FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – VII
PAPER –1
TEST DATE: 20-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

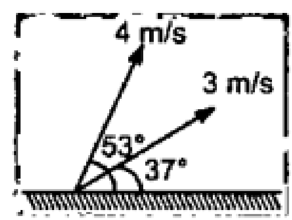
Section – A

1. AD

Sol. So, velocity of first particle

$= 3\cos 37°\,\hat{i} + 3\sin 37°\,\hat{j}$

$= \frac{12}{5}\hat{i} + \frac{9}{5}\hat{j}$



Velocity of second particle

$= 4\cos 53^{o}\,\hat{i} + 4\sin 53^{o}\,\hat{j}$

$= \frac{12}{5}\hat{i} + \frac{16}{5}\hat{j}$

So, relative horizontal velocity is zero. So their relative velocity is in vertical direction only. Since, both particles are moving under gravity, so their relative acceleration is zero.

Their relative velocity $= \frac{16}{5} - \frac{9}{5} = \frac{7}{5} = 1.4$ m/s

2. BC

Sol. Just after BP is cut.

For block A no force has changed.

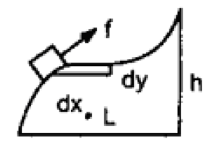
$\therefore$ acceleration of $m_1 = 0$

for m_2 downward force is being reduced

$\therefore$ m_2 will move upwards.

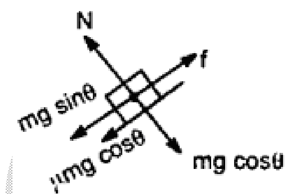
3. AC

Sol. $dw = \vec{f} \cdot \vec{ds}$



Since, body is hauled slowly, so

$f = mg \sin\theta + \mu mg \cos\theta$



$W = \int (mg\sin\theta + \mu mg\cos\theta)ds$

$= \int mgds\sin\theta + \int \mu mgds\cos\theta$

$= \int mgdy + \int \mu mgdx$

$= mgh + \mu mgL$

4. AC

Sol. $5 - F_1 = 1 \times 2 \Rightarrow F_1 = 3$ N

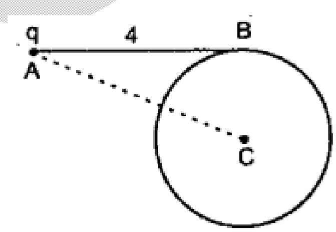
Taking torque about CM:

$5x = 3(\ell + x) \Rightarrow 2x = 3 \times 20 \Rightarrow x = 30$ cm

Length of rod $= 2 \times (\ell + x) = 100$ cm $= 1$ m

5. AC

Sol. AC = 5 m



$$V = \frac{kq}{AC} = \frac{9\times10^9 \times 1\times10^{-6}}{5}$$

$= 1.8 \times 10^3 = 1.8$ kV

$V_B = (V_B)_{\text{due to q}} + (V_B)_i$

$(V_B)_i = -0.45$ kV

So, (A) and (C) are correct.

6. BC

Sol. For a bulb $R = \dfrac{V^2}{W}$

$\Rightarrow R_B \le R_A$

when switch is open $I_A = I_B$

$P_A = R_A I_A^2$

$P_B = R_B I_B^2$

$\Rightarrow P_B < P_A$ and $V_A > V_B$

$\Rightarrow V_A > 12V$ and $V_B < 12V$

After closing the switch

$V_A = V_B = 12$ V

7. A

Sol. (I) 

$$P_i = P_f \Rightarrow 60(1-V) = 100\,V$$

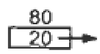
$$\Rightarrow \quad V = \frac{3}{8}$$ opposite to velocity of Ram i.e. $\frac{3}{8}$ m/s towards right.

(II) 80 V = 80 (1 – V)

$$\Rightarrow \; V = \frac{1}{2}\,m/sec \text{ left.}$$

(III) 80(1 + V) + 60(–1 + V) + 20V = 0

$$V = -\frac{1}{8}\,m/s$$

(IV) After jump of Ram 

$$\text{Now } (80+20)\frac{3}{8} = 80(1+V) + 20\,V$$

$$V = -\frac{17}{40}\,m/s$$

8. B

Sol. (I) Velocity of fish in air = $4 \times \frac{3}{4} = 3\uparrow$

Velocity of fish w.r.t. bird = 3 + 6 = 9↑

(II) Velocity of image of fish after reflection from mirror in air = $4 \times \frac{3}{4} = 3\downarrow$

w.r.t. bird = –3 + 6 = 3↑

(III) Velocity of bird in water = $6 \times \frac{4}{3} = 8\downarrow$

w.r.t. fish = 8 + 4 = 12↓

(IV) Velocity of bird in water after reflection from mirror = 8↑
w.r.t. fish = 8 – 4 = 4↑

9. D

Sol. $$I = \int_0^r 2\pi r dr br = \frac{2\pi b r^3}{3}$$

Use ampere law for B.

