FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2025
FULL TEST – VII
PAPER –2
TEST DATE: 20-04-2025

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

Section – A

1. C

Sol. For upper portion:

$$f_1 = \frac{\mu_2 - 1}{\frac{\mu_2}{\mu_1} - 1}(f_a) = \frac{1.5 - 1}{\frac{1.5}{1.2} - 1} \times 20 = 40 \text{ cm}$$

For lower portion, $f_1 = \dfrac{\mu_2 - 1}{\frac{\mu_2}{\mu_3} - 1}(f_a)$

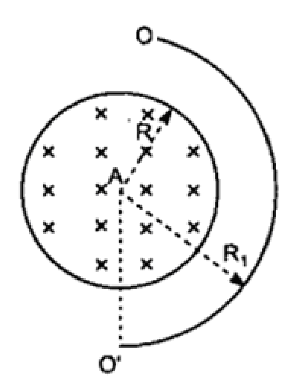
$$= \frac{1.5 - 1}{\frac{1.5}{2.5} - 1} \times 20 = -25 \text{ cm}$$

$\because$ Object is at infinity, images will be form at focal points.

Hence the distance between two images will = $40 + |-25| = 65$ cm

2. B

Sol. $E = \dfrac{R^2}{2R_1}\left(\dfrac{dB}{dt}\right)$



$$\text{e.m.f} = \frac{\theta}{2\pi}\frac{R^2}{2R_1}\left(\frac{dB}{dt}\right)2\pi R_1$$

$$= \frac{\theta}{2}R^2\left(\frac{dB}{dt}\right)$$

3. D

Sol. As $\gamma > 1$

for $TV^{\gamma-1}$ = constant $\frac{dT}{dv} < 0$

and for $T^{\gamma} = KP^{\gamma-1}$

$\frac{dT}{dP} > 0$

4. A

Sol. $y = \frac{0.8}{3(x+2t)^2+4}$

v = 2 m/s

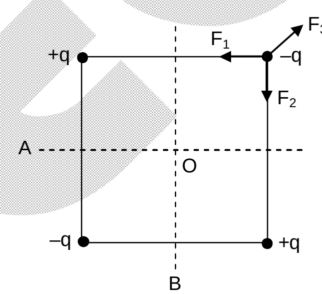
A = 0.2 m

5. AC

Sol. AOB is zero potential surface

Induce charge = –q

$F = \frac{kq^2}{a^2}\sqrt{2} - \frac{kq^2}{2a^2}$



6. AC

Sol. $I_1\left[\frac{n(n+1)}{2}r\right] + Ix = nE$ where x external resistance

$I_m\left(\frac{n(n+1)}{2}r\right) + I_x = mnE$

$I_1 + I_2 + I_3 + \dots\dots\dots + I_m = I$

$\Rightarrow \quad I = \frac{\frac{n(m+1)E}{2}}{x + \frac{n(n+1)}{2m}r} = \frac{V}{x+R}$

7. CD

Sol. $\frac{dQ}{dt} = \frac{KA(\Delta T)}{x} = \frac{KA[0-(-\theta)]}{x}$; $\frac{dQ}{dt} = \frac{KA\theta}{x}$

8. ACD

Sol. y (x, t) = –cos kx sin ωt

y(0.05, 0.05) = $-4\cos\left(\frac{2\pi}{0.4}0.05\right)\sin\left(\frac{2\pi}{0.2}0.005\right)$

$= -4\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{2}\right) = -2\sqrt{2}$ cm

$v = f\lambda = \frac{0.4}{0.2}$ = 2 m/s

$\frac{\partial y}{\partial t} = -A\,\omega \cos(kx)\cos(\omega t)$

$= -4\left(\frac{2\pi}{0.2}\right)\cos\left(\frac{2\pi}{0.4}\frac{1}{15}\right)\cos\left(\frac{2\pi}{0.2}0.1\right)$

$\frac{\partial y}{\delta t}$ = 20 π cm/sec = particle velocity

9. AC

Sol. In SHM, time taken to move from x = 0 to $x = \frac{A}{2}$ is $\frac{T}{12}$ and time taken to move from extreme to $x = \frac{A}{2}$ is $\frac{T}{6}$.

If $\frac{T}{12} = 1$ $\Rightarrow$ T = 12

If $\frac{T}{6} = 1$ $\Rightarrow$ T = 6.

10. ABC

Sol. $E_s = \frac{-GMm}{R}$; $E_1 = \frac{-GMm}{2(2R)}$; $E_2 = \frac{-GMm}{2(3R)}$

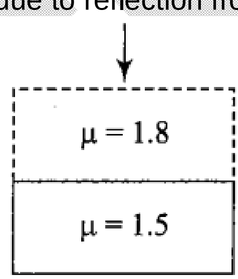
$E_1 - E_s = \frac{3}{4}mgR$

$E_2 - E_1 = \frac{mgR}{12}$

Section – B

11. 9

Sol. Path difference between rays reflected from upper and lower faces of layer = 2μt cosr = 2 μt (for normal incidence). But there is change in path of λ/2 of light at upper surface due to reflection from denser medium. So actual path difference is 2 μt – λ/2.



