TEST CODE : 113104

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IIT-JEE

Batch – GROWTH (JUNE) | MAJOR Test-02 (PAPER – II)

Time: 3 Hours **Test Date: 24th November 2024** **Maximum Marks: 180**

Name of the Candidate: ____ Roll No. ____________

Centre of Examination (in Capitals): ____

Candidate's Signature: ______________________ Invigilator's Signature: ______________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

2. This Test Booklet consists of 54 questions.

3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 18 questions each and every **PART** has three sections.

(i) **Section-I** contains **8** Numerical Value questions.

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

(ii) **Section-II** contains **6 Questions** Multiple Choice Option with more than one correct answer.

Marking scheme: (+4 for correct answer, 0, if not attempted and +1 partial marking —2 in all other cases.

(iii) **Section-III** contains **4** questions the answer to only 4 questions, is an only one correct.

Marking scheme: +3 for correct answer, 0 if not attempted and —1 in all other cases.

4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.

5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.

7. **For integer-based questions, the answer should be in decimals only not in fraction.**

8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

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TEST SYLLABUS

Batch – GROWTH (June) | Major Test-02 (Paper – II)

24th November 2024

Mathematics	:	Compound Angle & Trigonometric Eq Quadratic Eq St. Line
Physics	:	NLM & Friction WEP Circular Motion, Centre of Mass, Momentum & Collision
Chemistry	:	Chemical Bonding Thermodynamics-1 Thermochemistry & Thermodynamics-2

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821$ Lit atm $K^{-1}mol^{-1}$
		$= 1.987 \approx 2$ Cal $K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}$ Js
		$= 6.25 \times 10^{-27}$ erg.s
1 Faraday		= 96500 Coulomb
1 calorie		= 4.2 Joule
1 amu		$= 1.66 \times 10^{-27}$ kg
1 eV		$= 1.6 \times 10^{-19}$ J

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:

Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I (Single Digit Integer)

1. Value of 1 + $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} +$ .... to n terms is ____________

Ans. (1)

Sol. $E = \sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + ...$ n terms

$= \frac{\sin\left(n\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)} \sin\left(\frac{\text{1st Angle+Last Angle}}{2}\right) = 0 \qquad (\therefore\ \sin\pi = 0)$

2. Number of solutions of the equation $\tan\theta + \sec\theta = \sqrt{3}$ between 0 and 4π is :

Ans. (2)

Sol.

$\tan\theta + \sec\theta = \sqrt{3}$

$\Rightarrow \quad \sin\theta + 1 = \sqrt{3}\cos\theta \qquad (\cos\theta \neq 0)$

$\therefore \quad \sqrt{3}\cos\theta - \sin\theta = 1$

or $\quad \cos\left(\theta + \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

$\therefore \quad \theta + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \quad \theta = 2n\pi + \frac{\pi}{6}$ and $\theta = 2n\pi - \frac{\pi}{2}, n \in I$

$\therefore \quad \theta = 2n\pi + \frac{\pi}{6}, \theta \neq 2n\pi - \frac{\pi}{2} \qquad (\because \cos\theta = 0)$

$\Rightarrow \quad \theta = 2n\pi + \frac{\pi}{6}, n \in I$

$\therefore \quad \theta = \frac{\pi}{6}, \frac{13\pi}{6}$

3. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

Ans. (3)

Sol. $a^2 - 6\alpha - 2 = 0, \beta^2 - 6\beta - 2 = 0$

Now, $a_{10} - 2a_8 = \alpha^{10} + \alpha^{10} - 2(\alpha^8 + \beta^8)$

$= \alpha^8(\alpha^2 - 2) + \beta^8(\beta^2 - 2)$

$= \alpha^8(6\alpha) + \beta^8(6\beta)$

$= 6a^9$

$\Rightarrow \frac{a^{10} - 2a^8}{2a^9} = 3$

4. If the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents a pair of straight lines, then the value of λ =

Ans. (2)

Sol. $a = 12,\ h = -5,\ g = 11/2,\ b = 2,\ f = -5/2,\ c = \lambda$

Since the equation represents a pair of straight lines.

$$\begin{vmatrix} 12 & -5 & 11/2 \\ -5 & 2 & -5/2 \\ 11/2 & -5/2 & \lambda \end{vmatrix} = 0$$

$$= 12\left(2\lambda - \frac{25}{4}\right) + 5\left(-5\lambda + \frac{55}{4}\right) + \frac{11}{2}\left(\frac{25}{2} - 11\right) = 0$$

$$\Rightarrow \quad -\lambda - \frac{25}{4} + \frac{33}{4} = 0 \Rightarrow \lambda = 2$$

5. If $\sin\alpha + \sin\beta + \sin\gamma = -3, \alpha, \beta, \gamma \in (0, 2\pi)$, then $|\cos 2\alpha + \cos 4\beta + \cos 6\gamma|$ is equal to:

Ans. (1)

Sol. $\sin\alpha + \sin\beta + \sin\gamma = -3$

Only possible when $\alpha = \beta = \gamma = \frac{3\pi}{2}$

$\therefore \cos 2\alpha + \cos 4\beta + \cos 6\gamma = \cos 3\pi + \cos 6\pi + \cos 9\pi = -1 + 1 - 1 = -1$

6. If α, β, γ are such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$, then $\frac{(\alpha^4 + \beta^4 + \gamma^4)}{3}$ is

Ans. (6)

Sol. We have

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\beta\gamma + \gamma\alpha + \alpha\beta)$$

$$\Rightarrow \quad 4 = 6 + 2(\beta\gamma + \gamma\alpha + \alpha\beta)$$

$$\Rightarrow \quad \beta\gamma + \gamma\alpha + \alpha\beta = -1.$$

Also, $\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

$$= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta)$$

$$\Rightarrow \quad 8 - 3\alpha\beta\gamma = 2(6 + 1)$$

$$\Rightarrow \quad 3\alpha\beta\gamma = 8 - 14 = -6 \text{ or } \alpha\beta\gamma = -2.$$

Now, $(\alpha^2 + \beta^2 + \gamma^2)^2 = \sum \alpha^4 + 2\sum \beta^2\gamma^2$

$$= \sum \alpha^4 + 2\left[\left(\sum \beta\gamma\right)^2 - 2\alpha\beta\gamma\left(\sum \alpha\right)\right]$$

$$\Rightarrow \quad \sum \alpha^4 = 36 - 2[(-1)^2 - 2(-2)(2)] = 18$$

$$\Rightarrow \quad \frac{(\alpha^4 + \beta^4 + \gamma^4)}{3} = \frac{18}{3} = 6$$

7. If $f(x) = \frac{ax + 1}{x^2 - 1}$ gives all real values, then find sum of square of all integral values of ' a ' given that $-2 \le a \le -1$

Ans. (5)

Sol. $yx^2 - y = ax + 1 \Rightarrow yx^2 - ax - y - 1 = 0$

$\Rightarrow\ a^2 + 4(y)(y+1) \ge 0$

$\Rightarrow 4y^2 + 4y + a^2 \ge 0$ it is true for all real y

$D = 16 - 4 \times 4a^2 \le 0 \Rightarrow 1 - a^2 \le 0 \Rightarrow a^2 - 1 \ge 0$

$a \in (-\infty, -1] \cup [1, \infty)$ but $-1 \ge a \ge -2$

so $a = -2, -1$

Sum of square of values of a = 5

Hence, the correct answer is (5).

