(Unit – 2)

Pharmaceutics – II

(Unit Operations I, including Engineering Drawing)

" Fluid Flow "

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❖ What is Fluid Flow?

Fluid Flow is a part of fluid mechanics and deals with fluid dynamics. Fluids such as gases and liquids in motion are called fluid flow. It involves the motion of a fluid subjected to unbalanced forces. This motion continues as long as unbalanced forces are applied.

For example, if you are pouring water from a mug, the velocity of water is very high over the lip, moderately high approaching the lip, and very low at the bottom of the mug. The unbalanced force is gravity, and the flow continues as long as the water is available and the mug is tilted.

❖ Types of Fluid

Following are the types of fluid:

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-Newtonian fluid
5. Ideal plastic fluid
6. Incompressible fluid
7. Compressible fluid

Ideal fluid:

A fluid is said to be ideal when it cannot be compressed and the viscosity doesn't fall in the category of an ideal fluid. It is an imaginary fluid which doesn't exist in reality.

Real fluid:

All the fluids are real as all the fluid possess viscosity.

Newtonian fluid:

When the fluid obeys Newton's law of viscosity, it is known as a Newtonian fluid.

Non-Newtonian fluid:

When the fluid doesn't obey Newton's law of viscosity, it is known as Non-Newtonian fluid.

Ideal plastic fluid:

When the shear stress is proportional to the velocity gradient and shear stress is more than the yield value, it is known as ideal plastic fluid.

Incompressible fluid:

When the density of the fluid doesn't change with the application of external force, it is known as an incompressible fluid.

Compressible fluid: When the density of the fluid changes with the application of external force, it is known as compressible fluid.

❖ Types of Fluid Flow

Fluid flow has all kinds of aspects — steady or unsteady, compressible or incompressible, viscous or non-viscous, and rotational or irrotational, to name a few. Some of these characteristics reflect the properties of the liquid itself, and others focus on how the fluid is moving.

Steady or Unsteady Flow: Fluid flow can be steady or unsteady, depending on the fluid's velocity:

- **Steady:** In steady fluid flow, the velocity of the fluid is constant at any point.
- **Unsteady:** When the flow is unsteady, the fluid's velocity can differ between any two points.

Viscous or Non-Viscous Flow: Liquid flow can be viscous or non-viscous.

Viscosity is used to measure of the thickness of a fluid, and very gloppy fluids such as motor oil or shampoo are called **viscous fluids**.

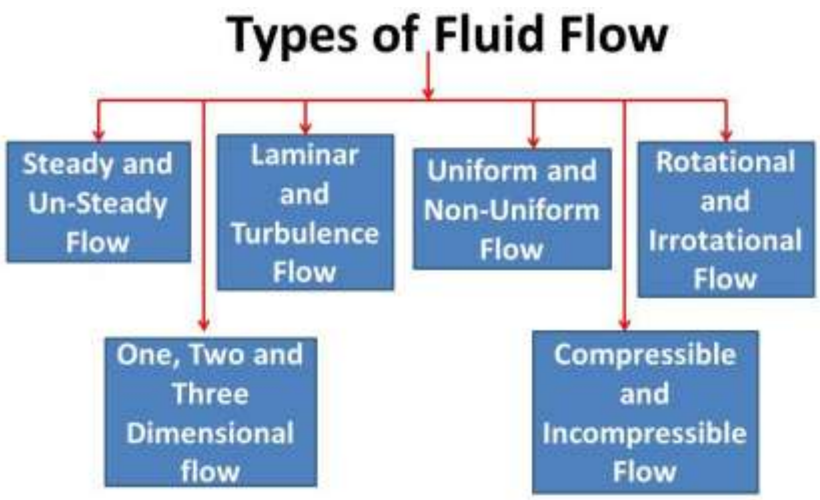
Fluid Flow Equation

The volume of fluid replaced in a given interval of time is called the fluid flow equation.

Mass flow rate=ρAV

Where,
ρ = density
V = Velocity
A = area

Flow rate=Area × Velocity



1. **Steady & Steady Flows: -**

steady Flows:

In which the fluid Characteristics Like velocity, pressure, density, etc. At a Point do not change with time

Unsteady Flow:

In which the fluid Characteristics Like velocity, pressure or density at a point changes with respect to time.

2. **Uniform & Uniform Flow**

Uniform Flow:

In which the velocity at given time does not change with respect to space (length of direction of the flow)

Non-Uniform Flow: -

In which the velocity at any time changes with respect to space. Changing in space

3. **Laminar Flow & Turbulent flows: -**

Laminar Flow:

In which the fluid particles move along well-defined paths or stream lines.

Turbulent flows:

- fluid moves in very irregular paths or zig – zag Way.
- velocity at a point fluctuates.

4. **Compressible Flows & Incompressible Flows**

Compressible Flows:

- In which the density of fluid changes from point to point.
- The density is not constant for the fluid.

