Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)
PART TEST
15-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY | PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	C	A	A		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,B,D	A,C	A,C,D			
SECTION-II (i)	Q.	1	2	3	4		
	A.	1.00	0.50	11.25	1.40		
SECTION-II (ii)	Q.	5	6	7	8	9	10
	A.	1	500	160	250	4	3

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4		
	A.	B	C	C	C		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,B,C,D	A,C,D	A,B			
SECTION-II (i)	Q.	1	2	3	4		
	A.	242.00	9.00	486.40	4.00		
SECTION-II (ii)	Q.	5	6	7	8	9	10
	A.	5	4	8	130	4	8

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	C	D	C		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,C	A,C,D	A,B,C			
SECTION-II (i)	Q.	1	2	3	4		
	A.	5.00	4.00	639.00	616.00		
SECTION-II (ii)	Q.	5	6	7	8	9	10
	A.	9	12	256	120	16	118

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (C)**

$F = IL \times B = 9.0 \times 10^{-3} e^{-0.2x} \hat{i}$

Then $F_a = -9.0 \times 10^{-3} e^{-0.2x} \hat{i}$ and

$$W = \int_0^2 (-9.0 \times 10^{-3} e^{-0.2x} dx)$$

$= -1.48 \times 10^{-2}$ J

The field moves the conductor, and therefore the work is negative. The power is given by

$$P = \frac{W}{t} = \frac{-1.48 \times 10^{-2}}{5 \times 10^{-3}} = -2.97 \text{ W}$$

2. Ans (C)

The proton and electron are attracted by the coulomb force,

$$F = \frac{Q^2}{4\pi\varepsilon_0 r^2}$$

which furnishes the centripetal force for the circular motion. Thus

$$\frac{Q^2}{4\pi\varepsilon_0 r^2} = m_e\omega^2 r \quad \text{or} \quad \omega^2 = \frac{Q^2}{4\pi\varepsilon_0 m_e r^2}$$

Now, the electron is equivalent to a current loop $I = (\omega/2\pi)Q$. The field at the center of such a loop is,

$$B = \mu_0 H = \frac{\mu_0 I}{2r} = \frac{\mu_0 \omega Q}{4\pi r}$$

Substituting the value of ω found above,

$$B = \frac{(\mu_0/4\pi)\,Q^2}{r^2\sqrt{4\pi\varepsilon_0 m_e r}}$$

$$= \frac{(10^{-7})\,(1.6\times10^{-19})^2}{(0.35\times10^{-10})^2\sqrt{\left(\frac{1}{9}\times10^{-9}\right)(9.1\times10^{-31})\,(0.35\times10^{-10})}}$$

$= 35$ T

3. Ans (A)

$J_c = \sigma E = 1250 \sin 10^{10}t\ (A/m^2)$

On the assumption that the field direction does not vary with time,

$$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0\epsilon_r\, 250 \sin 10^{10}t) = 22.1 \cos 10^{10}t\ (A/m^2)$$

For $J_c = J_D$.

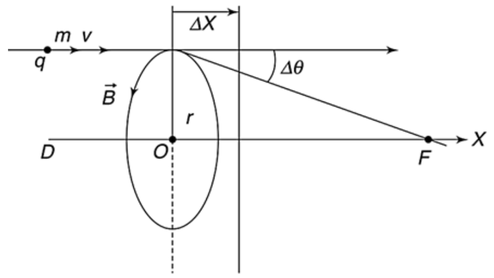
$$\sigma = \omega\,\epsilon \quad \text{or} \quad \omega = \frac{5.0}{8.854\times10^{-12}} = 5.65\times10^{11}\,\text{rad/s}$$

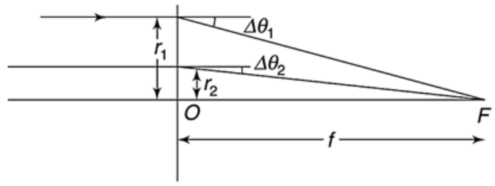
which is equivalent to a frequency

$f = 8.99 \times 10^{10}$ Hz $= 89.9$ GHz.

4. Ans (A)

Consider a charge particle at a distance r from the x-axis. Magnetic force on it is towards O (perpendicular) to its original direction of motion. This will cause the path of the charge to deviate by a small angle $\Delta\theta$.





Time required to cross the field is $\Delta t = \dfrac{\Delta x}{V}$

Impulse of magnetic force $= qV(B_0 r)\dfrac{\Delta x}{V} = qB_0 r\Delta x$

Change in momentum of the charge in crossing the field

$\Delta p = qB_0 r\Delta x$

This change is perpendicular to original direction of momentum.

