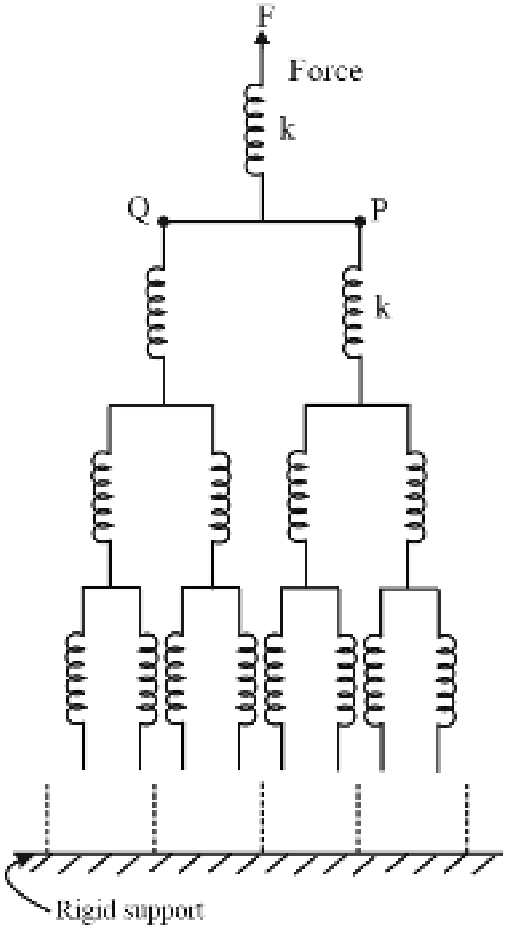
PART-1 : PHYSICS

SECTION-I

1) A geometric progression of very large number of identical springs of spring constant k is pulled with a force F. Assume bars & springs are massless. Last row is connected to rigid support. In



equilibrium choose the correct option(s)

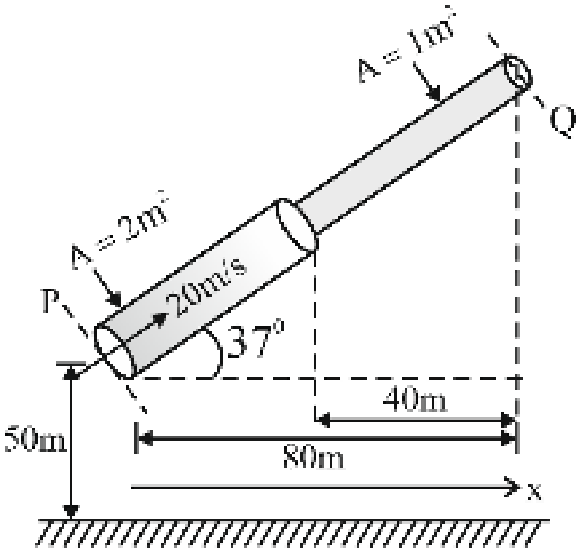
(A) Free end is displaced by amount $\frac{2F}{k}$.

(B) Free end is displaced by amount $\frac{F}{k}$.

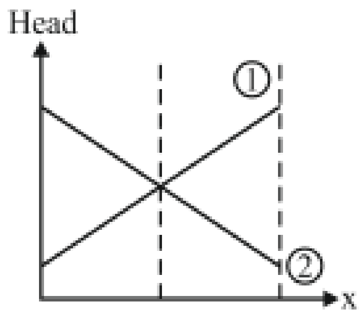
(C) Displacement of bar PQ is $\frac{F}{k}$.

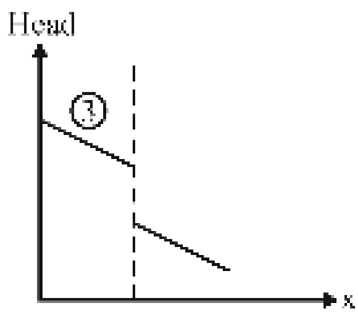
(D) Displacement of bar PQ is $\frac{F}{3k}$.

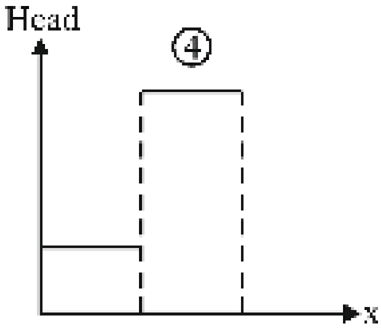
2) The figure below shows a practical situation of fluid flow where Bernoulli's principle holds good.

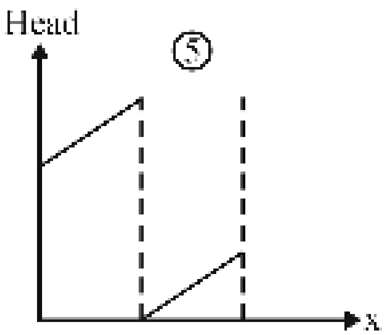


If $\frac{p}{\rho g}$ is pressure head, $\frac{v^2}{2g}$ is kinetic head, h is potential head, then variation of energy heads with respect to 'x' (coordinate) are as shown

(a) 

(b) 

(c) 

(d) 

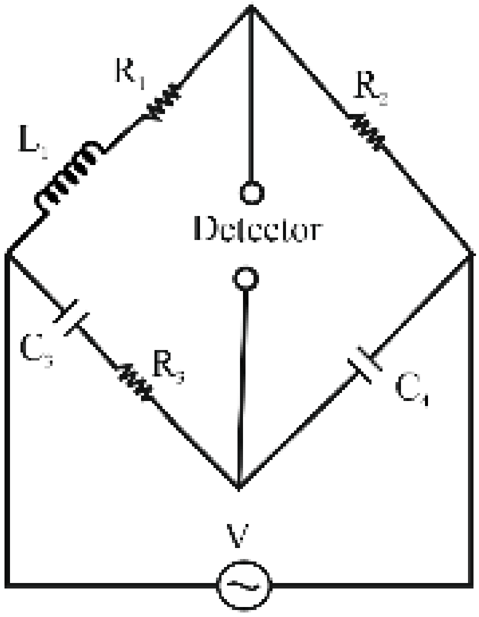
Choose correct option(s)

(A) Kinetic head v/s x will be best represented by curve (4)

(B) Potential head v/s x will be best represented by curve (2)

(C) Pressure head v/s x will be best represented by curve (3)

(D) $\left(\frac{p}{\rho g}\right)_P - \left(\frac{p}{\rho g}\right)_Q = 70$ m

3) 

A bridge for measuring the inductance L_1 & resistance R_1 of a coil is shown in figure. R_2 & R_3 are variable resistors & C_3 & C_4 are fixed capacitor. For bridge to be balanced, choose the correct option(s)

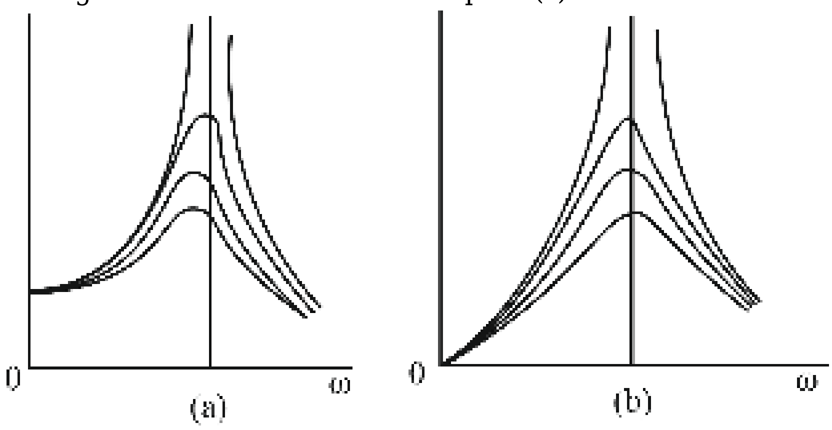
(A) $R_1 = \frac{R_2C_4}{C_3}$

(B) $L_1 = C_4R_3R_2$

(C) Conditions for the bridge to be balanced are independent of the frequency of the alternating voltage source V.

(D) Conditions for the bridge to be balanced are dependent of the frequency of the alternating voltage source V.

4) Following graphs represent the frequency dependence of the amplitude of displacements and frequency dependence of the velocity amplitude in forced oscillation on the frequency of the driving force. Choose the correct option(s)



(A) Figure (a) correspond to frequency dependence of the displacement amplitudes

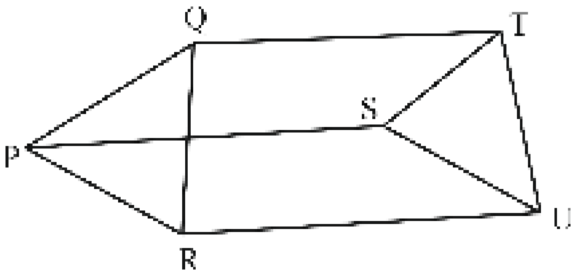
(B) Figure (b) correspond to frequency dependence of the displacement amplitudes

(C) Figure (b) correspond to the frequency dependence of the velocity amplitudes.

(D) Figure (a) correspond to the frequency dependence of the velocity amplitudes.

5)

Each wire has Resistance = R. Choose the correct option(s)



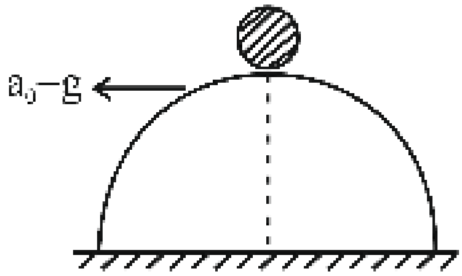
(A) Equivalent resistance between P & R is $\frac{8R}{15}$

(B) Equivalent resistance between Q & T is $\frac{3R}{5}$

(C) Equivalent resistance between P & R is $\frac{15R}{8}$

(D) Equivalent resistance between Q & T is $\frac{4R}{5}$

6) A small body is placed on the top of smooth hemisphere of radius R. The hemisphere is imparted a constant acceleration a_0 in horizontal direction as shown.



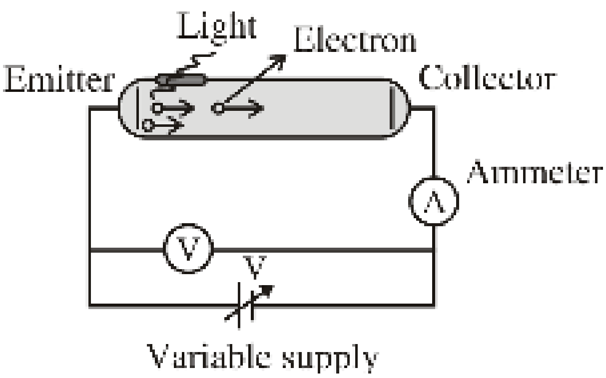
(A) The velocity of the body relative to sphere at the moment of break off is $V_0 = \sqrt{\frac{4gR}{3}}$

(B) The velocity of the body relative to sphere at the moment of break off is $V_0 = \sqrt{\frac{2gR}{3}}$

(C) The angle θ_0 between the vertical & the radius vector drawn from the centre of sphere to break off point is given by $\cos\theta_0 = \frac{2+\sqrt{14}}{6}$.

(D) The angle θ_0 between the vertical & the radius vector drawn from centre of the sphere to the break off point is given by $\cos\theta_0 = \frac{2}{3}$

7)



In a photoelectric effect setup, we have 4 metals

which can be used for emitter or collector. (hc = 1240 eVnm)
(1) Aluminium ϕ = 3.7 eV
(2) Platinum ϕ = 5.3 eV
(3) Silver ϕ = 4.3 eV
(4) Cesium ϕ = 1.9 eV
We use monochromatic light λ = 310 nm. In following options, choose the cases in which no photocurrent is detected. v is the voltage of the battery used with polarity as shown.

(A) Emitter : Al, collector Cs, V = 1V

(B) Emitter : Cs, collector Pt, V = 2V

(C) Emitter : Ag, collector Cs, V = 1V

(D) Emitter : Pt, collector Cs, V = –2V

8) The following equations represent transverse wave
$Z_1 = a\cos(kx - \omega t)$
$Z_2 = a\cos(kx + \omega t)$
$Z_3 = a\cos(ky - \omega t)$
Choose correct option(s)

(A) For the combination of Z_1 & Z_3, the wavefronts are represented by the planes x + y = constant

(B) $Z_1 + Z_3$ represents a wave going along the direction making an angle 45° with the positive x & positive y-direction

(C) $Z_1 + Z_2$ represents a standing wave.

