ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)

FULL SYLLABUS

02-03-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-II)

ANSWER KEY

PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4				
	A.	B	D	B	D				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	C,D	A,C	A,C	A,D	A,B,C,D	B,C,D		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	1	2	1	3	4	1	7	4

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4				
	A.	C	B	B	C				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,B,C,D	A,C,D	A,D	A,B,D	A,C,D	B,C,D		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	6	7	8	4	0	7	2	2

PART-3 : MATHEMATICS

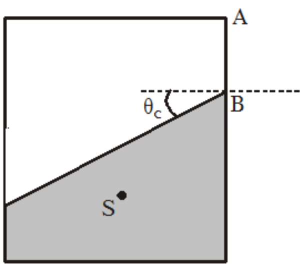
SECTION-I (i)	Q.	1	2	3	4				
	A.	A	C	C	C				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,B,C	B,C	A,B,C	A,D	B,D	A,C,D		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	4	9	5	1	3	6	8	7

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (B)**



$\sin C = \frac{1}{2}$; $C = 30°$

If s is anywhere in the shaded region, the light rays from S

will strike AB making an angle more than critical angle.

2. **Ans (D)**

$$\mu = \left(\frac{\mu_L - \mu_o}{L}\right)x + \mu_0$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{dx}{dt} = \frac{\sqrt{T}}{\left[\left(\frac{\mu_L - \mu_0}{L}\right)x + \mu_0\right]^{1/2}}$$

$$\int_0^L \left[\left(\frac{\mu_L - \mu_0}{L}\right)x + \mu_0\right]^{1/2} dx = \int_0^t \sqrt{T}\,dt$$

$$\frac{2L}{3}\frac{\left(\sqrt{\mu_L}\right)^3 - \left(\sqrt{\mu_0}\right)^3}{\mu_L - \mu_0} = \sqrt{T}\,t$$

3. Ans (B)

The number of atoms in 'm' kg of uranium $= \frac{m}{A} \times N_A$

Energy released $= \frac{m}{A} \times N_A \times 200\,\text{MeV}$

$= \left(\frac{1.5}{235} \times 10^3 \times 6.023 \times 10^{23} \times 200 \times 1.6 \times 10^{-19}\right)$

$= 1.2 \times 10^{14}\,\text{J}$

Mass of TNT $= \frac{1.2 \times 10^{14}}{4.1 \times 10^6} = 3 \times 10^7\,\text{kg}$

4. Ans (D)

Let T be the temperature at any time t.

Then rate of loss of heat $= -MS\left(\frac{dT}{dt}\right)$

The rate of loss of heat due to radiation

$= \sigma \times 4\pi r^2 (T^4 - 0)$

$-MS\frac{dT}{dt} = \sigma \times 4\pi r^2 \times T^4$

$\Rightarrow -\frac{4\pi}{3} r^3 \rho S \frac{dT}{dt} = \sigma \times 4\pi r^2 \times T^4$

$\Rightarrow \int_0^t dt = \frac{\rho rs}{3\sigma} \int_{T_1}^{T_2} \frac{dT}{T^4}$

$\Rightarrow t = 595165\,\text{s} = 165\,\text{hrs}\ 19\,\text{mins}$

PART-1 : PHYSICS

SECTION-I (ii)

7. Ans (A,C)

Moving piston up or down will have no effect on the net force

$\Rightarrow$ no change in spring compression.

If spring was compressed & the system is accelerated up, then final compression in spring will depend on the value of ρ_1, ρ_2 & ρ_{solid}.

If spring is uncompressed then accelerating the system will have no effect on net force

$\Rightarrow$ no change in compression of spring

10. Ans (B,C,D)

By symmetry, the centers of mass of each dumb-bell should have the same speed v' after collision and each dumb-bell should have the same angular velocity after collision. By energy conservation:

$\frac{1}{2}4mv^2 = 4mv'^2 + \frac{1}{2}m\ell^2 \times 2\omega^2 \times 2$

By angular momentum conservation:

$4mv\ell = 2m\ell^2 \times \omega \times 2$

Hence $v' = 0$ and $\omega = \frac{v}{\ell}$. Hence $T = \frac{mv^2}{\ell}$ and the dumb-bells rotate about their centers of mass. The next collision will occur after a time $\frac{\pi}{\omega}$.

