}{Z^2}$

$\therefore T_{Li^{++}} \propto \frac{3^3}{3^2}$ $\quad T_{He^+} \propto \frac{2^3}{2^2}$

$\frac{T_{Li^{++}}}{T_{He^+}} = \frac{3}{2} \Rightarrow \frac{t}{T_{He^+}} = \frac{3}{2}$

$T_{He^+} = \frac{2}{3} t\ s = 0.66t\ s$

41. C

Sol. $B = 801\ kJmol^{-1}$

$Al = 577\ kJmol^{-1}$

$In = 558\ kJmol^{-1}$

$Ga = 579\ kJmol^{-1}$
$T\ell = 589\ kJmol^{-1}$

42. A
Sol. $x^o = 101^o$
$y^o = 79^o$
$z^o = 118^o$

43. D
Sol. $NO_3^-(aq) \longrightarrow NO(g)$
$+5 \qquad\qquad +2$
$As_2S_3(s) \longrightarrow AsO_4^{3-}(aq) + SO_4^{--}(aq)$
$(+3)(-2) \qquad\qquad (+5) \qquad (+6)$
Net increase in oxidation state = 2 × 2 + 3 × 8 = 28
Net decrease in oxidation state = 3
Net increase = Net decrease
$28NO_3^-(aq) + 3As_2S_3(s) + 4H_2O(\ell) \longrightarrow 6AsO_4^{3-}(aq) + 28NO(g) + 9SO_4^{--}(aq) + 8H^+(aq)$
$\therefore x = 28,\ y = 3,\ z = 4$
$\Rightarrow x + y + z = 28 + 3 + 4 = 35$

44. A
Sol. $Ag_4[Fe(CN)_6](s) \rightleftharpoons 4Ag^+(aq) + [Fe(CN)_6]^{4-}(aq)$
$- \qquad\qquad 4s \qquad\qquad s$
$K_{SP} = (4s)^4(s)^1$
$K_{SP} = 256s^5$
$s = \sqrt[5]{\dfrac{K_{SP}}{256}}$ = Molarity of $[Fe(CN)_6]^{4-}$ in saturated solution.

45. A
Sol. $k = Ae^{-E_a/RT}$
$$\Rightarrow \ell nk = \ell nA - \frac{E_a}{RT}$$
$$\frac{d}{dT}(\ell nk) = \frac{d}{dT}\ell n(A) - \frac{d}{dT}\left(\frac{E_a}{RT}\right)$$
$$\Rightarrow \frac{d(\ell nk)}{dT} = \frac{E_a}{RT^2} \qquad \ldots (1)$$
Now, $\ell nk = x + y\ell nT - \dfrac{z}{T}$
$$\Rightarrow \frac{d(\ell nk)}{dT} = \frac{y}{T} + \frac{z}{T^2}$$
$$\Rightarrow \frac{d(\ell nk)}{dT} = \frac{yT + z}{T^2} \qquad \ldots (2)$$
From (1) and (2)
$$\frac{E_a}{RT^2} = \frac{yT + z}{T^2} \quad \Rightarrow E_a = yRT + zR$$

SECTION – B

46. 99

Sol. $\sqrt{\ell(\ell+1)}\frac{h}{2\pi} = 2\sqrt{5}\frac{h}{2\pi}$

$\sqrt{\ell(\ell+1)} = 2\sqrt{5}$

$\ell(\ell+1) = 20$

$\ell = 4$

x = Number of orbitals = $2\ell + 1 = 9$

$\therefore 11x = 11 \times 9 = 99$

47. 120

Sol.

$$\begin{array}{lcccc} & X(\ell) & \longrightarrow & Y(\ell) & + & Z(g) \\ t=0 & [R_o] & & - & & - \\ t=\infty & - & & [R_o] & & [R_o] \end{array}$$

$[R_o] \propto 80$ … (1)

$$\begin{array}{lcccc} & X(\ell) & \longrightarrow & Y(\ell) & + & Z(g) \\ t=0 & [R_o] & & - & & - \\ t=40 & [R_o - x] & & x & & x \end{array}$$

$x \propto 40$

$\Rightarrow [R_o - x] \propto 40$ … (2)

$$\begin{array}{lcccc} & X(\ell) & \longrightarrow & Y(\ell) & + & Z(g) \\ t=0 & [R_o] & & - & & - \\ t=40 & [R_o - y] & & y & & y \end{array}$$

$y \propto 70$

$\Rightarrow [R_o - y] \propto 10$ … (3)

Now, from Eq. (2) $= \frac{1}{40}\ell n\frac{[R_o]}{[R_o - x]}$

$K = \frac{1}{40}\ell n\frac{80}{40} = \frac{1}{40}\ell n2$

From, Eq. (3) $= \frac{1}{40}\ell n2 = \frac{1}{t}\ell n\frac{[R_o]}{[R_o - y]}$

$\frac{1}{40}\ell n2 = \frac{1}{t}\ell n\frac{80}{10} = \frac{3}{t}\ell n2$

t = 120 min.

