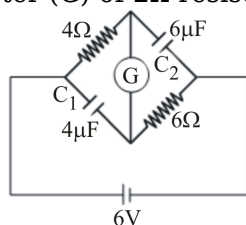


PART-1 : PHYSICS

SECTION-I

1) A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored



in C_1 and C_2 is :

- (A) $\frac{2}{3}$
- (B) $\frac{3}{2}$
- (C) 1
- (D) $\frac{1}{2}$

2) In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2Ω , the balance length changes by :

- (A) 22.5 cm
- (B) 20 cm
- (C) 62.5 cm
- (D) 65 cm

3) The dipole moment of a circular loop carrying a current, I is m and the magnetic field at the centre of the loop is B_1 . When the dipole moment is doubled by keeping the current constant, the

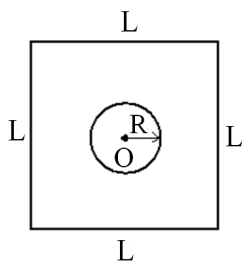
magnetic field at the centre of the loop is B_2 . The ratio $\frac{B_1}{B_2}$ is :

- (A) $\sqrt{3}$
- (B) $\sqrt{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) 2

4) A black box is connected across an ac source of emf $V = 100 \sin(100 \text{ s}^{-1}t)$ in volt. Phase of the current in the circuit is ahead of the phase of the source voltage by $\frac{\pi}{4}$. The peak current is observed to be $\sqrt{2}\text{A}$. The black box consists of :-

- (A) Only a resistance of 50Ω
- (B) Only a capacitance of 0.2 mF
- (C) A series combination of a resistance of 50Ω and an inductance of 0.2 mH
- (D) A series combination of a resistance of 50Ω and a capacitance of 0.2 mF

5) Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side L ($L \gg R$). The loops are coplanar and their centres coincide :



coincide :

- (A) $M = \frac{\sqrt{2}\mu_0 R^2}{L}$
- (B) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$
- (C) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$
- (D) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$

6) Given below are two statements:

Statement I: Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.

Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface. In the light of the above statements, choose the most appropriate answer from the options given below :

- (A) Statement I is incorrect but Statement II is correct
- (B) Both Statement I and Statement II are correct.
- (C) Both Statement I and Statement II are incorrect.
- (D) Statement I is correct but Statement II is incorrect.

7) In 10 complete rotations, the distance moved on the linear scale is 1 cm. There are 100 divisions on circular scale. When the gap between the screw is just closed, the 95th division of circular scale coincides with the reference line. While measuring the diameter of a wire, the linear scale reads 2 mm and 45th division on the circular scale coincides with the reference line. The cross-sectional area of the wire is :-

- (A) 3.8 cm^2
- (B) 0.049 cm^2
- (C) 0.098 cm^2
- (D) 0.45 cm^2

8) A photon has same wavelength as the de Broglie wavelength of electron. Given C = speed of light, v = speed of electron. Which of the following relation is correct?

[Here E_e = kinetic energy of electron,

E_{ph} = energy of photon,

P_e = momentum of electron and

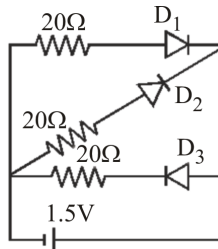
P_{ph} = momentum of photon] :-

- (A) $E_e / E_{ph} = 2C/v$
- (B) $E_e / E_{ph} = v/2C$
- (C) $P_e / P_{ph} = 2C/v$
- (D) $P_e / P_{ph} = v/2C$

9) With what potential an electron should be accelerated so that its de Broglie wave-length becomes equal to the wave length of second line of Balmer series for He^+ ion ?

- (A) $\frac{9R^2h^2}{32me}$
- (B) $\frac{162R^2h^2}{25me}$
- (C) $\frac{81R^2h^2}{32me}$
- (D) $\frac{32R^2h^2}{81me}$

10) In the circuit shown in the figure all the diodes are ideal. The current drawn from the battery of



1.5 voltage emf and 1Ω internal resistance is :-

- (A) $\frac{1.5}{\left(\frac{20}{3} + 1\right)}$ Amp.
- (B) $\frac{1.5}{21}$ Amp.
- (C) 0 Amp.
- (D) None

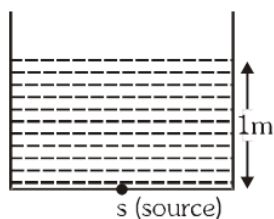
11) A cylindrical tube partially filled with water is in resonance with a tuning fork when the height of air column is 0.1m. When the level of water is lowered, the resonance is again observed at 0.35m. The end correction is :-

- (A) 0.025 m
- (B) 0.015 m
- (C) 0.001 m

(D) 0.002 m

12) In YDSE a light containing two wavelength 500 nm and 700 nm are used. Find the minimum distance where maxima of two wavelength coincide. Given $D/d = 10^3$, where D is the distance between the slits and the screen and d is the distance between the slits.

- (A) 7 mm
- (B) 14 mm
- (C) 3.5 mm
- (D) 6.5 mm



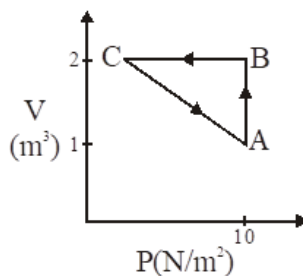
13)

A container is filled with water ($\mu = \frac{4}{3}$) upto height 1m. Find out diameter of disc at top of water surface from which light is coming out.

- (A) ($\mu = \frac{4}{3}$)
- (B) $\frac{2}{\sqrt{7}}$ m
- (C) $\frac{6}{\sqrt{7}}$ m
- (D) $\frac{3}{\sqrt{7}}$ m

14) An ideal gas is taken through the cycle

$A \rightarrow B \rightarrow C \rightarrow A$, as shown in figure. If the net heat supplied to the gas in the cycle is 5 J, the work



done by the gas in the process $C \rightarrow A$ is :-

- (A) -5 J
- (B) -10 J
- (C) -15 J
- (D) -20 J

15) Water flows into a large tank with flat bottom at the rate of $10^{-4} \text{ m}^3 \text{ s}^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is :

- (A) 4 cm
- (B) 2.9 cm
- (C) 1.7 cm
- (D) 5.1 cm

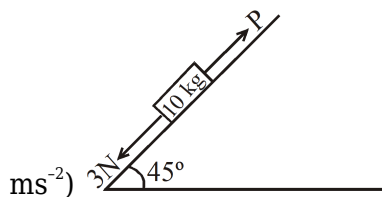
16) The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be :-

- (A) $\frac{2}{\sqrt{3}} \text{ s}$
- (B) $2\sqrt{3} \text{ s}$
- (C) $\frac{\sqrt{3}}{2} \text{ s}$
- (D) $\frac{3}{2} \text{ s}$

17) Two particles are projected from the same point with the same speed u such that they have the same range R , but different maximum heights, h_1 and h_2 . Which of the following is correct ?

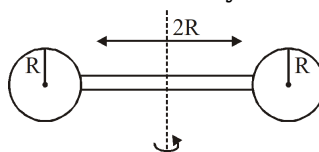
- (A) $R^2 = 2 h_1 h_2$
- (B) $R^2 = 16 h_1 h_2$
- (C) $R^2 = 4 h_1 h_2$
- (D) $R^2 = h_1 h_2$

18) A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P , such that the block does not move downward ? (take $g = 10 \text{ ms}^{-2}$)



- (A) 32 N
- (B) 25 N
- (C) 23 N
- (D) 18 N

19) Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing



perpendicularly through the centre of the rod is :

- (A) $\frac{152}{15}MR^2$
 (B) $\frac{17}{15}MR^2$
 (C) $\frac{137}{15}MR^2$
 (D) $\frac{209}{15}MR^2$

20) A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210$ s will be :

- (A) 2
 (B) $\frac{1}{9}$
 (C) 3
 (D) 1

SECTION-II

1) Five positive equal charges are placed at vertices of a regular hexagon and net electric field at the centre is E_1 . A negative charge having equal magnitude is placed at sixth vertex and then net

electric field is E_2 . Find $\frac{E_2}{E_1}$:-

2) A Zener diode connected across a source voltage of 12V such that 6V drops across it. The power drawn by zener diode is 2.4 mW, Then what is the maximum value of series resistance (in kilo ohm) connected in the circuit?

