Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Main)
PART TEST
15-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY — PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	C	C	D	B	A	C	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	C	D	B	A	B	A	A	D
SECTION-II	Q.	1	2	3	4	5					
	A.	68	7	2	25	3					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	B	D	B	A	C	B	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	D	B	B	A	C	B	A	D	C
SECTION-II	Q.	1	2	3	4	5					
	A.	03	06	3	4	21					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C	B	A	B	C	B	C	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	B	A	C	C	C	C	B	C
SECTION-II	Q.	1	2	3	4	5					
	A.	576	55	110	53	1680					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (C)**

The force on the conductor is

$F = IL\times B = ILa_z \times B_0a_r = B_0ILa_\phi$

so that the applied force is

$F_a = B_0IL(-a_\phi)$

The conductor is to be turned in the a_ϕ direction. Therefore, the work required for one full revolution is

$$W = \int_0^{2\pi} B_0IL\left(-a_\phi\right).rd\phi = -2\pi rB_0IL$$

Since N revolutions per minute is N/60 per second, the power is

$$P = -\frac{2\pi rB_0ILN}{60}$$

2. **Ans (A)**

For long coils of small cross section, H may be assumed constant inside the coil and zero for points just outside the coil. With the first coil carrying a current I_1,

$$H = \left(\frac{1000}{0.50}\right) I_1 \text{ (A/m) (in the axial direction)}$$

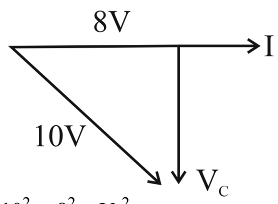
$B = \mu_0\, 2000\, I_1$ (Wb/m^2)

$\Phi = BA = (\mu_0\, 2000\, I_1)(\pi \times 10^{-4})$ (Wb)

Since H and B are zero outside the coils, this is the only flux linking the second coil.

$$M_{12} = N_2\left(\frac{\Phi}{I_1}\right) = (2000)\ 4(\pi \times 10^{-7})\ (2000)\ (\pi \times 10^{-4}) = 1.58 \text{ mH}$$

3. Ans (C)



$10^2 = 8^2 + V_C^2$

$V_C^2 = 100 - 64$

$V_C = \sqrt{36} = 6V$

4. Ans (C)

The quality factor Q measure sharpness and

$Q = \frac{\omega_0}{\omega_2 - \omega_1}$

Sharpness $= Q = \frac{\omega_0}{\omega_2 - \omega_1}$

Hence statement (2) is wrong

5. Ans (D)

The amplitude of forced oscillation is

$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - p^2) + 4\gamma^2\omega_0^2}}$,

At $p = \omega_0$, we get $A = \frac{\frac{F_0}{m}}{2\gamma\omega_0}$. To evaluate A, we need to know F_0, the amplitude of the driving force and γ which measures the damping of system.

F_0 = spring constant × displacement amplitude

$= (800\ N/m)\ (2\times10^{-3}\ m) = 1.6\ N$

where

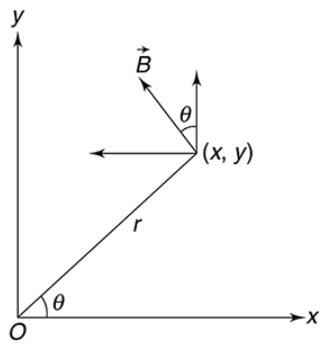
$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{800\ N/m}{2}} \Rightarrow \omega_0 = = 20$ rad/sec

$A = \frac{\frac{1.6}{2}}{2 \times 0.5 \times 20} = \frac{0.8}{20} = 0.04$ m $\quad A = 4$ cm

6. Ans (B)

With increased damping the resonant frequency shifts lower than the natural frequency.

7. Ans (A)



Field at any point (x,y) can be written as

$B = -B\sin\theta\hat{i} + B\cos\theta\hat{j}$

$= -\frac{k}{\sqrt{x^2+y^2}}\cdot\frac{y}{\sqrt{x^2+y^2}}\hat{i} + \frac{k}{\sqrt{x^2+y^2}}\cdot\frac{x}{\sqrt{x^2+y^2}}\hat{j}$

$= -\frac{ky}{(x^2+y^2)}\hat{i} + \frac{kx}{(x^2+y^2)}$

A small segment of the wire at point (x,y) can be taken as

$\vec{dl} = dx\hat{i} + dy\hat{j}$

Force on segment is

$\vec{dF} = I\vec{dl} \times \vec{B}$

$= I\frac{kxdx}{x^2+y^2}\hat{k} + I\frac{kydy}{x^2+y^2}\hat{k}$

∴ Force on wire AB is

$\vec{F} = \int \vec{dF} = KI\int_{A(x_1,y_1)}^{B(x_2,y_2)} \frac{xdx + ydy}{x^2+y^2}\hat{k}$

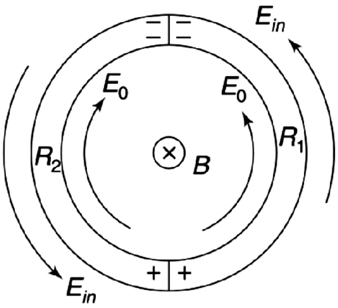
If $x^2 + y^2 = t$, $t_A = 2$, $t_B = 10$

then $2xdx + 2ydy = dt$;

$xdx + ydy = 1/2\ dt$

$\vec{F} = \frac{KI}{2}\int_A^B \frac{dt}{t}.\hat{k} = \frac{KI}{2}[\ell nt]_2^{10} = \left[\frac{KI}{2}\ell n5\right]$

8. Ans (C)



Current (and hence current density) in the entire loop must be same.

