Volatility Modeling

20 September 2024

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In [1]:
```
import numpy as np
import pandas as pd
from scipy.optimize import minimize
import matplotlib.pyplot as plt
import numpy as np
from scipy.optimize import minimize
from scipy.stats import norm, probplot
from arch import arch_model
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

Data

In [2]:
```
df = (
    pd
    .read_csv(
        'SPY ETF Stock Price History.csv',
        parse_dates=['Date']
    )
    .drop(columns='Change %')
    .rename(
        columns = {
            'Date': 'date',
            'Price': 'close',
            'Open': 'open',
            'High': 'high',
            'Low': 'low',
            'Vol.': 'volume'
        }
    )
    .set_index('date')
    .sort_index()
)
df['volume'] = df['volume'].str.slice(stop = -1).astype('float')
ser_price = df['close']
```

In [3]:
```
ser_returns = np.log(ser_price / ser_price.shift()).dropna()
```

Historical Volatility Models

In [4]:
```
ser_variance = ser_returns ** 2
df_historical_pred = pd.DataFrame({
    'Historical Average': np.sqrt(ser_variance.cumsum() / np.arange(1, len(ser_varianc
    'Simple Moving Average': np.sqrt(ser_variance.rolling(21).sum() / 21),
    'Exponential Moving Average': np.sqrt(ser_variance.ewm(span=len(ser_variance), adj
```

```
    'Exponential Weighted Moving Average': np.sqrt(ser_variance.ewm(span=21, adjust=Fa
})
```

In [5]:

```
test_size = 50
df_historical_pred[-test_size-1:-1].tail()
```

Out[5]:

date	Historical Average	Simple Moving Average	Exponential Moving Average	Exponential Weighted Moving Average
2024-08-23	0.010843	0.012649	0.010439	0.010979
2024-08-26	0.010841	0.012425	0.010436	0.010492
2024-08-27	0.010840	0.012428	0.010433	0.010013
2024-08-28	0.010839	0.012443	0.010431	0.009707
2024-08-29	0.010837	0.011935	0.010428	0.009255

In [6]:

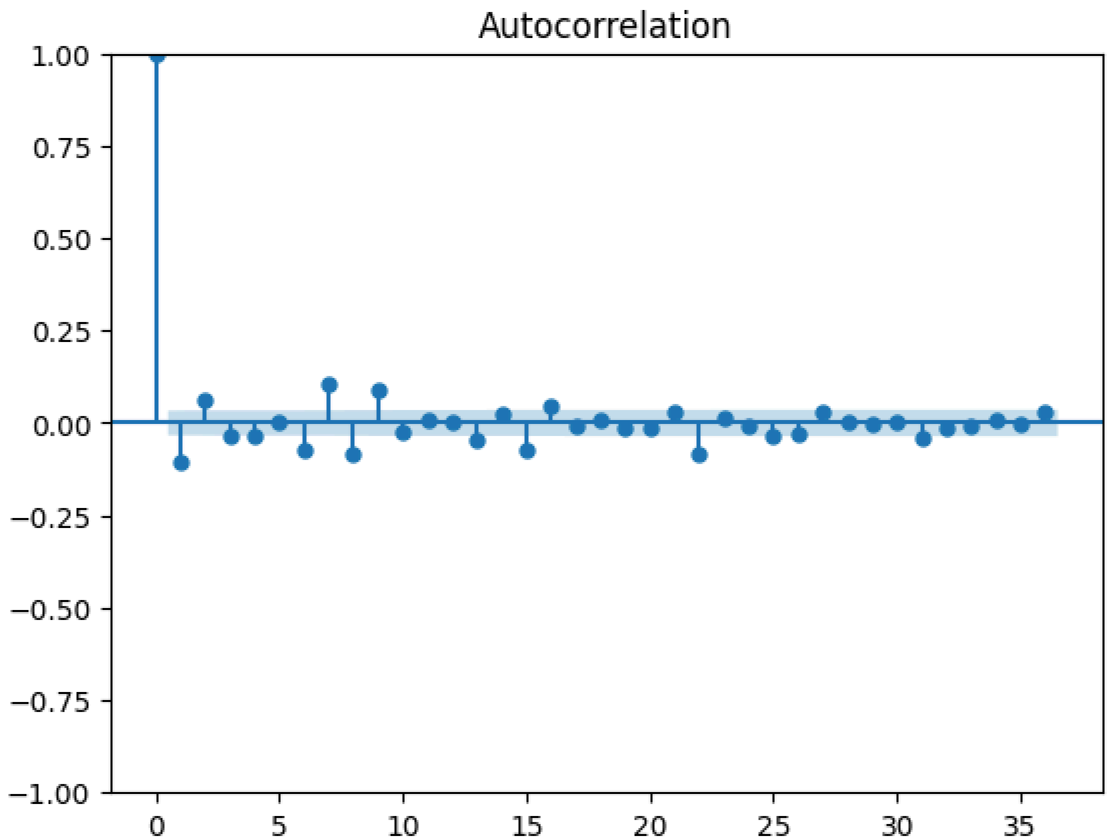
```
def historical_rolling_predictions(series, p=2, q=2, o=0, dist='normal', title=''):
    rolling_predictions = []
    for i in range(test_size):
        train = series[:-(test_size-i)]
        model = arch_model(train, p=p, q=q, o=o, dist=dist)
        model_fit = model.fit(disp='off')
        pred = model_fit.forecast(horizon=1)
        rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))
    plt.figure(figsize=(10,4))
    true, = plt.plot(series.rolling(window=21).std()[-test_size:].values)
    preds, = plt.plot(rolling_predictions)
    plt.title(title)
    plt.legend(['True Volatility', 'Predicted Volatility'])
```

ARCH/GARCH

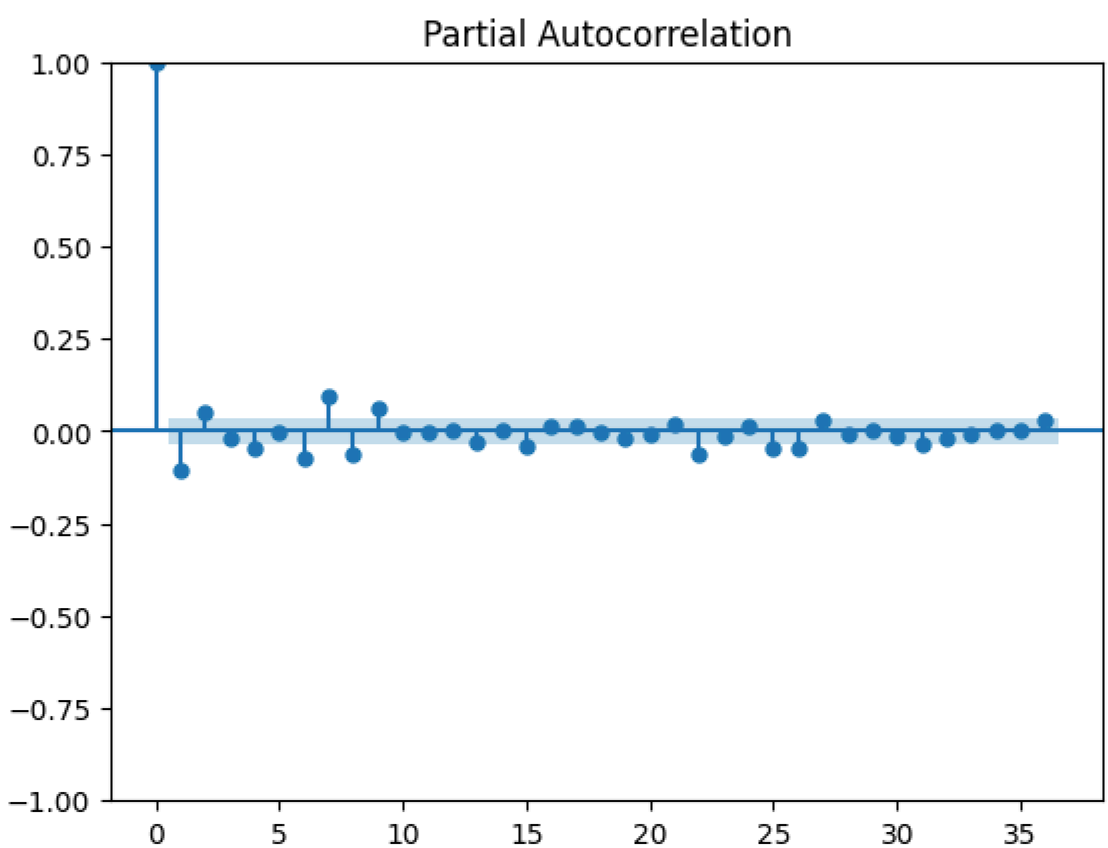
ACF/PACF Plots

In [7]:

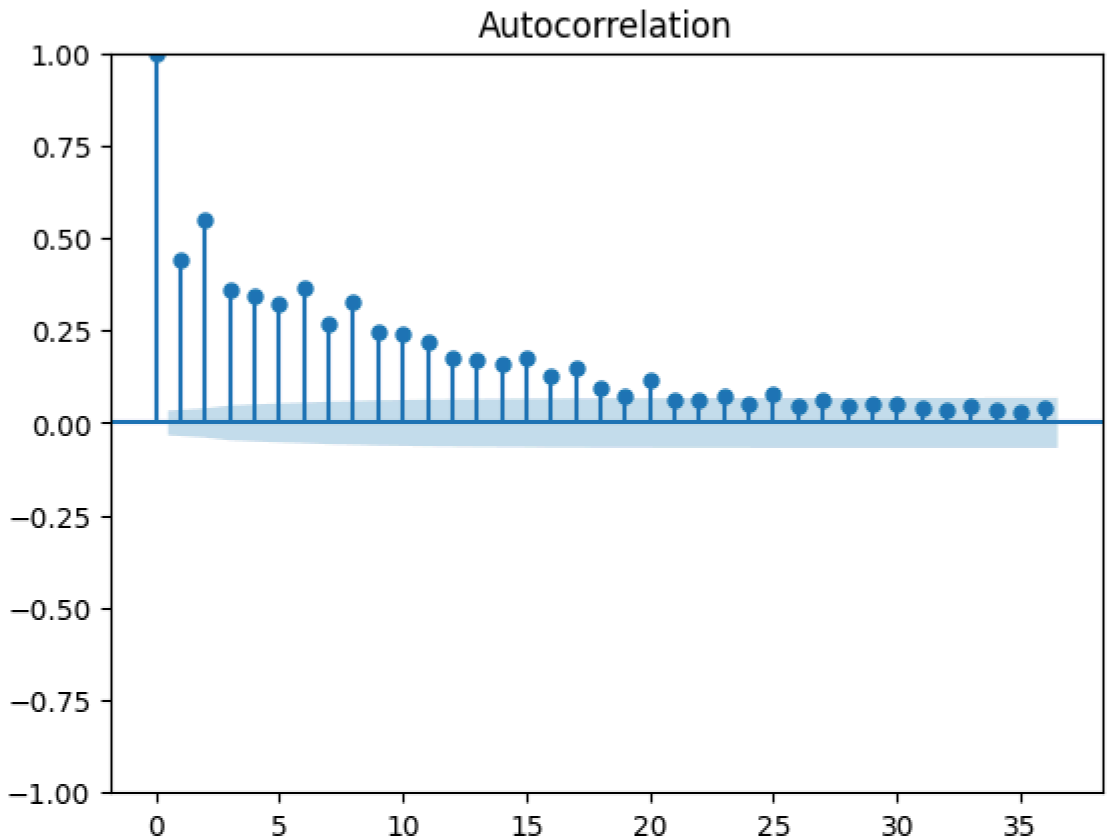
```
plot_acf(ser_returns)
plt.show()
```



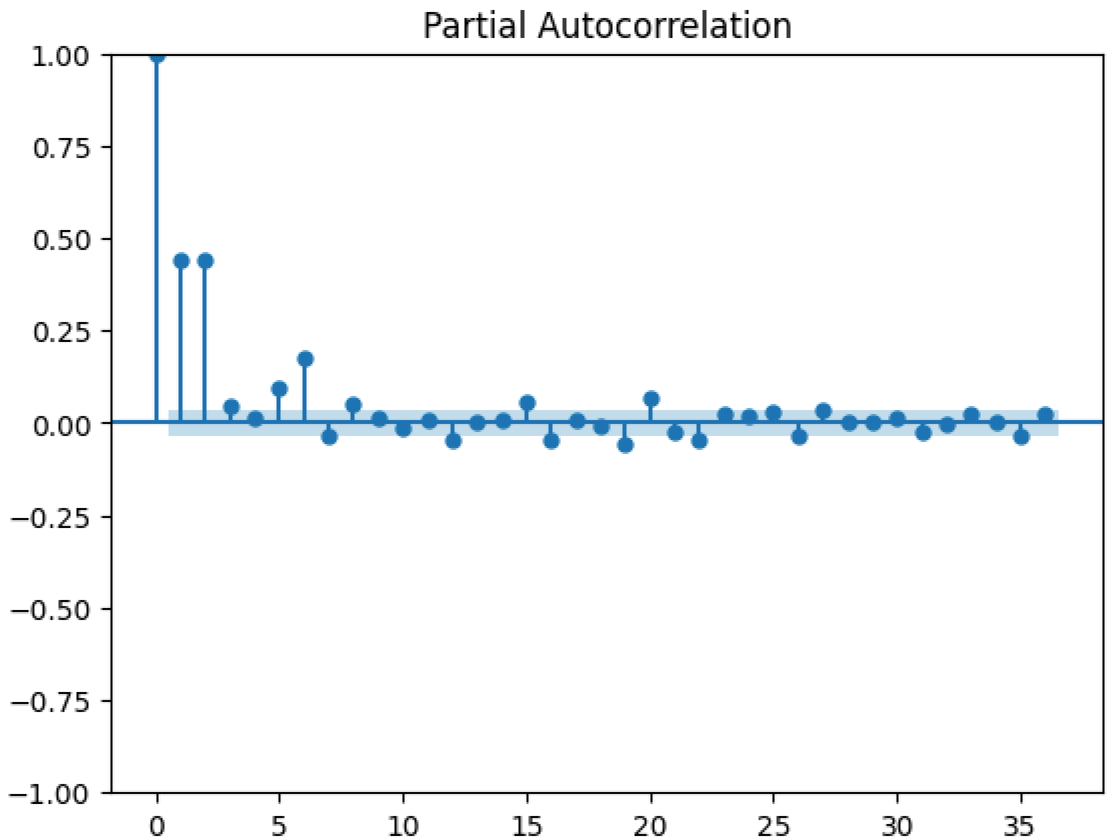
In [8]:
```
plot_pacf(ser_returns)
plt.show()
```



In [9]:
```
plot_acf((ser_returns - ser_returns.mean()) ** 2)
plt.show()
```



In [10]:
```
plot_pacf((ser_returns - ser_returns.mean()) ** 2)
plt.show()
```



Estimation

In [11]:
```
model_arch_1 = arch_model(ser_returns * 100, vol='Garch', p=1, q=0)
fitted_model_arch_1 = model_arch_1.fit()
model_garch_1_1 = arch_model(ser_returns * 100, vol='Garch', p=1, q=1)
fitted_model_garch_1_1 = model_garch_1_1.fit()
model_garch_2_2 = arch_model(ser_returns * 100, vol='Garch', p=2, q=2)
fitted_model_garch_2_2 = model_garch_2_2.fit()
```

