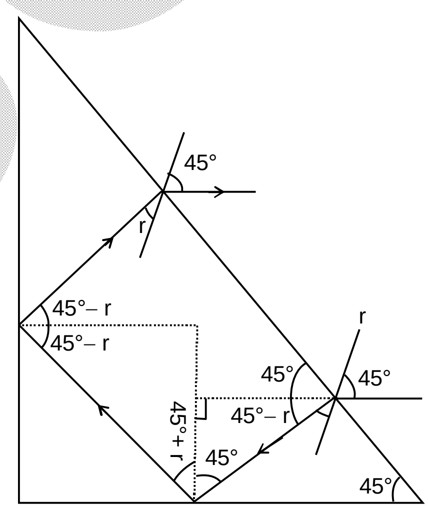
FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – V
PAPER –1
TEST DATE: 18-02-2025

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A



Sol. $\sin(45° - r) > \frac{1}{\mu}$ and

$\mu \sin r = 1 \sin 45° = \frac{1}{\sqrt{2}}$

$\sin r = \frac{1}{\mu\sqrt{2}}$

$\therefore \frac{1}{\sqrt{2}}\cos r - \frac{1}{\sqrt{2}}\sin r > \frac{1}{\mu}$

$\Rightarrow \sqrt{1-\sin^2 r} - \sin r > \frac{\sqrt{2}}{\mu}$

$\Rightarrow 1-\sin^2 r > \left(\frac{\sqrt{2}}{\mu} + \frac{1}{\mu\sqrt{2}}\right)^2$

$\Rightarrow \sin^2 r < 1 - \left(\frac{2+1}{\mu\sqrt{2}}\right)^2$ $\Rightarrow \frac{1}{\mu^2 2} < 1 - \frac{9}{\mu^2 2}$

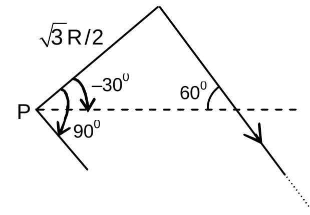
$\Rightarrow \mu^2 > 5$; $\mu > \sqrt{5}$

2. B

Sol. $100°C - Ir - \frac{I}{2}r - Ir = 0°C$

$\Rightarrow$ $Ir = 40°C$; $\therefore$ $t_F = 0° + Ir = 40°C$

3. C



Sol. $B = \frac{\mu_0 I}{4\pi\sqrt{3}R/2}\left[\sin 90^o + \sin(-30^o)\right]$

$= \frac{\mu_0 I}{4\sqrt{3}R}$

4. C

Sol. $\because$ E is continuously decreasing along radially outward.

$\therefore \frac{V_1 - V_2}{t_1} > \frac{V_2 - V_3}{t_2}$;

$\therefore t_1 < t_2$

5. AB

Sol. Displacement = area under V-t curves.
Distance = sum of magnitude of area under v-t curves.

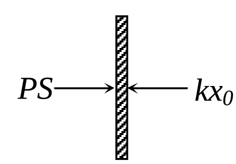
6. ACD

Sol. As $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and $P = \frac{h}{\lambda}$

Also, $E = \frac{hc}{\lambda}$

7. ABCD

Sol. Equilibrium of piston gives



$PS = kx_0$ or $P = \frac{kx_0}{S}$

Since, the chamber is thermally insulated $\Delta Q = 0$

$\therefore$ Elastic potential energy of spring = work done by gas

or work done by gas = $\frac{1}{2}kx_0^2$

This work is done in the expense of internal energy of the gas.

Therefore, internal energy of the gas is decreased by $\frac{1}{2}kx_0^2$.

Internal energy of an ideal gas depends on its temperature only. Internal energy of the gas is decreasing. Therefore, temperature of the gas will decrease.

