FIITJEE

ALL INDIA TEST SERIES

JEE (Advanced)-2025

FULL TEST – I

PAPER –1

TEST DATE: 26-12-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. C

Sol. $\frac{a}{u_x} + \frac{a}{eu_x} + \frac{a}{e^2 u_x} = \frac{2u_y}{g}$ ...(1)

Horizontal range:

$R = \frac{2.u_x u_y}{g}$ ...(2)

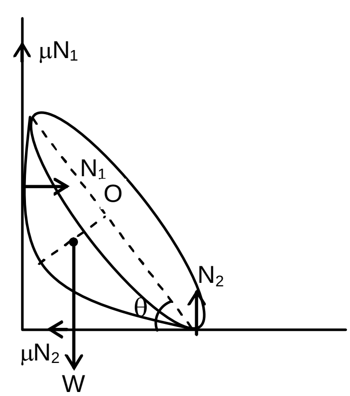
2. B

Sol. $N_1 = \mu N_2$ ...(1)

$W = \mu N_1 + N_2$ ...(2)

By (1) and (2)

$N_1 = \frac{\mu W}{1+\mu^2}$ ...(2)



Balancing torque about 'O'

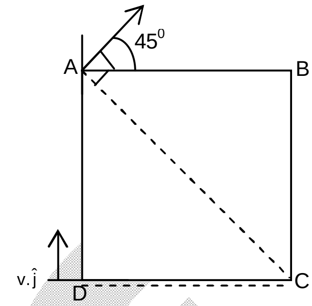
$\mu N_1 \times R + \mu N_2 \times R = W \times \frac{3R}{8} \sin\theta$

$\text{Sin}\,\theta = \frac{8}{3}\left[\frac{\mu+\mu^2}{1+\mu^2}\right]$

3. D

Sol. $v(\hat{i}+\hat{j})$

The motion can be considered as rotation about an instantaneous centre of rotation. To get it, draw normals to the velocities at A and D.



From the construction, C is the center of rotation so that

$V_C = 0$

$\frac{V_C}{V_B} = 0$

4. C

Sol.

$$v^2 = v_0^2 - 2gR(1-\cos\theta)$$

$$v^2 = 7gR + 2gR\cos\theta$$

$$T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = 7mg + 3mg\cos\theta$$

$$v_h = \sqrt{7gr} \Rightarrow T = 7mg$$

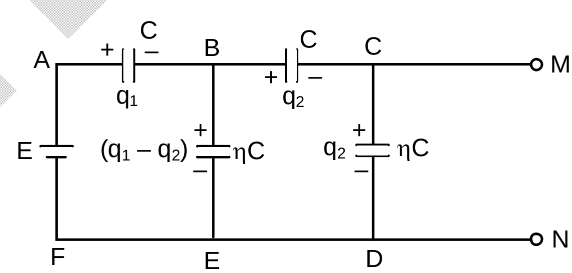
5. ABCD

Sol. Apply ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i$$

6. ABC

Sol. Loop ABEF



$$\frac{q_1}{c} + \frac{q_1 - q_2}{\eta c} = E \quad \ldots(1)$$

Loop ABCDEF

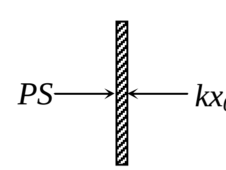
$$\frac{q_1}{c} + \frac{q_2}{c} + \frac{q_2}{\eta c} = E \quad \ldots(2)$$

Bu (1) and (2) we get

$$V_{MN} = \frac{q_2}{\eta c} = \frac{E}{\eta^2 + 3\eta + 1}$$

7. ABCD

Sol. Equilibrium of piston gives



$PS = kx_0$ or $P = \frac{kx_0}{S}$

Since, the chamber is thermally insulated $\Delta Q = 0$

$\therefore$ Elastic potential energy of spring = work done by gas

or work done by gas = $\frac{1}{2}kx_0^2$

This work is done in the expense of internal energy of the gas.

Therefore, internal energy of the gas is decreased by $\frac{1}{2}kx_0^2$.

