Test Pattern

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2024 - 2025)

JEE (Advanced)

PART TEST

22-12-2024

JEE(Main+Advanced) : ENTHUSIAST COURSE (SCORE-I)

ANSWER KEY

PAPER-2 (OPTIONAL)

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	D	A	B	C		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,B,C,D	A,C	A,B,C			
SECTION-I (iii)	Q.	8	9	10	11		
	A.	D	B	A	A		
SECTION-II	Q.	1	2	3	4	5	6
	A.	2	5	1	2	3	3

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4		
	A.	C	A	A	B		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,D	D	A,B,C			
SECTION-I (iii)	Q.	8	9	10	11		
	A.	A	B	D	A		
SECTION-II	Q.	1	2	3	4	5	6
	A.	4	50	737	120	11	7

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4		
	A.	A	A	B	B		
SECTION-I (ii)	Q.	5	6	7			
	A.	A,B,C	A,D	A,B			
SECTION-I (iii)	Q.	8	9	10	11		
	A.	D	A	D	C		
SECTION-II	Q.	1	2	3	4	5	6
	A.	3	6	3	6	466	6

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (D)**

Focal length for upper half is,

$$f_1 = \left(\frac{\mu - 1}{\mu/\mu_1 - 1}\right) f_{air} = \left(\frac{1.5 - 1}{\frac{1.5}{1.2} - 1}\right) 20 = 40\text{cm}$$

Focal length for lower half is,

$$f_2 = \left(\frac{\mu - 1}{\mu/\mu_1 - 1}\right) f_{air} = \frac{1.5 - 1}{\frac{1.5}{2.5} - 1} \times 20 = -25\text{cm}$$

If the object is at infinity, two will form at corresponding focuses.

So, the required separation is,

$x = |f_1| + |f_2| = 40 + 25 = 65$ cm

2. **Ans (A)**

The moment of inertia will be least about the centre of mass of the rod.

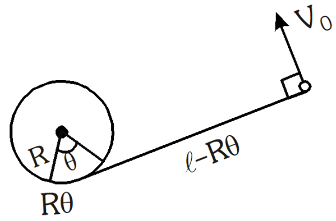
$$X_{cm} = \frac{1}{m}\int x dm = \frac{1}{m}\int_0^L (1 + kx)\,dx = \frac{L^2}{2} + \frac{KL^3}{3}$$

$$M = \int dm = \int_0^L (1 + kx)\,dx = \left(L + \frac{KL^2}{2}\right)$$

$$X_{cm} = \frac{\frac{L^2}{2} + \frac{KL^3}{3}}{L + \frac{KL^2}{2}} = \frac{\frac{L}{2} + \frac{KL^2}{3}}{1 + \frac{KL}{2}} = \frac{\frac{3L+2KL^2}{6}}{\frac{2+KL}{2}}$$

$$X_{cm} = \left(\frac{3L + 2KL^2}{6 + 3KL}\right)$$

3. **Ans (B)**



$$w = \frac{v_0}{\ell - R\theta}$$

$$\frac{d\theta}{dt} = \frac{v_0}{(\ell - R\theta)}$$

$$\int_0^{\ell/R} d\theta\,(\ell - R\theta) = \int_0^t v_0\,dt$$

$$\frac{\ell^2}{2R} = v_0 t$$

$$t = \frac{\ell^2}{2v_0 R}$$

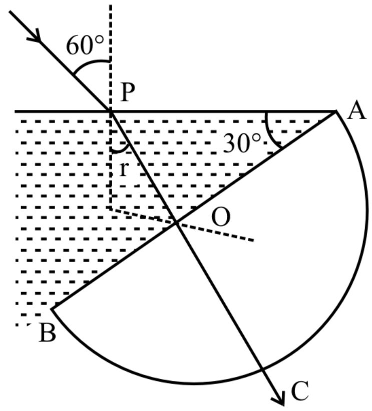
Distance travelled $= v_0 t = \dfrac{\ell^2}{2R}$ ...(i)

$$\alpha = w\frac{dw}{d\theta} = \frac{v_0}{\ell - R\theta} \times \frac{v_0 R}{(\ell - R\theta)^2} = \frac{v_0^2 R}{(\ell - R\theta)^3}$$

for $\theta = 0$

$$\alpha = \frac{v_0^2 R}{\ell^3}$$

4. **Ans (C)**



Ray is emerging from hemisphere at 0°, ray OC should be incident on curved surface radially means OC is coming from centre of hemisphere i.e., AO = R

