FIITJEE

ALL INDIA TEST SERIES

JEE (Advanced)-2025

FULL TEST – IV

PAPER –2

TEST DATE: 18-02-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

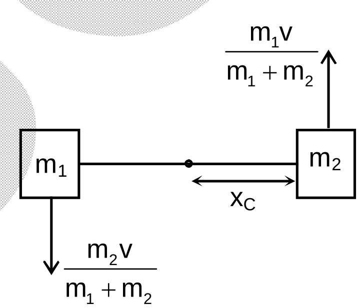
SECTION – A

1. D

Sol. Relative to COM frame, the rod is undergoing pure rotatory motion with a constant angular velocity, $\omega = \frac{v}{L}$

$x_C = \frac{m_1 L}{m_1 + m_2}$

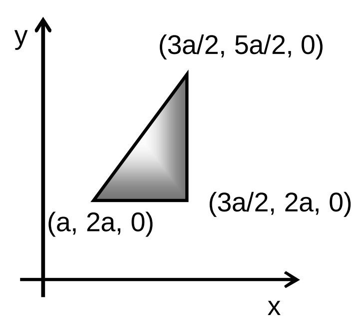
So, $t = \frac{2\pi L}{v}$



2. A

3. C

Sol. If a point charge is at $\left(a, 2a, \frac{a}{2}\right)$ then given surface is $\frac{1}{8}$th of a square surface of side 'a'



4. A

Sol. $i_1 = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$

$i_2 = -\frac{2\varepsilon}{R} e^{-\frac{t}{RC}}$

$i_3 = -\frac{\varepsilon}{R} e^{-\frac{t}{RC}}$

5. A, C, D

Sol. for circular orbit of satellite

$$E = \frac{U}{2} = -K \Rightarrow -dE = dK$$

$$\Rightarrow (kv)vdt = mvdv$$

$$\frac{dv}{dt} = \frac{kv}{m} \Rightarrow \int dt = -\frac{m}{k}\int_{v_i}^{v_f}\frac{dv}{v}$$

$$\Rightarrow t = \frac{m}{k}\ln\left(\frac{v_f}{v_i}\right) = \frac{m}{k}\ln\sqrt{\frac{GM/R}{GM/4R}} = \frac{m}{k}\ln 2$$

$$W_g = U_i - U_f = -\frac{GMm}{4R} - \left(-\frac{GmM}{R}\right) = \frac{3}{4}\frac{gR^2m}{R} = \frac{3}{4}mgR$$

$$W_g + W_r = \frac{1}{2}m\left(v_f^2 - v_1^2\right) = \frac{1}{2}m\left(\frac{GM}{R} - \frac{GM}{4R}\right)$$

$$\Rightarrow \frac{3}{4}mgR + W_r = \frac{3}{8}mgR \Rightarrow W_r = \frac{-3}{8}mgR$$

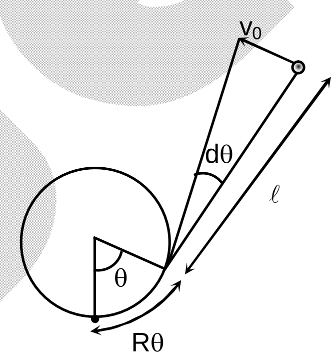
6. B, C, D

Sol. $T = \frac{mv_0^2}{\ell}$, ℓ decreases.

$$\ell = 2\pi R - R\theta$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{v_0}{\ell} \Rightarrow \frac{d\theta}{dt} = \frac{v_0}{2\pi R - R\theta}$$

$$\Rightarrow \int_0^{2\pi}(2\pi R - R\theta)d\theta = v_0\int_0^t dt$$



7. B, C

Sol. Condition for maximum intensity due to thin film

$$2\mu d - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu d = (2p+1)\frac{\lambda_1}{2} \Rightarrow (2p + 1)\lambda_1 = 4\,\mu d$$

