

# FIITJEE

## ALL INDIA TEST SERIES

### PART TEST – III

**JEE (Main)-2025**

**TEST DATE: 15-12-2024**

**Time Allotted: 3 Hours**

**Maximum Marks: 300**

#### **General Instructions:**

- The test consists of total 75 questions.
- Each subject (PCM) has 25 questions.
- This question paper contains **Three Parts**.
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.

**Section-A (01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

**Section-B (21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

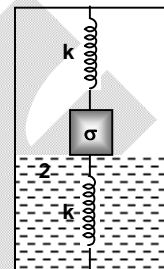
# Physics

## PART – A

### SECTION – A (One Options Correct Type)

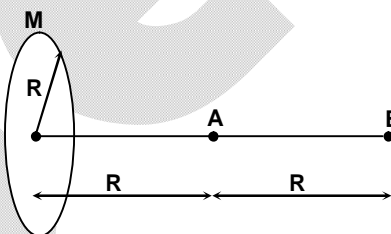
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A cubic block of side  $a$  is connected with two similar vertical springs as shown in the figure. Initially, bottom surface of the block of density  $\sigma$  touches the surface of the fluid of density  $2\sigma$  while floating. A weight is placed on the block so that it is immersed half in the fluid, then find the weight.



- (A)  $a\left(\frac{K}{2} + a^2\sigma g\right)$   
 (B)  $a(K + a^2\sigma g)$   
 (C)  $a\left(K + \frac{a^2}{2}\sigma g\right)$   
 (D)  $\frac{a}{2}(K + a^2\sigma g)$

2. A ring having non-uniform distribution of mass  $M$  and radius  $R$  is being considered. A point mass  $m_0$  is taken slowly towards the ring. In doing so, work done by the external force against the gravitational force exerted by ring is



- (A)  $\frac{GMm_0}{\sqrt{2}R}$   
 (B)  $\frac{GMm_0}{R} \left[ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}} \right]$   
 (C)  $\frac{GMm_0}{R} \left[ \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}} \right]$

(D) It is not possible to find the required work as the nature of distribution of mass is not known

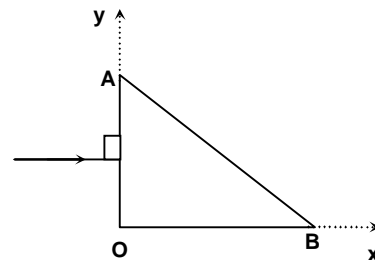
3. Two blocks of masses 40 kg and 20 kg are connected by a wire that has a linear mass density of 1g/m. These blocks are being pulled across horizontal frictionless floor by horizontal force  $F$  that is applied to 20 kg block. A transverse wave travels on the wire between the blocks with a speed of 400 m/s (relative to the wire). The mass of the wire is negligible compared to the mass of the blocks. The magnitude of  $F$  is

- (A) 160 N  
 (B) 240 N  
 (C) 320 N  
 (D) 400 N

4. A nucleus with mass number 220 initially at rest emits an  $\alpha$ -particle. If the  $Q$ -value of the reaction is 5.5 MeV, the kinetic energy of the  $\alpha$ -particle is (in MeV)

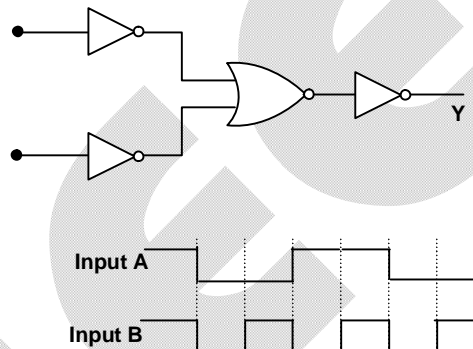
- (A) 2.7  
 (B) 7.4  
 (C) 5.4  
 (D) 19.0

5. A triangular medium has varying refracting index  $\mu = \mu_0 + ax$ , where  $x$  is the distance (in cm) along  $x$ -axis from origin and  $\mu_0 = 4/3$ . A ray is incident normally on face OA at the mid point of OA. Find the range of value of  $a$  so that light does not escape through face AB when it falls first time on the face AB. (OA = 4 cm, OB = 3 cm and AB = 5 cm) (surrounding medium is air)



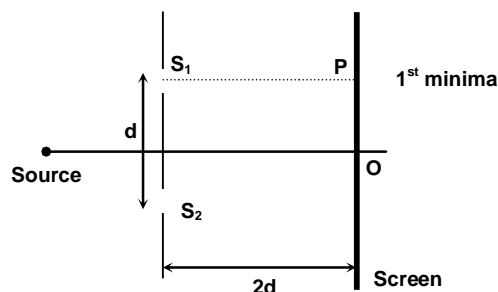
- (A)  $a > \frac{1}{9}$  (B)  $a < \frac{1}{9}$   
(C)  $a > \frac{2}{9}$  (D)  $a < \frac{2}{9}$

6. The logic circuit as shown has the input wave forms 'A' and 'B' as shown in the figure. Pick out the correct output waveform.



