unacademy

IIT-JEE

Batch – Growth (July) | Major Test – 3 |Paper - 1

Time: 3 Hours **Test Date: 12th January 2025** **Maximum Marks: 198**

Name of the Candidate: _____Roll No._____

Centre of Examination (in Capitals):_____

Candidate's Signature: _____ Invigilator's Signature:_____

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 54 questions.
3. This question paper is divided into three parts **PART A - PHYSICS, PART B - CHEMISTRY** and **PART C - MATHEMATICS** having 19 questions each and every **PART** has three sections.
   - **(i)** **Section-I** contains 4 Only One Correct Type questions.
     Marking scheme: +3 for correct answer, 0 if not attempted and –1 in all other cases.
   - **(ii)** **Section-II** 3 Paragraphs with 2 questions for each (Numerical Value).
     Marking scheme: +3 for correct answer, 0 if not attempted and 0 in all other cases.
   - **(iii)** **Section-III** contains 6 questions One or More than One Correct Answers
     Marking scheme: (+4 for correct answer, 0, if not attempted and +1 partial marking —2 in all other cases.
   - **(iv)** **Section-IV** contains 3 Non-Negative Integer questions.
     Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For integer-based questions, the answer should be in decimals only not in fraction.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

TEST CODE : 114105

unacademy

TEST SYLLABUS

Batch – Growth (July) | Major Test-03

12th January 2025

Mathematics: Circle Binomial Theorem Permutation & Combination, (Probability-NCERT)

Physics: Rotational Motion Gravitation Elasticity, Thermal Expansion, Calorimetry and Heat Transfer

Chemistry: Chemical Eq Ionic Eq Redox Reaction, Nomenclature

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821\,Lit\ atm\ K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}\,Js$
		$= 6.25 \times 10^{-27}\,erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66 \times 10^{-27}\,kg$
1 eV		$= 1.6 \times 10^{-19}\,J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:

Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I (Only One Correct Type)

1. The coefficient of x^2 in $(1+x)(1+2x)(1+4x).....(1+2^n x)$ is

(A) $\frac{(2^n+1)(2^n+2)}{3}$ (B) $\frac{(2^{n+1}-1)(2^{n+1}-2)}{3}$ (C) $\frac{(2^{n+1}+1)(2^{n+1}+2)}{3}$ (D) $\frac{(2^{n+1}+1)(2^{n+1}-2)}{5}$

Ans. (B)

Sol. Coefficient of $x^2 = \frac{1}{2}\left[\left(1+2+2^2.....+2^n\right)^2\right]$

$-\frac{1}{2}\left[1^2+2^2+....\left(2^n\right)^2\right]$

$=\frac{1}{2}\left[\left(\frac{2^{n+1}-1}{1}\right)^2-\left(\frac{\left(2^2\right)^{n+1}-1}{3}\right)\right]$

$=\frac{1}{2}\left[\left(2^{n+1}-1\right)\left[\left(2^{n+1}-1\right)-\frac{2^{n+1}+1}{3}\right]\right]$

$=\frac{\left(2^{n+1}-1\right)\left(2^{n+1}-2\right)}{3}$

Hence, the correct answer is (B).

2. The number of words that can be formed using the letters of the word ARTICLE which either begin with A or end with E or the middle letter I is

(A) 344 (B) 1824 (C) 720 (D) 252

Ans. (B)

Sol. n(A): number of words starting with A

$n(B)$: number of words starting with E

$n(C)$: number of words starting with middle term I

$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$

= 6! + 6! + 6! - 5! - 5! - 5! + 4! = 1824

Hence, the correct answer is (B).

3. In ΔABC, equation of side BC is $x-y=0$. Circumcentre and orthocentre of the triangle are $(2,3)$ and $(5,8)$, respectively. Equation of circumcircle of the triangle is:

(A) $x^2+y^2-4x+6y-27=0$ (B) $x^2+y^2-4x-6y-27=0$

(C) $x^2+y^2+4x+6y-27=0$ (D) $x^2+y^2+4x-6y-27=0$

Ans. (B)

Sol. $\because$ Reflection of (5,8) in BC will lie on circumcircle. $\because (8,5)$ will lie on circumcircle $\because$ Equation of circumcircle is

$(x-2)^2+(y-3)^2=(8-2)^2+(3-5)^2$.

$\Rightarrow x^2+y^2-4x-6y-27=0$

Hence, the correct answer is (B).

4. There is a five-volume dictionary among 50 books arranged on a shelf in random order. The probability that these volumes stand in increasing order from left to right(the volumes are not necessarily kept side-by-side) is

(A) $1/5^{50}$ (B) 1/5 (C) $1/50^5$ (D) 1/120.

Ans. (D)

Sol. Total ways $= 50!$

Favourable ways $= {}^{50}C_5 \times 45!$

$\therefore$ Probability $= \dfrac{{}^{50}C_5 \cdot 45!}{50!} = \dfrac{1}{120}$

Hence, the correct answer is (D).

SECTION – II Numerical Value

Paragraph-1 (Steam)

(If more than two decimal, truncate/roundoff the value two decimal places).

An equation of the family of circles passing through a given pair of points (x_1, y_1) and (x_2, y_2) can be taken as $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+k\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, k being a real parameter. If a member of this family satisfies some other conditions, then it enables us to determine k.

5. The number of values of $\lambda \in R$ for which there exists exactly one circle passing through the points (2,-3) and $(\lambda, 2\lambda - 1)$ and touching the line 16x-2y+27=0 is.

Ans. (2)

Sol. There will exist exactly one circle if the line passing through $A(2,-3)$ and $B(\lambda, 2\lambda-1)$ is parallel to the given line$16x - 2y + 27 = 0$. Also, if the point $B(\lambda, 2\lambda-1)$ lies on the line 16x-2y +27=0, then we will have exactly one circle. Thus, two values of λ are possible.

6. There exist exactly two circles that pass through (3,-5) and (5,-3) and touch the line $2x + 2y + 13 = 0$. Let the ratio of radii of the two circles be $\dfrac{m}{n}$ with $m(>0)$ and $n(>0)$ having no common factors except 1. Then the value of $(m+n)$ is.

Ans. (2)

Sol. The line joining (3,-5) and (5,-3) has slope 1 and thus it is perpendicular to $2x + 2y + 13 = 0$. Hence, the two circles will have same radii.

