FIITJEE RBT-6 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 1 | **QP Code: 100961** | **PAPER - 1**

Time Allotted: 3 Hours **Maximum Marks: 234**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-07) –** Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(i) **Part-A (08-13) –** Contains six (06) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **Part-B (01-08)** contains eight (08) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+4 marks** for correct answer and **there will be no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

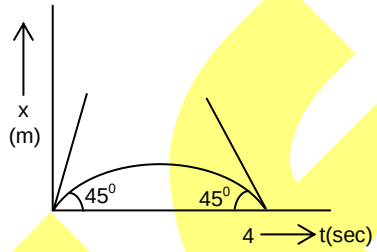
SECTION-1 : PHYSICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. A particle moves along x axis with constant acceleration and its x position depend on time 't' as shown in the following graph (parabola); then in interval 0 to 4 sec
(A) relation between x – coordinate & time is $x = t - t^2/4$
(B) maximum x – coordinate is 1 m
(C) total distance travelled is 2m
(D) average speed is 0.5 m/s

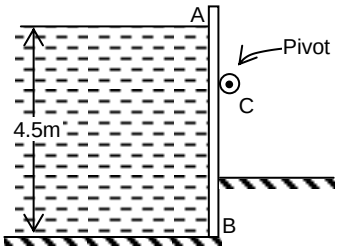


2. A vibrating string produces 2 beats per seconds when sounded with a tuning fork of frequency 256 Hz. increasing the tension in the string produces 3 beats per second. The initial frequency of the string may have been
(A) 253 Hz (B) 254 Hz (C) 258 Hz (D) 259 Hz

3. A sample of nuclei X contains 48×10^{23} nuclei at some time and reduced to 6×10^{23} nuclei in 3 minutes. The mean life of nucleus X is roughly?
(A) 0.693 min (B) 1.44 min (C) 2.079 min (D) 4.329 min

4 Air is introduced through a nozzle into a tank filled with water to form a stream of bubbles of diameter 2mm. Calculate by how much the pressure in the nozzle must exceed to that of the surrounding water? Assume surface tension of water = 0.075 N/m
(A) 133.4 Pa (B) 175.0 Pa (C) 75.0 Pa (D) 150.0 Pa

5. Figure shows a rectangular vertical board AB which is 4.5 m high and is pivoted at C. What must be the maximum height of C above B so that the vertical board will be on the verge of tipping about C when the water surface is at A?
(A) 2 m (B) 2.5 m
(C) 1.5 m (D) 3 m



Space For Rough Work

6. A convex lens made of glass (μ_g =3/2) has focal length *f* in air. The image of an object placed in front of it is inverted real and magnified. Now the whole arrangement is immersed in water (μ_w=4/3) without changing the distance between object and lens. Then
(A) the new focal length will become 4*f*
(B) the new focal length will become *f* / 4
(C) new image will be virtual and magnified
(D) new image will be real, inverted and smaller in size

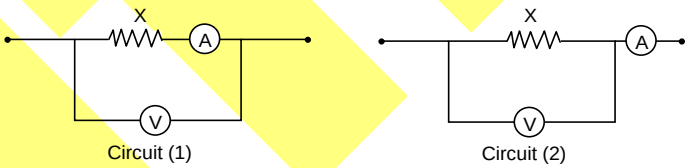
7. If the potential difference of Coolidge tube producing X-ray is increased, then choose the correct option(s).
(A) the interval between $\lambda_{k\alpha}$ and $\lambda_{k\beta}$ increases
(B) the interval between $\lambda_{k\alpha}$ and λ_0 increases
(C) the interval between $\lambda_{k\beta}$ and λ_0 increases
(D) λ_0 does not change
Here λ_0 is cutoff wavelength and $\lambda_{k\alpha}$ and λ_{kB} are wavelength of k_α and k_B characteristic X-rays.

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. An ammeter and a voltmeter are connected in series to a battery of emf 6 V. When a certain resistance is connected in parallel with the voltmeter the reading of the voltmeter decreases n = 2 times while the reading of the ammeter increases the same number of times. Then the ratio of voltmeter resistance to the ammeter resistance will be
(A) 1 : 2 (B) 2 : 3
(C) 2 : 1 (D) None of these

9. An ammeter and a voltmeter are connected as shown in figure (1) and figure (2). In order to measure an unknown resistance which connection is to be preferred,



(A) for low resistance circuit 2 and for high resistance circuit (1).
(B) for low resistance circuit (1) and for high resistance circuit 2
(C) For an y value of resistance circuit 1.
(D) For any value of resistance circuit 2.

Space For Rough Work

10. There is a 90^0 bend in a pipe line in which water is flowing at speed v. The cross sectional area of pipe is A. The force exerted by pipe walls on water at bend is

(A) $\sqrt{3}\,A\,v^2\rho$ (B) $\sqrt{2}\,Av^2\rho$

(C) $Av^2\rho$ (D) $\sqrt{5}\,Av\rho$

11. Calculate the capacitance of a parallel-plate capacitor of area S, separation d and dielectric constant varying linearly form ε_1 to ε_2 ($\varepsilon_2 > \varepsilon_1$) in the direction of the applied field.

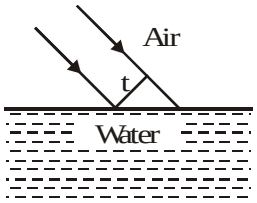
(A) $C = \frac{\varepsilon_o S}{d}\frac{(\varepsilon_2 - \varepsilon_1)}{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}$ (B) $C = \frac{\varepsilon_o S}{d}\frac{(\varepsilon_2 + \varepsilon_1)}{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}$

(C) $C = \frac{\varepsilon_o S}{2d}\frac{(\varepsilon_2 + \varepsilon_1)}{\ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right)}$ (D) none of these

12. A hoop of mass M and radius R is placed on an absolutely smooth level surface. An insect of mass m begins to slide along the hoop from the highest point with speed V relative to the hoop. Then the velocity of the centre of the hoop after time t will be

(A) $\frac{v\cos\left(\frac{vt}{R}\right)}{1+\frac{m}{M}}$ (B) $\frac{v\sin\left(\frac{vt}{R}\right)}{1+\frac{M}{m}}$ (C) $\frac{v\cos\left(\frac{vt}{R}\right)}{1-\frac{M}{m}}$ (D) $\frac{v\cos\left(\frac{vt}{R}\right)}{1+\frac{M}{m}}$

13. A monochromatic beam of width t is incident at 45º on an air water interface as shown in the figure. The refractive index of water is μ and that of air is 1. The width of the beam in water is,



(A)$(\mu - 1)$ t (B) μ t

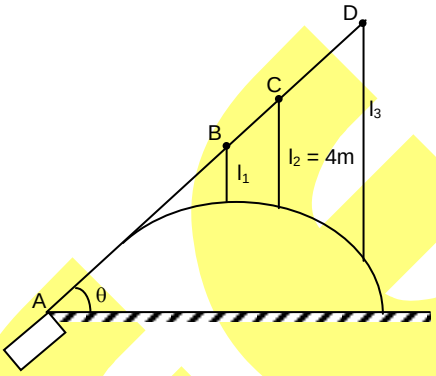
(C) $\frac{\sqrt{\mu^2 - 1}}{\mu}$t (D) $\frac{\left(\sqrt{2\mu^2 - 1}\right)}{\mu}$t

Space For Rough Work

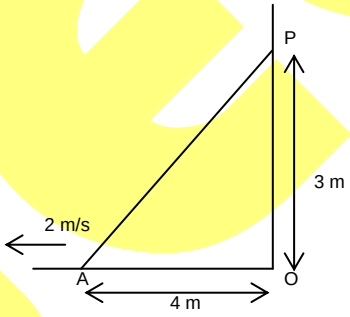
PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

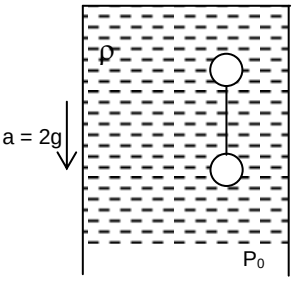
1. A stream of water coming out from a pipe at point A as shown in figure. A rod AD is along the projected velocity of stream. There are four points A,B,C,D on the rod (AB = BC = CD) from which string of length l_1, l_2, l_3 are hanging. These string are touching the trajectory of stream of water. If the length l_2 = 4m find l_3.



