FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
PART TEST – II
PAPER –1
TEST DATE: 08-12-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. D

Sol. $\int_B^C \vec{B}\cdot d\vec{\ell} = (\mu_0 I_0)\frac{15}{360} + (\mu_0 2I_0)\frac{15}{360} + (\mu_0 3I_0)\frac{15}{360} = \frac{\mu_0 I}{4}$

2. C

Sol. $\frac{1}{2}mv_r^2 = \frac{kQq}{2R} + \frac{m\omega^2R^2}{2}$

$\frac{1}{2}\times10^{-3}\times v_r^2 = \frac{9\times10^9\times10^{-6}\times10^{-6}\times\frac{1}{3}}{2\times1} + \frac{10^{-3}}{2}\times1\times1^2$

$v_r^2 = 3+1$

$\Rightarrow v_r = 2$ m/s

$v_t = r\omega = 1$ m/s

$v = \sqrt{2^2+1^2} = \sqrt{5}$

3. B

Sol. $\frac{E}{V} = \frac{12L}{2\times5L}$

$\frac{E}{V} = \frac{5}{6}$

$r = \left(\frac{E}{V}-1\right)R = 2\Omega$

When S_2 is open

$6V = ik\frac{L}{2}$

E =ikL = 12 V

4. B

Sol. As $TV^{\gamma-1} = \text{constant}$

$\Rightarrow \quad T \propto V^{1-\gamma}$

$\Rightarrow \dfrac{dT}{T} = (1-\gamma)\dfrac{dV}{V}$

$\Rightarrow \dfrac{dV}{dT} = \dfrac{-V}{(\gamma-1)T} = \tan 45°$

$\gamma = 1 + \dfrac{V_0}{T_0}$

Bulk modulus $K = \dfrac{dP}{-dV/V} = \gamma P = \left(1+\dfrac{V_0}{T_0}\right)P$

$= \left(1+\dfrac{V_0}{T_0}\right)\left(\dfrac{2RT_0}{V_0}\right) = 2R\left(1+\dfrac{T_0}{V_0}\right)$

5. A, C

Sol. $U_f = -q2\sqrt{2}a\sqrt{2}E$

$U_i = 0$

$W_E = U_i - U_f$

$\dfrac{1}{2}\cdot\dfrac{7}{5}mR^2\omega^2 = 4qaE$

$R^2\omega^2 = \dfrac{40qaE}{7m}$

$V = \sqrt{\dfrac{40qaE}{7m}}$

Friction force will be static and varying, when torque is equal to zero and friction will also be zero.

6. B, D

Sol. $(A\sigma T^4)\times 60 = m\times c\times(21-20)$

$A\sigma(2T)^4\times 60 = m\times c\times(T-20)$

$T = 36°C$

7. B, C, D

Sol. Across capacitor current will lead the voltage by $\pi/2$

$i = 20\sqrt{2}\sin\left(\omega t + \dfrac{3\pi}{4}\right)$

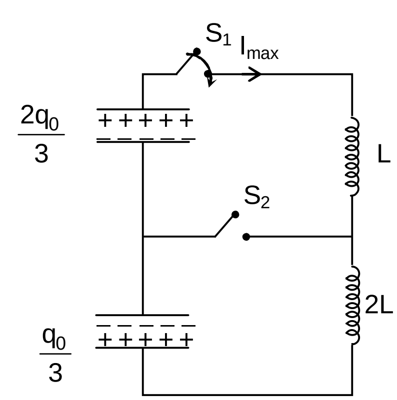
$V_L^2 + V_R^2 = V^2$

$\Rightarrow V_R = 120$ and $V_L = 160$

8. A

Sol. A equilibrium current in circuit will be maximum

By conservation of energy



$\dfrac{q_0^2}{4C} = \dfrac{1}{2}\times\dfrac{4q_0^2}{18C} + \dfrac{1}{2}\dfrac{q_0^2}{9C} + \dfrac{1}{2}(L+2L)I_0^2$

$I_0 = \dfrac{q_0}{3\sqrt{2LC}}$

Now, switch S_2 is connected

For loop -1

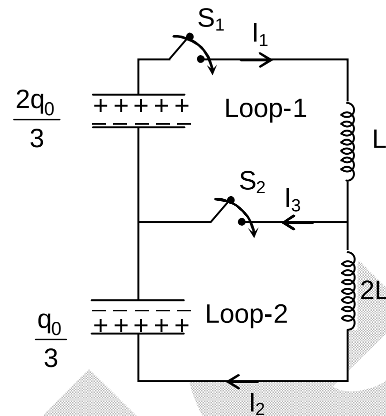
$\left(\dfrac{1}{2}\right)\dfrac{4q_0^2}{9\times 2C} + \left(\dfrac{1}{2}\right)\dfrac{Lq_0^2}{18\times 2C} = \dfrac{1}{2}L\left(I_1^2\right)$

$$I_1 = \frac{q_0}{3}\sqrt{\frac{5}{2LC}}$$

For loop -2

$$\left(\frac{1}{2}\right)\frac{q_0^2}{9C} + \left(\frac{1}{2}\right)\frac{2Lq_0^2}{18LC} = \frac{1}{2}2L\left(I_2^2\right)$$

$$I_2 = \frac{q_0}{3\sqrt{LC}}$$



Charge in capacitor . 2C

$$-\frac{2q_0}{3} = \frac{q_0\sqrt{5}}{3}\sin(\omega t + \phi_1)$$

$$\sin\phi_1 = \frac{-2}{\sqrt{5}}, \quad \cos\phi_1 = \frac{1}{\sqrt{5}}$$

Charge in capacitor 'C'

$$\frac{q_0}{3} = \frac{q_0\sqrt{2}}{3}\sin(\omega t + \phi_2)$$

$$\sin\phi_2 = \frac{1}{\sqrt{2}}, \quad \cos\phi_2 = \frac{1}{\sqrt{2}}$$

$$I_3 = I_1 - I_2$$

$$I_3^2 = (I_0\sqrt{5})^2 + (I_0\sqrt{2})^2 - 2I_0^2\sqrt{10}\cos(\phi_1 - \phi_2)$$

