Test Pattern

JEE (Main)
UNIT TEST # 06
13-10-2024

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME
(Academic Session : 2024 - 2025)

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	A	B	B	C	C	C	A	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	B	D	D	C	A	D	B	C	D
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	5	2	5880	2	1420	20	60	220	1	20

PART-2 : CHEMISTRY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	D	D	B	B	A	B	A	D	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	A	A	A	D	D	B	D	C	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	6	6	20	3	5	2	3	6	7	6

PART-3 : MATHEMATICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	C	C	B	A	A	B	D	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	D	B	C	C	A	B	B	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	27	114	26	64	2	4	4	2	2

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (C)**

$$F = mg\cos\theta = qVB$$

$$v = \frac{mg\cos\theta}{qB}$$

$$\ell = \frac{v}{g\sin\theta} = \frac{mg\cos\theta}{g\sin\theta\ qB} = \frac{m\cos\theta}{(\sin\theta)\ qB}$$

2. **Ans (A)**

$$\vec{B} = \frac{\mu_0}{4\pi}\oint\frac{I\vec{d\ell}\times\vec{r}}{r^3}$$

$$\vec{d\ell} = -df\,\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

3. **Ans (B)**

$$r = \frac{mv}{qB}$$

4. **Ans (B)**

$$\vec{\tau} = \vec{M}\times\vec{B} = MB\sin\theta$$

$$\tau = i\pi R^2 B\sin\theta$$

At equilibrium

$$i\pi R^2\ B\sin\theta = mg\ R\sin\theta$$

$$B = \frac{mg}{\pi iR}$$

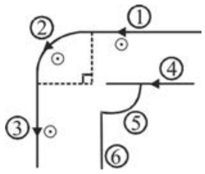
5. **Ans (C)**

$$U = -\vec{M}\,.\vec{B}$$

7. Ans (C)

$\odot B_1 = \frac{\mu_0 i}{4\pi R}, \overset{\odot}{B_2} = \frac{\mu_0 i}{8R}$

$\overset{\odot}{B_3} = \frac{\mu_0 i}{4\pi R}, \quad B_4 = 0, \quad B_6 = 0$



$\otimes B_5 = \frac{\mu_0 i}{8R}$

$B_{net} = \frac{\mu_0 i}{2\pi R}$

8. Ans (A)

$r = \frac{\sqrt{2mk}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

$r \propto \sqrt{\frac{m}{q}}$

$r_p \propto \sqrt{\frac{on}{e}}$

$r_d \propto \sqrt{\frac{2m}{e}}$

$r_\alpha \propto \sqrt{\frac{4m}{2e}}$

9. Ans (C)

Magnetic fore will act on the charge, So, it will follow helical path.

10. Ans (A)

$F_{net} = F_{AD} - F_{BC}$

11. Ans (A)

$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$= \frac{5(2\hat{i} - 7\hat{j} + 3\hat{k}) + 1(-10\hat{i} + 35\hat{j} - 3\hat{k})}{5+1} = 2\hat{k}\ \text{m/s}$

Thus, the centre of mass of the two-body system moves along the z-axis only

12. Ans (B)

In an inelastic collision, neutron (mass m) sticks with the α particle (mass 4m) after collision. Momentum remains conserved but the kinetic energy does not remain conserved.

From momentum conservation principle

$mv + 0 = (m + 4m)V$

where v is the velocity of neutron before collision and V is the velocity of composite particle (neutron + a- particle) after collision.

Thus, $V = \frac{v}{5}$

$K_1 = \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$

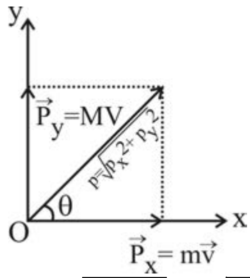
$K_2 = \frac{1}{2}(m + 4m)V^2 = \frac{1}{2}5m \times \left(\frac{v}{5}\right)^2$

$= \frac{1}{5}\left(\frac{1}{2}mv^2\right)$

Fraction of KE lost

$= \frac{K_1 - K_2}{K_1} = \frac{1 - \frac{1}{5}}{1} = \frac{4}{5}$

13. Ans (D)



$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(mv)^2 + (MV)^2}$

Thus, choice (3) is correct.

Also $\tan\theta = \frac{p_y}{p_x} = \frac{MV}{mv}$

or $\theta = \tan^{-1}\left(\frac{MV}{mv}\right)$

Thus, choice (2) is also correct

Further loss in kinetic energy $\Delta K = K_i - K_f$

$\Delta K = \left(\frac{1}{2}mv^2 + \frac{1}{2} + MV^2\right) - \frac{1}{2}\left(\frac{p^2}{m+M}\right)$

$= \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2\right) - \frac{1}{2}\left[\frac{(MV)^2 + (mv)^2}{m+M}\right]$

$\Delta K = \frac{1}{2}\left(\frac{Mm}{M+m}\right)(V^2 + v^2)$

Thus, choice (1) is also correct.

