PHYSICS

7

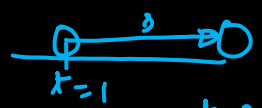
The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is

(A) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(B) $\frac{\alpha}{2} + \frac{\beta}{3}$

(C) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

(D) $3\alpha + 7\beta$



JEE 1

$$\frac{ds}{dt} = \alpha t + \beta t^2$$

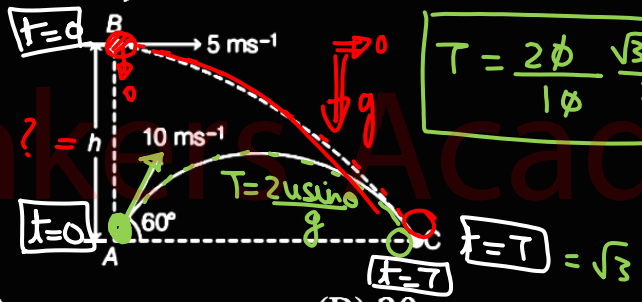
$$\int_0^s ds = \alpha \int_1^2 t \cdot dt + \beta \int_1^2 t^2 \, dt$$

$$s = \frac{\alpha t^2}{2}\Big|_1^2 + \frac{\beta t^3}{3}\Big|_1^2$$

$$s = \frac{3\alpha}{2} + \frac{7}{3}\beta$$

2

A particle A is projected from the ground with an initial velocity of $10\ ms^{-1}$ at an angle of $60°$ with horizontal. From what height h should an another particle B be projected horizontally with velocity $5\ ms^{-1}$, so that both the particles collide with velocity $5\ ms^{-1}$ on the ground at point C, if both are projected simultaneously? $(g = 10\ ms^{-2})$



(A) 10 m
(B) 30 m
(C) 15 m
(D) 25 m

Y

$h = \frac{1}{2} g T^2$

$T = \sqrt{\frac{2h}{g}} = \sqrt{3}$

$\frac{2h}{10} = 3$

$h = 15$

JEE 1

3

The force is given in terms of time t and displacement x by the equation

$F = A\cos(Bx) + C\sin(Dt)$

The dimensional formula of $\left[\frac{AD}{B}\right]$ is

(A) $[M^2 L^2 T^{-3}]$ (B) $[M^1 L^1 T^{-2}]$

(C) $[M^0 L T^{-1}]$ (D) $[ML^2 T^{-3}]$

$[x] = L$.

$[t] = T$

$M^0 L^0 T^0$

Ans $= \frac{[A][D]}{[B]} = ?$

$[B][x] = 1$

$[B]L = 1$

$[B] = L^{-1}$

$[D][t] = 1$

$[D] = T^{-1}$

$[F] = [MLT^{-2}] = [A] = [C]$

$$\text{Ans} = \frac{[A][D]}{[B]} = \frac{MLT^{-2}T^{-1}}{L^{-1}} = ML^2T^{-3}$$

JEE 1

4

A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

(A) 12.25%

(B) 16.5%

(C) 13.5%

(D) 14.5%

$z = a^{2} b^{2/3} c^{-1/2} d^{-3}$

Error

$\left|\frac{\Delta z}{z}\right| \times 100 = 2\left|\frac{\Delta a}{a}\right| \times 100 + \frac{2}{3}\left|\frac{\Delta b}{b}\right| \times 100 + \frac{1}{2}\left|\frac{\Delta c}{c}\right| \times 100 + 3\left|\frac{\Delta d}{d}\right| \times 100$

% error 'z' = (2) × 2% + (2/3)(3/2 %) + (1/2) 4% + (3) 2.5%

$\left|\frac{\Delta z}{z}\right| \times 100$

5

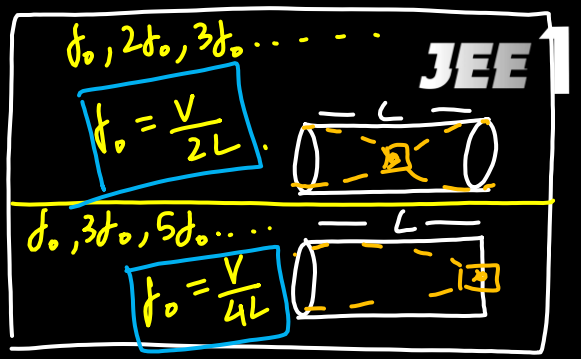
A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

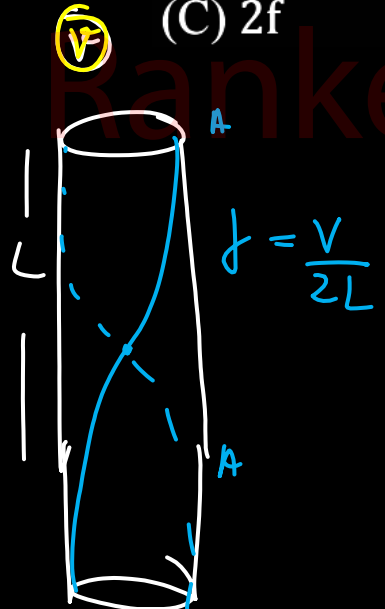
(A) $\frac{f}{2}$

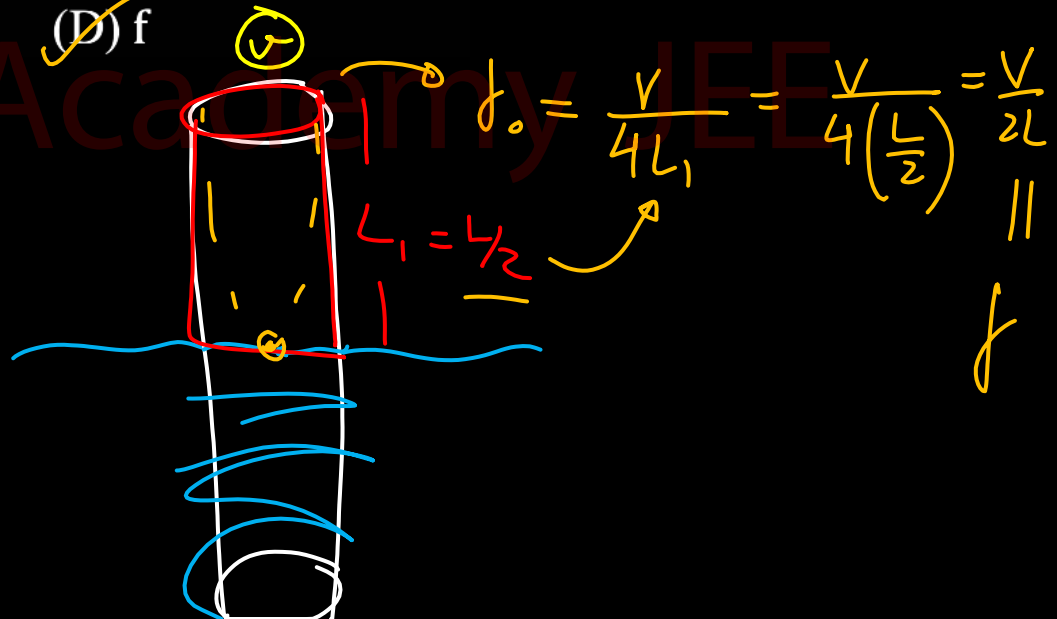
(B) $\frac{3f}{4}$

(C) 2f

(D) f

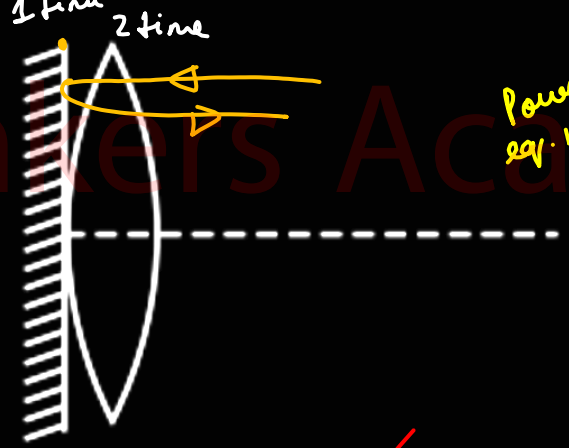






6

A thin convex lens of focal length f is put on a plane mirror as shown in the figure. When an object is kept at a distance a from the lens mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of a is

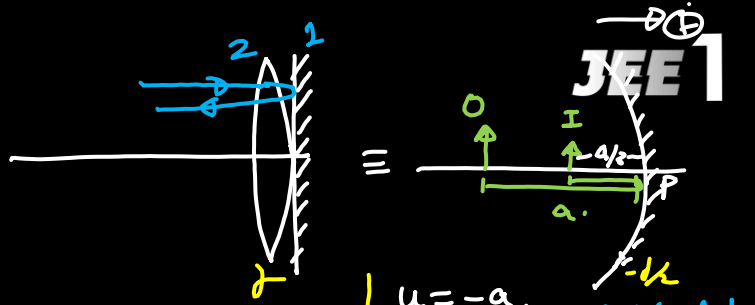


(A) $f = a$

(B) $2f = a$

(C) $3f = a$

(D) $\frac{3}{2}f = a$



$P_{eq} = 2P_L + P_M$

$P_{eq} = 2\left(\frac{1}{f}\right) + 0$

Power of eq. mirror.

$-\frac{1}{f_{eq}} = \frac{2}{f}$

$f_{eq} = -f/2$

$u = -a$

$v = -a/3$

$f = -f/2$

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

equivalent

$\frac{1}{(-f/2)} = \frac{1}{-a/3} + \frac{1}{-a}$

$\frac{2}{f} = \frac{4}{a}$

$a = 2f$

7

Two wires of same length (L) and radius (A) are joined end to end and loaded. The Young's moduli of the materials of the two wires are Y_1, and Y_2. The combination behaves as a single wire then its Young's modulus is

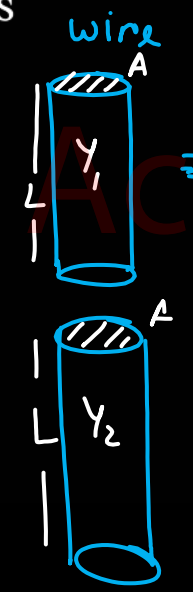
$K = \frac{YA}{L}$

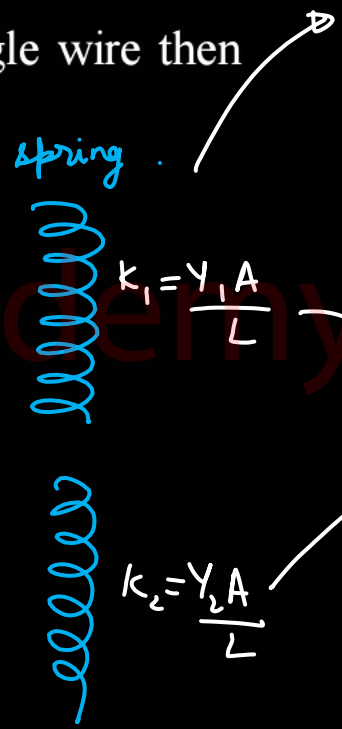
(A) $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$ ✓

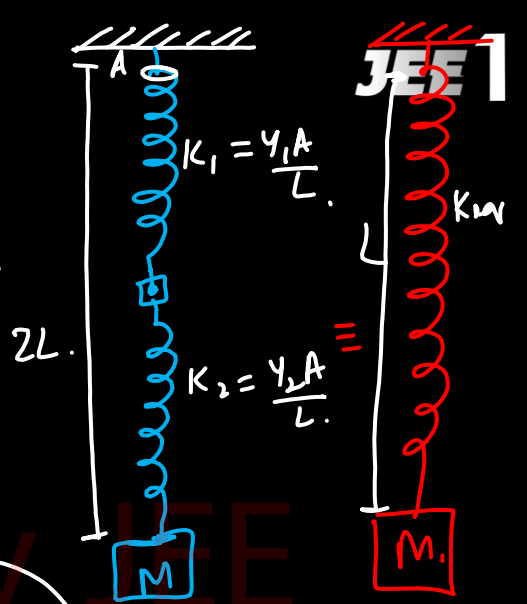
(B) $Y = \frac{Y_1Y_2}{Y_1+Y_2}$ ✗

(C) $Y = \frac{Y_1Y_2}{2(Y_1+Y_2)}$ ✗

(D) $Y = \frac{2Y_1Y_2}{3(Y_1+Y_2)}$







$K_{eq} = \frac{K_1K_2}{K_1+K_2}$

$\frac{Y(2A)}{2L} = \frac{\left(\frac{A}{L}\right)^2 Y_1Y_2}{\left(\frac{A}{L}\right)(Y_1+Y_2)}$

$Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

8

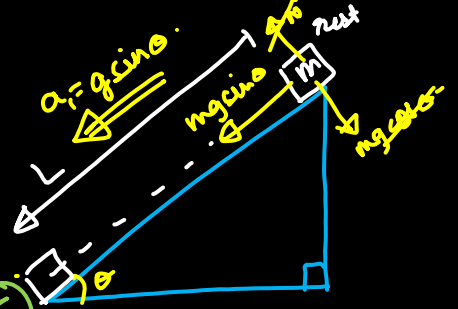
A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

$\theta = 45°$

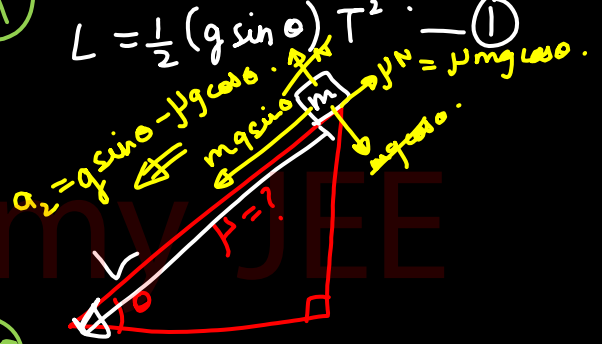
$\sin\theta = \cos\theta$

(A) $\sqrt{1-\frac{1}{n^2}}$ (B) $1-\frac{1}{n^2}$

(C) $\frac{1}{2-n^2}$ (D) $\sqrt{\frac{1}{1-n^2}}$



$L = \frac{1}{2}(g\sin\theta)T^2$ — (1)



