PART-1 : PHYSICS

SECTION-I (i)

1) Two plane electromagnetic waves with the same amplitude E_0 and frequency ω travel in opposite directions along the z-axis, creating a standing wave. The electric field vectors of these waves are given by :

$\vec{E}_1(z,t) = E_0\cos(kz - \omega t)\hat{i}$, $\vec{E}_2(z,t) = E_0\cos(kz + \omega t + \phi)\hat{i}$

The superposition of these two waves forms a standing wave. Select the correct statements.

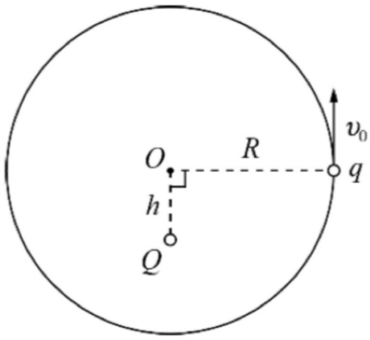
(A) When $\phi = \frac{\pi}{2}$, the resulting wave is elliptically polarized and the polarization changes with position along the z-axis.

(B) If $\phi = \pi$, the resulting wave is linearly polarized along a fixed direction in the xy-plane.

(C) The magnetic field vector of the resulting wave oscillates perpendicular to the plane of polarization at each point.

(D) For $\phi = \pi$, nodes of the electric field occur at positions $z = \frac{n\pi}{k}$, where n is an integer.

2) A small bead with mass m and charge +q can slide without friction along a non-conducting ring with radius R. In the plane of the ring, at a distance h from the center of the ring (h < R), a charge



+Q is fixed. Initially (see figure), the velocity of the bead is v_0.

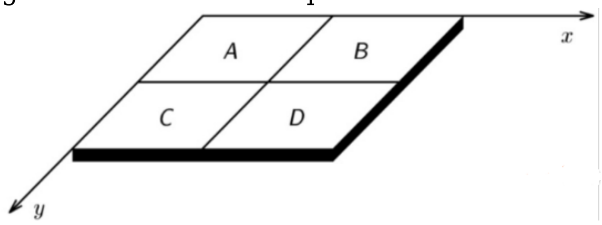
(A) The maximum speed reached by the bead during the motion is $\sqrt{v_0^2 + \frac{2kqQ}{m}\left(\frac{1}{\sqrt{R^2+h^2}} - \frac{1}{R+h}\right)}$

(B) The normal force acting on the bead when the speed is maximum, is zero.

(C) The initial velocity required to complete a full revolution around the ring is $\sqrt{\frac{2kqQ}{m}\left(\frac{1}{R-h} - \frac{1}{\sqrt{R^2+h^2}}\right)}$.

(D) The initial velocity required to complete a full revolution around the ring is $\sqrt{\frac{2kqQ}{m}\left(\frac{1}{R-h} + \frac{1}{\sqrt{R^2+h^2}}\right)}$

3) A thin homogeneous square plate with side length a and mass m has its center of mass at its geometric center. The plate is divided into four equal parts A,B,C,D _ See Fig.



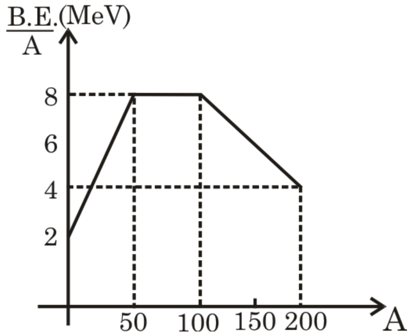
(A) The co-ordinates of center of mass of system after removing D is $\left(\frac{5a}{12}, \frac{a}{12}\right)$.

(B) The co-ordinates of center of mass of system after removing D and placing it on part A is $\left(\frac{3a}{8}, \frac{a}{8}\right)$.

(C) The co-ordinates of center of mass of system after removing D and placing it on part A is $\left(\frac{3a}{8}, \frac{3a}{8}\right)$.

(D) The co-ordinates of center of mass of system after removing D and placing it on part B is $\left(\frac{a}{2}, \frac{3a}{8}\right)$.

4) If the graph of binding energy $\left(\frac{B.E.}{A}\right)$ per nucleon versus mass number (A) looks like as shown in figure, then using the curve choose correct options :-



(A) Fusion of two nuclei with mass number 30 and 45 into one nucleons of mass number 75 will release energy.

(B) Fission of nuclei with mass number 80 into two nuclei of equal mass number will have Q-value 256 MeV.

(C) Fission of nuclei with mass number 150 into two nuclei of equal number will release energy.

(D) Fusion of two nuclei with mas number 10 and 20 into a nucleus of mass number 30 will release energy.

5) You have three tuning forks, A, B, and C. Fork B has a frequency of 441 Hz; when A and B are sounded together, a beat frequency of 3 Hz is heard. When B and C are sounded together, the beat frequency is 4 Hz.

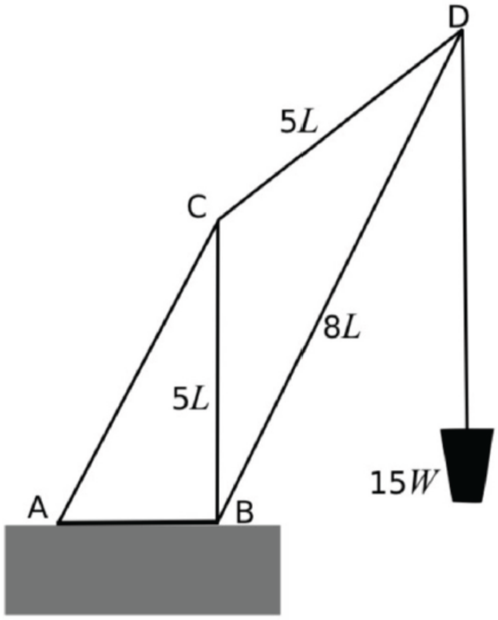
(A) The frequency of A can be 438 Hz.

(B) The frequency of C can be 444 Hz.

(C) The beat frequency between A and C can be 1 Hz.

(D) The beat frequency between A and C can be 6 Hz.

6) A winch is used to haul sand up from the bottom of a dry dock using a crane constructed from a very light framework of girders. The crane geometry can be seen in the diagram of figure. The crane is fixed to the horizontal dock side at points A and B. The dimensions of the sides CB, CD and BD are as shown, the girder CB is vertical, and the winch is hauling a load of weight 15W.



(A) The force exerted on D by BD in 24W.

(B) Tension in the rod CD is 15W.

(C) If the force on the clamp at A by ground acts vertically downwards and has a magnitude 36W, the distance AB is 2L.

(D) None of the above.

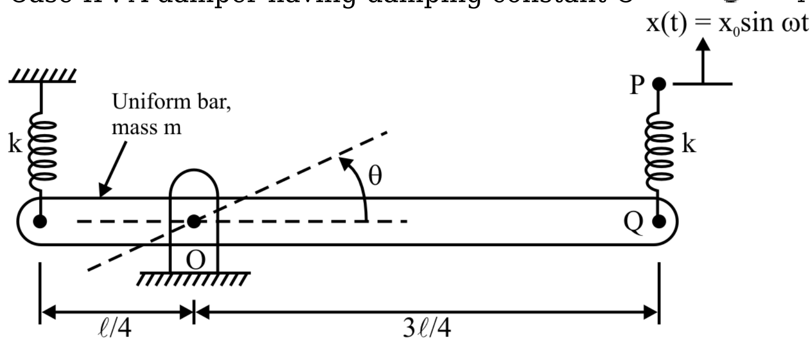
SECTION-I (ii)

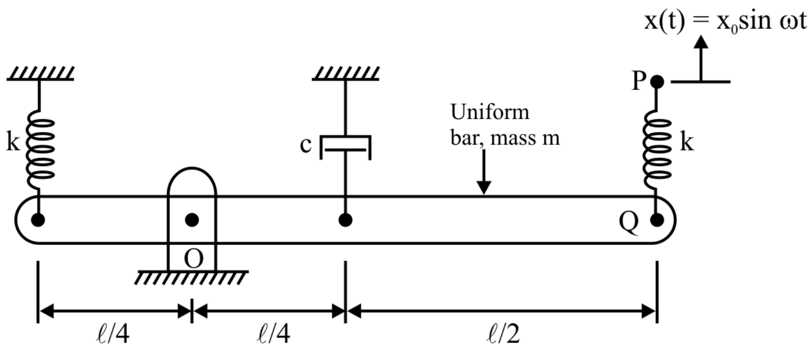
Common Content for Question No. 1 to 2

A uniform bar of mass m is pivoted at point O and supported at the ends by two springs, as shown in figure. End P of spring PQ is subjected to a sinusoidal displacement, $x(t) = x_0 \sin \omega t$. (ℓ = 1 m, k = 1000 N/m, m = 10 kg, x_0 = 1 cm, and ω = 10 rad/s.)

