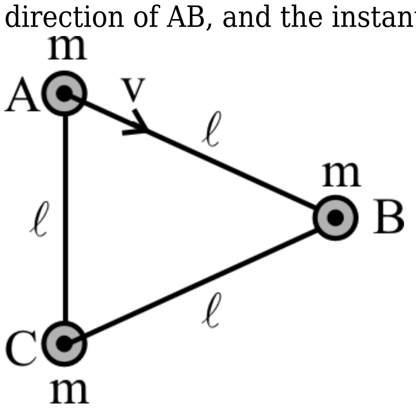
PART-1 : PHYSICS

SECTION-I

1) A system of three small spheres, each with mass m, connected by inextensible wires of negligible mass into the shape of a horizontal equilateral triangle with side length ℓ (see Fig.), slides without friction on a horizontal icy surface. At a certain moment, sphere A is moving with velocity v in the direction of AB, and the instantaneous velocity of sphere B is parallel to line BC.



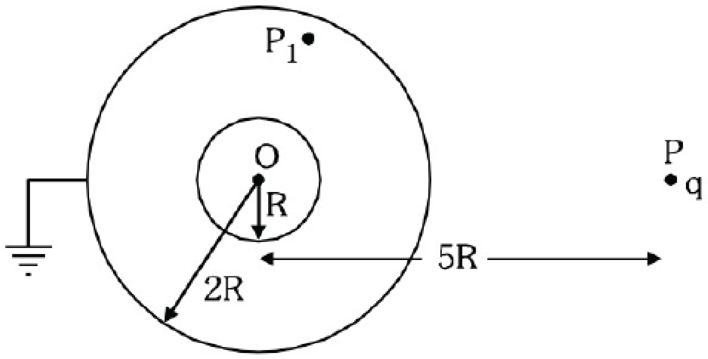
(A) The instantaneous speed of B is $v\sqrt{5}$.

(B) The angular velocity of the system is $\frac{v}{2\ell}$.

(C) The magnitude of force acting on the wires connecting the spheres is $\frac{2mv^2}{\ell}$.

(D) None of the above

2) Consider a solid conducting sphere of radius 2R. It has a spherical cavity of radius R has its centre at centre of solid sphere. The sphere is grounded. A point charge q is fixed at point P at distance of 5R. Choose the correct statement(s):



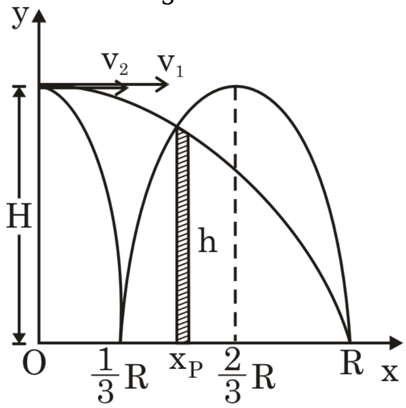
(A) The modulus of charge on outer surface of solid sphere is $\frac{2Q}{5}$.

(B) The electric field energy per unit volume at point P_1 is zero.

(C) The magnitude of net electric force experienced by "q" due to charge on the sphere is $\left[\frac{2Kq^2}{125R^2}\right]$.

(D) If the connection of sphere from ground is now removed then the potential energy of the system will increase.

3) As shown in the figure, balls 1 and 2 are both projected horizontally from the same point at a height H above the ground. Their horizontal initial velocities are v_1 and v_2 respectively, with $v_1 > v_2$. Ball 1 is projected and barely clears the top of a vertical obstacle located at position x_P, and then falls on the ground at point R. The distance between point R and point O is R. After being projected, ball 2 falls to the ground and undergoes an elastic collision, then bounces back over the obstacle and lands again at point R.



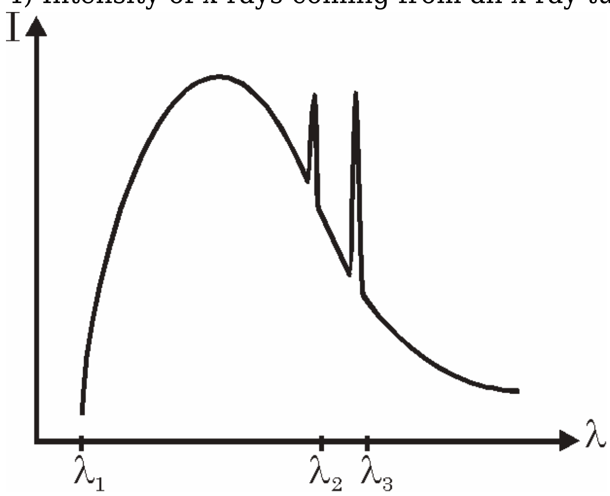
(A) The ratio of $\frac{v_1}{v_2}$ is 2.

(B) The position of obstacle x_P is $\frac{R}{2}$.

(C) The height of obstacle h is $\frac{3H}{4}$.

(D) None of the above.

4) Intensity of x-rays coming from an x-ray tube is plotted against wavelength as shown :-



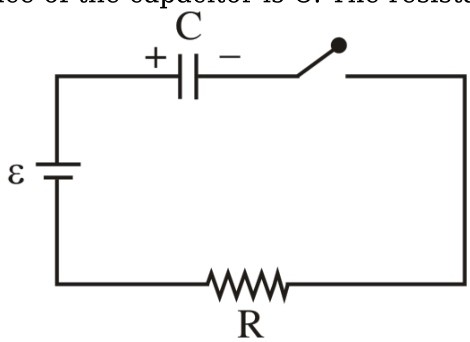
(A) If accelerating potential is changed, keeping target same, then sharp peaks (λ_2 & λ_3) will shift to new positions.

(B) If accelerating potential is changed keeping the target same, then minimum wavelength of spectrum (λ_1) will shift to new location.

(C) In the radiation coming out of the tube, photons corresponding to wavelength λ_3 will have more energy than those corresponding to λ_2.

(D) In the radiation coming out of the tube, number of photons corresponding to wavelength λ_3 will be more in number than number of photons corresponding to λ_2.

5) An electrical circuit consists of an ideal voltage source with an EMF ε, a resistor with resistance R, a switch, and a capacitor charged to a voltage of $\frac{\varepsilon}{3}$ (the polarity is indicated in the diagram). The capacitance of the capacitor is C. The resistance of the wires and the switch is negligible. The switch is closed.



(A) The maximum rate of change of energy in the capacitor is $\frac{\varepsilon_0^2}{4R}$.

(B) The amount of heat released in this circuit by the time the maximum rate of change of the capacitor's energy is reached is $\frac{5}{72}C\varepsilon_0^2$.

(C) The total amount of heat released in the resistor over a very long period of time after the switch is closed is $\frac{2}{9}C\varepsilon_0^2$.

(D) The total amount of heat released in the resistor over a very long period of time after the switch is closed is $\frac{1}{9}C\varepsilon_0^2$.

6) Becquerel performed an experiment in which the particles emitted after radioactive decay were passed through a magnetic field. He measured magnetic rigidity which is a product of magnetic field and the radius of curvature ρ of the particle in the magnetic field. The particles were emitted perpendicular to the magnetic field. Consider the reactions mentioned in options below. In each of the radioactive decay, a statement about the magnetic rigidity is made. Select the option(s) in which the statement correctly describes the outcome of the experiment.

