FIITJEE RBT-3 for (JEE-Advanced)

PHYSICS, CHEMISTRY & MATHEMATICS

Pattern – 2 | **QP Code: 100956** | **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 195**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. All the section can be filled in **PART-A** of OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For Only One Part.

(i) **Part-A (01-07) –** Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases**.
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(ii) **Part-A (08-14) –** Contains seven (07) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(iii) **Part-A (15-18) -** This section contains Two paragraphs. Based on each paragraph, there are Two multiple choice questions. Each question has only one correct answer and carries **+4 marks** for the correct answer and **– 2 marks** for wrong answer.

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

BATCHES – Two Year CRP325 batches

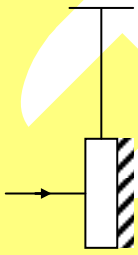
SECTION-1 : PHYSICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. A small mirror is suspended by a thread as shown in figure. A short pulse of monochromatic light rays is incident normally on the mirror and gets reflected. Which of the following statements are correct?



(A) mirror will starts oscillating

(B) wavelength of reflected rays will be greater than that of incident rays

(C) wavelength of reflected rays may be less than that of incident rays

(D) mirror will be at rest after some time

2. A particle of mass m is moving in a field where the potential energy is given by $U(x) = U_0(1-\cos ax)$, where U_0 and a are constants and x is the displacement from mean position . Then (for small oscillations)

(A) the time period is $T = 2\pi\sqrt{\frac{m}{aU_0}}$ (B) the speed of particle is maximum at $x = 0$

(C) the amplitude of oscillations is $\frac{\pi}{a}$ (D) the time period is $T = 2\pi\sqrt{\frac{m}{a^2U_0}}$

3. Two different coils have self inductances L_1 = 8 mH and L_2 = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At this time the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and U_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and U_2 respectively. Then

(A) $\frac{i_1}{i_2} = \frac{1}{4}$ (B) $\frac{i_1}{i_2} = 4$ (C) $\frac{U_2}{U_1} = 4$ (D) $\frac{V_2}{V_1} = \frac{1}{4}$

Space For Rough Work

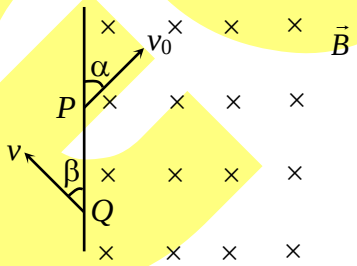
4. The magnetic field perpendicular to the plane of conducting ring of radius r changes at the rate $\frac{dB}{dt} = \alpha$. Then

(A) Emf induced in the ring is $\pi r^2 \alpha$
(B) Emf induced in the ring is $2\pi r\alpha$
(C) the potential difference between diametrically opposite points on the ring is half of induced emf
(D) all points on the ring are at same potential

5. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$ then

(A) the work function of A is 2.25 eV (B) the work function of B is 4.20 eV
(C) T_A = 2.00 eV (D) T_B = 2.75 eV

6. A particle of charge $-q$ and mass m enters a uniform magnetic field $\vec{B}$ (perpendicular to paper inwards) at P with a velocity v_0 at an angle α and leaves the field at Q with velocity v at angle β as shown in the figure. Then



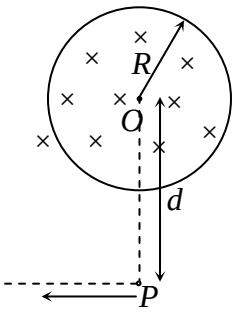
(A) $\alpha = \beta$

(B) $v = v_0$

(C) $PQ = \frac{2mv_0 \sin\alpha}{Bq}$

(D) particle remains in the field for time $t = \frac{2m(\pi - \alpha)}{Bq}$

7. In a cylindrical region of radius R, there exists a time varying magnetic field B such that $\frac{dB}{dt} = k(>0)$. A charged particle having charge q is placed at the point P at a distance d (> R) from its centre O. Now, the particle is moved in the direction perpendicular to OP (see figure) by an external agent upto infinity so that there is no gain in kinetic energy of the charged particle. Choose the correct statement/s.



(A) Work done by external agent is $\frac{q\pi R^2}{4}k$ if d = 2R

(B) Work done by external agent is $\frac{q\pi R^2}{8}k$ if d = 4R

(C) Work done by external agent is $\frac{q\pi R^2}{4}k$ if d = 4R

(D) Work done by external agent is $\frac{q\pi R^2}{4}k$ if d = 6R

Space For Rough Work

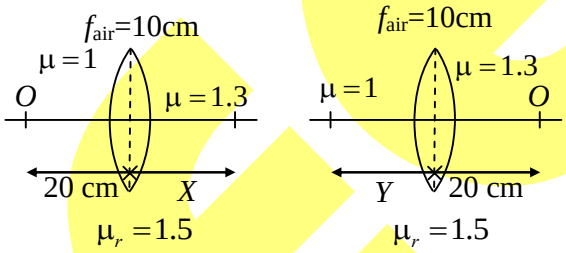
(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. A diatomic molecule having atoms of masses m_1 and m_2 has its potential energy function about the equilibrium position r_0 as given by $U(r) = -A + B(r - r_0)^2$ where A and B are constants. When the atom vibrate at high temperature condition, the square of angular frequency of vibration will be

(A) $\frac{2B}{m_1}$ (B) $\frac{2B}{m_2}$ (C) $\frac{2B(m_1+m_2)}{m_1m_2}$ (D) $\frac{B(m_1+m_2)}{2m_1m_2}$

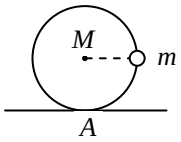
9. An equiconvex lens made up of a material of refractive index 1.5 has focal length of 10 cm when placed in air as shown in the figure. One side of the medium is replaced by another medium of refractive index 1.3. If X and Y are the image distances when the object is placed at a distance of 20cm from optical centre in the medium with refractive index 1 and 1.3 respectively, then