For constructive interference $2\,\mu t - \frac{\lambda}{2} = n\lambda$

$t = \frac{(2n+1)\lambda}{4\mu}$. For least thickness n = 0.

$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4 \times 1.8}$nm = 90 nm

12. 5

Sol. $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$, so $hf = +\frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{16} - \frac{1}{25}\right]$

$\therefore \quad f = \frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\left[\frac{9}{16 \times 25}\right]$

and frequency $f_4 = \frac{Z^2e^2m}{4\varepsilon_0^2 n^3 h^3} = \frac{Z^2e^4m}{4\varepsilon_0^2 (4)^3 h^3}$

$\therefore \quad \frac{f}{f_4} = \frac{18}{25}$, so m = 5.

13. 4

Sol. When the two spheres come in contact, they exert equal and opposite impulse.

$$\int Fa\, dt = I_1(\omega_0 - \omega_1) \qquad \ldots(i)$$

$$\int Fb\, dt = I_2\omega_2 \qquad \ldots(ii)$$

Finally $\quad \omega_1 a = \omega_2 b \qquad \ldots(iii)$

$$I_1(\omega_0 - \omega_1) = I_2 \frac{a^2}{b^2}\omega_1$$

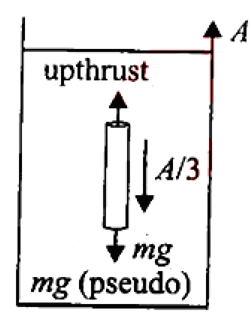
$$\omega_1 = \frac{I_1\omega_0}{I_1 + \frac{a^2}{b^2}I_2} = 4 \text{ rad/s.}$$

14. 5

Sol. Let the volume of the cylinder be V. When the cylinder is floating , upthrust = weight. Hence,

$\rho\left(\frac{3}{4}V\right)g = mg \quad \Rightarrow \quad V = \frac{4m}{3\rho}$

Let the acceleration of the particle vessel be A (upwards). In the reference frame of the vessel, the acceleration of the cylinder is A/3.



$\therefore \quad$ mg + mA – upthrust = $m\left(\frac{A}{3}\right)$

mg + mA – ρVg′ = $m\left(\frac{A}{3}\right)$

where g′ = g + A = effective value of g for upthrust.

$\therefore \quad$ mg + mA – ρV (g + A) = $m\left(\frac{A}{3}\right)$

$\Rightarrow \quad mg - \frac{4}{3}m(g + A) = -m\frac{2}{3}A$

$$A = \left(-\frac{g}{2}\right) \text{upwards}$$

The acceleration of the vessel should be $\frac{g}{2} = \frac{10}{2}$ = 5 ms^{-2} (downwards).

15. 6

Sol. Let the pressures in wide and narrow limbs be P_1 and P_2, respectively. If R_1 and R_2 be the radii of meniscus in wide and narrow limb, pressure just below the meniscus of wide tube = $P_1 - \frac{2T}{R_1}$ and pressure just below the meniscus of narrow limb = $P_2 - \frac{2T}{R_2}$.

Therefore, difference of these pressures

$$\left(P_1 - \frac{2T}{R_1}\right) - \left(P_2 - \frac{2T}{R_2}\right) = h\rho g$$

Therefore, true pressure difference,

$$P_1 - P_2 = h\rho g - 2T\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

For the water and glass surface, taking the angle of contact θ to be zero, we have $R_1 = \frac{r_1}{\cos\theta} = r_1$ and $R_2 = \frac{r_2}{\cos\theta} = r_2$ where r_1 and r_2 are radii of wide and narrow limbs, respectively.

$\therefore$

$$P_1 - P_2 = h\rho g - 2T\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3} \times \left(\frac{1}{7.2 \times 10^{-4}} - \frac{1}{1.44 \times 10^{-3}}\right)$$

= $1.96 \times 10^3 - 0.10 \times 10^3 = 1.86 \times 10^3$ N/m^2 = 1860 N/m^2

$\therefore$ N = 6.

