ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME

(Academic Session : 2024 - 2025)

JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	A	A	A	A	B	B	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	A	B	B	A	C	C	C	A	A
SECTION-II	Q.	1	2	3	4	5					
	A.	1	18	9	220	4					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	C	B	A	B	A	D	C	A	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	D	A	D	B	D	B	C	A
SECTION-II	Q.	1	2	3	4	5					
	A.	15	-1450	4	3	6					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	A	D	B	A	A	A	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	A	C	A	C	D	C	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	6	56	2	-2	84					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (C)

Intensity of EM wave is given by

$$I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$$

$$E_0 = \sqrt{2} \times 10^3 \text{ kV/m} = 1.4 \text{ kV/m}$$

2. Ans (A)

$$i_1 = \frac{V}{R_{eq}} = \frac{V}{1.5R}$$

$$i_2 = \frac{V}{R'_{eq}} = \frac{V}{\frac{4R}{3}}$$

$$\therefore i_2 > i_1$$

3. Ans (A)

$$B_x = \frac{\mu_0 N i}{2r} = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 16 \times 10^{-2}}$$

$$= 4\pi \times 10^{-4} \text{ T, East}$$

$$B_y = \frac{\mu_0 N i}{2r} = \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 10 \times 10^{-2}}$$

$$= 9\pi \times 10^{-4} \text{ T, West}$$

$$\therefore\ B = (B_y - B_x) = 5\pi \times 10^{-4}$$

$$= 5 \times 3.14 \times 10^{-4}$$

$$1.6 \times 10^{-3} \text{ T, West}$$

4. **Ans (A)**

$t = 0, B = -C$

$t = \frac{C}{K}$

$B = \frac{KC}{K} - C = 0 \quad \therefore \Delta B = C$

$\therefore \Delta\phi = A\Delta B = \pi a^2 C$

$q = \frac{\Delta\phi}{R} = \frac{\pi a^2 C}{R}$

5. **Ans (A)**

$\omega = \frac{1}{\sqrt{LC}}$

$i_0 = \frac{V_0}{R}$

6. **Ans (A)**

$dQ = dU + dW$

$5 = 0 + W_{AB} + W_{CA}$

$5 = 10 + W_{CA}$

$W_{CA} = -5J$

7. **Ans (B)**

$\frac{V\,rms_A}{V\,rms_B} = \sqrt{\frac{M_B}{M_A}} = \frac{1}{2}$

8. **Ans (B)**

$W_{AB} = 0,\ W_{BC} = R\,2T_0 \ell n\left(\frac{4P_0}{2P_0}\right)$

9. **Ans (B)**

$P_0T_0 = \frac{P_0T}{2}$

$T = 2T_0$

$\Delta U = \frac{f}{2} nR\,\Delta T$

$= \frac{3}{2}(2)R(2T_0 - T_0) = 3RT_0$

$P_0V_0 = nRT_0$

$T_0 = \frac{P_0V_0}{nR} = \frac{P_0V_0}{2R}$

$\Delta U = (3R)\left(\frac{P_0V_0}{2R}\right) = \frac{3P_0V_0}{2}$

10. **Ans (A)**

Refractive index $\mu = \frac{v_{air}}{v_{medium}} = \frac{\frac{x}{t_1}}{\frac{10\,x}{t_2}} = \frac{t_2}{10t_1}$

also $\sin\theta_c = \frac{1}{\mu} = \frac{10\,t_1}{t_2} \Rightarrow \theta_c = \sin^{-1}\left(\frac{10t_1}{t_2}\right)$

11. **Ans (A)**

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

12. **Ans (A)**

For 1^{st} resonance, $\ell_1 + e = \frac{\lambda}{4}$

For 2^{nd} resonance, $\ell_2 + e = \frac{3\lambda}{4}$

$\therefore e = \frac{\ell_2 - 3\ell_1}{2} = 0.025$ m

13. **Ans (B)**

$n = \frac{P\lambda}{hc}$

14. Ans (B)

$$\lambda = \frac{h}{\sqrt{2mqV_0}}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{\frac{h}{\sqrt{2maq\times50}}}{\frac{h}{\sqrt{2\times4m\times q\times2500}}} = 14.14$$

