FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
CONCEPT RECAPITULATION TEST – III
PAPER –1
TEST DATE: 24-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

Section – A

1. AD

Sol. $\vec{R}_1$ is vertical and $\vec{R}_2$ is also vertical

2. C

Sol. t_0 can be found by equating pseudo force on block to static friction
Eqn. for block w.r.t. plank :

$$m\frac{dv}{dt} = m\left(\frac{kt}{2m}\right) - \left(\frac{3\mu}{4}\right)mg \text{ (after relative slipping starts)}$$

$$m\int_0^v dv = \int_{t_0}^{2t_0}\left[m\left(\frac{kt}{2m}\right) - \frac{3\mu mg}{4}\right]dt$$

$$\Rightarrow v = \frac{9\mu^2 mg^2}{2k} \text{ (towards left)}$$

3. BD

Sol. In frame of observer magnitude of initial velocity of block = 20 m/s & final velocity of block when it reacts to horizontal floor is 0 m/s so change in kinetic energy of the block will be 400 J as observed by observer

Work done by force due to gravity = mgh
= 2 x 10 x 20
= 400 J

Say work done by normal reaction = W_n

So net work done by all the forces = 400 J + W_n

From work energy theorem

$400J + W_n = 0 – 400\ J$

$W_n = – 800\ J$

4. BC

Sol. $v_L = v_C = v_R$;

$\Rightarrow x_L = x_C = R$

when inductor is short circuited

$Z = \sqrt{R^2 + x_C^2} = \sqrt{2}R$

$\therefore I = \frac{30}{Z} = \frac{30}{\sqrt{2}R}$

$\therefore V_L = ix_L = \frac{30}{\sqrt{2}R} \times R = \frac{30}{\sqrt{2}}$

$\therefore$ (A) is incorrect and with similar calculations (B) will be correct.

Here f_0 is the resonance frequency as $v_L = v_C$

$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$

and $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\frac{x_L}{x_C} = \frac{\omega L}{1/\omega C} = \omega^2 LC$

Given $f = 2f_0$

$\Rightarrow \omega = 2\omega_0$

$\therefore \frac{x_L}{x_C} = 4$

$\therefore$ (C) is also correct.

5. CD

Sol. $f = \frac{nv}{2\ell}$

$v_I > v_{II}$

$\therefore$ for $f_1 = f_{II}$; $n_I < n_{II}$

6. BCD

Sol. $f = 3, n = 1 \quad P_0V_0 = nRTo$

$H_{AB} = P_o(2V_o - V_0) + \frac{3}{2}P_o(2V_o - V_o) = \frac{5}{2}P_oV_o = \frac{5RT_0}{2}$

$H_{BC} = \frac{fn}{2}R(T_c - T_B) = \frac{3}{2}(2P_o2V_o - P_o2V_o) = 3P_oV_o = 3RTo$

$H_{ABC} = \frac{5}{2}RT_O + 3RT_O = \frac{11}{2}RT_O$

$W_{ABC} = P_oV_o + 0 = RT_O$

$\Delta UA - B - C = H_{ABC} - W_{ABC} = \frac{9}{2}RT_O$

7. B

Sol. Draw FBD of block & check condition of equilibrium.

8. A

Sol. Heat lost by water = Heat gained by ice.

9. C

Sol. $(f_2)_A = (f_2)_B$

$\Rightarrow \dfrac{3}{2l_A} v_{sound} = \dfrac{5}{4l_B} v_{sound}$

$\Rightarrow \dfrac{l_A}{l_B} = \dfrac{6}{5}$; $\dfrac{f_{0A}}{f_{0B}} = \dfrac{\frac{1}{2l_A} v_{sound}}{\frac{1}{4l_B} v_{sound}}$; $T = \dfrac{1}{f}$

10. B

Sol. $T = 2\pi\sqrt{\dfrac{3}{4\pi G\lambda}} \Rightarrow \dfrac{T}{4} = \sqrt{\dfrac{3\pi}{16G\lambda}}$; $V = \omega A = (\sqrt{\pi G\lambda})r$; $N = \dfrac{4}{3}\pi G \times m\dfrac{r}{2}$

Section – B

11. 1.00

Sol. Since the disc was rolling, the horizontal component of the velocity of the top point P of the disc at every instant is zero and the vertical component of the velocity of the point P is equal to the vertical component of velocity of the CM of disc.

$\Rightarrow \sqrt{2 \times 2g(R - 0.1)} = 6$

$\Rightarrow 4 \times 10 (R - 0.1) = 36$

$R = 1$ m.

