ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)
PART TEST
08-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	B,C	A,B,D	A,C	A,B,D	A,C	B,C
SECTION-I (ii)	Q.	7	8	9	10		
	A.	A	A	B	A		
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	2.40 to 2.43	1.57	0.78 to 0.79	10.00	8.65 to 8.68	1.14 to 1.17
SECTION-II (ii)	Q.	7	8	9			
	A.	3	28	2			

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	A,B,D	B,D	B	A,B,D	C,D	A,D
SECTION-I (ii)	Q.	7	8	9	10		
	A.	D	D	A	B		
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	2.00	3.00	104.50	41.66 or 41.67	2.00	9.00
SECTION-II (ii)	Q.	7	8	9			
	A.	8	1270	433			

PART-3 : MATHEMATICS

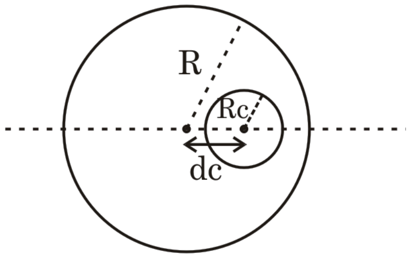
SECTION-I (i)	Q.	1	2	3	4	5	6
	A.	B,D	A,B,D	A,C	A,C,D	C	B,D
SECTION-I (ii)	Q.	7	8	9	10		
	A.	D	B	B	C		
SECTION-II (i)	Q.	1	2	3	4	5	6
	A.	0.13	1.00	2.00	0.00	6.00	0.88 or 0.89
SECTION-II (ii)	Q.	7	8	9			
	A.	5	0	4			

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. Ans (B,C)



$$g_0 = \frac{GM}{R^2},\ n_1 g_0 = \frac{GM}{R^2} - \frac{G.P.\frac{4}{3}\pi R_C^3}{(R-d)^2}$$

$$n_2 g_0 = \frac{GM}{R^2} - \frac{G.P.\frac{4}{3}\pi R_C^3}{(R+d_C)^2}$$

$$\frac{GP.\frac{4}{3}\pi R_C^3}{(R-d_C)^2} = g_0(1-n_1)$$

$$\frac{GP.\frac{4}{3}\pi R_C^3}{(R+d_C)^2} = g_0(1-n_2)$$

$$\frac{(R+d_C)^2}{(R-d_C)^2} = \frac{1-n_1}{1-n_2} \Rightarrow \frac{R+d_C}{R-d_C} = \frac{\sqrt{1-n_1}}{\sqrt{1-n_2}}$$

$$\left(\sqrt{1-n_2}\right)(R+d_C) = \left(\sqrt{1-n_1}\right)(R-d_C)$$

$$\left(\sqrt{1-n_2}-\sqrt{1-n_1}\right)R = \left(-\sqrt{1-n_1}-\sqrt{1-n_2}\right)d_C$$

$$d_C = \frac{\left(\sqrt{1-n_1}-\sqrt{1-n_2}\right)}{\left(\sqrt{1-n_1}+\sqrt{1-n_2}\right)}R$$

$$\frac{GP.\frac{4}{3}\pi R_C^3}{(R-d_C)^2} = g_0(1-n_1)$$

$$\frac{\frac{GM}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi R_C^3}{(R-d_C)^2} = \frac{GM}{R^2}(1-n_1)$$

$$\frac{R_C^3}{(R-d_C)^2}.\frac{1}{R} = (1-n_1)$$

$$R_C^3 = R(1-n_1)(R-d_C)^2$$

$$R_C^3 = R(1-n_1)\left(R - \frac{\sqrt{1-n_1}-\sqrt{1-n_2}}{\sqrt{1-n_1}+\sqrt{1-n_2}}R\right)^2$$

$$R_C^3 = R^3(1-n_1)\left(\frac{2\sqrt{1-n_2}}{\sqrt{1-n_1}+\sqrt{1-n_2}}\right)^2$$

$$R_C^3 = \frac{4R^3(1-n_1)(1-n_2)}{\left(\sqrt{1-n_1}+\sqrt{1-n_2}\right)^2}$$

$$R_C = 2^{2/3}R\frac{(1-n_1)^{1/3}(1-n_2)^{1/3}}{\left(\sqrt{1-n_1}+\sqrt{1-n_2}\right)^{2/3}}$$

2. Ans (A,B,D)

$$U = \int_a^{2a}\frac{1}{2}\varepsilon_0 E^2 4\pi r^2 dr$$

$$= \frac{1}{2}\varepsilon_0 4\pi \int_a^{2a}\left(\frac{K2Q}{r^2}\right)^2 r^2 dr$$

$$= \frac{1}{2}\varepsilon_0 4K^2 4Q^2 \int_a^{2a}\frac{1}{r^2}dr$$

$$= \frac{1}{8\pi\varepsilon_0}4Q^2\left[\frac{1}{a}-\frac{1}{2a}\right] = \frac{4Q^2}{16\pi\varepsilon_0 a} = \frac{Q^2}{4\pi\varepsilon_0 a}$$