10. C

Sol. $$\oint \bar{B} \cdot \overline{d\ell} = -A\left(\frac{dB}{dt}\right)$$

Section – B

11. 7.50

Sol. The centre of the wheel is moving with constant speed on a circular path of radius 6R. Hence, it has a centripetal acceleration of $a_c = \frac{V^2}{6R}$ directed towards the centre of curvature of the convex surface.

With respect to the centre of the wheel, the contact point has acceleration equal to $\frac{V^2}{R}$ directed towards the centre of the wheel.

$\therefore$ Acceleration of the contact point in reference frame of ground is

$$a_P = \frac{V^2}{R} - \frac{V^2}{6R} = \frac{5V^2}{6R}.$$

12. 2.56

Sol. For a jump of h_0 = 1 m on the earth, speed required is given by

$$\frac{1}{2}mV^2 = mgh_0 \quad \Rightarrow \quad V = \sqrt{20}\ m/s$$

Escape speed on the surface of a planet is

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{8\pi GR^2\rho}{3}}$$

$$\therefore \quad \frac{V_{esc-planet}}{V_{esc-earth}} = \frac{R_{planet}}{R_{earth}}$$

We want $V_{esc}^{planet} = \sqrt{20}\ m/s$

And it is given that V_{esc}^{planet} = 11.2 km/s

$$R_{planet} = \frac{\sqrt{20} \times (6400\ km)}{11200} = 2.56\ km$$

13. 400.00

Sol. If force exerted on piston of area A_2 is F_2 then, the force acting on the other piston will be

$$= \frac{F_2}{A_2} \cdot A_1 \qquad \text{[Pascal's law]}$$

$$= 5F_2 \qquad \because \left[\frac{A_1}{A_2} = 5\right]$$

To raise the load $5F_2$ = 20000 $\Rightarrow$ F_2 = 4000 N

Since lever bar is light, net torque on it (about the hinge) must be zero.

$\therefore$ $F_2a = F(a + b)$

$$\therefore \quad F = \frac{F_2 a}{a+b} = \frac{4000 \times 4}{4+36} = 400\ N$$

14. 283.40

Sol. The product nucleus ^{198}Hg is in excited state and possesses extra 1.088 MeV energy. If ^{198}Hg would had been in ground state, the kinetic energy available to electron and antineutrino must have

$$Q = (m_{Au} - m_{Hg})\ 931\ MeV$$

= (197.968233 – 197.966760) 931 MeV
= 1.3714 MeV

Since ^{198}Hg is in excited state, actual kinetic energy available to electron and antineutrino is

K = (1.3714 – 1.088) MeV
= 0.2834 MeV

As β-ray and antineutrino has continuous spectrum starting from zero value, therefore, this is also the maximum kinetic energy of the electron emitted.

15. 7.00

Sol. $\Delta x = (\mu_1 - 1)t_1 - (\mu_2 - 1)t_2$
$= (2 - 1)200 - (1.5 - 1)200$
$= 200 - 100 = 100$ nm

$$\therefore\ \phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{600}\times 100 = \frac{\pi}{3}$$

$$\therefore\ P = P_1 + P_2 + 2\sqrt{P_1P_2}\cos\theta$$

$$= I_0\pi r_1^2 + I_0\pi r_2^2 + 2\sqrt{I_0\pi r_1^2 \times I_0\pi r_2^2}\cos\frac{\pi}{3}$$

$= 7\ \mu W$

16. 4.01

Sol. As here volume of gas remains constant,
$(\Delta Q)_v = \mu C_V \Delta T$, Here $C_v = 5$ cal/mol K
And $\Delta T = (400 - 300) = 100$ K
And so for ideal gas $PV = \mu RT$,

$$\mu = \frac{(10)^5 \times (0.2)}{8.31 \times 300} = 8.0224$$

$(\Delta Q)_v = \mu \times 5 \times 100 = 4.01$ kcal.