8. C

Sol. $I = \frac{2+3-5+4+6}{2+3+5+4+6} = \frac{1}{2}A$

$V_D = 2 - \frac{1}{2}\times 2 = 1V$; $V_C = V_D + 3 - \frac{1}{2}\times 3 = 2.5V$

$V_B = V_C - 5 - \frac{1}{2}\times 5 = -5V$; $V_A = V_B + 4 - \frac{1}{2}\times 4 = -3V$

9. D

Sol. According to Lenz's law induced current flows in such a way that it is opposing the change in magnetic flux.

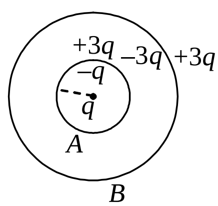
10. B

Sol. For closed pipe, $\nu = \dfrac{(2n+1)v}{4L}$ and for first overtone, n = 1

For open pipe, $\nu = \dfrac{nv}{2L}$ and for fourth harmonic, n = 4.

11. A

Sol. When switch is open



SECTION – B

12. 5

Sol. $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{0.5}{\dfrac{1\times10^{-3}}{20\times10^{-2}}}}$ = 10 m/s

$\lambda = \dfrac{v}{f} = \dfrac{10}{100} = \dfrac{1}{10}\,m = 10\,cm$; $\ell = \dfrac{\lambda}{2} = 5$

13. 5

Sol. At stable equilibrium, U is minimum.

$\dfrac{dU}{dx} = 0$ and $\dfrac{d^2U}{dx^2} > 0$

$= \dfrac{1}{dx}\left(\dfrac{x^3}{3} - \dfrac{ax^2}{2} + 20x\right) = 0.$

$\Rightarrow \quad x^2 - 9x + 20 = 0. \quad \Rightarrow \quad (x-5)(x-4) = 0.$

x = 5 and x = 4 are points of equilibrium.

And U minimum when $\dfrac{d^2U}{dx^2} > 0$. i.e. at x = 5.00

14. 5

Sol. $\sqrt{5gl_2} = \sqrt{gl_1} \Rightarrow \dfrac{l_1}{l_2} = 5$

15. 8
Sol. μ_w d sin 30 = $(\mu_g - 1) \times t$

16. 3
Sol. $PT^2 = k$

$\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right)$ ...(i) ; $\left(\frac{nRT}{V}\right)T^2 = k$

$\Rightarrow \frac{T^3}{V} = k'$

$\Rightarrow V = \frac{T^3}{k'}$ ...(ii)

$\frac{dV}{dT} = \frac{3T^2}{K'}$ ...(iii)

From (i), (ii) and (iii)

$\gamma = \left(\frac{k'}{T^3}\right)\times\left(\frac{3T^2}{k'}\right) = \frac{3}{T}$

17. 3
Sol. Initially the rod will be in equilibrium if

$2T_o = Mg$ with $T_o = kx_o$ ...(i)

when the current I is passed through the rod, it will experience a force
F = BIL vertically up,
In equilibriums
2T + BIL = Mg with T = kx ...(ii)
from (i) & (ii)

$\frac{T}{T_o} = \frac{Mg - BIL}{Mg}$ i.c. $\frac{x}{x_o} = 1 - \frac{BIL}{Mg}$

or, $B = \frac{Mg(x_o - x)}{I \;\; L \;\; x_0}$

Putting the values we get $B = 1.5 \times 10^{-2}\,T$.

Chemistry PART – II

SECTION – A

18. C
Sol. It is a linear molecule and contains three lone pairs.

19. A
Sol. $p^{K_a} = -\log K_a,\ K_b = \dfrac{k_w}{k_a}$

20. B
Sol. Benzyl halides undergo S_N1 and S_N2 reactions with equal extent. Br^- is a better leaving group than F^-

21. B
Sol. It is called carbylamine reaction.

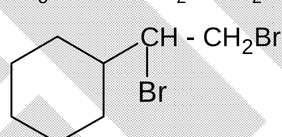
22. AB
Sol. The overall order of the reaction is 2.

