ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
UNIT TEST # 01
07-07-2024

# JEE(Main) : LEADER TEST SERIES / JOINT PACKAGE COURSE

## ANSWER KEY

### PART-1 : PHYSICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | B | C | B | B | D | C | B | C | B | D |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | A | D | C | C | B | A | C | A | B | B |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 2 | 12 | 9 | -10 | 40 | 13 | 25 | 20 | 0 | 7 |

### PART-2 : CHEMISTRY

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | D | C | C | C | B | D | C | C | A | A |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | B | A | B | B | B | A | C | D | B | A |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 5 | 350 | 300 | 6 | 50 | 100 | 200 | 50 | 146 | 8 |

### PART-3 : MATHEMATICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-I | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | C | A | B | C | D | B | B | A | A | C |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | C | C | B | B | A | B | C | B | D | B |
| SECTION-II | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 6 | 1 | 7 | 6 | 753 | 8 | 3 | 1 | 79 | -1 |

## HINT – SHEET

## PART-1 : PHYSICS

### SECTION-I

**1. Ans ( B )**

$m_e \times m_0 = 35$

$m_e \times 7 = 35$

$m_e = 5$

**2. Ans ( C )**

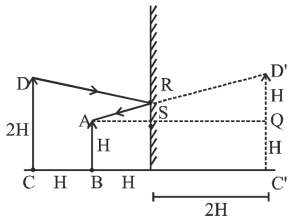
$V_I = -m^2 \times V_0$

$m = \frac{f}{f-u} = \frac{-24}{-24-(-60)} = \frac{-24}{36} = \frac{-2}{3}$

$V_I = -\left(\frac{-2}{3}\right)^2 \times 9$ ;

$V_I = -4$ cm/sec (toward the mirror)

**3. Ans ( B )**




In $\Delta$ AD'Q & $\Delta$ ARS $\frac{RS}{D'Q} = \frac{H}{3H}$ $RS = \frac{1}{3}H$

To sec his own(child) image & father image, minimum length of mirror required is $\frac{H}{2} + \frac{H}{3}$ i.e. $\frac{5H}{6}$

**4. Ans ( B )**

$\delta_1 + \delta_2 + \delta_3 = 30 + 60 - 30 = 60°$

**6. Ans ( C )**

Mirror shifting x = 6 = 2cm

$\because$ OA' = IA' = 32 cm

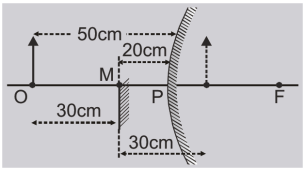
$\therefore$ Image from actual mirror IA = 30 cm

**7. Ans ( B )**

It is clear that virtual image in plane mirror is 30 cm behind it and there is no parallax so images formed by two mirrors will coincide and u = – 50 cm the distance of image formed by plane mirror from convex mirror

v = PI = MI – MP = MO – MP = 30 – 20 = 10 cm [MI = MO]

Since this image coincides with image formed by convex mirror, so for

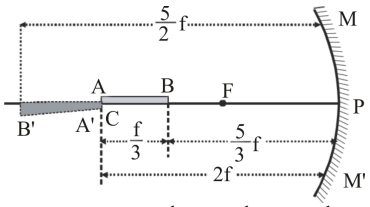



convex mirror $\frac{1}{+10}+\frac{1}{-50}=\frac{1}{f}$

$\Rightarrow f=\frac{50}{4}=12.5$ cm so R = 2f = 25 cm

**9. Ans ( B )**

Image is real and enlarged, the object must be between C and F. One end A' of the image coincides with the end A of rod itself.




So $v_A=u_A$, $\frac{1}{v_A}+\frac{1}{v_A}=\frac{1}{-f}$ i.e., $v_A=u_A=-2f$ so it clear that the end A is at C. Q the length of the rod is $\frac{f}{3}$

∴ Distance of the other end B from P is

$u_B=2f-\frac{f}{3}=\frac{5}{3}f$

If the distance of image of end B from P is $v_B$ then

$\frac{1}{v_B}+\frac{1}{-\frac{5}{3}f}=\frac{1}{-f}\Rightarrow v_B=-\frac{5}{2}f$

∴ the length of the image

$|v_B|-|v_A|=\frac{5}{2}f-2f=\frac{1}{2}f$ and magnification

$m=\frac{|v_B|-|v_A|}{|u_B|-|u_A|}=\frac{\frac{1}{2}f}{-\frac{1}{3}f}=-\frac{3}{2}$