$$I_3 = 3I_0 = \frac{q_0}{\sqrt{2LC}}$$

9. C

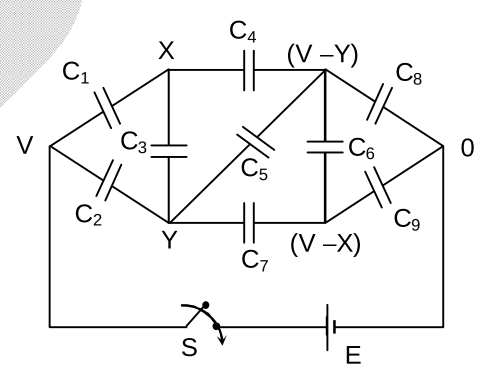
Sol. (V –X) = C (X – V + Y) + C (X – Y)

V – X = X – Y V + Y+ X – Y

$$\Rightarrow X = \frac{2V}{3}$$

C(V – Y) + (X – Y)C = (Y – V + Y)C + (Y –V + X)C

$$\Rightarrow Y = \frac{3V}{5}$$



$$\frac{1}{2}CV_5^2 = 22\mu J$$

$$V_5^2 = \frac{44}{11}\mu J$$

$$V_5 = 2V$$

(P) $\frac{3V}{5} - V + \frac{3V}{5} = 2$ volt

$$\frac{V}{5} = 2V$$

V = 10V

(Q) $\left(V - \frac{3V}{5}\right)C = \frac{2CV}{5} = \frac{2\times 11\times 10}{5} = 44\ \mu C$

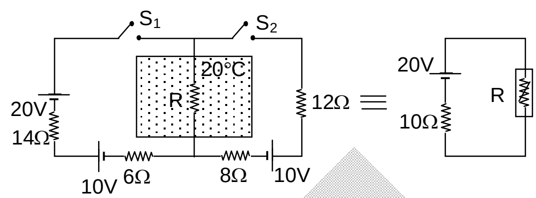
(R) $q = 11\times 4 + 11\times\frac{10}{3} = \frac{242}{3}\mu C$

(S) $\frac{242}{3}\mu C \times 10 = \frac{2420}{3}\mu J$

10. B

Sol. Power is maximum when R is 10 Ω

For(Q): R is 10Ω when drop in temperature is 20°C so temperature is 30°C



For(R): $-\frac{d\theta}{dt} = \frac{\ell n3}{100}(\theta - 20°)$

For(S): $\int_{50°}^{30°} -\frac{d\theta}{(\theta - 20°)} = \frac{\ell n3}{100}\int_0^t dt$

$\Rightarrow$ t = 100 sec

By applying loop law we can calculate current when R is 10Ω

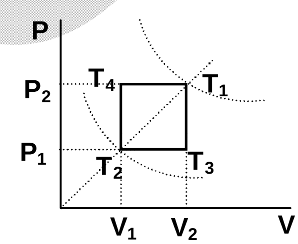
11. D

Sol. (P) $\frac{(100 - T)KA}{2} = \frac{(T - 0)2KA}{1}$

$\Rightarrow T = \frac{100}{5} = 20°C$

(Q) $P = \frac{nN_a m V_{rms}^2}{3A\ell} = 24 \times 10^4$ Pa

(R) $W = P_2(V_2 - V_1) - P_1(V_2 - V_1)$



$nRT_2 + nRT_1 - P_2V_1 - P_1V_2$

$\frac{P_1}{V_1} = \frac{P_2}{V_2}$

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

$W = nRT_2 + nRT_1 - \frac{nRT_2}{V_2}V_1 - \frac{nRT_1}{V_1}V_2 = nR(\sqrt{T_2} - \sqrt{T_1})^2$

$W = 1 \times \frac{25}{3}(20 - 17)^2 = 75$ J

(S) $\frac{2K \times 4R \times 2R(100 - T)}{R} = \frac{K \times 4 \times 2R \times 4R(T - 0)}{2R}$

T = 50°C

SECTION – B

12. 200

Sol. $\lambda_m T$ = constant

$\ell n\lambda_m + \ell nT = C$

$\frac{d\lambda_m}{\lambda_m} + \frac{dT}{T} = 0$

$\frac{d\lambda_m}{\lambda_m} = \frac{dT}{T}$

Now, $\frac{d\lambda_m}{\lambda_m} = -\frac{1}{2}\% = -\frac{1}{200}$ (–ve sign indicates decrease)

dT = 1 (given)

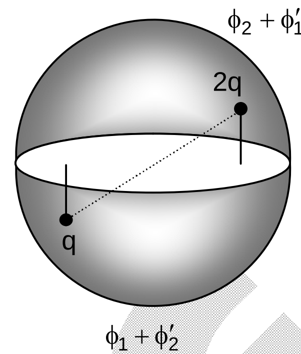
T = 200 K

13. 5

Sol. $\phi_1 = \frac{q}{3\varepsilon_0}, \quad \phi_1' = \frac{2q}{3\varepsilon_0}$

$\phi_2 = \frac{2q}{3\varepsilon_0}, \quad \phi_2' = \frac{4q}{3\varepsilon_0}$

$\phi = \phi_1 + \phi_2' = \frac{q}{3\varepsilon_0} + \frac{4q}{3\varepsilon_0} = \frac{5q}{3\varepsilon_0}$