From microscopic form of Ohm's law we can write

$\sigma_1 E_1 = \sigma_2 E_2$

$\Rightarrow R_2E_1 = R_1E_2 \qquad \Rightarrow \dfrac{E_1}{E_2} = \dfrac{R_1}{R_2}$

The induced electric field must be uniform ever)'where in the circular conductor. It is given by

$2\pi a E_{in} = \pi a^2 \qquad \Rightarrow E_{in} = E_{in} = \dfrac{a}{2}\left(\dfrac{dB}{dt}\right)$

There is accumulation of charge at the junctions which produce additional electric field in the conductor. If $R_1 > R_2$ then $E_1 > E_2$. In this case the charge at the upper junction is negative and charge at the lower junction will be positive. In figure E_n is electric field due to charge.

$E_S + E_{in} = E_1$

$E_{in} - E_0 = E_2$

$E_{in} = \dfrac{E_1 + E_2}{2}$, $E_0 = \left(\dfrac{E_1 - E_2}{2}\right)$

9. Ans (B)

$V_2^2 = V_R^2 + V_C^2$, $V_3 = (V_L - V_C)$

$V_1^2 = V_R^2 + (V_L - V_C)^2 \Rightarrow 260^2 = V_R^2 + 240^2$

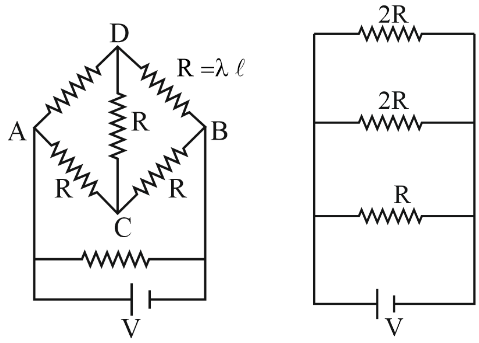
$V_R = 100$ V, $V_C^2 = 125^2 - 100^2 \Rightarrow V_C = 75V$

$V_L = 315$ V

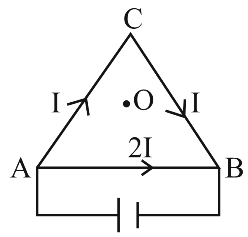
$\dfrac{V_L}{V_C} = \dfrac{I.X_L}{I.X_C} \Rightarrow \dfrac{X_L}{X_C} = \dfrac{315}{75} = 4.2$

10. Ans (A)

Drawing the equivalent circle

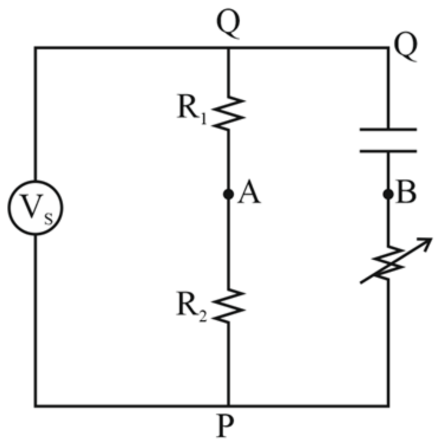


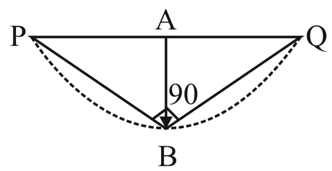
(This is balanced wheatstone Bridge)



At the centroid 0. The field produced will be zero.

11. Ans (A)





PQ = Phasor of source

PB = Phasor of voltage across R

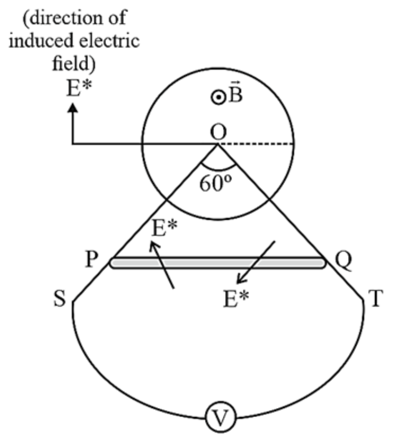
BQ = Phasor of voltage across C

AB = Phasor voltage between A & B

here pointer B moves on a circle of radius $\dfrac{V_S}{2}$ when R is change

$\dfrac{V_S}{2}$ = maximum inst. voltage between A & B which is constant.

12. Ans (D)



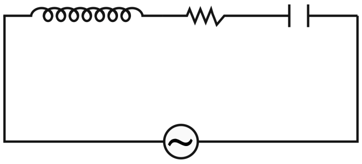
$\phi_{loop} OPQO = \frac{\pi R^2}{2 \times 3} ct; \ \frac{d\phi}{dt} = \frac{C\pi R^2}{6}$

$\therefore$ induced emf across two ends of rod $= \frac{C\pi R^2}{6}$

Here in loop PQTSP has zero current

$\therefore$ deflection in voltmeter must be zero.