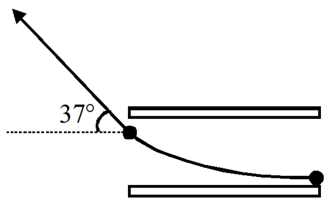
PART-1 : PHYSICS

SECTION-III

1. Ans (1)

$\tan 37° = \frac{V_y}{V_x} = \frac{V_y}{V_0}$

$V_y = a_y t = \frac{qE_y}{m} \times \frac{\ell}{V_0}$



$E_y = \frac{V}{d} = \frac{iR}{d} = \frac{\varepsilon R}{(R+r)d}$

$\Rightarrow \frac{3}{4}V_0 = V_y = \frac{q\ell}{mv_0} \times \frac{\varepsilon R}{(R+r)d}$

$\frac{3}{4}V_0^2 = \frac{16}{91} \times 10^{12} \times \frac{3 \times R}{(R+2)10^{-3}} \times 0.182$

$2.5R + 5 = 3R$

$5 = 0.5R$; $R = 10\Omega$

2. Ans (2)

$\cos(90+\theta) = \frac{x^2 + \left(10\sqrt{3}\right)^2 - R^2}{20\sqrt{3}x} = -\sin\theta = \frac{-1}{\sqrt{3}}$

4. **Ans (3)**

For a common maxima for both the colours,

we can use $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

$\Rightarrow 3n_1 = 5n_2$, so $n_1 = 5$, $n_2 = 3$

$n_2 = 10$, $n_2 = 6$

and so on.

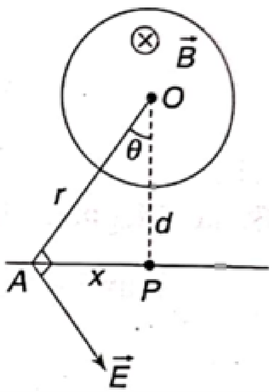
For the 2nd closest point we will consider $n_1 = 10$ on $n_2 = 6$.

$$y = \frac{n_1\lambda_1 D}{d} = \frac{10 \times 300 \times 10^{-9} \times 1}{10^{-3}}$$

$= 3 \times 10^{-3}$ m

$\Rightarrow y = 3$ mm

5. **Ans (4)**



Induced electric field at a point A shown in the figure can be calculated as

$$E \cdot 2\pi r = \pi a^2 \frac{dB}{dt}$$

$$\Rightarrow E = \frac{a^2\alpha}{2r}$$

Electric force on the charge along AP is

$$F_e = qE\cos\theta = \frac{qa^2\alpha \cdot d}{2r^2}$$

The external agent must apply equal and opposite force to keep the charge moving without gaining any kinetic energy. Work done by the external agent in small displacement dx will be

$$dW = \frac{qa^2\alpha d}{2r^2}dx$$

But $x = d\tan\theta \Rightarrow dx = d\sec^2\theta d\theta$ and $r = d\sec\theta$

$$dW = \frac{qa^2\alpha d}{2(d\sec\theta)^2}.d\sec^2\theta d\theta = \frac{qa^2\alpha}{2}d\theta$$

$$W = \frac{qa^2\alpha}{2}\int_0^{\pi/2} d\theta = \frac{\pi qa^2\alpha}{4}$$

6. **Ans (1)**

$$U = U_0 \ell nr - U_0 \ell nr_0$$

$$\Rightarrow \frac{dU}{dr} = \frac{U_0}{r}$$

Force between the particles is

$$\therefore F = -\frac{dU}{dr} = -\frac{U_0}{r}$$

Negative sign indicates attraction

$$\therefore \frac{mv^2}{r} = \frac{U_0}{r} \quad .....(i)$$

$$\text{and } mvr = \frac{nh}{2\pi} \quad ......(ii)$$

From (i) and (ii), $r = \frac{nh}{2\pi\sqrt{mU_0}}$

Kinetic energy of electron $K = \frac{1}{2}mv^2 = \frac{U_0}{2}$

Energy of electron in nth orbit is

$$E_n = KE + PE = \frac{U_0}{2} + U_0\ell n\left(\frac{r}{r_0}\right)$$

$$= \frac{U_0}{2} + U_0\ell n\left[\frac{nh}{2\pi\sqrt{mU_0}}.\frac{1}{r_0}\right]$$

$$= \frac{U_0}{2}\left[1 + \ell n\left(\frac{n^2h^2}{4\pi^2 mU_0 r_0^2}\right)\right]$$

$$\Delta E_{nm} = E_n - E_m$$

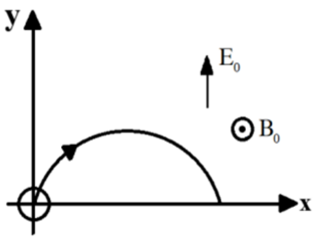
$$= \frac{U_0}{2}\ell n\left(\frac{n^2h^2}{4\pi^2 mU_0 r_0^2}\right) - \frac{U_0}{2}\ell n\left(\frac{m^2h^2}{4\pi^2 mU_0 r_0^2}\right)$$

$$= \frac{U_0}{2}\ell n\left(\frac{n}{m}\right)^2 = U_0\ell n\left(\frac{n}{m}\right)$$

$$\frac{\Delta E_{12}}{\Delta E_{24}} = \frac{\ell n\left(\frac{1}{2}\right)}{\ell n\left(\frac{2}{4}\right)} = 1$$

