

Rankers Academy JEE

(1001CJA101021240024)

Test Pattern



CLASSROOM CONTACT PROGRAMME (Academic Session : 2024 - 2025)

JEE (Main)

PART TEST

01-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	A	A	A	A	A	A	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	C	A	A	A	C	D	A	B
SECTION-II	Q.	1	2	3	4	5					
	A.	3	7	2	21	7					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	B	D	C	A	A	B	D	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	C	D	A	C	C	D	C
SECTION-II	Q.	1	2	3	4	5					
	A.	80	4	7	6	60					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B	A	C	B	A	B	B	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	A	C	A	A	B	B	C	B
SECTION-II	Q.	1	2	3	4	5					
	A.	5	2	3	5	1					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (C)

Radius of circular path described by a charged

particle in a magnetic field is given by

$$r = \frac{\sqrt{2mK}}{qB};$$

Where K = Kinetic energy of electron

$$\Rightarrow K = \frac{q^2 B^2 r^2}{2m} = \left(\frac{e}{m}\right) \frac{e B^2 r^2}{2}$$

$$= \frac{1}{2} \times 1.7 \times 10^{11} \times 1.6 \times 10^{-19} \times \left(\frac{1}{\sqrt{17}} \times 10^{-5}\right)^2 \times (1)^2$$

$$= 8 \times 10^{-20} \text{ J} = 0.58 \text{ eV}$$

$$\text{By using } \Rightarrow W_0 = E - K_{\max}$$

$$= \left(\frac{12375}{2475}\right) \text{ eV} - 0.5 \text{ eV} = 4.5 \text{ eV}$$

2. Ans (B)

Given,

$$m_y = 2m_x$$

$$\Rightarrow Z_y = 2Z_x$$

$$\left(\because \left(\frac{A}{Z} \right)_x = \left(\frac{A}{Z} \right)_y = 2 \Rightarrow n = p \text{ for both } x \text{ and } y \right)$$

$$\text{Let } Z_x = Z \text{ and } Z_y = 2Z$$

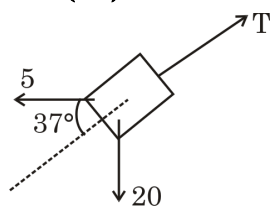
$$\text{Energy of first line balmer} \rightarrow \frac{13.6 \times Z^2 \times 5}{36}$$

$$\therefore \frac{13.6 \times 5}{36} \frac{13.6 \times 5}{36} \times [(2Z)^2 - Z^2] = \frac{17}{3}$$

$$\Rightarrow Z = 1$$

3. Ans (A)

Both A and R are true and R is the correct explanation of A.

5. Ans (A)


$$T = 5 \cos 37^\circ + 20 \cos 53^\circ = 16 \text{ N}$$

$$Y = \frac{F/A}{\frac{\Delta L}{L}}$$

$$\Rightarrow \Delta L = \frac{FL}{YA} = 8 \times 10^{-5} \text{ m}$$

6. Ans (A)

$$VT^{-2} = \text{constant}$$

$$\Rightarrow PV^{\frac{1}{2}} = \text{constant}$$

$$Q = \frac{nR\Delta T}{\gamma - 1} + \frac{nR\Delta T}{1 - \gamma}$$

$$= \frac{nR50}{\frac{7}{5} - 1} + \frac{nR50}{1 - \frac{1}{2}}$$

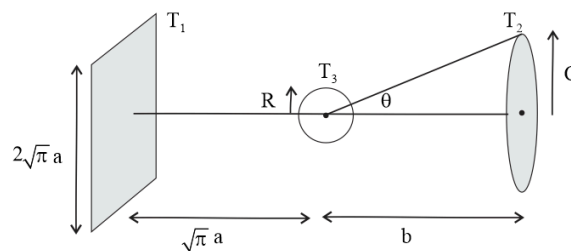
$$= 225R$$

7. Ans (A)

$$\text{Thermal resistance of rod A is } R_1 = \frac{3L}{4KA}$$

$$\text{similarly, } R_2 = \frac{2L}{3KA} \text{ and } R_3 = \frac{5L}{K_C A}$$

$$\text{If resistance of rod C is } R_3 \text{ then } \frac{R_1 + R_2}{R_3} = \frac{2}{3}$$

8. Ans (A)


$$P_{\text{total}} = \sigma (4\pi R^2) T^4$$

$$P_{\text{cross-section}} = P_{\text{square}} + P_{\text{circle}}$$

$$= \frac{1}{6} \{ \sigma (4\pi R^2) T^4 \} + \frac{2\pi}{4\pi} (1 - \cos \theta)$$

$$\{ \sigma (4\pi R^2) T^4 \}$$

$$= \left(\frac{1}{6} + \frac{1}{10} \right) \{ \sigma (4\pi R^2) T^4 \}$$

$$\therefore \frac{P_{\text{cross-section}}}{P_{\text{total}}} = \frac{4}{15}$$

9. Ans (D)

$$\text{In process CA } PV^{-2} = \text{constant}$$

$$\Rightarrow TV^{-3} = \text{constant}$$

$$T_A = T_C \times \left(\frac{V_A}{V_C} \right)^3 = 2400K$$

$$W_{CA} = \frac{nR(T_A - T_C)}{1 - (-2)} = 700R$$

In process AB (Isobaric),

$$T_B = T_A \times \frac{V_B}{V_A} = 1200K$$

$$W_{AB} = nR(T_B - T_A) = -1200R$$

$$\Delta U_{BC} = nC_V(T_C - T_B) = -2250R$$

$$Q_{AB} = nC_P(T_B - T_A) = -4200R$$

11. **Ans (C)**

$$\frac{dN}{dt} = \alpha t^3 - \lambda N$$

At $t = t_1$, 'N' is minimum.

