ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Advanced)
FULL SYLLABUS
02-03-2025

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-II)

ANSWER KEY

PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,B,C,D	B,C	B,C,D	A	A,C	C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	C	C	C	C				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	0.41	1.50	6.25	1.25	12.75	4.00	3.00	0.80

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,B	A,C	A,B,D	B,D	B,C	A,C		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	C	D	C	A				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	0.06	5.00	5.00	3.00	6.00	3.00	6.00	7.00

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	B,C,D	B,D	B,D	B	A,B,D	B,C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	A	B	A	A				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	4.00	2688.00	17.00	2.00	5.00	47.50	2.00	2.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (A,B,C,D)**

$$E\,4\pi r^2 = \frac{q_0 - Q_t}{\epsilon_0}$$

$$J = \frac{E}{\rho} = \frac{q_0 - Q_t}{4\pi\epsilon_0\rho r^2}$$

$$I = J\,A = \frac{1}{\epsilon_0\rho}(q_0 - Q)$$

$$I = \frac{dQ}{dt} \Rightarrow \frac{dQ}{q_0 - Q} = \frac{dt}{\epsilon_0\rho} \Rightarrow \frac{Q}{q_0} = 1 - e^{-t/\epsilon_0\rho}$$

2. **Ans (B,C)**

Initial pressure at the bottom

$$= \rho g \times 2H + 2\rho \times g \times H = 4\,\rho g H$$

$$\frac{\rho \times A \times 2H + 2\rho \times 2A \times H}{A \times 2H + 2A \times H} = \frac{3}{2}\rho$$

Final pressure $= \frac{3}{2}\,\rho \times g \times 3H = \frac{9}{2}\,\rho g H.$

3. **Ans (B,C,D)**

For adiabatic process (A ⟶ B)

$P_A V_A^{\gamma} = P_B V_B^{\gamma}$

$10^5 \times (0.8)^{\frac{5}{3}} = 3 \times 10^5 (V_B)^{\frac{5}{3}}$

$\Rightarrow V_B = 0.8 \times \left(\frac{1}{3}\right)^{0.6} = 0.4\ m^3$

Work done in process A ⟶ B

$W_{AB} = \dfrac{P_A V_A - P_B V_B}{\gamma - 1}$

$\Rightarrow W_{AB} = \dfrac{10^5 \times 0.8 - 3 \times 10^5 \times 0.4}{\frac{5}{3} - 1}$

$\Rightarrow W_{AB} = -60\ kJ = \Rightarrow |W_{AB}| = 60\ kJ$

Work done in process B ⟶ C (Isothermal process)

$W_{BC} = nRT\,\ell n\dfrac{V_C}{V_B} = P_B V_B\,\ell n\dfrac{V_C}{V_B}$

$\Rightarrow W_{BC} = 3 \times 10^5 \times 0.4\ell n\dfrac{0.8}{0.4}$

$\Rightarrow W_{BC} = 84\ kJ$

Work done in process C ⟶ A

$W_{CA} = P\Delta V = 0\ (\because \Delta V = 0)$

So total work done in the process A ⟶ B ⟶ C

$W_{ABC} = W_{AB} + W_{BC} + W_{CA} = -60 + 84 + 0$

$W_{ABC} = 24\ kJ$

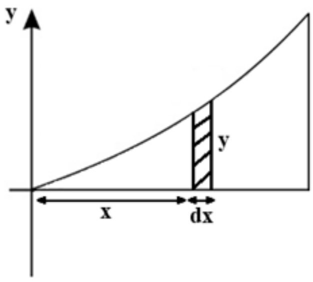
4. **Ans (A)**

(B) For dipole $E_{min} = \dfrac{KP}{r^3}$ and $E_{max} = \dfrac{2KP}{r^3}$.

So maximum value of E may be 10 N/C

(C), (D) net force or net torque on dipole in non-uniform electric field may be zero.

5. **Ans (A,C)**



Moment of inertia of the plate about y-axis is

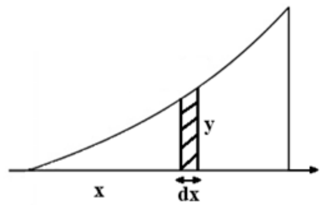
$I_y = 2\int dm.\,x^2$

$= 2\int \rho.\,y\,dx.\,x^2$

$= 2\int_0^a \rho\,2x^4.\,dx$

$= \left[\dfrac{4\rho x^5}{5}\right]_0^a = \dfrac{4\rho a^5}{5}$

Now,



Moment of inertia of the plate about x-axis

$I_x = 2\int \dfrac{dmy^2}{3}$

$= \dfrac{2}{3}\int \rho y.\,dx.\,y^2$

$= \dfrac{16}{3}\rho\left(\dfrac{x^7}{7}\right)_0^a = \dfrac{16}{3}\rho\dfrac{a^7}{7}$

