PART-1 : PHYSICS

SECTION-I

1) The vessel volume is V, the initial pressure is P_0. The process is assumed to be isothermal, and the evacuation rate is constant & equal to C and independent of pressure. [The evacuation rate is the gas volume being evacuated per unit time with that volume being measured under the gas pressure attained by that moment]. Choose the **CORRECT** option(s).

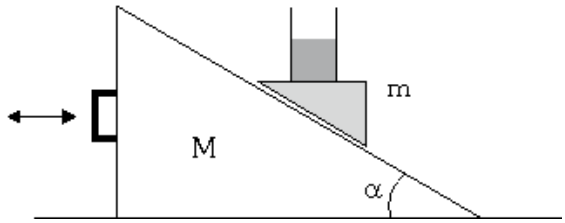
(A) Pressure P of gas at time 't' will be $P_0 e^{-\frac{Ct}{V}}$

(B) Pressure P of gas at time 't' will be $P_0 - \frac{Ct}{V}$

(C) No. of moles evacuated in time 't' will be linearly proportional to time t.

(D) No. of moles evacuated in time 't' will be exponentially dependent on time t.

2) A glass partially filled with water is fastened to a wedge that slides, without friction, down a large plane inclined at an angle α (as shown in the figure). The mass of the inclined plane is M, the combined mass of the wedge, the glass and the water is m. If there were no motion the water surface would be horizontal. Choose the **CORRECT** option(s).



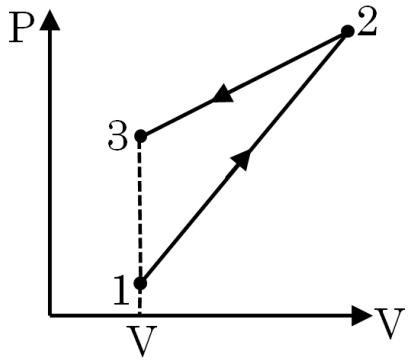
(A) If the inclined plane is fixed, then the angle free surface of water will make with the inclined plane in steady state will be zero.

(B) If the inclined plane is fixed, then the angle surface of water will make with the inclined plane in steady state will depend on m.

(C) If the inclined plane can move freely in the horizontal direction, then the angle surface of water will make with the inclined plane in steady state will be zero.

(D) If the inclined plane can move freely in the horizontal direction, then the angle surface of water will make with the inclined plane in steady state will depend on m & M.

3) In an old record found in a laboratory, a process 1→2→3 was shown. Over the time, ink faded and it became impossible to see the pressure and volume axes. However, descriptions given there reveal that the states 1 and 3 lie on an isochore corresponding to a volume V, amount of heat supplied during the whole process 1→2→3 is zero and the gas involved is one mole of helium, then choose the



CORRECT option(s).

(A) Volume occupied by the gas in the state 2 will depend on pressure at 2.

(B) Volume occupied by the gas in the state 2 is independent of pressure at 2.

(C) Volume occupied by the gas in the state 2 is 4V.

(D) Heat supplied will be positive if we complete cycle 1→2→3→1.

4) A point mass of 1 kg moving at constant speed of 5 m/s on an elliptical path experiences a centripetal force of 20N when at either end point of the major axis and a similar force of 2.5 N at each end of minor axis. Then :-

(A) length of semi major axis is 5 m

(B) length of semi major axis is 10 m

(C) length of semi minor axis is 2.5 m

(D) length of semi minor axis is 5 m

SECTION-II (i)

Common Content for Question No. 1 to 2

There are three sound sources producing sound with slight different frequency. The equation of sound waves when received by detector is given by

$$\Delta P_1 = \Delta P_0 \sin 298\pi \left(t - \frac{x}{v}\right)$$

$$\Delta P_2 = \Delta P_0 \sin 300\pi \left(t - \frac{x}{v}\right)$$

$$\Delta P_3 = \Delta P_0 \sin 302\pi \left(t - \frac{x}{v}\right)$$

Where ΔP is change in pressure and v is speed of sound in medium. Let the detector is placed at x = 0 and intensity due to each source is I_0 at the place of detector.

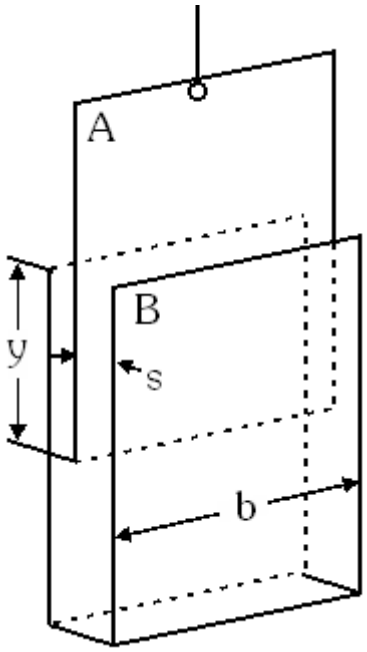
1) Number of maxima local as well as global of loudness in every one second will be :

2) If maximum intensity of resulting sound is $9I_0$ and detector cannot record an intensity lower than $4I_0$, If the duration of time for which detector remains idle during first one second is τ(sec), find 3τ.

Common Content for Question No. 3 to 4

A flat conducting sheet A is suspended by an insulating thread at distance ‘s’ from both the surfaces formed by the bend conducting sheet B as shown in the figure. The entire space between bent sheet B is filled with medium of dielectric constant K. The sheets A and B are oppositely charged, the

difference in potential is constant and equal to ΔV. This causes a force F, pulling A downward. (Neglect gravity)



3) If the modulus of work done by external agent needed to slowly increase the inserted distance by Δy is $\frac{P(\Delta V)^2 bK\varepsilon_0 \Delta y}{s}$. Find the value of P.

4) An expression for the difference in potential ΔV in terms of F and relevant dimensions shown in the figure is $\sqrt{\frac{PsF}{\epsilon_0 bK}}$. Find the value of P.

Common Content for Question No. 5 to 6

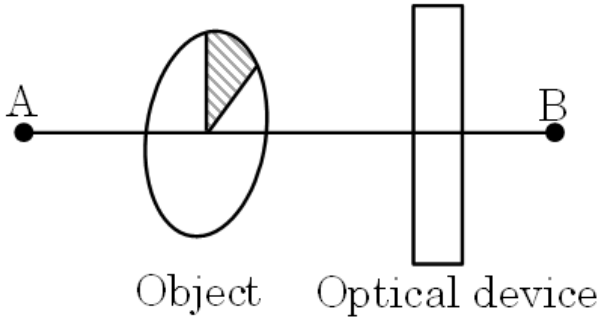
Hydrogen atoms in ground state are excited by means of a monochromatic radiation of wavelength 969·69 Å. After absorbing the energy of radiation, hydrogen atoms go to the excited state. After 10^{-8} s, the hydrogen atom will come to the ground state by emitting the absorbed energy. Different wavelengths are possible in the spectrum. (Take : R = 1.1×10^7 / m)

5) The electron of excited hydrogen atom is in which of the energy state n?

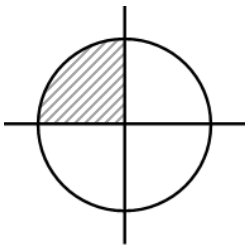
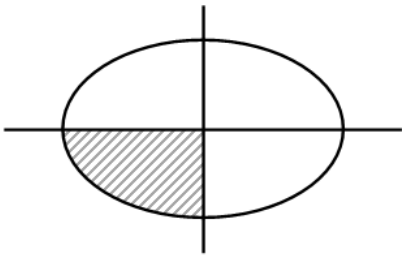
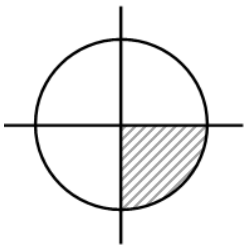
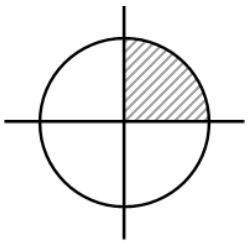
6) Number of different wavelengths present in the spectrum is :-

SECTION-II (ii)

1) A very small circular object is kept infront of an optical device as shown in figure. The plane of object is parallel to the optical device. One quadrant of circle is hatched as shown. Match the images

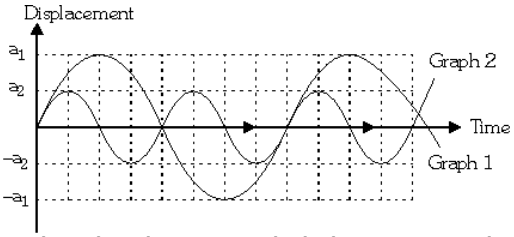
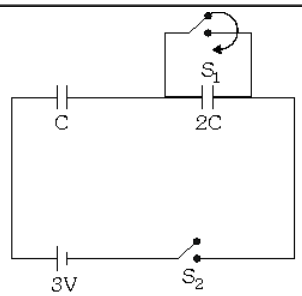
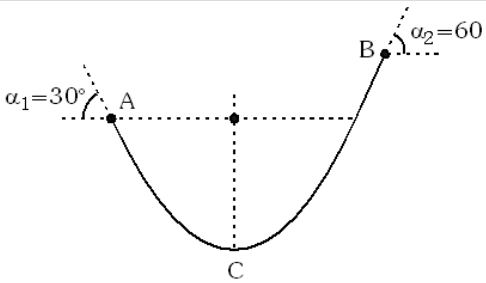
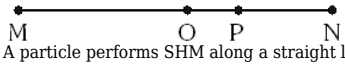


as seen by the observer (ignoring magnification).