8. Consider the family of lines

$5x + 3y - 2 + \lambda(3x - y - 4) = 0$ and

$x - y + 1 + \mu(2x - y - 2) = 0.$ Equation of straight line that belong to both families is $ax + by - 7 = 0,$

then a + b is

Ans. (3)

Sol. Lines $5x + 3y - 2 + \lambda(3x - y - 4) = 0$ are concurrent at (1,-1) and lines $x - y + 1 + \mu(2x - y - 2) = 0$ are concurrent at (3,4).

Thus equation of the line common to both family is

$$y + 1 = \frac{4+1}{3-1}(x-1)$$

or $\quad 5x - 2y - 7 = 0$

$\therefore \quad a = 5, b = -2 \Rightarrow a + b = 3$

SECTION-II (One or More than One Correct)

9. If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi,$ then,

(A) $x = \dfrac{y+1}{y-1}$

(B) $x = \dfrac{y-1}{y+1}$

(C) $y = \dfrac{1+x}{1-x}$

(D) $xy + x - y + 1 = 0$

Ans. (B), (C), (D)

Sol. We have

$$x = \frac{1-\sin\phi}{\cos\phi}, y = \frac{1+\cos\phi}{\sin\phi}$$

Multiplying, we get $xy = \frac{(1-\sin\phi)(1+\cos\phi)}{\cos\phi\sin\phi}$

$$\Rightarrow \quad xy + 1 = \frac{1 - \sin\phi + \cos\phi - \sin\phi\cos\phi + \sin\phi\cos\phi}{\cos\phi\sin\phi}$$

$$= \frac{1-\sin\phi + \cos\phi}{\cos\phi\sin\phi}$$

and $\quad x - y = \frac{(1-\sin\phi)\sin\phi - \cos\phi(1+\cos\phi)}{\cos\phi\sin\phi}$

$$= \frac{\sin\phi - \sin^2\phi - \cos\phi - \cos^2\phi}{\cos\phi\sin\phi}$$

$$= \frac{\sin\phi - \cos\phi - 1}{\cos\phi\sin\phi} = -(xy+1)$$

Thus, $xy + x - y + 1 = 0$.

$$\Rightarrow \quad x = \frac{y-1}{y+1} \text{ and } y = \frac{1+x}{1-x}.$$

10. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then which of the following is/are correct?

(A) $\sum \alpha^2 = p^2 - 2q$ (B) $\sum \alpha^2 = p^2 - q$

(C) $\sum \alpha^3 = p^3 + 3pq + 3r$ (D) $\sum \alpha^3 = p^3 - 3pq + 3r$

Ans. (A,D)

Sol. Since α, β, γ are the roots $x^3 - px^2 + qx - r = 0$

$\therefore \quad \sum\alpha = p, \sum\alpha\beta = q$ and $\alpha\beta\gamma = r$

$\because \Sigma\alpha \cdot \Sigma\alpha = p \cdot p$

$\Rightarrow \quad (\alpha+\beta+\gamma)(\alpha+\beta+\gamma) = p^2$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = p^2$

or $\quad \Sigma\alpha^2 + 2\Sigma\alpha\beta = p^2$

or $\quad \Sigma\alpha^2 = p^2 - 2q$

$\because \Sigma\alpha^2 \cdot \Sigma\alpha = (p^2 - 2q)\cdot p$ [from result (i)]

$\Rightarrow \quad (\alpha^2 + \beta^2 + \gamma^2)(\alpha + \beta + \gamma) = p^3 - 2pq$

$\Rightarrow \quad \alpha^3 + \beta^3 + \gamma^3 + (\alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \gamma^2\alpha + \gamma^2\beta) = p^3 - 2pq$

$\Rightarrow \quad \Sigma\alpha^3 + \Sigma\alpha^2\beta = p^3 - 2pq$

$\Rightarrow \quad \Sigma\alpha^3 + pq - 3r = p^3 - 2pq$ [from result (ii)]

or $\quad \Sigma\alpha^3 = p^3 - 3pq + 3r$

11. The distance of the point (1, 2) from the line x + y + 5 = 0 measured along the line parallel to 3x – y = 7 is equal to

(A) $\frac{4}{\sqrt{10}}$ (B) 40

(C) $\sqrt{40}$ (D) $10\sqrt{2}$

Ans. (C)

Sol. Let equation of line parallel to 3x – y = 7 be 3x – y = λ.

It passes through (1, 2).

$\therefore 3 - 2 = \lambda \Rightarrow \lambda = 1$

$\therefore$ Line is $3x - y = 1$

The point of intersection of

$x + y + 5 = 0$ and $3x - y = 1$ is $(-1, -4)$.

$\therefore$ Distance between (1, 2) and $(-1, -4)$

$= \sqrt{(2)^2 + (6)^2} = \sqrt{40}$

12. The equation $3\sin^2 x + 10\cos x - 6 = 0$ is satisfied if

(A) $x = n\pi + \cos^{-1}(1/3), (n \in I)$ (B) $x = n\pi - \cos^{-1}(1/3), (n \in I)$

(C) $x = 2n\pi + \cos^{-1}(1/3), (n \in I)$ (D) $x = 2n\pi - \cos^{-1}(1/3), (n \in I)$.

Ans. (C,D)

Sol. The given equation is equivalent to

$$3(1 - \cos^2 x) + 10\cos x - 6 = 0$$

$$\Rightarrow 3\cos^2 x - 10\cos x + 3 = 0$$

$$\Rightarrow (3\cos x - 1)(\cos x - 3) = 0$$

Therefore $\cos x = 1/3$ (because $\cos x \neq 3$)

Hence $x = 2n\pi \pm \cos^{-1}(1/3)$.

13. If the reflexion of (α, β) in the line $\alpha x + \beta y = p + q$ is (p + q), then

(A) $\alpha^2 + \beta^2 + p\alpha + q\beta = 2(p + q)$ (B) $\alpha^2 + \beta^2 - p\alpha - q\beta = 0$

(C) $p\beta - q\alpha = 0$ (D) $p\alpha - q\beta = 0$

Ans. (A,C)

Sol. Line joining (α, β) and (p, q) is perpendicular to $\alpha x + \beta y = p + q$ and is bisected by it

So $\frac{\alpha(\alpha + p)}{2} + \frac{\beta(\beta + q)}{2} = p + q$

$\Rightarrow \alpha^2 + \beta^2 + p\alpha + q\beta = 2(p + q)$

and $\frac{q - \beta}{p - \alpha} = \frac{\beta}{\alpha}$

$\Rightarrow q\alpha - p\beta = 0$

14. If roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ lie in the interval (– 2, 4), then

(A) – 1 < m < 3 (B) 1 < m < 5

(C) 1 < m < 3 (D) – 1 < m < 5

Ans. (A)

Sol. $(x-m)^2=1 \Rightarrow x=m\pm 1$

$\therefore -2<m-1, m+1<4$

$\Rightarrow -1<m<3.$

SECTION-III (Only One Correct Type)

15. If the lines joining the origin to the intersection of the line y = mx + 2 and the curve $x^2 + y^2 = 1$ are at right angles, then

(A) $m^2=1$

(B) $m^2=3$

(C) $m^2=7$

(D) $2m^2=1$

Ans. (C)

Sol. Joint equation of the lines joining the origin and the point of intersection of the line y = mx + 2 and the curve $x^2 + y^2 = 1$ is

$$x^2+y^2=\left(\frac{y-mx}{2}\right)^2$$

$\Rightarrow x^2\left(4-m^2\right)+2mxy+3y^2=0$

Since these lines are at right angles

$4-m^2+3=0 \Rightarrow m^2=7.$

16. The smallest positive root of the equation tan x – x = 0 lies in

(A) $(0,\pi/2)$

(B) $(\pi/2,\pi)$

(C) $(\pi,3\pi/2)$

(D) $(3\pi/2,2\pi)$

Ans. (C)

Sol. Let $f(x)=\tan x-x$

We know, for $0<x<\frac{\pi}{2}$

$\Rightarrow \tan x>x$

$\therefore f(x)=\tan x-x$ has no root in $(0,\pi$

So, $f(x)=0$ has no root in $\left(\frac{3\pi}{2},2\pi\right)$.

