Test Pattern

JEE (Main)
AIOT
12-01-2025

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE(Main+Advanced) : ENTHUSIAST & LEADER COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	A	A	B	B	B	B	A	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	D	A	A	B	C	D	D	B	A
SECTION-II	Q.	1	2	3	4	5					
	A.	25	3	5	125	4					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	C	C	C	B	C	C	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	D	B	C	B	B	D	D	C
SECTION-II	Q.	1	2	3	4	5					
	A.	2	2	6	264	6					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	D	A	B	A	A	B	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	A	A	C	C	A	A	A	C
SECTION-II	Q.	1	2	3	4	5					
	A.	3	8	1	101	16					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (B)**

$P = VI$

$10 = 5\,I_1 \Rightarrow I_1 = 2\text{ mA}$

Across load resistance

$5 = I_2 1 \Rightarrow I_2 = 5\text{ mA}$

$I_s = 7\text{ mA}$

$Vs = 3V \Rightarrow R_S = \frac{3}{7}k\Omega$

2. **Ans (B)**

$I = 4t = 0 \le t \le 1$

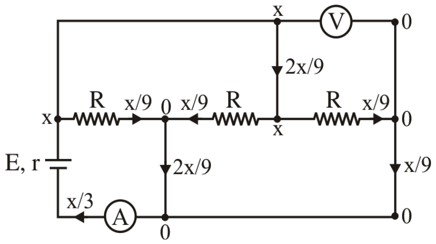
$$I_{rms}^2 = \frac{\int_0^1 (4t)^2 dt}{\int_0^1 dt} = \frac{16}{3}$$

$$I_{rms} = \frac{4}{\sqrt{3}}$$

3. **Ans (A)**

Fact

4. Ans (A)



$x = 4 - \frac{x}{3} \times 1$

$x = 3$

5. Ans (B)

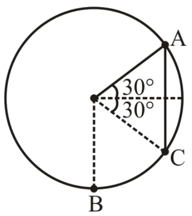
$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{g}{4}$

$\Rightarrow 4m_1 - 4m_2 = m_1 + m_2$

$\Rightarrow 3m_1 = 5m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{5}{3}$

6. Ans (B)



Velocity just before,

$C = \sqrt{2g\ell} = 20\text{m/s}$

Velocity just after,

$C = 20 \cos 30° = 10\sqrt{3}$ m/s

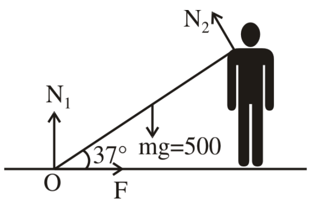
Velocity at B $= \sqrt{(10\sqrt{3})^2 + 2g\frac{\ell}{2}} = 10\sqrt{5}$ m/s

7. Ans (B)

$I = I_{cone} + I_{hemisphere}$

$= \frac{1}{2}MR^2 + \frac{2}{5}MR^2 = 0.9\ MR^2$

8. Ans (B)



Torque about O = 0

$500\left(\frac{\ell}{2}\cos 37^\circ\right) - N_2 \times \ell = 0$

$\Rightarrow N_2 = 500 \times \frac{1}{2} \times \frac{4}{5}$

$\Rightarrow N_2 = 200$ N

9. Ans (A)

$F = YA\frac{\Delta\ell}{\ell} = Y.\Delta\ell.\frac{A.A}{A.\ell} = Y.\Delta\ell.\frac{A^2}{V}$

$F \propto A^2$

$\Rightarrow \frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = 4 : 1$

10. Ans (A)

$V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}}$

$\frac{V_1}{V_2} = \sqrt{\frac{32}{2}}$

$\Rightarrow \frac{4}{V_2} = 4$

$\Rightarrow V_2 = 1$ km/sec

11. Ans (D)

9 MSD = 10 VSD

$1\text{VSD} = \frac{9}{10}$ MSD

1VSD = 0.9 × 2 mm = 1.8 mm

LC = 1 MSD – 1 VSD

= 2 mm – 1.8 mm = 0.2 mm

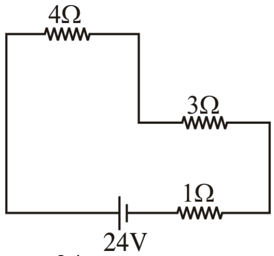
12. Ans (D)

$\frac{1}{2}mv^2 + \frac{KQq}{R} = \frac{KQq}{2R}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$