3) A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image (in cm) from the surface of the sphere is :

4) Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in B is :- (in K)

5) A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

PART-2 : CHEMISTRY

SECTION-I

1) Which of the following has been arranged in order of decreasing dipole moment ?

- (A) $\text{CH}_3\text{Cl} > \text{CH}_3\text{F} > \text{CH}_3\text{Br} > \text{CH}_3\text{I}$
- (B) $\text{CH}_3\text{F} > \text{CH}_3\text{Cl} > \text{CH}_3\text{Br} > \text{CH}_3\text{I}$
- (C) $\text{CH}_3\text{Cl} > \text{CH}_3\text{Br} > \text{CH}_3\text{I} > \text{CH}_3\text{F}$
- (D) $\text{CH}_3\text{F} > \text{CH}_3\text{Cl} > \text{CH}_3\text{I} > \text{CH}_3\text{Br}$

2) Select appropriate ligand for given complex $[\overset{\text{III}}{\text{Co}}(\text{.....})_6]^{\pm x}$; $\mu = 0 \text{ BM}$

- (A) $\text{C}_2\text{O}_4^{2-}$
- (B) en
- (C) H_2O
- (D) F^-

3) Oxidation state of Hg in Nessler's Reagent is:-

- (A) 5
- (B) 2
- (C) 6
- (D) 0

4) K_2MnO_4 can be converted into KMnO_4 by :-

- (A) Passing CO_2 gas
- (B) By passing Cl_2
- (C) Electrolytic oxidation
- (D) All of these

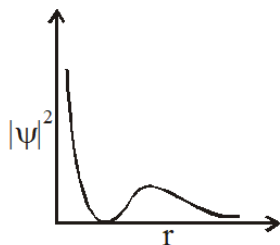
5) Among the oxo-acids of chlorine, the correct order of acid strength is :-

- (A) $\text{HClO}_4 < \text{HClO} < \text{HClO}_2 < \text{HClO}_3$
- (B) $\text{HClO}_3 < \text{HClO}_2 < \text{HClO}_4 < \text{HClO}$
- (C) $\text{HClO}_4 > \text{HClO}_3 > \text{HClO}_2 > \text{HClO}$
- (D) $\text{HClO}_4 < \text{HClO}_3 < \text{HClO}_2 < \text{HClO}$

6) Which of the following processes involves absorption of energy ?

- (A) $\text{S}_{(\text{g})} + \text{e}^- \rightarrow \text{S}_{(\text{g})}^-$
- (B) $\text{O}_{(\text{g})}^- + \text{e}^- \rightarrow \text{O}_{(\text{g})}^{2-}$
- (C) $\text{Cl}_{(\text{g})} + \text{e}^- \rightarrow \text{Cl}_{(\text{g})}^-$
- (D) $\text{O}_{(\text{g})} + \text{e}^- \rightarrow \text{O}_{(\text{g})}^-$

7) The graph between $|\psi|^2$ and r(radial distance) is shown below. This represents :-

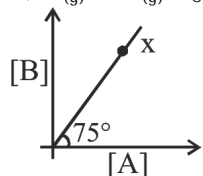


- (A) 3s orbital
- (B) 1s orbital
- (C) 2p orbital
- (D) 2s orbital

8) In a hydrogen like sample ion are in particular excitation state. If electron make transition to first excited state and produces maximum 15 different spectral line, then excited state will be :-

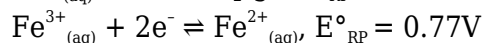
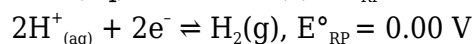
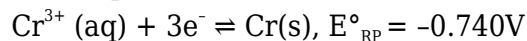
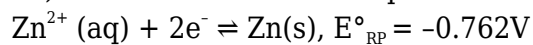
- (A) First excited state
- (B) 5th excited state
- (C) 4th excited state
- (D) 6th excited state

9) $A_{(g)} \rightleftharpoons B_{(g)}$ $K_c = 1.732$ at 298 K, which of the following statement is true at point 'X' in the graph



- (A) Equilibrium will be set up.
- (B) Reaction will go in forward direction till it attains equilibrium
- (C) Reaction will go in backward direction till it attains equilibrium.
- (D) Can't predict.

10) The standard reduction potentials at 25°C for the following half reactions are :



Which is the strongest reducing agent ?

- (A) Zn
- (B) Cr
- (C) $\text{H}_2(\text{g})$
- (D) $\text{Fe}^{2+}(\text{aq})$

11)

A weak acid of dissociation constant 10^{-5} is being titrated with aqueous NaOH solution. The pH at

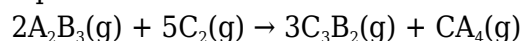
the point of one-third neutralization of the acid will be:

- (A) $5 + \log 2 - \log 3$
- (B) $5 - \log 2$
- (C) $5 - \log 3$
- (D) $5 - \log 6$

12) 0.15 gm of a substance dissolved in 15 gm of solvent, boiled at a temperature higher by 0.216°C than that of the pure solvent. Find out the molecular weight of the substance. (Molal elevation constant is 2.16°C) :-

- (A) 1.01
- (B) 10.0
- (C) 100
- (D) 10

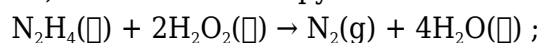
13) In a closed vessel 50 ml of A_2B_3 completely reacts with 200 ml of C_2 according to the following equation



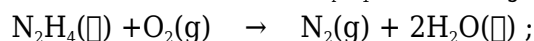
The composition of gaseous mixture in the system will be :-

- (A) 100 ml C_2 , 50 ml C_3B_2 , 50 ml CA_4
- (B) 25 ml C_2 , 75 ml C_3B_2 , 25 ml CA_4
- (C) 75 ml C_2 , 75 ml C_3B_2 , 25 ml CA_4
- (D) 10 ml C_2 , 25 ml C_3B_2 , 100 ml CA_4

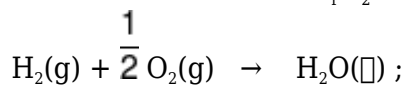
14) Determine enthalpy of formation for $\text{H}_2\text{O}_2(\text{l})$, using listed enthalpies of reaction :



$$\Delta_r H_1^\circ = -818 \text{ kJ/mol}$$



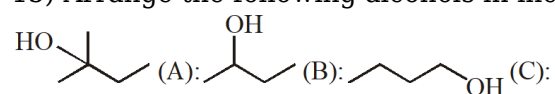
$$\Delta_r H_2^\circ = -622 \text{ kJ/mol}$$



$$\Delta_r H_3^\circ = -285 \text{ kJ/mol}$$

- (A) -383 kJ/mol
- (B) -187 kJ/mol
- (C) -498 kJ/mol
- (D) None of these

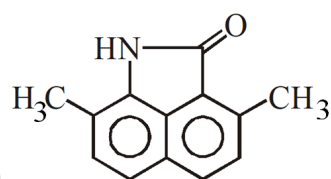
15) Arrange the following alcohols in increasing order of acid-catalyzed esterification by ethanol.



