FIITJEE RBT-5 for (JEE-Advanced)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 4 | **QP CODE: 100960** | **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 186**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Part.

(i) **PART–A (01–08)** contains (8) Multiple Choice Questions which have **One or More Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(ii) **Part-A (09-12) –** This section contains Two (02) List-Match Sets, each List-Match set has Two (02) Multiple Choice Questions. Each List-Match set has two lists: List-I and List-II. FOUR options are given in each Multiple Choice Question based On List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question. Each question carries **+3 Marks** for correct combination chosen and **-1 marks** for wrong options chosen.

(iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

BATCH – Two Year CRP325 batches

SECTION – I : PHYSICS

(PART – A)

(One or More Than One Options Correct Type)

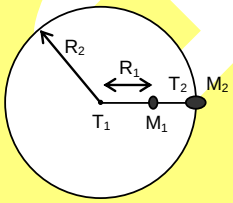
This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. At a given instant, an observer stationary on the ground sees a package falling with speed v_1 at some angle with the vertical. A pilot flying at a constant horizontal speed relative to the ground sees the package falling vertically with a speed v_2 at the same instant. What is the speed of the pilot relative to the ground?

(A) $v_1 + v_2$ (B) $\sqrt{|v_2^2 - v_1^2|}$

(C) $\sqrt{v_1^2 - v_2^2}$ (D) $\sqrt{v_1^2 + v_2^2}$

2. Two different masses are connected to two strings as shown in the figure. Both masses rotate about a central fixed point with angular velocity 10 rad/s on a smooth horizontal plane. The ratio of tension T_1/T_2 in the string is



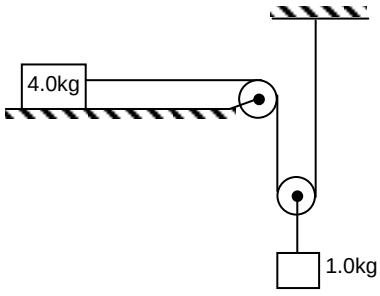
$M_1 = 0.25$ kg
$M_2 = 1.0$ kg
$R_1 = 5$ cm
$R_2 = 10$ cm

(A) The ratio of tension T_1/T_1 in the string is 11/9
(B) The ratio of tension T_1/T_2 in the string is 9/8
(C) The ratio of tension T_2/T_1 in the string is 8/9
(D) The ratio of tension T_2/T_2 in the string is 1

3. At time t = 0, car moving along a straight line has a velocity of 16 m/s. It slows down with an acceleration of $-0.5t\ m/s^2$, where t is in second. Mark the correct statement(s)

(A) The direction of velocity changes at t = 8 sec
(B) The distance travelled in 4 sec is 58.67 m
(C) The distance travelled by the particle in 10 sec is 94 m
(D) The velocity at t_4 = 10 sec is - 9 m/s

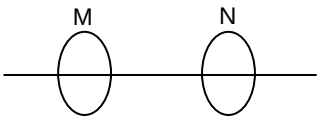
4. Considering the situation shown in the figure. The block of mass 1.0 kg is released from rest and it is found to have a speed of 0.3 m/s^{-1} after it has descended through a distance of 1 m. Which of the following statements are correct?



(A) Loss in gravitational potential energy is 10J
(B) kinetic energy of 1kg block is 0.045 J
(C) 4 kg block travels a distance of 2 m to acquire a velocity of 0.6 ms^{-1}
(D) Work done by friction is –80 μJ where μ is coefficient of kinetic friction in moving the block of mass for 4 kg through 2 m.

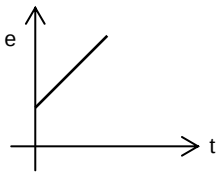
Space For Rough Work

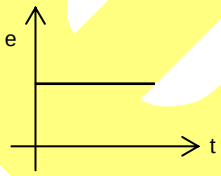
5. Two identical circular coils M and N are arranged coaxially as shown in the figure. Separation between the coils is large as compared to their radii. The arrangement is viewed from left along the common axis. The sign convention adopted is that currents are taken to be positive when they appear to flow in clockwise direction. Then

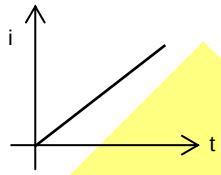


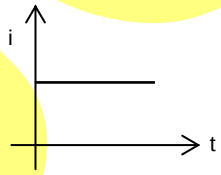
(A) if M carries a constant positive current and is moved towards N, a positive current is induced in N
(B) if M carries a constant positive current and N is moved towards M, a negative current is induced in N
(C) if a positive current in M is switched off, a positive current is momentarily induced in N
(D) if both coils carry positive currents, they will attract each other

6. Two parallel long straight conductors lie on a smooth horizontal surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists in vertical direction. Now all the four conductors start moving outwards with a constant velocity v. The induced e.m.f. e and induced current i will vary with time t as

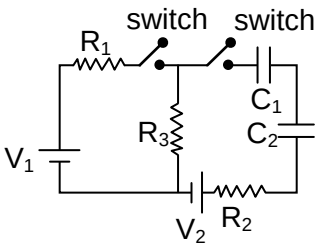
(A)


(B)


(C)


(D)


7. In the given circuit $C_1 = C$, $C_2 = 2C$, $V_1 = V_2 = V$ & $R_1 = R_2 = R_3 = R$ and all switches-are closed at t = 0 then choose the incorrect statement at time t = RC (initially all capacitors are uncharged)



(A) Current through V_1 is less than $\frac{V}{2Re}[e-1]$

(B) Power delivered by V_2 is less than $\frac{V^2}{3eR}$

(C) Current through V_1 is greater than $\frac{V}{2Re}[e-1]$

(D) Power delivered by V_2 is greater than $\frac{V^2}{3eR}$

Space For Rough Work

8. A metal cylinder of mass 0.5 kg is heated electrically by a 12 W heater in a room at 15°C. The cylinder temperature rises to 25°C in 5 min and finally becomes constant at 45°C. Assuming that rate of heat loss is proportional to the excess temperature over the surroundings, which of the following statements is/are correct.
(A) the rate of loss of heat of the cylinder to surrounding at 20°C is 2W.
(B) the rate of loss of heat of the cylinder to surrounding at 45°C is 12W.
(C) specific heat capacity of metal is $\frac{240}{\ell n(3/2)}$ $Jkg^{-1}{}^{\circ}C^{-1}$.
(D) specific heat capacity of metal is $\frac{240}{\ell n2}$ $Jkg^{-1}{}^{\circ}C^{-1}$.

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1.(μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while List-II lists the magnitude of some quantity.

List – I		List – II	
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	$1/2$
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	$3/16$
		(U)	$1/16$

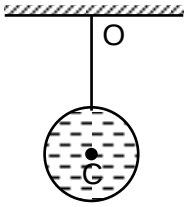
9. If the tension in each string is T_0 the correct match for the highest fundamental frequency in f_0 units will be.
(A) I→P, II→R, III→S, IV→Q
(B) I→Q, II→S, III→R, IV→P
(C) I→P, II→Q, III→T, IV→S
(D) I→Q, II→P, III→R, IV→T

10. The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th and 4th harmonies, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.
(A) I→P, II→Q, III→T, IV→U
(B) I→P, II→Q, III→R, IV→T
(C) I→P, II→R, III→T, IV→U
(D) I→T, II→Q, III→R, IV→U

Space For Rough Work

List Match Set (11-12)

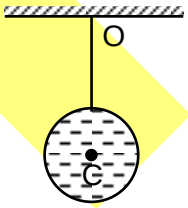
11. A pendulum is constructed as a light thin walled sphere of radius R filled up with water of mass m and suspended at the point O. The distance between centre of the sphere and point O is ℓ. Consider small oscillations of such a pendulum. (Consider water as ideal fluid).



