FIITJEE

ALL INDIA TEST SERIES

CONCEPT RECAPITULATION TEST – I

JEE (Main)-2025

TEST DATE: 20-01-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. C

Sol. $10^{-3}(1 + R) = 10$

$\therefore R = 9999\Omega$

2. B

Sol. $\Delta V = \frac{B\omega^2(\ell)}{2} = \frac{B\omega^2(\sqrt{3}R)}{2} = V_B - V_A$.

3. A

Sol. $W = \int Fvdt$

4. C

Sol. Since, D_1 will be acting as short circuit, D_2 will act as open circuit so,

Current through 5Ω resistance and diode D_1 is $\frac{20}{5} = 4$ A

5. B

Sol. At steady state.

Rate of generation of A = Rate of decay of A.

$q = \lambda N_A$

$\Rightarrow N_A = \frac{q}{\lambda}$

6. C

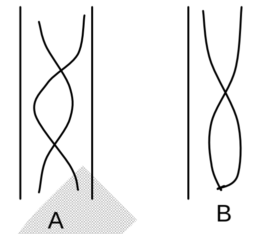
Sol. (a) $2\dfrac{V_A}{2L_A} = 3\dfrac{V_B}{4L_B}$

$\Rightarrow \dfrac{V_A}{V_B} = \dfrac{3}{4}$ $[\because L_A = L_B]$

$V_A = \sqrt{\dfrac{\gamma_A RT}{M_A}}, \quad V_B = \sqrt{\dfrac{\gamma_B RT}{M_B}}$

$\Rightarrow \sqrt{\dfrac{\gamma_A}{\gamma_B}\dfrac{M_B}{M_A}} = \dfrac{3}{4}, \qquad \gamma_A = \dfrac{5}{3}, \qquad \gamma_B = \dfrac{7}{5}$

$\Rightarrow \dfrac{M_A}{M_B} = \dfrac{400}{189}$



7. A

Sol. $\omega = \dfrac{v}{\ell \sin\theta}$

$V_c = \dfrac{v}{2\sin\theta}$

8. B

Sol. $\tan\theta = \dfrac{X_c - X_L}{X_R}$

and $\cos\theta$ = power factor.

9. C

Sol. Potential energy of particle at the centre of earth is $U = -\dfrac{3}{2}\dfrac{GMm}{R_e}$

So, $V_e = \sqrt{\dfrac{3GM}{R_e}} = \sqrt{3gR_e}$

10. D

Sol. $V = 2(x^2 - y^2)$

Equipotential surfaces will be hyperbolic and $\vec{E}$ everywhere will be $\perp$ to them.

11. C

Sol. For charging $q = CE(1 - e^{-t/RC})$

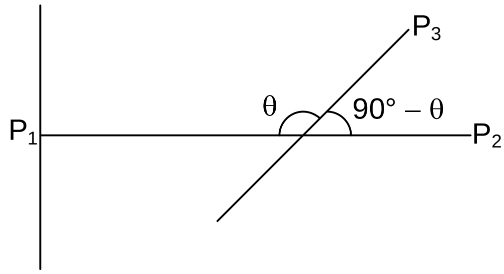
Charge at t = RC $\Rightarrow q_o = CE(1 - e^{-1})$

At t = RC discharging starts

$\Rightarrow q = q_o(e^{-t/RC}) = CE(1 - e^{-1}) \times \dfrac{1}{e} = CE\left(\dfrac{1}{e} - \dfrac{1}{e^2}\right)$

12. A

Sol. No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other



Let the initial intensity of light is I_0. So Intensity of light after transmission from first polaroid $= \frac{I_0}{2}$.

Intensity of light emitted from P_3

$$I_1 = \frac{I_0}{2}\cos^2\theta$$

Intensity of light transmitted from polaroid i.e., from

$$P_2 = I_1\cos^2(90° - \theta) = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

$$P_2 = \frac{I_0}{8}(2\sin\theta\cos\theta)^2 = \frac{I_0}{8}\sin^2 2\theta$$

13. D

Sol. No heat will be produced as no charge flows through S_2 when it is closed.

14. A

Sol. The resistivity of pure silicon is 2300 Ω m
and μ_e = 0.135 m^2/v-s, μ_h = 0.048 m^2/v-s.
Using
$\sigma = 1/\rho = (n_I \mu_e + n_I \mu_h)e$
$(2300)^{-1} = n_I (0.135 + 0.048) \times 1.6 \times 10^{-19}$
$n_I = 1.5 \times 10^{16}$ /m^3.
Is the intrinsic electron & hole concentration. The resistivity of a specimen doped with 10^{19} P-atoms/m^3 can be found from :
σ (conductivity) = $n_e e.\mu_e$ $(\because n_e = 10^{19}/m^3 >> n_I)$
$= 10^{19} \times 1.6 \times 10^{-19} \times 0.135 = 0.216$ mho / m.

