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IIT-JEE

Batch – Growth (July) | Major Test – 3 |Paper - 2

Time: 3 Hours **Test Date: 12th January 2025** **Maximum Marks: 180**

Name of the Candidate: _____Roll No._____

Centre of Examination (in Capitals):_____

Candidate's Signature: _____ Invigilator's Signature:_____

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space)
   on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 54 questions.
3. This question paper is divided into three parts **PART A - PHYSICS, PART B - CHEMISTRY** and **PART C - MATHEMATICS** having 18 questions each and every **PART** has three sections.
   - **(i)** **Section-I** contains **6** Question One or More than One Correct
     Marking scheme: +4 for correct answer, +1 if not attempted and -2 in all other cases.
   - **(ii)** **Section-II** contains **6 Question** Numerical Value Paragraph-1 (Steam), (If more than two decimal, truncate/roundoff the value two decimal places).
     Marking scheme: (+2 for correct answer, O, if not attempted and 0 partial marking 0 in all other cases.
   - **(iii)** **Section-III** contains **4 Question** Paragraph-1 Based (Single Correct Type)
     Marking scheme: +3 for correct answer, -1 if not attempted and in all other cases.
   - **(iv)** **Section-IV** contains **3 Question** Non-Negative Integer
     Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For integer-based questions, the answer should be in decimals only not in fraction.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

TEST CODE : 114106

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TEST SYLLABUS

Batch – Growth (July) | Major Test-03

12th January 2025

Mathematics: Circle Binomial Theorem Permutation & Combination, (Probability-NCERT)

Physics: Rotational Motion Gravitation Elasticity, Thermal Expansion, Calorimetry and Heat Transfer

Chemistry: Chemical Eq Ionic Eq Redox Reaction, Nomenclature

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821$ Lit atm $K^{-1}mol^{-1}$
		$= 1.987 \approx 2$ Cal $K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}$ Js
		$= 6.25 \times 10^{-27}$ erg.s
1 Faraday		= 96500 Coulomb
1 calorie		= 4.2 Joule
1 amu		$= 1.66 \times 10^{-27}$ kg
1 eV		$= 1.6 \times 10^{-19}$ J

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:

Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I (One or More than One Correct)

1. If $n = 4^5.3^7.5^{11}$ then which of the following is/ are true

(A) The number of divisors of n which are perfect squares is 144

(B) The number of proper divisors of n is 1054

(C) The number of odd divisors of n is 96

(D) The number of even divisors of n is 958

Ans. (A, B, C)

Sol $n = 2^{10}\ 3^7\ 5^{11}$

Total divisors = (10 + 1) (7 + 1) (11 + 1) = 1056

(A) For perfect square we can choose from $(2^0, 2^2, 2^4, 2^6, 2^8, 2^{10}, 3^0, 3^2, 3^4, 3^6, 5^0, \ldots 5^{10})$

= 6 × 4× 6 = 144

(B) Proper divisiors = 1056-2= 1054

(C) Odd divisors = (7 + 1) (11 + 1) = 96

(D) Even divisors = 1056-96 = 960

Hence, (A), (B) and (C) are correct.

2. The value of $^{1000}C_{50} + {}^{999}C_{49} + {}^{998}C_{48} + \ldots\ldots + {}^{950}C_0$ is

(A) $^{1001}C_{50}$

(B) $^{1002}C_{951} - {}^{1001}C_{51}$

(C) $^{1001}C_{951}$

(D) $^{1002}C_{51} - {}^{1001}C_{950}$

Ans. (A, B, C, D)

Sol. $^{1000}C_{50} + {}^{999}C_{49} + {}^{998}C_{48} + \ldots\ldots + {}^{950}C_0$ = coefficient of x^{950} in $\left\{(1+x)^{950} + (1+x)^{951} + \ldots\ldots + (1+x)^{1000}\right\}$

= coefficient of x^{950} in $(1+x)^{950}\ \dfrac{\left\{(1+x)^{51} - 1\right\}}{1+x-1}$

= coefficient of x^{950} in $\dfrac{\left\{(1+x)^{1001} - (1+x)^{950}\right\}}{x}$

$= {}^{1001}C_{951} = {}^{1001}C_{50} = {}^{1002}C_{951} - {}^{1001}C_{51} = {}^{1002}C_{51} - {}^{1001}C_{950}$,

Since $^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$.

Hence, the correct answer is (A, B, C, D).

3. $C_1 : x^2 + y^2 = 25$ and $C_2 : x^2 + y^2 - 2x - 4y - 7 = 0$ be two circles intersecting at the points A and B.

(A) Equation of common chord must be $x + 2y - 9 = 0$

(B) Equation of common chord must be $x + 2y + 7 = 0$

(C) Tangents at A and B to the circle C_1 intersect at $\left(\dfrac{25}{9}, \dfrac{50}{9}\right)$

(D) Tangents at A and B to the circle C_1 intersect at $(1,2)$

Ans. (A, C)

Sol. Let the tangents intersect at (α, β), then the line joining the two points at which tangents are drawn is:

(i) Common chord of C_1 and C_2 is $C_1 - C_2 = 0 \Rightarrow 2x + 4y - 18 = 0$.

(ii) Chord of contact when tangents are drawn from (α, β) to C_1. $\Rightarrow$ Equations $2x + 4y - 18 = 0$ and $ax + by - 25 = 0$ must be identical on comparing, we get $\frac{\alpha}{2} = \frac{\beta}{2} = \frac{25}{18}$

Hence, (A) and (C) are correct.

4. If $\left(4 + \sqrt{15}\right)^n = I + f$ where n is odd natural number, I is an integer and $0 < f < 1$ then

(A) I is a natural number

(B) I is an even integer

(C) $(I + f)(1 - f) = 1$

(D) $(I + f)(2 - f) = 1$

Ans. (A, C)

Sol. $I + f = \left(4 + \sqrt{15}\right)^n$

Let $f' = \left(4 - \sqrt{15}\right)^n \quad 0 < f' < 1$

$I + f = C_0 4^n + C_1 4^{n-1}\sqrt{15} + C_2 4^{n-2}.15 \ldots\ldots\ldots\ldots$

$I + f + f' = \left[{}^nC_0' 4^n + {}^nC_2 4^{n-2} 15 + \ldots\right] =$ an even integer

$f + f' = 1 \Rightarrow f' = 1 - f$

$I \rightarrow$ odd Integer

$(I + f)(1 - f) = 1$

Hence, the correct answer is (A, C).

5. Consider the following statements

(i) In a 12 storeyed house, 10 people enter the lift cabin at ground floor. It is known that they will leave lift in groups of particular 2, 3 and 5 people at different storey. The number of ways this can be done if the lift does not stop at first and second floors is 720.