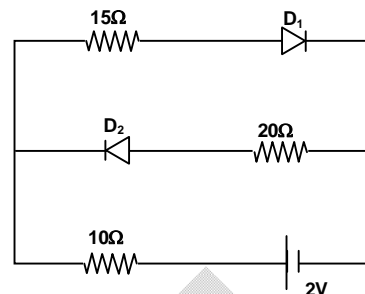
- (A) (B)   
(C) (D)

7. Consider a Young's double slit experiment as shown in figure. What should be the slit separation  $d$  in terms of wavelength  $\lambda$  such that the first minima occurs directly in front of the slit  $S_1$ ?



- (A)  $\frac{\lambda}{2(\sqrt{5}-2)}$   
(B)  $\frac{\lambda}{(\sqrt{5}-2)}$   
(C)  $\frac{\lambda}{2(5-\sqrt{2})}$   
(D)  $\frac{\lambda}{(5-\sqrt{2})}$

8. The current  $I$  through  $10\ \Omega$  resistor in the given circuit is (diodes are ideal)
- (A) 50 mA (B) 20 mA  
(C) 40 mA (D) 80 mA



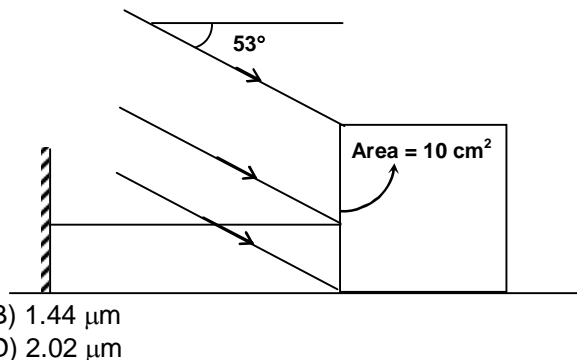
9. If the magnetic field of a plane electromagnetic wave is given by

$$B = 200 \times 10^{-6} \sin \left[ 2\pi \times 2 \times 10^{15} \left( t - \frac{x}{c} \right) \right]$$

Then the maximum electric field associated with it is: (The speed of light =  $3 \times 10^8$  m/s)

- (A)  $6 \times 10^4$  N/C (B)  $3 \times 10^4$  N/C  
(C)  $4 \times 10^4$  N/C (D)  $4.5 \times 10^4$  N/C
10. A closed organ pipe of length 99.4 cm is vibrating in its first overtone and is always in resonance with a tuning fork having frequency  $f = (300 - 2t)$  Hz, where  $t$  is time in second. The rate by which radius of organ pipe changes when its radius is 1 cm, is (speed of sound in organ pipe = 320 m/s)  $\frac{1}{b}$  m/s. The value of  $b$  is
- (A) 24 (B) 36  
(C) 48 (D) 72
11. Two polaroids are placed in the path of unpolarized beam of intensity  $I_0$  such that the intensity of light emitted from the second polaroid is  $\frac{9}{50} I_0$ . If a third polaroid whose polarization axis makes an angle  $\theta$  with the polarization axis of first polaroid, is placed between these polaroids, such that the intensity of light emerging from the last polaroid is zero, then angle  $\theta$  is
- (A)  $37^\circ$  (B)  $53^\circ$   
(C)  $60^\circ$  (D)  $30^\circ$

12. A block of mass 1 kg is connected with an elastic string of stiffness constant  $K = 10^5$  N/m. Now a light pulse of intensity  $I = 20$  W/m<sup>2</sup> strikes the block at its vertical surface of surface area 10 cm<sup>2</sup> at an angle  $53^\circ$  as shown in figure. If surface of block is 100% absorbing the light and the duration of light pulse is 6 ms, then the maximum displacement of block from its mean position is
- (A) 1.02  $\mu$ m (B) 1.44  $\mu$ m  
(C) 1.84  $\mu$ m (D) 2.02  $\mu$ m

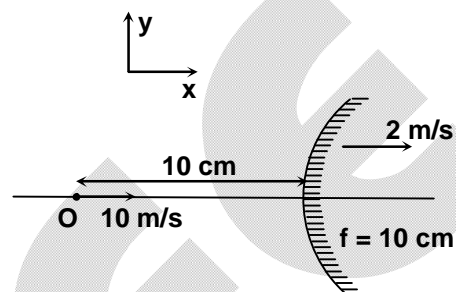


13. If mass density of earth varies with distance 'r' from centre of earth as  $\rho = kr$  and 'R' is radius of earth, then find the orbital velocity of an object revolving around earth at a distance '2R' from its centre.