$$\rho = \frac{1}{\sigma} = 4.6\ \Omega\text{-m}$$

15. B

Sol. Power = 10W

$\Rightarrow$ Force = $\frac{P}{C}$ (for perfectly absorbing surface)

$$F = \frac{10}{3\times10^8 m/s} = \frac{1}{3}\times10^{-7}$$

16. A

Sol. Given $\eta_1 = \frac{1}{6}$, $\eta_2 = \frac{1}{3}$

If the temperatures of the source and the sink between which the cycle is working are T_1 and T_2, then the efficiency in the first case will be

$$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{6}$$

In the second case $\eta_2 = 1 - \frac{T_2 - 65}{T_1} = \frac{1}{3}$

Solving T_1 = 390 K and T_2 = 325 K.

17. C

Sol. $m\frac{dv}{dt} = -6\pi\eta rv$

$$(1)\frac{dv}{dt} = 6\pi\left(\frac{1}{18\pi}\right)(1)v$$

$$\frac{dv}{dt} = -\frac{v}{3}$$

$$\frac{dv}{v} = -\frac{dt}{3}$$

$$[\ln v]_2^{0.5} = \left[-\frac{t}{3}\right]_{t_1}^{t_2}$$

$$\ln 4 = \frac{\Delta t}{3}$$

$$\Delta t = 3 \ln 4$$

18. B

Sol. muy = mVy + $\frac{3}{2}MR^2\omega$ where u is speed of ball before collision, V speed of ball after collision and ω is angular speed of cylinder after collision.

mu = mV + MRω (COM)

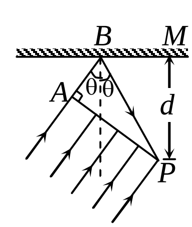
$\therefore$ $y = \frac{3R}{2}$.

19. B

Sol. $d = \frac{mv_0}{qB} + \frac{2mv_0}{2qB}$

20. B

Sol. $BP = \frac{MP}{\cos\theta}$



$$AB = BP\cos 2\theta = \frac{MP}{\cos\theta}\cos 2\theta$$

$$AB + BP = n\lambda + \frac{\lambda}{2}$$

$$\frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} = \frac{d}{\cos\theta}(2\cos^2\theta - 1 + 1) = 2d\cos\theta = n\lambda + \frac{\lambda}{2}$$ for n = 1, $\cos\theta = \frac{3\lambda}{4d}$

SECTION – B

21. 0

Sol. $V_A - V_B = i(5\Omega) + 10V + L\frac{di}{dt}$

$= 5(5) + 10 + L(-10^{+3})$

$V_A - V_B = 35 - 35 \times 10^{-3} \times 10^{+3} = 0.$

22. 15

Sol. Initially the rod will be in equilibrium if

$2T_o = Mg$ with $T_o = kx_o$ …(i)

when the current I is passed through the rod, it will experience a force

F = BIL vertically up,

In equilibriums

2T + BIL = Mg with T = kx …(ii)

from (i) & (ii)

$\frac{T}{T_o} = \frac{Mg - BIL}{Mg}$ i.c. $\frac{x}{x_o} = 1 - \frac{BIL}{Mg}$

or, $B = \frac{Mg(x_o - x)}{I \quad L \quad x_0}$

Putting the values we get $B = 1.5 \times 10^{-2}$ T.

23. 100

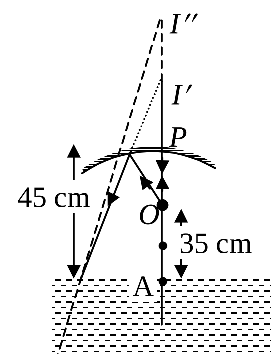
Sol. For the reflection at the concave mirror,

$u = -10$ cm; v = ?; $f = -15$ cm

From the mirror formula, we have

$$v = \frac{uf}{u-f} = \frac{(-10)\times(-15)}{-10+15} = \frac{150}{5} = +30 \text{ cm}$$

The positive sign indicates that the image is formed on the other side of the concave mirror,



Now, the image formed by the concave mirror serves as a virtual object for refraction at water surface which takes placed from air to water. So,

$$\mu = \frac{\text{Apparent height}}{\text{Real height}}$$

∴ AI″ = Apparent height = μ × real height $= \frac{4}{3} \times 75$ = 100 cm.

