Risk Management — Assignment 5

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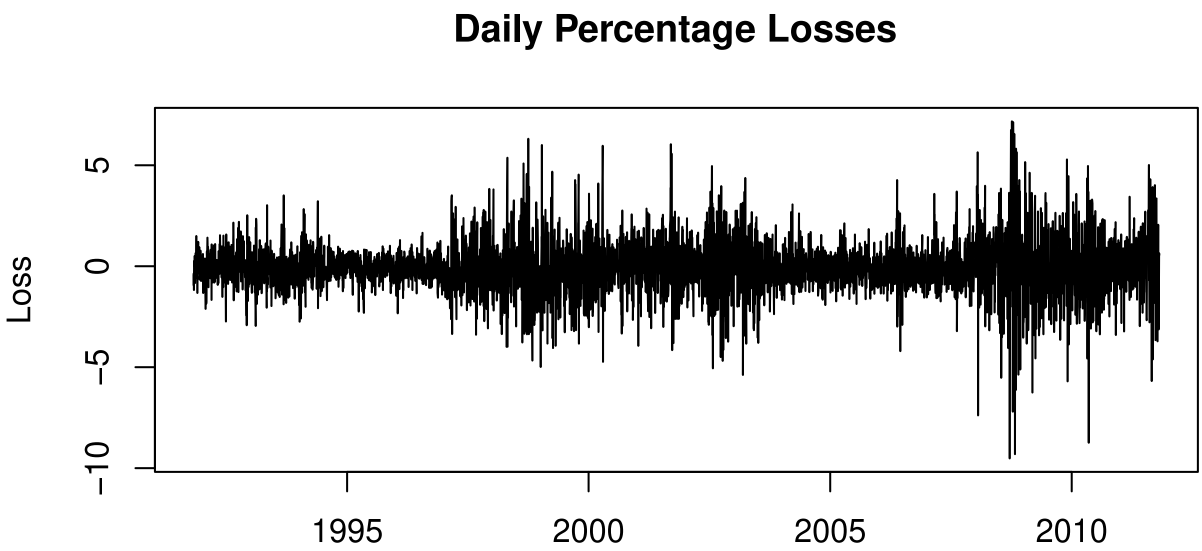
Question 1

```
## (a) values
values <- (prices$Netherlands+prices$Greece)/2

## (b) losses
losses <- 100*(1-Quot(values))  ## see package DescTools

## (c) plot
> plot(prices$Date[-1], losses, type="l", xlab="", ylab="Loss",
+      main ="Daily Percentage Losses", ylim=range(losses))
```



The daily percentage losses do not appear to be an *i.i.d* sample given the volatility clustering that is apparent in the data.

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Question 2

```
maxima <- NA
blocks <- split(losses, ceiling(seq_along(losses)/65))
> for(i in 1:80) {maxima[i] <- max(unlist(blocks[i]))}
```

Question 3

```
parameters <- fit.GEV(maxima)
xi.GEV <- parameters$par.ests[1]
mu.GEV <- parameters$par.ests[2]
sigma.GEV <- parameters$par.ests[3]
```

Our estimate of $\xi = 0.09469 > 0$, which would indicate that the distribution of the percentage losses is heavy-tailed.

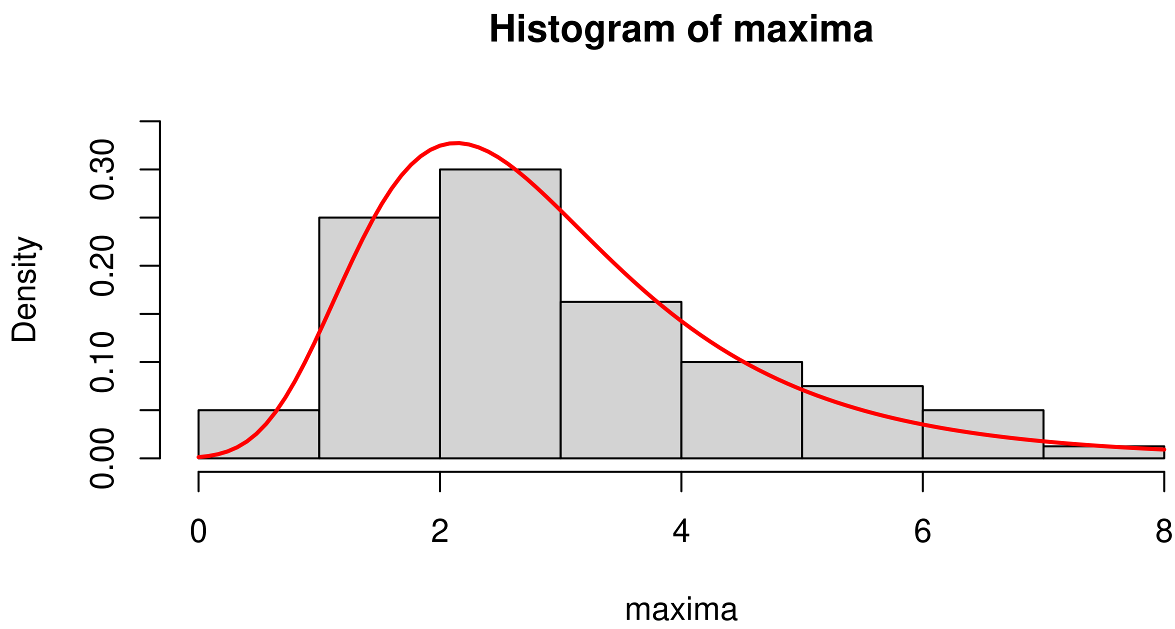
Question 4

```
neg.loglik <- function(par) {-sum(log(dGEV(maxima, par[1], par[2], par[3])))}
x <- 0; m <- 0; s <- 1
o1 <- optim(c(x,m,s), neg.loglik)
xi.o1 <- o1$par[1]; mu.o1 <- o1$par[2]; sigma.o1 <- o1$par[3];

> round(abs(c(xi.o1-xi.GEV, mu.GEV-mu.o1, sigma.GEV-sigma.o1)), 5)
     xi      mu   sigma
0.00001 0.00018 0.00005
```

P.T.O

Question 5

```
> hist(maxima, prob=T, ylim=c(0,0.35))
> curve(dGEV(x, xi=xi.GEV, mu=mu.GEV, sigma=sigma.GEV), lwd=2, col="red", add=T)
```



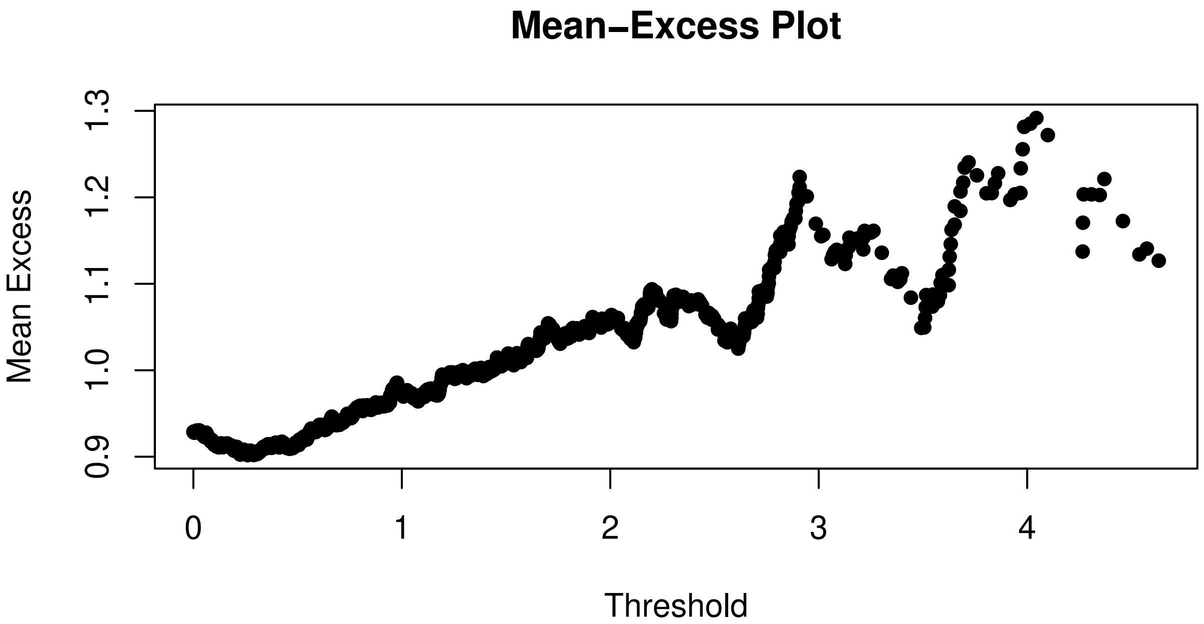
The GEV distribution provides a reasonable fit, although our estimate of ξ may need to be scrutinized given the excess empirical density towards the tail of the distribution.

Question 6

```
> 1/(1-pGEV(10, xi.GEV, mu.GEV, sigma.GEV))
200.4142
```

We see that the return period (in quarter-years) of a maximum daily loss of 10% or more is 200, hence we expect this to happen approximately once every 50 years.

Question 7



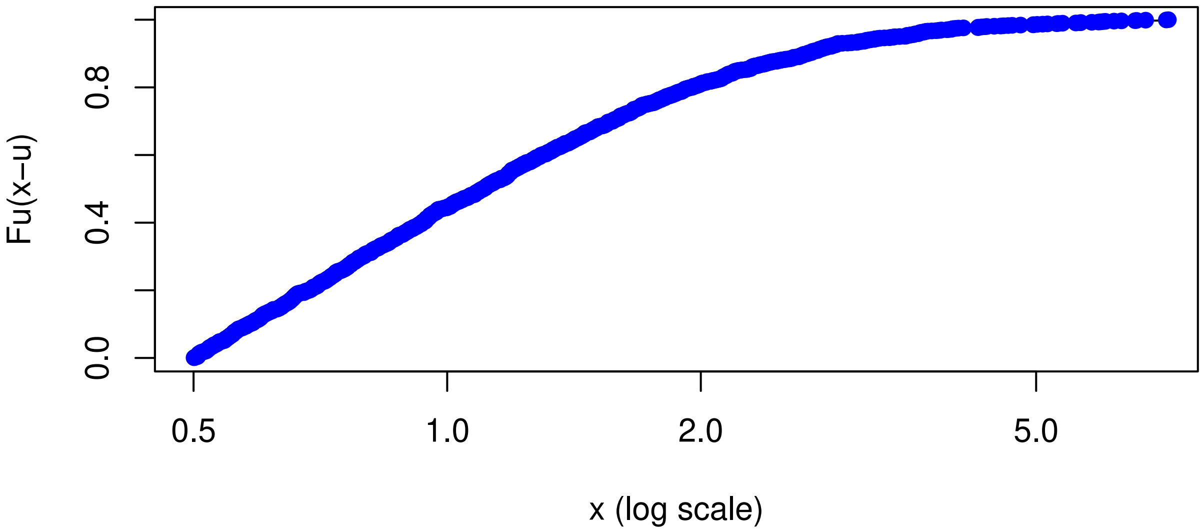

```
fitted.gpd <- fit.GPD(losses, threshold=0.5)
xi.GPD <- fitted.gpd$par.ests[1]    ## 0.08468195
beta.GPD <- fitted.gpd$par.ests[2]  ## 0.8380901
```

Our new estimate of $\xi = 0.0847$, which is slightly lower than the GEV maximum likelihood estimate. However, we would still consider this to be indicative of a distribution that exhibits heavy tails.

Question 8

```
excesses <- losses[losses>0.5]-0.5
neg.loglik <- function(par) {-sum(log(dGPD(excesses, par[1], par[2])))}
x <- 0; b <- 1;
o2 <- optim(c(x,b), neg.loglik)
xi.o2 <- o2$par[1]; beta.o2 <- o2$par[2];

> round(abs(c(xi.o2-xi.GPD, beta.o2-beta.GPD)), 5)
     xi    beta
0.00014 0.00003
```

Question 9

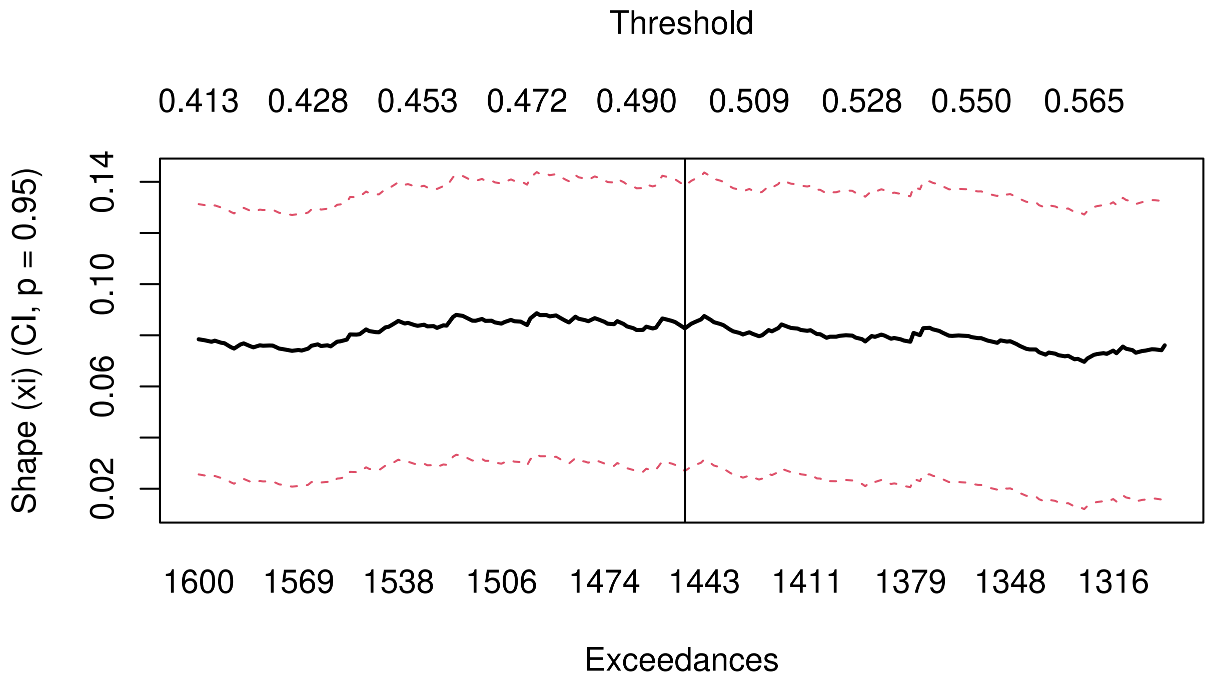


The GPD appears to fit the empirical distribution of the excesses above the threshold $u = 0.5$ very well.

Question 10

```
## (a)
u <- 0.5; xi <- xi.GPD; beta <- beta.GPD; p <- 0.995;
n <- length(losses); cap.N <- length(losses[losses>u])
val.risk <- u + (beta/xi)*((n*(1-p)/cap.N)^(-xi)-1)

> val.risk
[1] 4.510376

> quantile(losses, probs=.995)      ## verify
[1] 4.571094

## (b)
exp.short <- as.numeric(val.risk/(1-xi) + (beta-xi*u)/(1-xi))
> exp.short
[1] 5.797029

q <- as.numeric(quantile(losses, probs=.995)) ## verify
empirical.es <- mean(losses[losses>q])
> empirical.es
[1] 5.672348
```

Question 11



We note that, over the plotted range, our estimator $\hat{\xi}$ is insensitive to the chosen threshold ($u = 0.5$) *i.e.* it does not vary much at all, which is a positive result given the heuristic method we used to determine u.