13. Ans (C)



$\frac{i_{max}}{\sqrt{2}} = \frac{V_0}{z}; \ \frac{V_0}{R\sqrt{2}} = \frac{V_0}{\sqrt{\left(W_L - \frac{1}{W_C}\right) + R^2}}$

$\frac{V_0^2}{R^2 \times 2} = \frac{V_0^2}{\left(W_L - \frac{1}{W_C}\right) + R^2}$

$\left(W_L - \frac{1}{W_C}\right) = \pm R \ ....(1)$

$\tan\phi = \frac{\left(W_L - \frac{1}{W_C}\right)}{R_1} = \pm 1$

$\therefore$ At $w = w_1$ or $w = w_2 \qquad \phi = \pm\frac{\pi}{4}$

On solving (1)

$W_2 = \frac{RC + \sqrt{C^2R^2 + 4LC}}{2LC} \ .....(2)$

$W_1 = \frac{-RC + \sqrt{C^2R^2 + 4LC}}{2LC} \ .... (3)$

$\therefore$ At $w = w_1 \ \ \phi = \pm\frac{\pi}{4}$ i.e. circuit in capacitive

At $w = w_2 \ \ \phi = \pm\frac{\pi}{4}$ i.e. circuit is inductive power factor

$\therefore \cos\phi = \frac{1}{\sqrt{2}}$ at w_1 & w_2

(2) & (3)

$w_1 w_2 = \frac{4LC}{4L^2C^2}; \ w_1 w_2 = \frac{1}{LC} = w_r^2$

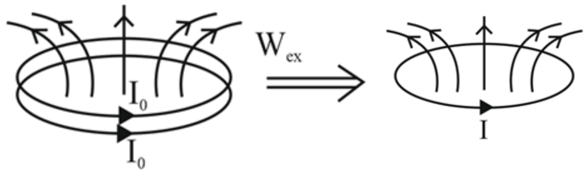
$w_r = \sqrt{w_1 w_2}$

At $w_r \ \phi = 0^\circ \ \cos\phi = 1$

14. Ans (D)

$\phi_{\text{each loop}} = q_{\text{self}} + f_{\text{mutual}}$

$= LI_0 + MI_0 = 2LI_0$



final

$(\phi_{\text{each}})_{\text{initial}} = \phi_{\text{each final}}$

$2LI_0 = LI$

$I = 2I_0$

$U_i = \frac{1}{2}LI_0^2 + \frac{1}{2}LI_0^2 + MI_1I_2$

$= LI_0^2 + LI_0^2 = 2LI_0^2$

or $U_i = \frac{1}{2}\text{Leff}^{LI_0^2}$

$= \frac{1}{2}[4LI_0^2) = 2LI_0^2$

$U_f = \frac{1}{2}LI^2 \times L$

$= L \times (2I_0)^2 = 4LI_0^2$

$W_{ex} = U_f - U_i$

$\boxed{W_{ex} = 2LI_0^2}$

15. Ans (B)

Both statement are true but statement-II is not correct explanation of statement-I

there work done by induced field is path dependent.

16. Ans (A)

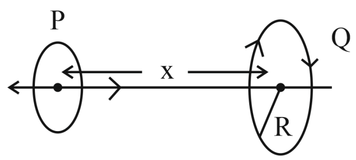
$$\int EdqR = \int \mu dmgR$$

$$\frac{R}{2}\frac{dB}{dt}\lambda R^2 d\theta = \mu \text{ mg } R$$

$$\frac{3t^2}{3}\frac{\lambda R^2 2\pi}{2} = \mu mg$$

$$t = \sqrt{\frac{\mu mg}{\lambda R^2 \pi}}$$

17. Ans (B)



Magnetic field due to Q and x is given by

$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$$

$$|E| = \frac{d}{dt}(B.\pi r2)$$

$$|E| = \frac{\mu_0 I R^2}{2}\frac{(2xy)}{(R^2+x^2)^{5/2}}$$

to Maximize |E|

$$\frac{d|E|}{dx} = 0 \quad \Rightarrow x = R/2$$

$$\therefore x = \frac{4}{2} = 2 \text{ m (from coil B)}$$

18. Ans (A)

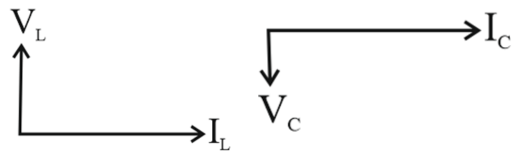
$$\frac{100\sqrt{2}}{2} = 100\sqrt{2}\sin\omega t$$

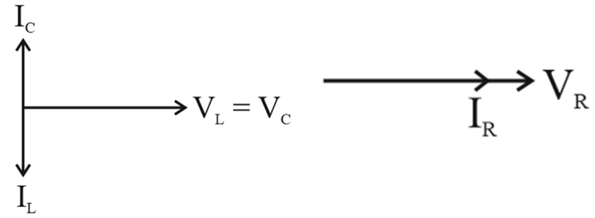
$$t = \frac{T}{12}$$

$$V_L = I_0 X_L \sin(\omega t + \pi/4)$$

VL = 96.5 V.

