UNIT=5
[ROTATIONAL MOTION]

-: Chapter-1 → [centre of Mass, torque angular mom

-: Rotational motion: If an object moves about a fix point or ~~time~~ line then the motion is called Rotational motion. The fixed line is known as Axis.
Ex- Motion of a fan.

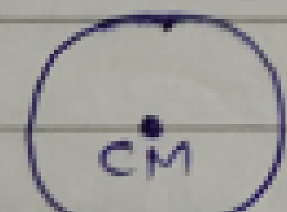
-: System: A system is a group of very large no. of particles. All paticles apply forces on each other.

* Internal force: These are the forces which act b/w particles of a system. Internal forces b/w two particles always act opp. to each other. So, their net effect will be zero.

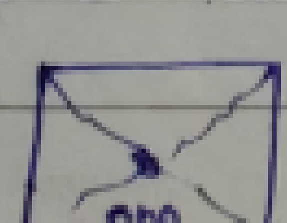
* External force: These are the forces which act from outside on the system. External forces are responsible for motion of object

* Centre of Mass: The point where whole mass of a body is supposed to be concentrated is called centre of mass.

i) The centre of mass lies at the centre of a circle.



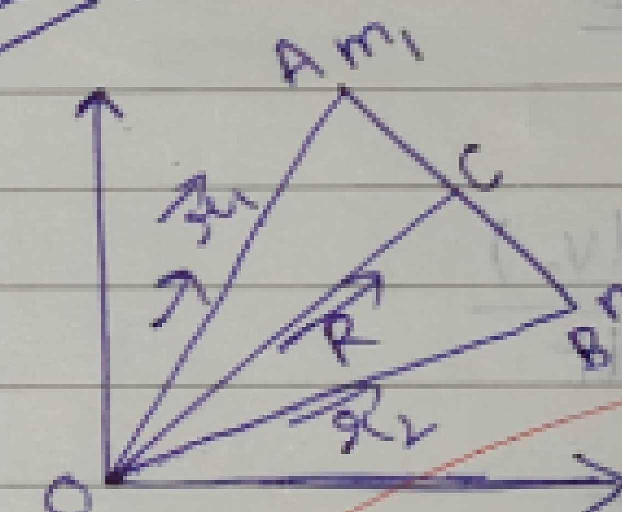
ii) In square and rect. centre of mass lies at the intersection point of it's diagonal.



iii) In Δ, centre of mass lies at centroid.

* Properties of centre of mass.

i) It is the point where whole mass of body is supposed to be concentrated.
ii) It may lie inside the body or outside the body.
iii) For two paticle system, centre of mass will lie line joining them.
iv) For two paticle system, If two masses are equal then centre of mass lies in middle. But, if masses are not equal then centre of mass will shift towards heavier mass.



Consider a system of two paticles having mass m_1 and m_2. Let $\vec{r_1}$ and $\vec{r_2}$ be the position vector of paticles. A and B w.r.t origin

$\vec{R}$ be the position vector of centre of mass

$\vec{f_{12}}$ → force A due to B
$\vec{f_{21}}$ → force on B due to A are internal forces.

$\vec{F_1}$ and $\vec{F_2}$ be the external forces act on A and B resp.
Then P_1 and P_2 be the linear mom. of A and B
Then, force on particle A = $\vec{F_1} + \vec{f_{12}}$
$= \vec{F_2} + \vec{f_{21}}$

Total force on System = $\vec{F_1} + f_{12} + \vec{F_2} + \vec{f_{21}}$
as $\vec{f_{12}} + \vec{f_{21}} = 0$

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Acc. to Newton's 2nd law
$\vec{F_1} = \frac{d\vec{p_1}}{dt}$ and $\vec{F_2} = \frac{d\vec{p_2}}{dt}$

$\Rightarrow \vec{F_1} + \vec{F_2} = \frac{d\vec{p_1}}{dt} + \frac{d\vec{p_2}}{dt}$ — i)

let $\vec{F}$ be the total force on a system
$\vec{F} = \frac{d\vec{p_1}}{dt} + \frac{d\vec{p_2}}{dt}$

as $\vec{p_1} = m_1\vec{v_1}$, $\vec{p_2} = m_2\vec{v_2}$

Where $\vec{v_1}$ and $\vec{v_2}$ be the velocity of particle A and B.

$$\vec{F} = \frac{d(m_1\vec{v_1})}{dt} + \frac{d(m_2 v_2)}{dt}$$

$$= \frac{m_1(d\vec{v_1})}{dt} + \frac{m_2(dv_2)}{dt}$$

as, $\vec{v_1} = \frac{dr_1}{dt}$, $\vec{v_2} = \frac{d\vec{r_2}}{dt}$

Total

$$\vec{F} = m_1 \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) + m_2 \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right)$$

$$\vec{F} = m_1 \frac{d^2\vec{r_1}}{dt^2} + m_2 \frac{d^2\vec{r_2}}{dt^2} \quad \text{— ii)}$$

$$\vec{F} = \frac{d^2}{dt^2}(m_1\vec{r_1} + m_2\vec{r_2})$$

$$\therefore \vec{F} = m\vec{a}$$

Where m is mass of system

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$\Rightarrow * \quad \vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

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From eq (iv) and (v)

$$\vec{R} = \frac{M_1\vec{r_1} + M_2\vec{r_2}}{M_1+M_2}$$

★ If system having 'n' number of particles then

$$\vec{R} = \frac{M_1\vec{r_1} + M_2\vec{r_2} + M_3\vec{r_3} + \cdots + m_n\vec{r_n}}{M_1 + M_2 + M_3 + \cdots + M_n}$$

$$\vec{R} = \sum_{i=1}^{n} \frac{m_i\vec{r_i}}{m_i}$$

⇒ In Case of two dimension motion:-

Let x_1 and x_2 be the position vector of mass m_1 and m_2 along x-axis. y_1 and y_2 be the position vector along y-axis.

'x' and 'y' be the position vector of centre of mass respectively. Then

$$x = \frac{m_1x_1 + m_2x_2}{m_1+m_2}$$

$$Y = \frac{m_1y_1 + m_2y_2}{m_1+m_2}$$

★ In Case of three dimension:-

z_1 and z_2 be the position vector of mass m_1 and m_2 along z-axis respectively.

$$Z = \frac{m_1z_1 + m_2z_2}{m_1+m_2}$$

$\tau_z = x F_y - y F_x$

Special Case:-

i) If $\theta = 90°$, $\vec{r} \perp \vec{F}$

$\tau = rF \sin 90°$

$(\tau = rF]$ - maximum.

ii) If $\theta = 0°, 180°$ then

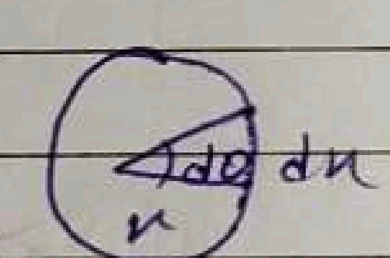
$\tau = rF \sin 0$	$\tau = rF \sin 180°$
$(\tau = 0)$ minimum	$(\tau = 0)$ minimum

★ Dimensional formula of $\vec{\tau} = [M^1 L^2 T^{-2}]$

⇒ Work done by torque:-

Let 'dx' be the small displacement, the work done is also small 'dw'.

$dw = F \cdot dx$ — i)



$\theta = \frac{\text{length of arc}}{\text{Radius}} = \frac{l}{r}$

$d\theta = \frac{dx}{r}$

⇒ $dx = r \cdot d\theta$

Put this value in eq i)

$dw = F \cdot r d\theta$

as, $r \cdot F = \tau$

$dw = \tau d\theta$

for total work done

$W = \int dw = \int \tau d\theta \Rightarrow W = \tau \cdot \theta$

★ Power delivered by torque:-

Let 'P' be the power, 'W' be the workdone.

$P = \frac{W}{t}$

Let 'dw' be the small work done in a small intervals of time dt.

$P = \frac{dw}{dt}$

as we know, $dw = \tau \cdot d\theta$

$P = \frac{\tau d\theta}{dt}$

$P = \tau \cdot \frac{d\theta}{dt}$

$P = \tau \cdot \omega$

where $\omega = \frac{d\theta}{dt}$ is called angular velocity.

Numerical

i) Find the torque of a force $(\hat{i} - 3\hat{j} - 5\hat{k})$ ab. the origin which act on a particle whose position vector is $r = \hat{i} + \hat{j} - \hat{k}$

Sol:- τ (torque) $= \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & -3 & -5 \end{vmatrix}$$

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$[\theta]_0^\theta = \omega_0 [t]_0^t + \alpha\left[\frac{t^2}{2}\right]_0^t$

$[\theta - 0] = \omega_0[t-0] + \alpha\left[\frac{t^2}{2} - \frac{0^2}{2}\right]$

$\theta = \omega_0 t + \alpha\frac{t^2}{2}$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

3> $\omega^2 - \omega_0^2 = 2\alpha\theta$

Proof:-

$\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{d\omega}{dt} \times \frac{d\theta}{d\theta}$

$\alpha = \frac{d\theta}{dt} \times \frac{d\omega}{d\theta}$

as $\frac{d\theta}{dt} = \omega$

$\alpha = \omega \cdot \frac{d\omega}{d\theta}$

$\alpha \cdot d\theta = \omega \cdot d\omega$

Integrating both side

$\int_0^\theta \alpha \cdot d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega$

$\alpha[\theta]_0^\theta = \left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega}$

$\alpha[\theta - 0] = \left[\frac{\omega^2}{2} - \frac{\omega_0^2}{2}\right]$

$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}$

$2\alpha\theta = \omega^2 - \omega_0^2$

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$\omega^2 - \omega_0^2 = 2\alpha\theta$

Torque:- Turning effect of force is called torque.

⇒ It is denoted by τ (tau). It is also known as movement of force.

⇒ It is measured by cross product of force and ⊥ distance from axis of rotation.

$\vec{\tau} = \vec{r} \times \vec{F}$

where $\vec{r}$ = ⊥ distance from axis of rotation.