Applying Snell's law at P

$\sin 60^\circ = \sqrt{3}\sin r \Rightarrow r = 30^\circ$

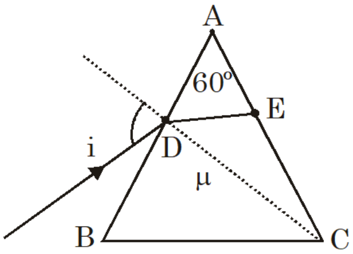
So $\angle POA = 90^\circ$

$$\Rightarrow PA = \frac{2R}{\sqrt{3}}$$

PART-1 : PHYSICS

SECTION-I (ii)

5. Ans (A,B,C,D)



c_r = critical Angle $= \sin^{-1}\left(\frac{1}{\mu}\right) = 45°$

For minimum distance of E from A, r, should be

max possible $\Rightarrow r_1 = c_r = 45$

So $\angle ADE = 45$

$\angle DAE = 60$

$AD = \frac{d}{2}$

$AE = ?$

By sine rule

$$\frac{AE}{\sin 45} = \frac{AD}{\sin 75}$$

$$AE = \frac{d}{2}\frac{\sin 45}{\sin 75} = \frac{d}{2}\frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= \frac{d}{2}\left(\sqrt{3}-1\right)$$

(B) $\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin(A/2)} \Rightarrow \delta_{min} = 30$

at $\mu = \sqrt{2}$

(C) with $\mu = 2$, at 90° Angle of incidence, their is emergent ray present.

6. Ans (A,C)

$$\frac{\mu}{v} - \frac{1}{\infty} = \frac{(\mu-1)}{10}$$

$$v = \frac{10\mu}{\mu-1} = \frac{10}{1-\frac{1}{\mu}}$$

$v_v < v_r$

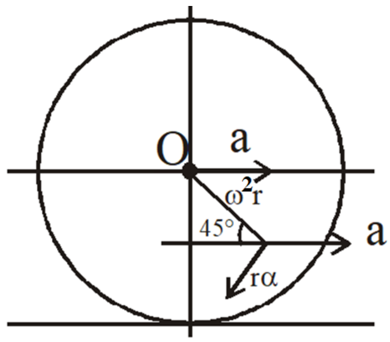
$$v_v = \frac{10}{0.615} \times 1.615$$

$$v_r = 10 \times \frac{1.6}{0.6}$$

$$v_v - v_r = \frac{16.15}{0.615} - \frac{16}{0.6} = \frac{16.15 \times 0.6 - 16 \times 0.615}{0.6 \times 0.615}$$

$= 0.40$cm

7. Ans (A,B,C)



As shown in figure, $a_{net} = 0$ when

$r\alpha = \omega^2 r \qquad \ldots(1)$

$\Rightarrow r.\frac{a}{R} = \omega^2 r$

$\Rightarrow \frac{a}{R} = \alpha^2 t^2 = \frac{a^2}{R^2}t^2$

$\Rightarrow at^2 = R \Rightarrow S = \frac{1}{2}at^2 = \frac{R}{2}$

Also , $\frac{r\alpha}{\sqrt{2}} + \frac{\omega^2 r}{\sqrt{2}} = a \Rightarrow r\alpha\sqrt{2} = a$

$\Rightarrow r\frac{a}{R}\sqrt{2} = a \Rightarrow \frac{R}{r} = \sqrt{2}$

Also, $\theta = \frac{1}{2}\alpha t^2 = \frac{a}{2R}t^2 = \frac{1}{2}$rad $= 0.5$rad

PART-1 : PHYSICS

SECTION-I (iii)

8. Ans (D)

(P) $\frac{\mu_s'}{v}-\frac{\mu_s}{u}=\frac{\mu_L-\mu_s}{R_1}-\frac{\mu_L-\mu_s'}{R_2}$

$\frac{1}{f}=\frac{\mu_L-\mu_s}{\mu_s' R_1}-\frac{\mu_L-\mu_s'}{\mu_s' R_2}$

$\frac{1}{f}=\frac{1}{R}\left[\frac{1.5-1.0}{1.4}+\frac{1.5-1.4}{1.4}\right]$

$\frac{1}{f}=\frac{1}{1.4R}[0.5+0.1]$

$\frac{1}{f}=\frac{6}{14R}=\frac{3}{7R}$

$f=\frac{7R}{3}$

(Q) $\frac{1}{(f_{eq})_m}=\frac{1}{f_m}-\frac{2}{f_L}=-\frac{2}{R}-\frac{2}{f_L}$

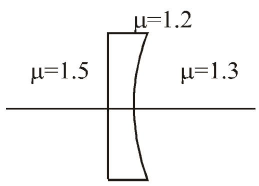
$\frac{1}{f_L}=\left(\frac{\mu_L}{\mu_s}-1\right)\frac{2}{R}=\left(\frac{1.5}{1.4}-1\right)\frac{2}{R}=\frac{0.1}{1.4}\times\frac{2}{R}$

