Risk Management — Assignment 2

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Question 1 – Non-parametric Methods

We begin by loading the data and computing the log-returns:

```
# import data
Data <- read.table(file="RMIP Data Tutorial.txt", header=TRUE)

# convert to date format
Data$Date <- as.Date(Data$Date, '%m/%d/%Y')

# nl log returns
netherlands.lr <- diff(log(Data$Netherlands))

# gr log returns
greece.lr <- diff(log(Data$Greece))
```

Next, we consider the losses of an investment portfolio consisting of €100 in Dutch and Greek equity respectively.

```
# losses

gr.losses <- greece.lr * -1 * 100

nl.losses <- netherlands.lr * -1 * 100

combined.losses <- nl.losses + gr.losses
```

For the remainder of the exercise we will consider a time horizon $h = 1$ and $\mathrm{VaR}_{.95}$ as $\varrho(L)$.

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(a) The Haircut Allocation

By definition, we have:

$$\mathrm{AC}_j = \gamma \cdot F_{L_j}^{-1}(p)$$

for γ satisfying

$$\sum_{j=1}^{d} \mathrm{AC}_j = \varrho(L).$$

Stated more directly,

$$\mathrm{AC}_i = \frac{\varrho(L)}{\sum_{j=1}^{d} F_{L_j}^{-1}(p)} \cdot F_{L_i}^{-1}(p).$$

For this exercise,

$$\mathrm{VaR}_\alpha(L) \equiv F_L^{-1}(\alpha) = \varrho(L).$$

In R:

```
# 1-day 95% VaR
var.gr <- quantile(gr.losses, probs = c(.95))
var.nl <- quantile(nl.losses, probs = c(.95))
var.combined <- quantile(combined.losses, probs = c(.95))

# allocation
allocate.nl <- ((var.combined)/(var.nl + var.gr)) * var.nl
allocate.gr <- ((var.combined)/(var.nl + var.gr)) * var.gr

# results
> allocate.nl
    95%
1.76199

> allocate.gr
     95%
2.415319
```

(b) Covariance Allocation

Principle:

$$\mathrm{AC}_i = \frac{\mathrm{Cov}(L_i, L)}{\mathrm{Var}(L)} \cdot \varrho(L)$$

In R:

```
# covariance method
variance.losses <- var(combined.losses)
cov.greece <- cov(gr.losses, combined.losses)
cov.nl <- cov(nl.losses, combined.losses)

# allocation 2
allocate2.gr <- (cov.greece / variance.losses) * var.combined
allocate2.nl <- (cov.nl / variance.losses) * var.combined

# results
> allocate2.nl
     95%
1.633856

> allocate2.gr
     95%
2.543454
```

(c) Quantile Allocation

We need to find p^* such that

$$\mathrm{AC}_i = F_{L_i}^{-1}(p^*)$$

and

$$\sum_{j=1}^{d} \mathrm{AC}_j = \varrho(L).$$

From the lecture, we define:

$$S^c = \sum_{i=1}^{d} F_{L_i}^{-1}(U)$$

where $U \sim \mathrm{Uniform}(0,1)$.

In R:

```
# loss vectors ordered
gr.ordered <- gr.losses[order(gr.losses)]
nl.ordered <- nl.losses[order(nl.losses)]

# comonotone sum
comonotone <- nl.ordered + gr.ordered
```

```
# recall the VaR of combined losses variable
var.combined                                # [1] 4.17731

# quantiles of comonotone sum
a <- quantile(comonotone, probs = c(0.93))  # [1] 4.090931
b <- quantile(comonotone, probs = c(0.94))  # [1] 4.446706
```

Having used the hint, we have found values for $p^* = .93$ and $\hat{p} = .94$ meeting the specified conditions.

Therefore, it suffices to solve

$$\alpha \cdot F_{L_{nl}}^{-1}(p^*) + (1-\alpha) \cdot F_{L_{nl}}^{-1}(\hat{p})$$

and

$$\alpha \cdot F_{L_{gr}}^{-1}(p^*) + (1-\alpha) \cdot F_{L_{gr}}^{-1}(\hat{p}).$$

for $\alpha \in [0,1]$ such that there is a full allocation.

