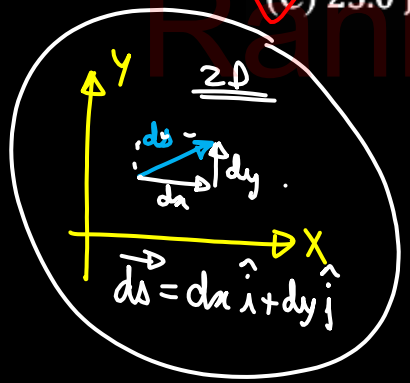
7

A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal $x-y$ plane. Assume distance in meters and force is newton. If the particle moves from point (1,2) to point (2,3) in the $x-y$ plane, then kinetic energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J




$$W = \int \vec{F} \cdot \vec{ds}$$

JEE 1

variable

$\vec{F} = 4x\hat{i} + 3y^2\hat{j}$

$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int (4x\,dx + 3y^2\,dy) + 0$$

$$W = 4\int_1^2 x\,dx + 3\int_2^3 y^2\,dy$$

$$W = 4\left.\frac{x^2}{2}\right|_1^2 + 3\left.y^3\right|_2^3$$

$$W = (8-2) + (27-8) = 25\,J$$

2

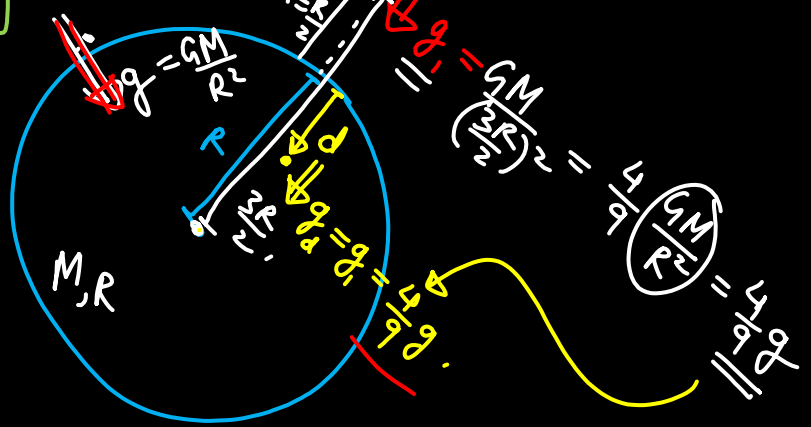
The value of the acceleration due to gravity is $g_1$ at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to $g_1$ at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals

(A) 4/9 (B) 5/9

(C) 1/3 (D) 7/9

$h \ll R$

$g_h = g\left(1 - \frac{2h}{R}\right)$




$g_d = g\left(1 - \frac{d}{R}\right) = g_1 = \frac{4g}{9}$

$g\left(1 - \frac{d}{R}\right) = \frac{4}{9}g$

$\frac{5}{9} = \frac{d}{R}$

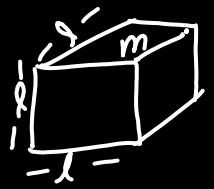


JEE 1

3

The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(A) 2.5%
(B) 3.5%
(C) 4.5%
(D) 6%




$$\rho = \frac{\text{mass}}{\text{vol}} = \frac{m}{l^3}$$

$$\rho = m^{1} l^{-3}$$

$$\left|\frac{\Delta \rho}{\rho}\right| \times 100 = 1\left|\frac{\Delta m}{m}\right| \times 100 + 3\left|\frac{\Delta l}{l}\right| \times 100$$

error

$$\left|\frac{\Delta m}{m}\right| \times 100 = 1.5\%$$

$$\left|\frac{\Delta l}{l}\right| \times 100 = 1\%$$

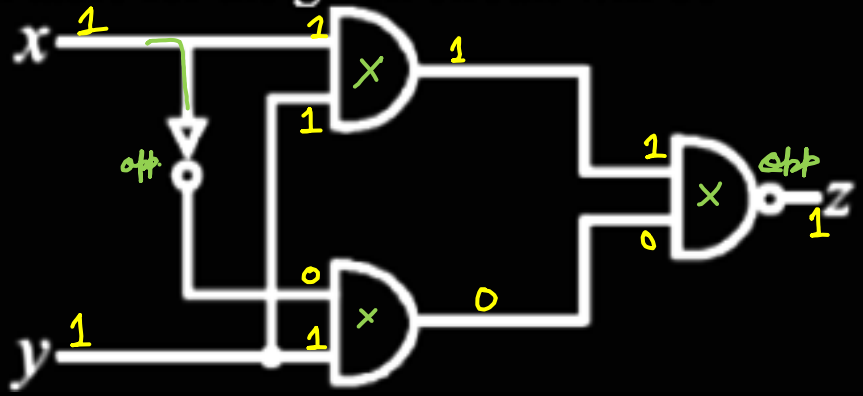
Ans $\left|\frac{\Delta \rho}{\rho}\right| \times 100 = ?$

$$\%\ \text{error}\ \rho = 1(1.5\%) + 3(1\%)$$

$$= 4.5\%$$

4

Truth table for the given circuit will be




Binary

AND.

$1 \times 1 = 1$

$1 \times 0 = 0$

$0 \times 0 = 0$

$0 \times 1 = 1$

JEE 1

OR

$0+0 = 0$

$0+1 = 1$

$1+0 = 1$

$1+1 = 1$

NOT

$\bar{0} = 1$

$\bar{1} = 0$

(A)

| x | y | z |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(B)

| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(C)

| x | y | z |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(D)

| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Ans

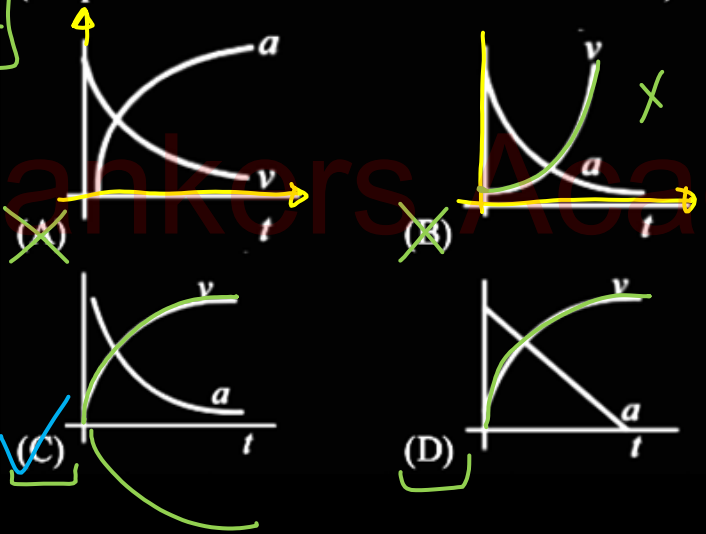
| x | y | z |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

Option C

5

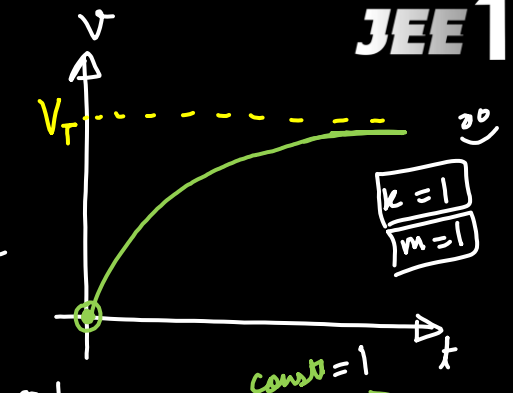
Which of the following options correctly describes the variation of the speed v and acceleration a of a point mass falling vertically in a viscous medium that applies a force $F_v = -kv$, where k is a constant, on the body? (Graphs are schematic and not drawn to scale)

~ $F = 6\pi\eta r v$










const = 1

$F_{net} = W - F_B - F_v$

$ma = 1 - (-v)$

$a = 1 - v$

5

$a = 1 - v$

$\frac{dv}{dt} = 1 - v$

$\int_0^v \frac{dv}{v-1} = -\int_{t=0}^{t} dt$

$\ln(v-1)\Big|_0^v = -t\Big|_0^t$

PHY

$\hookrightarrow \ln(v-1) - \ln(-1) = -t$

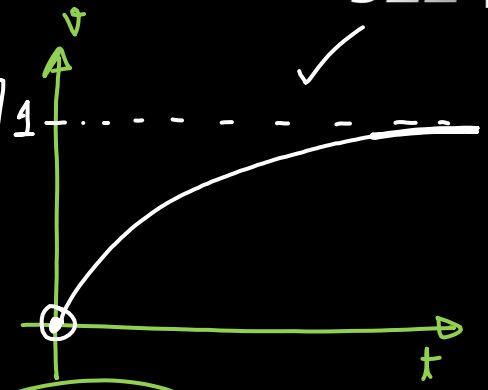
$\ln\left(\frac{v-1}{-1}\right) = -t$

$\ln(1-v) = -t$

$1 - v = e^{-t}$

$\boxed{v = 1 - e^{-t}}$

$1 - \left(\frac{1}{e}\right)^t$



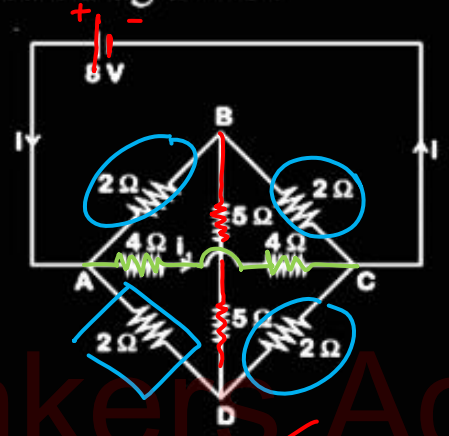


$a = 1 - (v)$

$a = 1 - (1 - e^{-t}) = e^{-t}$

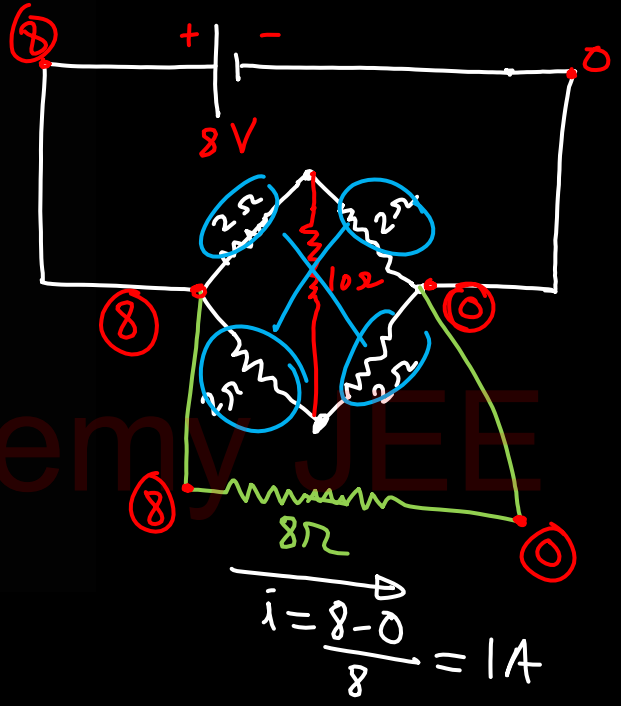
6

The value of current $i_1$ flowing from A to C in the circuit diagram is :




(A) 2 A
(B) 4 A
(C) 1 A
(D) 5 A

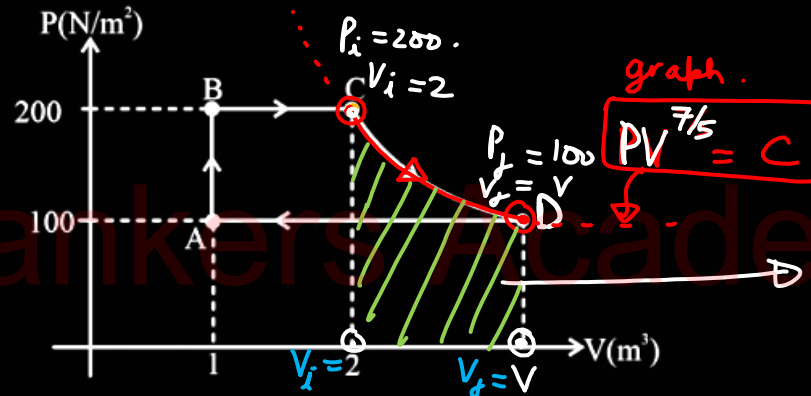
• BRIDGE



7

S.I.

The P – V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use $\gamma = 1.4$) [given $2^{5/7} = 1.64$]




(A) –360 J (B) 360 J

(C) –180 J (D) 180 J

$\gamma = 5$

$\gamma = \frac{f+2}{f} = \frac{7}{5} = 1.4$

$1 \to 2$.

$W_{adia} = \frac{nR\Delta T}{1-\gamma} = \frac{(P_2V_2 - P_1V_1)}{1-\gamma}$

graph.