(D) For the combination of Z_2 & Z_3, the wavefronts are represented by the planes y – x = constant

9) A nonconducting solid sphere A and a nonconducting hollow sphere B are placed on rough insulated surface at a very large distance as shown. Spheres start moving under the mutual force of attraction and collide elastically. (Assume no slipping any where)

(A) Velocity of A just before collision is $\sqrt{\frac{126}{335}\frac{KQ^2}{mR}}$

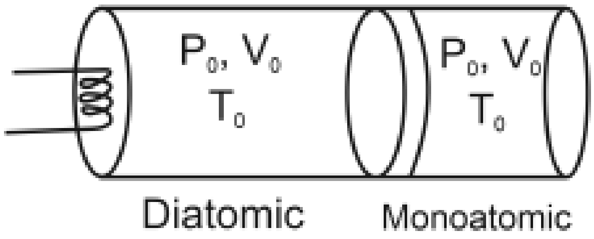
(B) Velocity of A just before collision is $\frac{25}{42}\sqrt{\frac{126}{335}\frac{KQ^2}{mR}}$

(C) Velocity of B just before collision is $\sqrt{\frac{126}{335}\frac{KQ^2}{mR}}$

(D) Velocity of B just before collision is $\frac{25}{42}\sqrt{\frac{126}{335}\frac{KQ^2}{mR}}$

10) A non conducting cylinder is divided in two equal parts by a non conducting movable piston. One part contains monoatomic gas and other part contains diatomic gas, each of 1 mole. Now heat is

supplied to the left part such that pressure of the right part becomes $\frac{243P_0}{32}$. Then : -



(A) Final temperature of gas in right chamber will be $\frac{27}{8}T_0$

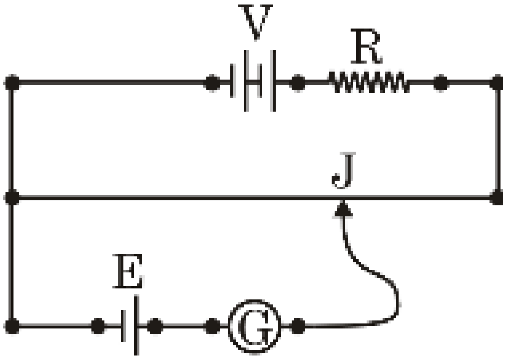
(B) Final temperature of gas in left chamber will be $\frac{207}{16}T_0$

(C) Work done by gas in the right chamber will be $\frac{-15}{8}P_0V_0$

(D) Heat supplied by heater will be $\frac{1015}{32}P_0V_0$

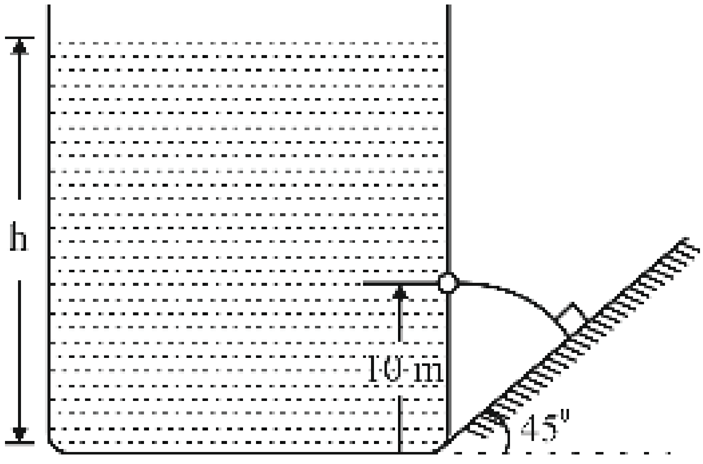
SECTION-II

1) In the potentiometer circuit shown, the battery has emf V = 10 V and internal resistance of 5 Ω. R is a resistance box whose resistance can be any integral value from 1 Ω to 100 Ω. Resistance of potentiometer wire is 50 Ω and length of 5 m. The emf E of unknown battery can be anywhere between 5 V to 7 V. For most accurate measurement of emf E, what should be value of R (in ohm)?

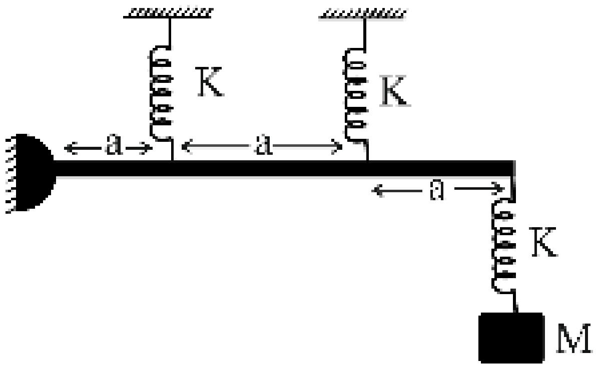


2) A rectangular tank of height h filled with water is placed near the bottom of a plane mirror inclined at an angle 45° with the horizontal. At height 10 m from bottom a hole is made such that stream coming out from hole strikes the plane mirror normally. Then at the time of hitting, speed of image of water droplet hitting the plane mirror, formed in plane mirror (in m/s) is (Take g = 10 m/s^2

) : -



3) Consider a block suspended with the light rod being horizontal in equilibrium as shown. All springs are light. Neglect friction everywhere. The time period for small oscillation of the block is

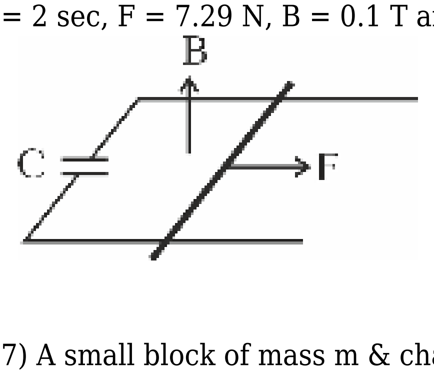


$2\pi\sqrt{\frac{\alpha M}{5K}}$. Find α.

4) Atoms of type A fly along the axis of a cylindrical pipe of radius R and collide with practically motionless atoms of type B. The kinetic energy of atoms A is equal to the threshold energy, so that a central impact forms a molecule AB, which then moves with velocity v. In the case of an off-centre impact, the reaction does not occur, i.e. the atoms collide elastically. In what minimum time (in sec) after the collision will atoms of type B be able to get from the axis of the cylinder to the wall of the cylinder? (R = 10 cm and v = 40 cm/s)

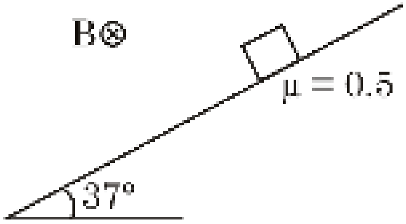
5) The density of sphere is directly proportional to depth below the surface of the sphere. If the maximum value of acceleration due to gravity is N times of the surface value, then the value of N is

6) A rod of resistance R, length ℓ and mass m is placed on two smooth parallel rails which are resistanceless. The rails are connected by capacitor of capacitance C. A uniform magnetic field B is present in vertical direction. The rod is pulled at it's center by a constant force F. If $m = B^2\ell^2C$, RC = 2 sec, F = 7.29 N, B = 0.1 T and ℓ = 2 m, what is current (in A) in rod at t = 2 sec. (e = 2.7)



7) A small block of mass m & charge q is released on a long rough inclined plane as shown. In space,

there is a uniform magnetic field of 0.5 T away from the reader. Find the maximum velocity (in m/s) of the mass during the motion. ($B = 0.5$ T, $q = 4\mu C$, $m = 8$ mg, $g = 10\ m/s^2$)



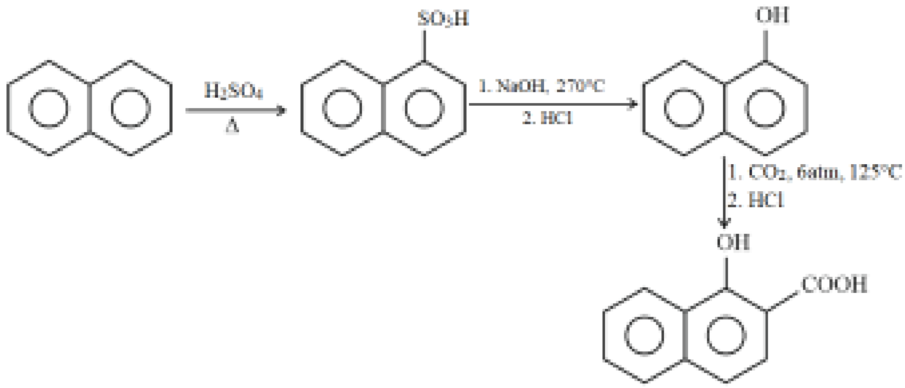
8) A well lagged wire of length $L = 2$ m of cross sectional area $A = 4\ m^2$ has it's ends maintained at temperatures $T_1 = 40^oC$ & $T_2 = 10^oC$. The thermal conductivity of the wire is given by $K = (3 + 6T)$, $\frac{W}{m^oC}$, where T is the temperature (in °C). The rate of flow of heat (in J/s) along the wire is:

PART-2 : CHEMISTRY

SECTION-I

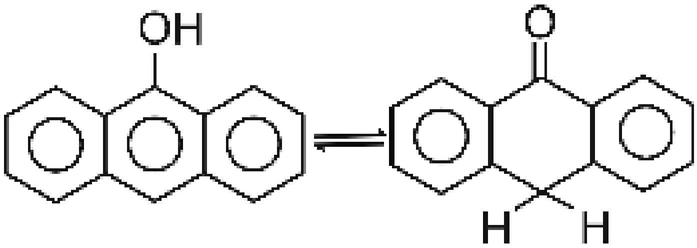
1) Which of the following statement(s) is/are **incorrect** ?

(A)



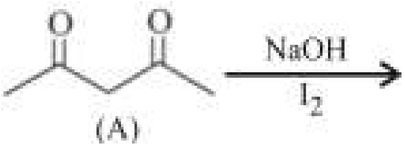
(B) 1,3,5-trihydroxybenzne reacts with NH_2OH but 1,2,4 -trihydroxybenzene and 1,2,3-trihydroxybenzene do not react with NH_2OH.

(C)



equilibrium lies in backward direction.

(D)



number of Iodoform molecules produced per molecule of A is 3.

2) Which epimer or epimers of D-glucose gives meso aldaric acid as product with conc. HNO_3.

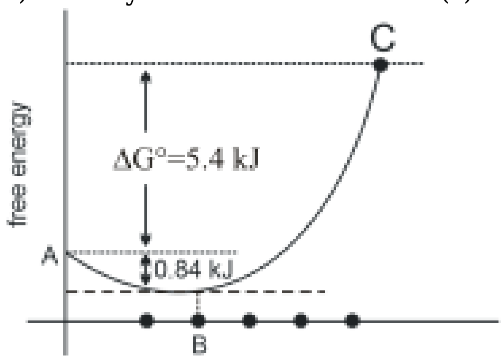
(A) C-2 Epimer

(B) C-3 Epimer

(C) C-4 Epimer

(D) C-5 Epimer

3) Identify the correct statement(s) from the following, at constant temperature :



A = Standard free energy of 1 mole of N_2O_4

C = standard free energy of 2 mole of NO_2

B = Equilibrium point

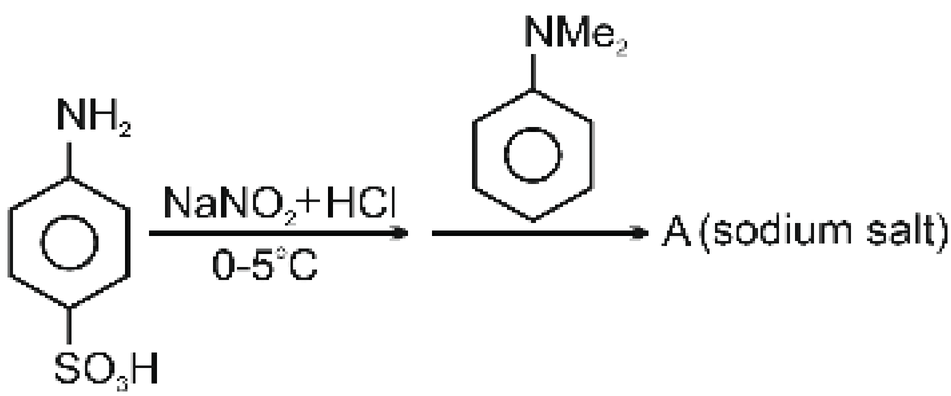
(A) 100% conversion of dimer to monomer is spontaneous.

(B) At point B, free energy of system becomes constant with time.

(C) In the region between A and B, reaction has tendency to move in forward direction.

(D) In the region between B and C, ΔG for the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ is positive.

4) Which of the following statements regarding A is/are incorrect ?



(A) Benzenoid form of anion of A is pinkish red in colour.

(B) Quinonoid form of anion of A is yellow in colour.

(C) Benzenoid form of anion accepts a proton and gets converted into a more deep coloured quinonoid form.

(D) A is used as indicator in weak base v/s strong acid titration.

5) In CaF_2 type crystal, which of following statement(s) is/are true?

(A) Anion is inside a cubical void made by cation.