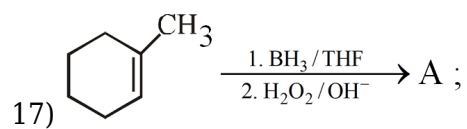
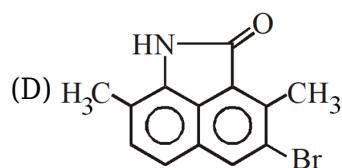
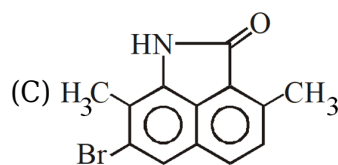
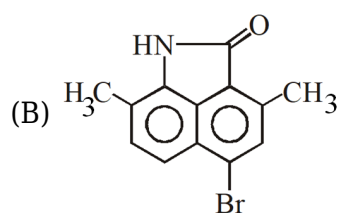
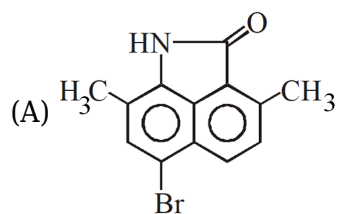
- (A) $\text{A} < \text{B} < \text{C}$
- (B) $\text{C} < \text{B} < \text{A}$

(C) $B < C < A$

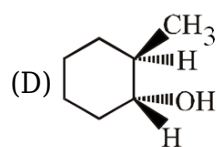
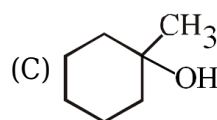
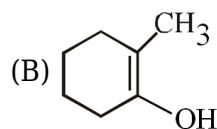
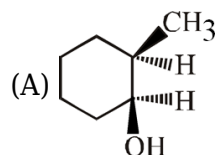
(D) $C < A < B$

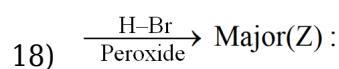
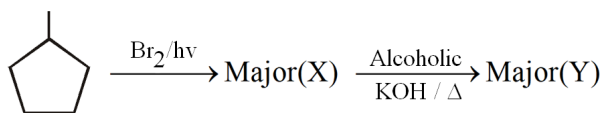


16) The major product obtained when Br_2/Fe is treated with

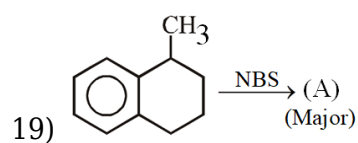
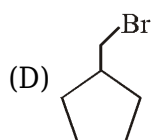
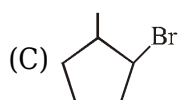
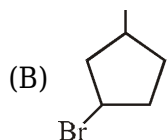
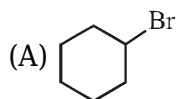


Product A is :

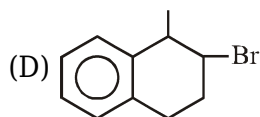
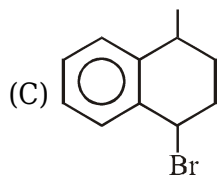
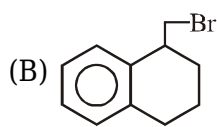
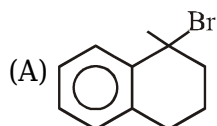




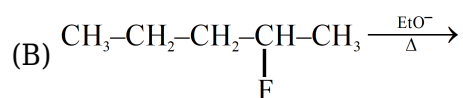
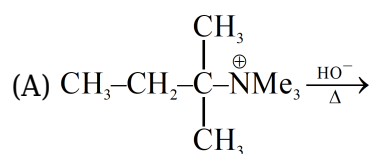
Product (Z) is :

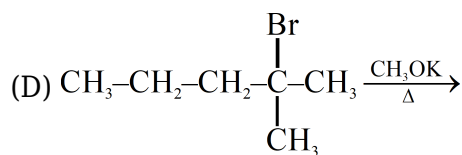
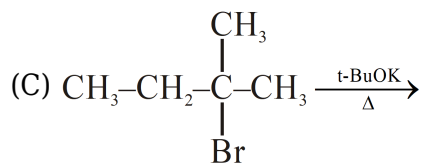


Major product (A) is :-



20) In which of the following reaction saytzeff alkene is major product ?





SECTION-II

1) Consider y-axis as internuclear axis, how many of following will lead to π -bond formation :

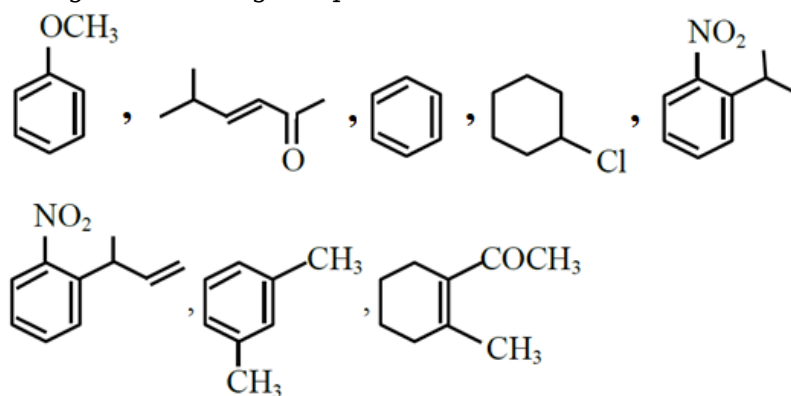
- (i) $p_y - p_y$ (ii) $p_x - p_x$
 (iii) $p_z - p_z$ (iv) $d_{xy} - d_{xy}$
 (v) $d_{yz} - d_{yz}$ (vi) $p_x - d_{xy}$
 (vii) $d_{xy} - p_z$ (viii) $d_{xz} - d_{xz}$

2) The number of ambidentate ligands among the following.



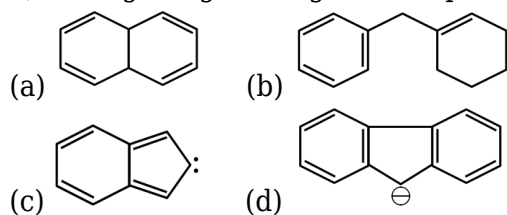
3) The half-life of decomposition of gaseous CH_3CHO at initial pressure of 364 mm and 182 mm of Hg were 440 sec and 880 sec respectively. The order of the reaction is :-

4) How many compounds among the following compounds show inductive, mesomeric as well as



hyperconjugation effects?

5) Among the given organic compounds, the total number of aromatic compounds is



PART-3 : MATHEMATICS

SECTION-I

1) $f(x) = (\sin^2 x) \cdot e^{-2 \sin^2 x}$; $\max. f(x) - \min. f(x) = ?$

- (A) $\frac{1}{e^2}$
- (B) $\frac{1}{2e} - \frac{1}{e^2}$
- (C) $\frac{1}{2e}$
- (D) 0

2) The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2 \log_e x} - (2x+3)}$, $x \in \mathbb{R}$

- (A) $\mathbb{R} - \{1, 3\}$
- (B) $(2, \infty) - \{3\}$
- (C) $(-1, \infty) - \{3\}$
- (D) $\mathbb{R} - \{3\}$

3) If $f(x) = \begin{cases} x + \{x\} + x \sin \{x\} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$

where $\{x\}$ denotes the fractional part function, then :-

- (A) f is continuous and differentiable at $x = 0$
- (B) f is continuous but not differentiable at $x = 0$
- (C) f is continuous and differentiable at $x = 2$
- (D) None

4) The number of way of selection of three numbers a, b, c from the set $\{1, 2, 3, \dots, 100\}$ such that $a \leq b \leq c$ is equal to

- (A) $^{102}C_3$
- (B) $^{100}C_3$
- (C) $^{101}C_2 + ^{100}C_3$
- (D) $^{100}C_1 + 3(^{100}C_3) + 6(^{100}C_3)$

5) The number of non negative integral solution of equation $x + y + z = 25$ if $x \geq 2, 3 < y \leq 5, 1 \leq z < 8$

- (A) 14
- (B) 24
- (C) 34
- (D) None

6) Consider the system of linear equation

$$x + y + z = 4\mu, \quad x + 2y + 2\lambda z = 10\mu,$$

$$x + 3y + 4\lambda^2 z = \mu^2 + 15, \quad \text{where } \lambda, \mu \in \mathbb{R}.$$

Which one of the following statements is NOT correct ?

