ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Advanced)
PART TEST
01-12-2024

JEE(Main + Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY — PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4				
	A.	B	C	C	A				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	B,C	A,C	B,C	A,B	B,C	A,C		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	6	3	3	3	5	4	5	3

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4				
	A.	B	C	D	C				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,B,D	A,B,C	B,D	A,B,C	A,B,C,D	B,C		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	5	7	2	5	7	9	3	9

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4				
	A.	B	B	A	B				
SECTION-I (ii)	Q.	5	6	7	8	9	10		
	A.	A,C	A,B	A,B,C	A,C	A,C	A,C		
SECTION-III	Q.	1	2	3	4	5	6	7	8
	A.	5	2	8	7	3	6	4	5

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. Ans (B)

Fission of a nucleus of feasible only if the binding energy of daughter nuclei is more than the parent nucleus.

A = 70 will have same BE as 110 but A = 40 will have more B.E.

A = 100 will have same BE as 110 but A = 10 will have lesser B.E.

A = 90 will have same BE as 110 but A = 20 will have lesser B.E.

2. Ans (C)

$$k_{B\ell}^{x}\frac{\pi\left(b^2-a^2\right)\Delta T}{\ell}=k_{in}\frac{\pi a^2\Delta T}{\ell}$$

$$\Rightarrow \frac{b^2}{a^2}-1=\frac{1}{3}$$

$$\frac{b}{a}=\frac{2}{\sqrt{3}}$$

3. Ans (C)

Energy lost = Area of parallelogram OABC

$= 500 \times 10^6 \times 0.02 = 10 \times 10^6$ J/m^3

= 10 J/cc

PART-1 : PHYSICS

SECTION-I (ii)

5. Ans (B,C)

Segment S_1 has lower wavelength photons, so $E_1 > E_2$ and Intensity is equal, so $n_2 > n_1$.

6. **Ans (A,C)**

Heat required to melt ice $= 4 \times 80 = 320$ Kcal

heat released by condensing steam

$= 1 \times 540 = 540$ Kcal

$320 + 4 \times 1 (T - 0) = 540 + 1 \times 1 \times (100 - T)$

$T = 64°C$

$$\frac{d\theta}{dt} = \frac{0.5 \times 540 + 0.5 \times 1 \times (100 - 0)}{108}$$

$$= \frac{2 \times 80}{t}$$

Half of ice melts in 54 sec.

7. **Ans (B,C)**

$$E_0 z^2 \left(1 - \frac{1}{9}\right) - E_0 z^2 \left(\frac{1}{4} - \frac{1}{9}\right) = 3E_0$$

$z = 2$

$\lambda_1/\lambda_2 = 3$

$$KE_1 = E_0 \left(1 - \frac{1}{9}\right) - \phi$$

$$KE_2 = E_0 z^2 \left(1 - \frac{1}{4}\right) - \phi$$

$$KE \propto \frac{1}{\lambda^2} = 8.5 \text{ eV}.$$

8. **Ans (A,B)**

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$\Rightarrow 20 \times (25)^{\frac{4}{3}} = P_2 (200)^{\frac{4}{3}}.$$

$$\Rightarrow P_2 = \frac{20}{16} = \frac{5}{4} = 1.25 \text{ atm}$$

Work – done by gas

$$= \frac{P_1 V_1 - P_2 V_2}{r - 1}$$

$$= \frac{\left[20(25) - \frac{5}{4}(200)\right] \times 10^5 \times 10^{-6}}{\frac{4}{3} - 1}$$

$= 75$ J

9. **Ans (B,C)**

$^{11}C_6 \longrightarrow {}^{11}B_5 + e^+ + v$

$\Delta m = 11.011434 + 11.009305 - 2 \times 0.000549]$

$= 0.001031$ u

$Q = 0.001031\ 5\ 930 = 0.959$ MeV

10. **Ans (A,C)**

Energy available for excitation = E/2

$\therefore$ If $E < 20.4$ eV excitation will not take place.

For $E < 20.4$ eV collision is perfectly elastic and neutron must come to rest.

