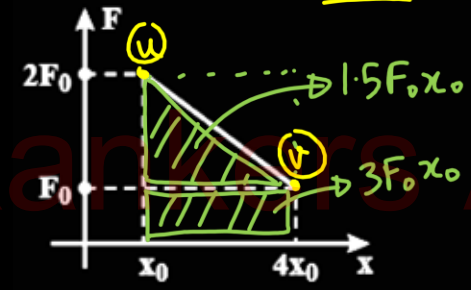

PHYSICS
Rankers Academy JEE

7

A particle of mass m moving along a straight line experiences force F which varies with the distance A travelled as shown in the figure. If the velocity of the particle at x_0 is $\sqrt{\frac{2F_0x_0}{m}}$, then velocity at $4x_0$ is:



(A) $2\sqrt{\frac{2F_0x_0}{m}}$

(B) $2\sqrt{\frac{F_0x_0}{m}}$

(C) $\sqrt{\frac{F_0x_0}{m}}$

(D) none of these

JEE 1

Area = $W = \Delta KE$

$u = \sqrt{\frac{2F_0x_0}{m}}$

$Area_{F-x} = W = \frac{9}{2}F_0x_0 = \frac{1}{2}m(v^2 - u^2)$

$\frac{9F_0x_0}{m} = v^2 - \frac{2F_0x_0}{m}$

$v = \sqrt{\frac{11F_0x_0}{m}}$

2

The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of E(t) at t = 5 s is

(A) 2 (B) 4

(C) 2.5 (D) 4.5

$\left|\frac{\Delta A}{A}\right| \times 100 = 1.25\%$

$\left|\frac{\Delta t}{t}\right| \times 100 = 1.5\%$

JEE 1

$\left|\frac{\Delta E}{E} \times 100\right|_{t=5} = ?$

$E = A^2 e^{-t/5}$

diff - 't'

$\frac{dE}{dt} = 2A \cdot \frac{dA}{dt} \cdot e^{-t/5} + A^2 e^{-t/5}\left(-\frac{1}{5}\right) \cdot dt$

$\frac{dE}{E} = \frac{2A e^{-t/5} \cdot dA}{A^2 e^{-t/5}} - \frac{1}{5}\frac{A^2 e^{-t/5} dt}{E}$

$\frac{dE}{E} = 2\frac{dA}{A} - \frac{t}{5}\frac{dt}{t}$

error

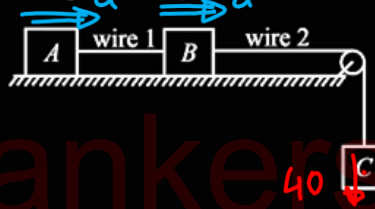
$\left|\frac{\Delta E}{E}\right| \times 100 = 2\left|\frac{\Delta A}{A}\right| \times 100 + \frac{t}{5}\left|\frac{\Delta t}{t}\right| \times 100$

$\left|\frac{\Delta E}{E} \times 100\right|_{t=5} = 2(1.25\%) + \frac{5}{5}(1.5\%)$

$= 4\%$

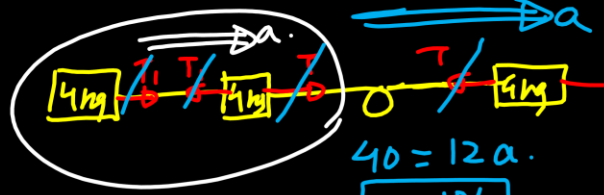
3

Three blocks A, B and C each of mass 4 kg are attached as shown in figure. Both the wires have equal cross sectional area $5 \times 10^{-7}\ m^2$. The surface is smooth. If the longitudinal strain in wire 2 is $\frac{a}{3} \times 10^{-4}$, then the a will be (if Young modulus of both the wires is $2 \times 10^{11}\ N/m^2$) (take $g = 10\ m/s^2$)



(A) 2
(C) 8
(B) 4
(D) 14

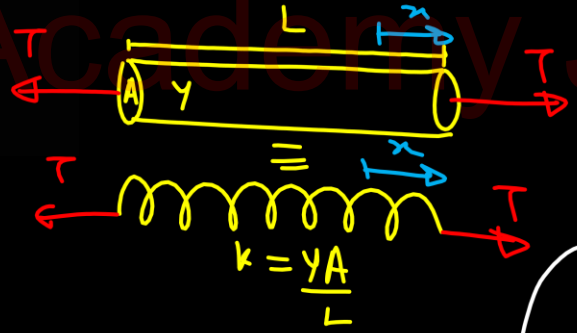
JEE 1



$40 = 12a$.

$a = 10/3$

$T = 8\left(\frac{10}{3}\right) = \frac{80}{3}$



$T = kx$

$T = \frac{YA}{L} \cdot x$

$\frac{80}{3} = (2 \times 10^{11})(5 \times 10^{-7})\left(\frac{x}{L}\right)$

$\text{strain} = \frac{x}{L} = \frac{8}{3} \times 10^{-4}$

$a = 8$

4

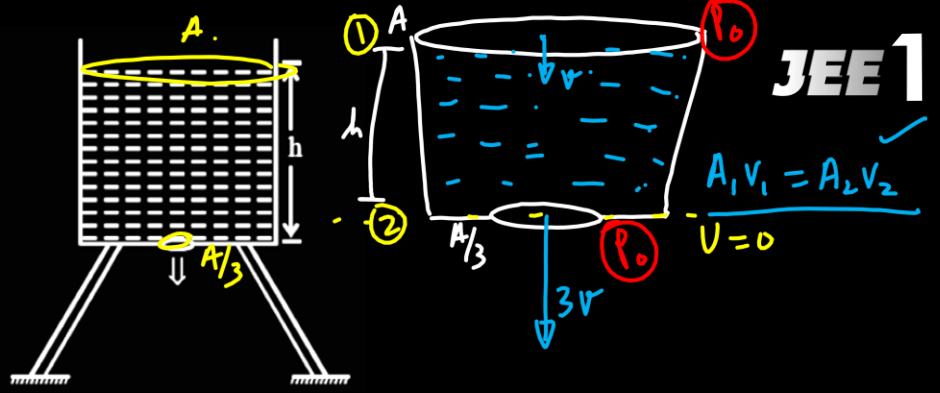
A small hole of area a is at the bottom of a container of area A . The liquid is filled up to height h from base. As liquid comes out then $\left[a = \frac{A}{3}\right]$.

(A) Level of liquid in container falls at rate of $\sqrt{\frac{gh}{4}}$ m/s.

(B) Magnitude of acceleration of top surface of liquid is $\frac{g}{9}$ m/s^2.

(C) Acceleration of top surface of liquid is $\frac{g}{3}$ m/s^2.

(D) None of these



B.T. ① → ②

$$\rho g h + P_0 + \frac{1}{2}\rho v^2 = 0 + \frac{1}{2}\rho(3v)^2 + P_0$$

$$2gh = 8v^2$$

$$v = \sqrt{\frac{gh}{4}}$$

$$v = \sqrt{\frac{g}{4}}\, h^{1/2}$$

$$\frac{dv}{dh} = \frac{1}{2}\sqrt{\frac{g}{4}}\, h^{-1/2}$$

$$a = v\frac{dv}{dh}$$

$$a = \frac{1}{2}\cdot\frac{g}{4} = \frac{g}{8}$$

5

Given below are two statements.

Statement-I: The law of gravitation holds good for any pair of bodies in the universe.

Statement-II: The weight of any person becomes zero when the person is at the centre of the earth.

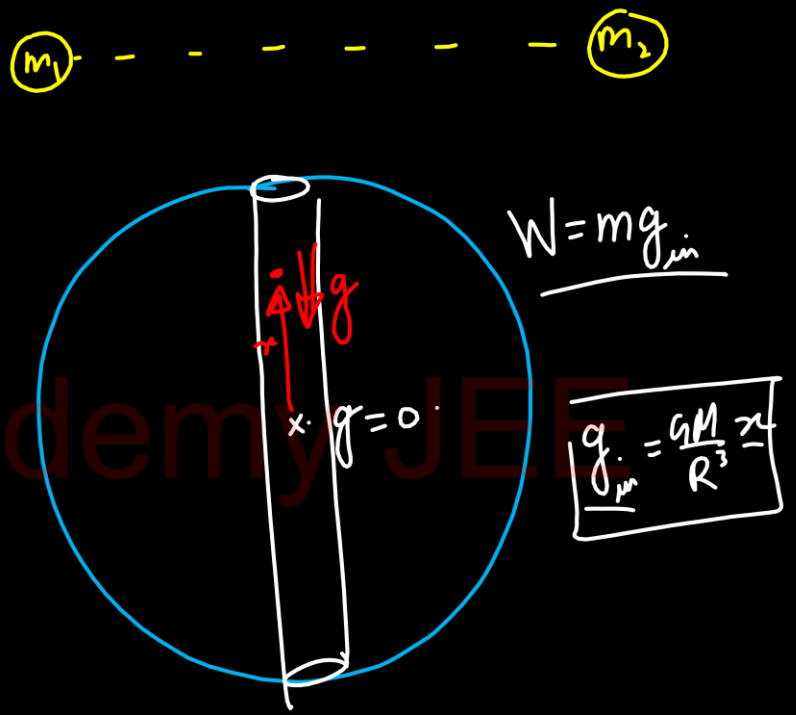
In the light of the above statements, choose the correct answer from the option given below.

(A) Both Statement-I and Statement-II are true

(B) Both Statement-I and Statement-II are false

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true



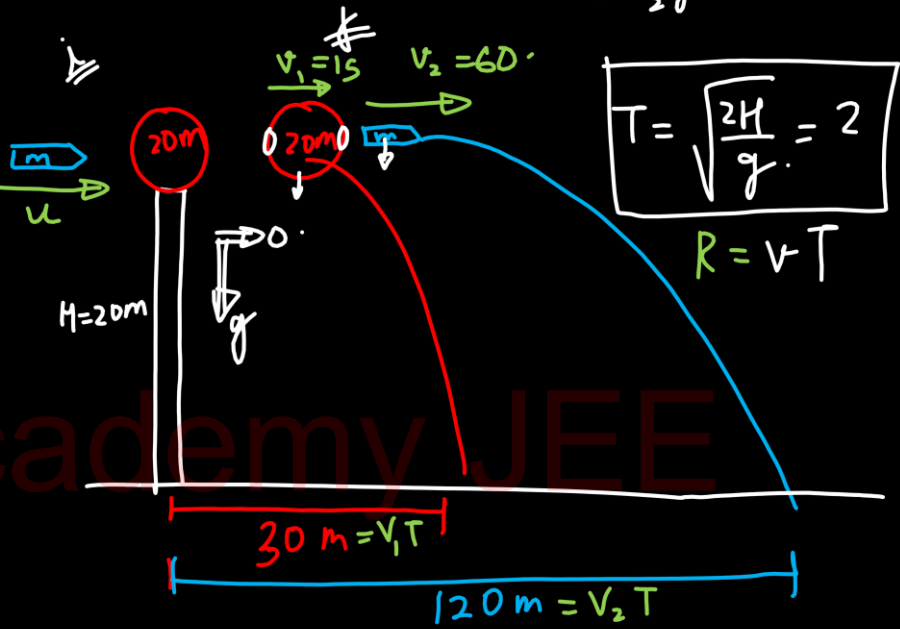
6

A ball of mass 200 g (20m) rests on a vertical post of height 20 m . A bullet of mass 10 g (m), travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if g = 10 m/s^2):

(A) 120 m/s (B) 60 m/s

(C) 400 m/s (D) 360 m/s

$H = \frac{1}{2}gT^2$ JEE 1



$T = \sqrt{\frac{2H}{g}} = 2$

$R = VT$

+

P

$mu = 20m(15) + m(60)$

$u = 300 + 60$

$u = 360$

$V_1(2) = 30$

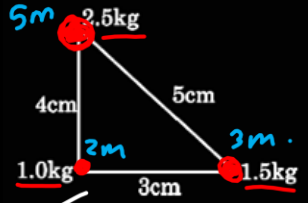
$V_1 = 15$

$V_2 = 60$

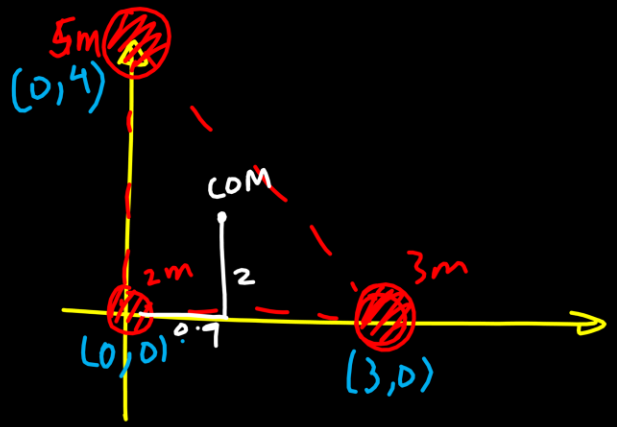
7

$m = 0.5\,kg$

Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg gare placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point :-



(A) 0.9 cm right and 2.0 cm above 1 kg mass.
(B) 1.5 cm right and 1.2 cm above 1 kg mass.
(C) 0.6 cm right and 2.0 cm above 1 kg mass.
(D) 2.0 cm right and 0.9 cm above 1 kg mass.



$$X_{COM} = \frac{0 + 9m + 0}{10m} = 0.9$$

$$Y_{COM} = \frac{0 + 0 + 20m}{10m} = 2$$

JEE 1

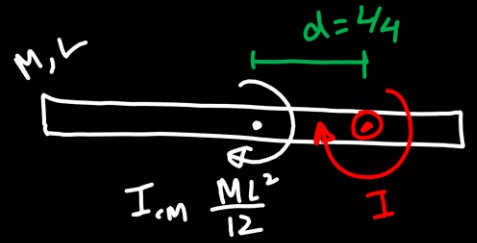
8

The radius of gyration of a uniform rod of length ℓ, about an axis passing through a point on rod, $\frac{\ell}{4}$ away from the centre of the rod, and perpendicular to rod is:-

(A) $\frac{1}{8}\ell$ (B) $\frac{1}{4}\ell$

(C) $\sqrt{\frac{7}{48}}\ell$ (D) $\sqrt{\frac{3}{8}}\ell$

$\ell = L$



$I = I_{CM} + Md^2$

$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7}{48}ML^2$

$I = mk^2$

$\frac{7}{48}mL^2 = mk^2$

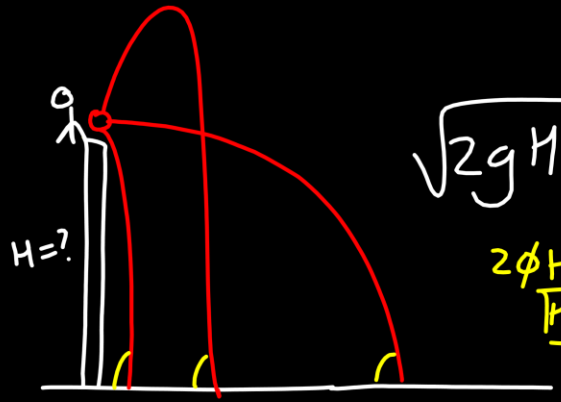
$k = \sqrt{\frac{7}{48}}L$

9

BRILLIANT!

