ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
TEST # 11
24-11-2024

JEE(Main+Advanced) : LEADER TEST SERIES / JOINT PACKAGE COURSE

Test Type : Review (Unit Test # 06, 07 & 08)

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	C	D	D	A	B	C	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	B	A	A	B	C	A	C	C
SECTION-II	Q.	1	2	3	4	5					
	A.	2	5	2	3	4					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	A	D	B	B	A	B	D	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	B	A	C	B	D	B	B	B
SECTION-II	Q.	1	2	3	4	5					
	A.	420	4	1	5	8					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	C	A	A	C	A	B	B	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	A	C	B	A	A	D	C	A	A
SECTION-II	Q.	1	2	3	4	5					
	A.	8	4	3	6	8					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (B)

After 1st collision, speed = (2 × 5) + 3 = 13 m/s

After 2nd collision, speed = (2 × 3) + 13 = 19 m/s

Ans. 19 m/s

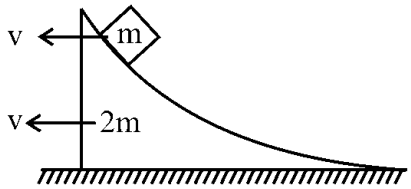
2. Ans (B)

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L (2x + x^2)\,dx}{\int_0^L (2 + x)\,dx} = \frac{\frac{2L^2}{2} + \frac{L^3}{3}}{2L + \frac{L^2}{2}}$$

Putting L = 3m ;

$$x_{cm} = \frac{12}{7}m$$

3. Ans (C)



$3mv = mu \Rightarrow v = \frac{u}{3}$

also $\frac{1}{2}mu^2 = \frac{1}{2}(3m)\left(\frac{u}{3}\right)^2 + mgh$

$\Rightarrow u = \sqrt{3gh}$

5. Ans (D)

$I_1 = I_2 = I_3 = I_4 = \frac{2}{3}mR^2$

$I_2 = I_{cm} + m(d)^2 = I_4$

$d = \frac{r}{2}$

6. **Ans (A)**

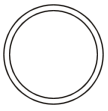
$N \times \frac{a}{2} = F \times a \sin 60$

$\frac{mg}{2} = F \times \frac{\sqrt{3}}{2}$

$F = \frac{mg}{\sqrt{3}}$

7. **Ans (B)**

$\text{Strain} = \frac{\Delta\ell}{\ell} = \left(\frac{2\pi R - 2\pi r}{2\pi r}\right)$



$\text{Strain} = \left(\frac{R-r}{r}\right)$

$Y = \frac{R/A}{\frac{\Delta\ell}{\ell}}$, $Y\frac{\Delta\ell}{\ell}A = F$

$F = AY\left(\frac{R-r}{r}\right)$

8. **Ans (C)**

$345 \times 10^6 \times 2 \times 10^{-2} \times 2 \times 10^{-2} = F = mg$

$13800\ \text{kg} = m$

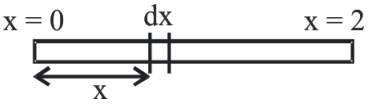
9. **Ans (A)**

Rate of temp change

$\frac{dT}{dt} = -\frac{e\sigma A}{ms}T_s^3(T - T_s)$

Gives temperature as function of time.

11. **Ans (C)**



Take element of length dx at distance x from one end.

Let dy is the expansion of length dx then $dy = dx\alpha\Delta T$

Integrate from x = 0 to x = 2 for net expansion.