Negative sign implies that image is inverted with respect to object and so it is real

**10. Ans ( D )**

$\frac{1}{f_{lens}}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

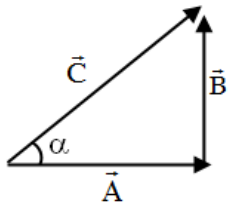
$=0.5\left[\frac{1}{10}-\frac{1}{20}\right]=\frac{1}{40}$

∴ $f_{lens}=40$ cm

Thus image will be formed at +2f

The ray will retrace if d = 2f or d = 2f + 30

**11. Ans ( A )**




$\cos\alpha=\frac{|\vec{A}|}{|\vec{C}|}$

$\cos\alpha=\frac{10}{20}=\frac{1}{2}$

$\alpha=60°$

$|\vec{C}|^2=|\vec{A}|^2+|\vec{B}|^2$

$|\vec{B}|=10\sqrt{3}$

**12. Ans ( D )**

Given relation

$f=cm^xk^y$ where f = frequency of vibration

$[T^{-1}]=[M]^x[MT^{-2}]^y$ m = mass suspended from a spring

x + y = 0 and 2y = 1 k = spring constant

$y=\frac{1}{2}$ $x=\frac{-1}{2}$ c = Dimensionless quantity

**13. Ans ( C )**

Pressure $p=\frac{\text{Force}}{\text{Area}}=\frac{\text{Mass}\times\text{Acceleration}}{\text{Area}}$

$[p]=\frac{ML^{+1}T^{-2}}{L^2}=[M^1L^{-1}T^{-2}]=[M^aL^bT^c]$

a = 1, b = –1, c = –2

**14. Ans ( C )**

So. $\frac{dy}{dx} = (x+1)^{-2}\frac{d}{dx}(x^3) + x^3\frac{d}{dx}(x+1)^{-2}$

$= \frac{3x^2}{(x+1)^2} - \frac{2x^3}{(x+1)^3}$

**15. Ans ( B )**

$\int_0^{\pi}\left(\frac{\pi t}{2} - \frac{t^2}{2}\right)dt$

$\left(\frac{\pi}{2}\left(\frac{t^2}{2}\right) - \frac{t^3}{6}\right)^{\pi} = \frac{\pi^3}{4} - \frac{\pi^3}{6} = \frac{\pi^3}{12}$

**16. Ans ( A )**

$2^\circ = \frac{\pi}{90}\text{rad}$

$x = \sin\left(\frac{\pi}{90}\right)\cos\left(\frac{\pi}{90}\right)$

$\sin\frac{\pi}{90} \approx \frac{\pi}{90}\ \cos\frac{\pi}{90} \approx 1$

$x = \frac{\pi}{90}$

**17. Ans ( C )**

$m \propto E^a v^b F^c$

$a = 1, b = -2, c = 0$

**18. Ans ( A )**

$\vec{A} = \vec{B} + \vec{C}$

$\vec{B} = \vec{A} - \vec{C}$

$B^2 = A^2 + C^2 - 2AC\cos\theta$

$\cos\theta = \frac{3}{5}$

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

**19. Ans ( B )**

$\vec{C} = \vec{A} + \vec{B}$ gives;

$C^2 = A^2 + B^2 + 2AB\cos\theta$

But $C^2 = A^2 + B^2$

$\therefore\ 2AB\cos\theta = 0$

or $\cos\theta = 0,\ \theta = \frac{\pi}{2}$

**20. Ans ( B )**

Let $\theta$ be angle between vectors $\vec{A}$ and $\vec{B}$.

$|\vec{A} + \vec{B}| = n\,|(\vec{A} - \vec{B})|$ (Given)

