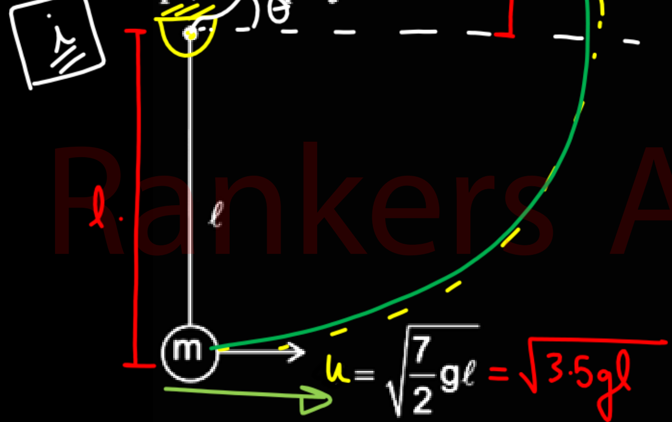
PHYSICS

A small ball is given a velocity $\sqrt{\frac{7}{2}g\ell}$ at the lowest point of vertical circle. If point of suspension is at rest then, find the maximum height reached by the ball from the point of projection as shown in the figure.



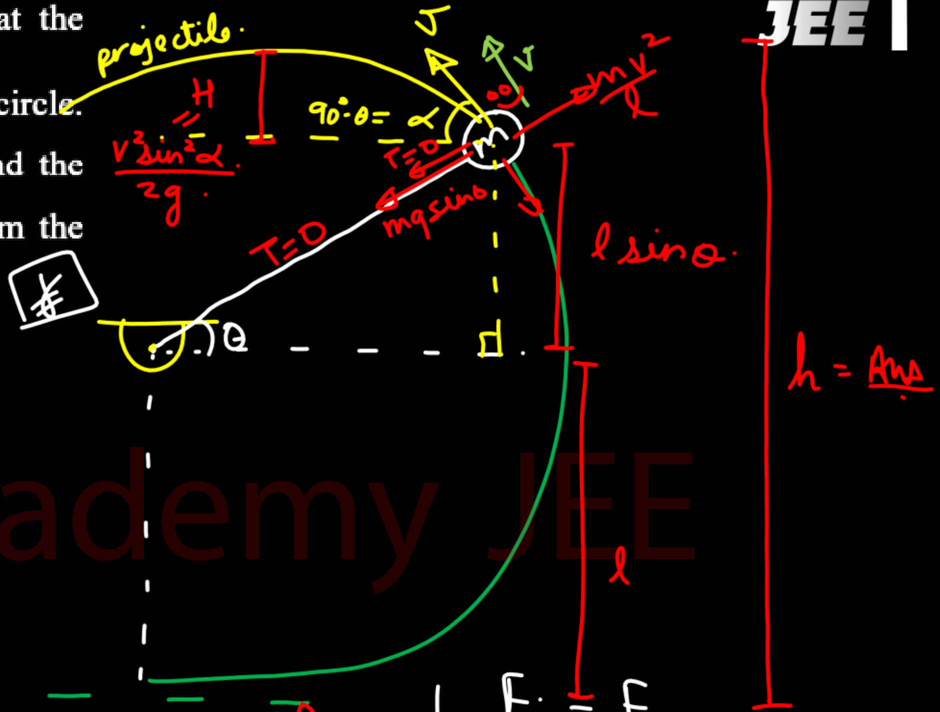
$u = \sqrt{\frac{7}{2}g\ell} = \sqrt{3.5g\ell}$

(A) $\frac{5\ell}{4}$

(B) $\frac{27\ell}{18}$

(C) $\frac{27\ell}{16}$ ✓

(D) $\frac{27\ell}{14}$



$\cancel{m}g\sin\theta + \cancel{T}^{0} = \frac{\cancel{m}v^2}{\ell}$

$\boxed{v = \sqrt{g\ell\sin\theta}}$ — ①

$E_i = E_f$

$\cancel{\frac{1}{2}}\cancel{m}(\sqrt{3.5g\ell})^2 = \cancel{m}g\ell(1+\sin\theta)^{2} + \cancel{\frac{1}{2}}\cancel{m}v^2$

$\boxed{\sqrt{\frac{3}{2}g\ell - 2g\ell\sin\theta} = v}$ — ②

① & ②

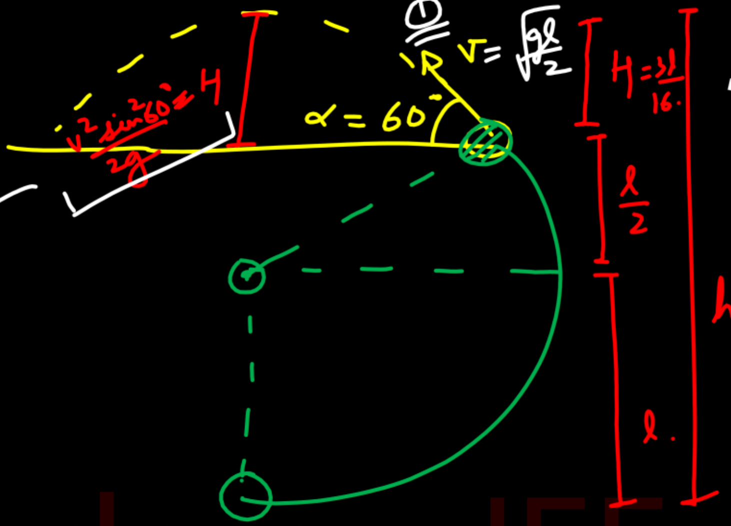
$\cancel{gl}\sin\theta = \frac{3}{2}\cancel{gl} - 2\cancel{gl}\sin\theta$

$3\sin\theta = \frac{3}{2}$

$\boxed{\sin\theta = \frac{1}{2}}$

$\boxed{\theta = 30^\circ}$

$H = \dfrac{\frac{\cancel{g}l}{2}\cdot\frac{3}{4}}{2\cancel{g}} = \frac{3l}{16}$



$\text{Ans} = h = l + \frac{l}{2} + \frac{3l}{16}$

$h = \frac{16}{16}\cdot l + \frac{8}{16}l + \frac{3l}{16}$

$h = \frac{27}{16}l$ Ans

2

A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\boxed{\theta(t) = 5t^2 - 8t}$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t . How much power is delivered by the applied torque, when t = 2 s ?

(A) $72MR^2$

(B) $8MR^2$

(C) $108MR^2$

(D) $60MR^2$ ✓

$I = \frac{1}{2}MR^2$.

$\theta = 5t^2 - 8t$

$\boxed{\omega = \frac{d\theta}{dt} = 10t - 8}$

$\boxed{\alpha = \frac{d\omega}{dt} = 10}$

$\tau = I\alpha = \frac{1}{2}MR^2 \times 10$.

const.

$\boxed{\tau = 5MR^2}$

Tx

$\boxed{P = \vec{F} \cdot \vec{v}} \rightarrow$

Rx

t = 2.

$\boxed{P = \vec{\tau} \cdot \vec{\omega}}$

$P_{t=2} = \tau_{t=2}\, \omega_{t=2}$.

$P_{t=2} = (5MR^2)(20-8) = 60\,MR^2$

Rankers Academy JEE

JEE 1

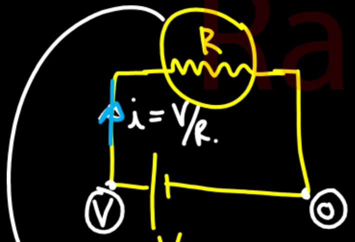
3

'n' identical light bulbs, each designed to draw power of P watts from a certain voltage supply are joined in series and that combination is connected across that supply. The power consumed by one bulb (in watts) will be

~~(A)~~ nP ~~(B)~~ P

(C) P/n (D) P/n^2

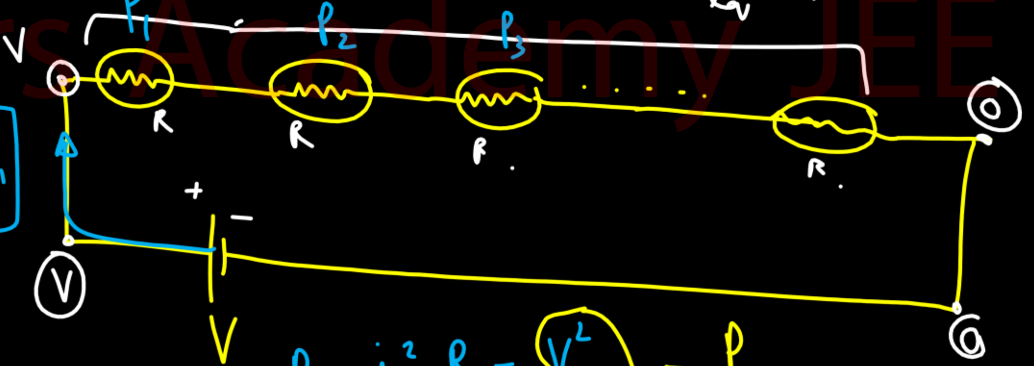
for 1 Bulb



$$\boxed{P = \frac{V^2}{R} = i^2 R}$$

'n' bulbs.

$R_{eq} = nR$



$$\boxed{\frac{V}{nR} = i_1}$$

$$P_1 = i_1^2 \cdot R = \frac{V^2}{n^2 R} = \frac{P}{n^2}$$

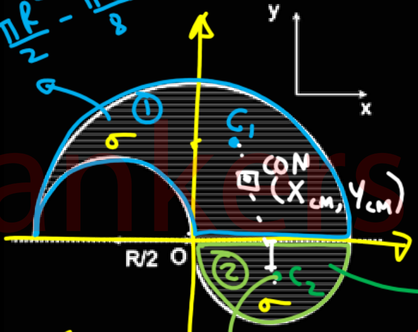
4

σ kg/m²

#JEE1

From a uniform semicircular thin disc of radius R , a semicircular portion of radius R/2 is removed and placed in a new position as shown in figure. Taking O as origin and x and y axis as shown in the figure, centre of mass of new setup is at

$A_1 = \frac{\pi R^2}{2} - \frac{\pi R^2}{8} = \frac{3}{8}\pi R^2$



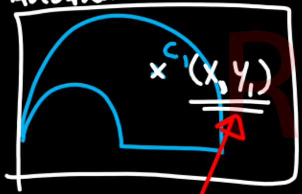
$A_2 = \frac{\pi R^2}{8}$

$\left(\frac{R}{2}, \frac{-4(R/2)}{3\pi}\right)$

(A) $\left(\frac{R}{2}, \frac{3R}{2\pi}\right)$ (B) $\left(\frac{R}{2}, \frac{R}{\pi}\right)$

(C) $\left(\frac{R}{4}, \frac{3R}{2\pi}\right)$ (D) $\left(\frac{R}{4}, \frac{R}{\pi}\right)$

Another Quesⁿ.



$X_1 = ?$

$Y_1 = ?$

$M_1 = \sigma\left(\frac{3}{8}\pi R^2\right) = 3m$

$M_2 = \sigma\left(\frac{\pi R^2}{8}\right) = m$

$X_1 = ?$

$Y_1 = ?$

JEE 1

Ans

$X_{CM} = \frac{M_1 X_1 + M_2 X_2}{M_1 + M_2}$

$X_2 = \frac{R}{2}$

$Y_2 = -\frac{2R}{3\pi}$

$Y_{CM} = \frac{M_1 Y_1 + M_2 Y_2}{M_1 + M_2}$

next page

$Ans_{①} = X_{com} = \frac{(3\cancel{m})\left(\frac{R}{6}\right) + (\cancel{m})\left(\frac{R}{2}\right)}{4\cancel{m}}$

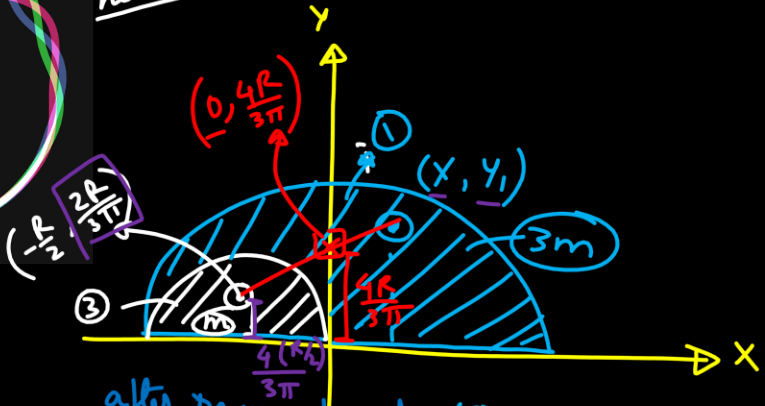
$\boxed{X_{com} = \frac{R}{4}}$

$Ans_{②} = Y_{com} = \frac{(\cancel{3m})\left(\frac{4R}{\cancel{3}\pi}\right) + (\cancel{m})\left(\frac{-2R}{3\pi}\right)}{4\cancel{m}}$

$Y_{COM} = \frac{12R}{12\pi} = \frac{R}{\pi}$

$\boxed{Y_{COM} = \frac{R}{\pi}}$

new ans



$x_1 = ?$

$y_1 = ?$

after reconstruct (1 & 3)

X-coordinate

$$0 = \frac{(3\cancel{m}) \cdot x_1 + (\cancel{m})\left(-\frac{R}{2}\right)}{4\cancel{m}}$$

$$\boxed{x_1 = \frac{R}{6}}$$

Y-coordinate

$$\frac{4R}{3\pi} = \frac{(3\cancel{m})\, y_1 + (\cancel{m})\left(\frac{2R}{3\pi}\right)}{4m}$$

$$\frac{16R}{3\pi} = 3y_1 + \frac{2R}{3\pi}$$

$$\boxed{y_1 = \frac{14R}{9\pi}}$$

5

JEE 1

If E, L, M and G denote the quantities as energy, angular momentum, mass and universal gravitational constant respectively, then the dimensions of A in the formula $A = \boxed{EL^2M^{-5}G^{-2}}$ are:

(A) $[ML^0 T^0]$ (B) $[M^{-1}L^2T^{-2}]$

(C) $[ML^2 T^0]$ (D) $[M^0L^0T^0]$ ✓

$E = mc^2$

$L = mvr$

$F = \frac{Gm_1m_2}{r^2}$

$\boxed{[E] = ML^2T^{-2}}$

$\boxed{[L_1] = ML^2T^{-1}}$

$\boxed{[M] = M^1}$

$[G] = \frac{[F][r^2]}{[m][m]} = \frac{MLT^{-2}.L^2}{M^2}$

$\boxed{[G] = M^{-1}L^3T^{-2}}$

$\frac{[E][L_1]^2}{[M]^5[G]^2} = \frac{\cancel{M}\cancel{L^2}\cancel{T^{-2}} . \cancel{M^2}\cancel{L^4}\cancel{T^{-2}}}{\cancel{M^5} . \cancel{M^{-2}}\cancel{L^6}\cancel{T^{-4}}}$

$= 1 = M^0L^0T^0$

6

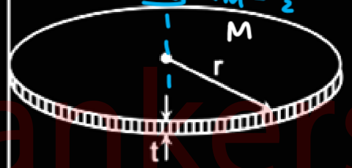
A solid sphere of radius R has moment of inertia I about its diameter. It is melted into a disc of radius r and thickness t. If its moment of inertia about the tangential axis (which is perpendicular to plane of disc) is also equal to I, then value of t and r are respectively.

