FIITJEE RBT-10 for (JEE-Advanced)

BATCH – Two Year CRP325 batches

PHYSICS, CHEMISTRY & MATHEMATICS

QP CODE: 100971 **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 186**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Part.

(i) **PART–A (01–08)** contains (8) Multiple Choice Questions which have **One or More Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(ii) **Part-A (09-12) –** This section contains Two (02) List-Match Sets, each List-Match set has Two (02) Multiple Choice Questions. Each List-Match set has two lists: List-I and List-II. FOUR options are given in each Multiple Choice Question based On List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question. Each question carries **+3 Marks** for correct combination chosen and **-1 marks** for wrong options chosen.

(iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :____

Batch :____________________ **Date of Examination :**____________________

Enrolment Number :____

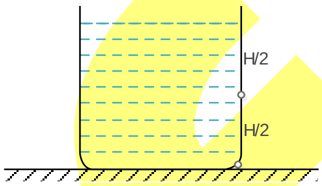
SECTION – I : PHYSICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

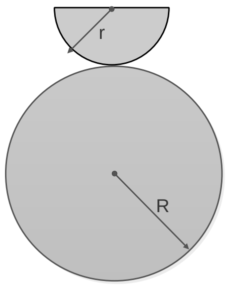
1. There is a cylindrical container of height H and base area A. It is filled with a fluid and two small holes of area a each are made at mid-height and at bottom of the container. $\frac{A}{a}\sqrt{\frac{H}{g}} = 600\,s$. Then



(A) Time taken to half-empty the tank lies between 150 s and 160 s
(B) Time taken to half-empty the tank lies between 155 s and 170 s
(C) Time taken to empty the tank lies between 760 s and 775 s
(D) Time taken to empty the tank lies between 755 s and 770 s

2. Choose the correct statement about X-rays
(A) Probability of K_α radiation is more than K_β radiation.
(B) A K_α photon is more energetic than a K_β photon.
(C) A photon of cutoff wavelength is least energetic.
(D) As we move down the periodic table, the energy of K_α X-ray photon goes on decreasing.

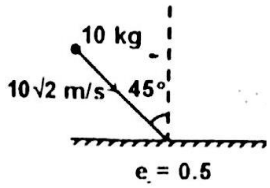
3. A solid hemisphere of radius r = 20 cm is placed on top of a fixed, very rough sphere of radius R in equilibrium such that curved surface of hemisphere is in contact with fixed sphere. There may not be any slipping between the surfaces. A very slight angular displacement is given to the hemisphere. Then



(A) Hemisphere cannot oscillate if $\frac{r}{R} < 0.2$
(B) Hemisphere cannot oscillate if $\frac{r}{R} > 0.6$
(C) If $\frac{r}{R} = 0.2$ then time period of oscillation is $0.24\pi\,s$
(D) If $\frac{r}{R} = 0.5$ then time period of oscillation is $\pi\,s$

Space For Rough Work

4. A ball of mass 10 Kg hits a smooth horizontal surface with a speed of $10\sqrt{2}\,m/s$ at an angle 45° as shown in figure. The co-efficient of restitution between the ball and the surface is 0.5 and the ball remains in contact with the surface for 0.1 s. $\vec{F}$ is the instantaneous force exerted by the surface on the ball at any time t during the collision.



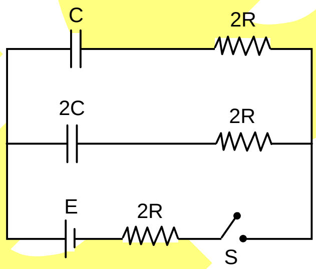
(A) The speed with which ball rebounds is $\sqrt{125}\,m/s$

(B) During collision $|\vec{F}|$ may be less than or equal to 1800 N

(C) During collision $|\vec{F}|$ may be less than or equal to 100 N

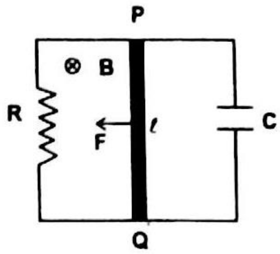
(D) The average force exerted by ground during collision is 1600 N

5. In the circuit shown in figure, switch S is closed at time t = 0. Select the correct statement(s)



(A) rate of increase of charge is same in both capacitors at t = 0
(B) ratio of charge stored in C to that in 2C at any instant is 1:2
(C) time constant of both the capacitors are equal
(D) steady state charge in capacitors C and 2C are in ratio 1:2

6. A conducting rod PQ of length l and mass m is dragged with a constant force F along two parallel smooth rails separated by a distance l. There is a magnetic field B in the region as shown in the adjacent figure. Then choose the correct statement(s). [Also, $m = B^2L^2C$ and at t = 0, velocity = 0].



(A) terminal velocity of rod, $V_T = \frac{2FR}{B^2l^2}$

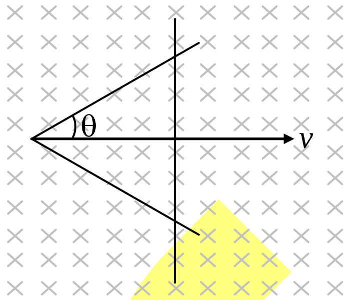
(B) terminal velocity of rod, $V_T = \frac{FR}{B^2l^2}$

(C) Maximum charge on the capacitor, $q_{max} = \frac{FCR}{Bl}$

(D) At t = 2RC, current through rod is $\frac{F}{B\ell}\left(1-\frac{1}{2e}\right)$

Space For Rough Work

7. Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails start at the vertex at $t = 0$ and moves with a constant velocity *v* along them as shown. A magnetic field *B* is directed into the page. The induced emf in the circuit at any time *t* is proportional to

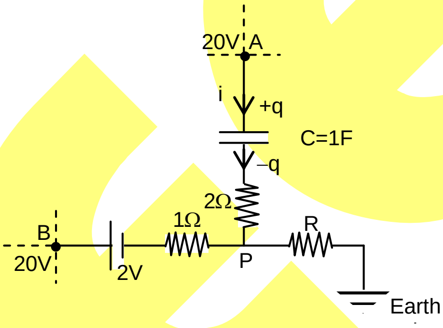


(A) t^0 (B) t

(C) v (D) v^2

8. The network shown in the figure is part of some bigger circuit. Charge on capacitor (C = 1 F) at any time t is q = 3(1 - e^{-t}) in coulombs. If at t = 0 the potentials of point A and B is 20 V, then at t = 0 match the following.

