FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
PART TEST – I
PAPER –2
TEST DATE: 17-11-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. C

Sol. Following graph shows the relative velocity of ‘A’ w.r.t ‘B’. Area under the graph represents the separation between the particles.

At t=2s, separation between the particles

$= \frac{1}{2} \times 2 \times 4 = 4\text{ m}$

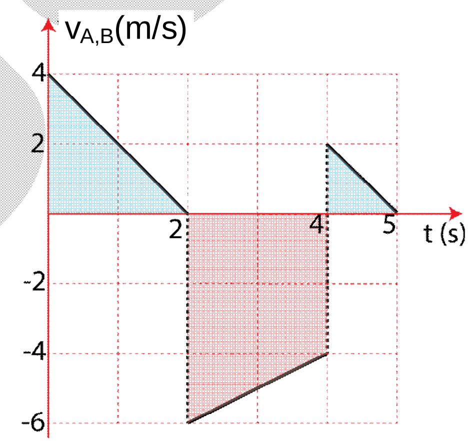
At t=4s, separation between the particles

$= \left|4 - \frac{1}{2}(6+4) \times 2\right| = 6\text{ m}$

= At t=5s, separation between the particles

$= \left|-6\text{m} + \frac{1}{2} \times 1 \times 2\right| = 5\text{ m}$

Hence, the maximum separation between the particles is 6 m.



2. D

Sol. $mgh = \mu mgd$

$\Rightarrow \mu = \frac{h}{d} = \frac{1}{2} = 0.5$

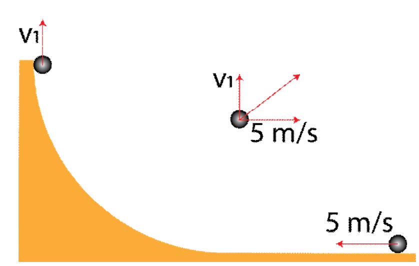
3. D

Sol. By Energy conservation from the frame of the truck, the velocity of the sphere when it leaves the truck can be calculated as:

$mgR + \frac{1}{2}mv_1^2 = \frac{1}{2}m \times 5^2 \Rightarrow v_1 = \sqrt{5}$ m/s upwards relative to truck.

From the ground frame, the speed of the sphere,

$v = \sqrt{(\sqrt{5})^2 + 5^2} = \sqrt{30}$ m/s



4. B

Sol. The centre of mass of m_B and m_D must lie at O. So, m_B=m_D=m. Now replacing the two masses at B and D with a single mass of 2m at O, we can conclude that m_A=2m, as the centre of mass of 2m and m_A lies at the midpoint of the two. Now we can replace the three masses with a mass of 4m at P. As the centre of mass of all four masses lies at Q, we can now write $4m \times 2d = m_C \times d$ $\Rightarrow m_C = 8m$. (d = AP = PO = OQ = QC)

5. B, C, D

Sol. $r_{CM} = \dfrac{2mR - mR + 0}{5m} = \dfrac{R}{5}$

For a pure rolling motion, $a = \alpha R$

Now,

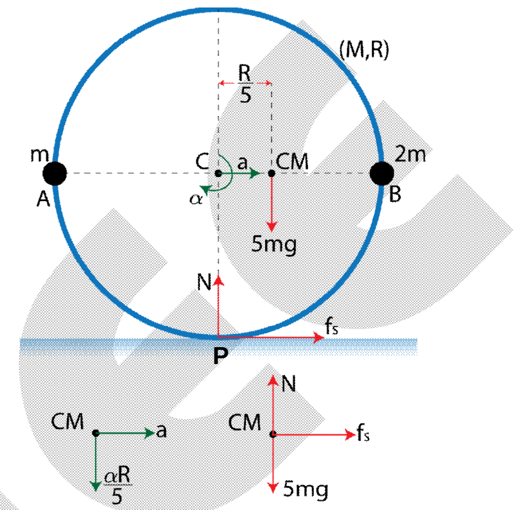
$I_P = 2MR^2 + 4mR^2 + 2mR^2 = 10mR^2$

Now, $\tau_p = I_P\alpha \Rightarrow 5mg\dfrac{R}{5} = 10mR^2\alpha$

$\alpha = \dfrac{g}{10R}$, $a = \alpha R \Rightarrow a = \dfrac{g}{10}$

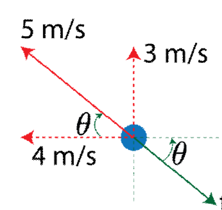
$f_s = 5ma = \dfrac{mg}{2}$

$5mg - N = 5m \times \dfrac{\alpha R}{5} = \dfrac{mg}{10} \Rightarrow N = \dfrac{49mg}{10}$



6. B, D

Sol. Kinetic friction acts opposite to the relative velocity of the disc w.r.t the belt. The figure shows the relative velocity of the disc w.r.t the belt and the direction of friction.



