Antenna Theory

National Yang Ming Chiao Tung University

Final-Term Assignment
To be delivered by 5 January 2024

1. Rectangular Apertures

Consider two rectangular radiating apertures, one located in a plane perfect electric conducting (PEC) ground plane, and the other in an aperture plane in free space. Assume that the aperture fields in both cases are uniformly distributed over the aperture, i.e.,

$$\vec{E}_a(x,y) = E_0\hat{y} \quad \text{and} \quad \vec{H}_a(x,y) = -\frac{1}{\eta}E_0\hat{x}$$

for $|x| < a/2$ and $|y| < b/2$, where η is the free space wave impedance, and $\hat{y}$ is the direction of the polarization.

(a) Write down the expressions for the far-field functions for both apertures.

(b) Find the expressions for the co- and cross polar far-field functions for the two cases for a desired linear y-polarization.

(c) What are the levels in dB of the relative cross-polar field at $\theta = 10^\circ$ and $\theta = 20^\circ$ in the $\phi = 45^\circ$ plane for each of the two cases of rectangular radiating apertures?

2. Pyramidal Horn

Consider a y-polarized large pyramidal horn antenna with a rectangular aperture.

(ai)Assuming a spherical wavefront across the outermost aperture and making use of paraxial approximation, write down the far-field function of this horn antenna. The answer may be left as an expression that involves integrals, but must be in a form that can be evaluated.

(aii) What is the cross-polar radiation?

(b) Assuming now that the phase is constant across the outermost planar aperture parallel to the xy plane, write down the far-field function of this pyramidal horn, this time in closed-form. Express your answer in terms of the sinc function defined as: $\operatorname{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$.

(c) The same horn is excited for circular polarization in such a way that there is no cross-polarization at the center of the horn aperture. As was in part b), again assume that the phase over the planar outermost aperture of the pyramidal horn is constant for this part.

(ci) Assuming right-hand circular polarization (RHCP), what is the maximum relative cross-polar level in the aperture (i.e., relative to the co-polar maximum)?

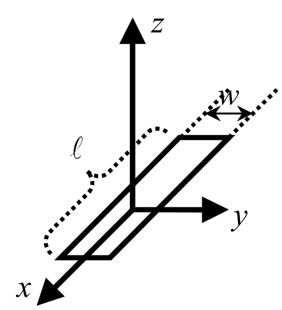
(cii) Use the Huygen's equivalent to derive an expression for the radiation field.

(ciii) Give the expressions for the co- and cross-polar radiation functions.

3. Linear Array of Open-Ended Rectangular Waveguides

Element

Consider first, an open-ended rectangular waveguide terminated in an aperture in an infinite perfect electric conducting (PEC) ground plane. Assume that the width $w << \lambda$ and the length $\ell = \lambda/2$. The coordinate system is shown in the figure below.



(a) Write down the approximate E-field distribution over the aperture.

(b) Introduce an equivalent current source and find the radiation field function of the aperture.

(c) What is the shape of the radiation field function in E-plane for $0 < \theta < 180^\circ$?

Linear Array

We want to use an equi-spaced linear array of such open rectangular waveguides radiating from a PEC ground plane to generate a beam which can be phase-steered in E-plane between broadside and 20° from broadside. The excitation of the array is uniform in amplitude and with a linearly progressing phase. In order to simplify the analysis, you are recommended to introduce the angle α which the radial unit vector $\hat{r}$ towards the direction of field observation makes with the y-axis, i.e., $\cos\alpha = \hat{r} \cdot \hat{y} = \sin\theta \sin\phi$.

Assume that the array factor (AF) of this linear array is given by:

$$AF = \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

where $\psi = kd\cos\alpha + \beta$

with β = phase progression from one element to the next,

and d = period (inter-element spacing/separation or unit-cell size) of the array.

(d) How long must the array be (along the direction of the periodicity) in wavelengths in order to get a main beam which at broadside has a beam width of 2° measured between the first two nulls in E-plane?

(e) Assume that a grating lobe is radiating in the end-fire direction: $\alpha = 180^{\circ}$ in E-plane. For the same array length of the previous part d), what is the width of this grating lobe between its maximum and the first null?

(f) What is the level in dB of this grating lobe relative to the main lobe?

(g) Write down the radiation field function of the element when we use the free space aperture formulas instead of the PEC aperture formulas.

(h) For this same case of the preceding part g) whereby the free-space aperture formulas are used instead, how large is now the relative level of a grating lobe in the $\alpha = 180^{\circ}$ direction if the main beam points at $\alpha = 70^{\circ}$?

(i) For the same main-beam direction of $\alpha = 70^{\circ}$ as part h) and the same length of the array as found in part d), how many array elements are needed so that there will be no radiation of grating lobes in visible space? (i.e., just the main-beam radiates and not even a single grating lobe would appear in visible space).

~ End of Assignment ~