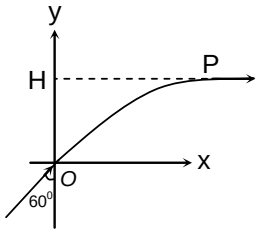
2. A ladder AP of length 5m is inclined to a vertical wall is slipping over a horizontal surface with velocity of 2 m/s when P is at a distance 3m from ground what is the velocity of centre of mass at this moment (in m/s) _____.



3. Two identical shaped spheres of specific gravity 0.8 & 1.6 are connected through string & placed in liquid container as shown in figure. if spheres are in equilibrium wrt container then tension in the string (take volume of each sphere v_0 = 250 CC) is xn, find the value of 'x'.



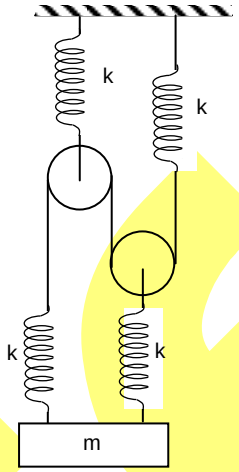
4. A system of coordinates is drawn in a medium whose refractive index varies as $\mu = \frac{2}{1+y^2}$, where $0 \le y \le 1$. A ray of light is incident at origin at an angle 60^0 with *y* –axis as shown in the figure. At point P ray becomes parallel to x-axis. Find the value of 'H'.



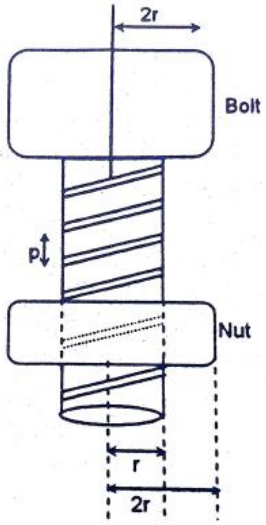
Space For Rough Work

5. The refractive index for water w.r.t. air for a sound wave if the velocities of sound in water and air are 1440 ms^{-1} and 340ms^{-1}, is

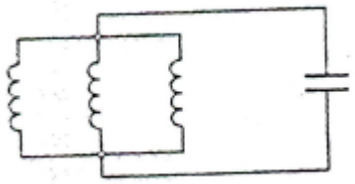
6. All pulleys & ropes are ideal find out period of small oscillation of block of mass m as shown in figure. Take $\frac{M}{k}=\left(\frac{9}{10}\right)$



7. A bolt with shaft radius r has pitch of its threading equal to ρ. The nut has frictionless movement on threads. The bolt is rotated anti-clockwise while looking from top with an angular acceleration α such that the nut moves up by an acceleration g. Find α in terms of kilo radian/sec^2. (take g = 10 m/s^2 ; ρ = 2MM)



8. A circuit shown consists of three coils of inductances 1.0 H each and a 1.0 μF capacitance. At an instant, when charge on the capacitor is zero, currents of 1.0 A, 2.0 A and 4.0 A are flowing through the coils in the same direction. Find the maximum charge on the capacitor. (in MC)



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SECTION-2 : CHEMISTRY

PART – A

(Multi Correct Choice Type)

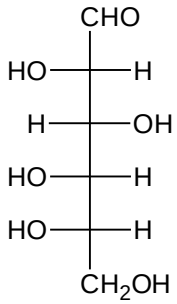
This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. Consider the following reaction

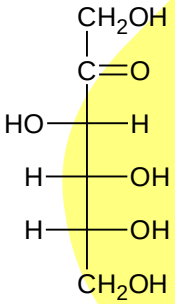
$$\text{D-glucose} \xrightarrow[\text{Cat } H^+]{\text{PhNHNH}_2\text{(3 eq)}} X$$

Among the following the compound(s) whose derivative(s) will have the same melting point as that of X is(are)

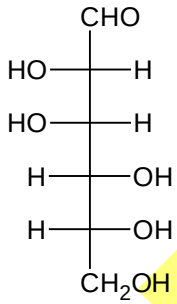
(A)



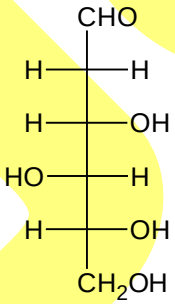
(B)



(C)



(D)



2. The type(s) of intersection(s) that hold layers of graphite together is(are)

(A) π-π stacking (B) van der Waal's

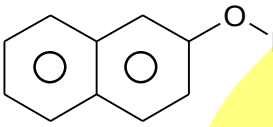
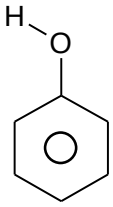
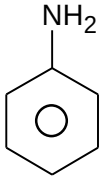
(C) hydrogen bonding (D) coulombic

Space For Rough Work

3. Which is/are correct statement about diazo-coupling reaction?

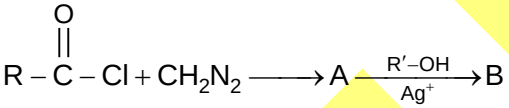
(A) Benzene diazonium ion is a strong electrophile $\left(PhN_2^+\right)$.

(B) For diazocoupling reaction of phenol pH optima should be slightly basic.
(C) pH diazo coupling reaction of Aniline pH optima should be slightly acidic.

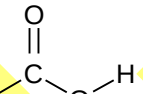
(D) The colour of diazocoupling product with β-napthol  is red, with  is orange and with  is yellow.

4. Which is/are correct about allotropes of carbon?
(A) Graphite is thermodynamically most stable allotrope.
(B) Molar entropy order is Fullerene > Graphite > Diamond
(C) Fullerene is aromatic
(D) Buck minister fullerene C_{60} has 12 five member and 20 six member rings

5. The intermediate formed in the reaction



Which is correct statements about above reaction?
(A) Intermediate involve is both carbene as well as carbanion.

(B) A can be formed by reaction of  (Carboxylic acid) with $AlPO_4$
(C) The nucleophile(CH_2N_2) is resonance stablised
(D) Final product 'B' is an ester

6. When B_2H_6 is treated with excess NH_3 at low temperature. The nature of bond present in the product is/are
(A) Ionic (B) Covalent
(C) Co-ordinate (D) Metallic

Space For Rough Work

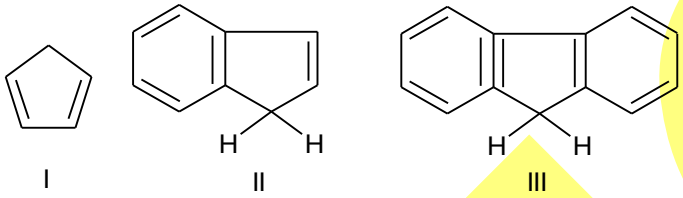
7. Spin isomerism is found in
(A) H_2 (B) D_2
(C) N_2 (D) F_2

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8 The VO_4^{3-}, CrO_4^{2-} and MnO_4^- ions exhibit intense ligand to metal charge transfer transition the wavelength of this transition follows the order
(A) $CrO_4^{2-} < VO_4^{3-} < MnO_4^-$ (B) $MnO_4^- < VO_4^{3-} < CrO_4^{2-}$
(C) $VO_4^{3-} < CrO_4^{2-} < MnO_4^-$ (D) $CrO_4^{2-} < MnO_4^- < VO_4^{3-}$

9. Correct acidic strength of



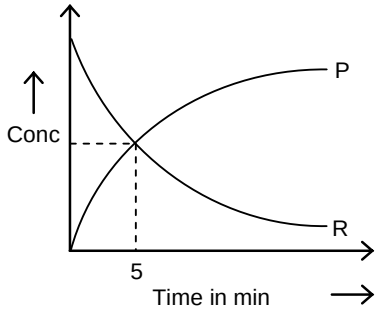
I II III
(A) I > II > III (B) III > II > I
(C) II > III > I (D) II > I > III

10. If EAN of metal is 36 in $[M(CO)_2(\sigma C_5H_5)(\pi C_5H_5)]$, then what is the atomic number of metal M?
(A) 23 (B) 24
(C) 25 (D) 26

11. A mineral consists of an equimolar mixture of the carbonates of two bivalent metals. One metal is present to the extent of 12.5% by mass. 2.8 g of the mineral on heating lost 1.32 g of CO_2. What is the % by mass of the other metal.
(A) 87.5 (B) 35.71
(C) 65.11 (D) 23.21

12. The CFSE for octahedral $[CoCl_6]^{4-}$ IS 18000 cm^{-1}. The CFSE for tetrahedral $[CoCl_4]^{2-}$ will be
(A) 18000 cm^{-1} (B) 16000 cm^{-1}
(C) 8000 cm^{-1} (D) 20000 cm^{-1}