A boy throws several balls out of the window of his house at different angles to the horizontal. All balls are thrown at speed $u = \sqrt{10}$ m/s and it was found that all of them hit the ground making an angle of 60° or larger than that with the horizontal. Find the height (in meter) of the window above the ground. [$g = 10\ m/s^2$]

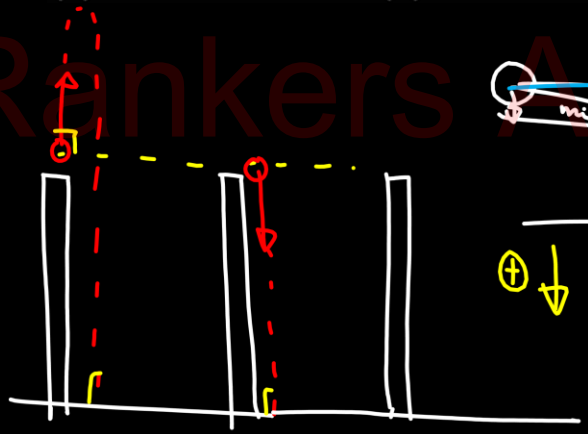
(A) 1.75 (B) 1.5
(C) 1.85 (D) 2

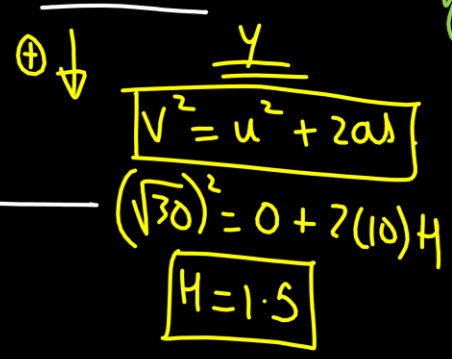


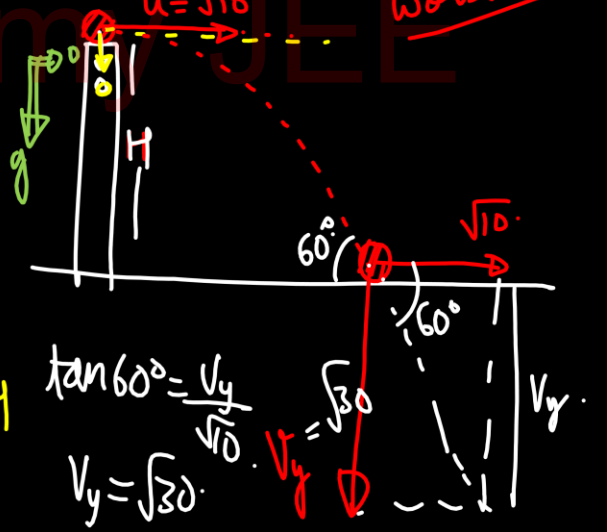
$\sqrt{2gH} = \sqrt{30}$

$2\phi H = 3\phi$

$H = 1.5$

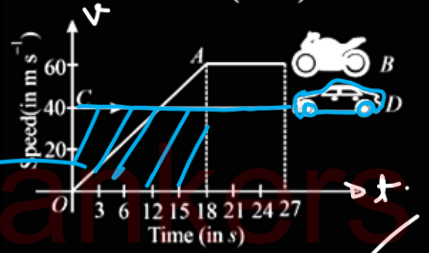






10

At the instant a motor bike starts from rest in a given direction, a car overtakes the motor bike, both moving in the same direction. The speed time graphs for motor bike and car are represented by OAB and CD respectively. Then at $t = 18\ s$ find the distance between then motor bike and car (in m).



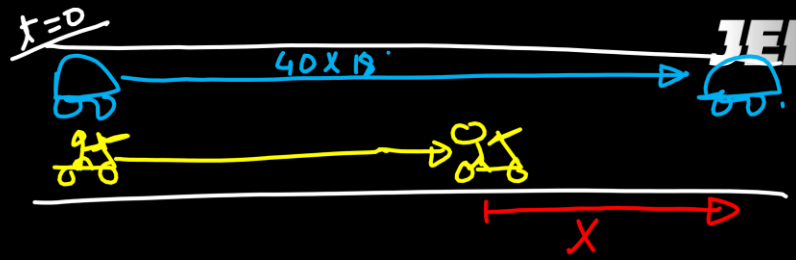
(A) 240 (B) 180

(C) 320 (D) 160

$s_B = 30 \times 18$

$s_C = 40 \times 18$

Area $v-t = s$



$X = (40 \times 18) - (30 \times 18)$.

$X = 180$

11

C.G.S

A particle of mass 1 kg, executing simple harmonic motion of period 20 s, crosses the mean position at $t = 0$ with velocity π cm/s.

(A) the maximum acceleration of the particle is 10 cm/s²

(B) when it is 4 cm from the mean position, its velocity is $6\pi/10$ cm/s

(C) the kinetic energy of the particle when its displacement is 5 cm from the mean position is $\frac{3\pi^2}{8\times10^6}$ J

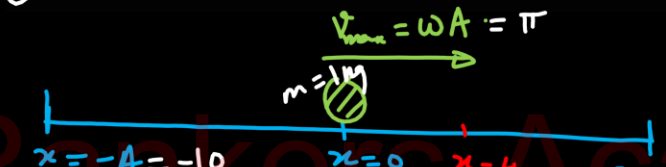
(D) its velocity at displacement 6 cm from the mean position is $8\pi/10$ cm/s

$m = 1000\,g$

$v = \omega\sqrt{A^2 - x^2}$

S.I.

T ω



$T = 20$

$\omega = \frac{2\pi}{T} = \pi/10$

$\omega A = \pi$

$\frac{\pi}{10} A = \pi$

$A = 10$

(A) $a_{max} = \omega^2 A = \frac{\pi^2}{100} \times 10 = \frac{\pi^2}{10}$

(B) $v\big|_{x=4} = \frac{\pi}{10}\sqrt{100 - 16} = \frac{\pi}{10}\sqrt{84}$

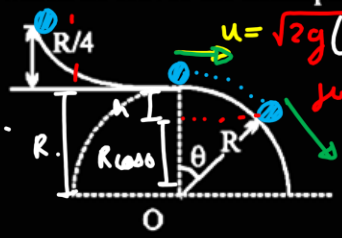
(C) $KE\big|_{x=5} = \frac{1}{2} m \omega^2 (A^2 - x^2)$

$= \frac{1}{2}(1000)\frac{\pi^2}{100}(100 - 25) = \frac{3}{8}\pi^2 \times 10^3$ erg

(D) $v\big|_{x=6} = \frac{\pi}{10}\sqrt{100 - 36} = \frac{8\pi}{10}$.

12

A skier plans to ski a smooth fixed hemisphere of radius R. He starts from rest from a curved smooth surface of height (R/4). The angle θ at which he leaves the hemisphere is



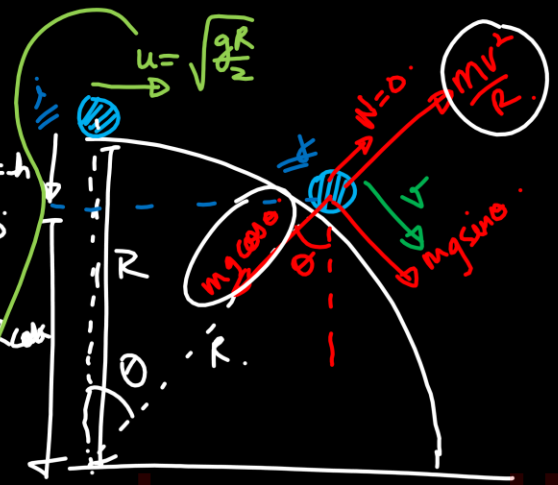
$u = \sqrt{2g(R/4)} = \sqrt{\frac{gR}{2}}$

$h = R(1-\cos\theta)$.

$mg\left(\frac{R}{4}\right) = \frac{1}{2}mu^2$

(A) $\cos^{-1}(2/3)$ (B) $\cos^{-1}(5/\sqrt{3})$

(C) $\cos^{-1}(5/6)$ (D) $\cos^{-1}(5/2\sqrt{3})$



$h = R(1-\cos\theta)$

$E_i = E_f$.

$\frac{1}{2}mu^2 + mgR(1-\cos\theta) = \frac{1}{2}mV^2$

$\sqrt{\frac{gR}{2} + 2gR(1-\cos\theta)} = V$ — ②

$\frac{mv^2}{R} = mg\cos\theta$.

$V = \sqrt{gR\cos\theta}$ — ①

final calc. ① & ②

$gR\cos\theta = \frac{gR}{2} + 2gR(1-\cos\theta)$.

$\cos\theta = \frac{1}{2} + 2 - 2\cos\theta$

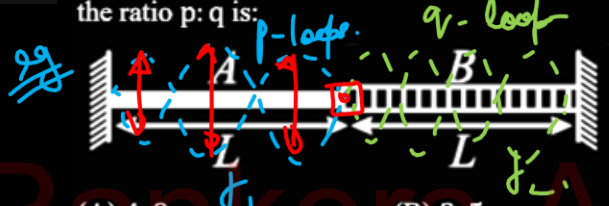
$3\cos\theta = \frac{5}{2}$

$\cos\theta = 5/6$

JEE 1

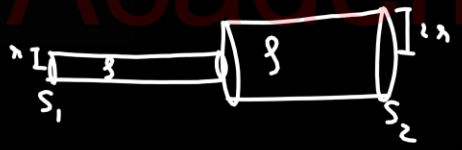
13

A wire of length 2L, is made by joining two wires A and B of same lengths but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of anti-nodes in wire A is p and that in B is q then the ratio p: q is:



(A) 4:9 (B) 3:5
(C) 1:4 (D) 1:2

$T_1 = T_2$



$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

p-loops: $f = pf_0$

q-loops: $f = qf_0$

$f_1 = f_2$

$\frac{p}{2L}\sqrt{\frac{T}{\mu_1}} = \frac{q}{2L}\sqrt{\frac{T}{\mu_2}}$

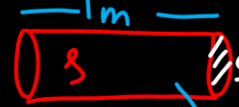
$\frac{p}{q} = \sqrt{\frac{\mu_1}{\mu_2}}$

$\frac{p}{q} = \sqrt{\frac{\rho S_1}{\rho S_2}} = \sqrt{\frac{\pi(r)^2}{\pi(2r)^2}}$

$= \sqrt{\frac{1}{4}} = \frac{1}{2}$ Ans

JEE 1

uniform.



$\mu = \rho S$

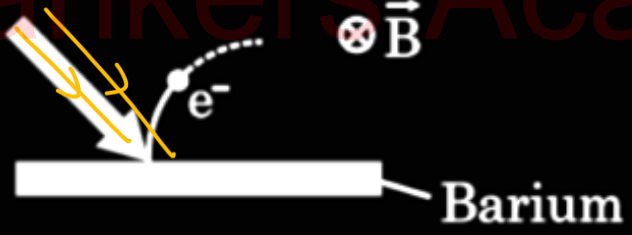
mass $= \mu = \rho(S.1)$

14

JEE 1

Light of wavelength 2475Å is incident on barium. Photoelectrons emitted describe a circle of maximum radius 100 cm by a magnetic field of flux density $\frac{1}{\sqrt{17}} \times 10^{-5}$ Tesla. Work function of the barium is nearly

(Given $\left(\frac{e}{m}\right) = 1.7 \times 10^{11}$), hc = 12375(eV − Å)



(A) 1.8 eV

(B) 2.1 eV

(C) 4.5 eV

(D) 3.3 eV

$K = \frac{hc}{\lambda} - \varphi$

$\varphi = \frac{hc}{\lambda} - K$

$R = \frac{mv}{qB}$

$mv = qBR$

$K = \frac{mv^2}{2} = \frac{q^2B^2R^2}{2m}$

$\varphi = \frac{hc}{\lambda} - \frac{q^2B^2R^2}{2m}$

$= \frac{12375\ \text{eV-Å}}{2475\text{-Å}} - \frac{\left(17 \times 10^{11} \times \frac{1}{17} \times 10^{-10} \times 1^2\right) \times e}{2}$

$= 5\text{eV} - 0.5\text{eV}$

$= 4.5\text{eV}$

15

JEE 1

In an a.c circuit, the instantaneous e.m.f and current are given by $e = 100\sin 30ti = 20\sin\left(30t + \frac{\pi}{4}\right)$. In one cycle of a.c the average power consumed by the circuit and the wattless current are, respectively :

(A) $\frac{50}{\sqrt{2}}, 0$

(B) 50,0

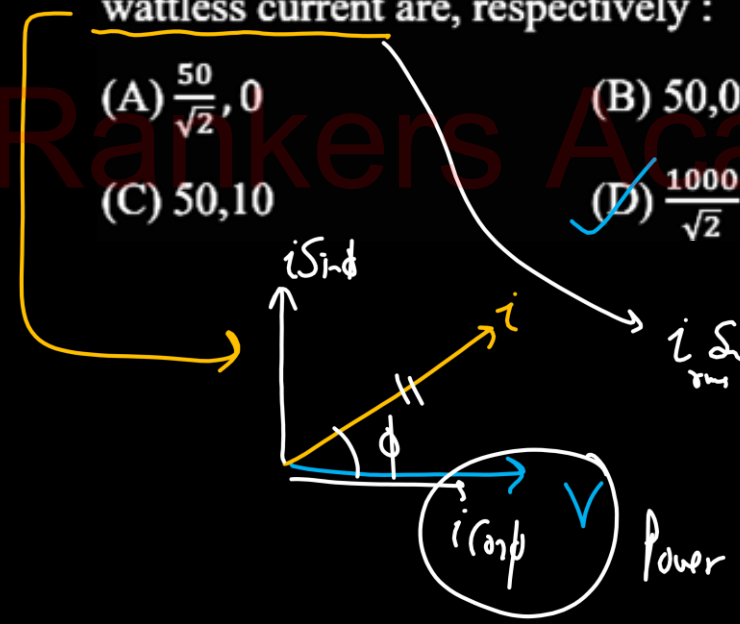
(C) 50,10

(D) $\frac{1000}{\sqrt{2}}, 10$

$\bar{P} = \frac{V_0 i_0}{2}\cos\phi$

$= \frac{100 \times 20}{2}\cos\frac{\pi}{4}$

$= \frac{1000}{\sqrt{2}}$ — (1)



$i_{rms}\sin\phi = \frac{20}{\sqrt{2}}\sin 45^\circ = 10A$ — (2)

16

One mole of diatomic gas is taken through below cyclic process. The process CA is defined as $P = (\text{constant})V^2$. Temperature at C is 300 K . Match the magnitudes of the quantities in Column I to those in Column II.

