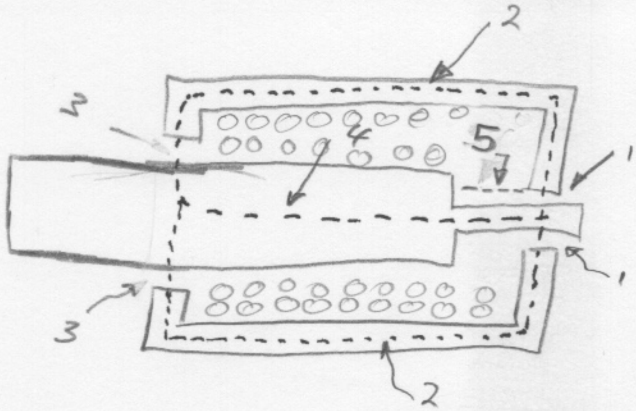
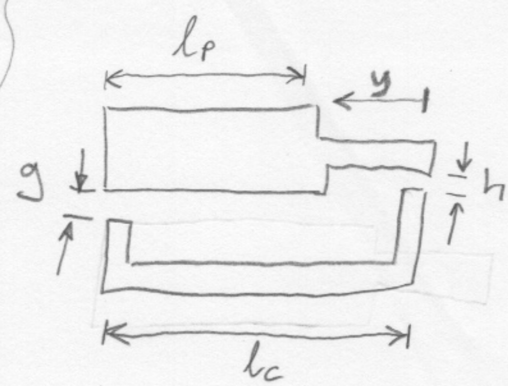
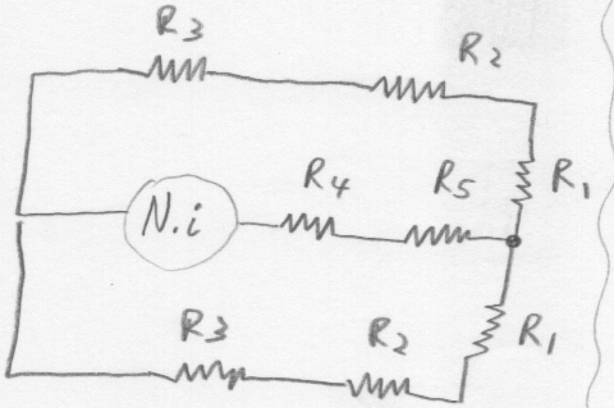
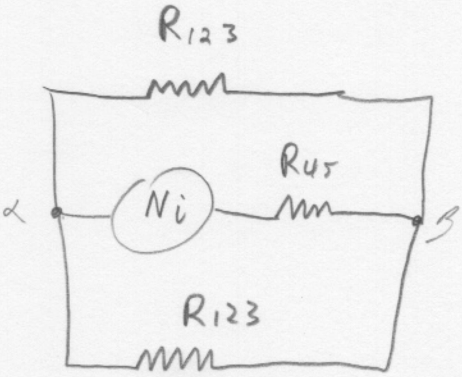


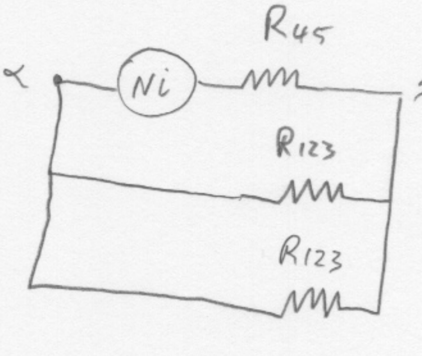




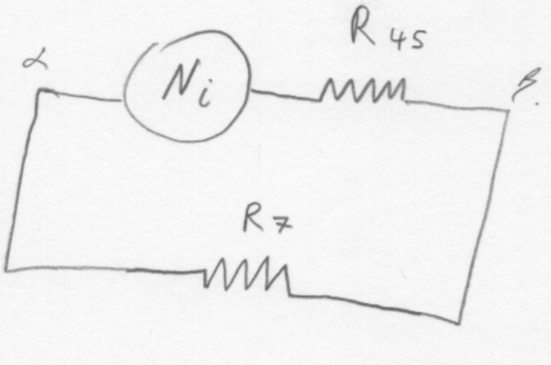
$\equiv$



$\equiv$



$\equiv$



<u>let</u>:-

- $A_2 = \frac{1}{2} A_c$
- $A_1 \equiv \frac{1}{2} A_h$
- $A_3 \equiv \frac{1}{2} A_g$
- $A_4 = A_p$
- $A_5 = A_y$

