FIITJEE RBT-7 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 3 | **QP CODE: 100963** | **PAPER - 1**

Time Allotted: 3 Hours **Maximum Marks: 204**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A & B** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-04) –** Contains **Four (04)** multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **PART–A (05–12)** contains Eight (8) Multiple Choice Questions which have **One or More Than One Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **PART – B (1 – 3):** This section contains **Three (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.** Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(iv) **Part-B (4 – 8)** contains **Five (05)** Numerical based questions, the answer of which maybe positive or negative numbers or decimals **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :______________________________

Batch :______________ **Date of Examination :**______________

Enrolment Number :______________________________

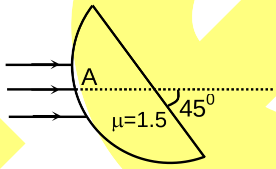
SECTION – I : PHYSICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

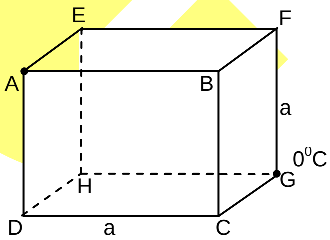
1. The diameter of a wire is measured with a screw gauge having 50 divisions on circular scale and by one complete rotation of circular scale, main scale moves 0.5 mm. If reading of screw gauge is 0.250 cm. The minimum percentage error in the reading will be
(A) 0.4 (B) 0.8 (C) 4 (D) 5

2. A parallel narrow beam of light is incident on the surface of a transparent hemisphere of radius R and refractive index μ =1.5 as shown. The distance (from A) of the image formed by refraction at the spherical surface only is



(A) $\frac{R}{2}$ (B) 3R (C) $\frac{R}{3}$ (D) 2R

3. A cubical frame is made by connecting 12 identical uniform conducting rods as shown in the figure. In the steady state the temperature of junction A is 100^0 C while that of the G is 0^0C. Then,



(A) B will be Hotter than H (B) Temperature of F is 40^0C
(C) Temperature of D is 66.67^0C (D) Temperature of E is 50^0C

4. The figure shows a charge q placed inside a cavity in an uncharged conductor. Now if an external electric field is switched on:
(A) only induced charges on outer surface will redistribute.
(B) only induced charges on inner surface will redistribute.
(C) both induced charges on outer and inner surfaces will redistribute.
(D) force on charge q placed inside the cavity will change.

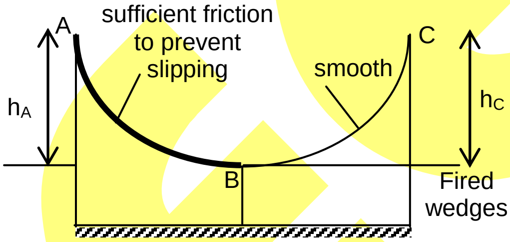
Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

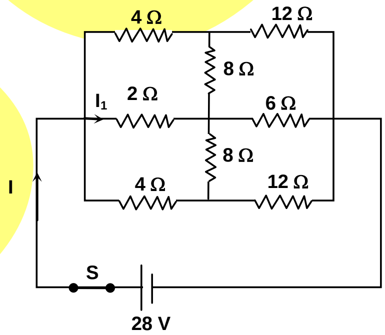
5. A sphere A moving with a speed u and rotating with an angular velocity ω, makes a head on elastic collision with an identical stationary sphere B. There is no friction between the surfaces of A and B. Disregard gravity. Then
(A) A will stop translating but continue to rotate with an angular speed ω.
(B) A will come to rest and stop rotating.
(C) B will move with a speed u without rotating.
(D) B will move with a speed u and rotate with an angular velocity ω.

6. The spherical ball is released from point A. If C be the point upto which the ball goes on smooth surface then (assuming h_A, h_c represent the height of point A and C from horizontal level B and k_A, k_C are the kinetic energies of spheres at points A and C respectively

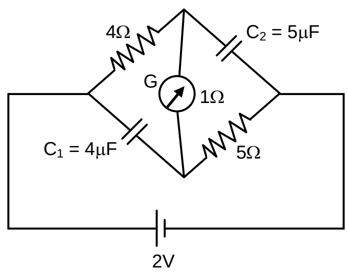


(A) $h_A > h_C$ and $k_A < k_C$
(B) $h_A > h_C$ and $k_B > k_C$
(C) $h_A = h_C$
(D) $h_A > h_C$ & $k_A > k_C$

7. In the circuit shown, the current
(A) I = 8 A
(B) I = 7 A
(C) I_1 = 3.5 A
(D) I_1 = 4 A

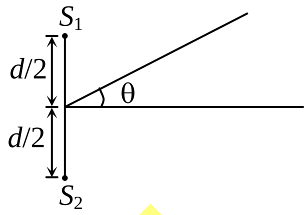


8. In the circuit shown, the cell is ideal with emf = 2V. The resistance of the coil of the galvanometer G = 1 Ω. Then in steady state
(A) No current flows in G.
(B) 0.2 A current flow in G.
(C) Potential difference across C_1 = 1.2
(D) Potential difference across C_2 = 2 V



Space For Rough Work

9. In an interference arrangement, similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by distance d = 150.0 m. The intensity $I_{(\theta)}$ is measured as a function of θ, where θ is defined as shown in the figure. If I_0 is maximum intensity, then $I_{(\theta)}$ for $0 \le \theta \le 90$ is given by



(A) $I_{(\theta)} = I_0$ for $\theta = 0°$

(B) $I_{(\theta)} = (I_0/2)$ for $\theta = 30°$

(C) $I_{(\theta)} = (I_0/4)$ for $\theta = 90°$

(D) $I_{(\theta)}$ is constant for all values of θ

10. The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. If the time period of the electron in the initial state is eight times that in the final state then the possible values of n_1 and n_2 are

(A) $n_1 = 4, n_2 = 2$
(B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$
(D) $n_1 = 6, n_2 = 3$

11. A string of length l is stretched along the x-axis and is rigidly clamped at x = 0 and $x = l$. Transverse vibrations are produced in the string. For nth harmonic which of the following equations may represents the shape of the string at any time t. (Assume amplitude at antinodes to be a)

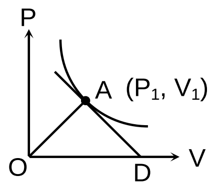
(A) $y = a\cos(\omega t)\cos\left(\frac{n\pi x}{l}\right)$

(B) $y = a\sin(\omega t)\cos\left(\frac{n\pi x}{l}\right)$

(C) $y = a\cos(\omega t)\sin\left(\frac{n\pi x}{l}\right)$

(D) $y = a\sin(\omega t)\sin\left(\frac{n\pi x}{l}\right)$

12. n moles of an ideal gas undergo an isothermal process at temperature T. P-V graph of the process is as shown in the figure. A point A (P_1, V_1) is located on the P-V curve. Tangent at point A, cuts the V-axis at point D. AO is the line joining the point A to the origin O of PV diagram. Then,



(A) coordinates of points D is $\left(\frac{3V_1}{2}, 0\right)$

(B) coordinates of points D is $(2V_1, 0)$

(C) area of the triangle AOD is nRT

(D) area of the triangle AOD is $\frac{3}{4}nRT$

Space For Rough Work

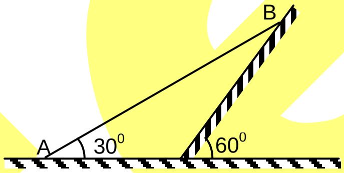
PART – B (Maximum Marks: 9)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. A bus is moving towards a huge wall with a velocity of 5 m/s. The driver sounds a horn of frequency 200Hz. The frequency of the beats heard by a passenger of the bus will be (In Hz) nearly (velocity of sound in air = $338\ m/s$)

2. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is A/T where T is temperature in kelvin. Find the value of A.

3. In the figure shown, the instantaneous speed of end A of the rod is v to the left. The angular velocity of the rod of length L is equal to $n\frac{v}{\ell}$ then find the value of 'n'.