$\therefore\ \Delta p \simeq p\Delta\theta;\ qB_0 r\Delta x = mV\Delta\theta$

$$\Rightarrow \Delta\theta = \frac{qB_0 r\Delta x}{mV}$$

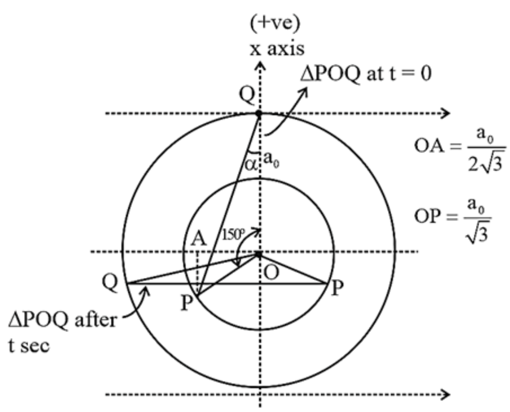
Because $\Delta\theta\ \alpha\ r$, all ions will get focused at one point F or the axis. (see figure)

$$f = \frac{r_1}{\Delta\theta_1} = \frac{r_2}{\Delta\theta_2};\ f = \frac{r}{\Delta\theta} = \frac{mV}{qB_0\Delta x}$$

PART-1 : PHYSICS

SECTION-I (ii)

5. Ans (A,B,D)



PART-1 : PHYSICS

SECTION-I (ii)

5. Ans (A,B,D)

by Sin law

$$\frac{\sin\alpha}{\frac{a_0}{\sqrt{3}}} = \frac{\sin(30-\alpha)}{a_0}$$

$$\sqrt{3}\sin\alpha = \frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha$$

$$\frac{3\sqrt{3}}{2}\sin\alpha = \frac{1}{2}\cos\alpha$$

$$\tan\alpha = \frac{1}{3\sqrt{3}}$$

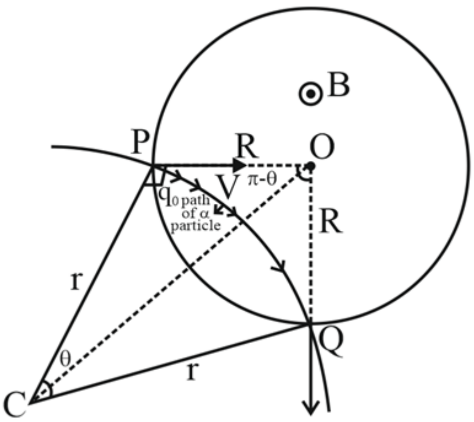
6. Ans (A,C)

(1) **True :** As the driving frequency approaches the natural frequency, the amplitude increases significantly (resonance effect)

(2) **False :** Increasing the damping coefficient b reduces the maximum amplitude of oscillation.

(3) **True :** The phase difference is affected by both driving frequency and damping coefficient.

7. Ans (A,C,D)



POAC in a cyclic quadrilateral

$$\tan\frac{\theta}{2} = \frac{R}{r};\ r = \frac{mv}{qB}$$

$$\theta = 2\tan^{-1}\frac{BqR}{mv}$$

$$\theta = \omega t;\ t = \theta/\omega$$

$$t = \frac{2qB}{m}\tan^{-1}\left(\frac{BqR}{mv}\right)$$

PART-1 : PHYSICS

SECTION-II (i)

1. Ans (1.00)

$$F = q\left\{E\hat{j} + \left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right) \times \left(B\hat{k}\right)\right\}$$

$$a_y = \frac{(qE - v_xqB)\hat{j}}{m};\ \frac{dv_y}{dt} = \frac{qE}{m} - \frac{BqV_x}{m}$$

$$a_x = \frac{qv_yB\hat{i}}{m};\ \frac{d^2v_y}{dt^2} = \frac{-Bq}{m} - \frac{dV_x}{dt}$$

$$\frac{dv_x}{dt} = \frac{Bq}{m}v_y;\ \frac{d^2v_y}{dt^2} = \frac{-B^2q^2}{m^2}v_y$$

