Remedial Mathematics:

1. Algebra :

Equations reducible to quadratics, simultaneous equations (linear and quadratic), Determinants, properties of solution of simultaneous equations. by cramer's rule, matrices, definition of special kinds of matrices, arithmetic operations on matrices, inverse of a matrix, solution of simultaneous equations by matrices, pharmaceutical applications of determinants and matrices. Evaluation of Σn^1, Σn^2 and Σn^3; mensuration and its pharmaceutical applications.

2. Measures of central value :

Objectives and pre-requisites of an ideal, measure, mean, mode and median.

3. Trigonometry :

Measurement of angle, T-ratios, addition, subtraction and transformation formula, T-ratios of multiple, submultible, allied and certain angles, Application of logarithms in pharmaceutical computations.

4. Analytical Plane Geometry :

Certain co-ordinates, distance between two points, area of triangle, a locus of point, straight line, slope and intercept form, double-intercept form, normal (perpendicular form), slope-point and two-point form, general equation of first degree.

5. Calculas :

Differential :

Limits and functions, definition of differential coefficient, differentiation of standard functions, including function of a function (chain rule). Differentiation of implicit functions, logarithmic differentiation, parametric differentiation, successive differention.

Integral :

Integration as inverse of differentiation, indefinite integrals of standard forms, integration by parts, substitution and partial fractions, formal evaluation of definite integrals.

Matrix :

A matrix is an ordered rectangular array of numbers or functions.

The numbers or functions are called the elements or the entries of the matrix. It is denoted by capital letter. It is denoted by ( ), [ ], || ||.

$$A = \begin{bmatrix} 8 & 10 \\ 10 & 15 \\ 18 & 20 \end{bmatrix}$$

Order of a matrix

A matrix having m rows and n columns is called a matrix of order m×n (read as an m by n matrix).

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1j} & \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2j} & \cdots a_{2n} \\ \vdots & \vdots & & & \\ a_{i1} & a_{i2} & a_{i3} \cdots & a_{ij} & \cdots a_{in} \\ \vdots & \vdots & & & \\ a_{m1} & a_{m2} & a_{m3} \cdots & a_{mj} & \cdots a_{mn} \end{bmatrix}$$

Q. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i-3j|$.

⇒ In general a 3×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

Now $a_{ij} = \frac{1}{2}|i - 3j|$, $i = 1, 2, 3$ and $j = 1, 2$.

Therefore. $a_{11} = \frac{1}{2}|1 - 3 \times 1| = 1$.

$a_{12} = \frac{1}{2}|1 - 3 \times 2| = \frac{5}{2}$

$a_{21} = \frac{1}{2}|2 - 3 \times 1| = \frac{1}{2}$

$a_{22} = \frac{1}{2}|2 - 3 \times 2| = \frac{4}{2} = 2$.

$a_{31} = \frac{1}{2}|3 - 3 \times 1| = 0$.

$a_{32} = \frac{1}{2}|3 - 3 \times 2| = \frac{3}{2}$.

Hence the required matrix is given by

$$A = \begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \\ 0 & 3/2 \end{bmatrix}$$

Types of Matrices:

(i) Column matrix

A matrix is said to be a column matrix if it has only one column.

$$\begin{array}{cl} A = \begin{bmatrix} 8 \\ 15 \\ 20 \end{bmatrix}_{3 \times 1} & \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array} \\ \downarrow & \\ C_1 & \end{array}$$

(ii) Row matrix :

A matrix is said to be a row matrix if it has only one row.

$$A = \begin{bmatrix} 1 & 4 & 6 \end{bmatrix}_{1\times 3} \rightarrow R_1$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array}$$

(iii) Square matrix:

A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix.

$$A = \begin{bmatrix} 8 & 13 & 5 \\ 3 & 2 & 15 \\ 18 & 20 & 13 \end{bmatrix}_{3\times 3}$$

$\boxed{\text{where } m = n}$

(iv) Diagonal matrix.

A square matrix $B = [b_{ij}]_{m\times n}$ is said to be a diagonal matrix if all its non-diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m\times n}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

(v) Scalar matrix

A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal.

$$A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

(vi) Identity matrix:

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(vii) Zero matrix:

A matrix is said to be zero matrix or null matrix if all its elements are zero.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equality of matrices.

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if.

(i) they are of the same order.

(ii) each element of A is equal to the corresponding element of B, ~~the~~ that is $a_{ij} = b_{ij}$ for all i and j.