List - I		List - II	
(I)	Time period of oscillation will be	P.	$2\pi\sqrt{\frac{\ell}{g}}$
(II)	Moment of inertia about O will be	Q.	$2\pi\sqrt{\frac{\ell}{g}\left(1+\frac{2}{5}\frac{R^2}{\ell^2}\right)}$
(III)	The water in ground frame will	R.	$m\ell^2$
(IV)	The water with respect to its centre of mass will	S.	$\frac{3}{2}mR^2+m\ell^2$
		T.	Rotate about C
		U.	Will not rotate about C

(A) I – Q; II – S; III – T; IV – T
(B) I – P; II – R; III – U; IV – U
(C) I – P; II – R; III – T; IV – U
(D) I – Q; II – S; III – U; IV – T

12. A pendulum is constructed as a light thin walled sphere of radius R filled up with water of mass m and suspended at the point O. The distance between centre of the sphere and point O is ℓ. Consider small oscillations of such a pendulum with water frozen. Match the following:



List - I		List - II	
(I)	Time period of oscillation will be	P.	$2\pi\sqrt{\frac{\ell}{g}}$
(II)	Moment of inertia about O will be	Q.	$2\pi\sqrt{\frac{\ell}{g}\left(1+\frac{2}{5}\frac{R^2}{\ell^2}\right)}$
(III)	The water in ground frame will	R.	$m\ell^2$
(IV)	The water with respect to its centre of mass will	S.	$\frac{3}{2}mR^2+m\ell^2$
		T.	Rotate about C
		U.	Will not rotate about C

(A) I – Q; II – S; III – T; IV – T
(B) I – P; II – R; III – U; IV – U
(C) I – P; II – R; III – T; IV – U
(D) I – Q; II – S; III – U; IV – T

Space For Rough Work

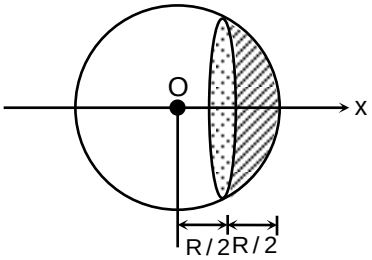
(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

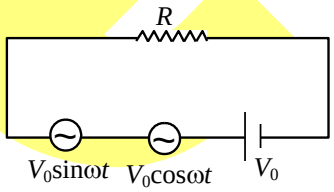
1. The moment of inertia about X-axis of the solid spherical segment of mass M is $\frac{53MR^2}{40X}$.

   Then, the value of ρ is

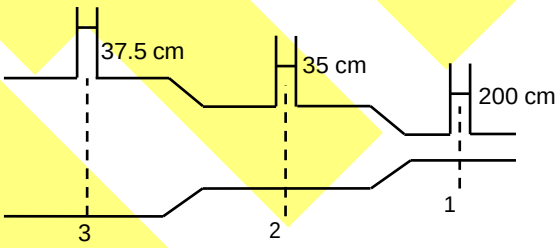


2. At what angle θ_0 (is π/n) should a shell be fired if at the top of its trajectory its path has a radius of curvature equal to twice the maximum height of the trajectory. Then 'n' is

3. Three sources of emf $V_0 \sin \omega t$, $V_0 \cos \omega t$ and V_0 are connected in series as shown. Find the value of rms current in the circuit.
   [where V_0 = 200 V and R = 5Ω]



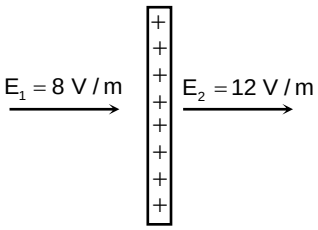
4. Water is flowing in varying cross-section pipe. The areas of cross-sections 1, 2 and 3 are 1 cm^2, 2 cm^2 and A cm^2, respectively. Water levels are shown in different vertical tubes of manometers. The speed of water at cross-section 3 is $\frac{1}{\sqrt{x}}$ m/s. The value of x is



Space For Rough Work

5. If the K_α radiation of Mo (Z = 42) has a wavelength of 0.71 Å, calculate wavelength of the corresponding radiation of Cu. i.e. K_α for Cu (Z = 29). Find the value of nearest integer value (in Å) of wavelength.

6. The electric field on two sides of a charged plate is shown in the figure. If the charge density on the plate is $n\varepsilon_0$, then 'n' is



Space For Rough Work

SECTION – II : CHEMISTRY

(PART – A)

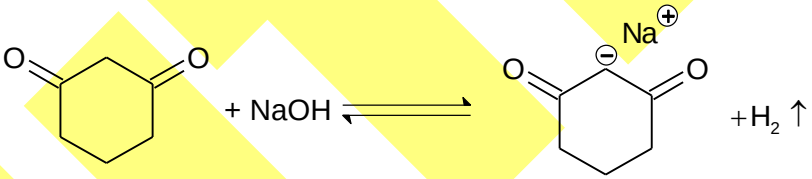
(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Which of the following statement(s) is/are incorrect?
   (A) For every cyclic process, the final state of surrounding is same as the initial state of surrounding.
   (B) $C_{v,m}$ is independent of temperature for a perfect gas
   (C) A thermodynamic process is specified by specifying initial and final state of system
   (D) A process in which final temperature is same as initial temperature, must be an isothermal process

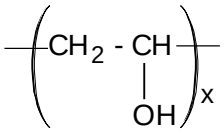
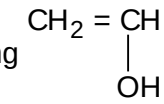
2. Which is/are not the necessary condition(s) for a compound to be optically active?
   (A) Presence of chiral carbon atom (or atleast one asymmetric atom)
   (B) Absence of centre of symmetry
   (C) Non super imposable mirror image
   (D) Absence of plane of symmetry

3. Which of the following has(have) transition metal?
   (A) Haemoglobin
   (B) Vitamin B_{12}
   (C) Cisplatin
   (D) Chlorophyll

4. Which of the following is/are thermodynamically possible?
   (A) $CH_3 - C \equiv C - H + NaOH \rightleftharpoons CH_3 - C \equiv CNa + H_2O$
   (B) $CH_3CH_2OH + NaHCO_3 \rightleftharpoons CH_3CH_2ONa + H_2CO_3$
   (C) $H_2O + NaNH_2 \rightleftharpoons NaOH + NH_3$
   (D) 

5. Which of the following statement(s) is/are correct?
   (A) When lead nitrate is heated at 400°C, only brown gas is released
   (B) $N_2O_5(g) + 2\,NaOH(aq) \longrightarrow 2NaNO_3 + H_2O$
   (C) Antimony on reaction with conc HNO_3 give antimonic acid
   (D) CCl_4 undergo normal hydrolysis to give H_2CO_3 & HCl

Space For Rough Work

6. Identify the correct statement(s).
(A) Novolac is a linear polymer
(B) Glyptal is a polymer of ethylene glycol and phthalic acid
(C) In manufacture of tyre rubber, 10% sulphur is used as a cross-linking agent during vulcanization
(D)  polymer is prepared using  (vinyl alcohol) as a monomer unit & made by addition polymerization

7. The correct statement(s) about interhalogen compound(s) ICl_3 is/are
(A) it exists as dimer
(B) geometry around the iodine is tetrahedral in solid state
(C) it decomposes as ICl and Cl_2 in gas phase
(D) liquid ICl_3 conducts electricity

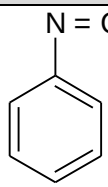
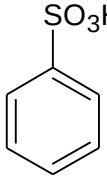
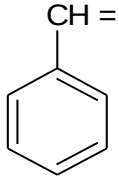
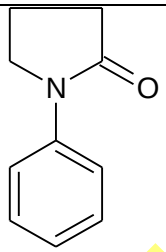
8. An amount of 1 mole of an non-volatile solute solid is dissolved in 200 moles of water. The solution is cooled to a temperature 'T' K (Lower than the freezing point of solution) to cause ice formation. After removal of ice, the remaining solution is heated at 373 K, where vapour pressure of solution is observed to be 740 mm Hg. Identify correct statement(s) (K_f of water = 2 K Kg mol^{-1})
(A) 2.934 Kg of ice is formed
(B) TK = 270 K
(C) freezing point of original solution is $\frac{-10^{o}}{18}$ C
(D) relative lowering of vapour pressure of final solution is $\frac{1}{38}$

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Match the lists and answer the question.

List– I		List – II	
(I)		(P)	o - p directing
(II)		(Q)	m-directing
(III)		(R)	Activating
(IV)		(S)	deactivating

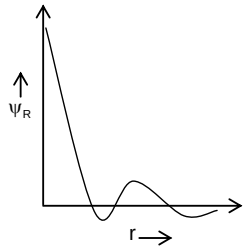
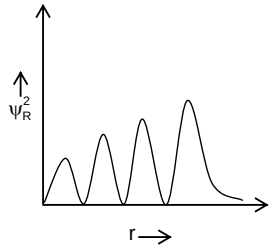
9. Which of the following is correct matched?
(A) I → PR (B) II → QR
(C) III → PS (D) IV → PS

10. Which of the following is correct matched?
(A) I → QS (B) II → PS
(C) III → QS (D) IV → PR

Space For Rough Work

List Match Set (11-12)

Match the lists and answer the question.