17. 3.14

Sol. $\frac{F}{A} = Y\frac{\Delta L}{L} \Rightarrow \frac{mg}{A} = Y(\alpha\Delta\theta)$

$$m = \frac{AY\alpha(\Delta\theta)}{g} = \frac{\pi r^2 Y\alpha(\Delta\theta)}{g}$$

$$= \frac{\pi(10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi$$

18. 30.00

Sol. $f \propto \sqrt{T}$ for strings.
On increasing the tension by 1%
$f' = \sqrt{1.01\,T}$

$$\frac{f'}{f} = \frac{\sqrt{1.01\,T}}{\sqrt{T}} = (1 + 0.01)^{\frac{1}{2}} = 1 + \frac{1}{200}$$

Beat frequency, $f' - f = f\left(\frac{f'}{f} - 1\right) = 1$

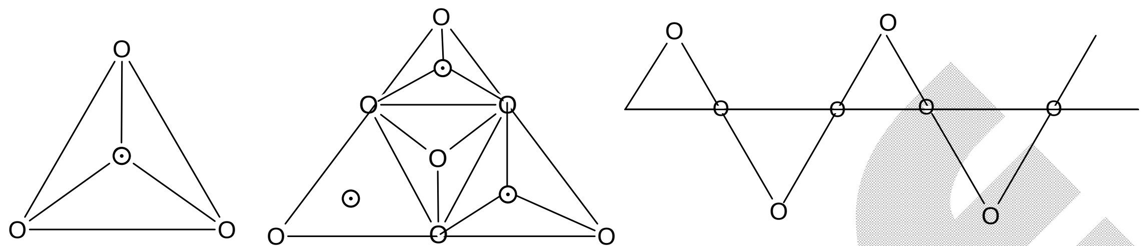
Number of beats in 30 seconds = 1 × 30 = 30.

Chemistry

PART – II

Section – A

19. A

Sol.


20. BC

Sol. $\underset{(s)}{2C} + \underset{(g)}{3H_2} \rightarrow \underset{(g)}{C_2H_6}$

$\Delta H_f^0 = \left[2 \times \Delta H^0_{sub} + 3 \times B.E(H-H)\right] - \left[B.E.(C-C) + 6 \times B.E(C-H)\right]$

$\Rightarrow -85 = \left[(2 \times 718) + (3 \times 436)\right] - (x + 6y)$

$\therefore x + 6y = 2829$ 1

Similarly for $C_3H_8(g)$

$2x + 8y = 4002$ 2

Solving (1) & (2), x = 345

y = 414

21. AB

Sol. $k = Ae^{-E_a/RT}$ or $\ell nk = \ell nA \dfrac{-E_a}{RT}$

22. ABC

Sol. The outermost electronic configuration of Yb is $4f^{14}6s^2$

$\therefore$ Yb^{2+} has full-filled $4f^{14}$ configuration radius of $Yb^{3+} < Yb^{2+}$

23. AD

Sol. $Be_2C \xrightarrow{H_2O} Be(OH)_2 + CH_4$

$Al_4C_3 \xrightarrow{H_2O} Al(OH)_3 + CH_4$

24. ABCD

Sol. $\Delta S = \dfrac{\Delta H}{T} = \dfrac{-40,600}{373} = -108.84\ j/K$ for one mole

for $\dfrac{9}{18}$mole, $\Delta S = \dfrac{-108.84}{2} = 54.42\ j/K$

25. A

Sol. I – can undergo Nu^- substitution, elimination

II – can undergo Nu^- substitution, esterification, dehydrogenation & oxidation.

III – can undergo Nu^- addition, esterification and oxidation

IV – can undergo Nu^- substitution

26. A

Sol. Fact based

27. C

Sol. Elements having filled d-orbitals are not true transition elements.