7. Ans (7)

The particle moves in xy plane. Let its velocity at any instant be



$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$\vec{F} = q\vec{E} + q(\bar{V} \times \bar{B})$

$= qE_0\hat{j} + q(V_x\hat{i} + V_y\hat{j}) \times (B_0\hat{k})$

$= q(E_0 - V_xB_0)\hat{j} + qB_0V_y\hat{i}$

$\frac{mdV_x}{dt} = qB_0V_y$ .....(1)

and $m\frac{dV_y}{dt} = q(E_0 - V_xB_0)$ .....(2)

Differentiating (2) w.r.t. time and substituting for $\frac{dV_x}{dt}$ from (1), we get

$\frac{d^2V_y}{dt^2} = \left(\frac{qB_0}{m}\right)^2 V_y$

This equation is of the form

$\frac{d^2x}{dt^2} = -\omega^2 x$

$\therefore\ V_y = V_{y0}\sin(\omega t + \delta)$ $\left[\omega = \frac{qB_0}{m}\right]$

at $t = 0;\ V_y = 0 \Rightarrow \delta = 0$

$\therefore\ V_y = V_{y0}\sin\left(\frac{qB_0}{m}t\right)$

$a_y = \frac{dV_y}{dt} = V_{y0}\frac{qB_0}{m}\cos\left(\frac{qB_0}{m}t\right)$

at $t = 0;\ a_y = \frac{qE_0}{m}$

$\frac{qE_0}{m} = \frac{qB_0}{m}V_{y0} \Rightarrow V_{y0} = \frac{E_0}{B_0}$

$V_y = \frac{E_0}{B_0}\sin\left(\frac{qB_0}{m}t\right)$

$\frac{dy}{dt} = \frac{E_0}{B_0}\sin\left(\frac{qB_0}{m}t\right)$

$\int_0^y dy = \frac{E_0}{B_0}\int_0^t \sin\left(\frac{qB_0}{m}t\right)dt$

$y = \frac{mE_0}{qB_0^2}\left[1 - \cos\left(\frac{qB_0}{m}t\right)\right]$

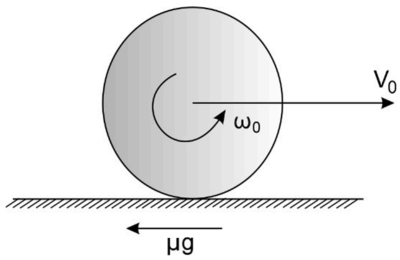
$y_{max} = \frac{2mE_0}{qB_0^2}$

$2R = \frac{2mE_0}{qB_0^2}$

$R = \frac{mE_0}{qB_0^2}$

8. Ans (4)

Ball will come back to the initial position if its angular velocity is greater than zero in the same direction (in which it was released) at the moment its' linear velocity becomes zero. In this condition ball would return back.

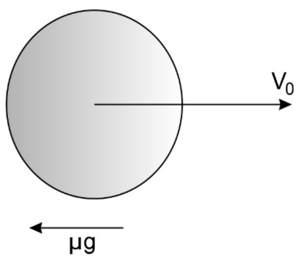


For linear motion

$0 = v_0 - (\mu g)t$

$t = \frac{v_0}{\mu g}$ (time when ball stops)

For rotation motion



$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{\mu mg \times R}{\frac{2}{5}MR^2} = \frac{5\mu g}{2R}$

Using $\omega_f = \omega_0 - \alpha t$

$\omega_f > 0$

$\Rightarrow \omega_0 > \alpha t$

$\Rightarrow \omega_0 > \frac{5\mu g}{2R}\cdot\frac{v_0}{\mu g}$ for limiting condition.

$\Rightarrow \lambda_0 = \frac{2}{5}$

PART-2 : CHEMISTRY

SECTION-I (i)

1. Ans (C)

$\Delta T_f = 273.15 - 272.15 = 1\ K$

$K_f = 1.86\ K\ kg\ mol^{-1}$

$\therefore \Delta T_f = kg \times \text{molality} \times i$

$$1 = 1.86 \times \frac{0.1}{\frac{(31\times18)}{1000}} \times i = \frac{1.86 \times 0.1 \times 1000}{31 \times 18} \times i$$