(A) current in branch AP is 3A
(B) current through R is 7A
(C) value of R is 2Ω
(D) value of R is 1Ω

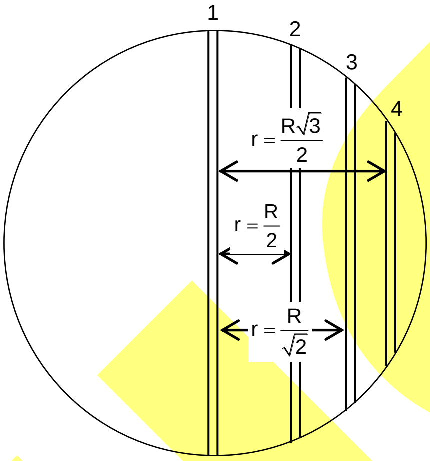


Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Consider a planet of radius R having density ρ. The four tunnels (1, 2, 3 and 4) are dug side at the distances from its centre are r = 0, $r = \frac{R}{2}$, $r = \frac{R}{\sqrt{2}}$ and $r = \frac{R\sqrt{3}}{2}$ respectively as shown. Four objects each of mass 1 kg are dropped inside the tunnel from the surface of plant at t = 0.



List-I gives the above four tunnels while List-II the magnitude of some quantity.

List – I		List – II	
(I)	Tunnel '1'	(P)	0
(II)	Tunnel '2'	(Q)	$\frac{2\pi G\rho R}{3}$
(III)	Tunnel '3'	(R)	$\frac{2\sqrt{2}\pi G\rho R}{3}$
(IV)	Tunnel '4'	(S)	$\frac{2\pi G\rho R}{\sqrt{3}}$
		(T)	$\frac{4\pi G\rho}{3}R$

9. The normal reaction between objects and mid of tunnels, the correct match is
(A) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ S
(B) I $\rightarrow$ S, II $\rightarrow$ R, III $\rightarrow$ Q, IV $\rightarrow$ P
(C) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ S
(D) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ S, IV $\rightarrow$ R

10. The magnitude of maximum acceleration of the objects in the tunnels
(A) I $\rightarrow$ T, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ Q
(B) I $\rightarrow$ S, II $\rightarrow$ T, III $\rightarrow$ R, IV $\rightarrow$ Q
(C) I $\rightarrow$ T, II $\rightarrow$ S, III $\rightarrow$ S, IV $\rightarrow$ R
(D) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ T

Space For Rough Work

List Match Set (11-12)

11. A ball is attached to a string and moves in vertical circle. There is no resistive force. Match the column-I to column-II.

List – I		List – II	
(I)	Minimum	(P)	Tension in the string when ball is at the lowest point.
(II)	Maximum	(Q)	Tension in the string when ball is at the highest point.
(III)	Towards the centre	(R)	The angle between tension and weight of ball when string becomes horizontal.
(IV)	Perpendicular	(S)	The direction of tension in the string
		(T)	Magnitude of tangential acceleration when string becomes horizontal.
		(U)	Magnitude of centripetal acceleration when string becomes horizontal.

(A) I $\to$ Q, II $\to$ P, III $\to$ U, IV $\to$ RS
(B) I $\to$ T, II $\to$ R, III $\to$ SU, IV $\to$ Q
(C) I $\to$ Q, II $\to$ PT, III $\to$ SU, IV $\to$ R
(D) I $\to$ Q, II $\to$ PT, III $\to$ S, IV $\to$ U

12. A small bob of mass m = 0.01 kg is hanging from the roof of a moving car in equilibrium making angle θ with vertical. If initial velocity of car was zero and acceleration of car is a = 2 m/s^2.

List – I		List – II	
(I)	Work done by tension in ground frame in first 10 second	(P)	20 m/s
(II)	Work done by tension in the frame of car in first 10 second	(Q)	2J
(III)	Velocity of bob in ground frame after 10sec	(R)	0J
(IV)	Work done by gravity	(S)	–ve
		(T)	+ve
		(U)	10 m/s

(A) I $\to$ QT, II $\to$ R, III $\to$ PT, IV $\to$ R
(B) I $\to$ QR, II $\to$ S, III $\to$ PT, IV $\to$ Q
(C) I $\to$ Q, II $\to$ P, III $\to$ S, IV $\to$ T
(D) I $\to$ U, II $\to$ P, III $\to$ S, IV $\to$ R

Space For Rough Work

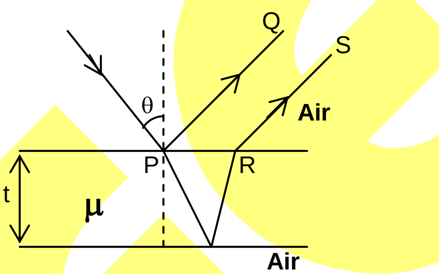
(PART – B)

(Numerical Type)

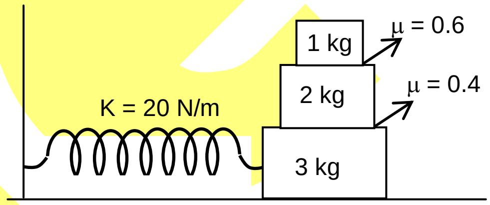
Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. Two spherical bodies A (of radius 6 cm) and B (of radius 36 cm) are at temperatures T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500nm. Considering them to be black bodies, the ratio of the rate of total energy radiated by A and B will be

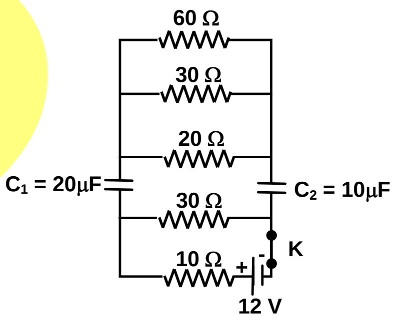
2. Rays PQ and RS (Wavelength λ) interfere constructively for small θ and corresponding minimum thickness of the slab is $\dfrac{n\lambda}{12\mu}$,then n =



3. What can be the maximum amplitude (in m) of the system so that there is no slipping between any of the blocks

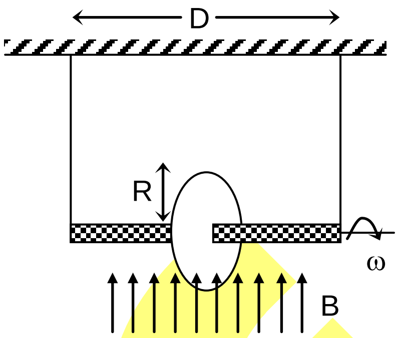


4. For the circuit arrangement shown in the figure, find the potential difference (in V) across C_2 in the steady state condition.

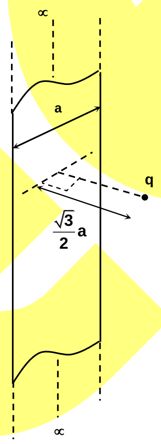


Space For Rough Work

5. The axle of a circular wheel of radius R is held horizontally by two identical strings of equal lengths separated by a distance D. The tension in each string is T_0. The rim of the wheel carries a total charge + Q distributed uniformly on it. The wheel is vertical and is kept in a uniform vertical magnetic field $\vec{B}$. It is now rotated at an angular speed ω. The string break at a tension of $3T_0/2$. If the maximum possible value of ω at which the wheel can be rotated without breaking a string is $\frac{4KT_0D}{5QR^2B}$. Then find the value of K.