```
Iteration:      1,   Func. Count:      5,   Neg. LLF: 23615.909671883626
Iteration:      2,   Func. Count:     14,   Neg. LLF: 8036.601651099558
Iteration:      3,   Func. Count:     22,   Neg. LLF: 5076.826483704974
Iteration:      4,   Func. Count:     27,   Neg. LLF: 4817.021531015172
Iteration:      5,   Func. Count:     32,   Neg. LLF: 4815.70958913206
Iteration:      6,   Func. Count:     36,   Neg. LLF: 4815.709565603178
Iteration:      7,   Func. Count:     39,   Neg. LLF: 4815.709565603053
Optimization terminated successfully    (Exit mode 0)
            Current function value: 4815.709565603178
            Iterations: 7
            Function evaluations: 39
            Gradient evaluations: 7
Iteration:      1,   Func. Count:      6,   Neg. LLF: 44203.00668218716
Iteration:      2,   Func. Count:     17,   Neg. LLF: 19923.260267375175
Iteration:      3,   Func. Count:     27,   Neg. LLF: 6759.323797102466
Iteration:      4,   Func. Count:     34,   Neg. LLF: 8754.764056310923
Iteration:      5,   Func. Count:     40,   Neg. LLF: 4767.975877147105
Iteration:      6,   Func. Count:     47,   Neg. LLF: 4374.8151661637
Iteration:      7,   Func. Count:     53,   Neg. LLF: 4368.328183919182
Iteration:      8,   Func. Count:     58,   Neg. LLF: 4368.327309282418
Iteration:      9,   Func. Count:     63,   Neg. LLF: 4368.32727161125
Iteration:     10,   Func. Count:     67,   Neg. LLF: 4368.327271611428
Optimization terminated successfully    (Exit mode 0)
            Current function value: 4368.32727161125
            Iterations: 10
            Function evaluations: 67
            Gradient evaluations: 10
Iteration:      1,   Func. Count:      8,   Neg. LLF: 24494.051833385416
Iteration:      2,   Func. Count:     20,   Neg. LLF: 15902.97738529019
Iteration:      3,   Func. Count:     32,   Neg. LLF: 6643.266347720565
Iteration:      4,   Func. Count:     41,   Neg. LLF: 6249.508413671872
Iteration:      5,   Func. Count:     49,   Neg. LLF: 4703.501475965731
Iteration:      6,   Func. Count:     57,   Neg. LLF: 4520.062365936684
Iteration:      7,   Func. Count:     65,   Neg. LLF: 4365.077607685911
Iteration:      8,   Func. Count:     72,   Neg. LLF: 4400.914463316514
Iteration:      9,   Func. Count:     80,   Neg. LLF: 4364.455073773498
Iteration:     10,   Func. Count:     87,   Neg. LLF: 4364.442919574739
Iteration:     11,   Func. Count:     94,   Neg. LLF: 4364.4419253000415
Iteration:     12,   Func. Count:    101,   Neg. LLF: 4364.441795791126
Iteration:     13,   Func. Count:    108,   Neg. LLF: 4364.441751193601
Iteration:     14,   Func. Count:    115,   Neg. LLF: 4364.441750121934
Iteration:     15,   Func. Count:    121,   Neg. LLF: 4364.441750121633
Optimization terminated successfully    (Exit mode 0)
            Current function value: 4364.441750121934
            Iterations: 15
            Function evaluations: 121
            Gradient evaluations: 15
```

In [12]:

```
fitted_model_arch_1
```

Out[12]:

```
                     Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                  close   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -4815.71
Distribution:                  Normal   AIC:                           9637.42
Method:            Maximum Likelihood   BIC:                           9655.85
                                        No. Observations:                 3442
Date:                Thu, Sep 19 2024   Df Residuals:                     3441
Time:                        21:11:22   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0844  1.707e-02      4.945  7.610e-07 [5.096e-02,  0.118]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.6953  4.520e-02     15.385  2.065e-53 [  0.607,  0.784]
alpha[1]       0.4066  6.365e-02      6.388  1.684e-10 [  0.282,  0.531]
==========================================================================

Covariance estimator: robust
ARCHModelResult, id: 0x13874933890
```

In [13]:

```
fitted_model_garch_1_1
```

Out[13]:

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  close   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -4368.33
Distribution:                  Normal   AIC:                           8744.65
Method:            Maximum Likelihood   BIC:                           8769.23
                                        No. Observations:                 3442
Date:                Thu, Sep 19 2024   Df Residuals:                     3441
Time:                        21:11:22   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0775  1.261e-02      6.149  7.806e-10 [5.281e-02,  0.102]
                               Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0387  7.692e-03      5.028  4.965e-07 [2.360e-02,5.375e-02]
alpha[1]       0.1811  2.275e-02      7.959  1.727e-15     [  0.137,  0.226]
beta[1]        0.7862  2.181e-02     36.042 1.851e-284     [  0.743,  0.829]
============================================================================

Covariance estimator: robust
ARCHModelResult, id: 0x13874966480
```

In [14]:

```
fitted_model_garch_2_2
```

Out[14]:

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                    close   R-squared:                       0.000
Mean Model:               Constant Mean   Adj. R-squared:                  0.000
Vol Model:                        GARCH   Log-Likelihood:               -4364.44
Distribution:                    Normal   AIC:                           8740.88
Method:              Maximum Likelihood   BIC:                           8777.75
                                          No. Observations:                 3442
Date:                  Thu, Sep 19 2024   Df Residuals:                     3441
Time:                          21:11:22   Df Model:                            1
                                  Mean Model
=========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
-------------------------------------------------------------------------
mu             0.0765  1.259e-02      6.079  1.212e-09 [5.185e-02,  0.101]
                               Volatility Model
=========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
-------------------------------------------------------------------------
omega          0.0730  1.427e-02      5.118  3.081e-07 [4.508e-02,  0.101]
alpha[1]       0.1511  2.815e-02      5.369  7.896e-08 [9.596e-02,  0.206]
alpha[2]       0.1811  2.623e-02      6.904  5.042e-12   [  0.130,  0.232]
beta[1]    2.8262e-03  9.687e-02  2.917e-02      0.977   [ -0.187,  0.193]
beta[2]        0.6017  8.119e-02      7.412  1.246e-13   [  0.443,  0.761]
=========================================================================

Covariance estimator: robust
ARCHModelResult, id: 0x13874d086e0
```

In [15]:

```
# Fit a GJR-GARCH model
model_gjr_garch_t = arch_model(
    ser_returns * 100, vol='Garch', p=2, q=2,
    o=1, dist='t'
)
fitted_model_gjr_garch_t = model_gjr_garch_t.fit(disp='off')
```

In [16]:

```
fitted_model_gjr_garch_t
```

Out[16]:

```
                     Constant Mean - GJR-GARCH Model Results                      
==================================================================================
Dep. Variable:                        close   R-squared:                     0.000
Mean Model:                   Constant Mean   Adj. R-squared:                0.000
Vol Model:                        GJR-GARCH   Log-Likelihood:               -4223.98
Distribution:      Standardized Student's t   AIC:                         8463.96
Method:                  Maximum Likelihood   BIC:                         8513.11
                                              No. Observations:               3442
Date:                      Thu, Sep 19 2024   Df Residuals:                   3441
Time:                              21:11:22   Df Model:                          1
                               Mean Model                               
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0653  1.108e-02      5.899  3.651e-09 [4.364e-02,8.706e-02]
                              Volatility Model                              
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0292  5.199e-03      5.609  2.036e-08  [1.897e-02,3.935e-02]
alpha[1]       0.0000  3.864e-02      0.000      1.000 [-7.573e-02,7.573e-02]
alpha[2]       0.0204  3.555e-02      0.575      0.565 [-4.924e-02,9.010e-02]
gamma[1]       0.2849  4.121e-02      6.913  4.748e-12      [  0.204,  0.366]
beta[1]        0.8139      0.100      8.102  5.388e-16      [  0.617,  1.011]
beta[2]    1.1748e-10  8.717e-02  1.348e-09      1.000      [ -0.171,  0.171]
                              Distribution                              
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu             6.3524      0.714      8.903  5.447e-19 [  4.954,  7.751]
========================================================================

Covariance estimator: robust
ARCHModelResult, id: 0x138749b0470
```

Predict

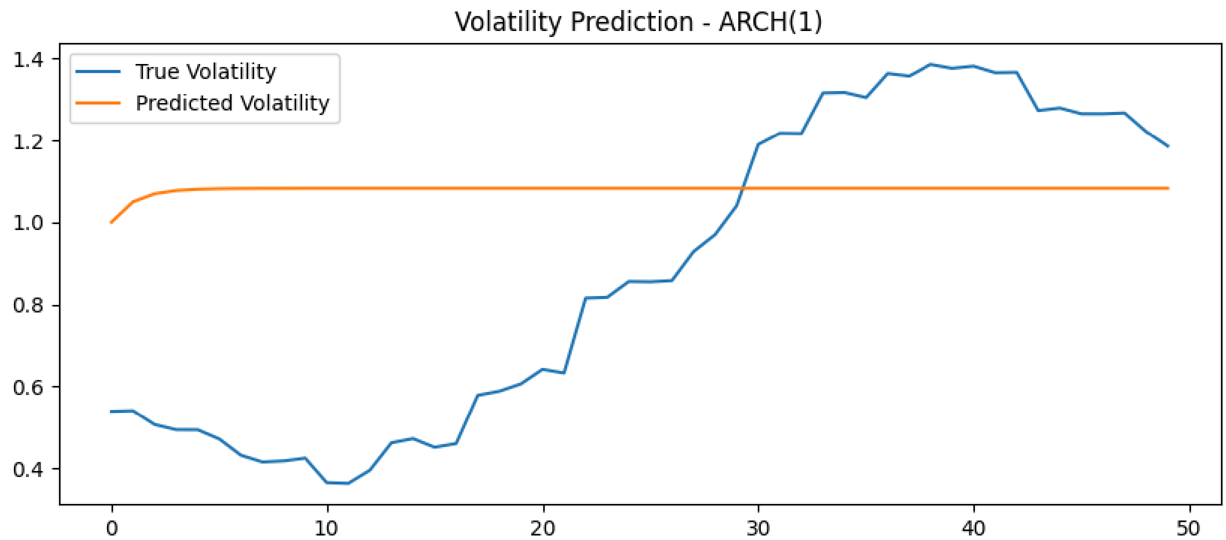
Source: https://github.com/ritvikmath/Time-Series-Analysis/blob/master/GARCH%20Model.ipynb

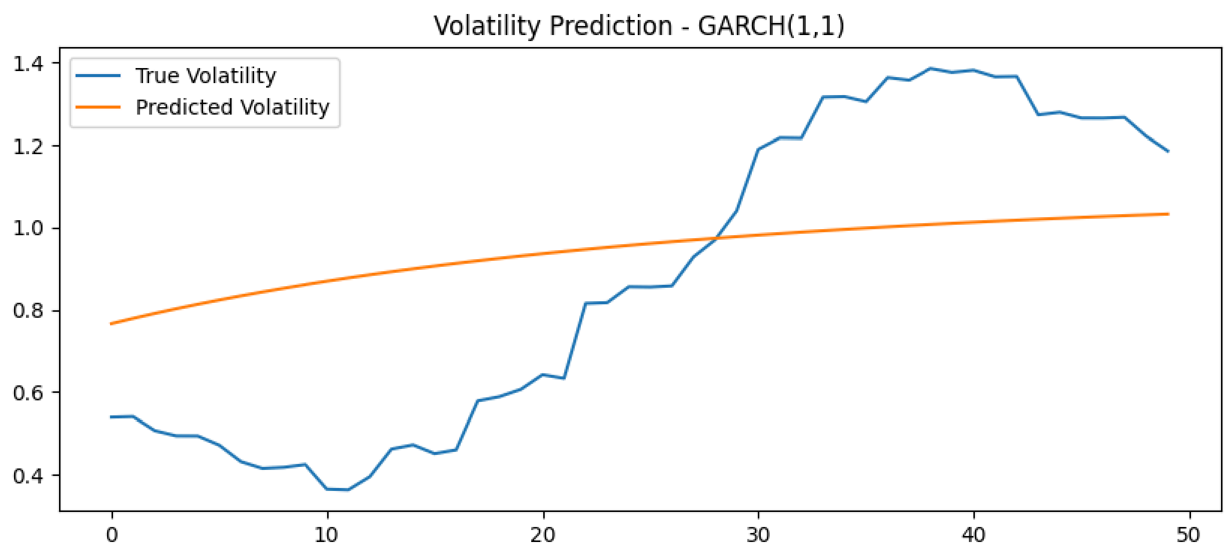
In [17]:

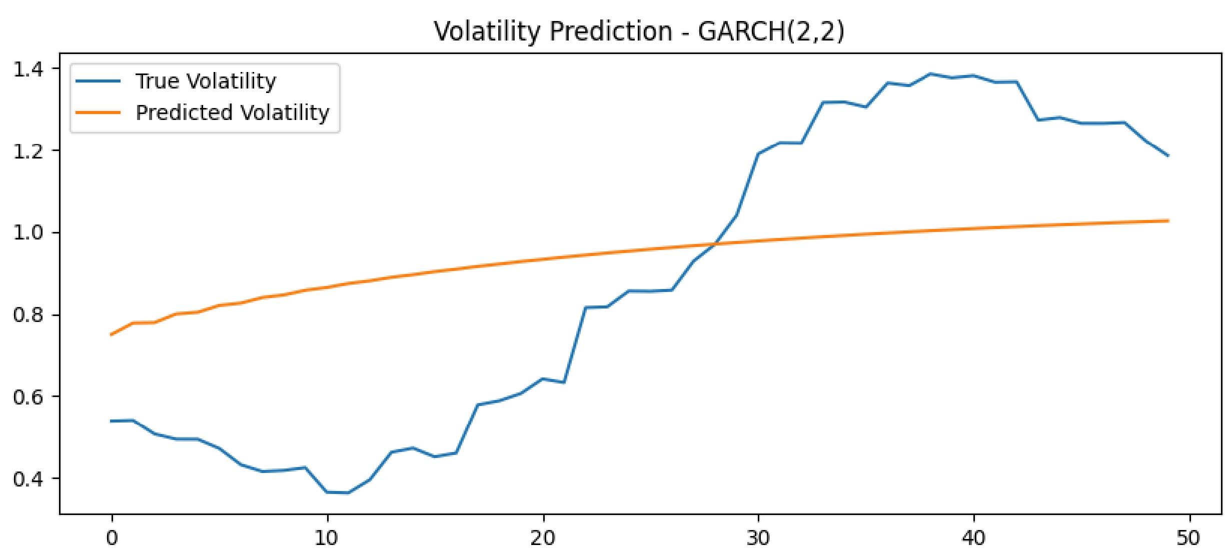
```
def display_prediction(model_fit, title = ''):
    predictions = model_fit.forecast(horizon=test_size)
    plt.figure(figsize=(10,4))
    true, = plt.plot((ser_returns * 100).rolling(window=21).std().values[-test_size:])
    preds, = plt.plot(np.sqrt(predictions.variance.values[-1, :]))
    plt.title(title)
    plt.legend(['True Volatility', 'Predicted Volatility'])
    return np.sqrt(predictions.variance.values[-1, :])
```

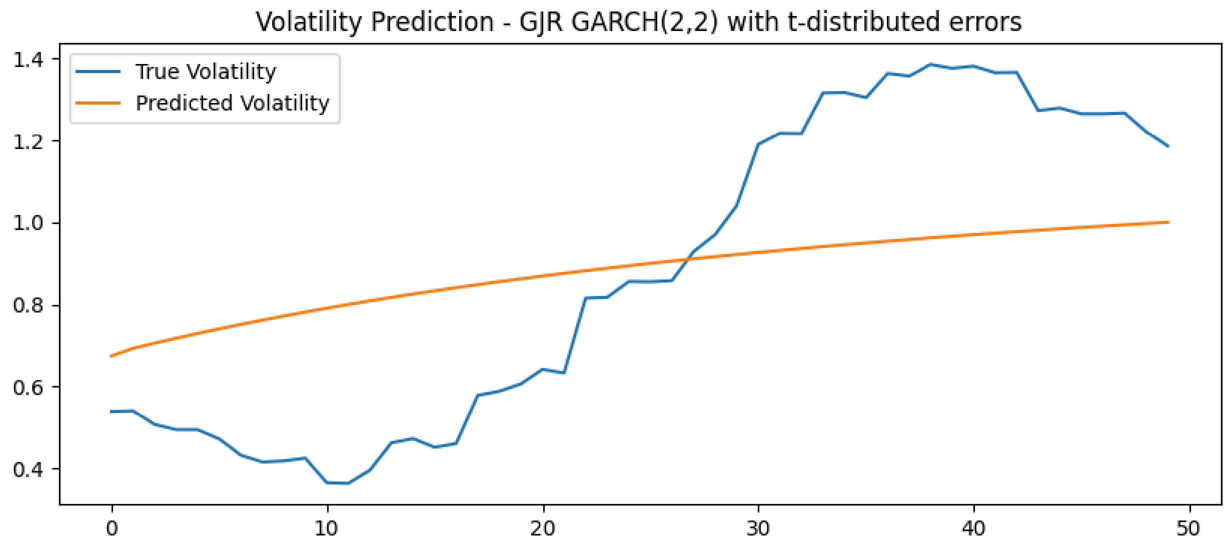
In [18]:

```
pred_arch_1 = display_prediction(fitted_model_arch_1, title = 'Volatility Prediction -
pred_garch_1_1 = display_prediction(fitted_model_garch_1_1, title = 'Volatility Predic
pred_garch_2_2 = display_prediction(fitted_model_garch_2_2, title = 'Volatility Predic
pred_gjr_garch_t = display_prediction(fitted_model_gjr_garch_t, title = 'Volatility Pr
```


Volatility Prediction - ARCH(1)
True Volatility
Predicted Volatility
1.4
1.2
1.0
0.8
0.6
0.4
0
10
20
30
40
50


Volatility Prediction - GARCH(1,1)
True Volatility
Predicted Volatility
1.4
1.2
1.0
0.8
0.6
0.4
0
10
20
30
40
50


Volatility Prediction - GARCH(2,2)
True Volatility
Predicted Volatility
1.4
1.2
1.0
0.8
0.6
0.4
0
10
20
30
40
50

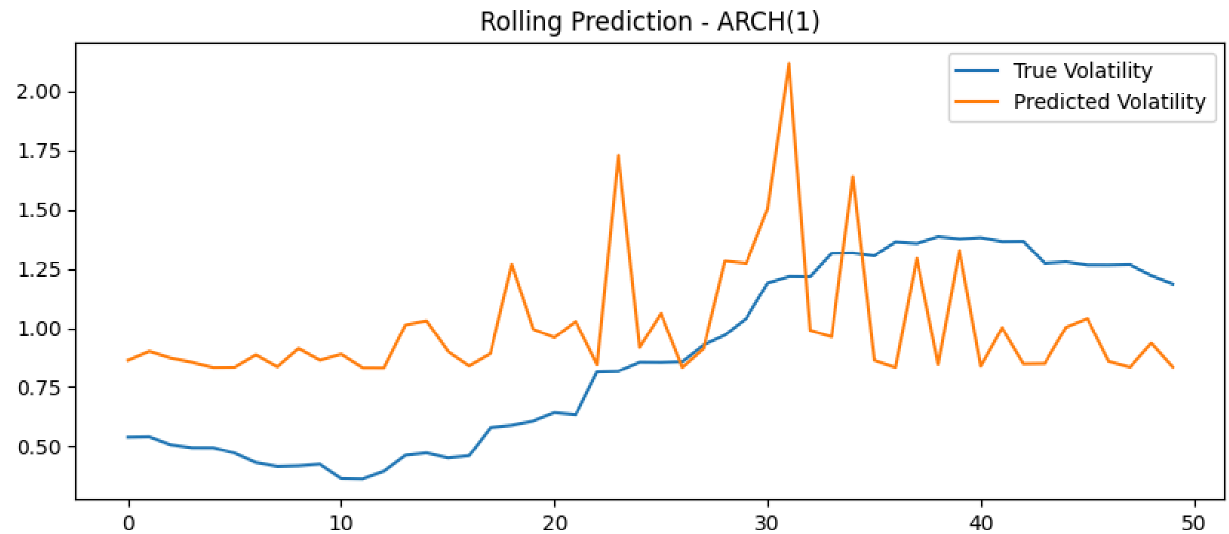


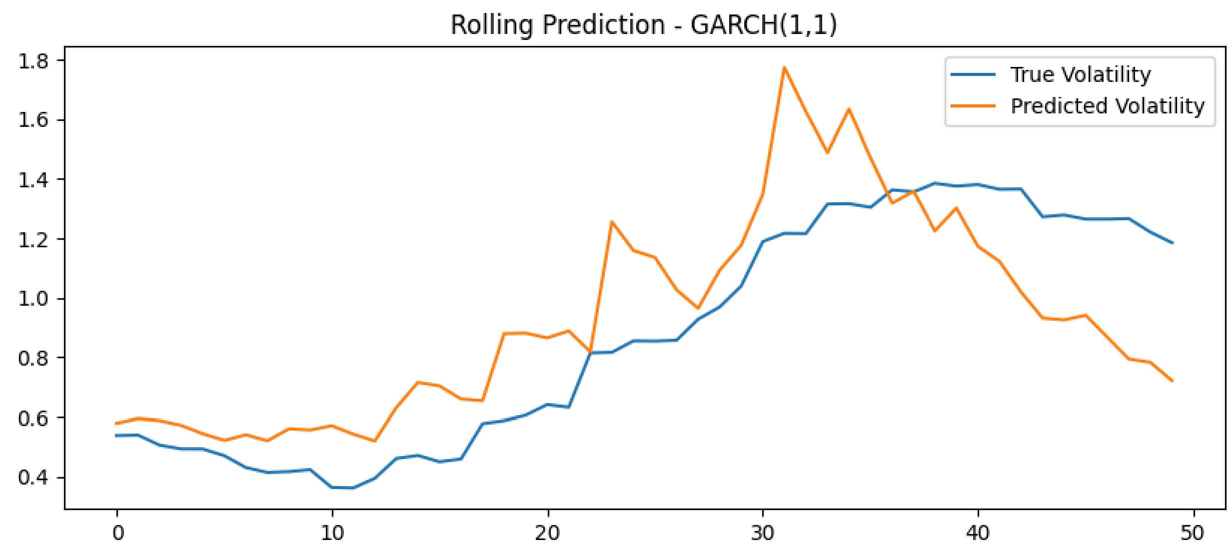
Rolling Forecast Origin

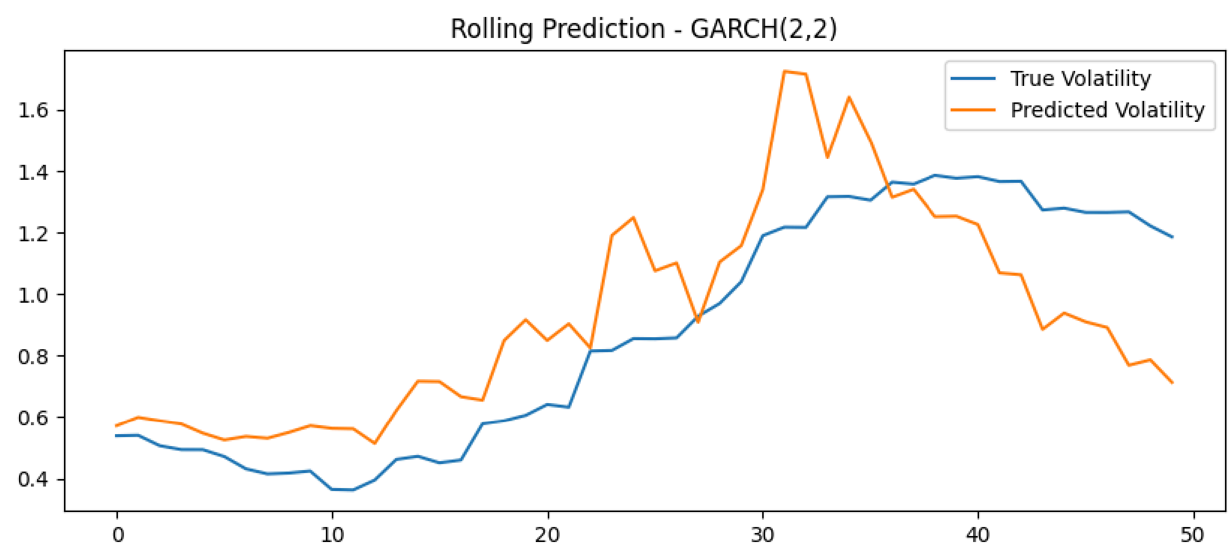
Source: https://github.com/ritvikmath/Time-Series-Analysis/blob/master/GARCH%20Model.ipynb

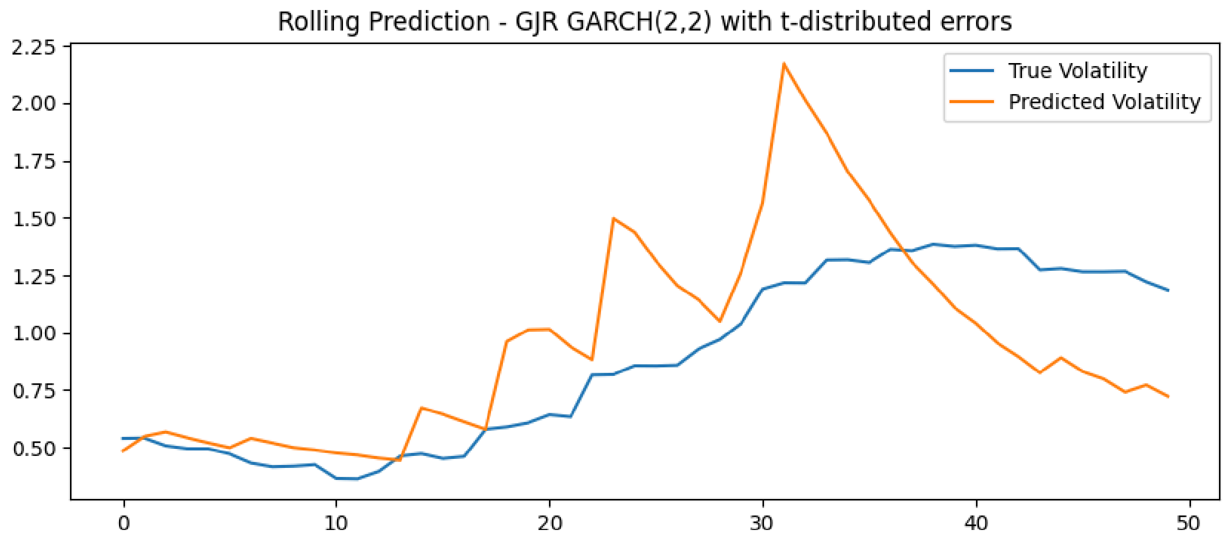
In [19]:
```
def display_rolling_predictions(series, p=2, q=2, o=0, dist='normal', title=''):
    rolling_predictions = []
    for i in range(test_size):
        train = series[:-(test_size-i)]
        model = arch_model(train, p=p, q=q, o=o, dist=dist)
        model_fit = model.fit(disp='off')
        pred = model_fit.forecast(horizon=1)
        rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))
    plt.figure(figsize=(10,4))
    true, = plt.plot(series.rolling(window=21).std()[-test_size:].values)
    preds, = plt.plot(rolling_predictions)
    plt.title(title)
    plt.legend(['True Volatility', 'Predicted Volatility'])
    return rolling_predictions
```

In [20]:
```
rolling_pred_arch_1 = display_rolling_predictions(series=ser_returns * 100, p = 1, q =
rolling_pred_garch_1_1 = display_rolling_predictions(series=ser_returns * 100, p = 1,
rolling_pred_garch_2_2 = display_rolling_predictions(series=ser_returns * 100, p = 2,
rolling_pred_gjr_garch_2_2 = display_rolling_predictions(series=ser_returns * 100, p =
```


Rolling Prediction - ARCH(1)
True Volatility
Predicted Volatility
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0
10
20
30
40
50


Rolling Prediction - GARCH(1,1)
True Volatility
Predicted Volatility
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0
10
20
30
40
50


Rolling Prediction - GARCH(2,2)
True Volatility
Predicted Volatility
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0
10
20
30
40
50



Comparison

In [21]:
```
def rmse(y_pred, y_true = (ser_returns * 100).rolling(window=21).std().values[-test_si
    return np.sqrt(((y_pred - y_true) ** 2).mean())