① $\boxed{I = \frac{2}{5}MR^2}$

M, R.

$\boxed{I = \frac{3}{2}Mr^2}$ — ②

$I_{cm} = \frac{1}{2}Mr^2$



$\boxed{\begin{array}{l} t = ? \\ r = ? \end{array}}$

(A) 5R, $\frac{2R}{15}$

(B) 5R, $\frac{2R}{\sqrt{15}}$

(C) 2R, $\frac{2R}{15}$

(D) 2R, $\frac{2R}{\sqrt{15}}$

① & ②

$\frac{2}{5}\cancel{M}R^2 = \frac{3}{2}\cancel{M}r^2$

$\sqrt{\frac{4}{15}R^2} = r$

$\boxed{r = \frac{2R}{\sqrt{15}}}$

$Vol_i = Vol_f$

$\frac{4}{3}\cancel{\pi}R^3 = \cancel{\pi}r^2 \boxed{t}$

$\frac{\cancel{4}R^{\cancel{3}}}{\cancel{3}} = \frac{\cancel{4}}{\cancel{15}_{5}}\cancel{R^2} \cdot \boxed{t}$

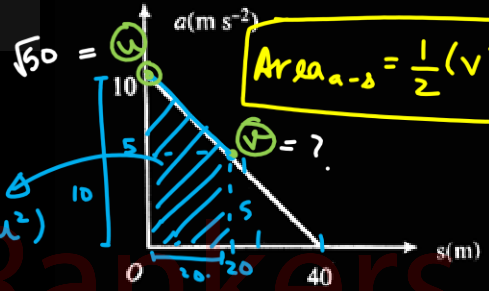
$\boxed{t = 5R}$

7

JEE 1

Referring to a-s diagram as shown in figure. The velocity of the particle when the particle just covers 20 m ($v_0 = \sqrt{50}\ ms^{-1}$). The velocity v is $\sqrt{n}$ m/s. Find n .

$u =$



$u = \sqrt{50}$

$\sqrt{50} = u$

$\boxed{Area_{a-s} = \frac{1}{2}(v^2 - u^2)}$

$\boxed{Area} = \frac{1}{2}(v^2 - u^2)$

$\cancel{\frac{1}{2}}(15)(20) = \cancel{\frac{1}{2}}(v^2 - 50)$

$300 + 50 = v^2$.

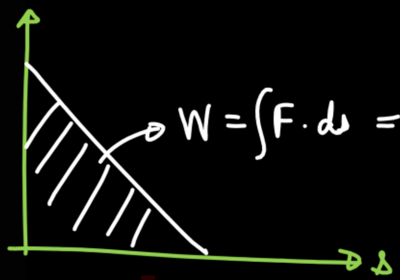
$\boxed{v = \sqrt{350}} = \sqrt{n}$ m/s.

$n = 350$

(A) 70 (B) 50

(C) 300 (D) 350

$F = ma$.



$W = \int F \cdot ds = \Delta KE = \boxed{\frac{1}{2}m(v^2 - u^2) = Area_{F-s}}$

$$\boxed{Area_{a-s} = \int a \cdot ds = \int_u^v v \cdot \frac{dv}{\cancel{ds}} \cdot \cancel{ds} = \frac{1}{2}(v^2 - u^2)}$$

8

JEE 1

A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy and what is the work done by the skater?

(A) angular momentum remains the same while kinetic energy increases and work done is positive. ✓

~~(B)~~ angular momentum remains the same while kinetic energy decreases and work done is negative.

~~(C)~~ both angular momentum and kinetic energy remain the same and work done is zero.

~~(D)~~ angular momentum increases while kinetic energy remains the same and work done may be positive or negative.

R_x

$\boxed{L = I\omega}$

$\boxed{KE = \frac{1}{2} I\omega^2}$

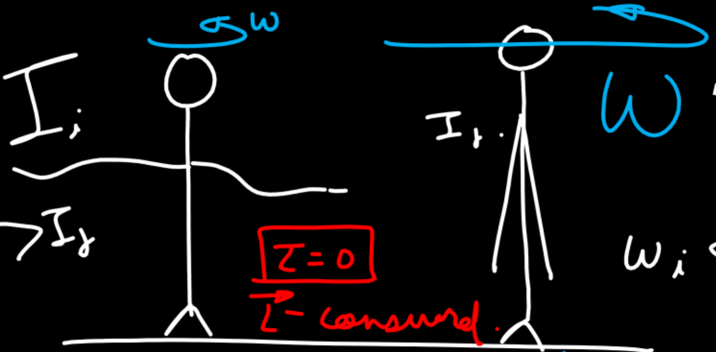
$\boxed{K.E = \frac{L^2}{2I}}$

$\boxed{p = mv}$

$\boxed{KE = \frac{1}{2} mv^2}$

$\boxed{KE = \frac{p^2}{2m}}$

$$\boxed{KE = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{(I^2\omega^2)}{I} = \frac{L^2}{2I}}$$



$I_i > I_f$

$\boxed{\tau = 0}$

L - conserved.

$\omega_i < \omega_f$

$I_i \omega_i = L$

$I_f \omega_f = L$

$\boxed{L_i = L_f}$

$$\boxed{KE_i = \frac{L^2}{2I_i}} < \boxed{KE_f = \frac{L^2}{2I_f}}$$

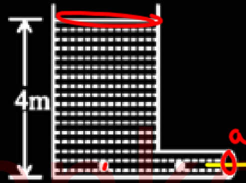
$\boxed{W = \Delta KE} = +ve.$

9

A vent tank of large cross-sectional area has a horizontal pipe 0.12 m in diameter at the bottom. This holds a liquid whose density is 1500 kg/m^3 to a height of 4.0 m . Assume the liquid is an ideal fluid in laminar flow. In figure, the velocity with which fluid flows out is :-

$A >> a$

TORICELLI

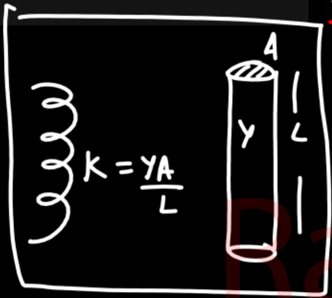


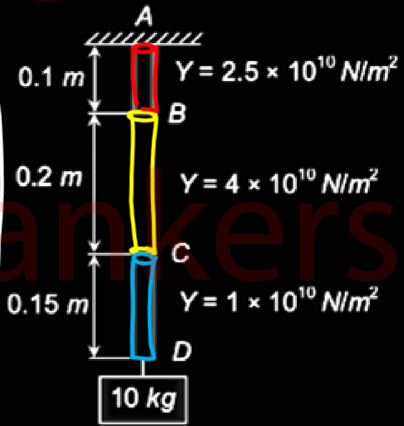
$V_{efflux} = \sqrt{2gh} = \sqrt{20 \times 4} = \sqrt{80} = 4\sqrt{5}$

(A) $2\sqrt{5}$ m/s (B) $\sqrt{5}$ m/s

(C) $4\sqrt{5}$ m/s (D) $\sqrt{10}$ m/s

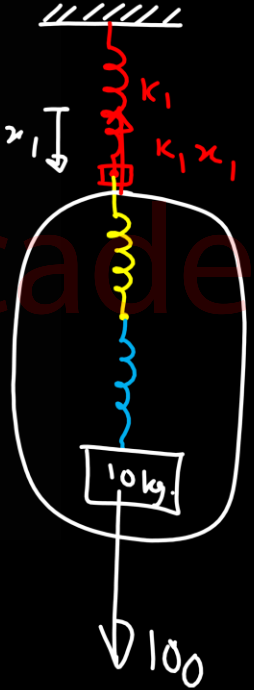
A light rod with uniform cross-section of $10^{-4}\ m^2$ is shown in the adjoining figure. The rod consists of three different materials whose lengths are $0.1\ m, 0.2\ m$ and $0.15\ m$ respectively and whose Young's modulii are $2.5 \times 10^{10}\ N/m^2, 4 \times 10^{10}\ N/m^2$ and $1 \times 10^{10}\ N/m^2$ respectively. The displacement of point B will be





(A) 24×10^{-6} m
(B) 9×10^{-6} m
(C) 4×10^{-6} m
(D) 1×10^{-6} m

=



$K_1 x_1 = 100$

$\frac{Y_1 A}{L_1} \cdot \boxed{x_1} = 100$

$\frac{\frac{\cancel{5}}{2} \times 10^{10} \cdot \times 10^{-4}}{\frac{1}{\cancel{10}}} \boxed{x_1} = \frac{2}{\cancel{100}}$

$\boxed{x_1 = 4 \times 10^{-6}}$

If the radius of the earth were to shrink by one per cent, its mass remaining the same, the value of g on the earth's surface would

(A) increase by 0.5%

(B) increase by 2%

~~(C) decrease by 0.5%~~

~~(D) decrease by 2%~~

R → reduce by 1%

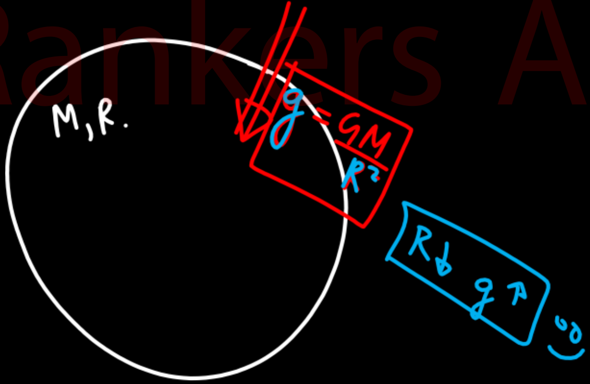
error trick

$g = \underset{\text{const.}}{GM} \cdot R^{-2}$

$\left|\frac{\Delta g}{g}\right| \times 100 = 2 \left|\frac{\Delta R}{R}\right| \times 100$

% error of R.

$\underbrace{\left|\frac{\Delta g}{g}\right| \times 100}_{\text{\% error g.}} = 2(1\%) = 2\%$



12

Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?

~~(A)~~ The one with the farthest range.

(B) The one which reaches maximum height.

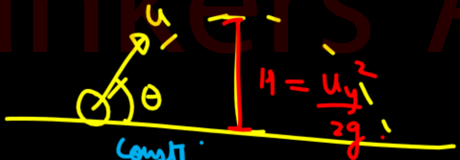
~~(C)~~ The one with the greatest initial velocity.

~~(D)~~ The one leaving the bat at 45° with respect to the ground.

when if: $\boxed{U_y \to \text{max.}}$

$H \to$ max

$T \to$ max.



const.

$T = \frac{2u}{g} \sin\theta$

90°

1

max

max.

$\boxed{H = \frac{U_y^2}{2g}}$

max

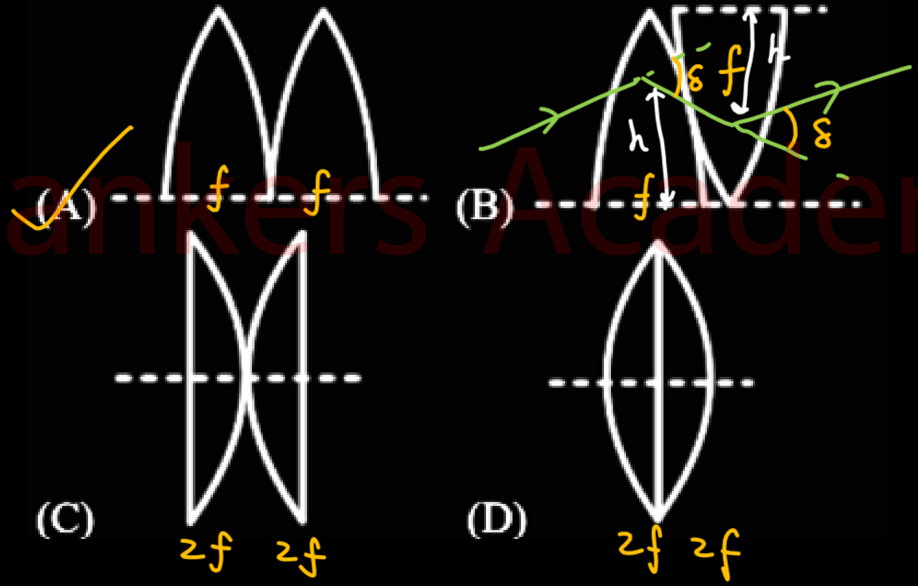
$\boxed{T = \frac{2U_y}{g}}$

max.

max

13

A convex lens is cut into two parts in different ways that are arranged in four manners, as shown. Which arrangement will give maximum optical power?



$P_A = \frac{1}{f} + \frac{1}{f} = \frac{2}{f}$ Maximum

$P_B = 0 \quad f_B = \infty$

$\delta_{net} = \frac{h}{f} + \frac{-h}{f} = 0$

$P_C = P_D = \frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$

14

Statement - 1: At thermal equilibrium between the He gas and H_2 gases both have equal $\boxed{\text{average translational KE.}}$

$\boxed{\text{Statement - 2 :}}$ Molar heat capacity of CO_2 is higher at higher temperatures.

(A) Both statements are true.

(B) statement - 1 is true and statement - 2 is false

(C) statement -2 is true and statement -1 is false

(D) Both statements are false

$K_{trans} = \frac{f_{trans}}{2} nRT = \frac{3}{2} nRT$

transl^n $f_{He} = f_{H_2} = 3$

same.