PART-1 : PHYSICS

SECTION-III

1. **Ans (6)**

$$v_1 = \frac{hC}{e\lambda} - \frac{\phi}{e} \qquad i = n\ell \frac{IA\lambda}{hC}$$

$$1.5 = 4 - \phi \Rightarrow \phi = 2.5 \text{ eV} \qquad \frac{i_1}{i_2} = \frac{\lambda_1}{\lambda_2}$$

$$\Rightarrow \lambda_2 = \frac{\lambda_1 i_2}{i_1} = 310 \times 1.5 = 465$$

$$v_2 = \frac{hc}{\lambda_2} = \frac{\phi}{e}$$

$$v_2 = \frac{1240}{1.5 \times 310} - 2.5 = \frac{1}{6} \text{ volts}$$

2. **Ans (3)**

At 60° Rate of heat loss = 18 W

$$\text{At } 35° \text{ Rate of heat loss} = \frac{10 \times 5}{30} = 3 \text{ W}$$

3. **Ans (3)**

$\rho u^{\eta} = C$

$\Rightarrow V^{-1} T^{\eta} = C$

$\Rightarrow T V^{-\frac{1}{\eta}} = C$

$\Rightarrow P V^{1-\left(\frac{1}{\eta}\right)} = C$

Compare with $PV^x = C$

$$\Rightarrow x = 1 - \frac{1}{\eta}$$

Now $\Delta Q \propto \left(\frac{R}{r-1} + \frac{R}{1-x}\right)$ and $\Delta W \propto \left(\frac{R}{1-x}\right)$

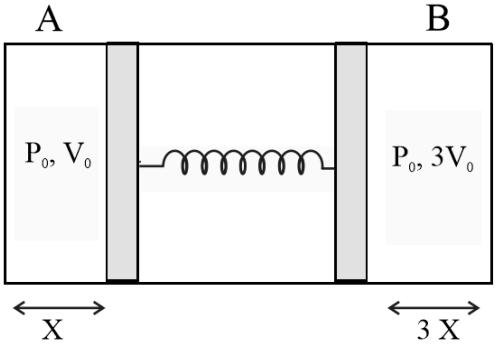
$$\Rightarrow \frac{\Delta W}{\Delta Q} = \frac{\left(\frac{R}{1-x}\right)}{\frac{R}{r-1} + \frac{R}{1-x}} = \left(\frac{r-1}{r-x}\right) = \frac{2}{3}$$

$$= \frac{\frac{5}{3} - 1}{\frac{5}{3} - x} = \frac{2}{3}$$

$$\Rightarrow x = \frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\Rightarrow \eta = 3$$

4. Ans (3)



Let $x = \frac{V_0}{A}$ $\Rightarrow$ compression = 4x

$\therefore P_0A = 4KX \Rightarrow K = \frac{P_0A^2}{4V_0}$

Initial energy in spring $= \frac{1}{2}K(4x)^2$

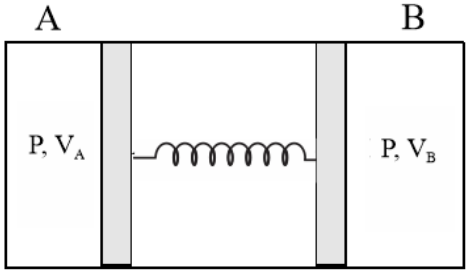
$= 8KX^2$

$= (8KX)X$

$= 2P_0A \times \frac{V_0}{A}$

$= 2P_0V_0$

$T_A = \frac{P_0V_0}{nR} = T_B\frac{P_0(3V_0)}{3nR} = T_0$



$P\,V_A = nR\,(3T_0)$ $\quad P\,V_B = 3nR(3T_0)$

$P_0\,V_A = nR\,T_0$ $\quad P_0\,(3V_0) = 3nRT_0$

$\Rightarrow P\,V_A = 3P_0V_0$ $\quad P\,V_B\,9P_0V_0$

Also, $PA = K\left(\frac{V_A + V_B}{A}\right)$

and $P_0A = K\left(\frac{4V_0}{A}\right)$

$\Rightarrow \frac{P}{P_0} = \frac{V_A + V_B}{4V_0} = \frac{12P_0V_0}{4PV_0}$

$\Rightarrow V_A = \sqrt{3}V_0$ and $V_B = 3\sqrt{3}V_0$

$\Rightarrow \Delta V_A = n\left(\frac{5R}{2}\right)\Delta T$

$= \frac{5}{2}\left[\left(\sqrt{3}P_0\right)\left(\sqrt{3}V_0\right) - P_0V_0\right]$

