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Tutorials in Advanced Control
Bureaux d'étude - Robust Control of Mechatronics systems -10h
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Part I
H_∞ control of a clutch flexible system

We consider here drive train dynamics. The system under consideration is an actuator connected to a load through a torsional spring which represents the joint flexibility. For simplicity we take the motor torque u as input. The equations of motion are easily derived, using the generalized coordinates θ_l and θ_m, the link (load) angle and the motor angle.

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) = 0 \quad (1)$$
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u \quad (2)$$

where J_l and J_m are the load and motor inertias, B_l and B_m are the load and motor damping constants, with $k = 0.8NM/rad$, $J_m = 4.10^{-4}NMs^2/rad$, $J_l = 4.10^{-4}NMs^2/rad$, $B_m = 0.015NMs/rad$, $B_l = 0NMs/rad$.
The output to be controlled is of course the load angle θ_l. The open loop transfer function between u and θ_l is then:

$$\frac{\theta_l}{u} = \frac{k}{p_l(s)p_m(s) - k^2}$$

with $p_l(s) = J_l s^2 + B_l s + k$ and $p_m(s) = J_m s^2 + B_m s + k$.
In this Lab study, the aim is to design an H_∞ control. The performance specifications are:

- Robustness : module margin > 0.5
- Settling time lower than than $0.5sec$, tracking steady state error lower than $1/1000$.
- Attenuation of input disturbance step, with "no" steady state error. This type of disturbance is some load torque on the input u.
- Sensitivity of the control input to a measurement noise of sinusoïdal type at 800 rad/s : lower than 10%.
- Actuator constraint : maximal controller gain $= 3$.

1 Performance analysis and specifications

A given PD control with load angle feedback is considered, as :

$$K(s) = K_d s + K_p$$

with $K_p = 0.3$ and $K_d = 0.0033$

1. Provide stability and performance analysis of this controller, through the suitable sensitivity functions, according to the performance specifications
2. Propose the different templates for the sensitivity functions and check the above analysis.

2 H_∞ control

In this part, an H_∞ control will be designed to solve the problem.

2.1 Frequency-domain Performance specifications

- Choose a control loop scheme, including the weights,adequate to solve the problem.
- Use **`Matlab/sysic`** function to get the general control configuration.

2.2 Design

Provide an insightful analysis of the design procedure, by following the steps:

- 1^{st} test: use of weight on tracking error only
- 2^{nd}test: weight the tracking error as well as the control input
- 3^{rd} (if time) Add the following filter on noise input:

$$W_n = \frac{3s + 150}{s + 2000}$$

For each test, give explanations and interpretations of the obtained results (improvement if any), according to the performance and robustness specifications. Then choose the best controller.

2.3 Validation

For the chosen solution controller:

1. Use simulink to show pertinent simulations.
2. Compare with PD control (NB. for time-domain simulations with the PD controller, use the block ”PID Controller (with Approximate Derivative):” of **`Simulink Extras`**)

2.4 Robustness study

1. Give the values of the robustness margins. Conclude.
2. Now we consider that the system parameters may vary of 10%. Evaluate the relative error between the plant and nominal model to get a frequency-domain representation of multiplicative unstructured uncertainties.
3. Is the robustness template, corresponding to the application of the Small Gain theorem, satisfied, either for the PD controller, or for the H_∞ one?
4. If not, give the way to meet this template through a new control design of an H_∞ controller (using suitable weights).

Remarks

With SIMULINK use the commande **`Random Number`** for noise generations.
You could build separate programs for design and analysis. A plot program for extracting the Simulink figures is also advised.

Part II
Chassis Control : roll and bounce

An half-car model of a front axle is considered.

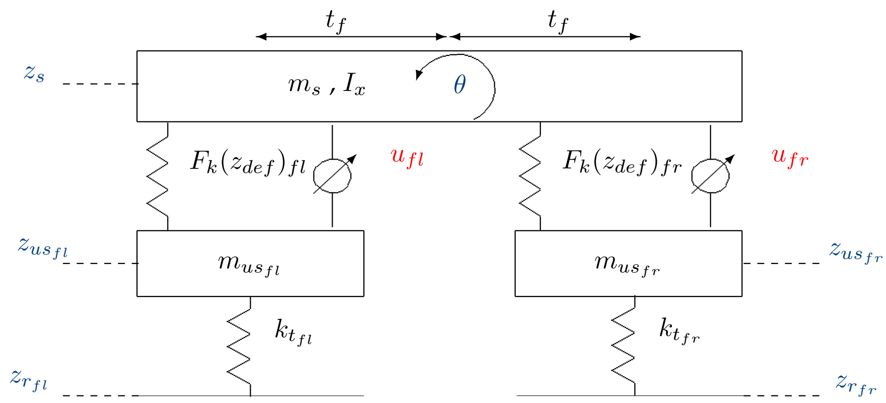

Figure 1: Half vehicle semi active model.

