

Q = 21] Derive the exp. for tension ( $T$ ) and acc. ( $a$ ) produced in string in connected motion.

Ans-

Let us consider two masses  $m_1$  and  $m_2$  connected with a string having mass  $T$  over a pulley. Let  $m_2 > m_1$ , then acc. of mass  $m_2$  is in upward direction while acc. of  $m_1$  is in downward.

$$m_1 a = T - m_1 g \quad \text{--- (1)}$$

$$m_2 a = m_2 g - T \quad \text{--- (2)}$$

Add (1) and (2)

$$m_1 a + m_2 a = T - m_1 g + m_2 g - T$$

$$(m_1 + m_2)a = m_2 g - m_1 g$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Divide eqn (1) by (2)

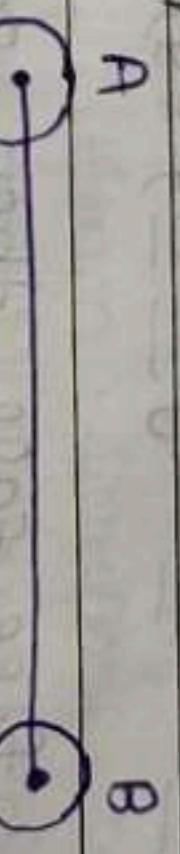
$$\frac{m_1 a}{m_2 a} \times \frac{T - m_1 g}{m_2 g - T}$$

$$m_1 m_2 a^2 g - m_1 a T = m_2 a T - m_1 m_2 a g$$

$$m_1 m_2 a g = a T (m_1 + m_2)$$

$$T = \frac{g m_1 m_2}{m_1 + m_2}$$

### Explanation $\rightarrow$



Let us consider two bodies A and B having

mass  $m_1$  and  $m_2$  respect. 'r' be the dist. b/w their centres. Then acc. due to gravitational

law of force is  
i) Directly prop. to product of their masses i.e.  $f \propto m_1 m_2$  - (1)

ii) Inversely prop. to square of distance b/w their centres i.e.

$$f \propto \frac{1}{r^2} \quad \text{--- (2)} \quad \left[ G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} \right]$$

From (1) and (2)

$$f = \frac{G m_1 m_2}{r^2}$$

where G is

Gravitational constant  $\uparrow$

### \* Gravitational force:

It is the attractive force

which come in any two bodies due to their masses is called gravitational force.

### \* Newton's law of gravitation:

Acc. to this law gravitational force b/w two mass bodies is directly prop. to product of their masses and inversely prop. to square of distance b/w their centres.

Equation of ① and ②

$$G = \frac{F_{\text{at}}}{m_1 m_2} = \frac{MLT^{-2} \times L^2}{[ML][ML]} = \frac{L^3 T^{-2}}{M^2}$$

$$G = [M^{-1} L^3 T^{-2}]$$

\* Newton's gravitation law obey Newton 3rd law of motion i.e.....

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

\* Acceleration due to gravity (g) :

When an object

moves under free fall then some acc. produced in its motion is called acc. due to gravity

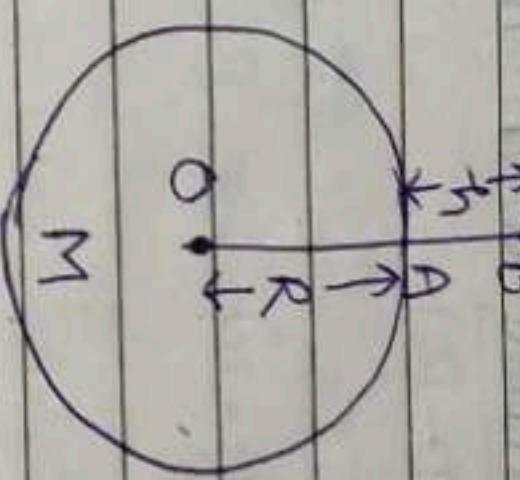
It is denoted (g).