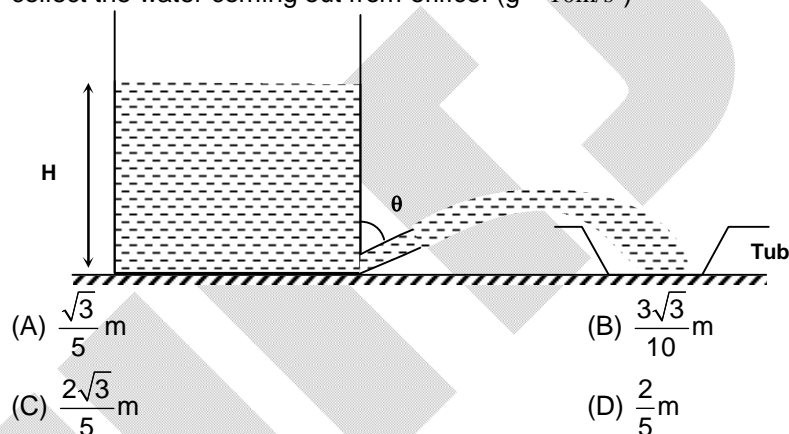
(A)  $\sqrt{\frac{\pi k R^3 G}{4}}$  (B)  $\sqrt{\frac{\pi k R^3 G}{2}}$   
 (C)  $\sqrt{\frac{\pi k R^3 G}{8}}$  (D)  $\sqrt{\pi k R^3 G}$

14. Consider a particle moving towards convex mirror along its principal axis with velocity 10m/s as shown in figure. The velocity of the mirror is 2m/s. The velocity of particle with respect ground is.

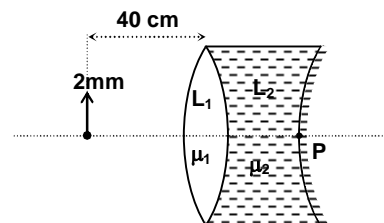
- (A) 12 m/s  
 (B) 8 m/s  
 (C) 2 m/s  
 (D) zero



15. There is a tank of cross-section area  $A_1$  with inclined orifice at its bottom with cross-section area  $A_2$  as shown in the figure. If height of water column in tank is 0.3 m and angle of inclination with vertical  $\theta = 30^\circ$  and  $\frac{A_1}{A_2} = 2$ , then at this instant find the position where tub is to be placed to collect the water coming out from orifice. ( $g = 10 \text{ m/s}^2$ )



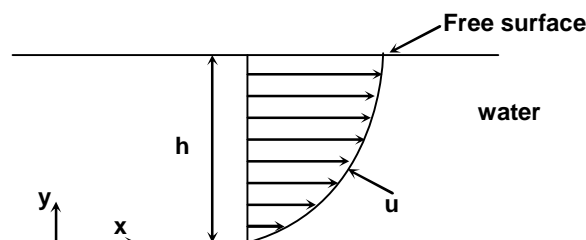
16. Consider a system of two lenses  $L_1$  and  $L_2$  which are combined as shown in figure having refractive index  $\mu_1 = 1.5$  and  $\mu_2 = 2.0$  respectively. All curved surfaces have equal radius of curvature of 20 cm. One side of  $L_2$  is silvered. An object of length 2 mm is at a distance of 40 cm. Consider paraxial rays. All lenses are thin. The final image formed by system is at a distance of (from point P).



- (A)  $\frac{40}{9} \text{ cm}$  (B)  $\frac{20}{3} \text{ cm}$   
 (C)  $\frac{10}{3} \text{ cm}$  (D)  $\frac{20}{9} \text{ cm}$

17. For the situation shown in figure, water flows on the surface of a fixed plate. The velocity of water a function of distance 'y' is given as  $u = \alpha \left[ \frac{y}{h} - 2 \left( \frac{y}{h} \right)^2 \right]$ .

Determine the magnitude of shear stress that the water applies at the base of plate. Coefficient of viscosity is  $\eta$ .



- (A)  $\frac{3\alpha\eta}{h}$  (B)  $\frac{2\alpha\eta}{h}$   
 (C)  $\frac{4\alpha\eta}{h}$  (D)  $\frac{\alpha\eta}{h}$
18. Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of hydrogen atom becomes -1.7 eV. Now a photoelectric plate having work function 2.3 eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de-Broglie wavelength of the ejected photoelectrons (in Å).  
 (A) 1.9 Å (B) 3.0 Å  
 (C) 3.8 Å (D) 2.2 Å
19. A particle is moving on X-axis and has potential energy  $U = 2 - 20x + 5x^2$  joule, where  $x$  is position. The particle is released at  $x = -3$ . If the mass of the particle is 0.1 kg, then the maximum velocity (in m/s) of the particle is  $25\beta$ . Find the value of  $\beta$ .  
 (A) 1 (B) 2  
 (C) 3 (D) 4
20. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of -0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is (in mm)  
 (A) 1.38 (B) 2.38  
 (C) 3.38 (D) 2.54

