Problem Set Two

EC201

Problem 1. True/False: We cannot compose a function with a random variable because a random variable is not a function

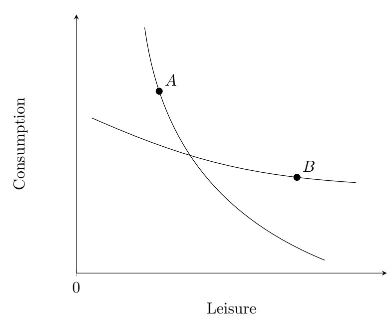
Answer. False – As we have discussed in class, a random variable is a function. We'll begin to make use of random variables as we start to model firms and consumers making decisions under uncertainty.

Problem 2. True/False: Price is a component of the firm's choice set

Answer. True – Firms decide what the price to set for their products

Problem 3. What would it mean if indifference curves of a consumer intersect?

Answer. – It would mean that the consumer does not have consistent preferences. For example, consider the choice set pictured below with the two indifference curves. Since the A and B are on different indifference curves, the consumer prefers one bundle to the other. If the consumer had consistent preferences, though, the consumer must be indifferent between A and B because they are indifferent between B and the intersection bundle and indifferent between the intersection bundle and A so if they have consistent preferences they must be indifferent between A and B. Hence we can conclude that if the indifference curves intersect the consumer cannot have consistent preferences.



Problem 4. Explain why a firm might not be trying to maximize their current profit?

Answer. It was reported today by the WSJ that ByteDance's operating losses exceeded \$7 billion last year. The rationale for such an "outcome" is that ByteDance is making current decisions with the aim of maximizing its long term growth. Such an aim differs significantly from the initial model of the firm that we introduced last week.

Problem 5. In class, we modeled a firm as producing a single good which they can sell for at some price $p > 0$. How might we extend this model to account for situations where in addition to producing a good or service that the firm can sell, they also produce wastes that they may have to pay to dispose of?

Answer - We can introduce a function that captures the amount of waste associated with any quantity of output and an associated price which captures how expensive it is to get rid of each unit of waste: p_w.

$$\text{WasteFunction} : \mathbb{R} \to \mathbb{R}$$

The profit function could then be expressed as follows

$$\text{Profit}(q) = \text{Revenue}(q) - \text{Cost}(q) - p_w \text{WasteFunction}(q)$$

Problem 6. Consider the following Utility Function:

$$U(x_1, x_2) = 2.x_1^{0.5} x_2^{0.5}$$

At the bundle (2,3), a consumer with the above utility function is approximately willing to trade-off one additional unit of x_1 for how many units of x_2? Would this tradeoff vary if we changed the consumer's utility function by composing it with a strictly monotonic function?

$$V = m \circ U, \quad \frac{d}{dx} m(x) > 0 \ \forall x$$

Answer - This tradeoff is captured by the slope of the indifference curve at (2,3), which is also referred to as the Marginal Rate of Substitution. Via the **Implicit Function Theorem**, we can compute this slope as the ratio of the marginal utilities

$$\text{MRS}(2,3) = -\frac{\partial_{x_1} U(2., 3.)}{\partial_{x_2} U(2., 3.)} = 1.5$$

And no, via the **chain-rule** we know that composing the utility function with a strictly increasing function will have no effect on the MRS.

Problem 7. Consider the following endowment: $(m_1, m_2) = (4, 10)$, where m_1 represents the endowment in the first period and m_2 represents the endowment in the second period. Given borrowing rates of 0.15 and a savings rate of (0.01) what is the present value of the bundle? Does the present value of the bundle increase as we increase the savings rate?

Answer - We know the present value of m_1 is 4, so the challenge that we need to tackle in this problem is how to compute the present value of m_2. Intuitively we can probably reason that the present value of m_2 is the amount of money in the current period such that if we paid that amount back plus interest we would have m_2. We can express this reasoning via the following equation. Solving for v we find the present value of m_2

$$v(1 + r_{\text{borrowing}}) = m_2$$

Therefore the present value of the bundle is

$$m_1 + \frac{m_2}{1 + r_{\text{borrowing}}} = 4 + \frac{10}{1.15}$$

As indicted by its absence in the above expression, the savings rate does not effect the present value of the endowment.