Internal energy of an ideal gas depends on its temperature only. Internal energy of the gas is decreasing. Therefore, temperature of the gas will decrease.

8. B

Sol. Just after cutting force of cut spring will be zero whereas the force of other spring will be unchanged.

9. A

Sol. (P) $\frac{mg}{2}\frac{\ell}{4} = T\frac{3\ell}{4} \Rightarrow T = \frac{mg}{6}$

(Q) $f_r = mg\frac{\sqrt{3}}{2}$

(R) $N = \frac{mg}{2} \times \frac{4}{7} = \frac{2mg}{7}$

(S) $N = \frac{mg}{2} - \frac{mg}{6} = \frac{mg}{3}$

10. B

Sol. From the graph, first velocity increases then decreases and further change its direction (DE).

11. C

Sol. $\oint \overline{B} \cdot \overline{d\ell} = -A\left(\frac{dB}{dt}\right)$

SECTION – B

12. 8

Sol. $\frac{4\ell_1}{3} = \frac{2\ell_2}{4}$

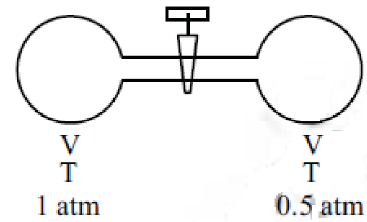
13. 2

Sol. For pulse $\frac{dx}{dt} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3-10kt}{\mu}}$

$\Rightarrow \int_o^L dx = \int_o^{t_o} \sqrt{\frac{3-10\,kt}{\mu}}\,dt$

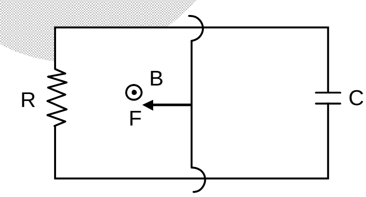
And $3 - 10\,k\,t_o = 0$

14. 6
Sol. After opening of at equilibrium temperature and pressure of whole gas is T_1 and P_1

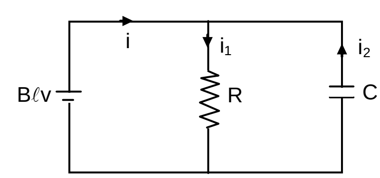


$$n_1 = \frac{1\times V}{RT},\ n_2 = \frac{0.5\times V\times 4}{RT}$$
$n_1 + n_2 = n$
$$\frac{V}{RT} + \frac{V\times 4}{2RT} = \frac{5VP_1}{RT_1}$$
$$\frac{3V}{RT} = \frac{5VP_1}{RT_1}\ ;\ \frac{P_1}{T_1} = \frac{0.6}{T}$$
$\Delta Q = 0,\ \ \Delta W = 0$
$\therefore\ \Delta U = 0$
$n_1C_VT + n_2C_VT = (n_1 + n_2)C_VT_1$
$T_1 = T$
$$\frac{P_1}{T} = \frac{0.6}{T}$$
$P_1 = 0.60$ atm $\Rightarrow$ $10P = 6$.

15. 4
Sol. $B\ell v = i_1R$ …(1)



$B\ell v = \frac{q}{c}$ but $\frac{dq}{dt} = i_2$
$q = B\ell vc$
$i_i = B\ell ca$ …(2)
$F - (i_1 + i_2)\,B\ell = ma$
Solving this we can find terminal velocity



$$v = \frac{FR}{B^2\ell^2} = \frac{FR}{4FR} = 0.25\ ;\ \frac{1}{v} = 4.$$

16. 2
Sol. Use formula in terms of reduced mass
$$\frac{1}{\lambda} = \frac{R_\infty z^2}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$\therefore\ \frac{\lambda_D}{\lambda_H} = \frac{1+\frac{m}{M_D}}{1+\frac{m}{M_H}}$$
$$\frac{M_D}{M_H} \approx 2.0$$