$PV^{7/5} = C$

$W_{CD} = \frac{P_fV_f - P_iV_i}{1-\gamma}$

$W_{CD} = \frac{100V - 400}{-0.4} = \frac{400 - 100V}{0.4}$

$200(2)^{7/5} = 100V^{7/5} = C$

$2^{5/7} \times (2)^1 = V^1$

$V = 3.28$

$W_{CD} = \frac{72}{0.4} = \frac{72}{4} \times 10 = 180$

8

At a height $h = 2R$, the length of a second pendulum from earth surface is (Given R = Radius of earth and acceleration due to gravity at the surface of earth, $g = \pi^2 ms^{-2}$)

$g \approx \pi^2 \approx 10$

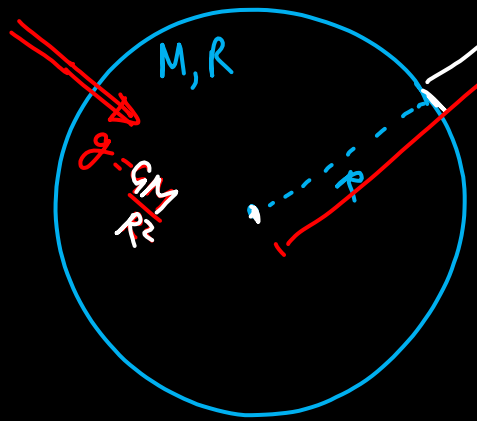
(A) $\frac{2}{9}$ m

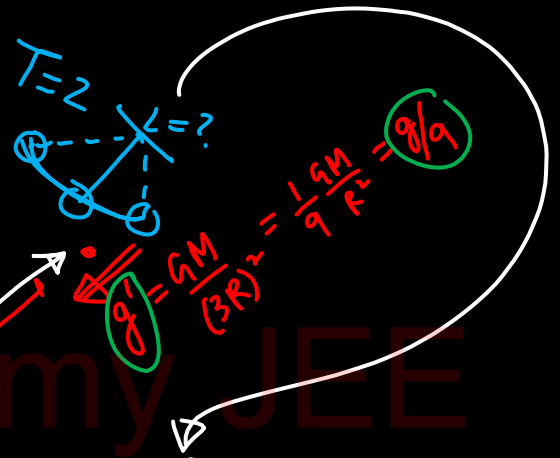
(B) $\frac{4}{9}$ m

(C) $\frac{8}{9}$ m

(D) $\frac{1}{9}$ m

$T_{pendulum} = 2\pi\sqrt{\frac{L}{g}}$




$T = 2\pi\sqrt{\frac{L}{g'}} = 2$

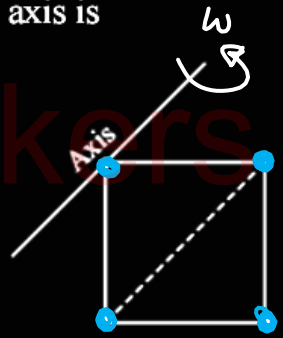
$2\pi\sqrt{\frac{L}{g/9}} = \frac{1}{2}$

$\pi^2 \frac{9L}{g} = 1$

$L = \frac{1}{9}$

9

Four point masses, each of mass m, are fixed at the corners of square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is



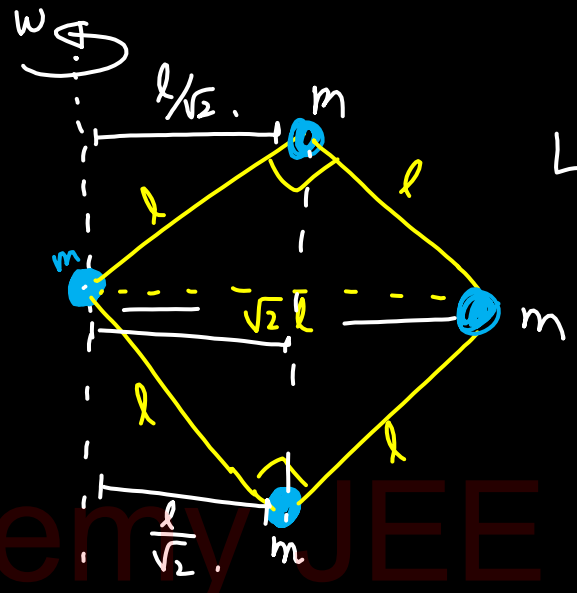

(A) $ml^2\omega$

(B) $4m^2\omega$

(C) $3ml^2\omega$

(D) $2ml^2\omega$






JEE 1

$L = I\omega$

$$I = 2\left(m\left(\frac{l}{\sqrt{2}}\right)^2\right) + m(\sqrt{2}l)^2$$

$$I = 3ml^2$$

$$L = I\omega = 3ml^2\omega$$

10

An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(A) $\frac{P}{3\alpha K}$ (B) $\frac{P}{\alpha K}$

(C) $\frac{3\alpha}{PK}$ (D) $3PK\alpha$



C1:

$$B = \frac{P_{ext}}{\left|\frac{\Delta V}{V}\right|}$$

$$B = \frac{P}{\left|\frac{\Delta V}{V}\right|} \quad -(1)$$

C2:

$\alpha : \beta : \gamma$

$1 : 2 : 3$

$\frac{\alpha}{\gamma} = \frac{1}{3}$

$\gamma = 3\alpha$

C3:

$\Delta V = V\gamma\Delta T$

Ans

$\frac{\Delta V}{V} = 3\alpha\Delta T$

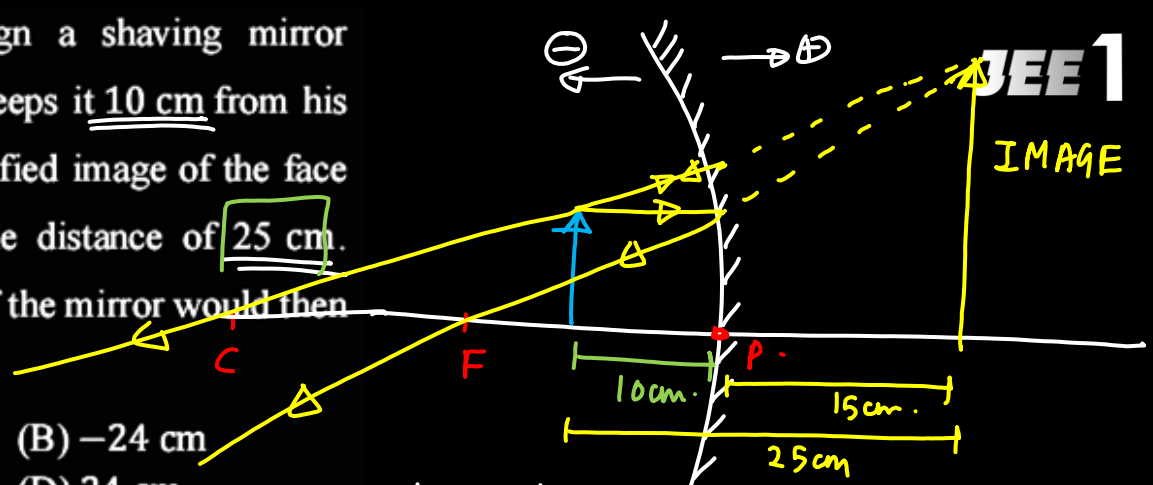
$B = \frac{P}{3\alpha\Delta T}$

$\Delta T = \frac{P}{3\alpha B}$

11

You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be :

(A) 60 cm
(B) −24 cm
(C) −60 cm
(D) 24 cm






$u = -10$

$v = +15$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

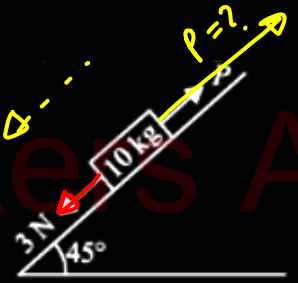
$$\frac{1}{f} = \frac{1}{15} - \frac{1}{10} = \frac{-5}{150}$$

$$f = -30$$

$$R = 2f = -60$$

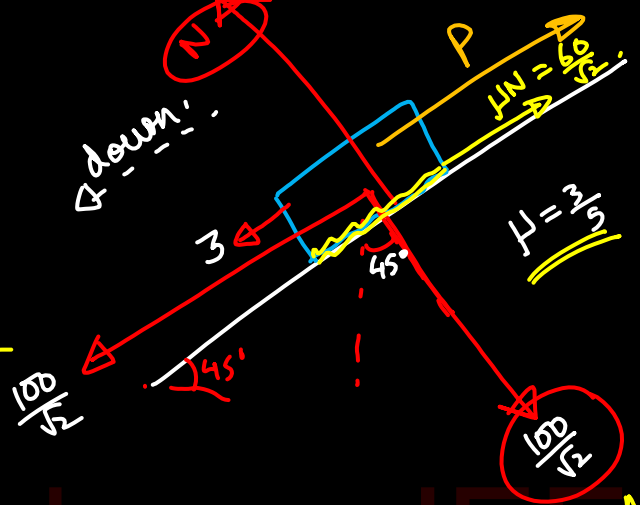
12

A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6 . What should be the minimum value of force P , such that the block does not move downward?




(A) 25 N (B) 23 N

(C) 18 N (D) 32 N




JEE 1

$\mu = \frac{3}{5}$ $f_m = \mu N = \frac{60}{\sqrt{2}}$

$N = \frac{100}{\sqrt{2}}$

$\frac{100}{\sqrt{2}} + 3 = P + \frac{60}{\sqrt{2}}$

$P = \frac{40}{\sqrt{2}} + 3 = 20\sqrt{2} + 3$

$P = 20(1.4) + 3$

$P = 28 + 3 \simeq 31$

13

Ratio of the ranges of the bullets fired from a gun at angle $\theta, 2\theta \& 4\theta$ is found in the ratio $x:2:2$, then the value of x will be (Assume same speed of bullets)

(A) 1 (B) 2

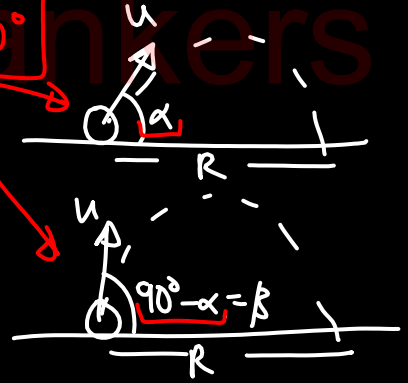
(C) $\sqrt{3}$ (D) none of these

$x = ?$

$x = \frac{2}{\sqrt{3}}$

$R = \frac{u^2}{g}\sin(2\theta)$

$\alpha + \beta = 90°$




$\frac{2}{\sqrt{3}} : 2 : 2$

Ratio: $\theta : 2\theta : 4\theta$

Range: $x : 2 : 2$

$\alpha + \beta = 90°$

$2\theta + 4\theta = 90°$

$6\theta = 90°$

$\theta = 15°$

Ratio: $15° \quad 30° \quad 60°$

Ranges: $\frac{u^2}{g}\sin(30°) : \frac{u^2}{g}\sin(60°) : \frac{u^2}{g}\sin(120°)$

$\frac{1}{2} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}}{2}$

$\frac{1}{\sqrt{3}} : 1 : 1$



JEE 1

14

Expression for time in terms of G (universal gravitational constant), h (Planck's constant) and c (speed of light) is proportional to

(A) $\sqrt{\frac{hc^5}{G}} = G^{-1/2} h^{1/2} c^{5/2}$ (B) $\sqrt{\frac{Gh}{c^3}}$

(C) $\sqrt{\frac{Gh}{c^5}}$ (D) $\sqrt{\frac{c^3}{Gh}}$

$[G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{MLT^{-2} \cdot L^2}{M^2}$

$[G] = M^{-1} L^3 T^{-2}$

$[c] = LT^{-1}$

$h = \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}}$

$[h] = ML^2T^{-1}$

$[t] = T^1$

$[t] = G^x h^y c^z$

$M^0 L^0 T^1 = (M^{-1} L^3 T^{-2})^x (M^1 L^2 T^{-1})^y (LT^{-1})^z$

$M^0 L^0 T^1 = M^{-x+y} L^{3x+2y+z} T^{-2x-y-z}$

$x = y$

$5x + z = 0$

$-3x - z = 1$

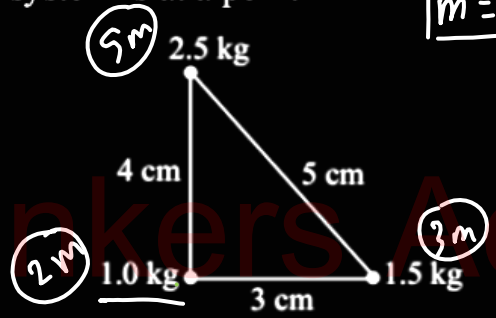
$2x = 1$

$x = 1/2$ $y = \frac{1}{2}$ $z = \frac{-5}{2}$

$[t] = G^{1/2} h^{1/2} c^{-5/2}$

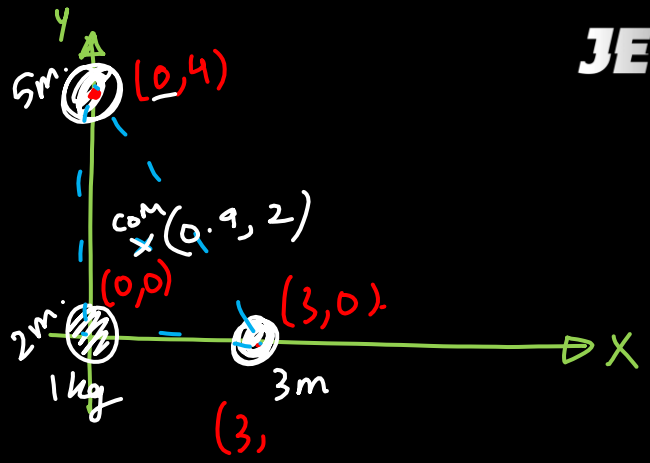
15

Three point particles of masses 1kg, 1.5kg and 2.5kg are placed at three corners of a right angle triangle of sides 4.0 cm and 5.0 as shown in the figure. The center of mass of the system is at a point




(A) 2.0 cm right and 0.9 cm above 1 kg mass
(B) 0.9 cm right and 2.0 cm above 1 kg mass
(C) 0.6 cm right and 2.0 cm above 1 kg mass
(D) 1.5 cm right and 1.2 cm above 1 kg mass






$$X_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$X_{COM} = \frac{0 + 0 + 9m}{10m} = 0.9$$