$L = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)(nT)^2$ — (2)

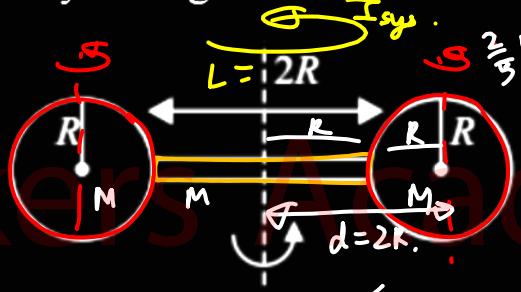
$\frac{1}{2}g\sin\theta T^2 = \frac{1}{2}g(\sin\theta - \mu\cos\theta)n^2T^2$

$\frac{1}{n^2} = 1-\mu$

$\mu = 1-\frac{1}{n^2}$

9

Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is



(A) $\frac{209}{15}MR^2$

(B) $\frac{137}{15}MR^2$

(C) $\frac{152}{15}MR^2$

(D) $\frac{17}{15}MR^2$

$I_{sys} = I_{rod} + 2\,I_{sphere}$

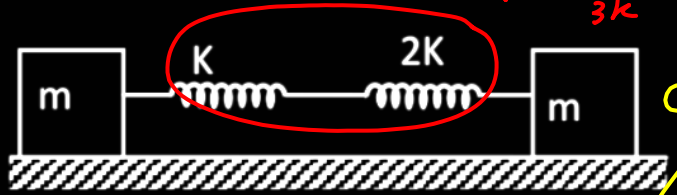
$I_{sys.} = \frac{ML^2}{12} + 2\left[\frac{2}{5}MR^2 + Md^2\right]$

$I_{sys} = \frac{4MR^2}{12} + 2\left[\frac{2}{5}MR^2 + M(4R^2)\right]$

$I_{sys} = \frac{137}{15}MR^2$

10

A system is shown in the figure. The time - period for small oscillations of the blocks will be



$K_{eq} = \frac{K(2K)}{3K} = \frac{2K}{3}$

$m_{red} = \frac{m \cdot m}{m+m} = \frac{m}{2}$

$T = 2\pi\sqrt{\frac{m_{red}}{K_{eq}}}$

(A) $2\pi\sqrt{\frac{3m}{K}}$

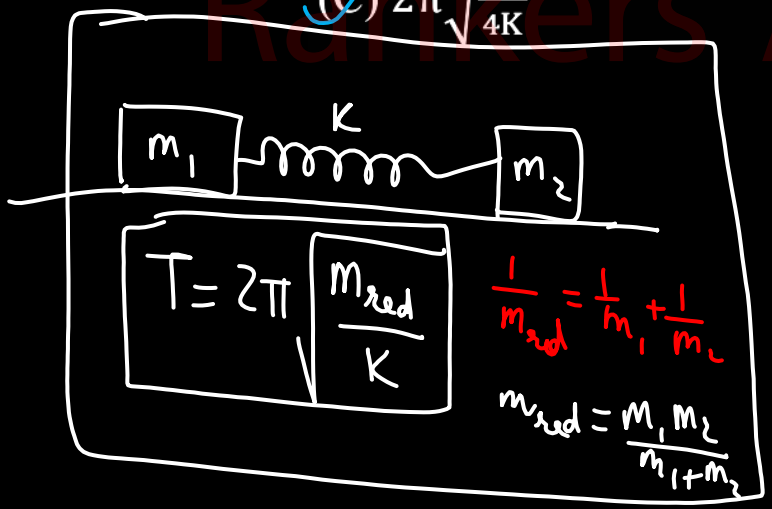
(B) $2\pi\sqrt{\frac{3m}{2K}}$

(C) $2\pi\sqrt{\frac{3m}{4K}}$

(D) $2\pi\sqrt{\frac{3m}{8K}}$

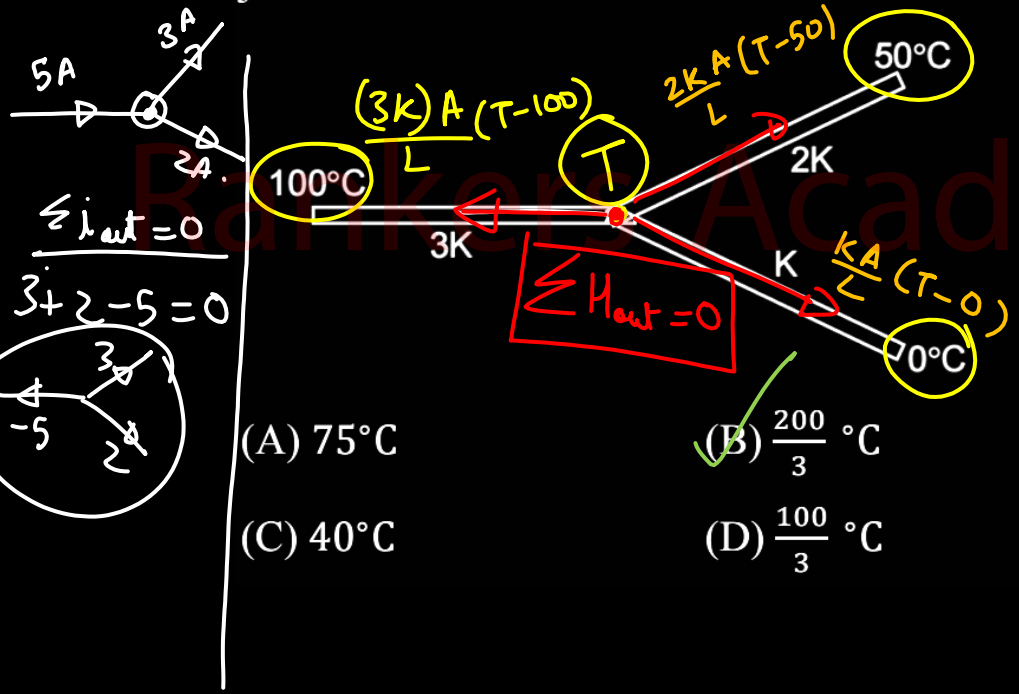
Ans = $T = 2\pi\sqrt{\frac{\left(\frac{m}{2}\right)}{\left(\frac{2K}{3}\right)}}$

$= 2\pi\sqrt{\frac{3}{4}\frac{m}{K}}$.



11

Three rods of the same dimensions have thermal conductivities 3 K, 2 K and K. They are arranged as shown, with their ends at 100°C, 50°C and 0°C. The temperature of their junction is



(A) 75°C

(B) $\frac{200}{3}$ °C

(C) 40°C

(D) $\frac{100}{3}$ °C

Heat Current

JEE 1

$$H=\left(\frac{KA}{L}\right)\Delta T$$

$\Sigma H_{out}=0$.

$$\frac{3KA}{L}(T-100)+\frac{2KA}{L}(T-50)+\frac{KA}{L}T=0$$

$$3T-300+2T-100+T=0$$

$$6T=400$$

$$T=\frac{200}{3}$$

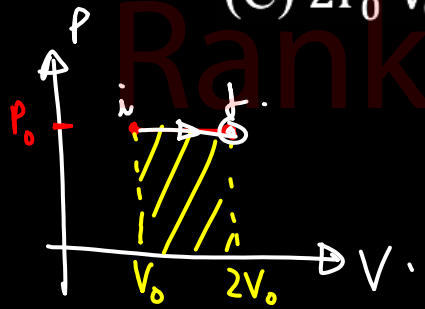
12

The internal energy of a gas is given by $U = 5 + 2PV$. It expands from V_0 to $2V_0$ against a constant pressure P_0. The heat absorbed by the gas in the process is

$Q = ?$

(A) $-3P_0V_0$ (B) $3P_0V_0$

(C) $2P_0V_0$ (D) P_0V_0



$U = 5 + 2PV$

$\Delta U = U_f - U_i$

$U_f = 5 + 2P_0(2V_0)$

$U_i = 5 + 2P_0(V_0)$

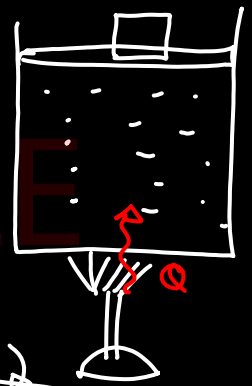
$\Delta U = 2P_0V_0$

isobaric process.

$W = P\Delta V$

$W = P_0(2V_0 - V_0)$

$W = P_0V_0$



Ans

$Q = \Delta U + W$

$Q = 2P_0V_0 + P_0V_0$

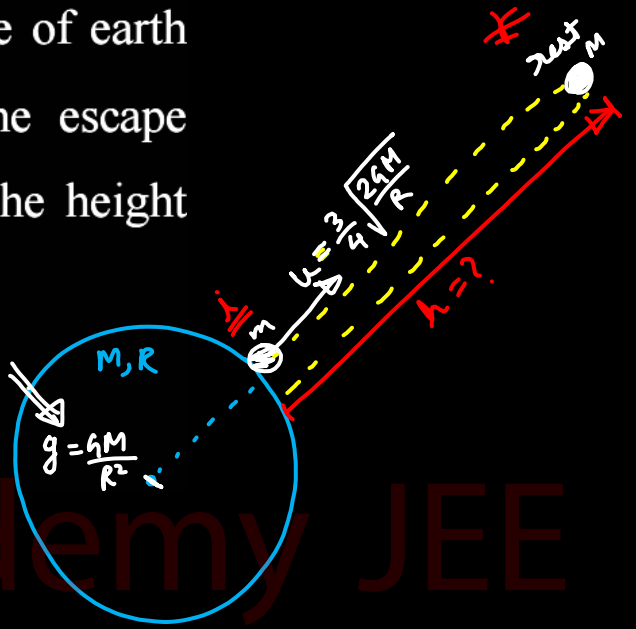
$Q = 3P_0V_0$

13

A body is thrown up from the surface of earth with a velocity equal to $3/4^{th}$ of the escape velocity from the surface of earth. The height attained by the body is

(A) 7R/9 (B) 8R/9

(C) 11R/17 (D) 9R/7



$$V_{esc} = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$$

$$E_i = E_f$$

$$\frac{1}{2}m\left(\frac{3}{4}\sqrt{\frac{2GM}{R}}\right)^2 + \left(-\frac{GMm}{R}\right) = \left(-\frac{GMm}{R+h}\right)$$

$$\frac{9}{16}\left(\frac{1}{R}\right) = \left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\frac{1}{R+h} = \frac{7}{16R}$$

$$16R = 7R + 7h.$$

$$\frac{9R}{7} = h$$

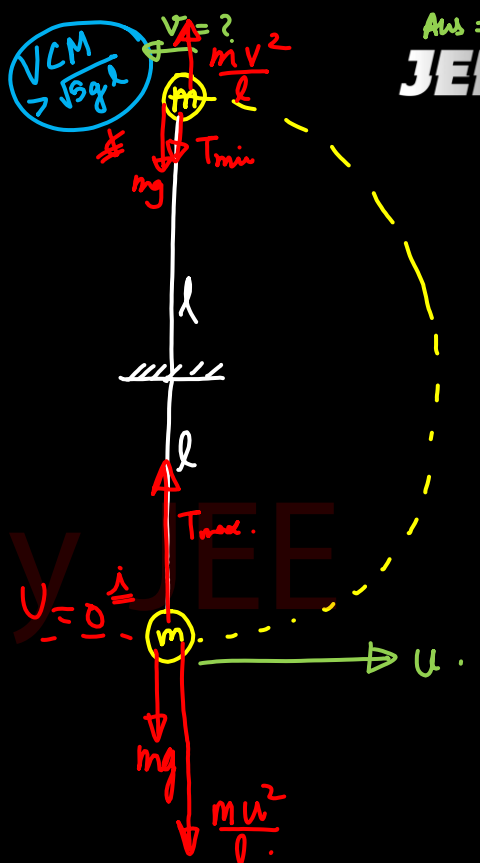
14

A stone of mass 1 kg tied to a light string of length $\frac{10}{3}$ m is whirled in a vertical circle. If the ratio of maximum tension to minimum tension is 4 and ($g = 10\ m/s^2$) then the speed of the stone at the highest point of circle is:

(A) $5\sqrt{3}$ m/s (B) 10 m/s

(C) $10\sqrt{3}$ m/s (D) 20 m/s

$\frac{T_{max}}{T_{min}} = \frac{4}{1}$



Energy

$E_i = E_f$

$\frac{1}{2}mu^2 = mg(2l) + \frac{1}{2}mv^2$

$u^2 = 4gl + v^2$ ☺ — (1)

Force

$T_{max} = mg + \frac{mu^2}{l}$

$T_{min} = \frac{mv^2}{l} - mg$

$\frac{T_{max}}{T_{min}} = \frac{4}{1} = \frac{gl + u^2}{v^2 - gl}$ — (2)

(1) in (2)

$\frac{4}{1} = \frac{5gl + v^2}{v^2 - gl}$

$4v^2 - 4gl = v^2 + 5gl$

$3v^2 = 9gl$.

