

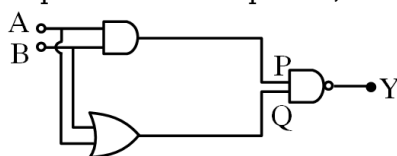
## PART-1 : PHYSICS

### SECTION-I

1) A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

- (A)  $\sqrt{\frac{2ghv^2 + 1}{h^2}}$   
 (B)  $\sqrt{2ghv^2 + h^2}$   
 (C)  $\sqrt{\frac{2v^2h}{g} + h^2}$   
 (D)  $\sqrt{\frac{2gh}{v^2} + h^2}$

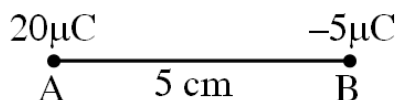
2) In the following logic circuit the sequence of the inputs A, B are (0, 0), (0,1), (1, 0) and (1, 1). The



output Y for this sequence will be :

- (A) 1, 0, 1, 0  
 (B) 0, 1, 0, 1  
 (C) 1, 1, 1, 0  
 (D) 0, 0, 1, 1

3) Two particles A and B having charges  $20 \mu\text{C}$  and  $-5 \mu\text{C}$  respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not



experience a net electric force?

- (A) At 5 cm from  $20 \mu\text{C}$  on the left side of system  
 (B) At 5 cm from  $-5 \mu\text{C}$  on the right side  
 (C) At 1.25 cm from  $-5 \mu\text{C}$  between two charges  
 (D) At midpoint between two charges

4) The average kinetic energy of a molecule of the gas is

- (A) proportional to absolute temperature

- (B) proportional to volume
- (C) proportional to pressure
- (D) dependent on the nature of the gas

5) An object is placed at the focus of concave lens having focal length  $f$ . What is the magnification and distance of the image from the optical centre of the lens?

- (A) 1,  $\infty$
- (B) Very high,  $\infty$
- (C)  $\frac{1}{2}$ ,  $\frac{f}{2}$
- (D)  $\frac{1}{4}$ ,  $\frac{f}{4}$

6) Find the binding energy per nucleon for  $^{120}_{50}\text{Sn}$ . Mass of proton  $m_p = 1.00783 \text{ U}$ , mass of neutron  $m_n = 1.00867 \text{ U}$  and mass of tin nucleus  $m_{\text{Sn}} = 119.902199 \text{ U}$ . (take  $1\text{U} = 931 \text{ MeV}$ )

- (A) 8.5 MeV
- (B) 7.5 MeV
- (C) 8.0 MeV
- (D) 9.0 MeV

7) A coil having  $N$  turns is wound tightly in the form of a spiral with inner and outer radii ' $a$ ' and ' $b$ ' respectively. Find the magnetic field at centre, when a current  $I$  passes through coil :

- (A)  $\frac{\mu_0 IN}{2(b-a)} \log_e \left( \frac{b}{a} \right)$
- (B)  $\frac{\mu_0 I}{8} \left[ \frac{a+b}{a-b} \right]$
- (C)  $\frac{\mu_0 I}{4(a-b)} \left[ \frac{1}{a} - \frac{1}{b} \right]$
- (D)  $\frac{\mu_0 I}{8} \left( \frac{a-b}{a+b} \right)$

8) A body of mass  $M$  moving at speed  $V_0$  collides elastically with a mass ' $m$ ' at rest. After the collision, the two masses move at angles  $\theta_1$  and  $\theta_2$  with respect to the initial direction of motion of the body of mass  $M$ . The largest possible value of the ratio  $M/m$ , for which the angles  $\theta_1$  and  $\theta_2$  will be equal, is :

- (A) 4
- (B) 1
- (C) 3
- (D) 2

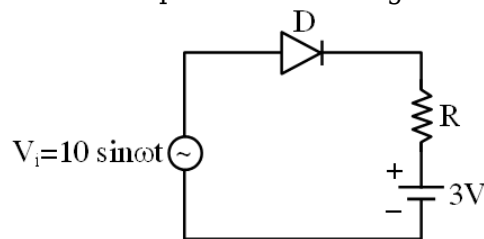
9) The masses and radii of the earth and moon are  $(M_1, R_1)$  and  $(M_2, R_2)$  respectively. Their centres are at a distance ' $r$ ' apart. Find the minimum escape velocity for a particle of mass ' $m$ ' to be projected from the middle of these two masses:

- (A)  $V = \frac{1}{2} \sqrt{\frac{4G(M_1 + M_2)}{r}}$
- (B)  $V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$
- (C)  $V = \frac{1}{2} \sqrt{\frac{2G(M_1 + M_2)}{r}}$
- (D)  $V = \frac{\sqrt{2G(M_1 + M_2)}}{r}$

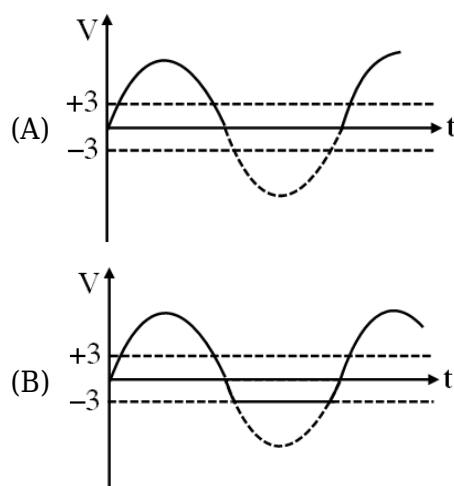
10) A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn ( $b \gg a$ ). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

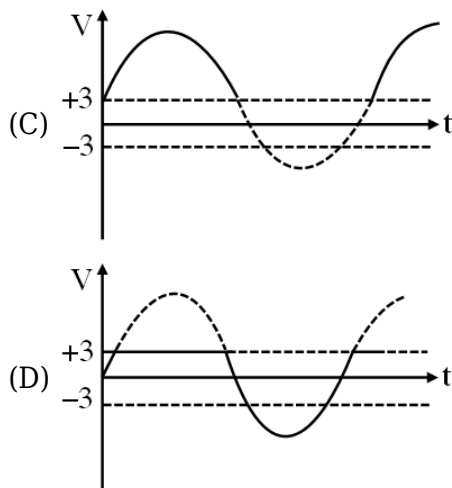
- (A)  $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$
- (B)  $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$
- (C)  $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$
- (D)  $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

11) Choose the correct waveform that can represent the voltage across R of the following circuit,



assuming the diode is ideal one:

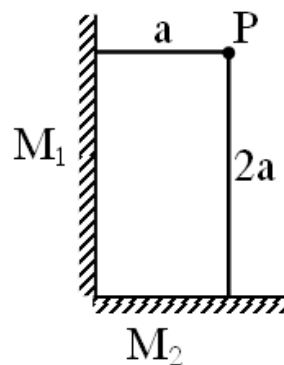




12) A uniform heavy rod of weight  $10 \text{ kg ms}^{-2}$ , cross-sectional area  $100 \text{ cm}^2$  and length  $20 \text{ cm}$  is hanging from a fixed support. Young modulus of the material of the rod is  $2 \times 10^{11} \text{ Nm}^{-2}$ . Neglecting the lateral contraction, find the elongation of rod due to its own weight.

- (A)  $2 \times 10^{-9} \text{ m}$   
 (B)  $5 \times 10^{-8} \text{ m}$   
 (C)  $4 \times 10^{-8} \text{ m}$   
 (D)  $5 \times 10^{-10} \text{ m}$

13) Two plane mirrors  $M_1$  and  $M_2$  are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from  $M_1$  and  $M_2$  respectively. The shortest distance between the images



thus formed is : (Take  $\sqrt{5} = 2.3$ )

- (A)  $3a$   
 (B)  $4.6 a$   
 (C)  $2.3 a$   
 (D)  $2\sqrt{10}a$

14) Match **List-I** with **List-II**.

List-I		List-II	
(a)	Torque	(i)	$\text{MLT}^{-1}$
(b)	Impulse	(ii)	$\text{MT}^{-2}$

(c)	Tension	(iii)	$ML^2T^{-2}$
(d)	Surface Tension	(iv)	$MLT^{-2}$

Choose the **most appropriate** answer from the option given below :

- (A) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)  
 (B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)  
 (C) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)  
 (D) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

15) For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation  $\frac{dp}{dv} = -ap$ . If  $p = p_0$  at  $v = 0$  is the given boundary condition, then the maximum temperature one mole of gas can attain is :

(Here R is the gas constant)

- (A)  $\frac{p_0}{a e R}$   
 (B)  $\frac{ap_0}{e R}$   
 (C) infinity  
 (D)  $0^\circ C$

16) Which of the following equations is dimensionally incorrect ?

Where t = time, h = height, s = surface tension,  $\theta$  = angle,  $\rho$  = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done,  $\Gamma$  = torque,  $\epsilon$  = permittivity, E = electric field, J = current density, L = length.

- (A)  $v = \frac{\pi p a^4}{8 \eta L}$   
 (B)  $h = \frac{2s \cos \theta}{\rho r g}$   
 (C)  $J = \epsilon \frac{\partial E}{\partial t}$   
 (D)  $W = \Gamma \theta$

17) Angular momentum of a single particle moving with constant speed along circular path :

- (A) changes in magnitude but remains same in the direction  
 (B) remains same in magnitude and direction  
 (C) remains same in magnitude but changes in the direction  
 (D) is zero

18) In an ac circuit, an inductor, a capacitor and a resistor are connected in series with  $X_L = R = X_C$ . Impedance of this circuit is :

- (A)  $2R^2$   
 (B) Zero

(C) R

(D)  $R\sqrt{2}$

19) A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

(A)  $K_p < K_e$  and  $P_p = P_e$

(B)  $K_p = K_e$  and  $P_p = P_e$

(C)  $K_p < K_e$  and  $P_p < P_e$

(D)  $K_p > K_e$  and  $P_p = P_e$

20) Consider a galvanometer shunted with  $5\Omega$  resistance and 2% of current passes through it. What is the resistance of the given galvanometer ?