After sliding stops, the velocity of the disc relative to ground will be $4\hat{i}$ m/s.

By Work Energy theorem, work done by friction = $\dfrac{1}{2} \times 2(4^2 - 3^2) = 7\,J$

Relative to the belt, the disc will keep sliding until the relative velocity is 0. So,

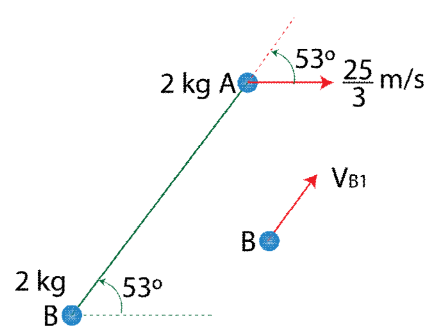
$0 = 5 - 0.25 \times 10t \Rightarrow t = 2s$

The radius of curvature initially = $\dfrac{3^2}{0.25 \times 10\cos\theta} = 4.5m$

7. B, C

Sol. Consider the collision between the balls, ‘A’ and ‘B’ first. The figure illustrates moments just before and after the string imparts an impulse to A and B. Since this is an elastic collision of the same mass, their velocities along the string must get exchanged. So,

$v_{B_1} = \dfrac{25}{3}\cos 53^\circ = 5$ m/s



Now consider collision between ‘B’ and ‘C’. The figure depicts just before and after the string imparts an impulse to B and C.

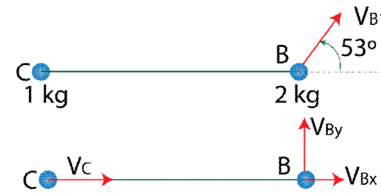
Using conservation of momentum,

$2v_{B_1}\cos 53^\circ = v_C + 2v_{Bx}$

$\Rightarrow v_C + 2v_{Bx} = 6$ …(i)

Equation for coefficient of restitution,

$$e = 1 = \frac{v_C - v_{Bx}}{v_{B_1}\cos 53^\circ} \Rightarrow v_C - v_{Bx} = 3 \qquad \ldots\text{(ii)}$$



$$v_{B_y} = v_{B_1}\sin 53^\circ = 4\text{ m/s}$$

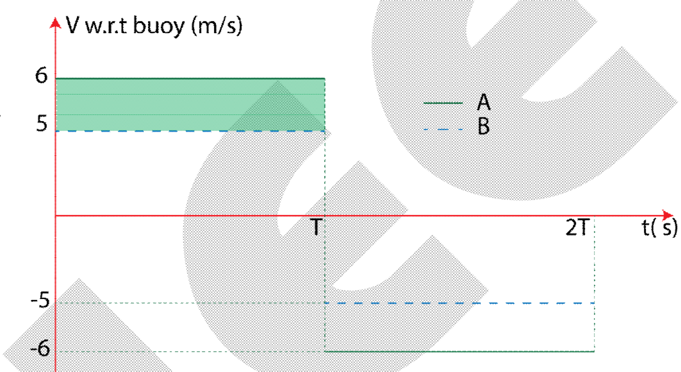
Solving the above equations, we get $v_C = 4\text{ m/s}$ and $v_{Bx} = 1$ m/s

So, $v_C = 4$ m/s and $v_{Bx} = 1$ m/s. So, $v_B = \sqrt{4^2 + 1^2} = \sqrt{17}$ m/s

SECTION – B

8. 400

Sol. Let at t=0, the buoy is dropped. Motorboat 'A' can move at a speed of 6 m/s and 'B' can move at a speed of 5 m/s relative to water. The graph below shows the relative speed of the boats with respect to the buoy.



After a time T, both the boats change direction and turn off the engine when they cross the buoy again. So, the Area under the graph of any of the curves must be zero. So, they cross the buoy again at t=2T.

The Area of the shaded portion gives a maximum distance of A w.r.t. B (After time T it will decrease as relative velocity is towards B). So

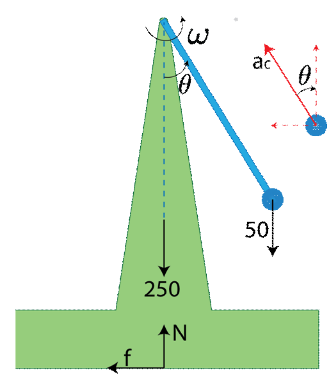
$1 \times T = 100$

$\Rightarrow T = 100$ s

The buoy has travelled for a time of 2T=200 s. Distance travelled by the buoy in that time period = $2 \times 200 = 400$ m

9. 5

Sol. When the rod makes an angle θ with the vertical, acceleration of centre of mass of the system in the vertical direction:



$$a_y = \frac{5\omega^2 \times 0.8\cos\theta}{25+5} = \frac{4\omega^2\cos\theta}{30}$$