$v = \sqrt{3gl} = \sqrt{30 \times \frac{10}{3}} = \boxed{10}$ Ans

15

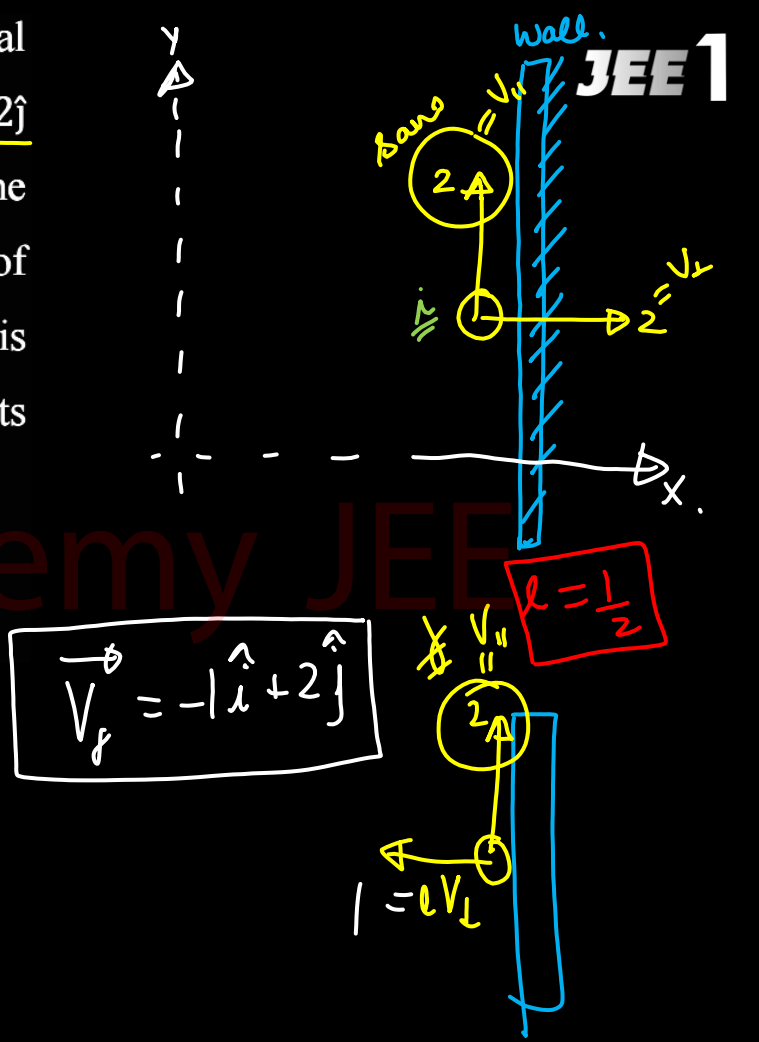
A smooth sphere is moving on a horizontal surface with velocity vector $2\hat{i} + 2\hat{j}$ immediately before it hits a vertical wall. The wall is parallel to $\hat{j}$ vector and the coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity of the sphere after it hits the wall is

(A) $\hat{i} - \hat{j}$

(B) $-\hat{i} + 2\hat{j}$

(C) $-\hat{i} - \hat{j}$

(D) $2\hat{i} - \hat{j}$



16

A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is:

(A) 2:1
(B) 2:3
(C) 1:2
(D) 1:4

JEE 1

$E_1 = \frac{1}{2} C V^2 = \frac{1}{2} 2 \times V^2 = V^2$ — (1)

$E_2 = \frac{1}{2} C_{eq} \left(\frac{V}{2}\right)^2$

$= \frac{1}{2} 2C \frac{V^2}{4} = \frac{V^2}{2}$ — (2)

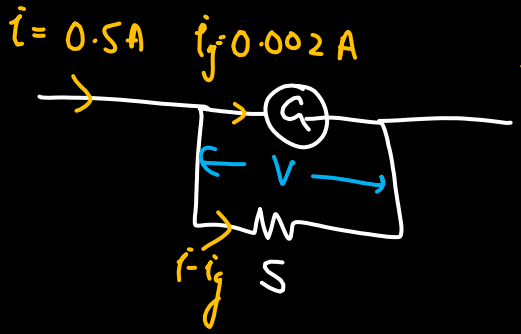
$\frac{E_2}{E_1} = \frac{V^2/2}{V^2} = \frac{1}{2}$ //

17

JEE 1

The resistance of a galvanometer is 50ohm and the maximum current which can be passed through it is i_g 0.002A. What resistance must be connected to it in order to convert it into an ammeter of range $0 - 0.5$ A?

(A) 0.002ohm
(B) 0.02ohm
(C) 0.2ohm
(D) 0.5ohm



$V = i_g G = (i - i_g) S$

as $i_g << i$

$i_g G = i S$

$\Rightarrow (0.002)50 = (0.5) S \Rightarrow S = \frac{0.002 \times 500}{0.5} = 0.2\,\Omega$

18

JEE 1

A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = 1.6×10^{-19}C)

(A) 2.0×10^{-24} kg (B) 1.6×10^{-19} kg

(C) 1.6×10^{-27} kg (D) 9.1×10^{-31} kg

$R = \frac{mv}{qB}$ -① | $E = Bv$ -②

$E = B\left(\frac{qBR}{m}\right)$ → $m = \frac{qRB^2}{E}$

$= \frac{1.6 \times 10^{-19} \times 5 \times 10^{-3} \times 1}{100} \times \frac{1}{0.4}$

$= 2 \times 10^{-24}$ kg

19

The correct match between the entries in column I and column II are

I Radiation		II Wavelength	
(A)	Microwave	(i)	100 m
(B)	Gamma rays	(ii)	10^{-15} m
(C)	A.M. radio waves	(iii)	10^{-10} m
(D)	X-rays	(iv)	10^{-3} m

(A) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)

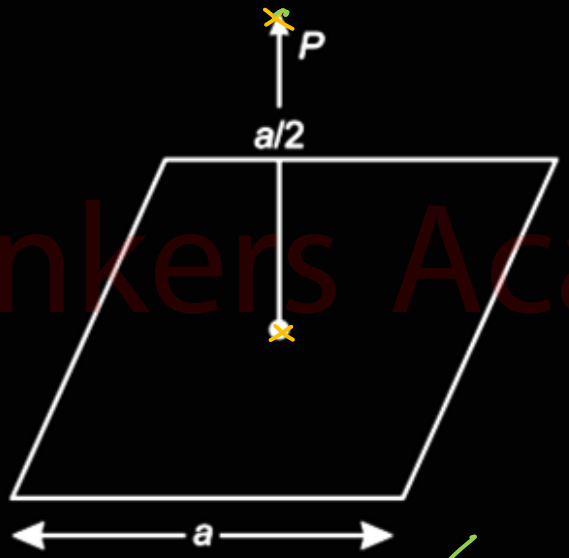
(B) (A)-(i), (B)-(iii)/(C)-(iv), (D)-(ii)

(C) (A)-(iii), (B)-(ii), (C)-(i), (D)-(iv)

(D) (A)-(iv), (B)-(ii), (C)-(i), (D)-(iii)

20

A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is

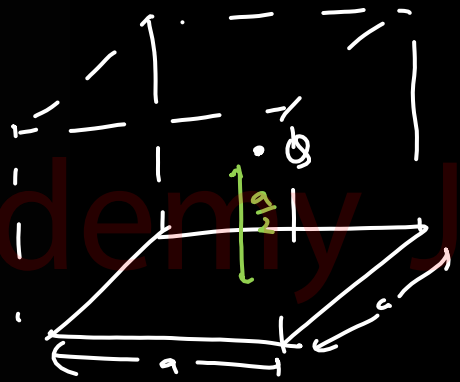


(A) $\frac{Q}{3\varepsilon_0}$

(B) $\frac{Q}{6\varepsilon_0}$

(C) $\frac{Q}{\varepsilon_0}$

(D) $\frac{Q}{2\varepsilon_0}$



$\phi_{cube} = \frac{Q}{\varepsilon_0}$

$\phi_{face} = \frac{1}{6}\phi_{cube}$

$= \frac{Q}{6\varepsilon_0}$

21

A ray of light passing through a prism ($\mu = \sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, angle of prism is _____ (in degrees).

JEE 1

$i = 2r$

$1 \cdot \sin i = \sqrt{3} \sin r$

$\sin(2r) = \sqrt{3} \sin r$

$r = 30°$

δ_{min} $r = r'$

also $r + r' = A$

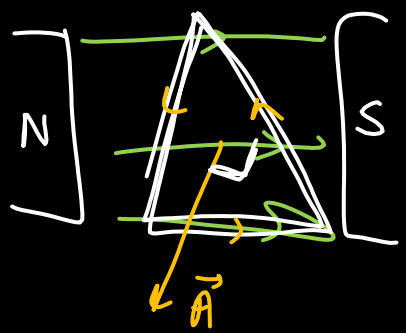
$r = r' = \frac{A}{2}$

$30° = \frac{A}{2}$

$A = \boxed{60°}$

22

A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5}$ N m. The value of x is ____



$\tau = MB\sin 90°$

$= iAB$

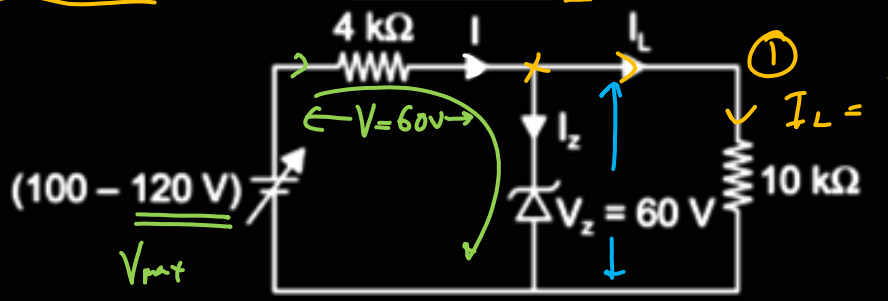
$= 0.2 \times \frac{\sqrt{3}}{4} \times (0.1)^2 \times 20 \times 10^{-3}$

$= \sqrt{3} \times 10^{-5}$

JEE 1

23

In the circuit shown below, maximum zener diode current will be _____ mA.



① $I_L = \frac{V_z}{R_L} = \frac{60\,V}{10\,k\Omega} = 6\,mA$

② $I = \frac{V}{R} = \frac{60\,V}{4k\Omega} = 15\,mA$

③ $I = I_L + I_z$

$I_z = I - I_L = 15 - 6$

$= 9\,mA$

24

A capacitor of capacitance 500μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50mH to form an LC circuit. The maximum current in LC circuit will be____ A.

JEE 1

$U_L = U_C$

$\frac{1}{2} L i_0^2 = \frac{1}{2} C V^2$

$i_0 = \sqrt{\frac{CV^2}{L}} = \sqrt{\frac{C}{L}} V$

$i_0 = Q_0 \omega$

$= \frac{CV}{\sqrt{LC}}$

$= \sqrt{\frac{C}{L}} V$

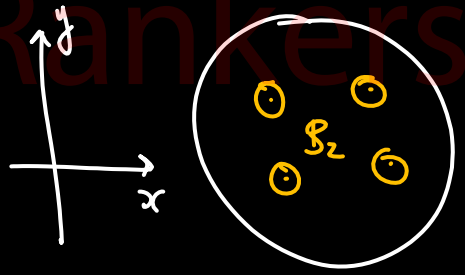
$\sqrt{\frac{500 \times 10^{-6}}{50 \times 10^{-3}}} \times 100$

$= 10^{-1} \times 100 = \boxed{10}$ A

25

A conducting circular loop is placed in X – Y plane in presence of magnetic field $\vec{B} = (3t^3\hat{j} + 3t^2\hat{k})$ in SI unit. If the radius of the loop is 1 m, the induced emf in the loop, at time t = 2 s is nπV. The value of n

JEE 1



$\phi = \vec{B} \cdot \vec{A}$

$= B_2(\pi r^2)$

$\phi = 3t^2 \pi r^2$ — (1)

$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = 6t\pi r^2 = 6(2)\pi(1^2) = \boxed{12}\pi\ V$

CHEMISTRY

1

The wavenumber of the first Balmer line of Li^{2+} ion is 1,36,800 cm^{-1}. The wavenumber of the first line of Balmer series of hydrogen atom is (in cm^{-1})

(A) 68,400 (B) 15,200

(C) 76,000 (D) 30,800

JEE 1

$$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = R_H (3)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 136800\ cm^{-1}$$

$$\frac{1}{\lambda} \text{ for H atom} = \frac{136800\ cm^{-1}}{9} = 15,200\ cm^{-1}$$

Pick out the disproportionation reaction of the following:

(A) $2\ N_2O(g) \longrightarrow 2\ N_2\ (g) + O_2\ (g)$

(B) $6Li(s) + N_2\ (g) \longrightarrow 2Li_3\ N(\ s)$

(C) $AgNO_3\ (aq) + NaCl(aq) \longrightarrow AgCl(s) + HNO_3$

(D) $2NO_2\ (g) + H_2O(\ell) \longrightarrow HNO_2(aq) + HNO_3(aq)$

$N^{+4} \longrightarrow N^{+3} + N^{+5}$

The degree of dissociation of PCl_5 at one atmosphere is 0.3 . The pressure at which PCl_5 is dissociated to 50% is

(A) 2.73 atm (B) 0.3 atm

(C) 0.273 atm (D) 1.67 atm

JEE 1

$K_p = \frac{P_{PCl_3} \times P_{Cl_2}}{P_{PCl_5}}$

$K_p = \frac{\left(\frac{\alpha}{1+\alpha}\right)P \left(\frac{\alpha}{1+\alpha}\right)P}{\left(\frac{1-\alpha}{1+\alpha}\right)P} = \frac{\alpha^2}{1-\alpha^2}$

$PCl_5 \rightleftharpoons PCl_3 + Cl_2$

	PCl_5	PCl_3	Cl_2
$t=0$	1	0	0
$t=t$	$1-\alpha$	α	α
mole fraction	$\frac{1-\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$
Partial Pressure	$\left(\frac{1-\alpha}{1+\alpha}\right)P$	$\left(\frac{\alpha}{1+\alpha}\right)P$	$\left(\frac{\alpha}{1+\alpha}\right)P$

So total moles $= 1+\alpha$

$K_p = \frac{(0.3)^2}{1-(0.3)^2} \times 1 = \frac{(0.5)^2}{1-(0.5)^2} P$

$P = 0.3$ atm

4

2 moles of a perfect gas at 27°C is compressed reversibly and isothermally from a pressure of $1.01 \times 10^5 Nm^{-2}$ to $5.05 \times 10^6 Nm^{-2}$.

