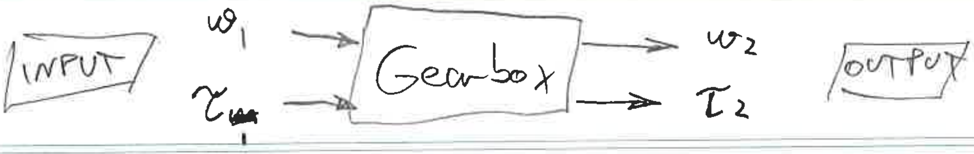
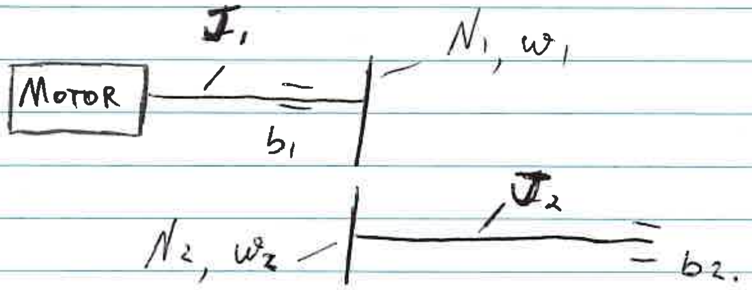


- $\omega_2 = \omega_1 / G$
- $\tau_2 = \tau_1 \cdot G$

$\Big\}$ ∴ $G > 1$ means increase in output Torque.

SPUR GEARS



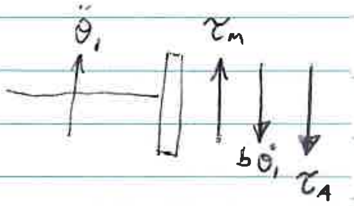
$$\omega_1 N_1 = -\omega_2 N_2$$

$$G \triangleq \frac{\omega_1}{\omega_2} = \frac{\mp N_2}{N_1} = \frac{R_2}{R_1}$$

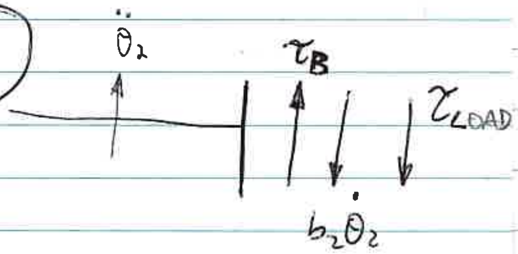
$$\omega_1 T_1 = \omega_2 T_2$$

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

$$J_1 \ddot{\theta}_1 = \tau_m - b_1 \dot{\theta}_1 - \tau_A.$$



$$J_2 \ddot{\theta}_2 = \tau_B - b_2 \dot{\theta}_2 - \tau_L$$



Say:- $\tau_A = F.R_1$
$\tau_B = F.R_2$ $\Big\}$ ∴

$$\boxed{\frac{\tau_B}{\tau_A} = \frac{R_2}{R_1}} \Rightarrow \tau_B = G \cdot \tau_A.$$

$$G = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{R_2}{R_1} = \frac{T_2}{T_1}$$

$$J_1\ddot{\theta}_1 = \tau_m - b_1\dot{\theta}_1 - \tau_A \tag{1}$$

$$J_2\ddot{\theta}_2 = \tau_B - b_2\dot{\theta}_2 - \tau_L \tag{2}$$

And:-

$$\left.\begin{aligned} \dot{\theta}_1 &= G.\dot{\theta}_2 \\ \ddot{\theta}_1 &= G\,\ddot{\theta}_2 \\ \tau_B &= G.\,\tau_A \end{aligned}\right] \tag{2b}$$

$$\cancel{J_1\ddot{\theta}_1 = \tau_m - b_1\dot{\theta}_1 - \tau_A}$$

$$J_2\frac{\ddot{\theta}_1}{G} = G\tau_A - b_2\frac{\dot{\theta}_1}{G} - \tau_L$$

$$\boxed{\therefore\ J_2.\frac{\ddot{\theta}_1}{G^2} = \tau_A - b_2\frac{\dot{\theta}_1}{G^2} - \frac{\tau_L}{G}} \tag{3}$$

$\therefore$ substitute (3) into (1)

$$J_1\ddot{\theta}_1 = \tau_m - b_1\dot{\theta}_1 - \left(J_2\frac{\ddot{\theta}_1}{G^2} + b_2\frac{\dot{\theta}_1}{G^2} + \frac{\tau_L}{G}\right)$$

$$\therefore\ \left(J_1 + \frac{J_2}{G^2}\right)\ddot{\theta}_1 = \tau_m - \left(b_1 + \frac{b_2}{G^2}\right)\dot{\theta}_1 - \frac{\tau_L}{G} \tag{4}$$

$J_1 \equiv J_{motor}$ $\theta_1 = \theta_{motor}$

$J_2 \equiv J_{shaft} + J_{wheels}$ $\theta_2 = \theta_{wheels}$

Similarly we could substitute 2b into 4 :-

$$\left(J_1 + \frac{J_2}{G^2}\right)\ddot{\theta}_1 = \tau_m - \frac{\tau_L}{G} - \left(b_1 + \frac{b_2}{G^2}\right)\dot{\theta}_1 \quad (5)$$

$$\therefore \left(J_1 + \frac{J_2}{G^2}\right) G\ddot{\theta}_2 = \tau_m - \frac{\tau_L}{G} - \left(b_1 + \frac{b_2}{G^2}\right) G\dot{\theta}_2$$

$$\therefore \left(J_1 + \frac{J_2}{G^2}\right)\ddot{\theta}_2 = \frac{\tau_m}{G} - \frac{\tau_L}{G^2} - \left(b_1 + \frac{b_2}{G^2}\right)\dot{\theta}_2$$

