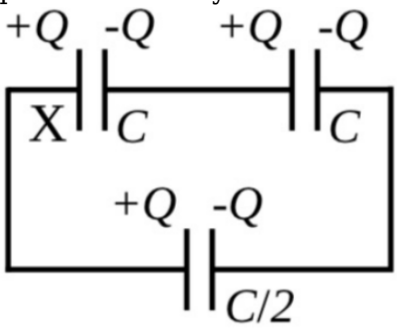
PART-1 : PHYSICS

SECTION-I (i)

1) An electric circuit consists of three charged capacitors. The capacitances of the capacitors and the values of their charges are given in the diagram. At some moment, the plates of capacitor X are released, allowing them to move freely toward each other. As a result of inelastic collisions, the plates eventually touch. What amount of heat is released in the system as a result?



(A) $\frac{Q^2}{2C}$

(B) $\frac{2}{3}\frac{Q^2}{C}$

(C) $\frac{Q^2}{C}$

(D) $\frac{2}{5}\frac{Q^2}{C}$

2) Match the curve in figure-1 to the curve in figure-2 having the same quality factor in LCR series circuit with AC voltage. Here ϕ is phase difference between current and voltage.

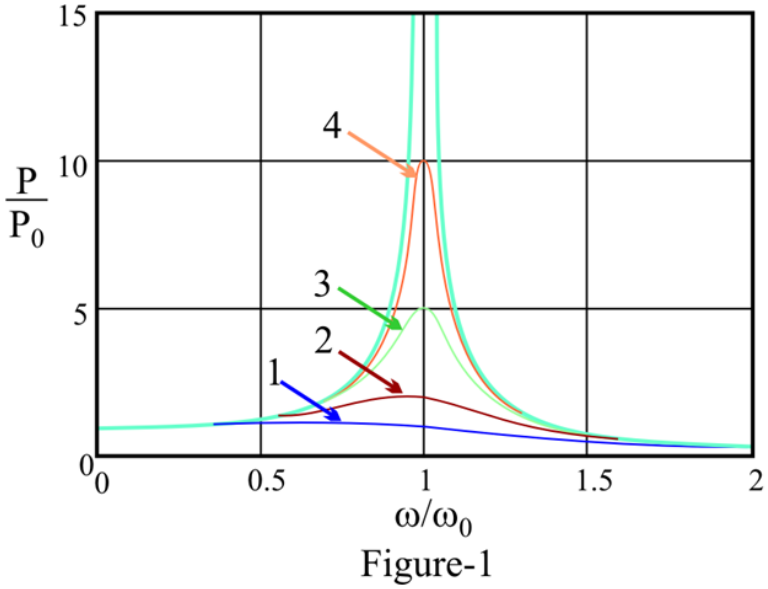


Figure-1

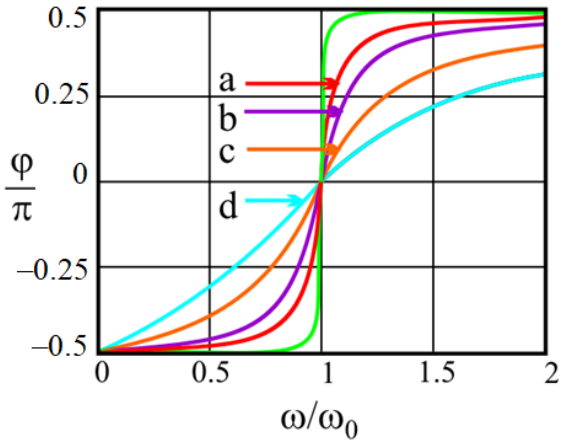


Figure-2

(A) 1 → a, 2 → b, 3 → c, 4 → d

(B) 1 → d, 2 → c, 3 → b, 4 → a

(C) 1 → a, 2 → b, 3 → d, 4 → c

(D) 1 → d, 2 → c, 3 → c, 4 → d

3) A circular table has radius R and N > 2 equally spaced legs of length h attached to its perimeter. Suppose the table has a uniform mass density with total mass m. Assuming the table does not slip, the minimum horizontal force needed to tip over the table is :-

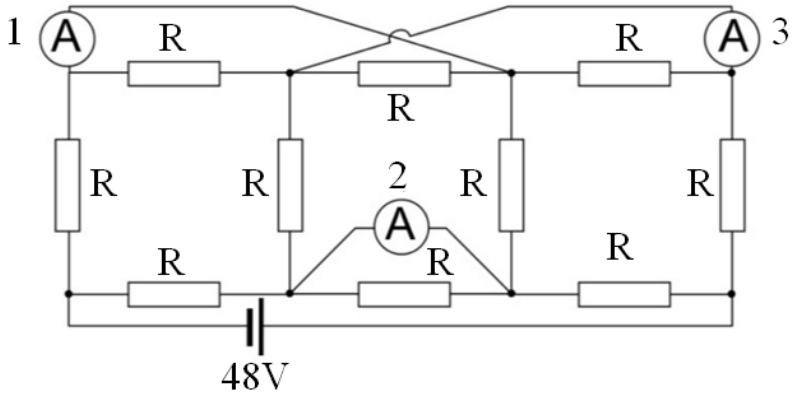
(A) $\frac{mgR}{h}$

(B) $\frac{mgR}{h}\cos\left(\frac{\pi}{N}\right)$

(C) $\frac{mgR}{h}\tan\left(\frac{N-2}{2N}\pi\right)$

(D) $\frac{mgR}{h}\sin\left(\frac{\pi}{2N}\right)$

4) The schematic provided has all resistors with a resistance of three ohms, and the battery voltage is 48 volts. All ammeters are ideal.
Statement-1 : Readings of all ammeters is same.
Statement-2 : Reading of ammeter 1 is 5A.
Statement-3 : Reading of ammeter 2 is 7A.
Statement-4 : Reading of ammeter 3 is 7A.



(A) All statements are correct.

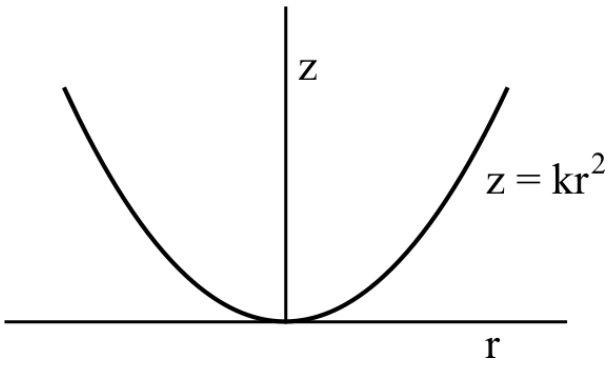
(B) Only statement-1 is correct.

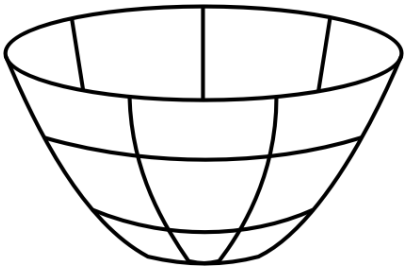
(C) Statement-1 & statement-4 are correct.

(D) Statement-2 & statement-3 are correct.

SECTION-I (ii)

1) A particle is constrained to move on the inner surface of a frictionless parabolic bowl whose cross-section has equation $z = kr^2$. The particle begins at a height z_0 above the bottom of the bowl with a horizontal velocity v_0 along the surface of the bowl. The acceleration due to gravity is g.





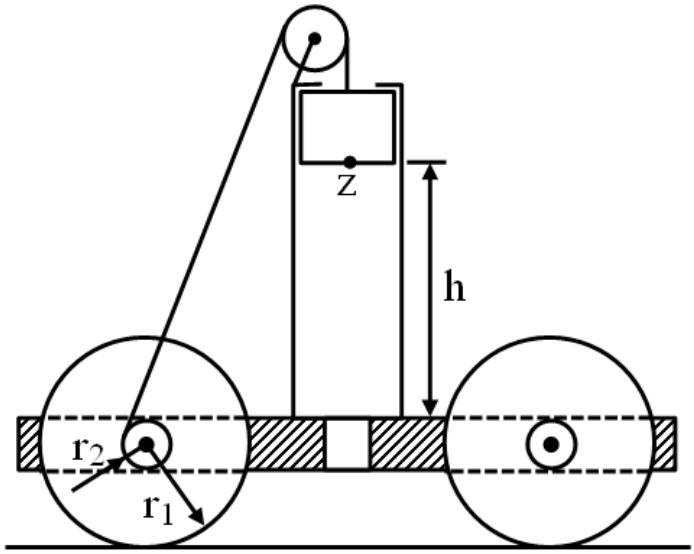
(A) The value of v_0 for which the particle moves in a horizontal circle is $\sqrt{gz_0}$.

(B) For $v_0 > \sqrt{2gz_0}$, the maximum height reached by the particle is $\frac{v_0^2}{2g}$

(C) If $v_0 = 0$ and z_0 is very small then the time in which the particle returns to its initial point is $4\sqrt{\frac{2z_0}{g}}$.

(D) If $v_0 = 0$ and z_0 is very small, then the time in which the particle returns to its initial point is $\frac{\pi\sqrt{2}}{\sqrt{kg}}$.

2) A cart for studying uniformly accelerated motion is driven by a weight, as shown in Fig. A descending weight of mass m rotates a rear axle via a rope passing over a pulley, with the rope sufficiently wound around the axle. Initially, the weight is at a height h above the cart. When released, the cart starts moving. Each of the four wheels of the cart has a mass m_1 and a radius r_1. The rear and front axles have a mass m_2 and a radius r_2. The wheels and axles are made as solid homogeneous cylinders. The motion of the weight is guided by a smooth vertical tube. The mass of the cart without the weight, axles, and wheels is negligible. The mass of the pulley, friction of the weight inside the tube, and rolling resistance of the wheels are considered negligible.



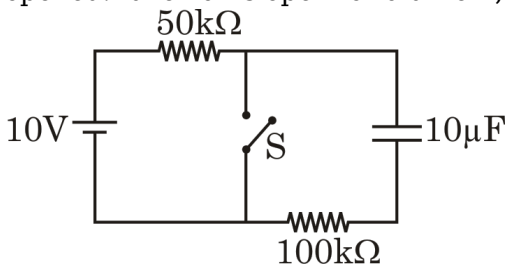
(A) The acceleration of the cart is $\frac{mgr_2}{r_1\left[m\left(1+\frac{r_2^2}{r_1^2}\right)+6m_1+m_2\left(2+\frac{r_2^2}{r_1^2}\right)\right]}$

(B) The speed of the cart till the weight falls on the cart is $\sqrt{\frac{2mgh}{m\left(1+\frac{r_2^2}{r_1^2}\right)+6m_1+m_2\left(2+\frac{r_2^2}{r_1^2}\right)}}$

(C) The tension in the string while the weight is falling is $m\left(g-a\frac{r_2}{r_1}\right)$, where a is the acceleration of the cart.

