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IIT-JEE

Batch – Growth (May) | Minor Test-06

Time: 3 Hours | **Test Date: 15th September 2024** | **Maximum Marks: 300**

Name of the Candidate: ____Roll No.____________

Centre of Examination (in Capitals):____

Candidate's Signature:__________________________Invigilator's Signature:__________________________

READ THE INSTRUCTIONS CAREFULLY

1. The candidates should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
2. This Test Booklet consists of 90 questions.
3. This question paper is divided into three parts **PART A - MATHEMATICS, PART B - PHYSICS** and **PART C - CHEMISTRY** having 30 questions each and every **PART** has two sections.
   (i) **Section-I** contains 20 multiple choice questions with only one correct option. Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
   (ii) **Section-II** contains 10 questions the answer to only 5 questions, is an INTEGERAL VALUE.
   Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc., except the Identity Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/Hall. However, the candidate is allowed to take away this Test Booklet with them.
7. **For integer-based questions, the answer should be in decimals only not in fraction.**
8. **If learners fill the OMR with incorrect syntax (say 24.5. instead of 24.5), their answer will be marked wrong.**

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TEST SYLLABUS

Batch – Growth (May) | Minor Test-06

15^{th} September 2024

Mathematics:	Quadratic Eq
Physics:	WEP
Chemistry:	Thermodynamics-1

Useful Data Chemistry:

Gas Constant	R	$= 8.314\ JK^{-1}mol^{-1}$
		$= 0.0821\ Lit\ atm\ K^{-1}mol^{-1}$
		$= 1.987 \approx 2\ Cal\ K^{-1}mol^{-1}$
Avogadro's Number	N_a	$= 6.023 \times 10^{23}$
Planck's Constant	h	$= 6.626 \times 10^{-34}\ Js$
		$= 6.25 \times 10^{-27}\ erg.s$
1 Faraday		$= 96500$ Coulomb
1 calorie		$= 4.2$ Joule
1 amu		$= 1.66 \times 10^{-27}\ kg$
1 eV		$= 1.6 \times 10^{-19}\ J$

Atomic No:
H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 2, O = 8, Au = 79.

Atomic Masses:
He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 2, Cl = 35.5, Sn = 118.6

Useful Data Physics:
Acceleration due to gravity $g = 10\ m/s^2$

PART-A: MATHEMATICS

SECTION-I

1. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then -

(A) p = 1 and q = 56

(B) p = 1 and q = -56

(C) p = -1 and q = 56

(D) p = -1 and q = -56

Ans. (D)

Sol. $\alpha + \beta = 5$

$\alpha\beta = 16$

Now $(\alpha^2 + \beta^2) + \frac{\alpha\beta}{2} = -p$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta + \frac{\alpha\beta}{2} = -p \Rightarrow -p = (\alpha + \beta)^2 - \frac{3}{2}\alpha\beta$

$\Rightarrow p = -25 + \frac{3}{2}16 = -25 + 24 = -1$

Now $(\alpha^2 + \beta^2) \times \left(\frac{\alpha\beta}{2}\right) = q$

$q = [(\alpha + \beta)^2 - 2\alpha\beta]\left[\frac{\alpha\beta}{2}\right]$

$= [(5)^2 - 2 \times 16]\left[\frac{16}{2}\right] = -56$

2. A quadratic equation with rational coefficients whose one root is $\frac{1}{2+\sqrt{5}}$ will be

(A) $x^2 + 4x - 1 = 0$

(B) $x^2 - 4x - 1 = 0$

(C) $x^2 + 4x + 1 = 0$

(D) None of these

Ans. (A)

Sol. Given root $= \frac{1}{2+\sqrt{5}} = \sqrt{5} - 2$

So, the other root = $-\sqrt{5} - 2$ Then sum of the roots = -4, product of the roots = -1.

Hence, the equation is $x^2 + 4x - 1 = 0$

3. If α, β are roots of the equation $ax^2 + bx + c = 0$ then the value of $\frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$ is

(A) $\frac{b^2-2ac}{ac}$

(B) $\frac{b^2-2ac}{a^2c^2}$

(C) $\frac{2ac-b^2}{ac}$

(D) $\frac{b^2}{a^2c}$

Ans. (B)

Sol. Since α, β are the roots of the equation $ax^2 + bc + c = 0$ then $a\alpha^2 + b\alpha + c = 0 \Longrightarrow \alpha(a\alpha + b) + c = 0$

$(a\alpha + b) = \frac{-c}{\alpha}$ ..... (1)

Similarly

$(a\beta + b) = \frac{-c}{\beta}$

$\therefore \frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2} = \frac{1}{\left(\frac{-c}{\alpha}\right)^2} + \frac{1}{\left(\frac{-c}{\beta}\right)^2}$

$\Rightarrow \frac{\alpha^2}{c^2} + \frac{\beta^2}{c^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{c^2} = \frac{\frac{b^2}{a^2}-\frac{2c}{a}}{c^2} = \frac{b^2-2ac}{a^2c^2}$

4. The roots of the equation $x^2 - 2\sqrt{2}x + 1 = 0$ are

(A) imaginary and different

(B) Real and different

(C) Real and equal

(D) Rational and different

Ans. (A)

Sol. The discriminant of the equation $(-2\sqrt{2})^2 - 4(1)(1) = 8 - 4 = 4 > 0$ and a perfect square, so roots are real and different, but we can't say that roots are rational because coefficients are not rational, therefore.

$\alpha, \beta = \frac{2\sqrt{2}\pm\sqrt{(2\sqrt{2})^2-4}}{2} = \frac{2\sqrt{2}\pm2}{2} = \sqrt{2} \pm 1$ this is irrational.

5. A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common is

(A) $-\sqrt{2}$

(B) $-i\sqrt{3}$

(C) $i\sqrt{5}$

(D) 2

Ans. (B)

Sol. Given that $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common.

The formula to find the common root is given below.

If a is a common root of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

Then,

$a^2/(b_1c_2 - b_2c_1) = a/(a_2c_1 - a_1c_2) = 1/(a_1b_2 - a_2b_1) \ldots (i)$

Comparing with given equations, we get

$a_1 = 1, b_1 = b, c_1 = -1$
$a_2 = 1, b_2 = 1, c_2 = b$
$a^2/(b_1c_2 - b_2c_1) = a/(a_2c_1 - a_1c_2) = 1/(a_1b_2 - a_2b_1)$

Substituting the values in (i) we get

$a^2/(b^2+1) = a/(-1-b) = 1/(1-b)$
$\Rightarrow a^2 = (b^2+1)/(1-b) \ldots$ (ii)
$a = (1+b)/(b-1) \ldots$ (iii)

Put a in (ii)

$(1+b)^2/(b-1)^2 = (b^2+1)/(1-b)$
$\Rightarrow (1+b)^2(1-b) = (b-1)^2(b^2+1)$
$\Rightarrow -(b-1)(1+b)^2 = (b-1)^2(b^2+1)$
$\Rightarrow -(1+b)^2 = (b-1)(b^2+1)$
$\Rightarrow (-1-2b-b^2) = (b^3-b^2+b-1)$
$\Rightarrow -3b = b^3$
$\Rightarrow b^2 = -3$

Taking square root

$\Rightarrow b = \pm i\sqrt{3}$

6. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(A) $\frac{1}{4}$

(B) 41

(C) 1

(D) $\frac{17}{7}$

Ans. (B)

Sol. $y = \frac{3x^2+9x+17}{3x^2+9x+7}$

$3x^2(y-1) + 9x(y-1) + 7y - 17 = 0$

$D \geq 0$ as x is real

$81(y-1)^2 - 4 \times 3(y-1)(7y-17) \geq 0$

$81(y-1)^2 - 4 \times 3(y-1)(7y-17) \geq 0$

$\Rightarrow (y-1)(y-41) \leq 0$

$\Rightarrow 1 \leq y \leq 41$

$\therefore$ Max value of y is 41

7. Determine the nature of the roots of the equation $2x^2 + 5x + 5 = 0$

(A) imaginary and distinct

(B) real and equal

(C) imaginary and equal

(D) real and distinct

Ans. (A)

Sol. Given:

The equation $2x^2 + 5x + 5 = 0$

Concept Used:

The quadratic equation $Ax^2 + Bx + C = 0$

If roots are imaginary, $B^2 - 4ac < 0$

Calculation:

Comparing of given equation

Now, A = 2, B = 5 & C = 5

$\Rightarrow (5)^2 - 4 \times 2 \times 5 < 0$

$\Rightarrow 25 - 40 < o$

$\Rightarrow -15 < 0$

$\therefore$ The roots are imaginary and distinct.