For B :

$$V_{inner} = \frac{K2Q}{a} - \frac{KQ}{2a} = \frac{3}{2}\frac{KQ}{a}$$

For C :

Assuming charge on outer surface of inner shell after closing the switch is Q_1 then

$$\frac{KQ_1}{a} - \frac{KQ}{2a} = 0$$

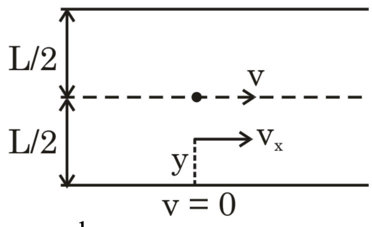
$$Q_1 = \frac{Q}{2}$$

So charge flow is $\frac{3Q}{2}$.

For D :

There is no electric field due to outer charge at common center.

5. Ans (A,C)



$$v_r = \frac{dy}{dt}$$

$$y = v_r t$$

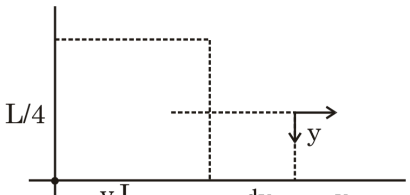
$$v_x = \frac{v_0}{L/2}.y = \frac{2v_0}{L}.v_r t$$

$$\frac{dx}{dt} = \frac{2v_0 v_r}{L}t$$

$$x = \frac{2v_0 v_r}{L}.\frac{t^2}{2} = \frac{v_0 v_r t^2}{L}$$

for trajectory :- $x = \frac{v_0 v_r}{L}.\left(\frac{y}{v_r}\right)^2 = \frac{v_0 y^2}{L v_r}$

for $y = \frac{L}{4}$, $x = \frac{v_0}{L v_r}\frac{L^2}{16} = \frac{v_0 L}{v_r 16} = \frac{v_0}{16}\frac{L}{v_r}$



$\frac{v_0L}{16v_r}$ $\quad \frac{dy}{dt} = -\frac{v_r}{2}$

$$\left(\frac{L}{4}-y\right) = \frac{v_r}{2}t$$

$$y = \frac{L}{4} - \frac{v_r}{2}t \quad \text{for } y = 0,\ t = \frac{L}{2v_r}$$

$$\frac{v_rt}{2} = \frac{L}{4} - y \Rightarrow t = \frac{2}{v_r}\left(\frac{L}{4}-y\right)$$

$$t = \frac{L}{2v_r} - \frac{2y}{v_r}$$

$$v_x = \frac{dx}{dt} = \left(\frac{v_0}{L/2}\right)y = \frac{2v_0}{L}\left(\frac{L}{4} - \frac{v_r}{2}t\right)$$

$$\frac{dx}{dt} = \frac{v_0}{2} - \frac{v_0v_rt}{L}$$

$$x - \frac{v_0L}{16v_r} = \frac{v_0t}{2} - \frac{v_0v_r}{L}\cdot\frac{t^2}{2}$$

for return trajectory,

$$x - \frac{v_0L}{16v_r} = \frac{v_0}{2}\left(\frac{L}{2v_r} - \frac{2y}{v_r}\right)$$

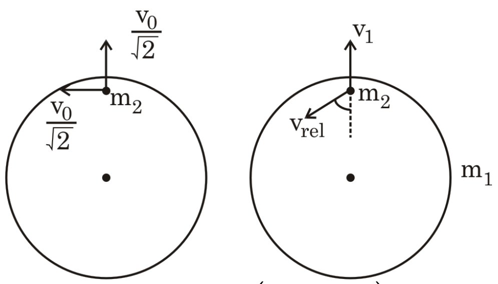
$$-\frac{v_0v_r}{2L}\left(\frac{L}{2v_r} - \frac{2y}{v_r}\right)^2$$

$$= \frac{v_0L}{4v_r} - \frac{v_0y}{v_r} - \frac{v_0v_r}{2L}\cdot\left(\frac{L^2}{4v_r^2} + \frac{4y^2}{v_r^2} - 2\left(\frac{L}{2v_r}\right)\frac{2y}{v_r}\right)$$

$$= \frac{v_0L}{4v_r} - \frac{v_0y}{v_r} - \frac{v_0L}{8v_r} - \frac{2v_0y^2}{v_r^2} + \frac{v_0y}{v_r}$$

$$x = \frac{v_0L}{8v_r} - \frac{2v_0}{v_rL}y^2 + \frac{v_0L}{16v_r} = \frac{3v_0L}{16v_r} - \frac{2v_0y^2}{v_rL}$$