$\vec{F}$ = Force applied

or, $\tau = rF\sin\theta$

S.I unit of τ = Nm

Let position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{\tau} = \tau_x\hat{i} + \tau_y\hat{j} + \tau_z\hat{k}$

as, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$= \hat{i}(y \cdot F_z - z \cdot F_y) + \hat{j}(z F_x - x F_z) + \hat{k}(x F_y - y F_x)$

Then,

$\tau_x = yF_z - zF_y$

$\tau_y = zF_x - xF_z$

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★ Angular displacement:- It is the displacement travelled by a body along a circular path. It is denoted by "θ"

★ S.I unit is radian.

Relation b/w Linear displacement and angular displacement is given by $\theta = \frac{l}{r}$

where θ = angular displacement
l = length of arc or linear displacement

Angular Velocity:- It is defined as the ratio of angular displacement and time. It is denoted by ω

$$\omega = \frac{d\theta}{dt}$$

Relation b/w linear velocity and angular velocity is given by

$$V = r\omega$$

S.I unit is = $rad\, s^{-1}$.

Angular acceleration:- It is a ratio of of angular velocity to the time. It is denoted by α

$$\alpha = \frac{d\omega}{dt}$$

S.I unit = $rad\, s^{-2}$

Relation b/w linear acc. and angular acc.

$$a = r\alpha$$

Equation of rotational motion:-

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i) $\omega = \omega_0 + \alpha t$

Derivation:-

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha \cdot dt$$

Integrating both sides

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha \cdot dt$$

$$[\omega]_{\omega_0}^{\omega} = \alpha [t]_0^t$$

$$\omega - \omega_0 = \alpha (t - 0)$$

$$\omega - \omega_0 = \alpha t$$

$$\omega = \omega_0 + \alpha t$$

2) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

Proof:

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = \omega \cdot dt \quad \text{— i)}$$

as we know

$$\omega = \omega_0 + \alpha t$$

Put the value of ω in eq i)

$$d\theta = (\omega_0 + \alpha t) \cdot dt$$

Integrating both side

$$\int_0^{\theta} d\theta = \int_0^t (\omega_0 + \alpha t) \cdot dt$$

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

If both particle having same mass i.e $m_1 + m_2 = m$

$$\vec{R} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

$$\vec{R} = \frac{m\vec{r_1} + m\vec{r_2}}{m+m}$$

$$\vec{R} = \frac{m(\vec{r_1} + \vec{r_2})}{2m}$$

$$\vec{R} = \frac{\vec{r_1} + \vec{r_2}}{2}$$

then centre of mass lies b/w the mid point of line joining with the particles.

⇒ Velocity vector of centre of mass:-

Let $\vec{v}$ be the velocity of centre of mass $\vec{v_1}, \vec{v_2}$ ---- $\vec{v_n}$ be the velocity of each particle respectively. $\vec{r_1}, \vec{r_2}$ ---- r_n be the position vector of each particle.

$\vec{R}$ be the position vector of centre of mass

$$\vec{R} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} + \cdots m_n \vec{r_n}}{m_1 + m_2 \cdots m_n}$$

Differentiating both side w.r.t dt

$$\frac{d\vec{R}}{dt} = \frac{m_1 \frac{d\vec{r_1}}{dt} + m_2 \frac{d\vec{r_2}}{dt} + \cdots m_n \frac{d\vec{n}}{dt}}{m}$$

where $m = m_1 + m_2 + m_3 + \cdots m_n$ be the mass of the system.

$$\vec{v} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2} + \cdots m_n \vec{v_n}}{m_1 + m_2 + m_3 \cdots m_n}$$

or $$\vec{v} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2} + \cdots m_n \vec{v_n}}{m}$$

⇒ Acceleration of centre of mass

Let $\vec{a}$ be the acc. produced at the centre of mass $\vec{a_1}, \vec{a_2}$ ---- $\vec{a_n}$ be the acc. of each particle.

$$\vec{v_1} = \frac{m_1 \vec{v_1} + m_2 \vec{v_2} + \cdots m_n \vec{v_n}}{m_1 + m_2 + m_3 \cdots m_n}$$

Now differentiating w.r.t dt

$$\frac{d\vec{v}}{dt} = \frac{m_1 \frac{d\vec{v_1}}{dt} + m_2 \frac{d\vec{v_2}}{dt} + \cdots m_n \frac{d\vec{v_n}}{dt}}{m_1 + m_2 + m_3 + \cdots m_n}$$

$$\vec{a} = \frac{m_1 \vec{a_1} + m_2 \vec{a_2} + \cdots + m_n \vec{a_n}}{m_1 + m_2 + m_3 + \cdots m_n}$$

* **Equilibrium of a rigid body:** A body is said to be equilibrium if both it's linear mom. and angular mom. are conserved.

For translatory equilibrium: Net external force acting on a rigid body is zero, then linear mom. will be constant.

$$f_{ex} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0$$

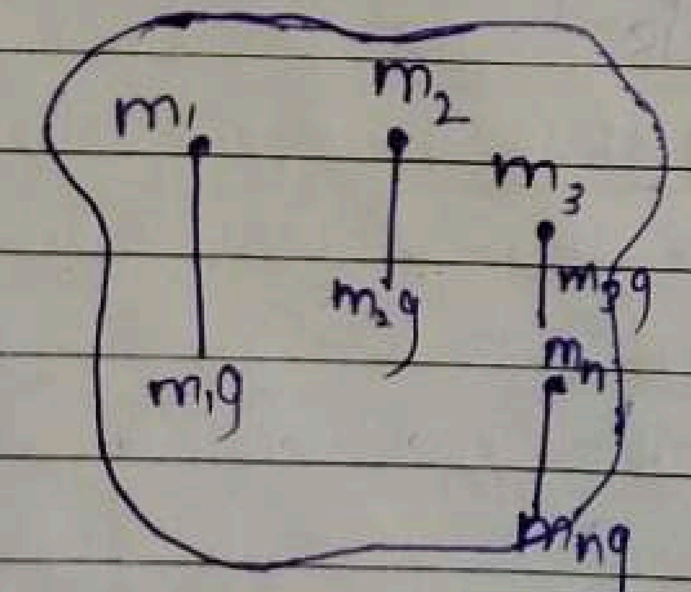
$$\vec{p} = \text{Constant}$$

For rotational Equilibrium: Net external torque acting on a body is zero then, angular mom. will be conserve.

$$\vec{\tau}_{ex} = 0, \quad \frac{d\vec{l}}{dt} = 0$$

$$[\vec{l} = \text{Constant}]$$

* **Centre of Gravity:** A point where the weight of body is supposed to be concentrated is called centre of gravity.



Consider body having n no. of particles let $m_1, m_2, m_3 \ldots\ldots m_n$ be the mass of each particle resp.
Then weight of each particle act in downward direction.
$\vec{W}$ be the weight of body.

$$\vec{W} = m_1\vec{g} + m_2\vec{g} + \cdots m_n\vec{g}$$

$$\vec{W} = (m_1 + m_2 + m_n)\vec{g}$$

as, $m_1 + m_2 + m_3 \cdots\cdots + m_n = M$

Then,

$$\boxed{\vec{W} = M\vec{g}}$$

In terms

* ~~Inertia~~ of mom. of inertia:

1) **Torque:**

let 'τ' be the torque acting on a body

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF\sin\theta$$

For max. value

$$\tau = rF$$

as, $F = ma$

$$\tau = r \times ma$$

Where 'a' is called linear accelerati

as,

$$a = r \cdot \alpha$$

Where α is called angular acc.

$$\tau = r \times mr \cdot \alpha$$

$$= mr^2\alpha$$

as, $mr^2 = I$ is called moment of inertia.

then,

$$\tau = I \cdot \alpha$$

Then,
M.I about radius gyration = Total M.I of all particles

$$MK^2 = m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2$$

let $m_1 = m_2 = m_n = m$

then,

$$MK^2 = mr_1^2 + mr_2^2 + \cdots mr_n^2$$
$$MK^2 = m(r_1^2 + r_2^2 + \cdots r_n^2)$$
$$MK^2 = \frac{m \times n}{n}(r_1^2 + r_2^2 + \cdots r_n^2)$$

as, $M = m \times n$ (Mass of System)

$$\cancel{M}K^2 = \frac{\cancel{M}(r_1^2 + r_2^2 + \cdots r_n^2)}{n}$$

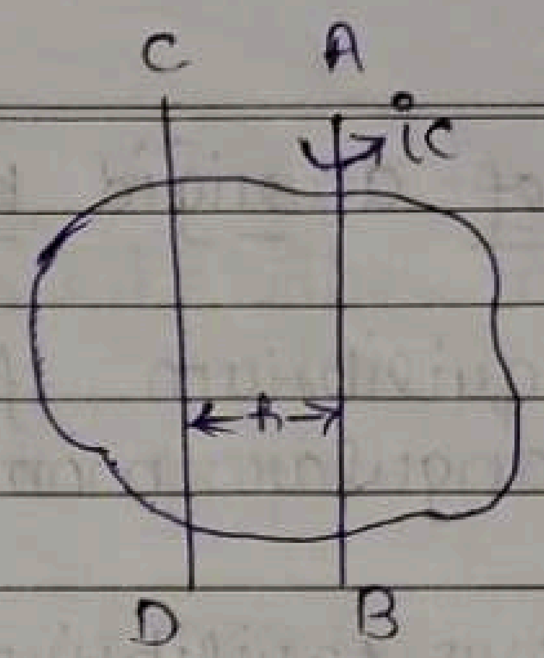
$$K^2 = \frac{r_1^2 + r_2^2 + \cdots r_n^2}{n}$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \cdots r_n^2}{n}}$$

Thus, radius of gyration is the Root mean Sq. dist of particles from axis of rotation.

