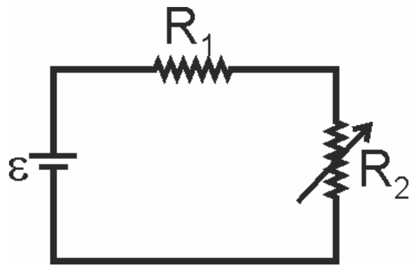
PART-1 : PHYSICS

SECTION-I



1) For what value of R_2, power loss through R_1 will be maximum?

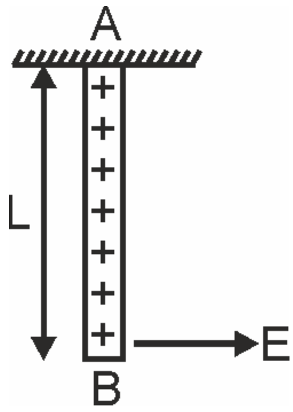
(A) $R_1 = R_2$

(B) $R_2 = 2R_1$

(C) $R_2 = 0$

(D) $R_2 = 4R_1$

2) A rod AB of length L and mass M is uniformly charged with a charge Q, and is suspended from end A as shown in figure. The rod can freely rotate about A in the plane of the figure. A uniform electric field E is switched on in the horizontal direction due to which the rod gets turned by a



maximum angle of 90º. The magnitude of E is equal to

(A) $\frac{Mg}{Q}$

(B) $\frac{Mg}{2Q}$

(C) $\frac{2Mg}{Q}$

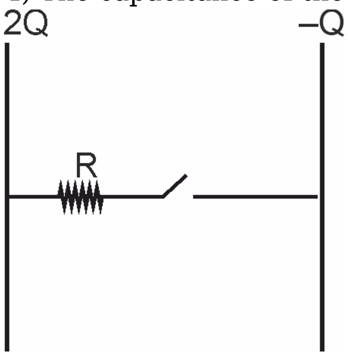
(D) $\frac{4Mg}{Q}$

3) **Assertion (A) :** When a longitudinal pressure wave is reflected at the open end of an organ pipe, the compression pressure wave pulse becomes rarefaction pressure wave pulse during the reflection.

Reason (R) : The phase of the wave changes by π when reflected at the open end.

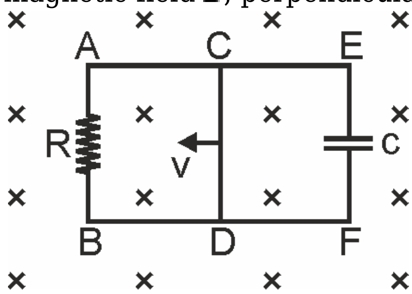
(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true but R is not correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

4) The capacitance of the system is C. If the key is closed, the total energy loss is equal to :



(A) $\frac{Q^2}{C}$
(B) $\frac{2Q^2}{C}$
(C) $\frac{9Q^2}{8C}$
(D) None of these

5) In the circuit shown, a conducting wire CD is moved with constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the circuit in inward direction. Current in CEFD is :-



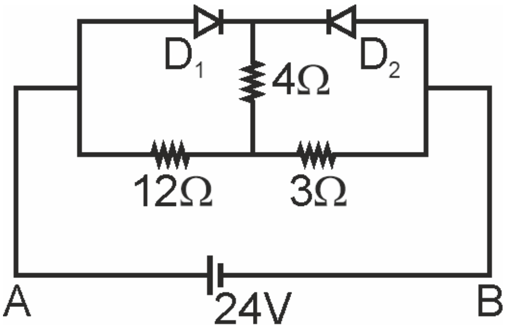
(A) Clock wise
(B) Anti-clockwise
(C) Alternating
(D) Zero

6) The magnetic field of a plane electromagnetic wave is given by $\vec{B} = 3 \times 10^{-8} \sin[200\pi (y + ct)]\hat{i}$ T, where c = 3 × 10^8 m/s. The corresponding electric field is :-

(A) $\vec{E} = 9 \sin[200\pi (y + ct)]\hat{k}$ V/m
(B) $\vec{E} = -10^{-6} \sin[200\pi (y + ct)]\hat{j}$ V/m
(C) $\vec{E} = 3 \times 10^{-8} \sin[200\pi (y + ct)]\hat{j}$ V/m

(D) $\vec{E} = -9\sin[200\pi(y+ct)]\hat{k}\,V/m$

7) In the circuit with ideal diodes as shown, current (in A) through battery is :-



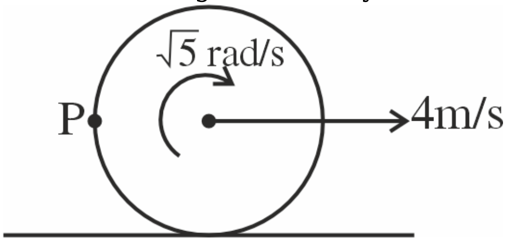
(A) 3

(B) 5

(C) 6

(D) 4

8) The centre of mass of disc of radius $\frac{8}{\sqrt{5}}m$ is moving with a velocity of 4 m/s on a horizontal surface. Its angular velocity is $\sqrt{5}$ rad/sec. The radius of curvature of point P as shown in figure is :-



(A) 2m

(B) 2.5m

(C) 4m

(D) 5m

9) In elliptical orbit of a planet, as the planet moves from aphelion position to perihelion position, match the following :-

	Column-I		Column-II
(P)	Speed of planet	(1)	Remains same
(Q)	Distance of planet from center of sun	(2)	Decreases
(R)	Potential energy	(3)	Increases
(S)	Angular momentum of planet about the centre of the sun	(4)	Can not say

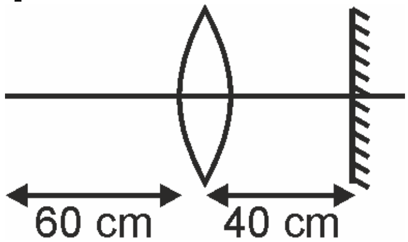
(A) $P \to 3; Q \to 4; R \to 2; S \to 1$

(B) $P \to 3; Q \to 4; R \to 2; S \to 3$

(C) $P \to 3; Q \to 1; R \to 3; S \to 2$

(D) $P \to 3; Q \to 2; R \to 2; S \to 1$

10) A point object is placed at a distance of 60 cm from a convex lens of focal length 30 cm. If a plane mirror is placed perpendicular to the principal axis of the lens at a distance of 40 cm from it, the final image would be formed at a distance of



(A) 20 cm from the lens, it would be a real image.

(B) 30 cm from the lens, it would be a real image.

(C) 30 cm from the plane mirror, it would be a virtual image.

(D) 20 cm from the plane mirror, it would be a virtual image.

11) A copper wire and a steel wire of the same diameter and length are connected end to end. A force is applied which stretches their combined length by 1 cm. Then the wires have:

(A) the same stress and strain

(B) the same stress but different strain

(C) the same strain but different stress

(D) different stress and strain

12) 20 gm of ice at –40ºC is mixed with 4 gm of steam at 100ºC. The final temperature of mixture is approximately (Latent heat of fusion is 80 cal/gm, Latent heat of vaporisation is 540 cal/gm, Specific heat of water is 1 cal/gm-ºC, specific heat of ice is 0.5 cal/gmºC)

(A) 30ºC

(B) 23ºC

(C) 28ºC

(D) 35ºC

13) A uniform string of length '□' is suspended from a rigid support. A short wave pulse is introduced at its lower end. It starts moving up the string. Choose the correct option.

(A) Acceleration of pulse increases

(B) Acceleration of pulse decreases

(C) Acceleration of pulse remains constant

(D) None of the above

14) A plane polarised electromagnetic wave is incident on a medium of refractive index $\sqrt{3}$ from air. There is no partially reflected wave. So the angle of refraction of electromagnetic wave is :-

(A) 45º

(B) 60º

(C) 30º

(D) 90º

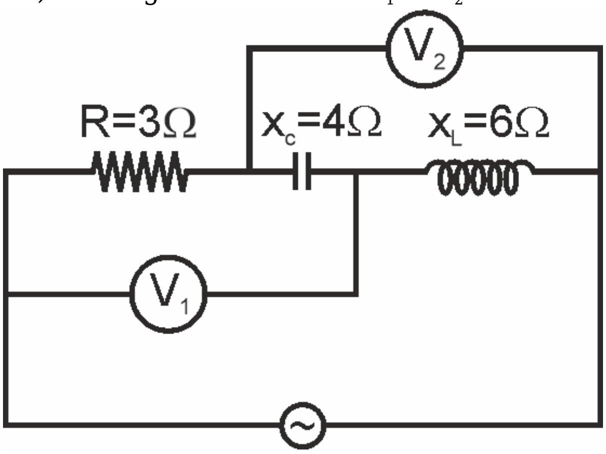
15) **Statement-I :** If Gaussian surface does not enclose any charge, then electric field at any point on the Gaussian surface must be zero.
Statement-II : No net charge enclosed by Gaussian surface means net electric flux passing through the Gaussian surface is zero.

(A) Both Statement I and II are correct.
(B) Statement I is correct but statement II is incorrect.
(C) Statement I is incorrect but statement II is correct.
(D) Both Statement I and II are incorrect.

16) If two SHM's whose equations are $x_1 = 3 \sin \omega t$ and $x_2 = - 4 \cos \omega t$ along the same line are superimposed, the equation of resultant SHM is :-

(A) $\sin(\omega t)$
(B) $5 \sin(\omega t - 53^\circ)$
(C) $5 \sin(\omega t + 53^\circ)$
(D) $5 \sin(\omega t - 37^\circ)$

17) In the given AC circuit V_1 & V_2 are ideal voltmeter. The ratio of reading of V_1 to V_2.