14. 2

Sol. $E = -\frac{dV}{dx} = \frac{2\times10^4}{10\times10^{-2}} = 2\times10^5$

$2\times a = 20\times10^{-6}\times2\times10^5$

$\Rightarrow a = 2\ m/s^2$

15. 87

Sol. $2 \times 4200\ (T - 27) = 1000 \times 600 - 160 \times 600$

$14T - 14 \times 27 = 1000 - 160$

$14T = 840 + 14 \times 27$

$T = 60 + 27 = 87°$

16. 300

Sol. $\frac{dV}{dT} = \frac{C}{P}$

$PdV = CdT$

$\int PdV = C(T_2 - T_1)$

$\Rightarrow C(600 - 300) = 300C$

17. 50

Sol. The speed of particle as (x, y) $\Rightarrow$ 5 m/s

$\frac{1}{2}mv^2 = qE_0x$

$\frac{1}{2}\frac{mv^2}{qE_0} = x$

$x = \frac{1}{2}\frac{25}{\alpha E}$

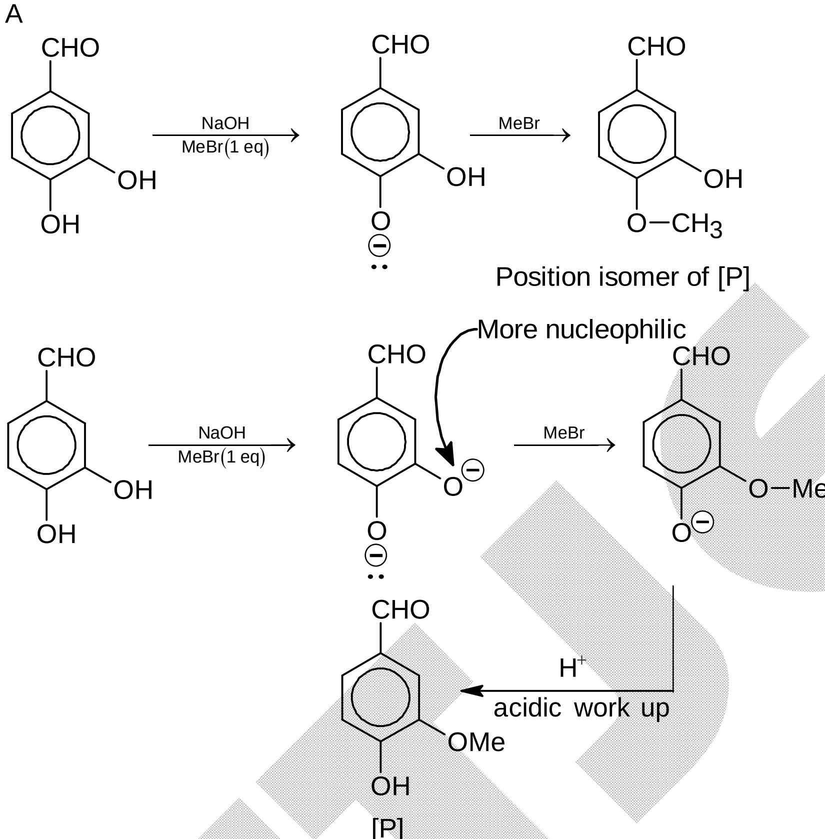
Chemistry

PART – II

SECTION – A

18. A

Sol.

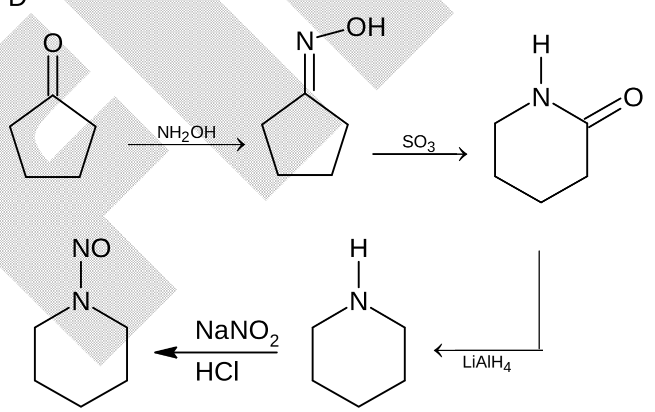


19. C

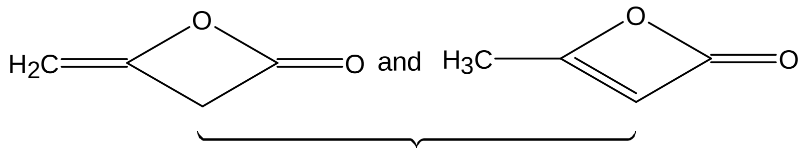
Sol. Cumulenes with even number of double bond do not show geometrical isomerism.

20. D

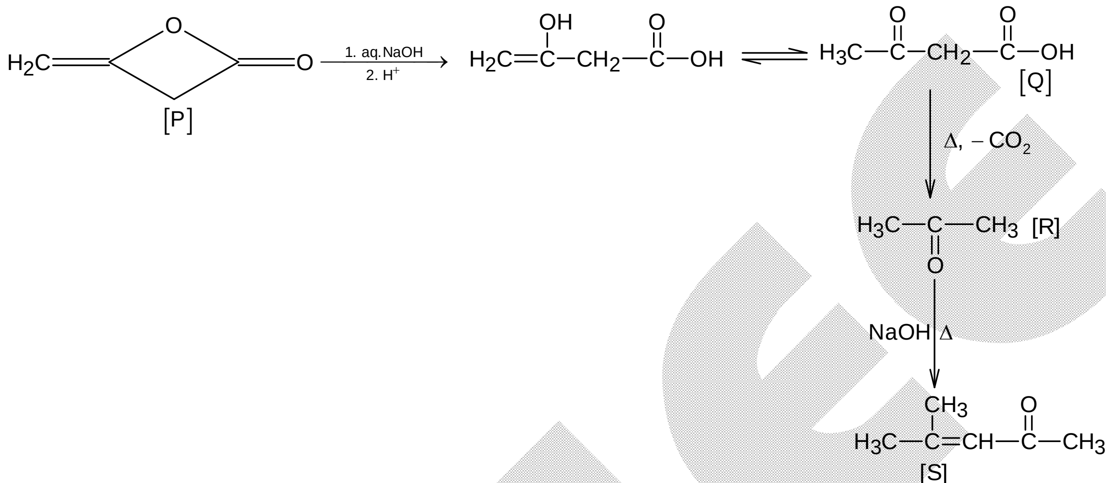
Sol.