$= \dfrac{16\rho a^7}{21}$

6. Ans (C,D)

$f = -Kv$

$$m \cdot \frac{dv}{dt} = -Kv$$

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{k}{m}\int_0^t dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{K}{m} \cdot t$$

$$v = v_0 e^{-\frac{K}{m}\cdot t}$$

$$R = \frac{mv}{qB} = \frac{mv_0}{qB} e^{-\frac{K}{m}\cdot t}$$

From f– = Kv

$$m.\, v\frac{dv}{dx} - = Kv$$

$$\int_{v_0}^{v} dv = -\frac{K}{m} \cdot \int_x^0 dx$$

$$v = v_0 - \frac{K}{m} \cdot x = v_0 - \frac{K}{m} \cdot \frac{mv_0}{2K}$$

$$v = \frac{v_0}{2}$$

$$R = \frac{mv}{qB} = \frac{mv_0}{2qB}$$

PART-1 : PHYSICS

SECTION-I (ii)

9. Ans (C)

The equation of rotating surface is given by $y = \frac{\omega^2 x^2}{2g}$

also the radius of curvature at any point is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$y = \frac{\omega^2 x^2}{2g};\ \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\frac{d^2y}{dx^2} = \frac{\omega^2}{g}$$

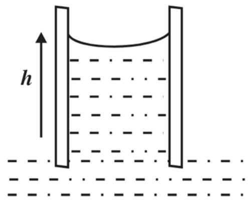
$$\therefore\ \rho = \frac{\left[1 + \frac{\omega^2 x^2}{g^2}\right]^{3/2}}{\omega^2/g}$$

$$\rho\left(\frac{r_0}{2}\right) = \frac{\left[1 + \frac{\omega^4 r_0^2}{4g^2}\right]^{3/2}}{\omega^2/g}$$

$$h\left(\frac{r_0}{2}\right) = \frac{\omega^2 \frac{r_0^2}{4}}{2g}$$

10. Ans (C)

$2SW = P(dhW)g$



$$h = \frac{2S}{\rho g d}$$

$$W_{ST} = (2SW)h = \frac{4S^2}{\rho g d}$$

$$W_g = (\rho d h W g)\frac{h}{2} = \frac{2S^2}{\rho g d}$$

$$W_{ST} - W_g = \Delta H$$

Force of attraction

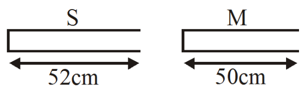
$$P_0 hW - (P_0 - \frac{2S}{d} + \rho g \frac{h}{2})hW = F$$

$$F = \frac{2S^2 W}{\rho g d^2}$$

PART-1 : PHYSICS

SECTION-II

3. Ans (6.25)



$v = 325$ m/s

$$v = \frac{V}{4L} \qquad \therefore\ v_{10} = \frac{325}{4 \times 0.52}$$

$$v_{20} = \frac{325}{4 \times 0.50}$$

$$v_{10} - v_{20} = \frac{325}{4}\left(1 - \frac{1}{0.264}\right) = \frac{325}{4}\left(\frac{0.004}{0.260 \times 0.0264}\right)$$

$$= \frac{0.325}{0.260 \times 0264} = 6.25\text{Hz}$$

4. Ans (1.25)

$\Delta X = 2\ \mu t$

For destructive interference

$2\mu t = \lambda/2$

$$\mu = \frac{\lambda}{4t} = 1.25$$

5. Ans (12.75)

For first refraction from lens

$$\frac{1}{v}-\frac{1}{30}=\frac{1}{20}\Rightarrow\frac{1}{v}=\frac{1}{20}-\frac{1}{30}$$

v = +60 cm (image on principal axes)

2nd refraction from lens

$$\frac{1}{v}-\frac{1}{+60}=\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{20}+\frac{1}{60}\Rightarrow\frac{1}{v}=\frac{3+1}{60}$$

V = + 15 cm

Now for second lens principal axis is 3mm below the original principal axis

Hence $\frac{h_i}{h_o}=\frac{v}{u}\Rightarrow\frac{h_i}{+3}=\frac{+15}{+60}$

$h_i=\frac{3}{4}$ mm

$\therefore$ y coordinate $=-\left(3-\frac{3}{4}\right)=-\frac{9}{4}$mm

Coordinates = $\left(15,\frac{-9}{4}\right)$

6. Ans (4.00)

$C_{eq}=3\,\mu F$

P.D. across R_1 in steady state $=\frac{30}{100+50}\times 100=20\,V$

$\therefore U=\frac{1}{2}(1+2)\times 10^{-6}\times 400=6\times 10^{-4}J$

After switch is opened, heat is generated is R_1 and R_2 only.