(A)  $300\Omega$

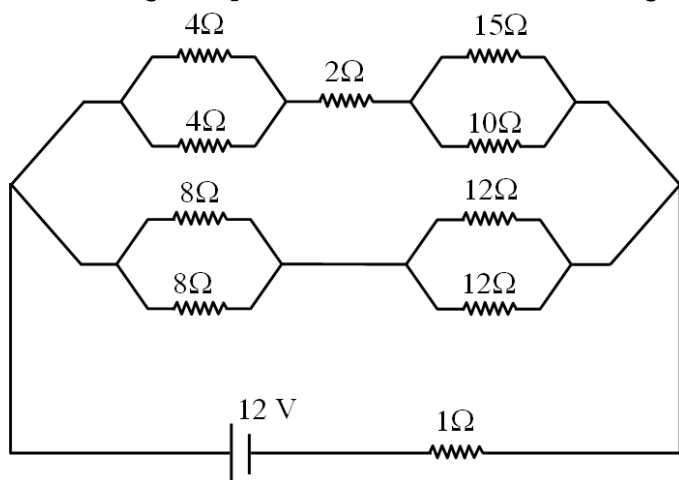
(B)  $344\Omega$

(C)  $245\Omega$

(D)  $226\Omega$

## SECTION-II

1) The voltage drop across  $15\Omega$  resistance in the given figure will be \_\_\_\_\_ V.



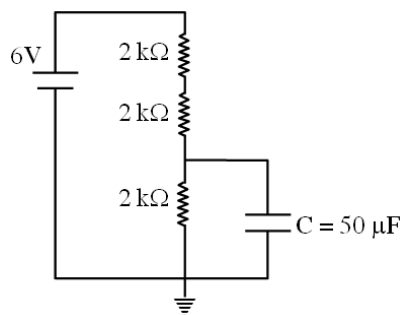
2) A block moving horizontally on a smooth surface with a speed of  $40\text{ ms}^{-1}$  splits into two equal parts. If one of the parts moves at  $60\text{ ms}^{-1}$  in the same direction, then the fractional change in the kinetic energy will be  $x : 4$  where  $x =$  \_\_\_\_\_.

3) The electric field in an electromagnetic wave is given by  $E = (50\text{ NC}^{-1}) \sin \omega(t-x/c)$

The energy contained in a cylinder of volume V is  $5.5 \times 10^{-12}\text{ J}$ . The value of V is \_\_\_\_\_  $\text{cm}^3$ .

(given  $\epsilon_0 = 8.8 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$ )

4) A capacitor of  $50\mu\text{F}$  is connected in a circuit as shown in figure. The charge on the upper plate of



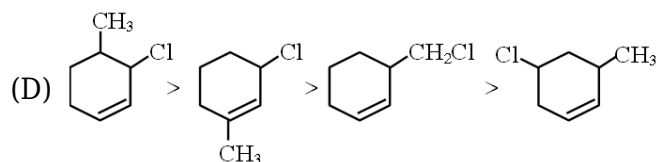
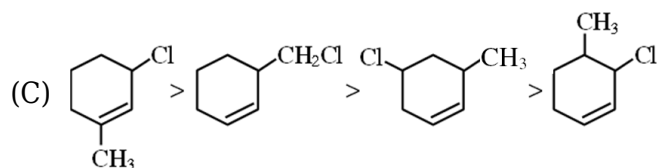
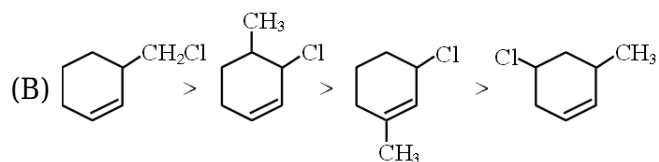
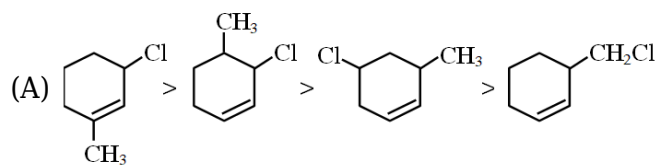
the capacitor is \_\_\_\_\_  $\mu\text{C}$ .

5) A car is moving on a plane inclined at  $30^\circ$  to the horizontal with an acceleration of  $10 \text{ ms}^{-2}$  parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is \_\_\_\_\_. (Take  $g = 10 \text{ ms}^{-2}$ )

## PART-2 : CHEMISTRY

### SECTION-I

1) The **correct** order of reactivity of the given chlorides with acetate in acetic acid is :



2) For the reaction of  $\text{H}_2$  with  $\text{I}_2$ , the rate constant is  $2.5 \times 10^{-4} \text{ dm}^3 \text{ mol}^{-1} \text{ s}^{-1}$  at  $327^\circ\text{C}$  and  $1.0 \text{ dm}^3 \text{ mol}^{-1} \text{ s}^{-1}$  at  $527^\circ\text{C}$ . The activation energy for the reaction, in  $\text{kJ mol}^{-1}$  is: ( $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ )

- (A) 72
- (B) 166
- (C) 150
- (D) 59

3) Which one among the following metals is the weakest reducing agent ?

- (A) K
- (B) Rb
- (C) Na
- (D) Li

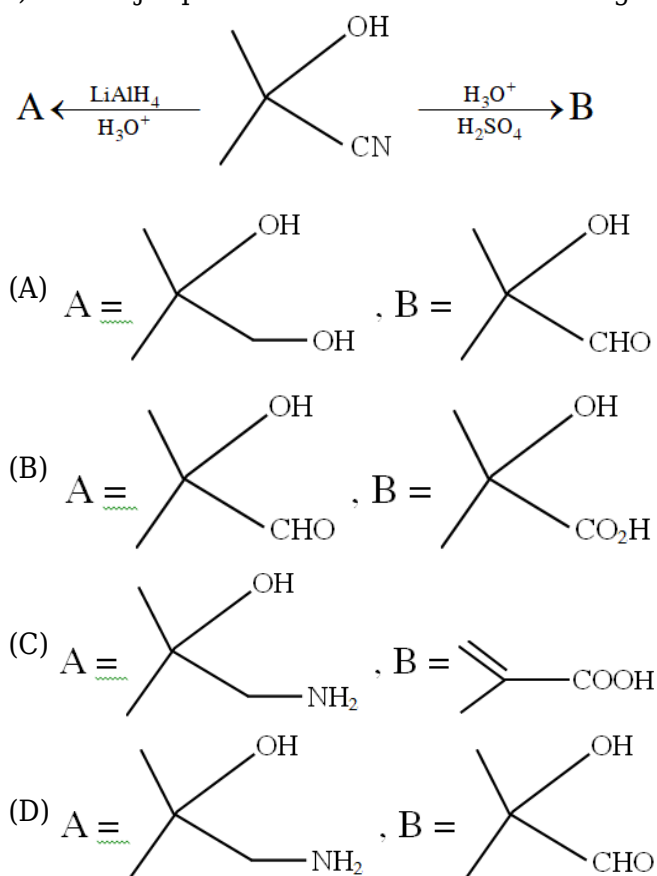
4) In the structure of the dichromate ion, there is a :

- (A) linear symmetrical Cr-O-Cr bond.
- (B) non-linear symmetrical Cr-O-Cr bond.
- (C) linear unsymmetrical Cr-O-Cr bond.
- (D) non-linear unsymmetrical Cr-O-Cr bond.

5) Which one of the following compounds contains  $\beta$ -C<sub>1</sub>-C<sub>4</sub> glycosidic linkage ?

- (A) Lactose
- (B) Sucrose
- (C) Maltose
- (D) Amylose

6) The major products A and B in the following set of reactions are :



7) Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature ?  
(Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58)

- (A) Nd
- (B) Yb

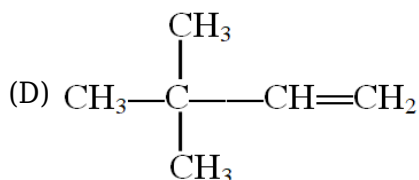
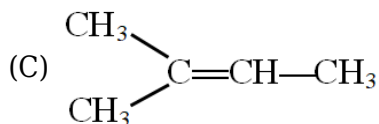
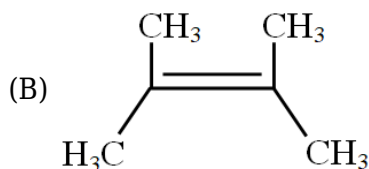
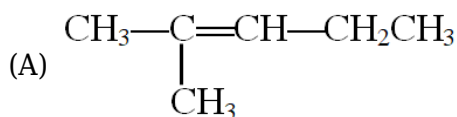
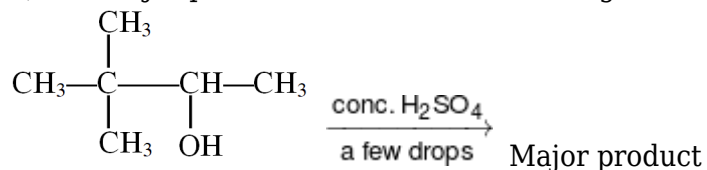


- (C) La  
(D) Ce

8) The complex cation which has two isomers is :

- (A)  $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$   
(B)  $[\text{Co}(\text{NH}_3)_5\text{Cl}]^{2+}$   
(C)  $[\text{Co}(\text{NH}_3)_5\text{NO}_2]^{2+}$   
(D)  $[\text{Co}(\text{NH}_3)_5\text{Cl}]^+$

9) The major product formed in the following reaction is :



10) Monomer of Novolac is :

- (A) 3-Hydroxybutanoic acid  
(B) phenol and melamine  
(C) o-Hydroxymethylphenol  
(D) 1,3-Butadiene and styrene

11) During the change of  $\text{O}_2$  to  $\text{O}_2^-$ , the incoming electron goes to the orbital :

- (A)  $\sigma^* 2P_z$   
(B)  $\pi 2P_y$   
(C)  $\pi^* 2P_x$   
(D)  $\pi 2P_x$

12) Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

**Assertion (A)** : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

**Reason (R)** : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

- (A) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**
- (B) **(A)** is false but **(R)** is true
- (C) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
- (D) **(A)** is true but **(R)** is false

13) The denticity of an organic ligand, biuret is :

- (A) 2
- (B) 4
- (C) 3
- (D) 6

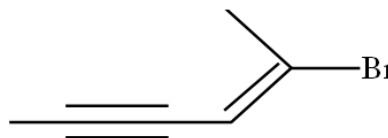
14) Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

**Assertion (A)** : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

**Reason (R)** : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

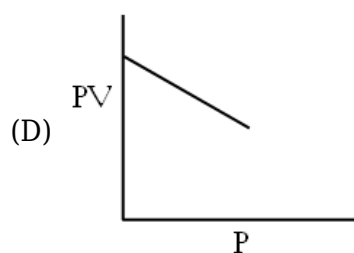
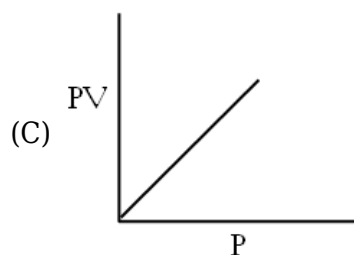
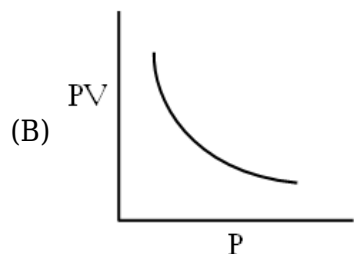
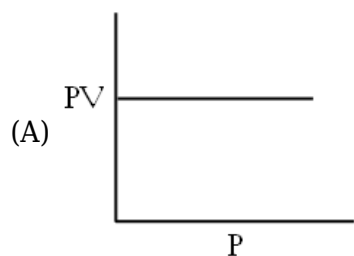
- (A) **(A)** is false but **(R)** is true.
- (B) **(A)** is true but **(R)** is false
- (C) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
- (D) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**



15) Choose the **correct** name for compound given below :

- (A) (4E)-5-Bromo-hex-4-en-2-yne
- (B) (2E)-2-Bromo-hex-4-yn-2-ene
- (C) (2E)-2-Bromo-hex-2-en-4-yne
- (D) (4E)-5-Bromo-hex-2-en-4-yne

16) Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas ?  
(P and V stand for pressure and volume of the gas respectively)



17) Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)** :

**Assertion (A)** : A simple distillation can be used to separate a mixture of propanol and propanone.

**Reason (R)** : Two liquids with a difference of more than  $20^{\circ}\text{C}$  in their boiling points can be separated by simple distillations.

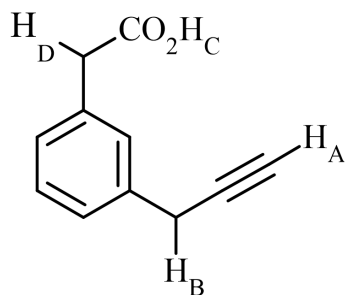
In the light of the above statements, choose the **most appropriate** answer from the options given below :

- (A) **(A)** is false but **(R)** is true.
- (B) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**
- (C) **(A)** is true but **(R)** is false
- (D) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

18) Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

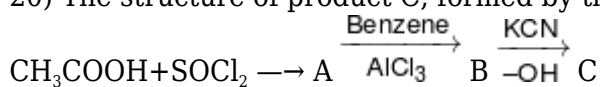
- (A) hydrazine
- (B) glucose
- (C) glycine
- (D)  $\text{KHSO}_4$

19) What is the correct order of acidity of the protons marked A-D in the given compounds ?



- (A)  $H_C > H_D > H_B > H_A$   
 (B)  $H_C > H_D > H_A > H_B$   
 (C)  $H_D > H_C > H_B > H_A$   
 (D)  $H_C > H_A > H_D > H_B$

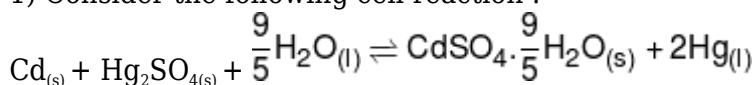
20) The structure of product C, formed by the following sequence of reactions is :



- (A)
- (B)
- (C)
- (D)

## SECTION-II

1) Consider the following cell reaction :



The value of  $E_{\text{cell}}^0$  is 4.315 V at 25°C. If  $\Delta H^\circ = -825.2 \text{ kJ mol}^{-1}$ , the standard entropy change  $\Delta S^\circ$  in  $\text{J K}^{-1}$  is \_\_\_\_\_. (Nearest integer) [Given : Faraday constant =  $96487 \text{ C mol}^{-1}$ ]

2) The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ( $\text{H}_2\text{C}_2\text{O}_4 \cdot 2\text{H}_2\text{O}$ ) in 250 mL of water in  $\text{mol L}^{-1}$  is  $x \times 10^{-2}$ . The value of x is \_\_\_\_\_. (Nearest integer) [Atomic mass : H :

1.0, C : 12.0, O : 16.0]

3) Consider the sulphides HgS, PbS, CuS, Sb<sub>2</sub>S<sub>3</sub>, As<sub>2</sub>S<sub>3</sub> and CdS. Number of these sulphides soluble in 50% HNO<sub>3</sub> is \_\_\_\_\_.

4) The total number of reagents from those given below, that can convert nitrobenzene into aniline is \_\_\_\_\_. (Integer answer)

I. Sn - HCl

II. Sn - NH<sub>4</sub>OH

III. Fe - HCl

IV. Zn - HCl

V. H<sub>2</sub> - Pd

VI. H<sub>2</sub> - Raney Nickel

5) The number of halogen/(s) forming halic (V) acid is \_\_\_\_\_.

## PART-3 : MATHEMATICS

### SECTION-I

1) Let  $f_1(x)$  and  $f_2(x)$  be even and odd function respectively, where  $x^2 f_1(x) - 2f_1\left(\frac{1}{x}\right) = f_2(x)$ , then value of  $f_1(3) =$

(A) 0

(B) 3

(C) 5

(D) 7

2) The number of real roots of the equation  $e^{4x} + 2e^{3x} - e^x - 6 = 0$  is :

(A) 2

(B) 4

(C) 1

(D) 0

3) The sum of 10 terms of the series  $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots$  is :

(A) 1

(B)  $\frac{120}{121}$

(C)  $\frac{99}{100}$

(D)  $\frac{143}{144}$

4) Let the equation of the plane, that passes through the point (1, 4, -3) and contains the line of intersection of the planes  $3x - 2y + 4z - 7 = 0$  and  $x + 5y - 2z + 9 = 0$ , be  $\alpha x + \beta y + \gamma z + 3 = 0$ , then  $\alpha + \beta + \gamma$  is equal to :

- (A) -23
- (B) -15
- (C) 23
- (D) 15

5) Let  $f$  be a non-negative function in  $[0, 1]$  and twice differentiable in  $(0, 1)$ . If  $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$ ,  $0 \leq x \leq 1$  and  $f(0) = 0$ , then  $\lim_{x \rightarrow 0} \frac{1}{x^2} \int_0^x f(t) dt$  :

- (A) equals 0
- (B) equals 1
- (C) does not exist
- (D)  $\frac{1}{2}$

6) Let  $\vec{a}$  and  $\vec{b}$  be two vectors such that  $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$  and the angle between  $\vec{a}$  and  $\vec{b}$  is  $60^\circ$ . If  $\frac{1}{8}\vec{a}$  is a unit vector, then  $|\vec{b}|$  is equal to :

- (A) 4
- (B) 6
- (C) 5
- (D) 8

7) The function  $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 - 12x + 4|}$  is not differentiable at exactly :

- (A) four points
- (B) three points
- (C) two points
- (D) one point

8) Three numbers are in an increasing geometric progression with common ratio  $r$ . If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference  $d$ . If the fourth term of GP is  $3r^2$ , then  $r^2 - d$  is equal to :

- (A)  $7 - 7\sqrt{3}$
- (B)  $7 + \sqrt{3}$

(C)  $7 - \sqrt{3}$

(D)  $\frac{7 + 3}{\sqrt{3}}$

9) Which of the following is **not** correct for relation R on the set of real numbers ?

(A)  $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$  is neither transitive nor symmetric.

(B)  $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$  is symmetric and transitive.

(C)  $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$  is reflexive but not symmetric.

(D)  $(x, y) \in R \Leftrightarrow |x - y| \leq 1$  is reflexive and symmetric.

10) The integral  $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}} dx$  is equal to :  
(where C is a constant of integration)

(A)  $\frac{3}{4} \left( \frac{x+2}{x-1} \right)^{\frac{1}{4}} + C$

(B)  $\frac{3}{4} \left( \frac{x+2}{x-1} \right)^{\frac{5}{4}} + C$

(C)  $\frac{4}{3} \left( \frac{x-1}{x+2} \right)^{\frac{1}{4}} + C$

(D)  $\frac{4}{3} \left( \frac{x-1}{x+2} \right)^{\frac{5}{4}} + C$

11) If p and q are the lengths of the perpendiculars from the origin on the lines,  
 $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$  and  $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$   
respectively, then  $k^2$  is equal to :

(A)  $4p^2 + q^2$

(B)  $2p^2 + q^2$

(C)  $p^2 + 2q^2$

(D)  $p^2 + 4q^2$

12)  $\operatorname{cosec} 18^\circ$  is a root of the equation :

(A)  $x^2 + 2x - 4 = 0$

(B)  $4x^2 + 2x - 1 = 0$

(C)  $x^2 - 2x + 4 = 0$

(D)  $x^2 - 2x - 4 = 0$

13) If the following system of linear equations

$$2x + y + z = 5$$

$$x - y + z = 3$$

$x + y + az = b$   
has no solution, then :

(A)  $\frac{a}{7} = -\frac{1}{3}, b \neq$

(B)  $\frac{a}{7} \neq \frac{1}{3}, b =$

(C)  $\frac{a}{7} \neq -\frac{1}{3}, b =$

(D)  $\frac{a}{7} = \frac{1}{3}, b \neq$

14) The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S (>R) respectively from the origin, is :