(D) The tension in the string while the weight is falling is $m\left(g-a\frac{r_2^2}{r_1^2}\right)$, where a is the acceleration of the cart.

3) In the circuit shown, the switch S has been closed for a long time. At time t = 0, the switch is opened. It remains open for a time T, after which it is closed again.



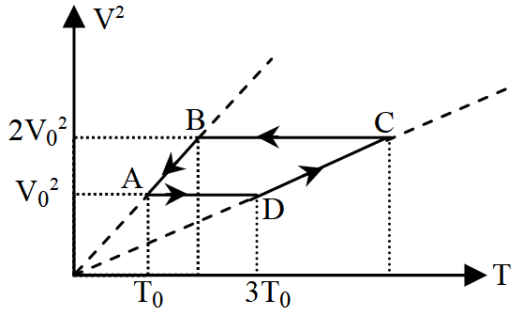
(A) Voltage drop across 100 kΩ resister is $10e^{-t/1.5}$ V for t < T.

(B) Voltage drop across 100 kΩ resister is $\frac{20}{3}e^{-t/1.5}$ V for t < T.

(C) Voltage drop across 100 kΩ resister is $10(e^{T-t} - e^{(T-3t)/3})$V for t > T.

(D) The time constant of the circuit is 1.5 sec for t > T.

4) One mole of ideal monoatomic gas follows cyclic process as shown in the figure. Choose the correct option(s) :



(A) $\frac{T_B+T_C}{2} = 4T_0$

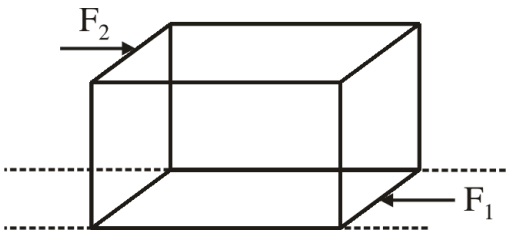
(B) Molar heat capacity for process B → A is 2R

(C) Molar heat capacity for process D → C is 4R

(D) Work done by gas during D to C is $\frac{3RT_0}{2}$

5) A cubical container has 2 fixed vertical walls and two vertical walls can freely rotate about one of the horizontal edges. The right wall can rotate about upper edge and the left wall can rotate about the bottom edge. Water is filled upto the top and forces F_1 and F_2 (perpendicular to the walls) are applied to keep the walls at from opening as shown in figure. Given that the cube is at rest on a

rough horizontal surface. (Ignore atmospheric pressure)



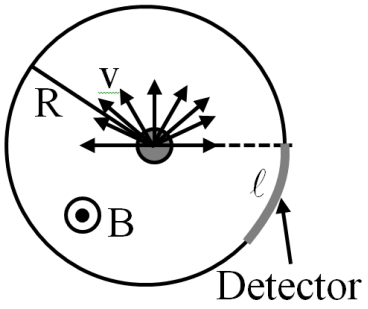
(A) Friction on the box from the surfaces is to the left.

(B) Friction on the box from the surfaces is to the right.

(C) Normal reaction is shifted towards left wall.

(D) Normal reaction passes through the center.

6) At the center of a tube with radius R, a source emits charged particles (see diagram). Each particle has a charge q > 0 and mass m. All particles are emitted from the source in the plane of the drawing with the same speed v, evenly distributed in the upper half of the tube. A detector of length ℓ, which registers the total number of particles that reach it in one unit of time, is fixed on the surface of the tube as shown in the diagram. Along the entire tube (in the direction "toward us"), a uniform magnetic field of induction B is applied. For what values of B will the detector readings be at their maximum possible values? The interaction between charged particles and radiation is negligible. [Take ℓ = $\frac{\pi R}{6}$]



(A) $\frac{mv}{qR}$

(B) $\frac{3}{2}\frac{mv}{qR}$

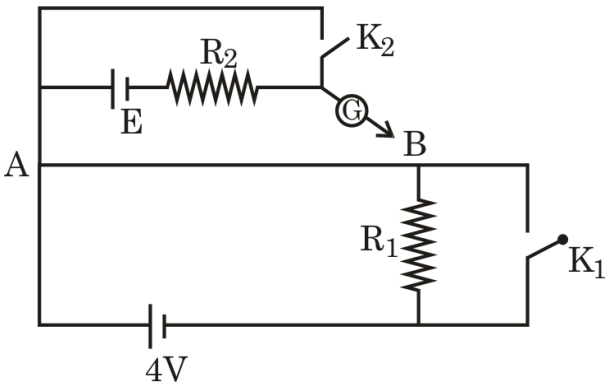
(C) $\frac{2mv}{qR}$

(D) $\frac{5}{2}\frac{mv}{qR}$

SECTION-II (i)

Common Content for Question No. 1 to 2

Figure shows the circuit of a potentiometer. The length of the potentiometer wire AB is 50 cm. The emf of the battery is 4 volt, having negligible internal resistance. Values of resistances R_1 and R_2 are 15 ohm and 5 ohm respectively. When both the keys are open, the null point is obtained at a distance of 31.25 cm from end A. Given resistance of wire AB = 10 Ω.



1) Potential gradient of wire AB at null point when K_2 is open and K_1 is open is _____ V/cm

2) The balance length when key K_2 is open and K_1 is closed is given by ____ cm.

Common Content for Question No. 3 to 4

The blue of the sky and the red of the sunset are due to a phenomenon called "scattering". When sunlight passes through the earth's atmosphere, much of the light is picked up by the air molecules and given out again in some other direction. Experiments show, in agreement with the theory of scattering, that the shortest waves are scattered more readily then longer waves. To more specific, the scattering is inversely proportional to the fourth power of the wavelength.

scattering $\propto \frac{1}{\lambda^4}$

For every ten violet wave ($\lambda = 4 \times 10^{-7}$ m) scattered from a beam, there is only one red wave ($\lambda = 7 \times 10^{-7}$ m). Scattering of different colours will be in the ratio as given below :

Violet	Blue	Green	Yellow	Orange	Red
10	6	3	2.5	2	1

3) Wavelength of yellow light will be approximately $\alpha \times 10^{-7}$ m. Write α.

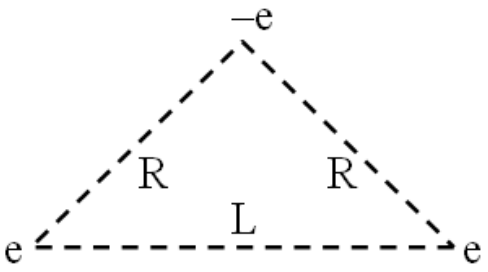
4) Scattering of wavelength corresponding to threshold wavelength of photoelectric setup of work function 6.2 eV will be nearly η times that violet colour. The value of η is (Give hC = 12400 eV Å)

Common Content for Question No. 5 to 6

Two particles with positive charge e and mass M, and a third particle with negative charge – e and mass $m = \frac{M}{2}$, move in circular orbits, maintaining the configuration of an isosceles triangle (see the figure). The planes of the particles' orbits are perpendicular to the base of the triangle.

5) Find the ratio $\frac{R_-}{R_+}$ where R_- is the radius of the orbit of the negatively charged particle, and R_+ is the radius of the orbit of the positively charged particles. Round your answer to the nearest whole number.

6) Find the ratio of the base length L of the triangle to the length of its side R. Round your answer to

two decimal places. 

SECTION-II (ii)

1) A steel girder is planted securely between two sides of a ravine in order to provide a bridge. The total cross-sectional area of the girder is 30 cm^2, and the length of the girder is 4.0 m. Installed at a temperature of 5°C, the temperature now rises to 20 °C. Calculate the force exerted (in N) by the girder due to the change in temperature, assuming the ends do not move. Young modulus of steel = 2.0×10^{11} Pa. Coefficient of linear expansion is 1.2×10^{-7} $°C^{-1}$.

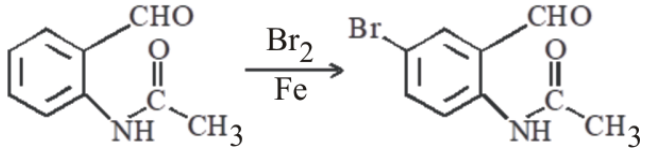
2) A closed cylindrical soft drink can of height 12cm and radius 4cm is filled up to height 9cm. Volume above liquid has air at a pressure 10 kPa. Soft drink is incompressible and has specific gravity 0.8. Taken far away from all celestial bodies in a spaceship where can is rotated about its axis with constant angular velocity $\frac{250}{\sqrt{3}}$rad/s, it exerts pressure (10000 × N) Pascal on curved surface of can. N is ……

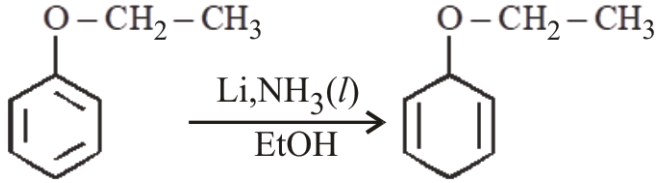
3) An ornament is lying at the bed of a pond of depth $8\sqrt{3}$m. Pond is full of water whose refractive index is $\sqrt{\frac{3}{2}}$. A diver at the surface of pond is trying to look at ornament such that her line of vision makes 30º with the surface of pond. Perpendicular distance of image of ornament from surface of water in metres is…….