* **Th^m of ∥ (Parallel) axis:** The moment of inertia of a body about any axis parallel to the axis of centre of mass is equal to the moment of inertia about that axis and sum of product of mass and square of ⊥ dist b/w both axis. i.e..

$$I = I_c + Mh^2$$

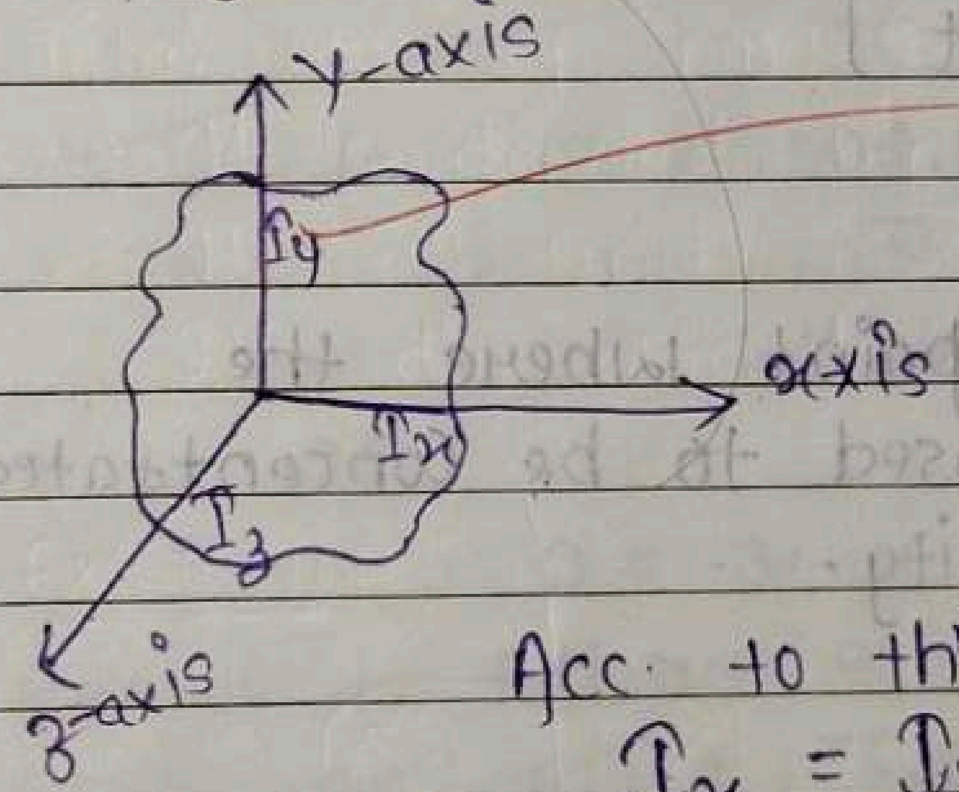


Consider an axis AB passes through the centre of mass of system. 'M' be the mass of system. Another axis CD which is parallel to AB passes through the system. 'h' be the ⊥ dist. between both axis then acc. to th^m of ∥ axis.

$$I_{CD} = I_{AB} + Mh^2$$

or $\boxed{I = I_c + Mh^2}$

* **Th^m of ⊥ axis:** The moment of inertia about an axis will be sum of moment of inertia of two mutually ⊥ axis passing through point of intersection of given axis.



let I_x, I_y and I_z be the moment of inertia about X-axis, Y axis and Z-axis respectively.

Acc. to th^m of ⊥ axis,

$$I_x = I_y + I_z$$
$$I_y = I_z + I_x$$
$$I_z = I_x + I_y$$

* **Law of Conservation of Angular mom.**-

Acc. to this law if external torque acting on a system is zero than angular mom. will be conserve i.e.--

If $\vec{\tau}_{ex} = 0$

then, $\frac{dL}{dt} = 0$

So, $\boxed{\vec{L} = \text{constant}}$

* Find the angular mom. If $\vec{p} = \hat{i} - \hat{j}$, $\vec{r} = \hat{i} + \hat{j} - \hat{k}$

Sol^n $\vec{p} = \hat{i} - \hat{j}$

$\vec{r} = \hat{i} + \hat{j} - \hat{k}$

$\vec{L} = \vec{r} \times \vec{p}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

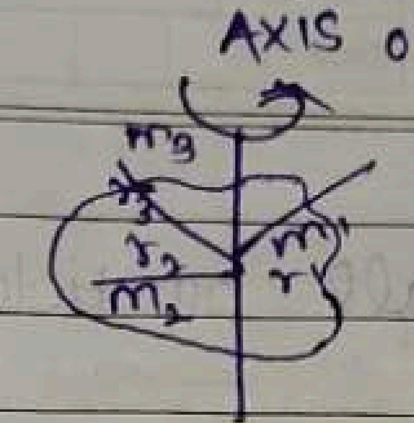
$\hat{i}(-1\mp0) + \hat{j}(-1\mp0) + \hat{k}(-1\mp1)$

$\Rightarrow -\hat{i} - \hat{j} - 2\hat{k}$

* **Moment of Inertia**: Moment of Inertia is the some of product of masses ~~wh~~ with square of dist. from axis of rotation. It is denoted by I.

$I = mr^2$, where m is mass of system r is dist. from axis of rotation

Moment of Inertia in rotational motion plays same role as that mass in linear motion.



Let a system having 'n' no. of particles $m_1, m_2, \dots m_n$ be the mass and $r_1, r_2, \dots r_n$ be the dist. of each particle from axis of rotation.

then,

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 \dots m_nr_n^2$

$I = \sum_{i=1}^{n} m_i r_i^2$

S.I unit is Kgm^2.

Dimensional formula = $[M^1L^2T^0]$

It is scalar quantity

* **Radius of gyration** : (K)

It is the perp. distance from axis for which moment of inertia must be equal to total moment of inertia of body due to particles. Such that,

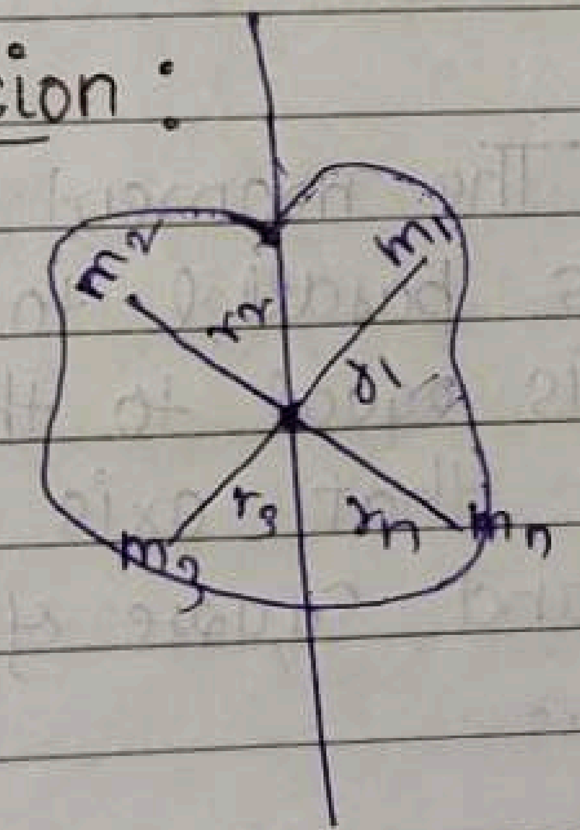
Moment of inertia due to radius of gyration = Total M.I of particles of system.

It is denoted by K

It's S.I unit is m.

It's dimensional formula = $[M^0L^1T^0]$

Expression :



Consider a system of n no. of particles having mass $m_1, m_2 \dots m_n$ and dist. $r_1, r_2 \dots r_n$ respect.

'M' be the mass of system.

'K' be the radius of gyration

$\tau = \hat{i}(-5-3) + \hat{j}(-7+5) + \hat{k}(-3-7)$

$\Rightarrow -8\hat{i} - 2\hat{j} - 10\hat{k}$

* **Angular momentum :** It is called moment of momentum. It is the cross product of position vector ($\vec{r}$) and linear momentum ($\vec{p}$). It is denoted by $\vec{L}$

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = rp\sin\theta$$

* Special cases : i) If $\vec{r} \perp \vec{p}$ then put $\theta = 90°$

$$\vec{L} = rp\sin 90°$$
$$\boxed{\vec{L} = rp} \text{ max.}$$

ii) If $\theta = 0°$, then

$$\vec{L} = rp\sin 0°$$
$$\boxed{\vec{L} = 0} \text{ min.}$$

iii) If $\theta = 180°$ (Antiparallel)

$$\vec{L} = rp\sin 180°$$
$$\boxed{\vec{L} = 0}$$

S.I unit of Angular mom. (L) = kgm^2s^{-1}

Dimensional formula =

$$\vec{L} = L \times MLT^{-1}$$
$$[M^1L^2T^{-1}]$$

If $\vec{L} = L_x\hat{i} + L_y\hat{j} + L_z\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$\hat{i}(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - yp_x)$$

$$\boxed{\begin{aligned} L_x &= yp_z - zp_y \\ L_y &= zp_x - xp_z \\ L_z &= xp_y - yp_x \end{aligned}}$$

* **Relation b/w torque and angular momentum :**

Let $\vec{L}$ be the angular momentum of a body rotating about an axis having position vector $\vec{r}$ and linear mom. ($\vec{p}$)

$$\vec{L} = \vec{r} \times \vec{p}$$

Differentiating w.r.t time both side

$$\frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt}$$
$$= \vec{r} \times \frac{d\vec{p}}{dt} + \vec{p} \times \frac{d\vec{r}}{dt}$$
$$= \vec{r} \times \vec{F} + \vec{p} \times \vec{v} \quad \left[\because \frac{d\vec{p}}{dt} = F, \frac{d\vec{r}}{dt} = \vec{v}\right]$$
$$= \vec{r} \times \vec{F} + 0 \quad [\because \vec{p} \parallel \vec{v}]$$

as, $\vec{r} \times \vec{F} = \vec{\tau}$ (torque)

