ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
UNIT TEST # 09
08-12-2024

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	C	A	C	B	C	A	C	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	A	B	A	B	B	B	D	A	A
SECTION-II	Q.	1	2	3	4	5					
	A.	11	3	3	48	15					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	C	D	C	A	A	C	D	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	D	B	C	A	C	D	C	C
SECTION-II	Q.	1	2	3	4	5					
	A.	27	2	3	3	13					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	B	D	A	A	B	C	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	A	A	D	C	B	B	B	A	B
SECTION-II	Q.	1	2	3	4	5					
	A.	13	20	5	6	88					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (B)**

$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1} = \frac{600}{400} = \frac{3}{2}$$

$\Rightarrow \beta_2 > \beta_1$

$$y = m_1\frac{D\lambda_1}{d} = m_2\frac{D\lambda_2}{d}$$

$$\frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{3}{2}$$

$\Rightarrow m_1 > m_2$

$$\frac{(y_{3r})_{max}}{(y_{5^{th}})_{min}} = \frac{\frac{3D\lambda_2}{d}}{\frac{9}{2}D\frac{\lambda_1}{d}} = \frac{2}{3} \times \frac{600}{400} = 1$$

Angular separation $\propto \lambda$

2. **Ans (D)**

Fringe width $\beta = \dfrac{D\lambda}{d}$

$$\lambda' = \frac{\lambda}{\mu}$$

β decreases

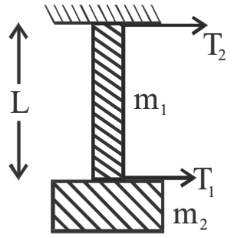
4. **Ans (A)**

$$\ell = \ell_1 + \ell_2 + \ell_3$$

$$n \propto \frac{1}{\ell}$$

$$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$$

6. **Ans (B)**



$T_1 = m_2 g$

$T_2 = (m_1 + m_2)g$

Velocity $\propto \sqrt{T}$

$\Rightarrow \lambda \propto \sqrt{T}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1 + m_2}{m_2}}$

7. **Ans (C)**

$x_n = \frac{nf\lambda}{a}$

$\Rightarrow \lambda = \frac{ax_n}{fn}$

$= \frac{6 \times 10^{-4} \times 10 \times 10^{-3}}{3 \times 2} = 10000Å$

$[\because \ n = 3]$

8. **Ans (A)**

$U_{av} = E_0 E^2{}_{rms}$

9. **Ans (C)**

Refractive index of medium is $= \frac{c}{v}$

where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}}$

$\therefore \ \mu = \frac{1/\sqrt{\mu_0 \varepsilon_0}}{1/\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = \sqrt{\mu_r \varepsilon_r}$

Given $\mu_r = \mu_0$ and $\varepsilon_r = \varepsilon_0$ then $\mu = \sqrt{\mu_0 \varepsilon_r}$

12. **Ans (A)**

$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$\frac{n_1 + n_2}{0.5} = \frac{n_1}{\frac{5}{3} - 1} + \frac{n_2}{\frac{7}{5} - 1}$

$\therefore n_1 = n_2$

14. **Ans (A)**

$V_{rms} = \sqrt{\frac{3RT}{M}} \qquad V_{mp} = \sqrt{\frac{2RT}{M}}$

$\frac{V_{rms}}{V_{mp}} = \sqrt{\frac{3}{2}}$

$V_{mp} = \sqrt{\frac{2}{3}} \times 1260$

$= 1028.78$ m/s.

16. **Ans (B)**

In isothermal process $P_1V_1 = P_2V_2$

$\Rightarrow PV = P_2 \times 4V \quad \because P_2 = P/4$

In adiabatic process

$P_2V_2{}^{\gamma} = P_3V_3{}^{\gamma}$

$\Rightarrow \frac{P}{4} \times (4V)^{1.5} = P_3V^{1.5} \Rightarrow P_3 = 2P$

17. **Ans (B)**

Let the initial pressure of the three samples be P_A, P_B and P_C, then $P_A(V)^{3/2} = (2V)^{3/2}P$, $P_B = P$ and $P_C(V) = P(2V)$

$\Rightarrow P_A : P_B : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$

18. **Ans (D)**

BC :