(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) $\frac{5}{2}$
(D) $\frac{3}{4}$

18) In a young's double slit experiment, light of wavelength 600 nm is used. P is the nearest point from the central maxima where intensity is $\frac{3}{4}^{th}$ of maximum intensity. What percentage of the maximum intensity is observed at P if light is replaced by the light of wavelength 400 nm?

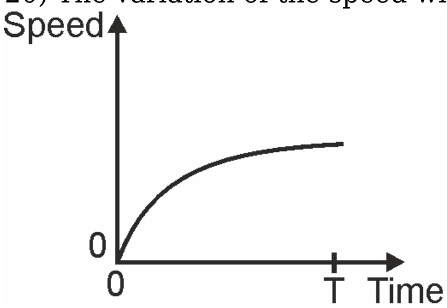
(A) 50%
(B) 75%
(C) 25%

(D) 100%

19) A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its magnetic susceptibility at 300 K is :-

(A) 3.672×10^{-4}
(B) 3.726×10^{-4}
(C) 3.267×10^{-4}
(D) 2.672×10^{-4}

20) The variation of the speed with time, of a ball falling in air is shown in below.

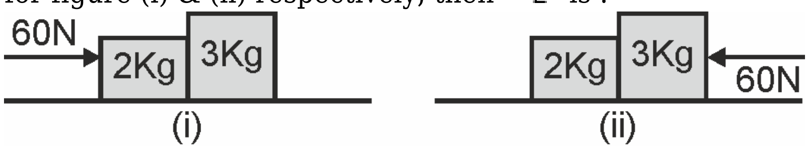


During the time from 0 to T, the ball gains some kinetic energy and loses gravitational potential energy ΔE_P. Which of the following statements must be correct?

(A) ΔE_P is equal to gain in kinetic energy
(B) ΔE_P is equal to the work done against air resistance
(C) ΔE_P is greater than the gain in kinetic energy
(D) ΔE_P is less than the work done against air resistance

SECTION-II

1) The blocks are kept on smooth horizontal surface. If N_1 and N_2 are contact force between blocks for figure (i) & (ii) respectively, then $\frac{2N_1}{N_2}$ is :-



2) A liquid of density 750 kg m^{-3} flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.2 \times 10^{-2}$ m^2 to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow section of pipe is 4500 Pa. The rate of flow of liquid is $n \times 10^{-3}$ m^3 s^{-1}, find n.

3) Radiation of photon energy 5eV is falling on a photo sensitive surface of work function 3eV. Consider an e^- which makes two collisions before coming out of the surface and there is a loss of 20% of energy in each collision. If K.E. of photoelectron coming out of surface is xeV, then 100x is :-

4) An ideal monoatomic gas undergoes a process in which its internal energy U and density ρ are

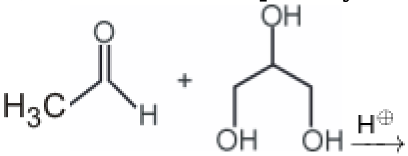
related as Uρ = constant. The ratio of change in internal energy and the work done by gas is 0.5x. Find the value of x.

5) The dimensions of the rectangular block measured with a vernier caliper having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of block is :-

PART-2 : CHEMISTRY

SECTION-I

1) The number of optically active isomers formed in the following reaction is



(A) 2

(B) 4

(C) 6

(D) 7

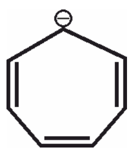
2) Among the following, the correct statement for thionyl tetrafluoride is

(A) the geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond on the trigonal plane.

(B) the geometry of thionyl tetrafluoride is trigonal bipyramidal having the sulphur-oxygen bond perpendicular to the trigonal plane.

(C) the geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond on the square plane.

(D) the geometry of thionyl tetrafluoride is square pyramidal having the sulphur-oxygen bond perpendicular to the square plane.

3) **Statement I :-**  Compound is antiaromatic in nature.
Statement II :- Compounds having (4n) π-electrons and planer with a cyclic conjugated π-system is anti aromatic.

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are true

(C) Both Statement I and Statement II are false

(D) Statement I is true but Statement II is false

4) A pink coloured aqueous solution of $Co(NO_3)_2$ turns blue on addition of HCl gradually. This colour

change happens due to the formation of

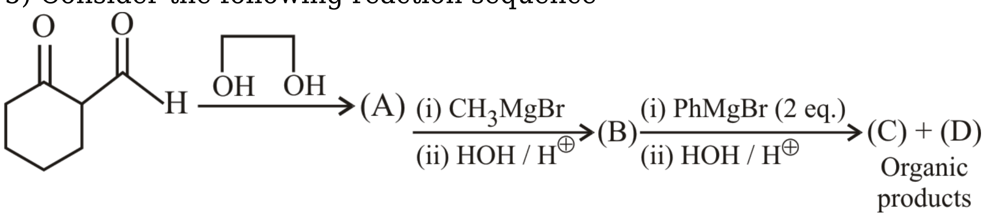
(A) $[CoCl_4]^{2-}$

(B) $[CoCl_6]^{4-}$

(C) $[Co(H_2O)_4Cl_2]$

(D) $[Co(H_2O)_2Cl_4]^{2-}$

5) Consider the following reaction sequence



How many sp^3 carbons are present in product (D), (assuming product (C) is benzene)

(A) 7

(B) 8

(C) 6

(D) 9

6) Three different electrolytic cells containing $AgNO_3$, $CuSO_4$, $AuCl_3$, respectively, are electrolysed. Find the simplest ratio of mass of metal deposited if the cells are connected in parallel and the ratio of resistances is 3 : 2 : 1 respectively.
given :- Mw(Ag) = 108, Mw(Cu) = 64 & Mw(Au) = 197

(A) 208 : 48 ; 197

(B) 108 : 48 : 197

(C) 108 : 64 : 197

(D) 208 : 64 : 197

7) Which of the following compound gives only one product on ozonolysis ?

(A) Ethene

(B) Propene

(C) 1-Butene

(D) 2-Methyl Propene

8) Which of the following compound does not undergo a nucleophilic substitution reation?

(A) 1-Chlorobutene

(B) Chlorobenzene

(C) Benzyl chloride

(D) 2-Chloropropane

9) Match the column and choose the correct option

Column-I (Properties)		Column-II (Order)	
(A)	Electronegativity	(1)	B < C < N < O
(B)	Cationic size	(2)	Li > Mg > Be
(C)	Metallic Character	(3)	K > Mg > Al
(D)	Electron affinity	(4)	Cl > F > Br > I

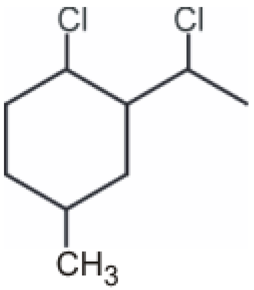
(A) A – 1, B – 2, C – 3, D – 4

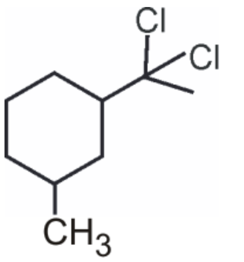
(B) A – 4, B – 3, C – 2, D – 1

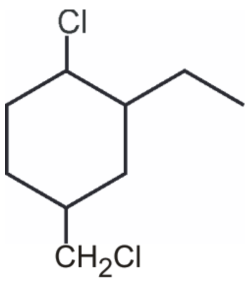
(C) A – 2, B – 3, C – 4, D – 1

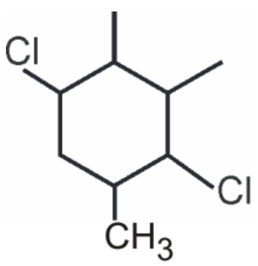
(D) A – 3, B – 2, C – 4, D – 1

10) Which of the following compounds has the highest number of chiral centres?

(A) 

(B) 

(C) 

(D) 

11) The magnetic moments in BM and geometry of complex $[Ni(CN)_4]^{4-}$

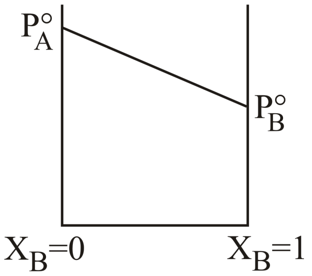
(A) 0.0 , TD

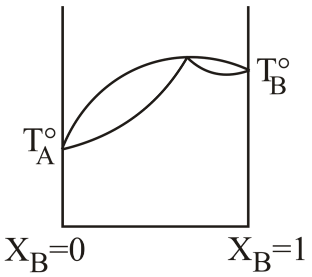
(B) 3.87, TBP.

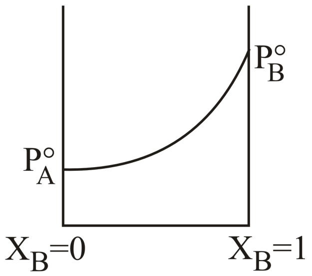
(C) 1.73, SP

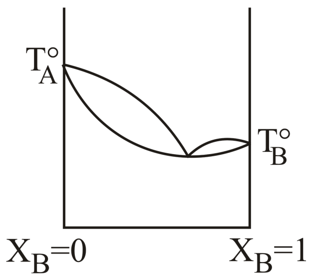
(D) 3.87, SP.

12) Which of the following plot represent positive deviation in a binary solution of liquids 'A' and 'B'.