List– I		List – II	
(I)		(P)	4s
(II)		(Q)	$5p_z$
(III)	Angular wave function independent of ϕ	(R)	$5d_{xy}$
(IV)	Nodal plane present	(S)	$6d_{z^2}$

11. Identify correct option among them?
(A) I $\rightarrow$ Q
(B) II $\rightarrow$ R
(C) III $\rightarrow$ Q
(D) IV $\rightarrow$ S

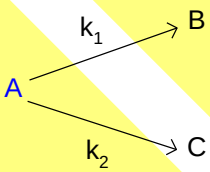
12. Identify correct option among them?
(A) I $\rightarrow$ S
(B) II $\rightarrow$ Q
(C) III $\rightarrow$ R
(D) IV $\rightarrow$ P

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. Consider the reaction



The rate constant for two parallel reactions were found to be 10^{-2} dm^3 $mol^{-1}s^{-1}$ and 4×10^{-2} dm^3 $mol^{-1}s^{-1}$. If corresponding energies of activation of parallel reaction are 100 & 120 kJ/mol. What is the net energy of activation(E_a) of disappearance of A(in kJ/mol)

Space For Rough Work

2. A mixture of NaCl & NaBr weighing 4 g was dissolved & treated with excess of $AgNO_3$ to precipitate all of chlorides & bromides as AgCl & AgBr. The washed precipitate was treated with KCN to dissolve the precipitate. The resulting solution was electrolysed, the weight of Ag deposited is 6 g.
(Mol.wt of NaCl = 58.5, NaBr = 103, Ag = 108)
$K\left[Ag(CN)_2\right] + KOH \xrightarrow{\text{Electrolysis}} Ag + KCN + O_2 + H_2O$
If x = mass of NaCl in mixture(in g)
y = mass of O_2 gas produced (in g) during electrolysis. What is the value of (x + y)?

3. At 27°C, the rate of forward reaction at any time t, is e^4 times greater than that of reverse reaction for the reaction $A + B \rightleftharpoons P$. The standard free energy change involved at that time(in cal) is _____?
(R = 2 Cal K^{-1} mol^{-1})

4. On passing electricity through nitrobenzene solution, it converted into azobenzene. The mass of azobenzene (in g) produced, if same quantity of electricity produces oxygen(by electrolysis of water), just sufficient to burn 96 g of fullerene(C_{60}) is ____?

5. Find the number of correct statement(s) among the following.
(i) $LiHCO_3$ does not exist in solid state.
(ii) $NaNO_3$ on heating gives $Na_2O + NO_2 + O_2$
(iii) Lithium is strongest reducing agent among alkali metal in aqueous solution.
(iv) A positive catalyst decrease the ΔH of a reaction.
(v) $A(g) \rightleftharpoons B(g)$, on increasing the pressure at constant temperature concentration of [A] & [B] do not change at equilibrium.
(vi) $4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$
(vii) H_3PO_2 reduces $AgNO_3$ solution to metallic silver(Ag).
(viii) Au(Gold) dissolves in concentration HCl due to formation of complex $H[AuCl_4]$.
(ix) XeF_6 undergo redox hydrolysis with water.
(x) "β" or Monoclinic sulphur(thermodynamically less stable at room temperature than rhombic sulphur), exist in Cyclo-S_6 chair form.

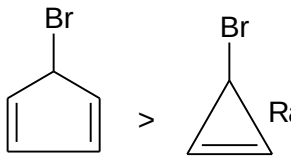
Space For Rough Work

6. Find the number of correct statement(s) among the following.

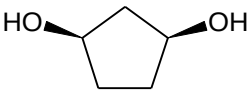
(i) The amount of bromine(in g) required for estimation of 42.3 g of phenol is 108 g(Given: atomic weight of Br = 80).

(ii) Adiabatic free expansion is isoentropic process.

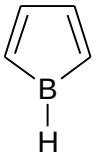
(iii) Maximum number dipeptides that could be obtained by reaction of phenylalanine with leucine is 4.

(iv) 

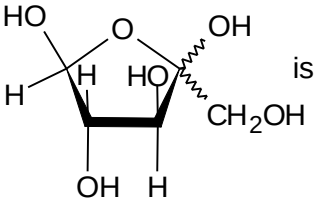
Rate of solvolysis

(v) 

With 1 equivalent of NaH & followed by CH_3I gives a single enantiopure compound

(vi) 

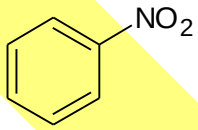
is aromatic in nature

(vii) 

is a reducing sugar

(viii) The complementary strand of 5' - G - A - A - T - T - C - 3' is 5' - C - T - T - A - A - G - 3' in double staranded DNA.

(ix) Compound $NH_2-\overset{\oplus}{N}H_3HSO_4^-$ gives test for both nitrogen and sulphur with sodium fusion extract.

(x) Nitrogen in pyridine & 

can be measured quantitatively using Kjeldahl's method.

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. If a, b, c, d are non – zero real numbers such that c, d are the roots of the equation $x^2+ax+b=0$ and a, b are the roots of the equation $x^2+cx+d=0$, then which of the following relations is(are) correct

(A) $a^2+c^2=2$ (B) $b^2+d^2=4$
(C) $a-b+c-d=6$ (D) $bd-ac=1$

2. Consider a sequence $\{a_n\}$ with $a_1=2$ and $a_n=\frac{a_{n-1}^2}{a_{n-2}}$ for all $n\geq 3$, terms of the sequence being distinct. Given that a_2 and a_5 are positive integers and $a_5\leq 162$, then the possible value(s) of a_5 can be:

(A) 2 (B) 32
(C) 64 (D) 162

3. In an acute triangle ABC, if the coordinates of orthocentre 'H' are (4, b) centroid 'G' are (b, 2b – 8) are circumcentre 'S' are (–4, 8), then 'b' cannot be:

(A) 4 (B) 8
(C) 12 (D) –12

4. Consider the circles:

$C_1: x^2+y^2-4x+6y+8=0$

$C_2: x^2+y^2-10x-6y+14=0$

Which of the following statement(s) hold good in respect of C_1 and C_2?

(A) C_1 and C_2 are orthogonal

(B) C_1 and C_2 touch each other

(C) Radical axis between C_1 and C_2 is also one of their common tangent

(D) Middle point of the line joining the centres of C_1 and C_2 lies on their radical axis.

Space For Rough Work

5. If C_r's denote the combinatorial coefficients in the expansion of $(1+x)^{50}$ and $N = C_0^2 + C_1^2 + C_2^2 + ... + C_{50}^2$ then the correct statement (s) is (are):

(A) N is equal to the coefficient of the middle term in the expansion of $(1+x)^{100}$.

(B) Number of zero at the end of N is 1.

(C) If $N = 2^m \cdot I$ where I is an odd integer then m = 3.

(D) N is also equal to ${}^{60}C_{10} \cdot {}^{40}C_{40} + {}^{60}C_{11} \cdot {}^{40}C_{39} + {}^{60}C_{12} \cdot {}^{40}C_{38} + ... + {}^{60}C_{50} \cdot {}^{40}C_0$.

6. Let f, g and h be three functions defined as follows: $f(x) = \frac{32}{4 + x^2 + x^4}$, $g(x) = 9 + x^2$ and $h(x) = -x^2 - 3x + k$. Identify which of the following statement(s) is (are) correct?

(A) Number of integers in the range of $f(x)$ is 8.

(B) Number of integral values of k for which $h(f(x)) > 0$ and $h(g(x)) < 0 \; \forall \; x \in R$ is 20.

(C) Number of integral values of k for which $h(f(x)) > 0$ and $h(g(x)) < 0 \; \forall \; x \in R$ is 19.

(D) Maximum value of $g(f(x))$ is 73.