24. 2

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}$; $q_1 + q_2 = 2Q_0$

$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}$; $C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$

$$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$$

$$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$$

$$q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$$

$$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 2 \text{ amp}$$

25. 750

Sol. $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$1 = \frac{v_2 - (-2)}{u_1}$$

$$u_1 = v_2 + 2$$

$$u_1 = 1(-2) + 5(u_1 - 2)$$

$$u_1 = -2 + 5u_1 - 10$$

$$u_1 = \frac{12}{4} = 3m/s$$

$$v_2 = 1\ m/s$$

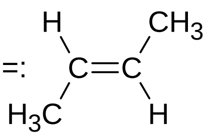
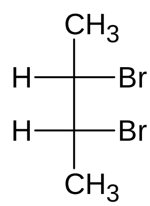
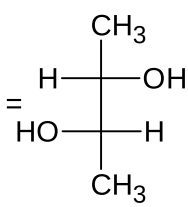
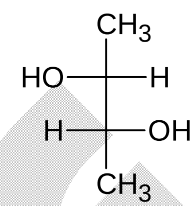
Kinetic energy of the centre of mass = $\frac{1}{2} \times (1+5) \times \left(\frac{3}{1+5}\right)^2 = \frac{3}{4} J$.

Chemistry

PART – B

SECTION – A

26. D

Sol. P =:  Q =  R or S =  or 

27. C

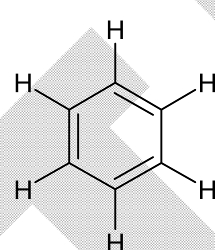
Sol. $2\pi r = n\lambda \Rightarrow \lambda = \frac{2\pi r}{n} = \frac{2\pi n^2 r_0}{n \times z} = 2\pi n r_0 = 2\pi \times 4 \times r_0 = 8\pi r_0$

28. B

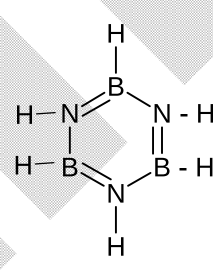
Sol. $CO_3^{2-} + H^+ \longrightarrow HCO_3^-$ (Weak base does not dissociate much). Therefore, the reaction proceeds to forward direction by removing CO_3^{2-} as HCO_3^-.

29. D

Sol. Organic benzene



Inorganic benzene



30. D

Sol. The nearer the –I groups towards COOH group, stronger is the acid.

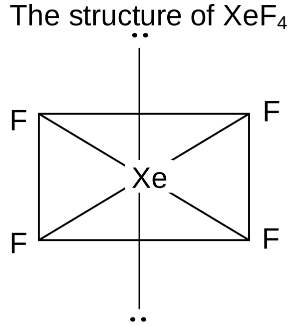
31. C

Sol. $Na_2B_4O_7 \xrightarrow{\Delta} 2NaBO_2 + B_2O_3$

32. B

Sol. A, C, D will show geometrical isomerism. (B) will show geometrical as well as optical isomerism.

33. C
Sol. The structure of XeF_4 is:



34. C
Sol. For zero order reaction,
Rate = k
Both rate and rate constant are independent of concentration.

35. D
Sol. pH of $NaH_2PO_4 = \frac{p^{K_{a_1}} + p^{K_{a_2}}}{2} = \frac{7.2 + 10.6}{2} = 8.9$

36. C
Sol. In NH_4Cl, the oxidation state of nitrogen is -3.

37. B
Sol. In case of weak acid heat of neutralization is less than –13.7 KCal / mole because extra energy is required to dissociate the acid.

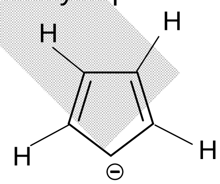
38. D
Sol. Sulphur forms two sigma bonds with two lone pairs.

39. B
Sol. $\frac{\Lambda_m}{\Lambda_e} = 3 \Rightarrow \Lambda_m = 3\Lambda_e$
n-factor of $MgCl_2.KCl.6\ H_2O$ = 3.

40. A
Sol. Aliphatic amines are stronger base than aromatic amines

41. B
Sol. In (B) the NH_2 group which is at a larger distance from the COOH group, should be more basic it should accept H^+ ion from acid.

42. C
Sol. In cyclopentadienyl anion, each carbon is sp^2 hybridized



43. B
Sol. Halogen atoms except fluorine form compounds with valency 1, 3, 5 and 7.

44. C

Sol. The fraction is $\frac{2}{3}$ because out of three chlorine atoms only two are precipitated.

45. C

Sol. Increases in pressure or addition of C affect the second equilibrium hence concentration of A_2 changes which further affect the first equilibrium. Addition of inert gas at constant volume does not affect the equilibrium.

SECTION – B

46. 4

Sol. $pH = \frac{1}{2}(pK_a - \log C)$

$= \frac{1}{2}\left(5 - \log 10^{-3}\right) = \frac{1}{2}(5+3) = 4$

47. 4

Sol. $A(g) + 2B(g) \rightleftharpoons 2C(g) + D(g)$

a 1.5 a 0 0

(a - x) 1.5a – 2x 2x x

Now, a – x = x

$\Rightarrow$ x = a/2

$$\therefore \ = K_c \frac{[C]^2(D)}{[A][B]^2} = \frac{\left(\frac{a}{2}\right)(a)^2}{\left(\frac{a}{2}\right)(0.5a)^2} = 4$$