List-I (Device)		List-II (Probable Image)	
(P)	Plane mirror (The observer is at A)	(1)	 Circle
(Q)	Concave mirror (The observer is at A) object between F and 2F	(2)	 Ellipse
(R)	Convex mirror (The observer is at A)	(3)	 Circle
(S)	Convex lens (The observer is at B) object between F and optical centre	(4)	 Circle

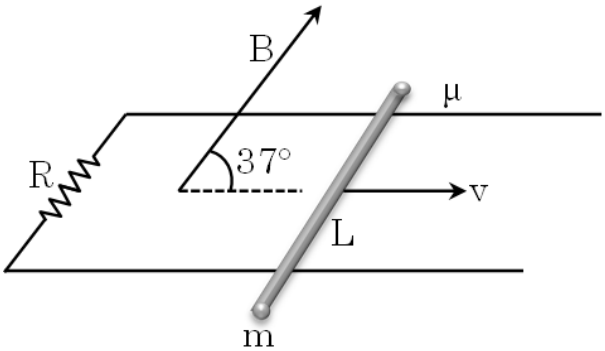
2) Match statement of List-I with List-II and select the correct answer using the code given below the list.

	List-I		List-II

(P)	 Graph-1 and Graph-2 represents the displacement time graph of two identical particles, performing SHM with same total energy. The value of a_1/a_2 is ____ .	(1)	0
(Q)	 A circuit contains two capacitors as shown in the figure. Initially switch S_1 is closed and S_2 is open. At t = 0, S_2 is closed keeping S_1 closed, and a charge Q_1 is supplied by the battery to the circuit. After long time, S_1 is opened keeping S_2 closed. Now battery supplied charge Q_2. The value of Q_2/Q_1 is	(2)	1
(R)	 A massive rope of uniform mass density and length □ is suspended from points A and B in vertical plane as shown in the figure. Point C is the lowest point of the chord. Given length of rope from A to C = □$_1$ and length of rope from B to C = □$_2$. The value of □$_2$/□$_1$ is	(3)	2
(S)	 A particle performs SHM along a straight line between the points M and N about O(mean position) as shown in figure. t_1 and t_2 are the time taken by the particle from M to P and P to N respectively. Find t_1/t_2 (given OP = PN)	(4)	3

SECTION-III

1) Parallel conducting rails are located horizontally at a distance L from each other and placed in a uniform magnetic field whose induction vector B is directed at an angle of 37° to their plane as shown. The rails are connected by a resistor of high resistance R. Far from the resistor on the rails lies massive conducting rod of mass m and length L. It is perpendicular to the rails. What force (in N) do you need to exert on the rod in the horizontal direction so that it slides along the rails with constant speed v? The surface is rough with coefficient of friction μ = 0.5 and B = 0.5 T, L = 30 cm,



m = 1 kg, R = 27 mΩ, v = 10 m/s, g = 10 m/s^2.

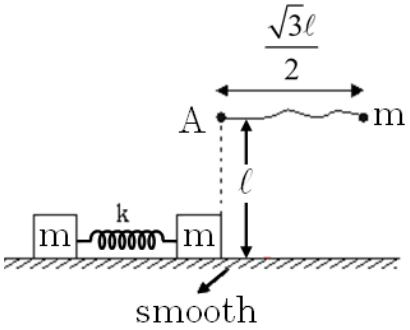
2) An ac voltmeter with large impedance is connected one by one across the inductor, the capacitor, and the resistor in a series circuit having an alternating emf of 125 V (rms); the meter gives the same reading in volts in each case. The angular frequency of the AC source is $4\sqrt{3}$ rad/s. Now the

circuit is disconnected from the AC source, the capacitor is separately charged and reconnected to the inductor and the resistor, all three elements in series. The circuit is found to undergo underdamped oscillations. What is the angular frequency of oscillation in rad/s ? Round off to nearest integer if necessary.

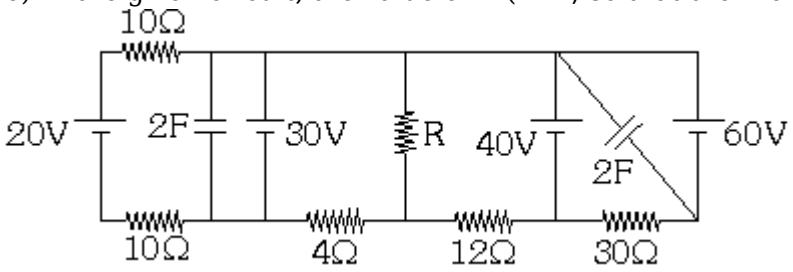
3) ^{4}He with energy E_0 bombard stationary ^{7}Li nuclei. They become composite nuclei ^{11}B after completely inelastic collision. It further splits into ^{10}B and neutron ^{1}n. The reaction equation is $^{4}He + {}^{7}Li \rightarrow (^{11}B) \rightarrow {}^{10}B + {}^{1}n$. Assume that the energy E_0 is minimum. What is the corresponding neutron kinetic energy E_n (in MeV)? Mass of helium = 4.0026 u, Mass of Lithium = 7.0160 u, Mass of ^{11}B = 11.0093 u, mass of neutron = 1.0087 u, Mass of ^{10}B = 10.0129 u, 1 amu = 930 MeV. Find 50 E_n. Round off to nearest integer if required.

4) Intensity observed in an interference pattern is $I = I_0 \sin^2 \theta$. At $\theta = 30°$, intensity $I = 5 \pm 0.0020$ W/m^2. If $I^0 = 20$ W/m^2 and percentage error in angle θ is $\frac{n}{\pi}\sqrt{3} \times 10^{-2}\%$, then the value of n is:

5) A simple pendulum of inextensible string of length □ and mass m is hinged at point A in a vertical plane as shown in the figure. The ball is released from the same horizontal level as that of A and at a distance $\frac{\sqrt{3}\ell}{2}$ from point 'A'. At the lowest point bob hits a block of identical mass m elastically. Find the maximum compression in the spring ($\frac{m\ell}{k} = \frac{56}{10}\text{kg} - \text{m}^2/\text{N}$ and ground is smooth). Height of A from ground is equal to □.

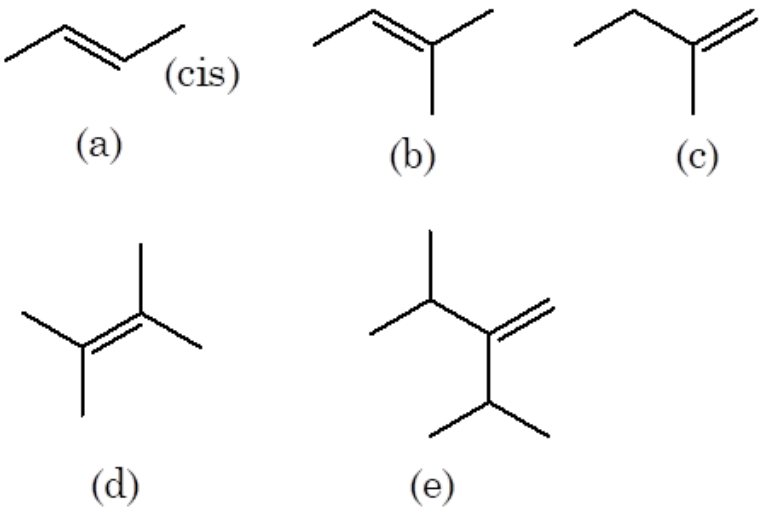


6) In the given circuit, the value of R (in Ω) so that thermal power generated in R will be maximum.



PART-2 : CHEMISTRY

SECTION-I



1)
Select correct statements for all above alkenes.

(A) Compound ‘e’ shows the highest heat of combustion.

(B) Compound ‘d’ is the thermodynamically least reactive towards catalytic hydrogenation.

(C) On ozonolysis using O_3/Me_2S, only 2 alkenes give smallest non–cyclic ketone.

(D) Only 2 alkenes among these give chiral alcohol product(s) on reactive with B_2H_6/THF followed by H_2O_2/OH^-.