$\therefore \boxed{A = B}$ $\quad A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$

Operations on Matrices.

Addition of matrices.

Q. $A = \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix}$, $B = \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$$A+B = \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$ Ans

Properties of matrix addition.

(i) Commutative Law:

If $A = [a_{ij}]$, $B = [b_{ij}]$ are matrices of the same order, say $m \times n$, then $A+B = B+A$.

eg. → $[a_{ij}] + [b_{ij}] = [b_{ij}] + [a_{ij}]$.

(ii) Associative law:

For any three matrices $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$ of the same order, say $m \times n$, then

$$(A+B)+C = A+(B+C)$$

$$([a_{ij}] + [b_{ij}]) + [c_{ij}] = [a_{ij}] + ([b_{ij}] + [a_{ij}])$$

(iii) Existence of additive identity.

~~Let $A = [a_{ij}]$ be an $m \times n$ matrix, then we have another matrix as $-A = [-a_{ij}]$ $m \times n$ such that $A +$~~

Let $A = [a_{ij}]$ be an $m \times n$ matrix and O be an $m \times n$ zero matrix, then $A + O = O + A = A$. In other words, O is the additive identity for matrix addition.

(iv) The Existence of additive inverse:

Let $A = [a_{ij}]_{m \times n}$ be any matrix, then we have another matrix as $-A = [-a_{ij}]_{m \times n}$ such that $A + (-A) = (-A) + A = O$. So $-A$ is the additive inverse of A or negative of A.

$$8 + (-8) = 0 = (-8) + 8.$$

Multiplication of Matrices:

$$A \times B = \begin{bmatrix} 3 & 8 \\ 2 & 10 \end{bmatrix} \times \begin{bmatrix} 5 & 7 \\ 4 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 5 + 8 \times 4 & 3 \times 7 + 8 \times 15 \\ 2 \times 5 + 10 \times 4 & 2 \times 7 + 10 \times 15 \end{bmatrix}$$

$$= \begin{bmatrix} 15+32 & 21+120 \\ 10+40 & 14+150 \end{bmatrix}$$

$$= \begin{bmatrix} 47 & 141 \\ 50 & 164 \end{bmatrix}$$

Definition.

The product of two matrices, A and B defined if the number of columns of A is equal to the no. of rows of B. Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{jk}]$ be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$.

29/11/21

Q. $A = \begin{bmatrix} 3 & 2 \\ 8 & 6 \end{bmatrix}_{2\times 4}$

$B = \begin{bmatrix} 2 & 8 & 3 & 4 \\ 3 & 4 & 8 & 7 \end{bmatrix}_{2\times 4}$

$A \times B = \begin{bmatrix} 6+6 & 24+8 & 9+16 & 12+14 \\ 16+18 & 64+24 & 24+48 & 32+42 \end{bmatrix}$

$A \times B = \begin{bmatrix} 12 & 32 & 25 & 26 \\ 34 & 88 & 72 & 74 \end{bmatrix}_{2\times 4}$

Transpose of matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{21} & a_{23} \end{bmatrix}$$

$$A' \text{ or } A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & 3 & 28 \\ 3 & 18 & 13 \end{bmatrix}$$

$$A' = \begin{bmatrix} 8 & 3 \\ 3 & 18 \\ 28 & 13 \end{bmatrix}$$

Definition.

If $A = [a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A' or (A^T).

In other words, if $A = [a_{ij}]_{m \times n}$, then

$A' = [a_{ji}]_{n \times m}$.

$$A = \begin{bmatrix} 8 & 3 & 18 \\ 3 & 28 & 13 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 8 & 3 \\ 3 & 28 \\ 18 & 13 \end{bmatrix}$$

Property of Transpose of matrices.

For any matrices A and B is of suitable orders, we have.