12. **Ans (D)**

$B = \frac{\mu_0 Ni}{2\pi R}$

$\phi = \pi b^2 \times B \times N$

$\phi = Li$

$L = \frac{\phi}{i} = \frac{\mu_0 N^2 b^2}{2R}$ with $b <<< R$

$\text{energy} = \frac{1}{2}Li^2 = \frac{\mu_0 N^2 I^2 b^2}{4R}$

13. **Ans (B)**

$\text{K.E.} = eV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$p = \sqrt{2mKE} = \sqrt{2meV}$

$r = \frac{mv}{qB} = \frac{m}{eB} \times \sqrt{\frac{eV \times 2}{m}} = \sqrt{\frac{2vm}{eB^2}}$

14. **Ans (A)**

$DA = 2\cos 30^0\hat{i} - 2\sin 30^0\hat{k} = (-\sqrt{3}\hat{i} - \hat{k})$

$\overrightarrow{AB} = 2\hat{j}$ $\therefore \overrightarrow{M} = i(\overrightarrow{DA} \times \overrightarrow{AB})$

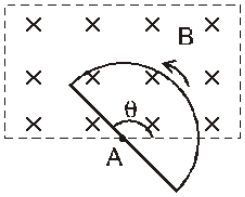
$= \frac{1}{2}\left[(-\sqrt{3}\hat{i} - \hat{k}) \times (2\hat{j})\right]$

$= -\sqrt{3}\hat{k} + \hat{i} = (i - \sqrt{3}\hat{k})A - m^2$

15. **Ans (A)**

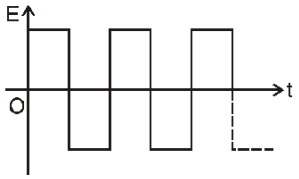
The flux through loop = $f = B(\frac{1}{2}r^2\theta)$

$\therefore$ Induced emf in loop $= \frac{d\varphi}{dt} = \frac{1}{2}Br^2\omega$

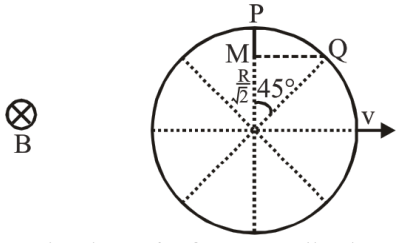


$\because \omega$ = constant, emf shall be constant in magnitude. Since magnetic flux increases for halfcycle and decreases for the other half. Hence emf changes sign every half cycle.

$\therefore$ The correct graph is



16. **Ans (B)**



Projection of PQ perpendicular to velocity is PM

$= R - \frac{R}{\sqrt{2}}$

emf across PQ $= Bv\left(R - \frac{R}{\sqrt{2}}\right)$

17. **Ans (C)**

Due to induced electric field

19. **Ans (C)**

$$\text{p.f.} = \cos\phi = \frac{1}{\sqrt{2}} = \frac{R}{Z}$$

20. **Ans (C)**

$\tau = RC$,

$\therefore \omega = \frac{1}{RC}$;

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = \sqrt{2}R$$

PART-1 : PHYSICS

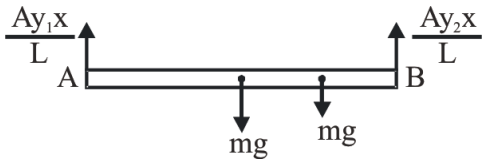
SECTION-II

1. **Ans (2)**

$$\varepsilon = \frac{d\phi}{dt} = \frac{d\left(BN\pi R^2 \cos\omega t\right)}{dt}$$

$$i = \frac{\varepsilon}{\eta}$$

2. **Ans (5)**

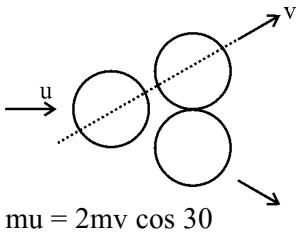


$$\frac{Ax}{L}(Y_1 + Y_2) = (M + m)\,g$$

$$\left(\frac{Ay_1x}{L}\right)\frac{\ell}{2} + mg \times \frac{\ell}{4} = \left(\frac{Ay_2x}{L}\right)\frac{\ell}{2}$$

