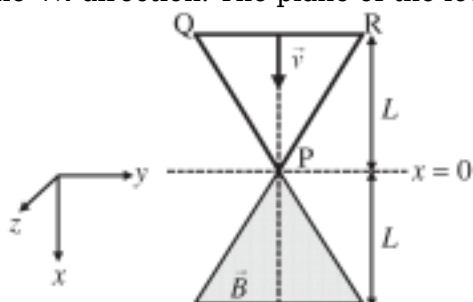


PART-1 : PHYSICS

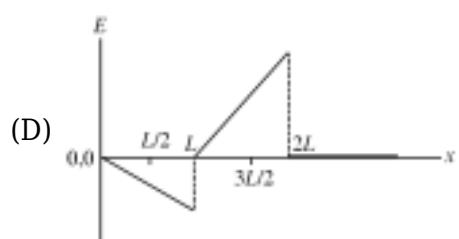
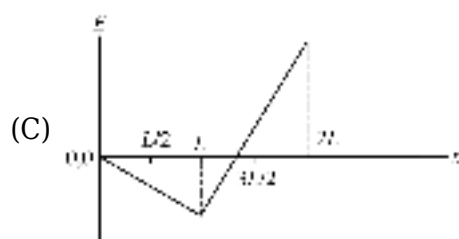
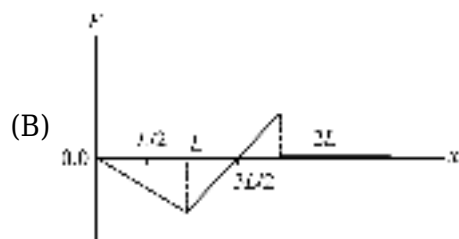
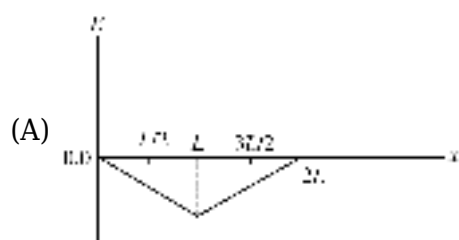
SECTION-I (i)

1) A region in the form of an equilateral triangle (in x-y plane) of height L has a uniform magnetic field \vec{B} pointing in the +z-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L , is placed in the x-y plane with its vertex P at $x = 0$ in the orientation shown in the figure. At $t = 0$, the loop starts entering the region of the magnetic field with a uniform velocity \vec{v} along the +x-direction. The plane of the loop and its orientation remain unchanged throughout its



motion.

Which of the following graph best depicts the variation of the induced emf (E) in the loop as a function of the distance (x) starting from $x = 0$?



2) A particle of mass m is under the influence of the gravitational field of a body of mass M ($\gg m$). The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M . Then, the particle is subjected to an additional central force, corresponding to the potential energy $V_c(r) = m\alpha/r^3$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational

potential due to M and $V_c(r)$, but with a new time period T_1 , then $\frac{T_1^2 - T_0^2}{T_1^2}$ is given by :- [G is the gravitational constant]

(A) $\frac{\alpha}{2GMr_0^2}$

(B) $\frac{2\alpha}{GMr_0^2}$

(C) $\frac{\alpha}{GMr_0^2}$

(D) $\frac{3\alpha}{GMr_0^2}$

3) A metal target with atomic number $Z = 91$ is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α -line and the cut-off is found to be $r = 8$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be

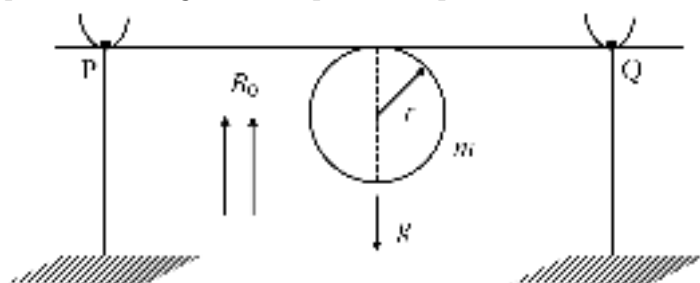
(A) 12.64

(B) 40.5

(C) 0.63

(D) 1.58

4) A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field B_0 , as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g , on two conducting supports at P and Q. When a current I is passed through the loop, the loop turns about the line PQ by maximum angle θ given by :-



(A) $\tan \theta = \frac{\pi r I B_0}{mg}$

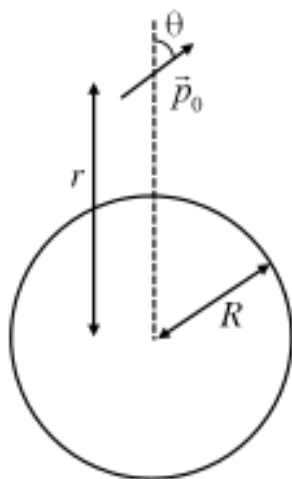
(B) $\tan \theta = \frac{2\pi r I B_0}{mg}$

(C) $\tan \frac{\theta}{2} = \frac{\pi r I B_0}{mg}$

(D) $\tan \frac{\theta}{2} = \frac{2\pi r l B_0}{mg}$

SECTION-I (ii)

1) A small electric dipole \vec{p}_0 , having a moment of inertia I about its center, is kept at a distance r from the center of a spherical shell of radius R . The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance r , the dipole is free to rotate about its center. If released from rest, then which of the following statement(s) is(are) correct? [ϵ_0 is the permittivity of free space.]



(A) The dipole will undergo small oscillations at any finite value of r .

(B) The dipole will undergo small oscillations at any finite value of $r > R$.

(C) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{\sigma p_0}{4\epsilon_0 I}}$ at $r = 2R$.

(D) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{\sigma p_0}{10\epsilon_0 I}}$ at $r = 10R$.

2) A table tennis ball has radius $(3/2) \times 10^{-2}$ m and mass $(22/7) \times 10^{-3}$ kg. It is slowly pushed down into a swimming pool to a depth of $d = 0.7$ m below the water surface and then released from rest. It emerges from the water surface at speed v , without getting wet, and rises up to a height H . Which of the following option(s) is(are) correct? [Given: $\pi = 22/7$, $g = 10 \text{ ms}^{-2}$, density of water = $1 \times 10^3 \text{ kg m}^{-3}$, viscosity of water = $1 \times 10^{-3} \text{ Pa-s}$.]

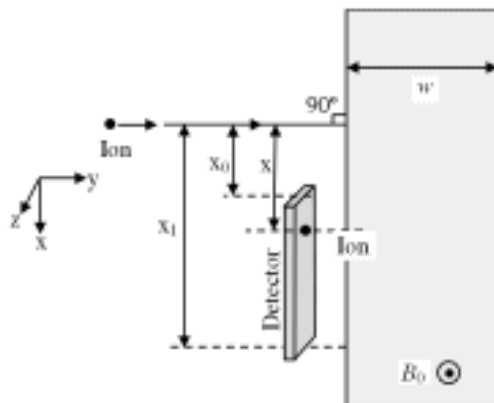
(A) The work done in pushing the ball to the depth d is 0.077 J.

(B) If we neglect the viscous force in water, then the speed $v = 7 \text{ m/s}$.

(C) If we neglect the viscous force in water, then the height $H = 1.4 \text{ m}$.

(D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is 250.

3) A positive, singly ionized atom of mass number A_M is accelerated from rest by the voltage 192 V. Thereafter, it enters a rectangular region of width w with magnetic field $\vec{B}_0 = 0.1\hat{k}$ Tesla, as shown in the figure. The ion finally hits a detector at the distance x below its starting trajectory. Which of the following option(s) is(are) correct? [Given: Mass of neutron/proton = $(5/3) \times 10^{-27} \text{ kg}$, charge of



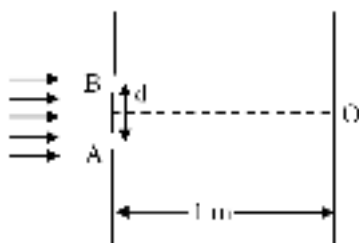
the electron = 1.6×10^{-19} C.]

- (A) The value of x for H^+ ion is 4 cm.
- (B) The value of x for an ion with $A_M = 144$ is 48 cm.
- (C) For detecting ions with $1 \leq A_M \leq 196$, the minimum height ($x_1 - x_0$) of the detector is 52 cm.
- (D) The minimum width w of the region of the magnetic field for detecting ions with $A_M = 196$ is 28 cm.

SECTION-II (i)

Common Content for Question No. 1 to 2

In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time t is given by $d = (0.8 + 0.04 \sin \omega t)$ mm, where $\omega = 0.08 \text{ rad s}^{-1}$. The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is 5000 \AA . The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe)

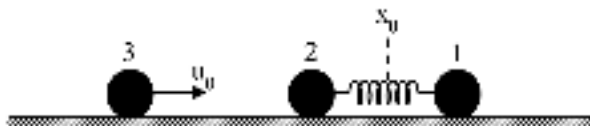


remains fixed at point O.

- 1) The 8th bright fringe above the point O oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer (μm), is _____.
- 2) The maximum speed in $\mu \text{ m/s}$ at which the 8th bright fringe will move is _____.