8. It is given that the roots of the equation $x^2 - 2x - \log_2 K = 0$ are real. For this, the minimum value of K is

(A) 1

(B) 1/2

(C) 1/4

(D) 1/16

Ans. (B)

Sol. Given: $x^2 - 2x - \log_2 K = 0$

General form of equation: $ax^2 + bx + c = 0$

On comparing a = 1, b = - 2, c = - $\log_2 K$

Given that the roots are real

So, Discriminant ≥ 0

$\Rightarrow b^2 - 4ac \geq 0$
$\Rightarrow (-2)^2 - 4 \times 1 \times (-\log_2 \mathrm{K}) \geq 0$
$\Rightarrow 4 + 4\log_2 K \geq 0$
$\Rightarrow 4\log_2 \mathrm{K} \geq -4$
$\Rightarrow \log_2 \mathrm{K} \geq -1$
$\Rightarrow K \geq (2)^{-1}$
$\therefore K \geq 1/2$

The minimum value of K is 1/2.

9. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c ∈ R, have a common root, then a : b : C is

(A) 1 : 2 : 3
(B) 3 : 2 : 1
(C) 1 : 3 : 2
(D) 3 : 1 : 2

Ans. (A)

Sol. Given equations are

$x^2 + 2x + 3 = 0$ .... (i)

$ax^2 + bx + c = 0$ .... (ii)

Roots of equation (i) are imaginary roots.

According to the question (ii) will also have both roots same as (i)

Thus $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda(say)$

$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$

Hence, required ratio is 1 : 2 : 3

10. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to:

(A) $\frac{p^2+q^2}{2}$
(B) $p^2 + q^2$
(C) $2(p^2 + q^2)$
(D) $p^2 + q^2 + r^2$

Ans. (B)

Sol. Given,

$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$
$\Rightarrow \frac{x+p+x+q}{(x+p)(x+q)} = \frac{1}{r}$
$\Rightarrow (2x + p + q)r = x^2 + px + qx + pq$
$\Rightarrow x^2 + (p + q - 2r)x + pq - pr - qr = 0$

Let α and β are the roots,

$\therefore \alpha + \beta = -(p + q - 2r)$
and $\alpha\beta = pq - pr - qr$
Given that, $\alpha = -\beta \Rightarrow \alpha + \beta = 0$
$\therefore -(p + q - 2r) = 0$
Now, $\alpha^2 + \beta^2$
$= (\alpha + \beta)^2 - 2\alpha\beta$
$= (-(p + q - 2r))^2 - 2(pq - pr - qr)$
$= p^2 + q^2 + 4r^2 + 2pq - 4pr - 4qr - 2pq + 2pr + 2qr$
$= p^2 + q^2 + 4r^2 - 2pr - 2qr$
$= p^2 + q^2 - 2r(p + q - 2r)$
$= p^2 + q^2 - 2r(0)$
$= p^2 + q^2$

11. The minimum value of the expression $4x^2 + 2x + 1$ is -
(A) 1/4
(B) 1/2
(C) 3/4
(D) 1

Ans. (C)

Sol. Since a = 4 > 0 therefore its minimum value = $-\frac{D}{4a} = \frac{4(4)(1)-(2)^2}{4(4)} = \frac{16-4}{16} = \frac{12}{16} = \frac{3}{4}$

12. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is:
(A) 3
(B) 2
(C) 4
(D) 1

Ans. (B)

Sol. Given, α and β and be the roots of $x^2 - 6x - 2 = 0$
$\alpha + \beta = 6$
$\alpha\beta = -2$
and $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$
$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$
$\frac{a_{10}-2a_8}{3a_9} = \frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{3(\alpha^9-\beta^9)}$
$= \frac{(\alpha^{10}-2\alpha^8)-(\beta^{10}-2\beta^8)}{3(\alpha^9-\beta^9)}$
Now, $= \frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{3(\alpha^9-\beta^9)}$
$= \frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{3(\alpha^9-\beta^9)} = \frac{6(\alpha^9-\beta^9)}{3(\alpha^9-\beta^9)} = \frac{6}{3} = 2$

13. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15°, respectively, then the value of 2 + q — p is
(A) 2
(B) 3
(C) 0
(D) 1

Ans. (B)

Sol. $x^2 + px + q = 0$

Sum of roots $= \tan 30° + \tan 15° = -p$

Products of roots = $\tan 30° \cdot \tan 15° = q$

$\tan 45° = \frac{\tan 30°+\tan 15°}{1-\tan 30°\cdot\tan 15°}$

$= \frac{-p}{1-q} = 1$

$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$

$\therefore 2 + q - p = 3$

14. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(A) (5, 6]

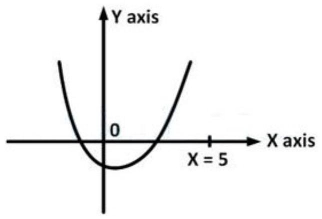
(B) (6, ∞)

(C) (-∞, 4)

(D) [4, 5]

Ans. (C)

Sol.



both roots are less than 5,

then (i) Discriminant ≥ 0

(ii) $p(5) > 0$

(iii) $\frac{\text{Sum of roots}}{2} < 5$

$Hence, (i)\ 4k^2 - 4(k^2 + k - 5) \geq 0$

$4k^2 - 4k^2 - 4k + 20 \geq 0$

$4k \leq 20 \Rightarrow k \leq 5$

(ii) $f(5) > 0; 25 - 10k + k^2 + k - 5 > 0$

$or\ k^2 - 9k + 20 > 0$

or $k(k-4) - 5(k-4) > 0$

or $(k-5)(k-4) > 0$

$\Rightarrow k \in (-\infty, 4) \cup (5,\ \infty)$

(iii) $\frac{\text{Sum of roots}}{2}$

$= -\frac{b}{2a} = \frac{2k}{2} < 5$

The intersection of $(i), (ii)(iii)$ gives

$k \in (-\infty, 4)$

15. Find the value of x for which the expression $2 - 3x - 4x^2$ has the greatest value.

(A) $-\frac{41}{16}$

(B) $\frac{3}{8}$

(C) $-\frac{3}{8}$

(D) $\frac{41}{16}$

Ans. (C)

Sol. $-4x^2 - 3x + 2 = 0$

greatest value at $x = -\frac{b}{2a} = \frac{3}{-8} = -3/8$

16. If (1 – p) is a root of quadratic equation x^2 + px + (1 – p) = 0 then its root are

(A) -1, 2

(B) -1, 1

(C) 0, -1

(D) 0, 1

Ans. (C)

Sol. Let the second root be α.