$$\Rightarrow N_{t_1} = \frac{\alpha t_1^3}{\lambda} = \frac{\alpha t_1^3}{3}$$

At $t = t_2$, $\frac{dN}{dt}$ is minimum,

$$\frac{d^2N}{dt^2} = 3\alpha t^2 - \lambda \frac{dN}{dt}$$

$$= 3\alpha t^2 - \lambda (\alpha t^3 - \lambda N) = 0$$

$$\Rightarrow \frac{\lambda \alpha t_2^3 - 3\alpha t_2^2}{\lambda^2} = N_{t_2}$$

$$\Rightarrow \frac{3\alpha (t_2^3 - t_2^2)}{9} = N_{t_2}$$

$$\therefore \frac{N_{t_1}}{N_{t_2}} = \frac{t_1^3}{t_2^3 - t_2^2}$$

12. **Ans (D)**

Inelastic collision

$$Q = \frac{1}{2} \frac{M}{2} \times V^2 = \frac{KE}{2}$$

$$KE_{\max} = 2Q_{\max} = 2 \times 13.6 = 27.2 \text{ eV.}$$

13. **Ans (C)**

$$\text{K.E.} = 2E_0 - E_0 = E_0 \text{ (for } 0 \leq x \leq 1)$$

$$\Rightarrow \lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

$$\text{K.E.} = 2E_0 \text{ (for } x > 1)$$

$$\Rightarrow \lambda_2 = \frac{h}{\sqrt{4mE_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

14. **Ans (A)**

$$\text{Given, } \frac{\lambda_1}{\lambda_2} = 5 \quad \frac{M_1}{M_2} = \frac{1}{3}$$

$$\therefore \frac{P_1}{P_2} = \frac{\lambda_2}{\lambda_1} = \frac{1}{5} \Rightarrow P_2 = 5P_1$$

$$\text{Also, } P_1^2 + P_2^2 + 2P_1P_2\cos 53^\circ = P^2$$

$$\Rightarrow 26P_1^2 + 10P_1^2 = \left(\frac{3}{5}\right) = P^2$$

$$\Rightarrow 32P_1^2 + P^2 \Rightarrow P_1 = \frac{P}{\sqrt{32}}$$

$$\therefore P_2 = \frac{5P}{\sqrt{32}}$$

$$\begin{aligned} \frac{K_f}{K_i} &= \frac{\frac{P_1^2}{2M_1} + \frac{P_2^2}{2M_2}}{\frac{P^2}{2M}} \\ &= \frac{\left(\frac{P^2}{32}\right)}{2 \times \left(\frac{M}{4}\right)} + \frac{\left(\frac{25P^2}{32}\right)}{2 \times \left(\frac{3M}{4}\right)} \\ &= \frac{\left(\frac{P^2}{2M}\right)}{\left(\frac{P^2}{2M}\right)} \end{aligned}$$

$$= 4 \left[\frac{1}{32} + \frac{25}{3(32)} \right] = \frac{7}{6}$$

$$\therefore \frac{\Delta K}{K} = \frac{K_f}{K_i} - 1 = \frac{1}{6}$$

15. **Ans (A)**

Based on theory

16. **Ans (A)**

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \Rightarrow \frac{1}{\lambda} = R \left[\frac{1}{9} - \frac{1}{25} \right]$$

$$\Rightarrow \lambda = \frac{225}{16R}$$

17. Ans (C)

'D₁' and 'D₄' in forward bias,

'D₂' and 'D₃' in reverse bias,

$$V - 10i - 0.7 - 10(i - 0.5) - 0.7 = 0$$

$$\Rightarrow i = \frac{V + 3.6}{20}$$

For zener diode,

$$\text{Given } P = 1\text{W}, V_Z = 2\text{V}$$

$$\Rightarrow R_Z = \frac{V_Z^2}{P} = 4\Omega, i_Z = 0.5\text{A}$$

\therefore Voltage across " 10Ω ",

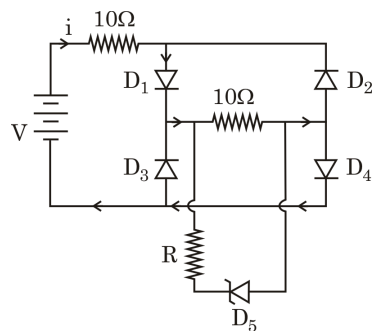
$$V_1 = i \left[\frac{10(R+4)}{(R+14)} \right]$$

$$V_1 = i = \left(\frac{V + 3.6}{20} \right) \left[\frac{10(R+4)}{(R+14)} \right]$$