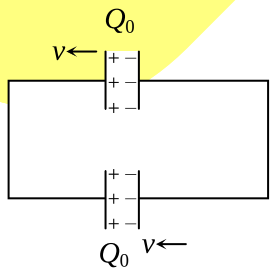


PART – B (Maximum Marks: 15)

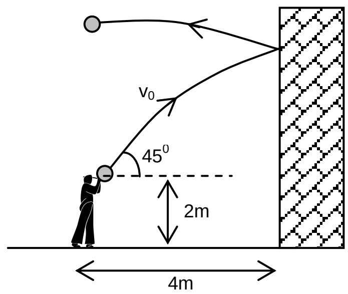
(Numerical **Type)**

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. Two identical capacitor connected as shown and having initial charge Q_0. Separation between plates of capacitor is d_0. Suddenly the left plate of upper capacitor and right plate of lower capacitor start moving with speed v towards left while other plate of capacitor remains fixed. (given $\frac{Q_0 v}{2d}=1$ amp). Find the value of current (in amp) in the circuit.

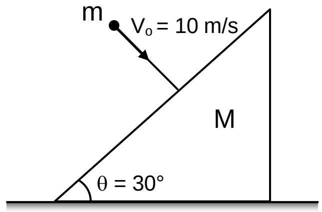


5. A boy stand l = 4 m away from a vertical wall and throws a ball. The ball leaves the boy's hand at h = 2m above the ground with initial velocity $v_0 = 10\sqrt{2}$ m/s at an angle 45^0 from the horizontal. After striking the wall elastically the ball rebounds and hit the ground at 4K (in m). Find K (take g = 9.8 m/s^2)

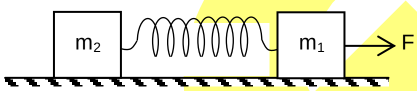


Space For Rough Work

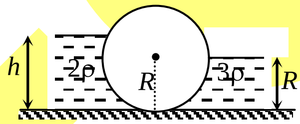
6. A ball of mass 1 kg moving with velocity 10 m/s collides perpendicularly on a smooth stationary wedge of mass 2 kg placed on smooth horizontal surface. If the coefficient of restitution is $e = 7/20$ then find the velocity of ball after the collision. [in ms^{-1}]



7. Two blocks of masses m_1 = 1kg and m_2 = 2kg are connected by a non deformed light spring. They are lying on a rough horizontal surface. The coefficient of friction between the blocks and the surface is 0.4, what minimum constant force F (in N) has to be applied in horizontal direction to the block of mass m_1 in order to shift the other block? ($g = 9.8\ m/s^2$)



8. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. If the height h for the equilibrium of cylinder is equal to be $R\sqrt{x}$. Then find the value of 'x'.



Space For Rough Work

SECTION – II : CHEMISTRY

PART – A (Maximum Marks: 12)

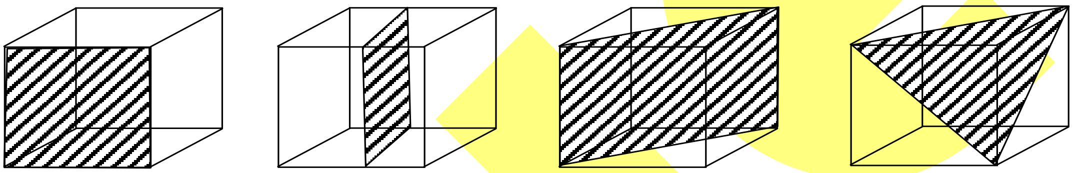
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. Copper reduces NO_3^- into NO and NO_2 depending upon concentration of HNO_3 in solution. Assuming $[Cu^{2+}]$ = 0.1 M, and P_{NO} = $P_{NO_2} = 10^{-3}$ bar, at which concentration of HNO_3, thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same?

$\left[\text{Given}: E^o_{Cu^{2+}|Cu} = +0.34 \text{ volt}, \quad E^o_{NO_3^-|NO} = +0.96 \text{ volt}, \quad E^o_{NO_3^-|NO_2} = +0.79 \text{ volt}\right]$

(A) $10^{1.23}$ M
(B) $10^{0.56}$ M
(C) $10^{0.66}$
(D) $10^{0.12}$ M

2. Following four planes (M_1, M_2, M_3, M_4) in an ideal Zinc Blend type unit cell are shown in the figure below. Consider the following statements and choose which of the following statement is incorrect



M_1 M_2 M_3 M_4

(A) Effective number of anions present on lattice sites on M_1 is 1.
(B) If ions along M_2 are removed, the formula of solid changes.
(C) All the cations are either located on lattice sites on M_3 or touches M_3.
(D) All the ions present at lattice sites on M_4 are closely packed.

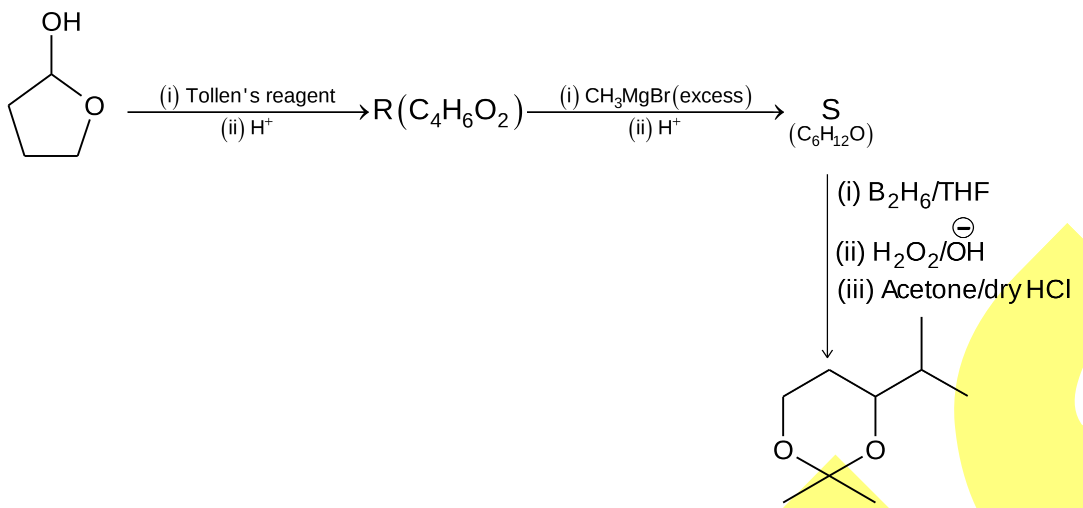
3. $$\text{Nucleic acid} \xrightarrow{Ba(OH)_2} P$$

$$\Big\downarrow \; MgO / \Delta$$

$$PO_4^{3-} + Q \xrightarrow{HCl} R + S + \text{Sugar}$$

In the scheme shown above P, Q, R and S are
(A) P = purine base, (Q) = pyrimidine base, (R) = nucleotides, (S) = nucleosides
(B) P = (Q) = nucleotides, (R) = pyrimidine base, (S) = purine base
(C) P = nucleosides, (Q) = nucleotides, (R) = (S) = purine base
(D) P = nucleotides, (Q) = nucleosides, (R) = pyrimidine base, (S) = purine base

Space For Rough Work

4. 

Choose the incorrect statement(s)
(A) R cannot be prepared by reacting cycloprapanone with m-CPBA.
(B) S contains 11 H-atoms bonded to sp^3-hybridized atoms
(C) Both R & S are cyclic compounds.
(D) Both R & S contains atleast one sp^2-hybridized C-atom.

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)
(A) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1.
(B) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic
(C) The work done by the gas is more when it is expanded reversibly form V_1 to V_2 under adiabatic conditions as compared to that when expanded reversible from V_1 to V_2 under isothermal conditions.
(D) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$ and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$.

6. When O_2 is adsorbed on a metallic surface, electron transfer occurs form the metal to O_2. The TRUE statement(s) regarding this adsorption is(are)
(A) O_2 is physisorbed
(B) heat is released
(C) occupancy of π^*_{2p} of O_2 is increased
(D) bond length of O_2 is decreased

Space For Rough Work

7. Which is/are not correctly stated about graphite?
(A) Electrical conductivity perpendicular to the planes is high.
(B) Bond order of C – C bond is 3/2.
(C) Electrical conductivity parallel to the planes increases as temperature is raised.
(D) It can act as either electron donor or an electron acceptor.