Then α + (1 – p) = -p ⇒ α = -1

Also α. (1 – p) = 1 – p

$\Rightarrow (\alpha - 1)(1 - p) = 0$

$\Rightarrow p = 1$ [as $\alpha = -1$]

∴ Roots are α = -1 and p – 1 = 0

17. All the values of *m* for which both roots of the equation x^2 – 2mx + m^2 – 1 = 0 are greater than – 2 but less than 4, lie in the interval

(A) -2 < m < 0

(B) m > 3

(C) -1 < m < 3

(D) 1 < m < 4

Ans. (C)

Sol. Equation $x^2 - 2mx + m^2 - 1 = 0$

$(x - m)^2 - 1 = 0$

or $(x - m + 1)(x - m - 1) = 0$

$x = m - 1, m + 1$

$m - 1 > -2$ and $m + 1 < 4$

$\Rightarrow m > -1$ and $m < 3$

or $-1 < m < 3$

18. If x^2 – px + 4 > 0 for all real value of x, then which one of the following is correct?

(A) |p| < 4

(B) |p| ≤ 4

(C) |p| > 4

(D) |p| ≥ 4

Ans. (A)

Sol. Given: x^2 – px + 4 > 0 ∀ real values of X

On comparing the given equation with the standard quadratic equation

$ax^2 + bx + c = 0$, we get

a = 1, b = -p and c = 4

Given quadratic equation is more than zero that means it has no real roots If roots are not real (Then only the quadratic equation will not intersect the x-axis)

$\therefore b^2 - 4ac < 0$
$\Rightarrow (-p)^2 - 4(1)(4) < 0$
$\Rightarrow (p)^2 - 16 < 0$
$\Rightarrow (p)^2 - 4^2 < 0$
$\Rightarrow (p+4)(p-4) < 0$

(Using $a^2 - b^2 = (a+b)\cdot(a-b)$)

This inequality holds if p ranges from -4 to 4 i.e., p = (-4, 4)

Only |p| < 4 satisfy the above condition.

19. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has:

(A) infinite number of real roots

(B) no real roots

(C) exactly one real root

(D) exactly four real roots

Ans. (B)

Sol. Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation,

we get $t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$

$= \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2}$

$= 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$(as $t = e^{\sin x}$)

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and

$e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$

and $\sin x = \ln(2 + \sqrt{5}) > 1$ So, rejected

Hence given equation has no solution.

$\therefore$ The equation has no real roots.

20. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then the value of (a + b) is

(A) 1

(B) -1

(C) 2

(D) 0

Ans. (B)

Sol. $x^2 + ax + b = 0$ ...(i)

$x^2 + bx + a = 0$ ...(ii)

If α is a common root of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

$a^2/(b_1c_2 - b_2c_1) = a/(a_2c_1 - a_1c_2) = 1/(a_1b_2 - a_2b_1)$ ... (iii)

Comparing (i) and (ii) with above equations, we get

$a_1 = 1, b_1 = a, c_1 = b$
$a_2 = 1, b_2 = b, c_2 = a$
$\Rightarrow a^2/(a^2 - b^2) = a/(b - a) = 1/(b - a)$
$\Rightarrow a = 1$
So $1/(a^2 - b^2) = 1/(b - a)$
$\Rightarrow (a - b)(a + b) = (b - a)$
$\Rightarrow a + b = -1$

SECTION-II

21. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

Ans. (1)

Sol. When $2^x \geq 1$

$5 + 2^x - 1 = 2^x(2^x - 2)$
Let $2^x = t$
$\Rightarrow 5 + t - 1 = t(t - 2)$
$\Rightarrow t = 4, -1$ (rejected)
$\Rightarrow 2^x = 4$
$\Rightarrow x = 2$

New when, $2^x < 1$

$5 + 1 - 2^x = 2^x(2^x - 2)$
Let $2^x = t$
$\Rightarrow 5 + 1 - t = t(t - 2)$
$\Rightarrow 0 = t^2 - t - 6$
$\Rightarrow 0 = (t - 3)(t - 2)$
$\Rightarrow t = 3, -2$
$2^x = 3, 2^x = -2$(rejected)

$\therefore$ Only one real root.

22. Find the values of k so that one root of $5x^2 + 13x + k = 0$ is reciprocal of the other:

Ans. (5)

Sol. $\alpha \cdot \frac{1}{\alpha} = \frac{k}{5} \Rightarrow 5$

23. Find the maximum value of quadratic expression $-4(x - 2)^2 + 2$.

Ans. (2)

Sol. Since the value of 'a' is negative, the given quadratic equation will have a maximum value. Hence, the maximum value of the quadratic equation $-4(x — 2)^2 + 2$ is 2.

24. The number of common roots of the equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is

Ans. (1)

Sol. Step 1: - For, $x^2 - 7x + 10 = 0$, roots are 2 and 5

Step 2: - For, $x^2 - 10x + 16 = 0$, roots are 2 and 8.

Step 3: - Thus, common root is only one, i.e. is 2.

25. If the product of the roots of the equation $x^2 — 3kx + 2e^{2\log k} — 1 = 0$ is 7, then the roots are real for k equal to

Ans. (2)

Sol. Given: $x^2 - 3kx + 2e^{2\log k} - 1 = 0$

$\Rightarrow x^2 - 3kx + 2k^2 - 1 = 0\left(\text{ since } n\log x = \log x^n, e^{\log x} = x\right)$

Product of roots = 7

$\Rightarrow 2k^2 - 1 = 7$
$\Rightarrow 2k^2 = 8$
$\Rightarrow k^2 = 4$
$\Rightarrow k = 2$, or $k = -2$.

For real roots, k > 0.

So, k = 2

26. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to ____.

Ans. (45)

Sol. Given that,

$e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$
$\Rightarrow 2e^{3x} - 22e^{2x} - 90 + 81e^x = 0$
$\Rightarrow 2(e^x)^3 - 22(e^x)^2 + 81e^x - 90 = 0$
Let $e^x = y$
$\Rightarrow 2y^3 - 22y^2 + 81y - 90 = 0$

Product of roots (y_1, y_2, y_3)

$y_1 \cdot y_2 \cdot y_3 = \frac{-(-90)}{2} = 45$

Let x_1, x_2, and x_3 be roots of given equation

$\Rightarrow e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45$
$\Rightarrow e^{x_1+x_2+x_3} = 45$
$\Rightarrow x_1 + x_2 + x_3 = \log_e 45 = \log_e p$
$\Rightarrow p = 45$

27. The minimum value of the sum of the squares of the roots of $x^2 + (3 - a)x + 1 = 2a$ is:

Ans. (6)

Sol.

$$x^2 + (3 - a)x + 1 = 2a \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

$\alpha + \beta = a - 3, \alpha\beta = 1 - 2a$
$\Rightarrow \alpha^2 + \beta^2 = (a - 3)^2 - 2(1 - 2a)$
$= a^2 - 6a + 9 - 2 + 4a$
$= a^2 - 2a + 7$
$= (a - 1)^2 + 6$

So, $\alpha^2 + \beta^2 \geq 6$

28. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is:

Ans. (3)

Sol. For rational D must be perfect square

$D = 121 - 24\alpha$

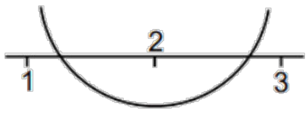
for $121 - 24\alpha$ to be perfect square α must be 3, 4, 5

so, Ans $\alpha = 3$

29. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3) is:

Ans. (1)

Sol.



$f(1) > 0 \Rightarrow k > 6$
$f(2) < 0 \Rightarrow k < 8$
$f(3) > 0 \Rightarrow k > 6$
$k \in (6,8)$

Only 1 integral value of k is 7.

30. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

Ans. (2)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64, \alpha\beta = 256$

$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$

$= \frac{\alpha^{\frac{3}{8}}}{\beta^{\frac{5}{8}}} + \frac{\beta^{\frac{3}{8}}}{\alpha^{\frac{5}{8}}}$

$= \frac{\alpha+\beta}{(\alpha\beta)^{\frac{5}{8}}}$

$= \frac{64}{(256)^{\frac{5}{8}}}$

= 2

PART-B: PHYSICS

SECTION-I

31. If the unit of force and length each be increased by four times, then the unit of work is increased by

(A) 16 times

(B) 2 times

(C) 4 times

(D) 8 times

Ans. (A)

Sol. Joule = (Newton) (Metre) = $= \frac{4\ \text{Newton}}{4} \times \frac{4\ \text{Metre}}{4} = \frac{\text{Joule}}{16}$

Hence, 1 Joule = 16 joule (Joule is new unit of force)

32. A body of 5 kg mass is raised vertically to a height of 10 m by a force of 120 N. Find the final velocity of the body:

(A) $\sqrt{280}$ m/s

(B) $\sqrt{200}$ m/s

(C) 20 m/s

(D) None of these

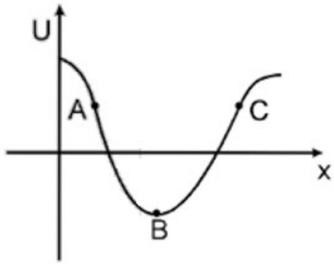
Ans. (A)

Sol. $120 - 50 = 5a \Rightarrow a = 14m/s$

$v^2 = 0^2 + 2(14)(10)$

$\Rightarrow v = \sqrt{280}\text{m/s}$

33. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A, B and C is respectively.



(A) Zero, Positive, Positive
(B) Positive, Zero, Negative
(C) Negative, Zero, Positive
(D) Negative, Zero, Negative

Ans. (B)

Sol. $\left.\frac{dU}{dx}\right|_{x=A} = -ve, \left.\frac{dU}{dx}\right|_{x=B} = +ve$

So, F_A = positive, F_B = negative

34. The negative of the work done by the conservative internal forces on a system equals the change in its

(A) total energy
(B) kinetic energy
(C) potential energy
(D) None of these

Ans. (C)

Sol. Follows from definition.

35. A body is dropped from a certain height. When it loses U amount of its energy it acquires a velocity 'v'. The mass of the body is:

(A) $2U/v^2$
(B) $2v/U^2$
(C) $2v/U$
(D) $U^2/2v$

Ans. (A)

Sol. $U_i + 0 = U_f + \frac{1}{2}mv^2$

$U_i - U_f = \frac{1}{2}mv^2$

$U = \frac{1}{2}mv^2$

$m = \frac{2U}{v^2}$

36. A rigid body is acted upon by a horizontal variable force which is inversely proportional to the distance covered from its initial position 's'. The work done by this force will be proportional to:

(A) s

(B) s^2

(C) $\sqrt{s}$

(D) None of these

Ans. (D)

Sol. $W_F = \int \left(\frac{K}{S}\right) ds = K\ln s + C$

37. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by:

(A) 100 J

(B) 200 J

(C) 300 J

(D) 400 J

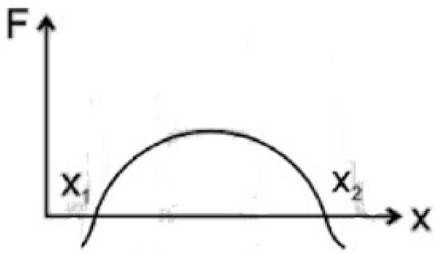
Ans. (C)

Sol. $100 = \frac{1}{2}K(2\text{cm})^2, E = \frac{1}{2}K(4\text{cm})^2$

So, $\frac{E}{100} = 4, E = 400\text{J}$

$\therefore E - 100 = 300\text{J}$

38. The force acting on a body moving along x-axis varies with the position of the particle as shown in the figure.



The body is in stable equilibrium at

(A) $x = x_1$

(B) $x = x_2$

(C) both x_1 and x_2

(D) Neither x_1 nor x_2

Ans. (B)

Sol. At $x = x_2$, as x increases, F acts along negative x-direction.

$x = x_2$

39. A stone is projected vertically up with a velocity u, reaches upto a maximum height h. When it is at a height of 3h/4 from the ground, the ratio of KE and PE at that point is : (consider PE = O at the point of projection)

(A) 1 : 1

(B) 1 : 2

(C) 1 : 3

(D) 3 : 1

Ans. (C)

Sol. $\frac{1}{2}mu^2 = mgh, u^2 = 2gh \ldots..(i)$

$mg\left(\frac{3h}{4}\right) + \text{K.E.} = mgh$

$\text{K.E.} = \frac{mgh}{4}$

$\frac{\text{K.E.}}{\text{P.E.}} = \frac{mgh/4}{3mgh/4} = \frac{1}{3}$

40. A particle moves from position $\vec{r}_1 = (3\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$ m to position $\vec{r}_2 = (14\hat{\imath} + 13\hat{\jmath} + 9\hat{k})$ m under the action of force $(4\hat{\mathbf{i}} + \hat{\jmath} + 3\hat{k})$N. The work done by this force will be

(A) 100 J

(B) 50 J

(C) 200 J

(D) 75 J

Ans. (A)

Sol. $W = \vec{F}'.(\bar{r}_2 - \bar{r}_1) = 100\text{J}$

41. The ratio of work done by the internal forces of a car in order to change its speed from O to V and from V to 2V is (Assume that the car moves on a horizontal road):

(A) 1

(B) 1/2

(C) 1/3

(D) 1/4

Ans. (C)

Sol. Work done in changing in speed from 0 to V is

$W_1 = \frac{1}{2}mV^2$

work done in changing the speed from V to 2V is

$W_2 = \frac{1}{2}m(2V)^2 - \frac{1}{2}mV^2 = \frac{1}{2}3mV^2$

$\therefore \frac{W_1}{W_2} = \frac{1}{3}$

42. A man pushes wall and fails to displace it. He does

(A) Negative work

(B) Positive but not maximum work

(C) No work at all

(D) Maximum work

Ans. (C)

Sol. W = (force) (displacement) = (force) (zero) = 0

43. The potential energy of a particle in a field is $U = \frac{a}{r^2} - \frac{b}{r}$, where a and b are constant. The value of r in terms of a and b where force on the particle is zero will be:

(A) $\frac{a}{b}$

(B) $\frac{b}{a}$

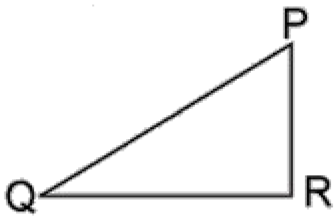
(C) $\frac{2a}{b}$

(D) $\frac{2b}{a}$

Ans. (C)

Sol. $\frac{du}{dr}=0, -\frac{2a}{r^3}+\frac{b}{r^2}=0, r=\frac{2a}{b}$

44. For the path PQR in a conservative force field (fig.), the amount of work done in carrying a body from P to Q & from Q to R are 5 J & 2 J respectively. The work done in carrying the body from P to R will be —



(A) 7 J
(B) 3 J
(C) √21 J
(D) ZERO

Ans. (A)

Sol. $W_3 = W_1 + W_2 = 5 + 2 = 7$

45. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(A) 850 J
(B) 950 J
(C) 900 J
(D) 875 J

Ans. (C)

Sol. $x = 3t^2 + 5$

$v = \frac{dx}{dt}$

$v = 6t + 0$

at $t = ov = o$

$t = 5\text{sec} \;\; v = 30\text{m/s}$

$\text{W.D.} = \Delta KE$

$\text{W.D.} = \frac{1}{2}mv^2 - o = \frac{1}{2}(2)(30)^2 = 900J$

46. The potential energy of a system of two particles is given by U(x) = a/x² — b/x. Find the minimum potential energy of the system, where x is the distance of separation and a, b are positive constants.

