CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)
FULL SYLLABUS
02-02-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE ALL STAR BATCH (SCORE-II)

ANSWER KEY PAPER (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,C	A,C	A,B,C,D	A,B,C	B,D	A,B		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	B	A	B	D				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	600.00	2.00	10.00	0.14	2.66 to 2.67	2.18 to 2.19	9.55 to 9.65	28.00

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	B,C,D	A,C,D	A,B	A,B,C	A,C,D	B,C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	D	B	A	D				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	1.50 to 1.51	7.68 to 7.71	96.00	9.00	4.00	80.00	14.00	5.75

PART-3 : MATHEMATICS

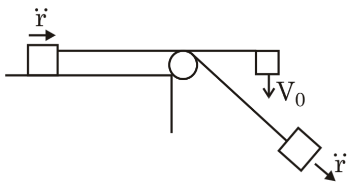
SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,B,C	A,C,D	A,B,C,D	A,B,C,D	A,B,C	A,B,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	B	C	D	A				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	582.00	5120.00	9.00	3.00	2.00	5.00	84.00	3.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. Ans (A,C)



$$-I_m\hat{r} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$I_m = \ddot{r}$$

$$V_0\ell = r^2\dot{\theta}$$

$$2\ddot{r} = r\dot{\theta}^2 = r\frac{V_0^2\ell^2}{r4^3}$$

$$2v_r\frac{dv_r}{dr} = \frac{V_0^2\ell^2}{r^3}$$

$$2\int_0^{v_r} v_r dv_r = V_0^2\ell^2\int_\ell^{2\ell}\frac{dr}{r^3}$$

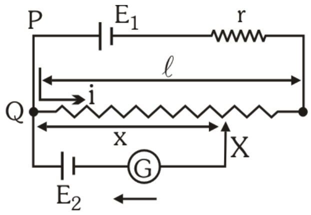
$$v_r^2 = \frac{V_0^2\ell^2}{2}\left(\frac{1}{\ell^2} - \frac{1}{4\ell^2}\right) = \frac{3V_0^2}{8}$$

$$v_r = \frac{\sqrt{3}V_0}{2\sqrt{2}}$$

$$\frac{1}{2}mV_r^2 + \frac{1}{2}mV_A^2 = \frac{1}{2}mV_0^2$$

$$V_A = \sqrt{\frac{5}{8}}V_0$$

4. **Ans (A,B,C)**



Let us assume, the resistance of external resistor is R and its length is ℓ_0; $i = \frac{E}{R+r}$

When current flows through galvanometer

$$V_{QX} < E_2 \text{ or } \frac{E_1}{(R+r)} \times \frac{Rx}{\ell} < E_2$$

For the above condition to hold, (A, B or C) may be correct.

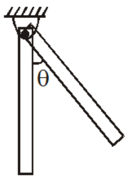
5. **Ans (B,D)**

$E \propto m$ and $r \propto \frac{1}{m}$

PART-1 : PHYSICS

SECTION-I (ii)

8. **Ans (A)**

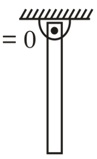


$$mg\frac{L}{2}(1-\cos\theta) = 0 + \frac{1}{2} \times \frac{mL}{3} \times \frac{3g}{L}$$

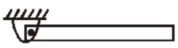
$1 - \cos\theta = 1$

$\cos\theta = 90°$

$F_x = 0$



$$F_y = mg + m \times \frac{3g}{2} = \frac{5mg}{2}$$



$\omega = 0$

$F_x = 0$

$$\frac{mL^2}{3} = \frac{mgL}{2}$$

$$mg - F_y = m \times \frac{L}{2} \times \frac{3g}{2L} \Rightarrow F_y = \frac{mg}{4}$$