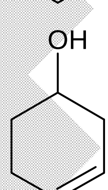
23. CD
Sol. Ni^{2+} contains eight and Zn^{2+} contains 10 electros.
$\therefore\ Ni^{2+} = t_{2g}^6 e_g^2$ and $Zn^{2+} = t_{2g}^6 e_g^4$

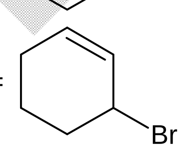
24. ACD
Sol. It has three geometrical isomers.

25. C
Sol. The products of (P), (Q), (R) and (S) respectively are

$$(P) = CH_3 - \overset{\overset{\displaystyle OH}{|}}{CH} - CH_2 - CH_2 - CH_2 - CH_2 - CH_2OH$$

(Q) = 

(R) = 

(S) = 

26. C

Sol. σ^*_{2p} has three nodal planes. Two planes are passing through two nuclei and the third one is passing through the negative overlapping region.

27. C

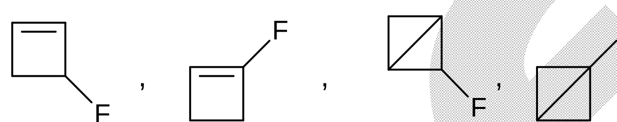
Sol. If pressure is decrease in isochoric process, temperature decreases. So heat is transferred to the surrounding. So entropy change of surrounding increases and that of system decreases.

28. D

Sol. $H_3BO_3 \xrightarrow{\Delta} H_2B_4O_7 \xrightarrow{\Delta} HBO_2 \xrightarrow{\Delta} B_2O_3$

SECTION – B

29. 4

Sol. The isomers are 

30. 9212

Sol. It is an isothermal process because PV remains constant by changing volume.

$W = -2.303nRT \log\frac{V_2}{V_1}$

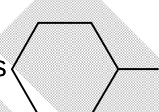
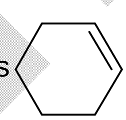
$= -2.303 \times PV \log\frac{1}{10}$

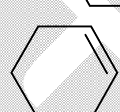
$= -2.303 \times 4000 \times \log 10^{-1} = 9212$ J

31. 30

Sol. All the electrons will have the magnetic quantum numbers varying from -2 to +2.

32. 4

Sol. X is  $-\overset{+}{N}(CH_3)_3\bar{I}$, Y is , Z is $(CH_3)_3N$

 $\xrightarrow[H^+]{MnO_4^-} HOOC-(CH_2)_4-COOH$

33. 70

Sol. X is $H_2B_4O_7$, Y is B_2O_3 & Z = H_3BO_3
Molar mass of B_2O_3 is 70

34. 600

Sol. $\Delta T_b = K_b im = 0.52 \times 1.8 \times 1 \times \frac{1000}{W}$ or, $1.56 = 0.52 \times 1.8 \times \frac{1000}{W}$, or W = 600 g

Mathematics

PART – III

SECTION – A

35. C

Sol. a, b > 0 and c < 0

angle between OA and OB is $\frac{\pi}{2}$.

Since z_2 lie in 2nd quadrant

$\Rightarrow \bar{z}_2$ will lie in 3rd quadrant.

$\Rightarrow \sqrt{2} < |z_1 - \bar{z}_2| \le 2$.

Also true if z_2 lies in 3rd quadrant.

36. B

Sol. Writing r as linear combination of a, b and $a \times b$, we have $r = xa + yb + z(a \times b)$ for solats x, y, z

$0 = r.a = x|a|^2 + ya.b$

$1 = r.b = xa.b + y|b|^2$

Solving we get $y = \frac{|a|^2}{|a|^2|b|^2 - (a.b)^2} = |a|^2$ and $x = \frac{a.b}{(a.b)^2 - |a|^2|b|^2} = a.b$