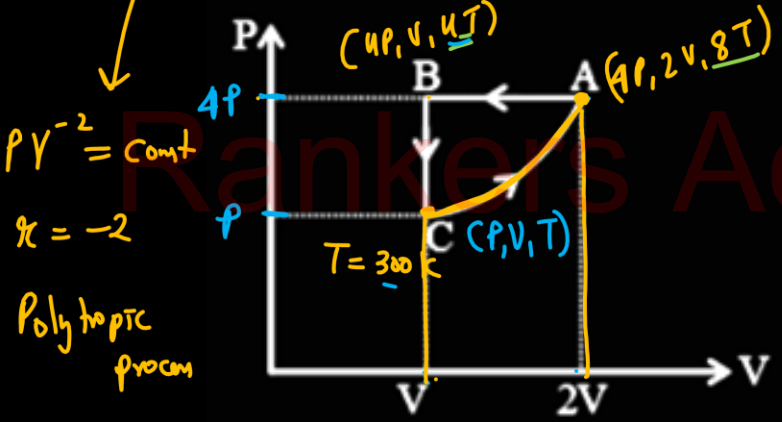
Column-I		Column-II	
(I)	Work done in path CA is	(P)	1200 R
(II)	Work done in path AB is	(Q)	2250 R
(III)	Change in internal energy in path BC is	(R)	700 R
(IV)	Heat transferred in path AB is	(S)	4200 R

(A) (I) → P, (II) → R, (III) → Q, (IV) → S

(B) (I) → P, (II) → R, (III) → S, (IV) → Q

(C) (I) → R, (II) → P, (III) → S, (IV) → Q

(D) (I) → R, (II) → P, (III) → Q, (IV) → S



$PV^{-2} = \text{Const}$

$n = -2$

Polytropic process

(2) $W_{CA} = \dfrac{P_1V_1 - P_2V_2}{n-1} = \dfrac{nR(T_1 - T_2)}{n-1}$

$= \dfrac{1 \times R \times (T - 8T)}{-2-1} = +\dfrac{7RT}{3} = \dfrac{7 \times 300}{3} R = 700R$ — (1)

16

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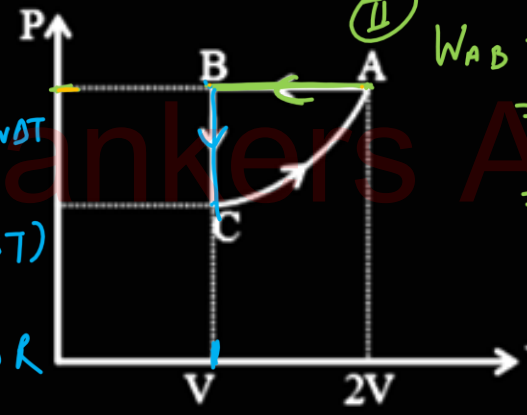

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(B) (I) → P, (II) → R, (III) → S, (IV) → Q

(C) (I) → R, (II) → P, (III) → S, (IV) → Q

(D) (I) → R, (II) → P, (III) → Q, (IV) → S

(II) $W_{AB} = P\Delta V = nR\Delta T = 1 \times R \times (4T - 8T) = -4R \times 300 = -1200R$ (2)

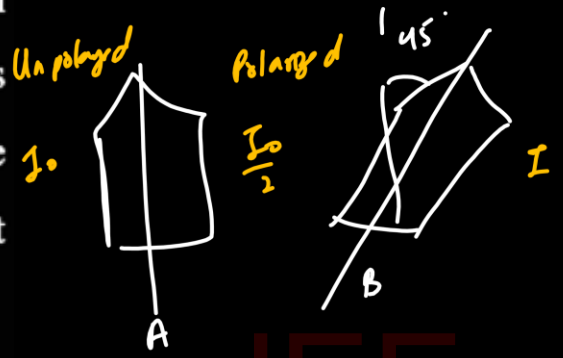
(3) $\Delta U_{BC} = nC_V\Delta T = 1 \times \frac{5R}{2} \times (3T) = \frac{15}{2} \times 300R = 2250R$

(4) $Q_{AB} = nC_P\Delta T = 1 \times \frac{7R}{2} \times (4T - 8T) = -14R \times 300 = -4200R$

17

JEE 1

A beam of unpolarized light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is :



(A) I_0

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{4}$

(D) $\frac{I_0}{8}$

Malus's Law $I = \left(\frac{I_0}{2}\right)\cos^2 45°$

$= \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{I_0}{4}$

18

JEE 1

Statement -1 : At thermal equilibrium between the He gas and H_2 gases both have equal average translational KE.

$f_{trans} = 3$ → $K_{tr} = \frac{3}{2}nRT = \frac{3}{2}pV$ ✓

Statement - 2 : Molar heat capacity of CO_2 is higher at higher temperatures. ✓

CO_2 → Linear

Low temp $f = 5$

High temp $f = 5 + 2$ (vibration) $= 7$

(A) Both statements are true.

(B) statement - 1 is true and statement - 2 is false

(C) statement - 2 is true and statement - 1 is false

(D) Both statements are false

$C_p = \left(\frac{f+2}{2}\right)R$

$C_v = \frac{fR}{2}$

19

In a hydrogen atom, electron jumps from 4th excited state to 2nd excited state. Wavelength of photon emitted is [R : Rydberg constant]

(A) $\frac{225}{16R}$ (B) $\frac{225}{4R}$

(C) $\frac{100}{21R}$ (D) $\frac{100}{4R}$

$n=5$ $n=3$

$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{5^2}\right]$

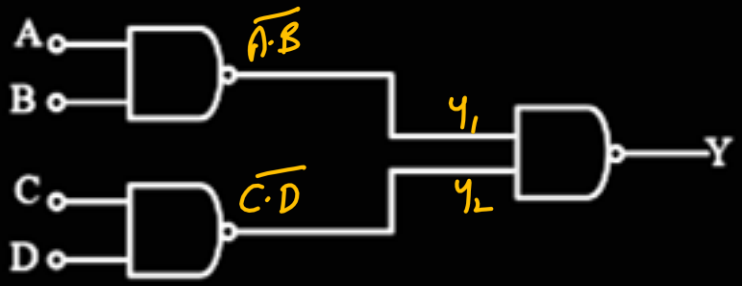
$\frac{1}{\lambda} = R\frac{(25-9)}{25\times 9}$

$\lambda = \frac{225}{16R}$

20

JEE 1

In the logic circuit, Y is given by :-



$Y = \overline{Y_1 \cdot Y_2}$

$= \overline{(\overline{A \cdot B}) \cdot (\overline{C \cdot D})}$

$= \overline{\overline{A \cdot B}} + \overline{\overline{C \cdot D}}$

$= A \cdot B + C \cdot D$

(A) $Y = ABCD$

(B) $Y = (A + B)(C + D)$

(C) $Y = A + B + C + D$

(D) $Y = AB + CD$

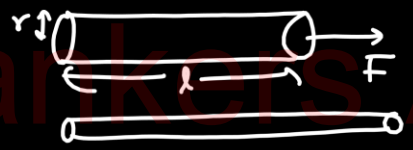
21

A hanging rod is pulled with force F which leads to its elongation. However it is found that there is no change in its volume. Its Poisson's ratio is $\frac{1}{m}$. Find m .

JEE 1

$$\sigma = \frac{\varepsilon_r}{\varepsilon_l} = -\frac{\frac{\Delta r}{r}}{\frac{\Delta l}{l}} \quad *$$

ε : strain



$V = \pi r^2 l$

$\Rightarrow dV = \pi\, d(r^2 l)$

$\Rightarrow 0 = \pi (2r\,dr)\, l + \pi r^2\, dl$

$\Rightarrow 2\frac{dr}{r} = -\frac{dl}{l}$

$\sigma = \frac{-\frac{dr}{r}}{\frac{dl}{l}}$

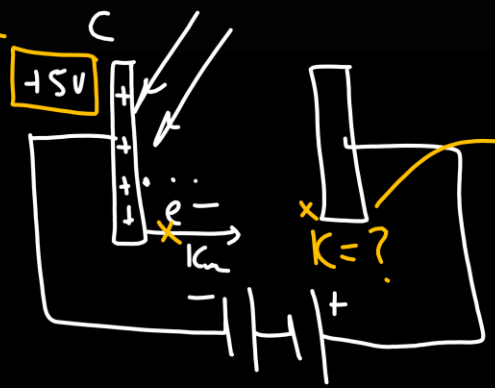
$= \frac{1}{2}$

Ans.

22

JEE 1

Photons with energy 6 eV are incident on cathode plate in photoelectric experiment. The maximum energy of emitted photoelectrons is 2 eV . When light of double frequency is incident and emitter plate is kept at +5 V with respect to collector plate, determine maximum KE of photo electrons in eV striking the collector plate.



$K_{max} = E - \phi$

$2eV = 6eV - \phi$

$\phi = 4eV$ — (1)

$E \rightarrow 2E$

$6eV \rightarrow 12eV$

$K_{max} = 2E - \phi$

$= 12eV - 4eV$

$= 8eV$

$K = K_{max}$ eV

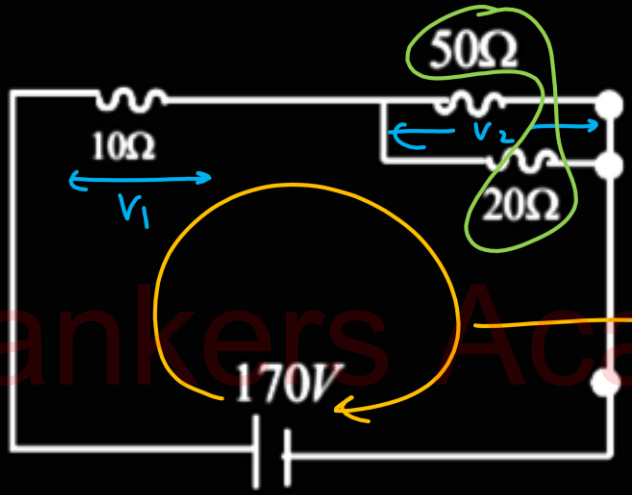
$= 8eV - e(5)$

$= 3$ eV

Ans

23

The voltage across the 10Ω resistor in the given circuit is x volt.



$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{10\,\Omega}{\frac{50\times 20}{50+20}\,\Omega} = \frac{70}{100} \quad \text{—(1)}$$

KVL $\quad V_1 + V_2 = 170\,V$ —(2)

from (1) & (2)

$V_1 = 70\,V$

24

$C_v = \frac{5R}{2}$

One mole of an ideal diatomic gas is expanded. During expansion, volume and temperature of gas vary such that $\frac{V}{T^2} = \text{constant}$. heat transfer during expansion if temperature of the gas increases by 50 K is nR, where R is ideal gas constant. Then find n.

$\frac{V}{\left(\frac{PV}{nR}\right)^2} = \text{const}$

$P^2V = \text{const}$

$PV^{\frac{1}{2}} = \text{const}$

Polytropic process

$C = C_v - \frac{R}{n-1}$ ×

$= \frac{5R}{2} - \frac{R}{\frac{1}{2}-1}$

$= \frac{9R}{2} - (1)$

$Q = nC\Delta T$

$= n\frac{9R}{2}\Delta T$

$= 1 \times \frac{9R}{2} \times 50$

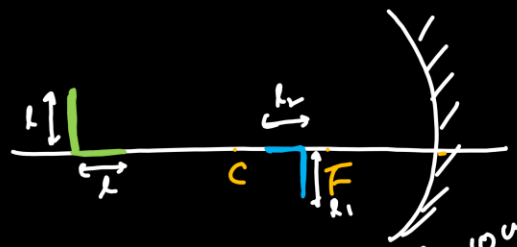
$= 225R$

JEE 1

25

JEE 1

A small piece of wire bent into an L shape with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 cm . If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is n: 1. Find n.



$R = 10\,cm$

$$m = -\frac{v}{u} = \frac{f}{f-u} = \frac{-5}{-5-(-20)}$$

$$m = \frac{-1}{3}$$

Lateral magnified

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2}dv - \frac{1}{u^2}du = 0$$

$$\Rightarrow \boxed{\frac{dv}{du} = -\frac{v^2}{u^2} = -m^2}$$

$$\frac{l_2}{l} = -m^2 = -\left(\frac{-1}{3}\right)^2 \quad \text{—(2)}$$

$$\frac{l_1}{l_2} = \frac{-\frac{1}{3}}{-\frac{1}{9}} = 3:1 \quad \text{Ans}$$

$$\frac{l_1}{l} = \frac{-1}{3} \quad \text{—(1)}$$

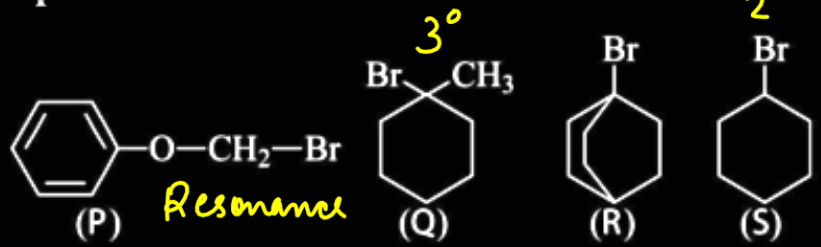
CHEMISTRY

7

Rate of reaction of the given compounds with aqueous ethanol follows the order:

Carbocation stability

JEE 1



(A) P > Q > S > R

(B) Q > P > R > S

(C) P > R > Q > S

(D) R > P > S > Q

2

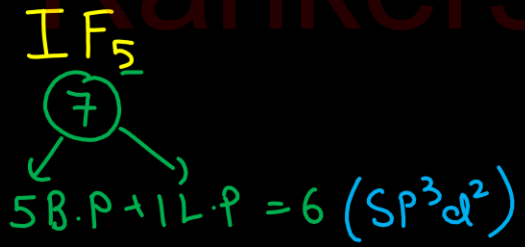
The hybridisation of central iodine atom in IF_5, I_3^- and I_3^+ are respectively :

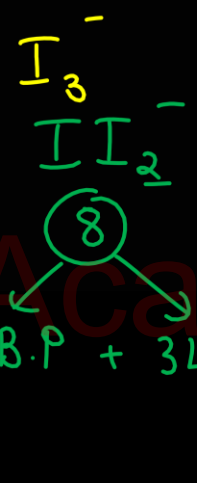
(A) sp^3d^2, sp^3d, sp^3

(B) sp^3d, sp^3d, sp^3

(C) sp^3d^2, sp^3d^2, sp^3

(D) sp^3d, sp^3d^2, sp^3





3

Incorrect statement is :

