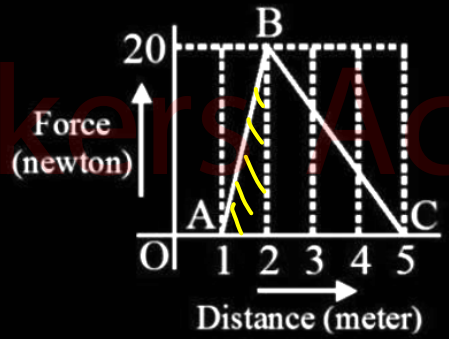
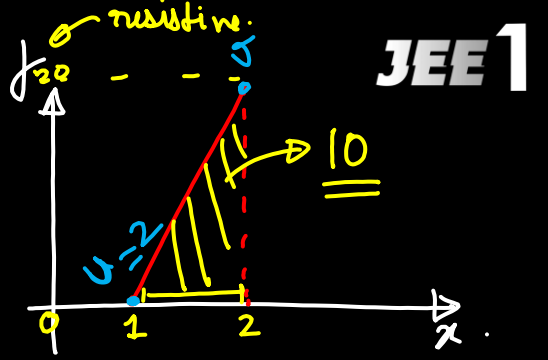
PHYSICS

7

The resisting force F acting on a body versus distance traversed by the body is plotted in figure. The mass of the body is 25 kg and the initial speed is 2 m/s. The kinetic energy (in joule) of the body at distance 2.0 m is



(A) 10
(B) 20
(C) 30
(D) 40



JEE 1

$W = -10 = \Delta KE$

$-10 = KE_f - KE_i$

$-10 = KE_f - \frac{1}{2}(25)(2)^2$

$KE_f = 40$

2

The physical quantities not having same dimensions are

(A) speed and $(\mu_0\varepsilon_0)^{-1/2}$

(B) torque and work

(C) momentum and Planck's constant

(D) stress and Young's modulus

LT^{-1} avoid. $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

Fx same

Ans — (C)

same.

stress. stress

strain

$\frac{\Delta L}{L}$

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$ JEE 1

$[p] = MLT^{-1}$

$[h] = \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}}$

$B = \frac{1}{4\pi\mu_0} \frac{I}{2\pi R}$

$F = BIL$

3

A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s². He reaches the ground with a speed of 3 m/s. At what height, did he bail out :

(A) 111 m

(B) 293 m

(C) 182 m

(D) 91 m

$v^2 = u^2 + 2as$

$3 \quad \sqrt{1000} \quad -2$

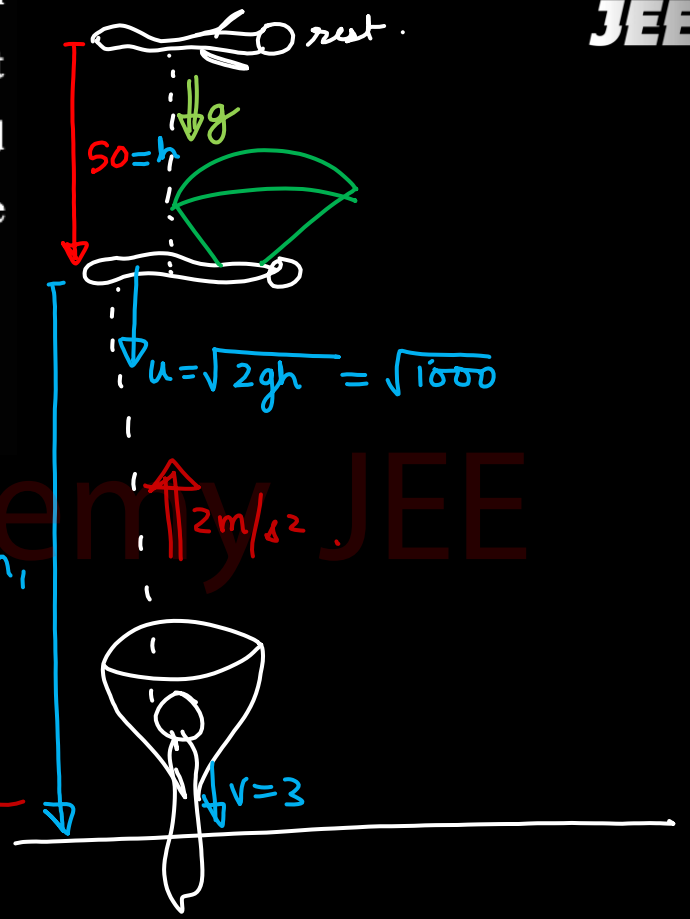
$9 = 1000 + 2(-2)h_1$

$4h_1 = 991$

$h_1 = \frac{991}{4} = 247.75$

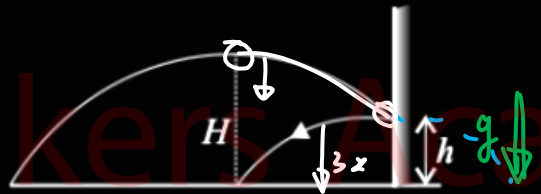
Ans $= 50 + h_1$

$= 293$



4

A stone is projected from a horizontal plane. It attains maximum height H and strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assuming the collision to be elastic, the height of the point on the wall where ball will strike is:

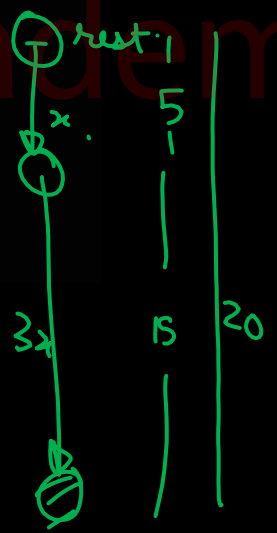


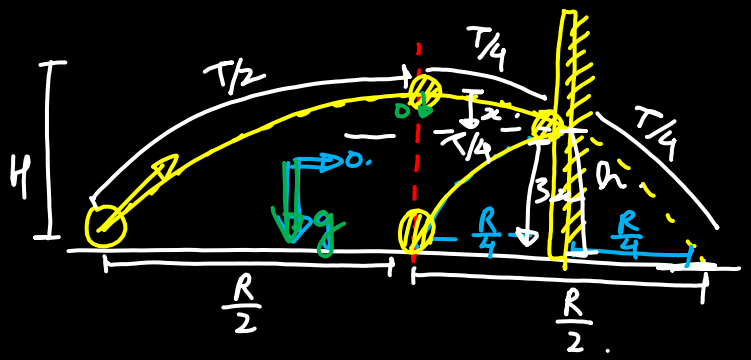
(A) $\frac{H}{4}$

(B) $\frac{H}{2}$

(C) $\frac{3H}{4}$

(D) $\frac{7H}{8}$





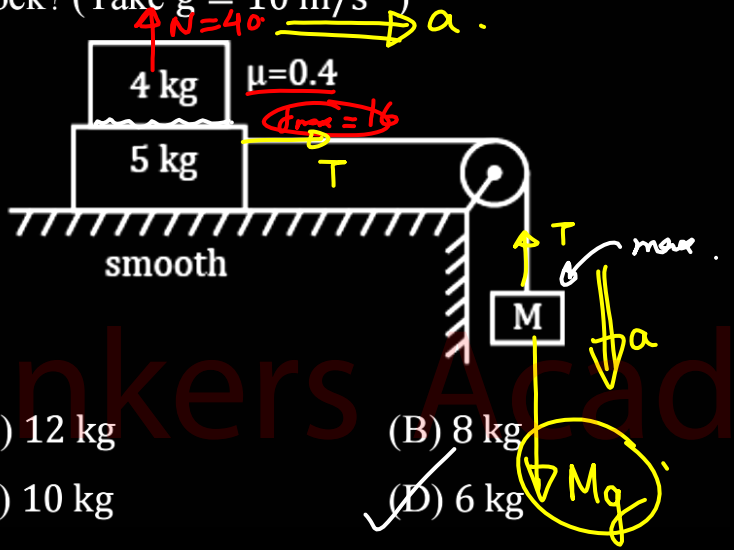
$x + 3x = H$

$x = H/4$

$h = 3x = \frac{3H}{4}$

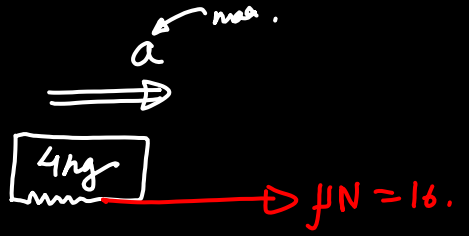
5

What should be the maximum value of M so that the 4 kg block does not slip over the 5 kg block? (Take g = 10 m/s^2)



(A) 12 kg (B) 8 kg

(C) 10 kg (D) 6 kg



$16 = 4a$

$a = 4$ ✓

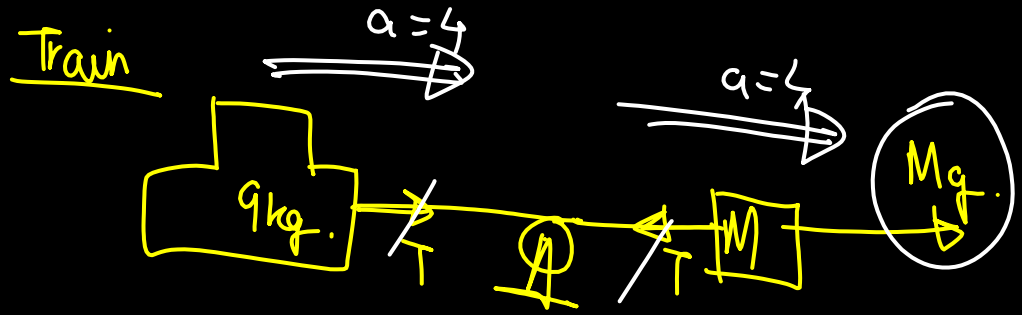
$Mg = (M+9)\,4$

$2.5M = M + 9.$

$1.5M = 9$

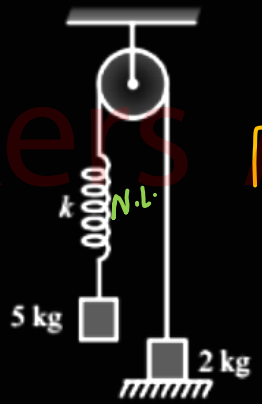
$\frac{3}{2}M = 9$

$M = 6$



6

System shown in the figure is released from rest when spring is unstretched. Pulley and spring are massless and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with ground is: (Take force constant of spring k = 40 N/m and g = 10 m/s²



(A) $\sqrt{2}$ m/s

(B) $2\sqrt{2}$ m/s

(C) 2 m/s

(D) $4\sqrt{2}$ m/s

$V = \sqrt{8}$

JEE 1

rest

HAND. 5kg = m

N.L.

rest

2kg

U=0

T=kx

Kx = 20

40x = 20

x = 1/2

T=20 = kx

N=0

rest

2kg

fly off.

20

x

5kg

?=v

$E_i = E_f$

$mgx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$50\left(\frac{1}{2}\right) = \frac{5}{2}v^2 + 20\left(\frac{1}{4}\right)$