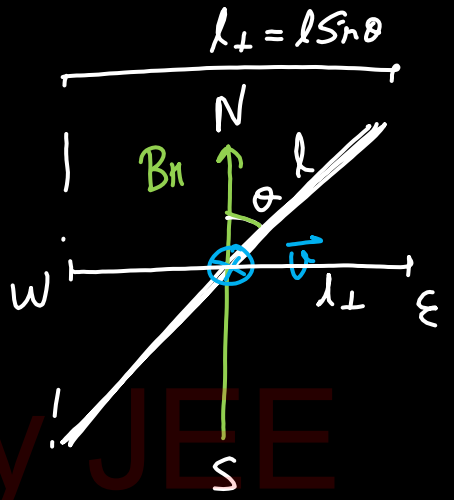
$$Y_{COM} = \frac{0 + 5m(4) + 0}{10m}$$

16

A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 m $s^{-1}$, at right angles to the horizontal component of the earth's magnetic field of $0.3 \times 10^{-4}$ Wb/$m^2$. The value of the induced emf in wire is

(A) $0.3 \times 10^{-3}$ V (B) $2.5 \times 10^{-3}$ V

(C) $1.5 \times 10^{-3}$ V (D) $1.1 \times 10^{-3}$ V






$$\mathcal{E} = B l_\perp v = B(l\sin\theta)v$$

$$= 0.3\times10^{-4} \times 10\text{m} \times \sin 45^\circ \times 5$$

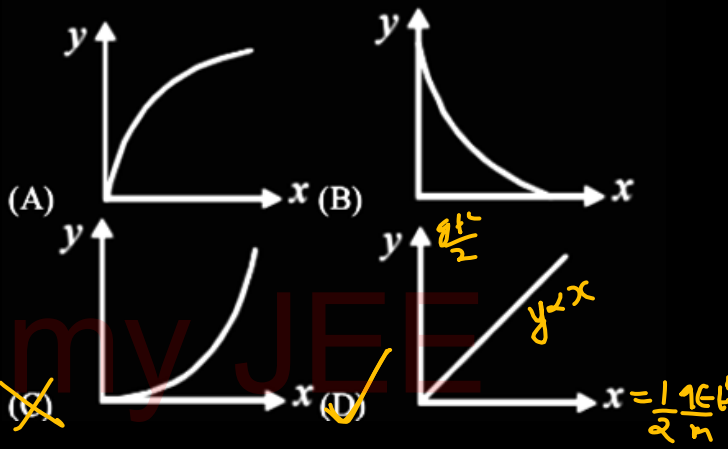
$$= \frac{15}{\sqrt{2}} \times 10^{-4} = \left(\frac{1.5}{1.414}\right)\times 10^{-3}\ \text{V} = 1.1\times10^{-3}\ \text{V}$$

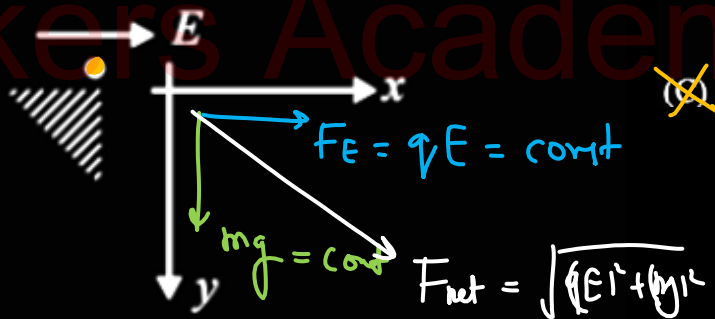
17

JEE 1

A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).

$u = 0$




$u = 0$

$x = \frac{1}{2}a_x t^2 = \frac{1}{2}\frac{qE}{m}t^2$ — (1)

$a_{net}$

$y = \frac{1}{2}gt^2$ — (2)

$y \propto x \propto t^2$

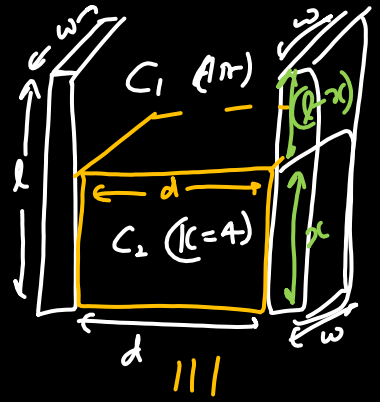
Straight line

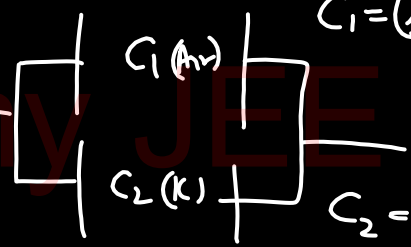
18

A parallel plate capacitor has plate of length 'l' width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant $K = 4$ is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?

(A) 2l/3 (B) l/3

(C) l/4 (D) l/2




$C_1 = \frac{(l-x)w\varepsilon_0}{d}$ — ①

$C_2 = \frac{K(xw)\varepsilon_0}{d}$ — ②

$\frac{A\varepsilon_0}{\left(d - t + \frac{t}{K}\right)}$ ✗

$U_f = 2U_i$

$\frac{1}{2}C_q(V^2) = 2 \times \frac{1}{2}C_{Air}(V^2)$

$C_q = 2C_{Air} = 2\frac{A\varepsilon_0}{d} = 2\frac{(lw)\varepsilon_0}{d}$

$C_1 + C_2 = 2C$

$(l-x)\frac{w\varepsilon_0}{d} + (Kx)\frac{w\varepsilon_0}{d} = 2l\left(\frac{w\varepsilon_0}{d}\right)$

$l - x + 4x = 2l$

$3x = l$

$x = \frac{l}{3}$

19

A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is (Assume that the current is flowing in the clockwise direction.)

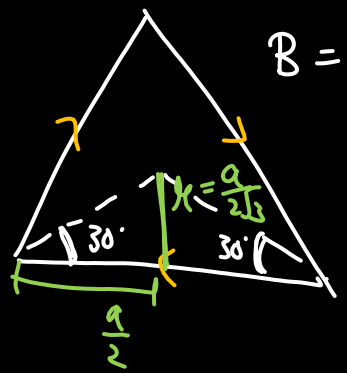
(A) $2\sqrt{3} \times 10^{-5}$ T, inside the plane of triangle

(B) $2\sqrt{3} \times 10^{-7}$ T, outside the plane of triangle

(C) $3 \times 10^{-7}$ T, outside the plane of triangle

(D) $3 \times 10^{-5}$ T, inside the plane of triangle.






$$B = 3B_{side} = 3 \times \frac{\mu_0 i}{4\pi x}(2\cos 30^\circ)$$

$$= 3\,\frac{\mu_0 i}{2\pi\left(\frac{9}{2\sqrt{3}}\right)} \times \frac{\sqrt{3}}{2}$$

$$B_s = \frac{9\mu_0 i}{2\pi a}$$

$$= \frac{9 \times 4\pi \times 10^{-7} \times 3/2}{2\pi \times 9 \times 10^{-2}}$$

$$= 3 \times 10^{-5}\ \text{T}$$

20

Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference 4 V. The de-Broglie wavelengths are $\lambda_e$ and $\lambda_p$ for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by : (given $m_e$ is mass of electron and $m_p$ is mass of proton).

(A) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$

(B) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$

(C) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$

(D) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$ ✓



JEE 1

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\lambda_e = \frac{h}{\sqrt{2m_e eV}} \quad -(1)$$

$$\lambda_p = \frac{h}{\sqrt{2m_p e(4V)}} \quad -(2)$$

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{4m_p}{m_e}} = 2\sqrt{\frac{m_p}{m_e}}$$

21

A plane electromagnetic wave, has frequency of $2.0 \times 10^{10}$ Hz and its energy density is $1.02 \times 10^{-8}\ J/m^3$ in vacuum. The amplitude of the magnetic field of the wave is close to __ nT ($= 10^{-9}$)

$\left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}\right.$ and speed of light $\left. = 3 \times 10^8\ ms^{-1}\right)$: → standard value

$B_0 = ?$

$u = \frac{B_0^2}{2\mu_0}$

$B_0 = \sqrt{2\mu_0 u}$

$= \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}}$

$\rightarrow \sqrt{8\pi \times 1.02 \times 1000 \times 10^{-18}}$

$B_0 = \sqrt{8\pi \times 1020}\ nT$

$= \sqrt{(25\,00\,0+)}\ nT$

$= 160\ nT$

If different

$B_0 = \frac{E_0}{c} = \frac{1}{c}\sqrt{\frac{2u}{\varepsilon_0}}$

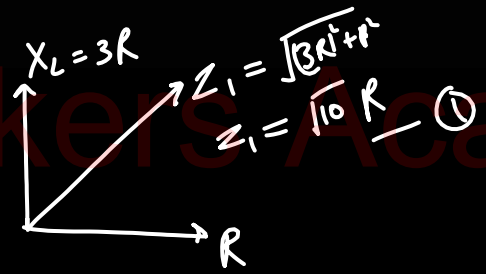


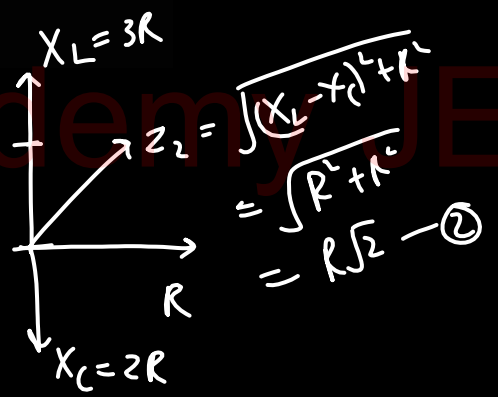
JEE 1

22

JEE 1

An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5}: x$ The value of x is



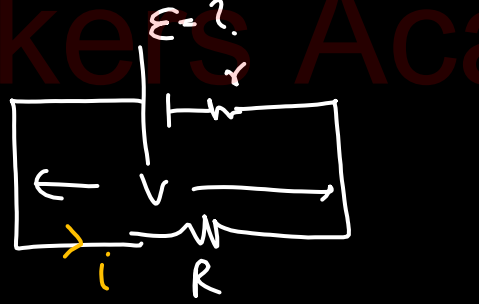

$$\frac{\cos\phi_2}{\cos\phi_1} = \frac{\frac{R}{Z_2}}{\frac{R}{Z_1}} = \frac{Z_1}{Z_2} = \frac{\sqrt{10}R}{\sqrt{2}R} = \sqrt{5} : \boxed{1}$$

23

In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is _____

JEE 1






$V = iR = \frac{\varepsilon}{(r+R)} R$

$1.25 = \frac{(\varepsilon)5}{r+5}$ — (1)

$1 = \frac{\varepsilon \times 2}{r+2}$ — (2) $\rightsquigarrow \varepsilon = \frac{(r+2)}{2}$

Divide

$1.25 = \frac{2.5\,(r+2)}{(r+5)}$

$r+5 = 2r+4$

$r = 1$

$= \left(\frac{1+2}{2}\right)$

$= \frac{3}{2}$

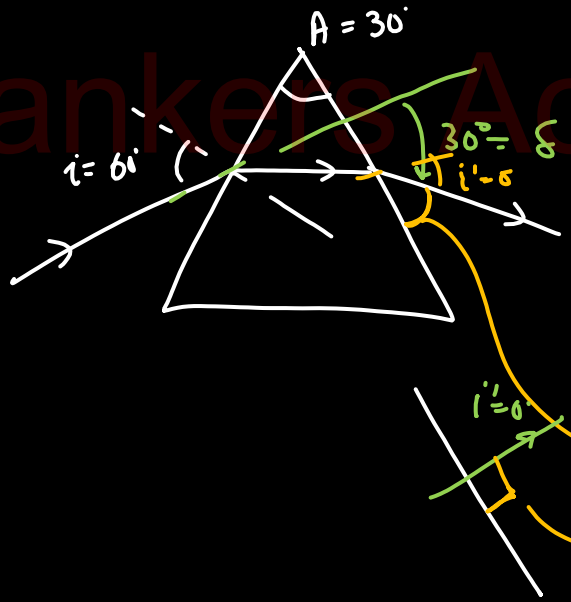
$= 1.5$

$= \frac{15}{10}$ Ans

24

A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The emergent ray of light makes an angle of 30° with incident ray. The angle made by the emergent ray with second face of prism will be:_____ degrees.

$\delta = 30°$




$\delta = i + i' - A$

$\Rightarrow i' = \delta + A - i$

$= 30° + 30° - 60°$

$i' = 0°$

25

If radius of the $^{27}_{13}Al$ nucleus is estimated to be 3.6 fermi then the radius of $^{125}_{52}Te$ nucleus be nearly_____ fermi

JEE 1

$$R = R_0(A^{\frac{1}{3}})$$ *



$$\frac{R_{Te}}{R_{Al}} = \left(\frac{125}{27}\right)^{\frac{1}{3}}$$

$$R_{Te} = 3.6 \times \frac{5}{3}$$

$$= \boxed{6} \text{ fermi}$$

# CHEMISTRY


A mixture of methane and ethene in the mole ratio of x : y has a mean molecular mass = 20. What would be the mean molecular mass, if the same gases are mixed in the mole ratio of y: x ?