12. 12.56

Sol. $\int_{+\infty}^{-\infty} \vec{B}.d\vec{x} = \mu_0 i_{inclosed} = \mu_0 (2 - 1) = \mu_0$

13. 3.00

Sol. $X_{cm} = \dfrac{1 \times \frac{(4)^2}{2 \times 1} - \frac{1 \times (2)^2}{2 \times 1}}{1 + 1} = 3$

14. 1.41

Sol. $T' = T\sqrt{\dfrac{1}{1 - \frac{r}{\rho}}} = \sqrt{2}T$

15. 86.67

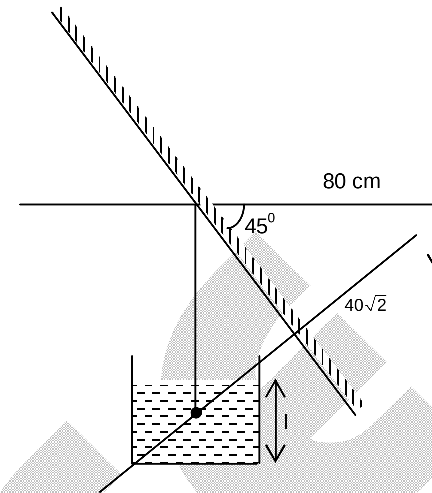
Sol. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{36}=\frac{1}{30} \Rightarrow \frac{1}{v}=\frac{1}{30}-\frac{1}{36}=\frac{1}{180}$

v = 180 cm

v – l = 80

$80 = H - d + \frac{d}{\mu}$

$H = \frac{260}{3}$ cm



16. 2.40

Sol. $R_x = \frac{R_1R_2}{R_1+R_2} = 2.4$

17. 3.14

Sol. kinetic energy of portion of length λ = $\frac{1}{2}\times 0.01\times a^2\times 200\times \pi^2\times \lambda$

$= a^2\pi^2\lambda$

$= \frac{2\pi^2}{2\pi} = 3.14$

18. 6.75

Sol. $\Delta l = \frac{2Fl}{AY}+\frac{3Fl}{AY}+\frac{4Fl}{AY}=\frac{9Fl}{AY}$

Chemistry

PART – II

Section – A

19. ABC

Sol. $KO \xrightarrow{H_2O} O_2$ gas

$CaC_2 \xrightarrow{H_2O} C_2H_2$ gas

$Mg_3N_2 \xrightarrow{H_2O} NH_3$ gas

$Ba(NO_3)_2 \xrightarrow{H_2O} Ba^{2+} + 2NO_3^-$

20. ABC

Sol. Neither the lone pair of oxygen nor the pi – bonds are in conjugation in (D).

21. ABCD

Sol. The boiling point of water is 100^{o}C.
$\therefore$ Dissolution of any non-volatile solute of any quantity in water will cause elevation in boiling point. So, the boiling point of aqueous solution of any non-volatile solute is greater than 100^{o}C.

22. BD

Sol. Le-Chatelier's principle.

23. ACD

Sol. $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ (Enthalpy is a state function and hence additive)

$\Delta H_{x\to y\to z} = \Delta H_{x\to z}$ (State function, depend in initial and final state)

$W_{x\to y\to z} = W_{x\to y}$ (work done in $y \to z$ is zero as it is an isochoric process)

24. BC

Sol. X is acetal, has no free hemiacetal, hence a non-reducing sugar while Y has a free hemiacetal group, it is reducing sugar. Also, glucosidic linkage of X is 'α' while that of Y is β-linkage.

25. A

Sol. (I) $Fe^{2+} + 2KCN \longrightarrow Fe(CN)_2 \downarrow \xrightarrow[\text{excess}]{4KCN} K_4\left[Fe(CN)_6\right]$

$Fe(OH)_2$ is not soluble in excess NaOH and excess NH_4OH

(II) HgO is not soluble in excess NaOH and excess NH_4OH

$Hg^{2+} + 2KI \longrightarrow HgI_2 \downarrow \xrightarrow{2KI} K_2\left[HgI_4\right]$

(III) $Pb^{2+} + 2KI \longrightarrow PbI_2 \downarrow \xrightarrow{2KI} K_2\left[PbI_4\right]$