$\frac{1}{f_L}=\frac{1}{7R}$

$\frac{1}{(f_{eq})_m}=-\frac{2}{R}-\frac{2}{7R}=-\frac{16}{7R}$

$\Rightarrow (f_{eq})_m=-\frac{7R}{16}$

(R)



$\frac{1}{f}=\left[\frac{1.2-1.5}{1.3\times\infty}-\frac{(1.2-1.3)}{1.3\times R}\right]$

$\frac{1}{f}=\frac{1}{13R}$

(S) $f=13\,R$

$\frac{1}{f_1}=\left(\frac{1.2}{1.3}-1\right)\left(-\frac{2}{R}\right)=+\frac{0.1}{1.3}\left(\frac{2}{R}\right)=\frac{2}{13R}$

$\frac{1}{f_2}=\left(\frac{1.4}{1.3}-1\right)\left(\frac{2}{R}\right)=\frac{2}{13R}$

$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{4}{13R}$

$\frac{1}{f_{eq}}=\frac{13R}{4}$

9. Ans (B)

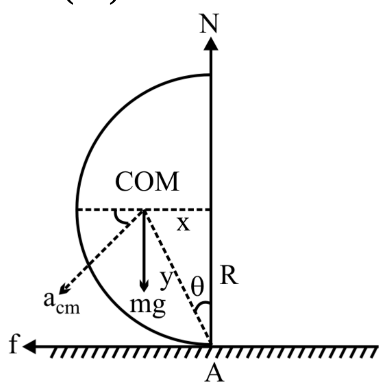
$\frac{\frac{4}{3}-1}{-1}=\frac{4}{3V}-\frac{1}{-2}\Rightarrow V=-\frac{8}{5}$

We know, $m=\frac{\mu_1 v}{\mu_2 u}=\frac{3}{5}$

Since $v=-ve$ & $|m|<1$

Hence image is virtual & diminished.

10. Ans (A)



A is instantaneous centre of rotation.

$mgx=I_A\alpha\Rightarrow\alpha=\frac{mgx}{I_A}$ $I_A=I_{cm}+mR^2$

$a_{cm}=\alpha y=\frac{mgxy}{I_A}$

$mg-N=ma_{cm}\sin\theta\Rightarrow N=mg-ma_{cm}\frac{x}{y}$

and $f=ma_{cm}\cos\theta\Rightarrow f=ma_{cm}\frac{R}{Y}$

$\mu_{min}=\frac{f}{N}$

For semicircular homogenous ring

$x=\frac{2R}{\pi}, I_A=2mR^2, y=R\sqrt{1+\frac{4}{\pi^2}}$

$\alpha=\frac{g}{\pi R}$

$a_{cm}=\frac{g}{\pi}\sqrt{1+\frac{4}{\pi^2}}$

$N=\frac{4}{5}mg=10N$

$f=\frac{mg}{\pi}=3.98N$

$\mu_{min}=\frac{f}{N}=\frac{5}{4\pi}$

For semicircular homogenous disc

$x=\frac{4R}{3\pi}, I_A=\frac{3}{2}mR^2, y=R\sqrt{1+\frac{16}{9\pi^2}}$

$\alpha=\frac{8g}{9\pi R}$

$a_{cm}=\frac{8g}{9\pi}\sqrt{1+\frac{16}{9\pi^2}}$

$N = \frac{119}{135}mg = 10N$

$f = \frac{8mg}{9\pi} = 3.21N$

$\mu_{min} = \frac{f}{N} = \frac{120}{119\pi}$

For homogenous hollow sphere

$x = \frac{R}{2}, I_A = \frac{5}{3}mR^2, y = R\frac{\sqrt{5}}{2}$

$\alpha = \frac{3g}{10R}$

$a_{cm} = \frac{3g}{4\sqrt{5}}$

$N = \frac{17}{20}mg = 10N$

$f = \frac{3mg}{10} = 3.53N$

$\mu_{min} = \frac{f}{N} = \frac{6}{17}$

For homogenous solid sphere

$x = \frac{3R}{8}, I_A = \frac{7}{5}mR^2, y = R\frac{\sqrt{73}}{8}$

$\alpha = \frac{15g}{56R}$

$a_{cm} = \frac{15\sqrt{73}}{448}g$

$N = \frac{403}{448}mg = 10N$

$f = \frac{15mg}{56} = 2.98N$

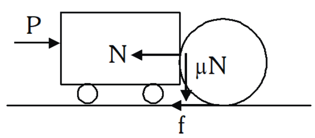
$\mu_{min} = \frac{f}{N} = \frac{120}{403}$

∴ (A) - (p), (r); (B) - (q), (r); (C) - (r), (s); (D) - (r), (t)