7. If $f(x) = \tan^{-1}\left(\frac{\sqrt{3}x - 3x}{3\sqrt{3} + x^2}\right) + \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, $0 \le x \le 3$, then:

(A) the least value $f(x)$ is $-\frac{\pi}{3}$

(B) the greatest value of $f(x)$ is $\frac{\pi}{4}$

(C) the least value of $f(x)$ is 0

(D) the greatest value of $f(x)$ is $\frac{\pi}{3}$

8. If variable line $x(3+\lambda) + 2y(2-\lambda) - (7-\lambda) = 0$ always passes through a fixed point (a, b) where λ is parameter and $\ell = \lim\limits_{x \to (a-b)^-} \frac{[\sin x - 2] + \{\cos x\}}{x - [x] - 1}$ where[y] and {y} denotes greatest integer $\le y$ and fractional part of y respectively, then:

(A) $a + 2b = 3$

(B) $a - b + 2l = 2$

(C) $l = 1$

(D) l does not exist

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Column-I		Column-II	
(I)	If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$ where $g(x)=\int_0^{\cos x}(1+\sin t^2)dt$ then the value of $f'\left(\frac{\pi}{2}\right)$	(P)	3
(II)	If $f(x)$ is a non – zero differentiable function such that $\int_0^x f(t)dt=(f(x))^2$ for all x, then $f(2)$ equals	(Q)	2
(III)	If $\int_a^b(2+x-x^2)dx$ is maximum then $(a+b)$ is equal to	(R)	1
(IV)	If $\lim_{x\to0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$ then $(3a+b)$ has the value equal to	(S)	–1

9. Which is correct option?
(A) I $\rightarrow$ R
(B) II $\rightarrow$ R
(C) III $\rightarrow$ S
(D) IV $\rightarrow$ P

10. Which is correct option?
(A) I $\rightarrow$ P
(B) II $\rightarrow$ Q
(C) III $\rightarrow$ R
(D) IV $\rightarrow$ S

Space For Rough Work

List Match Set (11-12)

Match the List-I with the List-II for question no. 11 and 12.

Column-I		Column-II	
(I)	Four different movies are running in a town. Ten students go to watch these four movies. The number of ways in which every movie is watched by at least one student, is: (Assume each way differs only by number of students watching a movie)	(P)	11
(II)	Consider 8 vertices of a regular octagon and its centre. If T denotes the number of triangles and S denotes the number of straight lines that can be formed with these 9 points then the value of $(T-S)$ equals	(Q)	36
(III)	In an examination, 5 children were found to have their mobiles in their pocket. The Invigilator fired them and took their mobiles in his possession. Towards the end of the test. Invigilator randomly returned their mobiles. The number of ways in which at most two children did no get their own mobiles is	(R)	52
(IV)	The product of the digit of 3214 is 24. The number of 4 digit natural number such that the product of their digits is 12, is	(S)	60
(V)	The number of ways in which a mixed double tennis game can be arranged from amongst 5 married couple if no husband and wife plays in the same game, is	(T)	84

11. Which is correct option?
(A) I $\rightarrow$ P
(B) II $\rightarrow$ R
(C) III $\rightarrow$ S
(D) IV $\rightarrow$ T

12. Which is correct option?
(A) I $\rightarrow$ R
(B) III $\rightarrow$ Q
(C) IV $\rightarrow$ P
(D) V $\rightarrow$ S

Space For Rough Work

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. Let P_1 be a plane passing through two points A (1, 2, 0) and B (2, –1, 1) and perpendicular to $\vec{r}.(i+j-3k)=1$. Let L be a line through (1, 1, 1) and perpendicular to line of intersection of planes P_1 and P_2 where equation of plane P_2 is $\vec{r}.(i+j+k)=5$. If the equation of line L is $\frac{x+1}{-1}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$, then find the value of $\left(\frac{2y_1+z_1}{bc}\right)$.

2. PQ, a variable chord of the parabola $y^2=4x$ subtends a right angle at the vertex. The tangents at P and Q meet at T and the normals at those points meet at N. If the locus of the mid point of TN is a parabola, then find its latus rectum

3. Let P_i and P'_i be the feet of the perpendiculars drawn from foci S, S' on a tangent T_i to an ellipse whose length of semi – major axis is 20. If $\sum_{i=1}^{10}(SP_i)(S'P'_i)$, then find the value of $100e$. (Where e denotes eccentricity).

4. A persons flips 4 fair coins and discard those which turn up tails. He again flips the remaining coin and then discards those which turn up tails. If the probability that he discards atleast 3 coins $=\frac{m}{n}$ where m and n are coprime then find $(m+n)$?

5. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ are such that $\min(f(x))<\max(g(x))$ then minimum integral value of $\frac{b^2}{c^2}(c\neq 0)$ is:

6. Let $S=\frac{1}{\sin 8^o}+\frac{1}{\sin 16^o}+\frac{1}{\sin 32^o}+...+\frac{1}{\sin 4096^o}+\frac{1}{\sin 8192^o}$. If $S=\frac{1}{\sin\alpha}$ where $\alpha\in(0, 90^o)$, then find α (in degree).

Space For Rough Work

Q. P Code: 100960

Answers

SECTION – I : PHYSICS

(PART – A)

1.	BC	2.	BCD	3.	ABCD	4.	ABCD
5.	BCD	6.	AD	7.	ABD	8.	ABC
9.	A	10.	A	11.	B	12.	A

(PART – B)

1.	5.00	2.	4	3.	56.56 (range: 56.20 to 57.00)
4.	2.00	5.	2.00	6.	4.00

SECTION – II : CHEMISTRY

(PART – A)

1.	ABCD	2.	ABD	3.	ABC	4.	CD
5.	BC	6.	AB	7.	ACD	8.	ABCD
9.	C	10.	D	11.	C	12.	B

(PART – B)

1.	116 (Range 115 to 117)	2.	2.71(2.65 to 2.75)
3.	-2400(Range: -2410 to -2390)	4.	728 (Range 725 to 730)
5.	4	6.	2

SECTION – III : MATHEMATICS

(PART – A)

1.	AC	2.	BD	3.	ABCD	4.	BC
5.	ACD	6.	ACD	7.	BC	8.	AD
9.	B	10.	C	11.	B	12.	D

(PART – B)

1.	9	2.	25	3.	60	4.	445
5.	1	6.	4				

Answers & Solutions
SECTION – I : PHYSICS

(PART – A)

1. **BC**

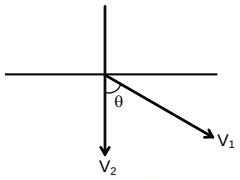
Sol. $\vec{V}_{pk} = V_1 \sin\theta \hat{i} - V_1 \cos\theta \hat{j}$

$\vec{V}_{pk,p\ell} = -V_2 \hat{j}$

$\vec{V}_{p\ell} = \vec{V}_{pk} - \vec{V}_{pk,p\ell} = V_1 \sin\theta \hat{i} + (V_2 - V_1 \cos\theta)\hat{j}$

But $V_2 - V_1 \cos\theta = 0 \Rightarrow \cos\theta = \frac{V_2}{V_1}$

$\vec{V}_{p\ell} = V_1 \sin\theta \hat{i}, \quad V_{p\ell} = \sqrt{V_1^2 - V_2^2}$

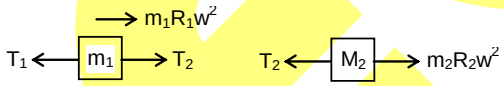


2. **BCD**

Sol. $T_1 = T_2 + m_1R_1w^2 \quad \ldots(1)$

$T_2 = m_2R_2w^2 \quad \ldots(2)$

$\frac{T_1}{T_2} = \frac{(m_1R_1 + m_2R_2)w^2}{m_2R_2w^2} = \frac{9}{8}$



3. **ABCD**

Sol. At the time when velocity changes direction, v = 0.

4. **ABCD**

Sol. $\Delta U = mgh = 10$

$E = \frac{1}{2}mv^2 = 0.045$

Work done by friction = $-\mu \times 4 \times 10 \times 2$

5. **BCD**

Sol. Apply Faraday law for direction of induced current.