$= 5\,P_0V_0$

$= \Delta V_B = 3n\left(\frac{5R}{2}\right)\Delta T$

$= \frac{5}{2}\left[\left(\sqrt{3}P_0\right)\left(3\sqrt{3}V_0\right) - P_0\,(3V_0)\right]$

$= 15\,P_0V_0$

Energy stored in spring

$= \frac{1}{2}K\left(\frac{2\sqrt{3}V_0}{A}\right)^2$

$= \frac{1}{2}\left(\frac{P_0A^2}{4V_0}\right) \times \frac{(16 \times 3)\,V_0^2}{A^2}$

$= 6P_0V_0$

Work-done by gas

in 'A' & 'B'

$= \Delta U_{spring} = 5P_0V_0$

$\therefore dQ = dV + dW$

$\therefore dQ = 15P_0V_0 + 5P_0V_0 + 4P_0V_0$

$= 24\,P_0V_0$

$4k = \frac{24}{2}$

$k = 3$

5. **Ans (5)**

Power required = $\frac{40 \times 4.2 \times 50}{35 \times 60 \times 0.8} = 5$ KW

6. **Ans (4)**

Power = 40 kV 10 mA = 400 W.

99% of it is produced as heating power

Heating power = 396 W = 396 Joules per second.

7. **Ans (5)**

$\Delta E = 3.4eV - \left(-13.6\frac{z^2}{n^2}\right)$

$= 17eV$

$\frac{hc}{\lambda_1} = 17eV$ ..... (1)

2^{nd} condition $\Delta E' = E_2 - E_1$

$= \frac{-13.6 \times 4}{2^2}\left(+\frac{13.6 \times 4}{1}\right)$

$\Delta E' = 40.8eV$

$\Delta E' = \frac{hc}{\lambda_2} = 40.8eV$

$\lambda_1 / \lambda_2 = 2.4$

8. **Ans (3)**

$\frac{m_1}{m_2} = \frac{1}{3}$ $\frac{V_1}{V_2} = \frac{3}{1}$

Relative velocity of 1 W.r.f $2 = V_{12} = V_1 + \frac{V_1}{3}$

$= \frac{4V_1}{3}$

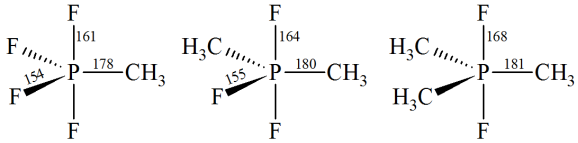
$\lambda = \frac{h}{m_1 v_1}$ $\lambda' = \frac{h}{m_1 \times \frac{4}{3}v_1}$

$\Rightarrow \lambda' = \frac{3}{4}\lambda$

PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (B)**

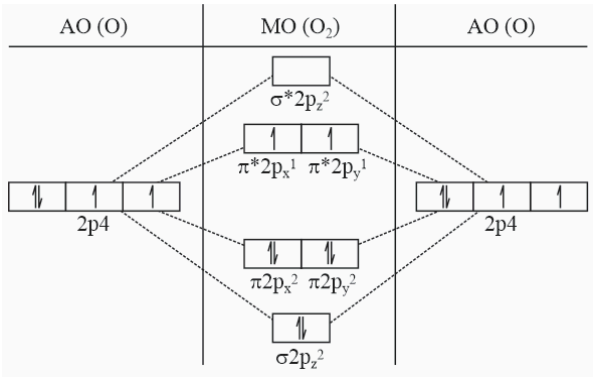


As %s character in a bond increase the bond length decreases.

2. **Ans (C)**

$O_2 : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2,$

$\pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1 = \pi^* 2p_y^1$



p : Energy of last e^- of O_2 is greater than O.

So

$IE(O) > IE(O_2)$

$\Rightarrow$ As Oxygen molecule has two unpaired electrons so it is paramagnetic.

