ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE (Main)
PART TEST
22-12-2024

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY PAPER-1 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	A	C	C	A	B	B	A	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	B	B	D	A	C	B	D	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	1	43	9	2	1					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	D	C	B	D	B	D	D	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	A	B	D	C	D	B	B	A	A
SECTION-II	Q.	1	2	3	4	5					
	A.	0	2	6	5	1					

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	A	B	B	B	B	A	A	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	A	D	B	C	C	B	C	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	2016	10	3	19	12					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (C)

Prism will not cause any rotation of emergent beam and mirror will make emergent beam rotate by $2\omega_2$

2. Ans (C)

$$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right) \text{.....(1)}$$

as mass is constant $\Rightarrow m = \rho V = \text{constant}$

$\pi^2 RI = \text{constant} \Rightarrow R^2 L = \text{constant}$

$$2RL = R^2\frac{dL}{dR} = 0 \text{ .....(2)}$$

From equation (1)

$$\frac{R}{2} + \frac{L}{6}\frac{dL}{dR} = 0$$

Substituting value of $\frac{dL}{dR}$ from equation (2)

$$\frac{R}{2} + \frac{L}{6}\left(\frac{-2L}{R}\right) = 0$$

$$\frac{R}{2} = \frac{L^2}{3R} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

5. Ans (C)

$\delta = \Sigma(\mu - 1)A$

$= (1.5 - 1)1 + (2 - 1)2$

$= 0.5 + 2 = 2.5$

6. Ans (A)

$-\frac{1}{F} = P = 2P_{1_1} + 2P_{1_2} + P_m$ .....(1)

$P_{1_1} = \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{1_1} = [(1.5 - 1)]\left[-\frac{1}{10} - \frac{1}{15}\right] = -\frac{1}{12}$ .....(2)

$P_{1_2} = \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{1_2} = \left(\frac{4}{3} - 1\right)\left[\frac{2}{15}\right] = \frac{2}{45}$ .....(3)

$P_m = -\frac{1}{f} = +\frac{2}{15}$ .....(4)

$-\frac{1}{F} = P = 2\left[-\frac{1}{12} + \frac{2}{45}\right] + \frac{2}{15} = -\frac{1}{6} + \frac{4}{45} + \frac{2}{15} = \frac{1}{18}$

$F = -18$ cm.

Focus is negative means system will behave as concave mirror.

7. Ans (B)

Acceleration $= \frac{\sum F}{M} = \frac{F}{M}$

($\because$ friction force is zero)

8. Ans (B)

$d = i + e - A$

$30 = 90 - A$

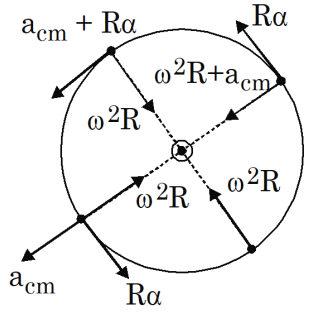
$A = 60°$

$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$

$\mu = \frac{\sin 45}{\sin 30} = \sqrt{2}$

10. Ans (D)

After one revolution acceleration represent in the figure.



11. Ans (B)

$\vec{V}_{I/m} = -\left(\frac{V}{u}\right)^2 (V_{01m})$

$\Rightarrow \vec{V}_I - 2 = -\left(\frac{60}{20}\right)^2 (V_0 - 2)$

$\vec{V}_I = 20$ m/s

$\frac{1}{V} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{V} - \frac{1}{20} = -\frac{1}{15}$

$\frac{1}{V} = -\frac{1}{15} + \frac{1}{20}$

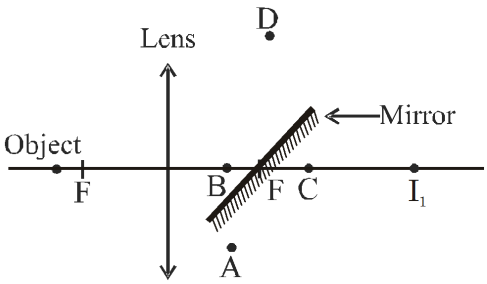
$V = -60$

13. Ans (B)

$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$, So, $v_0 = 36$ cm

Now, $m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = 32$

14. Ans (D)



I_1 is the image formed by lens.