We have, $f(\pi)=0-\pi<0$

and $f\left(\frac{3\pi}{2}\right)=\tan\frac{3\pi}{2}-\frac{3\pi}{2}>0$

$\therefore f(x)=0$ has atleast one root between π and $\dfrac{3\pi}{2}$

17. If l, m, n are real, $l+m\neq 0$, then the roots of the equation $(l+m)x^2-3(l-m)x-2(l+m)=0$ are

(A) real and unequal

(B) complex

(C) real and equal

(D) purely imaginary

Ans. (A)

Sol. Discriminant of the given equation is

$D=9(l-m)^2+8(l+m)^2$

As $l + m \neq 0$, $(l + m)^2 > 0$. Also, $(l - m)^2 \geq 0$.

Thus, $D > 0$.

Hence, roots of the given equation are real and unequal.

18. If $N = \sqrt{9\cos^2\theta + 16\sin^2\theta} + \sqrt{16\cos^2\theta + 9\sin^2\theta}$, then the sum of the maximum and minimum value of N^2 is __________

(A) 90 (B) 93

(C) 95 (D) 99

Ans. (D)

Sol. $N^2 = 9\cos^2\theta + 16\sin^2\theta + 16\cos^2\theta + 9\sin^2\theta$

$+2\left(\sqrt{(9\cos^2\theta + 16\sin^2\theta)\cdot(16\cos^2\theta + 9\sin^2}\right.$

$= 25 + 2\sqrt{(9 + 7\sin^2\theta)(9 + 7\cos^2\theta)}$

$= 25 + 2\sqrt{81 + 63\cos^2\theta + 63\sin^2\theta + 49\cos^2\theta\sin^2\theta}$

$= 25 + 2\sqrt{81 + 63 + \frac{49\sin^2 2\theta}{4}}$

$$N^2 = 25 + 2\sqrt{144 + \frac{49\sin^2 2\theta}{4}}$$

$$N^2_{max} = 25 + 2\sqrt{144 + \frac{49}{4}} = 25 + 2\sqrt{\frac{625}{4}} = 25 + 25 = 50$$

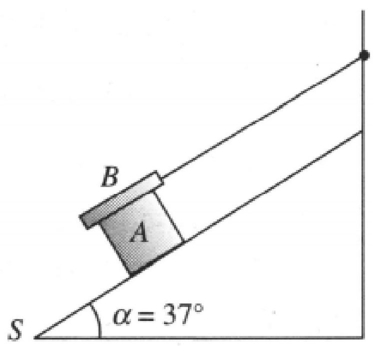
$$N^2_{min} = 25 + 2\sqrt{144} = 25 + 24 = 49$$

$$\text{Sum} = 50 + 49 = 99$$

PART-B: PHYSICS

SECTION-I (Single Digit Integer)

19. A block *A*, of weight *W*, slides down an inclined plane *S* of slope 37° at a constant velocity, while plank *B*, also of weight *W*, rests on top of A. The plank *B* is attached by a cord to the top of the plane. The coefficient of kinetic friction μ is the same between the surfaces A and B and between S and A. Determine the value of $1/\mu$.



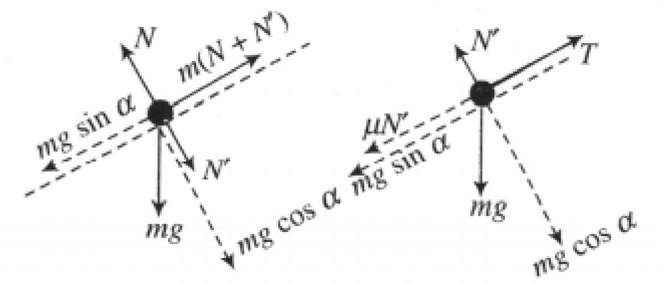
Ans. (4)

Sol. Since *A* tends to slip down, frictional forces act on it from both slides up the plane.

Let N be the reaction of the plank on A and N' be the mutual normal action – reaction between A and B.
From the free – body diagram of A,

$N' + mg\cos\alpha = N$ and $mg\sin\alpha = \mu\left(N + N'\right)$

From the free-body diagram of B



$N' = mg\cos\alpha$

$mg\sin\alpha + \mu N' = T$

$\therefore \quad 2mg\cos\alpha = N$

and $mg\sin\alpha = \mu\left(2mg\cos\alpha + mg\cos\alpha\right)$

or $\mu = \frac{1}{3}\tan\alpha = \frac{1}{3} \times \frac{3}{4} = 0.25$ or $\frac{1}{\mu} = 4$

20. A bob of mass m, suspended by a string of length l_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass *m* suspended by a string of length l_2 , which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio l_1 / l_2 is

Ans. (5)

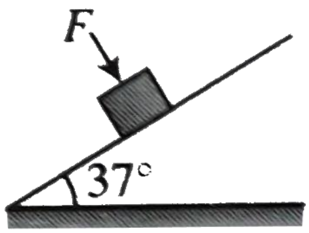
Sol. The initial speed of 1st bob (suspended by a string of length l_1) is $\sqrt{5gl_1}$.

The speed of this bob at highest point will be $\sqrt{gl_1}$

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

21. A block of mass $m = 2\text{kg}$ is resting on a rough inclined plane of inclination 37° as shown in figure. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force *F* (in newton) should be applied perpendicular to the plane on the block, so that the block does not slip on the plane?



