FIITJEE ALL INDIA TEST SERIES

JEE (Advanced)-2025
OPEN TEST – II
PAPER –2

TEST DATE: 13-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART - I

Section - A

$$Y = 0 \Rightarrow T = \frac{2u\sin\alpha}{g\cos 37^{\circ}}$$

$$\Rightarrow T = \frac{u \sin \alpha}{4}$$

Also at t = T, v is at 37° with normal on the plane i.e.

$$\tan 37^\circ = \frac{V_x}{-V_y}$$

$$\frac{3}{4} = \frac{u\cos\alpha - \frac{3}{5}gT}{-\left(u\sin\alpha - \frac{4}{5}gT\right)}$$

By solving $\tan \alpha = \frac{4}{9}$

Also,
$$T = \frac{1}{4}\sqrt{388} \sin \alpha = \sqrt{\frac{388}{4}} \times \frac{4}{\sqrt{97}} = 2 \sec \alpha$$

Velocity of particle just before it collides

$$v_v = u_v + a_v T = -8$$

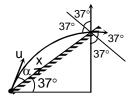
$$v_x = u_x + a_x T = 6$$

$$v = \sqrt{8^2 + 6^2} = 10 \text{ m/s}$$

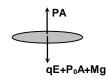
Velocity after collision = 10 m/s

$$\therefore T' = \frac{2 \times 10}{g} = 2 \sec$$

Total times in returning is 2 + 2 + 2 = 6 sec



2. A Sol.
$$PA = qE + P_0A + Mg$$
 $PV_0^{\gamma} = constant$



After displacing piston through small distance x in downward directions pressure changes. Let new pressure be P^\prime

So,
$$PV_0^{\gamma} = P'(V_0 - AX)^{\gamma}$$

$$PV_0^{\gamma} = P'V_0^{\gamma} \Biggl(1 - \frac{\gamma AX}{V_0}\Biggr)$$

$$\Rightarrow P = P' \left(1 - \frac{\gamma AX}{V_0} \right)$$

$$Mg + \frac{qE}{A} + P_0 = P' \Biggl(1 - \frac{\gamma AX}{V_0} \Biggr)$$

$$\omega = \sqrt{\frac{\left(\frac{qE+Mg}{A}+P_{0}\right)\gamma A^{2}}{MV_{0}}}$$

Sol.
$$\frac{dm}{dx} = kx$$

$$\Rightarrow \int_{0}^{m} dm = k \int_{0}^{\ell} x dx \Rightarrow k = \frac{2m}{\ell^{2}}$$

$$dI = x^2 dm = x^2 \frac{2m}{\ell^2} x dx$$

$$I = \frac{2m}{\ell^2} \int_0^\ell x^3 dx = \frac{m\ell^2}{2}$$

$$X_{cm} = \int xdm = \frac{2\ell}{3}$$

$$-(\text{mgsin30}^{\circ}) \ \frac{2\ell}{3} \theta = \frac{m\ell^2}{2} \alpha$$

$$\Rightarrow \alpha = -\frac{2g}{3\ell}\theta$$

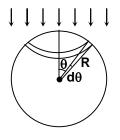
$$T=2\pi\sqrt{\frac{3\ell}{2g}}$$

Sol.
$$E.2\pi r = \pi r^2 \frac{dB}{dt}$$

$$\Rightarrow E = \frac{r}{2} \frac{dB}{dt}$$

$$rdq \cdot E = \frac{r^2}{2} dq \frac{dB}{dt}$$

$$d\tau = \frac{dB}{dt} \frac{r^2}{2} dq$$



$$\int d\tau = -\frac{dB}{dt} \int_{0}^{\pi^{2}} \frac{1}{2} R^{2} \sin^{2}\theta \cdot 2\pi R \sin\theta R d\theta \sigma \qquad (\because r = R \sin\theta)$$