Then, $|\vec{A} + \vec{B}|^2 = n^2\,|(\vec{A} - \vec{B})|^2$

$A^2 + 2AB\cos\theta + B^2$

$= n^2\,[A^2 - 2AB\cos\theta + B^2]$

$A^2 + 2A^2\cos\theta + A^2$

$= n^2\,[A^2 - 2A^2\cos\theta + A^2]$

$[\because\ |\vec{A}| = |\vec{B}|]$

$2A^2 + 2A^2\cos\theta$

$= n^2[2A^2 - 2A^2\cos\theta]$

$(n^2+1)\cos\theta = (n^2 - 1)$

$\cos\theta = \left(\frac{n^2-1}{n^2+1}\right)$

or $\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

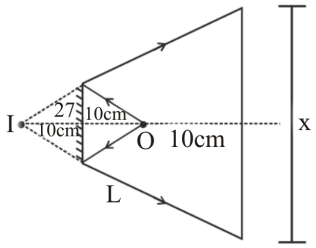
## PART-1 : PHYSICS

### SECTION-II

**1. Ans ( 2 )**

$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

**3. Ans ( 9 )**




$\frac{x}{27} = \frac{30}{10}$

$x = 81\text{cm}$

Image is visible upto a distance AB = 81cm

$S = \frac{1}{2}at^2$

$81 = \frac{1}{2} \times 2 \times t^2$

$t = \sqrt{81} = 9\ \text{sec}$

4. **Ans ( -10 )**

$$\frac{1}{f_{Net}} = \frac{1}{f_{mirror}} - \frac{2}{f_{lens}}$$

$$\frac{1}{f_{Net}} = \frac{1}{\infty} - \frac{2}{20} = -\frac{1}{10}$$

$$\boxed{f_{Net} = -10 \text{ cm}}$$

5. **Ans ( 40 )**

$$\frac{\mu_2}{v} - \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{u}{3f} - \frac{1}{\infty} = \frac{4/3 - 1}{10}$$

$$\boxed{f = 40}$$

6. **Ans ( 13 )**

$P^2 + Q^2 + 2PQ\cos\theta = 225$ .....(i)

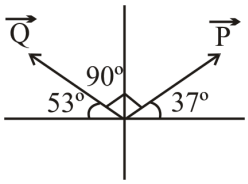
$P^2 + Q^2 - 2PQ\cos q = 113$ .....(ii)

by Adding (i) & (ii) $2(P^2+Q^2) = 338$

$P^2 + Q^2 = 169$

$\sqrt{P^2 + Q^2} = 13$

7. **Ans ( 25 )**




$\vec{P} \perp \vec{Q}$

$\therefore\ R = \sqrt{P^2 + Q^2} = \sqrt{400 + 225}$

$R = 25$

8. **Ans ( 20 )**

Least count of vernier scale is the different between one main scale division and vernier scale division.

$LC = 1\ MSD - 1VSD$

$LC = 1\ MSD - \frac{16}{28}MSD$

$LC = \frac{4}{20}\ MSD$

$LC = \frac{4}{20} \times 1mm \ldots\ldots = 0.2\ mm$

$0.2 = x \times 10^{-2}$

$20 \times 10^{-2} = x \times 10^{-2}$

$x = 20$

9. **Ans ( 0 )**

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{\ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} - \frac{2\Delta T}{T} = 2 - 2(1) = 0$$

10. **Ans ( 7 )**

$F = [MLT^{-2}]$

$a = 1, b = 1, c = -2$

$E = [ML^2T^{-2}]$

$x = 1, y = 2, z = -2$

$ax + by + cz = 1 + 2 + 4 = 7$

## PART-2 : CHEMISTRY

### SECTION-I

2. **Ans ( C )**

0.05 mol $XY_2$ = 5 gm

1 mol $XY_2 = \frac{5}{0.05} = 100$

$X + 2Y = 100$ ...(i)

$3.01 \times 10^{23}$ molecule $X_2Y_3$ = 85 gm

1 mol $X_2Y_3$ = $N_A$ molecule = 170 gm

$2X + 3Y = 170$ ...(ii)

On solving,

$X = 40, Y = 30$

3. **Ans ( C )**

Equal no. of moles will have same no. of atoms.

No. of moles of Fe $= \frac{560}{56} = 10$

No. of moles of N $= \frac{70}{14} = 5$, twice of this = 10

No. of moles of H $= \frac{20}{1} = 20$, half of this = 10

**4. Ans ( C )**

Mass of nitrogen in 194 amu caffeine

$= \frac{29}{100} \times 194 = 56.26$

$\therefore$ One molecule of caffeine 4 atoms of nitrogen

**6. Ans ( D )**

Molality is $\left[\frac{w}{W}\right]$ unit.

Hence it does not depend upon temp.

**7. Ans ( C )**

Mass of methanol = d × V = 0.8 × 125 = 100 g

$$m = \frac{100/32}{375/1000} = 8.33$$

**8. Ans ( C )**

$\Delta T_f = K_f \times$ molality

$2.8 = 1.86 \times \left[\frac{w/62}{1kg}\right]$

w = 93 g

**9. Ans ( A )**

$y_A = \frac{P_A^o x_A}{P}$ and $y_B = \frac{P_B^o x_B}{P}$

$\frac{y_A}{y_B} = \frac{P_A^o}{P_B^o} \times \frac{x_A}{x_B}$

$\Rightarrow \frac{4}{3} = \frac{1}{3} \times \frac{1 - x_B}{x_B}$

$x_B = \frac{1}{5}$

**10. Ans ( A )**

Both the solutions are isotonic as they will have same conc. of solute particles. The conc. of acetic acid is reduced to half due to dimerisation which conc. of NaCl doubled due to dissociation.