Also $1 = [r \quad a \quad b] = z|a \times b|^2$

$\Rightarrow \qquad z = \frac{1}{|a \times b|^2}$

Thus $r = \left((a.b)a - |a|^2 b\right) + \frac{a \times b}{|a \times b|^2}$

$= a \times (a \times b) + \frac{a \times b}{|a \times b|^2}$

37. D

Sol. $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.\,{}^nC_r\,{}^nC_{r+1}}{r+2}\right)$

$= \sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(r.\,{}^nC_r\right)\left(\frac{{}^nC_{r+1}}{r+2}\right)$

$= \sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(n.\,{}^{n-1}C_{r-1}\right)\left(\frac{{}^{n+1}C_{r+2}}{n+1}\right)$

$= \sum_{r=0}^{n-1} {}^{n-1}C_{r-1}\;{}^{n+1}C_{r+2}$

$= \sum_{r=0}^{n-1} {}^{n-1}C_{n-r}\;{}^{n+1}C_{r+2}$

$= {}^{n-1}C_n\,{}^{n+1}C_2 + {}^{n-1}C_{n-1}\,{}^{n+1}C_3 + ... + {}^{n-1}C_1\,{}^{n+1}C_{n+1}$
$= {}^{2n}C_{n+2} = {}^{2n}C_{n-2}$

38. B

Sol. $g'(0) = \lim_{h\to\infty} \dfrac{g(0+h)-g(0)}{h} = 0$

$\Rightarrow g(x)$ is differentiable $\forall x \in R$

$$g'(x) = \begin{cases} 2x\sin\left(\dfrac{\pi}{x}\right) - \pi\sin\left(\dfrac{\pi}{x}\right) + 2(x-1)\sin\left(\dfrac{\pi}{x-1}\right) - \pi\sin\left(\dfrac{\pi}{x-1}\right); & x \neq 1 \\ 0\ ; & x = 0,1 \end{cases}$$

But $\lim_{x\to 0} g'(x) =$ does not exist $\neq g'(0) \Rightarrow g'(x)$ is discontinuous at x = 0

Similarly $\lim_{x\to 1} g'(x) =$ does not exist.

39. AD

Sol. By intermediate value property $\dfrac{f(0)+f(2)}{2} = f(c),\ 0 < c < 2$

By mean value theorem

$f(1) - f(0) = f'(c_1),\ 0 < c_1 < 1$

$f(2) - f(1) = f'(c_2),\ 1 < c_2 < 2$

By subtraction

$f(0) + f(2) - 2f(1) = f'(c_2) - f'(c_1)$

$= (c_2 - c_1)f''(c),\ c_1 < c < c_2 \Rightarrow f(0) + f(2) - 2f(1) < 0$

$\Rightarrow f(0) + f(2) < 2f(1)$

40. AB

Sol. Volume $= \left|\left[2\vec{b}\times\vec{c}\quad 3\vec{c}\times\vec{a}\quad 4\vec{a}\times\vec{b}\right]\right| = 18$

$\Rightarrow \quad 24\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2 = 18$

$\Rightarrow \quad \left|\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]\right| = \dfrac{\sqrt{3}}{2}$

$$\text{Now, } \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = \begin{vmatrix} (1+\sin\theta) & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\dfrac{2\pi}{3}\right) & \cos\left(\theta+\dfrac{2\pi}{3}\right) & \sin\left(2\theta+\dfrac{4\pi}{3}\right) \\ \sin\left(\theta-\dfrac{2\pi}{3}\right) & \cos\left(\theta-\dfrac{2\pi}{3}\right) & \sin\left(2\theta-\dfrac{4\pi}{3}\right) \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ and expanding

$\left|\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]\right| = \sqrt{3}\left|\cos 3\theta\right| = \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \cos 3\theta = \pm\frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$$

41. ABC

Sol. $8\sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

$\Rightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x$

$4(1-\cos 2x)\cos x = \sqrt{3}\sin x + \cos x$

$3\cos x - 2(\cos 3x + \cos x) = \sqrt{3}\sin x$

$\cos x - 2\cos 3x = \sqrt{3}\sin x$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos 3x$