19. Ans (A)





20. Ans (D)

$$I_0 = \sqrt{I_1^2 + I_2^2}$$

$$I_0 = V_0\sqrt{\frac{1}{25}+\frac{1}{25}}$$

$$I_0 = \sqrt{2}$$

$$I_{rms} = 1A$$

PART-1 : PHYSICS

SECTION-II

1. Ans (68)

$\varepsilon = B\ell\upsilon = 4V$ = constant.

charge on the capacitor, $Q = C\varepsilon = 2C$

Work required to charge the capacitor,

$W = Q.\ \varepsilon = 8$ J

$$i_R = \frac{\varepsilon}{R} = 2A,\ L\frac{di}{dt} = 4 \Rightarrow 2i_L = 4t$$

$$i_L = 2t,\ F_{ext} = (2+2t)\times 2\times 2 = 4(2+2t)$$

$$w_{ext} = 8 + \int_0^3 4(2+2t).1.dt$$

$$= 8 + 4[2t+t^2]_0^3 = 68 \text{ J}$$

2. Ans (7)

Moment of inertia of combined body,

$$I = \frac{1}{2}\times mR^2 + \frac{2}{3}mR^2 = \frac{7}{6}mR^2$$

$L = I.\omega$ = angular momentum.

$$\frac{M}{L} = \frac{q}{2\times 2m} \Rightarrow M = \frac{7}{6}mR^2\omega \times \frac{q}{4m}$$

$$M = \frac{7}{24}\times 3\times 200\times 0.04 = 7 \text{ A.m}^2$$

3. Ans (2)

$v_R + v_L + v_C = v$

$v_R = i_R\ ,\ v_L = L\frac{di}{dt},\ v_C = \frac{q}{c} \Rightarrow \frac{dv_c}{dt} = \frac{i}{c}$

$v_R = R_C\frac{dv_c}{dt},\ v_L = L_C\frac{d^2v_c}{dt^2}$

$R_C\frac{dv_c}{dt} + L_C\frac{d^2v_c}{dt^2} + v_C = v$

$\frac{d^2v_c}{dt^2} + \frac{R}{L}.\frac{dv_c}{dt} + \frac{1}{LC}.v_c = \frac{200}{LC}.\sin 60t$

We can compare this equation with equation of forced damped oscillation. For very low damping and in steady state, amplitude of v_c would be maximum when $\omega = \sqrt{\omega_0^2 - 2\gamma^2}$

where, $\omega = 60$ rad/sec, $\omega_0^2 = \frac{1}{LC} = 10^4$

and $\gamma = \frac{R}{2L}$

$3600 = 10000 - \frac{R^2}{2L^2}$

$\Rightarrow \frac{R^2}{2L^2} = 6400 \Rightarrow R = 40\sqrt{2}\ \Omega$

4. Ans (25)

If the imaginary part of impedance a admitance is zero, then the circuit will behave purely resistive

$\frac{1}{z} = \frac{1}{\frac{1}{j\omega C}} + \frac{1}{R + j\omega L}$

$\frac{1}{z} = j\omega C + \frac{(R - j\omega L)}{R^2 + \omega^2L^2}$

$\ell n\left(\frac{1}{z}\right) = \left[\omega C - \frac{\omega L}{R^2 + \omega^2L^2}\right] = 0$

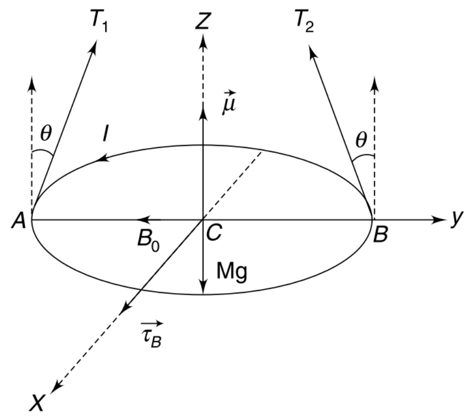
$\Rightarrow \omega C = \frac{\omega L}{R^2 + \omega^2L^2}$

$C = \frac{L}{R^2 + \omega^2L^2}$

$C = \frac{5 \times 10^{-3}}{(10)^2 + \left(2000 \times 5 \times 10^{-3}\right)^2}$

$= \frac{5 \times 10^{-3}}{2 \times 10^2} = 25\ \mu F$

5. Ans (3)



Magnetic dipole moment $\vec{\mu} = I.\pi R^2\hat{k}$

Magnetic torque $\vec{\tau}_B = \vec{\mu} \times \vec{B}$

$= I\pi R^2 B_0 (\hat{k} \times (-\hat{j})) = I\pi R^2 B_0 (\hat{i})$

To counterbalance this torque we must have $T_1 > T_2$

Torque about centre C

$-(T_1\cos\theta - R - T_2\cos\theta R)\hat{i} + I\pi R^2 B_0\hat{i} = 0$

$T_1 - T_2 = \frac{I\pi RB_0}{\cos\theta}$

And $(T_1 + T_2)\cos\theta = Mg$

$\Rightarrow (T_1 + T_2) = \frac{Mg}{\cos\theta}$

Solving (i) and (ii)

$T_1 = \frac{1}{2\cos\theta}(Mg + \pi IRB_0) = Mg + \pi IRB_0$ and T_2

$= Mg - \pi IRB_0$

$\frac{T_{AO}}{T_{BO}} = \frac{Mg + \pi IRB_0}{Mg - \pi IRB_0};\ \frac{10 + 5}{10 - 5} = 3$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (C)