(A)  $4(S + R)$

(B)  $2(S - R)$

(C)  $4(S - R)$

(D)  $2(S + R)$

$$f(x) = \begin{cases} \frac{1}{x} \log_e \left( \frac{1+\frac{x}{a}}{1-\frac{x}{b}} \right) & , x < 0 \\ k & , x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1} & , x > 0 \end{cases}$$

15) If the function  
to :

is continuous at  $x = 0$ , then  $\frac{1}{a} + \frac{1}{b} + \frac{4}{k}$  is equal

(A) -5

(B) 5

(C) -4

(D) 4

16) If  $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$ ,  $y(0) = 1$ , then  $y(1)$  is equal to :

(A)  $\log_2(2 + e)$

(B)  $\log_2(1 + e)$

(C)  $\log_2(2e)$

(D)  $\log_2(1 + e^2)$



17)  $\lim_{x \rightarrow 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$  is equal to :

- (A)  $\pi^2$
- (B)  $2\pi^2$
- (C)  $4\pi^2$
- (D)  $4\pi$

18) A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

- (A)  $12\sqrt{15}$
- (B)  $12\sqrt{10}$
- (C)  $8\sqrt{10}$
- (D)  $6\sqrt{10}$

19) If  $a_r = \cos \frac{2r\pi}{9} + i \sin \frac{2r\pi}{9}$ ,  $r = 1, 2, 3, \dots, i = \sqrt{-1}$ , then the determinant  $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$  is equal to :

- (A)  $a_2 a_6 - a_4 a_8$
- (B)  $a_9$
- (C)  $a_1 a_9 - a_3 a_7$
- (D)  $a_5$

20) The line  $12x \cos \theta + 5y \sin \theta = 60$  is tangent to which of the following curves?

- (A)  $x^2 + y^2 = 169$
- (B)  $144x^2 + 25y^2 = 3600$
- (C)  $25x^2 + 12y^2 = 3600$
- (D)  $x^2 + y^2 = 60$

## SECTION-II

1) Let  $[t]$  denote the greatest integer  $\leq t$ . Then the value of  $8 \cdot \int_{-\frac{1}{2}}^1 ([2x] + |x|) dx$  is \_\_\_\_\_.

2) A point  $z$  moves in the complex plane such that  $\arg \left( \frac{z-2}{z+2} \right) = \frac{\pi}{4}$ , then the minimum value of

$|z - 9\sqrt{2} - 2i|^2$  is equal to \_\_\_\_\_.

3) The square of the distance of the point of intersection of the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$  and the plane  $2x - y + z = 6$  from the point  $(-1, -1, 2)$  is \_\_\_\_\_.

4) If 'R' is the least value of 'a' such that the function  $f(x) = x^2 + ax + 1$  is increasing on  $[1, 2]$  and 'S' is the greatest value of 'a' such that the function  $f(x) = x^2 + ax + 1$  is decreasing on  $[1, 2]$ , then the value of  $|R - S|$  is \_\_\_\_\_.

5) The mean of 10 numbers  $7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \dots$  is \_\_\_\_\_.

## ANSWER KEYS

### PART-1 : PHYSICS

#### SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	A	C	A	A	C	B	A	C	D	B	A	A	A	B	C	A	C

#### SECTION-II

Q.	21	22	23	24	25
A.	6	1	500	100	30

### PART-2 : CHEMISTRY

#### SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	A	B	C	B	A	C	B	C	B	C	C	C	A	B	C	A	D	D	B	A

#### SECTION-II

Q.	46	47	48	49	50
A.	25	20	4	5	3

### PART-3 : MATHEMATICS

#### SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	A	C	B	A	D	C	C	B	B	C	A	D	D	C	A	B	C	B	C	B

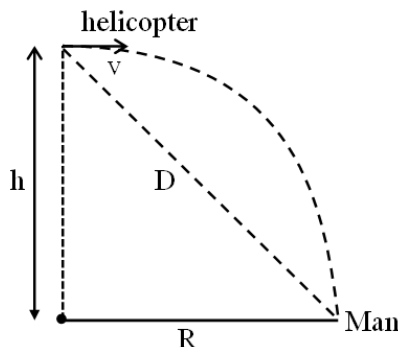
#### SECTION-II

Q.	71	72	73	74	75
A.	5	98	61	2	398

# SOLUTIONS

## PART-1 : PHYSICS

1)



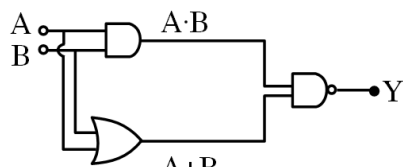
$$R = \sqrt{\frac{2h}{g}} \cdot v$$

$$D = \sqrt{R^2 + h^2}$$

$$= \sqrt{\left(\sqrt{\frac{2h}{g}} \cdot v\right)^2 + h^2}$$

$$D = \sqrt{\frac{2hv^2}{g} + h^2}$$

2)



$$Y = (A \cdot B) \cdot (A + B)$$

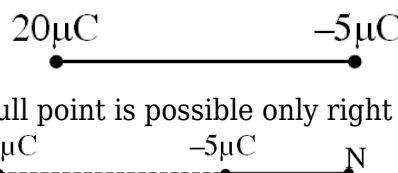
$$Y_{(0,0)} = 1$$

$$Y_{(0,1)} = 1$$

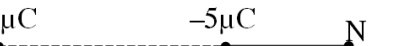
$$Y_{(1,0)} = 1$$

$$Y_{(1,1)} = 0$$

3)



Null point is possible only right side of  $-5\mu\text{C}$

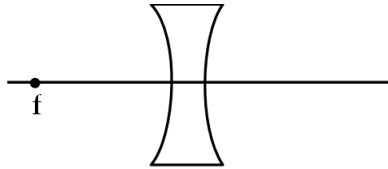


$$E_N = +\frac{k(-5\mu\text{C})}{x^2} + \frac{k(20\mu\text{C})}{(5+x)^2} = 0$$

$$x = 5 \text{ cm}$$

4) Basic theory

Translational  $K.E$  on average of a molecule is  $\frac{3}{2} KT$  which is independent of nature, pressure and volume.



5)

$$U = -f$$

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = -\frac{2}{f}$$

$$V = \frac{-f}{2}$$

$$m = \frac{U}{V} = \frac{1}{2} f$$

$$\text{distance} = \frac{f}{2}$$

Option (3)

$$6) \text{ B.E.} = [\Delta m].c^2$$

$$M_{\text{expected}} = ZM_p + (A - Z)M_n$$

$$= 50 [1.00783] + 70 [1.00867]$$

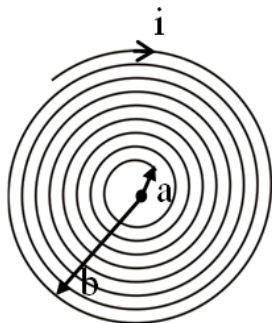
$$M_{\text{actual}} = 119.902199$$

$$\text{B.E.} = [50 [1.00783] + 70 [1.00867] - 119.902199]$$

$$\times 931$$

$$= 1020.56$$

$$\frac{\text{BE}}{\text{nucleon}} = \frac{1020.56}{120} = 8.5 \text{ MeV}$$



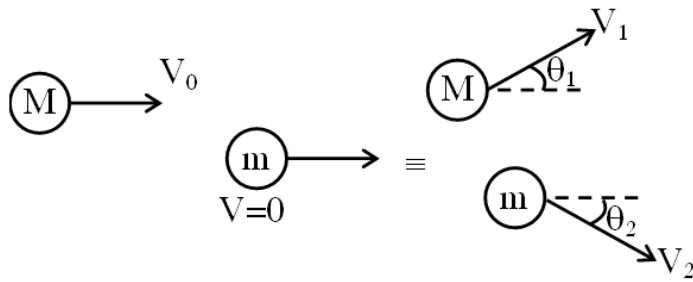
7)

$$\text{No. of turns in } dx \text{ width} = \frac{N}{b-a} dx$$

$$\int dB = \int_a^b \left( \frac{N}{b-a} \right) dx \frac{\mu_0 i}{2x}$$

$$B = \frac{N \mu_0 i}{2(b-a)} \ln \left( \frac{b}{a} \right)$$

Option (1)



8)

given  $\theta_1 = \theta_2 = \theta$

from momentum conservation

in x-direction  $MV_0 = MV_1 \cos \theta + mV_2 \cos \theta$

in y-direction  $0 = MV_1 \sin \theta - mV_2 \sin \theta$

Solving above equations

$$V_2 = \frac{MV_1}{m}, V_0 = 2V_1 \cos \theta$$

From energy conservation

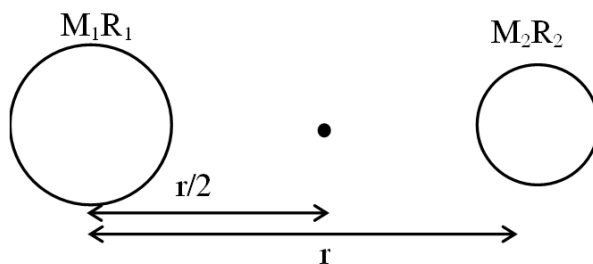
$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}mV_2^2$$

Substituting value of  $V_2$  &  $V_0$ , we will get

$$\frac{M}{m} + 1 = 4\cos^2 \theta \leq 4$$

$$\frac{M}{m} \leq 3$$

Option (3)

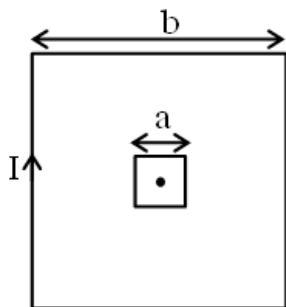


9)

$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$V = \sqrt{\frac{4G(M_1 + M_2)}{r}}$$

Option (2)



