|          | Matrix:   |
|----------|---|
|          | May distribute the literature of the said           |
|          | A matrix is an ordered rectangular array of         |
|          | numbers or functions.                               |
|          |   |
|          | The numbers or functions are called the elements or |
|          | the entries of the matrix. It is denoted by         |
|          | Capital letter. It is denoted by (), [], 11 11.     |
|          |   |
|          | 8 10  |
|          | 8 10<br>A = 10 15<br>18 20                          |
|          | -8 . 20   |
|          | Order of a matrix                                   |
|          | Doues of a marsix                                   |
|          | A matrix having m rows and n columns is called      |
|          | a matrix of order mxn (read as an mby n matrix).    |
|          | · · · · · · · · · · · · · · · · · · ·               |
|          | a11 a12 a13 aij a1n                                 |
|          | a21 a22 a23 a2j a2n                                 |
|          |   |
|          | A = ail ail ail ail ail ail                         |
|          | Tapas of Matrices and the same                      |
|          | ant amz amj amn                                     |
|          | (a) Column matrix                                   |
| 0.       | Constauct a 3x2 matoix whose elements are given     |
|          | by aij = 1 [i-3j].                                  |
| <b>カ</b> | an an   |
| 2        |   |
|          | In general a 3x2 matrix is given by A = a21 a22     |
|          | 931 932   |

| Now | aii | = 1 | 1-31 | ,1= | 1,2,3 | and | j = 1 | , 2. |
|-----|-----|-----|------|-----|-------|-----|-------|------|
|     | )   | 7   | )    |     |       |     | )     |      |

$$Q_{12} = \frac{1}{2} |1 - 3 \times 2| = 5$$

$$a_{21} = \frac{1}{2} |2 - 3 \times 1| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2} |2 - 3 \times 2| = 4 = 2.$$

$$\frac{a_{31}}{2} = \frac{1}{2} |3 - 3x| = 0$$
.

$$a_{32} = \frac{1}{2} |3-3\times 2| = \frac{3}{2}$$

Hence the required matrix is given by

|     | 1    | 5/2 |   |
|-----|------|-----|---|
| A 2 | 3110 | 2 1 |   |
| 20  | 0 0  | 3/2 |   |
| 1   |      |     | - |

## Types of Matrices!

#### (i) Column matrix

A matrix is said to be a column matrix if it has only one column.

$$A = 15 \longrightarrow R_1$$

$$20 \quad 3x_1 \longrightarrow R_3$$

Ci

| (ii) | Row matrix      |
|------|-----------------|
| _    | - UNITED TO THE |

A matrix is said to be a DOW matrix if it has only.

one row.

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 1 & 2 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 2 & 4 \end{bmatrix}$$

### (iii) Square matrix:

A matrix in which the numbers of rows are equal to the number of columns, is said to be a square matrix.

|    | 8  | 13 | 5  | \ |
|----|----|----|----|---|
| A= | 3  | 2  | 15 |   |
|    | 18 | 20 | 13 | 3 |

## (iv) Diagonal matrix.

A square motrix  $B = [bij]_{man}$  is said to be a diagonal matrix if all its non-diagonal elements are zero, that is a matrix  $B = [bij]_{man}$  is said to be a diagonal matrix if bij = 0, when itj

### (V) Scalar matoix:

A diagonal matrix is said to be a scalar matrix is its diagonal elements are equal.

|     | 8 | 0 | U |
|-----|---|---|---|
| A = | 0 | 8 | 0 |
|     | 0 | ٥ | 8 |

| (vi) | Identity | matrix: |
|------|----------|---------|
|      |          |         |

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix.

|     | _ |   | - |
|-----|---|---|---|
| A = | 1 | 0 | 0 |
|     | 0 | 1 | 0 |
|     | 0 | 0 | 1 |

## (vii) Zero matrix:

A matrix is said to be zero matrix or null matrix if all its elements are zero.

# Equality of matrices.

Two matrices A = [aij] and B = [bij] are said to be equal if.