For He^+ ion $E_n = -13.6 \times \frac{2^2}{n^2} = -KE_n = \frac{PE_n}{2}$

For $h = 2,\ E_2 = -13.6$ eV

P.E. = – 27.6 eV

K.E = 13.6 eV

2. **Ans (A)**

$L_{n,z} = \frac{nh}{2\pi}$

4. **Ans (D)**

Number of Nodal surfaces = n - 1

5. **Ans (B)**

$K_{eq} = \frac{K_1}{K_{-1}} = \frac{[O_2][O]}{[O_3]}$

$r = K_2 [O_3][O] = K_2 . [O_3] . \frac{K_1}{K_{-1}} . \frac{[O_3]}{[O_2]}$

$= K . [O_3]^2 [O_2]^{-1}$

6. **Ans (A)**

no. of nodes = n -1

7. **Ans (C)**

For zero order: $t_{3/4} = 1.5\ t_{1/2}$

8. **Ans (B)**

$-\frac{1}{x}\frac{d[A]}{dt} = +\frac{1}{y}\frac{d[B]}{dt}$

9. **Ans (C)**

Total number of lector in a shell = $2n^2$

Number of electron in a shell having

$(m_s = + 1/2) = n^2$

10. **Ans (A)**

theory based

11. **Ans (D)**

(P) = CH_3 - COOEt

(Q) $CH_3CH_2OH + CH_3CHO$

12. **Ans (D)**

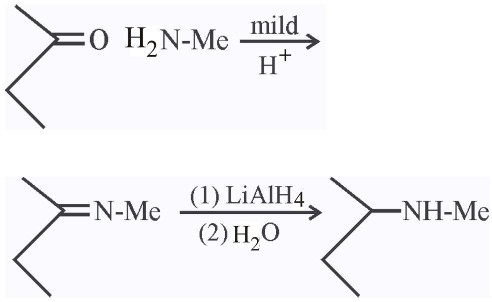
$CH_3-\overset{O}{\overset{||}{C}}-NH-\overset{O}{\overset{||}{C}}-CH_3 \xrightarrow{NaOH+Br_2} CH_3COO^- + CH_3-NH_2$

$CH_3-\overset{O}{\overset{||}{C}}-NH_2 \xrightarrow{NaOH+Br_2} CH_3-NH_2$

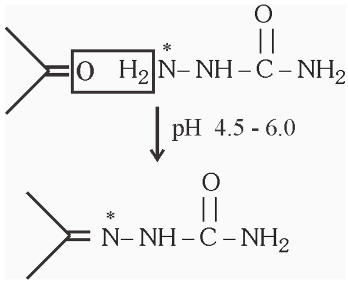
13. **Ans (B)**

Acetylation of salicylic acid gives aspirine

14. **Ans (B)**



15. **Ans (A)**

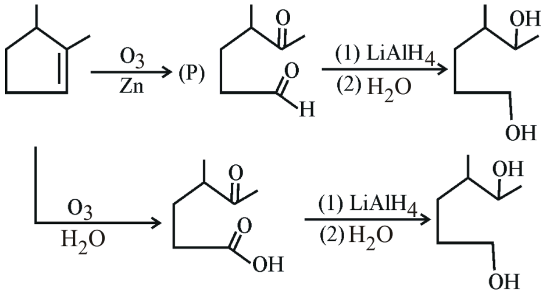


Both Statements are correct.

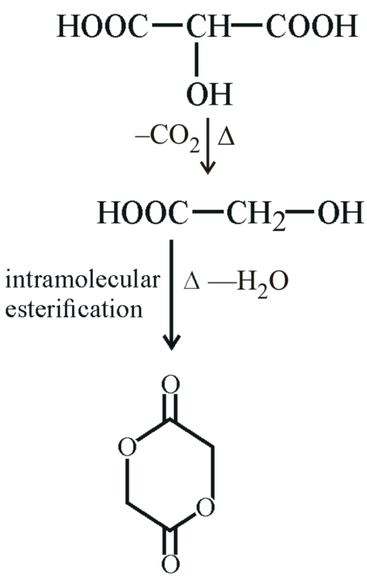
16. **Ans (C)**

$NaBH_4$ does not reduce - COOH group

17. **Ans (B)**



18. **Ans (A)**



19. Ans (D)

$$CH_3CONH_2 \xrightarrow[(2)H_2O]{(1)LiAlH_4} \underset{(P)}{CH_3CH_2-NH_2}$$

$$CH_3CONH-CH_3 \xrightarrow[(2)H_2O]{(1)LiAlH_4} \underset{(Q)}{CH_3CH_2NH-CH_3}$$

$$CH_3CH_2NH_2 \xrightarrow{(1)CHCl_3,KOH}$$

$$CH_3CH_2NC \xrightarrow[H_2O]{LiAlH_4} \underset{(Q)}{CH_3CH_2NHCH_3}$$

20. Ans (C)

$$PhCOOCH_3 \xrightarrow{H_3O^+} PhCOOH + CH_3OH$$

$$PhCOOCH \xrightarrow{Ca(OH)_2} (PhCOO)_2Ca \xrightarrow[(CH_3COO)_2Ca]{\text{Dry distillation}}$$

$$PhCOCH_3$$

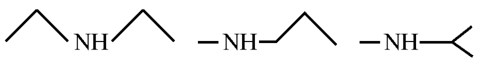
$$PhCOCH_3 \xrightarrow{m-CPBA} CH_3\text{-}COOPh\ (T)$$

