*** Rule II** -

In multiplication/ Division the final result have minimum no. of significant figure or number. → 2.342×2.1

$\Rightarrow 4.9182$

$\Rightarrow 4.918 \rightarrow 4.92$

$\Rightarrow \underline{4.9}$

Q = 1) The mass of a box measured is 2.3 kg. Two gold peices of mass 20.15g and 20.17g are added to the box

i) what is total Mass of box?

ii) what is the diff. of mass in gold upto significan figure.

→ Total Mass = 0.02015
0.02017
+ 2.3
2.34032

V. Good
Grade
8/5/23

Unit - 2
Kinematics

Chapter-1 (Motion and vector)

*** Motion** :-

A body is said to be in motion if its position changes continuosly with respect to a referance.

*** Refrance** :-

A refrence is a thing with respect to which a body is in motion or in rest. Refrence is taken for comparision.

*** Types of motion** :-

There are three types of motion.

i) **Rectilinear or Translatory motion** :-

If a body travels along a straight line then the motion is called Rectilinear motion.

ii) **Circular or Rotational motion** :-

If a body travels along a circular path, then the motion is called circular motion.
If a body moves about a fix point or line then the motion is called rotational motion.
The fix point or line is called axis.
For example- Motion of a fan.

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iii) Vibrating and Oscillatory motion ÷

The to and fro motion is called Vibratory or osscillatory motion. For ex – Motion of pendulam.

* Point object ÷

If an object covers much larger distance as compare to its size, then the object is said to be point object.

* Scalar ÷

Scalar are those quantities which have magnitude only. For ex – Speed, distance.

* Vector ÷

Vector are those quantities which have both magnitude and direction. For ex – Velocity, displacement

Vector can be represented as $\vec{V}$.

* Types of vector ÷

1) Position vector ÷ The vector which represent the position of a body with respect to origin is called position vector.

The position is represented by x, y, z

Position vector $(\vec{A}) = \underbrace{x}_{\text{Magnitude along x-axis}}\underbrace{\hat{i}}_{\text{Direction.}} + y\hat{j} + z\hat{k}$

Thus Coordinate of A will be (x, y, z)

Magnitude of a given vector ÷

$$(\vec{A}) = \sqrt{(\text{cofficient } \hat{i})^2 + (\text{co. } \hat{j})^2 + (\text{coff } \hat{k})^2}$$

$$(\vec{A}) = \sqrt{x^2 + y^2 + z^2}$$

for example –

$$\vec{A} = 2\hat{i} + 2\hat{j} + -3\hat{k}$$

$$(\vec{A}) = \sqrt{(2)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{4+4+9}$$

$$(\vec{A}) = \sqrt{17}$$

* Unit vector ÷

A vector whose magnitude is one unit is called unit vector. It is used to describe the direction of a given vector. It is denoted by $\hat{A}$ (A cap)

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\text{vector}}{\text{Magnitude}}$$

For ex – $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

* Equal vector ÷

Two vector are said to be equal If they both have same magnitude and direction.

For example – $\vec{A}$ $\vec{B}$ $|\vec{A}| = |\vec{B}| = 5m$

* Unequal vector ÷

Two vector are said to be unequal vector if either their magnitude or direction are not same.

For example ÷ $\vec{A}$ $|\vec{A}| = 5m$

$\vec{B}$ $|\vec{B}| = 7m$

* opposite vector ÷

Two vectors are said to be opposite vector if they have same length but acting in opposite direction.

For example ÷

$\vec{A} \rightarrow$ = $|\vec{A}| = |\vec{B}| = 7m$

$\leftarrow \vec{B}$

* Null vector | zero vector ÷

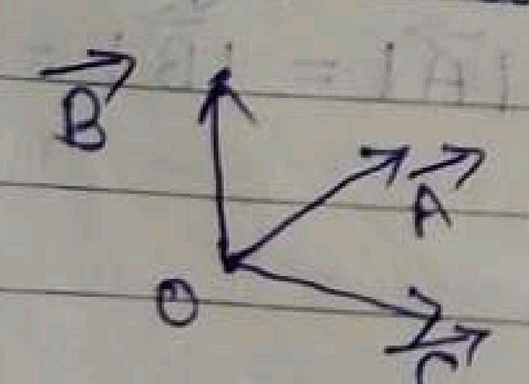
A vector whose magnitude is zero ~~vector~~ is called zero vector. It is denoted by $\vec{O}$

If two opposite vector are added with each other then their resultant will be zero vector.

* Co- initial vector ÷

Co- Common initial point.

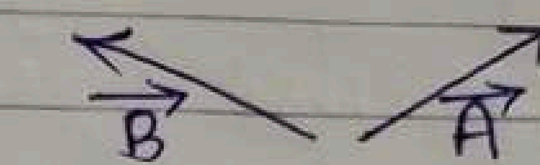
The vector who has common initial point are called co- initial vector.



$\vec{OA}, \vec{OB}, \vec{OC}$ are co- initial vector.

* Co- planor vector ÷

The vector which are in same plane are called co- planar vector.

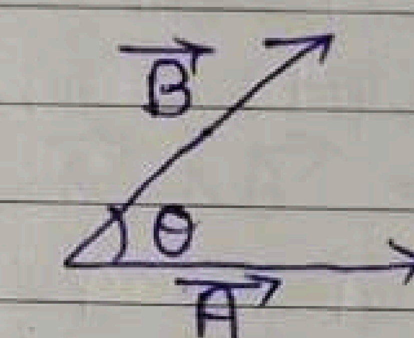


* Axial or polar vector ÷

The vector whose direction is along axis of rotation is called axial vector.

* Multiplication of two vectors ÷

i) Dot product (scalar product)



$\vec{A} \cdot \vec{B} = AB\cos\theta$

special case,

(a) when $\theta = 0$

$\boxed{\vec{A} \cdot \vec{B} = AB}$

$\Rightarrow \hat{i} \cdot \hat{i} = 1$

$\hat{j} \cdot \hat{j} = 1$

$\hat{k} \cdot \hat{k} = 1$

(b) where $\theta = 90°$

$\vec{A} \cdot \vec{B} = AB\cos 90°$

$\vec{A} \cdot \vec{B} = 0$

$\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{k} = 0$

$\hat{k} \cdot \hat{i} = 0$

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j} - \hat{k})(\hat{i} + \hat{j} + \hat{k})$

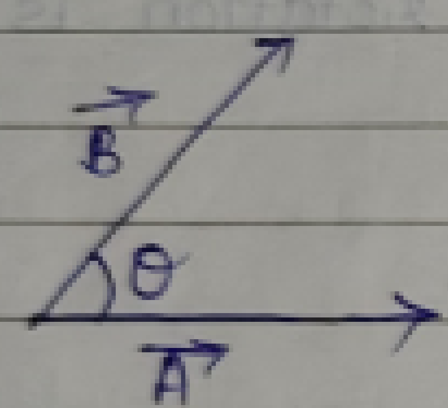
$= 2 \times 1(\hat{i} \cdot \hat{i}) + 3 \times 1(\hat{j} \cdot \hat{j}) + (-1)(1)\, \hat{k} \cdot \hat{k}$

$= 2 + 3 - 1$

$5 - 1 \Rightarrow \boxed{4}$

$$\vec{B}\cdot\vec{A} = (\hat{i}+\hat{j}+\hat{k})\,(2\hat{i}+3\hat{j}-\hat{k})$$
$$= 1\times2 + 1\times3 + 1\times(-1)$$
$$= 2+3-1$$
$$= 5-1$$
$$= 4$$
$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

ii) Cross product (vector product)



$$\vec{A}\times\vec{B} = AB\sin\theta$$

Special case

(a) When $\theta = 0$

$$\vec{A}\times\vec{B} = AB\sin\theta$$
$$\vec{A}\times\vec{B} = 0$$

$$\hat{i}\times\hat{i} = 0$$
$$\hat{j}\times\hat{j} = 0$$
$$\hat{k}\times\hat{k} = 0$$

(b)

When $\theta = 90^\circ$

$$\vec{A}\times\vec{B} = AB\sin\theta$$
$$\vec{A}\times\vec{B} = AB$$

$\hat{i}\times\hat{j} = \hat{k}$	$\hat{j}\times\hat{i} = -\hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$	$\hat{k}\times\hat{j} = -\hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$	$\hat{i}\times\hat{k} = -\hat{j}$

let $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{A}\times\vec{B}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$= \hat{i}\,(y_1z_2 - y_2z_1) + \hat{j}\,(x_2z_1 - x_1z_2) + \hat{k}\,(x_1y_2 - x_2y_1)$$

$$\vec{A} = 5\hat{i} - \hat{j} + 7\hat{k}$$
$$\vec{B} = \hat{i} - \hat{j} - \hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & -1 & -1 \end{vmatrix}$$

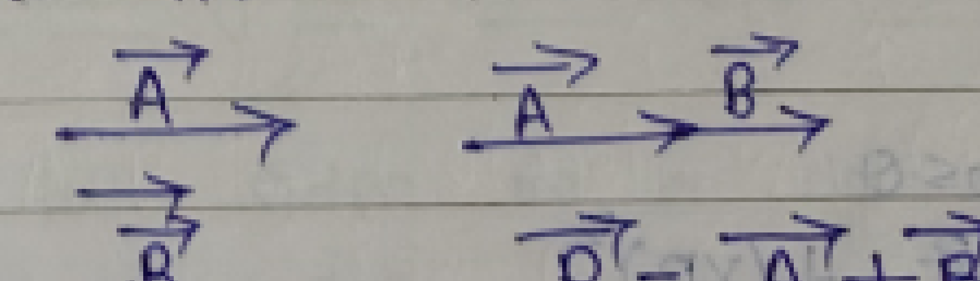
$$\hat{i}\,(1+7) + \hat{j}\,(7+5) + \hat{k}\,(-5+1)$$
$$\vec{A}\times\vec{B} = 8\hat{i} + 12\hat{j} - 4\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(8)^2+(12)^2+(4)^2}$$
$$= \sqrt{224}$$

Resultant Vector of two ~~factor~~ vectors :

A vector will said to be resultant vector if it produces same effect as produced by individual ~~factor~~ vector together. It is represented by $\vec{R}$.

i) When two vectors are in same direction.



$$\vec{R} = \vec{A} + \vec{B}$$

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* **Uniform Speed** :- A body is said to be uniform if it cover equal distance in equal intervals of time.

* **Constant Speed** :- A body is said to be moving with constant speed if it's does not change with time.

* **Non-Uniform Speed** :- A body is said to be moving with non-uniform speed if it cover equal distance with unequal intervals of time.

* **Average Speed** :- Average Speed is that constant speed with which a body cover equal distance as it covers with variable (Non-uniform) speed.
Or
It is the ratio of total distance to the total time taken.

