

$$\eta = \frac{1 - \frac{213}{313}}$$

$$\Rightarrow \left(\frac{313 - 213}{313} \right) \times 100 = \frac{100 \times 100}{313} = \frac{10000}{313} = 31.94\%$$

* A Carnot engine takes 4200 J of heat from a source at 42°C and exhausts it to sink at 27°C.

1) What is the efficiency of heat?

2) Find the heat exhausted to sink.

3) Find work done by system.

Soln
 $\eta = \left(1 - \frac{T_2}{T_1} \right)$

$$\frac{100 - 300}{100} \Rightarrow \frac{400 \times 100}{100}$$

$$\rightarrow \frac{400}{1} = 57.1$$

ii) $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow$

$$= \frac{300 \times 4200}{100} = 12600 \text{ J}$$

iii) $Q_1 - Q_2 = 4200 - 1800 \Rightarrow 2400 \text{ J}$

Handwritten note:
 10000/313 = 31.94%
 10000/313 = 31.94%

Unit-9

Kinetic Theory of Gases

* Ideal gas / Perfect gas :- An ideal gas is that gas which obeys gas laws and ideal gas equation.

$$PV = nRT$$

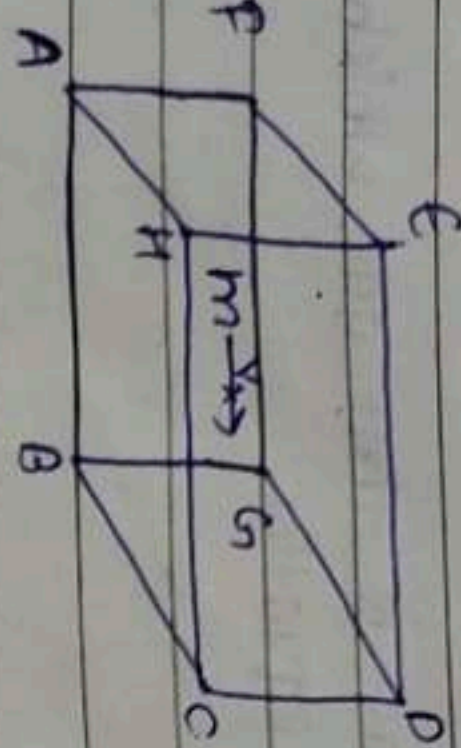
* Characteristics of ideal gas :-

- 1) The size of molecules of ideal gas is taken zero.
- 2) The molecules of ideal gas are very far apart from each other so that force b/w them becomes negligible at low pressure and high temp. Some real gases becomes ideal gas.

* Postulates
Assumptions of kinetic theory of gases :-

- 1) A gas has a very large no. of identical molecules which are elastic sphere.
- 2) The molecule of gas are in continuous random motion.
- 3) The size of gas molecules are very small and gas molecules are very far apart. So, the size of gas molecules is negligible as compared to distance b/w them.
- 4) They do not exert force on each other except collision.
- 5) The collision of molecules with each other and with container walls are elastic collision.
- 6) The molecular density of gas is assumed to be uniform.
- 7) A molecule moves along a straight line b/w any two successive collision.
- 8) The collisions are instantaneous such that the time of contact during collision is negligible.

* Expression for pressure exerted by gas:



consider a container ABCDEFGH in shape of a cuboid containing a gas having no. of molecule ' n '. ' m ' be the mass of each molecule

Let ' v_x ' be the velocity of molecule along X-axis.