$$= \left(\frac{V + 3.6}{2} \right) \left[\frac{(R+4)}{(R+14)} \right]$$

$$\text{As, } V_{\max} = 17.4\text{V}$$

$$\Rightarrow V_{1\max} = \frac{21}{2} \left(\frac{R+4}{R+14} \right)$$



For safe operation,

$$R = \frac{V_{1\max} - V_Z}{i_Z}$$

$$R = \frac{\left[\frac{21}{2} \left(\frac{R+4}{R+14} \right) \right] - 2}{0.5}$$

$$\Rightarrow R^2 - 3R - 28 = 0$$

$$\Rightarrow R = 7\Omega.$$

18. Ans (D)

$$y = \overline{(\overline{AB})} \cdot \overline{(\overline{CD})} = \overline{(\bar{A} + \bar{B})} \cdot \overline{(\bar{C} + \bar{D})}$$

$$= \overline{(\bar{A} + \bar{B})} + \overline{(\bar{C} + \bar{D})} = AB + CD$$

20. Ans (B)

$$\frac{dN_X}{dt} = -(2\lambda) N_X \Rightarrow N_X = N_0 e^{-2\lambda t}$$

$$\frac{dN_Y}{dt} = +3(2\lambda) N_X - \lambda N_Y$$

$$"N_Y" \text{ is max at } t = \frac{\ell n 10}{\lambda}$$

$$\Rightarrow 0 = 6\lambda N_X - \lambda N_{Y\max}$$

$$\Rightarrow N_{Y\max} = 6 N_X$$

$$\Rightarrow N_{Y\max} = 6 M e^{-2\lambda} \left(\frac{\ell n 10}{\lambda} \right)$$

$$\Rightarrow N_{Y\max} = \frac{6M}{100} = 0.06M$$

PART-1 : PHYSICS

SECTION-II

1. Ans (3)

$$\phi = 6 - 2 = 4\text{eV}$$

$$KE_{\max} = 2 \times 6 - 4 = 8\text{eV at emitter}$$

At collection

$$KE = 8 - 5 = 3\text{eV}$$

3. Ans (2)

$$V = A \ell$$

$$\frac{dV}{V} = \frac{dA}{A} + \frac{d\ell}{\ell}$$

$$0 = -2\mu \varepsilon + \varepsilon.$$

$$\Rightarrow \mu = \frac{1}{2}$$

4. **Ans (21)**

$$f = f_0 \frac{z^2}{n^3} \Rightarrow \frac{z^2}{n^3} = 4 \dots\dots (1)$$

$$E = E_0 \frac{z^2}{n^2} \Rightarrow \frac{z^2}{n^2} = 16 \dots\dots (2)$$

$$(1) \text{ and } (2) \Rightarrow n = 4 \text{ and } z = 16$$

$$\text{Now, } L = mvr = \frac{nh}{2\pi}$$

$$\Rightarrow \tau = \frac{\Delta L}{\Delta t} = \frac{\Delta n}{\Delta t} \frac{h}{2\pi}$$

$$= \frac{3}{(15 \times 10^{-9})} \times \frac{(2.1 \times 10^{-34})}{2}$$

$$= 2.1 \times 10^{-26} \text{ N-m}$$

5. **Ans (7)**

$$\text{Max KE converted} = \frac{1}{2} \frac{4}{5} m_x V^2$$

$$= \frac{4}{5} \times 2.5 = 2 \text{ MeV}$$

$$BE_y = \frac{8A + 5 - 2}{A + 1} = 7 \text{ MeV}$$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (C)**

$$\text{Slope of OA} = \frac{2}{\frac{1}{5.6}} = 11.2$$

$$\text{Slope of OB} = \frac{4}{\frac{1}{2.8}} = 11.2$$

If slope of OA is equal to slope of OB then AB

is isothermal reversible process.

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$= -PV \ln \frac{V_2}{V_1}$$

$$= -2 \times 5.6 \ln \frac{2.8}{5.6}$$

$$= 7.84 \text{ ltr-atm}$$

$$= 784 \text{ Joule}$$

2. **Ans (B)**

$$\Delta H = nC_p \Delta T$$

C_p is highest for XeF_4 .

(Non-linear polyatomic gas)

3. **Ans (B)**

$$\Delta H_{\text{Reaction}}^0 = (3 \times 380) - 3(150 + 250)$$

$$= 1140 - 1200$$

$$= -60 \text{ KJ/mol}$$

$$\text{Resonance energy} = \Delta H_{f(\text{exp})}^0 - \Delta H_{f(\text{theo})}^0$$

$$-120 = \Delta H_{f(\text{exp})}^0 - (-60)$$

$$\Delta H_{f(\text{exp})}^0 = -180 \text{ KJ/mol}$$

5. **Ans (C)**

$\Delta_f H^0 = 0$ for elements in their reference elemental states and for $\text{H}^+(\text{aq})$.