**11. Ans ( B )**

$n_A = 3$ mol, $n_B = 2$ mol

$X_A = \frac{3}{5}$, $X_B = \frac{2}{5}$

$P_S = p_A^0 X_A + p_B^0 X_B$

$184 = 200 \times \left(\frac{3}{5}\right) + p_B^0 \times \left(\frac{2}{5}\right)$

$p_B^0 = 160$ torr

**12. Ans ( A )**

$$X_{solute} = \frac{n_{solute}}{n_{solute} + n_{solvent}} = \frac{\frac{10}{60}}{\left(\frac{10}{60} + \frac{1000}{M.M_{solvent}}\right)}$$

$M.M_{solvent}$ is minimium then $X_{solute}$ is minimum

i.e. for water

**13. Ans ( B )**

At 90°C pressure of soln = 1 atm

$P_s = P_A^o X_A + P_B^o X_B$

$1 = P_A^o 0.2 + P_B^o (0.8)$

$P_A^o = \frac{P_B^o}{2}$

$P_B^o = 2P_A^o$

$1 = 60.2\ P_A^o + 2\ P_A^o(0.8)$

$1 = 1.8$

$\frac{1}{1.8} = P_A^o = \frac{10}{18} = \frac{5}{9}$

**14. Ans ( B )**

Decamolal $CH_3COOH \longrightarrow 10$ mol

$CH_3COOH$/kg solvent

mass of $CH_3COOH$

$= 10 \times 60 = 600$ gm

mass of solution

$= 600 + (1\ kg = 1000) = 1600$ gm

$ppm = \frac{600}{1600} \times 10^6 = 3.75 \times 10^5$

**15. Ans ( B )**

$1 = 1 + \alpha(n - 1)$

$3.4 = 1 + \alpha(4 - 1)$

$\alpha = \frac{3.4 - 1}{3} = 0.80$

**16. Ans ( A )**

B.P. > pure solute or pure solvent

V.P. < pure solute or pure solvent

Negative deviation

**18.** **Ans ( D )**

$$\frac{\Delta P}{P^0} = \frac{in_B}{in_B + n_A} \qquad n_B=1,\ n_A=3$$

$$0.4 = \frac{in}{in+3} = \frac{i}{i+3} = 0.4 \quad \boxed{i = 2}$$

$AB \rightleftharpoons A^+ + B^- \quad i = 1 + \alpha = 1 - \alpha\ \alpha\ \alpha$

$\boxed{\alpha = 1}$

**19.** **Ans ( B )**

R.L.V.P. is colligative properties.

$\Delta P = P°X_B$ (depends on P°)

**20.** **Ans ( A )**

$BaCl_2 \longrightarrow Ba^{2+} + 2Cl^- \quad i = 3$

$\Delta T_f = i\, K_f \times m = 3 \times 1.86 \times 0.05 = 0.279$

$\therefore\ T_f = 0 - \Delta T_f$

$T_f = -0.279$

## PART-2 : CHEMISTRY

### SECTION-II

**1.** **Ans ( 5 )**

$$P_s = \left(\frac{80}{100}\right) P°,\ w = ?$$

Msolute = 40, W = 57 gm, Msolvent = 114

$$\because \frac{P^o - P_s}{P_s} = \frac{w \times M_{solvent}}{M_{solute} \times W}$$

$\therefore\ w = 5g$

**2.** **Ans ( 350 )**

$$\left(200 = \frac{P_i}{4} + \frac{P_P \times 3}{4}\right) \quad ....(1)$$

$$\left(300 = \frac{3P_i}{4} + \frac{P_P}{4}\right) \times 3 \quad ....(2)$$

$$700 = \frac{9}{4}P_i - \frac{P_i}{4} = 2P_i$$

$$P_i = \frac{700}{2} = 350 \text{ mm of Hg}$$

**4.** **Ans ( 6 )**

$PT = P_A x_A + P_B x_B$

$$450 = 300\left(\frac{2}{x+2}\right) + 500\left(\frac{x}{x+2}\right)$$

$$\Rightarrow 45 = \frac{60}{x+2} + \frac{50x}{x+2}$$

$\Rightarrow (x + 2)45 = 60 + 50x$

$x = 6$

**6.** **Ans ( 100 )**

$$\frac{\overset{o}{P} - P_s}{\overset{o}{P}} = x_{solute}$$

$$\frac{\overset{o}{P} - 80}{\overset{o}{P}} = 0.2$$

$\overset{o}{P} = 100$ torr.