48. 4

Sol. Since $\Delta G = \Delta G^o + RT\ell nQ_C$

$\Rightarrow \Delta G = -RT\ell nK_{eq} + RT\ell nQ_C$

$\Rightarrow \Delta G = RT\ell n\frac{Q_C}{K_{eq}}$

$\Rightarrow \Delta G = RT\ell n\frac{[Z]}{[X][Y]} \times \frac{K_b}{K_f}$

$\Rightarrow \Delta G = RT\ell n\frac{r_b}{r_f} = RT\ell n\frac{1}{e^4} = -4RT$

$\Rightarrow x = |\Delta G| = 4RT = 4\times2\times300 = 2400$ cal.

$\therefore \frac{x}{600} = \frac{2400}{600} = 4$

49. 8

Sol. $[H^+]_{HX} = [H^+]_{HY}$

$\Rightarrow \sqrt{C\times1.8\times10^{-5}} = \sqrt{0.6\times2.4\times10^{-4}}$

$\Rightarrow C = \frac{0.6\times2.4\times10^{-4}}{1.8\times10^{-5}} = 0.8\times10 = 8$

C = 8 M

50. 2

Sol. $\underset{(+4)}{MO_2} + \underset{(+4)}{MO_2} \longrightarrow \underset{(+7)}{MO_4^-} + \underset{(x)}{M^{x+}}$

Net increase = 3, Net decrease = (4 – x)

Net increase = Net decrease

$\underset{\text{(Oxidation part)}}{(4-x)MO_2} + \underset{\text{(Reduction part)}}{3MO_2} \longrightarrow (4-x)MO_4^- + 3x^+$

Now, $\frac{4-x}{3} = \frac{2}{3}$

$x = 2$

Mathematics

PART – C

SECTION – A

51. B

Sol. Total function = $5^4 = 625$

Total one-one function = ${}^5C_4 \times 4!$

$= 5 \times 24 = 120$

Total many-one function = $625 - 120 = 505$

52. A

Sol. From Venn diagram

$n(C - B) = 34$, $n(C - A) = 35$

$n(A \cap B \cap C) = 1$

$\Rightarrow n(C - B) + n(C - A) + n(A \cap B \cap C) = 70$

53. D

Sol. It is given that the subset must have exactly one element from A, B and C. Say {1, 4, 7} is one such subset. If cannot include any number from {2, 3, 5, 6, 8, 9} only number left is 10.

So, there are two possible combinations {1, 4, 7} and {1, 4, 7, 10}.

Similarly we can have $3 \times 3 \times 3 = 27$

Such combination from A, B and C. For each there are two possibilities.

Hence total number of subsets = $27 \times 2 = 54$

54. C

Sol. R is not reflexive as $(2, 2) \notin R$, $(3, 3) \notin R$. Also R is not transitive as $(2, 1) \in R$ and $(1, 2) \in R$ but $(2, 2) \notin R$. However R is symmetric.

55. D

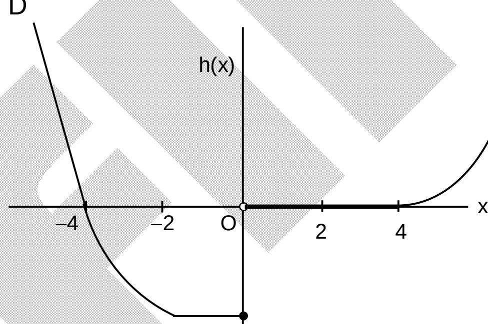
Sol. From analysis of the graph of f(x), we observe that f(x) = 6 has 2 real and distinct solutions.

56. D

Sol. Applying $e^{\lim_{x \to a}[f(x)-1]g(x)}$

57. D

Sol.



From the graph of h(x) we can observe.