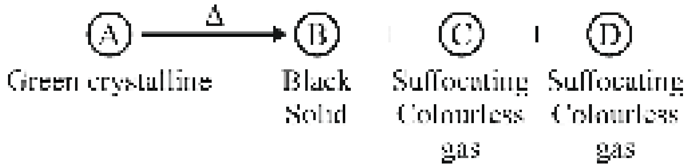
(B) For each cation, there will be 12 cations which are its second nearest neighbours.

(C) For each cation, there will be 16 third nearest as anionic neighbours.

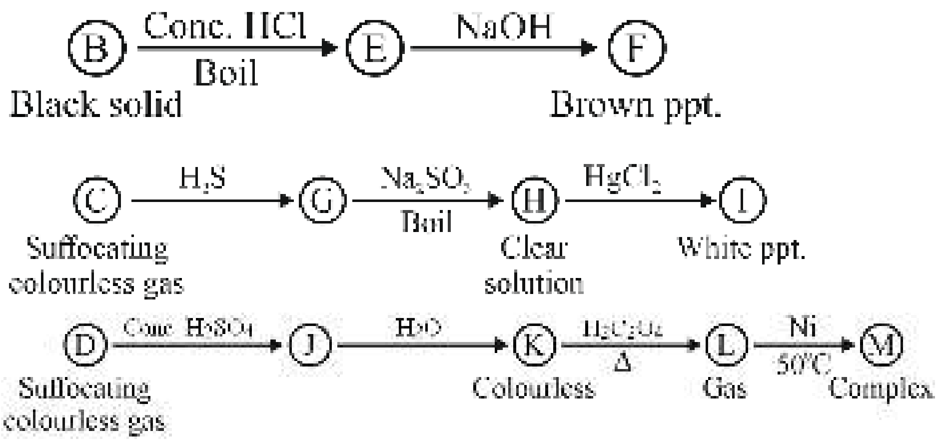
(D) The third nearest neighbour of cation is at a distance of $\frac{\sqrt{15}}{4}a$.

6) $[Co(H_2O)_6]^{2+} + Cl^{\ominus} \rightleftharpoons "X" + 6H_2O$
Correct statement(s) about the reaction is/are

(A) X is blue coloured compound with octahedral structure.
(B) Increasing the temperature favours forward reaction.
(C) Addition of Ca^{2+} to the equilibrium mixture favours forward reaction.
(D) Addition of Zn^{2+} ions to the equilibrium mixture favours forward reaction.



7)
Note: C and D are oxide of same element.



Which of the following option(s) is/are correct ?

(A) Hybridization of metal in complex M is dsp^2.

(B) Basicity of compound K is two.

(C) Compound C is H_2SO_4

(D) Oxidation state of central metal ion in compound F is +3 .

8) α - maltose $(C_{12}H_{22}O_{11})$ can be hydrolysed to glucose $(C_6H_{12}O_6)$ according to the following reaction: $C_{12}H_{22}O_{11}\,(aq) + H_2O\,(\ell) \rightarrow 2\,C_6H_{12}O_6\,(aq)$ **Given:**
Standard enthalpy of formation of $C_{12}H_{22}O_{11}\,(aq) = -2238$ kJ/mol
Standard enthalpy of formation of $H_2O\,(\ell) = -285$ kJ/mol
Standard enthalpy of formation of $C_6H_{12}O_6\,(aq) = -1263$ kJ/mol

Time(min)	0	50	100
Conc. of α - maltose (M)	4.0	1.0	0.25

which of the following statement(s) is/are true?

(A) The hydrolysis of α - maltose is exothermic.

(B) Heat liberated in combustion of 1 mole of α- maltose(s) must be greater than the heat liberated in combustion of 2 moles of glucose(s).

(C) Increasing temperature will increase the degree of hydrolysis of α - maltose.

(D) The hydrolysis of α - maltose follows 1st order kinetics.

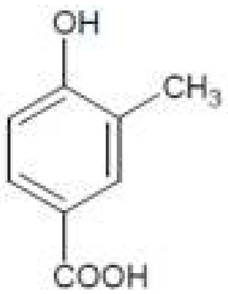
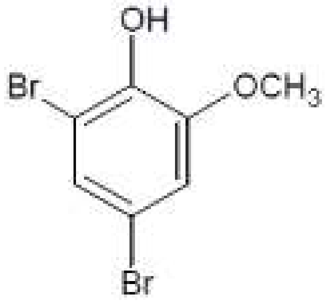
9) Vanillin (L), $C_8H_8O_3$, is isolated from vanilla beans. It give an intense colour with $FeCl_3$ and positive Tollen's test. It is not steam-distilled and does not react with HCl. It goes through following steps :

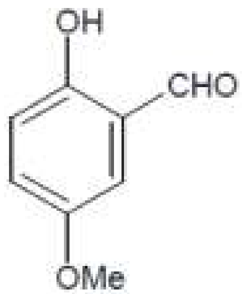
$$L \xrightarrow[\text{1 equivalent of } CH_3I]{OH^-} M,(C_9H_{10}O_3) \xrightarrow{KMnO_4} N,(C_9H_{10}O_4),(HCO_3^- \text{ soluble})$$

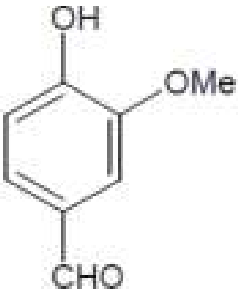
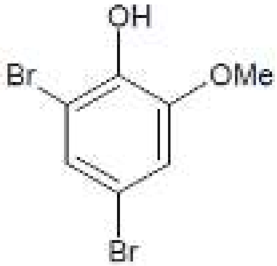
$$N \xrightarrow{conc.\ HI\ \Delta} \text{3,4-dihydroxy benzoic acid}$$

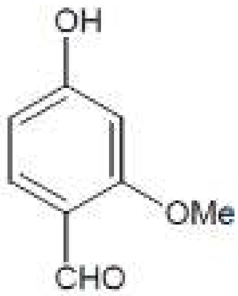
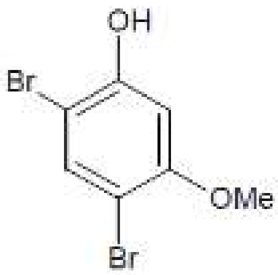
$$L \xrightarrow{[Ag(NH_3)_2]^-} O \xrightarrow{2.\ Br_2} P,(C_7H_6Br_2O_2)$$

Which of the following is correct statement about various products?

(A) L is  and P is 

(B) L is  and P is 

(C) L is  and P is 

(D) L is  and P is 

10) White phosphorous reacts with NaOH to produce gas (X) and the compound (Y).
'X' + HCl → 'Z'
What are the changes take place when 'X' is converted to Z.

(A) State of hybridisation of central atom.

(B) % s-character of bonds associated to central atom.

(C) Oxidation state of central atom

(D) Number of lone pair(s) on central atom.

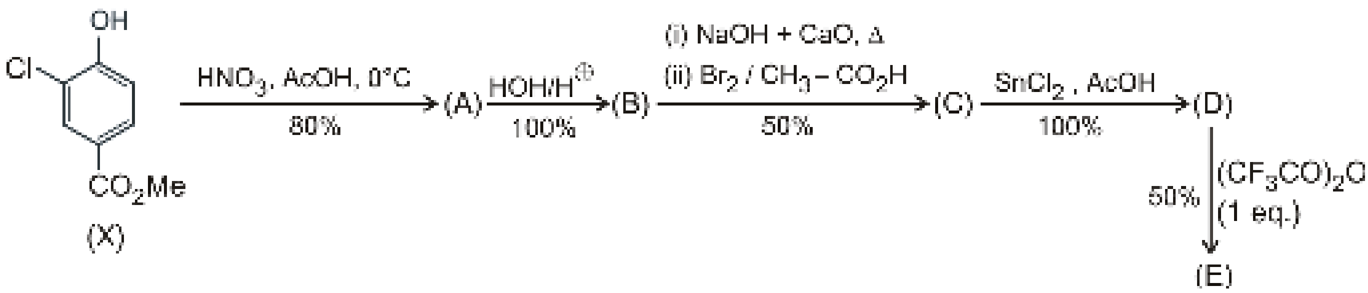
SECTION-II

1) If number of molecules per unit volume reduces by 20% and radius of molecule increased by 20%, the ratio of new mean free path to the old mean free path is x : 1. The value of 'x' is.

2) How many of the following statement(s) is/are true with respect to preparation of sodium hydroxide by electrolysis of brine solution with graphite anode ?
(i) H_2 is released at cathode, if we use Fe at cathode .
(ii) With Hg cathode, Na^+ is deposited instead of H^+ ion.

(iii) Standard reduction potential data suggests that it is easier to deposite H^+ than Na^+ ion.
(iv) Hg has a low hydrogen overvoltage while sodium has a very high overvoltage on Hg.
(v) Cl_2 is released at anode due to high overvoltage of O_2.
(vi) It is advised not to use an open cell for the above mentioned electrolysis.

3)



Calculate the gm of aroamtic product (E),if 1865 gm of reactant (X) is used for above reaction sequence.
[**Given, atomic weight :** H = 1, C = 12, O = 16, F = 19, Cl = 35.5, Br = 80]

4)

Number of moles of periodic acid required to oxidise one mole of glucose is X and number of moles of formic acid formed when one mole of fructose get oxidised by enough periodic acid is Y. Find the value of (X – Y).

5)

How many of following ions will form white precipitate with $K_4[Fe(CN)_6]$ (before aerial oxidation)?
Co^{2+}, Ca^{2+}, Fe^{2+}, Fe^{3+}, Mn^{2+}, Ag^+, Zn^{2+}

6) How many of the following statements are correct?
(1) Micellization is entropy driven process.
(2) A catalyst has no effect on ΔG, K_{eq} and ΔH of a reaction.
(3) On increasing non-polar chain length of $R-COO^-Na^+$ CMC increases.
(4) Heamoglobin is a positive sol.
(5) Charcoal is negative sol.

7) For the reaction : 4A(g) + 2B(g) → 3C(g); $\Delta_r S°$ = –40 cal/K-mol at 300 K. Calculate $\Delta_r S°$ for the reaction at 400 K, in cal K^{-1} mol^{-1}.
(Given : $C_{P,m}$ [/Cal K^{-1} mol^{-1}] : A(g) = 3.0, B(g) = 5.0; C(g) = 7.0; ln2 = 0.7; ln3 = 1.1)

8) NH_3 is not obtained as product from how many of the following reactions.
(a) $(NH_4)_2HPO_4 \xrightarrow{\Delta}$
(b) $CaCN_2 + H_2O \rightarrow$
(c) $NH_4ClO_4 \xrightarrow{\Delta}$
(d) $B_3N_3H_6 + H_2O \rightarrow$
(e) $Ba(N_3)_2 \xrightarrow{\Delta}$

(f) $Ba_3N_2 + H_2O \rightarrow$

PART-3 : MATHEMATICS

SECTION-I

1) Which of the following is/are true

(A) $\int_0^{\pi/2} \frac{\sin(7x)}{\sin x} dx = \frac{\pi}{2}$

(B) $\int_0^{\pi/2} \frac{\sin(10x)}{\sin x} dx = \frac{526}{315}$

(C) $\int_0^{\pi} \frac{\sin(9x)}{\sin x} dx = \pi$

(D) $\int_0^{1} x^4(1-x)^4 dx = \frac{1}{630}$

2) Let A and B are two non-zero $n \times n$ matrices then the value of $\frac{\operatorname{Tr}\left[(AB-BA)^n\right]}{\det\,(AB-BA)}$ equals (where Tr (x) and det (x) represents Trace and determinant value of matrix-x) (where $\det(AB-BA) \neq 0$)

(A) -2 If $n = 2$

(B) 3 If n = 3

(C) $\frac{1}{3}$ If n = 3

(D) $\frac{1}{2}$ If n = 2

3) Consider a tetrahedron is formed by planes y + z = 0, z + x = 0, x + y = 0 and x + y + z = 1 which of the following is/are true

(A) volume of tetrahedron is $\frac{4}{3}$

(B) centroid of tetrahedron is $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

(C) Shortest distance between two of non intersecting edges of tetrahedron is $\sqrt{\frac{2}{3}}$

(D) One of the edge of tetrahedron is $\frac{x-1}{0}=\frac{y+1}{2}=\frac{z-1}{-2}$

4) A curve $y=f(x)$ passes through point P(1,1), the normal to curve at P is $a(y-1)+(x-1)=0$. If slope of tangent at any point on the curve is proportional to ordinate of that point then correct option is/are -

(A) Equation of the curve is $y=e^{a(x-1)}$ (whre a ∈ R)

(B) Area bounded by y-axis, curve and normal to curve at P is $\left|1+\frac{e^{-a}}{a}-\frac{1}{2a}\right|$

(C) Area bounded by f(x), x-axis in $x \in (0,1)$ for some value of 'a' is $\left(1-\frac{1}{e}\right)$

(D) Area bounded by y-axis, curve and normal to curve at P is $\left|\int_0^1\left(1+\frac{1}{a}-\frac{x}{a}-e^{a(x-1)}\right)dx\right|$

5) Let the mirror image of the curve $\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$, with respect to the line $x+y-4=0$ is shifted 1 unit right and 2 unit up and the resulting curve is C which of the following regarding the curve C is/are correct?