(A) $X > 1.3Y$ (B) $X < 1.3Y$

(C) $X = 1.3Y$ (D) cannot be determined

10. A uniform body of mass M of radius R has a small mass m attached at edge as shown in the figure. The system is placed on a perfectly rough horizontal surface such that mass m is at the same horizontal level as the centre of body. It is assumed that there is no slipping at point A. If I_A is the moment of the inertia of combined system about point of contact A then the normal reaction at point A just after the system is released from rest is (M = 6 kg, m = 2 kg, I_A = 4 kg m^2, R = 1 m, g = 10 m/s^2)



(A) 60 N (B) 80 N (C) 75 N (D) 70 N

11. A circuit consists of a capacitor and a resistor having resistance R = 220 Ω connected in series. When an alternating e.m.f. of peak voltage V_0 = 220 $\sqrt{2}$ V is applied to the circuit, the peak current in steady state is observed to be I_0 = 1A. The phase difference between the current and the voltage is

(A) 30° (B) 45° (C) 60° (D) 90°

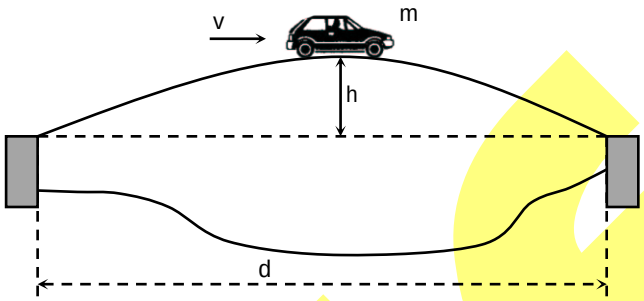
Space For Rough Work

12. A stone of mass m tied to one end of thread of length ℓ. The diameter of thread is 'd' and it is suspended vertically. The stone is now rotated in horizontal plane and makes an angle 'θ' with vertical. The increase in length of wire is (young's modulus of wire is y)

(A) $\frac{4mg\ell}{\pi d^2 y\cos\theta}$ (B) $\frac{4mg\ell}{\pi d^2 y\sin\theta}$

(C) $\frac{4mg\ell}{\pi d^2 y}$ (D) $\frac{4mg\ell}{\pi d^2 y\sec\theta}$

13. There is a parabolic-shaped bridge across a river of width 100 m. The highest point of the bridge is 5 m above the level of the banks. A car of mass 1000 kg is crossing the bridge at a constant speed of 20 ms^{-1}.

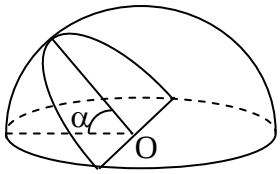


Using the notation indicated in the figure, find the force exerted on the bridge by the car when it is at the highest point of the bridge (Ignore air resistance and take g as 10 ms^{-2}.)

(A) 6.4 KN (B) 7.2 KN

(C) 8.4 KN (D) 9.2 KN

14. The strength of the electric field produced by charges uniformly distributed over the surface of a hemisphere at its centre O is E_0. A part of the surface is isolated from this hemisphere by two planes passing through the same diameter and forming an angle α with each other. Determine the electric field strength E produced at the same point O by the charges located on the isolated surface (on the "mericarp").



(A) $\frac{E_0}{2}\sin\frac{\alpha}{2}$ (B) $\frac{2E_0}{3}\sin\frac{\alpha}{2}$ (C) $E_0\sin\frac{\alpha}{2}$ (D) $2E_0\sin\frac{\alpha}{2}$

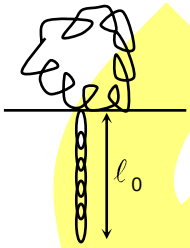
Space For Rough Work

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

A rope of mass m, length 'ℓ' is lying on a table with ℓ_0 length lying outside a hole on the table on which it is kept. It is kept at t = 0. Answer the following questions:



15. Speed of the free end of the chain when it is at a distance of 'y' from the table is?

(A) $v(y)=\sqrt{\frac{2g}{3}\left(y-\frac{\ell_0^3}{y^2}\right)}$ (B) $v(y)=\sqrt{\frac{g}{3}\left(y-\frac{2\ell_0^3}{y^2}\right)}$

(C) $v(y)=\sqrt{2g\left(y-\frac{\ell_0^3}{y^2}\right)}$ (D) $v(y)=\sqrt{g\left(y-\frac{2\ell_0^3}{y^2}\right)}$

16. If $\ell_0=0$, the time in which the entire rope will slip off the table is?

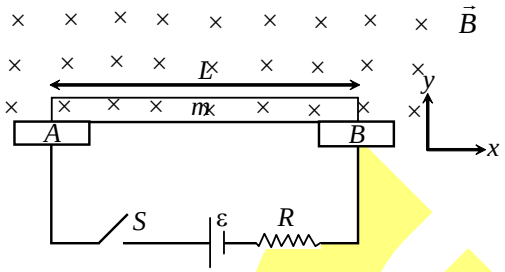
(A) $\sqrt{4\ell/g}$ (B) $\sqrt{12\ell/g}$

(C) $\sqrt{\frac{6\ell}{g}}$ (D) $\sqrt{\frac{2\ell}{g}}$

Space For Rough Work

Paragraph for Question no. 17 to 18

In a vertical plane, a metal rod of length L and mass m is placed over two conducting platforms A and B. A region of magnetic field $\vec{B} = -B_0\hat{k}$ starting from line joining A and B and lying over it, is switched on. Now, at t = 0 switch S is closed such that the charge q is passes through the rod in time dt due to which magnetic field exert an impulsive force which causes the rod to jump with certain velocity and since the gravitational field is present the rod comes back to AB after some time and collides inelastically and the process is repeated.