Ans. (8)

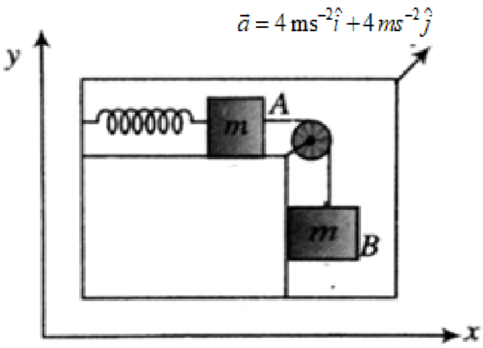
Sol. Since mg sin 37°> μmg cos 37°, the block has a tendency to slip downwards.
Let F be the minimum force applied on it, so that it does not slip. Then,
N = F + mg cos 37°

$$\therefore\ mg\sin 37^\circ = \mu N = \mu(F + mg\cos 37^\circ)$$

or $$F = \frac{mg\sin 37^\circ}{\mu} - mg\cos 37^\circ$$

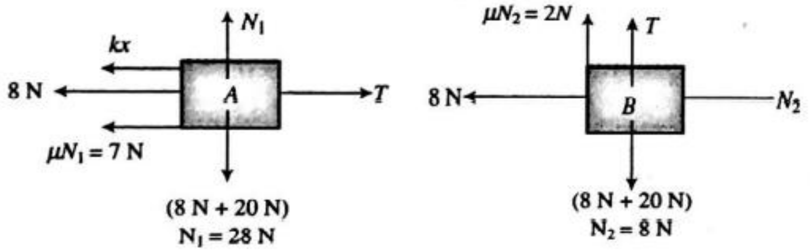
$$= \frac{(2)(10)(3/5)}{0.5} - (2)(10)\left(\frac{4}{5}\right) = 8N$$

22. The arrangement shown in figure is at rest. An ideal spring of natural length l_0 having spring constant k=220Nm^{-1}, is connected to block A. Blocks A and B are connected by an ideal string passing through a frictionless pulley. The mass of each block A and B is equal to m=2kg. When the spring was in natural length, the whole system is given an acceleration $\vec{a}$ as shown. If coefficient of friction of both surfaces is μ=0.25, then the maximum extension is 10k (in cm) of the spring. (g = 10ms^{-2}). Find k.



Ans. (1)

Sol. Let the maximum extension in spring be x.



Apply work-energy theorem: (Net work done by tension will be zero)

$$-\frac{1}{2}kx^2 - 8x - 7x + 28x - 2x = \Delta KE = 0$$

$$\Rightarrow -\frac{1}{2}\times 220x^2 + 11x = 0$$

$\Rightarrow$ x=0.1m=10cm

23. A railway flat car, whose mass together with the artillery gun is M = 2m moves at a speed V. The gun barrel makes an angle $\alpha = 60^\circ$ with the horizontal. A shell of mass m leaves the barrel at a speed v = 12ms^{-1}, relative to the barrel. Find the speed of the flat car V (in ms^{-1}) in order that it may stop after the firing.

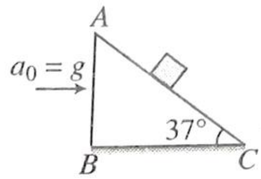
Ans. (2)

Sol. Using conservation of linear momentum

$$(2m+m)V = 2m\times 0 + mv\cos\alpha$$

$$V = \frac{mv\cos\alpha}{3m} = \frac{1}{3}v\cos\alpha = \frac{1}{3}\times 12\times\cos 60^\circ = 2ms^{-1}$$

24. A block is placed on an inclined plane moving towards right horizontally with an acceleration $a_0 = g$. The length of the plane AC = 1m. Friction is absent everywhere. Find the time taken (in seconds) by the block to reach from C to A.



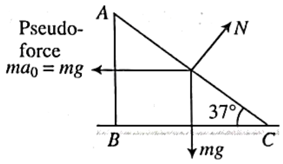
Ans. (1)

Sol.

Drawing the free-body diagram of block with respect to plane.
Acceleration of the block up the plane is

$$a = \frac{mg\cos 37^\circ - mg\sin 37^\circ}{m}$$

$$= g\left(\frac{4}{5}-\frac{3}{5}\right) = 2\ \text{ms}^{-2}$$



Applying, $s = \frac{1}{2}at^2$ we get

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{2}} = 1\ \text{s}$$

25. A water pump, rated 400 W, has an efficiency of 75%. If it is employed to raise water through a height of 40 m, find the volume of water drawn in 200 min. (in m^3)

Ans. (9)

Sol. Output power $=\eta\times$ input power

$=0.75\times 400 = 300W$

Now,

$$P = \frac{dW}{dt} = \frac{d(mgh)}{dt}$$

$$\frac{dm}{dt} = \frac{P}{gh} = \frac{300}{10\times 40} = \frac{3}{4}kg\ s^{-1} = \frac{3}{4}\times 60\ kg\ \ min^{-1} = 45kg\ \ min^{-1}$$

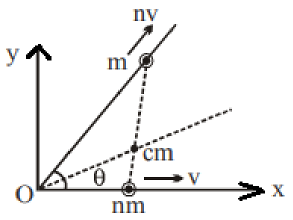
Hence, water drawn in 200 min = 45 × 200 = 9000 kg

Since density of water is 1000 kgm^{-3}, therefore, from

$$m = \rho V$$

$$V=\frac{m}{\rho}=\frac{9000}{1000}=9\text{m}^3$$

26. Two masses, *nm* and *m*, start simultaneously from the intersection of two straight lines with velocities *v* and nv respectively. It is observed that the path of their centre of mass is a straight line bisecting the angle between the given straight lines. Find the magnitude of the velocity of centre of mass. [Here θ = angle between the lines]. Given $\theta=60^\circ, n=3, v=\frac{4}{\sqrt{3}}ms^{-1}$



Ans. (3)

Sol.

$$\vec{v}_{cm}=\frac{m_1\vec{v}_1+m_1\vec{v}_2}{m_1+m_2}$$

$$\vec{v}_{cm}=\frac{nm\,v\,\hat{i}+m\,nv\cos\theta\,\hat{i}+nmv\sin\theta\,\hat{j}}{m+nm}$$

$$v_{cm}=\frac{\sqrt{(nmv+mnv\cos\theta)^2+(nmv\sin\theta)^2}}{m(1+n)}$$

$$v_{cm}=\frac{nmv\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}}{m(1+n)}$$

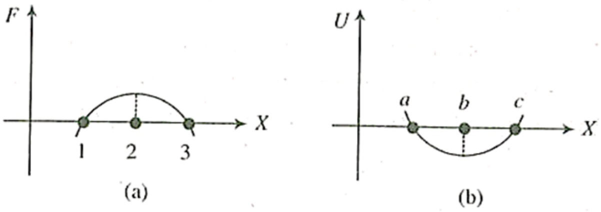
$$=\frac{nv\sqrt{1+\cos^2\theta+2\cos\theta+\sin^2\theta}}{(1+n)}$$

$$=\left(\frac{nv}{n+1}\right)\sqrt{\left(2\cos\frac{\theta}{2}\right)^2}$$

$$v_{cm}=\frac{2nv\cos\frac{\theta}{2}}{n+1}=3$$

SECTION-II (One or More than One Correct)

27. Referring the graphs, which of the following is/are correct?

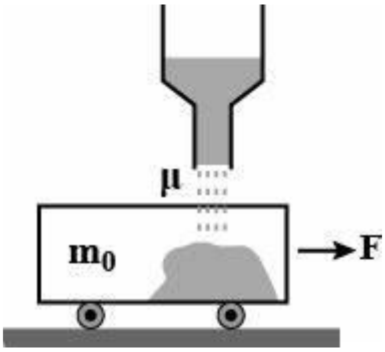


(A) The particle has stable equilibrium at points 3 and b.
(B) The particle is in neutral equilibrium at points b and 2.
(C) No power is delivered by the force on the particle at points 1, 3, and b.
(D) The particle has least kinetic energy at position 1.

Ans. (A,C,D)

Sol. Option (a) is correct because at 3 , force is opposite to displacement.
At (b),u is minimum.
Option (b) is incorrect because at 2 , there is net force and hence no equilibrium.
Option (c) is correct because at 1,3 and b force is zero.
At (b), $\frac{du}{dx} = -F = 0$
Option (D) is correct. Positive work is done after position 1.
Hence, KE will increase. So KE is least at 1 .