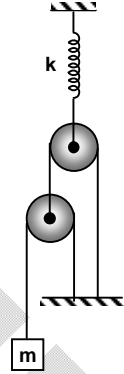
## SECTION – B

### (Numerical Answer Type)

This section contains 05 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

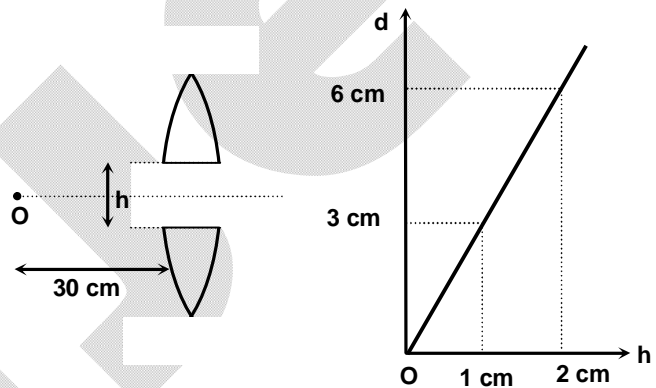
21. A compound microscope has a magnification of 30. The focal length of its eyepiece is 5 cm. Assuming the final image to be formed at least distance of distinct vision (25cm). The magnification produced by the objective is.
22. Air of density  $\rho$  is blown on a soap film of surface tension  $T$  by a pipe of radius  $r$  with its opening right next to film. The film is deformed and a bubble detaches from the film when shape of deformed surface is hemisphere. Given that the dynamic pressure on the film due to air blown at speed  $v$  as  $2\rho v^2$ , the speed at which the bubble is formed is  $\sqrt{\frac{XT}{\rho r}}$ , calculate value of  $X$ .

23. A block of mass  $m$  is connected with two ideal pulleys and a massless spring of spring constant  $K$  as shown in figure. The block is slightly displaced from its equilibrium position. If the time period of oscillation is  $\mu\pi\sqrt{\frac{m}{K}}$ , then find the value of  $\mu$ .



24. Interference fringes were produced using white light in a double slit arrangement. When a mica sheet of uniform thickness of refractive index 1.6 (relative to air) is placed in the path of light from one of the slits, the central fringe moves through some distance. This distance is equal to the width of 30 interference bands if light of wavelength  $4800\text{\AA}$  is used. The thickness (in  $\mu\text{m}$ ) of mica sheet is:

25. A convex lens cut symmetrically into two equal halves and separated laterally by a distance  $h$  as shown in figure. A point object  $O$  is placed at a distance  $30\text{ cm}$ , from the lens halves. The cut parts of the lens forms two real images separated by a distance  $d$ . A graph  $d$  versus  $h$  is plotted as shown in figure. The focal length of the lens (in  $\text{cm}$ ) is





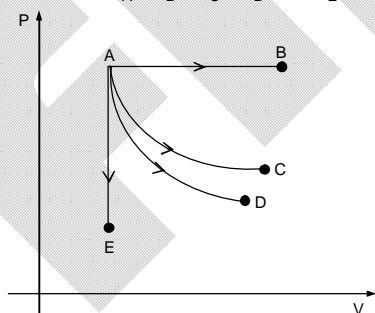
# Chemistry

## PART – B

### SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices (A), (B), (C) and (D)**, out of which **ONLY ONE** option is correct.