$O = C = O$

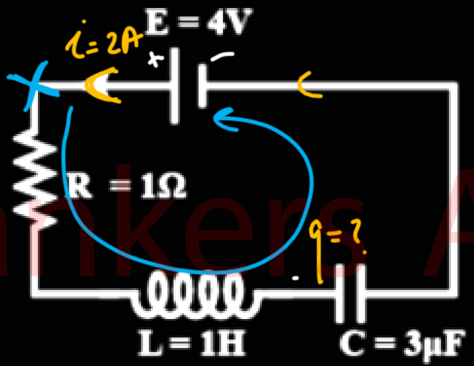
as T increases

vibration degree ↑

C ↑

15

The current in the gives circuit is increasing with a rate of 4amp/s . the charge on the capacitor at an instant when the current in the circuit is 2 amp will be :



(A) 4μC (B) 5μC

(C) 6μC (D) None of these

By KVL

$$+iR + L\frac{di}{dt} + \frac{q}{c} - \mathcal{E} = 0$$

$$iR + L\frac{di}{dt} + \frac{q}{c} = \mathcal{E}$$

$$2\times 1 + 1\times(\cancel{+}4) + \frac{q}{3\mu F} = \cancel{4}$$

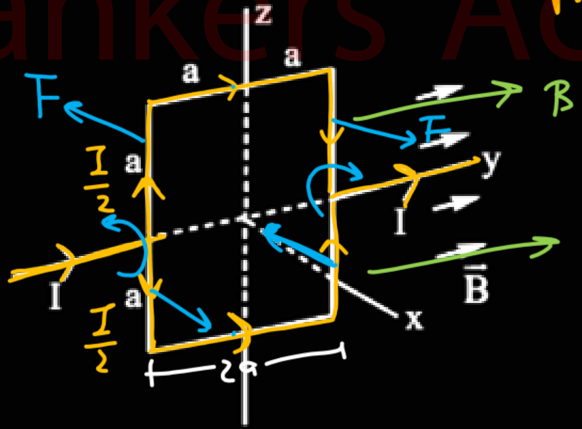
$$q = -6\mu C$$

16

Current I in a long wire along y-axis is passed through a square metal frame of side 2a oriented in the y – z plane as shown. The linear mass density of the frame is λ. A uniform magnetic field B is now switched on along y-axis. Then the instantaneous angular acceleration of the frame will be

(A) $\frac{4IB}{\lambda a}$ (B) $\frac{12IB}{\lambda a}$

(C) $\frac{4IB}{3\lambda a}$ ~~(D)~~ zero



M#1

Due to symmetry $\tau_{net} = 0$

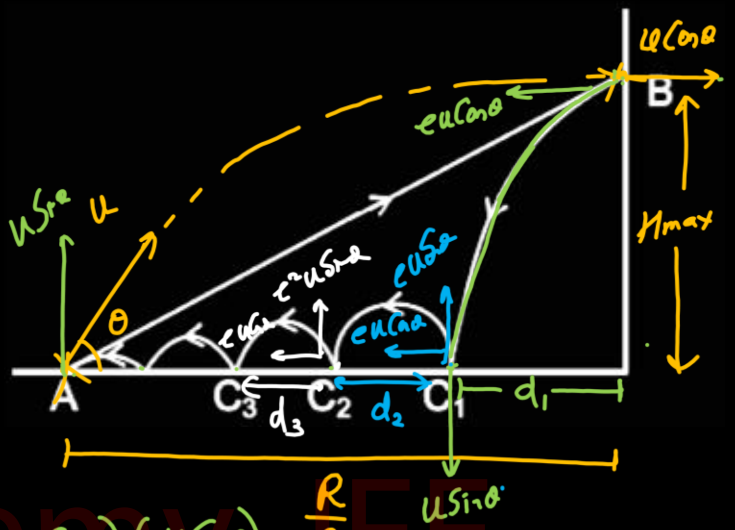
$\Rightarrow \alpha = 0$

M#2 $\vec{m}_{net} = -i(2a^2)\hat{i} + i(2a^2)\hat{i}$

$\vec{m}_{net} = \vec{0} \Rightarrow \vec{\tau} = \vec{0}$

17

A ball was projected from point A and it hits a vertical wall at point B horizontally. From point B the ball goes to points $C_1, C_2, \ldots$, and finally gets back to point A. The coefficient of restitution is 'e' at any point. Find the possible value of e, if the ball does not bounce up at all when it gets back to point A. $\boxed{v_y = 0}$



(A) $e = \sqrt{2} - 1$

(B) $e = \sqrt{3} - 1$

(C) $e = \frac{\sqrt{3}-1}{2}$

(D) $e = \frac{1}{\sqrt{2}}$

$$d_1 = (eu\cos\theta)\frac{T}{2} = (eu\cos\theta)\left(\frac{u\sin\theta}{g}\right)$$

$$= e\left[\frac{u^2\sin\theta\cos\theta}{g}\right] \quad \text{— (1)}$$

$$d_2 = v_{Horiz} \times T'$$

$$= (eu\cos\theta) \times \frac{2(eu\sin\theta)}{g} = 2e^2\left[\frac{u^2\sin\theta\cos\theta}{g}\right] \quad \text{— (2)}$$

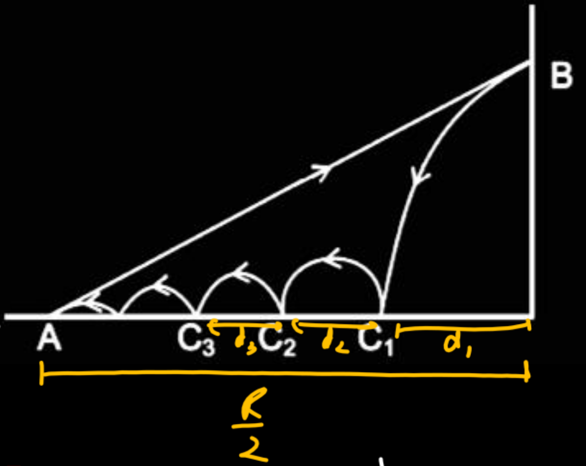
$$d_3 = (eu\cos\theta)\,T''$$

$$= (eu\cos\theta)\left[\frac{2e^2u\sin\theta}{g}\right]$$

$$= \frac{2e^3u^2\sin\theta\cos\theta}{g} \quad \text{— (3)}$$

17

$$\frac{R}{2} = d_1 + d_2 + d_3 + \ldots$$



$$\frac{u^2 \sin\theta \cos\theta}{g} = \left[e + 2e^2 + 2e^3 + \ldots\right]\frac{u^2 \sin\theta \cos\theta}{g}$$

$$1 = e + 2e^2\left[1 + e + e^2 + \ldots\right]$$

$$1 = e + 2e^2\left(\frac{1}{1-e}\right)$$

$$\Rightarrow (1-e)^2 = 2e^2$$

$$\rightarrow 1 + e^2 - 2e = 2e^2$$

$$e^2 + 2e - 1 = 0$$

$$e = \frac{-2 + \sqrt{4+4}}{2}$$

$$= \sqrt{2} - 1$$

(A) $e = \sqrt{2} - 1$ ✓

(B) $e = \sqrt{3} - 1$

(C) $e = \frac{\sqrt{3}-1}{2}$

(D) $e = \frac{1}{\sqrt{2}}$

18

JEE 1

When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet, if the central fringe has to shift to the position occupied by 20th bright fringe

$= t_1$

$t_2 = ?$

(A) 3.8 mm (B) 1.6 mm

(C) 7.6 mm (D) 3.2 mm

$\Delta = N\beta$

$(\mu-1)\frac{tD}{d} = N\frac{\lambda D}{d}$

$\boxed{(\mu-1)t = N\lambda}$

$(\mu-1)t_1 = 30\lambda$ — (1)

$(\mu-1)t_2 = 20\lambda$ — (2)

$\frac{t_2}{t_1} = \frac{20}{30} \Rightarrow t_2 = \frac{2}{3} \times 4.8$

$= 3.2\,mm$

19

A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

(A) 4.4 MeV

(B) 5.4 MeV

(C) 5.6 MeV

(D) 6.5 MeV

Rest

$A \rightarrow (A-4) + \alpha$

u_1, K_1 ; u_2, K_2

Energy Conservation

$Q = K_1 + K_2$ — ①

Cons. of Linear Momentum

$m_1 u_1 = m_2 u_2$

$\sqrt{2m_1K_1} = \sqrt{2m_2K_2}$

$(A-4)K_1 = 4K_2$ — ②

$Q = \frac{4}{A-4}K_2 + K_2$

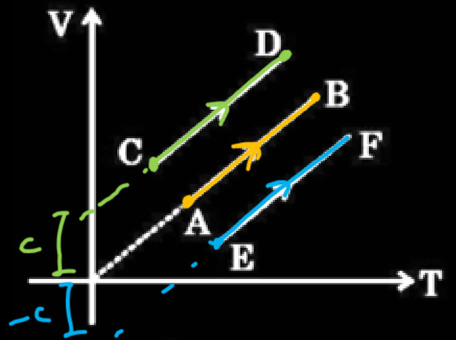
$\boxed{K_2 = \left(\frac{A-4}{A}\right)Q}$ *

$K_2 = \frac{216}{220} \times 5.5 = 5.4\,\text{MeV}$

20

JEE 1

The V-T graph shows some process for an ideal gas. The INCORRECT statement is:



(A) In process CD, pressure continuously increases.

(B) In process EF, pressure continuously decreases.

(C) Pressure remains constant in all process AB, CD and EF

(D) Internal energy of gas increases in all three processes.

$U \propto T \quad T\uparrow \quad U\uparrow$

AB: $V \propto T \Rightarrow V = kT \Rightarrow p = \text{const}$

CD: $V \propto T \Rightarrow V = mT + C$

$\frac{nRT}{P} = mT + C$

$P = \frac{nRT}{mT+C} = \frac{nR}{m + \frac{C}{T}}$ —— (1)

$T\uparrow \quad P\uparrow$

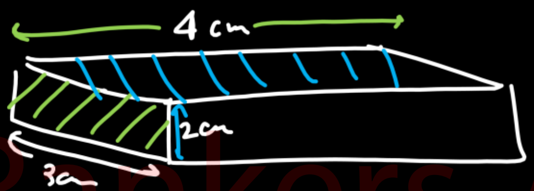
EF: $V = mT - C$

$\frac{nRT}{P} = mT - C$

$P = \frac{nRT}{mT - C} = \frac{nR}{m - \frac{C}{T}} \quad T\uparrow \quad P\downarrow$

21

The sides of rectangular block are 2 cm, 3 cm and 4 cm . The ratio of the maximum to minimum resistance between its parallel faces is



$$R_{max} = \frac{\rho\, l_{max}}{A_{min}} = \frac{\rho\,(4\,cm)}{3 \times 2\ cm^2} \quad \text{— (1)}$$

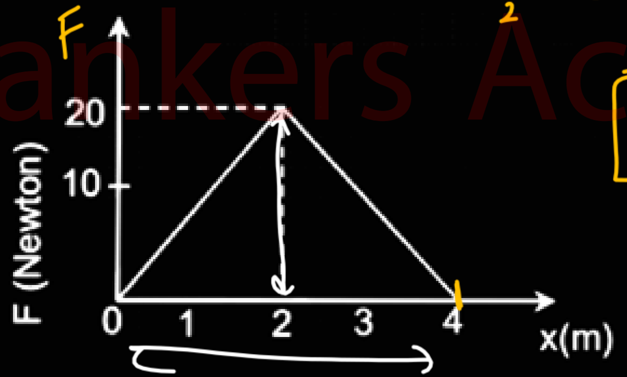
$$R_{min} = \frac{\rho\, l_{min}}{A_{max}} = \frac{\rho\ 2\,cm}{(3 \times 4\ cm^2)} \quad \text{— (2)}$$

$$\frac{R_{max}}{R_{min}} = \frac{\frac{4}{6}}{\frac{2}{12}} = 2 \times 2 = 4$$

22

The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4 m, its kinetic energy would be (in Joule)

$\frac{1}{2}mv_2^2 = ?$



$\boxed{\text{Area} = \int F\,dx = \text{Work}}$

$\text{Area} = \text{Work} = \frac{1}{2} \times 4 \times 20$

$= 40\,J$ — (1)

JEE1

$\boxed{W_{all} = \Delta K}$

$W = K_2 - K_1$

$\Rightarrow -40\,J = K_2 - \frac{1}{2}mv_1^2$

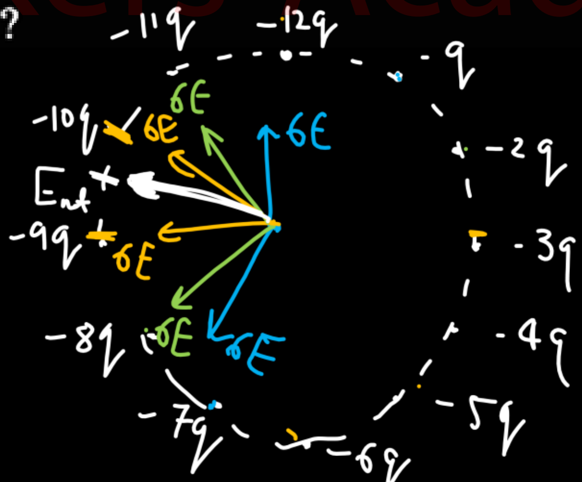
$K_2 = \frac{1}{2} \times 25 \times 2^2 - 40$

$= 10\,J$

23

JEE 1

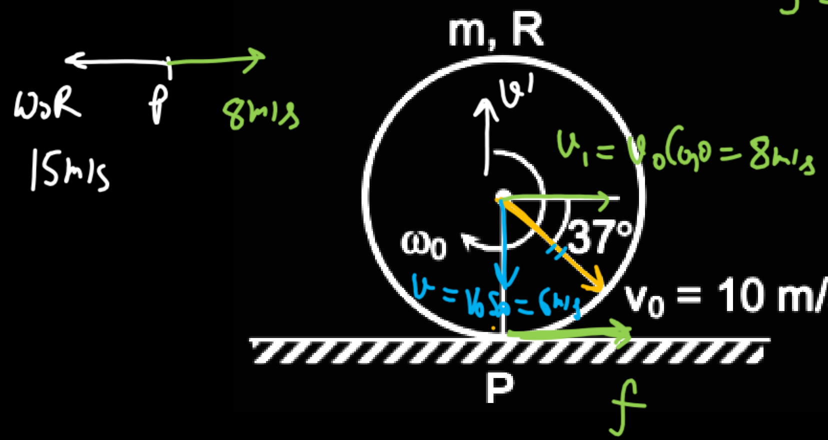
A clock face has negative charges $-q, -2q, -3q, \ldots\ldots -12q$ fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. Time at which the hour hand point in the same direction as electric field at the centre of the dial is X hours and 30 minutes, find X = ?



E_{net} is along $\boxed{9}$ hrs 30 mins.

24

A ball of mass $m = 4$ kg and radius $R = 0.5$ m having initial angular velocity $\omega_0 = 30$rad/s and initial velocity $v_0 = 10$ m/s collides with a rough horizontal surface with $\boxed{e = 0.5}$ as shown in the figure. The coefficient of friction between the ball and surface is $\boxed{\mu = 0.5}$. If the ball starts pure rolling after the collision, find the impulse (in N − s) on the ball due to friction during the collision.



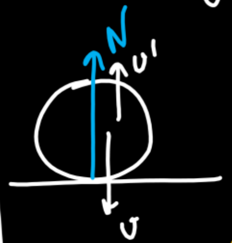
$J = \int f\,dt = \Delta p$

$J = P_2 - P_1$

$= m(v_2 - v_1)$ — (1)

$J_N = \int N\,dt = m(v' - (-v))$

$\int N\,dt = m(v' + v)$ — (2)



along N

$e = \frac{V_{sep}}{V_{app}} = \frac{v'}{v}$

$v' = ev = 0.5 \times 6$

$= 3$ m/s — (3)

from (2)

$J_N = m(v' + v)$

$= 4\,kg\,(3 + 6)$

$= 36$ N-s — (4)

Rankers Academy JEE

24

A ball of mass m = 4 kg and radius R = 0.5 m having initial angular velocity $\omega_0 = 30 \text{rad/s}$ and initial velocity $v_0 = 10$ m/s collides with a rough horizontal surface with e = 0.5 as shown in the figure. The coefficient of friction between the ball and surface is $\mu = 0.5$. ~~If the ball starts pure rolling after the collision~~, find the impulse (in N − s) on the ball due to friction during the collision.

$$J_f = \int f\,dt = \mu \int N\,dt$$

$$J_f = 0.5\, J_N$$

$$= 0.5(36)$$

$$= \boxed{18}\ \text{N-s}$$ — S

25

The electric current in a circuit is given by $I = I_0\left(\frac{t}{T}\right)^2$ for some time. The r.m.s current for the period $t = 0$ to $t = T$ is $\frac{I_0}{\sqrt{N}}$ then N=

JEE1

$$\boxed{i_{rms} = \sqrt{\frac{\int i^2 dt}{T}}}\ *$$

$$i^2_{rms} = \frac{\int i^2 dt}{T} = \frac{\int \left[i_0\left(\frac{t}{T}\right)^2\right]^2 dt}{T} = \frac{\frac{i_0^2}{T^4}\int_0^T t^4\, dt}{T}$$

$$= \frac{i_0^2\ \cancel{T^5}}{5\ \cancel{T^5}} \Rightarrow i_{rms} = \frac{i_0}{\sqrt{\boxed{5}}}\ \text{Ans}$$

CHEMISTRY

Which of the statements is not true?