6. There is an infinite long straight surface having width 'a'. A point charge q is placed at a perpendicular distance $\frac{\sqrt{3}}{2}a$ from the surface symmetrically as shown in the figure. If the flux linked with this infinitely long surface due to charge q is $\frac{q}{k\varepsilon_0}$. Then find the value of k.



Space For Rough Work

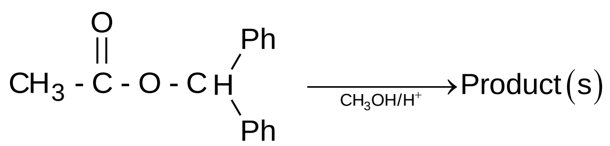
SECTION – II : CHEMISTRY

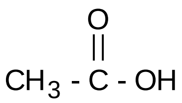
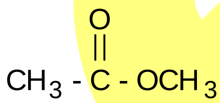
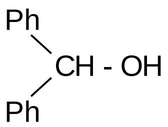
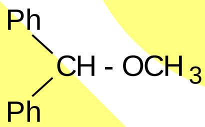
(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Which statements about corrosive sublimate($HgCl_2$) are correct?
(A) It is prepared by heating mercury in chlorine
(B) It oxidizes stannous chloride
(C) It is highly poisonous
(D) It sublimes on heating

2. 

(A) 
(B) 
(C) 
(D) 

3. Select the correct match
(A) $Fe^{3+} + [Fe(CN)_6]^{4-} \rightarrow$ Blue colour ppt.
(B) $Fe^{3+} + [Fe(CN)_6]^{3-} \rightarrow$ Red Brown colouration
(C) $Fe^{2+} + [Fe(CN)_6]^{3-} \rightarrow$ Blue colour ppt.
(D) $Fe^{2+} + [Fe(CN)_6]^{4-} \rightarrow$ Red Brown colouration

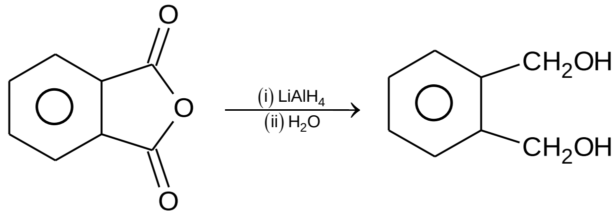
4. Let the height of hcp unit cell is ‘h’. The height of octahedral voids form the base is
(A) $\frac{h}{2}$
(B) $\frac{h}{3}$
(C) $\frac{h}{4}$
(D) $\frac{3h}{4}$

5. The significance of Heisenberg uncertainty principle in the atom is/are
(A) it rules out the concept of orbit
(B) it rules out the existence of nucleus
(C) it introduces the probability of position and momentum of an electron in an atom
(D) it introduces the probability of finding e^- in a given orbit

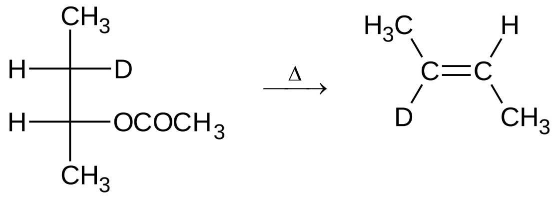
Space For Rough Work

6. Which of the following is(are) incorrect?

(A)



(B)



(C) $Ph-CH=CH-CHO \xrightarrow{LiAlH_4} PhCH=CH-CH_2OH$

(D) $Me-\overset{\overset{\large O}{||}}{C}-OMe \xrightarrow{NaBH_4} MeCH_2OH + MeOH$

7. Select correct statement(s)

(A) liquefication of gases in a continuous transition

(B) Boyle temperature $T_B = \dfrac{2a}{Rb}$

(C) van der Waal's equation is not very accurate near the critical states

(D) for an ideal gas $T_C = 0$

8. Which of the following factors is/are responsible for the extent of positive deviation?

(A) Difference in polarity of the molecules

(B) Difference in the length of hydrocarbon chain in case of their solution

(C) Difference in intermolecular forces of attraction

(D) Association of either of the constituent in the liquid state

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Match the Lists and answer the following question.

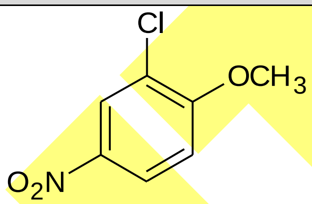
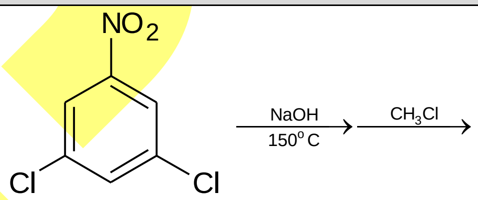
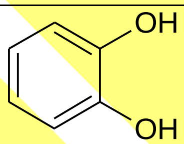
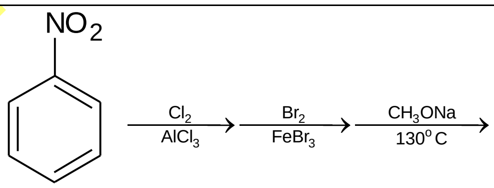
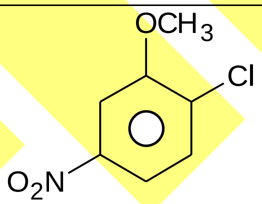
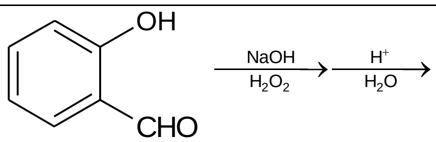
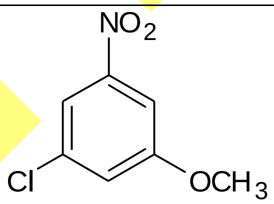
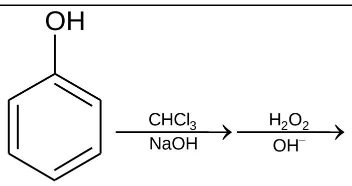
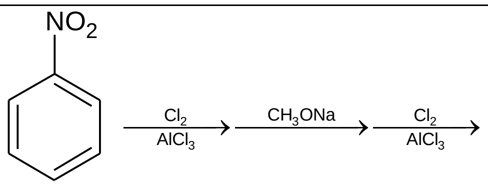
List – I		List – II	
(I)	Cubic	(P)	$a = b \neq c \quad \alpha = \beta = \gamma = 90^o$
(II)	Tetragonal	(Q)	$a = b = c \quad \alpha = \beta = \gamma = 90^o$
(III)	Orthorombic	(R)	$a \neq b \neq c \quad \alpha = \gamma = 90^o, \beta \neq 90^o$
(IV)	Monoclinic	(S)	$a \neq b \neq c \quad \alpha = \beta = \gamma = 90^o$

9. Which combination is correct according to the above list?
(A) I $\rightarrow$ Q
(B) II $\rightarrow$ S
(C) III $\rightarrow$ R
(D) IV $\rightarrow$ P

10. Which combination is correct according to the above list?
(A) I $\rightarrow$ S
(B) II $\rightarrow$ Q
(C) III $\rightarrow$ P
(D) IV $\rightarrow$ R

List Match Set (11-12)

Match the Lists and answer the following question.