$$\omega = \frac{Bq}{m}$$

$$v_y = A\sin\omega t + B\cos\omega t$$

At $t = 0;\ v_y = 0 = B$

$$\frac{dv_y}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$$

At $t = 0;\ \frac{dv_y}{dt} = \frac{qE}{m} = A\omega$

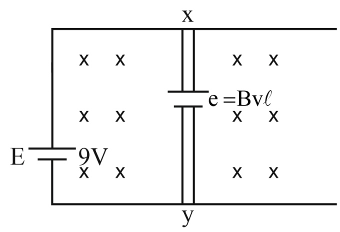
$$A = \frac{qE}{m\omega}$$

$$v_y = \frac{qE}{m\omega}\sin\omega t$$

2. Ans (0.50)

$$y = \frac{qE}{m\omega^2}[1 - \cos\omega t]$$

3. Ans (11.25)



Suppose e = induced emf in rod

$E - e = i.R$

Due to i wire experienced the force

$F = i\ell B = ma$

$$E - BV\ell = \frac{ma}{B\ell}R$$

$$a = \frac{(9 - 0.8V)}{3}0.8$$

for terminal speed a = 0

$$9 = 0.8\,V_T \Rightarrow V_T = \frac{90}{0.8}$$

$= 11.25$ m/s

4. Ans (1.40)

$v = 6$

$$a = \frac{(9 - 0.8 \times 6)}{3} 0.8$$

$$= \frac{9 - 4.8}{3} \times 0.8$$

$$= \frac{4.2}{3} \times 0.8 = 1.12$$

$$\therefore\ i = \frac{ma}{B\ell} = \frac{1}{0.8} \times \left(\frac{4.2}{3} \times 0.8\right) = 1.4\ A$$

PART-1 : PHYSICS

SECTION-II (ii)

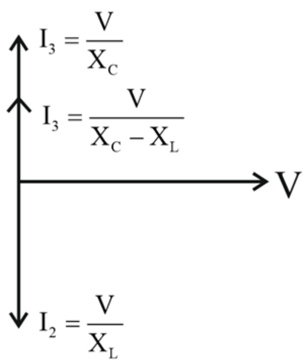
5. Ans (1)

$$t_{min} = \frac{2\pi\sqrt{LC}}{4}$$

$$= \frac{\pi}{2}\sqrt{1 \times 400 \times 10^{-6}}$$

$$= \frac{20\pi}{2} \times 10^{-3} = \frac{\pi}{100} sec$$

6. Ans (500)



$X_L = 10\ \Omega$, $XC = 20\ \Omega$

Let V is voltage across combination of inductors and capacitors. Phasor diagram for the combination

$$I = I_3 + I_1 - I_2$$

$$I = V\left[\frac{1}{X_C} + \frac{1}{X_C - X_L} - \frac{1}{X_L}\right]$$

$$I = \frac{V}{20}$$

$$V_m^2 = V_R^2 + V^2 \Rightarrow 40000 = I^2R^2 + 400I^2$$

$$\Rightarrow I = \frac{200}{20\sqrt{2}} = 5\sqrt{2}A$$

$$I_{rms} = 5A$$

$$P_{avg} = I_{rms}^2 . R = 500\ W$$

7. Ans (160)

First we will calculate torque of resistance force on the disc by considering an element of dr width at radius r.

$$d\tau = \frac{v}{\pi} \times 2\pi r dr \times r = \frac{2\pi\omega r^3 dr}{\pi} = 2\omega r^3 dr$$

$$\tau = \frac{1}{2}\omega\left(\frac{1}{2}\right)^4 = \frac{1}{32}.\frac{d\theta}{dt} \text{ (resistive torque)}$$

$$\tau_{net} = -200\theta - \frac{1}{32}.\frac{d\theta}{dt} \Rightarrow \tau_{net} = I.\propto$$

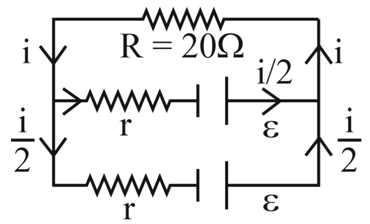
$$-200\,\theta - \frac{1}{32}.\frac{d\theta}{dt} = \frac{1}{8}.\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{1}{4}.\frac{d\theta}{dt} + 1600\theta = 0$$

$$2\beta = \frac{1}{4} \text{ and } \omega_0^2 = 1600 \Rightarrow \omega_0 = 40\frac{rad}{sec}$$

for very low damping, $Q = \frac{\omega_0}{2\beta} = 160$

8. Ans (250)



where $\varepsilon = b\ell v$, $r = 10\ \Omega$

$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{B\ell v}{20 + 5} = \frac{B\ell v}{25}$$

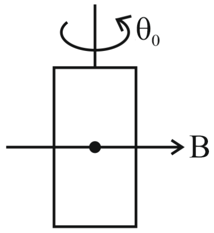
$$M\ell g = iB = Ma$$

$$i\ell B = \frac{Mg}{2}\ (a = g/2)$$

$$i = 2.5A$$

$$V = \frac{25i}{B\ell} = \frac{25 \times 2.5}{4} = \frac{62.5}{4} = 15.625\ m/s.$$