Hence, the correct answer is (D)

14. **Ans (D)**

Before collision

$u = \sqrt{6^2 + 2g \times 3.2} = 10\text{m/s.}$

After collision $v = \sqrt{2g(3.2)} = 8$ m/s

Therefore $e = \frac{v}{u} = 0.8$

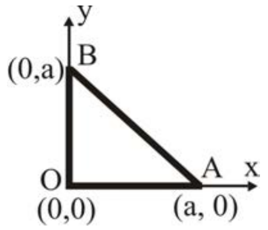
15. **Ans (C)**

$P_i = P_f$

$m(V) = \frac{m}{4}(O) + \frac{3m}{4}(V^1)$

$\Rightarrow V^1 = 4V/3$

16. **Ans (A)**



Centre of Mass of rod OA is at $\left(\frac{a}{2}, 0\right)$

Centre of Mass of rod OB is $\left(0, \frac{a}{2}\right)$

Centre of Mass of rod AB is $\left(\frac{a}{2}, \frac{a}{2}\right)$

For system $X_{cm} = \frac{m \times \frac{a}{2} + m \times 0 + m \times \frac{a}{2}}{m + m + m} = \frac{a}{3}$

& $Y_{cm} = \frac{m \times 0 + m \times \frac{a}{2} + m \times \frac{a}{2}}{m + m + m} = \frac{a}{3}$

17. **Ans (D)**

$\vec{I} = m(\vec{V}_2 - \vec{V}_1)$

$I = 2mv$

where $v = \sqrt{2gh'}$ $\left(h' = \frac{3\ell}{4}\sin\theta = \frac{3}{4}h\right)$

$I = 2m\sqrt{2g\frac{3}{4}h} = m\sqrt{6gh}$

18. **Ans (B)**

In elastic collision component of veclocity perpendicular to line of impact remains unchanged (SR is line of impact)

19. **Ans (C)**

Momentum conservation

$5 \times 10^3 \times 1.2 = (5 \times 10^3 + 10^3)V$

$V = 1$ m/s

20. **Ans (D)**

$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} \leqslant 1$

PART-1 : PHYSICS

SECTION-II

1. **Ans (5)**

Magnetic force on rod = $BI\ell$

Weight of the rod = mg

For no tension in wire, $BI\ell = mg$

or $I = \frac{mg}{Bl} = \frac{1 \times 10}{2 \times 1} = 5A$

3. **Ans (5880)**

$B = \mu_0 ni$

$\Rightarrow B = \mu_0 \frac{N}{\ell} i \quad \Rightarrow \quad N = \frac{B\ell}{\mu_0 i}$

$L = N \times 2\pi r = \frac{B\ell \times 2\pi r}{\mu_0 i}$

$= \frac{0.168 \times 1.4 \times 2\pi \times 0.01 \times 10}{4\pi \times 10^{-7} \times 2} = 5880$ m

4. **Ans (2)**

$F = \int dF = \int_x^{x+L} i\left(\frac{\mu_0 I}{2\pi x}\right)dx$

$= \frac{\mu_0 iI}{2\pi}\ell n\left(\frac{x+L}{x}\right)$

$N = 2$

5. **Ans (1420)**

$f = \frac{eB}{2\pi m}$

$f = \frac{e(\mu_0 ni)}{2\pi m}$

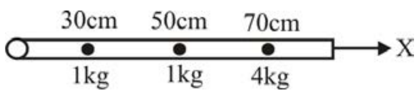
$n = \frac{2\pi mf}{e\mu_0 i}$

6. **Ans (20)**

$\vec{I} = m(\vec{V}_2 - \vec{V}_1)$

In perfect elastic collision between two equal masses, bodies will exchange their velocities.

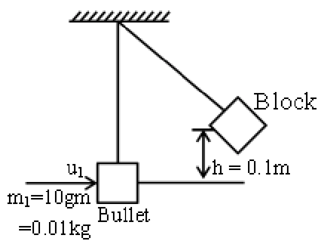
7. Ans (60)



$$X_{CM} = \frac{30 \times 1 + 50 \times 1 + 70 \times 4}{1 + 1 + 4} = 60 \text{ cm}$$

8. Ans (220)

Let V_1 and V_2 are the velocities of the bullet and the block after collision. Since the block rises to a height of h = 0.1 m so all its kinetic energy is converted into its potential energy thus by conservation of energy $\frac{1}{2}m_2v_2^2 = m_2gh$



$$v_2 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1} = 1.4 \text{ m/s}$$

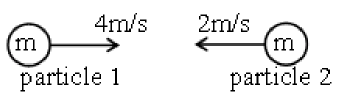
If u_1 is the initial velocity of the bullet then applying the law of conservation of momentum along the initial direction of bullet

$$m_1u_1 = m_1v_1 + m_1v_2 \quad v_1 = \frac{m_1u_1 - m_2v_2}{m_1}$$

$$v_1 = \frac{0.01 \times 500 - 2 \times 1.4}{0.01} = 220 \text{ m/s}$$

9. Ans (1)

Given that



As we know that the velocity of centre of mass of two particle system

$$V_C = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$V_C = \frac{m(4) + m(-2)}{m + m}$$

$$V_C = \frac{4m - 2m}{2m}$$

$$V_C = \frac{2m}{2m} = 1 \text{ m/s}$$

10. Ans (20)

From the conservation of energy

$$mgh = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times 0.04 \times 9.8 \times 4.9}{400}}$$

$$x = 9.8 \text{ cm}$$

PART-2 : CHEMISTRY

SECTION-I

1. Ans (D)

$LiAlH_4$ reduced

$\rightarrow$ –CH=O, –COOH, $-\overset{\|}{\underset{O}{C}}-$, $-\overset{}{\underset{O}{\overset{\|}{C}}}-O-$