PART-1 : PHYSICS

SECTION-II

1. **Ans (2)**



$P - N = 10a$

$N - f = 2a$

$(f - \mu N)R = \frac{1}{2}(2)(0.1)^2\left(\frac{a}{R}\right)^2$

Solving P = 32N

3. **Ans (1)**

Lei the angular acceleration of the smaller ball be α_1 that of the larger one α_2, their common horizontal acceleration being a_1 and the acceleration of the cart a_2. As the balls are rolling without slipping,

We have $R\alpha_2 = a_2 - a_1$

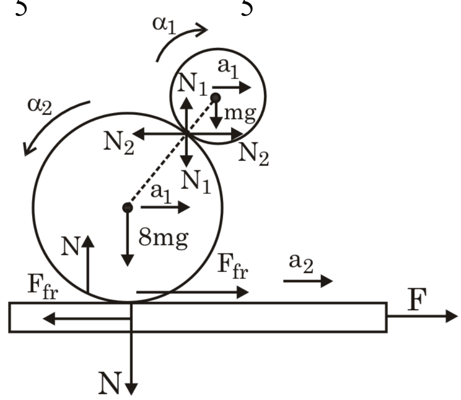
and $R\alpha_2 = r\alpha_1$

and, because $R = 2r$

$\alpha_1 = 2\alpha_2 = \frac{a_2 - a_1}{r}$

The moment of inertia of the smaller ball is $\frac{2}{5}mr^2$, while that of the larger one with the same density is

$\frac{2}{5} \times 8m \times (2r)^2 = \frac{64}{5}mr^2$



Using the free body diagram of the figure, we can write the following equations of motion :

$F - F_{fr} = Ma_2$

$8mg + N_1 - N = 0, F_{fr} - N_2 = 8\ ma_1$

$mg - N_1 = 0, N_2 = ma_1$

$N_1 r\cos\phi - N_2 r\sin\phi = \frac{2}{5}mr^2\alpha_1$

$2rF_{ft} + 2rN_2 r\sin\phi - 2rN_t r\cos\phi = \frac{64}{5}mr^2\alpha_2$

From these equations we can express the force F as

$F = \left(9m + \frac{7}{2}M\right)\frac{\cos\phi}{1+\sin\phi}g = 79N$

The acceleration of the balls relative to the cart is

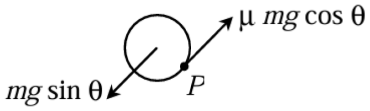
$\Delta a = a_2 - a_1 = \frac{5}{2}\frac{\cos\phi}{1+\sin\phi}g$

At the time t when the balls fall from the cart, the distance they have moved relative to the cart is L/2. As their initial velocities are zero,

$t = \sqrt{\frac{L}{\Delta a}} = 0.55s$

$\Rightarrow t + 0.45 = 1s$

4. **Ans (2)**



Coefficient of friction for rolling is

$$\mu \geq \frac{\tan\theta}{1+\frac{mR^2}{I}} . \mu >= 0.3$$

Here $\mu = 0.2$ so the sphere will slide.

$$a_{cm} = g(\sin\theta - \mu\cos\theta)\theta,\ \alpha = \frac{3}{2}\frac{\mu g\cos\theta}{k}$$

$$V_{cm} = g(\sin\theta - \mu g\cos\theta)\,t,\ \omega = \frac{3}{2}\frac{\mu g\cos\theta}{R}t$$

$$V_p = V_{cm} - R\omega = 2t$$

5. **Ans (3)**

Due to first refraction image will be formed at $\frac{4}{3}d$ from plane surface. This will act as an object for curved surface.

$$\frac{1}{v} + \frac{4/3}{\left(4+\frac{4}{3}d\right)} = \frac{1-\frac{4}{3}}{-4}$$

For image at infinity, $v = -3R$

$$\Rightarrow -\frac{1}{3\times(4)} + \frac{4/3}{4\left(1+\frac{d}{3}\right)} = \frac{1}{12}$$

$$\frac{1}{3\left(1+\frac{d}{3}\right)} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

$$\Rightarrow 1 + \frac{d}{3} = 2$$

$d = 3\text{cm}$

6. **Ans (3)**

Torque acting on the rod is $\tau = \mu mg\frac{\ell}{2}$

Now work done = change in K.E.

$$= \frac{-1}{2}I\omega^2$$

From (1) and (2),

$$\mu mg\frac{\ell}{2} \times \theta = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\therefore \theta = \frac{\omega^2\ell}{3\mu g} = \frac{\omega^2\ell}{n\mu g}(\text{given})$$

PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (C)**

Since ice has least entropy the sublimation of ice would be experiencing the greatest entropy increase.

2. **Ans (A)**

Sulphur dioxide is responsible for stiffness in flower buds.