2) How many of these solutions (of equal volume) on mixing create a Buffer solution?

(A) 0.1 M NaOH, 0.3 M HCN

(B) 0.3 M H_2SO_4, 0.8 M $CH_3COO^-Na^+$

(C) 0.2 M $Ba(OH)_2$, 0.4 M HCN

(D) 0.3 M H_2CO_3, 0.1 M NaOH

3) Among the following solutions, which two solutions will show ‘highest’ and ‘lowest’ increase in temperature on mixing?
Assume that initially all solutions at 25°C and density of solutions are identical.

(A) 0.4 M, 50 mL NaOH with 0.5 M, 100 mL HNO_3

(B) 0.2 M, 75 mL H_2SO_4 with 0.6 M, 25 mL NaOH

(C) 0.3 M, 20 mL HBr with 0.3 M, 80 mL $Ba(OH)_2$

(D) 0.9 M, 40 mL $Ba(OH)_2$ with 0.7 M, 60 mL HI

4) Select correct for metallurgical process which is/are involve in extraction metal from galena.

(A) Reduction of roasted ore with fresh galena

(B) Copper impurities can be remove by liquation

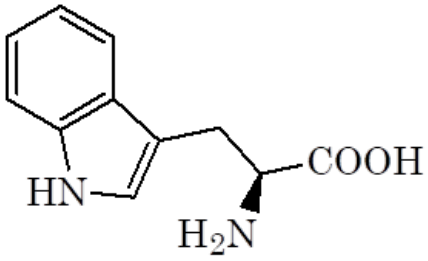
(C) Alkali metal cynide selectively prevent galena coming to the froth

(D) Oxidation of silver impurity in molten metal by O_2

SECTION-II (i)

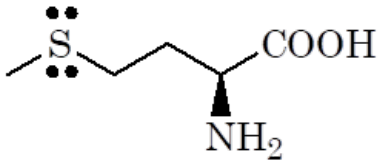
Common Content for Question No. 1 to 2

The following amino acids are used to form dipeptides :



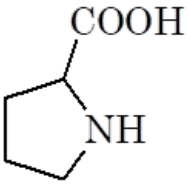
Tryptophan

(A)



Methionine

(B)



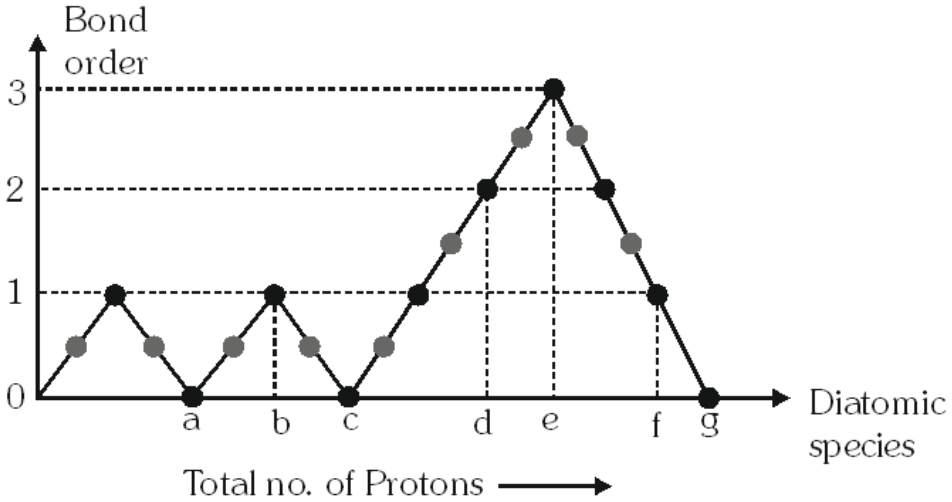
Proline

(C)

1) A dipeptide 'X' is formed which has maximum possible degree of unsaturation. The number of π–bonds in that dipeptide is 2n. Find the value of n.

2) Another dipeptide 'Y' with only one Sulphur atom and two nitrogen atoms is formed. Let 'P' is the number of oxygen atoms in 'Y' and 'Q' is the number of chiral centres in 'Y'. Then report the value of (P + Q)

Common Content for Question No. 3 to 4



Consider the following graph of 'Bond order' vs 'total number of protons' in diatomic species 'a' to 'f' According to molecular orbital theory, if bond order is zero at 'a', 'c' and 'g' (protons in 'g' = 20') then species is highly unstable & most likely non–existent.

3) Among the following species, how many will either relate to 'd' or 'e'?

$Ne_2^+, NO^+, N_2^+, N_2, O_2, O_2^{2+}, O_2^-, CN^-, BN$

4) If X_2 diatomic species relates to 'b' and Y_2 relates to 'f', then total number of electrons in σ*2s of both X_2 & Y_2 species for ground state only.

Common Content for Question No. 5 to 6

16 g of Sulphur is reacted with 'X' g of O_2 to produce both SO_2 & SO_3 gases. The actual proportion of SO_2 & SO_3 produced depends on value of 'X' (relative to mass of Sulphur taken). It is best to associate combustion of Sulphur to SO_2 & SO_3 using two separate chemical equations. (S = 32, O =

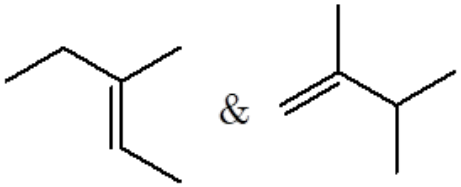
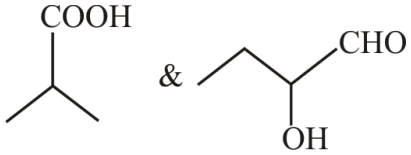
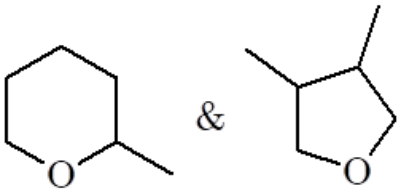
16)

5) Find mass % of SO_2 in product mixture if X = 25 (refer initial data in comprehension) and no sulphur is left behind.

6) If value of X = 20, then the ratio of moles of SO_2 & SO_3 formed as products, if none of the reactant is left behind is a : 1. The value of 'a' is.

SECTION-II (ii)

1) Correct matching from the two lists.

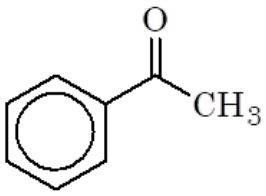
	List-I		**List-II**
(P)		(1)	Positional isomers
(Q)	$CH_3CH_2CH_2OH$ & $CH_3OCH_2CH_3$	(2)	Metamers
(R)		(3)	Chain isomers
(S)		(4)	Functional isomers

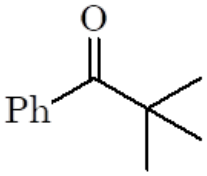
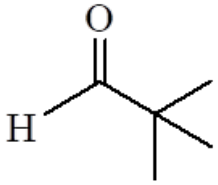
2) Correct matching from the two lists.

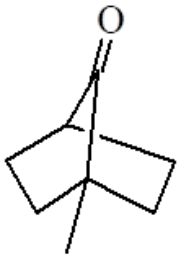
	List-I		**List-II (Type of π-bonds)**
(P)	NO_3^-	(1)	Only pπ–dπ bonding
(Q)	$(CH_3)_2O$	(2)	No pπ–dπ and pπ–pπ bonding
(R)	SO_3	(3)	Only pπ–pπ bonding
(S)	SO_4^{2-}	(4)	Both pπ–pπ and pπ–dπ bonding

SECTION-III

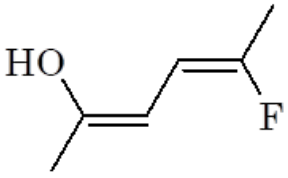
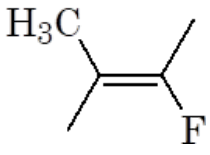
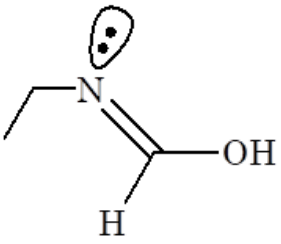
1) How many of these do not give self-aldol reaction?

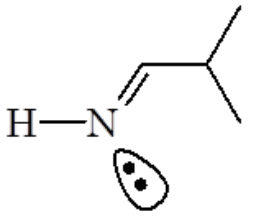
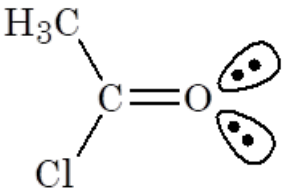
(i) CH_3CHO (ii) $CH_3COCH_2CH_3$ (iii) CD_3COCD_3 (iv) 

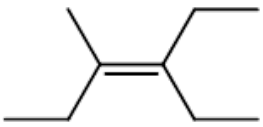
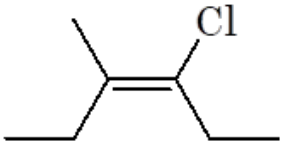
(v) Ph–CHO (vi) HCHO (vii)  (viii) 

(ix) 

2) How many of these can exhibit geometrical isomerism?