9. **Ans (4)**



Talking the Area vector out of the plane

$\phi = B\ell^2\cos(90-\theta) = B\ell^2\sin\theta$

$\varepsilon = \frac{d\phi}{dt} = B\ell^2\cos\theta\ \ \dot{\theta}$ (θ is very small)

so $\cos\theta \approx 1$

$\frac{\varepsilon}{R} = i =$ current

$\therefore i = \frac{B\ell^2}{R}$

Torque due to the magnetic field

$\tau = Bi\ell^2\sin(90-\theta) \approx Bi\ell^2$

Substituting for current

$\tau_{mag} = \frac{B^2\ell^4}{R}\dot{\theta}$

$\tau_{mech} = C\theta$, Where C is the torsional constant

(writing the differential equation of motion)

So, $I\frac{d^2\theta}{dt^2} + \frac{B^2\ell^4}{R}\dot{\theta} + C\theta = 0$

$\frac{d^2\theta}{dt^2} + \frac{B^2\ell^4}{RI}\dot{\theta} + \frac{C}{I}\theta = 0$

$\ddot{\theta} + \frac{B^2\ell^4}{R}\dot{\theta} + \frac{C}{F}\theta = 0$

Comparing this equation with the standard differential equation of damped oscillation

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

$x(t) = A_0 e^{\frac{-b}{2m}t}\cos(wt+\phi)$

we have $\frac{b}{bm} = \frac{B^2\ell^4}{2RI}$, $\theta_0 = A_0$

So we want

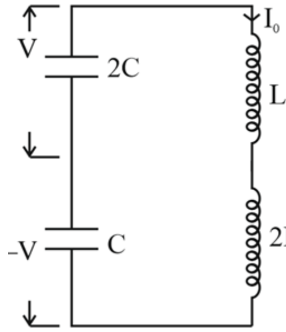
$\theta_0 e^{\frac{-b}{2m}t} = \frac{\theta_0}{e} \quad \Rightarrow t = \frac{2RI}{B^2\ell^4}$

Substituting the given values

$\frac{2 \times 0.01 \times 0.02}{(10^{-1})^4 \times (1)^2} = \frac{4 \times 10^{-4}}{10^{-4}} = 4\ S$

10. **Ans (3)**

At maximum current in the inductors. The total voltage across the inductors is zero, and also applying conservation of energy (Let I_0 be the maximum current)



As per charge conservation

$2CV + CV = q_0$

$\therefore V = \frac{q_0}{3C}$

Applying energy conservation

$\frac{q_0^2}{2\,(2C)} = \frac{LI_0^2}{2} + \frac{2LI_0^2}{2} + \frac{CV^2}{2} + \frac{2CV^2}{2}$

$\Rightarrow I_0 = \frac{q_0}{3\sqrt{2LC}}$

$\Rightarrow I_0 = \frac{36 \times 10^{-6}}{3\sqrt{2 \times 1 \times 10^{-3} \times 8 \times 10^{-9}}} = 3A$

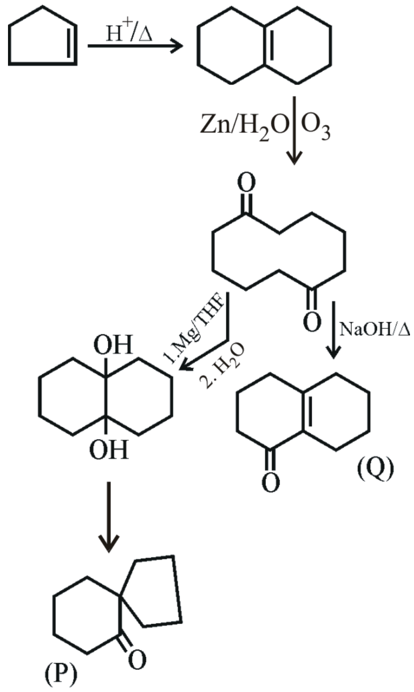
PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (B)**

Rectivity of RCOCl > RCl

2. **Ans (C)**



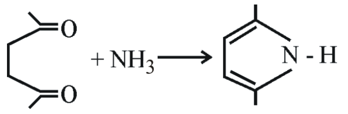
3. **Ans (C)**
For s-orbital, $\ell = 0$ and it is spherically symmetrical about the nucleus

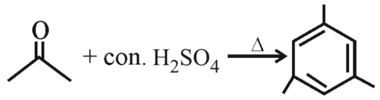
4. **Ans (C)**
For 1st order reaction
$P_{NO_2} = 2P_0(1 - e^{-kt})$

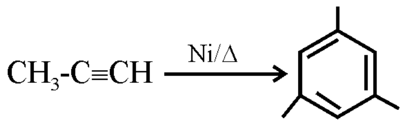
PART-2 : CHEMISTRY

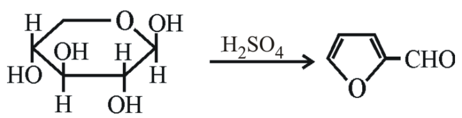
SECTION-I (ii)