## (i) they are of the same order.

(ii) each element of A is equal to the corresponding element of B. that that is air = bij for all i and J.

| <br>A=B]   | A= | 2 | 3  | B = | 2 | 3 |
|--|----|---|----|-----|---|---|
| The second secon |    | 0 | 11 |     | ٥ | 1 |

|      | Operations on Matrices.  |
|------|--|
|      |  |
|      | Addition of matrices.  |
|      | the same time of the same and the same and   |
| 0.   | $A = Cos^2x Sin^2x B = Sin^2x Cos^2x$  |
|      | $Sin^2x$ $Cos^2x$ $Cos^2x$ $Sin^2x$ .  |
|      | Let A - [act be as materix and a bear  |
|      | A+B= Cos2x+Sin2x Sin2x+Cos2x   |
|      | Sin2x + Cog2x (os2x + sin2x)   |
|      | Contract of the contract of th |
|      | A+B=11   |
|      | 11. 1 is Anisha to making the (vi)   |
|      |  |
|      | Let A = [oj] area be an motor then we  |
|      | Properties of matrix addition.   |
|      | AL A - 2 ,0 - A - (A-) - (A-) + A to AL  |
| (12  | Commutative Law!   |
|      |  |
|      | If A= [aij], B= [bij] are matrices of the  |
|      | same order, say mon, then A+B=B+A.   |
|      | : caristom lo mittaridithom  |
|      | eg. > [aij] + [bij] = [bij] = [aij].   |
|      |  |
| (11) | Associative low:   |
|      | Associative law:   |
|      | Fox any three matrices A: [qii] B: [Lii]   |
|      | For any three matrices A: [aii], B: [bij],<br>C: [Cij] of the same order, say mxn, then.   |
|      | Jag Inxh, then.  |
|      | (A+B)+C=A+(B+C)  |
|      |  |
|      | ([ais]+(bij])+[(ij] = [ais]+([bis]+[ais])  |
|      | [ [ [ [ [ [ [ [ [ [ [ [ [ [ [ [ [ [ [  |
|      |  |

| (iii) | Existence | of | additive | identil | 7 |   |
|-------|-----------|----|----------|---------|---|---|
|       |           |    |          |         | 0 | i |

Let A = [aij] be an mxn matrix, then we have another matrix as -A = [-aij] mxn
such that Ar

Let A = [aij] be an mxn matrix and 0 be an mxn zero matrix, then A+0=0+A=A. In other words, 0 is the additive identity for matrix addition.

### (iv) The Existence of additive inverse:

Let A = [aij] mxn be any matrix, then we have another matrix as -A = [-aii] mxn such that A + (-A) = (-A) + A = 0. So -A is the additive inverse of A or negative of A.

### Multiplication of Matrices:

|       | [3 | 8) |   | (3) | 7   |
|-------|----|----|---|-----|-----|
| AXB = |    |    | × |     | 160 |
|       | 2  | 10 |   | 4   | 15  |

$$3x5+8x4$$
  $3x7+8x15$  =  $2x5+10x4$   $2x7+10x15$ 

|   | 15+32 | 21+120  |
|---|-------|---------|
| = | 10+90 | 14 +150 |

Definition

0.

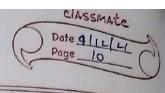
The product of two matrices, A and B defined if the number of columns of A is equal to the no. of rows of B. Let A = [aij] be an mixn matrix and B = [bjk] be an nxp matrix.

Then the product of the matrices A and B is the matrix C of order mxp.

$$A = \begin{bmatrix} 3 & 2 \\ 8 & 6 \end{bmatrix} 2 \times 4$$

 $A \times B = 6 + 6$   $2 + 7 \cdot 8$  9 + 16 12 + 19 16 + 18 6 + 4 + 24 24 + 98 32 + 42

 $A \times B = \begin{bmatrix} 12 & 32 & 25 & 26 \\ 34 & 88 & 72 & 79 & 2 \times 41 \end{bmatrix}$ 



### Transpose of matrix.

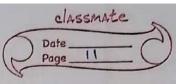
$$A = \begin{bmatrix} a_{11} & q_{12} & a_{13} \\ a_{21} & a_{21} & a_{23} \end{bmatrix}$$