(A) $-\frac{b^2}{4a}$
(B) $\frac{b^2}{4a}$
(C) $\frac{2a}{b}$
(D) $-\frac{2a}{b}$

Ans. (A)

Sol. $U(x) = a/x^2 - b/x, a > 0, b > 0$

$dU/dx = \frac{-2a}{x^3} + \frac{b}{x^2} = 0 \Rightarrow \frac{2a}{b} = x$

$d^2U/dx^2 = \frac{6a}{x^4} - \frac{2b}{x^3} = \frac{6a\times b^4}{16a^4} - \frac{2b\times b^3}{8a^3} > 0$

$\therefore U$ is min. at $x = \frac{2a}{b}$ \& min. value of U is

$U_{\text{min.}} = \frac{ab^2}{4a^2} - \frac{bb}{2a} = -\frac{b^2}{4a}$

47. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is:

(A) 1 : 2 : 3

(B) 1 : 4 : 9

(C) 1 : 3 : 5

(D) 1 : 5 : 3

Ans. (C)

Sol. $S_1 = \frac{1}{2}g1^2, S_2 = \frac{1}{2}g2^2, S_3 = \frac{1}{2}g3^2$

$S_2 - S_1 = \frac{1}{2}$ g $3, S_3 - S_2 = \frac{1}{2}$g5

$W_1 = (\text{mg})S_1, W_2 = (\text{mg})(\text{S}_2 - \text{S}_1), W_3 = (\text{mg})(\text{S}_3 - \text{S}_2)$

$\text{W}_1 : \text{W}_2 : \text{W}_3 = 1:3:5$

48. Two springs A and B (k_A = $2k_B$) are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, then in B is (assume equilibrium):

(A) E/2

(B) 2E

(C) E

(D) E/4

Ans. (B)

Sol. $W_F + W_S = 0, W_F - \Delta U = 0, W_F = \Delta U = E$

$\text{E} = \frac{1}{2}\text{K}_A x_A^2, \text{F}_\text{A} = \frac{1}{2}\text{K}_A x_{A^2}$

$\frac{2\text{F}}{\text{K}_A} = \text{X}_\text{A}, \frac{2\text{F}}{\text{K}_A} = \sqrt{\frac{2E}{\text{K}_A}}, \text{K}_\text{A} = \frac{2\text{F}^2}{E}$ (i)

Similarly $\text{K}_B = \frac{2\text{F}^2}{\text{E}_\text{B}}, \because \text{K}_\text{A} = 2\text{K}_\text{B} \therefore \frac{2\text{F}^2}{\text{E}} = 2\left(\frac{2\text{F}^2}{\text{E}_\text{B}}\right)$

$\therefore E_B = 2E$

49. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is

(A) 0°

(B) 30°

(C) 60°

(D) 90°

Ans. (C)

Sol. $25 = 5 \times 10 \times \cos\,\theta$ so $\theta = 60^\circ$

50. The potential energy for a force field $\vec{\text{F}}$ is given by U(x, y) = sin(x + y). The magnitude of force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is

(A) 1

(B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Ans. (A)

Sol. $\frac{\partial U}{\partial x} = \cos(x+y)$

$\frac{\partial U}{\partial y} = \cos(x+y)$

$\vec{F} = -\cos(x+y)\hat{\imath} - \cos(x+y)\hat{\jmath}$

$= -\cos\left(0+\frac{\pi}{4}\right)\hat{\imath} - \cos\left(0+\frac{\pi}{4}\right)\hat{\jmath}$

$|\vec{F}| = 1$

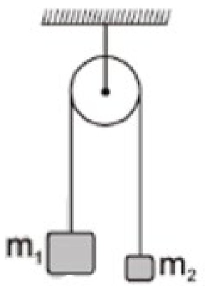
SECTION-II

51. A particle moves along the x-axis from x = 0 to x = 5 m under the influence of a force F (in N) given by F = $3x^2$ - 2x + 7. Calculate the work done by this force.

Ans. (135)

Sol. $W = \int F dx = \int_0^5 (3x^2 - 2x + 7)dx = 135J$

52. The heavier block in an Atwood machine has a mass twice that of the lighter one. The tension in the string is 16.0 N when the system is set into motion. Find the decrease in the gravitational potential energy (in Joule) during the first second after the system is released from rest. (Take g = 10 m/s^2)



Atwood machine

Ans. (20)

Sol. Let $m_1 = 2m_2$

$\therefore a = \frac{(m_1-m_2)}{(m_1+m_2)}g = \frac{m_2}{3m_2}g = g/3$

So, distance travelled by each block $= \frac{1}{2}at^2 = g/6$

Also, $T = \frac{2m_1m_2g}{m_1+m_2} = \frac{4m_2g}{3} = 16 \Rightarrow m_2 = \frac{12}{g}$

Hence, loss in gravitation P.E. during first second

$= (m_1 - m_2)gh$

$= (2m_2 - m_2)g \times \frac{g}{6} = \frac{12}{g} \times g \times \frac{g}{6} = 2g$

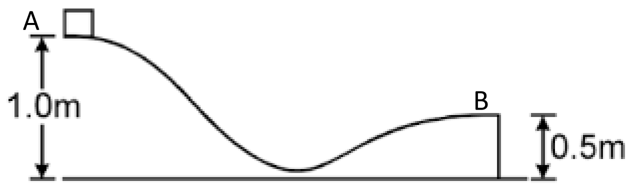
53. The potential energy for a force field $\vec{F}$ is given by $u(x,y) = -x^2y$. The magnitude of force acting on the particle of mass m at (1,0) is

Ans. (1)

Sol. $\vec{F} = (x, y) = -\nabla U(x, y)$

$\vec{F} = -[-2xy\hat{\imath} - x^2\hat{\jmath}]$
$F = 0\hat{\imath} + \hat{\jmath}$
$|F| = 1\,N$

54. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far (in meter) away from the track will the particle hit the ground?



Ans. (1)

Sol. Apply mechanical energy conservation,

$(mg \times 1) + 0 = mg\,(0.5) + \frac{1}{2}mV^2$

Where $(mg \times 1) = P.E_{initial}$; $o = K.E_{initial}$; $mg(0.5) = P.E_{final}$

. $mg = \frac{mg}{\Sigma} + \frac{mV^2}{\Sigma}$

. $\frac{mg}{\Sigma} = \frac{mV^2}{\Sigma}$

. $V = \sqrt{gm/s}$

Range = R = Vt

t = Time of flight.

$\because H = \frac{1}{2gt^2}$

$\therefore t = \sqrt{\frac{2H}{g}}$

$\therefore R = \sqrt{g} \cdot \sqrt{\frac{2H}{g}}$

$R = 1m$

55. A particle moves with a velocity $\vec{v} = (5\hat{\imath} - 3\hat{\jmath} + 6\hat{k})m/s$ under the influence of a constant force $\vec{F} = (10\hat{\imath} + 10\hat{\jmath} + 20\hat{k})N$. The instantaneous power (in watt) applied to the particle is:

Ans. (140)

Sol. $\vec{P} = \vec{F} \cdot \vec{V}$

$P = (10\hat{\imath} + 10\hat{\jmath} + 10\hat{k}) \cdot (5\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$
$P = (50 - 30 + 120)$
$P = 140J/sec$

56. A rigid body of mass 5 kg initially at rest is moved by a horizontal force of 20 N on a frictionless table. Calculate the work done by the force (in Joule) on the body in 10 second.