26. What is the correct order for the properties mentioned in bracket  
 (A)  $\text{Sc} < \text{V} > \text{Ni} < \text{Zn}$  ( $1^{\text{st}}$  ionization enthalpy)  
 (B)  $\text{Sc} > \text{V} > \text{Co} < \text{Zn}$  (Atomic radius)  
 (C)  $\text{Sc} < \text{V} < \text{Ni} < \text{Zn}$  (Density ( $\text{gm}/\text{cm}^3$ ))  
 (D)  $\text{Sc} < \text{V} > \text{Mn} > \text{Zn}$  (Enthalpy of atomization)
27. In stainless steel, which element is not present  
 (A) Fe (B) Cr  
 (C) Ni (D) Al
28. Which one of the following has not  $\text{dsp}^2$  hybridization  
 (A)  $[\text{PtClBr}(\text{PEt})_2]$  (B)  $[\text{AuBr}_4]^-$   
 (C)  $\text{K}[\text{Cu}(\text{NH}_3)_4]$  (D)  $[\text{Ni}(\text{dmg})_2]$
29.  $\lambda_m^\circ$  for NaCl, HCl and NaAc are 126.4, 425.9 and  $91.0 \text{ Scm}^2 \text{ mole}^{-1}$  respectively. If  $\lambda_m$  of  $0.1 \text{ M}$   $\text{CH}_3\text{COOH}$  is  $39.05 \text{ Scm}^2 \text{ mole}^{-1}$ , find its pH  
 (A) 1.5 (B) 1  
 (C) 2 (D) 2.5
30. For  $\text{Cu}^{+2}(\text{aq}) + 2\text{e}^- \longrightarrow \text{Cu}(\text{s})$   $E_{\text{Cu}^{+2}/\text{Cu}}^\circ$  is  $0.34 \text{ V}$ . If concentration of  $\text{Cu}^{+2}$  is decreased to  $\frac{1}{100}$  times the initial concentration then value of difference in reduction potential will be  
 (A)  $+ 0.0591 \text{ V}$  (B)  $0.591 \text{ V}$   
 (C)  $0.02955 \text{ V}$  (D)  $0.1182 \text{ V}$
31. For 1 mole of monoatomic ideal gas P – V diagram is given below. Four process are isothermal, adiabatic, isochoric and isobaric. (These are reversible processes which are given in random order)  $T_A, T_B, T_C, T_D$  &  $T_E$  are temperature at points A, B, C, D and E respectively.



Find correct order:

- (A)  $T_A = T_C > T_B > T_D > T_E$  (B)  $T_A < T_B < T_C < T_D < T_E$   
 (C)  $T_D < T_A = T_C < T_B$  (D)  $T_D > T_A = T_C < T_B$



32. Identify the **INCORRECT** order with the properties  
 (A)  $F^- > Cl^- > Br^- > I^-$  (Enthalpy of hydration)  
 (B)  $H_2O > H_2S < H_2Se < H_2Te$  (Melting point)  
 (C)  $H_2O < H_2Te < H_2Se < H_2S$  (Boiling point)  
 (D)  $PH_3 < AsH_3 < SbH_3 < NH_3$  (Boiling point)
33. For 0.01 M monobasic acid  $pK_a$  is 4, find it's van't Hoff factor (i) [Use  $K_a = C\alpha^2$ ]  
 (A) 1.2 (B) 1.1  
 (C) 1.5 (D) 1.4
34. On electrolysis of 2 litre 0.5 M  $ZnSO_4$  aq. solution using Pt electrode in electrolytic cell, find out **INCORRECT** statement. (Assume volume remain constant)  
 (A)  $[Zn^{+2}]$  will decrease  
 (B)  $[H^+]$  will increase  
 (C) pH will increase  
 (D) Pt will not liberate at anode
35. From Column-I to Column-II, which one is **INCORRECT** match
- | Column-I (Compound)    | Column-II (Central metal) |
|------------------------|---------------------------|
| (A) Vitamin $B_{12}$   | Cobalt                    |
| (B) Blood pigment      | Iron                      |
| (C) Chlorophyll        | Calcium                   |
| (D) Wilkinson catalyst | Rhodium                   |
36.  $[Cr(H_2O)_6]Cl_2$  and  $[Fe(H_2O)_6]Cl_2$  are reducing in nature because  
 (A)  $Cr^{+2}$  and  $Fe^{+2}$  will form stable  $Cr^{+3}$  and  $Fe^{+3}$  stable ion  
 (B) Both Cr and Fe will form +3 and +4 oxidation states respectively  
 (C) Due to  $Cl^-$  both will be oxidised  
 (D) Due to mixture of  $H_2O$  and  $Cl^-$
37. Electronic configuration  $[Xe]4f^7 5d^1 6s^2$  is of  
 (A) Gd (B) Tb  
 (C) Dy (D) Eu
38. Identify **CORRECT** statement in the following  
 (A) Mischmetal is used Hg-based alloy to produce bullet, shell and lighter flint  
 (B) IUPAC name of No is unnilbium  
 (C) Order of atomic radius is  $Pa > Cm > Tm > Lr$  due to actinoid contraction  
 (D) Oxidation states of U, Np, Pu, Am can be +6
39. 10 mole of Zn metal reacts with 0.25 M NaOH, produced  $H_2$  gas reacts with 28 litre of  $O_2$  at N.T.P. completely. Find volume of NaOH required for this reaction  
 (A) 20 lit (B) 10 lit  
 (C) 25 lit (D) 5 lit
40. Correct solubility order for Ar,  $CO_2$ ,  $CH_4$  and HCHO in water at 298 is  
 (A)  $Ar < CO_2 < CH_4 < HCHO$  (B)  $Ar < CH_4 < CO_2 < HCHO$   
 (C)  $CH_4 < Ar < HCHO < CO_2$  (D)  $Ar < CH_4 < HCHO < CO_2$