Then mom. of molecule before striking the wall = $m \cdot v_x$

As after collision molecule will move in opposite direction so $v_x = -v_x$

So mom. will be = $-m v_x$
change in mom. = $-m v_x - (m v_x)$

$$= -2m v_x$$

' p_x ' be the mom. along X-axis

' f_x ' be the force.

$$F_x = \frac{dp_x}{dt}$$

$$\text{or, } F_x = \frac{p_x}{t}$$

$$F_x = \frac{-2m v_x}{t}$$

$$\text{as, } v_x = \frac{\Delta x}{t}$$

' Δ ' be the distance travel by molecule from one end to another.

$$F_x = \frac{-2m v_x \Delta x}{t}$$

$$F_x = \frac{-2m v_x^2}{\Delta x}$$

$$F_x = \frac{-m v_x^2}{\Delta x}$$

' n ' no. of molecule exert force

$$F_x = -n \times \frac{m v_x^2}{t}$$

$$F_x = \frac{-M v_x^2}{\Delta x} \quad (\because n \times m = M)$$

$$\text{Now pressure } [p_x] = \frac{F_x}{A}$$

$$p_x = \frac{M v_x^2}{\Delta x A}$$

$$p_x = \frac{-M v_x^2}{V}$$

$$\text{as, } \frac{M}{V} = \rho \text{ (density)}$$

$$p_x = \rho \cdot v_x^2$$

Let p_y and p_z be the pressure exerted along Y-axis and Z-axis respectively. f_y and f_z be the density

$$p_y = \rho \cdot v_y^2$$

$$p_z = \rho \cdot v_z^2$$

as, $f_x = f_y = f_z = \rho$ is called density of gas

' P ' be the total pressure exerted by gas

$$P = \frac{p_x + p_y + p_z}{3}$$

$$P = \frac{1}{3} \rho (V_x^2 + V_y^2 + V_z^2)$$

Where $V_x^2 + V_y^2 + V_z^2 = V^2$ is called average velocity of gas.

$$P = \frac{1}{3} \rho V^2$$

* Relation between pressure and K.E of gas:

Let 'P' be the pressure exerted by gas

$$P = \frac{1}{3} \rho V^2 \quad (1)$$

K.E of gas is given by

$$K.E = \frac{1}{2} M V^2$$

K.E per unit vol^m (E) is given by

$$E = \frac{K.E}{V} = \frac{\frac{1}{2} M V^2}{V}$$

$$E = \frac{1}{2} \rho V^2 \quad (\because \frac{M}{V} = \rho) \quad (2)$$

Divide eq (1) by (2)

$$\frac{P}{E} = \frac{\frac{1}{3} \rho V^2}{\frac{1}{2} \rho V^2}$$

$$\frac{P}{E} = \frac{2}{3}$$

$$P = \frac{2}{3} E$$

$$\left[\text{or } E = \frac{3}{2} P \right]$$

* Average K.E per molecule of gas:

we know

$$P = \frac{2}{3} E$$

$$\text{Where } E = \frac{K.E}{V}$$

$$P = \frac{2}{3} \frac{K.E}{V}$$

$$P V = \frac{2}{3} \times K.E \quad (1)$$

Using Ideal gas eqn

$$P V = n R T$$

If n = 1 mole

$$P V = R T \quad (2)$$

$$\frac{2}{3} K.E = R T$$

$$\left[K.E = \frac{3}{2} R T \right]$$

This is K.E of all molecules of gas as 1 mole of gas contain avogadro no. of molecule (N)

$$N = 6.022 \times 10^{23}$$

$$\text{So, average K.E per molecule} = \frac{3}{2} \frac{R T}{N}$$

As, $\frac{R}{N} = k_B$ is called Boltzmann's constant

$$\text{So, } \left[\text{Average K.E} = \frac{3}{2} k_B T \right]$$

* Dependence of K.E on temperature of gas:

$$\text{As, average K.E per molecule} = \frac{3}{2} R T$$

So, $\propto T$

But we have

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M}{V} \cdot v^2$$

$$P = \frac{2}{3} \times \frac{1}{2} M \cdot v^2$$

$$P = \frac{2}{3} \cdot \frac{KE}{V}$$

$$P = \frac{2}{3} \cdot E$$

$$As, E = \frac{1}{2} M v^2$$

$$So, v^2 \propto T$$

Velocity of gas molecule increase with increase in temp. of gas.