(A) Centre of C is (7,5)

(B) Latus rectum of C is 9/2

(C) The curve C also passes through $\left(7+\frac{3}{\sqrt{2}}, 5+\sqrt{2}\right)$

(D) The point (4, 5) lies inside C.

6) Consider a curve mirror $y=f(x)$ passing through (8, 6) having property that all rays emanating from (0, 0) after getting reflected from mirror becomes parallel to x-axis then correct option is/are

(A) Length of latus rectum of $f(x)$ can be 4

(B) Length of latus rectum of $f(x)$ can be 36

(C) Directrix of f(x) can be x = –2

(D) Directrix of f(x) can be x = 16

7) Consider the set $A=\{1,2,3,4,5\}$.
Let the number of functions defined from A to A for which $f(f(i))=i$ for at least one $i \in A$ is N then

(A) N has four divisors

(B) N is odd

(C) N is even

(D) N is of the form $4\lambda+1$ (where λ ∈ N)

8) Which of the following statement is/are true?

(A) If f(g(x)) in injective then g(x) must be injective

(B) If f(g(x)) is surjective then f(x) must be surjective

(C) If f(g(x)) is injective g(x) may not be injective

(D) If f(g(x)) is surjective f(x) may not be surjective

9) Let x, y and z are three integers satisfying the system of simultaneous equations $xy + z = 100$, $yz + x = 87$ and $zx + y = 60$.
Which of the following option is/are correct?

(A) x + y + z = – 29

(B) x + y + z = –28

(C) xy + yz + zx = 276

(D) xy + yz + zx = 275

10) Let 'f' be a polynomial function such that $f(f(x)) - 8x^m = \frac{80}{f''(x)} - 64x^2 + 100 \ \forall x \in R$ and $m \in N$.
If $f(|x|)$ is differentiable for all $x \in R$ then choose the correct option(s).

(A) m = 4

(B) m = 2

(C) $\left[\frac{f(3)}{m}\right] = 2$, (where [.] denotes greatest integer function)

(D) f(x) is a polynomial of odd degree.

SECTION-II

1) Number of permutations of 1, 2, 3, 4, 5, 6 such that for each k with 1 ≤ k ≤ 5, at least one of the first k terms of the permutation is greater than k.

2) Let f(x,y,z) be a Homogenous polynomial of degree-4 in real variables x, y, z satisfying the following conditions
(a) $f(1, 2, 3) = 1/10$
(b) $f(x, y, z) = f(z, x, y)$ = f(y, z, x) for all x, y, z ∈ R
(c) $f(x, x, z) = 0$ for all x, y, z ∈ R
Value of f(1,4,9) equals

3) Let T_n = 1 + 2 + 3 + ... + n and $M_n = \frac{T_2}{T_2 - 1} \times \frac{T_3}{T_3 - 1} \times \frac{T_4}{T_4 - 1} \times \dots \times \frac{T_n}{T_n - 1}$ for n = 2, 3, 4, ...,
Value of $\frac{13}{25} M_{50}$ equals

4) $\dfrac{\tan^6\left(\frac{\pi}{9}\right)+27\tan^2\left(\frac{\pi}{9}\right)}{1+11\tan^4\left(\frac{\pi}{9}\right)}=\sqrt{P}-1$ (where $P \in N$)
then the remainder when P is divided by 7 is____________

5) Consider the function
$f(x)=\max\left\{x^3-\dfrac{1}{2},-\sqrt{16-x^2},\dfrac{x}{|x|}\right\}, x \in [-4,4]$
m = number of points where the function is discontinuous
n = number of points where the function is continuous but not differentiable
find (m + n)

6) In a circle 15 persons are seated and it is known that each person knows the two immediate neighbor (one on the left and one on the right) only. If 5 persons are selected at random, then the probability that the selected persons are unknown to each other is $\dfrac{p}{q}$, where p, q are coprime then sum of digits of p is______

7) If $(1+x)^n=P_0+P_1x+P_2x^2+P_3x^3+\ldots\ldots\ldots, (n \in N)$
$P_0-P_2+P_4-\ldots\ldots\ldots\ldots=\alpha$, $P_1-P_3+P_5-\ldots\ldots\ldots=\beta$ then $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\alpha^2+\beta^2}$ is

8) If $A(\vec{a}), B(\vec{b}), C(\vec{c}), D(\vec{d})$ are position vector of a cyclic quadrilateral ABCD,
$\alpha=\dfrac{\left|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{d})+(\vec{d}\times\vec{a})\right|}{\left|(\vec{b}-\vec{a}).(\vec{d}-\vec{a})\right|}$, $\beta=\dfrac{\left|(\vec{b}\times\vec{c})+(\vec{c}\times\vec{d})+(\vec{d}\times\vec{b})\right|}{\left|(\vec{b}-\vec{c}).(\vec{d}-\vec{c})\right|}$ then . $\left|\dfrac{3\alpha}{2\beta}\right|$ is

ANSWER KEYS

PART-1 : PHYSICS

SECTION-I

Q.	1	2	3	4	5	6	7	8	9	10
A.	A,C	A,C	A,B,C	A,C	A,B	B,C	B,C,D	A,B,C,D	B,C	B,C,D

SECTION-II

Q.	11	12	13	14	15	16	17	18
A.	16.00	11.50 to 11.55	14.00	0.25	1.33	15.72 to 15.73	16.00	9180.00

PART-2 : CHEMISTRY

SECTION-I

Q.	19	20	21	22	23	24	25	26	27	28
A.	A,C,D	B,C	B,C,D	A,B	B,C	B,C	B,D	A,D	C	A,B,D

SECTION-II

Q.	29	30	31	32	33	34	35	36
A.	0.86 or 0.87	5.00	637.00	2.00	5.00	4.00	-40.30	2.00

PART-3 : MATHEMATICS

SECTION-I

Q.	37	38	39	40	41	42	43	44	45	46
A.	A,B,C,D	A,B	B,C,D	A,B,C,D	A,C	A,B,C	A,B,D	A,B	A,C	A,C

SECTION-II

Q.	47	48	49	50	51	52	53	54
A.	461.00	14.00	1.50	2.00	5.00	9.00	2.00	1.50

SOLUTIONS

PART-1 : PHYSICS

1)

Net force on PQ must be zero, if top spring stretches by x, then next two stretches by $\frac{x}{2}$.

$\therefore$ Displacement of top end is,

$\Delta x = x + \frac{x}{2} + \frac{x}{4} + ...\infty$

$\Rightarrow x\left(\frac{1}{1-\frac{1}{2}}\right)$

$\Rightarrow 2x = \frac{2F}{k}$

So, displacement of PQ is,

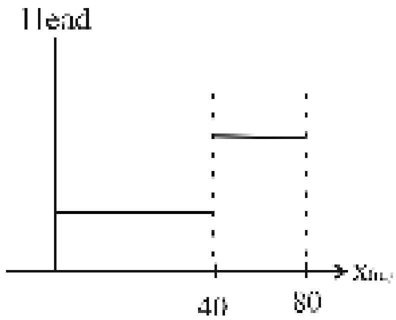
$x = \frac{F}{k}$

2)

(1) Kinetic head

$\frac{v^2}{2g} \Rightarrow \left[\frac{v^2}{2g}\right]_P = \frac{(20)^2}{2\times 10} = 20$ m [upto x = 40]

$\left[\frac{v^2}{2g}\right]_Q = \frac{(40)^2}{20} = 80$ m [x = 40 to 80]



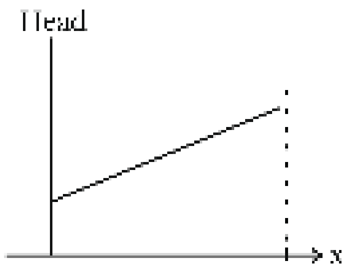
Hence option 1 is correct.

(2) Potential head (h)

h at P = 50 m

h at x = h at P + x tan 37^0

h at x $= 50 + \frac{3x}{4}$



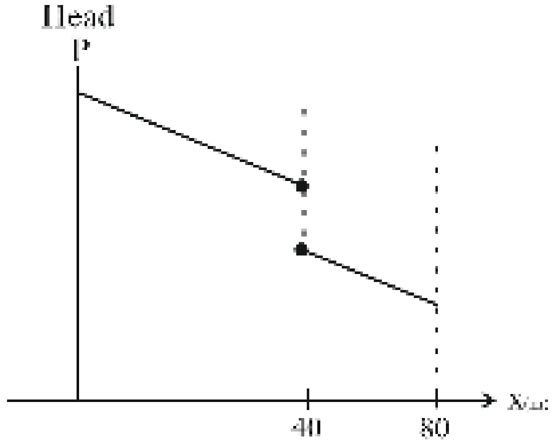
Hence option 2 is incorrect.

(3) Δ Pressure head = – [Δ Kinetic head + Δ Potential head]

$0 \leqslant x < 40$, Δ Pressure head $= -\left[0+\frac{3x}{4}\right] = \frac{-3x}{4}$

40< x < 80, Δ Pressure head $= -\left[0+\frac{3x}{4}\right] = \frac{-3x}{4}$

at x = 40 ΔPressure head $= -[80-20] = -60$ m



Hence option 3 is correct.

(4) $\left\{\left[\frac{p}{\rho g}\right]+\left[\frac{v^2}{2g}\right]+h\right\}_P = \left\{\left[\frac{p}{\rho g}\right]+\left[\frac{v^2}{2g}\right]+h\right\}_Q$

$\left[\frac{p}{\rho g}\right]_P + 20 + 50 = \left\{\frac{p}{\rho g}\right\}_Q + 80 + 60 + 50$

$\left\{\frac{p}{\rho g}\right\}_P - \left\{\frac{p}{\rho g}\right\}_Q = 120$ m

3) $Z_1 = R_1 + i\omega L$,

$Z_2 = R_2$

$Z_3 = R_3 + \frac{I}{i\omega C_3}$

$Z_4 = \frac{1}{i\omega C_4}$

For balanced bridge

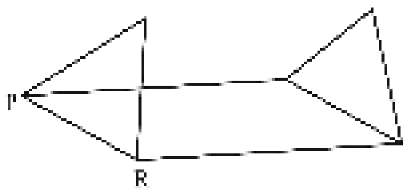
$Z_1Z_4 = Z_2Z_3$

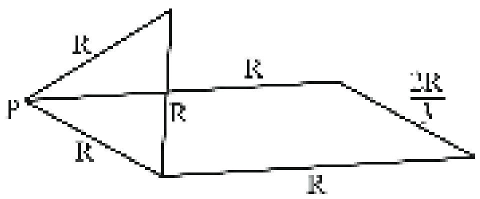
So $\frac{R_1}{i\omega C_4} + \frac{L_1}{C_4} = R_2R_3 + \frac{R_2}{i\omega C_3}$

$\Rightarrow \frac{L_1}{C_4} = R_2R_3, \frac{R_1}{\omega C_4} = \frac{R_2}{\omega C_3}$

4) The frequency dependence of the displacement amplitude in forced oscillations is given by the formula $A = \frac{F_0}{m\sqrt{\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2}}$, while the frequency dependence of the velocity

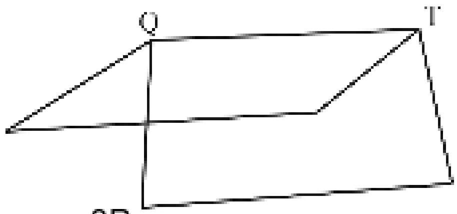
amplitude is given by the formula $v_m = \dfrac{F_0\omega}{m\sqrt{\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2}}$ In the first case, $\omega = 0$ at the amplitude A does not vanish but becomes equal to $F_0/m\omega_0^2$, or F/k, so that the curve cuts off a segment on the vertical axis, which segment is the displacement under a constant force. The velocity, of course, is zero in this case. Thus, the curves in Figure (a) correspond to frequency dependence of the displacement amplitudes, while the curves in Figure (b) correspond to the frequency dependence of the velocity amplitudes.