15. Ans (A)

$$\frac{6}{0.125} = R + \frac{1}{\left(\frac{1}{8}+\frac{1}{16}+\frac{1}{16}\right)}$$

16. Ans (C)

By conservation of momentum

$$m_1(40) = (m_1 + m_2)\,30$$

$$\frac{40}{30} = 1 + \frac{m_2}{m_1}$$

$$\frac{m_1}{m_2} = \frac{30}{10} = 3$$

19. Ans (A)

$$\frac{dV}{dt} = -a\sqrt{v}$$

$$\frac{dV}{v^{1/2}} = -adt$$

$$\int v^{-1/2}dv = \int -adt$$

$$2\sqrt{v} = -at + c$$

at $t = 0$ $v = V0$

$$2\sqrt{V_0} = c$$

$$\therefore \quad 2\sqrt{v} = -at + 2\sqrt{V_0}$$

put $v = 0$

$$t = \frac{2\sqrt{V_0}}{a}$$

PART-1 : PHYSICS

SECTION-II

1. Ans (1)

$$E_{Arc} = \frac{2k\lambda_2}{R}\sin\frac{\theta}{2}(\theta = 180^o)$$

$$E_{Infinite\ wire} = \frac{2k\lambda_1}{d} \quad (d = R)$$

$$\lambda_1 = \lambda_2$$

2. Ans (18)

Heat added to the gas in cylinder A is at constant pressure while that in cylinder B at constant volume. Therefore,

$$Q_A = \mu C_p(\Delta T)_A$$

$$Q_B = \mu C_v(\Delta T)_B$$

Given that, $Q_A = Q_B$

$$\therefore\ \mu C_p(\Delta T)_A = \mu C_v(\Delta T)_B$$

$$(\Delta T)_B = \frac{C_p}{C_v}(\Delta T)_A = 1.4 \times 30 = 42\text{ K}$$

3. Ans (9)

$$I_{max} = k$$

$$I_1 = I_2 = \frac{k}{4}$$

$$\Delta x = \frac{\lambda}{6} \Rightarrow \Delta\phi = \frac{\pi}{3}$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

$$I = \frac{K}{4} + \frac{K}{4} + 2\times\frac{K}{4}\times\frac{1}{2}$$

$$\frac{K}{2} + \frac{K}{4} = \frac{3K}{4} = \frac{9K}{12}$$

$$\boxed{n = 9}$$

4. Ans (220)

$\Delta E = (80 \times 7 + 120 \times 8) - (200 \times 6.5)$

$= 220$ MeV

5. Ans (4)

$W = \Delta K$

$= \frac{1}{2} mv^2 \left(1 + \frac{K^2}{R^2}\right)$

$= mv^2$

$= 100\,(0.2)^2 = 4J$

PART-2 : CHEMISTRY

SECTION-I

3. Ans (B)

$(N_1V_1)_{H_2O_2} = (N_2V_2)_{KMnO_4}$

$M \times 2 \times 20\text{ mL} = 0.05 \times 5 \times 80\text{ mL}$

$M = 0.5$

Volume strength of $H_2O_2 = 11.2 \times M = 5.6$

4. Ans (A)

$\overset{\infty}{\Lambda} AgCl = \overset{\infty}{\Lambda} Ag^+ + \overset{\infty}{\Lambda} Cl^-$