So,

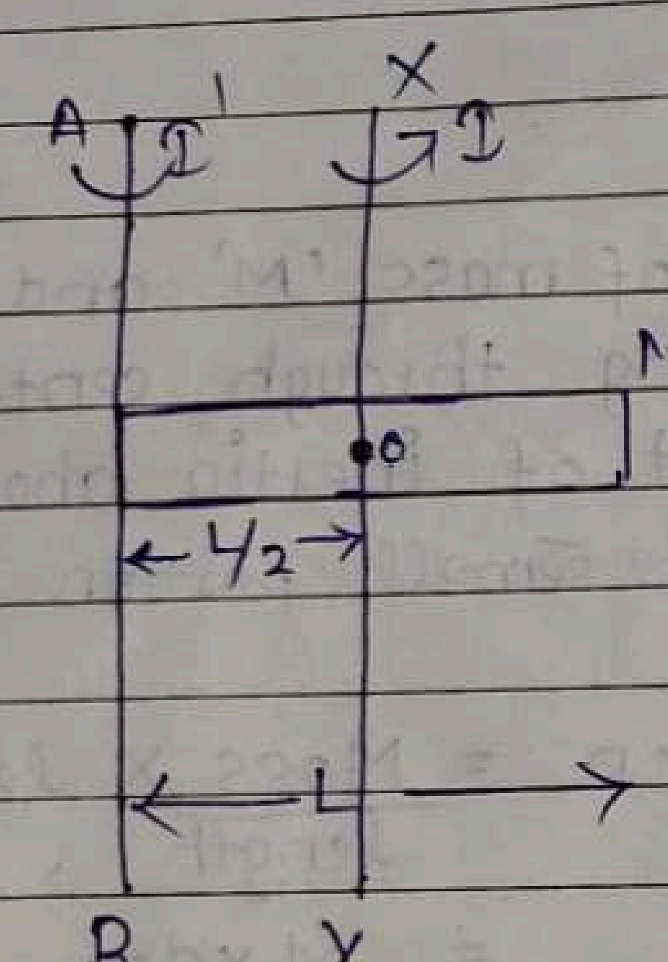
$$\boxed{\vec{\tau} = \frac{dL}{dt}}$$

$$\frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(\frac{-L}{2}\right)^3\right]$$

$$\frac{M}{3L}\left[\frac{L^3}{8} + \frac{L^3}{8}\right] = \frac{M \times \cancel{2}L^3}{3L \times \cancel{8}\,4}$$

$$\boxed{I = \frac{1}{12} ML^2}$$

ii) <u>About an axis passing through one end of rod:</u>



Let AB be any axis passing through one end of rod. I' be the moment of inertia about ~~axis~~ this axis.
As XY and AB are parallel to each other then using th^m of ∥ axis

$$I' = I + M\left(\frac{L}{2}\right)^2$$

as $I = \frac{1}{12} ML^2$

$$I' = \frac{1}{12} ML^2 + \frac{ML^2}{4}$$

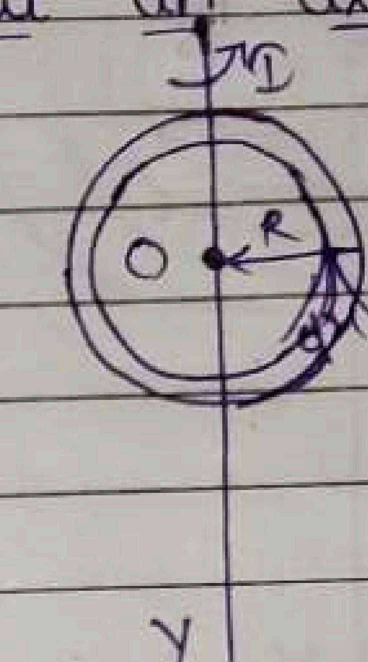
$$= \frac{ML^2 + 3ML^2}{12}$$

$$\Rightarrow \frac{4ML^2}{12\,3} = \frac{ML^2}{3}$$

$$\left(I' = \frac{1}{3} ML^2\right)$$

* <u>Moment of inertia of a circular ring:</u>

i) <u>About an axis passing through centre of ring:</u>



Consider a circular ring of Mass 'M' and having radius 'R'. Let 'O' be the centre of ring. XY be any axis passes through point 'O'. 'I' be the moment of inertia about this axis. 'dx' be the small portion of ring.

Then mass of small portion $= \frac{\text{Mass}}{\text{length}} \times$ length of small portion

$$= \frac{M}{2\pi R} \times dx$$

'dI' be the moment of inertia of small portion then,

$$dI = \frac{M \cdot dx}{2\pi R} \times R^2$$

Total mom of inertia will be

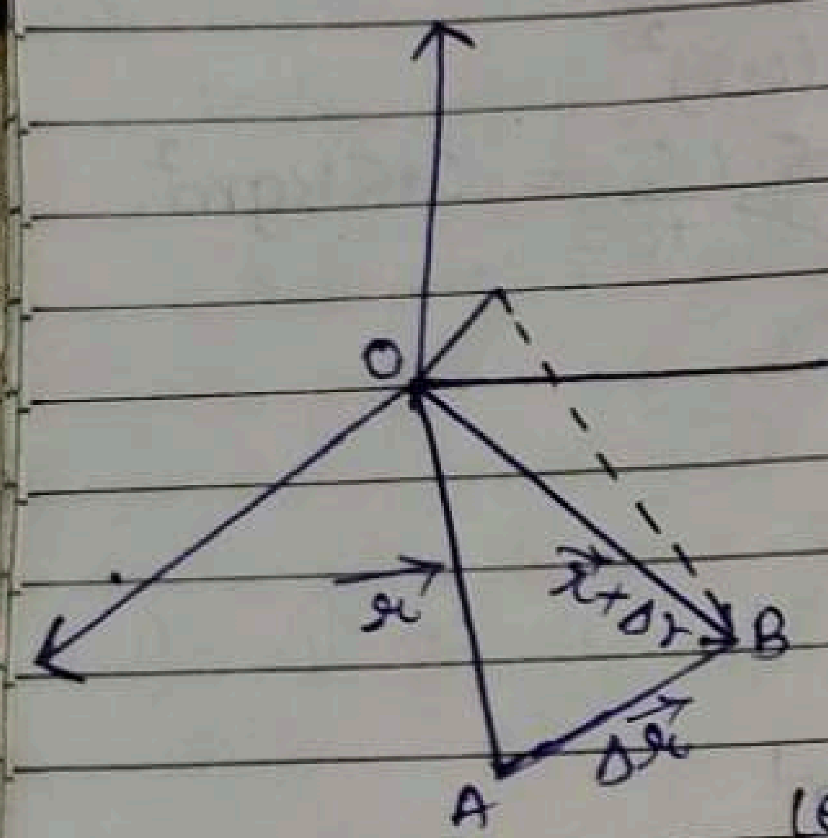
$$I = \int dI = \int_0^{2\pi r} \frac{MR^2\,dx}{2\pi r}$$

$$I = \frac{MR^2}{2\pi r}\int_0^{2\pi r} dx = \frac{MR^2}{2\pi R}\left[x\right]_0^{2\pi R}$$

$$I = \frac{M \times R^2}{2\pi R}\left[2\pi R - 0\right] \qquad I = \frac{M \times R^2 \times \cancel{2\pi R}}{\cancel{2\pi R}}$$

$$\boxed{I = MR^2}$$

* Physical meaning of angular mom. :-



Consider an obj. having position vector '$\vec{r}$' at point A. After small interval of time 'Δt', $\Delta\vec{r}$ be the change in position.

Now in ΔOAB,

Area of $\Delta = \frac{1}{2} \times B \times H$

let $\vec{\Delta A}$ be the area vector of Δ

$$\vec{\Delta A} = \frac{1}{2} \times \vec{r} \times \Delta\vec{r}$$

$\vec{v}$ be the velocity of obj.

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\text{or, } \Delta\vec{r} = \vec{v} \times \Delta\vec{t}$$

$$\Rightarrow \vec{\Delta A} = \frac{1}{2} \times \vec{r} \times \vec{v} \times \Delta t$$

Now, $$\Delta A = \frac{1}{2} \times \vec{r} \times \vec{v} \cdot \Delta t \times \frac{m}{m}$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{m \times \vec{r} \times v}{m}$$

as, $m v r = \vec{L}$ is called angular mom.

$$\frac{\Delta A}{\Delta t} = \frac{\vec{L}}{2m}$$

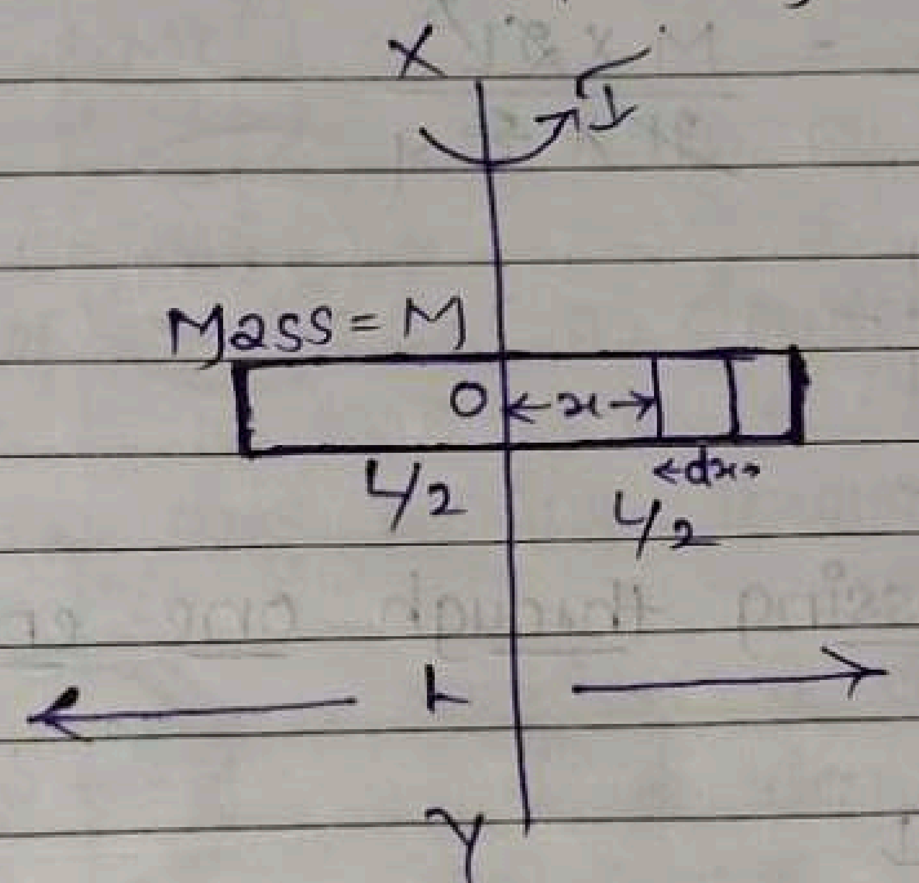
$$\vec{L} = 2m \cdot \frac{\Delta A}{\Delta t}$$

Where $\frac{\vec{\Delta A}}{\Delta t}$ is called areal velocity. So,

angular mom. is equal to twice of product of mass and areal velocity of an object.