Paragraph-2 (Steam)

If m,n,r are positive integers and if $r<m, r<n$ then ${}^{m}C_{r}+{}^{m}C_{r-1}\,{}^{n}C_{1}+{}^{m}C_{r-2}\,{}^{n}C_{2}+\ldots\ldots\ldots\ldots+{}^{n}C_{r}=$ coefficient of x^r in $(1+x)^m(1+x)^n=$ coefficient of x^r in $(1+x)^{m+n}={}^{m+n}C_{r}$

Based on above information answer the following

7. The value of $r(0\le r\le 30)$ for which ${}^{20}C_{r}.{}^{10}C_{0}+{}^{20}C_{r-1}.{}^{10}C_{1}+\ldots\ldots\ldots\ldots+{}^{20}C_{0}.{}^{10}C_{r}$ is minimum, is

Ans. (0)

Sol. ${}^{20}C_{r}.{}^{10}C_{0}+{}^{20}C_{r-1}.{}^{10}C_{1}+\ldots\ldots\ldots+{}^{20}C_{0}.{}^{10}C_{r}$

$=$ Coefficient of x^r in $(1+x)^{20}.(1+x)^{10}$

$=$ Coefficient of x^r in $(1+x)^{30}={}^{30}Cr$

${}^{30}Cr$ is minimum if $r=0$

8. If $S_n={}^{n}C_{0}.{}^{n}C_{1}+{}^{n}C_{1}.{}^{n}C_{2}+\ldots\ldots+{}^{n}C_{n-1}.{}^{n}C_{n}$ and if $\frac{S_{n+1}}{S_n}=\frac{15}{4}$ then sum of values of n is

Ans. (6)

Sol. $S_n={}^{n}C_{0}.{}^{n}C_{1}+{}^{n}C_{1}.{}^{n}C_{2}+\ldots\ldots+{}^{n}C_{n-1}.{}^{n}C_{n}$

$={}^{n}\mathbf{C}_{0}.{}^{n}\mathbf{C}_{n-1}+{}^{n}\mathbf{C}_{1}.{}^{n}\mathbf{C}_{n-2}+\ldots\ldots\ldots+{}^{n}\mathbf{C}_{n-1}.{}^{n}\mathbf{C}_{0}$

$=$ Coefficient of x^{n-1} in $(1+x)^n(1+x)^n={}^{2n}C_{n-1}$

$\therefore\frac{S_{n+1}}{S_n}=\frac{{}^{(2n+2)}C_{n}}{{}^{2n}C_{n-1}}=\frac{15}{4}\Rightarrow n=2,4$

Hence, 6 is the answer.

Paragraph-3 (Steam)

Let' S' be the set of first 18 natural numbers. The number of ways of selecting from 'S'

9. Three numbers such that they form an AP is

Ans. (72)

Sol. If a,b,c are in AP $\Rightarrow 2b=a+c$

$\therefore$ Both a,c are odd (or)both are even

$\Rightarrow$ number of ways $=2\left({}^{9}C_{2}\right)=72$

10. Two number such that the sum of their cubes is divisible by 3 is ...

Ans. (51)

Sol.

R_1	1	4	7	10	13	16
R_2	2	5	8	11	14	17
R_3	3	6	9	12	15	18

x^3+y^3 is divisible by '3'

$\Rightarrow$ we can select both x,y from R_3(or) one element from R and another element from R_2 and it can be done is ${}^{6}C_{2}+{}^{6}C_{1}\;\;{}^{6}C_{1.}=15+36=51$

SECTION-III (One or More than One Correct)

11. In a ΔABC, let $BC = 13, CA = 14$ and $AB = 15$. If I is the incentre of the ΔABC and if the incircle meets the sides AB, BC and CA at the points P, Q and R, respectively then which of the following options are correct?

(A) AP = 8

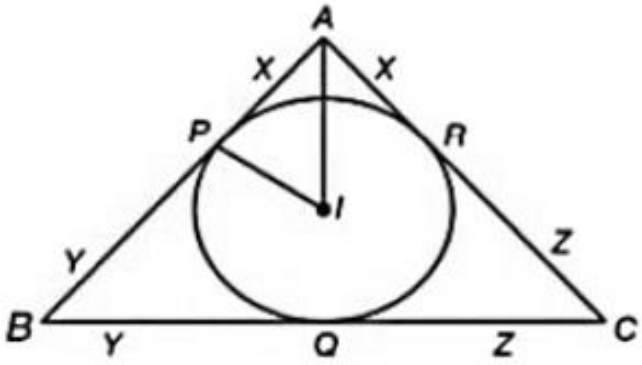
(B) Area of the ΔAPR is $\frac{128}{5}$ sq. units

(C) Area of the quadrilateral APIR is 32 sq. units

(D) $AI = 4\sqrt{5}$

Ans. (A, B, C, D)

Sol.



Let, AP = AR = x

BP = BQ = y

CQ = CR = z

Then, $x + y + z = \frac{a+b+c}{2} = 21$

$y + z = 13, z + x = 14, x + y = 15$

$\Rightarrow x = 8, y = 7, z = 6$

Also, area of the ΔABC is

$\Delta = \sqrt{S(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6} = 84$

The inradius is $r = \frac{\Delta}{s} = \frac{84}{21} = 4$

Area of the ΔAPR is $\frac{rL^s}{L^2 + r^2} = \frac{4.(8)^\circ}{64 + 16} = \frac{128}{5}$ sq. units

Area of the quadrilateral $APIR = rL = 4 \times 8 = 32$ sq. units

$AI = \sqrt{AP^2 + PI^2} = \sqrt{64 + 16} = 4\sqrt{5}$

Hence, (A), (B), (C) and (D) are correct.

12. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are the tangents to the same circle, then:

(A) radius of the circle $= \frac{3}{4}$

(B) radius of the circle= $\frac{3}{2}$

(C) centre of the circle lies on 12x- 16y+ 1= 0

(D) centre of the circle lies on 12x- 16y+ 31= 0

Ans. (A, C)

Sol. The given lines being parallel tangents to a circle, the diameter of the circle is equal to the distance between these lies, so that the required radius is $\frac{1}{2}\times\frac{\left|4+\frac{7}{2}\right|}{\sqrt{9+16}}=\frac{1}{2}\times\frac{15}{2}\times\frac{1}{5}=\frac{3}{4}$

The centre of the circle lies on the line parallel to the given lines at a distance of $\frac{3}{4}$ from each of them So, let the equation be 3x-4y+k=0 ....(1)

Then, $\left|\frac{k-4}{\sqrt{9+16}}\right|=\frac{3}{4}$

$\Rightarrow\ k=4\pm\left(\frac{15}{4}\right)$

$\Rightarrow\ \frac{31}{4}$

For k = $\frac{1}{4}$, distance of (1) from the other line is $\frac{3}{4}$.

Thus, the centre lies on the line 12x - 16y + 1 = 0.

Hence, (A) and (C) are correct.

13. A cubical die is thrown 9 times and the numbers obtained are written as a 9-digit number. The probability that the number.

(A) begins with 246 is $1/6^3$

(B) ends with 135 is $1/6^3$

(C) begins with 246 and ends with 135 is $1/6^6$

(D) begins with 246 or ends with 135 is $431/6^6$

Ans. (A, B, C, D)

Sol. (A) Fixing 2,4,6 in the first 3 places; we can fill the remaining 6 places in 6^6 ways

Hence probability $\frac{6^6}{6^9}=\frac{1}{6^3}$

(B) Same as (A) fix number 135 in the end and fill the remaining 6 places in 6^6 ways $\mathbf{P}=\frac{6^6}{6^9}=\frac{1}{6^3}$

(C) Fix 246 and 135 at respective places. Now fill the remaining 3 places in 6^3 ways $\mathbf{P}=\frac{6^3}{6^9}=\frac{1}{6^6}$

(D) Begin with 246 or end with 135 $P(E\cup F)=P(E)+P(F)-P(E\cap F)=\frac{1}{6^3}+\frac{1}{6^3}-\frac{1}{6^6}=\frac{431}{6^6}$

Hence, the correct answer is (A, B, C, D).

14. If S = {1,3,5,7} then which of the following is/are true?

(A) Sum of all four-digit numbers if repetition of digits is not allowed is equal to 3!.16(1111)

(B) Sum of all four-digit numbers if repetition of digits is allowed is equal to 432(1111)

(C) Sum of all three-digit numbers if repetition of digits is not allowed is equal to 5328

(D) Sum of all the digits in the units place of all four digited numbers if repetition of digits is allowed is equal to 433.