(i)  (ii)  (iii) 

(iv)  (v)  (vi) $(CH_3)_2C{=}CHMe$

(vii)  (viii)  (ix) 

3) A weak acid HA aqueous solution is found to have conductivity value of 4.5×10^{-4} $ohm^{-1}cm^{-1}$. Its concentration was found to be 0.01 molar and conductivity of water is found to be 3×10^{-4} Scm^{-1}. If K_a of this acid HA is $y \times 10^{-6}$, report value of 'y'? (Answer to the nearest integer)

Given: $\Lambda^0_{m(HA)} = 500\,Scm^2mol^{-1}$ [Limiting molar conductance]

4) A sample containing fixed amount of ideal gas mass, is subjected to 50% increase in T & 25% decrease in pressure. If the ratio of initial density over final density of gas sample is x : 1, the value of 'x' is?

5) If reaction $A \xrightarrow{K} B$ has K = 6.93×10^{-3} min^{-1} at 20°C, then half-life of 'A' at 70°C is $\frac{25x}{16}$ min. Find value of 'x', if temperature coefficient of reaction is assumed to have a constant value of 2 ?

6) SO_3 decomposes into SO_2 & O_2 according to this equation in a rigid closed vessel of volume 1 litre.

$$SO_3(g) \longrightarrow SO_2(g) + \frac{1}{2}O_2(g)$$

At time = 20 min, rate of appearance of O_2 was 20 g L^{-1} per minute and $\frac{d[SO_3]}{dt}$ at that instant is reported as $\frac{-x}{3} g L^{-1} s^{-1}$. Then value of 'x' is? [Atomic mass of S = 32]

PART-3 : MATHEMATICS

SECTION-I

1) Which of the following is/are correct

(A) ${}^{30}C_0 \cdot {}^{90}C_{60} - {}^{30}C_1 \cdot {}^{89}C_{60} + {}^{30}C_2 {}^{88}C_{60} + \ldots + {}^{30}C_{30} {}^{60}C_{60} = {}^{60}C_{30}$

(B) ${}^{30}C_0 \cdot {}^{90}C_{30} - {}^{30}C_1 \cdot {}^{89}C_{30} + {}^{30}C_2 {}^{88}C_{30} + \ldots + {}^{30}C_{30} {}^{60}C_{30} = 0$

(C) ${}^{30}C_0 \cdot {}^{90}C_{30} - {}^{30}C_1 \cdot {}^{88}C_{30} + {}^{30}C_2 {}^{86}C_{30} + \ldots + {}^{30}C_{30} {}^{30}C_{30} = (2)^{30}$

(D) ${}^{30}C_0 \cdot {}^{90}C_{30} - {}^{30}C_1 \cdot {}^{87}C_{30} + {}^{30}C_2 {}^{84}C_{30} + \ldots + {}^{30}C_{20} {}^{30}C_{30} = 1$

2) Let $-x + y + 2 = 0$ and $2x - y + 6 = 0$ be tangent of parabola having focus at (1, 3) then :

(A) Equation of directrix of parabola is $3x + 4y = 11$

(B) Length of latus rectum of parabola is $\left(\frac{4}{5}\right)$

(C) Equation of tangent at vertex is $3x + 4y - 13 = 0$

(D) Equation of axis of parabola is $4x - 3y + 5 = 0$

3) Which of the following is (are) correct (where [.] represent GIF and {.} represent fractional part function)

(A) Let $f(x) = \left\{\frac{x}{3}\right\} + \left[\frac{x}{5}\right]$, $x \in (0, 301)$ then number of points where f(x) is discontinuous is 120

(B) Let $f(x) = [3\sin x + 4\cos x]$, $x \in [0, 2\pi]$ then number of points where f(x) is discontinuous is 18

(C) $\int_0^{\pi} |3\sin x + 4\cos x|\, dx = 10$

(D) Let $f(x) = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3} + \frac{1}{x-4}$ then number of real roots of equation f(x) = 0 is 3

4) If A_i is the area bounded by $|x - a_i| + |y| = b_i$, $i \in N$ where $a_{i+1} = a_i + \frac{3}{2}b_i$ and $b_{i+1} = \frac{b_i}{2}$, $a_1 = 0$, $b_1 = 32$ then

(A) $A_3 = 128$

(B) $A_3 = 256$

(C) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$

(D) $\frac{A_6}{A_{10}} = 256$

SECTION-II (i)

Common Content for Question No. 1 to 2

Let $x^2 - x - 1 = 0$ is a factor of $ax^9 + bx^8 + 1 = 0$ then answer the following questions :

1) The value of '2a+b' is :

2) Number of real roots of equation $ax^9 + bx^8 + 1 = 0$ is:

Common Content for Question No. 3 to 4

Let 'z_1' and 'z_2' be any two distinct complex number which satisfy $z^{120} = 1$.

3) The probability that $|z_1 - z_2| \geq \frac{\sqrt{5}+1}{2}$ is $\frac{p}{q}$ (where p and q are co-prime) then the value of q – 2p is :

4) The number of possible ordered pair (z_1, z_2) such that $|z_1 - z_2| \leq \sqrt{2-\sqrt{3}}$

Common Content for Question No. 5 to 6

Consider all the permutation of the word 'MAHAKUMBH'

5) Number of permutation of the word in which vowels are in alphabetical order is P then the value of $\frac{P}{30}$ is :

6) Number of permutation of the word if no two alike letters are together is N then total number of divisor of N is :

SECTION-II (ii)

1) Let A be any 3 × 3 matrix such that |A| = 2, then match the column

	List-I		List-II
(P)	$\lvert 3\,\text{adj}(-2\text{adj}(\text{adj}(A)))\rvert$	(1)	$2^{14}.3^3$
(Q)	$\lvert 2\,\text{adj}(3\text{adj}(\text{adj}(2A)))\rvert$	(2)	$3^{12}.2^8$

(R)	$\lvert \text{adj}(\text{adj}(\text{adj}(3A))) \rvert$	(3)	$2^{35} \cdot 3^6$
(S)	$\lvert \text{adj}(\text{adj}(3\text{adj}(A))) \rvert$	(4)	$3^{24} \cdot 2^8$

2) Match the following

	List-I		List-II
(P)	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = \sqrt{6}$,then $2\lvert\vec{a}\rvert$ is equal to	(1)	3
(Q)	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular $\vec{a}+\vec{b}$, where $\lvert\vec{a}\rvert = 2$, $\lvert\vec{b}\rvert = 3$ and $\lvert\vec{c}\rvert = 6$ then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert - 2$ is equal to	(2)	2
(R)	$\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{d} = 3\hat{i}+2\hat{j}+\hat{k}$, then $\frac{1}{7}(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to	(3)	4
(S)	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 2$ and $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 2$ then $[\vec{a}\vec{b}\vec{c}]\cos 45^\circ$ is equal to	(4)	5

SECTION-III

1) If $\lim\limits_{n\to\infty}\sum\limits_{n=1}^{n}\tan^{-1}\left(\frac{2n}{n^4+n^2+2}\right) = L$, then the value of [8L] (where [.] is represent G.I.F)

2) Let $f(x) = \sqrt{7-3x}$, $x \leqslant \frac{7}{3}$ then number of solutions of the equation $f(x) = f^{-1}(x)$ is :

3) Number of solution of the equation $\log_k x = \cos(\pi x)$ is 3 , then sum of all possible integers value of 'k' is :

4) The value of integral $\int_0^1 \frac{12x^{11}}{3x^6-3x^3+1}dx$ is :

5) Let $P = (1)^{1/4}+(2)^{1/4}+....+(625)^{1/4}$ and $\left[\frac{P}{5}\right] = N$ then total number of even divisor of N is (where [.] represents greatest integer function).