(i) $A = (A')'$

If $A = \begin{bmatrix} 3 & 2 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$

Now $A' = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 6 & 8 \end{bmatrix}$

then $(A')' = \begin{bmatrix} 3 & 2 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$

(ii) $(KA)' = KA'$ where K is constant.

$$A = \begin{bmatrix} 5 & 6 & 13 \\ 3 & 8 & 10 \end{bmatrix}$$

$$KA = \begin{bmatrix} 5K & 6K & 13K \\ 3K & 8K & 10K \end{bmatrix}$$

$$(KA)' = \begin{bmatrix} 5K & 3K \\ 6K & 8K \\ 13K & 10K \end{bmatrix}$$

$$= k \begin{bmatrix} 5 & 3 \\ 6 & 8 \\ 13 & 10 \end{bmatrix}$$

$$= KA'$$

Then $A' = \begin{bmatrix} 5 & 3 \\ 6 & 8 \\ 13 & 10 \end{bmatrix}$

$$\Rightarrow \boxed{(KA)' = KA'}$$

3. $(A+B)' = A' + B'$

$A = \begin{bmatrix} 13 & 8 \\ 5 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$

$A' = \begin{bmatrix} 13 & 5 \\ 8 & 10 \end{bmatrix}$, $B' = \begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$

$$L.H.S = \begin{bmatrix} 13 & 8 \\ 5 & 10 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 13 \\ 11 & 17 \end{bmatrix}$$

$$R.H.S = A' + B$$

$$= \begin{bmatrix} 13 & 5 \\ 8 & 10 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 11 \\ 13 & 17 \end{bmatrix}$$

$R.H.S = L.H.S$ proved.

4. $(AB)' = B'A'$

$$A = \begin{bmatrix} 5 & 6 \\ 3 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 10 & 15 \\ 2 & 4 \end{bmatrix}$$

$$A' = \begin{bmatrix} 5 & 3 \\ 6 & 7 \end{bmatrix}, \quad B' = \begin{bmatrix} 10 & 2 \\ 15 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 6 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 10 & 15 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 50+12 & 75+24 \\ 30+14 & 45+28 \end{bmatrix} = \begin{bmatrix} 62 & 99 \\ 44 & 73 \end{bmatrix}$$

$$L.H.S = (AB)' = \begin{bmatrix} 62 & 44 \\ 99 & 73 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 10 & 2 \\ 15 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 50+12 & 30+14 \\ 75+24 & 45+28 \end{bmatrix} = \begin{bmatrix} 62 & 44 \\ 99 & 73 \end{bmatrix}$$

$L.H.S = R.H.S$ proved.

Theorem 1:

For any square matrix A with real numbers entities, $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.

Let $P = A + A'$

Now $P' = (A + A')'$

$\Rightarrow P' = A' + (A')'$

$\Rightarrow P' = A' + A$ (For commutative law)

$\Rightarrow P' = P$

$\Rightarrow P = P'$

Hence $A + A'$ is symmetric Matrix.

Let $Q = A - A'$

Now $Q' = (A - A')'$

$\Rightarrow Q' = A' - (A')'$

$\Rightarrow Q' = A' - A$

$\Rightarrow Q' = -(-A' + A)$

$\Rightarrow Q' = -(A - A')$ (From commutative law)

$\Rightarrow Q' = -Q$

Therefore $A - A'$ is a skew symmetric matrix.

Theorem 2:

Any square matrix can be expresed as the sum of a symmetric and a skew symmetric matrix.

Let A be a square matrix, then we can write:

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A').$$

Q.10 $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\Rightarrow A = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$ Ans

Elementary operation (Transformation) of Matrix:

1. The interchange of any two rows or two columns. Symbollically the interchange of ith and j^{th} rows is denoted by $R_i \leftrightarrow R_j$ and interchange of i^{th} and j^{th} column is denoted by $C_i \leftrightarrow C_j$.

For example :-

(a)
$$A = \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array}$$

$$R_1 \leftrightarrow R_2$$

$$A = \begin{bmatrix} 11 & 18 \\ 8 & 3 \end{bmatrix}$$

(b)
$$A = \begin{array}{cc} \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \\ \begin{array}{cc} \downarrow & \downarrow \\ C_1 & C_2 \end{array} \end{array}$$

$$C_1 \leftrightarrow C_2$$

$$A = \begin{bmatrix} 3 & 8 \\ 18 & 11 \end{bmatrix}$$

2. The multiplication of the elements of any row or column by a non zero number.

Symbolically, the multiplication of each element of the i^{th} row by K, where $K \neq 0$ is denoted by $R_i \rightarrow KR_i$.

$$\boxed{C_i \rightarrow KC_i}$$

(a)
$$A = \begin{array}{c c} \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} & \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array} \end{array}$$

$$R_2 \rightarrow KR_2$$

$$A = \begin{bmatrix} 8 & 3K \\ 11 & 18K \end{bmatrix}$$

where K is non-zero constant.