Ans. (A)

Sol. (A) 3! (11 1 1) (1+3+5+7)

(B) $(1111)(1+3+5+7).4^3$

(C) ${}^3P_2\ (1\ 1\ 1)\left(1+3+5+7\right) \neq 5328$

(D) $4^3\left(1+3+5+7\right) = 432$

Hence, the correct answer is (A).

15. The 9 horizontal and 9 vertical lines on an 8×8 chessboard form 'r' rectangles and's' squares. Then which of the following is/are true?

(A) No. of rectangles = 1296

(B) No. of squares = 204

(C) No. of non – congruent rectangles = 36

(D) No. of non – congruent squares = 1

Ans. (A, B, C)

Sol. No. of squares are

$$S = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 8^2 = \frac{8(9)(17)}{6} = 204$$

No. of rectangles $r = {}^9C_2 \cdot {}^9C_2 = 1296$

No. of non-congruent rectangles $= \sum_0^8 r = 36$

No. of non-congruent squares=8

Hence, (A), (B)and(C)are correct.

16. Which of the following is true?

(A) If $x^2 < 1$, the coefficient of x^n in $\frac{(1+2x)^n + (1+3x)^n}{1-x}$ is $3^n + 4^n$

(B) The value of $\sum_{r=0}^{n}(-1)^r \mathbf{C}_r \left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \ldots\ldots\infty\right)$ is equal to $\frac{1}{2^n - 1}$

(C) The remainder obtained when 1!+2!+3!+....+95! is divisible by 15 is 3

(D) The coefficient of $x^2 \sin\left(1 + x + x^2\right)^{10}$ is ${}^{12}C_2$.

Ans. (A, B, C)

Sol. $(A)\, x^2 < 1.$ Given expression $\left((1+2x)^n + (1+3x)^n\right)(1-x)^{-1}$

$$\therefore x^n = \sum_{r=0}^{n} {}^nC_r .2^r + \sum_{r=0}^{n} {}^nC_r .3^r = 3^n + 4^n.$$

(B) $$\sum_{r=0}^{n}(-1)^r\, {}^nC_r \left(\frac{1}{2^r} + \left(\frac{3}{4}\right)^r + \left(\frac{7}{8}\right)^r + \ldots.\right) = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \ldots. = \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{2n} + \ldots. = \frac{1/2^n}{1-1/2^n} = \frac{1}{2^n - 1}$$

(C) $1! + 2! + 3! + 4! = 33$ and $15 / n!$ for $n \geq 5$ Required remainder 3.

(D) $$\frac{10!}{2!8!} + \frac{10!}{1!9!} = {}^{10}C_2 + {}^{10}C_1$$

$$= {}^{11}C_2$$

Hence,the correct answer is (A,B,C).

SECTION – IV (Non-Negative Integer)

17. If number of numbers greater than 3000, which can be formed by using the digits 0,1,2,3,4,5 without repetition, is n then $\frac{n}{230}$ is equal to

Ans. (6)

Sol. No. of 4 digit numbers $= 3\times5\times4\times3 = 180$

No. of 5 digit numbers=5 x 5 x 4 x 3 x 2 =600

No. of 6 digit numbers= 5× 5×4×3×2= 600

$n = 1380$

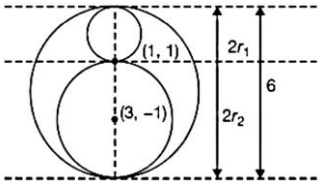
$\Rightarrow \frac{n}{230} = 6$

Hence, the correct answer is (6).

18. Circles are drawn through (1,1) touching the circle $x^2 + y^2 - 6x + 2y + 1 = 0$. If r_1 and r_2 are the radii of the smallest and the largest circles, then find the value of $\left(r_2 + r_1\right)^2 - \left(r_2 - r_1\right)^2$.

Ans. (1)

Sol.



$2r_1 + 2r_2 = 6$

$\Rightarrow r_1 + r_2 = 3$

$\text{Also}, 2r_2 - 2\sqrt{2} = 3$

$\Rightarrow r_2 = \frac{3+2\sqrt{2}}{2}$

$\therefore r_1 = \frac{3-2\sqrt{2}}{2}$

$G.E = 4r_1r_2 = 4\times\frac{1}{4} = 1$

Hence, the correct answer is (1).

19. If the coefficient of x^{24} in $\left(\frac{C_1}{C_0} - x\right)\left(x - 2^2.\frac{C_2}{C_1}\right)\left(x - 3^2.\frac{C_3}{C_2}\right)$ ..... $\left(x - 25^2.\frac{C_{25}}{C_{24}}\right)$ where C_r stands for $^{25}C_r$ is 3000-25K then K is

Ans. (3)

Sol. $\frac{C_1}{C_0} + 2^2.\frac{C_2}{C_1} + 3^2.\frac{C_3}{C_2} + ...25^2.\frac{C_{25}}{C_{24}} =$

$1\times25 + 24\times2 + 23\times3 ..... 1\times25 = \sum_{r=1}^{25} r\left(26 - r\right) = 2925 = 3000 - 75$

$= 2925 = 3000 - 75$

Hence, the correct answer is (3).

PART-B: PHYSICS

SECTION-I (Only One Correct Type)

20. A rod of length L is pivoted at an end. The linear mass density of the rod λ varies with the distance x from this end as $\lambda = ax^2 + b$ kgm^{-1}, where a and b are positive constants. Find the moment of inertia of the rod about the axis passing through this end and perpendicular to its length.

(A) $\frac{2aL^5}{5} + \frac{2bL^3}{3}$ (B) $\frac{2aL^5}{3} + \frac{2bL^3}{5}$ (C) $\frac{aL^5}{7} + \frac{bL^3}{3}$ (D) $\frac{aL^5}{5} + \frac{bL^3}{3}$

Ans. (D)

Sol. Consideran element of length dx at a distance x from the pivoted end. At the location of element $\lambda = ax^2 + b$.

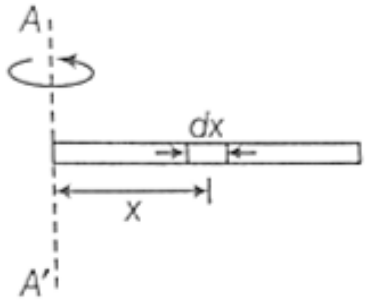
Mass of element is

$dm = \lambda dx = \left(ax^2 + b\right)dx$

$\left(dI\right)_{AA'} = \left(dm\right)x^2 = \left(ax^2 + b\right)x^2 dx$

$I_{AA'} = \int_0^L \left(ax^4 + bx^2\right)dx$

$= \frac{aL^5}{5} + \frac{bL^3}{3}$



21. The acceleration due to gravity at the surface of moon is only one-sixth of that of the earth. If the earth and moon are assumed to have same density, then ratio of the radius of the moon to radius of the earth is

(A) $\frac{1}{6}$ (B) $\frac{1}{(6)^{1/3}}$ (C) $\frac{1}{36}$ (D) $\frac{1}{(6)^{2/3}}$

Ans. (A)