Here λ_1 = 400 nm and n = p $\Rightarrow$ An integer

Let λ' = 1150 nm. Since only two wavelength give maximum intensity, so

$$\frac{4\mu d}{2(p-1)+1} < \lambda' < \frac{4\mu d}{2(p-2)+1}$$

$$\frac{\lambda_1(2p+1)}{2(p-1)+1} < \lambda' < \frac{\lambda_1(2p+1)}{2(p-2)+1}$$

$$\Rightarrow \frac{\lambda_1(2p+1)}{2(p-1)+1} < \lambda' \text{ or } \lambda' < \frac{\lambda_1(2p+1)}{2(p-2)+1}$$

$$\Rightarrow p > \frac{1}{2}\left(\frac{\lambda' + \lambda_1}{\lambda' - \lambda_1}\right) = \frac{1}{2}\left(\frac{1150+400}{1150-400}\right) = 1......, \text{ or}$$

$$p < \frac{1}{2}\left(\frac{3\lambda' + \lambda_1}{\lambda' - \lambda_1}\right) = \frac{1}{2}\left(\frac{3\times 1150+400}{1150-400}\right) = 2......$$

The only integer which satisfies both inequalities is 2, so

$$d = \frac{(2\times2+1)400\times10^{-9}}{4\times1} = 500 \text{ nm}$$

$$\lambda_2 = \frac{4\mu d}{2(p-1)+1} = \frac{4\times1\times500\times10^{-9}}{2+1} = 666.7 \text{ nm}$$

$$\Delta T = \frac{d}{\alpha h} = \frac{500\times10^{-9}}{8\times10^{-6}\times2\times10^{-2}} \approx 3.1\,°C$$

SECTION – B

8. 90

Sol. The maximum displacement on each side decreases by $\frac{2\mu mg}{k} = \frac{2\times0.4\times1\times10}{100} = 8$ cm

It stops completely if comes to rest between $\pm\frac{\mu mg}{k} = \pm4$ cm

$\therefore$ S = 27 + (19 + 19) + (11 + 11) + 3 = 90 cm

9. 5

Sol. For A, $1V = \frac{2S}{8} = 0.25$ mm

Least count of A = 1 – 0.25 × 3 = 0.25 mm

For B, $1V = \frac{3S}{5} = 0.6$ mm

Least count of B = 2 – 0.6 × 3 = 0.2

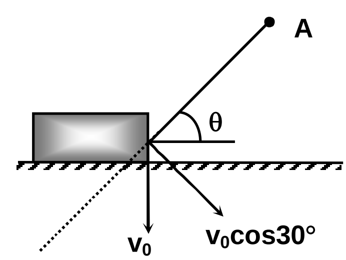
Difference = 0.25 – 0.2 = 0.05 mm

10. 4

Sol. When the block leaves the surface

T sin 30° = mg

At the moment the block leaves surface it moves in circular path about the end A of the string with velocity $v_0\cos30°$



$$\Rightarrow T - mg\sin30° = \frac{m}{\ell}(v_0\cos30°)^2$$

$$\Rightarrow \frac{mg}{\sin30°} - mg\sin30° = \frac{m}{\ell}v_0^2\cos^2 30°$$

$$\Rightarrow v_0 = \sqrt{\frac{g\ell}{\sin30°}} = 4 \text{ m/s}$$

11. 75

Sol. $F\cos\theta - mg - \mu N = m\frac{dv}{dt}$

$$\Rightarrow F\cos\theta - mg - \mu F\sin\theta = m\frac{dv}{dt}$$

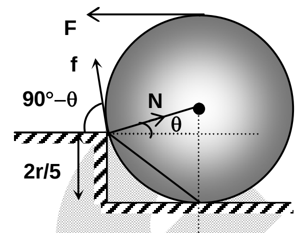
$$\Rightarrow m\int_{\theta}^{0} dv = \int_{0}^{\pi/2k}(F\cos kt - mg - \mu F\sin kt)dt$$