6. **AD**

Sol. $\frac{d(emf)}{dt} = Bu\frac{d\ell}{dt} = \text{constant}$

$i = \frac{Bu}{\lambda} = \text{constant}$

7. **ABD**

Sol. $q = \frac{CV}{3}\left(1 - e^{-t/RC}\right)$

Current through $V_2 \qquad i_2 = \frac{V}{3R}\left(e^{-t/RC}\right)$

Current through $V_1 \qquad i_1 = \frac{V}{2R} - \frac{V}{6R}e^{-1/RC}$

at t = RC, power delivered by $V_2 = Vi_2 = \frac{V^2}{3Re}$

8. **ABC**

Sol. Rate of loss of heat = K (T – T_0)

$12 = K \times 30 \Rightarrow k = \frac{2}{5}$

$$ms\frac{dT}{dt} = 12 - \frac{2}{5}(T - 15°)$$

$$\Rightarrow \int_{15}^{25} \frac{dT}{90 - 2T} = \frac{1}{(2.5)s}\int_{0}^{300} dt$$

$$\Rightarrow s = \frac{240}{\ln\left(\frac{3}{2}\right)}$$

9. **A**

Sol. For fundamental mode

$$\frac{\lambda}{2} = L \quad \therefore \ \lambda = 2L$$

$$f = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (i)

$$f_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (ii)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_o}{\sqrt{2}}$$

Similarly for other strings.

10. **A**

Sol. Length of string = L_o

It is vibrating in first harmonic.

$$f_o = \frac{1}{2L_o}\sqrt{\frac{T_o}{\mu}}$$

Length = $3L_o/2$ (3^{rd} harmonic)

$$f_0 = \frac{3V}{2L}$$

$$f_o = \frac{3}{2\left(\frac{3L_o}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$$

$$T_2 = \frac{T_o}{2}$$

For string 3

Length = $\frac{5L_o}{4}$

$$f_o = \frac{5V}{2L}$$

Similarly for string 4.

Length $\frac{7L_o}{4}$

So, $T_4 = \frac{T_o}{16}$

11. **B**

Sol. Assume a physical pendulum where the sphere mid of water does not rotate. So, $I_{CM} = 0$.

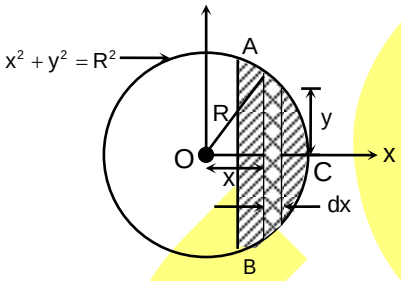
12. **A**

Sol. Assume a physical pendulum where the sphere mid of water does not rotate. So, $I_{CM} = \frac{2MR^2}{5}$.

(PART – B)

1. **5.00**

Sol. For the moment of inertia of the about disc of mass dm at distance x about the given axis, we will use



$$d\ell = \frac{1}{2}dmy^2, dm = \rho dV$$

Where, y will be radius

Density of spherical segment is

$$\rho = \frac{M}{V} = \frac{M}{\int_{\frac{R}{2}}^{R}\pi y^2 dx} = \frac{M}{\int_{R/2}^{R}\pi(R^2 - x^2)dx}$$

$$\Rightarrow \quad \rho = \frac{M}{\frac{5\pi R^3}{24}} = \frac{24M}{5\pi R^3}$$

$$d\ell = \frac{1}{2}dm \cdot y^2 d\ell$$

$$d\ell = \frac{1}{2}\rho\pi y^2 dx \times y^2 = \frac{\rho\pi}{2}y^2 dx$$

Total moment of inertia of segment of sphere is

$$\ell = \int d\ell = \frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R} y^2 dx$$

$$\ell = \frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R}(R^2 - x^2)^2 dx$$

$$= \frac{\rho\pi}{2}\int_{\frac{R}{2}}^{R}(R^4 - 2R^2x^2 + x^4)dx$$

$$\ell = \frac{24M}{5\pi R^3} \times \frac{\pi}{2} \times \left[R^4 x - \frac{2R^2x^3}{3} + \frac{x^5}{5}\right]_{R/2}^{R}$$

$$= \frac{24M}{5\pi R^3} \times \frac{\pi}{2} \times \frac{53R^5}{480} \quad \ell = \frac{53}{200}MR^2$$

$$\ell = \frac{53}{200}MR^2 = \frac{53mR^2}{40x}$$

$$\Rightarrow \quad x = 5$$

2. **4**

Sol. At maximum height $R = \frac{u^2\cos^2\theta_0}{g}$

Maximum height of projectile $H = \frac{u^2\sin^2\theta_0}{2g}$

Given: $R = 2H$

$$\frac{u^2\cos^2\theta}{g} = 2\left(\frac{u^2\sin^2\theta}{2g}\right) \quad \Rightarrow \quad \tan\theta = 1 \quad \Rightarrow \quad \theta = 45°.$$

3. **56.56**

(Range: 56.20 to 57.00)

Sol. $(V_{net})_{rms} = \sqrt{V_{dc}^2 + \frac{V_{rms}^2}{2}}$

$= \sqrt{V_0^2 + \frac{(V_0\sqrt{2})^2}{2}}$

$= V_0\sqrt{2}$

4. **2.00**

Sol. Apply Bernoulli's equations at pipe (1), (2) and (3)

$$p_3 + \frac{1}{2}\rho v_3^2 = p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \rho/2 v_1^2$$

$$p_0 + \rho g h_3 + \frac{1}{2}\rho v_3^2 = p_0 + \rho g h_2 + \frac{\rho}{2}v_2^2 = p_0 + \rho g h_1 + \frac{\rho}{2}v_1^2$$

$$gh_3 + \frac{v_3^2}{2} = gh_2 + \frac{v_2^2}{2} = gh_1 + \frac{v_1^2}{2} \quad ...(i)$$

$$v_3 A_3 = v_2 A_2$$

$$v_3 \times A = v_2 \times 2 \quad ....(ii)$$

$$v_2 \times 2 = v_1 \times 1 \quad ....(iii)$$

From Equation (i), we get

$$gh_2 + \frac{v_2^2}{2} = gh_1 + \frac{v_1^2}{2}$$

$$10 \times \frac{35}{100} + \frac{v_2^2}{2} = 10 \times \frac{20}{100} + \frac{v_1^2}{2}$$

$$\frac{15}{10} = \frac{v_1^2 - v_2^2}{2}$$

$$= \frac{4v_2^2 - v_2^2}{2}$$

$$= \frac{3v_2^2}{2}$$

$v_2 = 1$ m/s, $v_1 = 2$ m/s

From equation (i), we get

$$gh_3 + \frac{v_3^2}{2} = gh_2 + \frac{v_2^2}{2}$$

$$= 10 \times \frac{37.5}{100} + \frac{v_3^2}{2}$$

$$= 10 \times \frac{35}{100} + \frac{1}{2}$$

$$\frac{v_3^2}{2} = \frac{1}{2} + \frac{35}{10} - \frac{37.5}{10} = \frac{1}{2} - \frac{2.5}{10} = \frac{1}{2} - \frac{25}{100}$$

$$\frac{v_3^2}{2} = \frac{1}{4} \Rightarrow v_3^2 = \frac{1}{2}$$

$\Rightarrow$ $v_3 = \frac{1}{\sqrt{2}}, X = 2$

5. **2.00**

Sol. $\sqrt{v} = a(Z - b)$

$\sqrt{v} = a(Z - 1)$

$\Rightarrow$ $(Z-1)^2 \propto v$

or $(Z-1)^2 \propto \frac{1}{\lambda}$

$$\Rightarrow \quad \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2}=\frac{\lambda_{Cu}}{\lambda_{Mo}}$$

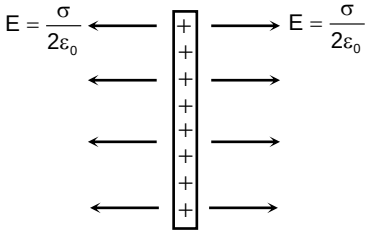
$$\Rightarrow \quad \lambda_{Cu}=\lambda_{Mo}\frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2}$$

$$=0.71\times\left(\frac{41}{28}\right)^2=1.52\text{Å}$$

λ = 2 Å

6. **4.00**

Sol. The electric field produced by thin charged sheet is given by $\frac{\sigma}{2\varepsilon_0}$.



[Where, σ = surface charge density]

$E_0+E=12$ V/m ...(i)

$E_0-E=8$ V/m ....(ii)

Solving Equations (i) and (ii), we get

$E=2$ V/m

Now, electric field due to plate $=\frac{\sigma}{2E_0}=2$

$s=4e_0$

$\therefore$ n = 4

SECTION – II : CHEMISTRY

(PART – A)

1. ABCD

Sol. (A) This is incorrect for irreversible process.
(B) $C_{v,m}$ depends on temperature, but it is assumed to be constant in very small range of temperature.
(C) It is correct for irreversible process, but incorrect for reversible process.
(D) In isothermal process, the temperature remains constant throughout the process. ∴ All four options are incorrect.