$$\tau = \frac{dB}{dt} \frac{4}{3} \sigma \pi R^4$$

$$\int \tau dt = \frac{4\sigma\pi R^4}{3} \int\limits_{B}^{0} dB$$

$$I\omega - 0 = \frac{4}{3}B\sigma\pi R^4$$

Conservation of angular momentum

$$I\omega = I\frac{v}{R} + mvR$$

$$\Rightarrow v = \frac{I \omega R}{I + mR^2} = \frac{5QBR}{21m}$$

Sol.
$$F - T - \mu mg \cos \theta - mg \sin \theta = ma_1$$

$$\Rightarrow$$
 106 - T - 40 = 4a₁ ...(i)

$$10g \sin \theta - T - f_S = 10a_C$$

$$\Rightarrow 60 - T - f_S = 10 a_C \qquad ...(ii)$$

$$\Rightarrow 60 - T - f_S = 10 a_C$$

$$f_S R + r T = I\alpha$$

$$\Rightarrow 2f_S + T = 10\alpha \qquad ...(iii)$$

$$r\alpha - a_C = a_1 \qquad \qquad \dots \text{(iv)}$$

and
$$a_C = R\alpha$$
 ...(v)

Solving the equations $a_C = 2 \text{ m/s}^2$, $a_1 = -1 \text{ m/s}^2$

$$Sol. \qquad \epsilon - i_1 r + \epsilon - i_1 r = 0$$

$$\Rightarrow i_1 = \frac{\varepsilon}{r} = 5A$$

$$\varepsilon - i_2 r - i_2 R - i_2 R = 0 \Rightarrow i_2 = \frac{\varepsilon}{r + 2R} = 1A$$

$$I = I_1 + I_2 = 6A$$

Charges on capacitors 2C, C is zero since current in resistance R is zero

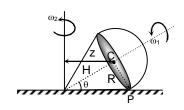
$$\therefore \varepsilon - i_2 R = \frac{q}{4C}$$

$$q = 16\mu c$$

Sol. Velocity of point P is
$$\omega_2 \times 2R - \omega_1 R = 0$$

$$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{1}{2}$$

$$f_S = \frac{mv_0^2}{z} = \frac{2}{3} \frac{mv_0^2}{R}$$



Sol. Velocity of particle after collision in vertical direction is 10 m/s
$$\mu N \Delta t = \mu \times 20$$
 m For disc, $\mu N \Delta t = m v_C$

$$\therefore v_C = 4 \text{ m/s}$$

$$\mu N \Delta t \frac{R}{2} = \frac{MR^2}{2} (\omega - 0)$$

$$\Rightarrow \omega = 20 \text{ rad/s}$$

Sol. Initial tension in spring
$$kx = \frac{4mg}{7}$$

FBD of block just after string is cut Since, v = 0

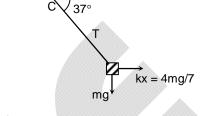
.. acceleration along string is zero

$$T = k\cos 37^{\circ} + mgsin37^{\circ} = \frac{37}{7}N$$

Acceleration is only perpendicular to spring

$$ma = mg \cos 37^{\circ} - kx \sin 37^{\circ}$$

$$\Rightarrow$$
 a = $\frac{32}{7}$ m/s²



10. A, C

Sol. Using conservation of energy and momentum when particle leaves end B, velocity of the block will be zero.

At lower most point vertical acceleration is $\frac{v^2}{R}$

∴
$$5v_1 = 10v_2$$

and
$$5 \times g \times 6 = \frac{1}{2} \times 5v_1^2 + \frac{1}{2} \times 10 \times v_2^2$$

