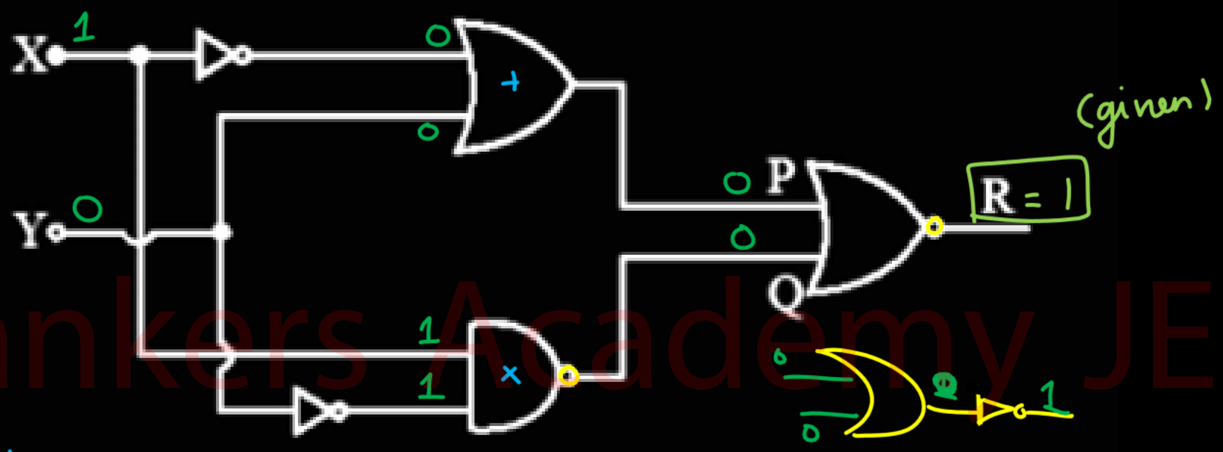
PHYSICS

To get output ' 1 ' at R , for the given logic gate circuit the input values must be :



$0+0=0$
$1+0=1$
$0+1=1$
$1+1=1$

$0\times0=0$
$0\times1=0$
$1\times0=0$
$1\times1=1$

check.

~~(A)~~ X = 0, Y = 1

(B) X = 1, Y = 1

(C) X = 0, Y = 0

check

(D) $\boxed{X = 1, Y = 0}$ ✓

2

A closed organ pipe and an open organ pipe have their first overtones identical in frequency. Their length are in the ratio :

(A) 1: 2 ~~(B)~~ 2: 3

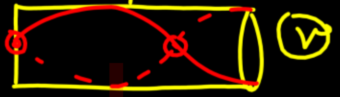
(C) 3: 4 (D) 4: 5

$v/4L$

$f_0, 3f_0, 5f_0 \ldots$

I overtone.

l_1

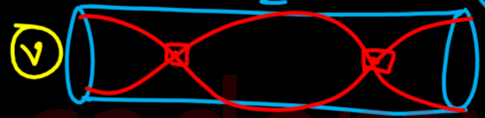


$v/2L$

$f_0, 2f_0, 3f_0 \ldots$

I overtone.

l_2



$$3\left(\frac{\cancel{v}}{4l_1}\right) = \cancel{2}\left(\frac{\cancel{v}}{\cancel{2}l_2}\right)$$

$$\boxed{\frac{3}{4} = \frac{l_1}{l_2}}$$

3

A tube of small uniform cross section is used to siphon the water from the vessel. Then choose correct alternative(s) : (density of water, $\rho_{water} = 10^3\ kg/m^3, g = 10\ m/s^2, P_{atm} = 10^5\ Pa$)

$a << A$

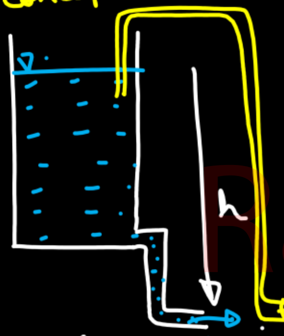
(A) Water will come out from section B with a velocity of $\sqrt{80}$ m/s

(B) Water will come out from section B with a velocity of $10\sqrt{2}$ m/s

(C) The greatest value of h_1 for which the siphon will work is 10 m

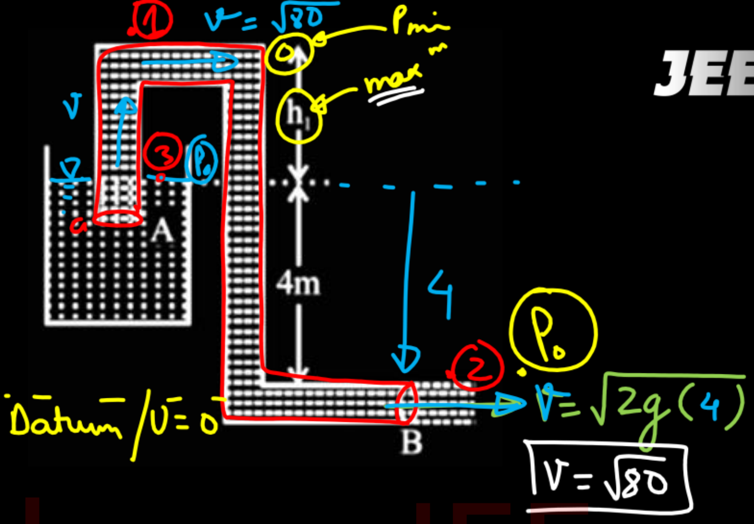
(D) The greatest value of h_1 for which the siphon will work is 8 m

Concept:



$v_{efflux} = \sqrt{2gh}$

$\boxed{P_{min} = 0} \neq -ve$



BT ① → ②

$$\underbrace{\rho g(h_1+4)}_{P.E_1} + \underbrace{\cancel{\tfrac{1}{2}\rho v^2}}_{KE_1} + \underbrace{\boxed{0}}_{P_1 = P_{min}} = \underbrace{0}_{P.E_2} + \cancel{\tfrac{1}{2}\rho v^2} + P_0$$

$$\rho g(h_1 + 4) = 10^5$$

$$\boxed{h_1 = 6}$$

JEE 1

4

A physical quantity P is related to four observables A, B, C and D as $\boxed{P = 4\pi^2 A^3 B^2/(\sqrt{C}D)}$. The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Value of P is calculated 3.763 , then % error is

(A) 12% (B) 14%

(C) 10% (D) 16%

error trick

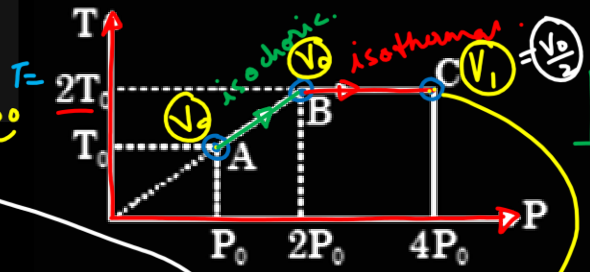
const

$$P = 4\pi^2 A^{3} B^{2} C^{-1/2} D^{-1}$$

1% 3% 2% 4%

% error P $= 3(1\%) + 2(3\%) + \frac{1}{2}(2\%) + 1(4\%) = 14\%$

One mole ($n=1$) of an ideal gas is taken through the process ABC as shown in the figure. The total work done on the gas is : $-W$ = Ans



(A) zero

(B) $2RT_0\ell n2$

(C) $-2RT_0\ell n2$

(D) $4RT_0\ell n2$

$\boxed{P_0V_0 = nRT_0}$

$P = \left(\frac{nR}{V}\right)T$ const

$\boxed{y = m x}$

Vol = const

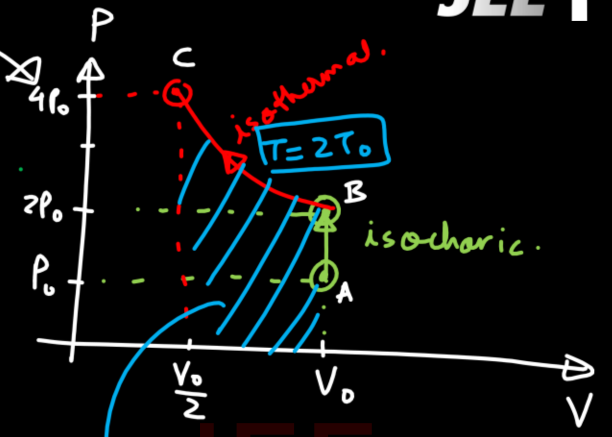
$(4P_0)V_1 = nR(2T_0)$

$V_1 = \frac{nRT_0}{2P_0}$

$\boxed{V_1 = \frac{V_0}{2}}$

$\boxed{Q = \Delta U + W}$

$\Delta U \rightarrow nC_v\Delta T$, $W \rightarrow \int P\,dV$



$\boxed{W_{AB} = 0}$

$W_{BC} = nRT\ln\left(\frac{V_f}{V_i}\right)$

$W = \cancel{W_{AB}}^{\,0} + W_{BC} = nRT\ln\left(\frac{V_0/2}{V_0}\right)$

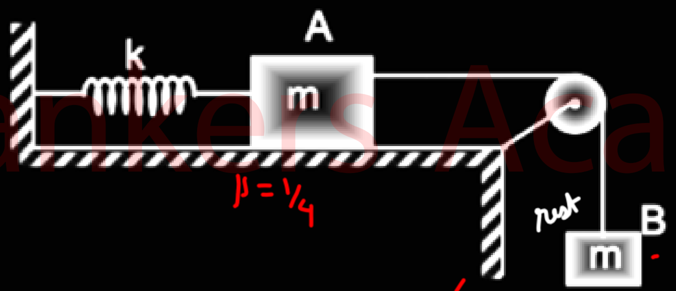
$= -R(2T_0)\ln(2)$

$\boxed{W = -2RT_0\ln(2).}$

$W_{on\,gas} = -W = 2RT_0\ln(2)$

6

The system is held with the spring at its relaxed length and then released. Find the maximum elongation of spring if coefficient of friction between the block A and the horizontal surface is $\frac{1}{4}$. (Take g = acceleration due to gravity and k = stiffness of spring, $m_A = m_B = m$)



(A) $\frac{3mg}{k}$

(B) $\frac{3mg}{2k}$ ✓

(C) $\frac{2mg}{3k}$

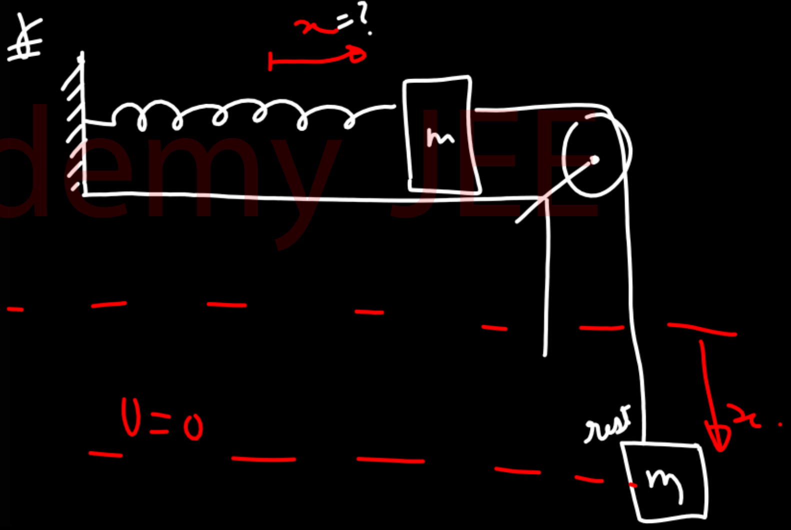
(D) $\frac{2mg}{k}$

$\boxed{\text{Energy}}$

$$\boxed{mgx} = \underbrace{\mu mgx}_{} + \frac{1}{2}kx^2 \qquad \mu = 1/4$$

$$\frac{3}{4_2}mg = \frac{1}{\cancel{2}}kx$$

$$\boxed{x = \frac{3mg}{2k}}$$



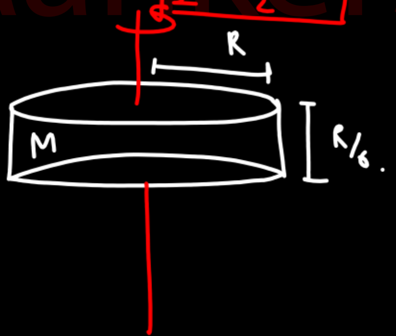
7

A circular disc of radius R and thickness $\frac{R}{6}$ has moment of inertia I about an axis passing through its centre and perpendicular to its plane. It is melted and recasted into solid sphere. What is the moment of inertia of the sphere about an axis passing through its diameter?

(A) $\frac{2I}{5}$ (B) $\frac{I}{5}$

(C) $\frac{I}{10}$ (D) $\frac{3I}{5}$

$\boxed{I = \frac{1}{2}MR^2}$



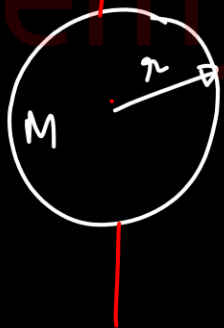
melt

$Vol_i = Vol_f$

$\cancel{\pi} R^2 \frac{R}{\cancel{6}_2} = \frac{4}{\cancel{3}} \cancel{\pi} r^3$

$\frac{R^3}{8} = r^3$

$\boxed{r = R/2}$

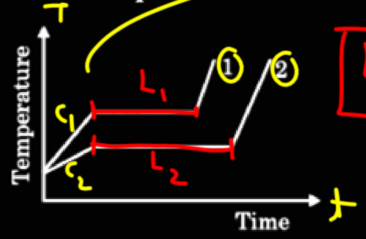


$I_1 = \frac{2}{5} M r^2 = \frac{\cancel{2}}{5} M \frac{R^2}{\cancel{4}_2} = \frac{MR^2}{10}$

$\frac{I_1}{I} = \frac{\frac{MR^2}{10}}{\frac{MR^2}{2}} = \frac{1}{5}$

$\boxed{I_1 = \frac{I}{5}}$

Two solid objects of the same mass are supplied with heat at the same rate $\Delta Q/\Delta t$. The temperature of the first object with latent heat L_1 and specific heat capacity c_1 changes according to graph 1 on the diagram. The temperature of the second object with latent heat L_2 and specific heat capacity c_2 changes according to graph 2 on the diagram. Based on what is shown in the graph, the latent heats L_1 and L_2, and the specific heat capacities c_1 and c_2 in solid state obey which of the following relationships?