pd.Series({
    'HA':              rmse(df_historical_pred[-test_size-1:-1]['Historical Average']
    'MA':              rmse(df_historical_pred[-test_size-1:-1]['Simple Moving Averag
    'EMA':             rmse(df_historical_pred[-test_size-1:-1]['Exponential Moving A
    'EWMA':            rmse(df_historical_pred[-test_size-1:-1]['Exponential Weighted
    'ARCH(1)':         rmse(rolling_pred_arch_1),
    'GARCH(1,1)':      rmse(rolling_pred_garch_1_1),
    'GARCH(2,2)':      rmse(rolling_pred_garch_2_2),
    'GJR GARCH(2,2)':  rmse(rolling_pred_gjr_garch_2_2),
})
```

Out[21]:
```
HA                0.943535
MA                0.944166
EMA               0.943911
EWMA              0.944333
ARCH(1)           0.417951
GARCH(1,1)        0.249971
GARCH(2,2)        0.253689
GJR GARCH(2,2)    0.352851
dtype: float64
```

GBM

Simulation

In [22]:
```
# Simulate GBM
def simulate_gbm(s0, mu, sigma, horizon = 1, timesteps = 252, n_sims = 1000):

    # Set the random seed for reproducibility
    np.random.seed(50)

    # Set
```

```
    S0 = s0
    T = horizon             # usually = # years
    n = n_sims

    # define dt
    dt = 1 / timesteps      # length of time interval

    # simulate 'n' asset price path with 't' timesteps
    S = np.zeros((T * timesteps, n))
    S[0] = S0

    for i in range(0, T * timesteps - 1):
        W = np.sqrt(dt) * np.random.standard_normal(n)
        S[i+1] = S[i] + mu*S[i]*dt + sigma*S[i]*W
    return S
```

In [23]:

```
ser_gbm_price = pd.Series(np.squeeze(simulate_gbm(s0 = 100, mu = .05, sigma = .2, n_si
```

Estimation

In [24]:

```
# Estimate GBM parameters using MLE

# Log returns from the simulated stock prices
ser_gbm_returns = np.diff(np.log(ser_gbm_price))

# Define the negative log-likelihood function for MLE
def nll_gbm(params, returns, dt = 1/252):
    mu, sigma = params
    if sigma <= 0:
        return np.inf  # Penalize non-positive sigma
    # n = len(returns)
    drift = (mu - 0.5 * sigma**2) * dt
    diff = sigma * np.sqrt(dt)
    # Negative log-likelihood
    ll = norm.logpdf(returns, drift, diff).sum()
    nll = - ll
    # nll = 0.5 * n * np.log(2 * np.pi * diff ** 2) + np.sum((returns - drift)**2) / (
    return nll  # We minimize the negative log-likelihood

# Initial guess for parameters
initial_guess = [0.05, 0.2]  # Initial guess for mu and sigma

# Perform MLE using the minimize function
result = minimize(nll_gbm, initial_guess, args=(ser_gbm_returns),
                  bounds=[(-1, 1), (1e-5, 1)])  # Bounds to ensure reasonable results

# Extract estimated parameters
mu_est, sigma_est = result.x

# Print the estimated parameters
mu_est, sigma_est
```

Out[24]:

```
(np.float64(-0.12938366782938748), np.float64(0.20543400281336172))
```

In [25]:

```
mu_est - sigma_est ** 2 / 2, sigma_est
```

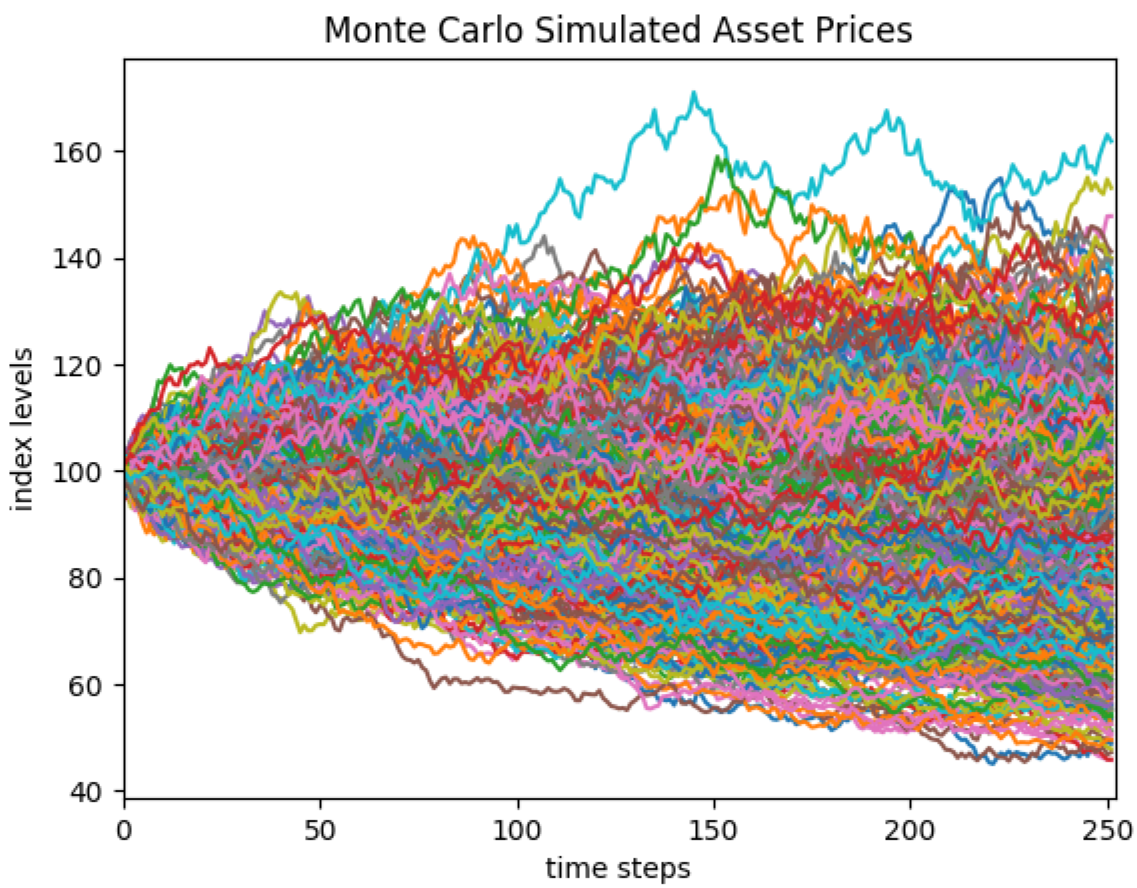
Out[25]:
```
(np.float64(-0.15048523258534763), np.float64(0.20543400281336172))
```

In [26]:
```
ser_gbm_returns.mean() * 252, ser_gbm_returns.std()*np.sqrt(252)
```

Out[26]:
```
(np.float64(-0.1504882190442562), np.float64(0.20543406320813795))
```

In [27]:
```
df_gbm = pd.DataFrame(simulate_gbm(s0 = 100, mu = mu_est, sigma = sigma_est))
```

In [28]:
```
# Plot initial 100 simulated path using matplotlib
plt.plot(df_gbm)
plt.xlabel('time steps')
plt.xlim(0, 252)
plt.ylabel('index levels')
plt.title('Monte Carlo Simulated Asset Prices');
```



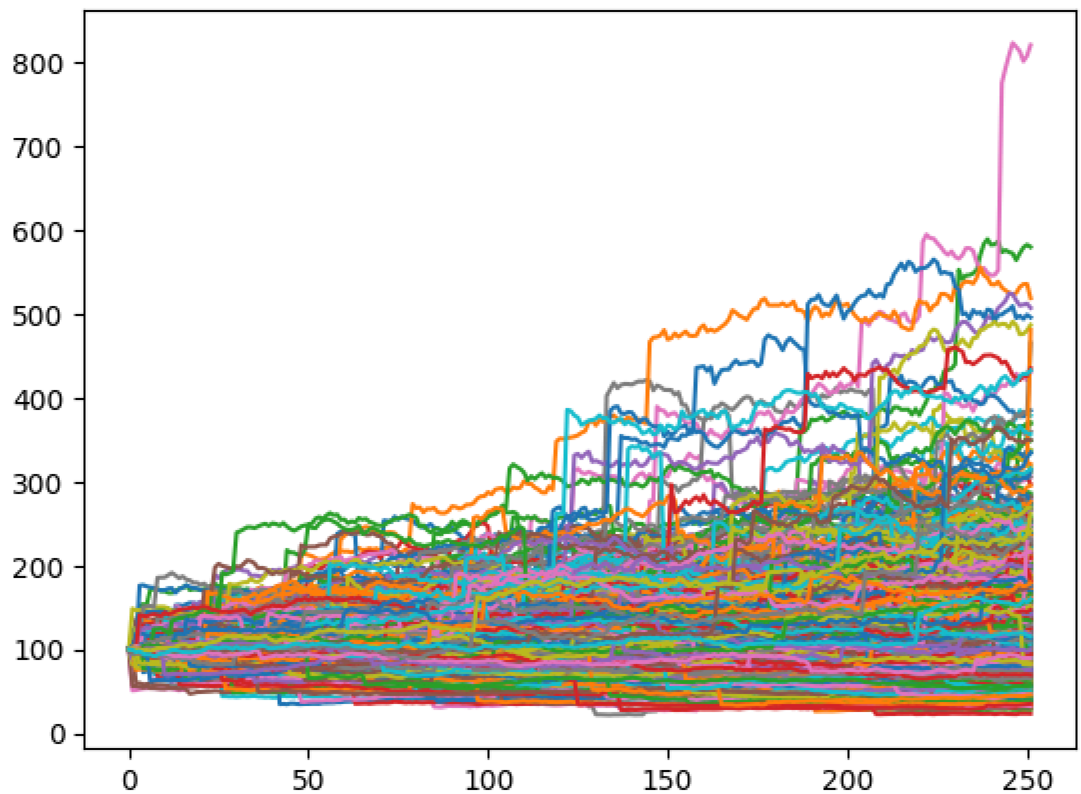
In [29]:
```
# Use this for TP/SL
percentiles = np.percentile(df_gbm.iloc[-1], [1, 5, 10, 90, 95, 99])
percentiles
```

Out[29]:
```
array([ 52.47153066,  60.80671992,  66.55406171, 112.36273401,
       121.18995043, 134.9613784 ])
```

PJD

Simulation

In [30]:
```python
# Simulate PJD
def simulate_pjd(s0, mu, sigma, lam, alpha, delta, horizon = 1, timesteps = 252, n_sim

