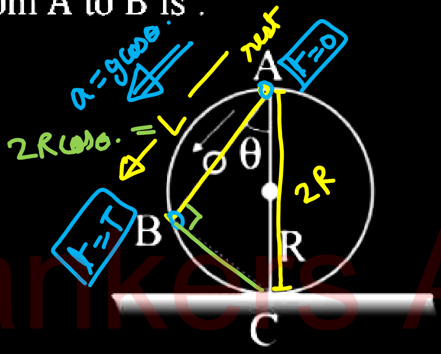
PHYSICS

A frictionless wire AB is fixed on a sphere of radius R. A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is :



(A) $\dfrac{2\sqrt{gR}}{g\cos\theta}$

(B) $2\sqrt{gR}\dfrac{\cos\theta}{g}$

(C) $\boxed{2\sqrt{\dfrac{R}{g}}}$

(D) $\dfrac{gR}{\sqrt{g\cos\theta}}$

$$L = \frac{1}{2} a T^2$$

$$2R\cancel{\cos\theta} = \frac{1}{2} g \cancel{\cos\theta} T^2$$

$$2\sqrt{\frac{R}{g}} = T$$

The depth d at which the value of acceleration due to gravity becomes $1/n$ times the value at the surface, is [R = radius of the earth]

(A) $\frac{R}{n}$

(B) $R\left(\frac{n-1}{n}\right)$

(C) $\frac{R}{n^2}$

(D) $R\left(\frac{n}{n+1}\right)$

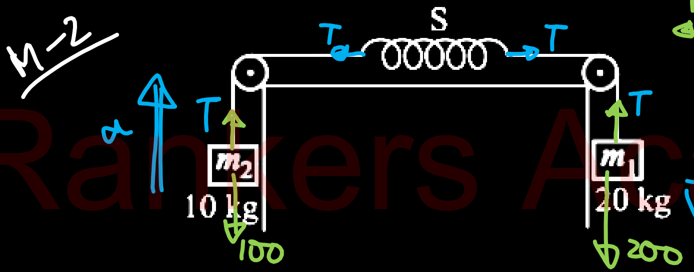
$$g_d = g\left[1-\frac{d}{R}\right]$$

$$\frac{\cancel{g}}{n} = \cancel{g}\left[1-\frac{d}{R}\right]$$

$$\frac{1}{1}-\frac{1}{n}=\frac{d}{R}$$

3

In the arrangement shown, the pulleys are fixed and ideal, the strings are light. $m_1 > m_2$ and S is a spring balance which is itself massless. The reading of S (in unit of mass) is : ($g = 10\ m/s^2$)



(A) 100 N

(B) 200 N

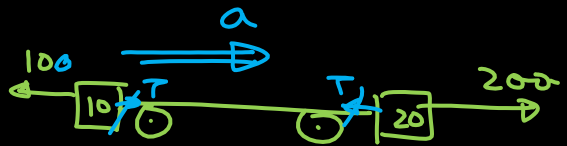
(C) $\frac{200}{3}$ N

(D) $\boxed{\frac{400}{3}}$ N

M-1

$$\boxed{T = \frac{2m_1 m_2 g}{m_1 + m_2}}$$

$$\frac{20 \times 20 \times 1\cancel{0}}{3\cancel{0}}$$



$10\cancel{0} = 3\cancel{0}\,a$

$$\boxed{a = \frac{10}{3}}$$

$$200 - T = 20\left(\frac{10}{3}\right)$$

$$T = 200 \times \frac{2}{3} = \frac{400}{3}$$

4

If the constant of gravitational constant (G) and Plank's constant (h) and the velocity of light (c) be chosen as fundamental units. The dimensions of the radius of gyration is :

(A) $h^{1/2}c^{-3/2}G^{1/2}$ (B) $h^{1/2}c^{3/2}G^{1/2}$

(C) $h^{1/2}c^{-3/2}G^{-1/2}$ (D) $h^{-1/2}c^{-3/2}G^{1/2}$

~~MLT~~

hGC

$[k] = L^1$

$[G] = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$[h] = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

$[c] = LT^{-1}$

$[K] = h^x G^y c^z$

$M^0L^1T^0 = [ML^2T^{-1}]^x [M^{-1}L^3T^{-2}]^y [LT^{-1}]^z$

$M^0L^0T^0 = M^{x-y} L^{2x+3y+z} T^{-x-2y-z}$

$\boxed{x = y}$

$5x + z = 1$

$-3x - z = 0$

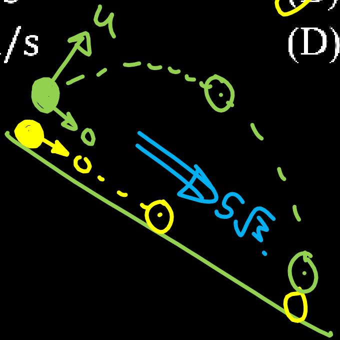
$2x = 1$ $\boxed{x = 1/2 = y = 1/2}$

$\boxed{z = -\frac{3}{2}}$

A particle P is projected from a point on the surface of long smooth inclined plane and Q starts moving down the plane from the same position. P and Q collide after 4 second. The speed of projection of P is : ($g = 10\ m/s^2$)

$u = ?$

(A) 5 m/s (B) 10 m/s
(C) 15 m/s (D) 20 m/s



$$T = \frac{2U_y}{g_y} = 4$$

$$\frac{2u}{5} = 4$$

$$\boxed{u = 10}$$

$U_x = 0$

$$\boxed{R = \cancel{U_x T} + \frac{1}{2} a_x T^2}$$

$$R = \frac{1}{2} 5\sqrt{3} (4)^2$$

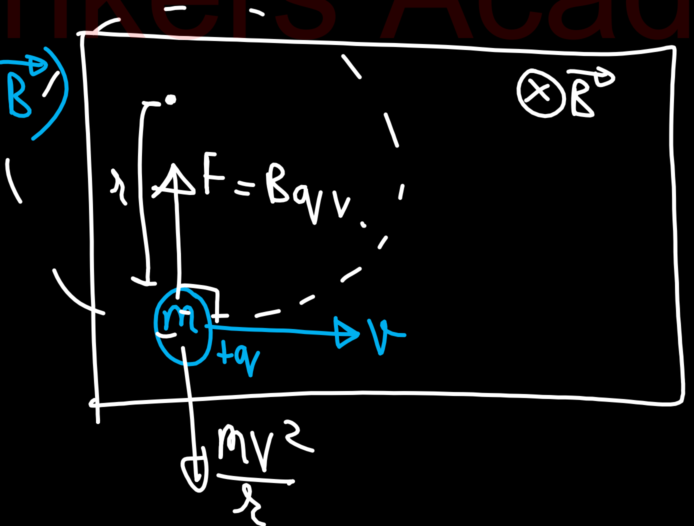
6

A proton, a deuteron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then

(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha > r_d > r_p$

(C) $r_\alpha = r_d > r_p$ (D) $r_p = r_d = r_\alpha$

$\vec{F} = q(\vec{v} \times \vec{B})$



$$\frac{mv^{\cancel{2}}}{r} = Bq\cancel{v}$$

$$\boxed{r = \frac{mv}{qB}} = \frac{p}{qB}$$

$$r = \frac{\sqrt{2m\boxed{K}}}{qB}$$

$$r = \left(\frac{\sqrt{m}}{q}\right)\frac{\sqrt{K}}{B}$$

proton	deutron	α-particle
p	p n	p p n n
m	$2m$	$4m$
e	e	$2e$

$$\boxed{r = \frac{\sqrt{m}}{q}\,\frac{\sqrt{K}}{B}}$$

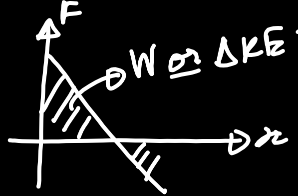
$$r \propto \frac{\sqrt{m}}{q}$$

$$\boxed{r_p = \frac{\sqrt{m}}{e}\,\frac{\sqrt{K}}{B}} \quad \boxed{r_d = \frac{\sqrt{2m}}{e}\,\frac{\sqrt{K}}{B}} \quad \boxed{r_\alpha = \frac{\cancel{2}\sqrt{m}}{\cancel{2}e}\,\frac{\sqrt{K}}{B}}$$

$$r_p = r_\alpha < r_d$$

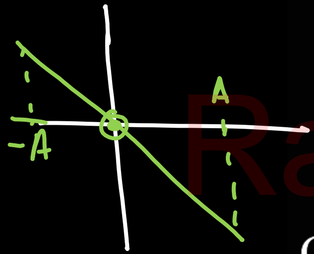
7

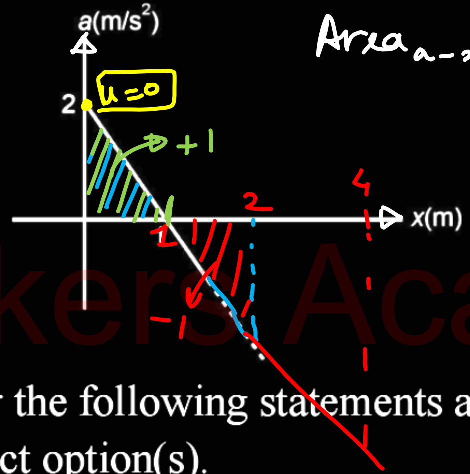
A particle (initially at rest and at origin) is to move along x-axis according to the following acceleration position $(a - x)$ graph:



$$\text{Area}_{a-x} = \frac{1}{2}(v^2 - \cancel{u^2})$$

$$\boxed{a = -\omega^2 x}$$





$$\text{Area}_{a-x} = \int_u^v \boxed{a}\, dx$$

$$\text{Area}_{a-x} = \int_u^v v\frac{dv}{\cancel{dx}}\cdot \cancel{dx}$$

$$\text{Area}_{a-x} = \frac{1}{2}(v^2 - u^2)$$

Consider the following statements and choose the correct option(s).

(A) It again comes to rest at $x = 2$ m

(B) It again comes to rest at $x = 4$ m

(C) It returns to origin with a speed of 4 m/s

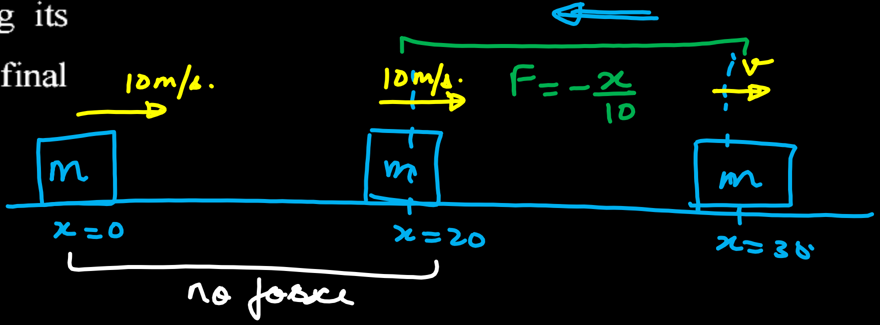
(D) It returns to origin with a speed of 2 m/s

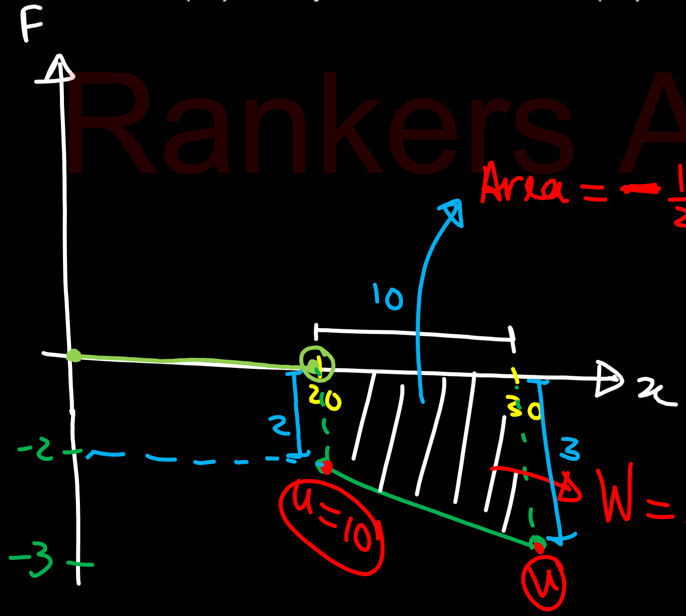
8

A block of mass 10 kg is moving in x-direction with a constant speed of 10 m/s. It is subjected to a retarding force $F = -0.1x$ J/m during its travel from $x = 20$ m to $x = 30$ m. Its final kinetic energy will be :

(A) 475 J (B) 450 J

(C) 275 J (D) 250 J





$W = \Delta KE$

$$\text{Area} = -\frac{1}{2}(2+3)10 = -25 = \frac{1}{2}mv^2 - \frac{1}{2}m(10)^2$$

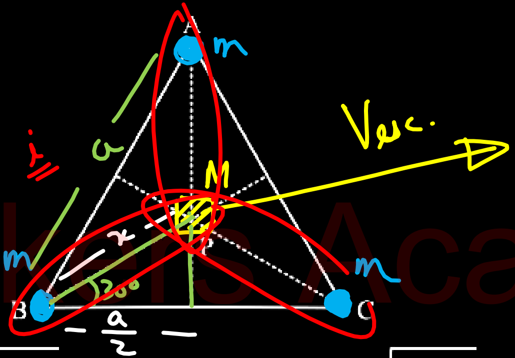
$$\boxed{475 = KE_f}$$

9

Three equal masses (each m) are placed at the corners of an equilateral triangle of side ' a '. Then the escape velocity of an object from the circumcentre P of triangle is :

$$\frac{\sqrt{3}x}{\cancel{2}} = \frac{a}{\cancel{2}}$$

$$\boxed{x = \frac{a}{\sqrt{3}}}$$



$V_{esc.}$

∞

$KE_f = 0.$

$V_f = 0$

$E_i = E_f$

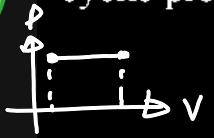
$$3\left(-\frac{G\cancel{M}m}{a/\sqrt{3}}\right) + \frac{1}{2}\cancel{M}V_{esc}^2 = 0 + 0$$