28. A flat car of mass m_0 starts moving to the right due to a constant horizontal force F. Sand pills on the flat car from a stationary hopper. The rate of loading is constant and equal to μ kg/s. The friction is negligibly small.



(A) The velocity of flat car at any time is $\frac{Ft}{m_0 + \mu t}$

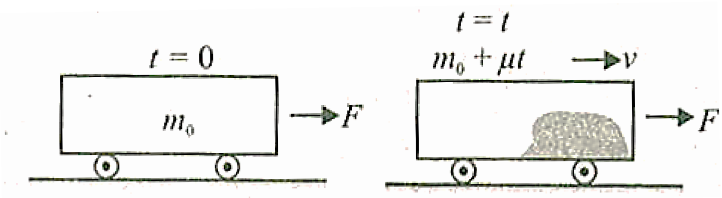
(B) The velocity of flat car at any time t is $\frac{2Ft}{(m_0 + \mu t)}$

(C) The instantaneous acceleration of flat car is $\frac{Fm_0}{(m_0 + (\mu t))^2}$

(D) The instantaneous acceleration of flat car is $\frac{Fm_0}{(m_0 + 2\mu t)^2}$

Ans. (A,C)

Sol. Initial velocity of the flat car is zero. Let v be its velocity at time t and m be its mass at that instant. Then



At $t = 0,\ v = 0$ and $m = m_0$

At $t = t,\ v = v$ and $m = m_0 + \mu t$

Here, $v_r = u - v = 0 - v = -v$

Here u is the velocity of mass (being added) in the horizontal direction which is zero.

$\frac{dm}{dt} = \mu$

Apply $F + v_{rel}\dfrac{dm}{dt} = \dfrac{mdv}{dt}$, we get

$$F + (-v)\mu = (m_0 + \mu t)\frac{dv}{dt} \text{........(i)}$$

or $$\int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\Rightarrow -\frac{1}{\mu}\left[\ln(F - \mu v)\right]_0^v = \frac{1}{\mu}\left[\ln(m_0 + \mu t)\right]_0^t$$

$$\Rightarrow \ln\left(\frac{F}{F - \mu v}\right) = \ln\left(\frac{m_0 + \mu t}{m_0}\right)$$

$$\Rightarrow \frac{F}{F - \mu v} = \frac{m_0 + \mu t}{m_0}$$

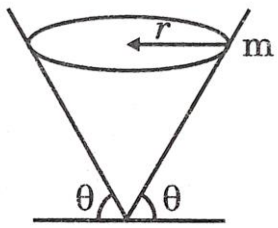
or $$v = \frac{Ft}{m_0 + \mu t} \text{........(ii)}$$

From Eq. (i),

$$a = \frac{F - \mu v}{m} = \left(\frac{F - \dfrac{F\mu t}{m_0 + \mu t}}{m_0 + \mu t}\right)$$

or $$a = \frac{Fm_0}{(m_0 + \mu t)^2}$$

29. A ball of mass 'm' is rotating in a circle of radius 'r' with speed '*v*' inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



(A) N = mg cos θ

(B) $g\ sin\theta = \frac{v^2}{r} cos\theta$

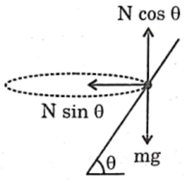
(C) $N\ sin\theta - \frac{mv^2}{r} = 0$

(D) none of these

Ans. (B,C)

Sol.

Figure (1)



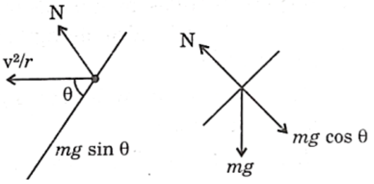
Resolving in horizontal and vertical

$$N\sin\theta = \frac{mv^2}{r} \quad ...(i)$$

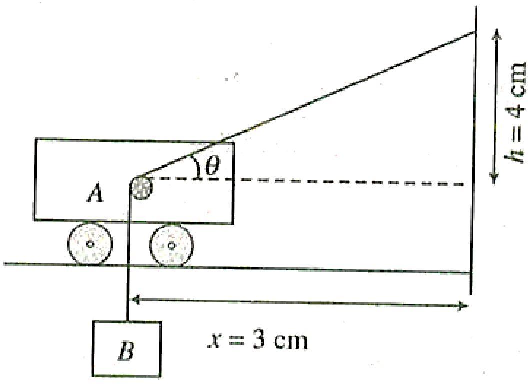
Resolving along and perpendicular to incline gives

$$mg\sin\theta = \frac{mv^2}{r}\cos\theta \quad ...(ii)$$

Figure (2)



30. The string shown in Fig. is passing over small smooth pulley rigidly attached to trolley A. If the speed of trolley is constant and equal to v_A towards right, speed and magnitude of acceleration of block B at the instant shown in figure are



(A) $v_B = v_A, a_B = 0$

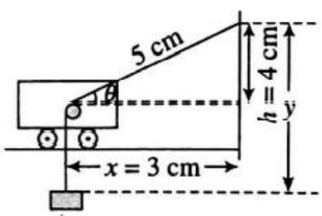
(B) $a_B = 0$

(C) $v_B = \frac{3}{5}v_A$

(D) $a_B = \frac{16v_A^2}{125}$

Ans. (C,D)

Sol.



$(y-h)+\sqrt{x^2+h^2} = l$ or $\frac{dy}{dt}+\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$ =0

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{3}{5}(-v_A)$$

$$v_B = \frac{3}{5}v_A$$

$$\frac{d^2y}{dt^2}=\frac{v_A^2h^2}{(x^2+h^2)^{3/2}}\Rightarrow a_B=v_A^2\frac{16}{(5)^3}=\frac{16}{125}v_A^2 \text{.....(ii)}$$

31. One of the forces acting on a particle is conservative, then

(A) Its work is zero when the particle moves exactly once around any closed path.

(B) Its work equals the change in the kinetic energy of the particle.

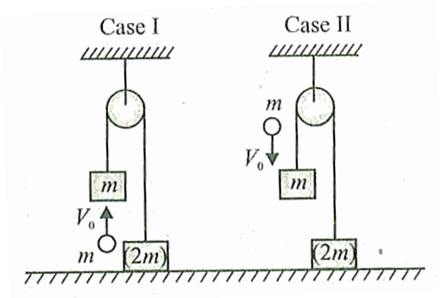
(C) It does not obey Newton's second law.

(D) Its work depends on the end points of the motion, not on the path in between.

Ans. (A,D)

Sol. It is the definition of a conservative force. Increase in KE will be due to work done by other forces also.

32. Two masses 2*m* and *m* are connected by an inextensible light string. The string is passing over a light frictionless pulley. The mass 2*m* is resting on a surface and mass m is hanging in air as shown in Fig. A particle of mass m strikes the mass m from below in case (I) with a velocity v_o and in case (II) strikes mass m with a velocity v_o from top and sticks to it.



(A) The conservation of linear momentum can be applied in both the cases just before and just after collision.

(B) The conservation of linear momentum can be applied, in case I but cannot be applied in case II just before and just after collision.

(C) The ratio of velocities of mass m just after collision in first and second cases is 1/2.