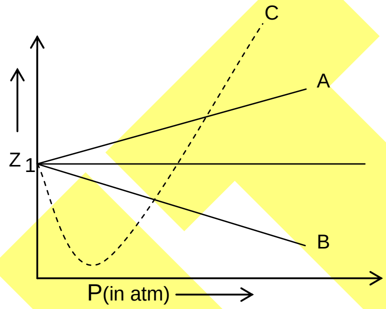
8. Which is/are correct statement?
(A) There cannot be two probabilities for an electron at any position in atom
(B) Probability cannot change abruptly form one point to the other
(C) For large distance from the nucleus probability should be smaller
(D) The total probability of electron some where in space around the nucleus approaches to1

9. When sodium is dissolved in liquid ammonia it will produce
(A) Ammoniated electron
(B) Sodium ion
(C) Amide ion
(D) Hydroxyl ion

10. When sulphur is heated with $NaOH_{(aq)}$ the compounds formed is/are
(A) Na_2SO_4
(B) Na_2S
(C) $Na_2S_2O_3$
(D) $NaHSO_3$

11. What volume of 0.1 M $KMnO_4$ in acidic medium is required for complete oxidation of 100 ml of 0.1 M FeC_2O_4 and 100 Ml of 0.1 M Ferric oxalate separately?
(A) 60 ml of $KMnO_4$ with FeC_2O_4
(B) 40 ml of $KMnO_4$ with FeC_2O_4
(C) 40 ml of $KMnO_4$ with ferric oxalate
(D) 120 ml of $KMnO_4$ with ferric oxalate

12. The given graph represents the variation of compressibility factor, Z Vs P for three real gases A, B and C at 0°C.



Identify the correct statement.
(A) For gas A, a = 0 and its dependence on P is linear at all pressures
(B) For gas B, b = 0 and its dependence on P is linear at all pressures
(C) For gas C, neither a nor b = 0. By knowing the minima and point of intersection with Z = 1, a and b can be calculate
(D) At high pressure, the slope is positive for all real gases

Space For Rough Work

PART – B (Maximum Marks: 9)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0 % to the total weight of the hydrolyzed products. The number of glycine units present in the decapeptide is

2. The number of unpaired electrons in the complex $[Mn(acac)_3]$ is
(Atomic number of Mn = 25)

3. The number of all possible isomers for the molecular formula C_6H_{14} is

PART – B (Maximum Marks: 15)

(Numerical **Type)**

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. Hydrolysis of 15.45 g of benzonitrile produced 10.98 g of benzoic acid. The percentage yield of acid formed is __________

5. The total number of isomers possible for the molecular $[Co(NH_3)_4Cl(NO_2)]^+$ is _____

6. Specific rotation of the (R)-enantiomer of a chiral compound is 48. The specific rotation of a sample of this compound which contains 25% of (s)-enantiomer is

7. If the percentage of the number of coordinate-covalent bonds present in the dimer of aluminium chloride is x, what is the value of $\frac{x}{10}$?

8. Consider the complexes of manganese
$[Mn(H_2O)_6]^{2+}$ (I) $[Mn(CN)_6]^{4-}$ (II)
If a = Number of electrons present in the t_{2g} orbitals in complex(II)
b = Number of unpaired electrons of metal ion in complex(I)
& c = Number of electrons present in the e_g orbital of complex(I)
What is the value of $\frac{a+b+c}{10}$?

Space For Rough Work

SECTION – III : MATHEMATICS

PART – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. Let function $g(x)$ be differentiable and $g'(x)$ is continuous in $(-\infty, \infty)$ with $g'(2)=14$, then $\lim\limits_{x\to 0}\dfrac{g(2+\sin x)-g(2+x\cos x)}{x-\sin x}$ is equal to:

(A) 7
(B) 14
(C) 28
(D) 56

2. $\displaystyle\int \frac{(2x+1)}{(x^2+4x+1)^{3/2}}dx$

(A) $\dfrac{x^3}{(x^2+4x+1)^{1/2}}+C$
(B) $\dfrac{x}{(x^2+4x+1)^{1/2}}+C$
(C) $\dfrac{x^2}{(x^2+4x+1)^{1/2}}+C$
(D) $\dfrac{1}{(x^2+4x+1)^{1/2}}+C$

3. Given a function g continuous on R such that $\displaystyle\int_0^1 g(t)dt=2$ and $g(1)=5$. If $f(x)=\dfrac{1}{2}\displaystyle\int_0^x (x-t)^2 g(t)dt$, then the value of $(f'''(1)-f''(1))$ is equal to:

(A) 0
(B) 3
(C) 5
(D) 7

4. If $r,k,p \in W$, then $\displaystyle\sum_{r+k+p=10} {}^{30}C_r \cdot {}^{20}C_k \cdot {}^{10}C_p$ is equal to

(A) $\begin{pmatrix}60\\50\end{pmatrix}$
(B) $\begin{pmatrix}60\\30\end{pmatrix}$
(C) $\begin{pmatrix}60\\20\end{pmatrix}$
(D) $\begin{pmatrix}30\\10\end{pmatrix}\begin{pmatrix}30\\20\end{pmatrix}$

Space For Rough Work

PART – A (Maximum Marks: 32)

*This section contains **EIGHT (08)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. If sides AB, BC and CA of a triangle ABC are represented by $x+2=0,\ 3x+y=0$ and $x+3y+2=0$ respectively, then identify the correct statement.

(A) $\sum \tan A = \frac{4}{3}$

(B) $\Pi \tan A = -\frac{4}{3}$

(C) $\sum \tan A \tan B = -\frac{41}{9}$

(D) $\sin^2(A+B)+\cos^2 C = \frac{5}{4}$

6. In triangle ABC, a=4 and $b=c=2\sqrt{2}$. A point P moves within the triangle such that the square of its distance from BC is half the area of rectangle contained by its distances from the other two sides. If D be the centre of locus of P, then

(A) locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

(B) locus of P is a hyperbola with eccentricity $\sqrt{\frac{3}{2}}$

(C) area of the quadrilateral ABDC=$\frac{16}{3}$ sq.units

(D) area of the quadrilateral ABDC= $=\frac{32}{3}$ sq. units

7. $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}.\vec{a}=\vec{b}.\vec{b}=\vec{c}.\vec{c}=3$ and $\left|\vec{a}+\vec{b}-\vec{c}\right|^2+\left|\vec{b}+\vec{c}-\vec{a}\right|^2+\left|\vec{c}+\vec{a}-\vec{b}\right|^2=36$ then

(A) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{9}{2}$

(B) $\vec{a},\vec{b},\vec{c}$ are co – planar

(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq 0$

(D) $\vec{a},\vec{b},\vec{c}$ represents sides of triangle

8. If $|\sin x|+|\cos x|\geq 1$ then x can lie in the interval

(A) $\left[n\pi-\frac{\pi}{4},\ n\pi\right]$

(B) $\left[n\pi,\ n\pi+\frac{\pi}{4}\right]$

(C) $\left[2n\pi,\ 2n\pi+\frac{\pi}{4}\right]$

(D) $\left[(4n+1)\frac{\pi}{4},(4n+3)\frac{\pi}{4}\right]$ where $n\in$ integer.

Space For Rough Work

9. If the sum of n terms of series $\frac{5}{1.2.3^1}+\frac{7}{2.3.3^2}+\frac{9}{3.4.3^3}+...$ is S_n then

(A) $S_3=\frac{107}{108}$ (B) $S_n=1-\frac{1}{3^n.n}$

(C) $U_{n\to\infty}S_n=1$ (D) $U_{n\to\infty}S_n=3$

10. If $f:R\to\left[-\frac{\pi}{4},\frac{\pi}{2}\right)$ is a function defined by $f(x)=\tan^{-1}\left(x^4-x^2-\frac{7}{4}+\tan^{-1}\alpha\right)$ and f is surjective then:

(A) $\cos^{-1}\left(\frac{1-\alpha^2}{1+\alpha^2}\right)=2$ (B) $\alpha+\frac{1}{\alpha}=2\operatorname{cosec}2$

(C) $\sin^{-1}\left(\frac{2\alpha}{\alpha^2+1}\right)=\pi-2$ (D) $\tan^{-1}\left(\frac{2\alpha}{\alpha^2-1}\right)=2-\pi$

11. The possible value(s) of k for which $\lim\limits_{x\to\infty}\frac{2x^3-\left(\tan^{-1}x\right)^3}{\frac{8}{\pi}x^3\cot^{-1}|kx|+k^2x^6\sin\frac{1}{x^3}-3kx^3}=\frac{1}{2}$, is:

(A) 0 (B) –1
(C) 2 (D) 4

12. Let function $y=f(x)$ satisfies the differential equation $x^2\frac{dy}{dx}=y^2e^{\frac{1}{x}}\ (x\neq 0)$ and $\lim\limits_{x\to 0^-}f(x)=1$. Identify the correct statement(s):

(A) Range of $f(x)$ is $(0,1)-\left\{\frac{1}{2}\right\}$ (B) $f(x)$ is bounded

(C) $\lim\limits_{x\to 0^+}f(x)=1$ (D) $\int_0^e f(x)dx>\int_0^1 f(x)dx$

Space For Rough Work

PART – B (Maximum Marks: 9)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

1. If A, B and C are angles of a triangle, then the minimum value of

$$E=\frac{\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)}+\frac{\cos\left(\frac{C-A}{2}\right)}{\cos\left(\frac{C+A}{2}\right)}+\frac{\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}$$

2. Let $f(x)$ is a polynomial function and $(f(\alpha))^2+(f'(\alpha))^2=0$, then find $\lim\limits_{x\to\alpha}\frac{f(x)}{f'(x)}\left[\frac{f'(x)}{f(x)}\right]$, where [.] denotes greatest integer function, is ___________

3. α,β are roots of equation $x^2-p(x+1)-c=0$ then find the value of

$$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}$$

PART – B (Maximum Marks: 15)

(Numerical Type)

This section contains ***Five (05)*** Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **TWO** decimal places (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

4. Let $A=\{1,2,3,4\}$ and $B=\{3,4,5,6,7\}$ are two sets. Let m is the number of one – one functions $f:A\to B$ such that $f(i)\neq i\,\forall\, i\in A$ f and n is the number of one – one functions $f:A\to B$ such that $|f(i)-i|\leq 3\,\forall\, i\in A,$ then find the value of $(m+n)$

5. Let $A(1,-1), B(4,-2)$ and $C(9,3)$ be the vertices of the triangle ABC. A parallelogram AFDE is drawn with vertices D, E and F on the line segments BC, CA and AB respectively. Find the maximum area of parallelogram AFDE.

Space For Rough Work

6. If the maximum area of the region enclosed by the curves $y=|x|e^{|x|}$ and the line $y=a(0\le a\le e)$ in $x\in[-1,1]$ is A, then find the value of [A].
[Note : [k] denotes greatest integer function less than of equal to k.]

7. Let $a_r=r\left({}^7C_r\right), b_r=(7-r)\left({}^7C_r\right)$ and $A_r=\begin{bmatrix} a_r & 0 \\ 0 & b_r \end{bmatrix}$. If $A=\sum_{r=0}^{7} A_r=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, then the value of $a+b$ must be $(k+895)$ then k is

8. The least possible value of a for which $\dfrac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\dfrac{a}{30}=0$ does not have a real solution is

Space For Rough Work

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Answers & Solutions

SECTION – I : PHYSICS

PART – A

1.	A	2.	B	3.	B	4.	A
5.	AC	6.	AB	7.	BC	8.	B
9.	AB	10.	AD	11.	CD	12.	BC

PART – B

1.	6	2.	3	3.	1	4.	2
5.	4.50	6.	2	7.	7.64	8.	1.50

SECTION – II : CHEMISTRY

PART – A

1.	C	2.	C	3.	D	4.	C
5.	AB	6.	BC	7.	ABC	8.	ABCD
9.	ABD	10.	BC	11.	AD	12.	ACD

PART – B

1.	6	2.	4	3.	5	4.	60
5.	4	6.	24	7.	2.5	8.	1.2

SECTION – III : MATHEMATICS

PART – A

1.	C	2.	B	3.	B	4.	A
5.	BC	6.	AC	7.	BCD	8.	ABCD
9.	AC	10.	ABC	11.	ABD	12.	ABD

PART – B

1.	6	2.	1	3.	1	4.	94
5.	5	6.	3	7.	1	8.	5

Answers & Solutions

SECTION – I : PHYSICS

PART – A

1. **A**

Sol. $LC = \dfrac{0.5\,\text{mm}}{50} = 0.01\text{ mm}$

Percentage error $= \dfrac{0.01\times10^{-3}}{0.25\times10^{-2}}\times100$

$= 0.4$

2. **B**

Sol. $\dfrac{1.5}{V} - \dfrac{1}{-\infty} = \dfrac{1.5-1}{R}$

V = 3R.

3. **B**

Sol. $100°C - Ir - \dfrac{I}{2}r - Ir = 0°C$

$\Rightarrow \quad Ir = 40°C \quad ; \quad \therefore \; t_F = 0° + Ir = 40°C$

4. **A**

Sol. Due to electrostatic shielding.

5. **AC**

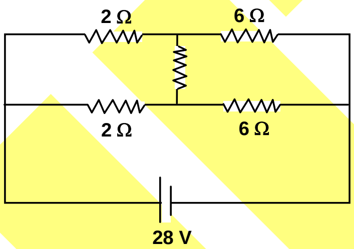
Sol. After elastic collision, translational velocities get interchanged.

6. **AB**

Sol. Apply energy conservation.

7. **BC**

Sol.



$\Rightarrow \quad i = \dfrac{28}{4} = 7A$

8. **B**

Sol. $i = \dfrac{V}{R_{ed}} = \dfrac{2}{10} = 0.2\text{ A}$

9. **AB**

Sol. $I_{(\theta)} = I_0\ \text{Cos}^2\left(\dfrac{\phi}{2}\right),\ \phi = \dfrac{d\sin\theta}{\lambda}\times2\pi$

For $\theta = 0°,\ \phi = 0,\ I_{(\theta)} = I_0$

For $\theta = 30°,\ \phi = \dfrac{\pi}{2},\ \ I_{(\theta)} = \dfrac{I_0}{2}$

10. **AD**

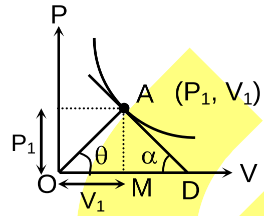
Sol. $T \propto n^3$

11. **CD**

Sol. at x = 0, $\text{Sin}\left(\frac{n\pi x}{l}\right) = 0$

12. **BC**

Sol. $\tan\alpha = -$ [slope at (P_1, V_1)] $= \frac{P_1}{V_1}$



$\tan\theta = \frac{P_1}{V_1}$

$\therefore\ \alpha = \theta$

$OM = MD = V_1$

$\therefore\ OD = 2V_1$

Area of triangle *AOD* $= \frac{1}{2}(AM \times OD) = \frac{1}{2}(P_1 \times 2V_1) = P_1V_1 = nRT$

PART – B

1. **6**

Sol. $\Delta f = f\left(\frac{338+5}{338-5} - 1\right) = 6$

2. **3**

Sol. $PT^2 = k$

$\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right)$ …(i)

$\left(\frac{nRT}{V}\right)T^2 = k$

$\Rightarrow \frac{T^3}{V} = k'$

$\Rightarrow V = \frac{T^3}{k'}$ …(ii)

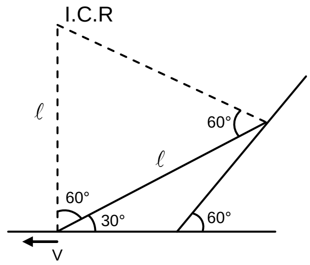
$\frac{dV}{dT} = \frac{3T^2}{K'}$ …(iii)

From (i), (ii) and (iii)

$\gamma = \left(\frac{k'}{T^3}\right) \times \left(\frac{3T^2}{k'}\right) = \frac{3}{T}$

3. **1**

Sol. $\omega = \frac{v}{L}$



4. **2**

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}$; $q_1 + q_2 = 2Q_0$

$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}$; $C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$