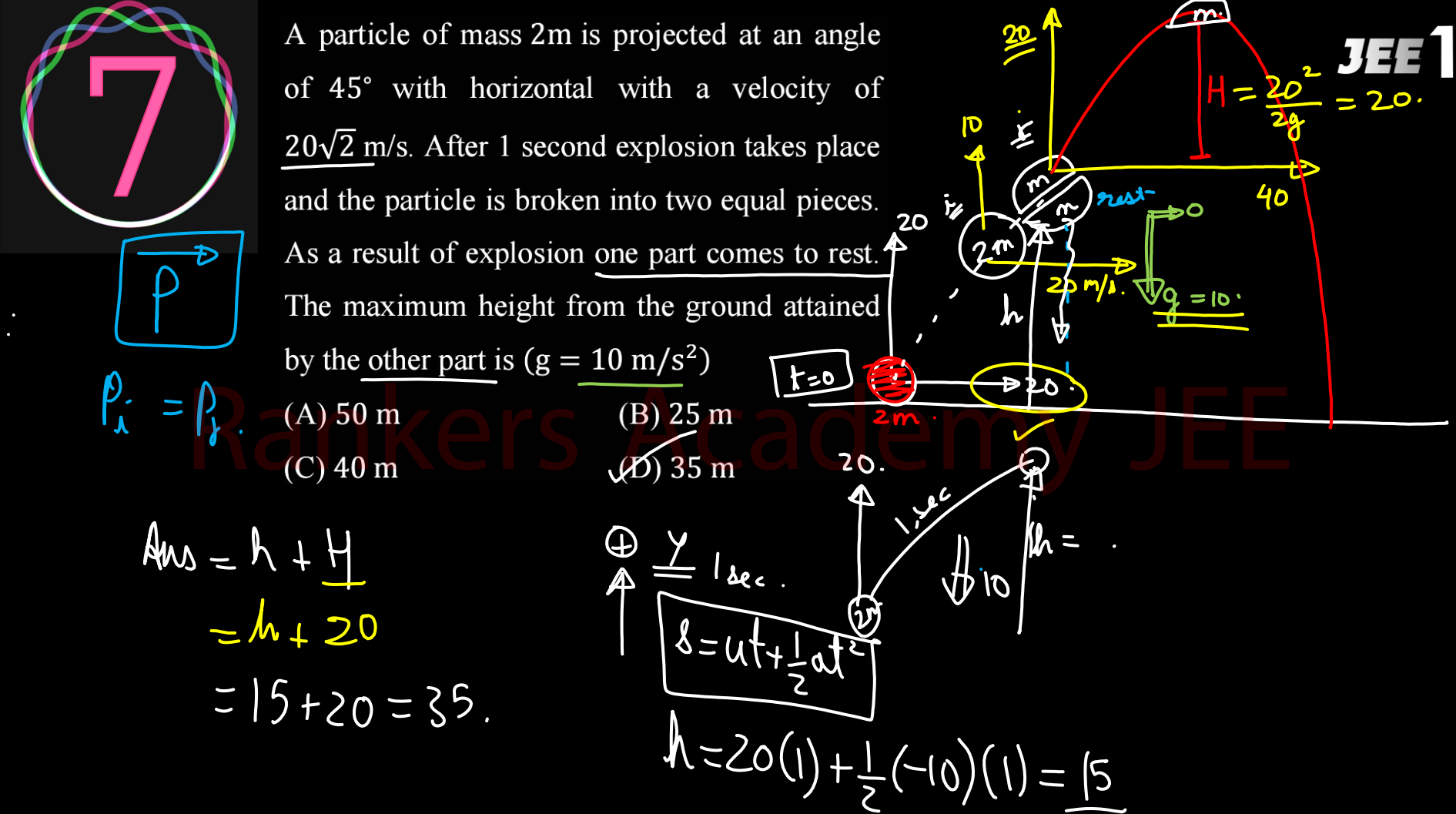
$20 = \frac{5}{2}v^2$

7

A particle of mass 2m is projected at an angle of 45° with horizontal with a velocity of $20\sqrt{2}$ m/s. After 1 second explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. The maximum height from the ground attained by the other part is ($g = 10\ m/s^2$)

(A) 50 m (B) 25 m

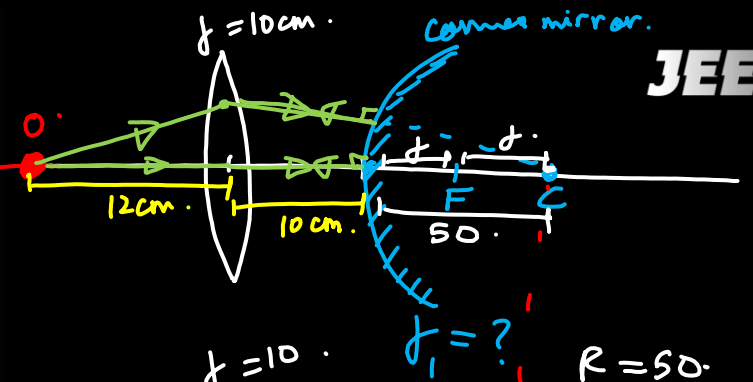
(C) 40 m (D) 35 m

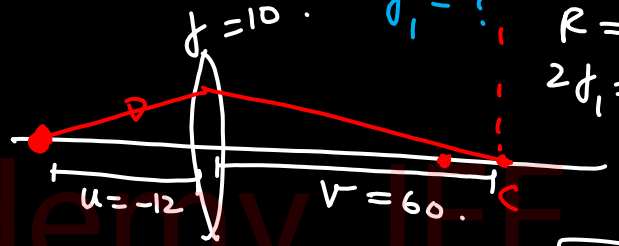


8

A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. Focal length of convex mirror is

(A) 52 cm
(B) 25 cm
(C) 32 cm
(D) 15 cm





$R = 50$.

$2f_1 = 50$.

$f_1 = 25$

$$\frac{1}{f} = \frac{1}{V} - \frac{1}{u} \rightarrow \oplus.$$

$$\frac{1}{10} = \frac{1}{V} - \frac{1}{-12}$$

$$\frac{1}{V} = \frac{1}{10} - \frac{1}{12} = \frac{2}{120}$$

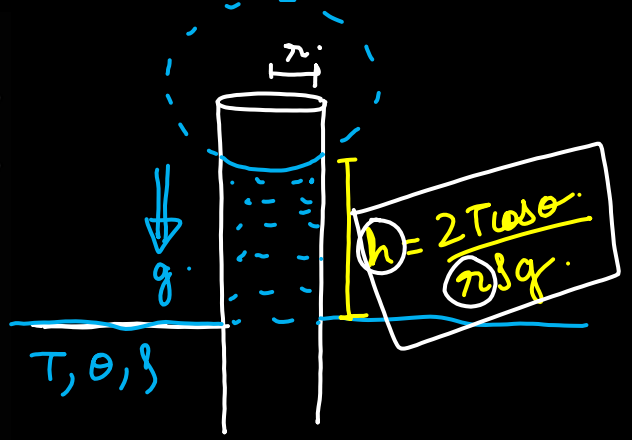
$V = 60$

9

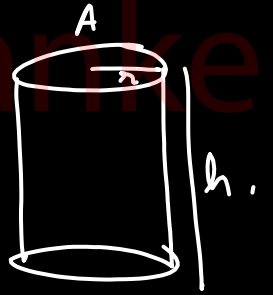
A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in the capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will now be :

(A) M (B) 2M

(C) M/2 (D) 4M

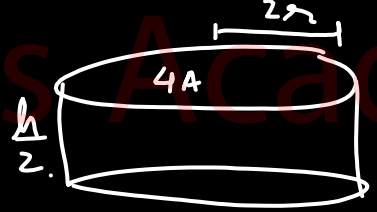


JEE 1



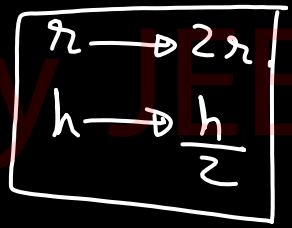
$V = Ah$

M



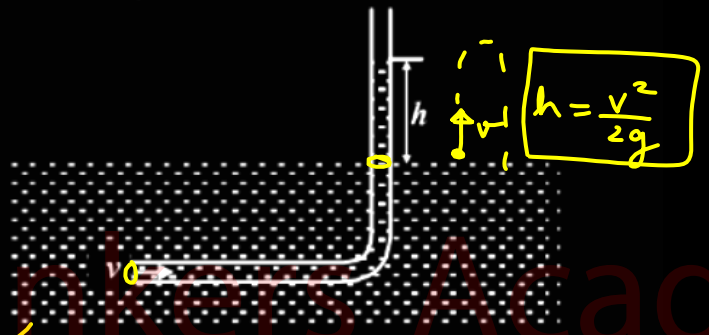
$V' = 4A\left(\frac{h}{2}\right) = 2V$

2M.



10

An L-shaped glass tube is just immersed in flowing water such that its opening is pointing against flowing water. If the speed of water current is v, then

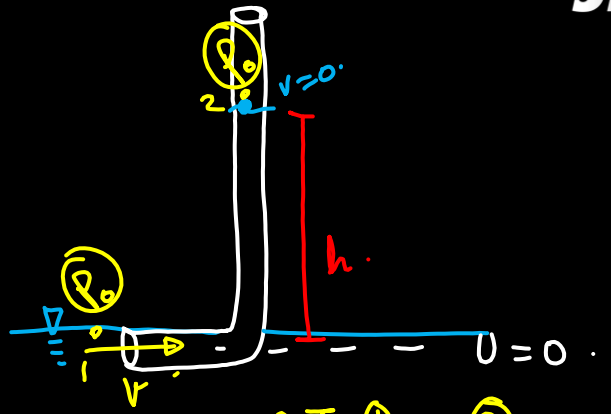


(A) The water in the tube rises to height $\frac{v^2}{2g}$

(B) The water in the tube rises to height $\frac{g}{2v^2}$

(C) The water in the tube does not rise at all

(D) None of these



BT ① → ②

$P_0 + \frac{1}{2}\rho v^2 + 0 = P_0 + \rho g h$

$v^2 = 2gh$

$h = \frac{v^2}{2g}$

11

The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000t)$. This expression may be considered to be a result of the superposition of independent harmonic motions

(A) Two (B) Three

(C) Four (D) Five

JEE 1

$y = 2[2\cos^2(t/2)]\sin(1000t)$

$y = 2[1+\cos t]\cdot\sin 1000t$

$y = 2\sin 1000t + 2\sin 1000t\cdot\cos t$

$y = 2\sin(1000t) + \sin(1001t) + \sin(999t)$

12

v_1 and v_2 are the velocities of sound at the same temperature in two monoatomic gases of densities ρ_1 and ρ_2 respectively. If $\frac{\rho_1}{\rho_2} = \frac{1}{4}$, then the ratio of velocities v_1 and v_2 will be

(A) 1: 2
(B) 4: 1
(C) 2: 1
(D) 1: 4

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma P}{\rho}}$$

JEE 1

$$PM = \rho RT$$
$$\frac{P}{\rho} = \frac{RT}{M}$$

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}}$$

$$v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

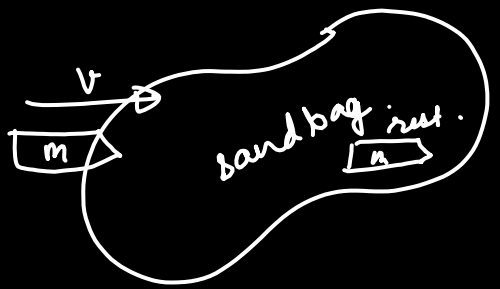
$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = 2$$

13

A 5.0 g bullet (specific heat of material of bullet = 128 J/kg°C) moving with a velocity of 200 m/s enters a sand bag and stops. If the entire kinetic energy of the bullet is changed into heat energy that is added to the bullet, then the rise in the temperature of the bullet is:

(A) 312.5°C (B) 156°C

(C) 500°C (D) 624°C



$\frac{1}{2}mv^2 = mS\Delta T$

$\frac{200 \times 200}{2} = 128\,\Delta T$

$\Delta T = 156^{\circ}C$

14

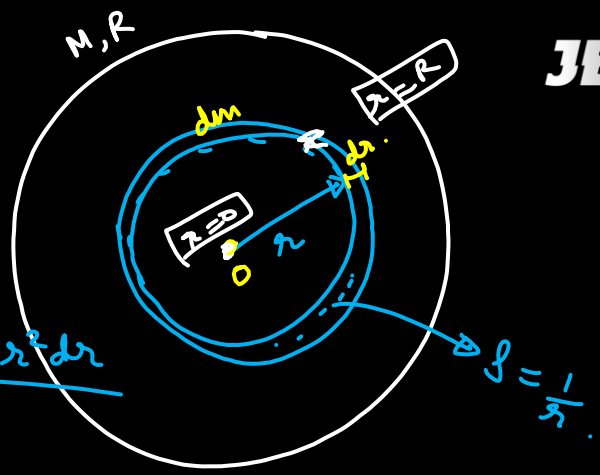
A solid sphere of mass M and radius R has density ρ varies with r as $\rho = \frac{1}{r}$, where r(r < R) is the distance from centre of sphere. The gravitational potential at the centre of sphere is

(A) $-\frac{GM}{R} = V_0$

(B) $-\frac{2GM}{R} = V_0$

(C) $-\frac{2GM}{3R} = V_0$.

(D) $-\frac{3GM}{2R}$



JEE 1

$dV = 4\pi r^2 dr$

$dm = \rho . dV$

$dm = \frac{1}{r} . 4\pi r^2 dr$.

$M = \int dm$.