58. B

Sol. Here, g(x) is an odd function, so, $g(\alpha) + g(-\alpha) = 0$

Also, g(x) is monotonically increasing function so, if $\alpha + \beta > 0$

$\Rightarrow \alpha > -\beta$

$\Rightarrow g(\alpha) > g(-\beta)$

$\Rightarrow g(\alpha) + g(\beta) > 0$

59. C

Sol. The given differential can be written as

$$\frac{2y}{(1-y^2)^2}\frac{dy}{dx}+\frac{y^2}{1-y^2}\frac{1}{x}=\frac{1}{x^3}$$

Put z = $\frac{y^2}{1-y^2} \Rightarrow \frac{2y}{(1-y^2)^2}\frac{dy}{dx}=\frac{dz}{dx}$

$\Rightarrow \quad \frac{dz}{dx}+\frac{z}{x}=\frac{1}{x^3}$ which is linear in z, after solving, we get

$x^2y^2 = (cx - 1)(1 - y^2)$

60. D

Sol. $E_n=\frac{1}{n^2+n+2}+\frac{2}{n^2+n+4}+.....\frac{n}{n^2+n+2n}$

Now, $\frac{1}{n^2+3n}+\frac{2}{n^2+3n}+.......+\frac{n}{n^2+3n} < E_n < \frac{1}{n^2+n+2}(1+2+.....+n)$

$\Rightarrow \frac{1}{2}\le \lim\limits_{n\to\infty} E_n \le \frac{1}{2}$

61. B

Sol. Let $(3, \alpha)$ be the point on $y = h(x)$ $\Rightarrow (\alpha, 3)$ lies on $y = f(x) \Rightarrow \alpha = 1$

Also, $x = f(h(x))$

$\Rightarrow h'(3) = \frac{1}{f'(h(3))}=\frac{1}{f'(1)}=\frac{1}{4}$

slope of normal at x = 3 is –4

Equation of normal

$y - 1 = -4(x - 3)$

$\Rightarrow \quad y - 1 + 4x - 12 = 0$

$4x + y - 13 = 0$

62. B

Sol. Assume

$$E=\lim_{x\to 0}\frac{1}{x}g\left(\frac{x}{n}\right)$$

$$=\lim_{x\to 0}\frac{g(x/n)-g(0)}{x-0}$$

Let $\frac{x}{n}=\alpha$

$$E=\lim_{\alpha\to 0}\frac{g(\alpha)-g(0)}{\alpha n}$$

$$E=\lim_{\alpha\to 0}\frac{g(\alpha)-g(0)}{\alpha n}=\frac{1}{n}g'(0)=\frac{1}{n}$$

Hence, $\lim\limits_{x\to 0}\frac{1}{x}\sum\limits_{n=1}^{\infty}(-1)^n g\left(\frac{x}{n}\right)=\sum\limits_{n=1}^{\infty}(-1)^n\frac{1}{n}$

$$=-\sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{1}{n}=-\ln 2$$

63. C

Sol. $I = \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}$

$= 2\tan^{-1}\sqrt{\frac{x^2+x+1}{x}}+c$

$= 2\left(\frac{\pi}{2}-\cot^{-1}\sqrt{\frac{x^2+x+1}{x}}\right)+c$

$= -2\cot^{-1}\sqrt{\frac{x^2+x+1}{x}}+c$

$\Rightarrow |\alpha| = 2$

64. B

Sol. $\int_0^1\left(\int_z^1 e^{x^2}dx\right)dz = \int_0^1 f(z)\cdot 1\,dz$

Integrating by parts, we get

$\left(f(z)\int 1dz - \int f'(z)z\,dz\right)_0^1$

$= \left(zf(z)+\int ze^{z^2}dz\right)_0^1 = \frac{e-1}{2}$

65. A

Sol. Assume $F(\beta) = \int_0^\infty \frac{\tan^{-1}\beta x - \tan^{-1}x}{x}dx$

$\frac{dF}{d\beta} = \int_0^\infty \frac{1\cdot x}{(1+\beta^2x^2)x}dx = \int_0^\infty \frac{dx}{1+\beta^2x^2}$

$= \frac{1}{\beta^2}\int_0^\infty \frac{dx}{x^2+\frac{1}{\beta^2}} = \frac{1}{\beta^2}\left(\beta\tan^{-1}\beta x\right)_0^\infty = \frac{\pi}{2\beta}$