I_1 behaves as object for mirror.

Final image is formed at D.

15. Ans (A)

$\vec{\tau}$ will change the direction of angular momentum at a constant angle from $\vec{A}$.

16. **Ans (C)**

$\mu mgR = \frac{1}{2}mR^2\alpha \Rightarrow \alpha = \frac{2\mu g}{R}$

$\mu mg = ma \Rightarrow a = \mu g$

$v = at = \mu gt$

$\omega = \omega_0 - \alpha t = \omega_0 - \frac{2\mu gt}{R}$

$v = R\omega$

$\Rightarrow \mu gt = \omega_0 R - 2\mu_g t$

$t = \frac{\omega_0}{3\mu g}R$

17. **Ans (B)**

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20}$

$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} = \frac{2}{3} \Rightarrow 3f_1 = -2f_2$

So $f_1 = -10$ and $f_2 = \frac{20}{3}$

18. **Ans (D)**

$f = \frac{100}{-5} = -20cm$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-22} - \frac{1}{\infty}$

$f = -22$ cm

PART-1 : PHYSICS

SECTION-II

1. **Ans (1)**

$x = \frac{y^2}{2}$

$\frac{dy}{dx} = \frac{1}{y}$, at $y = 1$, slope $= 1$

$\angle i = 45°$, $\therefore \angle r = 45°$ & deviation in first reflection $= 90°$.

Similarly for second reflection, Net deviation $= 180°$.

2. **Ans (43)**

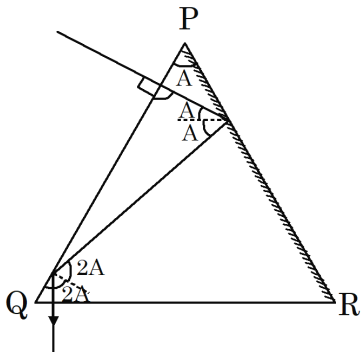
Angular Momentum about hinge

$L_i = L_f$

$mu\left(\frac{3\ell}{4}\right) = \left(\frac{m\ell^2}{3} + m\left(\frac{3\ell}{4}\right)^2\right)\omega$

$\omega = \frac{36u}{43\ell}$

3. **Ans (9)**

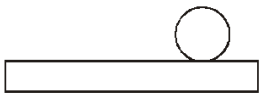


$\frac{\pi}{2} - \frac{A}{2} + \frac{\pi}{2} - 2A = \frac{\pi}{2}$

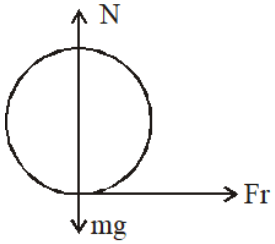
$\frac{5A}{2} = \frac{\pi}{2}$

$A = \frac{\pi}{5} = 36°$

4. **Ans (2)**



For ring



$f_r = M_2a$ ....(i)

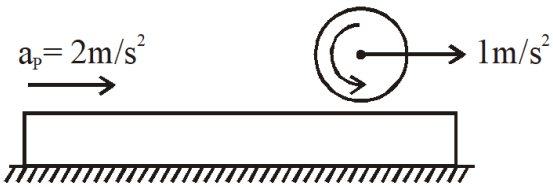
$\tau = I\alpha$

$f_rR = M_2R^2\alpha$

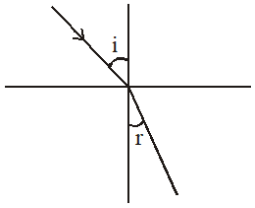
$\alpha = \frac{f_r}{M_2R}$ ....(ii)

From equation (i) and (ii)

$\Rightarrow a = \alpha R$



5. **Ans (1)**



$1 \sin i = \mu \sin r$

$i \times \sin 60 = \sqrt{3} \sin r$

$\sin r = \frac{1}{2}$

$r = 30$

$\sin i = \mu \sin r$

$\cos i \frac{di}{dt} = \mu \cos r \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\mu}\left(\frac{\cos i}{\cos r}\right)\frac{di}{dt}$