When $\theta = \frac{\pi}{2}$, $kt = \frac{\pi}{2}$ and $t = \frac{\pi}{2k}$

$$\Rightarrow F = \frac{\pi mg}{2(1-\mu)} = \frac{\pi mg}{2\left(1-\frac{1}{3}\right)} = \frac{3\pi mg}{4}$$

12. 1

Sol. Since the cylinder moves very slowly, it remains in equilibrium.



$$\Rightarrow F + f\frac{3}{5} = N \times \frac{4}{5}$$

Balancing torque about centre

$Fr = fr \Rightarrow f = F$

Further $F\left(r + \frac{3r}{5}\right) = mg\frac{4r}{5}$

$$F = \frac{mg}{2}$$

$$\Rightarrow f = \frac{mg}{2}$$

$$N = \frac{5}{4}\left(F + \frac{3f}{5}\right) = \frac{5}{4}\left(\frac{mg}{2} + \frac{3mg}{10}\right) = mg$$

The cylinder will not slip

If $f \le \mu N$

$$\Rightarrow \frac{mg}{2} \le \mu mg \Rightarrow \mu \ge \frac{1}{2}$$

$$\Rightarrow \mu_{min} = \frac{1}{2}$$

13. 5

Sol. Using symmetry equivalent resistance is $\frac{8}{5}R$

SECTION – C

14. 5.00

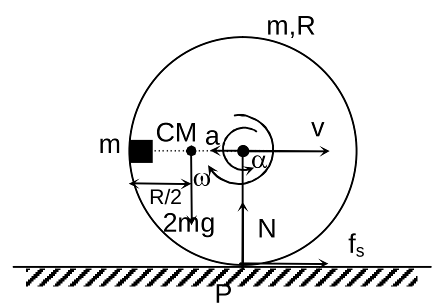
15. 12.50

Sol. (Q.14-15.)

For rolling without slipping,

$v = \omega R$ and $a = \alpha R$

Using conservation of energy



$$mv_0^2 = mv^2 + \frac{1}{2}m(v\sqrt{2})^2 + mgR$$

$$mv_0^2 = 2mv^2 + mgR$$

$$5mgR = 2mv^2 + mgR$$

$$v = \sqrt{2gR} \text{ and } \omega = \frac{v}{R} = \sqrt{\frac{2g}{R}} \qquad \ldots(i)$$

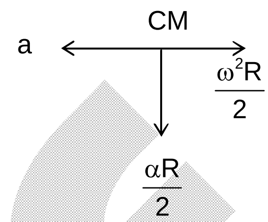
$$I_P = 2mR^2 + m\left(R\sqrt{2}\right)^2 = 4mR^2$$

$$\tau_P = I_P\alpha$$

$$2m(g+\omega^2R)\frac{R}{2} = 4mR^2\alpha$$

$$m(g+2g)R = 4mR^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{4R} \qquad \ldots\text{(ii)}$$



$$f_s = 2m\left(\frac{\omega^2R}{2} - a\right)$$

$$f_s = 2m\left(\frac{\omega^2R}{2} - \alpha R\right)$$

$$f_s = 2m\left(g - \frac{3g}{4}\right)$$

$$f_s = \frac{mg}{2}$$

Also, $2mg - N = \frac{2m\alpha R}{2}$

$$2mg - N = m\alpha R$$

$$2mg - N = \frac{3mg}{4}$$

$$\Rightarrow N = 2mg - \frac{3mg}{4}$$

$$\Rightarrow N = \frac{5mg}{4}$$

16. 0.13

17. 3.37

Sol. (Q.16-17).

$$\text{Pitch} = \frac{5\text{ mm}}{10} = 0.5\text{mm}$$

$$\text{Least count} = \frac{0.5}{100} = 0.005\ \text{ mm}$$

Negative zero error = 26 $\times$ 0.005 = 0.13 mm

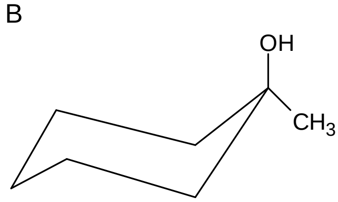
Diameter of the wire measured, d = 6 $\times$ 0.5 mm + 48 $\times$ 0.005 + 26 $\times$ 0.005 = 3.37 mm

Chemistry

PART – II

SECTION – A

18. B

Sol. 