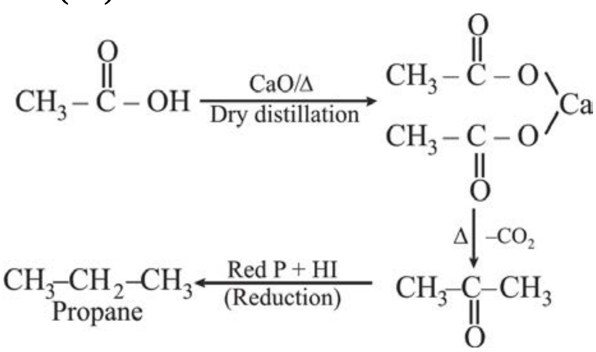
$\rightarrow$ 4 group

$NaBH_4$ reduced

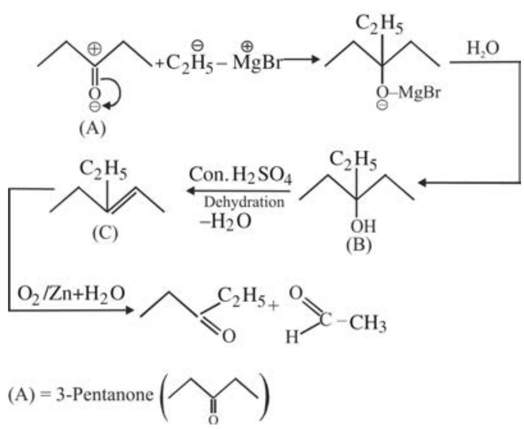
$\rightarrow$ Only, –CH=O, $-\underset{O}{\overset{\|}{C}}-$ group

$\rightarrow$ 2 group

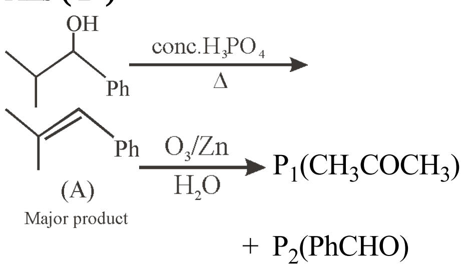
2. Ans (D)



3. Ans (B)

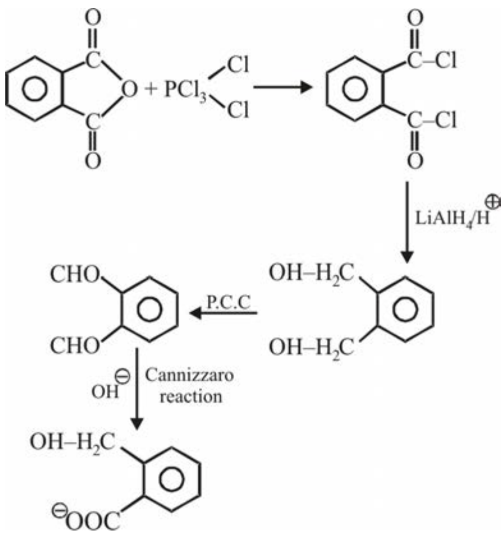


4. Ans (B)



Both compounds P_1 and P_2 reacts with $NaHSO_3$.
P_2 reduces tollens reagent but can not reduce Fehling's solution.
Only P_1 can undergo iodoform test.
Compound P_2 can undergo positive 2,4-DNP test as well as Cannizzaro reaction.

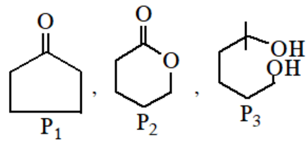
5. Ans (A)



6. Ans (B)

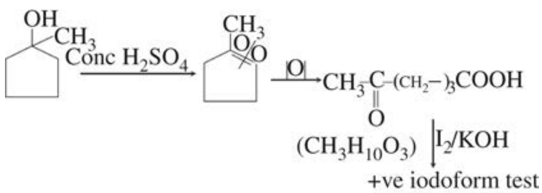
Reactivity for N.A.R. $\propto \frac{-M/-H/-I}{+M/+H/+I}$

7. Ans (A)



8. Ans (D)

A⟶Must be 3° alcohol because dichromate test do not given by 3° alcohol



9. Ans (B)

I ⇒ Octet complete (Negative charge on nitrogen)

II ⇒ Incomplete octet (Negative charge on nitrogen)

III ⇒ complete octet (Negative charge on carbon)

IV ⇒ Incomplete octet (Positive charge on nitrogen)

I > III > II > IV

10. Ans (C)

A.S ∝ Stability of conjugate base

$$\frac{\alpha - M + 1/-H/-I}{+M/+H/+I}$$

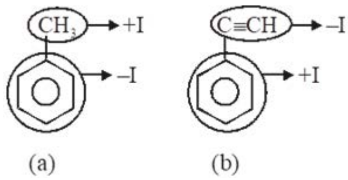
$-CH_3 \Rightarrow +H$

$-Cl \Rightarrow +-I$

$-OCH_3 \Rightarrow +M$

$-NO_2 \Rightarrow -M$

11. Ans (B)



Since I-effect is relative effect

12. Ans (A)

Heat of hydrogenation µ reactivity

Reactivity $\propto \frac{1}{\text{Stability of alkene}}$

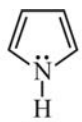
Stability of alkenes ∝ number of α–H(due to +H)

(P)	(Q)	(R)	(S)
1α–H	4α–H	10α–H	7α–H

13. Ans (A)

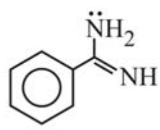
(I)

ℓp localised
more basic

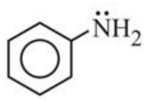


(II)

ℓp delocalized also
participate in aromatization
more stable, least basic



(III)
Phenylimidine
most basic



(IV)
ℓp delocalized
less basic

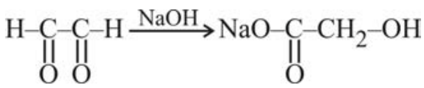
14. **Ans (A)**

$$-\overset{\oplus}{N}H_3 > \xrightarrow[\downarrow -I \text{ effect}]{-NO_2 > -CN > -COOH}$$