$= \frac{1}{\sqrt{3}} \frac{\cos 60}{\cos 30} \times 3 = 1$ rad/sec

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (C)**

$K_h = \frac{(1.6 \times 10^{-4})^2}{0.01} = 2.56 \times 10^{-6}$

$K_h = \frac{K_w}{K_b} \Rightarrow K_b = 3.9 \times 10^{-9}$

2. **Ans (A)**

$nM = Mn$

$1 \quad —$

$1 - \beta \quad \beta/n$

$i = 0.9 = \frac{1 - \beta + \beta/n}{1}$

$\frac{1}{9} = \frac{\beta/n}{1 - \beta + \beta/n} = \frac{\beta/n}{0.9} \Rightarrow \beta = 0.1\,n$

$0.9 = 1 - 0.1\,n + 0.1 \Rightarrow n = 2$

3. **Ans (D)**

(A) As K_{SP} of hydroxide of Al^{3+}, Fe^{3+} & Cr^{3-} are low and NH_4Cl suppresses the ionisation of NH_4OH.

(B) $(NH_4)_2SO_4 + Ba^{2+} \longrightarrow BaSO_4 \downarrow$ (white) $+ 2NH_4^+$.

(C) $SO_4^{2-} + Ba^{2+} \longrightarrow BaSO_4 \downarrow$ (white) as it contains SO_4^{2-} as anion.

4. **Ans (C)**

$S = [Ag^+] + [Ag(CN)_2^-]$

$= \frac{K_{sp}}{[CN^-]} + K_f . K_{sp} . [CN^-]$

For minimum solubility: $\frac{dS}{d[CN^-]} = 0$

or, $-\frac{K_{sp}}{[CN^-]^2} + K_f . K_{sp} = 0$

$\Rightarrow [CN^-] = \sqrt{\frac{1}{K_f}} = 2.58 \times 10^{-9}$M

5. **Ans (B)**

$2AB_2(g) \rightleftharpoons 2AB(g) + B_2(g)$

$1 \quad 0 \quad 0$ Initially

$(1-x) \quad x \quad \frac{x}{2}$ At equilibrium

Total moles at equilibrium

$= 1 - x + x + \frac{1}{2} = 1 + \frac{x}{2} = 1$

[$\because$ x is small in comparision to unity]

$p_{AB_2} = (1-x)P \;\; p_{AB} = xP \;\; p_{B_2} = \frac{xP}{2}$

$K_p = \frac{x^3P^3}{2(1-x)^2P^2} = \frac{x^3P}{2}$ [$\because (1-x) \approx 1$]

$K_p = \frac{x^3P}{2}$

$x^3 = \frac{2K_p}{P}$

$x = \sqrt{\frac{2K_p}{P}}$

6. **Ans (D)**

$\frac{3}{10} \times 360 + 24 \times \frac{7}{10} = 124.8$ torr for ideal behaviour.

But the solution of acetone and water show positive deviation.

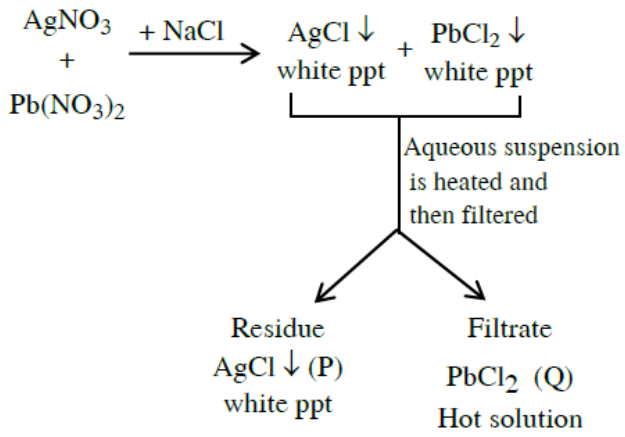
7. **Ans (B)**

Theory based.

8. **Ans (D)**

Simple salt is NaCl.