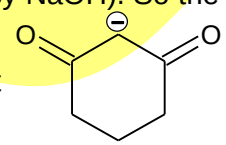
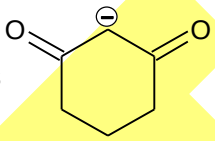
2. ABD

Sol. Non super imposable mirror image is the only necessary condition.

3. ABC

Sol. (A) contains Fe, (B) contains Co, (C) contains Pt, (D) contains Mg

4. CD

Sol. Thermodynamically possible reactions are those which produce stable products which are produced at high temperature. $H_2O + NaNH_2 \rightleftharpoons NaOH + NH_3$
Here NH_3 is a stable product which can't react with OH^-(produced by NaOH). So the forward reaction is favourable. In the reaction give in option(D), the product  and H_2 are stable species  gets stability due to delocalization of negative charge by resonance. H_2 is stable as it escapes as a gas. Both of them favour forward reaction.

5. BC

Sol. Heating of $Pb(NO_3)_2$ produces NO_2 and O_2 gases. CCl_4 does not hydrolysis at normal condition due to strong C – Cl bond.

6. AB

Sol. 5% sulphur is used in vulcanization. $CH_2 = CH - OH$ will tautomerize to CH_3CHO.

7. ACD

Sol. I_2Cl_6 exists as ICl_2^+ and ICl_4^- in solid state ICl_2^+ has angular shape and ICl_4^- has disorted square planar shape.

8. ABCD

Sol. $$\frac{P^o - P}{P} = \frac{n_1}{n_2} = \frac{760 - 740}{740} = \frac{1}{n_2} \Rightarrow n_2 = 37$$

$\therefore$ Moles of ice formed = 200 - 37 = 163

$\therefore$ Mass of ice = $\frac{163 \times 18}{1000} = 2.934\,Kg$

$\Delta T_f = K_f \times m = 2 \times \frac{1000}{37 \times 18} = 3$

$\Delta T_f = T_f^o - T$

or, 3 = 273 – T

or, T = 273 – 3 = 270 K

9. C

Sol. Mesomeric effect makes the electrophile attact at o- and p- positions.

10. D

Sol. N acts as electron donating.

11. C

Sol. For p_z orbitals, $\psi_{p_z} = \sqrt{\frac{3}{4\pi}}\cos\theta$

12. B

Sol. For $5p_z$, the number of radial node = (5 - 1 - 1) = 3. Also the ψ_R^2 graph contains three radial nodes.

(PART – B)

1. 116 (Range 115 to 117)

Sol. $E_a = \frac{k_1E_{a_1} + k_2E_{a_2}}{k_1 + k_2} = 116\,\text{kJ mol}^{-1}$

2. 2.71(2.65 to 2.75)

Sol. Let mass of NaCl = xg

$\therefore$ Mass of NaBr = (4 – x)g

Moles of Cl^- + moles of Br^- = Moles of Ag^+

$$\frac{x}{58.5} + \frac{4-x}{103} = \frac{6}{108}$$

On solving x = 2.26 g

$\therefore$ Mass of NaCl = 2.26 g

One mole of electron required to electrolyse one mole Ag

Four mole of electron required to electrolyse one mole O_2

Since 6 g Ag deposited the no. of Faradays passed = $\frac{6}{108}$F

$\therefore$ 4F deposits one mole O_2 $\left(2O^{2-} \longrightarrow O_2 + 4e^-\right)$

1 F $\rightarrow$ ¼ mole O_2

$\frac{6}{108}F \rightarrow \left(\frac{6}{108\times 4}\right)$ mole of O_2

Mass of O_2 = $\frac{6}{108\times 4}\times 32 = 0.44$ g

$\therefore$ x = 2.26, y = 0.44

$\therefore$ x + y = 2.70

3. -2400(Range: -2410 to -2390)

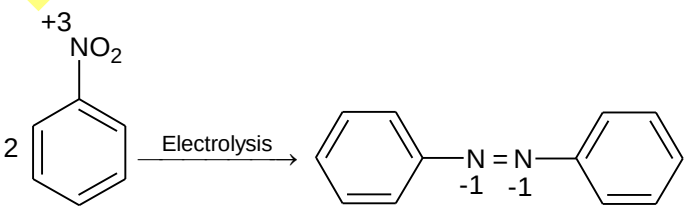
Sol. $K_{eq} = \frac{K_f}{K_b} = \frac{r_f}{r_b} = \frac{e^4 \times r_b}{r_b}$

$\therefore K_{eq} = e^4$

$\Delta G^o = -RT\ell nK_{eq} = -RT\ell ne^4 = -2 \times 300 \times 4\ell ne = -2400$ cal mol^{-1}

4. 728 (Range 725 to 730)

Sol.



n-factor, $(2 \times (+3)) - 2(-1) = 8$
8 F electricity produces one mole azobenzene
Molar mass of azobenzene = 182 g mol^{-1}

$$C_{60} + 60O_2 \longrightarrow 60CO_2$$

Mass of C_{60} = 96g

Mass of $C_{60} = \frac{96}{60 \times 12} = 0.133$

Moles of O_2 required to burn one mole C_{60} = 60
Moles of O_2 required to burn 0.133 mole $C_{60} = 0.133 \times 60 = 7.98$

$$2O^{2-} \longrightarrow O_2 + 4F$$

Electricity required to form one mole O_2 = 4F
Electricity required to form 7.98 mole $O_2 = (4 \times 7.98)$ F = 31.92 F
8F required to form one mole azobenzene

31.92 F required to form $\left(\frac{1}{8} \times 31.92\right) = 3.99$ mole azobenzene

$\therefore$ Mass of azobenzene = $3.99 \times 182 = 726.18$ g

5. 4
Sol. 1, 3, 6 & 7 are correct statements.

6. 2
Sol. 3rd & 7th are correct statements

SECTION – III : MATHEMATICS

(PART – A)

1. AC

Sol. $x^2+cx+d=0$

$x^2+ax+b=0$

$a+b=-c \Rightarrow a+b+c=0$ .....(i)

and $c+d=-a \Rightarrow a+c+d=0$ .....(ii)

From equation (i) and (ii), we get $b=d$

Now $ab=d$ .....(iii)

and $cd=b$ .....(iv)

Since, b = d hence from equation (iii) and (iv) a = c = 1

From equation (i) and (ii), we get $b=d=-2$ (Since a = c = 1)

Now, verify alternatives.

2. BD

Sol. Given $a_1=2; \frac{a_n}{a_{n-1}}=\frac{a_{n-1}}{a_{n-2}}$

$\Rightarrow a_1,a_2,a_3,a_4,a_5,....$ in G.P.

Let $a_2=x,$ then for $n=3,$ we have

$\Rightarrow \frac{a_3}{a_2}=\frac{a_2}{a_1}$

$\Rightarrow a_1^2=a_1a_3$

$\Rightarrow a_3=\frac{x^2}{2}$

i.e., $2, x, \frac{x^2}{2}, \frac{x^3}{4}, \frac{x^4}{8},...$ with common ratio $r=\frac{x}{2}$.

Given $\frac{x^4}{8} \le 162$

$\Rightarrow x^4 \le 1296$

$\Rightarrow x \le 6$

Also x and $\frac{x^4}{8}$ are integers.

$\Rightarrow$ x must be even then only $\frac{x^4}{8}$ will be an integer.

Hence, possible values of $a_5=\frac{x^4}{8}$ is $\frac{4^4}{8}, \frac{6^4}{8}, 32, 162$

$\Rightarrow (b),(d)$

3. ABCD

Sol. As, H, G and C are collinear

$$\therefore \begin{vmatrix} 4 & b & 1 \\ b & 2b-8 & 1 \\ -4 & 8 & 1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 4 & b & 1 \\ b-4 & b-8 & 0 \\ -(b+4) & 16-2b & 0 \end{vmatrix}=0$$

$\Rightarrow (b-4)(16-2b)+(b+4)(b-8)=0$

$\Rightarrow 2(b-4)(8-b)+(b+4)(b-8)=0$

$\Rightarrow (8-b)\{(2b-8)-(b+4)\}=0$

$\Rightarrow (8-9)(b-12)=0$

Hence $b=8$ or 12, which is wrong because collinearity does not explain centroid, orthocentre and circumcentre.