Case-I : No damper is present.

Case-II : A damper having damping constant C = $\frac{1000\sqrt{2}}{3}$ N-s/m is attached. (Neglect gravity)





1) The angular displacement of bar in case-I is

(A) 0.01565 sin (10t) rad
(B) 0.01565 sin (10t + π) rad
(C) 0.01565 sin $(10t + \frac{\pi}{2})$ rad
(D) 0.0313 sin (10t + π) rad

2)

The angular amplitude of bar in case-II is

(A) 0.008 rad
(B) 0.0131 rad
(C) 0.0262 rad
(D) 0.028 rad

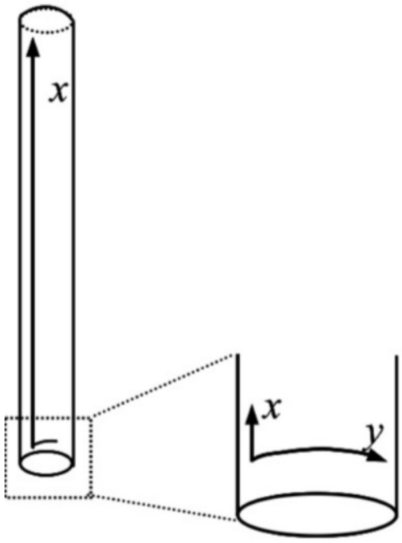
Common Content for Question No. 3 to 4
Consider a one-dimensional standing electromagnetic wave in the form of $E(x) = A \sin(k_x x)$ along the x-direction confined within the space between x = 0 and x = a. The wave must vanish at these two end points.

3)

Find the allowed values of k_x, (n = 1, 2, 3......)

(A) $k_x = \frac{n\pi}{a}$
(B) $k_x = \frac{2n\pi}{a}$
(C) $k_x = \frac{n\pi}{2a}$
(D) $k_x = \frac{4n\pi}{a}$

4) The String Theory predicts that our space is more than three-dimension, and the additional hidden dimensions are folded up like the dimension y on the surface of a thin cylinder shown in the figure. Suppose the radius of the cylinder is b (<< a), and the electromagnetic wave on the surface now takes the form $E(x,y) = A \sin(k_x x)\cos(k_y y)$, where y is the coordinate of the folded space around



the cylinder. Find the allowed values of k_y. (m = 1, 2, 3........)

(A) $k_y = \frac{m}{b}$

(B) $k_y = \frac{m}{2b}$

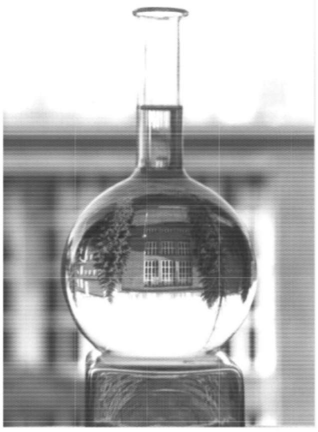
(C) $k_y = \frac{m}{\pi b}$

(D) $k_y = \frac{2m}{\pi b}$

SECTION-II (i)

Common Content for Question No. 1 to 2

In the window of the physics laboratory, an inverted image of the neighboring school was photographed, created by a spherical glass flask with a radius of R = 60 mm filled with water. Take



$\mu_{water} = \frac{4}{3}$.

1) At what distance d_1 (in cm) from the center of the flask is the center of this image located?

2) Since the camera is focused on the image of the building created by the flask, the actual school building is captured only as a blur. However, you can quite accurately compare the viewing angle under which you see the window of the building from the location where the center of the lens is located with the viewing angle under which you see, from the same location, the image of the window created by the flask. In the photograph, the height of the window appears as H = 50 mm, and the height of the image of the window created by the flask as h = 6 mm. What is the distance

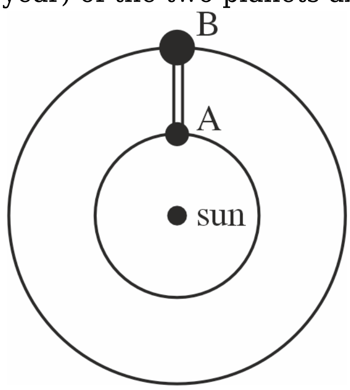
d_2 (in cm) from the center of the lens to the center of the image created by the flask ?

Common Content for Question No. 3 to 4

Consider two planets, A and B, of masses $M_A = 1.0 \times 10^{24}$ kg and $M_B = 2.0 \times 10^{24}$ kg. in circular orbits of radius $R_A = 1.0 \times 10^{11}$ m and $R_B = 4 \times 10^{11}$ m around a common star of mass $M_S = 24 \times 10^{31}$ kg. The two planets are orbiting in the same direction. You may assume that the planets are point masses and have no rotation about their own individual axes. (Take : G = $\frac{20}{3} \times 10^{-11}$ N-m^2/kg^2)

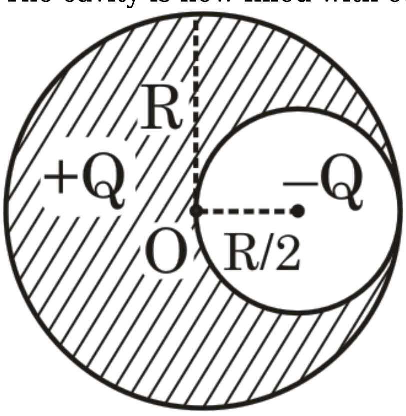
3) The planets have angular momentum L_A and L_B about the star. The value of $\frac{L_A}{L_B}$ is :

4) The inhabitants of the planets have constructed a light indestructible link bridge, which they connect when planets A and B are closest to each other. This joins the two planets instantaneously. Determine the angular velocity ω (in rad/year) of the two planets about their centre of mass at the instant just after the connection is made.



Common Content for Question No. 5 to 6

Consider a non-conducting sphere of radius R with centre O. A spherical cavity of radius $\frac{R}{2}$ is created with it's centre $\frac{R}{2}$ from O. The remaining sphere has charge Q uniformly distributed in it. The cavity is now filled with equal negative charge –Q, uniformly. [Take Q = 14 μC, R = 1cm.]

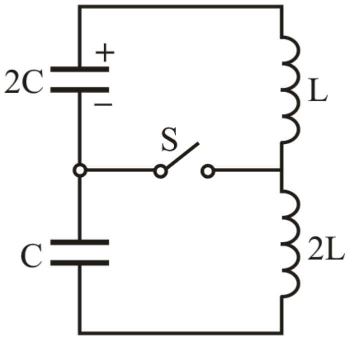


5) Find magnitude of potential energy of interation between positive and negative charges.

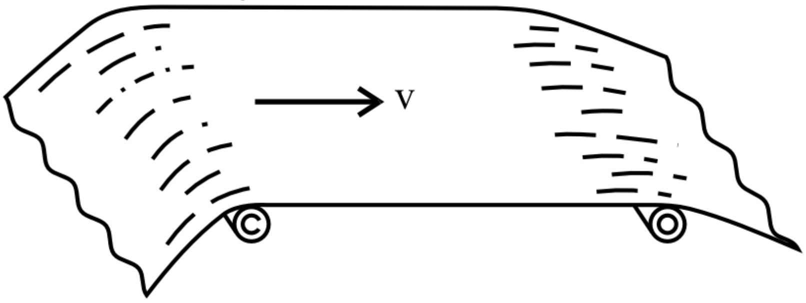
6) Find the net force on interaction between positive and negative charge (in kilo Newton)

SECTION-II (ii)

1) Initially, a switch S open in the circuit shown in the figure, a capacitor of capacitance 2C carries the electric charge q_0, a capacitor of capacitance C is uncharged, and there are no electric currents in both coils of inductance L and 2L, respectively. The capacitor starts to discharge and at the moment when the current in the coils reaches its maximum value, the switch S is instantly shorted. If the maximum current I_{max} through the switch S thereafter is $\frac{q_0}{\sqrt{\alpha LC}}$, the value of α is



2) In the production of polyethylene film, a wide strip is drawn over rollers at a speed of v = 45m/s as shown in the figure. During the processing (mainly due to friction), the surface of the film acquires a uniformly distributed charge. Find the maximum values of magnetic field induction B_{max} near the surface of the film, taking into account that at an electric field strength E = 20 kV/cm, an electrical discharge occurs in the air. The value of B_{max} is $\alpha \times 10^{-9}$ T. The value of α is