(A) On passing H_2S through acidified $K_2Cr_2O_7$ solution, a milky colour is observed.

(B) $Na_2Cr_2O_7$ is preferred over $K_2Cr_2O_7$ in volumetric analysis

(C) $K_2Cr_2O_7$ solution in acidic medium is orange.

(D) $K_2Cr_2O_7$ solution becomes yellow increasing the pH beyond 7 .

$$Cr_2O_7^{2-} \underset{H^+}{\overset{OH^-}{\rightleftharpoons}} CrO_4^{2-}$$

$$K_2Cr_2O_7 + H_2S + H_2SO_4 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + S + H_2O$$

Colloidal Sulphur (Milky)

Assertion : Phenol yields a mixture of ortho and para nitrophenols with dilute nitric acid at low temperature (298 K). ✓

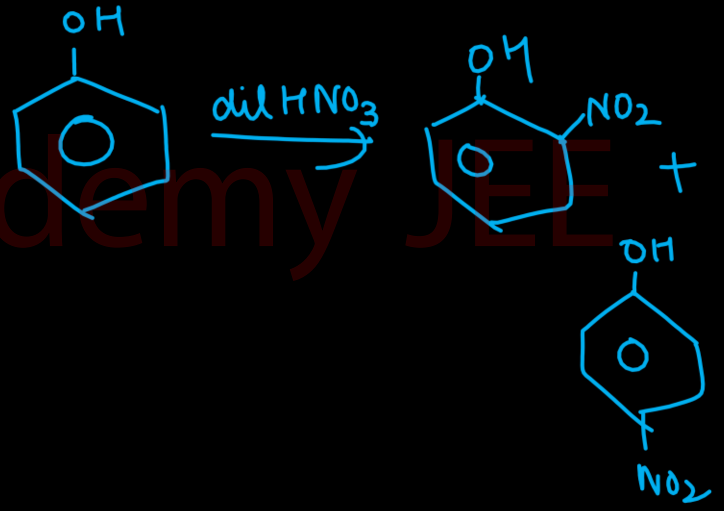
Reason : The ortho and para isomers of nitrophenol can be separated by steam distillation. ✓

(A) Assertion and reason both are correct and reason is correct explanation of assertion.

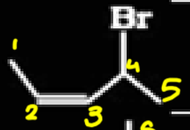
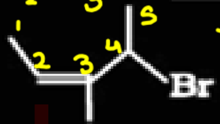
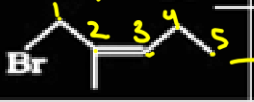
(B) Assertion and reason both are correct statements but reason is not correct explanation of assertion.

(C) Assertion is correct statement but reason is wrong statement.

(D) Assertion is wrong statement but reason is correct statement.



. Match the structures given in Column I with the names in Column II.

Column-I	Column-II
(i) 	(a) 4-Bromopent-2-ene
(ii) 	(b) 4-Bromo-3-methylpent-2-ene
(iii) 	(c) 1-Bromo-2-methylbut-2-ene
(iv) 	(d) 1-Bromo-2-methylpent-2-ene

(A) i – a, ii – b, iii-c, iv-d

(B) i-b, ii-a, iii-d, iv-c

(C) i – d, ii-b ,iii-c, iv-a

(D) i – a, ii-d, iii-c, iv-b

Match the element in list-I with the density in list-II.

LIST-I (Element)		LIST -II (Density gcm	
(P)	Ga	(I)	11.85
(Q)	In	(II)	5.90
(R)	Al	(III)	2.70
(S)	B	(IV)	7.31
(T)	TI	(V)	2.35

Choose the correct answer from the option given below :

(A) P-II; Q-IV; R-III; S-V; T-I

(B) P-III; Q-V; R-I; S-IV; T-II

(C) P-II; Q-V; R-IV; S-I; T-III

(D) P-I; Q-IV; R-III; S-II; T-V

5

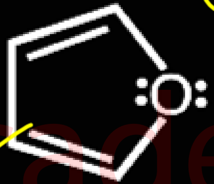
JEE 1

Which of the following compound is aromatic according to Huckel's rule?

(A) 

4nπ

Antiaromatic

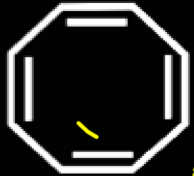
(C) 

$(4n+2)\pi$

Aromatic

(B) 

AA

(D)  

NA

6

The most stable oxidation states of elements among O, S, Se, Te, Cl, F, Br with the least negative electron gain enthalpy will be

(A) −2,2,4,6

(B) -1

(C) −2, −1,1,2

(D) −1, +3; +5, +7

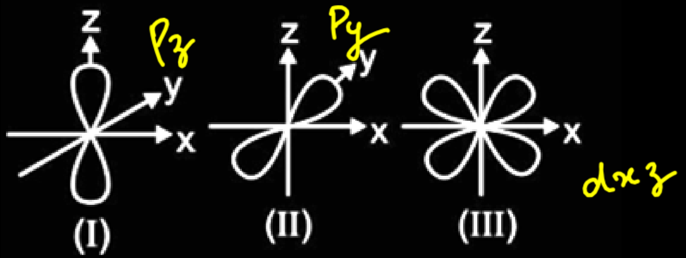
Least (−) e^- gain enthalpy

= oxygen

−1, −2, +1, +2

7

Consider the following three orbital :



INCORRECT statement regarding above orbitals is.

(A) If internuclear axis is ' x ' then combination of (I) and (III) orbitals can form π-bond ✓

$p\pi - d\pi$ bonds are formed perpendicular to internuclear axis.

(B) Orbital (III) can form δ-bond with other similar orbital having identical orientation of lobes if y axis is internuclear axis. ✓

(C) If internuclear axis is ' x ' then combination of (I) and (II) orbitals can form π-bond

(D) If internuclear axis is x or y or z orbitals (I) and (II) can never form any type of covalent bond.

8

Identify the product " Z " in the following series of reactions:

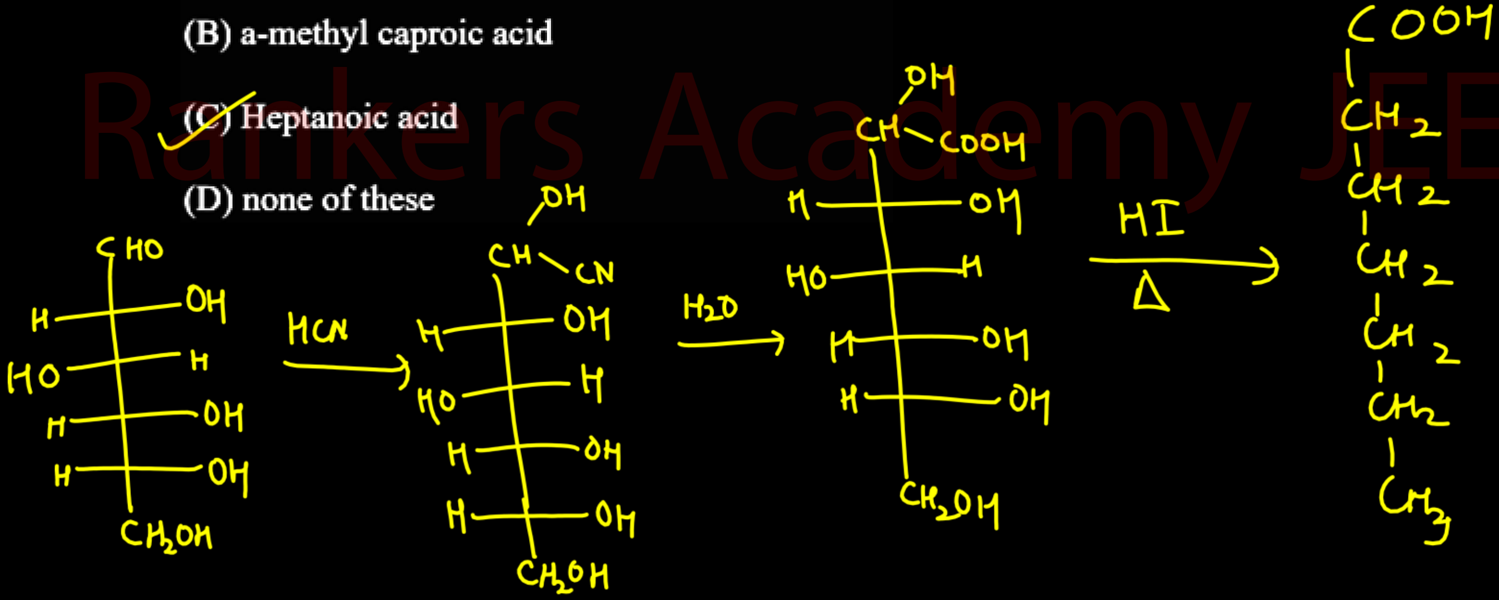
$C_6H_{12}O_6$ (glucose) $\xrightarrow{HCN} X \xrightarrow{H_2O} Y \xrightarrow{HI} Z$:

(A) hexanoic acid

(B) a-methyl caproic acid

(C) Heptanoic acid

(D) none of these



JEE1

9

Which of the following is incorrect combination of group of cations and group reagent.

Cations precipitated	Group reagent
(A) Cu^{2+}, Cd^{2+}	H_2S + dil. HCl
(B) Pb^{2+}, Ag^{+}	Dil. HCl
(C) $Fe^{3+}, Cr^{3+}, Al^{3+}$	$\boxed{NH_4Cl + NH_4OH}$ $+ (NH_4)_2CO_3$ ✗
(D) $Ni^{2+}, Mn^{2+}, Zn^{2+}$	H_2S in the presence of $NH_4OH + NH_4Cl$

10

JEE1

Which set of quantum numbers is not possible?

(A) $n = 2, \ell = 1, m = 0, s = -\frac{1}{2}$

(B) $n = 4, \ell = 3, m = -2, s = -\frac{1}{2}$

(C) $n = 4, \ell = 3, m = -3, s = \frac{1}{2}$

(D) $n = 3, \ell = 2, m = -3, s = \frac{1}{2}$

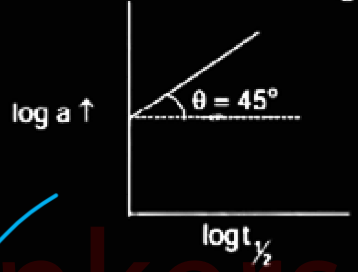
$m = -3$ not possible with $l = 2$

Possible values of $m = -2$ to $+2$ only

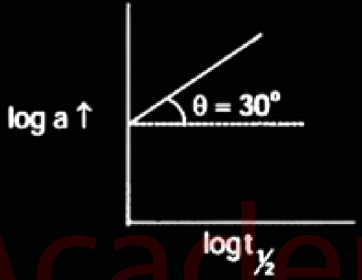
Which of the following is correct graph for the reaction?

$$2\,N_2O \xrightarrow{Pt(hot)} 2\,N_2 + O_2$$

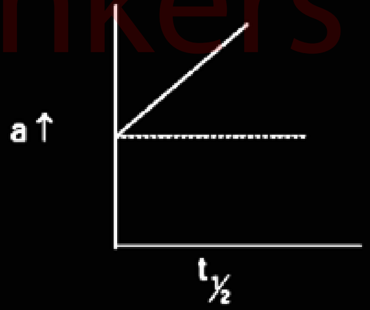
(a is initial concentration of N_2O)



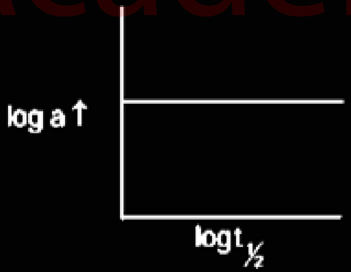
(A)



(C)



(B)



(D)

$$t_{1/2} = \frac{a}{2K}$$

$$a = t_{1/2} \times 2K$$

$$\log a = \log t_{1/2} + \log 2K$$

$$y = mx + c$$

Slope = 1

12

JEE1

Assertion (A): Nitration of aniline can be conventionally done by protecting the amino group by acetylation.

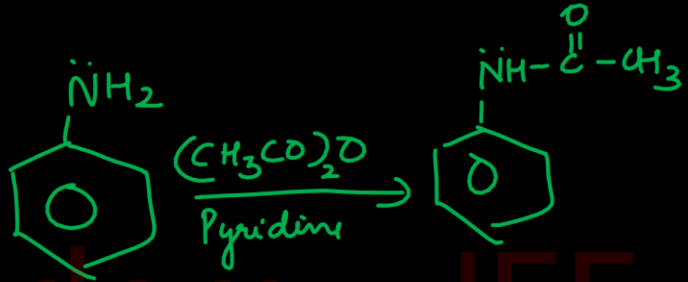
Reason (R): Acetylation increases the electron density in benzene ring.

(A) Assertion and reason both are correct and reason is correct explanation of assertion.

(B) Assertion and reason both are correct statements but reason is not correct explanation of assertion.

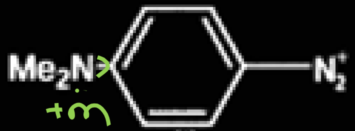
(C) Assertion is correct statement but reason is wrong statement.

(D) Assertion is wrong statement but reason is correct statement.

