Clustering-based Pair trading using reinforcement learning

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Data I used:

+ Target: Stock under S&P500
+ Training Period: 2017-2021
+ Testing Period: 2021-Mar 2023

Clustering method used:

- K-means
- Hierarchical Clustering
- Affinity Propagation
- DBSCAN
- Gaussian Mixture Model
- Ordering Points to Identify the Clustering Structure(OPTICS)
- Agglomerative Hierarchy clustering

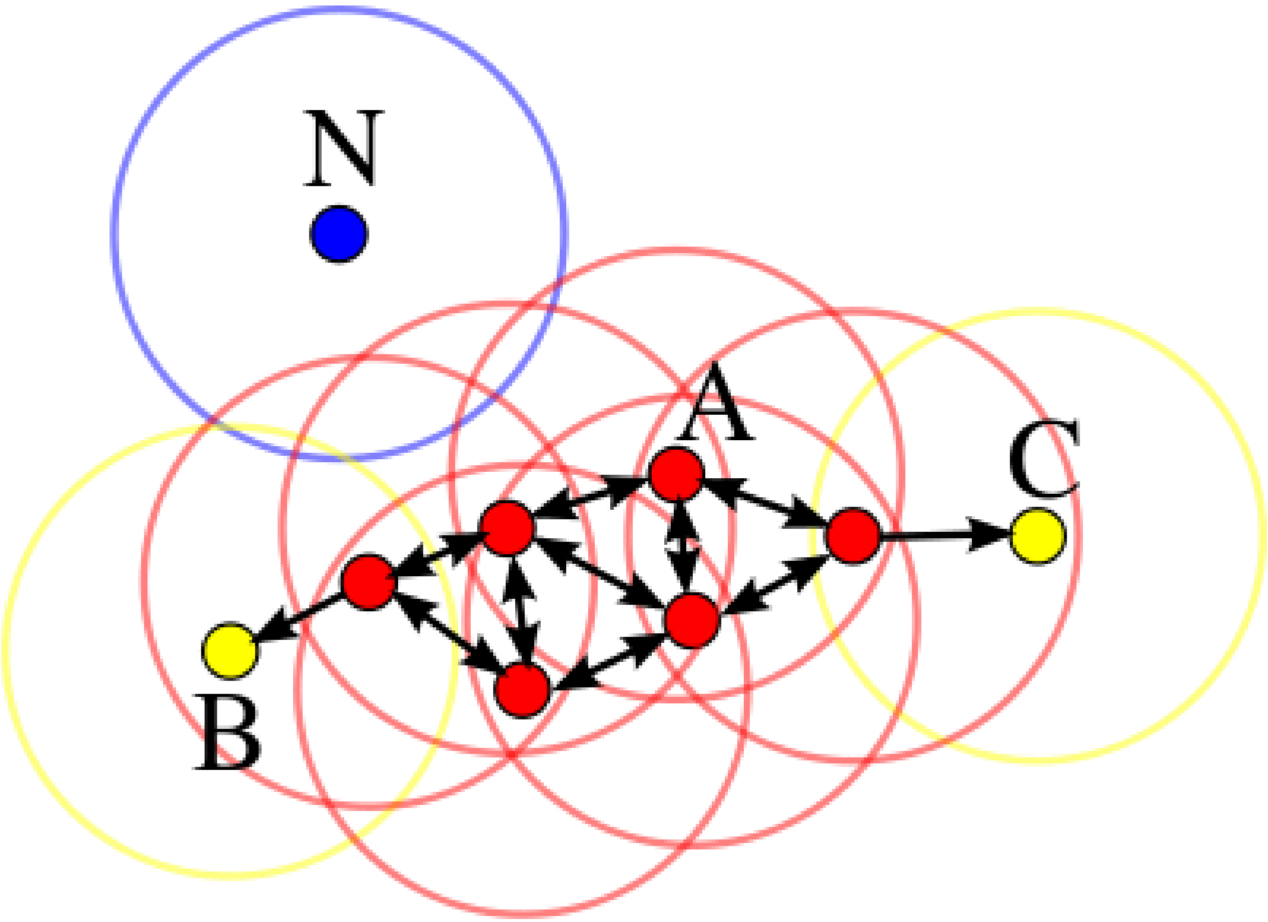
$$\arg\min_{\mathbf{S}} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \left\| \mathbf{x} - \boldsymbol{\mu}_i \right\|^2 = \arg\min_{\mathbf{S}} \sum_{i=1}^{k} |S_i| \operatorname{Var} S_i$$