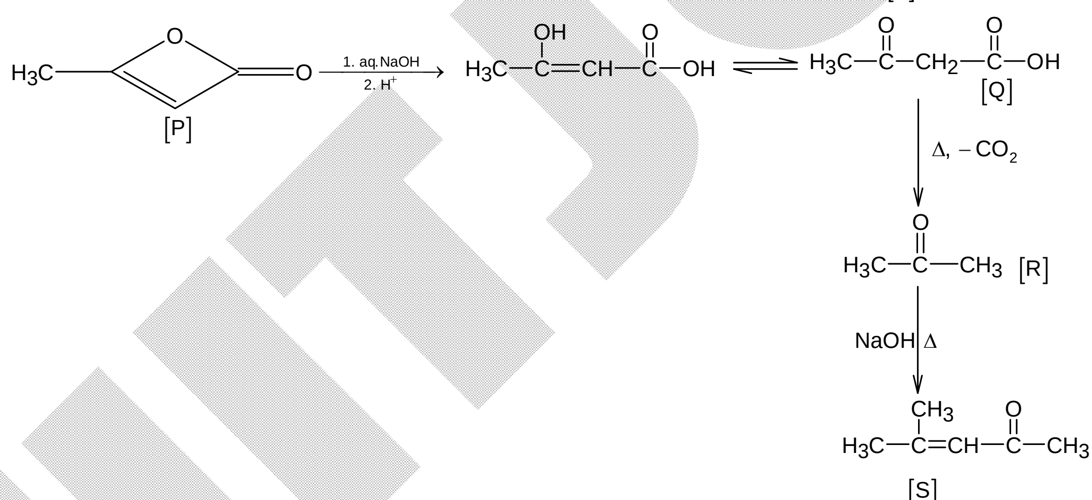


21. C

Sol. [P] can be 

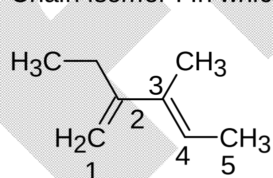
(Two structural isomers)



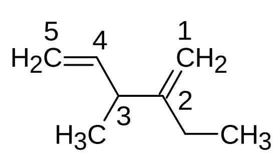


22. A, B, C

Sol. Option (D) has longest chain of 5C-atom which is not the chain isomer
Chain isomer : In which only main chain will differ



(Given compound)



(D)

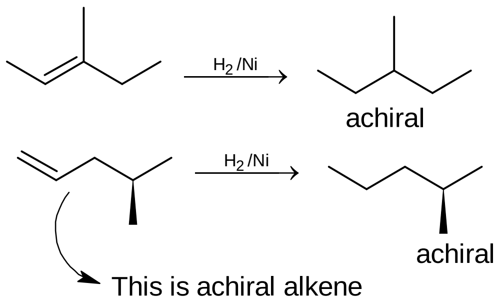
23. B, C, D

Sol. Neoprene is a polymer of chloroprene.

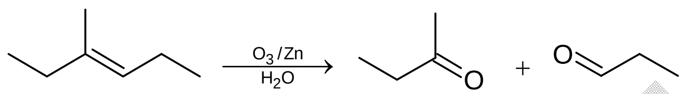
24. A, D

Sol. Alkene [K] has $8\alpha-$Hydrogens.

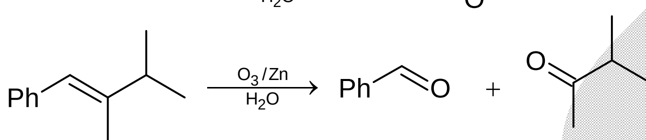
Alkene [L] has $2\alpha-$Hydrogens.



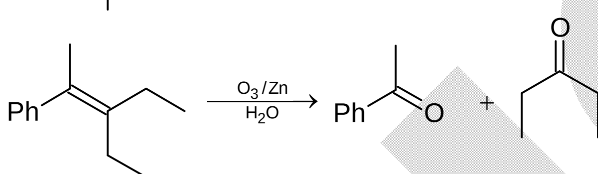
25. A
Sol. (P)



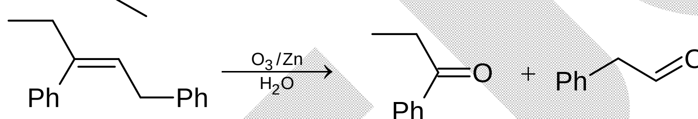
(Q)



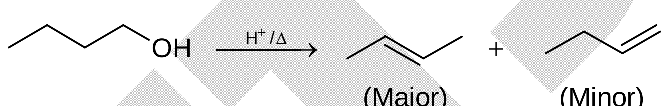
(R)



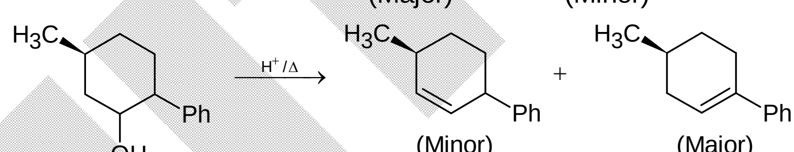
(S)



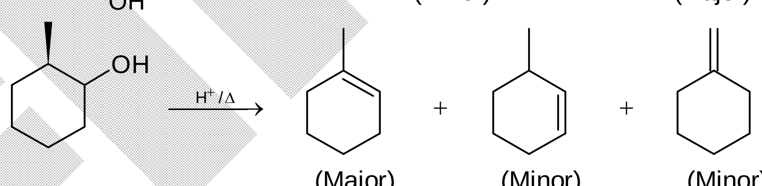
26. C
Sol. (P)