Incompressible Flows:

- In which the density of the fluid changes from point to point.
- The density is constant for the fluid.

5. **Rotational Flow & Irrotational Flows: -**

Rotational Flow:

In which the fluid particles while flowing along stream lines, also rotate about their own axis.

Irrotational Flows:

In which the fluid particles while flowing along stream lines, do not rotate about their own axis.

6. **One, Two & Three-Dimensional Flows**

One Dimensional Flow:

In which the flow parameter such as velocity is a function of time and one space co-ordinate only, is called.

Two-Dimensional Flow:

In which the flow parameter such as velocity is a function of time and two rectangular space co-ordinates, is called.

Three-Dimensional Flow:

In which the flow parameter such as velocity is a function of time and Three mutually perpendicular directions, is called.

❖ Reynolds number:

Reynolds number is a dimensionless value which is applied in fluid mechanics to represent whether the fluid flow in a duct or pat a body is steady or turbulent. This value is obtained by comparing the inertial force with the viscous force.

The Reynolds number id denoted by Re.

Reynolds number is given by,

Reynolds Number = Inertial Force / Viscous Force

The Reynolds number formula is expressed by,

$$R_e = \frac{\rho V L}{\mu}$$

Where,

ρ = density of the fluid,

V = velocity of the fluid,

μ = viscosity of fluid,

L = length or diameter of the fluid.

Reynolds number formula can be used in the problems to calculate the Velocity **(V)**, density **(ρ)**, Viscosity **(μ)** and diameter **(L)** of the liquid.

Flow type	Reynolds Number Range
Laminar regime	Less than 2000
Transition regime	2300-4000
Turbulent regime	Above 4000

❖ Viscosity:

Viscosity of often referred to as the thickness of a fluid. You can think of water (low viscosity) and honey (high viscosity). However, this definition can be confusing when we are looking at fluids with different densities. At a molecular level, viscosity is a result the interaction between the different molecules in a fluid. This can be also understood as friction between the molecules in the fluid. Just like in the case of friction between moving solids, viscosity will determine the energy required to make a fluid flow.

- **Shear stress** is the force per unit area required to move one layer of fluid in relation to another.
- **Shear rate** is the measure of the change in speed at which intermediate layers move with respect to one another.

$$\textbf{Viscocity} = \frac{\textbf{Shear Stress}}{\textbf{Shear Rate}}$$

Newtonian vs. Non-Newtonian Fluids:

Isaac Newton, the man to discover this formula, thought that, at a given temperature and shear stress, the viscosity of a fluid would remain constant regardless of changes to the shear rate.

He was only partly right. A few fluids, such as water and honey, do behave this way. We call these fluids Newtonian fluids. Most fluids, however, have viscosities that fluctuate depending on the shear rate. These are called Non-Newtonian fluids.

There are five types of non-Newtonian fluids: thixotropic, rheopectic, pseudoplastic, dilatant, and plastic. Different considerations are required when measuring each of these fluid types.

Some of the standard methods of measuring viscosity include:

- **Capillary Viscometer**: One of the oldest methods of measuring viscosity, the capillary viscometer measures the time between the volume of liquid/sample to pass through the length of the capillary tubes.
- **Rotational Viscometer**: Measures the torque required to revolve an object within the volume of liquid.
- **Falling Sphere Viscometer**: Measures the viscosity by dropping a sphere of a specific weight & density and measures the time it takes the sphere to reach designated junctures.
- **Zahn Cup Method**: Measures by observing the time it takes the volume of liquid to empty the cup though a small hole in the bottom of a container/cups.
- **Vibrational Viscometer**: By measuring the vibrational waves using a vibrating rod submerged in fluid, viscosity is calculated by analyzing the dampening of the vibration.
- **VROC Viscometer**: This viscometer is pressure driven using a pumping system that the laminar flow to push the liquid into a rectangular slit with pressure sensors, measuring the viscosity of a fluid through the change in pressure after passing each pressure sensor within the microchip.