5. **Ans (A,B,C,D)**

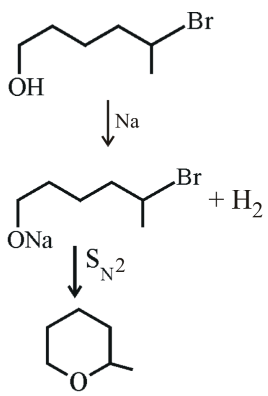








6. **Ans (A,C,D)**



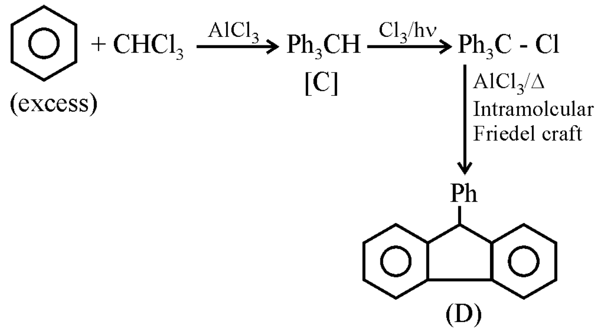
7. **Ans (A,B)**
Theory based

PART-2 : CHEMISTRY

SECTION-II (i)

2. **Ans (9.00)**

$$\underset{(A)}{CH_3-\underset{OH}{CH}-CH_3} \xrightarrow[H_2O]{CaOCl_2} \underset{(B)}{CHCl_3} + (CH_3COO)_2Ca$$



$$\underset{(B)}{CH_2Cl_2} + PhNH_2 \xrightarrow{KOH} \underset{(E)}{PhNC} \xrightarrow{Na/EtoH} \underset{(F)}{Ph\text{-}NH\text{-}CH_3}$$

3. **Ans (486.40)**

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{4^4}\right] = R_H\left[\frac{3}{16}\right]$$

$$\lambda = \frac{16}{3R_H} = 4864\text{Å} = 486.4\,\text{nm}$$

4. **Ans (4.00)**

For 2nd line of Balmer series

$$\frac{1}{\lambda_1} = R_H\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \Rightarrow \lambda_1 = \frac{16}{3R_H}$$

For 1st line of Lyman series

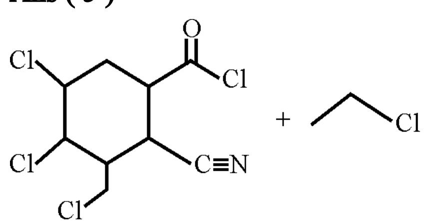
$$\frac{1}{\lambda_2} = R_H\left[1 - \frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{4}{3R_H}$$

$$\frac{\lambda_1}{\lambda_2} = 4$$

PART-2 : CHEMISTRY

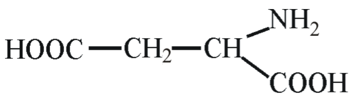
SECTION-II (ii)