15. **Ans (D)**

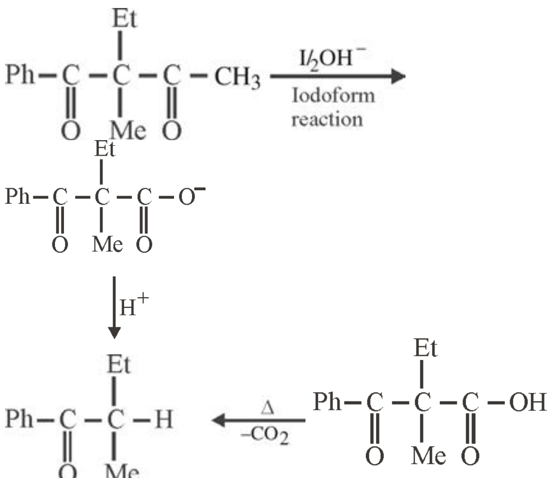
Reactivity $\propto$ (+)I, (+)M, $\propto \dfrac{1}{(-)I,\ (-)M}$

16. **Ans (D)**



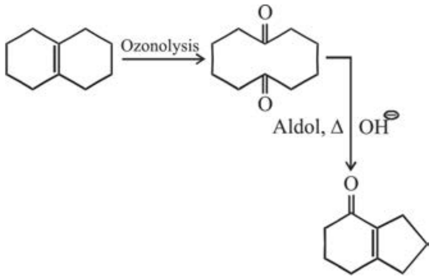
(Intramolecular cannizzaro r^{n})

17. **Ans (B)**

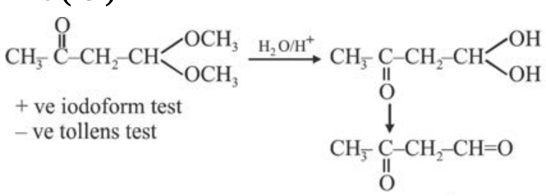


(B-Keto acid)

18. **Ans (D)**



19. **Ans (C)**



+ve Tollens test (+ve)

20. **Ans (B)**

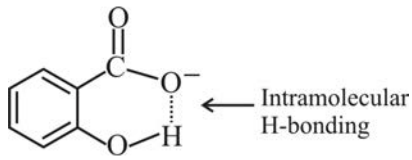
Acid strength $\propto -1 \propto \dfrac{1}{+1}$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (6)**

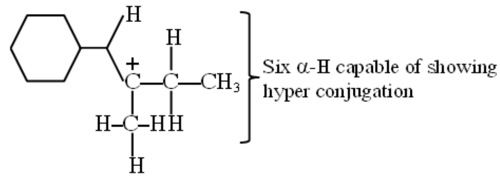
Acids, I, III, IV, VII, VIII and IX are all stronger than benzoic acid. I is stronger because of stabilisation of conjugate base by intramolecular H-bonding. III is stronger because from *meta* position, –OH exert only –I-effect, its electron donating resonance effect has no role on acidic strength.



IV is stronger acid due to loss of planarity of –COOH with phenyl ring, hence absence of electron donating resonance effect as phenyl rings on –COOH increases acidic strength. VII is stronger because a sulphonic acid is stronger than a carboxylic acid. VIII is stronger because electron withdrawing inductive effect of one –COOH over other increases acidic strength. IX is stronger due to only –I-effect of methoxy group operate from *meta* position but not its electron donating resonance effect.

2. **Ans (6)**

The given carbocation has six-H that can take part in hyperconjugation as :



no of hyperconjugated = a –H + 1

structure = 6 + 1 = 7

3. **Ans (20)**

x = 5

2, 3, 7, 11, 12 (Aromatic compound)

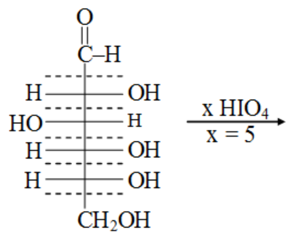
y = 4

1, 2, 5, 6 (Anti Aromatic compound)

Thus Þ x × y = 5 × 4 = 20

4. **Ans (3)**

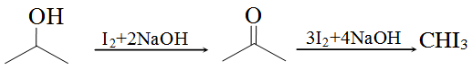
5. **Ans (5)**



Each C–C bond cleavage require 1 HIO_4

6. **Ans (2)**

$2NaOH + I_2 \longrightarrow NaOI + NaI + H_2O \longrightarrow NaI + [O]$



$+CH_3-COONa$

$a = 4I_2$, $b = 6\ NaOH$

b-a=6-4=2

7. **Ans (3)**

$CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-H$

8. **Ans (6)**

$NaBH_4$ can reduce only following group.

$-CH=O \longrightarrow -CH_2OH$

$>C=O \rightarrow >CH-OH$

$-\underset{\underset{O}{\|}}{C}-Cl \rightarrow -CH_2-OH$

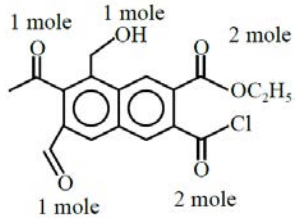
$R-x \longrightarrow R-H$

($2^o/3^o$ halide)