Now $dF = \frac{\pi}{2\beta}d\beta$

$\int dF = \frac{\pi}{2}\int\frac{d\beta}{\beta}$

$F(\beta) = \frac{\pi}{2}\ln\beta + C$

But $F(1) = 0 \Rightarrow C = 0$

Hence $F(\beta) = \frac{\pi}{2}\ln\beta$

66. C

Sol. We can arrange this $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n}\right)\frac{\left(\left(\frac{r}{n}\right)^2-\frac{\alpha}{n^3}\right)^{1/3}}{\left(\frac{r}{n}\right)}$

$$= \int_0^1 \frac{x^{2/3}}{x} dx = \int_0^1 x^{-1/3} dx = \frac{3}{2}$$

67. C

Sol. $\int_0^1 \left(2y^3 - f(y)\right) f(y)\, dy = \int_0^1 y^6 dy$

$\Rightarrow \quad \int_0^1 \left(f(y) - y^3\right)^2 dy = 0$

$\Rightarrow \quad f(y) = y^3 \Rightarrow f(x) = x^3$

Hence area $= \int_2^3 f(x)\, dx = \int_2^3 x^3 dx = \frac{65}{4}$ sq. unit

68. B

Sol. Area $= \int_2^6 g^{-1}(x)\, dx$

$= \int_2^6 t\, g'(t)\, dt$

$= tg(t)\Big|_2^6 - \int_2^6 g(t)\, dt$

$= 36 - 4 - 12 = 20$

69. A

Sol. To fix up the focus of hyperbola we need two effective parameters. Thus order of corresponding differential equation will be 2.

70. D

Sol. Consider a function $h(x) = x\, g(x)$, here, $h(x)$ is continuous in [0, 1] and differentiable in (0, 1) as $g(1) = 0 \Rightarrow h(0) = 0 = h(1)$

Hence, Rolle's theorem is applicable for $h(x)$

hence, for some $\alpha \in (0, 1)$

$h'(\alpha) = 0 \Rightarrow \alpha g'(\alpha) + g(\alpha) = 0$

SECTION – B

71. 4

Sol. Assume $I = \int_0^{4\pi} \ln\left|\left(13\sin y + 3\sqrt{3}\cos y\right)\right| dy$

$= \int_0^{4\pi} \ln\left|14\sin(y+\beta)\right| dy \quad$ where $\cos\beta = \frac{13}{14}$

$= 4\left(\int_0^{\pi} \ln\left|\sin(y+\beta)\right| dy + \pi \ln 14\right)$

Assume $y + \beta = t$

$I = 4\int_{\beta}^{\pi+\beta} \ln \sin t\, dt + 4\pi \ln 14$

$= 4\pi \ln 14 + 4 \times 2 \times \left(-\frac{\pi}{2}\ln 2\right) = 4\pi \ln 7 \therefore \alpha = 4$

72. 5

Sol. Since differentiable at x = 0, hence continuous at x = 0

$\Rightarrow f(0^+) = f(0) \Rightarrow a = 1$

also $f'(0^-) = f'(0^+) \Rightarrow \dfrac{b}{2\sqrt{1-\dfrac{c^2}{4}}} = \dfrac{1}{8}$

$\Rightarrow 64b^2 + c^2 = 4 \Rightarrow 64b^2 + c^2 + a^2 = 5$

73. 20

Sol. From given information $\alpha + \beta + 1 = 0$

Also, $\dfrac{dy}{dx} = -\dfrac{(y+\alpha)}{x+\beta}$

Now, $\left.\dfrac{dy}{dx}\right|_{1,1} = \dfrac{-(1+\alpha)}{(1+\beta)} = 2$

$\Rightarrow \alpha + 2\beta + 3 = 0$

Solving for α and β, get $\beta = -2$, $\alpha = 1$

$\Rightarrow \dfrac{40(\alpha+\beta)}{\alpha\beta} = 40 \times \dfrac{1}{2} = 20$

74. 9

Sol. Equation of tangent at P(2, 3) is x = 2y – 4

Required area = $\int_0^3 \left[(y-2)^2 + 1 - (2y-4)\right] dy = 9$ sq. unit.

75. 1000

Sol. Using the series expansion of "sinx" in $\lim\limits_{x\to 0} \dfrac{\sin^2 x - x^2}{x^2 \sin^2 x} = -\dfrac{1}{3} = L$

$\Rightarrow |3000L| = 1000$