48. 50

Sol. The possible orbitals for the 9th period are 9s, 9p, 8d, 7f, 6g. The number of electrons that can be accommodated in the orbitals are 2 + 6 + 10 + 14 + 18 = 50

49. 450

Sol. $A(g) \longrightarrow B(g) + 3C(g) + 4D(g)$

100 – x x 3x 4x

After one half-life, 100 – x = 50

x = 50

$\therefore$ Total pressure after one half-life

= (100 - x) + x + 3x + 4x

= 100 + 7x = 100 + 7 $\times$ 50 = 450 mm of Hg

50. 12

Sol. (P) is $Fe_4[Fe(CN)_6]_3$

Upon ionization(P) produces

$$Fe_4\left[Fe(CN)_6\right]_3 \rightleftharpoons 4Fe^{3+} + 3\left[Fe(CN)_6\right]^{4-}$$

$$Fe^{3+} + 3OH^- \longrightarrow Fe(OH)_3$$

For four moles of Fe^{3+}, 12 moles of OH^- is needed.

Mathematics PART – C

SECTION – A

51. D

Sol. Let $f(n)=\left[\frac{1}{3}+\frac{3a}{100}\right]n$ where $[n]$ is greatest integer function $=\left[0.33+\frac{3n}{100}\right]n$

For $n=1,2,.....,22,$ we get $f(n)=0$ and for $n=23, 24,....,55,$ we get $f(n)=1\times n$

For $n=56,\ f(n)=2\times n$

So $\sum_{n=1}^{56} f(n)=1(23)+1(24)+.....+1(55)+2(56)$

$=(23+24+.....+25)+112=\frac{33}{2}[46+32]+112$

$=\frac{33}{2}(78)+112=1399$

52. A

Sol. Given equation is, $x^2+x\sin\theta-2\sin\theta=0$

$\alpha+\beta=-\sin\theta$ and $\alpha\beta=-2\sin\theta$

$$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$\therefore|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{\sin^2\theta+8\sin\theta}$$

$$\therefore\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$

53. B

Sol. Let $I=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$

$$I=\int\left(\frac{\sin x.\cos x}{\sin^3 x+\cos^3 x}\right)^2 dx$$

$$I=\int\left(\frac{\sin x.\cos x}{\cos^3 x(1+\tan^3 x)}\right)^2 dx=\int\left(\frac{\sin x.\sec^2 x}{(1+\tan^3 x)}\right)^2 dx$$

Put $1+\tan^3 x=t$

$dt=-3\tan^2 x\sec^2 x\,dx$ or $dx=\frac{dt}{3\tan^2 x\sec^2 x}$

$$\therefore I=\int\frac{\sin^2 x.\sec^4 x}{t^2}\times\frac{dt}{3\tan^2 x\sec^2 x}$$

$$I = \frac{1}{3}\int \frac{\sin^2 x.\sec^4 x}{t^2} \times \frac{dt}{\frac{\sin^2 x}{\cos^2 x} \times \sec^2 x}$$

$$= \frac{1}{3}\int \frac{\sin^2 x.\sec^4 x}{t^2} \times \frac{dt}{\sin^2 x \sec^4 x}$$

$$\therefore I = \frac{1}{3}\int \frac{dt}{t^2} = \frac{1}{3}\int t^{-2}dt$$

$$\Rightarrow I = \frac{1}{3}\left[\frac{t^{-2+1}}{-2+1}\right] + c = \frac{-1}{3}\left[\frac{1}{t}\right] + c$$

$$\text{or } I = -\frac{1}{3\left(1+\tan^3 x\right)} + c$$

54. D

Sol. $P\left(4t^2+3, 8t^3-1\right)$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = 3t \text{ (slope of tangent at P)}$$

Let $Q = \left(4\lambda^2+3,\ 8\lambda^3-1\right)$

slope of PQ = 3t

$$\frac{8t^3-8\lambda^3}{4t^2-4\lambda^2} = 3t \Rightarrow t^3 - 3\lambda^2 t + 2\lambda^3 = 0$$

$$\left(t-\lambda\right).\left(t^2+t\lambda-2\lambda^2\right) = 0$$

$$\left(t-\lambda\right)^2.\left(t+2\lambda\right) = 0 \Rightarrow t = \lambda \left(or\right) \lambda = \frac{-t}{2}$$

$$\therefore\ Q\left[t^2+3, -t^3-1\right]$$

55. D

Sol. For infinitely many solution

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$\Rightarrow 1(10) - 1(7) + k(-1) - 0 \Rightarrow k = 3$

$k = 3$, 2nd system is $4x+5y=7$ ………(i)

and $7x+8y=10$ ………(ii)