$M = \int_0^R 4\pi r\, dr = \frac{4\pi R^2}{2} = 2\pi R^2$

$M = 2\pi R^2$ $\quad 2\pi R = \frac{M}{R}$

$dV_0 = -\frac{G\, dm}{r}$

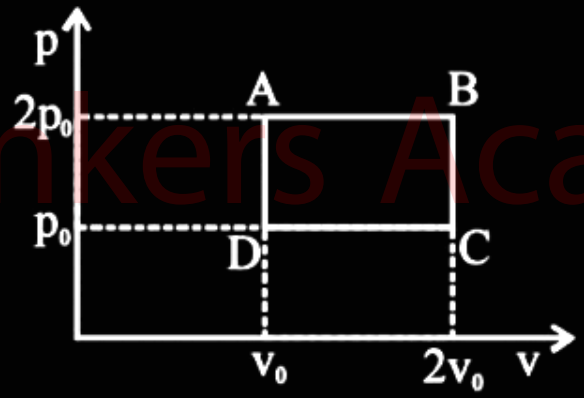
$V_0 = \int dV_0 = -\int_{r=0}^{r=R} \frac{G\, 4\pi r\, dr}{r}$

$V_0 = -4\pi G R = -G(4\pi R)$

$= -G\left(\frac{2M}{R}\right)$

15

The above $p-v$ diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is



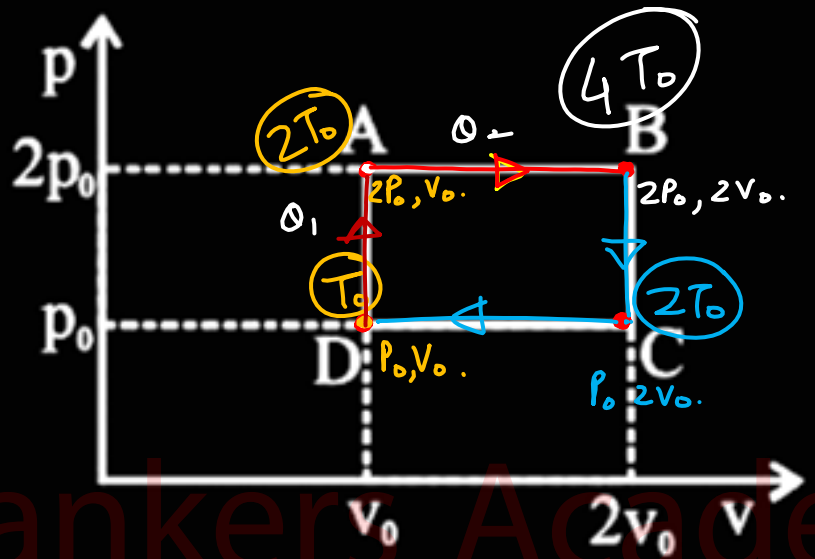
(A) p_0v_0

(B) $\left(\frac{13}{2}\right)p_0v_0$

(C) $\left(\frac{11}{2}\right)p_0v_0$

(D) $4p_0v_0$

15



$PV = nRT$

$\frac{PV}{nR} = T$

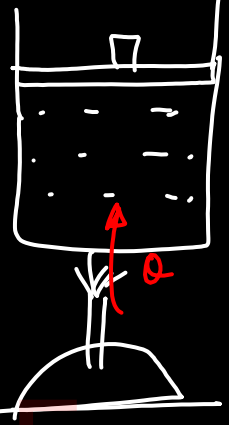
$C_p - C_v = R$

$C_v = \frac{fR}{2}$

$f = 3$

$C_v = \frac{3R}{2}$

$C_p = \frac{5R}{2}$



$Q = \Delta U + W$

$Q = nC_v\Delta T + nR\Delta T$ or $P\Delta V$

$\Delta W = Q_1 + Q_2$

$\Delta W = nC_v\Delta T_1 + nC_p\Delta T_2 = \left(\frac{3}{2}+5\right)P_0V_0$

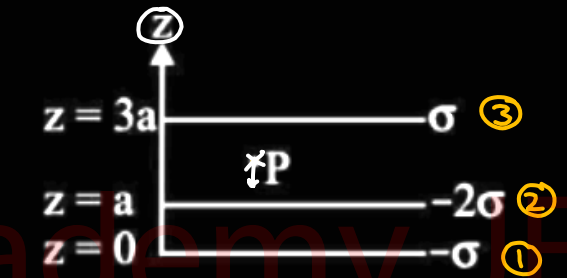
$= \frac{3}{2}nR(\Delta T_1) + \frac{5n}{2}R(2T_0) = \left(\frac{13}{2}\right)P_0V_0$

$= \frac{3}{2}nRT_0 + 5nRT_0$

16

JEE 1

Three infinite long charged sheets of charge densities $-\sigma, -2\sigma$ and σ are placed parallel to xy-plane at $z = 0, z = a, z = 3a$. Electric field at point P is



$$E_{net} = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)(-\hat{k})$$

$$= \frac{4\sigma}{2\varepsilon_0}(-\hat{k}) = \frac{2\sigma}{\varepsilon_0}\hat{k}$$

(A) $\frac{-2\sigma}{\epsilon_0}\hat{k}$

(B) $\frac{2\sigma}{\epsilon_0}\hat{k}$

(C) $\frac{-4\sigma}{\epsilon_0}\hat{k}$

(D) $\frac{4\sigma}{\epsilon_0}\hat{k}$

17

JEE 1

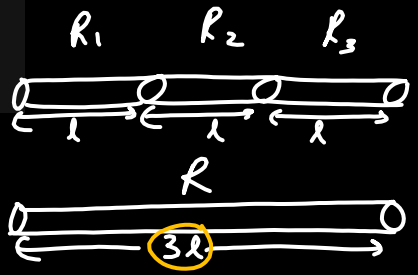
If $\sigma_1, \sigma_2, \sigma_3$ are the conductances of three conductors, of equal size then their equivalent conductance, when they are joined in series, will be

(A) $\sigma_1 + \sigma_2 + \sigma_3$

(B) $\frac{1}{\sigma_1} + \frac{1}{\sigma_2} + \frac{1}{\sigma_3}$

(C) $\frac{3\sigma_1\sigma_2\sigma_3}{\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_1\sigma_3}$

(D) none of these



$R = R_1 + R_2 + R_3$

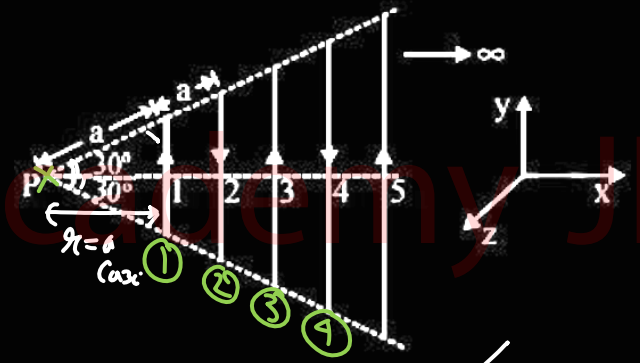
$$\frac{3l}{\sigma_q A} = \frac{l}{\sigma_1 A} + \frac{l}{\sigma_2 A} + \frac{l}{\sigma_3 A}$$

$$\frac{3}{\sigma_q} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2} + \frac{1}{\sigma_3} \Rightarrow \sigma_q = \frac{3\sigma_1\sigma_2\sigma_3}{\Sigma\sigma_1\sigma_2}$$

18

JEE 1

Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Net magnetic field at point P is



(A) $\frac{\mu_0 I}{4\pi}\frac{\ell n 2}{\sqrt{3}a}\hat{k}$

(B) $\frac{\mu_0 I}{4\pi}\frac{\ell n 4}{\sqrt{3}a}\hat{k}$

(C) $\frac{\mu_0 I}{4\pi}\frac{\ell n 4}{\sqrt{3}a}(-\hat{k})$

(D) zero

$B_1 = \frac{\mu_0 i}{4\pi(a\cos 30^\circ)}(\sin 30^\circ + \sin 30^\circ)$

$\vec{B}_1 = \frac{\mu_0 i}{2\pi a\sqrt{3}}\odot$

$\vec{B}_{net} = (B_1 - B_2 + B_3 - B_4 \ldots)\odot$

$= \frac{\mu_0 I}{2\pi\sqrt{3}}\frac{1}{a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\right]$

$\vec{B}_{net} = \frac{(\mu_0 I)}{4\pi\sqrt{3}}(2)\ell n 2\,\hat{k} = \frac{1}{2}\ell n 4\,\hat{k}$

19

JEE 1

The electric field of light wave is given as

$$\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times 6\times10^{14}t\right)\hat{x}\,\frac{N}{C}$$

This light falls on a metal plate of work function 2eV. The stopping potential of the photo electrons is _____ Given, $E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})}$

(A) 0.48 V (B) 0.72 V

(C) 2.48 V (D) 2.0 V

$eV_0 = \frac{hc}{\lambda} - \phi$

$= \frac{12375}{5000} - 2eV$

$= 2.475 - 2$

$= 0.475eV$

20

JEE 1

In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes maximum Intensity, $\frac{I}{I_0}$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$ *

$\Delta\phi = 2\pi \times \frac{\lambda/6}{\lambda}$

$= \frac{2\pi}{6} = \frac{\pi}{3}$

$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$ *

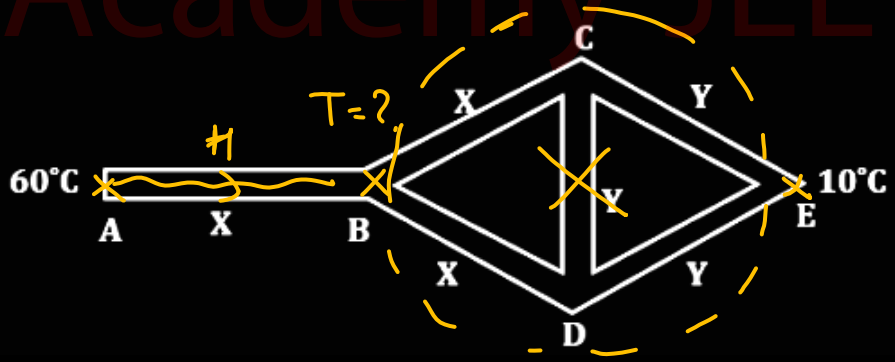
$= I_0 \cos^2\left(\frac{\pi}{6}\right)$

$\frac{I}{I_0} = \frac{3}{4}$

21

JEE 1

Three rods of material x and three of material y are connected as shown in figure. All the rods are identical in length and crosssectional area. If the end A is maintained at 60°C and the junction E at 10°C, the temperature of the junction B is ___ °C, The thermal conductivity of x is 800W/m – °C and that of y is 400W/m – °C.



$$H = \frac{\Delta T}{R}$$

$$H = \frac{60-T}{R_x} = \frac{T-10}{\frac{R_x+R_y}{2}}$$

$$\Rightarrow 1 + \frac{R_y}{R_x} = \frac{2T-20}{60-T}$$

$$\Rightarrow 1 + \frac{800}{400} = \frac{2T-20}{60-T}$$

$$180 - 3T = 2T - 20$$

$$200 = 5T$$

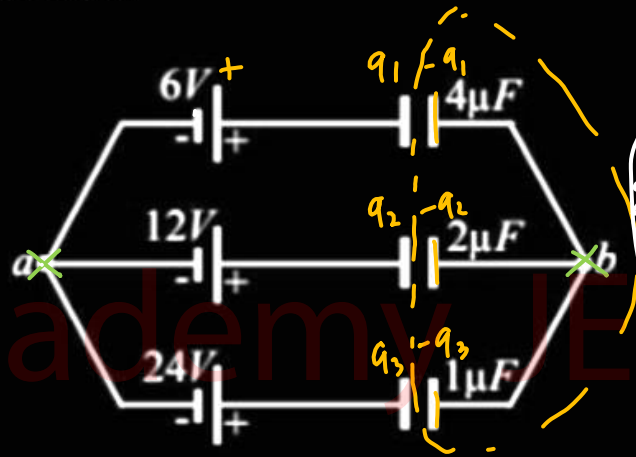
$$T = \frac{200}{5} = 40^\circ C$$

$$\frac{x}{2} + \frac{y}{2} = \frac{x+y}{2}$$

22

JEE 1

The potential difference $V_b - V_a$ between the points a and b shown in each part of the figure. is $\frac{x}{7}$ v. Find x



Method ①

$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$\varepsilon_{eq} = \frac{\varepsilon_1 C_1 + \varepsilon_2 C_2 + \varepsilon_3 C_3}{C_1 + C_2 + C_3} \quad *$$

$$= \frac{(6\times 4) + (12\times 2) + (24\times 1)}{4 + 2 + 1}$$

$$\varepsilon_{eq} = \frac{72}{7}\ V$$

Method ②

$$V_{ab} = \frac{q_1}{C_1} + \varepsilon_1 = \frac{q_2}{C_2} + \varepsilon_2 = \frac{q_3}{C_3} + \varepsilon_3$$

$q_1 = C_1V - \varepsilon_1 C_1$ | $q_2 = C_2V - \varepsilon_2 C_2$ | $q_3 = C_3V - \varepsilon_3 C_3$

$q_1 + q_2 + q_3 = 0$

$\Sigma(C_1V - \varepsilon_1) = 0$

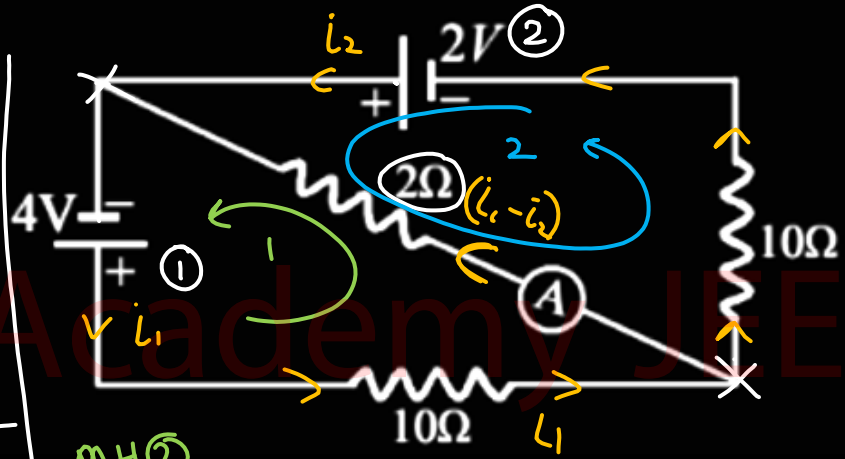
$\Sigma C_1V = \Sigma \varepsilon_1 C_1$

$$V = \frac{\varepsilon_1 C_1 + \varepsilon_2 C_2 + \varepsilon_3 C_3}{C_1 + C_2 + C_3}$$

23

JEE 1

The ammeter current in the circuit shown in figure is $\frac{1}{x}$ A. Find the value of x.