$P_0V_0 = 3P_0V1$

$V_1 = \frac{V_0}{3}$

AC

$\frac{P_0}{2} v_0^{\gamma} = 3P_0 \left(\frac{v_0}{3}\right)^{\gamma}$

$3^{\gamma} = 6$

$\gamma \ \ell n\, 3 = \ell n\, 6$

$\gamma = \frac{\ell n 6}{\ell n 3}$

19. **Ans (A)**

$dQ = dU + dW$

$= 1 \times \frac{3}{2} R \times (4T_0 - T_0) + 1 \times P_0 \times V_0$

$= \frac{9}{2}RT_0 + RT_0 = \frac{11}{2}RT_0$

20. **Ans (A)**

By adjoining graph $W_{AB} = 0$ and

$W_{BC} = 8 \times 10^4[5 - 2] \times 10^{-3} = 240J$

$\therefore \ W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240J$

Now,

$\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800J$

From FLOT $\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$

$\Rightarrow 800 = \Delta U_{AC} + 240 \Rightarrow \ \Delta U_{AC} = 560J.$

PART-1 : PHYSICS

SECTION-II

1. **Ans (11)**

Tension T = YA ∝ DQ

$$\Delta \ell = \alpha \ell \Delta \theta \Rightarrow Y = \frac{T/A}{\Delta\ell/\ell}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{\left(\frac{0.1}{1}\right)}} = 22\text{m/s}$$

∵ for fundamental note

$$\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\text{m} \Rightarrow f = \frac{v}{\lambda} = \frac{22}{2} = 11\text{Hz}$$

2. **Ans (3)**

$$v_1 = \frac{330}{2 \times l_1}$$

$$\Rightarrow \ell_1 = \frac{330}{2 \times 500} = 33 \text{ cm}$$

Similarly $\ell_2 = 2 \times 33 = 66$ cm

$\ell_3 = 3 \times 33 = 99$cm

$\ell_4 = 4 \times 33 = 132$cm

∴ Number of resonances will be 3.

3. **Ans (3)**

$I_A \cos^2 60° = I_B \cos^2 30°$

$$\frac{I_A}{I_B} = \frac{3/4}{\frac{1}{4}} = \frac{3}{1}$$

4. **Ans (48)**

dQ = dU + dW

$-30 = U_f - 60 - 18$

$U_f = 48$

5. **Ans (15)**

$Q_{ac} = \Delta U_{ac} + W_{ac}$

$30 = \Delta U_{ac} + 20$

$\Delta U_{ac} = 10$ J

$Q_{abc} = Q_{ab} + Q_{bc}$

$Q_{ab} = \Delta U_{ab} + W_{ab}$

$Q_{ab} = \Delta U_{ab}$

$Q_{bc} = \Delta U_{bc} + W_{bc}$

$Q_{abc} = \Delta U_{ab} + \Delta U_{bc} + W_{bc}$

$Q_{abc} = \Delta U_{ac} + W_{bc}$

$25 = 10 + W_{bc}$

$W_{bc} = 15\text{J} = W_{abc}$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (D)**

$P_{total} = P_{HNO_3} + P_{NO_2} + P_{H_2O} + P_{O_2}$

∵ $P_{NO_2} = 4P_{O_2}$ and $P_{H_2O} = 2P_{O_2}$

∴ $P_{total} = P_{HNO_3} + 7P_{O_2}$

$\Rightarrow 30 - 2 = P_{O_2} \times 7$

$$\Rightarrow P_{O_2} = \frac{28}{7} = 4$$

$$K_p = \frac{P^4_{NO_2} \cdot P_{H_2O} \cdot P_{O_2}}{P^4_{HNO_3}}$$

$$= \frac{(4 \times 4)^4 \times (2 \times 4)^2 \times 4}{2^4} = 2^{20}$$

$K_p = K_c (RT)^{\Delta n_g} = K_c (0.08 \times 400)^3$

$$\Rightarrow K_c = \frac{2^{20}}{(32)^3} = 32$$

2. **Ans (C)**

$$C_2H_6(g) \rightleftharpoons C_2H_4(g) + H_2(g)$$

at eqm $1-x$ x x

$$K_p = \frac{P_{C_2H_4} \cdot P_{H_2}}{P_{C_2H_6}} \Rightarrow \frac{x^2}{1-x} = 5 \times 10^{-2}$$