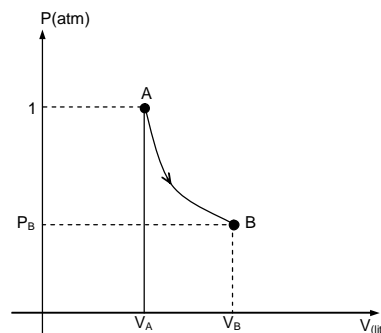
41. Identify **CORRECT** statement  
 (A) For ions  $\text{Co}^{+2}$ ,  $\text{Ni}^{+2}$ ,  $\text{Mn}^{+2}$  group reagent is  $(\text{NH}_4)_2\text{CO}_3$  in the presence of  $\text{NH}_4\text{OH}$   
 (B) Colour of  $\text{PbI}_2$  is black  
 (C) When  $\text{FeCl}_3$  reacts with  $\text{K}_4[\text{Fe}(\text{CN})_6]$  then prussian blue colour  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$  is formed  
 (D) When  $\text{MnCl}_2$  reacts with  $\text{NaOH}$ , pink colour  $\text{Mn}(\text{OH})_2$  is formed
42. When sodium nitroprusside is added to  $\text{Na}_2\text{S}$  (alkaline) then complex compound solution is formed whose colour is  
 (A) Red (B) Black  
 (C) Purple (D) White
43. When  $\text{I}_2$  comes in the contact of the compound blue colour complex is formed. That compound is  
 (A)  $\text{CH}_3\text{COOH}$  (B)  $\text{NaNO}_2$   
 (C)  $\text{KI}$  (D) Starch
44. Identify **CORRECT** statements  
 (A)  $\text{HNO}_3 + \text{H}_2\text{O}$  shows positive deviation  
 (B) Molal boiling contact ( $K_b$ ) order, water < Diethyl ether <  $\text{CHCl}_3$  <  $\text{CCl}_4$   
 (C) If concentration of saline solution is more than 0.9% w/v of  $\text{NaCl}$   
 (D) van't Hoff factor (i) decreases on dilution of  $\text{CH}_3\text{COOH}$
45. 45 gm of ethylene glycol ( $\text{C}_2\text{H}_6\text{O}_2$ ) is mixed with 600 gm of water. What is the difference between elevation in boiling point and depression in freezing point of the solution? (for water  $K_f = 1.86 \text{ K kg/mole}$  and  $K_b = 0.52 \text{ K kg/mole}$ )  
 (A) 1.608 K (B) - 2.232 K  
 (C) + 0.624 K (D) - 1.608 K

### SECTION – B

#### (Numerical Answer Type)

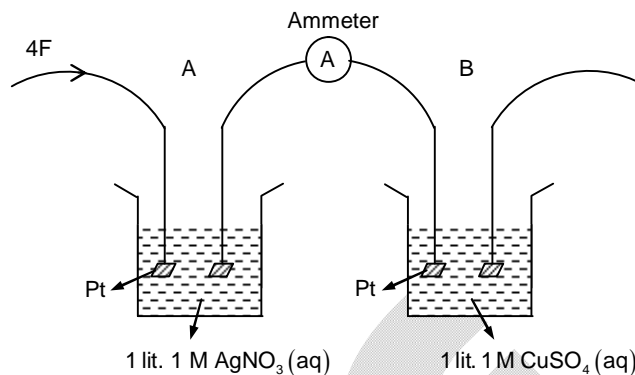
This section contains **05** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

46. An adiabatic reversible process AB is performed for 1 mole monoatomic ideal gas. If temperature at point A and B are 75 K and 300 K respectively. What will be pressure (atm) at B.



47.  $\text{CoSO}_4\text{Br} \cdot 5\text{H}_2\text{O}$  form two isomeric isomer A and B. Isomer 'A' reacts with  $\text{AgNO}_3$  form yellow precipitate but not with  $\text{BaCl}_2$ . Isomer 'B' gives white precipitate with  $\text{BaCl}_2$  but not with  $\text{AgNO}_3$ . If charge on cationic complex of both A and B are a and b respectively. What is the value of (a + b)?

48. 4 Faraday charge is passed through electrode A and B which are connected in series. Find out net mass (gm) deposited at electrodes A and B. (At. Mass Ag = 108, Cu = 63.5)



49. 2 mole of compound A (contain C & H only) required 480 gm of oxygen for complete combustion at standard state. If standard enthalpy of combustion of  $A(\ell)$  is 3400 kJ/mole, while enthalpy of formation  $\text{CO}_2(\text{g})$ ,  $\text{H}_2\text{O}(\ell)$  and  $A(\ell)$  are  $-400$ ,  $-300$  and  $100$  kJ/mole respectively. Then molar mass (gm/mole) of compound A is
50. Density of an aqueous glucose solution is 1.18 gm/mol. If it's concentration is 18% w/V then find out the difference between molarity and molality.