Space For Rough Work

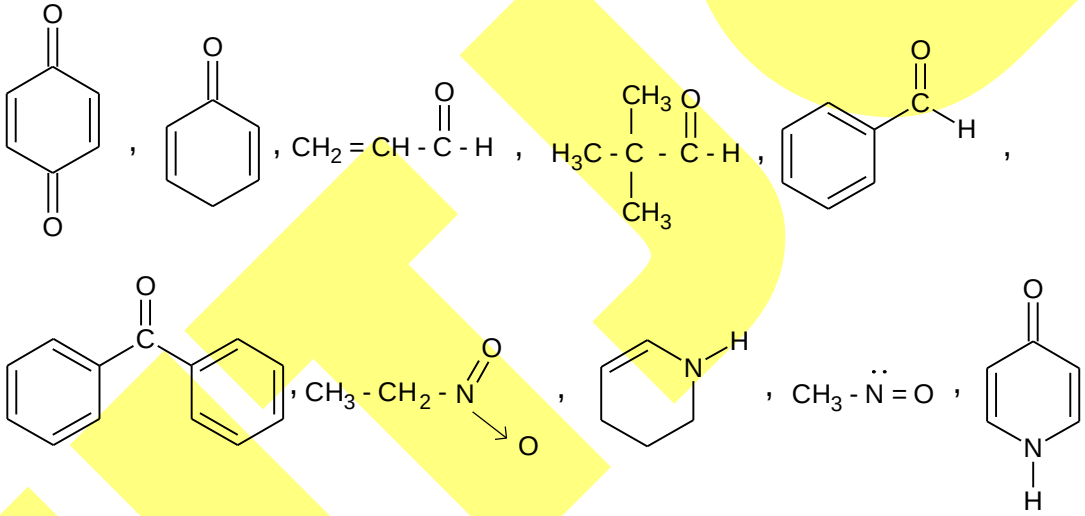
13. 

For a given 1st order reaction R → 3P
What is the time required for 20% completion of reaction in minute unit?
(A) 4 (B) 2
(C) 3 (D) 5

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. How many of the given molecule can show tautomerism?



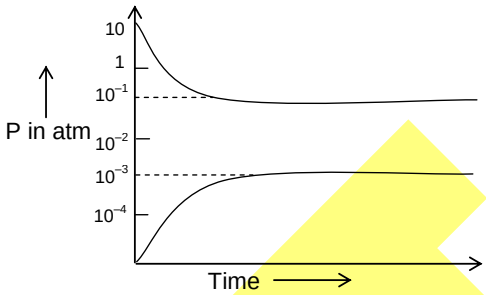
2. Number of geometrical square planar complex formed when $PtCl_2$ is treated with racemic mixture of alanine is

3. The molarity of 20% w/w sulphuric acid of density 1.14g cm^{-3} is

Space For Rough Work

4. How many of the given molecules have central atom sp^3 hybridized
SF_4, XeO_2F_2, $XeOF_2$, $XeOF_4$, XeO_4, CO_2, SiO_2, ClF_3, CH_3^-, CH_3^+

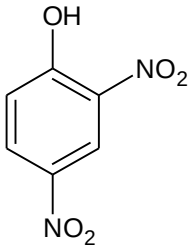
5. How many of the given statement(s) is/are correct?
(i) Metallic bond strength in 's'-block elements decreases down the group.
(ii) Metallic character increases down the group in 's'-block.
(iii) Reactivity of 's'-block elements increases down the group.
(iv) Solution of alkali metal in liquid NH_3 is blue in colour due to presence of solvated electron.
(v) The blue colour of alkali metals in liquid NH_3 slowly fades until it disappears owing to the formation of a metal amide and this reaction is catalysed by Fe or presence of impurities.
(vi) At concentrations about 3M, solution of alkali metal in liquid NH_3 are copper-bronze coloured and have a metallic lusture because metal ion clusters are formed.
(vii) Solutions of metals in liquid NH_3 conduct electricity better than any salt in any liquid and the conductivity is similar to that of pure metals
(viii) Conduction of alkali metal in liquid NH_3 is due to mainly to the presence of solvated electron.
(ix) Solutions of alkali metals in liquid NH_3 act as a powerful reducing agent and they will even reduce an aromatic ring.

6. 

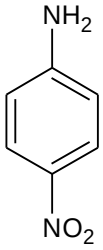
The plot of partial pressure of the product and reactant is given above. What is the value of ΔG^o in L atm unit at temperature 'T'?
[Assume 2.303 RT = 4 L atm mol^{-1}]

Space For Rough Work

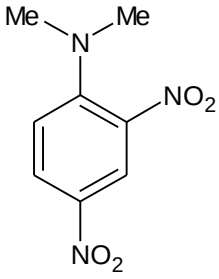
7. How many compounds can liberate CO_2 when treated with $NaHCO_3$

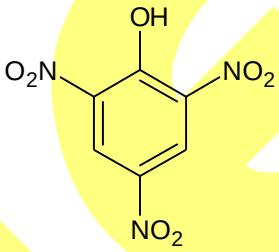
(a) 

(b) $CH_3CH_2NH_2$

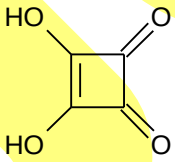
(c) 

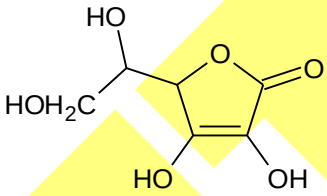
(d) CH_3COOH

(e) 

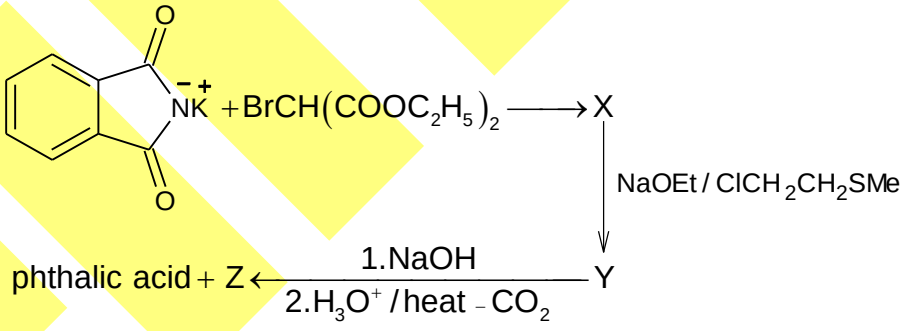
(f) 

(g) HCOOH

(h) 

(i) 

(j) CH_3CONH_2

8. 

A = $\frac{1}{4}$[Sum of all atoms present in Z] + no of optically active isomers of Z. Value of A is

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SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. Which of the following statement(s) is (are) correct?

(A) If $\left|\vec{v}\times\vec{w}\right|=0$ for all vector $\vec{w}$, then $\vec{v}=\vec{0}$

(B) If $\vec{a}+\vec{b}+\sqrt{3}\vec{c}$ is a null vector where $\vec{a}$ and $\vec{b}$ are unit vectors such that angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then the magnitude of $\vec{c}$ is 1.

(C) If $\left(\vec{u}+\vec{v}\right).\left(\vec{u}-\vec{v}\right)=0$, then $\vec{u}=\vec{v}$

(D) If $\vec{a}$ and $\vec{b}$ are two unit vectors then the maximum value of $\left|2\vec{a}+\vec{b}\right|$ is equal to 3.

2. Let $L_1:\frac{x+3}{-4}=-\frac{y-6}{3}=\frac{z}{2}$ and $L_2:\frac{x-2}{-4}=\frac{y+1}{1}=\frac{z-6}{1}$ be two lines in R^3.

Which of the following statement(s) is (are) correct?

(A) L_1, L_2 are coplanar.

(B) L_1, L_2 are skew lines

(C) Shortest distance between L_1 and L_2 is 9.

(D) $\left(\hat{i}-4\hat{j}+8\hat{k}\right)$ is a vector perpendicular to both L_1 and L_2.

3. Which of the following statement(s) is (are) correct?

(A) If $f(x)$ is differentiable $\forall x\in[0,1]$, then $f'(x)$ must be bounded in $[0,1]$

(B) There exist a bijective function $f:[0,1]\to[0,1]$ which is not continuous

(C) Let $f:[-1,2]\to R$ be defined as $f(x)=x^2-\cos\pi x+4$, then $f(c)=2\pi$ for some $c\in(-1,2)$

(D) If $f(x)=3+x+e^x$, then $\left(f^{-1}\right)'(4)$ is equal to $\frac{1}{2}$.