Solving equations $v_1 = \sqrt{80}$, $v_2 = \sqrt{20}$

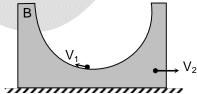
$$v_{P,B} = \sqrt{80} + \sqrt{20}$$

$$a_{P,B} = \frac{v_{P,B}^2}{R} = \frac{(\sqrt{80} + \sqrt{20})^2}{2}$$

$$\vec{a}_{P,g} = \vec{a}_{P,B} + \vec{a}_{B,g}$$

 $\vec{a}_{B,g} = 0$ (when particle is at lowest position)

$$\vec{a}_{P,q} = \vec{a}_{P,B} = 90 \text{ m/s}^2$$



Section - B

Sol.
$$T_1 = \mu M_B g$$

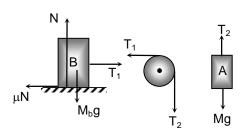
 $T_2 = mg$

due to friction on pull

in limiting case ,when block B is about to slide

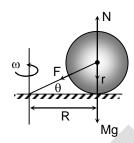
$$T_2 = T_1 e^{\mu \frac{\pi}{2}}$$

Solving equation m = 2kg



Sol.
$$F \cos \theta = m\omega^2 R$$

 $F \sin \theta + mg = N$
 $\therefore N = mg + m\omega^2 r = 36 N$

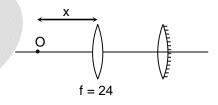


Sol.
$$\frac{q}{C} - \epsilon + \frac{q}{2C} = 0$$

$$q = \frac{2}{3}C\epsilon$$
 Loss energy H = $q\epsilon - \Delta U$
$$= \frac{3}{4}\frac{q^2}{C}$$
, where $C = 4\pi\epsilon_0 \times 9 = 30~\mu J$

Sol. Standing wave y = 2Asinkx cos
$$\omega t$$
 Has energy E = $\mu \omega^2 A^2 \ell$

Sol.
$$\frac{1}{v} - \frac{1}{-x} = \frac{1}{24}$$
$$\Rightarrow \frac{1}{v} = \frac{1}{24} - \frac{1}{x}$$
$$\frac{1}{v} = -\frac{1}{\left(\frac{24-x}{24x}\right)}$$



Object distance for silvered lens is $(14-x) + \frac{24x}{(24-x)}$ for image to be on object O, this distance

must be equal to equivalent radius of mirror. For (Reflecting lens is effectively mirror)

$$-\frac{2}{R_{eq}} = 2\left(\frac{3}{2} - 1\right)\left(\frac{1}{32} - \frac{1}{-32}\right) - \frac{3}{-32}$$

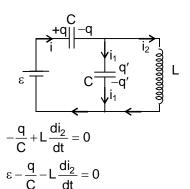
$$\Rightarrow$$
 R_{eq} = -16 cm

⇒
$$R_{eq} = -16 \text{ cm}$$

∴ $16 = (14 - x) + \frac{24x}{(24 - x)}$

$$\Rightarrow$$
 x = 6 cm

$$\epsilon - \frac{q}{C} - \frac{q'}{C} = 0$$



$$\begin{array}{c|c}
C & S \\
+\varepsilon C/2 & -\varepsilon C/2 & S \\
\hline
& C & -\varepsilon C/2 & S \\
\end{array}$$

Solving these equations and at t = 0, $i_2 = 0$ and $i_2 = \varepsilon \sqrt{\frac{C}{2l}} \sin \omega t$

$$\therefore i_{2(max)} = \epsilon \sqrt{\frac{C}{2L}} = 7A$$

- 17.
- Sol. Consider some resistance to the voltmeter and applying Kirchhoff's law
- 18. 4

Sol.
$$K_{emax} = h\nu - \phi = \frac{6.6 \times 10^{-34} \times 7.27 \times 10^{14}}{1.6 \times 10^{-19}} - 1 = 2 \text{ eV}$$