* **Special case** :-

i) If a body covers different distance with diff. intervals of time then
Let S_1 and S_2 -- S_n be the diff. distance cover in time t_1 and t_2 ---- t_n respectively then,

$$\text{Average Speed} = \frac{S_1+S_2+S_3 \text{----} S_n}{t_1+t_2+\text{----}t_n}$$

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ii) If a body cover diff. distance with diff. speed then,
Let S_1 and S_2 ---- S_n be the distance covered with diff. speed V_1 and V_2 ---- V_n then,

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

t_1 and t_2 ---- t_n be the time taken

$$\text{Average Speed} = \frac{S_1+S_2+\text{----}S_n}{\frac{S_1}{V_1}+\frac{S_2}{V_2}\text{----}\frac{S_n}{V_n}}$$

iii) If a body covers two equal distance $S_1=S_2$ with diff. speed V_1 and V_2 then

$$\text{Average Speed} = \frac{2V_1V_2}{V_1+V_2}$$

iv) If a body covers three equal distance $S_1=S_2=S_3$ with diff speed $V_1=V_2=V_3$

$$\text{Average Speed} = \frac{3V_1V_2V_3}{V_1V_2+V_2V_3+V_3V_1}$$

v) If a body moves with diff. speed in diff. time
let V_1, V_2 ---- V_n be the diff. speed

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total time}}$$

$S_1 = V_1t_1$, $S_2 = V_2t_2$ ---- $S_n = V_n \cdot t_n$

$$\text{Average Speed} = \frac{V_1t_1+V_2t_2 \text{----} + V_nt_n}{t_1+t_2\text{----}t_n}$$

vi) If a body move with two diff. speed in two

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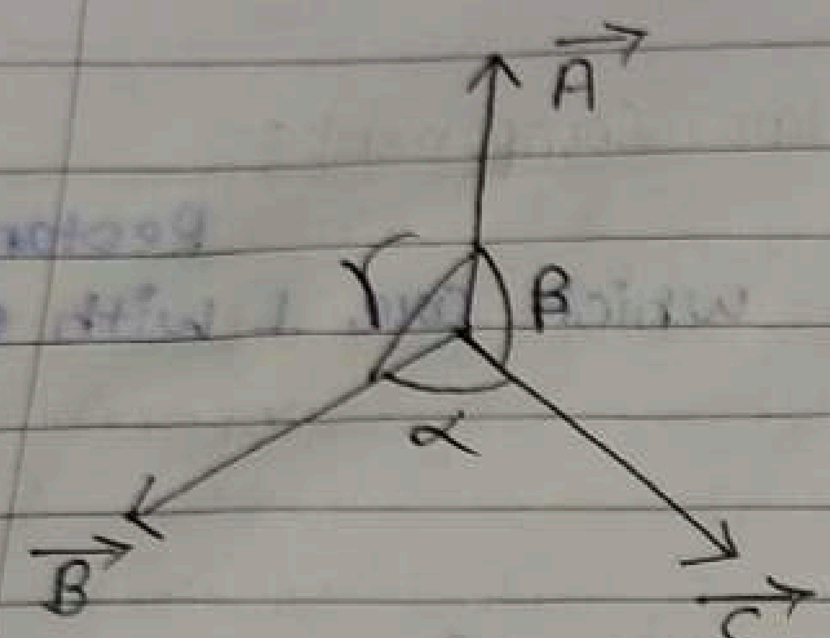


Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three balance forces α, β and γ be the angle BC, AC and AB respec.

Acc. to lami's theorem =

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Chapter-2

(One dimension motion)

*** Displacement ÷**

The shortest distance between initial and final position is called displacement. Displacement can be zero, −ve, +ve. It is vector quantity. It's SI unit is metre. It can be equal to or less than the distance travelled by object.

*** Distance ÷**

The total length covered by an object is called distance. It's SI unit is metre. It is +ve. It can never be less than displacement. It is a Scalar quantity.

*** Speed ÷**

The rate of change of distance is called speed. It is scalar quantity.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

It's S.I unit is ms^{-1}.

It's dimensional formula is $[M^0L^1T^{-1}]$

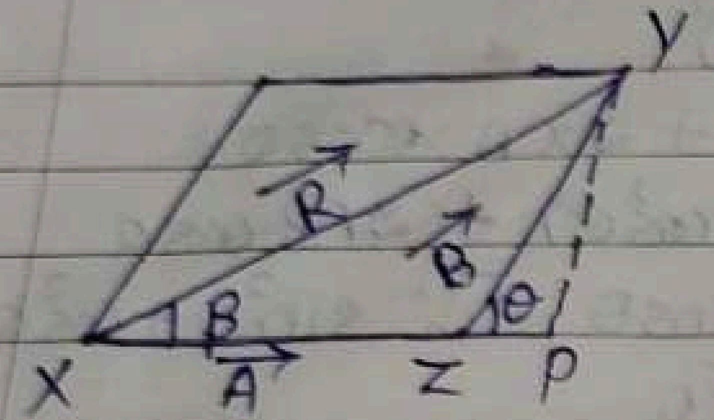
*** Velocity ÷**

The rate of change of displacement is called velocity. It is vector quantity.

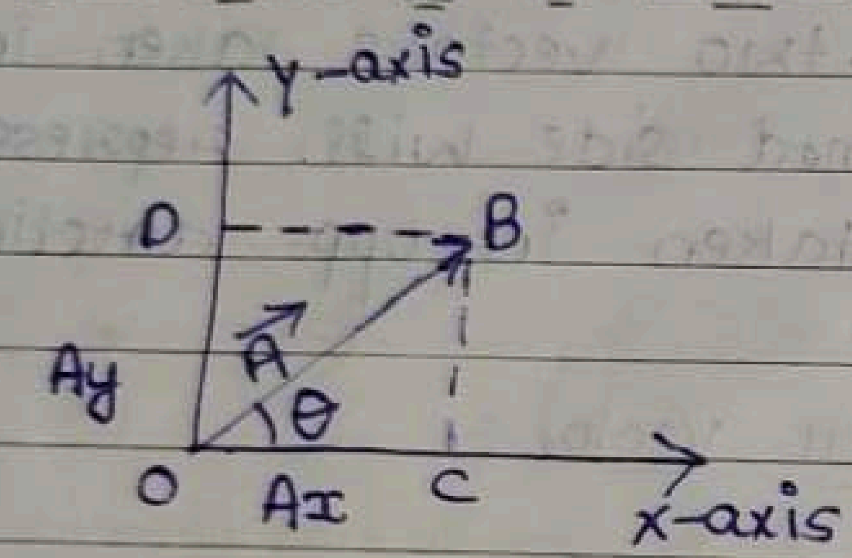
$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Dimensional formula $= [M^0L^1T^{-1}]$

It's SI unit is ms^{-1}.



* **Resolution of a vector** :-



let $\vec{A}$ be make an angle 'θ' with x-axis. Then Ax and Ay be the component of vector along x and y-axis respectively.

In ΔOBC

$$\cos\theta = \frac{OC}{OB}$$

$$OC = OB\cos\theta$$

$$A_x = A\cos\theta$$

$$\frac{BC}{OB} = \sin\theta$$

$$A_y = A\sin\theta$$

Thus, the component which is along 'θ' get cos θ and other will get sin θ.

Resultant of rectangular component :-

Rectangular component are those which are ⊥ with each other.

$$A = \sqrt{A_x^2 + A_y^2}$$

or $R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ}$

$$R = \sqrt{A^2 + B^2}$$

Rectangular component in three dimension :-

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

As,

$$\cos\alpha = \frac{A_x}{A} \Rightarrow A_x = A\cos\alpha$$

$$\cos\beta = \frac{A_y}{A}$$

$$\Rightarrow A_y = A\cos\beta.$$

and,

$$\cos\gamma = \frac{A_z}{A}$$

$$A_z = A\cos\gamma$$

or

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

* **Lami's theorem** :-

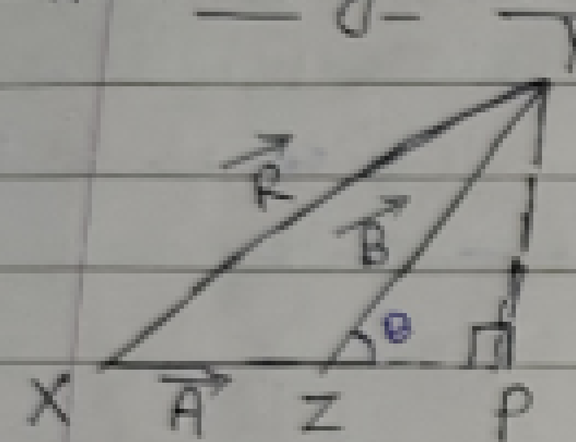
If resultant of three vector is zero, then the magnitude of vector is directly proportional ~~of~~ to sine of angle b/w other two vectors.

ii) When two vectors are in opposite direction.

$\overrightarrow{A}$ $\overrightarrow{B}$ $\overrightarrow{B}$ $\overrightarrow{A}$

$= \overrightarrow{R} = \overrightarrow{A} - \overrightarrow{B}$

* Triangle law of vector addition:



Consider two vector $\overrightarrow{A}$ and $\overrightarrow{B}$ inclined with each other. $\overrightarrow{R}$ be the resultant vector. Draw $YP \perp XP$ as ΔXYP and ΔZYP are right angle triangle:

In ΔXYP

$(XY)^2 = (YP)^2 + (XP)^2$ — (i)

$XP = XZ + ZP$

$XP = A + ZP$

$\cos\theta = \frac{ZP}{ZY}$ [$\because$ In ΔZYP]

$ZP = ZY\cos\theta$

$ZP = B\cos\theta$

$\Rightarrow XP = A + B\cos\theta$ — ii)

$\sin\theta = \frac{YP}{ZY}$

$YP = ZY\sin\theta$

$YP = B\sin\theta$ — iii)

Using ii) and iii) in eqn (i)

$XY = R$

$YP = B\sin\theta$

$XP = A + B\cos\theta$

$\Rightarrow (XY)^2 = (YP)^2 + (XP)^2$

$(R)^2 = (B\sin\theta)^2 + (A+B\cos\theta)^2$

$= B^2\sin^2\theta + A^2 + B^2 + 2 \times A \times B\cos\theta$

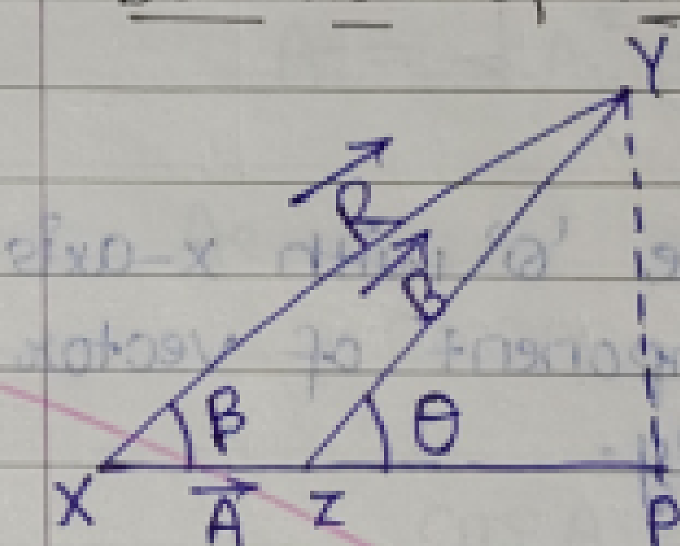
$(R)^2 = (A)^2 + (B)^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$

$(R)^2 = A^2 + (B)^2 + 2AB\cos\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Thus, Acc. to Δ law of Addition if two sides of a Δ represent two vectors taken in same order then the third side will represent the resultant vector taken in opp. direction.