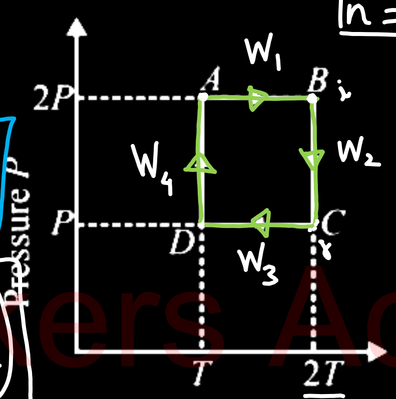
(A) $\sqrt{\frac{2\sqrt{3}Gm}{a}}$ (B) $\sqrt{\frac{\sqrt{3}Gm}{a}}$

(C) $\sqrt{\frac{6\sqrt{3}Gm}{a}}$ ✓ (D) $\sqrt{\frac{3\sqrt{3}Gm}{a}}$

$V_{esc} =$ ______

One mole of an ideal gas having initial volume V, pressure 2P and temperature T undergoes a cyclic process ABCDA as shown below

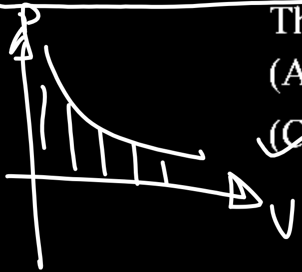




$\boxed{n=1}$

$$\boxed{W_{isobaric} = P\Delta V = \underline{nR\Delta T}}$$

$$\boxed{W_{isothermal} = nRT\ln\left(\frac{P_i}{P_f}\right)}$$



The net work done in the complete cycle is

(A) Zero

(B) 2RT ln 2

(C) RT ln 2 ✓

(D) $\frac{3}{2}$RT ln 2

$$\text{Ans} = W_{net} = W_1 + W_2 + W_3 + W_4$$

$$W_1 = (1)R(T) = RT$$

$$W_2 = (1)R(2T)\cdot\ln\left(\frac{2\cancel{P}}{\cancel{P}}\right) = 2RT\ln(2)$$

$$W_3 = (1)R(-T) = -RT$$

$$W_4 = (1)R(T)\cdot\ln\left(\frac{\cancel{P}\,1}{2\cancel{P}}\right)$$

$$W_{net} = 2RT\ln(2) - RT\ln(2)$$
$$= RT\ln(2)$$

A log of wood of mass $\boxed{120\text{ kg}}$ floats in water. The weight that can be put on the log to make it just sink, should be (density of wood = $\boxed{600\text{ kg/m}^3}$)

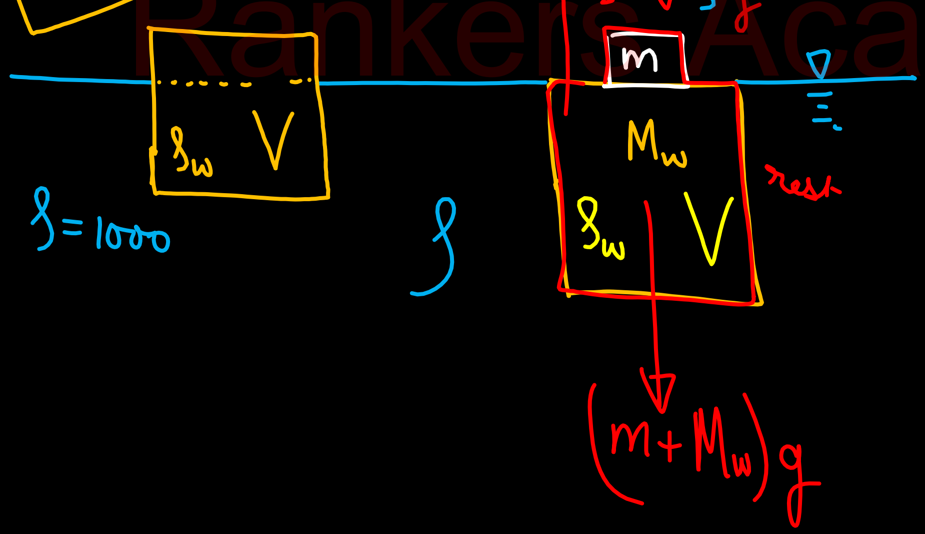
(A) 80 kg ✓ (B) 50 kg

(C) 60 kg (D) 30 kg

$V = \dfrac{M_w}{\rho_w}$

$\boxed{V = \dfrac{120}{600} = \dfrac{1}{5}}$

$\boxed{m = ?}$



$V\rho \cancel{g} = (m + M_w)\cancel{g}$

$\dfrac{1}{5} \times 1000 = (m + 120)$

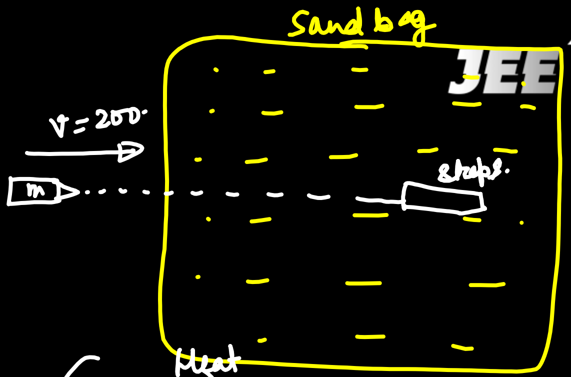
$\boxed{m = 80}$ kg.

12

A 5.0 g bullet (specific heat of material of bullet = $\boxed{128}$ J/kg$\boxed{°C}$) moving with a velocity of 200 m/s enters a sand bag and stops. If the entire kinetic energy of the bullet is changed into heat energy that is added to the bullet, then the rise in the temperature of the bullet is:

~~(A) 312.5°C~~ (B) 156°C

~~(C) 500°C~~ ~~(D) 624°C~~



Heat

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}S\boxed{\Delta T}$$

$$\frac{\cancel{200}^{100}\times 200}{\cancel{2}} = 128\,\Delta T$$

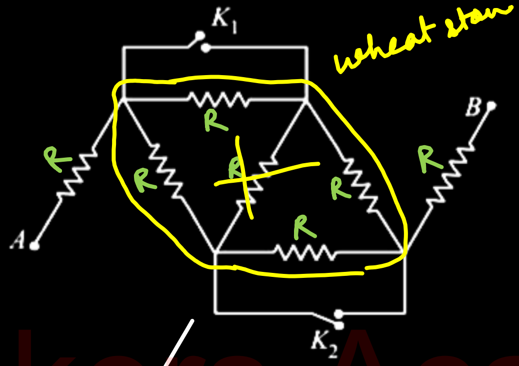
$$\frac{200}{128}\times 100 = \Delta T$$

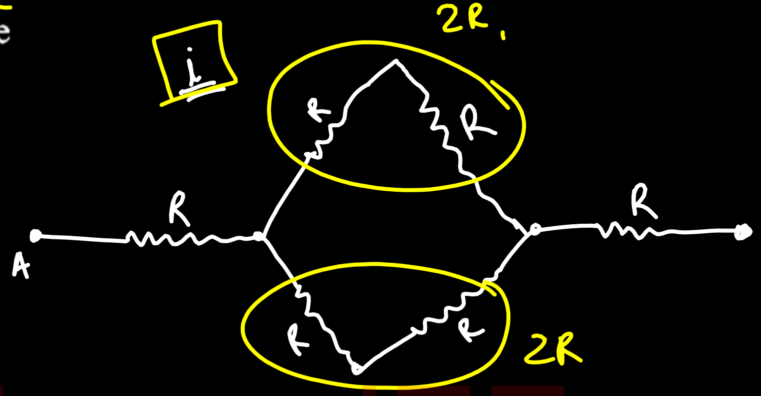
$$\Delta T = 100\left(\frac{24\ldots}{16}\right)$$

$\simeq 156$

13

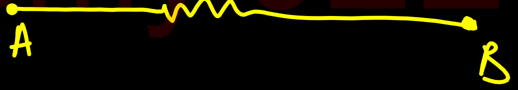
All wires have same resistance and equivalent resistance between A and B is $\boxed{R_0}$. Now keys are closed, then the equivalent resistance will become





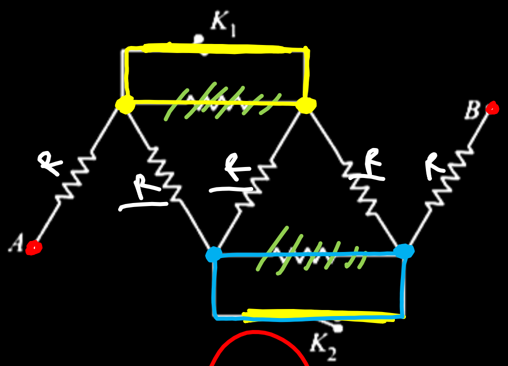
(A) $\frac{7R_0}{3}$ (B) $\frac{7R_0}{9}$ (C) $7R_0$ (D) $\frac{R_0}{3}$

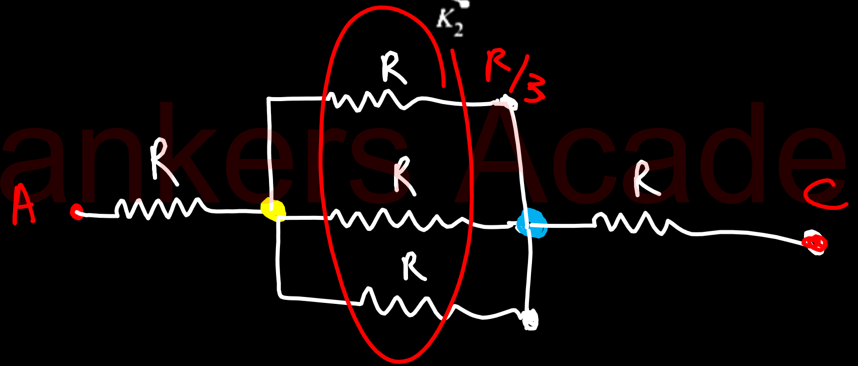
$\equiv$ $3R = R_0$.



$R_0 = 3R$

$R = \frac{R_0}{3}$





$$R_{eq} = 2R + \frac{R}{3} = \frac{7R}{3} = \frac{7}{3}\left(\frac{R_0}{3}\right)$$

14

A particle of charge $-q$ and mass m moves in a circular orbit of radius r about a fixed charge $+Q$. The relation between the radius of the orbit r and the time period $\boxed{T}$ is :

$\boxed{K = \frac{1}{4\pi\epsilon_0}}$

(A) $r = \frac{Qq}{16\pi^2\epsilon_0 m}T^2$

(B) $r^3 = \left(\frac{Qq}{16\pi^3\epsilon_0 m}\right)T^2$

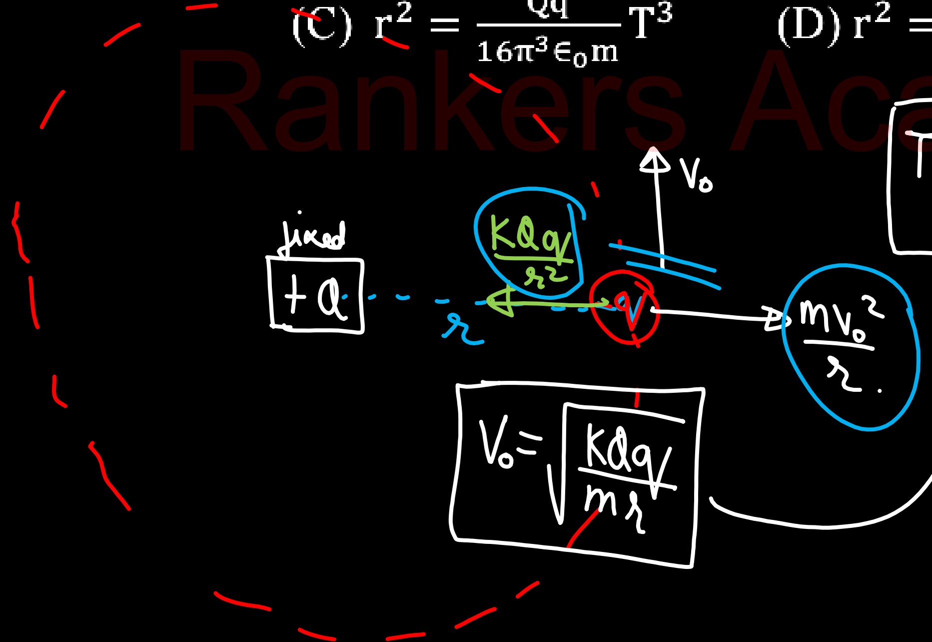
(C) $r^2 = \frac{Qq}{16\pi^3\epsilon_0 m}T^3$

(D) $r^2 = \frac{Qq}{4\pi^3\epsilon_0 m}T^3$

$\boxed{T^2 \propto r^3}$

$\boxed{T} = \frac{2\pi r\sqrt{r}}{\sqrt{\frac{KQq}{m}}}$

$\boxed{T^2} = \frac{4\pi^2 \boxed{r^3} m}{KQq}$



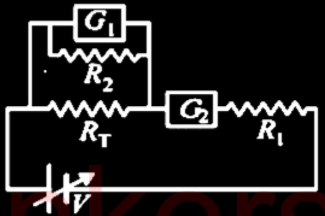
$\boxed{T = \frac{2\pi r}{V_0}}$

$\boxed{V_0 = \sqrt{\frac{KQq}{mr}}}$

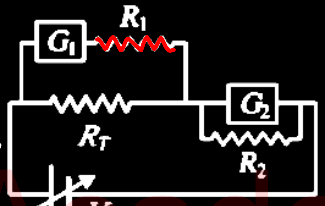
15

To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

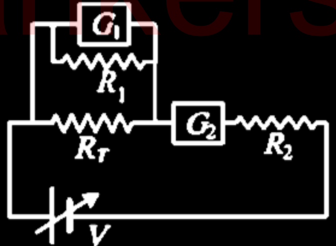
(A)