Sol. $g = \frac{GM}{R^2} = \frac{4}{3}\pi GR\rho$

$\frac{g_m}{g_e} = \frac{R_m}{R_e} \Rightarrow \frac{R_m}{R_e} = \frac{1}{6}$

22. Depth of sea is maximum at Mariana Trench in West Pacific Ocean. Trench has a maximum depth of about 11 km. At bottom of trench water column above exerts over 1000 atm pressures. Percentage change in density of sea water at such depth will be around (Given, B = 2×10^9 Nm^{-2} and $P_{atm} = 1 \times 10^5$ Nm^{-2})

(A) about 5% (B) about 10% (C) about 3% (D) about 7%

Ans. (A)

Sol. Change in volume,

$$\Delta V = \frac{-\Delta p \cdot V_i}{B}$$

Hence, density at depth of about 11 km is

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{\rho_0 \times V_i}{V_i - \frac{\Delta \rho V_i}{B}} = \frac{\rho_0 B}{(B - \Delta\rho)}$$

$$= \frac{\rho_0}{1 - \frac{\Delta p}{B}} = \frac{\rho_0}{1 - \frac{1\times 10^8.}{2\times 10^9}} = \frac{\rho_0}{1 - \frac{1}{20}} = \frac{\rho_0}{0.95}$$

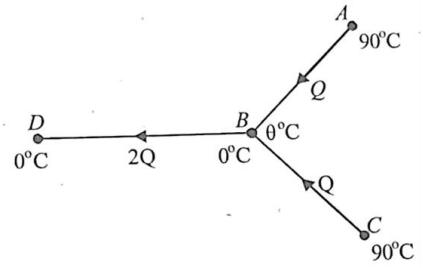
$$\rho = \frac{\rho_0}{0.95} \Rightarrow \rho_0 = 0.95\rho$$

23. Three rods made of same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0° C and 90°C respectively. The temperature of the junction of the three rods will be

(A) 45°C (B) 60°C (C) 30°C (D) 20°C

Ans. (B)

Sol. Let θ° C bethe of temperature junction at B. Let ℓ is the heat flowing per second from A at 90°C to B at θ°C on account of temperature difference.



$$\therefore Q = \frac{KA(90 - \theta)}{\ell} \quad ...(i)$$

And same for $\tilde{C}$ to B.

$$Q = \frac{KA(90 - \theta)}{l} \quad ...(ii)$$

$\therefore$ The heat flowing per second from B to D

$$2Q = \frac{KA(\theta - 0)}{l} \quad ...(ii)$$

Dividing eq. (ii) by (i)

$$2 = \frac{\theta}{90 - \theta} \Rightarrow \theta = 60^\circ$$

Hence temperature of the junction $\theta = 60^\circ$C

SECTION – II Numerical Value

Paragraph-1 (Steam)

(If more than two decimal, truncate/roundoff the value two decimal places).

The mean radius of the earth's orbit around the sun is 1.5×10^{11} m and that of the orbit of mercury is 6×10^{10} m.

24. The mercury will revolve around the sun in nearly $\left(\frac{n}{5}\right)^{3/2}$ yr then n is _____.

Ans. (2)

Sol. $T^2 \propto r^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 1\times\left(\frac{6}{15}\right)^{3/2} \text{ or } T_2 = \left(\frac{2}{5}\right)^{3/2} \text{ yr}$$

25. The ratio of the orbital velocity of mercury to that of the earth is $\sqrt{\frac{n}{4}}$ then n is _____

(assuming orbits to be circular)

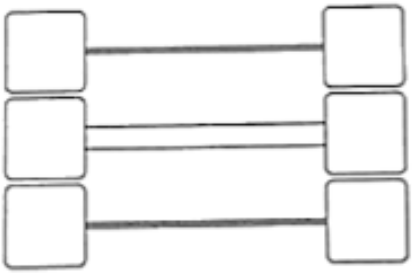
Ans. (10)

Sol. $v_0 \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{(v_0)_1}{(v_0)_2} = \sqrt{\frac{r_2}{r_1}}$

$$\frac{(v_0)_2}{(v_0)_1} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{15}{6}} = \sqrt{\frac{5}{2}} = \sqrt{\frac{10}{4}}$$

Paragraph-2 (Steam)

A steel bolt of cross-sectional area $A_b = 5\times10^{-5}$ m ^{2} is passed through a cylindrical tube made of aluminum. Cross-sectional area of the tube material is $A_t = 10^{-4}\,m^2$ and its length is $l = 50cm$. The bolt is just taut so that there is no stress in the bolt and temperature of the assembly increases through $\Delta\theta = 10^\circ$ C. The increase in length of the system is 0. 075 mm. Given, coefficient of linear thermal expansion of steel is $\alpha_b = 10^{-5^\circ}C^{-1}$,Young' s modulus of steels $Y_b = 2\times10^{11}Nm^{-2}$,Young' s modulus of aluminum is $Y_t = 10^{11}Nm^{-2}$ and coefficient of linear expansion of Al is αt = 2× 10-5 °C^{-1}.



26. The compressive strain in the tube is $n\times10^{-5}$ then n is _____

Ans. (5)

Sol. $\alpha_t > \alpha_b$.So, when the assembly is heated, tube tries to expand more than the bolt. But the tube is tightened by the bolt. Therefore, its expansion cannot be more than that of the bolt. Hence, a compressive stress is developed in the tube and a tensile stress is developed in the bolt. Let the initial length of the assembly be L and let its elongation be ΔL.

Elongation of the tube if it would be free to expand $= L\alpha_t\Delta\theta$ But its actual elongation is ΔL.So, elongation prevented in it $= L\alpha_t\Delta\theta - \Delta L$ So, compressive strain in the tube $= \dfrac{L\alpha_t\Delta\theta - \Delta L}{L}$

$= \dfrac{50\times2\times10^{-5}\times10 - 0.0075}{50} = 5\times10^{-5}$

27. The compressive stress in the tube is $n\times10^{6}\ Nm^{-2}$ then n is _____

Ans. (5)

Sol. Compressive stress in the tube is

$\sigma_t = \varepsilon_t Y_t$

Or $\sigma_t = \left(5\times10^{-5}\right)10^{11} = 5\times10^{6}\,Nm^{-2}$

Paragraph-3 (Steam)

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2.Both the discs rotate in the clockwise direction.

28. The ratio $\dfrac{x_1}{x_2}$ is $\dfrac{\sqrt{n}}{1}$ then, n is _____

Ans. (2)

Sol. $\dfrac{1}{2}I(2\omega)^2 = \dfrac{1}{2}kx_1^2 \quad \cdots(i)$

$\dfrac{1}{2}(2I)(\omega)^2 = \dfrac{1}{2}kx_2^2 \cdots(ii)$

From Eqs. (i) and (ii), we have

$\dfrac{x_1}{x_2} = \sqrt{2}$

29. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is $\dfrac{2I\omega}{xt}$ then, x is _____

Ans. (3)

Sol. Let ω' be the common velocity. Then, from conservation of angular momentum, we have

$(1+2I)\omega' = I(2\omega) + 2I(\omega)$

$\omega' = \dfrac{4}{3}\omega$

From the equation,

Angular impulse=Change in angular momentum

For any of the disc, we have

$$\tau t = I(2\omega) - I\left(\frac{4}{3}\omega\right) = \frac{2I\omega}{3}$$

$$\therefore \ \tau = \frac{2I\omega}{3t}$$

SECTION-III (One or More than One Correct)

30. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelenth corresponding to maximum spectral radiancy in the radiation from A , by 1.00 μm. If the temperature of A is 5802 K