    # Set the random seed for reproducibility
    np.random.seed(50)

    # Set
    S0 = s0
    T = horizon              # usually = # years
    n = n_sims

    # define dt
    dt = 1 / timesteps       # length of time interval

    # simulate 'n' asset price path with 't' timesteps
    S = np.zeros((T * timesteps, n))
    S[0] = S0

    for i in range(0, T * timesteps - 1):
        W = np.sqrt(dt) * np.random.standard_normal(n)
        J = np.random.normal(alpha, delta, n)
        N = np.random.poisson(lam * dt, n)
        S[i+1] = S[i] + mu*S[i]*dt + sigma*S[i]*W + J * S[i] * (N > 0)
    return S

S = simulate_pjd(s0=100, mu=0.05, sigma=0.2, lam=5, alpha=.05, delta=.2)
plt.plot(pd.DataFrame(S))
plt.show()
```



Estimation

In [31]:
```
# Log-likelihood function for jump diffusion
def nll_pjd(params, returns, dt = 1/252):
    mu, sigma, lam, alpha, delta = params

    # Ensure parameters are positive to maintain valid distributions
    if sigma <= 0 or lam <= 0 or delta <= 0:
        return np.inf  # Return a large number to penalize invalid parameters

    # Precompute constants
    prob_no_jump = np.exp(-lam * dt)
    prob_jump = 1 - prob_no_jump  # For small dt, approx lam * dt

    # Density for no jump
    drift_no_jump = (mu - 0.5 * sigma**2) * dt
    diff_no_jump = sigma * np.sqrt(dt)
    pdf_no_jump = norm.pdf(returns, drift_no_jump, diff_no_jump)

    # Density for jump
    drift_jump = drift_no_jump + alpha
    diff_jump = np.sqrt(sigma ** 2 * dt + delta**2)
    pdf_jump = norm.pdf(returns, drift_jump, diff_jump)

    # Total density
    total_pdf = prob_no_jump * pdf_no_jump + lam * dt * prob_jump * pdf_jump

    # Avoid log(0) by setting a minimum value
    total_pdf = np.maximum(total_pdf, 1e-300)

    # Compute log-likelihood
    log_likelihood = np.sum(np.log(total_pdf))

    nll = - log_likelihood
    return nll
```

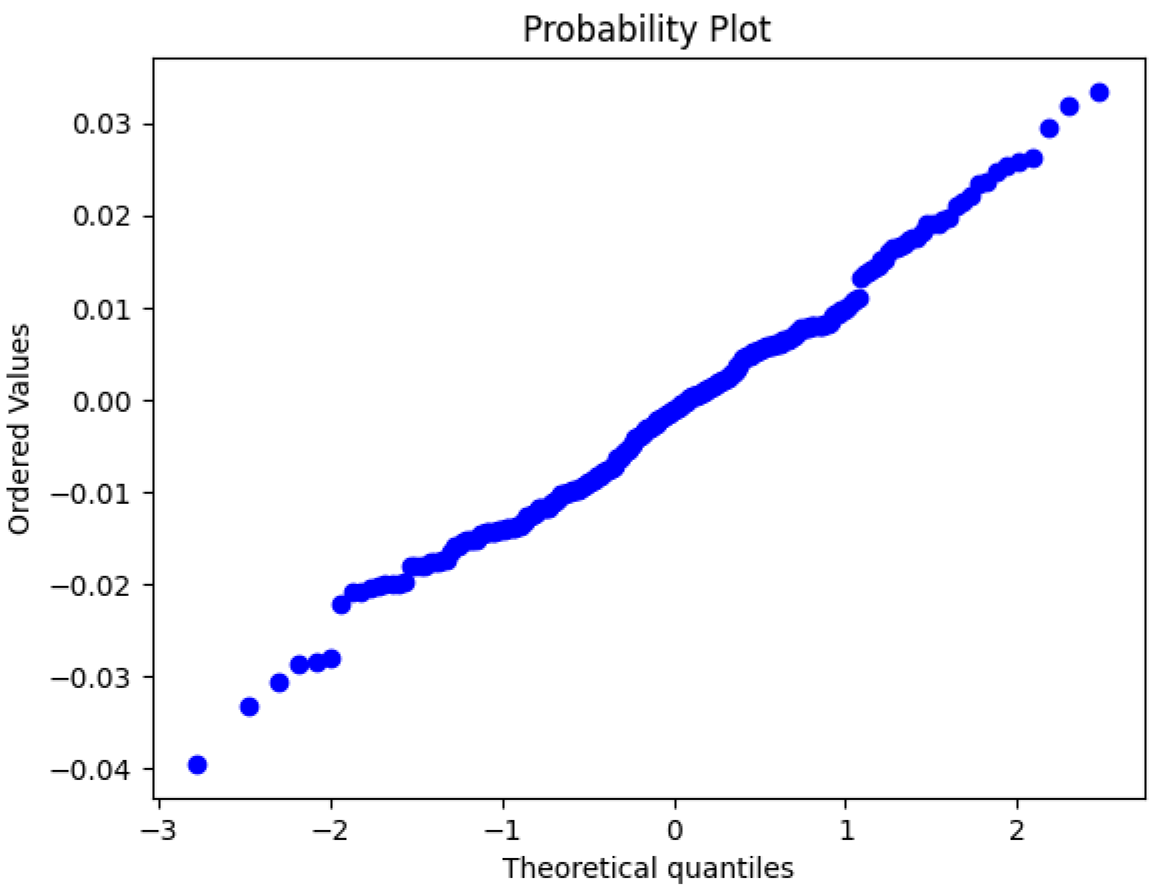
In [32]:
```
ser_pdj_price = pd.Series(np.squeeze(simulate_pjd(s0=100, mu=0.05, sigma=0.2, lam=5, a
ser_pdj_returns = np.diff(np.log(ser_pdj_price))
```

In [33]:
```
# Initial parameter guesses
params_init = [0.05, 0.2, 5, .05, .2]

# Bounds to ensure parameters remain in a valid range
bounds = [(None, None), (1e-6, None), (1e-6, None), (None, None), (1e-6, None)]

# Optimize the log-likelihood
result = minimize(nll_pjd, params_init, args=(ser_pdj_returns), method='L-BFGS-B', bou
mu_est, sigma_est, lam_est, alpha_est, delta_est = result.x

print("Estimated parameters:")
print(f"mu = {mu_est}")
print(f"sigma = {sigma_est}")
print(f"lambda = {lam_est}")
print(f"alpha = {alpha_est}")
print(f"delta = {delta_est}")
```

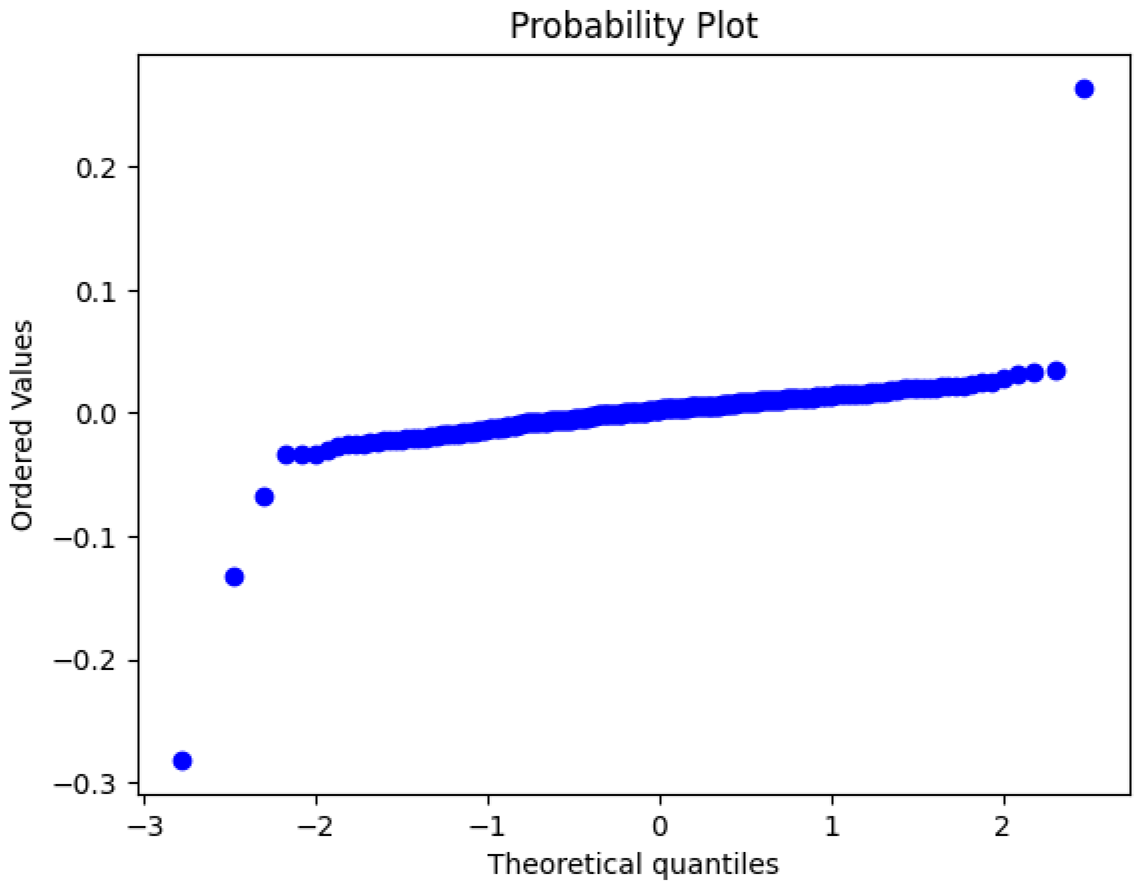
```
Estimated parameters:
mu = -0.45215183968607353
sigma = 0.2028481813245381
lambda = 14.635637551960489
alpha = 0.04936470850350823
delta = 0.16609659931096019
```

In [34]:
```python
df_pjd = pd.DataFrame(simulate_pjd(s0=100, mu=0.05, sigma=0.2, lam=5, alpha=.05, delta
```

In [35]:
```python
probplot(np.log(df_gbm / df_gbm.shift()).iloc[:, 0], dist="norm", plot=plt)
plt.show()
```



In [36]:
```python
df_pjd = pd.DataFrame(S)
probplot(np.log(df_pjd / df_pjd.shift()).iloc[:, 0], dist="norm", plot=plt)
plt.show()
```



In [37]:
```
# Use this for TP/SL
percentiles = np.percentile(df_pjd.iloc[-1], [1, 5, 10, 90, 95, 99])
percentiles
```

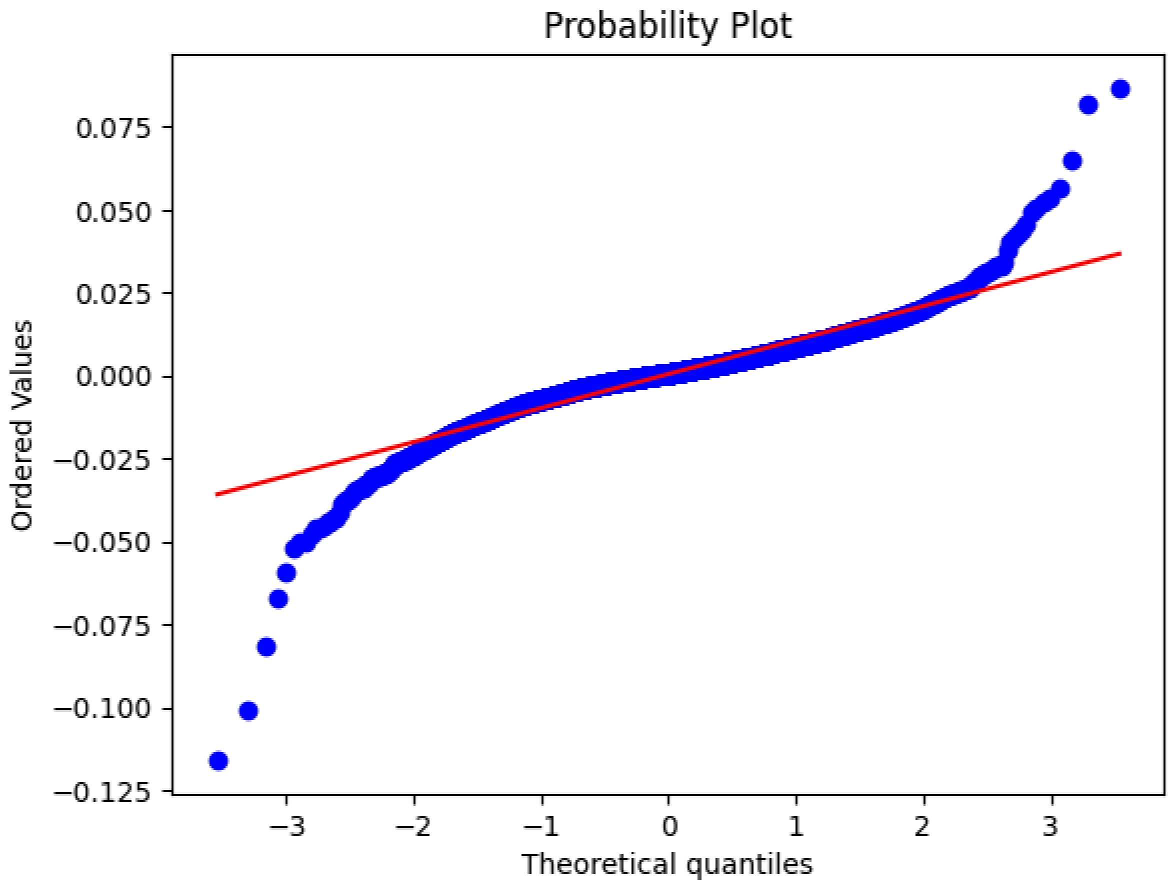
Out[37]:
```
array([ 35.60509287,  52.25280488,  63.98129322, 220.01820911,
       253.74799726, 430.25923803])
```

In [38]:
```
# Compare with GBM
np.percentile(df_gbm.iloc[-1], [1, 5, 10, 90, 95, 99])
```

Out[38]:
```
array([ 52.47153066,  60.80671992,  66.55406171, 112.36273401,
       121.18995043, 134.9613784 ])
```

Using data

In [39]:
```
probplot(ser_returns, dist="norm", plot=plt)
plt.show()
```



In [40]:
```
initial_guess = [0.05, 0.2]  # Initial guess for mu and sigma