(ii) Each of three ladies have brought their one child for admission to a school. The principal wants to interview the six persons one by one, subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is 90.

(iii) The number of ways in which one can put three balls numbered 1, 2, 3 in three boxes labelled a, b, c such that at most one box is empty is equal to 18.

(iv) A box contains5 different red balls and 6 different white balls. The total number of ways in which 4 ball can be selected, taking atleast 1 ball of each colour is 310.

(A) Statements (i),(ii) are correct.

(B) Statements (ii) and (iv) are correct.

(C) Statements (i) and (iii) are correct.

(D) All statements are correct.

Ans. (A, B)

Sol. (A) The number of ways $= {}^{10}C_3 \times 3! = 720$

(B) Each lady and her child can be arranged in a fixed order only.

$\therefore$ The toal number of ways in which interview can be held $= \frac{6!}{2!2!2!} = 90$

(C) Case l :

No box emply

Then he number of ways $= 3! = 6$

Case ii :

If one of the boxes is empty, then number of ways $= {}^{3}C_{1}\left(2^{3}-2\right) = 18$

$\therefore$ total rumber of ways $= 6 + 18 = 24$

(D) Total-(All red)-(All white)

${}^{11}C_{4} - {}^{5}C_{4} - {}^{6}C_{4} = 330 - 5 - 15 = 310$

Hence,(A)and(B)are correct.

6. Equations of the circles concentric with the circle $x^2 + y^2 - 2x - 4y = 0$ and touching the circle $x^2 + y^2 + 2x = 1$, will be:

(A) $x^2 + y^2 - 2x - 4y = 0$

(B) $x^2 + y^2 - 2x - 4y + 3 = 0$

(C) $x^2 + y^2 - 2x - 4y - 13 = 0$

(D) $x^2 + y^2 - 2x - 4y - 1 = 0$

Ans. (B, C)

Sol. Centre of the circle $C_1 = (1,2)$

Centre of 2nd circle $= (-1,0)$ and radius $= \sqrt{2}$

The distance between centres $= 2\sqrt{2}$,which must be either $r_1 + r_2$ or $r_1 - r_2$

$\Rightarrow r = \sqrt{2}$ or $3\sqrt{2}$

Hence,(B)and(C)arecorrect.

SECTION – II Numerical Value

Paragraph-1 (Steam)

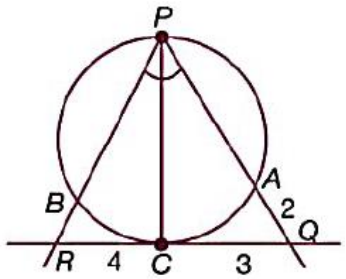
(If more than two decimal, truncate/roundoff the value two decimal places).

In a circle with centre O′,PA and PB are two chords drawn from point P which lies on circle PC is the chord that bisects the $\angle APB$. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R, respectively. If $QC = 3$, $QA = 2$ and $RC = 4$, then answer the following questions.

7. It the length of PQ is p then value of 2p is

Ans. (9)

Sol.



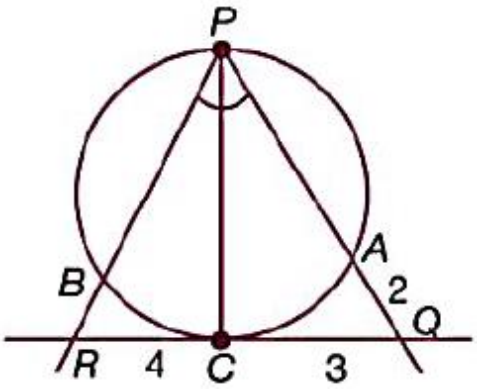
QA.QP = QC.QC

$2.QP = 3^2 \Rightarrow PQ = \frac{9}{2}$

8. The value of PQ:PR is p then value of 12p is

Ans. (9)

Sol.



$\because$ PQ : PR = QC : RC

PQ : PR = 3 : 4

Paragraph-2

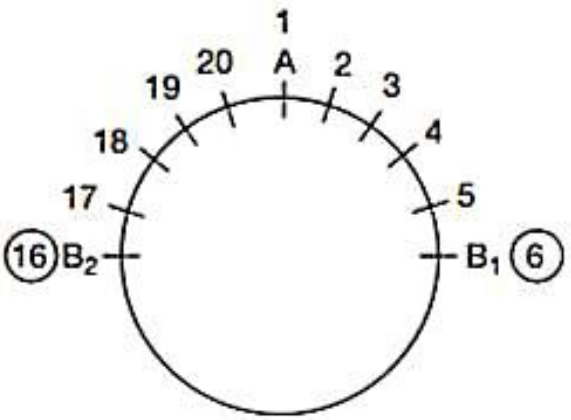
Twenty persons among whom are A and B, sit at random at a round table.

9. The probability that there are exactly 4 persons between A and B is p, then 19p is

Ans. (2)

Sol. If A occupies any place, B has 19 places to choose. But if there are to be exactly 4 persons between A and B, B can choose only two places $B_1(6), B_2(16)$ marked in the Figure.

$\therefore P = \frac{2}{19}$



10. The probability that there are at least 4 persons between A and B is p then 19p is

Ans. (11)

Sol. Now B can occupy any one of the 11 places 6^{th} to 16^{th}

$\therefore P = \frac{11}{19}$

Paragraph-3

Suppose there are 5 mangoes, 4 apples and 3 oranges in a bag, fruits of same variety being identical. Then

11. The number of ways few fruits are selected is

Ans. (119)

Sol. Req selections $= 6 \times 5 \times 4 - 1 = 119$

12. The number of ways can a selection of fruits be made if at least 2 mangoes be included is

Ans. (80)

Sol. Req. selections $= 4 \times 5 \times 4 = 80$

SECTION – III

Paragraph-1 Based (Single Correct Type)

Two circles touch each other externally. Equation of the pair of direct common tangents is $x^2 + 4xy + y^2 = 0$ and the equation of the smaller circle is $x^2 + y^2 - 4x + 4y + 6 = 0$.