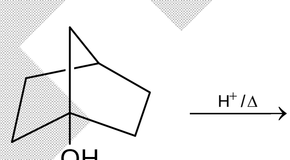
(Q)



(R)

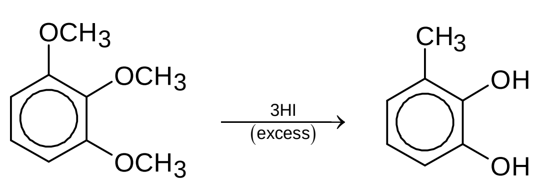


(S)

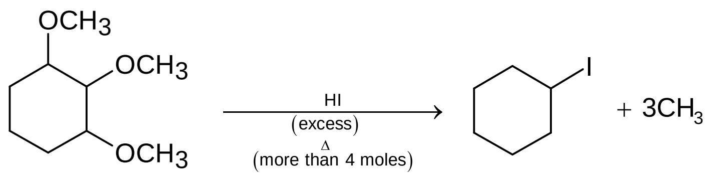


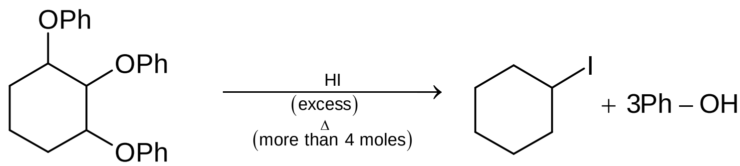
cannot undergo dehydration as bridge head C-atom can never be sp^2-hybridised.

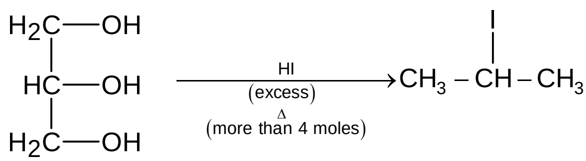
27. C
Sol. (P)



$+ 3CH_3I$

(Q)


(R)


(S)


28. B

Sol. Sucrose $\xrightarrow{\text{Hydrolysis}}$ α – D – glucose + β-D-fructose

Lactose $\xrightarrow{\text{Hydrolysis}}$ β – D – glucose + β-D-galactose

Amylose $\xrightarrow{\text{Hydrolysis}}$ α – D – glucose + α-D-glucose+.....

Mannose cannot be hydrolysed.
Lactose and mannose can reduce Fehling's solution due to presence of hemiacetal and aldehyde group.

SECTION – B

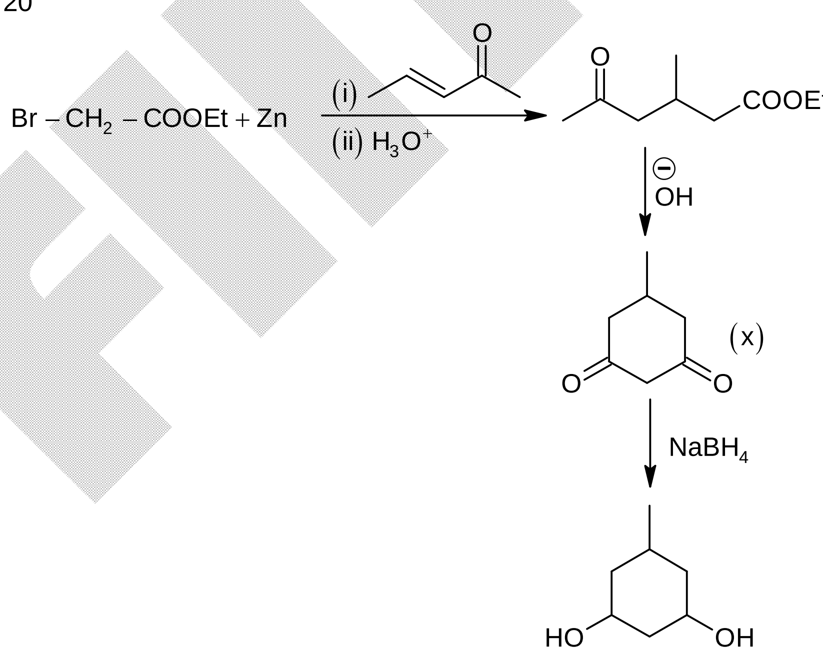
29. 7

Sol. 'A' is $CH_3 - CH = C = CH - CH_3$, optically active with minimum weight.

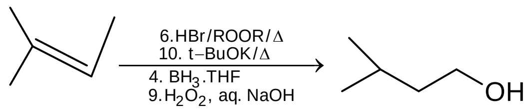
$\therefore x = 5, y = 2$.

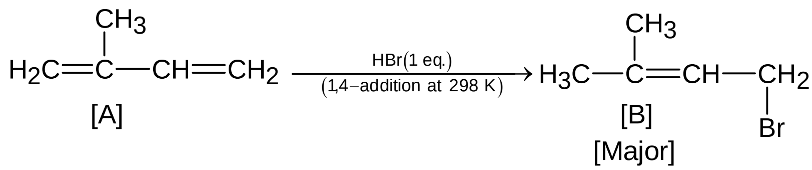
30. 20

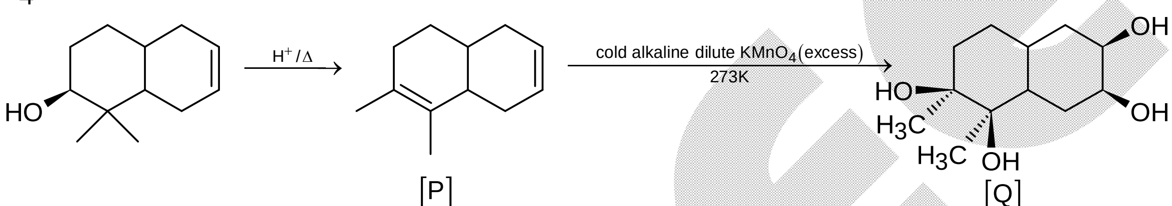
Sol.