5. **Ans (5)**

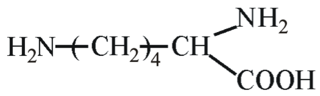


6. Ans (4)

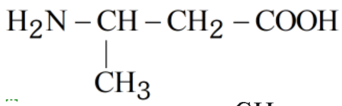
on Hydrolysis it produces



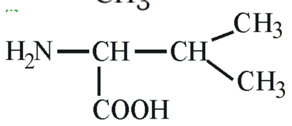
Aspartic acid



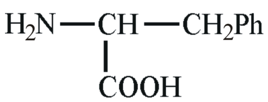
Lysine



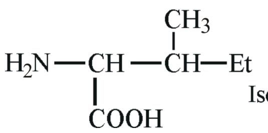
Not α-amino acid



Valine

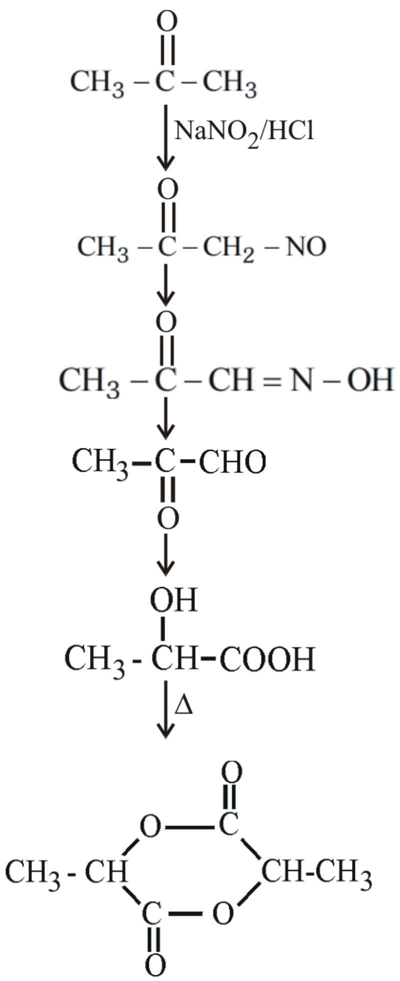


phenyl alanine



Isoleucine

7. Ans (8)



8. Ans (130)

$E_a = 300 \text{ kJ mol}^{-1}$

$$\frac{E_a}{T} = \frac{E_a'}{T'}$$

(Since rate of catalysed and uncatalysed reaction is same)

$$\frac{300}{600} = \frac{E'_{a,f}}{300}$$

$$E'_{a,f} = 150$$

$$20 = 150 - E'_{a,b}$$

$$E'_{a,b} = 130$$

9. Ans (4)

spin multiplicity = 2S + 1

where S = total spin

$= (\text{number of unpaired electron}) \times 1212\frac{1}{2}$

nitrogen atom, there are 3 unpaired electrons

10. Ans (8)

CsCl is 8 : 8 type structure

therefore, C.N. of cation is 8

PART-3 : MATHEMATICS

SECTION-I (i)

1. Ans (C)

Let $x_i - 5 = d_i$

$$\sigma_x^2 = \sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2$$

$$= \frac{125}{10} - \left(\frac{5}{10}\right)^2 = \frac{25}{2} - \frac{1}{4} = \frac{49}{4}$$

2. Ans (C)

$(1-x)^{-15}(1-x)^{-11}$

$= ({}^{14}C_{14} + {}^{15}C_{14}x + {}^{16}C_{14}x^2 + {}^{17}C_{14}x^3 \ldots\ldots\ldots)$

$({}^{10}C_{10} + {}^{11}C_{10}x + {}^{12}C_{10}x^2 + {}^{13}C_{10}x^3 \ldots.)$

coeff of x^{10} in $(1-x)^{-26}$ that is ${}^{26+10-1}C_{10}$

3. **Ans (D)**

First, we get rid of logs by taking powers :

$xyz - 3 + \log_5 x = 2^5$ and so on

and adding all we get

$3xyz + \log_5 xyz = 378$

$xyz = 125$

solving for x, y, z by substituting xyz = 125 in each equation

we get $\log_5 x = -90$, $\log_5 y = -41$, $\log_5 z = 134$

4. **Ans (C)**

Let E_1, E_2, E_3, E_4 be the events that first two drawn books are (math, math) (math, phy) (phy, math) and (phy, phy) respectively and A be the event that third drawn book is of maths.

Here $P(E_1) = \frac{3}{5} \times \frac{5}{7} = \frac{3}{7}$,

$P(E_2) = \frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$,

$P(E_3) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$

$P(E_4) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

Also $P(A/E_1) = \frac{7}{9}; P(A/E_2)$

$= 5/6; P(A/E_3)$

$= \frac{5}{6}$; $P(A/E_4) = 1$

Now by Baye's theorem

$$P(E_4/A) = \frac{P(E_4).P(A/E_4)}{P(A)}$$

$$= \frac{\frac{1}{10} \times 1}{\frac{3}{7} \times \frac{7}{9} + \frac{6}{35} \times \frac{5}{6} + \frac{3}{10} \times \frac{5}{6} + \frac{1}{10} \times 1}$$

$$= \frac{1}{\frac{10}{3} + \frac{10}{7} + \frac{5}{2} + 1}$$

$$= \frac{42}{140 + 60 + 105 + 42} = \frac{42}{347}$$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. **Ans (A,C)**

Here we are given

$n_1 = 100$, $\bar{x}_1 = 15$ and $\sigma_1 = 3$

$n = n_1 + n_2 = 250$, $\bar{x} = 15.6$, and $\sigma = \sqrt{13.44}$

We want σ_3

Obviously $n_2 = 250 - 100 = 150$ we have

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 15.6 = \frac{100 \times 15 + 150 + \bar{x}_2}{250}$$

$\Rightarrow 150\bar{x}_2 = 250 \times 15.6 - 1500 = 2400$

$\therefore \bar{x}_2 = \frac{2400}{150} = 16$

Hence $d_1 = \bar{x}_1 - \bar{x} = 15. - 15.6 = -0.6$

And $d_2 = \bar{x}_2 - \bar{x} = 16 - 15.6 = 0.4$

The variance σ^2 of the combined group is given by the formula :

$$(n_1 + n_2)\sigma^2 = n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)$$