$$1 = \frac{1}{3} \times i \quad \Rightarrow i = 3$$

$\therefore$ Chemical formula of compound

$= [Co(NH_3)_x Cl_y]\,Cl_2$

In case of cobalt (III),

$y + z = 3$

$y + 2 = 3$

$\therefore y = 1$

In case of octahedral salt $x + y = 6$

$\Rightarrow x + 1 = 6$

$\therefore x = 5$

$\therefore x + y + z = 5 + 1 + 2 = 8$

2. Ans (B)

$NaH + D_2O \rightarrow NaOH + HD$

$HDO \xrightarrow{\text{electrolysis}} H_2 + D_2 + HD + O_2$

$H_2 + D_2 \xrightarrow[\text{arc}]{\text{electric}} H_2 + D_2 + HD$

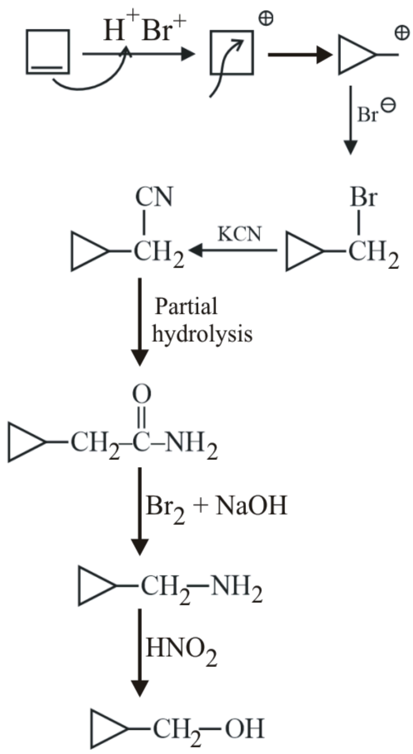
3. Ans (B)

$Ni \rightarrow [Ar]\,3d^8 4s^2$

$Pd \rightarrow [Kr]\,4d^{10} 5s^0$

$Pt \rightarrow [Xe]\,5d^9 6s^1$

4. Ans (C)



PART-2 : CHEMISTRY

SECTION-I (ii)

5. Ans (A,B,C,D)

(A) $C_6H_6 \rightleftharpoons C_6H_6\,(g)$

at eqb. $\Delta G = 0$

(B) $\Delta G^\circ = -RT \ln K_{eq}$

$K_{eq} = 1$ bar

$\Delta G^\circ = 0$

(C) $\Delta H^\circ = +ve$ in given direction

(D) ΔG at 2 atm and 80.1°C is +ve

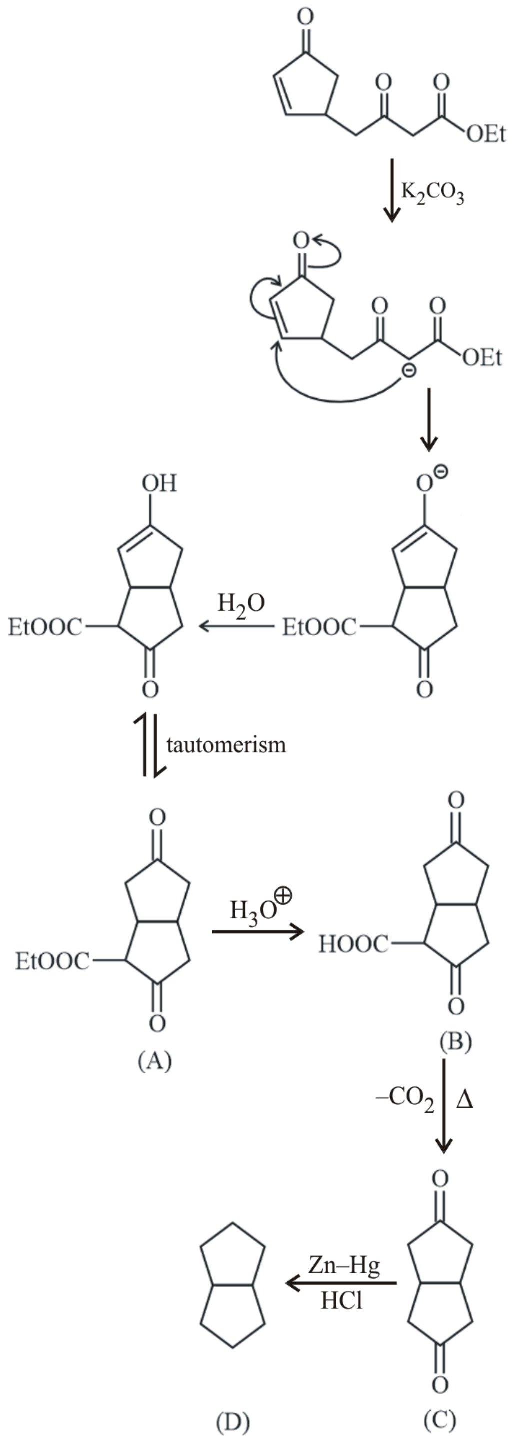
7. Ans (A,D)

$[Ru(bpy)_3]^{2+}$, $[Mn(acac)_3]$ does not have any plane or centre of symmetry.

8. Ans (A,B,D)

Tungston (W) is a high melting metal therefore liquation is not used for refining.