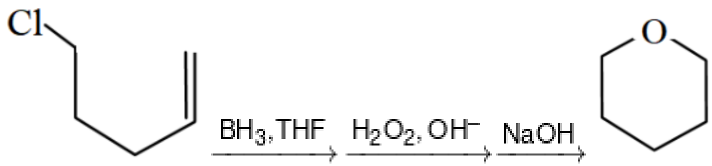


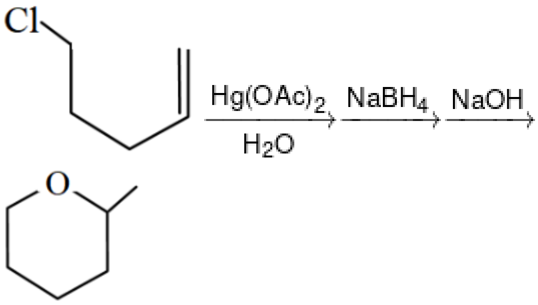
3) A certain planet of radius R is composed of a uniform material that, through radioactive decay, generates a net power P. This results in a temperature difference between the inside and outside of the planet as heat is transferred from the interior to the surface. The thermal conductivity of the material is k. The temperature difference between the surface of the planet and the center of the planet is equal to $\frac{P}{\alpha \pi k R}$. The value of α is

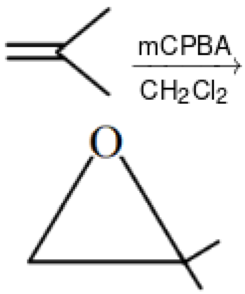
PART-2 : CHEMISTRY

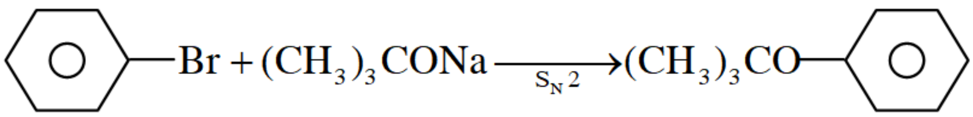
SECTION-I (i)

1) Select the correct major products

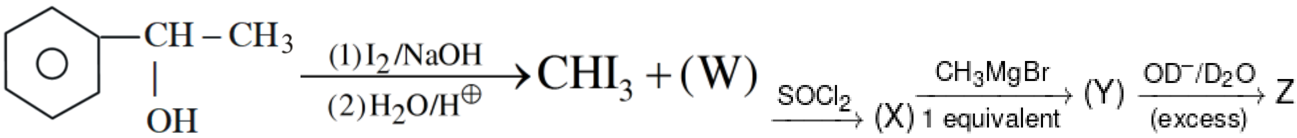
(A) 

(B) 

(C) 

(D) 

2) Which of the following option(s) have one of the product(s) obtained during this reaction sequence?



(A) $C_6H_5-\overset{\overset{\large O}{||}}{C}-OH$

(B) $C_6H_5-\underset{\underset{\large O}{||}}{C}-CD_3$

(C) $C_6H_5-\underset{\underset{\large O}{||}}{C}-Cl$

(D) $C_6H_5-\underset{\underset{\large O}{||}}{C}-CH_3$

3) Select the correct option(s) regarding complex $K_2Zn_3[Fe(CN)_6]_2$:

(A) It is insoluble in water but dissolves in exc. NaOH.

(B) Complex species is paramagnetic.

(C) Central metal uses d^2sp^3 hybrid orbitals for bond formation.

(D) Metal ligand linkage has π-character.

4) Select the correct statement(s) :

$XeF_6 \xrightarrow{\text{Hydrolysis}}$ 'X' + 2HF

'X' $\xrightarrow{\text{Hydrolysis}}$ 'Y' + 2HF

'Y' $\longrightarrow$ 'Z' + 2HF

(A) 'Y' is see-saw shaped.

(B) Central atom uses total three 'd-orbitals' for bonding in 'X'.

(C) 'Z' is planar but 'X' is non-planar.

(D) Z is white hygroscopic solid.

5) A metal cation M^{+2} gives black precipitate with acidified H_2S which dissolves in hot conc. HNO_3. Solution of M^{+2} gives brown precipitate on treatment with Potassium ferrocyanide. Select the correct statement(s) :

(A) M^{+2}(aq.) on reaction with exc. KCN produces a gas.

(B) Metal 'M' in its extraction process from its most common sulphide ore is obtained from auto-reaction process.

(C) $E^{\circ}_{M^{+2}/M}$ is positive.

(D) $M^{2+}_{(aq.)}$ produce dark blue solution with excess Cl^-

6) pH of which of the following solution in close to 5 ? (pka of CH_3COOH = 5, K_a of NH_4^+ = 5.6 × 10^{-10})

(A) 50 mL of 0.01 M CH_3COOH + 50 mL of 0.01 M CH_3COONa

(B) 0.1 M solution of NH_4Cl

(C) 0.1 M weak monoprotic acid having $K_a = 1 \times 10^{-9}$

(D) 5×10^{-6} M $Ba(OH)_2$

SECTION-I (ii)

Common Content for Question No. 1 to 2

1g of methane diffused in 20 seconds under certain conditions. Under the same conditions, $\sqrt{20}$g of an open chain hydrocarbon (A) diffused in 40 seconds. A 10 mg of sample of (A) consumes 8.40 ml of H_2 gas measured at 0°C and 760 mm pressure. One of the open chain isomers of (A) on ozonolysis gives only formaldehyde & glyoxal.

1)

A six membered stable monocyclic isomer of (A) on reductive ozonolysis can **NOT** give which of the following products

(A) OHC – CHO
(B) OHC–CH_2–CHO
(C) OHC–CH_2–CH_2–CHO
(D) OHC–CH_2–CH_2–CH_2–CHO

2) Identify the incorrect statement.

(A) One of the isomers of (A) exhibits geometrical isomerism
(B) One of the isomers of (A) gives white precipitate with Tollen 's reagent
(C) One of the isomers of (A) is optically active.
(D) One of the isomers of (A) on reductive ozonolysis gives only acetone.

Common Content for Question No. 3 to 4
Consider the following series of reactions

$$\underset{\text{2mole}}{NaN_3(s)} \xrightarrow[T<300°C]{\Delta} \underset{\text{(solid)}}{A} + \underset{\text{(gas)}}{B}$$

$$\underset{\text{(gas)}}{B} + \underset{\text{excess}}{H_2(g)} \rightarrow C(gas)$$

C(g) + bleaching powder solution → B(g) + D + H_2O

3) Select the correct statement’s.
(i) B is a diamagnetic gas having π bond order equal to 2.
(ii) In molecular orbital diagram of (B) gas H.O.M.O. contains 1 nodal plane.
(iii) Compound C is a weak acid.
(iv) Compound 'D' is obtained as byproduct of Solvay process.

(A) only (iv)
(B) (i), (iii) and (iv)
(C) (i) and (iv)
(D) (i), (ii) and (iii)

4) Compound 'D' can be used to dry :-

(A) NH_3
(B) C_2H_5OH
(C) Both (A) and (B)
(D) Neither 'A' nor 'B'

SECTION-II (i)

Common Content for Question No. 1 to 2
An acid A which is an important constituent of vineagar, on reaction with red P and Br_2 gives a monobromoderivative (B) which on reaction with NH_3 gives white solid (C). However ethanal on reaction with a mixture of ammonium chloride and sodium cyanide undergoes Strecker synthesis (synthesis of amino acids by the reaction of an aldehyde with cyanide in presence of ammonia) to give a product which on acidic hydrolysis gives another high melting solid (D). Let x and y are the

total number of optical isomers of (C) and (D) respectively while z is the total number of dipeptides (excluding stereoisomers) that (C) and (D) can form.

1) The value of x + y is _______.

2) The value of z is ________.

Common Content for Question No. 3 to 4

Consider the following sequence of reactions.

(i) $PbO_2 + \text{conc. } HNO_3 \rightarrow Pb(NO_3)_2 + H_2O + \underline{X}(g)$

(ii) $2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + H_2O + \underline{Y}(g)$

(iii) $FeS_{2(8)} + X(g) \rightarrow Fe_2O_3 + \underline{Z}(g)$

3) Assume ideal behavior of all three gases, 0.5 moles of each X and Y gas are taken in a container and expanded reversibly and adiabatically from 1 dm^3 to 4 dm^3 starting from 27°C. Find final temperature (K).
Given $(4)^{0.4} = 1.74$

4) If equal moles of X,Y,Z (assuming ideal behavior of the gas) are expanded adiabatically and reversibly from the same initial state to same final volume. The magnitude of work is maximum for a gas. What is the value of γ for that gas, if all the degree of freedom are active.

Common Content for Question No. 5 to 6

The following reacations are occurring in solvay process used for the manufacture of sodium carbonate.