28. B

Sol.
$$SO_3^{2-} + dil.H_2SO_4 \longrightarrow H_2O + SO_2 + SO_4^{2-}$$
$$CO_3^{2-} + dil.H_2SO_4 \longrightarrow H_2O + CO_2 + SO_4^{2-}$$
$$\overset{+}{N}H_4 + NaOH \longrightarrow Na^+ + H_2O + NH_3$$
$$S^{2-} + H_2SO_4 \longrightarrow H_2S + SO_4^{2-}$$

Section – B

29. 162.00

Sol.
$$K_f\left(\text{of camphor}\right) = \frac{R\left(T_f^o\right)^2 \times M_2}{1000 \times \Delta H_f^o} = \frac{2\times 152\times\left(450\right)^2}{1000\times 1.52\times 1000} = 40.5$$
$$\Delta T_f = i \times K_f \times m$$
$$450 - 430 = 1\times 40.5 \times \frac{0.04\times 1000}{M_{solute}\times 0.5}$$
$$M_{solute} = \frac{0.04\times 1000\times 40.5}{0.5\times 20} = 162$$

30. 81.81

Sol.

	NaOH	+ HCl	$\longrightarrow$ NaCl	+ H_2O
Millimoles	0.1	.001 x	0	0
	-0.001 x	-0.001x		
t = t	0.1 - .001 /x	x		

10^{-3} M NaOH have pH = 11

New pH after adding acid be 10

Hence, 10^{-4} M (100 + x) = 0.1 - .001 x

$$100 + x = \frac{0.1 - 0.001x}{10^{-4}}$$

$\therefore$ 11x = 1000 – 100 = 900

$$\therefore\ x = \frac{900}{11} = 81.81\ \text{mL}$$

31. 390.55

Sol. $\Lambda_m^0 CH_3COOH = \dfrac{\left(\Lambda_m^0 Ca\left(CH_3COO\right)_2 + 2\Lambda_m^0 HCl - \Lambda_m^0 CaCl_2\right)}{2}$

$= \dfrac{200.8 + 2\left(425.95\right) - 271.6}{2} = 390.55$

32. 1.50

Sol. Vander waals equation for 1 mole of real gas, when b = 0

$\Rightarrow \left(P + \dfrac{a}{V^2}\right)(V) = RT$

$\therefore\ PV = -a \times \dfrac{1}{V} + RT$

$y = mx + c$

Slope = tan(π–θ) = –*a*

So, tan θ = a = $\dfrac{21.6 - 20.1}{3 - 2} = 1.5$

33. 11.00

Sol. The product contains three phenyl group and two multiple bonds

$\therefore$ Total number of pi-bonds = (3 $\times$ 3) + 2 = 11

34. 2920.40

Sol. Meq of MnO_4^- = Meq of Fe^{2+}

$V \times M \times n = \dfrac{w}{E} \times 1000$

or, $104.3 \times 1000 \times 0.1 \times 5 = \dfrac{w}{56/1} \times 1000$

On solving w = 2920.4 g

35. 8.33

Sol. The average O.S of iron in $Fe_{0.96}O$ is $\dfrac{200}{96}$

Let the % of Fe(II) be x

$x \times (+3) + (100 - x) \times (+2) = \dfrac{100 \times 200}{96}$

3x + 200 – 2x = 208.33

$\therefore$ x = 8.33

36. 1.41

Sol. $d = \dfrac{Z \times M}{N_a \times a^3}$

$\dfrac{45}{16} = \dfrac{z \times 27}{6.02 \times 10^{23} \times \left(4 \times 10^{-8}\right)^3}$

$\Rightarrow$ Z = 4 (so fcc)

$r = \dfrac{\sqrt{2}\,a}{4} = 1.41\ \overset{o}{A}$

Mathematics PART – III

Section – A

37. AC

Sol. $V = \left|\vec{a}.\left(\vec{b}\times\vec{c}\right)\right| \le \sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}\sqrt{c_1^2+c_2^2+c_3^2}$ ....(i)

$\left[\left(a_1+a_2+a_3\right)\left(b_1+b_2+b_3\right)\left(c_1+c_2+c_3\right)\right]^{1/3}$ (Using A.M. $\ge$ G.M.)