17. 7

Sol. For process AB $T_A = 300\text{ K},\ T_B = 600\text{ K}$

$W = nR\Delta T = nR(T_B - T_A) = 300\text{ nR} = 600R.$

$$Q = n\,Cp\,\Delta T = 2 \times \frac{5}{2}R\ (300) = 1500R.$$

For process BC $W = nRT\ln\frac{v_f}{v_c} = nRT\ln\frac{p_i}{p_f} = nRT\ln 2 = 1200R\ln 2$

$Q = W = 1200R\ln 2$

For process CA $W = \int P\,dV = \int_{600}^{300} \frac{K}{T}\frac{2nRT}{K}dT.$

$= -2nR(300) = -1200R.$

$Q = nC_V\ \Delta T + W$

$= 2 \times \frac{3}{2}R(-300) - 1200R.$

$= -900R - 1200R = -2100R$

$$\eta = \frac{600R + 1200R\ln 2 - 1200R}{1500R + 1200R\ln 2}$$

$$= 1 - \frac{21}{12\ln 2 + 15} \Rightarrow x = 7.$$

Chemistry

PART – II

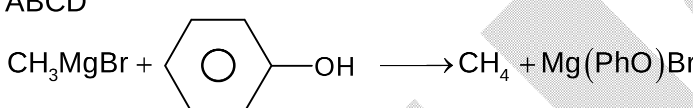
SECTION – A

18. B
Sol. $P_4O_{10} + 4HNO_3 \longrightarrow 2N_2O_5 + 4HPO_3$

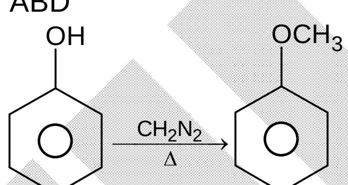
19. B
Sol. Each photon strikes out one electron form the metal surface. Intensity will increase the number of photons, so number of photoelectrons increase.

20. C
Sol. In(C), the complex ions have zero charge. So there is no attraction between the two.

21. B
Sol. $\alpha = \dfrac{\Lambda_m}{\Lambda_m^0}$

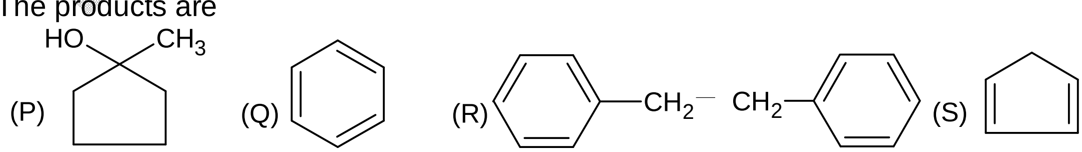
22. ABCD
Sol.


23. AB
Sol. $PH_4Cl \xrightarrow{\Delta} PH_3 + HCl$
$4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$

24. ABD
Sol.


25. B
Sol. CsCl has b.c.c unit cell, which coordination number equal to eight.

26. D
Sol. Due to half-filled $t_{2g}^3 e_g^2$ configuration $[FeF_6]^{3-}$ complex is colourless.

27. C
Sol. The products are


28. C
Sol. $Fe^{3+} \rightarrow Fe(SCN)_3$
$Fe^{2+} \rightarrow$ Green sulphate
$Fe^{4+}(n = 4)$, $Fe^{2+}(n = 4)$
$Fe \rightarrow Fe(CO)_5$ hybridization(dsp^3)

SECTION – B

29. 864
Sol. $P_{equilibrium} = p_B + 2p_C = 18$ atm
Since $p_B + 2p_C = 3x = 18$
or $x = 6$
$\therefore\ p_B = 6$ atm, $p_C = 12$ atm
$\therefore\ K_p = p_B \times (2p_C)^2 = 6 \times (12)^2 = 864$

30. 88
Sol.
$$\begin{array}{l} \qquad\qquad\quad CH_3 \quad O \\ \qquad\qquad\quad\ | \qquad\ \| \\ \text{X is } CH_3 - CH - C - OH \end{array}$$