## Definition.

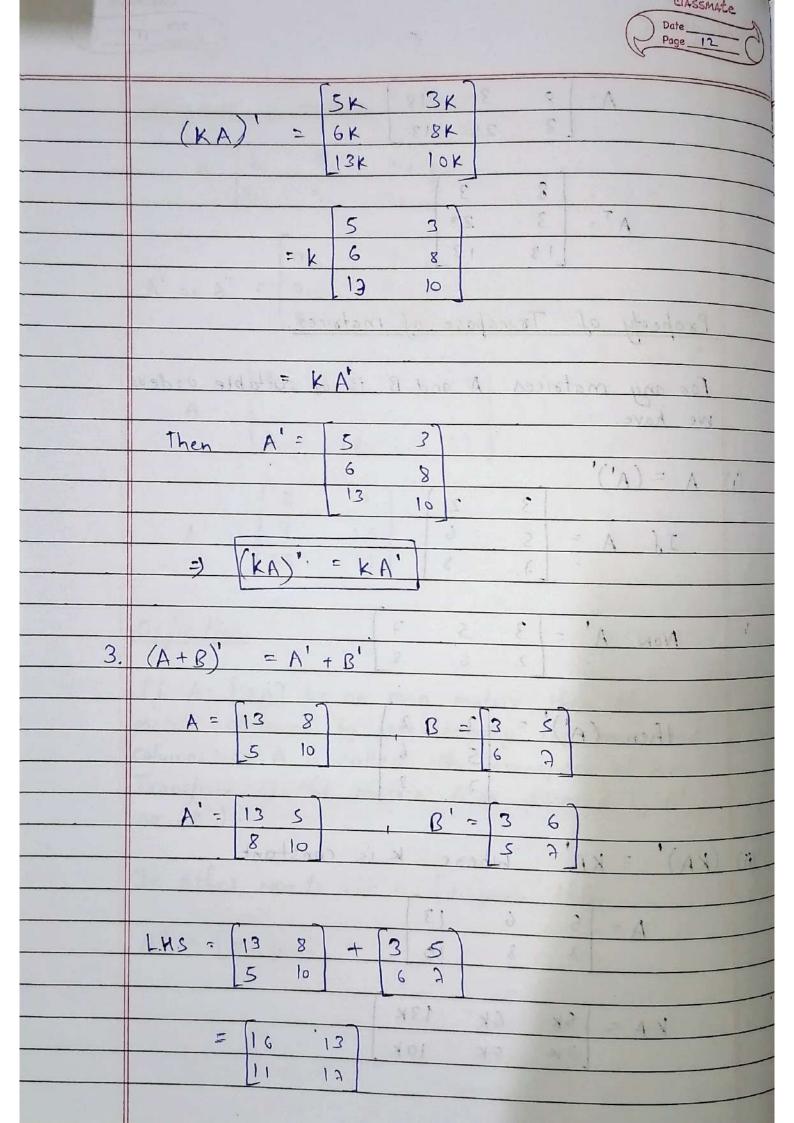
If A = [aij] be an man matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or (AT).

In other words, if A = [aij]mxn, then

A' = [aji]nxm.



|      |             |       |         |   |         | Page 11    | =9 |
|------|-------------|-------|---------|---|---------|------------|----|
|      | A = \       | 8     | 3 18    | 1 32  |         |            |    |
|      |             |       | 28 1 13 | 40  | . 17    | (11)       |    |
|      |             |       | l vol   | 421   |         |            |    |
|      |             | 8     | 3       | -   |         |            |    |
|      | AT=         | 3     | 28 -    | 2   |         |            |    |
|      |             | 18    | 13      | 2 V   | 1       |            |    |
|      |             |       | 01      | 61  |         |            |    |
|      | Property of | Trans | bose of | matrice                                       | 1       |            |    |
|      | 100         |       | 7       | , , , , , , , , , , ,                         |         |            |    |
|      | For any mat | rices | A and I | 3 is lox                                      | suitabl | e orders.  |    |
|      | we have.    |       |         |   |         |            |    |
|      |             |       | 18      | 2   | = 'A.   | asdt 1     |    |
| (i)  | A = (A')'   |       | 8       | 8   | I- YA   |            |    |
|      |             | 3     | 2)      | 5.1   |         |            |    |
|      | If A =      | 3     | 6       |   |         |            |    |
|      |             | L-3   | 8       | ANT   | TANY    | <i>t</i> - |    |
|      |             |       |         |   |         |            |    |
|      | NOW A'      |       |         |   |         |            |    |
|      |             | 2     | 6 9     | 3 19 + 1                                      | A =     | 79+A       | 3, |
|      |             |       |         |   |         |            |    |
|      | then (A     | ) = 5 | 3 3 2   | 1 8   | 3 21    | - A        |    |
|      |             | 6 3   | 5 6     | . 0   | l al    |            |    |
|      |             |       | 3       |   |         |            |    |
|      | , , ,       | 1     | - '8    | <u>, , , , , , , , , , , , , , , , , , , </u> | 13 5    | 'A         |    |
| (ii) | (KA)' = KA  | h     | here 1  | s is cor                                      | nstant. |            |    |
|      |             |       |         |   |         |            |    |
|      | A = 5       |       | 13      |   |         |            |    |
|      | 3           | . 8   | 10      |   | 8 81    | 2 14 ]     |    |
|      |             |       |         | -   | d a     |            |    |
|      |             |       | K 13K   |   |         |            |    |
|      |             | 3K g  | K 101   | ( ) []  | 21      |            |    |