13. Length of the direct common tangents of the two circles is:

(A) $4\sqrt{2}$

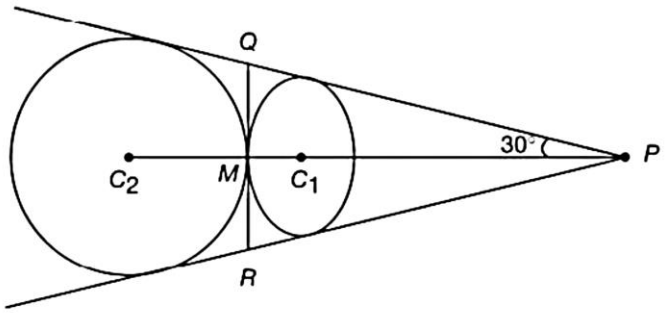
(B) $6\sqrt{3}$

(C) $2\sqrt{6}$

(D) $3\sqrt{2}$

Ans. (C)

Sol. Angle between the pair of lines is 60°.
The triangle formed by the common tangents is equilateral.



The given circle is the incircle of the $\Delta PQR \Rightarrow r = \sqrt{2}$

Radius of the other circle is $r_1 = 3\sqrt{2}$

Length of the direct common tangents $= 2\sqrt{rr_1} = 2\sqrt{6}$

Hence, the correct answer is (C).

14. Area of the triangle formed by the common tangents of the two circles is:

(A) $2\sqrt{6}$ sq. units

(B) $6\sqrt{3}$ sq. units

(C) $4\sqrt{3}$ sq. units

(D) $4\sqrt{2}$ sq. units

Ans. (B)

Sol. $\sin 30^\circ = \frac{r}{C_1P}$ or $C_1P = 2r$, M is the midpoint of QR Area of the triangle is

$$\frac{1}{2}QR \cdot PM = \frac{1}{2}(2QM)\cdot(3r) = \frac{3r}{\sqrt{3}}\cdot(3r) = 6\sqrt{3}$$

Hence, the correct answer is (B).

Paragraph-2

If $(1+x)^n = C_0x^0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$ then (where C_r denotes ${}^nC_r$)

15. Value of $\frac{(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) + \ldots\ldots\ldots\ldots (C_{n-1} + C_n)}{C_1C_2 \ldots\ldots\ldots C_{n-1}C_n}$ is

(A) $\frac{(n+1)^n}{n!}$

(B) $\frac{n+1}{n!}$

(C) $\frac{(n+1)^{n-1}}{n!}$

(D) $\frac{(n+1)^n}{n}$

Ans. (A)

Sol. $\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\cdots\left(1+\frac{C_{n-1}}{C_n}\right) = \prod_{r=1}^{n}\left(1+\frac{C_{r-1}}{C_r}\right)$

$$= \prod_{r=1}^{n}\left(1+\frac{r}{n-r+1}\right) = \prod_{r=1}^{n}\left(\frac{n+1}{n-r+1}\right)$$

$$= \frac{n+1}{n}\cdot\frac{n+1}{n-1}\cdots\frac{n+1}{1} = \frac{(n+1)^n}{n!}$$

Hence, the correct answer is (A).

16. Value of $C_0 + (C_0 + C_1) + \ldots\ldots\ldots\ldots + (C_0 + C_1 + C_2 + \ldots\ldots\ldots\ldots + C_{n-1})$ is

(A) $(n+1).2^n$

(B) $n.2^{n-1}$

(C) $(n.+1)2^{n-1}$

(D) $n.2^n$

Ans. (B)

Sol. $n\times C_0 + (n-1)\times C_1 + \ldots\ldots\ldots + 1\times(C_{n-1}) = n.2^{n-1}$.

Hence, the correct answer is (B).

SECTION – IV (Non-Negative Integer)

17. X wants to put 5 different rings into three different pockets so that no pocket is empty. Then the probability of such event occurring is $\frac{1}{P}$. Hence, $[P]$ is (where [.] is the greatest integer function)

Ans. (1)

Sol. There are two cases

Case 1:-3 1 (one pocket has 3 rings)

Number of ways $= {}^3C_1 \times {}^5C_3 \times {}^2C_1 = 60$

Case 2:-2 2 1 (one pocket has 1 ring, 2 pockets have 2 each)

Number of ways $= {}^5C_2 \times {}^3C_2 \times {}^3C_1 = 90$

Number of total ways $= 60 + 90 = 150$

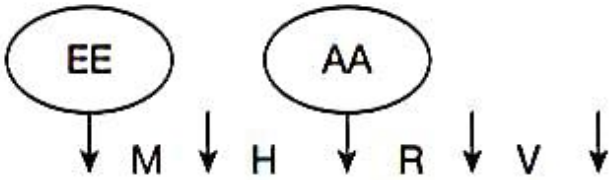
Number of ways without restriction $= 3^5 = 243$

Required probability $= \frac{150}{243} = \frac{1}{P} \Rightarrow [P] = 1$

Hence, the correct answer is (1).

18. If the number of different words that can be formed using all the letters of the word "MAHAVEER" where two of the vowels are together and the other two are also together but separated from the first two is S, then S-1432 is

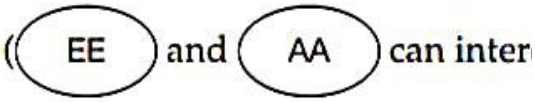
Ans. (8)

Sol. 

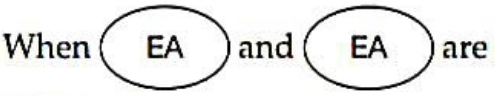
M,H,R,V can be arranged in 4! ways. Now, there are 5 places where the 2 packages of 2 vowels each can be put. Numbers of ways $= {}^5C_2$

Case – I

No. of ways ${}^5C_2 \times 2! \times 4!$



Case – II



No. of ways ${}^5C_2 \times 4! \times 2! \times 2!$

$\therefore$ Total number of ways=

${}^5C_2 \times 4! \times 2! (1+2)$

$$= \frac{5!}{2!3!} \times 4! \times 2! \times 3 = \frac{5! \times 4!}{2!}$$

$$= 120 \times 12 = 1440$$

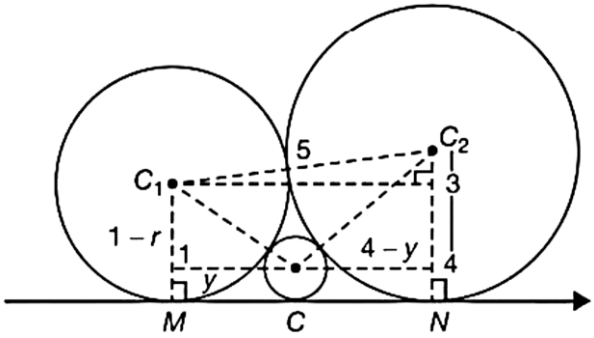
Hence, the correct answer is (8).

19. Two circles of radii 1 and 4 touch each other externally. Another circle of radius r touches both circles externally and also one direct common tangent of the two circles. Then, $\left[\frac{1}{r}\right] =$ (where[.] denotes greatest integer function).