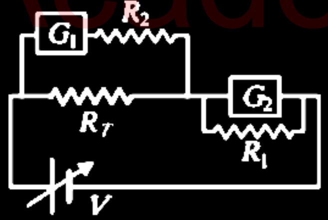
(C)

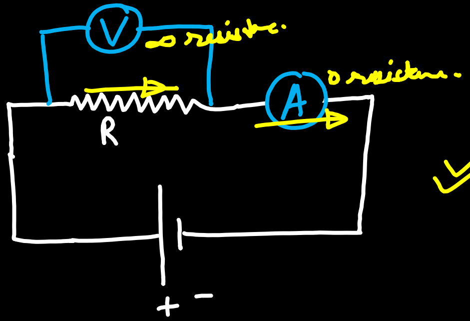


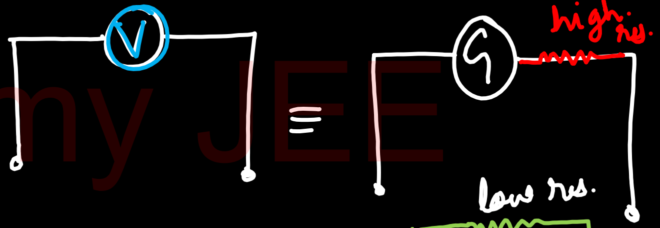
(B)

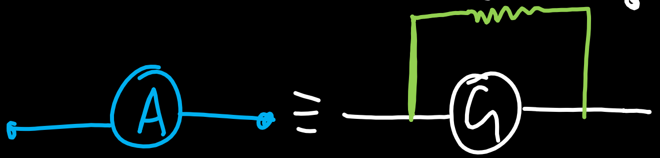


(D)









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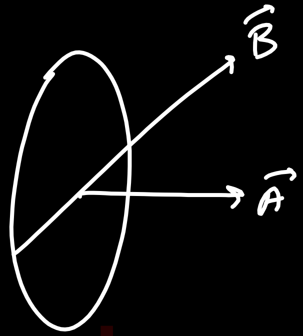


$\phi = BA\cos\theta$
1 2

A circular wire loop of radius r is placed in a region of magnetic field B such that the plane of the loop makes an angle θ with the direction of $\vec{B}$. Consider the following in this regard:

1 Change in B with time
2 Change in r with time
3 B being non-uniform in space
4 Change in θ with time

The conditions for an induced emf in the loop would include

(A) 1 and 4
(B) 1,2 and 4
(C) 1 and 3
(D) 2,3 and 4

17

JEE 1

Condenser A has a capacity of 15μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity of 1μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric medium being removed. The common potential now is

(A) 400 V (B) 800 V

(C) 1200 V (D) 1600 V

$C_1 = 1\mu F$

$KC_1 = 15\mu F$

$C_2 = 1\mu F$

$Q_1 = KC_1V$

$= 1500\mu C$

$Q_2 = 100\mu C$

$Q = Q_1 + Q_2 = 1600\mu C$

$$V' = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{1600}{(1+1)} = 800\,V$$

18

JEE 1

The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be

(A) $\frac{16}{25}\lambda_0$

(B) $\frac{20}{27}\lambda_0$

(C) $\frac{27}{20}\lambda_0$

(D) $\frac{25}{16}\lambda_0$

$$\frac{1}{\lambda_0} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad \text{— (1)}$$

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \quad \text{— (2)}$$

(1) ÷ (2)

$$\frac{\lambda}{\lambda_0} = \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4} - \frac{1}{16}} = \frac{\frac{5}{\cancel{36}\,9}}{\frac{\cancel{12}\,3}{\cancel{64}\,4}} = \frac{20}{27}$$

19

JEE 1

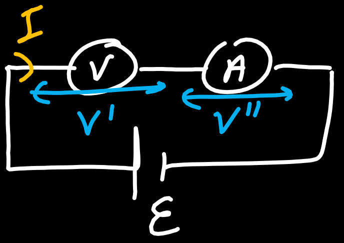
An ammeter and a voltmeter are connected in series to a battery. When a certain resistance is connected in parallel to voltmeter reading of voltmeter becomes half while reading of ammeter becomes double. What is the ratio of voltmeter resistance and ammeter resistance?

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

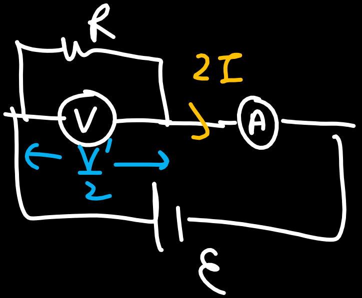
(C) $\frac{2}{1}$

(D) $\frac{3}{1}$



$$I = \frac{\varepsilon}{V+A} \quad \text{—} \ ①$$

$$V' = \frac{\varepsilon V}{V+A} \quad \text{—} \ ②$$



$$2I = \frac{\varepsilon}{A + \frac{RV}{R+V}}$$

for ①

$$\Rightarrow \frac{2\varepsilon}{V+A} = \frac{\varepsilon}{A + \frac{RV}{R+V}} \Rightarrow 2AR + 2AV + 2RV = V^2 + AR + AV + RV$$

$$(AR + RV) + AV = V^2 \quad \text{—} \ ③$$

19

JEE 1

An ammeter and a voltmeter are connected in series to a battery. When a certain resistance is connected in parallel to voltmeter reading of voltmeter becomes half while reading of ammeter becomes double. What is the ratio of voltmeter resistance and ammeter resistance?

(A) $\frac{1}{3}$ (B) $\frac{3}{2}$

(C) $\frac{2}{1}$ (D) $\frac{3}{1}$

$$\frac{V'}{2} = \frac{\varepsilon\left(\frac{VR}{R+V}\right)}{\frac{RV}{R+V} + A}$$

from (2) $$\frac{\cancel{\varepsilon} V}{2(V+A)} = \frac{\cancel{\varepsilon} VR}{RV+AR+VA}$$

$$RV+AR+VA = 2VR+2RA$$

$$VA = VR+RA$$

$$VA = V^2-AV \quad (\text{from } 3)$$

$$2VA = V^2$$

$$\Rightarrow \boxed{\frac{V}{A} = \frac{2}{1}}$$

20

If one mole of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for the mixture is

JEE 1

(A) 1.40

(B) 1.50

(C) 1.53

(D) 3.07

$$\boxed{\gamma_{mix} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}}$$

$$= \frac{1 \times \frac{5R}{2} + 1 \times \frac{7R}{2}}{1 \times \frac{3R}{2} + 1 \times \frac{5R}{2}}$$

$$= \frac{12}{8} = \frac{3}{2} = 1.50$$

21

Integer

A convex lens of focal length 40 cm is held co-axially 12 cm above a mirror of focal length 18 cm. An object held x cm above the lens gives rise to an image coincident with it. Then x is equal to :

JEE 1

$v = -(36-12)$

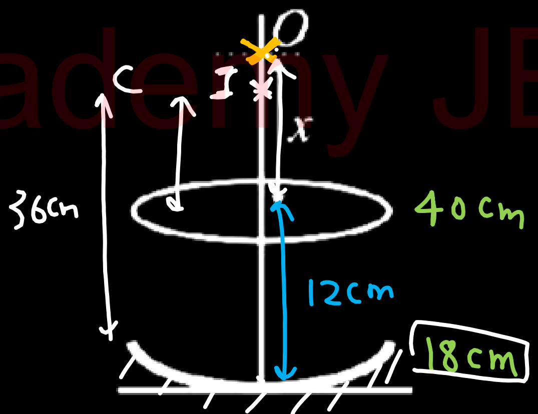
$= -24\,cm$ —— (1)

Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{-24} - \frac{1}{(-x)} = \frac{1}{40}$

$\frac{1}{x} = \frac{1}{40} + \frac{1}{24}$

$x = \frac{120}{3+5} = \frac{120}{8} = \boxed{15\,cm}$



22

The binding energy of deuteron $_1^2H$ is 1.047 MeV per nucleon and an α-particle He^4 has a binding energy of 7.047 MeV per nucleon. Then in the fusion reaction $_1^2H + _1^2H \rightarrow _2^4He + Q$, the energy Q released is ______ MeV

JEE 1

$$\boxed{Q = (B.E.)_P - (B.E.)_R}$$

$$= 4(7.047) - 2(2 \times 1.047)$$

$$= 4(7.047 - 1.047)$$

$$= 4 \times 6 = \boxed{24}\ \text{MeV}$$

23

JEE 1

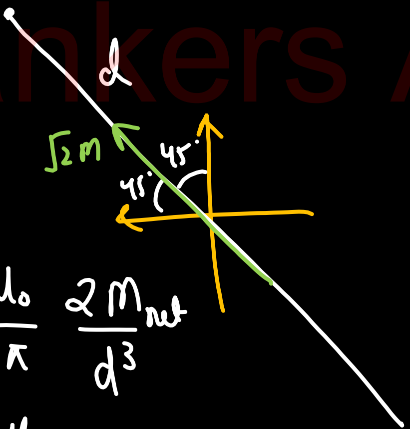
A generator supplies 100 V to the primary-coil of a transformer of 50 turns. If the secondary coil has 500 turns, then the secondary voltage is ______ volts.

$$\frac{V_0}{V_1} = \frac{N_0}{N_1}$$

$$\Rightarrow \frac{V_0}{100\,V} = \frac{500}{50} = \frac{10}{1}$$

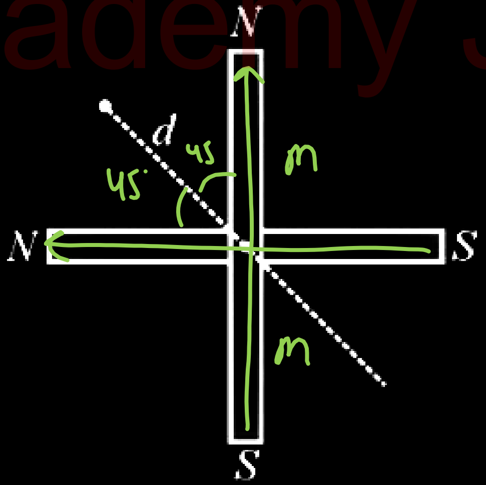
$$V_0 = 1000\,V$$

Two short magnets of equal dipole moment M are fastened perpendicularly at their centres. The magnitude of the magnetic field at a distance d from the centre of the bisector of the right angle is $\frac{\mu_0}{4\pi}\frac{\sqrt{n}M}{d^3}$ find n

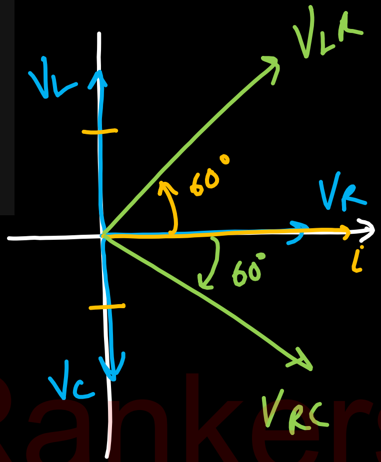


$$B_{axis} = \frac{\mu_0}{4\pi}\frac{2M_{net}}{d^3}$$

$$= \frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3} = \frac{\mu_0}{4\pi}\frac{\sqrt{\boxed{8}}\,M}{d^3}$$



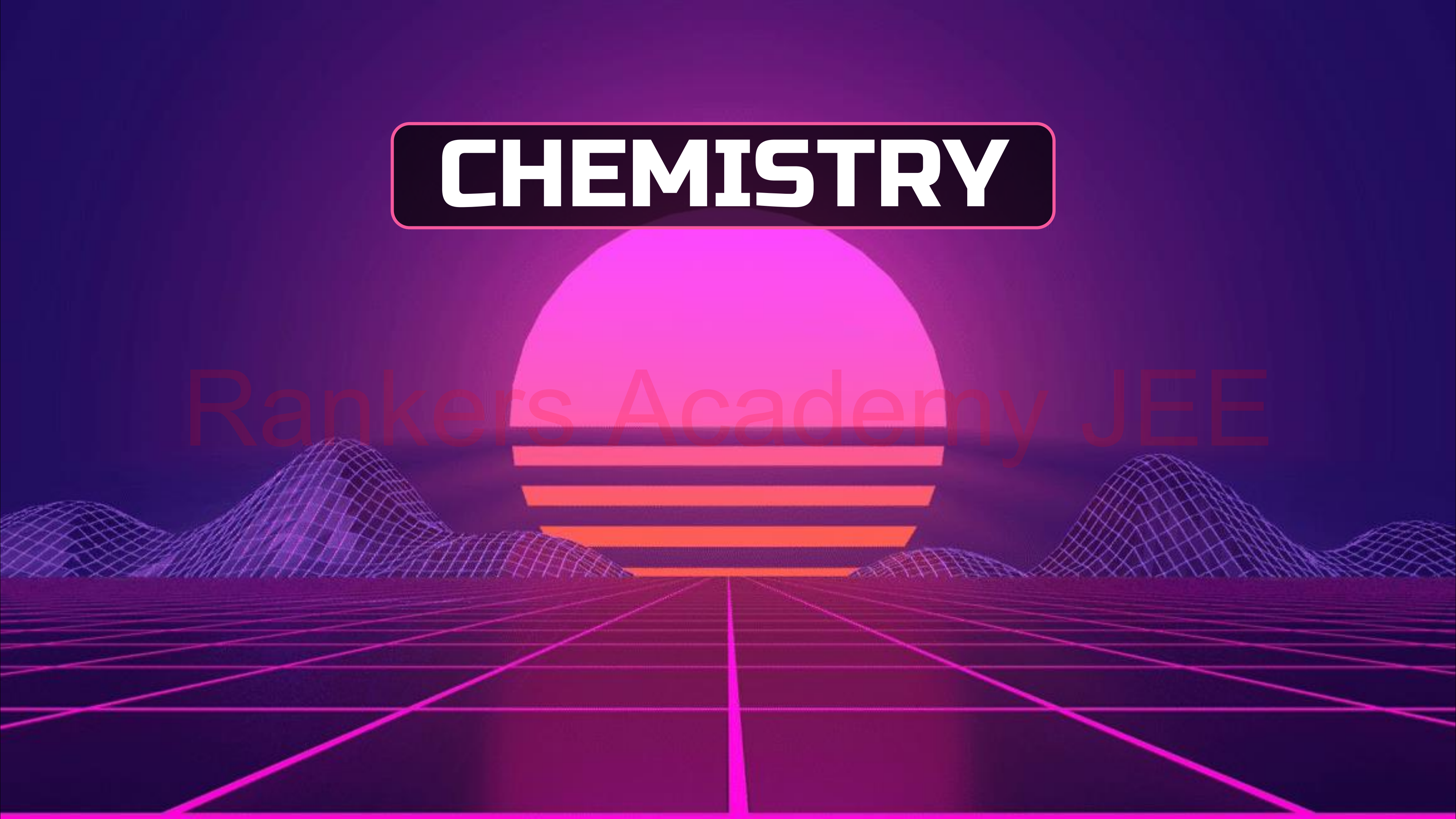
An LCR series circuit with 100Ω resistance is connected to an AC source of 200 V and angular frequency 300rad/s. When only the capacitance is removed, the current lags behind the voltage by $60°$. When only the inductance is removed, the current leads the voltage by $60°$. the current in the LCR circuit. is _____ A.