6. Ans (B,C)

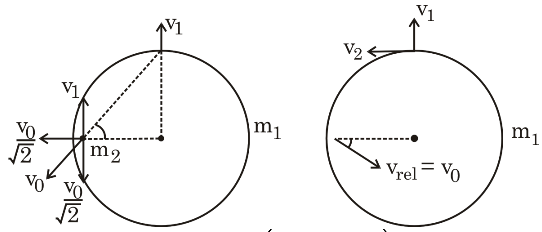


$$\frac{m_2v_0}{\sqrt{2}} = m_1v_1 + m_2\left(v_1 - \frac{v_{rel}}{\sqrt{2}}\right)$$

$$e = 1 = \frac{\frac{v_{rel}}{\sqrt{2}}}{\frac{v_0}{\sqrt{2}}} \Rightarrow v_{rel} = v_0$$

$$\frac{m_2v_0}{\sqrt{2}} = (m_1+m_2)v_1 - \frac{m_2v_0}{\sqrt{2}}$$

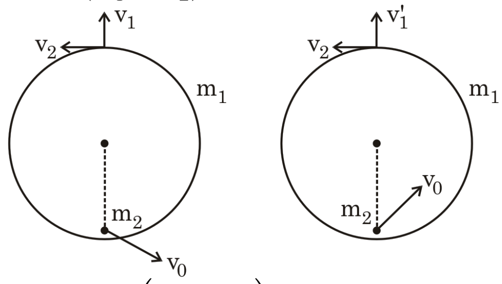
$$v_1 = \frac{\frac{2m_2v_0}{\sqrt{2}}}{m_1+m_2} = \frac{m_2v_0\sqrt{2}}{m_1+m_2}$$



$$\frac{m_2v_0}{\sqrt{2}} = m_1v_2 + m_2\left(v_2 - \frac{v_0}{\sqrt{2}}\right)$$

$$\frac{2m_2v_0}{\sqrt{2}} = (m_1+m_2)v_2$$

$$v_2 = \frac{m_2v_0\sqrt{2}}{(m_1+m_2)}$$



$$m_1v_1 + m_2\left(v_1 - \frac{v_0}{\sqrt{2}}\right)$$

$$= m_1v_1' + m_2\left(v_1' + \frac{v_0}{\sqrt{2}}\right)$$

$$(m_1+m_2)v_1 - \frac{m_2v_0}{\sqrt{2}}$$

$$= (m_1+m_2)v_1' + \frac{m_2v_0}{\sqrt{2}}$$

$$(m_1+m_2)\frac{m_2v_0\sqrt{2}}{(m_1+m_2)} - \frac{m_2v_0}{\sqrt{2}}$$

$$= (m_1+m_2)v_1' + \frac{m_2v_0}{\sqrt{2}}$$

$$\frac{m_2v_0}{\sqrt{2}} = (m_1+m_2)v_1' + \frac{m_2v_0}{\sqrt{2}}$$

$$v'_1 = 0$$

$$v_{m_1} = -v_2 = -\frac{m_2v_0\sqrt{2}}{m_1+m_2}$$

$$v_{m_2}^2 = \left(\frac{v_0}{\sqrt{2}}\right)^2 + \left(v_2 - \frac{v_0}{\sqrt{2}}\right)^2$$

$$= \frac{v_0^2}{2} + \left(\frac{m_2v_0\sqrt{2}}{(m_1+m_2)} - \frac{v_0}{\sqrt{2}}\right)^2$$

$$= \frac{v_0^2}{2} + \frac{v_0^2}{2} + \frac{2m_2^2v_0^2}{(m_1+m_2)^2} - \frac{2v_0}{\sqrt{2}}\cdot\frac{m_2v_0\sqrt{2}}{m_1+m_2}$$

$$= v_0^2 - \frac{2m_2v_0^2}{(m_1+m_2)} + \frac{2m_2^2\cdot v_0^2}{(m_1+m_2)^2}$$

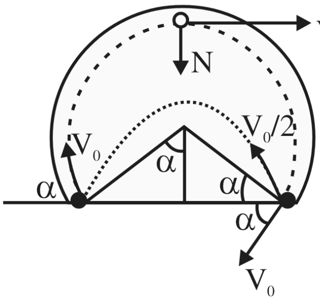
$$v_{m_2}^2 = v_0^2\left(\frac{2m_2^2}{(m_1+m_2)^2} - \frac{2m_2(m_1+m_2)}{(m_1+m_2)^2} + 1\right)$$

$$= v_0^2\left(\frac{(m_1+m_2)^2 - 2m_1m_2}{(m_1+m_2)^2}\right)$$

PART-1 : PHYSICS

SECTION-I (ii)