$$\frac{Y_1}{Y_2} = \frac{2M + m}{2M + 3m}$$

3. **Ans (2)**



$mu = 2mv\cos 30$

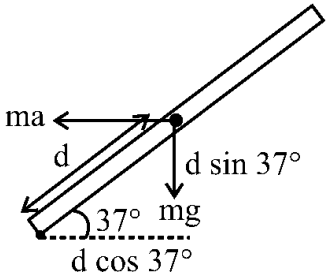
$$v = \frac{u}{\sqrt{3}}$$

$$\text{Apply} = e = \frac{v_2 - v_1}{u_1 - u_2}$$

4. **Ans (3)**

$$Q = \int mSdT = \frac{mT^4}{5} \Rightarrow \frac{Q}{m} = \frac{15}{5} = 3$$

5. **Ans (4)**



with respect to O,

Net $\tau = 0$

$ma\ d\sin 37 = mg\ d\cos 37$

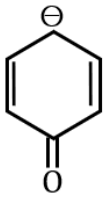
then $F = ma$

PART-2 : CHEMISTRY

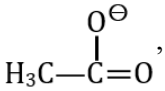
SECTION-I

1. **Ans (B)**

More Stable due to Resonance.



2. **Ans (D)**

In 

, negative charge resonates on more electronegative oxygen atom so, D is most stable.

3. **Ans (A)**

(I) 2° carbocation.

(II) 3° carbocation.

(III) 1° carbocation.

(IV) 2° and resonace stabilised carbocation.

(V) 3° and resonance stabilized carbocation.

Therefore, correct stability order for given carbocations is : V > IV > II > I > III

7. **Ans (A)**

$NaBH_4$ reduces reactant to

$$CH_3-\underset{\underset{OH}{|}}{C}H-CH_2-CH_2-CH_2-CO_2H$$

which forms ester.

17. **Ans (D)**

$\frac{x}{m} = kP^{\frac{1}{n}}$ (Freundlich Adsorption isotherm)