(A) $AlF_3 > MgO > MgF_2$: Lattice energy

(B) Li > Na > Al > Mg : Electron affinity

(C) $SF_6 > PF_5 > SiF_4$: Lewis acidic character

(D) $SiCl_4 > SiBr_4 > SiI_4$: Decreasing order of electronegativity of Si

4

JEE 1

Values of p, q, r, s and t are in the following redox reaction

$$pBr_2 + qOH^- \longrightarrow rBr^- + sBrO_3^- + tH_2O$$

	p	q	r	s	t
(a)	3	6	1	5	3
(b)	3	6	5	3	1
(c)	3	6	5	1	3
(d)	3	5	1	6	3

$$Br_2^0 \longrightarrow 2BrO_3^{-} + 10e^-$$

$$(Br_2^0 + 2e^- \longrightarrow 2Br^{-1}) \times 5$$

$$6Br_2 \longrightarrow 2BrO_3^- + 10Br^-$$

$$+ 12OH^- \qquad + 6H_2O$$

$$3Br_2 + 6OH^- \longrightarrow BrO_3^- + 5Br^- + 3H_2O$$

5

0.1mole CH_3NH_2 ($K_b = 6 \times 10^{-4}$) is added to 0.08mole of HCl (L.R) and the solution is diluted to 1L. The hydrogen-ion concentration in the solution is

(A) 5×10^{-5}M

(B) 8×10^{-2}M

(C) 1.6×10^{-11}M

(D) 6.7×10^{-11}M

Basic Buffer

JEE 1

$BOH \rightleftharpoons B^+ + OH^-$

$K_b = \frac{[B^+][OH^-]}{[BOH]}$

$K_w = [H^+][OH^-]$

$[H^+] = \frac{K_w}{OH^-}$

$[OH^-] = \frac{[K_b][Base]}{[Salt]}$

CH_3NH_2	+	HCl (L.R)	→	Salt	+	water
0.1		0.08		0		0
0.02		0		0.08		

$= \frac{[6 \times 10^{-4}][0.02]}{[0.08]} = 1.5 \times 10^{-4}$

$[H^+] = \frac{10^{-14}}{1.5 \times 10^{-4}} = 6.7 \times 10^{-11}$ M

6

JEE 1

The standard enthalpies of formation at 300 K for $CCl_4(\ell)$, $H_2O(g)$, $CO_2(g)$ and $HCl(g)$ are $-107, -242, -394$ and -93 kJ mol^{-1} respectively. The value of ΔU^0_{300} for the reaction

$CCl_4(l) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$ is

(A) -170 kJ mol^{-1} (B) -175 kJ mol^{-1}

(C) -182.5 kJ mol^{-1} (D) -282.5 kJ mol^{-1}

$\Delta n_g = 5 - 2 = 3$

$\Delta H = \Delta U + \Delta n_g RT$

$\Delta U = \Delta H - \Delta n_g RT$

$= -175 - (3)(8.314 \times 10^{-3})(300)$

$= -182.5$ kJ/mol

$\Delta H_r = \{\Delta H_{f(CO_2)} + 4\Delta H_{f(HCl)}\} - \{\Delta H_{f(CCl_4)} + 2\Delta H_{f(H_2O)}\}$

$= \{-394 + 4(-93)\} - \{-107 + 2(-242)\}$

$\Rightarrow -175$ kJ/mol

7

Among $Ni(CO)_4$, $[Ni(CN)_4]^{2-}$ and $[Ni(Cl)_4]^{2-}$:

(A) $Ni(CO)_4$ and $NiCl_4^{2-}$ are diamagnetic and $Ni(CN)_4^{2-}$ is paramagnetic ✗

(B) $NiCl_4^{2-}$ and $Ni(CN)_4^{2-}$ are diamagnetic and $Ni(CO)_4$ is paramagnetic ✗

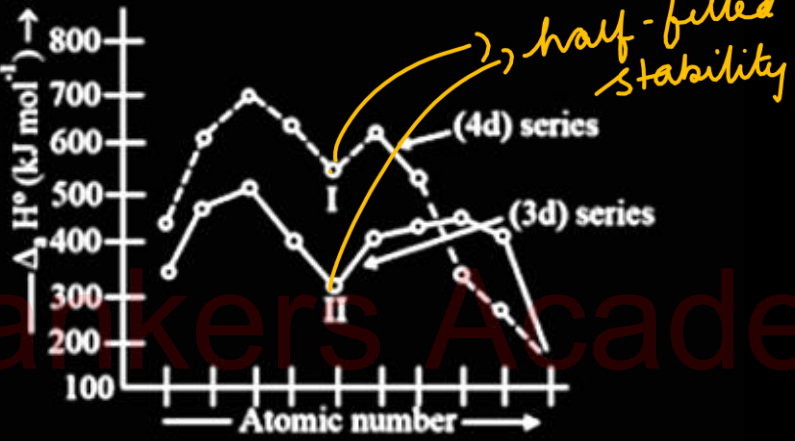
(C) $Ni(CO)_4$ and $Ni(CN)_4^{2-}$ are diamagnetic and $NiCl_4^{2-}$ is paramagnetic

(D) $Ni(CO)_4$ is diamagnetic and $NiCl_4^{2-}$ and $Ni(CN)_4^{2-}$ are paramagnetic

	$Ni(CO)_4$	$[Ni(CN)_4]^{2-}$	$[NiCl_4]^{2-}$
O.S	0	+2	+2
Hyb	sp^3	dsp^2	sp^3
Geo	Td	Square Planer	Td
Mag. nature	Di	Di	Para

8

The given graph represents the trend in melting point of transition metals.



(A) I – Mo, II – Cr

(B) I – Ru, II – Fe

(C) I- Tc, II -Mn

(D) None of these

JEE 1

9

JEE 1

Unknown salt 'A' $+K_2Cr_2O_7 +$ conc. $H_2SO_4 \rightarrow$ Reddish brown fumes. Which is the correct statements regarding the above observation?

(A) It confirms the presence of Cl^- ion.

(B) It confirms the presence of Br^- ion.

(C) It confirms the presence of both.

(D) It neither confirms Cl^- nor Br^- unless it is passed through NaOH solution.

$Cl^- + Cr_2O_7^{2-} \rightarrow CrO_2Cl_2$ Brown vapours

$CrO_2Cl_2 + OH^- \rightarrow CrO_4^{2-}$ Yellow

$Br^- + Cr_2O_7^{2-} \rightarrow Br_2$ Brown vapours

$Br_2 + OH^- \rightarrow$ No colour.

10

Bond dissociation enthalpy of $E-H(E=\text{element})$ bonds is given below. Which of the following compounds will act as the strongest reducing agent?

Compounds	NH_3	PH_3	AsH_3	SbH_3
$\Delta_{diss}(E-H)/kJmol^{-1}$	389	322	297	255

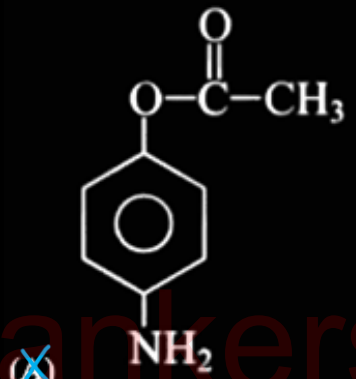
(A) NH_3 (B) PH_3

(C) AsH_3 (D) SbH_3

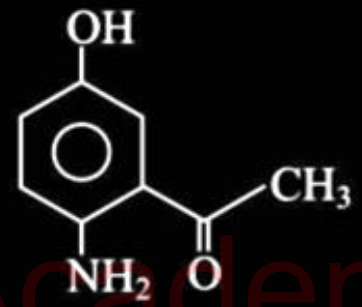
Reducing nature $\propto \frac{1}{B.D.E}$ (For Hydrides)

11

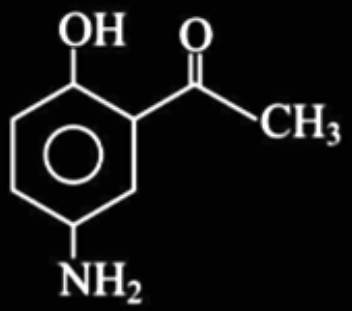
The reaction of p-aminophenol with one mole of acetyl chloride in presence of pyridine gives



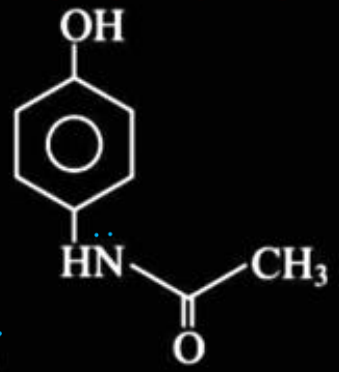
(A)



(C)

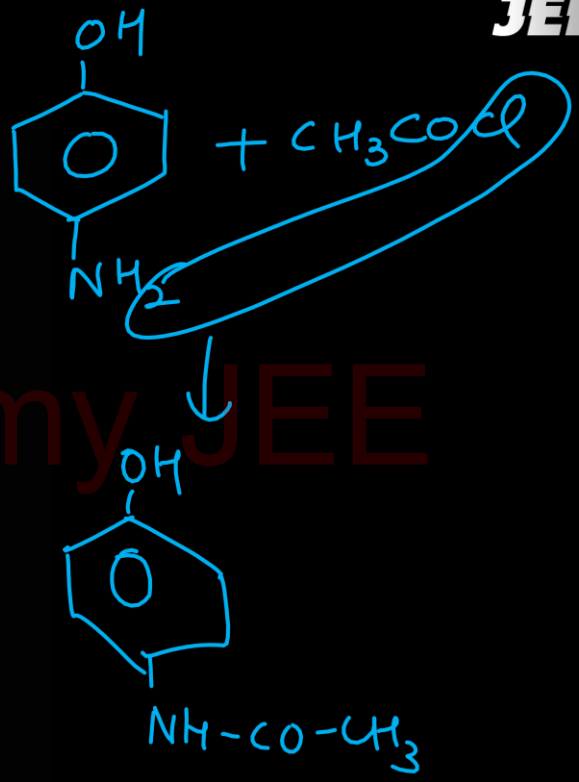


(B)



(D)

JEE 1



12

Which among the following compounds will give maximum enol content in solution?

(A) $C_6H_5-\overset{O}{\overset{\|}{C}}-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

(B) $CH_3-\overset{O}{\overset{\|}{C}}-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

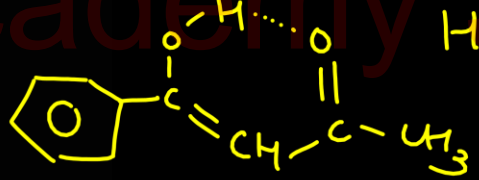
(C) $CH_3-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-CH_3$

(D) $CH_3-\overset{O}{\overset{\|}{C}}-CH_2-COOC_2H_5$

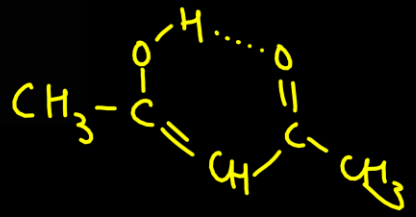
JEE 1

enol content

diketo > Ketoester > monoketo



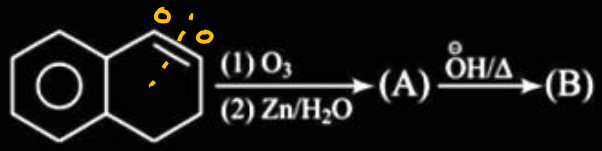
H-bond + Aromatic ring

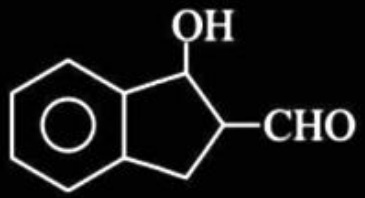


only H-bonding

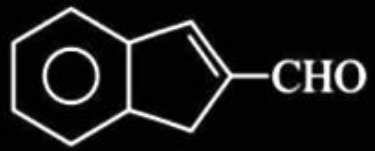
13

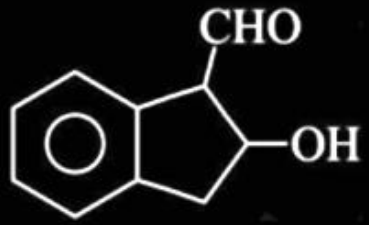
In the given reaction sequence, B is

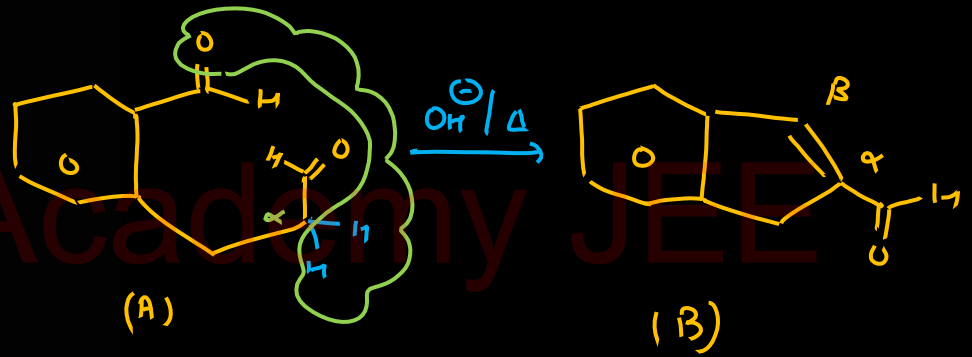


(A) 

(B) 

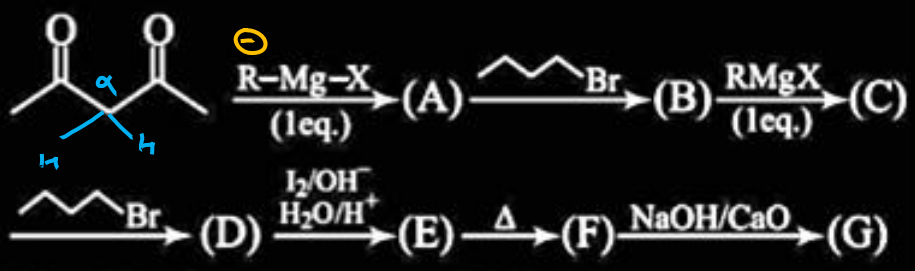
(C) 

(D) 



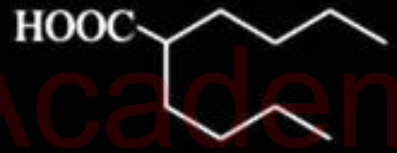
14

Identify the structure of ' G '.