Maximum work done on the gas in this process is [log 5 = 0.6990]

(A) -1.95×10^4 Joule (B) $+1.95 \times 10^4$ Joule

(C) -1.95×10^3 Joule (D) $+1.95 \times 10^3$ Joule

JEE 1

log = 1.6990

$$W = 2.303\, nRT \log \frac{P_2}{P_1}$$

$$= 2.303 \times 2 \times 8.314 \times 300 \log \frac{5.05 \times 10^6}{1.01 \times 10^5}$$

$$= 2.303 \times 2 \times 8.314 \times 300 \log 50$$

$$= 1.95 \times 10^4 \text{ Joule}$$

5

Arrange the elements S, P, As in the order of increasing ionization energy

(A) S < P < As

(B) P < S < As

(C) As < S < P

(D) As < P < S

JEE 1

P S As

Order of I.E = P > S > As.

(half-filled)

6

Which of the following statements is not correct?

(A) van der Waal radius of Iodine is more than its covalent radius.

(B) IE_1 of nitrogen is higher than that of oxygen while IE_2 of oxygen is higher than that of nitrogen.

(C) All isoelectronic ions belong to the same period of periodic table.

(D) The electron affinity of nitrogen is zero while for phosphorus it is 74.3 kJ mol^{-1}.

Na^+ Mg^{2+}

Ne

Ar

N : $1s^2 2s^2 2p^3 \xrightarrow{IE_1} 1s^2 2s^2 2p^2 \xrightarrow{IE_2}$

O : $1s^2 2s^2 2p^4 \xrightarrow{IE_1} 1s^2 2s^2 2p^3 \xrightarrow{IE_2}$ ✓

(half filled)

7

Which one of the following carbocations is most stable?

(A) $CH_2 = CH - \overset{\oplus}{C}H_2$

(B) $CH_3 - CH_2 - \overset{\oplus}{C}H_2$

(C) $C_6H_5 - \overset{\oplus}{C}H - CH = CH_2$

(D) $CH_3 - \overset{\oplus}{C}H - CH_3$

8

Identify the correct order of wavelength of light absorbed for the following complex ions.

$[Co(H_2O)_6]^{3+}; [Co(CN)_6]^{3-}; [Co(I)_6]^{3-}; [Co(en)_3]^{3+}$

I II III IV

(A) III > I > IV > II

(B) II > IV > I > III

(C) III > I > II > IV

(D) I > III > IV > II

JEE 1

Ligand strength:

$CN^- > en > H_2O > I^-$

$\lambda \propto \frac{1}{\text{Energy gap}}$

order of λ

$CN^- < en < H_2O < I^-$

II < IV < I < III

9

JEE 1

Which of the following is true for cation of coinage metal?

COLUMN - I		COLUMN - II		COLUMN - III	
(P)	$Cr^{2+}(aq)$	(I)	d^4, 4.90BM	(a)	Pseudo inert gas configuration
(Q)	$Fe^{2+}(aq)$	(II)	d^{10}, 0.00BM	(b)	Undergo disproportionation in aqueous solution
(R)	$Cu^{+}(aq)$	(III)	d^6, 4.90BM	(c)	Act as reducing agent
(S)	$Mn^{3+}(aq)$	(IV)	d^3, 5.93BM	(d)	Act as oxidizing agent

(A) (R) - (II) - (b)
(B) (P) - (I) - (c)
(C) (S) - (I) - (d)
(D) (R) - (III) - (a)

$Cu(29): 3d^{10}4s^1$

$Cu^{+1} = 3d^{10}4s^0$

$Cu^{+} \xrightarrow{aq\ med} Cu + Cu^{2+}$

10

The incorrect match in the below options is:

(A) Ag – $4d^{10}5s^1$; Mo – $4d^5 5s^1$

(B) Pd – $4d^{10}5s^0$; Ni – $4s^2 3d^8$

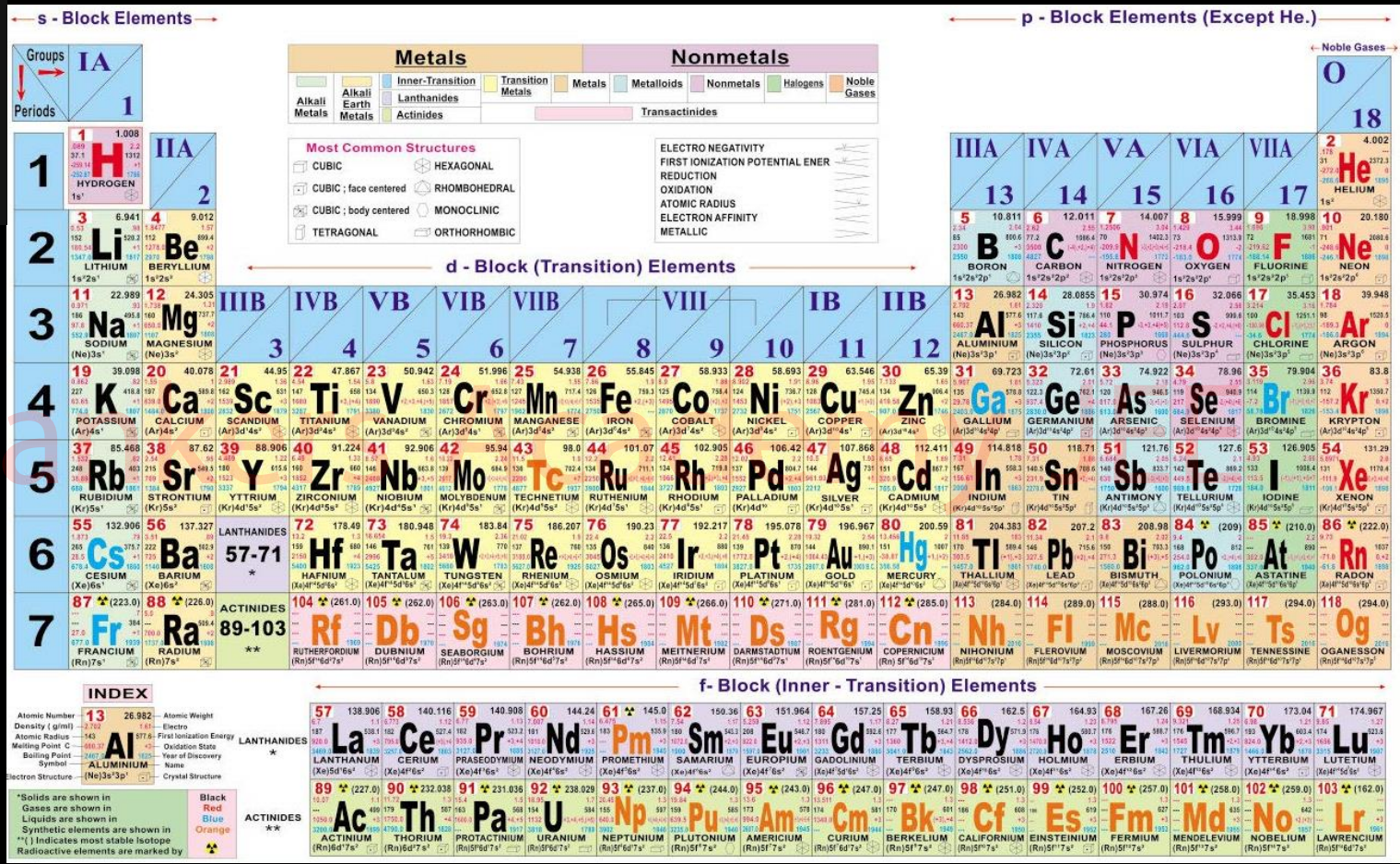
(C) Au – $5d^{10}6s^1$; W- $6s^2 5d^4$

(D) Pt – $5d^{10}6s^0$; W – $6s^1 5d^5$

JEE 1

$Pt = [Xe]\,4f^{14}\,5d^9\,6s^1$

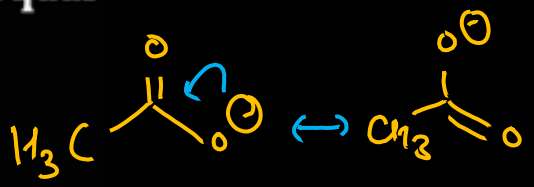
$W = [Xe]\,4f^{14}\,5d^4\,6s^2$


s - Block Elements
p - Block Elements (Except He.)
Metals
Nonmetals
Noble Gases
Groups
Periods
Alkali Metals
Alkali Earth Metals
Inner-Transition
Lanthanides
Actinides
Transition Metals
Metals
Metalloids
Nonmetals
Halogens
Noble Gases
Transactinides
Most Common Structures
CUBIC
CUBIC ; face centered
CUBIC ; body centered
TETRAGONAL
HEXAGONAL
RHOMBOHEDRAL
MONOCLINIC
ORTHORHOMBIC
ELECTRO NEGATIVITY
FIRST IONIZATION POTENTIAL ENER
REDUCTION
OXIDATION
ATOMIC RADIUS
ELECTRON AFFINITY
METALLIC
d - Block (Transition) Elements
LANTHANIDES
57-71
ACTINIDES
89-103
INDEX
Atomic Number
Density (g/ml)
Atomic Radius
Melting Point C
Boiling Point
Symbol
Electron Structure
Atomic Weight
Electro
First Ionization Energy
Oxidation State
Year of Discovery
Name
Crystal Structure
*Solids are shown in
Gases are shown in
Liquids are shown in
Synthetic elements are shown in
** ( ) Indicates most stable Isotope
Radioactive elements are marked by
Black
Red
Blue
Orange
f- Block (Inner - Transition) Elements
LANTHANIDES
ACTINIDES

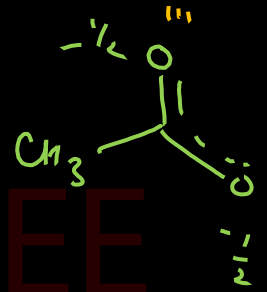
11

JEE 1

Assertion : Carbon-oxygen bonds are of equal length in acetate ion.



Reason : Bond length decreases with the multiplicity of bond between two atoms.



(A) Assertion is True, Reason is True; Reason is a correct explanation for Assertion.

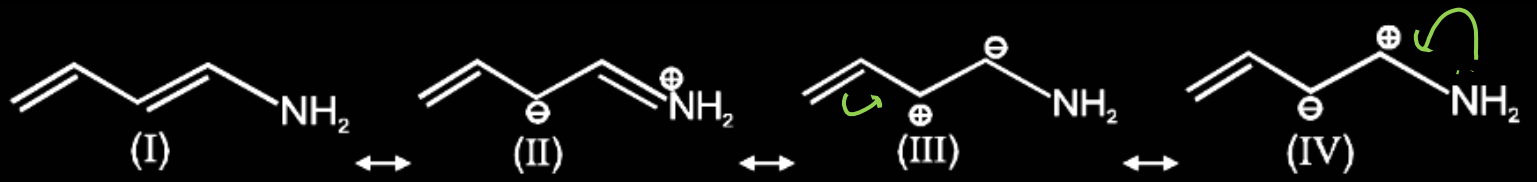
(B) Assertion is True, Reason is True; Reason is NOT a correct explanation for Assertion

(C) Assertion is True, Reason is False

(D) Assertion is False, Reason is True

12

The correct decreasing order of stability of following resonating structure :



(A) I > II > III > IV (B) I > II > IV > III

(C) I > IV > III > IV (D) I > IV > II > III

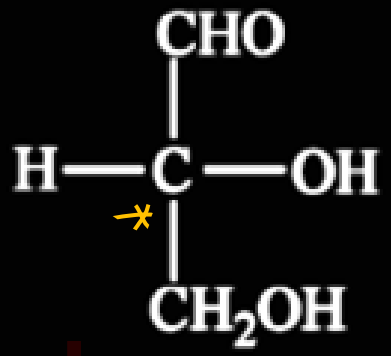
① Check no. of π-bond

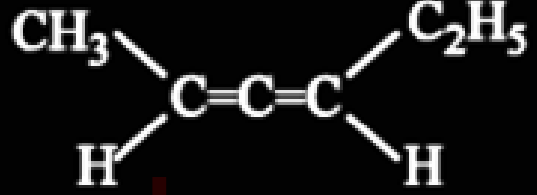
② Neutral > Charged

13

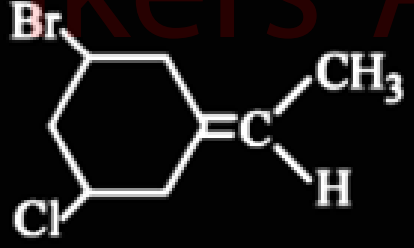
Which of the following compounds are chiral?

optically Activate
↓
Assymmetric.

(A) 

(B) 

non planar

(C) 

(D) All Of These

14

Dissolution of a non-volatile solute into a liquid leads to the –

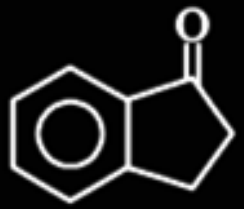
(A) decrease of entropy

(B) increase in tendency of the liquid to freeze

(C) increases in tendency to pass into the vapour phase.