- (A) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$
- (B) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$
- (C) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$
- (D) The system is consistent if $\lambda \neq \frac{1}{2}$

7) If $a, b \in \mathbb{R}$ which satisfying the relations

$$19a^2 + 99a + 1 = 0 \text{ and } b^2 + 99b + 19 = 0 \text{ and } ab \neq 1. \text{ Then the value of } \left| \frac{ab + 1 + 4a}{b} \right| \text{ is equal to}$$

- (A) 5
- (B) 13
- (C) 11
- (D) None

8) Let A be a square matrix of order 3 such that

$$A + A^T = \begin{bmatrix} 10 & 4 & 6 \\ a_{21} + a_{12} & 6 & a_{23} + a_{32} \\ a_{31} + a_{13} & 8 & 4 \end{bmatrix},$$

where a_{12}, a_{23}, a_{31} are positive real roots of equation $x^3 - 6x^2 + px - 8 = 0 \quad \forall p \in \mathbb{R}$, then the absolute value of $|A|$ is equal to :

- (A) 2
- (B) 4
- (C) 6
- (D) 8

9) $\tan \left(\sum_{n=1}^{\infty} \tan^{-1} \left(\frac{6^n}{2^{2n+1} + 3^{2n+1}} \right) \right) = ?$

- (A) 2
- (B) 3
- (C) 5
- (D) None

10)

The number of roots of the equation $x + 2 \tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is -

- (A) 1
- (B) 2
- (C) 3
- (D) infinite

11) The value of expression
$$\frac{2(\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 89^\circ)}{2(\cos 1^\circ + \cos 2^\circ + \dots + \cos 44^\circ) + 1}$$
 equals:-

- (A) $\sqrt{2}$
- (B) $1/\sqrt{2}$
- (C) $1/2$
- (D) 0

12) Let the observations $x_i (1 \leq i \leq 10)$ satisfy the equations, $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \dots, x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

- (A) (6, 6)
- (B) (3, 6)
- (C) (6, 3)
- (D) (3, 3)

13) $\int x \left(\frac{\ln a^{x/2}}{3a^{5x/2}b^{3x}} + \frac{\ln b^{bx}}{2a^{2x}b^{4x}} \right) dx$
(where $a, b \in \mathbb{R}^+$) is equal to

- (A) $\frac{1}{6 \ln a^2 b^3} a^{2x} b^{3x} \ln \frac{a^{2x} b^{3x}}{e} + k$
- (B) $\frac{1}{6 \ln a^2 b^3} \frac{1}{a^{2x} b^{3x}} \ln \frac{1}{e a^{2x} b^{3x}} + k$
- (C) $\frac{1}{6 \ln a^2 b^3} \frac{1}{a^{2x} b^{3x}} \ln (a^{2x} b^{3x} e) + k$
- (D) $-\frac{1}{6 \ln a^2 b^3} \frac{1}{a^{2x} b^{3x}} \ln (a^{2x} b^{3x} e) + k$

14) If $[]$ denotes the greatest integer function, then $\int_{\pi}^{2\pi} [2 \sin x] dx$ is equal to :-

- (A) $-\pi$

- (B) -2π
- (C) $-5\pi/3$
- (D) $5\pi/3$

15) Let $f(x) = \min. (x+1, \sqrt{1-x})$ for all $x \leq 1$. Then the area bounded by $y = f(x)$ and the x-axis is :-

- (A) $\frac{7}{3}$ sq. units
- (B) $\frac{1}{6}$ sq. units
- (C) $\frac{11}{6}$ sq. units
- (D) $\frac{7}{6}$ sq. units

16) If $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x\right] dx$ and $y(1) = \pi/2$, then $\cos(y/x)$ is equal to

- (A) x
- (B) $1/x$
- (C) $\log x$
- (D) e^x

17) The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\sqrt{\frac{3}{2}}$
- (C) $\sqrt{6}$
- (D) $3\sqrt{6}$

18) A circle C_1 of radius 2 touches both x-axis and y-axis. Another circle C_2 whose radius is greater than 2 touches circle C_1 and both the axes. Then the radius of circle C_2 is-

- (A) $6 - 4\sqrt{2}$
- (B) $6 + 4\sqrt{2}$
- (C) $6 - 4\sqrt{3}$
- (D) $6 + 4\sqrt{3}$

19) On the portion of line $\frac{x}{3} + \frac{y}{4} = 1$ intercepted between the axes a square is constructed away from the origin. Co-ordinates of the vertex of square which is farthest from origin is :

- (A) (3, 8)
- (B) (6, 4)
- (C) (7, 3)
- (D) (4, 7)

20) Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

- (A) (2, 3]
- (B) (3, ∞)
- (C) (3/2, 2]
- (D) (1, 3/2]

SECTION-II

1) $f(x) = \frac{x^2 - x}{x^2 + 4x}$, then $\frac{d}{dx}(f^{-1}(x))$ at $x = 2$

2) From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ is equal to

3) If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to :

4) If $\int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx = k \left(\sin \frac{x}{a} - \cos \frac{x}{b} \right) + C$;
then value of $(k + a + b) = ?$

5) If a line passing through (2, 1, 4) cuts off an intercept of minimum length between two non coplanar lines $x - 6 = \frac{y}{\alpha} = -z$ and $x = \frac{y}{0} = z$, then α is :

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| A. | D | A | B | D | C | B | B | B | A | B | A | C | C | A | D | B | B | A | C | C |

SECTION-II

| Q. | 21 | 22 | 23 | 24 | 25 |
|----|----|----|----|----|-----|
| A. | 2 | 15 | 6 | 18 | 900 |

PART-2 : CHEMISTRY

SECTION-I

| Q. | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A. | A | C | B | D | C | B | D | D | C | A | B | C | C | B | A | A | D | C | A | D |

SECTION-II

| Q. | 46 | 47 | 48 | 49 | 50 |
|----|----|----|----|----|----|
| A. | 5 | 3 | 2 | 4 | 3 |

PART-3 : MATHEMATICS

SECTION-I

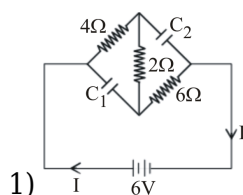
| Q. | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| A. | C | B | D | A | A | B | A | C | D | C | A | D | D | C | D | C | B | B | D | B |

SECTION-II

| Q. | 71 | 72 | 73 | 74 | 75 |
|----|----|----|----|----|----|
| A. | 5 | 71 | 18 | 24 | 2 |

SOLUTIONS

PART-1 : PHYSICS



1)

In steady state

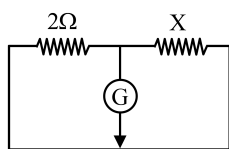
$$R_{eq} = 12\Omega ; I = \frac{6}{12} = 0.5A$$

$$\text{P.D across } C_1 = 3V$$

$$\text{P.D across } C_2 = 4V$$

$$q_1 = C_1 V_1 = 12 \mu C ; q_2 = C_2 V_2 = 24 \mu C$$

$$\frac{q_1}{q_2} = \frac{1}{2}$$



2)

$$\text{First case } \frac{2}{40} = \frac{X}{60} \Rightarrow X = 3\Omega$$

$$\text{In second case } X' = \frac{2 \times 3}{2 + 3} = 1.2\Omega$$

$$\frac{2}{\ell} = \frac{1.2}{100 - \ell}$$

$$200 - 2\ell = 1.2\ell$$

$$\ell = \frac{200}{3.2} = 62.5\text{cm}$$

Balance length changes by 22.5 cm

$$3) m = iA = i\pi r^2$$

$$2m = i\pi(r')^2$$

$$2 i\pi r^2 = i\pi(r')^2$$

$$r' = \sqrt{2}r$$

$$B_1 = \frac{\mu_0 i}{2r}$$

$$B_2 = \frac{\mu_0 i}{2(\sqrt{2}r)} = \frac{B_1}{\sqrt{2}}$$

$$\therefore \frac{B_1}{B_2} = \sqrt{2}$$

4) Since current is leading the source voltage, this shows that circuit is of capacitive nature.