* A gas exert a pressure of $5 \times 10^6 \text{ Nm}^2$ on container wall. find the speed of gas molecule of density of gas is 1500 kg. per m^3

Soln $P = \frac{1}{3} \rho v^2$

$$5 \times 10^6 = \frac{1}{3} \times 1500 \times v^2$$

$$\frac{5 \times 10^6 \times 3}{1500} = v^2$$

$$\frac{1}{1500} \times 15 \times 10^6 = v^2$$

$$10^4 = v^2$$

$$10000 = v^2$$

$$\sqrt{10000} = v$$

$$100 \text{ m/s} = v$$

* A gas exert pressure of 10^8 on container wall find K.E per unit vol^m of gas.

Soln

$$P = \frac{2}{3} E$$

$$\frac{10^8 \times 3}{2} = E$$

$$1.5 \times 10^8 = E$$

$$1.5 \times 10^8 \text{ Jm}^{-3} = E$$

* calculate average K.E of a gas at 100°C

$$\Rightarrow \frac{3}{2} RT$$

$$= \frac{3}{2} \times 8.31 \times 313$$

$$= 461.8 \text{ J}$$

* The pressure of a gas decrease to $\frac{1}{4}$ at constant temp on basis of kinetic theory of gases. Find effect on vol^m

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M}{V} \cdot v^2$$

$$P \propto \frac{1}{V}$$

$$as, v \propto \frac{1}{P}$$

it will be 4 times increases.

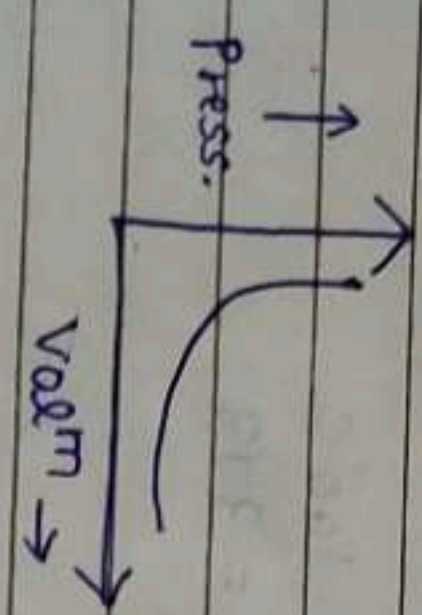
* Ideal gas laws:

1) Boyle's law:- According to this law if temp. is constant then vol^m of a gas is inversely prop. to pressure.
Using ideal gas eqⁿ

$$PV = RT \text{ (For 1 mole)}$$

$$PV = \text{constant}$$

$$\text{or } V \propto \frac{1}{P}$$



We know,

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$PV = \frac{1}{3} M v^2$$

As, $v \propto T$

$$PV \propto T$$

Here ' T ' is constant

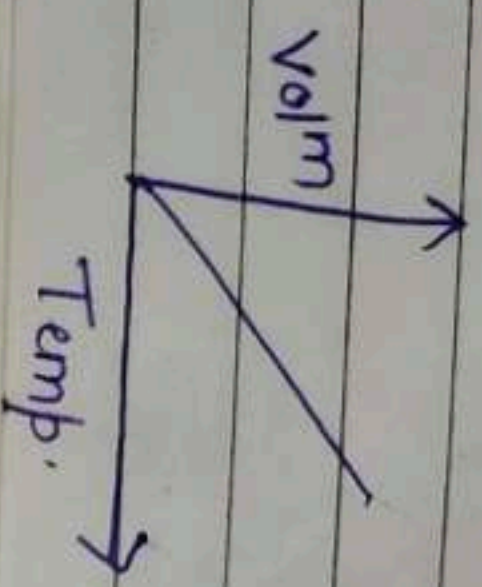
$$PV = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

* Charles's law: According to this law if pressure of a gas is constant then the vol^m of gas is directly prop. to temperature.