Ans. (2)

Sol. Let r be the radius of the rew circle.

Let y be the perpendicular distancefrom C to C_1M from the figure



$$(1-r)^2 + y^2 = (1+r)^2$$

$$\Rightarrow y^2 = 4r$$

$$(4-r)^2 + (4-y)^2 = (4+r)^2$$

$$\Rightarrow (4-y)^2 = 16r$$

$$\Rightarrow (4-y)^2 = 4y^2$$

$$\Rightarrow y = \frac{4}{3}$$

$$\Rightarrow r = \frac{4}{9}$$

Hence, the correct answer is (2).

PART-B: PHYSICS

SECTION-I (One or More than One Correct)

20. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct?

(A) Both cylinders P and Q reach the ground at the same time.

(B) Cylinders P has larger linear acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

Ans. (D)

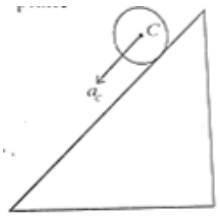
Sol. As we know, acceleration of the center of mass of cylinders rolling down an inclined plane

$$a_c = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$$

In case of P the mass is concentrated away from the axis,

So $I_P > I_Q$

$\therefore a_p < a_q \Rightarrow \upsilon_p < \upsilon_Q \Rightarrow \omega_P < \omega_Q$



21. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$

(A) T^2 is proportional to R^3

(B) T^2 is proportional to $R^{7/2}$

(C) T^2 is proportional to $R^{3/2}$

(D) T^2 is proportional to R^3 / 73

Ans. (B)

Sol. The centripetal force is provided by the gravitational force of attraction

So, $\Rightarrow \frac{mR\times 4\pi^2}{T^2} = \frac{GMm}{R^{5/2}} \Rightarrow T^2 \propto R^{7/2}$

22. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_c and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is.

(A) proportional to ΔT

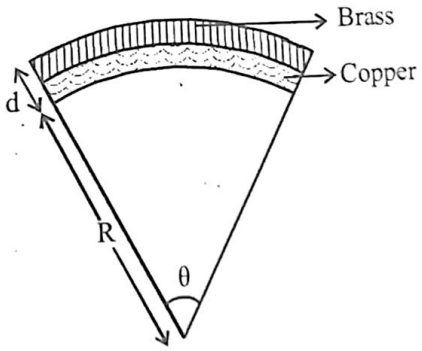
(B) inversely proportional to ΔT

(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

Ans. (B, D)

Sol. Let L0 be the orignal length of the strip.



Co-efficient of linear expansion of brass is greater than

that of copper i.e., $\alpha_B > \alpha_C$.

$L_B = L_0(1+\alpha_B\Delta T)(R+d)\theta$

$= L_0(1+\alpha_B\Delta T)$

Again, $L_C = L_0(1+\alpha_C\Delta T) = R\theta$

$\therefore \dfrac{(R+d)\theta}{R\theta} = \dfrac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}$

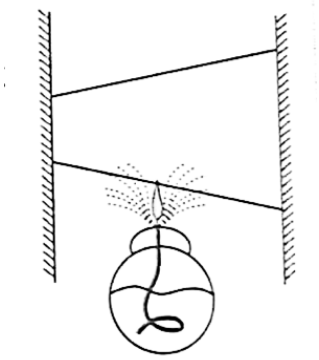
or, $\dfrac{R+d}{R} = (1+\alpha_B\Delta T)(1-\alpha_C\Delta T)$ [By binomial expansion]

or, $1+\dfrac{d}{R} = 1+(\alpha_B-\alpha_C)\Delta T-\alpha_B\alpha_C(\Delta T)^2$

or, $\dfrac{d}{R} = (\alpha_B-\alpha_C)\Delta T$ or $R = \dfrac{d}{(\alpha_B-\alpha_C)\Delta T}$ (ΔT^2 is very small)

$\therefore R \propto \dfrac{1}{\Delta T}$ and $R \propto \dfrac{1}{|\alpha_B-\alpha_C|}$

23. Arod is made of uniform material and has non-uniform cross-section. It is fixed at both the ends as shown and heated at mid-section. Which of the following statements are not correct?



(A) Force of compression in the rod will be maximum at mid-section
(B) Compressive stress in the rod will be maximum at left end
(C) Since the rod is fixed at both the ends, its length will remain unchanged. Hence, no strain will be induced in it
(D) Compressive stress in the rod will be maximum at right end

Ans. (A, C, D)

Sol. Force at the left end is same as force at right end. But area of cross-section at left end is minimum. So, stress at the left end is maximum.

24. Two satellites A and B of the same mass are orbiting the earth at altitudes R and $3R$, respectively, where Ris the radius of the earth. If K and U represent kinetic and potential energies, respectively. Choose the correct option(s)

(A) $\frac{K_A}{K_B} = 2$

(B) $\frac{U_A}{U_B} = 2$

(C) $\frac{K_A}{K_B} = \frac{1}{2}$

(D) $\frac{U_A}{U_B} = \frac{1}{2}$

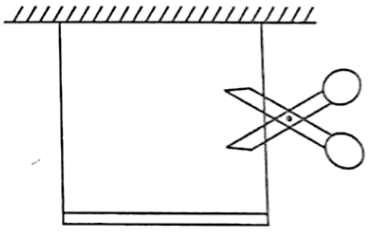
Ans. (A, B)

Sol. $(a,b) r_A = R + R = 2R, r_B = R + 3R = 4R$

$$K = \frac{GMm}{2r} \Rightarrow K \propto \frac{1}{r} \Rightarrow \frac{K_A}{K_B} = \frac{r_B}{r_A} = 2$$

$$U = -\frac{GMm}{r} \Rightarrow U \propto \frac{1}{r} \Rightarrow \frac{U_A}{U_B} = \frac{r_B}{r_A} = 2$$

25. A uniform rod of mass m and length L is held horizontally by two vertical strings of neligible mass as shown in the figure. Immediately after the string is cut, then



(A) linear acceleration of free end of the rod is $\frac{3}{2}g$

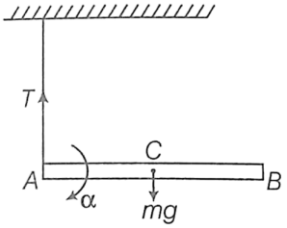
(B) linear acceleration of centre of mass of the rod is $\frac{3}{4}g$

(C) tension in the left string is $\frac{m}{g}4$

(D) tension in the left string is $\frac{mg}{2}$

Ans. (A, B, C)

Sol. $\alpha = \frac{\tau}{I} = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L}$



$(i) a_B = L\alpha = \frac{3}{2}g$

$(ii) a_C = \frac{L}{2}\alpha = \frac{3}{4}g$

$(iii) mg - T = ma_C$

$T = mg - ma_c$

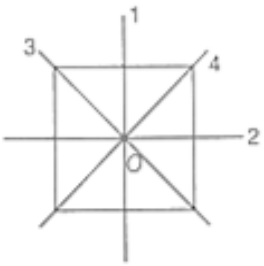
or $T = mg - m \cdot \frac{3}{4}g = \frac{mg}{4}$

SECTION – II Numerical Value

Paragraph-1 (Steam)

(If more than two decimal, truncate/roundoff the value two decimal places).