Common Content for Question No. 3 to 4

Two particles, 1 and 2, each of mass m , are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0 , are oscillating with amplitude a and angular frequency ω . Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) - a \sin \omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2, at time t_0 . Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω .



3) If the collision occurs at time $t_0 = \pi/2\omega$, the value of $v_{cm}/(a\omega)$ will be _____.

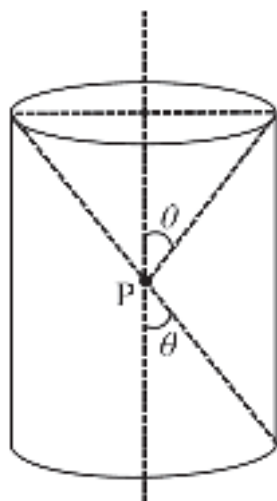
4) If the collision occurs at time $t_0 = 0$, then the value of $4b^2/a^2$ will be _____.

SECTION-II (ii)

1) The dimensions of a cone are measured using a scale with a least count of 1 mm. The diameter of the base and the height are both measured to be 10.0 cm. The maximum percentage error in the determination of the volume is _____.

2) A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v at an angle θ_0 from the +x-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the +x-direction with a suitable initial speed. For a fixed v_0 , when $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$, the stone hits the ball after time T_1 , and when $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$, it hits the ball after time T_2 . In such a case, $(T_1/T_2)^4$ is _____.

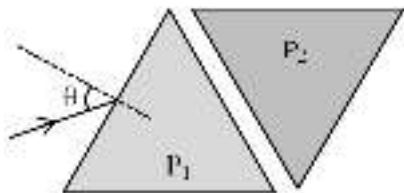
3) A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P, as shown in the figure. When $\theta = 60^\circ$, then the electric flux through the curved surface of the cylinder is Φ . If $\theta = 30^\circ$, then the electric flux through the curved surface becomes $\sqrt{n}\Phi$, where the



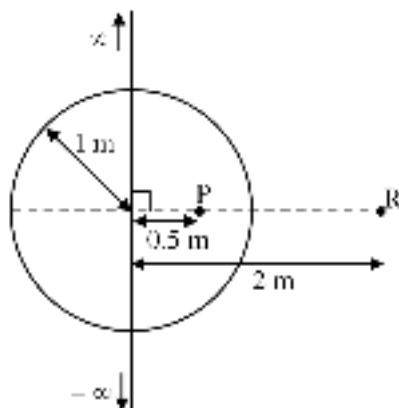
value of n is _____.

4) Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other and very close in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes maximum deviation in prism P_2 . If the respective refractive

indices of P_1 and P_2 are $\frac{5}{3}$ and 2, then $\theta = \sin^{-1} \left(\frac{5}{3} \sin(\beta) \right)$, where the value of β (in degree) is _____.



5) An infinitely long thin wire, having a uniform charge density per unit length of 5 nC/m , is passing through a spherical shell of radius 1 m , as shown in the figure. A 2 nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is _____. [Given: In SI units $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, $\ln 2 = 0.7$. Ignore the area pierced by the wire.]



6) A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5 \text{ Pa}$ has a certain radius so that the excess pressure inside the bubble is $\Delta P = 36 \text{ Pa}$. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is _____.

PART-2 : CHEMISTRY

SECTION-I (i)

1) According to Bohr's model, the lowest kinetic energy is associated with the electron in the :

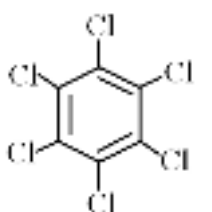
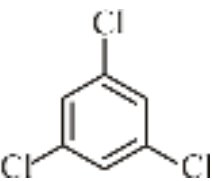
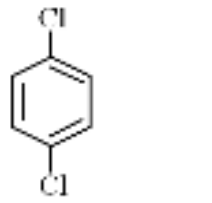
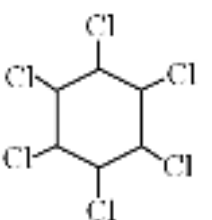
- (A) first orbit of H atom
- (B) first orbit of He^+
- (C) third orbit of He^+
- (D) second orbit of Li^{2+}

2) In a metal deficient oxide sample, $\text{M}_x\text{Y}_2\text{O}_4$ (M and Y are metals), M is present in both +2 and +3 oxidation states and Y is in +3 oxidation state. If the fraction of M^{2+} ions present in M is $\frac{3}{4}$, the value of X is _____.

- (A) 0.25
(B) 0.89
(C) 0.67
(D) 0.75

3) In the following reaction sequences, the major product **R** is: D-Glucose $\xrightarrow[\text{10-20 atm}]{\text{ii) Cr}_2\text{O}_3, 775\text{K}, \text{i) HI, } \Delta}$ **P**

$\xrightarrow[\text{UV}]{\text{Cl}_2(\text{excess})}$ **Q** $\xrightarrow[\Delta]{\text{Alc. KOH}}$ **R** (Major)

- (A) 
- (B) 
- (C) 
- (D) 

4) At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid (HNO_2) gives the species

- (A) H_3O^+ , NO_3^- and N_2O
(B) H_3O^+ , NO_3^- and NO_2
(C) H_3O^+ , NO^- and NO_2
(D) H_3O^+ , NO_3^- and NO

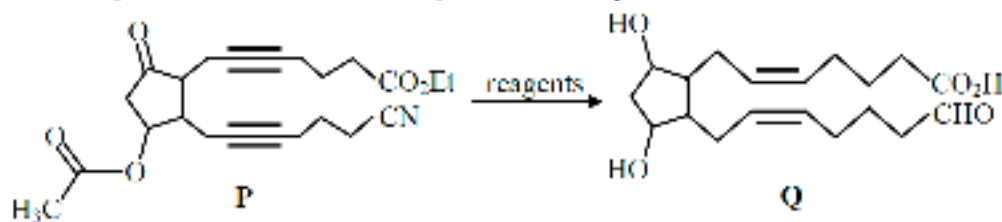
SECTION-I (ii)

1) An aqueous solution of hydrazine (N_2H_4) is electrochemically oxidized by O_2 , thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is $\text{N}_2(\text{g})$.

Choose the correct statement(s) about the above process :

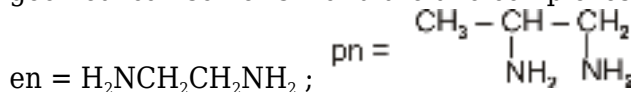
- (A) OH^- ions react with N_2H_4 at the anode to form $\text{N}_2(\text{g})$ and water, releasing 4 electrons to the anode.
- (B) At the cathode, N_2H_4 breaks to $\text{N}_2(\text{g})$ and nascent hydrogen released at the electrode reacts with oxygen to form water.
- (C) At the cathode, molecular oxygen gets converted to OH^- .
- (D) Oxides of nitrogen are major by-products of the electrochemical process.

2) The option(s) with correct sequence of reagents for the conversion of **P** to **Q** is(are) :



- (A) i) NaBH_4 ; ii) SnCl_2/HCl ; iii) H_3O^+ ; iv) Lindlar's catalyst, H_2
- (B) i) Lindlar's catalyst, H_2 ; ii) H_3O^+ ; iii) SnCl_2/HCl ; iv) NaBH_4
- (C) i) Lindlar's catalyst, H_2 ; ii) SnCl_2/HCl ; iii) NaBH_4 ; iv) H_3O^+
- (D) i) Lindlar's catalyst, H_2 ; ii) NaBH_4 ; iii) SnCl_2/HCl ; iv) H_3O^+

3) Among the following options, select the option(s) in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.



- (A) **Set-I** : $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$ and $[\text{PdCl}_2(\text{PPh}_3)_2]$
Set-II : $[\text{Co}(\text{en})_2(\text{NH}_3)\text{NO}_3] \text{SO}_4$ and $[\text{Co}(\text{en})_2(\text{NH}_3)\text{SO}_4] \text{NO}_3$
- (B) **Set-I** : $[\text{Co}(\text{en})(\text{NH}_3)_2\text{Cl}_2]$ and $[\text{PdCl}_2(\text{PPh}_3)_2]$
Set-II : $[\text{Co}(\text{NH}_3)_6] [\text{Cr}(\text{CN})_6]$ and $[\text{Cr}(\text{NH}_3)_6] [\text{Co}(\text{CN})_6]$
- (C) **Set-I** : $[\text{Pt}(\text{py})_2(\text{NH}_3)_2\text{Cl}_2]\text{Br}_2$ and $[\text{Co}(\text{en})\text{Br}_2(\text{NH}_3)_2]$
Set-II : $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{SO}_4$ and $[\text{Co}(\text{NH}_3)_5(\text{SO}_4)]\text{Cl}$
- (D) **Set-I** : $[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$ and $[\text{Co}(\text{pn})_3]\text{Cl}_3$
Set-II : $[\text{Pt}(\text{NH}_3)_4\text{Cl}_2]\text{Br}_2$ and $[\text{Pt}(\text{NH}_3)_4\text{Br}_2]\text{Cl}_2$

SECTION-II (i)

Common Content for Question No. 1 to 2

An organic compound **P** with molecular formula $\text{C}_9\text{H}_{18}\text{O}_2$ decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with H_2O_2 gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

Complete copolymerization of 510 moles of **Q** and 510 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass : H = 1, C = 12, O = 16]

1) Sum of number of sp^3 carbon atoms in **S** and **T** is _____.

