FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Main)-2025

TEST DATE: 15-12-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. B

Sol. Initially, W – 2Kx =0 …(i)

Finally, $W' - 2K\left(x + \frac{a}{2}\right) - a \cdot a \frac{a}{2} 2\sigma \cdot g = 0$ …(ii)

$W' = W + W_0$

$W + W_0 - 2Kx - Ka - a^3\sigma g = 0$ [from equation (ii)]

From equation (i), $W_0 = Ka + a^3\sigma g = a(K + a^2\sigma g)$

2. C

Sol. Even though the distribution of the mass is unknown, we can find the potential due to the ring on any axial point because from any axial point the entire mass is at the same distance (whatever would be the nature of distribution).

Potential at A due to the ring is $V_A = -\frac{GM}{\sqrt{2}R}$

Potential at B due to the ring is $V_B = -\frac{GM}{\sqrt{5}R}$

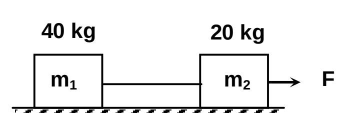
$\Delta U = U_f - U_i = U_A - U_B = m_0(V_A - V_B)$

$W_{ext} = \Delta U = \frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right]$

3. B

Sol. Tension T in the wire $= v^2\rho = (400)^2 \times 10^{-3} = 160$ N

Force applied $F = \frac{T(m_1 + m_2)}{m_1} = 160 \times \frac{(40+20)}{40} = 240$ N



4. C

Sol. Let K_1, K_2 and P_1, P_2 are K.E. and momentum of α particle and remaining nucleus, then

$K_1 + K_2 = 5.5$ MeV …(i)

$P_1 = P_2$ …(ii)

$\sqrt{2K_1 \times 4m} = \sqrt{2K_2 \times 216m}$

$\Rightarrow K_1 = 54K_2$

$\therefore$ by equation (i)

$K_1 = \frac{55 \times 5.4}{55} = 5.4$ MeV

5. C

Sol. PM = 3/2 cm

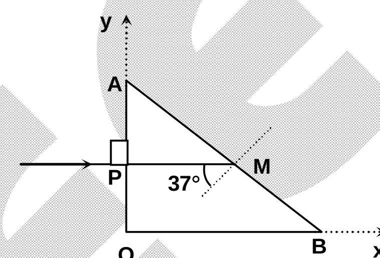
$37° > C$

$\Rightarrow 37° > \sin^{-1}\left(\frac{1}{\mu_0 + a\left(\frac{3}{2}\right)}\right)$

$\Rightarrow \frac{3}{5} > \frac{1}{\mu_0 + \frac{3}{2}a} \Rightarrow 3\mu_0 + \frac{9a}{2} > 5$

$\frac{9a}{2} > 5 - 3 \times \frac{4}{3}$

$a > \frac{2}{9}$

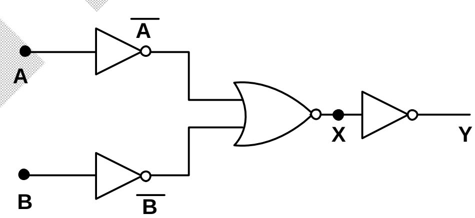


6. B

Sol. Output equation $y = \overline{\overline{A} + \overline{B}} = \overline{A \cdot B}$

= NAND GATE

A	B	$\overline{A}$	$\overline{B}$	X	Y
0	0	1	1	0	1
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	1	0



7. A

Sol. $S_2P - S_1P = \frac{\lambda}{2}$

$\sqrt{5}d - 2d = \frac{\lambda}{2}$

8. D

Sol. Here D_2 is reverse biased while D_1 is forward biased. So no current flows across D_2. Current will flow through D_1 only.

$I = \frac{V}{R} = \frac{2}{25}$ A

9. A

Sol. Using formula $E_0 = B_0 \times C$

$= 200 \times 10^{-6} \times 3 \times 10^{8} = 6 \times 10^{4}$ N/C

10. D

Sol. $\frac{3\lambda}{4} = \ell + e$

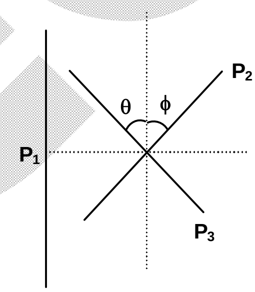
$\lambda = \frac{4(\ell + e)}{3} \Rightarrow f = \frac{3v}{4(\ell + e)}$