~~(A)~~ $L_1 > L_2; c_1 < c_2$ (B) $L_1 < L_2; c_1 < c_2$
(C) $L_1 < L_2; c_1 > c_2$ ~~(D)~~ $L_1 > L_2; c_1 > c_2$

8

say $\boxed{\frac{dQ}{dt} = 100\ J/s}$

$\boxed{Q = mc\Delta T}$

$\boxed{Q = mL}$

$\text{slope}_{T-t} = \frac{dT}{dt}$

$\boxed{L_1 < L_2}$

$\text{slope}_1 > \text{slope}_2$

$Q = mc\Delta T$

$\boxed{\text{diff - 't'}}$

same

slope

$$\frac{dQ}{dt} = m\,c\,\frac{dT}{dt}$$

same 1 & 2

$$\left(c \cdot \frac{dT}{dt}\right) = \text{const for 1 \& 2}$$

$$c_1 \cdot \text{slope}_1 = c_2\,\text{slope}_2$$

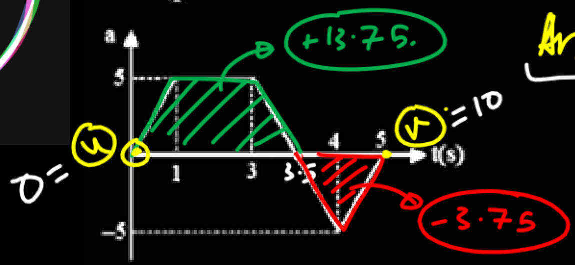
$$\frac{c_1}{c_2} = \frac{\text{slope}_2}{\text{slope}_1} < 1$$

$$\boxed{c_1 < c_2}$$

9

JEE 1

The acceleration of an object, starting from rest and moving along a straight line is as shown in the figure below.



Other than at t = 0, when is the velocity of the object equal to zero?

(A) At = 3.5 s

(B) During interval from 1 s to 3 s

(C) at t = 5 s

(D) at no other time on this graph

$\boxed{v = u = 0}$ $(v-u) = 0$

$\text{Area}_{a-t} = v-u = 0$

$v_{t=5} - u_{t=0}$

$\boxed{t = 0 \rightarrow 5}$

$v-u = 13.75 - 3.75 = 10$

$\boxed{v - 0 = 10}$

10

A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches 0 O under gravitational attraction only. Its speed at 0 O will be

(A) zero

(B) $\sqrt{\frac{2GM}{R}}$

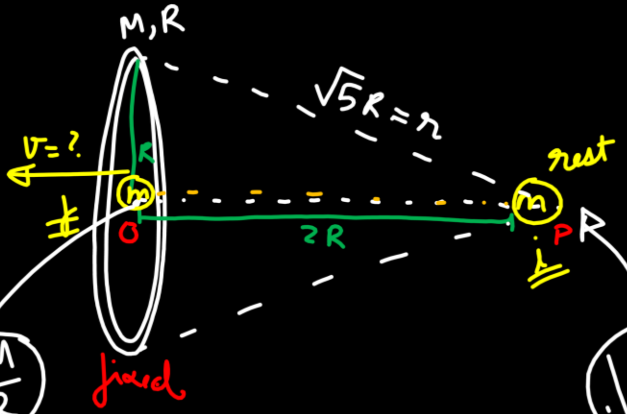
(C) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$

(D) $\sqrt{\frac{2GM}{R}\left(1-\frac{1}{\sqrt{5}}\right)}$

$\boxed{U = mV}$

$V \rightarrow$ Grav. Potential

$v \rightarrow$ velocity



$V_f = -\frac{GM}{R}$

$V_i = -\frac{GM}{\sqrt{5}R}$

$E_i = E_f$

$$\cancel{m}\left(-\frac{GM}{\sqrt{5}R}\right) + 0 = \cancel{m}\left(-\frac{GM}{R}\right) + \frac{1}{2}\cancel{m}v^2$$

$$\frac{2GM}{R} - \frac{2GM}{\sqrt{5}R} = v^2$$

$$v = \sqrt{\frac{2GM}{R}\left(1-\frac{1}{\sqrt{5}}\right)}$$

11

JEE 1

Starting from the mean position body oscillates simple harmonically with a period of $\boxed{2\text{ s}}$ ($T = 2$). After what time will its kinetic energy be 75% of the total energy

(A) $\frac{1}{6}$ s ✓
(B) $\frac{1}{4}$ s
(C) $\frac{1}{3}$ s
(D) $\frac{1}{12}$ s

$\boxed{TE = \frac{1}{2}m\omega^2A^2}$

at $x = x$ given.

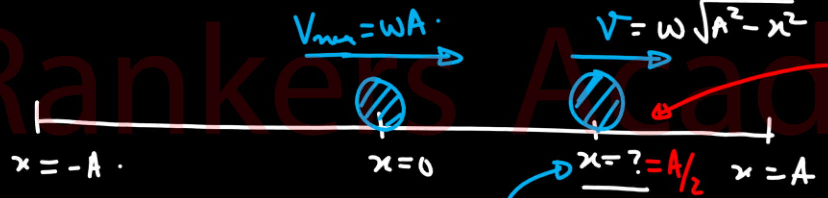
$KE = \frac{3}{4}TE$

$\cancel{\frac{1}{2}}\cancel{m}\cancel{\omega^2}(A^2 - x^2) = \frac{3}{4}\left(\cancel{\frac{1}{2}}\cancel{m}\cancel{\omega^2}A^2\right)$

$A^2 - \frac{3}{4}A^2 = x^2$

$\boxed{x = A/2}$

$t = \frac{T}{12} = \frac{1}{6}$

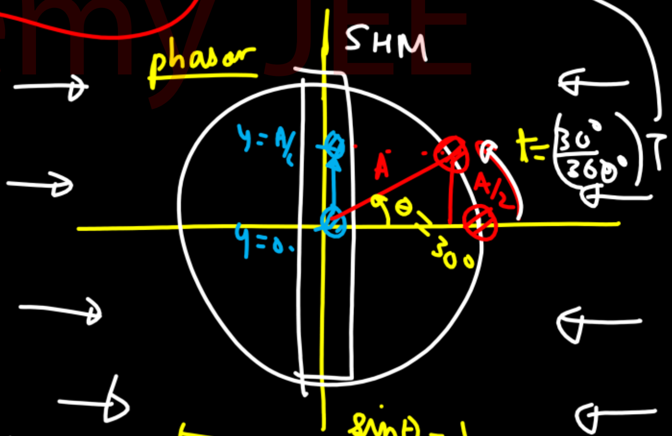


75% of TE = KE

$T/12$ $T/6$

$t = \frac{1}{6}$ s.

$\boxed{V = \omega\sqrt{A^2 - x^2}}$



$t = \left(\frac{30°}{360°}\right)T$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\theta = \omega t$

$\pi/6 = \frac{2\pi}{T} t$

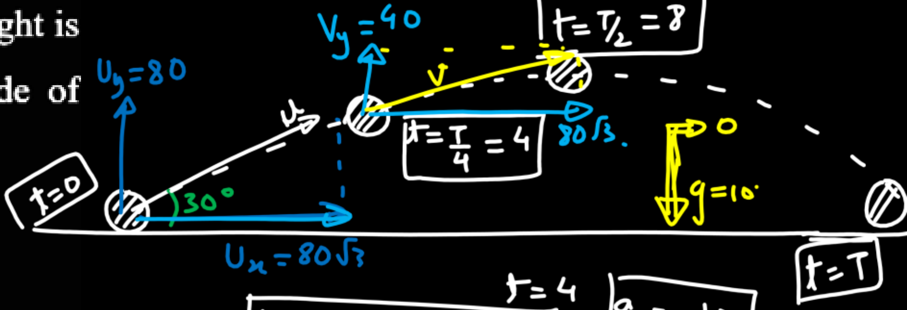
$\boxed{t = T/12 = \frac{1}{6}}$

A projectile is fired at an angle of 30° to the horizontal such that the vertical component of its initial velocity is 80 m/s. Its time of flight is T. Its velocity at $\boxed{t = T/4}$ has a magnitude of nearly (approx.)

~~(A) 200 m/s~~ ~~(B) 300 m/s~~

(C) 140 m/s ~~(D) 100 m/s~~

$$\boxed{T = \frac{2U_y}{g} = \frac{2(80)}{10} = 16}$$



$t = 4$ $\quad \boxed{a_y = -10}$

$$\boxed{V_y = U_y + a_y t}$$

$$V_y = 80 - 40 = 40$$

$$\tan\theta = \frac{U_y}{U_x}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{U_x}$$

$$\boxed{U_x = 80\sqrt{3}}$$

$$V = \sqrt{(40)^2 + (80\sqrt{3})^2}$$

$$V = 40\sqrt{1 + 12}$$

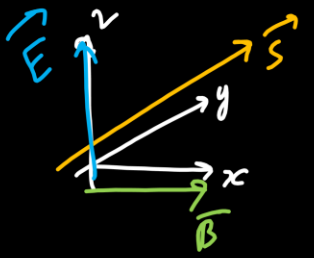
$$V = 40\sqrt{13} \approx 144\ldots$$

$\sqrt{9}$ $\quad \sqrt{13}$ $\quad \sqrt{16}$

3 $\quad$ 3.6 $\quad$ 4

13

A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by



(A) $\vec{E} = \sqrt{\frac{2I}{\epsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = +\frac{1}{c}E\hat{i}$

(B) $\vec{E} = \sqrt{\frac{2I}{\epsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y + ct)\right]\hat{k}; \vec{B} = \frac{1}{c}E\hat{i}$

(C) $\vec{E} = \sqrt{\frac{I}{\epsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = \frac{1}{c}E\hat{i}$

(D) $\vec{E} = \sqrt{\frac{I}{\epsilon_0 c}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{i}; \vec{B} = \frac{1}{c}E\hat{k}$

$I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c \Rightarrow E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

$B_0 = \frac{E_0}{c}$

$\vec{B} = \frac{E}{c}\hat{i} = \frac{1}{c}\sqrt{\frac{2I}{\varepsilon_0 c}} \cos \frac{2\pi}{\lambda}(y - ct)\hat{i}$

14

An orbital electron in the ground state of hydrogen has magnetic moment μ_1. This orbital electron is excited to 3^{rd} excited state by some energy transfer to the hydrogen atom. The new magnetic moment of the electron is μ_2, then

$n = 4$

(A) $\mu_1 = 4\mu_2$ (B) $2\mu_1 = \mu_2$

(C) $16\mu_1 = \mu_2$ (D) $4\mu_1 = \mu_2$

M#1

$$\mu = iA = \frac{e}{T}\pi r^2 = \frac{e}{\left(\frac{2\pi r}{v}\right)}\pi r^2$$

$$\mu = \frac{evr}{2} \propto \left(\frac{Z}{n}\right)\left(\frac{n^2}{Z}\right) \propto n$$

$$\boxed{\mu \propto n}$$

$$\frac{\mu_2}{\mu_1} = \frac{4}{1} \Rightarrow \mu_2 = 4\mu_1$$

M#2

$$\frac{M}{L} = \left(\frac{q}{2m}\right)$$

$$\mu = M = \left(\frac{q}{2m}\right) l = \frac{e}{2m} \times mvr$$

15

JEE 1

Assertion (A): If dQ and dW represent the heat supplied to the system and the work done 'on' the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$

$dU = dQ - dW$
if work done 'by' system

$dU = dQ + dW$
if work done 'on' system

Reason (R): First law of thermodynamics is based on law of conservation of energy.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true and (R) is NOT the correct explanation of (A).

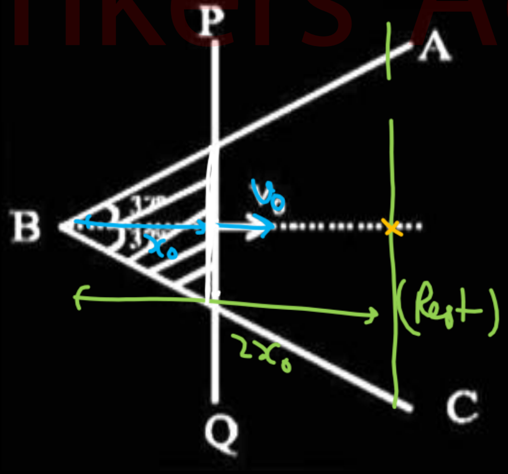
(C) (A) is true but (R) is false.

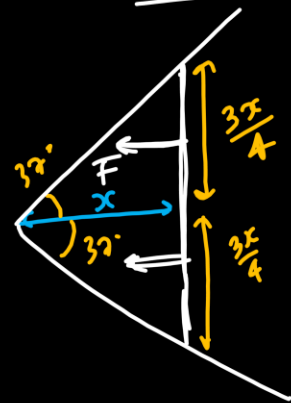
(D) (A) is false but (R) is true.