$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$

$$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$$

$$q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$$

$$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 2 \text{ amp}$$

5. **4.50**

Sol. The velocity component at the point of projection are

$u_x = 10 m/s$, $u_y = 10 m/s$

t = 0.4 s

$v_x = 10\ m/s$, $v_y = 6.1 m/s$

h = 3.2 m

H = 5.2 m

The time taken by the ball to hit the ground

$$5.2 = -6.1T + \frac{1}{2}gT^2$$

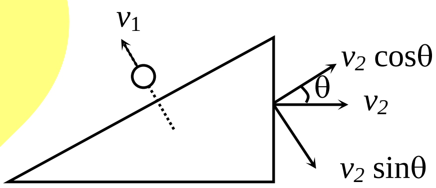
T=1.8s

$x = v_x T = 10 \times 1.8 = 18 m.$

4K = 18 ; $\therefore$ K = 4.5

6. **2**

Sol. Equation of Newtons collision law



After collision

$$e = \frac{v_1 + v_2 \sin\theta}{v_0}, \qquad e = \frac{v_1 + \frac{v_2}{2}}{v_0}$$

$$2v_1 + v_2 = 7 \qquad \ldots \text{(i)}$$

From momentum conservation

$$mv \sin 30 = -mv_1 \sin 30 + mv_2$$

$$5 = -\frac{v_1}{2} + 2v_2 \qquad \ldots \text{(ii)}$$

Solving v_1 = 2 m/s

7. **7.64**

Sol $$F.x - \mu m_1 gx - \frac{1}{2}kx^2 = 0$$

kx = $\mu m_2 g$ for just shifting m_2

$$F.x - \mu m_1 gx - \frac{1}{2}\mu m_2 gx = 0$$

$$F = \mu\left(m_1 + \frac{m_2}{2}\right)g = 0.4\left(1 + \frac{2}{2}\right)(9.8) = 7.64 \text{ N}$$

8. **1.50**

Sol. ρh^2 = constant

SECTION – II : CHEMISTRY

PART – A

1. C

Sol. $Cu + 4H^{+} + 2NO_3^{-} \longrightarrow Cu^{2+} + 2NO_2 + 2H_2O ----(1)$

$3Cu + 8H^{+} + 2NO_3^{-} \longrightarrow 3Cu^{2+} + 2NO + 4H_2O ----(2)$

According to question

ΔG of equation (1) = ΔG of equation (2)

$\therefore E_{Cell}$(equation -1) = E_{Cell}(equation -2)

or,($E_{cathdoe}$ - E_{anode})(equ^n.1) = ($E_{cathdoe}$ - E_{anode})(equ^n.2)

$\therefore E_{NO_3^-/NO_2} - E_{Cu^{2+}/Cu} = E_{NO_3^-/NO} - E_{Cu^{2+}/Cu}$

$\therefore E_{NO_3^-/NO_2} = E_{NO_3^-/NO}$

$$\text{or, } E^0_{NO_3^-/NO_2} - \frac{0.0591}{2}\log\frac{(p_{NO_2})^2}{[H^+]^4[NO_3^-]^2} = E^0_{NO_3^-/NO} - \frac{0.0591}{6}\log\frac{(p_{NO})^2}{[NO_3^-]^2[H^+]^8}$$

$$\text{or, } 0.79 - \frac{0.0591}{2}\log\frac{(10^{-3})^2}{(x^4)(x^2)} = 0.96 - \frac{0.0591}{6}\log\frac{(10^{-3})^2}{(x^2)(x)^8}$$

On solving x = $10^{0.66}$

2. C

Sol. (A) Number of corner atoms at four corners = 4. Effective number of atoms = $4 \times \frac{1}{4} = 1$

Each corner in the plane touches four unit cells.

(B) M_2 contains the face centre ions. $\therefore$ Formula will be changed.

(C) Incorrect as cations are present in tetrahedral voids.

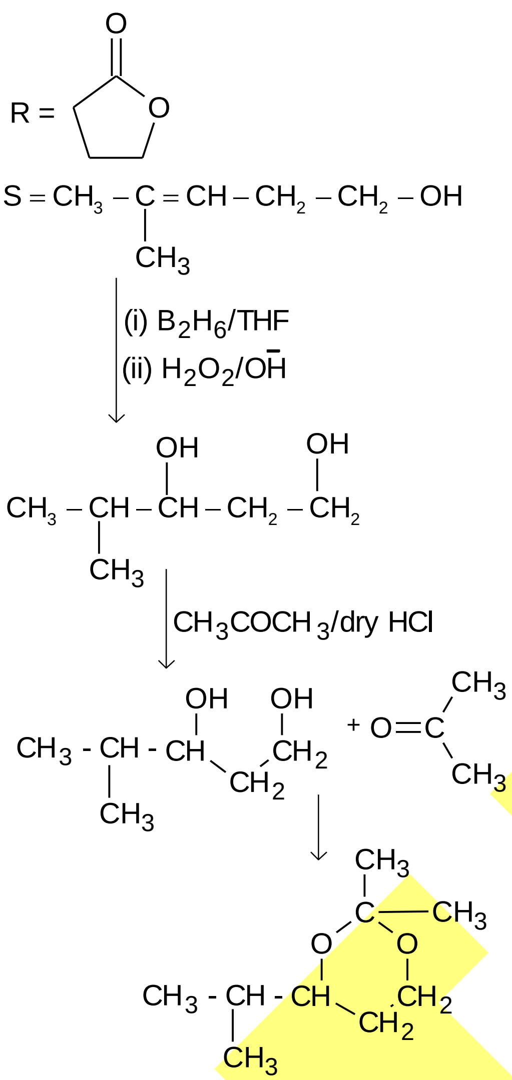
(D) Close packing takes place along face diagonals.

3. D

Sol. P is nucleotides and Q is nucleoside, R and S are bases.

4. C

Sol.



5. AB

Sol. $(p_1, V_1, T_1) \xrightarrow{\text{Expansion}} (p_2, V_2, T_2)$

$\therefore V_2 > V_1$ and $p_1 > p_2$

(A) $(p_2, V_2) \xrightarrow[\text{compression}]{\text{Irreversible}} (p_1, V_1)$

Work done = $-p_1(V_1 - V_2)$

Since $V_2 > V_1$, W_{irr} = +ve and $p_1 > p_2$

This is called maximum work because p_{ext} is p_1 which is larger than p_2.

$\therefore$ This is correct.

(B) For free expansion, $p_{ext} = 0$ and $W = 0$

In isothermal process

$\Delta T = 0$, $\Delta U = 0$

$\Delta U = q - W$

or, $q - W = 0$

Since q is zero for free expansion

$W = 0$, or $p_{ext}\Delta V = 0$

$\therefore$ $p_{ext} = 0$

In adiabatic process

$\Delta U = q - W$

Since $q = 0$, $\Delta U = -W$

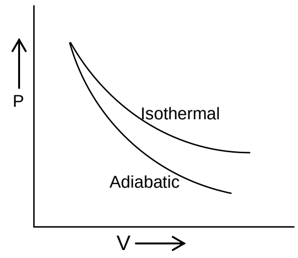
For free expansion, $\Delta U = 0$

$\therefore W = 0$

$p_{ext}\Delta V = 0$, $p_{ext} = 0$

(C) is incorrect

Work done in isothermal process is higher than that in adiabatic process



Work is area under the curve. So area under isothermal curve is higher than that in adiabatic curve.

(D) is incorrect

For isothermal process, the change in internal energy = 0 as $\Delta T = 0$

For adiabatic process, expansion of gas decreases internal energy, which is used to do work.

$\therefore \Delta U < 0$ in adiabatic process.

6. BC

Sol. Adsorption of O_2 on metal surface is called chemisorption. Since electrons are transferred from metal to oxygen, heat will be released as electron affinity of oxygen. The electrons are absorbed in the outermost molecular orbital $\left(\pi_y^*\right)$ of O_2.

7. ABC

Sol. (A) is incorrect, conduction of electricity taking place along the plane of graphite

(B) is incorrect, bond order of graphite = $\frac{4}{3}$

(C) is incorrect, decrease with temperature

(D) is correct, carbon can lose and accept electrons.

8. ABCD

Sol. Probability depends on distance form the nucleus. The probability is maximum($\psi^2 = 1$) around the nucleus and decreases away from it.

9. ABD

Sol. Liqour ammonia is aqueous solution of NH_3.

$\therefore NH_3 + H_2O \longrightarrow NH_4^+ + OH^-$ ions

$Na + NH_3 + H_2O \longrightarrow NaOH + NH_4^+ + H_2$

10. BC

Sol. Disproportionation of sulphur takes place

$S_8 \xrightarrow{OH^-} S^{2-} + S_2O_3^{2-} + H_2O$

11. AD

Sol. For FeC_2O_4(n = 3)

M_{eq} of FeC_2O_4 = M_{eq} of MnO_4^-

or, $100 \times 0.1 \times 3 = V \times 0.1 \times 5$

$\therefore V = 60$ mL

For $Fe_2(C_2O_4)_3$, n = 6

M_{eq} of $Fe_2(C_2O_4)_3$ = M_{eq} of MnO_4^-

or, $100 \times 0.1 \times 6 = V \times 0.1 \times 5$

$\therefore V = 120$ mL

12. ACD

Sol. Gas B is incorrectly represented. When a = 0, only positive deviation takes place.