PART-2 : CHEMISTRY

SECTION-I (ii)

5. **Ans (A,D)**

Hydrogen resembles more with halogens due to high ionisation energy and covalency
Hydrogen has three isotopes
Tritium emits low energy beta rays
Dihydrogen is quite stable due to high bond energy

PART-2 : CHEMISTRY

SECTION-I (iii)

8. **Ans (A)**

Theory based.

9. **Ans (B)**

Theory based.

10. **Ans (D)**

Benzene + toluene form ideal liquid mixture.
Acetic acid dimerises in benzene
Water + nitric acid & Chloroform + Acetone show negative deviation from Raoult's law

11. **Ans (A)**

Acetate shows hydrolysis.
Sodium ion is stable.
Chloride ion is stable.

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (4)**

Canary yellow precipitate is $(NH_4)_3PO_4.12MoO_3$

2. **Ans (50)**

75% of sunlight reaches the earth surface while 25% is absorbed by the greenhouse gases.

3. **Ans (737)**

$$SO_4^{2-} + H_2O \rightleftharpoons HSO_4^- + OH^-; \quad K_h = \frac{K_{a_1}}{K_{a_2}}$$

$$\left(\frac{0.2}{3} - X\right)M \qquad XM \qquad XM$$

$$\frac{10^{-14}}{1.2\times10^{-2}} = \frac{x.x}{\left(\frac{0.2}{3}\right)} \Rightarrow x = \sqrt{\frac{10^{-12}}{18}}$$

$pOH = -\log x = 6.63$

$\therefore pH = 14 - 6.63 = 7.37$

4. **Ans (120)**

$AgCl(s) \leftrightharpoons Ag^+(aq) + Cl^-(aq)$

$Q = [Ag^+][Cl^-]$

$$= \left(\frac{25\times4\times10^{-5}}{100}\right)\times\left(\frac{75\times2\times10^{-5}}{100}\right)$$

$= 1.5\times10^{-10} < K_{sp}$ (No precipitation)

Hence, final total effective concentration

$$= 2\times\left(\frac{25\times4\times10^{-5}}{100}\right) + 2\times\left(\frac{75\times2\times10^{-5}}{100}\right)$$

$= 5\times10^{-5}$ M

Now, $\pi = CRT = 5\times10^{-5}\times0.08\times300$

$= 1.2\times10^{-3}$ bar = 120 Pa.

5. **Ans (11)**

$$B(OH)_2 \quad + \quad H^+ \quad \rightleftharpoons B(OH)^+ + H_2O$$

$$25\times0.2 \qquad 6.25\times0.4$$

$$= 5 \text{ mmole} \qquad = 2.5 \text{ mmole}$$

Final 2.5 m mole 0 2.5 m mole

$$pOH = pK_{b_1} + \log\frac{[B(OH)^+]_0}{[B(OH)_2]_0}$$

$$\text{or, } 14 - 9.0 = pK_{b_1} + \log\frac{2.5}{2.5} \Rightarrow pK_{b_1} = 5.0$$

$$B(OH)_2 \quad + \quad H^+ \quad \rightleftharpoons \quad B(OH)^+ + H_2O$$

$$25\times0.2 \qquad 12.5\times0.4$$

$$= 5 \text{ mmole} \qquad = 5 \text{ mmole}$$

Final 0 0 5 mmole

$$pOH = \frac{1}{2}(pK_{b_1} + pK_{b_2})$$

$$\text{or, } 14 - 6.0 = \frac{1}{2}(5 + pK_{b_2})$$

$\therefore pK_{b_2} = 11$

6. **Ans (7)**

$$Ag_2O + 4NH_3 \rightarrow \underset{\text{Soluble}}{[Ag(NH_3)_2]^+}$$

$$Hg^{2+} + \text{dil. HCl} \rightarrow \underset{\text{Soluble}}{HgCl_2}$$

In all other cases, precipitates are formed.

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (A)**

We have $f(x+y) + f(x-y)$

$$= \frac{1}{2}\left[a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}\right]$$

$$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right]$$

$$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y}) = 2f(x)f(y)$$

2. **Ans (A)**

Line passing through the point (1, 2, –4) is

$$\frac{x-1}{1} = \frac{y-2}{m} = \frac{z+4}{n}$$

Now, according to question, $31 - 16m + 7n = 0$ and

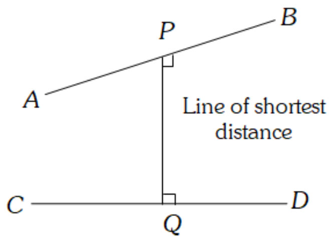
$31 + 8m - 5n = 0$

Hence required line is, $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$.