16

ABC is a smooth horizontal wire frame. PQ is a wire of mass m, which can slide on frame. There is a 'film of a liquid' of surface tension T covering the triangular part. Wire PQ is given a velocity v_0 when it is at a distance x_0 from B. Find v_0 if wire PQ is just able to reach upto a distance $2x_0$ from B.

$v_f = 0$





$l = \frac{6x}{4} = \frac{3x}{2}$

$F = T(2l)$

$F = T(3x)$ — ①

$W = \int F\,dx$

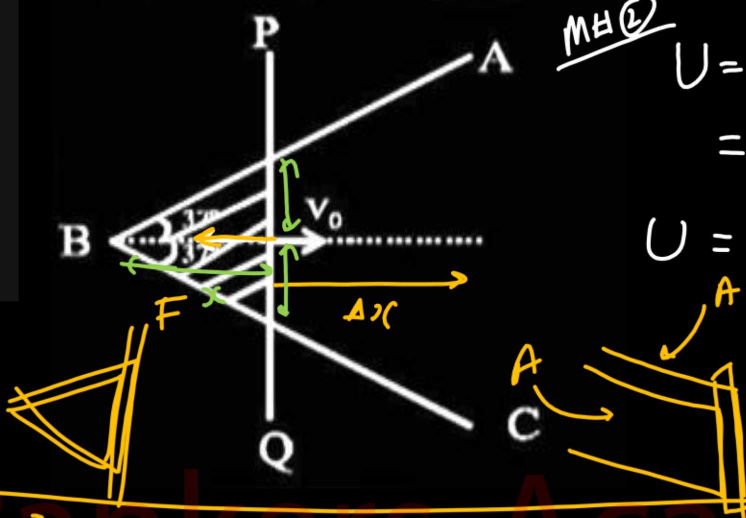
$= \int T\,3x\,dx = 3T\int_{x_0}^{2x_0} x\,dx$

(A) $x_0\sqrt{\frac{T}{m}}$

(B) $2x_0\sqrt{\frac{T}{m}}$

(C) $3x_0\sqrt{\frac{T}{m}}$

(D) $4x_0\sqrt{\frac{T}{m}}$



M#②

surface on both side

$U = T(2A)$

$= 2T\,\frac{1}{2}\left(\frac{6x}{4}\right)x$

$U = \frac{3Tx^2}{2}$

$U_i + K_i = U_f + K_f$

$\frac{3Tx_0^2}{\cancel{2}} + \frac{1}{\cancel{2}}mv_0^2 = \frac{3T(2x_0)^2}{\cancel{2}} + 0$

$mv_0^2 = 3T(3x_0^2)$

$V_0 = 3x_0\sqrt{\frac{T}{m}}$

$\vec{F} \parallel -\vec{x}$

$W_{all} = \Delta K$

$-3T\left[\frac{x^2}{2}\right]_{x_0}^{2x_0} = 0 - \frac{1}{2}mv_0^2$

$\Rightarrow 3T(3x_0^2) = mv_0^2 \longrightarrow v_0 = 3x_0\sqrt{\frac{T}{m}}$

(A) $x_0\sqrt{\frac{T}{m}}$

(B) $2x_0\sqrt{\frac{T}{m}}$

(C) $3x_0\sqrt{\frac{T}{m}}$

(D) $4x_0\sqrt{\frac{T}{m}}$

17

An electron having charge e and mass m starts from lower plate of two metallic plates separated by a distance d, if the potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by (ignore gravity)

(A) $\sqrt{\frac{2md^2}{eV}}$ (B) $\sqrt{\frac{md^2}{eV}}$

(C) $\sqrt{\frac{md^2}{2eV}}$ (D) $\frac{2md^2}{eV}$

Electrostatics

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{eE}{m} = \frac{e(V/d)}{m} \quad \text{—(1)}$$

$$s = ut + \frac{1}{2}at^2$$

$$d = 0 + \frac{1}{2}\left(\frac{eV}{md}\right)t^2$$

$$t = \sqrt{\frac{2md^2}{eV}}$$

18

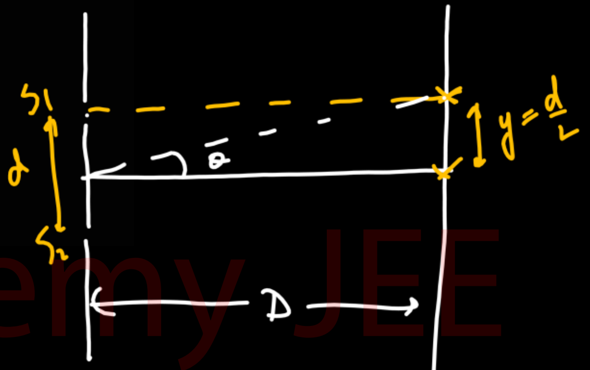
In a double slit experiment if 5^{th} dark fringe is formed opposite to one of the slits, the wavelength of light is

(A) $\frac{d^2}{6D}$

~~(B)~~ $\frac{d^2}{5D}$

(C) $\frac{d^2}{15D}$

(D) $\frac{d^2}{9D}$



$\Delta x = d\sin\theta$

$(n-\frac{1}{2})\lambda = d\left(\frac{y}{D}\right)$

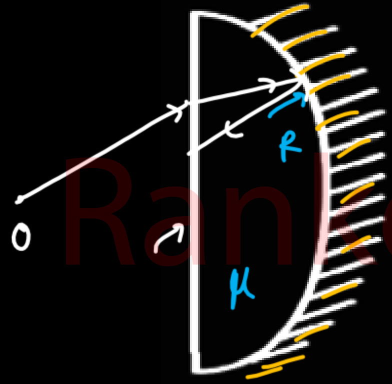
$(5-\frac{1}{2})\lambda = \frac{d\left(\frac{d}{2}\right)}{D}$

$\frac{9\lambda}{2} = \frac{d^2}{2D}$

$\lambda = \frac{d^2}{9D}$

19

Given a plano convex lens of curvature of radius R and refractive index μ. What will be the focal length if its curved surface is mirrored?



$P_{eq} = P_m + 2P_L$

$-\frac{1}{f_{eq}} = \frac{-1}{f_m} + \frac{2}{f_L}$

$\boxed{\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_L}}$

$\frac{1}{f_{eq}} = \frac{1}{\left(\frac{-R}{2}\right)} - 2(\mu-1)\left[\frac{1}{\infty} - \frac{1}{-R}\right]$

$= \frac{2}{-R} - \frac{(2\mu-2)}{R}$

$\frac{1}{f_{eq}} = \frac{-2-2\mu+2}{R}$

$f_{eq} = -\left(\frac{R}{2\mu}\right)$

concave mirror

(A) $\frac{R}{\mu}$

(B) $\frac{2R}{\mu}$

(C) $2R\mu$

(D) $\frac{R}{2\mu}$ ✓

20

A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the rim and the iron ring are 5.243 m and 5.231 m respectively at $\boxed{27°C}$ To what temperature should the ring be heated so as to fit the rim of the wheel ?

(α for iron = 1.2×10^{-5} / °C)

(A) 191°C

(B) 254°C

(C) 218°C

(D) 164°C

$D = D_0(1 + \alpha \Delta T)$

$5.243 = 5.231(1 + 1.2 \times 10^{-5} \times \Delta T)$

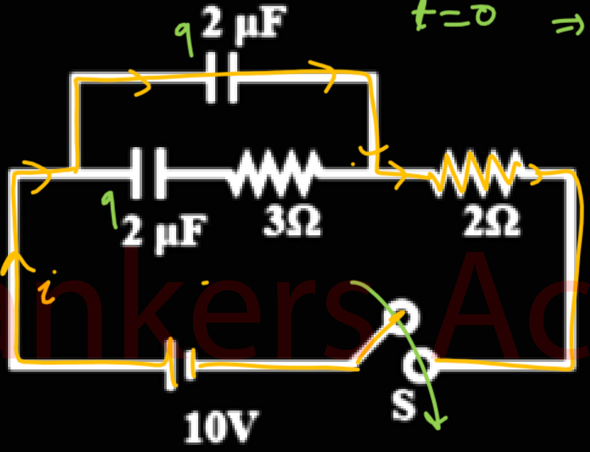
$\Rightarrow \Delta T = 191$

$\Rightarrow T - 27 = 191$

$\Rightarrow T = 218°C$

21

Current through the battery at the instant, when the switch " s " is closed is ______ A.



$t=0 \Rightarrow \boxed{q=0} \Rightarrow V = \frac{q}{C} = 0$

$\Rightarrow$ capacitor is a "conductor" (allows current through it)

$i = \frac{\varepsilon}{R_{eq}} = \frac{10V}{2\Omega} = 5A$

22

A proton and an α-Particle are accelerated through a potential difference of 100 V . The ratio of wavelength associated with the proton to that of α-particle is $\sqrt{x}$, find the value of x.

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}}$$

$$\lambda_p = \frac{h}{\sqrt{2meV}}$$

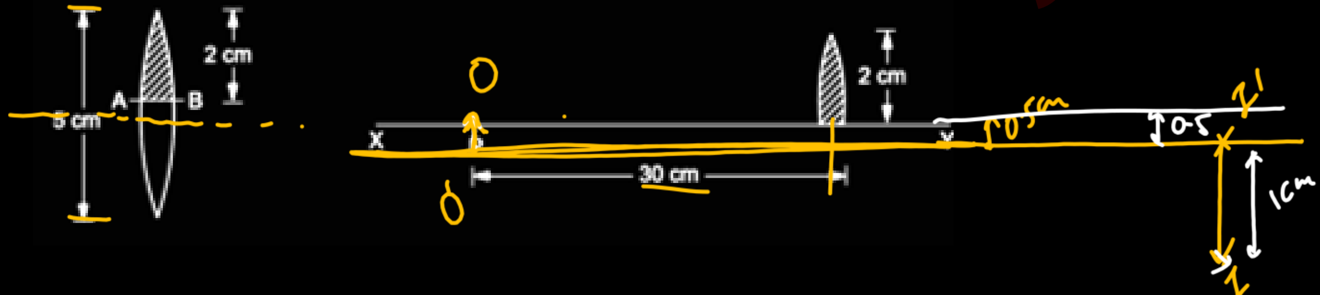
$$\lambda_\alpha = \frac{h}{\sqrt{2(4m)(2e)V}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{8}$$

23

JEE1

A converging lens of 20 cm focal length and 5 cm diameter is cut along the line AB. The part of the lens shown shaded in the diagram is now used to form an image of a point object P placed 30 cm away from it on the line XY, which is perpendicular to the plane of the lens. The distance of image of P will be formed from XY line is N/2 cm, then N is ______



M-①

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{+20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}$$

$$v = +60$$

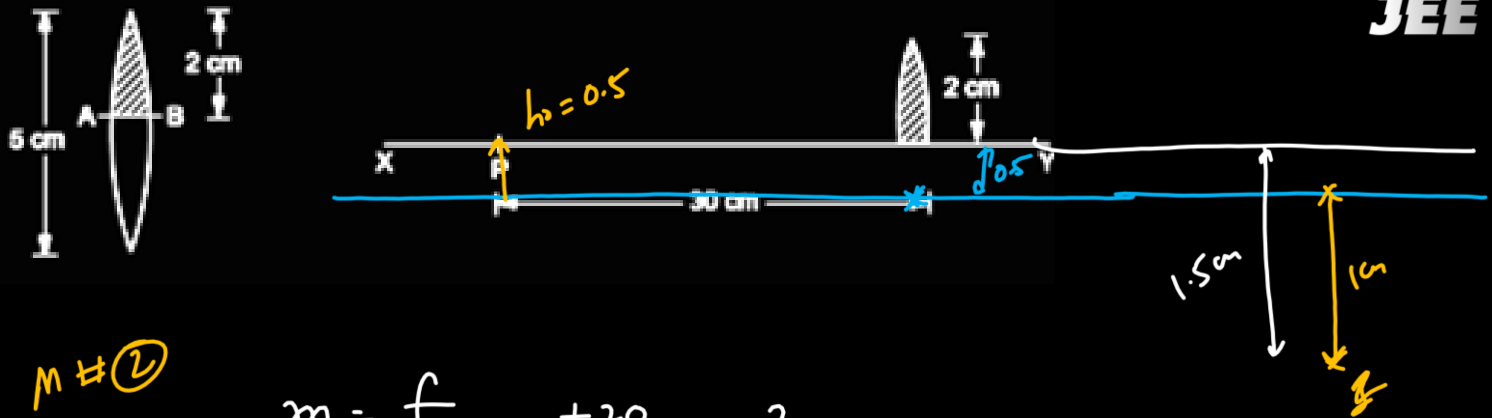
$$m = \frac{v}{u} = \frac{+60}{-30} = -2$$

$$\frac{h_i}{h_o} = -2$$

$$h_i = -2(0.5)$$

$$= -1 \text{ cm}$$

$$I \text{ from } XY = 1 + 0.5 = \frac{\boxed{3}}{2} \text{ cm}$$



M # 2

$$m = \frac{f}{f+u} = \frac{+20}{+20-30} = -2$$

$$h_i = -2h_o$$

$$= -2 \times 0.5$$

$$= -1 \text{ cm}$$

$1.5 = \boxed{\frac{3}{2}}$ cm

24

The magnet suspended in uniform magnetic field is heated so as to reduce its magnetic moment by 19% by doing this, time period of magnet will increases by ______ %

(nearest integer value)