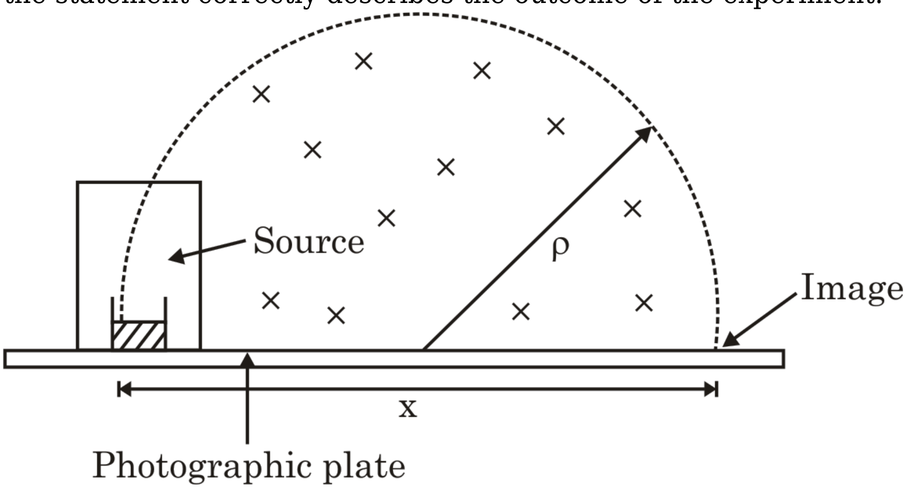


Figure : In Becquerel's arrangement β rays from a radioactive source were deflected in a semicircular arc by a magnetic field into the plane of the paper

(A) When ${}^{37}_{18}Ar$ decays to ${}^{37}_{17}C\ell$, the radiation has magnetic rigidity which is found to vary from a certain non-zero minimum value to a certain maximum value

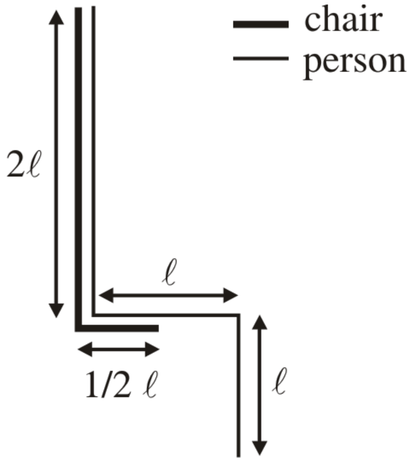
(B) When ${}^{238}_{92}U$ decays to ${}^{234}_{90}Th$, the radiation has magnetic rigidity which is the same for all the radiation

(C) When ${}^{19}_{8}O$ decays to ${}^{19}_{9}F$, the radiation has magnetic rigidity which is found to vary from zero to a certain maximum value

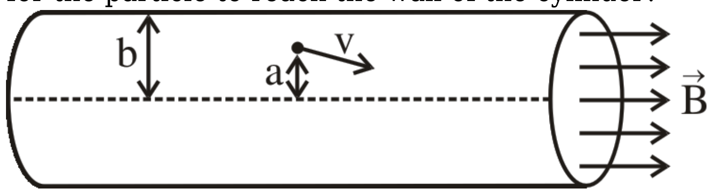
(D) When ${}^{25}_{13}A\ell$ decays to ${}^{25}_{12}Mg$, the radiation has magnetic rigidity which is found to vary from zero to a certain maximum value

SECTION-II

1) Shyam is sitting on a fixed bus seat with his feet off the floor. Here, he is modelled as 3 thin, rigid rods of uniform mass density joined to each other, with dimensions as shown in the figure. His seat consists of a base and a backrest. The coefficient of static friction between Shyam and the seat is μ = 0.5. The bus driver suddenly brakes at a constant deceleration and Shyam finds himself crashing into the seat in front of him. Fuming, Shyam decides to calculate the maximum bus deceleration a at which he would have remained stationary. What is the value of a (in m/s^2)?

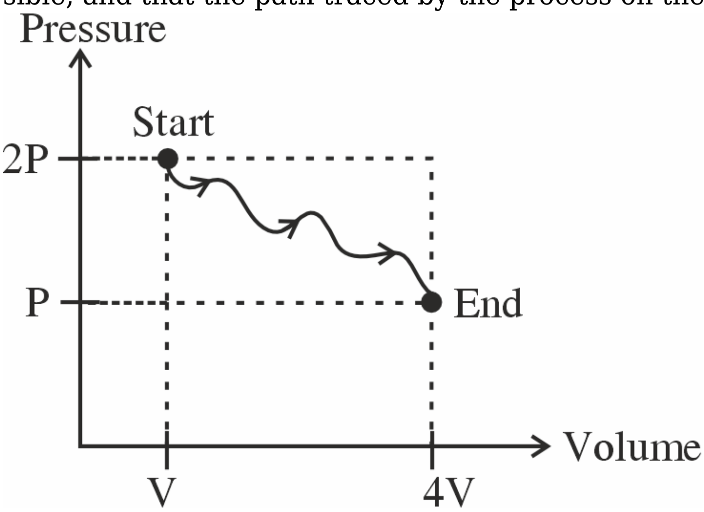


2) A uniform magnetic field of field strength B = 0.5 T runs parallel to the axis of a long insulating cylindrical shell of radius b = 35.0 m. A charged particle with mass m = 0.05 kg and charge q = 0.1 C is initially positioned at a distance a = 10.0 m away from the axis of the cylinder. The particle is launched with speed v = 25 m/s in an arbitrary direction. What is the minimum time taken t (in sec) for the particle to reach the wall of the cylinder?



3) Ideal monatomic gas is contained within a sealed syringe. The piston in the syringe can be moved such that the volume can be varied between V and 4V. Simultaneously, valves in the syringe allow the pressure to be controlled between P and 2P. The gas starts out at pressure 2P and volume V. It is to be brought to final pressure P and volume 4V. Depending on how the piston and the valves are controlled over time, the heat absorbed by the gas in the process can range between Q_{min} and Q_{max}.

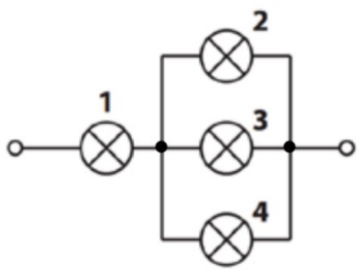
Find the ratio $\frac{Q_{max}}{Q_{min}}$. Assume that the piston and the valves are controlled such that the process undergone by the gas is reversible, and that the path traced by the process on the pressure-volume graph does not self-intersect.



4) There is 5 litres water at a temperature of 20 °C in a pot of base area 10 cm^2. The density of water is 998 kg/m^3. The water is heated to 80 °C. The coefficient of volume expansion of water can be considered constant between the temperature values of 20 °C and 80 °C, and it is $\beta_{water} = 4 \times 10^{-4}$ K^{-1}. The pot is made of stainless steel whose coefficient of volume expansion is $\beta_{steel} = 5 \times 10^{-5}$ K^{-1}. Neglect the vaporization of water. How much does the level of water change (in mm) due to the heating?