17. The maximum height reached by the rod is

(A) $\frac{2q^2L^2B_0^2}{m^2g}$ (B) $\frac{q^2L^2B_0^2}{2m^2g}$

(C) $\frac{q^2L^2B_0^2}{m^2g}$ (D) $\frac{q^2L^2B_0^2}{4m^2g}$

18. The maximum emf induced in the rod is

(A) $\frac{B_0^2L^2q}{m}$ (B) $\frac{B_0^2L^2q}{2m}$

(C) $\frac{2B_0^2L^2q}{m}$ (D) $\frac{4B_0^2L^2q}{m}$

Space For Rough Work

SECTION-2 : CHEMISTRY

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1. Which one of the following reactions of Xenon compounds is/are feasible ?

(A) $XeO_3 + 6HF \longrightarrow XeF_6 + 3H_2O$

(B) $3XeF_4 + 6H_2O \longrightarrow 2Xe + XeO_3 + 12HF + 1.5O_2$

(C) $2XeF_2 + 2H_2O \longrightarrow 2Xe + 4HF + O_2$

(D) $XeF_6 + RbF \longrightarrow Rb[XeF_7]$

2. Which of the following reactions form racemic mixture?

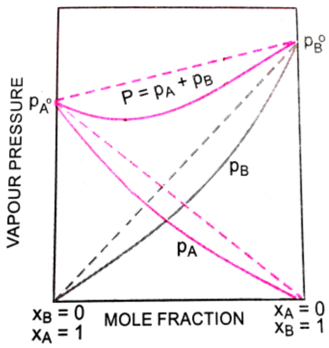
(A) $CH_3CH_2CH=CH_2 \xrightarrow{Br_2/CCl_4}$

(B) $CH_3CH_2CHO \xrightarrow[NaOH]{HCN}$

(C) $CH_3CH_2CH=CH_2 \xrightarrow[CCl_4]{HCl}$

(D) $CH_3CH_2CHO \xrightarrow{NH_2OH/H^+}$

3.



Which of the following statement(s) is/are correct for a solution of liquid A and B as given in the above figure?

(A) Enthalpy change of mixing of A with B(ΔH_{mix}) < 0

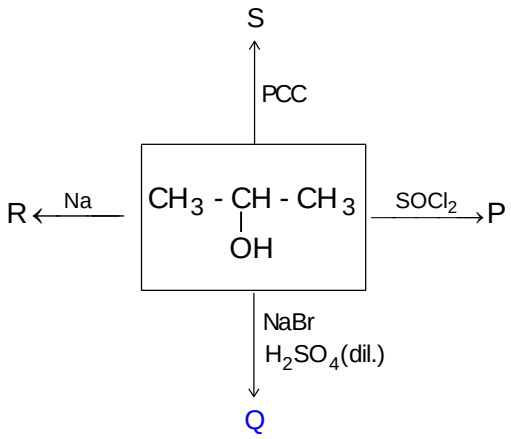
(B) Volume of mixing(ΔV_{mix}) < 0

(C) $p < p_A^0$

(D) Entropy of mixing ΔS_{mixing} < 0

Space For Rough Work

4.



P, Q, R and S are the organic products of above reactions. Choose correct statements.

(A) 'P' is less reactive than 'Q' towards dehydrohalogenation reaction in presence of alcoholic KOH through E_1 path.

(B) Formation of 'R' is accompanied with emission of H_2 gas.

(C) Reaction of 'S' with CH_3MgBr/H_3O^+ forms a tertiary alcohol.

(D) 'R' is more soluble in water than 'P'.

5. $Na_2CrO_4 \xrightarrow{Pb(CH_3COO)_2}$ Yellow ppt. $\begin{cases} \xrightarrow{H^+} \text{Soluble (P)} \\ \xrightarrow{OH^-} \text{Soluble (Q)} \end{cases}$

The anions present in the soluble compounds of (P) and (Q) in the above reaction is/are

(A) $[Pb(OH)_4]^{2-}$ (B) $Cr_2O_7^{2-}$

(C) $[Pb(CrO_4)_2]^{2-}$ (D) PbO_4^{2-}

6. $FeTiO_3$ contains Fe^{2+}, Ti^{4+} and O^{2-} ions. It's crystal contains hcp unit cells in which O^{2-} form the hcp array.

Choose the correct statements

(A) Fe^{2+} ions occupy $\frac{1}{3}^{rd}$ of octahedral voids

(B) Ti^{4+} ions occupy $\frac{1}{3}^{rd}$ of octahedral voids

(C) It's colour is due to only Fe^{2+} ions

(D) It is a paramagnetic substance

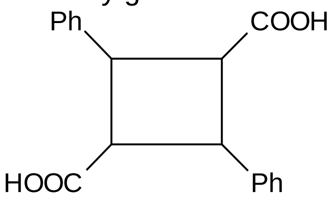
Space For Rough Work

7. The value of $\Delta H_{transition}$ of C (graphite) $\longrightarrow$ C (diamond) is 1.9 kJ/mol at 25°C entropy of graphite is higher than entropy of diamond. This Implies that :
(A) C (diamond) is more thermodynamically stable than C (graphite) at 25°C
(B) C (graphite) is more thermodynamically stable than C (diamond) at 25°C
(C) diamond will provide more heat on complete combustion at 25°C
(D) $\Delta G_{transition}$ of C (diamond) $\longrightarrow$ C (graphite) is –ve

(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8. How many geometrical isomers are possible for the following compound?



(A) 3 (B) 4
(C) 5 (D) 6

9. $SO_4^{2-} \xrightarrow{BaCl_2} (P)\downarrow$ (White ppt.)