$m' = (1-0.19)m$ —— (1)

$$\boxed{T = 2\pi\sqrt{\frac{I}{mB}}}$$

$$T' = 2\pi\sqrt{\frac{I}{m'B}} = 2\pi\sqrt{\frac{I}{0.81mB}} = \frac{2\pi}{(0.9)}\sqrt{\frac{I}{mB}}$$

$T' = \frac{T}{0.9}$

$T' = \frac{10}{9}T$

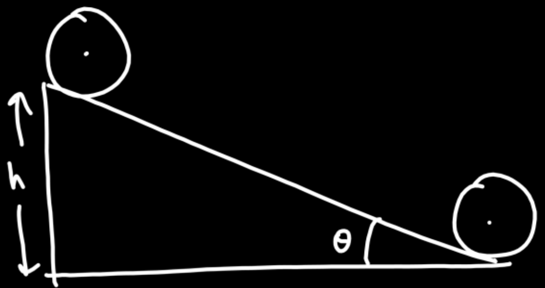
$\frac{\Delta T}{T}\times 100 = \frac{T'-T}{T}\times 100$

$= \frac{100}{9}$

$= \boxed{11}.111\%$

25

A thick-walled hollow sphere has outer radius R. It rolls down on an inclined plane without slipping and its speed at bottom is v_0. Now the incline is waxed so that the friction becomes zero. The sphere is observed to slide down without rolling and the speed now is $(5v_0/4)$. The radius of gyration of the hollow sphere about the axis through center is $\frac{nR}{4}$. Then the value of n is



$$mgh = \frac{1}{2}m\left(\frac{5v_0}{4}\right)^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$$

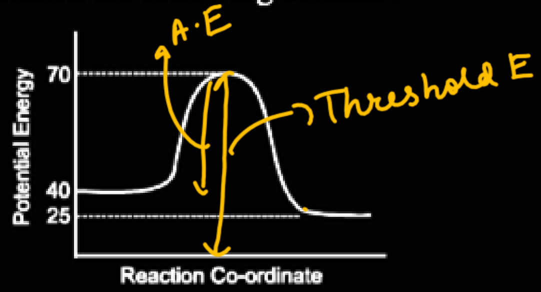
$$= \frac{1}{2}mv_0^2 + \frac{1}{2}mK^2 \times \left(\frac{v_0}{R}\right)^2$$

$$\frac{1}{2}mv_0^2\left(\frac{25}{16}\right) = \frac{mv_0^2}{2}\left(1 + \frac{K^2}{R^2}\right)$$

$$\frac{25}{16} - 1 = \frac{K^2}{R^2} \Rightarrow K = \frac{\boxed{3}R}{4}$$

CHEMISTRY

Match the following column :



Column-I	Column-II
(P) Activation energy of forward reaction 30	(1) 70 kJ mol^{-1}
(Q) Threshold energy 70	(2) 30 kJ mol^{-1}
(R) Activation energy for backward reaction 45	(3) 15 kJ mol^{-1}
(S) Enthalpy change of the reaction 15	(4) 45 kJ mol^{-1}

(A) P Q R S
1 4 3 2

(B) P Q R S
2 1 4 3

(C) P Q R S
1 2 3 4

(D) P Q R S
2 1 3 4

Select the incorrect statement

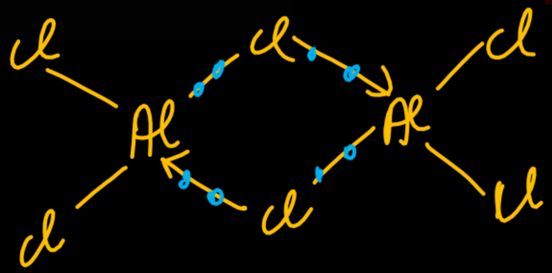
(A) The ionization energy of $X^-(g)$ is equal to positive value of electron affinity of $X(g)$

(B) Group 1 elements are better reducing agent then their corresponding group 2 elements

(C) Palladium (atomic no = 46) has d^{10} configuration. $[Kr]4d^{10}$

(D) Al_2Cl_6 (dimer of $AlCl_3$) contains two ' $3c-2e^-$ bond.

$3c-4e^-$



Which is the correct match for all?

(a)	Sucrose	(p)	$\beta-(1,4)$-Glycosidic linkage
(b)	Maltose	(q)	$\alpha,\beta-(1,2)$ - Glycosidic linkage
(c)	Starch	(r)	Natural biopolymer
(d)	Cellulose	(s)	Silver mirror with tollen's reagent

Code:

	a	b	c	d
(A)	s	p	r	q
(B)	q	s	p	r
(C)	q	s	r	p
(D)	r	s	q	p

Starch Cellulose

α β (unbranched)

Glycogen

α

1,2 S α β

1,4 M α α

1,4 L β β

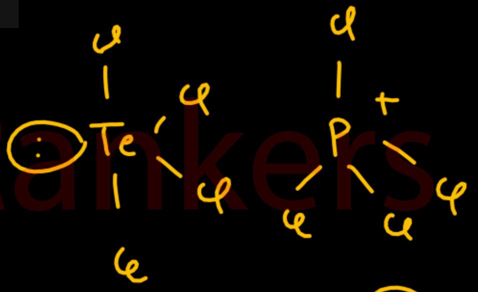
In which of the following pairs, the number of lone pair(s) and molecular geometry is same?

(A) $TeCl_4$, PCl_4^+ (B) ClF_3, PCl_3

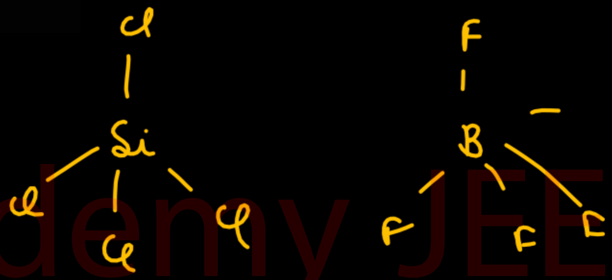
(C) $SiCl_4$, BF_4^- (D) BrF_5, XeF_5^-

0 L.P & both Td.

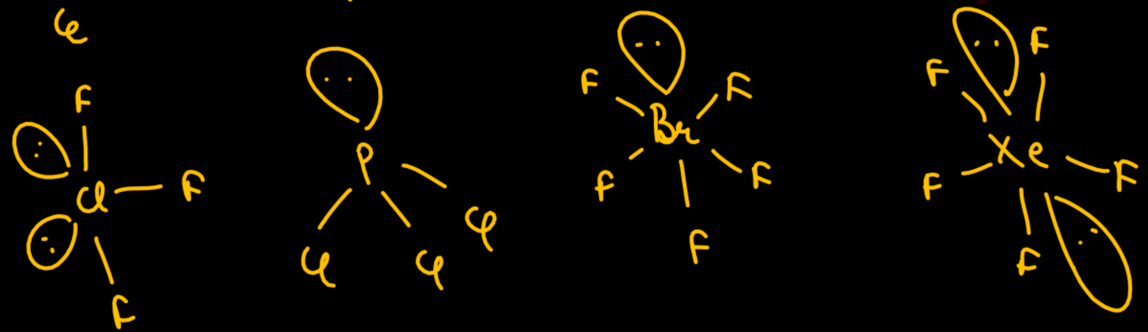
(A)



(C)



(B)



5

Statement-1: $K_3[Fe(CN)_6]$ is an inner orbital complex where as $K_3[FeF_6]$ is an outer orbital complex.

SL.

WL

Statement-2: CN^- being a strong ligand, pairs electrons of Fe^{3+} and we get d^2sp^3 hybridization. But F^- being weak ligand do not pair electrons in Fe^{3+}.

(A) Statement-1 is True, Statement- 2 is True; Statement- 2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement- 1 is True, Statement- 2 is False.

(D) Statement- 1 is False, Statement-2 is True.

$Fe^{3+} : 3d^5$

$SL : d^2sp^3$

$WL : sp^3d^2$

6

If the solubility of AgCl($K_{sp} = 1 \times 10^{-10}$) in water and 0.1M $AgNO_3(aq)$ is S_1 and S_2 respectively. Then,

(A) $S_1/S_2 = 10^{-4}$

(B) $S_1/S_2 = 10^{4}$

(C) $S_1/S_2 = 10^{-3}$

(D) $S_1/S_2 = 10^{-2}$

$$AgCl \rightleftharpoons \underset{S}{Ag^+} + \underset{S}{Cl^-}$$

$$K_{sp} = S^2$$

$$S_1 = \sqrt{K_{sp}} = 10^{-5}$$

For calculating S_2,

$$K_{sp} = [Ag^+][Cl^-]$$

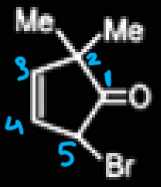
$$10^{-10} = [0.1][Cl^-]$$

$$\frac{10^{-10}}{10^{-1}} = 10^{-9} = [Cl^-] = S_2$$

$$\frac{S_1}{S_2} = \frac{10^{-5}}{10^{-9}} = 10^4$$

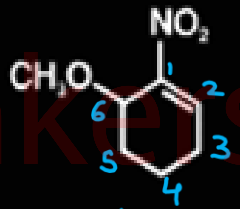
JEE 1

Which of the following structure has incorrect IUPAC name :

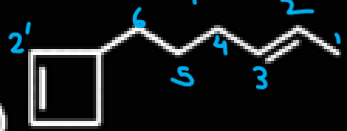
(A) 

2-Bromo-5,5-dimethylcyclopent-3-en-1-one

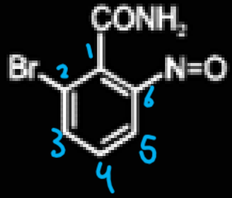
5-Bromo-2,2-dimethylcyclopent-3-en-1-one

(B) 

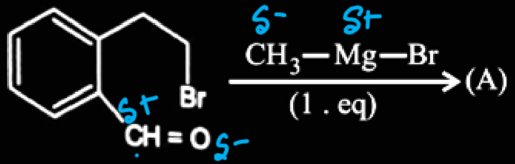
6-Methoxy-1-nitrocyclohex-1-ene

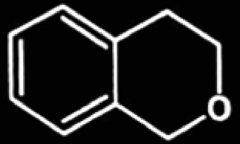
(C) 

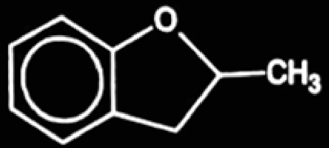
6-(Cyclobut-2-enyl)hex-2-ene

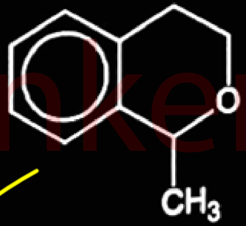
(D) 

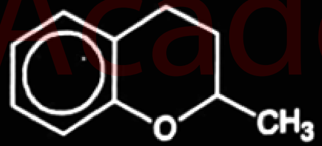
2-Bromo-6-nitrosobenzenecarboxamide

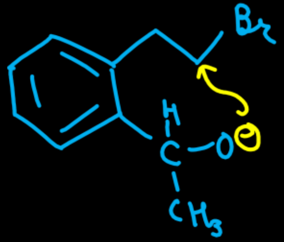


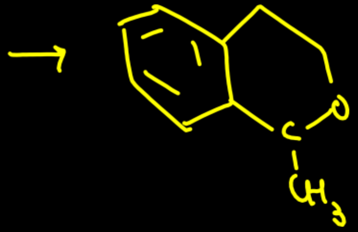
(A) 

(C) 

(B) 

(D) 





9

JEE 1

50 ml of 20.8%w/v $BaCl_2$(aq) and 100 ml of 9.8%w/v H_2SO_4(aq) solution are mixed. The molarity of $\underbrace{SO_4^{2-} \text{ in final solution is}}$:

(A) 0.66 M

(B) 0.5 M

(C) 1 M

(D) 0.33 M ✓

(L.R)

$$BaCl_2 + \underset{\checkmark}{H_2SO_4} \rightleftharpoons BaSO_4 + 2HCl$$

$\frac{20.8}{100} \times \frac{50}{208}$ $\qquad$ $\frac{9.8}{100} \times \frac{100}{98}$

Initially $= 0.05$ moles $\qquad = 0.1$ mol $\qquad 0 \qquad 0$

at time T $\qquad 0 \qquad 0.05 \qquad 0.05 \qquad 0.1$

Molarity of $[SO_4^{2-}] = \frac{0.05 \times 1000}{150} = 0.33$ M

10

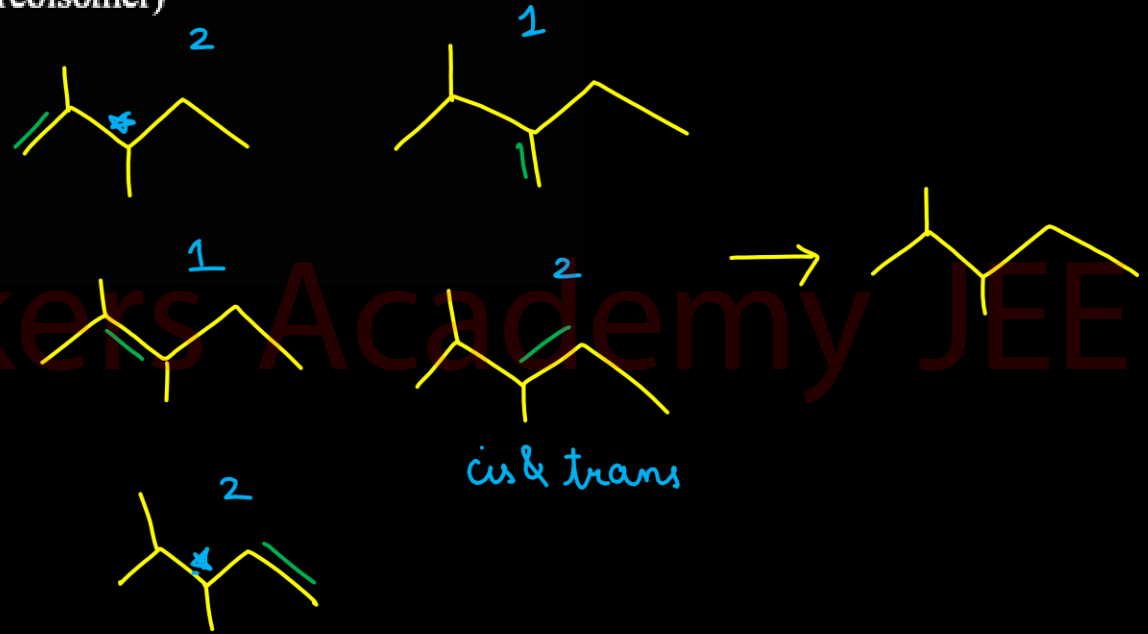
How many possible isomeric alkenes give 2,3-Dimethylpentane on catalytic hydrogenation. (include stereoisomer)