5) 

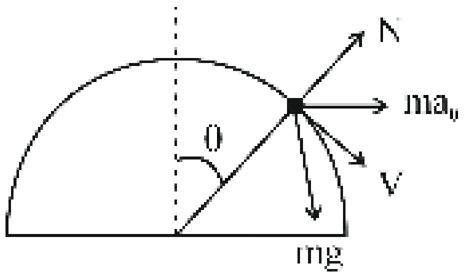
= 

$\text{Req} = \dfrac{8R}{15}$

B/w QT : -



$R_{eq} = \dfrac{3R}{5}$



$$mg\cos\theta - N - ma_0\sin\theta = \frac{mv^2}{R}$$

6) With respect to sphere

At the break off point. N = 0, $\theta = \theta_0$, $V = V_0$

$\Rightarrow \dfrac{V_0^2}{R} = g\cos\theta_0 - a_0\sin\theta_0$

$= g\left(\cos\theta_0 - \sin\theta_0\right)$ ..... (i)

from W.E.T

$mgR\left(1-\cos\theta_0\right) + ma_0R\sin\theta_0 = \dfrac{mv_0^2}{2}$

$\dfrac{V_0^2}{2R} = g\left(1-\cos\theta_0\right) + a_0\sin\theta_0$

$= g\left(1-\cos\theta_0 + \sin\theta_0\right)$ ...... (ii)

After solving eq (i) & (ii)

$V_0 = \sqrt{\dfrac{2gR}{3}}$ & $\cos\theta_0 = \dfrac{2+\sqrt{14}}{6}$

7) 2 criteria

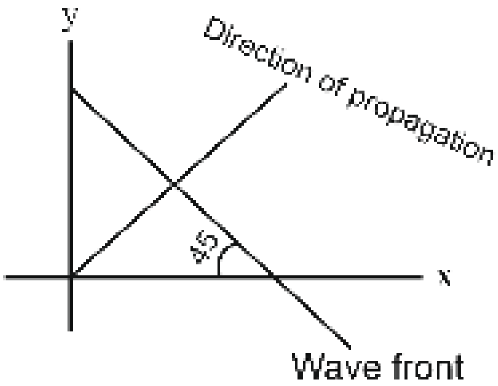
$\frac{hc}{\lambda} > \phi_E$ and $hV_S = \frac{hc}{\lambda} - \phi_C$

8) $Z_1 + Z_3 = a\cos(Kx - \omega t) + a\cos(ky - \omega t)$

$2a\cos\left[\frac{k}{2}(x+y) - \omega t\right]\cos\frac{k}{2}(x-y)$

phase of vibration is is $\phi = \frac{k}{2}(x+y) - \omega t$

The phase is constant where x+y = constant



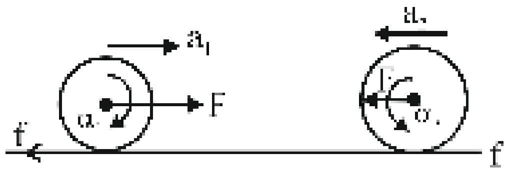
Thus wave fronts are represented by x+ y = constant

Similarly,

$Z_2 + Z_3 = 2a\cos\frac{k}{2}(x+y)\cos\left[\frac{k}{2}(y-x) - wt\right]$

9) $V_1 = \omega R_1,\ V_2 = \omega R_2$

Torque about ICOR



F.R. $= \frac{7}{5}.2mR^2\alpha_1$ & $a_1 = \alpha_1 R$

$\Rightarrow F = \frac{14}{5}ma_1$ .... (i)

$\Rightarrow FR = \frac{5}{3}mR^2\alpha_2$ & $a_2 = \alpha_2 R$

$\Rightarrow F = \frac{5}{3}ma_2$ .... (ii)

From (1) & (ii)

$\frac{14}{5}ma_1 = \frac{5}{3}ma_2$

$42a_1 = 25a_2$

$\Rightarrow 42v_1 = 25v_2$ ..... (iii)

$U_i + K_i = U_f + K_f$

$0+0 = \frac{KQ.(-Q)}{2R} + \frac{1}{2}(2m)v_1^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\omega_2^2$

$\frac{KQ^2}{2R} = \frac{1}{2}\left(\frac{14}{5}m\right)v_1^2 + \frac{1}{2}\left(\frac{5}{3}m\right)v_2^2$ ...... (iv)

from (iii) & (iv)

$$\frac{KQ^2}{2R} = \frac{1}{2}\left(\frac{14}{5}m\right)\left(\frac{25}{42}v_2\right)^2 + \frac{1}{2}\left(\frac{5}{3}m\right)v_2^2$$

$$= \frac{1}{2}m\left[\frac{125}{126}v_2^2 + \frac{5}{3}v_2^2\right]$$

$$= \frac{1}{2}m\left[\frac{125 + 5 \times 42}{126}\right]v_2^2 ; \frac{KQ^2}{2R} = \frac{1}{2}m\left[\frac{335}{126}\right]v_2^2$$

$$\Rightarrow v_2 = \sqrt{\frac{126}{335}\frac{KQ^2}{mR}} \Rightarrow v_1 = \frac{25}{42}\sqrt{\frac{126}{335}\frac{KQ^2}{mR}}$$

10) For right chamber ⇒ gas = Monoatomic

$$PV^{\gamma} = C \Rightarrow P^{1-\gamma}T^{\gamma} = C, \quad \gamma = \frac{5}{3}$$

$$P_0^{1-\frac{5}{3}}T_0^{\frac{5}{3}} = \left(\frac{243}{32}P_0\right)^{1-\frac{5}{3}}T^{\frac{5}{3}}$$

$$\Rightarrow T = \left[\left(\frac{32}{243}\right)^{-\frac{2}{5}} \times T_0^{\frac{5}{3}}\right]^{\frac{3}{5}} = \left(\frac{32}{273}\right)^{-2/5} . T_0$$

$$T = \frac{9}{4}T_0 \;\&\; P_0V_0^{5/3} = \frac{243}{32}P_0V^{5/3} \Rightarrow V = \frac{8}{27}V_0$$

$$W = -nC_V\Delta T$$

$$= -1 \times \frac{3}{2} \times R\left(\frac{9}{4}T_0 - T_0\right)$$

$$= -\frac{15}{8}T_0 = -\frac{15}{8}P_0V_0$$

⇒ for left chamber

$$V = 2V_0 - \frac{8}{27}V_0 = \frac{46}{27}V_0$$

$$P_1 = P_2 = \frac{243}{32}P_0$$

$$\frac{P_0V_0}{T_0} = \frac{\frac{243}{32}P_0 \times \frac{46}{27}V_0}{T}$$

$$T = \frac{207}{16}T_0$$

$$Q = \Delta U_1 + \Delta U_2$$

$$= \frac{5}{2} \times 1 \times R \times \left(\frac{207}{16}T_0 - T_0\right) + \frac{3}{2}R\left(\frac{9}{4}T_0 - T_0\right)$$

$$= \frac{5}{2} \times \frac{191}{16}RT_0 + \frac{15}{8}RT_0$$

$$= \frac{955 + 60}{32}RT_0 = \frac{1015}{32}P_0V_0$$

11) Null point should be close to end

⇒ V = 7V

10 = i(50 + 5 + R)

50i = 7

i = 0.14 A

10 = 0.14(55 + R)

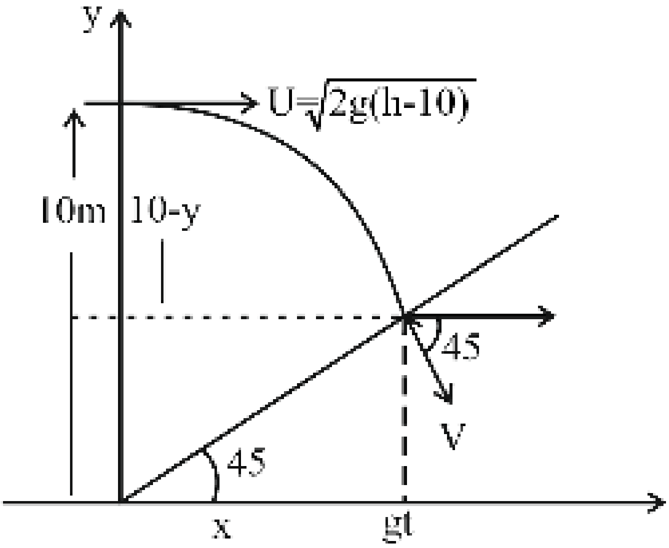
$\frac{100}{0.14} - 55 = R$

71.4 - 55 = R

R = 16.4 Ω ⇒ R = 16 Ω

12)

Along the inclined plane



$O = U\cos 45 - gt\cos 45$

$\boxed{U = gt}$

$x = Ut = \frac{U^2}{g}$ .... (i)

$\tan 45 = \frac{y}{x}$

$\boxed{x = y}$ ....(ii)

$\Rightarrow 10 - y = \frac{1}{2}gt^2$

$y = 10 - \frac{1}{2}gt^2$ ....(iii)

Eq. (i), (ii) & (iii)

$\frac{U^2}{g} = 10 - \frac{1}{2}g.\frac{U^2}{g^2}$

$\frac{3}{2}\frac{U^2}{g} = 10$

$\frac{3}{2 \times g}.2g\,(h - 10) = 10$

3h =40

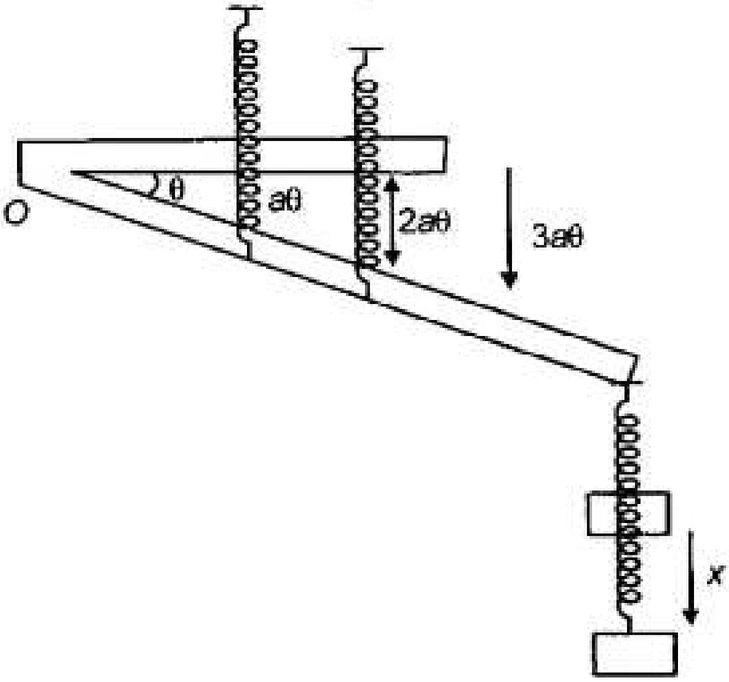
$h = \frac{40}{3}$

h = 13.33m ...... (iv)

$U = \sqrt{2 \times 10 \times 3.33}$

= 8.16 m/s

v= $\sqrt{2}$ U = 11.54



13)

For rotational equilibrium about hinge of rod

$(Ka\theta)\,a + (2Ka\theta)\,2a = K\,(x - 3a\theta)\,3a$

$a\theta + 4a\theta = 3x - 9a\theta$

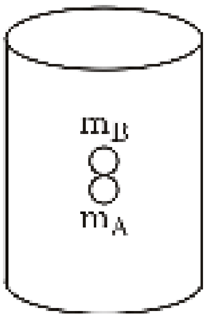
$14a\theta = 3x$

$a\theta = \dfrac{3x}{14}$

Restoring force on block = $K\,(x - 3a\theta)$

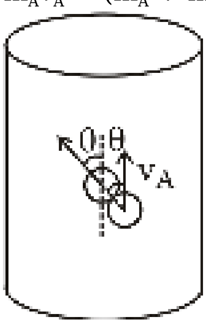
$= K\left(x - \dfrac{9x}{14}\right) = K\left(\dfrac{5x}{14}\right)$; $\dfrac{5}{14}Kx = ma$

$\dfrac{x}{a} = \left|\dfrac{14M}{5K}\right|$; $T = 2\pi\sqrt{\dfrac{14m}{5K}}$



14)

$m_A v_A = (m_A + m_B)v$



$m_A v_A \cos\theta = m_B v_B + m_A v'_{A\perp}$

$e = 1 = \dfrac{v_B - v'_{A\perp}}{v_A \cos\theta}$

$$v'_{A\perp} = v_A \cos\theta - \frac{m_B}{m_A} v_B$$

$$2v_A \cos\theta = \frac{(m_A + m_B)}{m_A} v_B$$

$$v_B = \frac{2m_A v_A \cos\theta}{m_A + m_B} = 2v\cos\theta$$

$$v_{B_x} = v_B \sin\theta = v\sin 2\theta$$

$$t_{min} \Rightarrow v_{B_x} = max = v$$

$$t = \frac{R}{v_{B_x}} = \frac{R}{v} = 0.25$$

15) Let R be the radius of the sphere and ρ the density at a distance r from the centre of the sphere.

$\therefore$ By the problem, $\rho = k(R - r)$, where k = proportionality constant.