$\Rightarrow$ LUMO : $\sigma^* 2p_z$

$\Rightarrow$ Bond order = $\frac{N_B - N_A}{2}$

$= \frac{10-6}{2} = 2$

3. **Ans (D)**

$B_2H_6(g) \longrightarrow 2B(s) + 3H_2(g)$ $\Delta H = -35$ KJ

$2B(s) + \frac{3}{2}O_2(g) \longrightarrow B_2O_3(s)$ $\Delta H = -1250$ KJ

$3 \times \left[H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g)\right]$ $\Delta H = -235$ KJ

$B_2H_6(g) + 3O_2(g) \longrightarrow B_2O_3(s) + H_2O(g)$

$\Delta H = -35 - 1250 - (3 \times 235)$

$= -1990$ KJ

PART-2 : CHEMISTRY

SECTION-I (ii)

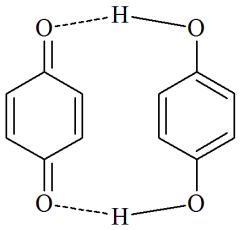
5. **Ans (A,B,D)**

Theoretical

6. **Ans (A,B,C)**

(A) $CF_3 - \overset{\delta^+}{C}H = \overset{\delta^-}{O} \ldots \overset{\delta^+}{H} - \ddot{O} - H$

(B) R_3N: ... H – O – C_2H_5



7. **Ans (B,D)**

$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

5 mol 10 mol [LR]

$\Delta n_g = -2$

$\Delta U = -170$ KJ/mol

$\Delta H = \Delta U + \Delta n_g RT$

For 3 mol H_2,

$\Delta H = -170 + (-2)\frac{25}{3} \times \frac{600}{1000}$

$= -180$ KJ

For 10 mol H_2,

$\Delta H = -180 \times \frac{10}{3}$

$= -600$ KJ

8. **Ans (A,B,C)**

A : PhCOOH(aq) + NaOH(aq) $\longrightarrow$ PhCOONa + H_2O

$\Delta_r H = (\Delta_N H) + (\Delta_{ion} H)$ PhCOOH

$= -13.7 + 3.6 = -10.1$ Kcal/mol

B : PhCOOH(g) + NaOH(g) $\longrightarrow$ PhCOONa + H_2O

$\Delta_r H = \Delta_{solution} H$(PhCOOH) $\longrightarrow$

$\Delta_{solution} H$(NaOH) + $\Delta_N H$ + $\Delta_{ion} H$(PhCOOH)

$= -16.8 - 4.8 - 13.7 + 3.6$

$= -31.7$ Kcal/mol

C : PhCOOH(g) + (aq) $\longrightarrow$ $PhCOO^-$(aq) + H^+(aq)

$\Delta_r H = \Delta_{solution} H + \Delta_{ion} H$

$= -16.8 + 3.6$

$= -13.2$ Kcal/mol

D : As NaOH is strong base no heat required for ionisation

10. **Ans (B,C)**

$U_{MP} = \sqrt{\frac{2RT}{M}}$

$U_{MP} \propto \frac{1}{\sqrt{M}}$

Rate of diffusion $\propto \frac{1}{\sqrt{M}}$

$\frac{r_{Gas\ 1}}{r_{Gas\ 2}} = \sqrt{\frac{M_2}{M_1}}$

$\frac{x}{1.2x} = \sqrt{\frac{M_2}{M_1}}$

$\frac{M_2}{M_1} = \frac{1}{1.44}$

$M_1 = 1.44 \times M_2$

$\frac{r_2}{r_3} = \frac{U_2}{U_3} = \frac{1.2x}{1.5x}$

$r_2 = 0.8\ r_3$

$U_{MP} = \sqrt{\frac{2RT}{M}}$

$U_{MP} = \sqrt{\frac{2RT}{M}} = x$

$\frac{RT}{M} = \frac{x^2}{2}$

$U_{rms} = \sqrt{\frac{3RT}{M}}$

$= \sqrt{3\frac{x^2}{2}}$

$= x\sqrt{\frac{3}{2}}$

$U_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$

$= \sqrt{\frac{8}{\pi}\frac{4}{2}\frac{x^2}{2}}$

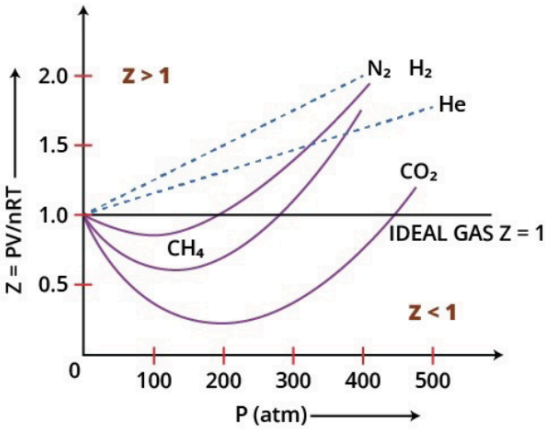
$= \frac{2x}{\sqrt{\pi}}$

PART-2 : CHEMISTRY

SECTION-III

1. Ans (5)

(i), (ii), (vi), (vii), (viii) are correct.