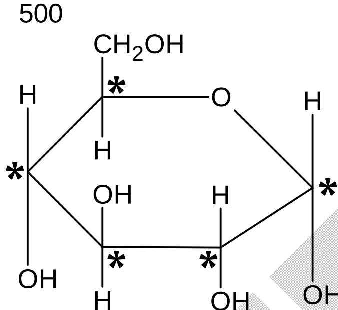


Number of stereoisomers = 4.

31. 29
Sol.


32. 8
Sol.


33. 4
Sol.


34. 500
Sol.


Mathematics

PART – III

SECTION – A

35. C

Sol. Let f(x) = 3 sin x + 4 cosec x = $3\sin x+\dfrac{4}{\sin x}$

Let x_1, x_2 be two values such that $\dfrac{\pi}{2}\ge x_2>x_1>0$

Then, $f(x_2)-f(x_1)=3(\sin x_2-\sin x_1)+4\left(\dfrac{1}{\sin x_2}-\dfrac{1}{\sin x_1}\right)$

$=(\sin x_2-\sin x_1)\left(3-\dfrac{4}{\sin x_2\sin x_1}\right)$

Since, $\dfrac{\pi}{2}\ge x_2>x_1>0$, $\sin x_2>\sin x_1$

Also, $\dfrac{4}{\sin x_2\sin x_1}\ge 4$

$\therefore$ $f(x_2)-f(x_1)<0$ or $f(x_2)<f(x_1)$ or

f(x) is decreasing, and minima of f(x) will be achieved for $x=\dfrac{\pi}{2}$

Similarly, let g(x) = 5 $\sin^2$ x + 6 cosec^2 x = $5\sin^2 x+\dfrac{6}{\sin^2 x}$

Minima for g(x) will be achieved for $x=\dfrac{\pi}{2}$

Minimum value = 3 + 4 + 5 + 6 = 18

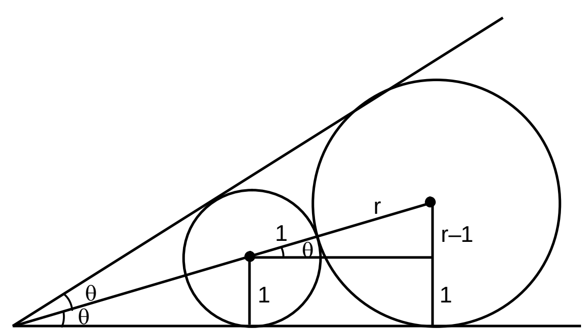
36. D

Sol. Let angle between the tangents = 2θ

$\tan(2\theta)=\left|\dfrac{2\sqrt{h^2-ab}}{a+b}\right|=\left|\dfrac{2\sqrt{15}-3}{1+3}\right|=\sqrt{3}$

$\therefore$ $\tan\theta=\dfrac{1}{\sqrt{3}}\Rightarrow\ \sin\theta=\dfrac{1}{2}$

Now, from the figure, $\sin\theta=\dfrac{r-1}{r+1}\Rightarrow r=3$



37. A

Sol. AB – AC = 2, AB + AC= 12

$\Rightarrow$ AB = 7, AC = 5

$\therefore$ $\Delta=\sqrt{10(3)(5)(2)}=10\sqrt{3}$

$r=\dfrac{\Delta}{s}=\sqrt{3}$

$$\tan\left(\frac{B}{2}\right) = \frac{r}{s-b} = \frac{\sqrt{3}}{5}$$

$$\therefore \quad \tan B = \frac{\frac{2\sqrt{3}}{5}}{1-\frac{3}{25}} = \frac{5\sqrt{3}}{11}.$$

38. A

Sol. ΔOPQ is right angled at O

Let equation of OP be y = mx

P lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\therefore \quad OP^2 = \frac{(1+m^2)(a^2b^2)}{b^2 - a^2m^2}$$

Similarly, $OQ^2 = \frac{(1+m^2)a^2b^2}{b^2m^2 - a^2}$

$$\Rightarrow \quad \frac{1}{OP^2} + \frac{1}{OQ^2} = \frac{1}{a^2} - \frac{1}{b^2} \Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{2} + \frac{1}{3} \ \& \ 1 + \frac{b^2}{a^2} = 6$$

39. A, B, D

Sol. Using trigonometric identities, $16\cos^4\theta - 8\cos^3\theta - 12\cos^2\theta + 4\cos\theta + 1 = 0$

Simplifies to $\cos\theta - \cos(2\theta) + \cos(3\theta) - \cos(4\theta) = 1/2$ … (i)

Using $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$, we get $8\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{5\theta}{2}\right)\cos\theta = 1$

Now, multiplying both sides of equation (i) by 2 $\sin\theta$ (where $\sin\theta \neq 0$), and using 2 sin A cos B = sin(A + B) + sin(A – B), we get $\sin(5\theta) = \sin(4\theta)$

$$\Rightarrow \theta = \frac{\pi}{9}, \frac{3\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

40. A, D

Sol. The diagonals intersect at M(1, 2). Now, MD = MB = 10

Slope of BD = 4/3 = tan $\theta \Rightarrow \cos\theta = \frac{3}{5}$ & $\sin\theta = \frac{4}{5}$

Slope of $AC = \frac{5}{12} = \tan\alpha \Rightarrow \cos\alpha = \frac{12}{13}$ & $\sin\alpha = \frac{5}{13}$

If BD & AC intersect at acute angle β, Then $\tan\beta = \left|\frac{\frac{5}{12} - \frac{4}{3}}{1 + \frac{5}{12}\cdot\frac{4}{3}}\right| = \frac{33}{56} \Rightarrow \sin\beta = \frac{33}{65}$