* Moment of Inertia of a straight rod :

i) About an axis passing through centre of rod.



Consider a straight rod of mass 'M' and length 'L'. XY be any axis passing through centre O of the rod. I be the Moment of inertia about this axis. 'dx' be the length of small portion

Then,

$$\text{Mass of small portion} = \frac{\text{Mass}}{\text{length}} \times \text{length of small portion}$$

$$= \frac{M \times dx}{L}$$

Moment of Inertia of small portion will be =

$$dI = \frac{M}{L} dx \times x^2$$

Total mom. of inertia of rod will be

$$I = \int dI = \int_{-L/2}^{+L/2} \frac{M}{L} x^2 dx$$

$$I = \frac{M}{L} \int_{-L/2}^{+L/2} x^2 dx$$

$$I = \frac{M}{L} \left(\frac{x^3}{3}\right)_{-L/2}^{+L/2}$$

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increases.

ii) When a cat starts jumping, see compresses her body while during landing on the floor, she streches her body even her tail.

Acc. to conservation of Angular mom.

$$L = \text{Constant}$$
$$I \cdot \omega = \text{constant}$$
$$\omega = \frac{\text{Constant}}{I}$$
$$\text{Thus,} \quad \omega \propto \frac{1}{I}$$

At the start of jump, due to compression her moment of inertia decreases and her speed increases so, that see takes less time to land.

iii) A Diver jumps from board into water, he performs diff. action. at the start of jump he compresses his body, By minimising size Due to this moment of Inertia decreases and his speed increases. While landing he streches his body straight. Due to this moment of Inertia increases and Angular speed decreases. So, he will get less hurt while landing.

1) A mass of 10 kg is rotating in circular path of radius 0.1m with an angular velocity of 44 radian per sec.

1) find M.I of mass.

2) If the radius of path becomes 0.5m finds it new M.I and new angular velocity.

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Soln i) Mr_1^2

$= 10 \times (0.1)^2$

$= \cancel{10} \times \frac{1}{\cancel{10}} \times \frac{1}{10}$

$I_1 = 0.1\ kgm^2$

$r_2 = 0.5$

$I_2 = mr_2^2$

$I_2 = 10 \times (0.5)^2$

$\frac{\cancel{10} \times 5 \times 5}{\cancel{10}\ \ 10^2} = 2.5\ kgm^2$

$$I_1\omega_1 = I_2\omega_2$$
$$\frac{I_1\omega_1}{I_2} = \omega_2$$
$$\frac{0.1 \times 44}{2.5} = \frac{\cancel{44}^{1.76}}{\cancel{25}_1} \Rightarrow 1.76\ rad s^{-1}$$

(2) An ice ~~skatter~~ skater with arms outstreched at 2 round per sec. having moment of Inertia $1.5\ kgm^2$. If the new moment of I. is $0.8\ kgm^2$ then what will be the new rotation speed.

Soln $\nu = 2$ rps

$\omega = 2\pi\nu$

$I_1\omega_1 = I_2\omega_2$

$$\frac{I_1\omega_1}{I_2} = \omega_2$$
$$\frac{1.5 \times \cancel{2\pi} \times 2}{0.8_{\,4}} = \cancel{2\pi}\nu_2$$
$$\frac{15}{4} = \nu_2$$

ii) **Angular mom.** :- Let $\vec{L}$ be the angular mom.

$\vec{L} = \vec{r} \times \vec{p}$

then,

$L = rp$

as, $p = mv$

$L = rmv$

Where v is called linear velocity

as, $v = r\omega$

ω is angular velocity

$L = r \times m \cdot r\omega$

$L = mr^2\omega$

$L = I\omega$

iii) **K.E** :- as K.E is given by

$K.E = \frac{1}{2}mv^2$

Where $v = r\omega$

$K.E = \frac{1}{2}m \cdot r\omega$

$\frac{1}{2} mr^2\omega^2$

as, $I = mr^2$ is called moment of inertia

$K.E = \frac{1}{2}I\omega^2$

* **Kinetic Energy of rolling motion** : K.E of rolling motion is equal to sum of K.E due to translatory motion and rotational motion.

K.E of rolling motion = $K.E_{Translatory} + K.E_{rotational}$

as, $K.E_{translatory} = \frac{1}{2}mv^2$

$K.E_{rotational} = \frac{1}{2}I\omega^2$

Thus,

$K.E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

as, $\omega = \frac{v}{r}$

$K.E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{Iv^2}{r^2}$

as, $I = mk^2$ (~~gneutta~~ terms of centre of mass)

$K.E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mk^2v^2}{r^2}$

$K.E_{rolling} = \frac{1}{2}mv^2\left[1 + \frac{k^2}{r^2}\right]$

* **Applications of Conservation of Angular mom.** →

i) A Ballet Dancer folds her hand to increase her speed of rotation while stretches her hands outward to decrease her rotational speed.

Acc. to conservation of Angular mom.

$\vec{L}$ = Constant

$I\omega$ = constant

$\omega = \frac{\text{constant}}{I}$

Thus, $\omega \propto \frac{1}{I}$

If hands are folded, radius of her turn decreases so, mom. of inertia decreases and angular speed

x) In elastic collision
Ans- Both conserved.

xi) Dimensional formula of force is
Ans- $[MLT^{-2}]$

xii) What does area of velocity - time graph represent
Ans- Displacement

xiii) Inertia depend upon which physical quantity?
Ans- Mass.

xiv) if two vectors ⊥ each other, then
Ans- $\vec{A}\cdot\vec{B} = 0$

Q=2] check the correctness of formula $\frac{1}{2}mv^2 = mgh$ by method of Dimension.
Ans- $\frac{1}{2}mv^2 = mgh$

L.H.S formula = $[M^1L^2T^{-2}]$

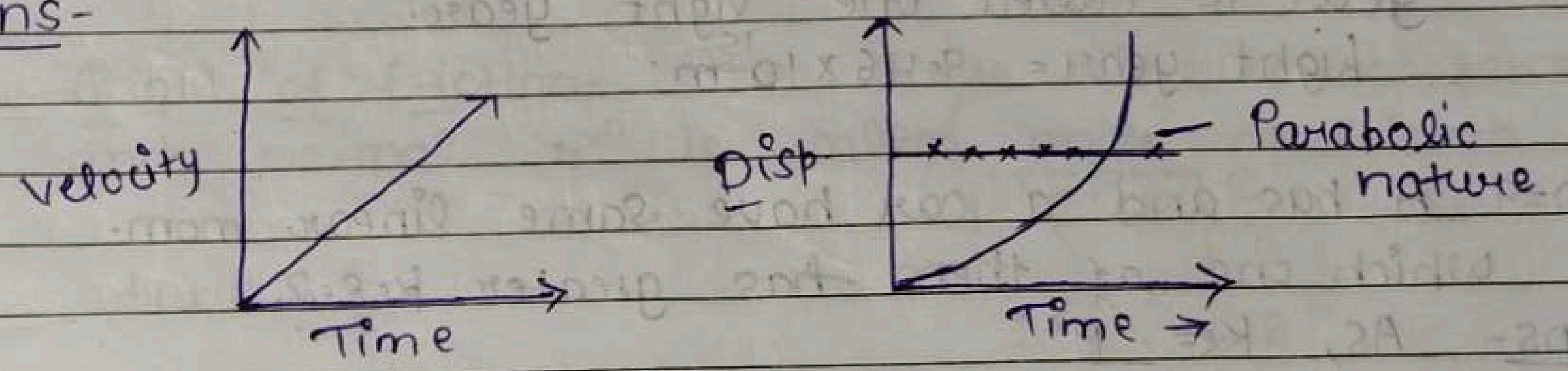
R.H.S formula =
$$[M][LT^{-2}][L]$$
$$\Rightarrow [ML^2T^{-2}]$$

L.H.S = R.H.S

Q=3] A cricter moving his hand backward while holding a catch Why?
Ans- A cricter moving his hand backward while catching because by doing this time of interval increases and force decreases. So, he will get less hurt/harm.

Q=4 Define Gravitational P.E ?
Ans- Energy possessed by body due to change it's position above the surface of earth.
P.E = mgh.

Q=5 An obj moving with constant acc. Draw time - velocity and time - displac. graph for obj
Ans-



Q=6 if a physical quantity $P = \frac{a^3 b^2}{c}$ and % error in measurement of a, b, c 1%, 2% and 3% resp. find max. percentage error in measurement

Ans- $\frac{\Delta P}{P}\times 100 = \frac{\Delta a}{a}\times 100 + \frac{\Delta b}{b}\times 100 + \frac{\Delta c}{c}\times 100$

% error in P = 3x% error in a + 2x% in b + % error in c
$= 3\times 1 + 2\times 2 + 3\times 1$
$= 3+4+3$
$= 10\%$

Q=7] What do you meant by friction? write laws of friction.
Ans- Friction is defined as a force which opposes motion and applied force. Friction always act opp. to the direction of motion.

$$a\left(m+\frac{I}{r^2}\right) = mg\sin\theta$$

$$a = \frac{mg\sin\theta}{m+\frac{I}{r^2}}$$

M.I of cylinder $= I = \frac{1}{2}mr^2$

then,

$$a = \frac{Mg\sin\theta}{m+\frac{\frac{1}{2}m\cancel{r^2}}{\cancel{r^2}}}$$

$$a = \frac{mg\sin\theta}{\frac{3}{2}m}$$

$$a = \frac{2}{3}g\sin\theta$$

Put this value of I and (α) in eq^n ii)

$$f = \frac{\frac{1}{\cancel{2}}m\cancel{r^2}\times\frac{2}{3}g\sin\theta}{r^2}$$

$$f = \frac{1}{3}mg\sin\theta$$

Good

Reaction (R) = $mg\cos\theta$

'μ' be the coff. of friction

$$\mu = \frac{f}{R}$$

$$\mu = \frac{\frac{1}{3}\cancel{mg}\sin\theta}{\cancel{mg}\cos\theta}$$

$$\boxed{\mu = \frac{1}{3}\tan\theta}$$

V.G.