2) The value of $\frac{U}{18}$ is _____
where **U** is molecular weight of copolymer.

Common Content for Question No. 3 to 4

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of ferric chloride to **P** in a slightly acidic medium results in insoluble complex **Q**.

3) If 3 moles of potassium iodide are taken then **X** moles of **P** and **Y** moles of iodine is formed. Find the value of $(X+4Y)$.

4) The number of iron present in the molecular formula of **Q** is _____.

SECTION-II (ii)

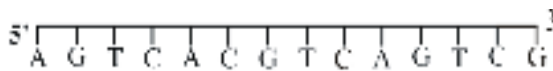
1) To form a complete monolayer of acetic acid on 1g of charcoal, 100 mL of 0.6 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $P \times 10^{-23} \text{ m}^2$ surface area on charcoal, the value of **P** is _____.

[Use given data : Surface area of charcoal = $1.5 \times 10^2 \text{ m}^2 \text{ g}^{-1}$; Avogadro's number (N_A) = $6.0 \times 10^{23} \text{ mol}^{-1}$]

2) Vessel-1 contains w_2 g of a non-volatile solute **X** dissolved in w_1 g of water. Vessel-2 contains w_2 g of another non-volatile solute **Y** dissolved in w_1 g of water. Both the vessels are at the same temperature and pressure. The molar mass of **X** is 60% of that of **Y**. The van't Hoff factor for **X** is 20% more than that of **Y** for their respective concentrations.

The elevation of boiling point for solution in Vessel-1 is _____ % of the solution in Vessel-2.

3) For a double strand DNA, one strand is given below:
The amount of energy required to split the double strand DNA into two single strands is _____ kcal mol^{-1} .



[Given : Average energy per H-bond for A-T base pair = $1.0 \text{ kcal mol}^{-1}$, G-C base pair = $1.5 \text{ kcal mol}^{-1}$, and A-U base pair = $1.25 \text{ kcal mol}^{-1}$. Ignore electrostatic repulsion between the phosphate groups.]

4) A sample initially contains only U-238 isotope of uranium. With time, some of the U-238 radioactively decays into Pb-206 while the rest of it remains undisintegrated.

When the age of the sample is $P \times 10^8$ years, the ratio of moles of Pb-206 to that of U-238 in the

sample is found to be 15. The value of **P** is _____.
 [Given : Half-life of U-238 is 4.5×10^9 years]

5) Among $[\text{Co}(\text{CN})_4]^{4-}$, $[\text{Co}(\text{CO})_3(\text{NO})]$, XeF_4 , $[\text{PCl}_4]^+$, SF_4 , $[\text{ICl}_4]^-$, $[\text{Cu}(\text{CN})_4]^{3-}$, $[\text{Ni}(\text{CO})_4]$, $[\text{CdCl}_4]^{2-}$, C_2H_4 and P_4 the total number of species with tetrahedral geometry is _____.

6) An organic compound **P** having molecular formula $\text{C}_6\text{H}_6\text{O}_3$ gives ferric chloride test and does not have intramolecular hydrogen bond. The compound **P** reacts with 3 equivalents of NH_2OH to produce oxime **Q**. Treatment of **P** with excess methyl iodide in the presence of KOH produces compound **R** as the major product. Reaction of **R** with excess *methyl*magnesium bromide followed by treatment with H_3O^+ gives compound **S** as the major product. The total number of methyl ($-\text{CH}_3$) group(s) in compound **S** is _____.

PART-3 : MATHEMATICS

SECTION-I (i)

1) Considering only the principal values of the inverse trigonometric functions, the value of $\tan \left(\sin^{-1} \left(\frac{4}{5} \right) - 2\cos^{-1} \left(\frac{2}{\sqrt{5}} \right) \right)$ is

- (A) $\frac{7}{24}$
- (B) $\frac{-7}{24}$
- (C) $\frac{-5}{24}$
- (D) 0

2) Let $S = \left\{ (x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8} \right\}$. If the area of the region S is $17\sqrt{2}(\alpha)$, then α is equal to

- (A) $\frac{17}{2}$
- (B) $\frac{17}{4}$
- (C) $\frac{1}{3}$
- (D) $\frac{17}{3}$

3) Let $k \in \mathbb{R}^+$. If $\lim_{x \rightarrow 0^+} \left(\sin(\sin k^2 x) + \cos x - x \right)^{\frac{2}{x}} = e^6$, then the value of k is

- (A) 1

- (B) 2
(C) 3
(D) 4

4) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$. Then which of the following statements is TRUE ?

- (A) $f(x) = 0$ has finitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$
 (B) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$
 (C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite
 (D) $f(x) = 0$ has no solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$

SECTION-I (ii)

1) Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that $\lim_{x \rightarrow \infty} \frac{\sin(x^2) (\log_e x)^\alpha \tan\left(\frac{1}{x^3}\right)}{x^{\alpha\beta} (\log_e(1+x))^\beta} = 0$. Then which of the following is (are) correct ?

- (A) $(-1, 3) \in S$
 (B) $(-1, 1) \in S$
 (C) $(1, -1) \in S$
 (D) $(1, -2) \in S$

2) A straight line drawn from the point $P(1, 3, 2)$, parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q . Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R . Then which of the following statements is (are) **TRUE** ?

- (A) The length of the line segment PQ is $\sqrt{6}$
 (B) The perimeter of the triangle PQR is $\sqrt{13} + \sqrt{6} + \sqrt{11}$
 (C) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$
 (D) The coordinates of R are $(1, 6, 3)$

3) Let A_1, B_1, C_1 be three points in the xy -plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1 , respectively. If $O = (0, 0)$ and $C_1 = (-8, 0)$, then which of the following statements is (are) **TRUE** ?

- (A) The length of the line segment OA_1 is $8\sqrt{2}$
 (B) The length of the line segment A_1B_1 is 16
 (C) The orthocentre of the triangle $A_1B_1C_1$ is (0, 0)
 (D) The orthocentre of the triangle $A_1B_1C_1$ is (-8, 0)

SECTION-II (i)

Common Content for Question No. 1 to 2

Let $S = \{1, 2, 3, 4, 5, 6\}$ and X be the set of all relations R from S to S that satisfy both the following properties :

- i. R has exactly 6 elements.
 ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$ and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$

Let $n(A)$ denote the number of elements in a set A .

1) If $n(X) = {}^mC_p$, then the maximum value of $m + p$ is

2) If the value of $n(Z) - n(Y)$ is k^2 , then $\sqrt{|k|}$ is

Common Content for Question No. 3 to 4

Let $f : \left[0, \frac{\pi}{2}\right] \rightarrow [0, 1]$ be the function defined by $f(x) = \cos^2 x$ and let $g : \left[0, \frac{\pi}{2}\right] \rightarrow [0, \infty)$ be the function defined by $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$.

3) The value of $2 \int_0^{\frac{\pi}{2}} f(x) g(x) dx - \int_0^{\frac{\pi}{2}} g(x) dx = \alpha$, then $\alpha + 2$ is

4) The value of $\frac{64}{\pi^3} \int_0^{\frac{\pi}{2}} f(x) g(x) dx$ is

SECTION-II (ii)

1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and $g : \mathbb{R} \rightarrow (0, \infty)$ be a function such that $g(x + y) = g(x) g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 9\right) g(0)$ is

2) A bag contains N balls out of which 3 balls are white, 6 balls are green and the remaining balls

are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let W_i , G_i and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{1}{4N}$ and the conditional probability $P(B_3 | W_1 \cap G_2) = \frac{1}{8}$, then N equals

3) Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x (x^{2025} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 5)} + \frac{4 (x^{2025} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 5)}.$$

Then the number of solutions of $f(x) = 0$ in \mathbb{R} is

4) Let $\vec{p} = 2\hat{j} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α , β and γ , we have $5\hat{i} + 2\hat{j} + \hat{k} = \alpha (2\vec{p} - \vec{q}) + \beta (\vec{p} + 2\vec{q}) + \gamma (\vec{p} \times \vec{q})$, then the value of 19γ is

5) A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B . Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB . If $r : s = 1 : 32$, then the value of $24a$ is

6) Let the function $f : [1, \infty) \rightarrow \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1} 2 & , \text{ if } t = 2n - 1, n \in \mathbb{N} \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1) & , \text{ if } 2n-1 \leq t \leq 2n+1, n \in \mathbb{N} \end{cases}$$

Define $g(x) = \int_1^x f(t) dt$, $x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval $(1, 10]$ and $\beta = \lim_{x \rightarrow 1^+} \frac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is equal to

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4
A.	C	D	B	C

SECTION-I (ii)

Q.	5	6	7
A.	B,C	A,B	A,B,C,D

SECTION-II (i)

Q.	8	9	10	11
A.	501.25	20.00	0.25	0.25

SECTION-II (ii)

Q.	12	13	14	15	16	17
A.	3	4	3	23	135	24

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	18	19	20	21
A.	C	B	B	D

SECTION-I (ii)

Q.	22	23	24
A.	A,C	A,D	A,C,D

SECTION-II (i)

Q.	25	26	27	28
A.	5.00	5271.00	9.00	7.00

SECTION-II (ii)