$\Rightarrow v = \sqrt{\frac{3KQq}{4mR}}$

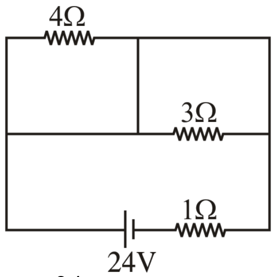
13. Ans (A)

Just after



$I = \frac{24}{8} = 3A$

Long time after



$I = \frac{24}{1} = 24A$

14. Ans (A)

Lyman : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$;

First Lyman line means n = 2, shortest Lyman line means $n \longrightarrow \infty$

Balmer : $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

First Balmer line means n = 3, shortest Balmer line means $n \longrightarrow \infty$

15. Ans (B)

$\Delta x_0 = (\mu - 1)t = \left(\frac{3}{2} - 1\right) 3\mu m = 1.5\mu m = 3\lambda$

$\Rightarrow$ There is constructive interference at O.

$\Rightarrow 4I_0$

16. Ans (C)

$\hat{E} = \frac{-\hat{i} + 2\hat{j}}{\sqrt{5}}$

$\hat{V} = -\hat{K}$

$\hat{B} = \hat{V} \times \hat{E} = \frac{\hat{j} + 2\hat{i}}{\sqrt{5}}$

17. Ans (D)

$g\left(1 - \frac{2h}{R}\right) = 0.99\, g$

$\Rightarrow h = 0.005\, R = 32$ km

18. Ans (D)

As magnetic flux is into the plane and increasing, induced current will be anti-clockwise, thereby creating out of the plane magnetic flux.

20. Ans (A)

$$\frac{KE_1}{KE_2} = \frac{5}{1} = \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi}$$

$$5\left(\frac{hc}{\lambda_2} - \phi\right) = \frac{hc}{\lambda_1} - \phi$$

$$\frac{5hc}{\lambda_2} - \frac{hc}{\lambda_1} = 4\phi$$

$$\phi = \frac{1}{4}\left[\frac{5hc}{\lambda_2} - \frac{hc}{\lambda_1}\right]$$

$$\phi = \frac{1}{4}\left[\frac{5 \times 12400}{6200} - \frac{12400}{3100}\right]$$

$$\phi = \frac{1}{4} \times [10 - 4] = \frac{6}{4} = \frac{3}{2} = 1.5$$

PART-1 : PHYSICS

SECTION-II

1. Ans (25)

$$\frac{\frac{Q^2}{2C_i} - \frac{Q^2}{2C_F}}{\frac{Q^2}{2C_i}} \times 100$$

$$\frac{\frac{d}{\varepsilon_0 A} - \frac{3d}{4\varepsilon_0 A}}{\frac{d}{\varepsilon_0 A} - \frac{1}{4}} \times \frac{\frac{1}{C_i} - \frac{1}{C_F}}{\frac{1}{C_i}} \times 100$$

$$= \frac{\frac{1}{C_0} - \frac{3}{4C_0}}{\frac{1}{C_0}} \times 100$$

$$\frac{1}{4} \times 100 = 25\%$$

2. Ans (3)

PV^x = constant

$$W = \frac{nR\Delta T}{1 - x}$$

$$W = \frac{P_2V_2 - P_1V_1}{1 - \frac{5}{3}}$$

$$= \frac{P_2V_2 - P_1V_1}{-\frac{2}{3}}$$

$$= \frac{3}{2}(P_1V_1 - P_2V_2)$$

3. Ans (5)

$F_B = F_g$

$\Rightarrow V_1\rho_w g + V_1\rho_x g = (V_1 + V_2)\, P_I g$

$\Rightarrow V_1 + V_2\dfrac{\rho_x}{\rho_w} = (V_1 + V_2)\dfrac{\rho_I}{\rho_W}$

$\Rightarrow V_1 + V_2 \times 0.4 = (V_1 + V_2)\, 0.9$

$\Rightarrow 0.1\, V_1 = 0.5V_2$

$= \Rightarrow \dfrac{V_1}{V_2} = \dfrac{5}{1}$

5. Ans (4)

$Q = (M_{Ne} - M_{Na})c^2$

$= 0.0047 \times 931.5 \text{ MeV} = 4.378 \text{ MeV}$

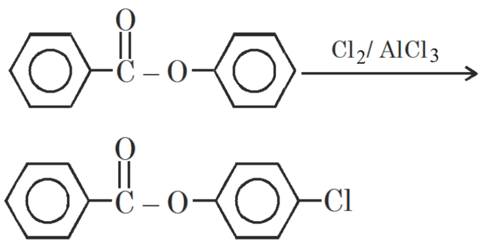
PART-2 : CHEMISTRY

SECTION-I

1. Ans (C)

Wolff-Kishner reduction is not suitable for base sensitive group.