$Pb^{2+} + H_2S \longrightarrow PbS \downarrow + 2H^+$

$3PbS + 8HNO_3 \longrightarrow 3Pb(NO_3)_2 + 2NO + 3S + 4H_2O$

(IV) $Ag^+ + KCN \longrightarrow AgCN \downarrow \xrightarrow{KCN} K\left[Ag(CN)_2\right]$

$$Ag^{+} + H_2S \longrightarrow Ag_2S\downarrow + 2H^{+}$$
$$3Ag_2S + 8HNO_3 \longrightarrow 6AgNO_3 + 2NO + 3S + 4H_2O$$

26. B

Sol. (I) $pH = p^{K_a} + \log\frac{[HCOONa]}{[HCOOH]} = 3.74 + \log\frac{0.1}{0.2} = 3.74$

$\therefore$ pH < 7

(II) pH = -log [H^+] = -log10^{-1} = 1, $\therefore$ pH < 7

CH_3COOH cannot dissociated in presence of HCl due to common ion effect.

(III) $CH_3COOH + NH_4OH \longrightarrow CH_3COONH_4 + H_2O$

Hydrolysis of CH_3COONH_4 will take place

$$pH = \frac{1}{2}\left(p^{K_w} + p^{K_a} - p^{K_b}\right) = \frac{1}{2}\left[14 + 4.74 - (14 - 9.26)\right]$$

= ½ [14 + 4.74 – 4.74] = 7. $\therefore$ pH = 7 and the solution is a buffer

(IV) $HCOONa + HCl \longrightarrow HCOOH + NaCl$

	HCOONa	HCl	HCOOH	NaCl
Initial meq	30	10	0	0
Meq. After reac^n	20	0	10	10

It is a buffer, $pH = p^{K_a} + \log\frac{[HCOONa]}{[HCOOH]} = 3.74 + \log\frac{20}{10} = 4.04, < 7$

27. A

Sol. In E_2 mechanism, loss of Hydrogen takes place from that carbon atoms which are adjacent to the carbon atoms that hold chlorine. Carbocations are formed in E_1 mechanism.

28. A

Sol. (I) forms monosubstituted product, it is more reactive than benzene due to presence two activating CH_3 groups.

(II) forms monosubstituted product because one position is meta w.r.t electron withdrawing groups and para with respect to CH_3 group. It is less reactive than benzene due to two deactivating and one activating group.

(III) forms more than one monosubstituted product and is more reactive than benzene due to two activating OH groups. The OH groups exert –I and +R effect.

(IV) forms more than one monosubstituted product and is less reactive than benzene. The Cl atoms exert –I as well as +R effect.

Section – B

29. 4.00

Sol. The products are $CH_3CH_2CH_2NO_2$, $CH_3\underset{\underset{NO_2}{|}}{C}HCH_3$, $CH_3CH_2NO_2$ and CH_3NO_2 because C–C and C – H bond cleavage takes place in this reaction.

30. 0.95

Sol. $HIn \rightleftharpoons H^{+} + In^{-}$

$$K_a = \frac{\left[H^+\right]\left[In^-\right]}{[HIn]} = \frac{\left[H^+\right][base]}{[Acid]}$$

$$or, \left[H^+\right] = Ka\frac{[Acid]}{[Base]}$$

For 75% red, $\left[H^+\right] = \frac{Ka \times 75}{25} = \frac{3 \times 10^{-5} \times 75}{25} = 9 \times 10^{-5}; pH = 4.05$

For 75% blue, $\left[H^+\right] = \frac{\left(3 \times 10^{-5}\right) \times 25}{75} = 1 \times 10^{-5}; pH = 5$

The change in pH = 5-4.05 = 0.95

31. 122.50

Sol. X is $CH_3 - \underset{}{\overset{CH_3}{\overset{|}{C}H}} - \overset{O}{\overset{||}{C}} - OH$

32. 2.00

Sol. The structure of H_2SO_5 is:

$$HO - O - \underset{\underset{O}{||}}{\overset{\overset{O}{||}}{S}} - OH$$

33. 418.52

Sol. Moles of Sn^{2+} = 523.15×10^{-3}

Moles of MnO_4^- required = $\frac{2}{5} \times 523.15 \times 10^{-3}$

Volume = $\frac{\frac{2}{5} \times 523.15 \times 10^{-3} \times 1000}{0.5}$ = 418.52 mL

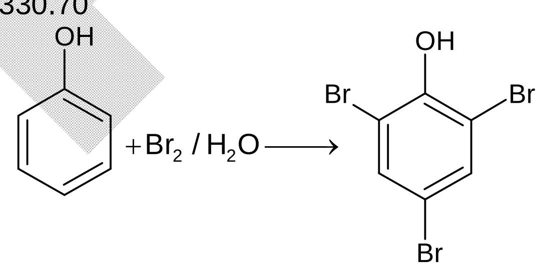
34. 2.50

Sol. Na^+, Mg^{2+}, O^{2-}, F^- & Cl^- will have more radius

So, $\frac{x}{2} = 2.5$

35. 330.70

Sol.