$(P)\downarrow \xrightarrow[\text{(ii) } Na_2CO_3]{\text{(i) Carbon}} (Q)$

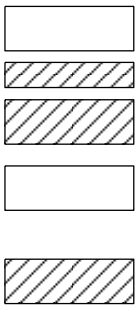
Compound (Q) containing sulphur is
(A) Na_2SO_4 (B) Na_2S
(C) Na_2SO_3 (D) CS_2

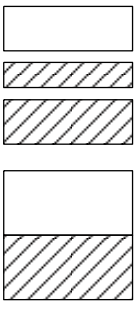
10. A solution of 'P' and 'Q' with 30 mole percent of 'P' is in equilibrium with its vapour which contains 60 mole percent of 'P'. The solution and the vapour behave ideally. If the ratio of vapour pressure of pure 'P' to pure 'Q' is expressed as x : y, the value of x + y is
(A) 3.5 (B) 9
(C) 18 (D) 7

11. A mixture of NaOH and Na_2CO_3 requires 100 mL of 3 M HCl for complete reaction upto phenolphthalein indicator. How many gram of NaOH is present in the mixture which contains 10.6 g of Na_2CO_3?
(A) 8 g (B) 12 g
(C) 4 g (D) 12 mg

Space For Rough Work

12. The bond structure of n-type semiconductor is

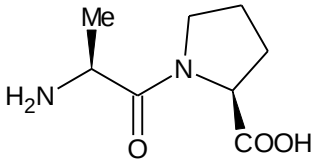
(A) 

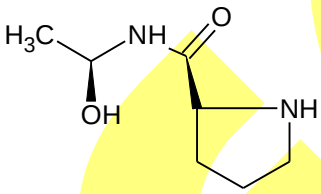
(B) 

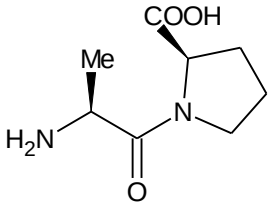
(C)

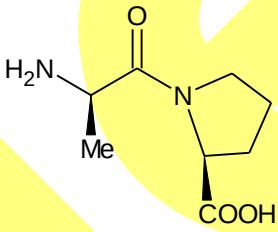
(D)

13. The structure of dipeptide “Ala-Pro” derived from Natural amino acid is

(A) 

(B) 

(C) 

(D) 

14. In the isoelectronic series VO_4^{3-}, CrO_4^{2-} & MnO_4^-, all members have CT transition. The incorrect statement is

(A) CT transition attributed to excitation of electrons from ligand to metal

(B) MnO_4^- exhibits charge transfer at shortest wavelength among three

(C) CT transition lapporte & spin allowed transition

(D) Charge on metal nucleus increases in order : $VO_4^{3-} < CrO_4^{2-} < MnO_4^-$

Space For Rough Work

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Lucas test is used to distinguish the different types of alcohols i.e., the primary, secondary and tertiary alcohols. Lucas reagent is anhydrous $ZnCl_2$ and conc.HCl. The alcohols are converted to alkyl chlorides in this test. The alcohols are distinguished through the rate of their reactions with HCl. Anhydrous $ZnCl_2$ is used as a catalyst. It removes the alcoholic OH to form a carbocation. The speed or rate of reaction of the different alcohols is

$3^o > 2^o > 1^o$

Formation of alkyl chlorides by the alcohols is observed due to formation of white turbidity.

Answer the following questions on the basis of above write up.

15. Which alcohol can form the white turbidity instantaneously?

(A) $CH_3CH_2CH_2CH_2OH$ (B) $CH_3CH_2\underset{\displaystyle OH}{\underset{|}{C}}HCH_3$

(C) $(CH_3)_3COH$ (D) CH_3CH_2OH

16. Which alcohol mostly require the catalyst anhy. $ZnCl_2$ to form the corresponding alkyl chloride?

(A) CH_3CH_2OH (B) $PhCH_2OH$ (C) $(CH_3)_2CHOH$ (D) $(CH_3)_3COH$

Paragraph for Question no. 17 to 18

Silicates are the oxo-anions of silicon, which contains Si – O sigma bonds. The anions are associated with metal cations. So, they are called ionic crystals. The silicate anions are very simple like ortho silicates SiO_4^{4-} and complex like cyclic or sheet silicates. The SiO_4^{4-} has no sharing oxygen atoms. The higher silicate contain shared oxygen atoms. For example $Si_2O_7^{6-}$ is called pyrosilicate in which one oxygen atom is shared. $(Si_3O_9)^{6-}$ is called cyclic silicate in which three oxygen atoms are shared.

Answer the following questions on the basis of above write up.

17. How many oxygen atom(s) is/are shared in the silicate anion $(Si_6O_{15})^{6-}$?

(A) 3 (B) 6 (C) 9 (D) 8

18. How many O^- ions per tetrahedra are present in pyrosilicate?

(A) 2 (B) 3 (C) 1 (D) 4

Space For Rough Work

SECTION-3 : MATHEMATICS

PART – A

(Multi Correct Choice Type)

This section contains 7 **multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

1 If $I_1 \equiv (1, -2)$ and $I_2 = (-3, 2)$ are the ex-centers of ΔABC and ex-centre I_3 lies on the circle $x^2 + y^2 + 3x + y - 6 = 0$, then

(A) locus of in-centre of ΔABC is given by $x^2 + y^2 + x - y - 5 = 0$

(B) locus of in-centre of ΔABC is given by $x^2 + y^2 + x - y + 5 = 0$

(C) locus of locus lies on the line y = -x

(D) origin lies inside the locus

2. Let N be the number of 7-digit numbers the sum of whose digits is even.
(A) The number of positive divisors of N is 126
(B) The number of positive divisors of N of the form 4k+1 , k ≥ 0 is 14
(C) The number of positive divisors of N of the form 4k+2 , k ≥ 1 is 9
(D) The number of positive divisors of N of the form 4k+2 , k ≥ 0 is 12

3. Consider $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + 3x^2 + 2x + 1 = 0, a_5, a_4, a_3 \in R - \{0\}$ .Then
(A) f(x)=0 will have atlest one non-real root
(B) f(x)=0 will have atleast one real root
(C) for $a_5 = 2021$ and $a_3 = 2019$, f(x)=0 has atleast one positive root
(D) for $a_5 = 1$, $a_4 = 9$ and $a_3 = 16$, the above equation has atleast one negative root