However, current leads in phase by $\frac{\pi}{4}$, therefore, box also consists of resistance in series with the capacitance.

$$\tan \frac{\pi}{4} = \frac{X_C}{R}$$

$$\Rightarrow X_C = \frac{1}{\omega C} = R \dots\dots\dots$$

$$I_{\max} = \frac{V_{\max}}{\sqrt{R^2 + X_C^2}} = \sqrt{2}A \dots\dots\dots$$

$$5) \phi = Mi$$

$$\phi = (BA)$$

$$= \pi R^2 \left(4 \frac{\mu_0}{4\pi} \frac{i}{\left(\frac{L}{2}\right)} \sqrt{2} \right)$$

$$\phi$$

$$\Rightarrow M = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

$$6) \frac{1}{2} \epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\therefore E = CB \text{ and } C = \frac{1}{\mu_0 \epsilon_0}$$

$$7) \text{Pitch} = \frac{1 \text{ cm}}{10} = 0.1 \text{ cm}$$

$$LC = \frac{0.1 \text{ cm}}{100} = 0.001 \text{ cm}$$

$$\text{zero error} = -0.005 \text{ cm}$$

Measured diameter

$$= \text{MSD} + \text{CD} \times \text{LC}$$

$$= 0.2 \text{ cm} + 45 \times 0.001 \text{ cm}$$

$$= 0.245 \text{ cm}$$

Actual diameter

$$= 0.245 - (0.005) \text{ cm}$$

$$= 0.250 \text{ cm}$$

Cross - Sectional area

$$\pi r^2 = \frac{\pi d^2}{4} = 0.049 \text{ cm}^2$$

$$8) \lambda_{ph} = \lambda_e = \frac{h}{mv}$$

$$\frac{E_e}{E_{ph}} = \frac{\frac{1}{2}mv^2}{\left(\frac{hc}{\lambda}\right)} = \frac{1}{2} \frac{mv^2}{hc} \times \lambda$$

$$= \frac{1}{2} \frac{mv^2}{hc} \times \frac{h}{mv} = \frac{v}{2c}$$

$$\begin{aligned}
 9) \frac{1}{\lambda} &= R(4) \left[\frac{1}{4} - \frac{1}{16} \right] \\
 \frac{1}{\lambda} &= R(4) \times \frac{3}{16} \\
 \lambda &= \frac{3R}{4} = \frac{h^2}{\sqrt{2meV}} \\
 \frac{16}{9R^2} &= \frac{2meV}{9R^2 h^2} \\
 V &= \frac{32me}{9}
 \end{aligned}$$

10) D_1 - Reverse bias

D_2 - Reverse bias

D_3 - Forwards bias

$$V = iR$$

$$1.5 = 21 \times i$$

$$i = \frac{1.5}{21} \text{ Amp.}$$

$$11) \text{ For 1}^{\text{st}} \text{ resonance, } \lambda_1 + e = \frac{\lambda}{4}$$

$$\text{For 2}^{\text{nd}} \text{ resonance, } \lambda_2 + e = \frac{3\lambda}{4}$$

$$\lambda e = \frac{\lambda_2 - 3\lambda_1}{2} = 0.025 \text{ m}$$

$$12) n = \frac{n\lambda D}{d} = \text{same}$$

$$n_1 \lambda_1 = n_2 \lambda_2$$

$$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{700}{500} = \frac{7}{5}$$

500 nm की 7th Bright, 700 nm की 5th Bright से coincide

500 nm की 14th Bright, 700 nm की 10th Bright से coincide

$$x = 7\beta = \frac{7\lambda D}{d} = 7 \times 500 \times 10^{-9} \times 10^3$$

$$= 35 \times 10^{-4} \text{ meter} = 3.5 \text{ mm}$$

$$13) r = \frac{h}{\sqrt{\mu^2 - 1}} = \frac{1}{\sqrt{\frac{16}{9} - 1}} = \frac{3}{\sqrt{7}} \text{ m}$$

$$\text{Diameter } 2r = \frac{6}{\sqrt{7}} \text{ m}$$

14)

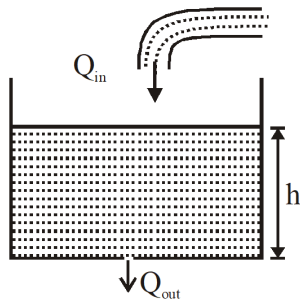
$$dQ = dU + dW$$

$$5 = 0 + W_{AB} + W_{CA}$$

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$$5 = 10 + W_{CA}$$

$$W_{CA} = -5J$$



15)

Since height of water column is constant therefore, water inflow rate (Q_{in}) = water outflow rate

$$Q_{in} = 10^{-4} \text{ m}^3\text{s}^{-1}$$

$$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$$

$$10^{-4} = 10^{-4} \sqrt{20 \times h}$$

$$h = \frac{1}{20} \text{ m}$$

$$h = 5 \text{ cm}$$

□ correct answer is (D)

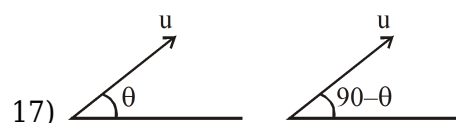
$$16) \because g = \frac{GM}{R^2}$$

$$\frac{g_p}{g_e} = \frac{M_e}{M_e} \left(\frac{R_e}{R_p} \right)^2 = 3 \left(\frac{1}{3} \right)^2 = \frac{1}{3}$$

$$\text{Also } T \propto \frac{1}{\sqrt{g}}$$

$$\Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$$

$$\Rightarrow T_p = 2\sqrt{3} \text{ s}$$



17)

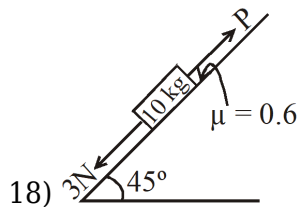
For same range angle of projection will be θ & $90 - \theta$

$$R = \frac{u^2 2 \sin \theta \cos \theta}{g}$$

$$h_1 = \frac{u^2 \sin^2 \theta}{g}$$

$$h_2 = \frac{u^2 \sin^2 (90 - \theta)}{g}$$

$$\frac{R^2}{h_1 h_2} = 16$$

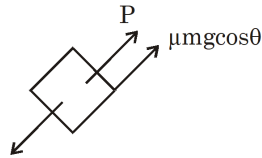


18) $\frac{100}{\sqrt{2}}$

$$mg \sin 45^\circ = \frac{100}{\sqrt{2}} = 50\sqrt{2}$$

$$\mu mg \cos \theta = 0.6 \times mg \times \frac{1}{\sqrt{2}} = 0.6 \times 50\sqrt{2}$$

$$P = 31.28 \approx 32 \text{ N}$$



$$73.7 = 3 + mg \sin \theta$$

19) For Ball

using parallel axis theorem.