36. 4.00

Sol. $\Delta T_f = K_f \times n \times \frac{1000}{W}$

Or, $4 = 1.86 \times \frac{6.2}{62} \times \frac{1000}{W}$

$\therefore$ W = 46.5 g

$\therefore$ Mass of water converted to ice = 50.5 – 46.5 = 4 g

Mathematics

PART – III

Section – A

37. ACD

Sol. (A) $(AB)^T = B^T A^T = -BA$

$= AB$

$(B,C)\ A^{-1}B^{-1} = (BA)^{-1}$

$= (-AB)^{-1}$

$A^{-1}B^{-1} = B^{-1}A^{-1}$

(D) $-AB = BA$

$A^{-1}AB = -A^{-1}BA$

$B = -A^{-1}BA$

$BA^{-1} = -A^{-1}BA\ A^{-1}$

$BA^{-1} = -A^{-1}B$

38. BC

Sol. $A = A^T \Rightarrow A = \text{adj}(2A) \Rightarrow A = 2^{3-1}(\text{adj}\,A)$

$|A| = |2A|^2 = |A| = 64|A|^2$

$|A| = \dfrac{1}{64}$

Now, $A^{-1} = \dfrac{\text{adj}\,A}{|A|} = \dfrac{A}{4|A|} = \dfrac{A}{4}t64 = 16A$

$\text{adj}(A^{-1}) = \text{adj}(16A) = 16^{3-1}(\text{adj}\,A)$

$= 256\left(\dfrac{A}{4}\right) = 64A$

39. ACD

Sol. $a_{n+1}^2 - a_n^2 = 1$. By Telescopic sum $a_n = \sqrt{n-1+a_1^2}$ now

$a_{2n_0} = 3a_{n_0} \Rightarrow n_o = \dfrac{8}{7}(1-a_1^2) < \dfrac{8}{7}$

But n_o is a positive integer $\Rightarrow n_o = 1 \Rightarrow 1 = \dfrac{8}{7}(1-a_1^2) \Rightarrow a_1^2 = \dfrac{1}{8}$

$\Rightarrow a_n = \sqrt{n - \dfrac{7}{8}}$

$\Rightarrow \sqrt{\dfrac{8}{8a_n^2+7}} = \dfrac{1}{\sqrt{n}},\ \sqrt{n+1} - \sqrt{n} < \dfrac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$

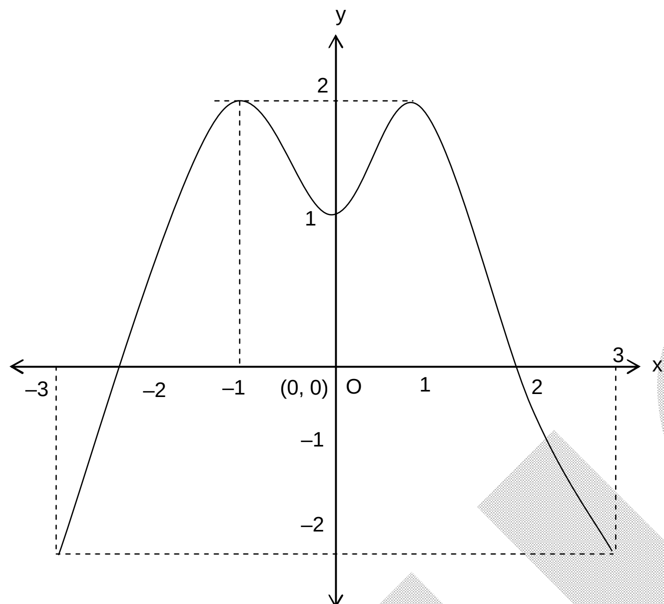
$\Rightarrow \sqrt{50} - 1 < \dfrac{1}{2}\sum_{n=1}^{49}\sqrt{\dfrac{8}{8a_n^2+7}} < 7$

40. AB

Sol. After 5 second bug can reach at (5, 0), (4, 1), (3, 2), (2, 3), (1, 4) and (0, 5) along 1, 5, 10, 5, 1 different paths.