(A) 

(B) 

(C) 

(D) 

13) If the binding energy of an electron in 3rd excited state of a H-like atom is 2eV, then 1st excitation energy for that atom :

(A) 36 eV

(B) 12 eV

(C) 48 eV

(D) 24 eV

14) **Statement-1 :** The hydrolysis of ethyl acetate is a second order reaction, in reality.
$CH_3COOC_2H_5 + H_2O \rightarrow CH_3COOH + C_2H_5OH$
$r = K[CH_3COOC_2H_5]\,[H_2O]$
Statement-2 : As the concentration of water does not get altered much during the course of reaction, the reaction behaves as first order reaction with respect to ester.

(A) Statement-1 and Statement 2 are correct and Statement 2 explains Statement 1

(B) Statement-1 and Statement-2 are correct and Statement -2 doesn't explain Statement-1

(C) Statement-1 is correct and Statement-2 is incorrect

(D) Statement-1 and Statement-2 both are incorrect.

15) A solution at 25ºC is 0.01 M in acetic acid and 0.05 M in sodium acetate. Then the pOH of solution is.

$$\begin{pmatrix} K_a(\text{acetic acid}) = 1.8\times10^{-5}; \log 5 = 0.7; \log 2 = 0.3 \\ \log 3 = 0.47 \ \ \& \log 3.47 = 0.54 \end{pmatrix}$$

(A) 4.45

(B) 5.45

(C) 8.55

(D) 9.55

16) For oxidation of iron : $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$; $\Delta_rH°_{300}$ = –1650 kJ mole^{-1} and $\Delta_rS°_{300}$ = –550 JK^{-1} mol^{-1}. Calculate $\Delta_rS°_{total}$ at 300 K :

(A) 0

(B) +4950 J K^{-1} mol^{-1}

(C) –6050 J K^{-1} mol^{-1}

(D) –1485 J K^{-1} mol^{-1}

17) 3 mol of gaseous phosphorus atoms are combined to formed $P_4(g)$. Calculate ΔH for this process. Given B.E.(P–P) = 300 kJ/mol

(A) +1350 kJ/mol.

(B) –1250 kJ/mol

(C) –1050 kJ/mol

(D) –1350 kJ/mol

18) Which of the following do not decolorize bromine water?

(A) 2-Butene

(B) Propane

(C) Ethene

(D) Cyclohexene

19) Mac Arthur forest process is applied for extraction of which of the following metals. (I) Zn (II) Au (III) Cu (IV) Ag (V) Al

(A) I only

(B) II only

(C) IV and V only

(D) II and IV only

20) For which of the element, the highest possible oxidation state shown by the element is not equal to its valence shell electron.

(A) Ru

(B) Fe

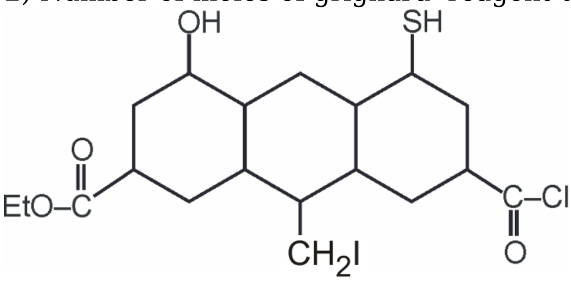
(C) V

(D) Ti

SECTION-II

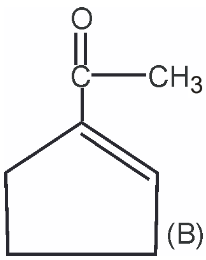
1) 300g mixture of C_2H_6 and HCHO is burned completely with oxygen and required 400gm O_2. Find the mass percentage of HCHO in original sample?

2) Number of moles of grignard reagent used for 1 mole of given compound is.



3) A first order reaction completes 75% in 40 minutes. In what time (in hours), the concentration of reactant will reduce from 0.64 M to 0.01 M ?

4) The reactant 'A' undergoes intramolecular aldol condensation reaction when heated in presence of NaOH to obtain the product 'B'



(B)

The number of methylene ($-CH_2-$) units in 'A' is

5) The total possible number of co-ordination isomers possible for $[Mo(en)_2Br_2][AgF_4]$

PART-3 : MATHEMATICS

SECTION-I

1) If the equation $\left|x^2+4x+3\right|-mx+2m=0$ has exactly three solutions, then the value(s) of m is equal to :-

(A) $-8 \pm 2\sqrt{15}$

(B) $-8-2\sqrt{15}$

(C) 3

(D) $-8+2\sqrt{15}$

2) The number of solution(s) of the equation $z^2 = 4z + |z|^2 + \frac{16}{|z|^3}$ is (where z = x + iy, x, y ∈ R, $i^2 = -1$ and x ≠ 2)

(A) 0

(B) 1

(C) 2

(D) 3

3) **Assertion (A) :** Slope of the curve given as $y = x^2$ at x = 1 is not defined. **Reason (R) :** Slope of the curve given as $y = x^2$ at x = 1 is $\pm \frac{1}{2}$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false.

(D) Both A and R are false.

4) Consider two sets A ={1, 2, 3, 4, 5} and B = {1, 2, 3, ... 9, 10}. Out of all the functions which can be defined from A → B, one functions randomly selected. The probability that the functions selected is injective, lies in the interval.

(A) $\left(\frac{3}{25}, \frac{4}{25}\right)$

(B) $\left(\frac{4}{25}, \frac{5}{25}\right)$

(C) $\left(\frac{5}{25}, \frac{6}{25}\right)$

(D) $\left(\frac{7}{25}, \frac{8}{25}\right)$

5) If $\cos\theta = \frac{ab+bc+ca}{a^2+b^2+c^2}$, where a, b, c are sides of a ΔABC, then θ can not be equal to :-

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{5\pi}{12}$

6) Let $S_n = \sum_{k=0}^{n} \left({}^{n+k}C_k\right) k.$ If $S_{100} = \frac{A!}{B!}\left(\frac{1}{101!} - \frac{1}{102!}\right)$ then the value of A + B is

(A) 150

(B) 300

(C) 450

(D) 600

7) There are two groups of students A and B in a sports academy which offers three games volleyball, basketball, baseball. The ratio of students playing volleyball, basketball and baseball is 3:4:5 in both the group. If the ratio of total number of students in Group A and B is 2:3 and there are 40 more students in Group B than Group A who plays baseball, then the total number of students in the sports academy is

(A) 360

(B) 720

(C) 480

(D) 960

8) A boat rowed with velocity V directly across a stream of width 'b'. If velocity of current is directly proportional to product of distances from two banks, find the distance downstream to the point where it lands.

(A) $\frac{kb^2}{2V}$

(B) $\frac{kb^3}{2V}$

(C) $\frac{kb^3}{6V}$

(D) $\frac{kb^2}{6V}$

9)

	List-I		List-II
(A)	If $x\frac{dy}{dx} = x^2 + y - 2$, $y(1) = 1$, then $y(2)$ equals	(I)	1
(B)	Let $S = \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n}+\sqrt{n+1}\right)\left(\sqrt[4]{n}+\sqrt[4]{n+1}\right)}$, then $\sqrt{S}$ equals	(II)	2
(C)	Let S be the area bounded by the curve y = sinx $(0 \le x \le \pi)$ and the x-axis and T be the area bounded by the curves y = sinx $\left(0 \leqslant x \leqslant \frac{\pi}{2}\right)$, y = a cosx $\left(0 \leqslant x \leqslant \frac{\pi}{2}\right)$ and the x-axis (where $a \in R^+$). The value of (3a) such that $S : T = 1 : \frac{1}{3}$, is	(III)	3

(D)	If the curve C in the xy plane has the equation $x^2 + xy + y^2 = 1$, then the square of greatest distance of a point on the curve C from the origin, is	(IV)	4

(A) (A)-II; (B)- III; (C)-IV ; (D)-I

(B) (A)-I; (B)-III; (C)-IV ; (D)-II

(C) (A)-II; (B)-III; (C)-IV ; (D)-II

(D) (A)-III; (B)-II; (C)-IV ; (D)-I

10) Sum of digits of the number of rational terms in the expansion of $\left(\sqrt{2}+\sqrt{3}+\sqrt[3]{5}\right)^{20}$, is :-

(A) 9

(B) 8

(C) 7

(D) 6

11) For an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, with vertices A & A′, tangent drawn at the point 'P' in the first quadrant meets the y-axis in Q and the chord A′P meets the y-axis in M. If 'O' is the centre of ellipse, then $OQ^2 - MQ^2$ is equal to,

(A) 9

(B) 13

(C) 4

(D) 5

12) Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, B = [a b c] and $C = \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+2)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$ be three given matrices, where a, b, c and x ∈ R, Given that tr.(AB) = tr.(C) ∀ x ∈ R, (where tr.(X) denotes trace of square matrix X). If $\int_0^\infty \frac{\ln x}{cx^2+ax+b}dx = \frac{\pi \ln 2}{\sqrt{q}}$, then the value of q is

(A) 29

(B) 27

(C) 26

(D) 28

13) Find the value of the definite integral $2^{2010}\frac{\int_0^1 x^{1004}(1-x)^{1004}dx}{\int_0^1 x^{1004}\left(1-x^{2010}\right)^{1004}dx}$.

(A) 1005

(B) 2010

(C) 4020

(D) 8040

14)

A line L passing through the point P (1, 4, 3), is perpendicular to both the lines $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$. If the position vector of point Q on L is (a_1, a_2, a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be:

(A) 2

(B) 3

(C) 15

(D) 16

15) $f(x) = \left|\{x\} - \frac{1}{2}\right|$ & g(x) = sinx where {.} is fractional part.

Statement I: The composite function f(g(x)) is non-differentiable in x ∈ (0, 2π) at 5 points.