Now multiply Both sides by G^2

$$\left(G^2 . J_1 + J_2\right)\ddot{\theta}_2 = G.\tau_m - \tau_L - \left(b_1 . G^2 + b_2\right)\dot{\theta}_2 \quad (6)$$

$$\boxed{\tau_m = K.i}$$

$J_1 \equiv J_{motor}$

$J_2 \equiv J_{shaft} + J_{wheels}$

$\theta_1 \equiv$ motor

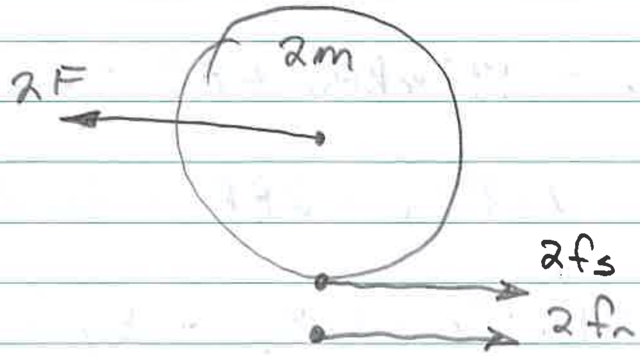
$\theta_2 \equiv$ wheel.

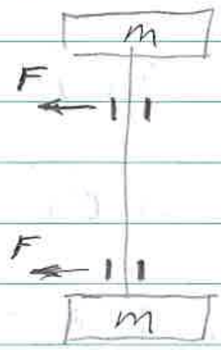
Consider the LEFT & RIGHT wheels:-

- f_s = friction $= |f_s| \cdot \text{sign}(\dot{x})$
- f_r = rolling resistance $= -1 \cdot |f_r| \cdot \text{sign}(\dot{x})$
- F = reaction force from car on wheel axle
- m = mass of 1 wheel
- $x = R\theta$
  $\dot{x} = R\dot{\theta}$
  $\ddot{x} = R\ddot{\theta}$

Assume that wheels share a common axle

∴



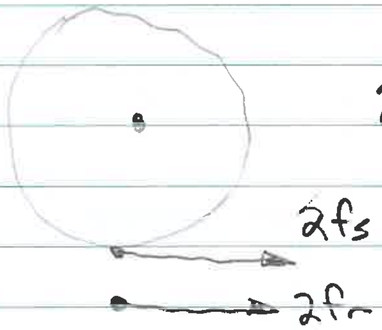


∴ Newton's Law says

$$2m\ddot{x} = \sum F$$
$$= 2f_s + 2f_r - 2F$$

∴ $m R\ddot{\theta} = f_s + f_r - F$ — (7)

Now the TORQUE LOAD on the combined wheel + axle system is:



$$\tau_{LOAD} = R.(2f_s + 2f_r)$$

$\therefore$ using (7) we can write:

$$\tau_{LOAD} = 2R(f_s + f_r)$$

$$\tau_{LOAD} = 2R(mR\ddot{\theta} + F) \quad - (8)$$

So let's substitute (8) into (6)

$$\underbrace{(G^2 J_1 + J_2)}_{J_{eq}} \ddot{\theta}_w = G.\tau_m - \tau_{LOAD} - \underbrace{(b_1 G^2 + b_2)}_{b_{eq}} \dot{\theta}_w$$

$$J_{eq}.\ddot{\theta}_w = G.\tau_m - \tau_{LOAD} - b_{eq}.\dot{\theta}_w$$

$$\therefore \quad J_{eq}.\ddot{\theta}_w = G.\tau_m - 2R(mR\ddot{\theta}_w + F) - b_{eq}\dot{\theta}_w$$

$$\therefore \quad J_{eq}\ddot{\theta}_w = G.\tau_m - 2mR^2\ddot{\theta}_w - 2RF - b_{eq}.\dot{\theta}_w$$

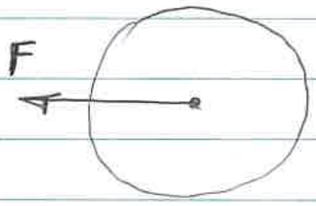
$$\therefore \quad 2RF = G.\tau_m - \ddot{\theta}_w(J_{eq} + 2mR^2) - b_{eq}\dot{\theta}_w$$

$$\therefore \quad F = \frac{G.\tau_m - b_{eq}\dot{\theta}_w - \ddot{\theta}_w.(J_{eq} + 2mR^2)}{2R}$$

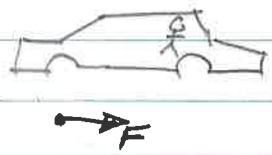
(9)

$$\therefore \quad \tau_m = \frac{2RF + \ddot{\theta}_w\left(J_{eq} + 2mR^2\right) + b_{eq}\cdot\dot{\theta}_w}{G} \tag{10}$$

AND REMEMBER OUR Sign Convention



WHEEL



CAR