10. Ans (A)



$AgCl + 2NH_3$ solution $\longrightarrow [Ag(NH_3)_2]Cl$
(P) (excess) clear solution

$AgCl + 2Na_2S_2O_3$ sol. $\longrightarrow Na_3[Ag(S_2O_3)_2] + NaCl$
(P) (excess) clear solution

$PbCl_2 + 2KI \longrightarrow PbI_2 \downarrow + 2KCl$
Hot solution (yellow ppt)
(Q)

11. Ans (A)

$\pi = \frac{n_B RT}{V}$; $n_B = \frac{w_B}{M_B}$

$\pi = \frac{w_B}{M_B} \times \frac{RT}{V}$

$M_B = \frac{w_B}{V} \times \frac{RT}{\pi} = \frac{2 \times 0.0821 \times 300 \times 760}{0.3 \times 20}$

$= 6239.6 \text{ gm mol}^{-1}$

12. Ans (A)

Solubility $\propto \frac{1}{K_H}$

13. Ans (B)

Option (A) :

$MgCl_2 + SO_4^{2-} \rightarrow MgSO_4$ Soluble

Option (B) :

$MgCl_2 + HCO_3^- \rightarrow \underset{\text{Soluble}}{Mg(HCO_3)_2} \xrightarrow{\Delta} \underset{\text{White}}{MgCO_3} + H_2O + CO_2$

Option (C) :

$MgCl_2 + CO_3^{2-} \rightarrow MgCO_3$(white ppt)

Option (D) :

$MgCl_2 + NO_3^- \rightarrow Mg(NO_3)_2$(Soluble)

14. Ans (D)

Moles of PCl_5 dissociated $= \frac{2 \times 35}{100} = 0.7$

Moles of PCl_5 left undissociated

$= 2 - 0.7 = 1.3$ mol

$[PCl_5] = \frac{1.3}{5}$M, $[PCl_3] = \frac{0.7}{5}$M, $[Cl_2] = \frac{0.7}{5}$M

$K = \frac{[PCl_3]\,[Cl_2]}{[PCl_5]} = \frac{\left(\frac{0.7}{5}\right)\left(\frac{0.7}{5}\right)}{\left(\frac{1.3}{5}\right)} = 0.075$

15. Ans (C)

Given millimoles of salt/compound

$= 40 \times 0.05 = 2$ mms

(i) Using Hph (phenolphthalein)

2 mms of HCl consumed to convert only Na_2CO_3 portion to $NaHCO_3$.

(ii) Using MeOH (Methyl orange)

6 mms of HCl consumed to convert entire salt to H_2CO_3.

So, $X = \frac{2}{0.05} = 40$ mL $Y = \frac{6}{0.05} = 120$ mL

Hence, $\frac{|Y - X|}{10} = \frac{120 - 40}{10} = 8$

20. Ans (A)

(i) $[Cu(C_2H_5NH_2)_4]^{2+} \Rightarrow$ Blue

(ii) $[Cu(CN)_4]^{-3} \Rightarrow$ Colourless

(iii) $Cu_2[Fe(CN)_4] \Rightarrow$ Chocolate Brown

(iv) $Cu_2I_2 + I_3^- \rightarrow$ Brown

PART-2 : CHEMISTRY

SECTION-II

1. Ans (0)

$Co^{2+} + H_2S \longrightarrow CoS \downarrow$ (Black)

(A)

$CoS + \text{Aqua-regia} \longrightarrow Co^{2+}(aq) + NOCl + S + H_2O$

(A) (B)

$CoCl_2 + 7KNO_2 + 2CH_3COOH \longrightarrow$

$K_3[Co(NO_2)_6] + 2KCl + 2CH_3COOK + NO + H_2O$

(C)

In $K_3[Co(NO_2)_6]$, $Co^{+3} : 3d^6\ 4s^0$

$Co^{3+} : d^2sp^3$ Hybridisation

Number of unpaired $e^- = 0$

Magnetic moment $= \sqrt{n(n+2)} = 0$ B.M

2. Ans (2)

Increase in temperature favours endothermic direction and increase in the pressure favours the direction of decreases in volume (moles of gases). Only (C) and (F) are correct.