Clearly, they have a unique solution

(ii) – (i) $\Rightarrow 3x+3y=3 \Rightarrow x+y=1$

56. C

Sol. Give $\cos^{-1}(2x) - 2\cos^{-1}\sqrt{1-x^2} = \pi$

$$\cos^{-1}(2x) - \cos^{-1}\left(2\left(1-x^2\right)-1\right) = \pi$$

$\cos^{-1}(2x)-\cos^{-1}(1-2x^2)=\pi$

$-\cos^{-1}(1-2x^2)=\pi-\cos^{-1}(2x)$

Taking cos both sides we get

$\cos\left(-\cos^{-1}(1-2x^2)\right)=\cos\left(\pi-\cos^{-1}(2x)\right)$

$1-2x^2=-2x \Rightarrow 2x^2-2x-1=0$

On solving, $x=\frac{1\pm\sqrt{3}}{2}$

As $x=\left[\frac{-1}{2},\frac{1}{2}\right]$, $x=\frac{1+\sqrt{3}}{2}=$ rejected

So $x=\frac{1-\sqrt{3}}{2}\Rightarrow x^2-1=\frac{-\sqrt{3}}{2}$

$=2\sin^{-1}(x^2-1)=2\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{-2\pi}{3}$

57. B

Sol. Since, $a_1+a_3=10=a_1+d\Rightarrow 5$ and $\frac{1}{6}\sum_{1=1}^{6}a_1=\frac{19}{2}$

$a_1+a_2+a_3+a_4+a_5+a_6=57$

$\Rightarrow \frac{6}{2}[a_1+a_6]=57\Rightarrow a_1+a_6=19$

$\Rightarrow 2a_1+5d=19$ and $a_1+d=5\Rightarrow a_1+2,\ d=3$

Numbers : 2, 5, 8, 11, 14, 17

Then, variance $=\sigma^2=$ mean of squares – square of mean

$=\frac{2^2+5^2+8^2+(11)^2+(14)^2+(17)^2}{6}-\left(\frac{19}{2}\right)^2$

$=\frac{699}{6}-\frac{361}{4}=\frac{105}{4}$; So, $8\sigma=210$

58. C

Sol. Given $\beta=\lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$

$$\beta=\lim_{x\to 0}\frac{1+\alpha x-\left[1+3x+\frac{9x^2}{2!}+....\right]}{(\alpha x)\frac{(e^{3x}-1)}{3x}3x}$$

$$\beta=\lim_{x\to 0}\frac{(\alpha x-3x)-\frac{9x^2}{2!}-.....}{3\alpha x^2}$$

For existence of limit $\alpha-3=0,\ \alpha=3$

Limit $\beta = \frac{-3}{2\alpha}, \beta = -\frac{1}{2}$

Now, $\alpha + \beta = \frac{5}{2}$

59. D

Sol. Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$

Then, $e_1 = \sqrt{1 - \frac{b^2}{25}}$

The equation of hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$

Then, $e_2 = \sqrt{1 + \frac{b^2}{16}}$, $e_1 e_2 = 1$

$\Rightarrow (e_1 e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25 \times 16} = 1$

$\Rightarrow \frac{9}{16.25} b^2 - \frac{b^4}{25.16} = 0 \Rightarrow b^2 = 9$ $\quad \therefore e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

And, $e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Distance between focii of ellipse $= \alpha = 2ae_1 = 2(5)(e_1) = 8$

Distance between focii of hyperbola

$= \beta = 2ae_2 = 2(4)(e_2) = 10 \therefore (\alpha, \beta) = (8, 10)$

60. A

Sol. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b$, $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y = mx \pm \sqrt{a^2 m^2 + b^2}$

Comparing with $\equiv y = \frac{-x}{6} + \frac{4}{3}$

$m = \frac{-1}{6}$ and $a^2 m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} + \frac{4}{3} = \frac{16}{9} \Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3} = \frac{4}{9}$

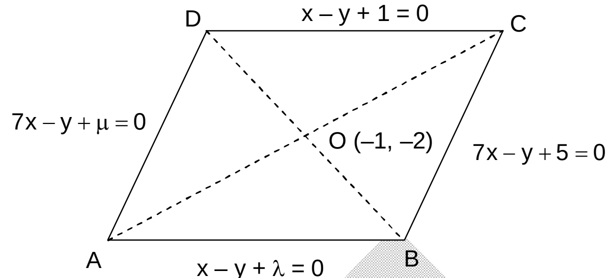
$\Rightarrow a^2 = 16 \Rightarrow a = \pm 4$

Now, eccentricity of ellipse $(e) = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}$

61. A

Sol. Let other two sides of rhombus are $x-y+\lambda=0$ and $7x+y+\mu=0$ then O is equidistant from AB and DC and from AD and BC



$\therefore |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda=-3$ and

$|-7+2-5| = |-7+2+\mu| \Rightarrow \mu=15$

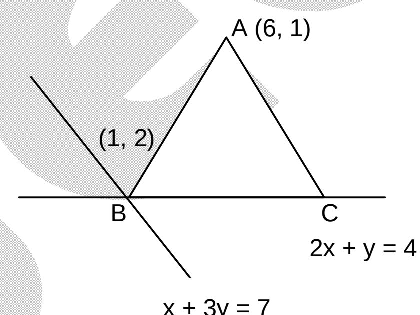
$\therefore$ Other two sides are $x-y-3=0$ and $7x-y+15=0$

$\therefore$ On solving the equations of sides pair wise, we get the vertices as

$\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$

62. C

Sol. Intersection of two lines in point B with coordinate (1, 2). As C lies on $2x+y=4$, suppose $x=h$ then k = 4 – 2h.