$= 62 + 76 = 138 = \overset{\infty}{\Lambda} = \frac{K \times 1000}{S}$

$S = \frac{1.26 \times 10^{-6} \times 1000}{138}$

$S = 9.13 \times 10^{-6}$

$\because K_{sp} = S^2 = 8.3 \times 10^{-11}$

7. Ans (D)

$\Delta T_b = i.\,k_b\,.\,m$

$1.518 = i \times 2.53 \times 1$

$i = 0.6$

$\because i = 1 + a\left(\frac{1}{7} - 1\right)$

$0.6 = 1 + \alpha\left(\frac{1}{2} - 1\right)$

$\alpha = 0.8 = 80\%$

8. Ans (C)

O_2 have 2 unpaired el^-

O_2^- & O_2^+ have 1 unpaire el^-

9. Ans (A)

EAN = At. No. – O.S + 2 C.N.

$[Ni(NH_3)_4]SO_4$

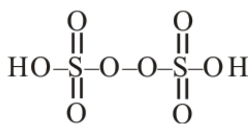
EAN = 28 – 2 + 2(4)

= 26 + 8 = 34

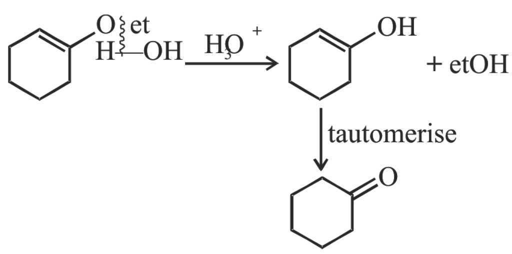
12. Ans (D)

Complex forming tendency $\propto \frac{1}{\text{Size of ion}}$

13. Ans (D)

Peroxodisulphuric acid is, 

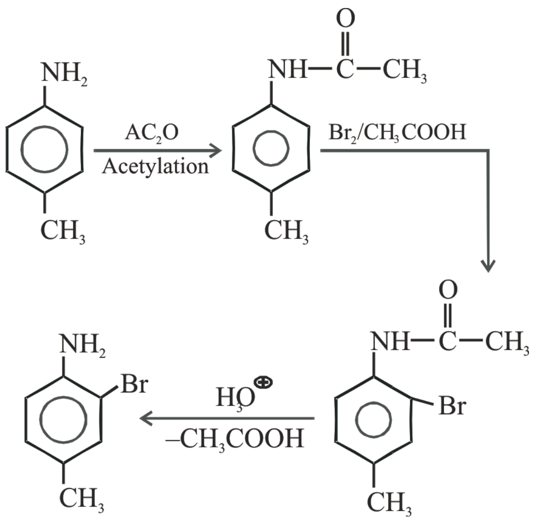
15. Ans (D)



16. **Ans (B)**

$$Me-\underset{\underset{O}{\|}}{C}-NH_2 \xrightarrow{LiAlH_4/H^{\oplus}} Me-CH_2-NH_2 :$$

17. **Ans (D)**



18. **Ans (B)**

2° amine will give +ve nitrosamine test

$$CH_3-CH_2-CH_2-\overset{\overset{CH_3}{|}}{C}H-NH-Et$$

N-ethyl-2-pentamine
(2° amine)

19. **Ans (C)**

In RDS of haloform reaction carbanion obtained as intermediate by cleavage of C–H bond

20. **Ans (A)**

$$AS \Rightarrow CH_3-\underset{\underset{O}{\|}}{C}-OH > C_6H_5OH > CH_3-NH_2$$

PART-2 : CHEMISTRY

SECTION-II

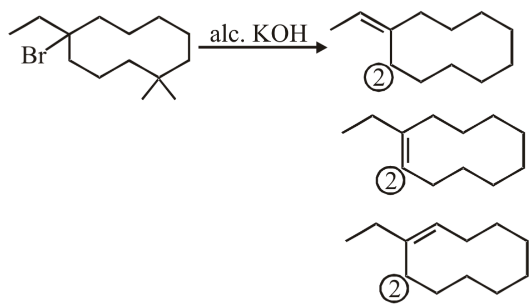
2. **Ans (-1450)**

$$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$$

$$\Delta H = 2(-400 - 300) - 50$$

$$= -1450 \text{ kJ/mole}$$

5. **Ans (6)**



Total (6)