3. Ans (C)



15. Ans (C)

Number of radoa; mpdes = $(n - \ell - 1)$

Number of angular nodes = ℓ

16. Ans (B)

Solubility of $CaC_2O_4 = \sqrt{K_{sp}} = 5 \times 10^{-4}$ M

gm equivalent of $CaC_2O_4 \equiv KMnO_4$

$5 \times 10^{-4} \times 2 = \dfrac{n}{1000} \times 5$

$n = 0.2$

17. Ans (B)

S is a state function so ΔS will be same is path

$A \longrightarrow B$ and $A \longrightarrow C \longrightarrow B$

18. Ans (D)

$CN^- + H_2O \rightleftharpoons HCN + OH^-$

$0.1-x \qquad\qquad x \qquad x$

$\dfrac{10^{-14}}{K_a} = \dfrac{x^2}{0.1 - x} \qquad\qquad x = 10^{-2}$

$K_a = 9 \times 10^{-12}$

20. Ans (C)

$m = 3 \Rightarrow$ If $w_{solvent} = 1000$ gm

Then moles of NaOH = 3

$\%\,(w/v) = \dfrac{3 \times 40}{\frac{(1000+120)}{1.12}} \times 100 = 12$

PART-2 : CHEMISTRY

SECTION-II

3. Ans (6)

Product P is benzene

4. Ans (264)

$\dfrac{1}{2}H_2 \rightarrow H^+ + e^\ominus$

$e^\ominus + AgCl_{(s)} \rightarrow Ag_{(s)} + Cl^\ominus$

$\frac{1}{2}H_2 + AgCl_{(s)} \rightarrow H^+_{(aq)} + Ag_{(s)} + Cl^\ominus_{(aq)}$

$E = \varepsilon^0 - \dfrac{.06}{1}\log\dfrac{[H^+][Cl^\ominus]}{P_{H_2}^{\frac{1}{2}}}$

$E = 0.22 - .06\log\dfrac{(10^{-1})\,(10^{-1})}{1^{\frac{1}{2}}}$

$E = 0.22 + .12 = .34$ volt

$\Rightarrow$ total energy of photon will be (for Na)

$= 2.3 + 0.34 = 2.64$ eV

5. Ans (6)

$\dfrac{p^o - p_s}{p_s} = \dfrac{n}{N}$

or, $\dfrac{2}{98} = \dfrac{\frac{w}{60}}{\frac{88.2}{18}} \Rightarrow w = 6$

PART-3 : MATHEMATICS

SECTION-I

1. Ans (A)

$f'(x) = c[e^{-x} - xe^{-x}] - x + 1$

$= ce^{-x}(1 - x) + (1 - x)$

$f'(x) = (1 - x)(ce^{-x} + 1) \leq 0$

$ce^{-x} + 1 \leq 0$

$c \leq -e^{x} \ \forall \ x \leq 0$

$c \in (-\infty, -1]$

Least value of $c^2 = 1$

2. Ans (D)

$Y - y = m(X - x)$ For x-intercept $y = 0$

$X = x - \frac{y}{m}$ $\quad \therefore \ x - \frac{y}{m} = y$

$\frac{dy}{dx} = \frac{y}{x - y}$

$xdy - ydy = ydx \Rightarrow \frac{-ydy}{y^2} = \frac{ydx - xdy}{y^2}$

$-\frac{dy}{y} = d\left(\frac{x}{y}\right)$

$-\ell ny = \frac{x}{y} + C$

$x = 1, y = 1, C = -1$

$-\ell ny = \frac{x}{y} - 1$ or $\ell ny = \frac{-x}{y} + 1$

$y = e \cdot e^{-\frac{x}{y}}$

3. Ans (D)

Assertion is False but Reason is True.