It is vector quantity.

Its value is  $9.8 \text{ ms}^{-2}$  at the surface of earth

Its value changes as we move from surface of earth.

\* Effect of height on 'g'



Consider earth as a sphere of radius 'R' and Mass 'M' placed on point 'B' at height 'h' from surface of earth.

Let 'g' and 'g'' be the value of acc. due to gravity at point A and B respectively. Then, as we know :-

$$g = \frac{GM}{R^2}$$

At point B

$$g' = \frac{GM}{(R+h)^2}$$

Divide eqn ② by ①

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \cdot \frac{GMr^2}{GM} = \left(1 + \frac{h}{R}\right)^{-2}$$

Then weight of body having mass 'm' and radius 'R'

Gravitational  $\Rightarrow F = \frac{GMm}{R^2} - \text{ weight of body : -}$

$$F = mg$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

Using binomial thm

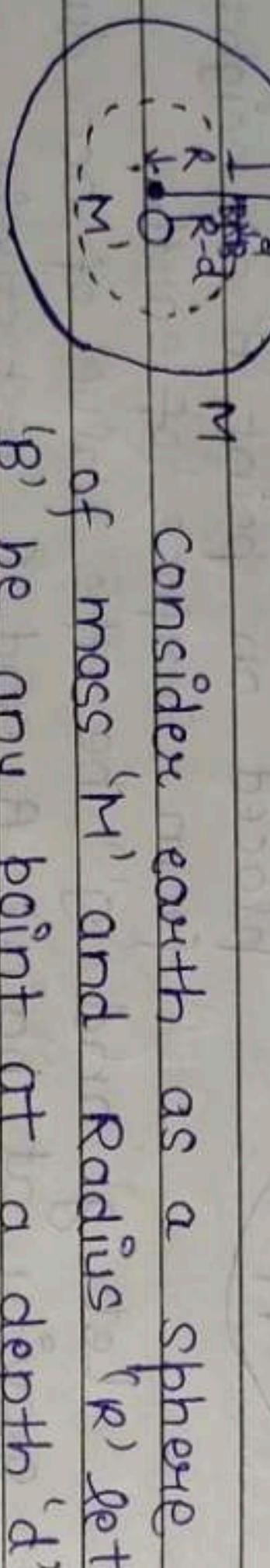
$$g' = g \left(1 - \frac{2h}{R}\right)$$

or,

$$g' = g - g \frac{2h}{R}$$

$[g' < g]$   
Thus, acc. due to gravity decrease with increase in height from earth's surface.

\* Effect of depth on  $g'$ :



Consider earth as a sphere of mass 'M' and Radius 'R'. Let 'B' be any point at a depth 'd' below surface of earth.  $g$  and  $g'$  be the value of acc. due to  $g$  at point A and B resp.

$$g = \frac{M}{\sqrt{R^2 + d^2}}$$

$$M = g \frac{4\pi R^3}{3} \quad (\text{eqn i})$$

$$g = \frac{GM}{R^2} \quad (\text{eqn ii})$$

from eqn (i)

$$g = \frac{G \cdot \frac{4}{3}\pi R^3 \cdot f(R)}{R^2}$$

$$g = \frac{4\pi G f(R)}{3} \quad (\text{eqn iii})$$

M be the mass of sphere having radius  $(R-d)$

$$M' = \frac{4}{3}\pi (R-d)^3$$

$$g' = \frac{4\pi G f(R-d)}{(R-d)^2}$$

$$g' = \frac{4\pi G f(R-d)}{3} \cdot \frac{1}{(R-d)^2}$$

$$g' = \frac{4}{3}\pi G f(R-d) \quad (\text{eqn iv})$$

$$\text{Divide eqn iii by iv}$$

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G f(R-d)}{\frac{4}{3}\pi G f(R)}$$