Give 3° carbocation as intermediate while other gives 2° carbocation.

19. D

20. A

Sol. $KI + I_2 \longrightarrow KI_3$

21. A

22. A, B, C

23. A, B, D

24. A, C, D

SECTION – B

25. 5

Sol. $d(kE) = mv.dv = mv\dfrac{h}{4\pi m\Delta x}$

$= \dfrac{3\times10^{8}}{3}\times\dfrac{6.62\times10^{-34}}{4\times\pi\times\dfrac{3.31}{\pi}\times10^{-12}}$

$= 5 \times 10^{-16}$J

26. 5

Sol. $T_B = \dfrac{a}{Rb} = \dfrac{3.6}{0.08\times0.6} = 75k$

$\dfrac{75}{15} = 5K$

27. 6

Sol. $NaCN$, Na_3PO_4, Na_2CO_3, $NaHCO_3$, $Na_2C_2O_4$, Na_2HPO_4

28. 1

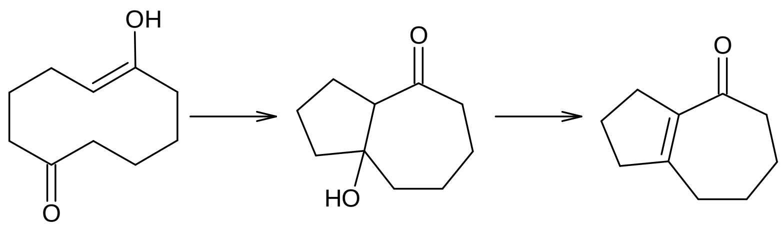
Sol. $\Delta G° = -2.303\ RT \log K$

29. 4

Sol. Fact

30. 4

Sol.



SECTION – C

31. 11.70

32. 8.71

33. 0.32

34. 0.16

Sol. $\kappa = \Lambda_m.C = \frac{\Lambda_m \times M}{1000} = \frac{200 \times 0.04}{1000}$

$= 8 \times 10^{-3}$ S cm^{-1}

$\kappa = G\left(\frac{\ell}{A}\right) \Rightarrow 8 \times 10^{-3} = G\left(\frac{0.50}{2}\right)$

G = 0.032 S, V = IR $\Rightarrow$ I/G

I = 5 $\times$ 0.032 = 0.16 A

Mathematics

PART – III

SECTION – A

35. B

Sol. $\int \frac{1-\cos x - x\sin x}{(x-\sin x)^2+\cos^2 x}dx = \int \frac{1}{1+\left(\frac{\cos x}{x-\sin x}\right)^2} \times \frac{1-\cos x - x\sin x}{(x-\sin x)^2}dx$

Let $\frac{\cos x}{x-\sin x} = t \Rightarrow \int \frac{1}{1+t^2}dt = \tan^{-1}t + c = \tan^{-1}\left(\frac{\cos x}{x-\sin x}\right)+c$

36. D

Sol. Around x = 4 $\Rightarrow$ f(x) = 3 – sin(x – 2)