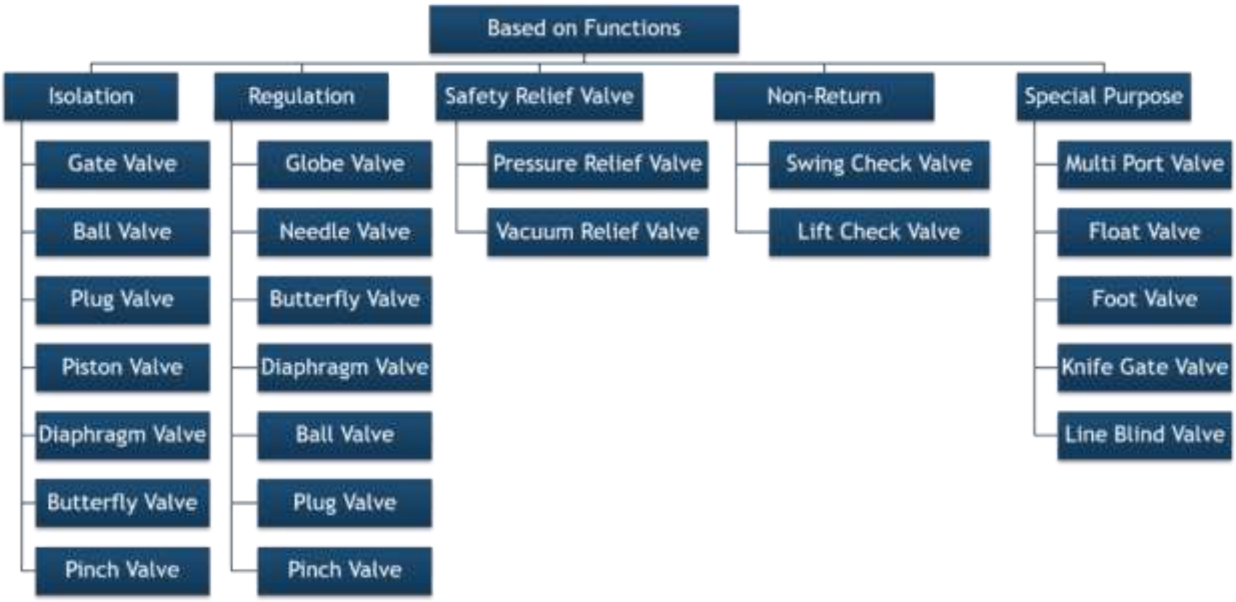
Valves:

Valve is a device that regulates, controls or directs the flow of a fluid by opening, closing, or partially obstructing fluid flow. A valve is a mechanical device that controls the flow and pressure of fluid within a system or Process. So basically, it controls flow & pressure.

Types of Valves

In piping following types of valves are used depending on the requirements. The cost of Valve in the piping system is up to 20 to 30% of the overall piping cost. And the cost of a given type and size of the valve can vary 100%. It means that if you choose ball valve over butterfly valve for the same function. It can cost you more. So, the selection of valves is essential to the economics, as well as operation, of the process plants.

- Gate Valve
- Globe Valve
- Check Valve
- Plug valve
- Ball Valve
- Butterfly Valve
- Needle Valve
- Pinch Valve
- Pressure Relief Valve



1. Gate valve

Gate valve is the most common type of valve in any process plant. It is a linear motion valve used to start or stop fluid flow. In service, these valves are either in fully open or fully closed position. Gate valves are used in almost all fluid services such as air, fuel gas, feedwater, steam, lube oil, hydrocarbon, and all most any services. Gate valve provides good shutoff.

2. Globe Valve

Globe valve is used to stop, start, and regulate the fluid flow. Globe Valves are used in the systems where flow control is required and leak tightness is also necessary. Globe valve provides better shut off as compared to gate valve and it is costlier than gate valve.

3. Check Valve

The check valve prevents backflow in the piping system. The pressure of the fluid passing through a pipeline opens the valve, while any reversal of flow will close the valve.

4. Plug Valve

Plug valve is Quarter-turn rotary motion Valve that uses a tapered or cylindrical plug to stop or start the flow. The disk is in plug shape, which has a passage to pass the flow. Plug valve used as on-off stop valves and capable of providing bubble tight shutoff. Plug valve can be used in vacuum to high-pressure & temperature applications

5. Ball Valve

A Ball valve is a quarter-turn rotary motion valve that uses a ball-shaped disk to stop or start the flow. Most ball valves are of the quick-acting type, which requires a 90° turn of the valve handle to operate the valve. The ball valve is Smaller and lighter than a gate valve of same size and rating.

6. Butterfly Valve

A Butterfly valve is a quarter-turn rotary motion valve, that is used to stop, regulate, and start the flow. Butterfly valve has a short circular body. Butterfly Valve is suitable for large valve applications due to Compact, lightweight design that requires considerably less space, as compared to other valves.

7. Needle Valve

Needle valves are similar to a globe valve in design with the biggest difference is the sharp needle like a disk. Needle valves are designed to give very accurate control of flow in small diameter piping systems. They get their name from their sharp-pointed conical disc and matching seat.