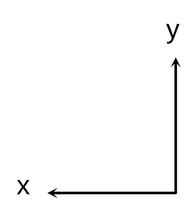
Acceleration of centre of mass of the system in the horizontal direction:

$$a_x = \frac{5\omega^2 \times 0.8\sin\theta}{25+5} = \frac{4\omega^2\sin\theta}{30}$$

Considering the whole as a system, we can write

$N - 300 = 30a_y = 4\omega^2 \cos\theta$

$f = 30a_x = 4\omega^2\sin\theta$



For limiting condition, $f = \mu N = \dfrac{N}{2\sqrt{2}}$

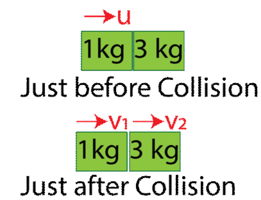
Solving the above equations, we get

$$\omega = \sqrt{\frac{300}{8\sqrt{2}\sin\theta - 4\cos\theta}}$$

$$\omega_{min} = \sqrt{\frac{300}{\sqrt{\left(8\sqrt{2}\right)^2 + 4^2}}} = \sqrt{\frac{300}{12}} = 5 \text{ rad/s}$$

10. 25

Sol. 25



The deceleration of the blocks due to friction is $\mu_k g = 8\ \text{m/s}^2$

The velocity of 1 kg block just before the collision

$$u = \sqrt{3^2 - 2\times 8\times 0.5} = 1\ \text{m/s}$$

By conserving momentum during the collision, we can write

$1u = 1v_1 + 3v_2 \Rightarrow v_1 + 3v_2 = 1$ …(i)

As the coefficient of restitution is 1, we can write

$$\frac{v_2 - v_1}{1} = 1 \Rightarrow v_2 - v_1 = 1 \quad \text{…(ii)}$$

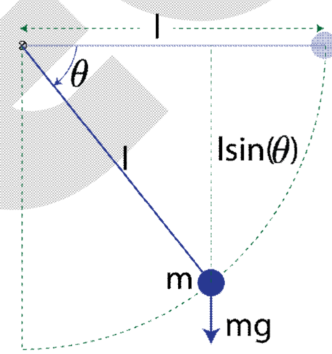
Solving the above equations, we get $v_1 = -0.5$ m/s and $v_2 = 0.5$ m/s.

The separation between the two blocks when they come to rest,

$$S = \frac{(0.5)^2}{2\times 8} + \frac{(0.5)^2}{2\times 8} = \frac{1}{32}\text{m} = \frac{100}{32}\text{cm} = \frac{25}{8}\text{cm}$$

11. 8

Sol. By conservation of energy, the angular speed of the rod after it rotates through an angle θ can be written as



$$\frac{1}{2}\times m\ell^2\omega^2 = mg\ell\sin\theta$$

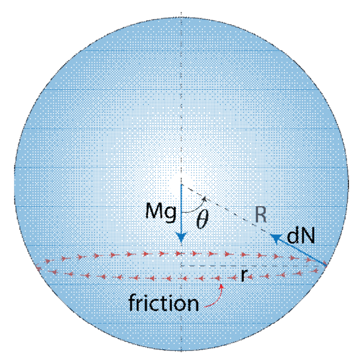
When $\ell = \ell_0$, $\omega_1(\theta) = \sqrt{\frac{2g}{\ell_0}\sin\theta}$

When $\ell = 4\ell_0$, $\omega_2(\theta) = \sqrt{\frac{2g}{4\ell_0}\sin\theta} = \frac{\omega_1(\theta)}{2}$

Angular velocity at any θ becomes half times the angular velocity in the first case. So, the time taken will be 2 times the time taken in the first case i.e., 8 s.

12. 15

Sol. $\int dN\cdot\cos\theta = Mg$



Torque due to friction about the vertical diameter

$$\tau = \int \mu dNR\sin\theta = \int \mu dNR\cos\theta\tan\theta = \mu MgR\tan\theta$$

Angular retardation, $\alpha = \frac{\mu MgR\tan\theta}{I}$

If initial angular speed is ω_0, time taken for it to completely stop can be calculated as

$$0 = \omega_0 - \alpha\Delta t$$

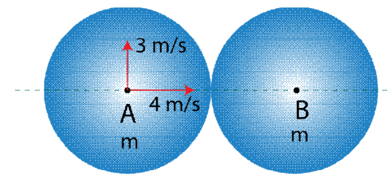
$$\Rightarrow \Delta t = \frac{\omega_0}{\alpha} = \frac{\omega_0 I}{\mu MgR\tan\theta}$$

$$\frac{\Delta t_1}{\Delta t_2} = \frac{\tan\theta_2}{\tan\theta_1} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