The value of g at a distance x from the centre (x<R) due to the sphere of radius x and is given by

$$g = \frac{G}{x^2}\int_0^{\pi} 4\pi\rho r^2 dr = \frac{4\pi kG}{x^2}\int_0^{x}(R - r)\, r^2 dr$$

$$\Rightarrow \frac{4\pi kG}{x^2}\left(\frac{x^3 R}{3} - \frac{x^4}{4}\right)$$

$$= 4\pi kG\left(\frac{xR}{3} - \frac{x^2}{4}\right)$$

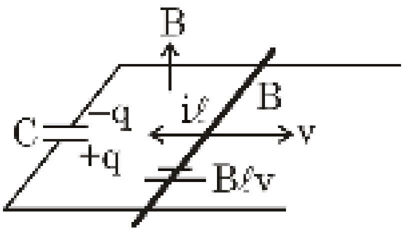
Now, the maximum value of g, g_{max} corresponds to $\frac{dg}{dx} = 0 \Rightarrow \frac{d}{dx}\left(xR/3 - x^2/4\right) = 0 \Rightarrow$ $\frac{R}{3} - \frac{x}{2} = 0 \Rightarrow x = \frac{2}{3}R$ and d^2g/dx^2 at the above x-value is negative.

$\therefore$ g is maximum at a depth $\frac{1}{3}R$ and the maximum value is given by

$$g_{max} = 4\pi kG\left[\frac{2}{3}R.\frac{R}{3} - \left(\frac{2}{3}R\right)^2.\frac{1}{4}\right] = \frac{4\pi kG}{9}R^2$$

But, $$g_{surface} = 4\pi kG\left[R.\frac{R}{3} - \frac{R^2}{4}\right] = \frac{1}{3}\pi kGR^2$$

$$\therefore g_{max} = \frac{4}{3}g_{surface}$$

16) 

$$B\ell v - iR = \frac{q}{C} = 0$$

$$F - i\ell B = m\frac{dv}{dt} = \frac{m}{B\ell} \times \frac{d}{dt}\left(iR + \frac{q}{C}\right)$$

$$F = \left(\frac{m}{B\ell C} + B\ell\right)i + \frac{mR}{B\ell}\frac{di}{dt}$$

$$F - \left(\frac{m + B^2\ell^2C}{B\ell C}\right) i = \frac{mR}{B\ell}\frac{di}{dt}$$

$$\frac{B\ell}{mR}\int_0^t dt = \int_0^i \frac{di}{F - \frac{m+B^2\ell^2C}{B\ell C}i}$$

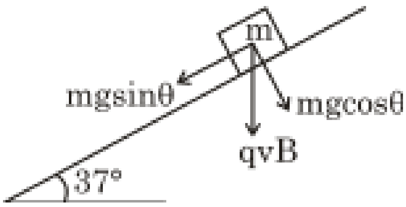
$$\frac{B\ell t}{mR} = \frac{-B\ell C}{m + B^2\ell^2C}\ell n\left(\frac{F - \frac{m+B^2\ell^2C}{B\ell C}i}{F}\right)$$

$$1 - \frac{m + B^2\ell^2C}{B\ell CF}i = e^{-\left(1+\frac{B^2\ell^2C}{m}\right)\frac{t}{RC}}$$

$$i = \frac{B\ell CF}{m + B^2\ell^2C}\left(1 - e^{-\left(1+\frac{B^2\ell^2C}{m}\right)\frac{t}{RC}}\right)$$

$$= \frac{mF}{2m \times B\ell}(1 - e^{-2})$$

$$= \frac{7.29}{0.1 \times 2}\left(1 - \frac{1}{e^2}\right) = 31.45 \text{ A}$$

17) 

$N = mg\cos\theta + qvB$

$$mg\sin\theta - \mu N = \frac{mdv}{dt}$$

$$\Rightarrow mg\sin\theta - \mu(mg\cos\theta + qvB) = \frac{mdv}{dt}$$

$$mg\sin\theta - \mu mg\cos\theta - \mu qvB = \frac{mdv}{dt}$$

at $v_{max}\ \frac{dv}{dt} = 0$

$$\Rightarrow v_{max}\frac{mg(\sin\theta - \mu\cos\theta)}{\mu qB}$$

$$= \frac{8 \times 10^{-6} \times 10}{0.5 \times 4 \times 10^{-6} \times 0.5}\left(\frac{3}{5} - 0.5 \times \frac{4}{5}\right) = 16 \text{ m/s}$$

18) Since wire is well lagged. We may assume that no heat enters or leaves it except at the ends,

So, $\frac{dQ}{dt} = \text{constant}$

Let

$$D = \frac{1}{A}\frac{dQ}{dt}, \quad \frac{dQ}{dt} = -KA.\frac{dT}{dx}$$

$\therefore K = 3 + 6T$

$$\Rightarrow (3 + 6T).\frac{dT}{dx} = -D$$

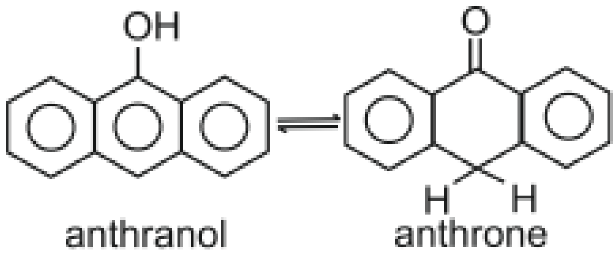
$$\int_{T_1}^{T_2}(3 + 6T)\,dT = -D\int_0^L dx$$

$\Rightarrow$ After solving

$$\frac{dQ}{dt} = \frac{A}{L}(T_1 - T_2)[3 + 3(T_1 + T_2)]$$
$$= \frac{4}{2}(30)(3 + 3 \times 50)$$
$$= 9180.00$$

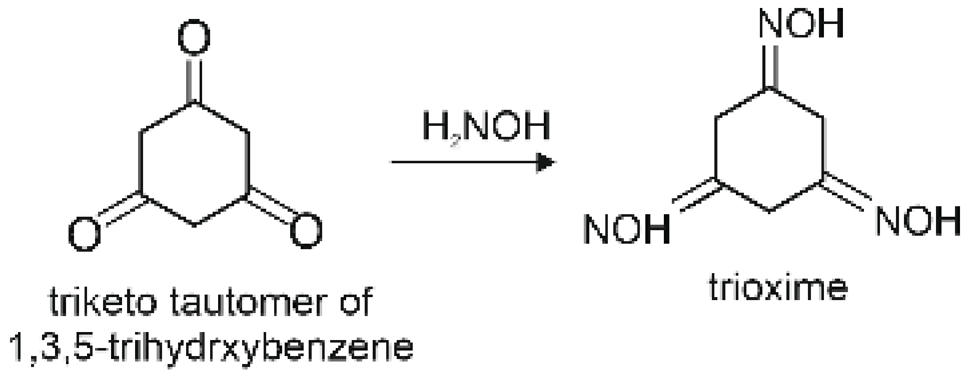
PART-2 : CHEMISTRY

19) At high temperature in napthelene sulphonation occurs at β position

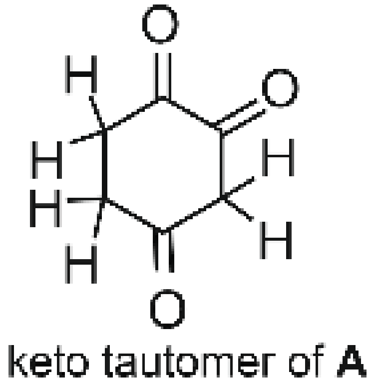


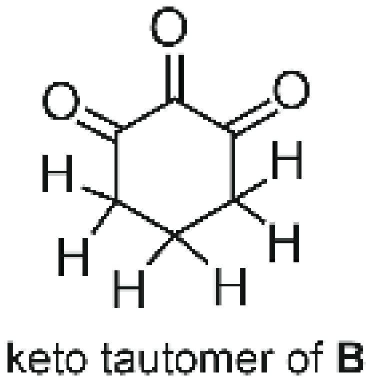
The equilibrium depends on the relative resonance energies of the tautomers. Although ring aromaticity usually stabilizes the phenolic tautomer, in this case the reverse is true. Anthrone has two distinct intact aromatic rings in addition to the C=O, making it more stable than anthranol, which suffers a loss of aromaticity per ring because its three rings are fused.

The triketo tautomer of this isomer has considerable stability because of the large resonance energy of its three C= O groups. Therefore it has sufficient concentration in the phenol-keto equilibrium to react to give a trioxime.



The triketo tautomers of 1,2,4 -trihydroxybenzene(A) and 1,2,3-trihydroxybenzene(B) are destabilized by the adjacent C = O's and have an insufficient concentration to react with H_2NOH.

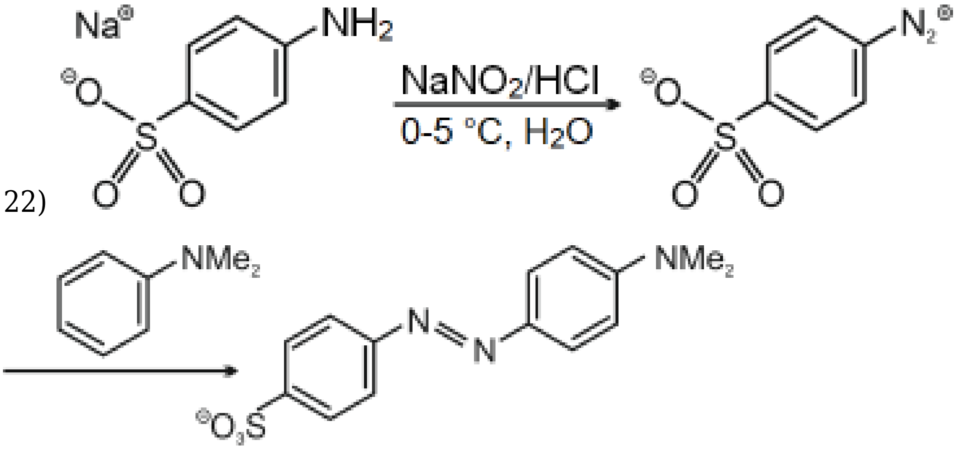




21) $\Delta G > 0$ for non-spontaneous processes.

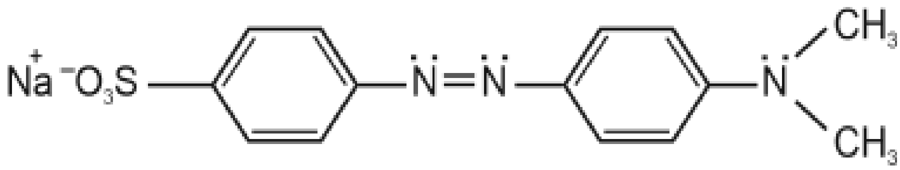
$\Delta G < 0$ for spontaneous processes.
$\Delta G = 0$ for processes at equilibrium.

22)

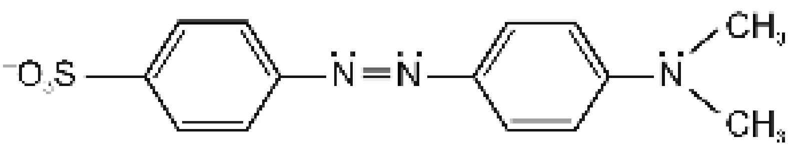


sodium salt is methyl orange

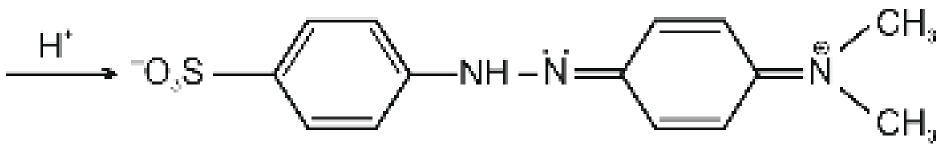
Methyl orange is a weak base and is yellow in colour in the unionised form. Sodium salt of methyl orange is represented as follows :



The anion formed from the indicator is an active species, which on accepting a proton (i.e acting as Bronsted Lowry base) changes from the benzenoid form to the quinonoid form. The quinonoid form is deeper in colour and thus is responsible for the colour change at the end point. This is illustrated in the following manner :



Benzenoid form of the anion (Yellow in colour)



Quinonoid form of the anion
(Pinkish red in colour)

23)

For each cation Ca^{2+} there will be 12 cations Ca^{2+} which are its second nearest neighbour at a distance of $\frac{a}{\sqrt{2}}$.

For each cation Ca^{2+} there will be 24 third nearest anionic neighbours at a distance of $\frac{\sqrt{11}}{4}a$.