9. Ans (A,C,D)



PART-2 : CHEMISTRY

SECTION-III

1. Ans (6)

$N_2O_4 \rightleftharpoons 2NO_2$

Initial moles $\quad 1 \quad\quad 0$

At eqm mole $\quad 1-\alpha \quad\quad 2\alpha$

$$K_P = \frac{(2\alpha)^2 P}{(1-\alpha^2)} = \frac{4(0.5)^2 \times 1}{1-(0.5)^2} = \frac{4}{3}$$

$\Delta G^\circ = -RT \ln K_P$

$= -2 \times 1000 \ln \left(\frac{4}{3}\right)$

$= -2 \times 1000 \times 0.3 = -600$ cal.

$|\Delta G^\circ| = 600.$

2. Ans (7)

$$E^o_{Cl^-/AgCl/Ag} = E^o_{Ag^+/Ag} + \frac{0.059}{1} \log K_{sp}$$

$0.21 = 0.80 + \frac{0.059}{1} \log K_{sp}$

$\because K_{sp} = 10^{-10}$

Let solubility of AgCl in 0.01 M NaCl be x

$AgCl \rightleftharpoons [Ag^+][Cl^-]$

$10^{-10} = x(x + 0.01)$

$x = 10^{-8}$ mol/lit

Moles of AgCl dissolved in 10 L = $10^{-8} \times 10 = 10^{-7}$

$\because y = 7$

3. Ans (8)

T = 300 K

V = 164 L

P = 2.25 atm

$Br_2(l) + Cl_2(g) \rightleftharpoons 2BrCl(g)$

10 mole

(10 – x) $\quad$ 2x

$2 \times 164 = (10 - x + 2x) \times 0.082 \times 300$

or $x = \frac{10}{3}$

$$n_{Br_2} = \frac{PV}{RT} = \frac{0.25 \times 164}{0.082 \times 300} = \frac{5}{3}$$

Required amount of Br_2

$= \frac{10}{3} + \frac{5}{3} = 5.00$ mole

$= 5 \times 160 = 800$ g

4. Ans (4)

Pyrolusite $\rightarrow MnO_2$

Rutile $\rightarrow TiO_2$

Chromite $\rightarrow FeCr_2O_4$

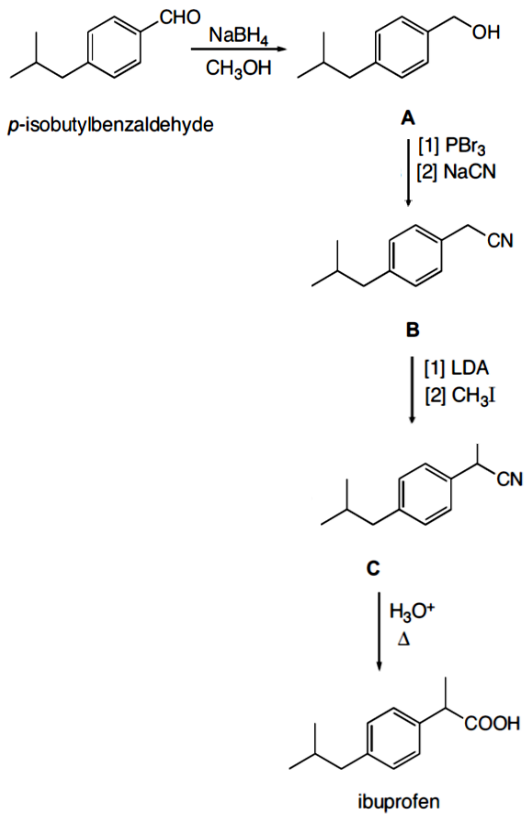
Limonite $\rightarrow Fe_2O_3.3H_2O$

5. Ans (0)

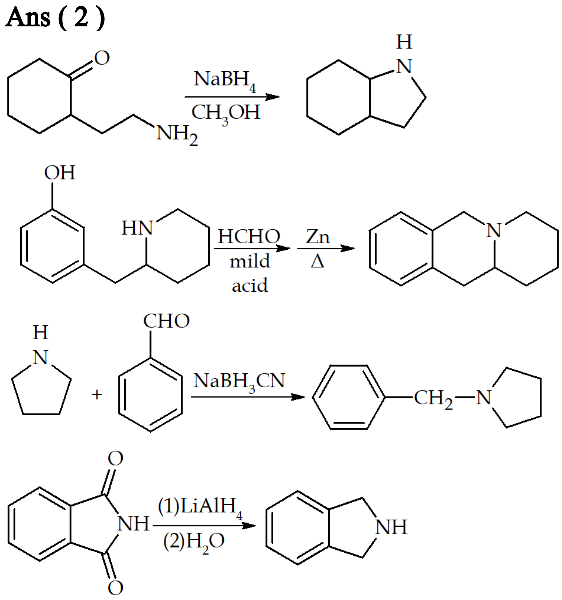
$Ni^{+2} + KCN \rightarrow K_2[Ni(CN)_4]$