PART-2 : CHEMISTRY

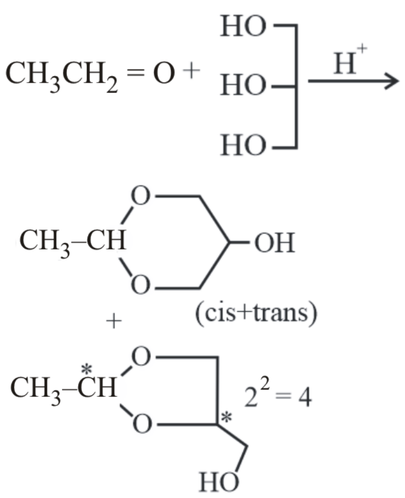
SECTION-II

1. Ans (03)

2° - amines form white ppt that is insoluble in alkali



2. Ans (06)



4. Ans (4)

$$\frac{t_{99.99\%}}{t_{90\%}} = \frac{\frac{2.303}{k}\log\left(\frac{100}{0.01}\right)}{\frac{2.303}{k}\log\left(\frac{100}{10}\right)} = 4$$

5. Ans (21)

Correct order is : 8s 5g 7f 7d 8p

PART-3 : MATHEMATICS

SECTION-I

1. Ans (A)

$$^{12}C_7 \times \frac{7!}{2!2!} \times 1 = (198)7!$$

2. Ans (C)

Sample space = S = {(1, 1), (1, 2).........(6, 6)?

n(s) = 36

For distinct and real roots D > 0

$4b^2 - 28a > 0 \Rightarrow 7a < b^2$

Favourable cases E = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 5), (3, 6), (4, 6), (5, 6)}

n(E) = 11

Required probability $= \frac{n(E)}{n(S)} = \frac{11}{36}$

3. Ans (B)

$p_n = p_{n-1}\,p(T) + p_{n-2}p(T)p(H)$; $p_n = \frac{p_{n-1}}{2} + \frac{p_{n-2}}{4}$

; $p_1 = 1, p_2 = \frac{3}{4}$; $p_3 = \frac{3}{8} + \frac{1}{4}$

$p_3 = \frac{5}{8}$; $p_4 = \frac{p_3}{2} + \frac{p_2}{4} = \frac{8}{16}$

Alternatively : Clearly $p_1 = 1$ and $p_2 = 1 - p(H\,H)$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Now for $n \geq 3$,

Compute $p_3 = \frac{5}{8}$; $p_4 = \frac{1}{2}$

Hence $p_2 = \frac{12}{16}$; $p_3 = \frac{10}{16}$; $p_4 = \frac{8}{16} \Rightarrow p_2, p_3, p_4$ are in A.P.

$$p_n = \underbrace{(1-p)}_{T}\,p_{n-1} + \underbrace{p}_{H}\,\underbrace{(1-p)}_{T}\,p_{n-2}$$

4. **Ans (A)**

$\log_x^3 . \log_{3x}^3 . \log_3^{9x} > 1$

$$\frac{2+\log_3^x}{\log_3^x(1+\log_3^x)} > 1 \text{ put } \log_3^x = t$$

$$\frac{t+2}{t^2+t} > 1 \Rightarrow \frac{t^2+t-t-2}{t^2+t} < 0$$

$$\Rightarrow \frac{\left(t-\sqrt{2}\right)\left(t+\sqrt{2}\right)}{t(t+1)} < 0$$

$$\Rightarrow -\sqrt{2} < t < -1 \text{ or } 0 < t < \sqrt{2}$$

$$-\sqrt{2} < \log_3^x < -1 \text{ or } 0 < \log_3^x < \sqrt{2}$$

$$3^{-\sqrt{2}} < x < \frac{1}{3} \text{ or } 1 < x < 3^{\sqrt{2}}$$

$\Rightarrow$ smallest natural value of x = 2

5. **Ans (B)**

Maximum value at $x = \frac{\pi}{4}$ only

$\left(\sqrt{2}\right)^2 = 2$

6. **Ans (C)**

Product of the three numbers appearing is even

$\Rightarrow$ atleast one of the throw is even.

Possible cases: $E_1 \equiv$ Exactly one even and 2 odd.

$E_2 \equiv$ Exactly two even and 1 odd.

$E_3 \equiv$ All three even.

Now $P(E_1 \cup E_2 \cup E_3) = 1 - P$(all three throws are odd)

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{7}{8}$$

Hence $P(E_1 \text{ or } E_3 / E_1 \cup E_2 \cup E_3)$

$$= \frac{P(E_1 \cup E_3)}{P(E_1 \cup E_2 \cup E_3)} = \frac{P(E_1) + P(E_3) - P(E_1 \cap E_3)}{P(E_1 \cup E_2 \cup E_3)}$$

$$= \frac{3\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) - 0}{\frac{7}{8}} = \frac{4}{8}\left(\frac{8}{7}\right) = \frac{4}{7}$$