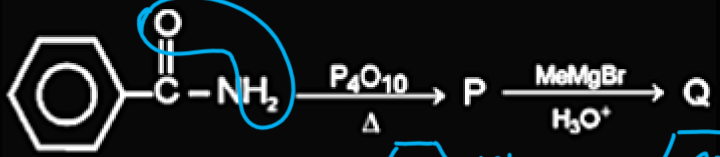
(A) 4

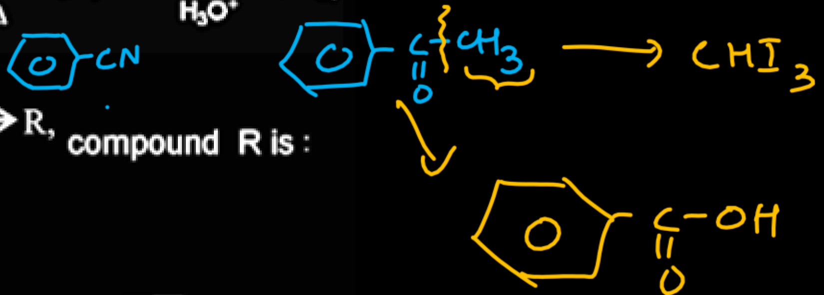
(B) 6

(C) 8

(D) 5

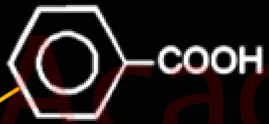


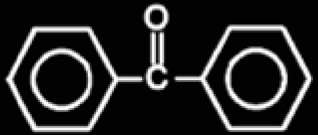




Q $\xrightarrow[\text{I (Iodoform reagent)},\ H^+]{Ca(OH)_2}$ R, compound R is :

(A) 

(B) 

(C) 

(D) 

12

Lanthanoid ions with $4f^6$ configuration are

(1) Sm^{2+}

(2) Eu^{2+}

(3) Eu^{3+}

(4) Tb^{3+}

(5) Tb^{4+}

Choose the correct answer from the Options given below:

(A) (2) and (3) only

(B) (1) and (2) only

(C) (1) and (3) only

(D) (1) and (5) only

Atomic Number	Name	Symbol	Electronic configurations*			Radii/pm		
			Ln	Ln^{2+}	Ln^{3+}	Ln^{4+}	Ln	Ln^{3+}
57	Lanthanum	La	$5d^1 6s^2$	$5d^1$	$4f^0$		187	106
58	Cerium	Ce	$4f^1 5d^1 6s^2$	$4f^2$	$4f^1$	$4f^0$	183	103
59	Praseodymium	Pr	$4f^3 6s^2$	$4f^3$	$4f^2$	$4f^1$	182	101
60	Neodymium	Nd	$4f^4 6s^2$	$4f^4$	$4f^3$	$4f^2$	181	99
61	Promethium	Pm	$4f^5 6s^2$	$4f^5$	$4f^4$		181	98
62	Samarium	Sm	$4f^6 6s^2$	$4f^6$	$4f^5$		180	96
63	Europium	Eu	$4f^7 6s^2$	$4f^7$	$4f^6$		199	95
64	Gadolinium	Gd	$4f^7 5d^1 6s^2$	$4f^7 5d^1$	$4f^7$		180	94
65	Terbium	Tb	$4f^9 6s^2$	$4f^9$	$4f^8$	$4f^7$	178	92
66	Dysprosium	Dy	$4f^{10} 6s^2$	$4f^{10}$	$4f^9$	$4f^8$	177	91
67	Holmium	Ho	$4f^{11} 6s^2$	$4f^{11}$	$4f^{10}$		176	89
68	Erbium	Er	$4f^{12} 6s^2$	$4f^{12}$	$4f^{11}$		175	88
69	Thulium	Tm	$4f^{13} 6s^2$	$4f^{13}$	$4f^{12}$		174	87
70	Ytterbium	Yb	$4f^{14} 6s^2$	$4f^{14}$	$4f^{13}$		173	86
71	Lutetium	Lu	$4f^{14} 5d^1 6s^2$	$4f^{14} 5d^1$	$4f^{14}$	–	–	–

13

Given the following molar conductivities at 25°C; HCl, $426 \Omega^{-1} cm^2 mol^{-1}$; NaCl, $126 \Omega^{-1} cm^2 mol^{-1}$; NaC (sodium crotonate), $83 \Omega^{-1} cm^2 mol^{-1}$. What is the ionization constant of crotonic acid? If the conductivity of a 0.001 M crotonic acid solution is $3.83 \times 10^{-5} \Omega^{-1} cm^{-1}$?

(A) 10^{-5}

(B) 1.11×10^{-5}

(C) 1.11×10^{-4}

(D) 0.01

$\Lambda_m = \frac{1000 k}{c}$

$= \frac{1000 \times 3.83 \times 10^{-5}}{10^{-3}}$

$\Lambda_m = 38.3$

$\Lambda^0_m = \lambda^0_{C^-} + \lambda^0_{H^+}$

$= \Lambda^0_{NaC} + \Lambda^0_{HCl} - \Lambda^0_{NaCl}$

$= 83 + 426 - 126$

$\Lambda^0_m = 383$

$\alpha = \frac{\Lambda_m}{\Lambda^0_m} = \frac{38.3}{383} = \frac{1}{10} = 0.1 \rightarrow \boxed{10\%}$

$HC \rightleftharpoons H^+ + C^{\ominus}$

$C - C\alpha \quad C\alpha \quad C\alpha$

$K_a = \frac{C\alpha^2}{1-\alpha}$

$= \frac{10^{-3}(10^{-1})^2}{1-0.1} = \frac{10}{9} \times 10^{-5}$

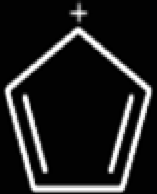
$= 1.1 \times 10^{-5}$

14

In the following pair, select the option in which II is more stable then I

(A)  > or  $4\pi e^-$

$2\pi e^-$ Aromatic I AA II

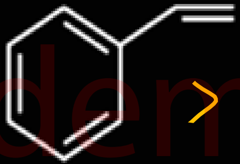
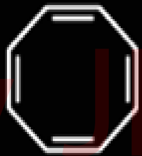
(C)  or > 

$6\pi e$ A I II $4\pi e$ AA

(B)  or < 

I II

$8\pi e$ NA $6\pi e^-$ A

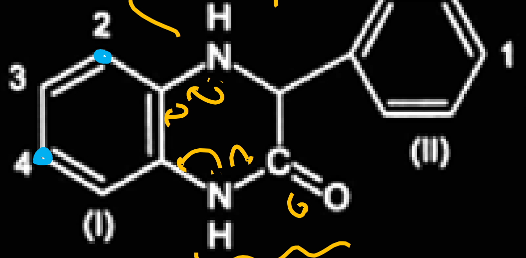
(D)  > or 

I II

A NA

15

Strong Activate.



$\xrightarrow{NO_2^+\text{(Mononitration)}}$?

the nitration major occurs at position?

Mild Activate.

(A) 1 (B) 2

(C) 3 (D) 4

16

Column-I	Column-II
(1) XeO_2F_4	(p) Central atom is sp^3 hybridized with no lone pair on it.
(2) SO_2Cl_2	(q) Central atom is sp^2 hybridized.
(3) $COCl_2$	(r) Central atom involves $d_{x^2-y^2}$ orbital in hybridization.
(4) $HClO_3$	(s) Contain central atom in oxidation state, two less than the highest.

(A) 1-p, 2-q, 3-r, 4-s

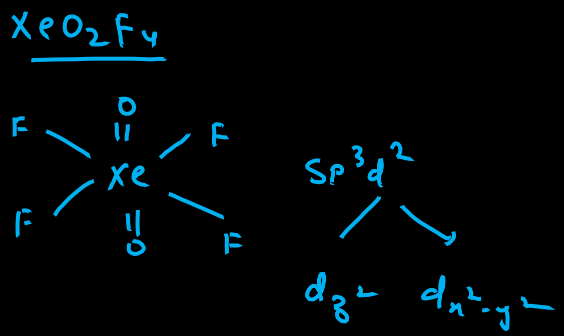
(B) 1-r, 2-s, 3-q, 4-s

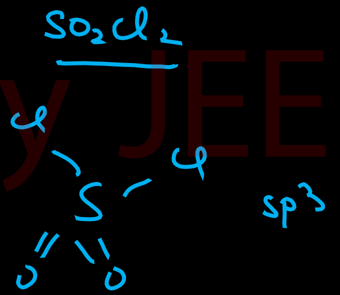
(C) 1-r, 2-p, 3-q, 4-s

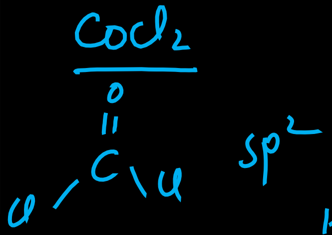
(D) 1-s, 2-p, 3-q, 4-r

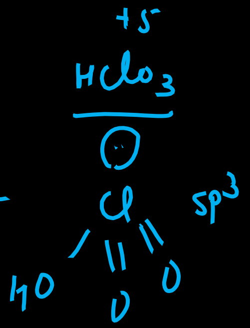
1 r   3 q

2 p   4 s









17

2 litres of an acidified solution of $KMnO_4$, containing 1.58 g of $KMnO_4$ per litre, is decolourised by passing sufficient amount of SO_2 gas. If whole of the sulphur from x g of FeS_2 is converted into SO_2 to be used in above reaction, calculate the value of x :

(A) x = 1.5 (B) x = 3

(C) x = 4.5 (D) x = 6

$Mn = 55$
$'O' = 16$

$n_f = 5$ $\quad n_f = 2$

$KMnO_4 + \overset{+4}{SO_2} \xrightarrow{H^{(+)}} Mn^{+2} + \overset{+6}{SO_4^{-2}}$

$n = \frac{2 \times 1.58}{158}$ $\quad n$

$n = \frac{2}{100}$

$FeS_2 \longrightarrow SO_2$

$n_1 = \frac{x}{120}$ $\quad n_2 = n$

Apply POAC

$\left(\frac{x}{120}\right) \times 2 = n_{SO_2} \times 1$

$\boxed{n_{SO_2} = \frac{x}{60}}$

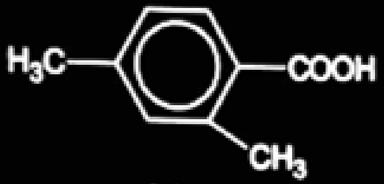
equi. of $KMnO_4$ = Eq of SO_2

$\left(\frac{2}{100}\right) \times 5 = \left(\frac{x}{60}\right) \times 2$

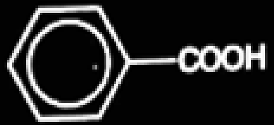
$\boxed{x = 3}$

18

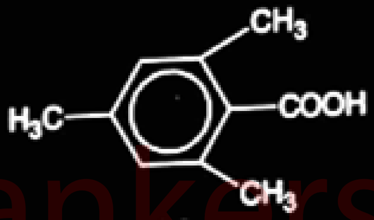
Given three Acids:



[x]



[y]



[z]

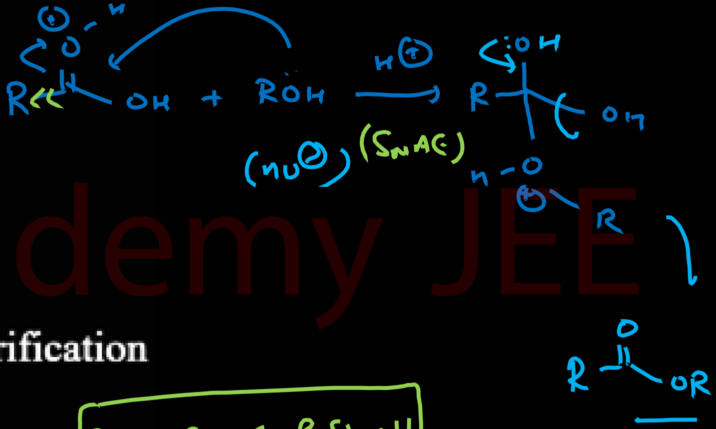
The correct order of acid catalysed esterification is:

(A) [x] > [z] > [y]

(B) [y] > [x] > [z]

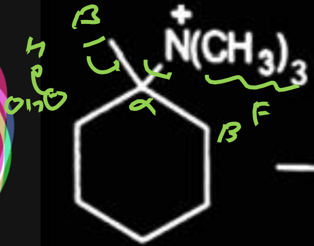
(C) [z] > [x] > [y]

(D) [y] > [z] > [x]



$\boxed{R \rightarrow EWG \ \& \ Small}$

19

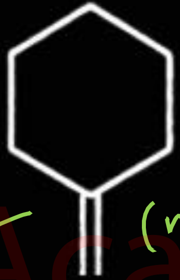


$\xrightarrow[\Delta]{OH^-}$ A (major)

(A)

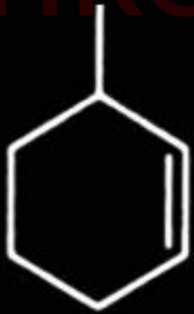
(minor)

(B)

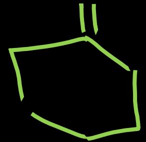
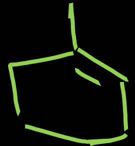


(major)

(C)



(D)

 + 

Hoffmann (major)

Saytzeff (minor)

20

Which of the following aryl amine undergoes diazotization most readily?