# Mathematics

## PART – C

### SECTION – A (One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices (A), (B), (C) and (D)**, out of which **ONLY ONE** option is correct.

51. Let  $\binom{n}{k}$  represents the combination of 'n' things taken 'k' at a time, then the value of the sum  $\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots + \binom{3}{1} + \binom{2}{0}$  equals
- (A)  $\binom{99}{97}$  (B)  $\binom{100}{98}$   
(C)  $\binom{99}{98}$  (D)  $\binom{100}{97}$
52. Let  $z$  be a complex number satisfying  $\frac{1}{2} \leq |z| \leq 4$ , then sum of greatest and least values of  $\left| z + \frac{1}{z} \right|$  is:
- (A)  $\frac{65}{4}$  (B)  $\frac{65}{16}$   
(C)  $\frac{17}{4}$  (D) 17
53. If both  $A - \frac{I}{2}$  and  $A + \frac{I}{2}$  are orthogonal matrix, then which of the following statements is correct? (where  $I$  is an identity matrix order same as that of  $A$ ).
- (A)  $A$  is skew-symmetric matrix of odd order. (B)  $A^2 = \frac{3}{4}I$   
(C)  $A$  is skew-symmetric matrix of even order. (D)  $A$  is orthogonal.
54. The determinant  $\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix} = 0$  for
- (A) only  $a+b+c+d=0$  (B) only  $ab+cd=0$   
(C) only  $ab(c+d)+cd(a+b)=0$  (D) any  $a, b, c, d$
55. Let  $\vec{a}, \vec{b}, \vec{c}$  be three vectors of magnitude 2, 3, 5 respectively, satisfying  $[\vec{a} \vec{b} \vec{c}] = 30$ . If  $(2\vec{a} + \vec{b} + \vec{c}) \cdot ((\vec{a} \times \vec{c}) \times (\vec{a} - \vec{c}) + \vec{b}) = k$ , then the value of  $\left( \frac{k}{103} \right)$  is:
- (A) 1 (B) 2  
(C) 3 (D) 4

56. If  $A^{-1} = \begin{bmatrix} \sin^2 \alpha & 0 & 0 \\ 0 & \sin^2 \beta & 0 \\ 0 & 0 & \sin^2 \gamma \end{bmatrix}$  and  $B^{-1} = \begin{bmatrix} \cos^2 \alpha & 0 & 0 \\ 0 & \cos^2 \beta & 0 \\ 0 & 0 & \cos^2 \gamma \end{bmatrix}$  where  $\alpha, \beta, \gamma$  are any real numbers and  $C = (A^{-5} + B^{-5}) + 5A^{-1}B^{-1}(A^{-3} + B^{-3}) + 10A^{-2}B^{-2}(A^{-1} + B^{-1})$  then  $|C|$ .
- (A) 0 (B) 1  
(C) 2 (D) 3
57. Three  $a$ 's, three  $b$ 's, and three  $c$ 's are placed randomly in a  $3 \times 3$  matrix. The probability that no row or column contain two identical letters can be expressed as  $\frac{p}{q}$ , where  $p$  and  $q$  are coprime then  $(p + q)$  equals to:
- (A) 151 (B) 161  
(C) 141 (D) 131
58. The equation of a plane passing through the line of intersection of the planes:  $x + 2y + z - 10 = 0$  and  $3x + y - z = 5$  and passing through the origin is:
- (A)  $5x + 3z = 0$  (B)  $5x - 3z = 0$   
(C)  $5x + 4y + 3z = 0$  (D)  $5x - 4y + 3z = 0$
59. Let set  $A = \{1, 2, 3, \dots, 22\}$ . Set  $B$  is a subset of  $A$  and  $B$  has exactly 11 elements, find the sum of elements of all possible subsets  $B$ .
- (A)  $252 {}^{21}C_{11}$  (B)  $230 {}^{21}C_{10}$   
(C)  $253 {}^{21}C_9$  (D)  $253 {}^{21}C_{10}$
60. If the equation  $x^4 - 4x^3 + ax^2 + bx + 1 = 0$  has four positive roots, then the value of  $(a + b)$  is:
- (A) -4 (B) 2  
(C) 6 (D) can not be determined
61. The product of uncommon real roots of the two polynomials  $P(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$  and  $Q(x) = x^3 + 4x^2 - x - 10$  is:
- (A) 4 (B) 6  
(C) 8 (D) 12
62. If the equation of the plane passing through the point  $(-1, 2, 0)$  and parallel to the lines  $\frac{x}{3} = \frac{y+1}{0} = \frac{z-2}{-1}$  and  $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-1}$  is  $ax + by + cz = 1$ , then the value of  $(a + b + c)$  is:
- (A) 3 (B) 4  
(C) 5 (D) 10
63. The number of three digit numbers of the form  $xyz$  such that  $x < y$  and  $z \leq y$  is:
- (A) 156 (B) 204  
(C) 240 (D) 276

64. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2$ . If  $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ , then angle between  $\vec{b}$  and  $\vec{c}$  is:
- (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{3}$   
(C)  $\frac{2\pi}{3}$  (D)  $\frac{5\pi}{6}$
65. If  $\hat{a}$  and  $\hat{b}$  are unit vectors then the vector defined as  $\vec{V} = (\hat{a} \times \hat{b}) \times (\hat{a} + \hat{b})$  is collinear to the vector:
- (A)  $\hat{a} + \hat{b}$  (B)  $\hat{b} - \hat{a}$   
(C)  $2\hat{a} - \hat{b}$  (D)  $\hat{a} + 2\hat{b}$
66. If the two lines represented by  $x + ay = b; z + cy = d$  and  $x = a'y + b'; z = c'y + d'$  be perpendicular to each other, then the value of  $aa' + cc'$  is:
- (A) 1 (B) 2  
(C) 3 (D) 4
67. Let  $y = \log_2 x + \log_4 x + \log_{16} x + \dots + \infty$  and  $4\log_4 x = \frac{5 + 9 + 13 + \dots + (4y + 1)}{1 + 3 + 5 + \dots + (2y - 1)}$ , then the value of  $x^2 y$  equals:
- (A) 20 (B) 22  
(C) 24 (D) 28
68. If matrix  $A = [a_{ij}]_{3 \times 3}$ , matrix  $B = [b_{ij}]_{3 \times 3}$ , where  $a_{ij} + a_{ji} = 0$  and  $b_{ij} - b_{ji} = 0 \forall i, j$ , then  $A^4 B^3$  is:
- (A) singular (B) zero matrix  
(C) symmetric (D) skew symmetric
69. Let  $\alpha_n, \beta_n$  be the distinct roots of the equation  $x^2 + (n+1)x + n^2 = 0$ . If  $\sum_{n=2}^{2021} \frac{1}{(\alpha_n + 1)(\beta_n + 1)}$  can be expressed in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are positive integer, the value of  $(b - a)$ , is:
- (A) 1 (B) 3  
(C) 6 (D) 9
70. Let  $S$  denote the sum of an infinite geometric sequence  $S > 0$ . If the second terms of this sequence is 1. Then the minimum possible value of  $S$ , is:
- (A) 2 (B) 4  
(C) 6 (D) 8

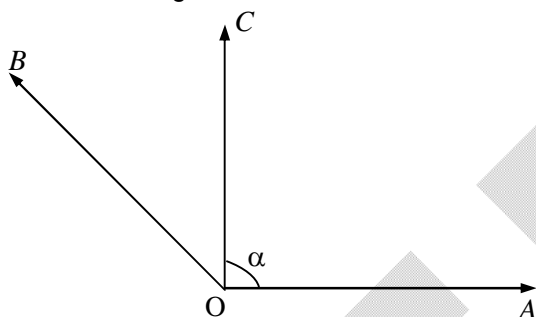
## SECTION – B

### (Numerical Answer Type)

This section contains 05 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. Let  $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$  where  $a_0, a_1, a_2, \dots, a_{20}$  are constant, then find the value of  $\frac{a_7}{a_{13}}$ .

72. If complex number  $z$  satisfies  $(z - \bar{z})^2 = 12|z|^2 - 4$  then find the maximum value of  $3\sqrt{3}\operatorname{Re}(z) + 8\operatorname{Im}(z)$ .
73. If number of words which can be formed using all the letters of the word "MIXIMA" in which no two alike letters are together is  $(12^m)$ , then find the value of  $m$ .
74. In throwing a dice thrice, getting numbers in order denoted by  $a, b, c$ , satisfying  $a^2 + 4b^2 + 4c^2 - 2ab - 4bc - 2ac = 0$ . If probability such that point  $(a, b, c)$  lies inside the tetrahedron formed by the plane  $x + y + z = 10$  and co-ordinate planes is  $\frac{6}{\lambda}$ , where  $\lambda \in \mathbb{N}$ , then  $\lambda$  is:
75. Vector  $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$  are on the same plane,  $|\overrightarrow{OA}| = 1, |\overrightarrow{OB}| = 1, |\overrightarrow{OC}| = \sqrt{2}$ , the relative position is shown in the figure.



If  $\tan \alpha = 7$  and  $\angle BOC = \frac{\pi}{4}$  and  $\overrightarrow{OC} = m\overrightarrow{OA} + n\overrightarrow{OB}$ , ( $m, n \in \mathbb{R}$ ), find the value of  $(m + n)$ .