$$\frac{g'}{g} = \frac{R-d}{R}$$

~~$$\frac{g'}{g} = \frac{R-d}{R}$$~~

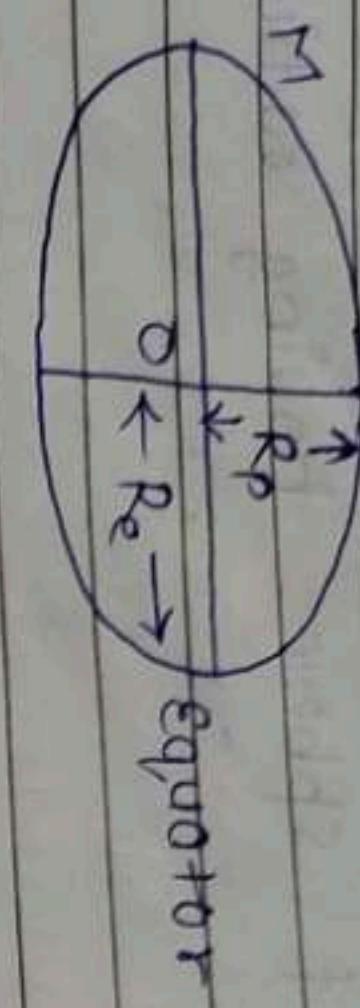
~~$$\frac{g'}{g} = \frac{R-d}{R}$$~~

~~$$\frac{g'}{g} = \frac{1-d}{R}$$~~

$$g' = g \left(1 - \frac{d}{R}\right)$$

as,  $\boxed{g' < g}$

\* Effect of shape of earth on 'g'



Consider earth as an ellipse of Mass ('M')  
R<sub>p</sub> and R<sub>e</sub> be the radius of pole and  
equator line.

g<sub>p</sub> and g<sub>e</sub> be the value of acc. due to g  
on pole and equator respect.

As we know,

$$g = \frac{GM}{R^2}$$

M

$$g \propto \frac{1}{R^2}$$

as,  $\Rightarrow R_e > R_p$

g<sub>t</sub> is denoted by T. g<sub>t</sub> is a vector quantity

$$\vec{T} = \frac{\vec{F}}{m}$$

where 'm' is mass of test body

$$\vec{T} = \frac{GMm}{r^2}$$

$$S.I. \text{ unit} = N \cdot kg^{-1}$$

g<sub>t</sub>'s direction is similar to the direction of gravitational force.

$$\text{Dimensional formula of } T = \frac{M^1 L^1 T^{-2}}{M} \Rightarrow [M^0 L^1 T^{-2}]$$

\* Gravitational potential :

So, g<sub>e</sub> < g<sub>p</sub>  
Thus value of acc. due to g will be maximum at pole.

amount of workdone in bringing a unit mass from infinite to a point against the gravitational force of earth. g<sub>t</sub> is denoted by 'V'.

$$V = \frac{W}{m}$$

body in which gravitational force due to that body can be felt/experienced is called gravitational field of that body.

$$S.I. \text{ unit is } N \cdot kg^{-1}$$

$$\text{Dimensional formula} = \frac{[M^1 L^1 T^{-2}]}{[M]} \Rightarrow [M^0 L^2 T^{-2}]$$

\* Intensity of gravitational field : The strength of gravitation field is called intensity of gravitational field.

g<sub>t</sub> is a scalar quantity.



Taking cube root both side

$$\left(\frac{GM \cdot T^2}{4\pi^2}\right)^{1/3} = R + h$$

$$h = \left(\frac{GM \cdot T^2}{4\pi^2}\right)^{1/3} - R$$

$$g = \left(\frac{GM \cdot T^2}{4\pi^2}\right)^{1/3} - R$$

In terms of 'g'