MH①

$$\mathcal{E}_{eq} = \frac{\frac{\mathcal{E}_1}{r_1} + \frac{\mathcal{E}_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$= \frac{\frac{4}{10} + \frac{-2}{10}}{\frac{1}{10} + \frac{1}{10}}$$

$\mathcal{E}_{eq} = 1\,V \quad r_{eq} = 5\,\Omega$

$$i = \frac{\mathcal{E}_{eq}}{R + r_{eq}} = \frac{1}{5+2} = \boxed{\frac{1}{7}}$$

MH②

$4 = 10i_1 + 2(i_1 - i_2)$ — ①

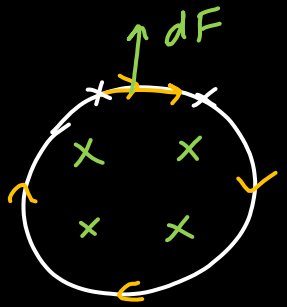
$2 = 10i_2 - 2(i_1 - i_2)$ — ②

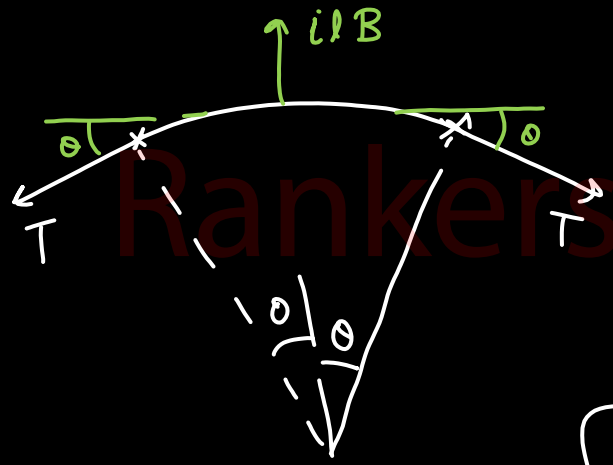
Solve for $i_A = (i_1 - i_2)$

24

JEE 1

A conducting loop is placed in a magnetic field of strength B perpendicular to its plane. Radius of the loop is r, current in the loop is i and linear mass density of the wire of loop is m. Speed of any transverse wave in the loop will be $\sqrt{\frac{Bir}{nm}}$. Find n





$2T\sin\theta = ilB$

$2T(\theta) = i(2\theta r)B$

$T = irB$

$v = \sqrt{\frac{I}{\mu}}$

$= \sqrt{\frac{I}{m}}$

$= \sqrt{\frac{irB}{m}}$

$n = 1$

25

JEE 1

In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16}/m^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}/m^3$. The electron density in the doped semiconductor in _____ $\times 10^9/m^3$.

Pure Semic n_i

Law of mass action

$n_i^2 = n_e n_H$ *

n_H

$n_e = ?$

$$n_e = \frac{n_i^2}{n_H}$$

$$= \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}}$$

$$= \frac{1.5 \times 1.5}{4.5} \times 10^{32-22} = 5 \times 10^9$$

CHEMISTRY

1

Consider the following sequence of reactions :

Compound [A] $\xrightarrow{\text{REDUCTTION}}$ [B] $\xrightarrow{HNO2}$ CH_3CH_2OH

The compound [A] is

(A) CH_3CH_2CN (B) CH_3NO_2

(C) CH_3NC (D) CH_3CN

(A) $CH_3CH_2C\equiv N \xrightarrow{[R]} CH_3CH_2CH_2NH_2 \xrightarrow{HNO_2} CH_3CH_2CH_2-OH$

(B) $CH_3NO_2 \xrightarrow{[R]} CH_3NH_2 \xrightarrow{HNO_2} CH_3OH$

(C) $CH_3NHCH_3 \xrightarrow{HNO_2} H_3C-\underset{\underset{NO}{|}}{N}-CH_3$

(B)

2°

2

In a set of the given reactions, acetic acid yielded a product C.

$$CH_3COOH + PCl_5 \longrightarrow A \xrightarrow[\text{anh.}AlCl_3]{C_6H_6} B \xrightarrow[\text{Ether}]{C_2H_5MgBr} C$$

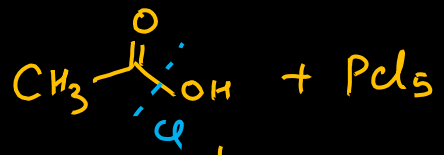
Product C would be -

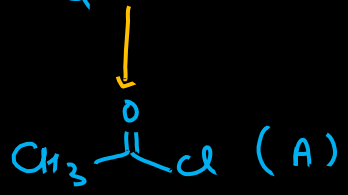
(A) $CH_3-\overset{\overset{C_2H_5}{|}}{C}(OH)C_6H_5$

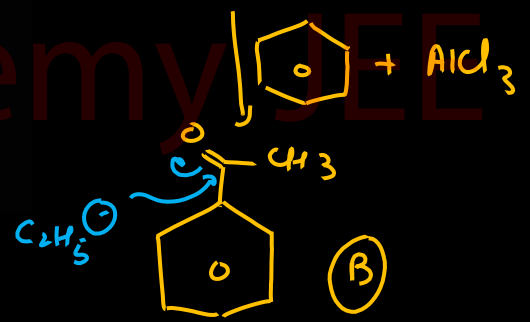
(B) $CH_3CH(OH)C_2H_5$

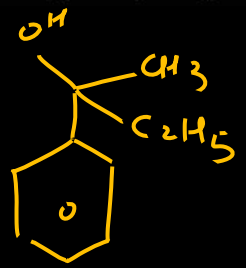
(C) $CH_3COC_6H_5$

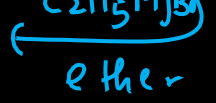
(D) $CH_3CH(OH)C_6H_5$











3

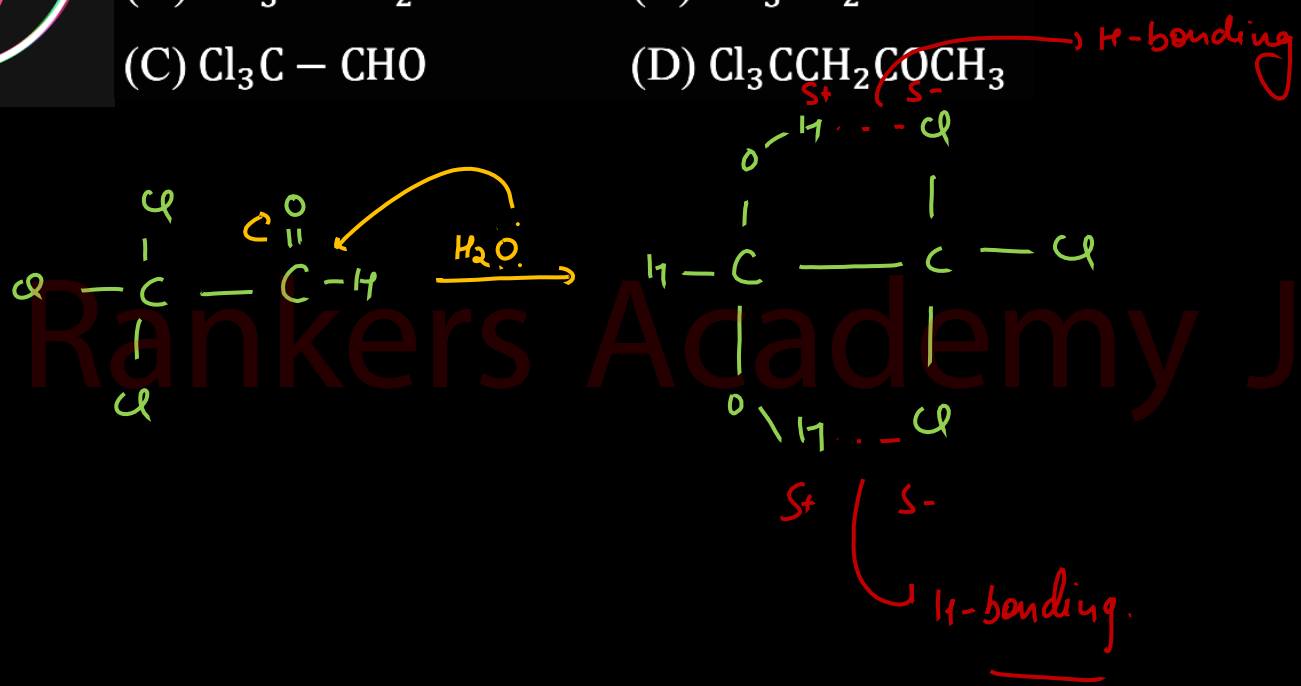
The compound that would undergo hydration very easily is :

(A) CH_3COCH_2Cl

(B) CH_3CH_2CHO

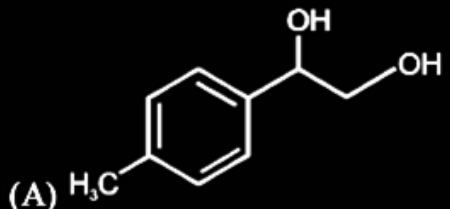
(C) Cl_3C-CHO

(D) $Cl_3CCH_2COCH_3$

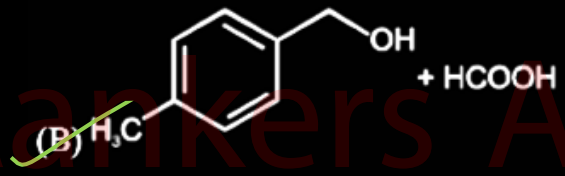


4

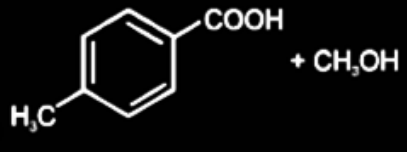
The reaction of 50% aq KOH on an equimolar mixture of 4-methylbenzaldehyde and formaldehyde followed by acidification gives –



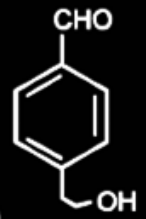
(A)



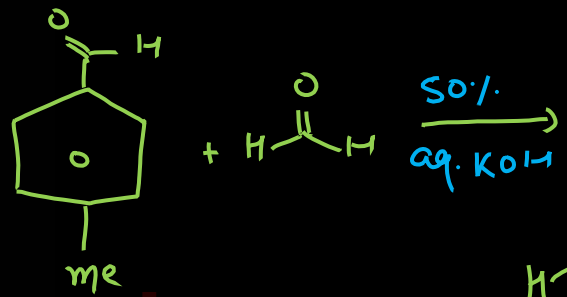
(B)

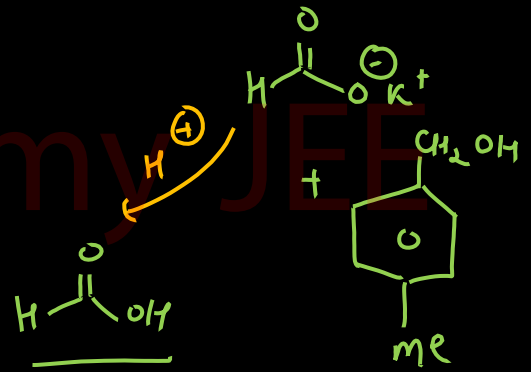


(C)



(D)

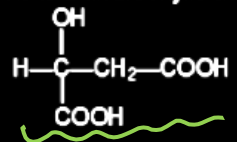




5

JEE 1

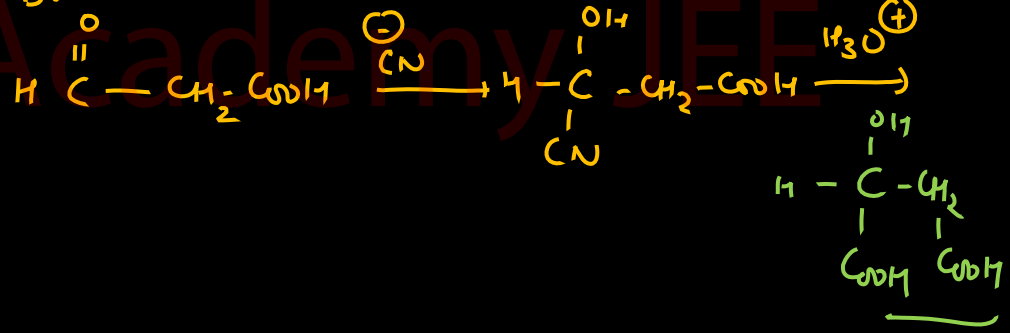
Malic acid is a dicarboxylic acid present in apples and it has the following structure



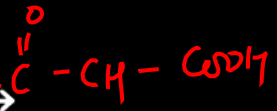
Which of the following synthetic routes will give (±) malic acid?