* Direction of resultant vector:



Let 'β' be the angle b/w resultant vector ($\overrightarrow{R}$) and $\overrightarrow{A}$. Then direction of resultant vector is given by

$\tan\beta = \frac{YP}{XP}$

$YP = B\sin\theta$, $XP = A + B\cos\theta$

$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$

$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$

* Parallelogram law of vector addition:

If two sides of a llgm represent two vectors taken in same order than the diagnol of llgm will represent the resultant vector taken in opp. directio

$x = 18t + 5t^2$ | $x_2 = 18t + 5t^2$
$t_1 = 2s, \quad t_2 = 3s$ | $t_2 = 3s$

$x_1 = 18 \times 2 + 5(2)^2$ | $x_2 = 18 \times 3 + 5(3)^2$
$x_1 = 36 + 20$ | $x_2 = 54 + 45$
$x_1 = 56$ | $x_2 = 99$

$$A.v = \frac{\text{final} - \text{initial}}{\text{Total time}} = \frac{99-56}{3-2} = \frac{43}{1}$$
$$= 43ms^{-1}$$

The displace. of a particle is given by $x = 3 - 5t + 2t^2$. find its
i) velocity at $t = 2s$
ii) Acc. at $t = 4s$

Soln

$$v = \frac{d(3-5t+2t^2)}{dt}$$

$$= \frac{d(3 - 5 \times t^0 + 2t)}{dt}$$

$$= \frac{d(0 - 5 \times 1 + 2t)}{dt}$$

$$= - \quad -5 + 4t$$

$v = 2 + 2 \times 2$
$v = -5 + 4 \times 2$
$v = -5 + 8$
$v = 3ms^{-1}$

$a = \frac{dv}{dt}$
$a = \frac{d(-5+2t)}{dt}$
$a = 4ms^{-2}$

(Lesson - 3)
[Uniformally Accelerated Motion]

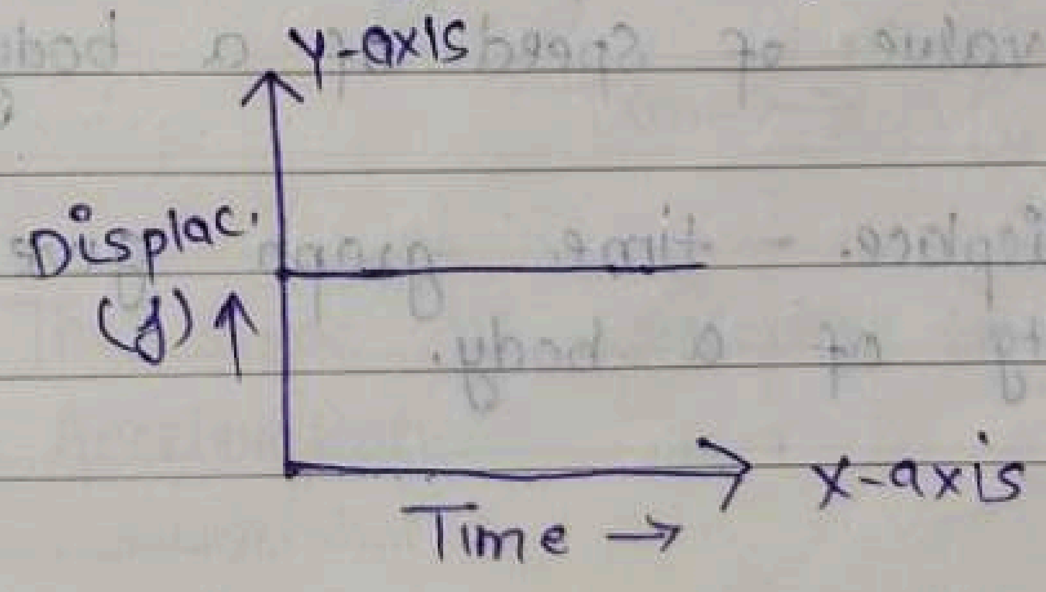
* **Uniform motion** :- A motion in which velocity of an object remains constant called uniform motion. For uniform motion v = constant
acceleration $(a) = 0$.

* **Non - Uniform motion** :- A motion in which velocity of an object does not remain constant is called Non- Uniform motion. In non- uniform motion velocity of a body may either increase or decrease.

* **Uniform Accelerated Motion** :- A motion in which acceleration of an object remains constant is called Uniform Accelerated motion. Acc. of a body can be constant only if velocity is either increasing or decreasing uniformally.
ex :- Motion of free fall of a body.

* **Position time graph** :-

i) For an obj. at rest (Stationary)



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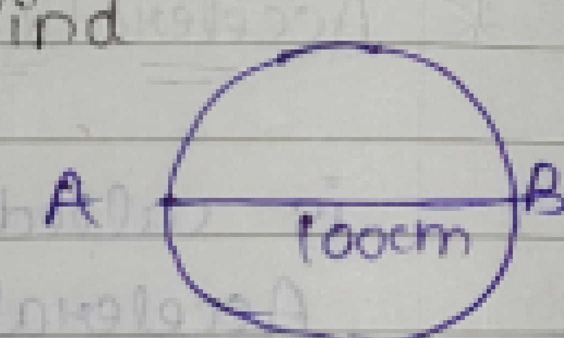
* **Non-Uniform Acceleration:** If a body covers equal change in velocity in unequal interval of time or unequal change in velocity with equal interval of time.

* **Retardation (-ve acceleration):** If velocity decreases with time then acceleration will be -ve. This -ve acceleration is called Retardation.

1) A body moves along circular track of radius 100cm. If it takes 1min. to move from A point diametrically opposite. find



1) Distance travel
2) Displacement
3) Average Speed
4) " velocity

Soln i) Distance = Speed x Time

$= 2\pi r$

$2 \times \frac{22}{7} \times 100 = \frac{2200}{7}$

ii) Displacement = AB

$= 2 \times 100$

$= 200cm.$

iii) Average Speed $= \frac{100}{60} = \frac{5}{3} = 200 \cdot \frac{22}{7} \times \frac{1}{60} = \frac{11}{210}$

iv) Average Velocity $= \frac{200}{60} = \frac{10}{3}$

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2) An object is sliding down an inclined plane. Its velocity changes at constant rate from $10ms^{-1}$ to $15m/s$ in 20 sec. find acc.

Soln Acceleration $= \frac{15-10}{20} = \frac{5}{20} = \frac{1}{4} ms^{-2}$

3) The displacement of a particle moving along x-axis is given by $x = 18t + 5t^2$. Calculate,
i) Instantaneous velocity at t = 2sec.
ii) Instantaneous Acceleration
iii) Average velocity b/w t = 2sec and t = 3sec.

Soln i) $v = \frac{d(18t+5t^2)}{dt}$

$= 18 + 5 \times 2t$

$v = 18 + 10t$

$v = 18 + 10 \times 2$

$v = 18 + 20$

$v = 38ms^{-1}$

ii) $a = \frac{dv}{dt}$

$a = \frac{d(18+10t)}{dt}$

$a = 0 + 10 \times t^0$

$0 + 10 \times 1$

$a = 10 ms^{-2}$

iii) Average velocity = $\frac{\text{Total displacement}}{\text{Time taken}}$

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5) A car moves with a speed of 30km/h for first 30 min and then speed of 40km/h for next 30min find average speed.

Sol^n Average Speed $= \frac{V_1t + V_2t}{2t} = \frac{30+40}{2} \rightarrow \frac{70}{2}$ (35)

$= 35\,km/h$.

* Velocity:–

The rate of change of displacement is called velocity.

OR

The ratio of displacement with respect to time is called velocity.

velocity $= \frac{\text{Displacement}}{\text{Time}}$

It is vector quantity
It's SI unit is ms^{-1}
It's dimensional formula $= [M^0L^1T^{-1}]$

* Uniform Velocity:–

A body is said to be moving with uniform velocity if it cover equal displace. in equal intervals of time.

* Non-uniform Velocity:–

A body is said to be moving with non-uniform velocity if it covers equal displacement in unequal interval of time.

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* Average velocity:–

It is the ratio of total displac. to total time during complete Journey.

Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

* Instantaneous velocity:–

The velocity of an object at any instant of time or at a particular time is called instantaneous velocity.

Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

* Acceleration:–

The rate of change of velocity is called acceleration.

Acceleration $= \frac{\text{change in velocity}}{\text{time taken}}$

Let during a time 't' velocity of a body change

Acceleration $= \frac{V_2 - V_1}{t}$ or $\frac{dv}{dt}$

It is denoted by 'a'
It is vector quantity.
It's SI unit is ms^{-2}.
Dimensional formula $= [M^0L^1T^{-2}]$

* Uniform acceleration –

The acceleration in which equal change in velocity in equal intervals of time

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Same time.

v_1, v_2 be the different speed

$t_1 = t_2 = t$

Average speed $= \frac{v_1 t + v_2 t}{2t} = \frac{\cancel{t}(v_1+v_2)}{2\cancel{t}} = \left(\frac{v_1+v_2}{2}\right)$

* **Instantaneous speed :-** The speed of an object at any instant of time during its journey is called Instantaneous speed.

The speed of a body at a particular time is called instantaneous speed.

Instantaneous speed $\rightarrow \lim_{\Delta t} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

1) A car covers 30km at a uniform speed of 60km/h find the time taken by the car?

Soln Distance = 30km/h

speed = 60

Time $= \frac{3\cancel{0}}{\cancel{60}_2} = \frac{1}{2}$ hr.

2) A car covers 40km at uniform speed of 80km/h and the next 30km at uniform of 40km/h

1) find total time taken

2) find its average speed

Soln Dis = 40

S = 80

Time $= \frac{4\cancel{0}}{\cancel{80}_2}$

Dis_2 — 30km/h

Sp = 40km

Time $= \frac{3\cancel{0}}{4\cancel{0}} = \frac{3}{4}$

Total time $= \frac{1}{2} + \frac{3}{4}$

$\frac{2+3}{4} = \frac{5}{4}$

ii) $\Rightarrow$ Total Dis = 70

Total time $= \frac{5}{4}$

Speed $= \frac{\cancel{70}^{14} \times 4}{\cancel{5}_1}$

$= 56$ km/h

3) Shyam home is 10km from market. He takes 1 hour to go to market finding the market close he turn back and reach home in 30 min. find average speed of his journey.

Soln Distance travelled by shyam = 10 + 10 = 20km

Time taken = 60 + 30 = 90min $= \frac{3}{2}$

Speed $= \frac{\text{Total distance}}{\text{Total time}}$

$\Rightarrow \frac{\cancel{20}^{10}}{3 \times \cancel{2}_1} \quad \frac{20 \times 2}{3} = \frac{40}{3}$

4) A covers first half of dist. with speed of 20km per hour And next half with 40km/h find average speed.