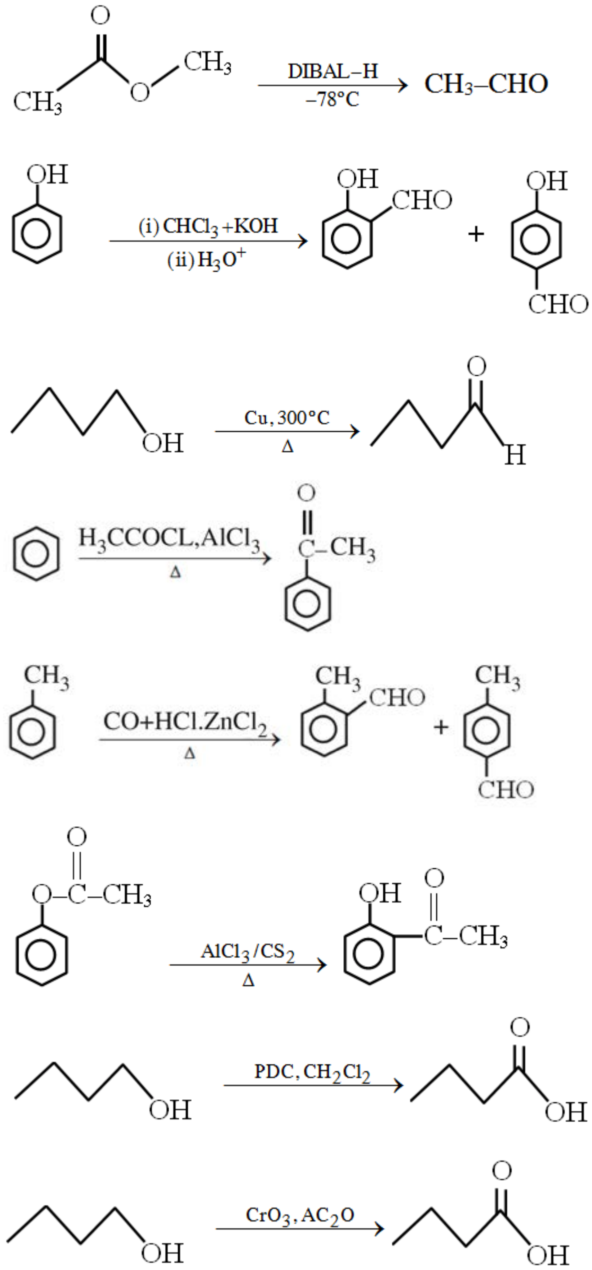
$-C=NH \longrightarrow -CH_2-NH_2$

Thus, (i), (iv), (vi), (viii), (ix), (x) are reduce by $NaBH_4$.

9. **Ans (7)**



10. **Ans (6)**



PART-3 : MATHEMATICS

SECTION-I

1. **Ans (A)**

$$A = \int_0^{\frac{\pi}{2}} ((\sin x + \cos x) - (|\cos x - \sin x|)dx$$

$$A = \int_0^{\frac{\pi}{2}} ((\sin x + \cos x) - (\cos x - \sin x))dx$$

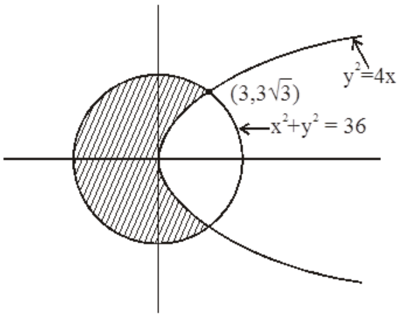
$$+ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} ((\sin x + \cos x) - (\sin x - \cos x))dx$$

$$A = 2\int_0^{\frac{\pi}{2}} \sin x\, dx + 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, dx$$

$$A = -2\left(\frac{1}{\sqrt{2}} - 1\right) + 2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$A = 4 - 2\sqrt{2} = 2\sqrt{2}\left(\sqrt{2} - 1\right)$$

2. **Ans (C)**



Required area

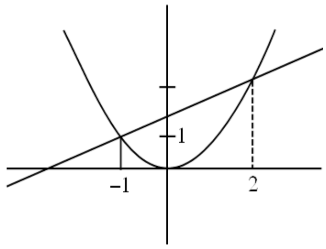
$$= \pi \times (6)^2 - 2\int_0^3 \sqrt{9x}dx - \int_3^6 \sqrt{36 - x^2}dx$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\frac{x}{6}\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right)$$

$$= 24\pi - 3\sqrt{3}$$

3. **Ans (C)**



$y - x = 2\ ,\ x^2 = y$

Now, $x^2 = 2 + x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x + 1)(x - 2) = 0$

$$\text{Area} = \int_{-1}^{2} (2 + x - x^2)$$

$$= \left|2x + \frac{x^2}{2} - \frac{x^3}{3}\right|_{-1}^{2}$$

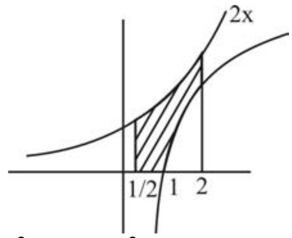
$$= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$$

$$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$$

4. **Ans (B)**

$R = \{(x, y) :$

$\max\{0, \log_e x\} \le y \le 2x, \frac{1}{2} \le x \le 2\}$



$$\int_{\frac{1}{2}}^{2} 2^x dx - \int_1^2 \ell nx\, dx$$

$$\Rightarrow \left[\frac{2^x}{\ell n2}\right]_{1/2}^{2} - [x\ell nx - x]_1^2$$

$$\Rightarrow \frac{(2)^2 - 2^{1/2}}{\log_e 2} - (2\ell n2 - 1)$$

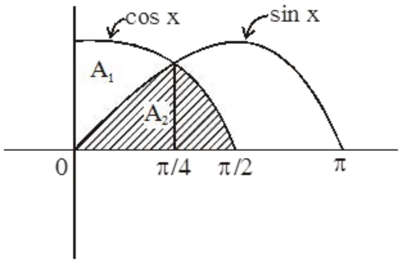
$$\Rightarrow \frac{\left(2 - \sqrt{2}\right)}{\log_e 2} - 2\ell n\, 2 + 1$$

$\therefore\ \alpha = 2^2 - \sqrt{2},\ \beta = -2,\ \gamma = 1$

$\Rightarrow (a + b + 2g)^2$

$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2 \ \Rightarrow\ (\sqrt{2})^2 = 2$