4. Ans (A)

$\int_{\ell n2}^{\ell n3} f(x)dx + \int_{8}^{27} g(y)dy = 27\ell n3 - 8\ell n2$

$\int_{\ell n2}^{\ell n3} f(x)dx = 12 - 12\ell n3 + 12\ell n2$ and

$\int_{8}^{27} g(y)dy = 39\ell n3 - 20\ell n2 - 12$

$a = 39 \quad b = 20 \quad c = 12$

$a - (b + c) = 7$

5. Ans (B)

$f(2 + x) = f(2 - x)$

$f'(2 + x) = -f'(2 - x)$

Put $x = 0 \quad f'(2) = 0$

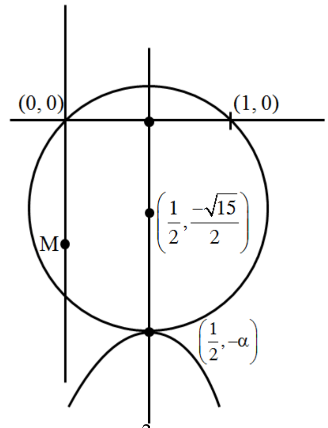
$x = -1 \quad f'(1) = -f'(3) = 0$

$x = -\frac{3}{2} \quad f'\left(\frac{1}{2}\right) = -f'\left(\frac{7}{2}\right) = 0$

$\therefore \ f'\left(\frac{1}{2}\right) = 0 = f'(1) = f'(2) = f'(3) = f'\left(\frac{7}{2}\right)$

minimum roots of $f''(x) = 0$

6. Ans (A)



$x(x - 1) + y^2 = 0$

$S_1 : x^2 + y^2 - x = 0$

Required circle

$x^2 + y^2 - x + \lambda y = 0$

$g = -\frac{1}{2} \quad f = \frac{\lambda}{2}$, radius = 2

$g^2 + f^2 - c = 4$

$\frac{1}{4} + \frac{\lambda^2}{4} = 4$

$1 + \lambda^2 = 16 \quad \therefore \ \lambda^2 = 15$

$\therefore \ \lambda = \sqrt{15}, -\sqrt{15}$

centre of circle $\left(\frac{1}{2}, \frac{-\lambda}{2}\right)$

parabola $y + \alpha = -\left(x - \frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = -(y + \alpha)$

$-\frac{\sqrt{15}}{2} - 2 = -\alpha$

$\alpha = 2 + \frac{\sqrt{15}}{2}$

$2\alpha = 4 + \sqrt{15}$

$(2\alpha - 4)^2 = 15$

7. Ans (A)

$2\sin x\cos x = \sin 2x$

$2\sin x\cos 3x = \sin 4x - \sin 2x$

$2\sin x\cos 5x = \sin 6x - \sin 4x$

$2\sin x\cos 7x = \sin 8x - \sin 6x$

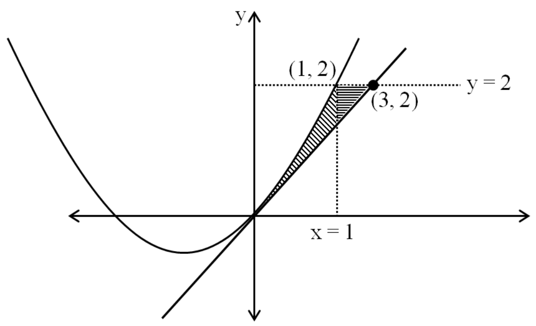
$2\sin x[\cos x + \cos 3x + \cos 5x + \cos 7x] = \sin 8x$

$$\int_0^{\frac{\pi}{2}} \frac{\sin 8x}{\sin x} = 2.\int_0^{\frac{\pi}{2}} \cos x + \cos 3x + \cos 5x + \cos 7x$$

$$2\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right]$$

$$= 2\left[\frac{105 - 35 + 21 - 15}{105}\right] = \frac{152}{105}$$

8. Ans (B)



$$A_1 = \int_0^1 \left(\frac{(2x+1)^2 - 1}{4} - x\right) dx = \frac{1}{3}$$

$$A_2 = \frac{1}{2}$$

9. Ans (B)

(A) $x = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots\infty$

$\frac{1}{2}x = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\infty$

$\frac{x}{2} = \frac{a_1}{2} + \left\{\frac{d}{2^2} + \frac{d}{2^3} + \ldots\infty\right\}$