Statement II: If f(x) is non-differentiable & g(x) is differentiable, then f(g(x)) is non-differentiable function.

(A) Both statements are true, statement-II is correct explanation of statement-I.

(B) Both statement are true, statement-II is not correct explanation of statement-I

(C) First statement true & II statement false.

(D) First statement false and II statement true.

16) If $\log^2 y + (2^{1+x} + 2^{1-x})(\log y) + 2^{2x+1} + 2^{1-2x} = 0$; where x, y ∈ R, then the value of log (x + y) is :-
(where base of log is e)

(A) –1

(B) 1

(C) –2

(D) 2

17) A line through the origin meets the circle $x^2 + y^2 = a^2$ at P and the hyperbola $x^2 - y^2 = a^2$ at Q. Then locus of the point of intersections of tangent to the circle at P with the tangent at Q to the hyperbola is :-

(A) $(a^6 + 2y^4)x^2 = a^4$

(B) $(a^4 + 2y^4)x^2 = a^3$

(C) $(a^6 + 4y^4)x^2 = a^4$

(D) $(a^4 + 4y^4)x^2 = a^6$

18) Area of the region in x-y plane satisfying $x^{10} - x^4 + y^2 \le 0$, is A, then [A] (where [x] represents greatest integer function)

(A) 1

(B) 2

(C) 3

(D) 4

19) Given $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \frac{1}{3}$. If the vector $\vec{V}$ can be expressed as linear combination of $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ and $\vec{a} \times \vec{b}$ as $\vec{V} = x(\vec{b} \times \vec{c}) + y(\vec{c} \times \vec{a}) + z(\vec{a} \times \vec{b})$ then (x + y + z) has the value equal to

(A) $\vec{V} \cdot \left(\vec{a} + \vec{b} + \vec{c}\right)$

(B) $3\vec{V} \cdot \left(\vec{a} + \vec{b} + \vec{c}\right)$

(C) $2\vec{V} \cdot \left(\vec{a} + \vec{b} + \vec{c}\right)$

(D) None

20) The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1.3.5.......(2r-1)}{2.4.6.......(2r+2)}$, is (where n ∈ N)

(A) $\frac{1}{2}$

(B) 0

(C) 1

(D) Does not exist

SECTION-II

1) Two friends have equal number of sons. There are 3 tickets for a cricket match which are to be distributed among the sons. The probability that two tickets go to the sons of one and one tickets goes to the sons of other is $\frac{6}{7}$. Then total number of boys is equal to (sum of son of each friend).

2) Let f(x) = $\cos x\left(\sin x + \sqrt{\sin^2 x + \sin^2 \theta}\right)$ where θ is given constant, then maximum value of f(x) is g(θ). The maximum value of $g^2(\theta)$, is

3) Let f(x) be a non-constant thrice differentiable function defined on (–∞, ∞) such that f(x) = f(6 – x) and f′(0) = 0 = f′(2) = f′(5). Determine the minimum number of zeroes of g(x) = $(f''(x))^2 + f'(x)\, f'''(x)$ in the interval [0, 6].

4) Let A = $\sin\frac{\pi}{81}.\sin\frac{2\pi}{81}...\sin\frac{80\pi}{81}$ and B = $\sin\frac{\pi}{27}.\sin\frac{2\pi}{27}...\sin\frac{26\pi}{27}$. If the value of the following expression $\sqrt[7]{\frac{12B}{A}}$ is 2^k, then k is

5) Given $2x - y + 2z = 1$; $x - 2y + z = -4$; & $x + y + \lambda z = 4$. Then the value of λ such that the given system of equation has no solution is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	C	A	A	C	D	D	D	D	D	D	B	B	C	C	C	B	C	A	C	C

SECTION-II

Q.	21	22	23	24	25
A.	3	24	20	3	5

PART-2 : CHEMISTRY

SECTION-I

Q.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A.	B	A	A	A	B	B	A	B	A	D	A	D	D	B	C	B	D	B	D	B

SECTION-II

Q.	46	47	48	49	50
A.	90	7	2	4	7

PART-3 : MATHEMATICS

SECTION-I

Q.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
A.	D	B	D	D	D	B	C	C	C	B	C	B	C	C	C	C	D	A	B	A

SECTION-II

Q.	71	72	73	74	75
A.	8	2	12	8	1

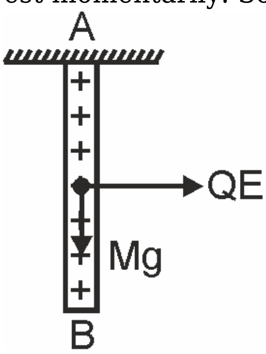
SOLUTIONS

PART-1 : PHYSICS

1) $P = I^2R_1 = \frac{\varepsilon^2}{(R_1 + R_2)^2}R_1$

P is maximum for $R_2 = 0$

2) After turning through 90º, the rod comes at rest momentarily. So there is no change in K.E.



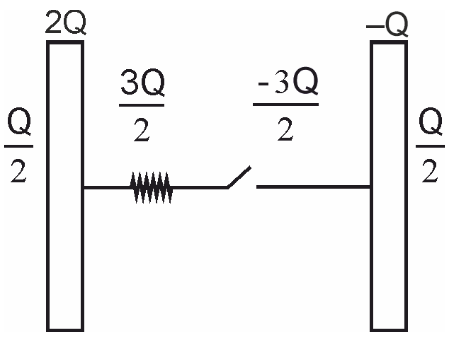
Net work done by all the forces should be zero.

$QE\frac{L}{2} - Mg\frac{L}{2} = 0$

$\Rightarrow E = \frac{Mg}{Q}$

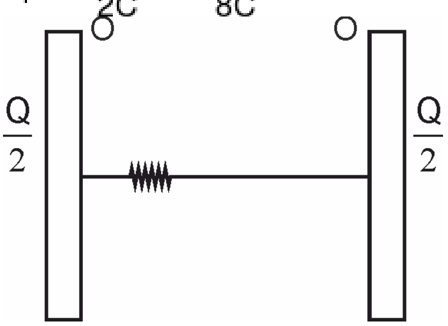
3) Conceptual.

4)



Before closing switch

$U_i = \frac{\left(\frac{3Q}{2}\right)^2}{2C} = \frac{9Q^2}{8C}$



after closing switch

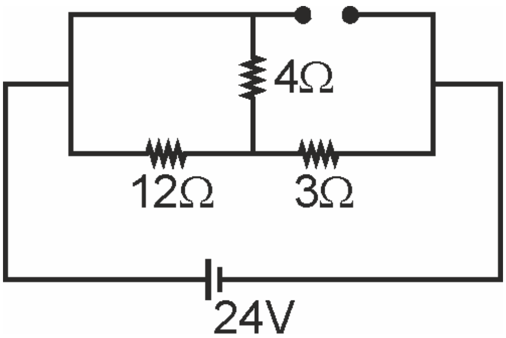
$U_f = \frac{(0)^2}{2C} = 0$

Loss of energy = U_i-U_f

$= \frac{9Q^2}{8C} - 0 = \frac{9Q^2}{8C}$

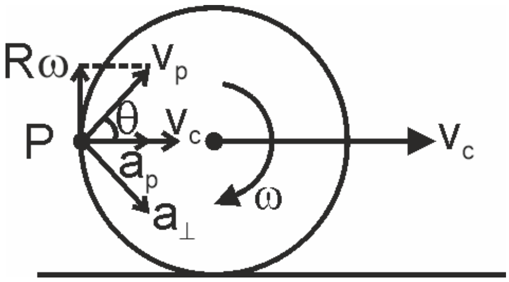
5) CD is moving with constant speed so constant emf is produced across CD, a constant charge will be there on capacitor so there is no current in CEFD.

6) $\vec{B}$ is along $\hat{i}$, & wave is travelling along –y direction.
so $\hat{v} = -\hat{j}$
$\hat{E} = \hat{B} \times \hat{v}$
$= \hat{i} \times (-\hat{j})$
$\hat{E} = -\hat{k}$
$|\vec{E}_0| = |\vec{v}|\,|\vec{B}_0|$, v = c
$= 3 \times 10^8 \times 3 \times 10^{-8} = 9$ V/m
So, $\vec{E} = 9 \sin[200\pi\,(y + ct)]\,(-\hat{k})$



7) Diode D_1 is forward bias & D_2 is reverse biased. So circuit looks like this
12Ω & 4Ω are in parallel & their combination is in series with 3Ω.
so equivalent resistance is
$= \frac{12 \times 4}{12 + 4} + 3 = 3 + 3 = 6\Omega$

So current through the battery = $\frac{24}{6} = 4A$

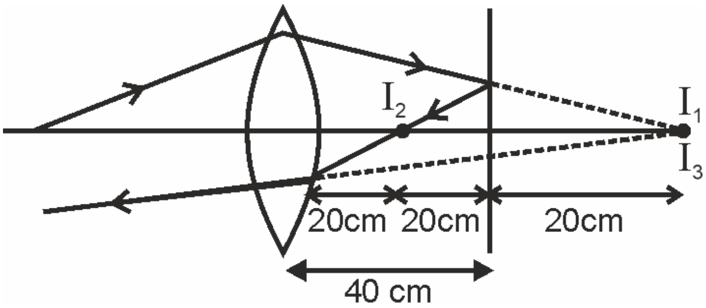


8) Radius of curvature = $\frac{v^2}{a_\perp}$
$v_P = \sqrt{v_c^2 + (R\omega)^2} = 4\sqrt{5}$ m/s
$a_P = R\omega^2 = \frac{5 \times 8}{\sqrt{5}} = 8\sqrt{5}$ m/s^2
$a_\perp = a_P \sin\theta = 8\sqrt{5} \times \frac{R\omega}{v_P} = \frac{8\sqrt{5} \times 8}{4\sqrt{5}} = 16$

radius of curvature $r = \frac{v_P^2}{a_\perp} = \frac{\left(4\sqrt{5}\right)^2}{16} = 5\text{m}$

9) At perigee position, planet is nearest to sun.