$\Rightarrow 250 \times 13.44 = 100(9 + 0.36) + 150\left(\sigma_2^2 + 0.16\right)$

$\therefore 150\sigma_2^2 = 250 \times 13.44 - 100 \times 9.36 - 150 \times 0.16$

$= 3360 - 936 - 24 = 2400$

$\therefore \sigma_2^2 = \frac{2400}{150} = 16$

Hence $\sigma_2 = \sqrt{16} = 4$

6. **Ans (A,C,D)**

Total number of triangles that are possible

= (Number of Non negative Integral Solutions of Eq $x_1 + x_2 \ldots\ldots\ldots + x_{11} = 3$)

- (Number of cases where triangle is not possible)

Number of solutions $= {}^{3+11-1}C_{11-1} = {}^{13}C_3 = 286\text{-}1$

$= 285$

7. **Ans (A,B,C)**

If the object took 4 steps,

two steps N (North) + 2 steps E (East)

$\Rightarrow \frac{4!}{2!2!} = 6$ ways

$\Rightarrow$ Probability $P_4 = \frac{6}{4^4}$

If the object took 6 steps, then its must be 2N + 2E steps and a pair of moves that would cancel out either N/S or W/E.

$\Rightarrow 2\left(\frac{6!}{3!2!1!} - \frac{4!}{2!2!} \times 2!\right) = 2 \times 48$ ways

$\Rightarrow$ Probability $P_6 = \frac{96}{4^6}$

We can clearly see that $P_5 = 0$

PART-3 : MATHEMATICS

SECTION-II (i)

1. **Ans (5.00)**

$\because \tan^4 x + \cot^4 x \geq 2$ and $\sin 2x \leq 1$

$\Rightarrow \tan^4 x + \cot^4 x = 2$

$\Rightarrow \tan^2 x = \pm 1 \Rightarrow x = n\pi \pm \frac{\pi}{4}$

$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Also $\sin 2x = 1 \Rightarrow 2x = 2n\pi + \frac{\pi}{2}$

$\Rightarrow x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Hence two solutions.

&

$1 + \cos x + \cos 2x + \cos 3x = 4$

$\Rightarrow \cos x + \cos 2x + \cos 3x = 3$

$\Rightarrow \cos x = \cos 2x = \cos 3x = 1$

$\Rightarrow x = 2n\pi$ & $2x = 2m\pi$ & $3x = 2r\pi$

$\Rightarrow$ common solution $x = 2n\pi \Rightarrow 0, 2\pi, 4\pi$

2. **Ans (4.00)**

$a^3 + b^3 + c^3 = 3abc$

$\Rightarrow$ either $a + b + c = 0$

$\Rightarrow \sin x + \cos y + 2 = 0$

$\Rightarrow \sin x = -1$ & $\cos y = -1$

$x = (4n-1)\frac{\pi}{2}$ & $y = (2n+1)\pi$

$$\left.\begin{array}{ll} \text{Or} & \sin x - \cos y = 0 \\ \& & \sin x - 2 = 0 \\ \& & \cos y - 2 = 0 \end{array}\right\} \text{which is not possible}$$

3. **Ans (639.00)**

The composition of the balls in the red box and in the green box and the sum suggested in the problem may be one of the following

Red box		Green box		Sum of green in red box and red in green box	Sum of red in red box and green in green box
Red ball	Green ball	Red ball	Green ball		
0	5	6	3	11	3
1	4	5	4	9	5
2	3	4	5	7	7
3	2	3	6	5	9
4	1	2	7	3	11
5	0	1	8	1	13

In the second last column the 2^{nd} and the last correspond to the sum being not a prime number.

Hence the required probability

$$P = \frac{{}^6C_1 \times {}^8C_4 + {}^6C_5 \times {}^8C_0}{{}^{14}C_5} = \frac{420+6}{2002} = \frac{213}{1001}$$

Thus 3003 × P = 639.

In the last column the 1^{st} and the 4^{th} correspond to the sum being multiple of 3.

Hence the required probability

$$Q = \frac{{}^6C_0 \times {}^8C_5 + {}^6C_3 \times {}^8C_2}{{}^{14}C_5} = \frac{616}{2002}$$

Thus 2002 × Q = 616.

4. **Ans (616.00)**

The composition of the balls in the red box and in the green box and the sum suggested in the problem may be one of the following

Red box		Green box		Sum of green in red box and red in green box	Sum of red in red box and green in green box
Red ball	Green ball	Red ball	Green ball		
0	5	6	3	11	3
1	4	5	4	9	5
2	3	4	5	7	7
3	2	3	6	5	9
4	1	2	7	3	11
5	0	1	8	1	13

In the second last column the 2^{nd} and the last correspond to the sum being not a prime number. Hence the required probability

$$P = \frac{{}^6C_1 \times {}^8C_4 + {}^6C_5 \times {}^8C_0}{{}^{14}C_5} = \frac{420+6}{2002} = \frac{213}{1001}$$

Thus 3003 × P = 639.