6) The value of $\int_0^2 \left(\sqrt{1+x^3}+\sqrt[3]{x^2+2x}\right)dx$ is equal to :

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4
A.	A,D	A,C	B,C	A,C

SECTION-II (i)

Q.	5	6	7	8	9	10
A.	2	2	1	1	4	6

SECTION-II (ii)

Q.	11	12
A.	1314	3143

SECTION-III

Q.	13	14	15	16	17	18
A.	6	6	7	4	7	3

PART-2 : CHEMISTRY

SECTION-I

Q.	19	20	21	22
A.	A,B,C	A,B,D	C,D	A,B

SECTION-II (i)

Q.	23	24	25	26	27	28
A.	5	5	2	2	0	1

SECTION-II (ii)

Q.	29	30
A.	3442	3241

SECTION-III

Q.	31	32	33	34	35	36
A.	5	5	9	2	2	5

PART-3 : MATHEMATICS

SECTION-I

Q.	37	38	39	40
A.	A,C	A,C,D	A,C,D	A,C,D

SECTION-II (i)

Q.	41	42	43	44	45	46
A.	8	3	3	2400	504	48

SECTION-II (ii)

Q.	47	48
A.	1342	2413

SECTION-III

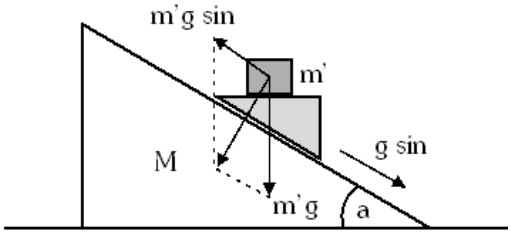
Q.	49	50	51	52	53	54
A.	6	3	3	2	8	6

SOLUTIONS

PART-1 : PHYSICS

2)

The forces acting on the wedge are its weight mg and a force K, perpendicular to the inclined plane; the magnitude of the latter may change with time. As a result of these two forces, the only component of the wedge's acceleration a parallel to the inclined plane is g sin α (as measured in an inertial frame). Newton's equations of motion remain valid in an accelerating frame of reference fixed to the wedge only if an 'inertial force' –m'a is added to the forces actually causing the motion of a body. Here m' is the mass of the body under examination (e.g. that of a small volume of water).



The resultant of the gravitational and inertial forces acting on the mass m' must be perpendicular to the inclined plane as the components parallel to it cancel each other. The bodies on the wedge (the glass and the water in it) 'feel' as if they were in a gravitational field perpendicular to the inclined plane, with the consequence that the surface of the water lies parallel to the plane.
This statement does not depend on the motion of the plane; it can be fixed or move freely or even–as the result of a small force – be shaken to and fro. As long as the friction between the inclined plane and the wedge is negligible and the wedge does not rise off the plane, the shape of the water surface cannot be other than a plane parallel to the inclined surface.

3) $\frac{1}{2} \times (P_3 - P_1) \times (V_2 - V) = \frac{1 \times 3}{2} R \times \frac{(P_3 - P_1)}{R} V$

$V_2 - V = 3V$

$V_2 = 4V$

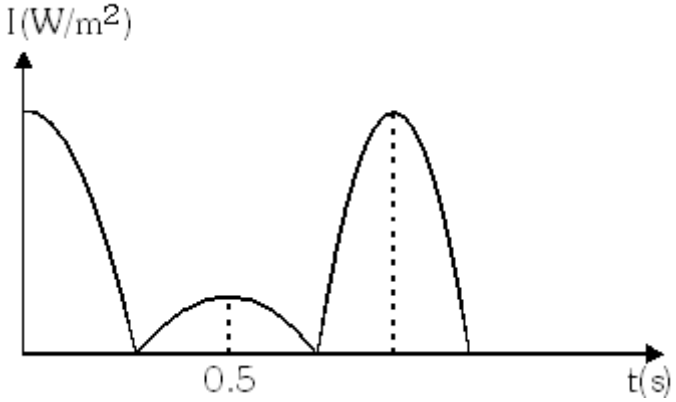
4)

Let '2a' & '2b' be the major axis & minor axis length.

$$\frac{mv^2}{\left(\frac{b^2}{a}\right)} = 20 \quad \text{....(i)}$$

$$\frac{mv^2}{\left(\frac{a^2}{b}\right)} = 2.5 \quad \text{....(ii)}$$

We get a = 5, b = 2.5



5) As observed from the graph we get 2 maxima in every one second.

6) $2A\cos\theta + A \leqslant 2A$

$\cos\theta \leqslant \frac{1}{2}$

we get, $\tau = \frac{2}{3}\,\text{sec}.$

7) $C = \left[\frac{K\varepsilon_0 by}{s} + \frac{K\varepsilon_0 by}{s}\right] = \frac{2K\varepsilon_0 by}{s}$

W.D. $dW_{ext.} = \frac{1}{2}dC\,V^2$

We get work done = $\frac{(\Delta V)^2 bK\varepsilon_0 \Delta y}{s}$

8) $F = \frac{dW_{ext}}{dy}$

We get, $F = \sqrt{\frac{sF}{\epsilon_0 bK}}$

10) $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$

$\Rightarrow n \simeq 4$

$k = nC_2 = \frac{4 \times 3}{2} = 6$

12) **For (P) :**

$T.E. \propto W^2A^2$

$(2W)^2 a_2^2 = W^2 a_1^2$

$\frac{a_1}{a_2} = 2$

For (Q) :

Initial condition $Q_1 = 3C$

$3 - \left(\frac{3C + Q_2}{C}\right) - \frac{Q_2}{2C} = 0$

$Q_2 = 0$

For (R) :

$T_1 \cos\theta_1 = T_2 \cos\theta_2 = T$ ....(1)

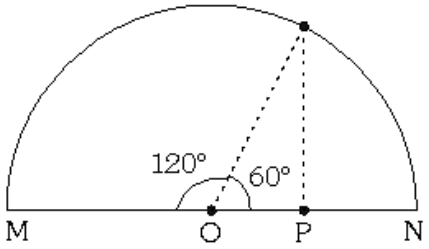
$T_1 \sin\theta_2 = m_1 g$ ....(2)

$T_2 \sin\theta_2 = m_2 g$ ....(3)

$$\Rightarrow \frac{m_1}{m_2} = \left(\frac{T_1}{T_2}\right)\frac{\sin\theta_2}{\sin\theta_2} = \left(\frac{\cos\theta_2}{\cos\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)$$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$$

For (S) :



$$t_1 = \frac{T}{4} + \frac{T}{12} = \frac{T}{3}$$

$$t_2 = \frac{T}{6}$$

$$\frac{t_1}{t_2} = 2$$

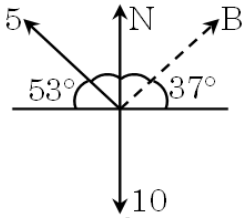
13) $E = B_\perp Lv = B\sin 37^\circ Lv$

$$= 0.5 \times \frac{3}{5} \times 0.3 \times 10 = 0.9V$$

$$i = \frac{0.9}{27\times 10^{-3}} = \frac{100}{3}A$$

$$F = i\vec{\ell}\times\vec{B} = \frac{100}{3} \times 0.3 \times 0.5 \times 1 = 5N$$

$N + 45^\circ \sin 53^\circ = mg = 40$



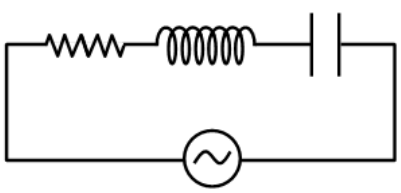
$$N + 5 \times \frac{4}{5} = 10$$

$N = 6$

$F = 5\cos 53^\circ + \mu N$

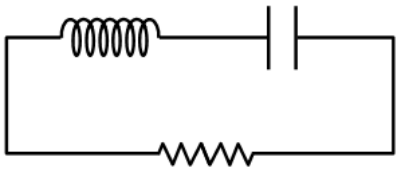
$$= 5 \times \frac{3}{5} + 0.5 \times 6 = 6$$

14)



$V_L = V_C$ = resonance

$$\omega L = \frac{1}{\omega C}$$



$$\omega' = \sqrt{\frac{1}{LC} = \frac{R^2}{4L^2}}$$

$V = iR \Rightarrow R = \frac{V}{i}$

$V = i\omega L \Rightarrow L = \frac{V}{i\omega}$

$V = \frac{i}{\omega C}; \; C = \frac{i}{V\omega}$

$$\omega' = \sqrt{\frac{1}{\frac{V}{i\omega} \times \frac{i}{V\omega}} - \frac{V^2}{V^2 \times 4V^2} \times i^2\omega^2}$$

$$= \frac{\sqrt{3}}{2}\omega = \frac{\sqrt{3}}{2} \times 4\sqrt{3} = 6 \text{ rad/s}$$