Soln Average speed $= \frac{2 \times 20 \times 40}{20 + 40}$

$= \frac{40 \times 4\cancel{0}^2}{\cancel{60}_3} = \frac{\cancel{80}^{26.6}}{\cancel{3}_1}$

$= 26.6$ km/h

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$s^{th} = un + \frac{1}{2}an^2 - [un - u + \frac{1}{2}a(n^2+1-2n)]$

$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{1}{2}an^2} - \frac{a}{2} + \frac{\cancel{2}an}{\cancel{2}}$

$= u - \frac{a}{2} + an.$

$= u + a(n - \frac{1}{2})$

$= u + a(\frac{2n-1}{2})$

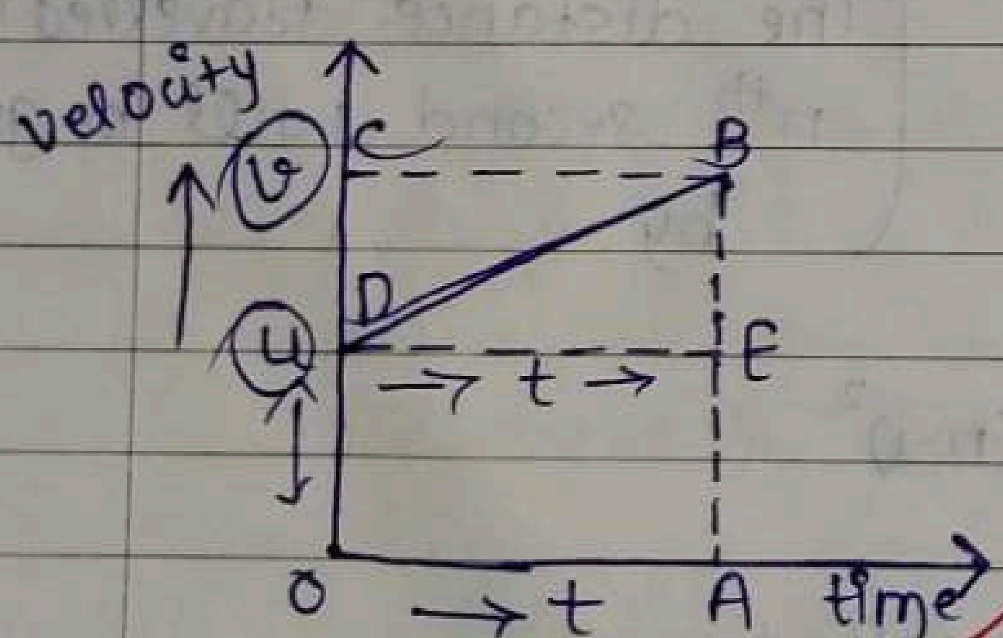
$= u + \frac{a}{2}(2n-1)$

* Equation of motion –

By graphical method

* i) velocity – time relation :-

i) $v = u + at$



let u and v be the initial and final velocity of particle
Acceleration = Slope of v-t graph

$a = \frac{BE}{DE}$ – i)

as =

$BE = BA - EA$

$BA = v$

$EA = u$

$BE = v - u$

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$DE = t$

put these value in eq^n (i)

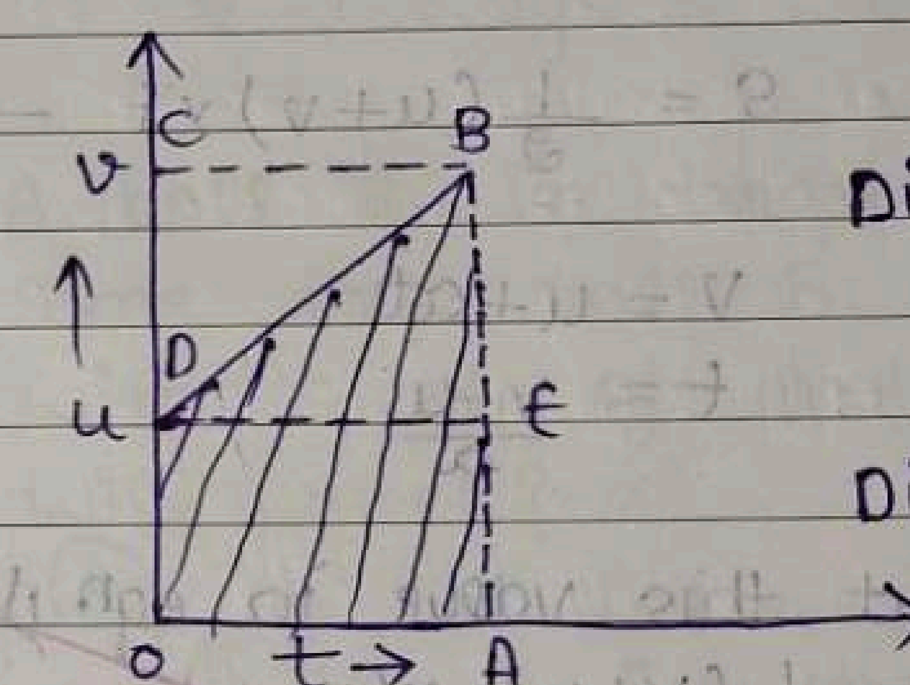
$a = \frac{v-u}{t}$

$at = v - u$

* $\boxed{v = u + at}$ *

ii) Displacement – time graph

$S = ut + \frac{1}{2}at^2$



Displacement = Area under v-t graph

Displacement = area of trapezium OAED

$= \frac{1}{2}$ (sum of ∥ side) × Altitude

$= \frac{1}{2}(OD + AB) \times DE$

$S = \frac{1}{2}(u+v) \times t$

as we know,

$v = u + at$

$S = \frac{1}{2}[u + (u+at)]t$

$= \frac{1}{2}[ut + ut + at^2]$

$S = \frac{1}{\cancel{2}}[\cancel{2}ut + at^2]$

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$$[x_2 - x_1] = u[t-0] + \frac{a}{2}[t^2 - 0]$$

$$[x_2 - x_1] = ut + \frac{1}{2}at^2$$

let $x_2 - x_1 = S$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

iii) velocity – displacement relation :-

$v^2 - u^2 = 2aS$

Acceleration = $\frac{\text{change in velocity}}{\text{time}}$

$$a = \frac{dv}{dt}$$

Multiply and divide by 'dx' in R.H.S

$$a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a \cdot dx = dv \cdot \frac{dx}{dt}$$

$$a \cdot dx = v \cdot dv \quad \left[\frac{dx}{dt} = v\right]$$

integrating both side

$$\int a \cdot dx = \int v \cdot dv$$

As,

$u \to v$

$x_1 \to x_2$

$$\int_{x_1}^{x_2} a \cdot dx = \int_u^v v \cdot dv$$

$$a[x]_{x_1}^{x_2} = \left[\frac{v^2}{2}\right]_u^v$$

$$a[x_2 - x_1] = \left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$2aS = v^2 - u^2$$

Hence,

$$* \boxed{v^2 - u^2 = 2aS} *$$

iv) Distance travelled in n^{th} second –

$S = ut + \frac{1}{2}at^2$ [Let S_n and S_{n-1} be the dist. travelled in 'n' sec and (n-1) second respect.

$S_n = un + \frac{1}{2}an^2$

$D^{th} = S_n - S_{n-1}$ — i) [The distance travelled in n^{th} second will be given by

As,

$S = ut + \frac{1}{2}at^2$

$S_{(n-1)} = u(n-1) + \frac{1}{2}a(n-1)^2$

put these value in eq^n 1

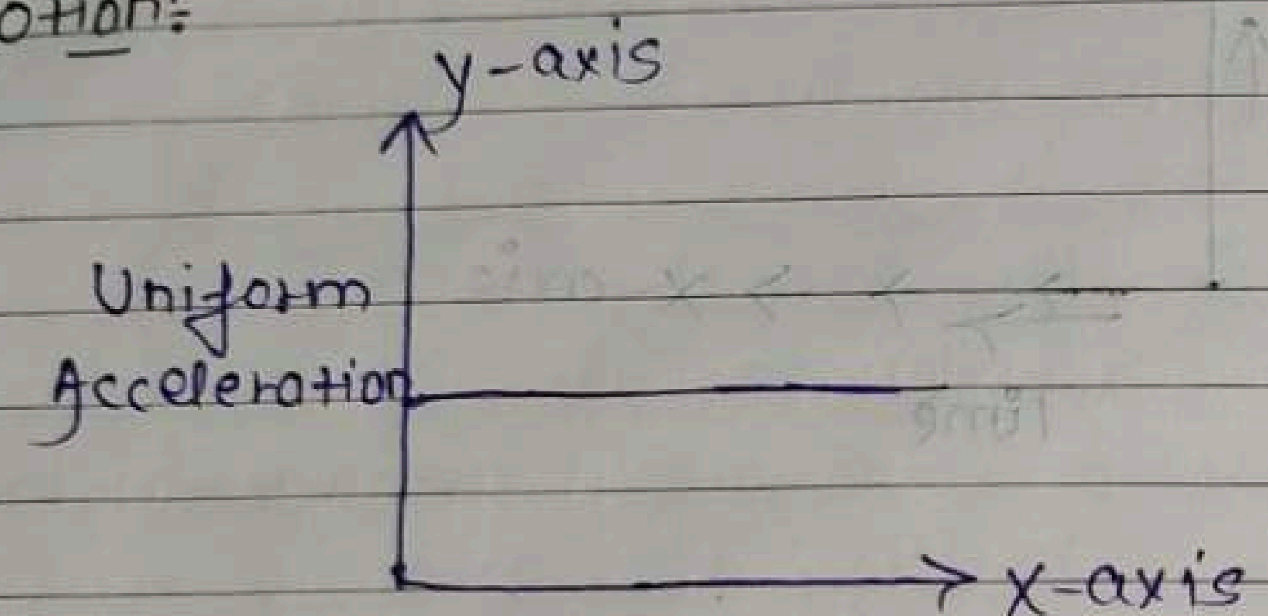
$$D^{th} = un + \frac{1}{2}an^2 - [u(n-1) + \frac{1}{2}a(n-1)^2]$$

$$un + \frac{1}{2}an^2 - [un - u + \frac{1}{2}a(n^2 + 1 - 2n)]$$

$$D^{nth} = \boxed{u + \frac{a}{2}(2n-1)}$$

→ Area of velocity-time graph give the value of displacement.

* Acceleration - time graph for uniformally accelerated motion:



* Equation of motion :

i) $v = u + at$
ii) $S = ut + \frac{1}{2}at^2$
iii) $v^2 - u^2 = 2as$

* Calculas method :
i) velocity time relation
$v = u + at$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$a = \frac{dv}{dt}$$

$$dv = a \cdot dt$$

Now, integrating both side

$$\int dv = \int a \cdot dt$$

$0 \longrightarrow t$
$u \longrightarrow v$

$$\int_u^v dv = \int_0^t a \cdot dt$$

$$(v)_u^v = a[t]_0^t$$

$$[v-u] = a[t-0]$$

$$v - u = at$$

* $\boxed{v = u + at}$ *

ii) Distance - time graph

$$S = ut + \frac{1}{2}at^2$$

$$v = \frac{dx}{dt}$$

$$dx = v \cdot dt$$

integrating both side

$$\int dx = \int v \cdot dt$$

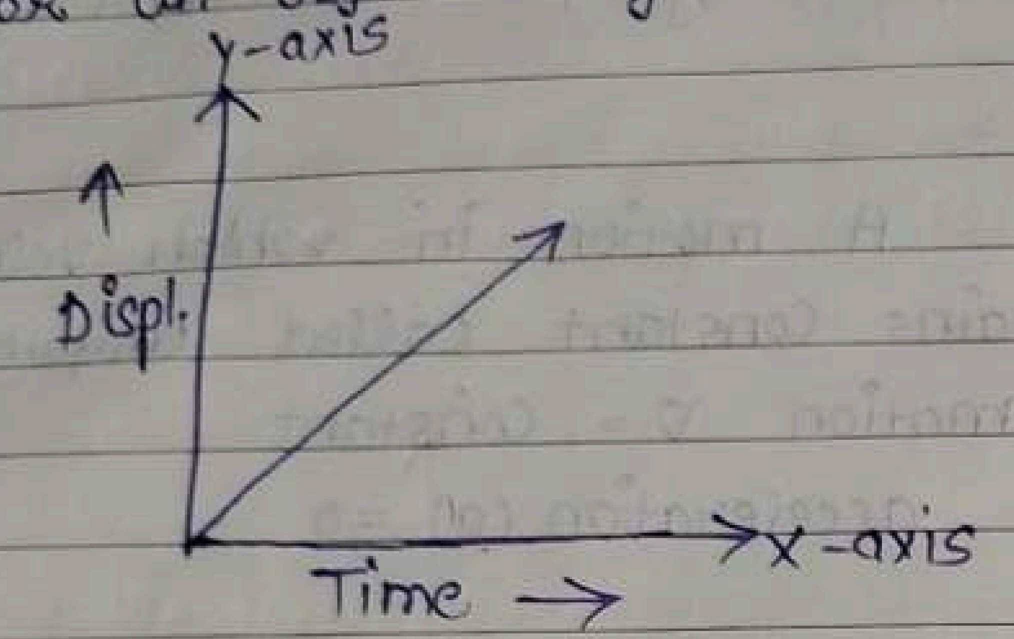
as,
$0 \longrightarrow t$
$x_1 \longrightarrow x_2$