PART – B

1. 6

Sol. Molar mass of decapeptide = 796 g mol^{-1}

Decapeptide contains nine peptide linkages. So one mole of decapeptide absorbs 9 moles of H_2O for hydrolysis.

$\therefore$Mass of all amino acids obtained after hydrolysis = 796 + 18(9) = 958 g mol^{-1}

Glycine contribute = 47%

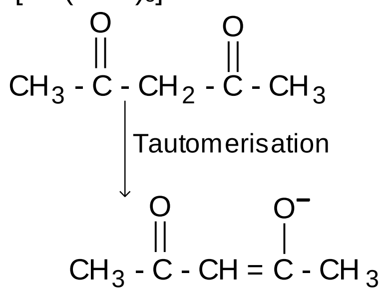
$\therefore$ Mass of glycine = $\frac{47}{100}\times 958 = 450.26$ g

Molar mass of glycine = 75 g mol^{-1}

$\therefore$ Number of glycine molecules = $\frac{450.26}{75} = 6$

2. 4

Sol. In $[Mn(acac)_3]$ the metal ion is Mn^{3+} and acac is a monodentate ligand.



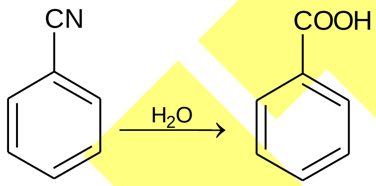
Mn^{3+} contains 7 – 3 = 4 unpaired electrons.

3. 5

Sol. C_6H_{14} has five chain isomers.

4. 60

Sol.



Molar mass of PhCN = 103

Mol. Mass of PhCOOH = 122

Moles of PhCN = $\frac{15.45}{103} = 0.15$

$\therefore$0.15 mole PhCN can produce 0.15 mole of PhCOOH

or, 15.45 g PhCN can produce(0.15 $\times$ 122) = 18.3 g PhCOOH

$\therefore$Product yield = $\frac{\text{Expt.yield}}{\text{Max.yield}}\times 100 = \frac{10.98}{18.3}\times 100 = 60\%$

5. 4

Sol. It produces two geometrical and two linkage isomers.

6. 24

Sol. Sp rotation of R-enantiomer = 48°

Sp rotation of S-enantiomer = -48°

25% S isomer contains 75% R isomer

$\therefore$100% R $\rightarrow$ 48°

$\therefore$75% R $\rightarrow \frac{48}{100}\times 75 = 36^o$

25% S $\rightarrow \frac{-48}{100}\times 25 = -12$

$\therefore$Sp rotation of sample = 36 - 12 = 24°

7. 2.5

Sol. Total number of bonds = 8

Number of coordinate bonds = 2

$\therefore$% of coordinate bonds = $\frac{2}{8} \times 100 = 25\%$

$$\frac{x}{10} = \frac{25}{10} = 2.5$$

8. 1.2

Sol. a = 5, b = 5, c = 2

$$\frac{a+b+c}{10} = \frac{12}{10} = 1.2$$

SECTION – III : MATHEMATICS

PART – A

1. C

Sol. $\lim\limits_{x\to 0}\left(\dfrac{g(2+\sin x)-g(2+x\cos x)}{\sin x - x\cos x}\right).\left(\dfrac{\sin x - x\cos x}{x-\sin x}\right)$

$=g'(2)\lim\limits_{x\to 0}\left(\dfrac{\sin x - x\cos x}{x-\sin x}\right)\left(\dfrac{0}{0}\right)$

$=g'(2)\lim\limits_{x\to 0}\left(\dfrac{x\sin x}{1-\cos x}\right)=2g'(2)$

2. B

Sol. $\displaystyle\int\frac{2x+1}{\left(x^2+4x+1\right)^{3/2}}dx=\int\frac{2x+1}{x^3\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}}dx=\int\frac{2x^{-2}+x^{-3}}{\left(1+\frac{4}{x}+\frac{1}{x^2}\right)^{3/2}}dx$

Now put $\dfrac{1}{x^2}+\dfrac{4}{x}+1=t^2$.

3. B

Sol. $f'(x)=\dfrac{1}{2}\displaystyle\int_0^x 2(x-t)g(t)dt+\dfrac{1}{2}(x-x)^2g(x)-0$

$=\displaystyle\int_0^x (x-t)g(t)dt$

$f''(x)=\displaystyle\int_0^x g(t)dt+(x-x)g(x)-0$

$f'''(x)=g(x)$

$f'''(1)-f''(1)=g(1)-\displaystyle\int_0^1 g(t)dt$

$=5-2=3$

4. A

Sol. Required sum of coefficient of x^{10} in the expansion of $(1+x)^{30}(1+x)^{20}(1+x)^{10}$ i.e. ${}^{60}C_{10}$.

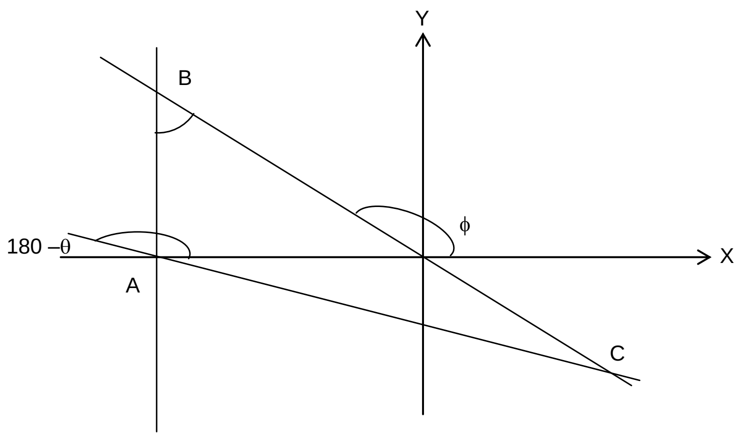
5. BC

Sol. $m_{AC}=\dfrac{-1}{3}=\tan\theta$

$\text{Tan}A=\text{Tan}(90+180-\theta)$

$=\text{Tan}(270-\theta)=\cot\theta=-3$

……..(1)



$\tan C=\left|\dfrac{m_{BC}-m_{AC}}{1+m_{BC}\,m_{AC}}\right|=\left|\dfrac{-3+\frac{1}{3}}{1+1}\right|=\dfrac{4}{3}$

…………..(2)

$\tan\phi=-3$

$B=\phi-90^o$

$\text{Tan}B=\text{Tan}(\phi-90)=-\cot\phi=\dfrac{1}{3}$

So $\tan A+\tan B+\tan C=-3+\dfrac{1}{3}+\dfrac{4}{3}=\dfrac{-4}{3}$

Now check Others.

6. AC

Sol. ΔABC is isosceles right angled at A

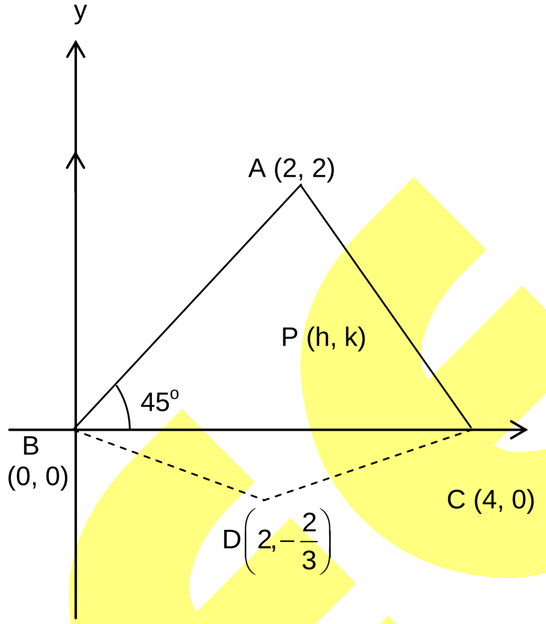
Let P be (h, k)