Now, $ar(\Delta MDC) = \frac{1}{4}ar(ABCD) = 66 = \frac{1}{2}$. MD. MC sin β

$\Rightarrow$ MC = 26 = MA

$\therefore$ co-ordinates of vertices of ABCD will be

$$\left(1\pm 10\left(\frac{3}{5}\right), 2\pm 10\left(\frac{4}{5}\right)\right), \left(1\pm 26\left(\frac{12}{13}\right), 2\pm 26\left(\frac{5}{13}\right)\right)$$
i.e., (7, 10), (–5, – 6), (25, 12), (–23, – 8)

41. B, D

Sol. Major and minor axis of the reflected ellipse will be the reflections of x-axis & y-axis respectively across y = 2x
Let reflection of (1, 0) across y = 2x be (h, k).
Then $\frac{h-1}{2} = \frac{k-0}{-1} = \frac{-2(2-0+0)}{2^2+(-1)^2} \Rightarrow h = -\frac{3}{5}, k = \frac{4}{5}$
$\therefore$ Equation of major axis of reflected ellipse is 3y + 4x = 0
Similarly, equation of minor axis of reflected ellipse is 3x – 4y = 0
Required curve $\frac{(3x-4y)^2}{5^2\times 9} + \frac{(3y+4x)^2}{5^2\times 4} = 1$
$\Rightarrow 36x^2 + 29y^2 + 2(12)xy = 180$

42. D

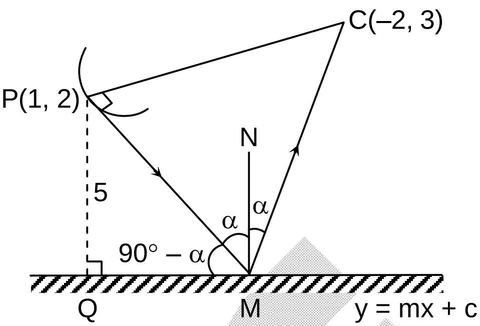
Sol. Let $P(\alpha, \beta) \equiv P(4t, t)$
A(1, 1), B(–11, –4), C(5, –2) are vertices of ΔABC
(P) $(4t-1)^2 + (t-1)^2 < (4t+11)^2 + (t+4)^2$ and $(4t-1)^2 + (t-1)^2 < (4t-5)^2 + (t+2)^2$
$\Rightarrow t \in \left(\frac{-135}{106}, \frac{27}{26}\right)$
(Q) $\left|\frac{3(4t)+4(t)-7}{\sqrt{3^2+4^2}}\right| < \left|\frac{5(4t)-12(t)+7}{\sqrt{5^2+12^2}}\right|$
$\Rightarrow \left(\frac{7}{31}, \frac{3}{4}\right)$
(R) ar(APB) + ar(BPC) + ar(CPA) = ar(ABC) = 28
For ar(APB) + ar(BPC) > ar(APC), we must have ar(APC) < 14
$\Rightarrow t \in \left(\frac{-21}{16}, \frac{35}{16}\right)$
(S) P and A must be on the same side of BC
$\Rightarrow$ (4t – 8t – 21)(1 – 8 – 21) > 0
P and B must be on the same side of AC
$\Rightarrow$ (12t + 4t – 7)(–33 – 16 – 7) > 0
P and C must be on the same side of AB
$\Rightarrow$ (20t – 12t + 7)(25 + 24 + 7) > 0
$\Rightarrow t \in \left(-\frac{7}{8}, \frac{7}{16}\right)$

43. B

Sol. Equation of normal is $y = mx - 2m - m^3$
It passes through (40, 40)
$\Rightarrow 40 = 38m - m^3 \Rightarrow m_1 + m_2 + m_3 = 0$
Let circle be $x^2 + y^2 + 2gx + 2fy + c = 0$
It passes through $(m^2, -2m)$
$\Rightarrow m^4 + 4m^2 + 2gm^2 - 4fm + c = 0$
$\therefore m_1 + m_2 + m_3 + m_4 = 0 \Rightarrow m_4 = 0$. Fourth point is (0, 0)
Using Vieta's relations in both cubic and quadratic equation in m, we get required equation of circle as $x^2 + y^2 - 42x - 20y = 0$

44. C

Sol. Radius of circle = $\sqrt{2^2+3^2-3}=\sqrt{10}$



$5=\left|\dfrac{m-2+c}{\sqrt{1+m^2}}\right|$

$\tan(2\alpha)=\dfrac{PC}{PM}=\dfrac{\sqrt{10}}{PM}$

$\sin(90°-\alpha)=\dfrac{5}{PM}\Rightarrow\dfrac{\tan(2\alpha)}{\cos\alpha}=\dfrac{\sqrt{10}}{5}$

$\Rightarrow\sin\alpha=\dfrac{\sqrt{18}-\sqrt{10}}{4}\Rightarrow\tan\alpha=\sqrt{\dfrac{\sqrt{5}-2}{3}}$

Tangent at (1, 2) on circle is $3x-y-1=0$

Slope of PM = 3

$\tan(\angle PMQ)=\left|\dfrac{m-3}{1+3m}\right|\Rightarrow\cot\alpha=\left|\dfrac{m-3}{1+3m}\right|$

$m=\dfrac{3+\cot\alpha}{1-3\cot\alpha}$ or $\dfrac{3-\cot\alpha}{1+3\cot\alpha}$

$m=\tan\left(\tan^{-1}3+\dfrac{\pi}{2}-\alpha\right)$ or $\tan\left(\tan^{-1}3-\dfrac{\pi}{2}+\alpha\right)$

$m=\cot(\alpha-\tan^{-1}3)$ or $-\cot(\alpha+\tan^{-1}3)$

Now, $c=2-m\pm5\sqrt{1+m^2}$

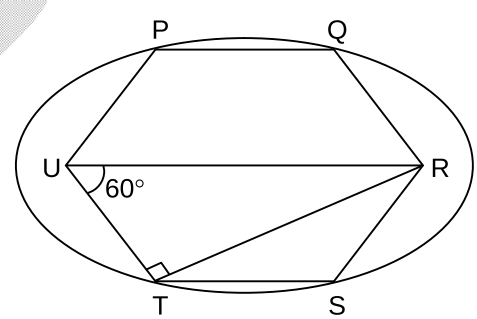
$\Rightarrow c=2-m\pm5\operatorname{cosec}(\alpha-\tan^{-1}3)$ or $2-m\pm5\operatorname{cosec}(\alpha+\tan^{-1}3)$