3. Ans (6)

$i = 1.25$

Original mole fraction

$= \frac{1}{n} = \frac{1}{1+(n-1)} = \frac{1.25}{1.25+(n-1)} = \frac{1}{5} \Rightarrow n = 6$

4. Ans (5)

$pH = pK_a + \log \frac{[Salt]}{[Acid]}$

$= pK_a + \log \frac{[HX]}{[X^-]}$

$= 14 - pK_b + \log 1$

$= 14 - 9 + 0$

$pH = 5$

PART-3 : MATHEMATICS

SECTION-I

1. Ans (B)

$y = \frac{x-1}{p-x^2+1} \Rightarrow x^2y + x - y(p+1) - 1 = 0$

As $x \in R$ so $D \geq 0 \Rightarrow 4y^2(p+1) + (4y+1) \geq 0$

Since $y \notin \left[-1, -\frac{1}{3}\right]$

So, $4y^2(p+1) + (4y+1) < 0 \quad \forall\ y \in \left[-1, -\frac{1}{3}\right]$

$\Rightarrow (2y+1)^2 + 4y^2p < 0$

$\Rightarrow p < -\left(\frac{2y+1}{2y}\right)^2 \quad \forall\ y \in \left[-1, -\frac{1}{3}\right]$

$\Rightarrow p < -\frac{1}{4}$

2. Ans (D)

$f(x) = 1 + \cos^2\left(\frac{x-\pi}{2\pi^2}\right)$

$\Rightarrow$ Period of $f(x) = \frac{\pi}{\frac{1}{2\pi^2}} = 2\pi^3$

3. Ans (A)

$6 - \lambda = 1 + \mu$

$2\lambda - 1 = 3\mu - 1 \Rightarrow \lambda = 3, \mu = 2$

$\Rightarrow$ so there exist values of 'λ' and 'μ' such that two values of r are same showing that lines intersect and hence they are coplanar

Thus **A** and **R** both are correct and **A** follows from **R**

4. Ans (B)

$\text{sgn}\left(\left[\frac{15}{1+x^2}\right]\right) = [1 + \{2x\}]$

$\Rightarrow 1 + x^2 \leq 15$

$\Rightarrow x^2 \leq 14$

Number of integral values of x are 7.

5. Ans (B)

$f'(x) = x^2 + 2(m-1)x + (m+5)$

$D = 4(m^2 + 1 - 2m - m - 5)$

$D = 4(m-4)(m+1) \leq 0$

$m \in \{-1, 0, 1, 2, 3, 4\}$

$k = 6$

6. Ans (B)

$f'(x) = (3x - 7)(x - 1)$

Many one but onto

7. Ans (B)

$f(x) = \frac{\pi}{2} + \sec^{-1}(x)$

$f(x) \in \left[\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right]$

As $x \in (-\infty, -1) \cup (1, \infty)$

8. Ans (A)

Range of $\tan^{-1}(2x - x^2 + \lambda) \in \left(-\frac{\pi}{2}, 0\right]$

$\Rightarrow \quad 2x - x^2 + \lambda \leq 0$

$\Rightarrow \quad D \leq 0$

$\Rightarrow \quad \lambda \leq -1$

9. Ans (A)

$S_n = 873 + 7I$ (I = integer)

$\frac{S_n}{7} = 124.71 + I$

$7\left[\frac{S_n}{7}\right] = 868 + 7I$

$S_n - 7\left[\frac{S_n}{7}\right] = 5$

Now $\Rightarrow$

(A) $\sin^{-1}(\sin 5) = 5 - 2\pi$

(B) $\cos^{-1}(\cos 5) = 2\pi - 5$

(C) $\tan^{-1}(\tan 5) = 5 - 2\pi$

(D) $\cot^{-1}(\cot 5) = 5 - \pi$

10. Ans (D)

$$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\frac{n}{(n^2 + r(r+1))}\right)$$

$$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\frac{\frac{r+1}{n} - \frac{r}{n}}{1 + \frac{r+1}{n} \cdot \frac{r}{n}}\right)$$

$$S_n = \sum_{r=0}^{n-1} \tan^{-1}\left(\frac{r+1}{n}\right) - \tan^{-1}\left(\frac{r}{n}\right)$$