(D) The ratio of velocities of mass m just after collision in first and second case is 2.

Ans. (B,D)

Sol. In the first case: $(m+m)v_1=mv_0\Rightarrow v_1=\frac{v_0}{2}$

In the second case:

$-\int Tdt=(m+m)v-mv_0$ ......(i)

$\int Tdt=2mv$ ......(ii)

From Eqs. (i) and (ii),

$4mv=mv_0\Rightarrow v=\frac{v_0}{4}$ and required ratio = 2

SECTION-III (Only One Correct Type)

33. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km h^{-1}. If the coefficient of static friction between the road and the tyre is 0.4, what is the ratio of possible speeds of a vehicle so that it neither slips down nor skids up?

(A) $\frac{3\sqrt{3}}{2}$ (B) $3\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$ (D) None of these

Ans. (A)

Sol. Angle of banking for designed speed,

$V = 36\ km\ h^{-1} = 10 ms^{-1}$

$\Rightarrow \tan\theta = \frac{\vartheta_0^2}{Rg} = \frac{10^2}{20\times 10} = \frac{1}{2}$

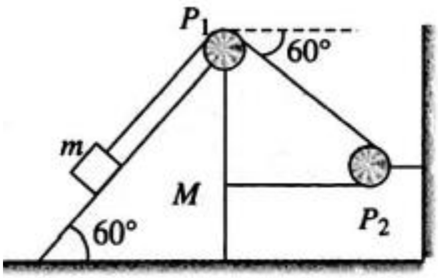
The vehicle may have the tendency to slide up or down depending on the speed of the vehicle. If speed of the vehicle is more it has tendency to slide up and vice-versa.

$\Rightarrow \left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right) = \frac{v_{\max}^2}{Rg}$ .....(i)

$\Rightarrow \left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right) = \left(\frac{0.5+0.4}{1-0.4\times 0.5}\right) = \frac{v_{\max}^2}{20\times 10} \Rightarrow v_{max} = 15\ ms^{-1}$

$\Rightarrow \left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right) = \frac{v_{\min}^2}{Rg} \Rightarrow v_{\min} = 10\sqrt{\frac{1}{6}} ms^{-1}$

34. In the arrangement shown in Fig., the block of mass m = 2kg lies on the wedge of mass M = 8kg. The initial acceleration of the wedge, if the surfaces are smooth, is

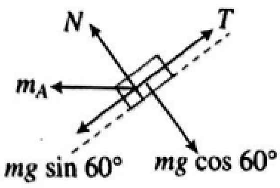


(A) $\frac{\sqrt{3}g}{23}$ ms^{-2} (B) $\frac{3\sqrt{3}g}{23}$ ms^{-2}

(C) $\frac{3g}{23}$ ms^{-2} (D) $\frac{g}{23}$ ms^{-2}

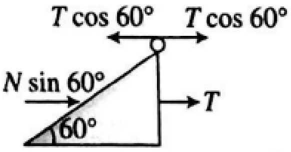
Ans. (B)

Sol. If initial acceleration of M towards right is A, then we can show that acceleration of m w.r.t. M down the incline is



$a = A(1+\cos\theta) = \frac{3A}{2}\left(\because \theta = 60^\circ\right)$

FBD of block m (w.r.t. M) is shown below:



FBD of M (Figure)

Equation of motion:

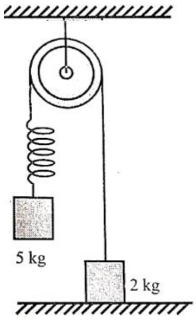
For m: $mg\dfrac{\sqrt{3}}{2}+mA\times\dfrac{1}{2}-T=m\dfrac{3}{2}A$......(i)

$$N+mA\frac{\sqrt{3}}{2}=mg\frac{1}{2}$$

For $M: T+N\frac{\sqrt{3}}{2}=MA$.......(ii)

From Eqs. (i),(ii) and (iii) $A=\dfrac{3\sqrt{3}g}{23}ms^{-1}$

35. The system shown in Fig. is released from rest with mass 2 kg in contact with the ground. Pulley and spring are massless, and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with the ground (force constant of the spring k = 40 N m^{-1} and g = 10 m s^{-2})



(A) $\sqrt{2}$ ms^{-1}

(B) $2\sqrt{2}$ ms^{-1}

(C) $2ms^{-1}$

(D) $\sqrt{2}$ ms^{-1}

Ans. (B)

Sol. Let x be the extension in the string when $2kg$ block leaves the contact with ground. Then tension in the spring should be equal to weight of $2kg$ block.

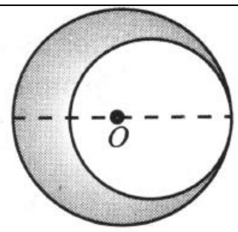
$$Kx=2g \text{ or } x=\frac{2g}{K}=\frac{2\times 10}{40}=\frac{1}{2}m$$

Now, from conservation of mechanical energy

$$mgx=\frac{1}{2}Kx^2+\frac{1}{2}mv^2$$

$$\Rightarrow v=\sqrt{2gx-\frac{Kx^2}{m}}=\sqrt{2\times 10\times\frac{1}{2}-\frac{40}{4\times 5}}=2\sqrt{2}\text{ ms}^{-1}$$

36. A circular plate of uniform thickness has a diameter of 28 cm. A circular portion of diameter 21 cm is removed from the plate as shown. O is the centre of mass of complete plate. The position of centre of mass of remaining portion will shift towards left from 'O' by

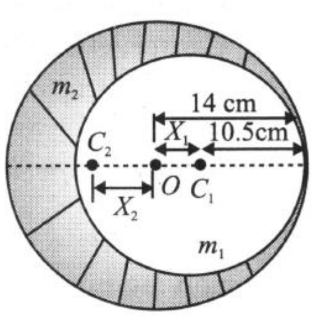


(A) 5 cm (B) 9 cm
(C) 4.5 cm (D) 5.5 cm

Ans. (C)

Sol. C_1 is the centre of mass of cut portion and C_2 that of remaining portion. We have to find x_2.

$$x_1 = 14 - 10.5 = 3.5 cm$$



Mass will be proportional to area. So mass of the whole disc is

$$M = k\pi(14)^2$$

Mass of cut portion $m_1 = k\pi(10.5)^2$

Mass of the remaining portion $m_2 = M - m_1 = k\pi(14^2 - 10.5)^2$

$= k\pi(24.5)\times(3.5)$

Now, $m_1x_1 = m_2x_2$

$$\Rightarrow x_2 = \frac{m_1x_1}{m_2} = \frac{k\pi(10.5)^2 \times 3.5}{k\pi(24.5)^2 \times 3.5} = 4.5 \text{ cm}$$

PART-C: CHEMISTRY

SECTION-I (Single Digit Integer)

37. How many 90° angles are present in BrF_5?

Ans. (0)

Sol. Lone pair repulsions distort the octahedral geometry.

38. How many of the following compounds violate octet rule ?

(i) BrF_5 (ii) SF_6 (iii) IF_7 (iv) $XeOF_4$ (v) ClF_2^- (vi) PCl_4^+

Ans. (5)

Sol. BrF_5 , SF_6 , IF_7 , $XeOF_4$, ClF_2^- have expanded octet of central atom.