$$\frac{x}{m} = ?$$

$$\text{Log}\ \frac{x}{m} = \log k + \frac{1}{n}\log P \qquad k = 10$$

$$\text{Slope} = \frac{1}{n} = \frac{1}{2}$$

$$\boxed{\frac{x}{m} = 7}$$

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (420)**

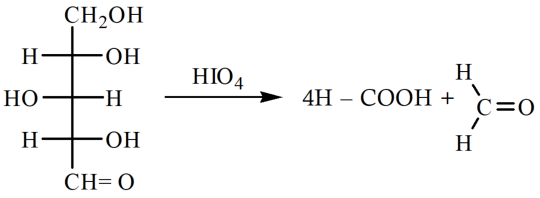
$P_{gas} = 760$

$$\text{final pressure when volume is doubled} = \frac{760}{2} + 40$$

$$= 380 + 40$$

$$= 420$$

3. **Ans (1)**



5. **Ans (8)**

$SrF_{2(s)} \rightleftharpoons Sr^{2+}_{(aq)} + 2F^{\Theta}_{(aq)}$

$$- \qquad s_1 \qquad 2s_1+0.1$$

$$k_{sp} = s_1 \times (2s_1+0.1)^2$$

$$= s_1 \times (0.1)^2 \ (2s_1 + 0.1 \approx 0.1)$$

$$1 \times 10^{-10} = s_1 \times (0.1)^2$$

$$s_1 = 10^{-8} = 1 \times 10^{-a} \ \text{ so, } a = 8$$

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (D)**

$$(1+x^2)\frac{dy}{dx} + xy = x^3$$

$$\text{or } \frac{dy}{dx} + \frac{x}{1+x^2}y = \frac{x^3}{1+x^2}$$

$$IF = e^{\int \frac{x}{1+x^2}dx} = \sqrt{1+x^2}$$

$$\Rightarrow \text{Solution is } y\sqrt{1+x^2}$$

$$= \sqrt{1+x^2}\left(\frac{x^2-2}{3}\right) + c$$

$$\text{or } y = \frac{x^2-2}{3} + \frac{c}{\sqrt{1+x^2}}$$

$$y(0) = -\frac{2}{3} \Rightarrow c = 0$$

$$\therefore 3y + 2 = x^2$$

2. **Ans (C)**

$$Q = -P^{-1}QP \qquad \therefore PQ = -QP$$

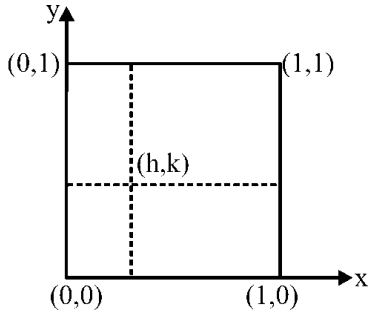
$$(P+Q)^2 = (P+Q)(P+Q)$$

$$= P^2 + PQ + QP + Q^2 = P^2 + Q^2$$

$$(P-Q)^2 = (P-Q)(P-Q)$$

$$= P^2 - PQ - QP + Q^2 = P^2 + Q^2$$

3. **Ans (A)**



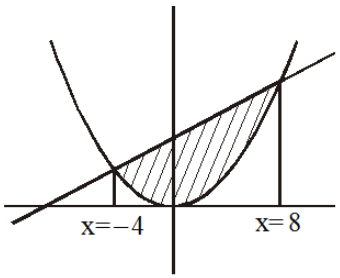
$$h^2 + (1-h)^2 + k^2 + (1-k)^2 = 3$$

$$2h^2 + 2k^2 - 2h - 2k - 1 = 0$$

$$x^2 + y^2 - x - y - \frac{1}{2} = 0$$

$$r = \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}} = 1$$

4. Ans (A)



Solving parabola $x^2 = 8y$

and line $x - 2y + 8 = 0$,

we get

$x^2 = 4(x + 8)$

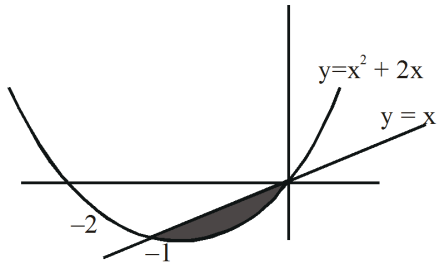
$\Rightarrow x = 8, -4$

$\therefore$ Shaded region

$$= \frac{1}{2}[8+2] \times 12 - \int_{-4}^{8} \frac{x^2}{8} dx$$

$$= 60 - \frac{1}{8}.\ \frac{x^3}{3}\Big|_{-4}^{8} = 60 - 24 = 36$$

5. Ans (C)



$$= \left| \int_{-1}^{0} (x^2 + 2x - x)\, dx \right|$$

$$= \frac{x^3}{3} + \frac{x^2}{2}\Big|_{-1}^{0}$$

$$= \left| 0 - \left(-\frac{1}{3} + \frac{1}{2} \right) \right|$$

6. Ans (A)

$$xdy + ydx = \frac{e^{xy}}{x^2}(xdy - ydx)$$

$$\Rightarrow \quad e^{-xy} d(xy) = d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad -e^{-xy} = \frac{y}{x} + C$$

7. Ans (B)

$$xdx = \frac{x^2}{y}dy - y^3 dy$$

$$x\frac{dx}{dy} = \frac{x^2}{y} - y^3$$

$$\frac{dx}{dy} = \frac{x}{y} - \frac{y^3}{x}$$

$$x\frac{dx}{dy} - \frac{x^2}{y} = -y^3$$

$x^2 = t$

$$2x\frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{1}{2}\frac{dt}{dy} - \frac{t}{y} = -y^3$$

$$\frac{dt}{dy} - \frac{2}{y}t = -2y^3$$

$$I.F = e^{\int -\frac{2}{y} dy} = e^{-2\ell ny} = \frac{1}{y^2}$$

$$\therefore \frac{t}{y^2} = \int -2ydy + c$$

$t = y^2(-y^2 + c)$

$x^2 = y^2(-y^2 + c)$

it satisfy (0,2)