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (C)**

$$\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$$

$$= \int \frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4} \cdot (x-1)^2}$$

$$\text{Put } \frac{x+2}{x-1} = t - \frac{1}{3}\int \frac{dt}{t^{5/4}}$$

$$= \frac{4}{3} \cdot \frac{1}{t^{1/4}} + C = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$$

2. **Ans (A)**

$$I = \int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{x/\pi\right\}}}dx = 100\int_0^{\pi} \frac{\sin^2 x}{e^{x/\pi}}dx$$

$$100\int_0^{\pi} e^{-x/\pi}\frac{(1-\cos 2x)}{2}dx$$

$$50\left\{\int_0^{\pi} e^{-x/\pi}dx - \int_0^{\pi} e^{-x/\pi}\cos 2x dx\right\}$$

$$I_2 = \int_0^{\pi} e^{-x/\pi}dx - \left[-\pi e^{-x/\pi}\right]_0^{\pi} = \pi\left(1-e^{-1}\right)$$

$$I_2 = \int_0^{\pi} e^{-x/\pi}\cos 2x dx$$

$$= -\pi e^{-x/x}\cos 2x]_0^{\pi} - \int -\pi e^{-x/x}(-2\sin 2x)\,dx$$

$$= \pi\left(1-e^{-1}\right) - 2\pi\int_0^{\pi} e^{-x/\pi}\sin 2x dx$$

$$\pi\left(1-e^{-1}\right)$$

$$-2\pi\left\{-\pi e^{-\pi/\pi}\sin 2x]_0^{\pi} - \int_0^{\pi} -\pi e^{-x/x} 2\cos 2x dx\right\}$$

$$= \pi\left(1-e^{-1}\right) - 4\pi^2 I_2$$

$$\Rightarrow \quad I_2 = \frac{\pi\left(1-e^{-1}\right)}{1+4\pi^2}$$

$$\therefore \quad I = 50$$

$$\therefore I = 50\left\{\pi\left(1-e^{-1}\right) - \frac{\pi\left(1-e^{-1}\right)}{1+4\pi^2}\right\}$$

$$\frac{200\left(1-e^{-1}\right)\pi^3}{1+4\pi^2}$$

3. **Ans (A)**

$$xdy = (y + x^3\cos x)dx$$

$$xdy = ydx + x^3\cos x dx$$

$$\frac{xdy - ydx}{x^2} = \frac{x^3\cos x dx}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x\cos x dx$$

$$\frac{y}{x} = x\sin x - \int 1.\sin x dx$$

$$\frac{y}{x} = x\sin x + \cos x + C$$

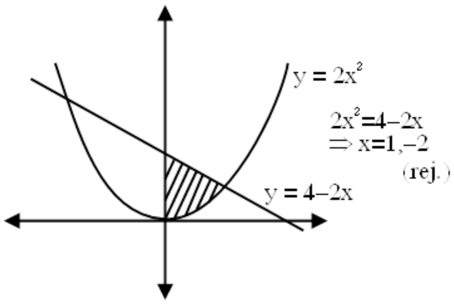
$$\Rightarrow 0 = -1 + C \Rightarrow C = 1,\ x = p,\ y = 0$$

$$\text{so } \frac{y}{x} = x\sin x + \cos x + 1$$

$$y = x^2\sin x + x\cos x + x \qquad x = \frac{\pi}{2}$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