Heat developed,

$$\frac{H_1}{H_2}=\frac{\int I^2R_1dt}{\int I^2R_2dt}=\frac{R_1}{R_2}$$

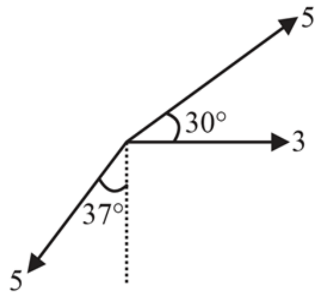
$$H_2=\frac{UR_2}{R_1+R_2}=6\times 10^{-4}\times\frac{200}{300}=4\times 10^{4}J$$

7. Ans (3.00)

$V_1=3\sin\omega t;$

$V_2=5\sin(\omega t+\phi_1);$

$V_3=5\sin(\omega t-\phi_2)$

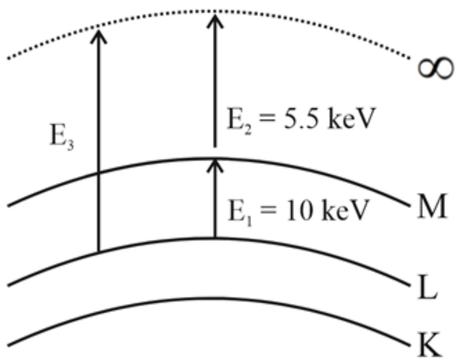


$$V_{max}=\sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2+(1.5)^2}=\sqrt{21}$$

$$I_{max}=\frac{V_{max}}{R}=\frac{\sqrt{21}}{\sqrt{\frac{7}{3}}}=\sqrt{\frac{21\times 3}{7}}=3A$$

8. Ans (0.80)

$\lambda_{L_\alpha}=124\,pm$



$$E_1=\frac{hc}{\lambda_{L_\alpha}}=10\,keV$$

$E_3=E_1+E_2$

$E_3=10+5.5=15.5$ keV

Energy required to ionise $e^{\ominus}$ from 'L' shell is 15.5 keV i.e. the minimum wavelength of characteristics x-ray we can produce

$$\lambda_{min}=\frac{hc}{15.5\,keV}=0.8\text{Å}$$

PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (A,B)**

$[Co(NH_3)_4 Cl(ONO)]Cl$ - Tetraammine chloridonitrito - o cobalt (III) chloride

$K_3[VF_6]$ - has unpaired d-electron therefore colored

2. **Ans (A,C)**

More is value of T_C, more will be adsorption in Cl_2. T_C is higher than H_2.

3. **Ans (A,B,D)**

(A) As the ratio of coordination numbers is 1 : 1, the positions of cations and anions can be interchanged.

(B) Void is surrounded by 6 face centre atoms.

(C) No. of octahedral voids = 4 per unit cell

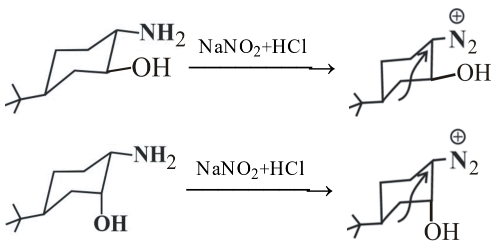
No. of tetrahedral voids = 8 per unit cell

(D) Ratio of $\frac{\text{No. of THV}}{\text{No. of OHV}} = \frac{2}{1}$

4. **Ans (B,D)**

B_2H_6 burns spontaneously in air to form oxide BN is bad conductor of electricity.

5. **Ans (B,C)**



When leaving group is present at equatorial position, ring contraction occurs.

6. **Ans (A,C)**

Conceptual

PART-2 : CHEMISTRY

SECTION-I (ii)

7. **Ans (C)**

(A) $\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) = \frac{4.606 \times 10^3}{2.303 \times 2}\left(\frac{1}{500} - \frac{1}{1000}\right)$

$\therefore \frac{k_2}{k_1} = 10$

(B) $K = 0.0693\ s^{-1}$

Order of reaction is first as per unit of k.

$\therefore\ t = \frac{2.303}{k}\log\frac{A_o}{A_t} = \frac{2.303}{0.0693}\log\frac{16}{2} = \frac{2.303}{0.0693}\log 8$

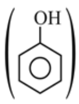
$= \frac{2.303 \times 3 \times 0.301}{0.0693} = 30\,s\ = 0.5\,min$

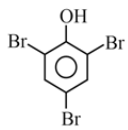
(C) $t_{1/2} \propto \frac{1}{(conc)^{n-1}}\quad \therefore n - 1 = 1$

$\Rightarrow n = 2$

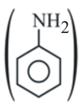
(D) Hydrolysis of ester in alkaline medium obey second order kinetics.

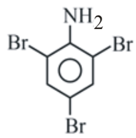
10. **Ans (A)**

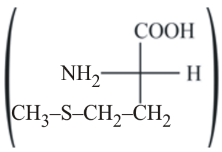
Phenol 

gives white precipitate of 

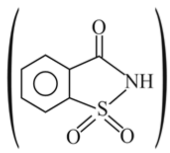
with bromine water & violet complex with $FeCl_3$.

Aniline 

gives white ppt of 

with bromine water and Lassagne's test for Nitrogen Methionine 

gives Lassagne's test for both sulphur and nitrogen and effervescence with $NaHCO_3$.

Saccahrin 

gives Lassagne's test for sulphur and nitrogen.

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (0.06)**

Let after 200 min,

x mole of A remained.