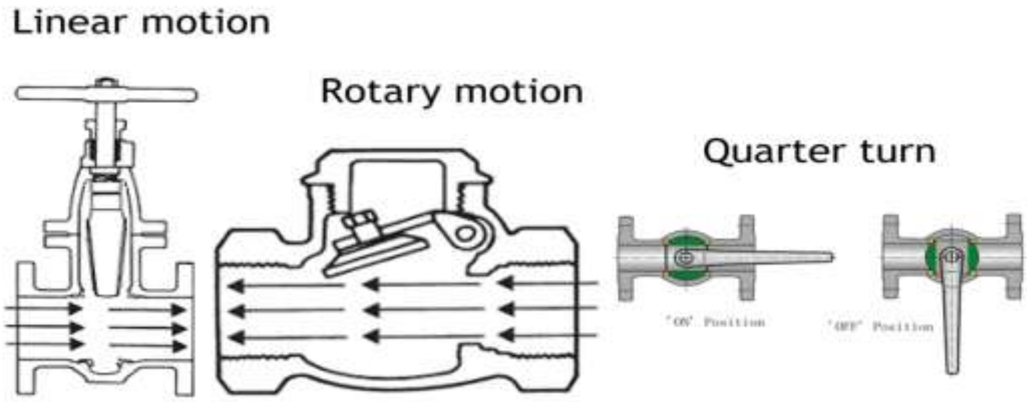
8. Pinch Valve

The pinch valve is also known as clamp valve. It is a linear motion valve. Used to start, regulate, and stop fluid flow. It uses a rubber tube, also known as a pinch tube and a pinch mechanism to control the fluid. Pinch Valve is ideally suited for the handling of slurries, liquids with large amounts of suspended solids, and systems that convey solid material pneumatically.

9. Pressure Relief Valve

A pressure Relief valve or pressure safety valve are used to protect equipment or piping system during an overpressure event or in the event of vacuum. This valve releases the pressure or vacuum at pre-defined set pressure.

In the image below, you can see the difference between opening methods of the valve

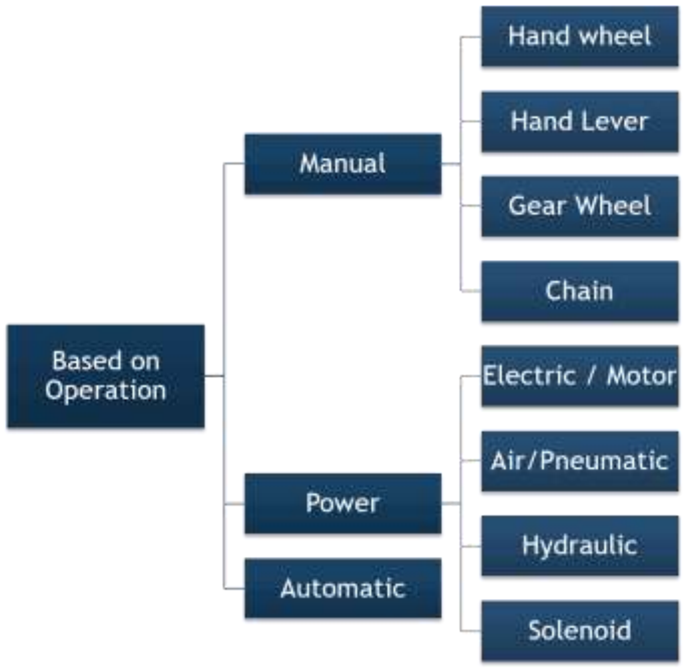


Applications/ Functions of Valves

Valve serve a various function within the piping system. Such as

- Stopping and starting a fluid flow. Depending on whether a valve is open or closed, it let pass the process fluid or halt the fluid.
- Throttling the fluid flow. Some of the valves let you throttle the fluid depending open % of total opening. Lesser the opening higher the throttling and otherwise.
- Controlling the direction of a fluid flow. Multiport valve lets you decide the way fluid will go.
- Regulating a flow or pressure within the piping system. Some of the automatic control valves maintain the flow and pressure within the system by adjusting opening and closing.
- Relieve pressure or vacuum from the piping system and equipment. Pressure and vacuum relief valve safeguard the process system from overpressure and during vacuum condition.

Classification of Valves Based on Types of Actuator It Used



WHAT IS A FLOW METER?

A flow meter is a device used to measure the volume or mass of a gas or liquid. Flow meters are referred to by many names, such as flow gauge, flow indicator, liquid meter, flow rate sensor, etc. depending on the particular industry. However, they all measure flow. Open channels, like rivers or streams, may be measured with flow meters. Or more frequently, the most utility from a flow meter and the greatest variety of flow meters focus on measuring gasses and liquids in a pipe. Improving the precision, accuracy, and resolution of fluid measurement are the greatest benefits of the best flow meters.

What is a flow meter and what does it do?

Flow meters are devices that can be used to measure the amount of liquid, or gas which passes through it. It is an instrument which is used to conduct the flow measurement. The flow rate of liquid and gases must be measured accurately for the better quality of the industrial process. Accurate measurement of gases is needed for the industrial process and it also does the control of flow rate. Flow meters measure the flow by two methods, some flow meters measure the flow as the amount of the liquid passes through the flow meter during a period of time. While other flow meters measure the flow by measuring the total amount of fluid that has passed through the flow meter.