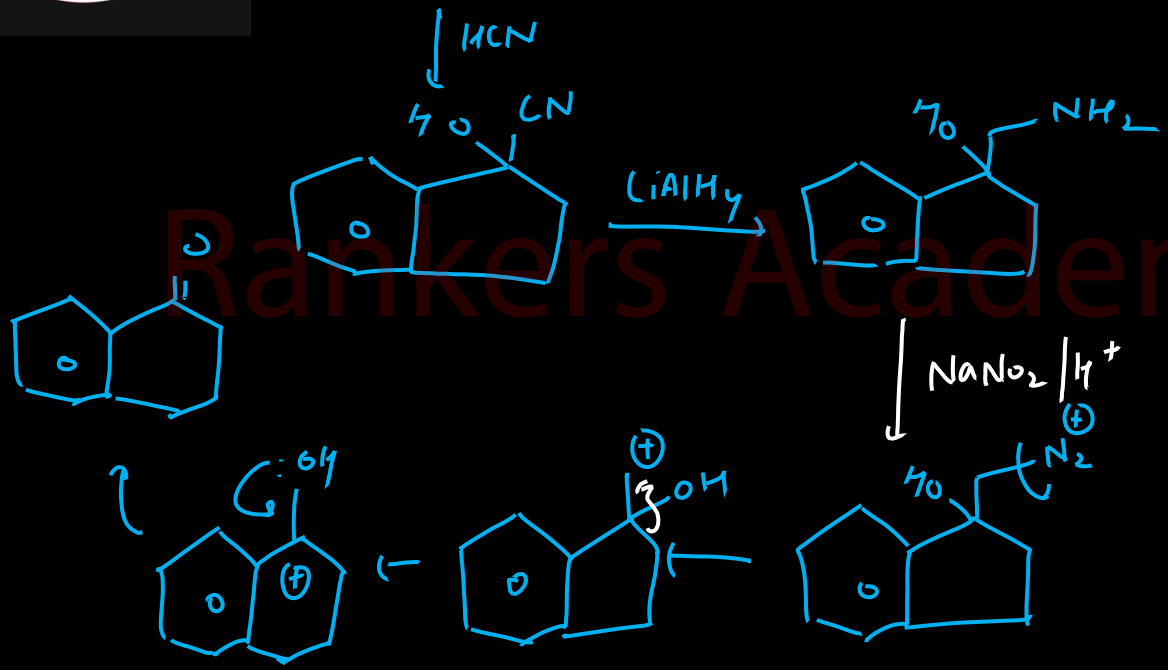
(D) decrease in tendency of the liquid to freeze

15

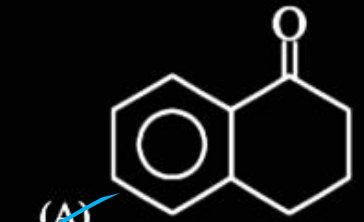


(i) HCN
(ii) $LiAlH_4$
(iii) $NaNO_2 / H^+$

(A) , Product A will be :
Major



(A)

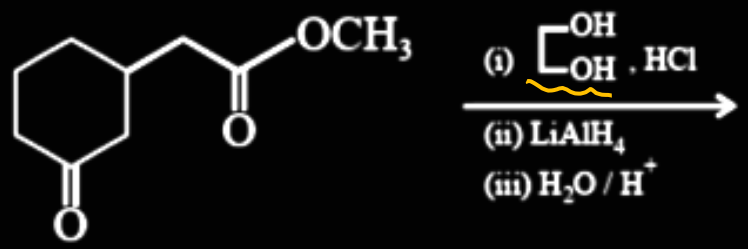


(B)

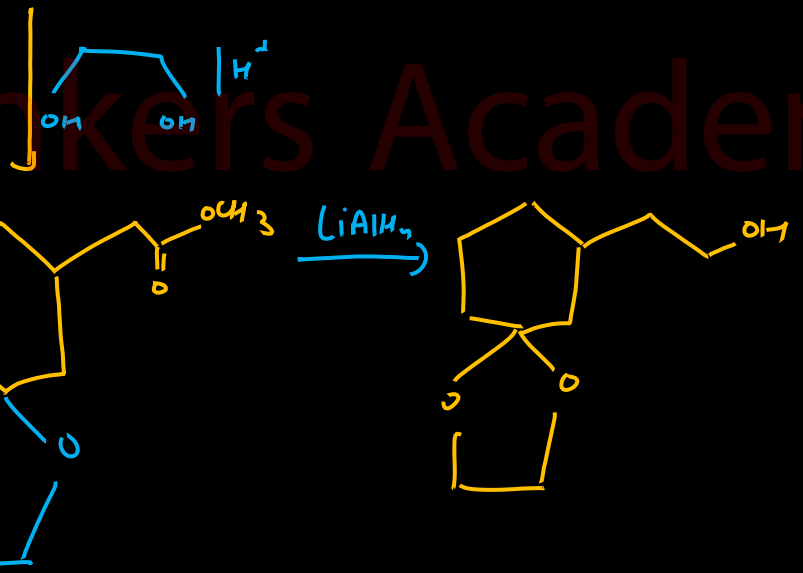
(C)

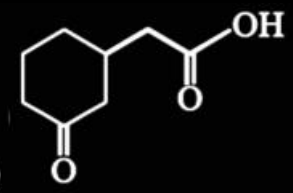
(D)

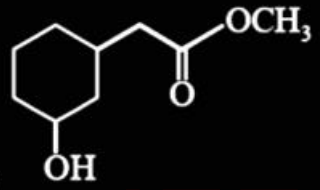
16

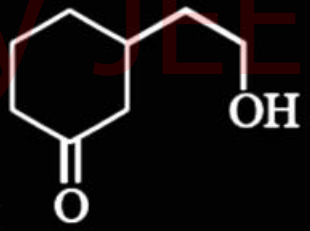


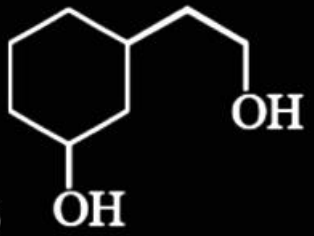
Final Major Product is ____



(A) 

(B) 

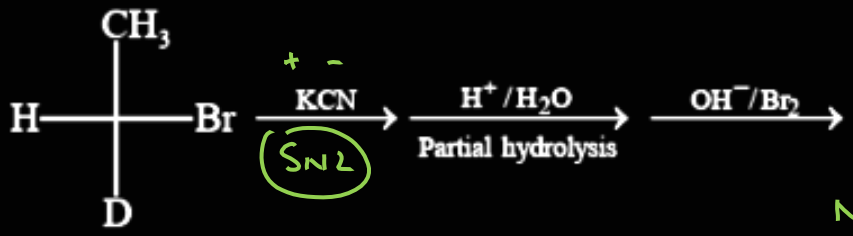
(C) 

(D) 

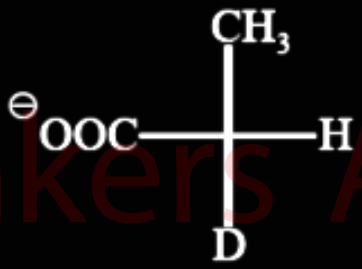
JEE 1

17

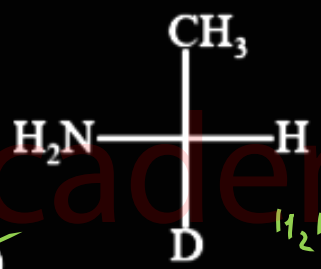
JEE 1



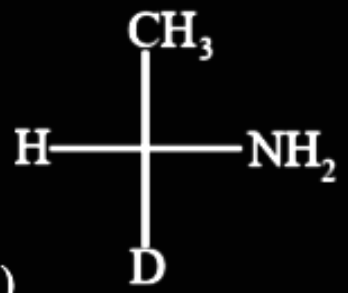
Final Product



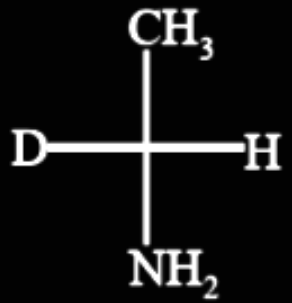
(A)



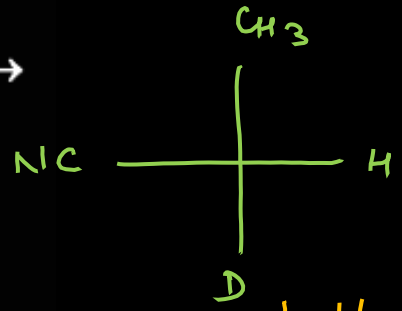
(B)



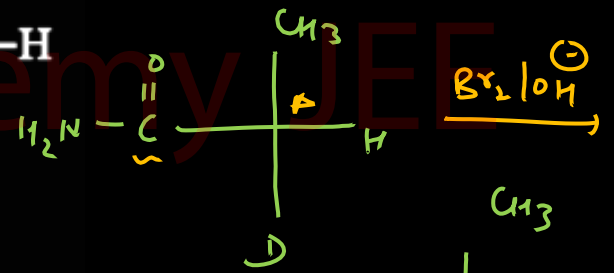
(C)

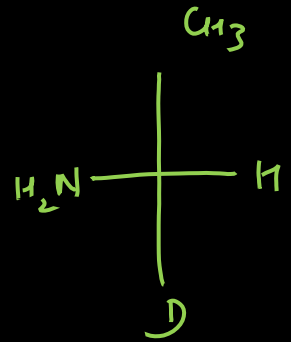


(D)



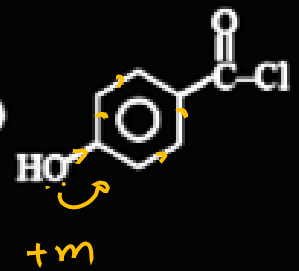
H+/H2O (Partial)

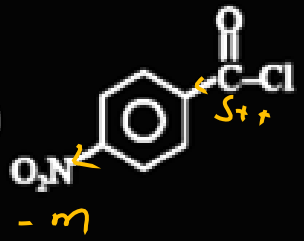


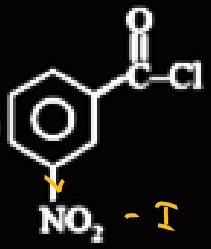


18

The order of rate of reaction with grignard reagent is

(I) 

(II) 

(III) 

(A) I > II > III

(B) II > III > I

(C) III > II > I

(D) II > I > III

19

JEE 1

Which of the following will give white ppt. with aq. $AgNO_3$.

AgCl ↓

(A) $Ph - Cl$

} no ppt

(B)

(C) $Ph - CH_2 - I$ → AgI (yellow)

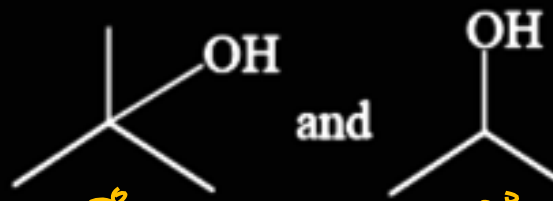
(D) $H_2N - CH_2CH_2CH_2CH_2 - Cl$ —heat→ AgCl ↓

White ppt.

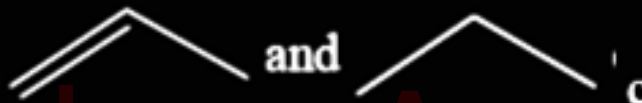
20

Which statement is incorrect?

JEE 1

(A) 

3° 2° can be differentiated by Lucas reagent.

(B)  can be differentiated by Br_2 water

(C) $Me - CH_2C \equiv CH$ and $Me - CH = CH_2$ can be differentiated by $NaNH_2$

(D) $CH_3-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\underset{|}{N}}}-CH_3$ and $H_3C-NH-CH_3$ can be differentiated by carbyl amine test

21

Number of electrons which give paramagnetic character for NO^{2+} ion is (1)

$NO^{2+} = 13e^-$

$7 + 8 - 2$

1

↑↓ ↑

22

How many statements are correct? 2

(A) Net rotation of the solutions, obtained on treatment of D-Glucose and D-Fructose with $NaBH_4$, is same.

(B) Glucose forms oxime with hydroxylamine while its penta-acetate form doesn't.

(C) L-Glucose is the mirror image of D-Glucose.

(D) α -Glucose is cyclic acetal structure of glucose.

D-Glucose $\xrightarrow{NaBH_4}$ D-Sorbitol.

D-Fructose $\xrightarrow{''}$ D-Sorbitol + D-Mannitol

1. Despite having the aldehyde group, glucose does not give Schiff's test and it does not form the hydrogensulphite addition product with $NaHSO_3$.
2. The pentaacetate of glucose does not react with hydroxylamine indicating the absence of free —CHO group.
3. Glucose is found to exist in two different crystalline forms which are named as α and β. The α-form of glucose (m.p. 419 K) is obtained by crystallisation from concentrated solution of glucose at 303 K while the β-form (m.p. 423 K) is obtained by crystallisation from hot and saturated aqueous solution at 371 K.

23

How many statements are correc

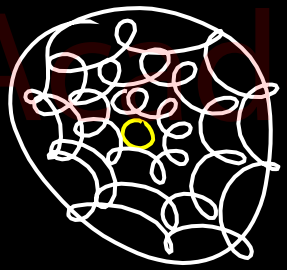
(A) Size of alkali metals decreases Li to Cs in aqueous medium

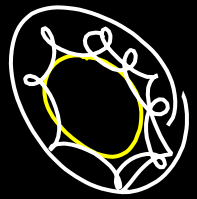
(B) NaF > NaCl > NaBr > NaI (Lattice energy)

(C) $CSO_2 > RbO_2 > KO_2 > NaO_2$ (stability)

(D) Lithium + Ions are most highly hydrated in aqueous solution

(4)





24

How many complexes among the following are paramagnetic

$[Mn(CN)_6]^{3-}$, $[Cr(H_2O)_6]^{3+}$ $[Co(en)_3]^{3+}$,

$[V(CO)_6]$, $[Ni(NH_3)_6]^{2+}$ $[Ni(dmg)_2]$

$[Pt(Cl)_2(NH_3)_2]$ $[Cu(NH_3)_4]^{2+}$

$[Cu(CN)_4]^{3-}$

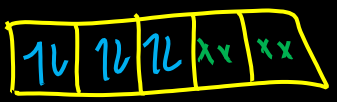
5

Paramagnetic : having unpaired e^-

$Mn^{3+} = 3d^4$

$Cr^{3+} = 3d^3$

$Co^{+3} = 3d^6$



25

For the mixture of ethyl alcohol and water at room temperature, among the following how many are correct?