16. 8

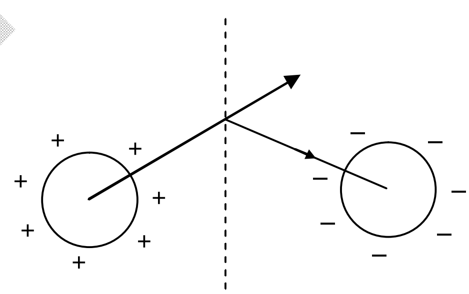
Sol. $J = \sigma E = \frac{\sigma q \ell}{2\pi\varepsilon_0 \varepsilon r^3}$

$$I = \int_0^\infty J\, 2\pi x dx = \frac{q}{\varepsilon_0 \varepsilon \rho}$$

$$\Delta V = \frac{q}{2\pi\varepsilon_0 \varepsilon a} = IR$$

$\Rightarrow$ $R = \frac{\rho}{2\pi a} = 0.50$

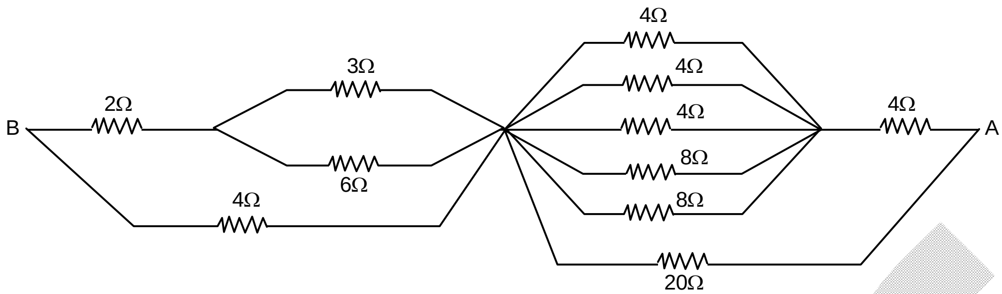
$\therefore$ n = 8.



17. 4

Sol. $$C_{eq} = \frac{2C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2 + 2C_3} = \frac{7}{5}$$

Where $C_1 = 1$, $C_2 = 2$, $C_3 = 1$



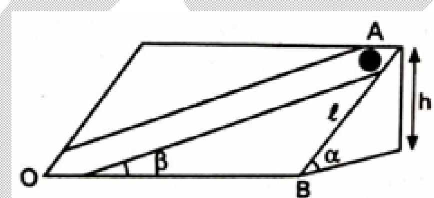
$R_{eq} = 6$

$\tau = RC = 8.4$

$\therefore$ n = 4.

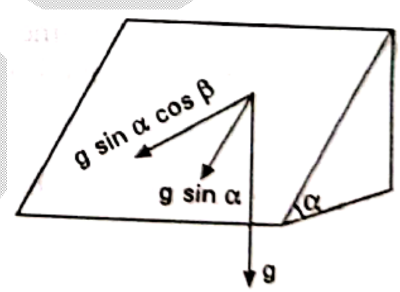
18. 2

Sol. Component of acceleration along incline is g sin α and effective acceleration along groove is a = sin α cos β.



From figure $\ell = \dfrac{h}{\sin\alpha} = OB$

OA is the groove making an angle $\beta = 45°$ with AB on the inclined surface.



From figure, $\dfrac{OB}{OA} = \cos\beta$

or $OA = \dfrac{h}{\sin\alpha\cos\beta}$

Since acceleration along groove is constant, we may use equation $s = \dfrac{1}{2}at^2$.

$\therefore$ k = 2.

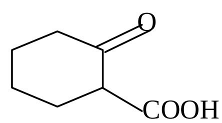
Chemistry

PART – II

Section – A

19. C

Sol.

Intermediate is 

Which loses CO_2 on heating (β-keto acid)

20. B

Sol. M absorbs some heat energy and prevents decomposition of O_3

21. C

Sol. $T_f = 4T_i$

$$\Delta S = nC_V \ln\frac{T_f}{T_i} + nR\ln\frac{v_f}{v_1}$$

$$0 = n\times\frac{3R}{2}\ln 4 + n.R\ln\frac{v_f}{v_1}$$

$$\frac{3}{2}\log 4 = \log\frac{v_i}{v_f}$$

$$4^{3/2} = \frac{v_i}{v_f}$$

$$8v_f = v_i$$

$$0 = \Delta S = nC_p \ln\frac{T_f}{T_i} + nR.\ln\frac{Pi}{P_f}$$

$$4^{5/2} = \frac{P_f}{P_i}$$

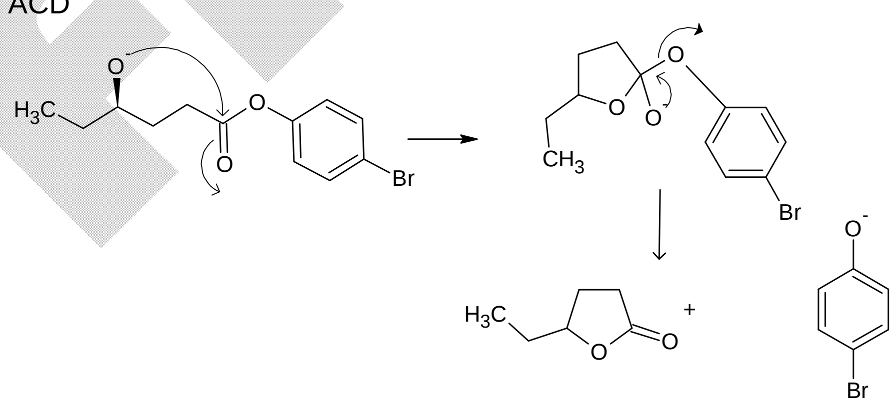
$$32P_i = P_f$$

22. D

Sol. At ¼ th and 3/4 th neutralization points the solution is buffer solution. Upon substitution we can get the ans.