12/9/23

Sept. Exam.

Physics

Q=1

i) Pick only vector quantity in the following

Ans- Impulse.

2) Two forces 5N each act at a point inclined $120°$ with each other. The magnitude of vector addition of these forces is

Ans- 5N.

3) When mom. of body increased by three, it K.E become

Ans- Nine times.

iv) One light year

Ans- 9.5×10^{15} m.

v) Number of significant fig. 4.304 are

Ans- 4.

vi) Find acc. when a mass of 10kg of force 25N is applied on the object.

Ans- $2.5\,ms^{-2}$

vii) What does speedometer of vechile measures

Ans- Instantaneous speed.

viii) The relation b/w linear velocity and angular velocity is

Ans- $V = r\omega$.

ix) What is range of coff. of restitution for inelastic

Ans- $0 < e < 1$.

$$I = \int dI = \int_0^R \frac{2M}{R^2} \cdot x^3 \cdot dx$$

$$I = \frac{2M}{R^2} \int_0^R x^3 dx$$

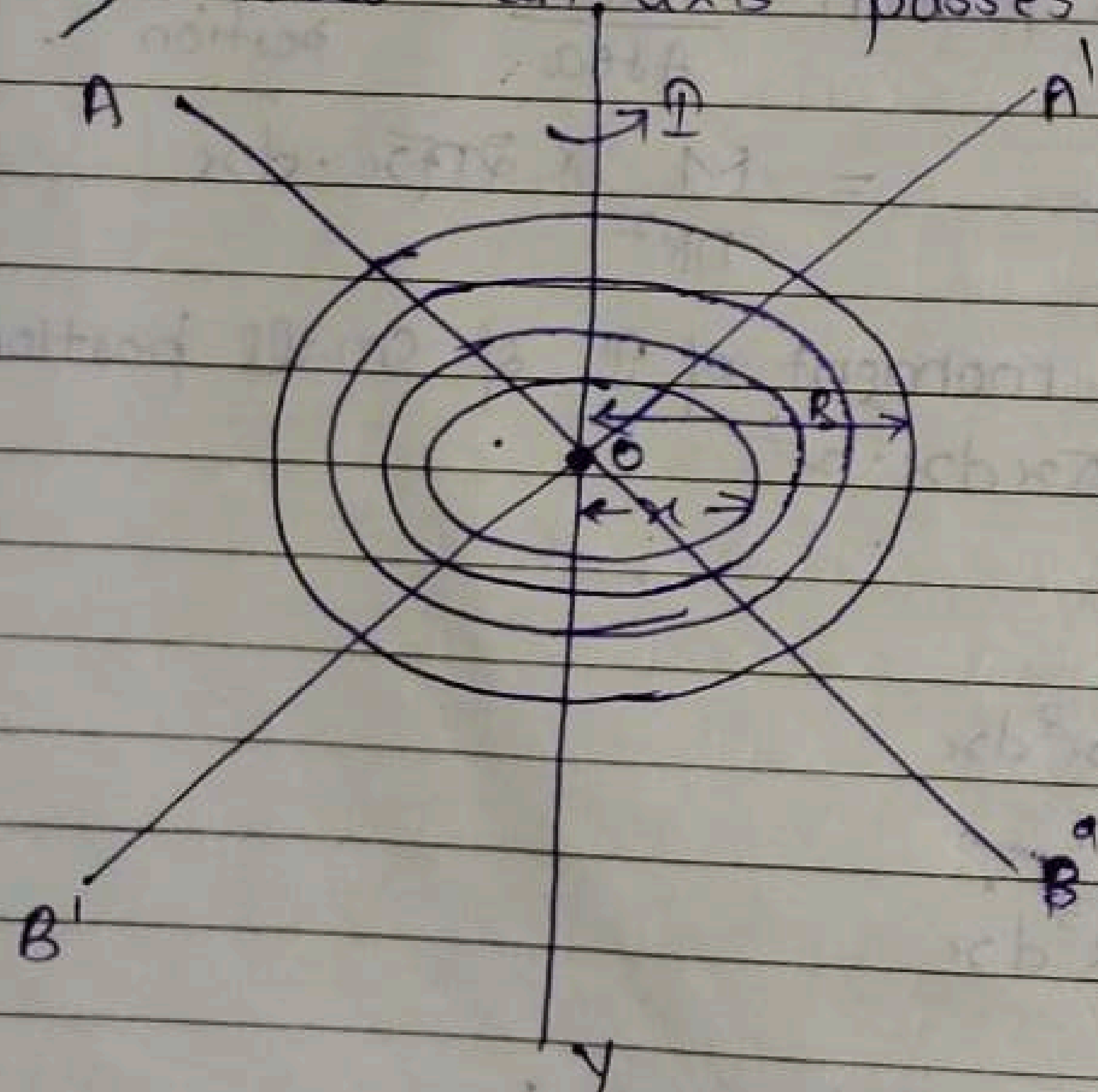
$$I = \frac{2M}{R^2} \left[\frac{x^4}{4}\right]_0^R$$

$$I = \frac{2M}{R^2} \left[\frac{R^4}{4} - \frac{(0)^4}{4}\right]$$

$$= \frac{2M}{R^2} \times \frac{R^4}{4}$$

$$\boxed{I = \frac{1}{2} MR^2}$$

ii) About an axis passes through diameter of disc



let AB and $A'B'$ be any two axis passes through diameter of disc.
'I_d' be the moment of inertia about this axis
Using thm of $\perp$ axis

$$I = I_d + I_d$$

$$2I_d = I$$

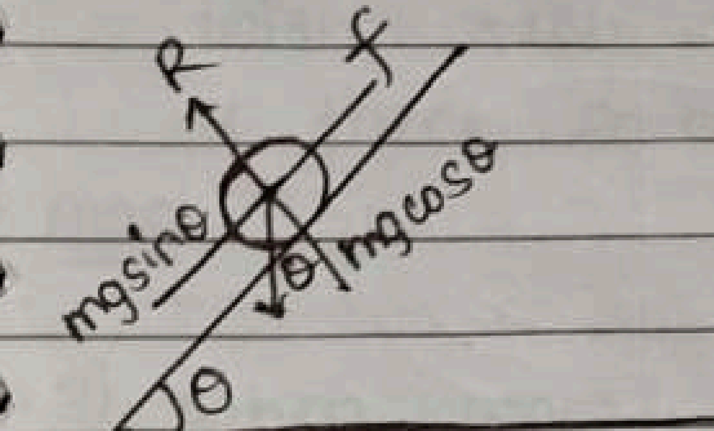
$$I_d = \frac{I}{2}$$

Where, $I = \frac{1}{2} MR^2$

$$I_d = \frac{1}{2} \times \frac{1}{2} MR^2$$

$$\boxed{I_d = \frac{1}{4} MR^2}$$

* Solid cylinder sliding down an inclined plane :



Consider a solid cylinder of mass 'm' and radius 'r' sliding down on an inclined plane. θ be the angle of inclination. weight mg split into two components.
$mg\cos\theta$ — vertically downward
$mg\sin\theta$ — along horizontal
'f' be the force of friction and 'R' be the normal reaction.
Let 'a' be the acc. produced

$$ma = mg\sin\theta - f \quad -(1)$$

let 'τ' be the torque acting on cylinder

$$\tau = r \cdot f$$

I be the moment of inertia
then,

$$r \cdot f = I \cdot \alpha$$

$$f = \frac{I \cdot \alpha}{r} \quad -(2)$$

as,

$$a = r \cdot \alpha$$

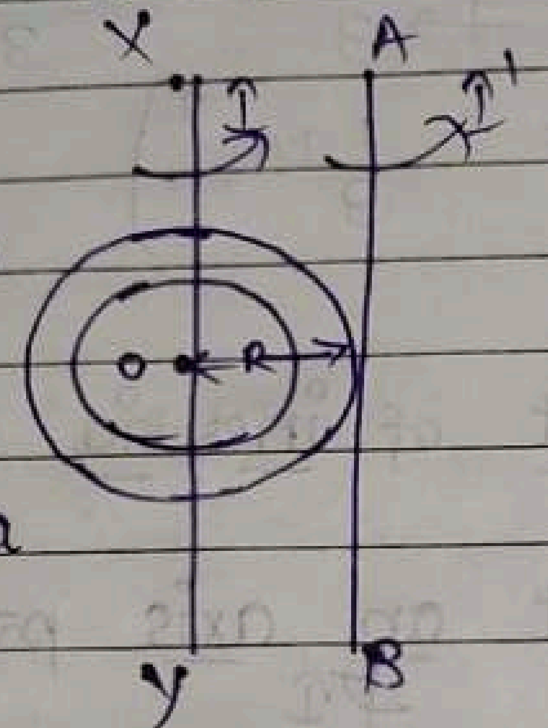
$$\alpha = \frac{a}{r}$$

put this value in eqn (i)

$$ma = mg\sin\theta - \frac{I \cdot a}{r^2} \quad -(3)$$

$$ma + \frac{I a}{r^2} = mg\sin\theta$$

ii) About a tangent axis parllel to the axis about centre of ring :



let AB be any tangent axis parllel to the xy passing through centre of any ring. I' be the moment of Inertia about axis AB.