3. **Ans (B)**

Let the two lines be *AB* and *CD* having equation $\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1} = \lambda$ and $\frac{x+a}{2} = \frac{y}{1} = \frac{z}{1} = \mu$

then $P \equiv (\lambda, \lambda - a, \lambda)$ and $Q = (2\mu - a, \mu, \mu)$



So according to question,

$$\frac{\lambda - 2\mu + a}{2} = \frac{\lambda - a - \mu}{1} = \frac{\lambda - \mu}{2}$$

$\Rightarrow$ m = a and $\lambda = 3a$

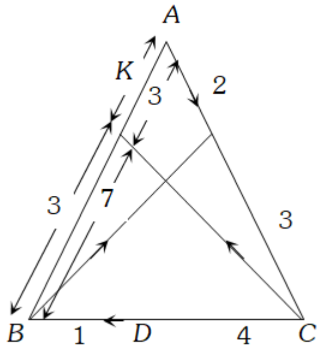
$\therefore$ $P \equiv (3a, 2a, 3a)$ and $Q \equiv (a, a, a)$

4. Ans (B)

Let $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AC} = \mathbf{b}$

So, $\overrightarrow{AD} = \dfrac{4a+b}{5}$, $\overrightarrow{AE} = \dfrac{2b}{5}$, $\overrightarrow{AF} = \dfrac{3a}{10}$ and

$\overrightarrow{AK} = \dfrac{a}{4}$



$$\frac{\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}}{\overrightarrow{CK}} = \frac{\frac{b+4a}{5}+\frac{2b-5a}{5}+\frac{3a-10b}{10}}{\frac{a-4b}{4}}$$

$$= \frac{6b-2a+3a-10b}{10(a-4b)} \times 4 = \frac{2}{5}$$

PART-3 : MATHEMATICS

SECTION-I (ii)

5. Ans (A,B,C)

A. $\sin\left(\tan^{-1}3+\tan^{-1}\dfrac{1}{3}\right) = \sin\dfrac{\pi}{2} = 1$

B. $\cos\left(\dfrac{\pi}{2}-\sin^{-1}\dfrac{3}{4}\right) = \cos\left(\cos^{-1}\dfrac{3}{4}\right) = \dfrac{3}{4}$

C. $\sin\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\dfrac{\sqrt{63}}{8} = \theta$

$\Rightarrow \sin\theta = \dfrac{\sqrt{63}}{8} \Rightarrow \cos\theta = \dfrac{1}{8}$

We have $\cos\dfrac{\theta}{2} = \sqrt{\dfrac{1+\cos\theta}{2}} = \dfrac{3}{4}$

$\Rightarrow \sin\dfrac{\theta}{4} = \sqrt{\dfrac{1-\cos\frac{\theta}{2}}{2}} = \dfrac{1}{2\sqrt{2}}$

Now, $\log_2 \sin\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right) = \log_2\dfrac{1}{2\sqrt{2}} = -\dfrac{3}{2}$

D. $\cos^{-1}\dfrac{\sqrt{5}}{3} = \theta \Rightarrow \cos\theta = \dfrac{\sqrt{5}}{3}$.

Hence, $\tan\dfrac{\theta}{2} = \dfrac{3-\sqrt{5}}{2}$ which is irrational.

6. Ans (A,D)

Let $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$

$= (\sin^{-1}x+\cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$

$= \dfrac{\pi^2}{4} - 2\sin^{-1}x\left[\dfrac{\pi}{2}-\sin^{-1}x\right]$

$= \dfrac{\pi^2}{4} - \pi\sin^{-1}x + 2(\sin^{-1}x)^2$

$= 2\left[\left(\sin^{-1}x\right)^2 - \dfrac{\pi}{2}\sin^{-1}x + \dfrac{\pi^2}{8}\right]$

$= 2\left(\sin^{-1}x-\dfrac{\pi}{4}\right)^2 + 2\left[\dfrac{\pi^2}{16}\right]$

Now, $-\dfrac{\pi}{2} \leqslant \sin^{-1}x \leqslant \dfrac{\pi}{2}$

$\Rightarrow \quad -\dfrac{3\pi}{4} \leqslant \sin^{-1}x - \dfrac{\pi}{4} \leqslant \dfrac{\pi}{4}$

$\Rightarrow \quad 0 \leqslant \left(\sin^{-1}x-\dfrac{\pi}{4}\right)^2 \leqslant \dfrac{9\pi^2}{16}$

$\Rightarrow \quad 0 \leqslant 2\left(\sin^{-1}x-\dfrac{\pi}{4}\right)^2 \leqslant \dfrac{9\pi^2}{8}$

$\Rightarrow \quad \dfrac{\pi^2}{8} \leqslant \left(\sin^{-1}x-\dfrac{\pi}{4}\right)^2 + \dfrac{2\pi^2}{16} \leqslant \dfrac{5\pi^2}{4}$

Hence, range of the given function is $\left[\dfrac{\pi^2}{8}, \dfrac{5\pi^2}{4}\right]$.