(i) $H_2C{=}CH{-}COOH \xrightarrow[(iii)\ HCN/H_3O^+]{(i)\ HBr;\ (ii)\ KOH\ (aq.)}$

(ii) $H_2C(Br){-}CH_2{-}COOH \xrightarrow[(iii)\ HCN/H_3O^+]{(i)\ KOH\ (aq.);\ (ii)\ PCC}$



(iii) $H_3C{-}CH_2{-}COOH \xrightarrow[(iv)\ HCN/H_3O^+]{(i)\ Red\ P/I_2;\ (ii)\ NaOH(aq.);\ (iii)\ [O]}$

(iv) $H_2C{=}CH{-}COOH \xrightarrow[(iii)\ HCN/H_3O^+]{(i)\ B_2H_6/H_2O_2;\ (ii)\ [O]}$ 

(A) i and ii

(B) ii and iv

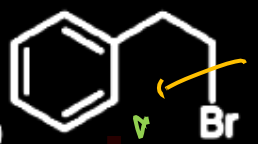
(C) ii and iii

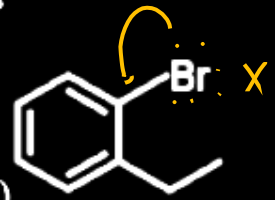
(D) i and iii

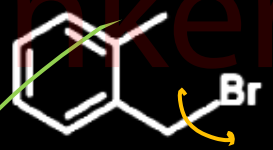
6

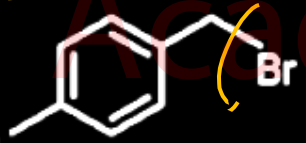
JEE 1

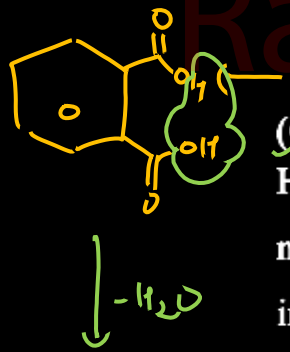
Compound 'Y' with molecular formula C_8H_9Br gives a precipitate on heating with alcoholic $AgNO_3$. Oxidation of 'Y' gives products 'Z' ($C_8H_6O_4$) which gives an anhydride upon heating. Compound 'Y' is

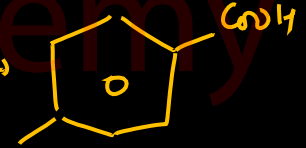
(A) 

(B) 

(C) 

(D) 





How many ml of 1 N HCl are required for X milimoles of Na_2CO_3 with methyl orange indicator

(A) X ml

(B) 2 X ml

(C) 3 X ml

(D) 4 X ml

7

How many ml of 1 N HCl are required for X milimoles of Na_2CO_3 with methyl orange indicator

(A) X ml

(B) 2 X ml

(C) 3 X ml

(D) 4 X ml

Me-OH

$Na_2Co_3 + HCl \longrightarrow H_2Co_3 + NaCl$

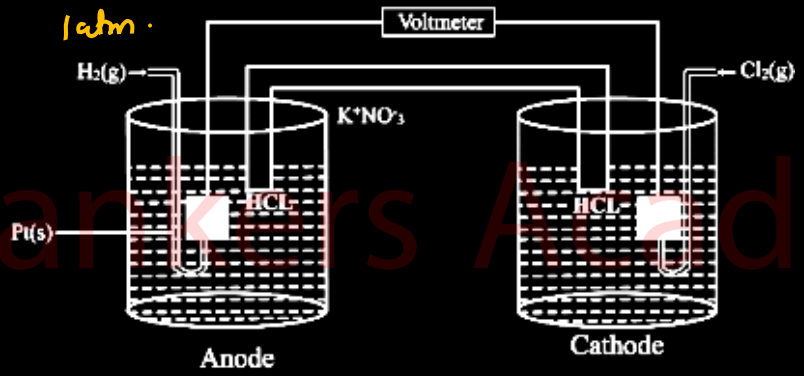
$n_f = 2 \quad n_f = 1$

$1 \times V = x \times 2$

$V = 2x$

8

Consider the following Galvanic cell as shown in figure. By what will value the cell voltage change when concentration of ions in anodic and cathodic compartments are both increased by factor of 10 at 298 K



(A) +0.591 V
(B) −0.0591 V
(C) −0.1182 V
(D) 0 V

anode

$H_2(g) \longrightarrow 2H^+ + 2e^-$

Cathode $Cl_2(g) + 2e^- \longrightarrow 2Cl^-$

$H_2(g) + Cl_2(g) \longrightarrow 2H^+ + 2Cl^-$

$E_1 = E^\circ - \frac{0.059}{2} \log [H^+]^2 [Cl^-]^2$ — ①

$E_2 = E^\circ - \frac{0.059}{2} \log \{10[H^+]\}^2 \{10[Cl^-]\}^2$

$E_2 = E^\circ - \frac{0.059}{2} \log [H^+]^2 [Cl^-]^2 - 4 \times \frac{0.059}{2}$ — ②

$E_2 - E_1 = -0.059 \times 2$

$= -0.118$

9

Consider the following reactions at 300 K.

A → B (uncatalysed reaction)

$A \xrightarrow{\text{catalyst}} B$ (catalyst reaction)

The activation energy is lowered by $8.314 KJmol^{-1}$ for the catalysed reaction. How many times the rate of this catalysed reaction greater than that of uncatalysed reaction? (Given $e^{3.33} = 28$)

(A) 15 times

(B) 38 times

(C) 22 times

(D) 28 times

$E_a' = E_a - 8.314\ KJ\ mol^{-1}$

$K = A e^{-\frac{E_a}{RT}}$ — (1)

$K' = A e^{-\frac{E_a'}{RT}}$ — (2)

$\frac{K'}{K} = e^{\frac{E_a - E_a'}{RT}}$

$= e^{\frac{8.314 \times 10^3}{8.314 \times 300}}$

$\boxed{\frac{K'}{K} = 28}$

10

The correct order of acidity of the following compounds is :

(I) CH_3COOH

(II) $ClCH_2COOH$

(III) O_2NCH_2COOH

(IV) $HOCH_2COOH$

(A) IV > II > III > I

(B) I > IV > II > III

(C) II > III > I > IV

(D) III > II > IV > I

JEE 1

11

JEE 1

A, B and C are three biomolecules. The results of the tests performed on them are given below:

	Molisch's Test	Barfoed Test	Biuret Test
A	Positive	Negative	Negative
B	Positive	Positive	Negative
C	Negative	Negative	Positive

Alanine = amino acid

Albumin = Protein

A, B and C are respectively :

(A) A = Glucose, B = Fructose, C = Albumin

(B) A = Lactose, B = Fructose, C = Alanine

(C) A = Lactose, B = Glucose, C = Alanine

(D) A = Lactose, B = Glucose, C = Albumin

12

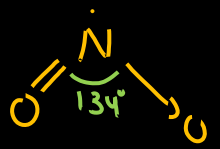
Which of the following is NOT correct for NO_2, NO_2^+ and NO_2^-

(A) NO_2 is paramagnetic in nature. ✓

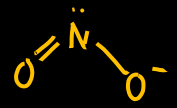
(B) NO_2^+ is linear, NO_2 is bent and bond angle is less than 120°. ✗

(C) NO_2^+ has the strongest bonds among the given species. ✓

(D) NO_2^- has the bond order of 1.5 ✓

NO_2 = 

NO_2^+ = $O=\overset{+}{N}=O$ B.O = 2

NO_2^- = 

B.O = 1.5

13

Assign the hybridization, shape and magnetic moment of $K_3[Cu(CN)_4]$:

(A) sp^3, tetrahedral, 1.73 B.M.

(B) dsp^2, square planar, 1.73 B.M.

(C) sp^3, tetrahedral, diamagnetic

(D) dsp^2, square planar, 2.44 B.M.

$K_3[Cu(CN)_4]$

$Cu^{+1} = 3d^{10}\ 4s^0$

SP^3, Di, Zoo, Td

14

Which one of the following statement is not correct?

(A) $La(OH)_3$ is less basic than $Lu(OH)_3$

(B) In lanthanide series, ionic radius of Ln^{3+} ions decreases

(C) La is actually an element of transition series rather than lanthanide series

(D) Atomic radii of Zr and Hf are same because of lanthanide contraction

15

Match the electronic configuration with their respective ΔH_{cg} (kJ/mole)/ value.

Column-I	Column-II
(P) $1s^2, 2s^2, 2p^6$	(1) -328
(Q) $1s^2, 2s^2, 2p^4$	(2) -122
(R) $1s^2, 2s^2, 2p^2$	(3) -141
(S) $1s^2, 2s^2, 2p^5$	(4) +116

Code:

	P	Q	R	S
(A)	3	1	2	4
(B)	2	4	1	2
(C)	2	4	3	1
(D)	4	3	2	1

16

Match the following:

JEE 1

List - I [Reaction]		List - II [Catalyst]	
(A)	$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$	(I)	Ni
(B)	$2H_2(g) + O_2(g) \longrightarrow 2H_2O(l)$	(II)	Pt
(C)	$CO(g) + 3H_2(g) \longrightarrow CH_4(g) + H_2O(g)$	(III)	$ZnO - Cr_2O_3$
(D)	$CO(g) + 2H_2(g) \longrightarrow CH_3OH(g)$	(IV)	Fe

(A) A-III, B-II, C-I, D-IV

(B) A-IV, B-III, C-II, D-I

(C) A-IV, B-II, C-I, D-III

(D) A-IV, B-I, C-III, D-II

17

JEE 1

In 200 g of water, 0.01 mole of NaCl and 0.02 mole of sucrose, are dissolved. Assuming solution to be ideal, the depression in freezing point of water (in °C) will be

(K_f of water =1.86°C/m)

(A) 0.37 (B) 0.67

(C) 0.186 (D) 0

$NaCl \rightarrow Na^+ + Cl^-$

1 1 1

0.01 0.01 0.01

0.02 m

$m = \frac{\text{Total moles}}{\text{mass of solvent in kg}}$

$\Delta T_f = i\, m\, k_f$

$\Delta T_f = \frac{(0.02 + 0.02) \times 1000}{200} \times 1.86$

$= 0.37\ ^\circ C$

18

The time required for an electron to make one complete revolution around nucleus of H-atom in a higher orbit n_2 is 8 times to that of a lower orbit n_1. Values of n_1 and n_2 cannot be

(A) 1 and 2 (B) 2 and 3 ✗

(C) 2 and 4 (D) 3 and 6

$T = \frac{2\pi r}{v} \propto \frac{n^2}{z/n}$

$T \propto \frac{n^3}{Z^2}$

$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 = \frac{8}{1}$

(A) $\frac{(2)^3}{(1)^3} = \frac{8}{1}$ ✓

(B) $\frac{(3)^3}{(2)^3} = \frac{27}{8}$ ✗

(C) $\frac{4^3}{2^3} = \frac{64}{8} = \frac{8}{1}$ ✓

(D) $\frac{6^3}{3^3} = \frac{216}{27} = \frac{8}{1}$ ✓

19

JEE 1

Given,

(i) $S + O_2 \rightarrow SO_2, \Delta H = -298.2$ kJ

(ii) $SO_2 + \frac{1}{2}O_2 \rightarrow SO_3, \Delta H = -98.7$ kJ

(iii) $SO_3 + H_2O \rightarrow H_2SO_4, \Delta H = -130.2$ kJ

(iv) $H_2 + \frac{1}{2}O_2 \rightarrow H_2O, \Delta H = -287.3$ kJ

$H_2 + S + 2O_2 \longrightarrow H_2SO_4$

Then the enthalpy of formation of H_2SO_4 at 298 K will be

(A) −814.4 kJ (B) +320.5 kJ

(C) −650.3 kJ (D) −933.7 kJ

$\Delta H_f(H_2SO_4) = -298 + (-98.7) + (-130.2) + (-287.3)$

$= -814.4$ kJ

20

JEE 1

Consider the equilibrium, $A(g) \rightleftharpoons B(g) + C(g)$

At a constant pressure of 1 atm, A dissociated to the extent of 50% at 500 K. Calculate the $\Delta G^\circ (kJmol^{-1})$ for the reaction

(A) 4.61 (B) 4608

(C) 6540 (D) 6.54

A	⇌	B	+	C
1		0		0
$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$

$$K_P = \frac{(P_B)(P_C)}{(P_A)} = \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3}} = \frac{1}{3}$$

$$\Delta G^\circ = -RT \ln K_P$$

$$= -2.303 \times 8.314 \times 500 \log \frac{1}{3}$$

$$= 2.303 \times 8.314 \times 500 \times 0.47$$

$$= 4499\,J = 4.5\,kJ$$

21

200 g impure $CaCO_3$ on heating gives 5.6 litre CO_2 gas at STP. Find the percentage (nearest integer) of calcium in the lime stone sample. (Take: 1 mole gas volume at STP equal to 22.4 litre, $Ca = 40, C = 12, O = 16$)