31. 801
Sol. $\mu_{max} = q \times d = 1.602 \times 10^{-19} \times 5 \times 10^{-10} = 8.01 \times 10^{-29}$ coulomb meter = b x 10^{-29}
$\therefore$ 100 b = 801

32. 120
Sol. X is mesitylene.
Y is Chloro mestiylene

33. 80
Sol. A = $Na_2SO_4.10H_2O$, B = Na_2SiO_3, C = SO_3, D = HCl, PPt = $BaSO_4$

34. 5
Sol. Moles of $Sn^{2+} = 523.15 \times 10^{-3}$
Moles of MnO_4^- required = $\frac{2}{5} \times 523.15 \times 10^{-3}$
$$\frac{\frac{2}{5} \times 523.15 \times 10^{-3} \times 1000}{418.52} = x$$
x = 0.5, 10x = 5

Mathematics PART – III

SECTION – A

35. B

Sol. $f(x) = 2\operatorname{cosec} 2x + \left(\sqrt{\sec x} - \sqrt{\operatorname{cosec} x}\right)^2 + \dfrac{2\sqrt{2}}{\sqrt{2\cos x . \sin x}}$

$\therefore\ f(x) = \dfrac{2}{\sin 2x} + \left(\sqrt{\sec x} - \sqrt{\operatorname{cosec} x}\right)^2 + \dfrac{2\sqrt{2}}{\sqrt{\sin 2x}}$

So, $f_{min}\left(x = \dfrac{\pi}{4}\right) = \dfrac{2}{1} + 0 + \dfrac{2\sqrt{2}}{\sqrt{1}} = 2\left(\sqrt{2}+1\right)$

36. D

Sol. $f(x) = \tan^{-1}\left(2x^2\right)$ and $g(x) = \left(-x^2\right)$

$\Rightarrow\ \lim\limits_{x\to 0} \dfrac{e^{2x^2} - e^{\tan^{-1} 2x^2}}{x^6} = \dfrac{8}{3}$

37. C

Sol. We have $f(x) = [3x] + 14 + |(2x+1)(2x-1)|(2x-1)(x+2) + \sin\left(\dfrac{\pi x}{2}\right)$

For $x \in [-4, 4]$

$-12 \le 3x \le 12$

$\therefore\ x = \underbrace{\dfrac{-12}{3}, \dfrac{-11}{3}, \dfrac{-10}{3}, ..., \dfrac{-1}{3}, 0, \dfrac{1}{3}, ..., \dfrac{11}{3}, \dfrac{12}{3}}_{25 \text{ points}}$

But $f(-4) = f\left(-4^+\right)$

$\Rightarrow f(x)$ is continuous at $x = -4$.

Here, $f(x)$ is continuous at $x = \dfrac{-1}{2}$ but non – derivable at $x = \dfrac{-1}{2}$.

As discontinuing $\Rightarrow$ non – differentiability

So, $f(x)$ is non – derivable at 25 points in $[-4, 4]$

38. C

Sol. Since $P(1) = 1 + b + c$ and $P(2) = 4 + 2b + c$ are both roots of P, we have, from Viet's that P (1) + P (2) = – b $\Rightarrow 4b + 2c = -5$ and $(1+b+c).(4+2b+c) = c$ substituting $b = -\dfrac{2c+5}{4}$, we get $\left(-\dfrac{1}{4} + \dfrac{c}{2}\right).\left(\dfrac{3}{2}\right) = c \Rightarrow \dfrac{4c-2}{8}.\dfrac{3}{2} = c \Rightarrow c = -\boxed{\dfrac{3}{2}}$