If pressure ' P ' is constant then

$$\frac{V}{T} = \text{constant}$$



As, we know

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$VP = \frac{1}{3} M v^2$$

$$PV \propto v^2$$

As, $v^2 \propto T$

$$PV \propto T$$

Hence, ' P ' is constant

So, $\left[\frac{V}{T} \right]$

$$\frac{V}{T} = \text{constant}$$

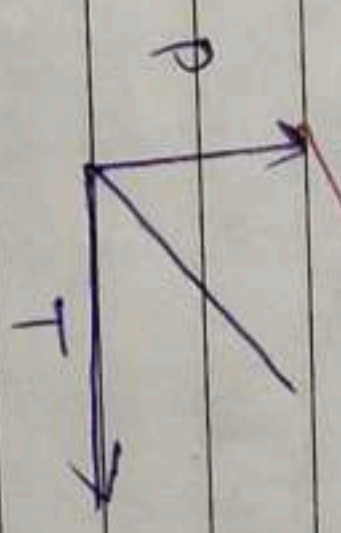
$$\left[\frac{V_1}{T_1} = \frac{V_2}{T_2} \right]$$

* Gay Lussac's law: According to this law if vol^m of a gas is constant then the pressure of gas will be directly prop. to the temperature.

If vol^m (V) is constant

$$P \propto T$$

$$\frac{P}{T} = \text{constant}$$



As, we know

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$\frac{P}{T} = \text{constant}$$

Here, ' v ' is constant
So, $P \propto T$

$$\left[\frac{P_1}{T_1} = \frac{P_2}{T_2} \right]$$

* Ideal gas eqⁿ or perfect gas eqⁿ:

A relation b/w pressure

, vol^m and temp. of an ideal gas is known as Ideal gas eqⁿ.

$$PV = nRT$$

where, $P = \text{Pressure}$

$$V = \text{Vol}^m$$

$n = \text{no. of moles of gas}$

$R = \text{universal gas constant}$

$T = \text{Temperature}$

As, we know

$$P = \frac{1}{3} \rho v^2$$

$$P = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$PV = \frac{1}{3} M v^2$$

$$PV \propto v^2$$

$$\text{as, } v^2 \propto T$$

$$PV \propto T$$

$$PV = RT$$

where R is called universal gas constant.

For ' n ' no. of moles:-

$$[PV = nRT]$$

* Universal gas constant:

$$PV = nRT$$

$$R = \frac{PV}{nT}$$

$$R = \frac{W}{nT} \quad [\because W = PV]$$

$$R = \frac{\text{Workdone}}{(\text{no. of mole})(\text{temp.})}$$

It is defined as workdone per mole per kelvin.

S.I unit of $R = \text{J mol}^{-1} \text{K}^{-1}$

Value of $R = 8.31 \text{ J mol}^{-1} \text{K}^{-1}$

as, 1 calorie = 4.2 J

$$R = \frac{8.31}{4.2} = 1.98 \text{ cal mol}^{-1} \text{K}^{-1}$$

* Avogadro law: According to this law equal vol^m of all

gases contain same no. of molecules if pressure and

temp. of all gases are same.

i.e. $P_1 = P_2$, $V_1 = V_2$ or $T_1 = T_2$ then $n_1 = n_2$

Consider a gas having vol^m (V) and pressure (P) at

temp. (T) then,

$$P = \frac{1}{3} \rho v^2 = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$\text{for gas 1st } P_1 = \frac{1}{3} \frac{M_1 \cdot v_1^2}{V_1}$$

$$\text{For gas 2nd } P_2 = \frac{1}{3} \frac{M_2 \cdot v_2^2}{V_2}$$

$$\text{as, } P_1 = P_2$$

$$\frac{1}{3} \frac{M_1 \cdot v_1^2}{V_1} = \frac{1}{3} \frac{M_2 \cdot v_2^2}{V_2}$$

as temp. same

$$v_1^2 = v_2^2$$

$$\Rightarrow M_1 = M_2$$

If (m) be the mass of each molecules of gas. n_1 and n_2 be the no. of molecules of gas 1st and 2nd respect.