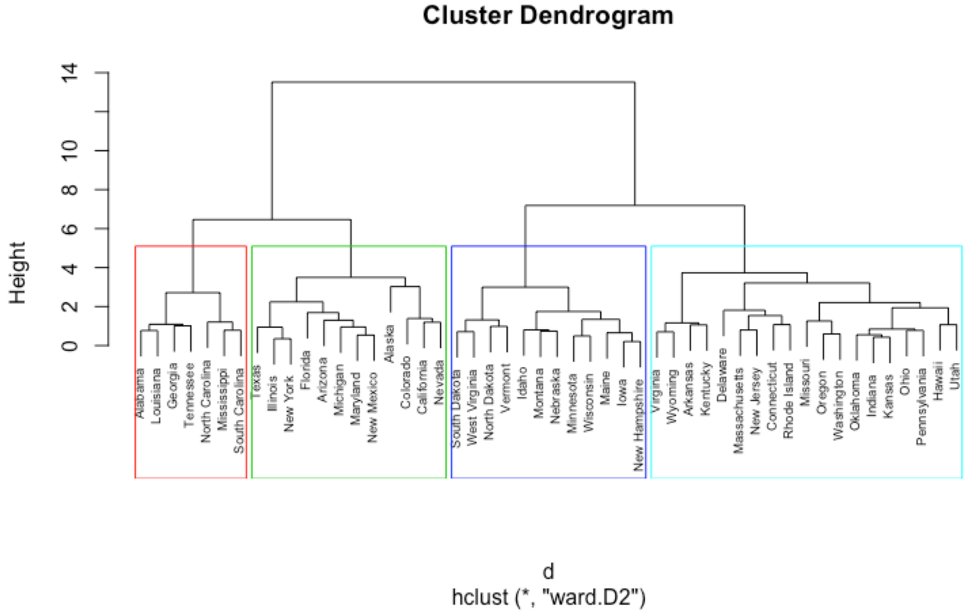
K-means

+ centroid-based algorithm
+ minimize the variance of data points within a cluster.

DBSCAN clustering & OPTICS

+ **density-based** spatial clustering of applications with noise
+ DBSCAN uses two parameters to determine how clusters are defined: **minPts** and **eps**.
+ OPTICS is a variation of DBSCAN, with Core Distance and Reachability Distance