39. BD

Sol. $$L = \lim_{x\to 0}\frac{\sin(x^2)+2\cos(bx)-ax^4-2}{e^{ax}-1-ax-2x^2-\frac{a^3x^3}{6}}$$

$$= \lim_{x\to 0}\frac{x^2-\frac{x^6}{3!}+....+2\left(1-\frac{b^2x^2}{2!}+\frac{b^4x^4}{4!}+....\right)-ax^4-2}{1+ax+\frac{a^2x^2}{2!}+\frac{a^3x^3}{3!}+\frac{a^4x^4}{4!}+....-1-ax-2x^3-\frac{a^3x^3}{6}}$$

$$= \lim_{x\to 0}\frac{(1-b^2)x^2+\left(\frac{b^2}{12}-a\right)x^4+.....}{\left(\frac{a^2}{2}-2\right)x^2+\frac{a^4x^4}{24}+....}$$

$$\frac{1-b^2}{\frac{a^2}{2}-2}=\frac{25}{8}\Rightarrow 16-16b^2=25(a^2-4)$$

$\Rightarrow\ 25a^2+16b^2=116$

$$\Rightarrow \left.\begin{array}{l} a^2=4 \text{ and } b^2=1 \\ a=\pm 2,\ b=\pm 1\end{array}\right\}(\because a,b\in I)$$

$$\frac{1-b^2}{\frac{a^2}{2}-2}=\frac{-23}{8}\Rightarrow 16-16b^2=-23(a^2-4)$$

$23a^2-16b^2=76\Rightarrow a^2=4$

$$\Rightarrow \left.\begin{array}{l} a^2=4 \text{ and } b^2=1 \\ a=\pm 2,\ b=\pm 1\end{array}\right\}(\because a,b\in I)$$

Now, $L=\dfrac{\frac{b^2}{12}-a}{\frac{a^4}{24}}=\dfrac{2(b^2-12a)}{a^4}$

For $a=2,\ b=\pm 1$

$\Rightarrow\ L=\dfrac{2.(1-24)}{16}=\dfrac{-23}{8}$

For $a=-2,\ b=\pm 1$

$\Rightarrow\ L=\dfrac{2.(1+24)}{16}=\dfrac{25}{8}$

40. ABD

Sol. $$f'(0)=0;\ f'(x)=\begin{cases} 2x\sin\frac{1}{x}-\cos\frac{1}{x}, & \text{for } 0<x\le 1 \\ 0, & \text{if } x=0\end{cases}$$

$\Rightarrow\ f(x)$ is differentiable in $[0, 1]$

$$\lim_{x\to 0}\frac{f(x)}{g(x)}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x^2}=\text{D.N.E.}$$

$$\lim_{x\to 0}f'(x)g'(x)=\lim_{x\to 0}\left(4x^2\sin\frac{1}{x}-2x\cos\frac{1}{x}\right)=0$$

Also, $\lim_{x\to 0}\frac{f'(x)}{g'(x)}$ does not exist.

41. ABCD

Sol. (A) Let $g(t)=e^{\frac{t^2}{2}}.f(t)$ for $t\in[a,b]$ here $g(a)=g(b)=0$

According to Rolle's theorem

$g'(x)=0$, for some $x\in(a,b)$

$\Rightarrow xe^{\frac{x^2}{2}}.f(x)+e^{\frac{x^2}{2}}.f'(x)=0$

$\Rightarrow xf(x)+f'(x)=0$

(B) Let $g(t)=e^{\frac{-t^2}{2}},f(t)$, for $t\in[a,b]$

here $g(a)=g(b)=0$

$\therefore$ According to Rolle's theorem

$g'(x)=0$ for some $x\in(a,b)$

$\Rightarrow -xe^{\frac{x^2}{2}}.f(x)+e^{\frac{-x^2}{2}}.f(x)=0$

$\Rightarrow xf(x)-f'(x)=0$

(C) Let $g(t)=e^{50t}f(t)$, $t\in[a,b]$

Here $g(a)=g(b)=0$

$\therefore$ According to Rolle's theorem

$g'(x)=0$ for some $x\in(a,b)$

$\Rightarrow 50e^{50x}f(x)+e^{50x}f'(x)=0\Rightarrow 50f(x)+f'(x)=0$

(D) Let $g(t)=50^t f(t)$, $t\in[a,b]$

Here $g(a)=g(b)=0$

$\therefore$ According to Rolle's theorem

$g'(x)=0$ for some $x\in(a,b)$

$\Rightarrow 50^x \ln 50f(x)+50^x f'(x)=0$

$\Rightarrow (\ln 50)f(x)+f'(x)=0$

42. B

Sol. (P) $A^2+B=I$ ....(1)