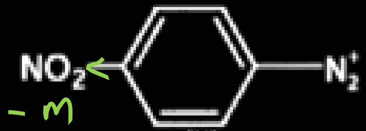


13

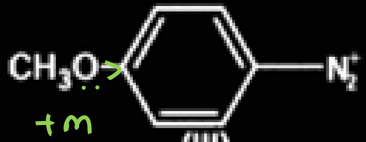
Consider the following diazonium ions:



(I)

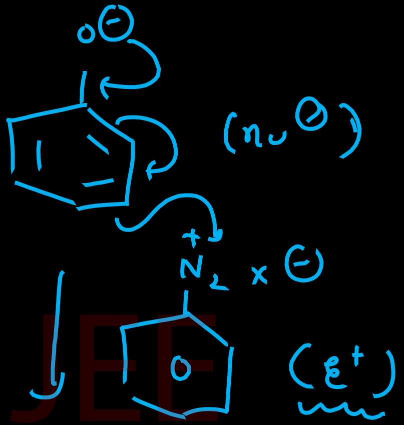


(II)



(III)

(IV)



The order of reactivity toward diazo-coupling with phenol in the presence of dil. NaOH is

(A) I < IV < II < III

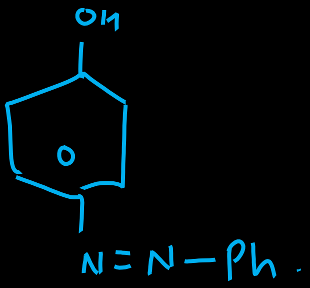
(B) I < III < IV < II

(C) III < I < II < IV

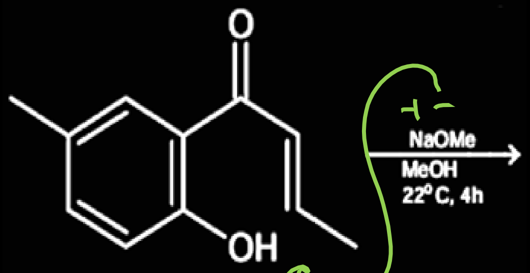
(D) III < I < IV < III

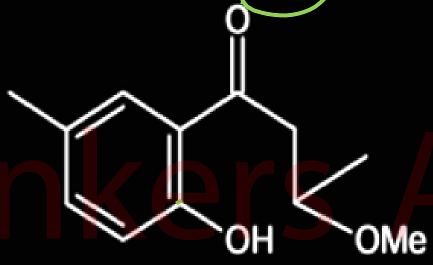
Power

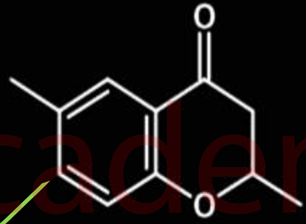
R > H > I

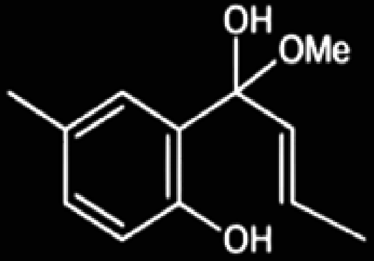


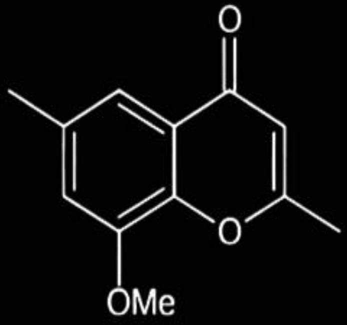
The major product of the following reaction is

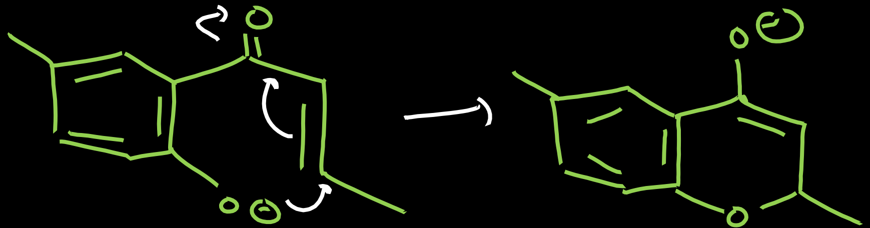


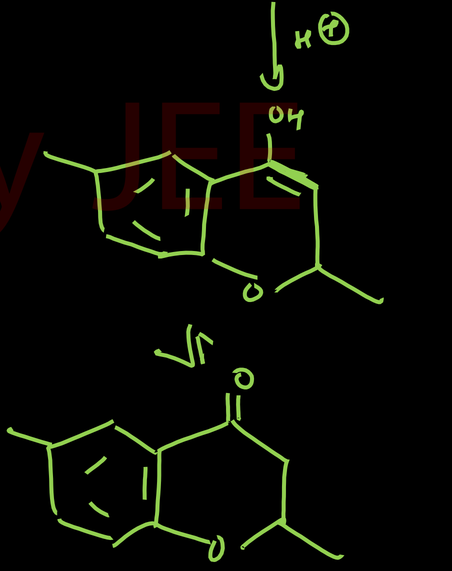
(A)


(C)


(B)


(D)


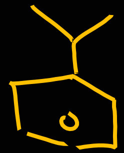




15

Which of the following is correct match for the given reactions?

(A) $C_6H_6 + CH_3 - CH = CH_2 \xrightarrow{H^+}$ [Gattermann-Koch reaction]



(friedel craft,

(B) $C_6H_6 + CO + HCl \xrightarrow{AlCl_3}$ [Etard reaction]

Ph-CHO (gattermann koch)

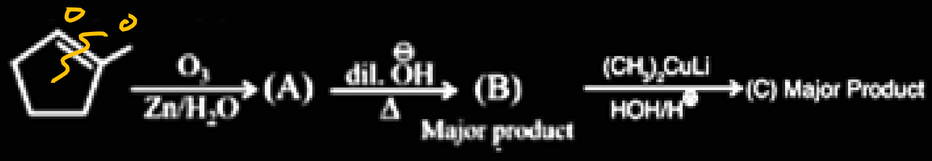
(C) $C_6H_5 - CH_3 + CrO_2Cl_2 \xrightarrow{CS_2}$ [Friedel Craft reaction]

Ph.CHO (Etards)

(D) $Ph\overset{+}{N}_2\overset{-}{Cl} + CuBr + HBr \longrightarrow$ [Sandmeyer reaction]

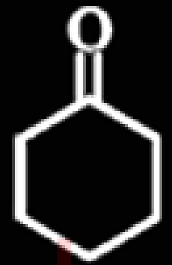
Ph-Br

Major product (C) is :

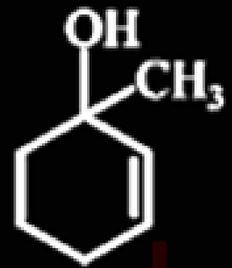


Priority 6>5>7>4

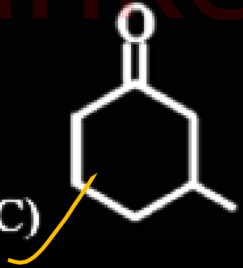
(A)



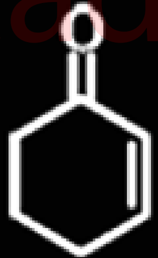
(B)

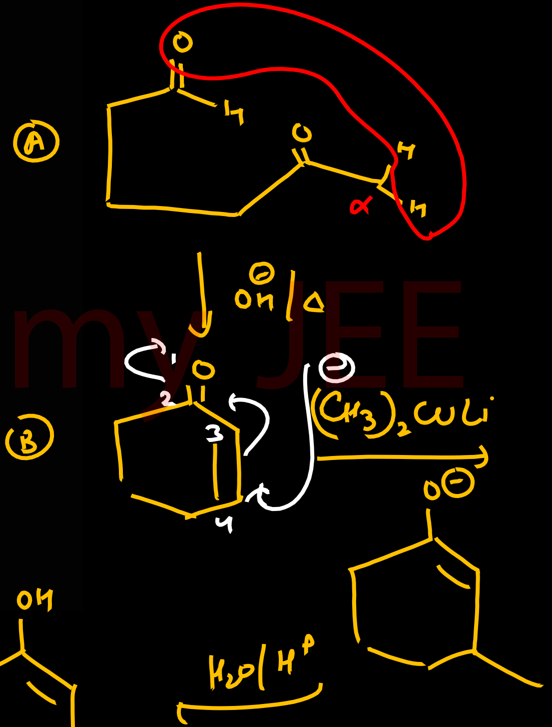


(C)

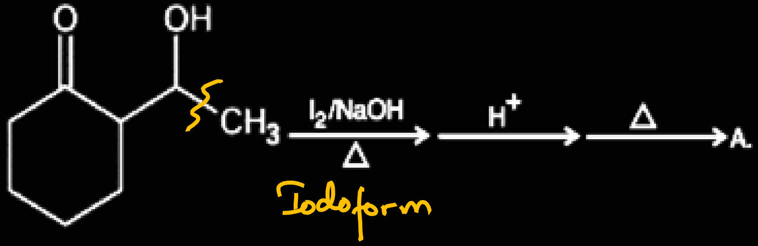


(D)





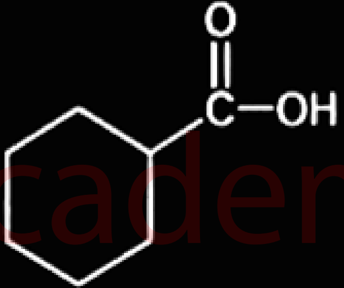
17



O
OH
CH3
I2/NaOH
Δ
H+
Δ
A
Iodoform

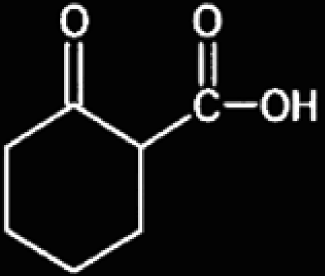
A is:

(A)



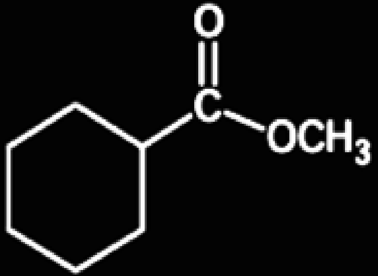
O
C—OH

(C)



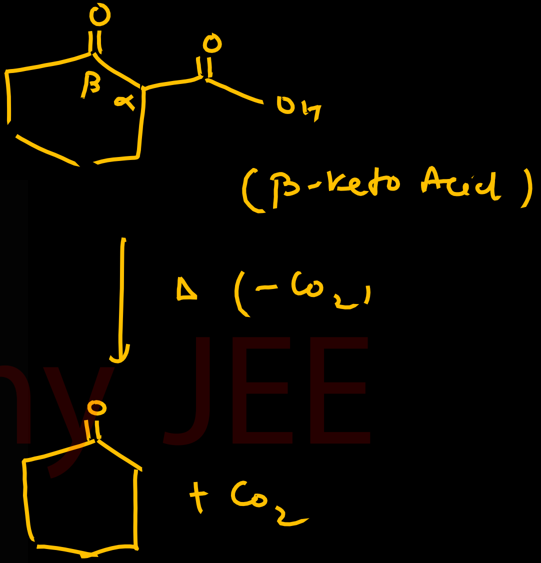
O
O
C—OH

(B)



O
C
OCH3

(D)



OH
(β-keto Acid)
Δ (−CO2)
+ CO2

18

JEE 1

Consider the following statement about the equilibrium.

$\Delta n_g < 0$

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g); \Delta H^0 = -198$ kJ

(I) On decreasing the temperature as well as pressure equilibrium shifts in forward direction.

(II) On increasing temperature and pressure equilibrium shifts in forward direction.

(III) On decreasing the temperature and increasing the pressure, equilibrium will shifts in forward direction.

Choose the correct statement.

(A) I and II (B) Only II

(C) Only III (D) I, II and III

Endothermic Rxn $\Delta n_g < 0$

T↑ forward P↑, forward

T↓ backward P↓, backward.

Exothermic Rxn $\Delta n_g < 0$

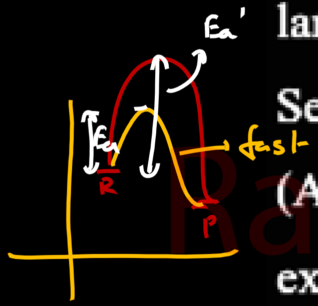
T↑ backward P↑ forward

T↓ forward, P↓ backward

19

Assertion (A) : The 1, 2-addition reaction of HBr with 1, 3-butadiene is found to undergo faster than 1, 4 addition.

Reason (R) : The 1, 2-addition reaction has larger activation energy than 1,4 addition.



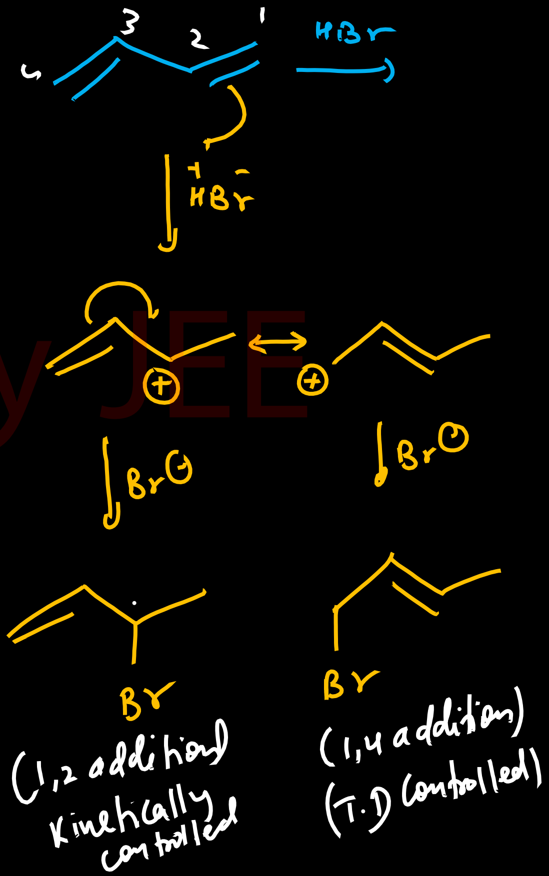
Select the correct answer.

(A) Both (A) and (R) are true and (R) is correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is not correct explanation of (A).

(C) (A) is true and (R) is false.

(D) (A) is false and (R) is true.



20

JEE 1

The pair in which both species have same magnetic moment is:

$\mu = \sqrt{n(n+2)}$

(A) $[Cr(H_2O)_6]^{2+}, [CoCl_4]^{2-}$

(B) $[Cr(H_2O)_6]^{2+}, [Fe(H_2O)_6]^{2+}$

(C) $[Mn(H_2O)_6]^{2+} \cdot [Cr(H_2O)_6]^{2+}$

(D) $[CoCl_4]^{2-} \cdot [Fe(H_2O)_6]^{2+}$

(C) Mn^{+2} : $3d^5$ $n = 5$

Cr^{+2} : $n = 4$

(A) Cr^{+2} : $3d^4$ ↑ ↑ ↑ ↑ $n = 4$

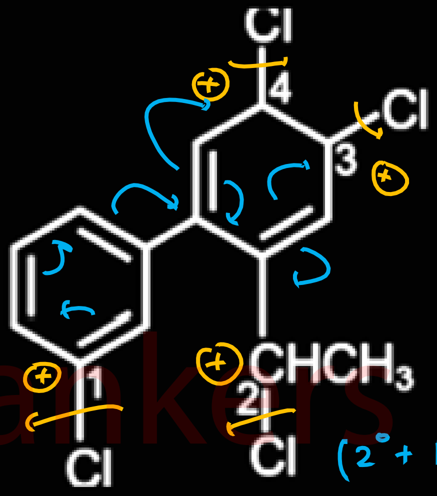
Co^{+2} : $3d^7$ ↑↓ ↑↓ ↑ ↑ ↑ $n = 3$

(B) Cr^{+2} : $\boxed{n = 4}$

Fe^{+2} : $3d^6$ ↑↓ ↑ ↑ ↑ ↑ $\boxed{n = 4}$

21

Write the number of that site which is most reactive for S_N1 reaction:



(2° + extended conj)

(2° + Resonance)

(4) > (3) > (2) > (1)

$\boxed{\text{Ans. 4}}$

22

JEE 1

The enthalpy change of a chemical reaction $A(g) + B(g) \rightleftharpoons 2C(g)$ is $70.4\ kJ\ mol^{-1}$. At what temperature in Kelvin will the reaction attain equilibrium if the entropies of A, B and C gases respectively are 250, 350 and $400\ J\ K^{-1}\ mol^{-1}$?