8. Ans (A)



For projectile part

$$\frac{V_0^2}{4}\frac{2\sin\alpha\cos\alpha}{g} = 2R\sin\alpha$$

$$V_0^2 = \frac{4gR}{\cos\alpha}$$

For top point of circular path $N \geq 0$ so

$$\frac{mV^2}{R} \geqslant mg \Rightarrow V^2 \geqslant gR$$

By energy conservation

$$\frac{mV_0^2}{2} = mgR(1+\cos\alpha) + \frac{mV^2}{2}$$

$$V^2 = V_0^2 - 2gR(1+\cos\alpha)$$

$$= \frac{4gR}{\cos\alpha} - 2gR(1+\cos\alpha)$$

$$\frac{4gR}{\cos\alpha} - 2gR(1+\cos\alpha) \geqslant gR$$

$$2\cos^2\alpha + 3\cos\alpha - 4 \leq 0$$

$$0 \leq \cos\alpha \leq \frac{\sqrt{41}-3}{4}$$

another condition for complete trajectory

$$h_{max} = \frac{V_0^2\sin^2\alpha}{8g} \leqslant R(1+\cos\alpha)$$

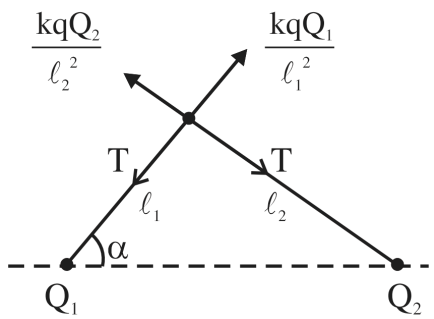
$$\frac{R\sin^2\alpha}{2\cos\alpha} \leqslant (1+\cos x)$$

$$3\cos^2\alpha + 2\cos\alpha - 1 \geq 0$$

$$\cos\alpha \geq \frac{1}{3}$$

So $\frac{1}{3} \leq \cos\alpha \leq \frac{\sqrt{41}-3}{4}$

9. Ans (B)



From the diagram

$$\frac{kqQ_1}{\ell_1^2} = \frac{kqQ_2}{\ell_2^2}$$

$$\frac{\ell_1}{\ell_2} = \sqrt{\frac{Q_1}{Q_2}} = x$$

$$\ell_1 + \ell_2 = 2a \Rightarrow \quad \ell_1 = \frac{2ax}{x+1},$$

$$\ell_2 = \frac{2a}{x+1}$$

by cosine law

$$\cos\alpha = \frac{a^2 + \ell_1^2 - \ell_2^2}{2a\ell_1}$$

Put $\ell_1 \& \ell_2$

$$\cos\alpha = \frac{5x^2+2x-3}{4x(x+1)}$$

For $\frac{Q_1}{Q_2} = 4 \Rightarrow x = 2$

$$\cos\alpha = \frac{5\times4+2\times2-3}{4\times2(2+1)} = \frac{21}{24}$$

For $\frac{Q_1}{Q_2} = 3,\ x = \sqrt{3}$

$$\cos\alpha = \frac{5(3)+2\sqrt{3}-3}{4\sqrt{3}\left(\sqrt{3}-1\right)}$$

$$= \frac{12+2\sqrt{3}}{4\sqrt{3}\left(\sqrt{3}+1\right)} = \frac{2\sqrt{3}\left(2\sqrt{3}+1\right)}{4\sqrt{3}\left(\sqrt{3}+1\right)}$$

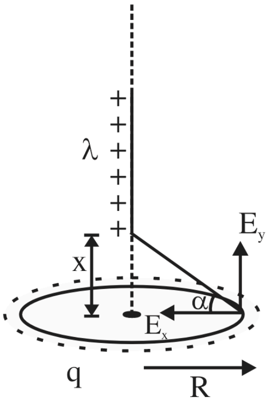
$$= \frac{2\sqrt{3}+1}{2\sqrt{3}+2} = 1 - \frac{1}{2\sqrt{3}+2}$$

$$= 1 - \frac{\left(\sqrt{3}-1\right)}{2\times2} = \frac{5-\sqrt{3}}{4}$$

PART-1 : PHYSICS

SECTION-II (ii)