$L^3 \ge \left(a_1+a_2+a_3\right)\left(b_1+b_2+b_3\right)\left(c_1+c_2+c_3\right)$

now $\left(a_1+a_2+a_3\right)^2 = a_1^2+a_2^2 = a_3^2+2\left(a_1a_2+a_2a_3+a_3a_1\right)$

$\left(a_1+a_2+a_3\right)^2 \ge a_1^2+a_2^2+a_3^2$

$\left(a_1+a_2+a_3\right) \ge \sqrt{a_1^2+a_2^2+a_3^2}$

similarly $\left(b_1+b_2+b_3\right) \ge \sqrt{b_1^2+b_2^2+b_3^2}$

$c_1+c_2+c_3 \ge \sqrt{c_1^2+c_2^2+c_3^2}$

$L^3 \ge \left[\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)\left(c_1^2+c_2^2+c_3^2\right)\right]^{1/2}$

$L^3 \ge V$

38. BC

Sol. We supposed to find m and n such that $\lim\limits_{x\to\infty} 3\sqrt[3]{8x^3+mx^2}-nx = 1$ or

$\lim\limits_{x\to-\infty} \sqrt[3]{8x^3+mx}-nx = 1.$

We compute

$$\sqrt[3]{8x^3+mx^2}-nx = \frac{\left(8-n^3\right)x^3+mx^2}{\sqrt[3]{\left(8x^3+mx^2\right)^2}+nx\sqrt[3]{8x^3+mx^2}+n^2x^2}.$$

$8-n^3$ must be equal to 0

$n=2$

Now $f(x) = \dfrac{m}{\sqrt[3]{\left(8+\dfrac{m}{x}\right)^2}+2\sqrt[3]{8+\dfrac{m}{x}}+4}.$

We see that $\lim\limits_{x\to\infty} f(x) = \dfrac{m}{12}$. For this to be equal to 1, m must be equal to 12. Hence the answer to the problem is (m, n) = (12, 2).

39. ABCD

Sol. $\int \frac{x^4+1}{x^6+1}dx = \int \frac{x^4-x^2+1}{x^6+1}dx \int \frac{x^2}{x^6+1}dx = \int \frac{1}{x^2+1}dx + \int \frac{1}{3}\frac{(x^3)'}{(x^3)^2+1}dx$

$= \arctan x + \frac{1}{3}\arctan x^3.$

To write the answer in the required form we should have

$3\arctan x + \arctan x^3 = \arctan \frac{P(x)}{Q(x)}$

Applying the tangent function to both sides, we deduce

$$\frac{\frac{3x-x^3}{1-3x^2}+x^3}{1-\frac{3x-x^3}{1-3x^2}.x^3} = \tan\left(\arctan\frac{P(x)}{Q(x)}\right).$$

From here $\arctan\frac{P(x)}{Q(x)} = \arctan\frac{3x-3x^5}{1-3x^2-3x^4+x^6}$, and hence

$P(x) = 3x-3x^5, Q(x) = 1-3x^2-3x^4+x^6$. The final answer is

$\frac{1}{3}\arctan\frac{3x-3x^5}{1-3x^2-3x^4+x^6}+C.$

40. AC

Sol. Denote the value of the integral by I. With the substitution $t = \frac{ab}{x}$ we have

$$I = \int_a^b \frac{e^{\frac{b}{t}}-e^{\frac{t}{a}}}{\frac{ab}{t}}.\frac{-ab}{t^2}dt = -\int_a^b \frac{e^{\frac{t}{a}}-e^{\frac{b}{t}}}{t}dt = -I.$$

Hence, I = 0.

41. AB

Sol. $3\tan 3x = \frac{3(3\tan x - \tan^3 x)}{1-3\tan^2 x} = \frac{3\tan^3 x - 9\tan x}{3\tan^2 x - 1}$

$= \frac{8\tan x}{3\tan^2 x - 1}.$

Hence

$\frac{1}{\cot x - 3\tan x} = \frac{\tan x}{1-3\tan^2 x} = \frac{1}{8}(3\tan 3x - \tan x)$ for all $x \neq k\frac{\pi}{2}, k \in \mathbb{Z}$.

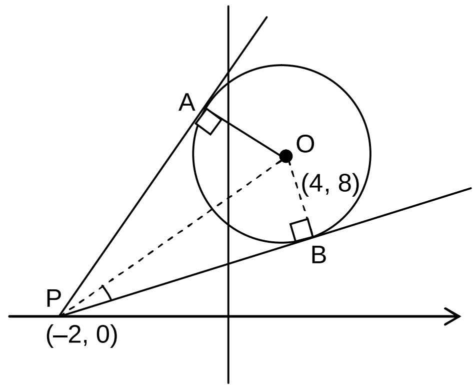
It follows that the left – hand side telescopes as

$\frac{1}{8}(3\tan 27^\circ - \tan 9^\circ + 9\tan 81^\circ - 3\tan 27^\circ + 27\tan 243^\circ - 9\tan 81^\circ + 81\tan 729^\circ - 27\tan 243^\circ)$

$$=\frac{1}{8}\left(81\tan 9^{o}-\tan 9^{o}\right)=10\tan 9^{o}.$$

42. BC

Sol.