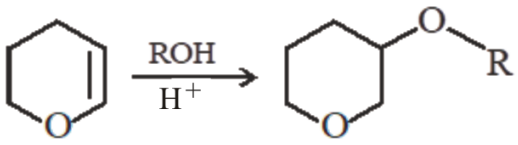
PART-2 : CHEMISTRY

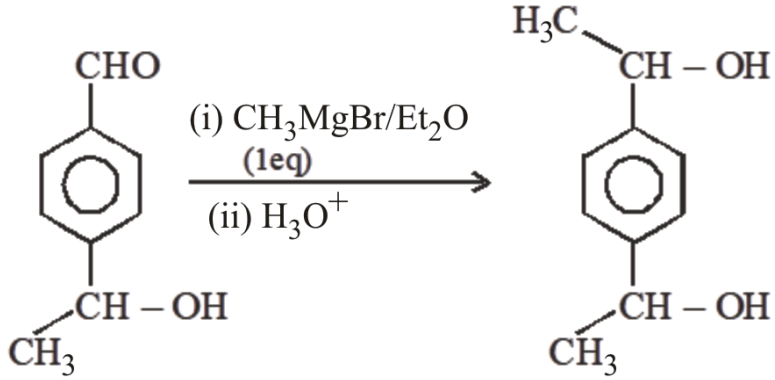
SECTION-I (i)

1) Which of the following reaction represents the correct major product :

(A) 

(B) 

(C) 

(D) 

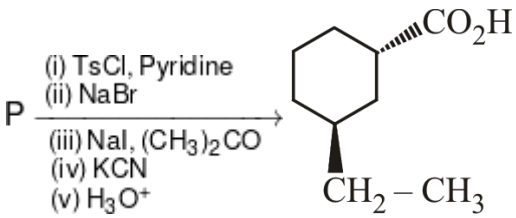
2) A solution of weak acid HA was titrated with strong base KOH. The weak acid is completely neutralized by adding 60 mL of 0.1 M KOH. Now, 20mL of 0.1 M HCl was added to titrated solution, the pH was found to be 5.3. What will be the pH of solution obtained by mixing 50 mL of 0.1 M KOH and 50 mL of 0.1 M HA? (log 2= 0.3)

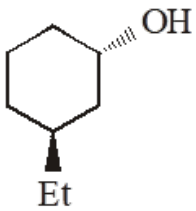
(A) 8.00

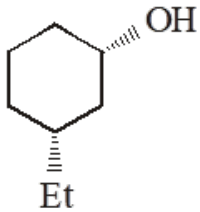
(B) 8.7

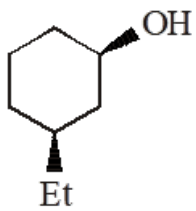
(C) 8.85

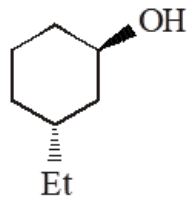
(D) 9.00

3) In the given reaction sequence the correct compound P is : 

(A) 

(B) 

(C) 

(D) 

4) Which of the following statement is correct?

(A) $[CoF_6]^{3-}$ is d^2sp^3 hybridized and diamagnetic in nature

(B) $[NiCl_4]^{2-}$ is sp^3 hybridized and diamagnetic in nature.

(C) $[Ni(CO)_4]$ is dsp^2 hybridized and diamagnetic in nature

(D) $[Cu(NH_3)_4]^{2+}$ is dsp^2 hybridized and paramagnetic in nature.

SECTION-I (ii)

1) Among the following, the correct statement(s) is/are :

(A) The two six-membered cyclic hemiacetal forms of D-(+)-glucose are called as C2 epimers.

(B) Invert sugar is prepared by acid catalyzed hydrolysis of maltose.

(C) DNA contains cytosine and thymine as pyrimidine bases.

(D) Glycine is an optically inactive amino acid.

2) Which of the following option is/are correct for ionic solids?

(A) In ZnS (Zinc blende) structure, distance between two nearest Zn^{+2} ions is 5Å, then distance between two nearest S^{2-} ions is 0.707Å.

(B) In NaCl (rock salt) structure, the second nearest cation for any anion placed at corner is present at a distance of $\frac{\sqrt{3}}{2}a$ (where 'a' = edge length of unit cell) and number of second nearest cations are 8.

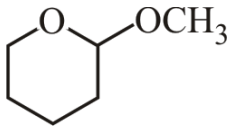
(C) In NaCl (rock salt) structure, the third nearest anion for any anion placed at corner is present at a distance of $\sqrt{\frac{3}{2}}a$ (where 'a' = edge length of unit cell).

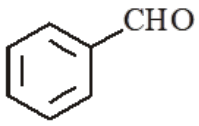
(D) The packing fraction of CsCl (8 : 8) structure is $\frac{\sqrt{3}\pi}{4}$ (approx.) (Given that : r_{Cs^+} = 170 pm, r_{Cl^-} = 180 pm)

3) Which of the following compound(s) will not give silver mirror on reaction with ammoniacal silver nitrate solution ?

(A) formic acid

(B) Propane

(C) 

(D) 

4) 2.84 g of mixture of $CaCO_3$ and $MgCO_3$ was dissolved in 250 mL of 0.4 M HCl(excess) solution which was then diluted to 400 mL. If 10 mL of resulting solution requires 10 mL of $\frac{M}{20}Na_2CO_3$ for neutralization. Then,

(A) The mass percentage of $CaCO_3$ in original mixture is 70.42 % (approx.)

(B) The mole percentage of $MgCO_3$ in original mixture is 33.33% (approx.)

(C) The number of equivalents of HCl used with original mixture was 0.06.

(D) The number of mole of HCl neutralized by $Na_2\ CO_3$ is 0.01.

5) The incorrect statement(s) related to metallurgical processes is (are)

(A) In leaching process of aluminium ore (Bayer process), a small amount of freshly prepared sample of hydrated Al_2O_3 is added to induce precipitation.

(B) Wrought iron is the purest form of commercial iron and is prepared from cast iron by reducing elemental impurities in a furnace lined with hematite.

(C) Copper matte primarily consists of Cu_2O and a smaller amount of FeS.

(D) Extraction of zinc from zinc blende involves calcination followed by reduction with carbon.

6) The incorrect statement(s) related to colloids is/are.

(A) The colloidion is a 4% solution of nitro-cellulose in a mixture of alcohol and ester.

(B) A mixture of milk and water appears red when viewed by the reflected light and blue when viewed by the transmitted light.

(C) Sols of starch, gum, clay and methylene blue are negatively charged colloidal sol.

(D) When the ionic strength of a colloidal solution is increased, thickness of double layer is decreased and the colloid gets precipitated.

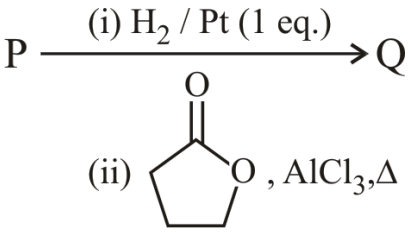
SECTION-II (i)

Common Content for Question No. 1 to 2

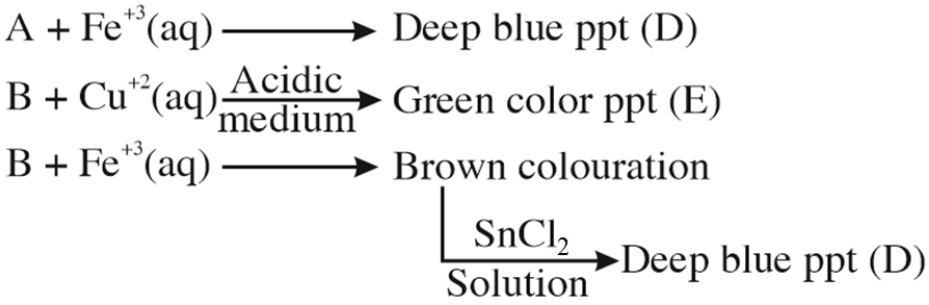
A compound P(C_9H_{10}) gives benzaldehyde and aliphatic aldehyde on reductive ozonolysis. This aliphatic aldehyde gives positive iodoform test. Identify P and answer the given questions.

1) What is the total number of major organic products formed [through most stable carbocation] when P is reacted with HBr/CCl_4 ?

2) Find degree of unsaturation of the product Q ?



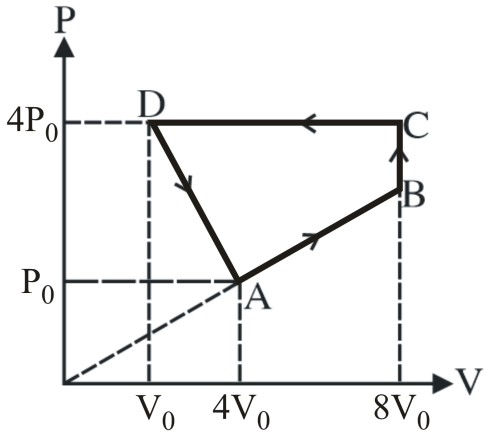
Common Content for Question No. 3 to 4
Question Stem



3) The number of unpaired electrons in compound 'B' is **'p'** and number of atoms in a molecule of compound E are **'q',** then |p – q| is:

4) When 'A' is treated with conc. H_2SO_4, a gas is produced which is colorless, odourless and poisonous. The molar mass of gas (g/mol) is 'x' and oxidation state of central metal in compound 'A' is 'y'. Then, value of |x + y| is :

Common Content for Question No. 5 to 6
Question Stem 0.06 moles of an ideal monoatomic gas undergoes a process as shown in figure. The temperature at point 'A' is 100 K.



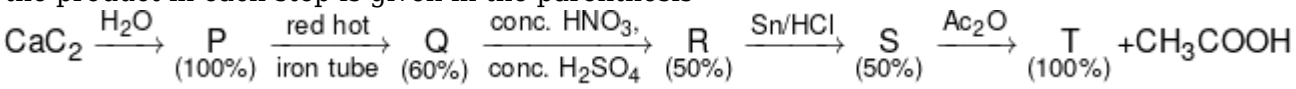
(Use : $R = \frac{25}{3}$J/mol.K, 1L.bar = 100J)

5) The magnitude of work done (in Joule) in cyclic process ABCD is

6) The magnitude of enthalpy change (in L.bar) in process CD is:

SECTION-II (ii)

1) In the following reaction sequence, the amount of T (in g) formed from 12 mole CaC_2 is___. [Atomic weight in $gmol^{-1}$: H = 1, C = 12, N = 14, O = 16, Ca = 40.] The yield (%) corresponding to the product in each step is given in the parenthesis



2) A sample of excited single electron ions can emit maximum 10 different spectral lines when de-excited to ground level. The maximum energy emitted during transition is 117.5 eV. Now, photons of minimum energy emitted during transition strikes on a metal having work function 1.254 eV emits photoelectrons. The de-Broglie wavelength of emitted photo electron (in Å) is:-

3) A sample of water is electrolysed during an experiment using 40 F electricity with 75% current efficiency. The released O_2 is used in a bomb calorimeter to burn 3.5 g C_2H_4 gas at 300 K. The temperature of the calorimeter was found to increase from 300 K to 300.50 K due to combustion process. The magnitude of heat of combustion at constant pressure (in kJ) for 5 moles of ethene is: (Given : Heat capacity of the calorimeter is 3 kJ/K, R = 8 J/mol.K)

PART-3 : MATHEMATICS

SECTION-I (i)

1) Straight line L: 2x + y = 4 is tangent to an ellipse whose foci are (1, 3) & (2, 4). Straight line L_1 parallel to line L also touches this ellipse. Point of contact of L_1 to this ellipse is