(A) 22 (B) 24

(C) 20 (D) 19.8

$20 = \frac{16x + 28y}{x + y}$

$x = 2y$



$n = \frac{\omega}{m}$

$m = \frac{\omega}{n}$

Mean molecular mass = $\frac{\text{Total given mass}}{\text{Total moles.}}$

Mean molecular mass = $\frac{28x + 16y}{x + y}$

$= \frac{28(2y) + 16y}{3y}$

$= 24$

2

1 g of a monobasic acid (HA) in 100 g water lowers the freezing point by 0.1 K. If 0.75 g of same acid requires 15ml of $\frac{1}{5}$ N NaOH solution for complete neutralisation, then % degree of ionization of acid in water is

($K_f$ of $H_2O = 1.86\ K\ kg\ mol^{-1}$)

(A) 0.22 (B) 0.25

(C) 0.34 (D) 0.50

$\alpha = \frac{i-1}{n-1}$

n for HA = 2

$HA \rightleftharpoons H^+ + A^-$

$\alpha = \frac{1.34-1}{2-1} = 0.34$

JEE 1



Law of equivalence

$\frac{0.75}{M} \times 1 \times 1000 = \frac{1}{5} \times 15$

$M = 250$

$\Delta T_f = i\, m\, K_f$

$0.1K = i \times \frac{1g \times 1000}{250 \times 100} \times 1.86$

$i = 1.34$

3

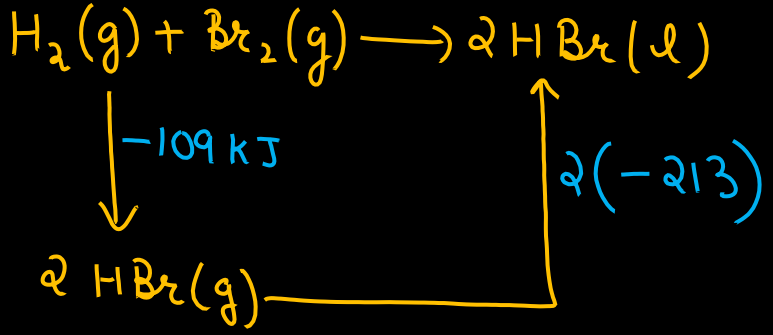
The change in enthalpy ($\Delta H$) for the reaction $H_2(g) + Br_2(g) \rightarrow 2HBr(\ell)$ when 2.0 moles each of $Br_2(g)$ and $H_2(g)$ react is

($H_2(g) + Br_2(g) \rightarrow 2HBr(g)$; $\Delta H_{reaction} = -109$ kJ; $\Delta H_{vap}$ of HBr = 213 kJ mol$^{-1}$)

(A) −644 kJ (B) 644 kJ

(C) −322 kJ (D) −1070 kJ






$\Delta H = -109\,kJ + 2(-213)$

$= -535$ kJ

For 2 moles

$= 2(-535)$ kJ

$= -1070$ kJ

4

An electrochemical cell was constructed with $Fe^{2+}/Fe$ and $Cd^{2+}/Cd$ at 25°C with initial concentrations of '$[Fe^{2+}] = 0.800M$ and $[Cd^{2+}] = 0.250M$. The EMF of the cell when $[Cd^{2+}]$ becomes 0.100M is

| Half cell | E°(V) |
|---|---|
| $Fe^{2+}(aq)/Fe(s)$ | -0.44 → anode |
| $Cd^{2+}(aq)/Cd(s)$ | -0.40 → cathode |

(A) 0.013 V (B) 0.011 V
(C) 0.051 V (D) 0.022 V

$Fe + Cd^{2+} \longrightarrow Fe^{2+} + Cd$

| | | |
|---|---|---|
| initial conc | 0.250 | 0.800 |
| | 0.250 − x | 0.800 + x |
| t = t | 0.1 | 0.95 |

At eqm $\Rightarrow 0.250 - x = 0.100$

$x = 0.150$

$Fe + Cd^{2+} \longrightarrow Fe^{2+} + Cd.$

$$E = E^\circ - \frac{0.059}{2}\log\frac{[Fe^{2+}]}{[Cd^{2+}]} \Rightarrow 0.04 - \frac{0.059}{2}\log\frac{0.95}{0.1}$$

$$= 0.011\,V$$

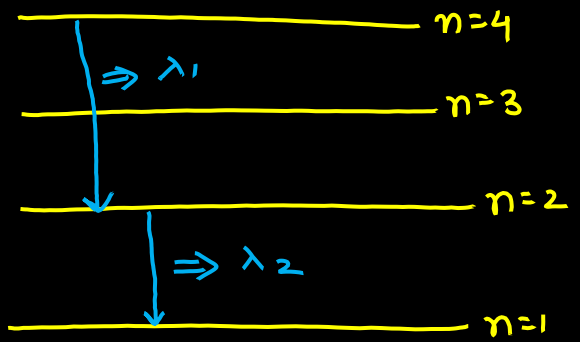
$$E^\circ = E^\circ_c - E^\circ_a = -0.40 - (-0.44) = 0.04$$

5

An electron present in the third exited state of a H atom returns of the first exited state and then to the ground state. If $\lambda_1$ and $\lambda_2$ are the wavelengths of light emitted in these two transitions respectively, $\lambda_1 : \lambda_2$ is

(A) 4:1 (B) 5:9

(C) 3:1 (D) 2:1




JEE 1



3rd Excited state $n=4$

1st Excited state $n=2$

Ground state $n=1$

$$\frac{1}{\lambda_1} = R_H\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R_H}{16}$$

$$\lambda_1 = \frac{16}{3R_H}$$

$$\frac{1}{\lambda_2} = R_H\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R_H}{4}$$

$$\lambda_2 = \frac{4}{3R_H}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{16}{3R_H} \times \frac{3R_H}{4}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{16}{4} = \frac{4}{1}$$

6

The set in which all the species are diamagnetic is

(A) $B_2, O_2, NO$

(B) $O_2, O_2^+, CO$

(C) $N_2, O_2^-, CN^-$

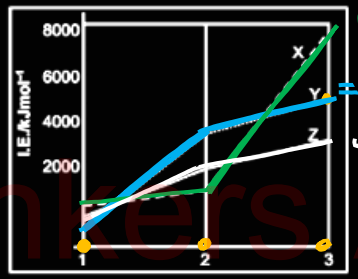
(D) $C_2, O_2^{2-}, NO^+$

12 18 14

Paramag 10, 16, Odd

7

The following qualitative plots depict the first, second and third ionization energies (I.E.) of Mg, Al and K. Among the following, the correct match of I.E. and the metal is




Mg
= K
Al

Mg : $1s^2 2s^2 2p^6 3s^2$

Al : $1s^2 2s^2 2p^6 3s^2 3p^1$

K : $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

(A) X – Al; Y – Mg; Z – K

(B) X – Mg; Y – Al; Z – K

(C) X – Mg; Y – K; Z – Al

(D) X-Al; Y-K; Z-Mg

For chromium element, how many electrons can have $n + \ell + |m| = 4$, with clockwise spin in ground state?

(I) 2
(II) 3
(III) 4
(IV) 6
(A) I, II
(B) II, III, IV
(C) II, III
(D) I, II, III, IV

JEE 1

$Cr(24) : 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1\ 3d^5$



$2p =$ ↑↓ | ↑ | (−1 0 +1)

$3p =$ | ↑ | (−1 0 +1)

$4s =$ 1

$n + \ell + |m| = 4$

for $2p = 2 + 1 + 1 = 4$
for $3p = 3 + 1 + 0 = 4$
for $4s = 4 + 0 + 0 = 4$

$= 2$
$= 1$

9

The pyrimidine bases present in DNA are

(A) cytosine and guanine

(B) cytosine and thymine

(C) cytosine and uracil

(D) cytosine and adenine

JEE 1



A G = Purines

C T U = Pyrimidine

C T → DNA

C U → RNA

10

Which is the correct code for the given name reactions ?

(p) $Ph-C\equiv N \xrightarrow[(ii)H_2O]{(i)SnCl_2/HCl}$ (i) Perkin reaction

(q) $Ph-CHO \xrightarrow{KCN/\Delta}$ (ii) Benzoin condensation

(r) $Ph-CHO \xrightarrow{NaOH}$ (iii) Stephen reduction

(s) $Ph-CHO \xrightarrow[(ii)H_3O^{\oplus}]{(i)(Ac)_2O/AcONa/\Delta}$ (iv) Canizzarro reaction

(1) p-iii, q-ii, r-iv, s-i

(2) p-iii, q-i, r-iv, s-ii

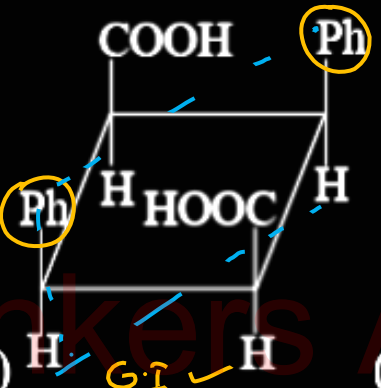
(3) p-ii, q-iii, r-iv, s-i

(4) p-iii, q-ii, r-i, s-iv

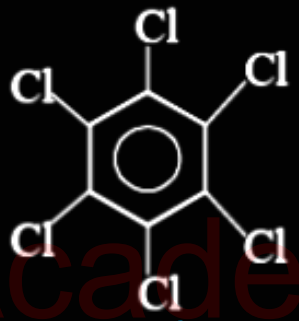
11

Which of the following will show optical as well as geometrical isomerism ?

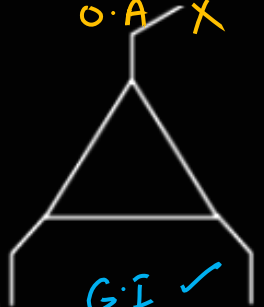
O.A ⟶ Assymmetric




(A) G.I ✓ O.A ✗




(B) no G.I & O.A ✗



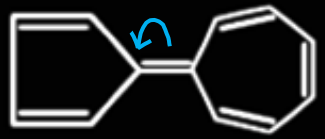
(C) G.I ✓ O.A ✗

(D) None Of The Above ✓

12

JEE 1

Select the correct statement about the following compounds.



(I)

(II)

(A) II has a greater dipole moment than I

(B) Covalent character of II is less than I

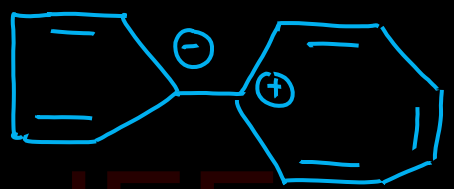
(C) I is more soluble in polar solvent than II

(D) None of these

$\mu = q \times d$ $q \uparrow \mu \uparrow$



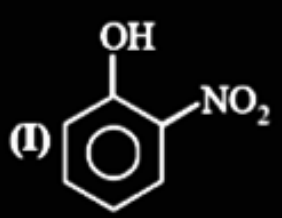
I



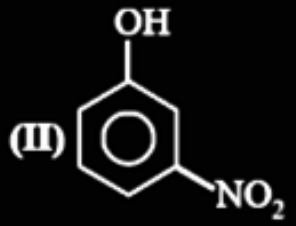

II

13

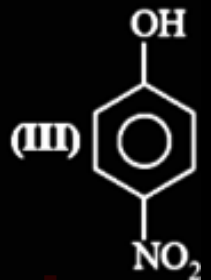
Three Nitrophenols are given. Select the correct statement.



(I)



(II)



(III)

(A) I is most acidic

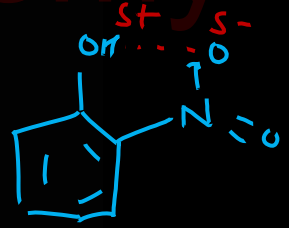
(B) II reacts fastest with Na metal

(C) I is steam volatile

(D) III reacts slowest with NaOH

Acidity

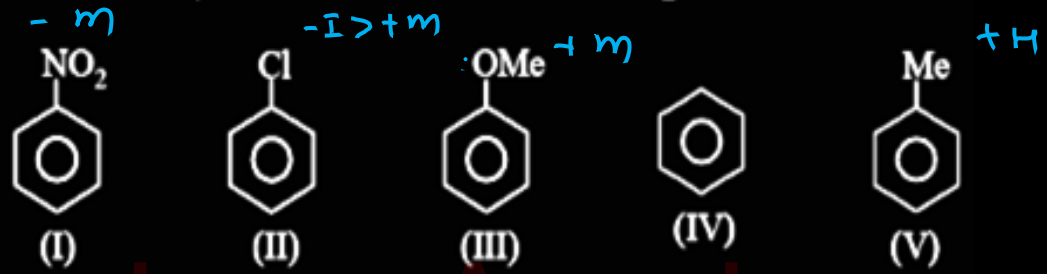
III > I > II



14

JEE 1

Arrange the following in the order of reactivity towards an electrophilic attack.