Steps to select a flow meter

1. Confirm the properties of the detection fluid
2. Confirm the purpose of measurement and determine the detection method
3. Confirm product specifications
4. Consider cost

How does a flow meter work?

Flow meter consists of devices such as transducer and transmitter, the transducer will sense the fluid that passes through the primary device. The transmitter will receive a signal from the transducer, so the transducer changes this signal into a usable flow signal. So a flow meter can be considered as the combination of these physical devices.

What are the factors that affect a flow meter selection?

- The phase of the fluid-like gas, liquid, steam
- Flow conditions and flow range, flow conditions like clean, dirty, abrasive, viscous will affect
- Process conditions like pressure and temperature
- Preferred material mostly in case of corrosive fluid
- Pipe size and accuracy
- Repeatability and cost

Different types of Monometers

1. Differential Pressure Flow Meters

Differential pressure flow meters measure the differential pressure across an orifice where flow is directly related to the square root of the differential pressure produced. There are also primary and secondary elements in differential flow meters. The primary element produces change in kinetic energy using either flow nozzle, pilot tube, orifice plate, or venture flow meters. The secondary element measures the differential pressure and provides the signal.

2. Positive Displacement Flow Meters

Positive displacement (PD) flow meters measure the volume filled with fluid, deliver it ahead and fill it again, which calculates the amount of fluid transferred. It measures actual flow of any fluid while all other types of flow meters measure some other parameter and convert the values into flow rate. In PD flow meters, output is directly related to the volume passing through the flow meter. PD flow meters include piston meters, oval-gear meters, notating disk meters, rotary vane type meters, etc.

Positive displacement flow meters are known for their accuracy. They are commonly used in the transfer of oils and fluids, like gasoline, hydraulic fluids as well as in-home use for water and gas applications.

3. Velocity Flow Meters

Velocity meters measure velocity of the stream to calculate the volumetric flow rate. These are less sensitive when the Reynolds number of fluid is higher than 10000. Velocity flow meters include turbine, paddlewheel, vortex shedding, electromagnetic and sonic/ultrasonic flow meters.

4. Mass Flow Meters

Mass flow meters are more effective in mass related processes as they measure the force that results from the acceleration of mass. More specifically, the force is measured as the mass moving per unit of time, instead of the volume per unit of time. Mass flow meters include Carioles mass meters and thermal dispersion meters.

Typical applications for mass flow meters are tied to chemical processes. In addition to the chemical and gas industries, typical industries using mass meters include pharma, power, mining and wastewater.

5. Open Channel Flow Meters

Measurement of liquid in open channels include v-notch, weirs and flumes. These dam-like structures, or overflows, allow for a limited or concentrated free-flow of liquids based on the unique shape and size of the structure. This type of flow meter allows for a reading of the flow rate to be calculated.

Common applications of open channel meters include free flowing liquids like streams, rivers, and irrigation channels systems.

- **Manometers**

Manometers are precision instruments that are used to measure pressure, which is the force exerted by a gas or liquid per unit surface area owing to the effects of the weight of that gas or liquid from gravity. Depending on the type and how they are configured, manometers can be set-up to provide a measurement of different pressure values. A common type of manometer with which most people are familiar is the one that physicians and medical professionals use to measure and monitor a patient’s blood pressure. This type of manometer is called a sphygmomanometer.

Following are the three different types of manometers:

1. Simple manometers
2. Differential manometers
3. Micro manometer

- **Simple Manometer**

A simple manometer has a glass tube that’s one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere.

Common types of simple manometers are:

1. Piezometer
2. U-tube manometer
    a) For gauge pressure
    b) For vacuum pressure
3. Single Column Manometer

4. Inclined tube manometer or Sensitive Manometer

1. Piezometer

For measuring the pressure inside a vessel or pipe in which liquid is there, a tube is attached to the walls of the container or pipe in which the liquid remains so liquid can rise in the tube. By determining the height to which liquid rises and using the relation $p = \rho gh$, a gauge pressure of the liquid can be determined.

Such a device is known as Piezometer. To avoid capillary forces, a piezometer tube has to be about 1/2 inch or more. It is essential that the opening of the instrument to be tangential to any fluid motion, otherwise an incorrect reading will result.

2. U-tube Manometer

As shown in the figure it consists of a glass tube bent in V-shape, with one end is connected to a point at which pressure is to be measured and the other end remains open to the atmosphere.

3. Single Column Manometer

Consider a vertical tube micro manometer connected to a pipe containing light liquid under very high pressure. The pressure in the pipe will force the lighter liquid in the basin to push the heavier liquid downwards.

Due to the larger area of the basin, the fall of a heavy liquid level will be very small. This downward movement of heavy liquid into the basin will result in a significant rise of heavy liquid in the right limb.

4. Inclined Tube Manometer

If the vertical tube of the micro manometer is made inclined as shown in figure then it is called inclined tube micro manometer.