It allows to take into account the bounce and the roll moment of the vehicle. The model is obtained by simply adding to two suspension equations, the dynamical equation of the chassis as follows.
The dynamic of the chassis (bounce and roll) is given by

$$\begin{array}{rcl} m_s\ddot{z}_s &=& F_{sz_{fl}} + F_{tz_{fr}} + F_{dz} \\ m_{us_{fl}}\ddot{z}_{us_{fl}} &=& -F_{sz_{fl}} + F_{tz_{fl}} \\ m_{us_{fr}}\ddot{z}_{us_{fr}} &=& -F_{sz_{fr}} + F_{tz_{fr}} \\ I_x\ddot{\theta} &=& -F_{sz_{fl}}t_f + F_{tz_{fr}}t_f + M_{dx} \end{array} \qquad \begin{array}{rcl} F_{tz_{fl}} &=& -k_{t_{fl}}(z_{us_{fl}} - z_{r_{fl}}) \\ F_{tz_{fr}} &=& -k_{t_{fr}}(z_{us_{fr}} - z_{r_{fr}}) \\ F_{sz_{fl}} &=& -k_{fl}(z_{s_{fl}} - z_{us_{fl}}) - c_{fl}(\dot{z}_{s_{fl}} - \dot{z}_{us_{fl}}) + u_l \\ F_{sz_{fr}} &=& -k_{fr}(z_{s_{fr}} - z_{us_{fr}}) - c_{fr}(\dot{z}_{s_{fr}} - \dot{z}_{us_{fr}}) + u_r \end{array}$$

where
$$\begin{array}{rcl} u_{fl} &=& c_{fl}(\dot{z}_{s_{fl}} - \dot{z}_{us_{fl}}) + u_l \\ u_{fr} &=& c_{fr}(\dot{z}_{s_{fr}} - \dot{z}_{us_{fr}}) + u_r \end{array} \qquad \begin{array}{rcl} z_{s_{fl}} &=& z_s - t_f sin(\theta) \\ z_{s_{fr}} &=& z_s + t_f sin(\theta) \\ \dot{z}_{s_{fl}} &=& \dot{z}_s - \dot{\theta}t_f cos(\theta) \\ \dot{z}_{s_{fr}} &=& \dot{z}_s + \dot{\theta}t_f cos(\theta) \end{array}$$

The parameters m_{fl}, m_{fr}, are the unsprung mass left and right respectively and m_s is the suspended mass (chassis). Then $F_{t..}$, $F_{s..}$, represent the forces delivered by the dampers and springs, where $k_{t..}$ is the stiffness of the tire (constant), $k_{..}$ the suspension stiffness and $c_{..}$ the damper coefficients. I_x is the chassis inertia on the roll axis. Then θ, z_s, $z_{u_{fl}}$ and $z_{u_{fr}}$ represent the roll angle, the chassis, unsprung mass left and right bounce respectively. F_{dz} and M_{dx} are the disturbances on the bounce and the roll angle respectively.
The main objective is here to improve the passenger comfort evaluated by the response of the sprung mass displacement $z_s(t)$ and of the roll angle $\theta(t)$ from the road signal $z_r(t)$ in low frequencies ([1-10] Hz).
The following characteristivs can then be considered

- both suspension deflections $z_{s_{fl}} - z_{us_{fl}}$ and $z_{s_{fr}} - z_{us_{fr}}$ are measured, with very low measurement noises.
- the vehicle variables to be controlled are: z_s and θ.
- the road behavior corresponds to irregularities of $10mm$.
- the control actuators have limited forces.

An H_∞ problem has been formulated such that the comfort of the car is improved, without deteriorating the road holding, and satisfying the suspension deflection constraints. The general control configuration uses very simple weighting functions, i.e either constant gains or first order filters ($k\frac{1}{\tau s+1}$ or $k\frac{\tau_1 s+1}{\tau_2 s+1}$).

Symbol	Value	Unit	Signification
m_s	1260	kg	sprung mass
$m_{us_{fl}}$	37.5	kg	front left unsprung mass
$m_{us_{fr}}$	37.5	kg	front right unsprung mass
I_x	250	kgm^2	roll inertia
t_f	0.7	m	half front axle length
k_{fl}	29500	N/m	front left suspension linearized stiffness
k_{fr}	29500	N/m	front right suspension linearized stiffness
c_{fl}	1500	$N/m/s$	front left suspension linearized damping
c_{fr}	1500	$N/m/s$	front right suspension linearized damping
$k_{t_{fl}}$	208000	N/m	front left tire stiffness
$k_{t_{fr}}$	208000	N/m	front right tire stiffness
z_{def}	$[-0.09; 0.05]$	m	suspension bounds

Table 1: Car parameters

The work to be done is only to use and complete the given **`Matlab`** files in order to provide a satisfactory solution according to the performance specifications, namely the comfort improvement evaluated through the sprung mass displacement $z_s(t)$ and of the roll angle $\theta(t)$ from the road signal $z_r(t)$ in low frequencies ([1-10] Hz).

Some time-domain simulations can be performed to complete the evaluation. For instance the road signal could be a 5cm bump of the left wheel (from $t = 0.5sec$ to $t = 1sec$.