**7.** **Ans ( 200 )**

Mass of 1 atom is $= 6 \times 10^{-23}$

Mass of 1 mole atom i.e. Atomic mass

$= 6 \times 10^{-23} \times 6 \times 10^{+23} = 36$

$$\text{No. of moles} = \frac{7.2 \times 10^3}{36} = \frac{7200}{36} = 200$$

**8.** **Ans ( 50 )**

$$PH_3 \longrightarrow P_{(s)} + \frac{3}{2}H_2(g)$$

100 ml $\longrightarrow$ 0 0

$$(100 - 100) \longrightarrow -\frac{3 \times 100}{2} = 150 \text{ mL}$$

$V_i = 100$ mL

$V_f = 150$ mL

$(V_f - V_i) = 50$ mL

**9.** **Ans ( 146 )**

Let, mol. mass = x

$$\%\text{ of Oxygen} = \frac{\text{Mass. of 'O'}}{x} \times 100$$

$$43.8 = \frac{(16 \times 4)}{x} \times 100$$

$$x = \frac{16400}{43.8} = 146$$

**10.** **Ans ( 8 )**

$$\frac{1.055}{24 + 254 + 18x} \times x \times 18 = (1.055 - 0.695)$$

$\Rightarrow x = 8$ Ans.

# PART-3 : MATHEMATICS

## SECTION-I

**1. Ans ( C )**

Given, $\frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$\Rightarrow \frac{-3x-2}{(x+1)(x+2)(2x+1)} > 0$

Equating each factor equal to 0, we have

$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$

It is clear $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$

**2. Ans ( A )**

$\log_5 5(x^2+1) > \log_5 (ax^2+4x+a)$

$\Rightarrow \quad 5(x2+1) > ax^2+4x+a > 0 \ \forall \ x \in R$

$\Rightarrow \quad ax^2+4x+a > 0 \ \forall \ x \in R \ \&$

$5x^2+5 > ax^2+4x+a > 0 \ \forall \ x \in R$

It is possible if

$a > 0 \ \& \ 16 - 4a^2 < 0$

$(a^2-4) > 0$

$a > 0 \cap (-\infty, -2) \cup (2,\infty)$

$a \in (2,\infty)$

and

.....

$(5-a)x^2 - 4x + (5-a) > 0 \ \forall \ x \in R$

$5 - a > 0 \ \& \ 16 - 4(5-a)^2 < 0$

$a < 5$

and

$(a-3)(a-7) > 0$

$(a \in (-\infty, 3) \cup (7,\infty)$

Now $a \in (2, 3)$

**3. Ans ( B )**

since $2^m - 2^n = 56$

$\Rightarrow \ 2n(2^{m-n} - 1) = 2^3 \times 7$

$\Rightarrow \ 2^n = 2^3, 2^{m-n} - 1 = 7$

$\Rightarrow \ n = 3, 2^{m-n} = 8$

$\Rightarrow \ m - n = 3 \quad \Rightarrow \quad m = 6$

**4. Ans ( C )**

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(-2, 2)\ (0, 0)\ (1, 1), (1, 2)\}$

$n(P(R)) = 2^4 = 16$

**5. Ans ( D )**

$\alpha^2 - 6\alpha - 2 = 0$

$\frac{a_{10} - 2a_8}{3a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$

$= \frac{\alpha^8(\alpha^2-2) - \beta^8(\beta^2-2)}{3(\alpha^9 - \beta^9)}$

$= \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$

**6. Ans ( B )**

$AB^2 + BC^2 = AC^2 = 32$

$\Rightarrow 2BC^2 = 32 \Rightarrow BC^2 = 16 \ \because AB=BC$

$\Rightarrow BC = 4$

Also $OC = 2\sqrt{2}$

In $\Delta$ BOC $OB^2 + OC^2 = BC2$

$OB^2 + 8 = 16$

$OB^2 = 8$

$OB = 2\sqrt{2}$

minimum value of $ax^2 + bx + c$ is $OB = -2\sqrt{2}$

**7. Ans ( B )**

Let the roots of

$x^3 + 5x^2 + px + q = 0$ are $\alpha_1, \beta_1, \gamma_1$ ....(i)