$\therefore$ It continuous and differentiable $f'(x) = -\cos(x-2)$

37. C

Sol. If x > 2 then $x^3 - 3x > 4x - x = x > \sqrt{x+2}$

$\Rightarrow |x| \le 2$ take $x = 2\cos\theta$ for some $\theta \in [0, \pi]$

$\Rightarrow 2\cos 3\theta = \sqrt{2(1+\cos\theta)}$

$\Rightarrow 2\sin\frac{7\theta}{4}\cdot\sin\frac{5\theta}{4} = 0$

$\Rightarrow \theta = 0, \frac{4\pi}{7}, \frac{4\pi}{5}$

$\therefore x = 2, 2\cos\frac{4\pi}{7}, -\frac{1}{2}\left(1+\sqrt{5}\right)$

38. B

Sol. $B^2 - tr(B)\cdot B + I = 0$

$\Rightarrow AB - (tr(B)A + AB^{-1} = 0$

$\therefore tr(AB) - tr(A)tr(B) + tr(AB^{-1}) = 0$

39. A, B, C, D

Sol. $A + B = ABAB = A^2 + A$

$\Rightarrow B = A^2$

$BAB = A^5 = A + I \Rightarrow A(A^4 - I) = I$

$B^5 - A^5 = (A^5)^2 - A^5 = (A+I)^2 - (A+I) = A^2 + A = A + B$

40. A, D

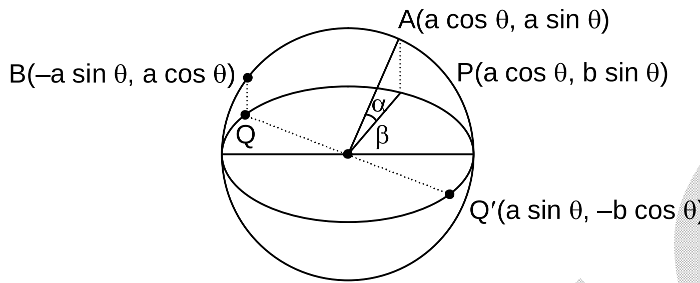
Sol. Let f(a) = 0 and $F(x) = \int_0^x |f(t)|dt$

$F'(x) = -f(x)$ $0 \le x < a$, $F'(x) = f(x)$ $a \le x \le 1$

$\Rightarrow \int_0^1 f(x)\left(\int_0^x |f(t)|dt\right)dx = \frac{(F(1))^2}{2} - (F(a))^2 = \frac{(S_1+S_2)^2}{2} - (S_2)^2 = 7$

41. B, C

Sol. $\tan\alpha = \dfrac{\tan\theta - \frac{b}{a}\tan\theta}{1+\frac{b}{a}\tan^2\theta} = \dfrac{1-\frac{b}{a}}{\cot\theta + \frac{b}{a}\tan\theta} \le \dfrac{1-\frac{b}{a}}{2\sqrt{\frac{b}{a}}} = \dfrac{a-b}{2\sqrt{ab}}$



$$\tan\beta = \frac{\frac{b}{a}\tan\theta + \frac{b}{a}\cot\theta}{1-\frac{b^2}{a^2}} \ge \frac{\frac{2b}{a}}{1-\frac{b^2}{a^2}} = \left(\frac{2ab}{a^2-b^2}\right)$$

SECTION – B

42. 9

Sol. Take $a_n = \dfrac{1}{(-2)^n t_n}$, on solving, we get $a_n = \dfrac{a_1}{a_1\left(\frac{2+(-2)^n}{6}\right)+(-2)^{n-1}}$,

for periodicity $a_1\left(\dfrac{2+(-2)^n}{6}\right)+(-2)^{n-1} = 1$,

for undefined $a_1\left(\dfrac{2+(-2)^n}{6}\right)+(-2)^{n-1} = 0$,

$\lim\limits_{n\to\infty} a_n = 0$

43. 1600

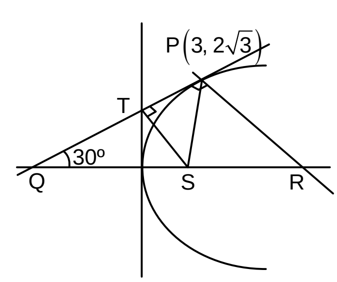
Sol. $x^6 - 2x^3 - 8 = (x - \alpha_1)(x - \alpha_2) \;.....(x - \alpha_6)$
$(2\omega)^6 - 2(2\omega)^3 - 8 = (2\omega - \alpha_1)(2\omega - \alpha_1) \;.....\; (2\omega - \alpha_6)$
$((2\omega^2)^6 - 2(2\omega^2)^3 - 8) = (2\omega^2 - \alpha_1) \;.....\; (2\omega - \alpha_6)$
On multiplying $(40)^2 = g(\alpha_1)g(\alpha_2) \;.....\; g(\alpha_6)$