Let ABC be triangle as shown in figure

Equation BC : y = 0

$AB: y - x = 0$

$AC: x + y - 4 = 0$



Given condition

$$k^2 = \frac{1}{2}\frac{|h+k-4|}{\sqrt{1+1}} \cdot \frac{|k-h|}{\sqrt{1+1}}$$

Replace (h, k) by (x, y)

$$\Rightarrow y^2 = |(x+y-4)(y-x)|$$

$$= -\frac{1}{4}(x+y-4)(x-y)$$

$$\Rightarrow 4y^2 = -\left(x^2 - y^2 - 4(x-y)\right)$$

$$x^2 + 3y^2 - 4x + 4y = 0$$

$$(x-2)^2 + 3\left(y^2 + \frac{4}{3}y + \frac{4}{9}\right) = 4 + \frac{4}{3}$$

$$(x-2)^2 + 3\left(y + \frac{2}{3}\right)^2 = \frac{16}{3}$$

$$\frac{(x-2)^2}{\frac{16}{3}} + \frac{\left(y+\frac{2}{3}\right)^2}{\frac{16}{9}} = 1$$

$$\frac{16}{9} = \frac{16}{3}\left(1 - e^2\right)$$

$$\frac{1}{3} = 1 - e^2 \Rightarrow e = \sqrt{\frac{2}{3}}$$

$$D = \left(2, \frac{-2}{3}\right)$$

$$\text{Area} = Ar(ABC) + Ar\,BCD$$

$$= \frac{1}{2}.4.2\sqrt{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times 4 \times \frac{2}{3}$$

$$= 4 + \frac{4}{3} = \frac{16}{3}$$

7. BCD

Sol. $|\vec{a} + \vec{b} - \vec{c}|^2 + |\vec{b} + \vec{c} - \vec{a}|^2 + |\vec{c} + \vec{a} - \vec{b}|^2 = 36$

$$\Rightarrow 3\left(\vec{a}.\vec{a} + \vec{b}.\vec{b} + \vec{c}.\vec{c}\right) - 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c} = 36$$

$$\Rightarrow 3(3+3+3) - 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 36$$

$$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-9}{2}$$

$$2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} + 9 = 0$$

$$2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} + \vec{a}.\vec{a} + \vec{b}.\vec{b} + \vec{c}.\vec{c} = 0$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$$

Now verify (B), (C), (D)

8. ABCD

Sol. $|\sin x| + |\cos x| \geq 1 \forall x \in R$

Note $1 \leq |\sin x| + |\cos x| < \sqrt{2}$ so $x \in R$

$\Rightarrow a, b, c, d$ all are correct.

9. AC

Sol. $$S_n = \sum_{r=1}^{n} \frac{2r+3}{r(r+1)3^r} = \sum_{r=1}^{n} \frac{3(r+1)-r}{r(r+1)3^r} = \sum_{r=1}^{n} \frac{1}{r\,3^{r-1}} - \frac{1}{(r+1)3^r}$$

$$= 1 - \frac{1}{(n+1).3^n}$$

10. ABC

Sol. $$f(x) = \tan^{-1}\left(x^4 - x^2 + \frac{1}{4} - 2 + \tan^{-1}\alpha\right) = \tan^{-1}\left(\left(x^2 - \frac{1}{2}\right)^2 + \tan^{-1}\alpha - 2\right)$$

For ‘f’ to be surjective, $\tan^{-1}\alpha - 2 = -1 \Rightarrow \alpha = \tan 1$

Now, verify the options.

11. ABD

Sol. When k = 0

$$\lim_{x\to\infty} \frac{2x^3 - \left(\tan^{-1}x\right)^3}{\frac{8}{\pi}x^3.\frac{\pi}{2} + 0 + 0} = \lim_{x\to\infty} \frac{2 - \left(\frac{\tan^{-1}x}{x}\right)^3}{4} = 2 - 1$$

$$= \frac{2-0}{4} = \frac{1}{2}$$

So k can be 0.

If $k \neq 0$ then $$\lim_{x\to\infty} \frac{2x^3 - \left(\tan^{-1}x\right)^3}{x^3\left[\frac{8}{\pi}\cot^{-1}|kx| + k^2\frac{\sin\frac{1}{x^3}}{\frac{1}{x^3}} - 3k\right]}$$

$$\lim_{x\to\infty} \frac{2 - \left(\frac{\tan^{-1}x}{x}\right)^3}{\frac{8}{\pi}\cot^{-1}(kx) + \frac{k^2\sin^{-1}\frac{1}{x^3}}{\frac{1}{x^3}} - 3k} = \frac{2-0}{\frac{8}{\pi}\times 0 + k^2 - 3k} = \frac{1}{2}$$

$\Rightarrow k^2 - 3k = 4$

$\Rightarrow k = 4, -1$

12. ABD

Sol. $$x^2\frac{dy}{dx} = y^2 e^{\frac{1}{x}} \Rightarrow \frac{dy}{y^2} = \frac{e^{\frac{1}{x}}}{x^2}dx$$

Integrating both sides,

$$\frac{-1}{y} = \int \frac{e^{\frac{1}{x}}}{x^2}dx + C \Rightarrow \text{ Putting } \frac{1}{x} = t \Rightarrow \frac{-1}{x^2}dx = dt \Rightarrow -\frac{1}{y} = -\int e^t\,dt + C \Rightarrow$$

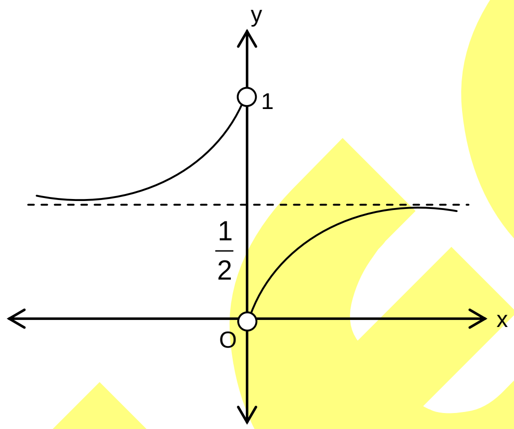
$$\frac{-1}{y}=e^{-t}+C \Rightarrow \frac{-1}{y}=-e^{\frac{1}{x}}+C$$

$$\because \lim_{x\to 0^-} f(x)=1 \Rightarrow -1=0+C \Rightarrow C=-1$$

$$\frac{-1}{y}=-e^{\frac{1}{x}}-1 \Rightarrow y=\frac{1}{1+e^{\frac{1}{x}}} \Rightarrow \frac{dy}{dx}=\frac{1}{\left(1+e^{\frac{1}{x}}\right)^2}e^{\frac{1}{x}}\left(\frac{-1}{x^2}\right)=\frac{e^{\frac{1}{x}}}{x^2\left(1+e^{\frac{1}{x}}\right)^2}$$

$$\therefore \quad \frac{dy}{dx}>0 \; \forall x\in R-\{0\}$$

$$\lim_{x\to\pm\infty}\frac{1}{1+e^{\frac{1}{x}}}=\frac{1}{2} \text{ and } \lim_{x\to 0^+}\frac{1}{1+e^{\frac{1}{x}}}=0$$



$\therefore$ Graph of the function is

From the graph options (A) (B) and (D) are correct.

PART – B

1. 6

Sol.
$$\frac{\cos\frac{B-C}{2}.\cos\frac{A}{2}}{\cos\left(90-\frac{A}{2}\right)\cos\frac{A}{2}}+\frac{\cos\left(\frac{C-A}{2}\right)\cos\frac{B}{2}}{\cos\left(90-\frac{B}{2}\right)\cos\frac{B}{2}}+\frac{\cos\left(\frac{A-B}{2}\right)\cos\frac{C}{2}}{\cos\left(90-\frac{C}{2}\right)\cos\frac{C}{2}}$$

$$\frac{2\cos\frac{B-C}{2}\sin\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}+\frac{2\cos\left(\frac{C-A}{2}\right)\sin\left(\frac{C+A}{2}\right)}{2\sin\frac{B}{2}\cos\frac{B}{2}}+\frac{2\cos\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)}{2\sin\frac{C}{2}\cos\frac{C}{2}}$$

$$\frac{\sin B+\sin C}{\sin A}+\frac{\sin A+\sin C}{\sin B}+\frac{\sin A+\sin B}{\sin C}$$

$$\frac{\sin B}{\sin A}+\frac{\sin C}{\sin A}+\frac{\sin A}{\sin B}+\frac{\sin C}{\sin B}+\frac{\sin A}{\sin C}+\frac{\sin B}{\sin C}\ge 6$$