(a) ΔG_{sys} is positive

(b) ΔS_{sys} is negative

(c) ΔS_{sur} is negative

(d) ΔS_{total} is positive

(e) ΔH_{mix} is positive

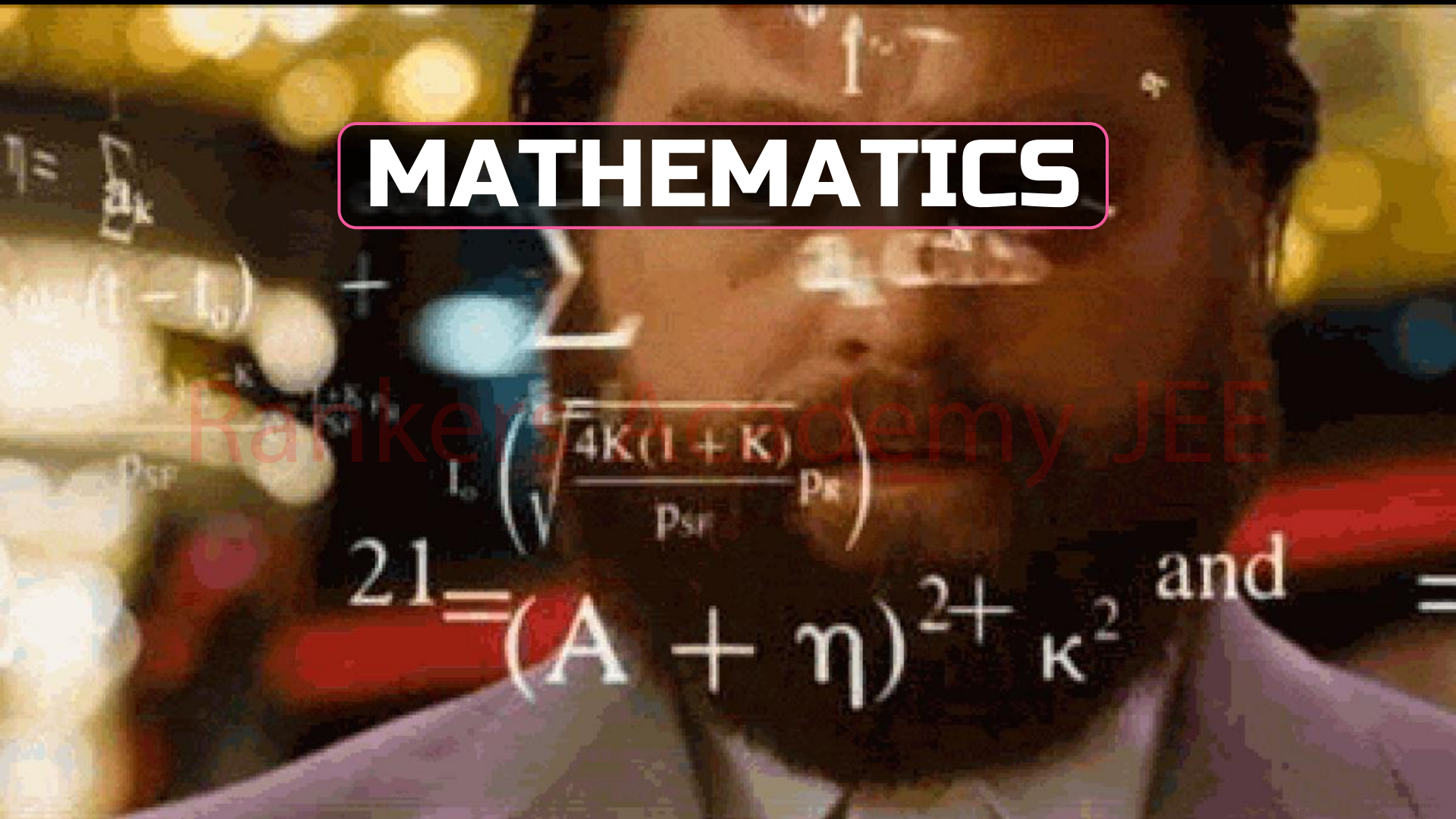
(f) ΔV_{mix} is negative

3

Ethyl Alcohol + water shows +ve deviation.

ΔH = +ve (endo)

ΔV = +ve


MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

If $f(x) = 2|x-1| + 3|x+1| - |x-3|$, then value of $(f'(5) + f'(0))$ is –

(A) 6 (B) 0

(C) 8 (D) 4

JEE 1

$x = 5$

$f(x) = 2(x-1) + 3(x+1) - (x-3)$

$= 4x + 4$

$f'(x) = 4$

$f'(5) = 4$

$x = 0$

$f(x) = 2(1-x) + 3(x+1) - (3-x)$

$= 2x + 2$

$f'(x) = 2$

$f'(0) = 2$

2

Given f is a twice differentiable function $\forall x \in R$ and $f'(x) \neq 0$. If $f(x) + f''(x) = -xh(x)f'(x)$ where $h(x) > 0 \forall x \in R$, $f(0) = -3, f'(0) = 4$ and $F(x) = (f(x))^2 + (f'(x))^2$ then $F(x)$ is monotonically strictly increasing function at

(A) $x = 2021$ (B) $x = 2019$

(C) $x = 5$ (D) $x = -5$

$F'(x) > 0$

$2ff' + 2f'f'' > 0$

$2f'(f + f'') > 0$

$\Rightarrow 2f'(-xhf') > 0$

$\Rightarrow -2h(f')^2 x > 0$

$\Rightarrow -x > 0 \Rightarrow \boxed{x < 0}$

3

If $1, \log_9(3^{1-x}+2), \log_3(4\cdot 3^x-1)$ are in A.P, then x is equal to

(A) $\log_3 4$ (B) $1-\log_3 4$ ✓

(C) $1-\log_4 3$ (D) $\log_4 3$

JEE 1

$a, b, c \to AP$: $\boxed{2b = a+c}$

$2\log_3(3^{1-x}+2) = 1+\log_3(4\cdot 3^x-1)$

$\Rightarrow 2\cdot\frac{1}{2}\log_3(3^{1-x}+2) = \log_3 3 + \log_3(4\cdot 3^x-1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3(3(4\cdot 3^x-1))$

$\Rightarrow 3^{1-x}+2 = 3(4\cdot 3^x-1)$

$\Rightarrow \frac{3}{3^x}+2 = 12\cdot 3^x-3$

Let $3^x = t$ ✓

$\Rightarrow \frac{3}{t}+2 = 12t-3$

$\Rightarrow 0 = 12t^2-5t-3$

$\Rightarrow 0 = 12t^2-9t+4t-3$

$\Rightarrow 0 = (3t+1)(4t-3)$

$\Rightarrow t = -\frac{1}{3}$ or $\frac{3}{4}$

$\Rightarrow 3^x = -\frac{1}{3}$ or $\frac{3}{4}$

✗ ✓

3

JEE 1

$3^x = \frac{3}{4}$

$x = \log_3 3/4$

$= \log_3 3 - \log_3 4$

$= 1 - \log_3 4$

4

Let n be the number of values of x satisfying $\left[\frac{3x^2-2x+1}{2}\right] = \frac{x+1}{2}$. Find the value of

$$\frac{1}{\pi}\left(\sin^{-1}\frac{1}{n} + \tan^{-1}\frac{1}{n} + \sec^{-1}\left(\frac{-1}{n}\right) + \frac{1}{2}\sin^{-1}\left(\frac{2n}{1+n^2}\right)\right)$$

[Note: [y] denotes the largest integer less than or equal to y.]

(A) 4 (B) 2
(C) ¼ (D) 1/2

[ ] = int

[ ] = int

int = $\frac{x+1}{2}$

x = odd int

x^2 = odd int

$3x^2$ = odd int

2x = even int

$3x^2 - 2x + 1$ = even = 2k

O − E + O

$[\text{int}] = \left[\frac{3x^2-2x+1}{2}\right] = \left[\frac{2k}{2}\right] = [k] = k$

JEE 1

4

JEE 1

$$\frac{3x^2-2x+1}{2} = \text{int}$$

$$\left[\frac{3x^2-2x+1}{2}\right] = \frac{3x^2-2x+1}{2}$$

$$\therefore \frac{3x^2-2x+1}{x} = \frac{x+1}{x}$$

$$3x^2 = 3x$$

$$x = 0, 1$$

$\times$ $\checkmark$

$x = 1$

$n = 1$

Ans: $\frac{1}{\pi}\left[\sin^{-1}1 + \tan^{-1}1 + \sec^{-1}(-1) + \frac{1}{2}\sin^{-1}(1)\right]$

$$\Rightarrow \frac{1}{\pi}\left[\frac{\pi}{2} + \frac{\pi}{4} + \pi + \frac{\pi}{2\cdot 2}\right]$$

$$\Rightarrow \frac{1}{\pi} \cdot 2\pi = 2$$

5

Let $f(x) = \cos^2 x + \frac{\sec x}{4}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; if m is the minimum value of f(x) then the value of $\frac{8}{3}$m is equal to

(A) 8/3 (B) 3/4

(C) 2 (D) 1/2

$\cos x > 0$

$\sec x > 0$

Max Min

AM GM

$AM \geq GM$

$$\Rightarrow \frac{\cos^2 x + \frac{\sec x}{8} + \frac{\sec x}{8}}{3} \geq \left(\cos^2 x \cdot \frac{\sec x}{8} \cdot \frac{\sec x}{8}\right)^{1/3}$$

$$\Rightarrow \frac{f(x)}{3} \geq \left(\frac{1}{64}\right)^{1/3}$$

$$\Rightarrow \frac{f(x)}{3} \geq \frac{1}{4}$$

$$\Rightarrow f(x) \geq 3/4$$

5

JEE 1

$m = 3/4$

Ans: $\frac{8}{3} m = \frac{8}{3} \cdot \frac{3}{4}$

$= 2.$

6

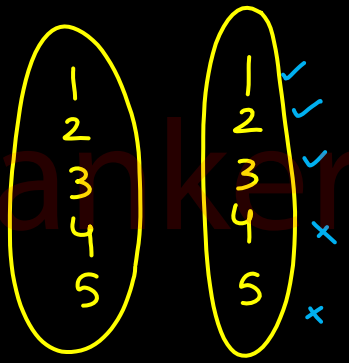
If the range of the function $f:\{1,2,3,4,5\} \to \{1,2,3,4,5\}$ assumes exactly 3 distinct values then if number of such functions are N then $\frac{N}{30}$ is

(A) 60 (B) 50

(C) 100 (D) 600

JEE 1

$5 \leq 2\,2\,1$ or $5 \leq 3\,1\,1$



$$N = {}^5C_3 \left[\frac{5!}{1!(2!)^2 .2!} + \frac{5!}{3!(1!)^2 2!} \right] \times 3!$$

$$= 10 \times 6 \left[15 + 10 \right]$$

$$N = 10 \times 6 \times 25$$

$$\frac{N}{30} = \frac{10 \times 6 \times 25}{30} = 50.$$

7

Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be nonzero. Then choose incorrect statement(s).

(A) $|z|^2 = 8$ ✓

(B) $|z|^2 + |z + \bar{z}|^2 = 8$

(C) $|z + 1|^2 = 7$

(D) $|z - \bar{z}|^2 = 12$

$z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

$\bar{z} = x - iy$

$z^2 = x^2 - y^2 + 2xyi$

$|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$

$\Rightarrow (x^2+y^2)^{3/2} + 2(x^2 - y^2 + 2xyi) + 4(x - iy) - 8 = 0 + 0i$

$\Rightarrow (x^2+y^2)^{3/2} + 2x^2 - 2y^2 + 4x = 8$ ✓

$4xy - 4y = 0$ ✓

$y = 0 \; ; \; x = 1$

(X) (✓)

7

JEE 1

$x = 1$

$\Rightarrow (1+y^2)^{3/2} + 2 - 2y^2 + 4 = 8\,2$

$\Rightarrow (1+y^2)^{3/2} = 2(1+y^2)$

$= (1+y^2)^{1/2} = 2 \Rightarrow y^2 = 3$

$y = \pm\sqrt{3}$

$\therefore (1, \sqrt{3})$ or $(1, -\sqrt{3})$

$|z|^2 = 1+3 = 4.$

Let $f: R - \{1\} \to R, y = f(x)$ satisfies $f(x) = 1 + \int_0^x ((t + f(t))^2 - 1)dt$. Then which of the following are incorrect

(A) $y = f(x)$ is a hyperbola

(B) $y = f(x)$ has local maxima at $x = 2$

(C) $[f(-3)] = 3$ ([.] is G.I.F)

(D) $y = 0$ is a horizontal tangent to $y = f(x)$ incorrect

$$f(x) = 1 + \int_0^x (t + f(t))^2 - 1 \, dt$$

diff wrt x

$$\Rightarrow f'(x) = (x + f(x))^2 - 1$$

$$\Rightarrow 1 + f'(x) = (x + f(x))^2$$

$$\Rightarrow \frac{1 + f'(x)}{(x + f(x))^2} = 1$$

$$\Rightarrow \int \frac{d(x + f(x))}{(x + f(x))^2} = \int 1$$

$$\Rightarrow \frac{-1}{x + f(x)} = x + c$$

$x = 0$

$f(0) = 1 + 0 \Rightarrow f(0) = 1$

$$\frac{-1}{0 + 1} = 0 + c \Rightarrow c = -1$$

8

JEE 1

$\Rightarrow \frac{-1}{x+y} = x-1$

$\Rightarrow \frac{-1}{x-1} = x+y$

$\Rightarrow y = \frac{-1}{x-1} - x$ ✓

$x = -3$

$y = \frac{-1}{-4} + 3 = 3\frac{1}{4}$

$[y] = 3$

$y' = \frac{1}{(x-1)^2} - 1 = 0$

$\Rightarrow (x-1)^2 = 1$

$\Rightarrow x-1 = 1, -1$

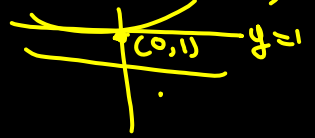
$\Rightarrow x = 2, 0$

$y'' = \frac{-2}{(x-1)^3} < 0$

Max.

$(x,y) \equiv (2,-3)$

$(x,y) \equiv (0,1)$



9

If $g(x) = \frac{2h(x)+|h(x)|}{2h(x)-|h(x)|}$ where $h(x) = \sin x - \sin^n x, n \in R^+$ and $f(x) = \begin{cases} [g(x)] & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3 & x = \frac{\pi}{2} \end{cases}$ where [ ] denotes greatest integer function. Then, which of the following is correct?