rate of EAS $\propto$ EDG

$\propto \frac{1}{EWG}$

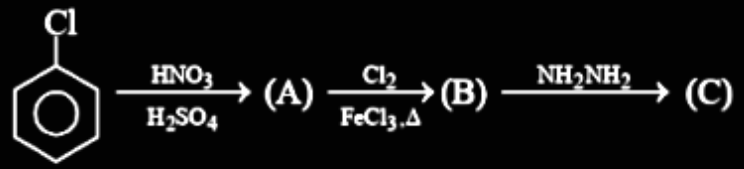
(A) V > IV > III > II > I

(B) III > V > IV > II > I

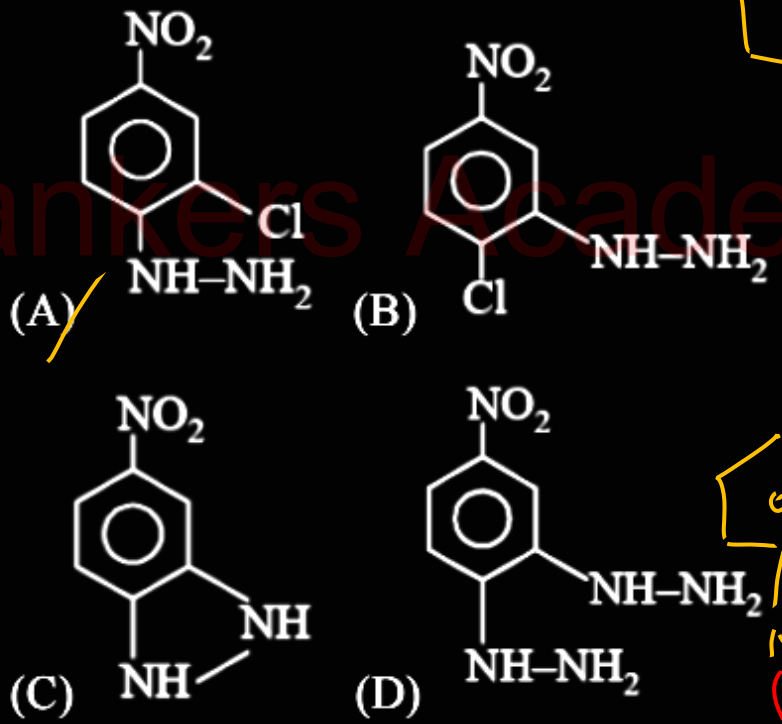
(C) III > IV > V > II > I

(D) V > IV > III > I > II

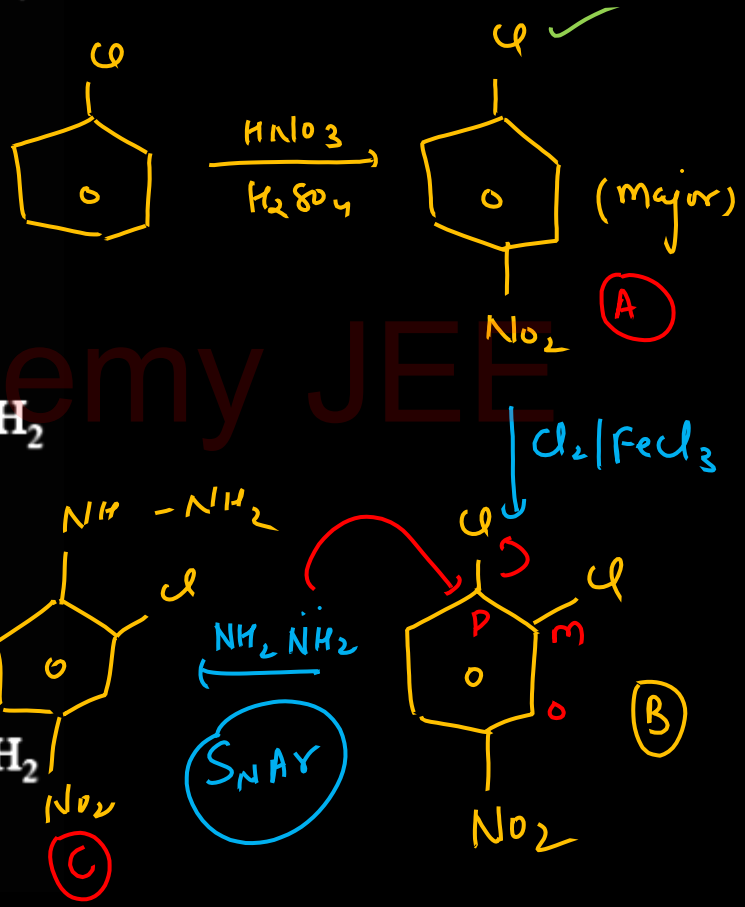
15



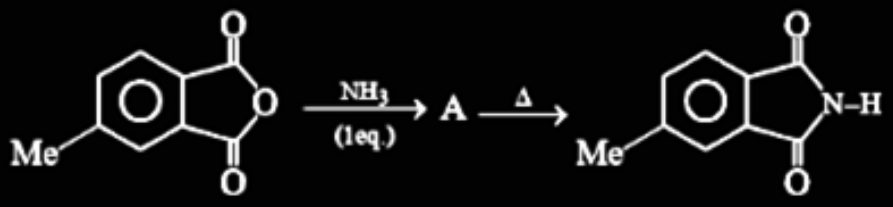

Product C ?





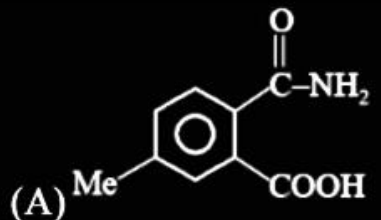



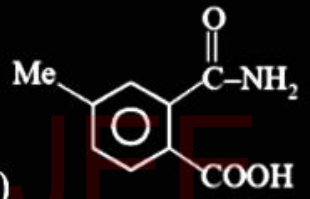
16

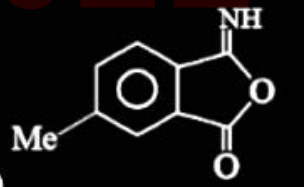


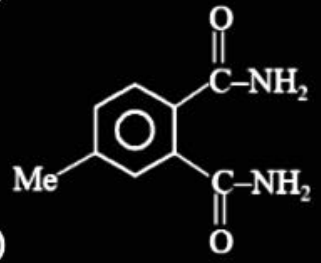

A is___

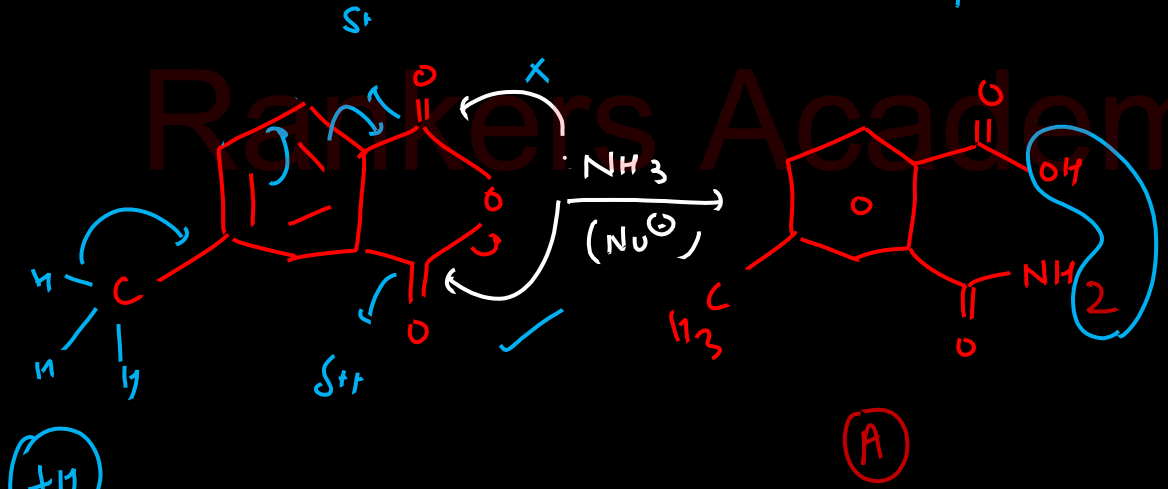
JEE 1

(A) 

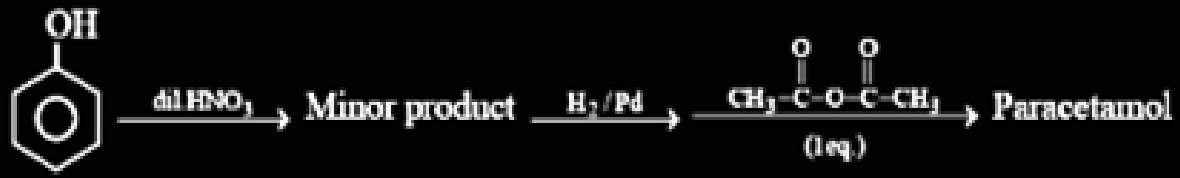

(B) 


(C) 


(D) 

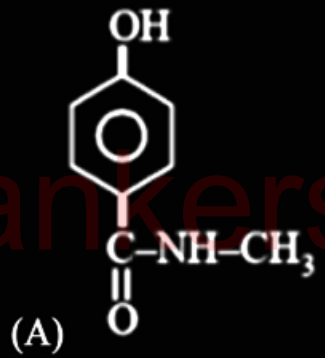



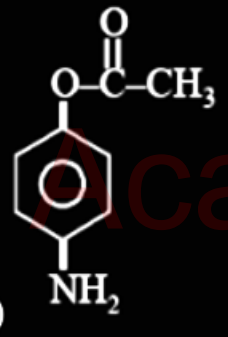
17

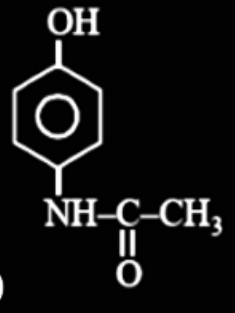


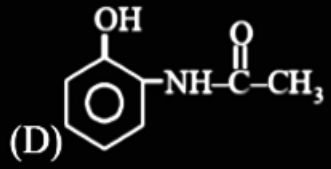

JEE 1

What is structure of paracitamol____

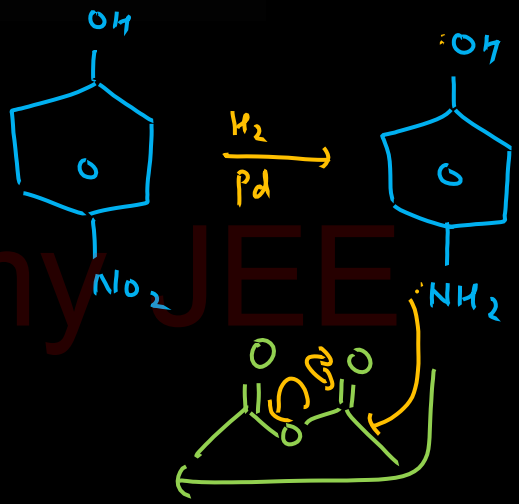


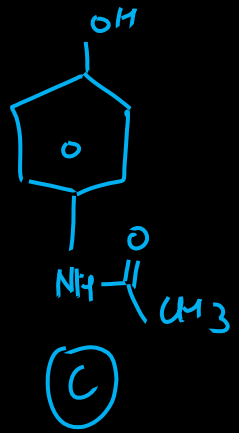



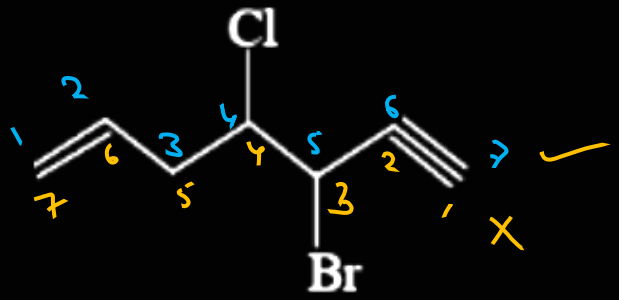




Minor



18




ene
yne
X

JEE 1

IUPAC name of given compound is -

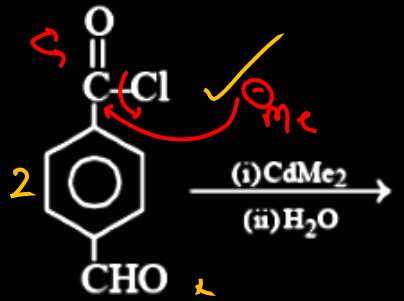
(A) 3 - Bromo 4-chlorohept - 1 - en - 6 - yne

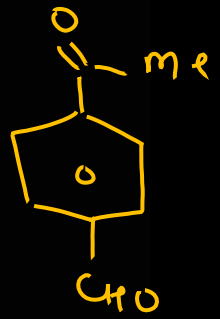
(B) 5 - Bromo 4-chlorohept - 1 - en - 6 - yne

(C) 3 - Bromo 4-chlorohept - 6 - en - 1 - yne

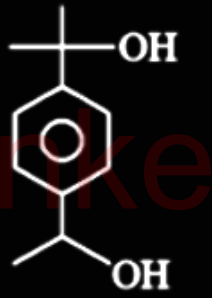
(D) 5 - Bromo 4-chlorohept - 6 - en - 1 - yne

19



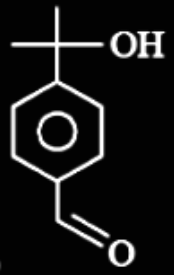

Product Is ____




(A)


(B)


(C)

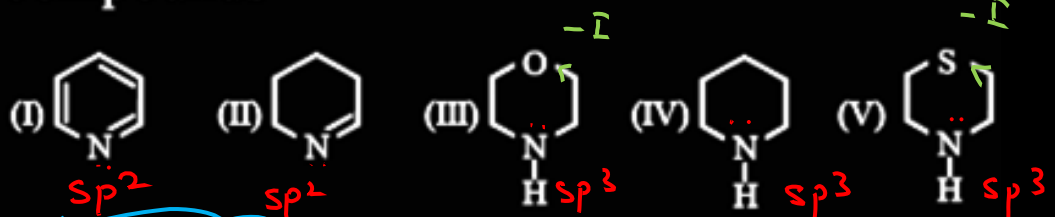



(D)

20

JEE 1

Correct basic strength order of given compounds




E·N
O > S

33.33 % S

25 % S

(A) I > II > IV > V > III

(B) IV > III > V > I > II

(C) II > I > III > V > IV

(D) IV > V > III > II > I

21


When $X_2$ and $Y_2$ are allowed to react, the equilibrium concentration of XY at 27°C is found to be 0.67M. What is the approximate value of equilibrium constant ($K_P$) for the following reaction?

$$X_{2(g)} + Y_{2(g)} \rightleftharpoons 2XY_{(g)}$$

After opening stopcock

Total V = 4 L

$[X_2] = \frac{2}{4} = \frac{1}{2}$ mol/L

$[Y_2] = \frac{4}{4} = 1$ mol/L.

$\Delta n_g = 0$

$K_p = K_c$

Ans 4

| | $X_2$ | + $Y_2$ | $\rightleftharpoons$ | $2XY$ |
|---|---|---|---|---|
| initial | 0.5 m | 1 mol | | 0 |
| | 0.5 − x | 1 − x | | 2x = 0.67 |

$$K_c = \frac{[XY]^2}{[X_2][Y_2]} = \frac{0.67 \times 0.67}{0.17 \times 0.67} = 4$$



JEE 1

22

What is the minimum pH required to prevent the precipitation of ZnS in a solution that is 0.01 M $ZnCl_2$ and saturated with 0.1 M $H_2S$ ?