(A) $\left(\frac{53}{14}, \frac{65}{14}\right)$

(B) $\left(\frac{61}{25}, \frac{103}{25}\right)$

(C) $\left(\frac{15}{7}, \frac{31}{25}\right)$

(D) $\left(\frac{63}{25}, \frac{103}{25}\right)$

2) $\int_1^2 (2^x + 2^{-x}) d(2^x) =$

(A) $\frac{5}{4} + \ln 2$

(B) $\frac{3}{2} + \ln 2$

(C) $\frac{3}{2} - \ln 2$

(D) $\frac{7}{8}+\ln 2$

3) $\lim_{x\to 0}\frac{xe^{\sin x}-e^{x}\sin^{-1}(\sin x)}{\sin^{2}x-x\sin x}=$

(A) –2

(B) –1

(C) 1

(D) 2

4) If the standard deviation of 20 obervations $x_1, x_2, x_3, \ldots\ldots, x_{20}$ is 4 and that of another 20 observations $y_1, y_2, y_3, \ldots\ldots, y_{20}$ is 3. Also $X_i = (x_i-\bar{x})(y_i-\bar{y})$ where $\overline{x}$ and $\overline{y}$ are means of $x_i's$ and $y_i's$ respectively. If $\sum_{i=1}^{20} X_i = 90$, then the standard deviation of the observations $x_1-y_1, x_2-y_2, x_3-y_3, \ldots\ldots, x_{20}-y_{20}$ is divisible by

(A) 5

(B) 4

(C) 3

(D) 7

SECTION-I (ii)

1) $\int\frac{x^4+32}{x^8(x^4-16)}dx=\frac{3}{\lambda}\ln\left|\frac{x-2}{x+2}\right|-\frac{\mu}{256}\tan^{-1}\frac{x}{2}+\frac{1}{\alpha x^3}+\frac{2}{7x^7}+c$

(A) λ = 512

(B) μ = 5

(C) μ = 3

(D) α = 15

2) Let A(3, 0, 4) and B(4, 6, 0) be two given points and P(α, 0, 0) be point on x-axis such that PA + PB is minimum, then

(A) If point Q is on the y-axis such that QA + QB is minimum then area of ΔPOQ is $\frac{21}{4}$ (unit)^2 where O is origin

(B) The volume of tetrahedron OPAB (where O is origin) is $\frac{68}{5}$ (unit)^3

(C) Perpendicular distance from the point A to the plane containing the points P, Q, B is 4 (units)

(D) If point Q is on the y-axis such that QA + QB is minimum then area of ΔPOQ is $\frac{17}{3}$ (unit)^2 where O is origin

3) If $(f(x)-2)(x^4+x^2+1)-(f(x)+2)(x^2+x+1)^2=0$ is true for all $x\in R-\{0\}$, then which of the

following statement(s) is/are true?

(A) f(x) has a local maxima at x = –1

(B) $|f(x)| \geq 4, \forall x \in R - \{0\}$

(C) f(x) has a local maxima at x = 1

(D) The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sec x) f(x) dx = 0$

4) If P_n denotes the number of ways in which n runners participating in a race can reach the finishing line, when one or more runners can reach at same time, then

(A) $P_4 = 74$

(B) $P_4 = 75$

(C) $P_5 = 541$

(D) $P_5 = 521$

5) Let roots of quadratic equation $8x^2 - 14x + 3 = 0$ are α and β where α < β. Which of the following equation has $\left(2\alpha + \sqrt{\frac{\beta}{2}}\, i\right)^{50}$ and $\left(2\alpha - \sqrt{\frac{\beta}{2}}\, i\right)^{50}$ as roots :

(A) $x^2 + x + 1 = 0$

(B) $x^3 + 1 = 0$

(C) $x^3 - 1 = 0$

(D) $x^2 - x + 1 = 0$

6) If $\ln x \leq \lambda x^3, \ \forall x \in (0, \infty)$ then possible value/s of λ is are

(A) $\frac{1}{5}$

(B) $\frac{1}{10}$

(C) 1

(D) $\frac{1}{e^2}$

SECTION-II (i)

Common Content for Question No. 1 to 2

Let a, b and c be real numbers such that the system of linear equations

10x + 11y + 12z = a

13x + 14y + 15z = b

16x + 17y + 18z = c – 3

is consistent. Let |x| denotes the determinant of matrix

$$X = \begin{bmatrix} a & 2 & 1 \\ b & 1 & 0 \\ c & 0 & -1 \end{bmatrix}$$

Let P be the plane containing all those (a, b, c) for which system of linear equations is consistent and D be the square of the distance of the point (–9, 0, 0) from the plane P.

1) The value of (|x| + 10) is _____

2) The value of D is _____

Common Content for Question No. 3 to 4

Let f be a polynomial function defined only over non negative real numbers such that $f(x)f(y) + 2 = f(x) + f(y) + f(xy)$ and f(x) is one-one function with f(0) = 1 and f '(1) = 2 then

3) The value of f(5) is

4) If $h(x) = \min\left\{\frac{2}{f(x)}, x^2, |1 - |x||\right\}$, where $x \geqslant 0$, then the number of points of non differentiability of h(x) is

Common Content for Question No. 5 to 6

Let S_n be the sum of first n terms of the sequence $\{a_n\}$. Let $a_n = 5S_n + 1$ holds for any $n \in N$. Let $b_n = \frac{4 + a_n}{1 - a_n}, n \in N$

5) $\left(\frac{4^{10}a_{10} + 5}{2}\right) =$

6) $\left(\frac{(4^{12} - 1)b_{12} - 1}{2^{27}}\right) =$

SECTION-II (ii)

1) A ship sends five different codes A, B, C, D, E for communication, but each week uses only one of them. The code used in a definite week is randomly selected with equal chance among the four ones that have not been used in the last week. Suppose the code used in the first week is A. If the probability that A is also used in the ninth week is $\frac{\alpha}{\beta}$ (where α,β are relatively prime positive integers) then the value of $\left[\frac{\alpha}{100}\right]$ = (where [.] is greatest integer function)

2) If z is a complex number such that |z| = 1. If maximum value of $|z^3 - z + 2|$ is 'a', then the sum of digit of a^2 is

3) If the area bounded by x = 0, y = 0, x = 2, y = 2, $y \leqslant e^x$ and $y \geqslant \ln x$ is (a – b ln2) sq. unit, where $a \in N,\ b \in N$ then value of $\left(\frac{a+b}{2}\right)$ is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	B	B	B	D

SECTION-I (ii)

Q.	5	6	7	8	9	10
A.	B,D	A,B,C	B,C	A,B,D	B	A,B,C

SECTION-II (i)

Q.	11	12	13	14	15	16
A.	0.03 to 0.04	12.50	5.60	16.00	4.00	1.25 to 1.27

SECTION-II (ii)

Q.	17	18	19
A.	1080	2	4

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	20	21	22	23
A.	A	C	C	D

SECTION-I (ii)

Q.	24	25	26	27	28	29
A.	C,D	B,C	B,C	A,B,C	B,C,D	A,B,C

SECTION-II (i)

Q.	30	31	32	33	34	35
A.	2.00	5.00	28.00	30.00	181.25	8.75

SECTION-II (ii)

Q.	36	37	38
A.	81	10	84

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	39	40	41	42
A.	**B**	**B**	**C**	**B**

SECTION-I (ii)

Q.	43	44	45	46	47	48
A.	**A,C**	**B,C,D**	**B,C,D**	**B,C**	**A,C**	**A,C,D**

SECTION-II (i)

Q.	49	50	51	52	53	54
A.	**7.00**	**24.00**	**26.00**	**3.00**	**3.00**	**0.50**

SECTION-II (ii)

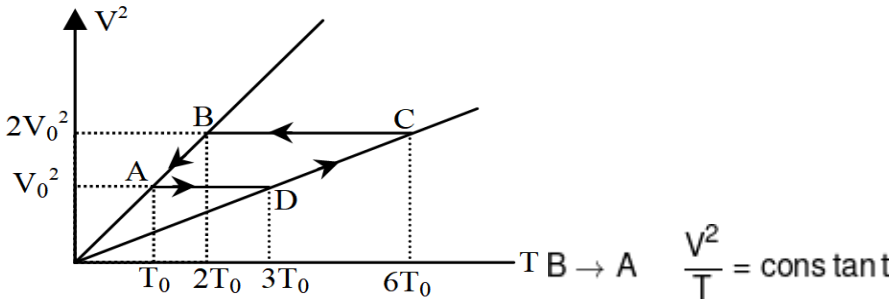
Q.	55	56	57
A.	**32**	**4**	**5**

SOLUTIONS

PART-1 : PHYSICS

7) When the switch 'S' is opened,
$\tau = RC = 1.5$ sec
Voltage across the capacitor,
$V_C = 10(1 - e^{-t/1.5})$ for $t < T$
The current, $i = \frac{dQ}{dt} = C\frac{dV_C}{dt}$
$= \frac{200}{3} \times 10^{-6}(e^{-t/1.5})$ for $t < T$
Voltage drop across 100 k μ resister,
$V = iR = \frac{20}{3}e^{-t/1.5}$ for $t < T$
After $t = T$,
$\tau = 100\ k\Omega \times 10\ \mu F = 1$ sec
Voltage across capacitor,
$V_C = V_0 t^{-[(t-T)/\tau]} = 10(1 - e^{-(T/1.5)}), e^{-[(t-T)/1]}$
The charge on capacitor,
$Q = V_C \cdot C$
The current through 100 k μ resister
$i = -C\frac{dV_C}{dt} = 10^3\left(1 - e^{-T/1.5}\right)\left(-e^{-(t-T)/t}\right)$

8)



$B \to A \quad \frac{V^2}{T} = constant$

$\therefore T_B = 2T_0$
$D \to C \quad \frac{V^2}{T} = constant$
$\Rightarrow T_C = 6T_0$
$\therefore \frac{T_B + T_C}{2} = 4T_0$
$B \to A \frac{V^2}{T} = constant$
$\frac{V^2}{\left(\frac{PV}{nR}\right)} = constant$
$PV^{-1} = constant$
polytropic index $m = -1$
$C = \frac{R}{1-m} + \frac{Rf}{2}$
$C = 2R$
(same for D → C)

