TEST CODE : 113108

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IIT-JEE
Batch – Growth (June) | Major Test - 4 (Paper-II)

Time: 3 Hours Test | **Date: 2nd March 2025** | **Maximum Marks: 180**

Name of the Candidate: ______________________________Roll No.____________

Centre of Examination (in Capitals):____

Candidate's Signature: ____________________Invigilator's Signature:____________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

2. This Test Booklet consists of 54 questions.

3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 18 questions each and every **PART** has four sections.

(i) **Section-I** contains **6** Question Single Digit Integer (0-9 both inclusive)

Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.

(ii) **Section-II** contains **6** Multiple Choice Option with more than one correct answer.

Marking scheme: +4 for correct answer, 0 if not attempted and +1 partial marking –2 in all other cases.

(iii) **Section-III** contains **6** Non-Negative Integer Value questions.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.

5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.

6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.

7. For integer-based questions, the answer should be in decimals only not in fraction.

8. If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.

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TEST SYLLABUS

Batch – Growth (June) | Major Test-04 (Paper II)

2nd March 2025

Mathematics:	FOM-1 (Real Numbers, Complex Numbers, Even Numbers, Odd Numbers Prime Numbers, Composite Numbers, Co-Prime Numbers/ Relatively Prime Numbers, Twin Prime Numbers, LCM and HCF, Indices, Polynomial in One Variable, Degree of Polynomials, Some Special Types of Polynomials, Value and Zeros of a Polynomial, Roots of a Polynomial Equation, Remainder Theorem, Factor Theorem, Factorization, Sets, Types of Sets, Laws of Algebra of Sets (Properties of Sets), INTERVALS AS A SUBSET OF R Venn Diagram), FOM-2 (LINEAR INEQUALITIES WAVY CURVE METHOD, Rational Inequalities, Irrational Inequalities, Modulus Inequalities, Logarithmic & Exponential Inequality), Logarithm & (Function - NCERT),Sequence & Series, Compound Angle & Trignometric Eq, Quadratic Eq St. Line, Circle, Binomial Theorem, Permutation & Combination, (Probability-NCERT), Parabola, Ellipse & Hyperbola, Statistics & Complex Number & (Limits - NCERT Level)
Physics:	Basic Mathematics (Except Vector),Basic Mathematics (Vector) ,Units & Dimension ,Kinematics -1D, Kinematics-2D,NLM & Friction, WEP, Circular Motion, Centre of Mass, Momentum & Collision, Rotational Motion, Gravitation, Elasticity, Thermal Expansion, Calorimetry and Heat Transfer, KTG & Thermodynamics, Fluid Mechanics, SHM & Waves
Chemistry:	Mole Concept & Concentration terms - 1 (Importance of chemistry, Nature of matter ,Sig. figure, Laws of chemical combination, Avogadro law, Dalton's atomic theory, Atomic and molecular masses, Till Average/ Mean Atomic Mass),Mole Concept & Concentration terms -2(Percentage composition, Stoichiometric, calculations, Limiting reagent & Concentration, terms Equivalent Concept) Atomic Structure, Periodic Table & Periodic Properties, Chemical Bonding, Thermodynamics-1, Thermochemisty & Thermodynamics-2,Chemical Eq,Ionic Eq, Redox Reaction, Nomenclature, GOC, Isomerism & Hydrocarbon, Hydrogen & its compound & S-block & Enviromental Chemistry

Useful Data Chemistry:

Gas Constant	R	$= 8.314\,JK^{-1}mol^{-1}$
		$= 0.0821\,Lit\ atm\,K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}\,Js$
		$= 6.25 \times 10^{-27}\,erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66 \times 10^{-27}\,kg$
1 eV		$= 1.6 \times 10^{-19}\,J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

PART – A : MATHEMATICS

Single Digit Integer (0-9 both inclusive)

1. The number of solutions of $\log_4(x-1)=\log_2(x-3)$ is

Ans. 1

Sol. For the given equation to be valid, we must have $x-1>0$ and $x-3>0$.

$\Rightarrow\ x>3$

We can write the given equation as

$$\frac{\log(x-1)}{\log 4}=\frac{\log(x-3)}{\log 2}$$
$$\Rightarrow \log(x-1)=2\log(x-3)\ (\because \log 4=2\log 2)$$
$$\Rightarrow (x-1)=(x-3)^2$$
$$\Rightarrow x^2-7x+10=0\Rightarrow(x-5)(x-2)=0\Rightarrow x=5 \text{ or } 2$$

As $x>3, x=5$.

2. For all complex numbers z_1,z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is

Ans. 2

Sol. We know $|z_1|=12$

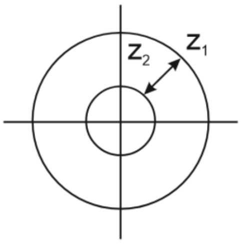
Now, $|z_2|=|z_2-(3+4i)+(3+4i)|$

$$\le|z_2-(3+4i)|+|3+4i|=5+5=10$$
$$\therefore\ -|z_2|\ge-10$$

Therefore $|z_1-z_2|\ge|z_1|-|z_2|=12-10=2$
Thus minimum value is 2.

Alternative solution : $|z_1|=12, |z_2-(3+4i)|=5$

Both represent circles one is completely contained in the other as shown in the figure.
Hence $|z_1-z_2|=12-2(5)=2$ units.



3. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals,

Ans. 7

Sol. $T_n={}^nC_3\ \therefore T_{n+1}-T_n=21$

${}^{n+1}C_3-{}^nC_3=21$

${}^nC_2+{}^nC_3-{}^nC_3=21\Rightarrow{}^nC_2=21$

$\frac{1}{2}n(n-1)=21\Rightarrow n^2-n-42=0$

$(n-7)(n+6)=0$

As $n\ge1$, we get $n=7$

4. The value of the expression $\sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ$ is equal to

Ans. 4

Sol. $GE = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{4\left[\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right]}{2\sin 20^\circ \cos 20^\circ}$

$= \frac{4\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = 4$

5. Let $p, q \in \{1,2,3,4\}$. The number of equations of the form $px^2 + qx + 1 = 0$ having real roots is

Ans. 7

Sol. For the equation $px^2 + qx + 1 = 0$ to have real roots $D \geq 0 \Rightarrow q^2 \geq 4p$

If $p = 1$ then $q^2 \geq 4 \Rightarrow q = 2,3,4$
If $p = 2$ then $q^2 \geq 8 \Rightarrow q = 3,4$
If $p = 3$ then $q^2 \geq 12 \Rightarrow q = 4$
If $p = 4$ then $q^2 \geq 16 \Rightarrow q = 4$
$\therefore$ No. of required equations $= 7$.

6. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

Ans. 2

Sol. The equation is $x^2 + y^2 + 2x + 4y - p = 0$ which can be written as $(x+1)^2 + (y+2)^2 = p + 5$

There are two cases possible, one with the circle passing through origin, i.e., $1^2 + 2^2 = p + 5 \therefore p = 0$
The other case is when the circle touch the x-axis and cuts y-axis.
For this to happen,

$p + 5 = 2^2 \Rightarrow p + 5 = 4 \therefore p = -1$

Please note that the case that the circle touch the y-axis and cut x-axis is not possible.
As, $f^2 - c = 0$ and $g^2 - c > 0$
Then $c = f^2 \Rightarrow -p = 4$.
But then $g^2 - c = 1 + p = 1 - 4 < 0$.
$\therefore$ Not possible.

One or More than One Correct

7. A circle S passes through the point $(0,1)$ and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8

(B) radius of S is 7

(C) centre of S is $(-7,1)$

(D) centre of S is $(-8,1)$

Ans. (B, C)

Sol. The circles $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 1 = 0$ has the radical axis $-2x - 14 = 0$. $\therefore x + 7 = 0$

Hence any circle orthogonal to them has its centre as $(-7, -\beta)$ Let its equation be

$S: x^2 + y^2 + 14x + 2\beta y + \lambda = 0$

Orthogonality with 2nd circle gives
$\lambda - 1 = 0$. $\therefore \lambda = 1$
Again, $(0,1)$ lies on $S \Rightarrow 1 + 2\beta + 1 = 0. \therefore \beta = -1$
Equation is $x^2 + y^2 + 14x - 2y + 1 = 0$
Radius $= \sqrt{7^2 + 1^2 - 1} = 7$

Alternative Solution :

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be orthogonal to $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 1 = 0$
Orthogonality condition requires $-2g = c - 15$ and $c - 1 = 0$
Also $(0,1)$ lies on circle $\Rightarrow 1 + 2f + c = 0$

Solving we get $c=1, g=7, f=-1$
Hence radius $=\sqrt{g^2+f^2-c}=7$, as before.

8. Let $S_1, S_2, \ldots \ldots$ be squares such that for each $n \geq 1$, the length of a side of S_n, equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm.
(A) 7
(B) 8
(C) 9
(D) 10

Ans. (B, C, D)

Sol. Length of the side of the square S_r is a_r Now $a_1=10, a_2=\frac{10}{\sqrt{2}}, a_3=\frac{10}{(\sqrt{2})^2}$ and so on $a_7=\frac{10}{(\sqrt{2})^6}=\frac{10}{8}>1$

$$a_8=\frac{10}{(\sqrt{2})^7}<1$$
$$a_9=\frac{10}{(\sqrt{2})^8}<1$$
$$a_{10}=\frac{10}{(\sqrt{2})^9}<1$$

As a_8, a_9, a_{10} are less than 1 , hence $S_8=a_7^2<1, S_9=a_8^2<1, S_{10}=a_9^2<1$
Hence b, c, d are the correct answers.

9. Let L be a normal to the parabola $y^2=4x$. If L passes through the point (9,6), then L is given by
(A) $y-x+3=0$
(B) $y+3x-33=0$
(C) $y+x-15=0$
(D) $y-2x+12=0$

Ans. (A, B, D)

Sol. The equation of normal is
$y=mx-2m-m^3$
As (9,6) lies on it, $6=9m-2m-m^3 \Rightarrow m^3-7m+6=0$
The roots are $m=1,2,-3$, So the normal are
$y=x-3, y=2x-12, y=-3x+33$

10. Let a hyperbola pass through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1 , then
(A) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$
(B) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$
(C) focus of hyperbola is (5, 0)
(D) vertex of hyperbola is $(5\sqrt{3}, 0)$

Ans. (A, C)

Sol. For the given ellipse

$$\frac{x^2}{25}+\frac{y^2}{16}=1, e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

$\Rightarrow$ Eccentricity of hyperbola $=\frac{5}{3}$
Let the hyperbola be $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ then

$B^2 = A^2\left(\frac{25}{9}-1\right) = \frac{16}{9}A^2; \frac{x^2}{A^2} - \frac{9y^2}{16A^2} = 1$

As it passes through (3,0) we get

$A^2 = 9 \Rightarrow B^2 = 16$

∴ Equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Focus of hyperbola is (5,0), vertex of hyperbola is (3, 0)

11. Let $a_1, a_2, a_3, ...$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, ...$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are true?