24) $\left[Co^{(+2)}(H_2O)\right]^{2+} + 4Cl^- \underset{Zn^{2+}}{\overset{Ca^{2+}}{\rightleftharpoons}} \left[Co^{(+2)}Cl_4\right]^{2-} + 6H_2O$ is $\Delta H = +ve$

Ca^{2+} is class-a metal, Zn^{2+} is class-b metal. Class-a metals form stable complexes with ligands N, O, F as donor atom, class - b metals form stable complexes with the ligands P, S, Cl as donor atoms

25)
$$\underset{\text{Green (A)}}{FeSO_4.7H_2O} \xrightarrow[-7H_2O]{300^\circ C} FeSO_4 \longrightarrow \underset{\text{Black}}{Fe_2O_3} + SO_2\uparrow + SO_3\uparrow$$

$$\underset{(B)}{Fe_2O_3} + 6HCl \xrightarrow{\text{boil}} \underset{(E)}{2FeCl_3} + 3H_2O$$

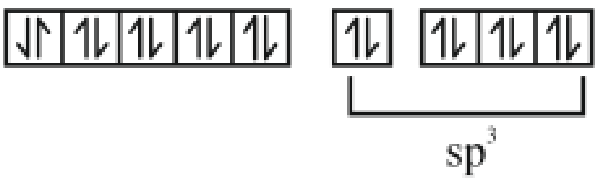
$$FeCl_3 + 3NaOH \longrightarrow \underset{(F)}{Fe(OH)_3}\downarrow + 3NaCl$$

$$\underset{(C)}{SO_2} + 2H_2S \longrightarrow \underset{(G)}{3S}\downarrow + 2H_2O$$

$$Na_2SO_3 + \underset{(G)}{S} \longrightarrow \underset{(H)}{Na_2S_2O_3} \xrightarrow{HgCl_2} \underset{(I)}{HgS_2O_3}\downarrow + 2NaCl$$

$$\underset{(D)}{SO_3} + H_2SO_4 \longrightarrow \underset{(J)}{H_2S_2O_7} \xrightarrow{H_2O} \underset{(K)}{2H_2SO_4} \xrightarrow{H_2C_2O_4} \underset{(L)}{CO}\uparrow + CO_2\uparrow + H_2O$$

$$Ni + \underset{(L)}{4CO}\uparrow \xrightarrow{40^\circ C} \underset{(M)}{[Ni(CO)_4]}$$

$[Ni(CO)_4] \longrightarrow Ni$ 

26)

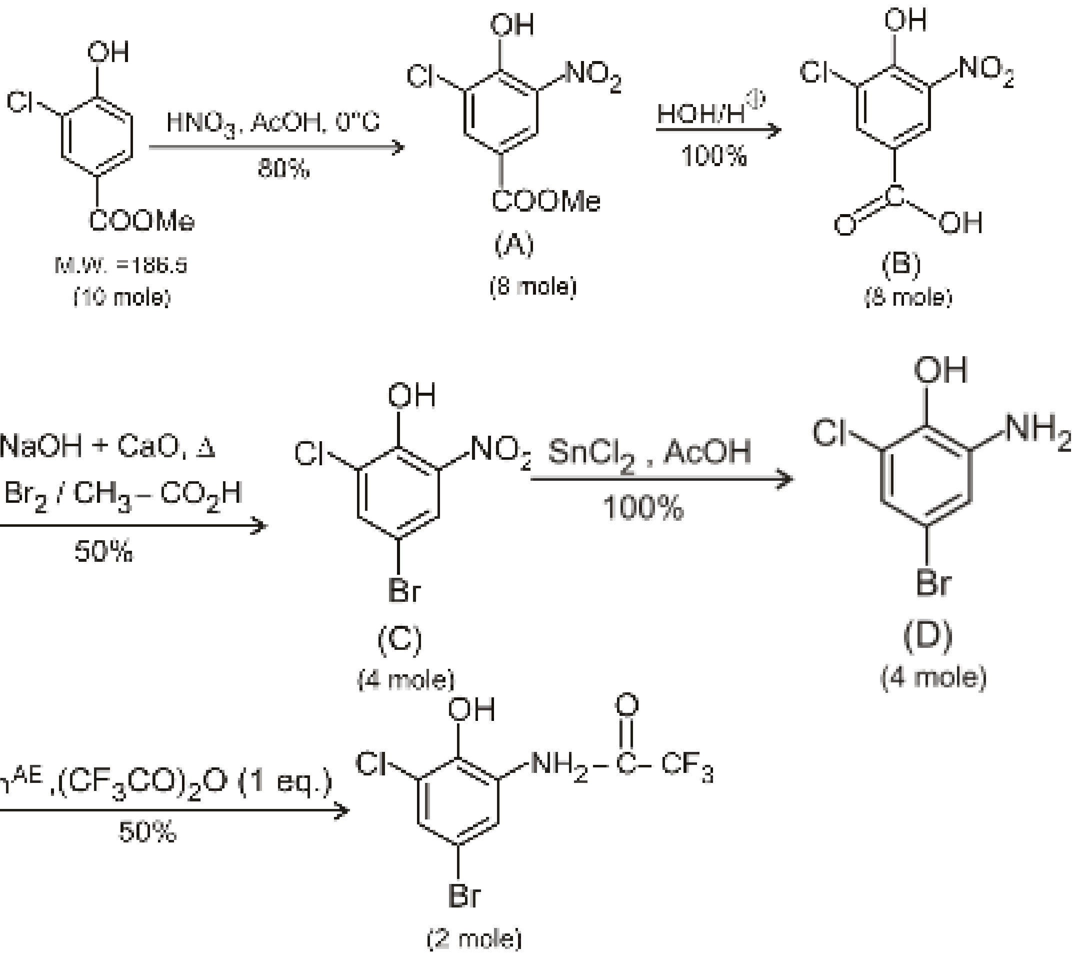
(A) $\Delta H = [2(-1263)] - [(-2238) + (-285)] = -3kJ$
(B) No comment without H data of enthalpies of solution of both solid.
(C) Hydrolysis is exothermic $T\uparrow h\downarrow$
(D) Concentration becomes one-forth is each equal time interval. Hence, 1st order

29) $\lambda_1 = \dfrac{1}{\sqrt{2}\pi nd^2} = \dfrac{1}{4\sqrt{2}\pi nr^2}$, n = N/V

$\lambda_2 = \dfrac{1}{4 \times \sqrt{2}\pi(0.8n) \times (1.2r)^2}$

$$\frac{\lambda_2}{\lambda_1} = \frac{4\sqrt{2}\,\pi\, n\, r^2}{4 \times \sqrt{2}\pi\,(0.8n) \times 1.44\, r^2}$$
$= 1.15$

30) (i), (ii), (iii), (v) and (vi) are true.
Castner-kellner process (sodium hydroxide). This is produced commercially by electrolysis of a sodium chloride solution using mercury cathode and carbon anode. Sodium is discharged on mercury and forms an amalgam. This amalgam is treated with water to produce sodium hydroxide.
An inspection of standard electrode potentials convinces us that it is more difficult to reduce Na^+ ion than H^+. ion. On electrolysis of a neutral brine solution we would expect release of H_2, at the cathode instead of deposition of sodium.
But proton transfer from water to a mercury surface needs a high activation energy.
Mercury has a high overvoltage while sodium has a very low overvoltage on mercury. So on electrolysis sodium is discharged on mercury in preference to hydrogen. The resulting amalgam is then reacted with water in the middle compartment of the cell to form sodium hydroxide (and H_2). It also follows from the above standard potentials that OH^- ion is a stronger reducing agent than Cl^- ion so that it will be easier to oxidize OH^- ion to O_2, at the anode than Cl^- to Cl_2. Once again, our prediction goes wrong. At the anode, chlorine is evolved in preference to oxygen. oxygen has a high overvoltage. The reader should note that if we use an open cell then the anode product and the cathode product would have inter-acted, in the cold to give NaOCl and in the hot $NaClO_3$

31)


M.F. $C_8H_4NO_2ClBrF_3$
M.W. = 318.5

318.5×2 mole = 637

32) $\underset{(1\text{mole})}{\text{Glucose}} + 5HIO_4 \rightarrow 5HCOOH + 1HCHO$

x = 5

$\underset{1\text{ mole}}{\text{Fructose}} + 5HIO_4 \rightarrow 2HCHO + 3HCOOH + CO_2$

y = 3

33) Ca^{2+}, Fe^{2+}, Mn^{2+}, Ag^{+}, Zn^{2+} these ions will form white precipitate.

34) (1) Micellization is entropy driven.

(2) A catalyst has no effect on ΔG, K_{eq} and ΔH of a process.

(3) On increasing non-polar chain length of $R-COO^{-}Na^{+}$ CMC decreases.

(4) Heamoglobin is a positive sol.

(5) Charcoal is negative sol.

35) $\Delta(\Delta_r S°) = \Delta_r C_P \, . \ln \frac{T_2}{T_2}$

or $\Delta_r S°_{400} - (-40) = [(3 \times 7.0) - (4 \times 3.0 + 2 \times 5.0)] \times \ln \frac{400}{300}$

$\therefore \Delta_r S°_{400} = -40.3$ cal/K-mol.

PART-3 : MATHEMATICS

37) Option A, B, C

Consider

$$I_n = \int \frac{\sin nx}{\sin x} dx$$

$\because \sin(nx) - \sin(n-2)x = 2\cos[(n-1)x] . \sin x$

$\Rightarrow \sin(nx) = 2\cos[(n-1)]. \sin x + \sin[(n-2)x]$

$$\Rightarrow I_n = \int \frac{2\cos[(n-1)x]. \sin x + \sin[(n-2)x]}{\sin x} dx$$

$$I_n = 2\int \cos(n-1)x.dx + \int \frac{\sin[(n-2)x]}{\sin x} dx$$

$$\Rightarrow I_n = \frac{2\sin[(n-1)x]}{(n-1)} + I_{n-2}$$

Therefore;

$$\int_0^{\pi/2} \frac{\sin(nx)}{\sin x} dx = \begin{cases} \frac{\pi}{2}; & \text{if n is odd} \\ \frac{2}{n-1}\sin\left[\frac{(n-1)\pi}{2}\right] + \int_0^{\frac{\pi}{2}} \frac{\sin((n-2)x)}{\sin x} dx & \text{if n is even} \end{cases}$$

Similarly;

$$\int_0^{\pi} \frac{\sin(nx)}{\sin x} dx = \int_0^{\pi} \frac{\sin[(n-2)x]}{\sin x} dx = \begin{cases} 0 \ ; \ \text{if n is even} \\ \pi \ ; \ \text{if n is odd} \end{cases}$$

option (D)

$$\int_0^1 x^4(1-x)^4 dx$$

Put $x = \sin^2\theta$

$$\int_0^{\frac{\pi}{2}} \sin^8\theta.\cos^8\theta(2\sin\theta.\cos\theta)d\theta$$

$$2\int_0^{\pi/2} \sin^9\theta.\cos^9\theta\, d\theta$$

Now using walli's formula:

$$= 2\left\{\frac{(8\times 6\times 4\times 2)^2}{(18\times 16\times 14\times \ldots\ldots \times 2)}\right\}$$

$$= \frac{1}{630}$$

38) Let $(AB - BA) = X$

case (1) : If $n = 2$

Then characteristic equation of matrix - X is;

$X^2 - [Tr(x)]\,x. + (\det(x))\, I_2 = 0$

$\because\ Tr(X) = Tr(AB - BA) = 0$

$\Rightarrow X^2 = -(\det X).I_2$

$\Rightarrow Tr(X^2) = Tr[-\det(X).I_2]$

$= -\det(X).Tr(I_2)$

$Tr\left(X^2\right) = -2.\det(X)$

$$\Rightarrow \frac{Tr(X^2)}{\det(X)} = -2$$

Or

$$\frac{Tr\left[(AB-BA)^2\right]}{\det(AB-BA)} = -2$$

Case (2):

If $n = 3$

Then characteristic equation of matrix - x is-

$X^3 - [Tr(X)].X^2 + (\alpha).X - (\det X).I_3 = 0$

$\Rightarrow X^3 = (Tr(X)).X^2 - (\alpha)X + (\det X)I_3$

$\Rightarrow Tr\left(X^3\right) = Tr(X).Tr\left(X^2\right) - \alpha.Tr(X) + (\det X).Tr(I_3)$

$= 0 - 0 + (\det X).(3)$

$$\Rightarrow \frac{Tr\left(X^3\right)}{\det(X)} = 3$$

Or

$$\frac{Tr\left[(AB-BA)^3\right]}{\det(AB-BA)} = 3$$

39) Solving y + z = 0, z + x = 0, x + y = 0, x + y + z = 1 vertices are (0, 0, 0), (-1, 1, 1), (1, -1, 1), (1, 1, -1)

volume = $\frac{1}{6}\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \frac{2}{3}$

edges : $\frac{x-0}{-1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$, $\quad \frac{x-1}{0} = \frac{y+1}{2} = \frac{z-1}{-2}$

$\vec{b}_1 = (-1, 1, 1)$

$\vec{b}_2 = (0, 2, -2)$

$\vec{a}_1 = (0, 0, 0)$

$\vec{a}_2 = (1, -1, 1)$

S.D = $\left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 0 & 2 & -2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(2) + \hat{k}(-2)$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j} + \hat{k}$

SD = $\left| \frac{-4+2-2}{\sqrt{24}} \right| = \frac{4}{\sqrt{24}} = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$

40)

slope of the line a (y -1) + (x -1) = 0

= $-\frac{1}{a}$

∴ slope of tangent =a

$\frac{dy}{dx} \propto y$

$\frac{dy}{dx} = ky$

Put (x, y) = (1, 1)

⇒ a = k

⇒ $\frac{dy}{dx} = ay$

⇒ y = Ce^{ax} this passes through (1, 1)