(A) 

$-m$

(B) 

$-I$

(C) 

$+m$

(D) 

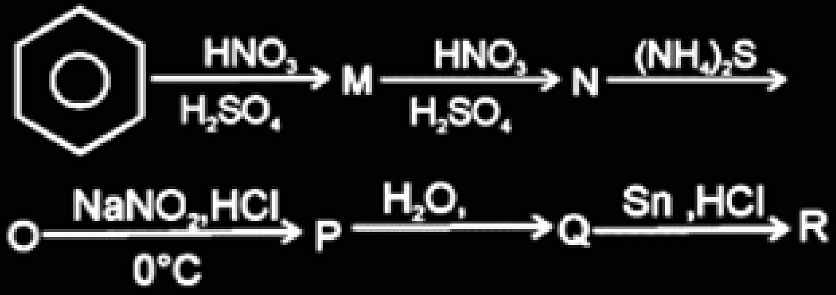
$+H$

$$R-\ddot{N}H_2 \xrightarrow{NaNO_2+HCl} R-\overset{+}{N_2}\,Cl^-$$

$Nu^{\ominus}$ $\overset{\oplus}{N}O$ (E^+)

e^- donating group

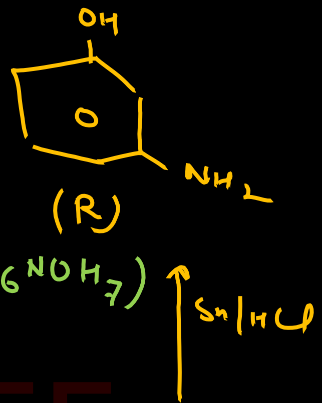
21

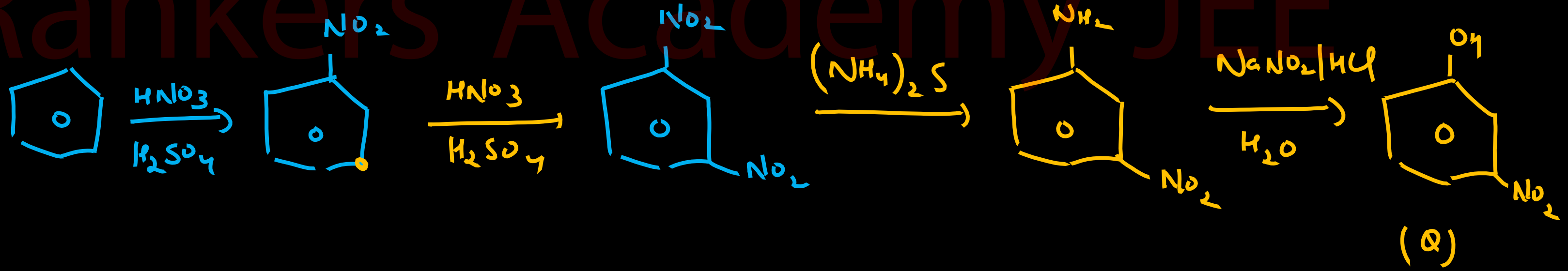


molecular weight of 0.1 mole of compound R is _______ (gram)

[Take C = 12, H = 1 N = 14, O = 16]

mol. wt
= 109





wt of 0.1 mole Product 'R' : 0.1 X 109 = 10.9 ~ 11

22

Find the number of oxides which is/are NOT amphoteric.

Mn_2O_7, MgO, BaO, BeO, CrO, Cr_2O_3, FeO, Fe_2O_3, $\boxed{ZnO}$, CuO, CrO_3, MnO, Li_2O, Cs_2O

JEE 1

Amphoteric : 3

Not Amphoteric : 11

Amphoteric

$\boxed{Zn}$ $\boxed{Be}$ Al Ga Sn Cr V Sb As Pb

$Cr \xrightarrow{+3} \boxed{Cr_2O_3}$

$V \xrightarrow{+5} V_2O_5$

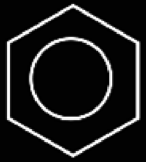
$Sb \xrightarrow{+3} Sb_2O_3$

$As \xrightarrow{+3} As_2O_3$

23

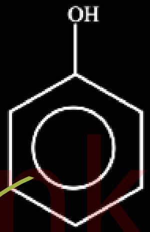
How many of the following compounds decolorise Br_2 water solution?

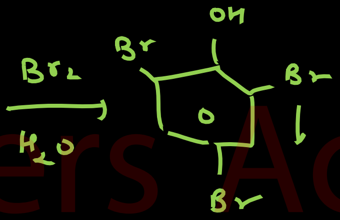
(I)



x

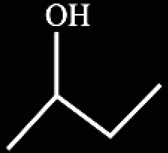
(II)





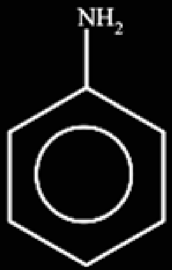
(III) $Me-C\equiv C-Me$ → $Me-\underset{Br}{\underset{|}{C}}=\underset{OH}{\underset{|}{C}}-Me$

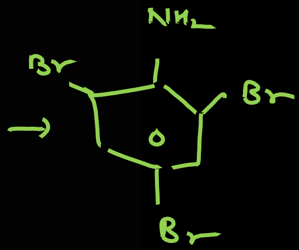
(IV)



x

(V)

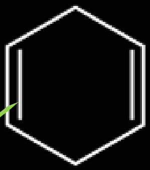




(VII)

(VIII) $Me-CH=CH-Et$

(IX)



5

24

JEE 1

How many of the following are diamagnetic?

$K_2[PtCl_6]$; $K_4[Fe(CN)_6]$; $[Ni(dmg)_2]$; $[Cu(NH_3)_4]^{2+}$

$Pt^{+4} : 5d^6$

$Fe^{+2} : 3d^6$

$Ni^{+2} : 3d^8$

$Cu^{+2} : 3d^9$

$n = 1$

1l	1l	1l			

5d d^2sp^3

1l	1l	1l			

d^2sp^3

1l	1l	1l	1l	

dsp^2

dsp^2

3d
4d
5d

Z_{eff}

always back pairing whether ligand is WFL | SFL

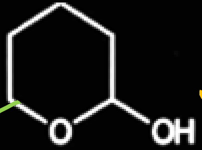
Ans. 3

25

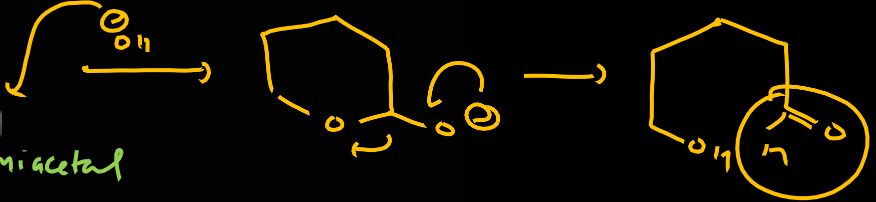
How many of the following give positive Tollen's Test?

(a) 

(b) 1 - butyne $H_3C-CH_2-C\equiv C-H$

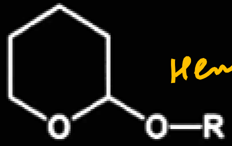
(c) 

cylic hemiacetal

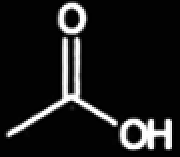


(d) 

(e) R - OH

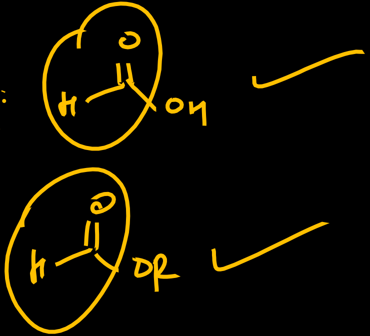
(f) 

hemiketal.

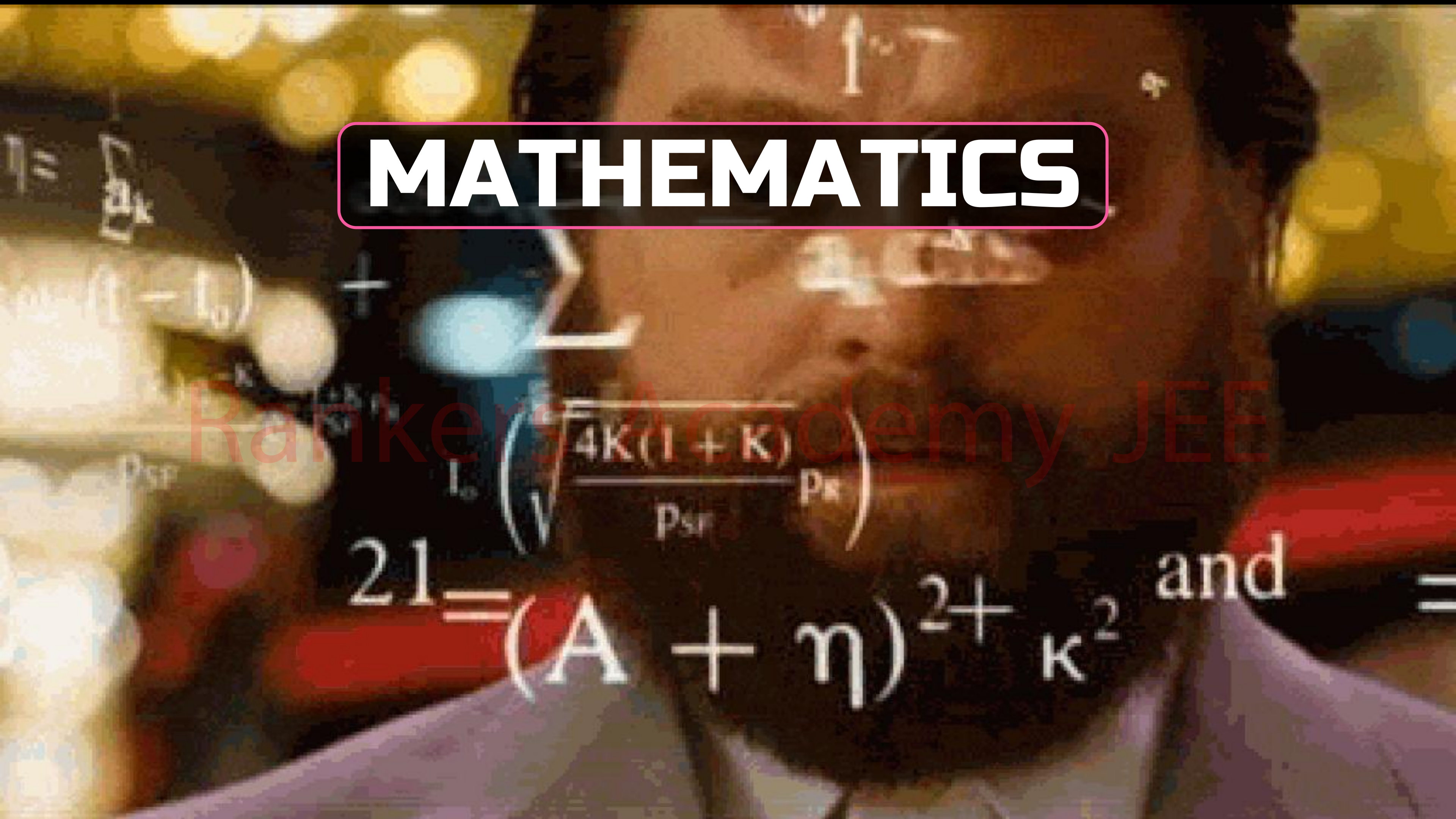
(g) 

3

Note:



MATHEMATICS



The number of real roots of the equation

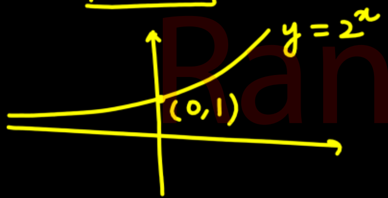
$5 + |2^x - 1| = 2^x(2^x - 2)$ is:

(A) 3 (B) 2

(C) 4 (D) 1 ✓

$2^x - 1 = 0$

$\boxed{x = 0}$



Case 1: $x < 0$

$0 < 2^x < 1$

$5 + (1 - 2^x) = 2^{2x} - 2.2^x$

Let $2^x = t$

$5 + 1 - t = t^2 - 2t$

$t^2 - t - 6 = 0$

$t = 2^x = 3, -2$

× ××

Case 2: $x > 0$

$2^x > 1$

$5 + (2^x - 1) = 2^{2x} - 2.2^x$

Let $2^x = t$

$5 + t - 1 = t^2 - 2t$

$t^2 - 3t - 4 = 0$

$t = 2^x = 4, -1$

✓ ×

$2^x = 4 = 2^2$

$\boxed{x = 2}$

Area bounded by the curves

$y=\sqrt{(4-x)^2}$, $x=\sqrt{3y}$ and x-axis is

(A) $\frac{1}{2}(1+2\pi/3-2\sqrt{3}/3)$

(B) $(1+2\pi/3-2\sqrt{3}/3)$

(C) $\frac{1}{2}(2\sqrt{3}/3-1-2\pi/3)$

(D) None of these

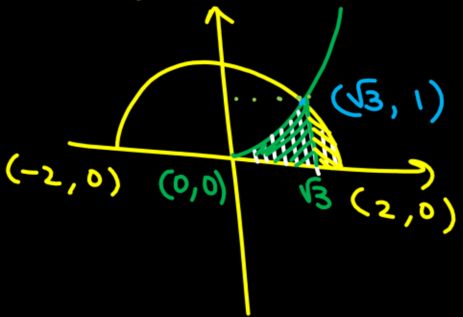
$y=\sqrt{\ \ } \geq 0$

$y^2=4-x^2$

$x^2+y^2=4$

$x^2=3y$

$x=\sqrt{\ \ } \geq 0$



$y^2+3y-4=0$

$y=-4, 1$

$$\text{Area} = \int_0^{\sqrt{3}} \frac{x^2}{3}\,dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

$$= \frac{1}{3}\cdot\sqrt{3}\cdot 1 + \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{\sqrt{3}} + 2\cdot\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2}+2\cdot\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{3}-\frac{1}{\sqrt{3}}$$

3

Vector $\hat{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b}$ and $\vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$. The value of $\hat{a}$ is :-

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

(B) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ ✓

(C) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$

(D) None

JEE 1

$\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$\vec{a} = \lambda(2\hat{i}+\hat{j}) + \mu(\hat{i}-\hat{j}+\hat{k})$

✓ $\vec{a} = (2\lambda+\mu)\hat{i} + (\lambda-\mu)\hat{j} + \mu\hat{k} = \mu\hat{i} - \mu\hat{j} + \mu\hat{k}$

$= \mu(\hat{i}-\hat{j}+\hat{k})$

$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\,|\vec{b}|} = \frac{\vec{a}\cdot\vec{d}}{\cancel{|\vec{a}|}\,\cancel{|\vec{d}|}}$

$2(2\lambda+\mu) + \cancel{1(\lambda-\mu)} = \cancel{1(\lambda-\mu)} + 2\mu$

$\boxed{\lambda = 0}$

$\vec{a} = \mu(\hat{i}-\hat{j}+\hat{k})$

Coefficient of x^2 in $(1 + x)^5(1 + 2x)^4$, is :-

(A) 74 ✓

(B) 88

(C) 96

(D) 108

$$(1+x)^5 \cdot (1+2x)^4$$

$${}^5C_r x^r \qquad {}^4C_r 2^r x^r$$

$$\left.\begin{array}{l} \boxed{{}^5C_0 x^0 \cdot {}^4C_2 2^2 x^2} = 24x^2 \\ {}^5C_1 x^1 \quad {}^4C_1 \cdot 2^1 \cdot x^1 = 40x^2 \\ \boxed{{}^5C_2 x^2 \quad {}^4C_0 \cdot 2^0 \cdot x^0} = 10x^2 \end{array}\right\} \Rightarrow 74x^2$$

5

JEE 1

The last common term to the sequences 1,11,21,31 ... (100 terms) and 31,36,41,46 … (100 terms) is :-

(A) 381 (B) 521

(C) 281 (D) None

$d_1 = 10$

$T_{100} = 1 + 99\times10$

AP1 : $1, 11, 21, \boxed{31} \ldots\ldots 991$;

$d_2 = 5$

$T_{100} = 31 + 99\times5$

$= 526$

AP2 : $\boxed{31}\, 36, 41, 46 \ldots\ldots 526$;

Common AP : $31, 41, 51 \ldots$

$d = LCM(d_1, d_2)$

$= LCM(10, 5) = 10$

$T_n \le \min(526, 991)$

$T_n \le 526$

$31 + (n-1)(10) \le 526$

$10n \le 505$

$n \le 50.5$

$\boxed{n = 50}$

$T_{50} = 31 + (50-1)(10)$

$= 31 + 490$

$= 521$

6

If $0 < \theta < \pi$, then minimum value of the expression $f(\theta) = 3\sin\theta + \operatorname{cosec}^3\theta$ is:-

(A) 4 (B) 3

(C) 5 (D) 6

JEE 1

M1

$f(\theta) = \sin\theta + \sin\theta + \sin\theta + \operatorname{cosec}^3\theta$

$AM \geq GM$

$\Rightarrow \dfrac{\sin\theta + \sin\theta + \sin\theta + \operatorname{cosec}^3\theta}{4} \geq \left(\sin\theta \cdot \sin\theta \cdot \sin\theta \cdot \operatorname{cosec}^3\theta\right)^{\frac{1}{4}}$

$\Rightarrow \dfrac{f(\theta)}{4} \geq 1$

$\Rightarrow f(\theta) \geq 4$

M2

$f'(\theta) = 3\cos\theta - 3\operatorname{cosec}^3\theta \cot\theta$

$= 3\cos\theta - 3\operatorname{cosec}^4\theta \cos\theta$

$= 3\cos\theta\,(1 - \operatorname{cosec}^4\theta)$

$= \underbrace{3\cos\theta}_{+ve}\,\underbrace{(1+\operatorname{cosec}^2\theta)}_{+ve}\,\underbrace{(1-\operatorname{cosec}^2\theta)}_{-ve}$

$\theta = \pi/2$

$f'(\theta)$: $-$ | $+$ at $\frac{\pi}{2}$

Min

Ans: $f(\pi/2) = 3(1) + (1)^3$

$= 4$

7

JEE1

x_1, x_2 & x_3 when divided by 4 leaves a remainder of 0,1&2 respectively find number of non-negative integral solution of the equation $x_1 + x_2 + x_3 = 35$, is :-

(A) 45

(B) 55

(C) 105

(D) 190

$x_1 = 4\lambda_1$

$x_2 = 4\lambda_2 + 1$

$x_3 = 4\lambda_3 + 2$

$x_1 + x_2 + x_3 = 35$

$4\lambda_1 + 4\lambda_2 + 1 + 4\lambda_3 + 2 = 35$

$\lambda_1 + \lambda_2 + \lambda_3 = 8$

Beggar's : ${}^{8+3-1}C_{3-1} = {}^{10}C_2 = 45$

8

JEE 1

Let $\lambda \in R$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for :

no sol :

(A) exactly one negative value of λ.

(B) exactly one positive value of λ.

(C) every value of λ.

(D) exactly two values of λ.

$$\Delta = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = -2\begin{vmatrix} 2 & 2 & \lambda \\ 1 & 3 & 1 \\ \lambda & 5 & 4 \end{vmatrix}$$

$$= +2(3\lambda+2)(\lambda-3) = 0$$

$$\lambda = 3, -\frac{2}{3}$$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = -2(\lambda-3)$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & \lambda \\ 1 & 2 & 1 \\ \lambda & 3 & 4 \end{vmatrix} = -2(\lambda-3)(\lambda+1)$$

$$\Delta_3 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 2 \\ \lambda & -10 & 3 \end{vmatrix} = -2(\lambda-3)$$

$\lambda = 3$: $\Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

∞ sol.

$\lambda = -\frac{2}{3}$: $\Delta = 0$

$\Delta_1, \Delta_2, \Delta_3 \neq 0$

no sol.

9

JEE 1

$\lim_{x\to\infty}\left(\boxed{\cancel{\frac{x^{2013}}{e^{3x}}}}+\underbrace{\left(\cos\frac{2}{x}\right)^{x^2}}_{1^\infty}\right)$ is

(A) 0

(B) e^3

(C) e^2

(D) e^{-2} ✓

$$e^{\lim_{x\to\infty} x^2\left(\cos\frac{2}{x}-1\right)}$$

$$e^{\lim_{x\to\infty} x^2\left(-2\sin^2\frac{1}{x}\right)}$$

$$= e^{-2}\boxed{\lim_{x\to\infty}\frac{\left(\sin\left(\frac{1}{x}\right)\right)^2}{\left(\frac{1}{x}\right)^2}} = 1$$

$$= e^{-2}$$

10

JEE 1

Let $f(x): Z \to Z$ be a function that satisfies $f(x) + f(y) = f(x+y) - xy$ and if $f(7) = 21$, then (Z is a set of integers)

(A) f(x) is onto and one-one

(B) f(x) is into and many one

(C) f(x) is into and one-one

(D) f(x) is onto and many one

$$f(x) + f(y) = f(x+y) - xy$$

$y=0$: $\cancel{f(x)} + f(0) = \cancel{f(x)} - 0$

$f(0) = 0$

$y=1$: $f(x) + f(1) = f(x+1) - x$

$f(x+1) - f(x) = f(1) + x$

$x=1$: $f(2) - f(1) = f(1) + 1$

$x=2$: $f(3) - f(2) = f(1) + 2$

$x=3$: $f(4) - f(3) = f(1) + 3$

$\vdots$

$x=6$: $f(7) - f(6) = f(1) + 6$

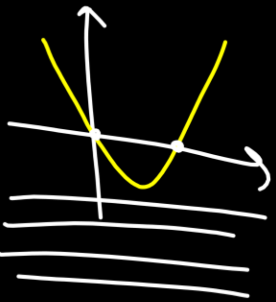
$x = x-1$: $f(x) - f(x-1) = f(1) + x - 1$

$\cancel{f(7)} - f(1) = 6 \cdot f(1) + \cancel{21}$

$\boxed{f(1) = 0}$

$f(x) - \cancel{f(1)} = \cancel{(x-1) \cdot f(1)} + \dfrac{x(x-1)}{2}$

$$\boxed{f(x) = \frac{x(x-1)}{2}}$$



M-1 $f(0) = f(1)$

into

f(x) can never -ive integral values

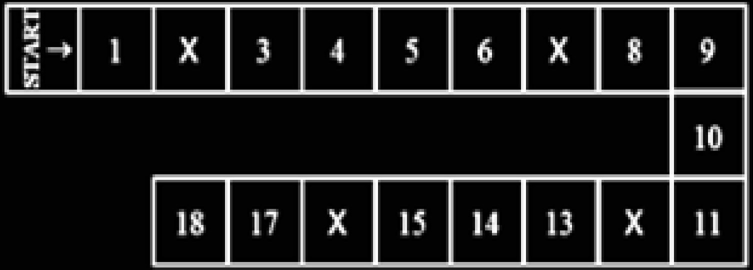
11

A game board is shown in diagram.

Player take turns to roll an ordinary die, then move their counter forward from 'START' a number of squares equal to the number rolled with the die. If a player's counter ends its move on a cross marked square, then it is moved back to START. Let α denotes the probability that player's counter is on START after rolling the die twice and let β denotes the probability that after rolling the die thrice, a player's counter is on square numbered 17 , then the value of $\frac{\alpha}{\beta}$ is

(A) 24 (B) 21

(C) 20 (D) 18



(1,1), (1,6)
(2,2), (3,4)
(4,3), (5,2)
(6,1), (6,6)

$\alpha = \frac{8}{36} = \frac{2}{9}$

(5,6,6)
(6,5,6)

$\beta = \frac{2}{216} = \frac{1}{108}$

$\frac{\alpha}{\beta} = \frac{2}{\cancel{9}} \times \cancel{108}^{12} = 24$

12

JEE1

For a statistical data $x_1, x_2, \ldots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$. He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

(A) 9 (B) 5

(C) 7 (D) 4

$\frac{\sum x_i}{10} = 5.5 \Rightarrow \sum x_i = 55$

$\sum x_i^2 = 371$

Correct Mean $= \frac{55 - 4 - 5 + 6 + 8}{10} = 6$

Correct var $= \frac{371 - 4^2 - 5^2 + 6^2 + 8^2}{10} - 6^2$

$= \frac{371 + 20 + 39}{10} - 36$

$= 43 - 36 = 7$

13

JEE 1

If A. adj $(A^2) = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then absolute value of sum of elements of adj A is (where adj (X) denotes adjoint of matrix X)

(A) 6 (B) 8

(C) 10 (D) 4

$$\text{adj}(A^2) = \text{adj}(A \cdot A) = \text{adj}A \cdot \text{adj}A = (\text{adj}A)^2$$

$$A \cdot (\text{adj}A^2) = A(\text{adj}A)^2 = \underbrace{A(\text{adj}A)}(\text{adj}A) = |A|I\,(\text{adj}A) = |A|(\text{adj}A)$$

$$\left| |A| \,\text{adj}A \right| = \left| \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \right|$$

$$|A|^3 \, |\text{adj}A| = -1$$

$$|A|^3 \, |A|^{3-1} = -1$$

$$|A|^5 = -1 \Rightarrow |A| = -1$$

$$-\text{adj}A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -1 & 0 & -2 \\ -2 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

Sum of ele $= -8$

14

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\left(2\tan^{-1} e^{x}-\frac{\pi}{2}\right)}{1+x^{2}} dx$ is equal to

(A) $\frac{\pi}{2}$

(B) $\frac{\pi^2}{4}$

(C) $e^{\frac{\pi^2}{2}}$

(D) 0 ✓

$$\int_0^{\pi/4} \left[\frac{2\tan^{-1}(e^{x})-\frac{\pi}{2}}{(1+x^2)} + \frac{2\tan^{-1}(e^{-x})-\pi/2}{(1+x^2)}\right] dx$$

$\cot^{-1} e^{x}$

$$\int_0^{\pi/4} \frac{2\left(\tan^{-1} e^{x}+\tan^{-1} e^{-x}\right)-\pi}{(1+x^2)}\, dx = 0$$