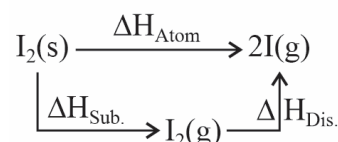
6. **Ans (A)**

$$q = n\Delta H_{\text{fusion}}$$

$$= \frac{10^3}{18} \times 9$$

$$= 500 \text{ KJ}$$

7. **Ans (A)**



$$\Delta H_{\text{Atom}} > \Delta H_{\text{Diss.}}$$

$$\Rightarrow \frac{a}{b} < 1$$

8. **Ans (B)**

NCERT

9. **Ans (D)**

At equilibrium $\Delta G = 0$, $\Delta^\circ G \neq 0$

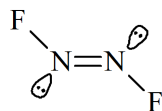
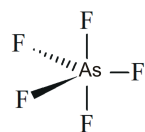
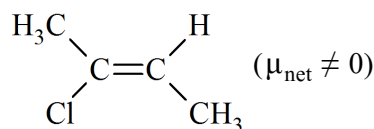
All the process which are spontaneous has

$\Delta G < 0$ and non-spontaneous has $\Delta G > 0$.

10. **Ans (B)**

$$\text{K } K^*, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2,$$

$$\pi^* 2p_y^0 = \pi^* 2p_x^0$$

11. **Ans (C)**

 XeO₄ (Tetrahedral)

 12. **Ans (C)**
 $(\Delta S)_{\text{syst}}$ for reversible adiabatic process is zero.

 as $q_{\text{rev}} = 0$

$$\Delta S = \frac{q_{\text{rev}}}{T} = 0$$

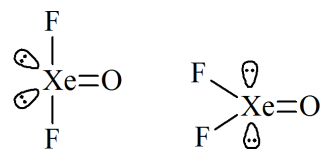
 13. **Ans (A)**

 XeOF₂

 Hybridisation sp³d

3-bond pair + 1 lone pair

Two possible structures :



(T-shape) (Trigonal planar)

 14. **Ans (C)**
 σ - molecular orbitals are symmetric whereas π -molecular orbitals are unsymmetric about the inter-nuclear axis.

 15. **Ans (D)**
 $Q_{\text{syst}} = q$ then $Q_{\text{surr}} = -q$
 $(\Delta G)_{\text{syst}} < 0$ for all process occurring spontaneously.

 16. **Ans (A)**
 $\text{HA} + \text{NaOH} \rightarrow \text{NaA} + \text{H}_2\text{O}$
 $\Delta H = -13.7 + 6.7$
 $= -7 \text{ Kcal}$

Energy release = 7 Kcal

 17. **Ans (C)**

 Basicity of acid is number of replaceable H⁺ ions.

 18. **Ans (C)**

Factual

 19. **Ans (D)**

 SO₂ is sp² hybridised.

 NH₃ and H₂O are sp³ hybridised.

 H₂S do not hybridise.

PART-2 : CHEMISTRY

SECTION-II

 1. **Ans (80)**

$$\Delta H_{\text{vap}} = \Delta H_{\text{sub}} - \Delta H_{\text{fus}}$$

$$= 38 - 6$$

$$= 32 \text{ kJ/mole}$$

$$\Delta H_{\text{vap}} - T_b \Delta S_{\text{vap}} = \Delta G_{\text{vap}} = 0$$

$$\therefore \Delta S_{\text{vap}} = \frac{\Delta H_{\text{vap}}}{T_b}$$

$$= \frac{8.32 \times 10^3}{300}$$

$$= 80 \text{ JK}^{-1} \text{ mol}^{-1}$$

 2. **Ans (4)**

Ans (4) : (1), (2), (3), (5) are correct.

 3. **Ans (7)**
 $PV^m = \text{constant}$

$$C_m = C_v - \frac{R}{m-1}$$

 Given $PV^{\frac{1}{2}} = K$

$$m = \frac{1}{2}$$

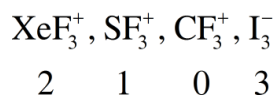
$$C_m = \frac{3}{2}R - \frac{R}{\left(\frac{1}{2} - 1\right)}$$

$$C_m = \frac{3}{2}R + 2R$$

$$C_m = \frac{7}{2}R$$

$$C_m = \frac{7}{2} \times 7 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$$C_m = 7 \text{ cal}$$

4. **Ans (6)**

 5. **Ans (60)**

$$\begin{aligned} \text{Resonance energy} &= (E)R_{S_1} - (E)R_H \\ &= -240 - (-300) \\ &= 60 \end{aligned}$$

PART-3 : MATHEMATICS

SECTION-I

 1. **Ans (D)**

$$\begin{aligned} f(x) &= A_0 + \sum_{k=1}^{20} A_k x^k = \sum_{k=0}^{20} A_k x^k \\ \therefore \sum_{r=0}^6 f(\alpha^r x) &= f(x) + f(\alpha x) + f(\alpha^2 x) + f(\alpha^3 x) \\ &\quad + f(\alpha^4 x) + f(\alpha^5 x) + f(\alpha^6 x) \\ &= \sum_{k=0}^{20} (A_k x^k + A_k (\alpha x)^k + A_k (\alpha^2 x)^k + \dots + A_k (\alpha^6 x)^k) \\ &= \sum_{k=0}^{20} \{A_k x^k (1 + \alpha^k + (\alpha^2)^k + (\alpha^3)^k + \dots + (\alpha^6)^k)\} \\ &= A_0 x^0 (7) + A_7 x^7 (7) + A_{14} x^{14} (7) = 7 \\ &\quad [A_0 + A_7 x^7 + A_{14} x^{14}] \\ \Rightarrow n &= 7 \end{aligned}$$