$$R = \frac{l}{\mu A} \equiv \frac{l}{\mu_0 \mu_r . A}$$

- μ_0 = permeability of vacuum $= 4\pi \times 10^{-7} \left(\frac{\text{Henry}}{\text{metre}}\right)$
- μ_r = relative permeability (dimensionless)
- μ = permeability of material $\left(\frac{\text{Henry}}{\text{metre}}\right)$
- A = cross sectional area (m^2)
- l = length (m)

$$\frac{1}{R_7} = \frac{1}{R_{123}} + \frac{1}{R_{123}} = \frac{2}{R_{123}}$$

$$\therefore \boxed{R_7 = \frac{R_{123}}{2}}$$

$$\therefore R_{TOTAL} = R_{45} + \frac{R_{123}}{2}$$

$$= \frac{l_4}{\mu_4 A_4} + \frac{l_5}{\mu_5 A_5} + \frac{1}{2}\left[\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3}\right]$$

$$= \frac{l_4}{\mu_4 A_4} + \frac{l_5}{\mu_5 A_5} + \cancel{\frac{1}{2}}\left[\frac{l_1 \cdot \cancel{2}}{\mu_1 A_h} + \frac{l_2 \cdot \cancel{2}}{\mu_2 A_c} + \frac{l_3 \cdot \cancel{2}}{\mu_3 \cdot A_g}\right]$$

$$\boxed{R_{TOTAL} = \frac{l_4}{\mu_4 A_4} + \frac{l_5}{\mu_5 A_5} + \frac{l_1}{\mu_1 A_h} + \frac{l_2}{\mu_2 \cdot A_c} + \frac{l_3}{\mu_3 \cdot A_g}}$$

<u>Let</u>:-
- $\mu_2 = \mu_4 = \mu_0 . 5000$ because casing and plunger are made of <u>IRON</u>
- $\mu_1 = \mu_3 = \mu_5 = \mu_0 = 4 \cdot \pi \times 10^{-7} \left(\frac{H}{m}\right)$

$$R_{TOTAL} = \boxed{\frac{l_p}{A_p . \mu_0 . 5000}} + \frac{y}{\mu_0 . A_y} + \frac{h}{\mu_0 A_h} + \boxed{\frac{l_c}{\mu_0 . 5000 . A_c}} + \frac{g}{\mu_0 . A_g}$$

So these terms are very small compared to the others.

$$\therefore R_{TOTAL} = \frac{y}{\mu_0 . A_y} + \frac{h}{\mu_0 A_h} + \frac{g}{\mu_0 . A_g}$$

<u>Next</u>:- Let's say
- $g = h$
- $A_g = A_h$

$$\therefore \quad R_{TOTAL} = \frac{y}{\mu_0 . A_y} + \frac{2 . g}{\mu_0 . A_g}$$

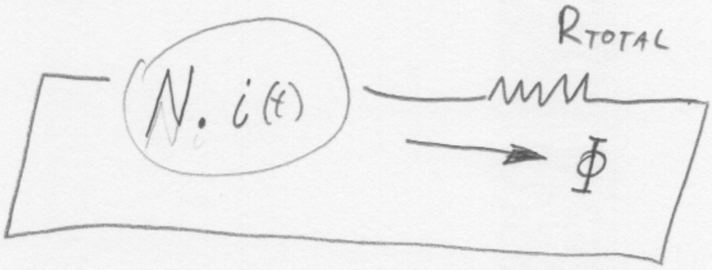
So to simplify further, let's just say that the

- $G = 2g$
- $A_G = A_g$

$$\therefore \quad R_{TOTAL} = \frac{y}{\mu_0 . A_y} + \frac{G}{\mu_0 . A_G}$$

$$\boxed{R_{TOTAL} = \frac{A_G . y + A_y . G}{\mu_0 . A_y . A_G}}$$

So



$$N . i = \Phi . R_{TOTAL}$$

where :- $\Phi \equiv$ Magnetic flux

$$\therefore \quad \Phi = \frac{N . i}{R_{TOTAL}}$$

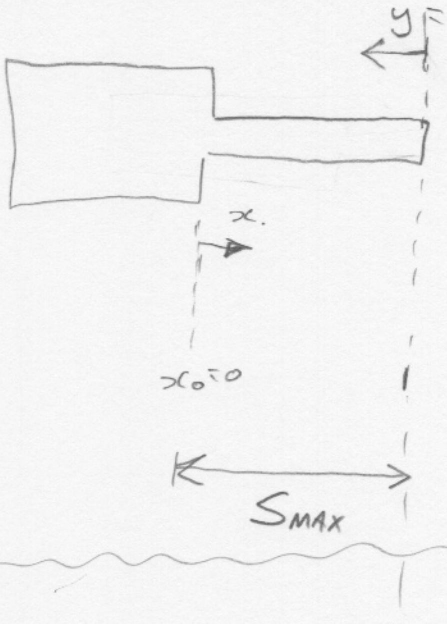
Also:-

$$L = \frac{N . \Phi}{i} = \frac{N . N . \cancel{i}}{\cancel{i} . R_{TOTAL}} = \frac{N^2}{R_{TOTAL}}$$

$$\therefore \quad L = \frac{N^2}{R_{TOTAL}} = \frac{N^2 . \mu_o . A_y . A_G}{A_G . y + A_y . G} = \frac{N^2 . \mu_o . A_G}{\left(\frac{A_G}{A_y}\right) . y + G}$$