$\xrightarrow{\text{K metal in liq. } NH_3} K_4[Ni(CN)_4]$

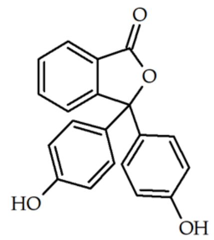
6. Ans (7)



7. Ans (2)



8. Ans (2)



PART-3 : MATHEMATICS

SECTION-I (i)

1. Ans (A)

Eq^n: $(\cot^2\theta + 1)(4\cot^2\theta + 9\cot\theta + 4) = 0$

$\cot\theta = \frac{-9 \pm \sqrt{17}}{8}$ both are negative and reciprocals of one another

$\Rightarrow \theta = \alpha, \frac{3\pi}{2} - \theta \in \left(\frac{\pi}{2}, \pi\right)$

and $\beta, \frac{7\pi}{2} - \alpha \in \left(\frac{3\pi}{2}, 2\pi\right)$

Total four solutions having sum $= 5\pi$

2. Ans (C)

Using AM $\geqslant$ GM

$$\frac{p^8 + q^8 + 6\left(\frac{r^2}{4}\right)}{8} \geqslant \left(p^8q^8\left(\frac{r^2}{4}\right)^6\right)^{1/8}$$

$$\Rightarrow \frac{2p^8 + 2q^8 + 3r^2}{16} \geqslant \frac{pqr^{3/2}}{2\sqrt{2}}$$

$$\Rightarrow \frac{(2p^8 + 2q^8 + 3r^2)^2}{p^2q^2r^3} \geqslant 32$$

3. Ans (C)

P : (False)

As trace $(AB - BA) \neq$ trace (I)

Q: (True) $|I + AB|\,|A^{-1}|\,|A|$

$|A^{-1}|\,|I + AB|\,|A|$

$|A^{-1} + A^{-1}AB|\,|A|$

$|A^{-1}A + BA|$

R: (True)

For skew symmetric matrix ‘A’ of non-zero determinant

$\exists$ P such that

$P^TAP = I$

$\Rightarrow |A| = |P^{-1}|^2$

S (True) $\Delta = a\{b(c + 1) + 1\} = 23$

$a = 1,\ b = 2,\ c = 10$

4. **Ans (C)**

$$\sum_{j=1}^{10} {}^{10}C_j \left(\sum_{i=1}^{10-j} {}^{10-j}C_i\right)$$

$$= \sum_{j=1}^{10} {}^{10}C_j \left(2^{10-j} - 1\right)$$

$$= \sum_{j=1}^{10} {}^{10}C_j\, 2^{10-j} - \sum_{j=1}^{10} {}^{10}C_j$$

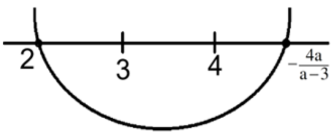
$$= \left(3^{10} - 2^{10}\right) - \left(2^{10} - 1\right)$$

$$= = 3^{10} - 2^{11} + 1$$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. **Ans (A,B,C)**



$[x](x-2) - 3(x-2) = 0$

$x = 2$ or $[x] = 3$

$(a-3)x^2 + 2(a+3) - 8a \leqslant 0$

$(x-2)((a-3)x + 4a) \leqslant 0$

Case I : $a - 3 > 0$

$$-\frac{4a}{a-3} \geqslant 4 \qquad \Rightarrow a \leqslant \frac{3}{2}$$

No solution

Case II : $a - 3 < 0$

$$-\frac{4a}{a-3} \leqslant 2 \qquad \Rightarrow a \leqslant 1$$

6. **Ans (B,C)**

$$f\left(0^-\right) = \lim_{x\to 0^-} \lim_{n\to\infty} \frac{\lambda \sin\left(n - \sqrt{n^2 - 8n}\right)x}{x}$$

$$= \lim_{\substack{x\to 0^- \\ n\to\infty}} \frac{\lambda \sin \frac{\left(n^2 - n^2 + 8n\right)x}{n + \sqrt{n^2 - 8n}}}{x}$$

$$= \lim_{x\to 0^-} 4\lambda \frac{\sin 4x}{4x} = 4\lambda$$

$$f\left(0^+\right) = \lim_{x\to 0^+} \frac{1 - \cos\left(\tan\left(\frac{\pi}{4} - x\right)\right)}{x^2}$$

$$= 2$$

7. **Ans (A,B,C)**

Vector normal to the plane

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\vec{V}_x = \hat{i};\ \vec{V}_y = \hat{j};\ \vec{V}_z = \hat{k}$

$$\cos(90 - \alpha) = \frac{\vec{V}_x \cdot \vec{n}}{|\vec{n}|} \quad \Rightarrow \quad \sin\alpha = \frac{1}{\sqrt{3}}$$

$$|||^{ly} \quad \sin\beta = \frac{1}{\sqrt{3}} \quad \text{and} \quad \sin\gamma = \frac{1}{\sqrt{3}}$$

Hence $\sum \sin^2\alpha = 1$ and $\sum \cos^2\alpha = 2$

Also plane is equally inclined with the coordinate axes.

$$\text{Also } A = \frac{1}{2}\sqrt{9^2 + 9^2 + 9^2} = \frac{9\sqrt{3}}{2}$$

$\Rightarrow$ (D) is not correct.