5) The lengths of the semi-major axis of two satellites orbiting around the Earth are the same. The ratio of the speeds of the two satellites when they are at perigee is 3/2, and the eccentricity of the orbit of that satellite which is faster at this point is 0.5. The eccentricity of the path of the other satellite is :-

6) Lamps with two different resistance values are connected as shown in the diagram. The circuit is connected to a voltage source, and the power dissipated in each of the lamps was found to be the same. The dependence of the lamp resistances on the current flowing through them can be neglected. Then, the lamp numbered 4 was swapped with the lamp numbered 1. The ratio $\frac{P_1}{P_2}$, where P_1 is the total power dissipated in the circuit initially, and P_2 is the total power dissipated in the circuit after swapping the lamps is

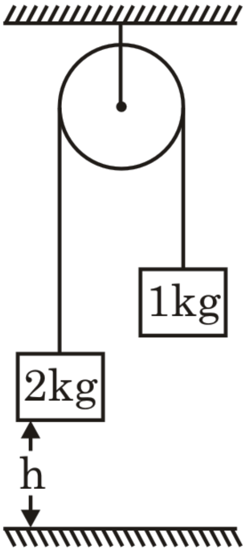


SECTION-III

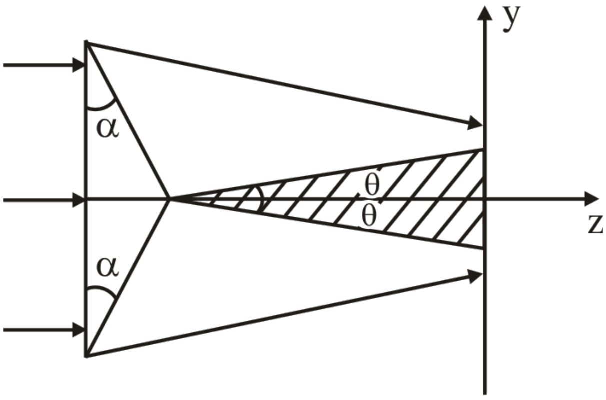
1) A uniform solid spherical ball of radius r floats in water. The ball is displaced vertically

downwards by a distance a << r and released from rest, after which it oscillates with period T. After a long time t_1 >> T, its amplitude of oscillations drops below a fixed value δ << a. If a ball of the same material and radius 2r is displaced by the same distance a, the time taken for its amplitude to drop below δ becomes t_2. Determine the ratio $\frac{t_2}{t_1}$. Assume that the damping force on the ball is given by Stokes' Law as F = –6πrηv; where η is the viscosity of the water and v is the velocity of the ball.

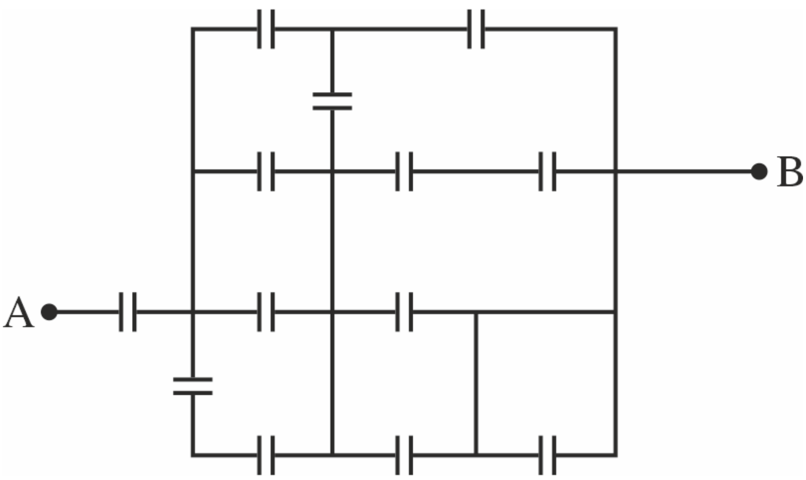
2) Two masses 2 kg and 1 kg are attached to the ends of an inextensible long light string passing through a fixed smooth massless pulley as shown in the Figure. The masses are released when 2 kg is at a height h above the ground. The 2kg block comes to rest instantly after colliding with ground. It is found that during the time interval $\Delta t = \sqrt{\frac{nh}{3g}}$, the string remained tensionless. Find the value of n. [g -acceleration due to gravity].



3) Monochromatic plane light with a wavelength of λ = 6000 Å is incident normally on the base of a thin prism with a small angle α and a refractive index n = 1.5. If the fringe spacing on the screen is 0.1 mm, find the prism base angle α (in radian). Fill the value of 1000α.



4) The following circuit consists of identical capacitors, each of capacitance C_0 = 22 μF. It's extremely wasteful, and Ram wishes to be prudent. Hence, he replaces this with a single capacitor of capacitance C_{eq} between A and B. The value of $\left(\frac{C_{eq}}{2}\right)$ (in μF) is



5) In a long cylindrical metal wire of radius r and of resistivity ρ, a current of strength I flows in a uniform distribution. The wire has a constant surface temperature T_0. It is known that the metal has thermal conductivity k. The temperature difference (in K) between the axis of the wire and surface of the wire is :- (Take I = 4A, ρ = 10 Ωm, r = 10 cm, k = 100 W/m-k, $\pi^2 = 10$)

6) A thick spherical shell having density 2σ and outer radius R and inner radius $r = \frac{R}{(3)^{1/3}}$ is thrown downward inside a tank, containing a liquid having density σ. Value of rate of change of speed of the shell when the speed of the shell becomes $v_0 = \frac{R^2 \sigma g}{9\eta}$ (where η is coefficient of viscosity and g is acceleration due to gravity) is g/k (m/s^2). Find the value of k.

PART-2 : CHEMISTRY

SECTION-I

1) The correct statement(s) for orthoboric acid is/are:

(A) when orthoboric acid is strongly heated, the residue left is boric anhydride
(B) orthoboric acid does not act as a proton donor but acts as a Lewis acid by accepting hydroxyl ion.
(C) orthoboric acid is obtained along with H_2 gas when diborane is hydrolysed
(D) orthoboric acid is obtained by acidifying an aqueous solution of borax

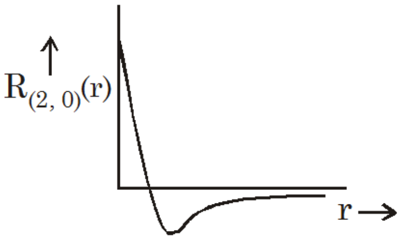
2) If pH of an aqueous solution of 0.01M monoprotic acid HA is found to be 3 at T kelvin, then the correct information(s) is/are

(A) Degree of dissociation of the acid would be 10%
(B) Osmotic pressure of the solution would be 0.011RT
(C) Theoretical molar mass of HA is 1.1 times of its observed molar mass
(D) K_b of A^-(aq.) is 10^{-4} (approx.) at 298K

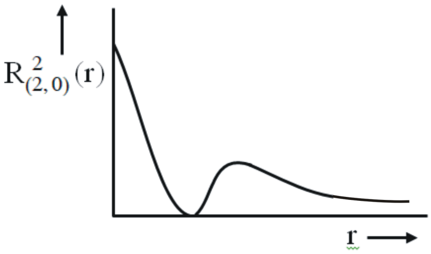
3) Choose the correct statement(s) among the following regarding hydrogen atom?
(All graphs given below are indicative in nature, not to scale)

(A) For an atomic orbital, radial wave function R(r) depends on principal, azimuthal and magnetic quantum number.