Now the energy stored in the INDUCTOR is:-

$\boxed{W_{mag} = \frac{1}{2} . L . i^2}$. This stored magnetic energy has the capacity to be converted into MECHANICAL WORK via the plunger.



$$W_{mech} = \int_{x=0}^{S_{MAX}} F . dx = \int_{S_{MAX}}^{0} -F . dy$$

$$y = S_{MAX} - x$$

$$\boxed{W_{mag} = \frac{1}{2} . i^2 . L}$$

$$\therefore \quad dW_{mag} = dW_{mech}$$

$$\therefore \quad dW_{mag} = \frac{\partial W_{mag}}{\partial y} . dy$$

$$\therefore \quad dW_{mech} = -F . dy$$

$$\therefore \quad F = -\frac{\partial W_{mag}}{\partial y}$$

$$\boxed{\therefore \quad F = -\frac{1}{2} . i^2 . \frac{\partial L}{\partial y}}$$

So? - So revisiting our expression for INDUCTANCE

$$L = \frac{N^2 . \mu_o . A_y . A_G}{A_G . y + A_y . G} = N^2 . \mu_o . A_y . A_G . \left(A_G . y + A_y . G\right)^{-1}$$

$$\therefore \frac{\partial L}{\partial y} = -N^2 . \mu_o . A_y . A_G^2 . \left(A_G . y + A_y . G\right)^{-2}$$

$$\therefore F = -\frac{1}{2} . i^2 . \frac{\partial L}{\partial y} = \frac{-\frac{1}{2} . i^2 . \left(-N^2\right) . \mu_o . A_y . A_G^2}{\left(A_G . y + A_y . G\right)^2}$$

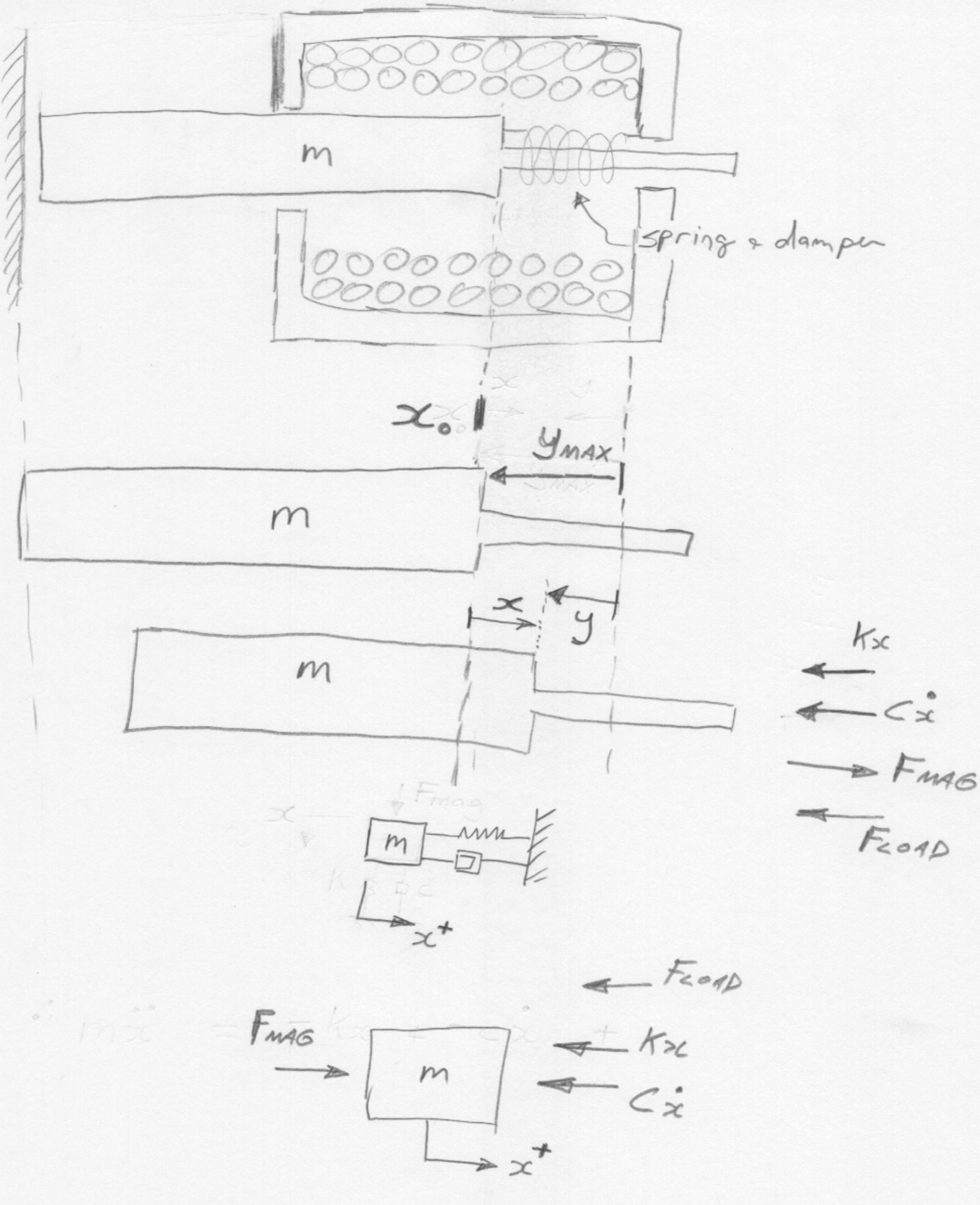
$$\therefore F = \frac{\frac{1}{2} . i^2 . N^2 . \mu_o . A_y . A_G^2}{\left(A_G . y + A_y . G\right)^2}$$