8. **Ans (A,D)**

$$I = \int \frac{(x-1)dx}{x^3\sqrt{2 - \frac{2}{x} + \frac{1}{x^2}}}$$

$$\text{Let } 2 - \frac{2}{x} + \frac{1}{x^2} = t^2 \text{ then } \left(\frac{2}{x^2} - \frac{2}{x^3}\right)dx = 2t\,dt$$

$$\frac{(x-1)}{x^3}dx = t\,dt$$

$$I = \int \frac{t\,dt}{\sqrt{t^2}} = \int dt = t + c$$

$$I = \sqrt{2 - \frac{2}{x} + \frac{1}{x^2}} + c$$

$$I = \frac{\sqrt{2x^2 - 2x + 1}}{x} + c$$

9. **Ans (B,D)**

$(2x^3 - y)dx + (x + 2yx^2)dy = 0$

$2x^3dx + 2yx^2\,dy + x\,dy - y\,dx = 0$

$$2x\,dx + 2y\,dy + \frac{x\,dy - y\,dx}{x^2} = 0$$

$$d\left(x^2 + y^2\right) + d\left(\frac{y}{x}\right) = 0$$

$$d\left(x^2 + y^2 + \frac{y}{x}\right) = 0$$

$$f(x, y) = x^2 + y^2 + \frac{y}{x}$$

$$\text{Now, } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z+1}{1} = \mu$$

$x = 2\mu + 1,\ y = 3\mu - 1,\ z = \mu - 1 = 0$

$\Rightarrow \mu = 1$

$x = 3,\ \ y = 2$

$$\lambda = f(3, 2) = 3^2 + 2^2 + \frac{2}{3} = 9 + 4 + \frac{2}{3} = \frac{41}{3}$$

10. **Ans (A,C,D)**

$$f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$$

Differentiate w.r.t. x while keeping y as constant:

$$\frac{1}{2}f'\left(\frac{x+y}{2}\right)=\frac{1}{2}f'(x)+0$$

$$\downarrow \begin{array}{l} x=0 \\ y=2x \end{array}$$

$$\frac{1}{2}f'(x)=\frac{1}{2}f'(0)$$

$$f'(x)=f'(0)=-1$$

$$f'(x)=-1$$

$$f(x)=-x+c$$

$$\downarrow f(0)=1$$

$$\boxed{f(x)=-x+1}$$

(A) $f(|x|)=-|x|+1$ is non-differentiable at $x=0$

(B) $f^{-1}(x)=-x+1=f(x)$

$\Rightarrow\ f(x)=f^{-1}(x)\ \forall x\in R$

(C) $\sum\limits_{r=0}^{10}(f(r))^2=\sum\limits_{r=0}^{10}(1-r)^2$

$=1^2+0^2+1^2+2^2+........+9^2$

$=1+\frac{9.10.19}{6}=1+3.5.19=286$

(D) $\tan^{-1}(f(x))=\tan^{-1}(1-x)$ is derivable $\forall x\in R$

PART-3 : MATHEMATICS

SECTION-III

1. **Ans (4)**

$3y^2=x^2(3-x)$

$y=\pm x\sqrt{\frac{3-x}{3}}$

$\text{Area}=2\int_0^3 x\sqrt{\frac{3-x}{3}}\,dx$

Let $\frac{3-x}{3}=t^2$

$-\frac{1}{3}dx=2t\,dt$

$=2\int_0^1(3-3t^2)\cdot t\cdot 6t\,dt$

$=36\left[\int_0^1 t^2-t^4\,dt\right]$

$=36\left[\frac{1}{3}-\frac{1}{5}\right]$

$=36\times\frac{2}{15}$

$=4.80$

2. **Ans (9)**

Let r be the radius of the circle and x be the side of the square.