7. Ans (3)



To lift the ring $qE_y = mg$

$$q\left(\frac{2k\lambda}{R}\cos\alpha\right) = mg$$

for maximum mass $\cos\alpha = 1$

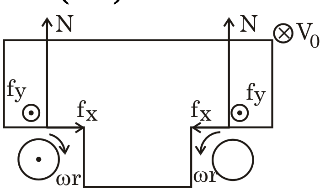
$$\frac{m_{max}}{m} = \frac{1}{\cos\alpha}$$

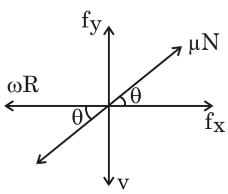
$$\frac{m_{max}}{m} = \frac{m}{\cos\alpha} = \frac{m\sqrt{R^2+x^2}}{R}$$

$$= \frac{m\sqrt{R^2+\frac{9R^2}{4}}}{R} = \frac{m\sqrt{13}}{2} = \frac{m\sqrt{(10+\alpha)}}{2}$$

$\alpha = 3$

8. Ans (28)





$$\tan\theta = \frac{v}{\omega R}$$

$$f_y = \mu N \sin\theta$$

$$f_y = \frac{\mu mg}{2}\cdot\frac{v}{\sqrt{v^2+(\omega R)^2}}$$

for steady velocity,

$$F = 2f_y = \frac{\mu mg}{\sqrt{2}} = \frac{0.2\times 20g}{\sqrt{2}} = \frac{40}{\sqrt{2}}N$$

9. Ans (2)

$$F = -p\frac{dE}{dx} = -(q\ell)\left(-\frac{2x}{L^2}\right)E_0$$

$$F = \frac{2q\ell}{L^2}E_0x = -ma$$

$$a = -\frac{2q\ell E_0}{mL^2}x$$

$$\omega = \sqrt{\frac{2q\ell E_0}{mL^2}}$$

$$v_0 = \sqrt{\frac{2q\ell E_0}{mL^2}}\sqrt{A^2-L^2}$$

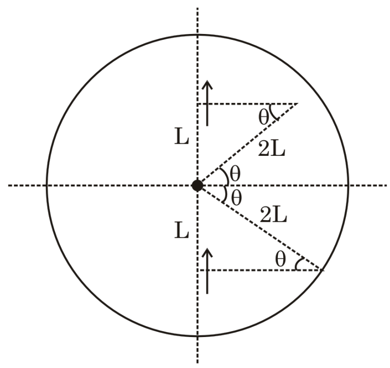
$$\frac{mv_0^2L^2}{2q\ell E_0} = A^2 - L^2 = \frac{6qE_0\ell L^2}{2q\ell E_0}$$

$$A^2 - L^2 = 3L^2$$

$$A = 2L$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$



$$t = \frac{2\theta}{\omega} = \frac{2\pi/6}{\sqrt{\frac{2q\ell E_0}{mL^2}}} = \frac{\pi/3}{\sqrt{\frac{2}{L^2}\frac{V_0^2}{6}}} = \frac{\pi/3}{\frac{v_0}{L}\cdot\frac{1}{\sqrt{3}}}$$

$$t = \frac{L}{v_0}\cdot\frac{\pi}{\sqrt{3}} = \frac{\pi}{\sqrt{3}}\frac{L}{v_0}$$

PART-2 : CHEMISTRY

SECTION-I (i)

4. Ans (A,B,D)

$[RhCl_6]^{3-}$, $[Co(H_2O)_6]^{3+}$, $[Fe(CN)_6]^{4-}$: Low spin complexes

$[RhCl_6]^{3-}$ ⇓ Central metal from 4d – series

$[Co(H_2O)_6]^{3+}$ ⇓ $\Delta_0 > P$

$[Fe(CN)_6]^{4-}$ ⇓ CN^- : strong ligand

5. **Ans (C,D)**

· Complex is low spin, diamagnetic.

· Central atom is d^2sp^3 hybridised.

· Complex has 2 Geometrical isomers, no one is optically active.

PART-2 : CHEMISTRY

SECTION-II (i)

1. **Ans (2.00)**

$$CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_3 \;\&\; CH_3-\overset{\overset{O}{\|}}{C}-\underset{\underset{CH}{|}}{CH}-CH_3$$

2. **Ans (3.00)**

$CH_3-CH_2-CH_2-CH_2-CHO$,

$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CHO \;,$$

$$CH_3-CH_2-\underset{\underset{CH_3}{|}}{CH}-CHO$$

3. **Ans (104.50)**

CFSE $= -460 - (-435) = -25$ Kcal/mol

$= -25 \times 4.18 = -104.5$ kJ/mol

4. **Ans (41.66 or 41.67)**

$M^{+2} = d^4 = t_{2g}{}^3\, e_g{}^1$

CFSE $= 3(-0.4\ \Delta) + 1(0.6\ \Delta)$

CFSE $= -0.6\ \Delta$

-25 Kcal/mol $= -0.6\ \Delta$

$\Delta = \dfrac{25}{0.6} = 41.666$ Kcal/mol

5. **Ans (2.00)**

$b = 6,\ a = 4$

PART-2 : CHEMISTRY

SECTION-II (ii)