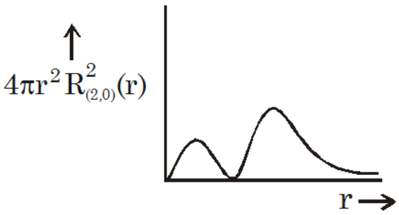
Plot of $R_{(2,0)}$ (r) versus r (distance from nucleus), for 2s electron is

(B) 

Plot of $R^2_{(2.0)}$ versus r (distance from nucleus), for 2s electron is

(C) 

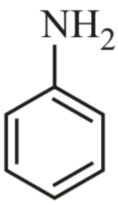
Plot of $4\pi r^2 R^2_{(2.0)}$ against r (distance from nucleus) is

(D) 

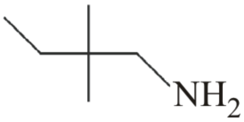
4) Compound $[Co(en)_2(NO_2)Cl]Br$ has :

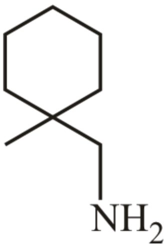
(A) three ionisation isomeric form
(B) two linkage isomeric form
(C) two geometrical isomeric form
(D) three stereoisomers

5) Which of the following amine(s) can not be prepared by Gabriel phthalimide reaction?

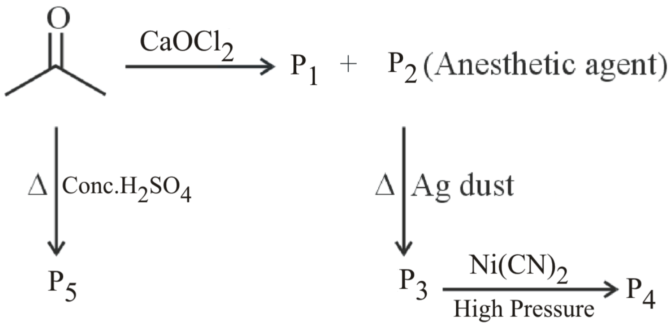
(A) 

(B) 

(C) 

(D) 

6) Choose the correct option(s) for the following set of reactions.



(A) Conjugated π electrons in product P_4 are 8

(B) Product P_4 can undergo electrophilic addition reaction

(C) Product P_4 is antiaromatic

(D) Degree of unsaturation in product P_5 is 4

SECTION-II

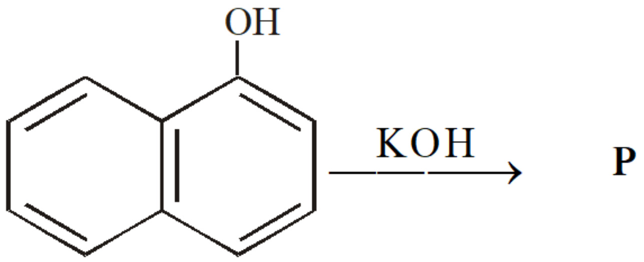
1) Total number of complex which diamagnetic and low spin in following complex compound.
$[Cr(H_2O)_6]^{3+}$, $[Co(H_2O)_6]^{3+}$, $[Ni(en)_3]^{2+}$, $[HgI_4]^{2-}$, $[Cu(CN)_4]^{3-}$, $[Fe(CN)_6]^{3-}$, $[Co(ox)_3]^{3-}$, $[NiF_6]^{2-}$

2) 0.1 mole of white phosphorus reacts with 0.6 mole $SOCl_2$ to form a pungent smelling gas(A) alongwith other products. Gas(A) is reacted with 0.2 mole $KMnO_4$ in acidic medium and the product mixture is treated with excess $BaCl_2$ solution. A white precipitate insoluble in dilute acids is formed. Let the weight of precipitate formed be x gram. Find the value of $\frac{x}{3}$. (Molecular weight of Ba = 137)

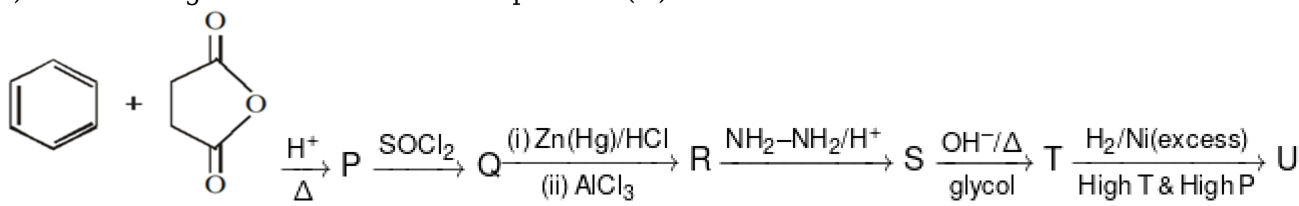
3) Solid $AgNO_3$ is added to an aqueous solution containing 0.2 M Br^- and 0.1 M I^- . Assuming that $AgNO_3$ addition does not change volume of this solution, calculate the percentage of $[I^-]$ precipitated, when $[Br^-]$ just starts precipitating : (Given : $k_{sp}(AgBr) = 10^{-13}\ M^2$; $k_{sp}(AgI) = 10^{-17}\ M^2$)

4) When an ideal gas having initial pressure P, volume V and temperature T, is allowed to expand adiabatically and reversibly until its volume becomes 5.66V, while its temperature falls to half. The workdone by the gas is represented by x times PV. Find the value of x.
($10^{0.3} = 2$, $10^{0.48} = 3$, $10^{0.75} = 5.66$)

5) The number of resonating structures for organic compound P is



6) Number of geometrical isomers for product (U) is



SECTION-III

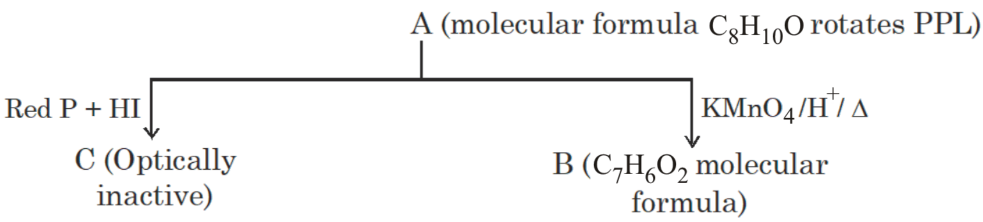
1) A solution of weak acid HA was titrated with base NaOH. The equivalence point was reached when 50 mL of 0.1 M NaOH has been added. 25 mL of 0.1 M HCl was added to titrated solution and the pH was found to be 5.0. The pk_a value of weak acid HA is ______ .

2) For the complex compound $[Pt(NH_3)(NO_2)py(ONO)]$, total number of geometrical isomers will be :

3) A list of species having the formula XZ_4 is given below :
XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$.
Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

4) Out of $N_2O, SO_2, I_3^+, I_3^-, CN_2^{2-}, NO_2^-, N_3^-, C_3^{4-}$, number of linear species is x, then find x :

5) Number of correct statement(s) is :
(i) Both D-Glucose & D-Fructose give positive tollen's and fehling's test
(ii) D-Glucose & D-Galactose form same osazone
(iii) Amylopectin is water insoluble while amylose is water soluble
(iv) Reaction of D-Glucose with HNO_3 yields Glucaric acid
(v) One mole of $PhNHNH_2$ reacts with 3 mol glucose to form osazone.
(vi) Reaction of D-Glucose with HI/Red P yields n-hexane
(vii) Vitamin B12 is a water soluble vitamin
(viii) Number of nitrogen atoms in one histidine molecule is 4
(ix) DNA is a polynucleoside



6)
A reacts with I_2 + NaOH to produce Yellow ppt.