Then, $\ell=2\pi r+4x$

Let S be the sum of the area of the circle & the square then

$S=\pi r^2+x^2$

$=\pi\left(\frac{\ell-4x}{2\pi}\right)^2+x^2$ .....(1)

$=\frac{1}{4\pi}(\ell-4x)^2+x^2$

$\therefore\ \frac{dS}{dx}=\frac{2}{4\pi}(\ell-4x)(-4)+2x$

$\frac{dS}{dx}=0$

$x=\frac{1}{4+\pi}$ .....(2)

From (1) & (2)

$x=2r$

$\therefore\ k=1$

3. Ans (5)

$$K = \lim_{x\to 0} \frac{\sin^{-1}x - \tan^{-1}x + (1+x^3)^{1/3} - 1}{x^3} \quad \left(\frac{o}{o} \text{ form}\right)$$

using L'Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2} + \frac{1}{3}\frac{3x^2}{(1+x^3)^{2/3}}}{3x^2} \quad \left(\frac{o}{o} \text{ form}\right)$$

$$= \lim_{x\to 0} \frac{(1-x^2)^{-1/2} - (1+x^2)^{-1}}{3x^2} + \frac{1}{3}(1+x^3)^{-2/3}$$

$$= \lim_{x\to 0} \frac{\left(1+\frac{1}{2}x^2\right) - (1-x^2)}{3x^2} + \frac{1}{3}(1+x^3)^{-2/3}$$

$$K = \lim_{x\to 0} \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$6K = 5$

4. Ans (1)

$$(ax-b)^2 + (bx-c)^2 + (cx-d)^2 = 0$$

$$x = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \Rightarrow b^2 = ac,\ c^2 = bd$$

$\Rightarrow$ $\ell na, \ \ell nb, \ \ell nc$ are in A.P.

5. Ans (3)

$$\lim_{n\to\infty} \left(\frac{n+7}{n}\right)^{2-\lambda} \frac{\sum_{r=1}^{n} \left(\frac{r}{n}\right)^{\lambda}}{\sum_{r=1}^{n} \lambda + \left(\frac{r}{n}\right)^2}$$

$$= \frac{\int_0^1 x^{\lambda} dx}{\int_0^1 (\lambda + x^2)\, dx} = \frac{\frac{1}{\lambda+1}}{\lambda + \frac{1}{3}} = \frac{3}{(\lambda+1)(3\lambda+1)}$$

$$\Rightarrow (\lambda+1)(3\lambda+1) = 40 \Rightarrow \lambda = 3$$

6. Ans (6)

Let $z = x + iy$

$$\arg\left(\frac{((x+\ell)+iy)(x+m-iy)}{(x+m)^2+y^2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \arg\left[\frac{(x+\ell)(x+m) + y^2 + iy((x+m)-(x+\ell))}{(x+m)^2+y^2}\right] = \frac{\pi}{6}$$

$$\Rightarrow \arg\left[\frac{x^2 + (\ell+m)x + \ell m + y^2 + iy(m-\ell)}{(x+m)^2+y^2}\right] = \frac{\pi}{6}$$

$$\Rightarrow \frac{y(m-\ell)}{x^2+y^2+(\ell+m)x+\ell m} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x^2 + y^2 + (\ell+m)x - \sqrt{3}(m-\ell)y + \ell m = 0$$

comparing $\ell + m = 3\sqrt{2},\ m - \ell = \sqrt{2},\ \ell m = 4$

$$\Rightarrow \ell = \sqrt{2},\ m = 2\sqrt{2}$$

$$\Rightarrow \ell^2 + m^2 - \ell m = 2 + 8 - 4 = 6$$

7. Ans (8)

Near $x = 2$, $\tan^{-1}(\tan x) = -\pi + x$

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = \pi - x$

$$\Rightarrow \tan^{-1}(\tan x) + \operatorname{cosec}^{-1}(\operatorname{cosec} x) = 0$$

$(y + x^3)^3 = x^6$

Differentiating both sides w.r.t x

$3(y+x^3)^2(y' + 3x^2) = 6x^5$

Put $x = 2$ & $y = -4$

$3(4)^2(y' + 12) = 6(2)^5$

$y' + 12 = 4 \Rightarrow y' = -8$

$m_1 = -8,\ m_2 = \frac{1}{8}$

$$[m_2 - m_1] = \left[\frac{1}{8} + 8\right] = 8$$

8. Ans (7)

Equation of plane :

$$\vec{r} = \hat{i} + 2\hat{j} + \lambda(\hat{j} - \hat{k}) + \mu(2\hat{i} - \hat{j})$$

$$\Rightarrow [\ \vec{r} - \hat{i} - 2\hat{j} \quad \hat{j} - \hat{k} \quad 2\hat{i} - \hat{j}\ ] = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z \\ 0 & 1 & -1 \\ 2 & -1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow x + 2y + 2z = 5$$

$$\frac{a-3}{1} = \frac{b-2}{2} = \frac{c-9}{2} = \frac{-2(3+2(2)+2(9)-5)}{1^2+2^2+2^2}$$

$$\Rightarrow b = \frac{-62}{9},\ c = \frac{1}{9}$$

$c - b = 7$