$\cos\left(x + \frac{\pi}{3}\right) = \cos 3x$

$3x = 2nx \pm \left(x + \frac{\pi}{3}\right)$

(+)	(–)
$3x = 2x\pi + x + \frac{\pi}{3}$	$3x = 2n\pi - x - \frac{\pi}{3}$
$2x = 2n\pi + \frac{\pi}{3}$	$4x = 2n\pi - \frac{\pi}{3}$
$x = nx + \frac{\pi}{6}$	$x = \frac{n\pi}{2} - \frac{\pi}{12}$
$[0, 2\pi]$	
$\frac{\pi}{6}$	$\frac{\pi}{2} - \frac{\pi}{12} = \frac{6\pi - \pi}{12} = \frac{5\pi}{12}$
$\frac{5\pi}{6}$	$\pi - \frac{\pi}{12} = \frac{11\pi}{12}$
	$\frac{3\pi}{2} - \frac{\pi}{12} = \frac{18\pi - \pi}{12} = \frac{17\pi}{12}$
	$2\pi - \frac{\pi}{12} = \frac{23\pi}{12}$

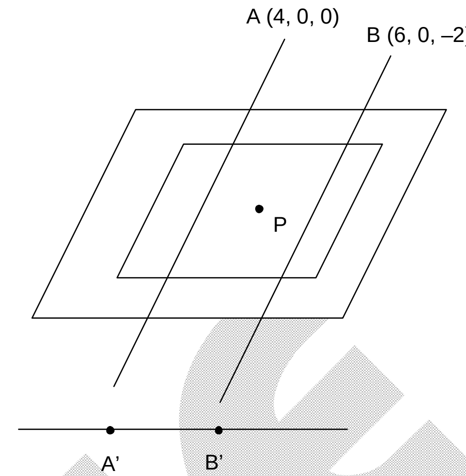
42. B

Sol. (P) Let $C(1,0,1), D(3,2,-1)$

$\vec{n} \perp$ to $\overrightarrow{CD} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ 2 & 2 & -2 \end{vmatrix} = -2\hat{i} + 0\hat{j} - 2\hat{k}$$

Equation of plane π : $-2x - 2z = -2 - 2 \Rightarrow x + z = 2$



(Q) $\overrightarrow{AB} = 2\hat{i} - 2\hat{k} \perp \vec{n}$ hence $\overrightarrow{AB}$ is parallel to plane π and both A and B are on same side of π

Mirror image of A (4, 0, 0) about π.

$$\frac{x-4}{1} = \frac{y-0}{0} = \frac{z-0}{1} = -2\left(\frac{4+0-2}{1+1}\right) = -2$$

$x = 2,\ y = 0,\ z = -2 \Rightarrow A' = (2, 0, -2)$

If PA + PB is minimum then P is intersection of plane π with BA'.

$BA': \frac{x-2}{4} = \frac{y-0}{0} = \frac{z+2}{0} = \alpha$ and π is $x + z = 2$

Let $P' = (4\alpha + 2,\ 0,\ -2)$ lies on $x + z = 2$

$4\alpha + 2 - 2 = 2 \Rightarrow \alpha = \frac{1}{2}$

So, $P' = (4, 0, -2) = (x_0, y_0, z_0)$

(R) $0 \le |PA - PB| < AB$

$0 \le |PA - PB| < \sqrt{4+4}$

$0 \le |PA - PB| < \sqrt{8}$

(S) Reflected line is parallel to AB i.e. $(2\hat{i} - 2\hat{k})$ and passes through $n'(2, 0, -2)$

So equation $\frac{x-2}{2} = \frac{y-0}{0} = \frac{z+2}{-2}$

$\Rightarrow \frac{x-2}{1} = \frac{y}{0} = \frac{z+2}{-1}$

So $\alpha = 0,\ \beta = 2$

43. A

Sol. (P) $S = a + (a+d) + (a+2d) + ... + (a+98d) = \frac{99}{2}(2a+98d) = 99(a+49d)$ ....(1)

$2550 = a + (a+2d) + (a+4d) + ... + (a+98d)$ (Odd numbered terms)

$= \frac{50}{2}(2a+98d) = 50(a+99d) \Rightarrow a + 49d = \frac{2550}{50} = 51$

$S = 99(a+49d) = 99 \times 51 = 5049$.