 2. **Ans (B)**

$$f(r) = \begin{bmatrix} 4r+1 & 4r^2+r^3 & 3r^3+r^5+r^4 \\ 3r+2 & 4r^2+r^3 & 2r^3+r^5+2r^4 \\ 4r+7 & 10r^2+r^3 & 3r^3+r^5+7r^4 \end{bmatrix}$$

$$|f(r)| = r^2 \cdot r^3 \begin{vmatrix} 4r+1 & 4+r & 3+r^2+r \\ 3r+2 & 4+r & 2+r^2+2r \\ 4r+7 & 10+r & 3+r^2+7r \end{vmatrix};$$

$$R_2 \rightarrow R_2 - R, \text{ \& } R_3 \rightarrow R_3 - R_1$$

$$|f(r)| = r^5 \begin{vmatrix} 4r+1 & 4+r & 3+r^2+r \\ -r+1 & 0 & -1+0+r \\ 6 & 6 & 0+0+6r \end{vmatrix}$$

$$|f(r)| = 0$$

 3. **Ans (A)**

$$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0 = \sin\theta_1 + \sin\theta_2 + \sin\theta_3$$

centroid and circum centre are origin.

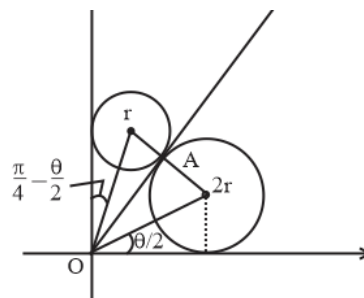
 ΔABC is equilateral. ortho centre is.

Also origin.

 4. **Ans (C)**

$$OA = r \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = 2r \cot \frac{\theta}{2}$$

$$\text{Let, } \tan \frac{\theta}{2} = t$$



$$\Rightarrow \frac{1+t}{1-t} = \frac{2}{t}$$

$$\Rightarrow t = \frac{-3 \pm \sqrt{17}}{2}$$

$$\therefore \tan \frac{\theta}{2} = \frac{\sqrt{17}-3}{2}$$

$$\Rightarrow a + b + c = 17 + 3 + 2 = 22$$

 5. **Ans (B)**

$$|2a - 1| = 3[a] + 2\{a\} = [a] + 2a$$

$$\text{case(i) If } a \geq \frac{1}{2}$$

$$2a - 1 = [a] + 2a$$

$$[a] = -1$$

$$a \in [-1, 0) \text{ Which is not possible}$$

$$\text{case(ii) If } a < \frac{1}{2}$$

$$-2a + 1 = [a] + 2a$$

$$4a - 1 = -[a]$$

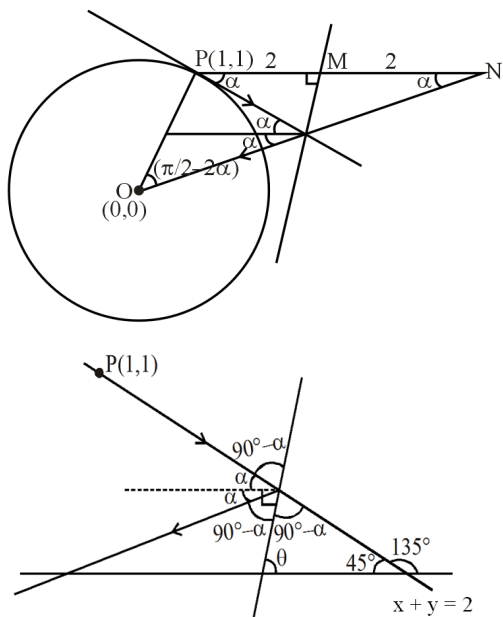
$$\text{Case-1 : } 0 \leq a < \frac{1}{2}$$

$$4a = 1$$

$$a = \frac{1}{4}$$

No further solution possible.

6. Ans (A)



$$\frac{\sin \alpha}{\sqrt{2}} = \frac{\cos(2\alpha)}{4}$$

$$\theta = 180^\circ - (90^\circ - \alpha + 45^\circ)$$

$$2\sqrt{2} \sin \alpha = 1 - 2\sin^2 \alpha$$

$$\Rightarrow \theta = 45^\circ + \alpha$$

$$2\sin^2 \alpha + 2\sqrt{2} \sin \alpha - 1 = 0$$

$$\sin \alpha = \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} + 4}{4} = -\frac{1}{\sqrt{2}} + 1$$

$$\sin \alpha = \left(\frac{-1 + \sqrt{2}}{\sqrt{2}} \right)$$

7. Ans (B)

Shaded area is the region traced by P,

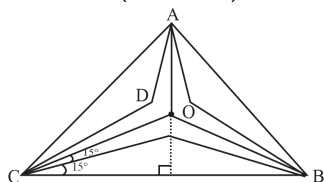
its area =

$$\Delta ABC - 3\Delta ADC$$

$$= \frac{\sqrt{3}}{4} a^2 - 3 \left(\frac{a}{2} \times \frac{a}{2} \tan 15^\circ \right)$$

$$= \frac{\sqrt{3}}{4} a^2 - \frac{3}{4} a^2 \tan 15^\circ$$

$$= \frac{\sqrt{3}}{2} a^2 \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right)$$