$$\sin\theta=\frac{2\sqrt{5}}{10}=\frac{1}{\sqrt{5}}$$

Slopes of PA and PB are $\tan(\alpha\pm\theta)$ where $\tan\alpha=\frac{8}{6}=\frac{4}{3}$

$$=\frac{\frac{4}{3}+\frac{1}{2}}{1-\frac{4}{3}.\frac{1}{2}},\frac{\frac{4}{3}-\frac{1}{2}}{1+\frac{4}{3}.\frac{1}{2}}$$

$$=\frac{11}{2},\frac{5}{10}$$

$$\therefore\ A,B\equiv\left(4+2\sqrt{5}\left(\frac{-11}{5\sqrt{5}}\right),8+2\sqrt{5}\left(\frac{2}{5\sqrt{5}}\right)\right),$$

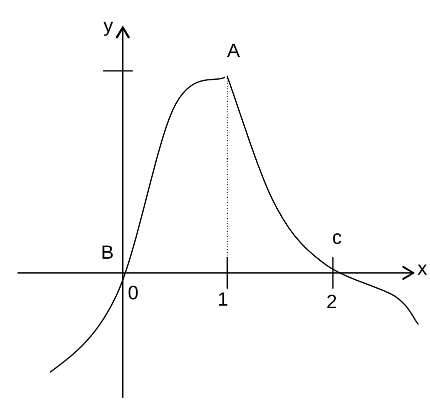
$$\left(4-2\sqrt{5}\left(\frac{-1}{\sqrt{5}}\right),8-2\sqrt{5}\left(\frac{2}{\sqrt{5}}\right)\right)=\left(\frac{-2}{5},\frac{44}{5}\right)$$

$$=(6,4)$$

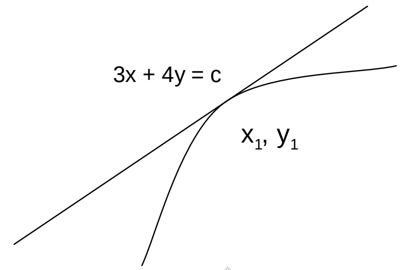
43. B

Sol. (I) A, B, C are the 3 critical points of $y=f(x)$.

$f''(x)=0$ for $x=2$ and fails to exists at x = 0.



(II) $x = \frac{1}{4}$ and 2. Make a quadratic in $\log_2 x$ and interpret the result.



(III) $\frac{dy}{dx} = -1 + 2x_1^3 = -\frac{3}{4} \Rightarrow x_1 = \frac{1}{2}$

$\Rightarrow \frac{1}{32} = \frac{1}{2} + y_1$ or $y_1 = -\frac{15}{32} \Rightarrow c = -\frac{3}{8}$

(IV) $f'(x) = 2x^3 - 3x + 1$ this is always positive in (1, 2).

$\therefore$ Increasing in [1, 2]

$\therefore$ $f(2)$ will be the greatest value.

44. D

Sol. Let $L = \lim_{n\to\infty} \left(\frac{n^2}{n-1}\right)^{\tan\frac{1}{\sqrt{n}}} \left(\infty^o\right)$

$$\ln L = \lim_{n\to\infty} \tan\frac{1}{\sqrt{n}} \ln\left(\frac{n^2}{n-1}\right) = \lim_{n\to\infty} \frac{2\ln - \ln(n-1)}{\cot\left(\frac{1}{\sqrt{n}}\right)} \left(\frac{\infty}{\infty}\right)$$

$$\text{Put } n = t^2 = \lim_{t\to\infty} \frac{4\ln t - \ln\left(t^2 - 1\right)}{\cot\left(\frac{1}{t}\right)} = \lim_{t\to\infty} \frac{2\left(t^2 - 2\right)}{t\left(t^2 - 1\right)\operatorname{cosec}^2\left(\frac{1}{t}\right)\left(\frac{1}{t^2}\right)}$$

$$= \lim_{t\to\infty} \frac{2\left(t^2 - 2\right)}{t\left(t^2 - 1\right)} \left(\frac{\sin\frac{1}{t}}{\frac{1}{t}}\right)^2 = 0$$

$L = 1 \Rightarrow (Q)$

(III) $\sin 2x = \pm\frac{\sqrt{3}}{2}$

$2x \in (0, 4\pi)$

Hence, 8 solutions.