(Q) $f(n) = f(n-1) + n$

$f(2) = f(1) + 2 = 1 + 2$

$f(3) = f(2) + 3 = 1 + 2 + 3$

$f(4) = f(3) + 4 = 1 + 2 + 3 + 4$

and so on

So $f(100) = 1 + 2 + 3 + 4 + ... + 100 = \frac{100 \times 101}{2} = 5050$.

(R) $f(n)=\frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 4}.....\frac{\log n}{\log(n-1)}=\frac{\log n}{\log 2}=\log_2(n)$

$\sum_{k=2}^{100} f(2^k)=\sum_{k=2}^{100}\log_2(2^k)=\sum_{k=2}^{100}k=\frac{100\times 101}{2}-1=5049$.

(S) Area = $\pi(r_2^2-r_1^2)+\pi(r_4^2-r_3^2)+\pi(\pi_6^2-r_5^2)+....+\pi(r_{100}^2-r_{99}^2)$

$=\pi(r_1+r_2)(r_2-r_1)+\pi(r_4+r_3)(r_4-r_3)+\pi(r_6-r_5)(r_6+r_5)+.....+\pi(r_{100}-r_{99})(r_{100}+r_{99})$

$=\pi(r_1+r_2+r_3+r_4+r_5+......+r_{100})$

$=\pi(1+2+3+4+5+......+100)=\pi\left(\frac{100\times 101}{2}\right)=5050\pi$

44. C

Sol. (P) Equation of tangent at $\left(\frac{\cos\theta}{2},\frac{\sin\theta}{3}\right)$ is $2x\cos\theta+3y\sin\theta=1$, Which is parallel to the given line $8x=9y$

$\therefore\ \cos\theta=\pm\frac{4}{5},\ \sin\theta=\mp\frac{3}{5}$

Hence, points are $\left(\frac{2}{5},-\frac{1}{5}\right)$ and $\left(-\frac{2}{5},\frac{1}{5}\right)$

Distance between the points is $\sqrt{\frac{16}{25}+\frac{4}{25}}=\frac{2}{\sqrt{5}}$,

(Q) The given equation is $\frac{(x+1)^2}{9}+\frac{(y+2)^2}{25}=1$

$\Rightarrow\ e^2=1-\frac{9}{25}=\frac{16}{25}\Rightarrow e=\frac{4}{5}$

Hence, the foci are $S, S'\equiv S(-1,2)$ and $S'(-1,-6)$.

The required sum of distances $=2+6=8$.

(R) Equation of normal at $(3\cos\theta, 2\sin\theta)$ is $3x\sec\theta-2y\,\mathrm{cosec}\,\theta=5$, Which is parallel to the given line $2x+y=1$. Therefore, $\cos\theta=\mp\frac{3}{5},\ \sin\theta=\pm\frac{4}{5}$

Hence, points are $\left(\frac{-9}{5},\frac{8}{5}\right)$ and $\left(\frac{9}{5},\frac{8}{5}\right)$.

The required sum of distances $=\frac{16}{5}$

(S) Consider any point $(t,t+2)$, $t\in R$, one the line $x-y+2=0$.

The chord of contact of ellipse w.r.t. this point is $xt+2y(t+2)=2$

$\Rightarrow\ (4y-2)+t(x+2y)=0$

This line passes through point of intersection of lines $4y-2=0$ and $x+2y=0$

Therefore, $x=-1$.