$\phi_1 = \phi_2$

$\Rightarrow \boxed{V_L = V_C}$ $\Rightarrow Z_{min} = R \Rightarrow i_{rms} = \dfrac{V_{rms}}{R} = \dfrac{200}{100} = \boxed{2}\text{ A}$

Resonance


CHEMISTRY

In which reaction incorrect product is formed?

(A) $CH_3-COCH_3 \xrightarrow[\text{conc.HCl}]{\text{Zn-Hg}} CH_3-CH_2-CH_3$

(B) $CH_3-CHO \xrightarrow[\Delta]{\text{dil.NaOH}} CH_3-CH=CH-CHO$

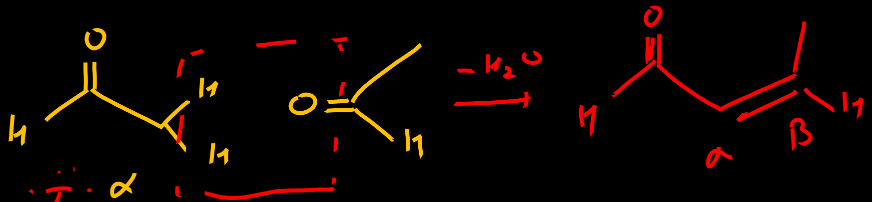
α

(C) $Ph-I + C_2H_5ONa \longrightarrow Ph-OC_2H_5$

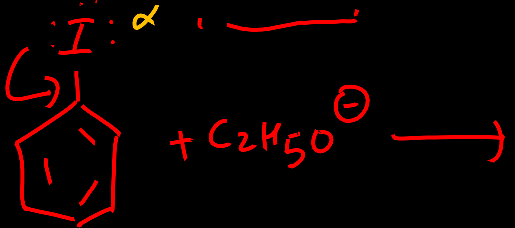
(D) $HCHO \xrightarrow{\text{conc.NaOH}} HCOONa + CH_3OH$

(A) C.R

(B)



(C)



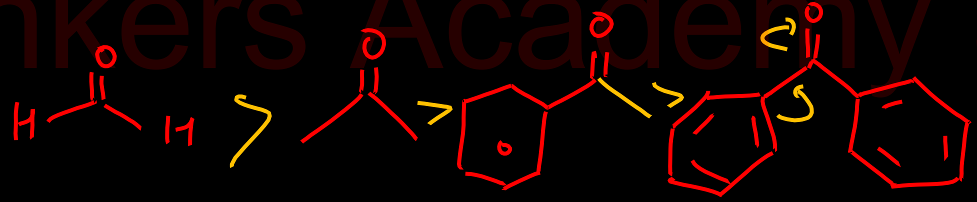
The increasing order of the rate of HCN addition to compound A - D is

(a) HCHO (b) CH_3COCH_3

(c) $PhCOCH_3$ (d) PhCOPh

(A) D < C < B < A (B) C < D < B < A

(C) A < B < C < D (D) D < B < C < A



At 25°C, if $E^0_{Sn^{2+}|Sn} = x$ volt and $E^0_{Sn^{4+}|Sn} = y$ volt then $E^0_{Sn^{2+}|Sn^{4+}}$ in volt will be :

(A) $x - y$ (B) $2x + 4y$

(C) $x - 2y$ (D) $4y - 2x$

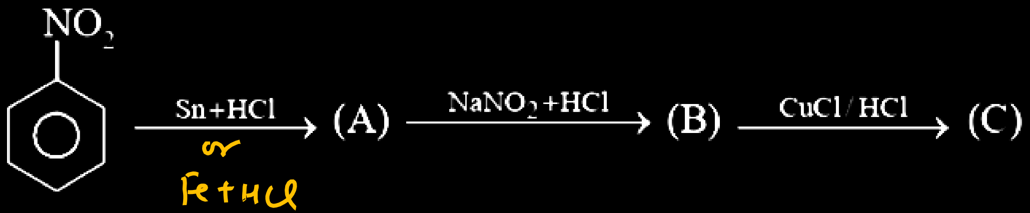
$$Sn^{+2} + 2e^- \longrightarrow \cancel{Sn} \quad E_1^0 = +x \text{ volt} \quad —(1)$$

$$\cancel{Sn} \longrightarrow Sn^{+4} + 4e^- \quad E_2^0 = -y \text{ volt} \quad —(2)$$

$$Sn^{+2} \longrightarrow Sn^{+4} + 2e^- \quad E_3^0 = ? \quad —(3)$$

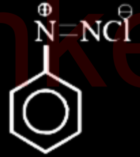
$$-2FE_3^0 = -2F(+x) - 4F(-y)$$

$$E_3^0 = x - 2y$$

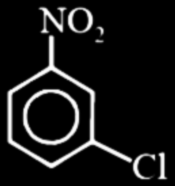


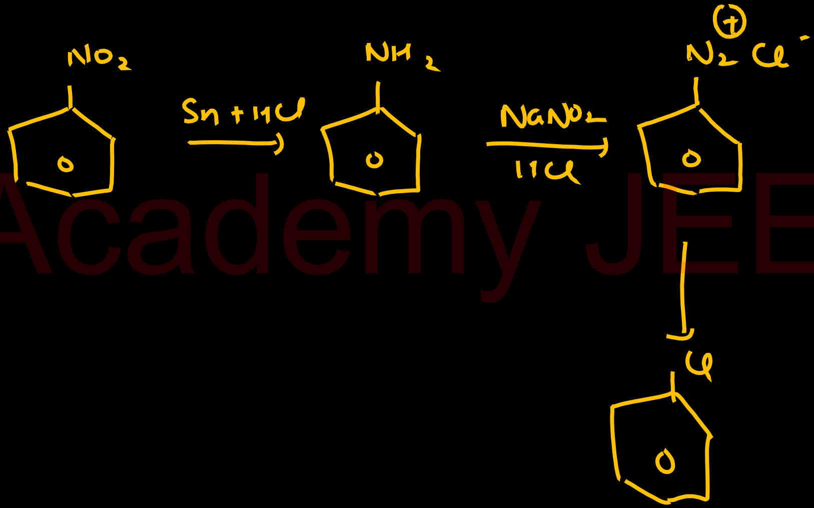
Final product C is :

(A) 

(B) 

(C) 

(D) 



5

JEE 1

Match the column-I with column-II

	Column-I (Reaction)		Column-II (Yield of product)
(a)	$2H_2 + O_2 \rightarrow 2H_2O$ 2 g 32 g	(i)	68 g
(b)	$N_2 + 3H_2 \rightarrow 2NH_3$ 2 mole 9 mole	(ii)	9 g
(c)	$S_8 + 8O_2 \rightarrow 8SO_2$ 0.1 mole 8 mole	(iii)	51.2 g
(d)	$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$ 1 g 16 g	(iv)	18 g

Choose the most appropriate answer from the options given below.

(A) (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

(B) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)

(C) (a) → (i), (b) → (iii), (c) → (iv), (d) → (i)

(D) (a) → (iii), (b) → (ii), (c) → (i), (d) → (iv)

L.R

(a) $2H_2 + O_2 \longrightarrow 2H_2O$

mole 1 1 1mole

$w_{H_2O} = 1 \times 18 = 18g$

(b) $n_{NH_3} = 4$ $wt_{NH_3} = 17 \times 4 = 68g$

(c) $n_{SO_2} = 0.8$ $\therefore wt_{SO_2} = 64 \times 0.8$

$= 51.2g$

6

JEE 1

Choose the correct statement:

(A) 9.8% H_2SO_4 by weight $\left(\text{density} = 1.8 \frac{gm}{ml}\right)$ have M = 3

(B) 9.8% H_3PO_4 by weight $\left(\text{density} = 1.22 \frac{gm}{ml}\right)$ have M = 1

(C) $\underbrace{1.8\ N_A \text{ molecules}}_{1.8 \text{ mole}}$ of HCl in 500ml have = 1.8 ~~gm~~ equivalent in solution.

(D) 250ml of 0.4 NNaOH + 250ml of 0.16M$Ca(OH)_2$ have pH = 13.44

(Use log 2 = 0.30, log 3 = 0.47)

(A) 9.8% (w/w)

9.8 g H_2SO_4 in 100g solution

$n_{H_2SO_4} = \frac{9.8}{98} = 0.1$ mole

Volume of sol. (ml) = $\frac{100}{1.8}$ ml

$M = \frac{0.1}{100/1.8} \times 1000 = 1.8\ M$

(B) $M = \frac{0.1}{100/1.22} \times 1000 = 1.22\ M$

(C) equivalent = mole × n-factor

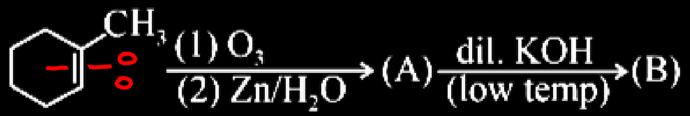
= 1.8 × 1

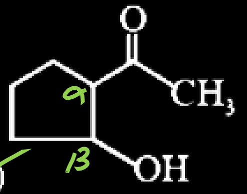
(D) $[OH^-] = \frac{N_1V_1 + N_2V_2}{V_1 + V_2} = \frac{(0.4 \times 250) + (0.16 \times 2 \times 250)}{250 + 250}$

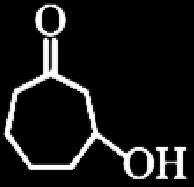
$pOH = -\log[36 \times 10^{-2}] = 2 - 2[\log 2 + \log 3]$ $= 0.36$

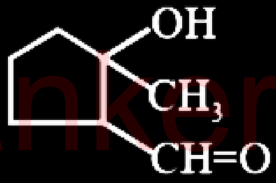
$pH = 14 - pOH$

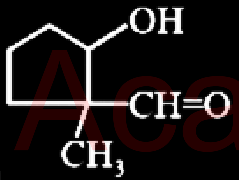
Find the major product of the given reaction.



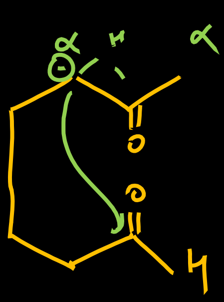
(A) 

(B) 

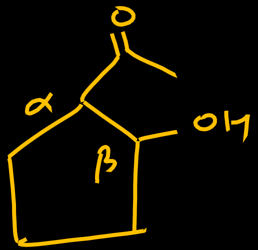
(C) 

(D) 

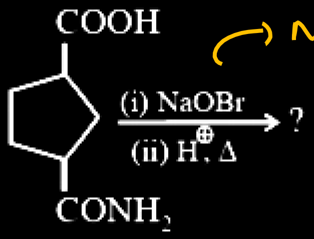
6 > 5 > 7 > 4 > 3



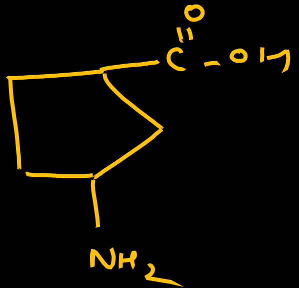
(A)

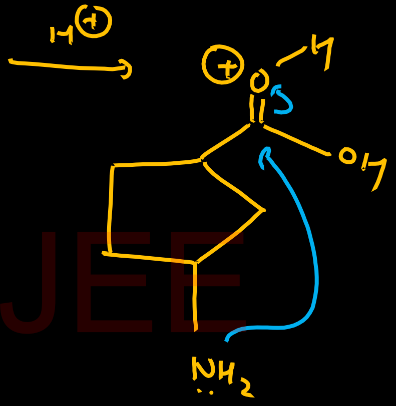


What is the final product of the following reaction?