# Bounds to ensure parameters remain in a valid range
bounds = [(-1, 1), (1e-5, 1)]

# Perform MLE using the minimize function
result = minimize(
    nll_gbm,
    initial_guess,
    args=(ser_returns.dropna()),
    bounds=bounds      # Bounds to ensure reasonable results
)

# Extract estimated parameters
mu_est_gbm, sigma_est_gbm = result.x

print("Estimated parameters:")
print(f"mu = {mu_est_gbm}")
print(f"sigma = {sigma_est_gbm}")
```

```
Estimated parameters:
mu = 0.12385240164472511
sigma = 0.1718913420984464
```

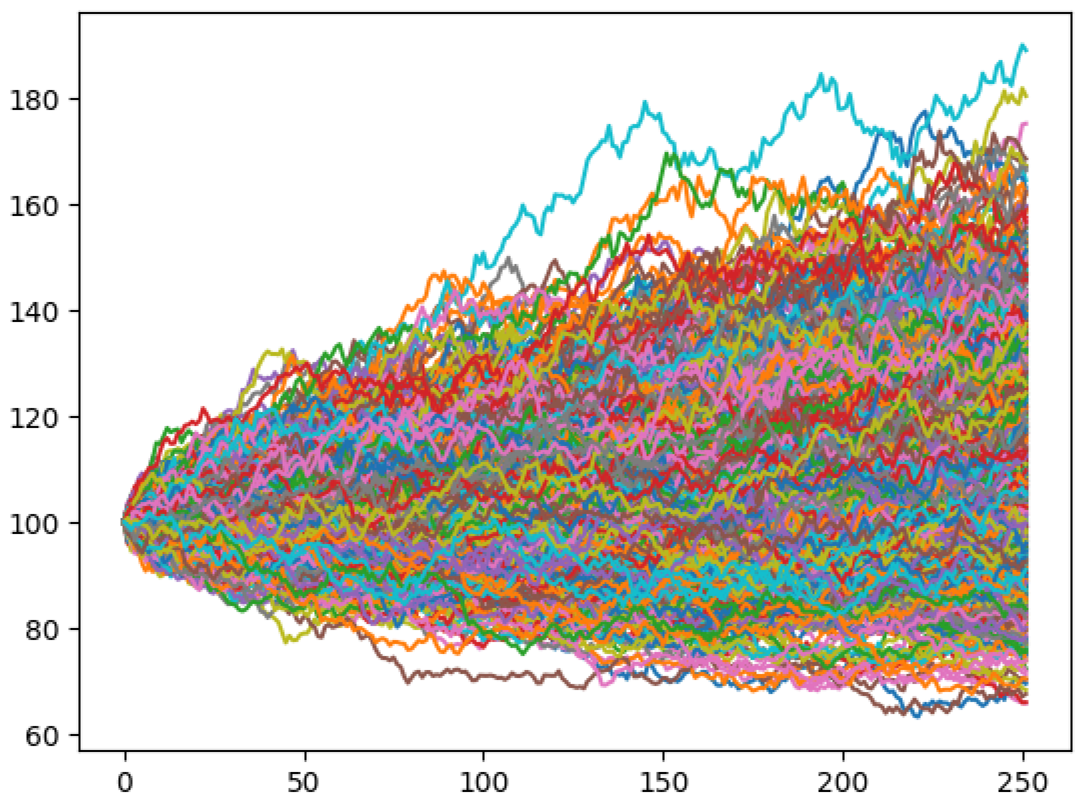
In [41]:
```
params_init = [0.05, 0.2, 5, .05, .2]   # Initial parameter guesses

# Bounds to ensure parameters remain in a valid range
bounds = [(None, None), (1e-6, None), (1e-6, None), (None, None), (1e-6, None)]

# Optimize the log-likelihood
result = minimize(
```

```
    nll_pjd,
    params_init,
    args=(ser_returns.dropna()),
    method='L-BFGS-B',
    bounds=bounds
)