then roots of $x^3 + 7x^2 + px + r = 0$ are $\alpha_1, \beta_1, \gamma_2$ ....(ii)

from (i) – (ii) $\Rightarrow -2x^2 + q - r = 0$

This equation has roots $\alpha_1, \beta_1 \Rightarrow \alpha_1 + \beta_1 = 0$

Now from (i) $\alpha_1 + \beta_1 + \gamma_1 = -5 \Rightarrow \gamma_1 = -5$

from (ii) $\alpha_1 + \beta_1 + \gamma_2 = -7 \Rightarrow \gamma_2 = -7$

$\therefore \ \gamma_1 + \gamma_2 = -12 \Rightarrow |\gamma_1 + \gamma_2| = 12$

**8. Ans ( A )**

$$\frac{ax^2+2(a+1)x+(9a+4)}{x^2-8x+32}<0\ \forall\ x\in R$$

$\Rightarrow [x^2-8x+32=(x-4)^2+16>0\ \forall\ x\in R]$

$\Rightarrow ax^2+2(a+1)x+(9a+4)<0\ \ \forall\ x\in R$

$\Rightarrow a<0 \quad \& \quad D<0$

$4(a+1)^2-4.9(9a+4)<0$

$a^2+2a+1-9a^2-4a<0$

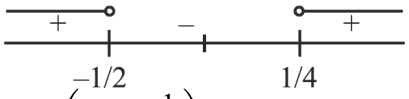
$-8a^2-2a+1<0$

$8a^2+2a-1>0$

$8a|+4a-2a-1>0$

$4a(2a+1)-1(2a+1)>0$

$(4a-1)(2a+1)>0$




$a\in\left(-\infty,-\frac{1}{2}\right)$

**9. Ans ( A )**

$\lambda x^2+x(1-\lambda)+5=0$

$\Rightarrow \alpha+\beta=\frac{\lambda-1}{\lambda},\alpha\beta=\frac{5}{\lambda}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}$ Given

$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{4}{5}$

$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{4}{5}$

$\Rightarrow \frac{\frac{(\lambda-1)^2}{\lambda^2}-2\times\frac{5}{\lambda}}{\frac{5}{\lambda}}=\frac{4}{5}$

$\Rightarrow \frac{(\lambda-1)^2-10\lambda}{5\lambda}=\frac{4}{5}$

$\Rightarrow \lambda^2-2\lambda+1-10\lambda=4\lambda$

$\Rightarrow \lambda^2-16\lambda+1=0$

$\Rightarrow \lambda_1+\lambda_2=16$ ....(1) $\quad \lambda_1\lambda_2=1$ ....(2)

So find

$=-245$ using (1) and (2)

$=256-247=9$

**10. Ans ( C )**

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(2^{r+1}+\frac{1}{2^r}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(\frac{2^r2^{r+1}+1}{2^r}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r+1}-2^r}{1+2^r\cdot 2^{r+1}}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\tan^{-1}(2^{r+1})-\tan^{-1}(2^r)$$

$$=\lim_{n\to\infty}(\tan^{-1}2^{n+1}-\tan^{-1}2)$$

$$=\frac{\pi}{2}-\tan^{-1}2=\cot^{-1}2$$

**11. Ans ( C )**

$(\sin^{-1}x+\cos^{-1}x)^3-3\sin^{-1}x\cos^{-1}x$

$(\sin^{-1}x+\cos^{-1}x)$

$\frac{\pi^3}{8}-3.(\sin^{-1}x\cos^{-1}x).\frac{\pi}{2}$

$\frac{\pi^3}{8}-\frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$

$=\frac{\pi^3}{8}-\frac{3\pi^2}{4}\sin^{-1}x+\frac{3\pi}{2}(\sin^{-1}x)^2$

$=\frac{\pi^3}{8}-\frac{3\pi}{2}\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x\right]=\frac{\pi^3}{8}+\frac{3\pi}{2}$

$\left[(\sin^{-1}x)^2-2.\sin^{-1}x.\frac{\pi}{4}+\frac{\pi^2}{16}-\frac{\pi^2}{16}\right]$

$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left(\sin^{-1}x-\frac{\pi}{4}\right)^2-\frac{3\pi^3}{32}$

$=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\underbrace{\sin^{-1}x}-\frac{\pi}{4}\right)^2=0$