$\rightarrow$ NaOH + Br_2 (HBD)

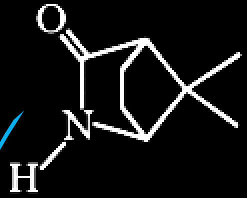




$-H_2O$

(A)

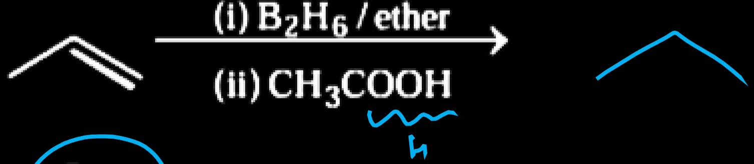

(B)


(C)


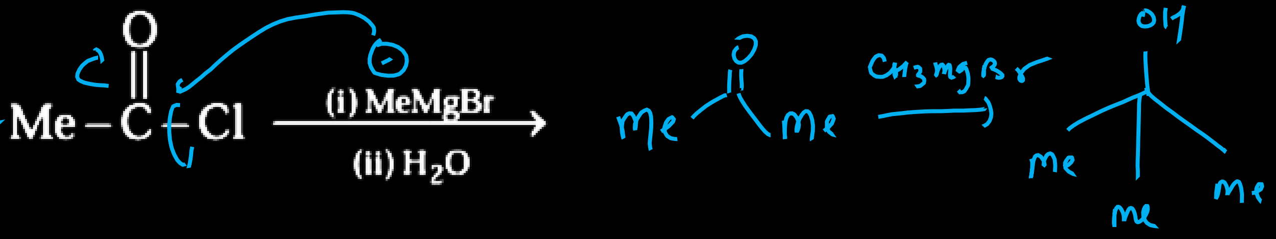
(D)

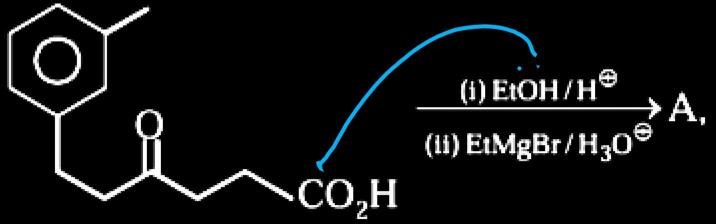

Which of the following reaction will not give propane?

(A) 

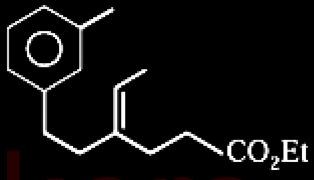
(B) 

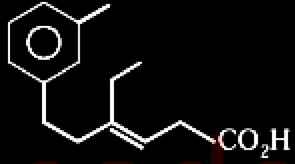
(C) 

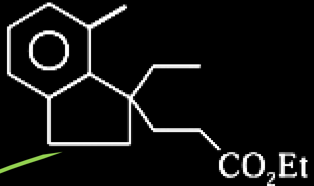
(D) 

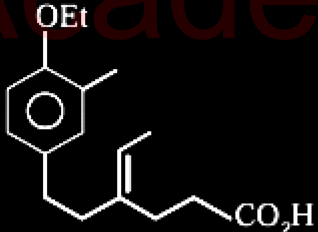


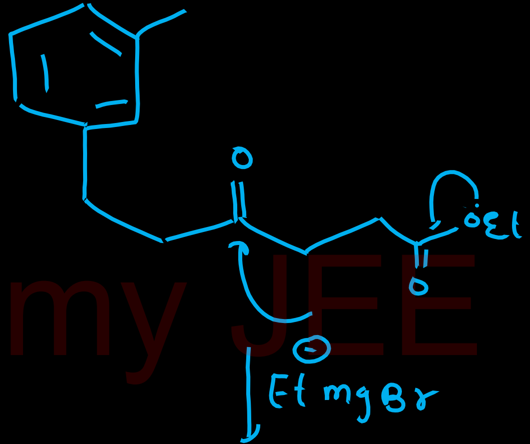
A will be:

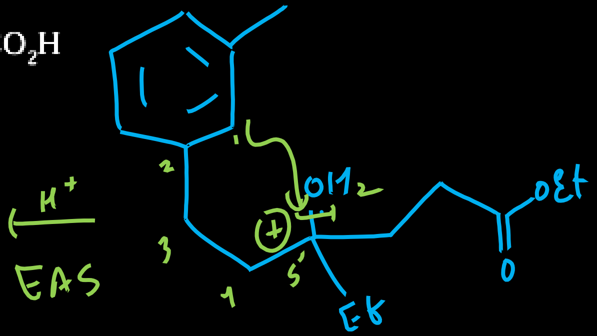
(A) 

(B) 

(C) 

(D) 





11

JEE 1

In which one of the following processes, magnetic nature of species does not change?

(A) $O_2^+ \longrightarrow O_2^{+2}$ (B) $CN^- \longrightarrow CN$

(C) $F_2 \longrightarrow F_2^+$ (D) $B_2 \longrightarrow B_2^+$

10, 16, odd : Para

even : Di

(A) $O_2^+ \longrightarrow O_2^{2+}$

15 Para, 14 di

(C) $F_2 \longrightarrow F_2^+$

18 Di, 17 Para

(B) $CN^- \longrightarrow CN$

14 Dimag, 13 Para

(D) $B_2 \longrightarrow B_2^+$

10 (Para), 9 (Para)

12

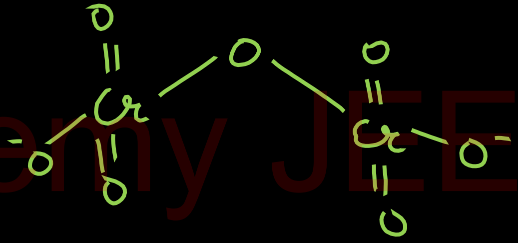
Which of the following statement is correct for $K_2Cr_2O_7$ structure?

(A) All Cr – O bonds have same bond length

(B) Bridge Cr – O bond has bond order equal to 2

(C) Hybridisation of Cr is dsp^2

(D) Two tetrahedron CrO_4 units having one common oxygen are found



13

JEE1

Velocity of electron in 2^{nd} orbit of He^{+} is

(A) 2.18×10^{6} m/s (B) 2.18×10^{3} m/s

(C) 4.36×10^{6} m/s (D) 4.36×10^{3} m/s

$$V = 2.18 \times 10^{6} \frac{Z}{n} \ m/s$$

$$= 2.18 \times 10^{6} \cdot \frac{\cancel{2}}{\cancel{2}} \ m/s$$

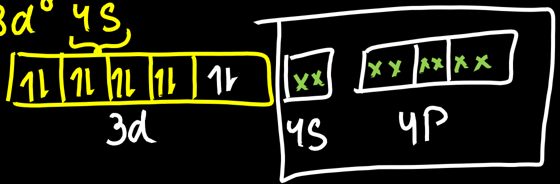
14

JEE 1

From the following, select the correct statement.

(~~A~~) $[Co(NH_3)_6]^{3+}$ is a high spin complex SL $3d^6$

(~~B~~) $[Ni(CO)_4]$ has tetrahedral geometry and is paramagnetic 0 $3d^8\ 4s^2$



(~~C~~) $[Fe(CN)_6]^{3-}$ is diamagnetic Fe^{+3}: $3d^5$

(D) $[Co(C_2O_4)_3]^{3-}$ is inner orbital complex having d^2sp^3 hybridisation Co^{+3}: $3d^6$

15

JEE 1

. Which of the following is correct combination?

Atomic Number	Name	Symbolw
(A) 105	Unnilpentium	Unp
(B) 107	Ununheptium	Uuh
(C) 109	Unnilennium	Unn
(D) 111	Unnilunium	Uuu

16

JEE 1

Which carbohydrate does not contain $C_1 - C_4$ glycosidic linkage?

(A) Maltose (B) Lactose

(C) Amylose (D) Sucrose

S 1, 2

M 1, 4

L 1, 4

17

JEE1

Amongst H_2O, H_2S, H_2Se and H_2Te the one with the highest boiling point is

(A) H_2O (B) H_2Te

(C) H_2S (D) H_2Se

For 16 th grp hydrides

18

Which of the following reaction is always non-spontaneous?

(A) $2H_2O_2(g) \rightarrow 2H_2O(g) + O_2(g), \Delta H = -211\ kJ,$
$\Delta S = 132\ J\ K^{-1}$

(B) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g), \Delta H = -92\ kJ,$
$\Delta S = -199\ J\ K^{-1}$

(C) $N_2O_4(g) \rightarrow 2NO_2(g), \Delta H = 57.2\ kJ,$
$\Delta S = 176\ J\ K^{-1}$

(D) $N_2(g) + 2O_2(g) \rightarrow 2NO_2(g), \Delta H = 66.36\ kJ, \Delta S = -122\ J\ K^{-1}$

$\Delta G = \Delta H - T\Delta S$

$-ve$	$-ve$	$+ve$	Alw spon
	$+ve$	$-ve$	" non-spon
	$-ve$	$> -ve$	spon
	$+ve$	$< +ve$	spon

19

JEE 1

In case of nitrogen, all oxidation states from +1 to +4 tend to disproportionate in acidic solution. Consider the disproportionation of HNO_2 which forms A and B

(Both are nitrogen containing compounds)

$HNO_2 \rightarrow A + B$

A has N in an oxidation state of x and B has N in an oxidation state of y where $x > y$. Select the correct statement.

(A) A cannot act as an oxidising agent

(B) B does not follow octet rule

(C) $x = 4$ and $y = 2$

(D) A is a gas whereas B is an oxoacid

$$HNO_2 \xrightarrow{H^+} \underset{(X)}{\overset{A}{\overset{+5}{HNO_3}}} + \underset{(Y)}{\overset{B}{\overset{+2}{NO}}}$$

$N=O$

20

Conversion of SO_2 to SO_3 finds great importance in the manufacture of sulphuric acid by contact process. Consider the following equilibrium

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g),\ K_c = 1.7 \times 10^{26}$$

Some statements are given w.r.t.the above process.

SI: The value of K_c is suggestive of reaction going to almost completion.

SII: Practically the oxidation of SO_2 to SO_3 is very slow.

SIII: Using catalyst such as V_2O_5 increases the rate of reaction.

SIV: Catalysts are of major help only for equilibriums having very small value of K_c ($K_c < 1$).

The correct statements from the ones given above are

(A) SI and SIII only

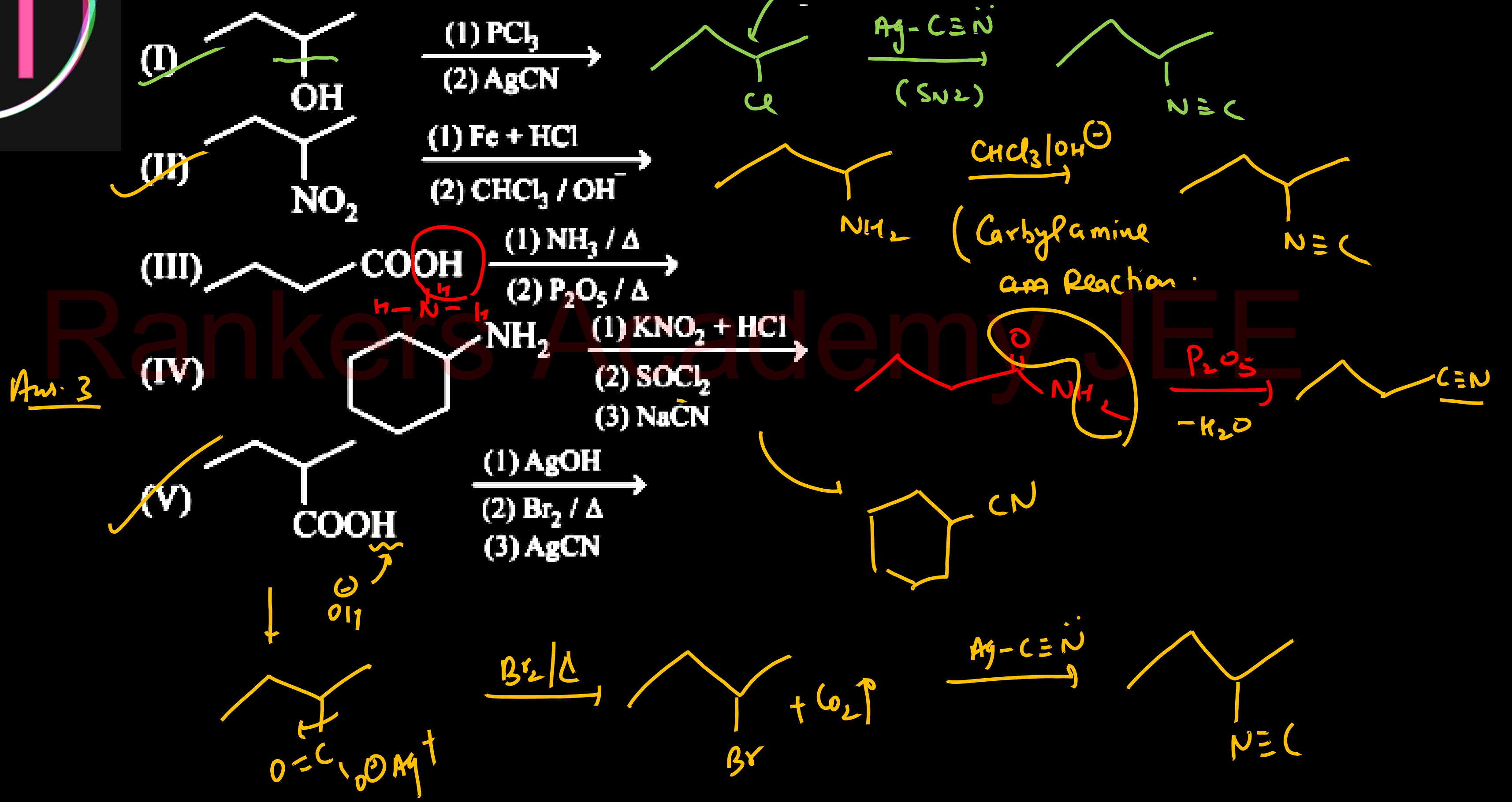
(B) SII and SIV only

(C) SI, SIII and SIV only

(D) SI, SII and SIII only

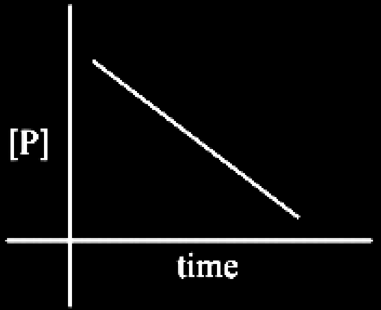
21

Total number of reactions in which isocyanides are formed



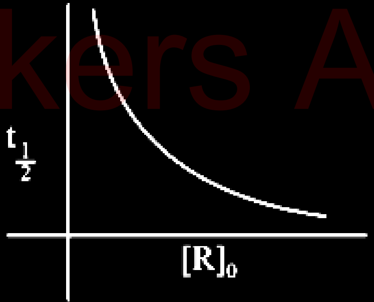
Find total order of a reaction $P + Q + R \longrightarrow$ product ,

for P



$[P]_t = [P]_0 - k_A t$ $\boxed{\text{Zero}}$

for R

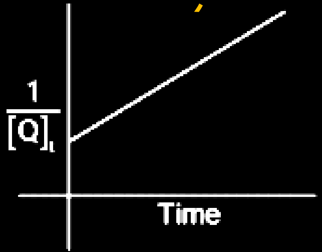


$t_{1/2} \propto [R]_0^{1-n}$ $n = 2$ $t_{1/2} \propto \frac{1}{[R]_0}$

$y \propto \frac{1}{x}$

$\boxed{2^{nd} \text{ order}}$

for Q



$\boxed{\frac{1}{[Q]_t} - \frac{1}{[Q]_0} = k_A t}$ $\rightarrow$ 2^{nd} order

23

JEE 1

Consider the following cell

Pt |

$H_2(P_1 \text{ atm})|H^+(M_1)||H^+(M_2)|H_2(P_2 \text{ atm})$ | Pt

Where P_1 and P_2 are pressures. M_1 and M_2 are molarities.