$BA^2+B^2=B$

$\Rightarrow B^2 = B$

$\therefore A^2 + B^2 = 1 \Rightarrow tr\left(A^2 + B^2\right) = 3$

(Q) $x\frac{dy}{dx} + y(\ln y) = 0 \Rightarrow \int\frac{dx}{x} + \int\frac{dy}{y(\ln y)} = C;$

$\ln(x\ln y) = C$. If $x = 1$ then $y = e$

Hence, $\Rightarrow \ln(\ln e) = c \Rightarrow c = 0$

$\Rightarrow \ln(x\ln y) = 0 \Rightarrow x\ln y = 1 \Rightarrow y = f(x) = e^{\frac{1}{x}}$

Now, $\lim_{x\to\infty} x(f(x) - 1) = \lim_{x\to\infty}\left(e^{\frac{1}{x}} - 1\right) = 1 = e^0$

$\Rightarrow \lambda = 0]$

(R) $\int_1^e\left(2^{\ln x} + e\right)dx + \int_{1+e}^{2+e} e^{\log_2(x-e)}\,dx$

$= e(2+e) - (1+e)$

$\Rightarrow \int_1^e 2^{\ln x}dx + \int_{1+e}^{2+e} e^{\log_2(x-e)}\,dx$

$= (2e - 1)$

$\therefore [\lambda] = 4$

(S) As two given lines are parallel, so we must have

$(\alpha + 1 + 2(\alpha - 3) - 1)(2(\alpha + 1) + 4(\alpha - 3) - 14) < 0$

$\Rightarrow (3\alpha - 6)(6\alpha - 24) < 0 \Rightarrow 2 < \alpha < 4$

Hence only one integral value of $\alpha = 3$ exist.

Note : $P(\alpha + 1, \alpha - 3)$ lies on line $x - y = 4$

43. C

Sol. Properties of modulus.

44. A

Sol. (P) x b b b x c x c x c x

Number of ways $= \frac{4!}{3!} \times {}^5C_4 = 20$

(Q) 2b, 1b; 2c, 1c or 2b, 1b ; 1c, 1c, 1c or 2b, 1b ; 3c
x bb x c x c x b x c x, bbcbcc ; bcbbcc, bccbbc (same way starting with c)
cbbcbc, cbcbbc
bbcccb, bcccbb
number of ways $= 12 \times {}^6C_4 = 180$

(R) bcbcbc $= {}^7C_4$

bccbcb or bcbccb $= 2 \times {}^6C_3$

bbccbc or bccbbc or bbcbcc or bcbbcc $= 4 \times {}^5C_2$

bcccbb or bbcccb $= 2 \times {}^4C_1$

bbbccc $= {}^4C_4$

Total ways $= 2(35 + 40 + 40 + 8 + 1) = 248$

(S) bcbcbc, cbcbcb
bccbcb, cbbcbc
bcbccb, cbcbbc
number of ways $= {}^{7}C_1 \times 2 + 4 = 18$

45. D

Sol. (P) $^{n}C_3\left(\frac{x}{a}\right)^{n-3}\left(\frac{1}{x}\right)^{3} = \frac{5}{2}$ $\Rightarrow$ $n-6=0$

$\Rightarrow$ $a^3 = 8$ $\Rightarrow$ $a = 2$

(Q) $$\sum_{p=1}^{4}\sum_{r=p}^{4}\frac{4!}{r!(4-r)!(r-p)!p!} = \sum_{p=1}^{4}{}^{4}C_p\sum_{r=p}^{4}\frac{(4-p)!}{(r-p)!(4-r)!}$$