∴ equation of curve is

$y = e^{-a}.e^{ax} \Rightarrow y = e^{a(x-1)}$

Area = $\int_0^1 \left(1 + \frac{1}{a} - \frac{x}{a} - e^{a(x-1)}\right) dx = \left(1 + \frac{e^{-a}}{a} - \frac{1}{2a}\right)$

at a = 1 $\quad$ area = $\int_0^1 e^{x-1} dx = \left(e^{x-1}\right)_0^1$

$= 1 - \frac{1}{e}$

41) Image of $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$ with respect to the line $x + y - 4 = 0$ is

$\frac{(x-6)^2}{9} + \frac{(y-3)^2}{4} = 1$

curve C is

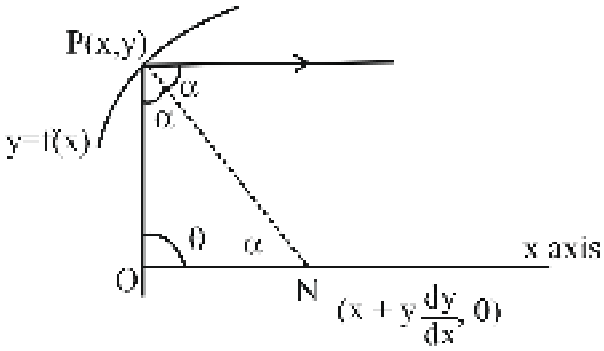
$\frac{(x-7)^2}{9} + \frac{(y-5)^2}{4} = 1$

Now verify the options

42)

PN is normal

Equation of normal at P $Y - y = -\frac{dx}{dy}(X - x)$



OP = ON

$x^2 + y^2 = \left(x + y\frac{dy}{dx}\right)^2$

$x^2 + y^2 = x^2 + y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$

$y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$

$\frac{dy}{dx} = \frac{-2x \pm \sqrt{4x^2 + 4y^2}}{2y}$ (Homogeneous D.E.) $\Rightarrow y^2 = 4(1 + x)$

$y^2 = 36(9 - x)$

43)

Number of functions = $(5^5 - 4^5) + ({}^5C_2.4^3) - {}^5C_4.3.4$

45)

Adding up first two equations.

$(x + z)(y + 1) = 187$

$\Rightarrow y + 1 = \pm 11, \pm 17$

$\Rightarrow y = -12, 10, -18, 16$

Subtracting equation-1 from equation-2 :

$(x - z)(y - 1) = 13$

$\Rightarrow y - 1 = \pm 1, \pm 13$

$y = 0, 2, -12, 14$

$\Rightarrow y = -12$

$\Rightarrow x + z = -17$

$\Rightarrow x + y + z = -29$

$\Rightarrow xy + yz + zx = 100 + 87 + 60 - (x + y + z)$

= 276

46) $\because f(|x|)$ is derivable $\forall\ x \in R$

⇒ f(x) must be a polynomial of even degree

let degree of f(x) is 'n'

clearly n = 2 only the possibility

let $f(x) = ax^2 + bx + c$

$$\Rightarrow a\left(ax^2 + bx + c\right)^2 + b\left(ax^2 + bx + c\right) + c - 8x^m = \frac{80}{2a} - 64x^2 + 100$$

on comparison

$a = 2$; $b = 0$; $c = -8$ and $m = 4$

$\Rightarrow f(x) = 2x^2 - 8$

$\Rightarrow f(3) = 2 \times 9 - 8 = 10$

$$\Rightarrow \left[\frac{f(3)}{m}\right] = \left[\frac{10}{4}\right] = 2$$

47) **Case (1) :** If the first number is 6, then there are no. restrictions. There are 5! or 120 ways to place the other 5 numbers.

Case (2) : If the first no is 5, Then 6 can go in four places, and there are 4! ways to place the other 4 numbers.

Total = 4(4!) = 96 ways

Case (3) : If the first number is 4, ...

46 _ _ _ _ ⇒ 24 ways

4_ 6_ _ _ ⇒ 24 ways

4_ 6 _ _ _ ⇒ 24 ways

4 _ _ _ 6 _⇒ 5 must go b/w 4 and 6, so there are 3.(3!) = 18 ways.

Total = 24 +24 +24 +18 = 90 ways.

Case (4) : If the first number is 3, ...

36 _ _ _ _ ⇒ 24 ways

3_ 6_ _ _ ⇒ 24 ways

31_ 6_ _ ⇒ 4 ways

32 _6 _ _ ⇒ 4 ways

34 _ 6 _ _ ⇒ 6 ways

35 _ 6 _ _ ⇒ 6 ways

35 _ 6 _ _ ⇒ 6 ways

3 _ 5_ 6_ ⇒ 6 ways

3 _ __ 56_ ⇒ 4 ways

Total = 24+24+4+4+6+6+6+6+4= 84 ways.

Case (5) : If the first no. is 2, ...

26 _ _ _ _ ⇒ 24 ways
2 _ 6 _ _ _ ⇒ 18 ways
23 _ 6 _ _ ⇒ 4 ways
24 _ 6 _ _ ⇒ 6 ways
25 _ 6_ _ ⇒ 6 ways
25 _ _6 _ ⇒ 6 ways
2 _ 5_ 6 _ ⇒ 4 ways
24 _ 56 _ ⇒2 ways
234561 ⇒ 1 way

Total = 71 ways
Grand total = 120 + 96 + 90 + 84 + 71 = 461.

48) Clearly $f(x, y, z)$ is cyclic polynomial of degree-4
Also $f(x, x, z) = 0 \Rightarrow (x-y)$ is the factor of 'f'
thus $(y-z)$ and $(z-x)$ also the factor of f (x, y, z)

$\Rightarrow f(x, y, z) = k(x-y)(y-z)(z-x)(x+y+z)$

$\Rightarrow \dfrac{f(1, 4, 9)}{f(1, 2, 3)} = \dfrac{k(1-4)(4-9)(9-1)(1+4+9)}{k(1-2)(2-3)(3-1)(1+2+3)}$

$= \dfrac{(-3)(-5)(8)(14)}{(-1)(-1)(2)(6)}$

= 140

$\Rightarrow f(1, 4, 9) = 140.f(1, 2, 3)$

$= 140\left(\dfrac{1}{10}\right)$

= 14

49) $T_r = \dfrac{r(r+1)}{2}$, $T_r - 1 = \dfrac{r(r+1)}{2} - 1 = \dfrac{r^2+r-2}{2} = \dfrac{(r-1)(r+2)}{2}$

$\dfrac{T_r}{T_r - 1} = \dfrac{r(r+1)}{(r-1)(r+2)}$

$M_{50} = \dfrac{T_2}{T_2-1} \times \dfrac{T_3}{T_3-1} \times \dfrac{T_4}{T_4-1} \times \ldots \dfrac{T_{48}}{T_{48}-1} \times \dfrac{T_{49}}{T_{49}-1} \times \dfrac{T_{50}}{T_{50}-1}$

$= \dfrac{2\times3}{1\times4} \times \dfrac{3\times4}{2\times5} \times \dfrac{4\times5}{3\times6} \times \dfrac{5\times6}{4\times7} \times \ldots \times \dfrac{48\times49}{47\times50} \times \dfrac{49\times50}{48\times51} \times \dfrac{50\times51}{49\times52}$

$= \dfrac{2\times3^2\times4^2\times5^2\times\ldots\times50^2\times51}{1\times2\times3\times4^2\times5^2\times\ldots49^2\times50\times51\times52}$

$= \dfrac{3\times50}{52}$

So 52 M_{50} = 150

50) $\theta = \dfrac{\pi}{9}$

$3\theta = \dfrac{\pi}{3}$

$\tan 3\theta = \sqrt{3}$

$\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \sqrt{3}$

squaring

$$\frac{9\tan^2\theta + \tan^6\theta - 6\tan^4\theta}{1 + 9\tan^4\theta - 6\tan^2\theta} = 3$$

$9\tan^2\theta + \tan^6\theta - 6\tan^4\theta$

$= 3 + 27\tan^4\theta - 18\tan^2\theta$

$\tan^6\theta - 33\tan^4\theta + 27\tan^2\theta = 3$

$\tan^6\theta + 27\tan^2\theta = 3(1 + 11\tan\theta)$

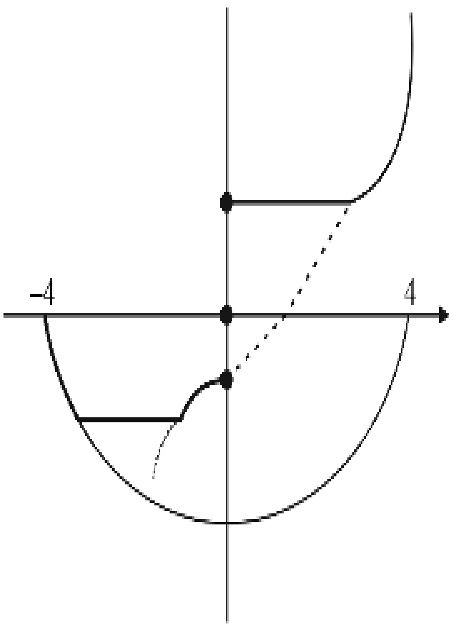
$$\frac{\tan^6\theta + 27\tan^2\theta}{1 + 11\tan^2\theta} = 3 = \sqrt{P} - 1$$

$P = 16$

when divided by 7

remainder = 2

51) $\max\left\{x^3 - \frac{1}{2} - \sqrt{16 - x^2}, \frac{x}{|x|}\right\}$



The fn. is discontinuous at one points $x = 0$

The fn. is continuous but not differentiable is three

m = 1, n = 3

m + n = 4

52) $n(S) = {}^{15}C_5$

$n(E) = {}^{11}C_5 - {}^{9}C_3$

$$P = \frac{\frac{11\times10\times9\times8\times7}{5\times4\times3\times2} - \frac{9\times8\times7}{3\times2}}{\frac{15\times14\times13\times12\times11}{5\times4\times9\times2}}$$

$$= \frac{11\times6\times7 - 12\times7}{7\times13\times3\times11} = \frac{6\times7[11-2]}{7\times13\times3\times11}$$

$$\frac{42\times9}{7\times13\times3\times11} = \frac{18}{143} = \frac{p}{q}$$

Sum of digit $= 18 + 1 = 9$

53) $x = i$

$(1 + i)^n = P_0 + P_1 i - P_2 - P_3 i + P_4$ .... (1)

x= -i

$(1-i)^n = P_0 - P_1 i - P_2 + P_3 i$ .... (2)

$\therefore \dfrac{(1)+(2)}{2}$

$P_0 - P_2 + P_4 .... = \dfrac{(1+i)^n + (1-i)^n}{2} = \dfrac{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^n + \left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^n}{2}$

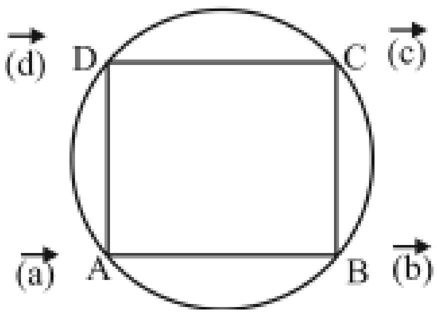
$= 2^{\frac{n}{2}} \dfrac{2\cos\frac{n\pi}{4}}{2} = \alpha$

$\dfrac{(1)-(2)}{2i} \quad P_1 - P_3 + P_5 .... = \dfrac{(1+i)^n - (1-i)^n}{2i} = \dfrac{2^{\frac{n}{2}}.2i\sin\frac{n\pi}{4}}{2i} = \beta$

$\therefore \sum_{n=1}^{\infty} \dfrac{n}{2^n} = \dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \dfrac{4}{2^4} + ...\infty$

$= \dfrac{1}{2}\left[1 + \dfrac{2}{2} + \dfrac{3}{2^1} + \dfrac{4}{2^3} + ...\infty\right]$

$= \dfrac{1}{2}\left[\dfrac{1}{1-\frac{1}{2}} + \dfrac{1.\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}\right] = \dfrac{1}{2}[2+2] = 2$



54)

Now $\left|\left(\vec{b}-\vec{a}\right).\left(\vec{d}-\vec{a}\right)\right| = AB.AD.\cos A$..........(1)

$\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\vec{d}\times\vec{a}\right| = \left|\left(\vec{b}-\vec{a}\right)\times\left(\vec{d}-\vec{a}\right)\right|$

$= AB.AD.\ \sin A$........(2)

$\left(\vec{b}-\vec{c}\right).\left(\vec{d}-\vec{c}\right) = BC.CD.\cos C$

$\left|\left(\vec{b}-\vec{c}\right)\times\left(\vec{d}-\vec{c}\right)\right| = BC.CD\sin C$

$\alpha = \tan A$

$\beta = \tan C = \tan(\pi - A) = -\tan A$

$\left|\dfrac{3\alpha}{2\beta}\right| = \dfrac{3}{2} = 1.50$