This type of inclined micro manometer is more sensitive than the vertical tube type. Due to inclination, the distance moved by the heavy liquid in the right limb is comparatively more. Thus it can give a higher reading for the given pressure.

- **Differential Manometer**

The differential manometer is a device used to measure the pressure difference between two points in a pipe or in two different pipes.

A differential manometer consists of a U-tube, containing a heavy liquid, with two ends connected by points whose pressure difference is to be measured:

Types of differential manometers are:

1. Two piezometer manometers

2. U-tube differential manometer
3. Inverted differential manometer

1. Two Piezometer Manometer

It consists of two piezometers mounted at two different gauge points where the pressure difference is to be measured. The pressure difference between two points can be simply measured by the difference in the level of liquid between the two tubes. It possesses some limitations in the form of piezometers.

2. U-tube Differential Manometer

It is a device that is used to measure the pressure difference between two points in a pipe or between two different pipes. this manometer is consists of a U shaped tube containing a heavy liquid.

The two ends are connected to the two desired points in the pipe whose difference of pressure is required. Let pressure at point A be more than at point B. Then the greater pressure at A will force the heavy liquid in U-tube to move downwards. This downwards movement of the heavy liquid in the left limb will cause a corresponding rise of the heavy liquid in the right limb.

3. Inverted Differential Manometer

In this type of manometer, the U-tube is inverted and contains a light liquid. The two ends of the tube are connected to the points whose pressure difference is to be measured.

It is used for measuring the difference in low pressures. The figure shows an inverted U-tube a differential manometer connected to the two points A and B. Let the pressure at point A is more than the pressure at point B.

Advantages of Manometers

Following are the main advantages of manometer:

1. It is simple to construct.
2. It has great accuracy.
3. Used to measure pressure, temperature, flow and other process variables.

Disadvantages of Manometers

Following are the main disadvantages of manometer:

1. The manometer has a smaller dynamic response.
2. They are fragile and therefore provide low portability.
3. They have small operational limits which are on the order of 1000 kN/m2.
4. The density of manometric fluid depends on temperature. Therefore, errors may occur due to change in temperature.

Manometer Applications

- Used in the maintenance of heating, ventilation, and air conditioning (HVAC) systems, low pressure pneumatic or gas systems.
- Construction of bridges, installing swimming pools and other engineering applications.
- Climate forecasting.
- Clinical applications like measuring blood pressure and in physiotherapy.
- Piezometers are used to measure the pressure in pipes where the liquid is in motion.

What is Pressure

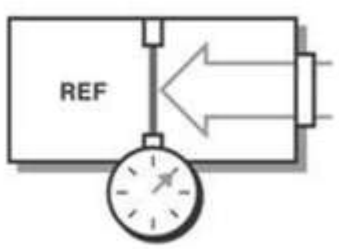
Pressure is defined as force per unit area that a fluid exerts on its surroundings. Pressure, P, is a function of force, F, and area, A:

$$P = F/A$$

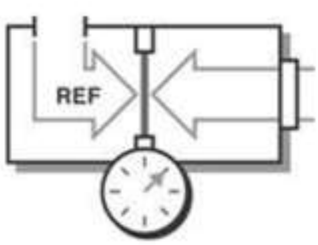
The SI unit for pressure is the Pascal (N/m^2), but other common units of pressure include pounds per square inch (psi), atmospheres (atm), bars, inches of mercury (in. Hg), millimeters of mercury (mm Hg), and torr.

Pressure measurement methods

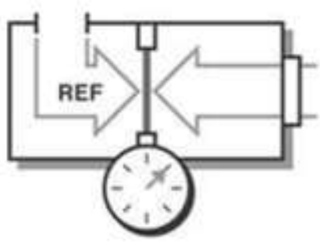
A pressure measurement can further be described by the type of measurement being performed. The three methods for measuring pressure are absolute, gauge, and differential. *Absolute pressure* is referenced to the pressure in a vacuum, whereas *gauge and differential pressures* are referenced to another pressure such as the ambient atmospheric pressure or pressure in an adjacent vessel.



1 Absolute Pressure



2. Gauge Pressure



3. Differential Pressure

1. Absolute Pressure

The absolute measurement method is relative to 0 Pa, the static pressure in a vacuum. The pressure being measured is acted upon by atmospheric pressure in addition to the pressure of interest. Therefore, absolute pressure measurement includes the effects of atmospheric pressure. This type of measurement is well-suited for atmospheric pressures such as those used in altimeters or vacuum pressures. Often, the abbreviations Paa (Pascal's absolute) or psia (pounds per square inch absolute) are used to describe absolute pressure.

2. Gauge Pressure

Gauge pressure is measured relative to ambient atmospheric pressure. This means that both the reference and the pressure of interest are acted upon by atmospheric pressures. Therefore, gauge pressure measurement excludes the effects of atmospheric pressure. These types of measurements include tire pressure and blood pressure measurements. Similar to absolute pressure, the abbreviations Pag (Pascal's gauge) or psig (pounds per square inch gauge) are used to describe gauge pressure.