$(A) \xrightarrow{\Delta} (B) + CO_2$

$(B) + H_2O \rightarrow (C)$

$(C) + NH_4Cl \rightarrow (D) + M$

$\downarrow H_2O$ [D is a gas which is soluble in H_2O]

(E) solution

$(E) + CO_2(\text{excess}) \rightarrow (F)$

$(F) + NaCl \rightarrow (G) \xrightarrow{\Delta}$ sodium carbonate.

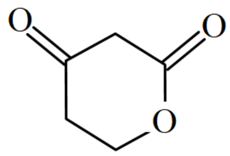
(Given that Atomic Mass : Na = 23, K = 39, Ca = 40, Mg = 24, Fe = 56, Cu = 63.5, O = 16, S = 32, H = 1, C = 12, Cl = 35.5, F = 19, Zn = 65)

5) If the molar mass of G is X g $mole^{-1}$ and that of (D) is Y g $mole^{-1}$. Then (X + Y) is__

6) The number of molecules of F are obtained on reaction of $\left(\frac{1}{2}\right)$ mole of (E) with $\left(\frac{1}{4}\right)$ mole of CO_2

is Q × 10^{21}. What is the value of Q ?

SECTION-II (ii)



1) In how many steps decarboxylation reaction is taking place.

$$\xrightarrow[\text{Step-1}]{H_3O^+}\xrightarrow[\text{Step-2}]{\Delta}\xrightarrow[\text{Step-3}]{I_2/OH^-}\xrightarrow[\text{Step-4}]{KMnO_4/H^+}\xrightarrow[\text{Step-5}]{\Delta}\xrightarrow[\text{Step-6}]{NaOH/CaO/\Delta}$$

2) Two mole of non-volatile weak electrolyte XY was dissolved in 20 dm^3 of water. If boiling point of the solution was found to be 100.08°C and K_b of H_2O is 0.5, then calculate the conductivity of the solution in S m^{-1} unit. Assuming molality is equal to molarity.

Λ°_{m} of XY = 500 Scm^2/mole

3) 200 cm^3, 0.2 M $BaCl_2$(aq.) is mixed with 0.1M, 600 cm^3 Na_2SO_4 solution. Find osmotic pressure of resulting solution (in atm). Round off up to nearest integer value.
R = 0.0821 L-atm K^{-1} mol^{-1}
T = 37°C

PART-3 : MATHEMATICS

SECTION-I (i)

1) Let f(x) be a polynomial function satisfying $0 < xf(y) < yf(x)\ \forall$ x, y such that $0 < x < y < 1$ and f (0) = 0 then :

(A) $f'(x) < f(1)$

(B) $f(1) < 2\int_0^1 f(x)\,dx$

(C) $3f\left(\frac{1}{3}\right) > 2f\left(\frac{1}{2}\right)$

(D) $6f\left(\frac{1}{6}\right) < 5f\left(\frac{1}{5}\right)$

2) Let a, b , c be distinct complex numbers with |a| = |b| = |c| = 1 and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $|z_1| = 1$. Also P and Q are the points representing the complex number z_1 and z_2 respectively in the complex plane with $\angle POQ = \theta$ (where O being the origin) then which of the following is/are correct?

(A) $b^2 = ac$

(B) $\theta = \frac{2\pi}{3}$

(C) $PQ = \sqrt{3}$

(D) $|z_1 + z_2| = 1$

3) Let $< T_n >$ be a sequence such that $T_n^3 + 2T_n = T_{n+1} \forall n \in N$, and $T_1 = 1$, then :

(A) $\sum_{n=1}^{100} (T_n^3 + T_n + 1) = T_{101} + 99$

(B) $\sum_{n=1}^{100} (T_n^3 + T_n + 1) = T_{101} + 100$

(C) $\prod_{n=1}^{100} \left(T_n^2 + 2\right) = T_{100}$

(D) $\prod_{n=1}^{100} \left(\frac{T_{n+1}}{T_n} - T_n^2\right) = 2^{100}$

4) Let straight line $y = mx + 4$ meets the curve $3x^2 - (1 - 3a) xy - ay^2 = 0$ at two points A and B such that $\angle AOB = 90° \forall m \in R - \{m_1, m_2\}$ where $m_1 < m_2$ and 'O' is the origin. Identify which of the following statements is/are correct ?

(A) $m_1 + m_2 = \frac{10}{3}$

(B) $am_1 + m_2 = 2$

(C) If $m = 2$, then area of $\Delta AOB = \frac{80}{7}$ sq. units

(D) If $m = 2$, then area of $\Delta AOB = \frac{85}{7}$ sq. units

5) Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ and $B = \begin{bmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ac - b^2 \end{bmatrix}$
be two non-singular matrices such that $(A^2 - 2I) B = O$ where $a > b > c > 0$, then which of the following statements is/are correct?
[Note: I is an identify matrix of order 3 and Tr. (P) and det. (P) denote trace and value of the determinant of square matrix P respectively.]

(A) $Tr.(AB) = 6\sqrt{2}$

(B) $Tr.(AB) = -6\sqrt{2}$

(C) $det.(A - \sqrt{2}B) = 54\sqrt{2}$

(D) $det.(A - \sqrt{2}B) = -54\sqrt{2}$

6) If $\lambda_1 > \lambda_2 > \lambda_3 > \ldots > \lambda_n$ be the values of 'λ' for which equation $x^3 - 3x + \lambda = 0$ has all integral roots,
if $h(x) = x^3 - 3x + |\lambda_1| + |\lambda_2| \ldots.. + |\lambda_n|$ for $x \in [1, \infty)$ and g(x) be the inverse of h(x) then:

(A) $(\lambda_1)^2 + (\lambda_2)^2 + \ldots.. + (\lambda_n)^2 = a$, number which is not the perfect square of an integer.

(B) Area bounded by g(x) and y-axis between limits $y = 2 \text{ to } y = 5 \text{ is } \frac{531}{4}$

(C) $\int_1^3 (x^3 - 3x)\,dx + \int_2^{22} g(x)\,dx = 64$

(D) $\int_1^3 (x^3 - 3x)\,dx + \int_2^{22} g(x)\,dx = 56$

SECTION-I (ii)

Common Content for Question No. 1 to 2

Consider, $f(x) = \lim_{n\to\infty} \frac{\text{sgn}(\sqrt{ac} - b)e^{nx} + x^2 + f}{2e^{nx+x} + x + d}$ where a > b > c > 0 and $d, f \in R$ [Note: sgn (y) denotes the signum function of y.]

1) If a, b and c are in A.P. and f (x) is continuous for all x ∈ R, then the value of (2f + d + 1) is equal to :

(A) 0

(B) 1

(C) –1

(D) $\frac{-1}{2}$

2) If a, b and c are in G.P. and f(x) is continuous for all x ∈ R (d < 0), then number of solutions of the equation f(x) = ||x – 4| – 2| – 1 is (are) :

(A) 0

(B) 2

(C) 3

(D) 4

Common Content for Question No. 3 to 4

Let B_n denotes the event that n fair dice are rolled once with $P(B_n) = \frac{1}{2^n}, n \in N,$ e.g. $P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{2^2}, P(B_3) = \frac{1}{2^3},$ ... and $P(B_n) = \frac{1}{2^n}.$ Hence B_1, B_2, B_3,...,B_n are pair wise mutually exclusive and exhaustive events as $n \to \infty$. The events A occurs with atleast one of the event B_1, B_2, B_3,..........B_n and denotes that the sum of the numbers appearing on the dice is S.

3) If even number of dice has been rolled, the probability that S = 4, is

(A) very closed to $\frac{1}{2}$

(B) very closed to $\frac{1}{4}$

(C) very closed to $\frac{1}{8}$

(D) very closed to $\frac{1}{16}$

4) Probability that greatest number on the dice is 4 if three dice are known to have been rolled, is

(A) $\frac{37}{216}$

(B) $\frac{64}{216}$

(C) $\frac{27}{216}$

(D) $\frac{31}{216}$

SECTION-II (i)

Common Content for Question No. 1 to 2

An ellipse whose major axis is parallel to x-axis such that segment of a focal chord are 1 and 2 units. The line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ are the chords of the ellipse such that a, c, b are in H.P. and are bisected by the point at which they are concurrent, the equation of auxiliary circle is $2x^2 + 2y^2 + 4px + 4qy - 16p + 1 = 0$ then

1) Find p + q

2) If length of latus rectum of ellipse is L, then find [L]. (where [.] represent Greatest integer function)

Common Content for Question No. 3 to 4

If n be a natural number define polynomial $f_n(x)$ of n^{th} degree as follows $f_n(\cos\theta) = \cos n\theta$ i.e. $f_2(x) = 2x^2 - 1$; $f_3(x) = 4x^3 - 3x$. Then.