$$\int_{x_1}^{x_2} dx = \int (u + at) \cdot dt$$

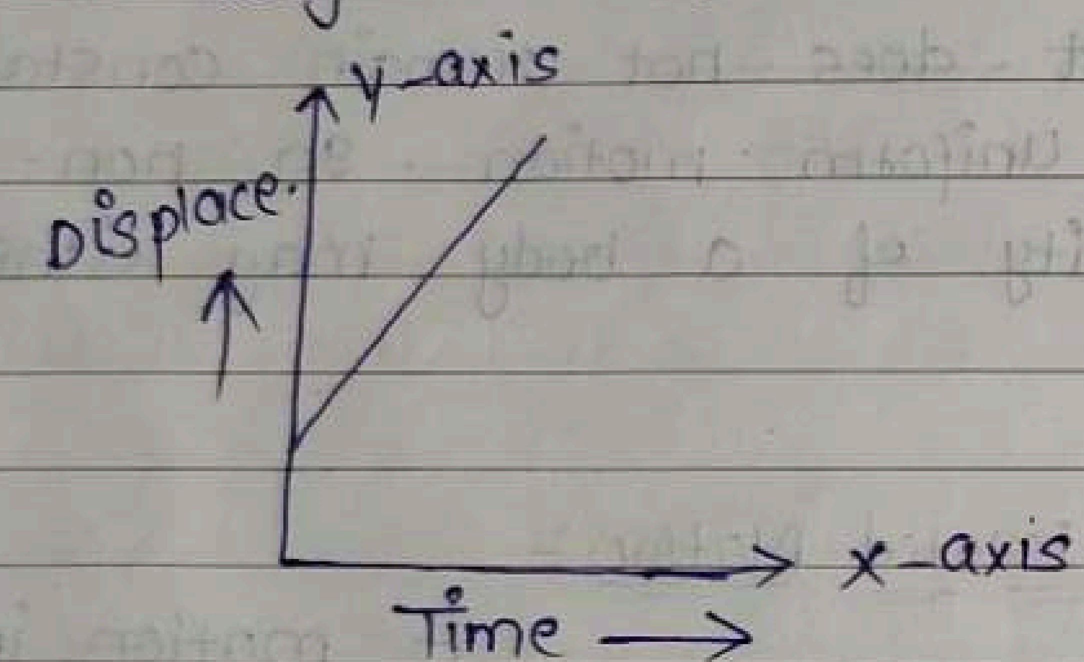
$$\int_{x_1}^{x_2} dx = \int_0^t u \cdot dt + \int_0^t at \cdot dt$$

$$(x)_{x_1}^{x_2} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

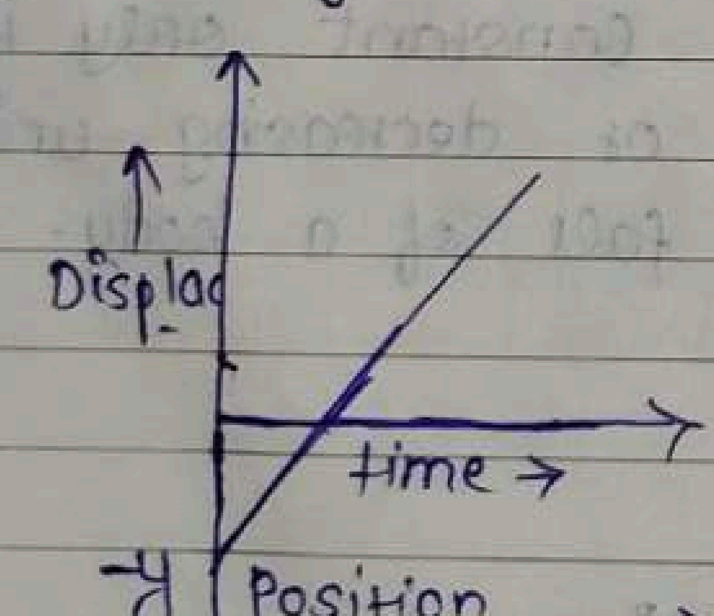
iii) For an obj. moving with uniform velocity/speed



a) If a body have some +ve displacement then,



b) If a body have some -ve initial value of displace.

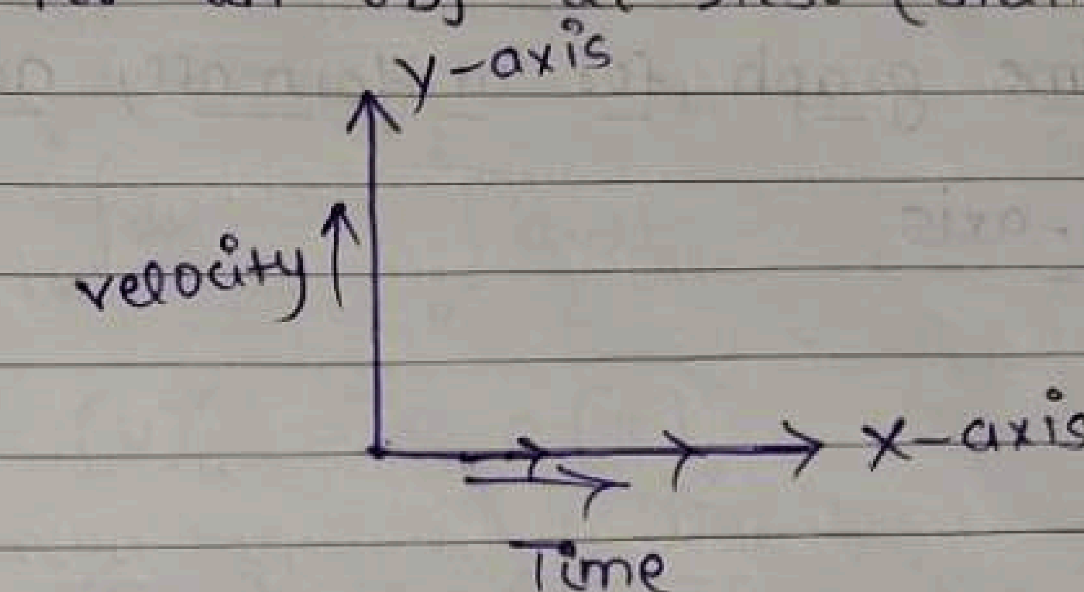


→ The slope of distance - time graph give the value of speed of a body

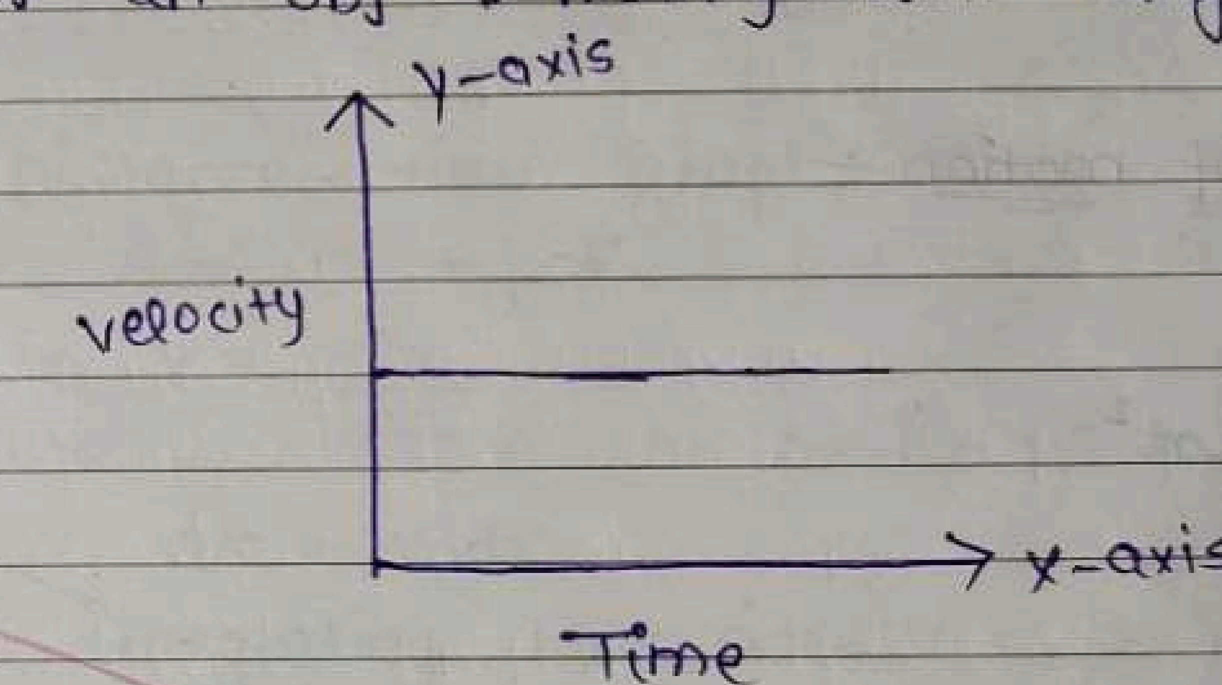
→ The slope of displace. - time graph give the value of velocity of a body.