10) u = -60, f = 30, $\nu = \frac{uf}{u+f} = \frac{-60 \times 30}{-60+30} = +60\text{cm}$



further, u = -20 cm, f = 20 cm

$v = \frac{-20 \times 30}{-20+30} = \frac{-600}{10} = -60\text{cm}$

final image is virtual & at a distance of 20 cm from mirror.

12)

20 gm ice + 4 gm steam

20 gm ice	4 gm steam
$-40^\circ C$	$100^\circ C$
$\downarrow$ $Q = ms\Delta T = 20 \times \frac{1}{2} \times 40 = 400$ cal	$\downarrow$ $Q = mL_V = 4 \times 540 = 2160$ cal
$0^\circ C$ ice	$100^\circ C$ water
$\downarrow$ $Q = 20 \times 80 = 1600$ cal	$\downarrow$ $Q = ms\Delta T = 4 \times 1 \times 100 = 400$ cal
$0^\circ C$ water	$0^\circ C$ water
$Q_{Total} = 2000$ cal	$Q_{Total} = 2560$ cal

ice melts completely

$\Delta Q = 2560 - 2000$

$= 560$ cal

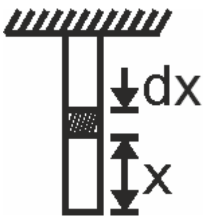
Let final temperature be T

$\Delta Q = mS\Delta T$

$560 = 24 \times 1 \times (T - 0)$

$T = \frac{560}{24} = 23.33^\circ C$

13)



$T = \frac{mx}{\ell} g$

Velocity of wave $= v = \sqrt{\frac{T}{\mu}}$

$$v = \sqrt{\frac{\frac{mx}{\ell}g}{\frac{m}{\ell}}}$$

$v = \sqrt{xg}$, $v^2 = xg$

$$2v\frac{dv}{dx} = g$$

$$v\frac{dv}{dx} = a = \frac{g}{2} = \text{constant}$$

14) since, there is no partially reflected wave so wave is incident at Brewster angle. Which is given by

$\tan i_p = \mu$

$i_p = \tan^{-1}(\mu)$

$i_p = \tan^{-1}\left(\sqrt{3}\right)$

$i_p = 60^\circ$

$\mu_1 \sin i = \mu_2 \sin r$

$\sin i_p = \sqrt{3}\sin r$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r$$

$$\sin r = \frac{1}{2}$$

$r = 30^\circ$

16) $x_1 = 3\sin\omega t$

$x_2 = -4\cos\omega t$

$= -4\sin\left(\frac{\pi}{2} - \omega t\right)$

$= 4\sin\left(\omega t - \frac{\pi}{2}\right)$

$x = x_1 + x_2$

$= 3\sin(\omega t) + 4\sin\left(\omega t - \frac{\pi}{2}\right)$

$= 5\sin\omega t - 53^\circ$

17) $V_1 = i_{rms}Z_1 = i_{rms}\sqrt{R^2 + X_c^2}$

$V_2 = i_{rms}Z_2 = i_{rms}\left|(X_L - X_C)\right|$

$$\frac{V_1}{V_2} = \frac{i_{rms}\sqrt{R^2 + X_C^2}}{i_{rms}\left|(X_L - X_C)\right|} = \frac{\sqrt{3^2 + 4^2}}{6 - 4} = \frac{5}{2}$$

18) $I = I_{max}\cos^2\frac{\phi}{2}$

$$\frac{3}{4}I_{max} = I_{max}\cos^2\frac{\phi}{2}$$

$$\cos\frac{\phi}{2} = \pm\frac{\sqrt{3}}{2}$$

$$\frac{\phi}{2} = \frac{\pi}{6} \Rightarrow \phi = \frac{\pi}{3}$$

$$\phi = \frac{2\pi}{\lambda}.(\Delta x)$$

$$\phi' = \frac{2\pi}{\lambda'}\Delta x$$

$$\frac{\phi}{\phi'} = \frac{\lambda'}{\lambda} = \frac{400}{600} = \frac{2}{3}$$

$$\phi' = \frac{3}{2}\phi = \frac{3}{2} \times \frac{\pi}{3}$$

$$\phi' = \frac{\pi}{2}$$

$$I = I_{max}\cos^2\frac{\phi}{2}$$

$$I_{max}\cos^2\frac{\pi}{4} = \frac{I_{max}}{2}$$

19) $\chi m \propto \frac{1}{T}$

$$\frac{\chi m_1}{\chi m_2} = \frac{T_2}{T_1}$$

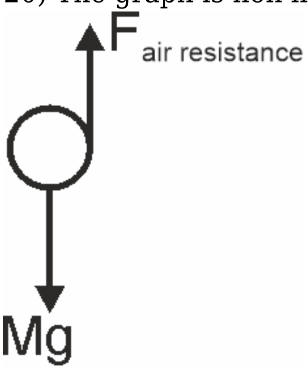
$$\frac{\chi m_1}{\chi m_2} = \frac{350}{300}$$

$$\chi m_1 = \frac{350}{300}\chi m_2$$

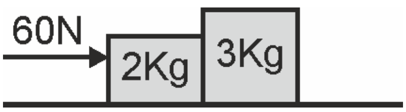
$$= \frac{350}{300} \times 2.8 \times 10^{-4}$$

$= 3.267 \times 10^{-4}$

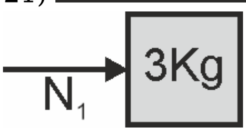
20) The graph is non linear i.e. there is some dissipative force apart from gravitational force



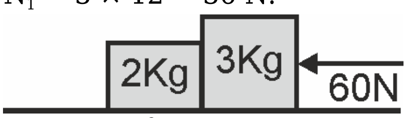
By conservation of energy, loss in gravitational P.E. = gain in K.E. + work done against air drag.

21) 

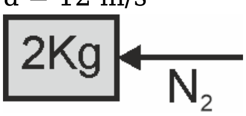
$a = 12\ m/s^2$



$N_1 = 3 \times 12 = 36$ N.



$a = 12\ m/s^2$



$N_2 = 2 \times 12 = 24$

$\frac{N_1}{N_2} = \frac{36}{24} = \frac{3}{2} = 1.5$

22) $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2$

$v_2 = \frac{A_1}{A_2} v_1$

$(P_1 - P_2) = \frac{1}{2}\rho \left(v_2^2 - v_1^2\right)$

$4500 = \frac{1}{2} \times 750 \left(\frac{A_1^2}{A_2^2} - 1\right) v_1^2$

$4500 = \frac{1}{2} \times 750 (4-1) v_1^2$

$450 = \frac{1}{2} \times 75 \times 3 \times v_1^2$

$\frac{900}{225} = v_1^2$

$v_1 = \frac{30}{15} = 2 \text{ m/s}$

Rate of flow $= A_1 v_1 = 1.2 \times 10^{-2} \times 2$

$= 2.4 \times 10^{-2} = 24 \times 10^{-3} \text{ m}^3 \text{ s}^{-1}$

23) Electron absorbs energy of 5eV from the photon. It loses 1eV in first collision, so its remaining kinetic energy is 4eV. It further loses 0.8eV in second collision, so its remaining kinetic energy is 3.2eV, out of this energy 3eV goes to come out of surface and remaining 0.2eV appears as the kinetic energy of electron.

24) $U\rho$ = constant.

$\because U \propto T$

$T\rho$ = constant

$\frac{T}{V}$ = constant

$V \propto T$

$\Rightarrow$ Isobaric process.

$\Delta U = nC_V \Delta T = \frac{3}{2} nR\Delta T$

$\Delta W = P\Delta V$

$\Delta W = nR\Delta T$

$\frac{\Delta U}{\Delta W} = \frac{3R}{2R} = \frac{3}{2} = 1.5$

$1.5 = 0.5x$

$x = 3$

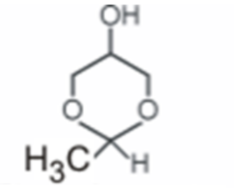
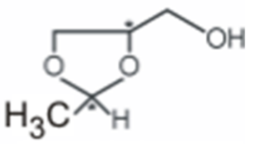
25) $v = l \times b \times h$

$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$

$= \left(\frac{0.1}{5} + \frac{0.1}{10} + \frac{0.1}{5}\right) \times 100$

= 5%

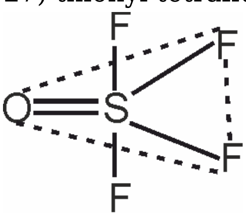
PART-2 : CHEMISTRY

26)  

Plane of symmetry (Optically inactive)

Two chiral centres (4-isomers)

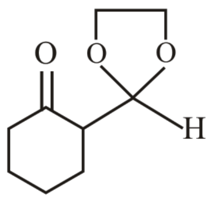
27) thionyl tetrafluoride ⇒ SOF_4



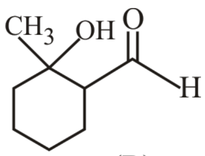
Hybridization of 'S' ⇒ sp^3d (trigonal bi-pyramidal)
(S=O) on trigonal plane because more space required between '=' bond and '–' bond to compress repulsion (ACC. to Bent's rule)

28) Given Compound is Non-Aromatic due to Non planer in Nature.