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

Ans. (B, C)

Sol. Given $T_{n+1} - T_n = a_n, n \geq 1$

Put $n = 1,2,$ in succession and adding, we get

$T_2 - T_1 = a_1; T_3 - T_2 = a_2; T_n - T_{n-1} = a_{n-1}$

So, $T_n = T_1 + a_1 + a_2 + \cdots . + a_{n-1} = 3 + 7 + 15 + \cdots$ to $(n-1)$ term $= 3 + (n-1)(4n-1)$

$= 4n^2 - 5n + 4, n \geq 1$

$$\text{Also, } \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (4k^2 - 5k + 4)$$

$$= \frac{4n(n+1)(2n+1)}{6} - 5\frac{n(n+1)}{2} + 4n$$

$$= \frac{n(n+1)}{6}\{4(2n+1) - 15\} + 4n = \frac{n(n+1)}{6}(8n-11) + 4n$$

Thus, $T_{20} = 4 \times 20^2 - 5 \times 20 + 4 = 1504$ and $T_{30} = 4 \times 30^2 - 5 \times 30 + 4 = 3454$

Also we can compute

$$\sum_{k=1}^{20} T_k = 10510; \sum_{k=1}^{30} T_k = 35615$$

12. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ?

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

(B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(C) $\left(0, \frac{1}{\sqrt{5}}\right)$

(D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Ans. (A, D)

Sol. We have $\alpha x^2 - x + \alpha = 0$

$D = 1 - 4\alpha^2$

For distinct real roots, $1 - 4\alpha^2 > 0$

i.e., $\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Now, $|x_1 - x_2| < 1$ i.e. $(x_1 - x_2)^2 < 1$

i.e., $\frac{b^2 - 4ac}{a^2} < 1 \Rightarrow 1 - 4\alpha^2 < \alpha^2$

$\Rightarrow \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Combining the two bounds, we have

$$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$
$$\therefore S = \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

Numerical Value (If more than two decimal, truncate/roundoff the value two decimal places).

13. Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is

Ans. 16000

Sol. Let $E = ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$

$$\begin{aligned}
&= a\{ca - c^2 + da - d^2\} + b\{cb - c^2 + db - d^2\} \\
&= a\{a(c+d) - (c^2 + d^2)\} + b\{b(c+d) - (c^2 + d^2)\} \\
&= a^2(c+d) - a(c^2 + d^2) + b^2(c+d) - b(c^2 + d^2) \\
&= (a^2 + b^2)(c+d) - (a+b)(c^2 + d^2) \\
&= \{(a+b)^2 - 2ab\}(c+d) - (a+b)\{(c+d)^2 - 2cd\} \\
&= (c+d)(a+b)^2 - 2ab(c+d) - (a+b)(c+d)^2 + 2cd(a+b)
\end{aligned}$$

From given equations, we have $a + b = -20, ab = -2020$

and $c + d = 20, cd = 2020$

$$\begin{aligned}
\therefore E &= 20(-20)^2 - 2(-2020)(20) + 20(20)^2 + 2(2020)(-20) \\
&= 2(20)(20)^2 = 16000
\end{aligned}$$

14. Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z} - z^2 = i(\bar{z} + z^2)$ is ________.

Ans. 4

Sol. $\Rightarrow z^2 = \frac{1-i}{1+i}\bar{z} \Rightarrow z^2 = -i\bar{z}$

Let $z = x + iy$

$\therefore \ (x^2 - y^2) + i(2xy) = -i(x - iy)$

So, $x^2 - y^2 + y = 0$
and $x(2y + 1) = 0$

$x = 0$, gives $y = 0,1$

and $y = -\frac{1}{2}$ gives $x = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$
So, we get 4 solutions of ordered pair (x, y).

15. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is ________.

Ans. 8

Sol. Using A.M. $\geq$ G.M. inequality, we have

$$\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq \sqrt[3]{3^{y_1}\cdot 3^{y_2}\cdot 3^{y_3}}$$

$\Rightarrow \quad 3^{y_1}+3^{y_2}+3^{y_3} \geq 3\cdot\sqrt[3]{3^{y_1+y_2+y_3}} = 3\cdot\sqrt[3]{3^9} = 3\cdot 3^3 = 3^4$

$\Rightarrow \quad \log_3\left(3^{y_1}+3^{y_2}+3^{y_3}\right) \geq 4$

$\therefore \quad m = 4.$

Again using A.M. $\geq$ G.M., we have

$$\frac{x_1+x_2+x_3}{3} \geq \sqrt[3]{x_1x_2x_3} \Rightarrow x_1x_2x_3 \leq 27 = 3^3$$

$\Rightarrow \quad \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

$\therefore \quad M = 3$

Now, $\log_2(m^3) + \log_3(M^2) = 3\log_2 m + 2\log_3 M$

$= 3\log_2 4 + 2\log_3 3 = 6 + 2 = 8$

16. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2+\frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^7$, then the value of $2b$ is

Ans. 3

Sol. Given, $\left(ax^2+\frac{70}{27bx}\right)^4$

The general term is given by

$$T_{r+1} = {}^4C_r(ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r$$

For coefficient of x^5, put $8-2r-r=5 \Rightarrow r=1$

$\therefore$ The coefficient x^5 in $\left(ax^2+\frac{70}{27bx}\right) = {}^4C_1\cdot\frac{a^3\cdot(70)^1}{27\cdot b}$

General term of $\left(ax-\frac{1}{bx^2}\right)^7$ is

$$T_{r+1} = {}^7C_r(ax)^{7-r}\left(-\frac{1}{bx^2}\right)^r$$

For coefficient of x^{-5}, put $7-r-2r=-5 \Rightarrow r=4$

$\therefore$ The coefficient of x^{-5} in $\left(ax-\frac{1}{bx^2}\right)^7 = {}^7C_4\frac{a^3(-1)^4}{b^4}$

Now, ${}^4C_1\frac{a^3(70)}{27(b)} = {}^7C_4\frac{a^3}{b^4}$

$$\Rightarrow \frac{4\times 70}{27(b)} = \frac{35}{b^4} \Rightarrow b^3 = \frac{27}{8} \Rightarrow b = \frac{3}{2} \therefore\ 2b = 3$$

17. A group of 9 students, $s_1, s_2, \ldots, s_9$, is to be divided to form three teams X, Y, and Z of sizes 2,3 , and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is __________.