$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

$\frac{1}{4}$ mole $\quad n = \frac{5.6}{22.4} = \frac{1}{4}$ mole

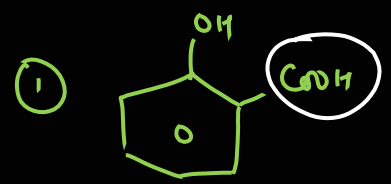
wt. of Ca in product $= \frac{1}{4} \times 40 = 10$ g

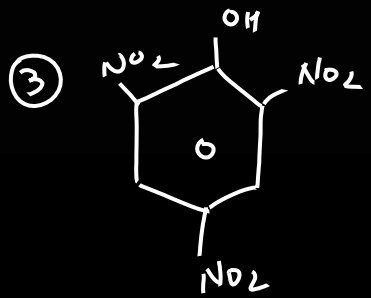
% wt of Ca in $CaCO_3 = \frac{10}{200} \times 100 = 5\%$

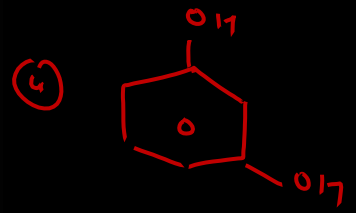
22

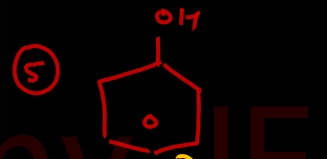
How many of the following compounds react with $NaHCO_3$ and liberate $CO_2(g)$

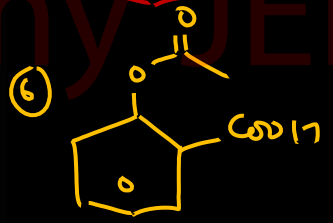
1. Salicylic acid
2. Pthalic acid
3. Picric acid
4. Resorcinol
5. Carbolic acid
6. Aspirin
7. Anisol
8. Tarteric acid

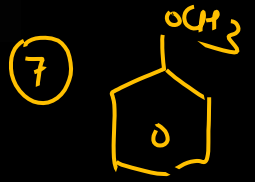










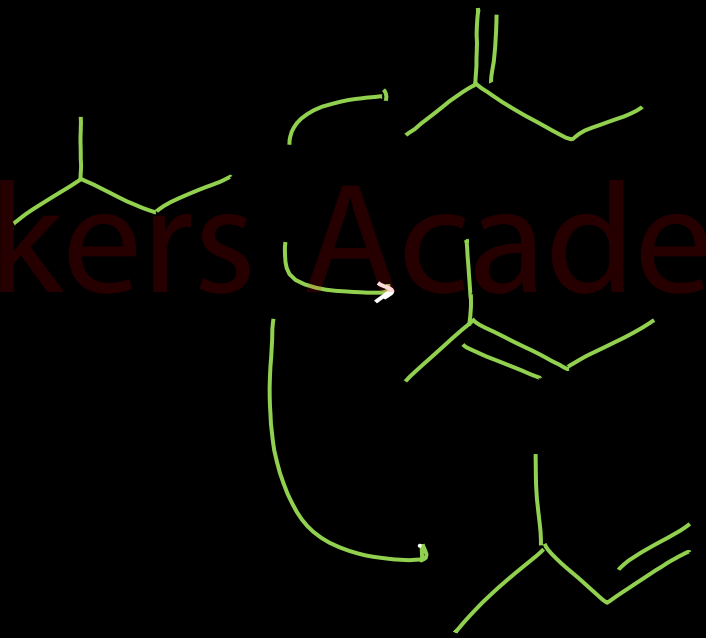


23

How many alkenes on catalytic hydrogenation give isopentane as a product (consider only structural isomers)?

JEE 1

3



24

JEE 1

Depression of freezing point of 0.01 molal aq. CH_3COOH solution is $0.02046°$. 1 molal urea solution freezes at $-1.86°C$. Assuming molality equal to molarity, pH of CH_3COOH solution is (nearest integer):

$\Delta T_f = 0.02046 = i K_f m$

$0 - (-1.86) = 1 \times K_f \times 1$

$K_f = 1.86$

$\therefore \frac{0.02046}{1.86 \times 0.01} = i$

$\frac{2.046}{1.86} = i$

$CH_3COOH \rightarrow CH_3COO^- + H^+$

Eqbm: $C - C\alpha$ $\quad C\alpha$ $\quad C\alpha$

$i = 1 + \alpha$

α ✓

$pH = -\log[C\alpha]$

$= -\log[10^{-3}]$

$= 3$

25

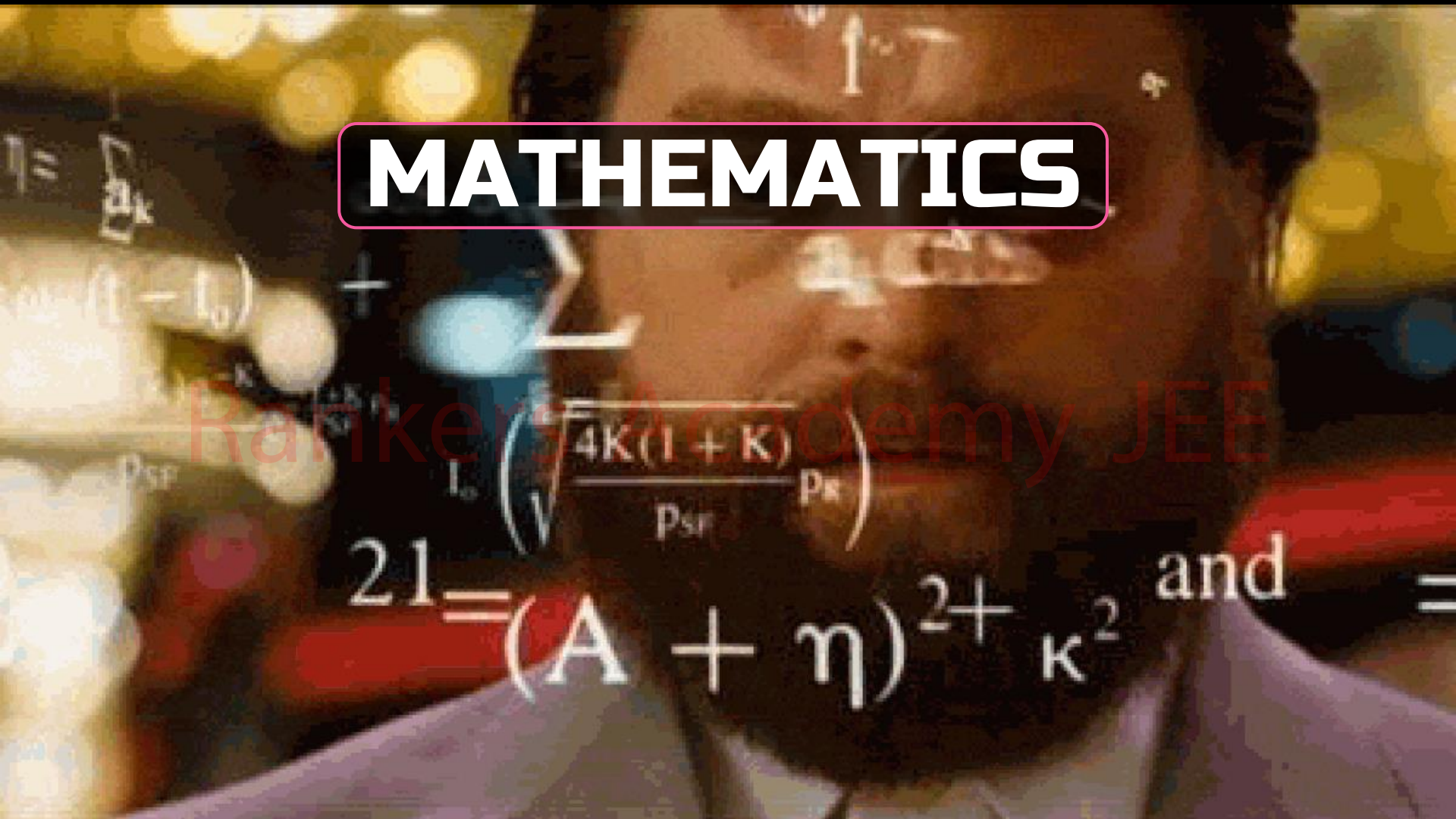
Molarity of H_2SO_4 is 18M . Its density is 1.8 g/cm^3, hence molality is (If your answer is ' x ' then, Report your answer x/500).

JEE 1

$$m = \frac{1000\,M}{1000d - MM'}$$

$x = 5\omega$

Ans. 1


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

The value of $\lim_{x\to\infty} x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right)\right]$ is

(A) 1
(B) -1
(C) $\frac{1}{2}$
(D) $-\frac{1}{2}$

JEE 1

$$= \lim_{x\to\infty} x \tan^{-1}\left(\frac{\frac{x+1}{x+2} - \frac{x}{x+2}}{1 + \frac{x+1}{x+2}\cdot\frac{x}{x+2}}\right)$$

$$= \lim_{x\to\infty} x \tan^{-1}\left(\frac{x+2}{2x^2+5x+4}\right)$$

$$= \lim_{x\to\infty} x\left(\frac{x+2}{2x^2+5x+4}\right)$$

$$= \lim_{x\to\infty} \frac{x^2+2x}{2x^2+5x+4} = \frac{1}{2}.$$

$\frac{\infty}{\infty} \to 0$

$\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

2

$f(x) = \int \left(2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}\right) dx$ then f is

(A) Increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$

(B) Increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$

(C) Increasing in $(-\infty, \infty)$

(D) Decreasing in $(-\infty, \infty)$

JEE 1

$f'(x) = 2 - \left(\frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}}\right) > 0$

$x^2+1 > 1$ $\qquad$ $\sqrt{x^2+1} > 1$

$\Rightarrow \frac{1}{x^2+1} < 1$ $\qquad$ $\frac{1}{\sqrt{x^2+1}} < 1$

$\Rightarrow \frac{1}{x^2+1} + \frac{1}{\sqrt{x^2+1}} < 2$

3

In a certain factory, machines A, B and C produce bolts. Of their production, machines A, B, and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts, machine B produces 25% and machine C produces 40%. A bolt is chosen at random from the factory's production and is found to be defective. The probability it was produced on machine C, is

(A) $\frac{6}{11}$ (B) $\frac{23}{45}$

(C) $\frac{24}{43}$ (D) $\frac{3}{11}$

$$P(C/def.) = \frac{\frac{40}{100}\cdot\frac{3}{100}}{\underbrace{\frac{35}{100}\cdot\frac{2}{100}}_{A} + \underbrace{\frac{25}{100}\cdot\frac{1}{100}}_{B} + \underbrace{\frac{40}{100}\cdot\frac{3}{100}}_{C}}$$

JEE 1

4

Let R be the relation on the set of all real numbers defined by aRb iff $|a - b| \leq \frac{1}{2}$. Then R is

(A) Reflexive and symmetric but not transitive

(B) Symmetric and transitive but not reflexive

(C) Transitive but neither reflexive nor symmetric

(D) Symmetric but neither reflexive nor transitive

JEE 1

Ref → $(a,a) \in R$ check

$|a-a| = 0 \leq \frac{1}{2}$ ✓

Sym → $(a,b) \in R$ ✓ $|a-b| \leq 1/2$ ✓

Check (b,a) $|b-a| \leq 1/2$ ✓

Trans: $a = 0$ $b = 1/2$

$|a-b| \leq 1/2$ ✓

$b = 1/2$ $c = 1$

$|b-c| \leq 1/2$ ✓

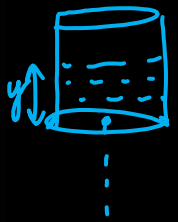
$|a-c| = 1 \not\leq 1/2$

5

Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = 1/15$ then the time (in minutes) to drain the tank if the water is 4 meter deep to start with is