2. Ans (7)

$HA \underset{}{\overset{K_a}{\rightleftharpoons}} H^+ + A^{\ominus}$

$K_a = \dfrac{C\alpha^2}{1-\alpha} \approx C\alpha^2$

$= 1 \times (0.01)^2 = 10^{-4}$

$\Delta G^\circ = -2.3\ R\ T \log K_a$

$= -2.3 \times 0 \times 300 \log 10^{-4}$

$= -2.3 \times 2500 \times (-4)$

$= 2.3 \times 10^4$

Thus, x = 2.3, y = 4

So, $\dfrac{1}{9}[23 + 40] = 7$

3. Ans (2)

$P_4 + SO_2Cl_2 \longrightarrow PCl_5 + SO_2$

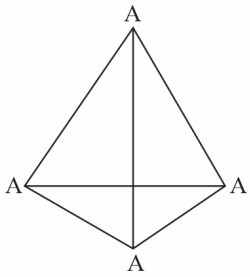
Hybridization of S in SO_2 is sp^2

4. Ans (5)

$A_4(s) \longrightarrow 4A(g) \quad \Delta_a H = 40$ Kcal/mol

$A_4(s) \longrightarrow A_4(g) \quad \Delta_{sub} H = 10$ Kcal/mol

$A_4(g) \longrightarrow 4A(g) \quad \Delta_r H = 30$ Kcal/mol



Number of bond (A – A) = 6

$6(BDE)_{A-A} = 30$

$(BDE)_{A-A} = 5$ Kcal/mol

5. Ans (7)

$T_B = \dfrac{a}{Rb}; V_c = 3b$

$\therefore a = RT_B \times \left(\dfrac{V_C}{3}\right)$

$= \dfrac{1}{12} \times 1260 \times \dfrac{0.2}{3}$

$= 7$

6. Ans (9)

For adiabatic reversible process,

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_2 = T_1 \left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$

For triatomic linear gas $\gamma = \dfrac{7}{5}$

$T_2 = 300 \times \left(\dfrac{1}{32}\right)^{2/5}$

$= 300 \times \dfrac{1}{4}$

$= 75$ K

$W = n\ C_V\ \Delta T$

$= 8 \times 5 \times (75 - 300)$

$= -9000$ cal

work-done = –9 Kcal

work-done by gas = 9 Kcal

7. **Ans (3)**

(i), (ii), (v)

8. **Ans (9)**

$2NaBH_4 + I_2 \longrightarrow 2NaI + B_2H_6 + H_2$

Moles of I_2 = moles of B_2H_6

$\frac{2.286 \times 1000}{254}$ = moles of B_2H_6

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (B)**

$G_1 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$G_2 = \left(\frac{2.\left(\frac{x_1+x_2}{2}\right) + 1 \times x_3}{3}, \frac{2 \times \left(\frac{y_1+y_2}{2}\right) + 1 + y_3}{3}\right)$

$\therefore G_2 = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

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Similarly

$G_{n-1} = \left(\frac{x_1 + x_2 + \ldots + x_n}{n}, \frac{y_1 + y_2 + \ldots + y_n}{n}\right)$