$$\cancel{\Delta G} = \Delta H - T\Delta S$$

$$0$$

$$T = \frac{\Delta H}{\Delta S} = \frac{70.4 \times 10^3}{[400 \times 2] - [250 + 350]} = \frac{704 \times 100}{[800 - 600]} = \frac{704}{2} = 352$$

23

$Zn(s)|Zn^{2+}(0.01M) \parallel H^{+}|H_2(1\ atm), Pt,$

The e.m.f of the above electrochemical cell is 0.6418 V. What is the pH of the cathode half-cell if $E^0_{Zn^{2+}|Zn} = -0.76\ V$ & $T = 298\ K$

JEE 1

$$Zn(s) + 2H^+ \longrightarrow Zn^{+2} + H_2(g)$$

? 0.01 m 1 atm

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log \frac{[Zn^{+2}]\,P_{H_2}}{[H^+]^2}$$

$$0.6418 = -(-0.76) - \frac{0.0591}{2} \log \frac{(0.01)\times(1)}{[H^+]^2}$$

$$\therefore 4 = \log \frac{10^{-2}}{[H^+]^2}$$

$$\frac{10^{-2}}{[H^+]^2} = 10^4$$

$$[H^+]^{+2} = 10^{-6}$$

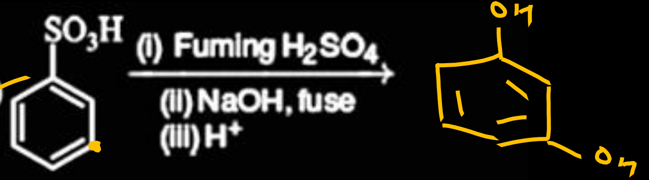
$$\therefore [H^+] = 10^{-3}$$

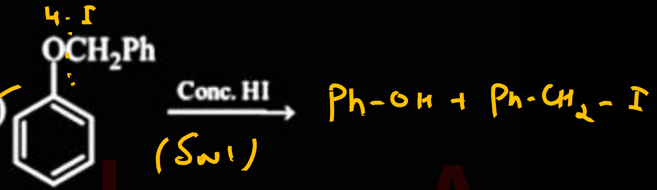
$$pH = -\log[H^+] = -\log(10^{-3})$$

$$\therefore \boxed{pH = 3}$$

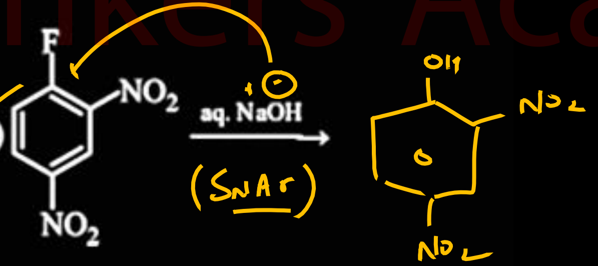
24

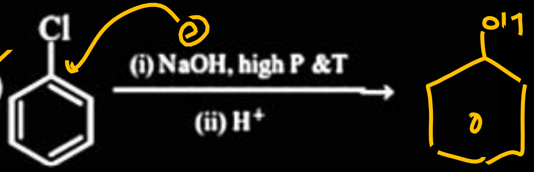
In how many reactions given below phenolic group is present in the major product:

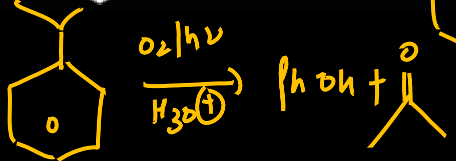
(i)


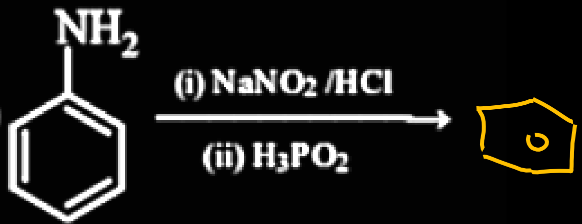
(ii)


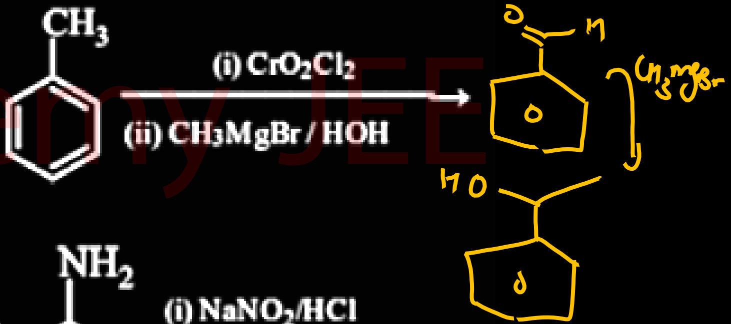
5

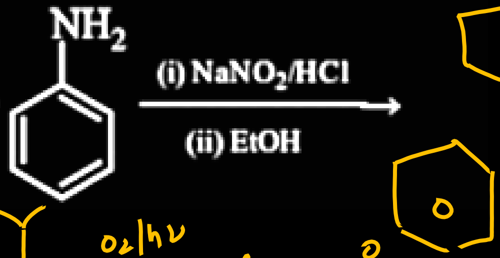
(iii)


(iv)


(v)


(vi)


(vii)


(viii)


25

For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2°C. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of solution is (take $K_b = 0.76\ K\ kg\ mol^{-1}$)

$\Delta T_b = i K_b m$

$2 = 1 \times 0.76 \times m$

$m = \frac{2}{0.76} = \frac{200}{76} = \frac{50}{19}$

Molality

$\frac{P^\circ - P_s}{P^\circ} = \frac{n}{N} = \frac{n}{W} \times \frac{1000}{1000} (\text{mol.wt})_{\text{solvent}}$

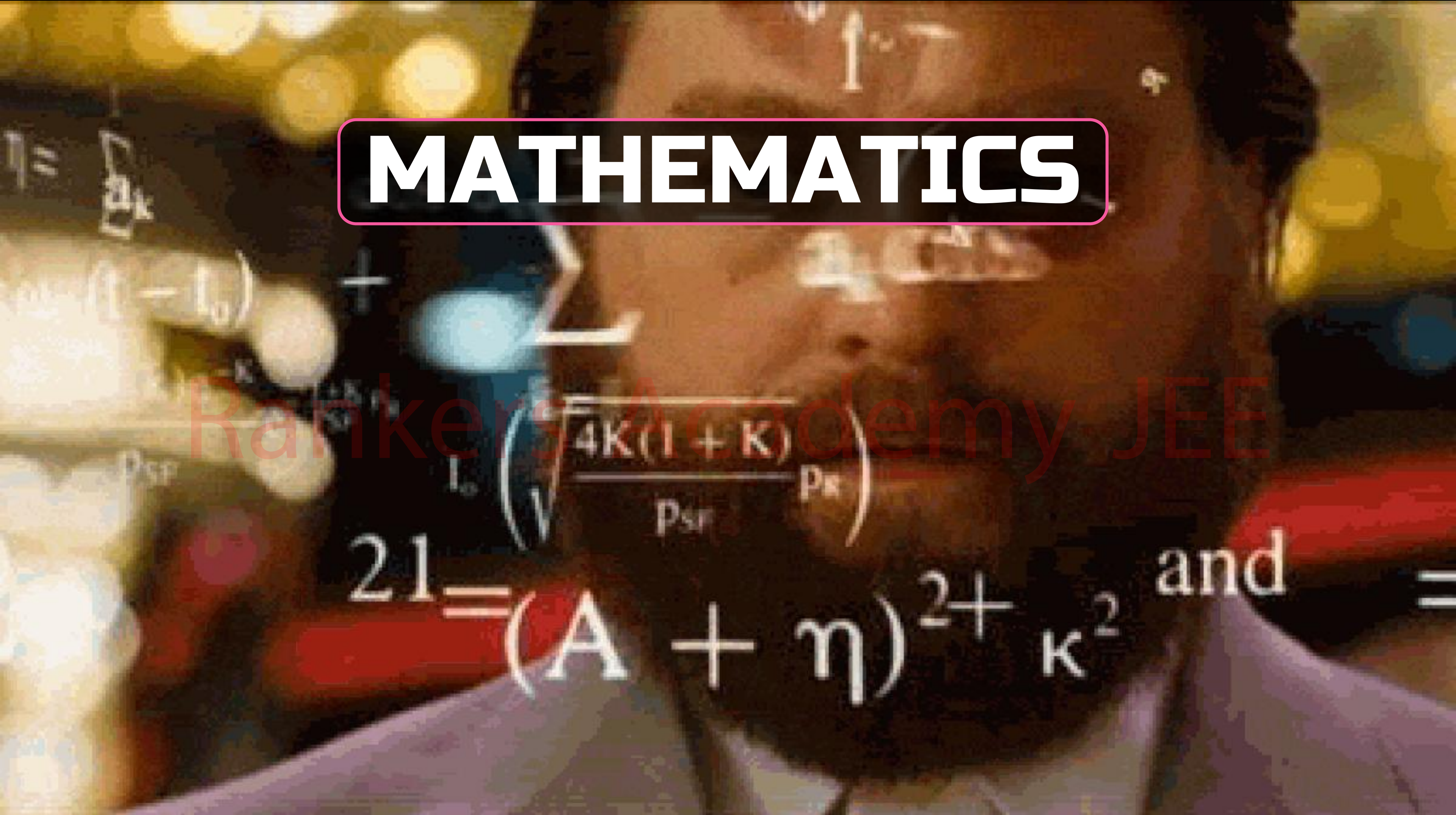
$\frac{P^\circ - P_s}{P^\circ} = \text{Molality} \times \frac{18}{1000}$

$\frac{760 - P_s}{\cancel{760}_{\,2}} = \frac{\cancel{50}}{\cancel{19}} \times \frac{18}{\cancel{1000}_{\,20}}$

$\therefore P_s = 760 - 36$

$\boxed{P_s = 724}$

MATHEMATICS



Let g be the differentiable function satisfying

$\int_0^x (x-t+1)g(t)dt = x^4 + x^2 \ \forall \ x \geq 0$, then

$\int \frac{12}{g(x)+g'(x)-14}dx$ is equal to

(A) $6\ln\left|\frac{x-1}{x+1}\right| + c$

(B) $12\ln\left|\frac{x+1}{x-1}\right| + c$

(C) $\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$

(D) none of these

$$\Rightarrow \int_0^x x\,g(t)dt - \int_0^x (t-1)g(t)dt = x^4 + x^2$$

$$\Rightarrow x\underbrace{\int_0^x g(t)dt}_{\text{diff wrt } x} - \int_0^x (t-1)g(t)dt = x^4 + x^2$$

$$\Rightarrow 1\int_0^x g(t)dt + \cancel{x\,g(x)} - (\cancel{x}-1)g(x) = 4x^3 + 2x$$

$$\Rightarrow g(x) + g'(x) = 12x^2 + 2$$

$$\int \frac{12}{(12x^2+2)-14}dx$$

$$= \int \frac{dx}{x^2-1}$$

$$= \frac{1}{2(1)}\ln\left|\frac{x-1}{x+1}\right| + c$$

If S_n is sum of n terms of the series

$2+4+7+11+16+\cdots$, then $\left(\lim_{n\to\infty}\frac{S_n}{n^3}\right)=$

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) 3 (D) 6

2 4 7 11 16

2 3 4 5

1 1 1

$T_n = 2 + 2\frac{(n-1)}{1!} + 1\frac{(n-1)(n-2)}{2!}$

$T_n = \frac{1}{2}(n^2+n+2)$

$S_n = \sum T_n$

$= \frac{1}{2}\left[\sum n^2 + \sum n + 2\sum 1\right]$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + 2n\right]$

$\frac{1}{2}\lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + 2n}{n^3}$

$\frac{1}{2} \quad \frac{\frac{2}{6}}{1} = \frac{1}{6}.$

3

Consider the integral

$I_1 = \int_1^e x(1+x)(x+\ell nx)^{2022}dx$ and

$I_2 = \int_{\sin^{-1}\left(\frac{1}{e}\right)}^{\frac{\pi}{2}} (1 + e\sin x + \ln \sin x)^{2023} \cos x dx$,

then $I_1 + \frac{eI_2}{2023}$ is equal to

(A) $\frac{(e+1)^{2023}}{2023}$ (B) $\frac{e(e+1)^{2023}-1}{2023}$

(C) $\frac{(e+1)^{2022}}{2022}$ (D) $\frac{e(e+1)^{2022}-1}{e\ 2022}$

JEE 1

Let $\sin x = t$

$\cos x\, dx = dt$

$I_2 = \int_{\frac{1}{e}}^{1} (1 + et + \ln t)^{2023} dt$

Let $et = z \Rightarrow e dt = dz$

$I_2 = \frac{1}{e}\int_1^e \left(1 + z + \ln\left(\frac{z}{e}\right)\right)^{2023} dz$

$I_2 = \frac{1}{e}\int_1^e (\cancel{1} + z + \ln z - \cancel{1})^{2023} dz$

$I_2 = \frac{1}{e}\int_1^e (z + \ln z)^{2023} dz$

Let $z + \ln z = w$

$\left(\cancel{1} + \frac{1}{z}\right) dz = dw$

$I_1 = x\frac{(x+\ln x)^{2023}}{2023}\Big|_1^e - \int_1^e 1 \cdot \frac{(x+\ln x)^{2023}}{2023} dx$

$I_1 = \frac{e(e+1)^{2023} - 1}{2023} - \frac{eI_2}{2023}$

$I_1 + \frac{eI_2}{2023} = \frac{e(e+1)^{2023} - 1}{2023}$

4

The area of smaller region bounded by the curve $9x^2 + 4y^2 - 36x + 16y + 16 = 0$ and the line $3x + 2y = 8$ is

(A) $\frac{3}{2}(\pi + 2)$ (B) $3(\pi - 2)$

(C) $\frac{3}{4}(\pi - 2)$ (D) $\frac{3}{2}(\pi - 2)$

JEE 1

$$9(x^2 - 4x \overset{+4-4}{}) + 4(y^2 + 4y \overset{+4-4}{}) + 16 = 0$$

$$\Rightarrow 9((x-2)^2 - 4) + 4((y+2)^2 - 4) + 16 = 0$$

$$\Rightarrow 9(x-2)^2 + 4(y+2)^2 = 36$$

$$\Rightarrow \boxed{\frac{(x-2)^2}{4} + \frac{(y+2)^2}{9} = 1}$$

$x - 2 = X$

$y + 2 = Y$

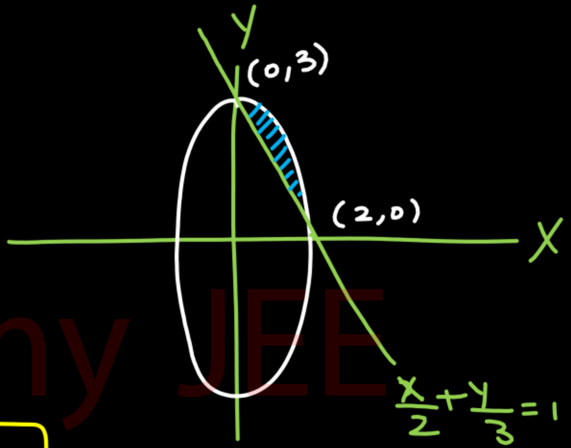
E: $\boxed{\frac{X^2}{4} + \frac{Y^2}{9} = 1}$