$$GM = gR^2 \cdot T^2$$

$$h = \left(\frac{gR^2 \cdot T^2}{4\pi^2}\right) - R$$

\* Angular mom. of Satellite: let  $\vec{L}$  be the angular mom. of satellite

let  $\vec{L}$  be the

angular mom. of Satellite:  
 $L = mv_o \cdot r$  —  
 as,

$$v_o = \sqrt{\frac{GM}{r}}$$

put this value in eqn 1<sup>st</sup>

$$L = m \sqrt{\frac{GM}{r}}$$

$$[L = m \sqrt{GM \cdot r}]$$

\* Binding Energy of Satellite: The energy required

$$- \cdot \left[ L = m \sqrt{GM(R+h)} \right] \cdot -$$

to remove a satellite from its orbit is called binding energy of satellite.

Binding Energy = -T. Energy

$$B.E = -T.E$$

Energy of satellite is equal to sum of its K.E and gravitational P.E.

$$E = K.E + P.E \quad \text{--- } \Theta$$

$$K.E = \frac{1}{2}mv_o^2$$

$$= v_o = \sqrt{\frac{GM}{R+h}}$$

$$K.E = \frac{1}{2}m \left( \frac{\sqrt{GM}}{R+h} \right)^2$$

$$K.E = \frac{GMm}{2(R+h)}$$

as, gravitational p.e is given by

$$(P.E) = -\frac{GMm}{(R+h)}$$

Put value in eqn (i)

$$E = GMm - \frac{GMm}{R+h}$$

(i) for some time

$$E = \frac{GMm}{R+h}$$

on putting E =

$$\left[ E = -\frac{GMm}{2(R+h)} \right]$$

$$\left[ P.E = \frac{1}{2}T.E \right]$$

\* Energy of Satellite: How much gainup?

Energy of satellite is equal to sum of its K.E and gravitational P.E.

Total energy is given by:

$$E = -\frac{GMm}{2(R+h)}$$

### \*Special case:

special case: When satellite is very close to earth's surface then  $R_{\text{eff}}$ ,  $\mu$  will be neglected then,

11  
G  
E

Here,  $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$   
 $M = 6 \times 10^{24} \text{ kg}$ .  
 $R = 6.4 \times 10^6 \text{ m}$ .  
 $V_o = 1.92 \text{ km s}^{-1}$

In terms of acc. due to gravity  
as,  $g = \frac{GM}{r^2}$ .

$G M = g R^2$

$$V_o = \frac{g R^2}{R} + g \cdot 0 = g R^2$$

where,  $g = 9.8 \text{ m/s}^2$

\* Time period of Satellite

As we know by satellite to complete one revolution around earth, it is denoted by 'T'.

As, we know  
Velocity =

$$V_0 = \frac{2\pi(R+h)}{T} - \textcircled{1}$$

where  $v_o$  is called orbital velocity.

$$\frac{GM}{R+h} = g\pi(R+h)$$

Special case. When satellite is very close to earth surface then,  $R \gg h$ ,  $\theta$  will be neglected.

$$T = 2\pi \left( R + h \right) \sqrt{\frac{R+h}{GM}}$$

in terms of  $\sigma$ )

$$T = \frac{2\pi}{g} \sqrt{\frac{R}{g}}$$

11

$$T = \frac{2\pi}{\omega}$$

\* Height of Satellite:

is given by

$$T = \frac{2\pi}{GM} \sqrt{\frac{(R+h)}{g}}$$

Squatting both side

$$V_0 = 2\pi(R+h) - \Theta$$

Put value in eqn

\* The gravitational field intensity at a point 20,000 km from centre of earth is 9.4 N/kg

calculate the G.P at that point.

$$\text{Soln } \frac{T}{m} = \frac{GmM}{r^2} = \frac{GM}{r^2}$$

$$\text{or, } V = -\frac{GM}{r}$$

$$\Rightarrow \frac{V}{T} = -\frac{GM}{r^2} = \frac{-k^2}{r} = -\omega$$

$$[\sqrt{\omega} = V]$$

Consider a very high tower on the surface of earth whose height is more than height of earth atmosphere. Let a body is given some horizontal velocity from the top of tower. The body will follow a parabolic path due to gravity and hit earth surface following path 1st. But if we go increasing horizontal velocity the body will follow path 2, 3, 4. Finally body will start revolving around the earth. There are two necessary