10)

$$B = \left[ \frac{\mu_0 I}{4\pi b/2} \times 2 \sin 45 \right] \times 4$$

$$\phi = 2\sqrt{2}\frac{\mu_0 I}{\pi b} \times a^2$$

$$\therefore M = \frac{\phi}{I} = \frac{2\sqrt{2}\mu_0 a^2}{\pi b} = \frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$$

Option (1)

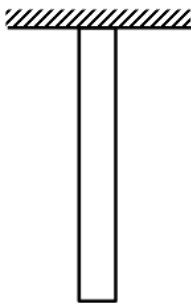
11)

When  $V_i > 3$  volt,  $V_R > 0$

Because diode will be in forward biased state

When  $V_i \leq 3$  volt,  $V_R = 0$

Because diode will be in reverse biased state



12)

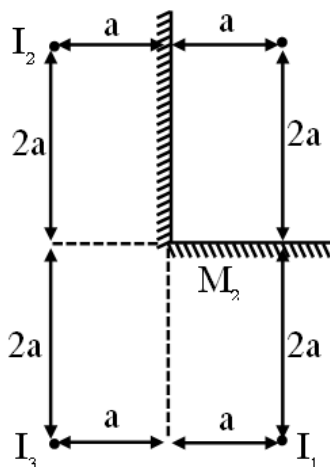
We know,

$$\Delta \ell = \frac{WL}{2AY}$$

$$\Delta \ell = \frac{10 \times 1}{2 \times 5} \times 100 \times 10^{-4} \times 2 \times 10^{11}$$

$$\Delta \ell = \frac{1}{2} \times 10^{-9} = 5 \times 10^{-10} \text{ m}$$

Option (4)



13)

Shortest distance is  $2a$  between  $I_1$  &  $I_3$

But answer given is for  $I_1$  &  $I_2$

$$\sqrt{(4a)^2 + (2a)^2}$$

$$a\sqrt{20}$$

$$4.47 a$$

Option (2)

14)

torque  $\tau \rightarrow ML^2T^{-2}$  (III)

Impulse  $I \Rightarrow MLT^{-1}$  (I)

Tension force  $\Rightarrow MLT^{-2}$  (IV)

Surface tension  $\Rightarrow MT^{-2}$  (II)

$$\int_{p_0}^p \frac{dp}{p} = -a \int_0^v dv$$

$$\ln \left( \frac{p}{p_0} \right) = -av$$

$$p = p_0 e^{-av}$$

For temperature maximum p-v product should be maximum

$$T = \frac{pv}{nR} = \frac{p_0 v e^{-av}}{R}$$

$$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R} \{ e^{-av} + v e^{-av} (-a) \}$$

$$\frac{p_0 e^{-av}}{R} \{ 1 - av \} = 0$$

$$v = \frac{1}{a}, \infty$$

$$T = \frac{p_0 \cdot 1}{Rae} = \frac{p_0}{Rae}$$

at  $v = \infty$

$$T = 0$$

Option (1)

16)

$$(i) \frac{\pi p a^4}{8\eta L} = \frac{dv}{dt} = \text{Volumetric flow rate (poiseuille's law)}$$

$$(ii) h\rho g = \frac{2s}{r} \cos \theta$$

$$(iii) \text{RHS} = \frac{\epsilon \times \frac{1}{4\pi\epsilon_0} \frac{a}{r^2} \times \frac{1}{\epsilon} = \frac{q}{t} \times \frac{1}{r^2} \\ = \frac{I}{L^2} = IL^{-2}$$

LHS

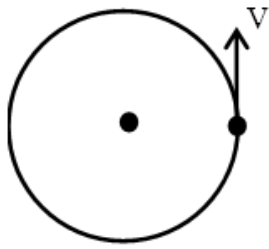
$$T = \frac{I}{A} = IL^{-2}$$

$$(iv) W = \tau\theta$$

Option (1)

17)





$$|\vec{L}| = mvr$$

And direction will be upward & remain constant

Option (2)

$$18) Z = \sqrt{(X_L - X_C)^2 + R^2} = R \quad \because X_L = X_C$$

Option (3)

$$19) \lambda_P = \frac{h}{P_P} \quad \lambda_e = \frac{h}{P_e}$$

$$\because \lambda_P = \lambda_e$$

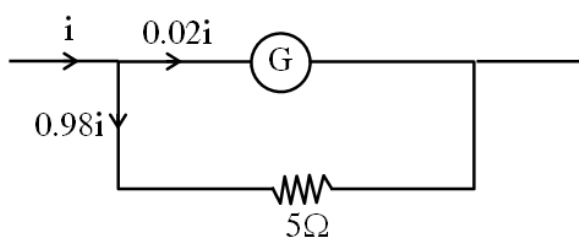
$$\Rightarrow P_P = P_e$$

$$(K)_P = \frac{P_P^2}{2m_P}$$

$$(K)_e = \frac{P_e^2}{2m_e}$$

$$K_P < K_e \text{ as } m_P > m_e$$

Option (1)

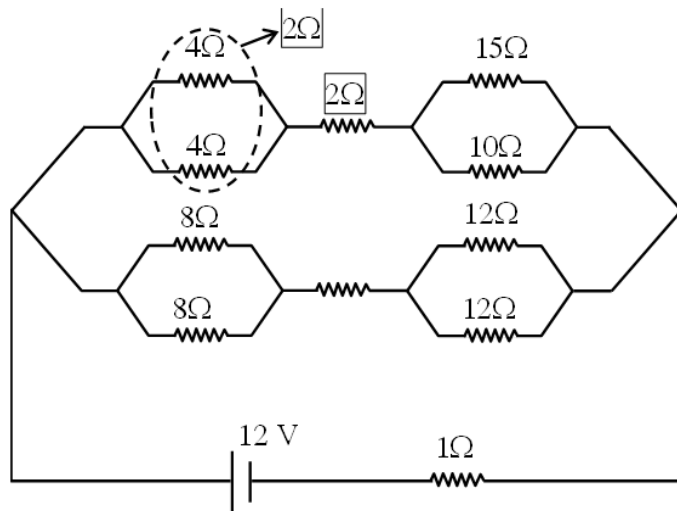


20)

$$0.02i R_g = 0.98i \times 5$$

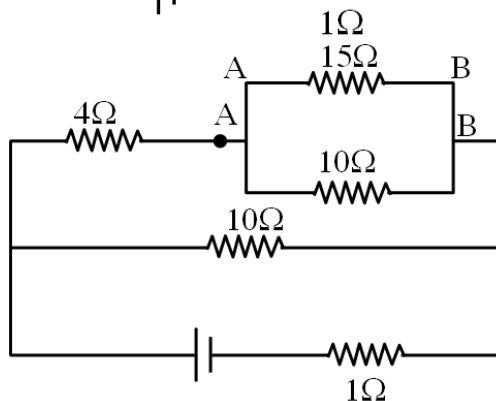
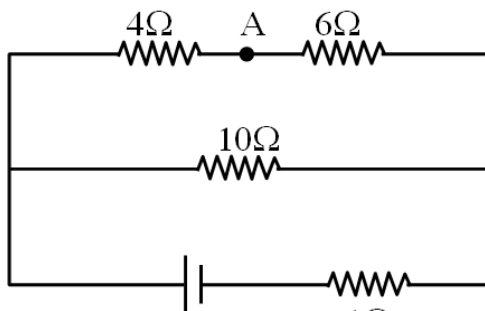
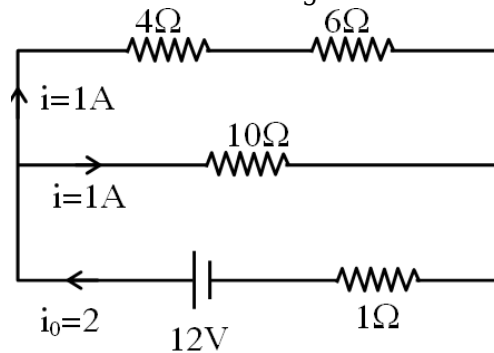
$$R_g = 245 \Omega$$

Option (3)



21)

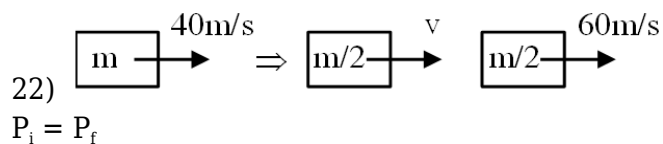
⇒ effective circuit diagram will be



⇒

Point drop across  $6\Omega = 1 \times 6 = 6 = V_{AB}$

⇒ Hence point drop across  $15\Omega = 6 \text{ volt} = V_{AB}$



$$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60$$

$$40 = \frac{v}{2} + 30$$

$$\Rightarrow v = 20$$

$$(K.E.)_i = \frac{1}{2} m \times (40)^2 = 800m$$

$$(K.E.)_f = \frac{1}{2} \frac{m}{2} \cdot (20)^2 + \frac{1}{2} \cdot \frac{m}{2} (60)^2 = 1000m$$

$$|\Delta K.E.| = |1000m - 800m| = 200m$$

$$\frac{\Delta K.E.}{(K.E.)_i} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4}$$

$$x = 1$$

$$23) E = 50 \sin \left( \omega t - \frac{\omega}{c} \cdot x \right)$$

$$\text{Energy density} = \frac{1}{2} \epsilon_0 E_0^2$$

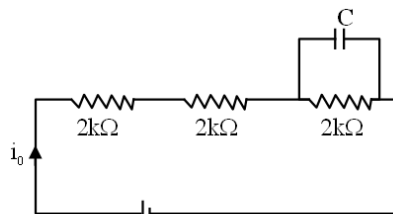
$$\text{Energy for volume } V = \frac{1}{2} \epsilon_0 E_0^2 \cdot V = 5.5 \times 10^{-12}$$