5. Ans (A)



$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$A_1 = (\sin x + \cos x)_0^{\pi/4} = \sqrt{2} - 1$$

$$A_2 = \int_0^{\pi/4} \sin x\ dx + \int_{\pi/4}^{\pi/2} \cos x\ dx$$

$$= (-\cos x)_0^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2}$$

$$A_2 = \sqrt{2}\left(\sqrt{2} - 1\right)$$

$$A_1 : A_2 = 1 : \sqrt{2},\ A_1 + A_2 = 1$$

6. Ans (A)

$$xdy = (y + x^3\cos x)dx$$

$$xdy = ydx + x^3\cos x dx$$

$$\frac{xdy - ydx}{x^2} = \frac{x^3 \cos x\, dx}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x\, dx$$

$$\Rightarrow \frac{y}{x} = x\sin x - \int 1.\sin x\, dx$$

$$\frac{y}{x} = x\sin x + \cos x + C$$

$$\Rightarrow 0 = -1 + C \Rightarrow C = 1,\ x = p,\ y = 0$$

$$\text{so } \frac{y}{x} = x\sin x + \cos x + 1$$

$$y = x^2\sin x + x\cos x + x \quad x = \frac{\pi}{2}$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

7. Ans (B)

$$\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$$

$$2^y \frac{dy}{dx} = 2^x\left(2^y - 1\right)$$

$$\int \frac{2^y}{2^y - 1} dy = \int 2^x\ dx$$

$$\frac{\ell n\left(2^y - 1\right)}{\ell n\, 2} = \frac{2^x}{\ell n\, 2} + C$$

$$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$$

$$\because y(0) = 1 \Rightarrow 0 = \log_2 e + C$$

$$C = -\log_2 e$$

$$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2 e$$

put $x = 1$, $\log_2(2^y - 1) = \log_2 e$

$$2^y = e + 1$$

$y = \log_2(e + 1)$ Ans.

8. Ans (D)

$$\alpha.\ R = \frac{|3(2) + 4(-3) - 5|}{5} = \frac{11}{5}$$

$$(x - h)^2 = \frac{11}{5}(y - k)$$

differentiate w.r.t 'x' :

$$2\,(x - h) = \frac{11}{5}\frac{dy}{dx}$$

again differentiate

$$2 = \frac{11}{5}\frac{d^2y}{dx^2}$$

$$\frac{11 d^2y}{dx^2} = 10$$

9. Ans (D)

$$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt \quad 0 \le x \le 1$$

differentiating both the sides

$$\sqrt{1-(f'(t))^2} = f(x) \Rightarrow 1-(f'(x))^2 = f^2(x)$$

$$\frac{f'(x)}{\sqrt{1-f^2(x)}} = 1$$

$$\sin^{-1} f(x) = x + C$$

$$\because\ f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$$

$$\text{Now } \lim_{x\to 0} \frac{\int_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right) = \frac{1}{2}$$

10. Ans (D)

$$\frac{dP}{dt} = 0.5P - 450$$

$$\Rightarrow \int_0^t \frac{dp}{P-900} = \int_0^t \frac{dt}{2}$$

$$\Rightarrow [\ell n\,|P(t)-900|]_0^t = \left[\frac{t}{2}\right]_0^t$$

$$\Rightarrow \ell n\,|P(t)-900| - \ell n\,|P(0)-900| = \frac{t}{2}$$

$$\Rightarrow \ell n\,|P(t)-900| - \ell n\,|50| = \frac{t}{2}$$

for $P(t) = 0$

$$\Rightarrow \ell n\left|\frac{900}{50}\right| = \frac{t}{2} \Rightarrow t = 2\,\ell n 18$$

11. Ans (B)

$$\frac{dy}{dx} + \frac{y}{x} = bx^3$$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = x$$

So, solution of D.E. is given by

$$y.x = \int b.x^3.x\,dx + c$$

$$y = \frac{c}{x} + \frac{bx^4}{5}$$

Passes through (1, 2)

$$2 = c + \frac{b}{5} \qquad ...(1)$$

$$\int_1^2 f(x)dx = \frac{62}{5}$$

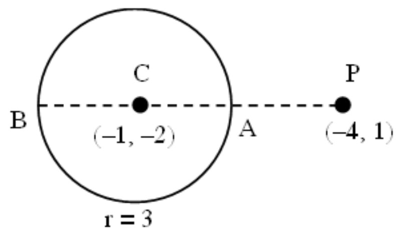
$$\left[c\ell nx + \frac{bx^5}{25}\right]_1^2 = \frac{62}{5}$$

$$c\,\ell n2 + \frac{31\,b}{25} = \frac{62}{5} \qquad ...(2)$$

By equation (1) & (2)

$c = 0$ and $b = 10$

12. Ans (C)



Centre of smallest circle is A

Centre of largest circle is B

$$r_2 = |CP - CA| = 3\sqrt{2} - 3$$

$$r_1 = CP + CB = 3\sqrt{2} + 3$$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{\left(3\sqrt{2}+3\right)^2}{9}$$

$$= \left(\sqrt{2}+1\right)^2 = 3 + 2\sqrt{2}$$

$a = 3,\ b = 2$

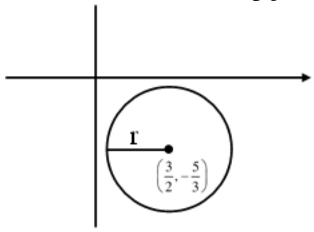
13. Ans (D)

$S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$

Centre $\equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$

radius $= r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$



Now,

$\Rightarrow r < \frac{3}{2}$

$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow C > 100$ .....(1)

Now point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$

$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$

$\boxed{C < 156}$ .....(2)

From (1) & (2)