15) $Q = \left(M_{He} + M_{Li} - M_{B_{10}} - M_n\right) C^2 = -2.79 \text{ MeV}$

$$\frac{1}{2} \times \frac{4 \times 7}{11}u^2 = 2.79 \text{ MeV}$$

$4u = 11v$

$$v = \frac{4u}{11}$$

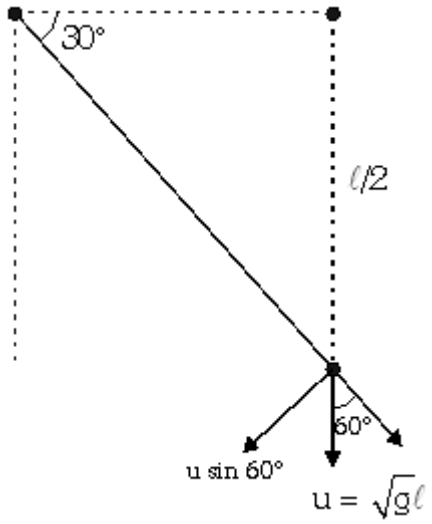
$$K_n = \frac{1}{2} \times 1 \times \frac{16u^2}{121} = \frac{8}{121} \times \frac{22 \times 2.79}{28}$$

$$= \frac{4 \times 2.79}{77} \times 50 \approx 7 \text{ MeV}$$

16) $\frac{dI}{d\theta} = I_0 \sin 2\theta$

$$\Rightarrow \frac{d\theta}{\theta} \times 100 = \frac{dI}{\theta(I_0 \sin 2\theta)} \times 100$$

17) $\frac{1}{2}mu^2\sin^2 60^\circ = \frac{1}{2}mv^2 + \frac{mg\ell}{2}$

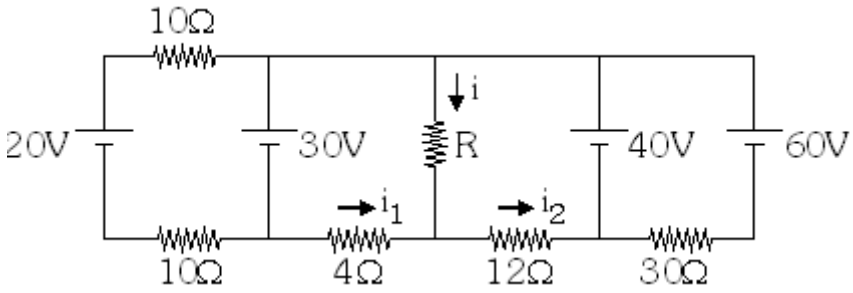


$\Rightarrow v = \sqrt{\frac{7g\ell}{2}}$

$\frac{1}{2}kx^2 = \frac{1}{2}\frac{m \times m}{2m}\frac{7g\ell}{4}$

$\Rightarrow x\sqrt{\frac{70}{8}\left(\frac{ml}{K}\right)}$, m = 0.1kg and k = 10/8

⇒ □ = 7m



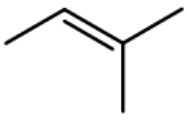
18)

$R_{eq} = R_{in} = \frac{4 \times 12}{4 + 12} = 3\Omega$

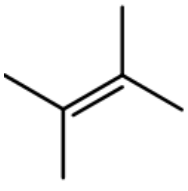
PART-2 : CHEMISTRY

19)

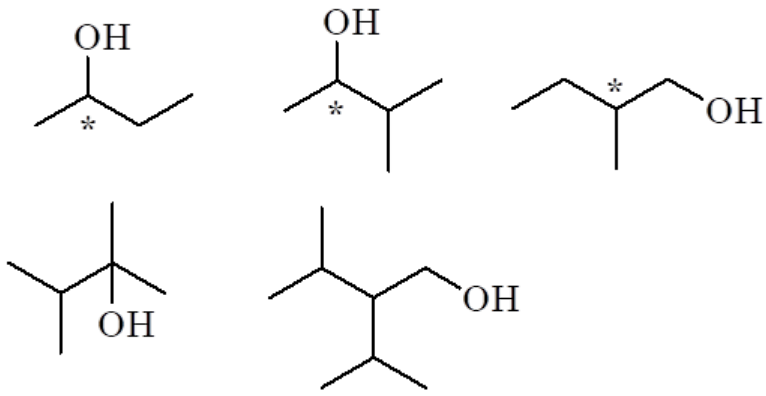
(A) Compound 'e' has maximum carbon content, so maximum heat of combustion.
(B) Compound 'd' is most stable alkene. (Maximum hyperconjugable H)



$\xrightarrow[Me_2S]{O_3}$ $CH_3CHO + CH_3COCH_3$ (Acetone)



(C) $\xrightarrow[Me_2S]{O_3}$ $2 \times CH_3COCH_3$ (Acetone)

(D) 

20)

(A) $\left.\begin{matrix} HCN \longrightarrow 0.2V \\ NaCN \longrightarrow 0.1V \end{matrix}\right\} \longrightarrow \text{imme conju.} \} \longrightarrow \frac{C_1}{C_2}$ ratio ok $\longrightarrow$ Buffer formed

(B) $\left.\begin{matrix} CH_3COOH \longrightarrow 0.6V \\ CH_3COO^- \longrightarrow 0.2V \end{matrix}\right\} \longrightarrow \text{imme conju.} \} \longrightarrow \frac{C_1}{C_2}$ ratio ok $\longrightarrow$ Buffer formed

(C) CN^- only → imme. Conju. not formedNo buffer formed.

(D) $\left.\begin{matrix} H_2CO_3 \longrightarrow 0.2V \\ HCO_3^- \longrightarrow 0.1V \end{matrix}\right\} \begin{matrix} \text{left} \\ \text{formed} \end{matrix} \longrightarrow$ bircarbonate with carbonic acid $\longrightarrow$ Buffer formed

21) Heat of neutralization Q ∝ $n_{acid/base}$ (depends on LR)

$$\Delta T \propto \frac{Q}{V_{Solution}} \Rightarrow \Delta T \propto \frac{n_{A/B}}{V_{Solution}}$$

	n_{H^+}	n_{OH^-}	$V_{Solution}$	n/V	
(A)	n_1 = 50 m mol	n_2 = 20 m mol	150 mL	20/150	
(B)	n_1 = 30 m mol	n_2 = 15 m mol	100 mL	15/100	
(C)	n_1 = 6 m mol	n_2 = 48 m mol	100 mL	6/100	(lowest)
(D)	n_1 = 42 m mol	n_2 = 72 m mol	100 mL	42/100	(highest)

T_{Final} of 4 solutions: D > B > A > C

23) (Trp–Trp) is dipeptide with maximum DOU and total '10' π-bonds.

24) Met–Pro or Pro–Met is the dipeptide 'Y'.
P = 3
Q = 2

$$'d' \longrightarrow \text{B.O.} = 2$$

$$\underbrace{\text{with no. of protons} = 12}_{\text{BN species}}$$

$$'e' \longrightarrow \text{B.O.} = 3$$

$$\underbrace{\text{no. of protons} = 14}_{N_2 \text{ molecules}}$$

25)

26) $X_2 = Li_2 \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^0$

$Y_2 = F_2 \Rightarrow [KK] \sigma 2s^2 \sigma^* 2s^2 \sigma^2 2p_z \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^2 = \pi^* 2p_y^2 \sigma^* 2p_z^2$

27) Assume 'x' grams of 'S' converting to SO_2 gas, then

$$S + O_2 \longrightarrow SO_2$$

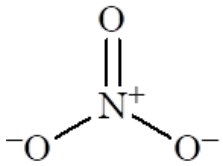
$$\frac{x}{32}\text{ mol} \quad \frac{x}{32}\text{ mol}$$

$$S + \frac{3}{2}O_2 \longrightarrow SO_3$$

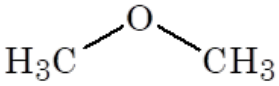
$$\frac{16-x}{32} \quad \frac{3}{2}\left(\frac{16-x}{32}\right)\text{mol}$$

$$\left[\frac{x}{32} + \frac{3}{2}\left(\frac{16-x}{32}\right)\right] \times 32 = 25 \Rightarrow x = -ve$$

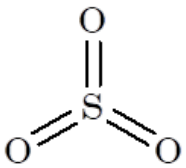
Hence all 'S' is converted to SO_3.



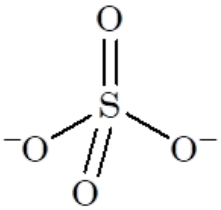
1 pπ–pπ



No π–bond



1 pπ–pπ
2 pπ–dπ



2 pπ–dπ

30)

31)

(i), (ii), (iii), (iv) will give self–aldol reaction.
(v), (vi), (vii), (viii) and (ix) will not give self–aldol reaction.

32) (i), (iii), (iv), (vii) and (ix) exhibit geometrical isomerism.