(A) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(B) f(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $n > 1$

(C) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(D) f(x) is continuous but not differentiable at $x = \frac{\pi}{2}$, when $n > 1$

JEE 1

$n > 1$

$0 < n < 1$

$\sin x > \sin^n x$

$\sin x < \sin^n x$

$n > 1 \longrightarrow h(x) = \sin x - \sin^n x = |h(x)|$

$g(x) = \frac{2h(x) + h(x)}{2h(x) - h(x)} = 3$

$0 < n < 1 \longrightarrow h(x) = \sin x - \sin^n x$

$|h(x)| = \sin^n x - \sin x$

$g(x) = \frac{2(s - s^n) + (s^n - s)}{2(s - s^n) - (s^n - s)} = \frac{1}{3}$

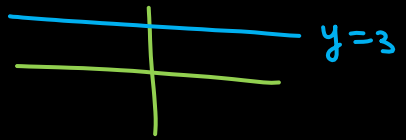
9

JEE 1

$n > 1$

$g(x) = 3$

$$f(x) = \begin{cases} 3 & x \in (0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi) \\ 3 & x = \pi/2 \end{cases}$$



c ✓

d ✓

$0 < n < 1$

$g(x) = 1/3$

$$f(x) = \begin{cases} 0 & \text{LHL} = \text{RHL} \text{ at } x = \frac{\pi}{2} \\ 3 & = f(\pi/2) \end{cases}$$

disco

non-diff

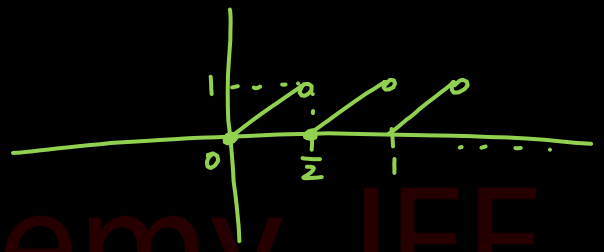
10

Let $f: R \to R$ be a differentiable non-constant function satisfying $f(x) = f(y)f(x-y)\ \forall x, y \in R$ and $f'(0) = \int_0^4 \{2x\}dx = 2$, where $\{.\}$de notes the fractional part function and . $f'(-3) = \alpha e^{\beta}$ Then $|\alpha + \beta|$ is equal to___

(A) 1 (B) 2

(C) 4 (D) 3

$f'(0) = \int_0^4 \{2x\}\,dx = 2$



$8 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = 2$

$f(x) = f(y) \cdot f(x-y)$

$x \to x+y$

$f(x+y) = f(y) \cdot f(x+y-y)$

$f(x+y) = f(x) \cdot f(y)$

$f(x) = e^{kx}$

$f'(x) = ke^{kx}$

$f'(0) = k = 2$

$\therefore f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f'(-3) = 2e^{-6}$

$\alpha = 2$, $\beta = -6$ $\Rightarrow \alpha + \beta = -4$

11

A number is selected at random from the set of numbers which are of 3m digits where m is natural number and using digits from the set {1,4,7}, then the probability that the number is divisible by 6

(A) 1/2 (B) 1/4

(C) 1/3 (D) 1/6

$m = 1$

div by 2 → — — 4

div by 3 →

Sum of digits = 3k

4 4 4
1 4 4
7 7 4

1 4 4
4 1 4
1 7 4
7 1 4
4 7 4
7 4 4

Ans: $\frac{3 \cdot 3 \cdot \times 1}{3 \cdot 3 \cdot 3} = \frac{1}{3}$

Or: $\frac{3^{3m-1} \times 1}{3^{3m}}$

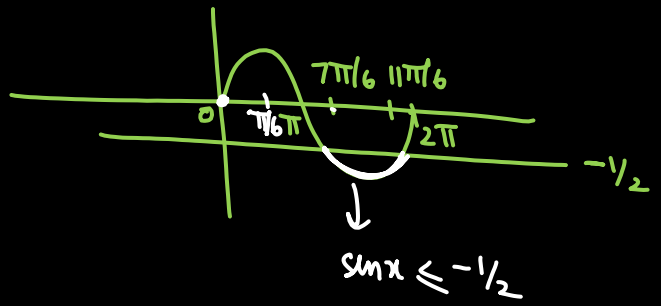
12

JEE 1

The function $f(x) = \cos^{-1}\left(\frac{2[|\sin x|+|\cos x|]}{\sin^2 x+2\sin x+\frac{11}{4}}\right)$ is defined if x does not belong to (where [.] denotes greatest integer function and |.|denotesabsolutevalue)

(A) $\left[0, \frac{7\pi}{6}\right]$ (B) $\left[0, \frac{\pi}{6}\right]$

(C) $\left[\frac{11\pi}{6}, 2\pi\right]$ (D) $\left[\frac{7\pi}{6}, \frac{11\pi}{6}\right]$.



$1 \le |\sin x| + |\cos x| \le \sqrt{2}$
1.4

$[\quad] = 1$

$f(x) = \cos^{-1}\left(\frac{2}{(\sin^2 x+2\sin x+11/4)}\right)$

$= [-1, 1]$ fraction.

$\Rightarrow D^r \ge 2$
or $D^r \le -2$

$$-1 \le \frac{2}{\sin^2 x + \sin x + 11/4} \le 1$$

$$-1 \le \frac{2}{\left(\sin x + \frac{1}{2}\right)^2 + \frac{5}{2}} \le 1$$

$\left(\sin x + \frac{1}{2}\right)^2 \ge 0$, $\left(\sin x + \frac{1}{2}\right)^2 + \frac{5}{2} \ge 2.5$

$-(\text{or}) \le 2$ or $2 \le \text{or}$

$(\text{or}) \ge -2$

13

Given $a, b \in \{0,1,2,3,4 \ldots 9,10\}$. Consider the system of equations

$x + y + z = 4$

$2x + y + 3z = 6$

$x + 2y + az = b$

Let L : denotes number of ordered pairs (a, b) so that the system of equations has unique solution, M: denotes number of ordered pairs (a, b) so that the system of equations has no solution and N : denotes number of ordered pairs (a, b) so that the system of equations has infinite solutions The value of $\left(\frac{L}{M} - 3N\right)$ is

(A) 14 (B) 11

(C) 8 (D) 7

$\frac{110}{10} - 3(1)$

8

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & a \end{vmatrix} = -a$

Unique sol: $\Delta \neq 0$

$-a \neq 0$

$a \neq 0$

$\therefore a = 1, 2 \ldots 10$

$b = 0, 1 \ldots 10$

$\therefore (a,b) \equiv (\underset{10}{\_\_\_}, \underset{11}{\_\_\_})$

$L \equiv 110$

JEE 1

13

JEE 1

∞ sol.

$\Delta = 0$

$\boxed{a = 0}$ ✓

$$\Delta_1 = \begin{vmatrix} 4 & 1 & 1 \\ 6 & 1 & 3 \\ b & 2 & 0 \end{vmatrix}$$

$= -24 + 3b + 12 - b$

$\Rightarrow 2(b-6) = 0$

$\therefore \boxed{b = 6}$

$(a, b) \rightarrow (\underset{\uparrow}{0}, \underset{\uparrow}{6}) = 1 = N$

no sol: $\Delta = 0 \Rightarrow \boxed{a = 0}$

$b \neq 6 \therefore b \in \{0, 1, 2, 3, 4, 5, 7, 8, 9, 10\}$

$\therefore$ 10 choices

$(a, b) = 10 = M$ (a: $\uparrow$ 1, b: $\uparrow$ 10)

$\Delta_1 = 2(b-6)$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 6 & 3 \\ 1 & b & 0 \end{vmatrix} = -3b + 12 + 2b - 6 \Rightarrow -(b-6)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 1 & 6 \\ 1 & 2 & b \end{vmatrix} \Rightarrow (b-2) - (2b-6) + 4(-3)$$

$= -b + 6 = -(b-6)$

14

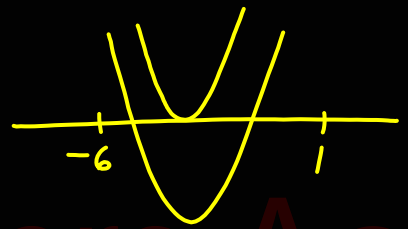
Let ' p ' be an integer for which both roots of the quadratic equation $x^2 + 2(p-3)x + 9 = 0$ lies in $(-6,1)$. If $2, g_1, g_2, \ldots, g_{19}, g_{20}, p$ are in G.P., then find the value of $g_4 g_{17}$.

(A) 6

(B) 12

(C) 72

(D) 36

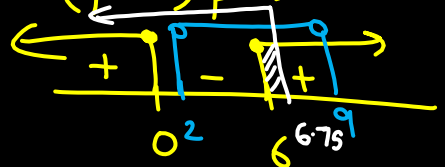


JEE 1

① $D \geq 0$

$4(p-3)^2 - 4(9) \geq 0$

$(p-6)p \geq 0$



② $-6 < \frac{-2(p-3)}{2(1)} < 1$

$-6 < 3-p < 1$

$p < 9 \quad 2 < p$

$\therefore p = 6$

③ $f(1) > 0$

$1 + 2p - 6 + 9 > 0$

$2p + 4 > 0$

$p > -2$

④ $f(-6) > 0$

$36 - 12(p-3) + 9 > 0$

$81 > 12p$

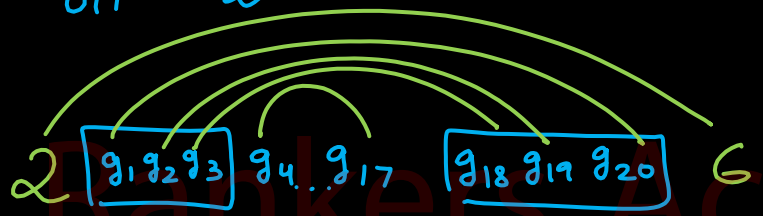
$6.75 = \frac{27}{4} > p$

14

JEE 1

$g_1\ g_2\ \cdots\ g_{20}$

GP : 2 ... 6



$g_4\ g_{17} = 2 \times 6$

$= 12$

15

If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1:7 and $a + n = 33$, then the value of n is

(A) 21 (B) 22

(C) 23 (D) 24

JEE 1

$A_1\ A_2\ - - -\ A_n$

$a \qquad 100$

$d = \frac{100 - a}{n+1}$ ✓ — (1)

$A_n = a + nd$ ✓

$A_1 = a + d$ ✓

$\Rightarrow \frac{A_1}{A_n} = \frac{1}{7}$

$\Rightarrow \frac{a+d}{a+nd} = \frac{1}{7}$ — (2)

$a + n = 33$ — (3)

$n = ? = 23$

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JEE 1

Let R be a relation on $\mathbb{R}$, given by $R = \{(a, b): 3a - 3b + \sqrt{7}$ is an irrational number$\}$. Then R is

(A) Reflexive but neither symmetric nor transitive

(B) Reflexive and transitive but not symmetric

(C) Reflexive and symmetric but not transitive

(D) An equivalence relation

① Reflexive : (a, a)

$3a - 3a + \sqrt{7}$ ✓

② Symm :

$(a, b) \equiv \left(\frac{\sqrt{7}}{3}, 0\right)$

$3\left(\frac{\sqrt{7}}{3}\right) - 3(0) + \sqrt{7}$

$= 2\sqrt{7}$ ✓

Now: $(b, a) : \left(0, \frac{\sqrt{7}}{3}\right)$

$3(0) - 3\left(\frac{\sqrt{7}}{3}\right) + \sqrt{7} = 0$ ✗

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JEE 1

(3) Transitive:

$(a,b) : \left(\frac{\sqrt{7}}{3}, 0\right) : \sqrt{7} - 0 + \sqrt{7} = 2\sqrt{7}$ ✓

$(b,c) : \left(0, \frac{2\sqrt{7}}{3}\right) : 0 - \frac{2\sqrt{7}}{3} + \sqrt{7} = -\sqrt{7}$ ✓

$(a,c) : \left(\frac{\sqrt{7}}{3}, \frac{2\sqrt{7}}{3}\right) : \sqrt{7} - 2\sqrt{7} + \sqrt{7} = 0$ ✗

17

If $y = f(x)$ is the solution of the differential equation $x(y^3 - x)dy = y(x + y^3)dx$ and $f(1) = (-2)^{1/3}$ and $f^{-1}(-2) = k$. Then $|k| =$ ____

(A) 2 (B) 4

(C) 8 (D) -4

JEE 1

$$xy^3dy - x^2dy = xydx + y^4dx$$

$$y^3[xdy - ydx] = x[ydx + xdy]$$

$$y^3 \frac{[xdy - ydx]}{x^2} = \frac{x[ydx + xdy]}{x^2}$$

$$y^3 d(y/x) = \frac{d(xy)}{x}$$

$$\frac{y^3}{xy} d\left(\frac{y}{x}\right) = \frac{d(xy)}{(xy)(xy)}$$

$$\int \left(\frac{y}{x}\right) d\left(\frac{y}{x}\right) = \int \frac{d(xy)}{(xy)^2}$$

$$\frac{(y/x)^2}{2} = \frac{-1}{(xy)} + c$$

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JEE 1

$$\frac{y^2}{2x^2} + \frac{1}{xy} = c$$

$$\frac{y^3 + 2x}{2x^2y} = c$$

$x = 1 \; ; \; y = (-2)^{1/3}$

$y^3 = -2$

$c = 0$

$$y^3 + 2x = 0$$

$y(k) = -2$

$x = k \; ; \; y = -2$

$-8 + 2(k) = 0$

$$k = 4$$

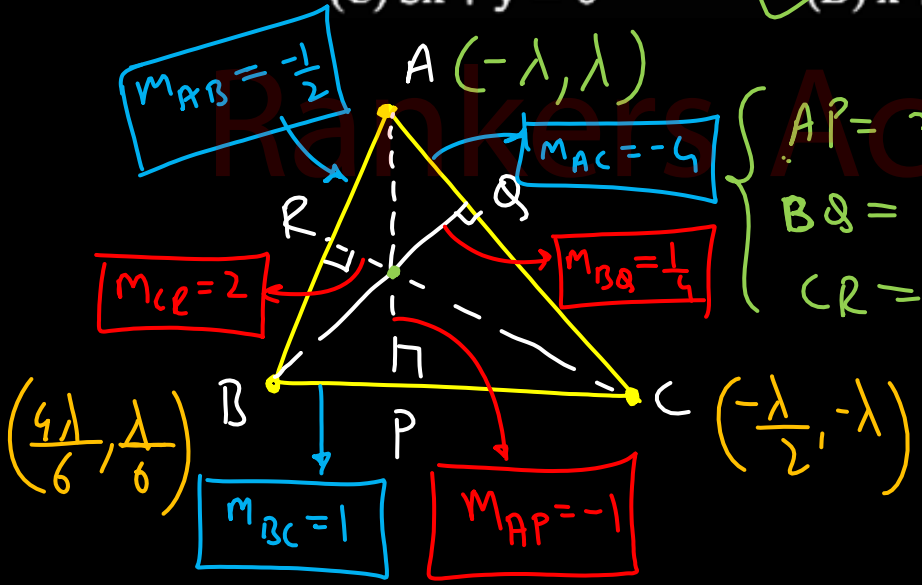
18

JEE 1

The lines $x+y=0, x-4y=0$ and $2x-y=0$ are the altitudes of a triangle. If one of the vertices has the coordinates $(-\lambda, \lambda)$, locus of the centroid of this triangle is