Maximum potential difference is 2V

$$U = \frac{1}{2}CV^2$$

Chemistry

PART - II

Section - A

- 19. A
- Sol. AB_2L_3 has linear shape but AB_3L_2 has T-shape.
- 20. B
- Sol. Total moles of air $=\frac{PV}{RT} = \frac{2 \times 49.26}{0.0821 \times 300} = 4$ mole

$$n_{O_2} = 4 \times 0.2 = 0.8 \text{ mole}$$

Moles of 'C' = 1

$$C + O_2$$

$$\longrightarrow$$
 CO + CO₂

1 mole 0.8 mole

x mole y mole

Apply POAC for C-atom

$$1 \times 1 = x \times 1 + y \times 1$$
 or $x + y = 1$

Also, apply POAC for O-atoms,

$$0.8 \times 2 = x \times 1 + y \times 2 \text{ or } x + 2y = 1.6$$

On solving Equation (1) and (2), we get

$$y = 0.6$$
 and $x = 0.4$

So, total amount of heat produced

$$= -0.4 \times (26) - 0.6 \times (94)$$

$$=-10.4-56.4$$

$$= -66.8 \text{ kcal}$$

- 21. A
- Sol. Since, decomposition of N₂O on hot platinum surface follows ZERO order kinetics:

So,
$$t_{1/2} = \frac{a}{2k_0}$$

or
$$a = (2k_0) \times t_{1/2}$$

$$\log a = \log(2k_o) + \log t_{1/2}$$

$$y = C + mx$$

$$m = 1 \text{ or } \tan \theta = 1 \text{ or } \theta = 45^{\circ}$$

$$C = log 2k_o = +ve$$

22. D

Sol.
$$\alpha = \frac{\theta}{\ell(\text{dm}) \times C(g/\text{ml.})}$$

$$-92.4^{\circ} = \frac{-27.7^{\circ}}{(1 \text{ dm}) \times \text{C}}$$

$$C = \frac{27.7}{1 \times 92.4} = 0.2997 \text{ g/ml}$$

So, mass of fructose dissolved in 100 mL solution

- $= 0.2997 \times 100$
- = 29.97 g.

Sol. Since, the crystal should be electrically neutral. So, empirical formula of the solid should be: $\left(\mathsf{AI}^{3+} \right)_{4 \times \frac{1}{2}} \left(\mathsf{Zn}^{2+} \right)_{8 \times \frac{1}{2}} \left(\mathsf{S}^{2-} \right)_4 \Rightarrow \mathsf{ZnAI}_2 \mathsf{S}_4$

Since, Zn^{2+} ions are present in tetrahedral void, so, its coordination number = 4. Also, Al^{3+} ions are present in octahedral void, so, its coordination number = 6.

Sol. Since, the solution contains a weak base (NH_4OH) and salt of this weak base with strong acid, i.e. $(NH_4)_2 SO_4$. So, it is a basic buffer.

By Henderson's Equation:

$$pOH = pK_b + log \frac{2 \times 0.01}{0.02} = pK_b$$

$$\therefore pH = 14 - pOH$$

(Q)

$$pH = 14 - (5 - \log 2) = 9 + \log 2$$

Since, it's a buffer, so dilution does not affect the pH.

Sol. (A)
$$XeF_6 + RbF \longrightarrow Rb^+ + XeF_7^-$$

(B)
$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$$

(C)
$$2XeF_6 + SiO_2 \longrightarrow 2XeOF_4 + SiF_4$$

(D)
$$XeF_6 + 6HCI \longrightarrow 6HF + Xe + 3CI_2$$

Sol.
$$H_3C$$
 H_3C $H_$

Sol.
$$r = \frac{2[A]}{3+[A]}$$

So, integrated rate equation of the above reaction is

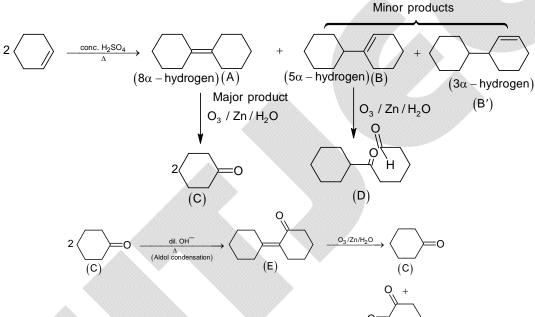
$$t = \frac{1}{2} \left[\left[A \right]_o - \left[A \right] + 3 \ell n \frac{\left[A \right]_o}{\left[A \right]} \right]$$

So,
$$t_{1/2} = \frac{1}{4} [A]_o + \frac{3}{2} \ell n2$$

$$t_{3/4} = \frac{3\big[A\big]_o}{8} + 3\ell n2 = \frac{3}{8} \times 8 + 3 \times 0.7 = 5.1 \text{ min utes}$$

A, B, C 28.