Now, $\frac{-8+4}{3}=b \Rightarrow b=-\frac{4}{3}$

and $\frac{16+b}{3}=2b-8 \Rightarrow 8$

But no common value of 'b' is possible

$\Rightarrow A,B,C,D$

4. BC

Sol. $C_1(2,-3); \quad r_1=\sqrt{4+9-8}=\sqrt{5}$

$C_2(5,3); \quad r_2=\sqrt{25+9-14}=2\sqrt{5}$

$C_1C_2=\sqrt{9+36}=\sqrt{45}=3\sqrt{5}$

$\therefore\ C_1C_2=r_1+r_2$

Hence, circle touch each other externally $\Rightarrow (b)$

Hence, radical axis is $S_1-S_2=0$ i.e. $x+2y-1=0$ is also one of the common tangent

$\Rightarrow (c)$

(a) and (d) are obviously not correct.

5. ACD

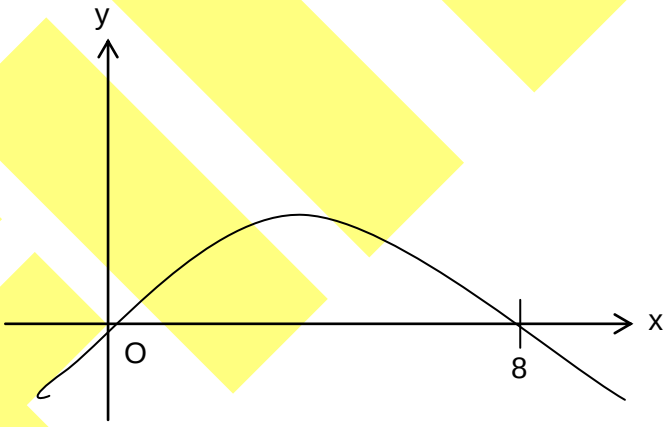
Sol. $N=C_0^2+C_1^2+C_2^2+...+C_{50}^2={}^{100}C_{50}$

Now verify answers are (a), (c), (d).

(b) is wrong as correct answer is no zero.

6. ACD

Sol. $f(x)=\frac{32}{4+x^2+x^4}; g(x)=9+x^2; h(x)=-x^2-3x+h$



(a) Range of f is $(0,8]$

(b) $h(f(x))>0$ and $h(g(x))<0$

$h(0)\geq 0$

$\Rightarrow \quad k\geq 0$

$h(8)>0$

$\Rightarrow -64-24+k>0$

$\Rightarrow \quad k>88$

$h(9)<0$

$\Rightarrow -81-27+k<0$

$\Rightarrow \quad k < 108$

Number of integral values of k is 19.

(d) Maximum value of $g(f(x))$ is $g(8) = 64 + 9 = 73$.

7. BC

Sol. $f(x) = \tan^{-1}\left(\dfrac{\dfrac{x}{3} - \dfrac{x}{\sqrt{3}}}{1 + \dfrac{x^2}{3\sqrt{3}}}\right) = \tan^{-1}\dfrac{x}{3} - \tan^{-1}\dfrac{x}{\sqrt{3}}; \; 0 \le x \le 3$

Hence, $f(x) = \tan^{-1}\left(\dfrac{x}{3}\right)$

$\Rightarrow$ Range of $f(x)$ is $\left[0, \dfrac{\pi}{4}\right]$

8. AD

Sol. The given variable line can be expressed as

$\Rightarrow 3x + 4y - 7 + \lambda(x - 2y + 1) = 0$

$\Rightarrow L_1 + \lambda L_2 = 0$

$L_1 = 3x + 4y - 7 = 0, \; L_2 = x - 2y + 1 = 0$

Point of intersection of $L_1 = 0$ and $L_2 = 0$ is (1. 1)

$\therefore a = b = 1 \Rightarrow a + 2b = 3$

Now, $\ell = \lim\limits_{x \to 0^-} \dfrac{[\sin(0-h)] - 2 + \{\cos(0-h)\}}{\{0-h\} - 1}$

$= \lim\limits_{h \to 0} \dfrac{[\sin(0-h)] - 2 + \{\cos(0-h)\}}{\{0-h\} - 1}$

$= \dfrac{-1 - 2 + 1}{1 - 1} = \dfrac{-2}{0}$

Hence I does not exist.

9. B

10. C

Sol (9 & 10)

(I) $f'(x) = \dfrac{g'(x)}{\sqrt{1 + x^3(x)}}$ and $g'(x) = \left[1 + \sin\left(\cos^2 x\right)\right](-\sin x)$

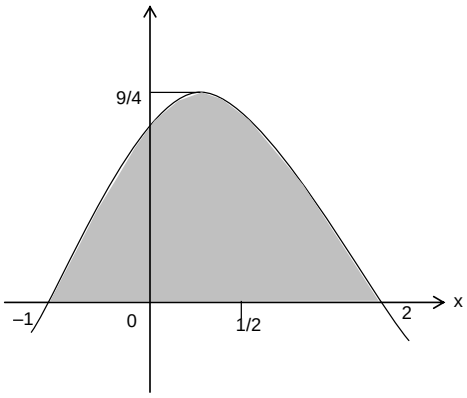
Hence, $f'(x) = \dfrac{\left[1 + \sin\left(\cos^2 x\right)\right](-\sin x)}{\sqrt{1 + g^3(x)}}$

$f'\left(\dfrac{\pi}{2}\right) = \dfrac{1 + 0}{\sqrt{1 + g^3\left(\dfrac{\pi}{2}\right)}} = \dfrac{-1}{1 + 0} = -1$ as $g\left(\dfrac{\pi}{2}\right) = 0$

$\therefore f'\left(\dfrac{\pi}{2}\right) = -1$

(III) Maximum when $a=-1; b=2$

$\Rightarrow a+b=1$



(IV) If $\lim\limits_{x\to0}\dfrac{\sin 2x}{x^3}+a+\dfrac{b}{x^2}=0$

$\lim\limits_{x\to0}\dfrac{\sin 2x+ax^3+bx}{x^3}=0$

For limit to exist $2+b=0\Rightarrow b=-2$

$\therefore \lim\limits_{x\to0}\dfrac{\sin 2x+ax^3-2x}{x^3}=0$

Apply LHS rule,

$\lim\limits_{x\to0}\dfrac{2\cos 2x+3ax^2-2}{3x^2}=0$

$\therefore \quad a=\lim\limits_{x\to0}\dfrac{2(1-\cos 2x)}{3x^2}$

$a=\dfrac{4\sin^2 x}{3x^2}=\dfrac{4}{3}$

$\therefore \quad 3a+b=3.\dfrac{4}{3}-2=2$

11. B

12. D

Sol (11 & 12)

(I) $x+y+z+t=10$

4 movies 10 people

(People as like object and movies as 4 different beggars) give one to each x, y, z and t and hence the number of ways = then number of non – negative integral solution of the equation $x+y+z+t=6$

Using Beggar method $\underbrace{00000}\theta\theta\theta$

Number of ways is ${}^9C_3=84$

(II) $\left({}^9C_3-4\right)-{}^8C_2=52$.

(III) At most two children got mobile of the other children.

$\Rightarrow$ Exactly 3R and 2W + all 5 got their own mobile

$={}^5C_3.1+1=11$

(IV) $12=2^2.3$

Hence groups of 4 that work out

$1,1,6,2=\dfrac{4!}{2!}=12$

$1,1,3,4=\dfrac{4!}{2!}=12$

$$2,2,3,1 = \frac{4!}{2!} = 12$$

$$\overline{\quad 36 \quad}$$

(V) $\quad {}^5C_2 . {}^3C_2 . 2! = 60$

(PART – B)

1. 9

Sol. Equation of plane P (with usual $\vec{n}_1$ which n to $\vec{n}$ and $\overrightarrow{AB}$)

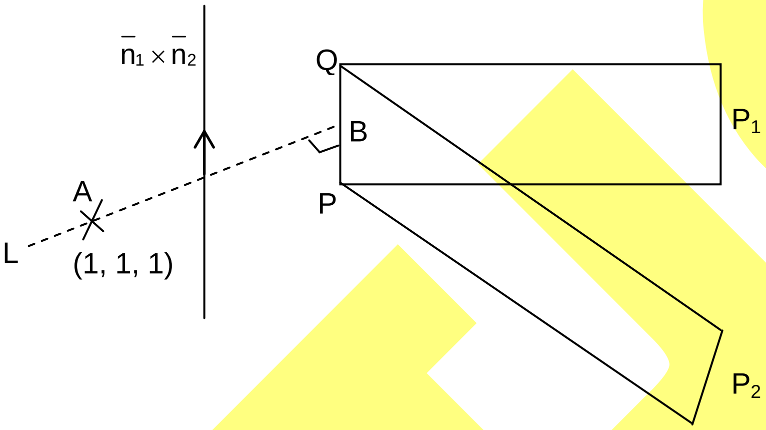
$\vec{n}_1 = \vec{n} \times \overrightarrow{AB} = (i + j - 3k) \times (i - 3j + k)$

$= 2i + j + k$

$\therefore$ Equation of plane $P_1 : 2(x-1) + 1(y-2) + 1(z-0) = 0$

$\Rightarrow P_1 : 2x + y + z = 4$

Also $P_2 : x + y + z = 5$



Line PQ is parallel to vector

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = k - j$$

$\therefore$ Equation PQ : $\frac{x+1}{0} = \frac{y-6}{-1} = \frac{z-0}{1} = \lambda$

(Contained P_1 and P_2 in symmetric form)

Let $B(-1, 6-\lambda, \lambda)$ be any point on line PQ.