15

JEE 1

If z and w are two complex numbers satisfying $|z-1|=2$ and $|w-5|=3$, then the maximum value of $|z-4w|$ is

(A) 14 (B) 15

(C) 33 (D) 31

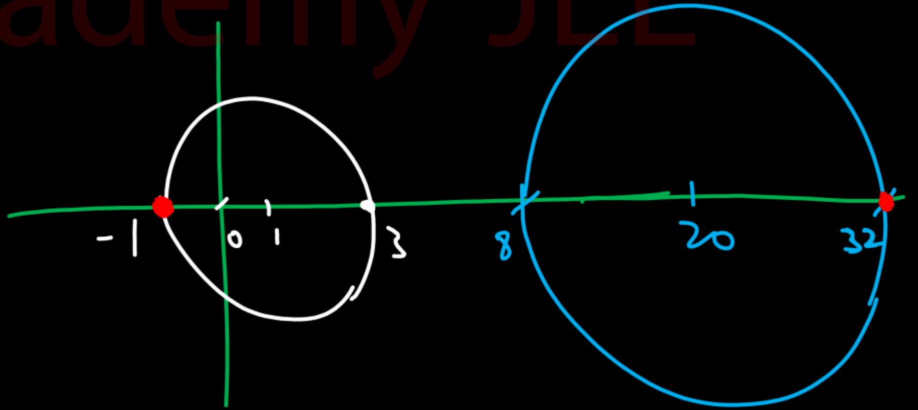
$|4w-20|=12$

$|z'-20|=12$

$|z-4w|$

$4w \to z'$

$|z-z'|$



16

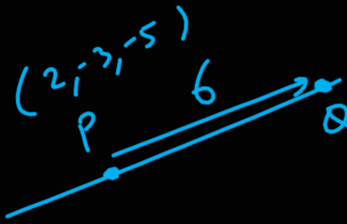
The point on the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+5}{-2} = \lambda$ at a distance of 6 from the point (2, −3, −5) is

(A) (3, −5, −3)

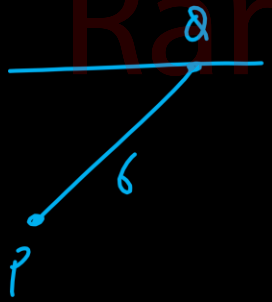
(B) (4, −7, −9) ✓

(C) (0,2, −1)

(D) (−3,5,3)



$Q(\lambda+2, -2\lambda-3, -2\lambda-5)$



$PQ = \sqrt{\lambda^2+4\lambda^2+4\lambda^2} = 6$

$9\lambda^2 = 36$

$\lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

17

JEE 1

If $I = \int \frac{dx}{x^4\sqrt{a^2+x^2}}$, then I equals

(A) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^3}\sqrt{a^2+x^2}\right] + C$

(B) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{3x^3}(a^2+x^2)^{\frac{3}{2}}\right] + C$

(C) $\frac{1}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{2\sqrt{x}}(a^2+x^2)^{\frac{3}{2}}\right] + C$

(D) $\frac{2}{a^4}\left[\frac{1}{x}\sqrt{a^2+x^2} - \frac{1}{x^3}\sqrt{a^2+x^2}\right] + C$

$$I = \int \frac{dx}{x^4 \cdot x\sqrt{a^2x^{-2}+1}}$$

Let: $a^2x^{-2}+1 = t^2$

$a^2(-\cancel{2}x^{-3}dx) = \cancel{2}t\,dt$

$$\int \frac{x^{-2}\cdot x^{-3}\,dx}{\sqrt{a^2x^{-2}+1}}$$

$$= \int \frac{\left(\frac{t^2-1}{a^2}\right)\left(-\frac{1}{a^2}\cancel{t}\,dt\right)}{\cancel{t}}$$

$$= \frac{1}{a^4}\left[\int (1-t^2)\,dt\right]$$

$$= \frac{1}{a^4}\left[t - \frac{t^3}{3}\right] + C$$

$$= \frac{1}{a^4}\left[\sqrt{a^2x^{-2}+1} - \frac{1}{3}\left(a^2x^{-2}+1\right)^{3/2}\right] + c$$

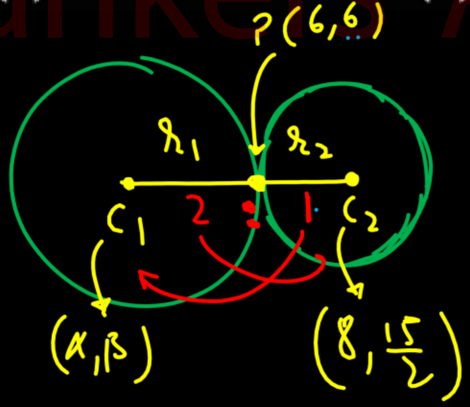
$$= \frac{1}{a^4}\left[\frac{\sqrt{a^2+x^2}}{x} - \frac{1}{3x^3}\left(a^2+x^2\right)^{3/2}\right] + c$$

18

Let the circles $C_1: (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2: (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6,6). If the point (6,6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2: 1, then $(\alpha+\beta) + 4(r_1^2 + r_2^2)$ equals

(A) 110 (B) 130

(C) 125 (D) 145



$$\begin{cases} \dfrac{16+\alpha}{3} = 6 \\ \Rightarrow \alpha = 2 \end{cases}$$

$$\begin{cases} \dfrac{15+\beta}{3} = 6 \\ \beta = 3 \end{cases}$$

Now: $r_1 = C_1P = \sqrt{16+9} = 5$

$r_2 = 5/2$

19

At how many points in the interval (0,2): $f(x) = x^2[2x] - x[x^2]$ is discontinuous (where $[x]$ represents greatest integer $\leq x$)

(A) 5 (B) 4

(C) 3 (D) infinite

$(0,2)$

$$f(x) = x^2[2x] - x[x^2]$$

$$\left\{\frac{1}{2}, 1, \frac{3}{2}\right\} \qquad \left\{1, \sqrt{2}, \sqrt{3}\right\}$$

DC DC DC DC

Check

$x=1$

$f(1) = (1)(2) - (1)(1)$
$= 1$

$f(1^+) = (1+h)^2(2) - (1+h)(1)$
$h \to 0$
$= 2 - 1 = 1$

$f(1^-) = (1-h)^2(1) - \cancel{(1-h)(0)}$
$h \to 0$
$= 1$

$\Rightarrow$ Continuous at $x=1$

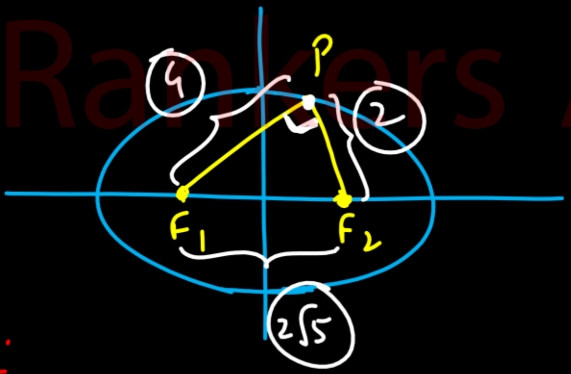
20

F_1, F_2 are two foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let P be a point on the ellipse such that $PF_1 = 2PF_2$, then area of $\Delta PF_1 F_2$ is –

(A) 3 (B) 4

(C) $\sqrt{5}$ (D) $\left(\frac{\sqrt{13}}{2}\right)$

$$\left\{\begin{array}{l} a^2 = 9 \rightarrow a = 3 \\ b^2 = 4 \rightarrow b = 2 \end{array}\right\}$$



$$PF_1 + PF_2 = 2(3)$$

$$2(PF_2) + PF_2 = 6$$

$$3(PF_2) = 6$$

$$\boxed{PF_2 = 2} \Rightarrow \boxed{PF_1 = 4}$$

$$F_1F_2 = 2ae$$

$$= 2(3)\sqrt{1-\frac{4}{9}}$$

$$= 2(\cancel{3})\frac{\sqrt{5}}{\cancel{3}}$$

$$= 2\sqrt{5}$$

$\Rightarrow 90^\circ$ at point $\boxed{P}$

$$\therefore \Delta = \frac{1}{2}(2)(4) = 4$$

if req.

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

21

Let f be a function defined on the interval $[0,2\pi]$ such that $\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\tan t\,dt$ and $f(0) = 1$. If the maximum value of f(x) is λ, find 8λ.

JEE 1

$$\int_0^x (f'(t) - \sin 2t)dt = \int_x^0 f(t)\cdot \tan t \cdot dt$$

$$(f'(x) - \sin 2x)(1) = 0 - (f(x)\cdot \tan x)(1)$$

Let: $\left\{ y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) \right.$

$$\Rightarrow \frac{dy}{dx} - \sin 2x = -y\tan x$$

JEE 1

$$\frac{dy}{dx} + (\tan x)y = (2\sin x\cos x)$$

$$I.F. = e^{\int \tan x\, dx}$$

$$= e^{\ln \sec x}$$

$$= \sec x$$

Sol^n of DE:

$$y(\sec x) = \int (2\sin x \cancel{\cos x})(\cancel{\sec x})\,dx$$

$$\boxed{y(\sec x) = -2\cos x + C}$$

$x = 0;\ y = 1$

$1 = -2 + C$

$\Rightarrow C = 3$

$\Rightarrow y\sec x = -2\cos x + 3$

$\Rightarrow y = (-2\cos^2 x + 3\cos x)$

$\Rightarrow y = -2\left(\cos^2 x - \frac{3}{2}\cos x\right)$

$y = -2\left(\left(\cos x - \frac{3}{4}\right)^2 - \frac{9}{16}\right)$

$y = 2\left(\frac{9}{16} - \left(\cos x - \frac{3}{4}\right)^2\right)$

$\Rightarrow y_{max} = \frac{9}{8}$

$\lambda = \frac{9}{8}$

$$\boxed{8\lambda = 9}$$

22

If $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ & $(I + A)^{20} - 19A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$, then value of $\alpha + \beta + \gamma + \delta$ is equal to (Here I is 2×2 unit matrix)

$$A^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow (I + A)^{20} = (I + 20A)$$

$$\Rightarrow (I + A)^{20} - 19A = \boxed{I + A} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \alpha + \beta + \gamma + \delta = 4$$

23

JEE 1

Let $S = 1 + 2.2 + 3.2^2 + 4.2^3 + \cdots + 100.2^{99}$. If sum can be expressed as $a(b)^c + 1$ where $a, b, c \in N$ & b is prime number then value of $\frac{2(c-a)}{b}$ is equal to

$$S = 1 + 2(2) + 3(2)^2 + 4(2)^3 + \cdots + 100(2)^{99}$$

$$2S = \boxed{}\ 1(2) + 2(2)^2 + 3(2)^3 + \cdots + 99(2)^{99} + 100(2)^{100}$$

$$-S = \left[1 + 2 + 2^2 + 2^3 + \cdots + 2^{99}\right] - 100(2)^{100}$$

$$-S = \frac{1(2^{100} - 1)}{(2-1)} - 100(2)^{100}$$

$$-S = -99(2)^{100} - 1$$

$$S = 99(2)^{100} + 1$$

$\therefore$ Ans:

$$\frac{2(100-99)}{2} = 1$$

24

Let f(x) be a real valued function such that

$f(x) = \frac{2x-1}{x-2}\ \forall x \in (2, \infty)$ and

$g(x) = \frac{x^2+1}{x} + \frac{(f(x))^2+1}{f(x)}\ \forall x > 2$, then minimum value of g(x) is ______ .

JEE 1

M-1 : calculate g(x)

↓

$g'(x) = 0$

↓

Critical points.

↓

min = ?

M-2 :

$$f(f(x)) = \frac{2f(x)-1}{f(x)-2}$$

$$= \frac{2\left(\frac{2x-1}{x-2}\right)-1}{\left(\frac{2x-1}{x-2}\right)-2}$$

$$= \frac{4x-\cancel{2}-x+\cancel{2}}{\cancel{2x}-1-\cancel{2x}+4} = \frac{3x}{3} = x$$

JEE 1

Now:

$$g(x) = \left(x + \frac{1}{x}\right) + \left(f(x) + \frac{1}{f(x)}\right)$$

$$g(f(x)) = \left(f(x) + \frac{1}{f(x)}\right) + \left(f(f(x)) + \frac{1}{f(f(x))}\right)$$

$$g(f(x)) = \left(f(x) + \frac{1}{f(x)}\right) + \left(x + \frac{1}{x}\right)$$

$$g(f(x)) = g(x)$$

$$\boxed{f(x) = x}$$

$$\frac{2x-1}{x-2} = x$$

$$2x - 1 = x^2 - 2x$$

$$x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{16-4}}{2}$$

$$x = 2 \pm \sqrt{3}$$

$$\because x > 2 \Rightarrow \boxed{x = 2 + \sqrt{3}}$$

$g(x)$ min at $x = 2+\sqrt{3}$

$$2\left(x + \frac{1}{x}\right)$$

$$2\left(2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}}\right)$$

$$= \boxed{8}$$

25

There are fifteen coupons numbered 1 to 15 in a bag. Three of them are selected at random. If it is given that their sum is odd and probability that all selected numbers are odd is ' p ', then 4p is equal to

$$\begin{cases} O \to 8 \\ E \to 7 \end{cases}$$

$$\frac{OOO}{EEO + OOO} = \frac{{}^{8}C_{3}}{{}^{7}C_{2}\,{}^{8}C_{1} + {}^{8}C_{3}} = \frac{1}{4} \Rightarrow 4p = 1$$