Using th^m of parllel axis

$I' = I + Mh^2$

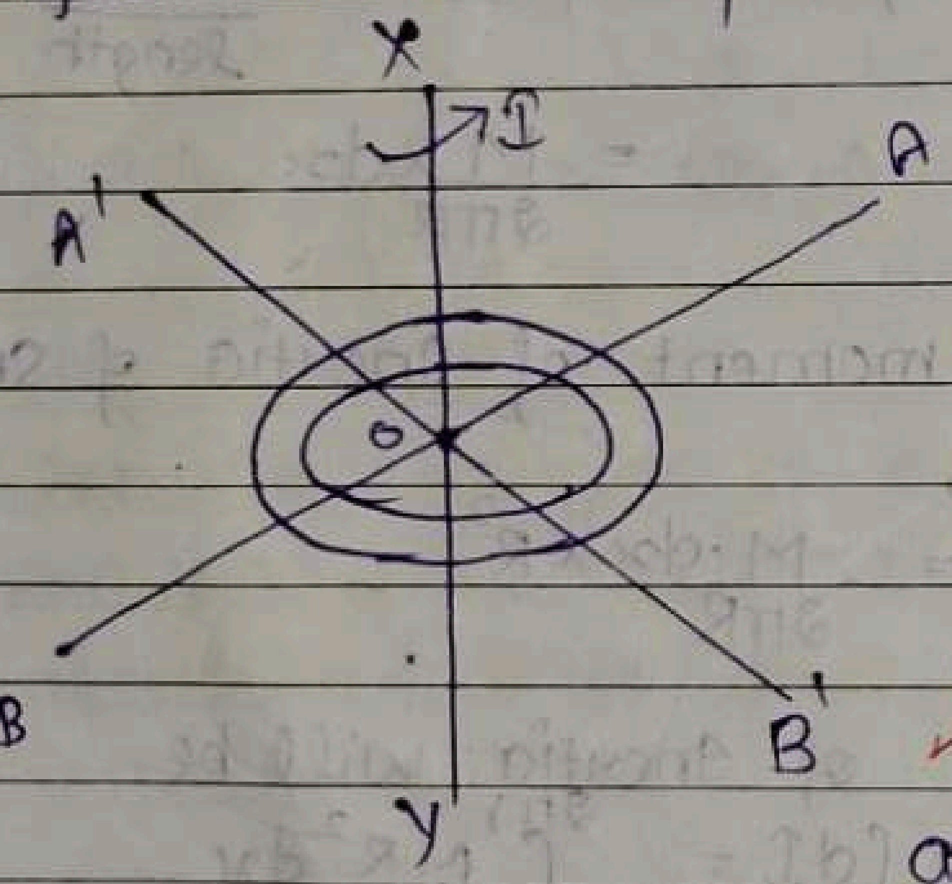
as, $I = MR^2$

$h = R$

So, $I' = MR^2 + MR^2$

$I' = 2MR^2$

iii) About an axis passes through Diameter of ring



let AB and A'B' be two axis passes through diameter of ring. 'I_d' be the moment of Inertia about the axis.

as, axis of diameter is ⊥ to the axis of centre of ring.

Then using th^m of ⊥ axis –

$I = I_d + I_d$

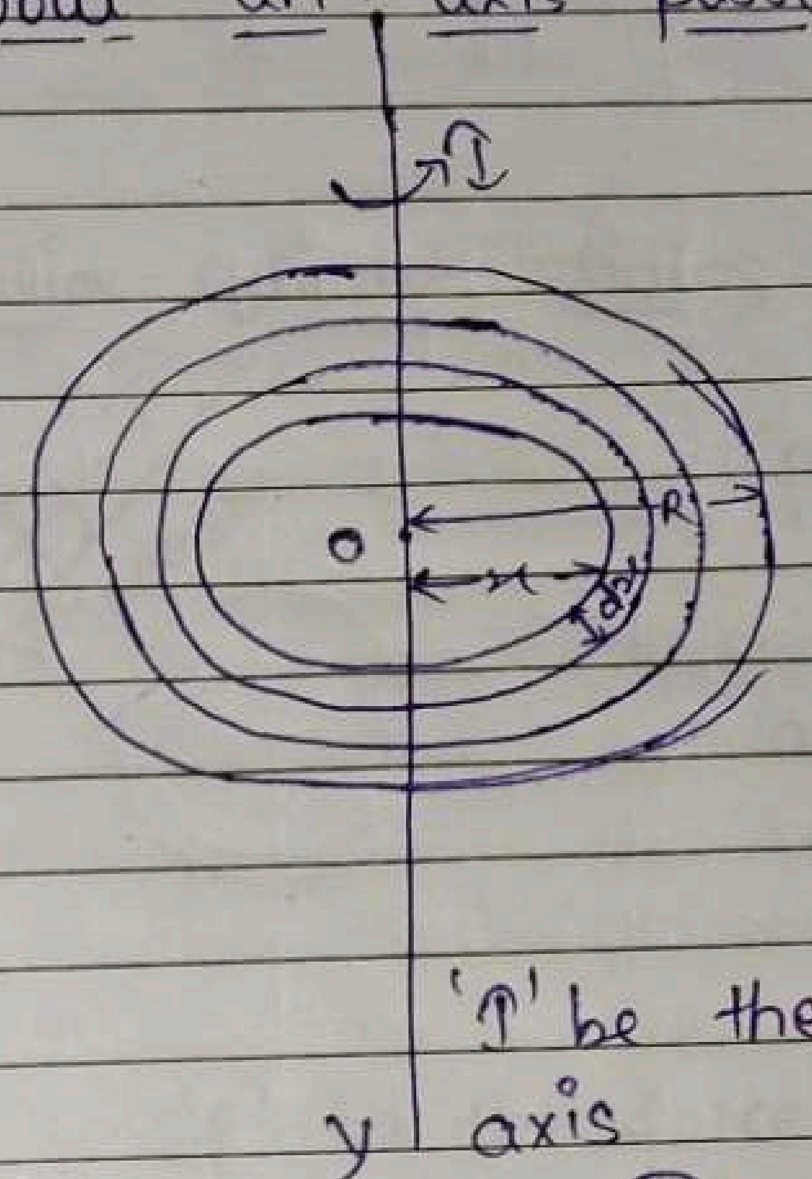
$2I_d = I$

or, $I_d = \frac{I}{2}$

as, $I = MR^2$ → $\boxed{I_d = \frac{1}{2} MR^2}$

* M.I of a circular disc :

1) About an x axis passes through centre of disc.



A circular disc consist of a no. of circular ring. let 'R' be the radius of circular disc and 'M' be the mass. xy be the axis passes through centre 'O' of disc.

'I' be the moment of Inertia about this axis

Now consider a small portion (circular ring) of radius 'x'

Mass of small portion $= \frac{Mass}{Area} \times$ Area of small portion

$= \frac{M}{\pi R^2} \times 2\pi x \cdot dx$

'dI' be the moment of I of small portion

$dI = \frac{M}{\pi R^2} \cdot 2\pi x\, dx \cdot x^2$

$dI = \frac{M \cdot 2\cancel{\pi} x^3 dx}{\cancel{\pi} R^2}$

$dI = \frac{2M}{R^2} \cdot x^3 dx$

Total mom. of Inertia will be

Q=8 Define i) Astronomical unit ii) Light year

Ans- Astronomical unit : The average dist. b/w Sun and earth is called Astronomical unit.

$1\ A.U = 1.496 \times 10^{11}\,m$.

Light year - Dist. travelled by light in one year is called one light year.

Light year $= 9.46 \times 10^{15}\,m$.

Q=9 A bus and a car have same linear mom. which one of them has greater K.E ?

Ans- As, $KE = \frac{p^2}{2m}$

Above the statement, K.E is inversely prop. to mass of the body. But have same linear mom. So, car has greater K.E.

Q=10 State principle of conservation of linear mom.

Ans- If external force acting on a body is zero So, linear mom. will be conserved.

$\vec{P} = \text{constant}$

$f_{ext} = \frac{d\vec{p}}{dt}$

Q=11 State newton's third law?

Ans- Acc. to this law if any action takes place So, it will have opp. reaction.

$\vec{f_{12}} = -\vec{f_{21}}$

28/9/23

Q=12) Establish relation b/w coff of friction and angle of friction?

Ans- Coff of friction -

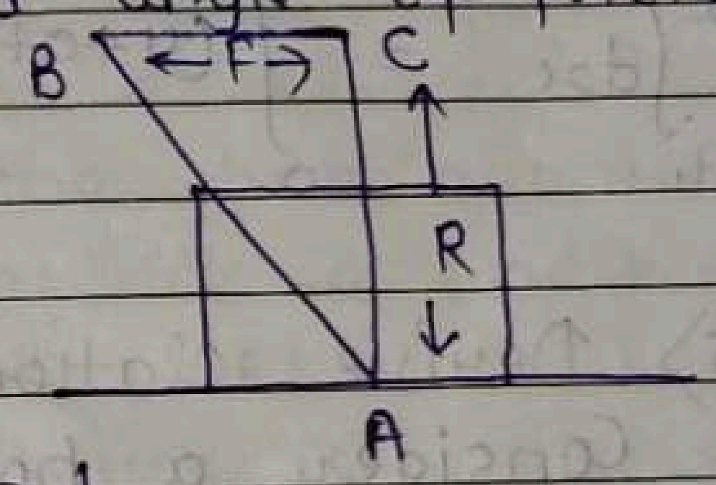
$f \propto R$

$f = \mu R$

f where μ is coff. of friction.

It has no unit and no dimensional formula.

Angle of friction- The angle b/w contact force and normal r^{xn} is called angle of friction



$\tan\theta = \frac{f}{R}$

Where, $\mu = \frac{F}{R}$

then - $\left(\tan\theta = \mu = \frac{f}{R}\right)$

Q=13) Name and define various absolute or gravitional units of force ?

Ans- Absolute units -

S.I unit = $f = ma$

$N = kg\,ms^{-2}$

When 1 kg mass of an obj produced an acc. of $1\,ms^{-2}$ then force will be 1N.

C.G.S -

$F = ma$

$dyne = g \times cms^{-2}$

When 1 g mass of an obj produced an acc. of $1\,cms^{-2}$ then force will be 1 dyne.

Q=14) Obtain eq^n of motion $S = ut + \frac{1}{2}at^2$ by calculus

<u>Ans</u>- Here, $v = \frac{dx}{dt}$

$v \cdot dt = dx$

Integrating both side $x_1 \to x_2$, $0 \to t$

$$\int_{x_1}^{x_2} dx = \int_0^t v \cdot dt \Rightarrow \int_{x_1}^{x_2} dx = \int u\,dt + \int a\,dt^2$$

$$u\int dt + a\int \frac{dt^2}{2}$$

$$\int_{x_1}^{x_2} dx = \int_0^t (u+at) \cdot dt \Rightarrow \boxed{S = ut + \frac{1}{2}at^2}$$

Q=15) Derive relation b/w K.E and linear mom.

<u>Ans</u>- Consider a body of mass 'm' with velocity v, p be the linear mom.