$$n_1 \times m = n_2 \times m$$

$$n_1 = n_2$$

* Graham's law of diffusion: Acc. to this law, the rate of diffusion of two gases will be inversely prop. to square root of their densities. If 'g' is the density of gas and 'x' is rate of Diffusion then

$$x \propto \frac{1}{\sqrt{g}}$$

As, we know

$$P = \frac{1}{3} \rho v^2$$

$$= \frac{1}{3} \frac{M \cdot v^2}{V}$$

Let A and B are two gases mix with each other

$$P_1 = \frac{1}{3} \frac{M_1 \cdot v_1^2}{V_1}$$

$$P_2 = \frac{1}{3} \frac{M_2 \cdot v_2^2}{V_2}$$

$$\text{or, } P_1 = \frac{1}{3} \rho_1 v_1^2$$

$$P_2 = \frac{1}{3} \rho_2 v_2^2$$

$$\text{If } P_1 = P_2$$

$$\frac{1}{3} \rho_1 v_1^2 = \frac{1}{3} \rho_2 v_2^2$$

$$\frac{\rho_1}{\rho_2} = \frac{v_2^2}{v_1^2} \quad \text{--- (i)}$$

As rate of diffusion is directly prop. to the Velocity of molecule of gas then,

$$\frac{x_1}{x_2} = \frac{v_1}{v_2} \quad \text{--- (ii)}$$

From eqn (i) and (ii) we get,

$$\frac{\rho_1}{\rho_2} = \frac{x_2^2}{x_1^2}$$

$$\text{or } x \propto \frac{1}{\sqrt{\rho}}$$

$$\frac{x_2}{x_1} = \sqrt{\frac{\rho_1}{\rho_2}} \quad \checkmark$$

This is called Graham's law of diffusion.

* Dalton's law of Partial Pressure: Acc. to this law total pressure exerted by mixt. of gases (which do not interact with each other) will be equal to sum of partial pressure of each gas if each gas occupy same volm as a given temperature.

Let 'P' be the total pressure and P_1, P_2, P_3, \dots be the partial pressure exerted by each gas respect.

As, we know

$$P = \frac{1}{3} \rho M \cdot v^2$$

$$P_1 = \frac{1}{3} \rho_1 v_1^2, P_2 = \frac{1}{3} \rho_2 v_2^2, P_3 = \frac{1}{3} \rho_3 v_3^2, \dots$$

$$\frac{1}{3} \frac{M_1 \cdot v_1^2}{V}, \frac{1}{3} \frac{M_2 \cdot v_2^2}{V}, \frac{1}{3} \frac{M_3 \cdot v_3^2}{V}, \dots$$

$$\text{As, } V_1 = V_2 = V_3 = V$$

as temp is same

$$v_1 = v_2 = v_3 = v$$

$$P_1 + P_2 + P_3 + \dots = \frac{1}{3} \frac{M_1 \cdot v^2}{V} + \frac{1}{3} \frac{M_2 \cdot v^2}{V} + \frac{1}{3} \frac{M_3 \cdot v^2}{V} + \dots$$

$$= \frac{1}{3} v (M_1 + M_2 + M_3 + \dots) v^2$$

$$\text{As, } M_1 + M_2 + M_3 + \dots = M \quad \text{(total mass of gas)}$$

$$P_1 + P_2 + P_3 + \dots = \frac{1}{3} \frac{M \cdot v^2}{V}$$

$$P_1 + P_2 + P_3 + \dots = \frac{1}{3} \rho v^2 \quad \text{--- (i)}$$

$$\text{As, } P = \frac{1}{3} \rho v^2 \quad \text{--- (2)}$$

$$\left[P = P_1 + P_2 + P_3 + \dots \right]$$

* Mean or average speed: It is the average speed with which gas molecule moves is called mean speed. Let $v_1, v_2, v_3, \dots, v_n$ be the speed of each molecule resp. Then mean speed (\bar{v}) is given by:

$$\bar{v} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$$

$$\bar{v} = \frac{\sqrt{\frac{8k_B T}{\pi m}}}{\sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{8PV}{\pi M}}$$

where m = mass of each molecule of gas

M = mass of gas

k_B = Boltzmann's constant.