[Given $K_{sp}(ZnS) = 10^{-21}$ & $K_{a_1} \times K_{a_2}(H_2S) = 10^{-20}$]

$ZnS \rightleftharpoons Zn^{2+} + S^{2-}$

$K_{sp} = [Zn^{2+}][S^{2-}]$

$10^{-21} = [10^{-2}][S^{2-}]$

$[S^{2-}] = 10^{-19}$

$H_2S \rightleftharpoons HS^- + H^+$

$K_{a_1} = \frac{[HS^-][H^+]}{[H_2S]}$

$HS^- \rightleftharpoons [S^{2-}][H^+]$

$K_{a_2} = \frac{[S^{2-}][H^+]}{[HS^-]}$

$K_{a_1} \times K_{a_2} = \frac{[HS^-][H^+]}{[H_2S]} \times \frac{[S^{2-}][H^+]}{[HS^-]}$

$10^{-20} = \frac{[H^+]^2[S^{2-}]}{[H_2S]} = \frac{[H^+]^2\, 10^{-19}}{10^{-1}}$

$[H^+]^2 = 10^{-2}$

$[H^+] = 10^{-1}$ so $pH = 1$

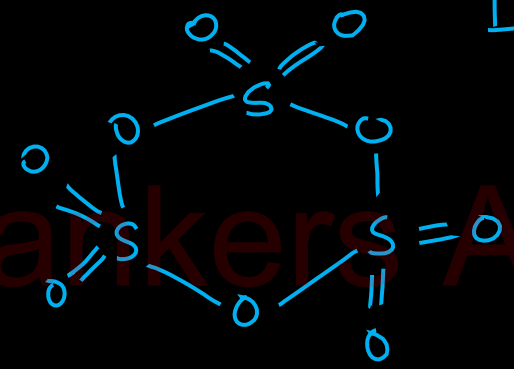


JEE 1

23

Calculate the number of S – S bonds in cyclic trimer of sulphur trioxide.

Zero



24

JEE 1

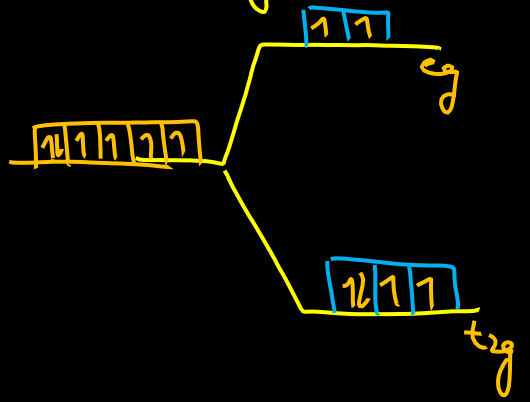
In the complex $[CoF_6]^{3-}$ ion, find the number of electrons present in its $e_g$ level.

Ans = 2

$[CoF_6]^{3-}$

$Co^{+3}$ : $3d^6 4s^0$

Now ligand WF

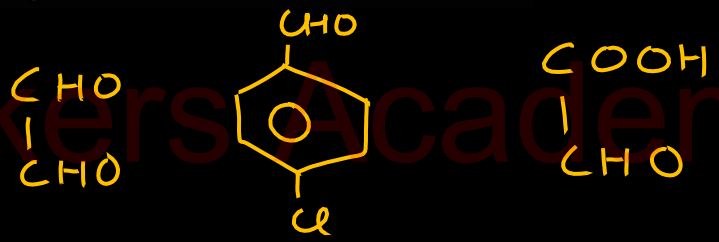


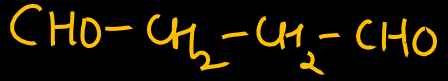
25

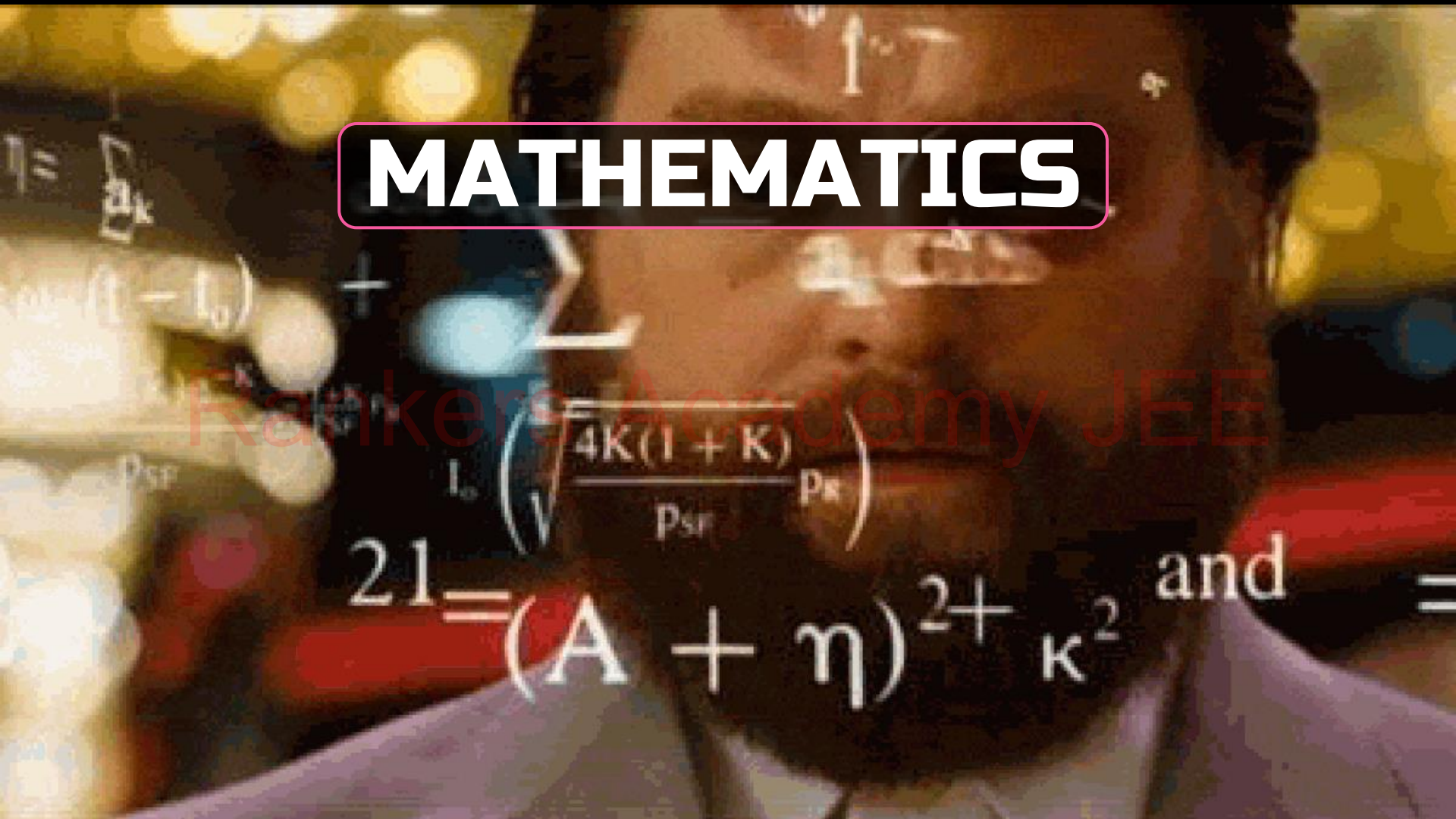
Of the following the number of compounds which answer Fehling's test is ___

Glucose, $\alpha$-hydroxy ketone, glyoxal, p-chlorobenzaldehyde, glyoxalic acid, succinaldehyde.

3






MATHEMATICS
Rankers Academy JEE
$21 = (A + \eta)^2 + \kappa^2$ and

7

Let $f(x) = \begin{cases} \frac{x}{2x^2+|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then $f(x)$ has

(A) Continuity but non differentiability at $x = 0$

(B) Differentiability at $x = 0$

(C) Discontinuity at $x = 0$

(D) None of these



$f(x) = \begin{cases} \frac{x}{2x^2-x} = \frac{1}{2x-1}; & x < 0 \longrightarrow LHL = \frac{1}{2(0)-1} = -1 \\ 1; & x = 0 \longrightarrow f(0) = 1 \\ \frac{x}{2x^2+x} = \frac{1}{2x+1}; & x > 0 \longrightarrow RHL = \frac{1}{2(0)+1} = 1 \end{cases}$

2

The sum of the real values of ' x ' which satisfy the equation

$\frac{1}{x^2+4x-50}+\frac{1}{x^2+4x-66}=\frac{2}{x^2+4x-90}$ is

(A) -60 (B) 4

(C) -4 (D) 12

Let $x^2+4x-66=t$

$x^2+4x-50=t+16$

$x^2+4x-90=t-24$

$\Rightarrow \frac{1}{t+16}+\frac{1}{t}=\frac{2}{t-24}$

$\Rightarrow \frac{2t+16}{t^2+16t}=\frac{2}{t-24}$

$\Rightarrow \frac{t+8}{t^2+16t}=\frac{1}{t-24}$

$\Rightarrow t^2-16t-8\times 24=t^2+16t$

$\Rightarrow 32t=-8\times 24$

$\boxed{t=-6}$

$\Rightarrow x^2+4x-66=-6$

$\Rightarrow x^2+4x-60=0$

$\Rightarrow x=-10,6$

Sum $=-\frac{(4)}{1}$



JEE 1

3

Let $I = \int_0^{\pi/2} \frac{\cos x+4}{3\sin x+4\cos x+25}dx$ and

$J = \int_0^{\pi/2} \frac{\sin x+3}{3\sin x+4\cos x+25}dx$.

If $(11I + 2J) = a\pi + b\ln\frac{c}{d}$ where a, b, c and $d \in N$ and $\frac{c}{d}$ is not a perfect square of a rational then find the value of $(a + b + c + d - 55)$.

(A) 10 (B) 4

(C) 6 (D) 12



$$4I = \int_0^{\pi/2} \frac{4\cos x+16}{3\sin x+4\cos x+25}dx$$

$$3J = \int_0^{\pi/2} \frac{3\sin x+9}{3\sin x+4\cos x+25}dx$$

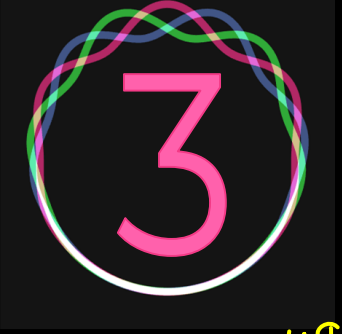
$$4I+3J = \int_0^{\pi/2} dx = \pi/2.$$

$$3I = \int_0^{\pi/2} \frac{3\cos x+12}{3\sin x+4\cos x+25}dx$$

$$-4J = \int_0^{\pi/2} \frac{-4\sin x-12}{3\sin x+4\cos x+25}dx$$

$$3I-4J = \int_0^{\pi/2} \frac{3\cos x-4\sin x}{3\sin x+4\cos x+25}dx$$

$$= \ln|3\sin x+4\cos x+25|_0^{\pi/2}$$




JEE 1

$4I + 3J = \pi/2$ — ① $\longrightarrow$ $12I + 9J = 3\pi/2$

$3I - 4J = \ln\frac{28}{29}$ — ② $\longrightarrow$ $12I - 16J = 4\ln\frac{28}{29}$

$25J = \frac{3\pi}{2} - 4\ln\frac{28}{29}$

$J = \frac{3\pi}{50} - \frac{4}{25}\ln\frac{28}{29}$

$4I = \frac{\pi}{2} - 3J$

$= \frac{\pi}{2} - 3\left(\frac{3\pi}{50} - \frac{4}{25}\ln\frac{28}{29}\right)$

$= \frac{25\pi}{50} - \frac{9\pi}{50} + \frac{12}{25}\ln\frac{28}{29}$

$4I = \frac{8\pi}{25} + \frac{12}{25}\ln\frac{28}{29}$

$I = \frac{2\pi}{25} + \frac{3}{25}\ln\frac{28}{29}$ ✓

3

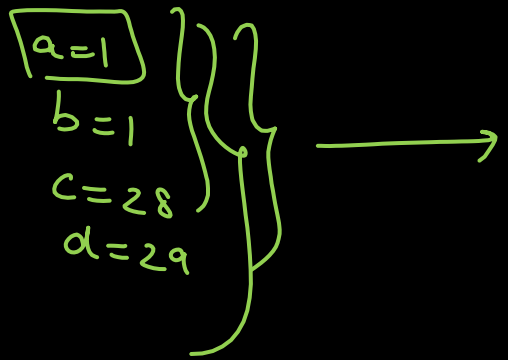
JEE 1

$$11\mathcal{I} + 2J = \frac{22\pi}{25} + \frac{33}{25}\ln\frac{28}{29} + \frac{6\pi}{50} - \frac{8}{25}\ln\frac{28}{29}$$

$$= \pi + \ln\frac{28}{29}$$

$$= a\pi + b\ln\frac{c}{d}$$





4

$y = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$

then y

(A) increases in $[0, \infty)$ only

(B) decreases in $[0, \infty)$

(C) neither increases nor decreases in $[0, \infty)$

(D) increases in $(-\infty, \infty)$



$$y' = 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2} - x} \cdot \left(\frac{1}{2\sqrt{1+x^2}} \cdot 2x - 1\right)$$

$$= 2 - \left(\frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}}\right)$$

Badi Chhoti

$> 0$

$x \in R$

$1 + x^2 \geq 1$

$\frac{1}{(1+x^2)} \leq 1$

$\frac{1}{\sqrt{1+x^2}} \leq 1$

$\frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}} \leq 2$

5

If $y = \tan^{-1}(\sec x^3 - \tan x^3), \frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