$x = a_1 + d = 4$

$\therefore a_2 = 4$

(C) $x + y + z = 21$

$(x+1) + (y+3) + (z+4) = 13$

$x + y + z = 13 \quad \therefore {}^{13+3-1}C_{3-1} = {}^{15}C_2 = 105$

(D) $\frac{1}{16}$, a, b in G. P. $\therefore a^2 = \frac{b}{16}$

$\frac{1}{a}, \frac{1}{b}$, 6 in A. P. $\quad \therefore 2a = b + 6ab$

$$16a^2 = \frac{2a}{1+6a}$$

$(12a - 1)(4a + 1) = 0 \quad \therefore a = \frac{1}{12}, \frac{-1}{4}$

$$b = 16.\frac{1}{12}.\frac{1}{12} = \frac{1}{9}$$

$$72\left[\frac{1}{12} - \frac{1}{9}\right] = 6 + 8 = 14$$

10. Ans (A)

START →	1	X	3	4	5	6	X	8	9
									10
	18	17	X	15	14	13	X	11	

To be on start after two throws

(1, 1), (2, 2), (1, 6), (6, 1), (5, 2), (3, 4), (4, 3), (6, 6)

$$\alpha = 8\cdot\left(\frac{1}{6}\times\frac{1}{6}\right) = \frac{2}{9}$$

To be on square marked 17. He needs to throw sum of 17 in three throws

$$= (6, 5, 6), (5, 6, 6) = 2\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\right) = \frac{1}{108}$$

Value of $\frac{\alpha}{\beta} = \frac{2}{9}\times\frac{108}{1} = 24$

11. Ans (C)

$(\sin^{-1}a)^2 = \frac{\pi^2}{4}$, $(\cos^{-1}b)^2 = \pi^2$

$(\sec^{-1}c)^2 = \pi^2$, $(\operatorname{cosec}^{-1}d)^2 = \frac{\pi^2}{4}$

12. Ans (A)

$$\frac{x^2}{2} - \frac{y^2}{3} = 1$$

$y = mx \pm \sqrt{2m^2 - 3}$

$(\beta - m\alpha)^2 = 2m^2 - 3$

$(\alpha^2 - 2)m^2 - 2\alpha\beta m + \beta^2 + 3 = 0$

$\tan\theta\tan\phi = 2$

$m_1m_2 = 2$

$\frac{\beta^2 + 3}{\alpha^2 - 2} = 2 \quad \therefore \beta^2 = 2\alpha^2 - 7$

13. **Ans (A)**

$$\frac{\tan 3^\circ}{1-3\tan^2 3^\circ}+\frac{\tan 3^\circ}{8}-\frac{\tan 3^\circ}{8}$$

$$\Rightarrow \frac{3}{8}\tan 9^\circ-\frac{\tan 3^\circ}{8}$$

Again $\frac{3}{8}\tan 9^\circ+\frac{3\tan 9^\circ}{1-3\tan^2 9^\circ}=\frac{9}{8}\tan 27^\circ$

and so on

$$\frac{81\tan 243^\circ}{8}-\frac{\tan 3^\circ}{8}$$

$$\frac{81}{8}\cot 27^\circ-\frac{1}{8}\cot 87^\circ$$

$$x+y=\frac{80}{8}$$

$$4(x+y)=40$$

14. **Ans (A)**

$P(x)=x^3+ax^2+bx+c$

$P(-3)=-27+9a-3b+c=0 \quad ...(i)$

$P(2)=8+4a+2b+c=0 \quad ...(ii)$

$-35+5a-5b=0$

$a-b=7 \; ...(iii)$

$P'(x)=3x^2+2ax+b$

$P'(-3)=27-6a+b<0$

$27-6(a-b)-5b<0$

$27-6(7)-5b<0$

$-15-5b<0$

$b+3>0 \quad \therefore\ b>-3$

$a=7+b$

$a>4$

$8+(\geq 16)+2(\geq -3)+c=0$

$18+c_{max}=0 \quad c_{max}=-18$

$c<-18$

15. **Ans (C)**

$\text{adj}(N^{-1}BM^{-1})=(\text{adj } M^{-1})(\text{adj } B)(\text{adj } N^{-1})$

