STATISTICS

Dispersion :-

It is a mesure of the variations. It measures the spread of the observed values of a distribution around the central value.

Measures of Dispersion :-

The measures of dispersion commonly used are

(1.) Range

(2.) Quartile deviation OR Semi inter quartile range.

(3.) Mean deviation

(4.) Standard ~~devi~~ deviation.

Measures of despersion are also called averages of the second kind.

1. Range :-

We find the difference of maximum observation and minimum observation values of each series.

This difference is called the 'Range' of the data.

Thus,

$\boxed{\text{Range of a series = Max value − Mini value}}$

Example:-

The runs scored by two batsmen in their last ten matches as follows.

Batsman A :- 30, 91, 0, 64, 42, 80, 30, 5, 117, 71

Batsman B :- 53, 46, 48, 50, 53, 53, 58, 60, 57, 52

Solu^n:-

Mean of Batsman A $= \frac{\sum n}{n}$

$= 53$

Mean of Batsman B $= 53$

Therefore,

Range of Batsman A $= 117 - 0$

$= 117$

Range of Batsman B $= 60 - 46$

$= 14$

$\boxed{\therefore \text{ Range of A > Range of B}}$

2. Quartile Deviation :-

We shall study ~~in~~ the Quartile Deviation for higher Degree.

OR

We have not read in this class.

3. Mean Deviation :-

Mean deviation of a distribution is the arithmetic mean of the absolute ~~dev~~ deviations of the terms of the distribution from its statistical Mean, Median or Mode.

Mean Deviation for ungrouped Data

i) For Mean OR About Mean

$$\boxed{M.D(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{N}}$$

Example :-

Find the mean deviation about the mean for the following data.

6, 7, 10, 12, 13, 4, 8, 12

Solu^n:-

Given Data is

6, 7, 10, 12, 13, 4, 8, 12

$\therefore$ Mean $(\bar{x}) = \frac{\sum x_i}{N}$

$= \frac{6+7+10+12+13+4+8+12}{8}$

$= \frac{72}{8} = 9$

x_i	$\lvert x_i - \bar{x} \rvert$
6	$\lvert -3 \rvert = 3$
7	$\lvert -2 \rvert = 2$
10	1
12	3
13	4
4	5
8	1
12	3

$\sum \lvert x_i - \bar{x} \rvert = 22$

$\therefore$ M.D $(\bar{x}) = \frac{\sum \lvert x_i - \bar{x} \rvert}{N}$

$= \frac{22}{8}$

$= 2.75$ Ans

(iii) Mean Deviation about Median.

$$\boxed{M.D(M) = \frac{\sum |x_i - M|}{N}}$$

If N is odd

Then Median (M) = $\left(\frac{N+1}{2}\right)^{th}$ observation

If N is even

Then Median (M) = $\frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$ observation

Note:-

Data Arrange in ascending order for median.

$$\boxed{\text{median} = l + \left(\frac{\frac{N}{2} - C.F}{f}\right) \times h}$$

where Median class = $\frac{N}{2}$

l = Lower limit of median class

N = Number of observation.

$C.F$ = Cumulative Frequency of class Preceding the median class

f = Frequency of median class

h = Class size

Example:-

Find the mean deviation about the median for the following data.

3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21

Solun:-

Data Arrange in Ascending order.

3, 3, 4, 5, 7, $\boxed{9,}$ 10, 12, 18, 19, 21

∴ Number of observations is = 11

∴ ~~Mean~~

∴ Median(M) = $\left(\frac{N+1}{2}\right)^{th}$ observation

$= \left(\frac{11+1}{2}\right)^{th}$

= 6 th observation

= 9.

x_i	$\lvert x_i - M \rvert$
3	$\lvert 3-9 \rvert = \lvert -6 \rvert = 6$
3	$\lvert -6 \rvert = 6$
4	5
5	4
7	2
9	0
10	1
12	3
18	9
19	10
21	12
	$\sum \lvert x_i - M \rvert = 58$

$$\therefore MD(M) = \frac{\sum |x_i - M|}{N}$$

$$= \frac{58}{11}$$

$$= 5.27 \quad \underline{\text{Ans}}$$

Mean Deviation For Grouped Data

We know that data can be grouped into two ways.

(a.) Discrete frequency distribution.

(b.) Continuous frequency distribution.

a. Discrete Frequency Distribution

Mean Deviation about Mean

$$\boxed{MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}}$$

Example:—

Find mean deviation about the mean for the following Data.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5.

Solu^n:—

Table

x_i	f_i	$f_i x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
2	2	4	$\lvert 2-7.5\rvert = \lvert -5.5\rvert = 5.5$	11
5	8	40	$\lvert -2.5\rvert = 2.5$	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	$\Sigma f_i = 40$	$\Sigma f_i x_i = 300$		$\Sigma f_i\lvert x_i-\bar{x}\rvert = 92$

$$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{30\cancel{0}}{4\cancel{0}}$$

$$= 7.5$$

$$\therefore MD(\bar{x}) = \frac{92}{40}$$

$$= 2.3$$

Ans

Mean Deviation about Median

$$\boxed{MD(M) = \frac{\sum f_i |x_i - M|}{N}}$$

Example:-

Find the mean deviation about the median for the following data.