4. Ans (D)



Required area

$$= \int_0^1 (4-2x-2x^2)\,dx = 4x - x^2 - \frac{2x^3}{3}\Big|_0^1$$

$$= 4 - 1 - \frac{2}{3} = \frac{7}{3}$$

6. Ans (A)

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$$

$\Rightarrow\ \det(A) = (a-b)^2 (b-c)^2 (c-a)^2$

& $\det(4I) = 64$

$\Rightarrow\ (a-b)(b-c)(c-a) = \pm 8$

$\because\ (a-b) + (b-c) + (c-a) = 0$

$\therefore\ (a-b)^3 + (b-c)^3 + (c-a)^3$

$= 3(a-b)(b-c)(c-a) = \pm 24$

8. Ans (A)

$$\left| z_1 z_2 z_3 \left(\frac{2|z_1|^2}{z_1} + \frac{3|z_2|^2}{z_2} + \frac{4|z_3|^2}{z_3} \right) \right|$$

$|z_1||z_2||z_3|\ |2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3|$

$2 \times 3 \times 4 \times 9 = 216$

9. Ans (A)

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 2(3\lambda + 2)(\lambda - 3)$$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them is zero so equations are inconsistant

$$\boxed{\therefore\ \lambda = -\frac{2}{3}}$$

10. Ans (C)

$$\frac{\frac{3\left(\sqrt{5}+1\right)}{4} + 5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right) - 3\left(\frac{\sqrt{5}-1}{4}\right)} = \frac{8\sqrt{5}-2}{2\sqrt{5}+8}$$

$$= \frac{4\sqrt{5}-1}{\sqrt{5}+4} \times \frac{\sqrt{5}-4}{\sqrt{5}-4}$$

$$= \frac{20 - 16\sqrt{5} - \sqrt{5} + 4}{-11}$$

$$= \frac{17\sqrt{5} - 24}{11} \Rightarrow a = 17,\ b = 24,\ c = 11$$

$a + b + c = 52$

11. Ans (C)

x	C	2C	3C	4C	5C	6C
f	2	1	1	1	1	1

$$\bar{x} = \frac{(2+2+3+4+5+6)C}{7} = \frac{22C}{7}$$

$$\text{Var}(x) = \frac{c^2\left(2+2^2+3^2+4^2+5^2+6^2\right)}{7}$$

$$-\left(\frac{22c}{7}\right)^2$$

$$= \frac{92c^2}{7} - c^2 \times \frac{484}{49}$$

$$= \frac{(644-484)c^2}{49} = \frac{160c^2}{49}$$

$$160 = \frac{160 \times c^2}{49} \Rightarrow c = 7$$

12. Ans (A)

$$\frac{\sum x_i}{6} = 2 \text{ and } \frac{\sum x_i^2}{N} - \mu^2 = 23$$

$\alpha + \beta = 10$

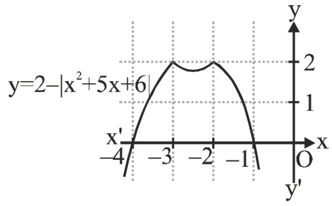
$\alpha^2 + \beta^2 = 52$

solving we get $\alpha = 4$, $\beta = 6$

$$\frac{\sum |x_i - \bar{x}|}{6} = \frac{5+2+5+8+2+4}{6} = \frac{13}{3}$$

13. Ans (A)

f(x) will have maxima at $x = -2$ only if $a^2 + 1 \geq 2$ or or $|a| \geq 1$.



14. Ans (C)

Given $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$

Replacing x by 3x and y by zero,

then $f(x) = \frac{f(3x)+f(0)}{3}$

$\Rightarrow f(3x) - 3f(x) = -f(0)$

and $f'(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{f\left(\frac{3x+3h}{3}\right) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{f(3x)+f(3h)}{3} - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(3x)+f(3h)-3f(x)}{3h}$$

$$= \lim_{h\to 0} \frac{f(3h)-f(0)}{3h} \quad \text{[from Eq. (i)]}$$

$= f'(0) = 3$

$\therefore f(x) = 3x + c$

$\Rightarrow f(0) = 0 + c = 3$

$\therefore c = 3$

Then, $f(x) = 3x + 3$

Hence, f(x) is continuous and differentiable everywhere.