(IV) Since $g(x)$ is differentiable $\forall x \in R$

$\therefore$ $f(x)$ must have the factor $x(x-1)(x-2)$ atleast once.

$\therefore$ minimum 3 roots of $f(x) = 0 \Rightarrow (R)$

45. A

Sol. x b b b x c x c x c x

Number of ways $= \frac{4!}{3!} \times {}^{5}C_4 = 20$

(II) 2b, 1b; 2c, 1c or 2b,1b; 1c,1c,1c or 2b, 1b; 3c

(same way starting with c)

cbbcbc, cbcbbc

bbcccb, bcccbb

Number of ways $= 12 \times {}^{6}C_4 = 180$

(III) bcbcbc $= {}^{7}C_4$

bccbcb or bcbccb $= 2 \times {}^{6}C_3$

bbccbc or bccbbc or bbcbcc or bcbbcc $= 4 \times {}^{5}C_2$

bcccbb or bbcccb $= 2 \times {}^{4}C_1$

bbbccc $= {}^{4}C_4$

Total ways $= 2(35 + 40 + 40 + 8 + 1) = 248$

(IV) bcbcbc, cbcbcb

bccbcb, cbbcbc

bcbccb, cbcbbc

bcbccb, cbcbbc

number of ways $= {}^{7}C_1 \times 2 + 4 = 18$

46. D

Sol. (I) $P = \frac{1 \times {}^{11}C_5}{{}^{12}C_6} = \frac{1}{2}$

(II) $P = \frac{{}^{2}C_1 \times {}^{10}C_5}{{}^{12}C_6} = \frac{6}{11}$

(III) $P = \frac{{}^{10}C_4}{{}^{12}C_5} = \frac{5}{22}$

(IV) $P = \frac{10}{11}$

Section – B

47. 252.00

Sol. $x = \left(\sqrt{3}+1\right)^{2018}$ and $y = \left(\sqrt{3}-1\right)^{2018}$

$[x]+\{x\}+y = \left(\sqrt{3}+1\right)^{2018} + \left(\sqrt{3}+1\right)^{2018}$

$[x]+1 = 2^{1009}\left[\left(2+\sqrt{3}\right)^{1009} + \left(2-\sqrt{3}\right)^{1009}\right]$

$= 2^{1009}\times 2\left[{}^{1009}C_0 2^{1009} + {}^{1009}C_2 2^{1007} 3^1 + ... + {}^{1009}C_{1006} 2^3 3^{503} + {}^{1009}C_{1008} 2\times 3^{504}\right]$

$[x]+1 = 2^{1011}\underbrace{\left[{}^{1009}C_0 2^{1008} + ... + {}^{1009}C_{1006} 2^2 3^{503} + {}^{1009}C_{1008} 3^{504}\right]}_{\text{odd}}$

$[\because \{x\} + y \in (0, 2) \Rightarrow \{x\} + y = 1]$

$\Rightarrow$ N is divisible by 2^{1011}, hence divisible by $(16)^{252}$

48. 13.00

Sol. Coefficient of $x^{\frac{n(n+1)}{2}-7} = -7 + (1\cdot 6 + 2\cdot 5 + 3\cdot 4) - (1\cdot 2\cdot 4) = 13$

49. 18.00

Sol. We have $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2(\beta\gamma+\gamma\alpha+\alpha\beta)$

$\Rightarrow \quad 4 = 6 + 2(\beta\gamma+\gamma\alpha+\alpha\beta)$

$\Rightarrow \quad \beta\gamma+\gamma\alpha+\alpha\beta = -1.$

Also, $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$

$=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\beta\gamma-\gamma\alpha-\alpha\beta)$

50. –1.59

Sol. The critical point of f are solutions to the system of equations

$$\frac{\partial f}{\partial x}(x,y) = 4x^3 + 12xy^2 - \frac{9}{4} = 0,$$

$$\frac{\partial f}{\partial y}(x,y) = 12x^2y + 4y^3 - \frac{7}{4} = 0$$

divide the two equations by 4 and then add, respectively, subtract them, we obtain $x^3+3x^2y+3xy^2+y^3-1=0$ and $x^3-3x^2+3xy^3-y^3=\frac{1}{8}$. We write these as $(x+y)^3=1$ and $(x-y)^3=\frac{1}{8}$, from which we obtain $x+y=1$ and $x-y=\frac{1}{2}$. We find a unique critical point $x=\frac{3}{4}, y=\frac{1}{4}$. The minimum of f is attained at this point, and it is equal to $f\left(\frac{3}{4},\frac{1}{4}\right) = -\frac{51}{32}$.