$P = X_A . P_A^o + X_B . P_B^o$

$400 = \dfrac{x}{21+x} \times 300 \dfrac{20}{21+x} \times 500$

$\therefore x = 16$

Now, $K = \dfrac{1}{t} . \ln \dfrac{n_A^o}{n_A}$

$= \dfrac{1}{200/60} \times \ln \dfrac{20}{16}$

$= 0.06 \text{ hr}^{-1}$

3. **Ans (5.00)**

$$\text{Cane sugar} + H_2O(\ell) \longrightarrow \text{Glucose} + \text{Fructose}$$

$t=0 \quad 0.05 \text{ mole} \qquad 0 \qquad 0$

$t \quad 0.05-0.05\alpha \qquad 0.05\alpha \qquad 0.05\alpha$

Total moles $= 0.05 - 0.05\alpha + 0.05\alpha + 0.05\alpha = 0.05(1+\alpha)$

$\Delta T_f = iK_f m$

$0.279 = (1+\alpha)\, 1.86 \times \left(\dfrac{0.05}{500/1000}\right)$

$(1+\alpha) = \dfrac{0.279 \times 5}{1.86 \times 0.5} = 1.5$

$\alpha = 0.5$

Moles of cane sugar left $= 0.05 - 0.05\alpha = 0.025$

Mass of cane sugar left $= 342 \times 0.025 = 8.55$ gm.

4. **Ans (3.00)**

$\dfrac{2\pi r_{n_1}}{2\pi r_{n_2}} = \dfrac{n_1\lambda_1}{n_2\lambda_2}$ ....(i)

Also we know that

$r_n = \dfrac{0.529n^2}{z}$

Putting the value in equation (i)

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{\lambda_1}{\lambda_2}$

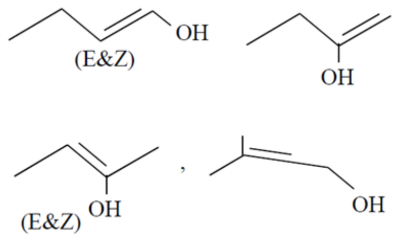
Also $n_1 = 1$ and if we provide energy equal to 12.09 eV, then electron will jump to $n_2 = 3$

$\Rightarrow \dfrac{\lambda_1}{\lambda_2} = \dfrac{1}{3}$

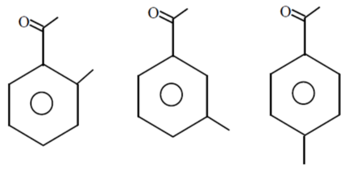
$\Rightarrow \lambda_2 = 3\lambda_1$

So, wavelength increases 3 times.

5. **Ans (6.00)**



6. **Ans (3.00)**



All aldehyde isomers of 'A' and $PhCOCH_2CH_3$ and $PhCH_2COCH_3$ does not give iodoform

7. **Ans (6.00)**

(i), (ii), (iii), (v), (vii) & (viii) gives aldehydes.

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (B,C,D)**

$T_r = \dfrac{r^2}{(2r-1)(2r+1)} = \dfrac{1}{4}\left(\dfrac{r}{2r-1} + \dfrac{r}{2r+1}\right)$

$4S = \sum_{r=1}^{500}\left(\dfrac{r}{2r-1} + \dfrac{r}{2r+1}\right)$

$= 1 + \left(\dfrac{1}{3} + \dfrac{2}{3}\right) + \left(\dfrac{2}{5} + \dfrac{3}{5}\right) + \left(\dfrac{3}{7} + \dfrac{4}{7}\right) + \ldots + \dfrac{500}{999} + \dfrac{500}{1001}$

$\Rightarrow 4S = 1 + 499 + \dfrac{500}{1001} \Rightarrow [S] = 125$

2. **Ans (B,D)**

Let $\vec{w} = \lambda_1 \vec{u} + \lambda_2 \vec{v} + \lambda_3(\vec{u} \times \vec{v})$

Taking dot product with $\vec{u} \times \vec{v}$, we get $\lambda_3 = \dfrac{5}{3}$.

$\vec{w} . \vec{u} = \lambda_1 ; \ \vec{w} . \vec{v} = \lambda_2$

$\Rightarrow 3\lambda_1 + 2\lambda_2 = 0$

Given : $\displaystyle\int_{-2\lambda_1}^{2\lambda_1} \dfrac{x^3+1}{x^2+1} dx = 2\int_0^{2\lambda_1} \dfrac{dx}{x^2+1}$

$\Rightarrow \lambda_1 = \dfrac{1}{2}, \ \lambda_2 = -\dfrac{3}{4}$

$\vec{w} . \vec{w} = \dfrac{19}{8}$

3. Ans (B,D)

$A + B^T = \text{adj}B\ ;\ B + A^T = \text{adj}A$

$\Rightarrow\ A^T + B = (\text{adj}\,B)^T$

$\Rightarrow\ \text{adj}\,A = (\text{adj}\,B)^T$

$\Rightarrow\ |B|^2 = |A|^2$

$|B| = \pm|A|$

(a) $|A| = |B|$

$(B^{-1})^T = A^{-1}$

$\Rightarrow\ A = B^T$

$\text{adj}\,B = 2B^T$

$\Rightarrow\ |B|^2 = 8|B|$

$\Rightarrow\ |B| = |A| = 8$

(b) $|A| = -|B|$

similarly $\Rightarrow\ A = -B^T$

$\therefore\ \text{adj}\,B = 0$ (Not possible)