$S_{100} = \frac{\pi}{4}$

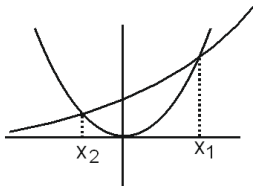
11. Ans (A)

$$f(x) = \left(\sin^{-1}(\sin x) - \frac{1}{2}\right)^2 - \frac{1}{4}$$

For maximum value of f(x), $\sin^{-1}(\sin x) = -\frac{\pi}{2}$

$$f(x)_{max} \Rightarrow \left(\frac{\pi}{2} + \frac{1}{2}\right)^2 - \frac{1}{4}$$

12. Ans (A)



$-1 \leqslant e^x \leqslant 1$

$\Rightarrow x \in (-\infty, 0]$

$-1 \leqslant x^2 \leqslant 1$

$\Rightarrow \quad x \in [-1, 1]$

$\Rightarrow \quad e^x = x^2$ for $x \in [-1, 0]$

$\because$ x_1 is +ve (not acceptable)

$\Rightarrow$ only 1 solution

13. Ans (D)

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$,

then $\vec{a}.\vec{a} + (\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) = \vec{b}.\vec{b} + (\vec{c} - \vec{c}) \cdot (\vec{c} - \vec{a})$

$\Rightarrow -2\vec{b}.\vec{c} = -2\vec{c}.\vec{a}$

$\vec{c}.(\vec{b} - \vec{a}) \Rightarrow \overrightarrow{BA} \cdot \overrightarrow{OC} = 0$

Hence $\overrightarrow{AB} \perp \overrightarrow{OC}$, similarly

$\overrightarrow{BC} \perp \overrightarrow{OA}$ and $\overrightarrow{CA} \perp \overrightarrow{OB}$

14. Ans (B)

Let $\bar{r}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$\bar{r}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$|\vec{r}_1 \times \vec{r}_2|^2 \leqslant |r_1|^2|r_2|^2$ ...(1)

$$\bar{r}_1 \times \bar{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 3 & 4 & 5 \end{vmatrix}$$

$\Rightarrow \hat{i}(5b - 4c) + \hat{j}(3c - 5a) + \hat{k}(4a - 3b)$ from (1)

$(5b - 4c)^2 + (3c - 5a)^2 + (4a - 3b)^2 \leqslant 50$

15. Ans (C)

$\left|\left(\hat{a}+\hat{b}\right)+2\left(\hat{a}\times\hat{b}\right)\right|=2, \theta\in(0,\pi)$

$\left(\left(\hat{a}+\hat{b}\right)+2\left(\hat{a}\times\hat{b}\right)\right).\left(\left(\hat{a}+\hat{b}\right)+2\left(\hat{a}\times\hat{b}\right)\right)=4$

$\left|\hat{a}+\hat{b}\right|^2+4\left|\left(\hat{a}\times\hat{b}\right)\right|^2+0=4$

Let the angle be θ between $\hat{a}$ and $\hat{b}$

$2+2\cos\theta+4\sin^2\theta=4$

$2+2\cos\theta-4\cos^2\theta=0$

Let $\cos\theta=t$ then

$2t^2-t-1=0$

$\Rightarrow t=-\frac{1}{2}$ or $t=1$

$\cos\theta=-\frac{1}{2}$ or $\cos\theta=1$

$\theta=\frac{2\pi}{3}$ Not possible $\theta\in(0,\pi)$

S_1 $2\left|\vec{a}\times\vec{b}\right|=2\sin\left(\frac{2\pi}{3}\right)$

$\left|\hat{a}-\hat{b}\right|=\sqrt{1+1-2\cos\left(\frac{2\pi}{3}\right)}=\sqrt{2-2\times\left(-\frac{1}{2}\right)}$