(A) the temperature of B is 1934 K

(B) $\lambda_B = 1.5\mu m$

(C) the temperature of B is 11604 K

(D) the temperature of B is 2901 K

Ans. (A, B)

Sol. Energy emitted per second by body A and B are same. $\therefore = \varepsilon_A \sigma T_A^4 A = \varepsilon_B \sigma T_B^4 A$

$$T_B = \left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{1/4} \times T_A = 1934\ K$$

According to Wein's displacement law $\lambda_m \propto \frac{1}{T}$

$$(\lambda_m)_A\, T_A = (\lambda_m)_B\, T_B \Rightarrow \frac{(\lambda_m)_A}{(\lambda_m)_B} = \frac{T_B}{T_A} = \frac{1934}{5802} \ldots (i)$$

Since temperature of A is more therefore $(\lambda_m)_A$ is less

$$\therefore (\lambda_m)_B - (\lambda_m)_A = 1\times 10^{-6} m (given) \qquad \ldots (ii)$$

Solving eq.(i)and (ii), we get

$\lambda_B = 1.5 \times 10^{-6} m.$

31. The magnitudes of the gravitational filed at distance r1 and r2 from the center of a uniform sphere of radius R and mass m are F1 and F2 respectively. Then:

(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Ans. (A, B)

Sol. For $r > R$, the gravitational field, $F = \frac{GMm}{r^2}$

$F_1 = \frac{GMm}{r_1^2}$ and $F_2 = \frac{G}{M} m r_2^2$ or, $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

For $r < R$, the gravitational field, $F = \frac{Gm}{R^3} \times r$

$T_1 = \frac{GMm}{R^3} \times r_1$ and $F_2 = \frac{GMm}{R^3} \times r_2$

$\frac{F_1}{F_2} = \frac{r_1}{r_2}$

32. Consider a body of mass 1.0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = \left(\alpha t \hat{i} + \beta \hat{j}\right)$ is applied on the body, where $\alpha = 1.0$ Ns $^{-1}$ and $\beta = 1.0$N. The torque acting on the body about the origin at time $t = 1.0$s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \frac{1}{3}$Nm

(B) The torque τ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at $t = 1$s is $\vec{v} = \frac{1}{2}\left(\hat{i} + 2\hat{j}\right)$ms^{-1}

(D) Them magnitude of displacement of the body at $t = 1$s is $\frac{1}{6}$m

Ans. (A, C)

Sol. Given $\vec{F} = \alpha t \hat{i} + \beta \hat{j}$ or $\vec{F} = t\hat{i} + \hat{j}$

$\left(\because \alpha = 1 \text{NS}^{-1} \text{ and } \beta = 1\text{N}\right)$

$\frac{m d\vec{v}}{dt} = t\hat{i} + \hat{j}$

$d\vec{v} = t dt \hat{i} + dt\hat{j} \left[\because m = 1\right]$

$\int_0^v d\vec{v} = \int_0^t t dt \hat{i} + \int_0^t dt \hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At $t = 1\text{s}, \vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}\left(\hat{i} + 2\hat{j}\right)$ms^{-1}

Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$

$d\vec{r} = \frac{t^2}{2} dt \hat{i} + t dt \hat{j}$

or, $\int_0^r d\vec{r} = \int_0^t \frac{t^2}{2} dt \hat{i} + \int_0^t t dt \hat{j}$

$\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

$$\text{Att} = 1, \vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j} \therefore |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times \left(\hat{i} + \hat{j}\right) \quad (\text{at } t = 1s)$$

$$\text{or}, \vec{\tau} = -\frac{1}{3}\hat{k} \quad \therefore \quad |\vec{\tau}| = \frac{1}{3}\text{Nm}$$

33. For a satellite to orbit around the earth, which of the following must be true?

(A) It cannot pass over the poles at any time

(B) It must be above the equator at sometime

(C) Its period of rotation must be greater than $2\pi\sqrt{\frac{R}{g}}$

(D) Its height above the surface of earth cannot exceed 36000 km

Ans. (B, C)

Sol. A satellite while orbiting around the earth, it must be above the equator at sometime. The period of revolution of satellite is

$$T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} > 2\pi\sqrt{\frac{R}{g}}$$

34. Two small balls A and B, each of mass m, are attached rigidly to the ends of a light rod of length d. The structure rotates about the perpendicular bisector of the rod at an angular speed ω. Choose the correct option(s).

(A) The angular momentum of individual ball about the axis is $\frac{1}{4}m\ \omega d^2$

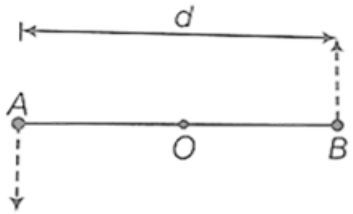
(B) The angular momentum of individual ball about the axis is $\frac{1}{2}m\ \omega d^2$

(C) The angular momentum of the system about the axis is $\frac{1}{2}m\ \omega d^2$

(D). The angular momentum of the system about the axis is $m\omega d^2$

Ans. (A, C)

Sol. Consider the situation shown in the figure. The structure rotates about the perpendicular bisector of the rod at an angular speed ω. The velocity of ball A w.r.t. centre O is $v = \omega\frac{d}{2}$.



The angular momentum of ball A w.r.t. axis is

$$L_1 = mvR = m\left(\frac{\omega d}{2}\right)\left(\frac{d}{2}\right) = \frac{1}{4}m\omega d^2$$

The same is the angular momentum L_2 of the second ball. The angular momentum of the system is equal to the sum of these two angular momenta, i.e. $L = \frac{1}{2}m\omega d^2$

35. Two wires A and B have equal lengths and are made of the same material, but diameter of wire A is twice that of wire B. Then, for a given load,

(A) the extension of B will be four times that of A

(B) the extensions of A and B will be equal

(C) the strain in B is four times that in A

(D) the strains in A and B will be equal

Ans. (A, C)

Sol. $\text{Area of cross} - \text{seation is } A = \frac{\pi d^2}{4}$

$$\Delta L = \frac{F}{\pi d^2 / 4} \cdot \frac{L}{Y}$$

$$\Rightarrow \Delta L \alpha \frac{1}{d^2}$$

$$\Delta L \alpha \frac{1}{d^2}$$

$$\Delta L_B \alpha \frac{(2d)^2}{(d)^2}$$

$$\Delta L_A \alpha \frac{(2d)^2}{(d)^2}$$

$$\Delta L_B = 4\Delta L_A$$

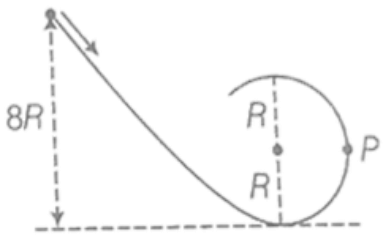
$$\text{Strain} = \frac{\Delta L}{L} = \frac{4F}{\pi d^2 Y}$$

$$\Rightarrow \text{Strain } \alpha \frac{1}{d^2}$$

$$\therefore \text{Gtrain})_B = 4(\text{Strain})_A$$

SECTION – IV (Non-Negative Integer)

36. A small solid ball (mass =0.1kg) rolls without slipping along the track shown in the figure. The radius of the circular track is R. If the ball starts from rest at a height 8 R above the bottom, then the horizontal force acting on it at point P is 5x newton. Find the value $0f \times .(\text{Given}, g = 10 ms^{-2})$



Ans. (2)

Sol. At point P,

$$\text{Decrease in GPH} = \text{Increase in}(K)_T + \text{Increase in}(K)_R$$

$$mg \cdot 7R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right) \cdot \frac{v^2}{r^2}$$

$= \frac{7}{10}mv^2$ (r is radius of ball)

$\therefore\ v^2 = 10gR$

Centripetal force is $\frac{mv^2}{R} = \frac{m(10gR)}{R} = 10mg = 10(0.1)(10)$

$= 10N \Rightarrow x = 2$

37. There are two bodies of masses 10^3 kg and 10^5 kg separated by a distance of 1 km. At a distance $\frac{p}{11}$ km from the smaller body, the intensity of gravitational field is zero. Find the value of p.