$$I_{\text{ball}} = \frac{2}{5} MR^2 + M(2R)^2 = \frac{22}{5} MR^2$$

$$2 \text{ Balls so } \frac{44}{5} MR^2$$

$$I_{\text{rod}} = \text{for rod } \frac{M(2R)^2}{3} = \frac{MR^2}{3}$$

$$I_{\text{system}} = I_{\text{Ball}} + I_{\text{rod}}$$

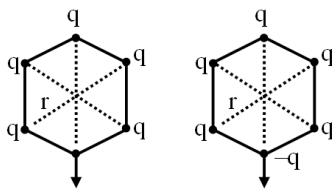
$$= \frac{44}{5} MR^2 + \frac{MR^2}{3} = \frac{137}{15} MR^2$$

$$20) k = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$$

$$U = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t$$

$$\frac{k}{U} = \cot^2 \omega t = \cot^2 \frac{\pi}{90} (210) = \frac{1}{3}$$

No answer is matching as correct answer is 1/3. Hence ratio is 3 (most appropriate)



21)

$$E = \frac{Eq}{r^2} \quad E = \frac{Eq}{r^2} + \frac{Eq}{r^2} = \frac{2Eq}{r^2}$$

$$22) V_s = 12 \text{ Volt}$$

$$V_{\text{drop}} = 6 \text{ Volt}$$

$$\text{So, remaining voltage } V = V_s - V_{\text{drop}} = 12 - 6$$

$$= 6 \text{ Volt}$$

Power drawn by Zener diode $P = 2.4 \text{ MW}$

As we know that

$$P = I^2 R = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{(6)^2}{2.4 \times 10^{-3}}$$

$$R = \frac{36 \times 10^3}{2.4}$$

$$R = 15 \text{ k}\Omega$$

$$23) u = -6 \text{ cm} \quad R = -6 \text{ cm}$$

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{1.5}$$

$$\frac{\mu}{v} - \frac{1}{\mu} = \frac{\mu - 1}{R}$$

$$\frac{1/1.5}{v} - \frac{1}{(-6)} = \frac{1/1.5 - 1}{(-6)}$$

$$\frac{\mu}{v} - \frac{(1)}{(-6)} = \frac{1/1.5 - 1}{(-6)}$$

$$v = -6 \text{ cm}$$

$$24)$$

Heat added to the gas in cylinder A is at constant pressure while that in cylinder B at constant volume. Therefore,

$$Q_A = \mu C_p (\Delta T)_A$$

$$Q_B = \mu C_v (\Delta T)_B$$

$$\text{Given that, } Q_A = Q_B$$

$$\mu C_p (\Delta T)_A = \mu C_v (\Delta T)_B$$

$$(\Delta T)_B = \frac{C_p}{C_v} (\Delta T)_A = 1.4 \times 30 = 42 \text{ K}$$

$$25) x = 3t^2 + 5$$

$$v = \frac{dx}{dt}$$

$$v = 6t + 0$$

$$\text{at } t = 0 \quad v = 0$$

$$t = 5 \text{ sec} \quad v = 30 \text{ m/s}$$

$$W.D. = \Delta KE$$

$$W.D. = \frac{1}{2} mv^2 - 0 = \frac{1}{2} (2) (30)^2 = 900 \text{ J}$$

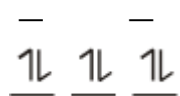
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PART-2 : CHEMISTRY

27) $[\text{Co}(\text{H}_2\text{O})_6]^{+3}$

C.N. = 6

$\text{Co}^{+3} \quad \text{H}_2\text{O} \quad \text{act SFL}$



$\mu = \text{BM} = \sqrt{n(n+2)}$

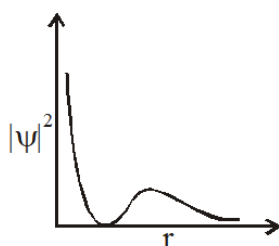
no. of unpaired $e^- = 0$

$\text{Co}^{+3} = [\text{Ar}] 3d^6 4s^0$

29) By using all these methods K_2MnO_4 can be converted into KMnO_4 .

30) $(\text{HO})\text{Cl} < (\text{HO})\text{ClO} < (\text{HO})\text{ClO}_2 < (\text{HO})\text{ClO}_3$

31) Addition of electron in anion is endothermic process.



32)

Represent s-orbital containing 1 radial node

33)

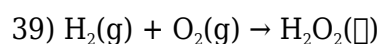
$5 + 4 + 3 + 2 + 1 = 15 \quad (7 - 2 = 5)$

34) $Q_P = \frac{[B]}{[A]} = \text{slope} = \tan(75^\circ) > 1.732$

$Q_c > K_c \leftarrow \text{backward}$

36) $\text{pH} = \text{pK}_a + \log \frac{1/3}{2/3}$

37) $m = \frac{K_f \times w \times 1000}{W \times \Delta T_b} = \frac{2.16 \times 0.15 \times 1000}{15 \times 0.216} = 100$

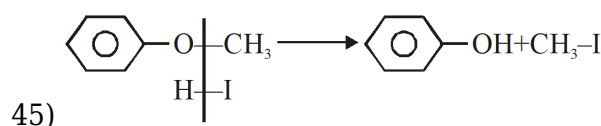
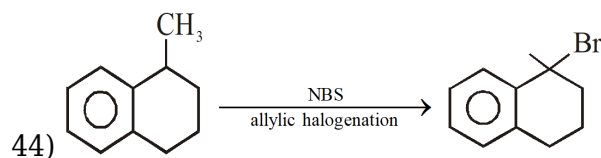
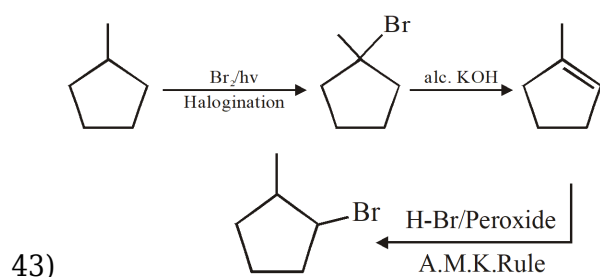


$$\Delta_f H^\circ(\text{H}_2\text{O}_2, \square) = \Delta_f H_3^\circ + \frac{\Delta_r H_2^\circ}{2} - \frac{\Delta_r H_1^\circ}{2}$$

40) Esterification involves formation of alkoxide ion. More stable is alkoxide ion more is reactivity of corresponding alcohol.

41) The phenyl ring having H-N < group is activated, while another one is deactivated due to $\text{—}\overset{\text{O}}{\underset{\text{||}}{\text{C}}}\text{—}$, so electrophilic aromatic bromination will occur at p-position with respect to H-N < group inactivated ring.

42) Due to formation of cyclic transition state syn addition of $\overset{+}{\text{H}}\text{—}\overset{-}{\text{O}}\text{H}$ take place in HBO reaction according to A.M.K. Rule.



46) (i) σ (ii) π (iii) π (iv) π
(v) π (vi) π (vii) no bond (viii) δ

47) CN^- , SCN^- , NO_2^- are ambidentate ligands.