4. Ans (B)

$Z^9 - 9 = (Z - Z_1)(Z - Z_2)\ldots\ldots\ldots(Z - Z_9)$

Put $Z = -Z_1, -Z_2, \ldots, -Z_9$ and multiply

$-18 = -2Z_1(Z_1 + Z_2)(Z_1 + Z_3)\ldots(Z_1 + Z_9)$

$-18 = -(Z_9 + Z_1)(Z_9 + Z_2)\ldots\ 2(Z_9)$

$(18)^9 = 2^9 Z_1 Z_2 \ldots\ldots Z_9 \ldots\ldots\ldots\ldots \prod\limits_{i \neq j}(z_i + z_j)$

$= 2^9.9\,(\lambda^2)$

$\lambda^2 = 9^8$

5. Ans (A,B,D)

$$A = \int_0^1 e^{-\left(z^2+\frac{1}{z^2}\right)}\,dz + \int_1^\infty e^{-\left(z^2+\frac{1}{z^2}\right)}\,dz$$

Let $z = \dfrac{1}{t}$

$$A = \int_1^\infty \frac{1}{t^2} e^{-\left(t^2+\frac{1}{t^2}\right)}dt + \int_1^\infty e^{-\left(z^2+\frac{1}{z^2}\right)}dz$$

$$A = \int_1^\infty \left(1 + \frac{1}{x^2}\right) e^{-\left(x^2+\frac{1}{x^2}\right)}dx \quad \text{let } x - \frac{1}{x} = u$$

$$= \int_0^\infty e^{-(u^2+2)}du = \frac{B}{e^2}$$

6. Ans (B,C,D)

Let $x = \tan\theta$

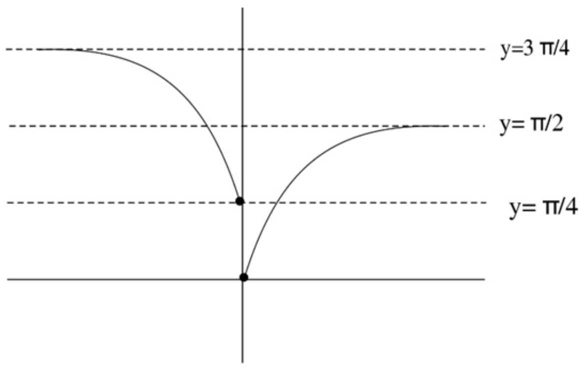
$x \leqslant 0\ \left(\theta \in \left(-\frac{\pi}{2}, 0\right]\right)$

$$f(x) = \cos^{-1}\left(\frac{\sin\theta + \cos\theta}{\sqrt{2}}\right)$$

$$f(x) = \cos^{-1}\left(\cos\left(\theta - \frac{\pi}{4}\right)\right)$$

$$f(x) = -(\theta - \pi/4) = \frac{\pi}{4} - \tan^{-1}x$$

$$f(x) = \begin{cases} \frac{\pi}{4} - \tan^{-1}x, & x \leqslant 0 \\ \tan^{-1}x, & x > 0 \end{cases}$$



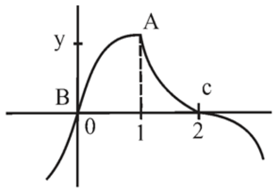
PART-3 : MATHEMATICS

SECTION-I (ii)

7. Ans (A)

(I) A, B, C are the 3 critical points of y = f(x)

$f''(x) = 0$ for x = 2 and fails to exists at x = 0

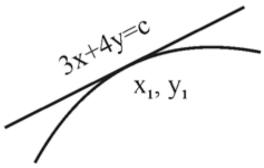


(II) x = 1/4 and 2. Make a quadratic in $\log_2 x$ and interpret the result.

(III) $\frac{dy}{dx} = -1 + 2x_1^3 = -\frac{3}{4} \quad \Rightarrow x_1 = \frac{1}{2}$

$\Rightarrow \frac{1}{32} = \frac{1}{2} + y_1$ or

$y_1 = -\frac{15}{32} \quad \Rightarrow c = -\frac{57}{32}$



(IV) $f'(x) = 2x^3 - 3x + 1$ this is always positive in (1, 2)

∴ increasing [1, 2]

∴ f(2) will be greatest value.

8. Ans (B)

$f(x) = 2x^2 - 10px + 7p - 1$

$D = (-10p)^2 - 4.2.(7p-1)$

$= 100p^2 - 8(7p-1)$

$= 4(25p^2 - 14p + 2)$

∵ For the equation $25p^2 - 14p + 2 = 0$

$\Rightarrow$ a − 25 > 0 and

$D = (-14)^2 - 4 \cdot 25 = 2 - 4 < 0$

$\Rightarrow 4(25p^2 - 14p + 2) > 0$ for all $p \in R$

$\Rightarrow D > 0$

Now,

(I) Both roots of f(x) = 0 are confined in (−1, 1)

$\Rightarrow$ (i) $D \geqslant 0$ (for all $p \in R$)

(ii) $-1 < \frac{-b}{2a} < 1$

$\Rightarrow -1 < -\left(\frac{-10p}{4}\right) < 1 \quad \Rightarrow p \in \left(\frac{-2}{5}, \frac{2}{5}\right)$