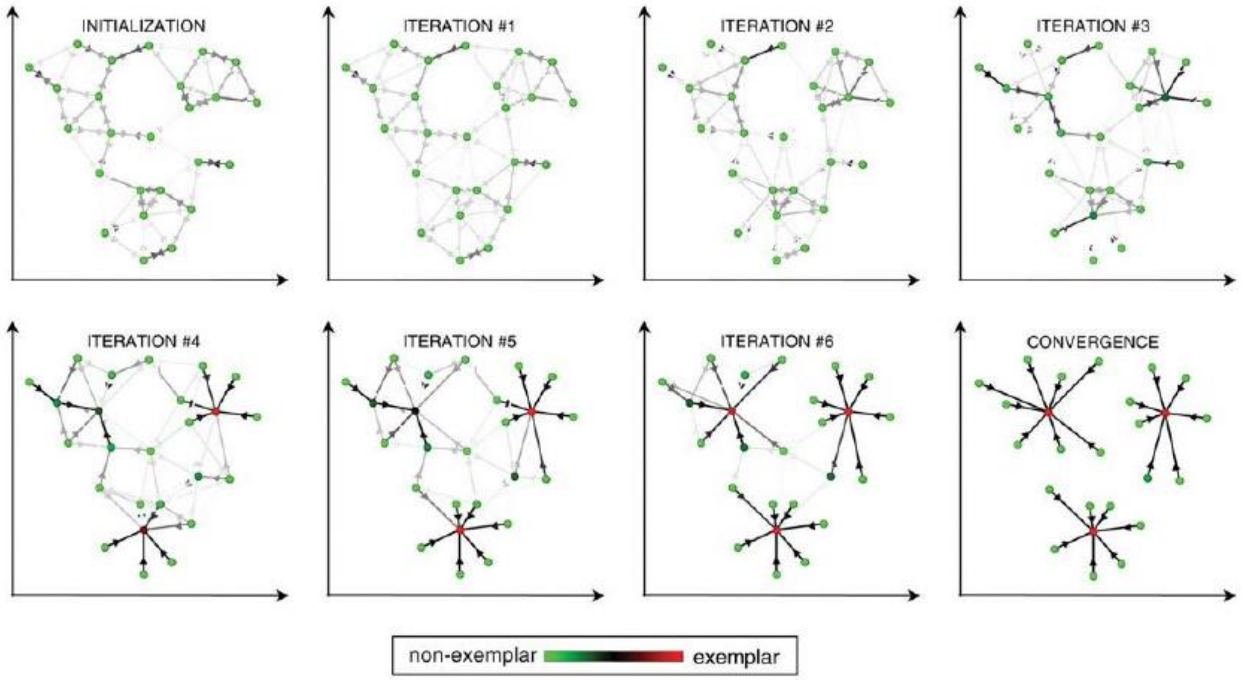
Maximum or complete-linkage clustering	$\max_{a\in A,\, b\in B} d(a,b)$
Minimum or single-linkage clustering	$\min_{a\in A,\, b\in B} d(a,b)$
Unweighted average linkage clustering (or UPGMA)	$\frac{1}{\lvert A\rvert \cdot \lvert B\rvert} \sum_{a\in A} \sum_{b\in B} d(a,b).$
Weighted average linkage clustering (or WPGMA)	$d(i\cup j,k) = \frac{d(i,k)+d(j,k)}{2}.$

Hierarchical Clustering & Agglomerative Hierarchy clustering

+ Top-down approach: All observations start in one cluster, and splits are performed recursively as one moves down the hierarchy.

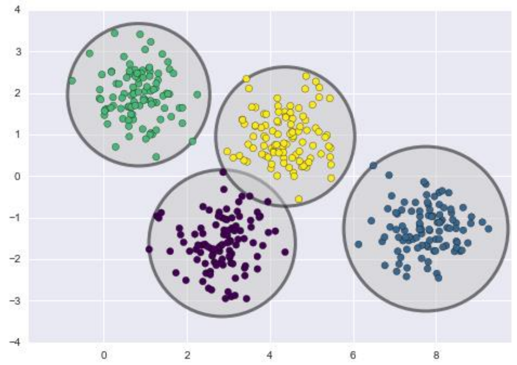
Affinity Propagation

- each data point sends messages to all other points informing its relative attractiveness.
- Each target then responds to all senders with a reply
- Senders reply to the targets with messages informing each target of the target’s revised relative attractiveness to the sender.
- The message-passing procedure proceeds until a consensus is reached.

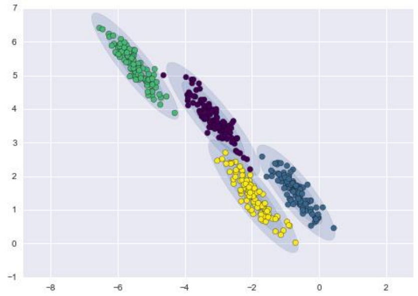


Gaussian Mixture Models

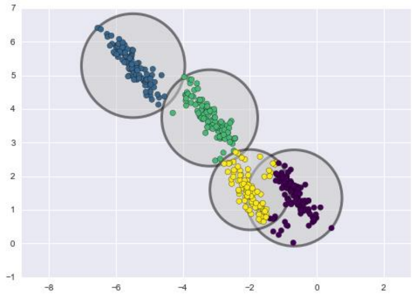
+ One way to think about the k-means model is that it places a circle (or, in higher dimensions, a hyper-sphere) at the center of each cluster. This works fine for when data is circular.
+ In contrast, Gaussian mixture models can handle even very oblong clusters.