41. ABCD

Sol. (A) Graph of $y = f(-|x|)$ is



$\therefore$ Range is $[-2, 2]$

(B) Domain of $y = f(|x|)$ is $-3 \le |x| \le 2 \Rightarrow -2 \le x \le 2$

(C) Domain of $y = f(|x|+1)$ is $-3 \le |x|+1 \le 2 \Rightarrow -4 \le |x| \le 1$

42. CD

Sol. $f_n =$ number of subset in which n appears + number of subset in which n does not appears.

$S = \{1, 2, 3, ... (n-2), (n-1), n\}$ when n appears obviously (n – 1) will not appear and when n does not appear up to (n – 1) will appear

$\Rightarrow \quad f_n = f_{n-2} + f_{n-1}$

$f_1 = 2$

$f_2 = 3$

$\Rightarrow \quad f_3 = f_1 + f_2 = 2 + 3 = 5$

$f_4 = f_2 + f_3 = 3 + 5 = 8$

43. D

Sol. (I) $\left|\lambda_1 a_1\omega + \lambda_2 a_2\omega^2 + ... + \lambda_n a_n \omega^n\right| \le \left|\lambda_1 a_1 \omega\right| + \left|\lambda_2 a_2 \omega^2\right| + ... + \left|\lambda_n a_n \omega^n\right|$

$= |\lambda_1||a_1||\omega| + |\lambda_2||a_2||\omega|^2 + ... + |\lambda_n||a_n||\omega|^n \qquad (\because |\omega| = 1, \lambda_i \ge 0)$

$= \lambda_1|a_1| + \lambda_2|a_2| + ... + \lambda_n|a_n| \qquad ....(i)$

$< \left(\lambda_1 + \lambda_2 + \ldots + \lambda_n\right) \qquad \left(\because |a_i| < 1\right)$

$= 1 < 2(Q)$

$\therefore \left|\lambda_1 a_1 \omega + \lambda_2 a_2 \omega^2 + \ldots + \lambda_n a_n \omega^n\right| < 1$ (S)

Also, as $\lambda_i \geq 0$ and $\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n = 1$, none of λ_i can exceed 1, thus $0 \leq \lambda_i \leq 1$ therefore, from equation (i), we get

$\left|\lambda_1 a_1 \omega + \lambda_2 a_2 \omega^2 + \ldots + \lambda_n a_n \omega^n\right| < |a_1| + |a_2| + - + |a_n|$ (T)

(II) $\because \left|1 + z + z^2 + \ldots + z^n\right|$

$= \left|\frac{z^{n+1} - 1}{z - 1}\right| < \left|\frac{z^{n+1} - 1}{z}\right| \leq |z|^n + \frac{1}{|z|}$ (P) $\qquad \left(\because \operatorname{Re}(z) < 0, |z - 1| > |z|\right)$

(III) $\because 1 + 2x + 3x^2 + \ldots + 3nx^{3n-1}$

$$= \frac{d}{dx}\left(x + x^2 + x^3 + \ldots + x^{3n}\right)$$

$$= \frac{d}{dx}\left\{\frac{x\left(1 - x^{3n}\right)}{1 - x}\right\}$$

$$= \frac{(1 - x)\left(1 - (3n + 1)x^{3n}\right) + x\left(1 - x^{3n}\right)}{(1 - x)^2}$$

Put $x = \omega$, then $1 + 2\omega + 3\omega^2 + \ldots. + 3n\omega^{3n-1}$

$$= \frac{(1 - \omega)\left(1 - (3n + 1)\right) + \omega(1 - 1)}{(1 - \omega)^2}$$

$$= \frac{-3n}{(1 - \omega)}$$

$$\therefore \left|1 + 2\omega + 3\omega^2 + \ldots\ldots + 3n\omega^{3n-1}\right| = \left|\frac{-3n}{1 - \omega}\right| = n\sqrt{3}$$

$$\therefore \frac{1}{\sqrt{3}}\left|1 + 2\omega + 3\omega^2 + \ldots + 3n\omega^{3n-1}\right| = \left|\frac{-3n}{1 - \omega}\right| = n\sqrt{3}$$

$$\therefore \frac{1}{\sqrt{3}}\left|1 + 2\omega + 3\omega^2 + \ldots.. + 3n\omega^{3n-1}\right| = n(R)$$

(IV) $\log_2\left|1 + \omega + \omega^2 + \omega^3 - \omega^4\right| = \log_2\left|2\omega^4\right| = 2$