(ii) $f(-1) > 0$ and $f(1) > 0$

$2(-1)^2 - 10p(-1) + 7p - 1 > 0$ and

$2(1)^2 - 10p(1) + 7p - 1 > 0$

$17p + 1 > 0$ and $-3p + 1 > 0$

$\Rightarrow p \in \left(\frac{-1}{17}, \frac{1}{3}\right)$

From (i), (ii) and (iii) $p \in \left(\frac{-1}{17}, \frac{1}{3}\right)$

(II) Exactly one roots of f(x) = 0 lies in (−1, 1)

$f(-1) \cdot f(1) < 0$

$(17p+1)(-3p+1) < 0$

$(17p+1)(3p-1) > 0$

$\Rightarrow p \in \left(-\infty, \frac{-1}{17}\right) \cup \left(\frac{1}{3}, \infty\right)$

Also when $f(-1) = 0 \quad \Rightarrow p = \frac{-1}{17}$

$f(x) = 2x^2 + \frac{10}{17}x - \frac{24}{17}$

$f(x) = 34x^2 + 10x - 24$

Here, $f(1) = 34(1)^2 + 10(1) - 24 > 0$

⇒ Other root is lies in (−1, 1)

∴ $p = \frac{-1}{17}$ is also possible.

Again $f(1) = 0 \Rightarrow \quad -3p + 1 = 0 \quad \Rightarrow p = \frac{1}{3}$

$f(x) = 2x^2 - \frac{10}{3}x + \frac{4}{3} = 6x^2 - 10x + 4$

$f(-1) = 6(-1)^2 - 10(-1) + 4 > 0$

⇒ Other root lies in (−1, 1)

∴ So exactly one root of f(x) = 0 lies in (−1, 1)

If $p \in \left(-\infty, \frac{-1}{17}\right] \cup \left[\frac{1}{3}, \infty\right)$

(III) Both roots of f(x) = 0 are greater than I.

(i) $D \geqslant 0$ (always)

(ii) $\frac{-b}{2a} > 1 \quad \Rightarrow \frac{5p}{2} > 1 \quad \Rightarrow p = \left(\frac{2}{5}, \infty\right)$

(iii) $f(1) > 0 \quad \Rightarrow -3p + 1 > 0 \quad \Rightarrow p < \frac{1}{3}$

$\Rightarrow p \in \left(-\infty, \frac{1}{3}\right)$

From (i), (ii) and (iii) $p \in \phi$

(IV) One root of f(x) = 0 is greater than 1 and other root of f(x) = 0 is less then −1.

$f(1) < 0$ and $f(-1) < 0$

$\Rightarrow f(1) < 0$ and $f(-1) < 0$

$\Rightarrow 17p + 1 < 0$ and $-3p + 1 < 0$

$\Rightarrow p < \frac{-1}{17}$ and $p > \frac{1}{3}$

$\Rightarrow p \in \phi$

9. Ans (A)

(I) We have $f(x) = \sqrt[5]{x} + \sin^{-1}x$

Clearly domain of $f(x) = [-1, 1]$.

Also, $f(x)$ is increasing so $f(x)$ is one-one function

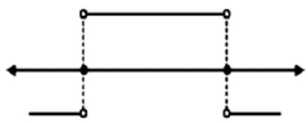
Ans. P, S

(II) $f(x) = \operatorname{sgn} \dfrac{(1-|x|)}{(1+|x|)}$

$D_f = R$

$R_f = \{-1, 0, 1\}$ even function

Ans. Q, R, T



(III) $f(x) = \dfrac{2^{[-x]}}{2^{\{x\}}} - 2^{|x|} = 2^{-x} - 2^{-x} = 0 \ \forall \ x \leqslant 0$

Ans. T

(IV) For domain of $f(x)$ we must have

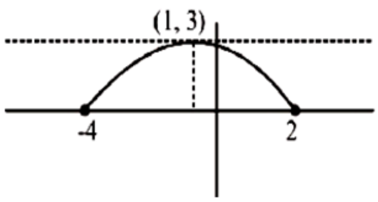
$8 - 2x - x^2 \geqslant 0$

$\Rightarrow \ x^2 + 2x - 8 \leqslant 0$

$\Rightarrow \ (x+4)(x-2) \leqslant 0$

$\Rightarrow \ x \in [-4, 2]$

$\Rightarrow \ R_f = [0, 3]$



10. Ans (A)

(I) $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

Mid point (0, 3)

$T = S_1$

$x(0) + y\left(\dfrac{3}{16}\right) - 1 = \dfrac{0}{9} + \dfrac{9}{16} = 1$

$\boxed{y = 3}$

$\dfrac{x^2}{9} + \dfrac{9}{16} = 1$

$x = \pm\dfrac{3}{4}\sqrt{7}$

Length of chord $= \dfrac{3}{2}\sqrt{7}$

(II) Equation of chord of parabola is

$y(t_1 + t_2) = 2x + 2at_1t_2$

$y(t+1) = 2x + 2t$

$\dfrac{2x = (t+1)y}{-2t} = 1 \qquad .....(1)$

$y^2 = 4x(1)$

$y^2 = 4x\left(\dfrac{2x - (t+1)y}{-2t}\right)$

$-2 + y^2 = 8x^2 - 4(t+1)xy$

$8x^2 + 2 + y^2 - 4(t+1)xy = 0$

As it subtend 90° at origin

$8 + 2t = 0$

$\boxed{t = -4}$

(III) As all three given lines are parallel hence no circle will touch all the lines.