3) $\left(x+\sqrt{x^2-1}\right)^{10} + \left(x-\sqrt{x^2-1}\right)^{10}$ is equal to $\lambda f_k(x)$. Then find value of $\lambda + k$.

4) $f_6(x)$ is equal to $px^6 + qx^4 + rx^2 + s$ then find value of $7(p + q + r - s)$

Common Content for Question No. 5 to 6

Let $P_1 : x + y + 2z - 3 = 0$ and $P_2 : x - 2y + z = 4$ be two planes. Let A(1,3,4) and B(3,2,7) be two points in R^3.

5) The equation of plane P_3 through line of intersection of P_1 and P_2 and length of projection upon which of the line segment AB is greatest is equal to $ax + by + cz + 1 = 0$, then find the value of a + b + c. (where $a,b,c \in I$)

6) The equation of plane P_4 through the line of intersection of P_1 and P_2 and the length of projection upon which of the line segment AB is least is equal to $ax + by + cz - 7 = 0$, then find the value of a + 2b + c. (where $a,b,c \in I$)

SECTION-II (ii)

1) The sum of infinite series

$\frac{1}{15} + \frac{1}{30} + \frac{1}{50} + \frac{1}{75} + \ldots\ldots\ldots\infty$ is $\frac{k}{20}$, then the value of k is

2) Given that for $a, b, c, d \in R$, if $a \sec(200°) - c \tan(200°) = d$ and $b \sec(200°) + d \tan(200°) = c$ then

$\frac{a^2 + b^2 + c^2 + d^2}{ac - bd} = p\operatorname{cosec}(200°)$ find the value of p______.

3) If $I_n = \int_0^{\infty} e^{-x}(\sin x)^n dx \, (n > 1),$ then the value of $\frac{101 I_{10}}{I_8}$ is equal to __________.

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I (i)

Q.	1	2	3	4	5	6
A.	B,C,D	A,C	C,D	A,C,D	A,C	A,B,C

SECTION-I (ii)

Q.	7	8	9	10
A.	B	B	A	A

SECTION-II (i)

Q.	11	12	13	14	15	16
A.	12.00	100.00	0.25	121.14 to 121.15	201.60	10.08

SECTION-II (ii)

Q.	17	18	19
A.	2	1	8

PART-2 : CHEMISTRY

SECTION-I (i)

Q.	20	21	22	23	24	25
A.	A,C	A,B,C,D	A,C,D	A,B,D	A,B,C	A,B,C

SECTION-I (ii)

Q.	26	27	28	29
A.	D	D	C	D

SECTION-II (i)

Q.	30	31	32	33	34	35
A.	2.00	4.00	172.41	1.15	101.00	150.55

SECTION-II (ii)

Q.	36	37	38
A.	3	3	7

PART-3 : MATHEMATICS

SECTION-I (i)

Q.	39	40	41	42	43	44
A.	**B,C**	**A,B,C,D**	**A,D**	**B,C**	**B,D**	**A,B,D**

SECTION-I (ii)

Q.	45	46	47	48
A.	**B**	**D**	**D**	**A**

SECTION-II (i)

Q.	49	50	51	52	53	54
A.	**1.00**	**2.00**	**12.00**	**21.00**	**4.00**	**3.00**

SECTION-II (ii)

Q.	55	56	57
A.	**4**	**2**	**90**

SOLUTIONS

PART-1 : PHYSICS

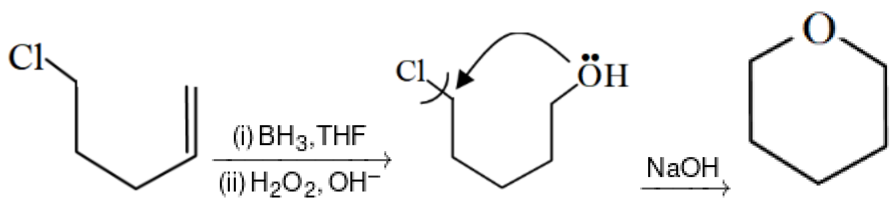
16) F = QE = $Q\left(\frac{\rho R}{6\epsilon_0}\right)$

= $\frac{QR}{6\epsilon_0}\left(\frac{Q}{\frac{4\pi}{3}\left(R^3-\frac{R^3}{8}\right)}\right) = \frac{Q^2}{7\pi\epsilon_0 R^2}$

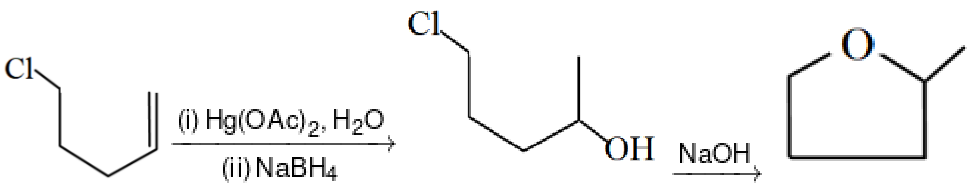
19) For the first question, apply the Boltzmann equation, and
$P = \sigma A T_s^4$
where A is the surface area of the planet, and T_s the temperature at the center. Then
$T_s = \left(\frac{P}{4\pi\sigma R^2}\right)^{\frac{1}{4}}$
For the second question, it is reasonable to assume that the temperature depends on the distance form the center only. Then the definition of k gives for a spherical shell of thickness dr
$k = \frac{\Delta Q}{\Delta t}\frac{1}{4\pi r^2}\frac{dr}{dT}$.
The heat through the shell depends on the power radiated from within the shell. Since the planet is uniform, this depends on the volume according to
$\frac{\Delta Q}{\Delta t} = P\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = P\frac{r^3}{R^3}$
so that rearrangement yields
$dT = \frac{P}{4\pi k R^3} r\, dr$
Integrating between the center and the surface,
$\Delta T = \frac{P}{8\pi k R}$,
which could be used to find the temperature of the interior.

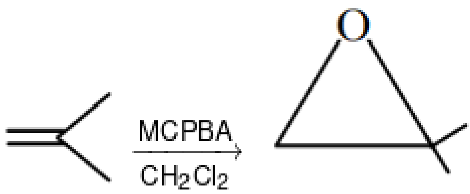
PART-2 : CHEMISTRY

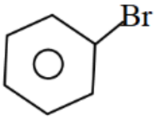
20) (1)

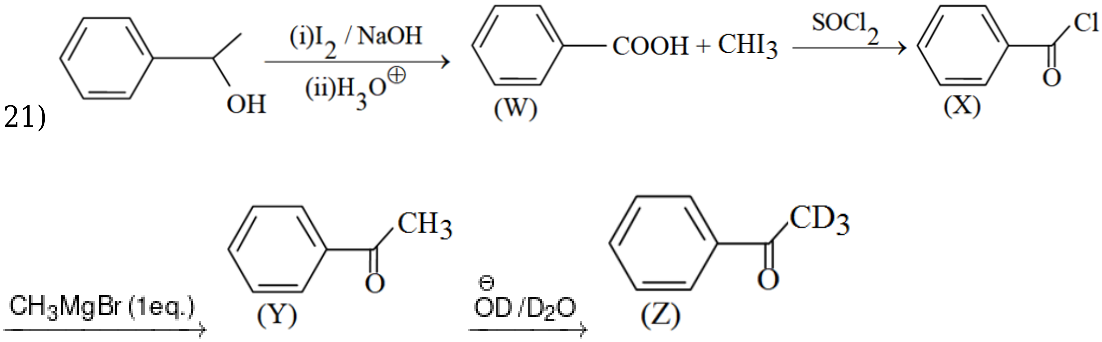


(2)



(3) 

(4)  + $(CH_3)_3CO^-Na^+ \rightarrow$ No reaction

21) 

22)
$$\underbrace{K_2Zn_3[\overset{+2}{Fe}(CN)_6]_2}_{\text{diamagnetic}} \xrightarrow{NaOH} K^+ + Zn(OH)_4^{2-} + [Fe(CN)_6]^{4-}$$

$\rightarrow$ Synergic bonding b/w Fe^{+2} and $\bar{C}N$.