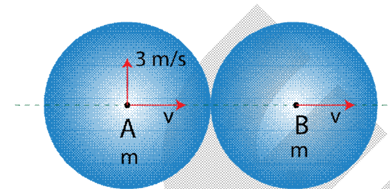
$$\Rightarrow \Delta t_2 = 3\times\Delta t_1 = 15\text{ sec}$$

13. 4

Sol. At the maximum deformation, the components of velocities of the balls along the line of impact must be equal.
Using the conservation of momentum of the system,
$mv_0 \cos 37 = 2mv$
$\Rightarrow v = \frac{2v_0}{5} = 2 \text{ m/s}$
The Kinetic energy of the striking ball that has turned into elastic potential energy at the maximum deformation
$= \frac{1}{2} \times 1\left[5^2 - (3^2 + 2^2 + 2^2)\right] = 4 \text{ J}$



Just before collision



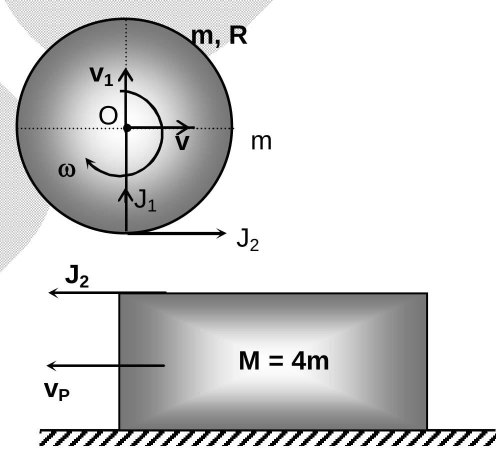
At the maximum deformation

SECTION – C

14. 12.50

15. 4.80

Sol. (for Q.14-15).
J_1= Impulse due to normal force on the sphere
J_2 = Impulse due to friction on the sphere
$v_1 = ev_0 \Rightarrow v_1 = 0.6v_0$
$J_1 = \int N dt = m(v_1 + v_0)$
$J_1 = 1.6\, mv_0$



Maximum impulse due to friction that can act on the sphere
$J_{max} = \int \mu N dt = \mu \int N dt = \mu J_1$
$J_{max} = 0.4 \times 1.6 mv_0 = 0.64 mv_0$
Let the sphere starts pure rolling motion during collision
$\omega R - v = v_P$ ...(i)
Using conservation of momentum of the system in horizontal direction
$mv - 4mv_P = 0$
$v = 4v_P$ ...(ii)
using conservation of angular momentum of the hollow sphere about an axis passing through contact point and fixed to the ground.
$\frac{2}{3} mR^2 \omega_0 = \frac{2}{3} mR^2 \omega + mvR$
$\Rightarrow 2v_0 = 6\omega R + 9v$ ...(iii)
Solving equation (i) and (ii), we get
$\omega R = \frac{5v}{4} \Rightarrow \omega = \frac{5v}{4R}$ ...(iv)
Solving equation (iii) and (iv)
$2v_0 = 6 \times \frac{5v}{4} + 9v$

$$2v_0 = \frac{15v}{2} + 9v$$

$$\Rightarrow v = \frac{4v_0}{33} \Rightarrow \omega = \frac{5v_0}{33R} = \frac{5\times16.5}{33\times0.2} = 12.5 \text{ rad/s}$$

The velocity of plank just after collision

$$v_P = \frac{v}{4} \Rightarrow v_P = \frac{v_0}{33} = \frac{1}{2} \text{ m/s}$$

Impulse due to friction, $J_2 = mv = \frac{4mv_0}{33} = \frac{4\times2.4\times16.5}{33} = 4.80$ N-s

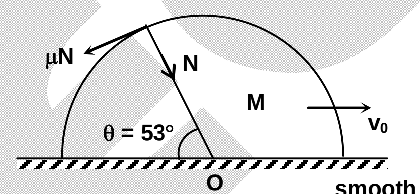
Since, $J_2 < J_{max}$

The sphere will start pure rolling motion during collision.

16. 5.00

17. 15.00

Sol. (for Q. 16-17).



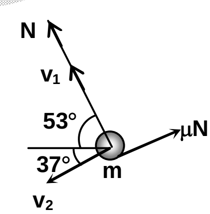
$$\int N dt = m(v_1 + 20) \qquad \ldots(i)$$

$$\mu\int N dt = m(15 - v_2) \qquad \ldots(ii)$$

From (i) and (ii)

$$\mu m(v_1 + 20) = m(15 - v_2)$$

$$\frac{3}{29}(v_1 + 20) = 15 - v_2 \qquad \ldots(iii)$$



Using conservation of momentum of the system along horizontal direction

$$m(v_1\cos53° + v_2\cos37°) = Mv_0$$

$$\frac{3v_1 + 4v_2}{5} = 3v_0 \Rightarrow 3v_1 + 4v_2 = 15v_0 \qquad \ldots(iv)$$

$$e = \frac{v_0\cos53° + v_1}{u\cos37°}$$

$$\Rightarrow 0.6 = \frac{0.6v_0 + v_1}{20}$$

$$\Rightarrow v_0 = \left(\frac{12 - v_1}{0.6}\right) \qquad \ldots(v)$$

Solving equation (iv) and (v) we get

$$3v_1 + 4v_2 = 15\left(\frac{12 - v_1}{0.6}\right)$$

$28v_1 + 4v_2 = 300$

$\Rightarrow 7v_1 + v_2 = 75$ ...(vi)