7. **Ans (8)**

Phosphodiester linkage is present between 5' and 3' carbon atoms

8. **Ans (1270)**

$Cu_2O + H_2SO_4 \longrightarrow Cu\downarrow + CuSO_4 + H_2O$

0.02 mole   moles obtained $= 0.02$

$x = 0.02 \times 63.5 = 1.27$g

9. **Ans (433)**

$$[(H_2O)_5\overset{III}{Co}-O-O-\overset{III}{Co}(H_2O)_5]^{4+} \xrightarrow[\text{into}]{\text{oxidised}} [(H_2O)_5\overset{III}{Co}-O_2-\overset{III}{Co}(H_2O)_5]^{5+}$$

(A) peroxide ion (O_2^{2-}), unpaired e^-s = 0, $\mu = 0$

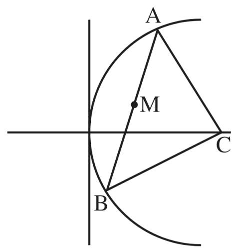
(B) superoxide ion (O_2^{-}), unpaired e^-s = 1, $y = \sqrt{3}$, $y = 1.732$ BM

$250y = 433$

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (B,D)**



$(2x-4)^2 = 4x$

$\Rightarrow x^2 - 5x + 4 = 0$

$\dfrac{x_1 + x_2}{2} = \dfrac{5}{2}$

x coordinate of m $= \dfrac{5}{2}$

$\therefore$ y coordinate of m $= 2(2.5) - 4 = 1$

Equation of perpendicular at M

$y - 1 = -\dfrac{1}{2}\left(x - \dfrac{5}{2}\right)$

Put $y = 0$ $x = \dfrac{9}{2}$

$\therefore C\left(\dfrac{9}{2}, 0\right)$

2. **Ans (A,B,D)**

$D_1 = 4(c^2 - ab)$

$D_2 = 8(ab - c^2)$

$D_1 > 0 \Rightarrow D_2 < 0$

$D_2 > 0 \Rightarrow D_1 < 0$

$D_1 = 0 \Rightarrow D_2 = 0$

3. Ans (A,C)

Let tanA = x, tanB = y, tanC = z

$\therefore$ xy + xz = 44,

yz + xy = 50

yz + zx = 54

$\therefore$ xy + yz + zx = 74

$\therefore$ yz = 30, xz = 24, xy = 20

$\therefore (xyz)^2 = 30.24.20$

$\therefore |xyz| = 120$

$\therefore |\tan(B+C)| = \left|\frac{y+z}{1-yz}\right| = \frac{|y+z|}{29}$

$= \frac{44}{29|x|} = \frac{44}{29.4} = \frac{11}{29}$

$|\tan(C+A)| = \left|\frac{x+z}{1-xz}\right| = \left|\frac{\frac{50}{z}}{1-24}\right|$

$= \frac{50}{23|z|} = \frac{50}{23.5} = \frac{10}{23}$

$\therefore |\tan(C+A)| > |\tan(B+C)|$

$\because |x| = 4, |y| = 5$ and $|z| = 6$

$\therefore |y| + |z| > |x|$ & $|x| + |y| > |z|$

4. Ans (A,C,D)

let m(h,k) be the mid-pint of the chord

$\therefore T = S_1, \frac{xh}{4} - yk = \frac{h^2}{4} - k^2$

Put (–2, 0) $\Rightarrow \frac{-h}{2} = \frac{h^2}{4} - k^2$ .....(i)

Now,

$k = mh + \frac{1}{m} \Rightarrow m^2h - mk + 1 = 0$

$\Rightarrow k = \frac{m^2h+1}{m}$ put in (i)

$\frac{-h}{2} = \frac{h^2}{4} - \frac{(m^2h+1)^2}{m^2}$, make a quadratic in h

and $D > 0 \Rightarrow m^2 < \frac{4}{7} \Rightarrow m \in \left(-\frac{2}{\sqrt{7}}, \frac{2}{\sqrt{7}}\right)$