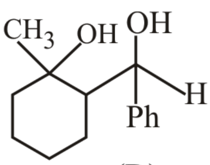
29) $[CoCH_2O]^{+3} + 4Cl^- \rightleftharpoons [CoCl_4]^{-2}$ Blue + $6H_2O$
Fact based question Co^{+2} ⇒ Blue
Co^{+3} ⇒ Pink

30) 

(A)



(B)



(D)

8 sp^3 carbons are present in Product (D)

31) By apply faraday's law $W = \frac{EQ}{96500}$

Charge $\alpha \frac{1}{\text{resis tan ce}}$

Charge ratio = $\frac{1}{3} : \frac{1}{2} : \frac{1}{1}$

= 2 : 3 : 6

$W_1 : W_2 : W_3 = E_1Q_1 : E_2Q_2 : E_3Q_3$

$$= \frac{108}{1} \times 1 : \frac{64}{2} \times 3 : \frac{197}{3} \times 6$$

$= 108 \times 2 : 32 \times 3 : 197 \times 2$

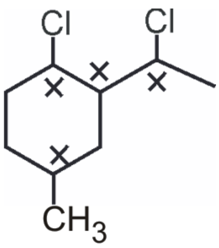
$= 108 : 48 : 197$

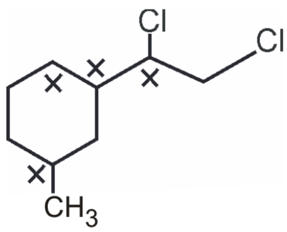
32) Ozonolysis cleaves double bonds to form carbonyl compounds. Ethene produces only one product, formaldehyde (HCHO) Because the molecule is summetrical . While other compound like propene and 2-butene produce two different carbonyl products.

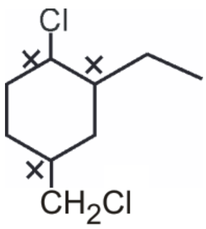
33) Chlorobenzene

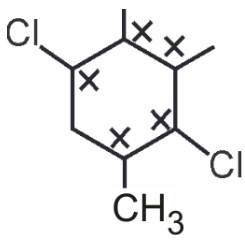
In chlorobenzene, the electron density of the aromatic ring makes the carbon-chlorine bond resistant to nucleophilic attack.

35)

(a) 

(b) 

(c) 

(d) 

36) Ni ⇒ [Ar] $3d^8, 4s^2$ but in this complex, Ni ⇒ [Ar]$3d^{10}$, $6s^0$

and d^{10} system forms Td only under C.N. = 4

37)

Solution showing positive deviation has low boiling point

38) $$E = 13.6\,Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$2 = 13.6\,Z^2 \left(\frac{1}{4^2} - \frac{1}{\infty^2}\right) \text{.......(1)}$$

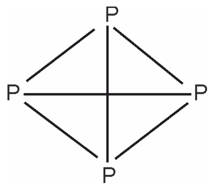
$$E = 13.6\,Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right) \text{.......(2)}$$

$$\frac{E}{2} = \frac{\frac{3}{4}}{\frac{1}{16}} \Rightarrow \boxed{E = 24eV}$$

39)

$C(s) + O_2(g) \rightarrow CO_2(g)$
$\Delta H = \Delta U + \Delta n_g RT$
$\Delta n_g = 0 \Rightarrow \Delta H = \Delta U$

40) $pH = pK_a + \log\left[\frac{0.05}{0.01}\right]$
$pOH = 14 - pH$

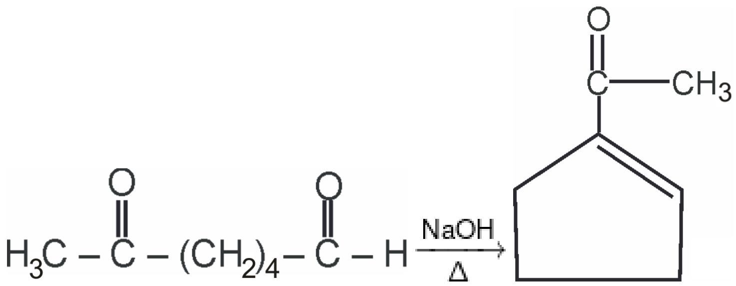
42) 
$4\,P(g) \rightarrow P_4(g)$
1 mol of P_4 contains 6 mol P–P bond.
$\Delta Hr = O - 6 \times 300 = -1800$ kJ/mol
For 3mol $= \frac{1800}{4} \times 3 = \ -1350 kJ/mol.$

46) Moles of $O_2 = \frac{900}{32} = 12.5 mol$
$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$
Xmol $\frac{7}{2}x$
$HCHO + O_2 \rightarrow CO_2 + H_2O$
ymol y
$Mw(C_2H_6) = 30$, $Mw(HCHO) = 30$
$30x + 30y = 300$ gm
$x+y = 10$
$\frac{7}{2}x + y = 12.5$
$7x + 2y = 25$
using (i) & (ii)
$7x+2y=25$
$2x+2y=20$
$5x = 5 \Rightarrow x = 1$
$y = 9$
$HCHO = 9 \times 30 = 270g$
$HCHO = \frac{270}{300} \times 100 = 90 HCHO = \frac{270}{300} \times 100 = 90$

47) 2 moles used for each ester.
2 moles used for each acid halide.
1 mole used for –OH.
1 mole used for –SH

1 mole used for CH_2–I gives SN^2 reaction.

48) $t_{75\%} = 2 \times t_{1/2} = 40$ min.
$\Rightarrow t_{1/2} = 20$ min.
Now, $t_{req} = 6 \times t_{1/2} = 120$ min. = 2 hrs.

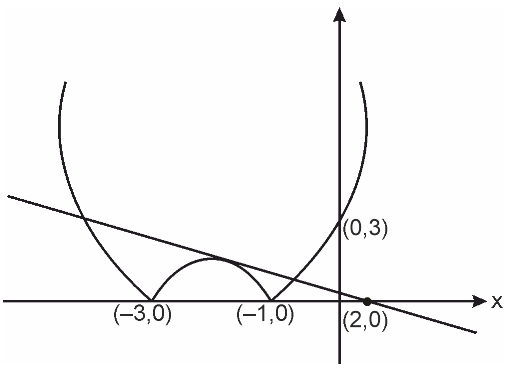
49) 

Number of – CH_2 – unit '4'

PART-3 : MATHEMATICS

51)

If $|x^2 + 4x + 3| = mx - 2m$ has exactly three solution, then the curves $y = |x^2 + 4x + 3|$ and $y = m(x - 2)$ intersect at exactly three points $\Rightarrow y = m(x - 2)$ is tangent to $y = -x^2 - 4x - 3$.



$\Rightarrow m(x - 2) = -x^2 - 4x - 3$ has equal roots
$\Rightarrow (m + 4)^2 - 4(3 - 2m) = 0$
$\Rightarrow m = -8 \pm 2\sqrt{15}$
∴ $m = -8 + 2\sqrt{15}$ $(m \neq -8 - 2\sqrt{15})$

52)

We have
$|z|^2 + \frac{16}{|z|^3} = z^2 - 4z = (\bar{z})^2 - 4\bar{z} \Rightarrow (z - \bar{z})(z + \bar{z} - 4) = 0$
$\Rightarrow z = \bar{z} = x$ $(x \neq 2)$
So, $x^2 = 4x + x^2 + \frac{16}{|x|^3} \Rightarrow x = \frac{-4}{|x|^3} \Rightarrow x = -\sqrt{2}$
∴ $z = -\sqrt{2}$
Hence only one z will satisfy above equation.

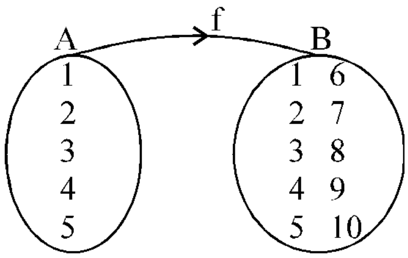
53) $\frac{dy}{dx} = 2x\frac{dy}{dx} = 2$

54)

Total functions = 10^5

Number of injective functions = ${}^{10}C_5 \times 5!$

 Probability of injective functions = $\frac{{}^{10}C_5 \times 5!}{10^5} = \frac{252 \times 120}{10 \times 10^4} = \frac{189}{625}$



and $\frac{175}{625} < \frac{189}{625} < \frac{200}{625} \Rightarrow$ (D) is correct

55)

In a triangle ABC, cos A < 1 $\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} < 1$

$\Rightarrow b^2 + c^2 - a^2 < 2bc$ ...(1)

Similarly $c^2 + a^2 - b^2 < 2ca$ ...(2)

$a^2 + b^2 - c^2 < 2ab$ ...(3)

Also, $a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$\Rightarrow \frac{ab + bc + ca}{a^2 + b^2 + c^2} > \frac{1}{2}$

Also, $\frac{ab + bc + ca}{a^2 + b^2 + c^2} \le 1$

$\Rightarrow \frac{1}{2} < \cos\theta \leqslant 1 \Rightarrow \theta \in \left[0, \frac{\pi}{3}\right)$

56)

$S_n = \sum_{k=0}^{n} {}^{n+k}C_k . k = 1.{}^{n+1}C_n + 2.{}^{n+2}C_n + \ldots + n.{}^{2n}C_n.$

$\left({}^nC_r = {}^nC_{n-r}\right)$

= coefficient of x^n in

$\left\{(1 + x)^{n+1} + 2(1 + x)^{n+2} + 3(1 + x)^{n+3} + \ldots n(1 + x)^{2n}\right\}$

= coefficient of x^n in

$\left\{\frac{n(1 + x)^{2n+1}}{x} + \frac{(1 + x)^{n+1} - (1 + x)^{2n+1}}{x^2}\right\}$

$= n\,{}^{2n+1}C_{n+1} - {}^{2n+1}C_{n+2}$

$S_{100} = \frac{100.201!}{101!100!} - \frac{201!}{102!99!} = \frac{201!}{99!}\left(\frac{1}{101!} - \frac{1}{102!}\right)$

$\Rightarrow A + B = 300$

57)

	Volleyball	Basketball	Baseball
Let number of student in Group A	6x	8x	10x
Let number of student in Group B	9x	12x	15x

⇒ 15x - 10x = 40

⇒ x = 8

Total number of students = (6 + 8 + 10 + 9 + 12 + 15) × 8

= 480

58)

Take origin at point from where boat starts.