23. ACD

Sol.



24. AC
Sol. Azeotropic mixture has fixed composition.

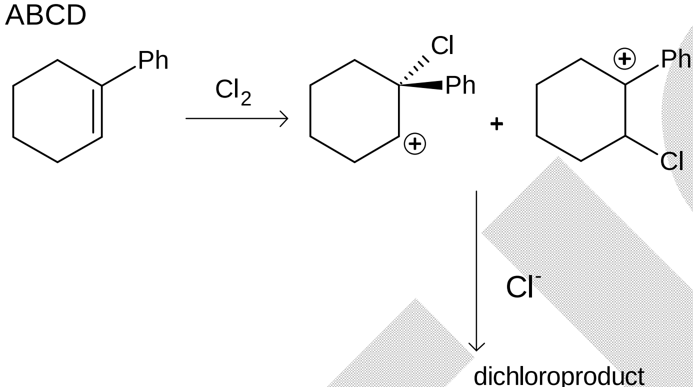
25. ABC
Sol. Rate = $k[X][Y]^0 \Rightarrow 2 \times 10^{-8} = k[0.1]\,[0.1]^0 \Rightarrow k = 2 \times 10^{-7} s^{-1}$

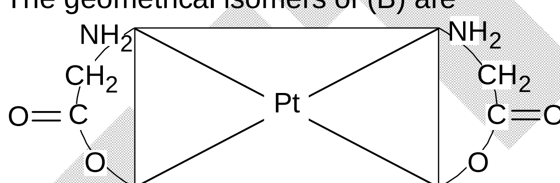
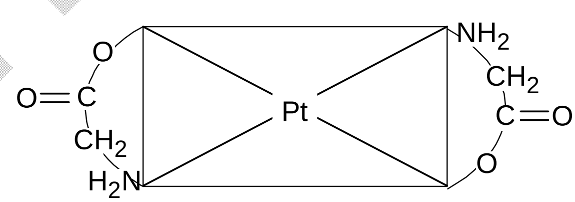
26. ABC
Sol. $CaC_2 + 2H_2O \longrightarrow Ca(OH)_2 + C_2H_2$

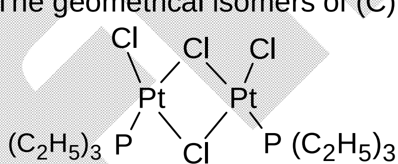
$Mg_3N_2 + 6H_2O \longrightarrow 3Mg(OH)_2 + 2NH_3$

$Na_2S + 2H_2O \longrightarrow 2NaOH + H_2S$

$KNO_3 + H_2O \longrightarrow$ No reaction

27. ABCD
Sol.


28. BCD
Sol. The geometrical isomers of (B) are

Cis

trans

The geometrical isomers of (C) are

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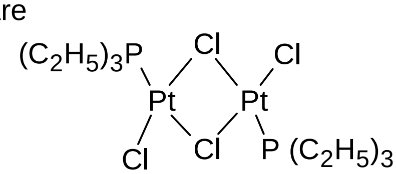
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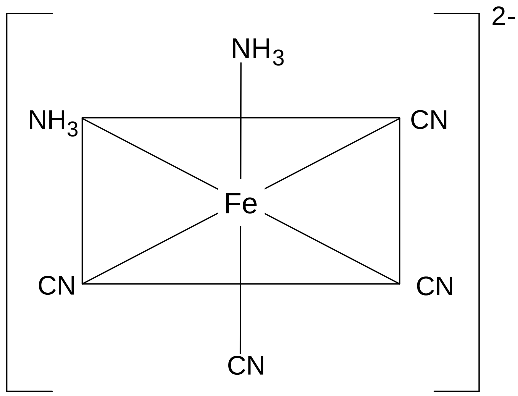
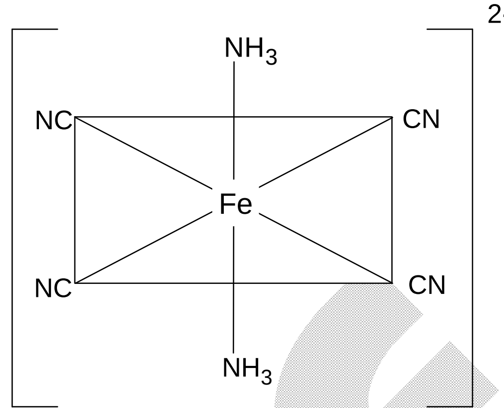
The geometrical isomers of (D) are