(IV) $x^2 + y^2 - 7x + 9y + 10 = 0$

$\left(x - \dfrac{7}{2}\right)^2 + \left(y + \dfrac{9}{2}\right)^2 = \left(\dfrac{7}{2}\right)^2 + \left(\dfrac{9}{2}\right)^2 - 10$

$= \dfrac{90}{4} = \dfrac{45}{2}$

$\boxed{x^2 + y^2 = \dfrac{45}{2}}$

PART-3 : MATHEMATICS

SECTION-II

1. Ans (4.00)

$$l = \lim_{x\to\infty} x \ln\left(\frac{e(1+(1/x))}{(1+(1/x))^x}\right) \quad (\infty \times 0 \text{ form})$$

$$= \lim_{x\to\infty} x\left(1 + \ln\left(1+\frac{1}{x}\right) - x\ln\left(1+\frac{1}{x}\right)\right)$$

put $x = \frac{1}{t}$; as $x \longrightarrow \infty$, $t \longrightarrow 0$

$$\text{Hence } l = \lim_{t\to 0} \frac{1}{t}\left(1 + \ln(1+t) - \frac{\ln(1+t)}{t}\right)$$

$$= \lim_{t\to 0}\left[\ln(1+t)^{1/t} + \frac{t-\ln(1+t)}{t^2}\right]$$

$$= 1 + \lim_{y\to 0}\left(\frac{e^y - 1 - y}{y^2}\right) \text{ where } \ln(1+t) = y;\ 1+t$$

$= e^y$, hence $t = e^y - 1$

$$= 1 + \frac{1}{2} = \frac{3}{2} = \frac{m}{n}$$

2. Ans (2688.00)

8 ⟨ 1, 2, 5 or 1, 3, 4

So, total number of ways to distribute chocolates among his grand children

$$= \left(\frac{8!}{1!\,2!\,5!} + \frac{8!}{1!\,3!\,4!}\right)3!$$

$= 2688$

3. Ans (17.00)

P(G G $\underline{G}$ $\underline{G}$)

$$\downarrow \downarrow$$

$$= \frac{4}{8} \times \frac{3}{7} = \frac{3}{14}$$

4. Ans (2.00)

If $|z| = |z-2|$

$$z.\bar{z} = z\bar{z} - 2z - 2\bar{z} + 4$$

$$z.\bar{z} = 2$$

$\therefore\ |z+\bar{z}| = 2$

If $|z| = |z+2|$

$$z.\bar{z} = z\bar{z} + 2z + 2\bar{z} + 4$$

$$z + \bar{z} = -2$$

$\therefore\ |z+\bar{z}| = 2$

5. Ans (5.00)

$y = 2 + (1-x) - \sqrt{x^2 - 2x + 1},\ x < 1$

$= 2 + (1-x) - (1-x)$

$\boxed{y = 2}$

A, $y = 2$ $x^2 + 4 = 13$

$x^2 = 9$

$x = 3, -3, \quad x < 1$

$\boxed{x_1 = -3}$

A(−3, 2, 0)

$x_2^2 + y_2^2 < 13$

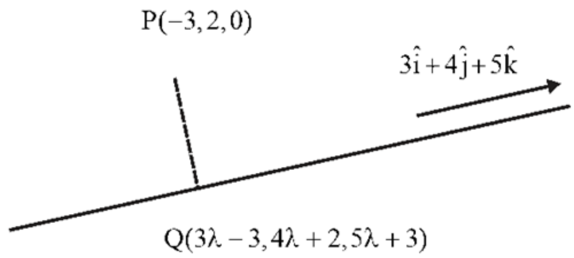
$x_2^2 < 9$

$-3 < x_2 < 3, \quad x_2 < 1$

$\text{Max}(x_2) = 0$

Equation of line

$$\frac{x+3}{3} = \frac{y-2}{4} = \frac{z-3}{5}$$



$3(3\lambda) + 4(4\lambda) + 5(5\lambda + 3) = 0$

$50\lambda = -15$

$$\lambda = \frac{-3}{10}$$

$$PQ = \sqrt{\left(\frac{9}{10}\right)^2 + \left(\frac{12}{10}\right)^2 + \left(\frac{15}{10}\right)^2}$$

$$= \frac{1}{10}\sqrt{81+144+225}$$

$$= \frac{\sqrt{450}}{10} = \frac{3\times 5\sqrt{2}}{10} = \frac{3}{\sqrt{2}}$$

A = 3, b = 2

$\boxed{a + b = 5}$

6. Ans (47.50)

$$I(a, b) = \int \frac{1}{2} x^{b-1} \frac{2x}{(1+x^2)^a} dx$$

$$= \frac{1}{2}x^{b-1} \frac{1}{1-a} \times \frac{1}{(1+x^2)^{a-1}} - \int \frac{b-1}{2} x^{b-2} \times \frac{1}{1-a} \frac{1}{(1+x^2)^{a-1}} dx$$