Solving equation (iii) and (vi), we get

$$\frac{3}{29}(v_1 + 20) = 15 - (75 - 7v_1)$$

$$3v_1 + 60 = (7v_1 - 60)29$$

$\Rightarrow v_1 = 9$ m/s

From equation (vi),

$v_2 = 75 - 7v_1$

$\Rightarrow v_2 = 75 - 63$

$v_2 = 12$ m/s

from equation (v)

$$v_0 = \left(\frac{12 - v_1}{0.6}\right)$$

$$v_0 = \left(\frac{12 - 9}{0.6}\right)$$

$v_0 = 5$ m/s

The velocity of hemispherical body just after collision, $v_0 = 5$ m/s

The velocity of the particle just after collision,

$$v = \sqrt{v_1^2 + v_2^2} = \sqrt{(9)^2 + (12)^2}$$

v = 15 m/s

Chemistry
PART – II

SECTION – A

18. B

Sol. Pyrosilicates have general formula $(Si_2O_7)^{6-}$

19. A

Sol. $B_2O_3 + P_2O_5 \longrightarrow 2BPO_4$

20. C

Sol. Atomic radius Ga (135 pm), Al (143 pm).
Oxidation state of Thallium in $T\ell I_3$ is +1
Boron does not form BF_6^{3-} ion due to non-availability of d-orbitals.
H_3BO_3 is a monobasic acid.

21. C

Sol.
$$r_1 = k\left[\frac{a}{V}\right]^2\left[\frac{b}{V}\right]$$
$$r_2 = k\left[\frac{4a}{V}\right]^2\left[\frac{4b}{V}\right]$$
$$r_2 = 64k\left[\frac{a}{V}\right]^2\left[\frac{b}{V}\right]$$
$$r_2 = 64r_1$$

22. A, D

Sol. Peroxide reacts with dilute H_2SO_4 to form hydrogen peroxide.
$$SnO_2 + 2H_2SO_4 \longrightarrow Sn(SO_4)_2 + 2H_2O$$
$$2MnO_2 + 2H_2SO_4 \longrightarrow 2MnSO_4 + 2H_2O + O_2$$

23. A, C

Sol. Bond angle $NF_3(102^\circ)$, $NH_3(107^\circ)$
$H_2O(104.5^\circ)$, $H_2S(92^\circ)$
$F_2O(103^\circ)$, $Cl_2O(111^\circ)$

24. A, B, D

Sol.
$$Be_2C + 4H_2O \longrightarrow 2Be(OH)_2 + CH_4$$
$$CaC_2 + N_2 \xrightarrow{1100^\circ C} \underset{\text{Calcium cyanamide}}{CaCN_2} + C$$

SECTION – B

25. 8

Sol.
$$PCl_3(g) \quad + Cl_2(g) \rightleftharpoons PCl_5(g)$$
$$P(1-0.75) \quad P(1-0.75) \quad 0.75P$$
$$K_P = \frac{0.75P}{0.25P \times 0.25P}$$
$$3 = \frac{12}{P}$$
P = 4
Initial total pressure of the mixture = 2P = 2 × 4 = 8 atm

26. 40

Sol.
$$\frac{t_1}{t_2} = \frac{\ell n\frac{1}{1/3}}{\ell n\frac{1}{1/9}}$$
$$\frac{20}{t_2} = \frac{\ell n3}{2\ell n3}$$
$$t_2 = 40 \text{ minutes}$$

27. 3

Sol. In the following molecules/ions the hybridization of central atom is sp^3d^2
SF_6, XeF_4 and BrF_5

28. 4

Sol. $H_2S_2O_5$, $H_2S_2O_4$, $H_2S_2O_6$, $H_2S_4O_6$

29. 6

Sol. NH_3 and ClO_4^- does not undergo disproportionation.