Re-writing these equations as:

$$\alpha \cdot w + (1-\alpha) \cdot x$$

and

$$\alpha \cdot y + (1-\alpha) \cdot z,$$

we find the closed-form expression for α to be:

$$\alpha = \frac{\varrho(L) - (x+z)}{(w+y) - (x+z)}$$

Plugging in and solving in R, we have:

```
#  quantile allocation continued
w <- quantile(nl.losses, probs = c(0.93))
x <- quantile(nl.losses, probs = c(0.94))
y <- quantile(gr.losses, probs = c(0.93))
z <- quantile(gr.losses, probs = c(0.94))

# results
alpha <- (var.combined - (x + z)) / ((w + y) - (x + z))
allocationC.nl <- alpha * w + (1 - alpha) * x
allocationC.gr <- alpha * y + (1 - alpha) * z

> alpha
[1] 0.7572107

> allocationC.nl
[1] 1.755398

> allocationC.gr
[1] 2.421911
```

(d) CTE Allocation

By definition,

$$\mathrm{CTE}_\alpha(L) = \sum_{i=1}^{d} \mathrm{E}[L_i \mid L > \mathrm{VaR}_\alpha(L)]$$

with allocation

$$\mathrm{AC}_i = \frac{\varrho(L)}{\mathrm{CTE}_\alpha(L)} \cdot \mathrm{E}[L_i \mid L > \mathrm{VaR}_\alpha(L)].$$

To simplify matters, we will be using the shorthand formula mentioned in the tutorial:

$$\mathrm{E}(A \mid B) = \frac{\mathrm{E}(A \cdot \mathbb{1}B)}{\mathbb{P}(B)}$$

Consider:

```
# (L > VaR)
events <- which(combined.losses > var.combined)
greece.cte <- gr.losses[events]
nl.cte <- nl.losses[events]

# denominator
cte <- (1 / 0.05) * (mean(greece.cte) + mean(nl.cte))

# allocations
cte.allocation.nl <- (var.combined / cte) * (mean(nl.cte)/.05)
cte.allocation.gr <- (var.combined / cte) * (mean(greece.cte)/.05)

# results
> cte.allocation.nl
     95%
1.733956

> cte.allocation.gr
     95%
2.443354
```

Question 2 – Parametric Methods

(a) The Haircut Allocation

We now assume that the losses are bivariate Gaussian *i.e*:

$$\begin{pmatrix} L_{nl} \\ L_{gr} \end{pmatrix} \sim \mathrm{N}(\mu, \mathbf{\Sigma})$$

where

$$\mu = \begin{pmatrix} \mu_{nl} \\ \mu_{gr} \end{pmatrix}, \quad \mathbf{\Sigma} = \begin{pmatrix} \sigma^2_{nl} & \sigma_{nl,gr} \\ \sigma_{gr,nl} & \sigma^2_{gr} \end{pmatrix}.$$

I will also use the following result:

Suppose $(X_1, \ldots, X_n)$ are jointly multivariate normally distributed.

Consider the linear transformation $\mathbf{Y} = \mathbf{a}^T\mathbf{X}$, where $\mathbf{a} = (a_1, \ldots, a_n)^T$, $\mathbf{X} = (X_1, \ldots, X_n)^T$.

Then:

$$\mathbf{Y} = \mathbf{a}^T\mathbf{X} \sim \mathrm{N}(\mathbf{a}^T\mu, \mathbf{a}^T\mathbf{\Sigma}\mathbf{a})$$

Consider $L = L_{nl} + L_{gr}$.

```
# preliminaries
mu <- c(mean(nl.losses), mean(gr.losses))
covariance.l <- cov(nl.losses, gr.losses)
nl.variance <- var(nl.losses)
gr.variance <- var(gr.losses)

# establish distribution parameters
a <- c(1,1)
Covmat <- rbind(c(nl.variance, covariance.l),
                c(covariance.l, gr.variance))

l.mu <- t(a) %*% mu
l.sig2 <- t(a) %*% Covmat %*% a
```

It follows that $L \sim \mathrm{N}(-0.0067, 6.8344)$.