10. **Ans (D)**

(P) : $\Delta x_1 = CP_1\frac{d}{D} = 0.3\mu m = \frac{3}{4}\lambda$

$\Delta x_2 = CP_2\frac{d}{D} = 1.2\mu m = 3\lambda$

$\Delta x_C = 0$

$\Rightarrow I_C = 4I_0, I_{P_1} = 2I_0, I_{P_2} = 4I_0$

(Q) : $\Delta x_C = 0 \Rightarrow I_C = 4I_0$

$\Delta x_1 = \mu CP_1\frac{d}{D} = 0.4\mu m = \lambda$

$I_{P_1} = 4I_0$

$\Delta x_2 = \mu CP_2\frac{d}{D} = 1.6\mu m = 4\lambda$

$I_{P_2} = 4I_0$

(R) : $\Delta x_C = (\mu - 1)t = 0.4\ \mu m = \lambda$

$I_C = 4I_0$

$\Delta x_1 = 0.3\ \mu m - 0.4\ \mu m = -0.1\ \mu m = \frac{-\lambda}{4}$

$\Rightarrow I_{P_1} = 2I_0$

$\Delta x_2 = 0.4\ \mu m - 0.4\ \mu m = 0$

$\Rightarrow I_{P_2} = 4I_0$

(S) : $\Delta x_C = d\sin\theta = d\theta = 0.1\ \mu m = \frac{\lambda}{4}$

$\Rightarrow I_C = 2I_0$

$\Delta x_1 = 0.3\ \mu m - 0.1\ \mu m = 0.2\ \mu m = \frac{\lambda}{2}$

$\Rightarrow I_{P_1} = 0$

$\Delta x_2 = 0.4\ \mu m - 0.1\ \mu m = 0.3\ \mu m = \frac{3\lambda}{4}$

$\Rightarrow I_{P_2} = 3I_0$

PART-1 : PHYSICS

SECTION-II

2. **Ans (2.00)**

Energy incident on cesium metal $= \frac{1240}{400} = 3.1$eV

$K_{mx} = 3.1 - 1.9 = 1.2$ eV

(k_{mx} = maximum kinetic energy of photoelectron from the metal cesium)

So kinetic energy of photoelectron

$0 \le k \le k_{mx}$.

(K. E) gain by E. F.

$$K = \left(\frac{\sigma}{\epsilon_0}\right).d\ \text{eV} = 1.2\,\text{eV}$$

$$\therefore \frac{K_{max}}{K_{min}} = \frac{1.2 + 1.2}{0 + 1.2} = 2$$

4. Ans (0.14)

$$\frac{mg}{4\times\pi R^2}=\frac{2S}{x}$$

$$x=\frac{8\pi R^2 S}{mg}$$

$$\pi R^2 x=\frac{4}{3}\pi r^3$$

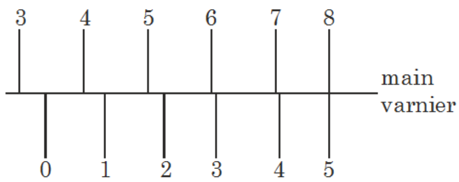
$$\pi R^2=\frac{4\pi r^3}{3x}$$

$$x=\frac{8S}{mg}\times\frac{4\pi r^3}{3x}$$

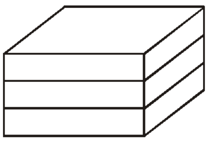
$$x=\sqrt{\frac{32\pi r^3 S}{3mg}}=\sqrt{\frac{32\pi\times10^{-9}\times0.465}{3\times80\times10^{-4}\times10}}$$

$$=10^{-4}\sqrt{\frac{32\pi\times0.465}{24}}=0.14$$

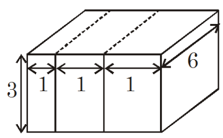
5. Ans (2.66 to 2.67)



6. Ans (2.18 to 2.19)



$$C_1=\frac{\epsilon_0\times18}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}}=\frac{\epsilon_0\times18}{\frac{6+3+2}{6}}=\epsilon_0\times\frac{108}{11}$$

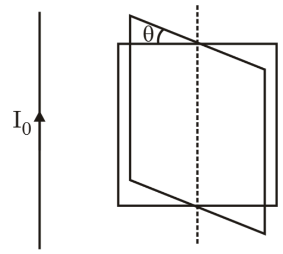


$$C_2=\frac{1\times\epsilon_0\times1\times6}{3}+\frac{2\times\epsilon_0\times1\times6}{3}+\frac{3\times\epsilon_0\times1\times6}{3}$$

$$=\epsilon_0\times12$$

$$\Rightarrow\frac{C_2-C_1}{\epsilon_0}=12\left[1-\frac{9}{11}\right]=\frac{24}{11}=2.18$$