$G_{n-1} = \left(\frac{\sum x_i}{n}, \frac{\sum y_i}{n}\right)$

2. **Ans (B)**

$\sum z_k = \sum_{k=1}^{5} W_k = 3 + 504i$

Now Z_i lies on line $y = mx + 3$

$z_1 = x_1 + (mx_1 + 3)i$

$z_2 = x_2 + (mx_2 + 3)i$

$z_5 = x_5 + (mx_5 + 3)i$

$\sum_{k=1}^{5} x_k + (mx_k + 3)\, i = 3 + 504i$

$\sum_{k=1}^{3} x_k = 3$

$\sum_{k=1}^{5} m_1 x_k = 489$

So, m = 163

3. **Ans (A)**

Let S be the desired value. Note that

$z_1z_2+z_3 -1= z_1z_2 + 1 -z_1 -z_2 = (z_1 -1)(z_2 -1)$

Similarly, $z_2z_3+z_1 -1= (z_2 - 1) (z_1 -1)$

And $z_3z_1+z_2 -1= (z_3 - 1) (z_2-1)$

Hence

$S = \frac{1}{(z_1 - 1)(z_2 - 1)} + \frac{1}{(z_2 - 1)(z_3 - 1)} + \frac{1}{(z_3 - 1)(z_1 - 1)} = K$

$= \frac{z_1 + z_2 + z_3 - 3}{(z_1 - 1)(z_2 - 1)(z_3 - 1)} = \frac{-1}{(z_1 - 1)(z_2 - 1)(z_3 - 1)}$

$= \frac{-1}{z_1z_2z_3 - (z_1z_2 + z_2z_3 + z_3z_1)\, z_1 + z_2 + z_3 - 1}$

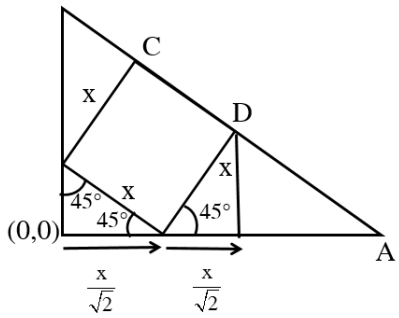
But $2(z_1z_2 + z_2z_3 + z_3z_1) = (z_1 + z_2 + z_3)^2$ —

$(z_1^2 + z_2^2 + z_3^2) = 1$

Therefore $S = -\frac{2}{9}$

$= \frac{-1}{5 - (z_1z_2 + z_2z_3 + z_3z_1)}$

4. **Ans (B)**



$D = \left(\frac{2x}{\sqrt{2}}, \frac{x}{\sqrt{2}}\right)$

$OD = \sqrt{\left(\frac{2x}{\sqrt{2}}\right)^2 + \left(\frac{x}{\sqrt{2}}\right)^2}$

$R = \sqrt{\frac{5}{2}}x$

$R = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{3}{\sqrt{10}}$

$= \frac{3}{2} = 1.5$

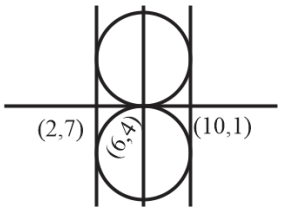
$R^2 = \frac{9}{4} \Rightarrow 2.25$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. Ans (A,C)

$$r = \left| \frac{C_1 - C_2}{\frac{\sqrt{a^2+b^2}}{2}} \right| = 5$$



L_4 always passes through (12, 12)

$$\frac{1+\lambda}{\lambda-1} < \frac{1}{2} \text{ or } \frac{1+\lambda}{\lambda-1} > \frac{11}{2}$$

So, $\frac{1+\lambda}{\lambda-1} = 6$

$$\lambda = \frac{7}{5}$$

6. Ans (A,B)

$$T_n = \begin{vmatrix} n-1 & n-2 & n-6 \\ 2n-4 & 2n-6 & 2n-11 \\ 3n-9 & 3n-12 & 3n-18 \end{vmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$

$$T_n = \begin{vmatrix} n-1 & -1 & -4 \\ 2n-4 & -2 & -5 \\ 3n-9 & -3 & -6 \end{vmatrix}$$

$$R_2 \to R_2 - \frac{(R_1 + R_3)}{2}$$

$$T_n = \begin{vmatrix} n-1 & -1 & -4 \\ 1 & 0 & 0 \\ 3n-9 & -3 & -6 \end{vmatrix}$$