Ans. 665

Sol. We make cases :

$X \to 2$ members
$Y \to 3$ members
$Z \to 4$ members

Case I: $S_1 \notin X, S_2 \in X$, (i.e., S_2 is in X, S_1 can be in Y or Z)

$$1\cdot {}^7C_1\cdot\frac{7}{\underline{3}\lfloor 4} = 7\cdot\frac{7\cdot 6\cdot 5}{6} = 245$$

S_2 is there.

Case II: $S_1, S_2 \notin X, S_2 \notin Y$ (i.e. $S_2 \in Z, S_1$ can be in Y or Z)

$${}^7C_2\cdot\frac{6}{3\lfloor 3} = 21\cdot 20 = 420$$

Hence, the total number of ways to make up team

$= 245 + 420 = 665$

18. A number is chosen at random from the set $\{1,2,3, \ldots\ldots,2000\}$. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7 . Then the value of $500p$ is ________.

Ans. 214

Sol. Let $|M(k)|$ = Number of multiples of k in the given set. Then

$$|M(3)| = \left[\frac{2000}{3}\right] = 666; |M(7)| = \left[\frac{2000}{7}\right] = 285$$

$$|M(21)| = \left[\frac{2000}{21}\right] = 95$$

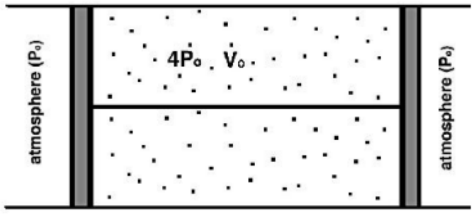
The probability, $p = \frac{666+285-95}{2000} = \frac{856}{2000} = \frac{214}{500}$

Thus, $500p = 214$

PART-B: PHYSICS

Single Digit Integer (0-9 both inclusive)

19. A monoatomic gas at pressure $4P_0$ and volume V_0 is contained between a conducting cylinder and two circular smooth adiabatic pistons. Both the pistons are tied to each other with the help of an ideal string. Now the gas is slowly heated by pouring hot water over the curved surface of the cylinder until tension in the string is doubled. Then, the heat supplied to the gas is $\frac{\eta}{2}P_0V_0$. The value of η is ___________



Ans. 9

Sol. Gas will undergo isochoric process $T_{\text{initial}} = 3\mathbf{P}_0A$ $T_{final} = 6P_0A$

$$\mathbf{P}_{\text{final}} = 7\mathbf{P}_0$$

$$Q = \Delta U = \frac{3}{2}[P_2V_2 - P_1V_1] = \frac{3}{2}[7P_0V_0 - 4P_0V_0] = 4.5P_0V_0$$

20. The equation of traveling wave in a uniform string of mass per unit length μ is given as $y = A\sin(\omega t - kx)$ The total energy transferred through the origin in time interval from t = 0 to t = $\pi/12\omega$ is $\frac{(\pi+3)\mu\omega^2A^2}{3nk}$. The value of n is ___________

Ans. 8

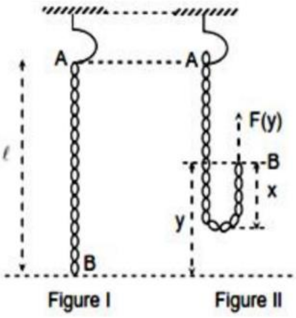
Sol.

$$\text{E= energy stored in the string} = \mu A^2\omega^2\int_0^{\Delta\pi}\cos^2 kx dx$$

Where Δx is distance traveled by the wave in the $\frac{\pi}{12\omega} = \frac{\pi}{12\omega}\frac{\omega\lambda}{2\pi} = \frac{\lambda}{24}$

So, $E = \frac{(\pi+3)\mu A^2\omega^2}{24k}$

21. A uniform chain of mass m and length l hangs from a hook in ceiling as shown in figure I. The bottom of link is now raised vertically with the help of upward external force $F(y) = \frac{mgy}{nl}$ very slowly as shown in figure II. The value of n is



Ans. 2

Sol. As x=y/2

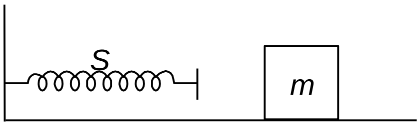
Force applied is mgx/l = mgy/2l

22. On a cricket field, the batsman is at the origin of coordinates and a fielder stands in position given as $\left(46\hat{i}+28\hat{j}\right)$ m. The batsman hits the ball so that it rolls along the ground with constant velocity given by $\left(7.5\hat{i}+10\hat{j}\right)$ m/s. The fielder can run with a speed of 5 m/s. If he starts to run immediately when the ball is hit, what is the shortest time (in seconds) in which he could intercept the ball

Ans. 4

Sol. Find shortest distance from point (46, 28) to the line along $\left(7.5\hat{i}+10\hat{j}\right)$ which is 20 m. Hence, least time is 20/5 = 4s

23. A light spring *S* is attached to a wall as shown. The spring constant *k* is 300 N/m. The mass *m* = 120 g shown in the figure initially moves to the left at a speed of 8.0 m/s. It strikes the spring and becomes attached to it. Maximum compression in the spring will be 2x cm. Find the value of x.