Least value is $\frac{\pi^3}{32}$

For maximum value put

$\sin^{-1}x=-1\Rightarrow x=-\frac{\pi}{2}$

$=\frac{\pi^3}{32}+\frac{3\pi}{2}\left[-\frac{\pi}{2}-\frac{\pi}{4}\right]^2$

$=\frac{\pi^3}{32}+\frac{3\pi}{2}.\frac{9\pi^2}{16}=\frac{28\pi^3}{32}=\frac{7\pi^3}{8}$

**12. Ans ( C )**

$$\frac{2x^2+4}{1+x^2} = 2 + \frac{2}{1+x^2}$$

$$\therefore\ 2 < \frac{2x^2+4}{1+x^2} \leqslant 4$$

$$\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right) < \pi - 3$$

$$\Rightarrow\ \pi - \frac{2x^2+4}{1+x^2} < \pi - 3$$

$$\Rightarrow\ 2 + \frac{2}{1+x^2} > 3$$

$$\Rightarrow\ \frac{2}{1+x^2} > 1$$

$$\Rightarrow\ 2 > 1 + x^2$$

$$\Rightarrow\ x^2 < 1$$

$$\Rightarrow\ x \in (-1, 1)$$

**13. Ans ( B )**

$$(\cot^{-1}x)^2 - 5\cot^{-1}x + 6 > 0$$

$$(\cot^{-1}x - 3)(\cot^{-1}x - 2) > 0$$

$$\cot^{-1}x < 2 \ \&\ \cot^{-1}x > 3$$

$$x > \cot 2 \ \&\ x < \cot 3$$

$$x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$$

**14. Ans ( B )**

$$f(x) = \sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)} \quad -1 \leq \frac{1-|x|}{2} \leq 1$$

$$\Rightarrow -2 - 1 \leq -|x| \leq 2 - 1$$

$$\Rightarrow -3 \leq -|x| \leq 1$$

$$\Rightarrow -1 \leq |x| \leq 3 \Rightarrow x \in [-3, 3]$$

**15. Ans ( A )**

$$e^{f(x)} = \frac{10+x}{10-x}$$

$$f(x) = \log\left(\frac{10+x}{10-x}\right)$$

$$\therefore\ f\left(\frac{200x}{200+x^2}\right) = \log\left(\frac{10 + \frac{200x}{100+x^2}}{10 - \frac{200x}{100+x^2}}\right)$$

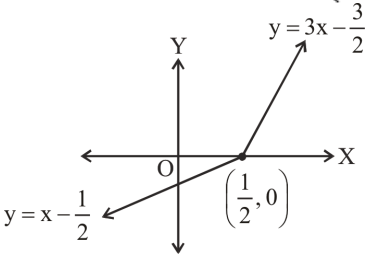
$$= \log\left(\frac{1000+200x+10x^2}{1000-200x+10x^2}\right) = \log\left(\frac{10+x}{10-x}\right)^2$$

$$= 2\log\left(\frac{10+x}{10-x}\right) = 2f(x)$$

$$f\left(\frac{200x}{200+x^2}\right) = 2.kf\left(\frac{200x}{200+x^2}\right);\ k = 1/2$$

**16. Ans ( B )**

$$f(x) = 2x - 1 + \left|x - \frac{1}{2}\right| = \begin{cases} 3x - \frac{3}{2}, & x \geqslant \frac{1}{2} \\ x - \frac{1}{2}, & x < \frac{1}{2} \end{cases}$$