Q.	29	30	31	32	33	34
A.	1250	200	48	180	7	9

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	35	36	37	38
A.	D	C	B	A

SECTION-I (ii)

Q.	39	40	41
A.	A,B,C,D	A,B,C	A,B

SECTION-II (i)

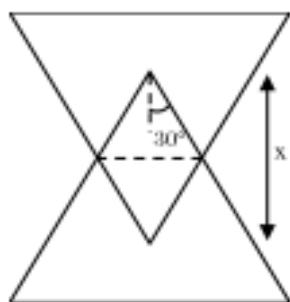
Q.	42	43	44	45
A.	34.00	6.00	2.00	1.00

SECTION-II (ii)

Q.	46	47	48	49	50	51
A.	50	10	1	26	24	6

SOLUTIONS

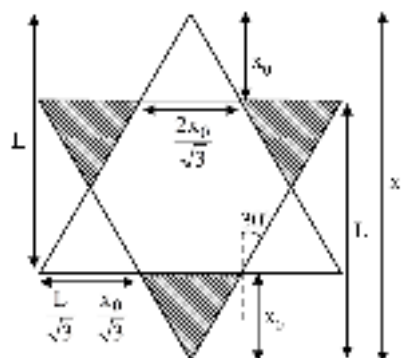
PART-1 : PHYSICS



1) 0 to L

$$\varepsilon = B \ell_{\text{eff}} v = B \times \frac{x}{\sqrt{3}} v$$

L to 2L



$$|\text{emf}| = B \left(\frac{L}{\sqrt{3}} - \frac{x_0}{\sqrt{3}} \right) v - B \frac{2x_0}{\sqrt{3}} v$$

$$= \frac{BvL}{\sqrt{3}} - \sqrt{3}Bvx_0$$

$$= Bv \left[\frac{L}{\sqrt{3}} - \sqrt{3}(x-L) \right]$$

$$= \frac{Bv}{\sqrt{3}} [L - 3x + 3L]$$

$$= \frac{Bv}{\sqrt{3}} [4L - 3x]$$

$$\text{at } x = \frac{4L}{3}$$

$$\text{emf} = 0$$

$$2) \quad F_1 = \frac{GMm}{r_0^2}$$

$$F_2 = \frac{GMm}{r_0^2} - \frac{3m\alpha}{r_0^4}$$

$$\frac{\omega_1^2}{\omega_0^2} = \frac{F_2}{F_1} = \frac{\frac{GM}{r_0^2} - \frac{3\alpha}{r_0^4}}{\frac{GM}{r_0^2}}$$

For More Material Join: @JEEAdvanced_2025

$$\frac{T_0^2}{T_1^2} = 1 - \frac{3\alpha}{GM r_0^2}$$

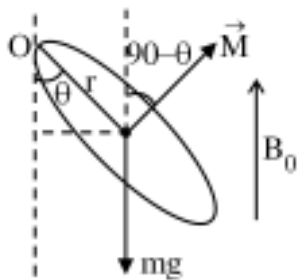
$$\frac{T_1^2 - T_0^2}{T_1^2} = \frac{3\alpha}{GM r_0^2}$$

$$r = \frac{\lambda_{k\alpha}}{\lambda_0} \propto \frac{1}{(z-1)^2}$$

3)

$$\frac{r_2}{r_1} = \frac{(z_1-1)^2}{(z_2-1)^2} = \frac{(90)^2}{(40)^2}$$

$$r_2 = \frac{81}{16} \times 8 = 40.5$$



4) Let loop makes angle θ with vertical.

$$mg(r - r \cos \theta) - I \pi r^2 B_0 \sin \theta = 0$$

$$\tan \frac{\theta}{2} = \frac{I \pi r B_0}{mg}$$

5) The electric field inside sphere is zero. So dipole will oscillate when $r > R$.

$$\text{For } r > R; E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{P_0 \sigma R^2}{I \epsilon_0 r^2}}$$

when $r = 2R$

$$\omega = \sqrt{\frac{P_0 \sigma}{4 I \epsilon_0}}$$

when $r = 10 R$

$$\omega = \sqrt{\frac{P_0 \sigma}{100 I \epsilon_0}}$$

6)

$$(A) w_{\text{all}} = k_f - k_i = 0$$

$$w_g + w_B + w_v + w_{\text{ext}} = 0$$

$$mgd - \rho_w v g d - 6\pi\eta r v d + w_{\text{ext}} = 0$$

(slowly $v = 0$)

$$w_{\text{ext}} = \rho_w v g d - mgd = \left(1000 \times \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^3 - \frac{22}{7} \times 10^{-3}\right) g d$$

$$w_{\min} = R = \frac{1}{0.1} \sqrt{\frac{2 \times 196 \times \frac{5}{3} \times 10^{-27} \times 192}{1.6 \times 10^{-19}}} = 28 \text{ cm}$$

$$8) \quad y = n \cdot \left(\frac{\lambda D}{d} \right)$$

for 8th fringe

$$y = 8 \frac{\lambda D}{d}$$

$$y_{\max} = 8 \frac{\lambda D}{d_{\min}}$$

$$y_{\min} = 8 \frac{\lambda D}{d_{\max}}$$

$$y_{\max} - y_{\min} = 8 \lambda D \left[\frac{1}{d_{\min}} - \frac{1}{d_{\max}} \right]$$

$$\lambda = 5000 \text{ \AA}$$

$$D = 1 \text{ m}$$

$$d_{\max} = 0.34 \text{ mm}$$

$$d_{\min} = 0.76 \text{ mm}$$

$$y_{\max} - y_{\min} = 8 \times 5000 \times 10^{-10} \times 1 \left[\frac{1}{0.76 \times 10^{-3}} - \frac{1}{0.84 \times 10^{-3}} \right]$$

$$= 8 \times 5 \times 10^{-4} \times \left[\frac{0.08}{0.76 \times 0.84} \right] = 501.25 \mu\text{m}$$

$$9) \quad y = n \cdot \frac{\lambda D}{d}$$

$$v = \frac{dy}{dt} = -n \cdot \frac{\lambda \cdot d}{d^2} \cdot \frac{d(d)}{dt}$$

$$d = 0.8 + 0.04 \sin \omega t$$

$$\frac{d(d)}{dt} = 0.04 \omega \cos \omega t$$

$$\text{for } v \rightarrow \max \Rightarrow \frac{d(d)}{dt} \rightarrow \max.$$

$$\text{For } \frac{d(d)}{dt} \rightarrow \max.$$

$$\cos \omega t = 1 \Rightarrow \sin \omega t = 0$$

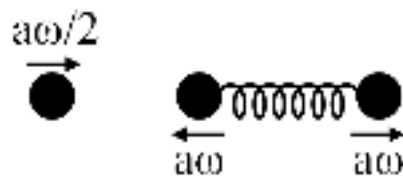
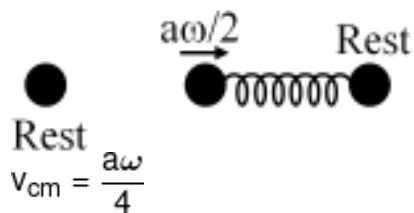
$$\Rightarrow \left(\frac{d(d)}{dt} \right)_{\max} = 0.04$$

$$\Rightarrow d = 0.8 \text{ mm } v_{\max} = \frac{8 \times 5000 \times 10^{-10} \times 1 \times 0.04 \times 0.08}{0.8 \times 0.8 \times 10^{-6} \times 10^{-3}} = 20 \mu\text{m/s}.$$

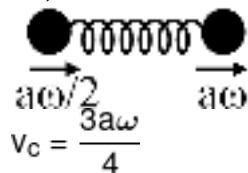
10) At T $t_0 = 0$

Before collision

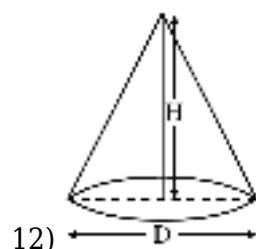
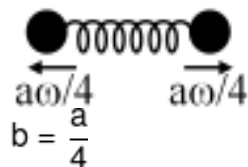




11) Particles at mean



w.r.t. to cm



$$V = \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 H$$

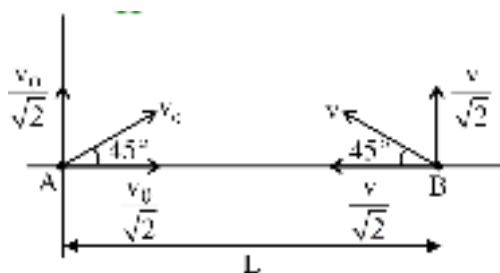
□ % Error in $V = 2(\% \text{ error in } D) + \% \text{ error in } H$.