Ans. (1)

Sol. $\frac{G\times 10^3}{x^2} = \frac{G\times 10^5}{(1-x)^2}$

$\Rightarrow \frac{1-x}{x} = 10$

$\Rightarrow 11x = 1$

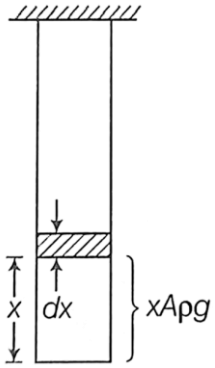
$x = \frac{1}{11}km$

$p = 1$

38. A wire having a length L and cross-sectional area A is suspended at one of its ends from a ceiling. Density and Young's modulus of material of the wire are ρ and Y, respectively. Its strain energy due to its own weight is $\frac{\rho^2 g^2 AL^3}{\alpha Y}$. Find the value of α.

Ans. (6)

Sol. Consider an element as shown in the figure.



Stress in the element $= \frac{\text{Force}}{\text{Area}} = \frac{xApg}{A} = x\,pg$

Now, elastic potential energy stored in the wire is

$dU = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume})$

$= \frac{1}{2}\cdot\frac{(\text{Stress})^2}{Y}(\text{Volume})$

$dU = \frac{1}{2}\cdot\frac{(xpg)^2}{Y}Adx = \frac{1}{2}\cdot\frac{p^2g^2A}{Y}x^2dx$

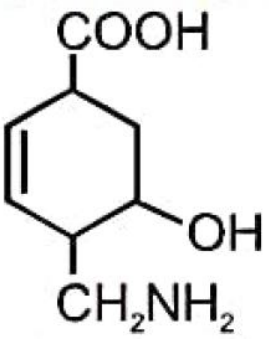
Total elastic potential energy $= \frac{1}{2} \cdot \frac{\rho^2 g^2 A}{Y} \int_0^L x^2 dx$

$= \frac{\rho^2 g^2 A L^3}{6Y}$

PART-C: CHEMISTRY

SECTION-I (Only One Correct Type)

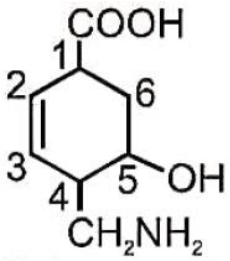
39. The correct IUPAC name of following compound is



(A) 4-Aminomethyl-3-hydroxycyclohex-5-ene-1-carboxylic acid
(B) 2-Aminomethyl-5-carboxycyclohex-3-en-1-ol
(C) 4-Aminomethyl-5-hydroxycyclohex-2-ene-1-carboxylic acid
(D) 3-Hydroxy-4-aminomethylcyclohex-5-en-1-oic acid

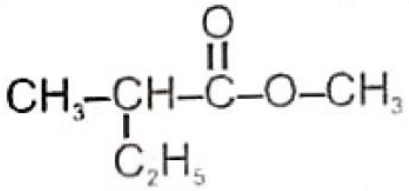
Ans. (C)

Sol.



4-Aminomethyl-5-hydroxycyclohex-2-ene-1-carboxylic acid

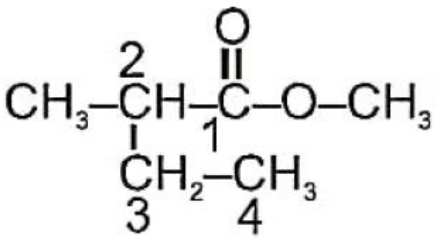
40. The correct IUPAC name of following compound is:



(A) Methyl-2-ethylpropanoate
(B) Methyl butane-2-carboxylate
(C) Methyl-2-methylbutanoate
(D) Methoxypentanone

Ans. (C)

Sol.



Methyl-2-methylbutanoate

41. In alkaline medium, MnO^-_4 oxidises I^- to

(A) I_2

(B) IO^-_3

(C) IO^-

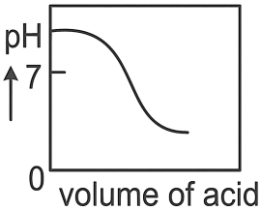
(D) IO^-_4

Ans. (B)

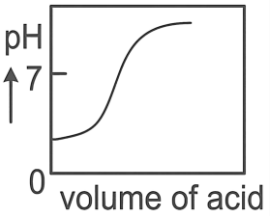
Sol. $2MnO^-_4 + H_2O + I^- \xrightarrow{\text{alkaline medium}} 2MnO_2 + 2OH^- + IO^-_3$

42. The plot of pH-metric titration of weak base NH_4OH vs strong acid HCl looks like:

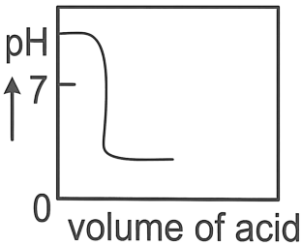
(A)



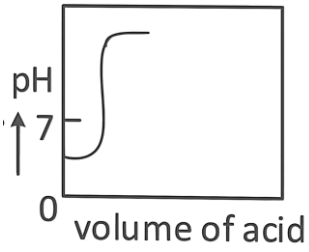
(B)



(C)

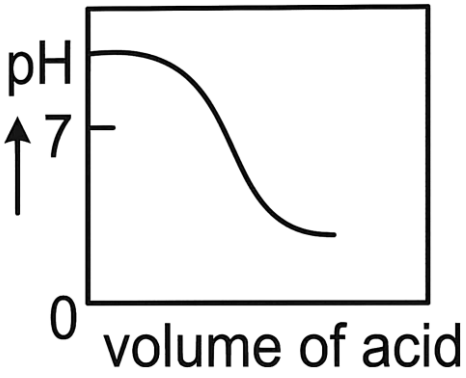


(D)



Ans. (A)

Sol. NH_4OH is a weak base and HCl is a strong acid.
With the addition of HCl to NH4OH, pH of solution will decrease gradually.
So, the correct graph should be



SECTION – II Numerical Value

Paragraph-1 (Steam)

(If more than two decimal, truncate/roundoff the value two decimal places).

Some amount of "20V" H_2O_2 at STP is mixed with excess of acidified solution of KI. The iodine so liberated required 200 mL of 0.1 N $Na_2S_2O_3$ for titration.

43. The volume of H_2O_2 solution required for reaction with excess of KI solution is 56×10^{-x} mL then find value of x.