# Extract estimated parameters
mu_est_pjd, sigma_est_pjd, lam_est_pjd, alpha_est_pjd, delta_est_pjd = result.x

print("Estimated parameters:")
print(f"mu = {mu_est_pjd}")
print(f"sigma = {sigma_est_pjd}")
print(f"lambda = {lam_est_pjd}")
print(f"alpha = {alpha_est_pjd}")
print(f"delta = {delta_est_pjd}")
```

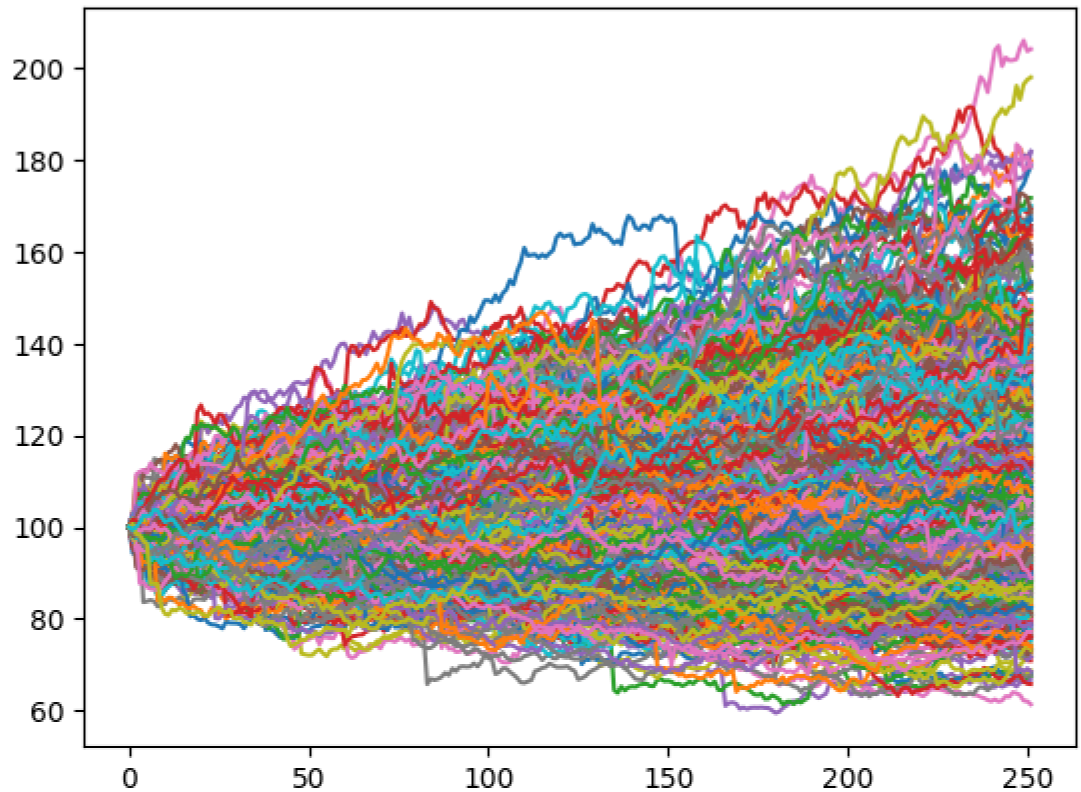
```
Estimated parameters:
mu = 0.1437043217062225
sigma = 0.15117423353681886
lambda = 3.5413052943392973
alpha = -0.013148080684250742
delta = 0.060179584918791405
```

In [42]:

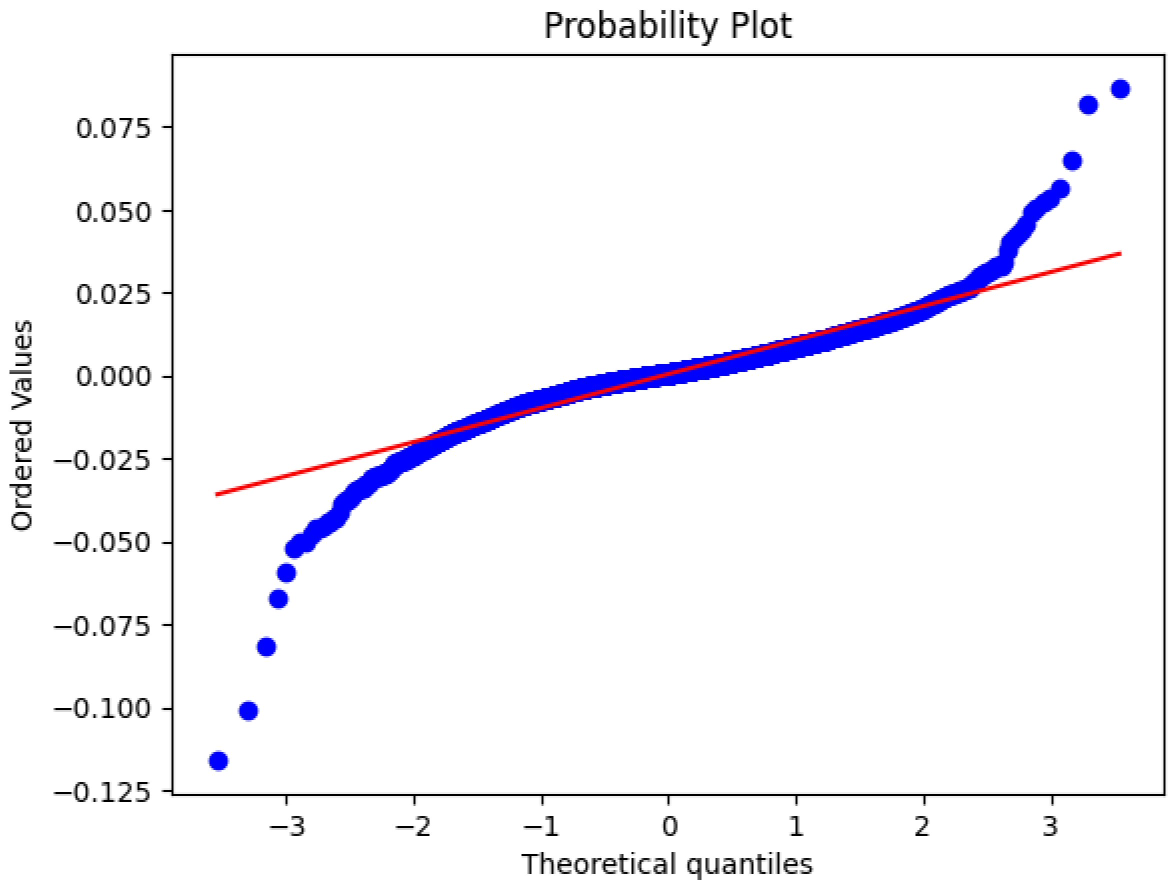
```
simulate_gbm_results = simulate_gbm(
    s0=100,
    mu=mu_est_gbm,
    sigma=sigma_est_gbm
)

plt.plot(pd.DataFrame(simulate_gbm_results))
plt.show()
```



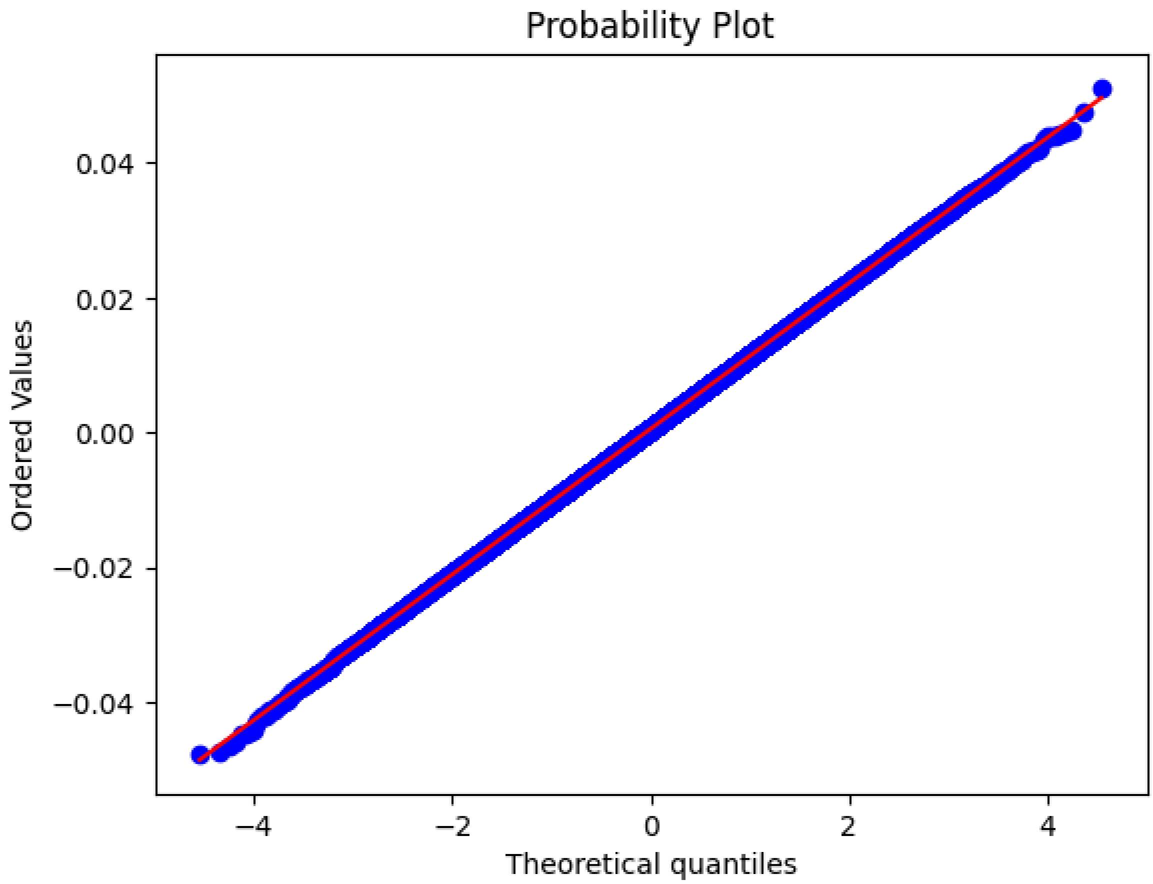
In [43]:
```
simulate_pjd_results = simulate_pjd(
    s0=100,
    mu=mu_est_pjd,
    sigma=sigma_est_pjd,
    lam=lam_est_pjd,
    alpha=alpha_est_pjd,
    delta=delta_est_pjd
)

plt.plot(pd.DataFrame(simulate_pjd_results))
plt.show()
```



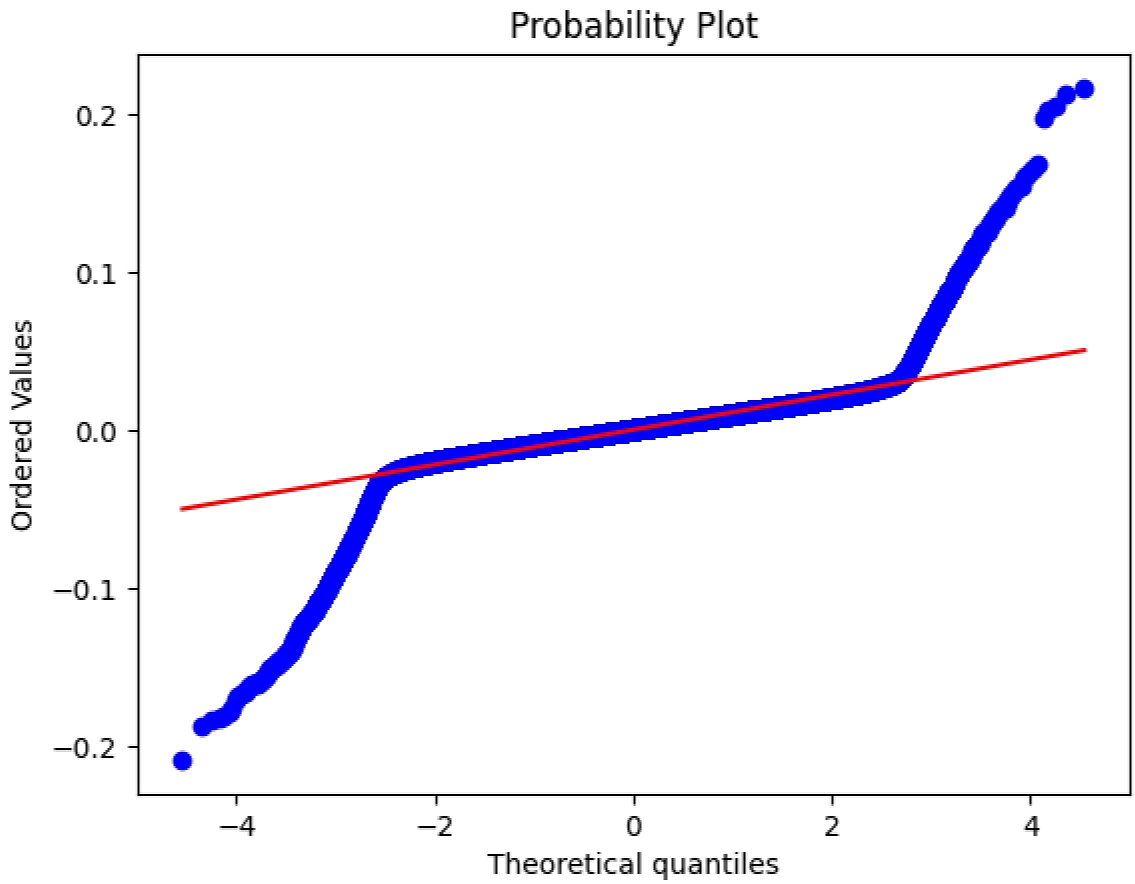
In [44]:
```
probplot(ser_returns.dropna(), dist="norm", plot=plt, )
plt.show()
```



In [45]:
```
probplot(pd.DataFrame(simulate_gbm_results).pct_change().dropna().values.flatten(), di
plt.show()
```



In [46]:
```
probplot(pd.DataFrame(simulate_pjd_results).pct_change().dropna().values.flatten(), di
plt.show()
```



Probability Plot
Ordered Values
Theoretical quantiles
0.2
0.1
0.0
−0.1
−0.2
−4
−2
0
2
4