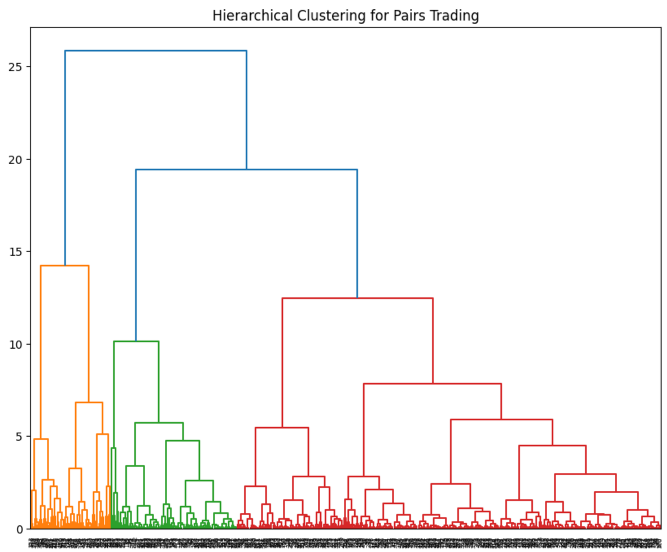
K-means

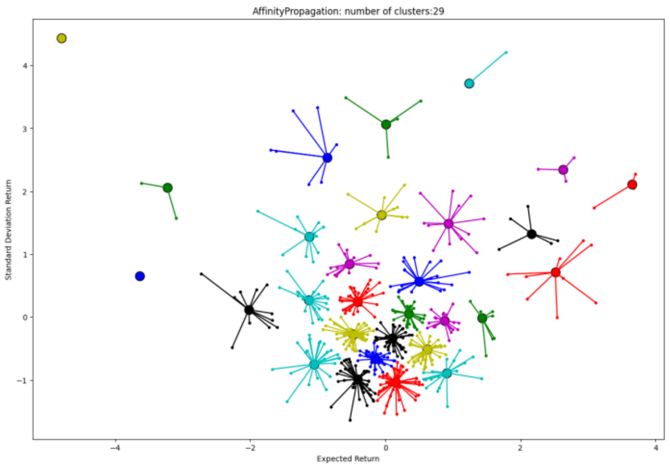


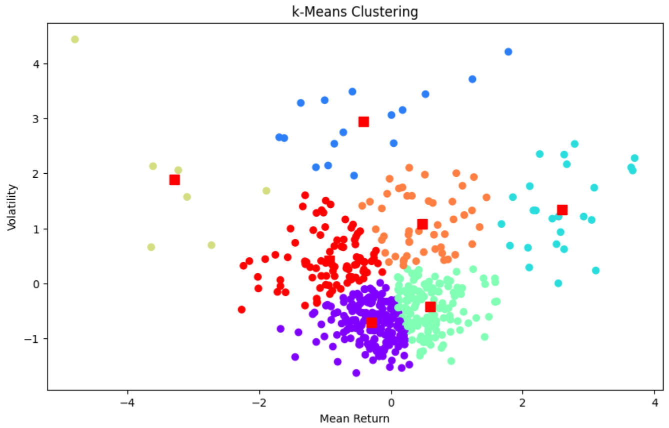
Gaussian Mixture Models



K-means







Example of clustering results

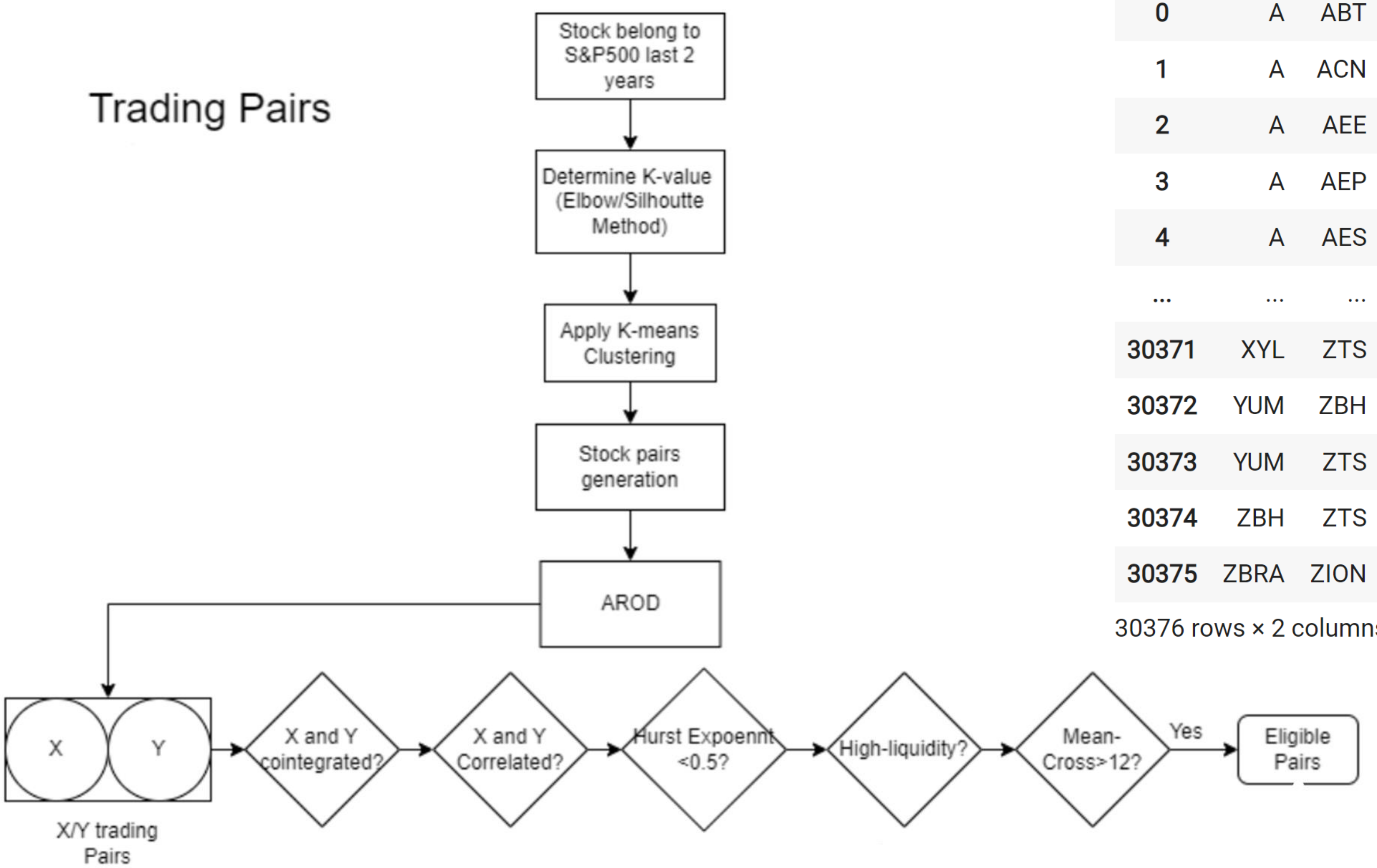
How can we know which clustering algorithms works the best?

+ Silhouette Score = (b-a)/max(a,b) , [-1,1]
+ where a= average intra-cluster distance i.e the average distance between each point within a cluster.
+ b= average inter-cluster distance i.e the average distance between all clusters.
+ 1: Means clusters are well apart from each other and clearly distinguished.
+ -1: Means clusters are assigned in the wrong way.

Comparison

Silhouette Score comparison	
Algorithm	Silhouette Score
K-means	0.389
Hierarchical Clustering	0.325
Affinity Propagation	0.312
DBSCAN	0.365
OPTICS	0.375
Gaussian Mixture Model	0.395