7. Ans (9.55 to 9.65)



$$\tau=\frac{\mu_0 i_0}{2\pi}\frac{i_1 a}{\left(ha-\frac{a}{2}+\frac{a}{2}(1-\cos\theta)\right)}\frac{a}{2}\sin\theta$$

$$+\frac{\mu_0 i_0}{2\pi}\frac{i_1 a}{\left[ha+\frac{a}{2}\cos\alpha\right]}\frac{a}{2}\sin\theta$$

$$W=\int_0^\pi\left(\frac{\mu_0 i_0 i_1 a^2\sin\theta}{4\pi\left[ha-\frac{a}{2}\cos\theta\right]}+\frac{\mu_0 i_0 i_1 a^2\sin\theta}{4\pi\left[ha+\frac{a}{2}\cos\theta\right]}\right)d\theta$$

$$=\frac{\mu_0 i_1 i_0 a^2}{4\pi}\int_0^\pi\frac{\sin\theta\,d\theta}{ha-\frac{a}{2}\cos\theta}+\int_0^\pi\frac{\sin\theta\,d\theta}{ha+\frac{a}{2}\cos\theta}$$

$$\left[\frac{10^{-7}\times4.5\times64}{100\times100}\right]\left[\int_0^\pi\frac{\sin\theta\,d\theta}{1.2-0.04\cos\theta}+\int_0^\pi\frac{\sin\theta\,d\theta}{1.2+0.04\cos\theta}\right]$$

$$\left[10^{-11}\times4.5\times64\right]\left[\int\frac{dt}{t\times0.04}+\int-\frac{dt}{0.04t}\right]$$

[let $(1.2-0.04\cos\theta=t)$]

$$2.88\times10^{-9}\left[\frac{1}{0.04}\ell n[1.2-0.04\cos\theta]_0^\pi-\frac{1}{0.04}\ell n[1.2+0.04\cos\theta]_0^\pi\right]$$

$$=\frac{2.88\times10^{-9}}{0.04}\left[\ell n\left(\frac{1.2+0.04}{1.2-0.04}\right)-\ell n\left(\frac{1.2-0.04}{1.2+0.04}\right)\right]=9.60355$$

8. Ans (28.00)

$$\vec{P}=Q\left[(-1.2-1.4)\hat{i}+(1.1+1.3)\hat{j}\right]C-mm$$

$$=4\times10^{-3}\left[-2.6\hat{i}+2.4\hat{j}\right]C-m$$

$$\therefore\ T=\vec{P}\times\vec{E}=4\times10^{-3}\left(-2.6\hat{i}+2.4\hat{j}\right)\times\left(2500\hat{i}-5000\hat{j}\right)$$

$$=28\hat{k}\,(N-m)$$

PART-2 : CHEMISTRY

SECTION-I (i)

5. **Ans (A,C,D)**

$\pi = iCRT$

$i = \frac{\pi}{CRT} = \frac{96}{2 \times 0.08 \times 300} = 2$

$i = 1 + (n - 1)\alpha$

$\Rightarrow$ n = 2, hence, formula of compound is

$[Co(NH_3)_4 Cl_2] Cl$

(A) $[Co(NH_3)_4 Cl_2]Cl$	+ $AgNO_3$	$\rightarrow$ AgCl	+ $[Co(NH_3)_4Cl_2]^+$	+ NO_3^-
2M, 1litre	1M, 2litre			
0	0	2mole		
		= 287 gm	2mole	2 mole

(B) Osmotic pressure of final solution which contains 2mol of $[Co(NH_3)_4Cl_2]NO_3$

$\pi = i\,CRT = 2 \times \frac{2}{3} \times 0.08 \times 300 = 32$ atm

6. **Ans (B,C,D)**

Theory based.

PART-2 : CHEMISTRY

SECTION-I (ii)

10. **Ans (D)**

Theoretical.