$$= \frac{1}{2(1-a)} \frac{x^{b-1}}{(1+x^2)^{a-1}} - \frac{b-1}{2(1-a)} \int \frac{x^{b-2}(1+x^2)}{(1+x^2)^a} dx$$

$$= \frac{1}{2(1-a)} \frac{x^{b-1}}{(1+x^2)^{a-1}} - \frac{b-1}{2(1-a)} \int \frac{x^{b-2}}{(1+x^2)^a} dx - \frac{(b-1)I(a, b)}{2(1-a)}$$

$$I(a, b) \left(\frac{2-2a+b-1}{2(1-a)}\right) = \frac{x^{b-1}}{2(1-a)(1+x^2)^{a-1}} - \frac{b-1}{2(1-a)} I(a, b-2)$$

$$I(a, b)(1-2a+b) = \frac{x^{b-1}}{(1+x^2)^{a-1}} - (b-1) I(a, b-2)$$

$$I(a, b) = \frac{1}{b+1-2a} \frac{x^{b-1}}{(1+x^2)^{a-1}} - \frac{b-1}{b+1-2a} I(a, b-2)$$

$$A = \frac{1}{b+1-2a}, \quad B = \frac{1-b}{b+1-2a}$$

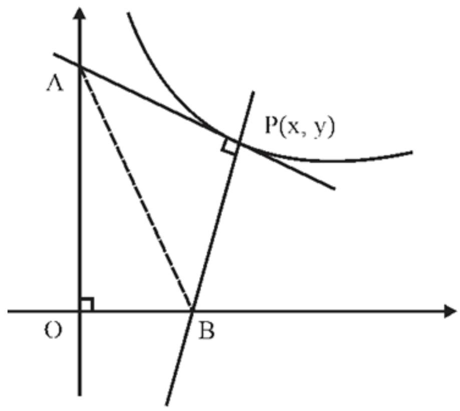
$b = 97, \quad a = 50$

$A = \frac{-1}{2}, \quad B = 48$

$A + B = 47.5$

7. Ans (2.00)

Let the equation of curve is $Y = F(x)$



Equation of tangent at point P(x, y) is

$Y - y = F'(x)$ for point A

$X = 0$

$\Rightarrow Y = y - xF'(x)$

Equation of Normal at P(x, y)

$$Y - y = -\frac{1}{F'(x)}(x - x)$$

For B, $Y = 0 \quad x = x + yF'(x)$

$B(x + tF'(x), 0)$

Circumcenter of ΔPAB is Midpoint of AB

$x + yF'(x) = y - xF'(x)$

$F'(x)(x + y) = y - x$

$$F'(x) = \frac{y-x}{y+x}$$

Let $y = tx$

$$t + x\frac{dt}{dx} = \frac{t-1}{t+1}$$

$$x\frac{dt}{dx} = \frac{t-1-t-t^2}{t+1}$$

$$\frac{(t+1)\,dt}{t^2+1} = \frac{-dx}{x}$$

$$\left(\frac{1}{2}\frac{2t}{t^2+1} + \frac{1}{t^2+1}\right) dt = -\frac{dx}{x}$$

$$\frac{1}{2}\ln(t^2+1) + \tan^{-1}t = -\ln x + C$$

$$\frac{1}{2}\ln\left(\frac{x^2+y^2}{x^2}\right) + \tan^{-1}\frac{y}{x} = -\ln x + C$$

$$\frac{1}{2}\ln(x^2+y^2) + \tan^{-1}\frac{y}{x} = C$$

$$f(1) = 0 \Rightarrow \frac{1}{2}\ln(1) + \tan^{-1}(0) = C \Rightarrow C = 0$$

$$\frac{1}{2}\ln(x^2+y^2) = -\tan^{-1}\left(\frac{y}{x}\right)$$

8. Ans (2.00)

$$f(k) = \lim_{z \to \infty} z^k \int_0^{\frac{1}{z}} t^{t+k-1}\, dt$$

Put $z = \frac{1}{x}$

$$= \lim_{x \to 0} \frac{\int_0^x t^{t+k-1}\, dt}{x^k}$$

$$= \lim_{x \to 0} \frac{x^x \cdot x^{k-1}}{K\, x^{k-1}}$$

$$= \frac{1}{K} \lim_{x \to 0} x^x$$

$$= \frac{1}{K} \lim_{x \to 0} e^{x \ln x}$$

$$= \frac{1}{K} \lim_{x \to 0} e^{\frac{\ln x}{1/x}}$$

$$f(k) = \frac{1}{K}$$

$$f(4) = \frac{1}{4}$$

$xy = 4, \quad x^2 + y^2 = 8$

$$x^2 + \frac{16}{x^2} = 8$$

$$x^4 - 8x^2 + 16 = 0$$

$$x^2 = 4 \Rightarrow x = \pm 2$$

$$y = \pm 2$$

"Two common points" (2, 2) and (−2, −2)