* Velocity-time graph :-

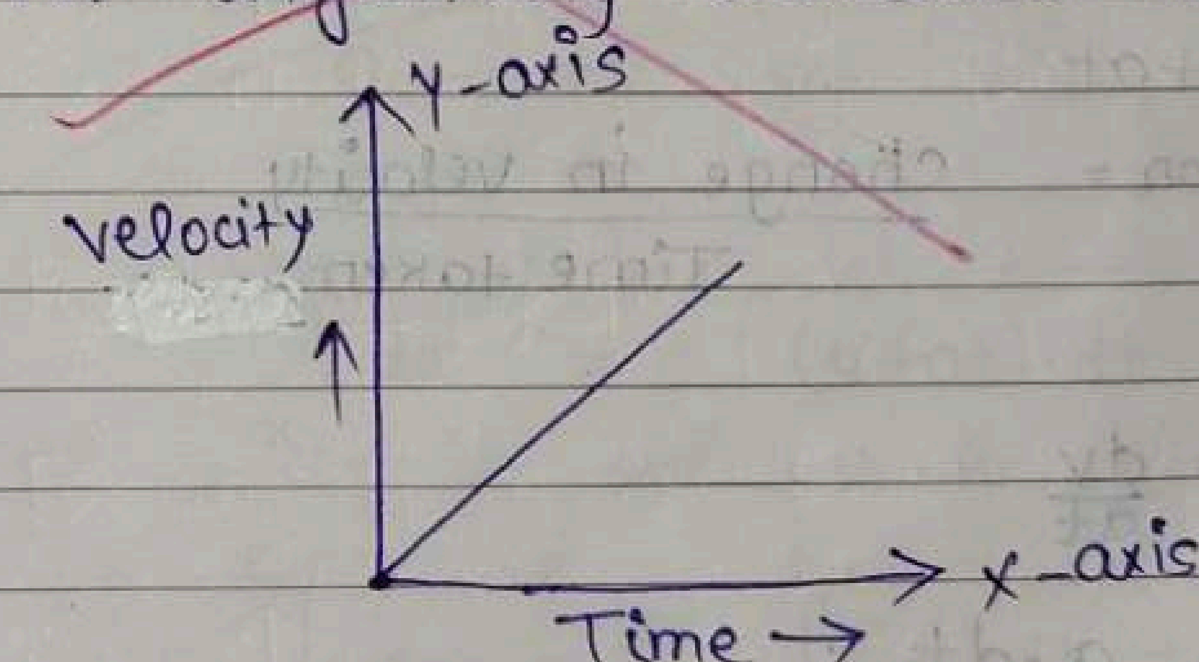
i) For an obj. at rest (stationary)



ii) For an obj. moving with uniform velocity.



iii) For uniformally accerlated motion



→ The slope of velocity time graph give the value of Acceleration.

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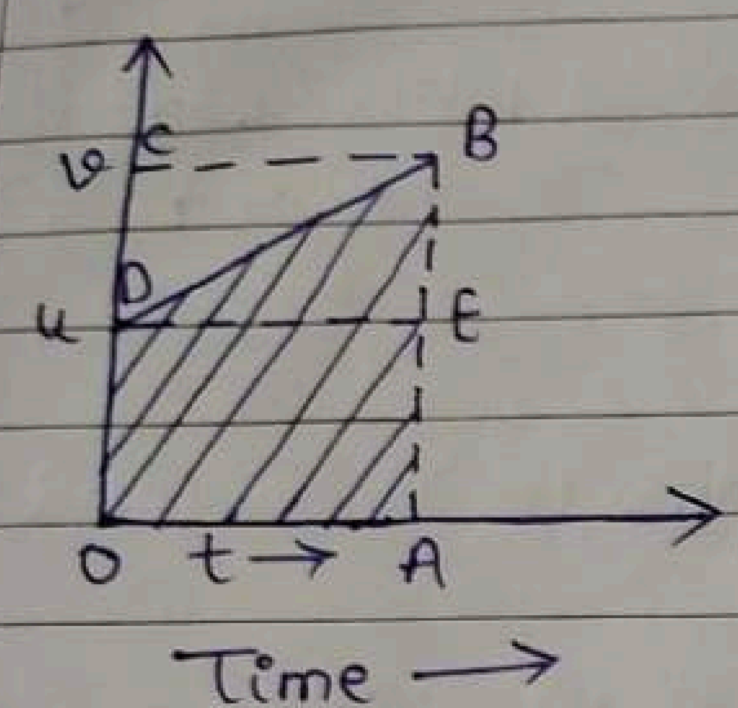
iii) Velocity - Distance relation :-

$v^2 - u^2 = 2as$



Distance = Area under velocity-time graph

Distance = Area of trapezium OAED

$= \frac{1}{2}(OD + BA) DE$

$S = \frac{1}{2}(u+v) \times t$ —i)

$v = u + at$

$t = \frac{v-u}{a}$

put this value in eqn i)

$S = \frac{1}{2}(v+u) \times \left(\frac{v-u}{a}\right)$

$S = \frac{v^2 - u^2}{2a}$

* $\boxed{v^2 - u^2 = 2as}$ *

* Motion under freefall :-

When a body falls freely then acceleration due to gravity (g) act on body. The value of g is $9.8 ms^{-2}$ at the surface of earth. Its value changes as we move from Surface of earth. When a body is falls down from a height then, g has +ve value while in case of a body thrown up then g has -ve value.

When a body fall down from a height then, its initial velocity is zero. When a body thrown up to some height then it has some value of initial velocity.

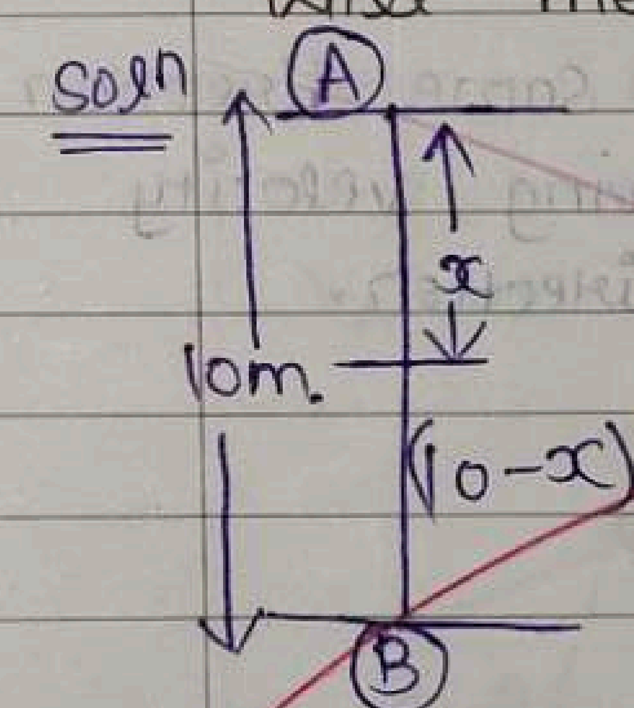
Then, eqn of motion becomes-

$a = g$

1) $v = u + gt$
2) $S = ut + \frac{1}{2}gt^2$
3) $v^2 - u^2 = 2gs$

1) A ball A is dropped from height 10m at the same time ball B is thrown up from ground with speed of 10m/s. When and where they will meet?

Soln



$S = ut + \frac{1}{2}gt^2$

$g = -9.8 ms^{-2}$

* $S = ut + \frac{1}{2}gt^2$

$x = 0 \times t + \frac{1}{2}(-9.8) \times t^2$

$x = 4.9 \times t^2$

* $(10-x) = 10 \times t + \frac{1}{2} \times (-\cancel{9.8})t^2$ (4.9)

$(10-x) = 10t - 4.9t^2$

$10 - \cancel{x} = 10t - \cancel{x}$

$t = 1$

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$x = 4.9t^2$
$= 4.9(1)^2$
$\Rightarrow 4.9$

* final - initial
  $10 - 4.9$
  $\Rightarrow \underline{5.1}$

* **Relative velocity :-**
The velocity of one body with respect to another body is known as relative velocity
$$\vec{V}_{rel} = \vec{V_1} - \vec{V_2}$$
The rate of change of position of one body w.r.t to another body is known as relative velocity.

* **Relative velocity in one dimension -**

* When two bodies are moving in same direction
let us consider two bodies having velocity V_1 and V_2 moving in same direction.

$\xrightarrow{V_1}$

$\xrightarrow[V_2]{}$

The relative velocity of 1st body wrt 2nd body is given by

$\underset{V_1}{\longleftrightarrow}\underset{V_2}{}$ 2nd body comes to rest

$\underset{V_2}{\longleftrightarrow}\underset{V_1}{} \Rightarrow V_1 - V_2$

So,
$\vec{V}_{rel} = \vec{V_1} - \vec{V_2}$ (towards right)

Or relative velocity of 2nd body w.r.t to 1st body
$\vec{V}_{rel} = \vec{V_2} - \vec{V_1}$ (towards left)

* When two bodies moving in opp. direction

$\xrightarrow[V_1]{}$

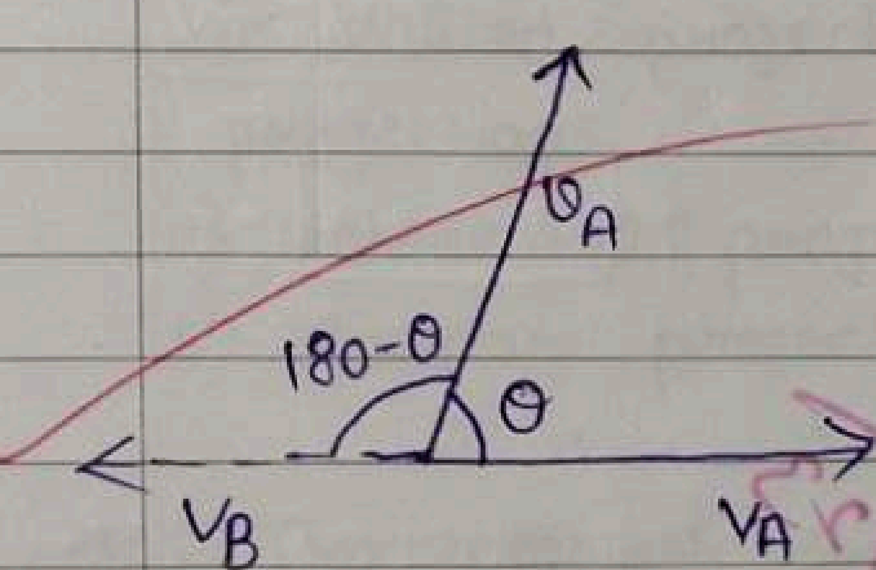
$\xleftarrow[V_2]{}$

Relative velocity of 1st w.r.t 2nd body

$\underset{V_2}{\longleftrightarrow}\underset{V_2}{}$ 2nd (Body comes rest)

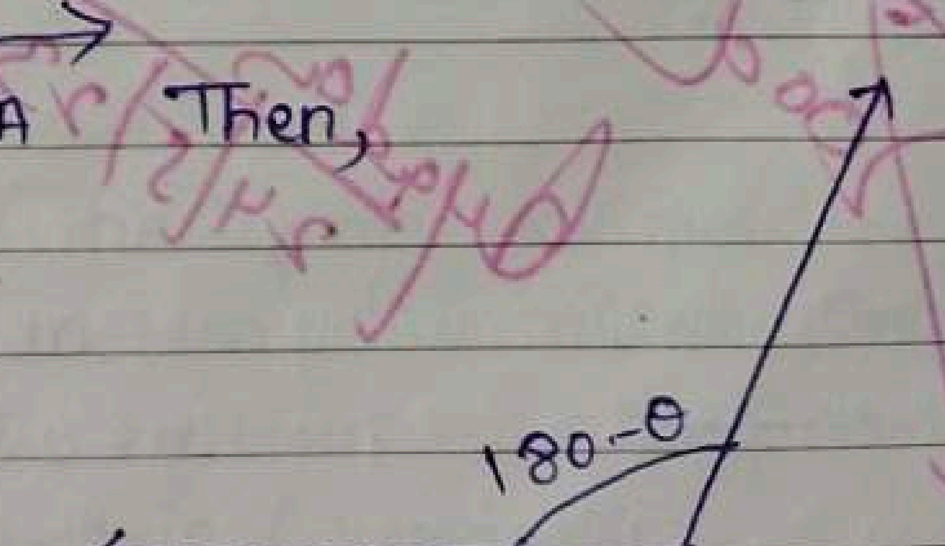
$\underset{V_1}{\longrightarrow}\underset{V_2}{\longrightarrow}$ $\vec{V_1} + \vec{V_2} = \vec{V}_{rel}$.