Δ is isosceles with base BC. Then, $AB=AC$

$\sqrt{25+1} = \sqrt{(6-h)^2+(3-2h)^2}$

$\sqrt{26} = \sqrt{36+h^2-12h+4h^2+9-12h}$

Take square both sides

$26 = 5h^2-24h+45 \Rightarrow 5h^2-24h+19=0$

$\Rightarrow 5h^2-5h-19h+19=0$

$\Rightarrow 5h^2-24h+19=0$

$h=\frac{19}{5}$ or $h=1$.

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid (α,β) of triangle

$$ABC = \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$$

$\alpha=\frac{54}{15}; \beta=\frac{-3}{15}$

Put the value of α and β in $15(\alpha+\beta)$.

$15(\alpha+\beta)=51$

63. C

Sol. By splitting given equation

$$\frac{1}{20}\left[\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\left(\frac{1}{40-a}-\frac{1}{60-a}\right)+....+\left(\frac{1}{180-a}-\frac{1}{200-a}\right)\right]=\frac{1}{256}$$

$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{256}$$
$$\Rightarrow (20-a)(200-a)=256\times 9$$
$$\Rightarrow a^2-220a+1696=0 \Rightarrow a=8,\ 212$$
$\Rightarrow$ Hence, maximum value of a is 212

64. C

Sol. $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+........+x^{10}$

$$=(1+x)^{10}\frac{\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$
$$\Rightarrow (1+x)^{11}-x^{11}$$
Coefficient of x^7 is ${}^{11}C_7=330$

65. B

Sol. Let $3^x=y$

$\therefore\ y(y-1)+2=|y-1|+|y-2|$

Case I : When $y>2$

$y^2-y+2=y-1+y-2 \Rightarrow y^2-3y+5=0$

$\because\ D<0$ [$\because$ Equation not satisfy]

Case II : When $1\le y\le 2$

$y^2-y+2=y-1-y+2;\ y^2-y+1=0$

$\because\ D<0$ [$\because$ Equation not satisfy]

Case III : When $y\le 1$

$y^2-y+2=-y+1-y+2 \Rightarrow y^2+y-1=0$

$\therefore y=\frac{-1+\sqrt{5}}{2}=\frac{-1-\sqrt{5}}{2}$ [$\because$ Equation not satisfy]

$\therefore$ only one $-1+\frac{\sqrt{5}}{2}$ satisfy equation

66. C

Sol. $\therefore \sum_{k=0}^{10}(3k+4)\,{}^{10}C_k=\sum_{k=1}^{10}3k.\frac{10}{k}\,{}^{9}C_{k-1}+4\sum_{k=0}^{10}{}^{10}C_k$

$=30\sum_{k=1}^{10}{}^{9}C_{k-1}+4\sum_{k=0}^{10}{}^{10}C_k$

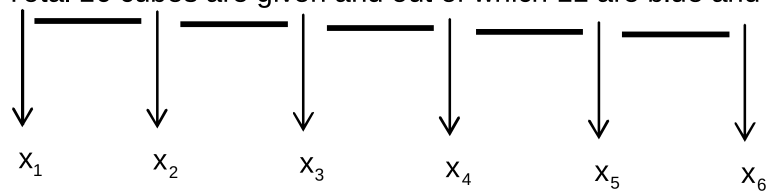
$=30.2^9+4.2^{10}\left[\because \sum_{r=0}^{n}{}^{n}C_r=2^n\right]$

$=19.2^{10}\Rightarrow \alpha=0$ and $\beta=19$.

So, $\alpha+\beta=19$

67. C

Sol. Total 16 cubes are given and out of which 11 are blue and 5 are red.



$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_1, x_6 \geq 0,\ x_2, x_3, x_4, x_5 \geq 2$

$x_2 = t_1 + 2 \Rightarrow x_3 = t_3 + 2 \Rightarrow x_4 = t_4 + 2 \Rightarrow x_5 = t_5 + 2$

$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$

Number of solutions $= {}^{6+3-1}C_3 = {}^{8}C_3 = 56$

68. A

Sol. Given, $\left|\vec{a} + \vec{b}\right| = \sqrt{3} \Rightarrow \cos\theta = \frac{1}{2}$ or $\theta = \frac{\pi}{3}$