(A) 20 (B) 30

(C) 60 (D) 120



$$\frac{dy}{dt} \propto -\sqrt{y}$$

$$\frac{dy}{dt} = -k\sqrt{y}$$

$$\boxed{\frac{dy}{dt} = -\frac{1}{15}\sqrt{y}}$$

$t = 0$; $y = 4$

$t = ?$; $y = 0$

5

JEE 1

$$\int_4^0 \frac{dy}{\sqrt{y}} = \frac{-1}{15}\int_0^t dt$$

$$2\left[\sqrt{y}\right]_4^0 = -\frac{t}{15}$$

$$2(0-2) = -\frac{t}{15}$$

$$t = 60$$

6

Let $f(x) = 1 + 4x - x^2, \forall x \in R$,

$$g(x) = \begin{cases} \max\{f(t): x \le t \le (x+1): 0 \le x < 3\} \\ \min\{(x+3): 3 \le x \le 5\} \end{cases}$$

then $g(x)$ is

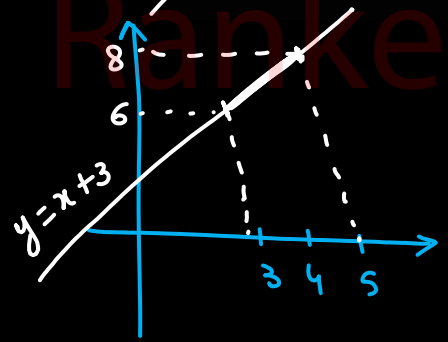
(A) Continuous $\forall x \in [0,5]$

(B) Continuous $\forall x \in [0,5] - \{3\}$

(C) Continuous $\forall x \in [0,4]$

(D) Discontinuous at all integral values of x

One of the toughies

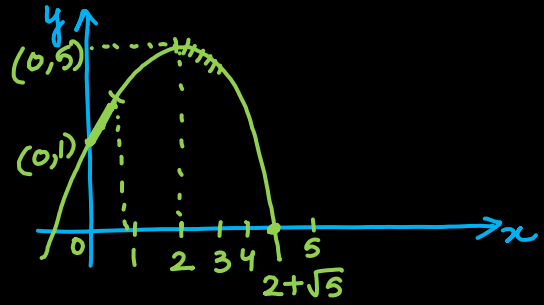


$$g(x) = \begin{cases} f(x+1) \\ = 1 + 4(x+1) - (x+1)^2 \\ = -x^2 + 2x + 4 \end{cases} ; 0 < x \le 1$$

$$5 \quad ; 1 < x \le 2$$

$$f(x) = 1 + 4x - x^2 \quad ; 2 < x < 3$$

$$6 \quad ; 3 < x < 5$$



$$1 + 4x - x^2 = 0$$

$$x^2 - 4x - 1 = 0$$

$$x = \frac{4 \pm \sqrt{20}}{2}$$

$$= 2 \pm \sqrt{5}$$

6

JEE 1

$$g(x) = \begin{cases} -x^2 + 2x + 4 & ; \ 0 < x < 1 \\ 5 & ; \ 1 < x < 2 \\ 1 + 4x - x^2 & ; \ 2 < x < 3 \\ 6 & ; \ 3 < x < 5 \end{cases}$$

At $x = 3$

$LHL = 1 + 4(3) - 3^2 = 4$

$RHL = 6$

$\therefore$ discont at $x = 3$

7

The value of $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is equal to

(A) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ ✓

(B) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + c$

(C) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$

(D) $\frac{1}{3}\left(\frac{x+1}{x-1}\right)^{1/4} + c$

JEE 1

$$= \int \frac{dx}{\left(\frac{(x-1)^3 (x+2)^8}{(x+2)^3}\right)^{1/4}}$$

$$= \int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4} \cdot (x+2)^2}$$

Let $\frac{x-1}{x+2} = t \Rightarrow \frac{(x+2)-(x-1)}{(x+2)^2} dx = dt$

$$= \int t^{-3/4} \frac{dt}{3}$$

$$= \frac{t^{1/4}}{1/4} \cdot \frac{1}{3} + c$$

$$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$$

8

Given α and β are the roots of the quadratic equation $x^2 - 4x + k = 0 (k \neq 0)$. If $\alpha\beta, \alpha\beta^2 + \alpha^2\beta, \alpha^3 + \beta^3$ are in geometric progression, then the value of $7k/2$ equals

(A) 8 (B) 7

(C) 16 (D) 16/7

$x^2 - 4x + k = 0 < \alpha, \beta$

$\alpha + \beta = 4$

$\alpha\beta = k$

$\Rightarrow (\alpha\beta^2 + \alpha^2\beta)^2 = \alpha\beta(\alpha^3 + \beta^3)$

$\Rightarrow (\alpha\beta(\alpha+\beta))^2 = \alpha\beta((\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta))$

$\Rightarrow (\alpha\beta)(\alpha+\beta) = (\alpha+\beta)^2 - 3\alpha\beta$

$\Rightarrow 4k = 16 - 3k$

$\Rightarrow k = \frac{16}{7}$

$\Rightarrow \frac{7k}{2} = 8$

9

If z is a variable point such that the $\arg\left(\frac{z-(1+3i)}{z-(3+i)}\right) = \pm\frac{\pi}{2}$, then centre of the curve represented by z is

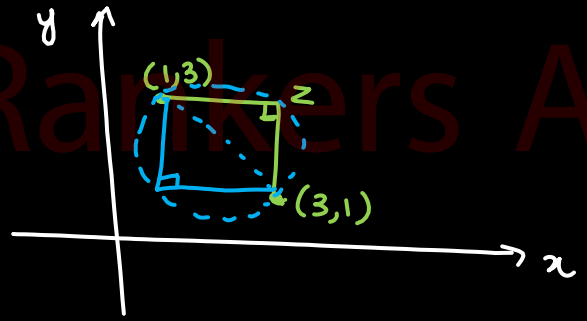
circle

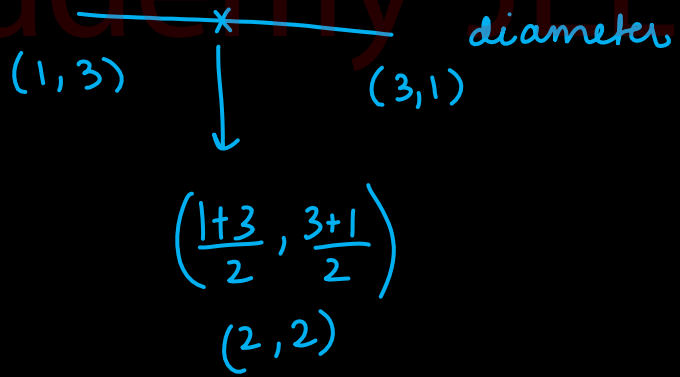
(A) $1 + i$

(B) $2 - 2i$

(C) $2 + 2i$

(D) $-2 + 2i$





9

JEE 1

M2 $\quad z = x + iy$

$$\arg\left(\frac{(x-1)+i(y-3)}{(x-3)+i(y-1)}\right)$$

$$\arg\left(\frac{((x-1)+i(y-3))\cdot((x-3)-i(y-1))}{(x-3)^2-(y-1)^2}\right)$$

$$\arg\left(\frac{(x-1)(x-3)-i(x-1)(y-1)+i(y-3)(x-3)+(y-1)(y-3)}{(x-3)^2-(y-1)^2}\right) = \pm\frac{\pi}{2}$$

9

JEE 1

$$\arg\left(\frac{\overbrace{(x-1)(x-3)+(y-1)(y-3)}^{=0}+i\left[(y-3)(x-3)-(x-1)(y-1)\right]}{(\quad)}\right)=\pm\frac{\pi}{2}$$

$(x-1)(x-3)+(y-1)(y-3)=0$

dia form of ⊙ ✓

10

If $a > b > c$ and the system of equations $ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$ has a non-trivial solution, then the quadratic equation $ax^2 + bx + c = 0$ has

(A) Atleast one positive root

(B) Roots of opposite signs

(C) Positive roots

(D) Imaginary roots

$$\Delta = 0 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\Rightarrow a(bc - a^2) - b(b^2 - ac) + c(ab - c^2) = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow a + b + c = 0$$

$$f(x) = ax^2 + bx + c$$

$$f(1) = a + b + c = 0$$

$\therefore x = 1$ is a root

11

Sum to n terms of the series

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\cdots$ is

(A) $\frac{n^2}{n^2+n+1}$

(B) $\frac{n(n+1)}{2(n^2+n+1)}$ ✓

(C) $\frac{n(n+1)}{(n^2+n+1)}$

(D) $\frac{n}{3}$

$$= \sum \frac{n}{1+n^2+n^4}$$

$$= \frac{1}{2}\sum \frac{2n}{(n^2+1+n)(n^2+1-n)}$$

$$= \frac{1}{2}\sum \frac{(n^2+1+n)-(n^2+1-n)}{(n^2+1+n)(n^2+1-n)}$$

$$= \frac{1}{2}\sum_{n=1}^{n} \frac{1}{n^2+1-n} - \frac{1}{n^2+1+n}$$

$$= \frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{7}+\cdots-\frac{1}{n^2+1+n}\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{n^2+1+n}\right]$$

$$= \frac{1}{2}\frac{n^2+n}{(n^2+1+n)}$$

12

JEE 1

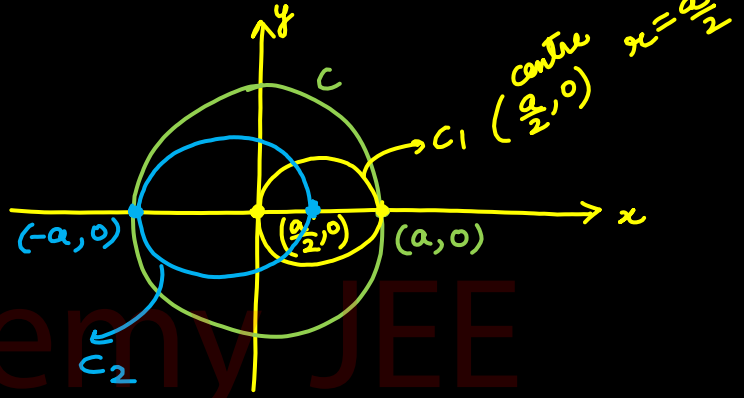
Let C be the circle $x^2 + y^2 = a^2$, and C_1 be the circle, with diameter along x-axis, which passes through the centre of C and the point where C cuts the positive x-axis. Further, let C_2 be the circle, with diameter along x-axis, which passes through the centre of C_1 and the point where C cuts the negative x-axis. The equation of C_2 is

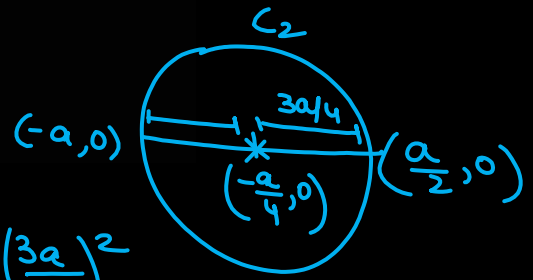
(A) $x^2 + y^2 + \frac{ax}{2} = \frac{a^2}{4}$

(B) $x^2 + y^2 + \frac{ax}{4} = \frac{a^2}{2}$

(C) $4(x^2 + y^2) + ax = a^2$

(D) $2(x^2 + y^2) + ax = a^2$





$\Rightarrow \left(x-\left(\frac{-a}{4}\right)\right)^2 + (y-0)^2 = \left(\frac{3a}{4}\right)^2$

$\Rightarrow x^2 + \frac{a^2}{16} + \frac{ax}{2} + y^2 = \frac{9a^2}{16}$

13

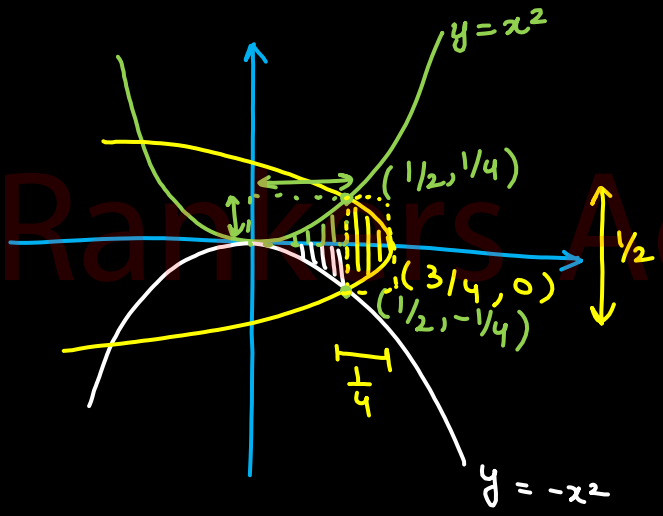
Area bounded between the curves $y = x^2, y = -x^2$ & $16y^2 + 4x - 3 = 0$ is

(A) $\frac{1}{6}$ ✓ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

$16y^2 = -4x + 3$

$= -4(x - \frac{3}{4})$



 $\rightarrow \frac{1}{3}(\frac{1}{2})(\frac{1}{4}) = \frac{1}{24}$

 $\rightarrow \frac{1}{24}$.

 $\rightarrow \frac{2}{3}(\frac{1}{4})(\frac{1}{2}) = \frac{2}{24}$.

Ans: $\frac{1}{24} + \frac{1}{24} + \frac{2}{24} = \frac{1}{6}$.