Ans. (4000)

Sol. Given, mass of the body, m = 5 kg;

Initial velocity of the body, u = o;

Force acting on the body, F = 20 N;

Time for which the force acts, t = 10 s

If a is the acceleration produced in the body,

$a = \frac{F}{m} = \frac{20}{5} = 4 m/s^2$

Let v be the velocity of the body after 10 s.

Clearly, v = u + at = o + (4) x (10) = 40 m/s

Kinetic energy acquired by the body,

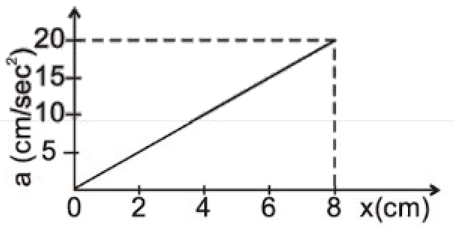
$E_k = \frac{1}{2} mv^2 = \frac{1}{2}(5) \times (40)^2 = 4000J$

57. A particle of mass m = 1 kg is moving along y-axis and a single conservative force F(y) acts on it. The potential energy of particle is given by U(y) = (y^2—6y+ 14) J where y is in meters. At y = 3 m the particle has kinetic energy of 15 J.
The total mechanical energy of the particle is

Ans. (20)

Sol. Total mechanical energy = kinetic energy + potential energy $= 15 + [3^2 - 6(3) + 14] = 15 + 5 = 20J$

58. A 10 kg mass moves along x-axis. Its acceleration as function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from x = O to x = 8 cm? If work done = N × 10^{-2}J, then find the value of N



Ans. (8)

Sol. $8 \times 10^{-2} J$

Work done = m x (area under the graph)

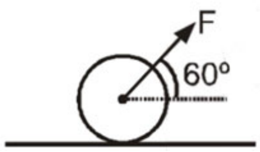
$= 10 \times \frac{1}{2} \times 20 \times 8 \times 10^{-4} J$

$= 8 \times 10^{-2} J$

59. A gardener pulls a lawn roller along the ground through a distance of 20 m. If he applies a force of 20 kg wt in a direction inclined at 60° to the ground, find the work done (In Joule) by him. (Take g = 10 m/s^2)

Ans. (2000)

Sol.

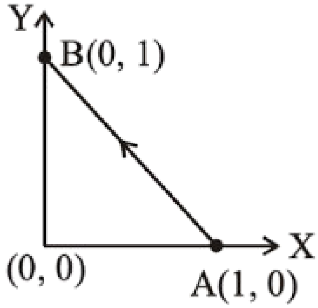


$W_F = Fs\cos 60°$

$= 200 \times 20 \times \frac{1}{2} J$

$= 2000J.$

60. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)



Ans. (1)

Sol. $W = \int \vec{F} \cdot d\vec{s}$

$= (-x\hat{\imath} + y\hat{\jmath}) \cdot (dx\hat{\imath} + dy\hat{\jmath})$

$\int_1^0 -xdx + \int_0^1 ydy$

$= -\frac{x^2}{2}\Big|_0^1 + \frac{y^2}{2}\Big|_1^0 = \left(0 + \frac{1}{2}\right) + \left(\frac{1}{2}\right)$

$= 1\text{J}$

PART-C: CHEMISTRY

SECTION-I

61. Specific heat capacity, molar heat capacity and heat capacity respectively are:

(A) extensive, intensive, intensive

(B) intensive, intensive, extensive

(C) intensive, intensive, intensive

(D) extensive, extensive, intensive

Ans. (B)

Sol. Specific heat capacity = $\frac{q}{m\Delta T}$ so Mass independent, intensive

Molar heat capacity = $\frac{q}{n\Delta T}$ so Intensive

heat capacity = $\frac{q}{\Delta T}$, extensive

62. 2 moles of ideal gas is expanded reversibly from 4 atm to 3 atm at constant temperature of 300 K. Calculate the work done (approx.)

[log4 = 0.6, log3 = 0.48]

(A) -273 cal

(B) -332 cal

(C) -402 cal

(D) -315 cal

Ans. (B)

Sol. $W = -2.303nRT\log\frac{p_1}{p_2}$

$= -2.303 \times 2 \times 2 \times 300\log\frac{4}{3}$

$= -2.303 \times 1200[0.6 - 0.48]$

$= -331.632$

$= -332\text{cal}$

63. Calculate the number of kJ of heat necessary to raise the temperature of 60.0 g of aluminum from 35° C to 55° C. Molar heat capacity of Al is 24 J mol^{-1} K^{-1}.

(A) 2 kJ

(B) 1.07 kJ

(C) 1.5 kJ

(D) 1.25 kJ

Ans. (B)

Sol. $q = nC_m\Delta T$

$= \left(\frac{60}{27}\right) \times \frac{24\times20}{1000} = 1.07\text{kJ}$

64. A thermodynamic state function is:

(A) one which obeys all the law of thermodynamics

(B) a quantity which is used in measuring thermal changes

(C) one which is used in thermochemistry

(D) a quantity whose value depends only on the state of system

Ans. (D)

Sol. Conceptual

65. The heat of combustion of sucrose $C_{12}H_{22}O_{11}$ (s) at constant volume is – 1348.9 kcal mol^{-1} at 25° C, then the heat of reaction at constant pressure, is

(A) -1348.9 kcal

(B) -1342.344 kcal

(C) 1250 kcal

(D) -1250 kcal

Ans. (A)

Sol. Heat of combustion of sucrose:

$C_{12}H_{22}O_{11}(s) + 12O_2(g) \longrightarrow 12CO_2(g) + 11H_2O(l)$

$\Delta n_g = 12 - 12$

$= 0$

The relation between heat of reaction at constant pressure and constant volume is

$\Delta H = \Delta U + RT\Delta n_g$

Where $\Delta H =$ Heat of reaction at constant pressure

$\Delta U =$ Heat of reaction at constant volume

$\text{R} =$ Universal gas constant $= 2\text{Cal/K/mol}$

$= 2 \times 10^{-3}\text{KCal}^{-1}\text{Kmol}^{-1}$

$\Delta H = -1348.9 + (2 \times 10^{-3} \times 298 \times 0) =$ -1348.9 kcal mol^{-1}

66. An ideal gas undergoes isothermal compression from 5 m^3 to 1 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mol of Al. If molar heat capacity of Al is 24 J mol^{-1} K^{-1}, the temperature of Al increases by

(A) $\frac{3}{2}\text{K}$

(B) 1 K

(C) 2 K

(D) $\frac{2}{3}\text{K}$

Ans. (D)

Sol. $w = -P_{ext}\,(V_+ - V_i)$

$= -4\text{Nm}^{-2}(1-5)\text{m}^3$

$= 16\text{Nm} \Rightarrow 16\text{J}$

For isothermal process,

$\Delta U = q + w \Rightarrow q = -w = -16\text{J}$

($\therefore \Delta U = 0$ for isothermal process)

From calorimetry, Heat given $= nC\Delta T$

$16 = \frac{1\times 24\text{J}\times\Delta\text{T}}{\text{molK}}$

$\Delta\text{T} = \frac{2}{3}\text{K}$

67. ΔU is equal to

(A) Adiabatic work

(B) Isothermal work

(C) Isochoric work

(D) Isobaric work

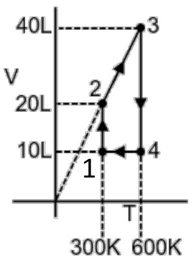
Ans. (A)