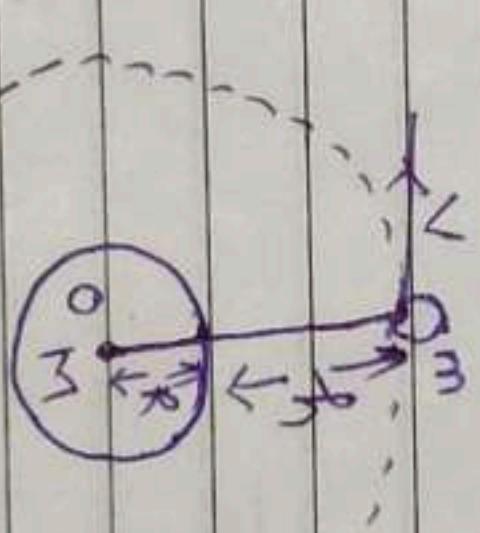
$$= -3.4 \times 10^6 \text{ J/kg}$$

- first take the satellite to a height then give a suitable horizontal velocity.

### \* Satellite:

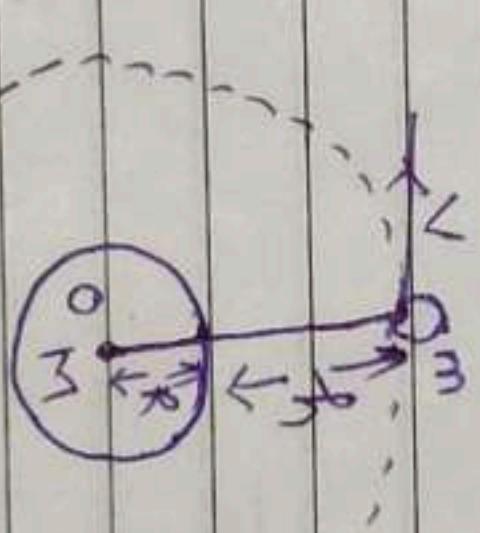
Satellite is body which is revolving continuously in an orbit around a comparatively much larger body. For ex: Moon is a satellite of earth. There are two types of satellite

i) Natural Satellite And ii) Artificial Satellite

 required to put satellite into its orbit around earth is called orbital velocity of satellite.  $g_t$  is denoted by  $V_o$ .

Moon is a natural satellite of earth. Anybhatta is a artificial satellite.