4. Let $f(x)$ be twice differentiable function such that $f''(x)<0$ in $[0,2]$. Then

(A) $f(0)+f(2)=2f(c),\ 0<c<2$ (B) $f(0)+f(2)=2f(1)$

(C) $f(0)+f(2)>2f(1)$ (D) $f(0)+f(2)<2f(1)$

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5. If the tangent at a point P_1 (other than (0, 0)) on the curve $ax^3 - y + b = 0$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3 and so on. If the abscissae of $P_1, P_2, P_3, \ldots\ldots P_n$ form a G.P. then (a, b) may be

(A) (1, 0) (B) (2, 7)
(C) (3, 5) (D) (4, 9)

6. If three planes $P_1 = 2x + y + z - 1 = 0, P_2 = x - y + z - 2 = 0$ and $P_3 = \alpha x - y + 3z - 5 = 0$ intersects each other at point P on XOY plane and at point Q on YOZ plane, where O is origin then identify the correct statement(s)?

(A) The value of α is 4

(B) Straight line perpendicular to plane P_3 and passing through P is $\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z}{3}$.

(C) The length of projection of $\overrightarrow{PQ}$ on x – axis is 1

(D) Centroid of the triangle OPQ is $\left(\frac{1}{3}, \frac{-1}{2}, \frac{1}{2}\right)$

7. Which of the following statement is(are) true?

(A) From any point on the directrix of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$, a pair of tangents are drawn to the auxiliary circle of the ellipse. The chord of contact will pass through the corresponding focus of the ellipse.

(B) If two tangents to a parabola $y^2 = 4ax$ intersect on the line $x = a$ then their chord of contact always passes through the (–a, 0).

(C) If $P(x, y)$ is such that it moves on a hyperbola $\left|\sqrt{(x-3)^2 + (y-4)^2} - \sqrt{x^2 + y^2}\right| = k^2 + 1$, then number of possible integral values of k is equal to 3.

(D) A circle passing through 3 co – normal points on the parabola $y^2 = 4x$ also passes through (1, 0).

(Single Correct Choice Type)

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. Find the locus of the centroid of an equilateral triangle inscribed in the given parabola $x^2 = 4ay$.

(A) $9y^2 = 4a(x - 8a)$ (B) $9x^2 = 4a(y - 6a)$
(C) $9x^2 = 4a(y - 8a)$ (D) $9x^2 = 4a(y + 8a)$

Space For Rough Work

9. A line through the origin divides parallelogram with vertices (10, 45), (10, 114), (28, 153) and (28, 84) into two congruent pieces. The slope of the line is

(A) $\frac{19}{99}$ (B) $\frac{99}{19}$

(C) $\frac{9}{19}$ (D) $\frac{19}{9}$

10. Equation of chord AB of circle $x^2+y^2=2$ passing through P (2, 2) such that $\frac{PB}{PA}=3$, is given by

(A) $x=3y$ (B) $x=y$

(C) $y-2=\sqrt{3}(x-2)$ (D) none of these

11. The number of ways in which 3 children can distribute 10 tickets out of 15 consecutively numbered tickets among themselves such that they get consecutive blocks of 5, 3 and 2 tickets is

(A) ${}^8C_5$ (B) ${}^5C_3 3!$

(C) ${}^8C_5 (3!)^2$ (D) none of these

12. The number of ordered triplets of positive a, b, c satisfying $a+b+c=24, a^2+b^2+c^2=210$ and $abc=440$ is

(A) 3 (B) 6

(C) 9 (D) 12

13. If $\int_0^{\sin^{-1}\frac{\pi}{4}} (\pi-4\sin\theta)\sin 2\theta \ln\left(1+\tan(\sin\theta)\right)d\theta$ is k, then $\frac{960k}{\pi^3 \ln 2}$ is equal to

(A) 11 (B) 10

(C) 12 (D) none of these

PART – B
(Numerical based)

This section contains **8 Numerical based questions**, the answer of which maybe positive or negative numbers or decimals to **two decimal places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)

1. A conical vessel is to be prepared out of a circular sheet of metal of unit radius. In order that the vessel has maximum volume, the sectorial area that must be removed from the sheet is A_1 and the area of the given sheet is A_2. Find $\frac{A_2}{A_1}$.

Space For Rough Work

2. Let $A=\{3,4,5,6\}$ and $B=\{1,2,3,...,10\}$ be two sets. Let a function f be defined from set A to set B such that $f(i)-f(j)\geq 2 \ \forall\ i>j$, where $i, j \in A$. If the number of such functions is N, then the value of N is equal to

3. If $a=1+\frac{x^3}{3!}+\frac{x^6}{6!}+.....\infty$, $b=x+\frac{x^4}{4!}+\frac{x^7}{7!}+.....\infty$, $c=\frac{x^2}{2!}+\frac{x^5}{5!}+\frac{x^8}{8!}+.....\infty$. Find the value of $a^3+b^3+c^3-3abc=$ _____

4. Four people sit round a circular table, and each person will roll a normal six sided die once. The probability that no two people sitting next to each other will roll the same number is $\frac{N}{1296}$. Then the value of N is

5. Let two non – collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=4$. A point P moves so that any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP}=(e^t+e^{-t})\vec{a}+(e^t-e^{-t})\vec{b}$. If the least distance of P from origin is $\sqrt{2}\sqrt{\sqrt{a}-b}$ where $a,b\in N$ then find the value of (a + b).

6. Lot A consists of 6 good and 4 defective articles. Lot B consists of 7 good and 3 defective articles. A new lot C is formed by taking 6 articles from lot A and 4 articles from lot B. The probability that an articles chosen at random from the lot C is defective, is $\frac{k}{25}$ then k is

7. If the dependent variable y is changed to 'z' by the substitution $y=\tan z$ and the differential equation $\frac{d^2y}{dx^2}=1+\frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $\frac{d^2z}{dx^2}=\cos^2 z+k\left(\frac{dz}{dx}\right)^2$, then the value of $(305.33)\times k$ equals

8. Rectangle ABCD has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Let perimeter of the rectangle ABCD is P, then 103.44P is

Space For Rough Work

QP Code: 100961

Answers

SECTION-1 : PHYSICS

PART – A

1.	ABCD	2.	BC	3.	B	4.	D
5.	C	6.	AC	7.	BC	8.	C
9.	A	10.	B	11.	A	12.	D
13.	D						

PART – B

1.	9	2.	1.67	3.	1	4.	0.39
5.	0.24	6.	6.28	7.	31.43	8.	4.04

SECTION-2 : CHEMISTRY

PART – A

1.	ABC	2.	AB	3.	BCD	4.	ABCD
5.	ABCD	6.	ABC	7.	ABCD	8.	C
9.	A	10.	D	11.	D	12.	C
13.	A						

PART – B

1.	5	2.	6	3.	2.32	4.	3
5.	9	6.	8	7.	6	8.	7

SECTION-3 : MATHEMATICS

PART – A

1.	ABD	2.	BCD	3.	BCD	4.	AD
5.	ABCD	6.	ABCD	7.	ABC	8.	C
9.	B	10.	B	11.	B	12.	B
13.	B						

PART – B

1.	5.45	2.	35.00	3.	1.00	4.	630.00
5.	467.00	6.	9	7.	610.66	8.	8275.20

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **ABCD**

Sol. Since the graph is like a parabola

$\therefore$ let $x(t) = At + Bt^2 + C$

From graph $x(0) = 0 \Rightarrow C = 0$

$$x(t) = Bt^2 + At$$

$$x(4) = 0 \Rightarrow 16B + 4A = 0$$

$$\left(\frac{dx}{dt}\right)_0 = 1 \Rightarrow (A + 2Bt)_{t=0} = 1$$

Put in (i), we get $B = -\frac{1}{4}$

$$x = t - \frac{t^2}{4}$$

max x coordinate = 1 (from max and min)

$\rightarrow$ Since motion is a straight line motion

$\rightarrow$ total distance travelled = 2 x 1 = 2 min

Average speed = $\frac{2}{4}$ = 0.5 m/sec

2. **BC**

Sol. I f – 256 | = 2.