$T_n = -1(-6) = 6$

7. Ans (A,B,C)

$A^2 = 4I \Rightarrow |A|^2 = 64 \Rightarrow |A| = -8$

$\because AB^T = adjA$

$\therefore A^2B^T = A\ adjA \Rightarrow A^2B^T = -8I$

$\Rightarrow B = -2I$

$AB^T = adjA \Rightarrow -2A = adjA$

$\Rightarrow |A + adjA| = |-A| = 8$

8. Ans (A,C)

$AP = 2I \Rightarrow |A| = 8$

$P = 2A^{-1} \Rightarrow P = \frac{2(adjA)}{|A|} \Rightarrow adjA = 4P$

$adj(adjA) = adj(4P) = 16adjP$

$Tr(adj(adjA)) = 48;$

$adjA.adj(adjA) = 16.4P.adjP$

$= 64|P|\ I_3 = 64I$

$\therefore \det(adjA.adj(adjA)) = (64)^3 = 2^{18}$

9. Ans (A,C)

$$\sum_{j=1}^{200}\sum_{k=1}^{200} \frac{Z_j}{Z_k} = (Z_1 + Z_2 + \ldots . Z_{200})$$

$$\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots . + \frac{1}{Z_{200}}\right)$$

$$\sum_{j=1}^{200}\sum_{k=1}^{200} \frac{Z_j}{Z_k} = 0$$

10. Ans (A,C)

O (0, 0) is orthocentre.

Let D, E be feet of perpendicular from A, C, S_B

passes the E & D.

also S_{AC} passes the E & D.

PART-3 : MATHEMATICS

SECTION-III

1. Ans (5)

$C\left(\frac{20-5t}{4}, t\right)$ $D(s, 0)$

$\Delta = \frac{1}{2}(5-s)t = 5$

$CD^2 = \left(\frac{20-5t}{4} - s\right)^2 + t^2$

$= \left[\left(\frac{10}{t}\right) - \frac{5t}{4}\right]^2 + t^2$

$= \frac{100}{t^2} + \frac{41}{16}t^2 - 25 \geqslant 5\sqrt{41} - 25$

2. Ans (2)

Let $x - \frac{2}{3} = a$, $y - \frac{2}{3} = b$ and $z - \frac{2}{3} = c$. We have $a + b + c = 0$. Also,

$1 = \left(a + \frac{2}{3}\right)\left(b + \frac{2}{3}\right) + \left(b + \frac{2}{3}\right)\left(c + \frac{2}{3}\right) + \left(c + \frac{2}{3}\right)\left(a + \frac{2}{3}\right) = ab + bc + ca + \frac{4}{3}$

Hence, $-\frac{1}{3} = ab + c(a + b) = ab - (a + b)^2$

$= -a^2 - b^2 - ab \Rightarrow a^2 + b^2 + ab = \frac{1}{3}$

Then, $\frac{4}{3} = 4a^2 + 4b^2 + 4ab \geqslant a^2 + b^2 - 2ab$

$= (a - b)^2 \Rightarrow \frac{2}{\sqrt{3}} \geqslant a - b = x - y$

Equality occurs when (x, y, z)

$= \left(\frac{2}{3} + \frac{1}{\sqrt{3}}, \frac{2}{3} - \frac{1}{\sqrt{3}}, \frac{2}{3}\right)$.

3. Ans (8)

$B + A^T = \text{adj } B$ ....(1)

$A + B^T = \text{adj } B$ ....(2)

$\Rightarrow A^T + B = (\text{adj}B)^T$

From (1) & (2) $A = B^T$, $A^T = B$

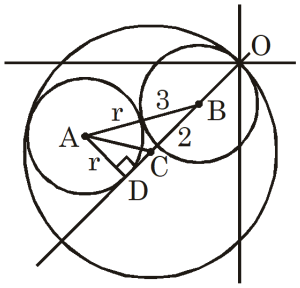
$2A = \text{adj } B = \text{Adj } (A^T)$

$|2A| = |\text{adj } A^T|$

$\Rightarrow 8|A| = |A|^2$

$\Rightarrow |A| = 8$

4. Ans (7)



$AB = r + 3$, $BC = 2$

$AC = 5 - r$.