PART-2 : CHEMISTRY

SECTION-II

6. **Ans (80.00)**

For adiabatic process, q=0

$\therefore \Delta U = w = -P_{ext}(V_2 - V_1)$

$= -6 \times (25 - 40) = +90L - bar$

Now, $\Delta H = \Delta U + \Delta(PV)$

$= 90 + (150 - 160) = 80L - bar$

7. **Ans (14.00)**

$n_{CaCO_3} + n_{BaCO_3} = n_{CO_2} = \frac{168}{22400} = 7.5 \times 10^{-3}$

$2BaCO_3 \rightarrow 2BaCrO_4 \xrightarrow{H^+} BaCr_2O_7 \xrightarrow{KI} I_2 + Na_2S_2O_3$

eq. of $Na_2S_2O_3$ = eq. ofI_2 = eqof$BaCr_2O_7$

$= \frac{20 \times 10^{-3} \times 0.05 \times 100}{10}$

$= 1 \times 10^{-2}$

Moles of $BaCr_2O_7 = \frac{1}{6} \times 10^{-2}$

Moles of $BaCrO_4 = \frac{2}{6}(1 \times 10^{-2})$

Moles of $BaCO_3 = \frac{1}{3} \times 10^{-2} = 3.33 \times 10^{-3}$

Weight of $BaCO_3$ = 0.656g

From equation (1) and (2) we get,

$n_{CaCO_3} = 4.17 \times 10^{-3}$

Weight of $CaCO_3 = 100 \times 4.17 \times 10^{-3} = 0.417$g

Weight of CaO = 1.249 − 0.656 − 0.417 = 0.176

Percentage of CaO $= \frac{0.176}{1.249} \times 100 = 14.09\%$

8. **Ans (5.75)**

$$1.0\times10^4\times\frac{1}{100}=e^{\frac{-Ea}{RT}}=e^{\frac{-Ea}{\frac{25}{3}\times300}}$$

$E_a = 34500\ \text{J mol}^{-1}$

Now,

$k = Ae^{-(Ea/RT)}$

$$\frac{dk}{dT}=k.\frac{E_a}{RT^2}$$

$$k=\frac{0.2\times\frac{25}{3}\times345\times345}{34500}=5.75\ s^{-1},$$

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (A,B,C)**

$P = 2^5.3^6.5^4.7^3$

form $(2n+3)\Rightarrow(6+1)(4+1)(3+1)-2$

$=138$

form $(4n+1)\Rightarrow 5$ can be taken any number of times.

while no. of 3 or 7 can be taken even no. of times in total.

$\Rightarrow(4+1)(6+8)=70$

form $(6n+3)\Rightarrow$ atleast one "3 must be selected and no "2" be selected

$=6\times(4+1)(3+1)=120$

form $(4n+3)\Rightarrow(4+1)[8+6]=70$

2. **Ans (A,C,D)**

(A) sum of a continuous & a discontinuous function is discontinuous.

(C) $Z = AB - BA$

$Z^T = B^TA^T - A^TB^T$

$= -BA + AB = Z$

3. **Ans (A,B,C,D)**

A	B	C
3R 3W	2R 3W	4R 1W

(A)

$$\left(\frac{{}^3C_2}{{}^3C_2+{}^3C_2+{}^4C_2}\right)\left(\frac{2}{3}\right)+\left(\frac{{}^2C_2}{{}^3C_2+{}^2C_2+{}^4C_2}\right)(1)+\left(\frac{{}^4C_2}{{}^3C_2+{}^2C_2+{}^4C_2}\right)\cdot\left(\frac{1}{3}\right)$$

$$=\left(\frac{3}{10}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{10}\right)\left(\frac{3}{3}\right)+\left(\frac{6}{10}\right)\left(\frac{1}{3}\right)=\frac{1}{2}$$

(B) Probability that the two drawn balls are

from bag C $=\frac{6}{10}$

and not from bag C $=\frac{4}{10}$

Reg. prob. $=\frac{6}{10}\times\frac{1}{3}+\frac{4}{10}\times\frac{1}{15}=\frac{1}{5}+\frac{2}{25}=\frac{7}{25}$

(C) Two balls, are from Bag B $=\frac{{}^2C_2}{{}^3C_2+{}^2C_2+{}^4C_2}=\frac{1}{10}$

(D) Two balls are from Bag A $=\frac{3}{10}$

4. **Ans (A,B,C,D)**

$$y=\frac{\sin x}{x}$$

$$\frac{dy}{dx}=\frac{x\cos x-\sin x}{x^2}=\frac{\cos(x-\tan x)}{x^2}$$

$\sin x < x < \tan x$

$$\frac{\sin(\sin x)}{\sin x}>\frac{\sin x}{x}>\frac{\sin(\tan x)}{\tan x}$$

$I_1 > I_2 > I_3$

$$\Rightarrow\frac{\sin x}{x}>\frac{1}{x}\left(x-\frac{x^3}{3!}\right)$$

$$I_1>\int_0^{\pi/2}\left(1-\frac{x^2}{6}\right)dx\Rightarrow I_1>\frac{\pi}{2}-\frac{\pi^3}{144}$$