$=\sqrt{3}$

S_1 is correct

S_2 projection of $\hat{a}$ on $\left(\hat{a}+\hat{b}\right)$

$\frac{\hat{a}.\left(\hat{a}+\hat{b}\right)}{\left|\hat{a}+\hat{b}\right|}=\frac{1+\cos\left(\frac{2\pi}{3}\right)}{\sqrt{2+2\cos\frac{2\pi}{3}}}=\frac{1-\frac{1}{2}}{\sqrt{1}}=\frac{1}{2}$

16. Ans (C)

Put $z=0$ in line equation

$\frac{x-2}{3}=\frac{y+1}{2}=\frac{0-1}{-1}$

$\Rightarrow$ $x=5, y=1$

Put these in $xy=c^2$ $\Rightarrow$ $c^2=5$ $\Rightarrow$ $c=\pm\sqrt{5}$

17. Ans (B)

$-2(b+c)^2-bc=0$

$\Rightarrow$ $2b+c=0$ or $b+2c=0$

If $2b+c=0$ $\Rightarrow$ $a=-(b+c)\Rightarrow a=b$

$\Rightarrow$ $a=b$ and $c=-2b$

$\frac{a}{1}=\frac{b}{1}=\frac{c}{-2}$

If $b+2c=0$ then $a=-(b+c)\Rightarrow a=c$

$\frac{a}{1}=\frac{b}{-2}=\frac{c}{1}$

$\cos\theta=\frac{1-2-2}{\sqrt{1+1+4}\sqrt{1+4+1}}=-\frac{1}{2}$ $\Rightarrow\theta=\frac{2\pi}{3}$

18. Ans (C)

Equation of plane is $(\vec{r}-\vec{a}).\left((\vec{a}-\vec{b})\times\vec{c}\right)=0$

$\Rightarrow$ $\vec{r}.\left((\vec{a}\times\vec{c})-(\vec{b}\times\vec{c})\right)=-\vec{a}(\vec{b}\times\vec{c})$

$\Rightarrow$ $\vec{r}.(\vec{b}\times\vec{c}+\vec{c}\times\vec{a})-[\vec{a}\ \vec{b}\ \vec{c}]=0$

Length of perpendicular from origin to this plane

$\left|\frac{0.(\vec{b}\times\vec{c}+\vec{c}\times\vec{a})-[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}\right|$

$\Rightarrow$ $\frac{\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]}{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}$

19. Ans (D)

$n(A\cup B\cup C)=\Sigma n(A)-\Sigma n(A\cap B)+(A\cap B\cap C)$

$Sn(A\cap B)=36$

Number of students who got exactly 2 medals

$\Rightarrow$ $36-15=21$

20. Ans (B)

R_1 is not transitive for x=+2, y = 0, z = –2 and

R_2 is not symmetrical as $a\geq b$ does not implies that

$b\geq a$. Both R_1 and R_2 are not equivalence relations

PART-3 : MATHEMATICS

SECTION-II

1. Ans (2016)

$$g(x) = \left(\frac{3}{2} - x^9\right)^{1/9}$$

$g(g(x)) = x$

$g(g(2016)) = 2016$

2. Ans (10)

$$\cos^{-1}\left(\cos\left(\frac{-14\pi}{5}\right)\right) = \cos^{-1}\left(\cos\left(\frac{4\pi}{5}\right)\right) = \frac{4\pi}{5}$$

$$\text{so, } \left(\frac{1}{2}\frac{4\pi}{5}\right) = \cos\left(\frac{2\pi}{5}\right) = \sin\left(\frac{\pi}{10}\right)$$

3. Ans (3)

$$\begin{vmatrix} \alpha & \alpha+\beta & \beta \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{\alpha}{\beta} = -3$$

4. Ans (19)

$$\sin\theta = \left(\frac{1.4 + (-3) + 1.5}{\sqrt{3}\sqrt{50}}\right) = \sqrt{\frac{6}{25}}$$

$b - a = 25 - 6 = 19$

5. Ans (12)

$n(A \times A) = 16$

Any reflexive relation must have (1, 1) (2, 2) ..... (m, m) i.e. (m) elements may contain any number of element out of (12)

$12C_0 + 12C_1 + 12C_2 + \ldots..+12C_{12} \Rightarrow 2^{12}$

$2^{\lambda} = 2^{12}$

$\lambda = 12$