$\boxed{100 < C < 156}$

14. Ans (B)

$PA = AQ = \lambda$

OA.AB

= AP.AQ

$\Rightarrow 1.12 = \lambda . \lambda$

$\Rightarrow \lambda = 2\sqrt{3}$

Area $\Delta PQB = \frac{1}{2} \times 2\lambda \times AB$

$\Delta = \frac{1}{2}.4\sqrt{3} \times 12$

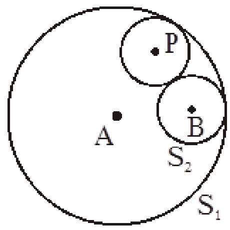
$24\sqrt{3}$

15. Ans (C)

$S_1 : x^2 + y^2 = 9$ $\left\langle \begin{array}{l} r_1 = 3 \\ A(0, 0) \end{array} \right.$

$S2 : (x - 2)^2 + y^2 = 1$ $\left\langle \begin{array}{l} r_2 = 1 \\ B(2, 0) \end{array} \right.$

$\because c_1c_2 = r_1 = r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius r

$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

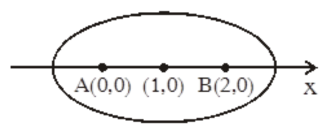
$\Rightarrow PA + PB = 4$ $(> AB)$

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is $2a = 4$,

$e = \frac{1}{2}$

$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$

if x-ellipse



$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

16. **Ans (C)**

$x^2 + y^2 - 10x - 10y + 41 = 0$

$A(5,5), R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

$B(11,5), R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point

17. **Ans (A)**

P be a point on $(x - 1)^2 + (y - 1)^2 = 1$

so $P(1 + \cos\theta, 1 + \sin\theta)$

$A(1, 4) \quad B(1, -5)$

$(PA)^2 + (PB)^2$

$= (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$

$= 47 + 6\sin\theta$

is maximum if $\sin\theta = 1$

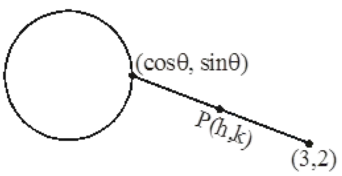
$\Rightarrow \sin\theta = 1, \cos\theta = 0$

$P(1, 1)\ A(1, 4)\ B(1, -5)$

P, A, B are collinear points

18. **Ans (B)**

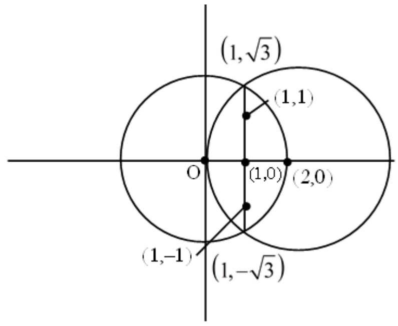
$$h = \frac{\cos\theta + 3}{2}$$



$$k = \frac{\sin\theta + 2}{2}$$

$$\Rightarrow \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4} \Rightarrow r = \frac{1}{2}$$

20. **Ans (B)**



$(x - 2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in C_1 & C_2 is 5

(0, 0) , (1, 0) , (2, 0) , (1, 1), (1, −1)

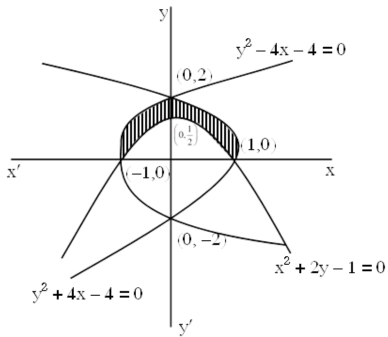
Similarly in C_2 & C_3 is 5

No. of relations $= 2^{5\times5} = 2^{25}$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (2)**

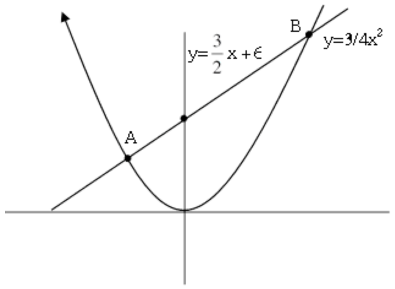


Required Area (shaded)

$$= 2\left[\int_0^2 \left(\frac{4-y^2}{4}\right) dy - \int_0^1 \left(\frac{4-x^2}{2}\right) dx\right]$$

$$= 2\left[\frac{4}{3} - \frac{1}{3}\right] = (2)$$

2. Ans (27)



For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

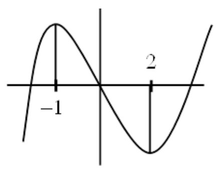
$$\text{Area} = \int_{-2}^{4}\left(\frac{3}{2}x + 6 - \frac{3}{4}x^2\right)dx$$

$$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = 27$$

3. Ans (114)

$f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$

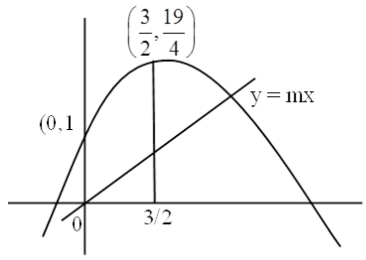
Point = (2,–20) & (–1,7)



$$A = \int_{-1}^{0}(2x^3 - 3x^2 - 12x)\,dx + \int_{0}^{2}(12x + 3x^2 - 2x^3)\,dx$$

$$A = \left(\frac{x^4}{2} - x^3 - 6x^2\right)_{-1}^{0} + \left(6x^2 + x^3 - \frac{x^4}{2}\right)_{0}^{2};\ 4A = 114$$