5. **Ans (A,B,C)**

ax + by + c = 0 are concurrent at centre

$\Rightarrow a - 2b + c = 0 \Rightarrow$ centre (1,–2)

$$\frac{b^2}{a} = \frac{2(1)(3)}{1+3} = \frac{3}{2}$$

Auxilary circle : $(x-1)^2 + (y+2)^2 = a^2$

$\Rightarrow x^2 + y^2 - 2x + 4y + 5 - a^2 = 0$

$\Rightarrow \alpha = -1, \beta = -1, 5 - a^2 = -2\alpha - 1$

$5 - a^2 = 1$

$a^2 = 4, b^2 = 3$

Equation of ellipse $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{3} = 1$

$$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

6. **Ans (A,B,D)**

Let P = cosy. cos2y.cos3y....cos999y

Q = siny sin2y sin3y..... sin999y

$2^{999}PQ$ = sin2y sin4y sin6y...... sin1998y

$2^{999}PQ = Q \Rightarrow P = \dfrac{1}{2^{999}}$

$7x = \pi \Rightarrow 3x = \pi - 4x$

sin3x = sin4x

$\Rightarrow 3\sin x - 4\sin^3 x = 4\sin x \cos x \cos 2x$

$3 - 4\sin^2 x = 4\cos x(2\cos^2 x - 1)$

$\Rightarrow 4\cos^2 x - 1 = 8\cos^3 x - 4\cos x$

$\Rightarrow 8\cos^3 x - 4\cos^2 x - 4\cos x = -1$

similarly $\tan 3x = \tan(\pi - 4x)$

$\tan 3x = -\tan 4x$

$$\Rightarrow \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} + \frac{2\tan 2x}{1 - \tan^2 2x} = 0$$

$\tan^6 x - 21\tan^4 x + 35\tan^2 x - 7 = 0$

$\Rightarrow \tan^2 x + \tan^2 2x + \tan^2 3x = 21$

PART-3 : MATHEMATICS

SECTION-I (ii)

7. **Ans (B)**

(I) $S = \sum_{k=1}^{n} (-1)^{k-1}\, {}^nC_k \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k}\right)$

$$= \sum_{k=1}^{n} \left((-1)^{k-1}\, {}^nC_k \sum_{r=1}^{k} \int_0^1 x^{r-1} x \right)$$

$$= \sum_{k=1}^{n} (-1)^{k-1}\, {}^nC_k \int_0^1 \frac{1-x^k}{1-x} dx$$

$$= -\int_0^1 \sum_{k=0}^{n} (-1)^k\, {}^nC_k \frac{(1-x^k)}{(1-x)} dx$$

$$= -\int_0^1 -\frac{(1-x)^n}{1-x} dx$$

$$= \int_0^1 (1-x)^{n-1} dx = \int_0^1 x^{n-1} dx = \frac{1}{n}$$

(II) $S = \sum_{r=0}^{9} \dfrac{{}^9C_r(-1)^r}{r+8}$

$$x^7(1-x)^9 = \sum_{r+8}^{9} {}^9C_r(-1)^r x^{r+7}$$

$$\int_0^1 x^7(1-x)^9 dx = \sum_{r=0}^{9} \frac{{}^9C_r(-1)^r}{r+8} = \frac{7!9!}{17!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10}$$

$$= \frac{1}{17 \times 16 \times 13 \times 11 \times 5}$$

(III) $N = 43^{43^{43}}$

$43^{43} = (40+3)^{43} = 4k + 3$

$N = 43^{4k+3} = (40+3)^{4k+3} = 40\lambda + 81^k.27$

Remainder = 27

(IV) $S_n = \sum_{k=0}^{n} \dfrac{{}^{n+k}C_n}{2^k}$

= coefficient of x^n in

$$\left((1+x)^n + \frac{(1+x)^{n+1}}{2} + \frac{(1+x)^{n+2}}{2^2} + \ldots + \frac{(1+x)^{n+n}}{2^n}\right)$$