Sol. For adiabatic process, q = 0

∴ As per 1st law of thermodynamics,

ΔU = W

68. What is the net work done (in calories) by 1 mol of monoatomic ideal gas in a process described by 1, 2, 3, 4 in given V-T graph. **(Use: R = 2 cal/mole K, In 2 = 0.7)**



(A) -600 cal

(B) -660 cal

(C) +660 cal

(D) +600 cal

Ans. (B)

Sol. $W_{12} = -nR\text{TIn}\,\frac{V_2}{V_1} = -420\text{cal}$

$W_{23} = 0P\Delta V = -nR\Delta T = -600\text{cal}$

$W_{34} = -1\times 2\times 600\times \text{In }4 = 1680$

W = 6600cal (i.e. work is done on the gas) work done by the gas = -660cal

69. A system has internal energy equal to U_1 J, 450 J of heat is taken out of it and 600 J of work is done on it. The final energy of the system will be:

(A) $(U_1 + 150)$

(B) $(U_1 + 1050)$

(C) $(U_1 - 150)$

(D) $(U_1 - 250)$

Ans. (A)

Sol. $w = +600\text{J}q = -450\text{J}$

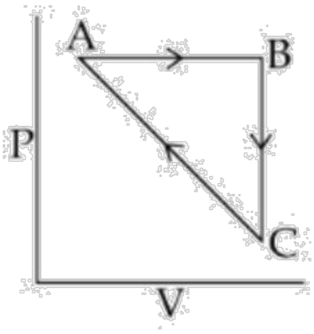
$\Delta U = U_2 - U_1 = q + w$
$\therefore U_2 = U_1 + (-450) + 600$
$\therefore U_2 = U_1 + 150J = (U + 150)J$

70. During compression of a spring the work done is 10 kJ and 2 kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is
(A) 12
(B) -12
(C) 8
(D) -8

Ans. (C)

Sol. w = 10 kJ
Q = -2kJ
ΔU = q + w = 10 − 2 = 8 kJ

71. An ideal gas undergoes a cyclic process as shown in Figure.
$\Delta U_{BC} = -5kJ\text{mol}^{-1}, q_{AB} = 2k\text{kmol}^{-1}$
$W_{AB} = -5\text{kJmol}^{-1}, W_{CA} = 3\text{kJmol}^{-1}$
Heat absorbed by the system during process *CA* is:

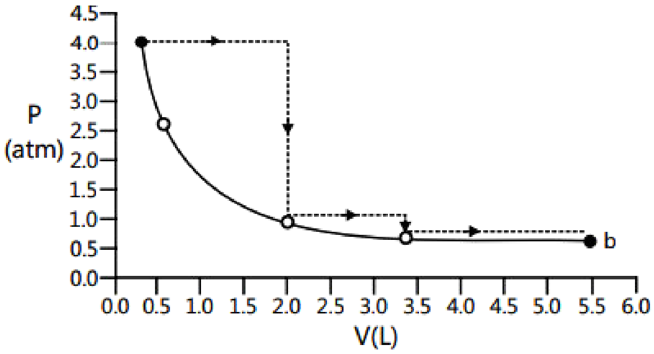


(A) -5kJmol^{-1}
(B) $+5\text{kJmol}^{-1}$
(C) 18kJmol^{-1}
(D) -18kJmol^{-1}

Ans. (B)

Sol. We have $\Delta U_{AB} = q_{AB} + w_{AB}$
$= 2 + (-5) = -3kJ\text{mol}^{-1}$
For a cyclic process, $\Delta U = 0$.
Therefore, $\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA}$
$\Delta U_{CA} = -\Delta U_{AB} - \Delta U_{BC}$
$= -(-3) - (-5) = 8\text{kJmol}^{-1}$
$\Delta U_{CA} = q_{CA} + w_{CA}$
$8 = q_{CA} + 3 \Rightarrow q_{CA} = 5\text{kJmol}^{-1}$

72. One mole of an ideal gas is taken from a to b along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is W_s, and that along the dotted line path is W_d, then the integer closest to the ratio W_d/W_s is



(A) 1
(B) 2
(C) 3
(D) 4

Ans. (B)

Sol. Work done along dashed path $|-W| = \Sigma P\Delta V$

$= 4 \times 1.5 + 1 \times 1 + \frac{2}{3} \times 2.5 = 8.65 \text{Latm}$

Work done along solid path - $W = \text{nRTln}\,\frac{V_2}{V_1} = p_1V_1\ln\frac{V_2}{V_1}$

$= 2 \times 2.3\log\frac{5.5}{0.5} = 2 \times 2.3\log\ 11 = 4.79$

$\Rightarrow \frac{W_d}{W_s} = \frac{8.65}{4.79} = 1.80 \approx 2$

73. Molar heat capacity of water in equilibrium with ice at constant pressure is
(A) Zero
(B) Infinity
(C) 40.45 kJ K^{-1} mol^{-1}
(D) 75.48 JK^{-1} mol^{-1}

Ans. (B)

Sol. ∞

$C_P = \frac{H_2 - H_1}{\Delta T} = \frac{\Delta H}{0} = \infty$

[$\Delta T = 0$ because two states liquid and solid of water are in equilibrium]

74. Two moles of an ideal monoatomic gases are allowed to expand adiabatically and reversibility from 300 K to 200 K. The work done by the system is (C_v = 12.5 J/K/mole).
(A) -12.5 kJ
(B) -2.5 kJ
(C) -625 kJ
(D) 500 kJ

Ans. (B)

Sol. $\Delta E = Q + W$

For Adiabatic Reversible process Q = 0

$\Delta E = W$

$\Rightarrow \Delta E = nC_v\Delta T$

$= 2 \times 12.5 \times (200 - 300)$

$= -2500$ J/mole $= -2.5$ KJ

$\Rightarrow \Delta E = -2.5$kJ

75. Adiabatic expansion of ideal gas into vacuum corresponds to

(A) w = 0

(B) $\Delta E = 0$

(C) $\Delta H = 0$

(D) All of these

Ans. (D)

Sol. $\Delta H = 0$

$\Delta E = 0$

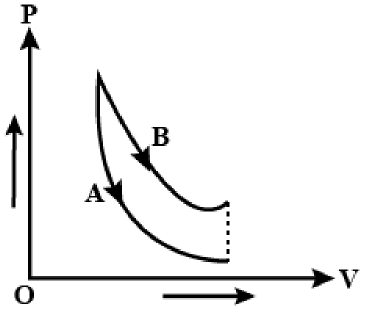
W = 0

At vacuum, P = 0

Hence, $w = - P \Delta V = 0$ .... (1)

In an adiabatic process, there will be no flow of heat. Hence, q = 0.

76. In figure, A and B are two adiabatic curves for two different gases. Then A and B corresponds to:



(A) Ar and He respectively

(B) He and H_2 respectively

(C) O_2 and H_2 respectively

(D) H_2 and He respectively

Ans. (B)

Sol. Slope of P-V adiabatic curve, $\frac{dP}{dV} = -\gamma \frac{P}{V}$

We know, $\gamma = \frac{C_P}{C_V}$

γ increases with atomicity.

Therefore, atomicity of B > atomicity of A.

Among the given options, atomicity,

Ar	He	H_2	O_2
1	1	2	2

Hence, graph A maybe Ar or He and graph B maybe H_2 or O_2.

Options B is the correct answers.

$\Rightarrow$ atomicity of H2 > atomicity of He,

$\Rightarrow \gamma$ of H_2 > γ of He.

