MS&E 211X/MS&E 111X/ENGR 62X Introduction to Optimization (Accelerated) HW4

Course Instructor: Yinyu Ye

Due Date: 12th November 2019, 5pm

Please submit your homework through canvas. Please note: late homework will not be accepted. Each problem below is worth 10 points.

Problem 1

For parts a)-e) below, label them as True or False. If true, provide a short reason; if false, provide reasoning or a counter example.

a) The shadow price of a non-binding constraint can be non-zero.

b) Suppose the primal feasible region F_p is non-empty. Then for any $x \in F_p$, we can have $c^T x \geq b^T y$ for any y.

c) For a LP problem, it is possible that the primal problem has an unbounded objective value, while the dual problem has a non-empty feasible region.

d) For a LP problem, it is possible that the primal problem has a finite optimal value, while the dual problem has no feasible solution.

e) We can simultaneously find A, b, x, y, such that $Ax = b, x \geq \mathbf{0}, A^T y \leq \mathbf{0}, b^T y > 0$.

Problem 2 (Lecture Notes #8):

Consider a variant of the Two-Person Zero-Sum Matrix Game in Slides 2-4 of Lecture Note #8, where the payoff matrix becomes:

$$P = \begin{bmatrix} 4 & -1 & -4 & -2 \\ -2 & 1 & 4 & 2 \\ 1 & -2 & 2 & -4 \end{bmatrix}$$

a) Write down the linear program for Player Row.

b) Write down the dual of the above linear program.

c) Give interpretations of the dual problem (with respect to the meaning of the dual variables and dual objective).

Problem 3 (Lecture Note #07):

Consider Problem 3 in Homework 1:
There are 5 securities available in a tournament assets market for open trading at fixed prices and pay-offs (see the table below). Here, for example, Security 1's pay-off is $1 if either Team A or Team D wins. The Price is the current purchasing price per share of each security. Assume there is no share limit (can be ∞) and short is allowed, that is, the decision variable can be both positive (buy) and negative (sell).

Security	Price	Team A	Team B	Team C	Team D
1	0.80	1	0	0	1
2	0.45	0	1	1	0
3	0.90	1	1	0	1
4	0.65	0	1	1	0
5	0.55	0	0	1	1

In order to decide how many shares of each security to purchase so as to maximize the worst-case (minimum) pay-off when the game is finally realized, we can formulate following linear program:

$$\begin{aligned} max \quad & z - \boldsymbol{\pi}^T \mathbf{x} \\ s.t. \quad & \mathbf{e}.z - A\mathbf{x} \leq \mathbf{0} \\ & z, \mathbf{x} \quad \text{free} \end{aligned}$$

where

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

$\pi^T = [0.80, 0.45, 0.90, 0.65, 0.55]$.

a) Write down its corresponding dual problem.

b) Use your favorite solver or by inspection to check if the dual problem is feasible or not. Does there exist an opportunity for arbitrage for the primal optimal value? Show your answer.

Problem 4

Consider a variant of the Reinforcement Learning and Markov Decision Process Problem in Slides 28-32 of Lecture Note #2. In particular, the cost of blue action at state 3 is 0.5 and the cost of red action at state 2 is 0.4.

You can interpret this problem in the following way. There is a robot struggling in a maze. The robot can stay in one of six different states in each time step. State 5 is the robot's final destination, at which he can stop exploring and rest. State 4 is a trap which will cost the robot $1 to get out of before arriving at the final destination (state 5). At states 0, 1, 2 or 3, the robot has two options: either go directly to the next state (red action), or take a short-cut by going to other states with uncertainty (blue action).

a) Formulate the problem as a linear program where the decision variables are the cost-to-go values of decision states and the discount factor is 0.9.

b) Write down its dual problem, solve it using your favorite solver, and give some interpretations about these dual variables

b) Give an optimal policy for this Reinforcement Learning problem (Hint: use the dual optimal solution and the complementarity conditions).

Problem 5 (Lecture Note #8)

Consider a variant of the Robust Portfolio Management Problem in Slides 21-25 of Lecture Note #8, where constraints on x_1, x_2 become:

$$x_1 + x_2 = 1$$

$$3x_1 - x_2 \geq 0$$

and constraints on μ_1, μ_2 become

$$\mu_1 + 3\mu_2 = 2$$

$$|\mu_1 - \mu_2| \leq 1$$

a) Write its inner problem as a linear program.

b) For fixed x_1 and x_2 (under the constraints of $x_1+x_2 = 1, 3x_1-x_2 \geq 0$), find the dual of the inner problem, and simplify the dual objective if possible.

c) Combine the objectives of the outer and inner problem into a joint single layer problem.