3. **B**

Sol. Number of nuclei became $\frac{1}{8^{th}} \Rightarrow$ 3 half lives passed

$$\Rightarrow \quad 3t_{1/2} = 3\,\text{min} \Rightarrow t_{1/2} = 1\,\text{min}$$

$$\therefore \quad t_{mean} = \frac{t_{1/2}}{\ln 2} = \frac{1}{0.693}\,\text{min} \approx 1.44\,\text{min}$$

4. **D**

Sol. $\frac{2T}{r} = 150$ Pa

5. **C**

Sol. For critical stability, the centre of equivalent force should be at point C. So BC = $\frac{AB}{3}$ = 1.5m

6. **AC**

Sol. $\frac{1}{f} = (\mu_r - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

7. **BC**

Sol. $\lambda_0 = \frac{hc}{eV}$

8. **C**
Sol. Use Kirchoff's loop law & Kirchhoff's junction law to make equations.

9. **A**
Sol. Voltage across resister = current × resistance.

10. **B**
Sol. $F=\frac{dp}{dt}=\left(\sqrt{2}\,V\right)\frac{dm}{dt}=\left(\sqrt{2}\,V\rho\right)\frac{d(\text{volume})}{dt}=\left(\sqrt{2}\,V\rho\right)(AV)$

11. **A**
Sol. $\frac{1}{c}=\int_0^d\frac{dx}{\varepsilon_0 S\frac{\varepsilon_1+\varepsilon_2-\varepsilon_{xy}}{d}}$

12. **D**
Sol. $V_0=\frac{MV\cos\left(\frac{vt}{R}\right)}{M+m}$

13. **D**
Sol. Apply Snell's law and use geometry.

PART – B

1. **9**
Sol. $\frac{\ell_2}{4}=\frac{\ell_3}{9}$

2. **1.67**
Sol. about ICR

$\omega\times 3=2\Rightarrow\omega=\frac{2}{3}$

$V_{cm}=\omega\times\frac{5}{2}=\frac{2}{3}\times\frac{5}{2}=\frac{5}{3}\text{m/s}$ = 1.67 m/s.

3. **1**
Sol. $F_b+m_1g+T=2m_1g$

$F_b+T=m_1g$

$F_b+m_2g=T+2m_2g$

$F_b-T=m_2g$

$\frac{F_b+T}{F_b-T}=\frac{m_1g}{m_2g}=\frac{\rho_1}{\rho_2}=\frac{1.6}{0.8}$

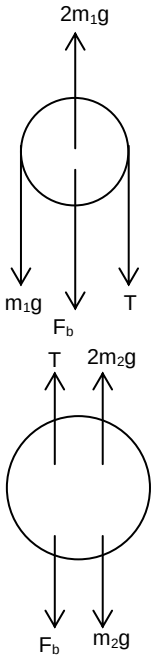
$F_b+T=2F_b-2T$

$3T=F_b$

$T=\frac{F_b}{3}=\frac{V\rho g}{3}$

$\rho=\frac{\rho_1+\rho_2}{2}=\frac{1.6\rho_\omega+0.8\rho_\omega}{2}=1.2\rho_\omega$

T = 1N



4. **0.39**

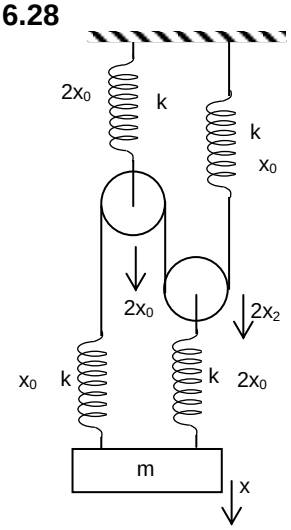
Sol. $\frac{2}{1+1+1^2} = 2\,\text{Sin}(60°)$

5. **0.24**

Sol. $\frac{\mu_w}{\mu_{air}} = \frac{340}{1440}$

6. **6.28**

Sol.



$2 \times 2x_0 + x_0 + x_0 = 2x_2 + x$

$6x_0 = 2x_2 + x$

$x - x_2 = 2x_0$

$3x = 10x_0$

$x_0 = \frac{3x}{10}$

$kx_0 + k2x_0 = k_{eq}x$

$3kx_0 = k_{eq}x$

$3k\frac{3x}{10} = k_{eq}x$

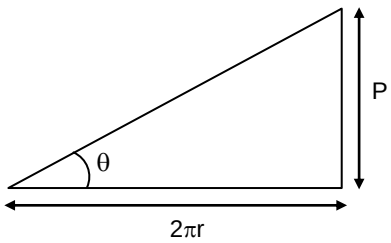
$k_{eq} = \frac{9k}{10}$

$T = 2\pi\sqrt{\frac{10m}{9k}}$

7. **31.43**

Sol. $\tan\theta = \frac{\rho}{2\pi r} = \frac{g}{r\alpha}$

$\Rightarrow \quad \alpha = \frac{2\pi g}{\rho}$



8. **4.04**

Sol. $q_{max} = 7\sqrt{\frac{CL}{3}} = \frac{7}{\sqrt{3}}\text{mA}$

SECTION-2 : CHEMISTRY

PART – A

1. **ABC**

Sol. C2-epimer and enantiomer will give osazone having same melting point.

2. **AB**

Sol. van der Waal's forces and π - π stacking.

3. **BCD**

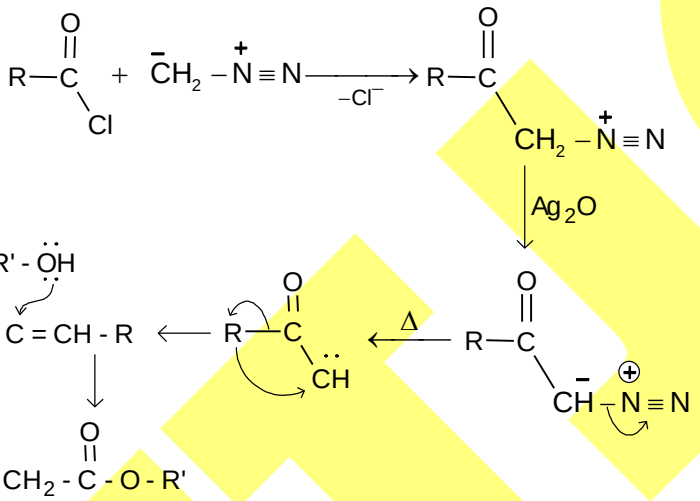
Sol. With phenol and phenol derivatives coupling occurs in weakly basic medium whereas with aniline and its derivatives it requires weakly acidic medium.

4. **ABCD**

Sol. Fact based.

5. **ABCD**

Sol.



6. **ABC**

Sol. $B_2H_6 + NH_3\,(excess) \xrightarrow{\text{low temp}} \left[H_3N \longrightarrow BH_2 \longleftarrow NH_3\right]^+ \left[BH_4\right]^-$

7. **ABCD**

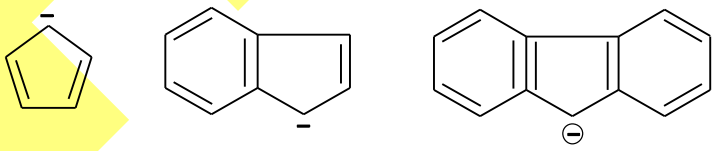
Sol.

8. **C**

Sol. Metal in high oxidation state will show efficient LMCT.

9. **A**

Sol.



Net aromatic stabilisation decreases if an aromatic ring shares aromatic character with an another ring.

10. **D**

Sol. $\left[M(CO)_2\left(\sigma\bar{C}_2H_5\right)\left(\pi\bar{C}_2H_5\right)\right]$

$2CO \longrightarrow 4e^-$

$\sigma C_5H_5^- \longrightarrow 2e^-$

$\pi C_5H_5^- \longrightarrow 6e^-$

This means M is in +2 oxidation state. We need 6 more electron to have 18 electrons Fe^{2+}

$$\frac{6e^-}{18e^-} = Fe(26)$$

11. **D**

Sol. Let metal are M and N

$MCO_3 \xrightarrow{\Delta} MO + CO_2$

$NCO_3 \longrightarrow NO + CO_2$

Carry out the required calculation

12. **C**

Sol. $\Delta_t = \frac{4}{3}\Delta_o$

$\Rightarrow \Delta_t = 8000 \times \frac{4}{9} cm^{-1}$

$\Delta_t = 8000\, cm^{-1}$

13. **A**

Sol.