8. Ans (B)

$$C_3 \rightarrow C_3 + C_2 - C_1$$

 Expand along C_3

$$f(n) = \frac{1}{(n+1)(n+2)^2} - \frac{1}{(n+2)(n+3)^2},$$

$$\text{So } \sum_{n=1}^{\infty} f(n) = \left(\frac{1}{2 \cdot 3^2} - \frac{1}{3 \cdot 4^2} \right) + \left(\frac{1}{3 \cdot 4^2} - \frac{1}{4 \cdot 5^2} \right) + \dots + \left(\frac{1}{(n+1)(n+2)^2} - \frac{1}{(n+2)(n+3)^2} \right)$$

$$\text{So, for } n = 7 \sum_{n=1}^7 f(n) = \frac{49}{900}$$

$$\text{For } n \rightarrow \infty \sum f(n) = \frac{1}{18}$$

9. Ans (A)

$$(A) \log_2 \frac{4}{5} \cdot \log_2 20 + (\log_2 5)^2$$

$$= (2 - \log_2 5)(2 + \log_2 5) + (\log_2 5)^2$$

$$= 4 - (\log_2 5)^2 + (\log_2 5)^2 = 4$$

$$(B) \log_6 \frac{2^{x+3}}{3^x - 2} = x$$

$$\Rightarrow 8 \cdot 2^x = 6^x \cdot (3^x - 2)$$

$$\Rightarrow 8 = (3^x)^2 - 2 \cdot 3^x$$

$$\Rightarrow (3^x - 4)(3^x + 2) = 0$$

$$\Rightarrow 3^x = 4 \Rightarrow x = \log_3 4 = a$$

$$\Rightarrow 9^{\log_3 4} = 16$$

$$(C) \log_6^3 4 + 6 \log_6 4 \cdot \log_6 9 + \log_6^3 9$$

$$= (\log_6 4 + \log_6 9)^3 = 2^3 = 8$$

$$(D) (2a + 1)^2 + (3b - 1)^2 + (c - 2)^2 = 0$$

$$\Rightarrow a = -\frac{1}{2}, b = \frac{1}{3}, c = 2$$

$$\Rightarrow 4a + 3b + c = -2 + 1 + 2 = 1$$

10. Ans (B)

$$z^3 + (-\alpha z_1)^3 + ((\alpha - 1)z_2)^3 = 3z(-\alpha z_1)((\alpha - 1)z_2)$$

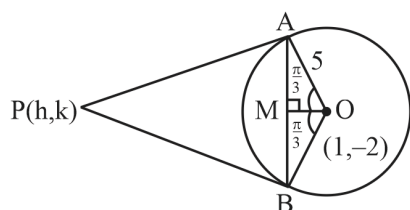
$$\Rightarrow z - \alpha z_1 + (\alpha - 1)z_2 = 0$$

$$z = \frac{\alpha z_1 + (1 - \alpha)z_2}{\alpha + (1 - \alpha)}$$

z lies on line joining z_1 and z_2 .

$|z|_{\min}$ = perpendicular distance of origin from line

joining z_1 and z_2 .

11. Ans (C)


eq. of circle $x^2 + y^2 - 2x + 4y - 20 = 0$

eq. of chord of central form point $P(h, k)$ to the circle is.

$$T = 0$$

$$\Rightarrow hx + ky - (h + x) + 2(k + y) - 20 = 0$$

$$\Rightarrow (h - 1)x + (k + 2)y - h + 2k - 20 = 0$$

In $\triangle OAM$,

$$\cos \frac{\pi}{3} = \frac{OM}{OA}$$

$$\Rightarrow OM = \frac{5}{2}$$

$$\Rightarrow \frac{|(h - 1) - 2(k + 2) - h + 2k - 20|}{\sqrt{(h - 1)^2 + (k + 2)^2}} = \frac{5}{2}$$

$$\Rightarrow (h - 1)^2 + (k + 2)^2 = 100$$

$$\Rightarrow \text{Locus is } (x - 1)^2 + (y + 2)^2 = 100$$

$$\text{Radius} = 10$$

12. Ans (C)

$$\text{Here } ABA^T = AB^T A^T = I$$

$$\text{So } C = A(ABA^T)^{25} (AB^T A^T)^{10} A^T = AA^T$$

$$= \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow \text{Trace of matrix } C = 6$$