51. 126.00

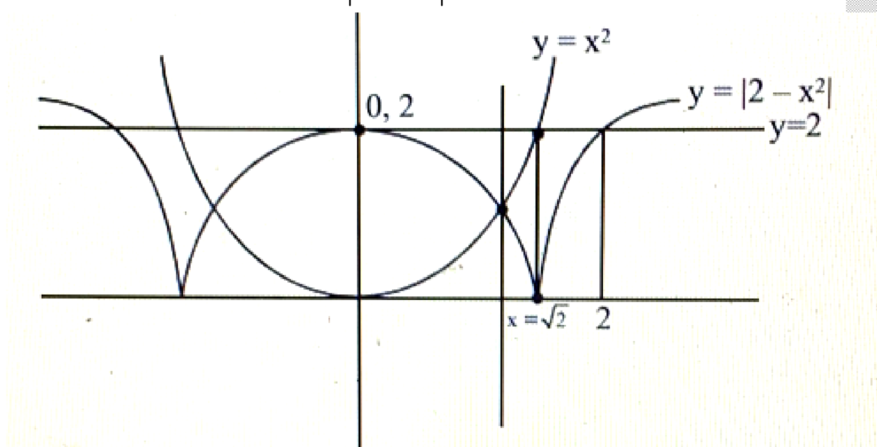
Sol. We have, $\lim\limits_{n\to\infty} gn(3) = \lim\limits_{n\to\infty} n\left(f\left(3+\frac{5}{n}\right) - f\left(3-\frac{2}{n}\right)\right)$

$= 5\lim\limits_{h\to 0^+}\frac{f(3+h)-f(3)}{h} + 2\lim\limits_{h\to 0^+}\frac{f(3)-f(3-h)}{h}$

$= 5f'(3) + 2f'(3) = 5\times 18 + 2\times 18 = 90 + 36 = 126$.

52. 1.01

Sol. $y = x^2$ $\qquad$ $y = |2 - x^2|$ $\qquad$ $y = 2$



Required area $\int\limits_1^{\sqrt{2}}\left[x^2 - (2-x^2)\right] + \int\limits_{\sqrt{2}}^{2}\left[2 - (x^2-2)\right]dx$

$= \frac{20}{3} - 4\sqrt{2} = 1.01$

53. 18.00

Sol. The points (x_1, x_2) and (y_1, y_2) lie on the circle of radius $2\sqrt{2} = c$ centered at the origin. We can write $(x_1, x_2) = (c\cos\phi, c\sin\phi)$ and $(y_1, y_2) = (a\cos\theta, c\sin\theta)$. Then

$s = 2 - c(\cos\phi + \sin\phi + \cos\theta + \sin\theta) + c^2(\cos\phi\cos\psi + \sin\phi\sin\psi)$

$= 2 + c\sqrt{2}\left(-\sin\left(\phi + \frac{\pi}{4}\right) - \sin\left(0 + \frac{\pi}{4}\right)\right) + c^2\cos(\phi - \theta)$ to 1 by choosing

$\phi = \theta = \frac{5\pi}{4}$. The maximum of S is $2 + 2c\sqrt{2} + c^2 = \left(c + \sqrt{2}\right)^2$.

54. 11.00

Sol. $\left[x-\frac{1}{2}\right]=1$ or $\left[x+\frac{1}{2=1}\right]$

$x-\frac{1}{2}\in[1,2)$ $x+\frac{1}{2}\in[1,2)$

$x\in\left[\frac{3}{2},\frac{5}{2}\right)$ ....(1) $x\in\left[\frac{1}{2},\frac{3}{2}\right)$ ....(2)

$(1)\cup(2)$

$x\in\left[\frac{1}{2},\frac{3}{2}\right)\cup\left[\frac{3}{2},\frac{5}{2}\right)$

$x_1^2+x_2^2+x_3^2+x_4^2=\frac{1}{4}+\frac{9}{4}+\frac{9}{4}+\frac{25}{4}=11$