3. Differential Pressure

Differential pressure is similar to gauge pressure; however, the reference is another pressure point in the system rather than the ambient atmospheric pressure. You can use this method to maintain relative pressure between two vessels such as a compressor tank and an associated feed line. Also, the abbreviations Pad (Pascal's differential) or PSID (pounds per square inch differential) are used to describe differential pressure.

Types of Pressure Measurement

Type	Definition	Application Examples
Gauge	Reference to atmospheric pressure.	• Car Tyre • Water Level Measurement • Chamber Pressure • Hydraulic Applications
Sealed	Referenced to a sealed chamber closed with atmospheric pressure (approximately 1bar).	• For use in Aggressive Media • Industrial Applications • Washdown Environments • Food and Beverage
Absolute	The reference is a vacuum (0bar or no pressure).	• Barometric • Weather Stations and Meteorological Applications
Differential	Measuring the difference between two pressure port readings.	• Filter and Pump Monitoring • Air Conditioner • HVAC Heating Ventilation and Air Conditioning • Clean Room Monitoring

There are 3 basic methods for pressure measurement;

I. The first method involves balancing the unknown pressure against the pressure produced by a column of liquid of known density.

II. The second method involves allowing the unknown pressure to act on a known area and measuring the resultant force either directly or indirectly.

III. The third method involves allowing the unknown pressure to act on an elastic material and measuring the resultant stress or strain.

Basic equations of Fluid Flow

The basic equations of fluid dynamics, the main task in fluid dynamics is to find the velocity field describing the flow in a given domain. To do this, one uses the basic equations of fluid flow, which we derive in this section. These encode the familiar laws of mechanics:

- Conservation of mass
- Conservation of momentum
- Conservation of energy

Conservation of momentum

The Cauchy equations

Consider a volume V bounded by a *material* surface S that moves with the flow, always containing the same material elements. Its momentum is $\int_V dV \rho \mathbf{v}$, so:

$$\text{rate of change of momentum} = \frac{d}{dt}\int_V dV \rho \mathbf{v} = \int_V dV \rho \frac{D\mathbf{v}}{Dt}.$$

(The mass $\rho\, dV$ of each material element is constant.) This must equal the net force on the element. Actually there are two different types of forces that act in any fluid:

- Long ranged external **body** forces that penetrate matter and act equally on all the material in any element dV. The only one considered here is gravity, $\rho \mathbf{g}\, dV$.
- Short ranged molecular forces, internal to the fluid. For any element, the net effect of these due to interactions with other elements acts in a thin **surface** layer. In 3D, each of the 3 sets of surface planes bounding an element experiences a 3-component force, giving 9 components in all. These form the stress tensor $[\Pi]$, defined so the force exerted per unit area across a surface element $\mathbf{dS} \equiv \hat{\mathbf{n}}\, dS$ (*by* the fluid on the side to which $\hat{\mathbf{n}}$ points *on* the fluid on the other side) is $\mathbf{f} = [\Pi]\cdot\hat{\mathbf{n}}$.

$$\begin{aligned}\text{Total force (body + surface)} &= \int_V dV \rho \mathbf{g} + \int_{\mathbf{S}} [\Pi]\cdot d\mathbf{S} \\ &= \int_V dV \left(\rho \mathbf{g} + \nabla\cdot[\Pi]\right).\end{aligned}$$

By Newton's second law, Eqns. 9 and 10 must be equal for any V, so we get finally

$$\textbf{the Cauchy equation:} \quad \rho\frac{D\mathbf{v}}{Dt} = \rho\mathbf{g} + \nabla\cdot[\Pi].$$

Conservation of mass

The continuity equation

Consider a volume V bounded by a surface S that is fixed in space. This mass inside it is given by $\int_V \rho\, dV$, so the

$$\text{rate of decrease of mass in } V = -\frac{d}{dt}\int_V \rho\, dV = -\int_V \frac{\partial \rho}{\partial t}\, dV. \quad (1)$$

If mass is conserved, Eqn. 1 must equal the total rate of mass flux out of V. How do we calculate this? The rate of outward mass flux across any small element $d\mathbf{S}$ of S is $\rho\,\mathbf{v}\cdot d\mathbf{S}$ where the magnitude of $d\mathbf{S}$ is equal to the element's area and we take $d\mathbf{S}$ along the outward normal. Integrating over the whole surface we have

$$\text{rate of mass flux out of } V = \int_{\mathbf{S}} \rho\,\mathbf{v}\cdot d\mathbf{S} = \int_V \nabla\cdot(\rho\mathbf{v})dV \quad (2)$$

where we used Green's formula to convert to a volume integral. The integrand $\nabla\cdot(\rho\mathbf{v})$ on the RHS is expressed in Cartesian coordinates $\mathbf{x} = (x, y, z)$, $\mathbf{v} = (u, v, w)$ as

$$\nabla\cdot(\rho\mathbf{v}) = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}. \quad (3)$$