(A) $xy'' + 2y' = 0$

(B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$

(D) $xy'' - 4y' = 0$

JEE 1



M2 $y = \tan^{-1}(\sec(x^3) - \tan(x^3))$ ; $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$

$$y' = \frac{1}{1+(\sec x^3 - \tan x^3)^2}\left(\sec(x^3)\tan(x^3)\cdot 3x^2 - \sec^2(x^3)\cdot 3x^2\right)$$

$$= \frac{3x^2\sec(x^3)}{1+(\sec x^3 - \tan x^3)^2}(\tan x^3 - \sec x^3)$$

$$= \frac{3x^2\sec(x^3)}{1+\tan^2 y}(-(\tan y))$$

5

JEE 1

M1 $y = \tan^{-1}\left(\frac{1}{\cos x^3} - \frac{\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{\left(\cos\frac{x^3}{2} - \sin\frac{x^3}{2}\right)^2}{\left(\cos\frac{x^3}{2} + \sin\frac{x^3}{2}\right)\left(\cos\frac{x^3}{2} - \sin\frac{x^3}{2}\right)}\right)$

$= \tan^{-1}\left(\frac{c-s}{c+s}\right) = \tan^{-1}\left(\frac{1-t}{1+t}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right)$

$= \frac{\pi}{4} - \frac{x^3}{2}$

$\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$

$-\frac{3\pi}{2} < -x^3 < -\frac{\pi}{2}$

$-\frac{3\pi}{4} < -\frac{x^3}{2} < -\frac{\pi}{4}$

$\frac{\pi}{4} - \frac{3\pi}{4} < \frac{\pi}{4} - \frac{x^3}{2} < \frac{\pi}{4} - \frac{\pi}{4}$

$-\frac{\pi}{2} < \quad < 0$

5

JEE 1

$$y = \frac{\pi}{4} - \frac{x^3}{2}$$

$$y' = -\frac{3x^2}{2}$$

$$y'' = -3x$$

$$x^2 y'' = -3x^3$$

$$-6y = -\frac{6\pi}{4} + 3x^3$$

$$x^2 y'' - 6y = -\frac{3\pi}{2}$$

6

If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(A) $y\log\left(\frac{x}{y}\right) = cx$

(B) $x\log\left(\frac{y}{x}\right) = cy$

(C) $\log\left(\frac{y}{x}\right) = cx$

(D) $\log\left(\frac{x}{y}\right) = cy$

$\frac{dy}{dx} = \frac{y}{x}\left(\ln(y/x) + 1\right)$

$\Rightarrow$ Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v(\ln v + 1)$

$\Rightarrow v + x\frac{dv}{dx} = v\ln v + v$

$\Rightarrow \int \frac{dv}{v\ln v} = \int \frac{dx}{x}$ ($\ln v = t$, $\frac{dv}{v} = dt$)

$\ln t$

$\ln|\ln v| = \ln|x| + \ln c$

$\ln|\ln v| = \ln|cx|$

$\therefore |\ln v| = |cx|$

6

JEE 1

$$\left|\ln \frac{y}{x}\right| = |cx|$$

7

If 7 divides $32^{32^{32}}$, the remainder is

(A) 1 (B) 0

(C) 4 (D) 6

JEE 1

$$\text{rem}\ \frac{32^{32^{32}}}{7}$$

$$= \text{rem}\ \frac{(35-3)^{32^{32}}}{7}$$

$$= \text{rem}\ \frac{3^{32^{32}}}{7}$$

$$= \text{rem}\ \frac{(3^{30}\cdot 3^2)^{32}}{7} \Rightarrow \frac{3^{30^{32}}\cdot 3^{2^{32}}}{7}$$

$$\Rightarrow \frac{((3^3)^{10})^{32}\cdot 3^{2^{32}}}{7}$$

$$\Rightarrow \frac{((28-1)^{10})^{32}\cdot 9^{32}}{7}$$

$$\Rightarrow \frac{9^{32}}{7} \Rightarrow \frac{3^{64}}{7}$$

$$\Rightarrow \frac{(3^3)^{21}\cdot 3}{7} = \frac{(28-1)^{21}\cdot 3}{7}$$

7

$$= \frac{-1 \cdot 3}{7}$$

$$\Rightarrow -3$$

$$\Rightarrow -3+7$$

$$\Rightarrow 4.$$

JEE 1

The solution of $\frac{1}{2} + \cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is –

(A) $x = \frac{2n\pi}{9}, n \in I, n \neq 9m, m \in I$

(B) $x = \frac{2n\pi}{9}, n \in I, n = 9m, m \in I$

(C) $x = \frac{n\pi}{9} + \frac{\pi}{2}, n \in I$

(D) $x = \frac{2n\pi}{3} + \frac{\pi}{6}, n \in I$



$(\cos x + \cos 2x + \cos 3x + \cos 4x) = -\frac{1}{2}$

$\Rightarrow \frac{\sin\left(4 \cdot \frac{x}{2}\right) \cos\left(x + (4-1)\frac{x}{2}\right)}{\sin \frac{x}{2}} = -1/2$

$\Rightarrow \frac{\sin 2x \cos 5x/2}{\sin x/2} = -1/2$

JEE 1

$$\Rightarrow \frac{1}{2}\cdot\left(\frac{2\sin 2x\cos 5x/2}{\sin x/2}\right)=-\frac{1}{2}$$

$$\Rightarrow \frac{1}{2\sin\frac{x}{2}}\left[\sin\frac{9x}{2}+\sin\left(-\frac{x}{2}\right)\right]=-\frac{1}{2}$$

$$\Rightarrow \frac{\sin 9x/2}{\sin x/2}=-1$$

$$\Rightarrow \boxed{\sin\frac{9x}{2}=0}$$

$$\frac{9x}{2}=n\pi$$

$$x=\frac{2n\pi}{9};\ n\in Z.$$

Why not $n=9m$

$$x=2m\pi$$

eg $\boxed{x=0}$ $\boxed{m=0}$

$$\frac{1}{2}+1+1+1+1\neq 0$$

9

If $\omega$ is a non-real cube root of unity then value of

$$\frac{6(5+\omega^2)}{(5+\omega)^{-1}} + \frac{7(6+\omega^2)}{(6+\omega)^{-1}} + \cdots\cdots\cdots \frac{21(20+\omega^2)}{(20+\omega)^{-1}}$$

is :

(A) 42015 (B) 44016

(C) 43015 (D) 45012

JEE 1

$\omega^3 = 1$

$1+\omega+\omega^2 = 0$

$1+\omega^n+\omega^{2n} = \begin{cases} 3 \\ 0 \end{cases}$

M3

$n = 1 \ldots 16$

$$T_n = \frac{(n+5)(n+4+\omega^2)}{(n+4+\omega)^{-1}}$$

M2

$n = 6 \ldots 21$ ✓

$$T_n = \frac{n(n-1+\omega^2)}{(n-1+\omega)^{-1}}$$

$$= n(n-1+\omega^2)(n-1+\omega)$$

$$= n\left((n-1)^2 + (n-1)(\omega+\omega^2) + \omega^3\right)$$

$$= n\left((n-1)^2 + (n-1)(-1) + 1\right)$$

$$= n\left(n^2 - 2n + 1 - n + 1 + 1\right)$$

$$= n(n^2 - 3n + 3)$$

9

JEE 1

$n = 5 \cdots 20$

M1 $T_n = \dfrac{(n+1)(n+\omega^2)}{(n+\omega)^{-1}}$

$= (n+1)(n+\omega^2)(n+\omega)$

$= (n+1)(n^2 + n(\omega+\omega^2) + \omega^3)$

$= (n+1)(n^2 - n + 1)$

$= n^3 + 1.$

$\sum_{n=5}^{20} n^3+1$

$\sum_{n=1}^{20} n^3+1 - \sum_{n=1}^{4} n^3+1$

$\Rightarrow \left(\left(\frac{20(21)}{2}\right)^2 + 20\right) - \left(\left(\frac{4\cdot 5}{2}\right)^2 + 4\right)$

$\Rightarrow 44100 - 100 + 16$

$= 44016$

10

JEE 1

Let

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right) & ;x\neq 0 \\ 0 & ;x=0 \end{cases},$$

then value of $\left[e^{3\lim_{x\to\infty} f(x)}\right]-11$ is

(Where [.] represent greatest integer function)

(A) 9 ✓ (B) 3

(C) 1 (D) 7

$\lim_{x\to\infty} f(x)$

$= \lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)$

Let $x = 1/y$ $\quad x\to\infty$ $y\to 0$

$= \lim_{y\to 0} \frac{1}{y}\sin y + \sin y^2$

$= 1 + 0 = 1$

Ans $\left[e^{3}\right]-11$

$\downarrow$

$20 - 11$

$\downarrow$

$9$

11

Let A, b, c be the sum, product and sum of the reciprocals of 37 consecutive terms of G.P. (common ratio of G.P. is not 1). If $\boxed{b^m = \left(\frac{A}{c}\right)^n}$, where m, n are natural numbers and m&n are co-prime, then the digit at units place in $(m^n + n^m)$ is

(A) 1 (B) 2

(C) 7 (D) None of these

$\frac{37-1}{2} = 18$



GP: $\frac{a}{r^{18}} \cdots \frac{a}{r^3} \quad \frac{a}{r^2} \quad \frac{a}{r} \quad a \quad ar \quad ar^2 \quad ar^3 \cdots ar^{18}$

Reci GP: $\frac{r^{18}}{a} \quad \frac{r^{17}}{a} \cdots \frac{1}{a} \quad \frac{1}{ar} \quad \frac{1}{ar^2} \cdots \frac{1}{ar^{18}}$

$$A = a + \left(\frac{a}{r} + \frac{a}{r^2} + \cdots + \frac{a}{r^{18}}\right) + (ar + ar^2 + \cdots + ar^{18}) = a\left(\frac{1}{r^{18}} + \frac{1}{r^{17}} + \cdots + \frac{1}{r} + 1 + r + \cdots r^{17} + r^{18}\right)$$

$$b = a^{37}$$

$$c = \frac{1}{a} + \left(\frac{1}{ar} + \frac{1}{ar^2} + \cdots + \frac{1}{ar^{18}}\right) + \left(\frac{r}{a} + \frac{r^2}{a} + \cdots + \frac{r^{18}}{a}\right) = \frac{1}{a}\left(\frac{1}{r^{18}} + \cdots \frac{1}{r} + 1 + r + \cdots + r^{18}\right)$$

$b^m = \left(\frac{A}{C}\right)^n$

$(a^{37})^m = \left(\frac{a}{1/a}\right)^n$

$a^{37m} = a^{2n}$

$\therefore 37m = 2n$

$m = 2$
$n = 37$

$2 \rightarrow 4 \rightarrow 8 \rightarrow 6 \rightarrow 2$

$2^{37} + 37^2$

$\underline{\quad 2 \quad} \quad \underline{\quad 9 \quad}$

$\underline{\qquad\qquad 1}$

12

Let A be the set of all real number. Then the relation $R = \{(a, b): a^4 - b^4 \geqslant 0\}$ on A is

(A) Reflexive and symmetric but not transitive

(B) Reflexive, transitive and symmetric

(C) Symmetric, transitive but not reflexive

(D) Reflexive and transitive but not symmetric

Ref : $a^4 - a^4 \geq 0$

Sym $a^4 - b^4 \geq 0 \Rightarrow a^4 \geq b^4$

$b^4 - a^4 \leq 0 \Rightarrow b^4 \leq a^4$.

Trans $a^4 \geq b^4$, $b^4 \geq c^4$

$\Rightarrow a^4 \geq c^4$

13

The domain of definition of the function

$$f(x) = \sqrt{\left[\sin^{-1}\left(\frac{x^2+1}{2x}\right)\right]} + \cos^{-1}\sqrt{\log_{[x]+1}\left(\frac{|x|}{x}\right)} = 1.$$

(where [.] denotes greatest integer function)

(A) $(0,1]$ (B) $\{-1,1\}$

(C) $\{1\}$ (D) $\phi$



① $x \neq 0$

$[x]+1 > 0$

$1+[x] \neq 1$

$[x] \neq 0$

$x \in [1,\infty)$

$\therefore |x| = x$

$\cos^{-1}\sqrt{\log_{[x]+1}|} = \cos^{-1} 0 = \pi/2$

$\frac{x^2+1}{2x} = \frac{1}{2}\left(x+\frac{1}{x}\right)$

$\geq 2$

$\geq 1$

$\sin^{-1}(\geq 1)$

$\therefore x = 1$

14

If $\lambda = \lim_{x\to 0} \frac{x\left(1-\frac{11}{2}\cos x\right)+\frac{9}{2}\sin x}{\sin^3 x}$, then

value of $\lim_{x\to\lambda} \frac{\sqrt{x^2-\lambda^2}}{\sqrt{x-\lambda}+\sqrt{x-\lambda}} = k$, then the value of $k^2$ is

(A) 2 (B) 4

(C) 1 (D) 1/2

$$\lambda = \lim_{x\to 0} \frac{x\left(1-\frac{11}{2}\left(1-\frac{x^2}{2!}\cdots\right)\right)+\frac{9}{2}\left(x-\frac{x^3}{3!}\cdots\right)}{x^3}$$

$$\frac{x\left(\left(1-\frac{11}{2}\right)+\frac{11}{4}x^2\cdots\right)+\frac{9}{2}x\left(1-\frac{x^2}{6}\cdots\right)}{x^2}$$

$$\lambda = \frac{\frac{11}{4}-\frac{9}{2}}{1} = 2$$

14

JEE 1

$$K = \lim_{x \to 2} \frac{\sqrt{x^2 - 2^2}}{\sqrt{x-2} + \sqrt{x} - \sqrt{2}}$$



$$\frac{\frac{2x}{2}\cdot\sqrt{x^2-4}}{\frac{1}{2\sqrt{x-2}} + \frac{1}{2\sqrt{x}}} = 2\left(\frac{x}{\sqrt{x^2-4}}\right)\frac{\sqrt{x}\sqrt{x-2}}{(\sqrt{x-2}+\sqrt{x})}$$

$$= 2\,\frac{x^{3/2}\sqrt{x-2}}{\sqrt{(x+2)(x-2)}\,(\sqrt{x-2}+\sqrt{x})}$$

$$= \frac{2\cdot 2^{3/2}}{2\cdot\sqrt{2}} = 2$$

15

Let $f: R \to R$ be a function defined by $f(x) = [x+2] + [x]^2 - 8$ (where [.] denotes GIF)

(A) One-one onto (B) Many-one onto

(C) One-one into (D) Many-one into

$f(x) = \text{int}$

Range $\in Z$

Codom $\in R$

$\therefore$ into.