13. Ans (A)

$$\log_2 a \cdot \log_2 2a - \log_2 2a$$

$$= \log_2 c \cdot \log_2 8c + \log_2 8c$$

$$\Rightarrow (\log_2 a - 1) \log_2 2a = \log_2 8c \cdot (\log_2 c + 1)$$

$$\Rightarrow (\log_2 a - 1)(\log_2 a + 1) = (\log_2 c + 3)(\log_2 c + 1)$$

$$\Rightarrow \log_2^2 a - 1 = \log_2^2 c + 4\log_2 c + 3$$

$$\Rightarrow (\log_2 a)^2 = (\log_2 c + 2)^2$$

$$\log_2 a = \log_2 c + 2 \Rightarrow \frac{a}{c} = 4$$

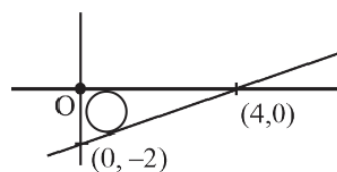
$$\text{or } \log_2 a = -2 - \log_2 c$$

$$\Rightarrow \log_2 ac = -2 \Rightarrow ac = \frac{1}{4}$$

$$\Rightarrow a, \frac{1}{2}, c \text{ are in GP.}$$

14. Ans (C)

$$xy(x - 2y - 4) = 0$$



$$r = \frac{\Delta}{s} = \frac{4}{3 + \sqrt{5}}$$

$$r_1 = \frac{\Delta}{s - a} = \frac{4}{1 + \sqrt{5}}$$

$$r_2 = \frac{4}{\sqrt{5} - 1}$$

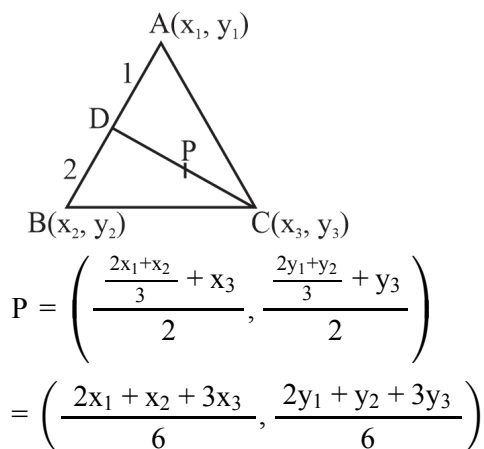
$$r_3 = \frac{4}{3 - \sqrt{5}}$$

$$\text{Sum} = \frac{1}{r} + \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{3 + \sqrt{5}}{2}$$

$$\text{so, } a = 3 \text{ \& } b = 2$$

15. Ans (A)

$$\left(\frac{2x_1 + x_2}{3} \cdot \frac{2y_1 + y_2}{3} \right)$$

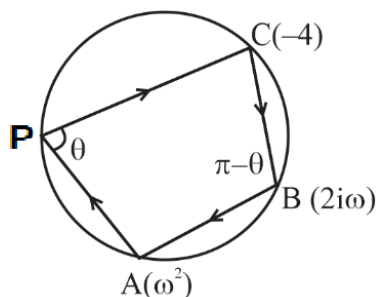


$$P = \left(\frac{\frac{2x_1+x_2}{3} + x_3}{2}, \frac{\frac{2y_1+y_2}{3} + y_3}{2} \right)$$

$$= \left(\frac{2x_1 + x_2 + 3x_3}{6}, \frac{2y_1 + y_2 + 3y_3}{6} \right)$$

 $\therefore P$ lies in side the ΔABC
 \Rightarrow Area of $\Delta PBC <$ area of ΔABC

16. Ans (A)


 Rotate \overrightarrow{PA} to get \overrightarrow{PC} and rotate \overrightarrow{BC} to get \overrightarrow{BA}

Applying rotation formula at P and B we get

$$\frac{p+4}{p-\omega^2} \times \frac{2i\omega-\omega^2}{2i\omega+4} = -1$$

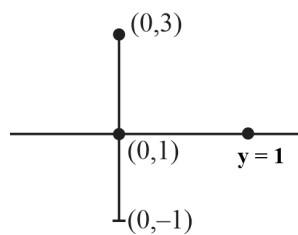
 $\Rightarrow p$

$$= \frac{2i-8i\omega+8\omega^2}{4i\omega-\omega^2+4} = \frac{2(i-4i\omega+4\omega^2)}{(-4\omega^2-1+4i\omega)} \times i\omega = -2i\omega$$