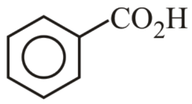
$$(A) \xrightarrow{TsCl} (D) \xrightarrow[KOH/\Delta]{alcoholic} (E)$$

How many statement(s) are correct regarding the above set of reactions ?

(a) (A) decolourises Br_2 water

(b) (A) reacts with Na metal to give colourless odourless gas

(c) (A) reacts with Lucas reagent to give turbidity after 5-10 minutes

(d) (B) is 

(e) (C) does not react with NBS/hν

(f) (D) is optically active and can undergo S_N2 reaction

(g) (E) decolourises Br_2/H_2O solution

(h) (E) can undergo electrophilic addition reaction as well as electrophilic aromatic substitution reaction

(i) (E) can undergo addition polymerisation reaction

PART-3 : MATHEMATICS

SECTION-I

1) Let n be a positive integer with f(n) = 1! + 2! + 3 ! + ...n! and g(x) and h(x) are polynomials such that f(n + 2) = g(n) f(n + 1) + h(n) f(n) ∀ n ≥ 1, then which of the following is/are correct

(A) g(100) = 103

(B) h(48) = -50

(C) g(50) = 52

(D) h(42) = -43

2) Let $\left(1+\frac{1}{n}\right)^{n+x_n} = e$ and $\lim\limits_{n\to\infty} x_n = k$

$$f(x) = \sum_{r=1}^{m} \left(x^r + x^{-r}\right)^{1/k},\ x \neq \pm 1$$

$$g(x) = \begin{cases} \lim\limits_{m\to\infty} (f(x) - 2m)\, x^{-2m-2}.\left(1 - x^2\right), & x \neq 1, -1, 0 \\ -1 \text{ for } x = \pm 1 \end{cases}$$

then g(x) is :

(A) discontinuous at x = –1

(B) continuous at x = 2

(C) limit exist at $x = 1$

(D) limit does not exist at $x = 1$

3) $\int \cos^{2n+1}x\,dx = \frac{\sin x}{a_1} - {}^nC_1\frac{\sin^3 x}{a_2} + {}^nC_2\frac{\sin^5 x}{a_3} - \ldots\ldots\ldots + (-1)^n\frac{\sin^{2n+1}x}{a_n} + c$ (where C is the constant of integration)

If $a_1 + a_2 + a_3 + \ldots.. + a_n = \lambda(n+1)^2$ and $\int \frac{dx}{(x^3+\lambda)^2} = A\int \frac{dx}{(x^3+\lambda)} + \frac{Bx}{x^3+\lambda}$

(A) $A + B = \lambda$

(B) $A - B = \frac{1}{3}$

(C) $A + 2B = \frac{4}{3}$

(D) $A = 2B = \frac{\lambda}{3}$

4) Let $f(k) = \sum_{r=1}^{k}\frac{1}{r}$ and $\sum_{r=1}^{2023} f(r) = af(b) + c$ and $b = 2023$, $a,c \in I$ then which of the following is/are true

(A) $b = c$

(B) $a - b = 1$

(C) $b + c = 0$

(D) $a + c = 0$

5) Given that number of solution of the equation $\frac{1}{5}\log_2 x = \sin 5\pi x$ is N then N is

(A) divisible by 3

(B) perfect square

(C) greater than 121

(D) less than 155

6) Let a_i, $i = 1,2,3,\ldots,n$ denote integers in the domain of $f(x) = \sqrt{\log_{1/2}\left(\frac{4x-25}{x-21}\right)}$ where $a_i < a_{i+1}$ $\forall i \in N$. If line $L : \frac{2x-a_1}{4} = \frac{y+a_1}{a_2} = \frac{z-z_3}{a_5}$ meets xy, yz, zx planes at A, B, C respectively and D is image of C in x axis then volume of tetrahedron OABD is V, where O is origin and Δ_1, Δ_2, Δ_3, Δ_4 are areas of faces and h_1, h_2, h_3, h_4 corresponding altitudes of any tetrahedron PQRS. (In case PQRS is regular tetrahedron, volume of PQRS is equal to volume of OABD). Assuming minimum value of $\sum_{1\le i\ne j\le 4}\sum \Delta_i h_j$ is λ, then which of the following is/are true

(A) $\lambda + 9V = 140$

(B) $\lambda - 40\,[V] = 8$
([.] is greatest integer function)

(C) sum of digit of 9V is 10

(D) sum of digits of λ is 4

SECTION-II

1) If number of positive terms in the sequence $a_n = \dfrac{195}{4\,{}^{n}P_n} - \dfrac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$ is a and $b_1 + b_3 + b_5 + b_7 + b_9 + b_{11} + b_{13} = b$ where b_k is digit in the units place of number $(1!+2!+....+k!)$, $k \in N$, then $\dfrac{b}{a}$ equals

2) If the locus of middle point of contact of tangent drawn to the parabola $y^2 = 8x$ and the foot of perpendicular drawn from its focus to the tangent is a conic then find latus rectum of the conic.

3) Three circles with radius 3 have centres at P (14, 92), Q (17, 76) and R (19, 84). A line passes through Q such that total area of the parts of the three circles to one side of the line is equal to the total area of the part of three circle to the other side of it. If slope of line is m then $\dfrac{|m|}{5}$ is equal to

4) Let real number x, y, z, w satisfy $\dfrac{x^2}{n^2-1} + \dfrac{y^2}{n^2-3^2} + \dfrac{z^2}{n^2-5^2} + \dfrac{w^2}{n^2-7^2} = 1$ for n = 2, 4, 6, 8.
Value of $\dfrac{x^2 + y^2 + z^2 + w^2}{10}$ equals

5) $N = \left[\sum \log_{10} K\right]$ where [.] greatest integer function and $\sum$ denotes summation of all values of $\log_{10}k$ where k is positive divisor of 1000,000 excluding 10^6 itself. Then $\dfrac{N}{100}$ is

6) There are 'm' red socks and 'n' blue socks in a bag. Where $m + n \leqslant 1991$. Let two socks taken out at random, probability that they have the same colour is $\dfrac{1}{2}$. If largest possible value of m is P then $\left(\dfrac{1000-P}{8}\right)$ equals

SECTION-III

1) The equation $\sec\theta + \operatorname{cosec}\theta = c$ has two solutions between 0 and 2π if $c^2 < k$ and four solutions if $c^2 > k$. Find number of integral solution of the equation $x^2 + k + 6x = 0$

2) Let matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ satisfies the equation $A^2 + aA + bI_2 = 0$ and value $\int_a^{4b} x^3 \cos x \, dx = \frac{ma + nb}{na - mb}$. (where m, n are least positive integers). Evaluate the sum of diagonal elements of the matrix $\begin{bmatrix} m+n-1 & m^2n^2 \\ -m & -n \end{bmatrix}^{2025}$.
(where I_2 is an identity matrix of order = 2)

3) E_1 and E_2 are two ellipses such that area of ellipse E_2 is one third of the area of quadrilateral formed by tangents at the ends of latus rectum of the ellipse $E_3 \equiv 5x^2 + 9y^2 = 45$. E_1 is inscribed in E_2 in such a way that both E_1 and E_2 touch each other at one end of their common major axis. Given length of major axis of E_1 equals length of minor axis of E_2 and eccentities of E_1, E_2, E_3 are equal then the area of E_2 lying outside E_1 equals