□ Least count is 2mm.

$$\square \% \text{ error in } D = \frac{1\text{mm}}{10\text{cm}} \times 100\% = 1\%$$

$$\& \% \text{ error in } H = \frac{1\text{mm}}{10\text{cm}} \times 100\% = 1\%$$

So % error in $V = 2 \times 1\% + 1\% = 3\%$



13) For case I :

$$a_{rel} = 0$$

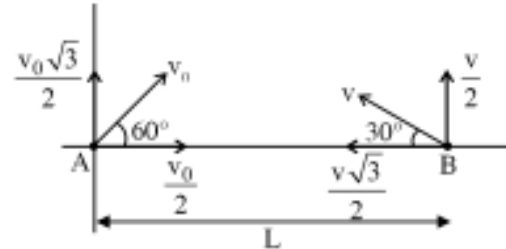
$$\text{For collision } \frac{v_0}{\sqrt{2}} = \frac{v}{\sqrt{2}}$$

$$\square v = v_0$$

$$\text{So } T_1 = \frac{\frac{L}{\frac{v_0}{\sqrt{2}} + \frac{v}{\sqrt{2}}}}{L}$$

$$\square T_1 = \sqrt{2}v_0 \dots\dots(1)$$

For case II,



$$a_{\text{rel}} = 0$$

$$\text{For collision, } \frac{v_0\sqrt{3}}{2} = \frac{v}{2}$$

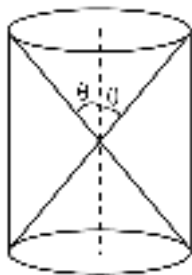
$$\square v = \sqrt{3}v_0$$

$$\text{So, } T_2 = \frac{\frac{L}{\frac{v_0}{2} + v\frac{\sqrt{3}}{2}}}{L}$$

$$T_2 = \frac{\frac{L}{\frac{v_0}{2} + \frac{3v_0}{2}}}{L}$$

$$\square T_2 = 2v_0 \dots\dots\dots(2)$$

$$\text{so, } \left(\frac{T_1}{T_2}\right)^2 = (\sqrt{2})^2 = 2 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = 2$$



14)

$$\text{Solid angle made by plane surfaces } \Omega = 2 \times 2\pi(1 - \cos \theta)$$

$$\Rightarrow \Omega = 4\pi - 4\pi \cos \theta$$

$$\text{So solid angle made by curved surface} = 4\pi - \Omega$$

$$= 4\pi - (4\pi - 4\pi \cos \theta) = 4\pi \cos \theta$$

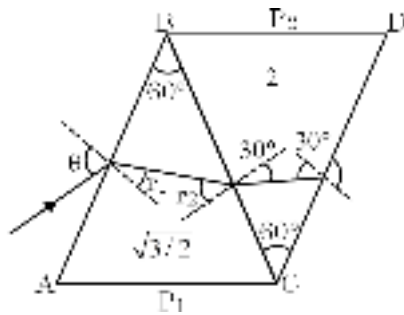
$$\phi_{30^\circ} = \phi = \frac{4\pi \cos 30^\circ}{4\pi} \frac{Q}{\epsilon_0} = \cos 30^\circ \frac{Q}{\epsilon_0}$$

$$\phi_{60} = \frac{4\pi \cos 60^\circ}{4\pi} \frac{Q}{\epsilon_0} = \cos 60^\circ \frac{Q}{\epsilon_0}$$

$$\frac{\phi_{30}}{\phi_{60}} = \frac{\cos 30^\circ}{\cos 60^\circ} = \sqrt{3}$$

$$\frac{\phi}{\phi_{60}} = \sqrt{3}$$

$$\phi_{60} = \frac{\phi}{\sqrt{3}} \Rightarrow n = 3$$



15) $2 \sin \theta = 1 \sin 90^\circ$

$$\theta = 30^\circ$$

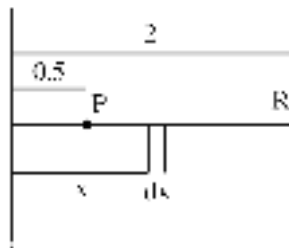
$$1 \sin \theta = \sin \phi$$

$$\sin 90^\circ = \frac{5}{3} \sin r$$

$$r = \sin^{-1} \left(\frac{3}{5} \right)$$

$$= \sin^{-1} \left(\frac{5}{3} \sin(60^\circ - 37^\circ) \right)$$

$$1 \sin \theta = \frac{5}{3} \sin 23^\circ$$



16)

due to wire

$$dV = -\vec{E} \cdot d\vec{x}$$

$$\int_{V_P}^{V_R} dV = - \int_{0.5}^2 \frac{2k\lambda}{x} dx$$

$$V_R - V_P = -2k\lambda \ln \frac{2}{0.5}$$

$$= -2 \times 9 \times 10^9 \times 5 \times 10^{-9} \times 2 \times 0.7 = -126V$$

due to sphere

$$V_R - V_P = \frac{kQ}{2} - \frac{kQ}{1} = -\frac{kQ}{2} = \frac{-9 \times 10^9 \times 8 \times 10^{-9}}{2}$$

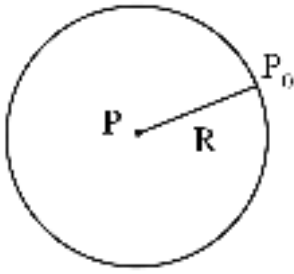
$$= -9V$$

$$V_R - V_P = -126 - 9 = -135V$$

$$V_P - V_R = 135V$$

17)

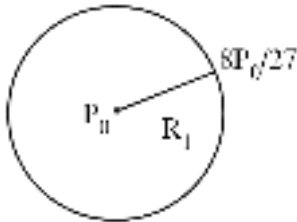
Case-1



$$P - P^0 = \Delta P = \frac{4T}{R}$$

$$P = \left(P_0 + \frac{4T}{R} \right)$$

Case-2



$$P_1 - \frac{8P_0}{27} = \Delta P_1 = \frac{4T}{R_1}$$

$$P_1 = \frac{4T}{R_1} + \frac{8P_0}{27}$$

Constant temperature process

$$PV = P_1 V_1$$

$$\left(P_0 + \frac{4T}{R} \right) \frac{4}{3} \pi R^3 = \left(\frac{4T}{R_1} + \frac{8P_0}{27} \right) \frac{4}{3} \pi R_1^3 ;$$

$$\left(\frac{4T}{R} \right), \left(\frac{4T}{R_1} \right) \rightarrow (\text{Neglected})$$

$$R = \frac{2}{3} R_1 \Rightarrow R_1 = \frac{3}{2} R$$

$$\Delta P_1 = \frac{4T}{R_1} = \frac{4T}{\frac{3}{2}R} \times 2 = \frac{2}{3} \times (36) = 24 \text{ Pa}$$

PART-2 : CHEMISTRY

$$18) \text{ KE} = +13.6 \times \frac{Z^2}{n^2}$$

$$(A) \text{ KE}_{1,H} = +13.6 \times \frac{1^2}{1^2} = 13.6 \text{ eV}$$

$$(B) \text{ KE}_{1,He^+} = +13.6 \times \frac{2^2}{1^2} = 13.6 \times 4 \text{ eV}$$

$$(C) \text{ KE}_{2,He^+} = +13.6 \times \frac{2^2}{3^2} = 13.6 \times \frac{4}{9} \text{ eV}$$

$$(D) \text{ KE}_{2,Li^{2+}} = +13.6 \times \frac{3^2}{2^2} = 13.6 \times \frac{9}{4} \text{ eV}$$

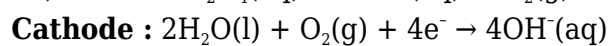
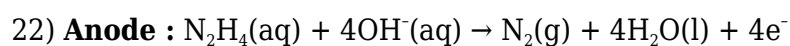
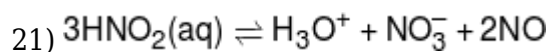
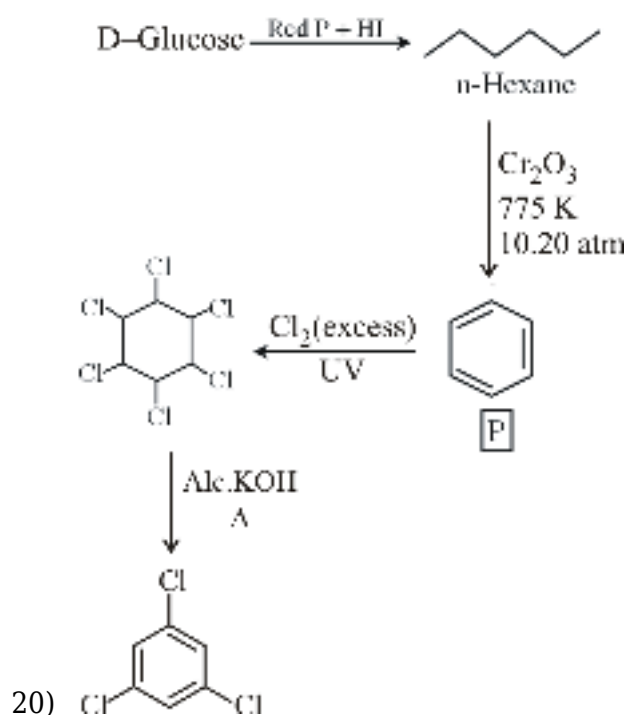
19)