The emf of cell at 25°C if $P_1 = P_2$, and M_1 is 50% higher than M_2 $\quad m_1 = 1.5\, m_2$

is $-x \times 10^{-4}$, find x (nearest integer)

[Take : $\frac{2.303RT}{F} = 0.06$ and $\log 3 = 0.48$, $\log 2 = 0.3$]

$H_2(g) \longrightarrow 2H^+ + 2e^-$
$\quad P_1 \qquad\quad M_1$

$2H^+ + 2e^- \longrightarrow H_2(g)$
$\quad M_2 \qquad\qquad\quad P_2$

$H_2(g) + 2H^+ \longrightarrow H_2(g) + 2H^+$
$\quad P_1 \qquad M_2 \qquad\quad P_2 \qquad M_1$

$E_{cell} = 0 - \frac{0.06}{2} \log\left(\frac{M_1}{M_2}\right)^2$

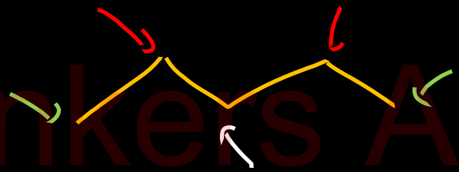
$= -0.06\left[\log 3 - \log 2\right]$

$E_{cell} = -108 \times 10^{-4}$

24 The total number of structural monochloroderivatives possible in C_5H_{12} is (exclude stereo)

JEE 1

$$DBE = (5+1) - \frac{12}{2} = 0$$



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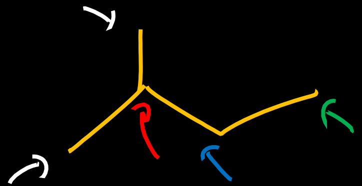

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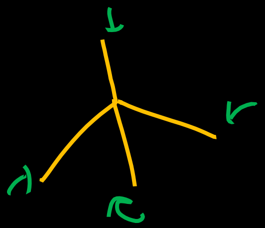
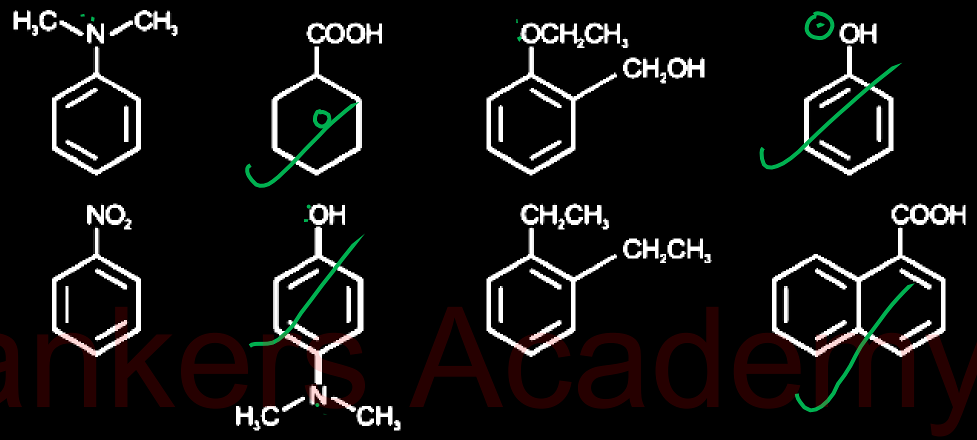


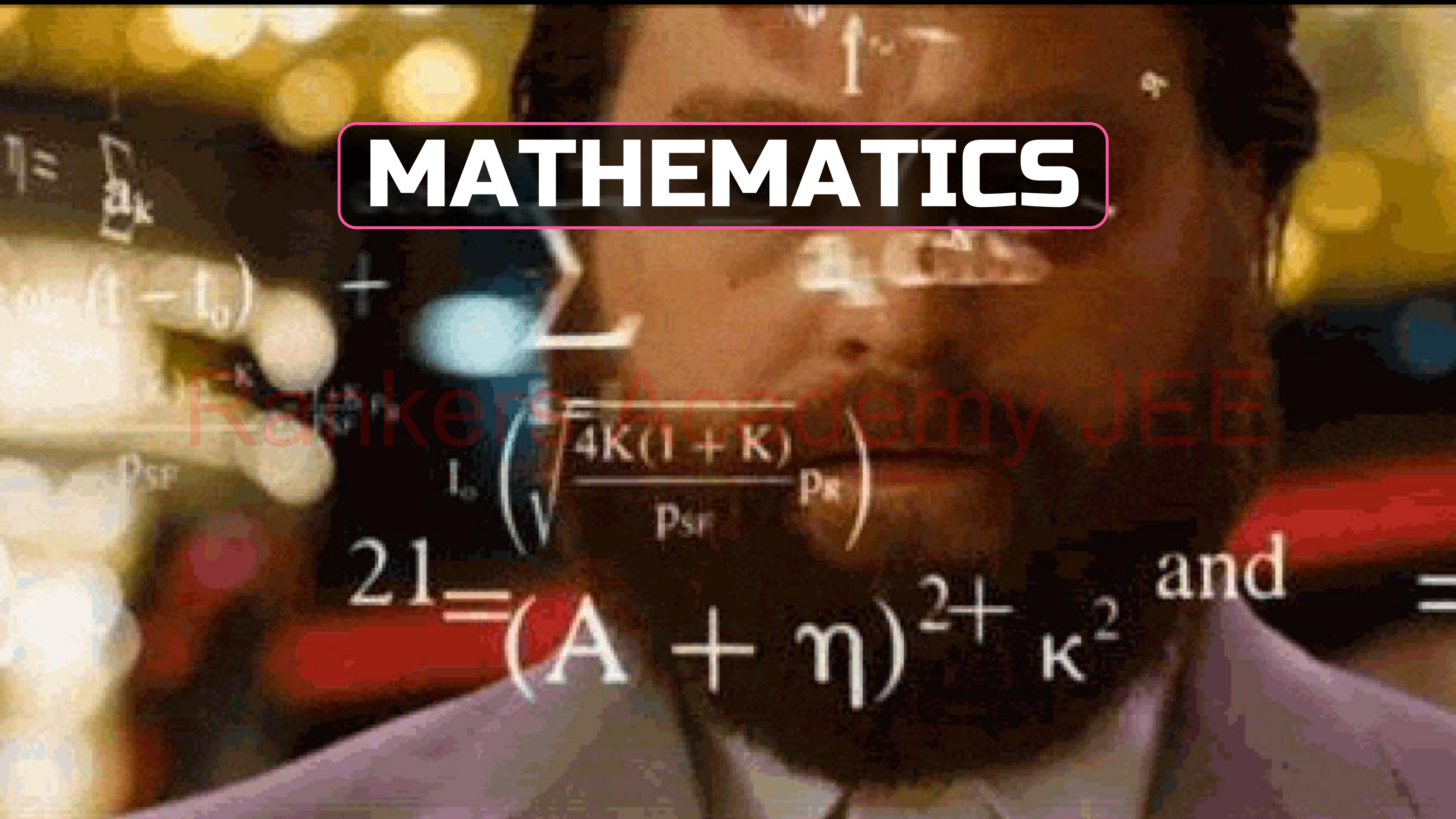
1

25

Amongst the following, the total number of compounds soluble in aqueous NaOH is



MATHEMATICS



Sum of global maxima and global minima of

$f(x) = 2x^3 - 9x^2 + 12x + 6$ in $[0,2]$

(A) 14 (B) 15

(C) 16 (D) 17 ✓

$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$

$\therefore x = 1, 2$

$f(0) = 6$ min

$f(1) = 11$ max

$f(2) = 16 - 36 + 24 + 6 = 10$

Ans: $6 + 11 = 17$

2

JEE 1

Letters of the word TITANIC are arranged to form all the possible words. What is the probability that a word formed starts either with a T or a vowel?

(A) $\frac{2}{7}$ (B) $\frac{4}{7}$

(C) $\frac{3}{7}$ (D) $\frac{5}{7}$

T I A N C
T I

$$D^r = \frac{7!}{2!\,2!} = \frac{7!}{4}$$

$\boxed{T}$ _ _ _ _ _ _ $\qquad \frac{6!}{2!}$

T, I, A N C
I

$\boxed{A}$ _ _ _ _ _ _ $\qquad \frac{6!}{2!\,2!}$

T I N C
T I

$\boxed{I}$ _ _ _ _ _ _ $\qquad \frac{6!}{2!}$

T I A N C
T

$$N^r = \underbrace{\frac{6!}{2!} + \frac{6!}{2!}}_{} + \frac{6!}{2!\,2!}$$

$$= 6! + \frac{6!}{4}$$

$$= \frac{5}{4} \cdot 6!$$

Let $f(x) = |x-1|([x]-[-x])$, then which of the following is correct, (where [.] denote greatest integer and |.| denote modulus function)

(A) $f(x)$ is continuous at $x = 1$

(B) $f(x)$ is derivable at $x = 1$

(C) $f(x)$ is discontinuous at $x = 1$

(D) None of these

$$f(x) = \begin{cases} -(x-1)([1-h]-[-1+h]) = -(x-1) & x<1 \\ 0 & ; \quad x=1 \\ (x-1)([1+h]-[-1-h]) & x>1 \end{cases}$$

$$= (x-1)(1-(-2))$$
$$= 3(x-1)$$

$$f(x) = \begin{cases} 1-x & ; \; x<1 \\ 0 & ; \; x=1 \\ 3(x-1) & ; \; x>1 \end{cases}$$

$f(1) = 0$

LHL $= 0$

RHL $= 0$

$$f'(x) = \begin{cases} -1 & ; \; x<1 \\ 3 & ; \; x>1 \end{cases}$$

LHD $\neq$ RHD

The least positive integer n, for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x \geq 0$ and $i = \sqrt{-1}$ is

~~(A) 0~~ (B) 1

(C) 2 (D) None

$x + \frac{1}{x} \geq 2$

$\frac{1}{2}\left(x+\frac{1}{x}\right) \geq 1$

$\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\underbrace{\frac{1}{2}\left(x+\frac{1}{x}\right)}_{\in[-1,1]}\right)$

At $x=1$ ✓

$\Rightarrow \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^n = \frac{2}{\pi}\sin^{-1}(1)$

$\Rightarrow \left(\frac{\cancel{1}+\cancel{i^2}+2i}{1-i^2}\right)^n = \frac{2}{\pi}\cdot\frac{\pi}{2}$

$\Rightarrow i^n = 1$

$\therefore \boxed{n=4}$

5

JEE 1

$\lim_{n\to\infty}\left(\sqrt{n^2+n+1}-\left[\sqrt{n^2+n+1}\right]\right)$,

(where [.] denote greatest integer function)

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

$n^2 < n^2+n+1 < n^2+2n+1$

$n < \sqrt{n^2+n+1} < n+1$

$\left[\sqrt{n^2+n+1}\right] = n$

$\lim_{n\to\infty} \sqrt{n^2+n+1} - n$

$\lim_{y\to 0} \sqrt{\frac{1}{y^2}+\frac{1}{y}+1} - \frac{1}{y}$

$\lim_{y\to 0} \frac{\sqrt{1+y+y^2}-1}{y}$ $\left(\frac{0}{0}\right)$

$\lim_{y\to 0} \frac{(1+2y)}{2\sqrt{1+y+y^2}} \Rightarrow \frac{1}{2}$ ✓

6

JEE 1

The shortest distance between the lines

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1} \text{ and } \frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3} \text{ is:}$$

(A) $\frac{18}{\sqrt{5}}$ ✓

(B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$

(D) $\frac{22}{3\sqrt{5}}$

$$sd = \left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$\vec{a_2}-\vec{a_1} = (-3\hat{i}+6\hat{j}+5\hat{k})-(3\hat{i}+2\hat{j}+\hat{k})$$

$$= -6\hat{i}+4\hat{j}+4\hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix} = 10\hat{i}-8\hat{j}-4\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{100+64+16} = \sqrt{180} = 3\sqrt{20} = 6\sqrt{5}$$

JEE 1

$$sd = \left| \frac{(-6)(10) + 4(-8) + 4(-4)}{\sqrt{180}} \right|$$

$$= \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}.$$

7

Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of second set. The value of m and n are :-

(A) 7,6

(B) 6,3

(C) 5, 1

(D) 8,7

$2^m - 2^n = 56$

8

If f(x) is a decreasing function and attain positive value, then set of values of ' K ' for which the major axis of ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$ is the x-axis is $(-a, b)$, then $a+b$ is