7. **Ans (B)**

Total selections of four consecutive natural number from first 50 natural numbers

n(s) = 47

$\because$ S={(1, 2, 3, 6), (2, 3, 4, 5)..........(47, 48, 49, 50)}

Selection of four consecutive natural numbers, if their sum is divisible by 3

n(E) = 15

$\because E=\{(3n, 3n+1, 3n+2, 3n+3) : n \in N\}$

E = {(3, 4, 5, 6), (6, 7, 8, 9), (9, 10, 11, 12), ... (45, 46, 47, 48)}

Required probability $= \frac{15}{47}$

8. **Ans (C)**

$$\sum_{r=0}^{n} a_r \frac{x^r}{(1+x)^{2r}} = \frac{\left(1+x^3\right)^n}{(1+x)^{3n}}$$

$$= \left\{\frac{1+x^3}{(1+x)^3}\right\}^n = \left\{\frac{1-x+x^2}{(1+x)^2}\right\}^n$$

$$\sum_{r=0}^{n} a_r \left\{\frac{x}{(1+x)^2}\right\}^r = \left\{1 - \frac{3x}{(1+x)^2}\right\}^n$$

Put $\frac{x}{(1+x)^2} = t$

$$\sum_{r=0}^{n} a_r t^r = \{1-3t\}^n$$

$$\sum_{r=0}^{n} {}^nC_r(-3)^r t^r$$

$a_r = {}^nC_r(-3)^r$

$a_4 = {}^nC_4(-3)^4$

$a_5 = {}^nC_5(-3)^5$

$$\Rightarrow \frac{a_5}{a_4} = \frac{{}^nC_5(-3)^5}{{}^nC_4(-3)^4} = \frac{-3(n-4)}{5}$$

$$= \frac{12-3n}{5}$$

9. **Ans (D)**

Obs. x_i : 5, 5, 7, 10, 12, 12, 14, 15

$$\bar{x} = \frac{\sum x_i}{8} = \frac{80}{8} = 10$$

$$\sum x_i^2 = 908$$

$$\text{Variance} = \sigma^2 = \frac{\sum x_i^2}{x} - \left(\frac{\sum x_i}{x}\right)^2$$

$$= \frac{908}{8} - \left(\frac{80}{8}\right)^2 = \frac{908}{8} - 100 = \frac{108}{8} = \frac{27}{2}$$

$\sigma = \sqrt{13.5}$, $[\sigma] = 3$, $[\sigma^2] = 13$

$$\text{Median M} = \frac{10+12}{2} = 11$$

x_i	$\lvert x_i - \bar{x}\rvert$	$\lvert x_i - M\rvert$
5	5	6
5	5	6
7	3	4
10	0	1
12	2	1
12	2	1
14	4	3
15	5	4

$$\sum |4 - \bar{x}| = 26$$

$$\sum |x_i - M| = 26$$

10. **Ans (D)**

$$(1+x+x^2)(1+x^2)^{50} = (1+x+x^2)\sum_{r=0}^{50} {}^{50}C_r x^{2r}$$

Coefficient of $x^{51} = {}^{50}C_{25}$

Coefficient of $x^{50} = {}^{50}C_{25} + {}^{50}C_{24}$

$$\text{Required ratio} = \frac{{}^{50}C_{25}}{{}^{50}C_{25} + {}^{50}C_{24}} = \frac{26}{51}$$

11. **Ans (C)**

$$r^2\left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)^2 = r^2\left\{\frac{n-r+1}{r}\right\}^2 = (n-r+1)^2$$

$$\therefore \sum_{r=1}^{n} r^2\left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)^2 = \sum_{r=1}^{n} .(n-r+1)^2$$

$$= n^2 = (n-1)^2 + ...+ 1^2$$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{50 \times 51 \times 101}{6}$$

$= 42925$

12. **Ans (D)**

$${}^5C_0\ {}^{95}C_{90} + {}^5C_1\ {}^{95}C_{91} + {}^5C_2\ {}^{95}C_{92} + {}^5C_3\ {}^{95}C_{93} + {}^5C_4\ {}^{95}C_{94} + {}^{95}C_{95}$$

$$= {}^5C_5\ {}^{95}C_{90} + {}^5C_4\ {}^{95}C_{91} + {}^5C_2\ {}^{95}C_{92} + {}^5C_3\ {}^{95}C_{92} + {}^5C_1\ {}^{95}C_{94} + {}^5C_0\ {}^{95}C_{95}$$

$$= {}^{5+95}C_{95} = {}^{150}C_{95}$$

$$= {}^{100}C_5 = \frac{100}{5} \times \frac{99}{4} \times {}^{98}C_3 = 495 \times {}^{98}C_3$$

13. **Ans (B)**

Take $\log x_{10} = t$, then equation will be :

$(2+t)^3 + (t-1)^3 = (1+2t)^3$

Here $(2+t) + (t-1) = 2t+1$

$\therefore a^3 + b^3 = (a+b)^3$ can be interpreted

$\therefore 3a^2b + 3ab^2 = 0$

$\therefore a = 0$ or $b = 0$ or $a + b = 0$

$\therefore t = -2, 1, -\frac{1}{2}$

$\Rightarrow \log_{10} x = -2, 1, -\frac{1}{2}$

$x = \frac{1}{100}, 10. \frac{1}{\sqrt{10}}$

14. **Ans (A)**

Prime as sum on throwing a die 2 times : 2, 3, 5, 7, 11

Such no. of cases = 15 lake (1, 1), (1, 2), (2, 1) and so on.