A thin square lamina has side L. Its moment of inertia about an axis passing through its centre of mass (O) and perpendicular to its plane is I_0.



Axes 1,2,3 and 4 are on the plane of the lamina as shown and moments of a inertia about them are I_1, I_2, I_3 and I_4, respectively.

26. $I_1 + I_2$ is equal to $n \times I_0$ then n is _____

Ans. (1)

Sol. Angle between 1 and 2 is 90° and angle between 3 and 4 is 90°. From perpendicular axes theorem, $I_0 = I_1 + I_2$ and $I_0 = I_3 + I_4$ From symmetry, $I_1 = I_2$ and $I_3 = I_4$. So, we can conclude that $I_1 = I_2 = I_3 = I_4$ in this case of square lamina

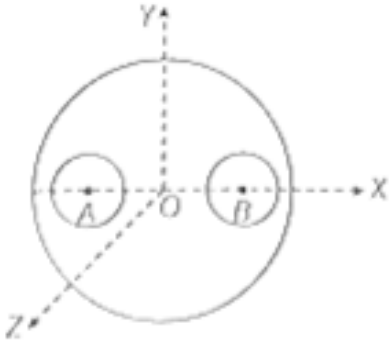
27. $I_3 + I_4$ is equal to $n \times I_0$ then n is _____

Ans. (1)

Sol. Angle between 1 and 2 is 90° and angle between 3 and 4 is 90°. From perpendicular axes theorem, $I_0 = I_1 + I_2$ and $I_0 = I_3 + I_4$ From symmetry, $I_1 = I_2$ and $I_3 = I_4$. So, we can conclude that $I_1 = I_2 = I_3 = I_4$ in this case of square lamina

Paragraph-2 (Steam)

A solid sphere of uniform density and radius 4 m is located with its centre at the origin O of the coordinate system (see figure). Two spheres of equal radius 1 m with their cavities at $A(-2,0,0)$ and $B(2,0,0)$ respectively are taken out leaving behind spherical cavities. The mass of each sphere taken out is M.



28. If a particle of mass m is released at point B, then its acceleration would be $\frac{31}{x^2}GM$ then x is _____

Ans. (4)

Sol. Mass of the sphere of radius 1 m= M (given)

$\therefore$ Mass of the whole sphere of radius 4m is

$$M' \frac{M\left(\frac{4}{3}\pi(4)^3\right)}{\frac{4}{3}\pi(1)^3} = 64M$$

$\therefore$ The gravitational field at B is

E_B = field due to whole sphere- field due to sphere A

$$= \frac{GM\uparrow}{R^3} - \frac{GM}{(AB)^2}$$

$$\text{or } E_B = \frac{G(64M)\times 2}{4^3} - \frac{GM}{4^2} = \frac{31}{16}GM$$

$\therefore$ Acceleration of Bis $a_B = \frac{31}{16}GM$

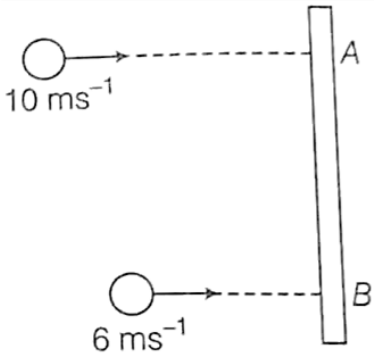
29. The gravitational field due to the sphere (with holes) shown at point O is _____

Ans. (0)

Sol. The gravitational field at O is zero due to symmetry.

Paragraph-3 (Steam)

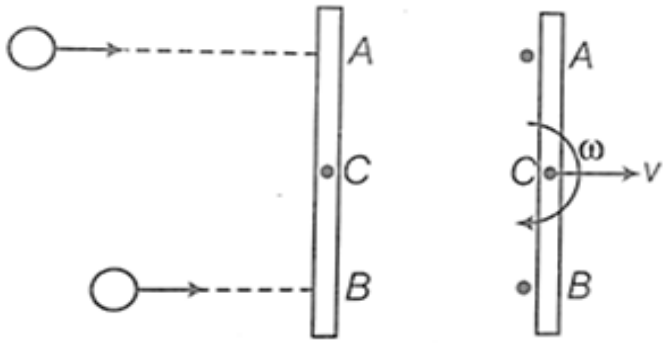
A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16kg and length is $\sqrt{3}$ m. Two particles, each of mass 0.08 kg are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity 10 ms^{-1} and the other with 6 ms^{-1} as shown in the figure. The first particle strikes the bar at point A and the other at point B. Each of A and B is at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar after collision.



30. The velocity of centre of mass of the system just after impact (in ms^{-1}) is _____

Ans. (4)

Sol. Let C be the centre of mass of (rod+ two particles) and ω be the angular velocity of the system and v its linear velocity, after



From law of conservation of linear momentum $(0.08)(10)+(0.08)(6)=(0.08+0.08+0.16)v$

$v = 4ms^{-1}$

31. The angular velocity of the system just after impact (in rad/s) is _____

Ans. (2)

Sol. $AC = BC = 0.5m$

From conservation of angular momentum about C

$(0.08)(10)(0.5)-(0.08)(6)(0.5)=I_{system}\omega$

where, $I_{system}=I_{rod}+I_{two\ particles}$

$$I_{system}=\frac{(0.16)(\sqrt{3})^2}{12}+2(0.08)\ (0.5)^2$$

$I_{system} = 0.08kg-m^2$

Using this in the equation above, we get $\omega = 2$ rad s $^{-}1$

SECTION – III (Single Correct Type)

Paragraph-1

On gradual Loading, stress-strain relationship for a metal wire is as follows.

With in proportionality limit, stress $\propto$ strain and so, $\frac{\text{Stress}}{\text{Strain}}$ = a constant for the material of wire.

32. Two wires of same material have length and radius (L,r) and $\left(2L,\frac{r}{2}\right)$. The ratio of their Young's moduili is

(A) 1:2

(B) 2:3

(C) 2:1

(D) 1:1

Ans. (D)

Sol. Young's modulus is a material constant.