$\therefore c=2-\cot(\alpha-\tan^{-1}3)\pm5\operatorname{cosec}(\alpha-\tan^{-1}3)$ or $2+\cot(\alpha+\tan^{-1}3)\pm5\operatorname{cosec}(\alpha+\tan^{-1}3)$

$\therefore$ at $x=1$, $y=m+c=2\pm5\operatorname{cosec}(\alpha+\tan^{-1}3)$ or $2\pm5\operatorname{cosec}(\alpha-\tan^{-1}3)$

45. B

Sol. Let E has eccentricity e



(P) UR = 2ae

UT = UR cos 60° = ae

TR = UT sin 60° = $ae\sqrt{3}$

$\Rightarrow ae+ae\sqrt{3}=2a$

$\Rightarrow e=\dfrac{2}{\sqrt{3}+1}=\sqrt{3}-1$

$\therefore UR=(2\sqrt{3}-2)a$

(Q) Latus rectum = $\dfrac{2b^2}{a}=2a(1-e^2)=(4\sqrt{3}-6)a$

(R) $\dfrac{\text{area of hexagen}}{\text{area of ellipse}}=\dfrac{6\times\frac{\sqrt{3}}{4}(ae)^2}{\pi ab}=\dfrac{3\sqrt{3}}{2\pi}\cdot\dfrac{e^2}{\sqrt{1-e^2}}=\dfrac{3^{\frac{5}{4}}}{2^{\frac{1}{2}}\pi}(\sqrt{3}-1)$

(S) Let E be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. Then as per figure, coordinates of Q will be (ae cos 60°, ae sin 60°)

i.e., $\left(\dfrac{ae}{2},\dfrac{ae\sqrt{3}}{2}\right)$

Equation of tangent at Q will be $\dfrac{x\left(\frac{ae}{2}\right)}{a^2}+\dfrac{y\left(\frac{ae\sqrt{3}}{2}\right)}{b^2}=1$

It's slope is $\sqrt{3}-2$

$\therefore$ Acute angle between it PQ will be 15°

$\therefore$ $\tan\theta = 2-\sqrt{3}$

SECTION – B

46. 13

Sol. Let B and C be foot of perpendiculars from P and Q on 3x + 4y = 7 respectively

$PB \cdot QC = b^2 \Rightarrow b = \dfrac{\sqrt{72}}{5}$

Now, $2ae = PQ = \sqrt{2}$ and $b^2 = \dfrac{72}{25} = a^2\left(1-e^2\right)$

Solving gives $a = \dfrac{13}{5\sqrt{2}}$. Now PA + QA = 2a = $\dfrac{26}{5\sqrt{2}}$

47. 8

Sol. $\cos^5 x\left(5+\cos^4 x\right) < \sin^5 x\left(5+\sin^4 x\right)$

The LHS only has a phase shift to $\dfrac{\pi}{2}$ from RHS

This will then be true whenever cosx < sinx

$\therefore \quad x \in \left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{9\pi}{4}, \dfrac{13\pi}{4}\right) \cup \left(\dfrac{17\pi}{4}, 5\pi\right]$

48. 23

Sol. Equation of auxiliary circle is $x^2 + y^2 = 36$

Equations of asymptotes is $y = \dfrac{10x}{9}, y = \dfrac{-10x}{9}$.

The required points must lie in I or II quadrants and inside the circle. Total points = (9 ×2) + 5=23.

49. 39

Sol. $\angle BIC = \dfrac{\pi}{2} + \dfrac{\angle A}{2} = \angle BOC = 2\angle A$

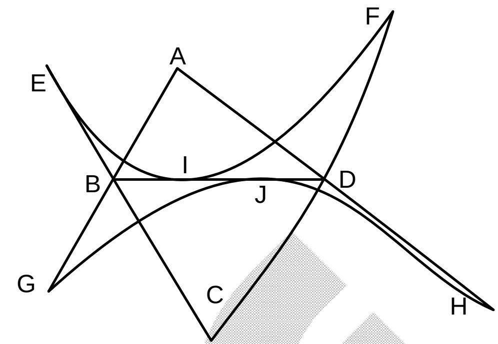
$\Rightarrow \quad \angle A = \dfrac{\pi}{3}$

Using cosine law, $AC = 2+\sqrt{13}$

$\therefore \quad \text{Area } (\Delta ABC) = \dfrac{1}{2} \cdot AB \cdot AC \sin\left(\dfrac{\pi}{3}\right) = \left(\sqrt{12}+\sqrt{39}\right)$ unit^2.

50. 18

Sol. Using parabola properties $\Delta ABI \sim \Delta BCJ$ &
$\Delta ADI \sim \Delta DCJ$



$\therefore \quad \frac{BI}{CJ} = \frac{AI}{BJ}$ & $\frac{DI}{CJ} = \frac{AI}{DJ}$

$\Rightarrow$ BI . BJ = DI . DJ

$\Rightarrow$ BI = DJ

51. 10

Sol. $(1 + \tan x)(1 + \tan^2 x) = 2$ or $1 + \tan x = 2\cos^2 x$

Now, $\cos^2 x(2 - \sin^2(2x)) = 4\cos^6 x - 4\cos^4 x + 2\cos^2 x$

$$= 2\left[\frac{(1+\tan x)^3}{4} - \frac{(1+\tan x)^2}{2} + \frac{1+\tan x}{2}\right]$$

$$= \frac{1}{2}\left(1+\tan x+\tan^2 x+\tan^3 x\right) = 1$$