List – I		List – II	
(I)		(P)	
(II)		(Q)	
(III)		(R)	
(IV)		(S)	
		(T)	

11. Which combination is correct according to the above list?
(A) I $\to$ ST (B) II $\to$ RS (C) III $\to$ PQ (D) IV $\to$ RT

12. Which combination is correct according to the above list?
(A) I $\to$ RS (B) II $\to$ Q (C) III $\to$ PQ (D) IV $\to$ P

(PART – B)

(Numerical Type)

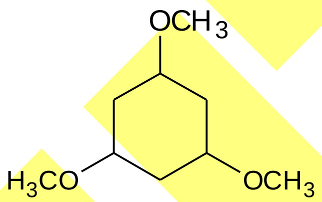
Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. pH of 0.1 M $NH_4^+HCO_3^-$ solution, given $K_b = 1.8 \times 10^{-5}$ K_1 of $H_2CO_3 = 4.2 \times 10^{-7}$ M and $K_2\left(HCO_3^{-1}\right) = 4.8 \times 10^{-11}$

2. Which of the following cations can form soluble complex when treated with conc.NaOH(excess)
Cu^{2+}, Pb^{2+}, Zn^{2+}, Ni^{2+}, Al^{3+}, Fe^{3+}, Sn^{2+}, Cd^{2+}, Co^{2+}

3. 3.24 g of mercuric nitrate is dissolved in 1 kg of water, the freezing point of the solution is found to be -0.0558ºC. Sum of degree of dissociation and van't Hoff factor for mercuric nitrate is
(Given: $K_{f_{H_2O}}$ = 1.86 mol^{-1} K kg; molecular mass of mercuric nitrate = 324.6)

4. In the following reactions,
$Na_2SO_3 + S \xrightarrow{\text{boiling water}} [X]$ (Colorless Solid)
$AgBr \xrightarrow{\text{excess}[X]} [Y]$ (soluble Complex)
$X + Cl_2 + H_2O \longrightarrow [Z] + HCl$
Sum of sulphur atoms in compound [X], [Y] and [Z]

5. 

How many maximum moles of HI is needed for complete reaction of one mole of the above compound?

6. 9 gm oleum completely neutralizes 100 mL 2 M NaOH. Percentage oleum is

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. If $y = f(x)$ is a solution of $\frac{dy}{d(x^2)} + \frac{y}{x^2} = \frac{\sin^{-1}x}{2x^2}$ and $f(0) = 0$, then

(A) $f(1) = \frac{\pi}{8}$

(B) $f(-1) = -\frac{\pi}{8}$

(C) $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\pi}{6}$

(D) $x^2 f(x) = \frac{1}{2}(2x^2 - 1)\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + \frac{1}{4}$

2. If $f(x) = \begin{vmatrix} 5+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 5+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 5+4\sin 2x \end{vmatrix}$, then which of the following is/are correct

(A) $f(x)$ is a non – monotonic function

(B) $f(x) = 0$ has no real roots

(C) $f'(x)$ has infinitely many maxima and minima

(D) Number of integers in the range of $f(x)$ is 201

3. If $\cos\frac{2\pi}{2019}.\cos\frac{4\pi}{2019}.\cos\frac{6\pi}{2019}...\cos\frac{2018\pi}{2019}$ is 2^{-k} where $k = \frac{p}{q}, p, q \in \mathbb{N}$ and p and q are relatively prime, then

(A) $(p+q)$ is a 4 – digit number

(B) sum of the digits in $(p+q)$ is 2

(C) $(p+q)$ is a 3 – digit number

(D) number of divisors of p is 8

4. For the curve $\sin x + \sin y = 1$, $\lim_{x \to 0}(x^{\alpha})\frac{d^2y}{dx^2}$ exists then α can be

(A) $\frac{3}{2}$

(B) 2

(C) $\frac{5}{2}$

(D) 1

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5. A point P moves inside a square with vertices A (1, 1), B (–1, 1), C(–1, –1) and D (1, –1) such that minimum $\{PA, PB, PC, PD\} \le 1$. If the area bounded by the curve traced out by moving point P is λ, then which of the following is/are correct

(A) λ is a rational number

(B) $[\lambda] = 3$ (where [.] represents greatest integral function)

(C) λ is an irrational number

(D) $[\lambda] = 0$ (where [.] represents greatest integral function)

6. $\lim\limits_{x\to 0} \dfrac{x^2 \sin\beta x}{\tan\alpha x - \sin x} = 4$ where $\alpha, \beta \in \mathbb{R}$, then which of the following is/are correct

(A) $\alpha = 1$ (B) $\beta = 2$

(C) $\alpha = 2$ (D) $\beta = 1$

7. Let A and B be two matrices different from *I* such that $AB = BA$ and $A^n - B^n$ is invertible for some positive integer *n*. If $A^n - B^n = A^{n+1} - B^{n+1} = A^{n+2} - B^{n+2}$, then

(A) $I - A$ is singular (B) $I - B$ is singular

(C) $A + B = AB + I$ (D) $(I - A)(I - B)$ is non – singular

8. $a, b \in \mathbb{R}, |a| \le 1, |b| \le 1,$ let $I(a,b) = \int_0^{\pi} (a\sin x + b\cos x)^3\, dx$ then

(A) $I(a,b)$ is independent of a (B) $I(a,b)$ is independent of b

(C) Maximum value of $I(a,1)$ is $\dfrac{10}{3}$ (D) minimum value of $I(-1, b)$ is $\dfrac{-10}{3}$

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

	List – I		List – II
(I)	If $(n-1)(2n-1)$ is the total number of odd numbers less than 1000 formed by using 0, 3, 5 and 7 (repetition of digits not allowed), then n is	(P)	8
(II)	Number of triangles formed by the vertices of the regular polygon of 10 sides is n and no side of the polygon is the sides of the triangle then $\frac{n}{10}$ is	(Q)	7
(III)	If five – digit numbers divisible by 4 can be formed by digit 1, 2, 3, 4 and 5 (the digits cannot be repeated in the same number) is $(n+2)(n-3)$, then n is	(R)	6
(IV)	A student writes 4 quizes of 4 marks each. If the number of ways of getting total 8 marks is n, then $\left[\frac{n}{16}\right]$ is ([.] denotes greatest integer function)	(S)	5
		(T)	4

9. Which of the following is the correct combination?
(A) I $\rightarrow$ T (B) I $\rightarrow$ Q (C) II $\rightarrow$ Q (D) II $\rightarrow$ U

10. Which of the following is the correct combination?
(A) I $\rightarrow$ Q (B) II $\rightarrow$ S (C) III $\rightarrow$ R (D) IV $\rightarrow$ T

List Match Set (11-12)