39. Out of I_3^-, ICl_2^-, $BeCl_2$, XeF_2, XeF_6, BrF_5, $CH_2 = CH_2$

How many will have linear shape?

Ans. (4)

Sol. I_3^-, ICl_2^-, $BeCl_2$ and XeF_2 are linear.

40. How many species show bond order 2.5 and paramagnetic character?

O_2^{2-}, $O_2^{\ominus}$, O_2, $O_2^{\oplus}$, F_2, Ne_2

Ans. (1)

Sol.

$O_2^{\oplus}$	2.5	Paramagnetic
O_2	2.0	Paramagnetic
$O_2^{\ominus}$	1.5	Paramagnetic
O_2^{2-}	1.0	Diamagnetic
F_2	1.0	Diamagnetic
Ne_2	0	Diamagnetic

41. When 500 J of heat energy is given to a sample of a gas in an isobaric process, work done by the gas is 143 J. What is the no. of atoms per molecule of the gas?

Ans. (2)

Sol. $w = -143J$

$q = +500\ J$

$\Delta U = 357\ J$

$w = -P\Delta V = -nR\Delta T$

$\Delta U = nC_{v,m}\Delta T$

$$\frac{500}{357} = \gamma = 1.4 = \frac{7}{5}$$

$$\frac{\Delta U}{w}=\frac{nC_{v,m}\Delta T}{-nR\Delta T}$$

$$\frac{\Delta U}{w}=\frac{C_{v,m}}{R}$$

$$\frac{C_{v,m}}{-R}=\frac{357}{-143}$$

$$\Rightarrow C_{v,m}=\frac{5R}{2}$$

Hence, the gas is diatomic

42. If the pressure and density of a diatomic gas (γ =7/5) in adiabatic reversible process changes from (P, d) to (P', d'), where p'/p is 128 then d'/d is 8x, find x?

Ans. (4)

Sol. $PV^{\gamma}=\text{constant} \Rightarrow (P/P')\times(V/V')^{\gamma}=1 \Rightarrow (P/P')=(V'/V)^{\gamma}$

$\Rightarrow (P/P') = (d/d')^{\gamma}$ (Since V ∝ /d)

$$\Rightarrow (d'/d) = (128)^{5/7} = 32$$

$$\Rightarrow 8x = 32$$

$$\Rightarrow x = 4$$

43. 1 mole of an ideal gas is allowed to expand isothermally and reversibly at 27°C until its volume is tripled. Calculate ΔS_{sys} (Give answer as nearest integer in SI units). $(\ln 3 = 0.477)$

Ans. (9)

Sol. In isothermal reversible process

$\Delta S = q_{rev}/T$

$q = -W = 2.303RT\log(V_2/V_1)$

$= 2.303 \times 8.314 \times 300 \log 3 = 2740.6 \text{ J mol}^{-1}$

$$\Delta S_{system}=\frac{q_{rev}}{T}=\frac{2740.6}{300}=9.135\,JK^{-1}mol^{-1} \cong 9$$

44. 3.00 mole of an ideal gas expands isothermally (in thermal contact with the surroundings at temperature 15°C) against a fixed external pressure of 1 bar. The initial and final volume of the gas are 10 litre and 30 litre respectively. How many statements are correct regarding above process?

(i) $\Delta S_{sys} > 0$ (ii) $\Delta S_{surr} < 0$ (iii) $\Delta U = 0$ (iv) $\Delta H = 0$ (v) $\Delta S_{surr} = -6.94$ J/K

Ans. (5)

Sol. For isothermal process on an ideal gas, $\Delta U = 0, \Delta H = 0$

$$\Delta S_{sys} > 0\left(V_{final} > V_{initial}\right)$$

$$\Delta S_{surr} = \frac{-q_{sys}}{T}$$

SECTION – II (Multiple Correct Questions)

45. Which of the following statements are not correct?

(A) Hydrogen bonds are not present in hydrogen fluoride.

(B) Both SO_4^{2-} and SO_3^{2-} have tetrahedral geometry of electron pairs.

(C) In ozone both the bond lengths between oxygen atoms are not identical because it contains one double bond and one coordinate-covalent bond.

(D) In CO_3^{2-}, all the three carbon-oxygen bond length are identical.

Ans. (A), (C)

Sol. (A) (Fluorine is the most electronegative element. A strong polarization of bonding electrons occurs in HF leading to hydrogen bonding).
(C) Due to the resonance, both bonds are identical.

46. Which of the following statements are not correct?

(A) Beryllium, like nitrogen, forms diatomic molecule Be_2.

(B) He_2 molecule does not exist but He_2^+ does exist.

(C) The dipole moment of HCl is greater than that of HBr.

(D) HBr is a stronger acid than HI.

Ans. (A), (D)

Sol. (A) Be_2 would contain equal number of bonding and antibonding electrons.
(B) There are more bonding electrons than antibonding electrons.
(D) Stronger the hydrogen bonding, weaker the base.

47. Which of the following statements are correct?

(A) The value of $C_{p,m} / C_{V,m}$ for a mixture of 1 mol of a monatomic gas (γ = 5/3) and 1 mol of a diatomic gas (γ = 7/5) is 1.5.

(B) The work done by an ideal gas during expansion from V_1 to V_2 is higher when done isothermally as compared to that carried out adiabatically.

(C) The internal energy of an ideal gas does not change in the isothermal process.

(D) The fraction of heat utilized in increasing internal energy of a diatomic gas is 3/5 when the heat is supplied under constant pressure condition.

Ans. (A), (B), (C)

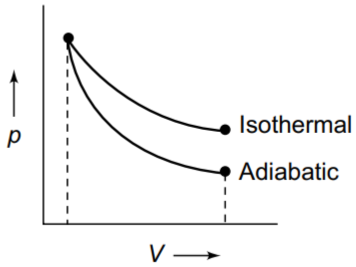
Sol. We have

$$C_{p,\mathrm{m}}(\text{mixture}) = \frac{1}{2}(C_{p1} + C_{p2}) = \frac{1}{2}\left(\frac{5}{2}R + \frac{7}{2}R\right) = 3\ R$$

$$C_{V,\mathrm{m}}(\text{mixture}) = \frac{1}{2}(C_{V1} + C_{V2}) = \frac{1}{2}\left(\frac{3}{2}R + \frac{5}{2}R\right) = 2\ R$$

$$\gamma = C_{p,\mathrm{m}}/C_{V,\mathrm{m}} = 3R/2R = 3/2$$

The choice b is correct as may be seen from the areas under the isothermal and adiabatic curves shown in Fig.



The choice c is correct as the internal energy of an ideal gas depends only on temperature.
The choice d is also incorrect as shown in the following.

Heat used in increasing internal energy = $C_V \Delta T$

Heat absorbed at constant pressure = $C_p \Delta T$

Fraction of heat used in increasing internal energy is $\phi = \frac{C_V \Delta T}{C_p \Delta T} = \frac{C_V}{C_p}$

For one mole of the gas $\phi = \frac{C_V, m}{C_p, m} = \frac{(5/2)R}{(7/2)R} = \frac{5}{7}$

48. Which of the following statements with regard to Kossel Lewis Approach are correct?

(A) Atoms achieve stable octet when they are linked by chemical bonds.

(B) Kernel is nucleus plus inner electrons

(C) Eight electrons of an octet are assumed to occupy the corners of a cube which surround 'Kernel'.