Sol.



Section - B

29. Sol. 8

has 3 planes of symmetry, i.e. x = 3

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(Terephthalic acid)

$$\begin{array}{c} \text{CH}_3\\ \text{H} \longrightarrow \text{CI}\\ \text{H} \longrightarrow \text{CI} \end{array}$$

It has one plane of symmetry and one centre of symmetry. So, y = 2.

(Meso-2,3-dichlorobutane)

Stereocentres

ĊH₃

Stereocentres (Pent-3-en-2-ol)

So, z = 3.

30.

Sol.
$$x= 2, y= 2, z= 4.$$

n = 1, m = 2

31.

Sol. Freezing point in Kelvin = (-1.8 m + 273) K

Boiling point in Kelvin = (373 + 0.5 m) K

Difference of boiling point and freezing point = 100 + 2.3 m = 100.46

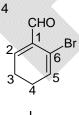
∴ M = 0.2

Now,
$$0.2 = \frac{P \times 1000}{60 \times 500}$$

∴ P = 6 gram

32. Sol.

For More Ma



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IV

Sol. (i) Na – Hg + H₂O
$$\longrightarrow$$
 NaOH + Hg + $\frac{1}{2}$ H₂

(ii)
$$Na_2CO_3 \xrightarrow{\Delta} X$$

(iii)
$$4NaOH + ZnCl_2 \longrightarrow Na_2ZnO_2 + 2NaCl + 2H_2O$$

$$(iv) \qquad \text{CuSO}_4 + \text{Na}_2 \text{CO}_3 + \text{H}_2 \text{O} \longrightarrow \qquad \underbrace{\text{CuCO}_3.\text{Cu}\left(\text{OH}\right)_2}_{2}$$

basic copper carbonate

(v) In case of alkali metals, for performing flame test, chlorides are preferred over carbonates as chlorides are more volatile.

Sol.
$$N_{H_2O_2} = \frac{22.4}{5.6} = 4$$

Let, mass of pure H₂S present in impure sample = 'w' gm

Then
$$4 \times 5 \times 10^{-3} = \frac{w}{\left(\frac{34}{2}\right)} \Rightarrow w = 0.34 \text{ gm}$$

So, % of pure H_2S in impure sample = $\frac{0.34}{0.40} \times 100 = 85\%$

$$\therefore 17x = 85$$

Sol. (i)
$$NH_4CI + Na_2B_4O_7 \xrightarrow{\text{red hot}} (BN)_y + NaCI + H_2O$$

(ii)
$$\mathsf{BCl}_3 + \mathsf{NH}_4\mathsf{Cl} \xrightarrow{140^{\circ}\mathsf{C}} \mathsf{B}_3\mathsf{N}_3\mathsf{H}_3\mathsf{Cl}_3 \xrightarrow{\mathsf{NaBH}_4} \mathsf{B}_3\mathsf{N}_3\mathsf{H}_6$$

(iii)
$$KBF_4 + K \xrightarrow{\Delta} B + KF$$

(iv)
$$B_2O_3 + Cr_2(SO_4)_3 \xrightarrow{\Delta} Cr(BO_2)_3$$

(v)
$$B_2H_6 \xrightarrow{CH_3NH_2} \left[BH_2 \left(CH_3NH_2\right)_2\right]^+ \left[BH_4\right]^-$$

(vi)
$$B_2H_6 \xrightarrow{(CH_3)_3N} 2BH_3.(CH_3)_3 N$$

Sol. Average velocity =
$$\sqrt{\frac{8RT}{\pi M}} = 4 \times 10^2$$

$$\therefore RT = 2\pi M \times 10^4$$

Total kinetic energy of 'He'
$$=\frac{3}{2}$$
nRT $=\frac{3}{2} \times \frac{6}{4} \times$ RT $=\frac{9$ RT $=\frac{3}{4} \times \frac{6}{4} \times \frac{1}{4} \times \frac{1}{4$