$P = mv$ —(1)

$K.E = \frac{1}{2}mv^2$ —(2)

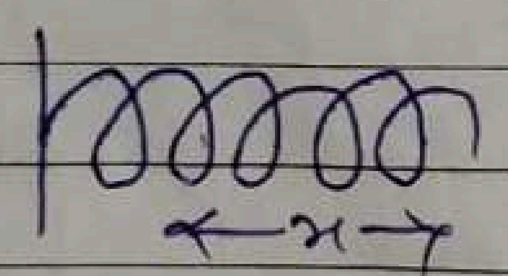
Divide & Multiply m on ~~bo~~ RHS

$$K.E = \frac{1}{2}mv^2 \times \frac{m}{m} \Rightarrow \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{(mv)^2}{m}$$

$$K.E \Rightarrow \frac{p^2}{2m}$$

Q=16) Derive an expression for elastic potential of Spring.

<u>Ans</u>-



Let a variable force acting on a spring. 'x' be the disp. Energy is stored as ~~kinetic~~ potential Energy of spring. 'dx' be the small disp. 'dw' be the small workdone.

$d\omega = \vec{F}dx$

as, $F = -Kx$

$d\omega = -(-Kx)dx$

$d\omega = Kx\,dx$

integrating for total ~~mass~~ work done

$\int d\omega = K\int \frac{x^2}{2}$

$$\boxed{\omega = \frac{1}{2}Kx^2}$$

Q=17] Write a short note note on propagation of Subtraction.

<u>Ans</u>- Let a and b be the two quantities $\pm\Delta a, \pm\Delta b$ be the error in a and b. $\pm\Delta x$ be the error in x

$x \pm \Delta x = a \pm \Delta a - b \pm \Delta b$

$x \pm \Delta x = a - b\ (\mp\Delta a \pm \Delta b)$

$\cancel{x} \pm \Delta x = \cancel{x}\ (\mp\Delta a \mp \Delta b)$

$\pm\Delta x = \mp\Delta a \mp \Delta b$

There are four possible quantities

$+\Delta a + \Delta b = \Delta x$

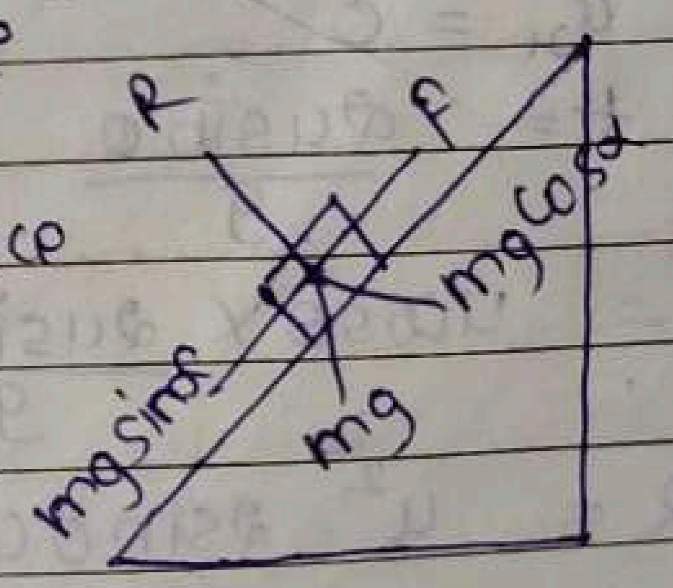
$-\Delta a - \Delta b = \Delta x$

$+\Delta a - \Delta b = \Delta x$

$-\Delta a + \Delta b = \Delta x$

Q=18] What do you meant by angle of Repose?

<u>Ans</u>- It is the minimum angle of inclination when placed on a horizontal surface Just begin to slide down. 'α' be the angle of Repose. Here weight mg split into two Component



$mg\sin\alpha - F$ —(1)
$mg\cos\alpha - R$ —(2)
Divide (1) and (2)

$$\frac{mg\sin\alpha}{mg\cos\alpha} = \frac{F}{R}$$

$$\tan\alpha = \frac{F}{R}$$

as, $\alpha = \theta$
then $\tan\theta = \frac{F}{R}$

$$\theta = \tan^{-1}\left(\frac{F}{R}\right)$$

Q=19 A projectile is thrown at an angle θ from horizontal with velocity u under the gravitational field. find max. height and horizontal range

Ans- Horizontal Range is total dist. covered by projectile along x-axis during Journey.
As, $S = ut + \frac{1}{2}at^2$

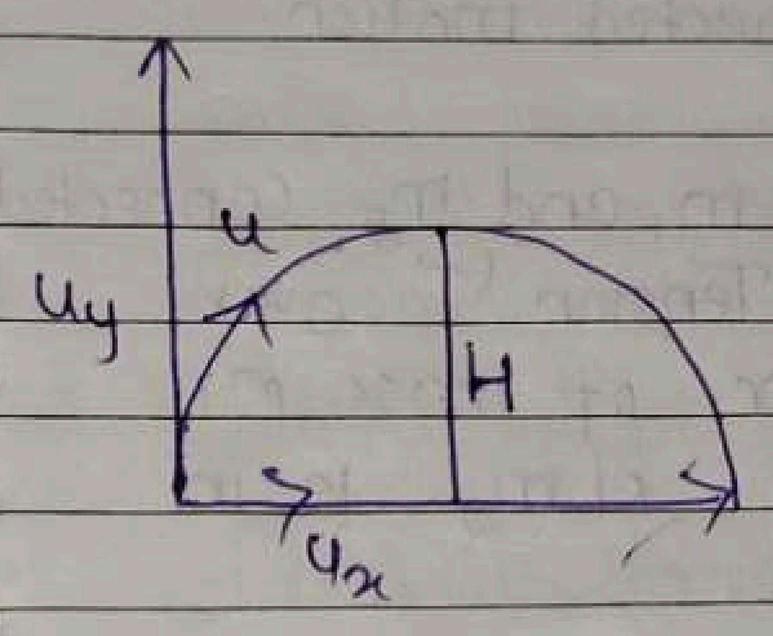
$S = R,\ u = u_x,\ a = a_x$
$R = u_x \cdot t + \frac{1}{2}a_x \cdot t^2$

$u_x = u\cos\theta$
$a_x = 0$
$t = \frac{2u\sin\theta}{g}$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} + \frac{1}{2}\times 0 \times \frac{2u\sin\theta}{g}$$

$$R = u^2 \cdot \frac{2\sin\theta\cos\theta}{g}$$

Max. height



As, $S = ut + \frac{1}{2}at^2$
$S = H,\ u = u_y\ \&\ a = a_y$

$H = u_y \cdot t + \frac{1}{2}a_y t^2$
$u_y = u\sin\theta,\ a_y = -g,\ t = \frac{u\sin\theta}{g}$

$$H = u\sin\theta \times \frac{u\sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$$

$$\frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \frac{2u^2\sin^2\theta - u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{g}$$

20] Work energy th^m by calculas method.

Ans- $dW = f \cdot dx$
$dW = ma \cdot dx$
$= \frac{m\,dv\,dx}{dt}$

$m\,dv \cdot \frac{dx}{dt}$

$dW = dv \cdot mv$
Integrating $u - v$

$$\int dW = \int_u^v mv\,dv$$

$$\int dW = \left[\frac{v^2}{2}\right]_u^v$$

$$W = m\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Q=21] Derive the exp. for tension (T) and acc. (a) produced in string in connected motion.

Ans- Let us consider two masses m_1 and m_2 connected with a string having ~~mass~~ Tension 'T' over a pulley. let $m_2 > m_1$ then acc. of mass m_1 in upward direct. while acc. of m_2 is in downward.

$m_1 a = T - m_1 g$ — (1)

$m_2 a = m_2 g - T$ — (2)

Add (1) and (2)

$m_1 a + m_2 a = T - m_1 g + m_2 g - T$

$(m_1 + m_2) a = m_2 g - m_1 g$

$a = \frac{(m_2 - m_1) g}{m_1 + m_2}$

Divide eq^n (1) by (2)

$\frac{m_1 a}{m_2 a} \times \frac{T - m_1 g}{m_2 g - T}$

$m_1 m_2 a g - m_1 a T = m_2 a T - m_1 m_2 a g$

$2 m_1 m_2 a g = a T (m_1 + m_2)$

$T = \frac{2 m_1 m_2 g}{m_1 + m_2}$

✓. Good

30/9/23

-:[Unit ~ 6 Gravitation]:-

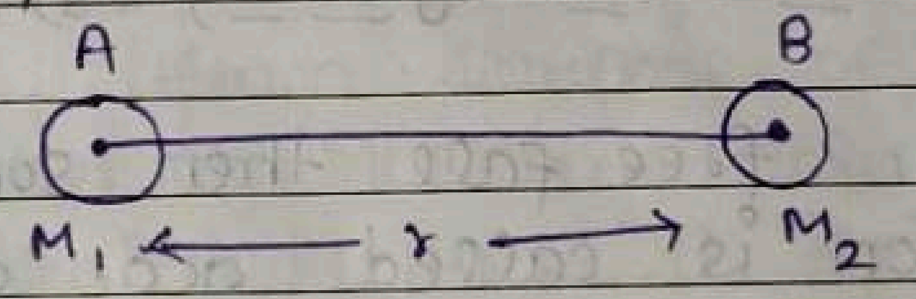
* Gravitational force:

It is the attractive force which come in any two bodies due to their masses is called gravitational force.

* Newton's law of gravitation:

Acc. to this law gravitational force b/w two mass bodies is directly prop. to product of their masses and inversly prop. to square of distance b/w their centres.

Explanation →



Let us consider two bodies A and B having Mass M_1 and M_2 respect. 'r' be the dist. b/w their centres. Then acc. due to gravitational law of force is:

i) Directly prop. to product of their masses i.e

$f \propto m_1 m_2$ — (1)

ii) Inversly prop. to square of distance b/w their centres i.e

$f \propto \frac{1}{r^2}$ — (2)

-: $\boxed{G = 6.67 \times 10^{-11} Nm^2/Kg^2}$:-

From (1) and (2)

$f = \frac{G m_1 m_2}{r^2}$

Where G is Gravitational constant ↑