4) If $f : R - \{-1\} \rightarrow R$, f(x) is a differentiable non-constant, non-identity function satisfies, $f(x + f(y) + x f(y)) = y + f(x) + y f(x) \; \forall \; x, y \in R - \{-1\}$, then $2025 (1 + f(2024))$ equals

5) Let x, y, z, w are non-zero complex numbers such that $w = x^{24} = y^{40} = x^{12}y^{12}z^{12}$, if $z = 1 + i$ find the remainder when |w| divided by 7 is

6) Let $x^2 + bx + 6b = 0$ has only integer roots. If number of real values of 'b', is n then value of $\frac{n}{2}$ equals

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6
A.	D	A,B	B,C	B,D	A,C	A,C

SECTION-II

Q.	7	8	9	10	11	12
A.	3.33	1.04 to 1.06	1.50	88.00 to 91.00	0.14 to 0.15	2.45 to 2.47

SECTION-III

Q.	13	14	15	16	17	18
A.	4	8	6	7	4	8

PART-2 : CHEMISTRY

SECTION-I

Q.	19	20	21	22	23	24
A.	A,B,C,D	A,B,C	B,C,D	A,B,C,D	A,B,C,D	B,D

SECTION-II

Q.	25	26	27	28	29	30
A.	3.00	23.30	99.98	1.25	10.00	2.00

SECTION-III

Q.	31	32	33	34	35	36
A.	5	3	4	5	5	6

PART-3 : MATHEMATICS

SECTION-I

Q.	37	38	39	40	41	42
A.	A,B	A,B,D	A,B,C	B,C	A,C	A,C,D

SECTION-II

Q.	43	44	45	46	47	48
A.	6.25	9.00	4.80	3.60	1.41	1.25

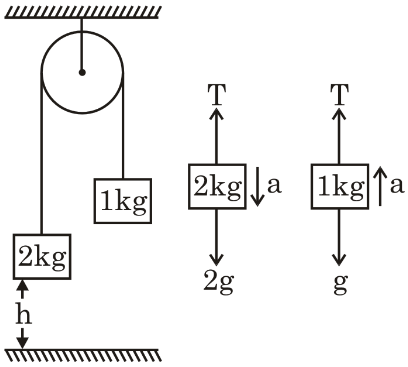
SECTION-III

Q.	49	50	51	52	53	54
A.	2	0	4	1	1	5

SOLUTIONS

PART-1 : PHYSICS

4) Spectrum peaks are due to characterstic X-rays, which depend on target, where as lmin is a property of accelerating voltage.



14)

$2g - T = 2a$; $T - g = a$ $\Rightarrow$ $a = \dfrac{g}{3}$

The velocity of 1 kg at the instant of 2 kg falling down a height h is

$v_0^2 = 0 + 2ah$ or $v_0 = \sqrt{\dfrac{2gh}{3}} \uparrow$

From this moment, 1 kg moves upward as a free body till its speed becomes zero and then comes down. The string is not in taut condition during the speed changes from v_0 to zero and from zero to v_0.

Hence $0 = v^0 - gt$ or $t = \dfrac{v_0}{g}$

String is not taut during $\Delta t = 2t$

$= \dfrac{2v_0}{g} = 2\sqrt{\dfrac{2h}{3g}} = \sqrt{\dfrac{8h}{3g}} = \sqrt{\dfrac{nh}{3g}}$

∴ n = 8

PART-2 : CHEMISTRY

20) pH = 3 ⇒ $[H^+] = 10^{-3}$ M

let the degree of dissociation be α

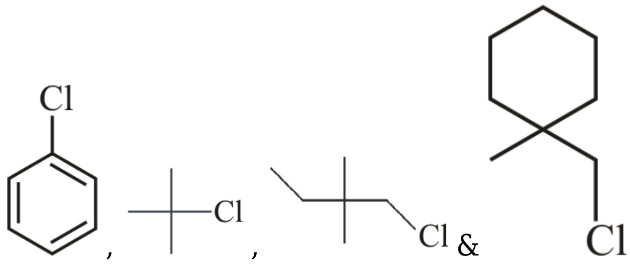
$\Rightarrow \alpha = \dfrac{10^{-3}}{0.01} = 0.1 \Rightarrow \%\ \alpha = 10$

⇒ π(osmotic pressure) = (1.1) (0.01)RT = 0.011 RT

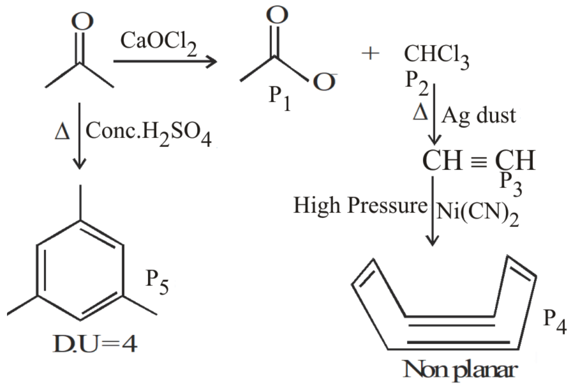
$\Rightarrow i = 1 + (h - 1)\alpha = 1.1$

HA = H+ + A–

Kb of at 25°C

23) 

Will not give S_N2 reaction with phthalimidate ion so (A), (B), (C), (D) can not be prepared by gabriel phthalimide reaction.

24) 

26) $\underset{0.1}{P_4} + \underset{0.6}{8SOCl_2} \rightarrow 4PCl_3 + \underset{-}{2S_2Cl_2} + \underset{0.3}{4SO_2(A)}$

$\underset{0.2}{2MnO_4^{-1}} + \underset{0.3}{5SO_2} + \underset{-}{2H_2O} \rightarrow 2Mn^{2+} + \underset{0.3}{5SO_4^{2-}} + 4H^+$

$\underset{0.3}{SO_4^{2-}} + Ba^{2+} \rightarrow \underset{0.3}{BaSO_4}$

$\Rightarrow x = 0.3 \times [137 + 32 + 64] = 69.9g$

27)

$[Ag+]$ for pptⁿ of I^- = $\dfrac{k_{sp}(AgI)}{[I^-]} = \dfrac{10^{-17}}{0.1} = 10^{-16}\,M$

$[Ag^+]$ for pptⁿ of Br^- = $\dfrac{k_{sp}(AgBr)}{[Br^-]} = \dfrac{10^{-13}}{0.2} = \dfrac{1}{2} \times 10^{-12} M$

$\Rightarrow [I^-]$ when AgBr starts precipitating = $\dfrac{k_{sp}(AgI)}{[Ag^+]} = \dfrac{10^{-17}}{10^{-12}} \times 2 = 2 \times 10^{-5}$

$\Rightarrow$ % $[I^-]$ remaining in solution = $2 \times \dfrac{10^{-5}}{0.1} \times 100$ for precipitation of AgBr

$\Rightarrow$ % $[I^-]$ precipitated = 100 – 0.02 = 99.98

28) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = T$, $T_2 = \dfrac{T}{2}$