Ans. 8

Sol. $\frac{1}{2}mv^2=\frac{1}{2}kx_{max}^2$ $\qquad \Rightarrow x_{max}=v\sqrt{\frac{m}{k}}=8\sqrt{\frac{0.12}{300}}=0.16$

24. The motion of an insect on table is given as $x=4t-2\sin t$ and $y=4-2\cos t$, where x and y are in metres and t is in seconds. Find the value of maximum velocity magnitude minus minimum velocity magnitude attained by the insect.

Ans. 4

Sol. $\frac{dx}{dt}=V_x=4-2\cos t$

$\frac{dy}{dt}=V_y=2\sin t$

$V=\sqrt{V_x^2+V_y^2}$

$v=\sqrt{\left(4-2\cos t\right)^2+4\sin^2 t}=\sqrt{20-16\cos t}$

$\Rightarrow v_{max}=\sqrt{36}=6$

$v_{min}=\sqrt{4}=2$

One or More than One Correct

25. For a particle along any arbitrary path $\vec{r},\vec{v}$ and $\vec{a}$ are position, velocity and acceleration vector respectively. Choose correct statement-

(A) $\left|\frac{d\vec{r}}{dt}\right|$ gives speed of particle

(B) $\frac{d|\vec{r}|}{dt}$ gives speed of particle

(C) $|\vec{a}| = \frac{d|\vec{v}|}{dt}$

(D) $\frac{d\vec{v}}{dt} = 0 \Rightarrow \frac{d|\vec{v}|}{dt} = 0$

Ans: (A),(D)

Sol: Change in length of vector is not same as length of change vector

26. Two particles A and B separated by a distance 2R are moving clockwise along the same circular path of radius R each with a uniform speed 'V'. At time t = 0, A is given a constant tangential acceleration of magnitude a= $\frac{32V^2}{25\pi R}$ in the same direction of initial velocity. Then

(A) The time elapse for the two bodies to collide is $\frac{5\pi R}{4V}$

(B) Angle covered by 'B' in this time is $\frac{5\pi}{4}$

(C) Angular acceleration of 'A' is $\frac{32V^2}{25\pi R^2}$

(D) Tangential acceleration of 'B' is zero

Ans. (A), (B), (C), (D)

$$S_{AB}=u_{AB}t+\frac{1}{2}a_{AB}t^2$$

$$\pi R = \frac{1}{2}\left(\frac{32V^2}{25\pi R}\right)t^2$$

$$\Rightarrow t = \frac{5}{4}\frac{\pi R}{V}$$

$$\Rightarrow a_t = R\alpha \qquad \Rightarrow \alpha = \frac{32V^2}{25\pi R^2}$$

For B: $\theta_B = \omega t = \frac{V}{R}t = \frac{5}{4}\pi$

Sol.

27. Choose the correct statement(s) regarding center of mass frame:

(A) If $\sum \vec{F}_{ext} = \vec{0}$ on a system, then center of mass frame is an inertial frame

(B) Centre of mass frame is a zero-momentum frame

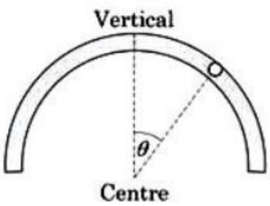
(C) Kinetic energy of a system is minimum in centre of mass frame

(D) None of these

Ans. (A), (B), (C)

Sol. Conceptual

28. A semicircular glass tube filled with water containing an air bubble is sealed at its ends. If the tube is held with its plane vertical and made to move in its plane with a constant acceleration, the bubble stays aside of the highest point as shown in the figure. What can you conclude about acceleration vector of the tube?



(A) It points towards left.

(B) It points towards right.

(C) Its magnitude is g tanθ .

(D) Its magnitude is g cotθ .

Ans. (B), (C)

Sol. Conceptual (effective acceleration due to gravity)

29. A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After the explosion, motion of the mass 4M.

(A) will be in a bound orbit

(B) will be in an unbound orbit

(C) will be in a hyperbolic orbit

(D) will be in elliptical orbit

Ans. (B), (C)

Sol. Let V = velocity of 5M, V_1 = velocity of 4M, V_2 = velocity of M.

As per the information: $V_2 = -V$

Applying the conservation of momentum, we get :

$5MV = 4MV_1 + M(-V)$

$6MV = 4MV_1$

$V_1 = 3/2\ V$

Now, for near earth orbit, if Vo = orbital velocity and Ve = escape velocity, then $Ve = \sqrt{2}\ Vo$. In a bound orbit the object is gravitation

Hence, $V_1 = \frac{3}{2}\frac{V_e}{\sqrt{2}}$. Hence, $V_1 > V_e$.

An unbound orbit is typically hyperbolic, and the object will escape from the source of gravity

30. A long cylinder of radius R_1 length "l " is displaced along its axis with constant velocity v_0 inside a stationary coaxial cylinder of radius R_2 . The space between the cylinders is filled with viscous liquid of coefficient of viscosity η . The flow is laminar. Choose correct options from following

(A) The velocity of liquid as a function of the distance "r" from axis of cylinders is

$$v = v_0\left\{\frac{ln(r/R_2)}{ln(R_1/R_2)}\right\}$$

(B) The velocity of liquid as a function of the distance "r" from axis of cylinders is

$$v = v_0\left\{\frac{ln(r/R_1)}{ln(R_2/R_1)}\right\}$$

(C) The magnitude of force required to move the cylinder with constant velocity is $F = \frac{2\pi l\eta v_0}{ln(R_2/R_1)}$

(D) The magnitude of force required to move the cylinder with constant velocity is $F = \frac{4\pi l\eta v_0}{ln(R_2/R_1)}$

Ans. (A), (C)

Sol.

$$F\frac{dr}{r} = 2\pi l\eta\, dv$$

$$\Rightarrow f\int\frac{dr}{r} = 2\pi l\eta\int dv$$

Numerical Value (If more than two decimal, truncate/roundoff the value two decimal places).