Cis

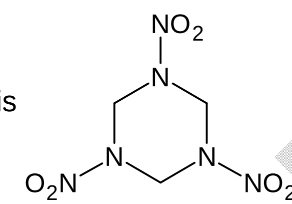


trans

Section – B

29. 6

Sol. Compound (Z) is



30. 4

Sol. $(4n + 2)\pi e^-$ rule

31. 3

Sol. $\eta^5 C_2H_5 - 5e^-$

$2\ CO - 4e^-$

$Mo - 6e^-$

$Mo - Mo - 3e^-$

32. 3

Sol. $S^{2-} + HCl \longrightarrow H_2S \xrightarrow[NaOH]{Na_2[Fe(CN)_5NO]} Na_4[Fe(CN)_5(NOS)]$

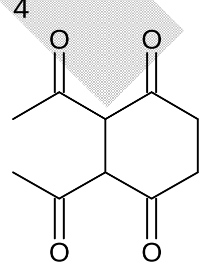
33. 5

Sol. $\dfrac{25\times10^{-3}(g)}{250} = \dfrac{16\times10^{-3}(g)}{250 - x18}$

X = 5

34. 4

Sol.



35. 4

Sol. (1) ala-phe-leu-gly
(2) ala-phe-gly-leu
(3) ala-leu-phe-gly
(4) ala-leu-gly-phe

36. 3

Sol. It exists in two geometrical isomeric forms in which trans isomer is optically active.

Mathematics PART – III

Section – A

37. A

Sol. Since for two square matrices A & B the trace of AB – BA is zero.

$\therefore$ let $AB - BA = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$

$\Rightarrow$ $(AB – BA)^2 = k\, I_2$

where $k = a^2 + bc$

and odd power of AB – BA is equal to a multiple of this matrix.

i.e. odd power it can not be equal to I_2

$\therefore$ 'n' is even.

38. B

Sol. $A^2B^2 = rB^4$

$B^2A^2 = rB^4$

$\Rightarrow A^2B^2 = B^2A^2$

& $rB^2A = rAB^2 = A^3$

Multiplying $pAB + qBA = I_n$ on the right and then on the let by B, we obtain

$pAB^2 + qBAB = B$

$pBAB + qB^2A = B$

also we have $B^2A = AB^2$

on subtraction $(p – q)(AB^2 – BAB) = 0$

if $p \neq q$ then $AB^2 = BAB$

$\therefore (p + q) AB^2 = (p + q) B^2A = B$

$\Rightarrow (p + q) A^2B^2 = AB$

& $(p + q)B^2A^2 = BA$

$\Rightarrow AB = BA$

which is a contradiction

$\therefore p = q$

39. A

Sol. Integrating the given differential equation, we have $\frac{dy}{dx} = \frac{-\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$ but

$y_1(0) = 1$ so $1 = \left(\frac{-1}{3}\right) + 1 + C_1 \Rightarrow C_1 = \frac{1}{3}$.

Again integrating, we get $y = \frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$

but $y(0) = 0$ so $0 = 0 + 1 + C_2 \Rightarrow C_2 = -1$. Thus $y = \frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

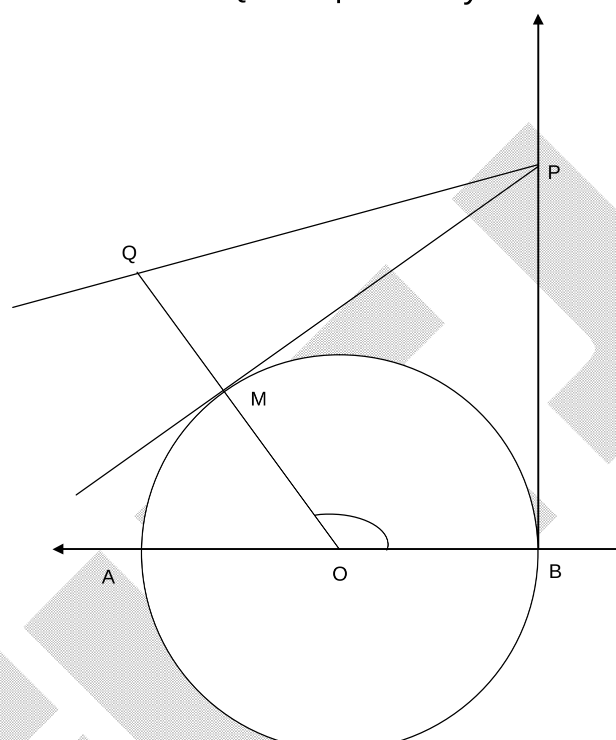
40. B

Sol. Let the radius of the circle is equal to 1. Set the origin at B with BA the positive x – semi axis and t the y – axis. If $\angle BOM = \theta$, then $BP = PM = \tan\frac{\theta}{2}$. In triangle PQM, PQ $= \dfrac{\tan\frac{\theta}{2}}{\sin\theta}$. So the coordinates of Q are $\left(\dfrac{\tan\frac{\theta}{2}}{\sin\theta}, \tan\dfrac{\theta}{2}\right) = \left(\dfrac{1}{1+\cos\theta}, \dfrac{\sin\theta}{1+\cos\theta}\right)$.