33) $K_{Solution} - K_{Water} = K_{Solute\ or\ Electrolyte)}$

$K_{HA} = K_{Solution} - K_{H_2O} \Rightarrow (4.5 \times 10^{-4} - 3 \times 10^{-4}) = 1.5 \times 10^{-4}\ Scm^{-1}$.

$$\Lambda_m = \frac{K_{HA} \times 1000}{M} = \frac{1.5 \times 10^{-4} \times 1000}{0.01} = 15\,Scm^2mol^{-1}$$

$$\alpha = \frac{\Lambda_m}{\Lambda_m^0} \Rightarrow \frac{15}{500} = 0.03$$ (3% dissociation)

$$K_a = C\alpha^2 \Rightarrow (0.01)(0.03)^2 = 9 \times 10^{-6}$$

34)

Initial **Final**

$P_1 = P$ $P_2 = (3/4)P = 0.75P$

$T_1 = T$ $T_2 = (3/2)T = 1.5T$

$d_1 = d$ $d_2 = ?$

$$\frac{P_1}{T_1.d_1} = \frac{P_2}{T_2.d_2} = \frac{R}{M} = \text{constant}$$

$$d_2 = \frac{P_2}{P_1}.\frac{T_1}{T_2}.d_1 \Rightarrow d_2 = \left(\frac{0.75}{1.5}\right) \times d_1 \Rightarrow \frac{d_1}{d_2} = 2$$

35) At 20°C $t_{1/2} = \frac{0.693}{0.00693} = 100\ min.$

$$\frac{K_{final}}{K_{initial}} = (\mu \times \mu \times .....) = 2^{(\Delta T/10)} = 2^{0.5} = 32$$

$$(t_{1/2})_{final} = \frac{1}{32}(t_{1/2})_{initial} = \frac{100}{32}\,min.,$$

Hence = 2.

36) $$\frac{d[O_2]}{dt} = \frac{20}{32}\,mol\,L^{-1}\,min^{-1} = \frac{1}{96}\,molar\,sec^{-1} = \frac{20}{32 \times 60}$$

$$-\frac{d[SO_3]}{dt} = \frac{2.d[O_2]}{dt} = ror. \Rightarrow \frac{d[SO_3]}{dt} = \frac{-1}{48}\,molar\,s^{-1}$$

$$\frac{-80}{48}\,g\,L^{-1}s^{-1} \Rightarrow \frac{-5}{3}\,g\,L^{-1}s^{-1}$$

PART-3 : MATHEMATICS

37)

Compare coefficient of x^{60} and x^{30}

Both side in expansion of $\left((1+x)^3 - (1+x)^2\right)^{30}$ to get A and B for (C) compare coefficient of x^{30} in

$\left((1+x)^3 - (1+x)^2\right)^{30}$

38)

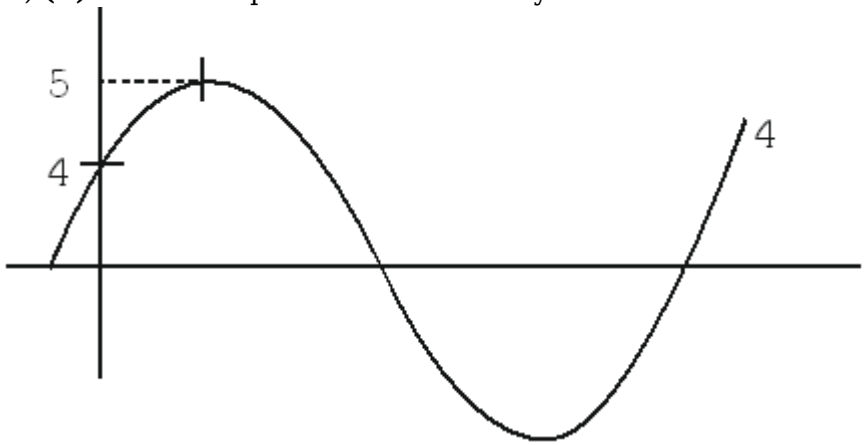
Reflection of (1, 3) in the tangent lines lie on directrix

Directrix passes through (5, –1) and (–3, 5)

So equation of directrix is $(y+1) = \frac{6}{8}(x-5)$

$3x + 4y = 11$

39) **(A)** Number of points of discontinuity $\left[\frac{301}{3}\right] + \left[\frac{301}{5}\right] - 2\left[\frac{301}{15}\right] = 100 + 60 - 40 = 120$



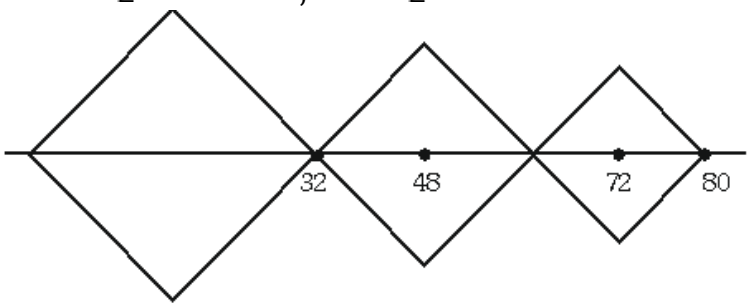
Number of points of discontinuity is 19

40)

$a_1 = 0,\ b_1 = 32,\ a_2 = a_1 + \frac{3}{2}b_1 = 48$

$b_2 = \frac{b_1}{2} = 16$

$a_3 = 48 + \frac{3}{2} \times 16 = 72,\ b_3 = \frac{16}{2} = 8$



Area of i^{th} loop (square) $= \frac{1}{2}(\text{diagonal})^2$

$A_i = \frac{1}{2}(2b_i)^2 = 2(b_i)^2$

$\frac{A_{i+1}}{A_i} = \frac{2(b_i+1)^2}{2(b_i)^2} = \frac{1}{4}$

$\lim_{n\to 0} \sum A_i = \frac{A_1}{1-r} = \frac{2(32)^2}{1-\frac{1}{4}} = \frac{8}{3} \times (32)^2$

41)

$$x^2 - x - 1 = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$$

$\alpha + \beta = 1, \alpha\beta = -1$

$a\alpha^9 + b\alpha^8 + 1 = 0$ ....(1)

$a\beta^9 + b\beta^8 + 1 = 0$ ....(2)

$$\Rightarrow \frac{a}{\alpha^8 - \beta^8} = \frac{-b}{\alpha^9 - \beta^9} = \frac{1}{\alpha^9\beta^8 - \alpha^8\beta^9} = \frac{1}{\alpha - \beta}$$

$$\Rightarrow a = \frac{\alpha^8 - \beta^8}{\alpha - \beta} = (\alpha + \beta)(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)$$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = 9 - 2 = 7$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^4 - 2\alpha^4\beta^4 = 49 - 2 = 47$

Hence $a = 1 \times 3 \times 7 = 21$

Also

$$b = -\frac{\alpha^9 - \beta^9}{\alpha - \beta}$$

$$= -(\alpha^8 + \alpha^7\beta + \alpha^6\beta^2 + \alpha^5\beta^3 + ... + \beta^8)$$

$$= -\left((\alpha^8 + \beta^8) - (\alpha^6 + \beta^6) + (\alpha^4 + \beta^4) - (\alpha^2 + \beta^2) + (\alpha^4.\beta^4)\right)$$

$= -(47 - 18 + 7 - 3 + 1) = -34$

$21x^9 - 34x^8 + 1 = 0$ have three real and distinct roots.

42)

$$x^2 - x - 1 = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$$

$\alpha + \beta = 1, \alpha\beta = -1$

$a\alpha^9 + b\alpha^8 + 1 = 0$ ....(1)

$a\beta^9 + b\beta^8 + 1 = 0$ ....(2)

$$\Rightarrow \frac{a}{\alpha^8 - \beta^8} = \frac{-b}{\alpha^9 - \beta^9} = \frac{1}{\alpha^9\beta^8 - \alpha^8\beta^9} = \frac{1}{\alpha - \beta}$$

$$\Rightarrow a = \frac{\alpha^8 - \beta^8}{\alpha - \beta} = (\alpha + \beta)(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)$$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = 9 - 2 = 7$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^4 - 2\alpha^4\beta^4 = 49 - 2 = 47$

Hence a= 1 × 3 × 7 = 21

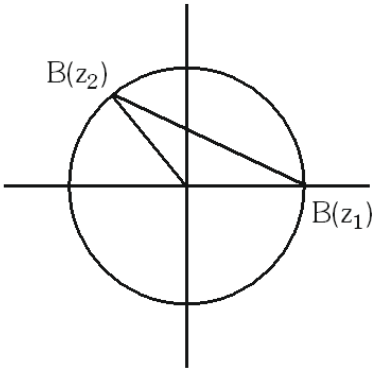
Also $b = -\frac{\alpha^9 - \beta^9}{\alpha - \beta}$

$= -(\alpha^8 + \alpha^7\beta + \alpha^6\beta^2 + \alpha^5\beta^3 + ... + \beta^8)$

$= -\left((\alpha^8 + \beta^8) - (\alpha^6 + \beta^6) + (\alpha^4 + \beta^4) - (\alpha^2 + \beta^2) + (\alpha^4.\beta^4)\right)$

$= -(47 - 18 + 7 - 3 + 1) = -34$

$21x^9 - 34x^8 + 1 = 0$ have three real and distinct roots.