$(\text{adj } M)^{-1}.\ A(\text{adj } N)^{-1}$

MAN $\quad |M|=1=|N|$

Note: $P \longrightarrow \text{adj } P^{-1}=|P^{-1}|In$

$$\text{adj } P^{-1}=\frac{P}{|P|} \quad ...(i)$$

$$P^{-1}=\frac{\text{adj } P}{|P|} \Rightarrow P=|P|(\text{adj } P)^{-1}$$

Also $M^{-1}=\frac{\text{adj } M}{|M|} \Rightarrow M^{-1}=\text{adj } M$

$(\text{adj } M)^{-1}=M$

Note $M^{-1}\text{ adj } M^{-1}=\frac{In}{|M|}$

$\text{adj } M^{-1}=M$

16. **Ans (C)**

$$f(x)=(x-3)(x+3)\,|(x-1)(x-2)(x-3)|+\frac{x}{1+|x|}$$

Not differentiable at $x=1, 2 \quad \therefore\ m=2$

For g(x)

$$\lim_{x\to -1^-}[x]\left|x^2-1\right|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1]$$

$$(-2)(0)+\sin\pi+1=1$$

$$\lim_{x\to -1^+}(-1)(0)+\sin\frac{\pi}{2}+0=1$$

Similarly $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to 1} g(x)$

$\therefore$ Discontinuous at $x=1$ and $x=0$

$\therefore\ n=2$

17. **Ans (A)**

Point of intersection of lines is (4, 3, 5). For plane to be at maximum distance from origin normal to plane will be $4\hat{i}+3\hat{j}+5\hat{k}$

Equation of plane $=4(x-4)+3(y-3)+5(z-5)=0$

$4x+3y+5z=50$

$-4x-3y-5z+50=0$

18. **Ans (A)**

$$f(x)=\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x\,du+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}|u|\,f(u)du$$

$$f(x)=\cos x+\pi\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}|u|\,f(u)du$$

$$f(x)=(1+\pi)\cos x+A\;;\;A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}|u|\,f(u)du$$

$$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}((1+\pi)\cos u+A)\cdot|u|\,du$$

$$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+\pi)|u|\cos u\,du+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}A|u|\,du$$

$$A=2\cdot\int_{0}^{\frac{\pi}{2}}(1+\pi)u\cos u+2A\cdot\int_{0}^{\frac{\pi}{2}}u\,du$$

$$= 2 \cdot (1+\pi)\left[\frac{\pi}{2}-1\right] + 2A \cdot \left[\frac{u^2}{2}\right]_0^{\frac{\pi}{2}}$$

$$A = 2(1+\pi)\left(\frac{\pi}{2}-1\right) + A\left[\frac{\pi^2}{4}-0\right]$$

$$A = \frac{-4(\pi+1)}{\pi+2}$$

$$f(x) = (1+\pi)\cos x - \frac{4(\pi+1)}{\pi+2}$$

$$f_{max} = \frac{(\pi+1)}{\pi+2}\cdot(\pi-2) \quad ;$$

$$f_{min} = -\left(\frac{\pi+1}{\pi+2}\right)[\pi+6]$$

$$\frac{M}{m} = \frac{2-\pi}{6+\pi}$$

19. Ans (A)

$\sigma_b^2 = 2$ (variance of boys) $\quad n_1$ = no. of boys

$\bar{x}_b = 12 \quad n_2$ = no. of girls

$\sigma_g^2 = 2$

$$\bar{x}_g = \frac{50\times 15 - 12\times n_1}{30} = \frac{750-12\times 20}{30} = 17 = \mu$$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1+n_2} + \frac{n_1 . n_2}{(n_1+n_2)^2}(\bar{x}_b - \bar{x}_g)^2$$

$$\sigma^2 = \frac{20\times 2 + 30\times 2}{20+30} + \frac{20\times 30}{(20+30)^2}(12-17)^2$$

$$\sigma^2 = 8$$

$$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$$

20. Ans (C)

$1+\alpha+\alpha^2 = 0 \ ; \ \alpha^3 = 1$

$1+\alpha^2 = -\alpha \quad ; \ 1+\alpha = -\alpha^2$

considering first three consecutive terms

$\therefore (1-\alpha+\alpha^2)(1-\alpha^2+\alpha^4)(1-\alpha^3+\alpha^6)$

$\Rightarrow (-2\alpha)(-2\alpha^2)(1) = 2^2$

$$\begin{cases} (2)^{2K}, & n = 3K \\ (2)^{2K+1}(-\alpha), & n = 3K+1 \\ (2)^{2K+2}, & n = 3K+2 \end{cases}$$

since (2, 12) is orthocentre $\therefore a^b = 2^{12}$

$2^{12} = 2^{2K} \quad \because K = 6 \longrightarrow n = 18$

$2^{12} = 2^{2K+2} \quad \because K = 5 \longrightarrow n = 17$

sum of possible values of n is 35

PART-3 : MATHEMATICS

SECTION-II

1. Ans (3)

SHREYANSH

Total words : $\frac{9!}{2!2!}$

for two alike letters together

n(A) : Two H together $= \frac{8!}{2!}$

n(B) : Two S together $= \frac{8!}{2!}$

n(A ∩ B) : H together, S together = 7!