The x and y coordinates are related as follows:

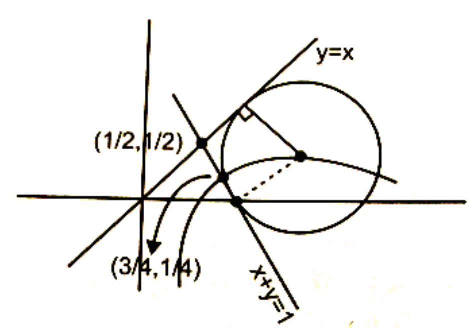
$$\left(\frac{\sin\theta}{1+\cos\theta}\right)^2 = \frac{1-\cos^2\theta}{(1+\cos\theta)^2} = \frac{1-\cos\theta}{1+\cos\theta} = 2\frac{1}{1+\cos\theta} - 1.$$

Hence the locus of Q is the parabola $y^2 = 2x - 1$.



41. BC

Sol. $4a = 2\left(\dfrac{|0-1|}{\sqrt{2}}\right) = \dfrac{2}{\sqrt{2}} = \sqrt{2}$



42. BCD

Sol. Put $(a - r\cos\theta, 2 - r\sin\theta)$ to equn. of ellipse

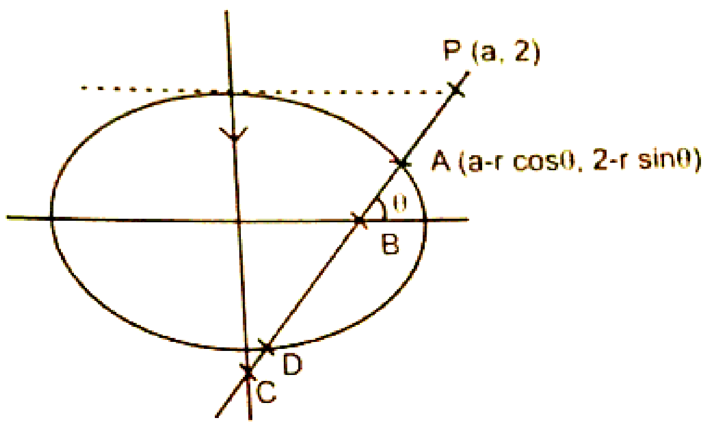
$$\Rightarrow \frac{a^2 + r^2\cos^2\theta - 2ar\cos\theta}{9}$$

$$+\frac{4 + r^2\sin^2\theta - 4r\sin\theta}{4} - 1 = 0$$

$$= \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta}$$

Put $(a - r\cos\theta, 2 - r\sin\theta)$ to equation $xy = 0$

$$\Rightarrow (PA)(PD) = \frac{\frac{a^2}{9}}{\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{4}}$$



$$\Rightarrow 2a + r^2\sin\theta\cos\theta - (a\sin\theta + 2\cos\theta)r = 0$$

$$\Rightarrow (PB)(PC) = \frac{2a}{\sin\theta\cos\theta}$$

$PA.PD = PB.PC$

$$\Rightarrow \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta} = \frac{2a}{\sin\theta.\cos\theta}$$

$$\Rightarrow a = \frac{4\cot\theta + 9\tan\theta}{2}$$

$$\Rightarrow |a| \ge \sqrt{4 + 9} = 6$$

$$\Rightarrow a \in (-\infty, -6] \cup [6, \infty)$$

43. CD

Sol. If α is a multiple of π, then $I(\alpha) = 0$. Otherwise, use the substitution $x = \cos\alpha + t\sin\alpha$. The indefinite integral becomes

$$\int \frac{\sin\alpha\, dx}{1 - 2x\cos\alpha + x^2} = \int \frac{dt}{1+t^2} = \text{art } \tan t + C.$$

$$\therefore I(\alpha) = \arctan\left(\frac{1-\cos\alpha}{\sin\alpha}\right) - \arctan\left(\frac{-1-\cos\alpha}{\sin\alpha}\right),$$

where the angles are to be taken between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. But

$$\frac{1-\cos\alpha}{\sin\alpha} \times \frac{-1-\cos\alpha}{\sin\alpha} = -1.$$

Hence the difference between these angles is $\pm\frac{\pi}{2}$. Notice that the sign of the integral is the same as the sign of α. Hence $I(\alpha) = \frac{\pi}{2}$ if $\alpha \in \left(2k\pi, (2k+1)\pi\right)$ and $-\frac{\pi}{2}$ if $\alpha \in \left((2k+1)\pi, (2k+2)\pi\right)$ for some integer k.