$$= \sum_{p=1}^{4}{}^{4}C_p\,2^{4-p} = 3^4 - 2^4 = 65$$

(R) Coefficient of x^{13} in $(1-x)(1-x^4)^4 = -\left(-{}^{4}C_3\right) = 4$

(S) $\sum_{r=0}^{4}{}^{4}C_r\,(r-2)^2 = 16$

SECTION – B

46. 2

Sol. $$S_n = \sum_{r=1}^{n}\tan^{-1}\left(\frac{2(2r-1)}{4+r^2(r^2-2r+1)}\right)$$

$$= \sum_{r=1}^{n}\tan^{-1}\left(\frac{2(2r-1)}{4+r^2(r-1)^2}\right)$$

$$= \sum_{r=1}^{n}\tan^{-1}\left(\frac{\frac{2r-1}{2}}{1+\frac{r^2}{2}\,\frac{(r-1)^2}{2}}\right)$$

$$= \sum_{r=1}^{n}\tan^{-1}\left(\frac{\frac{r^2}{2}-\frac{(r-1)^2}{2}}{1+\frac{r^2}{2}.\frac{(r-1)^2}{2}}\right)$$

$$= \sum_{r=1}^{n}\left(\tan^{-1}\frac{r^2}{2}-\tan^{-1}\frac{(r-1)^2}{2}\right)$$

$$S_n = \tan^{-1}\frac{n^2}{2} \Rightarrow \cot(S_n) = \frac{2}{n^2}$$

$$S_{n-1} = \tan^{-1}\left(\frac{(n-1)^2}{2}\right) \Rightarrow \cot(S_{n-1}) = \frac{2}{(n-1)^2}$$

$$\cot(S_{n-1})-\cot(S_n)=\frac{2}{(n-1)^2}-\frac{2}{n^2}$$

$$\lim_{n\to\infty}\sum_{n=2}^{n}\left(\cot(S_n)-\cot S_n\right)=\lim_{n\to\infty}\sum_{n=2}^{n}\left(\frac{2}{(n-1)^2}-\frac{2}{n^2}\right)$$

$$=\lim_{n\to\infty}\left[\left(\frac{2}{1^2}-\frac{2}{2^2}\right)+\left(\frac{2}{2^2}-\frac{2}{3^2}\right)+\left(\frac{2}{3^2}-\frac{2}{4^2}\right)+\left(\frac{2}{(n-1)^2}-\frac{2}{n^2}\right)\right]$$

$$=\lim_{n\to\infty}\left(2-\frac{2}{n^2}\right)=2$$

47. 45

Sol. We have, $f(x)=1-x^3$ and $g_5(x)=\dfrac{5}{\frac{1}{f(x)}+\frac{1}{f(2x)}+\ldots\ldots+\frac{1}{f(5x)}}$

$$1-\frac{5}{\frac{1}{f(x)}+\ldots.+\frac{1}{f(5x)}}$$

Now, $\displaystyle\lim_{x\to 0}\frac{1-g_5(x)}{x^3}=\lim_{x\to 0}\frac{1-\dfrac{5}{\frac{1}{f(x)}+\ldots..+\frac{1}{f(5x)}}}{x^3}$

$$=\lim_{x\to 0}\frac{\left(\frac{1}{f(x)}-1\right)+\left(\frac{1}{f(x)}-1\right)+\ldots..+\left(\frac{1}{f(5x)}-1\right)}{x^3\left(\frac{1}{f(x)}+\frac{1}{f(2x)}+\frac{1}{f(3x)}+\frac{1}{f(4x)}+\frac{1}{f(5x)}\right)}$$

$$=\lim_{x\to 0}\frac{\frac{(1-f(x))}{f(x)}+\frac{(1-f(2x))}{f(2x)}+\ldots.+\frac{(1-f(5x))}{f(5x)}}{5x^3}$$