$V_1 = V$ and $V = 5.66\ V$

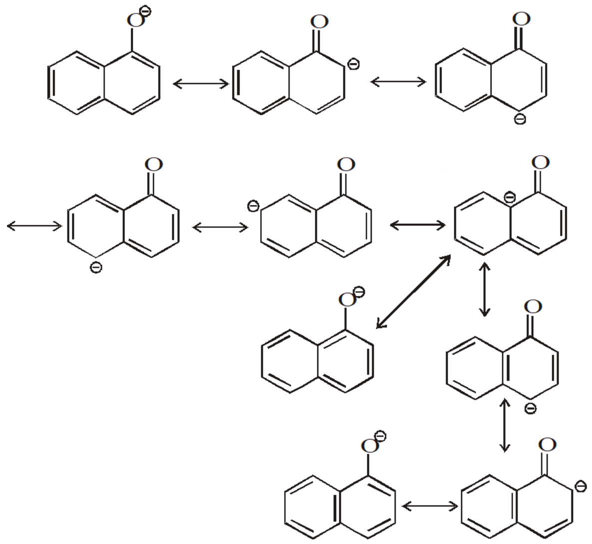
$TV^{\gamma-1} = \dfrac{T}{2}(5.66V)^{\gamma-1}$ Pv = nRT

$2 = (5.66)^{\gamma-1}$

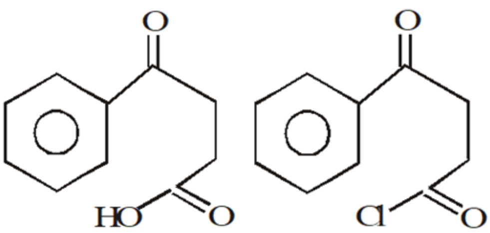
$\gamma = 1.4$

$$|w| = \left|\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right| = \frac{nRT}{2(\gamma - 1)} = \frac{1}{0.8}PV = 1.25PV$$

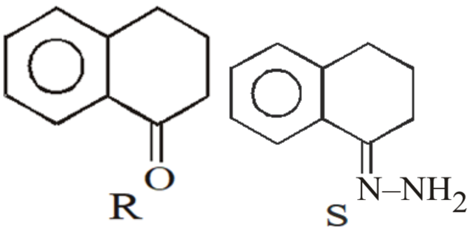
29)



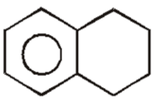
30)



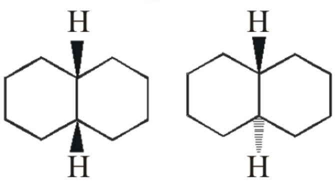
P Q



R S



T



Cis-Decalin Trans-Decalin

U

$\boxed{\text{Total Geometrical isomers} = 2}$

31)

HA + NaOH → NaA

Moles = $50 \times 10^{-1} \times 10^{-3}$

NaA + HCl → HA + NaCl

5 × 10-3 $25 \times 10^{-1} \times 10^{-3}$

moles 2 HA = 2.5×10^{-3}

moles 2 NaA = 2.5×10^{-3}

$$PH = Pka + \log\frac{[salt]}{[Acid]}$$

$\Rightarrow$ PH = pka = 5

35) (i), (iii), (iv), (vi), (vii) statements are correct.

PART-3 : MATHEMATICS

37) f(n + 2) – f(n + 1) = (n + 2)!

= (n + 2) [f(n + 1) – f(x)]

f(n + 2) = (n + 3) f(n +1) – (n + 2) f(n)

f(n) = n + 3, h(n) = – (n + 2)

38) $\left(1+\frac{1}{n}\right)^{n+x_n} = e \Rightarrow x + x_n = \frac{1}{\ln\left(1+\frac{1}{n}\right)}$

$x_n = \frac{1}{\ln\left(1+\frac{1}{n}\right)} - x,\ \text{Let}\frac{n+1}{n} = u$

$x_n = \lim\limits_{u\to 1}\left(\frac{1}{\ell nu} - \frac{1}{u-1}\right) = \frac{1}{2}$

$f(x) = \sum x^{2r} + \sum \frac{1}{x^{2r}} + 2n$

$= \frac{x^2(1-x^{2n})}{1-x^2} + \frac{1}{x^2}\frac{1-\frac{1}{x^{2n}}}{1-\frac{1}{x^2}} + 2n$

$(f(x)-2n)\left(1-x^2\right) = \left(1-x^{2n}\right)\left(x^2+\frac{1}{x^{2n}}\right)$

g(n) = $\lim\limits_{n\to\infty}(f(n)-2n)x^{-2n-2}(1-x^2)$ for $x \neq \pm 1$

$$= \begin{cases} -1 & \text{IF} \quad |x| > 1 \\ -\infty & \text{IF} \quad |x| < 1 \\ 0 & \text{If} \quad x = \pm 1 \end{cases}$$

has non removable infintie type discontinuity at $x = \pm 1$

39) $\int \cos^{2n+1} x dx = \int \left(1-\sin^2 x\right)^n . \cos x dx$

$= \int \left({}^nC_0 \cos x - {}^nC_1 \sin^2 x \cos x + ...\right) dx$

$= \frac{\sin x}{1} - {}^nC_1.\frac{\sin^3 x}{3} + {}^nC_2.\frac{\sin^5 x}{5}......$

$a_1 + a_2 + a_3 + ... = 1 + 3 + 5 + ... + 2n + 1$

$= (n+1)^2 \Rightarrow \lambda = 1$

$\int \frac{dx}{(x^3+1)^2} = \int \frac{x^3+1-x^3}{(x^3+1)}dx = \int \frac{dx}{x^3+1} + \int \frac{-x.x^2}{(x^3+1)}dx$

$= \int \frac{dx}{x^3+1} - \frac{1}{3}\int x.\frac{3x^2}{(x^3+1)}dx$

$= \int \frac{dx}{x^3+1} - \frac{1}{3}\left[x.\frac{-1}{x^3+1} - \int \frac{-1}{x^3+1}dx\right]$

$= \frac{2}{3}\int \frac{dx}{x^3+1} + \frac{1}{3}\frac{x}{x^3+1}$ $\quad A = \frac{2}{3}, B = \frac{1}{3}$

40) $\sum_{r=1}^{2023} f(r) = 1 + \left(1+\frac{1}{2}\right) + \left(1+\frac{1}{2}+\frac{1}{3}\right) + \ldots \left(1+\frac{1}{2}+\ldots\frac{1}{2023}\right)$ $= 2023 \times 1 + 2022 \times \frac{1}{2} + 2021 \times \frac{1}{3} + \ldots.. + \frac{1}{2023}$

$= 2023 \times 1 + (2023-1)\frac{1}{2} + (2023-2)\frac{1}{3} + \ldots (2023-2022)\frac{1}{2023}$

$= 2023\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{2023}\right) - \left(\frac{1}{2}+\frac{2}{3}+\ldots\frac{2022}{2023}\right)$

$= 2023f(2023) - \left[1-\frac{1}{2}+1-\frac{1}{3}+\ldots 1-\frac{1}{2023}\right]$

$= 2023f(2023) - [2022] + \left(\frac{1}{2}+\frac{1}{3}+\ldots.\frac{1}{2023}\right)$

$= 2023\ f(2023) - 2022 + f(2023) - 1$

$= 2024\ f(2023) - 2023$

41)

$\log x < 0$ on (0, 1) and > 0 for $x > 1$ for $x > 32$, $\frac{1}{5}\log_2 n > 1$ (Rejected)