15. Ans (A)

$f(x) = 3^{\alpha x} + 3^{\beta x}$

$f'(x) = \alpha 3^{\alpha x} \ln 3 + 3^{\beta x}\, b \ln 3$

$f''(x) = \alpha^2 3^{\alpha x} (\ln 3)^2 + 3^{\beta x}\ \beta^2 (\ln 3)^2$

Put it in given condition and solve

16. Ans (C)

$f(x) = f(-x)$

$\Rightarrow \left[\dfrac{x^4+1}{a}\right] = 0$

$\Rightarrow 0 \leqslant \dfrac{x^4+1}{a} < 1$

$\Rightarrow a > x^4 + 1$

$\Rightarrow a > 257$

$\therefore\quad a \in (257, \infty)$

17. Ans (D)

Distance of all the points from (0, 0) are 5 units. That means the circumcenter of the triangle formed by the given points is (0, 0). If $G \equiv (h, k)$ is the centroid of the triangle, then

$3h = 3+5(\cos\theta+\sin\theta),\ 3k=4+5(\sin\theta-\cos\theta)$.

If $H(\alpha, \beta)$ is the orthocenter then.

$OG : GH = 1 : 2$ or $\alpha = 3h,\ \beta = 3k$

$\cos\theta + \sin\theta = \dfrac{\alpha-3}{5},\ \sin\theta - \cos\theta = \dfrac{\beta-4}{5}$

or $\sin\theta = \dfrac{\alpha+\beta-7}{10},\ \cos\theta = \dfrac{\alpha-\beta+1}{10}$

Thus, the locus of (a, b) is

$(x+y-7)^2 + (x-y+1)^2 = 100$

18. Ans (C)

$\dfrac{3a^2-(a+1)+1}{-a^2-2a-2+5} > 0$

$\Rightarrow \dfrac{3a^2-a}{a^2+2a-3} < 0$

$\Rightarrow \dfrac{a(3a-1)}{(a+3)(a-1)} < 0$

$\Rightarrow a \in (-3, 0) \cup \left(\dfrac{1}{3}, 1\right)$

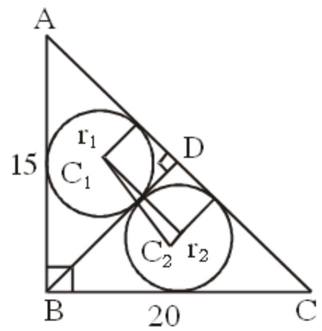
19. Ans (D)

$AC = \sqrt{15^2+20^2} = 25$

$BD = 12$

$AD = 9$

$CD = 16$



$r_1 = \dfrac{\Delta}{S} = \dfrac{\frac{1}{2}\times 12\times 9}{\frac{12+9+15}{2}} \Rightarrow \dfrac{9\times 12}{36} = 3$

$r_2 = \dfrac{\Delta}{S} = \dfrac{\frac{1}{2}\times 16\times 12}{\frac{20+16+12}{2}} \Rightarrow \dfrac{16\times 12}{48} = 4$

distance C_1C_2

$= \sqrt{(r_2-r_1)^2 + (r_1+r_2)^2} = \sqrt{1+49} = \sqrt{50}$

20. **Ans (B)**

Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

is equally inclined to $\vec{b}$ and $\vec{d}$, where $\vec{d} = \hat{j} + 2\hat{k}$.