* RMS Speed: (Root mean square) \div The square root of mean of square of speed of gas molecule is known as RMS speed.

If v_1, v_2, \dots, v_n be the speed of each molecule of gas then,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}}$$

$$v_{rms} = \sqrt{\frac{3k_B T}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

* Most Probable speed: It is the speed which is followed by maximum no. of molecules of gas. It is given by:

$$v_m = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

$$v_{rms} : v_m = \sqrt{\frac{3k_B T}{M}} : \sqrt{\frac{2k_B T}{M}} = \sqrt{\frac{3}{2}}$$

Ex: $v_{rms} : v_m = \sqrt{\frac{8}{\pi}} = \sqrt{3} : \sqrt{2}$

* A gas is in a container its molecule have a speed 1, 2, 4, 6, 2, 4, 2, 3, 1 ms^{-1} and mean speed of molecule

ii) RMS speed of molecule

iii) Most probable speed.

Soln: $\bar{v} = \frac{1+2+4+6+2+4+2+3+1}{9}$

$\Rightarrow \bar{v} = 3.6 \text{ms}^{-1}$

ii) $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \dots + v_n^2}{n}}$

$= \sqrt{\frac{1+4+16+36+4+16+4+9+9}{9}}$

$= \sqrt{13.9} \approx 3.7 \text{ms}^{-1}$

iii) Most probable speed is 2ms^{-1} .

1) A given mass of a gas at 27°C exerts a pressure of 2atm (atmospheric pressure) what pressure will exert at 127°C . If v_{rms} remains constant.

Soln: $T_1 = 27^\circ\text{C} = 273 + 27 \Rightarrow 300\text{K}$

$T_2 = 127^\circ\text{C} = 127 + 273 = 400\text{K}$

$P_1 = 2 \text{atm}$

$P_2 = \frac{P_1 \times T_2}{T_1} \Rightarrow \frac{2 \times 400}{300} = 2.66 \text{atm}$

* The ratio of density of two gases are 4:9 what will be the ratio of their rate of diffusion.

Soln $\rho \propto \frac{1}{\sqrt{f}}$

$$\frac{\rho_1}{\rho_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{\rho_1}{\rho_2} = \sqrt{\frac{9}{4}}$$

$$\frac{\rho_1}{\rho_2} = \frac{3}{2}$$

* Degree of freedom: No. of coordinates required to describe motion of an object is known as degrees of freedom.

→ If a particle move along x-axis only, Then its degree of freedom will be one.

→ If a particle moves in two dimensions then degree of freedom will be two.

→ Motion of a bee, mosquitoes, contain three degree of freedom.

→ Formula of degrees of freedom $\Rightarrow [N = 3A - R]$

where, N = no. of degree of freedom

A = no. of atom in given gas

R = no. of relation between atom of gas.

* For example:

i) For monoatomic gas - A=1, R=0

$$N = 3 \times 1 - 0$$

$$[N = 3]$$

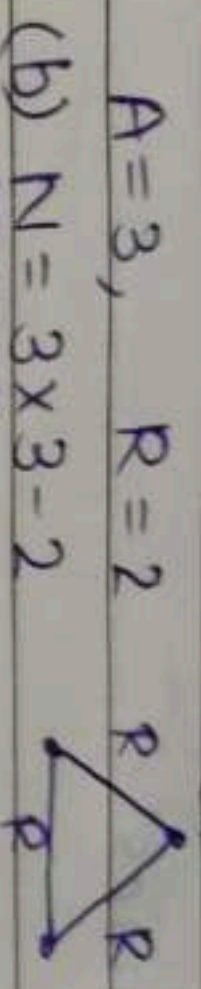
ii) Diatomic gas - A=2, R=1

$$N = 3 \times 2 - 1$$

$$[N = 5]$$

iii) For triatomic gas

(a)