43)

$|z_1 - z_2| \geqslant \frac{\sqrt{5} + 1}{2}$

$\Rightarrow \quad 2\sin\left(\frac{\theta}{2}\right) \geqslant 2\sin(54^\circ)$

$\Rightarrow \quad \theta \geqslant 108^\circ$

No of favourable cases $= \left(120 \times \frac{49}{2}\right)$

h(s) = $^{120}C_2$

$P(E) = \frac{49 \times 120}{120 \times 119} = \frac{49}{119}$

$= \frac{7}{17} \Rightarrow q - 2p = 3$

44) $|z_1 - z_2| \leqslant \sqrt{2 - \sqrt{3}} = \sqrt{\frac{4 - 2\sqrt{3}}{2}} = \frac{\sqrt{3} - 1}{\sqrt{2}}$

$= 2 \sin 15^\circ$

$\Rightarrow \theta \leq 30^\circ$

45) $^9C_3 \cdot \frac{\lfloor 6}{\lfloor 2 \lfloor 2} = \frac{9.8.7}{3.2.1} = \frac{\lfloor 6}{4}$

$= 126 \times 120$

46)

P : A are together

Q : M are together

R : H are together

$n(P' \cap Q' \cap' R') = n(P \cup Q \cup R)'$

$= \frac{\lfloor 9}{\lfloor 2.\lfloor 2.\lfloor 2} - \left[3.\frac{\lfloor 8}{\lfloor 2\,\lfloor 2} - 3.\frac{\lfloor 7}{\lfloor 2} + \lfloor 6\right]$

$= \lfloor 7\left(9-6+\frac{3}{2}\right) - 720 = 21960$

47) $3^{12}.3^8$

(P) $|3\,\text{adj}(-2\text{adj}(\text{adj}(A)))|$

$= 3^3\left|(-2)^{3-1}\,\text{adj}(\text{adj}(\text{adj}A))\right|$

$= 3^3 2^6.|A|^{(3-1)^3}$

$= 3^3.2^6.2^8 = 2^{14}.3^3$

(Q) $|2\,\text{adj}(3\text{adj}(\text{adj}(2A)))|$

$= \left|2\,\text{adj}\,(3.2^4.\,\text{adj}\,(\text{adj}(A)))\right|$

$= \left|2^9.3^2\,\text{adj}\,(\text{adj}(\text{adj}(A)))\right|$

$= 2^{27}.3^6.2^8 = 2^{35}.3^6$

(R) $|\text{adj}(\text{adj}(\text{adj}(3A)))|$

$= \left|3^8.\text{adj}(\text{adj}(\text{adj}(A)))\right|$

$= 3^{24}|A|^8 = 2^8.3^{24}$

(S) $|\text{adj}(\text{adj}(3\text{adj}(A)))|$

$= 3^{12}|\text{adj}(\text{adj}(\text{adj}\,A)))|$

$= 3^{12}.2^8$

48)

(P) $\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{6}$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} = 6$

$\square \quad |\vec{a}| = 1$

(Q) $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$

$\Rightarrow \quad \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0 \qquad \ldots.(i)$

$\vec{b}$ is perpendicular to $\vec{a}+\vec{c}$

$\Rightarrow \quad \vec{b}.\vec{c} + \vec{b}.\vec{c} = 0 \qquad \ldots.(ii)$

$\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$

$\Rightarrow \quad \vec{c}.\vec{a} + \vec{c}.\vec{b} = 0 \quad \ldots(iii)$

From (i), (ii) and (iii), we get

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\left|\vec{a}+\vec{b}+\vec{c}\right| = 7$

49) $\sum_{n=1}^{n} \tan^{-1}\left(\frac{(n^2+n+1)-(n^2-n+1)}{1+(n^2+n+1)(n^2-n+1)}\right)$

$L = \lim_{n\to\infty} \sum_{n=1}^{n} \tan^{-1}(n^2+n+1) - \tan^{-1}(n^2-n+1)$

$= \frac{\pi}{4}$, $8L = 2\pi$

50) $\sqrt{7-3x} = y$, $7 - 3x = y^2$

$\Rightarrow x = \frac{7-y^2}{3}$, $f^{-1}(x) = \frac{7-x^2}{3}$

Now $\sqrt{7-3x} = \left(\frac{7-x^2}{3}\right)$

$\Rightarrow (7-3x) = \left(\frac{7-x^2}{3}\right)^2$

$\Rightarrow (7-3x)\,9 = (7-x^2)^2$

$\Rightarrow x^4 - 14x^2 + 49 + 27x - 63 = 0$

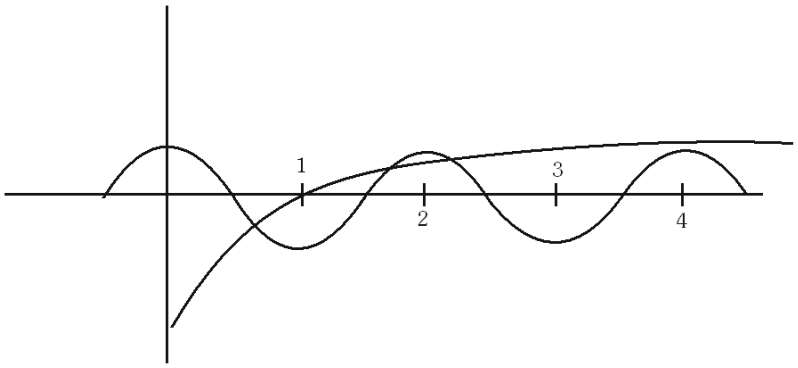
$\Rightarrow x^4 - 14x^2 + 27x - 14 = 0$

$\Rightarrow (x-1)(x^3 + x^2 - 13x + 14) = 0$

$\Rightarrow (x-1)(x-2)(x^2 + 3x - 7) = 0$

$\Rightarrow x = 1, 2, \frac{-3 \pm \sqrt{37}}{2}$

$\Rightarrow x = 1, 2, \frac{-3+\sqrt{37}}{2}, \left(\frac{-3-\sqrt{37}}{2}\right)$ (Rejected)



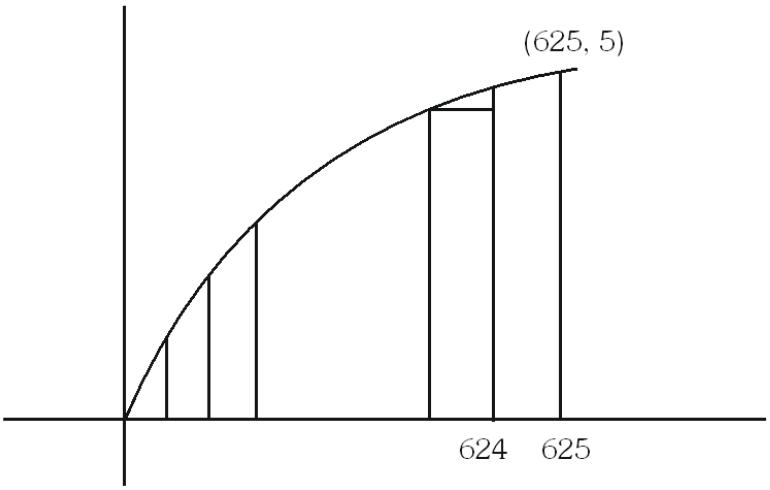
51) $\log_k 4 > 1$, $k < 4$ and $\log_k 2 < 1 \Rightarrow k > 2$

$\Rightarrow k = 3$

52)

Let $x^3 = t$, $3x^2\,dx = dt$

$I = 4\int_0^1 \frac{t^3}{3t^2-3t+1}\,dt = 4\int_0^1 \frac{t^3}{t^3+(1-t)^3}\,dt = 2$



53)

$$P > \int_0^{625} x^{1/4}\,dx = \left(\frac{4x^{5/4}}{5}\right)_0^{625}$$

$$= \frac{4}{5}(625)\,(5) = 2500$$

Also

$$P - 5 < \int_0^{625} (x)^{1/4}\,dx = 2500$$

$\Rightarrow$ P < 2505

$\Rightarrow$ $500 < \frac{P}{5} < 501$

54) Let $f:[0,2] \xrightarrow[\text{onto}]{\text{one-one}} [0,2]$

Defined as

$f(x) = \sqrt[3]{x^2 + 3x}$

$f^{-1}: [0, 2] \xrightarrow[\text{onto}]{\text{one-one}} [0, 2]$

Defined as $f^{-1}(x) = \left(\sqrt{1 + x^3} - 1\right)$

$$I = \int_0^2 \left(\sqrt{1 + x^3} + \sqrt[3]{x^2 + 2x}\right)$$

$$\int_0^2 \sqrt[3]{x^2 + 2x}\,dx + \int_0^{-2} \left(\sqrt{1 + x^3} - 1\right)dx + \int_0^2 (1)dx$$

= (4 - 0) + 2 = 6