23) X = $XeOF_4$, Y = XeO_2F_2, Z = XeO_3

24) M = Cu

$$Cu^{2+} \xrightarrow{K_4[Fe(CN)_6]} \underset{\text{(Brown)}}{Cu_2[Fe(CN)_6]} \downarrow$$

$$E^{\circ}_{Cu^{+2}/Cu} = \text{'+'Ve}$$

25) (A) pH = 5 + log 1 = 5 (Buffer solution)

(B) salt hydrolysis

(C) $[H^+] = \sqrt{CKa} = 10^{-5}$; pH = 5

(D) $[OH^-] = 2 \times 5 \times 10^{-6} = 10^{-5}$, pOH = 5, pH = 9

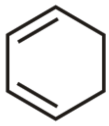
26) Rate of diffusion of methane = $\frac{1}{16 \times 20}$ mole sec^{-1}

Rate of diffusion of hydrocarbon = $\frac{\sqrt{20}}{M \times 40}$ moles sec^{-1}

Now, $\frac{r_{CH_4}}{r_{hyd}} = \sqrt{\frac{M_{hyd}}{16}}$

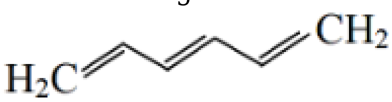
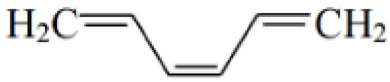
$$\frac{\frac{1}{16\times20}}{\frac{\sqrt{20}}{M\times40}} = \frac{\sqrt{M}}{4} \Rightarrow \frac{40\times M}{16\times20\times\sqrt{20}} = \frac{\sqrt{M}}{4}$$
$$\Rightarrow \sqrt{M} = 2\sqrt{20}$$
$$\Rightarrow M = 80\ \text{g mol}^{-1}$$

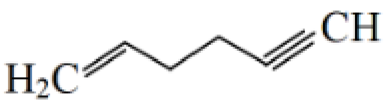
0.125 m mol of (A) consumes 0.375 m mol H_2. So, (A) has three π-bonds
Therefore : molecular formula of A is C_6H_8
An open chain isomer of A can be $CH_2 = CH - CH = CH–CH = CH_2$

A six membered stable monocyclic isomer of (A) can be  or 

27)

$\Rightarrow = M = 80\ \text{g mol}^{-1}$

 and 



$$H_3C - \overset{\overset{\displaystyle H}{|}}{\underset{\underset{\displaystyle C\equiv CH}{|}}{C^*}} - CH = CH_2$$

(Terminal alkynes responsed to tollen's test)

29) $\underset{\text{1mole}}{NaN_3(s)} \xrightarrow[T<300°C]{\Delta} \underset{\text{(solid)}}{Na} + \underset{\text{(gas)}}{\frac{3}{2}N_2}$

$N_{2(g)} + 3H_2 \rightarrow 2NH_3(g)$

$NH_3 + Ca(OCl)Cl \rightarrow N_2 + CaCl_2 + H_2O$

30)

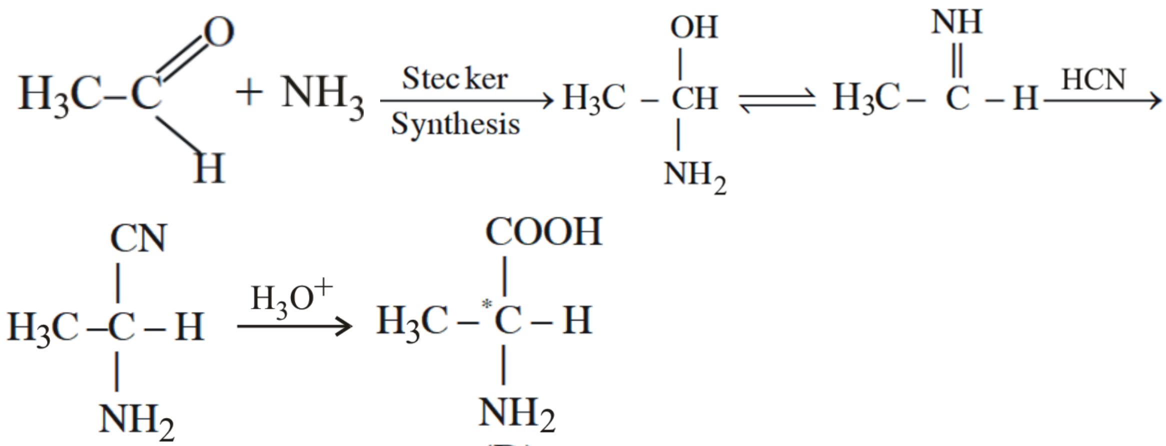
Acid (A) is ethanoic acid as it is an important constitutent of vineagar.

$$H_3C - \underset{(A)}{\overset{\overset{\displaystyle O}{\|}}{C}} - OH \xrightarrow[\text{(HVZ reaction)}]{P/Br_2} \underset{\underset{\displaystyle Br}{|}}{H_2C} - \underset{(B)}{\overset{\overset{\displaystyle O}{\|}}{C}} - OH \xrightarrow[\Delta]{NH_3} \underset{\underset{\underset{\text{(Glycine)}}{\displaystyle NH_2}}{|}}{H_2C} - \underset{(C)}{\overset{\overset{\displaystyle O}{\|}}{C}} - OH$$

Also,

$$NH_4Cl + NaCN \rightleftharpoons NH_4CN + NaCl$$

$$NH_4CN \rightleftharpoons NH_3 + HCN$$



(D)
Alanine

Now, since (C) has no chiral carbon. So, x = 0
And (D) has one asymmetric carbon, so y = 2

31)

Total number (z) of dipeptides which can be made by (C) and (D) are
gly - gly, ala - ala, gly - ala, ala - gly
Hence, z = 4

32) $Cv_{mix} = \left(\frac{n_1 Cv_1 + n_2 Cv_2}{n_1 + n_2}\right)$

$= \frac{\left(0.5 \times \frac{5R}{2}\right) + \left(0.5 \times \frac{5R}{2}\right)}{1}$

$Cv_{mix} = \frac{5R}{2}$

$Cp_{mix} = R + Cv_{mix}$

$= R + \frac{5R}{2}$

$= \frac{7R}{2}$

For reversible adiabatic process

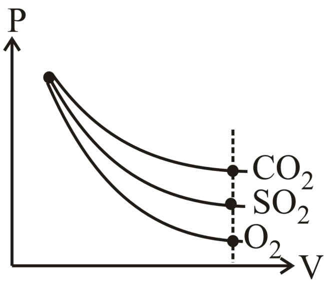
$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$300(1)^{r-1} = t_2(4)^{\frac{7}{5}}$

$300 \times 1 = T_2(4)^{\frac{2}{5}}$

$300 = T_2 \times 1.74$

$T_2 = (172.41\ K)$

33) 

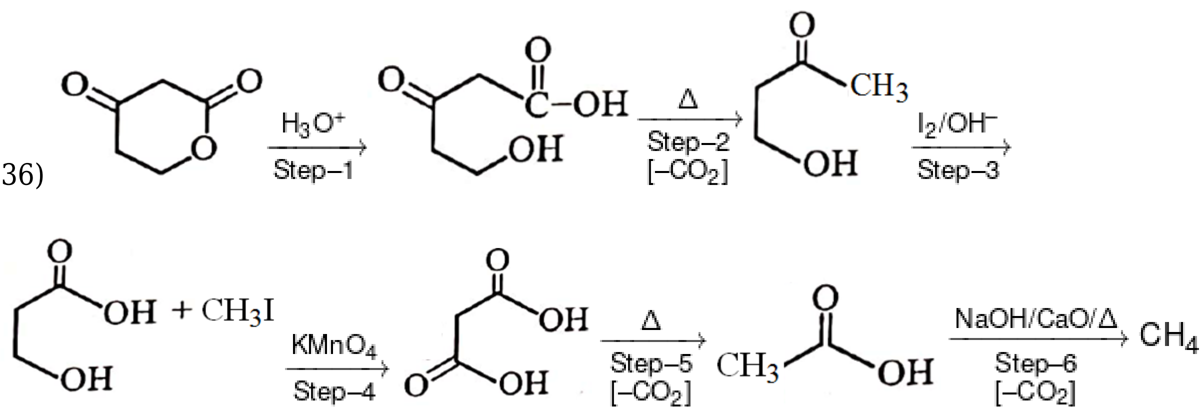
$X = O_2 \quad \gamma = \frac{9}{7} = 1.28$

$Y = CO_2 \quad \gamma = \frac{15}{13} = 1.15$

$Z = SO_2 \quad \gamma = \frac{7}{6} = 1.16$

34)

$G = NaHCO_3$, $D = NH_3$

36) 

37) $\Delta T_b = i.K_b.m$

$0.08 = i \times 0.5 \times \frac{2}{20}$

$\frac{0.08}{0.5} \times \frac{20}{2} = i$

$i = 1.6$

$\Lambda_m = \Lambda_m^\circ.C.\propto.$

$= 500\frac{Scm^2}{mole} \times \frac{2}{20}\frac{mole}{L} \times 0.6$

$\Lambda_m = 30\frac{Scm^2}{1000cm^3} = \frac{3}{100}Scm^{-1}$

$\Lambda_m = 3\ Sm^{-1}$

38) $BaCl_2 \quad + \quad Na_2SO_4 \rightarrow BaSO_4 \downarrow + 2NaCl$

0.2 M   0.1 M   ppt

0.04 mole   0.02 mole   0.08 mole

$\pi_{Na_2SO_4} = i_1C_1RT \quad \pi_{NaCl} = i_2C_2RT$

$\pi_{Na_2SO_4} + \pi_{NaCl} = (i_1C_1 + i_2C_2).RT$

$= 3 \times \left(\frac{0.02}{0.8}\right) + 2\left(\frac{0.08}{0.8}\right).R \times T$
= (0.075 + 0.2)RT
= 0.275 × 0.0821 atm × 300
= 6.99
= 7 atm