(A) 2
(B) 3
(C) 4
(D) 5

JEE 1

$f(k^2+2k+5) > f(k+11)$

$k^2+2k+5 < k+11$

$k^2+k-6<0$

$(k+3)(k-2)<0$

$k \in (-3,2)$

$a=3$

$b=2$

$a+b=5$

9

JEE 1

If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C, x > 0$ where C is the constant of integration, then value of $9(\sqrt{3}a + b)$ is equal to

(A) 15 (B) $15\sqrt{3}$

(C) $2\sqrt{3} + 5$ (D) $9\sqrt{3} + 27$

$a = \frac{4}{3\sqrt{3}}$ $b = \frac{1}{3}$

$9(\sqrt{3}a+b)$

$9\left(\frac{4}{3}+\frac{1}{3}\right) = 15.$

$\int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right)^2}$

Let $x+\frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$

$dx = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$

$\frac{2x+1}{\sqrt{3}} = \tan\theta$

$= \frac{\sqrt{3}}{2}\int \frac{\sec^2\theta\, d\theta}{\left(\frac{3}{4}\tan^2\theta+\frac{3}{4}\right)^2}$

$= \frac{\sqrt{3}}{2}\int \frac{\sec^2\theta\, d\theta}{\left(\frac{3}{4}\sec^2\theta\right)^2}$

$= \frac{\sqrt{3}}{2}\cdot\frac{16}{9}\int \cos^2\theta\, d\theta$

$\frac{4}{3\sqrt{3}}\int 1+\cos 2\theta\, d\theta$

$\frac{4}{3\sqrt{3}}\left[\theta + \frac{\sin 2\theta}{2}\right]+c$

$\frac{4}{3\sqrt{3}}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{(2x+1)\sqrt{3}}{4(x^2+x+1)}\right]+c$

$b = \frac{4}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{4} = \frac{1}{3}$

$$\frac{\sin 2\theta}{2} = \frac{2\tan\theta}{2\left(1+\tan^2\theta\right)}$$

$$= \frac{2\left(\frac{2x+1}{\sqrt{3}}\right)}{2\left(1+\left(\frac{2x+1}{\sqrt{3}}\right)^2\right)}$$

$$= \frac{2(2x+1)\sqrt{3}}{2\left(3+(2x+1)^2\right)}$$

$$= \frac{2(2x+1)\sqrt{3}}{2\left(3+4x^2+4x+1\right)}$$

$$= \frac{\cancel{2}(2x+1)\sqrt{3}}{\cancel{2}\,4\left(x^2+x+1\right)}$$

$$=$$

10

f(x) is differentiable function which satisfies the equation

$f(x) = -\int_0^x f(t)\tan t\,dt + \boxed{\int_0^x \tan(t-x)dt}$ → King $t \to 0 + x - t$

where $x \in (-\pi/2, \pi/2)$, passes through (0,0) then the maximum value of f(x) is

(A) 1 (B) -1

(C) 0 (D) 2

$\int_0^x \tan(x-t-x)\,dt$

$\int_0^x \tan(-t)\,dt$

$-\int_0^x \tan t\,dt$

$$\boxed{f'(x) = -f(x)\tan x - \tan x}$$

$$\boxed{\frac{dy}{dx} + y\tan x = -\tan x}$$

JEE 1

10

$$IF = e^{\int \tan x\, dx} = e^{\ln|\sec x|} = |\sec x| = \sec x$$

$$y(\sec x) = -\int \sec x \tan x\, dx$$

$$y(\sec x) = -\sec x + c$$

$$\boxed{y = -1 + c\cos x}$$

$x=0 \quad y=0$

$0 = -1 + c \Rightarrow c = 1$

$y = -1 + \underbrace{\cos x}$ ✓

$\max = 1$

$y = 0$

11

JEE 1

If $t_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1,2,3, \ldots\ldots\ldots$

then $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \cdots \ldots\ldots + \frac{1}{t_{2011}} = ?$

(A) $\frac{4022}{3021}$ (B) $\frac{2011}{3021}$

(C) $\frac{4006}{2011}$ (D) None

$$\sum_{n=1}^{2011} \frac{4}{(n+2)(n+3)} = 4\sum \frac{(n+3)-(n+2)}{(n+2)(n+3)} = 4\sum_{n=1}^{2011} \frac{1}{n+2} - \frac{1}{n+3}$$

$$= 4\left[\frac{1}{3} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} \ldots - \frac{1}{2014}\right]$$

$$= 4\left[\frac{1}{3} - \frac{1}{2014}\right]$$

$$= \frac{\cancel{4}^{2} \cdot 2011}{3 \cdot \cancel{2014}_{1007}}$$

$$\frac{4022}{3021}$$

The value of $\int_1^{\frac{1+\sqrt5}{2}} \frac{(x^2+1)}{(x^4-x^2+1)}\log\left(1+x-\frac{1}{x}\right)dx$ is

(A) $\frac{\pi}{8}\log_e 2$

(B) $\frac{\pi}{2}\log_e 2$

(C) $-\frac{\pi}{2}\log_e 2$

(D) None of these

$$\int_1^{\frac{1+\sqrt5}{2}} \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2-1+\frac{1}{x^2}\right)} \ln\left(1+\left(x-\frac{1}{x}\right)\right)(dx)$$

Let $x-\frac{1}{x}=t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

$$\int_0^1 \frac{\ln(1+t)}{t^2+1}\,dt$$

$t = \frac{\sqrt5+1}{2} - \frac{2}{\sqrt5+1}$

$= \frac{\sqrt5+1}{2} - \frac{\cancel{2}(\sqrt5-1)}{\cancel{4}\,2}$

$= 1$

12

JEE 1

Ans : $\displaystyle\int_0^1 \frac{\ln(1+t)}{t^2+1}\,dt$

Let $t = \tan\theta$

$dt = \sec^2\theta\, d\theta$

$= \displaystyle\int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{\cancel{\sec^2\theta}} \cdot \cancel{\sec^2\theta}\, d\theta$

$I = \text{Ans} = \displaystyle\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)\, d\theta$

King $\theta \longrightarrow 0 + \frac{\pi}{4} - \theta$

$I = \displaystyle\int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) d\theta$

$= \displaystyle\int_0^{\frac{\pi}{4}} \ln\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta$

$= \displaystyle\int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan\theta}\right) d\theta$

$= \displaystyle\int_0^{\frac{\pi}{4}} \ln 2 - \ln(1+\tan\theta)\, d\theta$

$I = \frac{\pi}{4}\ln 2 - I$

12

JEE 1

$$I = \frac{\pi}{4}\ln 2 - I$$

$$2I = \frac{\pi}{4}\ln 2$$

$$I = \frac{\pi}{8}\ln 2 \checkmark$$

13

JEE 1

. The value of

$$\lim_{n\to\infty} n^2 \left\{ \sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)\ldots\infty}}} \right\} \text{ is :}$$

(A) 1

(B) 2

(C) 0

(D) 1/2

Let $n = 1/y$

$$\lim_{y\to 0} \frac{1}{y^2} (1-\cos y)^{1/2} (1-\cos y)^{1/4} (1-\cos y)^{1/8} \ldots$$

$$= \lim_{y\to 0} \frac{1}{y^2} (1-\cos y)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots}$$

$$= \lim_{y\to 0} \frac{1}{y^2} (1-\cos y)^{\frac{1/2}{1-1/2}} = \lim_{y\to 0} \frac{1-\cos y}{y^2} = \frac{1}{2}.$$

14

JEE 1

If f: R → R is defined by :-

$$f(x) = \begin{cases} x + 4 & \text{for} \quad x < -4 \\ 3x + 2 & \text{for} \quad -4 \le x < 4 \\ x - 4 & \text{for} \quad x \ge 4 \end{cases}$$

then the correct matching of List I from List II is

	List-I		List-II
(a)	f(–5) + f(–4)	(i)	14
(b)	f(|f(–8)|)	(ii)	4
(c)	f(f(– 7) + f(3))	(iii)	–11
(d)	f(f(f(f(0)))) + 1	(iv)	–1
		(v)	1
		(vi)	0

	(a)	(b)	(c)	(d)
(A)	(iii)	(vi)	(ii)	(v)
(B)	(iii)	(iv)	(ii)	(v)
(C)	~~(iv)~~	~~(iii)~~	~~(ii)~~	~~(i)~~
(D)	(iii)	(vi)	(v)	(ii)

$f(-5) + f(-4) = (-5+4) + (3(-4)+2)$

$= (-1) + (-10) = -11$

$f(|f(-8)|) = f(|-8+4|) = f(4)$

$= 4-4 = 0$

$f((-3) + 11) = f(8) = 8-4 = 4$

15

JEE 1

If f: R ⟶ R&

$$f(x) = \cancel{\boxed{\frac{\sin([x]\pi)}{x^2+2x+3}}} + 2x - 1 + \sqrt{x(x-1)+\frac{1}{4}}$$

(where [x] denotes integral part of x), then f(x) is -

int = n $\sin(n\pi) = 0$

~~(A)~~ one-one but not onto

~~(B)~~ one-one & onto

(C) onto but not one-one

~~(D)~~ neither one-one nor onto

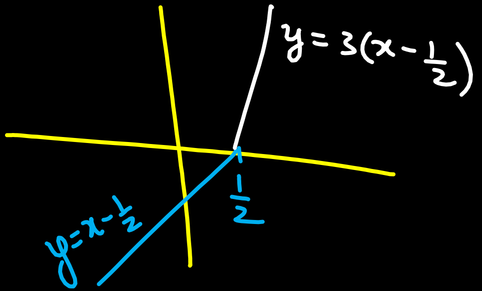
$$\therefore f(x) = (2x-1) + \sqrt{x^2 - x + \frac{1}{4}}$$

$$= 2x - 1 + \sqrt{\left(x-\frac{1}{2}\right)^2}$$

$$= 2x - 1 + \left|x - \frac{1}{2}\right|$$

$$= 2\left(x-\frac{1}{2}\right) + \left|x-\frac{1}{2}\right|$$

$$= \begin{cases} 2\left(x-\frac{1}{2}\right) - \left(x-\frac{1}{2}\right) = x - \frac{1}{2} & x \le \frac{1}{2} \\ 2\left(x-\frac{1}{2}\right) + \left(x-\frac{1}{2}\right) = 3\left(x-\frac{1}{2}\right) & x > \frac{1}{2} \end{cases}$$



16

If $f(x) = \sqrt{4 - x^2} + \sqrt{x^2 - 1}$, then range of f(x) is –

(A) $[\sqrt{3}, \sqrt{7}]$

(B) $[\sqrt{3}, \sqrt{5}]$

(C) $[\sqrt{2}, \sqrt{3}]$

(D) $[\sqrt{3}, \sqrt{6}]$

JEE 1

$x^2 = 4 \longrightarrow f(\ ) = \sqrt{3}$

$x^2 = 1 \longrightarrow = \sqrt{3}$

$x^2 = \frac{5}{2} \longrightarrow \sqrt{4 - \frac{5}{2}} + \sqrt{\frac{5}{2} - 1}$

$= \sqrt{\frac{3}{2}} + \sqrt{\frac{3}{2}}$

$= \sqrt{6}$

17

JEE1

Let $\vec{u}, \vec{v}, \vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}$, $\vec{a} \cdot \vec{u} = \frac{3}{2}$, $\vec{a} \cdot \vec{v} = \frac{7}{4}$ and $|\vec{a}| = 2$ then

Which option is incorrect?

$|u| = |v| = |w| = 1$

$\vec{a} \cdot \vec{w} = 3/4$

(A) $\boxed{\vec{u} \cdot \vec{v} = \frac{3}{4}}$

(B) $\vec{v} \cdot \vec{w} = 0$

(C) $\vec{u} \cdot \vec{w} = \frac{-1}{4}$

(D) $\vec{u} \cdot \vec{v} = 1$

$\vec{u} + \vec{v} + \vec{w} = \vec{a}$

dot with $\vec{a}$

$\vec{a} \cdot \vec{u} + \vec{a} \cdot \vec{v} + \vec{a} \cdot \vec{w} = \vec{a} \cdot \vec{a} = |\vec{a}|^2$

$\frac{3}{2} + \frac{7}{4} + \vec{a} \cdot \vec{w} = 4 \Rightarrow \vec{a} \cdot \vec{w} = 3/4$

17

JEE 1

$$\Rightarrow \vec{u} + \vec{v} = \vec{a} - \vec{w}$$

$$\Rightarrow |\vec{u} + \vec{v}|^2 = |\vec{a} - \vec{w}|^2$$

$$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u}\cdot\vec{v} = |\vec{a}|^2 + |\vec{w}|^2 - 2\vec{a}\cdot\vec{w}$$

$$1 + 1 + 2\vec{u}\cdot\vec{v} = 4 + 1 - 2\cdot\frac{3}{4}$$

$$\vec{u}\cdot\vec{v} = 3/4$$

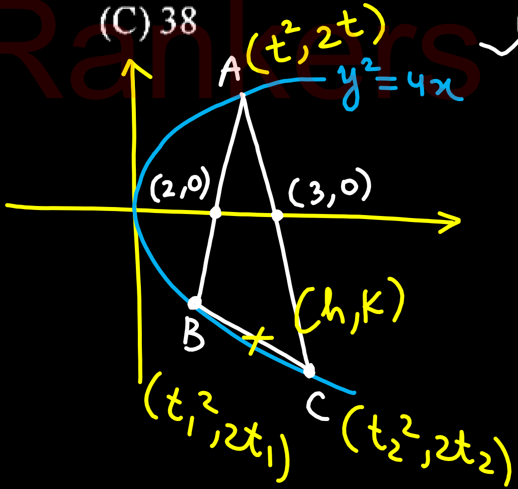
18

A $\triangle ABC$ is inscribed in the parabola $y^2 = 4x$ such that AB and AC always passes through (2,0) and (3,0) respectively. The locus of midpoint of side BC is another parabola whose latus rectum is $\frac{p}{q}$ (where p and q are coprime positive integers then the value of $p + q$ is

(A) 50 (B) 53

(C) 38 (D) 63 ✓

$a = 1$



$$\frac{2t-0}{t^2-2} = \frac{2t-2t_1}{t^2-t_1^2}$$

or

$$AB:\ y - 2t = \left(\frac{2t-2t_1}{t^2-t_1^2}\right)(x - t^2)$$

$$(2,0) \longrightarrow y - 2t = \left(\frac{2}{t+t_1}\right)(x - t^2)$$

$$0 - 2t = \frac{2}{(t+t_1)}(2 - t^2)$$

$$-\cancel{t^2} - tt_1 = 2 - \cancel{t^2}$$

$$tt_1 = -2 \ ✓$$

$$tt_2 = -3 \ ✓$$

$$\left.\begin{aligned} t\,t_1 &= -2 \\ t\,t_2 &= -3 \end{aligned}\right\}$$

$$h = \frac{t_1^2 + t_2^2}{2} = \frac{\frac{4}{t^2} + \frac{9}{t^2}}{2} = \frac{13}{2t^2}$$

$$K = \frac{2t_1 + 2t_2}{2} = t_1 + t_2 = -\frac{2}{t} - \frac{3}{t} = -\frac{5}{t}$$

$$t = \frac{-5}{K}$$

$$h = \frac{13}{2\left(-\frac{5}{K}\right)^2}$$

$$h = \frac{13}{50}K^2$$

$$x = \frac{13}{50}y^2$$

$$\boxed{y^2 = \frac{50}{13}x} = 4a\,x$$

19

JEE 1

For 2n observations

$a_1, -a_1, a_2, -a_2, a_3, -a_3, \ldots\ldots\ldots, a_n, -a_n$ where all a′ s are distinct. Let $\bar{x}, \sigma$ and M denotes mean, standard deviation and median, respectively. Then which of the following option is incorrect when n is an odd integer?