$c = 4$

$\therefore x^2 = y^2(-y^2 + 4)$

8. Ans (B)

$PF_1 = 2PF_2$

Also $PF_1 + PF_2 = 6$

Solving $PF_2 = 2$; $PF_2 = 4$

Also $F_1F_2 = 2\sqrt{5}$

$\Rightarrow$ ΔPF_1F_2 is right angle

$A(\Delta PF_1F_2) = \frac{1}{2} \times 2 \times 4 = 4$

9. Ans (C)

Equation will be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ its directrix is

$x = 2 = \frac{a}{e}$ also given $\frac{b^2}{a} = 12 = a(e^2 - 1)$

$\Rightarrow 12 = 2e \times (e^2 - 1)\ e^3 - e - 6 = 0$

$\Rightarrow (e - 2) \times (e^2 + 2e + 3) = 0 \Rightarrow \quad e = 2$

10. Ans (A)

$\lambda^2 - 3\lambda + 2 = 0 \quad \Rightarrow \lambda = \{1, 2\}$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \quad \Rightarrow \lambda = \{1, 2, 3\}$

$\tan \frac{\lambda\pi}{4} - 1 = 0 \Rightarrow \lambda = \{1\}$

All satisfy for $\lambda = 1$

11. Ans (B)

Diagonal elements can be $\begin{cases} 3,2,2 \rightarrow \frac{|3}{|2} \\ 3,3,1 \rightarrow \frac{|3}{|2} \\ 3,3,2 \rightarrow \frac{|3}{|2} \\ 3,3,3,1 \rightarrow \frac{|4}{|2} \end{cases}$

So total matrices $= 10 \times (7^6) \begin{bmatrix} . & 0 & 0 \\ 0 & . & 0 \\ 0 & 0 & . \end{bmatrix}$

12. Ans (A)

Centre = (1, −2) radius = 3

Image of centre (1, −2) about line

$x - y + 5 = 0$ is (−7, 6)

$\therefore$ Equation of circle

$(x + 7)^2 + (y - 6)^2 = (3)^2$

$x^2 + y^2 + 14x - 12y + 76 = 0$

13. Ans (C)

$3x^2 - 10x - 5y - 20 = 0$

$3\left(x^2 - \frac{10x}{3}\right) = 5(y + 4)$

$\left(x - \frac{5}{3}\right)^2 = \frac{5}{3}\left(y + \frac{17}{3}\right)$

$L(LR) = \frac{5}{3}$

14. Ans (B)

Perpendicular distance from focus (1, 0) to directrix is half of latus rectum

$LR = M = 2\left|\frac{3(1) + 2}{5}\right| = 2$

15. Ans (A)

$e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{12}{4} = 4 \Rightarrow e_1 = 2$

Now $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 \Rightarrow \frac{1}{e_2^2} = 1 - \frac{1}{e_1^2} = \frac{3}{4}$

$\Rightarrow e_2^2 = \frac{4}{3} \Rightarrow e_2 = \frac{2}{\sqrt{3}}$

16. Ans (A)

$\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad P \equiv (2\cos\theta, 3\sin\theta).$

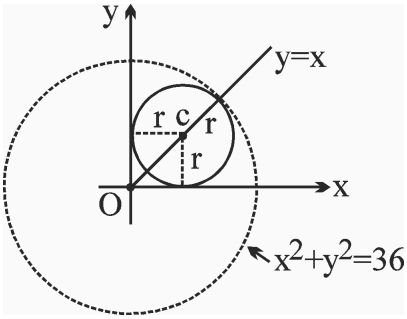
$\therefore 4\cos^2\theta + 9\sin^2\theta = \frac{31}{4}.$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2}$

$\therefore |2\cos\theta| = 1.$

17. **Ans (D)**



$r^2 + r^2 = (OC)^2$

$\Rightarrow OC = r\sqrt{2}$

$\therefore r\sqrt{2} + r = 6$

$r = \dfrac{6}{\sqrt{2}+1} = 6\left(\sqrt{2}-1\right)$

18. **Ans (C)**

Put cosy = t

$\dfrac{dt}{dx} = -\sin y \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dt}{dx} - \dfrac{t}{x} = -x^4t^2$