$$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}\cdot\vec{d}}{|\vec{a}||\vec{d}|}$$

$$\Rightarrow \frac{\left(\lambda\vec{b}+\mu\vec{c}\right).\vec{b}}{\sqrt{5}} = \frac{\left(\lambda\vec{b}+\mu\vec{c}\right)\cdot\vec{d}}{\sqrt{5}}$$

$$\frac{\left[\lambda\left(2\hat{i}+\hat{j}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)\right]\cdot\left(2\hat{i}+\hat{j}\right)}{\sqrt{5}}$$

$$= \frac{\left[\lambda\left(2\hat{i}+\hat{j}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)\right]\cdot\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}$$

or $\lambda(4+1) + \mu(2-1) = \lambda(1) + \mu(-1+2)$

or $4\lambda = 0$, i.e., $\lambda = 0$

$$\therefore \hat{a} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (6)**

$$\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)} + \int \frac{dx}{x^4+3x^2+1}$$

$$\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}$$

$$+\frac{1}{2}\int \frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\dfrac{1}{x}\right) = t$

$$\int \frac{dt}{t} + \frac{1}{2}\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5} - \frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put, $x-\dfrac{1}{x} = y,\ x+\dfrac{1}{x} = z$

$$\log_e t + \frac{1}{2}\int \frac{dy}{y^2+5} - \frac{1}{2}\int \frac{dz}{z^2+1}$$

$$= \log_e \tan^{-1}\left(x+\frac{1}{x}\right) + \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

$$\alpha = 1,\ b = \frac{1}{2\sqrt{5}},\ g = \frac{1}{\sqrt{5}},\ d = \frac{-1}{2}$$

$$\text{or } \alpha = 1,\ \beta = \frac{-1}{2\sqrt{5}},\ \gamma = \frac{-1}{\sqrt{5}},\ \delta = \frac{-1}{2}$$

$$10(\alpha+\beta\gamma+\delta) = 10\left(1+\frac{1}{10}-\frac{1}{2}\right) = 6$$

2. **Ans (56)**

16 cubes < 11 Blue, 5 Red

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

$x_1, x_6 \geqslant 0,\ x_2, x_3, x_4, x_5 \geqslant 2$

$x_2 = t_1 + 2$

$x_3 = t_3 + 2$

$x_4 = t_4 + 2$

$x_5 = t_5 + 2$

$x_1, t_2, t_3, t_4, t_5, x_6 \geqslant 0$

No. of solutions $= {}^{6+3-1}C_3 = {}^{8}C_3 = 56$

3. **Ans (2)**

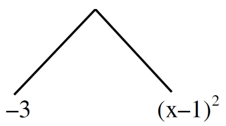
$\sin^2x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$

$\sin^2x - (x-1)^2 \sin x + 3\sin x - 3(x-1)^2 = 0$

$(\sin x + 3)(\sin x - (x-1)^2) = 0$

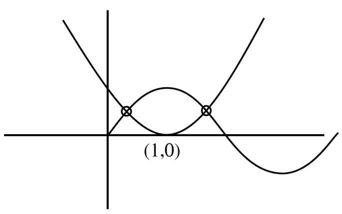
$\sin x = -3, (x-1)^2$

roots :



$\sin x = -3$ (reject) or $(x-1)^2$

$\sin x = (x-1)^2$



4. **Ans (-2)**

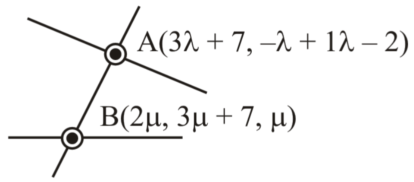
Ans. of first part $= -1$

second part $= 1$

third part $= -2$

$\therefore$ Ans. $= -2$

5. **Ans (84)**



DR's of AB

$(3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2)$

$$\frac{3\lambda - 2\mu + 7}{1} = \frac{-\lambda - 3\mu - 6}{-4} = \frac{\lambda - \mu - 2}{2}$$

Taking first (2) $-12\lambda + 8\mu - 28 = -\lambda - 3\mu - 6$

$\lambda - \mu + 2 = 0$

Taking second & third

$-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

After solving above two equation $\lambda = -5, \mu = -3$

$A = (-8, 6, 7)$

$B = (-6, -2, -3)$

$(AB)^2 = 4 + 64 + 16 = 84$