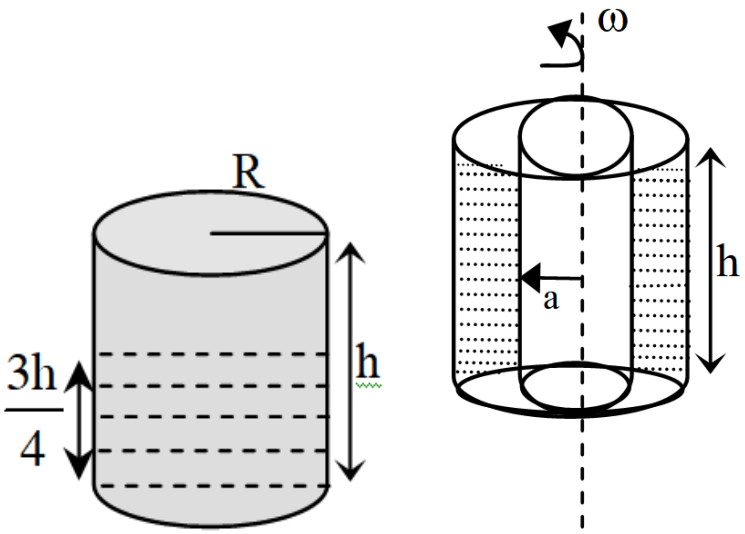
$D \rightarrow C : Q = \Delta U + W$

$nc\Delta T = nc_V\Delta T + W$

$1 \times 2R \times 3T_0 =$

$$1 \times \frac{3R}{2} \times 3T_0 + W$$

$$W = \frac{3RT_0}{2}$$



18)

g is absent

$g_{eff} = \omega^2 r \rightarrow$ (r = distance from axis)

volume of liquid = constant

$$\frac{3\pi R^2 h}{4} = \pi R^2 h - \pi a^2 h \quad (h = 12cm)$$

$$a = \frac{R}{2}$$

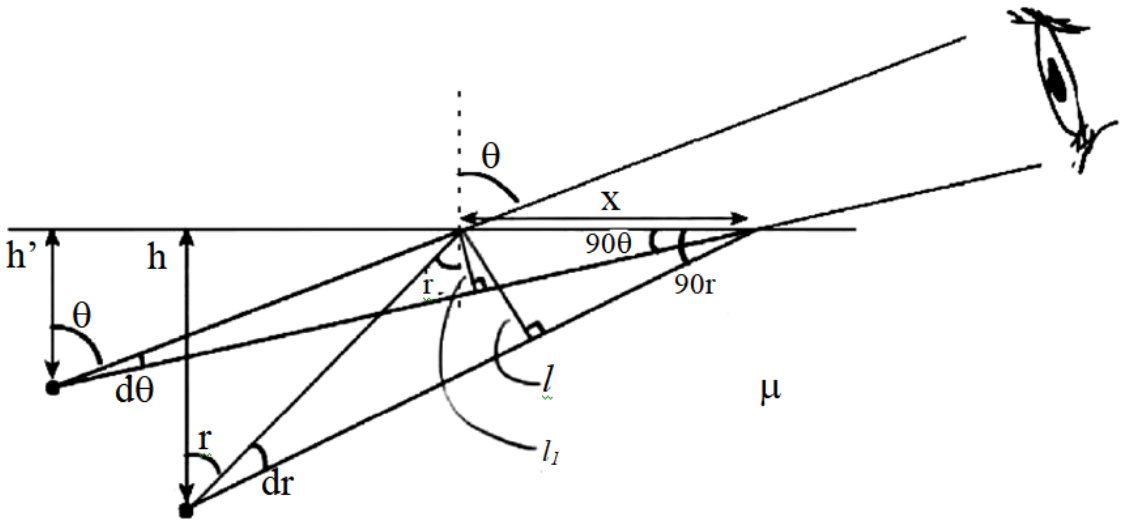
$$P_{wall} = P_0 + \rho \int_a^R \omega^2 r \, dr$$

$$P_{wall} = P_0 + \frac{\rho\omega^2}{2}(R^2 - a^2)$$

$$P_{wall} = P_0 + \frac{3\rho\omega^2 R^2}{8}$$

$$P_{wall} = 10^4 + \frac{3 \times 800 \times \left[\frac{250}{\sqrt{3}} \times 4 \times 10^{-2}\right]^2}{8}$$

$$P_{wall} = 2 \times 10^4 Nm^{-2}$$



19)

Rays are marginal. Shift is perpendicular to surface as well as along the surface.

$$\ell' = \left(\frac{h'}{\cos\theta}\right) d\theta = x \sin(90^\circ - \theta)$$

$$\ell = \left(\frac{h}{\cos r}\right) dr = x \sin(90^\circ - r)$$

$$\text{Divide : } \frac{h'}{h}\frac{d\theta}{dr} = \frac{\cos^2\theta}{\cos^2 r} \ldots.(1)$$

Also, $\sin\theta = \mu \sin r$

$\Rightarrow \cos d\theta = \mu \cos r\, dr$ ....(2)

$$\frac{h\cos^3\theta}{\mu\cos^3 r} = \frac{h\cos^3\theta}{\mu\left(1-\frac{\sin^2\theta}{\mu^2}\right)^{3/2}}$$

From 1,2 : h' =

put $h = 8\sqrt{3}m$

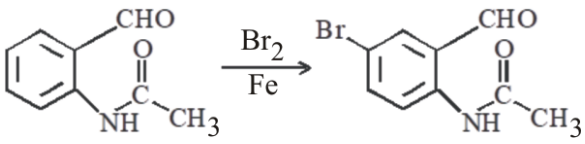
$\theta = 60º$

$\mu = \sqrt{3/2}$

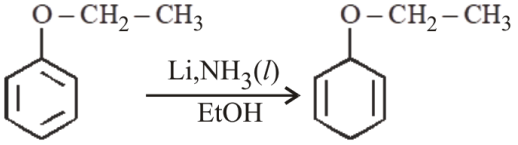
h' = 4 m

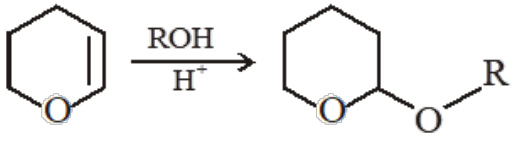
PART-2 : CHEMISTRY

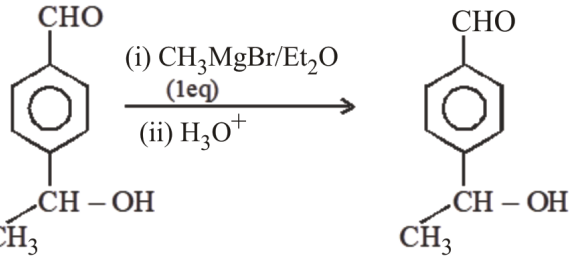
20)

(A) 

More activating so, bromination occurs at its para

(B) 

(C) 

(D) 

21)

$HA + KOH \rightarrow K^+A^- + H_2O$

At equivalence point,

Number of equivalents of HA

= Number of equivalents of KOH

= Number of equivalents of salt

$$= \frac{60}{1000} \times 0.1 \times 1 \Rightarrow 6 \times 10^{-3}$$

Now HCl is added to the salt solution

K^+A^- + HCl → HA + KCl t = 0 0.006 moles 0.002 moles

t = t 0.004 moles 0 0.002 moles

It is an acidic buffer solution

$$p^H = p^{Ka} + \log \frac{[A^-]}{[HA]}$$

$p^{Ka} = 5$

Now, p^H will be due to salt hydrolysis :

$$K_n = \frac{K_w}{K_a} = \frac{10^{-14}}{10^{-5}} = 10^{-9}$$

$$10^{-9} = \frac{Ch^2}{1-h}$$

Suppose h < < < 1

$$10^{-9} = \left(\frac{5 \times 10^{-3}}{10^{-1}}\right).h^2$$

$$\frac{1}{5} \times 10^{-7} = h^2$$

$2 \times 10^{-8} = h^2$

$h = \sqrt{2} \times 10^{-4}$

$[OH^-] = Ch$

$$= (5 \times 10^{-2})\sqrt{2} \times 10^{-4}$$

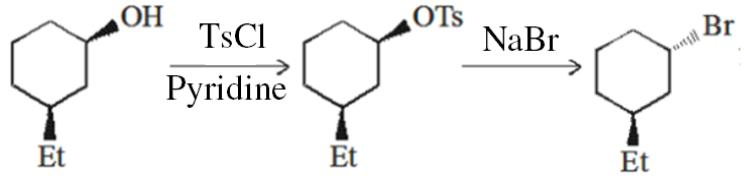
$$= 5\sqrt{2} \times 10^{-6}$$

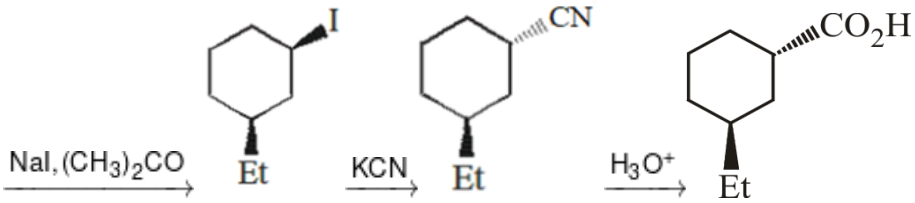
$$p^{OH} = 6 - \log 5\sqrt{2}$$

$$= 6 - \left\{\log 5 + \frac{1}{2}\log 2\right\}$$

= 6 - 0.85 = 5.15

p^H = 14 - 5.15 =8.85

22) 



23) $[CoF_6]^{3-}$ → sp^3d^2 and paramagnetic
$[NiCl_4]^{2-}$ → sp^3 and paramagnetic
$[Ni(CO)_4]$ → sp^3 and diamagnetic
$[Cu(NH_3)_4]^{2+}$ → dsp^2 and paramagnetic

25)

(A) The distance between two nearest Zn^{+2} ions is $\frac{a}{\sqrt{2}} = 5A^\circ$, then the distance between two nearest S^{2-} ions is $\frac{a}{\sqrt{2}} = 5A^\circ$.
(B) In NaCl (rock salt) structure; for any anion (placed at corner), distance of second nearest cation is $\frac{\sqrt{3}}{2}a$ (present at body centre) and coordination number is 8.
(C) The third nearest anion for any anion placed at corner is present at centre of opposite face at a distance of $\sqrt{\frac{3}{2}}\,a$.
(D) The packing fraction of CsCl is

$$P.F. = \frac{\frac{4}{3}\pi(r_+^3 + r_-^3)}{a^3}$$
$$= \frac{\frac{4}{3}\pi(r_+^3 + r_-^3)}{\frac{8}{3\sqrt{3}}(r_+ + r_-)^3}$$
$$= \frac{\sqrt{3}\pi}{2} \times \frac{10.745}{42.875} = \frac{\sqrt{3}\pi}{8}$$

27)

$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$
moles x 2x
$MgCO_3 + 2HCl \rightarrow MgCl_2 + H_2O + CO_2$
moles y 2y

Number of equivalents of HCl added = $\frac{250}{1000} \times 0.4 \times 1$
= 0.1 equivalents

Number of equivalents of HCl neutralized by Na_2CO_3 (in 10ml) =

$$\frac{10}{1000} \times \frac{1}{20} \times 2 = 0.001 \text{ equivalents}$$

Total equivalents of HCl neutralized by Na_2CO_3 in 400 ml = 0.001 × 40 = 0.04 equivalents

No. of moles of HCl used by $CaCO_3$ & $MgCO_3$ are = 0.1 - 0.04 = 0.06 moles

according to equations

$2x + 2y = 0.06$

$x + y = 0.03$ ----------(1)

and

$100x + 84y = 2.84$ ---------- (2)

Solving (1) & (2)

$x = 0.02$ moles

$y = 0.01$ moles

(A) % mass of $CaCO_3$ = $\frac{2}{2.84} \times 100 = 70.42\%$

(B) % moles of $MgCO_3$ = $\frac{0.01}{0.03} \times 100 = 33.33\%$

(C) Number of equivalents of HCl neutralised by Na_2CO_3 = 0.06

(D) Number of moles of HCl neutralised by Na_2CO_3 is 0.04

28)

(A) In leaching of aluminium ore, a small amount of freshly prepared sample of hydrated Al_2O_3 is used to cause precipitation (Bayer process)

(B) Wrought iron is purest commercial iron and impurities are oxidised with hematite.

$$Fe_2O_3 + C \xrightarrow{\Delta} Fe + CO_2$$

$$Fe_2O_3 + Si \xrightarrow{\Delta} Fe + SiO_2$$ (C) Copper matte consists of Cu_2S + FeS (very small amount)

(D) Extraction of zinc from ZnS involves roasting followed by carbon reduction.