(b)

$$\frac{R}{R} \cdot \frac{R}{R}$$

$$A=3, R=2$$

$$[N = 7]$$

$$(a) N = 3 \times 3 - 3$$

$$[N = 6]$$

* Law of Equipartition of energy: Acc. to this law energy given to a gas divides equally among its degree of freedom. each degree of freedom per molecule receive energy equal to $\frac{1}{2} k_B T$. where k_B = Boltzmann constant. T = Temp. of a gas.

Energy per degree of freedom per molecule = $\frac{1}{2} k_B T$

* Specific heat of monoatomic gas:

There are two

types of specific heat of gases -

(1) Specific heat at constant volm = C_v

(2) Specific heat at constant pressure = C_p

Acc. to law of equipartition of energy.

Energy per degree of freedom of molecule = $\frac{1}{2} k_B T$

For monoatomic gas degree of freedom = 3

Energy of one molecule of monoatomic = $3 \times \frac{1}{2} k_B T$

As, 1 mole of gas contains = Avogadro no. of molecule

So, energy of molecule gas of monoatomic = $N \times 3 \times \frac{1}{2} k_B T$

AS, $N \cdot k_B = R$ is called

universal gas constant

$$\left[U = \frac{3}{2} RT \right]$$

Using first law of thermodynamics

$$d\theta = du + dw \quad \text{--- (1)}$$

$$du = \frac{3}{2} R \cdot dT$$

let C_p and C_v be the specific heat at constant pressure and V_{vol} respect.

$$dQ = m \cdot s \cdot dT$$

where, m = mass of gas
 s = specific heat

at constant Vol^m :-

$$dw = v \cdot dv$$

$$\text{Here, } dv = 0$$

$$[dw = 0]$$

So,

$$d\theta = 1 \cdot C_v \cdot Cp$$

$$d\theta = C_v \cdot dT$$

Put these value in eqn (1)

$$C_v \cdot dT = \frac{3}{2} R \cdot dT + 0$$

$$C_v \cdot dT = \frac{3}{2} R \cdot dT$$

$$C_v = \frac{3}{2} R$$

$$\text{Or, } Cp - C_v = R$$

$$Cp = \frac{3}{2} R + R$$

$$Cp = R + \frac{3}{2} R$$

$$Cp = \frac{5}{2} R$$

Handwritten notes and corrections:
A large red 'X' is drawn over the equations for C_p .
Below the equations, there is a red scribble that looks like "Handwritten" or "Corrected".
There are also some red marks and numbers like "2" and "11" near the bottom of the equations.

Unit - 10

---: Oscillation and waves :-

* Oscillatory motion - A motion in which body moves to and fro about a fix position/point is called oscillatory motion - Ex - Motion of simple pendulum, vibration in spring

* Conditions for oscillatory motion: There are 2-conditions for oscillatory motion

- 1) It must be periodic means during oscillatory motion the body will repeat its path after a fixed time.
- 2) It must be bounded means b/w two points or limited

* Types of oscillatory motion | Harmonic motion:

There are

two types

1) Simple harmonic motion (S.H.M) - If only one harmonic (frequency) is used to represent oscillatory motion then the oscillatory motion is called Simple Harmonic motion

For ex: $y = a \sin \omega t$

$$y = a \sin \omega t \cdot a \cos \omega t$$

$$y = a \cos \omega t$$

* Non-Harmonic motion: If two or more harmonic

(fg) are used to represent oscillatory motion then the oscillatory motion is called non-harmonic motion

$$\text{Ex: } y = a \sin \omega t \sin 3\omega t, \text{ freq.} = \omega, 3\omega$$

$$y = a \sin \omega t \cdot \sin 5\omega t, \text{ freq.} = \omega, 5\omega$$

* Some important terms related with S.H.M: