Risk Management — Assignment 1

AJ*

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Question 1

(a) We use the formula:

$$\mathrm{VaR}_\alpha = \mu + \sigma\Phi^{-1}(\alpha)$$

Thus, the 10-day $\mathrm{VaR}_{.99}$ is given by:

```
sigma <- 5e6 * 0.01 * sqrt(10)
VaR <- 0 +  sigma * qnorm(.99)
```

So, $\mathrm{VaR}_{.99} =$ `367827.9`

(b) (1)

We have:

$$\sigma_m = 0.02, \sigma_a = 0.01, \rho = 0.3$$

Let

$$\mathbf{w} = \begin{pmatrix} w_m \\ w_a \end{pmatrix}$$

and

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_m^2 & \sigma_{ma} \\ \sigma_{am} & \sigma_a^2 \end{pmatrix}$$

Now,

$$\mathrm{E}[\mathbf{w}^T\mu] = 0$$

and

$$\begin{aligned} \sigma_p^2 &= \mathbf{w}^T\mathbf{\Sigma}\mathbf{w} \\ &= w_m^2\sigma_m^2 + w_a^2\sigma_a^2 + 2w_m w_a \rho\sigma_m\sigma_a \end{aligned}$$

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*Student number: ∞

in R:

```
mu.p <- 0
sigma.m <- 2e6 * 0.02 * sqrt(10)
sigma.a <- 5e6 * 0.01 * sqrt(10)
sigma.p2 <- ((2/7)^2 * (sigma.m)^2 + (5/7)^2 * (sigma.a)^2 +
               2*(2/7)*(5/7)*0.3*sigma.m*sigma.a)
```

Using the same formula from **(a)**, we see:

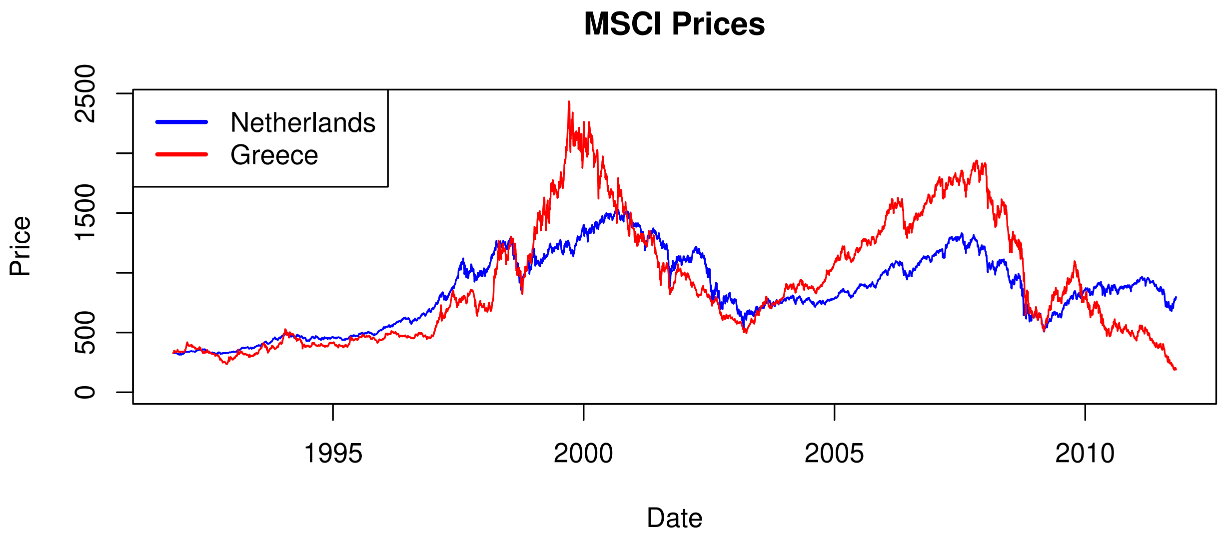
```
VaR.p <- mu.p + sqrt(sigma.p2) * qnorm(.99)
[1] 298917.2
```

(2) In this instance, the benefit is that by adding Microsoft stock to our portfolio, we have **increased** the overall size of our position, yet we have **reduced** the Value-at-Risk. By combining assets whose returns are not highly correlated, we have reduced the non-systematic risk component.

(3) In this case, VaR is subadditive. However, this does not hold more generally.

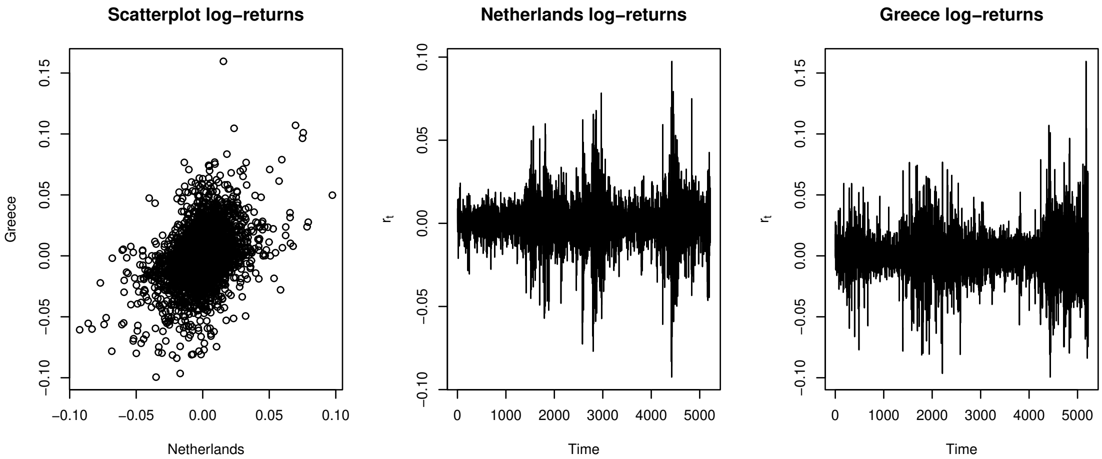
Question 2

```
# import data
Data <- read.table(file="RMIP Data Tutorial.txt", header=TRUE);

# convert to date format to be used as the X-axis
Data$Date <- as.Date(Data$Date, '%m/%d/%Y');

# Identifying largest Index for plotting purposes
index.max <- which.max(Data$Greece)
value.max <- Data$Greece[index.max]

# plot Dutch price dynamics
plot(Data$Date, Data$Netherlands, type = "l",
     col = "blue", xlab = "Date", ylab = "Price" ,
     main = "MSCI Prices", ylim = c(0, value.max));

# plot Greek price dynamics
lines(Data$Date, Data$Greece, type = "l", col = "red");

# include legends
legend("topleft", c("Netherlands","Greece"),
       lty=c(1,1), lwd=c(2.5,2.5), col=c(" blue" ,"red"))
```



(1)

```
# nl log returns
netherlands.lr <- diff(log(Data$Netherlands))

# gr log returns
greece.lr <- diff(log(Data$Greece))

# plot log returns
par(mfrow=c(1,3))

plot(netherlands.lr, greece.lr, main="Scatterplot log-returns",
     xlab = "Netherlands", ylab = "Greece")

plot(netherlands.lr, xlab="Time", type="l",
     ylab=expression('r'[t]), main="Netherlands log-returns")

plot(greece.lr, xlab="Time", type="l",
     ylab=expression('r'[t]), main="Greece log-returns")
```



(1) *continued:*

```
# 2.1 Compute 10 Day .99 VaR Netherlands
h <- 10
mean.nl <- mean(netherlands.lr)
sd.nl <- sd(netherlands.lr)
var.nl <- qnorm(.99, mean = mean.nl*h, sd = sd.nl * sqrt(h))
[1] 0.09966107
```

(2) Now, on the basis of a Gaussian model, we can compute the $\mathrm{ES}_{.99}$ analytically using:

$$\mathrm{ES}_\alpha = \mu + \sigma \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$$

In R:

```
# 2.2 Compute the 10 Day 99% Expected Shortfall Netherlands
alpha <- .99
es.nl <- (mean.nl*h +
   sd.nl * sqrt(h) * (dnorm(qnorm(alpha))/ (1-alpha)))
[1] 0.1139325
```

(3) We repeat these calculations for Greece:

```
# 2.3 Compute 10 Day .99 VaR, ES Greece

# VaR
mean.gr <- mean(greece.lr)
sd.gr <- sd(greece.lr)
var.gr <- qnorm(.99, mean = mean.gr*h, sd = sd.gr*sqrt(h))
[1] 0.1318504

# Expected Shortfall
alpha <- .99
es.gr <- (mean.gr*h +
             sd.gr*sqrt(h) * (dnorm(qnorm(alpha))/ (1-alpha)))
[1] 0.1512039
```

(3) *continued:*

We see that the Netherlands had a 10 day Value-at-Risk of approximately **9.97%** and Expected Shortfall of **11.4%**, compared to Greece's figures of **13.19%** and **15.12%** respectively.

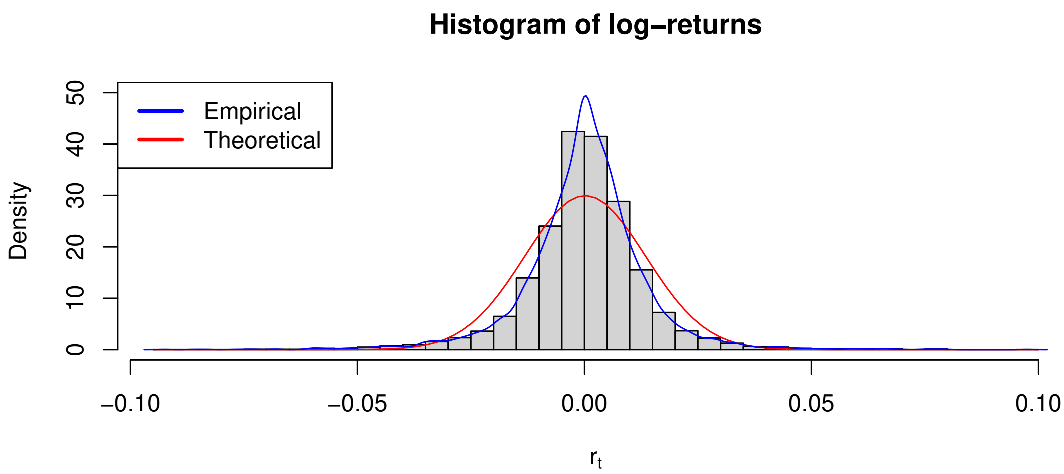
The data shows that Greece had more volatile returns over the period of observation, with the first implication (inferred using *Value-at-Risk*) being that larger losses occur with equal probably for Greece as opposed to the Netherlands.

Secondly, once this threshold was exceeded, the losses for Greece were expected to be more substantial than those expected for the Netherlands, as evidenced by the *Expected Shortfall* figures.

P.T.O

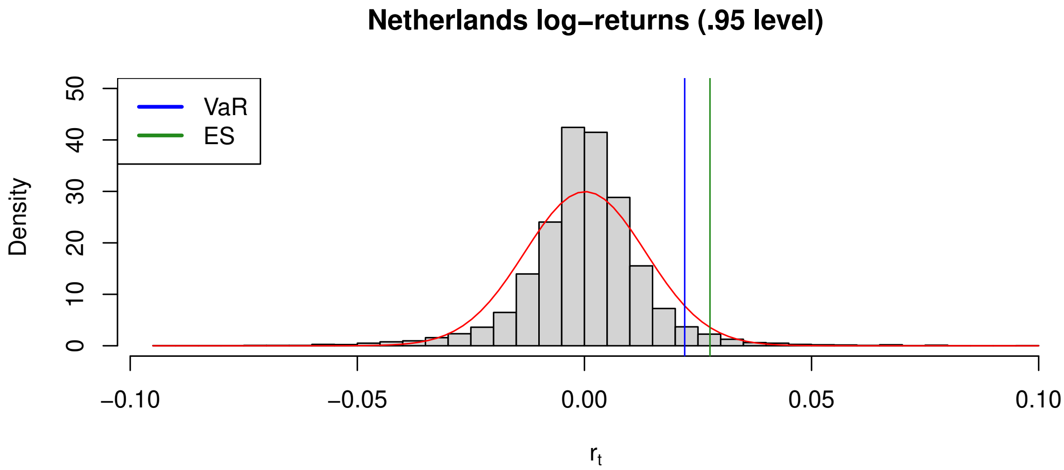
(4) Before assessing the sensitivity of our metrics, let's unpack the daily log returns (which we have assumed to be Gaussian):

```
# histogram vs density
hist(netherlands.lr, freq = FALSE, breaks=40, ylim = c(0,50),
     main="Histogram of log-returns", xlab =expression('r'[t]))
curve(dnorm(x, mean=mean(netherlands.lr), sd=sd(netherlands.lr)),
      add=TRUE, col="red")
lines(density(netherlands.lr), col="blue")

# include legends
legend("topleft", c("Empirical","Theoretical"),
       lty=c(1,1), lwd=c(2.5,2.5), col=c(" blue" ,"red"))
```