29)

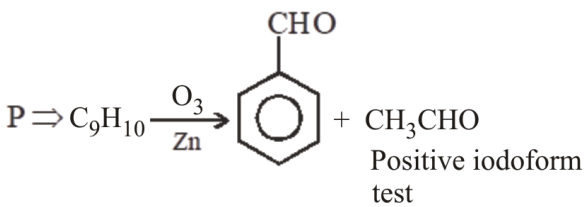
(A) Colloid-ion is a 4% solution of nitro-cellulose in a mixture of alcohol and ether. (NCERT)

(B) A mixture of milk and water appears blue when viewed by the reflected light and red when viewed by the transmitted light. (NCERT)

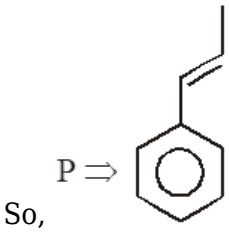
(C) Starch, gum, clay → Negatively charged sol.
Methylene blue → Positively charged sol. (NCERT)

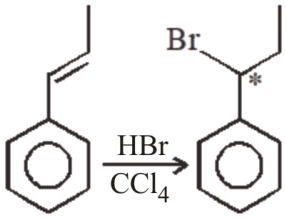
(D) Correct

30)



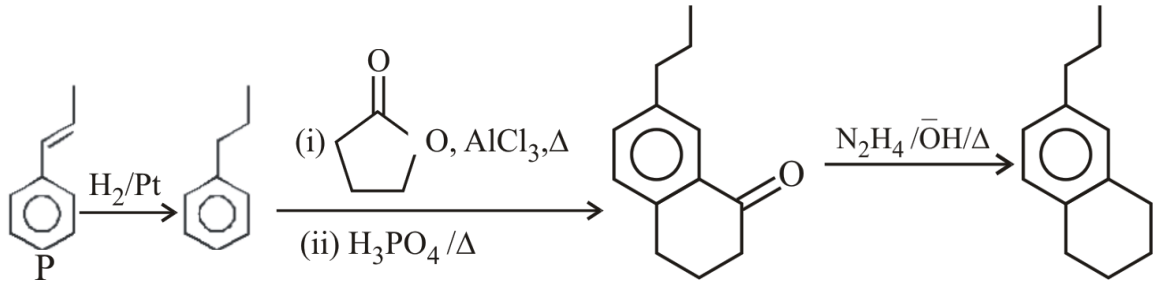
So,





Total products = 2

31)



32) $K_4[Fe(CN)_6] + Fe^{+3} \rightarrow Fe_4[Fe(CN)_6]_3$

(A) Deep blue ppt. (D)

$$K_3[Fe(CN)_6] + Cu^{2+} \xrightarrow[\text{medium}]{\text{Acidic}} Cu_3[Fe(CN)_6]_2$$

(B) Green colour ppt. (E)

$$K_3[Fe(CN)_6] + Fe^{+3} \rightarrow Fe[Fe(CN)_6] \xrightarrow{SnCl_2 \text{ solution}} Fe_4[Fe(CN)_6]_3$$

(B) Brown coloration Deep blue ppt. (D)

Number of unpaired electron in $K_3[Fe(CN)_6]$ is 'p' = 1

Number of atoms in a molecule of $Cu_3[Fe(CN)_6]_2$ is 'q'= 29

|p - q| = 28

33) $K_4[Fe(CN)_6] + Fe^{+3} \rightarrow Fe_4[Fe(CN)_6]_3$

(A) Deep blue ppt. (D)

$$K_3[Fe(CN)_6] + Cu^{2+} \xrightarrow[\text{medium}]{\text{Acidic}} Cu_3[Fe(CN)_6]_2$$

(B) Green colour ppt. (E)

$$K_3[Fe(CN)_6] + Fe^{+3} \rightarrow Fe[Fe(CN)_6]$$

(B) Brown coloration $\xrightarrow{SnCl_2 \text{ solution}} Fe_4[Fe(CN)_6]_3$

$K_4\left[Fe(CN)_6\right] + H_2SO_4(conc.) \rightarrow CO$

(x) = 28

oxidation state of central metal in 'A' is y = 2

|x + y| = 30

34) $W_{ABCDA} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$P \propto V \Rightarrow P = KV \text{ or } T = K_1V^2 \Rightarrow \frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)^2$

Along AB

At point A; P_0, $4V_0$, $T_A = 100K$

At point B; $2P_0$, $8V_0$, $T_B = 400K$

Along BC, volume is constant

$P \propto T$

At point C; $4P_0$, $8V_0$, $T_C = 800$ K

Along CD; pressure is constant

$V \propto T$

At point D; P_0, V_0, $T_D = 100K$

$W_{AB} = \frac{-1}{2}(P_0 + 2P_0) \times 4V_0 = -6P_0V_0$

$W_{BC} = 0$

$W_{CD} = +4P_0(8V_0 - V_0) = 28P_0V_0$

$W_{DA} = \frac{-1}{2}[4P_0 + P_0] \times 3V_0 = -7.5P_0V_0$

Net work done

$W_{ABCDA} = -6P_0V_0 + 28P_0V_0 - 7.5P_0V_0$

$= +14.5\,P_0V_0$

At point A;

$P_04V_0 = nR(100K)$

$P_0V_0 = 25nR$

$|W_{ABCDA}| = 14.5 \times 25 \times 0.06 \times \frac{25}{3}$

$= 181.25$ J

35)

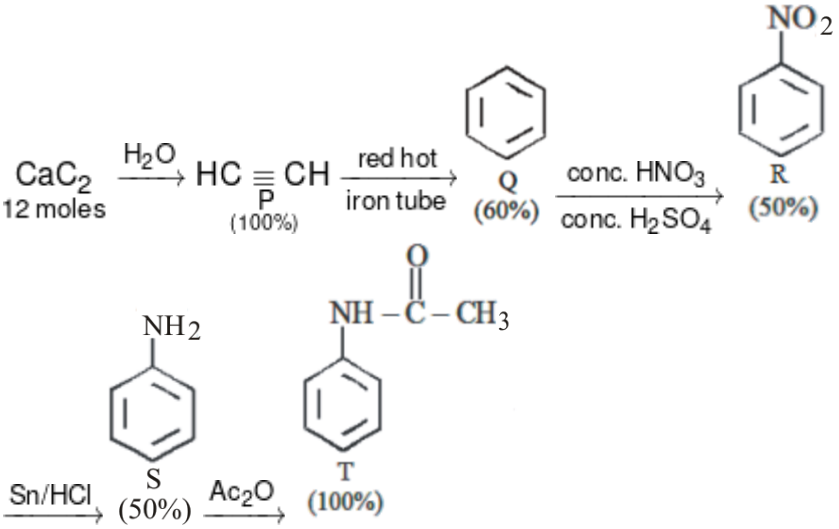
$\Delta H_{CD} = nC_{p,m}\,\Delta T$

$= 0.06 \times \frac{5}{2} \times \frac{25}{3}(100 - 800)$

$= -875J = -8.75$ Lbar

$|\Delta H_{CD}| = 8.75$Lbar

36)



Actual yield of P = 12 moles

3 moles of P form = 1 mole of Q

So actual yield of Q = 60% of 4 = 2.4 moles

Actual yield of R = 1.2 moles

Actual yield of S = 0.6 moles

Actual yield of T = 0.6 moles

Molar mass of T = 135 g/mol

Mass of T = 135 × 0.6 = 81 g

37) A sample can emit 10 different spectral lines when $n_2 = 5$ and $n_1 = 1$
maximum energy emitted is 117.5 eV

$$\Delta E = 13.6z^2\left(\frac{1}{1}-\frac{1}{25}\right) = 117.5eV \Rightarrow z = 3$$

Photon of minimum energy is emitted from 5 → 4 transition.

$$\Delta E = 13.6 \times 9 \times \left(\frac{1}{16}-\frac{1}{25}\right)$$

$$= \frac{13.6 \times 9 \times 9}{16 \times 25} = 2.754eV$$

Work function of metal = 1.254eV
K.E.$_{max.}$ of photo electrons = 1.5eV
de-Broglie wavelength of photo electron is

$$\lambda = \sqrt{\frac{150}{1.5}} = \sqrt{100}A^\circ$$

$$\lambda = 10A^\circ$$

38) $H_2O \rightarrow H_2 + \frac{1}{2}O_2$ (nf = 4)

Moles of O_2 produced = $\frac{40F \times 0.75}{F \times 4}$ = 7.5 moles

$$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(\ell)$$

moles $\frac{3.5}{28} = \frac{1}{8}$ 7.5

The heat released by $\frac{1}{8}$ moles of C_2H_4 is

= C.ΔT

= 3 × 0.5 = 1.5KJ

ΔU(KJ/mol) = $\frac{-1.5 \times 28}{3.5}$ = -12KJ/mol

$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(\ell)$

$\Delta H = \Delta U + \Delta ngRT$ (for one mole)

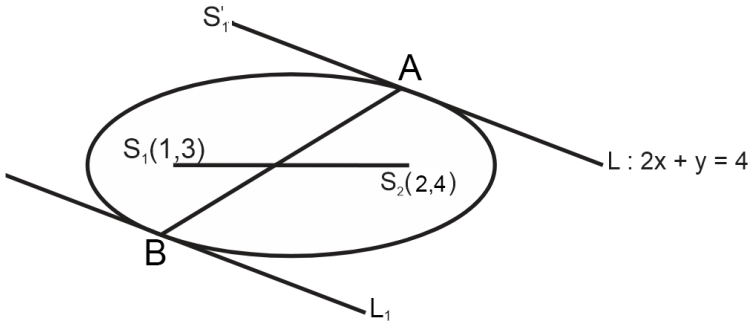
$= -12 + \frac{(-2) \times 8 \times 300}{1000}$