PART-3 : MATHEMATICS

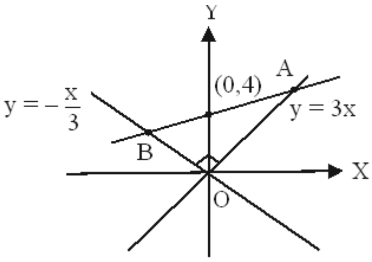
39)

$xf(y) < yf(x)\ \forall\ 0 < x < y < 1$
$\Rightarrow \quad \frac{f(y)}{y} < \frac{f(x)}{x}\ \forall x < y$
$\Rightarrow \quad g(x) = \frac{f(x)}{x}$ is decreasing
$\Rightarrow \quad g'(x) < 0 \Rightarrow xf'(x) < f(x)$
$\Rightarrow \quad f'(x) < \frac{f(x)}{x} \forall\ x, 0 < x < 1$
Hence, $f'(x) < g(x) \forall x, 0 < x < 1$
Hence, f'(x) is less than minimum value of g(x) which is g (1)
$\therefore f'(x) < f(1)$
$x f'(x) < f(x) \Rightarrow \int_0^1 x f'(x)\,dx < \int_0^1 f(x)\,dx$
$\Rightarrow x\ f(x)\ |_0^1 - \int_0^1 f(x)\ dx < \int_0^1 f(x)\ dx$
$\Rightarrow \quad f(1) < 2\int_0^1 f(x)\ dx$

40) $|z_1\ z_2| = \left|\frac{c}{a}\right| = 1, \quad |z_1 + z_2| = \left|\frac{-b}{a}\right| = 1$
$\Rightarrow \quad (z_1 + z_2) \times (\bar{z}_1 + \bar{z}_2) = 1 \Rightarrow (z_1 + z_2)\left(\frac{1}{z_1} + \frac{1}{z_2}\right) = 1$
$\Rightarrow \quad (z_1 + z_2)^2 = z_1\ z_2$
$\Rightarrow \quad \left(\frac{-b}{a}\right)^2 = \frac{c}{a} \Rightarrow b^2 = ac$
$|z_1 + z_2| = |z_1||1 + e^{i\theta}| = 2\cos\frac{\theta}{2} = 1 \Rightarrow \theta = \frac{2\pi}{3}$
$PQ = |z_2 - z_1| = |z_1||e^{i\theta} - 1| = 2\sin\frac{\theta}{2}$
$= 2\sin\frac{\pi}{3} = \sqrt{3}$

41) (a) $T_n^3 + T_n = T_{n+1} - T_n$
$\sum_{n=1}^{100}(T_n^3 + T_n + 1) = T_{101} - T_1 + 100 = T_{101} + 99$
(c) $T_n^2 + 2 = \frac{T_{n+1}}{T_n}$
$\prod_{n=1}^{100}(T_n^2 + 2) = \prod_{n=1}^{100}\frac{T_{n+1}}{T_n} = \frac{T_{101}}{T_1} = T_{101}$

(d) $\prod_{n=1}^{100}\left(\frac{T_{n+1}}{T_n}-T_n^2\right)=2^{100}$



42)

$(3x-y)(x+3y)=0$

$y=3x,\ y=\frac{-x}{3}$

(a) $m_1+m_2=3-\frac{1}{3}=\frac{8}{3}$

(b) $am_1+m_2=3\left(-\frac{1}{3}\right)+3=2$

(c) $y=2x+4$

$A\equiv(4,12),\ B=\left(\frac{-12}{7},\frac{4}{7}\right)$

$\text{Area}(\Delta AOB)=\frac{1}{2}\times\sqrt{16+144}\times\sqrt{\frac{144+16}{49}}=\frac{160}{2\times7}=\frac{80}{7}$

43) $(A^2-2I)B=0\Rightarrow A^2=2I$ and $B=\text{adj}\,A=|A|.\frac{A}{2}=-\sqrt{2}A\{\because|A|=-2\sqrt{2}\}$ AB = A (adj A) = |A|. I

$\text{tr}.(AB)=3|A|=-6\sqrt{2}$

and $\det.(A-\sqrt{2}B)=\det.(A+2A)=\det.(3A)=27(-2\sqrt{2})=-54\sqrt{2}$

44) $x^3-3x+\lambda=0$ let root = α,β,γ

$\therefore\alpha+\beta+\gamma=0,\ \alpha\beta+\beta\gamma+\gamma\alpha=-3,\ \alpha\beta\gamma=-\lambda$

$\therefore\alpha^2+\beta^2+\gamma^2=6,\ (\alpha,\beta,\gamma\in I)$

∴ Possibilities are

$$\left.\begin{aligned}\alpha=2,\beta=-1,\gamma=-1\\ \alpha=-2,\beta=1,\gamma=1\end{aligned}\right\}\begin{aligned}&\Rightarrow\lambda=2\ or\ -2\\ &\Rightarrow(\lambda_1)^2+(\lambda_2)^2=8\end{aligned}$$

$\therefore h(x)=x^3-3x+4$

Required area $=\int_2^5 h(x)\,dx=\int_2^5(x^3-3x+4)\,dx=\frac{531}{4}$

And $\int_1^3(x^3-3x)\,dx+\int_2^{22}g(x)\,dx=\int_1^3(x^3-3x+4)\,dx+\int_2^{22}g(x)\,dx-8=66-2-8=56$

45)

a, b and c are in A.P.

$\therefore\sqrt{ac}<b$

$$f(x) = \lim_{n\to\infty} \frac{-(e^x)^n + x^2 + f}{2e^x(e^x)^n + x + d} = \begin{cases} \frac{x^2+f}{x+d} & , \quad x < 0 \\ \frac{-1+f}{2+d} & , \quad x = 0 \\ \frac{-1}{2e^x} & , \quad x > 0 \end{cases}$$

For f(x) to be continuous

$$\frac{f}{d} = \frac{-1+f}{2+d} = \frac{-1}{2}$$

$$\therefore 2f + d = 0 \ \Rightarrow \ 2f + d + 1 = 1$$

46)

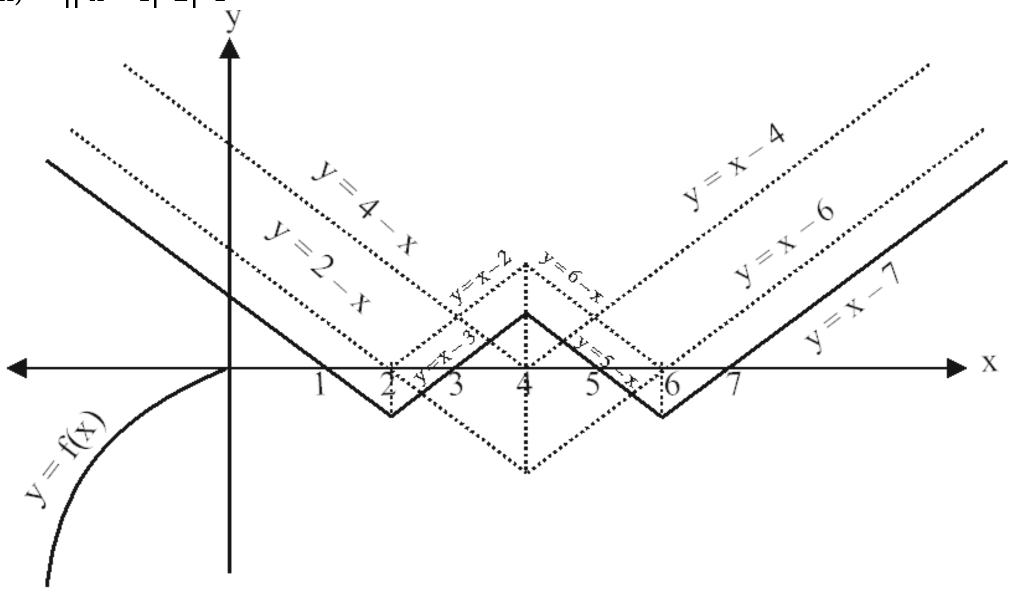
a, b and c are in G.P.

