PSET 4

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1\. a. False; from complementarity conditions, shadow price of non-binding constraint must be 0.

b. False; This problem is about weak duality theorem, It is not true "for any y". We can pick a y such that $b^T y > c^T x$.

c. False; If primal problem is unbounded, the dual problem must have infeasible solution instead of non-empty feasible solution.

d. False; If primal has a finite optimal value, dual must have feasible solution.

e. false; From Lec. 7 #25, if $Ax = b$, y has to be free. Hence only $Ax = b$, $x \geq 0$ or $A^T y \leq 0$, $b^T y > 0$ can be true at once.

2. a. maximize v over x_1, x_2, x_3, v

$$
\begin{array}{ll}
-4x_1 + x_2 + 4x_3 + 2x_4 + v \le 0 & y_1 \\
2x_1 - x_2 - 4x_3 - 2x_4 + v \le 0 & y_2 \\
-x_1 + 2x_2 - 2x_3 + 4x_4 + v \le 0 & y_3 \\
x_1 + x_2 + x_3 + x_4 = 1 & u \\
(x_1, x_2, x_3, x_4) \ge 0 &
\end{array}
$$

b. Dual: minimize u over y_1, y_2, u

$$
\begin{array}{l}
u - (4y_1 - 2y_2 + y_3) \ge 0 \\
u - (-y_1 + y_2 - y_3) \ge 0 \\
u - (-4y_1 + 4y_2 + 2y_3) \ge 0 \\
u - (-2y_1 + 2y_2 - 4y_3) \ge 0 \\
y_1 + y_2 + y_3 = 1 \\
(y_1, y_2, y_3) \ge 0
\end{array}
$$

c. The interpretations is that player would have $y_1, y_2,$ or y_3 probabilities to choose row 1, 2, or 3 respectively. We then choose the row that has the least expected payoff since we subtract u with expected payoff.

3. a. max $z - \pi^T x$ $\rightarrow$ min $0^T y$

s.t. $ez - Ax \le 0$ $\quad y$ $\qquad \begin{array}{c} z \\ x \end{array} \begin{bmatrix} e^T \\ -A^T \end{bmatrix} y = \begin{bmatrix} 1 \\ -\pi^T \end{bmatrix}$

free x, z

b. By inspection, dual problem is infeasible.

Because the dual is infeasible, the primal is unbounded. We can pick any x, z to exceed the value from the objective function.

Hence, there is no opportunity for arbitrage.

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4. a. max $\sum_{i=0}^{4} y_i$

	$y_4 = 1$	x_1
$y_3 - \gamma y_4 \le 0$	$y_3 \le \gamma y_4$	x_2
	$y_3 \le 0.5$	x_3
	$y_2 \le \gamma y_3 + 0.4$	x_4
	$y_2 \le \gamma (0.5 y_4)$	x_5
$y_1 - \gamma y_2 \le 0$	$y_1 \le \gamma \cdot y_2$	x_6
	$y_1 \le \gamma (0.5 y_3 + 0.25 y_4)$	x_7
	$y_0 \le \gamma y_1$	x_8
	$y_0 \le \gamma (0.5 y_2 + 0.25 y_3 + 0.125 y_4)$	x_9

$$b = \begin{bmatrix} 1 \\ 0 \\ 0.5 \\ 0.4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4.b. min $0.5x_3 + 0.4x_4 + x_1$

y_0) s.t. $x_8 + x_9 = 1$

y_1) $x_6 + x_7 - \gamma x_8 = 1$

y_2) $x_4 + x_5 - \gamma x_6 - \gamma 0.5 x_9 = 1$

$y_3)\quad x_2 + x_3 - \gamma x_4 - \gamma 0.5 x_7 - \gamma 0.25 x_9 = 1$

$y_4)\quad x_1 - \gamma x_2 - \gamma 0.5 x_5 - 0.25 \gamma x_7 - \gamma 0.125 x_9 = 1$

x_1 free $x_i \geq 0 \ \forall i \in \{2, \ldots, 9\}$

$(x_1, \ldots, x_9) =$

(from cvx) $(2.2195, 0, 1.0000, 0000, 2.7109, 1.9, 0.0000, 1, 1, 0)$

The positive values indicate the choice to go for. For example, at state 3, it is recommended to follow the blue (x_3) instead over to follow the red.

4.c. The positive values are binding constraints while zero values are non-binding.

The optimal policy is for each state to do as follows:

state 0 - red
1 - red
2 - blue
3 - blue
4 - cost 2

5. a. $\max\limits_{u_1, u_2} \quad -x_1 u_1 - x_2 u_2$

s.t. $u_1 + 3u_2 = 2 \qquad\qquad y_1$

$|u_1 - u_2| \leq 1 \Rightarrow \qquad u_1 - u_2 \leq 1 \qquad y_2$

$u_1 - u_2 \geq -1$

$\Downarrow$

$u_2 - u_1 \leq 1 \qquad y_3$

b. $\min \quad 2y_1 + y_2 + y_3$

s.t. $y_1 + y_2 - y_3 = -x_1$

$3y_1 - y_2 + y_3 = -x_2$

y_1 free

$y_2, y_3 \geq 0$

Problem session

$$-\frac{1}{4}(y_1 + y_2 + y_3) + \frac{3}{4}(3y_1 - y_2 + y_3) = 2y_1 - y_2 + y_3$$

$$= \frac{x_1}{4} - \frac{3}{4}x_2$$

$(2y_1 - y_2 + y_3) + 2y_2$

$y_2 \geq 0 \qquad y_2 = 0$

$$y_1 - y_3 = -x_1$$
$$3y_1 + y_3 = -x_2$$
$$\Rightarrow \quad y_1 = \frac{-x_1 - x_2}{4}$$

$$y_3 = \frac{3x_1 - x_2}{4}$$

$$y_3 \geq 0$$

c.

$$c_1 + 3c_2 = 2$$

$$c_1 - c_2 = 1$$

$$-c_1 + c_2 = 1$$

from above,

$$\therefore \quad \frac{x_1}{4} - \frac{3}{4}x_2$$