In the last column the 1^{st} and the 4^{th} correspond to the sum being multiple of 3.

Hence the required probability

$$Q = \frac{{}^6C_0 \times {}^8C_5 + {}^6C_3 \times {}^8C_2}{{}^{14}C_5} = \frac{616}{2002}$$

Thus 2002 × Q = 616.

PART-3 : MATHEMATICS

SECTION-II (ii)

5. **Ans (9)**

coefficient of x^2 in $(1 + x + 2x^2 + 3x^3)^4$

$(1 + x + 2x^2 + 3x^3)^4$

$$= \sum \frac{4!}{r_1!\, r_2!\, r_3!\, r_4!}(1)^{r_1}(x)^{r_2}(2x^2)^{r_3}(3x^2)^{r_4}$$

$$= \sum \frac{4!}{r_1!\, r_2!\, r_3!\, r_4!} 2^{r_3} 3^{r_4} x^{(r_2 + 2r_3 + 3r_4)}$$

Now $r_2 + 2r_3 + 3r_4 = 2$ and $r_1 + r_2 + r_3 + r_4$

r_1	r_2	r_3	r_4
3	0	1	0
2	2	0	0

Hence coefficient of x^2

$$\frac{4!}{3!1!}2! + \frac{4!}{2!2!} = 14$$

coefficient of x^2 in

$1 + x + 2x^2 + 3x^3 + 4x^4)^2$ is 5

a = 14 and b = 5

a − b = 9

6. **Ans (12)**

$4 - x > 0,\ 1 + x > 0 \Rightarrow x \in (-1, 4)$ .....(1)

Let $|x - 1| < 1 \Rightarrow x \in (0, 2)$ .....(2)

The inequality implies

$\Rightarrow \log_2(4 - x) < \log_2(1 + x)$

$\Rightarrow 4 - x < 1 + x$

$\Rightarrow x < \frac{3}{2}$ .....(3)

$(1) - (3) \Rightarrow x \in \left(0, \frac{3}{2}\right)$

Let $|x - 1| > 1 \Rightarrow x \in (-\infty, 0) \cup (2, \infty)$ ....(4)

The inequality implies

$\Rightarrow \log_2(4 - x) < \log_2(1 + x)$

$\Rightarrow 4 - x < 1 + x$

$\Rightarrow x > \frac{3}{2}$ ......(5)

(1), (4), (5) $\Rightarrow x \in (2, 4)$ Finally, we have

$x \in \left(0, \frac{3}{2}\right) \cup (2, 4)$

7. **Ans (256)**

Required number of ways $= (D_6 + D_5) - (D_5 + D_4)$

$= D_6 - D_4 = 256$

8. **Ans (120)**

$xyz = 72$ (where z is the dummy positive integer)

when $z = 1$, we get the solution of $xy = 72$

when $z = 2$, we get the solution of $xy = 36$

and so on

$\therefore$ the required number of solution will be obtained by number of solutions of xyz

$= 2^3.3^2$

${}^5C_2 \,.\, {}^4C_2 \times 2 = 120$

9. **Ans (16)**

$(1 + \sin^4 x)(2 + \cot^2 y)(4 + \sin 4z) \leq 12 \sin^2 x$

$\Rightarrow (\sin^2 x + \operatorname{cosec}^2 x)(2 + \cot^2 y)(4 + \sin 4z) \leq 12$

Now, $\sin^2 x + \operatorname{cosec}^2 x \geq 2,\ 2 + \cot^2 y \geq 2,\ 4 + \sin 4z \geq 3$

$\Rightarrow \sin^2 x = 1, \cot^2 y = 0, \sin 4z = -1$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, y = \frac{\pi}{2}, \frac{3\pi}{2}, z$

$= \frac{3\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}, \frac{7\pi}{8}$

Number of triplets $= 2 \times 2 \times 4 = 16$

10. **Ans (118)**

Total number of ways in which $n_1 + n_2 = 100$ is equal to 99.

Now, $n_1 . n_2 > 1600$

$\Rightarrow n_1 (100 - n_1) > 1600$

$\Rightarrow n_1{}^2 - 100n_1 + 1600 < 0$

$\Rightarrow (n_1 - 80)(n_1 - 20) < 0$

$\Rightarrow 20 < n_1 < 80$

$\Rightarrow 21 \leq n_1 \leq 79.$

Thus number of favourable ways $= 79 - 21 + 1 = 59$

Hence required probability $p = \frac{59}{99}$

$198p = 118$