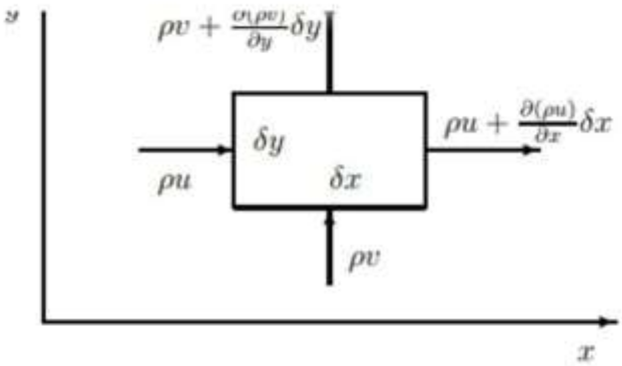


Figure 1: Mass fluxes entering and leaving an element.

See Fig. 1, which shows clearly that gradients in the flow field are required for non-zero net flux. For mass to be conserved everywhere, Eqns. 1 and 2 must be equal for any volume V and so we get

$$\textbf{the continuity equation:} \quad \frac{\partial \rho}{\partial t} + \nabla.(\rho\mathbf{v}) = 0. \quad (4)$$

Concept of boundary layer:

A boundary layer is a thin layer of viscous fluid close to the solid surface of a wall in contact with a moving stream in which (within its thickness δ) the flow velocity varies from zero at the wall (where the flow "sticks" to the wall because of its viscosity) up to U_e at the boundary, which approximately (within 1% error) corresponds to the free stream velocity (see Figure 1). Strictly speaking, the value of δ is an arbitrary value because the friction force, depending on the molecular interaction between fluid and the solid body, decreases with the distance from the wall and becomes equal to zero at infinity.

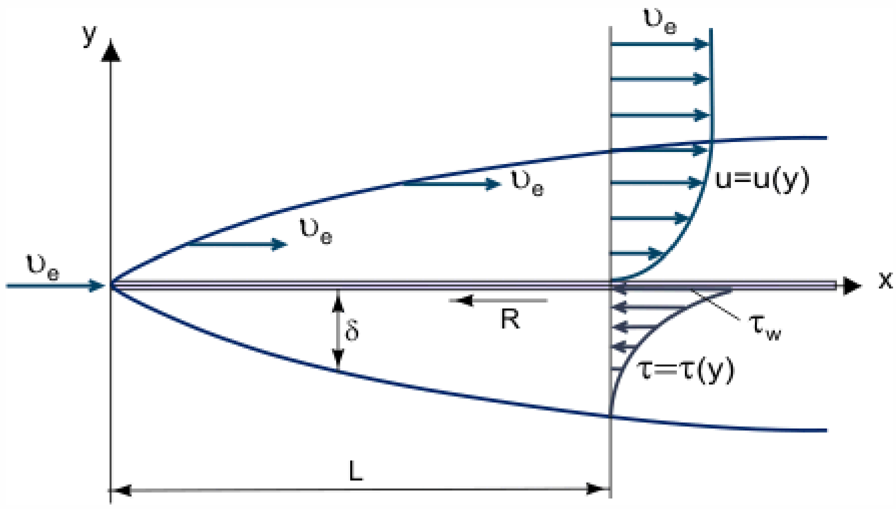


Figure 1. Growth of a boundary layer on a flat plate.

The fundamental concept of the boundary layer was suggested by L. Prandtl (1904), it defines the boundary layer as a layer of fluid developing in flows with very high **Reynolds Numbers** Re, that is with relatively low viscosity as compared with inertia forces. This is observed when bodies are exposed to high velocity air stream or when bodies are very large and the air stream velocity is moderate. In this case, in a relatively thin boundary layer,

Friction **Shear Stress** (viscous shearing force): $\tau = \eta[\partial u/\partial y]$

(where η is the dynamic viscosity; u = u(y) – "profile" of the boundary layer longitudinal velocity component, see **Figure 1**) may be very large; in particular, at the wall where u = 0 and $\tau_w = \eta[\partial u/\partial y]_w$ although the viscosity itself may be rather small.

It is possible to ignore friction forces outside the boundary layer (as compared with inertia forces), and on the basis of Prandtl's concept, to consider two flow regions: the boundary layer where friction effects are large and the almost **Inviscid Flow** core. On the premises that the boundary layer is a very thin layer (δ << L, where L is the characteristic linear dimension of the body over which the flow occurs or the channel containing the flow, its thickness decreasing with growth of Re, **Figure 1**), one can estimate the order of magnitude of the boundary layer thickness from the following relationship:

$$\delta / L = Re^{-0.5}$$