= -12-4.8 = -16.8 KJ/mol.

for 5 moles of ethene

$|\Delta H| = +16.8 \times 5 = 84KJ$

PART-3 : MATHEMATICS



39)

Let $S_1'(a, b)$

$\frac{a-1}{2} = \frac{b-3}{1} = \frac{-2(2+3-4)}{4+1}$

$\Rightarrow a = \frac{1}{5}, b = \frac{1}{5}$

$\Rightarrow S_1'\left(\frac{1}{5}, \frac{13}{5}\right)$

$S_1'S_2 : y - 4 = \frac{\frac{13}{5} - 4}{\frac{1}{5} - 2}(x - 2)$

Now

 7x - 9y = -22 ---------- (1)

Also L : 2x + y = 4 ------- (2)

By (1) and (2) : $A\left(\frac{14}{25}, \frac{72}{25}\right)$

Now O is mid point of A and B.

here $O\left(\frac{3}{2}, \frac{7}{2}\right)$

Let B(α, β)

$\frac{\alpha + \frac{14}{25}}{2} = \frac{3}{2} \Rightarrow \alpha = \frac{61}{25}$

$\frac{\beta + \frac{72}{25}}{2} = \frac{7}{2} \Rightarrow \beta = \frac{103}{25}$

$\therefore B\left(\frac{61}{25}, \frac{103}{25}\right)$

40) $I = \int_1^2 (2^x + 2^{-x})\, d(2^x)$

$= \int_0^1 (2^x + 2^{-x})\, 2^x \ell n 2 dx$

$= \ell n 2 \int_0^1 (4^x + 1)\, dx$

$= \ell n 2 \left[\frac{4^x}{\ell n 4} + x\right]_0^1$

$= \ell n 2 \left[\frac{4}{\ell n 4} + 1 - \frac{1}{\ell n 4}\right]$

$= \ell n 2 \left[\frac{3}{\ell n 4} + 1\right]$

$= \frac{3}{2} + \ell n 2$

41) Given limit

$= \lim_{x \to 0} \left(\frac{e^{\sin x} - e^x}{\sin x - x}\right)\left(\frac{x}{\sin x}\right)$

$= 1$

42) $\sum_{i=1}^{20} (x_i - \overline{x})(y_i - \overline{y}) = 90$

$\Rightarrow \sum x_i y_i - \overline{x} \sum y_i - \overline{y} \sum x_i + \overline{xy} \sum 1 = 90$

$\Rightarrow \sum x_i y_i - 20\overline{xy} = 90$

$\Rightarrow \sigma^2 = \frac{\sum_{i=1}^{20} (x_i - y_i)^2}{20} - (\overline{x} - \overline{y})^2$

$= \frac{\sum x_i^2}{20} + \frac{\sum y_i^2}{20} - \frac{2\sum x_i y_i}{20} - (\overline{x})^2 - (\overline{y})^2 + 2\overline{xy}$

$= 16 + 9 - \frac{2}{20}(90) = 16$

$\Rightarrow \sigma = 4$

43) $I = \int \frac{3x^4 - 2\left(x^4 - 16\right)}{x^8\left(x^4 - 16\right)} dx$

$= 3\int \frac{1}{x^4\left(x^4 - 16\right)} dx - 2\int \frac{1}{x^8} dx$

$$= \frac{3}{16}\int \frac{x^4-\left(x^4-16\right)}{x^4\left(x^4-16\right)}dx + \frac{2}{7}\cdot\frac{1}{x^7}$$

$$= \frac{3}{16}\int \frac{1}{x^4-16}dx - \frac{3}{16}\int \frac{1}{x^4}dx + \frac{2}{7x^7}$$

$$= \frac{1}{8}\times\frac{3}{16}\int \frac{\left(x^2+4\right)-\left(x^2-4\right)}{\left(x^2+4\right)\left(x^2-4\right)}dx + \frac{3}{16}\cdot\frac{1}{3x^3} + \frac{2}{7x^7}$$

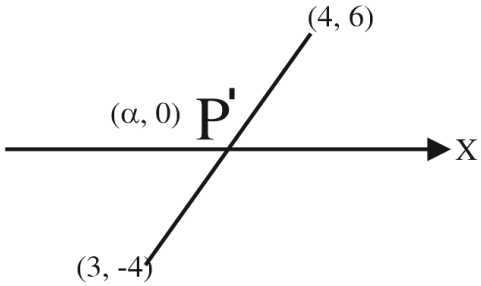
$$= \frac{3}{128}\int \left(\frac{1}{x^2-4}-\frac{1}{x^2+4}\right)dx + \frac{1}{16x^3} + \frac{2}{7x^7}$$

$$= \frac{3}{128}\left(\frac{1}{2.2}\ell n\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\tan^{-1}\frac{x}{2}\right) + \frac{1}{16x^3} + \frac{2}{7x^7} + C$$

$$= \frac{3}{512}\ell n\left|\frac{x-2}{x+2}\right| - \frac{3}{256}\tan^{-1}\frac{x}{2} + \frac{1}{16x^3} + \frac{2}{7x^7} + C$$

44) PA + PB = $\sqrt{(\alpha-3)^2+16} + \sqrt{(\alpha-4)^2+36}$

which is equivalent to P'A' + P'B' where P'(α, 0), A'(3, –4) & B'(4, 6) it will be minimum when



$$\frac{4}{\alpha-3} = \frac{10}{1} \Rightarrow 2 = 5\alpha - 15$$

$$\Rightarrow \alpha = \frac{17}{5}$$

So $P\left(\frac{17}{5}, 0, 0\right)$

let Q (0, λ, 0)

Now QA + QB = $\sqrt{\lambda^2+25} + \sqrt{(\lambda-6)^2+16}$

it will be minimum when $\lambda = \frac{10}{3}$

$Q\left(0, \frac{10}{3}, 0\right)$

Area of ΔOPQ

$$= \frac{1}{2}\left|\overrightarrow{OP}\times\overrightarrow{OQ}\right|$$

$$= \frac{1}{2}\left|\frac{17}{5}\hat{i}\times\frac{10}{3}\hat{j}\right|$$

$$= \frac{17}{3}$$

Volume of tetrahedron OPAB

$$= \frac{1}{6}\left|\left[\overrightarrow{OP}\ \overrightarrow{OA}\ \overrightarrow{OB}\right]\right|$$

$$= \frac{1}{6}\begin{vmatrix} \frac{17}{5} & 0 & 0 \\ 3 & 0 & 4 \\ 4 & 6 & 0 \end{vmatrix} = \frac{68}{5}$$

equation of plane passing there P, Q, B :

$$\begin{vmatrix} x-\frac{17}{5} & y & z \\ \frac{-17}{5} & \frac{10}{3} & 0 \\ \frac{3}{5} & 6 & 0 \end{vmatrix} = 0$$

$\Rightarrow Z = 0$

distance of A from above plane = 4

45) $(x^2 + x + 1)[(f(x)-2)(x^2 - x + 1) - (f(x)+2)(x^2+x+1)]=0$

$f(x)(x^2 - x + 1 - x^2 - x -1)-2x^2+2x - 2-2x^2-2x - 2 = 0$

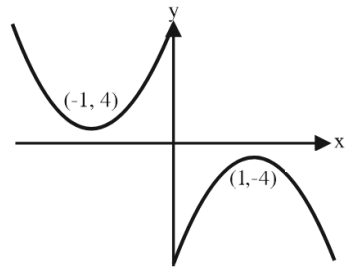
$f(x)(-2x) = 4x^2 + 4$

$f(x) = -2\left(x+\frac{1}{x}\right)$

$x+\frac{1}{x} \in (-\infty,-2] \cup [2, \infty)$

$f(x) \in (-\infty,-4] \cup [4, \infty)$

$f'(x) = -2\left(1-\frac{1}{x^2}\right) = \frac{-2}{x^2}(x+1)(x-1)$



46) The ways in which 4 runners can reach finishing line are as following :

(i) $1+1+1+1 \Rightarrow \lfloor\!\underline{4} = 24$ ways

(ii) $1+1+2 \Rightarrow \frac{\lfloor\!\underline{4}}{\lfloor\!\underline{1}\,\lfloor\!\underline{1}\,\lfloor\!\underline{2}\,\lfloor\!\underline{2}} \times \lfloor\!\underline{3} = 36$ ways

(iii) $1+3 \Rightarrow \frac{\lfloor\!\underline{4}}{\lfloor\!\underline{1}\,\lfloor\!\underline{3}} \times \lfloor\!\underline{2} = 8$ ways

(iv) $2 + 2 \Rightarrow \frac{\lfloor\!\underline{4}}{\lfloor\!\underline{2}\,\lfloor\!\underline{2}\,\lfloor\!\underline{2}} \times \lfloor\!\underline{2} = 6$ ways

$\therefore P_4 = 24 + 36 + 8 + 6 + 1 = 75$ ways

The ways in which 5 runners can reach finishing line are as following :

(i) $1 + 1 + 1 + 1 + 1 \Rightarrow \lfloor\!\underline{5} = 120$ ways

(ii) $1+1+1+2 \Rightarrow \frac{\lfloor\!\underline{5}}{\lfloor\!\underline{1}\,\lfloor\!\underline{1}\,\lfloor\!\underline{1}\,\lfloor\!\underline{2}\,\lfloor\!\underline{3}} \times \lfloor\!\underline{4} = 240$ ways

(iii) $1+1+3 \Rightarrow \frac{\lfloor\!\underline{5}}{\lfloor\!\underline{1}\,\lfloor\!\underline{1}\,\lfloor\!\underline{2}\,\lfloor\!\underline{3}} \times \lfloor\!\underline{3} = 60$ ways

(iv) $1+2+2 \Rightarrow \frac{\lfloor\!\underline{5}}{\lfloor\!\underline{1}\,\lfloor\!\underline{2}\,\lfloor\!\underline{2}\,\lfloor\!\underline{2}} \times \lfloor\!\underline{3} = 90$ ways

(v) $1+4 \Rightarrow \frac{\lfloor\!\underline{5}}{\lfloor\!\underline{1}\,\lfloor\!\underline{4}} \times \lfloor\!\underline{2} = 10$ ways

(vi) $2+3 \Rightarrow \frac{\lfloor\!\underline{5}}{\lfloor\!\underline{2}\,\lfloor\!\underline{3}} \times \lfloor\!\underline{2} = 20$ ways

(vii) 5 ⇒ 1 ways

$\therefore P_5 = 120 + 240 + 60 + 90 + 10 + 20 + 1 = 541$ ways

47) $8x^2-14x + 3 = 0$

$\Rightarrow x = \frac{1}{4}, \frac{3}{2}$

$\Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{2}$

Now $\left(2\alpha + \sqrt{\frac{\beta}{2}}i\right)^{50} = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{50} = \left(-w^2\right)^{50} = w$