~~(A)~~ $\bar{x} = 0$

~~(B)~~ $\bar{x} = M$

~~(C)~~ $\sigma > \bar{x}$

~~(D)~~ $\sigma < M$ ✓

Shortcut :

$-30, \cancel{-20}, \cancel{-10}, \cancel{10}, \cancel{20}, 30$

$-3, -2, -1, +1, 2, 3$

$$\bar{x} = \text{Mean} = \frac{(a_1)+(-a_1)+(a_2)+(-a_2)+\ldots+(a_n)+(-a_n)}{2n} = 0$$

$$\sigma = \sqrt{\frac{\sum x_i^2}{2n} - \cancel{(\bar{x})^2}} = \sqrt{\frac{\cancel{2}(a_1^2+a_2^2+\ldots+a_n^2)}{\cancel{2}n}}$$

$-a_n, -a_{n-1} \ldots -a_2, -a_1, a_1, a_2 \ldots a_{n-1}\ a_n$

$$\text{Median} = \frac{(-a_1)+(a_1)}{2} = 0$$

20

JEE 1

Area bounded by $f(x) = \min(\{x-2\}, \{3-x\})$, $x = 4$ and $x = -4$ and x-axis is equal to

($\{\cdot\}$ denotes fractional part function)

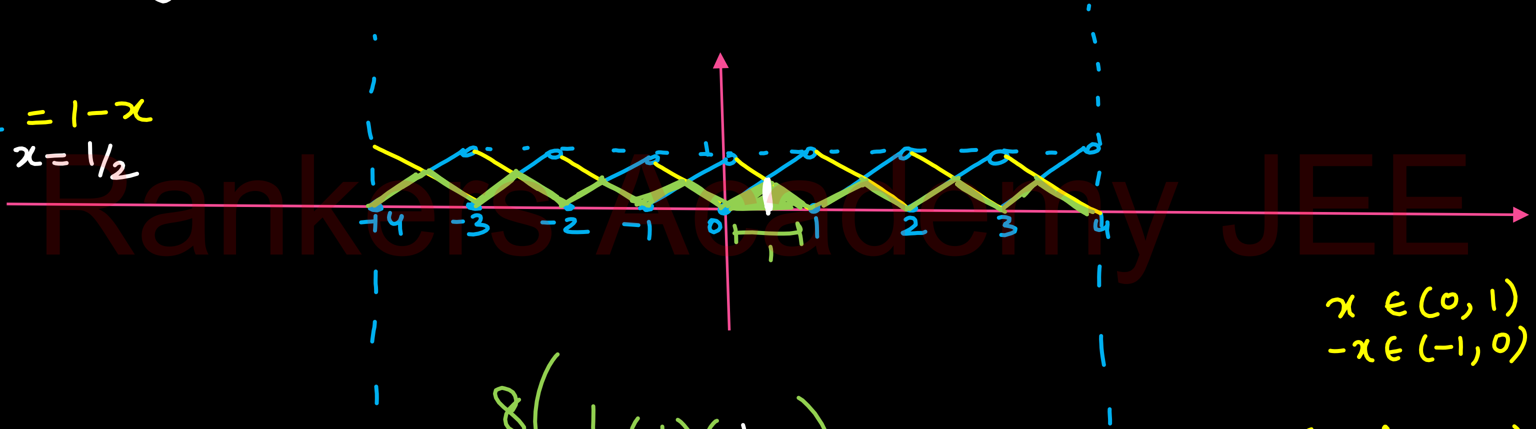
(A) 1 sq. unit
(B) 3/2 sq. units
(C) 2 sq. units
(D) 4 sq. units

$\{x - \cancel{2}\} = \{x\}$

$\{\cancel{3} - x\} = \{-x\}$

$x = 1 - x$

$x = 1/2$



$x \in (0, 1)$

$-x \in (-1, 0)$

$8\left(\frac{1}{2}(1)\left(\frac{1}{2}\right)\right)$

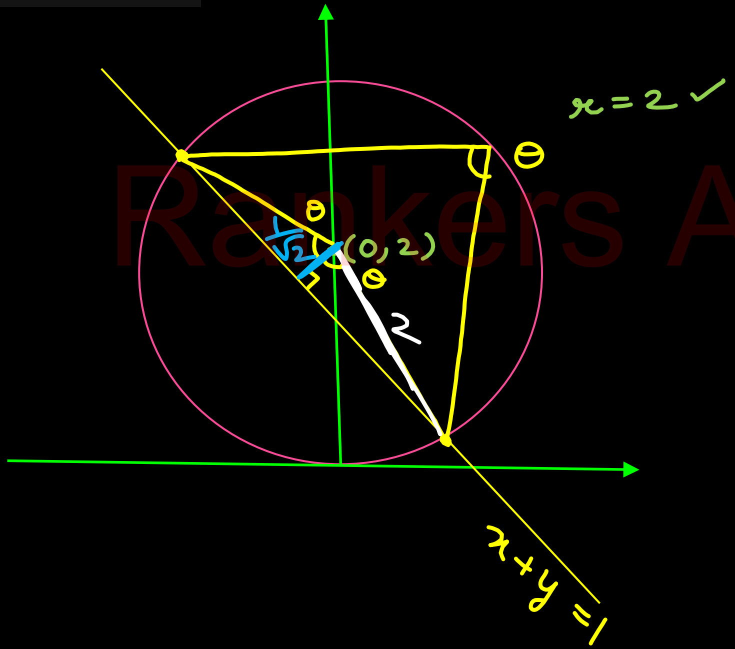
$\{-x\} = (-x) - [-x]$

$= (-x) - (-1)$

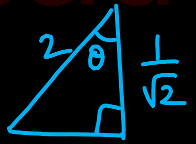
$= 1 - x$

21

If the angle subtended by the chord $x + y = 1$ of the circle $x^2 + y^2 - 4y = 0$. At the circumference of the larger segment be θ, then the value of $\sec^2\theta$ is equal to 8 (Ans.)



$$\left|\frac{0+2-1}{\sqrt{1^2+1^2}}\right| = \frac{1}{\sqrt{2}}$$



$$\cos\theta = \frac{1/\sqrt{2}}{2} = \frac{1}{\sqrt{8}}$$

$$\sec\theta = \sqrt{8}$$

22

If $\vec{a}, \vec{b}$ are any two perpendicular vectors of equal magnitude and $|3\vec{a} + 4\vec{b}| + |4\vec{a} - 3\vec{b}| = 20$, then $|\vec{a}|$ equals: $\Rightarrow 2$ Ans

JEE 1

$|\vec{a}| = |\vec{b}| = k$

$\vec{a} \cdot \vec{b} = 0$

$|3\vec{a} + 4\vec{b}|^2 = 9|\vec{a}|^2 + 16|\vec{b}|^2 + 24\,\cancel{\vec{a} \cdot \vec{b}} = 25k^2$

$|4\vec{a} - 3\vec{b}|^2 = 16|\vec{a}|^2 + 9|b|^2 - 24\,\cancel{\vec{a} \cdot \vec{b}} = 25k^2$

$\therefore\ 5k + 5k = 20 \Rightarrow k = 2$

23

JEE1

The value of $\int_0^{\pi/2} x\left|\sin^2 x - \frac{1}{2}\right| dx$ is equal to $\frac{a\pi}{b}$ where a, b are co-prime numbers, then a · b is 8 ✓ ______.

$\sin^2 x = \frac{1}{2}$

$x = \pi/4$ ✓

$$\text{ans} = -\int_0^{\pi/4} x\left(\sin^2 x - \frac{1}{2}\right) dx + \int_{\pi/4}^{\pi/2} x\left(\sin^2 x - \frac{1}{2}\right) dx$$

$$= +\frac{1}{2}\int_0^{\pi/4} x\cos 2x \;-\frac{1}{2}\int_{\pi/4}^{\pi/2} x\cos 2x\, dx$$

$$= \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_0^{\pi/4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_{\pi/4}^{\pi/2}$$

D	I
①	②
$+x$	$\cos 2x$
-1	$\frac{\sin 2x}{2}$
0	$-\frac{\cos 2x}{4}$

$$\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}$$

$$= \frac{1}{2}\left[\frac{\frac{\pi}{4}(1)}{2} - \cancel{\frac{1}{4}}\right] - \frac{1}{2}\left[-\cancel{\frac{1}{4}} - \frac{\frac{\pi}{4}}{2}\right]$$

$$= \frac{\pi}{8} \checkmark$$

$$\left.\begin{array}{l} a = 1 \\ b = 8 \end{array}\right\} \rightarrow ab = 8 \checkmark$$

24

JEE 1

Let a, b, c, d are distinct real numbers and a, b are the roots of the quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$, the sum of the digits of numerical values of $a + b + c + d$ is Ans: 3.

$x^2 - 2cx - 5d = 0 < \begin{matrix} a \\ b \end{matrix}$

$a + b = 2c$ (1)

$a^2 - 2ac - 5d = 0$ (5)

$x^2 - 2ax - 5b = 0 < \begin{matrix} c \\ d \end{matrix}$

$c + d = 2a$ (2)

$c^2 - 2ac - 5b = 0$ (6)

(1) + (2)

$\cancel{a} + b + \cancel{c} + d = \cancel{2}(a+c)$

$b + d = a + c$ ✓ — (3)

(1) − (2)

$(a-c) + (b-d) = 2(c-a)$

$b - d = 3(c-a)$ — (4)

$d - b = 3(a-c)$

$\therefore a+b+c+d = 2(a+c)$

$= 2(15) = 30$

$(5) - (6)$

$$(a^2 - c^2) - 5(d-b) = 0$$

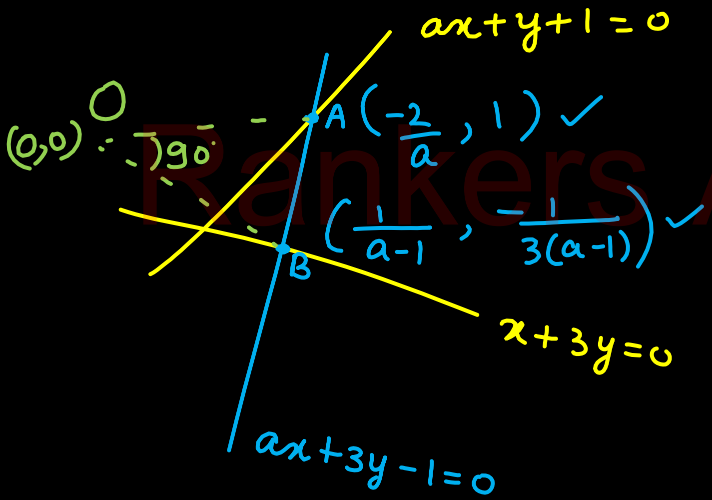
$$(a+c)(a-c) = 5(d-b)$$

$$(a+c)\cancel{(a-c)} = 5 \cdot 3\cancel{(a-c)}$$

$$a+c = 15$$

25

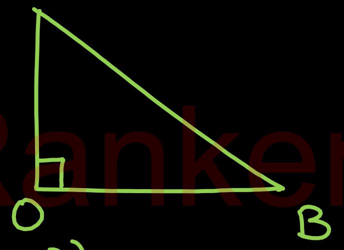
The portion of the line $ax + 3y - 1 = 0$, intercepted between the lines $ax + y + 1 = 0$ and $x + 3y = 0$ subtend a right angle at origin, then the value of $|a|$ is 6.



$$\begin{aligned} ax + y + 1 &= 0 \\ ax + 3y - 1 &= 0 \\ \hline 2y - 2 &= 0 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} x + 3y &= 0 \\ ax + 3y &= 1 \\ \hline (a-1)x &= 1 \\ x &= \frac{1}{a-1} \end{aligned}$$

$A\left(-\frac{2}{a}, 1\right)$



$(0,0)$

$\left(\frac{1}{a-1}, \frac{-1}{3(a-1)}\right)$

$$m_{AO} \cdot m_{BO} = -1$$

$$\left(\frac{1-0}{-\frac{2}{a}-0}\right)\left(\frac{\frac{-1}{3(a-1)}-0}{\frac{1}{a-1}-0}\right) = -1$$

$$\left(+\frac{a}{2}\right)\left(+\frac{1}{3}\right) = -1$$

$$a = -6$$