	List – I		List – II
(I)	A vector perpendicular to the line $x=2t+1, y=t+2$ and $z=-t-3$	(P)	$7\hat{i}+3\hat{j}+5\hat{k}$
(II)	A vector parallel to the planes $x+y+z-3=0$ and $2x-y+3z=0$	(Q)	$4\hat{i}-\hat{j}-3\hat{k}$
(III)	A vector along which the distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{-1}$ and $\vec{r}=(3\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+\hat{j}-2\hat{k})$ is the shortest	(R)	$-11\hat{i}+7\hat{j}+5\hat{k}$
(IV)	A vector normal to the plane $\vec{r}=-\hat{i}+4\hat{j}-6\hat{k}+\lambda(\hat{i}+3\hat{j}-2\hat{k})+\mu(-\hat{i}+2\hat{j}-5\hat{k})$	(S)	$\hat{i}+3\hat{j}+\hat{k}$
		(T)	

11. Which of the following is not the correct combination?
(A) II $\rightarrow$ Q (B) I $\rightarrow$ P (C) III $\rightarrow$ P (D) IV $\rightarrow$ R

12. Which of the following is the correct combination?
(A) I $\rightarrow$ R (B) II $\rightarrow$ R (C) III $\rightarrow$ P (D) IV $\rightarrow$ S

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(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. If the differential equation of all straight lines which are at a fixed distance of 10 units from origin is $(y-xy_1)^2 = 400A(1+y_1^2)$ then A is equal to ________units.

2. There are n different objects, *1, 2, 3,……, n*, distributed at random in n places marked 1, 2, 3….,n. If p is the probability that at least three of the objects occupy places corresponding to their number, then 6p is equal to

3. If $\int_0^{\pi/2} \sin\theta \log\sin\theta \, d\theta = \log\left(\frac{5A}{e}\right)$ then A is equal to

4. If p_1, p_2 are respectively the perpendicular distances of the points with position vectors $a = 3i - 5j + 8k$ and $b = 2i - 41j + 21k$ from the plane $r.(2i + 2j - k) = 12$, then $\frac{p_2}{37} + \frac{p_1}{32}$ is equal to

5. A variable plane at a unit distance fom the origin cuts the coordinate axes at points A, B, C. If the centroid (x, y, z) of the triangle ABC satisfies the equation $x^{-2} + y^{-2} + z^{-2} = k$, then the value of $\frac{k}{4}$ is

6. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim_{x \to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then the value of $p(2)$ is

Space For Rough Work

QP Code: 100971

Answers

SECTION – I : PHYSICS

(PART – A)

1.	**BCD**	2.	**A**	3.	**BC**	4.	**ABD**
5.	**AD**	6.	**BCD**	7.	**BD**	8.	**ABC**
9.	**A**	10.	**A**	11.	**C**	12.	**A**

(PART – B)

1.	**2.25**	2.	**3**	3.	**1.20**	4.	**6**
5.	**1.25**	6.	**6**				

SECTION – II : CHEMISTRY

(PART – A)

1.	**ABCD**	2.	**AD**	3.	**ABC**	4.	**CD**
5.	**AC**	6.	**BCD**	7.	**CD**	8.	**ABCD**
9.	**A**	10.	**D**	11.	**B**	12.	**D**

(PART – B)

1.	7.83	2.	4	3.	4	4.	**7**
5.	**6**	6.	108.8				

SECTION – III : MATHEMATICS

(PART – A)

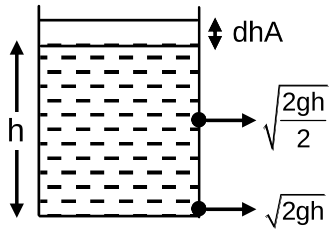
1.	AB	2.	ABCD	3.	AB	4.	ABC
5.	BC	6.	AB	7.	ABC	8.	CD
9.	A	10.	B	11.	B	12.	C

(PART – B)

1.	0.25	2.	0.17	3.	0.4	4.	1.25
5.	2.25	6.	0				

Answers & Solutions
SECTION – I : PHYSICS

(PART – A)

1. **BCD**
Sol.


Liquid coming out = $\sqrt{2g\frac{h}{2}}\,a.\,dt+\sqrt{2gh}\,adt$

$A(-dh) = \sqrt{gh}\,adt+\sqrt{2gh}\,\,adt$

2. **A**
Sol. Probability of K_α radiation is more than K_β radiation.

3. **BC**
Sol. 10 r > 6R

$\frac{r}{R}>\frac{6}{10};\frac{r}{R}>0.6$ cannot oscillate & if $\frac{r}{R}=0.2$

$\Rightarrow$ T = 0.24 π.

4. **ABD**
Sol. Apply in impulse momentum equation.

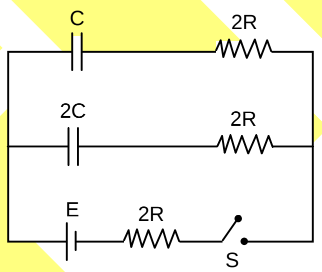
$\int_0^{0.1}(N-m_y)dt=mv_{yf}-mv_{y1}$

$\int_0^{0.1}Ndf=\frac{10\times10}{2}+10\times10+10\times10\times0.1$

$\Rightarrow\quad \int_0^{0.1}Ndt=160$

<N> $\Rightarrow\quad 160=\frac{\int Ndt}{0.1}$

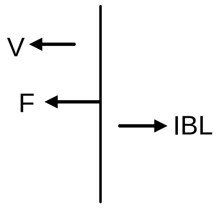
$\Rightarrow\quad$ 1600

5. **AD**
Sol.


$q=q_{max}\left(1-e^{-\frac{t}{CR_{eq}}}\right)$

$Q_{max}\ \rightarrow$ Max. charge at t = ∞
R_{eq} = Net resistance about capacitor keeping battery as short circuit.

6. **BCD**
Sol. At any instant motional emf = BVI
Charge on capacitor $\Rightarrow$ q = C(BIV)
Current I in rod = $\frac{d_q}{dt}+\frac{BVI}{R}$



when terminal velocity achieved F = IBL

$$F=\left(\frac{dq}{dt}+\frac{BVl}{R}\right)BL$$

$$F=\left(\frac{CBl\,dV}{dt}+\frac{BVl}{R}\right)BL$$

$$\frac{dV}{dt}=0$$

$$F=\frac{B^2Vl^2}{R}\quad ; \quad V_t=\frac{FR}{B^2l^2}$$

7. **BD**

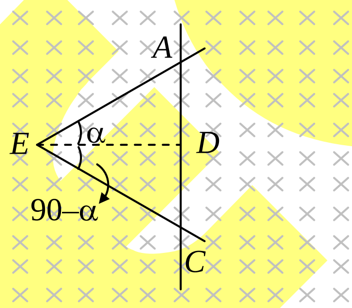
Sol. At time t, the distance traveled by the rod is

$$ED=vt$$

$$\tan\alpha=\frac{AD}{ED}\ \text{ or }\ AD=ED\tan\alpha$$

$$AD=vt\tan\alpha$$

$$\tan(90-\alpha)=\frac{DC}{ED}\ \text{ or }\ DC=at\cot\alpha$$



So, $AC=AD+DC=vt(\tan\alpha+\cot\alpha)$

Induced emf = $Bvl=Bv(AC)$

$=Bv.vt(\tan\alpha+\cot\alpha)$

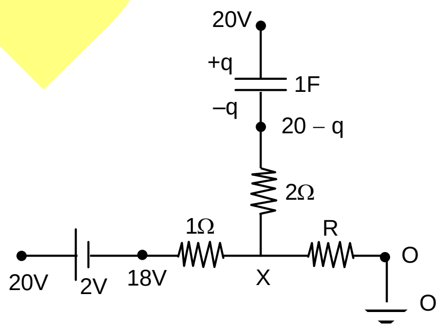
Induced emf $=Bv^2t(\tan\alpha+\cot\alpha)$

Hence, Induced emf $\propto t$ and v^2

Therefore, choices (B) and (D) are correct and choices (A) and (C) are wrong.