| R.H.S  | = | ۸۱ |   | 0  |
|--------|---|----|---|----|
| 1,11,2 | _ | A  | + | 13 |

Proved. 1 - A 19

('A) = 1A = 19 0

('A - A) = 'A CHO A

$$A = \begin{bmatrix} 5 & 6 \\ 3 & 7 \end{bmatrix}$$
  $B = \begin{bmatrix} 10 & 15 \\ 2 & 4 \end{bmatrix}$ 

$$A' = \begin{bmatrix} 5 & 3 \\ 6 & 7 \end{bmatrix}$$
 $B' = \begin{bmatrix} 10 & 2 \\ 15 & 4 \end{bmatrix}$ 

$$AB = \begin{bmatrix} 5 & 6 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 50+12 \\ 45+28 \end{bmatrix} \begin{bmatrix} 62 \\ 44 \end{bmatrix} \begin{bmatrix} 99 \\ 44 \end{bmatrix}$$

$$R.H.S = 10$$
 2  $S$  3 =  $50+12$   $30+14$  =  $62$   $44$   $15$  4  $6$  7  $75+24$   $45+28$   $99$  73

Intumpor month (4-4)

R. M. S 2. M. S

| - | -1 |    |    |   | 1 |  |
|---|----|----|----|---|---|--|
|   | he | 08 | 21 | n | 1 |  |

For any square matrix A with real numbers entities, A+A' is a symmetric matrix and A-A' is a skew symmetric matrix.

Let P = A+A'

NOW P' = (A+A')'

> p' = A' + (A')'

\* P' = A'+ A 1 (For commutative low)

=> P' = P

7 P = P'

Hence A+A' is symmetric Matrix.

EFLETTO = A - A 2 + 11 + 12 + 21 01 2 12 4 A

NOW Q' = (A - A')' PP (2) = '(8A) - 34.

= Q' = A' -(A')

1 0' =-(-A'+A)

 $\frac{1}{2} \frac{0}{0} = -\left(A - A'\right)$  (From commutative law)

Therefore A-A' is a skew symmetric matrix.

| TT   | 44   |     |   |
|------|------|-----|---|
| Ih.  | osen | . 2 | ı |
| 1116 | oren | 11  | , |

0.10

Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

Let A be a square matrix, then we can write:

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$A = \begin{bmatrix} -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A' = \begin{bmatrix} -1 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A = \frac{1}{2} (A + A') + \frac{1}{2} (A - A')$$

|       | [12 | 0 -4 | 4 | 1 / 8 | 0 | 3  | 0 - | AO |
|-------|-----|------|---|-------|---|----|-----|----|
| >A= = | -2  | 6    | 2 | + 2   | 0 | 11 | 0   | 0  |
|       | 2   | 2    | 6 |       | 0 |    | 0   | 0  |

|        | 6  | -2 | 2 | 3 3 | 0 | 9) | 0 |
|--------|----|----|---|-----|---|----|---|
| =) A = | -2 | 3  | 1 | +   | 0 | 0  | 0 |
|        | 2  | 1  | 3 |     | D | 00 | 0 |

|   | 6  | - 2 | 2 |
|---|----|-----|---|
| - | -2 | 3   | 1 |
|   | 2  |     | 3 |

| Elementary | oberation ( | (To ans, | (ormation) | of Mati | dx. |
|------------|-------------|----------|------------|---------|-----|
| 0          |             |          |            | 1       |     |

1. The interchange of any two rows or two columns.

Symbollically the interchange of ith and jth rows is denoted by Ri \rightarrow Rj and interchange of ith and jth column is denoted by C; \rightarrow Cj.