* When body inclined with each other.
let two bodies A and B having velocity V_A and V_B inclined with each other.



Then relative velocity of A w.r.t B is given by
Body B should being to rest

Then,



V_R is the resultant vector of relative velocity

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$$V_R = \sqrt{V_A^2 + V_B^2 + 2V_A \cdot V_B \cos(\theta - \overset{180-\theta}{180})}$$

$$V_R = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$

If both bodies move in same direction then put $\theta = 0$

$$V_R = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos 0^\circ}$$

$$= \sqrt{V_A^2 + V_B^2 - 2V_A V_B}$$

$$V_R = \sqrt{(V_A - V_B)^2}$$

$$\boxed{V_R = V_A - V_B}$$

When both bodies move in opp. direction put $\theta = 180^\circ$

$$V_R = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos 180^\circ}$$

$$V_R = \sqrt{V_A^2 + V_B^2 - 2V_A V_B (-1)}$$

$$V_R = \sqrt{(V_A + V_B)^2}$$

$$\boxed{V_R = V_A + V_B}$$

Good
24/5/23

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Chapter = 4

[Projectile]

* **Projectile** :
Anything which is given to some initial velocity and allow to travel in 2-Dimension or 3-D under the effect of gravity alone is called projectile.

* Conditions for projectile motion
1) There must be some initial velocity.
2) It must ideal travel in 2-D or 3-D.
3) The effect of gravity must be there.

* **Assumption** :
i) Air resistance (force) is taken as zero.
2) The value of gravity is assumed to be constant
3) Rotational effect of earth is not considered.

* **Projection** -
The process of launching a projectile is called projection. There are two types of projection
1) Horizental projection
2) Angular projection.

* **Trajectory** :
The path followed by a projectile during its motion is called Trajectory. There will be two motions - along x-axis (Horizontal) and along y-axis (vertical)

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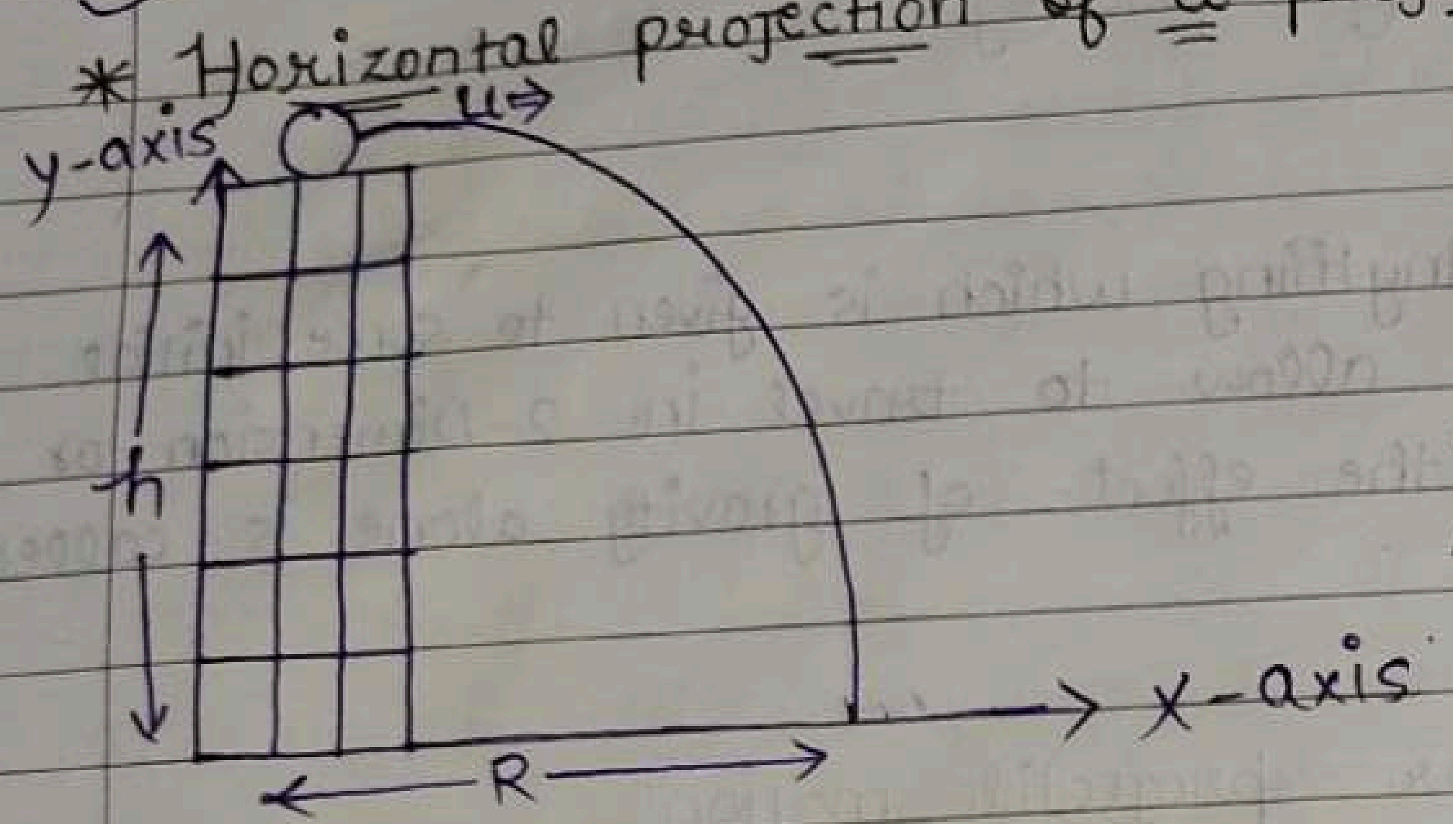
* Horizontal projection of a projectile :



If initial velocity is given in horizontal direction then the projection is called horizontal projection. Let us consider a building of height h. u be the initial velocity provide to a ball then,

Initial velocity along x-axis — $\boxed{u_x = u}$

final velocity along x-axis $\boxed{v_x = u}$

Acceleration along x-axis, $a_x = \frac{v_x - u_x}{t}$

Here, $v_x = u$

$a_x = \frac{v_x - v_x}{t}$

$\boxed{a_x = 0}$

* Along y-axis –

Initial velocity along y-axis

$u_y = 0$

final velocity along y-axis – v_y = maximum.

acceleration along y-axis

$a_y = +g$

as, $v = u + at$

$v_y = u_y + a_y \cdot t$

$v_y = 0 + gt$

$\boxed{v_y = gt}$

(imp...)

* Equation of Trajectory :

Let x be the displacement along x-axis. u_x and a_x be the initial velocity and acceleration along x-axis

As we know,

$S = ut + \frac{1}{2}at^2$

Along x-axis,

$x = u_x \cdot t + \frac{1}{2}a_x \cdot t^2$

Here,

$u_x = u$

$a_x = 0$

$x = ut + \frac{1}{2} \times 0 \times t^2$

$x = ut$

$\boxed{t = \frac{x}{u}}$

Along y-axis –

$y = u_y + \frac{1}{2}a_y \cdot t^2$

$u_y = 0, \quad a_y = +g$

$y = 0 \times t + \frac{1}{2} \times g \times t^2$

$y = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}}$

* Time of flight :

It is the time taken by projectile to complete its journey. It is denoted by T.

As,

$S = ut + \frac{1}{2}at^2$

* Horizontal Range :

Horizontal Range is the total distance covered by projectile along x-axis during it's Journey. It is denoted by 'R'.

As,

$$S = ut + \frac{1}{2}at^2$$

$$S = R, \quad u = u_x, \quad a = a_x$$

$$R = u_x \cdot t + \frac{1}{2} a_x \cdot t^2$$

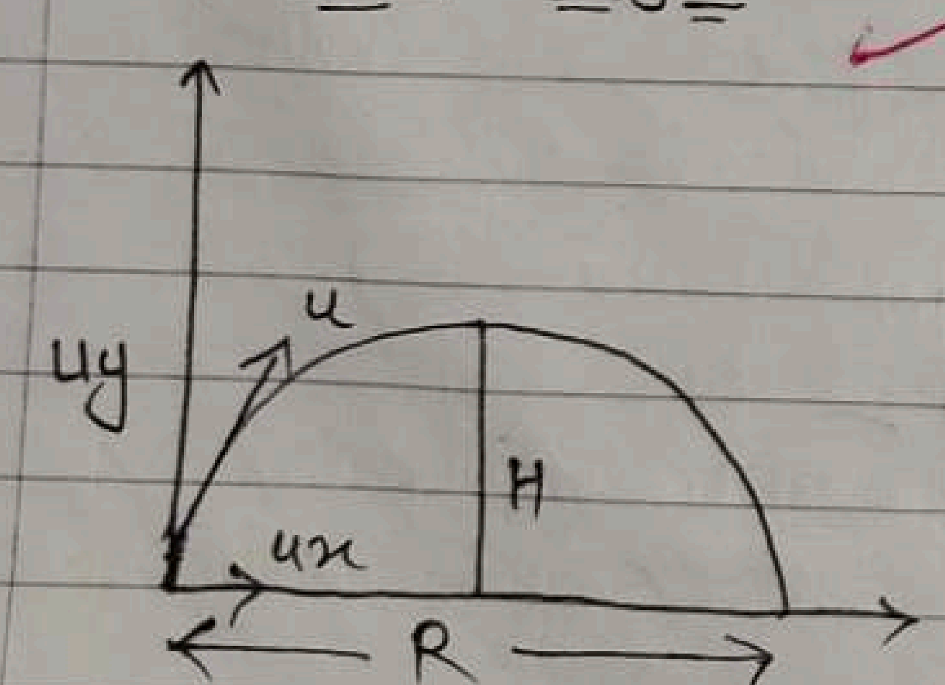
$$u_x = u\cos\theta$$
$$a_x = 0$$
$$t = \frac{2u\sin\theta}{g}$$

$$R = u\,\cancel{\sin\theta}\,\cos\theta \times \frac{2u\sin\theta}{g} + \frac{1}{2}\times 0 \times \frac{2u\sin\theta}{g}$$

$$R = u^2 \cdot \frac{2\sin\theta\cos\theta}{g}$$

$$R = u^2 \cdot \frac{\sin 2\theta}{g} \quad [2\sin\theta\cos\theta = \sin 2\theta]$$

* Maximum height :



As,

$$S = ut + \frac{1}{2}at^2$$

For maximum height,

$$S = H, \quad u = u_y, \quad a = a_y$$

$$H = u_y \cdot t + \frac{1}{2} a_y \cdot t^2 \quad \text{—(1)}$$

$$u_y = u\sin\theta, \quad a_y = -g, \quad t = \frac{u\sin\theta}{g}$$

Put these value in eqⁿ (i)

$$H = u\sin\theta \times \frac{u\sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$$

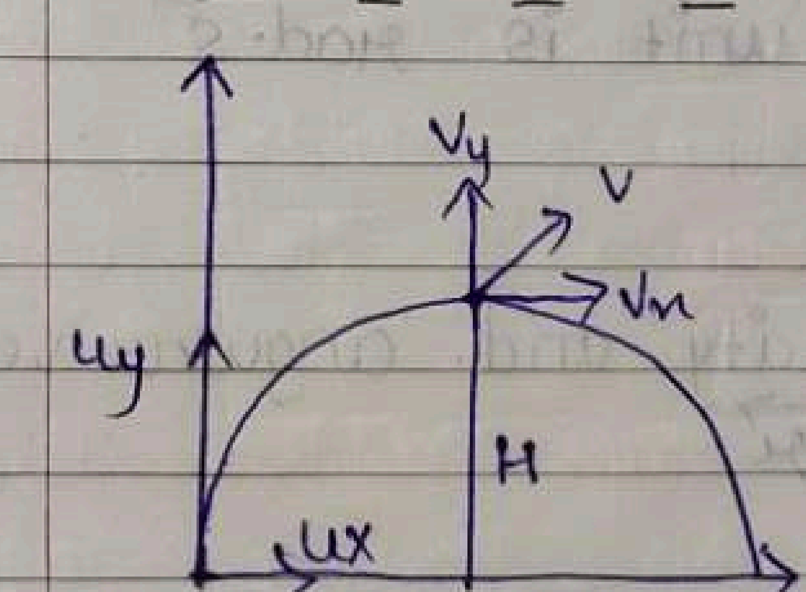
$$= \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\cancel{g} \times \frac{u^2\sin^2\theta}{g^{\cancel{2}}}$$

$$= \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\frac{u^2\sin^2\theta}{g}$$

$$= \frac{2u^2\sin^2\theta - u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{2g}$$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

* Velocity at any instant :



Let 'v' be the velocity at any instant

$$v = \sqrt{v_x^2 + v_y^2 + 2v_x v_y \cos 90°}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = u\cos\theta, \quad v_y = u\sin\theta - gt$$

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* Equation of Trajectory

As, $S = ut + \frac{1}{2}at^2$

Then along x-axis

$S = x,\quad u = u_x,\quad a = a_x$

$$x = u_x t + \frac{1}{2}a_x \cdot t^2$$

We have $u_x = u\cos\theta$

$a_x = 0$

$$x = u\cos\theta t + \frac{1}{2} \times 0 \times t^2$$

$$x = u\cos t$$

$$\boxed{t = \frac{x}{u\cos\theta}} \quad \text{—(i)}$$

⇒ Along y-axis

$S = y,\quad u = u_y,\quad a = a_y$

$$y = u_y \cdot t + \frac{1}{2}a_y t^2$$

as,

$$u_y = u\sin\theta,\quad a_y = -g$$

$$y = u\sin\theta \cdot t + \frac{1}{2}(-g)t^2$$

using eqn(1)

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2}$$

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* Time of flight

It is time taken by projectile to complete it's journey. It is denoted by 'T'. It is equal to sum of time of Ascent and time of decent.

$$T = t_{ascent} + t_{descent}$$

as, $t_{ascent} = t_{descent}$

$$T = 2t_{ascent}$$
$$= 2_{descent}$$

Along y-axis,

$$V_y = u_y + a_y \cdot t$$

$u_y = u\sin\theta$

$a_y = -g$

$V_y = 0$ [At heighest point]

$$0 = u\sin\theta - gt$$

$$gt = u\sin\theta$$

$$t_{ascent} = \frac{u\sin\theta}{g}$$

Time of flight

$$T = 2t_{ascent}$$

$$= 2 \times \frac{u\sin\theta}{g}$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

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* A projectile is fired horizon with a speed 196 m/s from top of a tower of height 490m. find
1) the distance of target from tower,
2) Time taken by projectile.
3) horizontal component of velocity = 10 sec.
4) vertical component of velocity = 10 sec.
5) velocity with which projectile hit target.

Soln 1) $R = u\sqrt{\frac{2h}{g}}$

$= \frac{196\sqrt{2} \times \cancel{490}^{5} \times 10}{\cancel{9.8}_{1}}$

$= 1960m$

ii) $T = \sqrt{\frac{2h}{g}} = \sqrt{2}\ \frac{\times \cancel{196}^{5} \times 10}{\cancel{98}_{1}}$

$= 10 sec$

iii) 196 m/s

iv) $V = \sqrt{V_x^2 + V_y^2} \qquad V_y = gt$

$\Rightarrow \sqrt{u + g^2 t^2}$

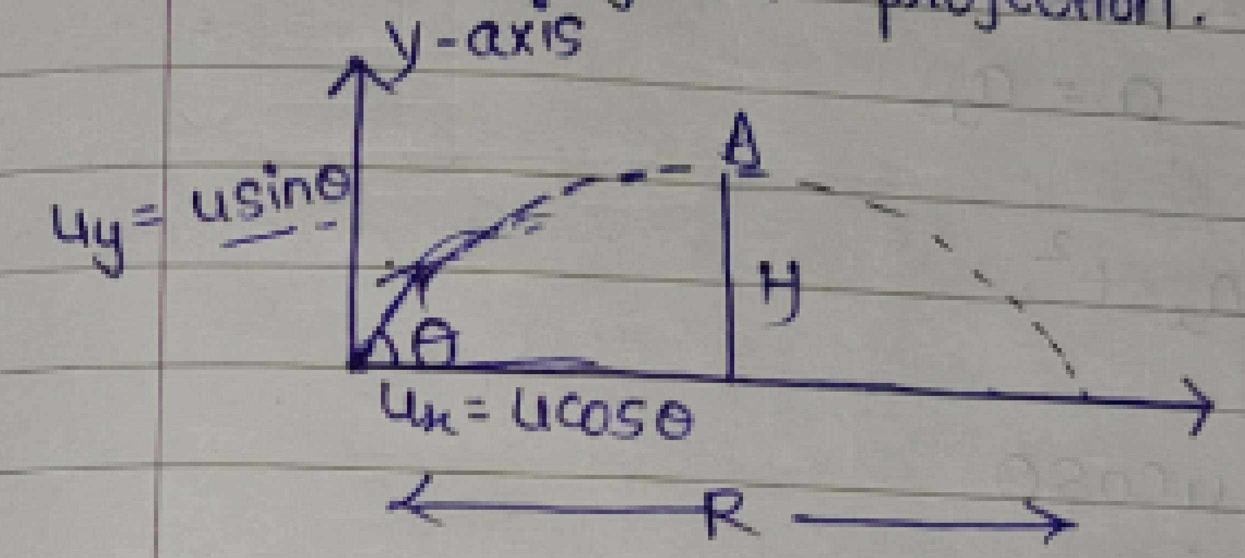
9.8×10

$\Rightarrow 98$

v)

* Angular projection of a projectile :-

If initial velocity is given at some angle then the projection is called angular projection.



Ascent $= u_x = u\cos\theta$
Decent $= u_x = u\cos\theta$
$a_x = \frac{V_x - u_x}{t} = 0$

y-axis
$u_y = u\sin\theta$
$v_y = u_y + a_y t$
$v_y = u\sin\theta - g t$

Along x-axis (Horizontal component) :-

Initial velocity $(u_x) = u\cos\theta$
final velocity $(v_x) = u\cos\theta$
Acceleration $(a) = \frac{V_x - u_x}{t} = \frac{u\cos\theta - u\cos\theta}{t} = 0$

→ Thus motion along x-axis is uniform motion

* Along y-axis -
Initial velocity $(u_y) = u\sin\theta$
final " $(v_y) = u_y + a_y t$
$= u\sin\theta - gt$

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Along y-axis,
$S = h,\ u = u_y = 0\quad a = a_y = +g\ ,\ t = T$
$h = 0 \times T + \frac{1}{2} g \times T^2$

$h = \frac{1}{2} g T^2$

$T^2 = \frac{2h}{g}$

$T = \sqrt{\frac{2h}{g}}$

* Maximum horizontal Range ÷
It is the total dist. covered by projectile along x-axis during its journey.

Along x-axis,
$S = ut + \frac{1}{2} a t^2$
$S = R\ ,\ u = u_x\ ,\ a = a_x = 0\ ,\ t = T$

$R = u_x \times T + \frac{1}{2} a_x \times T^2$

$u_x = u,\quad a_x = 0\quad , T = \sqrt{\frac{2h}{g}}$

$R = u\sqrt{\frac{2h}{g}} + \frac{1}{2} \times 0 \times \left(\sqrt{\frac{2h}{g}}\right)^2$

$\boxed{R = u\sqrt{\frac{2h}{g}}}$

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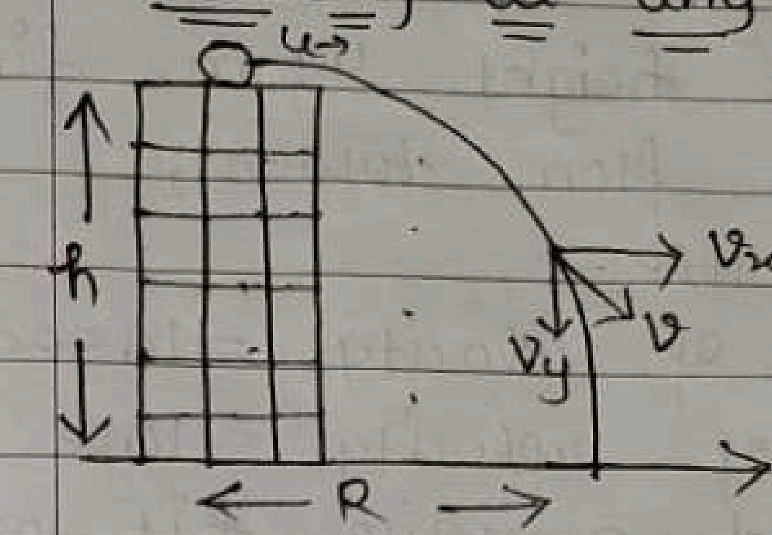
* Velocity at any instant ÷



Let 'v' be the velocity at any instant. v_x and v_y be the component of velocity along x-axis and y-axis respect.
Thus,
$v = \sqrt{v_x^2 + v_y^2 + 2 v_x v_y \cos 90^\circ}$

$v = \sqrt{v_x^2 + v_y^2}\quad [\cos 90^\circ = 0]$
as, $v_x = u$
$v_y = gt$
$v = \sqrt{u^2 + g^2 t^2}$

* A projectile fired horizontelly with a speed of 20m/s. It's a target in 10 sec

Soln $R = u\sqrt{\frac{2h}{g}} = ut$

$R = 20 \times 10$
$R = 200m$

* A stone is dropped from the window of bus moving at 20m/s the window is at height of 1.96m

Soln $R = u\sqrt{\frac{2h}{g}}$

$= 20\sqrt{2 \times 2}$
$= 20 \times 2$
$= 40m$

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$$v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$v = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

* Uniform circular Motion ÷

If a body moves along a circular path then its motion is called circular motion.

In circular motion there are two types of forces

1) centerpital force ii) centrifugal force.

* Angular Displacement ÷

It is the dist. travelled by a body during its circular motion. It is denoted by θ.

$\theta = \frac{l}{r}$, It has no unit.

* Angular velocity ÷

It is defined as the ratio of angular displacement and time. It is denoted by ω.

$\omega = \frac{d\theta}{dt}$, It's S.I unit is $rad \cdot s^{-1}$

Relation b/w linear velocity and angular velocity is given by $\Rightarrow \vec{v} = \vec{\omega} \times \vec{r}$

* Angular acc. →

It is defined as the ratio of

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angular velocity and time. It is denoted by 'α'.

as, $\alpha = \frac{d\omega}{dt}$

$\vec{\alpha} = \vec{a} \cdot \vec{r}$

Where, $\vec{u}$ = linear acc.

S.I unit is $rad\,s^{-2}$

Good

Ayadav
7/6/23

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