8. **ABC**

Sol. $\frac{x}{R}+\frac{x}{2}-10+\frac{q}{2}+x-18=0$



$$\frac{x-0}{R}+\frac{x-20+q}{2}+\frac{x-18}{1}=0$$

$$x\left(\frac{3}{2}+\frac{1}{R}\right)+\frac{q}{2}=28$$

$$x=\frac{28-\frac{q}{2}}{\frac{3}{2}+\frac{1}{R}}$$

Current in AP = $\frac{20-q-(x)}{2}$ = 3A

Current through R = $\frac{x}{R}=7A$ + R = 2Ω.

9. **A**

10. **A**

11. **C**

Sol. $T-mg=\frac{mv_A^2}{R}\quad ;\quad T+mg=\frac{mv_B^2}{R}$

12. **A**

Sol. (P) → Work done = Fs cosθ = (F cosθ) × s = 0.01 × 2 × ½ × 2 × 10^2 = 2 J

(Q) $\rightarrow$ Work done = 0 as bob is not getting displaced relative to car
(R) $\rightarrow$ v = u + at = 0 + 2 × 10 = 20
(S) $\rightarrow$ Work done = 0 as bob is not getting displaced in vertical direction

(PART – B)

1. **2.25**

Sol. $\lambda_m = \frac{b}{T}$

$\Rightarrow \quad 500 = \frac{b}{T_A}$

And $1500 = \frac{b}{T_B}$

$\Rightarrow \quad \frac{T_A}{T_B} = 3$

$\frac{U_A}{U_B} = \frac{\sigma(4\pi r_A^2)T_A^4}{\sigma(4\pi r_B^2)T_B^4} = \frac{9}{4} = 2.25$

2. **3**

Sol. $\mu.2\frac{n\lambda}{12u} = \frac{\lambda}{2}$

$\Rightarrow \quad n = 3$

3. **1.20**

Sol. $\omega = \sqrt{\frac{k}{m_1 + m_2 + m_3}} = \sqrt{\frac{24}{1+2+3}} = 2$

$0.4 \times g = \omega^2 A_{max}, \; A_{max} = 1.$

4. **6**

Sol. Current through 30 Ω is $\frac{12}{40}$ and apply loop rule then charge on the capacitor C_2 is 60 μC.

5. **1.25**

Sol. $B\int_0^R \frac{\omega}{2\pi} Q \frac{2\pi r}{\pi R^2} dr\, \pi r^2 = \frac{T_0 D}{2}$

$\Rightarrow \quad \omega = \frac{T_0 D}{QR^2 B}.$

6. **6**

Sol. Consider five more identical surfaces, such that a Hexagonal prism is formed with point charge at the centre. Flux will be equally divided between these surfaces.

SECTION – II : CHEMISTRY

(PART – A)

1. **ABCD**

Sol. $HgCl_2 + SnCl_2 \longrightarrow Hg_2Cl_2 \downarrow \longrightarrow Hg$

2. **AD**

Sol. Stable alkyl carbocation

3. **ABC**

Sol. Theoretical

4. **CD**

Sol. Conceptual

5. **AC**

Sol. Heisenberg contradict Bohr

6. **BCD**

Sol. Conceptual

7. **CD**

Sol. $T_B = \dfrac{a}{Rb}$

$T_C = \dfrac{8}{27}\dfrac{a}{Rb}$

8. **ABCD**

Sol. Positive deviation

$\begin{pmatrix} A----A \\ B----B \end{pmatrix} > (A---B)$

9. **A**

10. **D**

Sol. 9 & 10: 7 crystal system

11. **B**

12. **D**

Sol. 11 & 12: General organic reaction

(PART – B)

1. 7.83

Sol. pH = $pH = 7 + \dfrac{1}{2}\left(p_{K_a} - p_{K_b}\right)$

2. 4

Sol. Amphoteric

3. 4

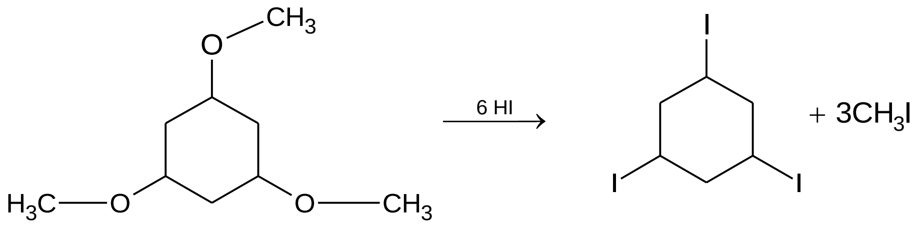
Sol. $-\Delta T_f = iK_f m \Rightarrow i = 1, \alpha = 1$

$\Rightarrow i + \alpha = 4$

4. **7**

Sol. $Na_2S_2O_3$, $[Ag(S_2O_3)_2]^{3-}$, Na_2SO_4

5. **6**

Sol. 

6. 108.8

Sol. % SO_3 = 39.5

$\%SO_3 = (\%\ \text{oleum} - 100) \times \dfrac{80}{18}$

%oleum = 108.88

SECTION – III : MATHEMATICS

(PART – A)

1. AB

Sol. $\dfrac{dy}{2x\,dx}+\dfrac{y}{x^2}=\dfrac{\sin^{-1}x}{2x^2}$

$\Rightarrow \dfrac{dy}{dx}+\left(\dfrac{2}{x}\right)y=\dfrac{\sin^{-1}x}{x}$

I.F. $e^{\int \frac{2}{x}dx}=e^{2\ell nx}=x^2$

Solution is given by $y\times x^2=\int \dfrac{\sin^{-1}x}{x}\times x^2dx$

$\Rightarrow y\times x^2=\dfrac{1}{2}\left(2x^2-1\right)\sin^{-1}x+\dfrac{1}{4}x\sqrt{1-x^2+C}$

2. ABCD

Sol. $f(x)=\begin{vmatrix}5+\sin^2x & \cos^2x & 4\sin 2x\\ \sin^2x & 5+\cos^2x & 4\sin 2x\\ \sin^2x & \cos^2x & 5+4\sin 2x\end{vmatrix}$