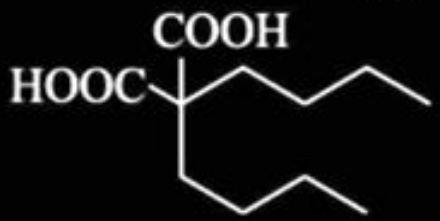


(A)

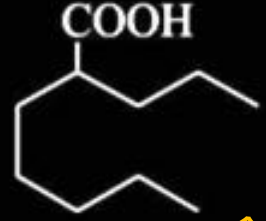
(B)

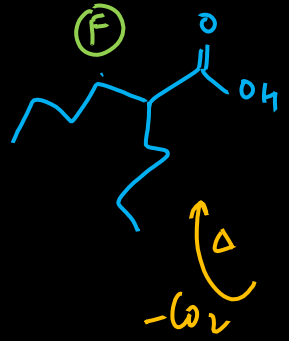


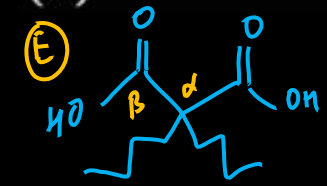
(C)

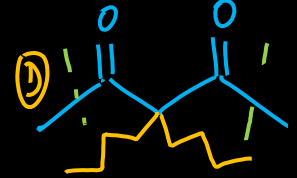


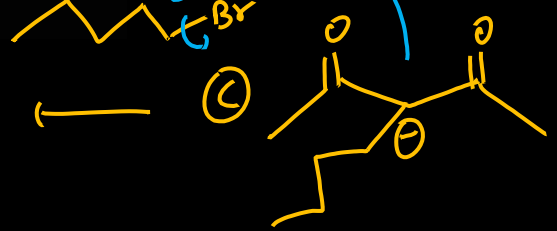
(D)

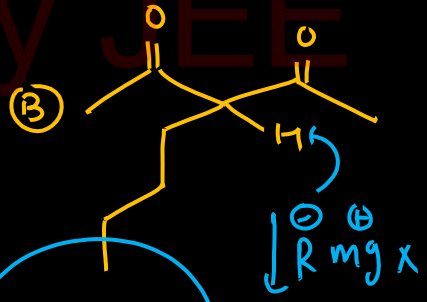


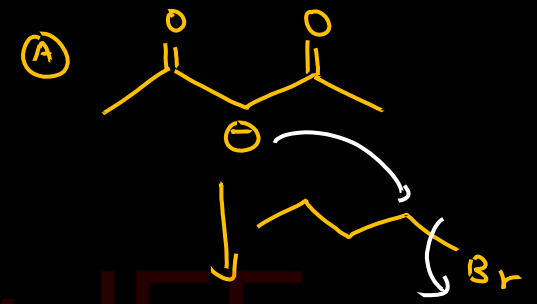






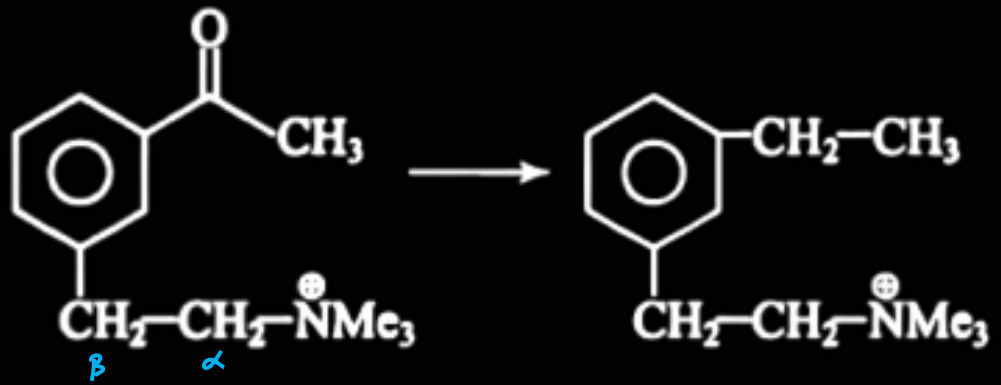






15

Above conversion can be achieved by



WKR

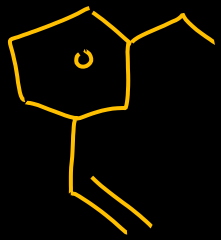
β α

(A) $Zn - Hg/HCl$

(B) $NH_2 - NH_2/NaOH$

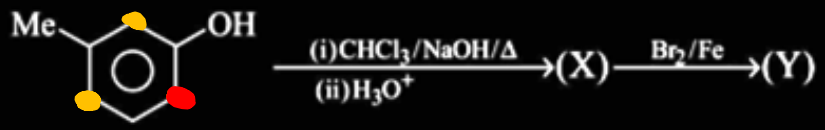
(C) $LiAlH_4$

(D) $NaBH_4$

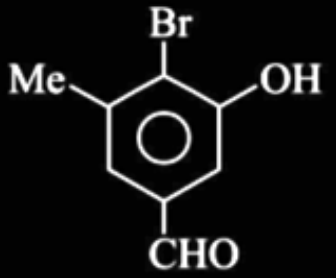


16

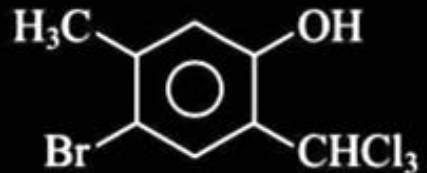
The product (Y) of the following sequence of reactions will be

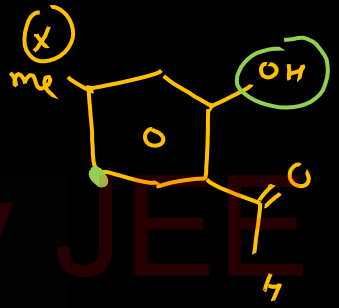


(A) 

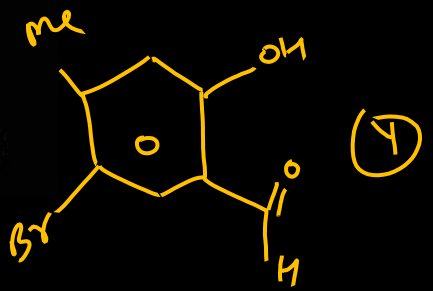
(B) 

(C) 

(D) 



↓ Br_2/Fe



17

The data for the reaction: $A + B \xrightarrow{k} C$.

Experiment	$[A]_0$	$[B]_0$	Initial rate
1	0.012	0.035	0.10
2	0.024	0.070	0.80
3	0.024	0.035	0.10
4	0.012	0.070	0.80

2 times

no change

8 times.

2 times

The rate law corresponding to the above data is

(A) Rate = $k[B]^3$

(B) Rate = $k[B]^4$

(C) Rate = $k[A][B]^3$

(D) Rate = $k[A]^2[B]^2$

$Rate = k[A]^x[B]^y$

order w.r.t A

$[B] \rightarrow$ Const.

$x = 0$

order w.r.t [B]

$[A] \rightarrow$ const.

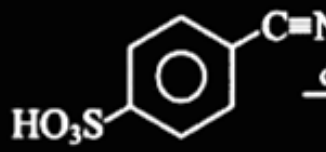
$(2)^y \propto 8 = 2^3$

$y = 3$

JEE 1

18

JEE 1

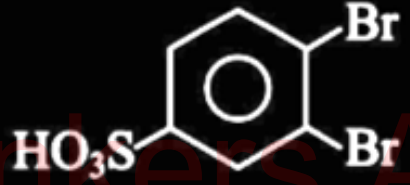


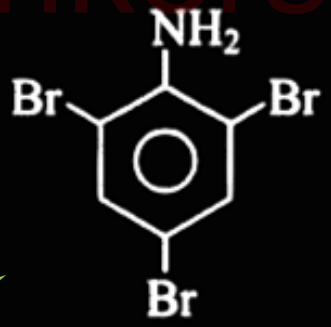
dilute NaOH/Δ → $(C_7H_7NSO_4)$

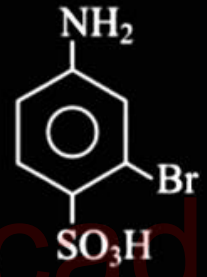
(Partial hydrolysis)

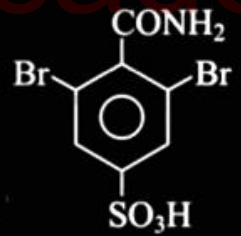
KOBr/Δ → product → Br_2/H_2O → end product.

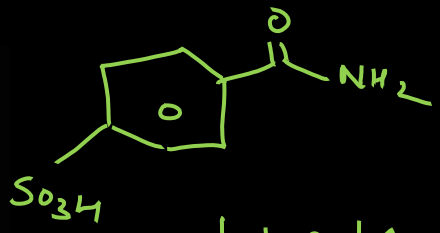
The end product is.

(A) 

(B) 

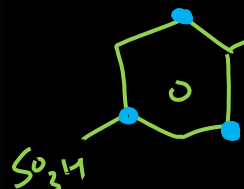
(C) 

(D) 

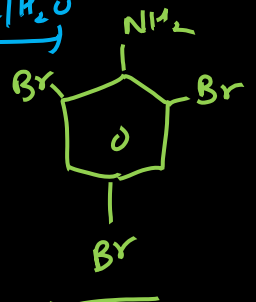


KOBr/Δ

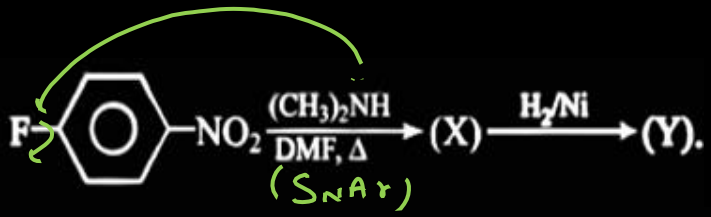
(HBD Rxn)



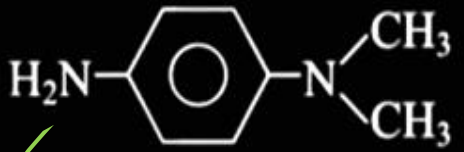
Br_2/H_2O



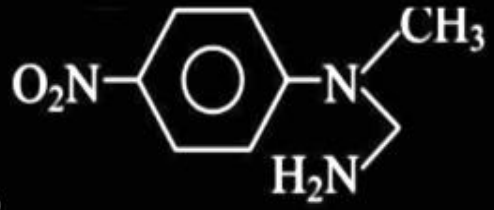
19



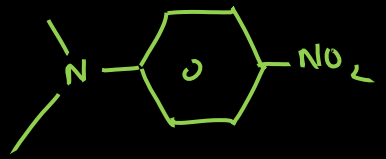
The product (Y) is

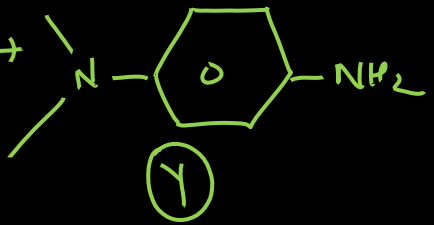
(A) 

(B) 

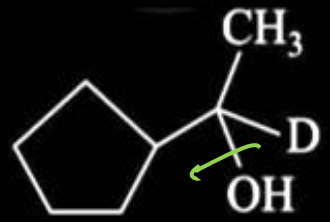
(C) 

(D) 

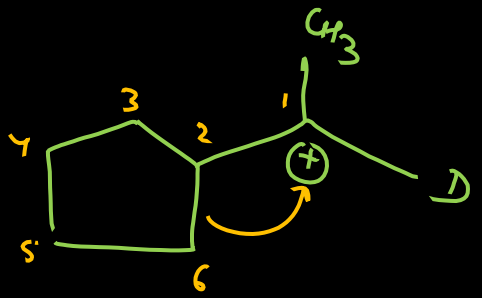
(X) 



20

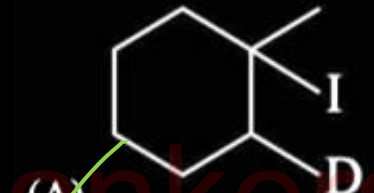


HI → Product.



Identify the major product.

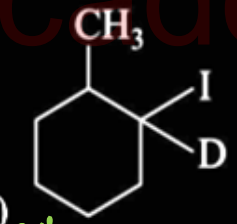
(A)

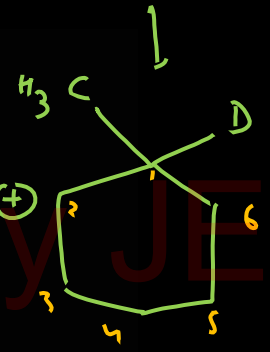


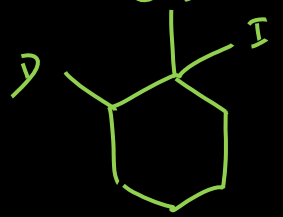
(B)

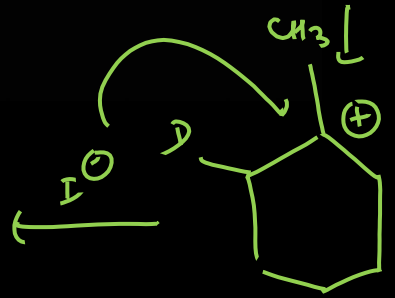
(C)

(D)









21

An unknown solution [mol.wt. of solute = 250] is 20%(w/w). Molarity of solution is. [Given $d_{solution} = 1.25$ g/ml]

JEE 1

20% w/w → 20g solute in 100g Solution.

$n_{solute} = \frac{20}{250} = \frac{2}{25}$

Volume of Solution = $\frac{mass}{density} = \frac{100}{1.25} = \frac{100 \times 100}{125} = 80ml$

Molarity = $\frac{n}{V(ml)} \times 1000 = \frac{2/25}{80/1000} = \frac{2}{25} \times \frac{1000}{80} = 1M$

22

From the given compounds if X number of compounds are acidic in water

$CaO, SO_2, SO_3, Fe_2O_3, Cl_2O_7, CO_2, Na_2O$

Find the value of X .