30. 64

Sol.
$$Ca_3(PO_4)_2(s) \rightleftharpoons 3Ca^{2+}(aq) + 2PO_4^{3-}(aq)$$
$$3S \qquad 2S$$
$$2S = 4 \times 10^{-5} \text{ M}$$
$$S = 2 \times 10^{-5} \text{ M}$$
$$K_{SP} = 108\ S^5$$
$$= 108 \times (2 \times 10^{-5})^5$$
$$= 108 \times 32 \times 10^{-25}$$
$$\therefore y = 108 \times 32$$
$$\frac{y}{54} = 64$$

SECTION – C

31. 61.80

32. 13.92

Sol. (for the Q. No. 31 and 32).

$$2BF_3 + 6NaH \xrightarrow{450K} \underset{(P)}{B_2H_6(g)} + \underset{(Q)}{6NaF}$$

$$\underset{(P)}{B_2H_6} + 6H_2O \longrightarrow \underset{(R)}{2H_3BO_3} + 6H_2$$

$$\underset{(P)}{B_2H_6} + 3O_2 \longrightarrow \underset{(M)}{B_2O_3} + \underset{(N)}{3H_2O}$$

Molecular weight of $R(H_3BO_3)$ is 61.8

27.6g B_2H_6 produces B_2O_3 = 69.6 g

5.52 g B_2H_6 produces $B_2O_3 = \frac{69.6 \times 5.52}{27.6}$

= 13.92 g

33. 5.34

Sol.

	CH_3COOH +	NaOH $\longrightarrow$	$CH_3COONa + H_2O$
Initial m. mol	$40 \times \frac{1}{10}$ $= 4$	$32 \times \frac{1}{10}$ $= 3.2$	0
Final m.mol	0.8	0	3.2

The resulting solution is an acidic buffer.

$$pH = pK_a + \log\frac{[CH_3COONa]}{[CH_3COOH]}$$

$$pH = 4.74 + \log\frac{3.2}{0.8}$$

pH = 4.74 + 0.6 = 5.34

34. 8.72

Sol.

	CH_3COOH +	NaOH $\longrightarrow$	$CH_3COONa + H_2O$
Initialm. mol	$40 \times \frac{1}{10}$ $= 4$	$40 \times \frac{1}{10}$ $= 4$	0
Final m.mol	0	0	4

$$[CH_3COONa] = \frac{4}{80} = 5 \times 10^{-2}\,M$$

$$\therefore pH = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log C$$

$$= 7 + \frac{4.74}{2} + \frac{1}{2}\log\left(5 \times 10^{-2}\right)$$

= 8.72

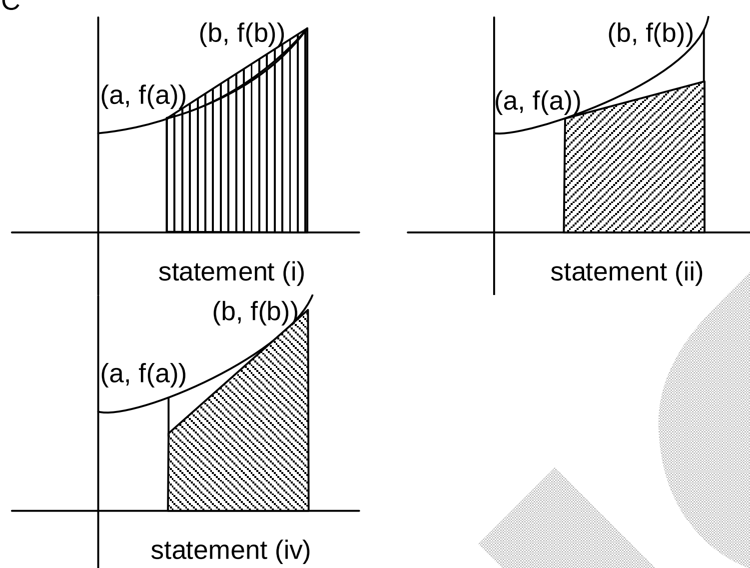
Mathematics

PART – III

SECTION – A

35. C

Sol.



Statement (iii) $f\left(\frac{a+b}{2}\right) = f\left(\frac{\lambda a+(1-\lambda)b+(1-\lambda)a+\lambda b}{2}\right) \le \frac{f(\lambda(a)+(1-\lambda)b)+f((1-\lambda)a+\lambda(b))}{2}$

$\Rightarrow \int_0^1 f\left(\frac{a+b}{2}\right)dx \le \frac{1}{2}\int_0^1 \left(f(\lambda a+(1-\lambda)b)+f((1-\lambda)a+\lambda(b))\right)d\lambda$

$= \int_0^1 f((1-\lambda)a+\lambda b)d\lambda = \frac{1}{b-a}\int_a^b f(x)dx$.

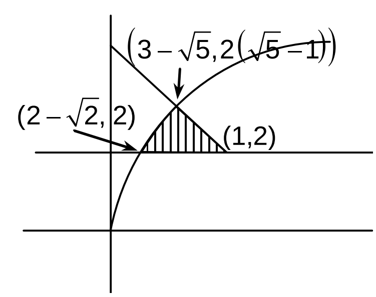
36. C

Sol.