$x^2 + 0.05x - 0.05 = 0$

$$x = \frac{-0.05 + \sqrt{(0.05)^2 + 4 \times 0.05}}{2} = 0.20 \text{ atm}$$

Particle pressure of C_2H_6

= mole fraction × total pressure

$\Rightarrow 0.80 =$ mole fraction × 1.2

$$\therefore \text{ mole \% of } C_2H_6 = \frac{0.8}{1.2} \times 100 = 66.66$$

3. **Ans (D)**

$\frac{K_P}{K_C}$ max means $\frac{K_C}{K_P}$ min

Δn_g max

4. **Ans (C)**

It is a heterogenous equilibrium. Addition of solid $CaCO_3$ has no effect on equilibrium concentration of CO_2.

5. **Ans (A)**

By decreasing the pressure of system reaction moves in direction of increasing volume.

6. **Ans (A)**

$$K_c = \frac{(0.2)^2\ (0.1)}{(1)\ \ (2)^2} = 0.001$$

7. **Ans (C)**

$2NO + O_2 \rightleftharpoons 2NO_2, \quad K_1 \quad ....(1)$

$NO_2 + SO_2 \rightleftharpoons SO_3 + NO \quad K_2 \quad ...(2)$

$2SO_3 \rightleftharpoons 2SO_2 + O_2, \quad K_3 \quad ...(3)$

(1) + (3)

$2NO + 2SO_3 \rightleftharpoons 2NO_2 + 2SO_2 \quad K_1 \times K_3 \quad ....(4)$

$\frac{1}{2}(4)^{-1}$

$$\boxed{\left(\frac{1}{K_1 \times K_3}\right)^{1/2} = K_2}$$

8. **Ans (D)**

By using all these methods K_2MnO_4 can be converted into $KMnO_4$.

9. **Ans (C)**

In $Cr_2O_7^{2-}$, two type of Cr–O bonds are present.

10. **Ans (A)**

$K_2Cr_2O_7 + H_2SO_4(conc) + NaCl$

$\longrightarrow KHSO_4 + NaHSO_4 + H_2O$

$+ \underset{\text{Chromylchloride(red)}}{CrO_2Cl_2} \uparrow$

$CrO_2Cl_2 + NaOH \longrightarrow + NaCl + H_2O$

11. **Ans (B)**

$KMnO_4 \xrightarrow{\text{acidic}} Mn^{+2}$

$KMnO_4 \xrightarrow{\text{neutral}} MnO_2$

$KMnO_4 \xrightarrow{\text{basic}} MnO_4^{2-}$

14. **Ans (B)**

Order of radius is :

$La^{3+} > Ce^{+3} > Pm^{3+} > Yb^{3+}$

(According to lanthanoid contraction)

17. **Ans (C)**

CaC_2, Al_4C_3 and $Be_2C \Rightarrow$ Ionic

SiC $\Rightarrow$ Covalent network

18. **Ans (D)**

$$\left.\begin{array}{l} H\overset{+3}{N}O_2 \\ \overset{-1}{H_2O_2} \\ \overset{+4}{S}O_2 \end{array}\right\} \text{They all behave as oxidising and reducing agent}$$

19. **Ans (C)**

CCl_3^- — $2p\pi$–$3d\pi$ back bonding from C to Cl–atom

$:CCl_2$ — $2p\pi$–$3p\pi$ back bonding from Cl to C–atom

20. **Ans (C)**

$Br_2 + F^- \longrightarrow Br^- + F_2$ reaction not possible

order of oxidising $\longrightarrow F_2 > Cl_2 > Br_2 > I_2$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (27)**

$$A(g) \rightleftharpoons 3B(g) \quad ; K_{P_1}$$

At equilibrium $1-\alpha \qquad 3\alpha$

$$K_{P_1} = \frac{(3\alpha)^3}{(1-\alpha)} \frac{P^2}{(1+2\alpha)^2}$$

$$P(g) \rightleftharpoons 2Q(g) + R(g) \ ; K_{P_2}$$

At equilibrium $1-\alpha \qquad 2\alpha \qquad \alpha$

$$K_{P_2} = \frac{(2\alpha)^2\alpha}{(1-\alpha)} \frac{P^2}{(1+2\alpha)^2}$$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{X}{4} = \frac{\frac{3^3\alpha^3P^2}{(1-\alpha)(1+2\alpha)^2}}{\frac{4\alpha^3P^2}{(1-\alpha)(1+2\alpha)^2}}$$