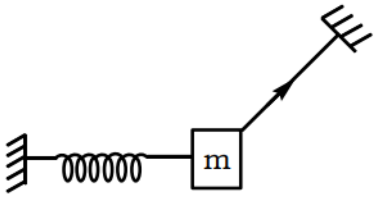
31. A capillary tube made of glass and internal radius 'r' kept vertically such that its lower end is dipped in a liquid. The angle of contact is found to be 53° and surface tension of liquid is T_1. Now the tube is replaced by a polymer tube with the same inner radius. Now, the contact angle changes

to 37° and surface tension of the liquid is T_2. Find $\frac{T_1}{T_2}$. (Temperature of the liquid is assumed to be constant).

Ans. 1.00

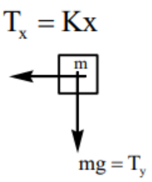
Sol. Surface tension of a liquid is independent of solid in contact.

32. In the fig shown a block of mass 2 kg is suspended in air by an ideal spring and an ideal string as shown. The spring is horizontal and tension in the string is 40 N. The acceleration of block just after the string is cut is ______ m/s^2 ($g = 10 m/s^2$)



Ans. 20.00

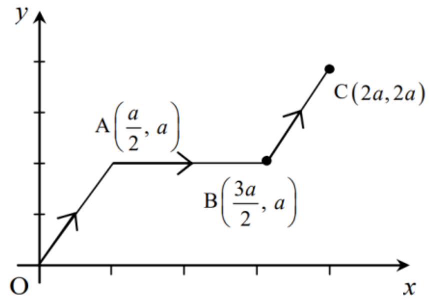
Sol.



just after string is cut acceleration $a = \frac{\sqrt{(kx)^2 + (mg)^2}}{M}$

$a = T/m = 40/2 = 20 m/s^2$

33. A force $\vec{F} = 2xy^3\hat{i} + 3x^2y^2\hat{j}$ on a body which travels from point 'O' (origin) to point C (2a, 2a) along the path OABC , as shown in the figure. If the work done by this force is xa^5, find the value of x.



Ans. 32.00

Sol. $W = \int_O^C \vec{F} \cdot d\vec{r} = \int_O^C \left(2xy^3\hat{i} + 3x^2y^2\hat{j}\right) \cdot \left(dx\hat{i} + dy\hat{j}\right)$

$= \int_O^C 2xy^3 dx + 3x^2y^2 dy$

$= \int_O^C d\left(x^2y^3\right)$

$= \left[x^2y^3\right]_0^{(2a)^2(2a)^3} = (2a)^5$

34. A particle undergoes SHM with a time period of 3 seconds. Minimum time taken by it, to travel from its mean position to a displacement equal to half of its amplitude is _________seconds.

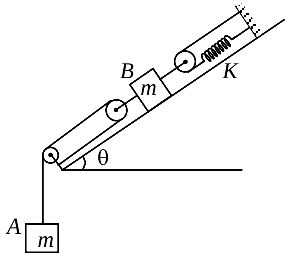
Ans. 0.25

Sol. $T = 2\pi/\omega = 3$ or $\omega = 2\pi/3$

For a particle undergoing SHM, starting from the origin, $x = a \sin \omega t$

For $x = a/2$, $a/2 = a \sin \omega t$ or $t = \frac{1}{4}$ s

35. Two blocks A and B, each of mass m are connected by means of a pulley-spring system on a smooth inclined plane of inclination θ as shown in the figure. All the pulleys and spring are ideal. Now, B is slightly displaced from its equilibrium position. It starts to oscillate. Time period of oscillation of B will be _________seconds (Take m = 4 kg, K = 5 N/m, π = 3.14)



Ans. 6.28

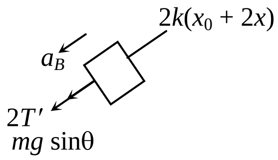
Sol. Let elongation of spring be x_0 in equilibrium. Then,

$$2T + mg\sin\theta = 2kx_0 \quad ...(i)$$

and $$T = mg \quad ...(ii)$$

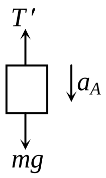
Let Block B is displaced by x down the inclination

F.B.D. of B



$$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta \quad ...(iii)$$

F.B.D. of A



$$mg - T' = ma_A$$

Also, $$a_A = 2a_B$$

$$T' = mg - 2ma_B$$

$$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$$

$$-ma_B = 4kx + 4ma_B$$

$$a_B = -\frac{4k}{5m}x$$

$\therefore$ $$T = 2\pi\sqrt{\frac{5m}{4k}}$$

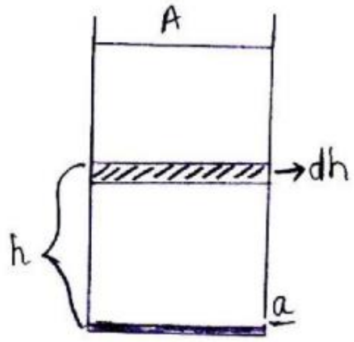
T = 6.28 s.