$g(x) = 4 - 2x,\ 0 \le x \le 2,\ f(x) = 4x - x^2$

required area $\int_{2-\sqrt{2}}^{3-\sqrt{5}} (4x - x^2)\,dx + \int_{3-\sqrt{5}}^{1} (4-2x)dx - 2(\sqrt{2}-1)$

$= \frac{17-10\sqrt{5}+4\sqrt{2}}{3}$.



37. C

Sol.

$P_n \equiv (n, n^2)$ and $P_{n+1} \equiv ((-n-1), (n+1)^2)$ when n is odd

or $P_n = (-n, n^2)$ and $P_{n+1} \equiv ((1+n), (n+1)^2)$ when is even

$\lim\limits_{n\to\infty} \frac{d_n}{n} = \lim\limits_{n\to\infty} \frac{\sqrt{8n^2+8n+2}}{n} = 2\sqrt{2}$.

38. C

Sol. R′ = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (1, 2), (2, 1), (4, 6), (6, 4), (5, 6), (6, 5), (4, 5), (5, 4)}

39. A, C

Sol. f(x) can't be constant function as $|f'(1)| = 3$. If y = z = 1, f(x) = f(x) f(1)^2

$\Rightarrow$ f(1) = ± 1 solving diff. eq. use get

$f(x) = x^3, x \in R$, $f(x) = -x^3 \ x \in R$

$$f(x) = \begin{cases} x^3, & x \ge 0 \\ -x^3, & x < 0 \end{cases}, \ f(x) = \begin{cases} -x^3, & x \ge 0 \\ x^3, & x < 0 \end{cases}.$$

40. A, B, C

Sol. If f′(x) = 0 equality holds. If $f'(x) \ne 0$, we apply mean value theorem taking $x + f'(x)$ and x as end pts of interval

$$\frac{f(x + f'(x)) - f(x)}{f'(x)} = f'(c)$$

$$f(x + f'(x)) - f(x) = f'(c)f'(x) > 0$$

$\Rightarrow$ $f(x + f'(x)) - f(x) > 0$

$\Rightarrow$ only possibility is f′(x) = 0.

41. A, B, C

Sol.
$$\int_0^1 \frac{\sqrt{1-x} + \sqrt{x}}{1 + \sqrt{2x}} f(x)\,dx = \int_0^1 \frac{\sqrt{2x} - 1}{\sqrt{x} - \sqrt{1-x}} f(x)dx$$

$$= \frac{1}{2}\int_0^1 \left(\frac{\sqrt{2x} - 1}{\sqrt{x} - \sqrt{1-x}} f(x) + \frac{\sqrt{2(1-x)} - 1}{\sqrt{1-x} - \sqrt{x}} f(1-x) \right) dx$$

$$= \frac{1}{2}\int_0^1 \frac{f(x)\sqrt{2}}{\sqrt{x} - \sqrt{1-x}} \left(\sqrt{x} - \sqrt{1-x}\right) dx \quad \text{if } (f(1-x) = f(x))$$

$$= \frac{1}{\sqrt{2}} \int_0^1 f(x)\,dx.$$

SECTION – B

42. 4

Sol.
$$p = \left(\lim_{y\to\infty} \left(\frac{2}{y^2} \left(\lim_{z\to\infty} \frac{1}{z^4} \lim_{t\to 0} \frac{(z^2+z+1)(y^2+y+1)\left((y^2+y+1)(z^2+z) - (y^2+y)(z^2+z+1)\right)}{2} \right) \right) \right)^y$$

$$p = \lim_{y\to\infty} \left(\frac{2}{y^2} \left(\lim_{z\to\infty} \left(\frac{(z^2+z+1)(y^2+y+1)}{z^4} \frac{\left((z^2+z) - (y^2+y)\right)}{2} \right) \right) \right)^y$$

$$\lim_{y\to\infty} \left(\frac{2}{y^2} \frac{(y^2+y+1)}{2} \right)^y = e^{\lim\limits_{y\to\infty} y\left(\frac{y^2+y+1}{y^2} - 1\right)} = e$$

$$q = \lim_{n\to\infty} \left(\prod_{r=1}^{n} \left(\frac{n+r}{n} \right) \right)^{1/n} = e^{\int_0^1 \ln(1+x)dx} = e^{\ln 4 - 1} \Rightarrow p.q = 4$$

43. 8

Sol. $g(x) = \frac{x^2-1}{x-2} + \frac{x^2-1}{2x^2-x}$

taking x – 2 = t, the expression is $\left(t+\frac{3}{t}+4\right)+\frac{t+\frac{3}{t}+4}{2\left(t+\frac{3}{t}\right)+7}$,

again putting $2\left(t+\frac{3}{t}\right)+7=p$, expression changes to $\frac{1}{2}\left(p+\frac{1}{p}+2\right)$ where $p \ge 4\sqrt{3}+7$