L: $3x + 2y = 8$

$\Rightarrow 3(X+2) + 2(Y-2) = 8$

$\Rightarrow 3X + 2Y = 6$

$\Rightarrow \boxed{\frac{X}{2} + \frac{Y}{3} = 1}$



Ans = Blue area

$= \frac{\pi ab}{4} - \Delta$

$= \frac{\pi}{4}(2)(3) - \frac{1}{2}\cdot(2)(3)$

$= 3\left(\frac{\pi}{2} - 1\right)$

5

If α, β be two roots of the equation

$x^2 + (24)^{1/4}x + 6^{1/2} = 0$, then $\alpha^8 + \beta^8$ is equal to

(A) 12 (B) 144

(C) 72 (D) 228

JEE 1

$$x^2 + 24^{1/4}x + 6^{1/2} = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$$

$$\text{Root} = \frac{-24^{1/4} \pm \sqrt{24^{1/2} - 4.6^{1/2}}}{2}$$

$$= \frac{-24^{1/4} \pm \sqrt{4^{1/2}6^{1/2} - 4.6^{1/2}}}{2}$$

$$= \frac{-24^{1/4} \pm 6^{1/4}\sqrt{2-4}}{2}$$

$$= \left(-6^{1/4}.4^{1/4} \pm 6^{1/4}2^{1/2}i\right)/2$$

$$\alpha, \beta = 6^{1/4}.2^{1/2}\frac{(-1 \pm i)}{2}$$

$$= 3^{1/4}.2^{1/4+\frac{1}{2}-1}(-1 \pm i)$$

$$= \left(\frac{3}{2}\right)^{1/4}(-1 \pm i)$$

$$\alpha^2, \beta^2 = \left(\frac{3}{2}\right)^{1/2}(-1 \pm i)^2$$

$$= \left(\frac{3}{2}\right)^{1/2}(\cancel{1} + (\cancel{-1}) \pm 2i)$$

$$\alpha^2, \beta^2 = \pm(6)^{1/2}i$$

$$\alpha^4, \beta^4 = -6$$

$$\alpha^8, \beta^8 = 36$$

$$\alpha^8 + \beta^8 = 72.$$

M2

$$\alpha, \beta = \left(\frac{3}{2}\right)^{1/4}\cdot\sqrt{2}\left(\frac{-1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right)$$

6

JEE 1

Let A be a square matrix of order 3 whose elements are real numbers and adj (adj (adj A)) = $\begin{bmatrix} 27 & 0 & -3 \\ 0 & 9 & 0 \\ 0 & 3 & 27 \end{bmatrix}$ then the value of $3(\mathrm{Tr}(A^{-1}))$ is equal to (adj A and Tr (A) denotes adjoint matrix and trace of matrix A respectively)

(A) 5 (B) 7

(C) 3 (D) 15

$|adj(adj(adj A))| = |A|^{(n-1)^3} = |A|^8$

$= 27[9 \times 27 - 0]$

$= 3^3 \cdot 3^2 \cdot 3^3 = 3^8$

$\therefore \boxed{|A| = \pm 3}$

$adj(adj(adj A)) = |adj A|^{n-2}(adj A)$

$= |adj A|(adj A)$

$= |A|^2 (adj A)$

$= 9(adj A)$

$tr(adj\,adj\,adj A) = 27+9+27 = 63 = 9\,tr(adj A)$

$\Rightarrow tr(adj A) = 7$

$3\,tr(A^{-1}) = 3\,tr\left(\frac{adj A}{|A|}\right)$

$= 3\,tr\left(\frac{adj A}{\pm 3}\right) = 7$

7

JEE 1

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 2|\vec{b}| = 4|\vec{c}|$ and $2\vec{a} - 3\vec{b} + 6\vec{c} = \vec{0}$. If θ is the angle between $\vec{a}$ and $\vec{b}$, then $\cos\theta$ equals:

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

$$|\vec{b}| = \frac{|\vec{a}|}{2}$$

$$|\vec{c}| = \frac{|\vec{a}|}{4}$$

$$|2\vec{a} - 3\vec{b}|^2 = |-6\vec{c}|^2$$

$$\Rightarrow 4|\vec{a}|^2 + 9|\vec{b}|^2 - 12|\vec{a}||\vec{b}|\cos\theta = 36|\vec{c}|^2$$

$$\Rightarrow 4\cancel{|\vec{a}|^2} + 9\frac{\cancel{|\vec{a}|^2}}{4} - 12\cancel{|\vec{a}|}\frac{\cancel{|\vec{a}|}}{2}\cos\theta = 36\frac{\cancel{|\vec{a}|^2}}{16}$$

$$\Rightarrow 4 + \cancel{9/4} - 6\cos\theta = \cancel{9/4}$$

$$\Rightarrow \cos\theta = 2/3$$

8

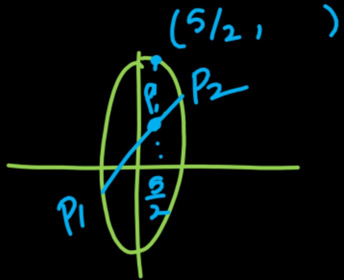
Consider an ellipse E: $\frac{x^2}{25}+\frac{y^2}{80}=1$ and a chord L of the ellipse E which is bisected at point $P\left(\frac{5}{2},\alpha\right)$; $\alpha \in I$. If the chord has integral slope and P_1 and P_2 be the position of P for which $|\alpha|$ is maximum, then

(A) number of possible values of α is/are 7

(B) number of possible values of α is/are 8

(C) number of possible values of α is/are 6

(D) number of possible values of α is/are 4



$T = S_1$

$$\Rightarrow \frac{x\left(\frac{5}{2}\right)}{25}+\frac{y(\alpha)}{80}-\cancel{1}=\frac{25/4}{25}+\frac{\alpha^2}{80}-\cancel{1}$$

$$\Rightarrow \frac{x}{10}+\frac{y\alpha}{80}=\frac{1}{4}+\frac{\alpha^2}{80}=\frac{\alpha^2+20}{80}$$

chord

$$\boxed{8x+y\alpha=\alpha^2+20} \Rightarrow m=-\frac{8}{\alpha}=\text{int.}$$

$$\frac{25/4}{25}+\frac{\alpha^2}{80}=1$$

$$\frac{\alpha^2}{80}=\frac{3}{4}$$

$$\alpha^2=60$$

$$\Rightarrow \alpha \in \{0,\pm1,\pm2,\pm3,\pm4,\pm5,\pm6,\pm7\}$$

$$\alpha=\pm1,\pm2,\pm4$$

9

If the function $f(x) = 2x^3 - (8 - a)x^2 + \left(a^2 + \frac{16}{9}\right)x - 12$ has local minima at some $x \in R^-$, then find the number of integers in the range of a.

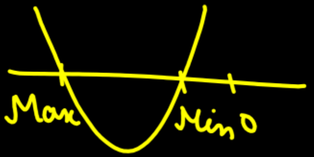
(A) 1 (B) 2

(C) 0 (D) 3

$$f'(x) = \underline{6}x^2 - 2(8-a)x + \left(a^2 + \frac{16}{9}\right) \begin{cases} \text{-ve root} \\ \text{-ve root} \end{cases}$$

Prod of roots

f'(x):



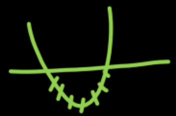
$D > 0$

$4(8-a)^2 - \cancel{24}^{\,6}\left(a^2 + \frac{16}{9}\right) > 0$

$64 + a^2 - 16a - 6a^2 - \frac{32}{3} > 0$

$-5a^2 - 16a + \frac{160}{3} > 0$

$\boxed{5a^2 + 16a - \frac{160}{3} < 0}$



$$a = \frac{-16 \pm \sqrt{256 + \frac{3200}{3}}}{10}$$

10

JEE 1

The vertices of a triangle in the argand plane are $\overset{A}{3+4i}, \overset{B}{4+3i}$ and $\overset{C}{2\sqrt{6}+i}$, then distance between orthocentre and circumcentre of the triangle is equal to, $= OC =$

(A) $\sqrt{137-28\sqrt{6}}$ (B) $\sqrt{137+28\sqrt{6}}$

~~(C)~~ $\frac{1}{2}\sqrt{137+28\sqrt{6}}$ ~~(D)~~ $\frac{1}{3}\sqrt{137+28\sqrt{6}}$

$|\vec{OA}| = 5 = |\vec{OB}| = |\vec{OC}|$

$C \equiv$ Circumcentre $\equiv (0,0)$

Centroid $\equiv \left(\frac{3+4+2\sqrt{6}}{3}, \frac{4+3+1}{3}\right)$

$G \equiv$

$\equiv \left(\frac{7+2\sqrt{6}}{3}, \frac{8}{3}\right)$

$O \equiv ?$

2 : 1

O — G — C $(0,0)$

$\underset{(x,y)}{?}$ $\left(\frac{7+2\sqrt{6}}{3}, \frac{8}{3}\right)$

$\frac{x}{3} = \frac{7+2\sqrt{6}}{3} \Rightarrow x = 7+2\sqrt{6}$

$\frac{y}{3} = \frac{8}{3} \Rightarrow y = 8$

$|\vec{OC}| = \sqrt{(7+2\sqrt{6})^2 + (8)^2}$

If $f''(x) > 0, \forall x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then $g(x)$ is increasing in

(A) $\left(0, \frac{\pi}{4}\right)$

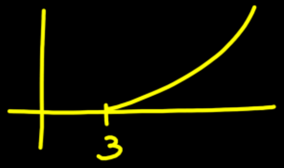
(B) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(C) $\left(0, \frac{\pi}{3}\right)$

(D) none of these ✓

$f'' > 0$

$f' \uparrow$ fn.



$g(x) = f(\tan^2 x - 2\tan x + 4)$

$g'(x) = f'(\tan^2 x - 2\tan + 4) \cdot (2\tan x \sec^2 x - 2\sec^2 x)$

$= \underbrace{f'((\tan x - 1)^2 + 3)}_{+ve} \cdot \underbrace{2\sec^2 x}_{+ve} \cdot (\tan x - 1) > 0$

$x = \pi/4$ $\quad$ $x > \frac{\pi}{4}$

$f'(3) = 0$ $\quad$ $f'(3^+) > 0$

$\tan x - 1 > 0$

$\tan x > 1 \Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

12

Let $f(x) = (x^2 - 4)|(x^3 - 6x^2 + 11x - 6)| + \frac{x}{1+|x|}$. The set of points at which the function f(x) is not differentiable is

(A) $\{-2, 2, 1, 3\}$ (B) $\{-2, 0, 3\}$

(C) $\{-2, 2, 0\}$ (D) $\{1, 3\}$

$$f(x) = \underbrace{(x+2)(x-2)}_{\Downarrow\, 2} \left| \underbrace{(x-1)}_{\downarrow\, 1}\underbrace{(x-2)}_{\downarrow\, 2}\underbrace{(x-3)}_{\downarrow\, 3} \right| + \frac{x}{1+|x|}$$

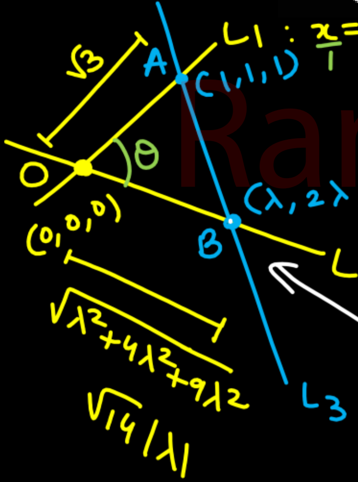
$x = 1, \cancel{2}, 3$

13

If lines $x = y = z, x = \frac{y}{2} = \frac{z}{3}$ and third line passing through (1,1,1) form a triangle of area $\sqrt{6}$ units then point of intersection of third line with second line will be

(A) (1,2,3) (B) (2,4,6)

(C) $\left(\frac{4}{3}, \frac{8}{3}, \frac{12}{3}\right)$ (D) none of these



$L_1 : \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

$L_2 : \frac{x}{1} = \frac{y}{2} = \frac{z}{3} = \lambda$

$\cos\theta = \frac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i}+2\hat{j}+3\hat{k})}{\sqrt{3}\sqrt{14}} = \frac{6}{\sqrt{3}\sqrt{14}} = \sqrt{\frac{6}{7}}$

$\sin\theta = \sqrt{1-\frac{6}{7}} = \frac{1}{\sqrt{7}}$

$\Delta = \sqrt{6} = \frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{14} |\lambda| \cdot \frac{1}{\sqrt{7}}$

$|\lambda| = 2$

$B = (2,4,6)$

14

The relation R defined on the set

$A = \{1,2,3,4,5\}$ is given by

$R = (x: y): |x^2 - y^2| < 16$ then

(A) R is reflexive and symmetric

(B) R is transitive

(C) R is not symmetric

(D) R is reflexive and transitive

$|x^2 - y^2| < 16$

R ✓

S ✓

T ✗

$(1,4) \rightarrow |1^2 - 4^2| = 15 < 16$ ✓

$(4,5) \rightarrow |4^2 - 5^2| = 9 < 16$ ✓

$(1,5) \rightarrow |1^2 - 5^2| = 24$ ✗

15

Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A + I)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then the value of a + b + c + d is :-

(A) 2 ✓ (B) 1

(C) 4 (D) None of these

$A^2 = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 = A^4 = - - - = A^{50} = 0$

$(A+I)^{50}$

$= \cancel{\left[{}^{50}C_0 A^{50} + {}^{50}C_1 A^{49} + - - - \right]} = 0 + {}^{50}C_{49} A^1 I^{49}$

$+ {}^{50}C_{50} I^{50}$

$= 50A + I$

16

Eccentricity of the hyperbola, satisfying the differential equation $2xy\frac{dy}{dx} = x^2 + y^2$, and passing through the point (2,1) is e, then $e^2 + 1$ is

(A) 2 (B) 3

(C) 4 (D) 5

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1+(y/x)^2}{2(y/x)}$$

Let:

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$x\frac{dv}{dx} = \left(\frac{1+v^2}{2v} - v\right)$$

$$x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\int\frac{2v}{1-v^2}dv = \int\frac{dx}{x}$$

$$-\ln(v^2-1) = \ln cx$$

JEE1

$$(v^2-1)^{-1} = cx$$

$$\left(\frac{y^2-x^2}{x^2}\right)^{-1} = cx$$

$$\frac{x^{\cancel{2}}}{y^2-x^2} = c\cancel{x}$$

$$x = c(y^2-x^2)$$

$(2,1)$: $2 = c(1-4)$

$$\boxed{c = -\frac{2}{3}}$$

$$\Rightarrow x = -\frac{2}{3}(y^2-x^2)$$

$$\Rightarrow -\frac{3x}{2} = y^2-x^2$$

$$\Rightarrow x^2-\frac{3x}{2} = y^2$$

$$\Rightarrow \left(x-\frac{3}{4}\right)^2-\frac{9}{16} = y^2$$

$$\Rightarrow \left(x-3/4\right)^2-y^2 = (9/16)$$

$$\Rightarrow \frac{(x-3/4)^2}{(9/16)}-\frac{y^2}{(9/16)} = 1$$

$$\therefore \frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$$

$a = b$ = rec. hyp.