36. A vessel of large area of cross-section opened at the top has a small hole at its bottom and is completely filled with water. When the hole at the bottom is opened ¾ of volume of water comes out in time t_1. The time (t_2) taken for the remaining ¼ of the water to come out is equal to xt_1. The value of x is

Ans. 1.00

Sol.

Let A, a be the area of cross-section of the vessel and hole respectively. Let h be the height of water level in the vessel at any instant during its fall. At this level velocity of efflux through hole $v=\sqrt{2gh}$.



Let the level falls further by dh in time dt, then rate of decrease of volume in the vessel is $A\frac{dh}{dt}$ which is equal to rate at which liquid comes out $a\sqrt{2gh}$.

i.e. $-A\frac{dh}{dt}=a\sqrt{2gh}$ $\Rightarrow -\frac{A}{2}\int_{h_1}^{h_2}\frac{dh}{\sqrt{2gh}}=\int_0^t dt$ $\Rightarrow -\frac{A}{a}\times\frac{1}{\sqrt{2g}}\left[2\sqrt{h}\right]_{h_1}^{h_2}=t$

$\Rightarrow \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{h_1}-\sqrt{h_2}\right)=t$

$\Rightarrow t_1=\frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{\frac{H}{4}}\right)$ $\Rightarrow t_2=\frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{\frac{H}{4}}-\sqrt{0}\right)$ $\Rightarrow t_2=t_1$

PART-C: CHEMISTRY

Single Digit Integer (0-9 both inclusive)

37. Find the quantum numbers of excited state of electrons in He^+ion which on transition to ground state and first excited state emit two photons of wavelengths, $\mathbf{30.4\ mm}$ and 108.5 nm respectively. $(R_h = 1.09678 \times 10^7\ m^{-1})$

Ans. 5

Sol. For transition to ground state, $n_1 = 1, n_2 = ?$

$$\frac{1}{30.4 \times 10^{-9}} = 1.09678 \times 10^7 \times 4\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$$

This gives, $n_2 = 2$
For $n_1 = 2$ (first excited state), $n_2 = ?$

$$\frac{1}{108.5 \times 10^{-9}} = 1.09678 \times 10^7 \times 4\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$$

This gives, $n_2 = 5$

38. The formula weight of an acid is $82.0.100\ cm^3$ of a solution of this acid containing 39.0 g of the acid per litre were completely neutralized by 95.0 cm^3 of aqueous NaOH containing 40.0 g of NaOH per litre. What is the basicity of the acid ?

Ans. 2

Sol. Normality of acid $= \frac{39}{\frac{82}{n} \times 1}$

Normality of NaOH $= \frac{40}{40} \times \frac{1000}{1000} = 1$

Now, Meq. of acid = Meq. of NaOH

$$\frac{39n}{82} \times 100 = 1 \times 95$$

$\therefore$ $n = 2$; i.e., acid is dibasic.

39. A certain buffer solution contains equal concentration of $X^-$and $HX(K_b$ for $X^-$is 10^{-10}) what is pH of buffer?

Ans. 4

Sol. For a conjugate acid -base pair we have

$$K_a(HX) \times K_b(X^-) = 10^{-4}$$

$$K_a = \frac{10^{-14}}{10^{-10}} = 10^{-4}$$

$$[HX^-] = [X^-]$$

So $pH = pK_a + \log\frac{[\text{salt}]}{[\text{acid}]}$

$= 4$

40. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

Ans. 7

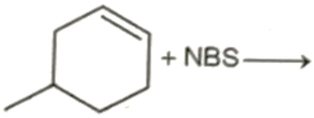
Sol.

Cyclic C_5H_{10}

1 2 3 4 5

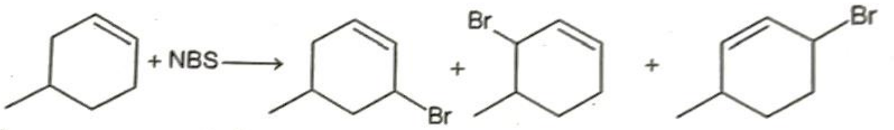
For 3rd structure 2 cis-trans and 1 optical isomer are possible.
Total 7 isomers.

41. Number of products in the following reaction:



Ans. 3

Sol.



42. A river contains 5 mg of lead in 1,000L of water. What is the concentration of lead in ppm?

Ans. 5

Sol. Using the formula:

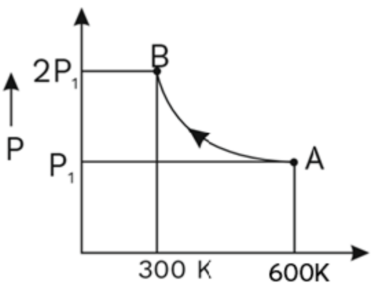
$$\text{ppm} = \left(\frac{\text{mass of solute (mg)}}{\text{volume of solution (L)}}\right) \times 10^6$$

$$= \left(\frac{5}{1000}\right) \times 10^6$$

$$= 5\text{ppm}$$

One or More than One Correct

43. 2 moles of an ideal monoatomic gas is taken from a state A to state B through a process AB in which PT = constant. The process can be represented on a P-T graph as follows : .