$h(p)=\frac{1}{2}\left(p+\frac{1}{p}+2\right), h'(p)=\frac{1}{2}\left(1-\frac{1}{p^2}\right)>0$

minimum h(p) = $h\left(4\sqrt{3}+7\right)=\frac{1}{2}\left(7+4\sqrt{3}+\frac{1}{7+4\sqrt{3}}+2\right)$ = 8

44. 4048

Sol. $e^{\frac{-x^2}{2}}P_{n+1}(x)=\left(P_n(x)e^{\frac{-x^2}{2}}\right)'$

$\Rightarrow \quad P_n(x)e^{-\frac{x^2}{2}}=\frac{d^n}{dx^n}\left(P_0(x)e^{-\frac{x^2}{2}}\right)$

$\Rightarrow \quad P_n(0)$ = constant term in $\frac{d^n}{dx^n}\left(e^{-\frac{x^2}{2}}\right)=\left(-\frac{1}{2}\right)^{n/2}\frac{n!}{\left(\frac{n}{2}\right)!}$ (where n is even).

45. 16

Sol. Let f(x) = ax^3 + bx and tangent at $\left(\alpha, a\alpha^3+b\alpha\right)$ intersect at $\left(\beta, a\beta^3+b\beta\right)$,

then $3a\alpha^2+b=a\left(\alpha^2+\beta^2+\alpha\beta\right)+b$

$\Rightarrow \quad (\alpha-\beta)(2\alpha+\beta)=0, \beta=-2\alpha$

$A_1 = \int_{-2\alpha}^{\alpha}\left(ax^3+bx-\left(3a\alpha^2+b\right)x+2a\alpha^3\right)dx = k\alpha^4$

$A_2 = \int_{-2\beta}^{\beta}\left(ax^3-3a\beta^2x+2a\beta^3\right)dx = k\beta^4 = 16k\alpha^4$.

46. 5

Sol. $p=2^{k_0}p_1^{k_1},p_2^{k_2}....p_n^{k_n}$ where $p_i \in$ odd prime

Total number of even divisors $k_0(k_1+1)(k_2+1)....(k_n+1)$

Total number of odd divisors $(k_1+1)(k_2+1)....(k_n+1)$

$f(p)=(k_0-1)(k_1+1)(k_2+1)....(k_n+1)=4$

$k_0 = 2, k_i = k_j = 1$, p = 4.3.5 = 60

$k_0 = 3, k_i = 1$, p = 24, 40, 56

$k_0 = 5$, p = 32.

47. 3

Sol. $\frac{1}{x^2}\frac{dx}{dy}+\frac{y}{x}=\left(1+\frac{1}{y^2}\right)$

Let $\frac{1}{x} = t$

$$\frac{dt}{dy} - yt = -\left(1+\frac{1}{y^2}\right),\ te^{-\frac{y^2}{2}} = -\int\left(1+\frac{1}{y^2}\right)e^{-\frac{y^2}{2}}dy$$

$$\frac{1}{x} = \frac{1}{y} + ce^{\frac{y^2}{2}},\ f(1, 2) = 0 \Rightarrow c = \frac{1}{2}e^{-2}$$

$$\frac{2(y-x)}{xy} = e^{\left(\frac{y^2}{2}-2\right)}$$

hyperbola $\frac{2(y-x)}{xy} = 1$ & $\frac{2(y-x)}{xy} = e^{\left(\frac{y^2}{2}-2\right)}$

intersect where y = ± 2 so point is (1, 2).

SECTION – C

48. 0.75

49. 25.25

Sol. $$I(n, \lambda) = \int_0^{\pi/2} (\sin x \cos x)^n \frac{(\cos^n x + \lambda \sin^2 x)}{(\lambda + \sin^n x + \cos^n x)} dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (\sin x \cos x)^n \frac{(\cos^n x + \lambda \sin^2 x + \sin^n x + \lambda \cos^2 x)}{\lambda + \sin^n x + \cos^n x} dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (\sin x \cos x)^n dx = \frac{1}{2^{n+1}}\int_0^{\pi/2} \sin^n 2x\ dx =$$

$$\frac{1}{2^{n+1}}\int_0^{\pi/2} \sin^n t\ dt = \frac{1}{2^{n+1}} \frac{(n-1)(n-3)....2}{n(n-2)(n-4)...1} = \frac{1}{4n\ {}^{n-1}C_{\frac{n-1}{2}}} \text{ (when n is odd).}$$

50. 82.50

51. 18.00

Sol. f(x) can have roots {0, 1} or {0, –1}

$f(x) = x(x + 1)(x - \alpha)$, $f'(x) = x(x+1) + (x - \alpha)(2x+1)$

$$f'\left(-\frac{1}{4}\right) = -\frac{1}{4} \Rightarrow \alpha = -\frac{1}{8}$$

$f(x) = x(x + 1)\left(x + \frac{1}{8}\right)$.