$$\frac{1}{2} 8.8 \times 10^{-12} \times 2500V = 5.5 \times 10^{-12}$$

$$V = \frac{5.5 \times 2}{2500 \times 8.8} = .0005m^3$$

$$= .0005 \times 10^6 (c.m)^3$$

$$= 500 (c.m)^3$$

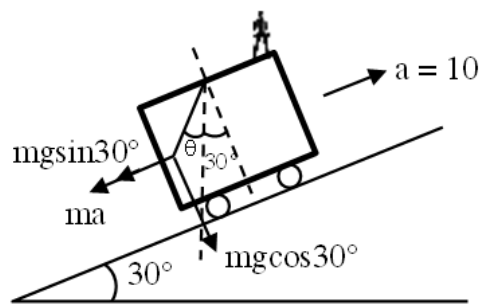


$$24) i_0 = 1 \text{ mA} \quad 6V$$

Pot. Diff. across each resistor = 2V

$$q = CV$$

$$= 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} = 100 \mu C$$



$$25) \tan(30 + \theta) = \frac{mg \sin 30^\circ + ma}{mg \cos 30^\circ}$$

$$\tan(30 + \theta) = \frac{5 + 10}{5\sqrt{3}} = \frac{1 + 2}{\sqrt{3}}$$

$$\frac{\tan \theta + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}} \tan \theta} = \sqrt{3}$$

$$\sqrt{3} \tan \theta + 1 = 3 - \sqrt{3} \tan \theta$$

$$2\sqrt{3} \tan \theta = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

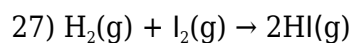
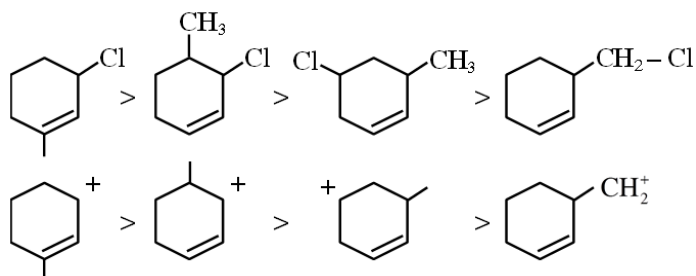
$$\theta = 30^\circ$$

## PART-2 : CHEMISTRY

26)

As it is example of  $\text{SN}^1$ .

so carbocation stability  $\uparrow$ , reaction rate  $\uparrow$



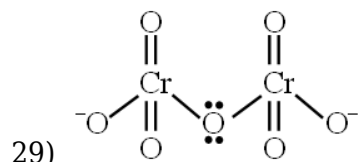
Apply Arrhenius equation

$$\log \frac{K_2}{K_1} = \frac{E_a}{2.303R} \left( \frac{1}{600} - \frac{1}{800} \right)$$

$$\log \frac{1}{2.5 \times 10^{-4}} = \frac{E_a}{2.303 \times 8.31} \left( \frac{200}{600 \times 800} \right)$$

$$\square E_a \approx 166 \text{ kJ/mol}$$

28) Sodium have lowest oxidation potential in alkali metals. Hence it is weakest reducing agent among alkali metals.

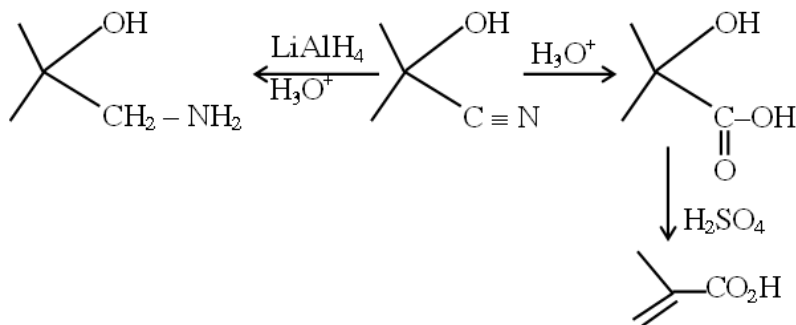


dichromate ion contain non-linear symmetrical Cr-O-Cr Bond

30)

In Lactose it is  $\beta$   $\text{C}_1 - \text{C}_4$  glycosidic linkage.

In Maltose, Amylose  $\alpha$   $\text{C}_1 - \text{C}_4$  glycosidic linkage is present.

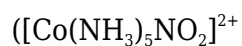


31)

32)

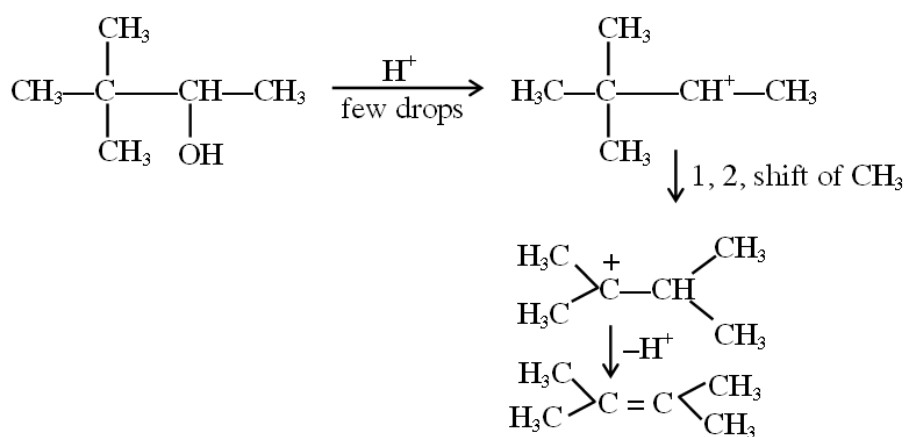
Ytterbium shows +2 oxidation state with diamagnetic nature. So ans is 2

33)



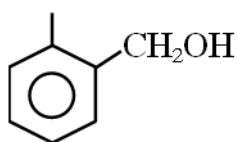
Two linkage isomers possible

$\text{NO}_2 \rightarrow$  Ambidentate ligand

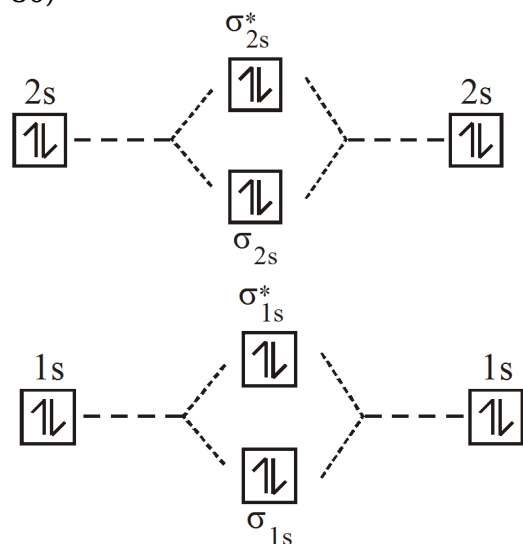
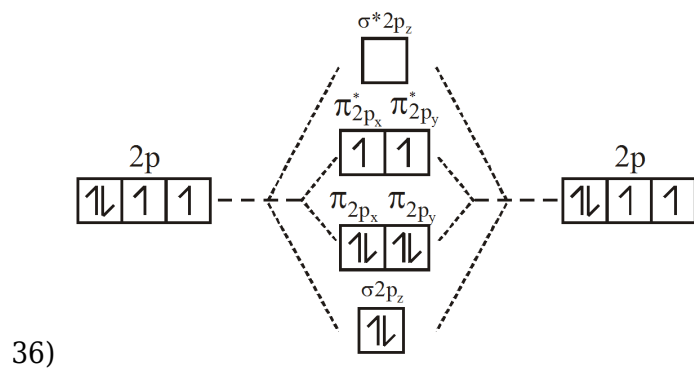


34)

35) Monomer of Novolac is

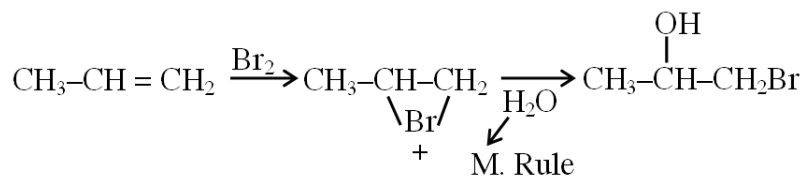


O-hydroxy methyl phenol



An incoming electron will go in  $\pi^*_{2p_x}$  orbital.

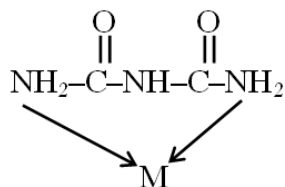
38)



Its IUPAC name 1-bromopropan-2-ol

A and R are true and (R) is the correct explanation of (A)

38)



Biuret :- Bidentate ligand

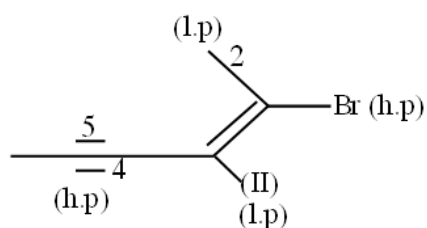
The denticity of organic ligand is 2.