At any time 't' boat be at

P(x, y) so that

$\frac{dx}{dt}$ = velocity of current = ky(b – y)

$\frac{dy}{dt}$ = V

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{v}{ky(b-y)}$

$$\int_0^y \left(by - y^2\right) dy = \int_0^x \frac{v}{k} dx$$

$$\frac{by^2}{2} - \frac{y^3}{3} = \frac{v}{k}x$$

Putting y = b we get, $\boxed{x = \frac{kb^3}{6v}}$

59) **(A)** Given $\frac{dy}{dx} - \frac{1}{x}y = \left(x - \frac{2}{x}\right)$

I.F. = $e^{-\int \frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

Now general solution is given by $\frac{y}{x} = \int \left(x - \frac{2}{x}\right)\frac{1}{x}dx \Rightarrow \frac{y}{x} = x + \frac{2}{x} + C$

As y(1) = 1 ⇒ C = -2

 $\frac{y}{x} = x + \frac{2}{x} - 2 \Rightarrow y = x^2 - 2x + 2$

Hence y(2) = $(2)^2$ – 2(2) + 2 = 2 **Ans.**

(B) Given

$$S = \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n} + \sqrt{n+1}\right)\left(\sqrt[4]{n} + \sqrt[4]{n+1}\right)}$$

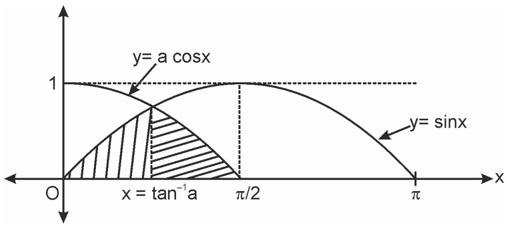
$$= \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n} + \sqrt{n+1}\right)\left(\sqrt[4]{n} + \sqrt[4]{n+1}\right)}\left(\frac{\sqrt[4]{n} - \sqrt[4]{n+1}}{\sqrt[4]{n} - \sqrt[4]{n+1}}\right)$$

$= \sum_{n=1}^{9999}\left((n+1)^{1/4} - n^{1/4}\right)$

$= \left(\left(2^{\frac{1}{4}} - 1\right) + \left(3^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) + \left(4^{\frac{1}{4}} - 3^{\frac{1}{4}}\right) + \dots + \left((9999+1)^{\frac{1}{4}} - (9999)^{\frac{1}{4}}\right)\right)$

$= \left(10^4\right)^{\frac{1}{4}} - 1 = 9$

Hence $\sqrt{S} = 3$ **Ans.**



(C) We have $S = \int_0^{\pi} \sin x dx = 2,$ so $T = \frac{2}{3}$ where a > 0.

Now $T = \int_0^{\tan^{-1}a} \sin x dx + \int_{\tan^{-1}a}^{\pi/2} a\cos x\, dx = \frac{2}{3}$,

i.e. $-\cos\left(\tan^{-1}a\right) + 1 + a\left(1 - \sin\left(\tan^{-1}a\right)\right) = \frac{2}{3},$

i.e. $-\frac{1}{\sqrt{1+a^2}} + 1 + a - \frac{a^2}{\sqrt{1+a^2}} = \frac{2}{3}$

$\Rightarrow (a+1) - \sqrt{a^2+1} = \frac{2}{3}$

$\Rightarrow a + \frac{1}{3} = \sqrt{a^2+1}$

$\Rightarrow a = \frac{4}{3}$

Hence 3a = 4 **Ans.**

(D) Let $x = r\cos\theta$, $y = r\sin\theta$

$\Rightarrow r^2(1+\cos\theta\sin\theta) = 1 \Rightarrow r^2 = \frac{2}{2+\sin 2\theta}$

Clearly $r^2|_{max.} = \frac{2}{1}$

$\Rightarrow r_{max.} = \sqrt{2}$ **Ans.**

60) Number of non-negative integral solution of equation 2x + 2y + 3z = 20

$= {}^{10+2-1}C_{2-1} + {}^{7+2-1}C_{2-1} + {}^{4+2-1}C_{2-1} + {}^{1+2-1}C_{2-1} = 26$

61) $T \equiv \frac{x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1$

$x = 0$, $y = 2\text{cosec}\theta$

chord A′P;

$y = \frac{2\sin\theta}{3(\cos\theta+1)}(x+3)$

$x = 0$, $OM = \frac{2\sin\theta}{(\cos\theta+1)}$

Now, $OQ^2 - MQ^2 = OQ^2 - (OQ - OM)^2$

$= 2(OM)(OQ) - OM^2$

$= OM[2OQ - OM]$

$= \frac{2\sin\theta}{(\cos\theta+1)}\left[\frac{4}{\sin\theta} - \frac{2\sin\theta}{1+\cos\theta}\right]$

$= \frac{2\sin\theta\left[4+4\cos\theta-2\sin^2\theta\right]}{(1+\cos\theta)^2\sin\theta}$

$= \frac{4\sin\theta\left[2+2\cos\theta-1+\cos^2\theta\right]}{(1+\cos\theta)^2\sin\theta}$

$=4$

62) **29.00**

We have $AB = \begin{bmatrix} 3ax^2 & 3bx^2 & 3cx^2 \\ a & b & c \\ 6ax & 6bx & 6cx \end{bmatrix}$

Now tr.(AB) = tr.(C) $\Rightarrow 3ax^2 + b + 6cx = (x+2)^2 + 2x + 5x^2\ \forall\ x \in R$ (Identify)

$\Rightarrow 3ax^2 + 6cx + b = 6x^2 + 6x + 4$

Consider $I = \int_0^\infty \frac{\ln x dx}{x^2+2x+4}$; (Put x = 2t and solve)

$= 2\int_0^\infty \frac{\ln 2 + \ln t}{4\left(t^2+t+1\right)}dt = \frac{\ln 2}{2}\underbrace{\int_0^\infty \frac{dt}{t^2+t+1}}_{I_1} + \frac{1}{2}\underbrace{\int_0^\infty \frac{\ln t dt}{t^2+t+1}}_{I_2 = \text{zero}}$

$I_2 = \int_0^\infty \frac{\ln t dt}{t^2+t+1}$ (Put $t = \frac{1}{y}$, and solve) $\Rightarrow I_2 = 0$

Now $I_1 = \int_0^\infty \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{2}{\sqrt{3}}\tan^{-1}\frac{\left(t+(1/2)^2\right)}{\sqrt{3}}\Bigg|_0^\infty = \frac{2}{\sqrt{3}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right] = \frac{2\pi}{3\sqrt{3}}$

Hence $I = \frac{\ln 2}{2}.\frac{2\pi}{3\sqrt{3}} = \frac{\pi\ln 2}{3\sqrt{3}}$

$\Rightarrow q = 27$

63) **4020**

Consider $I_2 = \int_0^1 x^{1004}\left(1-x^{2010}\right)^{1004}dx$; Put $x^{1005} = t \Rightarrow 1005\ x^{1004}\ dx = dt$

So, $I_2 = \frac{1}{1005}\int_0^1 \left(1-t^2\right)^{1004}dt$ ...(1)

Also $I_2 = \frac{1}{1005}\int_0^1 \left[1-(1-t)^2\right]^{1004}dt$ ...(2) (Using King)

$\Rightarrow I_2 = \frac{1}{1005}\int_0^1 (t(2-t))^{1004}dt = \frac{1}{1005}\int_0^1 t^{1004}(2-t)^{2004}dt$; Put $t = 2y \Rightarrow dt = 2dy$

So, $I_2 = \frac{1}{1005}\int_0^{1/2}(2y)^{1004}(2-2y)^{1004}2dy = \frac{1}{1005}2\cdot 2^{1004}\cdot 2^{1004}\int_0^{1/2}y^{1004}(1-y)^{1004}dy$

$I_2 = \frac{1}{1005}2^{2009}\int_0^{1/2}y^{1004}(1-y)^{1004}dy$ ...(3)

Now $I_1 = \int_0^1 x^{1004}(1-x)^{1004}dx = 2\int_0^{1/2}x^{1004}(1-x)^{1004}dx$ ...(4)

 From (3) and (4), we get

$I_2 = \frac{1}{1005}2^{2010}\frac{I_1}{4} \Rightarrow 2^{2010}\frac{I_1}{I_2} = 4020$ **Ans.**

64)

Equation of the line passing through P(1, 4, 3) is

$\frac{x-1}{a} = \frac{y-4}{b} = \frac{z-3}{c}$ **...(1)**

Since (1) is perpendicular to $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{4}$ and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$.