(b)
$$A = \begin{array}{c} \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \\ \begin{array}{cc} \downarrow & \downarrow \\ C_1 & C_2 \end{array} \end{array}$$

$$\boxed{C_1 \rightarrow KC_1}$$

$$A = \begin{bmatrix} 8K & 3 \\ 11K & 18 \end{bmatrix}$$

where K is non-zero constant.

3. The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non zero number.

Symbolically, the addition to the elements of i^{th} row, the corresponding elements of j^{th} row multiplied by K is denoted by $R_i \rightarrow R_i + kR_j$.

Example :-

(a) $A = \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array}$

$$R_1 \rightarrow R_1 + kR_2$$

$$A = \begin{bmatrix} 8+11K & 3+18K \\ 11 & 18 \end{bmatrix}$$

(b) $A = \begin{array}{c} \begin{bmatrix} 8 & 3 \\ 11 & 18 \end{bmatrix} \\ \begin{array}{cc} \downarrow & \downarrow \\ C_1 & C_2 \end{array} \end{array}$

$$C_2 \rightarrow C_2 + KC_1$$

$$A = \begin{bmatrix} 8 & 3+8K \\ 11 & 18+11K \end{bmatrix}$$

Inverse of matrices.

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is called the inverse matrix of A and it is denoted by A^{-1}.

eg. $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 2\times2 + 3\times-1 & 2\times3 + 2\times3 \\ 1\times2 + 2\times-1 & 1\times3 + 2\times2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} 4-3 & -6+6 \\ -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Theorem 4.

If A and B are ~~matrices~~ invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$

We know that :-

$$(AB)(AB)^{-1} = I$$

Multiplying A^{-1} on both side, we have,

$$\Rightarrow A^{-1}(AB)(AB)^{-1} = A^{-1} \cdot I$$

$$\Rightarrow (A^{-1}A)B(AB)^{-1} = A^{-1}$$

$$\Rightarrow IB(AB)^{-1} = A^{-1}$$

$$\Rightarrow B(AB)^{-1} = A^{-1}$$

Multiplication of B^{-1} on both side, we have,

$$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$$

$$I(AB)^{-1} = B^{-1}A^{-1}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$ proved.

Q.1.) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

We know that ow

$$A = AI$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By Row transformation.

$$R_2 \to R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$R_2 \to \tfrac{1}{5} \times R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2/5 & 1/5 \end{bmatrix} A$$

$$R_1 \to R_1 + R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix} A$$

Therefore, $A^{-1} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}$ Ans.

Determinants

Definition.

To every square matrix $A = [a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of the square matrix A, where $a_{ij} = (i,j)^{th}$ element of A.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$.

It is denoted by $\det(A)$, $|A|$ or Δ.

Note :-

For matrix A, $|A|$ is read as determinant of A and not modulus of A.

Determinant of a matrix of order one :-

Let $A = [a]$ be the matrix of order 1, then determinant of A is defined to be equal to a.

$$|A| \text{ or } \det(A) \text{ or } \Delta = |a| = a.$$

Determinant of a matrix of order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2

then the determinant of A is defined as :

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{11} a_{22} - a_{21} a_{12}$$

Q $A = \begin{vmatrix} 8 & 5 \\ 10 & 1 \end{vmatrix}$

Then $|A| = 8 - 50 = -42$ Ans.

Determinant of a matrix of order 3×3.

Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value as shown below:-

Consider the determinant of square matrix

$$A = [a_{ij}]_{3\times3}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expanding along R_1, we have.

$$|A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11} (a_{22} \cdot a_{33} - a_{23} a_{32}) - a_{12} (a_{21} a_{33} - a_{23} \cdot a_{31})$$
$$+ a_{13} (a_{21} a_{32} - a_{22} a_{31}).$$

Q. $|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding along R_1, we have

$$\Rightarrow (-1)^{1+1} a_{11} (a_{22} \cdot a_{33} - a_{23} a_{32}) - a_{12} (a_{21} a_{33} - a_{23} \cdot a_{31})$$
$$+ a_{13} (a_{21} a_{32} - a_{22} a_{31})$$

$$\Rightarrow 3(0.0 - (5)) - (-1)(0.0 - (-3)) + (-2)(0 - 0)$$

$$\Rightarrow 3 \times -5 + 1 \times 3 + (-2)$$
$$\Rightarrow -15 + 3 - 2$$
$$\Rightarrow -14 \text{ Ans.}$$