$$F = \frac{\frac{1}{2} . i^2 . C_1}{\left(C_2 . y + C_3\right)^2}$$

$$C_1 = N^2 . \mu_o . A_y . A_G^2$$
$$C_2 = A_G$$
$$C_3 = A_y . G$$

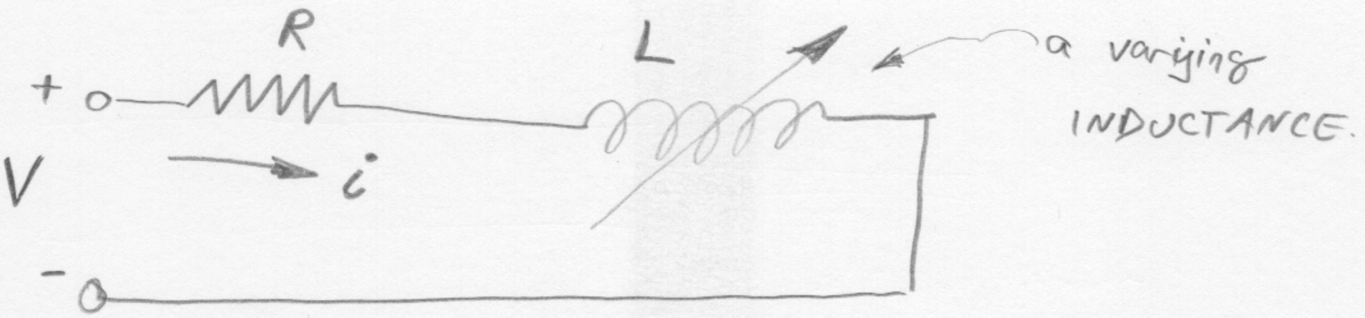
$$\therefore L = \frac{\left(\frac{C_1}{C_2}\right)}{C_2 . y + C_3}$$

Consider now the MECHANICAL DYNAMICS:



$$\boxed{\therefore\ m\ddot{x} + c\dot{x} + kx = F_{MAG} - F_{LOAD}}$$

Now consider the ELECTRICAL dynamics



$$V = i.R + \frac{d}{dt}(L.i)$$

$$V = (i.R) + \left(L.\frac{di}{dt}\right) + \left(i\,\frac{dL}{dy}.\frac{dy}{dt}\right)$$

UNITS:-

$$L = \frac{\left(\frac{C_1}{C_2}\right)}{C_2 \cdot y + C_3}$$

where:-

- $C_1 = N^2 \cdot \mu_0 \cdot A_y \cdot A_G^2$
- $C_2 = A_G$
- $C_3 = A_y \cdot G$
- $\mu_0 = 4\pi \times 10^{-7} \cdot \left(\frac{H}{m}\right)$

So:-

- $C_1 = \left[\frac{H}{m}\right] \cdot m^2 \cdot m^4 = H \cdot m^5$
- $C_2 = m^2$
- $C_3 = m^2 \cdot m = m^3$

$$\therefore L = \frac{\left(\frac{C_1}{C_2}\right)}{C_2 \cdot y + C_3} = \frac{\left(\frac{H \cdot m^5}{m^2}\right)}{m^2 \cdot m + m^3} = \frac{H \cdot m^5}{m^2 \times m^3} = (H) \checkmark$$