$\frac{df}{dt} = \frac{3v\left(-0.6\frac{dr}{dt}\right)}{4(\ell + 0.6r)^2} = -2$

$\therefore \frac{dr}{dt} = \frac{1}{72} m/s$

11. A

Sol. Let Initial intensity of light I_0. So intensity of light after transmission from first polaroid = $\frac{I_0}{2}$



Let ϕ be angle between 1st and 2nd polaroid

Hence, $\frac{9}{50}I_0 = \frac{I_0}{2}\cos^2\phi$

$\phi = 53°$

From figure $\phi + \theta = 90°$

$\therefore \theta = 37°$

12. B

Sol. Impulse on block = $\left(\frac{IA}{C}\right)\cos^2 53° \times (\Delta t)$

$= \frac{(20)(10 \times 10^{-4})}{3 \times 10^{8}} \times (0.6)^2 \times 6 \times 10^{-3}$

$= \frac{72}{5} \times 10^{-14}$ kg-m/s

Now we have

Impulse = mv

$\frac{72}{5} \times 10^{-14} = 1 \times 10^{-9} v$

$v = \frac{72}{5} \times 10^{-5}$ m/s

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

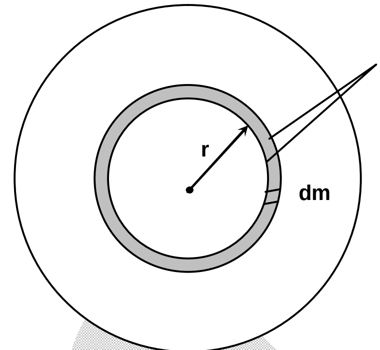
$10^{-5}x^2 = 10^{-9} \times \left(\frac{72}{5} \times 10^{-5}\right)^2$

$x = \frac{72}{5} \times 10^{-7}$ m

$N = \frac{7.2}{5} \mu m = 1.44\ \mu m$

13. B

Sol. Let 'M' be total mass of earth.
Consider a shell of thickness 'dr' and mass 'dm' at a distance 'r' from centre inside earth,



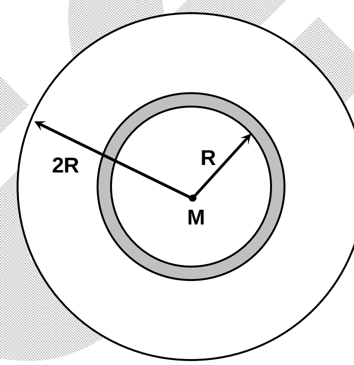
$\Rightarrow\ dm = \rho 4\pi r^2 dr$

$$M = \int dm = \int_0^r 4\pi k r^3 dr = \frac{4\pi k R^4}{4} = \pi k R^4$$

Let field due to earth's gravity at a distance '2R' from centre be I

$I \times A = 4\pi G m_{inside}$

$\Rightarrow\ I \times 4\pi(2R)^2 = 4\pi G(\pi k R^4)$



$$I = \frac{\pi k R^4 G}{4R^2}$$

$$\Rightarrow\ I = \frac{\pi k R^4 G}{4R^2}$$

For a satellite of mass ' m ' moving in orbit of ' 2R′ radius.

$$mI = \frac{mv^2}{(2R)}$$

$$I = \frac{v^2}{2R}$$

$$\frac{\pi k R^2 G}{4} = \frac{v^2}{2R}$$

$$\Rightarrow v = \sqrt{\frac{\pi k R^3 G}{2}}$$

14. D

Sol. $\frac{1}{v} + \frac{1}{-10} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{5};\ v = 5\text{ cm}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$-\frac{1}{v^2}\left(\frac{dv}{dt}\right) - \frac{1}{u^2}\left(\frac{du}{dt}\right) = 0$

$\vec{v}_I = -\frac{v^2}{u^2}\cdot\vec{v}_0$

$v_{I,M} = -2\hat{i}\ \text{m/s}$

$\vec{v}_{I,g} = \vec{v}_{I,M} + \vec{v}_{M,g} = 0$

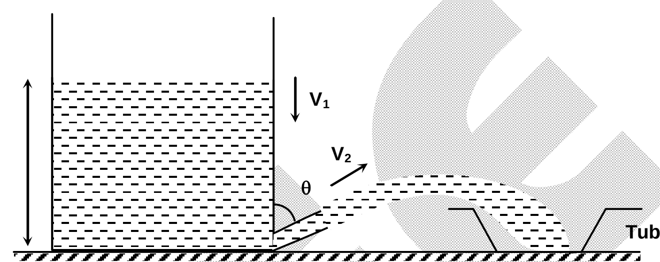
15. C

Sol. Applying Bernoulli's theorem between point on surface of water and point at orifice taking ground as reference,