For More Material Join: @JEEAdvanced_2025

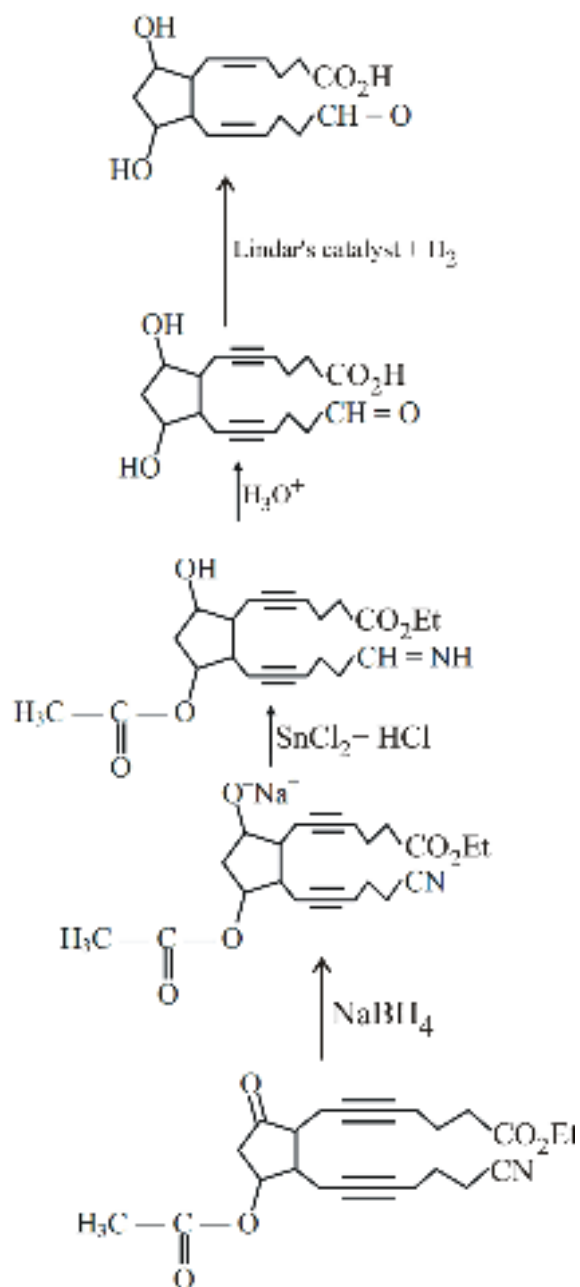
Charge balance

$$8 = \frac{x}{4} \times 3 + \frac{3x}{4} \times 2 + 6$$

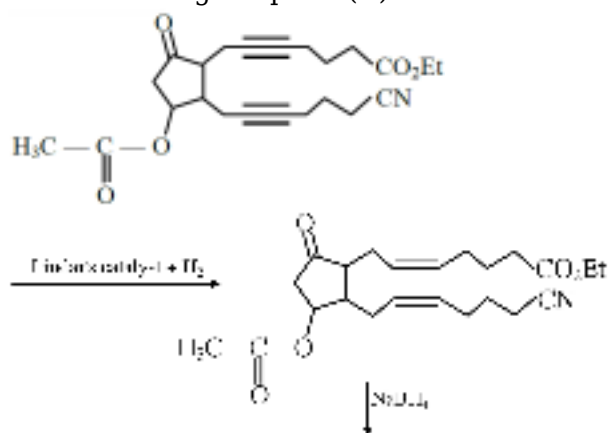
$$x = 0.89$$

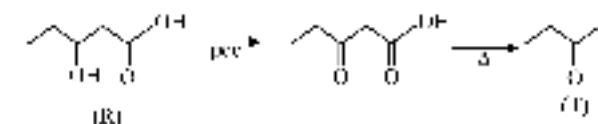
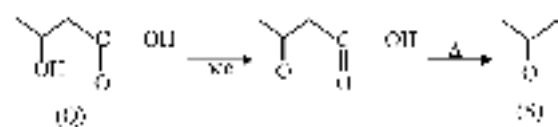
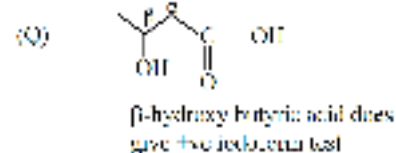
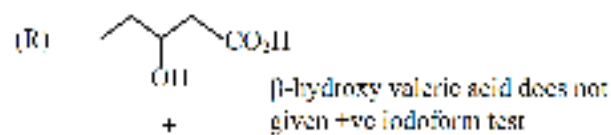
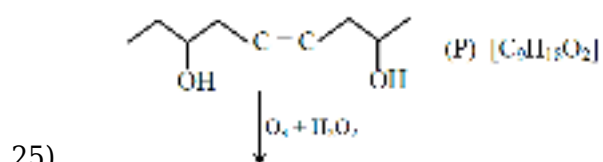
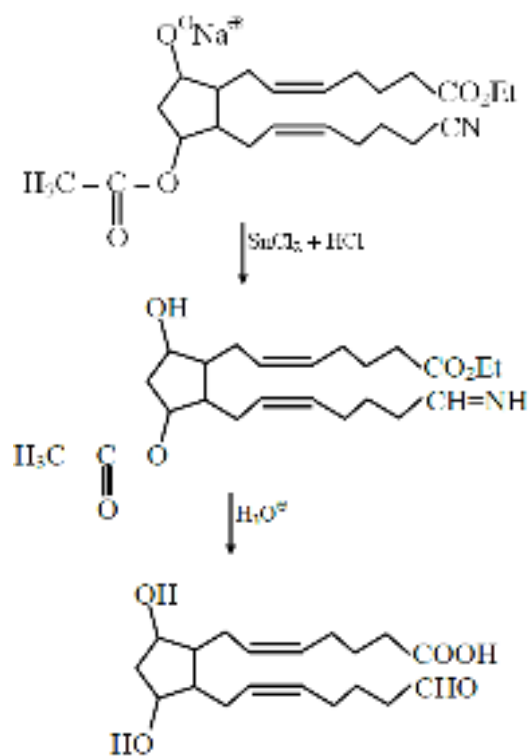


23) Path according to option (A)



Path according to option (D)



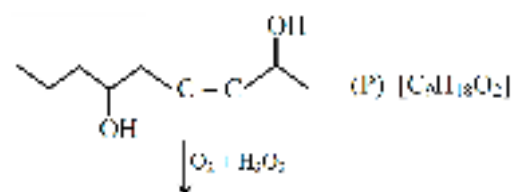


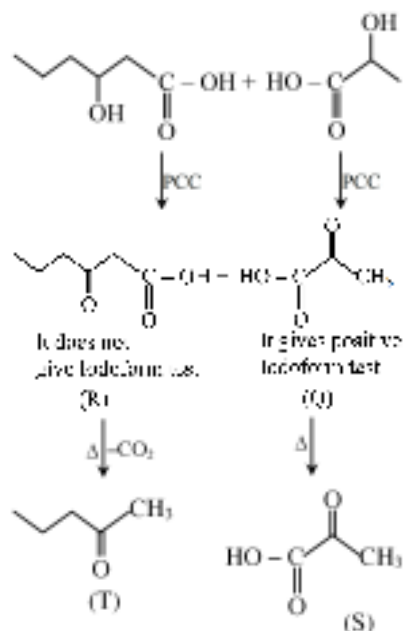
S & T shows +ve iodoform test.

Total sp^3 carbon atoms

present in S, T are = $2 + 3 = 5$

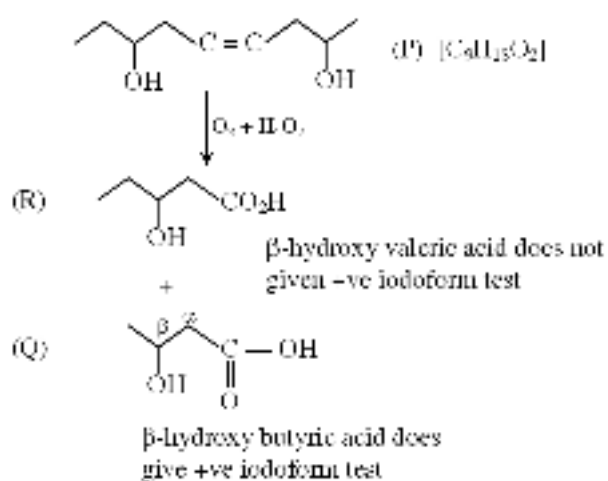
Ans. $\Rightarrow 5$



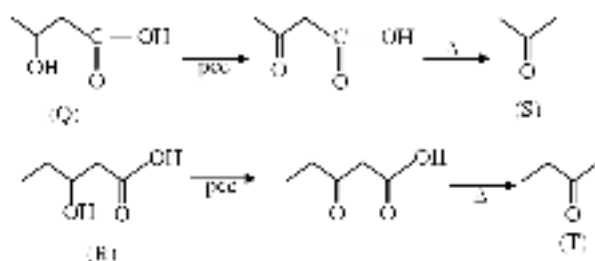


Both S and T show positive Iodoform

Total sp^3 carbon atoms present in S, T are = 4 + 1 = 5



26)



S & T shows +ve iodoform test.