33. Just on crossing the yield region, the material will have

(A) increased and breaking stress

(B) reduced and breaking stress

(C). constant stress

(D) None of the above

Ans. (B)

Sol. The yield region, the material will experience the breaking stress and further elongation causes reduction in stress and breaking of the wire.

Paragraph-2

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the escape speed from the earth. Now, the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth. The radius of the earth is R and acceleration due to gravity on earth's surface is g.

34. The height of the satellite above the surface of the earth is

(A) $\frac{R}{2}$

(B) $\frac{2R}{3}$

(C) R

(D) 2R

Ans. (C)

Sol. $V_0 = \frac{V_e}{2} = \frac{\sqrt{2gR}}{2} = \sqrt{\frac{gR}{2}}$

$\therefore \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR}{2}} \Rightarrow r = \frac{2GM}{gR} = \frac{2GM}{\frac{GM}{R^2}\cdot R} = 2R$

$\therefore h = r - R = 2R - R = R$

35. The speed with which it hits the surface of the earth is

(A) $\sqrt{gR}$

(B) $\sqrt{2gR}$

(C) $\sqrt{\frac{gR}{2}}$

(D) $\frac{\sqrt{gR}}{2}$

Ans. (A)

Sol. $(U+K)_{orbit} = (U+K)_{surface}$

$$-\frac{GMm}{(2R)} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{1}{2}v^2 = \frac{GM}{2R} = \frac{gR}{2} \Rightarrow v = \sqrt{gR}$$

SECTION – IV (Non-Negative Integer)

36. A Piece of ice (heat capacity = 2100J kg^{-1} $°C^{-1}$ and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at -5 °C atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Ans. (8)

Sol. As there is no other heat exchange in this process, So Heat supplied = Heat used in converting m grams of ice from-5°Cto0°C+Heatusedinconverting l gram ofice at $0^\circ C$ to water at $0^\circ C$

$$\text{or}, 420 = mc\Delta\theta + ml$$

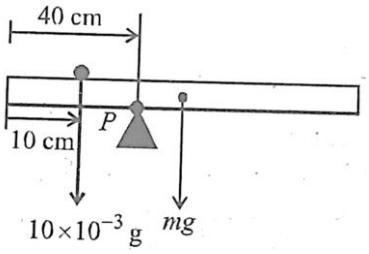
$$\Rightarrow 420 = m\times\frac{2100}{1000}\times5 + \frac{1\times3.36\times10^5}{1000}$$

$$420 = m\times10.5 + 336 \therefore m = \frac{84}{10.5} = 8g$$

37. A metre scale is balanced on knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be x × 10^{-2} kg. The value of x is

Ans. (6)

Sol.



=10^{-2}g

Let mass of scale be 'm'. Then,

About 'P' $\hat{\tau}$ net = 0

$\Rightarrow$ 2 × 10^{-2}g ×30 -mg×10 = 0

$\Rightarrow$ 6 × 10^{-1}g-mg×10 = 0

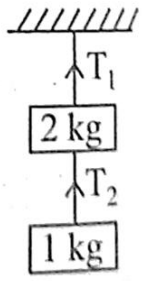
$$\Rightarrow m = \frac{0.6}{10} \Rightarrow m = 0.06kg$$

38. One end of metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. The ratio of longitudinal strain of upper wire to that of the lower wire will be _____

[Area of cross section of wire = 0.005 cm^2, Y = 2 × 10^{11} Nm-2 and g = 10 ms^{-2}]

Ans. (3)

Sol. Longitudinal strain is, $e = \frac{F}{AY} = \frac{T}{AY}$



$\therefore \frac{e_1}{e_2} = \frac{T_1}{T_2} = \frac{30N}{10N} = 3$

PART-C: CHEMISTRY

SECTION-I (One or More than One Correct)

39. For the reaction $PCl_5(g) \Leftrightarrow PCl_3(g) + Cl_2(g)$, the forward reaction at constant temperature is favored by:

(A) Introducing an inert gas at constant volume
(B) Introducing chlorine gas at constant volume
(C) Introducing an inert gas at constant pressure
(D) Increasing the volume of the container

Ans. (C, D)

Sol. Adding inert gas at constant pressure reaction shift in that direction where no. of moles of gases are more. On increasing volume the reaction shift in that direction where no. of moles of gases are more.

40. 200 mL of 0. 01 M HCl is mixed with 400 mL of 0. 01 M H_2SO_4. The pH of the mixture is _____

Given: $\log 2 = 0.30, \log 3 = 0.48, \log 5 = 0.70, \log 7 = 0.84, \log 11 = 1.04$

(A) 1.14
(B) 1.78
(C) 2.34
(D) 3.02

Ans. (B)

Sol. $\left[H^+\right] = \dfrac{0.01\times 200 + 2\times 0.01\times 400}{600}$

$= \dfrac{0.01 + 2\times 0.01\times 2}{3}$

$= \dfrac{0.01 + 0.04}{3}$

$= \dfrac{5}{3}\times 10^{-2}$

$pH = -\log\left[H^+\right]$

$= -\log\left(\dfrac{5}{3}\times 10^{-2}\right)$

$= -\left[\log\dfrac{5}{3} + \log 10^{-2}\right]$

$= -\left[\log 5 - \log 3 - 2\right]$

$= -0.7 + 0.48 + 2$

$= 2.48 - 0.7$

$= 1.78$

41. Dichromate ion in acidic medium oxidizes stannous ion as:

$xSn^{2+} + yCr_2O_7^{2-} + zH^+ \rightarrow aSn^{4+} + bCr^{3+} + cH_2O$

(A) the value of x:y is 1:3
(B) the value of x + y + z is 18
(C) a:b is 3:2
(D) The value of z-c is 7

Ans. (B, C, D)

Sol. $3Sn^{2+} + 14H^{+} + Cr_2O_7^{2-} \rightarrow 3Sn^{3+} + 2cr^{3+} + 7H_2O$

42. Two solid compounds X and Y dissociates at a certain temperature as follows

$X(s) \Leftrightarrow A(g) + 2B(g); K_{p_1} = 9 \times 10^{-3}$ atm^3

$Y(s) \rightleftharpoons 2B(g) + C(g); K_{p_2} = 4.5 \times 10^{-3}$ atm ^{3}

The total pressure of gases over a mixture of X and Y is:

(A) 4.5atm

(B) 0.45 atm

(C) 0.6 atm

(D) None of these

Ans. (B)

Sol. Let x is partial pressure of A and y is partial pressure of Ĉ whèn both equilibrium simultaneously established in a vessel