4. If A and B are different matrices satisfying $A^3 = B^3$, $A^2B = B^2A$ then which of the following is/are incorrect.
(A) $|A^2 + B^2| = 0$ (B) $|A - B| = 0$
(C) $|A^2 + B^2|$ and $|A - B|$ must be zero (D) At least $|A^2 + B^2|$ or $|A - B|$ must be zero

Space For Rough Work

5. Let f : R $\to$ R, g : R $\to$ R and h : R $\to$ R be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then

(A) $g'(2) = \frac{1}{3}$ (B) $h'(1) = 666$

(C) $h(0) = 16$ (D) $h(g(3)) = 38$

6. If lines $x + y - 3 = 0$ and $y = x + 1$ are two tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; then angle between the pair of tangents drawn from point $P\left(4, -\sqrt{5}\right)$ to the curve $\frac{x^2}{3a^2 + b^2} + \frac{y^2}{a^2 + 3b^2} = 1$ can be

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

7. Given that f is a real valued differentiable function such that $f(x).f'(x) \leq 0$, for all real x, there exists 'a' such that $f'(x) \neq 0 \ \forall \ x \in [a, \infty)$ then it follows that:
(A) $f^2(x)$ is non-decreasing function
(B) $f^2(x)$ is non-increasing function
(C) f(x) has no point of local maxima or minima
(D) f(x) = 0 does not have any real root

(Single Correct Choice Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

8 If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are three points on the parabola $y^2 = 4ax$ the normals at which meet in a point, then $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2}$ is equal to

(A) 4 (B) 8

(C) 0 (D) 1

9. Let P(x) be a polynomial with integer coefficients. It is known that P(x) takes the value 2015 for four distinct integers. Then the number of integral values of x for which P (x) equals 2022
(A) 1 (B) 7
(C) 2011 (D) 0

Space For Rough Work

10. If α is a root of the equation $x^2 - 3x + 1 = 0$, then $\frac{2\alpha^5 - 5\alpha^4 + 2\alpha^3 - 8\alpha^2}{\alpha^2 + 1}$ is

(A) –1 (B) –3

(C) $\frac{2}{3}$ (D) 1

11. If the points of intersection of the curves $x^2 - y^2 = a^2$ and $y = x^2$ lie on a unique circle, and S is the set of all such values of a, then S contains

(A) (–1, 1) (B) (0, 1)

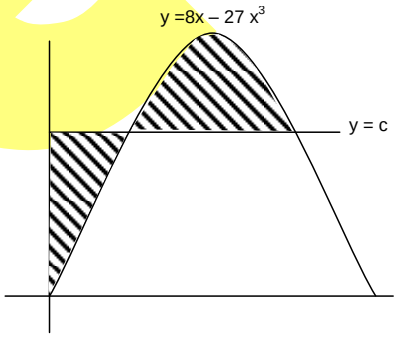
(C) (–1, 0) (D) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

12. A fair coin is repeated by tossed. If the probability that the first time heads is tossed, twice in a row, is on the 9th and 10th toss, is

(A) $\frac{17}{512}$ (B) $\frac{15}{512}$

(C) $\frac{11}{512}$ (D) $\frac{1}{2}$

13. The value of c such that areas of shaded region are equal.



(A) $\frac{32}{27}$ (B) $\frac{1}{7}$

(C) $\frac{16}{9}$ (D) none of these

14. If p, q are two real numbers satisfies the relations $2p^2 - 3p - 1 = 0$ and $q^2 + 3q - 2 = 0$ and $pq \neq 1$, then the value of $\frac{1 + 2p + pq}{q}$ is

(A) $\frac{-3}{4}$ (B) $\frac{\left(12 + \sqrt{17}\right)\left(3 + \sqrt{17}\right)}{24}$

(C) 1 (D) $\frac{1}{2}$

Space For Rough Work

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the paragraphs **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question no. 15 to 16

Tangents are drawn to the parabola $y^2 = 4x$ from the point P(6, 5) to touch the parabola at Q and R. C_1 is a circle which touches the parabola at Q and C_2 is a circle which touches the parabola at R. Both the circles C_1 and C_2 pass through the focus of the parabola.

15. Area of the ΔPQR equals

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) $\frac{1}{4}$

16. Radius of the circle C_2 is

(A) $5\sqrt{5}$ (B) $5\sqrt{10}$

(C) $10\sqrt{2}$ (D) $\sqrt{210}$

Paragraph for Question no. 17 to 18

If $f(x)=(x-\alpha)^n g(x)$, then $f(\alpha)=f'(\alpha)=f''(\alpha)=.....=f^{n-1}(\alpha)=0$ where f(x) and g(x) are polynomials for a polynomial f(x) with rational coefficients, answer the following questions

17. If f(x) touches x-axis at only one point, then the point of touching is

(A) always a rational number (B) may or may not be a rational number

(C) never a rational number (D) none of these

18 $f(\alpha)=f'(\alpha)=f''(\alpha)=0, f(\beta)=f'(\beta)=f''(\beta)=0$ and f(x) is a polynomial of degree 6, then