44. AD

Sol. $k_1u - k_2v = 0$ ....(i)

$k_1u + k_2v = 0$ ....(ii)

$\because$ equations of bisectors of the angles formed by lines (i) and (ii) are

$$\frac{k_1u - k_2v}{\sqrt{(ak_1 - bk_2)^2 + (k_1b + ak_2)^2}} = \frac{\pm(k_1u + k_2v)}{\sqrt{(k_1a + bk_2)^2 + (k_1b - ak_2)^2}}$$

$\Rightarrow k_1u - k_2v = \pm(k_1u + k_2v)$ ....(iii)

(i) by taking positive sign in (iii), we get

$k_1u - k_2v = k_1u + k_2v$

$2k_2v = 0 \quad \Rightarrow \quad v = 0$

(ii) by taking negative sign in (iii), we get

$u = 0$

45. BD

Sol. After we bring the function into the form $f(x)=\dfrac{\left(x-1+\frac{1}{x}\right)^3}{x^3-1+\frac{1}{x^3}}$, let $x+\dfrac{1}{x}=s$

$h(s)=\dfrac{(s-1)^3}{s^3-3s-1}=1+\dfrac{-3s^2+6s}{s^3-3s-1}$ over the domain $(-\infty,-2]\cup[2,\infty)$. Setting the first derivative equal to zero yields the equation $3(s-1)(s^3-3s^2+2)=0$.

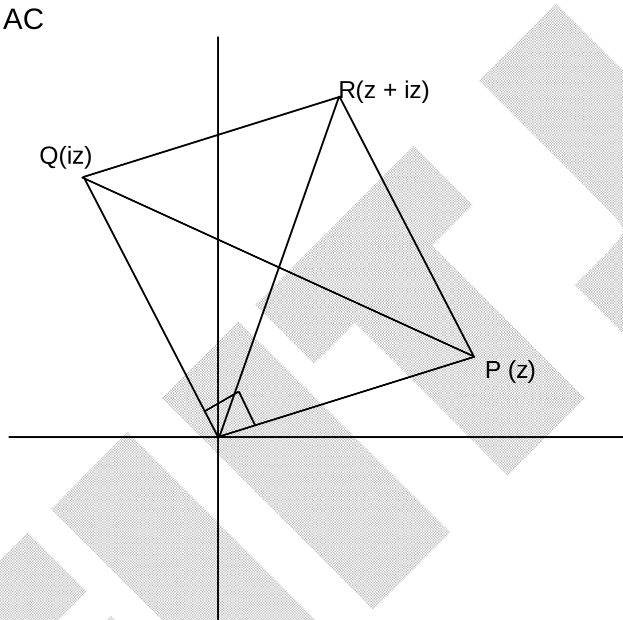
The roots are s = 1 (double root) and $s=1\pm\sqrt{3}$. Of these, only $s=1+\sqrt{3}$ lies in the domain of the function.

We compute

$$\lim_{x\to\pm\infty}h(s)=1,\ h(2)=1,\ h(-2)=9,\ h\left(1+\sqrt{3}\right)=\frac{\sqrt{3}}{2+\sqrt{3}}.$$

46. AC

Sol.



Section – B

47. 3

Sol. Equation of the chord of contact of a point $P(3\sec\theta, 2\tan\theta)$ on the hyperbola with respect to the circle is $(3\sec\theta)x+(2\tan\theta)y=9$ (1)

Let M (h, k) be the mid point of (1), then equation of (1) in terms of the mid – point is

$hx+ky=h^2+k^2$ (2)

Since (1) and (2) represent the same line.

$\sec\theta=\dfrac{3h}{h^2+k^2}$, $\tan\theta=\dfrac{9k}{2(h^2+k^2)}$s

$\Rightarrow$ locus of (h, k) is

$$\Rightarrow \frac{9x^2}{\left(x^2+y^2\right)^2}-\frac{81y^2}{4\left(x^2+y^2\right)^2}=1$$

or $4\left(x^2+y^2\right)^2=bx^2-cy^2$

$\Rightarrow \quad a=4,\ b=36,\ c=81$

$a^2+b^2+c^2=16+1296+6561=7873$

48. 5

Sol. Let $r=\lambda b+\mu c$ and $c=\pm(xi+yj)$. Since c and b are perpendicular, we have

$4x+3y=0 \qquad \Rightarrow c=\pm x\left(i-\frac{4}{3}j\right)$

$\pm 1=$ proj. of r on $b=\frac{r.b}{|b|}=\frac{(\lambda b+\mu c).b}{|b|}=\frac{\lambda b.b}{|b|}$