7. Ans (A,B)

$f(x) = [x]^2 + [x] + 1 - 3 = \{[x]+2\}\ \{[x]-1\}$

So, $x = 1, 1.1, 1.2$ ..... $\Rightarrow f(x) = 0$

PART-3 : MATHEMATICS

SECTION-I (iii)

8. **Ans (D)**

(P) In (0, cos 1), we have $\cos^{-1}x > \sin^{-1}x$

Also $\cos^{-1}x > 1$ and $\sin^{-1}x < 1$

The greatest is $(\cos^{-1}x)^{\cos^{-1}x} = t_4$ and the least is

$(\sin^{-1}x)^{\cos^{-1}x} = t_2$

and $(\sin^{-1}x)^{\sin^{-1}x} < (\cos^{-1}x)^{\sin^{-1}x}$

$\Rightarrow t_1 < t_3$

So, $t_4 > t_3 > t_1 > t_2$

(Q) Similarly, in $\cos 1 < x < \frac{1}{\sqrt{2}}$

$\cos^{-1}x > \sin^{-1}x$ and both are less than 1 so,

greatest is t_3 and least is t_2 and $t_4 > t_1$

Hence , $t_3 > t_4 > t_1 > t_2$

(R) for $\frac{1}{\sqrt{2}} < x < \sin 1$

We have, $1 > \sin^{-1}x > \cos^{-1}x$

so, greatest is t_2 and least is t_3 also $t_1 > t_4$

Hence, $t_2 > t_1 > t_4 > t_3$

(S) For $\sin 1 < x < 1$ we have

$\sin^{-1}x > 1 > \cos^{-1}x$

So, the greatest is t_1 and least is t_3 and $t_2 > t_4$

Hence, $t_1 > t_2 > t_4 > t_3$

9. **Ans (A)**

(P) for 1,3,5,7 ∈ A we have 4,3,2,1 choice respectively.

(Q) Image of $f(7)$ should be greater then 9.

(R) for 1,3,5,7 ∈ A we have 2,2,2,2 choice respectively.

(S) for any i ∈ A we have 3 choices.

10. **Ans (D)**

(P) $[\vec{a}\,\vec{b}\,\vec{c}] = 2$

$\Rightarrow [2\vec{a}\times\vec{b}\;\; 3\vec{b}\times\vec{c}\;\; \vec{c}\times\vec{a}] = 6[\vec{a}\,\vec{b}\,\vec{c}] = 6\times 4 = 24$

(Q) $[\vec{a}\,\vec{b}\,\vec{c}] = 5$

$\Rightarrow [3(\vec{a}+\vec{b})\vec{b} + \vec{c}2(\vec{c}+\vec{a})]$

$= 6[(\vec{a}+\vec{b})\vec{b} + \vec{c}(\vec{c}+\vec{a})] = 12[\vec{a}\,\vec{b}\,\vec{c}] = 60$

(R) Given $\frac{1}{2}|\vec{a}\times\vec{b}| = 20$

Now $\frac{1}{2}|2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})|$

11. **Ans (C)**

(P) If the required image is (x, y, z) then

$$\frac{x-3}{2} = \frac{y-5}{1} = \frac{z-7}{1} = \frac{-2(6+5+7+18)}{2^2+1^2+1^2} = -12$$

or $(x, y, z) \equiv (-21, -7, -5)$

(Q) point on line is of the form $(2-3\lambda, 1-2\lambda, 3+2\lambda)$

$\Rightarrow 2(2-3\lambda) + (1-2\lambda) - (3+2\lambda) - 3 = 0$

$\Rightarrow \lambda = -\frac{1}{10}$

$\Rightarrow$ required point is $\left(\frac{23}{10}, \frac{6}{5}, \frac{14}{5}\right)$

(R) If (x, y, z) is the required foot of the perpendicular, then

$$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = \frac{-(2-2+8+5)}{2^2+(-2)^2+4^2}$$

or $(x, y, z) \equiv \frac{-1}{12}, \frac{25}{12}, \frac{-2}{12}$

(S) Any point on the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

is $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$ which satisfies the line

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$$

or $\frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3}{1}$

or $\lambda = -1$

The required point is (–1, –1, –1)

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (3)**

$\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}$

$=\tan^{-1}\frac{x-y}{1+xy}+\tan^{-1}\frac{y-z}{1+yz}+\pi+\tan^{-1}\frac{z-x}{1+zx}$

$\left[\because \tan^{-1}\left(\frac{1}{x}\right)=\begin{cases}\cot^{-1}x, & \text{for } x>0\\ -\pi+\cot^{-1}x, & \text{for } x<0\end{cases}\right]$

$=\tan^{-1}x-\tan^{-1}y+\tan^{-1}y-\tan^{-1}z+\pi+$

$\tan^{-1}z-\tan^{-1}x$

$=\pi$

2. **Ans (6)**

Clearly, the domain of the function is $[-1, 1]$.