Hence, the point is $\left(-1,\frac{1}{2}\right)$, whose distance from $\left(2,\frac{1}{2}\right)$ is 3.

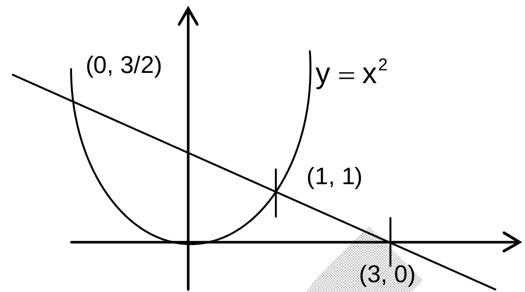
45. D

Sol. (P) $2a^2 + a - 3 = 0$

$(2a+3)(a-1) = 0$

$\Rightarrow a \in (0, 1)$

Number of integral values of a = 0



(Q) $a^2 + a - 2 = 0$

$\Rightarrow (a+2)(a-1) = 0$

$\Rightarrow \quad a = -2, 1$

$4a^2 + 4a - 3 = 0$

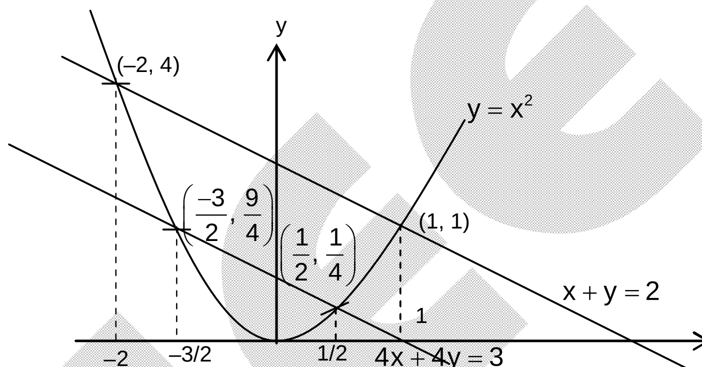
$(2a-1)(2a+3) = 0$

$\Rightarrow a = \frac{1}{2}, -\frac{3}{2}$

$\Rightarrow \; a \in \left(-2, \frac{3}{-2}\right) \cup \left(\frac{1}{2}, 1\right)$

Values of a of form $\frac{K}{3}$ are

$\frac{-5}{3}, \frac{2}{3}$



(R) Slope of line joining $(t-1, 2t+2)$ and $(2t+1, t)$ is $\frac{2t+2-t}{t-1-2t-1} = -1$

$\therefore$ Slope of perpendicular bisectors of point is 1.

(S) Images of A w.r.t. y = x and y = 0 lies on BC which are (2, 1), (1, –2)\

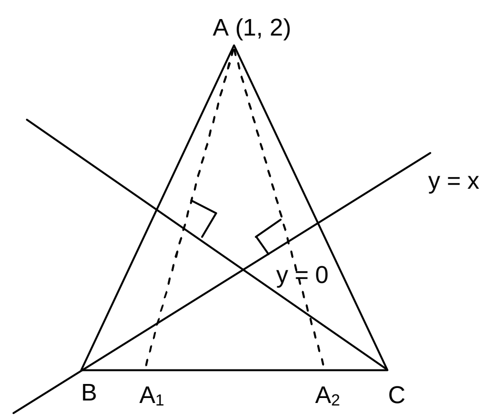
$\therefore$ Equation of BC is $y = 3x - 5$

Perpendicular distance of A from

$BC = \frac{|3-2-5|}{\sqrt{10}}$

$d(A,BC) = \frac{4}{\sqrt{10}}$

$\Rightarrow \sqrt{10}\, d(ABC) = 4$



SECTION – B

46. 29

Sol. E_1 : First bag is chosen, $P(E_1) = \frac{1}{2}$.

E_2 : Second bag is chosen, $P(E_2) = \frac{1}{2}$.