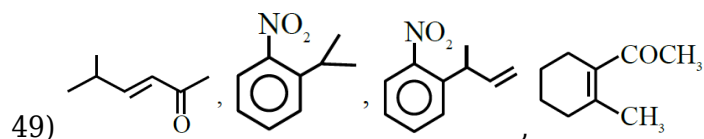
48)

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{P_2}{P_1}\right)^{n-1}$$

$$\frac{440}{880} = \left(\frac{182}{364}\right)^{n-1}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}$$

$$n - 1 = 1 ; n = 2$$



50) b, c and d are Aromatic

PART-3 : MATHEMATICS

51) Let $t = \sin^2 x$; $t \in [0, 1]$

$$f(x) = g(t) = t \cdot e^{-2t}$$

$$g'(t) = e^{-2t} - 2te^{-2t}$$

$$g'(t) = (1 - 2t) \cdot e^{-2t} = 0$$

$$t = \frac{1}{2} \in (0, 1) \quad (\because e^{-2t} \rightarrow +ve)$$

$$g(0) = 0 \text{ (min.)}$$

$$g\left(\frac{1}{2}\right) = \frac{1}{2}e^{-1} = \frac{1}{2e} \text{ (max.)}$$

$$g(1) = 1 \cdot e^{-2} = 1/e^2$$

$$\text{max. } f(x) - \text{min. } f(x) = \frac{1}{2e} - 0 = \frac{1}{2e}.$$

$$52) x - 2 > 0 \Rightarrow x > 2$$

$$x + 1 > 0 \Rightarrow x > -1$$

$$x + 1 \neq 1 \Rightarrow x \neq 0 \text{ and } x > 0$$

Denominator

$$x^2 - 2x - 3 \neq 0$$

$$(x - 3)(x + 1) \neq 0$$

$$x \neq -1, 3$$

$$\text{So Ans } (2, \infty) - \{3\}$$

$$53) f(0^-) = \lim_{h \rightarrow 0} -h + \{-h\} + (-h)\sin\{-h\}$$

$$= \lim_{h \rightarrow 0} -h + (1 - h) + (-h)\sin(1 - h) = 1$$

$$f(0^+) = \lim_{h \rightarrow 0} h + \{h\} + h\sin\{h\} = 0$$

Not continuous at $x = 0$

$$55) {}^{20}C_2 - ({}^{18}C_2 + {}^{13}C_2 - {}^{11}C_2) = 14$$

$$56) x + y + z = 4\mu, x + 2y + 2\lambda z = 10\mu,$$

$$x + 3y + 4\lambda^2 z = \mu^2 + 15,$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

For unique solution

$$\Delta \neq 0, 2\lambda - 1 \neq 0, (\lambda \neq \frac{1}{2})$$

$$\text{Let } \Delta = 0, \lambda = \frac{1}{2}$$

$$\Delta_y = 0, \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2 + 15 & 3 & 1 \end{vmatrix}$$

$$= (\mu - 15)(\mu - 1)$$

$$\text{For infinite solution } \lambda = \frac{1}{2}, \mu = 1 \text{ or } 15$$

58) $A + A^T$ is a symm. mat

$$\text{also } a_{12} + a_{23} + a_{31} = 6 ; a_{12} a_{23} a_{31} = 8$$

$$AM = GM \Rightarrow a_{12} = a_{23} = a_{31} = 2$$

$$a_{12} + a_{21} = 4 \Rightarrow a_{12} = a_{21} = 2$$

$$a_{31} + a_{13} = 6 \Rightarrow a_{13} = 4, a_{31} = 2$$

$$a_{23} + a_{32} = 8 \Rightarrow a_{23} = 2, a_{32} = 6$$

diagonal element

$$2a_{11} = 10 ; 2a_{22} = 6 ; 2a_{33} = 4$$

$$a_{11} = 5 ; a_{22} = 3 ; a_{33} = 2$$

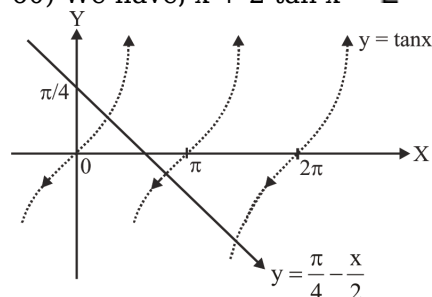
$$A = \begin{pmatrix} 5 & 2 & 4 \\ 2 & 3 & 2 \\ 2 & 6 & 2 \end{pmatrix}$$

$$|A| = 5(6 - 12) - 2(4 - 4) + 4(12 - 6)$$

$$= -30 + 24 = -6$$

$$|\det(A)| = 6$$

$$60) \text{ We have, } x + 2 \tan x = \frac{\pi}{2} \Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$$



Now, the graph of the curve $y = \tan x$ and

$y = \frac{\pi}{4} - \frac{x}{2}$, in the interval $[0, 2\pi]$ intersect at three points. The abscissa of these three points. The abscissa of these three points are the roots of the equation.
Hence, (C) is the correct answer.

$$61) \text{ Numerator} = 2[(\sin 1^\circ + \sin 89^\circ) + (\sin 2^\circ + \sin 88^\circ) + \dots + (\sin 44^\circ + \sin 46^\circ) + \sin 45^\circ]$$

$$\begin{aligned} & \frac{\text{Numerator}}{\text{Denominator}} \\ &= \frac{2 [\sin 45^\circ \{2(\cos 44^\circ + \cos 43^\circ + \dots + \cos 1^\circ) + 1\}]}{2(\cos 44^\circ + \cos 43^\circ + \dots + \cos 1^\circ) + 1} \\ &= 2 \sin 45^\circ = \sqrt{2} \end{aligned}$$

$$62) \sum_{i=1}^{10} (x_i - 5) = 10$$

$$x_i - 5 = \frac{1}{10} \sum_{i=1}^{10} (x_i - 5) = 1$$

\Rightarrow Mean of observation

$\Rightarrow \mu = \text{mean of observation } (x_i - 3)$

$= (\text{mean of observation } (x_i - 5)) + 2 = 1 + 2 = 3$

Variance of observation

$$x_i - 5 = \frac{1}{10} \sum_{i=1}^{10} (x_i - 5)^2 - (\text{Mean of } (x_i - 5))^2 = 3$$

$\Rightarrow \lambda = \text{variance of observation } (x_i - 3)$

$= \text{variance of observation } (x_i - 5) = 3$

$\therefore (\mu, \lambda) = (3, 3)$

$$63) I = \int x \left(\frac{\ln a a^{x/2}}{3a^{5x/2}b^{3x}} + \frac{\ln b b^x}{2a^{2x}b^{4x}} \right) dx = \int \frac{\ln a^{2x} b^{3x}}{6a^{2x}b^{3x}} dx$$

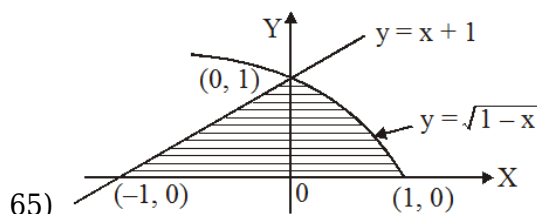
Let $a^{2x} b^{3x} = t$. Then $t \ln a^{2x} b^{3x} dx = dt$. Therefore,

$$\begin{aligned} I &= \int \frac{1}{6 \ln a^{2x} b^{3x}} \frac{\ln t}{t^2} dt \\ &= \frac{1}{6 \ln a^{2x} b^{3x}} \left(\frac{-\ln t}{t} - \int \frac{-1}{t^2} dt \right) \\ &= -\frac{1}{6 \ln a^{2x} b^{3x}} \left(\frac{\ln et}{t} \right) + k \\ &= -\frac{1}{6 \ln a^{2x} b^{3x}} \left(\frac{\ln a^{2x} b^{3x} e}{a^{2x} b^{3x}} \right) + k \end{aligned}$$

$$64) I = \int_{\pi}^{\pi + \pi/6} (-1) dx + \int_{\pi + \pi/6}^{\pi + \pi/2} (-2) dx$$

$$+ \int_{\pi + \pi/2}^{\pi + \pi/2 + \pi/3} (-2) dx + \int_{x + \pi/2 + \pi/3}^{2\pi} (-1) dx$$

$$= -\frac{\pi}{6} - 2\left(\frac{\pi}{2} - \frac{\pi}{6}\right) - 2\left(\frac{\pi}{3}\right) - \left(\frac{\pi}{2} - \frac{\pi}{3}\right) = -5\pi/3$$



