Computational Electromagnetics

Assignment 2

COMPUTING THE S-PARAMETERS OF A WAVEGUIDE USING THE FDTD METHOD

1.1 Modal representation of the electric field in a rectangular waveguide

In an air-filled, rectangular waveguide with the transverse dimensions L_x and L_y, the electric and magnetic fields can be divided into *transverse electric* (TE_{mn}) and *transverse magnetic* (TM_{mn}) modes. Each mode has its own propagation constant γ_{mn} given by:

$$\gamma_{mn} = \sqrt{k_c^2 - k^2} = \sqrt{k_c^2 - k_0^2} = \sqrt{k_c^2 - \left(\frac{\omega}{c}\right)^2} \tag{1}$$

since $k = k_0$ for the air-filled waveguide, and where k_c is the cutoff wavenumber, being expressed as

$$k_c^2 = \left(\frac{m\pi}{L_x}\right)^2 + \left(\frac{n\pi}{L_y}\right)^2 \tag{2}$$

For TE modes, m and n are non-negative integers and also $m + n > 0$. For TM modes, both m and n are positive integers.

Numbering the modes from 1 to ∞, the electric field of both TE and TM modes can be written as

$$\vec{E}(x, y, z, t) = \sum_{n=1}^{\infty} V_m(z, t)\vec{e}(x, y) \tag{3}$$

where $V_m(z,t)$ is the modal amplitude, or voltage, of mode m (which can be either a TE or a TM mode) and $\vec{e}(x, y)$ is its modal field.

The frequencies (ω) and the dimensions (L_x, L_y) of a waveguide are usually chosen so that γ_{mn} is real for all modes except the dominant TE_{10} mode. Only this TE_{10} mode will exist far away from any source or irregularity in the waveguide. The other modes will decay exponentially since V_m is proportional to $\exp(-\gamma z)$.

1.2 Computing the S-Parameters

The quantity that is to be computed in this assignment is the relation between the amplitude of the TE_{10} mode at the two ends, or *ports*, of the rectangular waveguide. In order to accomplish this, a wave is sent into the waveguide through one of the ports and the wave that comes out through each port is measured. This is illustrated in Fig. 1 that follows.

Figure 1: Illustration of incoming and outgoing waves for a rectangular waveguide

Let $s_{1in}(t)$ be the amplitude of the incoming TE_{10} wave at port 1 and let $s_{1out}(t)$ and $s_{2out}(t)$ be the amplitudes of the outgoing TE_{10} waves at ports 1 and 2 respectively. Taking the Fourier transform of these signals, we get $S_{1in}(\omega)$, $S_{1out}(\omega)$ and $S_{2out}(\omega)$. The relations between the amplitudes at the different ports are usually described by the so-called S-parameters:

$$S_{11}(\omega) = \frac{S_{1out}(\omega)}{S_{1in}(\omega)} \tag{4}$$

$$S_{12}(\omega) = \frac{S_{2out}(\omega)}{S_{1in}(\omega)} \tag{5}$$

1.3 Basic Task

You are given a MATLAB program that computes the S-parameters for an empty rectangular waveguide. The dimensions of the waveguide under consideration are as follows:

$$L_x = 40.0 \text{ mm}, L_y = 22.5 \text{ mm and } L_z = 80.0 \text{ mm} \tag{6}$$

The incoming signal is a Gaussian-modulated sinusoidal pulse containing frequencies in the interval 4 to 7 GHz.

Question 1: What are the expected values of S_{11} and S_{12} for an empty (air-filled) waveguide in the current frequency interval (of 4 to 7 GHz)? Run the code and see if the result is what you expected.

Question 2: What is the cutoff frequency of the TE_{10} mode?

Question 3: Which mode has the second lowest cutoff frequency and what frequency is that?

Your task is now to change the shape of the waveguide to the one shown in Figs. 2 and 3. The walls are assumed to be perfectly conducting. Your report should include a plot of $|S_{11}|$ and $|S_{12}|$ together with the modified code.

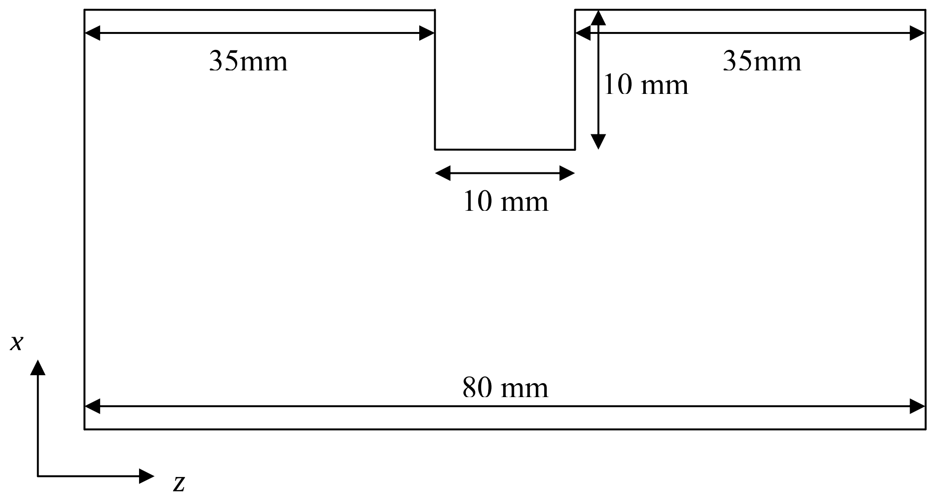

Figure 2: Modified geometry: *xz*-view

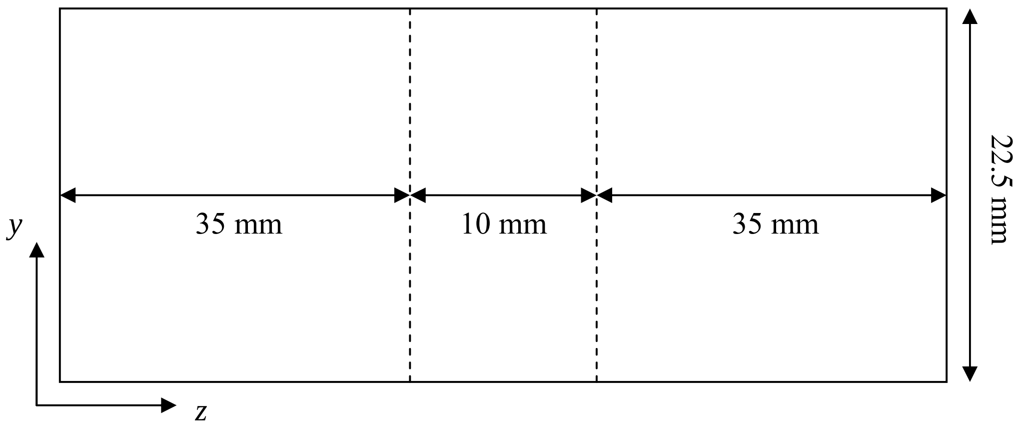

Figure 3: Modified geometry: *yz*-view