	R	→	3P
t = 0	a		0
t = 5	a – x		3x

$$K = \frac{2.303}{t}\log\frac{[A]_o}{[A]}$$

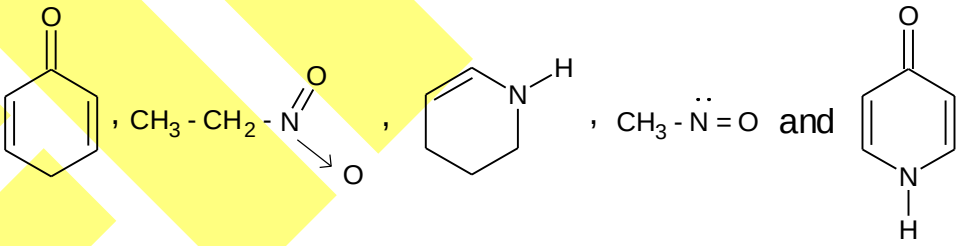
Now do the required calculation

a – x = 3x

x = a/4

PART – B

1. **5**

Sol. 

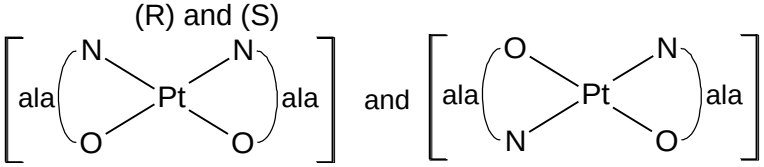
These are the molecules which can show tautomerism.

2. 6

Sol. $PtCl_2 + H_2N-CH(CH_3)-COOH \longrightarrow [Pt(ala)_2]$

(racemic) ±

(R) and (S)



3. **2.32**

Sol. 20 g H_2SO_4 in 100 g of solution

Volume = $\frac{100}{1.14}$ mL

$M = \frac{n_{H_2SO_4}}{V(l)}$

4. 3

Sol. XeO_4, SiO_2, CH_3^-

5. 9

Sol. All are correct (Fact based)

6. 8

Sol. $K_P = \frac{10^{-3}}{10^{-1}} = 10^{-2}$

ΔG^o = -2.303 RTlogK_P

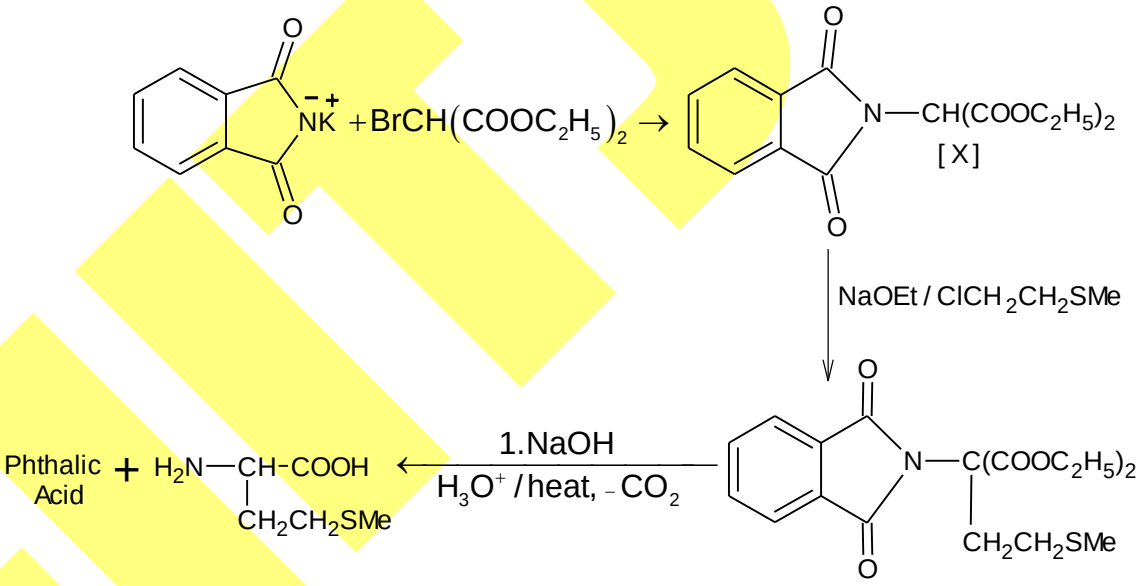
= -2.303 RTlog10^{-2} = (-4)(2 log10) = 8

7. **6**

Sol. Fact based

8. **7**

Sol.



SECTION-3 : MATHEMATICS

PART – A

1. ABD

Sol.

(A) If $|\vec{v}\times\vec{w}|=0\ \forall\vec{w}$ the $\vec{v}=\vec{0}$, otherwise the above relation is not possible $\Rightarrow$ True.

(B) Given, $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}=\vec{a}+\vec{b}=-\sqrt{3}\vec{c}$

$\therefore\ |\vec{a}+\vec{b}|^2=3|\vec{c}|^2\Rightarrow 2+2\cos\frac{\pi}{3}=3|\vec{c}|^2$

$\Rightarrow|\vec{c}|^2=1\Rightarrow|\vec{c}|=1\Rightarrow$ True.

(C) If $(\vec{u}+\vec{v}).(\vec{u}-\vec{v})=0\Rightarrow|\vec{u}|^2-|\vec{v}|^2=0\Rightarrow|\vec{u}|=|\vec{v}|$

But it does not imply that $\vec{u}=\vec{v}$.

e.g. Let $\vec{u}=\hat{i}$ and $\vec{v}=\hat{j}$

Now, $(\hat{i}+\hat{j}).(\hat{i}-\hat{j})=0$

But $\hat{i}\neq\hat{j}\Rightarrow$ False.

(D) Given, $|\vec{a}|=1=|\vec{b}|$

Let $\theta=\vec{a}\wedge\vec{b}$ where $\theta\in[0,\pi]$

Now, $|2\vec{a}+\vec{b}|^2=4|\vec{a}|^2+|\vec{b}|^2+4\vec{a}.\vec{b}=5+4(1)(1)\cos\theta=5+4\cos\theta$

$\therefore\quad|2\vec{a}+\vec{b}|^2_{max}=9$, when $\cos\theta=1$ i.e., $\theta=0^o$

$\Rightarrow$ Maximum value of $|2\vec{a}+\vec{b}|=3$.

2. BCD

Sol. Shortest distance $=\left|\frac{(\vec{a}_1-\vec{a}_2).(\vec{b}_1-\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

where, $\vec{a}_1=-3\hat{i}+6\hat{j},\ \vec{a}_2=2\hat{i}-\hat{j}+6\hat{k}$

$\vec{b}_1=-4\hat{i}+3\hat{j}+2\hat{k},\ \vec{b}_2=-4\hat{i}+\hat{j}+\hat{k}$

$\therefore$ S.D. $=\left|\frac{(-5\hat{i}+7\hat{j}-6\hat{k}).(\hat{i}-4\hat{j}+8\hat{k})}{|\hat{i}-4\hat{j}+8\hat{k}|}\right|=9$

Now, verify alternatives.

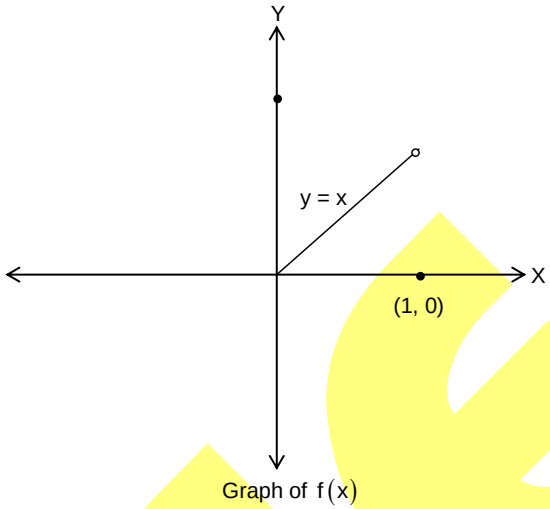
3. BCD

Sol.

(A) False, $f(x)=\begin{cases}x^2\sin\frac{1}{x^2} & x\neq0\\ 0 & x=0\end{cases}$

(B) This statement is true.

For example: $f(x)=\begin{cases} x & 0<x<1 \\ 1 & x=0 \\ 0 & x=1 \end{cases}$



Graph of $f(x)$

(C) $f(x)=x^2-\cos\pi x+4$

Now $f(-1)=1-(-1)+4=6$

and $f(2)=4-1+4=7$

As f is continuous $[-1, 2]$, so $f(c)=2\pi$, for some $c\in(-1, 2)$

(By using intermediate value theorem)

(D) $\left(f^{-1}\right)'(4)=\dfrac{1}{f'(0)}=\dfrac{1}{1+e^x}\Big|_{x=0}=\dfrac{1}{2}$.