Required area = shaded region = $\int_0^1 (x_2 - x_1) dy$

$$= \int_0^1 [(1 - y^2) - (y - 1)] dy = \frac{7}{6} \text{ sq. units}$$

66) From the given equation

$$\frac{dy}{dx} = \frac{y \sin(y/x) - x}{x \sin(y/x)}$$

Now $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

and transformed equation will be

$$v + x \frac{dv}{dx} = \frac{vx \sin v - x}{x \sin v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \sin v - 1}{\sin v} - v = -\frac{1}{\sin v}$$

$$\Rightarrow \sin v dv + \frac{dx}{x} = 0$$

$$\Rightarrow -\cos v + \log x = c \quad [\text{by integration}]$$

$$\log x - \cos(y/x) = c$$

But when $x = 1$, $y = \pi/2$, so

$$0 - \cos(\pi/2) = c \Rightarrow c = 0$$

$$\square \log x - \cos(y/x) = 0$$

$$\Rightarrow \cos(y/x) = \log x$$

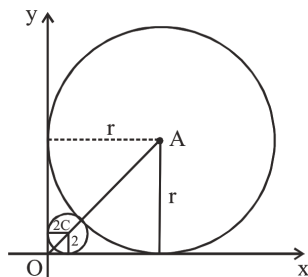
67) Vector perpendicular to plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ & $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

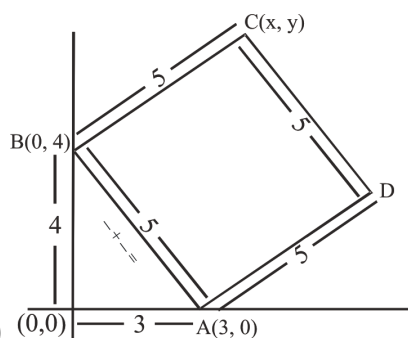
\square Required magnitude of projection

$$= \frac{|(2\hat{i} + 3\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + \hat{k}|}$$

$$= \frac{|2 - 6 + 1|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$



68) $OA^2 = 2r^2 = (OC + CA)^2$
 $2r^2 = OC^2 + CA^2 + 2 \cdot OC \cdot CA$
 $2r^2 = 8 + (r + 2)^2 + 2 \cdot 2\sqrt{2}(r + 2)$
 By solving this eq.
 $r = 6 + 4\sqrt{2}$



69) Slope of AB is $-4/3$
 $BC \perp AB$ n slope of BC $= 3/4 = \tan \theta$
 Co-ordinate of c $(x_1 + r \cos \theta, y_1 + r \sin \theta)$
 $c(0 + 5 \cdot \frac{3}{5}, 4 + 5 \cdot \frac{4}{5}) \Rightarrow c(4, 7)$

70) $e = \sqrt{1 + \tan^2 \theta} = \sec \theta$

As, $\sec \theta > 2 \Rightarrow \cos \theta < \frac{1}{2}$
 $\Rightarrow \theta \in (60^\circ, 90^\circ)$

Now, $\square(L \times R) = \frac{2b^2}{a} = 2 \frac{(1 - \cos^2 \theta)}{\cos \theta}$

$= 2(\sec \theta - \cos \theta)$

Which is strictly increasing, so

$\square(L, R) \in (3, \infty)$.

71) $f(x) = \frac{x^2 - x}{x^2 + 4x}$, to find $\frac{df^{-1}(x)}{dx}$ at $x = 2$

First we have to find

$$f^{-1}(x) =$$

$$\Rightarrow x = \frac{y^2 - y}{y^2 + 4y} = \frac{y(y-1)}{y(y+4)}$$

$$\Rightarrow y = \frac{-(1+4x)}{(x-1)} = \frac{(1+4x)}{(1-x)} = f^{-1}(x)$$

$$\frac{df^{-1}(x)}{dx} = \frac{(1-x)4 - (1+4x)(-1)}{(1-x)^2}$$

$$= \frac{4 - \cancel{4x} + 1 + \cancel{4x}}{(1-x)^2}$$

$$\left. \frac{df^{-1}(x)}{dx} \right]_{\text{at } x=2} = \frac{5}{(1-2)^2} = 5$$

$$72) \quad a = 1 - \frac{{}^3C_5}{{}^{12}C_5}$$

$$b = 3 \cdot \frac{{}^9C_4}{{}^{12}C_5}$$

$$c = 3 \cdot \frac{{}^9C_3}{{}^{12}C_5}$$

$$d = 1 \cdot \frac{{}^9C_2}{{}^{12}C_5}$$

$$u = 0.a + 1.b + 2.c + 3.d = 1.25$$

$$\sigma^2 = 0.a + 1.b + 4.c + 9.d - u^2$$

$$\sigma^2 = \frac{105}{176}$$

$$\text{Ans. } 176 - 105 = 71$$

73)

$$\text{Variance of first 'n' natural numbers} = \frac{n^2 - 1}{12} = 10$$

$$\Rightarrow n = 11$$

and variance of first 'm' even natural numbers

$$= 4 \left(\frac{m^2 - 1}{12} \right) \Rightarrow \frac{m^2 - 1}{3} = 16 \Rightarrow m = 7$$

$$m + n = 18$$

$$74) \int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx = \int \sqrt{1 + 2 \sin \frac{x}{8} \cos \frac{x}{8}} dx$$

$$\int \sqrt{\sin^2 \frac{x}{8} + \cos^2 \frac{x}{8} + 2 \sin \frac{x}{8} \cos \frac{x}{8}} dx$$

$$\int \sqrt{\left(\sin \frac{x}{8} + \cos \frac{x}{8}\right)^2} dx = \int \left(\sin \frac{x}{8} + \cos \frac{x}{8}\right) dx$$

$$= -8 \cos\left(\frac{x}{8}\right) + 8 \sin\left(\frac{x}{8}\right) + C$$

$$= 8 \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + C$$

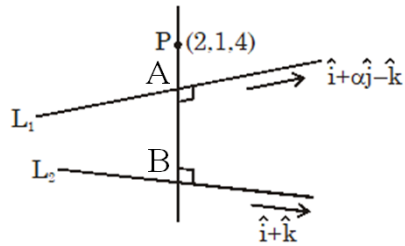
75) Line passes through P(2,1,4) lie along shortest distance between two non-coplanar lines

$$L_1 \equiv \frac{x-6}{1} = \frac{y}{\alpha} = \frac{z}{-1} = k_1$$

$$\Rightarrow A(6 + k_1, \alpha k_1, -k_1)$$

$$L_2 \equiv \frac{x}{1} = \frac{y}{0} = \frac{z}{1} = k_2$$

$$\Rightarrow B(k_2, 0, k_2)$$



PB perpendicular to $L_2 \Rightarrow k_2 = 3$

PA perpendicular to L_1

$$\Rightarrow (6 + k_1 - 2)1 + (\alpha k_1 - 1) \cdot \alpha + (-k_1 - 4)(-1) = 0$$

$$\alpha^2 k_1 - \alpha + 2k_1 + 8 = 0 \quad \dots (1)$$

AB perpendicular to L_1

$$(6 + k_1 - 3) + \alpha(\alpha k_1) + (-1)(-k_1 - 3) = 0$$

$$\alpha^2 k_1 + 2k_1 + 6 = 0 \quad \dots (2)$$

$$(1) - (2) \Rightarrow \alpha = 2$$