Problem 8. Explain the economic interpretation of the following function, where F is the production function of a firm.

$$\partial_K \partial_K \partial_L F(L, K)$$

Answer – I will work through this one function at a time

- $\partial_L F(L, K)$ – How output changes if we increase labor by a small amount

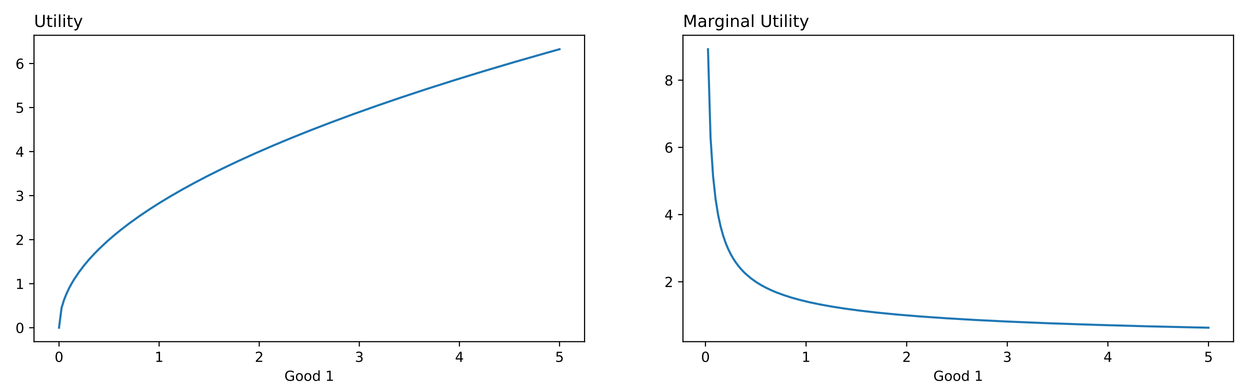
- $\partial_K \partial_L F(L, K)$ – How the the marginal product responds if we increase capital by a small amount

- $\partial_K \partial_K \partial_L F(L, K)$: How does the responsiveness of the MPL to capital vary as we increase capital

Problem 9. In class we discussed the idea of the law of diminishing marginal utility. What does this law suggest about the slope of indifference curves?

Answer – The law of diminishing marginal utility states that as we consume more of a good, the benefit from that additional consumption decreases. Mathematically, this suggests that the partial derivative of the utility function is a decreasing function. As we consume more of one good, in order to stay on the same indifference curve, we will consume less of the other good. The two points taken together suggest that as we consumer more of x_1, $\partial x_1 U(x_1, x_2)$ declines while $\partial x_2 U(x_1, x_2)$ increases. Since we can express the slope of the indifference curves in terms on the these partial derivatives, the above reasoning suggests that the MRS is declining in absolute value as we increase x_1.

$$\text{MRS}(x_1, x_2) = -\frac{\partial x_1 U(x_1, x_2)}{\partial x_2 U(x_1, x_2)}$$



(a) Graph of a Partially Evaluated Utility Function

(b) Graph of the Derivative (a)

Problem 10. On the first day of your job as an Economist at Uber, your manager comes to you with the following information: They estimated the relationship between the base price that a driver receives (p) and the number of hours a driver will work for (h).

$$W :: \text{Price} \to \text{Hours}$$

Your manager says that they would like to know the rate of change of costs with respect to number of hours worked. Is this feasible with the information provided? Explain.

Solution. It is feasible

Costs:

$$C(h) := W^{-1}(h)h$$

Aim: $\frac{d}{dh}C(h)$ We can start by differentiating both sides

$$\frac{d}{dh}C(h) = \frac{d}{dh}\big(W^{-1}(h)h)$$
$$\frac{d}{dh}C(h) = \frac{d}{dh}\big(W^{-1}(h))h + W^{-1}(h)$$

Aim: $\frac{d}{dh}\big(W^{-1}(h))$ We can start by writing the following equation

$$W(W^{-1}(h)) = h$$
$$\frac{d}{dh}W(W^{-1}(h)) = \frac{d}{dh}h$$
$$\frac{d}{dp}W\big(W^{-1}(h)\big)\frac{d}{dh}W^{-1}(h) = 1$$
$$\frac{d}{dh}W^{-1}(h) = \frac{1}{\frac{d}{dp}W\big(W^{-1}(h)\big)}$$

Therefore, we can compute this rate of change as follows:

$$\frac{d}{dh}C(h) = \frac{1}{\frac{d}{dp}W\big(W^{-1}(h)\big)}h + W^{-1}(h)$$

□

Problem Bonus. Some columnists have lamented that teachers are leaving positions in response to Covid-19 because the classroom environment has become more difficult. As Chief Economist of the Education Department, you are concerned about this potential issue, but what other effect of Covid-19 might have a longer lasting impact on the teacher shortage?