39)

From left to right in periodic table :-  
Metallic character decreases

Non-metallic character increases

⇒ It is due to increase in ionization enthalpy and increase in electron gain enthalpy.



h.p. ⇒ higher priority

l.p. ⇒ lower priority

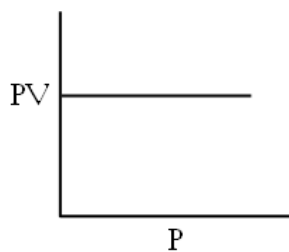
40)

2E -2- bromo hex -2- en-4-yne

41)

$PV = nRT$  (n,T constant)

$PV = \text{constant}$



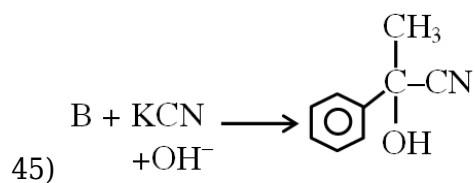
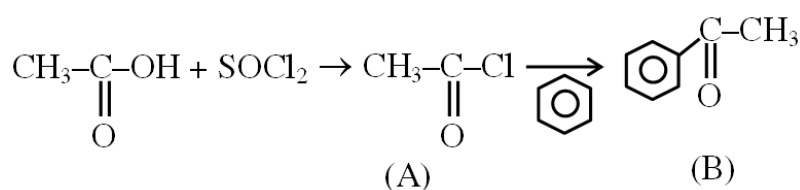
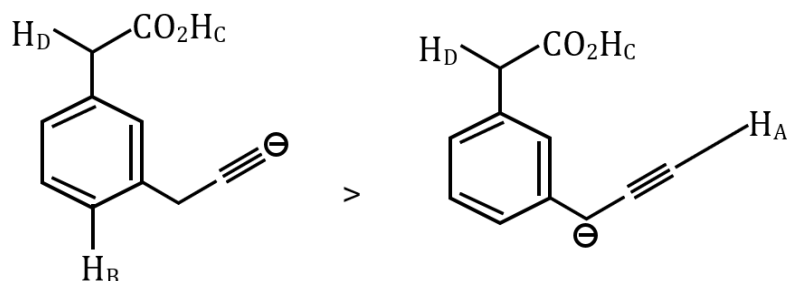
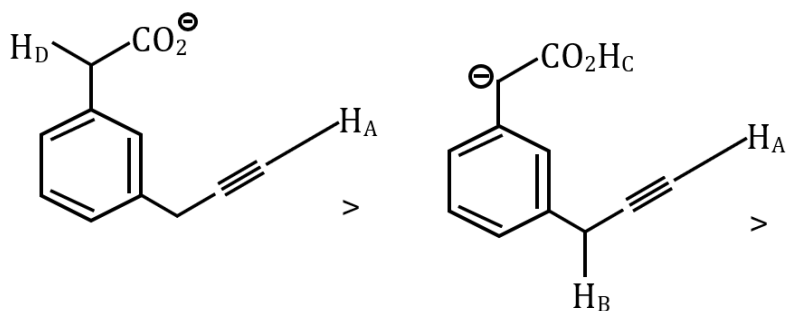
42) Both assertion & reason are correct & (R) is the correct explanation of (A)

43)

□ Van't Hoff factor is highest for  $\text{KHSO}_4$

□ colligative property ( $\Delta T_f$ ) will be highest for  $\text{KHSO}_4$

44) acidity of an acid depends upon the stability of its conjugate base



46)  $\Delta G^\circ = -nFE^\circ = \Delta H^\circ - T\Delta S^\circ$   
 $= \frac{\Delta H^\circ + nFE^\circ}{T}$   
 $= \frac{(-825.2 \times 10^3) + (2 \times 96487 \times 4.315)}{298}$   
 $= \frac{-825.2 \times 10^3 + 832.682 \times 10^3}{298}$   
 $= \frac{7.483 \times 10^3}{298} = 25.11 \text{ JK}^{-1}\text{mol}^{-1}$

□ Nearest integer answer is 25

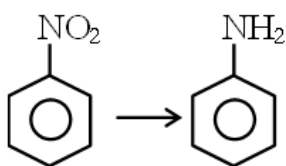
47)  $[\text{H}_2\text{C}_2\text{O}_4 \cdot 2\text{H}_2\text{O}] = \frac{\text{weight } M_\text{w}}{V(\text{L})}$   
 $= \frac{6.3/126}{250/1000}$   
 $\Rightarrow x \times 10^{-2} = 250/1000$   
 $x = 20$

48)



Pbs, CuS, As<sub>2</sub>S<sub>3</sub>, CdS are soluble in 50% HNO<sub>3</sub> HgS, Sb<sub>2</sub>S<sub>3</sub> are insoluble in 50% HNO<sub>3</sub>  
So Answer is 4.

49)



Reagents used can be

- (i) Sn + HCl
- (ii) Fe + HCl
- (iii) Zn + HCl
- (iv) H<sub>2</sub> - Pd
- (v) H<sub>2</sub> (Raney Ni)

50)

The number of halogen forming halic (V) acid



So Answer is 3

### PART-3 : MATHEMATICS

$$x^2 \cdot f_1(x) - 2f_1\left(\frac{1}{x}\right) = f_2(x) \quad \dots(1)$$

replace  $x \rightarrow -x$  and then add with (1)

$$\Rightarrow 2x^2 f_1(x) - 4f_1\left(\frac{1}{x}\right) = 0$$

$$(\because f_1(-x) = f_1(x) \text{ \& } f_2(-x) = -f_2(x))$$

$$\therefore x^2 f_1(x) - 2f_1\left(\frac{1}{x}\right) = 0 \quad \dots(2)$$

Now replace  $x \rightarrow \frac{1}{x}$  into (2)

$$\Rightarrow \frac{1}{x^2} f_1\left(\frac{1}{x}\right) - 2f_1(x) = 0 \quad \dots(3)$$

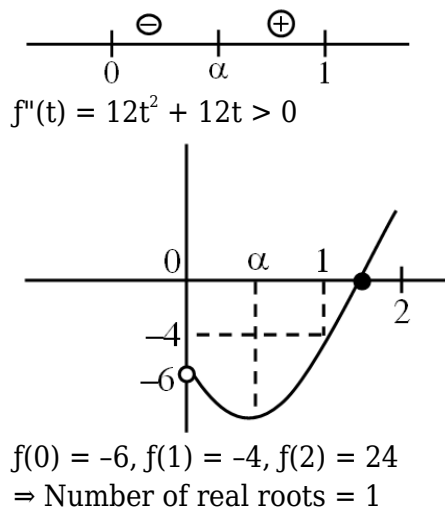
51) solve (2) and (3) to get  $f_1(x) = 0 \Rightarrow f_1(3) = 0$

52)

Let  $e^x = t > 0$

$$f(t) = t^4 + 2t^3 - t - 6 = 0$$

$$f'(t) = 4t^3 + 6t^2 - 1$$



53)

$$S = \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \dots$$

$$= \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] + \left[ \frac{1}{2^2} - \frac{1}{3^2} \right] + \left[ \frac{1}{3^2} - \frac{1}{4^2} \right] + \dots + \left[ \frac{1}{10^2} - \frac{1}{11^2} \right]$$

$$= 1 - \frac{1}{11^2}$$

$$= \frac{120}{121}$$

54)

Equation of plane is

$$3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$$

$$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$$

passing through (1, 4, -3)

$$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$$

$$\Rightarrow \lambda = \frac{2}{3}$$

$\Rightarrow$  equation of plane is

$$-11x - 4y - 8z + 3 = 0$$

$$\Rightarrow \alpha + \beta + \gamma = -23$$

$$55) \int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt \quad 0 \leq x \leq 1$$

differentiating both the sides

$$\sqrt{1 - (f'(x))^2} = f(x)$$

$$\Rightarrow 1 - (f'(x))^2 = f^2(x)$$

$$\frac{f'(x)}{\sqrt{1 - f^2(x)}} = 1$$

$$\sin^{-1} f(x) = x + C$$

$$\square f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$$

Now  $\lim_{x \rightarrow 0} \frac{\int_0^x \sin t \, dt}{x^2} \left( \frac{0}{0} \right) = \frac{1}{2}$

56)  $|3\vec{a} + \vec{b}|^2 = |2\vec{a} + 3\vec{b}|^2$   
 $(3\vec{a} + \vec{b}) \cdot (3\vec{a} + \vec{b}) = (2\vec{a} + 3\vec{b}) \cdot (2\vec{a} + 3\vec{b})$   
 $9\vec{a} \cdot \vec{a} + 6\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 4\vec{a} \cdot \vec{a} + 12\vec{a} \cdot \vec{b} + 9\vec{b} \cdot \vec{b}$   
 $5|\vec{a}|^2 - 6\vec{a} \cdot \vec{b} = 8|\vec{b}|^2$

$5(8)^2 - 6 \cdot 8 \cdot |\vec{b}| \cos 60^\circ = 8|\vec{b}|^2 \left( \begin{array}{l} \because \frac{1}{8} |\vec{a}| = 1 \\ \Rightarrow |\vec{a}| = 8 \end{array} \right)$

$40 - 3|\vec{b}| = |\vec{b}|^2$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$|\vec{b}| = -8, |\vec{b}| = 5$

(rejected)

57)  $f(x) = |(x-3)(x+1)| \cdot e^{(3x-2)^2}$   

$$f(x) = \begin{cases} (x-3)(x+1) \cdot e^{(3x-2)^2} & ; x \in (3, \infty) \\ -(x-3)(x+1) \cdot e^{(3x-2)^2} & ; x \in [-1, 3] \\ (x-3) \cdot (x+1) \cdot e^{(3x-2)^2} & ; x \in (-\infty, -1) \end{cases}$$

Clearly, non-differentiable at  $x = -1$  &  $x = 3$

58)

Let numbers be  $\frac{a}{r}, a, ar \rightarrow$  G.P

$\frac{a}{r}, 2a, ar \rightarrow$  A.P  $\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

$r = 2 \pm \sqrt{3}$

$4^{\text{th}}$  form of G.P  $= 3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}, a = 3, d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3}$

$= 7 + \sqrt{3}$

59)

Note that (1,2) and (2,3) satisfy  $0 < |x - y| \leq 1$

but (1,3) does not satisfy it so

$0 \leq |x - y| \leq 1$  is symmetric but not transitive

So, (2) is correct.