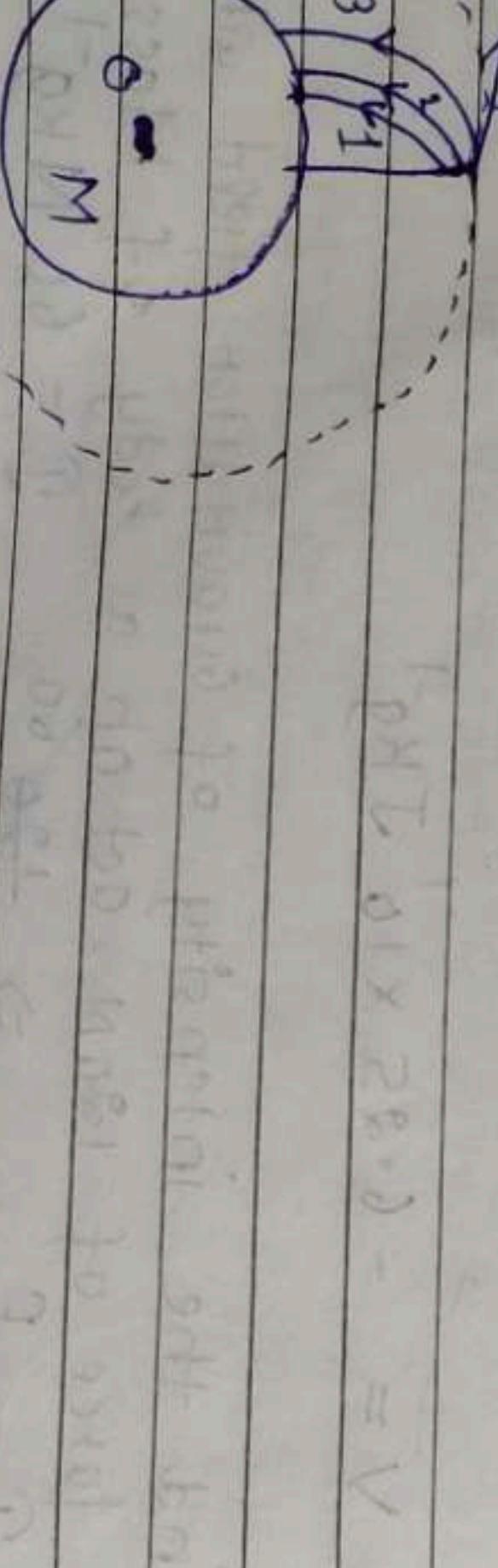
\* Principle of launching a satellite:

 consider earth as a sphere of mass 'M' and radius 'R'. A satellite of mass 'm' is placed at a height 'h' from earth's surface. 'V' be the orbital velocity of satellite. Centripetal force and gravitational force act on satellite.

centrifugal force = gravitational force

$$\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$$

$$mv^2 = \frac{GMm}{R+h} \Rightarrow V^2 = \frac{GM}{R+h}$$





5) An artificial satellite revolves in orbit at a height of 1000 km find its orbital velocity?

Soln

$$V_o = \sqrt{\frac{GM}{R+h}}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$M = 6 \times 10^{24} \text{ kg}$$

$$R = 6.4 \times 10^6 \text{ m}$$

$$H = 10^6 \text{ m.}$$

$$V_o = \sqrt{\frac{GM}{(R+h)^2}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6 + 10^6)^2}}$$

$$V_o = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6 + 10^6)^2}}$$

$$P.E = -\frac{GMm}{R+h}$$

$$= \frac{1}{2} \times 500 \times 6.67 \times 10^{-11} \times (6 \times 10^{24}) \times 500$$

$$= 7.4 \times 10^6$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7.4 \times 10^6}$$

$$T.E = k \cdot \epsilon + P.E = 10^{10} - 2 \times 10^{10} = -10^{10} \text{ Joule}$$

~~W.L.O.G~~

$$= 6.67 \times 6 \times 10^{-11} \times 10^6$$

$$= 66.7 \times 6 \times 10^{-5}$$

$$= 374 \times 10^{-5}$$

$$374 \Rightarrow 7353 \text{ m/s}$$

\* A geostationary satellite of 500 kg is revolving around earth at height of  $3.6 \times 10^6 \text{ m}$  find

i)  $k \cdot \epsilon$

ii)  $P.E$

iii) Total  $\epsilon$ .

iv) Binding  $\epsilon$ .

Soln Here,  $r = R+h = 6.4 \times 10^6 + 3.6 \times 10^6 = 10^7 \text{ m.}$

\* Weightlessness -

It is a situation where effective weight of a body becomes zero.

- i) At the centre of earth - The value of acc. due to gravity at the centre of earth is zero

As,

$$W = mg$$

$$W = 0$$

So,  $|W| = 0$  is called weightless.

ii) Freefalling lift:

$$W = mg$$

$$W = 0$$

So,  $|W| = 0$  called weightless.