Therefore,

```
# 1-day 95% VaR
alpha <- .95
var.nl <- qnorm(alpha, mean = mean(nl.losses), sd = sqrt(nl.variance))
var.gr <- qnorm(alpha, mean = mean(gr.losses), sd = sqrt(gr.variance))
var.losses <- qnorm(alpha, mean = l.mu, sd = sqrt(l.sig2))
```

```
# allocation
allocate.nl <- ((var.losses)/(var.nl + var.gr)) * var.nl
allocate.gr <- ((var.losses)/(var.nl + var.gr)) * var.gr

# results
> allocate.nl
[1] 1.810552

> allocate.gr
[1] 2.482786
```

(b) Covariance Allocation

We use the fact that:

$$\begin{aligned}
\mathrm{Cov}(L_{nl}, L) &= \mathrm{Cov}(L_{nl}, L_{nl} + L_{gr}) \\
&= \mathrm{Cov}(L_{nl}, L_{nl}) + \mathrm{Cov}(L_{nl}, L_{gr}) \\
&= \mathrm{Var}(L_{nl}) + \mathrm{Cov}(L_{nl}, L_{gr}).
\end{aligned}$$

In R:

```
# covariance method
variance.losses <- l.sig2
cov.nl <- nl.variance + covariance.l
cov.greece <- gr.variance + covariance.l

# allocation: covariance (parametric)
allocate.nl <- (cov.nl / variance.losses) * var.losses
allocate.gr <- (cov.greece / variance.losses) * var.losses

# results
> allocate.nl
[1] 1.679238

> allocate.gr
[1] 2.6141
```

(c) Quantile Allocation

For this question, we follow a very similar process to Question 1 part (c), with the difference being that the VaR and quantiles for the losses can be computed parametrically.

```
# recall the comonotone sum variable
comonotone

# and the risk measure to be allocated
var.losses                                  # 4.293338

# quantiles of comonotone sum
a <- quantile(comonotone, probs = c(0.93))  # 4.090931
b <- quantile(comonotone, probs = c(0.94))  # 4.446706

#  quantiles for equations
w <- qnorm(.93, mean = mean(nl.losses), sd = sqrt(nl.variance))
x <- qnorm(.94, mean = mean(nl.losses), sd = sqrt(nl.variance))
y <- qnorm(.93, mean = mean(gr.losses), sd = sqrt(gr.variance))
z <- qnorm(.94, mean = mean(gr.losses), sd = sqrt(gr.variance))

alpha <- (var.losses - (x + z)) / ((w + y) - (x + z))
allocation.nl <- alpha * w + (1 - alpha) * x
allocation.gr <- alpha * y + (1 - alpha) * z

# results
> alpha
[1] 2.334568

> allocation.nl
[1] 1.808212

> allocation.gr
[1] 2.485126
```

We have

$$\sum_{j=1}^{d} \mathrm{AC}_j = \varrho(L),$$

but $\alpha \notin [0,1]$, so we have been unable to find a solution.

(d) CTE Allocation

Using the formula in the hint:

```
# recall the variables from 2b)
cov.nl <- nl.variance + covariance.l
cov.greece <- gr.variance + covariance.l

# allocations
alpha <- .95
cte.nl <- (mean(nl.losses) + (cov.nl / sqrt(l.sig2)) *
             (dnorm(qnorm(alpha)) / (1-alpha)))

cte.gr <- (mean(gr.losses) + (cov.greece / sqrt(l.sig2)) *
             (dnorm(qnorm(alpha)) / (1-alpha)))

cte.total = cte.nl + cte.gr

cte.allocation.nl <- (cte.nl / cte.total) * var.losses
cte.allocation.gr <- (cte.gr / cte.total) * var.losses

# results
> cte.allocation.nl
[1] 1.667893

> cte.allocation.gr
[1] 2.625445
```