$CD = \sqrt{(5-r)^2 - r^2}$

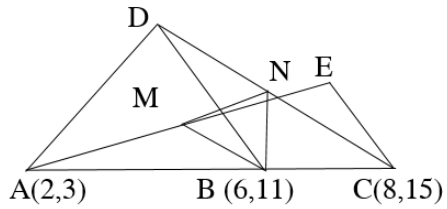
$AB^2 = AD^2 + BD^2$

$(r + 3)^2 = r^2 + \left(2 + \sqrt{5(5-2r)}\right)$

Sove & find r

$r = \frac{15}{8} \Rightarrow p - q = 7$

5. Ans (3)



Equation of AC : $(y-3)=\frac{12}{6}(x-2)$

$\Rightarrow 2x-y-1=0$

Point B divides AC in 2 : 1

So point B be (6, 11)

Point (1, 2) w.r. t line $2x-y-1=0$

$\Rightarrow 2\times 1-2-1<0$

So, Point D $\left(4-4\sqrt{3},7+2\sqrt{3}\right)$ and

Point E $\left(7-2\sqrt{3},13+\sqrt{3}\right)$

Now point M be $\left(\frac{9-2\sqrt{3}}{2},\frac{16+\sqrt{3}}{2}\right)$

and Point N be $\left(\frac{12-4\sqrt{3}}{2},\frac{22+2\sqrt{3}}{2}\right)$

Centroid of ΔBMN be (a, b)

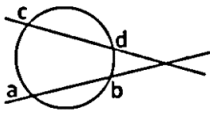
$$=\left(\frac{6+\frac{9-2\sqrt{3}}{2}+\frac{12-4\sqrt{3}}{2}}{3},\frac{11+\frac{16+\sqrt{3}}{2}+\frac{22+2\sqrt{3}}{2}}{3}\right)$$

$$=\left(\frac{11}{2}-\sqrt{3},10+\frac{\sqrt{3}}{2}\right)$$

$$\frac{2(a+2b)}{17}=3$$

6. Ans (6)

$$\frac{z-a}{\bar{z}-\bar{a}}=\frac{z-b}{\bar{z}-b}\Rightarrow\frac{z}{ab}+\frac{\bar{z}}{1}-\frac{(b+a)}{ab}=0 \quad ...(1)$$



Similarly

$$\frac{z-c}{\bar{z}-\bar{c}}=\frac{z-d}{\bar{z}-d}\Rightarrow\frac{z}{cd}+\frac{\bar{z}}{1}-\frac{(c+d)}{cd}=0 \quad ...(2)$$

Solving (1) & (2) for z gives

$$z=\frac{a^{-1}+b^{-1}-c^{-1}-d^{-1}}{a^{-1}b^{-1}-c^{-1}d^{-1}}$$

$\because c=d$

$$z=\frac{a^{-1}+b^{-1}-2c^{-1}}{a^{-1}b^{-1}-c^{-2}}$$

7. Ans (4)

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ adj $A=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$ & det

(A) = ad − bc = k (let)

Now, $|A+|A|\text{ adj }A|=0$

$$\Rightarrow\begin{vmatrix}a+kd & b-kb\\ c-kc & d+ka\end{vmatrix}=0$$

$\Rightarrow ad+k^2ad+kd^2+ka^2-bc(1-k)^2=0$

$\Rightarrow (1+k^2)(ad-bc)+k(a^2+d^2+2bc)=0$

$\Rightarrow k(1+k^2)+k(a^2+d^2+2ad-2k)=0$

$\Rightarrow k(k-1)^2+k(a+d)^2=0$

$\Rightarrow k=1$ and $a+d=0$ (As $K\neq 0$)

$\Rightarrow$ tr(A) = 0 and |A| = 1

$$|\text{adj }B|=|B|=|A-\text{adj }A|=\begin{vmatrix}a-d & 2b\\ 2c & d-a\end{vmatrix}$$

$=-a^2-d^2+2ad-4bc$

$=-(a+d)^2+4(ad-bc)$

$=0+4=4$

$\Rightarrow$ tr(A) + |adj AB| = 0 + |AB| = |A||B| = 4

8. Ans (5)

$\Rightarrow A^4 = A^2 - 4A + 4I$

$= A - 2I - 4A + 4I$

$\Rightarrow A^4 = -3A + 2I$

$\Rightarrow A^8 = 9A^2 - 12A + 4I$

$= 9(A - 2I) - 12A + 4I$

$\Rightarrow A^8 = -3A - 14I$

$\Rightarrow A^{10} = (-3A - 14I)(A - 2I)$

$= -3A^2 - 8A + 28I$

$= -3(A - 2I) - 8A + 28I$

$\Rightarrow A^{10} = -11A + 34I$

$\Rightarrow A^4 + A^8 + A^{10} = -17A + 22I$

$\Rightarrow a = -17, b = 22$