(A) atleast three roots of f"(x)=0 are always real

(B) atleast two roots of f"(x)=0 are always real

(C) exactly two roots of f"(x)=0 are real

(D) none of these

Space For Rough Work

Q.P. Code: 100956

Answers

SECTION-1 : PHYSICS

PART – A

1.	**ABD**	2.	BCD	3.	ACD	4.	**AD**
5.	ABC	6.	ABCD	7.	ACD	8.	C
9.	B	10.	D	11.	B	12.	A
13.	C	14.	C	15.	A	16.	B
17.	B	18.	A				

SECTION-1 : CHEMISTRY

PART – A

1.	BCD	2.	ABC	3.	AB	4.	ABCD
5.	AB	6.	ABCD	7.	BCD	8.	C
9.	B	10.	B	11.	A	12.	A
13.	A	14.	B	15.	C	16.	A
17.	C	18.	B				

SECTION-1 : MATHEMATICS

PART – A

1	ACD	2.	ABCD	3.	ABD	4.	ABC
5.	ABCD	6.	AB	7.	BCD	8.	C
9.	D	10.	A	11.	D	12.	A
13.	A	14.	D	15.	A	16.	B
17.	A	18.	B				

Answers & Solutions

SECTION-1 : PHYSICS

PART – A

1. **ABD**

2. **BCD**

Sol. $F = -\frac{\partial U}{\partial x}$

3. **ACD**

Sol. $U = \frac{1}{2}Li^2$

4. **AD**

Sol. $-\int \vec{E}\,\vec{dl} = -\frac{dB}{dt}$

5. **ABC**

Sol. $h\nu = \phi + KE$

6. **ABCD**

Sol. $R = \frac{mv}{qB}$

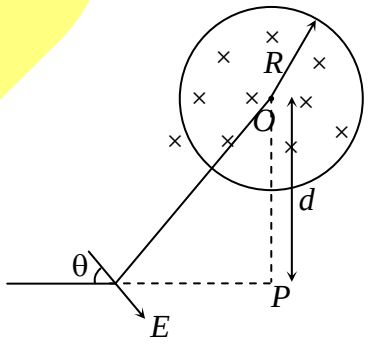
7. **ACD**

Sol. $\int \vec{E}\cdot d\vec{l} = A\frac{dB}{dt}$

$E2\pi\sqrt{x^2+d^2} = \pi R^2 k$

$E = \frac{\pi R^2 k}{2\sqrt{x^2+d^2}}$

$W_{ext} = \int_0^\infty q\vec{E}\cdot dx = \frac{q\pi R^2}{4}k$



8. **C**

Sol. $F = -\frac{\partial U}{\partial r}$

9. **B**

Sol. $\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, $\frac{\mu_3}{v} - \frac{\mu_2}{v'} = \frac{\mu_3 - \mu_2}{-R}$ $\Rightarrow$ $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{2\mu_2 - \mu_1 - \mu_3}{R}$

for Ist case: $\mu_3 = 1.3$, $\mu_2 = 1.5$, $\mu_1 = 1$, $u = -20$, $v = X$

we get $\frac{1.3}{X} + \frac{1}{20} = \frac{0.7}{R}$ … (i)

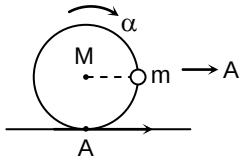
for IInd case: $\mu_3 = 1$, $\mu_2 = 1.5$, $\mu_1 = 1.3$, $u = -20$, $v = Y$

$\frac{1}{Y} + \frac{1.3}{20} = \frac{0.7}{R}$ … (ii)

From (i) and (ii), $1.3Y > X$

10. **D**

Sol. $(A_{CM})_x = \dfrac{mA + MA}{m + M} = A$



$(A_{CM})_y = \dfrac{M \times 0 + mR\alpha}{m + M} = \dfrac{mR\alpha}{m + M}$

$f = (M + m)A$ … (i)

$(M + m)g - N = (M + m)(A_{CM})_y$ … (ii)

$mgR = I_A\alpha$ … (iii)

$A = R\alpha$ … (iv)

$\therefore$ N = 70 N

11. **B**

12. **A**

Sol. $y = \dfrac{T\ell}{A\Delta\ell}, \Delta L = \dfrac{T\ell}{Ay} = \dfrac{mg\ell}{Ay\cos\theta}$

13. **C**

Sol. $mg - N = \dfrac{mv^2}{R}$

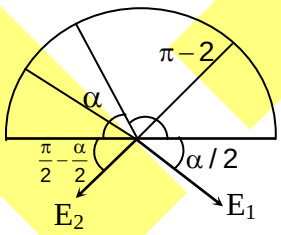
'R' can be measured by $\dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

14. **C**

Sol. $E_1 \cos\dfrac{\alpha}{2} = E_2 \sin\dfrac{\alpha}{2}$ …(i)

$E_1 \sin\dfrac{\alpha}{2} + E_2 \cos\dfrac{\alpha}{2} = E_0$ …(ii)

Solving we get E_1



15. **A**

Sol. $V_{cm} = \dfrac{yv}{\ell}$

$\lambda yg - \lambda v^2 = \lambda\ell.\dfrac{d}{dt}(V_{cm})$

$\dfrac{dy}{dt} = v.$

16. **B**

17. **B**

Sol. $H_{max} = \dfrac{v^2}{2g} = \dfrac{q^2L^2B_0^2}{2m^2g}$

18. **A**

Sol. $E_{max} = B_0Lv = \dfrac{B_0^2L^2q}{m}$ $(\because v_{max} = v)$

SECTION-2 : CHEMISTRY

PART – A

1. BCD

Sol. All fluorides of 'Xe' undergo hydrolysis. Perxenate ion is formed by XeF_6.

2. ABC

Sol. $CH_3CH_2CH(Br)CH_2Br$, $CH_3CH_2CH(OH)CN$ and $CH_3CH_2CH(Cl)CH_3$

contains chiral carbon atoms. So they form a pair of enantiomers.

3. AB

Sol. Negative deviation takes place.

4. ABCD

Sol. $P = CH_3CH(Cl)CH_3$, $Q = CH_3 - CH(Br) - CH_3$, $R = CH_3 - CH(O^-Na^+) - CH_3$, $S = CH_3 - C(=O) - CH_3$

contains chiral carbon atoms. So they form a pair of enantiomers.

5. AB

Sol. $P = Cr_2O_7^{2-}$ and $Q = Pb(OH)_4^{2-}$

6. ABCD

Sol. Formula(empirical) = $FeTeO_3$

As there are six oxide ions per hcp unit cell. The molecular formula will be $Fe_2Ti_2O_6$.