X = 27

2. **Ans (2)**

$K_3 = K_1 \,.\, K_2$

$$= \frac{(K_1)^1}{(K_2)^{-1}} \Rightarrow \begin{array}{l} x = 1 \\ y = -1 \end{array}$$

$\Rightarrow (x - y) = 1 + 1 = 2$

3. **Ans (3)**

CrO_2Cl_2 has d^3S hybridisation and all 3d-orbitals are non-axial which are d_{xy} ; d_{yz} and d_{xz}.

4. **Ans (3)**

Atomic number: 58, 64, 71

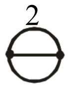
PART-3 : MATHEMATICS

SECTION-I

1. **Ans (B)**

$$C = \frac{Z_1 + Z_2}{2}$$

$$r = \frac{|z_1 - z_2|}{2}$$

(6, 1)  (4, –3)

5. Ans (A)

$|z_1| = 2, |z_2| = 3, |z_3| = 4$

$y = |z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$

$y = 2\left(|z_1|^2 + |z_2|^2 + |z_3|^2\right)$

$-\left(z_1\bar{z}_2 + z_2\bar{z}_1 + z_3\bar{z}_2 + z_2\bar{z}_3 + z_1\bar{z}_3 + z_3\bar{z}_1\right)$

$y = 58 - \left(z_1\bar{z}_2 + z_2\bar{z}_1 + z_3\bar{z}_2 + z_2\bar{z}_3 + z_1\bar{z}_3 + z_3\bar{z}_1\right)$

$y_{max} = 87$

6. Ans (A)

$$(x_1 \times x_3 \times x_5 \ldots.\infty) + \frac{1}{(x_2 \cdot x_4 \cdot x_6 \ldots. \infty)}$$

$$\Rightarrow \left\{\cos\left(\frac{\pi}{2} + \frac{\pi}{2^3} + \frac{\pi}{2^5} + \ldots.\infty\right) + i \sin\left(\frac{\pi}{2} + \frac{\pi}{2^3} + \frac{\pi}{2^5} + \ldots.\infty\right)\right\}$$

$$+ \frac{1}{\left\{\cos\left(\frac{\pi}{2^2} + \frac{\pi}{2^4} + \frac{\pi}{2^6} + \ldots\infty\right) + i \sin\left(\frac{\pi}{2^2} + \frac{\pi}{2^4} + \frac{\pi}{2^6} + \ldots\infty\right)\right\}}$$

$$\Rightarrow \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + \frac{1}{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$$

$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} + \cos\frac{\pi}{3} - i\sin\frac{\pi}{3} = 0$$

9. Ans (A)

$(z_1 - 3z_2)(\bar{z}_1 - 3\bar{z}_2) = (3 - z_1\bar{z}_2)(3 - \bar{z}_1 z_2)$

$\Rightarrow |z_1|^2 - 3z_1\bar{z}_2 - 3z_2\bar{z}_1 + 9|z_2|^2$

$= 9 - 3\bar{z}_1 z_2 - 3z_1\bar{z}_2 + |z_1|^2|z_2|^2$

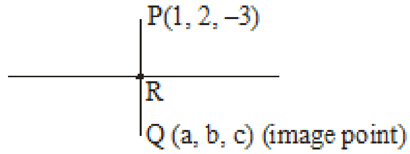
$\Rightarrow |z_1|^2 - |z_1|^2|z_2|^2 + 9|z_2|^2 - 9 = 0$

$\Rightarrow |z_1|^2(1 - |z_2|^2) - 9(1 - |z_2|^2) = 0$

$\Rightarrow (1 - |z_2|^2)(|z_1|^2 - 9) = 0 \;\Rightarrow |z_1| = 3$

11. Ans (B)

Line is $\dfrac{x+1}{2} = \dfrac{y-3}{-2} = \dfrac{z}{-1} = \lambda$: Let point R is $(2\lambda - 1, -2\lambda + 3, -\lambda)$