We have $\vec{c} = \vec{a} + 2\vec{b} + 3\left(\vec{a} \times \vec{b}\right)$

$$\left|\vec{c}\right| = \sqrt{\left(\vec{a}\right) + \left(2\vec{b}\right)^2 + \left(\frac{3\sqrt{3}}{2}\hat{n}\right)^2 + 4\vec{a}.\vec{b} + 6.\sqrt{3}\,\hat{n}.\vec{b} + 3\sqrt{3}\,\hat{n}\,\vec{a}}$$

$$= \sqrt{1 + 4 + \frac{27}{4}\left|\hat{n}\right|^2 + 4.\frac{1}{2} + 0 + 0} \qquad \left[\because \hat{n} \perp \vec{b} \text{ and } \hat{n} \perp \vec{a}\right]$$

$$\left(\vec{c}\right) = \frac{\sqrt{55}}{2}$$

69. B

Sol. Given $\vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$

Multiply $\vec{b}$ both sides,

$\vec{b}.\vec{c} = \vec{b}.\left(2\vec{a} \times \vec{b}\right) - 3\left|\vec{b}\right|^2$

Since, $\vec{b}.\left(2\vec{a} \times \vec{b}\right) = 0$ then $\vec{b}.\vec{c} = -3\left|\vec{b}\right|^2$

$\vec{b}.\vec{c} = -3 \times 16 = -48$

70. B

Sol. $\vec{a}.\vec{b} = 1 \Rightarrow \left(\alpha\hat{i} + \beta\hat{j} + 3\hat{k}\right)\left(-\beta\hat{i} - \alpha j - \hat{k}\right) = 1$

$\Rightarrow -\alpha\beta - \alpha\beta - 3 = 1 \Rightarrow \alpha\beta = -2$ ……(i)

$\vec{b}.\vec{c} = -3 \Rightarrow \left(-\beta\hat{i} - \alpha\hat{j} - \hat{k}\right)\left(\hat{i} - 2\hat{j} - \hat{k}\right) = -3$

$\Rightarrow -\beta + 2\alpha + 1 = -3 \Rightarrow 2\alpha - \beta = -4$ …….(ii)

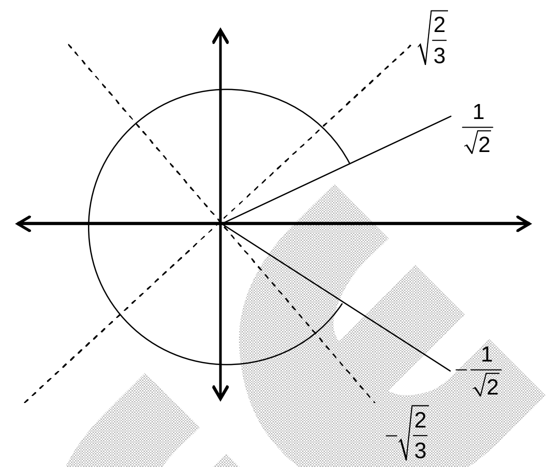
From (i) and (ii), $\alpha = -1,\ \beta = 2,$

$$\frac{1}{3}\left(\left(\vec{a} \times \vec{b}\right).\vec{c}\right) = \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = 2$$

SECTION – B

71. 4

Sol. $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$



Given equation is

$14\,\text{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$

$= 21 - 4\left(1 - \sin^2 x\right) = 17 + 4\sin^2 x$

$14\,\text{cosec}^2 x - 6\sin^2 x = 17$

Let $\sin^2 x = t$

$\frac{14}{t} - 6t = 17 \Rightarrow 14 - 6t^2 = 17t$

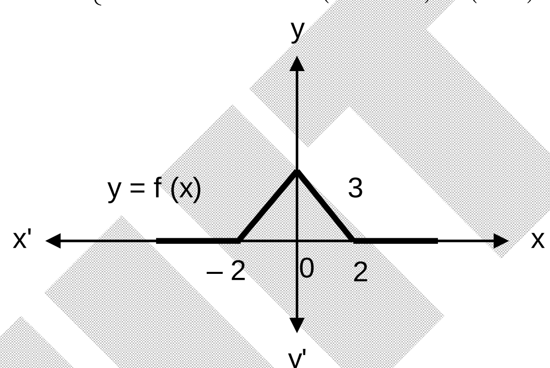
$6t^2 + 17t - 14 = 0 \Rightarrow t = -3, 5, \frac{2}{3} \Rightarrow \sin^2 x = \frac{2}{3}$

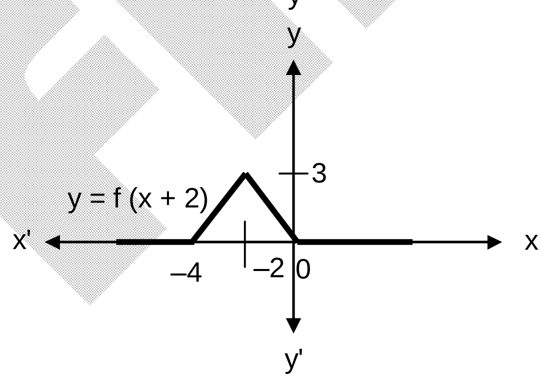
$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$