R mf $\text{C}_5\text{H}_{10}\text{O}_3$ (M. wt)_R = 70 + 48 = 118

Q mf $\text{C}_4\text{H}_8\text{O}_3$ (M. wt)_Q = 56 + 48 = 104

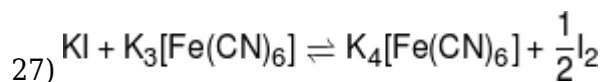
510 mole Q + 510 mole R $\xrightarrow[\text{polymer}]{\text{Condensation}}$ PHBV. (1 mole) V
Acyclic polymer

U molecular weight = (510 × 118 + 510 × 104) - 1019 × 18

U = 113220 - 18342 = 94878

Value of $\frac{U}{18} = \frac{94878}{18} = 5271$

Answer is = 5271



29) Millimole of acid taken = $100 \times 0.5 = 60$

Millimole of NaOH used = $40 \times 1 = 40$

Millimole of acid adsorbed = $60 - 40 = 20$

Molecules of acid adsorbed = $20 \times 10^{-3} \times 6 \times 10^{23} = 12 \times 10^{21}$

Surface area occupied per molecule = $\frac{1.5 \times 10^2}{12 \times 10^{21}} = 1250 \times 10^{-23}$

30) **Vessel - I :**

$$(\Delta T_b)_1 = i_1 \times K_b \times \frac{w_2/\text{GMM}_X}{w_1/1000}$$

Vessel - 2 :

$$(\Delta T_b)_2 = i_2 \times K_b \times \frac{w_2/\text{GMM}_Y}{w_1/1000}$$

$$\frac{(\Delta T_b)_1}{(\Delta T_b)_2} = \frac{i_1}{i_2} \times \frac{\text{GMM}_Y}{\text{GMM}_X} = \frac{1.2}{0.6} = 2$$

$$\left[\frac{(\Delta T_b)_1}{(\Delta T_b)_2} \right] \times 100 = \frac{2}{1} \times 100 = 200\%$$

31) A = T 2 H-bond

G \equiv C 3 H-bond

Number of A=T pair = 6

Number of G \equiv C pair = 8

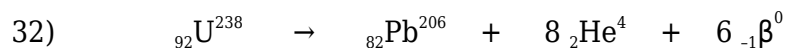
Number of H-bond involve in A = T = $6 \times 2 = 12$

Number of H-bond involve in G \equiv C = $8 \times 3 = 24$

Energy required for A = T = $12 \times 1 = 12$

Energy required for G \equiv C = $24 \times 1.5 = 36$

Total energy required $12 + 36 = 48$



$$t = 0 \quad N_0 = 1 + 15 = 16 \quad 0$$

$$t = t \quad N_t = 1 \quad 15$$

As per 1st order kinetics :

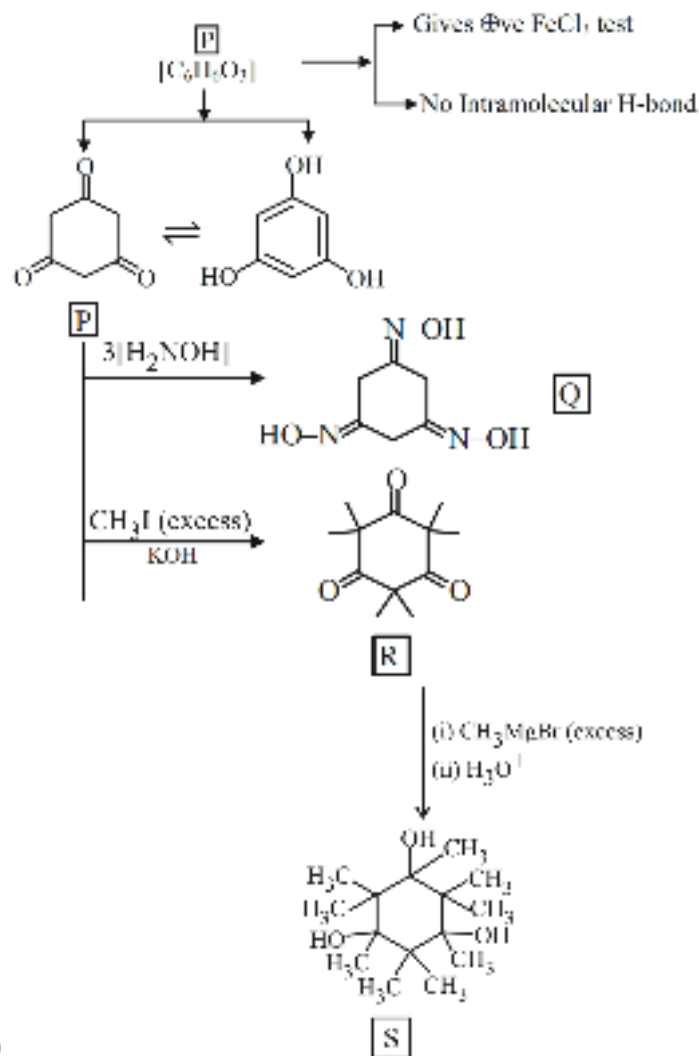
$$\lambda t = \ln \frac{N_0}{N_t}$$

$$f = \text{age} = 4 \times t_{1/2}$$

$$= 4 \times 4.5 \times 10^9 \text{ yrs.}$$

$$= 18 \times 10^9 \text{ yrs.}$$

$$= 180 \times 10^8 \text{ yrs.}$$



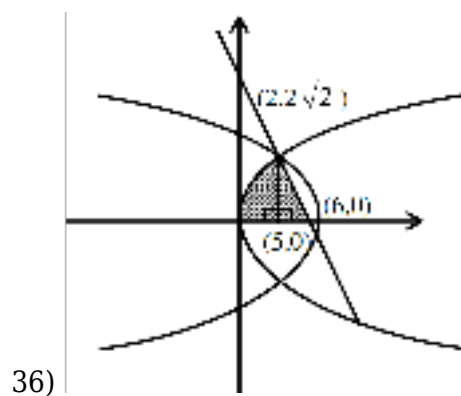
34)

No. of $-CH_3$ group (methyl group) in S is $\rightarrow 9$

PART-3 : MATHEMATICS

35) $\tan \left(\sin^{-1} \left(\frac{4}{5} \right) - 2\cos^{-1} \left(\frac{2}{\sqrt{5}} \right) \right)$

$\tan \left(\tan^{-1} \left(\frac{4}{5} \right) - \tan^{-1} \left(\frac{4}{3} \right) \right) = 0$



$$\text{Area} = \frac{2}{3} (2\sqrt{2}) (2) + \frac{1}{2} \cdot (3) (2\sqrt{2}) = \frac{8}{3}\sqrt{2} + 3\sqrt{2} = \frac{17}{3}\sqrt{2}$$

$$\begin{aligned} 37) \lim_{x \rightarrow 0^+} (\sin(\sin k^2 x) + \cos x - x)^{\frac{2}{x}} \\ = e^{\lim_{x \rightarrow 0^+} \frac{\sin(\sin k^2 x) + \cos x - x - 1}{x} \times 2} \quad (1^\infty \text{ form}) \\ e^{\lim_{x \rightarrow 0^+} \left[\left(\frac{\sin(\sin k^2 x)}{\sin k^2 x} \times \frac{\sin k^2 x}{k^2 x} \times k^2 \right) + \frac{(1 - \frac{x^2}{2} - x - 1)}{x} \right] \times 2} \\ \Rightarrow k^2 - 1 = 3 \Rightarrow k = 2 \end{aligned}$$

$$38) f(x) = x^2 \sin\left(\frac{\pi}{x^2}\right) = 0$$

$$\sin\left(\frac{\pi}{x^2}\right) = \sin \pi n$$

$$\frac{1}{x^2} = n$$

$$(A) \quad x \in \left(\frac{1}{10^{10}}, \infty\right) \quad \frac{1}{x} \in (0, 10^{10})$$

$$\frac{1}{x^2} \in (0, 10^{20}) \rightarrow \text{finite solutions}$$

$$(B) \quad \frac{1}{x} \in (0, \pi)$$

$$\frac{1}{x^2} \in (0, \pi^2) \rightarrow 9 \text{ solutions}$$

$$(C) \quad \frac{1}{x} \in (10^{10}, \infty) \rightarrow \text{infinite solutions}$$

$$(D) \quad \frac{1}{x} \in (\pi, \pi^2)$$

$$\frac{1}{x} \in (\pi^2, \pi^4) \rightarrow (9.8, 96.1) \text{ approx} \\ \text{more than 25 solutions}$$

$$\begin{aligned} 39) \lim_{x \rightarrow \infty} \frac{\sin(x^2)(\ln x)^\alpha \tan\left(\frac{1}{x^3}\right)}{x^{\alpha\beta+2}(\ln(1+x))^\beta \left(\frac{1}{x^2}\right)} \\ \lim_{x \rightarrow \infty} \frac{\sin(x^2)(\ln x)^{\alpha-\beta}}{x^{\alpha\beta+3}} \underbrace{\left(\frac{\ln x}{\ln(1+x)}\right)^\beta}_{\text{as } x \rightarrow \infty, \text{ tends to } 1} = 0 \end{aligned}$$

$$\text{Now, if } \alpha\beta + 3 > 0, \lim_{x \rightarrow \infty} \frac{(\ln x)^{\alpha-\beta}}{x^{\alpha\beta+3}} \rightarrow 0$$

□ limit = 0 if $\alpha\beta + 3 > 0$

40)

Line passing through 'P' is $\frac{(x-1)}{1} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$

$C_1 (\lambda + 1, 2\lambda + 3, \lambda + 2)$

put C_1 in $L \rightarrow (\lambda + 1) - (2\lambda + 3) + 3(\lambda + 2) = 6$

$\Rightarrow \lambda = 1 \Rightarrow Q(2, 5, 3)$

Also, line through Q is $\frac{(x-2)}{1} = \frac{(y-5)}{-1} = \frac{(z-3)}{3} = k$