So, $\overrightarrow{AB} = 2\hat{i} + (5-\lambda)\hat{j} + (\lambda - 1)\hat{k}$

Now, $\overrightarrow{AB} . \vec{n}_1 \times \vec{n}_2 = 0$

$\Rightarrow 0 - 5 + \lambda + \lambda - 1 = 0$

$\Rightarrow \lambda = 3$

$\therefore B \equiv (-1, 3, 3)$

So, equation of line L is $\frac{x+1}{1+1} = \frac{y-3}{1-3} = \frac{z-3}{1-3}$

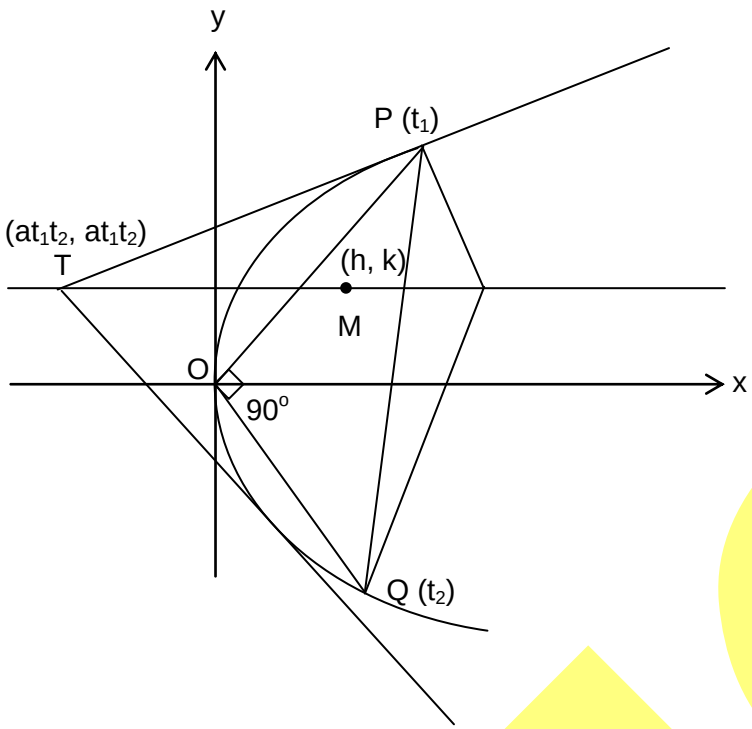
i.e. $\frac{x+1}{-1} = \frac{y-3}{1} = \frac{z-3}{1}$

Hence $\left(\frac{2y_1 + z_1}{bc} = \frac{6+3}{1}\right) = 9$

2. 25

Sol. $\frac{2at_1}{at_1^2} . \frac{2at_2}{at_2^2} = -1$

$t_1t_2 = -4$



Now $T = at_1t_2, a(t_1 + t_2)$ and $N = a(t_1^2 + t_2^2 + t_1t_2 + 2), -at_1t_2(t_1 + t_2)$

Hence $2h = a\left((t_1 + t_2)^2 + 2\right)$ and $2k = a(t_1 + t_2)(1 - t_1t_2)$

$$2k = 5a(t_1 + t_2)$$

$$\therefore\ t_1 + t_2 = \frac{2k}{5a}$$

$$\therefore\ 2h = \left[\left(\frac{2k}{5a}\right)^2 + 2\right]$$

$$2h = \left[\frac{4k^2 + 50a^2}{25a^2}\right]$$

$$50ah = 4k^2 + 50a^2$$

$$25ah = 2k^2 + 25a^2$$

$25.2(h-2) = 2k^2$ or $y^2 = 25(x-2)$

Hence latus rectum = 25

3. 60

Sol. We have $\sum_{i=1}^{10}(SP_i)(S'P_i')$

$\Rightarrow$ $10b^2 = 2560$

$\Rightarrow$ $b^2 = 256$

$\Rightarrow$ $a = 20$

Also $a = 20$ (Given)

$\therefore$ From $b^2 = a^2(1 - e^2)$, we get

$$256 = 400(1 - e^2)$$

$\Rightarrow$ $e^2 = \frac{9}{25}$

$\Rightarrow$ $e = \frac{3}{5}$

Hence, $100e = 100 \times \frac{3}{5} = 60$

4. 445

Sol. Let C_1, C_2, C_3, C_4 are coins.

4 coins tossed twice $\rightarrow$ each coin is tossed twice.

Let S : denotes the success that a coin is discarded

$P(S) = 1-$ coin is not discarded

$= 1 - P(HH) = 1 - \frac{1}{4} = \frac{3}{4}$

Hence S can take value 0, 1, 2, 3, 4

$P(S = 3 \text{ or } 4) = P(S = 3) + P(S = 4)$

$= {}^4C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right) + {}^4C_4 \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^3 \left(1 + \frac{3}{4}\right)$

$= \frac{(27)(7)}{256} = \frac{189}{256} = \frac{m}{n}$

$\therefore m + n = 445$

5. 1

Sol. Min $\left(f(x)\right) = \frac{D}{4a} = -\frac{4b^2 - 8c^2}{4} = 2c^2 - b^2$

Max $\left(g(x)\right) = -\frac{D}{4a} = -\frac{4c^2 + 4b^2}{-4} = b^2 + c^2$

$\therefore$ For $\min\left(f(x)\right) < \max\left(g(x)\right)$

$\Rightarrow \quad 2c^2 - b^2 < b^2 + c^2$

$\Rightarrow \quad 2b^2 > c^2$

$\Rightarrow \quad \frac{b^2}{c^2} > \frac{1}{2}$

$\Rightarrow$ Minimum integral value of $\frac{b^2}{c^2} = 1$

6. 4

Sol. $T_1 = \frac{1}{\sin 8^\circ} = \frac{\sin(8^\circ - 4^\circ)}{\sin 4^\circ \sin 8^\circ} = \cot 4^\circ - \cot 8^\circ$ and so on

Given $S = \frac{1}{\sin 8^\circ} + \frac{1}{\sin 16^\circ} + \frac{1}{\sin 32^\circ} + \ldots + \frac{1}{\sin 4096^\circ} + \frac{1}{\sin 8192^\circ}$

$= (\cot 4^\circ + \cot 8^\circ) + (\cot 8^\circ - \cot 16^\circ)$

$+ (\cot 16^\circ - \cot 32^\circ) + \ldots + (\cot 2048^\circ - \cot 4096^\circ) + (\cot 4096^\circ - \cot 8192^\circ)$

$= (\cot 4^\circ - \cot 8192^\circ)$

Also $\cot 8192^\circ = \cot(45 \times 180^\circ + 92^\circ)$

$= \cot 92^\circ \qquad (\text{As } 45 \times 180^\circ = 8100^\circ)$

$\therefore S = \cot 4^\circ - \cot 92^\circ = \frac{1 - \tan^2 2^\circ}{2 \tan 2^\circ} + \tan 2^\circ$

$= \frac{1 + \tan^2 2^\circ}{2 \tan 2^\circ} = \frac{\sec^2 2^\circ}{2 \tan 2^\circ}$

$= \frac{1}{\sin 4^\circ}$

$\Rightarrow \alpha = 4^\circ$