Hence 2a + b + 4c = 0

and 3a + 2b - 2c = 0

 $\frac{a}{-2-8} = \frac{b}{12+4} = \frac{c}{4-3} \Rightarrow \frac{a}{-10} = \frac{b}{16} = \frac{c}{1}$

Hence the equation of the line is $\frac{x-1}{-10} = \frac{y-4}{16} = \frac{z-3}{1}$ **...(2) Ans.**

Now any point Q on (2) can be taken as (1-10λ, 16λ + 4, λ + 3)

 Distance of Q from P (1, 4, 3) = $(10\lambda)^2 + (16\lambda)^2 + (\lambda)^2 = 357$

⇒ λ = 1 or –1

⇒ Q(-9, 20, 4) or (11, –12, 2)

65) $f(g(x)) = \left|\{\sin x\} - \frac{1}{2}\right|$

$\{\sin x\} = \frac{1}{2}$

sinx = $\frac{1}{2}, \frac{-1}{2}$

x = $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Also sinx = 0 ∈ x = π

66) $\log^2 y + 2\left(2^x + \frac{1}{2^x}\right)\log y + 2\left(2^{2x} + \frac{1}{2^{2x}}\right) = 0$

$\left(\log y + 2^x + \frac{1}{2^x}\right)^2 + 2\left(2^{2x} + \frac{1}{2^{2x}}\right) = 2^{2x} + \frac{1}{2^{2x}} + 2$

$$\underbrace{\left(\log y + 2^x + \frac{1}{2^x}\right)^2}_{\geqslant 0} = \underbrace{-\left(2^x + \frac{1}{2^x}\right) + 2}_{\leqslant 0}$$

$\Rightarrow x = 0; y = e^{-2}$

67) the equation of the tangents to the circle $x^2 + y^2 = a^2$ at P and the hyperbola $x^2 - y^2 = a^2$ at Q are

$x + my = a\sqrt{1+m^2}$

or $x - my = a\sqrt{1-m^2}$ respectively

Where $y = mx$ is intersecting line through (2, 0) Let (h, k) be the point of intersection of these two lines.

∴ $h + mk = a\sqrt{1+m^2}$ are $h - mk = a\sqrt{1-m^2}$

$\Rightarrow (h + mk)^2 = a^2(1 + m^2)$ and $(h - mK)^2 = a^2(1 - m^2)$

$\Rightarrow m^2(K^2 - a^2) + 2mhk + h^2 - a^2 = 0$

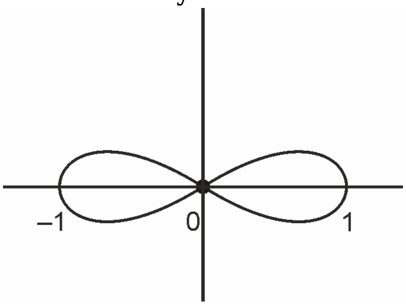
$\Rightarrow m^2(K^2 + a^2) - 2mhk + h^2 - a^2 = 0$

Elimination m from these two equation and we get

$\Rightarrow (a^4 + 4y^4)x^2 = a^6$

68) $y^2 = x^4 - x^{10}$

Curve is symmetric about x & y axis



Domain is [–1, 1] $\quad A = 4\int_0^1 \sqrt{x^4 - x^{10}}\,dx$

$= 4\int_0^1 \frac{3x^2}{3}\sqrt{1-(x^3)^2}\,dx$

$= \frac{4}{3}\int_0^1 \sqrt{1-t^2}\,dt$

$= \frac{4}{3}\cdot\frac{\pi}{4}$

$[A] = \left[\frac{\pi}{3}\right] = 1$

69) $\vec{V}.\vec{a} = x\left[\begin{matrix}\vec{a} & \vec{b} & \vec{c}\end{matrix}\right] = \frac{x}{3}$ ....(1)

$|||^{ly}$ $\vec{V}.\vec{b} = y\left[\begin{matrix}\vec{a} & \vec{b} & \vec{c}\end{matrix}\right] = \frac{y}{3}$ ....(2)

and $\vec{V}.\vec{c} = z\left[\begin{matrix}\vec{a} & \vec{b} & \vec{c}\end{matrix}\right] = \frac{z}{3}$ ....(3)

(1) + (2) + (3)

$\frac{x+y+z}{3} = \vec{V}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

$x + y + z = 3\vec{V}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

70) $T_r = \dfrac{1.3.5.........(2r-1)((2r+2)-(2r+1))}{2.4.6.8.......(2r+2)}$

$= \dfrac{1.3.5.........(2r-1)(2r+2)}{2.4.6.8.......(2r+2)} - \dfrac{1.3.5.........(2r+1)}{2.4.6.8......(2r+2)}$

$T_r = \dfrac{1.3.5.........(2r-1)}{2.4.6.8.......(2r)} - \dfrac{1.3.5.........(2r+1)}{2.4.6.8......(2r+2)}$

$T_1 = \dfrac{1}{2} - \dfrac{1.3}{2.4}$

$T_2 = \dfrac{1.3}{2.4} - \dfrac{1.3.5}{2.4.6}$ and so......on

Finally it comes out to be $\dfrac{1}{2}$

71) Let each friend has n sons 3 tickets can be distributed among 2n sons in ${}^{2n}C_3$ ways.
The number of ways distributing 3 tickets such that two tickets go to the sons of one and one tickets goes to sons of the other.
$= {}^nC_2 \times {}^nC_1 + {}^nC_1 \times {}^nC_2 = 2 \times {}^nC_1 \times {}^nC_2$
 probability that two tickets go to the sons of one and one tickets goes the sons of the other

$= \dfrac{2 \times {}^nC_1 \times {}^nC_2}{{}^{2n}C_3}$

$= \dfrac{6n^2(n-1)}{2n(2n-1)(2n-2)} = \dfrac{3n}{2(2n-1)}$

but from question

$\dfrac{6}{7} = \dfrac{3n}{2(2n-1)}$

 n = 4
Hence total number of boys = 8

72) $f(x) = \cos x(\sin x + \sqrt{\sin^2 x + \sin^2\theta})$
$\Rightarrow f(x)\sec x = \sin x + \sqrt{\sin^2 x + \sin^2\theta}$
$\Rightarrow (f(x)\sec x - \sin x)^2 = \sin^2 x + \sin^2\theta$
$\Rightarrow f^2(x)(1+\tan^2 x) - 2f(x)\tan x = \sin^2\theta$
$\Rightarrow f^2(x)\tan^2 x - 2f(x)\tan x + f^2(x) - \sin^2\theta = 0$
 tanx is Real $\Rightarrow D \geq 0$
$\Rightarrow 4f^2(x) \geq 4f^2(x)(f^2(x) - \sin^2\theta)$
$\Rightarrow f^2(x) \leq 1 + \sin^2\theta \Rightarrow |f(x)| \leq \sqrt{1+\sin^2\theta}$

73) $f(x) = f(6-x)$ ...... (1)
On differentiating (1) w.r.t. x, we get
$f'(x) = -f'(6-x)$ ...... (2)
Putting x = 0, 2, 3, 5 in (2), we get
$f'(0) = -f'(6) = 0$
|||ly $f'(2) = -f'(4) = 0$
$f'(3) = 0$

$f'(5) = -f'(1) = 0$

$f'(0) = 0 = f'(2) = f'(3) = f'(5) = f'(1) = f'(4) = f'(6)$

 $f'(x) = 0$ has minimum 7 roots in [0, 6]

Now, consider a function $y = f'(x)$

As $f'(x)$ satisfy Rolle's theorem in intervals [0, 1], [1, 2], [2, 3], [3, 4], [4, 5] and [5, 6] respectively.

So, by Rolle's theorem, the equation $f''(x) = 0$ has minimum 6 roots.

Now $g(x) = (f''(x))^2 + f'(x) f'''(x) = \frac{d}{dx}(f'(x).f''(x))$

$= h'(x)$, where $h(x) = f'(x) f''(x)$

Clearly $h(x) = 0$ has minimum 13 roots in [0, 6]

Hence again by Rolle's theorem, $g(x) = h'(x)$ has minimum 12 zeroes in [0, 6].

74) Therefore,

$\sin\alpha \sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right) = \frac{1}{4}\sin 3\alpha.$

$\sin\frac{\pi}{81}.\sin\frac{26\pi}{81}.\sin\frac{28\pi}{81} = \frac{1}{4}\sin\frac{\pi}{27},$

$\sin\frac{2\pi}{81}.\sin\frac{25\pi}{81}.\sin\frac{29\pi}{81} = \frac{1}{4}\sin\frac{2\pi}{27},$

$\vdots \quad \vdots \quad \vdots$

$\sin\frac{13\pi}{81}.\sin\frac{14\pi}{81}.\sin\frac{40\pi}{81} = \frac{1}{4}\sin\frac{13\pi}{27},$

$\sin\frac{\pi}{81}.\sin\frac{2\pi}{81}...\sin\frac{40\pi}{81} = \frac{\sqrt{3}}{2}.\left(\frac{1}{4}\right)^{13}\sin\frac{\pi}{27}...\sin\frac{13\pi}{27},$

$A = \left(\sin\frac{\pi}{81}.\sin\frac{2\pi}{81}....\sin\frac{80\pi}{81}\right) = \left(\sin\frac{\pi}{81}.\sin\frac{2\pi}{81}....\sin\frac{40\pi}{81}\right)^2$

$= \frac{3}{4^{27}}\left(\sin\frac{\pi}{27}.\sin\frac{2\pi}{27}....\sin\frac{13\pi}{27}\right)^2$

$= \frac{3}{4^{27}}\sin\frac{\pi}{27}.\sin\frac{2\pi}{27}....\sin\frac{26\pi}{27} = \frac{3}{4^{27}}.B.$

Thus, we obtain that

$\sqrt[7]{\frac{12B}{A}} = 256$

75) $\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = 1$

$\Delta_z = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & -4 \\ 1 & 1 & 4 \end{vmatrix} = 3 \neq 0$