$\therefore$ one-one & onto

**17. Ans ( C )**

Domain = $[-1, 1] \cap (-\infty, -1] \cup [1, \infty) \cap R$

$= \{-1, 1\}$

**18. Ans ( B )**

$$f(x) = \log_2 [3x - 3[x]]$$

$$f(x) = \log_2 [3\{x\}]$$

$$3\{x\} \in [0, 3)$$

$$\&\ [3\ \{x\}] = 0, 1, 2$$

$\therefore$ Range of f(x) = 0, 1

**19. Ans ( D )**

$f(x) = 1 - x^3$

And $\sqrt[3]{5^{\log_5 7} + \frac{1}{\sqrt{\log_{10} 10}}} = \sqrt[3]{8} = 2$

$\therefore f(2) = 1 - 8 = -7$

**20. Ans ( B )**

$g(f(x)) = (\sin x + \cos x)^2 - 1 = \sin 2x$

$2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

## PART-3 : MATHEMATICS

### SECTION-II

**1. Ans ( 6 )**

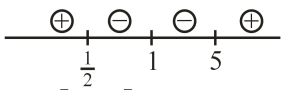
$(x^2 - 6x + 5)(2x^2 - 3x + 1) \le 0$

$(x^2 - 5x - x + 5)(2x^2 - 2x - x + 1) \le 0$

$(x(x-5) - 1(x-5))[2x(x-1) - 1(x-1)] \le 0$

$(x-1)(x-5)(2x-1)(x-1) \le 0$

$(x-5)(2x-1)(x-1)^2 \le 0$




$x \in \left[\frac{1}{2}, 5\right]$

**2. Ans ( 1 )**

Using $(a_1 c_2 - a_2 c_1)^2 = (b_1 c_2 - b_2 c_1)(a_1 b_2 - a_2 b_1)$

$a_1 = 1, b_1 = -a, c_1 = b$

$a_1 = 1, b_2 = b, c_2 = -a$

$\Rightarrow (-a - b)^2 = (a^2 - b^2)(b + a)$

$\Rightarrow -(a+b)(a+b) = (a-b)(a+b)(a+b)$

$a - b = 1$

**3. Ans ( 7 )**

For Real roots $q^2 \ge 4P$

| q | P |
|---|---|
| 2 | 1 |
| 3 | 1, 2 |
| 4 | 1, 2, 3, 4 |
| | |

7 different equations

**4. Ans ( 6 )**

If $f(x) = x^2 + 2(a-3)x + 9 = 0$

$D \ge 0 \Rightarrow a \in (-\infty, 0] \cup [6, \infty)$ ...(1)

$-6 < -\frac{B}{2A} < 1 \Rightarrow a \in (2, 9)$ ...(2)

& $f(-6) > 0 \Rightarrow a \in \frac{27}{4}$ ...(3)

$f(1) > 0 \Rightarrow a > -2$ ...(4)

from (1), (2), (3) & (4)

$a \in \left[6, \frac{27}{4}\right)$

$\therefore a_{min} = 6$

**5. Ans ( 753 )**

$8x^3 \mid 1-1x \mid 2008 = 0 \longrightarrow \alpha, \beta, \gamma$

S.R. $\Rightarrow \alpha+\beta+\gamma=0 \Rightarrow \alpha^3+\beta^3+\gamma^3 = 3\alpha\beta\gamma$ ...(1)

No, $(\alpha+\beta)^3 + (\beta+\gamma)^3 + (\gamma+\alpha)^3 = -\gamma^3 - \alpha^3 - \beta^3$

$\Rightarrow -[\alpha^3+\beta^3+\gamma^3] = -3\alpha\beta\gamma = -3\left(\frac{-2008}{8}\right)$

$\Rightarrow 753$

**6. Ans ( 8 )**

$\sin^{-1}x_1 + \sin^{-1}x_2 + .....+ \sin^{-1}x_{20} = 10\pi$

$\therefore \sin^{-1}x_1 = \frac{\pi}{2}, ..... ; \sin^{-1}x_{20} = \frac{\pi}{2}$

$x_1 = 1 = x_2 = ..... = x_{20}$

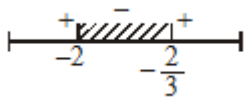
$\sum_{i=5}^{12} x_i = x_5 + x_6 + x_7 + x_8 + .... + x_{12} = 8$

**7.** **Ans ( 3 )**

$\because \cos^{-1}\left(-\frac{7}{25}\right) = \pi - \cos^{-1}\left(\frac{7}{25}\right)$

$\lambda = \pi$

**8.** **Ans ( 1 )**




$3x^2 + 8x < 2(\pi - 4) - (2\pi - 4)$

$3x^2 + 8x < -4$

$3x^2 + 8x + 4 < 0$

$(3x + 2)(x + 2) < 0$

$x \in \left(-2, -\frac{2}{3}\right)$ One solution only

$\because x \in z \Rightarrow x = -1$

**9.** **Ans ( 79 )**

$f(x) = x^2 - 2$

$f\left(\sqrt{11}\right) = 11 - 2 = 9$

$f\left(f\sqrt{11}\right) = 81 - 2 = 79$

**10.** **Ans ( -1 )**

$g(x) = f(f(x))$

$= f = \left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$

$h(x) = f(f(f(x))) = \frac{1}{1-\frac{x-1}{x}} = x$

$\therefore f(x).g(x).h(x) = \frac{1}{1-x}.\frac{x-1}{x}.x = -1$