$$f(x) = \lim_{n\to\infty} \frac{x^2 + f}{2e^x(e^x)^n + x + d} = \begin{cases} \frac{x^2+f}{x+d} & , \quad x < 0 \\ \frac{f}{2+d} & , \quad x = 0 \\ 0 & , \quad x > 0 \end{cases}$$

For f(x) to be continuous

$$\frac{f}{d} = \frac{f}{2+d} = 0 \qquad \Rightarrow \ f = 0$$

f(x) = || x – 4|–2|–1



No of solution = 4

47) $P\left(\frac{A}{B_E}\right) = \frac{P(A \cap B_E)}{P(B_E)}$, where A = sum is 4.

$$P(B_E) = P(B_2) + P(B_4) + \dots\dots\dots = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\dots\dots\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{4}.\frac{4}{3} = \frac{1}{3}$$

And

$$P(A \cap B_E) = P(A \cap B_2) + P(A \cap B_4) = P(B_2)P\left(\frac{A}{B_2}\right) + P(B_4)P\left(\frac{A}{B_4}\right)$$

$$= \frac{1}{4}\left(\frac{3}{36}\right) + \frac{1}{16}\left(\frac{1}{6^4}\right) = \frac{(4.3.6^2+1)}{16.6^4} = \frac{433}{16.6^4}$$

Hence $P\left(\frac{A}{B_E}\right) = \frac{433}{16.6^4}.3 = \frac{433}{32.216} \cong \frac{1}{16}$

48) n(S) = 6 × 6 × 6 = 6^3 = 216
Now greatest number is 4, so atleast one of the dice shows up 4.
$\therefore n(A) = 4^3 - 3^3 = 37$
Hence $P(A) = \dfrac{37}{216}$

49) a, c, b are in H.P, $\Rightarrow \dfrac{2}{c} = \dfrac{1}{a} + \dfrac{1}{b}$ Family of lines
$\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{1}{c} = 0 \Rightarrow \dfrac{x}{a} + \dfrac{y}{b} + \dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b}\right) = 0 \Rightarrow \dfrac{1}{a}\left(x + \dfrac{1}{2}\right) + \dfrac{1}{b}\left(y + \dfrac{1}{2}\right) = 0$
$\Rightarrow$ centre of ellipse $\equiv \left(\dfrac{-1}{2}, \dfrac{-1}{2}\right)$
$AC : x^2 + y^2 + 2px + 2qy - 8p + \dfrac{1}{2} = 0 \Rightarrow p = \dfrac{1}{2} : q = \dfrac{1}{2};$ radius of AC = 2
$\therefore \ell$ (major axis) = 2a = 4
$\Rightarrow$ We know, harmonic mean of segment of focal chord is equal to semi latus rectum.
semi latus rectum $= \dfrac{2.(1 \times 2)}{(1 + 2)} = \dfrac{4}{3}$

50) a,c, b are in H.P, $\Rightarrow \dfrac{2}{c} = \dfrac{1}{a} + \dfrac{1}{b}$
Family of lines : $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{1}{c} = 0 \Rightarrow \dfrac{x}{a} + \dfrac{y}{b} + \dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b}\right) = 0 \Rightarrow \dfrac{1}{a}\left(x + \dfrac{1}{2}\right) + \dfrac{1}{b}\left(y + \dfrac{1}{2}\right) = 0$
$\Rightarrow$ centre of ellipse $\equiv \left(\dfrac{-1}{2}, \dfrac{-1}{2}\right)$
$AC : x^2 + y^2 + px + 2qy - 8p + \dfrac{1}{2} = 0 \Rightarrow p = \dfrac{1}{2} : q = \dfrac{1}{2};$ radius of AC = 2
$\therefore l$ (major axis) = 2a = 4
$\Rightarrow$ We know, harmonic mean of segment of focal chord is equal to semi latus rectum.
Semi latus rectum
$= \dfrac{2.(1 \times 2)}{(1 + 2)} = \dfrac{4}{3}$
$\therefore$ latus rectum, L = $\dfrac{8}{3}$ = 2.67
[L] = [2.67] = 2

51)

Put x = cos θ
$1 - x^2 = \sin^2\theta. \Rightarrow \sqrt{x^2 - 1} = i\sin\theta$
$\left(x + \sqrt{x^2 - 1}\right)^{10} + \left(x - \sqrt{x^2 - 1}\right)^{10} = (\cos\theta + i\sin\theta)^{10} + (\cos\theta - i\sin\theta)^{10} = 2\cos 10\theta = 2\, f_{10}(x) = \lambda f_k(x)$
$\therefore \lambda = 2; k = 10 \Rightarrow \lambda + k = 12.$

52) f_6 (cos x) = cos 6 x
we know,
$(\cos x + i \sin x)^6$ = cos 6 x + i sin 6x
Equating real part both sides :
$\cos 6x = 6_{C_0}\cos^6 x - 6_{C_2}\cos^4 x \sin^2 x + 6_{C_4}\cos^2 x \sin^4 x - 6_{C_6}\sin^6 x$

$f_6(\cos x) = \cos 6x = \cos^6 x - 15\cos^4 x(1-\cos^2 x) + 15\cos^2 x(1-\cos^2 x)^2 - (1-\cos^2 x)^3$

$\therefore f_6(x) = x^6 - 15x^4(1-x^2) + 15x^2(1-x^2)^2 - (1-x^2)^3$

$\therefore p = 32$; $q = -48$; $r = 18$, $s = -1$.

$\therefore 7(p+q+r-s) = 7(32-48+18+1) = 7 \times 3 = 21$

53) Equation of family of planes containing the line of intersection of P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$

$\Rightarrow (1+\lambda)x + (1-2\lambda)y + (2+\lambda)z - 3 - 4\lambda = 0$ ....(1)

Projection of length of line segment AB is greatest, then AB must be parallel to plane (1)

$\therefore$ drs of AB = 2, – 1,3.

$2(1+\lambda) - 1(1-2\lambda) + 3(2+\lambda) = 0 \Rightarrow \lambda = -1$

$\therefore P_3 : 3y + z + 1 = 0. \Rightarrow a = 0, b = 3, c = 1$

54)

Projection of length of line segment is least if AB is perpendicular to plane (1).

$$\frac{1+\lambda}{2} = \frac{1-2\lambda}{-1} = \frac{2+\lambda}{3} \Rightarrow \lambda = 1$$

$\therefore$ Equation of plane P_4 is $2x - y + 3z = 7. \Rightarrow a = 2, b = -1, c = 3$

55) $\frac{1}{5}\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + ...\infty\right]$

$\frac{2}{5}\left[\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + ...\infty\right]$

$\frac{2}{5}\left[\frac{(3-2)}{6} + \frac{(4-3)}{12} + \frac{(5-4)}{20} + ...\infty\right]$

$\frac{2}{5} \times \frac{1}{2} = \frac{k}{20} \Rightarrow k = 4$

56) $\frac{a}{\cos(200^\circ)} - \frac{c\sin(200^\circ)}{\cos(200^\circ)} = d$ .... (1)

$\frac{b}{\cos(200^\circ)} + \frac{d\sin(200^\circ)}{\cos(200^\circ)} = c$ .... (2)

$a = d\cos(200^\circ) + c\sin(200^\circ)$ .... (3)

$b = c\cos(200^\circ) - d\sin(200^\circ)$ .... (4)

$\therefore a^2 + b^2 = c^2 + d^2$

$\therefore (a^2 + b^2 + c^2 + d^2) = 2(c^2 + d^2)$

$ac = dc\cos(200^\circ) + c^2\sin(200^\circ)$

$bd = cd\cos(200^\circ) - d^2\sin(200^\circ)$

Also ______________________________

$ac - bd = (c^2 + d^2)\sin(200^\circ)$

So $\frac{a^2+b^2+c^2+d^2}{ac-bd} = \frac{2(c^2+d^2)}{(c^2+d^2)\sin(200^\circ)} = 2\,\mathrm{cosec}\,(200^\circ)$

$\therefore p = 2$

57) $I_n = \left(\dfrac{e^{-x}(\sin x)^n}{-1}\right)\Big|_0^{\infty} + n\int_0^{\infty}[\ (\sin x)^{n-1}\cos x e^{-x}dx$

$= 0+n\left(\dfrac{(\sin x)^{n-1}\cos x\, e^{-x}}{-1}\right)\Big|_0^{\infty} + n\int_0^{\infty}[(\sin x)^{n-1}(-\sin x) + (\cos x)(n-1)\cos x(\sin x)^{n-2}\cos x]e^{-x}dx$

$= n\int_0^{\infty}\ (-(\sin x)^n + (n-1)(1-\sin^2 x)(\sin x)^{n-2})e^{-x}dx = \dfrac{n(n-1)}{n^2+1}I_{n-2}$

$101\dfrac{I_{10}}{I_8} = 101 \times \dfrac{10\times 9}{10^2+1} = 90.$