Direction ratio of PQ≡$(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is ⊥r to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is $(1, 1, -1)$

$\dfrac{a+1}{2} = 1 \Big| \dfrac{b+2}{2} = 1 \Big| \dfrac{c-3}{2} = -1$

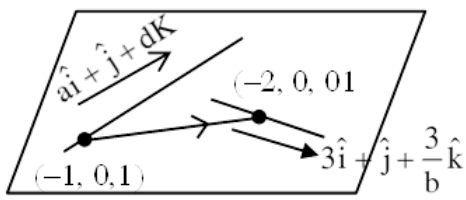
$a = 1 \qquad b = 0 \qquad c = 1$

$\Rightarrow a + b + c = 2$

12. Ans (A)

$\dfrac{x+1}{a} = y = \dfrac{z-1}{a}$

$\dfrac{x+2}{3} = y = \dfrac{z}{3/b}$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0$$

$\Rightarrow -\left(\dfrac{3}{b} - a\right) - 1(a - 3) = 0$

$a - \dfrac{3}{b} - a + 3 = 0$

$b = 1, a \in R - \{0\}$

14. Ans (D)

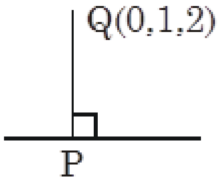
$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}=r$$

$\Rightarrow P(x, y, z) = (2r + 1, 3r - 1, -2r + 1)$

Since, $\overrightarrow{QP} \perp (2\hat{i}+3\hat{j}-2\hat{k})$

$\Rightarrow 4r + 2 + 9r - 6 + 4r + 2 = 0$

$\Rightarrow r = \dfrac{2}{17}$



$\Rightarrow P\left(\dfrac{21}{17}, \dfrac{-11}{17}, \dfrac{13}{17}\right)$

$\Rightarrow \overrightarrow{PQ} = \dfrac{21\hat{i}-28\hat{j}-21\hat{k}}{17}$

So, $\overrightarrow{QP} = \dfrac{x}{-3} = \dfrac{y-1}{4} = \dfrac{z-2}{3}$

15. Ans (C)

$n = \ell + m$

Now, $\ell^2 + m^2 = n^2 = (\ell + m)^2$

$\Rightarrow 2\ell m = 0$

If $\ell = 0 \Rightarrow m = n = \pm\dfrac{1}{\sqrt{2}}$

And, If $m = 0 \Rightarrow n = \ell = \pm\dfrac{1}{\sqrt{2}}$

So, direction cosines of two lines are

$\left(0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ and $\left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

Thus, $\cos\alpha = \dfrac{1}{2} \Rightarrow \alpha = \dfrac{\pi}{3}$

16. Ans (B)

(3,5,7) satisfy the line L_1

$$\frac{3-a}{\ell}=\frac{5-2}{3}=\frac{7-b}{4}$$

$$\frac{3-1}{\ell}=1 \ \& \ \frac{7-b}{4}=1$$

$a + \ell = 3 \qquad ...(1)$

$\& \ b = 3 \qquad ...(2)$

$\vec{v}_1 =< 4,3,8 >=< 3,5,7 >$

$\vec{v}_1 =< 1,-2,1 >$

$\vec{v}_2 =< \ell,3,4 >$

$\vec{v}_1 . \vec{v}_2 = 0 \Rightarrow \ell - 6 + 4 = 0 \Rightarrow \ell = 2$

$\alpha + \ell = 3 \Rightarrow a = 1$

$L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

$L_2 : \dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$

$A = <1, 2, 3>$

$B = <2, 4, 5>$

$\overrightarrow{AB} =< 1,2,2 >$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

Shortest distance $= \left|\dfrac{\overrightarrow{AB}.(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \dfrac{1}{\sqrt{6}}$

17. **Ans (B)**

P(3, –1, 2)

Q(1, 2, –4)