2. 1

Sol. $[f(\alpha)]^2+[f'(\alpha)]^2=0$ both $f(\alpha)$ and $f'(\alpha)$ are real so $f(\alpha)=0=f'(\alpha)$

$$\underset{x\to\alpha}{Lt}\frac{f(x)}{f'(x)}\left[\frac{f'(x)}{f(x)}\right]$$

$$\underset{x\to\alpha}{Lt}\frac{f(x)}{f'(x)}\left(\frac{f'(x)}{f(x)}-\left\{\frac{f'(x)}{f(x)}\right\}\right) \qquad \{.\} \text{ represents fraction part function}$$

$$=\underset{x\to\alpha}{Lt}\, 1-\frac{f(x)}{f'(x)}\left\{\frac{f'(x)}{f(x)}\right\}$$

$1-0\ x$ any number is $[0,1)$

3. 1

Sol. $\alpha+\beta=p$

$\alpha\beta = -p - c$

$\alpha\beta = -(\alpha+\beta) - c$

$\Rightarrow \alpha+\beta+\alpha\beta = -c$

$\alpha+\beta+\alpha\beta+1 = -c+1$

$\Rightarrow (\alpha+1)(\beta+1) = 1-c$

$$\frac{(\alpha+1)^2}{(\alpha+1)^2+c-1}+\frac{(\beta+1)^2}{(\beta+1)^2+c-1}$$

$$\Rightarrow \frac{(\alpha+1)^2}{(\alpha+1)^2-(\alpha+1)(\beta+1)}+\frac{(\beta+1)^2}{(\beta+1)^2-(\alpha+1)(\beta+1)}$$

$$=\frac{\alpha+1}{\alpha-\beta}+\frac{\beta+1}{\beta-\alpha}=1$$

4. 94

Sol. $m = {}^5C_4.4! = 5! = \text{Total}$

When exactly 2 elements of A maps to itself i.e. $f(3)=3, f(4)=4$

$\therefore$ From 5, 6, 7 select any 2 in ${}^3C_2 \times 2! = 6$

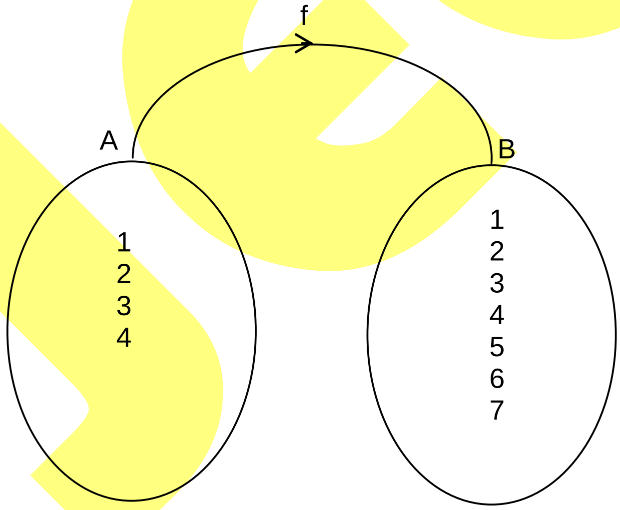
When exactly one element of A maps to itself say $f(3)=3$

Now 4 can be map in 3 ways and remaining elements $3\times 2 = 6$

$\therefore \quad {}^2C_1 \times 3 \times 6 = 36$

Total $= 36+6 = 42 \Rightarrow 5! - 42 = 78 = m$

$n = {}^2C_1.{}^2C_1.{}^2C_1.{}^2C_1 = 16$



5. 5

Sol. Let AF = x = DE and AE = y = DF

As ΔCAB is ΔCED

So, $\dfrac{CE}{CA}=\dfrac{DE}{AB}\Rightarrow\dfrac{b-y}{b}=\dfrac{x}{c}\Rightarrow y=b\left(1-\dfrac{x}{c}\right)$

(Here BC = a, AC = b and AB = c)

Now, area of parallelogram AFDE

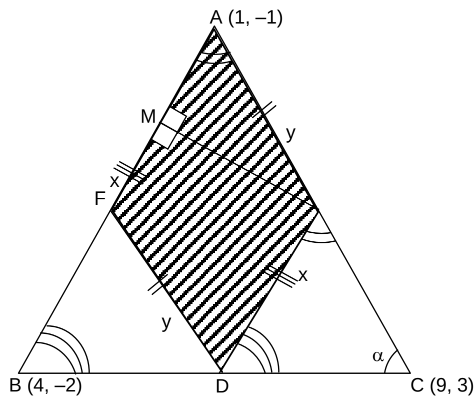
$= S = (AF)(EM) = xy\sin A \Rightarrow S = x.b\left(1-\dfrac{x}{c}\right)\sin A$

Note: $\sin A$ is fixed …(i)

Now, differentiating both sides of equation (i) with respect to x, we get

$\dfrac{dS}{dx}=\dfrac{b}{c}(c-2x)\sin A = 0 \Rightarrow x = \dfrac{c}{2}$

Also, $\left.\dfrac{d^2S}{dx^2}\right]_{x=\frac{c}{2}} = \dfrac{-2b}{c} < 0$



So, S is maximum when $x = \dfrac{c}{2}$.

Now, $S_{max} = \dfrac{1}{4}bc\sin A$

$$=\frac{1}{2}\left(\frac{1}{2}bc\sin A\right)=\frac{1}{2}\left[\text{area}(\Delta ABC)\right]=\frac{1}{4}\left\|\begin{matrix}1 & -1 & 1\\ 4 & -2 & 1\\ 9 & 3 & 1\end{matrix}\right\|=\frac{20}{4}=5$$

(square units.)

6. 3

Sol. $f(x)=\begin{cases} xe^{x}, & x\geq 0\\ -xe^{-x}, & x\leq 0\end{cases}$

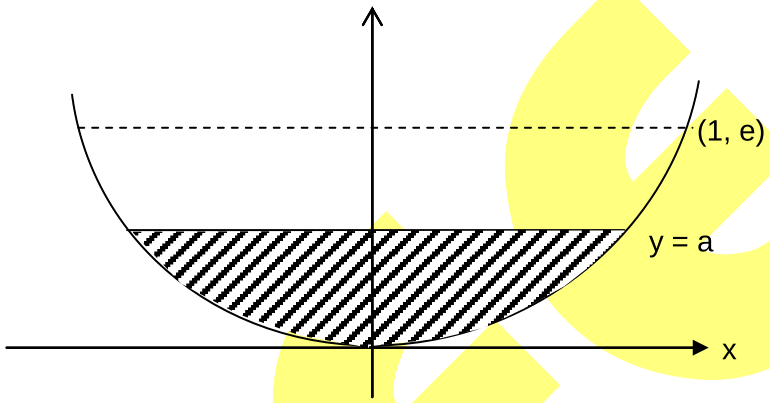
For maximum area $a=e$

$A_{max}=2\int_0^1\left(e-xe^x\right)dx$

$=2\left[ex-\left(xe^x-e^x\right)\right]_0^1$

$=2(e-1)$

Hence, $[A]=3$



7. 1

Sol. $\sum_{r=0}^{7}A_r=\begin{bmatrix}\sum_{r=0}^{7}r\left({}^7C_r\right) & 0\\ 0 & \sum_{r=0}^{7}(7-r)\,{}^7C_r\end{bmatrix}$

$=\begin{bmatrix}7.2^6 & 0\\ 0 & 7.2^7-7.2^6\end{bmatrix}$

$=\begin{bmatrix}7.2^6 & 0\\ 0 & 7.2^6\end{bmatrix}$

$a+b=2.7.2^6=14\times 64=896$

8. 5

Sol. $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\frac{a}{30}=0$

$\frac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}+\frac{a}{30}=0$

$\Rightarrow\frac{x-3}{x+4}+\frac{9}{30}=0,\ x\neq 1,2$

Above equation do no have solution then $-\frac{a}{30}$ should not belong to range of

$y=\frac{x-3}{x+4},\ x\neq 1,\ 2$

Range of above is $R-\left\{1,\frac{-2}{5},-\frac{1}{6}\right\}$

So, $\frac{-a}{30}=1,\ \frac{-2}{5},\ -\frac{1}{6}$

$\Rightarrow a=-30,\ 12,\ 5$

So least value of a = 5

$-\frac{a}{30}=\frac{x-3}{x+4}$