For example: - (A-A) 1 - ('A-A) 1 = A

| (a) | A | 1,1 | 8   | 3  | $\rightarrow R_1$ |
|-----|---|-----|-----|----|-------------------|
|     | 0 |     | 110 | 18 | -> Rz             |

R, AR

| (b) A = 0 | 8 . 3    | A 1-0- |          |
|-----------|----------|--------|----------|
| 0 0       | 11 0 128 | 3,621+ | - CANDAC |
| 10 40     | 4        | 2 6    | 4        |
|           | 7        |        |          |

CI CZ

$$A = \begin{bmatrix} 3 & 8 \\ 18 & 11 \end{bmatrix}$$

| 2. | The multiplication of the elements of any row or        |
|----|---|
|    | column by a non zero number.                            |
|    | TION ZEOU MUMBER.                                       |
|    | Sumbolically 18 Walls to                                |
|    | Symbolically, the multiplication of each element        |
|    | The JOW by K. Where K fo is denoted                     |
|    | by R; → KR; Ci→ KCi                                     |
|    | de MCi - KCi ) and haddelper work                       |
|    |   |
|    | (a) $A = 8$ 3 $\rightarrow R_1$ 11 18 $\rightarrow R_2$ |
|    | 11 18 - Ru - oldman                                     |
|    |   |
|    | R2 ->KR2 : A (1)  |
|    | 11 12 - 18 - 18 - 18 - 18 - 18 - 18 - 18                |
|    | A = 8 3K  |
|    | [11 18K] + 9 - 9  |
|    | where kis non-zero constant.                            |
|    | (XSI+C XII A) A   |
|    | (b) $A = \begin{bmatrix} 8 & 3 \end{bmatrix}$           |
|    | 11 18   |
|    | C, Cz i A (a)   |
|    |   |
|    | $C_1 \rightarrow KC_1$                                  |
|    |   |
|    | A = [8K :3]   |
|    | 11K 18  |
|    | 1 2 2 2 2 1 1   |
|    |   |
|    | Where K is non-zero constant.                           |
|    |   |
|    |   |
|    |   |
|    |   |

3. The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non zero number.

Symbolically, the addition to the elements of ith row, the corresponding elements of ith row multiplied by K is denoted by Ri > Ri+kRj.

Example : -

(a) 
$$A = \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \rightarrow R_1$$

R, + R, + KR2

C2 -> C2+KC1

Inverse of matrices.

If A is a square matrix of order m, and if
there exists another square matrix B of the
same order m, such that AB = BA = I, then
B is called the inverse matrix of A and
if is denoted by A'.

eg. 
$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$
  $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ 

$$AB = 2 \times 2 + 3 \times -1$$
  $2 \times 3^{\circ} + 2 \times 3$   $1 \times 2 + 2 \times -1$   $1 \times 3 + 2 \times 2$ 

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = T$$

$$BA = \begin{bmatrix} 4-3 & -6+6 \\ -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

| TI      | - 4 |
|---------|-----|
| Theorem | 4   |

If A and B are matrices invertible matrices of the same order, then (AB) = B-1A-1

We know that :-

(AB) (AB) = I Multiplying A on both side, we have

=  $A^{-1}(AB)(AB)^{-1} = A^{-1}I$ 

=> (A-A) B(AB)- = A-1

=) IB (AB) - = A-1 0

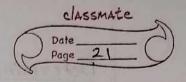
 $\beta (AB)^{-1} = A^{-1}$ 

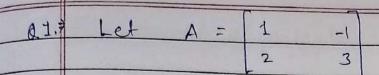
Multiplication of B" on both side we have

B-1 B (AB)-1 = B-1 A-1

I (AB) -1 = B-1 A-1

(AB) = B-1 A-1 proved.





We know that

OW

A = AI

 $\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ 

By Row transformation.

 $R_2 \rightarrow R_2 \rightarrow 2R_1$ 

1 -1 = 1 0 A 0 5 -2 1

R2 - 1 XR2

 $R_1 \rightarrow R_1 + R_2$ 

Therefore, A-1 = 3/5 1/5