&

$\left(2\alpha - \sqrt{\frac{\beta}{2}}i\right)^{50} = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{50} = (-w)^{50} = w^2$

48) $\ell nx \le \lambda x^3 \Rightarrow \lambda \geqslant \frac{\ell nx}{x^3}$

let $f(x) = \frac{\ell nx}{x^3}$

$f'(x) = \frac{x^2 - 3x^2\ell nx}{x^6}$

$= \frac{1-3\ell nx}{x^4}$

Maximum value of f(x)

$= f(e^{1/3}) = \frac{\ell n\left(e^{1/3}\right)}{e} = \frac{1}{3e}$

49) $D = \begin{vmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} a & 11 & 12 \\ b & 14 & 15 \\ c-3 & 17 & 18 \end{vmatrix} = \begin{vmatrix} a & 11 & 1 \\ b & 14 & 1 \\ c-3 & 17 & 1 \end{vmatrix} = \begin{vmatrix} a & 11 & 1 \\ b-a & 3 & 0 \\ c-b-3 & 3 & 0 \end{vmatrix}$

⇒ 3b – 3a – 3c + 3b + 9

⇒ 6b – 3a – 3c + 9

⇒ 3[2b – a – c + 3]

$D_2 = \begin{vmatrix} 10 & a & 12 \\ 13 & b & 15 \\ 16 & c-3 & 18 \end{vmatrix} = \begin{vmatrix} 10 & a & 12 \\ 3 & b-a & 3 \\ 3 & c-b-3 & 3 \end{vmatrix} = \begin{vmatrix} 10 & a & 12 \\ 3 & b-a & 3 \\ 0 & c-2b+a-3 & 0 \end{vmatrix}$

= –(c – 2b + a – 3)[–6]⇒ 6(c + a – 2b – 3)

$$D_3 = \begin{vmatrix} 10 & 11 & a \\ 13 & 14 & b \\ 16 & 17 & c-3 \end{vmatrix} = \begin{vmatrix} 10 & 1 & a \\ 13 & 1 & b \\ 16 & 1 & c-3 \end{vmatrix} = \begin{vmatrix} 10 & 1 & a \\ 3 & 0 & b-a \\ 3 & 0 & c-b-3 \end{vmatrix}$$

$= -1[3c - 3b - 9 - 3b + 3a]$

$= [6b - 3c - 3a + 9]$

$= 3[2b + 3 - a - c]$

system will be consistent if $D = D_1 = D_2 = D_3 = 0$

$\Rightarrow 2b + 3 - a - c = 0$

Now $|X| = a(-1) - 2(-b) + 1(-c)$

$= -a + 2b - c$

$= -3$

Now plane P : $-x + 2y - z + 3 = 0$

50) $D = \begin{vmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{vmatrix} = 0$

$$D_1 = \begin{vmatrix} a & 11 & 12 \\ b & 14 & 15 \\ c-3 & 17 & 18 \end{vmatrix} = \begin{vmatrix} a & 11 & 1 \\ b & 14 & 1 \\ c-3 & 17 & 1 \end{vmatrix} = \begin{vmatrix} a & 11 & 1 \\ b-a & 3 & 0 \\ c-b-3 & 3 & 0 \end{vmatrix}$$

$\Rightarrow 3b - 3a - 3c + 3b + 9$

$\Rightarrow 6b - 3a - 3c + 9$

$\Rightarrow 3[2b - a - c + 3]$

$$D_2 = \begin{vmatrix} 10 & a & 12 \\ 13 & b & 15 \\ 16 & c-3 & 18 \end{vmatrix} = \begin{vmatrix} 10 & a & 12 \\ 3 & b-a & 3 \\ 3 & c-b-3 & 3 \end{vmatrix} = \begin{vmatrix} 10 & a & 12 \\ 3 & b-a & 3 \\ 0 & c-2b+a-3 & 0 \end{vmatrix}$$

$= -(c - 2b + a - 3)[-6] \Rightarrow 6(c + a - 2b - 3)$

$$D_3 = \begin{vmatrix} 10 & 11 & a \\ 13 & 14 & b \\ 16 & 17 & c-3 \end{vmatrix} = \begin{vmatrix} 10 & 1 & a \\ 13 & 1 & b \\ 16 & 1 & c-3 \end{vmatrix} = \begin{vmatrix} 10 & 1 & a \\ 3 & 0 & b-a \\ 3 & 0 & c-b-3 \end{vmatrix}$$

$= -1[3c - 3b - 9 - 3b + 3a]$

$= [6b - 3c - 3a + 9]$

$= 3[2b + 3 - a - c]$

system will be consistent if $D = D_1 = D_2 = D_3 = 0$

$\Rightarrow 2b + 3 - a - c = 0$

Now $|X| = a(-1) - 2(-b) + 1(-c)$

$= -a + 2b - c$

$= -3$

Now plane P : $-x + 2y - z + 3 = 0$

51) $f(x)f(y) + 2 = f(x) + f(y) + f(xy)$ ....(1)

putting $x = y = 1$

$f^2(1) + 2 = 3f(1)$
$f^2(1) - 3f(1) + 2 = 0$
$f(1) = 1, 2$
since f(x) is one-one & f(0) = 1
so f(1) = 2
replace y with $\frac{1}{x}$ in (1)
f(x) $f\left(\frac{1}{x}\right) + 2 = f(x) + f\left(\frac{1}{x}\right) + 2$
$f(x) = 1 \pm x^n$
but f(1) = 2 & f′(1) = 2
$\Rightarrow f(x) = 1 + x^2$

52) $f(x)f(y) + 2 = f(x) + f(y) + f(xy)$ ....(1)
putting x = y = 1
$f^2(1) + 2 = 3f(1)$
$f^2(1) - 3f(1) + 2 = 0$
$f(1) = 1, 2$
since f(x) is one-one & f(0) = 1
so f(1) = 2
replace y with $\frac{1}{x}$ in (1)
f(x) $f\left(\frac{1}{x}\right) + 2 = f(x) + f\left(\frac{1}{x}\right) + 2$
$f(x) = 1 \pm x^n$
but f(1) = 2 & f′(1) = 2
$\Rightarrow f(x) = 1 + x^2$

53) When n = 1, $a_1 = 5S_1 + 1 = 5a_1 + 1$
$\Rightarrow a_1 = \frac{-1}{4}$
for $n \geq 1$
$a_{n+1} - a_n = 5\,S_{n+1} - 5S_n$
$\Rightarrow a_{n+1} - a_n = 5a_{n+1}$
$\Rightarrow 4a_{n+1} = -a_n \Rightarrow \frac{a_{n+1}}{a_n} = \frac{-1}{4}$
there fore $\{a_n\}$ is a GP with $a_1 = \frac{-1}{4}$ & common ratio $r = \frac{-1}{4}$
So $a_{10} = \left(\frac{-1}{4}\right)\left(\frac{-1}{4}\right)^9 = \frac{1}{4^{10}}$
Also $a_{12} = \left(\frac{-1}{4}\right)\left(\frac{-1}{4}\right)^{11} = \frac{1}{4^{12}}$
So $b_{12} = \frac{4 + a_{12}}{1 - a_{12}} \Rightarrow \frac{4 + \frac{1}{4^{12}}}{1 - \frac{1}{4^{12}}} \Rightarrow \frac{4^{13} + 1}{4^{12} - 1}$

54) When n = 1, $a_1 = 5S_1 + 1 = 5a_1 + 1$

$\Rightarrow a_1 = \frac{-1}{4}$

for $n \geq 1$

$a_{n+1} - a_n = 5\,S_{n+1} - 5S_n$

$\Rightarrow a_{n+1} - a_n = 5a_{n+1}$

$\Rightarrow 4a_{n+1} = -a_n \Rightarrow \frac{a_{n+1}}{a_n} = \frac{-1}{4}$

there fore $\{a_n\}$ is a GP with $a_1 = \frac{-1}{4}$ & common ratio $r = \frac{-1}{4}$

So $a_{10} = \left(\frac{-1}{4}\right)\left(\frac{-1}{4}\right)^9 = \frac{1}{4^{10}}$

Also $a_{12} = \left(\frac{-1}{4}\right)\left(\frac{-1}{4}\right)^{11} = \frac{1}{4^{12}}$

So $b_{12} = \frac{4 + a_{12}}{1 - a_{12}} \Rightarrow \frac{4 + \frac{1}{4^{12}}}{1 - \frac{1}{4^{12}}} \Rightarrow \frac{4^{13} + 1}{4^{12} - 1}$

55) Let P_k denotes the probability that code A is used in the k^{th} week.

$P_{k+1} = \frac{1}{4}(1 - P_k)$

$P_{k+1} - \frac{1}{5} = \frac{-1}{4}\left(P_k - \frac{1}{5}\right)$

as $P_1 = 1$, $\left\{P_K - \frac{1}{5}\right\}$ is a GP with $\frac{4}{5}$ as the first term & $\left(\frac{-1}{4}\right)$ as the common ratio.

So $P_k - \frac{1}{5} = \frac{4}{5}\left(-\frac{1}{4}\right)^{k-1}$

So $P_9 = \frac{4}{5}\left(-\frac{1}{4}\right)^8 + \frac{1}{5} = \frac{3273}{16384}$

56) $z = e^{i\theta}$

let $y = |z^3 - z + \alpha|$

$y = |(\cos 3\theta + i \sin 3\theta) - (\cos\theta + i\sin\theta) + 2|$

$y^2 = (\cos 3\theta - \cos\theta + 2)^2 + (\sin 3\theta - \sin\theta)^2$

$y^2 = 4\cos 3\theta - 2\cos 2\theta - 4\cos\theta + 6$

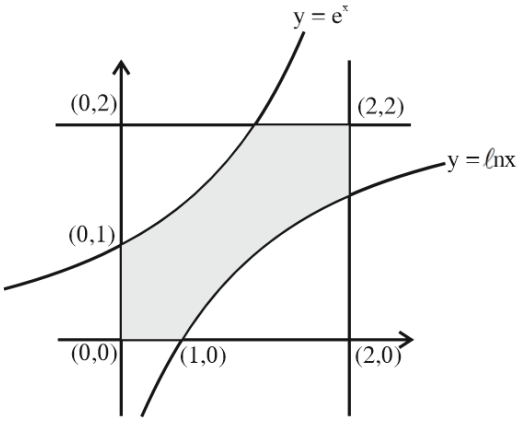
Let $\cos\theta = x$

$y^2 = 16x^3 - 4x^2 - 16x + 8$

so $x \in [-1, 1]$

$y^2_{max} = 13$

$y_{max} = \sqrt{13}$



57)

$$2\int_{1}^{2} \ell nx dx$$

Area = 4-

= 6 - 4ℓn2

a = 6, b = 4