7. BCD

Sol. Graphite is thermodynamically more stable than diamond.

8. C

Sol. Five isomers are possible.

9. B

Sol. $P = BaSO_4$ and $Q = Na_2S$

10. B

Sol. $\dfrac{X_P p_P^o}{X_Q p_Q^o} = \dfrac{y_P P_T}{y_Q P_T}$

$\dfrac{0.3\, p_P^o}{0.7\, p_Q^o} = \dfrac{0.6}{0.4}$

$\dfrac{p_P^o}{p_Q^o} = \dfrac{0.6}{0.4} \times \dfrac{0.7}{0.3} = \dfrac{7}{2} = \dfrac{x}{y}$

$\therefore x + y = 9$

11. A

Sol. M_{eq} of HCl = M_{eq} of NaOH + M_{eq} of Na_2CO_3(n = 1)

12. A

Sol. The conduction band contains more electrons than that of p-type semiconductor.

13. A

Sol. The amino acids are alanine and proline.

14. B

Sol. MnO_4^- exhibit charge transfer band at longer wavelength that that of the other three.

15. C

Sol. Reactivity of alcohols towards Lucas test is $3^o > 2^o > 1^o > CH_3$

16. A

Sol. Primary alcohols can't remove the OH group as water easily by HCl along. So they forms a co-ordinated complex with $ZnCl_2$ which fascilate the removal of OH group and form the carbocation, which then forms the alkyl chloride.

17. C

Sol. $(Si_6O_{15})^{6-}$ shares nine oxygen atoms.

18. B

Sol. In pyrosilicate $Si_2O_7^{6-}$ contain one sharing oxygen atom. So other three atom oxygen per tetrahedra carries 6-charge. Therefore per tetrahedral three O^- ions are present.

SECTION-3 : MATHEMATICS

PART – A

1 ACD

Sol. Clearly $x^2+y^2+3x+y-6=0$ is circumcircle of $\Delta I_1I_2I_3$ and in-centre of ΔABC is orthocenter of $I_1I_2I_3$ let (h,k)

As mirror image of orthocenter about any side of triangle lies on the circumcircle. Hence required locus is $x^2+y^2+x-y-5=0$

2. ABCD

Sol. The seven digit numbers are 1000000, 1000001, ……. 9999999

The sums of their digits are odd, even, odd, even, ….etc.

$\therefore \quad N=\frac{9\times 10^6}{2}=2^5\,3^2\,5^6$

Required no.number of divisors of N is 1.2.7=14

3. ABD

Sol. If x_1,x_2,x_3,x_4,x_5 be the five roots, then $\sum\frac{1}{x_i^2}=-2$

So, the equation has atleast one complex root and since it is an odd degree polynomial equations with coefficients real always have atleast one real root

4. ABC

Sol. $A^3-A^2B=B^3-B^2A$

$A^2(A-B)+B^2(A-B)$

$(A^2+B^2)(A-B)=0$

$\therefore$ Either $A^2+B^2=0$ or $A-B=0$

5. ABCD

Sol. g'(2) = $\frac{1}{f'(0)}=\frac{1}{3}$

h(g(g(x))) = x	let g(g(x)) = y
h(y) = f(f(y))	g(x) = f(y)
h'(y) = f'(f(x)).f'(y)	x = f(f(y))
h(0) = 16,	h'(1) = 666

6. AB

Sol. Tangents are perpendicular $\Rightarrow$ their point of intersection (1, 2) lies on director circle $x^2+y^2=a^2+b^2 \Rightarrow a^2+b^2=5$

Equation of director circle of ellipse $x^2+y^2=4(a^2+b^2)=20$

Clearly point $(4,-\sqrt{5})$ lies outside the director circle

$\Rightarrow$ Angle is acute

7. BCD

Sol. Case possible f(x) > 0 and f(x) decreasing or f(x) < 0, f(x) increasing and f(x) and f'(x) have to change sign simultaneously to maintain the condition which is not possible also $(f^2(x))' = 2f(x).f'(x) \le 0$

8. C

Sol. $\frac{x_1-x_2}{y_3}+\frac{x_2-x_3}{y_1}+\frac{x_3-x_1}{y_2}$

$$= \frac{a\left(t_1^2 - t_2^2\right)}{2at_3} + \frac{a\left(t_2^2 - t_3^2\right)}{2at_1} + \frac{a\left(t_3^2 - t_1^2\right)}{2at_2}$$

$$= \frac{\left(t_1 - t_2\right)\left(-t_3\right)}{2t_3} + \frac{\left(t_2 - t_3\right)\left(-t_1\right)}{2t_1} + \frac{\left(t_3 - t_1\right)\left(-t_2\right)}{2t_2} = 0$$

9. D

Sol. $P(x) - 2015 = q(x)(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

Let $P(\alpha) = 2022$

$\Rightarrow P(\alpha) - 2015 = q(\alpha)(\alpha - x_1)(\alpha - x_2)(\alpha - x_3)(\alpha - x_4)$

$\Rightarrow 7 = q(\alpha)(\alpha - x_1)(\alpha - \alpha_2)(\alpha - \alpha_3)(\alpha - \alpha_4)$

Impossible since 7 is prime

10. A

Sol. α is a root of $x^2 - 3x + 1 = 0$

$\Rightarrow \alpha^2 - 3\alpha + 1 = 0,$ yields

$\alpha^2 + 1 = 3\alpha\,(\alpha \neq 0)$

Now $\dfrac{2\alpha^5 - 5\alpha^4 + 2\alpha^3 - 8\alpha^2}{\alpha^2 + 1} = \dfrac{\left(\alpha^2 - 3\alpha + 1\right)\left(2\alpha^3 + \alpha^2 + 3\alpha\right) - 3\alpha}{\alpha^2 + 1}$

$= \dfrac{-3\alpha}{\alpha^2 + 1} = \dfrac{-3\alpha}{3\alpha} = -1$

11. D

Sol. The points of intersection lies on $(x^2 - y^2 - a^2) + \lambda(x^2 - y) = 0$

It represents a circle if $\lambda = -2$

$\therefore$ equation of circle is $x^2 + (y - 1)^2 = 1 - a^2$

$\Rightarrow 1 - a^2 > 0 \Rightarrow a \in (-1, 1)$

But both curves will intersect in real points if $y^2 - y + a^2 = 0$ for some real y

i.e. $a \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$.