4. AD

Sol. By Intermediate value property

$\dfrac{f(0)+f(2)}{2}=f(c),\ 0<c<2$

By mean value theorem,

$f(1)-f(0)=f'(c_1),\quad 0<c_1<1$

$f(2)-f(1)=f'(c_2),\quad 1<c_2<2$

By subtraction

$f(0)+f(2)-2f(1)=f'(c_2)-f'(c_1)$

$=(c_2-c_1)f''(c), c_1<c<c_2 \Rightarrow f(0)+f(2)-2f(1)<0$

$\Rightarrow\quad f(0)+f(2)<2f(1)$

5. ABCD

Sol. Given curve is $ax^3-y+b=0 \qquad \Rightarrow y=ax^3+b$

Let point P_1 be $\left(t_1, at_1^3+b\right)$

Slope of tangent $=\dfrac{dy}{dx}\Big|_{P_1}=3at_1^2$

$\therefore$ Equation of tangent is $y-\left(at_1^3+b\right)=3at_1^2(x-t_1)$

$\therefore$ Tangent meets curve at $P_2\left(t_2, at_2^3+b\right)$

$\therefore\ \left(at_2^3+b\right)-\left(at_1^3+b\right)=3at_1^2(t_2-t_1)$

$\Rightarrow a\left(t_2^3-t_1^3\right)=3at_1^2(t_2-t_1) \qquad \Rightarrow \qquad t_2^2+t_1^2+t_2t_1=3t_1^2 \qquad (\because t_1\neq t_2)$

$\Rightarrow t_2^2-t_2t_1-2t_1^2=0 \qquad \Rightarrow \qquad (t_2+2t_1)(t_2-t_1)=0$

$\Rightarrow t_2=-2t_1$

Similarly, $t_3=-2t_2$

$\therefore$ abscissae are in G.P. for all values of a and b.

6. ABCD

Sol. There planes meet at two points it means they have infinitely many solutions, so

So, $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ \alpha & -1 & 3 \end{vmatrix} = 0 \quad \Rightarrow \quad 2(-3+1)-1(3+1)+\alpha(1+1)=0 \Rightarrow \alpha = 4$

$P_1 : 2x + y + z = 1$

$P_2 : x - y + z = 2$

$P_3 : 4x - y + 3z = 5$

P on XOY plane $= (1, -1, 0)$ (which can be obtained by putting z = 0 in any two of the given planes).

Q on YOZ plane $= \left(0, \frac{-1}{2}, \frac{3}{2}\right)$ (which can be obtained by putting x = 0 in any two of the given planes)

$\therefore$ Straight line perpendicular to plane P_3 passing through P is –

$\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z}{3}$

$\overrightarrow{PQ} = \hat{i} - \frac{1}{2}\hat{j} - \frac{3}{2}\hat{k}$

Projection of $\overrightarrow{PQ}$ on x – axis $\Rightarrow \left|\frac{\overrightarrow{OP}.\hat{i}}{|\hat{i}|}\right| = 1$

Centroid of ΔOPQ is $\left(\frac{1}{3}, \frac{-1}{2}, \frac{1}{2}\right)$

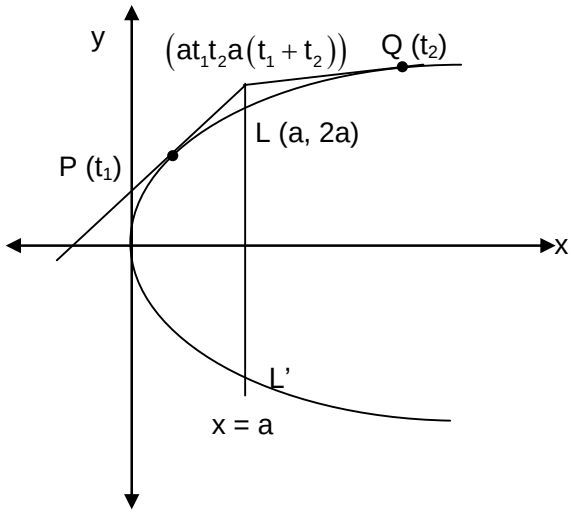
7. ABC

Sol.

(A) Let the point be $\left(\frac{a}{e}, \beta\right)$.

Chord of contact is T = 0

i.e., $x\left(\frac{a}{e}\right) + y\beta = a^2$, which is passing through (ae, 0)



(B) $T(at_1t_2, a(t_1+t_2)) \Rightarrow at_1t_2a \Rightarrow t_1t_2 = 1$

[Hence, equation of PQ is $2x - (t_1+t_2)y + 2a = 0$

$(x+a) - \lambda y = 0$]

$\Rightarrow$ chord PQ passes (–a, 0) which is foot of directrix

$\therefore$ (B) is also true.

(C) As $|PS_1 - PS_2| = \text{constant}$

So, for hyperbola

$0 < k^2 + 1 < 5 \quad \Rightarrow k^2 < 4 \quad \Rightarrow -2 < k < 2$

So, number of integral values of k are 3 (i.e. k = –1, 0, 1)

8. C

Sol. Let equation of circumcircle of triangle ABC is

$x^2+y^2+2gx+2fy+c=0$ .....(1)

Any point on the parabola is of the form $\left(2at, at^2\right)$,

Solving with (1),

$a^2t^4+2t^2a(f+2a)+4gat+c=0$.

$\sum t_1=0,\ \sum t_1t_2=\frac{2}{a}(f+2a),\ \sum t_1t_2t_3=\frac{-4g}{a},\ t_1t_2t_3t_4=\frac{c}{a^2}$

Since centriod and circumcentre coincides so

$-g=\frac{2a\left(t_1+t_2+t_3\right)}{3}=-\frac{2a}{3}t_4=\frac{3g}{2a}$ .....(2)

$-f=\frac{a}{3}\left(t_1^2+t_2^2+t_3^2\right)=\frac{a}{3}\left(t_1^2+t_2^2+t_3^2+t_4^2-t_4^2\right)=\frac{a}{3}\left(\left(\sum t_1\right)^2-2\sum t_1t_2-t_4^2\right)$

$\Rightarrow -f=\frac{a}{3}\left(0-\frac{4}{a}(f+2a)-t_4^2\right)$

Putting the value of t_4 from (2),

The required locus is $9x^2=4a(y-8a)$.

9. B

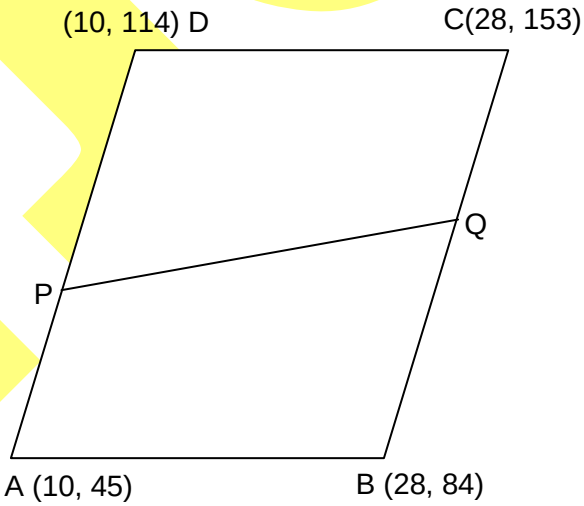
Sol. Let $AP=CQ=x$

$\Rightarrow \frac{45+x}{10}=\frac{153-x}{28}$

$\Rightarrow 1260+28x=1530-10x$

$\Rightarrow x=\frac{135}{19}$

$\Rightarrow$ slope of $PQ=\frac{45+\frac{135}{19}}{10}=\frac{99}{19}$.



10. B

Sol. Any line passing through (2, 2) will be of the form $\frac{y-2}{\sin\theta}=\frac{x-2}{\cos\theta}=r$

When this line cuts the circle $x^2+y^2=2$, $(r\cos\theta+2)^2+(r\sin\theta+2)^2=2$

$\Rightarrow r^2+4(\sin\theta+\cos\theta)r+6=0$

$\frac{PB}{PA}=\frac{r_2}{r_1}$, now if $r_1=\theta$, $r_2=3\alpha$,

then $4\alpha=-4(\sin\theta+\cos\theta), 3\alpha^2=6\Rightarrow\sin2\theta=1\Rightarrow\theta=\frac{r}{4}$.