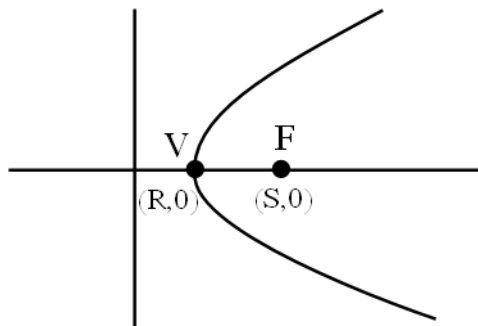
$$\begin{aligned}
 60) & \int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}} \\
 &= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4} \cdot (x-1)^2} \\
 & \text{put } \frac{x+2}{x-1} = t \\
 &= -\frac{1}{3} \int \frac{dt}{t^{5/4}} \\
 &= \frac{4}{3} \cdot \frac{1}{t^{1/4}} + C \\
 &= \frac{4}{3} \left(\frac{x-1}{x+2}\right)^{1/4} + C
 \end{aligned}$$

$$\begin{aligned}
 61) \text{ First line is } & \frac{x}{\sin \alpha} - \frac{y}{\cos \alpha} = \frac{k \cos 2\alpha}{\sin 2\alpha} \\
 & \frac{k}{2} \\
 \Rightarrow x \cos \alpha - y \sin \alpha &= \frac{k}{2} \cos 2\alpha \\
 \Rightarrow p = \left| \frac{k}{2} \cos \alpha \right| \\
 \Rightarrow 2p = |k \cos 2\alpha| \quad \dots(i) \\
 \text{second line is } & x \sin \alpha + y \cos \alpha = k \sin 2\alpha \\
 \Rightarrow q = |k \sin 2\alpha| \quad \dots(ii) \\
 \text{Hence } 4p^2 + q^2 &= k^2 \quad (\text{From (i) \& (ii)})
 \end{aligned}$$

$$\begin{aligned}
 62) \quad \operatorname{cosec} 18^\circ &= \frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5}-1} = \sqrt{5} + 1 \\
 \text{Let } \operatorname{cosec} 18^\circ = x &= \sqrt{5} + 1 \\
 \Rightarrow x - 1 &= \sqrt{5} \\
 \text{Squaring both sides, we get} \\
 x^2 - 2x + 1 &= 5 \\
 \Rightarrow x^2 - 2x - 4 &= 0
 \end{aligned}$$

$$\begin{aligned}
 63) \text{ Here } D &= \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + 1+1 \\
 &= 1-3a \\
 D_3 &= \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1(b-3) + 5(1+1) \\
 &= 7-3b
 \end{aligned}$$

for  $a = \frac{1}{3}, b \neq \frac{7}{3}$ , system has no solutions



64)

V → Vertex

F → focus

VF = S - R

So latus rectum = 4(S - R)

65) If  $f(x)$  is continuous at  $x = 0$ ,  $\text{RHL} = \text{LHL} = f(0)$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1} \cdot \frac{\sqrt{x^2 + 1} + 1}{\sqrt{x^2 + 1} + 1} \quad (\text{Rationalisation})$$

$$\lim_{x \rightarrow 0^+} -\frac{2\sin^2 x}{x^2} \cdot (\sqrt{x^2 + 1} + 1) = -4$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{1}{x} \ln \left( \frac{1 + \frac{x}{a}}{1 - \frac{x}{b}} \right)$$

$$\lim_{x \rightarrow 0^-} \frac{\ln \left( 1 + \frac{x}{a} \right)}{\left( \frac{x}{a} \right) \cdot a} + \frac{\ln \left( 1 - \frac{x}{b} \right)}{\left( -\frac{x}{b} \right) \cdot b}$$

$$= \frac{1}{a} + \frac{1}{b}$$

$$\text{So } \frac{1}{a} + \frac{1}{b} = -4 = k$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{4}{k} = -4 - 1 = -5$$

$$66) \frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y}$$

$$2^y \frac{dy}{dx} = 2^x (2^y - 1)$$

$$\int \frac{2^y}{2^y - 1} dy = \int 2^x dx$$

$$\frac{\ln(2^y - 1)}{\ln 2} = \frac{2^x}{\ln 2} + C$$

$$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$$

$$\square y(0) = 1 \Rightarrow 0 = \log_2 e + C$$

$$C = -\log_2 e$$

$$\Rightarrow \log_2(2^y - 1) = (2^x - 1) \log_2 e$$

$$\text{put } x = 1, \log_2(2^y - 1) = \log_2 e$$

$$2^y = e + 1$$

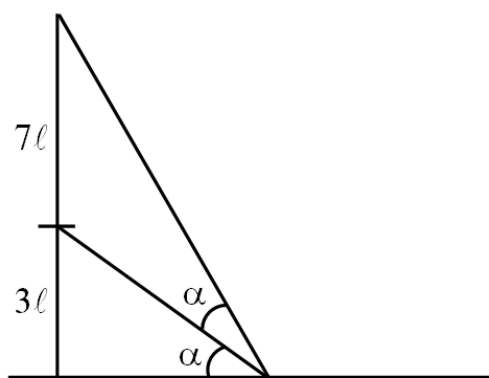
$$y = \log_2(e + 1)$$

$$67) \lim_{x \rightarrow 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2\pi \cos^4 x)}{2x^4}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2\pi - 2\pi \cos^4 x)}{[2\pi(1 - \cos^4 x)]^2} = 4\pi^2 \cdot \frac{\sin^4 x}{2x^4} (1 + \cos^2 x)^2$$

$$= \frac{1}{2} \cdot 4\pi^2 \cdot \frac{1}{2} (2)^2 = 4\pi^2$$



$$68) \quad 18$$

Let height of pole =  $10\sqrt{5}$

$$\tan \alpha = \frac{3l}{18} = \frac{l}{6}$$

$$\tan 2\alpha = \frac{10l}{18}$$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{10l}{18}$$

$$\text{use } \tan \alpha = \frac{l}{6} \Rightarrow \frac{l}{6} = \sqrt{\frac{72}{5}}$$

height of pole =  $10\sqrt{5} = 12\sqrt{10}$

$$69) a_r = e^{\frac{i2\pi r}{9}}, r = 1, 2, 3, \dots a_1, a_2, a_3, \dots \text{ are in G.P.}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_1^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = \begin{vmatrix} a_1 & a_1^4 & a_1^7 \end{vmatrix} \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

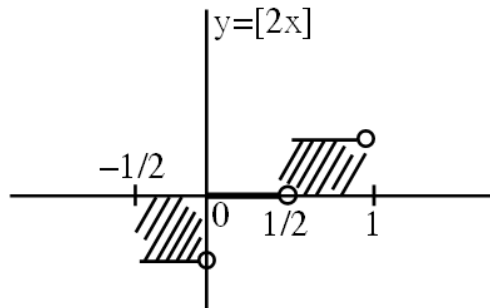
$$\text{Now } a_1 a_9 - a_3 a_7 = a_1^{10} - a_1^{10} = 0$$

$$70) 12x \cos \theta + 5y \sin \theta = 60$$

$$\frac{x \cos \theta}{5} + \frac{y \sin \theta}{12} = 1$$

is tangent to  $\frac{x^2}{25} + \frac{y^2}{144} = 1$   
 $144x^2 + 25y^2 = 3600$

71)  $I = \int_{-1/2}^1 ([2x] + |x|) dx$



$$\begin{aligned}
 &= \int_{-1/2}^1 [2x] dx + \int_{-1/2}^1 |x| dx \\
 &= 0 + \int_{-1/2}^0 (-x) dx + \int_0^1 x dx \\
 &= \left( -\frac{x^2}{2} \right)_{-1/2}^0 + \left( \frac{x^2}{2} \right)_0^1 \\
 &= \left( 0 + \frac{1}{8} \right) + \frac{1}{2} \\
 &= \frac{5}{8} \\
 8I &= 5
 \end{aligned}$$

72) Let  $z = x + iy$

$$\arg \left( \frac{x-2+iy}{x+2+iy} \right) = \frac{\pi}{4}$$

$$\arg(x-2+iy) - \arg(x+2+iy) = \frac{\pi}{4}$$

$$\tan^{-1} \left( \frac{y}{x-2} \right) - \tan^{-1} \left( \frac{y}{x+2} \right) = \frac{\pi}{4}$$

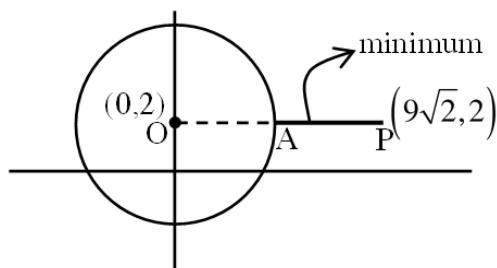
$$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \left( \frac{y}{x-2} \right) \cdot \left( \frac{y}{x+2} \right)} = \tan \frac{\pi}{4} = 1$$

$$\frac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$$

$$4y = x^2 - 4 + y^2$$

$$x^2 + y^2 - 4y - 4 = 0$$

locus is a circle with center  $(0, 2)$  & radius  $= 2\sqrt{2}$



$$\begin{aligned}\text{min. value} &= (AP)^2 = (OP - OA)^2 \\ &= (9\sqrt{2} - 2\sqrt{2})^2 \\ &= (7\sqrt{2})^2 = 98\end{aligned}$$

$$73) \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$$

$$x = 2\lambda + 1, y = 3\lambda + 2, z = 6\lambda - 1$$

for point of intersection of line & plane

$$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$$

$$7\lambda = 7 \Rightarrow \lambda = 1$$

point : (3, 5, 5)

$$\begin{aligned}(\text{distance})^2 &= (3 + 1)^2 + (5 + 1)^2 + (5 - 2)^2 \\ &= 16 + 36 + 9 = 61\end{aligned}$$

74)

$$f(x) = x^2 + ax + 1$$

$$f'(x) = 2x + a$$

when  $f(x)$  is increasing on  $[1, 2]$

$$2x + a \geq 0 \quad \forall x \in [1, 2]$$

$$a \geq -2x \quad \forall x \in [1, 2]$$

$$R = -4$$

when  $f(x)$  is decreasing on  $[1, 2]$

$$2x + a \leq 0 \quad \forall x \in [1, 2]$$

$$a \leq -2 \quad \forall x \in [1, 2]$$

$$S = -2$$

$$|R - S| = |-4 + 2| = 2$$

75)

$$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14 \dots\dots$$

$$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$$

$$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$$

$$\begin{aligned}S_{10} &= \sum_{n=1}^{10} T_n = 6 \sum_{n=1}^{10} n^2 + 26 \sum_{n=1}^{10} n + 24 \sum_{n=1}^{10} 1 \\ &= \frac{6(10 \times 11 \times 21)}{6} + 26 \times \frac{10 \times 11}{2} + 24 \times 10\end{aligned}$$

$$= 10 \times 11(21 + 13) + 240$$

$$= 3980$$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$