$f(x) = [x]^2 + [x] + 2 - 8$

$= [x]^2 + [x] - 6$

$= ([x]+3)([x]-2)$

M1

16

JEE 1

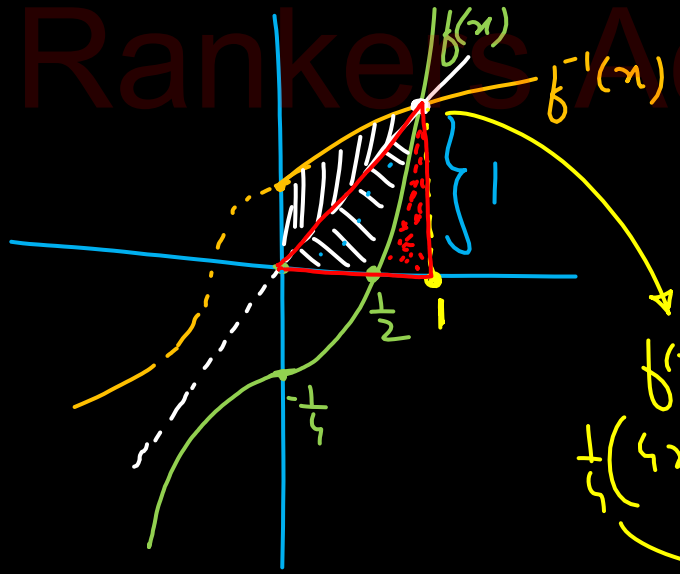
Let $f(x) = \frac{1}{4}(4x^3 + x - 1)$ is bijective function. If the area bounded by the curve $f(x)$, $f^{-1}(x)$, x-axis and y-axis in first quadrant is $\lambda$, then $16\lambda$ equals

(A) 19 (B) 19/2

(C) 19/4 (D) 9/2

$f'(x) = \frac{1}{4}(12x^2 + 1) > 0$ $f''(x) = \frac{1}{4}(24x)$

$f(x) \uparrow$




$y = f(x)$

$0 = \frac{1}{4}(4x^3 + x - 1)$

$x = \frac{1}{2} : \frac{1}{4}\left(4\left(\frac{1}{8}\right) + \frac{1}{2} - 1\right)$

$f(x) = x$

$\frac{1}{4}(4x^3 + x - 1) = x \Rightarrow 4x^3 + x - 1 = 4x$

$4x^3 - 3x - 1 = 0 \rightarrow x = 1$

16

JEE 1

$$A = 2\left[\frac{1}{2}(1\times 1) - \int_{1/2}^{1} f(x)\,dx\right]$$



$$A = \frac{19}{32}$$

$$16A = \frac{19}{2}$$

17

If the value of the integral $I = \int \frac{\sin 3x + 6\cos 3x}{10e^{-x/2} + 2\sin 3x} dx$ is equal to $\ln|\lambda + e^{\mu x}\sin \gamma x| + C$, (where C is any arbitary constant and $\lambda, \gamma \in N$ ), then the value of $\lambda + \mu + \gamma$ is

(A) 17 (B) 17/2

(C) 8 (D) 17/4

$$\int \frac{(\sin 3x)e^{x/2} + (6\cos 3x)e^{x/2}}{\left[10 + (2\sin 3x)e^{x/2}\right]} dx$$

$$t = 10 + (2\sin 3x)e^{x/2}$$

$$dt = \left[(2\cos 3x \times 3)e^{x/2} + (2\sin 3x)\left(\frac{1}{2}e^{x/2}\right)\right]dx$$

$$= \left[(6\cos 3x)e^{x/2} + (\sin 3x)e^{x/2}\right]dx$$

17

JEE 1

$$\int \frac{dt}{t}$$

$$\ln|t| + c$$

$$\ln\left|10 + (2\sin 3x)e^{x/2}\right| + c$$

$$\ln\left|2\cdot\left(5 + \sin 3x\, e^{x/2}\right)\right| + c$$

$$\ln\left|5 + (\sin 3x)e^{x/2}\right| + c'$$

$$\lambda = 5$$

$$\mu = \frac{1}{2}$$

$$\gamma = 3$$

$$\lambda + \mu + \gamma = 8.5$$

$$= \frac{17}{2}$$

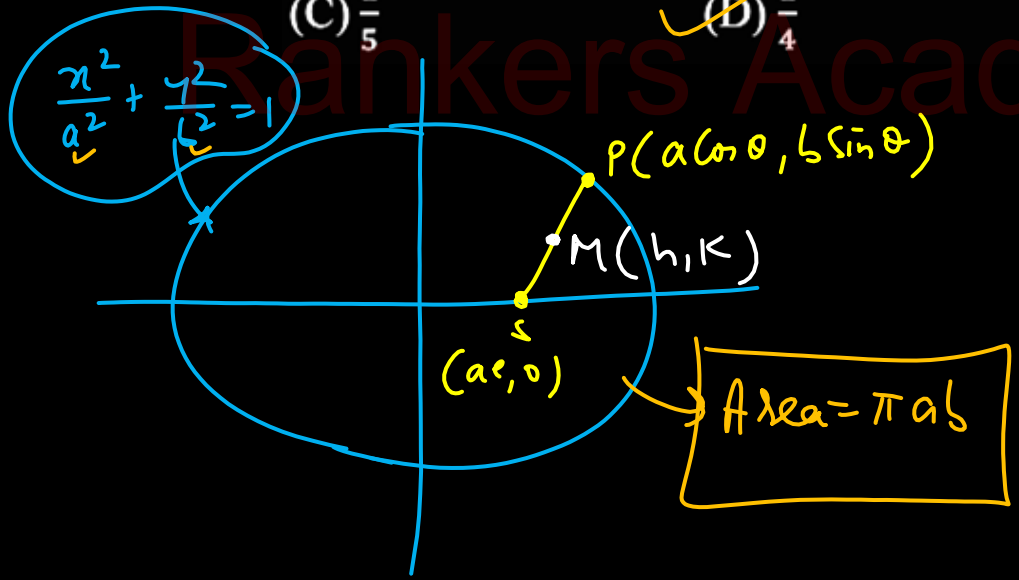
18

The ratio of the area enclosed by the locus of mid-point of PS and area of the ellipse where P is any point on the ellipse and S is the focus of the ellipse, is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{5}$ (D) $\frac{1}{4}$






Area = πab

$$M \equiv \begin{cases} h = \frac{a\cos\theta + ae}{2} \\ K = \frac{b\sin\theta + 0}{2} \end{cases}$$

$$\left(\frac{2h-ae}{a}\right)^2 + \left(\frac{2K}{b}\right)^2 = 1$$

$$\frac{\left(x-\frac{ae}{2}\right)^2}{\left(\frac{a}{2}\right)^2} + \frac{y^2}{\left(\frac{b}{2}\right)^2} = 1$$

18

JEE 1

Locus → Area = $\pi\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)$

$= \frac{\pi ab}{4}$

19

The locus of a point which moves such that its distance from the line $3x - 2y + 5 = 0$ is same as its distance from the point $(-1,1)$ is

(A) $2x + 3y - 1 = 0$

(B) $2x - 3y + 1 = 0$

(C) $2x + 3y = 0$

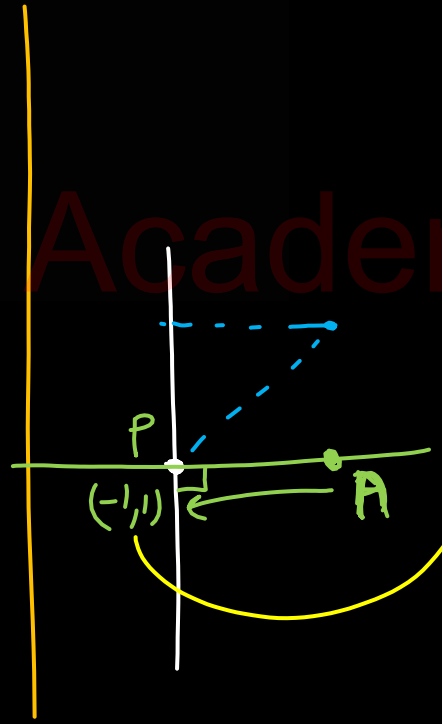
(D) $2x + y = 5$

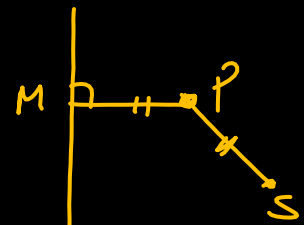
JEE 1



$L \equiv 3x - 2y + 5 = 0$

$P \equiv (-1,1)$ satisfy




$2x + 3y + \lambda = 0$

$2(-1) + 3(1) + \lambda = 0$

$\lambda = -1$

20

Let $A = \{1,2,3,4,5\}$. The number of different ordered pairs $(B, C)$ that can be formed such that $B \subseteq A$, $C \subseteq A$ and $B \cap C$ is not empty, is

(A) 175 (B) 781

(C) 243 (D) 1024

21

If $\alpha, \beta$ are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to



$x^2 + (5\sqrt{2})x + 10 = 0$

$P_n = \alpha^n - \beta^n$

(1) $P_n + (5\sqrt{2})P_{n-1} + (10)P_{n-2} = 0$

$P_n + 5\sqrt{2}P_{n-1} = -10P_{n-2}$

$$\frac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})}$$

$$= \frac{P_{17}(-10P_{18})}{P_{18}(-10P_{17})}$$

$= 1$

22

JEE 1

If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$, then value of

$\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is



$A \longrightarrow adjA$ = transpose of cofactor matrix

$|adjA| = |\text{cofactor matrix}|$

$|A| = 3$

$\Downarrow$

$|adjA| = |A|^{n-1}$

$= |A|^2$

$= (3)^2$

$= 9$

23

Let the hyperbola H: $\frac{x^2}{a^2} - y^2 = 1$ and the ellipse E: $3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If $e_H$ and $e_E$ are the eccentricities of H and E respectively, then the value of $2(e_H^2 + e_E^2)$ is equal to ____

7



JEE 1

$H: \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

$E: \frac{x^2}{4} + \frac{y^2}{3} = 1$

$\frac{2(1)}{a} = \frac{2(3)}{(2)}$

$a = 2/3$

$e_H = \sqrt{1 + \frac{1}{4/9}} = \sqrt{1 + \frac{9}{4}} = \sqrt{\frac{13}{4}}$

$e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$2(e_H^2 + e_E^2) = 2\left(\frac{13}{4} + \frac{1}{4}\right) = 2\left(\frac{14}{4}\right) = 7$

24

JEE 1

Let $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + \hat{j}, \bar{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to $\frac{6\sqrt{3}}{\lambda}$, then the value of $\lambda$ is

$|\bar{c} - \bar{a}|^2 = (2\sqrt{2})^2$

$\bar{a} \cdot \bar{c} = |\bar{c}|$

$|\bar{c}|^2 + |\bar{a}|^2 - 2(\bar{a} \cdot \bar{c}) = 8$

~~$|\bar{a}||\bar{c}|\cos\theta = |\bar{c}|$~~

$|\bar{c}|^2 + |\bar{a}|^2 - 2|\bar{c}| = 8$

$|\bar{a}|\cos\theta = 1$

$|\bar{c}|^2 + 9 - 2|\bar{c}| = 8$

$|\bar{c}|^2 - 2|\bar{c}| + 1 = 0$

$(|\bar{c}| - 1)^2 = 0$

$|\bar{c}| = 1$

Now:

$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$= \hat{i}(2) - \hat{j}(2) + \hat{k}(1)$

$\therefore |\bar{a} \times \bar{b}| = \sqrt{4+4+1} = 3$

24

$$|(\bar{a}\times\bar{b})\times\bar{c}| = |\bar{a}\times\bar{b}|\,|\bar{c}|\sin\left(\frac{\pi}{3}\right)$$

$$= 3\times 1\,\frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2}$$

$$= \frac{6\sqrt{3}}{4} \longrightarrow \lambda = 4$$


JEE 1

Let $f(x) = \left[x - \frac{1}{4}\right] + x[x] + |x(x - 4)\sin x| + (2x-1)^{1/3}$. Find the number of points in $(0, 2\pi)$ where $f(x)$ is non-derivable. → 15

→ $(0, 6.28)$

[Note: [ . ] denotes the greatest integer function.]



$f(x) = \left[x - \frac{1}{4}\right] + x[x] + |x(x-4)\sin x| + (2x-1)^{1/3}$

Discont. Discont. Cont. Cont.

$\left[x - \frac{1}{4}\right] \rightarrow \left\{\frac{1}{4}, 1+\frac{1}{4}, 2+\frac{1}{4}, 3+\frac{1}{4}, 4+\frac{1}{4}, 5+\frac{1}{4}, 6+\frac{1}{4}\right\}$

$[x] \rightarrow \{1, 2, 3, 4, 5, 6\}$

25

JEE 1

$(2x-1)^{1/3} \longrightarrow \{\frac{1}{2}\}$.



$|x(x-4)\sin x| \Rightarrow |x|\,|x-4|\,|\sin x|$

Non-Diff: ~~$x=4$~~ (already included)

$x=0$ ✗

$x=\pi$ ✓