77. The difference between ΔH and ΔU ($\Delta H - \Delta U$), when the combustion of one mole of heptane(l) is carried out at a temperature T, is equal to (T < 100°C)

(A) -3RT

(B) 4RT

(C) 3RT

(D) -4RT

Ans. (D)

Sol. $C_7H_{16}(l) + 11O_2(g) \rightarrow 7CO_2(g) + 8H_2O(l)$

Now,

$\triangle n_g = n_p - n_r = 7 - 11 = -4$
$\because \triangle H = \triangle U + \triangle n_g RT$
$\therefore \triangle H - \triangle U = -4RT.$

78. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is

(A) Zero

(B) – RT In V_2/V_1

(C) $C_V(T_2-T_1)$

(D) -RT (V_2-V_1)

Ans. (A)

Sol. Explanation for correct option:

From the formula of work done,

W = - P_{ext} ΔV

In expansion against vacuum, P_{ext} = 0

Hence,

W = 0

As it is free expansion against zero external pressure, work done is zero.

79. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1 bar. The work done in kJ is

(A) -9.0

(B) -0.9

(C) -2.0

(D) +10.0

Ans. (B)

Sol. According to the given conditions, the expansion is against constant external pressure.

So, the work done is given by following formula.

$W = -p_{ext}(V_2 - V_1)$
$= -1\text{bar}\ (10\text{L} - 1\text{L})$
$= -9\text{Lbar}$
$= -9 \times 100\text{J} = -0.9\text{kJ}$

80. For the combustion reaction at 298 K

$4Ag(s) + O_2(g) \longrightarrow 2Ag_2O(s)$

Which of the following relation will be true?

(A) ΔH = ΔE

(B) ΔH > ΔE

(C) ΔH < ΔE

(D) ΔH and ΔE bear no relationship with each other

Ans. (C)

Sol. We know that

$\Delta H = \Delta U + \Delta nRT$

Where $\Delta n = n_p - n_R$ for gaseous moles

$\therefore \quad \Delta n = -1$

$\therefore \quad \Delta H = \Delta U - 1RT$

$\therefore \quad \Delta U = \Delta H + RT$

$\therefore \quad \Delta U > \Delta H$

Or $\quad \Delta E > \Delta H$

SECTION-II

81. How many of the following are state function:

(i) Internal energy (ii) Heat

(iii) Enthalpy (iv) Entropy

(v) Pressure (vi) Temperature

(vii) Volume (viii) Work

(ix) Specific heat capacity (x) Molar heat capacity

Ans. (6)

Sol. Except heat, work, molar heat capacity and specific heat capacity rest are state functions.

82. Some properties of system are given below out of which x are number of extensive property and y are the number of intensive property. Find (x - y).

Boiling point, surface tension, heat capacity, Gibbs free energy, vapour pressure, refractive index, density, temperature, entropy, enthalpy,

mass, length, volume, area, internal energy.

Ans. (3)

Sol. Extensive properties $\longrightarrow$ Heat capacity. Gibbs free energy, entropy, enthalpy, mass, length, volume, area, internal energy

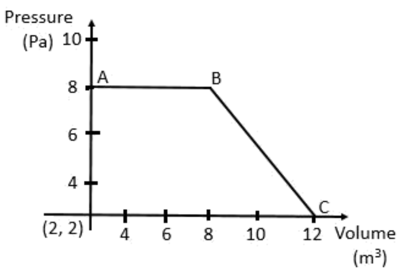
x = 9

Intensive properties $\longrightarrow$ Surface tension, boiling point, vapour pressure, refractive index, density, temperature

y = 6,

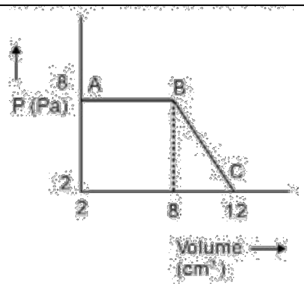
x - y = 3

83. The magnitude of work done by a gas that undergoes reversible expansion along the path ABC shown in the figure is _______.



Ans. (48.00)

Sol. Work done by a gas that undergoes a reversible expansion along the path ABC is given by



$W = (6 \times 6) + \left(\frac{1}{2} \times 4 \times 6\right) = 48.00\text{J}$

84. At constant volume, 4 mol of an ideal gas when heated from 300 K to 500 K changes its internal energy by 5000 J. The molar heat capacity at constant volume is ______. (Nearest Integer)

Ans. (6)

Sol. $\Delta U = nC_v\Delta T$

$5000 = 4 \times C_v(500 - 300)$

$C_v = 6.25\text{JK}^{-1}\text{mol}^{-1}$

85. A piston freely moves in a insulated cylinder from volume 5 lit to 10 lit then find value of work done during this expansion.

Ans. (0)

Sol. For expansion in vacuum p_{ext} = 0, W = 0.

86. The work done by a system is 8 joule, when 40 joule heat is supplied to it. What is the increase in internal energy of system in Joules.

Ans. (32)

Sol. $\Delta U = q + w$

$= 40 - 8$

$= 32\text{J}$

87. The work done in adiabatic compression of 2 mole of an ideal monoatomic gas by constant external pressure of 2 atm starting from initial pressure of 1 atm and initial temperature of 30 K (R = 2 cal/mol-degree)

Ans. (72)

Sol. $q = 0, \Delta U = W$

$nC_V(T_2 - T_1) = -P_{\text{ext}}\,(V_2 - V_1)$

$\frac{3}{2}nR(T_2 - 30) = -2\left(\frac{nRT_2}{2} - \frac{nR \times 30}{1}\right)$

$\frac{3}{2}(T_2 - 30) = (60 - T_2)T_2 = 42\text{K}$

$W = nC_v(T_2 - T_1) = 2 \times \frac{3}{2} \times 2(42 - 30) = 72\text{cal}$

88. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100°C, then the internal energy for vaporization in kJ mol^{-1} is ______. [nearest integer]

[Assume steam to be an ideal gas. Given ΔH^0_{vap} for water at 373 K and 1 bar is 41.1 kJ mol^{-1}; R = 8.31 JK^{-1} mol^{-1}]

Ans. (38)

Sol. $\underset{36g}{H_2O(l)} \longrightarrow \underset{36g}{H_2O(g)}$ (evaporation)

$n_{H_2O} = \frac{36}{18} = 2 \qquad \Delta n_g = 1 - 0 = 1$

$$\begin{aligned}\Delta U_{vap} &= \Delta H_{vap} - \Delta n_g RT \\ &= 41.1 - (1) \times 8.31 \times 10^{-3} \times 373 \\ &= 41.1 - 3.099 \\ &= 38 \text{kJ/mol}\end{aligned}$$

89. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is ________ K (Nearest Integer). Given C_v = 20 $Jmol^{-1}$ K^{-1}

Ans. (150.00)

Sol. $T_1 = 300\text{K} w = \frac{3\text{kJ}}{\text{mole}}$

$w = nC_v\Delta T$

$3000 = 1 \times 20 \times (300 - T_2)$

$300 - T_2 = 150$

$T_2 = 150\text{K}$

90. Find the magnitude of work done when one mole of the gas is expanded reversibly and isothermally from 5 atm to 1 atm at 25°C. (Nearest Integer)

Ans. (4)

Sol. $W = -2.303nRT\log \frac{p_i}{p_f}$

$= -\left(\frac{2.303 \times 1 \times 8.314 \times 298 \log 5}{1000}\right) \text{kJ}$

$= -3.99\text{kJ} \simeq -4\text{kJ}$

$|W| = 4\text{kJ}$

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