4. Ans (26)

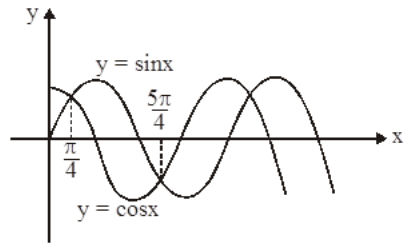


$$\text{Total area} = \int_{0}^{3/2}(1 + 4x - x^2)\,dx$$

$$= x + 2x^2 - \frac{x^3}{3}\Big|_{0}^{3/2} = \frac{39}{8}\ \&\ \frac{39}{16} = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}\,m$$

$$\Rightarrow 3m = \frac{13}{2} \Rightarrow 12m = 26$$

5. Ans (64)



$$A = \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)\,dx = (-\cos x - \sin x)\Big|_{\pi/4}^{5\pi/4}$$

$$= \left(-\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right)\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$$

$$\Rightarrow A = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 16 \times 4 = 64$$

6. **Ans (2)**

$$\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-1}\right)\right)dx = \sqrt{e^{2x}-1}\,dy$$

Put $\cos^{-1}(e^{-x})\ \theta$, $\theta \in [0, \pi]$

$$\cos\theta = e^{-x} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = e^{-x}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^x+1}{2e^x}}$$

$$\sqrt{\frac{e^{-x}+1}{2e^x}}dx = \sqrt{e^x-1}\,dx$$

$$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^x}\sqrt{x^x-1}} = \int dy$$

Put $e^x = t,\ \frac{dt}{dx} = e^x$

$$\frac{1}{\sqrt{2}}\int\frac{dt}{e^x\sqrt{e^x}\sqrt{x^x-1}} = \int dy$$

$$\int\frac{dt}{t\sqrt{t^2-t}} = \sqrt{2}\,y$$

Put $t = \frac{1}{z},\ \frac{dt}{dz} = -\frac{1}{z^2}$

$$\int\frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2}\,y - \int\frac{dz}{\sqrt{1-z}} = \sqrt{2}\,y$$

$$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$$

$$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$$

$$2\left(1-e^{-x}\right)^{1/2} = \sqrt{2y} + c \xrightarrow{(0,1)} \Rightarrow c = \sqrt{2}$$

$2(1\text{-}e^{-x})^{1/2} = \sqrt{2}\,(y+1)$, passes through $(\alpha, 0)$

$$2(1\text{-}e^{-\alpha})^{1/2} = \sqrt{2}$$

$$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1 - e^{-\alpha} = \frac{1}{2}$$

$$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$$

7. **Ans (4)**

$$y + 1 = Y \Rightarrow dy = dY$$

$$x + 2 = X \Rightarrow dx = dX$$

$$\Rightarrow \left(Xe^{\frac{Y}{X}} + Y\right)dX = XdY$$

$$\Rightarrow XdY - YdY - YdX = Xe^{Y/X}dX$$

$$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}} = \frac{dX}{X}$$

$$-e^{-Y/X} = \ell|X| + c$$

$$(3, 2) \longrightarrow -e^{-2/3} = \ell|3| + c$$

$$-e^{-\frac{Y}{X}} = \ell n|X| - e^{-\frac{2}{3}} - \ell n3$$

$$-e^{-\frac{Y}{X}} = e^{2/3} + \ell n3 - \ell n|X| \ > 0$$

$$\ell n|X| < \left(e^{2/3} + \ell n3\right)$$

Let $\lambda = \left(e^{2/3} + \ell n3\right)$

$$|x+2| < e^{\lambda}$$

$$-e^{\lambda} < x + 2 < e^{\lambda}$$

$$-e^{\lambda} - 2 < x < e^{\lambda} - 2$$

a b

$$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$$

Although $x = -2$ should be excluded from domain but according to the given problem it will the most appropriate solution.

8. Ans (4)

Let $e^y = t \Rightarrow \frac{dt}{dx} - (2\sin x)t = -\sin x \cos^2 x$

I.F. $= e^{2\cos x}$

$\Rightarrow t.\, e^{2\cos x} = \int e^{2\cos x}.\left(-\sin x\cos^2 x\right)\, dx$

$\Rightarrow e^y.\, e^{2\cos x} = \int 2^{2z}.\, z^2\, dz,\ \ z = e^{2\cos x}$

$\Rightarrow e^y.\, e^{2\cos x} = \frac{1}{2}.\cos^2 x.\, e^{2\cos x}$

$-\frac{1}{2}\cos x.\, e^{2\cos x} + \frac{e^{2\cos x}}{4} + C$

at $x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{4}$

$\Rightarrow e^y = \frac{1}{2}\cos^2 x - \frac{1}{2}\cos x + \frac{1}{4} + \frac{3}{4}.\, e^{-2\cos x}$

$\Rightarrow y = \log\left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4}e^{-2\cos x}\right]$

Put $x = 0$

$\Rightarrow y = \log\left[\frac{1}{4} + \frac{3}{4}e^{-2}\right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$

9. Ans (2)

$\sec y \frac{dy}{dx} = 2 \sin x \cos y$

$\sec^2 y\, dy = 2\sin x dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$\sec^2 y \frac{dy}{dx} = 2 \sin x$

$5\frac{dy}{dx} = 2$

10. Ans (2)

$\int e^{-y} dy = \int e^{\alpha x} dx$

$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c$ ......(i)

Put $(x,y) = (\ell n2, \ell n2)$

$\frac{-1}{2} = \frac{2^\alpha}{\alpha} + C$ .....(ii)

Put $(x,y) \equiv (0, -\ell n2)$ in (i)

$-2 = \frac{1}{\alpha} + C$ .....(iii)

(ii) – (iii)

$\frac{2^\alpha - 1}{\alpha} = \frac{3}{2}$

$\Rightarrow \alpha = 2$ (as $\alpha \in \mathbb{N}$)