A : Drawn number is 4.

Now, $P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)$

$= \frac{1}{2}.\frac{1}{4} + \frac{1}{2}.\frac{1}{6} = \frac{5}{24}$

47. 6

Sol. $\because f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$\therefore\ f'(x) = 3x^2 + 2x f'(1) + f''(2)$

Put $x = 1$

$\therefore\ f'(1) + f''(2) = -3$ ……………….(i)

Again, $f''(x) = 6x + 2f'(1)$, $f'''(x) = 6$

Put $x = 2$

$\therefore\ f''(2) = 12 + 2f'(1)$ ……………..(ii)

Solving equation (i) and (ii), we get

$f'(1) = -5$ and $f''(2) = 2$

$\therefore\ f(x) = x^3 - 5x^2 + 2x + 6$

$\therefore\ f(2) - f(1) = -6$

$= -f(0)$

48. 16

Sol. We have $f(x) - f(-x) = 6x$

$\therefore\ f(4) - f(-4) = 24$

$\Rightarrow\ N = f(4) = 24 + 2286 = 2310 = 2.3.5.7.11$

Hence number of divisors $= 2^{n-1} = 2^{5-1} = 16$

49. 9

Sol. Given,

$a + 19d = \log_{10} 20$ ……….(1)

$a + 31d = \log_{10} 32$ ……….(2)

(2) – (1)

$12d = \log_{10}\frac{32}{20} = \log_{10} 16 - 1$

$12d = 4\log_{10} 2 - 1$

$\log_{10} 2 = \frac{12d + 1}{4}$ ……….(A)

Again (2) + (1)

$2a + 50d = \log_{10} 640 = 6\log_{10} 2 + 1$

$\log_{10} 2 = \frac{2a + 50d - 1}{6}$ ……….(B)

$$\therefore \frac{12d+1}{4} = \frac{2a+50d-1}{6} \Rightarrow 36d+3 = 4a+100d-2$$

$$4a+64d=5$$

$$\underbrace{a+16d}_{17^{th}\text{ term}} = \frac{5}{4}$$

Hence, 17^{th} term is rational and its value is $\frac{5}{4} = \frac{p}{q} \Rightarrow (p+q)=9$

50. 3

Sol. Given, $z+\omega = i$ ……………(1)

and $z^2+\omega^2 = 1$ ……………..(2)

$\therefore$ From (1), on squaring we get

$z^2+\omega^2+2z\omega = -1 \quad \Rightarrow \quad 1+2z\omega = -1$ [Using (2)]

$\Rightarrow \ zw = -1$ …………..(3)

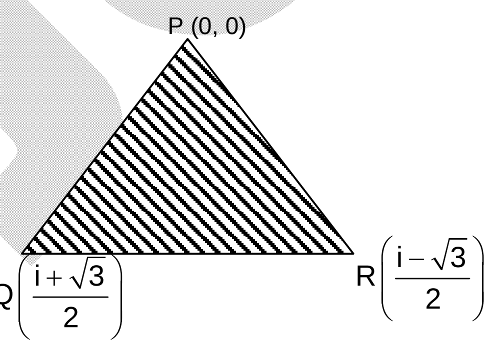
Now, let us consider a quadratic equation in x whose roots are z and ω.

$\Rightarrow \quad x^2 - ix - 1 = 0$

$$\therefore \quad x = \frac{i \pm \sqrt{i^2+4}}{2} = \frac{i \pm \sqrt{3}}{2}$$

Let x

$\omega = \frac{i+\sqrt{3}}{2}$ $\qquad$ $z = \frac{i-\sqrt{3}}{2}$



$$\text{So, ar.}(\Delta PQR) = \frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1 \end{matrix}\right\| = \frac{1}{2}\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{4}$$

51. 8

Sol. Area $= \int_0^1 (6-f(x))dx + \int_{-1}^0 (f(x)-(-2))dx$

$$= \frac{5}{4}$$