Limits And Derivatives

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New CH

Limit :

We say limit $x \to a^-$. $F(x)$ is the expected value of F at $x = a$ given the value of F near x to the left of a. This value is called the left hand limit of $F(x)$ at a

We say limit $x \to a^+$ $F(x)$ is the expected value of F at $x = a$ given the value of F near x to the right of a. This value is called the right hand limit of $F(x)$ at a.

If the right hand, left hand limit are equal to each other then the limit exists.

We called at common value as the limit of $F(x)$ at $x \to a$, and it is denoted by $\lim_{x \to a} F(x)$.

Algebra of limits.

Let f and g be two functions such that both $\lim_{x \to a} F(x)$ and $\lim_{x \to a} g(x)$ exists then

i) $\lim_{x \to a} [F(x) + g(x)] = \lim_{x \to a} F(x) + \lim_{x \to a} g(x)$

ii) $\lim_{x \to a} [F(x) - g(x)] = \lim_{x \to a} F(x) - \lim_{x \to a} g(x)$

iii) $\lim_{x \to a} [F(x) \cdot g(x)] = \lim_{x \to a} F(x) \cdot \lim_{x \to a} g(x)$

iv) $\lim_{x \to a} \left[\frac{F(x)}{g(x)}\right] = \frac{\lim_{x \to a} F(x)}{\lim_{x \to a} g(x)}$

Q. If $f(x) = 2x + 1$ where x belongs to $x \in R$ are find limit $\lim_{x \to 1} f(x)$

then check the lim.

Sol^n LHL $= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-} (2x+1)$

$= 2(1) + 1$

$= 3$.

$$RHL = \lim_{x \to 1^+} F(x)$$

$$= \lim_{x \to 1} (2x+1)$$

$$= 2+1$$

$$= 3.$$

$\because$ LHL = RHL

$\therefore$ Then the limit exists.

★ For any positive integer n

then $\boxed{\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}}$

★ **Limit of trigonometric functions**

(i) $\lim_{x \to 0} \frac{\sin x}{x} = 1.$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0.$

Example:

VVI

Q. $\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$

$\Rightarrow \lim_{x \to 1} \frac{x^{15} - 1^{15}}{x - 1} \cdot \frac{x - 1}{x^{10} - 1^{10}}$

$\Rightarrow \lim_{x \to 1} \frac{x^{15} - 1}{x - 1} \div \frac{x^{10} - 1}{x - 1}$

$\Rightarrow \lim_{x \to 1} \frac{x^{15} - 1}{x - 1} \div \lim_{x \to 1} \frac{x^{10} - 1}{x - 1}$

$= 15.(1)^{15-1} \div (10).(1)^{10-1}$

$= 15 \div 10 = \frac{15}{10} = \frac{3}{2}$ Ans

Q. Evaluate $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$.

$\Rightarrow \lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$

$\rightarrow \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{2x}{\sin 2x} \cdot 2$

$\Rightarrow 2\left[\lim_{x \to 0} \frac{\sin 4x}{4x} \div \frac{\sin 2x}{2x}\right]$

$\Rightarrow 2\left[\lim_{x \to 0} \frac{\sin 4x}{4x} \div \lim \frac{\sin 2x}{2x}\right]$

$\rightarrow 2.(1 \div 1.) \Rightarrow 2.$ Ans.

Q. Evaluate $\lim_{x \to 0} \frac{\tan x}{x}$

$\Rightarrow \lim_{x \to 0} \frac{\tan x}{x} \Rightarrow \lim_{x \to 0} \frac{\sin x}{\cos x} \cdot x.$

Ex: 13.1

1. $\lim_{x \to 3} x + 3 \Rightarrow 3 + 3 = 6$ Ans

(4) $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4} \Rightarrow \frac{3.4 - 2 - 10}{4 - 4} = \frac{0}{0} = \infty.$

Now, $\lim_{x \to 2} \frac{3x^2 - 6x + 5x - 10}{(x+2)(x-2)}$

$\Rightarrow \lim_{x \to 2} \frac{3x(x-2) + 5(x-2)}{(x+2)(x-2)}$

$\Rightarrow \lim_{x \to 2} \frac{(x-2)(3x+5)}{(x+2)(x-2)} = \frac{11}{4}$ Ans

Ex 13.1

(8) $\lim_{x \to 3} \frac{x^4 - 81}{2x^3 - 5x - 3}$

$\Rightarrow \lim_{x \to 3} \frac{(x^2)^2 - 9^2}{2x^2 - 6x + x - 3}$

$\Rightarrow \lim_{x \to 3} \frac{(x^2 + 9)(x^2 - 9)}{2x(x - 3) + 1(x - 3)}$

$\Rightarrow \lim_{x \to 3} \frac{(x^2 + 9)(x^2 - 3^2)}{(x - 3)(2x + 1)}$

$\Rightarrow \lim_{x \to 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{(x - 3)(2x + 1)}$

$= \frac{108}{7}$ Ans

(13) $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$

$\Rightarrow \frac{1}{\pi} \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$

$= \frac{1}{\pi} \cdot 1$

$= \frac{1}{\pi}$ Ans

(13) $\lim_{x \to 0} \frac{\sin ax}{bx} \times \frac{ax}{ax} = \lim_{x \to 0} \frac{\sin ax}{ax} \cdot \frac{a}{b}$

$= \frac{a}{b} \lim_{x \to 0} \frac{\sin ax}{ax}$

$= \frac{a}{b} \cdot 1 = \frac{a}{b}$ Ans.