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

Solu^n:-

Table

x_i	f_i	C.F	$\|x_i - M\|$	$f_i \|x_i - M\|$
3	3	3	$\|-10\| = 10$	30
6	4	7	7	28
9	5	12	4	20
12	2	14	1	2
13	4	18	0	0
15	5	23	2	10
21	4	27	8	32
22	3	30	9	27

Medin $N = 30$

$$\text{median} = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2} = \frac{15^{th}\text{ obs} + 16^{th}\text{ obs}}{2} = \frac{13+13}{2} = 13$$

$\therefore \sum f_i |x_i - M| = 149$

$\therefore MD(M) = \frac{149}{30}$

$= \frac{49.6}{10}$

$= 4.96$ Ans

b. Continuous Frequency Distribution

(i) Mean Deviation About Mean

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Example:-

Find the mean deviation about the mean for the following data.

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of students	2	3	8	14	8	3	2

Solu^n:-

C.I	x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10-20	15	2	30	30	60
20-30	25	3	75	20	60
30-40	35	8	280	10	80
40-50	45	14	630	0	0
50-60	55	8	440	10	80
60-70	65	3	195	20	60
70-80	75	2	150	30	60
		$\sum f_i = 40$	$\sum f_i x_i = 1800$		$\sum f_i \lvert x_i - \bar{x} \rvert = 400$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{1800}{40}$$

$$= 45$$

$$\therefore MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$= \frac{400}{40}$$

$$= 10 \text{ Ans}$$

Note:- If given continuous frequency distribution then change continuous frequency distribution into Discrete frequency distribution.

(ii) Mean deviation about median.

$$\boxed{MD(M) = \frac{\sum f_i |x_i - M|}{\sum f_i}}$$

$$\text{Median}(M) = l + \left(\frac{\frac{N}{2} - C.F}{f}\right) \times h$$

Example:-

Calculate the mean deviation about ~~Median~~ Median for the following data.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

Solu^n:-

Table

Class	f_i	x_i	C.F	$\lvert x_i - M\rvert$	$f_i\lvert x_i - M\rvert$
0-10	6	5	6	23	138
10-20	7	15	13	13	91
20-30	15	25	28	3	45
30-40	16	35	44	7	112
40-50	4	45	48	17	68
50-60	2	55	50	27	54

$N = 50$ $\qquad \sum f_i\lvert x_i - M\rvert = 508$

Median class $\left(\frac{N}{2}\right) = \left(\frac{N}{2}\right)^{th}$ observation $= 25$

$l = 20 =$ Lowest limit of median class

$\frac{N}{2} = 25$

$F = 15 =$ frequency of median class

$C.F = $ C.f of class preceding median $= 13$

$h =$ Class size $= 10$

$$\text{Median}(M) = 20 + \frac{25-13}{15}\times 10$$

$$= \cancel{20 + \frac{100}{13}}$$

$$= \cancel{20 + 7.7}$$

$$= 20 + \frac{\cancel{120}^{8}}{\cancel{15}}$$

$$= 28$$

$$\therefore MD(M) = \frac{\sum f_i|x_i - M|}{\sum f_i}$$

$$= \frac{508}{50}$$

$$= \frac{101.6}{10}$$

$$= 10.16$$

Ans

Variance and Standard Deviation

Variance :-

Mean of the squares of the deviations from mean is called the variance.

It is denoted by σ^2 (Read as Sigma Square)

$$\therefore \boxed{\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\boxed{Var(x) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

Example :-

Find the variance of the following data

6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Solu^n :-

Table

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	−9	81
8	−7	49
10	−5	25
12	−3	9
14	−1	1
16	1	1
18	3	9
20	5	25
22	7	49
24	9	81

$$Mean = \frac{150}{10} = 15$$

$$\Sigma(x_i - \bar{x})^2 = 330$$

$$\therefore \sigma^2 = Var(x) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

$$= \frac{330}{10}$$

$$= 33 \text{ Ans}$$

Standard Deviation :-

The proper measure of dispersion about the mean of a set of observations is expressed as positive square root of the variance is called standard Deviation.

It is denoted by σ.

$$\therefore \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$$

Example :-

Find the variance and standard deviation for the following data:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Solu^n:-

Table

x_i	f_i	$f_i x_i$	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	$\sum f_i = 30$	$\sum f_i x_i = 420$			$\sum f_i(x_i-\bar{x})^2 = 1374$

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{\cancel{420}^{14}}{\cancel{30}}$$

$$= 14$$

$$\therefore Var = \sigma^2 = \frac{\sum_{i=1}^{n} f_i(x_i-\bar{x})^2}{N}$$

$$= \frac{1374}{30}$$

$$= 45.8$$

Therefore,

$$\text{Standard Deviation}(\sigma) = \sqrt{\frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{45.8}$$

$$= 6.77 \quad \text{Ans}$$

For Continuous Frequency distribution

Example:-

Calculate the mean, variance and standard deviation for the following distribution :-

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Soluⁿ:-

Table

Class	frequency (f_i)	x_i	$f_i x_i$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178

$\sum f_i = 50$ $\qquad \sum f_i x_i = 3100$ $\qquad \sum f_i(x_i-\bar{x})^2 = 10050$

$$\text{Mean}(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{310\cancel{0}}{5\cancel{0}}$$

$$= 62 \quad \text{Ans}$$

$$\text{Variance}(\sigma^2) = \frac{\sum_{i=1}^{n} f_i(x_i-\bar{x})^2}{N}$$

$$= \frac{1005\cancel{0}}{5\cancel{0}}$$

$$= 201 \quad \text{Ans}$$

$\therefore$ Standard Deviation $(\sigma) = \sqrt{\dfrac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{n}}$

$= \sqrt{201}$

$= 14.18$ Ans