JEE 1

non metal oxide generally

CaO, Fe_2O_3, Na_2O (Basic)

SO_2, SO_3, Cl_2O_7, CO_2 (Acidic)

4

23

The molar conductance at infinite dilution of $CaC\ell_2$, $Ca(CH_3COO)_2$ and HCl are respectively 271.6, 200.8 and 425.95 S cm^2 mol^{-1}. What will be the molar conductance of CH_3COOH in S cm^2 mol^{-1} unit? (nearest integer)

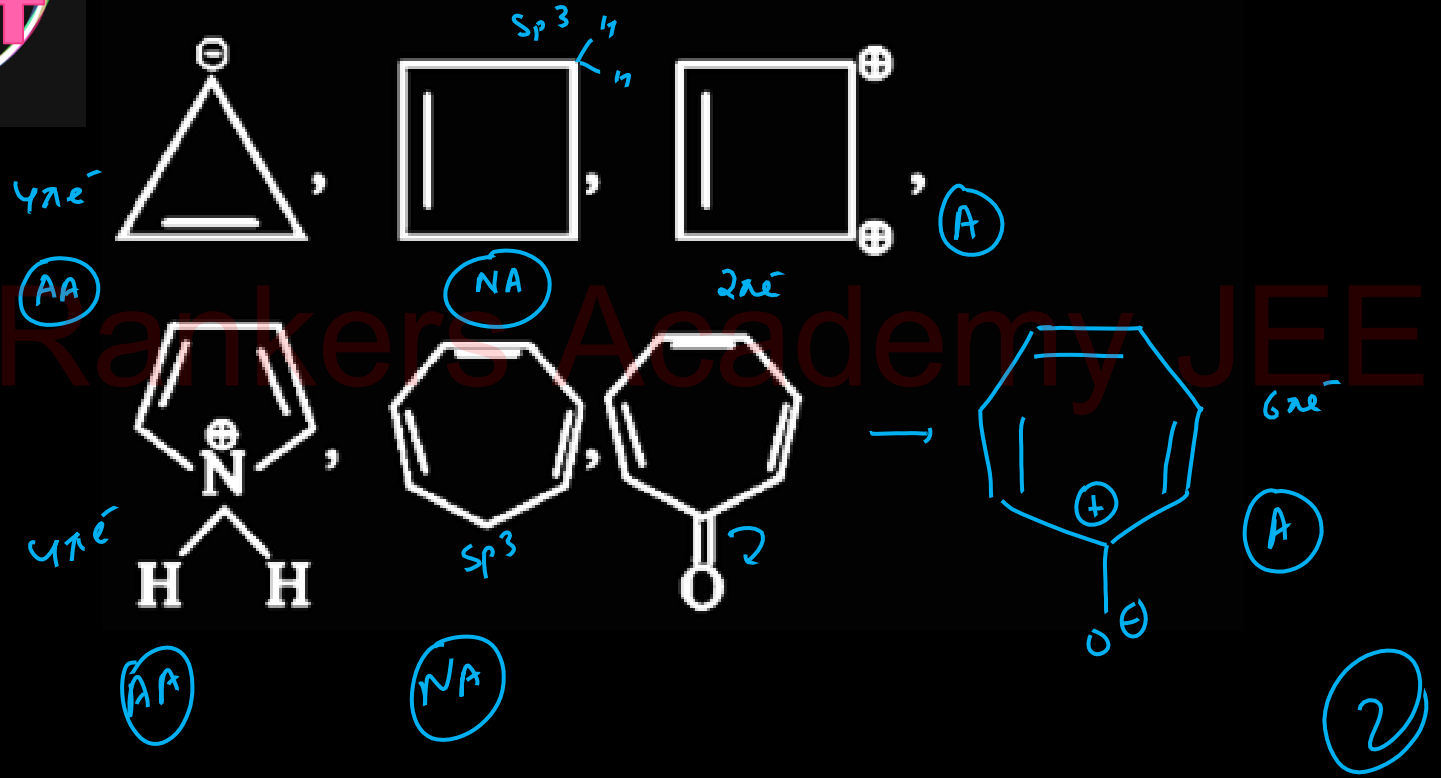
$$\Lambda^\infty(CH_3COOH) = \Lambda^\infty_{CH_3COO^-} + \Lambda^\infty_{H^+}$$

$$= \frac{1}{2}\Lambda^\infty\left(Ca(CH_3COO^-)_2\right) + \Lambda^\infty(HCl) - \frac{1}{2}\Lambda^\infty(CaCl_2)$$

$$= \frac{1}{2}(200.8) + (425.95) - \frac{1}{2}\times 271.6$$

24

How many species out of the following are aromatic?

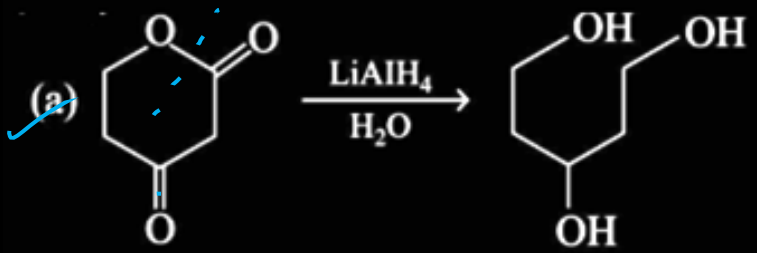


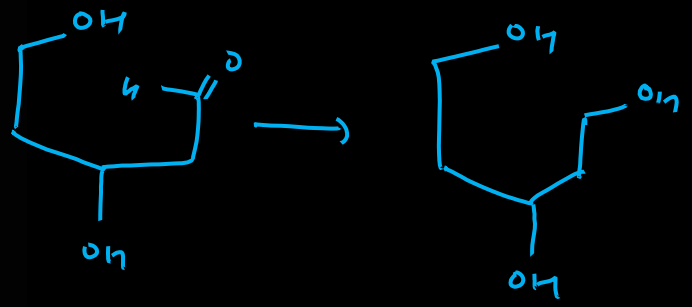
JEE 1

25

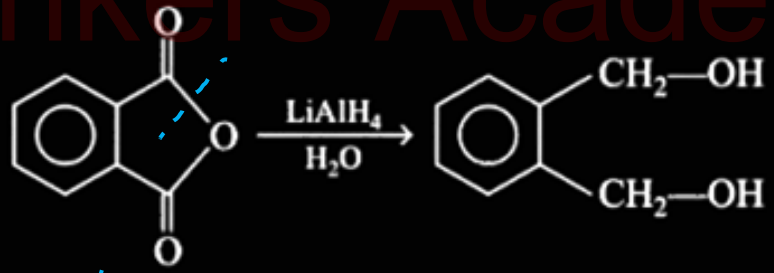
How many of the following reactions is/are correct?

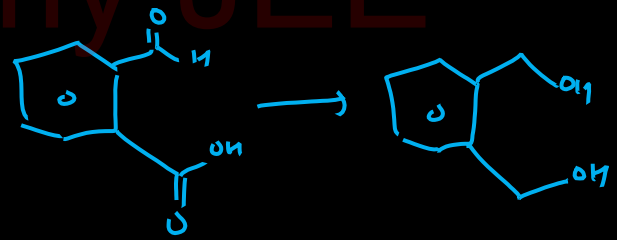
JEE 1

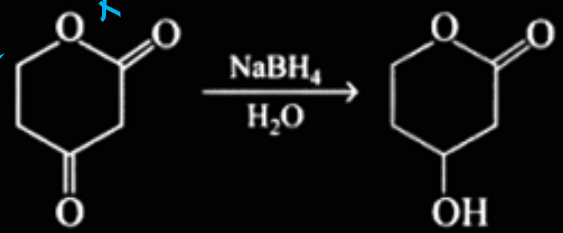
(a) 



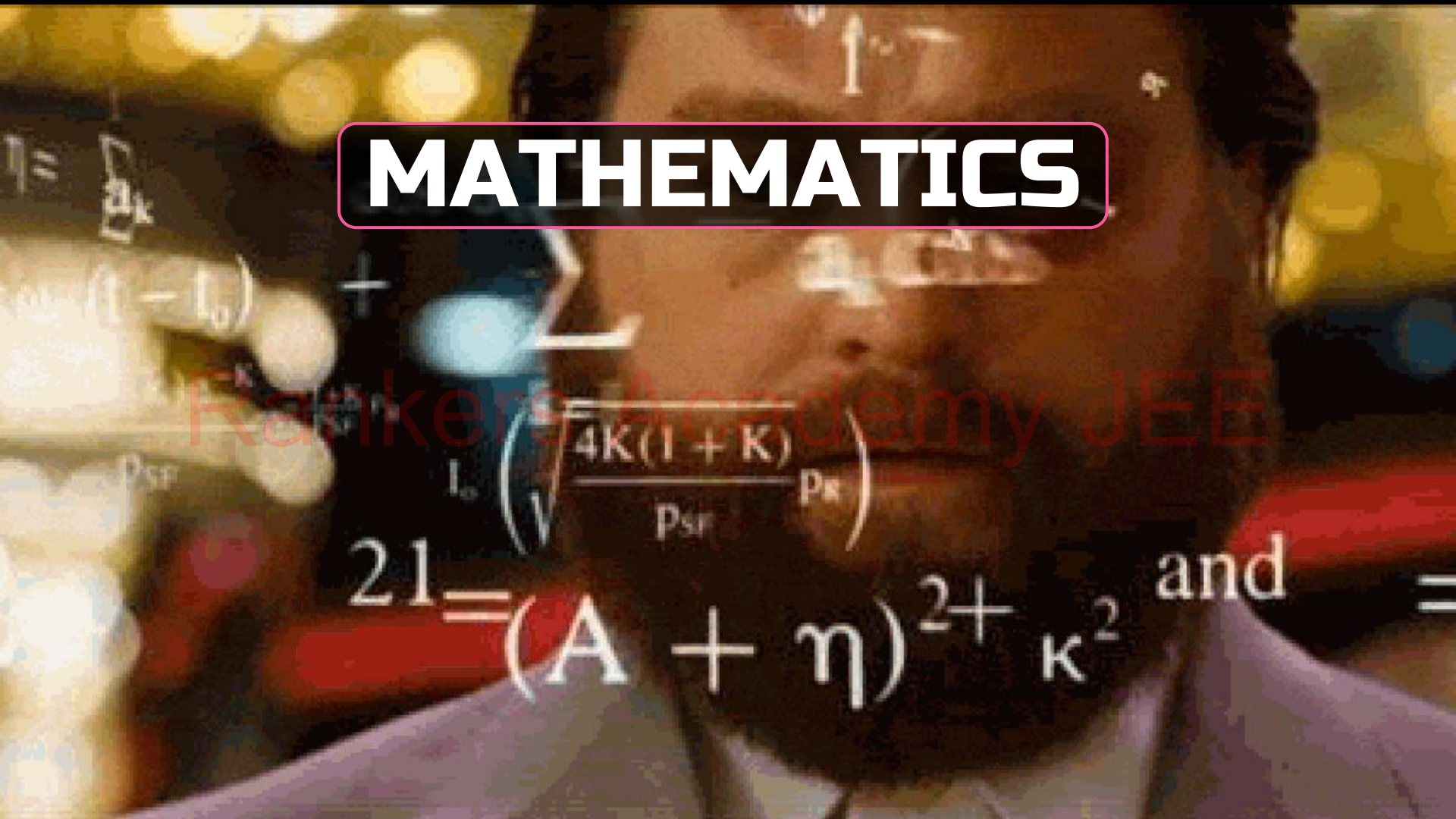
(b) $CH_3—C\equiv N \xrightarrow[H_2O]{LiAlH_4} CH_3—CH_2—NH_2$

(c) 



(d) 

4


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^{2+} \kappa^2$ and

7

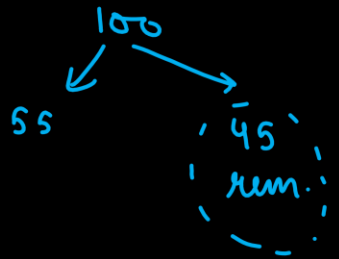
100 identical objects are distributed among 10 persons in which 1st person gets at least one object, 2nd person gets at least 2 objects, 3rd person gets at least 3 objects, ..., 10th person gets at least 10 objects, then total number of ways of distribution is

$45+10-1C_{10-1}$

(A) $^{55}C_{10}$ (B) $^{55}C_9$

(C) $^{54}C_9$ (D) $^{54}C_{44}$

$1+2+3+..+10 = 55$ ✓



2

If $\lim_{x\to 0}\int_0^x \frac{t^2 dt}{(x-\sin x)\sqrt{a+t}} = 1$, then the value of a is

(A) 4 ✓

(B) 2

(C) 1

(D) 8

JEE 1

$$\lim_{x\to 0} \frac{\int_0^x \frac{t^2\,dt}{\sqrt{a+t}}}{x-\sin x} = 1 \quad \left(\frac{0}{0}\right)$$

$$\Rightarrow \lim_{x\to 0} \frac{\frac{x^2}{\sqrt{a+x}}(1) - 0}{1-\cos x}$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\sqrt{a+x}} \left(\frac{x^2}{1-\cos x}\right) = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}} \cdot (2) = 1$$

$$\Rightarrow a = 4$$

3

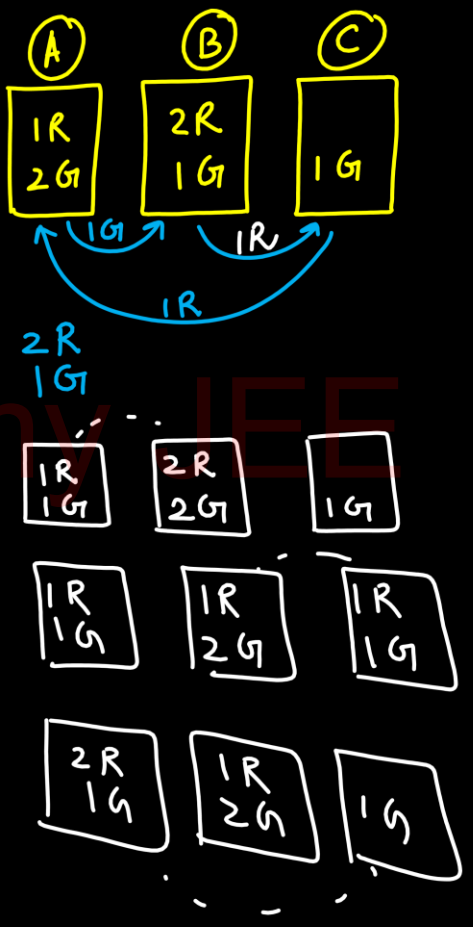
There are 3 bags A, B & C. Bag A contains 1 Red & 2 Green balls, bag B contains 2 Red & 1 Green balls and bag C contains only one green ball. One ball is drawn from bag A & put into bag B then one ball is drawn from B & put into bag C & finally one ball is drawn from bag C & put into bag A. When this operation is completed, probability that bag A contains 2 Red & 1 Green balls, is –