Total kinetic energy of 'He' =
$$\frac{9}{4} \times 2\pi M \times 10^4$$

$$= \frac{9}{4} \times 2\pi \times \left(4 \times 10^{-3}\right) \times 10^4$$

$$=180\pi$$
 Joule

Total kinetic energy of 'Ne²⁰' =
$$\frac{3}{2}$$
nRT = $\frac{3}{2} \times \frac{12}{20} \times$ RT

$$\begin{split} &= \frac{9}{10} \times 2\pi M \times 10^4 \\ &= \frac{9}{10} \times 2\pi \times \left(20 \times 10^{-3}\right) \times 10^4 \\ &= 360\pi \text{ Joule} \end{split}$$

Average kinetic energy of mixture per mole $=\frac{360\pi+180\pi}{1.5+0.6}$

$$= \frac{540}{2.1} \times 3.14$$

= 807.42 J

= 0.80742 kJ

So, x = 0.8

 $\therefore 10x = 8$.



Mathematics

PART - III

Section - A

- 37. D
- Sol. Favorable cases: For first diagonal we have 12 choices them for the second to be skew we have 5 choices. So number of favorable choices is $\frac{12 \times 5}{2} = 30$

Required probability =
$$\frac{30}{^{12}C_2} = \frac{5}{11}$$

- 38. C
- Sol. Since $\left[\sqrt{2066}\right] = \left[\sqrt{2067}\right] = \left[\sqrt{2068}\right] = \left[\sqrt{2069}\right] = 45$ and 2023 + 45 = 2068
- 39. C
- Sol. Let $z = -x \frac{\pi}{6}$, then $z \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ and $2z \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Then,
$$y = \cot z + \tan z + \cos z = \frac{2}{\sin 2z} + \cos z$$

Since, both $\frac{2}{\sin 2z}$ and $\cos z$ are monotonic decreasing in this case

Then
$$y_{max} = \frac{2}{\sin{\frac{\pi}{3}}} + \cos{\frac{\pi}{6}} = \frac{11\sqrt{3}}{6}$$

- 40. A
- Sol. $x^a = y^b = z^c = \lambda$, now x, y, z are in Geometric Progression

 $\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ therefore a, b, c are in Harmonic Progression

By G.M. > H.M.; $\sqrt{ac} > b \text{ for } a^3, b^3, c^3$

A.M. > G.M.;
$$\frac{a^3 + c^3}{2} > (\sqrt{ac})^3 > b^3$$

- 41. A, D
- Sol. Draw variable lines of slope 4x 3y that passes through the points on or inside the triangle
- 42. A, C
- Sol. $\left| \frac{1}{z} + z_0 \right| = \frac{1}{|z|}$ multiply both side by $\left| \frac{z}{z_0} \right|$, we have $\left| z \left(-\frac{1}{z_0} \right) \right| = \frac{1}{|z_0|}$

Circle with centre $\frac{-1}{z_0}$ and radius $\frac{1}{|z_0|}$, however we have to exclude the point z=0 from the locus

- 43. B, D
- Sol. If PA + PB is minimum \Rightarrow Area of \triangle PAB is minimum \Rightarrow C is mid-point of AB \Rightarrow Equation of AB is x + y = 10

Sol. Since,
$$\log_2 x - 1 \ge 0$$
; Let $\sqrt{\log_2 x - 1} = t$, we have $t - \frac{3}{2}t^2 + \frac{1}{2} > 0$ and $t \ge 0$
Solution of the above inequality is $0 \le t < 1 \Rightarrow 0 \le \log_2 x - 1 < 1 \Rightarrow 2 \le x < 4$

Sol.
$$f(x) = \frac{x^2}{2} + 3$$
 if x is even and $f(x) = \frac{x+1}{2}$ is x is odd