$$\text{So, } z = \frac{p+2i\omega}{2} = 0 \Rightarrow |z|^2 = 0$$

17. Ans (B)

$$|z+1| = |z-3i|$$



$$z = \alpha + i$$

$$w = z\bar{z} - 2z + 2$$

$$w = \alpha^2 + 1 - 2(\alpha + i) + 2$$

$$w = \alpha^2 - 2\alpha + 3 - 2i$$

$$\text{Re}(w) = \alpha^2 - 2\alpha + 3$$

$$= (\alpha - 1)^2 + 2$$

 at $\alpha = 1$
 $\text{Re}(w)_{\min} = 2$, so that

$$w = 2 - 2i$$

$$w = 2(1 - i)$$

$$w^n = 2^n (1 - i)^n$$

 so the minimum value of n is

$$n = 4$$

18. Ans (B)

$$\Delta_1 = 3abc - a^3 - b^3 - c^3$$

 Δ_2 is formed by cofactors of elements of Δ_1

19. Ans (C)

Equation of tangent at (6, -8) to the circle

$$x^2 + y^2 = 100 \text{ is :}$$

$$T = 0$$

$$\Rightarrow 6x - 8y = 100 \dots\dots\dots (1)$$

equation of normal at (6, -8) is

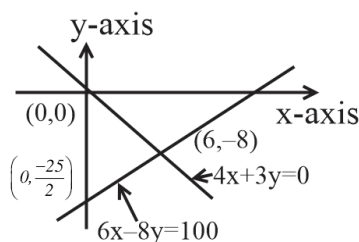
$$(y + 8) = -\frac{4}{3}(x - 6)$$

$$\Rightarrow 4x + 3y = 0 \dots\dots (2)$$

Area of triangle

$$= \frac{1}{2} \times \frac{25}{2} \times 6$$

$$= \frac{75}{2} \text{ sq units}$$


20. Ans (B)

$$A = \frac{1967 + 1686i \sin \theta}{7 - 3i \cos \theta}$$

$$= \frac{281(7 + 6i \sin \theta)}{7 - 3i \cos \theta} \times \frac{7 + 3i \cos \theta}{7 + 3i \cos \theta}$$

$$= \frac{281(49 - 18 \sin \theta \cos \theta + i(21 \cos \theta + 42 \sin \theta))}{49 + 9 \cos^2 \theta}$$

for positive integer

$$\text{Im}(A) = 0$$

$$21 \cos \theta + 42 \sin \theta = 0$$

$$\tan \theta = \frac{-1}{2}, \sin 2\theta = \frac{-4}{5}; \cos^2 \theta = \frac{4}{5}$$

$$\text{Re}(A) = \frac{281(49 - 9 \sin 2\theta)}{49 + 9 \cos^2 \theta}$$

$$= \frac{281 \left(49 - 9 \times \frac{-4}{5} \right)}{49 + 9 \times \frac{4}{5}} = 281 \text{ (+ve integer)}$$

PART-3 : MATHEMATICS
SECTION-II
1. Ans (5)

$$z = x + iy, \bar{z} = x - iy, (2iy)^2 = 12(x^2 + y^2) - 4$$

$$\Rightarrow 12x^2 + 16y^2 = 4 \Rightarrow 3x^2 + 4y^2 = 1$$

$$\Rightarrow \frac{x^2}{\frac{1}{3}} + \frac{y^2}{\frac{1}{4}} = 1$$

$$x = \sqrt{\frac{1}{3}} \cos \theta, y = \sqrt{\frac{1}{4}} \sin \theta$$

$$3\sqrt{3} \text{Re}(z) + 8 \text{Im}(z) = 3 \cos \theta + 4 \sin \theta$$

$$\max = 5$$

2. Ans (2)

Point may lie on the same side of the line or

atleast one on the line so

$$4[a^2 + 2b(a + b + c) + 4] \geq 0$$

$$a^2 + (2b)a + 2b(b + c) + 4 \geq 0$$

$$D \leq 0$$

$$4b^2 - 4[2b^2 + 2bc + 4] \leq 0$$

$$-b^2 - 2bc - 4 \leq 0$$

$$b^2 + 2bc + 4 \geq 0$$

$$D \leq 0; 4c^2 - 16 \leq 0$$

$$c^2 - 4 \leq 0$$

$$-2 \leq c \leq 2$$

$$\text{Max value of } c = 2$$

3. Ans (3)

$$\text{Hint : } XX^T = I$$

$$\text{Now, } PQ = AXB \quad BX^T A$$

$$= 3A^2$$

4. Ans (5)

$$\text{Let } f(x) = x^4 + x^3 + ax^2 + bx + c$$

$$\& x^2 + x + 1 = (x + \omega)(x + \omega^2)$$

$\therefore f(x)$ is divisible by $x^2 + x + 1$

$$f(\omega) = 0 \Rightarrow \omega + 1 + a\omega^2 + b\omega + c = 0 \dots(1)$$

$$f(\omega^2) = 0 \Rightarrow \omega^2 + 1 + a\omega + b\omega^2 + c = 0 \dots(2)$$

$$\text{eq. (1) - eq. (2)}$$

$$\Rightarrow \omega - \omega^2 - a(\omega - \omega^2) + b(\omega - \omega^2) = 0$$

$$\Rightarrow 1 - a + b = 0 \Rightarrow a - b = 1 \dots(3)$$

& eq. (1) + eq. (2)

$$\Rightarrow (\omega + \omega^2) + 2 + a(\omega + \omega^2) + b(\omega + \omega^2) + 2c = 0$$

$$\Rightarrow -a - b + 2c + 1 = 0$$

$$\Rightarrow 2c = a + b - 1 \dots(4)$$

$$\text{Now, } 5a - b - 4c = 5a - b - 2a - 2b + 2$$

$$= 3(a - b) + 2 = 3 + 2 = 5$$

5. Ans (1)

$$2x^4 + 1 - 2x^3 - x^2 = 1 - x^2 - 2x^3(1 - x) = (1 - x)$$

$$(1 + x) - 2x^3(1 - x)$$

$$= (1 - x)(x + 1 - 2x^3) = (1 - x)(x(1 - x^2) + 1 - x^3)$$

$$= (1 - x)(x(1 - x)(1 + x) + (1 - x)(1 + x + x^2))$$

$$= (1 - x)((1 - x)(x(1 + x) + 1 + x + x^2))$$

$$= (1 - x)^2((x + 1)^2 + x^2) \leq 0.$$

Equality occurs if and only if $x = 1$.