Ans. (1)

Sol. $\underset{\text{n–factor=1}}{H_2O_2} + KI_{excess} \rightarrow \underset{\text{n–factor=2}}{I_2} - (i)$

$\underset{\text{n–factor=2}}{I_2} + \underset{\text{n–factor=1}}{Na_2S_2O_3} \rightarrow I^- + Na_2S_4O_6 - (ii)$

(a) Normality of H_2O_2 solution = $\dfrac{\text{Volume Strength of } H_2O_2}{5.6}$

Normality of H_2O_2 solution = $\dfrac{20}{5.6}$ N

Now apply law of equivalence in (ii) equation

Milli eq. of I_2 = Milli eq. of $Na_2S_2O_3$

Milli moles of I_2 liberated × n-factor = Normality × volume (mL) of $Na_2S_2O_3$

Milli moles of I_2 liberated × 2 = 0.1 × 200

Milli moles of I_2 liberated = 10

Now apply law of equivalence in (i) equation

Milli Eq. of H_2O_2 reacted = Milli eq. of I_2 formed

Normality × Volume (mL) of H_2O_2 = Mill moles of I_2 × n-factor

$\dfrac{20}{5.6} \times \text{Volume}(mL) \text{ of } H_2O_2 = 10 \times 2$

$\text{Volume}(mL) \text{ of } H_2O_2 = 5.6 = 56 \times 10^{-1}$.

Hence, value of x is 1.

44. The mass (g) of $K_2Cr_2O_7$ needed to oxidise the above volume of H_2O_2 solution is 98×10^{-y}. Find the value of y. (Molar mass of $K_2Cr_2O_7$ = 294g/mol).

Ans. (2)

Sol. $\underset{\substack{(O.A)\\ \text{n–factor=6}}}{K_2Cr_2O_7} + \underset{\substack{(R.A)\\ \text{n–factor=2}}}{H_2O_2} \longrightarrow Cr^{+3} + O_2$

According to law of equivalence

Eq. of $K_2Cr_2O_7$ reacted = Eq. of H_2O_2 reacted

Moles of $K_2Cr_2O_7$ × n-factor of $K_2Cr_2O_7$ = Normality of H_2O_2 × volume of H_2O_2

Moles of $K_2Cr_2O_7$ × 6 = $\dfrac{20}{5.6} \times \dfrac{5.6}{1000}$

Moles of $K_2Cr_2O_7$ = $\dfrac{1}{300}$

Mass of $K_2Cr_2O_7$ reacted = Moles × Molar mass of $K_2Cr_2O_7$

Mass of $K_2Cr_2O_7$ reacted = $\frac{1}{300} \times 294 = 0.98g = 98 \times 10^{-2}$ g.

Hence, Value of Y = 2.

Paragraph-2 (Steam)

Phosphoric acid ionizes according to the following equations:

$H_3PO_4(aq) \rightleftharpoons H^+(aq) + H_2PO_4^-(aq); \quad K_1 = 7 \times 10^{-3}$

$H_2PO_4^-(aq) \rightleftharpoons H^+(aq) + HPO_4^{2-}(aq); \quad K_2 = 6 \times 10^{-8}$

$HPO_4^{2-}(aq) \rightleftharpoons H^+(aq) + PO_4^{3-}(aq); \quad K_3 = 4.5 \times 10^{-13}$

45. If you are asked to prepare a buffer with a pH =7.00, what may be the molar ratio of $\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$ the species that should be used in the solution? Report your final answer by multiplying the ratio by 10. (Take $10^{-0.22} = 0.6$).

Ans. (6)

Sol. $pH = pK_{a_2} + \log \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 7$

$\Rightarrow \log \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = -0.22 \Rightarrow \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 0.6$

Hence, final answer is 0.6 × 10 = 6

46. Assume 50 mL of the buffer prepared in the previous part is available in which more abundant species has a concentration of O.1 M. If to this solution, 20 mL O.1 M NaOH is added further, what will be the new pH? Report your final answer by dividing with 3.72.

Ans. (2)

Sol. A buffer solution show best buffering action when the pH of solution is near to Ka of acid component of buffer solution

There for the above buffer can be best prepared by taking $H_2PO_4^-$ and $HPO4_2^-$ In 50 mi buffer solution

$[HPO_4^{2-}] = 0.06 = 3$ millimole $\quad [H_2PO_4^-] = 0.1M = 5$milimole,

milimole of NaOH added $= 20 \times 0.01 = 2$

$H_2PO_4^-$	+	$OH-$	HPO_4^{2-}	+	H_2O
5		2	3		0
3		0	5		0

$pH = pK_{a2} + \log\left(\frac{5}{3}\right) = 7.44$

Hence final answer is $\frac{7.44}{3.72} = 2$

Paragraph-3 (Steam)

The pH of basic buffer mixtures is given by: $pH = pK_a + \log\frac{[Base]}{[Salt]}$.whereas pH of acidic buffer mixtures is given by: $pH = pK_a + \log\frac{[Salt]}{[Acid]}$. Appreciable of little acid or base although show no appreciable change in pH for all practical purposes, but since the ratio $\frac{[Base]}{[Salt]}$ for $\frac{[Salt]}{[Acid]}$ charges, a slight decrease or increase in pH results.

47. A solution containing 0.2 mole of dichloroacetic ($K_a = 5 \times 10^{-2}$) and 0.1 mole sodium dichloroacetate in one litre solution has $[H^+]$: equal to 5×10^{-y}. then find y.

Ans. (2)

Sol. Since $CHCl_2COOH$ is relatively Strongs acid havings more Ka.

$CHCl_2.COONa \rightarrow CHCl_2COO^- + Na^+$

0.1 0.1

$CHCl_2.COOH \rightleftharpoons CHCl_2COO^- + H^+$

0.2 0 0

$(0.2-x)$ $(x+0.1)$ x

$\therefore K_a = \frac{[CHCl_2COO^- \times [H^+]]}{[CHCl_2\ COOH]}$ or $5\times10^{-2} = \frac{[0.1+x][X]}{[0.2-x]}$.

$\therefore$ **x** $= 0.05 = 5 \times 10^{-2}$.

Hence, y = 2

48. The volume of O.2 M NaOH needed to prepare a buffer of pH 4.74 with 50 mL of 0.2 M acetic acid is: (pK_b of CH_3COO^- = 9.26)

Ans. (25)

Sol. Let V mL of NaOH be needed to give CH_3COONa.

NaOH	+	CH_3COOH	$\rightarrow$	CH_3COONa	$+H_2O$
0.2xV		50x0.2		0	0
		$[10-0.2V]$		0.2V	0.2V

$$pH = pK_a + \log\frac{[Salt]}{[Acid]} = pK_w - pK_0 + \log\frac{[Salt]}{[Acid]} = 14 - 9.26 + \log\frac{[Salt]}{[Acid]}$$

$$= 14 - 9.26 + \log\frac{\left[\frac{0.2V}{50+V}\right]}{\left[\frac{10-0.2V}{50+V}\right]}$$

$$4.74 = 4.74 + \log\left[\frac{0.2V}{10-0.2V}\right] \quad \therefore V = \frac{10}{0.4} = 25mL.$$

SECTION-III (One or More than One Correct)

49. 100 mL of 0.1 M $NaHC_2O_4$ is neutralised by V_2 mL of 0.1 M NaOH and V_3 mL of **'a'** M $KMnO_4$ separately, then for complete neutralisation:

(A) volume of NaOH required=200 mL

(B) if M of $KMnO_4$ is 0.1 M then $\frac{V_2}{V_3} = 5:2$

(C) if M of $KMnO_4$ is 0.1M then $V_3 = 40$ mL

(D) if M of $KMnO_4$ is 0.2M then $V_3 = 2$ mL

Ans. (B, C)

Sol. $NaHC_2O_4$ when react with NaOH then it is acid base titration.

