Derivative Definition

$$\frac{d}{dx}[f(x)] = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \tag{1}$$

Differentiation Rules

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg' \tag{2}$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2} \tag{3}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \tag{4}$$

Mean Value Theorem

If f is a continuous function over the closed interval $[a,b]$ differentiable over the open interval (a,b), and $c \in (a,b)$.

$$f'(c) = \frac{f(b)-f(a)}{b-a} \tag{5}$$

Derivatives and the Shape of a Graph

1. If f' changes sign from positive when $x < c$ to negative when $x > c$, then $f(c)$ is a local maximum of f.
2. If f' changes sign from negative when $x < c$ to positive when $x > c$, then $f(c)$ is a local minimum of f.
3. If f' has the same sign for $x < c$ and $x > c$, then $f(c)$ is neither a local maximum nor a local minimum of f.

Concavity and Points of Inflection

Sign of f'	Sign of f''	Is f inc. or dec.?	Concavity
+	+	Increasing	Up
+	-	Increasing	Down
-	+	Decreasing	Up
-	-	Decreasing	Down

Power Rule of Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \tag{6}$$

Riemann Sums

$$L_n \approx \sum_{i=1}^{n} f(x_{i-1})\Delta x \tag{7}$$

$$R_n \approx \sum_{i=1}^{n} f(x_i)\Delta x \tag{8}$$

$$A \approx \sum_{i=1}^{n} f(x_i^*)\Delta x \tag{9}$$

The Definite Integral

$$\int_a^b f(x)dx = \lim_{n\to} \sum_{i=1}^{n} f(x_i^*)\Delta x \tag{10}$$

where:

- The limit exists and $f(x)$ is continuous on $[a,b]$, then f is integrable on $[a,b]$.

Average of a Function If $f(x)$ is continuous over the interval $[a,b]$.

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx \tag{11}$$

Fermat's Theorem If f has a local extremum at c and f is differentiable at c, then

$$f'(c) = 0 \tag{12}$$

Linear Approximations Consider a differentiable function f such that $\lim_{x\to a} f(x) = 0$. For x near a, we can write

$$L(x) = f(a) + f'(a)(x-a) \tag{13}$$

Common Derivatives and Their Antiderivatives

Differentiation	Indefinite Integration
$\frac{d}{dx}(k) = 0$	$\int kdx = kx + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
$\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln\|x\| + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \tan x \sec x dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \cot x \csc x dx = -\csc x + C$
$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$
$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$	$\int -\frac{1}{\sqrt{1-x^2}}dx = \arccos x + C$
$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}dx = \arctan x + C$