44. B

Sol. (I) $\because f(x) = \max\{1+\sin x, 1, 1-\cos x\}$

$$= \begin{cases} 1+\sin x, & 0 \le x \le \dfrac{3\pi}{4} \\ 1-\cos x, & \dfrac{3\pi}{4} \le x \le \dfrac{3\pi}{2} \\ 1, & \dfrac{3\pi}{2} \le x \le 2\pi \end{cases}$$

$$g(x) = \max\{1, |x-1|\} = \begin{cases} 1-x, & x \le 0 \\ 1, & 0 \le x \le 2 \\ x-1, & x \ge 2 \end{cases}$$

$\therefore \ f(0) = 1 \Rightarrow g(f(0)) = g(1) = 1$

$\therefore g(f(0)) = 1(S)$ and $f(1) = 1+\sin 1$ $\quad \left(\because 0 < 1 < \dfrac{3\pi}{4}\right)$

$\therefore g(f(1)) = g(1+\sin 1) = 1$ $\quad (\because 1 < 1+\sin 1 < 2)$

$\therefore g(f(1)) = 1(p)$

(II) $\because f(g(x)) = \ln\left(\dfrac{1+g(x)}{1-g(x)}\right)$

$\therefore \ f(g(0)) = \ln\left(\dfrac{1+g(0)}{1-g(x)}\right) = \ln\left(\dfrac{1+0}{1-0}\right) = \ln 1 = 0(Q)$

and $g\left(f\left(\dfrac{e-1}{e+1}\right)\right) = g\left(\ln\left(\dfrac{1+\dfrac{e-1}{e+1}}{1-\dfrac{e-1}{e+1}}\right)\right) = g(\ln(e)) = e(1)$

$= \dfrac{3+1}{1+3} = \dfrac{4}{4} = 1(T)$

(III) $f(g(0)) = f(0) = 1+0^2 = 1(R)$

$gf(0) = g(1) = 1-1^2 = 0$

$g(f(1)) = g(2) = 2-2^2 = -2$

(IV) $f(x) = \dfrac{x}{\sqrt{1+3x^2}}$

45. B

Sol. (I) Let $I = \int (\tan x)^{1/3} dx$

Put $\tan x = t^3$

$\therefore \sec^2 x\, dx = 3t^2 dt$

$\Rightarrow dx = \dfrac{3t^2 dt}{(1+t^6)}$

(II) Let $I = \int \dfrac{(\sin x + \sin^3 x)dx}{\cos 2x}$

$= \int \dfrac{(1+\sin^2 x)\sin x\, dx}{(2\cos^2 x - 1)}$

$= \int \dfrac{(2-\cos^2 x)\sin x}{(\sqrt{2}\cos x)^2 - 1} dx$

Put $\sqrt{2}\cos x = t$

$\therefore \sin x\, dx = -\dfrac{dt}{\sqrt{2}}$

Then, $I = \int \dfrac{\left(2 - \dfrac{t^2}{2}\right)\left(-\dfrac{dt}{\sqrt{2}}\right)}{(t^2-1)}$

$= 3\int \dfrac{t^3 dt}{(1+t^2)^3}$

Put $t^2 = z$

$\Rightarrow 2t\, dt = dz$

$I = \dfrac{3}{2}\int \dfrac{z dz}{1+z^3} = \dfrac{3}{2}\int \dfrac{z dz}{(1+z)(1-z+z^2)}$

$= \dfrac{1}{4}\ln\left(\dfrac{t^4 - t^2 + 1}{(t^2+1)^2}\right) + \dfrac{\sqrt{3}}{2}\tan^{-1}\left(\dfrac{2t^2-1}{\sqrt{3}}\right) + c$

$\therefore A = \dfrac{1}{4}(P); B = \dfrac{\sqrt{3}}{2}(S)$

(III) Let $I = \int \dfrac{dx}{(x^2+1)(x^2+4)} = \dfrac{1}{3}\int\left(\dfrac{1}{x^2+1} - \dfrac{1}{x^2+4}\right)dx$

$$=\frac{1}{3}\left\{\tan^{-1}x-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right\}+c$$

$$=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$\therefore A=\frac{1}{3}(Q),B=-\frac{1}{6}$$

(IV) Integral reduces to $\int\frac{\sin 8x}{2}dx=-\frac{\cos 8x}{16}+C$

If m is the least value (global minimum) and M is the greatest value (global maximum) of the function $f(x)$ on the interval [a, b]. (estimation of an integral). Then

$$m(b-a)\le\int_a^b f(x)dx\le M(b-a).$$

46. A

Sol. (I) Let $f(x)=\left(\frac{5-x}{9-x^2}\right)$, $\therefore f'(x)=\frac{(x-9)(x-1)}{(9-x^2)^2}$