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{6}$

$= \frac{2}{3} \times \frac{2}{4} \times \frac{1}{2}$

$= \frac{1}{6}$

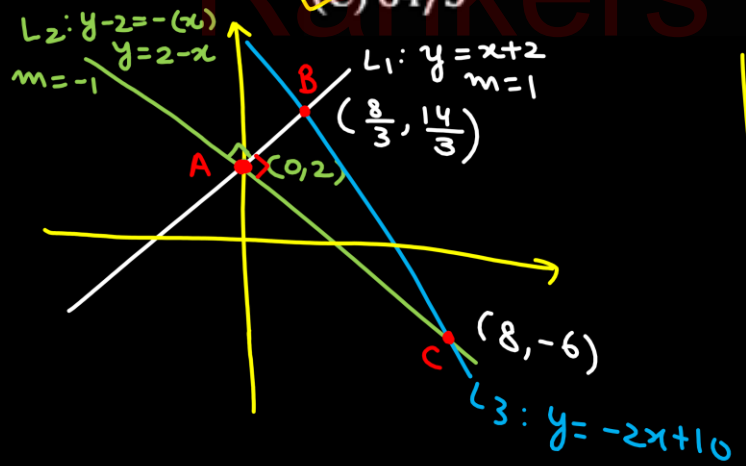


4

A line $L_1; y = x + 2$ is rotated through an angle of 90° in counter-clockwise direction about the point (0,2) in xy plane and let new position of the line be $L_2 = 0$. Let $L_3: y = -2x + 10$. The area of the triangle formed by lines $L_1 = 0, L_2 = 0$ and $L_3 = 0$ is equal to:

(A) 56/3 (B) 28/3

(C) 64/3 (D) 32/3



$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ \frac{8}{3} & \frac{14}{3} & 1 \\ 8 & -6 & 1 \end{vmatrix}$$

$$= \frac{64}{3}.$$

B ≡ $y = x+2$, $y = -2x+10$

$-2x+10 = x+2$

$x = 8/3$ $y = \frac{14}{3}$

C ≡ $y = 2-x$, $y = -2x+10$

$-2x+10 = 2-x$

$x = 8$

$y = -6$

5

Remainder when $2025^{2024^{2033}}$ is divided by 7 is

(A) 1 (B) 4

(C) 3 (D) 2

JEE 1

$= \frac{2025^{2024^{2023}}}{7}$

$= \frac{2^{2024^{2023}}}{7}$

$= \frac{2^{2^{2023}}}{7}$

$= \frac{2^{(23)\text{even}} \cdot 2^1}{7} = \frac{2^2}{7} = \frac{4}{7}$

$\text{rem}\ \frac{8}{7} = 1$

6

If $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 5 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ then $\text{tr}(A^4 \text{Adj} A^3)$ is

(Where $\text{tr}(A) = \Sigma a_{ii}$ and AdjAdenotes Adjoint of a matrix A)

(A) 984 (B) 576

(C) 720 (D) 1080

$A(\text{adj}A) = |A| I_n$

$A^3(\text{adj}\, A^3) = |A^3| I$

$\text{tr}(kA) = k\,\text{tr}(A)$

M1

$\text{tr}(A^4 \cdot \text{adj}\, A^3) =$

$= \text{tr}(A \cdot A^3 \cdot \text{adj}(A^3))$

$= \text{tr}(A \cdot |A^3| I)$

$= \text{tr}(|A|^3 \cdot A)$

$= \text{tr}(216 A) = 216\,\text{tr}(A)$

$= 216(5) = 1080$

$|A| = -2(0-3) = 6$

M2

$216A = \begin{bmatrix} 0 & 2\times216 & 0 \\ 1\times216 & 5\times216 & 1\times216 \\ 3\times216 & -1\times216 & 0 \end{bmatrix}$

$\text{tr}(216A) = 0 + 5\times216 + 0$

7

The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $a^2x^2 - y^2 = 4$ intersects at right angles then the equation of the circle through the points of intersection of two conic is

(A) $x^2 + y^2 = 5$

(B) $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$

(C) $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$

(D) $x^2 + y^2 = 25$

JEE 1

→ E & H are confocal

E : $4x^2+9y^2=36$

H : $4x^2-y^2=4$

Add

$8(x^2+y^2)=40$

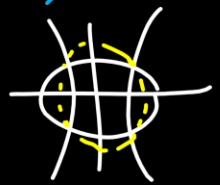
$x^2+y^2=5$ ✓

E : $\frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow e=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3} \Rightarrow$ Foci $=(\pm ae,0)\equiv\left(\pm 3\cdot\frac{\sqrt{5}}{3},0\right)=(\pm\sqrt{5},0)$

H : $\frac{x^2}{4/a^2}-\frac{y^2}{4}=1 \Rightarrow e=\sqrt{1+\frac{4}{4/a^2}}=\sqrt{1+a^2} \Rightarrow$ Foci $\equiv\left(\pm\frac{2}{a}\sqrt{1+a^2},0\right)$

$\Rightarrow \pm\sqrt{5}=\pm\frac{2}{a}\sqrt{1+a^2}$

$\Rightarrow 5a^2=4+4a^2 \Rightarrow a=2$



8

The solution of differential equation,

$e^{-x}(y+1)dy + y(\cos^2 x - \sin 2x)dx = 0$

subject to condition $y = 1$ when $x = 0$, is:

(A) $(y+1) + e^x \cos^2 x = 3$

(B) $\ell n(y+1) + e^x \cdot \cos^2 x = 1 + \ell n 2$

(C) $y + \ln y = e^x \cos^2 x$

(D) $y + \ln y + e^x \cdot \cos^2 x = 2$

$x=0, y=1$

$1+0 = -1+c$

$c = 2$

$\Rightarrow \frac{dy}{dx} = -\frac{e^x(\cos^2 x - \sin 2x)y}{(y+1)}$

$\Rightarrow \int\left(\frac{y+1}{y}\right)dy = -\int e^x(\cos^2 x - \sin 2x)\,dx$

$\hookrightarrow e^x(f(x)+f'(x))$

$\Rightarrow y + \ln|y| = -e^x \cos^2 x + c$

9

JEE 1

Let a function f(x) be defined in $[-2,2]$ as

$$f(x)=\begin{cases}\{x\}, & -2\le x<-1\\ |\text{sgn}(x)|, & -1\le x\le 1\\ \{-x\}, & 1<x\le 2\end{cases}$$

where $\{x\}$ denotes fractional part, then area bounded by graph of f(x) and x-axis is :

$\{x\} = x-[x]$

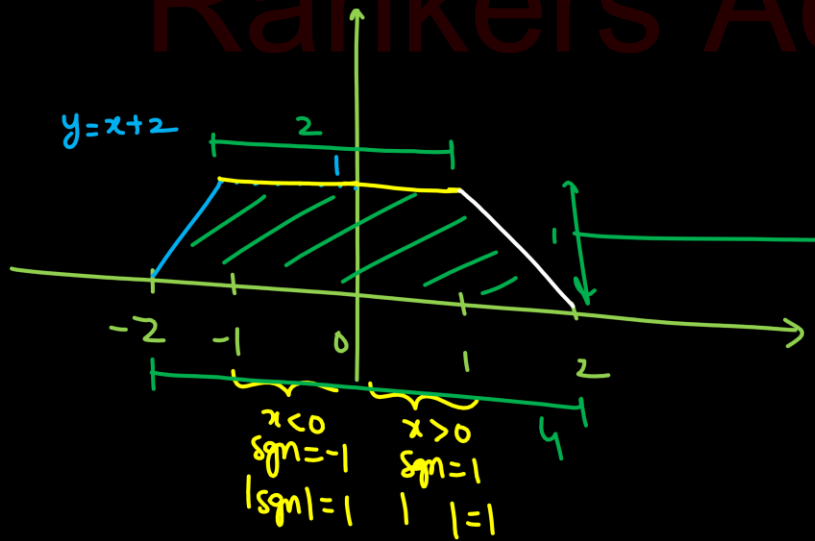
$= x-[-2] = x+2$

$\{-x\} = (-x)-[-x]$

$= (-x)-(-2) = 2-x$

(A) 2 (B) 3

(C) 4 (D) 5



trap $= \frac{1}{2}(2+4)(1) = 3$

10

If $\sum_{r=1}^{100} \sin^{-1}\left(\frac{1}{\sqrt{r^2+1}} \times \frac{1}{\sqrt{r^2+2r+2}}\right)$ is equal to $\tan^{-1}\left(\frac{p}{q}\right)$ where p and q are co-prime, then the value of $(p+q)$ is equal to :

(A) 99

(B) 100

(C) 101 ✓

(D) 102

JEE 1

$r=1 \longrightarrow \sin^{-1}\frac{1}{\sqrt{10}} = \tan^{-1}\frac{1}{3}$

$r=2 \longrightarrow \sin^{-1}\frac{1}{\sqrt{50}} = \tan^{-1}\frac{1}{7}$

$r=3 \longrightarrow \sin^{-1}\frac{1}{\sqrt{170}} = \tan^{-1}\frac{1}{13}$

$r=4 \longrightarrow \sin^{-1} =$

$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \cdots$ + upto 100 terms

$= \tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+6} + \tan^{-1}\frac{1}{1+12} + \cdots$

$= \tan^{-1}\frac{2-1}{1+(1)(2)} + \tan^{-1}\frac{3-2}{1+(2)(3)} + \tan^{-1}\frac{4-3}{1+(3)(4)} + \cdots$

$= \tan^{-1}2 - \tan^{-1}1$
$+ \tan^{-1}3 - \tan^{-1}2$
$+ \tan^{-1}4 - \tan^{-1}3$
$\cdots$
$\tan^{-1}101 -$

$\Rightarrow \tan^{-1}(101) - \tan^{-1}1$

$\Rightarrow \tan^{-1}\left(\frac{101-1}{1+(101)(1)}\right)$

$\Rightarrow \tan^{-1}\left(\frac{100}{102}\right)$

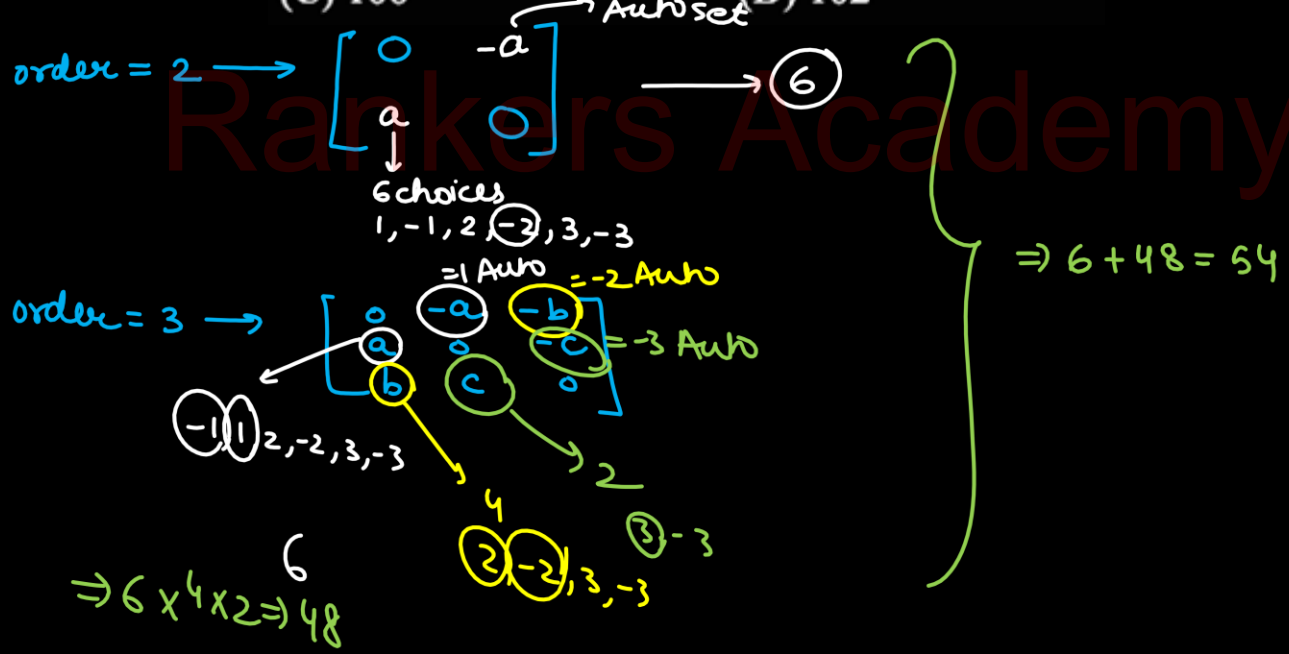
$= \tan^{-1}\left(\frac{50}{51}\right)$

11

The number of skew symmetric matrix of order at least 2 with distinct elements except principal diagonal formed by the elements $0, \pm1, \pm2, \pm3$ taking any number of elements at a time is

(A) 48 (B) 54

(C) 106 (D) 102



12

Consider the system of linear equations: $2x + (p^2 - 2)y + 6z = 8, x + 2y + (q-1)z = 5, x + y + 3z = 4$ where $p, q \in R$. Which of the following statements is NOT correct?

(A) The system will have unique solution if $p \in R - \{-2, 2\}, q \in R - \{4\}$.

(B) The system is inconsistent if $p = 2, q = 4$.

(C) The system will have infinite solutions if $p = -2, q \in R$.

(D) The system is consistent if $p \in R, q \in R - \{4\}$.

$$\Delta = \begin{vmatrix} 2 & p^2-2 & 6 \\ 1 & 2 & q-1 \\ 1 & 1 & 3 \end{vmatrix} = (p^2-4)(q-4)$$

$$\Delta_1 = \begin{vmatrix} 8 & p^2-2 & 6 \\ 5 & 2 & q-1 \\ 4 & 1 & 3 \end{vmatrix} = (4-p^2)(19-4q)$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q-1 \\ 1 & 4 & 3 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & p^2-2 & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} = p^2 - 4$$

13

JEE 1

Let $f(x) = \begin{cases} 3\sin x + a^2 - 10a + 30, & x \notin Q \\ 4\cos x, & x \in Q \end{cases}$

Which one of the following statement is correct?

(A) f(x) is continuous for all x when a = 5

(B) f(x) is discontinuous for all x when a = 5

(C) f(x) IS continuous for all $x = 2n\pi - \tan^{-1}\left(\frac{3}{4}\right), n \in I$ when a = 5

(D) f(x) is continuous for all $x = 2n\pi - \tan^{-1}\left(\frac{4}{3}\right), n \in I$ when a = 5

$\Rightarrow 3\sin x + a^2 - 10a + 30 = 4\cos x$

$\Rightarrow \underbrace{a^2 - 10a + 30}_{(a-5)^2 + 5 \geq 5} = \underbrace{4\cos x - 3\sin x}_{-5 \leq \; \leq 5} = 5$

$a = 5 \longrightarrow$ cont.