$$e = \sqrt{2}$$

$$\therefore e^2+1 = 3$$

17

JEE 1

A fair coin is tossed repeatedly until two consecutive heads is obtained. The probability that two consecutive heads occur on the seventh and eight flips is equal to :

(A) $\frac{11}{256}$ (B) $\frac{15}{256}$

(C) $\frac{13}{256}$ (D) $\frac{17}{256}$

$\frac{13}{2^8}$

$\underbrace{\_\ \_\ \_\ \_\ \_}\ \underline{T}\ \underbrace{\underline{H}\ \underline{H}}$

(i) 5 T : 1

(ii) 1H 4T : 5

(iii) 2H 3T : $^4C_2$

$\_\ T\ \_\ T\ \_\ T\ \_$ = 6

(iv) 3H 2T : 1

18

$S = 3^{10} + 3^9 + \frac{3^9}{4} + \frac{3^7}{2} + \frac{5.3^6}{16} + \frac{3^6}{16} + \frac{7.3^4}{64} + \cdots \ldots$ upto infinite terms, then $\left(\frac{25}{36}\right)$ S equals to

(A) 6^9

(B) 3^{10}

(C) 3^{11}

(D) 2.3^{10}

$$S = 3^{10} + 3^9 + \frac{3^9}{4} + \frac{3^7}{2} + \frac{5\cdot 3^6}{16} + \frac{2\cdot 3^6}{16\times 2} + \frac{7\cdot 3^4}{64}$$

$$S = \frac{1(3^{10})}{2^0} + \frac{2\cdot(3^9)}{2^1} + \frac{3\cdot(3^8)}{2^2} + \frac{4\cdot(3^7)}{2^3} + \frac{5\cdot(3)^6}{2^4} + \frac{6\cdot(3^5)}{2^5} + \frac{7\cdot(3^4)}{2^6}$$

$$S = \sum_{r=1}^{\infty} \frac{r \cdot 3^{11-r}}{2^{r-1}}$$

$$S = \sum_{r=1}^{\infty} \frac{r\,(3^{11})(3^{-r})}{2^{r}\,(2^{-1})}$$

$$S = 2(3^{11}) \sum_{r=1}^{\infty} \left(\frac{r}{6^{r}}\right) \rightarrow S_1$$

$$\left(\frac{25}{36}\right) S = \frac{\cancel{25}}{\cancel{36}} \times \cancel{2}(3^{11})\left(\frac{\cancel{6}}{\cancel{25}}\right) = 3^{10}$$

$$S_1 = \frac{1}{6^1} + \frac{2}{6^2} + \frac{3}{6^3} + \frac{4}{6^4} + \cdots$$

$$\frac{S_1}{6} = \frac{1}{6^2} + \frac{2}{6^3} + \frac{3}{6^4} + \cdots$$

$$\frac{5}{6}(S_1) = \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \cdots \infty$$

$$\frac{5}{6}(S_1) = \frac{1/6}{1 - 1/6} = \frac{1}{5}$$

$$\boxed{S_1 = \frac{6}{25}}$$

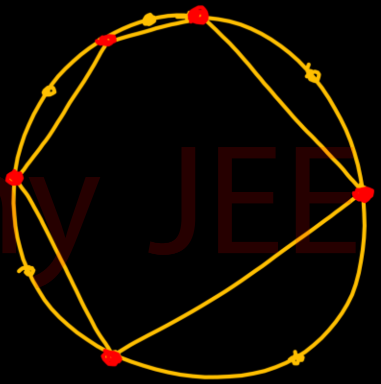
19

JEE1

Let $A_1, A_2, A_3 \ldots A_n$ are the vertices of a polygon of n sides. If the number of pentagons that can be constructed by joining those vertices such that none of the sides of the polygon is also a side of the pentagon is 36, then the value of $(n-3)$ is equal to.

A. 9
B. 10
C. 12
D. 15

$\left.\begin{matrix} n \\ r \end{matrix}\right\}$ non-consecutive



$$= \frac{n}{r}\left({}^{n-r-1}C_{r-1}\right)$$

$$36 = \frac{n}{5}\left({}^{n-6}C_{4}\right)$$

$$36 \times 5 = n\frac{(n-6)(n-7)(n-8)(n-9)}{4\times3\times2}$$

$$36 \times \underbrace{5 \times 4 \times 3 \times 2} = n \underbrace{(n-6)(n-7)(n-8)(n-9)}$$

$$12 \times 3 \times 5 \times 4 \times 3 \times 2$$

$$12\,(6 \times 5 \times 4 \times 3)$$

$$n = 12$$

20

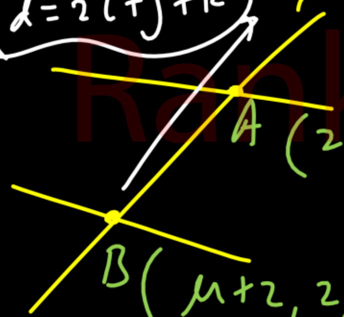
If a line with direction ratio 2:1:1 intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+2}{3}=\mu$ at A and B then $|\overrightarrow{AB}|$ is:

(A) $2\sqrt{3}$ (B) $2\sqrt{5}$

(C) $2\sqrt{6}$ (D) $2\sqrt{7}$

$\vec{d}=2\hat{i}+\hat{j}+\hat{k}$

$\langle 2,1,1 \rangle$



$A\,(2\lambda+1,\ 3\lambda+2,\ 4\lambda+3)$

$B\,(\mu+2,\ 2\mu-1,\ 3\mu-2)$

$\overrightarrow{AB} \equiv (\mu-2\lambda+1)\hat{i}+(2\mu-3\lambda-3)\hat{j}+(3\mu-4\lambda-5)\hat{k}$

$$\frac{\mu-2\lambda+1}{2}=\frac{2\mu-3\lambda-3}{1}=\frac{3\mu-4\lambda-5}{1}$$

solve

$\lambda=-1;\ \mu=1$

$A\equiv(-1,-1,-1)$

$B\equiv(3,1,1)$

$|\overrightarrow{AB}|=\sqrt{16+4+4}=\sqrt{24}=2\sqrt{6}$

21

JEE 1

If the remainder when $(13^{2023}+21^{2023})$ is divided by 51 is λ, then the remainder when λ is divided by 11 is $\boxed{1}$

$51 = 17 \times 3$

$$(17+4)^{2023} = {}^{2023}C_0(17)^{2023}(4)^0 + {}^{2023}C_1(17)^{2022}(4)^1 + {}^{2023}C_2(17)^{2021}(4)^2 + {}^{2023}C_3(17)^{2020}(4)^3 + \dots$$

$$(17-4)^{2023} = {}^{2013}C_0(17)^{2023}(-4)^0 + {}^{2023}C_1(17)^{2022}(-4)^1 + {}^{2023}C_2(17)^{2021}(-4)^2 + {}^{2023}C_3(17)^{2020}(-4)^3 + \dots$$

$$(13^{2023}+21^{2023}) = 2\left[{}^{2023}C_0(17)^{2023} + {}^{2023}C_2(17)^{2021}(4)^2 + \dots {}^{2023}C_{2022}(17)^1(4)^{2022}\right]$$

$$\underbrace{13^{2023}+21^{2023}}_{\downarrow} = 17(\underline{N})$$

$$(\cancel{12}+1)^{2023} + (\cancel{2}1)^{2023}$$

$\downarrow$

$\left(\boxed{1}\ \text{Rem when divided by } 3\right)$

$$= \begin{cases} 17(3M) \\ 17(3M+1) \\ 17(3M+2) \end{cases}$$

$$= \begin{cases} 51M \xrightarrow[\text{by } 3]{\text{Rem when divided}} 0 \ \times \\ 51M+17 \longrightarrow 2 \ \times \\ 51M+34 \longrightarrow \boxed{1} \ \checkmark \end{cases}$$

$$\therefore 13^{2023}+21^{2023}$$

$$= 51M+34$$

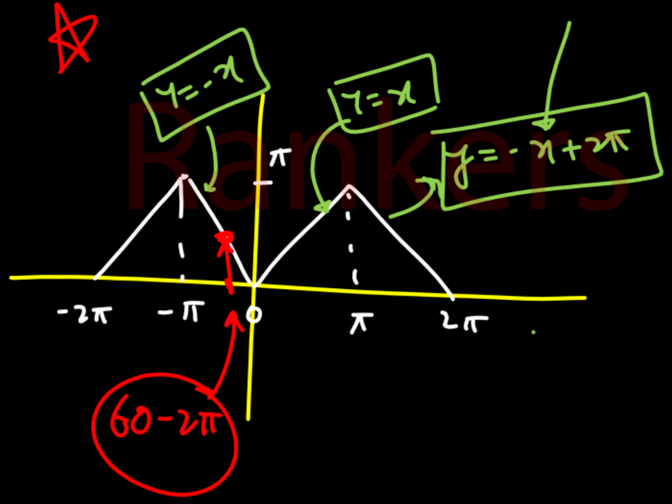
$$\Rightarrow \lambda = 34$$

Now Rem when λ is divided by 11 : $\boxed{1}$

22

Let $a = \cos^{-1}\cos \underline{60}, b = \sin^{-1}\sin 50, \underline{c} = \tan^{-1}\tan(a+b)$. If the greatest integer x for which $\cos^{-1}\cos x \ge |x + c - a - b|$ is equal to k, then k is ___4___

$\cos(2\pi + \frac{\pi}{3})$



$a = \cos^{-1}\underbrace{\cos(\underline{60})}$

$a = \cos^{-1}\cos(2\cancel{0}\pi + (60 - 20\pi))$

$a = \cos^{-1}\cos(60 - 20\pi)$

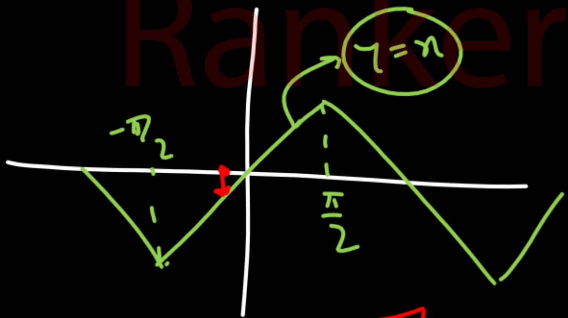
$a = -(60 - 20\pi)$

$\boxed{a = 20\pi - 60}$

$$b = \sin^{-1}(\sin 50)$$

$$b = \sin^{-1}\sin(\cancel{16\pi} + (50 - 16\pi))$$

$$b = \sin^{-1}\sin(\underline{50 - 16\pi})$$



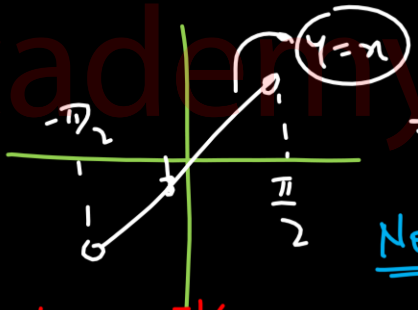
$$\boxed{b = 50 - 16\pi}$$

$$c = \tan^{-1}\tan(a+b)$$

$$c = \tan^{-1}\tan(20\pi - 60 + 50 - 16\pi)$$

$$c = \tan^{-1}\tan(4\pi - 10)$$

$$c = \tan^{-1}\tan(\cancel{\pi} + (3\pi - 10))$$



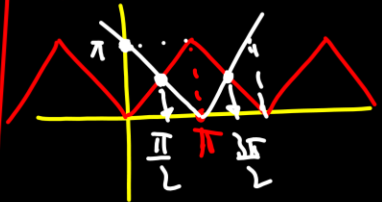
$$\Rightarrow \boxed{c = 3\pi - 10}$$

Now: $|x + c - (a+b)|$

$$|x + 3\pi - \cancel{10} - 4\pi + \cancel{10}|$$

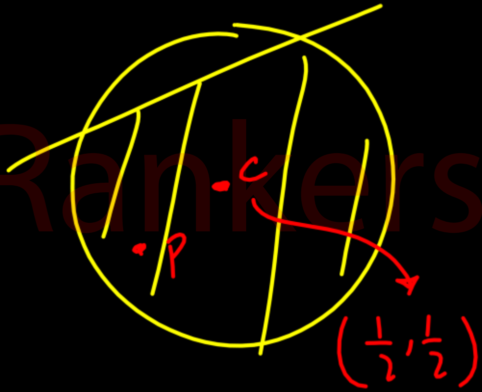
$$\boxed{|x - \pi|}$$

Now: $\cos^{-1}(\cos x) \geq |x - \pi|$



23

Find the number of integral values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$.



$$(\alpha-1)^2+(\alpha+1)^2-(\alpha-1)-(\alpha+1)-6<0$$

$$2\alpha^2+2-\alpha+\cancel{1}-\alpha-\cancel{1}-6<0$$

$$2\alpha^2-2\alpha-4<0$$

$$\alpha^2-\alpha-2<0$$

$$(\alpha-2)(\alpha+1)<0$$

$+ \quad - \quad +$

$-1 \quad 0 \quad 1 \quad 2$

1

$\left(\frac{1}{2}, \frac{1}{2}\right) \rightarrow L \rightarrow -ve$

$\left((\alpha-1), (\alpha+1)\right) \rightarrow L \rightarrow -ve$

$\alpha - \cancel{1} + \alpha + \cancel{1} - 2 < 0$

$2\alpha < 2$

$\boxed{\alpha < 1}$ — 2

$\alpha = \{0\}$

Only one value.

24

Let $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + 5x + 7$ is an injective function $\forall\, x \in R$, where $a, b \in \{1, 2, \ldots\ldots 9\}$, then number of ordered pair (a, b) is

one-one

$f'(x) = ax^2 + bx + 5 \geq 0$ $\left(\because a \text{ is } +ve\right)$

$\boxed{D \leq 0} \Rightarrow b^2 - 20a \leq 0$

$\boxed{b^2 \leq 20a}$

a	b	Count
1	1, 2, 3, 4	4
2	1, 2, 3, 4, 5, 6	6
3	1, 2, 3, 4, 5, 6, 7	7
4	1, 2, 3, 4, 5, 6, 7, 8	8
5		9
6		9
7		9
8		9
9		9

$\underbrace{10 + 15 + 45}$

$\boxed{70}$

25

Let

$$L = \lim_{x\to 0}\left(1 + \underbrace{e^{-\frac{1}{x^2}}\cdot \tan^{-1}\frac{1}{x^2} + x^2\cdot e^{-\frac{1}{x^2}}\cdot \sin\frac{1}{x^4}}\right)^{e^{\left(\frac{1}{x^2}\right)}}$$

then $[15\log_e L]$ is

(Where [.] represents greatest integer function)

$$L = e^{\lim_{x\to 0}\left(\cancel{e^{-1/x^2}}\tan^{-1}\left(\frac{1}{x^2}\right) + x^2\cancel{e^{-1/x^2}}\cdot\sin\frac{1}{x^4}\right)\cancel{e^{\left(\frac{1}{x^2}\right)}}}$$

$$L = e^{\frac{\pi}{2}+0}$$

$$\ln L = \frac{\pi}{2}$$

Now: $\left[15\left(\frac{\pi}{2}\right)\right] = 23$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JEE 1