(A) $5x-y=0$ (B) $x-5y=0$

(C) $5x+y=0$ (D) $x+5y=0$



$AP = x+y=0$

$BQ = x-4y=0$

$CR = 2x-y=0$

Eqn of AB:

$(y-\lambda) = -\frac{1}{2}(x+\lambda)$

Eqn of AC:

$(y-\lambda) = -4(x+\lambda)$

B: BQ & AB

$y-\lambda = -\frac{1}{2}(4y+\lambda)$

$2y-2\lambda = -4y-\lambda \Rightarrow y = \frac{\lambda}{6}$

18

JEE 1

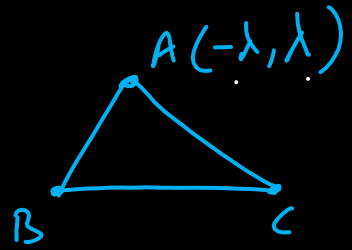
C : CR & AC

$(2x-\lambda) = -4(x+\lambda)$

$2x-\lambda = -4x-4\lambda$

$6x = -3\lambda$

$\boxed{x = -\frac{\lambda}{2}} \Rightarrow \boxed{y = -\lambda}$



Centroid: (h,k)

$h = \frac{-\lambda + \frac{2\lambda}{3} - \frac{\lambda}{2}}{3}$

$h = \frac{-6\lambda + 4\lambda - 3\lambda}{6 \times 3}$

$\boxed{h = \frac{-5\lambda}{18}}$ ✓

$k = \frac{\left(\lambda + \frac{\lambda}{6} - \lambda\right)}{3}$

$\boxed{k = \frac{\lambda}{18}}$ ✓

$h = -5k$

$x = -5y$

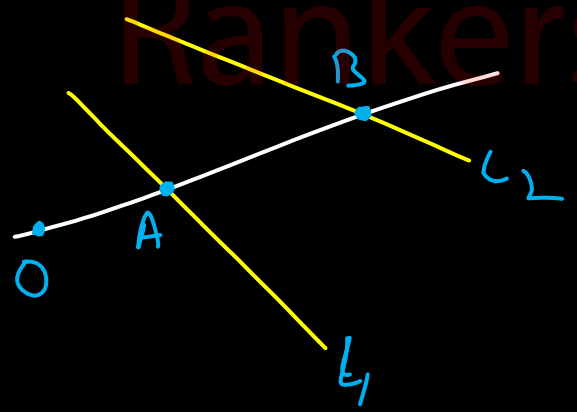
$\boxed{x + 5y = 0}$

19

JEE 1

A line passing through origin cuts the lines $\frac{x+1}{1}=\frac{y-2}{1}=\frac{z-1}{-2}=\lambda$ and $\frac{x-1}{1}=\frac{3y-8}{6}=\frac{z+3}{-1}=\mu$ at A and B respectively, then the square of length of AB is equal to

(A) 3
(B) 6
(C) 9
(D) 12



$A \equiv (\lambda-1, \lambda+2, -2\lambda+1)$

$B \equiv (\mu+1, \frac{6\mu+8}{3}, -\mu-3)$

O, A & B are collinear

$\vec{OA} = (\lambda-1)\hat{i} + (\lambda+2)\hat{j} + (-2\lambda+1)\hat{k}$

$\vec{OB} = (\mu+1)\hat{i} + (\frac{6\mu+8}{3})\hat{j} + (-\mu-3)\hat{k}$

19

JEE 1

$$\frac{\lambda-1}{\mu+1} = \frac{\lambda+2}{\frac{6\mu+8}{3}} = \frac{-2\lambda+1}{-\mu-3}$$

calculation

$$\begin{cases} \lambda = 3 \\ \mu = \frac{1}{3} \end{cases}$$

$$\begin{cases} A \equiv (2, 5, -5) \\ B \equiv \left(\frac{4}{3}, \frac{10}{3}, \frac{-10}{3}\right) \end{cases}$$

$$(AB)^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2 + \left(\frac{5}{3}\right)^2 = \frac{54}{9} = 6$$

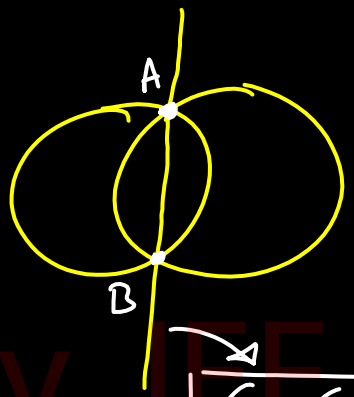
20

If two circles $x^2 + y^2 - 6x - 6y + 13 = 0$ and $x^2 + y^2 - 8y + 9 = 0$ intersects at A and B, then the focus of the parabola whose directrix is line AB and vertex at (0,0) is

(A) $\left(\frac{3}{5}, \frac{1}{5}\right)$ (B) $\left(\frac{-3}{5}, \frac{1}{5}\right)$

(C) $\left(\frac{-3}{5}, \frac{-1}{5}\right)$ (D) $\left(\frac{3}{5}, \frac{-1}{5}\right)$

JEE 1

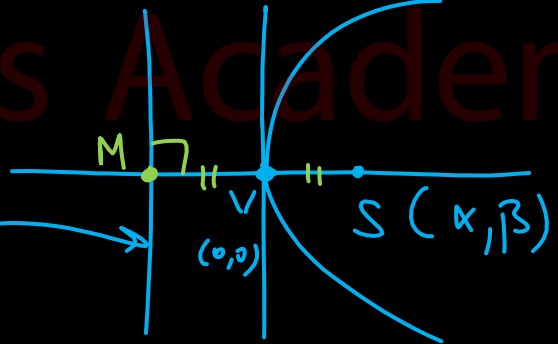


$C_1 - C_2 = 0$

Radical axis

$-6x + 2y + 4 = 0$

$3x - y - 2 = 0$



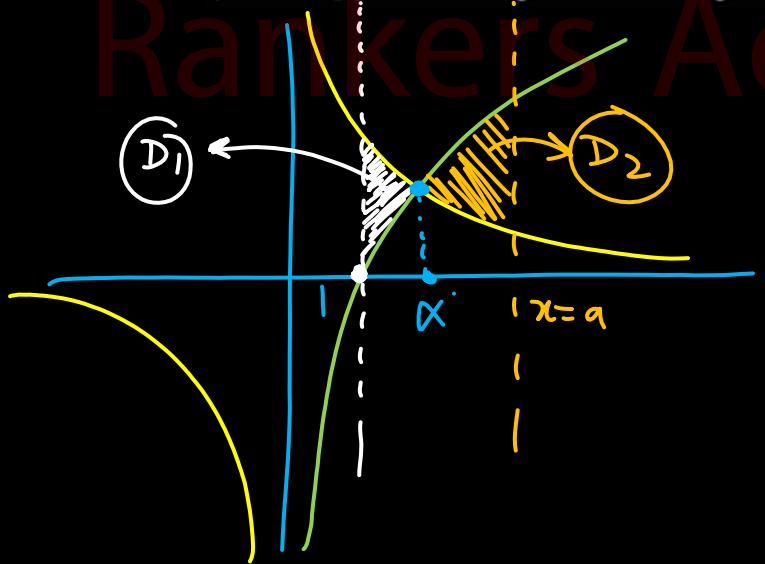
$M : \frac{x-0}{3} = \frac{y-0}{-1} = -\frac{(-2)}{10} = \frac{1}{5}$

$M \equiv \left(\frac{3}{5}, \frac{-1}{5}\right)$

21

Consider two curves $C_1: y = \frac{1}{x}$ and $C_2: y = \ln x$ on the xy plane. Let D_1 denotes the region bounded by C_1, C_2 and the line $x = 1$ and D_2 denotes the region surrounded by C_1, C_2 and the line $x = a$. If $D_1 = D_2$, then the value of [a] is equal to

(where [.] denotes greatest integer function)



JEE 1

$$\int_1^{\alpha} \left(\frac{1}{x} - \ln x\right) dx = \int_{\alpha}^{a} \left(\ln x - \frac{1}{x}\right) dx$$

$$\int_1^{\alpha} \left(\frac{1}{x} - \ln x\right) dx = -\int_{\alpha}^{a} \left(\frac{1}{x} - \ln x\right) dx$$

$$\int_1^{\alpha} \left(\frac{1}{x} - \ln x\right) dx + \int_{\alpha}^{a} \left(\frac{1}{x} - \ln x\right) dx = 0$$

JEE 1

$$\int_1^a \left(\frac{1}{x} - \ln x\right) dx = 0$$

$$\left(\ln x - (x\ln x - x)\right)\Big|_1^a = 0$$

$$(\ln a - a\ln a + a) - (1) = 0$$

$$\ln a(1-a) - 1(1-a) = 0$$

$$(1-a)(\ln a - 1) = 0$$

$$\ln a = 1$$

$$a = e$$

$$[a] = 2 \checkmark$$

22

Let B be a skew symmetric matrix of order 3×3 with real entries. If $A = (I + B)(I - B)^{-1}$ where $I + B, I - B$ are non-singular matrices, then sum of values of n satisfying the equation $|\sqrt[3]{n^2} \cdot A| - n\,|\sqrt[3]{5}\,.\,\text{adj}\,(\text{adj}\ A)| + 6|\text{adj}\ A|^3 = 0$ is equal to

(where |A| denotes determinant value of A.)

$B^T = -B$

$A = (I+B)(I-B)^{-1}$

$|A| = |I+B|\,|(I-B)^{-1}|$

$= \dfrac{|I+B|}{|I-B|}$

$= \dfrac{|(I+B)^T|}{|I-B|}$

$= \dfrac{|I-B|}{|I-B|} = 1$

$|n^{2/3} A| - n\,|5^{1/3}\,\text{adj adj} A|$

$n^2|A| - 5n|\text{adj adj} A| + 6|\text{adj} A|^3 = 0$

$n^2|A| - 5n|A|^4 + 6(|A|^2)^3 = 0$

$n^2 - 5n + 6 = 0$ → Sum = 5

23

Let $|\vec{a}| = 2, |\vec{b}| = 1$ and $|\vec{c}| = 3$ such that $\vec{a} \wedge \vec{b} = \frac{\pi}{3}, \vec{b} \wedge \vec{c} = \frac{\pi}{4}$ and $\vec{c} \wedge \vec{a} = \frac{\pi}{2}$. If $|\vec{a} - 2\vec{b} + \vec{c}|$ can be expressed as $\sqrt{p + q\sqrt{2}}$ (where $p, q \in I$), then $(p + q)$ is

JEE 1

$$|\bar{a} - 2\bar{b} + \bar{c}| = \sqrt{|\bar{a}|^2 + |2\bar{b}|^2 + |\bar{c}|^2 - 4\,\bar{a}\cdot\bar{b} - 4\bar{b}\cdot\bar{c} + 2\,\bar{a}\cdot\bar{c}}$$

$$= \sqrt{4 + 4 + 9 - 4 \times 2\left(\frac{1}{2}\right) - 4(3)\frac{1}{\sqrt{2}} + 2(6)(0)}$$

$$= \sqrt{13 - 6\sqrt{2}} \quad \Rightarrow p = 13,\ q = -6$$

$$\boxed{p + q = 7}$$

24

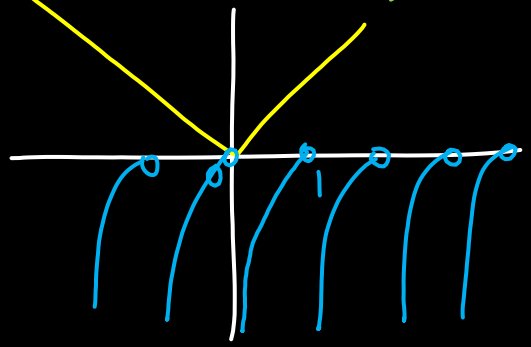
If α is the number of solutions of $|x| = \ln(x - [x])$ [$[.] \rightarrow$ G.I.F] and $\lim_{x\to\alpha} \frac{xe^{ax} - b\sin x}{x^3}$ is finite, then $a + b$ equals

JEE 1

$\alpha = ?$

$|x| = \ln(x - [x])$

$|x| = \ln(\{x\})$



$\because$ no common soln

$\Rightarrow \boxed{\alpha = 0}$

$$\lim_{x\to 0} \frac{xe^{ax} - b\sin x}{x^3}$$

$$\lim_{x\to 0} \frac{x\left(1 + \frac{ax}{1!} + \frac{(ax)^2}{2!}\right) - b\left(x - \frac{x^3}{6}\right)}{x^3}$$

$1 - b = 0$

$b = 1$

$a = 0$

$a + b = 1$

25

JEE 1

If $\begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A + Bx + Cx^2 + \cdots \ldots$, then $B + 1$ equals

For B = Diff & use $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = B$$

$$+ \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$\Rightarrow B = 0$

$B + 1 = 1$ ✓