Apply $C_1\to C_1+C_2+C_3$

$=\left(6+4\sin 2x\right)\begin{vmatrix}1 & \cos^2x & 4\sin 2x\\ 1 & 5+\cos^2x & 4\sin 2x\\ 1 & \cos^2x & 5+4\sin 2x\end{vmatrix}$

Apply $R_3\to R_3+R_1$

$=\left(6+4\sin 2x\right)\begin{vmatrix}1 & \cos^2x & 4\sin 2x\\ 1 & 5+\cos^2x & 4\sin 2x\\ 0 & 0 & 5\end{vmatrix}$

$=\left(6+4\sin 2x\right)5\times 5=25\times\left(6+4\sin 2x\right)$

$[50,\ 250]$

3. AB

Sol. $A=\cos\theta.\cos 2\theta.\cos 3\theta......\cos(n-1)\theta$

$B=\sin\theta.\sin 2\theta.\sin 3\theta.....\sin(n-1)\theta$ where $\theta=\dfrac{2\pi}{2019}$

$n=2019$

$2^{1009}\times A\times B=\left(2\sin\theta.\cos\theta\right)\left(2\sin 2\theta.\cos 2\theta\right)$

$.....\left(2\sin(n-1)\theta.\cos(n-1)\theta\right)$

$=\sin 2\theta.\sin 4\theta\sin 6\theta......\sin(2n-2)\theta$

$\Rightarrow 2^{1009}\times A\times B=B$

$A=\dfrac{1}{2^{1009}}=2^{1009}$

$\dfrac{p}{q}=\dfrac{1009}{1}\Rightarrow p+q=1010$

4. ABC

Sol. $\sin x+\sin y=1$

$$\Rightarrow \cos x + \cos y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\cos x}{\cos y}$$

$$\Rightarrow -\sin x - (\sin y)\left(\frac{dy}{dx}\right)^2 + (\cos y)\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\sin x + \sin y\left(-\frac{\cos x}{\cos y}\right)^2}{\cos y}$$

$$= \frac{\cos^2 y.\sin x + \sin y.\cos^2 x}{(\cos y)^3}$$

Put $\sin x = t, \sin y = 1 - t, \cos y = \sqrt{2t - t^2}$

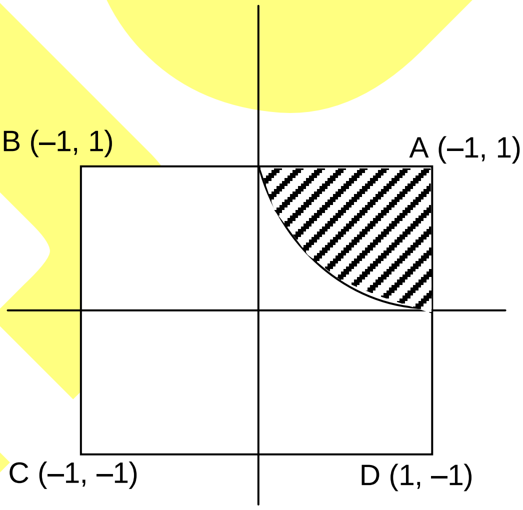
$$\lim_{x \to 0} x^{\alpha} \frac{d^2y}{dx^2} = \lim_{t \to 0} \frac{t^n\left[(2t - t^2)t + (1-t)(1-t^2)\right]}{\left(\sqrt{2t - t^2}\right)}$$

exists $\Rightarrow \alpha \geq \frac{3}{2}$

5. BC

Sol. Required area = 4 × area in quadrant 1

$$= 4 \times \left(1 - \frac{\pi}{4}\right) = 4 - \pi$$



6. AB

Sol. $$\lim_{x \to 0} \frac{x^2 \frac{\sin \beta x}{\beta x} \times \beta x}{\alpha n + \frac{(\alpha x)^3}{3} + \ldots - \left(x - \frac{x^3}{3} + \ldots\right)} = 4$$

$$\Rightarrow \lim_{x \to 0} \frac{x^3 \times \beta}{x(\alpha - 1) + \left(\frac{\alpha^3}{3} + \frac{1}{3!}\right)x^3 + \ldots} = 4$$

$$\Rightarrow \alpha = 1 \text{ and } \frac{\beta}{\frac{1}{3} + \frac{1}{3!}} = 4$$

$$\Rightarrow \beta = 4 \times \left(\frac{1}{3} + \frac{1}{6}\right)$$

$$= 4 \times \frac{1}{2} = 2$$

7. ABC

Sol. $A^{n+2} - B^{n+2} = (A+B)(A^{n+1} - B^{n+1}) - AB(A^n - B^n)$

$\Rightarrow A^n - B^n = (A+B)(A^n - B^n) - AB(A^n - B^n)$

$\Rightarrow I = A + B - AB$ [$\because A^n - B^n$ is invertible]

$\Rightarrow (I - A)(I - B) = O$.

As $A, B \neq I$, we get $I - A$ and $I - B$ are singular matrices.

8. CD

Sol. $I(a,b) = a^3 I_1 + 3a^2 b I_2 + 3ab^2 I_2 + b^3 I_3$ where

$$I_1 = \int^{\pi} \sin^3 x\, dx = 2\int_0^{\pi/2} \sin^3 dx = \frac{4}{3}$$

$$I_2 = \int_0^{\pi} \sin^2 x \cos x\, dx = 0$$

$$I_3 = \int_0^{\pi} \sin x \cos^2 x\, dx = -\frac{1}{3}\cos^3 x\Big]_0^{\pi} = \frac{2}{3}$$

$$I_4 = \int_0^{\pi} \cos^3 x\, dx = 0$$

Thus, $I(a,b) = \frac{4}{3}a^3 + 2ab^2$

$$I(a,1) = \frac{4}{3}a^3 + 2a \leq \frac{4}{3} + 2 = \frac{10}{3} = I(1,1)$$

and $I(-1, b) = \frac{-4}{3} + 2b^2 \geq \frac{-10}{3} = I(-1,1)$

9. A

10. B

Sol. (9 to 10)

(I) Odd numbers:

One – digit number $\boxed{\ } \rightarrow 3$

Two – digit numbers $\boxed{\text{two choices not zero}}\boxed{\text{(Choices 3, 5, 7)}} \rightarrow 3 \times 2 = 6$

Three – digit number $\boxed{\text{two non zero choices}}\boxed{\text{two choices}}\boxed{\text{Choices} = 3, 5, 7}$ $3 \times 2 \times 2 = 12$

$(n-1)(2n-1) = 3 + 6 + 12 = 21 \Rightarrow 2n^2 - 3n - 20 = 0$

$\Rightarrow n = 4,$ (Option S)

(II) Number of triangles $= {}^{10}C_3 - (10 + 10 \times 6)$

n = 50 (option – S)

(III) $$\underbrace{\boxed{}\boxed{}\boxed{}}_{3!}\boxed{\begin{matrix}12, & 52\\ 32, & 24\end{matrix}} \rightarrow 4 \text{ choices}$$

{3! (Last two digits)

$(n+2)(n-3) = 4 \times 3! = 24 \Rightarrow n^2 - n - 30 = 0 \Rightarrow n = 6$

Option (R)

(IV) Assumption: Each question carries one mark for correct and zero for other cases.