$NaHC_2O_4$ have 1 acidic H^+ that will react with NaOH so in this case n-factor of $NaHC_2O_4$ will be 1

According to law of equivalence

Milli eq. of $NaHC_2O_4$ = Milli eq. of NaOH required

$M_1 \times V_1 \times$ n-factor of $NaHC_2O_4 = M_2 \times V_2 \times$ n-factor of NaOH (n-factor of NaOH=1)

$0.1 \times 100 \times 1 = 0.1 \times V_2 \times 1$

$V_2 = 100\text{mL}$

$NaHC_2O_4$ when react with $KMnO_4$ then it is redox titration.

$$\underset{\substack{(0.A)\\ n-factor=5}}{\overset{+7}{KMnO_4}} + \underset{\substack{(R.A)\\ n-factor=2}}{\overset{+3}{NaHC_2O_4}} \xrightarrow{H^+} Mn^{+2} + \overset{+4}{CO_2}$$

According to law of equivalence

Milli eq. of $KMnO_4$ = Milli eq. of $NaHC_2O_4$

$M_3 \times V_3 \times$ n-factor of $KMnO_4 = M_1 \times V_1 \times$ n-factor of $NaHC_2O_4$

$a \times V_3 \times 5 = 0.1 \times 100 \times 2$

$\therefore V_3 = \frac{20}{5a}$

if $M_{KMnO_4} = 0.1M$, then $a = 0.1\,M$ then $V_3 = 40\text{mL}$

$\therefore \frac{V_1}{V_2} = \frac{100 \times 5a}{20}$

if $M_{KMnO_4} = 0.1M$, then $a = 0.1\,M$ and put this value in above equation we will get $\frac{V_1}{V_2} = 5:2$

50. Which one is not acidic salts?

(A) Na_2HPO_4

(B) NaH_2PO_2

(C) Na_2HPO_3

(D) NaH_2PO_4

Ans. (B, C)

Sol. H_3PO_2 is monobasic and H_3PO_3 is dibasic.

51. Plots of log K vs. $\frac{1}{T}$ plots shows an intercept of 2 on Y-axis with a slops of 45° for the studied reaction. Which of the following are correct assuming ΔH° and ΔS° as temperature independent.

(A) ΔS° = 4.606 cal

(B) ΔH° = -4.606 cal

(C) ΔG° = 2.75 Kcal

(D) ΔK = 100.8

Ans. (B, C, D)

Sol. $\Delta G^\circ = -2.303RT\log K$,

$\Delta H^\circ - T\Delta S^\circ = -2.303RT\log K$

$2.303\log K = -\frac{\Delta H^\circ}{R} + \frac{\Delta S^\circ}{R}$,

$\text{Slope} = -\frac{\Delta H^\circ}{2.303R} = \tan 45^\circ$

$\Delta H^\circ = -4.606\text{cal}$

$\text{Intercept} = \frac{\Delta S^\circ}{2.303R} = 2$

$\Delta S^\circ = 9.212\ \frac{\text{Cal}}{\text{K}}$

$\therefore \Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -4.606 - 298 \times 9.212$

$= -2.75\ \text{k cal}$

52. Mesityl oxide is a common name of

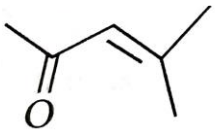
(A) 4-methylpent-3-en-2-one

(B) 2,4-dimethylpentan-3-one

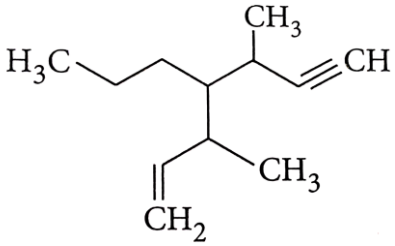
(C) 2-methylcyclohexanone

(D) 3-methylcyclohexane carbaldehyde.

Ans. (A)

Sol. 

53. The IUPAC name for the following compound is



(A) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene

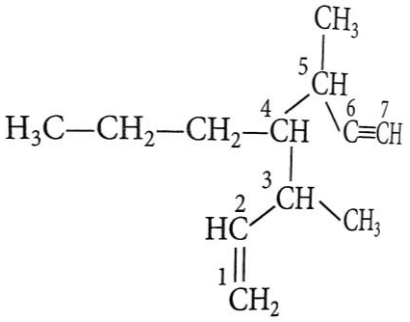
(B) 3,5-dimethyl-4-propylhept-6-en-1-yne

(C) 3,5-dimethyl-4-propylhept-1-en-6-yne

(D) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

Ans. (C)

Sol.



3, 5-Dimethyl-4-propylhept-1-en-6-yne

54. Which of the following statement(s) is/are correct?

(A) The pH of 1×10^{-8} M HCl solution is 8.

(B) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.

(C) K_w increases with increase in temperature.

(D) When a solution of a weak monoprotic acid is titrated against a strong base at half neutralisation point, $pH=\frac{1}{2}pK_a$.

Ans. (B, C)

Sol. (A) pH of 1×10^{-8} M HCl solution is 6.98.

(B) Conjugate base of $H_2PO_4^-$ is HPO_4^{-2}.

(C) Degree of ionisation increases with temperature, because dissociation of water is reversible and endothermic process so equilibrium constant increases thus ionic product of water (K_w) also increases.

(D) At half neutralisation point, pH= pK_a

SECTION – IV (Non-Negative Integer)

55. $A_{(g)} \rightleftharpoons 2B_{(g)} + C_{(g)}$ For the given reaction, if the initial pressure is 450 mm Hg and the pressure at time t is 720 mm Hg at a constant temperature T and constant volume V. The fraction of A(g) decomposed under these conditions Is $\mathbf{X}\times10^{-1}$. The value of **X** is _____ (nearest integer)

Ans. (3)

Sol. Given the reaction:

$$A_{(g)} \rightleftharpoons 2B_{(g)} + C_{(g)}$$

At t = 0	450mm Hg	0	0
At t = t	450-p	2p	p

Given that 450-p + 2p + p = 720

p = 135mm Hg

The fraction of A decomposed would then be this change in pressure divided by the initial pressure:

$$\text{fraction decomposed}=\frac{x}{\text{Initial pressure}}=\frac{135\text{mmHg}}{450\text{mmHg}}=0.3=3\times10^{-1}$$

56. Total number of species from the following which can undergo disproportion reaction is _____.

$H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+$

Ans. (6)

Sol. Intermediate oxidation state of element can undergo disproportionation.

$H_2O_2, ClO_3^-, P_4, Cl_2, Cu^{+1}, NO_2$

57. The pH at which $Mg(OH)_2 \left[K_{sp} = 1 \times 10^{-11}\right]$ begins to precipitate from a solution containing 0.10M Mg^{2+} ions is

Ans. (9)

Sol. Precipitation when $Q_{sp} = K_{sp}$

$$\left[Mg^{2-}\right]\left[OH^-\right]^2 = 10^{-11}$$

$$0.1 \times \left[OH^-\right]^2 = 10^{-11} \Rightarrow \left[OH^-\right] = 10^{-5}$$

$$\Rightarrow pOH = 5 \Rightarrow pH = 9$$