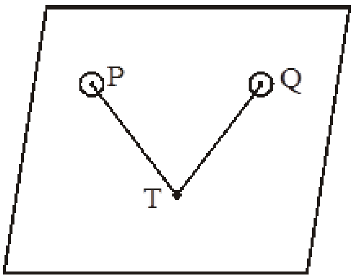
$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$

dr's of normal to the plane containing P, T & Q

will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$\therefore \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$

For point, T : $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$

T : $(4\lambda + 3, -\lambda - 1, 2\lambda + 2)$

$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$

$4\lambda + 3 = -2\mu + 1 \Rightarrow 2\lambda + \mu = -1$

$\lambda + \mu = -3 \Rightarrow \lambda = 2$

& $\mu = -5 \quad \lambda + \mu = -3 \Rightarrow \lambda = 2$

So point T : (11, –3, 6)

$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(\frac{2\hat{j} + \hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(2\hat{j} + \hat{k}\right)$

$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$

or $9\hat{i} - 5\hat{j} + 5\hat{k}$

$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$

18. **Ans (B)**

$\ell_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$\ell_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$

$\ell_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$

$\ell_1 \perp \ell_2 \Rightarrow \frac{|3 - \alpha + 0|}{\sqrt{13}\sqrt{1 + \frac{\alpha^2}{4} + 4}} = 0 \Rightarrow \alpha = 3$

angle between l_2 & l_3

$$\cos\theta = \frac{|1 \times (-3) + (-2)(\alpha/2) + 2 \times 4|}{\sqrt{1 + 4 + \frac{\alpha^2}{4}}\sqrt{9 + 16 + 4}}$$

$$\cos\theta = \frac{|-3 - \alpha + 8|}{\sqrt{5 + \frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

19. **Ans (A)**

$l + m - n = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$n = l + m$

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$(3 + c)\left(\frac{\ell}{m}\right)^2 + c\left(\frac{\ell}{m}\right) + 1 = 0 \quad ....(1)$

$\because$ lies are parallel

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

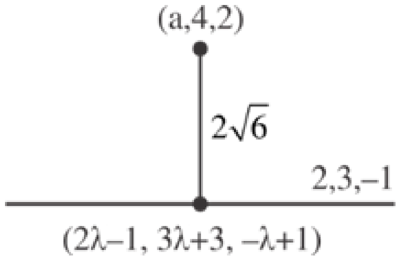
$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

$c = 6 \quad$ or $c = -2$

+ve value of c = 6

20. Ans (B)



$2(2\lambda - 1 - a)^2 + 3(3\lambda - 1) + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0 \Rightarrow a = 7\lambda - 2$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let (a_1, a_2, a_3) be reflection of point P

$\alpha_1 + 5 = 2 \quad \alpha_2 + 4 = 12 \quad \alpha_3 + 2 = 0$

$\alpha_1 = -3 \quad \alpha_2 = 8 \quad \alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$

PART-3 : MATHEMATICS

SECTION-II

2. Ans (20)

$|z| = 1, z\bar{z} = 1 \quad \text{so} \bar{z} = \frac{1}{2}$

Now $\left|3 + \frac{1}{z}\right|^2 + |3 - z|^2$

$= |3 + \bar{z}|^2 + |3 - z|^2$

$= 2\left(9 + |z|^2\right) = 2[10] = 20$

4. Ans (6)

If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$

then shortest distance between two lines is

$$L = \frac{(\vec{a}.\ \vec{c}).\left(\vec{b} \times \vec{d}\right)}{|b \times d|}$$

$\therefore \ \vec{a} - \vec{c} = \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$\frac{\vec{b} \times \vec{d}}{|b \times d|} = \frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3}$$

$$\therefore \left((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}\right).\frac{\left(2\hat{i} + 2\hat{j} + \hat{k}\right)}{3} = 9$$

or $\alpha = 6$

5. Ans (88)

P(a, 6, 9)

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$$

$Q = (20, b, -a - 9)$

$$\frac{\frac{20+a}{2} - 3}{7} = \frac{\frac{b+6}{2} - 2}{5} = \frac{-\frac{Q}{2} - 1}{-9}$$

$$\frac{14 + Q}{14} = \frac{b+2}{10} = \frac{a+2}{18}$$

$\Rightarrow a = -56$ and $b = -32$

$\Rightarrow |a + b| = 88$