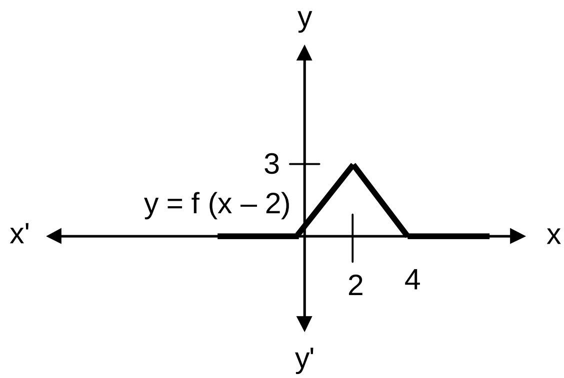
$\therefore$ Total 4 solutions

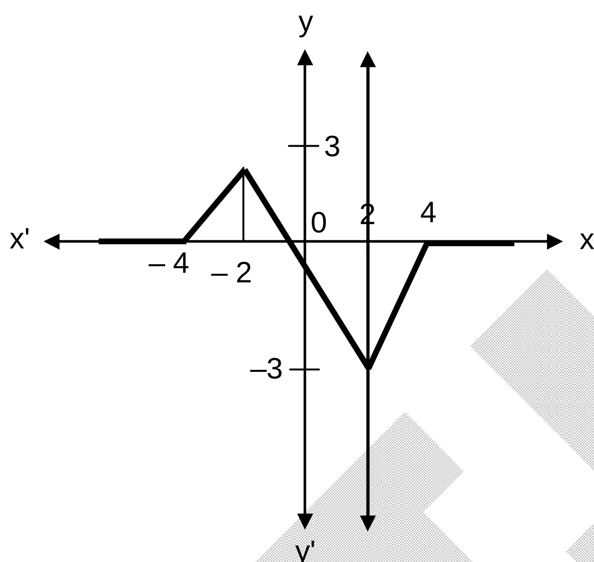
72. 4

Sol. $f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } -2 \le x \le 2 \\ 0 & \text{if } x \in (-\infty, -2) \cup (2, \infty) \end{cases}$









73. 4

Sol. $\therefore A^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix}, A^3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^3 & 0 \\ 3 & 0 & -1 \end{vmatrix}, A^4 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^4 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

So, $A^{20} + \alpha A^{19} + \beta A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{vmatrix} + \alpha \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{vmatrix} + \beta \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{vmatrix}$

$= \begin{vmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

On comparing, we get

$\alpha+\beta=0$ and $2^{20}+\alpha.2^{19}+2\beta=4 \Rightarrow \alpha=-2$ and $\beta=2$

So, $\beta-\alpha=2+2=4$

74. 19

Sol. $A=\{0,3,4,6,7,8,9,10\}$ $3,7,9 \to$ odd, $0, 4, 6, 8, 10 \underset{\text{even}}{\to}$

$R=\{x-y=\text{odd}+\text{ve or } x-y=2\}$

R = {(6, 4), (8, 6), (9, 7), (10, 8), (3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 4), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}
minimum ordered pairs to be added must be 15 + 4 = 19

75. 2

Sol. $x^2 + y^2 - 4x - 2y + 5 - \alpha = 0$

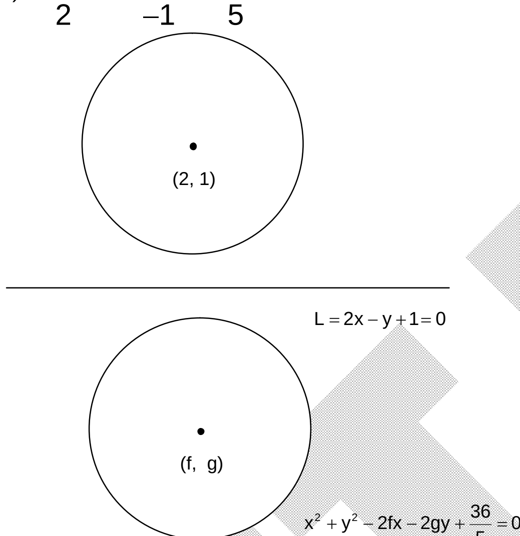
$C_1(2, 1)$ and $r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

Image of (2, 1) in given line will be

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$



$$\Rightarrow x = 2 - \frac{16}{5} = \frac{-6}{5},\ y = 1 + \frac{8}{5} = \frac{13}{5}$$

So, $x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$, $C_2(f, g)$ and $r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$

[radius of both circle will be same]

$$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5} \qquad \left[\because f = -\frac{6}{5},\ g = \frac{13}{5}\right]$$

$$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$$

$\therefore \alpha + r = 2$