So required chord will be $y-2=1(x-2)\Rightarrow y=x$.

11. B

Sol. Problem is same as arranging 8 things out of which 5 identical i.e. $\frac{8!}{5!}$ which gives total number of ways of selecting block and distributing them away 3 children i.e. $\frac{8!}{5!}3!$.

12. B

Sol. $ab+bc+ca=\frac{1}{2}\left(\left(a+b+c\right)^2-\left(a^2+b^2+c^2\right)\right)=183$

$\Rightarrow a,b,c$ are the roots of $x^3-24x^2+183x-440=0$

but it factorises as $(x-5)(x-8)(x-11)=0$

$\Rightarrow$ there are $3!$ solutions.

13. B

Sol. Let $I=\int_0^{\sin^{-1}\frac{\pi}{4}}(\pi-4\sin\theta)\sin2\theta\ln\left(1+\tan\left(\sin\theta\right)\right)d\theta$

put $\sin\theta=t,\cos\theta\,d\theta=dt$

$I=2\int_0^{\pi/4}(\pi-4t)t\ln(1+\tan t)dt$

$=2\int_0^{\pi/4}\left(\pi-4\left(\frac{\pi}{4}-t\right)\right)\left(\frac{\pi}{4}-t\right)\ln\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt$

$=2\int_0^{\pi/4}t(\pi-4t)\ln2dt-\int_0^{\pi/4}t(\pi-4t)\ln(1+\tan t)dt$

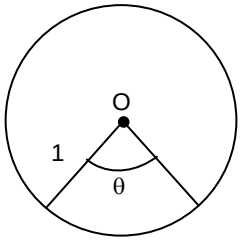
$I=\ln2\left[\frac{\pi}{2}.\frac{\pi^2}{16}-\frac{4}{3}\times\frac{\pi^3}{64}\right]=\frac{\pi^3}{32}\ln2\left[1-\frac{2}{3}\right]=\frac{\pi^3\ln2}{96}=k$

$\frac{960k}{\pi^3\ln2}=10.$

PART – B

1. 5.45

Sol. $\frac{A_2}{A_1}=\frac{\pi r^2}{\left(\frac{2\pi-\theta}{2\pi}\right).\pi r^2}=\frac{2\pi}{2\pi-\theta}$



$V=\frac{\pi}{3}\left(\frac{\theta}{2\pi}\right)^2\sqrt{1^2-\left(\frac{\theta}{2\pi}\right)^2}$

$\frac{dV}{d\theta}=0\Rightarrow\theta=\sqrt{\frac{8}{3}}\pi$

$\frac{A_2}{A_1}=\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}=3+\sqrt{6}$

2. 35.00

Sol. Let x_1 be the number of elements in B before $f(3)$, x_2 the number of elements between $f(3)$ and $f(4)$, x_3 the number of elements between $f(4)$ and $f(5)$, x_4 the number of elements between $f(5)$ and $f(6)$, and x_5 the number of elements after $f(6)$.

According to the conditions of the problem, $x_2, x_3, x_4 \geq 1$ and $x_1, x_5 \geq 0$.

$x_1+x_2+x_3+x_4+x_5=6$

$N = {}^{7}C_4$

3. 1.00

Sol. It is obvious

$\frac{da}{dx} = c, \frac{db}{dx} = a, \frac{dc}{dx} = b$

Let $f(x) = a^3 + b^3 + c^3 - 3abc$

Differentiate with respect to x on both sides

$f'(x) = 3a^2\frac{da}{dx} + 3b^2\frac{db}{dx} + 3c\frac{dc}{dx} - 3\left(\frac{da}{dx}bc + \frac{db}{dx}ac + \frac{dc}{dx}ab\right)$

$= 3a^2c + 3b^2a + 3c^2b - 3\left(bc^2 + ca^2 + ab^2\right)$

$f'(x) = 0 \Rightarrow f(x) = \lambda$ for every x

Now, $x = 0 \Rightarrow f(0) = 1 + 0 + 0 - 0 = 1$

$f(x) = 1$

4. 630.00

Sol. $n(S) = 6.6.6.6 = 6^4 = 1296$

Now A can throw die in 6 ways
B can throw only in 5 ways
D can also throw in 5 ways
Suppose C throws the same number as that of A
Hence 'C' can throw only in 1 ways.
$\therefore$ Number of ways = 6 . 5 . 5 . 1 = 150
Suppose C throws different number than A
Then A can throw in 6 ways
B can throw in 5 ways
Now C will throw different from A and B hence C can throw in 4 ways
Now D can not throws what A and C have thrown and therefore can throw in 4 ways.
$\therefore$ Number of ways = 6 . 5 . 4 . 4 = 480
Total ways = 630

5. 467.00

Sol. We have $\left(\overrightarrow{OP}\right)^2 = \left(e^t + e^{-t}\right)^2\left(\vec{a}\right)^2 + \left(e^t - e^{-t}\right)^2 + 2\left(e^t + e^{-t}\right)\left(e^t - e^{-t}\right)\left(\vec{a}.\vec{b}\right)$

$\left(\left(\vec{a}\right)^2 = \left|\vec{a}\right|^2 = 9, \left(\vec{b}\right)^2 = \left|\vec{b}\right|^2 = 16 \text{ and } \vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\frac{2\pi}{3}\right)$

$\Rightarrow \left|\overrightarrow{OP}\right|^2 = 9\left(e^t + e^{-t}\right)^2 + 16\left(e^t - e^{-t}\right)^2 + 2\left(e^{2t} - e^{-2t}\right).3.4.\left(-\frac{1}{2}\right) = 13e^{2t} + 37e^{-2t} - 14$

$\left|\overrightarrow{OP}\right| = \sqrt{13e^{2t} + 37e^{-2t} - 14}$

$\left|\overrightarrow{OP}\right|_{min} = \sqrt{2\sqrt{13 \times 37} - 14}$

$= \sqrt{2\sqrt{481} - 14}$

$a = 481, b = 14$

$a - b = 467$

6. 9

Sol. Apply Total Probability

7. 610.66

Sol. Given $y = \tan z$

$$\frac{dy}{dx} = \sec^2 z . \frac{dz}{dx} \qquad (1)$$

Now $\frac{d^2y}{dx^2} = \sec^2 z . \frac{d^2z}{dx^2} + \frac{dz}{dx} . \frac{d}{dx}\left(\sec^2 z\right)$ [Using product rule]

$$= \sec^2 z . \frac{d^2z}{dx^2} + \frac{dz}{dx} . \frac{d}{dz}\left(\sec^2 z\right)\frac{dz}{dx}$$

$$\frac{d^2y}{dx^2} = \sec^2 z . \frac{d^2z}{dx^2} + \left(\frac{dz}{dx}\right)^2 . 2\sec^2 z . \tan z \qquad (2)$$

Now $1 + \frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$

$$= 1 + \frac{2(1+\tan z)}{\sec^2 z} . \sec^4 z . \left(\frac{dz}{dx}\right)^2$$

$$= 1 + 2(1+\tan z) . \sec^2 z . \left(\frac{dz}{dx}\right)^2$$

$$= 1 + 2\sec^2 z \left(\frac{dz}{dx}\right)^2 + 2\tan z . \sec^2 z \left(\frac{dz}{dx}\right)^2 \qquad (3)$$

From (2) and (3), we have RHS of (2) = RHS of (3)

$$\sec^2 z . \frac{d^2z}{dx^2} = 1 + 2\sec^2 z \left(\frac{dz}{dx}\right)^2$$

$$\Rightarrow \quad \frac{d^2z}{dx^2} = \cos^2 z + 2\left(\frac{dz}{dx}\right)^2$$

$$\Rightarrow \quad k = 2 \text{ so } (305.33) \times k = 610.66$$

8. 8275.20

Sol. $(AB)(AD) = 200$

$AB + AD = 2a$

$BD^2 = AB^2 + AD^2 = (2ae)^2$

$(AB + AD)^2 = AB^2 + AD^2 + 2(AB)(AD)$

$\Rightarrow 4a^2 = 4a^2e^2 + 400$

$\Rightarrow b = 10$

$\Rightarrow a^2(1 - e^2) = b^2 = 100$

$\Rightarrow a = 20$

$P = 2(AB + AD) = 4a = 80$

Or $103.44P = 8275.20$