$C_2 (k + 2, 5 - k, 3k + 3)$

put in $L_2 \rightarrow k = -1 \Rightarrow R(1, 6, 0)$

41)

$ty = x + at^2 \rightarrow (-8, 0) \Rightarrow 0 = -8 + 2t^2 \Rightarrow t = \pm 2$

$P(at^2, 2at) = P(2t^2, 4t)$

$A_1(8, 8), B_1(8, -8), C_1(-8, 0), O(0, 0)$

(A) $OA_1 = 8\sqrt{2}$

(B) $A_1B_1 = 16$

(D) orthocentre is (4, 0)

42)

Number of elements in $S \times S = 36$

Number of possible ordered pairs which satisfy given condition = $36 - 16 = 20$

$n(X) = {}^{20}C_6$

$m = 20$

43)

Maximum number of pre-images of any elements is 4

therefore no six ordered pairs can have same images

$n(Y) = 0$

All functions have six ordered pairs

number of possible function form S to S

$n(Z) = 4 \times 3 \times 3 \times 3 \times 3 \times 4 = 4^2 \times 3^4$

$n(Y) + n(Z) = 4^2 \times 3^4 = k^2$

$k = 36$

44)

$$I_1 = \int_0^{\pi/2} f(x)g(x) dx = \int_0^{\pi/2} (\sin^2 x) \sqrt{x \left(\frac{\pi}{2} - x \right)} dx$$

Let ... (1)

$$I_1 = \int_0^{\pi/2} \cos^2 x \sqrt{\left(\frac{\pi}{2} - x \right)} x dx$$

... (2)

$$2I_1 = \int_0^{\pi/2} (\sin^2 x + \cos^2 x) \sqrt{x \left(\frac{\pi}{2} - x \right)} dx$$

$$2I_1 = \int_0^{\pi/2} \sqrt{x \left(\frac{\pi}{2} - x \right)} dx$$

$$2I_1 - I_2 = 0$$

$$45) \quad I_1 = \frac{1}{2} \int_0^{\pi/2} \sqrt{x \left(\frac{\pi}{2} - x \right)} dx$$

$$I_1 = \int_0^{\pi/4} \sqrt{x \left(\frac{\pi}{2} - x \right)} dx = \int_0^{\pi/4} \sqrt{\left(\frac{\pi}{4} - x \right) \left(\frac{\pi}{2} - \frac{\pi}{4} + x \right)} dx$$

$$I_1 = \int_0^{\pi/4} \sqrt{\left(\frac{\pi}{4} \right)^2 - x^2} dx$$

$$I_1 = \frac{x}{2} \int_0^{\pi/4} \sqrt{\left(\frac{\pi}{4} \right)^2 - x^2} + \frac{(\pi/4)^2}{2} \sin^{-1} \left(\frac{x}{\pi/4} \right) \Bigg|_0^{\pi/4}$$

$$I_1 = \left(0 + \frac{\pi^2}{32} \times \frac{\pi}{2} \right) - (0 + 0) = \frac{\pi^3}{64}$$

$$\frac{64}{\pi^3} I_1 = \frac{4}{4} = 1$$

46)

$$f(0) = 0$$

$$f\left(\frac{-3}{5} + \frac{3}{5}\right) = f\left(\frac{-3}{5}\right) + f\left(\frac{3}{5}\right) \Rightarrow f\left(\frac{3}{5}\right) = -12$$

$$f(2x) = 2f(x)$$

$$f(3x) = f(2x) + f(x) = 3f(x)$$

$$f(4x) = 4f(x)$$

$$f(5x) = 5f(x)$$

$$f(nx) = nf(x)$$

$$x = \frac{3}{5}, n = 5 \quad f(3) = 5f\left(\frac{3}{5}\right) = -60$$

$$n = 1, x = 1 \quad f(3) = 3f(1) = -60 \Rightarrow f(1) = -20$$

$$n = 4, x = \frac{1}{4} \quad f(1) = 4f\left(\frac{1}{4}\right) \Rightarrow -20 = 4f\left(\frac{1}{4}\right) \Rightarrow f\left(\frac{1}{4}\right) = -5$$

$$g(x + y) = g(x) g(y)$$

$$g(0) = g^2(0) \Rightarrow g(0) = 1$$

$$g(nx) = g^n(x)$$

$$n = 6, x = -\frac{1}{3} \quad g(-2) = g^6\left(-\frac{1}{3}\right) = (2)^6 = 64$$

$$\left(f\left(\frac{1}{4}\right) + g(-2) - 9\right)_{g(0)}$$

$$(-5 + 64 - 9) \times 1 = 50$$

47)

$$P(W_1 \cap G_2 \cap B_3) = \frac{1}{4N}$$

$$\frac{3}{N} \cdot \frac{6}{N-1} \cdot \frac{1}{N-2} = \frac{1}{4N}$$

$$\Rightarrow N = 65, 10$$

$$\text{Also } \frac{P(B_3 \cap W_1 \cap G_2)}{P(W_1 \cap G_2)} = \frac{1}{8}$$

$$\Rightarrow \frac{\frac{1}{4N}}{\frac{3}{N} \cdot \frac{6}{N-1}} = \frac{1}{8}$$

$$\Rightarrow N = 10$$

48)

$$f(x) = 0 \Rightarrow x^{2025} + 2024x + 2025 = 0$$

As LHS is strictly increasing, number of solutions is '1'

($\sin x + 4 \neq 0$ if $x \in \mathbb{R}$)

$$49) \vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{q} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(1+3) - \hat{j}(2-3) + \hat{k}(-2,-1) = 4\hat{i} + \hat{j} - 3\hat{k}$$

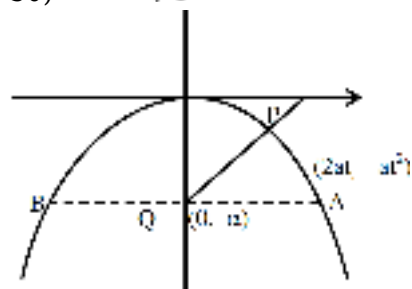
Given equation

$$5\hat{i} + 2\hat{j} + 3\hat{k} = \alpha(2\vec{p} - \vec{q}) + \beta(\vec{p} + 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$$

$$(4\hat{i} + \hat{j} - 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} + \hat{k}) = \gamma|\vec{p} \times \vec{q}|^2$$

$$\gamma = \frac{26}{19}$$

$$50) m_N = \frac{1}{\sqrt{6}}$$



$$x^2 = -4ay$$

$$2x = -4ay'$$

For More Material Join: @JEEAdvanced_2025

$$y' = \frac{x}{-2a} = \frac{2at}{-2a} = -t$$

$$m_N = \frac{1}{t} = \frac{1}{\sqrt{6}} \Rightarrow t = \sqrt{6}$$

$$P(2\sqrt{6}a, -6a)$$

$$\text{equation of normal } (y + 6a) = \frac{1}{\sqrt{6}}(x - 2\sqrt{6}a)$$

$$(y + 6a) = \frac{1}{\sqrt{6}}(x - 2\sqrt{6}a)$$

$$y + 6a = -2a$$

$$y = -2a - 6a = -8a$$

$$\alpha = 8a$$

Equation of line L

$$y = -8a$$

$$x^2 = -4ay$$

$$x^2 = +32a^2 \Rightarrow x = \pm 4\sqrt{2}a$$

$$AB = 8\sqrt{2}a$$

$$S = 128a^2$$

$$r = 4a$$

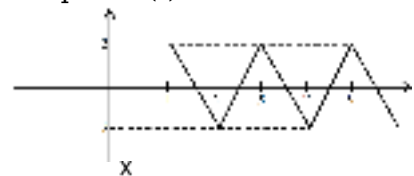
$$\frac{r}{S} = \frac{4a}{128a^2} = \frac{1}{32} \Rightarrow a = 1$$

$$51) f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ \frac{(2n+1-t)}{2}(-1)^{n+1} \cdot 2 + \frac{(t-2n+1)}{2}(-1)^{n+2} \cdot 2 & ; 2n-1 < t < 2n+1 \end{cases}$$

$$f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ (-1)^{n+1}[2n+1-t-t+2n-1] & ; 2n-1 < t < 2n+1 \end{cases}$$

$$f(t) = \begin{cases} (-1)^{n+1}2 & ; t = 2n-1 \\ (-1)^{n+1}2(2n-t) & ; 2n-1 < t < 2n+1 \end{cases}$$

Graph of $f(t)$



$$g(x) = \int_1^x f(t) dt$$

Number of solution of equation $g(x) = 0$ in $(1, 10]$ is 4

$$\alpha = 4$$

$$\beta = \lim_{x \rightarrow 1^+} \frac{g(x)}{x-1} = \lim_{h \rightarrow 0} \frac{g(1+h)}{h} = \lim_{h \rightarrow 0} \frac{\int_1^{1+h} f(t) dt}{h}$$

$$\text{Apply L'Hospital } \beta = \lim_{h \rightarrow 0} \frac{f(1+h)}{1} = 2$$

$$\alpha + \beta = 4 + 2 = 6$$