$\Rightarrow f'(x)=0$ or not defined (for critical points)

$\therefore x=1$

Then, $f(10)=\frac{5}{9},f(1)=\frac{1}{2},f(2)=\frac{3}{5}$

$\therefore\quad M=\frac{3}{5}$ and $m=\frac{1}{2}$

$\lambda=(b-a)M=(2-0)\frac{3}{5}=\frac{6}{5}$ and $\mu=(b-a)m=(2-0)\frac{1}{2}=1$

$\lambda+\mu=2.2\Rightarrow[\lambda+\mu]=2(R)$

$5\lambda-\mu=5\Rightarrow[5\lambda-\mu]=5(T)$

(II) Let $f(x)=\frac{\sin x}{x}$

$$\therefore\ f'(x)=\frac{x\cos x-\sin x}{x^2}$$

$$=\frac{(x-\tan x)\cos x}{x^2}<0\,\forall x\in\left[\frac{\pi}{4},\frac{\pi}{3}\right]$$

$\therefore f(x)$ is decreases on the interval $\left[\frac{\pi}{4},\frac{\pi}{3}\right]$

Then, $m = f\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}\right)}{\left(\frac{\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{\pi}{3}} = \frac{3\sqrt{3}}{2\pi}$ and $M = f\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}} = \frac{4}{\pi\sqrt{2}} = \frac{2\sqrt{2}}{\pi}$

$\therefore \quad \lambda = (b-a)M = \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \times \frac{2\sqrt{2}}{\pi} = \frac{\sqrt{2}}{6}$ and

$\mu = (b-a)m = \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \times \frac{3\sqrt{3}}{2\pi} = \frac{\sqrt{3}}{8}$

(III) Let $f(x) = \sqrt{(3 + x^3)}$

$\therefore \quad f'(x) = \frac{3x^2}{2\sqrt{(3+x^3)}} > 0 \; \forall \, x \in [1, 3]$

$\therefore f(x)$ is increasing on the interval $[1, 3]$

$\Rightarrow \; m = f(1) = \sqrt{4} = 2$ and $M = f(3) = \sqrt{30}$

$\therefore \; \lambda = (b-a)M = (3-1)\sqrt{30} = 2\sqrt{30}$ and $\mu = (b-a)m = (3-1)2 = 4$

$[\lambda - 2] = 8$ and $[\lambda - \mu] = 6$ (Q,S)

(IV) $\int_{1/10}^{2} x^x \, dx$

$\in \left(\frac{19}{10} e^{-1/e}, \frac{38}{5}\right)$

Section – B

47. 10.29

Sol. $P(n) = \prod_{r=3}^{n} \frac{(r^3 + 3r)^2}{r^6 - 64} = \prod_{r=3}^{n} \frac{r}{r-2} \prod_{r=3}^{n} \frac{r}{r+2} \prod_{r=3}^{n} \frac{r^2+3}{(r+1)^2+3} \prod_{r=3}^{n} \frac{r^2+3}{(r-1)^2+3}$

So, $P(n) = \frac{n(n-1)}{2} \cdot \frac{12}{(n+1)(n+2)} \cdot \frac{12}{(n+1)^2+3} \cdot \frac{n^2+3}{7}$

$\lim_{n\to\infty} P(n) = \frac{72}{7}$

48. 11.11

Sol. Let $N = \sum_{a=1}^{\infty}\sum_{b=1}^{\infty}\sum_{c=1}^{\infty} \frac{ab(3a+c)}{4^{a+b+c}(a+b)(b+c)(c+a)} \Rightarrow 6N = 3\sum_{a=1}^{\infty}\sum_{b=1}^{\infty}\sum_{c=1}^{\infty} \frac{1}{4^{a+b+c}}$

$$N=\frac{1}{2}\left(\sum_{a=1}^{\infty}\frac{1}{4^a}\right)^3=\frac{1}{54}\Rightarrow 600N=\frac{100}{9}=11.11$$

49. 16.00

Sol. $y=\dfrac{3x^2+mx+n}{x^2+1}$

$\Rightarrow x^2(y-3)-mx+y-n=0$

As $x\in R$,

$D\geq 0$

$\Rightarrow m^2-4(y-3)(y-n)\geq 0$

$\Rightarrow m^2-4\left(y^2-ny-3y+3x\right)\geq 0$ ....(1)