$S_n = 2^n \Rightarrow S_{12} = 4096$

$\dfrac{S_{12}}{40} \Rightarrow$ Remainder = 16

8. **Ans (C)**

(I) $f(x_1) > f(x_2) \Rightarrow N = {}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165 \Rightarrow 12$

(II) $f(x_1) \le f(x_2) \Rightarrow N = {}^{8-1}C_{4-1} = {}^{7}C_3 = 35 \Rightarrow 8$

(III) $N = {}^8C_4 \times 4! - {}^4C_1.{}^7C_3 3! + {}^4C_2.{}^6C_2 \times 2! - {}^4C_3.{}^5C_1 + 1 = 1001 \Rightarrow 2$

(IV) $N = {}^4C_2 \times [5 \times 5 - 4] = 126 \Rightarrow 9$

9. **Ans (D)**

(I) Let $g(x) = f(x)\, e^{\cos x}$

$\because$ g(x) is a decreasing function

$\therefore g(x) \le g(0) = 0$ i.e. $f(x)\, e^{\cos x} \le 0$ but $f(x) \ge 0$, $e^{\cos x} > 0 \Rightarrow f(x) = 0$

$\therefore f\left(\frac{3\pi}{2}\right) = 0$

(II) $(a-3)x^2 + 12x + 6 + a - (\sin^{-1}\sin 100 + \cos^{-1}\cos 100) < 0$

$\Rightarrow a < 3, D < 0$

$144 - 4(a+6)(a-3) < 0 \Rightarrow a < -9$ or $a > 6$.

(III) $P\left(\frac{B}{A \cup B^C}\right) = \frac{P(A \cap B)}{P(A \cup B^C)} = \frac{1}{5}$

(IV) $\cos^{-1}(4x^3 - 3x) - \frac{\pi}{2} \geqslant 0$

$\Rightarrow -1 \leqslant 4x^3 - 3x \leqslant 0$ i.e.

$x \in \left[-1, -\frac{\sqrt{3}}{2}\right] \cup \left[0, \frac{\sqrt{3}}{2}\right]$

also $\frac{(2x+3)!}{\sqrt{x+2}}$ is defined $\forall\ x > -2$ and $(2x+3) \in W$

$x = -1, 0, \frac{1}{2}$

10. **Ans (A)**

(I) Prob. $= \frac{4^4 - 3^4}{6^4}$

(II) Prob. $= \frac{4^4 - 3^4 - 3^4 + 2^4}{6^4}$

(III) Prob. $= \frac{{}^{8-1}C_{4-1}}{6^4} = \frac{35}{6^4}$

(IV) Prob.

$= \frac{{}^{16-1}C_{4-1} - {}^4C_1.{}^{10-1}C_{4-1} + {}^4C_2.{}^{4-1}C_{4-1}}{6^4}$

$= \frac{125}{6^4}$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (582.00)**

$z^{1997} = 1$

$Z = \frac{\cos 2k\pi}{1997} + i \sin \frac{2k\pi}{1997}$

Let. $v = 1$, $w = \text{Cis}\theta$

$|1 + \cos\theta + i\sin\theta| \geqslant \sqrt{2 + \sqrt{3}}$

$\left|2\cos\frac{\theta}{2}\left(i\frac{n\theta}{2} + i\sin\frac{\theta}{2}\right)\right| \geqslant \sqrt{2+\sqrt{3}}$

$4\cos^2\frac{\theta}{2} \geqslant 2 + \sqrt{3}$

$\Rightarrow 2(\cos\theta) \geqslant \sqrt{3}$

$\cos\theta \geqslant \frac{\sqrt{3}}{2}$

$-\frac{\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$

$-\frac{\pi}{6} \leqslant \frac{2k\pi}{1997} \leqslant \frac{\pi}{6}$

$-166.41 \le k \le 166.41$

$\Rightarrow$ Req. prob. $= \frac{332}{1996} = \frac{83}{494} = \frac{m}{n}$

$m + n = 582$

2. **Ans (5120.00)**

$P = \frac{1}{20(1+5x)\left(1+\frac{3y}{4x}\right)\left(1+\frac{6z}{5y}\right)\left(1+\frac{18}{z}\right)}$

Let $5x = \alpha$, $\frac{3y}{4x} = \beta$, $\frac{6z}{5y} = \gamma$, $\frac{18}{z} = \delta$

$P = \frac{1}{20(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)}$

$\alpha\beta\gamma\delta = 5 \times \frac{3}{4} \times \frac{6}{5} \times 18 = 81$

$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)$

$= 1 + \Sigma\alpha + \Sigma\alpha\beta + \Sigma\alpha\beta\gamma + \alpha\beta\gamma\delta \ge 256$

$P_{max} = \frac{1}{20 \times 256} \Rightarrow \left(\frac{1}{P}\right)_{min} = 5120$

3. Ans (9.00)

$A + \text{adj}A = A^{-1}$

$\Rightarrow A + |A|A^{-1} = A^{-1} \Rightarrow A = (1 - |A|)A^{-1}$

$|A| = (1 - |A|)^2 . \dfrac{1}{|A|} \Rightarrow |A|^2 = 1 - 2|A| + |A|^2$

$|A| = \dfrac{1}{2}$

$\Rightarrow |A^{-1}| = 2 \Rightarrow |2A^{-1}| = 8 = x$

$\Rightarrow P(\text{adj } P + \text{adj}Q) = Q$

$\Rightarrow P(P^{-1} + Q^{-1}) = Q$

$\Rightarrow I + PQ^{-1} = Q$

$\Rightarrow Q + P = Q^2 \Rightarrow |P + Q| = |Q^2| = 1$

$y = 1$

then $x + y = 9$

4. Ans (3.00)

Let first three points are , orthocentre = (α,β)

(x_1,y_1) (x_2,y_2) and (x_3,y_3)

circumcentre $(0,0)$, centroid

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

2 : 1

O G C

$\alpha = x_1 + x_2 + x_3,\ \beta = y_1 + y_2 + y_3$

centroid of (x_4,y_4), (x_5,y_5) and (x_6,y_6)

$$\gamma = \frac{x_4+x_5+x_6}{3}, \delta = \frac{y_4+y_5+y_6}{3}$$

$\alpha + 3\gamma = 8,\ \beta + 3\delta = 4$

P(h,k)

(α,β) 3 : 1 (γ,δ)

$$h = \frac{3\gamma+\alpha}{4}, k = \frac{3\delta+\beta}{4}$$

$h = 2, k = 1 \Rightarrow h + k = 3$

5. Ans (2.00)

$$f(x) = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

$$\ell_1 = \lim_{n\to\infty}\left[\frac{\frac{n^2+n}{2} - n}{\left(\frac{n(n+1)}{2}\right)^2 - \frac{n^3(n+2)}{4}}\right]$$

$$= \lim_{n\to\infty}\left[\frac{\frac{(n^2-n)}{2}}{\frac{n^2}{4}\left((n+1)^2 - n(n+2)\right)}\right]$$

$$= \lim_{n\to\infty}\left[2\left(\frac{n^2-n}{n^2}\right)\right] = 1$$

$P(x) = -x^3 + x^2 - x + 1,\ P'(x) = -3x^2 + 2x - 1$

$P'(x) < 0,\ \forall\ x \in R$

$$\Rightarrow \ell_2 = \lim_{x\to 1^+} g\left(g\left(1^+\right)\right) = \lim_{x\to 0^+} g(x) = 1$$

$\ell_1 + \ell_2 = 2$

6. Ans (5.00)

$$\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = \left|(\vec{a}\cdot\vec{c})\vec{b} - \left(\vec{b}\cdot\vec{c}\right)\vec{a}\right| = \frac{21}{4}$$

7. Ans (84.00)

$x + y + z < 10,$

$x, y, z > 0$

$\Rightarrow\ x + y + z + p = 10$

No. of integral points = ${}^{10-1}C_{4-1} = {}^{9}C_3$

$$= \frac{9\times8\times7}{6} = 84$$

8. Ans (3.00)

$\sqrt{7}\sqrt{y^2+z^2} + \sqrt{8}\sqrt{z^2+x^2} + 3\sqrt{x^2+y^2} = 12$

$\sqrt{7+8+9}\sqrt{2x^2+2y^2+2z^2} \geqslant 12$

$\sqrt{2x^2+2y^2+2z^2} \geqslant \sqrt{6}$

$x^2 + y^2 + z^2 \geqslant \sqrt{3}$