$x_1 + x_2 + x_3 + x_4 = 8$ $(0 \leq x_1, x_2, x_3 \leq x_4 \leq 4)$.

Number of non – negative solutions = Coefficient of x^8 in

$$\left(1+x+x^2+x\right)^4=\left(\frac{1-x^5}{1-x}\right)^4=\left(1+x^{10}-2x^5\right)^2\left(1-x\right)^{-4}$$

$$=\left(x^{20}+4x^{10}-4x^{15}-4x^5+2x^{10}+1\right)\left(1-x\right)^{-4}$$

Coefficient of $x^8=\left(-4\right)^{3+4-1}C_{4-1}+{}^{8+4-1}C_8={}^{11}C_8-4\,{}^{6}C_3=85$

$$n=85.\left[\frac{n}{16}\right]=\left[\frac{85}{16}\right]=5.$$

11. B

12. C

Sol. (11 & 12)

(I) Vector along the line is $2\hat{i}+\hat{j}-\hat{k}$

None of the vectors from (P) (Q) (R) (S) are perpendicular to $2i+\hat{j}-\hat{k}$

(II) Vectors perpendicular to the planes are

$\vec{n_1}=\hat{i}+\hat{j}+\hat{k}$ (Plane $x+y+z-3=0$)

$\vec{n_2}=2\hat{i}-\hat{j}+3\hat{k}$ (Plane $2x-y+3z=0$)

Vector parallel to both planes is along $\vec{n_1}\times\vec{n_2}$.

$$\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 3\end{vmatrix}=4\hat{i}-\hat{j}-3\hat{k}.\ \text{[Option (Q)]}$$

(III) $\frac{x}{2}=\frac{y}{-3}=\frac{z}{-1}$ (line – 1)

$\frac{x-3}{1}=\frac{y+1}{1}=\frac{z-1}{-2}$ (line – 2)

Vectors along the lines are $2\hat{i}-3\hat{j}-\hat{k}$ and $\hat{i}+\hat{j}-2\hat{k}$.

Vector along the shortest distance : $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & -1\\ 1 & 1 & -2\end{vmatrix}=7\hat{i}+3\hat{j}+5\hat{k}$ (Option P)

(IV) Vector normal to the plane is $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -2\\ -1 & 2 & -5\end{vmatrix}=-11\hat{i}+7\hat{j}+5\hat{k}$ (Option R)

(I) $\to$ none, (II) $\to$ (Q), (III) $\to$ (P) (IV) $\to$ (R)

(PART – B)

1. 0.25

Sol. The given family of lines can be represented by the equation

$x\cos\alpha+y\sin\alpha=10$ ....(1)

where α is an arbitrary constant. Differentiating we have

$\cos\alpha+\sin\alpha\, y_1=0$ ....(2)

Multiplying (2) by x and subtracting it from (1)

$$y\sin\alpha-x\sin\alpha\frac{dy}{dx}=10$$

$\Rightarrow$ $\left(y-xy_1\right)\sin\alpha=10$ ....(3)

Multiplying (1) by y_1 and (2) by y and subtract

$\Rightarrow \quad xy_1\cos\alpha - y\cos\alpha = 10y_1$ ....(4)

Square and add (3) and (4) we get, $(y - xy_1)^2 = 100\left(1 + y_1^2\right)$.

2. 0.17

Sol. Let E_i denote the event that the ith object goes to the ith place.

We have $P\left(E_i \cap E_j \cap E_k\right) = \dfrac{(n-3)!}{n!}$ for $i < j < k$

Since we can choose 3 places out of n in ${}^nC_3$ ways, the probability of the required event is

$$p = {}^nC_3 \cdot \frac{(n-3)!}{n!} = \frac{n!}{3!(n-3)!}\frac{(n-3)!}{n!} = \frac{1}{6}.$$

3. 0.4

Sol. Putting $t = \cos\theta$, we get $\int_0^{\pi/2} \sin\theta \log\sqrt{1-\cos^2\theta}\, d\theta$

$$= \frac{1}{2}\int_0^{\pi/2} \sin\theta \log\left(1-\cos^2\theta\right)d\theta$$

$$= -\frac{1}{2}\int_1^0 \log\left(1-t^2\right)dt$$

$$= \frac{1}{2}\int_0^1 \left[\log(1-t) + \log(1+t)\right]dt$$

$$= \frac{1}{2}\int_0^1 \log\left[1-(1-t)\right]dt + \frac{1}{2}\int_0^1 \log(1+t)dt$$

$$= \frac{1}{2}\int_0^1 \left[\log t + \log(1+t)\right]dt$$

$$= \frac{1}{2}\left(t\log t\Big|_0^1 - \int_0^1 dt + t\log(1+t)\Big|_0^1 - \int_0^1 \frac{1+t-1}{1+t}dt\right)$$

$$= \frac{1}{2}\left(-1 + \log 2 - 1 + \log 2\right) = \log 2 - \log e$$

$$= \log\left(\frac{2}{e}\right).$$

4. 1.25

Sol. $$p_1 = \frac{\left|12 - (3i - 5j + 8k).(2i + 2j - k)\right|}{\left|2i + 2j - k\right|}$$

$$= \left|\frac{12 - (6 - 10 - 8)}{\sqrt{4+4+1}}\right| = \frac{24}{3} = 8$$

Similarly, $p_2 = \dfrac{1}{3}\left|12 - (4 - 82 - 21)\right| = \dfrac{111}{3} = 37$.

5. 2.25

Sol. Let the equation of the plane be $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

$\Rightarrow A(a,0,0), B(0,b,0) C(0,0,c)$

$\Rightarrow (x,y,z) = \left(\dfrac{a}{3}, \dfrac{b}{3}, \dfrac{c}{3}\right)$

$k = x^{-2} + y^{-2} + z^{-2} = 9\left(a^{-2} + b^{-2} + c^{-2}\right)$

Since the distance of the plane from the origin is 1

$$\left|\frac{-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}\right|=1 \Rightarrow a^{-2}+b^{-2}+c^{-2}=1$$

Hence k = 9.

6. 0

Sol. $\lim\limits_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)=2 \Rightarrow \lim\limits_{x\to 0}\left(\frac{p(x)}{x^2}\right)=1$

Let $p(x)=a_0x^4+a_1x^3+a_2x^2+a_3x+a_4$. The given limit is finite is possible only if $a_3=a_4=0$ and is 1 if $a_2=1$. Thus $p(x)=a_0x^4+a_1x^3+x^2p(x)$ has extremum at $x=1,2$ if $p'(1)=0, p'(2)=0 \Rightarrow 4a_0+2a_1=-2, 32a_0+12a_1=-4$.

Solving we get, $a_0=\frac{1}{4}, a_1=-1$

$\therefore p(x)=\frac{1}{4}x^4-x^3+x^2 \Rightarrow p(2)=\frac{1}{4}(16)-8+4=0$.