5. Ans (C)

$yz(x^2 + 4x + 4) = 81$

$yz\,(x+2)^2 = 81$

$\frac{(x+2)+(x+2)+y+z}{4} \geqslant 1\left((x+2)^2yz\right)^{\frac{1}{4}}$

$\Rightarrow \frac{2x+y+z+4}{4} \geqslant 3 \Rightarrow 2x+y+z \geqslant 8$

if x + 2 = y = z = 3

6. Ans (B,D)

$2y + 4(y-a)^2 = 4$

$\Rightarrow 2y^2 + (1-4a)y + 2a^2 - 2 = 0$

$D \geq 0 \Rightarrow a \leqslant \frac{m}{8}$

$\because y = x^2$ at least one $y \geq 0 \therefore a \in \left[-1, \frac{17}{8}\right]$

$\therefore A \cap B \neq \phi\ \forall a \in \left[-1, \frac{17}{8}\right]$

PART-3 : MATHEMATICS

SECTION-I (ii)

7. Ans (D)

$a = \frac{10^{20}-1}{9}, b = \frac{4}{9}\left(10^{20}-1\right),$

$c = \frac{7}{9}\left(10^{10}-1\right), d = 10^{10}-1$

$\frac{a}{d} = \frac{10^{10}+1}{9}, \frac{c^2}{d} = \frac{49}{81}\left(10^{10}-1\right)$

$\frac{441a - 81c^2}{14d} = \frac{9\left(49a - 9c^2\right)}{14d}$

$= \frac{9}{14}\left(49\frac{\left(10^{10}+1\right)}{9} - 9.\frac{49\left(10^{10}-1\right)}{81}\right)$

$= \frac{49}{14}\left(10^{10}+1-\left(10^{10}-1\right)\right) = \frac{7}{2}.2 = 7$

8. Ans (B)

$\frac{d}{b} = \frac{9}{4\left(10^{10}+1\right)}$

$\frac{4d}{b}(d+2) = \frac{4.9}{4\left(10^{10}+1\right)}\left(10^{10}+1\right) = 9$

9. Ans (B)

Let $S = \cos^4\frac{5\pi}{14} + \cos^4\frac{\pi}{14} + \cos^4\frac{3\pi}{14}$

$\therefore \cos^4\theta = \frac{3 + 4\cos 2\theta + \cos 4\theta}{8}$

$\therefore S = \frac{9}{8} + \frac{1}{2}\left(\cos\frac{2\pi}{14} + \cos\frac{6\pi}{14} + \cos\frac{10\pi}{14}\right)$

$+\frac{1}{8}\left(\cos\frac{4\pi}{14} + \cos\frac{12\pi}{14} + \cos\frac{20\pi}{14}\right)$

$\Rightarrow \frac{9}{8} + \frac{1}{2}\left(\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7}\right)$

$+\frac{1}{8}\left(\cos\frac{2\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{10\pi}{7}\right)$

$= \frac{9}{8} + \frac{1}{4} - \frac{1}{16} = \frac{21}{16}$

$\therefore p = 21, q = 16$

10. **Ans (C)**

Let $S = \cos^4\frac{5\pi}{14} + \cos^4\frac{\pi}{14} + \cos^4\frac{3\pi}{14}$

$\therefore \cos^4\theta = \frac{3 + 4\cos 2\theta + \cos 4\theta}{8}$

$\therefore S = \frac{9}{8} + \frac{1}{2}\left(\cos\frac{2\pi}{14} + \cos\frac{6\pi}{14} + \cos\frac{10\pi}{14}\right)$

$+\frac{1}{8}\left(\cos\frac{4\pi}{14} + \cos\frac{12\pi}{14} + \cos\frac{20\pi}{14}\right)$

$\Rightarrow \frac{9}{8} + \frac{1}{2}\left(\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7}\right)$

$+\frac{1}{8}\left(\cos\frac{2\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{10\pi}{7}\right)$

$= \frac{9}{8} + \frac{1}{4} - \frac{1}{16} = \frac{21}{16}$

$\therefore p = 21, q = 16$

PART-3 : MATHEMATICS

SECTION-II (i)

1. **Ans (0.13)**

$\sin\alpha = \frac{(1 + 2\sin\beta) \pm \sqrt{-3(2\sin\beta - 1)^2}}{2}$

$\therefore \sin\beta = \frac{1}{2}$ an $\sin\alpha = 1$

$\therefore \beta = \frac{\pi}{6}, \alpha = \frac{\pi}{2}$

$\sin 2x + \cos 2y = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3} - 2}{2}$

$\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$

2. **Ans (1.00)**

$\sin\alpha = \frac{(1 + 2\sin\beta) \pm \sqrt{-3(2\sin\beta - 1)^2}}{2}$

$\therefore \sin\beta = \frac{1}{2}$ an $\sin\alpha = 1$

$\therefore \beta = \frac{\pi}{6}, \alpha = \frac{\pi}{2}$

$\sin 2x + \cos 2y = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3} - 2}{2}$

