**Lab W1D4**

**Question 1**

An ***array*** of size 100000000000 is filled with four different letters A, B, C and D. Assume that all four letters are equally likely to appear in the array S. However, there is no guarantee that all four letters are in the array.

***(a) What is the average number of array locations to inspect to find the first D? Give your answer using a formula or result mentioned in the class note. Please give the Slide number as a reference***

As per lesson 4, probability space of the experiment is 4, because all possibility of outcome is {A, B, C, D}, p = ¼ as it is successful to find first D, the expected number (average number) of trials for a success = 𝟏/p, therefore we need **4** trials to find first D

***(b) What is the average number of array locations to inspect to find 10 D’s? Give your answer using a formula or result mentioned in the class note. Please give the Slide number as a reference.***

We need 1/p is an average to find first D, so find 10 D’s, we need 10\*1/p = **40** trials.

**(c) What is the “average time complexity” to find k D’s in an array?**

The average time complexity to find k D’s in same to number of array locations needed to inspect to find k D’s, so it is k\*(1/p) = 4k, therefore time complexity is O(4k) = **O(n)**

**Question 2. Prove: 1 + 1/2 + 1/3 + …+ 1/n = O(log n).**

let f(n) = 1 + ½ + 1/3 + … + 1/n

**let n = 7, we have**

1 + ½ + 1/3 + ¼ + 1/5 + 1/6 + 1/7

<= 1 + (½ + ½) + (¼ + ¼ + ¼ + ¼)

= 1 + 1 + 1

= 3

= log(7 + 1)

**Let n = 17, we have**

1 + ½ + 1/3 + ¼ + 1/5 + 1/6 + 1/7 + 1/8 + 1/9 + 1/10 + 1/11 + 1/12 + 1/13 + 1/14 + 1/15 + 1/16 + 1/17

<= 1 + (1/2 + ½) + (1/5 + 1/5 + 1/5 + 1/5 + 1/5) + (1/9 + 1/9 + 1/9 + 1/9 + 1/9 + 1/9 + 1/9 + 1/9 + 1/9)

= 4

= log(16 + 1)

So f(n) <= log(n + 1), so f(n) <= logn

Therefore f(n) is **O(logn)**

**Question 3. Find the sum: 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + …**

Let S = **1/2 + 2/4 + 3/8 + 4/16 + 5/32 + 6/64**

S/2 = ¼ + 2/8 + 3/16 + 4/32 + 5/64 + …

S – S/2 = ½ + ¼ + 1/8 + 1/16 + 1/64 + .. + ½^(n -1)

= 1/(1 – ½) – 1

S/2 = 1

**S = 2**