Select the correct option(s):

(A) Heat evolved by the gas during process $AB = 2100R$

(B) Heat absorbed by the gas during process $AB = 2100R$

(C) $\Delta U = -900\text{R}$

(D) $W = 2000R$

Ans. (A, C)

Sol. $PT =$ constant $\Rightarrow PV^{1/2} =$ constant

$$W = \frac{P_2V_2 - P_1V_1}{x-1}$$

$$= \frac{nR(T_2 - T_1)}{x-1}$$

$= \frac{2 \times R(-300)}{-\frac{1}{2}}$

$= 1200R$

$\Delta U = nC_{v,m}\Delta T = 2 \times \frac{3}{2}R(300-600) = -900R$

$Q = \Delta U - q = -900R - 1200R = -2100R$

44. Which of the following is/are correct?

(A) For $A(g) + e^- \longrightarrow A^-(g)\Delta H$ may be negative

(B) For $A^-(g) + e^- \longrightarrow A^{2-}(g)\Delta H$ may be negative

(C) For $A^-(g) + e^- \longrightarrow A^{2-}(g)\Delta H$ must be positive

(D) For $A^{+3}(g) + e^- \longrightarrow A^{+2}(g)\Delta H$ must be negative

Ans. (A, C, D)

Sol. Fact.

45. Choose the correct relations on the basis of Bohr's theory

(A) Velocity of electron $\propto \frac{1}{n}$

(B) Frequency of revolution $\propto \frac{1}{n^3}$

(C) Radius of orbit $\propto n^2 Z$

(D) Force of electron $\propto \frac{1}{n^4}$

Ans. (A, B, D)

Sol. Fact.

46. In the reaction,

$$3Br_2 + 6CO_3^{2-} + 3H_2O \rightarrow 5Br^- + BrO_3^- + 6HCO_3^-$$

(A) bromine is oxidized and carbonate is reduced

(B) bromine is oxidized

(C) bromine is reduced

(D) it is disproportionation reaction or autoredox change

Ans. (B, C, D)

Sol. Fact.

47. What in general may be the general criteria of choosing a suitable indicator for a given titration?

(A) The indicator should have a broad pH range

(B) pH at the end point of titration should be close to neutral point of indicator

(C) Indicator should have neutral at $pH = 7$

(D) The indicator must show a sharp change in colour near the equivalence point of titration point

Ans. (B, D)

Sol. Fact.

48. A hydrazine molecule is split in NH_2^+ and NH_2^- ions. Which of the following statements is/are correct?

(A) NH_2^+ shows sp^2 - hybridisation whereas NH_2^- shows sp^3 – hybridization

(B) $Al(OH)_4^-$ has a regular tetrahedral geometry

(C) sp^2 - hybridized orbitals have equal s - and p – character

(D) Hybridized orbitals always form σ - bonds

Ans. (A, B, D)

Sol. Fact.

Numerical Value (If more than two decimal, truncate/roundoff the value two decimal places).

49. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid is ionized is y × 10^{-1}. Find Y

Ans. 95

Sol.

50. Calculate the bond length of $C-X$ bond, if $C-C$ bond length is $1.54Å$, $X-X$ bond length is $1.00Å$ and electronegativity values of C and X are 2.0 and 3.0 respectively

Ans. 1.18

Sol. (1) $C-C$ bond length $= 1.54Å$

$$r_C = \frac{1.54}{2} = 0.77Å$$

$$r_X = \frac{1.00}{2} = 0.50Å$$

(2) $C-X$ bond length

$$d_{C-X} = r_C + r_X - 0.09|X_X - X_C|$$
$$= 0.77 + 0.50 - 0.09|3-2| = 0.77 + 0.50 - 0.09 \times 1 = 1.27 - 0.09 = 1.18Å$$

Thus $C-X$ bond length is $1.18Å$

51. The number of benzylic hydrogen atoms in ethylbenzene is

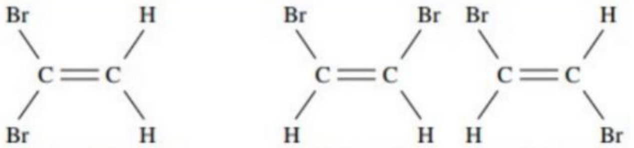
Ans. 2

Sol. $PH-\overset{*}{C}H_2-CH_3$ starred carbon is benylie carbons it has two hydrogen atoms.

52. The number of isomers of dibromoderivative of an alkene (molar mass 186 g mol^{-1}) is

Ans. 3

Sol. Compound is dibromoethane, it can have 3 isomers (two geometrical isomers (cis & trans) of 1, 2 dibromoethene and one is 1, 1 dibromoethene).



no geometric isomers
1,1-dibromoethene

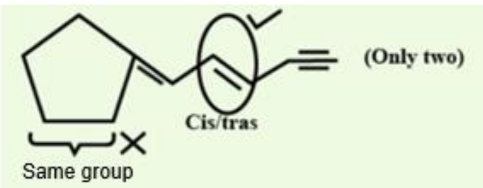
cis isomer *trans* isomer
1,2-dibromoethene

53. The number of cis-trans isomer possible for the following compound

Ans. 2

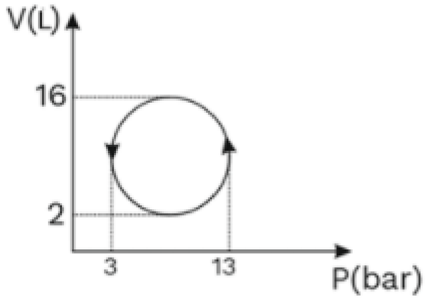
Sol. Cis-trans Isomers:- It is under the class of geometric isomer,

- It exists in alkenes (organic molecules which have double bonds).
- When two similar or higher priority groups are attached to the carbon on the same side, it is termed as cis isomer and when it is attached to the opposite side it is called as trans isomer.



54. Work done (in kJ) by the gas in the following cyclic process is

(Given 1 litre bar = 100K π =3.14)



Ans. 11

Sol. $|w| = \pi ab = \frac{22}{7} \times 7 \times 5 = 110$ bar lit. $= 11000\ J = 11\ kJ$

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