44. 62

Sol. $\sum\limits_{r=0}^{2014}\sum\limits_{k=0}^{r}(-1)^k(k+1)(k+2)\,{}^{2019}C_{r-k} = \sum\limits_{r=0}^{2014} 2\;{}^{2016}C_r = 2^{2017} - 4034$

45. 18

Sol. Area PRST = ΔPQR – ΔQST
$= \dfrac{1}{2}8\times 2\sqrt{3} - \dfrac{1}{2}\left(\dfrac{1}{2}\times 4\times 4\sin 150^\circ\right)$
$= 8\sqrt{3} - 2\sqrt{3} = 6\sqrt{3}$



46. 9

Sol. $\int_0^y \sqrt{x^4+(y(3-y))^2}\,dx \le \int_0^y (x^2+y(3-y))\,dx \le \frac{y^3}{3}+y^2(3-y)$

$f(y)=\frac{y^3}{3}+y^2(3-y)$

$f'(y) = y^2 + 6y - 3y^2 = 2y(3 - y) > 0$ for $0 \le y \le 3$ so maximum occurs at $y = 3$

47. 2000

Sol. Let $\vec{\alpha}=3\hat{i}+4\hat{j}+10\hat{k}$ and $\vec{\beta}=a\hat{i}+b\hat{j}+c\hat{k}$

$S=\vec{\alpha}\cdot\vec{\beta}=|\vec{\alpha}||\vec{\beta}|\cos\theta \le |\vec{\alpha}||\vec{\beta}| \le \sqrt{2000}$

SECTION – C

48. 240.00

49. 256.00

Sol. (Q.48-49)

$$D=\begin{vmatrix}1&1&1\\2&1&3\\1&2&p\end{vmatrix}=-p\,;\ Dx=\begin{vmatrix}4&1&1\\6&1&3\\q&2&p\end{vmatrix}=-2(p-q+6)\,;$$

$$Dy=\begin{vmatrix}1&4&1\\2&6&3\\1&q&p\end{vmatrix}=-2p-q+6\,;\ D3=\begin{vmatrix}1&1&4\\2&1&6\\1&2&q\end{vmatrix}=6-q$$

For unique solution $D \ne 0 \Rightarrow p \ne 0$ $\therefore$ total number of ordered pairs = 15 × 16 = 240 $\therefore$ A = 240
For no solution p = 0, $q \ne 6$ $\therefore$ total number of ordered pairs = 1 × 15 = 15 $\therefore$ B = 15
For infinite solution p = 0, q = 6 $\therefore$ total number of ordered pairs = 1 × 1 = 1 $\therefore$ C = 1

50. 3.00

51. 2.00

Sol. $f'(x) = 0 \Rightarrow f(x) = \text{constant}$ $\because$ $f(3) = \lambda$ $\therefore$ $f(x) = \lambda$

$\therefore\ \frac{f(1)+f(2)+f(3)}{f(3)}=3$

$$P^2=P\cdot P=\begin{bmatrix}\cos\frac{2\pi}{9}&\sin\frac{2\pi}{9}\\-\sin\frac{2\pi}{9}&\cos\frac{2\pi}{9}\end{bmatrix};\ P^n=\begin{bmatrix}\cos\frac{n\pi}{9}&\sin\frac{n\pi}{9}\\-\sin\frac{n\pi}{9}&\cos\frac{n\pi}{9}\end{bmatrix}$$

$$aP^6+bP^3+cI=a\begin{bmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{bmatrix}+b\begin{bmatrix}\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{bmatrix}+c\begin{bmatrix}1&0\\0&1\end{bmatrix}=0$$

$\Rightarrow -\frac{a}{2}+\frac{b}{2}+C=0\ ;\ \frac{\sqrt{3}}{2}(a+b)=0 \Rightarrow c = a \Rightarrow a = -b$