$n(A \cup B) = \frac{8!}{2!} + \frac{8!}{2!} - 7! \Rightarrow 8! - 7! = 7\times 7!$

Required prob $= \frac{7\times 7!}{9!}\times 2\times 2 = \frac{7\times 4}{9\times 8} = \frac{7}{18}$

For word SANIDHYA

Total words $= \frac{8!}{2!}$

n(X) : two A together 7!

Required probability $= \frac{7!\times 2}{8!} = \frac{1}{4}$

Final probability $= \frac{1}{2}\times\frac{7}{18} + \frac{1}{2}\times\frac{1}{4} = \frac{14+9}{72}$

$= \frac{23}{72}$

2. Ans (8)

Volume of tetrahedron $= \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} |\vec{a}|^2 & \frac{|\vec{a}||\vec{b}|}{2} & \frac{|\vec{a}||\vec{c}|}{2} \\ \frac{|\vec{a}||\vec{b}|}{2} & |\vec{b}|^2 & \frac{|\vec{b}||\vec{c}|}{2} \\ \frac{|\vec{c}||\vec{a}|}{2} & \frac{|\vec{c}||\vec{b}|}{2} & |\vec{c}|^2 \end{vmatrix}$$

$$= |\vec{a}|^2|\vec{b}|^2|\vec{c}|^2 \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix}$$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \frac{1}{2}|\vec{a}|\,|\vec{b}|\,|\vec{c}|^2$$

Now $\dfrac{4|\vec{a}|^2 + 3|\vec{b}|^2 + 2|\vec{c}|^2}{3} \geqslant \left(24|\vec{a}|^2|\vec{b}|^2|\vec{c}|^2\right)^2$

$$\frac{144}{3} \geqslant \left(24\left(|\vec{a}|\,|\vec{b}|\,|\vec{c}|\right)^2\right)^{\frac{1}{3}}$$

$$\frac{48 \times 48 \times 48}{24} \geqslant \left(|\vec{a}|\,|\vec{b}|\,|\vec{c}|\right)^2$$

$$|\vec{a}|\,|\vec{b}|\,|\vec{c}| \leqslant 48\sqrt{2}$$

$$V_{max} = \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \frac{1}{6}(\leqslant 48) \leqslant 8$$

$\therefore V_{max} = 8$

3. Ans (1)

$f(x) = 2\sin 2x - 3\cos^2 x - (a^2 + a - 7)x + 5$

$f'(x) = 4\cos 2x + 3\sin 2x - (a^2 + a - 7) \geq 0$

$a^2 + a - 7 \leq 4\cos 2x + 3\sin 2x$

$a^2 + a - 7 \leq -5$

$a^2 + a - 2 \leq 0$

$(a + 2)(a - 1) \leq 0 \quad a \in [-2, 1]$

$|p + q| = |-2 + 1| = 1$

4. Ans (101)

$$T_r = 7 \times 10^r + \frac{50}{9}\left(10^{r-1} - 1\right) + 7$$

$$\sum_{r=1}^{100} T_r = \frac{7\left(10\left(10^{100} - 1\right)\right)}{9}$$

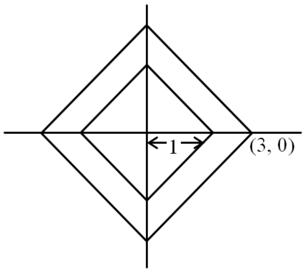
$$+\frac{50}{9}\left(\frac{10^{100} - 1 - 100}{9}\right) + 700$$

$$S = \frac{68.10^{101} + 11020}{81}$$

$\therefore \lambda = 101$

5. Ans (16)

$$1 \leqslant \left|\frac{x+y}{\sqrt{2}}\right| + \left|\frac{x-y}{\sqrt{2}}\right| \leqslant 3$$



$1 \leq |x| + |y| \leq 3$

Area $4\left(\frac{1}{2} \times 3 \times 3 - \frac{1}{2} \times 1 \times 1\right)$

$$4\left(\frac{9}{2} - \frac{1}{2}\right) = 16$$