- 2) A satellite revolving in an orbit has energy  $-17.4 \times 10^9 \text{ J}$  what will be the binding energy

$$\begin{aligned} \text{Soln} \quad B.E. &= -E \\ &= -(-17.4 \times 10^9 \text{ J}) \\ &= 17.4 \times 10^9 \text{ J} \end{aligned}$$

- 3) Find or what is relation b/w escape and orbital velocity of a satellite orbiting near earth's surface.

$$\begin{aligned} \text{Soln} \quad V_e &= \sqrt{gR} \\ V_e &= \sqrt{gR} \end{aligned}$$

$$\frac{V_e}{V_o} = \frac{\sqrt{gR}}{\sqrt{gR}}$$

$$\frac{V_e}{V_o} = \frac{1}{\sqrt{2}}$$

- 3) When the body is taken at null points. Null points are the points where gravitational forces due to diff. masses cancel out.

There will be no gravity. So, it is called weightless condition.

- 4) find the G.P.E b/w 2 masses of 8kg and 50kg placed 10m apart.

$$\text{Soln} \quad P.E. = -\frac{GMm}{r}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}$$

$$M_1 = 8 \text{ kg}$$

$$M_2 = 50 \text{ kg}$$

$$r = 10 \text{ m}$$

$$\text{Potential energy} = qF$$

$$= -q \times 8.1 \times 10^9$$

$$= -17.4 \times 10^9 \text{ J}$$

$$= -6.67 \times 10^{-11} \times 2 \times 8 \times 50$$

$$= -6.67 \times 10^{-10} \text{ J}$$

$$\omega = GMm \left[ \frac{2^{x-2+1}}{-2+1} \right]_R^{\infty}$$

$$\omega = GMm \left[ \frac{x^{-1}}{-1} \right]_R^{\infty}$$

$$= GMm \left[ \frac{-1}{x} \right]_R^{\infty}$$

$$\omega = GMm \left[ -\frac{1}{x} - \left( -\frac{1}{R} \right) \right]$$

$$\omega = GMm \left[ 0 + \frac{1}{R} \right]$$

$$\omega = GMm \left[ R - \left( -\frac{1}{R} \right) \right]$$

$$\omega = GMm \left[ R + \frac{1}{R} \right]$$

$$\omega = GMm \left[ \frac{2R}{R^2 + 1} \right]$$

$$v_e = \sqrt{\frac{GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R^2}}$$

$$v_e = \sqrt{\frac{2gR}{R}}$$

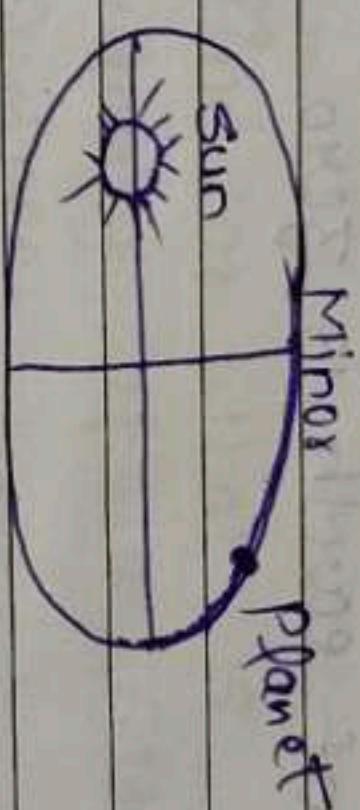
$$v_e = \sqrt{\frac{2g}{R}}$$

$$v_e = \sqrt{\frac{2g}{R}}$$

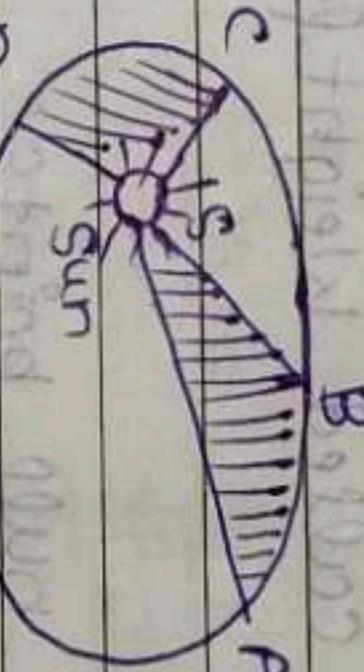
$$v_e = \sqrt{\frac{2g}{R}}$$

### \* Kepler's Laws:

1) 1st Law: According to this law, every planet revolve around sun in an elliptical orbit. This is also called law of orbit.