$$P_{atm} + \frac{1}{2}\rho v_1^2 + \rho gH = P_{atm} + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow v_2^2 - v_1^2 = 2gH$$

$$\Rightarrow v_2^2 - \left(\frac{A_2}{A_1}\right) v_2^2 = 2gH \qquad [\because A_1v_1 = A_2v_2]$$



$$\Rightarrow v_2^2 = \frac{2gH}{1 - \left(\frac{A_2}{A_1}\right)^2}$$

Substituting $\frac{A_2}{A_1} = \frac{1}{2}$, H = 0.3 m

$v_2 = 2\sqrt{2}$ m/s

If $\theta = 30°$

$$\text{Range} = \frac{v_2^2 \sin 2(90 - \theta)}{g}$$

$$= \frac{8 \times \frac{\sqrt{3}}{2}}{10} = \frac{2\sqrt{3}}{5} \text{ m}$$

16. A

Sol. $P_0 = 2P_{L_1} + 2P_2 + P_M$

$$P_{L_1} = \frac{1}{f_{L_1}} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-20}\right) = \frac{1}{20}$$

$$P_{L_2} = \frac{1}{f_{L_2}} = (2 - 1)\left(\frac{1}{-20} - \frac{1}{20}\right) = -\frac{1}{10}$$

$$P_M = -\frac{1}{f_M} = -\frac{1}{R/2} = -\frac{1}{10}$$

$$P_0 = 2\left(\frac{1}{20}\right) + 2\left(-\frac{1}{10}\right) - \frac{1}{10}$$

$$P_0 = -\frac{1}{5\text{ cm}} = \left[\frac{-1}{0.05\,\text{m}}\right] = -20 \text{ diopter}$$

$$f_0 = -\frac{1}{P_0} = 5 \text{ cm}$$

$f_0 = 5$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_0}, \qquad \frac{1}{v} + \frac{1}{-40} = \frac{1}{5}$$

$$\frac{1}{v} = \frac{1}{40} + \frac{1}{5} = \frac{9}{40} \text{ cm}$$

17. D

Sol. $\frac{F}{A} = \eta\frac{du}{dy}$ and $u = \alpha\left[\frac{y}{h} - 2\left(\frac{y}{h}\right)^2\right]$

$\frac{du}{dy} = \alpha\left[\frac{1}{h} - 4\frac{y}{h^2}\right]$

Strain at fixed plate y = 0

$\frac{F}{A} = \eta\alpha\left[\frac{1}{h} - \frac{4\times 0}{h^2}\right] = \frac{\eta\alpha}{h}$

18. C

Sol. U = −1.7eV

$\Rightarrow E = \frac{U}{2} = -0.85 \text{ eV} = \frac{-13.6}{n^2}$

$\Rightarrow$ n = 4

Ejected photoelectron will have minimum de-Broglie wavelength corresponding to transition from n = 4 to n = 1, so we have

$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75 \text{ eV}$

Using Einstein's Photo-Electric Equation, we get

$\Rightarrow K_{max} = \Delta E - W = 10.45$ eV

$\Rightarrow \lambda = \sqrt{\frac{150}{10.45}}$ Å {for an electron}

$\Rightarrow \lambda = 3.8$Å

19. B

Sol. $U = 2 - 20x + 5x^2$

$\frac{dU}{dx} = -20 + 10x$

$F = -\frac{dU}{dx} = 20 - 10x = -10(x-2)$

$\therefore$ The particle is executing SHM about mean position

x – 2 = 0 or x = 2

$\therefore$ k = 10

$\Rightarrow m\omega^2 = 10$

$\omega^2 = \frac{10}{m} = \frac{10}{0.1} = 100$

$\Rightarrow \omega = 10$ rad/s

By the given data amplitude (A) = 5 m

$V_{max} = A\omega = 5(10) = 50$ m/s

$\therefore \beta = 2$

20. C

Sol. Diameter = M.S.R. + (C.S.R. $\times$ LC) – zero error

$= 3 \text{ mm} + 35\times\left(\frac{0.5}{50}\right) + 0.03$

= 3.38 mm

SECTION – B

21. 5

Sol. Here $m = 30$, $f_e = 5$ cm, $D = 25$ cm

Magnifying power of a compound microscope is

$$m = m_0 \times m_e = m_0\left(1+\frac{D}{f_e}\right)$$

or $30 = m_0\left(1+\frac{25}{5}\right)$

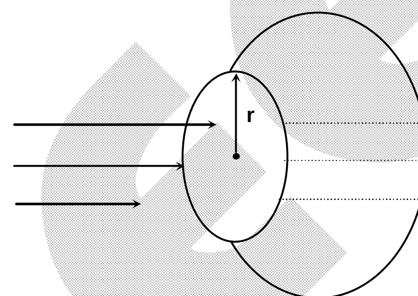
or $m_0 = 5$

22. 2

Sol. $(\pi r^2)(2\rho v^2) = 2(2\pi r)T$

$\Rightarrow v = \sqrt{\frac{2T}{\rho r}}$

$\Rightarrow X = 2$

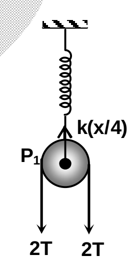


23. 8

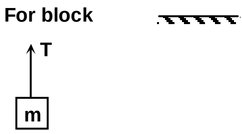
Sol. From equilibrium position, if block is displaced downward by x, pulley P_2 and P_1 moves $\frac{x}{2}$ and $\frac{x}{4}$ downward and spring further stretched by $\frac{x}{4}$

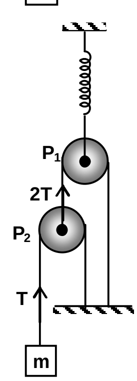
For pulley P1

$$4T = k\frac{x}{4};\ T = \frac{K}{16}x$$



For block





For block

$$F_{net} = T = -\frac{K}{16}x$$

Time period, $T = 2\pi\sqrt{\frac{m}{K/16}}$

24. 24

Sol. Shift of fringe pattern = $(\mu - 1)\frac{tD}{d}$

$\therefore \ \frac{30D(4800 \times 10^{-10})}{d} = (0.6)t\frac{D}{d}$

$30 \times 4800 \times 10^{-10} = 0.6t$

$t = \frac{30 \times 4800 \times 10^{-10}}{0.6} = 24 \times 10^{-6}$

25. 60

Sol. Using lens formula : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Here u =– 30 cm

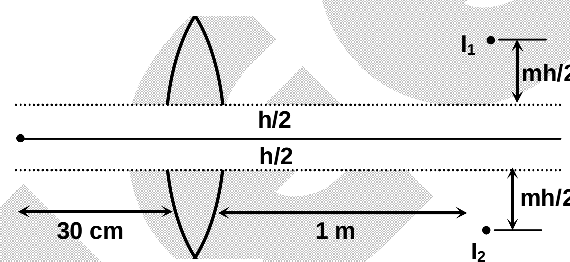
$\frac{1}{v} - \frac{1}{-30} = \frac{1}{f} \Rightarrow v = \frac{30f}{30 - f}$

and magnification, $m = \frac{v}{u} = \frac{-f}{30 - f}$

Hence separation between the images

$d = \frac{h}{2} + \frac{h}{2} + (m)\left(\frac{h}{2}\right) + m\left(\frac{h}{2}\right) = (m + 1)h$

$\Rightarrow \ d = \left(\frac{2f - 30}{f - 30}\right)h$



From given graph, the slope of the line = 6/2 = 3

$\frac{2f - 30}{f - 30} = 3$

$\Rightarrow$ f = 60 cm

Chemistry

PART – B

SECTION – A

26. B

Sol. 1st ionization enthalpy Zn > Ni > V > Sc

906 736 650 631 (kJ/mole)

Atomic radius Sc > V > Co > Zn

164 135 125 137 (pm)

Density Sc < V < Ni > Zn

343 607 8.9 7.1 (gm/cm^3)

Enthalpy of atomisation Sc < V > Mn > Zn

326 515 281 126 (gm/cm^3)

27. D

Sol. In stainless steel Fe, Cr, Ni are present. It is an alloy of Iron also known as inox or corrosion resistant steel.

28. C

Sol. $K\left[Cu(NH_3)_4\right]$ Cu has +1 Oxidation number.

$Cu^+ = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10}$

It means sp^3 hybridisation will form.

29. C

Sol. $\lambda^o_{(CH_3COOH)} = \lambda^o_{(HCl)} + \lambda^o_{(CH_3COONa)} - \lambda^o_{(NaCl)}$

= 425.9 + 91 – 126.4

= 390.5 $SCm^2\ mole^{-1}$

$\alpha = \frac{\lambda_m^C}{\lambda_m^o} = \frac{39.05}{390.5} = 0.1$

$\left[H^+\right] = C\alpha = 0.1 \times 0.1 = 10^{-2}$

$pH = -\log\left[H^+\right] = 2$

30. A

Sol. $Cu^{+2}(aq) + 2e \longrightarrow Cu(s)$

$E_{cell_1} = 0.34 - \frac{0.0591}{2}\log_{10}\left(\frac{1}{C}\right)$

$E_{cell_2} = 0.34 - \frac{0.0591}{2}\log_{10}\left(\frac{100}{C}\right)$

$E_{cell} - E_{cell} = \frac{0.0591}{2}\left(\log_{10}\frac{100}{C} - \log\frac{1}{C}\right)$

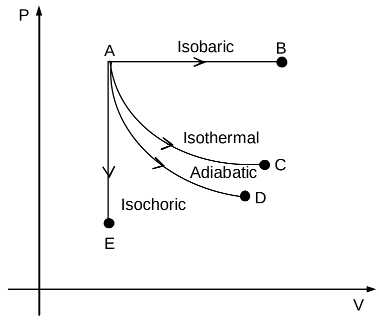
$= \frac{0.0591}{2}(\log_{10}100)$

= 0.0591 V

31. C

Sol. $T_A < T_B,\ T_A = T_C,\ T_A > T_D,\ T_A > T_E$

$T_D < T_A = T_C < T_B$



32. D

Sol.

(A)

	F^- >	Cl^- >	Br^- >	I^-	
$\Delta H_{hyd.}$	515	381	347	305	(kJ / mole)

(B)

	H_2O >	H_2S >	H_2Se >	H_2Te	
Melting point	273	188	208	222	K

(C)

H_2S <	H_2Se <	H_2Te <	H_2O	
213	232	269	373	K

(D)

PH_3	< AsH_3 <	NH_3 <	SbH_3	
185.5	210.6	238.5	254.6	K

33. B

Sol. $pK_a = -\log_{10} K_a = 4$

$K_a = 10^{-4}$

$K_a = C\alpha^2 \Rightarrow \alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{10^{-4}}{0.01}}$

$\alpha = 0.1$

For monobasic acid $i = 1 + \alpha(n-1)$

$= 1 + 0.1(2-1)$

$= 1.1$

34. C

Sol. $ZnSO_4(aq) \xrightarrow{\text{electrolysis}} \underbrace{Zn(s)}_{\text{Cathode}} + \underbrace{H^+}_{\text{Anode}} + SO_4^{-2}$

$[H^+]$ will increase hence pH decreases.

35. C

Sol. Chlorophyll - Magnesium

Rest are correct.

36. A

Sol. $Cr^{+2} \longrightarrow \underset{(\text{Stable})}{Cr^{+3}}$, $Fe^{+2} \longrightarrow \underset{(\text{Stable})}{Fe^{+3}}$

37. A

Sol. $Gd_{64)} = [Xe]4f^7 5d^1 6s^2$

38. D

Sol. U, Np, Pu and Am can have +6 oxidation states.

39. A

Sol. $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$

Volume of $H_2 = 2V_{O_2}$

= 2 × 28 = 56 lit

Mole of $H_2 = 2.5$ mole

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2$

$\because$ 1 mole H_2 form by 2 mole NaOH

$\therefore$ 2.5 mole H_2 form by $= \frac{2}{1} \times 2.5$

= 5 mole NaOH

$M = \frac{n}{V(\text{lit})} \Rightarrow V_{(\text{lit})} = \frac{n}{M} = \frac{5}{0.25}$

40. A

Sol.

Gas	Henry constant ($K_{H)}$) (K bar)
Ar	40
CO_2	1.67
CH_4	0.413
HCHO	1.83×10^{-5}

Solubility $\propto \frac{1}{K_H}$

41. C

Sol. $FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6] + 12KCl$

Ferric ferro cyanide

(Prussian blue colour)

42. C

Sol. $Na_2S + Na_2[Fe(CN)_5NO] \longrightarrow \underset{\text{(Purple)}}{Na_4[Fe(CN)_5NOS]}$

43. D

Sol. $I_2 + \text{Starch} \longrightarrow \text{Blue complex}$

44. B

Sol. $K_b(\text{Water}) = 0.52$

$K_b(\text{Diethyl ether}) = 2.02$

$K_b(CHCl_3) = 3.63$

$K_b(CCl_4) = 5.03$

45. D

Sol. Molality of ethylene glycol $\frac{45/62}{600/100} = 1.2$ mole/kg

$\Delta T_b - \Delta T_f = K_b m - K_f m$

$= (K_b - K_f)m$

$= (0.52 - 1.86)1.2$

$= -1.608$ K

SECTION – B

46. 32

Sol. $P_A^{1-\gamma} T_A^{\gamma} = P_B^{1-\gamma} T_B^{\gamma}$ $\left(\gamma = \frac{5}{3} \text{ for monoatomic}\right)$

$1 T_A^{5/3} = P_A^{-2/3} (300)^{5/3}$

$P_A^{2/3} = \left(\frac{300}{75}\right)^{5/3}$

$P_A = 32$ atm

47. 3

Sol. Isomer of $CoSO_4Br.5H_2O$ are $\underset{(A)}{\left[CoSO_4(H_2O)_5\right]Br}$

and $\underset{(B)}{\left[CoBr(H_2O)_5\right]SO_4}$

Ion isomers A and B complex ions are $\left[CuSO_4(H_2O)_5\right]^{+1}$ and $\left[CoBr(H_2O)_5\right]^{+2}$, so a = +1, b = +2

a + b = 3

48. 235

Sol. $\underset{1\text{ mole}}{Ag^+} + e \longrightarrow \underset{1\text{ mole}}{Ag(s)}$

Only 1 mole Ag can deposit at electrode A.

$Cu^{+2} + 2e \longrightarrow Cu$

$\because$ 2 mole Cu will deposit at electrode B.

Net mass = mass of Ag at A and mass of Cu at B

= 108 + 2 × 63.5 = 235 gm.

49. 78

Sol. $C_xH_y + \frac{1}{2}\left(2x + \frac{y}{2}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

$\frac{1}{2}\left(2x + \frac{y}{2}\right) = \frac{480}{2} \Rightarrow x + \frac{y}{4} = 7.5$ … (1)

Enthalpy of combustion $\Rightarrow -400x - 150y - 100 = 3400$

$40x + 15y = 330$ … (2)

Solve (1) and (2)

x = 6, y = 6

Compound A is C_6H_6

Molar mass of A = 12 × 6 + 1 × 6 = 78 gm/mole

50. 0

Sol. 18% w/V of aq. glucose solution means 100 ml soln contain 18 gm glucose.

$\therefore$ 1000 ml soln contain $= \frac{18}{100} \times 1000 = 180$ gm glucose

Mass of glucose = 180 gm V = 1000 ml

$d = \frac{W}{V} \Rightarrow 1.18 = \frac{W}{1000} \Rightarrow W_{\text{soln}} = 1180$ gm

$W_{\text{solvent}} = 1180 - 180 = 1000$ gm

Molarity $= \frac{180/180}{1000/1000} = 1$ M

Molality $= \frac{180/180}{1000/1000} = 1$ M

Molarity – Molality = 1 – 1 = 0

Mathematics PART – C

SECTION – A

51. D

Sol. $^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}$

$^{3}C_{0}+{}^{3}C_{1}+{}^{4}C_{2}+{}^{5}C_{3}+....+{}^{99}C_{97}={}^{100}C_{97}$

52. C

Sol. $\frac{1}{2}\le|z|\le 4$

We know that

$\left||z|-\frac{1}{|z|}\right|\le\left|z+\frac{1}{z}\right|\le|z|+\frac{1}{|z|}$

Maximum value of $|z|+\frac{1}{|z|}=\frac{17}{4}$ and minimum value of $\left||z|-\frac{1}{|z|}\right|=0$

53. C

Sol. Given that both the matrices

$A-\frac{I}{2}$ and $A+\frac{I}{2}$ are orthogonal that means

$\left(A-\frac{I}{2}\right)\left(A'-\frac{I}{2}\right)=I \qquad (\text{as } I'=I)$

$AA'-\frac{AI}{2}-\frac{IA'}{2}+\frac{I}{4}=I$ … (1) (as $I^2 = I$)

Also, $\left(A+\frac{I}{2}\right)\left(A+\frac{I}{2}\right)'=I$

$\left(A+\frac{I}{2}\right)\left(A'+\frac{I}{2}\right)=I$

$AA'+\frac{AI}{2}+\frac{IA'}{2}+\frac{I}{4}=I$ … (2)

subtracting Eqn. (1) from Eqn. (2), we get

$AI+IA'=0\Rightarrow A=-A'$

$\Rightarrow$ Hence, A is an skew-symmetric matrix.

Now, for order of matrix add Eqn. (1) and Eqn. (2), we get

Hence, $|A|^2\neq 0$ have this so even order.

54. D

Sol. $\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix}=\begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix}\begin{vmatrix} 1 & a+b & ab \\ 1 & c+d & cd \\ 0 & 0 & 0 \end{vmatrix}=0$

55. C

Sol. $\left|\left[\vec{a},\vec{b},\vec{c}\right]\right|=30$

$\left|abc\sin\theta\cos\phi\right| = 30 \Rightarrow \theta = \frac{\pi}{2}, \phi = 0 \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular.

$\left(2\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}\times\vec{c}\right)\times\left(\vec{a}-\vec{c}\right)+\vec{b}\right]$

$= \left(2\vec{a}+\vec{b}+\vec{c}\right).\left[a^2\vec{c}+c^2\vec{a}+\vec{b}\right]$

$= 50a^2 + b^2 + 4c^2 = 200 + 9 + 100 = 309$

$\therefore \frac{k}{103} = \frac{309}{103} = 3$

56. B

Sol. $A^{-1}B^{-1} = B^{-1}A^{-1} \Rightarrow C = \left(A^{-1}+B^{-1}\right)^5 = (I)^5$

57. C

Sol. Required probability $= \dfrac{3!\times 2}{\dfrac{9!}{3!3!3!}} = \dfrac{1}{140}$

58. B

Sol. We know that the equation of the plane passing through the line of intersection of planes $p_1 = 0$ and $p_2 = 0$ is

$p_1 + \lambda p_2 = 0$

That is,

$(x+2y+z-10)+\lambda(3x+y-z-5) = 0$ ... (i)

Since, this plane passes through the origin (0, 0, 0) satisfies this equation. This implies that

$(-10)+\lambda(-5) = 0$

$\Rightarrow \lambda = -2$

Substituting the value of λ in Eq. (1), we get

$(x+2y+z-10)-2(3x+y-z-5) = 0$

That is,

$-5x+3z = 0$

$\Rightarrow 5x-3z = 0$

59. D

Sol. $(1+2+3+.....+22)^{21}C_{10}$

60. B

Sol. A.M. $(\alpha,\beta,\gamma,\delta) = \frac{4}{4} = 1$

G.M. $(\alpha,\beta,\gamma,\delta) = 1 \Rightarrow \alpha = \beta = \gamma = \delta = 1$

So, equation is $(x-1)^4 = 0$

61. B

Sol. $P(x) = x^4 - 8x^2 + 15 + 2x^3 - 6x = \left(x^2-3\right)\left(x^2-5\right)+2x\left(x^2-3\right)$

$= \left(x^2-3\right)\left(x^2+2x-5\right)$

$Q(x) = (x+2)\left(x^2+2x-5\right)$

62. C

Sol. Normal vector of the plane $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$

$\vec{n} = 2\hat{i} + 2\hat{j} + 6\hat{k} = 2(\hat{i} + \hat{j} + 3\hat{k})$

$\therefore$ Equation of plane $1(x+1) + 1(y-2) + 3(z-0) = 0$

$P : x + y + 3z = 1$

Hence, $(a + b + c) = 1 + 1 + 3 = 5$

63. D

Sol. **Case 1:** $x < y$ and $y > z$

Two digits = ${}^{9}C_2$

Three digits = ${}^{9}C_3 \times \underline{|2} + {}^{9}C_2 \times 1$

Case 2: $x < y$ and $y = z$

Required ways = ${}^{9}C_2$

Total = 276

64. D

Sol. $|\vec{c}|^2 = 4|(\vec{a} \times \vec{b})|^2 + 9b^2 = 4\left(a^2b^2 - (\vec{a}.\vec{b})^2\right) + 9b^2 = 192$

$\vec{c} + 3\vec{b} = 2\vec{a} \times \vec{b} \Rightarrow c^2 + 9b^2 + 6\vec{b}.\vec{c} = 4\left(a^2b^2 - (\vec{a}.\vec{b})^2\right)$

$\Rightarrow 6.4.\sqrt{192}\cos\theta = -288 \Rightarrow\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$

65. B

Sol. $(\hat{a} \times \hat{b}) \times (\hat{a} + \hat{b}) = (\hat{a}.(\hat{a} + \hat{b}))\hat{b} - (\hat{b}.(\hat{a} + \hat{b}))\hat{a} = (1 + \hat{a}.\hat{b})(\hat{b} - \hat{a})$

66. A

Sol. Line represented by $x + ay - b = 0, cy + z - d = 0$ is parallel to

$(\hat{i} + a\hat{j}) \times (c\hat{j} + \hat{k}) = a\hat{i} - \hat{j} + c\hat{k}$

Line represented by $-x + a'y + b' = 0, c'y - z + d' = 0$ is parallel to

$(\hat{i} - a'\hat{j}) \times (c'\hat{j} - \hat{k}) = a'\hat{i} + \hat{j} + c'\hat{k}$

If these two lines are perpendicular, then

$aa' + cc' = 1$

67. C

Sol. $y = \log_2 x\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...\right) = 2\log_2 x$ … (1)

$4\log_4 x = \frac{5 + 9 + 13 + ... + (4y+1)}{1 + 3 + 5 + ... + (2y-1)}$

$2\log_2 x = \frac{2y^2 + 3y}{y^2} = y \Rightarrow y^2 = 2y + 3$

$\therefore y = 3\left(y = -1, \text{ rejected}\right)$

and $x = 2^{3/2}$

$\therefore x^2y = 24$

68. A

Sol. Clearly, *A* is skew symmetric and *B* is symmetric and $|A| = 0$

$\therefore \left|A^4B^3\right| = 0$

$\therefore$ Singular.

69. A

Sol. Replace *x* by *x* –1 in given equation, then we will get an equation whose roots are $(\alpha_n + 1)$ and $(\beta_n + 1)$ and product of roots will be $n(n-1)$.

$$\sum_{n=2}^{2021} \frac{1}{(\alpha_n + 1)(\beta_n + 1)} = \sum_{n=2}^{2021} \frac{1}{n(n-1)} = 1 - \frac{1}{2021} = \frac{2020}{2021} = \frac{a}{b}$$

$\therefore b - a = 1$

70. B

Sol. $S = \dfrac{1}{r(1-r)}$ where $r \in (0,1)$

$\therefore S_{max} = 4$

SECTION – B

71. 8

Sol. Replace $x \to \dfrac{2}{x}$

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x}\right)^r$$

$$2^{10}\left(2x^2 + 3x + 4\right)^{10} = \sum_{r=0}^{20} a_r.2^r.x^{20-r}$$

$$2^{10}.\sum_{r=0}^{20} a_r.x^r = \sum_{r=0}^{20} a_r.2^r.x^{20-r}$$

$\therefore$ Coefficient of x^7.

$2^{10}a_7 = a_{13}2^{13}$

$\dfrac{a_7}{a_{13}} = 2^3 = 8$

72. 5

Sol. Let $z = x + iy$

$\therefore \bar{z} = x - iy$

$\therefore (2iy)^2 = 12\left(x^2 + y^2\right) - 4 \Rightarrow 12x^2 + 16y^2 = 4$

$$3x^2 + 4y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{3}} + \frac{y^2}{\frac{1}{4}} = 1$$

$\therefore x = \sqrt{\frac{1}{3}}\cos\theta,\ y = \sqrt{\frac{1}{4}}\sin\theta$

$\therefore 3\sqrt{3}\,\text{Re}(z) + 8\,\text{Im}(z) = 3\cos\theta + 4\sin\theta$

$\therefore \max = 5$

73. 7

Sol. Required number of words = number of words in which M's are separated – number of words in which M's are separated but I's are together.

$= \frac{4!}{2!} \times {}^5C_2 - 3! \times {}^4C_2$

$= 120 - 36 = 84 = 12 \times 7$

74. 9

Sol. $a^2 + 4b^2 + 4c^2 - 2ab - 4bc - 2ac = 0$

$(a - 2b)^2 + (2b - 2c)^2 + (2c - a)^2 = 0$

$\Rightarrow a = 2b = 2c$

$\therefore$ Number of ordered triples satisfying are 3 i.e. (2, 1, 1), (4, 2, 2), (6, 3, 3).
Two points (2, 1, 1) and (4, 2, 2) lying inside the given tetrahedron.

$\therefore$ Required probability is $\frac{2}{3} = \frac{6}{\lambda} \Rightarrow \lambda = 9$

75. 3

Sol. $\overrightarrow{OC} = m\overrightarrow{OA} + n\overrightarrow{OB}$

$\vec{c} = m\vec{a} + n\vec{b}$ … (1)

Given $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = \sqrt{2}$, $\tan\alpha = 7$

Now take dot of equation (1) with $\vec{a}$ and $\vec{c}$ to get

$m = \frac{5}{4}; n = \frac{7}{4}$

$\therefore m + n = 3$