Also $\tan^{-1}x \in \left[-\frac{\pi}{4},\frac{\pi}{4}\right]$ for $x \in [-1, 1]$.

Now $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ for $x \in [-1, 1]$.

Thus $f(x)=\tan^{-1}x+\frac{\pi}{2}$, where $x \in [-1, 1]$.

Hence, the range is

$\left[-\frac{\pi}{4}+\frac{\pi}{2},\frac{\pi}{4}+\frac{\pi}{2}\right]=\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$.

3. **Ans (3)**

Let direction cosines be ℓ, ℓ, ℓ

$\ell^2+\ell^2+\ell^2=1 \Rightarrow \ell=\frac{1}{\sqrt{3}}$

equation of line

$\frac{x-2}{\frac{1}{\sqrt{3}}}=\frac{y+1}{\frac{1}{\sqrt{3}}}=\frac{z-2}{\frac{1}{\sqrt{3}}}=\lambda$ (say)

Let $Q\left(\frac{\lambda}{\sqrt{3}}+2,\frac{\lambda}{\sqrt{3}}-1,\frac{\lambda}{\sqrt{3}}+2\right)$

$\Rightarrow 2\left(\frac{\lambda}{\sqrt{3}}+2\right)+\left(\frac{\lambda}{\sqrt{3}}-1\right)+\left(\frac{\lambda}{\sqrt{3}}+2\right)=9$

$\Rightarrow \frac{4\lambda}{\sqrt{3}}=4 \Rightarrow \boxed{\lambda=\sqrt{3}}$

$\Rightarrow Q(3, 0, 3)$

$PQ=\sqrt{3}=\ell \quad \Rightarrow \boxed{\ell^2+3}$

4. **Ans (6)**

We have $f(x)=\sin^{-1}\left[\log_9\left(\frac{x^2}{4}\right)\right]$. Since the domain of $\sin^{-1}x$ is $[-1\ 1]$,

$f(x)=\sin^{-1}\left[\log_9\left(\frac{x^2}{4}\right)\right]$ is defined if

$-1\le\log_9\left(\frac{x^2}{4}\right)\le 1$

$\Rightarrow 9^{-1}\le\frac{x^2}{4}\le 9^1$

$\Rightarrow \frac{4}{9}\le x^2\le 36$

$\Rightarrow \frac{2}{3}\le |x|\le 6$

$\Rightarrow x\in\left[-6,-\frac{2}{3}\right]\cup\left[\frac{2}{3},6\right]$

$(\because a\le|x|\le b \Leftrightarrow x\in[-b,-a]\cup[a,b])$

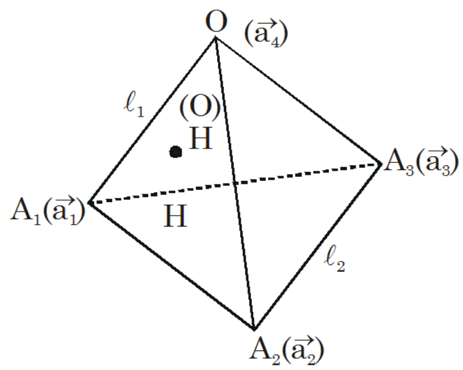
Hence, the domain of f(x) is $\left[-6,-\frac{2}{3}\right]\cup\left[\frac{2}{3},6\right]$

5. **Ans (466)**

$2^9-(1+1+2+3+4+5+6+7+8+9)$

$=512-46=466$

6. **Ans (6)**



$\vec{a}_1.(\vec{a}_2-\vec{a}_3)=0$

$\vec{a}_4.(\vec{a}_2-\vec{a}_3)=0$

$(\vec{a}_1-\vec{a}_4).(\vec{a}_2-\vec{a}_3)$

$OA_1 \longrightarrow \vec{r}=\vec{a}_4+x(\vec{a}_1-\vec{a}_4)$

$A_1A_3 \rightarrow \vec{r}=\vec{a}_2+\mu(\vec{a}_2-\vec{a}_3)$

$d=\frac{(\vec{a}_4-\vec{a}_2).\{(\vec{a}_1-\vec{a}_4)\times(\vec{a}_2-\vec{a}_3)\}}{\ell_1\ell_2}$

$\ell_1\ell_2 d=(\vec{a}_4-\vec{a}_2).\{-\vec{a}_1\times\vec{a}_2+\vec{a}_1\times\vec{a}_3-\vec{a}_4\times\vec{a}_2+\vec{a}_1\times\vec{a}_3\}$

$\ell_1\ell_2 d=-[\vec{a}_1\ \vec{a}_2\ \vec{a}_4]+[\vec{a}_1\ \vec{a}_3\ \vec{a}_4]$