$\Rightarrow 4\cos x - 3\sin x = 5$

$\Rightarrow \frac{4}{5}\cos x - \frac{3}{5}\sin x = 1$

$\frac{4}{5} = \cos\theta$, $\frac{3}{5} = \sin\theta$

$\Rightarrow \cos(x + \theta) = 1 = \cos 0$

$\Rightarrow x + \theta = 2n\pi \quad ; n \in Z$

$\Rightarrow x = 2n\pi - \theta$

$x = 2n\pi - \tan^{-1}\left(\frac{3}{4}\right)$

14

If (α, β, γ) is the image of the point $(1,6,3)$ with respect to the line $\frac{x}{2} = \frac{y-1}{4} = \frac{z-2}{6}$ then the value of $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

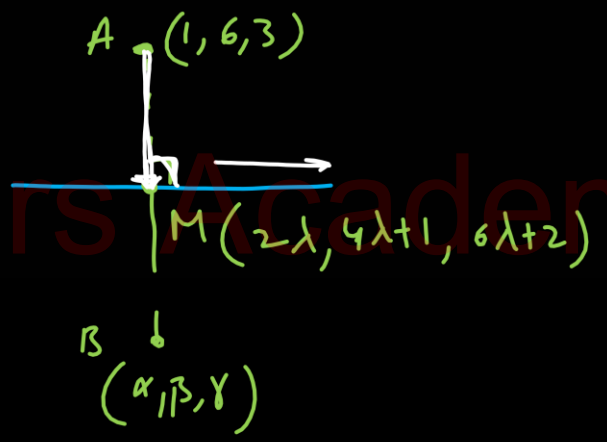
$= \lambda$

(A) 26

(B) 37

(C) 17

(D) 50



$\vec{AM} \cdot \vec{d} = 0$

$2(2\lambda - 1) + 4(4\lambda - 5) + 6(6\lambda - 1) = 0$

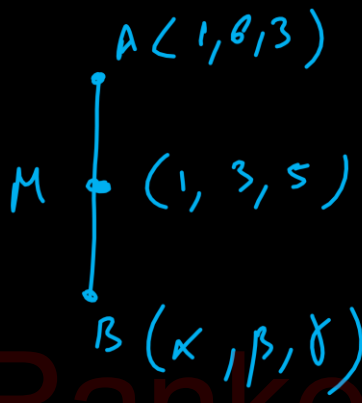
$2\lambda - 1 + 8\lambda - 10 + 18\lambda - 3 = 0$

$28\lambda = 14$

$\lambda = \frac{1}{2}$

$\vec{AM} = (2\lambda - 1)\hat{i} + (4\lambda - 5)\hat{j} + (6\lambda - 1)\hat{k}$

$\vec{d} = 2\hat{i} + 4\hat{j} + 6\hat{k}$



$\frac{\alpha+1}{2} = 1 \Rightarrow \alpha = 1$

$\frac{\beta+6}{2} = 3 \Rightarrow \beta = 0$

$\frac{\gamma+3}{2} = 5 \Rightarrow \gamma = 7$

$\Rightarrow \alpha^2+\beta^2+\gamma^2 = 50$

15

JEE 1

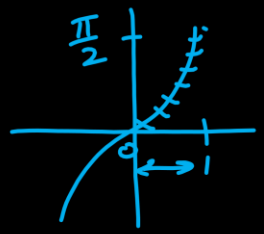
Let $f(x) = \left[\log\left(\sin^{-1}\left(\sqrt{x^2+3x+2}\right)\right)\right]$ where [.] is G.I.F. Which of the following is true?

(A) Domain of definition of f(x) is $\left[\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}\right]$.

(B) Domain of definition of f(x) is $\left[\frac{-3+\sqrt{5}}{2}, -1\right)$.

(C) Range of f(x) is the set of all non-positive integers.

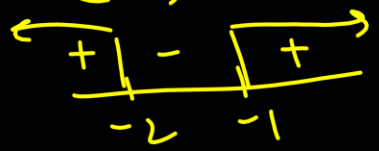
(D) Range of f(x) is the set of all positive integers.



$0 < \sqrt{x^2+3x+2} \leq 1$

P-1 :- $x^2+3x+2 > 0$

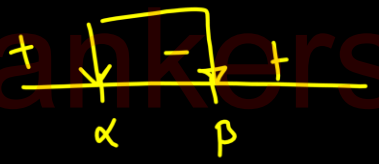
$(x+1)(x+2) > 0$



P-2: $x^2+3x+2 \leq 1$

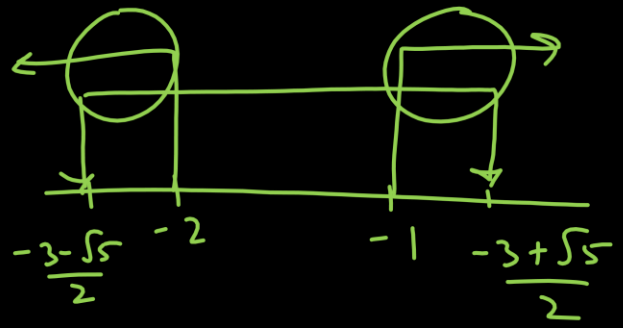
$x^2+3x+1 \leq 0$

$(x-\alpha)(x-\beta) \leq 0$



$x^2+3x+1=0$

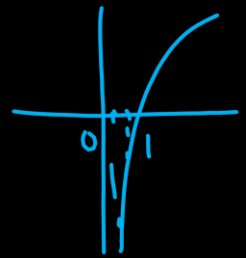
$x = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$



Now:

Range : min $[\log(\to 0)]$

max $\left[\log\left(\frac{\pi}{2}\right)\right]$



16

If Arg $\left(\frac{z^2-z_1}{z^2-z_2}\right) = \frac{\pi}{4}$ where $z_1 = 2 + 9i$ and $z_2 = 8 + 9i$ then which of the following is true

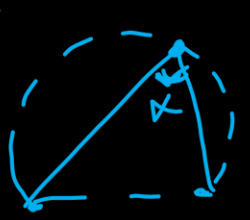
(A) Locus of z is a circle

(B) Locus of z is minor arc of a circle

(C) Max $|z^2|$ is $13 + 3\sqrt{2}$

(D) Min $|z^2|$ is $5 + 3\sqrt{2}$

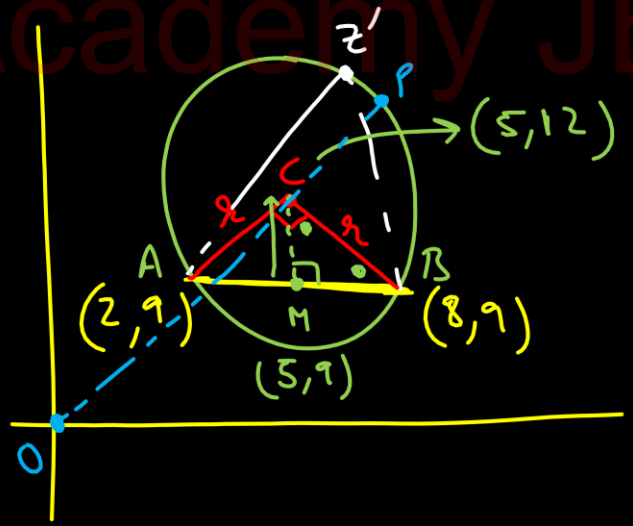
JEE 1



$\arg\left(\frac{z-z_1}{z-z_2}\right) = \alpha$

$z^2 = z'$

$\arg\left(\frac{z'-z_1}{z'-z_2}\right) = \frac{\pi}{4}$



$OC = 13$

$|z'|_{max} = 13 + 3\sqrt{2}$

17

In a set of real numbers a relation R is defined as xRy such that $|x| + |y| \leq 1$ then relation R is:

(A) reflexive and symmetric but not transitive

(B) symmetric but not transitive and reflexive

(C) transitive but not symmetric and reflexive

(D) none of reflexive, symmetric and transitive

R ✗ $(2,2)$ ✗

$\leq$ ✓

T ✗ $((0.9),(0.1))$

$((0.1),(0.8))$

18

The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1,2]$, is

(A) $\frac{3}{2}$ (B) $\frac{1}{4}$

(C) $\frac{5}{4}$ (D) $\frac{3}{4}$

$\begin{cases} x^2 - x + 1 = 0 \\ D < 0 \end{cases}$

$2x - 1 = 0$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = |\frac{1}{4} - \frac{1}{2} + 1| + [\frac{3}{4}]$

$= \frac{3}{4} + 0$

$= 3/4$

19

Let $\overrightarrow{AB} = 3\vec{i} + 4\vec{k}$ and $\overrightarrow{AC} = 5\vec{i} - 2\vec{j} + 4\vec{k}$. If the length of altitude from A to the side BC of Δ ABC is P then the greatest integer less than P^2 is equal to:

(A) 20 (B) 18

(C) 19 (D) 21

JEE 1

$$\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 4 \\ 5 & -2 & 4 \end{vmatrix}$$

$$= \sqrt{41}$$

$\vec{BC} = 2\hat{i} - 2\hat{j} \Rightarrow |\vec{BC}| = 2\sqrt{2}$

$\Delta = \frac{1}{2}(BC)(P)$

$\sqrt{41} = \frac{1}{2}(2\sqrt{2})(P)$

$\Rightarrow P = \sqrt{\frac{41}{2}}$

20

The mean deviation from the median of the given data is equal to:

Class-interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	5	3	6	2

(A) 7 (B) 7.5

(C) 6 (D) 6.5

CI	f_i	C.F.	x_i	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
0-6	4	4	3	11	44
6-12	5	9	9	5	25
12-18	3	12	15	1	3
18-24	6	18	21	7	42
24-30	2	20	27	13	26
					$\Sigma = 140$

$$M = \ell + \left(\frac{\frac{N}{2} - C}{f}\right) h$$

$\Sigma f_i = 20$

$$M = 12 + \frac{(10-9)}{3} \times 6$$

$M = 14$

$$M.D.(M) = \frac{\Sigma f_i \lvert x_i - M \rvert}{\Sigma f_i}$$

JEE 1

$$\frac{\Sigma |x_i - M|}{N}$$

21

JEE 1

Let $a = 5^2 + 11^2 + 19^2 + 29^2 + 41^2 + \cdots$..10 terms and $b = \sum_{n=1}^{10} n^4$. The value of $\left[\frac{a-b}{909}\right]$ (where [.] denotes G.I.F) is equal to

5, 11, 19, 29, 41

2^2+1, 3^2+2, 4^2+3, 5^2+4, 6^2+5

$$a = \sum_{n=1}^{10} \left[(n+1)^2 + n\right]^2$$

$$(a-b) = \sum_{n=1}^{10} \left(\left(n^2+3n+1\right)^2 - n^4\right)$$

$$= \sum_{n=1}^{10} \left(n^4 + 6n^3 + 6n + 2n^2 + 9n^2 + 1 - n^4\right)$$

$$= \sum_{n=1}^{10} \left(6n^3 + 11n^2 + 6n + 1\right)$$

$$= 6\left(\frac{10\times11}{2}\right)^2 + 11\left(\frac{10\times11\times21}{6}\right) + 6\left(\frac{10\times11}{2}\right) + 10$$

$$= 22725$$

Ans: $\left[\frac{22725}{909}\right] = 25$

22

The set of natural numbers is divided intoariay of rows and columns in the form of matrices as

$$A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$$

and so on. Let the trace of A_{10} be λ. Find unit digit of λ ?

$\{1^2, 2^2, 3^2, \ldots, 9^2\} \rightarrow \frac{9 \times 10 \times 19}{6} = 15 \times 19 = 285$

$$A_{10} = \begin{bmatrix} 286 & & \\ & 286+11 & \\ & & 286+11+11 \\ & & \ddots \end{bmatrix}$$

$$tr(A_{10}) = \frac{10}{2}\left(2(286) + 9\times 11\right)$$

units place $\rightarrow$ (5)

23

If the point on line $x + y + 13 = 0$, which is nearest to circle $x^2 + y^2 + 4x + 6y - 5 = 0$ is (p, q), then $|p + q|$ is

JEE 1

$$\frac{x-(-2)}{1} = \frac{y-(-3)}{1} = -\frac{(-2-3+13)}{2}$$

$x+y+13=0$

$x+2 = y+3 = -5$

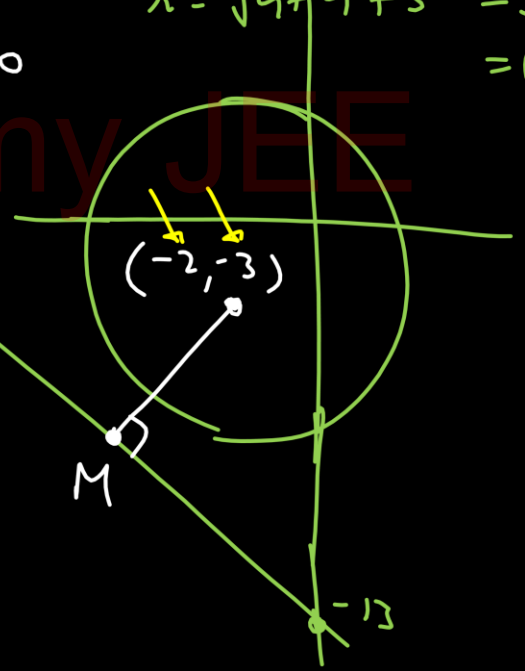
$(x, y) \equiv (-6, -7)$

$p \quad q$

$|p+q| = 13$

$C \equiv (-2, -3)$

$r = \sqrt{4+9+5} = \sqrt{18} = 3\sqrt{2}$



24

JEE 1

If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$

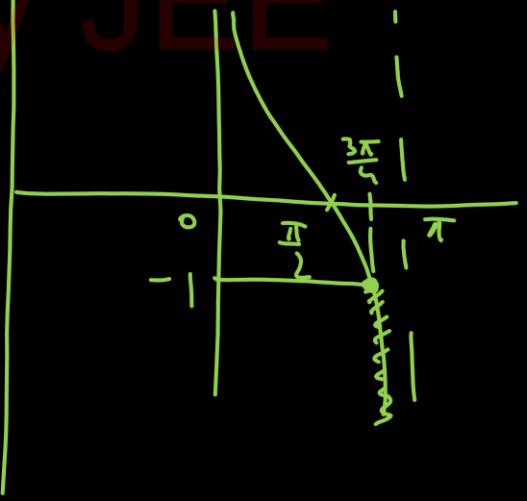
then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is

$$y(\alpha) = \sqrt{2\left(\frac{1}{\sin\alpha\cdot\cos\alpha}\right)+\cos^2\alpha+\operatorname{cosec}^2\alpha}$$

$$= \sqrt{2\cot\alpha+\cot^2\alpha+1}$$

$$= \sqrt{(\cot\alpha+1)^2}$$

$$y(\alpha) = |1+\cot\alpha|$$



$$y = -(1 + \cot\alpha)$$

$$y = -1 - \cot\alpha$$

$$\frac{dy}{d\alpha} = -(-\operatorname{cosec}^2\alpha)$$

$$= \operatorname{cosec}^2\alpha$$

$$\Rightarrow \operatorname{cosec}^2\left(\frac{5\pi}{6}\right) = 4$$

25

$16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to $2\sqrt{k}$, then k is

$\sin\theta \sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

$16\left(\frac{1}{4}\sin(3(20^\circ))\right)$

$4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$k = 3$