$\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$

3. **Ans (2.00)**

$(p + q + r)x^2 + 2(pq + qr + rp)x + 3pqr = 0$

$\frac{D}{4} = (pq)^2 + (qr)^2 + (rp)^2 - p^2qr - qp^2r - qrp^2$

$= \frac{1}{2}\left\{(pq - qr)^2 + (qr - rp)^2 + (rp - pq)^2\right\} > 0$

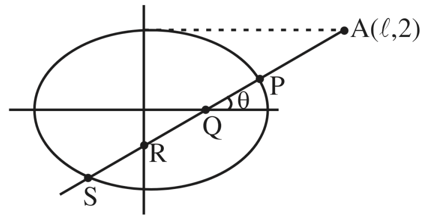
4. **Ans (0.00)**

$(p + q + r)x^2 + 2(pq + qr + rp)x + 3pqr = 0$

$\frac{D}{4} = (pq)^2 + (qr)^2 + (rp)^2 - p^2qr - qp^2r - qrp^2$

$= \frac{1}{2}\left\{(pq - qr)^2 + (qr - rp)^2 + (rp - pq)^2\right\} > 0$

5. **Ans (6.00)**



$\frac{x - \ell}{\cos\theta} = \frac{y - 2}{\sin\theta} = r$

Put in ellipse and get product of roots

$\therefore AP.AS = \frac{4\ell^2}{4 + 5\sin^2\theta}$

$\because AP.AS = AQ.AR = \frac{2\ell}{\sin\theta.\cos\theta}$

$\Rightarrow \ell = \frac{13 - 5\cos 2\theta}{2\sin 2\theta}$

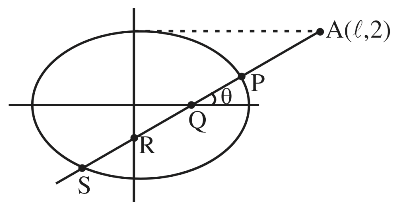
$\Rightarrow 2\ell\sin 2\theta + 5\cos 2\theta = 13$

$\therefore 13 \leqslant \sqrt{4\ell^2 + 25} \Rightarrow \ell \geqslant 6$

Slope of tangent (m)is maximum if $\ell = 6$

$\therefore\ m = \frac{24}{27} = 0.88$ or 0.89

6. **Ans (0.88 or 0.89)**



$\frac{x - \ell}{\cos\theta} = \frac{y - 2}{\sin\theta} = r$

Put in ellipse and get product of roots

$\therefore AP.AS = \frac{4\ell^2}{4 + 5\sin^2\theta}$

$\because AP.AS = AQ.AR = \frac{4\ell}{\sin\theta.\cos\theta}$

$\Rightarrow \ell = \frac{13 - 5\cos 2\theta}{2\sin 2\theta}$

$\Rightarrow 2\ell\sin 2\theta + 5\cos 2\theta = 13$

$\therefore 13 \leqslant \sqrt{4\ell^2 + 25} \Rightarrow \ell \geqslant 6$

Slope of tangent (m)is maximum if $\ell = 6$

$\therefore\ m = \frac{24}{27} = 0.88$ or 0.89

PART-3 : MATHEMATICS

SECTION-II (ii)

7. Ans (5)

$\alpha+\beta = a,\ \alpha\beta = b,$

$$\alpha + \beta + \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{4}{b},\ \left(\frac{(\alpha^2+\beta^2)^2}{\alpha\beta}\right) = \frac{4}{b}$$

Solving we get $|a| = 2,\ b = 3\ (\because\ b > 1)$

$\therefore\ |a| + b = 5$

8. Ans (0)

$a + b^2 = p,\quad ab^2 = 1$

$b + a^2 = q,\quad ba^2 = 8$

$\therefore\ (ab)^3 = 8 \quad \Rightarrow \quad ab = 2 \quad \Rightarrow \quad b = \frac{1}{2},\ a = 4$

$\therefore\ p = 4 + \frac{1}{4} = \frac{17}{4},\ q = \frac{1}{2} + 16 = \frac{33}{2}$

$D = p^2 - 4q = \frac{289}{16} - 66 < 0$

$\therefore$ Ans. 0

9. Ans (4)

$y = mx + \frac{2}{m} \ \Rightarrow m^2x - my + 2 = 0$

centre of circle is $(6,0)$, $r = \sqrt{32}$

$$\left|\frac{6m^2+2}{\sqrt{1+m^4}}\right| = \sqrt{32} \Rightarrow 4\left(3m^2+1\right)^2 = 32\left(1+m^4\right)$$

$\Rightarrow m = \pm 1$

$\therefore$ perpendicular tangent meet on directrix

Equation of directrix is $x = -2 \quad \therefore\ P(-2,0)$

$\therefore$ Ans. 4