$\sin 5\pi x = 0$ at $x = 0, \frac{1}{5}, \frac{2}{5}, \ldots$

for $n < 1$, $\log x$ lies in $(-1, 0)$ in the interval $\left(\frac{1}{5},\frac{2}{5}\right), \left(\frac{3}{5},\frac{4}{5}\right)$

$1 < x < 32$, LHS lies in (0, 1) in interval

$x \in \left(\frac{6}{5},\frac{7}{5}\right), \left(\frac{8}{5},\frac{9}{5}\right) \ldots \left(\frac{158}{5},\frac{159}{5}\right)$

Hence equation has roots

$x = 1$, two roots each in $\left(\frac{1}{5},\frac{2}{5}\right), \left(\frac{3}{4},\frac{4}{5}\right)$ two each in $\left(\frac{6}{5},\frac{7}{5}\right), \left(\frac{8}{5},\frac{9}{5}\right) \ldots. \left(\frac{158}{5},\frac{159}{5}\right)$

total = 159

42)

Domain of f(x) is $\left[\frac{4}{3},\frac{25}{4}\right)$ integers 2, 3, 4, 5, 6

Line L = $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{6} = r$ (let) at xy plane, $z = 0$ $\quad r = \frac{-2}{3}$

$A\left(-\frac{1}{3},-4,0\right), B\left(0,-\frac{7}{2},1\right), C\left(\frac{7}{3},0,-8\right)$

$V = \frac{1}{6}[\bar{a}\bar{b}\bar{c}] = \frac{28}{9}$

$\sum\sum \Delta_i\Delta_j = \Delta_1(h_2 + h_3 + h_4) + \ldots..$ min value = $12.\Delta.h$ (AM$\geqslant$GM)

$= 12 \times 3.\frac{28}{9} = 112 = \lambda$

43) $a_n = \frac{195}{4^n.n!} - \frac{(n+3)!}{n!(n+1)!} = \frac{195}{4n!} - \frac{(n+3)(n+2)}{n!}$

$a_n > 0$ means

$\frac{195}{4} - (n+3)(n+2) > 0 \Rightarrow n = 4$

also $b_1 = 1$, $b_2 = 3$, $b_3 = 9$, $b_4 = 3$ ..... $b_{13} = 3$

b = 25

$\frac{b}{a} = \frac{25}{4} = 6.25$

44) (h, k) is mid point of PT then

$h = \frac{at^2}{2}$; $k = \frac{2at + at}{2}$

$\Rightarrow 2h = a.\frac{4k^2}{9a^2}$

$\Rightarrow 2y^2 = 9ax$

a = 2

$y^2 = 9x$

L.R = 9

45)

line passes (17, 76) then it will pass (17 - x, 92) and $\left(17 - \frac{x}{2}, 84\right)$

$3 - x = 2 + \frac{x}{2} \Rightarrow x = \frac{2}{3}$

slope = $\frac{16}{-\frac{2}{3}} = -24$

46)

Let $n^2 = t$ and make it 4 degree equation in t.

$t^4 - (84 + x^2 + y^2 + z^2 + w^2)t^3 + \ldots\ldots. = 0$ where

t = 4, 16, 36, 64

Hence $84 + x^2 + y^2 + z^2 + w^2 = 120$

47)

$10^6 = 2^6.5^6$

$\sum\limits_{0 \leq m,\ n \leq 6} \log(2^m.5^n)$ excluding m = n = 6

= (7(1 + 2+ ... +6) -6) (log2 + log5)

= 141

48) $\dfrac{{}^mC_2+{}^nC_2}{{}^{m+n}C_2}=\dfrac{1}{2}\Rightarrow$ Solving $(m - n)^2 = m + n$

Put N = m + n, larger of m, n is $\dfrac{N+\sqrt{N}}{2}$

N must be largest square $\leqslant 1991 = 44^2$

possible values of m = 990, 946

49)

$\sin\theta + \cos\theta = c\sin\theta\cos\theta$

$c^2\sin^2\theta\cos^2\theta = 1 + 2\sin\theta\cos\theta$

take $\sin 2\theta = t$, $c^2t^2 - 4t - 4 = 0$, $t \in [-1, 1]$

Hence k = 8

50) $A^2=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$, $A^2-4A+I=0$

a = – 4, b = 1, $\displaystyle\int_{-4}^{4} x^3\cos x\,dx = 0$

$=\dfrac{a+4b}{4a-b}$

$m = 1, n = 4$

$\begin{bmatrix}4 & 16\\ -1 & -4\end{bmatrix}^{2025} =$ null matrix

51)

$5x^2 + 9y^2 = 45$ a = 3, $b=\sqrt{5},\ c=\dfrac{2}{3}$, end of LR is $\left(2,\dfrac{5}{3}\right)$

tangent 2x + 3y = 9, meets at $\left(\dfrac{9}{2},0\right)$ (0,3)

area = $4.\dfrac{1}{2}.\dfrac{9}{2}.3=27$

Let E_2 & E_3 $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1,\ \dfrac{x^2}{b^2}+\dfrac{y^2}{c^2}=1$

Required area = $\pi ab-\pi bc$

$b^2 = a^2(1 - e^2)$, $c^2 = b^2(1 - e^2) \Rightarrow a - c = ae^2$

Area = $\pi b(a-c)=\pi abe^2=9\times\dfrac{4}{9}=4$

52)

Differentiable wrt x, and y constant

$f'(x+f(y)+xf(y))(1+f(y))=f'(x)+yf'(x)$

now Differentiable wrt y

$f'(x + f(y) + xf(y))(1 + x)f'(y) = 1 + f(x)$

$\frac{(1+y)f'(y)}{1+f(y)} = \frac{1+f(x)}{(1+x)f'(x)} = \boldsymbol{\lambda} \Rightarrow \boldsymbol{\lambda} = \pm 1$

$\frac{f'(x)}{1+f(x)} = \pm\frac{1}{1+x}$, Integerate f(x) = $c(1+x)^{\pm+1} - 1$

put x = y = 0, f(f(0)) = f(0)

f(0) = c – 1 hence f(c – 1) = c – 1

c = 0, 1 (taking + sign)

where c = 1 (taking – sign)

wherec = 0, f(x) = – 1, when c = 1, f(x) = 1 + x – 1 = x

and $f(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x}$

$1 + f(x) = \frac{1}{1+x}$

$1 + f(2024) = \frac{1}{2025}$

Ans = 1

53)

$w^{10} = x^{120}\ y^{120}\ z^{120} = w^5 w^3 . z^{120}$

$\Rightarrow w = z^{60}$

$|w| = |z|^{60} = 2^{30} = (7 + 1)^{10}$

54)

b must be integer, $b^2 - 24b \geqslant 0$, b (b – 24) perfect square b = 0, 24 can work.

Now put B = b – 24, b < 24 and put B = – b for b < 0

If d = GCD (B, B + 24), d must be 1, 2, 3, 4, 6, 8, 12, 24 and B/d and $\frac{(B+24)}{d}$ are squares.

$b = 125 \quad x = -25, 30$

$b = -8 \quad x = -4, 12$

$b = -3 \quad x = -3, 6$

$b = -1 \quad x = -2, 3$

$b = 0 \quad x = 0$

$b = 24 \quad x = -12, -12$

$b = 25 \quad x = -15, -10$

$b = 27 \quad x = -18, -9$

$b = 32 \quad x = -24, -8$

$b = 49 \quad x = -42, -7$