$\dfrac{1}{t^2}\dfrac{dt}{dx} - \dfrac{1}{tx} = -x^4$

put $-\dfrac{1}{t} = z \Rightarrow \dfrac{dz}{dx} = \dfrac{1}{t^2}\dfrac{dt}{dx}$

$\dfrac{dz}{dx} + \dfrac{z}{x} = -x^4 \Rightarrow zx = -\dfrac{x^6}{6} + c$

$x(\sec y) = \dfrac{x^6}{6} + c$

19. **Ans (A)**

F (4, 0)

$y = m(x - 4)$

$mx - y - 4m = 0$

$p = r$

$\left|\dfrac{6m-4m}{\sqrt{1+m^2}}\right| = \sqrt{2}$

$4m^2 = 2(1 + m^2)$

$2m^2 = 2$

$m = \pm 1$

20. **Ans (A)**

$P \equiv (4 - 5\alpha, \alpha)$

$\dfrac{x(4-5\alpha)}{5} + \dfrac{y\alpha}{3} = 1$

$3x(4 - 5\alpha) + 5\alpha y = 15$

$(12x - 15) + \alpha(-15x + 5y) = 0$

$x = \dfrac{5}{4}$ and $y = 3x$

$(x, y) \equiv \left(\dfrac{5}{4}, \dfrac{15}{4}\right) = (\alpha, \beta)$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (8)**

$A.\text{adj}\left(A^2\right) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\because \quad A.(\text{adj}A)^2 = A(\text{adj}A).(\text{adj}A) = |A|\ I\ \text{adj}(A)$

$\Rightarrow \quad |A|\text{adj}(A) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

take determinant on both the sides

$\Rightarrow \quad |A|^5 = -1 \quad \Rightarrow \quad |A| = -1$

$\Rightarrow \quad \text{adj}A = -\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

2. **Ans (4)**

Tangent to circle is $y - 3 = m(x - 2) \pm \sqrt{1 + m^2}$

satisfy $(0, 0) \Rightarrow -3 = -2m \pm \sqrt{1 + m^2}$

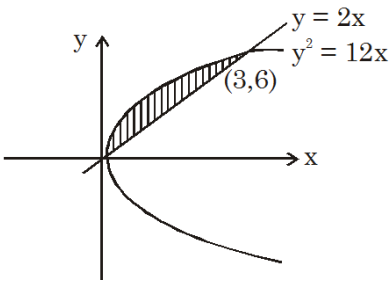
$2m - 3 = \sqrt{1 + m^2}$

square both side

$3m^2 - 12m + 8 = 0 \begin{cases} m_1 \\ m_2 \end{cases}$

$m_1 + m_2 = 4$ Ans.

3. **Ans (3)**



$$\text{Area bounded} = \int_0^3 \left(\sqrt{12x} - 2x\right)\, dx = 3$$

4. **Ans (6)**

$\frac{dy}{dx} + \left(-\frac{1}{x}\right) y = \left(x - \frac{2}{x}\right)$ (Linear differential equation)

$\Rightarrow f(x) = (x-1)^2 + 1$

$\therefore$ Required area

$$= \int_0^3 \left((x-1)^2 + 1\right) dx = 3 + 3 = 6$$

5. **Ans (8)**

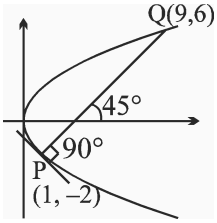
N : $y + tx = 2t + t^3$; slope of the normal is $-t$

hence $-t = 1 \Rightarrow t = -1$

$\Rightarrow$ coordinates of P are $(1, -2)$

Hence parameter at Q,

$t_2 = -t_1 - 2/t_1 = 1 + 2 = 3$



$\therefore$ Coordinates at Q are (9, 6)

$\therefore$ $l(PQ) = \sqrt{64 + 64} = 8\sqrt{2}$