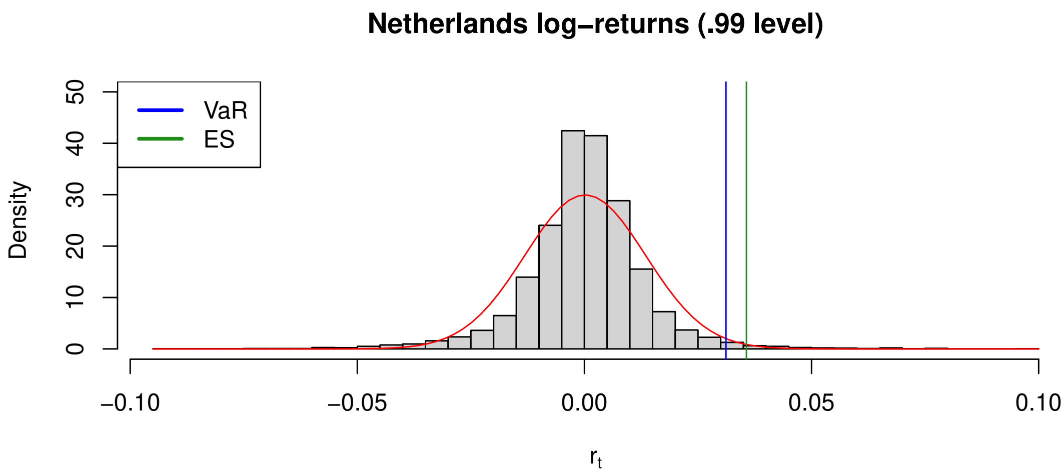


As expected, this assumption needs to be scrutinised a bit more, but for the sensitivity analysis we will proceed under the assumption of Gaussian log-returns.

```
# 2.4 Generate plots of VaR and ES for varying alpha
#  .95 Netherlands
alpha <- .95
mean.nl <- mean(netherlands.lr)
sd.nl <- sd(netherlands.lr)
var.nl <- qnorm(alpha, mean = mean.nl, sd = sd.nl)
es.nl <- (mean.nl +
             sd.nl * (dnorm(qnorm(alpha))/ (1-alpha)))
# plot
par(mfrow=c(1,1))
hist(netherlands.lr, freq = FALSE, breaks=40, ylim = c(0,50),
     main="Netherlands log-returns (.95 level)", xlab =expression('r'[t]))
curve(dnorm(x, mean=mean(netherlands.lr), sd=sd(netherlands.lr)),
      add=TRUE, col="red")
abline(v=var.nl, col="blue")
abline(v=es.nl, col="forestgreen")

# include legends
legend("topleft", c("VaR","ES"),
       lty=c(1,1), lwd=c(2.5,2.5), col=c("blue" ,"forestgreen"))
```



```
#  .99 VaR Netherlands
alpha <- .99
var.nl <- qnorm(alpha, mean = mean.nl, sd = sd.nl)
es.nl <- (mean.nl +
             sd.nl * (dnorm(qnorm(alpha))/ (1-alpha)))

# plot
par(mfrow=c(1,1))
hist(netherlands.lr, freq = FALSE, breaks=40, ylim = c(0,50),
     main="Netherlands log-returns (.99 level)", xlab =expression('r'[t]))
curve(dnorm(x, mean=mean(netherlands.lr), sd=sd(netherlands.lr)),
      add=TRUE, col="red")
abline(v=var.nl, col="blue")
abline(v=es.nl, col="forestgreen")

# include legends
legend("topleft", c("VaR","ES"),
       lty=c(1,1), lwd=c(2.5,2.5), col=c("blue" ,"forestgreen"))
```



```
#  .9975 VaR Netherlands
alpha <- .9975
var.nl <- qnorm(alpha, mean = mean.nl, sd = sd.nl)
es.nl <- (mean.nl +
             sd.nl * (dnorm(qnorm(alpha))/ (1-alpha)))

# plot
par(mfrow=c(1,1))
hist(netherlands.lr, freq = FALSE, breaks=40, ylim = c(0,50),
     main="Netherlands log-returns (.9975 level)", xlab =expression('r'[t]))
curve(dnorm(x, mean=mean(netherlands.lr), sd=sd(netherlands.lr)),
      add=TRUE, col="red")
abline(v=var.nl, col="blue")
abline(v=es.nl, col="forestgreen")

# include legends
legend("topleft", c("VaR","ES"),
       lty=c(1,1), lwd=c(2.5,2.5), col=c("blue" ,"forestgreen"))
```