$[\because b.c=0]$

$=\lambda|b|=5\lambda$. Hence $\lambda=\frac{1}{5}$

Also, $\pm 2=$ proj. of r on $c=\frac{r.c}{|c|}$

$=\frac{(\lambda b+\mu c).c}{|c|}=\mu|c|=\frac{5}{3}\mu x$

Thus, $\mu x=\pm\frac{6}{5}$. Therefore,

$r=\frac{1}{5}(4i+3j)+\frac{6}{5}\left(i-\frac{4}{3}j\right)=\pm(2i-j)$

$r=\frac{1}{5}(4i+3j)-\frac{6}{5}\left(i-\frac{4}{3}j\right)=\pm\left(-\frac{2}{3}i+\frac{11}{5}j\right)$

Thus there are four such vectors

$$\sum_{i=1}^{4}|r_i|^2=2|2i-j|^2+2\left|-\frac{2}{5}i+\frac{11}{5}j\right|^2=20$$

49. 1

Sol. Let $f(x)=\sin x+\tan x-2x$. Then $g(x)=f'(x)$

$=\cos x+\sec^2 x-2$

$\Rightarrow \quad g'(x)=-\sin x+2\sec^2 x\tan x$

$=\sin x\left(-1+\frac{2}{\cos^3 x}\right)$

$=\sin x\left(\frac{1-\cos^3 x+1}{\cos^3 x}\right)$

Since for $0 < x < \frac{\pi}{2}$, we have $0 < \cos^3 x < 1$, g is an increasing function. Hence $g(x) > g(0)$, i.e. $\cos x + \sec^2 x - 2 > 0$. Therefore f is an increasing function, so $f(x) > f(0)$ for $0 < x < \frac{\pi}{2}$. Hence $\sin x + \tan x > 2x$. Thus $g\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + 1$

50. 1

Sol. The inverse of 2 × 2 matrices C = $[C_{ij}]_{2\times2}$ with integer entries is a matrix with integer entries if and only if |c| = ± 1

$$\left\{C^{-1} = \frac{\text{adj}(C)}{|C|} \;\&\; |C^{-1}| = \frac{1}{|C|}\right\}$$

So lets take a polynomial.

$$P(x) = |A + xB|$$

as per the problem $P(0), P(1), P(2), P(3), P(4) \in \{-1, 1\}$

three of these must be same

and P(x) has degree at most 2.

$\therefore$ it is constt.

$\therefore |A + xB| = \pm 1, \forall x$

$\therefore |A + 5B| = \pm 1$

51. 2

Sol. $a_1 + a_3 + ... + a_{99} = 50$

$\Rightarrow a + (a + 2d) + (a + 4d) + ... + (a + 98d) = 50$

$\Rightarrow 50a + 2d(1 + 2 + ... + 49) = 50$

$\Rightarrow 50a + \frac{2d(50)(49)}{2} = 50$

$\Rightarrow a + 49d = 1$

$= -a_1 - a_3 + a_5 + a_7 - a_9 - a_{11} + ... + a_{93} + a_{95} - a_{97} - a_{99}$

(26 negative terms and 24 positive)

$\Rightarrow |-a_1 - a_{99}| = |-2a - 98d| \Rightarrow |-2| \Rightarrow 2$

52. 1

Sol. Given $a + b + c = 1$ ....(1)

$9a + 3b + c = 7$ ....(2)

$18 < 25a + 5b + c < 22$ ....(3)

$\Rightarrow$ From above (1), (2) and (3) $4 < 7a - b - c < 8$

$4 < 7a - b + a - 1 < 8$

$5 < 8a < 9$

$a = 1, b = -1, c = 1$

$a = 1, b = -1, c = 1$

For question (2) $h(x) = \ln(x^2 - x + 1) - x$

53. 4

Sol. Let $z_1 = z_1 e^{i\theta_1}$ and

$z_2 = r_2 e^{i\theta_2} \Rightarrow \dfrac{z_1\bar{z}_2 + \bar{z}_1 z_2 + z_1 z_2 + \bar{z}_1\bar{z}_2}{|z_1 z_2|} = 2\cos(\theta_1+\theta_2) + 2\cos(\theta_1+\theta_2)$

54. 3

Sol. $\dfrac{a+b+c}{b-a} = \dfrac{a+b-a+a+c}{b-a}$

$= 1 + \dfrac{2a+c}{b-a}$

Now: $f(x) = ax^2 + bx + c$

Given $f(x) \ge 0\ \forall x\ f(-2) \ge 0$

$4a - 2b + c \ge 0$

$2a + c \ge 2(b-a)$

$\dfrac{2a+c}{b-a} \ge 2$

$\therefore \dfrac{a+b+c}{b-a} \ge 3$