12. A

Sol. All 8 tails | T | T | T | T | T | T | T | T H H $\Rightarrow$ 1 way

Exactly 7 tails | T | T | T | T | T | T | T H H $\Rightarrow$ ${}^7C_1$ = 7 ways

7 gaps and one to be selected for a head

Exactly 6 tails | T | T | T | T | T | T H H $\Rightarrow$ ${}^6C_2$ = 15 ways

6 gaps and 2 to be selected for heads

Exactly 5 tails | T | T | T | T | T H H $\Rightarrow$ ${}^5C_1$ = 10 ways

5 gaps and 3 to be selected for heads

Exactly 4 tails | T | T | T | T H H $\Rightarrow$ ${}^4C_4$ = 1 ways

Total ways = 34

$\therefore$ Required probability $= \dfrac{34}{1024}\ \dfrac{17}{512} \equiv \dfrac{p}{512} \Rightarrow p = 17.$

13. A

Sol. $\displaystyle\int_0^a \left(c - \left(8x - 27x^3\right)\right)dx = \int_a^b \left(\left(8x - 27x^3\right) - c\right)dx$

$O = 4b^2 - 27b^4 - bc$

$O = 4b^2 - \dfrac{27}{4} - b\left(8b - 27b^3\right)$

$b^4\left(\dfrac{81}{4}b^2 - 4\right) = 0$

$b > 0 \;\; b^2 = \frac{4^2}{81} \qquad b = \frac{4}{81}$

$C = 8b - 27b^3$

$= \frac{32}{27}$

14. D

Sol. We can write the 2nd equation in form $2\left(\frac{1}{q}\right)^2 - 3\left(\frac{1}{q}\right) - 1 = 0$

then it is found that p and $\frac{1}{q}$ are both roots of equation $2x^2 - 3x - 1 = 0, (pq \neq 1)$ from viete

$p + \frac{1}{q} = \frac{3}{2}$ and $\frac{p}{q} = \frac{-1}{2}$

so, $\frac{1 + 2p + pq}{q} = \frac{1}{q} + p + \frac{2p}{q} = \frac{3}{2} - \frac{2}{2} = \frac{3}{2} - 1 = \frac{1}{2}$

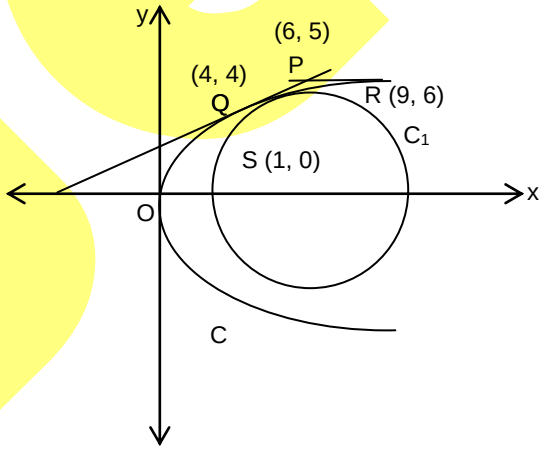
15. A

16. B

Sol. (15 to 16)

Equation of tangent of slope m to $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ....(1)



(i) As (1) passes through P(6, 5), so $5 = 6m = \frac{1}{m}$

$\Rightarrow 6m^2 - m + 1 = 0 \Rightarrow m = \frac{1}{2}$ or $m = \frac{1}{3}$

Points of contact are $\left(\frac{1}{m_1^2}, \frac{2}{m_1}\right)$ and $\left(\frac{1}{m_2^2}, \frac{2}{m_2}\right)$

Hence P (4, 4) and Q (9, 6)

Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 6 & 5 & 1 \\ 4 & 4 & 1 \\ 9 & 6 & 1 \end{vmatrix} = \frac{1}{2} \Rightarrow (A)$

(ii) $y = \frac{1}{2}x + 2 \Rightarrow x - 2y + 4 = 0$ ....(2)

And $y = \frac{1}{3}x + 3 \Rightarrow x - 3y + 9 = 0$

Now equation of circle C_2 touching $x - 3y + 9 = 0$ at (9, 6), is

$(x - 9)^2 + (y - 6)^2 + \lambda(x - 3y + 9) = 0$

As above circle passes through (1, 0), so

$64 + 36 + 10\lambda = 0 \quad \Rightarrow \quad \lambda = -10$

Circle C_2 is $x^2 + y^2 - 28x + 18y + 27 = 0$ ....(3)

Radius of C_2 is

$r_2^2 = 196 + 81 - 27 = 277 - 27 = 250 \Rightarrow \quad r_2 = 5\sqrt{10} \quad \Rightarrow$ (B)

17. A

Sol. If f(x) touches x-axis at only one irrational point, then $f(x) = (x-\alpha)^2 g(x)$, where α is irrational

$\Rightarrow$ coefficient of f(x) cant be rational

$\Rightarrow$ for f(x) with rational coefficient , then point of touching is rational

18. B

Sol. $f(x) = (x-\alpha)^3 (x-\beta)^3$

$\Rightarrow f'(x) = f(x-\alpha)^2 (x-\beta)^2 (2x-(\alpha+\beta))$

$\Rightarrow f''(x)$ has roots α, β and a root between $\left(\alpha, \frac{\alpha+\beta}{2}\right)$ and $\left(\frac{\alpha+\beta}{2}, \beta\right)$