Also given $(y+4)(y-3)\leq 0$

$\Rightarrow y^2+y-12\leq 0$ ......(2)

$\therefore$ compare (1) and (2) we get $\dfrac{4}{1}=\dfrac{4(n+3)}{1}=\dfrac{12n-m^2}{-12}$

$\Rightarrow m=0$ and $n=-4$

50. 0.75

Sol. Let $\dfrac{\pi}{7}=\theta$

Now $2\cos^3\theta-\cos^2\theta-\cos\theta$

$=\cos\theta\left(2\cos^2\theta-1\right)-\cos^2\theta$

$=\cos\theta\cos 2\theta-\cos^2\theta$

$=\cos\theta\left[\cos 2\theta-\cos\theta\right]$

$=-2\cos\theta.\sin\dfrac{3\theta}{2}\sin\dfrac{\theta}{2}$

$=-2\cos\dfrac{2\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{\pi}{14}$

$=-2\cos\dfrac{2\pi}{14}\cos\dfrac{4\pi}{14}\cos\dfrac{6\pi}{14}$

$=-2\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{3\pi}{7}$

$=2\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}$

$$=\frac{\sin 8\frac{\pi}{7}}{4\sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\pi + \left(\frac{\pi}{7}\right)\right)}{4\sin\left(\frac{\pi}{7}\right)} = -\frac{1}{4}$$

51. 0.25

Sol. $T = x^2 + 2xy + 3y^2 - 6x - 2y$

$\Rightarrow x^2 + 2(y-3)x + 3y^2 - 2y - t = 0$

$\Rightarrow 4(y-3)^2 - 4\left(3y^2 - 2y - t\right) \geq 0$ $\quad\quad D \geq 0$

$\Rightarrow t + 9 \geq 2\left(y^2 - 2y\right)$

$\Rightarrow t \geq 2(y+1)^2 - 11$

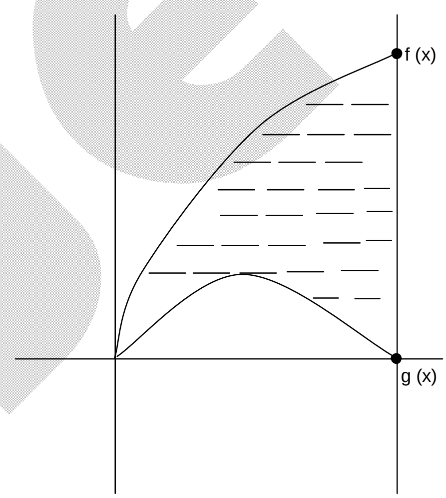
52. 0.80

Sol. $y - x = \pm x\sqrt{x} \Rightarrow y = x \pm x\sqrt{x}$

$f(x) = x + x\sqrt{x},\ g(x) = x - x\sqrt{x}$

Area $= \int_0^1 f(x) - g(x)\ dx = \int_0^1 2x^{3/2} dx$

Answer is $\frac{4}{5}$



53. 0.88

Sol. $\frac{f(x)}{1+x^2} = 1 + \int_0^x \frac{f^2(t)}{1+t^2} dt$ $\ (f(0)=1)$ Differentiate

$\Rightarrow \frac{\left(1+x^2\right)f'(x) - 2x f(x)}{\left(1+x^2\right)^2} = \frac{f^2(x)}{1+x^2}$

$\Rightarrow \frac{dy}{dx} - \frac{2x}{1-x^2}.y = y^2$ $\quad\quad$ L.D.E

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{2x}{\left(1-x^2\right)}\frac{1}{y} = 1$

Put $\frac{-1}{y} = T$

$\Rightarrow f(x) = \dfrac{-3(1+x^2)}{x^3+3x-3}$

Answer is $\dfrac{15}{17}$

54. 0.10

Sol. $f(x) = axe^{-bx}$ has a local maximum at the point (2, 10)

$\therefore\ f(2) = 10$

$\Rightarrow\ 2ae^{-2b} = 10$

$\Rightarrow\ ae^{-2b} = 5$ (i)

Now, $f'(x) = a\left[e^{-bx} - bx\,e^{-bx}\right]$

$f'(2) = 0$

$\Rightarrow\ a\left(e^{-2b} - 2be^{-2b}\right) = 0$

$\Rightarrow\ ae^{-2b}(1-2b) = 0$

$\Rightarrow\ b = \dfrac{1}{2}$

Putting $b = \dfrac{1}{2}$ in (i), we get

$a = 5e$

$\therefore\ a = 5e$ and $b = \dfrac{1}{2}$