$$=\frac{1^3+2^3+3^3+4^3+5^3}{5}$$

$$=\frac{\left(\frac{5(5+1)}{2}\right)^2}{5}=45$$

48. 118

Sol. $\dfrac{d}{dx}\left(f^3(x)-g^3(x)\right)=3f^2(x)f'(x)-3g^2(x)g'(x)=0$

So, $f^3(10)-g^3(10)=f^3(2)-g^3(2)$

$\Rightarrow f^3(2)=(5)^3-(2)^3+(1)^3=125-8+1=126-8=118$

49. 3

Sol. The given equation can be written as $\frac{x^2}{\alpha^2}+\frac{y^2}{4}=1$

Any point on the curve is $P(\alpha\cos\theta, 2\sin\theta)$

$\therefore\ PQ=\sqrt{\alpha^2\cos^2\theta+4(1+\sin\theta)^2}$

Let $S(\theta)=\alpha^2\cos^2\theta+4(1+\sin\theta)^2=(4+\alpha^2)+8\sin\theta+(4-\alpha^2)\sin^2\theta$

Now, $\frac{dS}{d\theta}=2\cos\theta\left[4+(4-\alpha^2)\sin\theta\right]$

$=\frac{ds}{d\theta}=0$ gives $\theta=\frac{\pi}{2},\frac{3\pi}{2}$ or $\sin\theta=\frac{4}{\alpha^2-4}$.

But $0<\alpha^2<8$ and $|\sin\theta|\le 1$, so $\sin\theta=\frac{4}{\alpha^2-4}$ is not possible.

Also, at $\theta=\frac{3\pi}{2}$, P becomes (0, –2) which is same as point Q. So PQ = 0.

Also, $\left.\frac{d^2S}{d\theta^2}\right]_{\theta=\frac{\pi}{2}}=-8\sin\theta+2(4-\alpha^2)\cos 2\theta$

$=-8+2(4-a^2)(-1)$

$=-2(8-\alpha^2)<0\,(\text{As } 0<\alpha^2<8)$

Hence $\theta=\frac{\pi}{2}$ corresponds to the maximum values of S.

So, P is (0, 2)

Clearly, reflection of P (0, 2) in the x – axis is R (0, –2).

So, least distance of R (0, –2) from the line $3x-4y+7=0$, is $\frac{|3(0)-4(-2)+7|}{\sqrt{(3)^2+(-4)^2}}=\frac{15}{5}=3$

50. 5

Sol. $P=\lim\limits_{n\to\infty}\prod_{r=1}^{n}\left(\sin\left(\frac{r\pi}{4n}\right)\cos\left(\frac{r\pi}{4n}\right)\right)^{\frac{1}{n}}$

$\ln P=\lim\limits_{n\to\infty}\sum_{r=1}^{n}\ln\left(\frac{\sin r\pi}{n}.\frac{\cos r\pi}{n}\right)$

$\ln P=\int_0^1\ln\left(\sin\frac{\pi x}{4}.\cos\frac{\pi x}{4}\right)dx$

$\ln P=\int_0^1\ln\left(\sin\frac{\pi x}{2}\right)dx-\ln 2$

Put $\frac{\pi x}{2}=t$

$\therefore\ \ln P=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\ell n(\sin t)\,dt-\ell n 2$

$= -2\ell n 2 = \ell n \frac{1}{4}$

$\therefore P = \frac{1}{4} \equiv \frac{a}{b} \Rightarrow a + b = 5$

51. 6

Sol. We have, $\left|x^2 - \left(7 + k^2\right)x + 7k^2\right| + \sqrt{(x-3)(x-3k+2)} = 0$

$\Rightarrow \left|(x-7)\left(x-k^2\right)\right| + \sqrt{(x-3)(x-(3x-2))} = 0$

The above equation has a solution if equations $(x-7)\left(x-k^2\right) = 0$ and $(x-3)(x-(3k-2)) = 0$ has at least one common root.

$\therefore k^2 = 3 \Rightarrow k = \pm\sqrt{3}$

or $k^2 = 3k - 2 \Rightarrow k^2 - 3k + 2 = 0$

$\Rightarrow (k-1)(k-2) = 0 \Rightarrow k = 1$ or 2

$3k - 2 = 7 \Rightarrow k = 3$

$\therefore$ Sum of all values of k is $= \sqrt{3} - \sqrt{3} + 1 + 2 + 3 = 6$