Prime as sum on drawing tokens 2 times : 2, 3, 5, 7, 11, 13, 17, 19

Such no. of cases = 37 lake (1, 1), (1, 2), (2, 1), (1, 4), (4, 0), (2, 3), (3, 2) etc.

$$\therefore \text{Req. probability} = \frac{\frac{1}{2} \times \frac{15}{36}}{\frac{1}{2} \times \frac{15}{36} + \frac{1}{2} \times \frac{37}{100}} = \frac{125}{236}$$

15. **Ans (C)**

Statement I :

$N = 500 = 2^2 \times 5^3$

$500 = 2d_1 \times 2d_2 \Rightarrow d_1d_2 = 125 = 5^3$

$\therefore$ 500 can be expressed as product

two even natural no. in $= \frac{3+1}{2} = 2$ ways

Statement II :

$x_1 + x_2 + x_3 + x_4 + x_5 = 250$

Number of non negative integer solutions

$= {}^{250+5-1}C_{5-1}$

$= {}^{254}C_4$

16. Ans (C)

AP : a, a + d, a + 2d, ..., a + 10d, d = -5

$$\text{Variance} = \frac{\sum (x_i^2)}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{\sum_{r=0}^{10} (a+rd)^2}{11} - \left(\frac{11a + d \times \sum_{r=0}^{10}(a+rd)}{11}\right)^2$$

$$= \frac{\left(11a^2 + 2ad\sum_{r=0}^{10} r + d^2 \sum_{r=0}^{10} r^2\right)}{11} - \left(\frac{11a + d \times \sum_{r=0}^{10} r11}{11}\right)^2$$

$$= \frac{\left(11a^2 + 2ad \times \frac{10\times11}{2} + d^2 \times \frac{10\times11\times21}{6}\right)}{11} - \left(\frac{11a + d \times \frac{10\times11}{2}}{11}\right)^2$$

$$= \left(a^2 + 10ad + 35d^2\right) - (a+5d)^2$$

$$= 10d^2 = 250 \text{ (as d = -5)}$$

17. Ans (C)

Variance of x_i = Variance of x_i - λ

$$= \frac{\sum (x_i - \lambda)^2}{10} - \left(\frac{\sum (x_i - \lambda)}{10}\right)^2$$

$$= \frac{3}{10} - \frac{9}{100} = \frac{121}{100} = 1.21$$

18. Ans (C)

No. of ways of selecting 3 squares of dim 1 × 1 both diagonals, where exactly & are connected

$= 2[2 \times 5 + 4 \times 5] = 60$

$$P = \frac{60}{^{64}C_3} = \frac{5}{3472} \quad \therefore\ a + b = 5 + 3472 = 3477$$

19. Ans (B)

Words with phrase "YUVZ" : $\boxed{\text{Y U V I}}$ R. A, J, S, N, G, H

Such no. of words = 8!

So no. of words with phrase YUVI but not IRA

n = 8! - 6! = 6! (8 × 7 - 1) = 95 × 6!

$$\frac{4}{5!} = \frac{55 \times 5! \times 6}{5!} = 55 \times 6 = 330$$

20. Ans (C)

$x^{100} + 2x^{99}(1+x) + 3x^{98}(1+x)^2 + \ldots\ldots\ldots + 101(1+x)^{100}$

$$= x^{100}\left[1 + 2\left(\frac{1+x}{x}\right) + 3\left(\frac{1+x}{x}\right)^2 + \ldots\ldots + 101\left(\frac{1+x}{x}\right)^{100}\right] \ldots.(1)$$

$$\underbrace{1 + 2\left(\frac{1+x}{x}\right) + 3\left(\frac{1+x}{x}\right)^2 + \ldots\ldots + 101\left(\frac{1+x}{x}\right)^{100}}_{\text{A.G.P.}}$$

$$x^{100} + 2x^{99}(1+x) + 3x^{98}(1+x)^2 + \ldots\ldots\ldots + 101(1+x)^{100}$$

$$= x^{102} - x(1+x)^{101} + 101(1+x)^{101}$$

$\therefore$ Coefficient of 99 $= -^{101}C_{98} + 101 \times {}^{101}C_{99}$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (576)

Numbers which have no 2 consecutive digits are

Same : $\bar{4} \times \bar{3} \times \bar{3} \times \bar{3} \times \bar{3} \times \bar{3} = 972$

But these numbers are also consist numbers in which all digit are not used.

Numbers formed with only 2 digit = $6 \times {}^4C_2 = 36$

Number formed with only 3 digit = ${}^4C_3 \times (3 \times 2^5 - 6)$ = 360

Required numbers = 972 - 396 = 576

2. Ans (55)

3 digit $\Rightarrow \frac{7/8}{2} \times \bar{4} \times \bar{4} = 32$

4 digit $\Rightarrow \frac{2}{1} \times \frac{2}{1} \times \bar{4} \times \bar{4} = 16$

$\frac{2}{1} \times \frac{6}{1} \times \frac{6}{1} \times \bar{4} = 4$

$\frac{2}{1} \times \frac{6}{1} \times \frac{7}{1} \times \bar{3} = 3$

$= 55$

3. Ans (110)

$k_1 = 25 + 3 \times 4 = 37$

$k_2 = 2^6 + 9 = 73$

$k_1 + k_2 = 110$

4. Ans (53)

$$\frac{p}{q} = \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{7}} = \frac{14}{39}$$

5. Ans (1680)

$$\underbrace{\_\ \_\ \_}_{^9C_3} \times \underbrace{\_\ \_\ \_}_{^6C_3} = 1680$$