(D) In the formation of a molecule, Kernels and Valence electrons take part in chemical combination.

Ans. (A,B,C)

Sol. Conceptual

49. Which of the following orders are correct?

(A) Dipole moment : H – F > H – Cl > H – Br

(B) Bond length : C – C > C = C > C – H > O – H

(C) Dipole moment : $NH_3 < NF_3$

(D) Dipole moment: $H_2O > HF > H_2S$

Ans. (A,B,D)

Sol.

Type of Molecule	Example	Dipole Moment, μ(D)	Geometry
Molecule (AB)	HF	1.78	linear
	HCl	1.07	linear
	HBr	0.79	linear
	HI	0.38	linear
	H_2	0	linear
Molecule (AB_2)	H_2O	1.85	bent
	H_2S	0.95	bent
	CO_2	0	linear
Molecule (AB_3)	NH_3	1.47	trigonal-pyramidal
	NF_3	0.23	trigonal-pyramidal
	BF_3	0	trigonal-planar
Molecule (AB_4)	CH_4	0	tetrahedral
	$CHCl_3$	1.04	tetrahedral
	CCl_4	0	tetrahedral

50. Which of the following statements regarding hybridization is incorrect?

(A) Orbitals present in the valence shell of the atom are hybridized.

(B) Promotion of electron is a necessary condition of hybridization.

(C) Orbitals undergoing hybridization must have equal energy

(D) Filled orbitals of valence shell may also take part in hybridization.

Ans. (B,C)

Sol. Conceptual

SECTION-III (Single Correct Questions)

51. If the bond energies of (C—H) = 413.4 kJ mol^{-1}, (C—C) = 347.7 kJ mol^{-1}, (C=C) = 615.1 kJ mol^{-1}, $\Delta_{sub}H(C,\text{graphite})$= 718.4 kJ mol^{-1} and $\Delta_f H(H,g)$ = 218 kJ mol^{-1}, the enthalpy of formation of gaseous isoprene $CH_2{=}CH{-}\underset{\underset{CH_3}{|}}{C}{=}CH_2$ will be about

(A) 206.4 kJ mol^{-1}

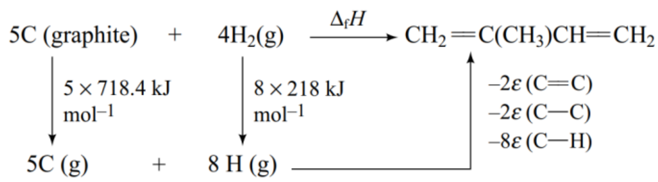
(B) – 206.4 kJ mol^{-1}

(C) 103.2 kJ mol^{-1}

(D) – 103.2 kJ mol^{-1}

Ans. (C)

Sol. The formation reaction is



Hence $\Delta_f H = 5\times718.4+8\times218-2\times\ 615.1-2\times347.7-8\times413.4)\ \text{kJ mol}^{-1}$

$$= 103.2\ \text{kJ mol}^{-1}$$

52. If $\Delta_f H^\circ(CO,g)$ = – 110.5 kJ mol^{-1} and $\Delta_f H^\circ(CO_2,g)$ = – 393.5 kJ mol^{-1}, what is the mass of oxygen consumed when 1 mol of graphite on combustion gives a mixture of CO(g) + CO_2(g) and liberates 313.8 kJ heat

(A) 24.0 g

(B) 25.5 g

(C) 27.5 g

(D) 29.0 g

Ans. (C)

Sol. Given: The data are

$$C(\text{graphite}) + \tfrac{1}{2}\ O_2(g) \rightarrow CO(g) \quad : \Delta_f H^\circ = -\ 110.5\ \text{kJ mol}^{-1}$$

$$C(\text{graphite}) + O_2(g) \rightarrow CO_2(g) \quad : \Delta_f H^\circ = -\ 393.5\ \text{kJ mol}^{-1}$$

$$C(\text{graphite}) + O_2(g) \rightarrow \text{a mixture of } CO(g) \text{ and } CO_2(g) \quad : \Delta_r H^\circ = -\ 313.8\ \text{kJ mol}^{-1}$$

If x is the amount fraction of CO in the mixture, then the amount of O_2 consumed will be

$$\frac{x}{2}+(1-x)=1-\frac{x}{2}$$

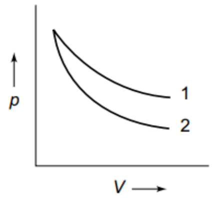
where x may be calculated from the expression

$$x\left(-\ 110.5\ kJ\ mol^{-1}\right) + (1-x)\left(-\ 393.5\ kJ\ mol^{-1}\right) = -\ 313.8\ kJ\ mol^{-1}$$

which gives x = 0.272.

Mass of O_2 consumed = $\left\{\left(1-\frac{0.272}{2}\right)\text{mol}\right\}(32\text{g mol}^{-1}) = 27.6\ g$

53. p–V plots for two gases during adiabatic processes are shown in the Fig. Plots 1 and 2 should correspond respectively to



(A) He and O_2 (B) O_2 and He

(C) He and Ar (D) O_2 and N_2

Ans. (B)

Sol. For an adiabatic process, $pV^{\gamma} = \text{constant}$. Also $\gamma\left(= C_{p,m}/C_{V,m}\right)$ for a diatomic gas (= 7/5) is smaller than that of a monatomic gas (= 5/3). Hence, for given volume.

$\left(V^{\gamma}\right)_{\text{diatomic gas}} < \left(V^{\gamma}\right)_{\text{monatomic gas}}$

Since at any instant, pV = constant, we will have $p_{diatomic\ gas} > p_{monatomic\ gas}$

Hence, the curve 1 is meant for a diatomic gas and the curve 2 for a monatomic gas.

54. The increasing bond length of O—O bond in O_2, $O_2[AsF_6]$, and KO_2 is

(A) $O_2[AsF_6] < KO_2 < O_2$

(B) $O_2[AsF_6] < O_2 < KO_2$

(C) $KO_2 < O_2[AsF_6] < O_2$

(D) $KO_2 < O_2 < O_2[AsF_6]$

Ans. (B)

Sol. In the given compounds, oxygen is present as O_2, O_2^+ and O_2^-, respectively. For these species, we have

O_2	KK $(\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p)^2\ (\pi 2p_x)^2\ (\pi 2p_y)^2\ (\pi^*2p_x)^1\ (\pi^*2p_y)^1$
O_2^+	KK $(\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p)^2\ (\pi 2p_x)^2\ (\pi 2p_y)^2\ (\pi^*2p_x)^1$
O_2^-	KK $(\sigma 2s)^2\ (\sigma^*2s)^2\ (\sigma 2p)^2\ (\pi 2p_x)^2\ (\pi 2p_y)^2\ (\pi^*2p_x)^2\ (\pi^*2p_y)^1$

The bond orders of these species as given by the expression

BO = (1/2) [Number of (bonding – antibonding) electrons]

are as follows.

For O_2 = $BO = \frac{1}{2}(8-4) = 2$

For O_2^+ = $BO = \frac{1}{2}(8-3) = 2.5$

For O_2^- = $BO = \frac{1}{2}(8-5) = 1.5$

Since bond length is inversely proportional bond order, the increasing order of bond length of O—O is

$O_2[AsF_6] < O_2 < KO_2$

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