$X(s) \rightleftharpoons A(g) + 2B(g)$

$x \quad (2x + 2y); K_{p_1} = P_A . P^2_{B(total)}$

$Y(s) \rightleftharpoons C(g) + 2B(g)$

$y \quad (2y + 2x); K_{p_2} = P_C . P^2_{B(total)}$

$\frac{K_{p_1}}{K_{p_2}} = \frac{x}{y} \Rightarrow x = 2y$

$K_{p_1} . = x(2x + 2y)^2$

$\Rightarrow x = 0.1$ atm;

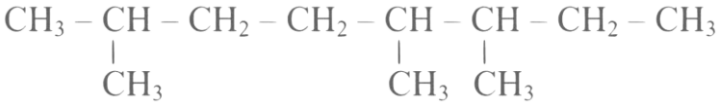
$\therefore y = 0.05$ atm

Total pressure of gases $= P_A + P_B + P_C$

$= 3(x + y)$

=0.45 atm.

43. IUPAC name of following hydrocarbon is:



(A).2,5,6-Trimethyloctane

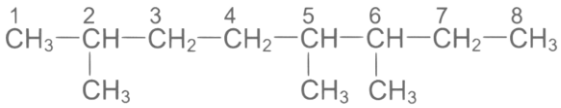
(B) 2-Ethyl-2,6-diethylheptane

(C) 3,4,7-Trimethyloctane

(D) 2-Ethyl-3,6-dimethylheptane

Ans. (A)

Sol.



2, 5, 6-trimethyloctane

44. The concentration of oxalic acid is 'x' mol litre^{-1}. 40 mL of this solution reacts with 16 mL of 0.05 M acidified $KMnO_4$. What is the pH of 'x' M oxalic acid solution? (Assume that oxalic acid dissociates completely.)

(A) 1.3

(B) 1.699

(C) 1

(D) 2

Ans. (C)

Sol. $KMnO_4 + C_2O_4^{2-} \xrightarrow{H^+} Mn^{2+} + CO_2$

$n_f = 5 \qquad n_f = 2$

16 mL 40 mL

0.05M XM

Milliequiv. of $C_2O_4^{2-}$ = milliequiv. of $KMnO_4$

$2(40)X = 5 \times 16 \times 0.05$

$x = 0.05M$

$\therefore [H^+] = 2X = 0.1M$

$pH = 1$

SECTION – II Numerical Value

Paragraph-1 (Steam)

(If more than two decimal, truncate/roundoff the value two decimal places).

Equivalent weight = $\dfrac{\text{Molecular weight / Atomic weight}}{\text{n – factor}}$

n-factor is very important in redox as well as non-redox reactions. With the help of n-factor we can predicts the molar ratio of the reactant species taking part in reactions. The reciprocal of n-factor's ratio of the reactants is the molar ratio of the reactants. In general n-factor of acid/base is number of moles of H'OH' furnished per mole of acid/base. n-factor of a reactant is no. of moles of electrons lost or gained per mole of reactant.

Example 1

1. In acidic medium: $KMnO_4 (n = 5) \rightarrow Mn^{2+}$
2. In neutral medium: $KMnO_4 (n = 3) \rightarrow MnO_2$
3. Inbasicmedium : $KMnO_4 (n = 1) \rightarrow MnO_4^{-2}$

45. Find number of moles of electron lost by 1 mole of FeC_2O_4. For given reaction

$FeC_2O_4 \rightarrow Fe^{3+} + 2CO_2$

Ans. (3)

Sol. Total no. of moles of e^- lost by 1 mole of FeC_2O_4

$= 1 + 1 \times 2 \Rightarrow 3$

46. n -factor of $Ba(MnO_4)_2$ in acidic medium is :

Ans. (10)

Sol. n -factor $= 5 \times 2 = 10$

Paragraph-2 (Steam)

10 mole of NH_3 is heated at 15atm from 27° C to 347°C assuming volume constant. The pressure at equilibrium is found to be 50 atm. The equilibrium constant for dissociation of NH_3.

$$2NH_{3(g)} \rightleftharpoons N_{2(g)} + 3H_{2(g)}; \Delta H = 91.94kJ$$

can be written as $K_p = \dfrac{P_{N_2} \times \left(P_{H_2}\right)^3}{\left(P_{NH_3}\right)^2} (atm)^2$

47. The % degree of dissociation of NH_3 is found to be 0.613×10^y find value of y

Ans. (2)

Sol.

$2NH_3$	$\rightleftharpoons$	N_2	+	$3H_2$	
10		0		0	at 300K
(10-2x)		x		3x	at 300K

Pressure increase due to increases in temperature as well as due to increase in moles

Initially $P \propto T$

$\therefore 15 \propto 300$

$P \propto 620$

$\therefore P = \dfrac{620}{300} \times 15 = 31$ atm of 10 mole of NH_3 at 620 K

Now, NH_3 is dissociated to attain 50 atm at 620 K

Thus, $P \propto n$

$10 + 2x \propto 50$

$2x = 6.13$

$\therefore \alpha = \dfrac{2x}{10} \times 100 = \dfrac{6.13 \times 100}{10} = 61.3\%$

Hence, x is 2

48. The equilibrium constant K_p for the reaction is found to be 15.28×10^z find value of z

Ans. (2)

Sol. $K_p = \dfrac{n_{N_2} \times \left(n_{H_2}\right)^3}{\left(n_{NH_3}\right)^2} \times \left[\dfrac{P}{\Sigma n}\right]^2$

$$= \frac{\frac{6.13}{2} \times \left[\frac{6.13 \times 3}{2}\right]^3}{\left[10 - 6.13\right]^2} \times \left[\frac{50}{10 + 6.13}\right]^2 = 1.528 \times 10^3 atm^2$$

$K_p = = 15.28 \times 10^2 atm^2$

Hence, z = 2

Paragraph-3 (Steam)

Solubility of a substance in its saturated solution can be derived from its K_{sp} values. Higher is the K_{sp} for same type of compound more is the solubility. If S is the solubility in mol / litre then K_{sp} of a compound A_xB_y is expressed as

$K_{sp} = x^x \cdot y^y[S]^{x+y}$

Consider a compound M(OH)x having = $K_{sp} = 27 \times 10^{-12}$ and solubility in pure water is 10^{-3} mol litre^{-1}

49. The value of x is:

Ans. (3)

Sol. $M(OH)_x \rightleftharpoons \underset{S}{M^{+x}} + \underset{xs}{xOH^-}$

$K_{sp} = x^x . (S)^{1+x} = 27 \times 10^{-12}$

$\therefore x = 3$

50. The solubility (in M) of $M(OH)_x$ in 0.1 M NaOH solution is found to be 2.7×10^{-x} find value of x

Ans. (8)