Sol. If
$$\lambda \ge 6$$
, then sum of solutions = 5π and if $\lambda \le -6$, then sum of solutions = 3π

Section - B

Sol.
$$\left(x + \frac{1}{x^{2023}} \right) \left(1 + x^2 + x^4 + \dots + x^{2022} \right) = 2024$$

$$\Rightarrow \left(x + x^3 + x^5 + \dots + x^{2023} \right) + \left(\frac{1}{x^{2023}} + \frac{1}{x^{2021}} + \dots + \frac{1}{x} \right) = 2024$$

$$(x + x^3 + x^5 + \dots + x^{2023}) + (\frac{1}{x} + \frac{1}{x^3} + \dots + \frac{1}{x^{2023}}) \ge 2 \times 1012 = 2024$$

Where the equality holds if and only if
$$x = \frac{1}{x}$$
, $x^3 = \frac{1}{x^3}$,, $x^{2023} = \frac{1}{x^{2023}} \Rightarrow x = 1$

Sol.
$$|z_1| = |z_2| = 3$$
 and $z_1\overline{z}_2 + \overline{z}_1z_2 = -9 \Rightarrow |z_1\overline{z}_2| = 9$

Let
$$z_1\overline{z}_2 = 9(\cos\theta + i\sin\theta)$$
, then $\overline{z}_1z_2 = 9(\cos\theta - i\sin\theta)$

$$\Rightarrow 18\cos\theta = -9 \Rightarrow \cos\theta = -\frac{1}{2}$$

$$z_1\overline{z}_2=-9\omega$$
 and $\overline{z}_1z_2=-9\omega^2$ where ω is cube root of unity other than 1

$$log_3 \left| \left(z_1 \overline{z}_2 \right)^{2023} + \left(\overline{z}_1 z_2 \right)^{2023} \right| = 2 \times 2023 = 4046$$

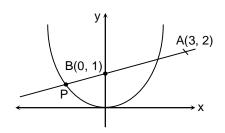
Sol.
$$f(x) = \sqrt{(x-3)^2 + (x^2-2)^2} - \sqrt{x^2 + (x^2-1)^2}$$

Let P be any point on the parabola $y = x^2$ and A(3, 2), B(0, 1)

By Δ inequality $|PA - PB| \leq AB$

$$|PA - PB| \le \sqrt{10}$$

$$\therefore k = \sqrt{10}$$



- 50.
- Sol. Locus of I is the tangent at the vertex \Rightarrow Locus is $x = \pm 3$
- 51. 8

Sol.
$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 62 \implies \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -10 \implies \vec{c} = -4\vec{a} = -2\vec{b}$$

52.

Sol.
$$ln(1-t^2) = \sum_{r=1}^{\infty} \left(-\frac{t^{2r}}{r}\right) \Rightarrow \int_{0}^{1} f(x) dx = \sum_{r=1}^{\infty} \frac{-1}{2r(r+1)} = -\frac{1}{2}$$

- 53.
- Sol. Let y = mx + c be the tangent $\Rightarrow x^4 2x^2 x (mx + c) = (x^2 + ax + b)^2$ $\Rightarrow a = 0, b = -1, m = -1, c = -1$ $\Rightarrow x + y + 1 = 0$ is tangent to the curve at (-1, 0) and (1, -2) $\Rightarrow |\alpha_1| + |\alpha_2| + |\beta_1| + |\beta_2| = 4$
- 54.
- Sol. A and B are (1, 23) and (2, 1, 2) respectively

If AC = d_1 and BC = d_2 , then volume of tetrahedron = $\frac{1}{6}$ · AB $d_1d_2 = \frac{\sqrt{3}}{6}d_1d_2$

Also, $CD^2 = d_1^2 + d_2^2 + 3 \implies d_1^2 + d_2^2 = 24$

Using AM, GM, $\frac{d_1^2 + d_2^2}{2} \ge d_1 d_2 \implies d_1 d_2 \le 12 \implies V = 2\sqrt{3}$