Trading Pairs



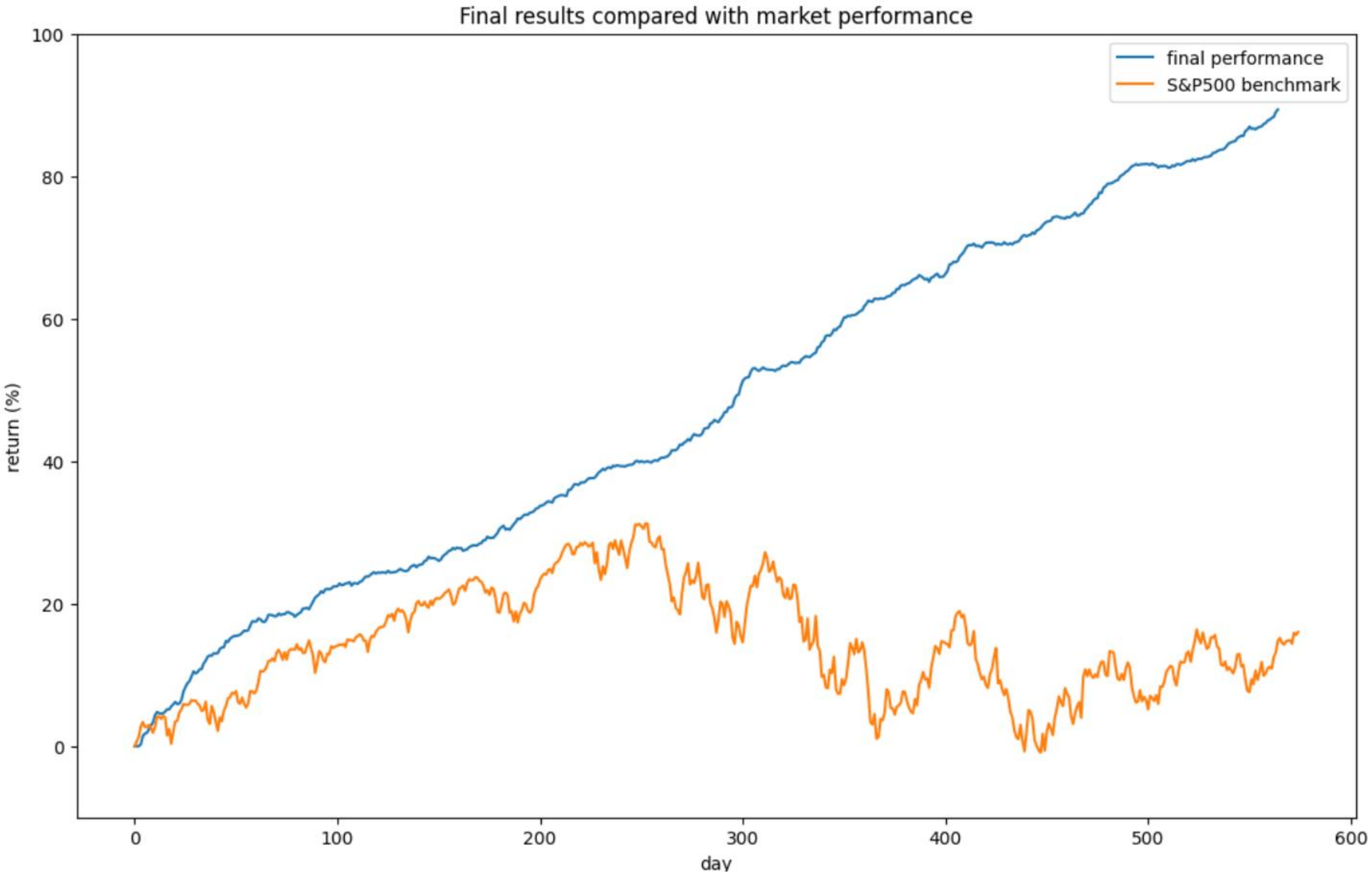
	0	1
0	A	ABT
1	A	ACN
2	A	AEE
3	A	AEP
4	A	AES
...	...	...
30371	XYL	ZTS
30372	YUM	ZBH
30373	YUM	ZTS
30374	ZBH	ZTS
30375	ZBRA	ZION

30376 rows × 2 columns

Apply Reinforcement learning

+ Network: Deep Q-learning
+ Action space: {-1,0,1} #Sell, Hold, Buy
+ Environment variables: spread of trading pairs, normalized price ratio, technical indicators (Moving average, Relative Strength Index), Bi-LSTM predicted price
+ Reward: profit/loss

Last Step: Back-testing



Sharpe ratio: 1.68

$$S_a = \frac{E[R_a - R_b]}{\sigma_a}$$

S_a = Sharpe ratio

E = expected value

R_a = asset return

R_b = risk free return

σ_a = standard deviation of the asset excess return

A Perfect Sharpe ratio?

Neglected the effect of short interest rate (that’s critical)

Transaction fee was not considered

My estimated Sharpe ratio is around 1.45

End