Sol. $K_{sp} = \left[M^{+x}\right]\left[OH^-\right]^x = \left[M^{+3}\right]\left[OH^-\right]^3$

$27 \times 10^{-12} = S \times \left[3S + 0.1\right]^3 \left[3S <<< 0.1\right]$

$\therefore \quad S = \dfrac{27 \times 10^{-12}}{(0.1)^3} = 2.7 \times 10^{-8}$

SECTION – III (Single Correct Type)

Paragraph-1

At 25C° K_b of CH_3COO^- is 5.26×10^{-10}. Calculate for 0.01 N solution of sodium acetate. (Take log 2.29=0.36)

51. Hydrolysis constant of CH_3COO^- is:

(A) 5.26×10^{-10}

(B) 5.26×10^{-11}

(C) 5.26×10^{-12}

(D) 5.26×10^{-9}

Ans. (A)

Sol. For $CH_3COONa + H_2O \rightleftharpoons CH_3COOH + NaOH$

Before hydrolysis	C	0	0
After hydrolysis	$C(1-h)$	Ch	Ch

At 25C° $K_a \times K_b = 10^{-14}$

K_b for $CH_3COO^- = 5.26 \times 10^{-10}$

$\therefore$ K_a for $CH_3COOH = 1.9 \times 10^{-5}$

$K_h = \dfrac{K_w}{K_a} = \dfrac{10^{-14}}{1.9 \times 10^{-5}} = 5.26 \times 10^{-10}$

52. pH of solution is:

(A) 8.56

(B) 5.44

(C) 8.36

(D) 9.56

Ans. (C)

Sol. From NaOH, a strong alkali we get concentration of $[OH^-]$ = Ch

h = Degree of hydrolysis = $\sqrt{\frac{K_h}{c}} = \sqrt{\frac{K_w}{K_a c}} = \sqrt{\frac{5.26\times10^{-10}}{0.1}} = 2.29\times10^{-4}$

$[OH^-] = 0.01\times2.29\times10^{-4} = 2.29\times10^{-6}M$

$\therefore pOH = 5.64 \quad \therefore pH = 8.36$

Paragraph-2

2.5 g sample of copper is dissolved in excess of H_2SO_4 to prepare 100 mL of0. 02 M $CuSO_4$ (aq). 10mL of 0.02 M solution of $CuSO_4$ (aq) is mixed with excess of KI to show the following changes.

$$CuSO_4 + 2KI \rightarrow K_2SO_4 + CuI_2$$

$$2CuI_2 \rightarrow Cu_2I_2 + I_2$$

The liberated iodine is titrated with hypo $Na_2S_2O_3$ and requires V mL of 0.1 M hypo solution for its complete reduction.

53. The volume (V) of hypo required is:

(A) 2 mL

(B) 20 mL

(C) 1.mL

(D) 10 mL

Ans. (A)

Sol. The reactions are:

$$2\left(Cu^{2+}\right) + 2e \rightarrow \left(Cu^{+}\right)_2, E = \frac{M}{1}$$

$$2\left(I^{-1}\right) \rightarrow I_2 + 2e, E_{I^-} = \frac{M}{1}$$

$$2\left(S^{2+}\right)_2 \rightarrow \left(S^{5/2+}\right)_4 + 2e,$$

$$E_{Na_2S2O_3} = \frac{M}{1}$$

Milli eq. **of CuSO**$_4$ = Milli eq. of $Na_2S_2O_3$

=Milli eq. of I_2 liberated

$10\times0.02\times1 = V\times0.1\times1 \quad V = 2$ mL

54. Percentage of purity of sample is:

(A) 10.16

(B) 5.08

(C) 2.54

(D) 1.27

Ans. (B)

Sol. Milli eq. of $CuSO_4$ in 100mL = $100 \times 0.02 = 2$

$\therefore$ weight of $CuSO_4 = \frac{2 \times 249.6}{100} = 0.499g$

$\therefore$ weight of $Cu = \frac{0.499 \times 63.6}{249.6} = 0.127g$

$\therefore$ % of $Cu = \frac{0.127}{2.5} \times 100 = 5.08$

SECTION – IV (Non-Negative Integer)

55. At 25C° K_a for CH_3COOH is 1.8×10^{-5} and K_b of NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be _____.

Ans. (7)

Sol. To find the pH of an ammonium acetate solution, we use the fact that ammonium acetate is a salt resulting from the neutralization of a weak acid (acetic acid, CH_3COOH) by a weak base (ammonium hydroxide, NH_4OH). The respective ionization constant values for the weak acid K_a and the weak base K_b are given as 1.8×10^{-5}.

Firstly, calculate the pK_a and pK_b values.

$pK_a = -\log(K_a) = -\log(1.8 \times 10^{-5})$

$pK_b = -\log(K_b) = -\log(1.8 \times 10^{-5})$

Given that the values of K_α and K_b are the same, their pK_a and pK_b values will also be the same, establishing a neutral condition where the effects of the acid and base neutralize each other.

The formula used to determine the pH of a solution of such a salt is

$$pH = \frac{1}{2}\left(pK_w + pK_a - pK_b\right)$$

Where pK_w is the negative log of ionic product of water, which is 14 at 25°C. In this specific case, since $pK_a = pK_b$, the formula simplifies to

$$pH = \frac{1}{2}\left(14 + pK_a - pK_a\right)$$

$$pH = \frac{1}{2}(14)$$

$$pH = 7$$

The pH of an ammonium acetate solution in this scenario is 7, indicating a neutral solution.

56. For the reaction $N_2O_4(g) \rightleftarrows 2NO_{2(g)}$, Kp = 0.492 atm at 300K. K_c for the reaction at same temperature is _____ $\times 10^{-2}$.

(Given : $\mathbf{R} = 0.082$L atm $mol^{-1}\mathbf{K}^{-1}$)

Ans. (2)

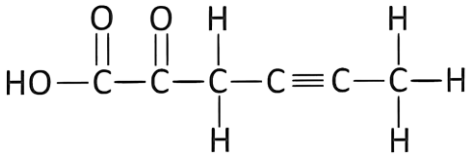
Sol. $K_p = K_C \cdot (RT)^{\Delta n_g}$

$\Delta n_g = 1$

$\Rightarrow K_c = \frac{K_p}{RT} = \frac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$

57. The total number of 'sigma' and 'pi' bonds in 2-oxohex-4-ynoic acid is _____

Ans. (18)

Sol. 

Total number of σ & π bonds=18