JEE 1

13

JEE 1

$16y^2 + 4x - 3 = 0$

$y = x^2$

$\rightarrow 16x^4 + 4x - 3 = 0$

$\rightarrow x = \frac{1}{2}$ ✓

14

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angles, then the value of k will be

(A) $-\frac{10}{7}$

(B) $-\frac{7}{10}$

(C) -10

(D) -7

$\cos\theta = 0 = a_1a_2 + b_1b_2 + c_1c_2$

$(-3)(3k) + (2k)(1) + (2)(-5) = 0$

$-7k = 10$

15

If $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + [\log(3-x)]^{-1}$, then its domain is

(A) $[-2,6]$

(B) $[-6,2) \cup (2,3)$

(C) $[-6,2]$

(D) $[-2,2) \cup (2,3]$

JEE 1

$\downarrow$

$\frac{1}{\log(3-x)}$

$3-x > 0 \Rightarrow x < 3$ — (1)

$\log(3-x) \neq 0$

$3-x \neq 1$

$x \neq 2$ — (2)

$-1 \leq \frac{2-|x|}{4} \leq 1$

$-4 \leq 2-|x| \leq 4$

$|x| \leq 6$ $\quad -2 \leq |x|$ ✓

$-6 \leq x \leq 6$ — (3)

16

Consider the differential equation $e^x(ydx - dy) = e^{-x}(ydx + dy)$. Let $y = f(x)$ be the particular solution of the differential equation which passes through $(0,2)$. Then range of $g(x) = \log_2 (f(x))$, is

(A) $(0,1)$ (B) $[1,\infty)$

(C) $(-1,0)$ (D) $(-\infty,-1)$

$$e^x y\,dx - e^x dy = e^{-x} y\,dx + e^{-x} dy$$

$$y(e^x y - e^{-x} y)\,dx = (e^{-x} + e^x)\,dy$$

$$\int \frac{(e^x - e^{-x})}{(e^x + e^{-x})}\,dx = \int \frac{dy}{y}$$

$$\ln(e^x + e^{-x}) = \ln(cy)$$

$$e^x + e^{-x} = cy$$

$c = 1$

$$y = f(x) = (e^x + e^{-x})$$

16

JEE 1

$g(x) = \log_2\left(e^x + \frac{1}{e^x}\right)$

≥ 2

Range $\in [1, \infty)$

17

JEE 1

Rectangular arrangements of natural numbers follows a pattern, such as

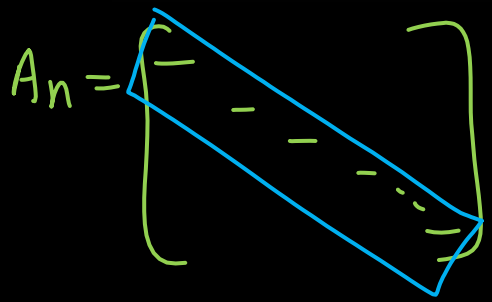
$A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ ..... and so on if $\text{tr}(A_n) = f(n)$, then $\sum_{n=1}^{6} \frac{6f(n)}{n}$ is

(A) 918 (B) 921

(C) 924 (D) 927

$1, 4, 9, \cdots, (n-1)^2$

$= \frac{(n-1)(n)(2n-1)}{6}$



$f(n) = \frac{n}{2}\left[2\left(\frac{(n-1)(n)(2n-1)}{6} + 1\right) + (n-1)(n+1)\right]$

$f(n) = \frac{n}{6}(2n^3 + n + 3)$

17

JEE 1

$$\sum_{n=1}^{6} (2n^3 + n + 3)$$

$$2\sum_{n=1}^{6} n^3 + \sum_{n=1}^{6} n + 3\sum_{n=1}^{6} 1 = 921$$

18

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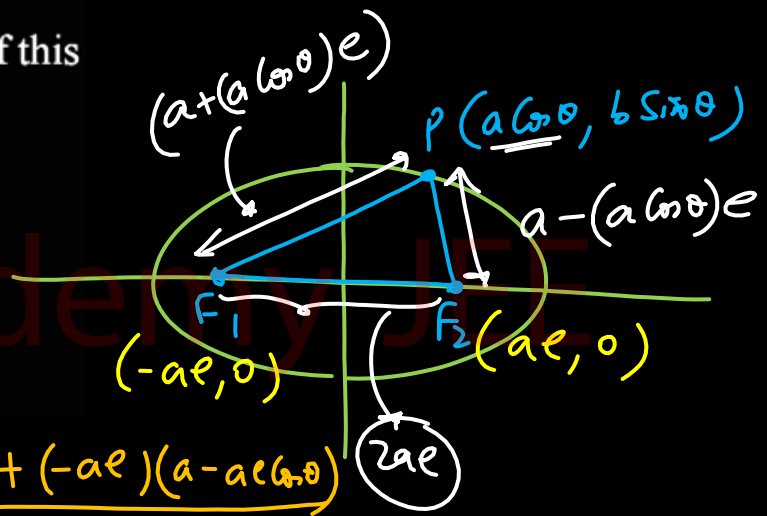
The equation of an ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$, a variable point P moves on the boundary of this ellipse. The eccentricity of the locus of the incentre of $\triangle PF_1F_2$ if F_1, F_2 are two foci of this ellipse is

(A) $\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{1}{3}}$

(C) $\frac{1}{2}$ (D) $\sqrt{\frac{3}{4}}$



$$h=\frac{(2ae)(a\cos\theta)+ae(a+ae\cos\theta)+(-ae)(a-ae\cos\theta)}{(2a+2ae)}$$

$$h=\frac{(2ae)(a\cos\theta)+ae(\cancel{a}+ae\cos\theta-\cancel{a}+ae\cos\theta)}{2a(1+e)}$$

18

JEE 1

$$h = \frac{(2ae)(a\cos\theta) + ae(2ae\cos\theta)}{2a(1+e)}$$

$$h = \frac{2a^2e\cos\theta(1+e)}{2a(1+e)}$$

$$\boxed{h = (a\cos\theta)e} \quad -(1)$$

$$K = \frac{(2ae)(b\sin\theta) + 0 + 0}{2a(1+e)}$$

$$\boxed{K = \frac{e(b\sin\theta)}{1+e}} \quad -(2)$$

$$\frac{h^2}{a^2e^2} + \frac{K^2}{\left(\frac{b^2e^2}{(1+e)^2}\right)} = 1$$

$$e' = \sqrt{1 - \frac{b^2}{(1+e)^2a^2}}$$

18

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$$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

$$e' = \sqrt{1 - \frac{b^2}{(1+e)^2 a^2}}$$

$$= \sqrt{1 - \frac{3}{\left(1+\frac{1}{2}\right)^2 (4)}}$$

$$= \sqrt{1 - \frac{3 \times 4}{9 \times 4}} = \sqrt{\frac{2}{3}}$$

19

If the sum of all the elements of the set $\{\alpha: \alpha \in \{1,2,3, \ldots \ldots 100\}$ and HCF $(\alpha, 24) = 1\}$ is N, then the units digit of N is

$2^3 \cdot 3^1$

(A) 6 (B) 9

(C) 3 (D) 4

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Total : $\frac{100 \times 101}{2} = 5050$

Unwanted

$n_2 = \left[\frac{100}{2}\right] = 50$; $S_2 = \frac{50}{2}(2+100)$

$n_3 = \left[\frac{100}{3}\right] = 33$; $S_3 = \frac{33}{2}(3+99)$

$n_6 = \left[\frac{100}{6}\right] = 16$; $S_6 = \frac{16}{2}(6+96)$

$\rightarrow S_2 + S_3 - S_6$

$= 51(50+33-16)$

$= 51(67)$

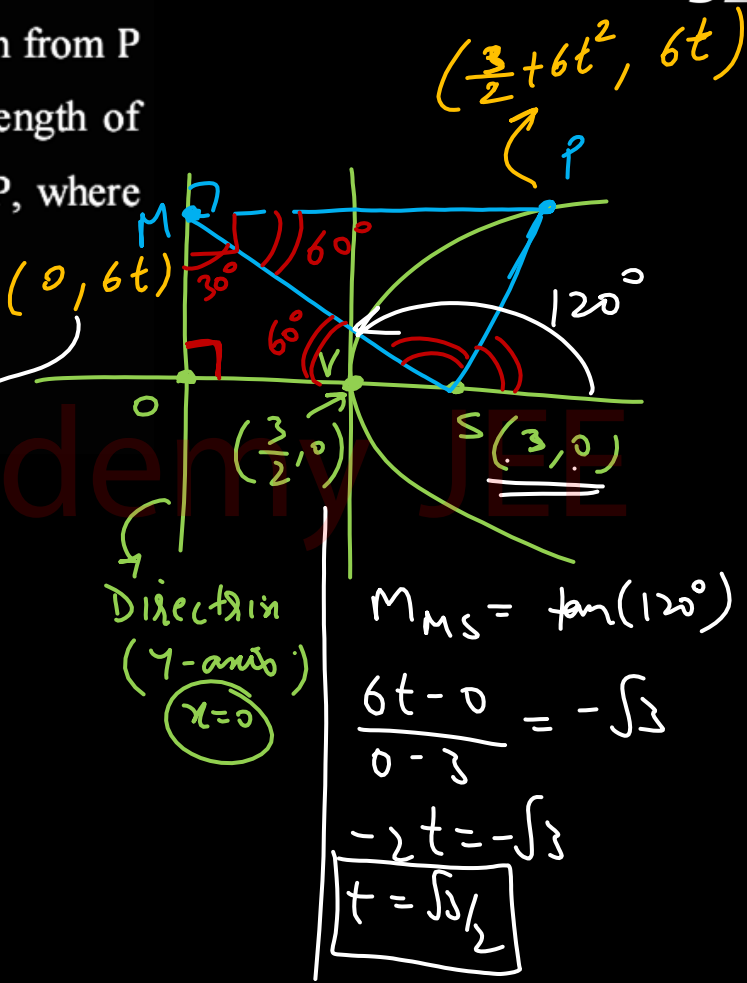
$= \boxed{\quad 7}$

20

If P be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from P on the directrix of the parabola, then length of each side of an equilateral triangle SMP, where S is focus of the parabola, is

(A) 2 (B) 4

(C) 6 (D) 8



$y^2 = 6\left(x - \frac{3}{2}\right)$

$y^2 = 4\left(\frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

$X = -A$; $x - \frac{3}{2} = -\frac{3}{2} \Rightarrow x = 0$

$M_{MS} = \tan(120°)$

$\frac{6t - 0}{0 - 3} = -\sqrt{3}$

$-2t = -\sqrt{3}$

$t = \sqrt{3}/2$

20

$$MS = \ell = \sqrt{27 + 9}$$

$$= 6$$

21

JEE 1

Let $T = \int_0^{\ln 2} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^{x}+1}\,dx$, then $4e^T$ equals

$$T = \int \frac{(3e^{3x}+2e^{2x}-e^{x}) - e^{3x} - e^{2x} + e^{x} - 1}{(e^{3x}+e^{2x}-e^{x}+1)}\,dx$$

$$T = \int \frac{(3e^{3x}+2e^{2x}-e^{x})}{(e^{3x}+e^{2x}-e^{x}+1)}\,dx - \int \frac{(e^{3x}+e^{2x}-e^{x}+1)}{(e^{3x}+e^{2x}-e^{x}+1)}\,dx$$

$$T = \left[\ln(e^{3x}+e^{2x}-e^{x}+1) - x\right]_0^{\ln 2}$$

21

JEE 1

$$T = \left(\ln(8+4-2+1) - \ln 2\right) - \left(\ln 2\right)$$

$$T = \ln\left(\frac{11}{4}\right)$$

$$4e^{T} = 4e^{\ln\left(\frac{11}{4}\right)}$$

$$= 4 \times \frac{11}{4} = \boxed{11}$$

22

JEE 1

If $\frac{T_2}{T_3}$ in the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then n is equal to

$$\frac{{}^nC_1\, a^{n-1} b^1}{{}^nC_2\, a^{n-2} b^2} = \frac{{}^{n+3}C_2\, a^{n+1} b^2}{{}^{n+3}C_3\, a^{n} b^3}$$

$$\frac{2}{n-1} = \frac{2+1}{(n+3)-2}$$

$$\frac{2}{n-1} = \frac{3}{n+1}$$

$$2n+2 = 3n-3$$

$$n = 5$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{n!}{(n-r)!\,r!} \times \frac{(n-r-1)!\,(r+1)!}{n!} = \frac{r+1}{n-r}$$

23

If $\sum_{r=0}^{\infty} \tan^{-1}\left(\frac{2.3^r}{1+3^{2r+1}}\right)$ equals $\frac{\pi}{K}$, the value of K is

$$\tan^{-1}\left(\frac{3^{n+1}-3^{n}}{1+3^{n}\cdot 3^{n+1}}\right)$$

$$\tan^{-1}\left(3^{n+1}\right)-\tan^{-1}\left(3^{n}\right)$$

$\tan^{-1}3-\tan^{-1}1$

$\tan^{-1}3^{2}-\tan^{-1}(3)$

$\tan^{-1}(3^{3})-\tan^{-1}(3^{2})$

$\tan^{-1}\left(3^{n+1}\right)-\tan^{-1}\left(3^{n}\right)$

$\frac{\pi}{2}-\frac{\pi}{4}\rightarrow\left(\frac{\pi}{4}\right)$

If A is a square matrix of order 3 such that $|A| = 2$, then $|(\text{adj } A^{-1})^{-1}|$ is

$$\frac{1}{|\text{adj } A^{-1}|} \quad | \quad |A|^2$$

$$\frac{1}{|A^{-1}|^2} \quad | \quad (2)^2$$

$$= 4$$

25

The number of symmetric matrices of order 3×3 can be made using five 1 's and four zeroes is

JEE 1