2) 2nd Law: Acc. to this law, the line joining planet and sun sweeps out equal area in equal interval of time. It is also called law of area.



So, circumferential velocity of a planet revolving around sun is constant.

3) 3rd Law: Acc. to this law, the square of time-period of revolution of planet around Sun is directly proportional to cube of length of semi-major axis.

Let 'T' be the time period and R be the length of semi major axis.

Then,

$$\left[ \frac{T^2}{R^3} \right]$$

$$\left[ \frac{T^2}{R^3} \right]$$

$$\left[ \frac{T^2}{R^3} \right]$$

$$\left[ \frac{T^2}{R^3} \right]$$

$$B.E = -\left[ \frac{-GMm}{2(R+h)} \right]$$

$$\therefore B.E = \frac{GMm}{2(R+h)}$$

$$[B.E = k.e] \text{ or } [B.E = -2P.E]$$

### \* Geostationary Satellite:

The satellite which appears to be at rest or at a fixed location w.r.t. to earth is called Geostationary Satellite.

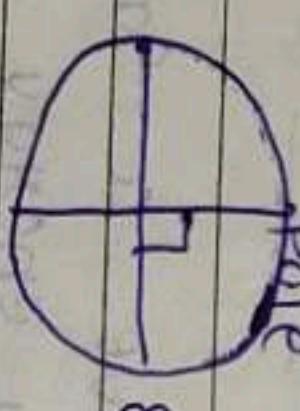
### (Conditions for geostationary Satellite):

- 1) The height of satellite is nearly 36000 km.
- 2) Orbital speed of satellite is nearly 3.1 km/sec
- 3) It's time period is 24 hours.
- 4) It should rotate in Anticlock wise direction such that from west to east.

### \* Polar Satellite:

The satellite which revolve in polar orbits around earth is called polar satellite. It's angle of inclination should be  $90^\circ$  with equatorial plane.

A polar satellite will pass over both north and south pole



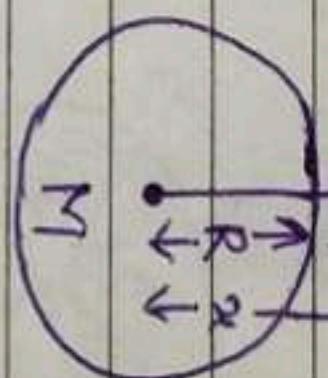
$90^\circ$

### \* Uses of Satellite:

- i) In weather forecasting
- ii) In communication radio, TV, mobile.
- iii) For getting information about atmosphere of earth.
- iv) In study of remote sensing.

### \* Escape Velocity:

The minimum velocity with which a body is thrown upward from the surface of earth so that it overcome the gravitational field and never return to earth surface is called escape velocity. It is denoted by ' $v_e$ '.



Consider earth as a sphere of Mass 'M' and radius 'R'. An obj. of mass 'm' is thrown upward with a velocity ' $v_0$ ' from earth's surface.

Let  $\rho dx$  be the small displacement from earth's surface.

Then workdone will also small i.e.  $-dw = F_o dx$  — (i)

From gravitational law:

$$F_o = \frac{GMm}{x^2}$$

Put this value in eqn (i)

$$dw = GMm \cdot \frac{dx}{x^2}$$

Total workdone will be

$$W = \int_R^{\infty} \frac{1}{x^2} dx = \frac{GMm}{R}$$

$$W = GMm \int_R^{\infty} x^{-2} dx$$