1.

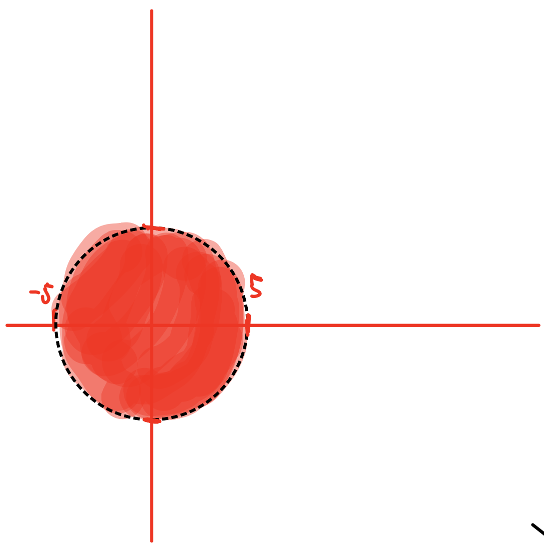
$$f(x,y) = \ln(25 - x^2 - y^2) + \arccos(1-x)$$

$\sqrt{x}$	$\langle 0, \infty)$	$\dots \geq 0$
$\frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\dots \neq 0$
$\ln(x)$	$(0, \infty)$	$\dots > 0$
tg	$\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$	
cotg	$\mathbb{R} \setminus \{\pi + k\pi,\ k \in \mathbb{Z}\}$	
arcsin	$\langle -1, 1 \rangle$	
arccos	$\langle -1, 1 \rangle$	

$$25 - x^2 - y^2 > 0$$

$$-x^2 - y^2 > -25$$

$$x^2 + y^2 < 25$$



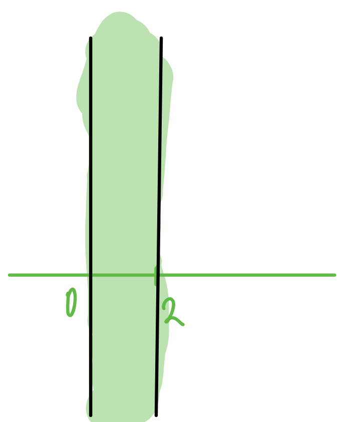
$$-1 \leq 1 - x \leq 1$$

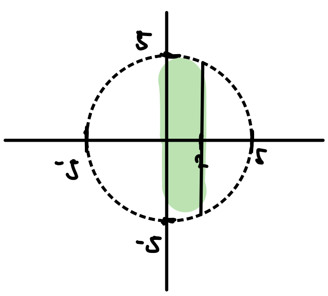
$$-1 \leq 1 - x \qquad 1 - x \leq 1$$

$$-2 \leq -x \qquad -x \leq 0$$

$$2 \geq x \qquad x \geq 0$$

$\times$





$$D(f) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 25;\ x \in \langle 0; 2 \rangle\}$$

2.

$f(x,y) = \sqrt{x^2 - \sin y}$ $\quad C = (-1, 0, \overset{z_0}{1})$

$\tau: z = f_0(x_0, y_0) + f_x'(x_0, y_0) \cdot (x - x_0) + f_y'(x_0, y_0) \cdot (y - y_0)$

$f_x' = \frac{\cancel{1}}{\cancel{2}} \cdot (x^2 - \sin y)^{-\frac{1}{2}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2 - \sin y}} \Rightarrow \frac{-1}{\sqrt{(-1)^2 - \sin 0}} = -1$

$f_y' = \frac{1}{2} \cdot (x^2 - \sin y)^{-\frac{1}{2}} \cdot \cos y = \frac{-\cos y}{2\sqrt{x^2 - \sin y}} \Rightarrow \frac{-1}{2 \cdot 1} = -\frac{1}{2}$

$z = 1 - 1 \cdot (x + 1) - \frac{1}{2} \cdot (y - 0)$

$z = 1 - x - 1 - \frac{1}{2} y = -x - \frac{1}{2} y \Rightarrow 1{,}05$

$\sqrt{(\underset{x}{-1{,}075})^2 - \sin(\underset{y}{0{,}05})}$

3. $f(x,y) = x^3 - 10x^2 - 4xy - 4y^2 + 20x - 8y + 32$

$f'_x = 3x^2 - 20x - 4y + 20$

$f_y' = -4x - 8y - 8 \Rightarrow -4x - 8y - 8 = 0$

$$-8y = 4x + 8$$

$$y = \frac{4x+8}{-8}$$

$$y = -\frac{1}{2}x - 1$$

$3x^2 - 20x - 4 \cdot \left(-\frac{1}{2}x - 1\right) + 20 = 3x^2 - 20x + 2x + 4 + 20$

$= 3x^2 - 18x + 24$

$= x^2 - 6x + 8$

$D = 36 - 4 \cdot 1 \cdot 8$

$= 36 - 32$

$= 4$

$$x_{1,2} = \frac{6 \pm 2}{2} = \begin{cases} 4 \\ 2 \end{cases}$$

$B_1\,[4, -3]$

$B_2\,[2, -2]$

$$y_1 = -\frac{1}{2} \cdot 4 - 1 = -3$$

$$y_2 = -\frac{1}{2} \cdot 2 - 1 = -2$$

$$f_{xx} = 6x - 20$$

$$f_{xy} = -4$$

$$f_{yy} = -8$$

B_1:

$$J = \begin{bmatrix} 6x-20 & -4 \\ -4 & -8 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & -8 \end{bmatrix}$$

$$= -32 - 16$$

není extrém

B_2

$$J = \begin{bmatrix} -8 & -4 \\ -4 & -8 \end{bmatrix} = 64 - 36$$

je extrém

ostré maximum

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4. $y'' + 7y' + 12y = 0$ $\quad y(0) = 0$

$y'(0) = -1$

$$\lambda^2 + 7\lambda + 12 = 0$$

$$D = 49 - 4 \cdot 1 \cdot 12$$

$$D = 49 - 48$$

$$D = 1$$

$$\lambda_{1,2} = \frac{-7 \pm 1}{2} = \begin{cases} -3 \ldots e^{\int \lambda_1} = e^{-3x} \\ -4 \ldots e^{\int \lambda_2} = e^{-4x} \end{cases}$$

$-3 \quad e^{-3x}$ common knowledge

$-3 \quad xe^{-3x}$

$$2\sqrt{-16} = 2 \pm 4i < \begin{matrix} e^{2x} \cdot \cos(4x) \\ e^{2x} \cdot \sin(4x) \end{matrix}$$

$y = C_1 \cdot e^{-3x} + C_2 \cdot e^{-4x}$; $\quad C_1, C_2 \in \mathbb{R}$

$y' = C_1 \cdot e^{-3x} \cdot (-3) + C_2 \cdot e^{-4x} \cdot (-4)$

$0 = C_1 \cdot e^{-3 \cdot 0} + C_2 \, e^{-4 \cdot 0}$

$0 = C_1 \cdot 1 + C_2 \cdot 1$

$0 = C_1 + C_2$

$C_1 = -C_2$ $\Rightarrow$ $C_1 = -1$

$-1 = c_1 \cdot 1 \cdot (-3) + c_2 \cdot 1 \cdot (-4)$

$-1 = -c_2 \cdot 1 \cdot (-3) + c_2 \cdot 1 \cdot (-4)$

$-1 = 3c_2 - 4c_2$

$-1 = -c_2$

$c_2 = 1$

$y = -e^{-3x} + e^{-4x}$

5.

$y' - 2y = 12x$

$$e^{\int -2\,dx} = e^{-2x}$$

$t = -2x$

$dt = -2\,dx$

$\frac{dt}{-2} = dx$

$\int e^{-2x}\,dx$

$\int e^{t}\,\frac{dt}{-2}$

$$\underbrace{e^{-2x} \cdot y' - 2y \cdot e^{-2x}}_{\int (y \cdot e^{-2x})'} = 12x \cdot e^{-2x}$$

$\int (y \cdot e^{-2x})' = \int 12x \cdot e^{-2x}$

$$-\frac{1}{2}\int e^{t}\,dt$$

$$u = x \qquad v' = e^{-2x}$$

$$u' = 1 \qquad v = -\frac{1}{2}\cdot e^{-2x}$$

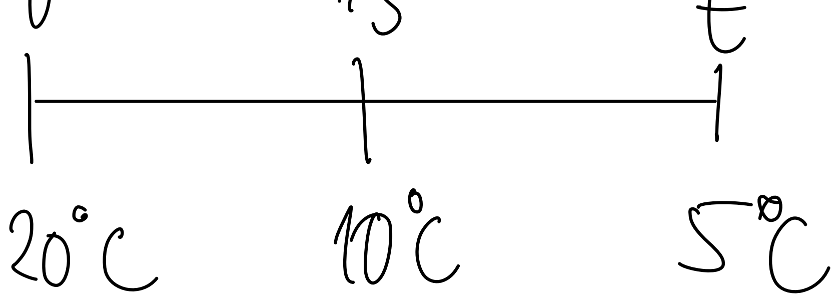
$$12\int x\cdot e^{-2x} = 12\cdot\left(x\left(-\frac{1}{\cancel{2}}\right)e^{-2x} - \int 1\cdot\left(-\frac{1}{\cancel{2}}\right)\cdot e^{-2x}\right)$$

$$= 6\cdot\left(xe^{-2x} + \int e^{-2x}\right)$$

$$-6xe^{-2x} - 3\cdot e^{-2x}$$

$$y\cdot e^{-2x} = -6xe^{-2x} - 3e^{-2x} + C$$

$$y = -6x - 3 + C\cdot e^{2x}$$



$y(0) = 20$

$y(15) = 10$

$y(t) = 5$

$$y' = -m y \qquad m \geq 0$$

$$\frac{1}{y} y' = -m$$

$$\int \frac{1}{y} = \int -m$$

$$\ln(y) = -mt + C$$

$$|y| = e^{-mt} + C$$

$$y = e^{-mt} \cdot e^{C}$$

$$y = e^{-mt} \cdot C_2$$

$$20 = e^{-m \cdot 0} \cdot C_2$$

$$20 = C_2$$

$$10 = e^{-m \cdot 15} \cdot 20$$

$$\frac{1}{2} = e^{-m \cdot 15}$$

$$\ln\left(\frac{1}{2}\right) = -k \cdot 15$$

$$-\frac{1}{15}\ln\left(\frac{1}{2}\right) = k$$

$$y = e^{\frac{1}{15}\ln\left(\frac{1}{2}\right)t} \cdot 20$$

$$5 = e^{\frac{1}{15}\ln\left(\frac{1}{2}\right)\cdot t} \cdot 20$$

$$\ln\left(\frac{1}{4}\right) = \frac{1}{15}\ln\left(\frac{1}{2}\right)\cdot t$$

$$t = 15\cdot\frac{\ln\left(\frac{1}{4}\right)}{\ln\left(\frac{1}{2}\right)} = 30\,\text{min}$$

$$u = \frac{dx}{du}$$

$$f_{(x,y)} = \operatorname{arctg}\left(\frac{2+x+y}{1+y}\right)$$

$$f_x' = \frac{1}{\left(\frac{2+x+y}{1+y}\right)^2+1}\cdot\frac{1}{1+y}$$

$$f_y' = \frac{1}{\left(\frac{2+x+y}{1+y}\right)^2+1}\cdot\frac{1\cdot(1+y)-(2+x+y\cdot 1)}{(1+y)^2}$$

arc sin

$$\frac{1}{\sqrt{1-x^2}}$$

arc cos

$$-\frac{1}{\ldots}$$

$\sqrt{1-x^2}$

arctg $\quad$ arccotg

$\frac{1}{1+x^2} \quad -\frac{1}{1+x^2}$

$$f'_x(1,0) = \frac{1}{\left(\frac{2+1+0}{1+0}\right)^2+1} \cdot \frac{1}{1+0} = \frac{1}{10}$$

$$f'_y(1,0) = \frac{1}{10} \cdot \frac{1\cdot 1-3}{1} = \frac{1}{10} \cdot -3 = -\frac{2}{10} = -\frac{1}{5}$$

$$\operatorname{grad} f(x_0,y_0) = \left(\frac{1}{10} ; -\frac{1}{5}\right)$$

$$\|\operatorname{grad} f(x_0,y_0)\| = \sqrt{\frac{1}{100}+\frac{1}{25}} = \sqrt{\frac{1+4}{100}} = \sqrt{\frac{5}{100}} = \frac{\sqrt{5}}{10}$$

$$u = \frac{\operatorname{grad} f}{\|\operatorname{grad} f\|} = \left(\frac{\frac{1}{10}}{\frac{\sqrt{5}}{10}} ; \frac{-\frac{1}{5}}{\frac{\sqrt{5}}{10}}\right) = \left(\frac{\cancel{10}}{\sqrt{5}\cdot\cancel{10}} ; -\frac{\cancel{10}^{\,2}}{\cancel{5}\sqrt{5}}\right)$$

6. $y' = -my \qquad m \geq 0$

$y(0) = 1$

$y(28{,}1) = \frac{1}{2}$ ... Poločas

$y(t_1) = \frac{1}{4}$

$$\frac{1}{y} y' = -m$$

$$\int \frac{1}{y}\,dy = \int -m\,dt$$

$$\ln|y| = -mt$$

$$y = e^{-mt} \cdot h$$

$$y_1(0) = 1 : \quad 1 = e^{0} \cdot h$$

$$1 = h$$

$$y_2(28{,}1) = \frac{1}{2} : \quad \frac{1}{2} = e^{-m \cdot 28{,}1} \cdot 1$$

$$\ln\left(\frac{1}{2}\right) = -m \cdot 28{,}1$$

$$-\frac{\ln\left(\frac{1}{2}\right)}{28{,}1} = m$$

$$y_3(t_1) = \frac{1}{4} : \quad \frac{1}{4} = e^{\frac{\ln\left(\frac{1}{2}\right)}{28{,}1} \cdot t} \cdot 1$$

$$t = \frac{\ln\left(\frac{1}{4}\right) \cdot 28{,}1}{\ln\left(\frac{1}{2}\right)}$$

B

$$t = \frac{-\ln(4)\cdot 28{,}1}{-\ln(2)} = 2\cdot 28{,}1 =$$

$$= 56{,}2 \text{ let}$$

C.

3.

$$f(x,y) = y^3 + 4x^2 + 4xy + 19y^2 - 28x + 51y + 20$$

$$f_x' = 8x + 4y - 28$$

$$f_y' = 3y^2 + 4x + 38y + 51$$

$$8x + 4y - 28 = 0$$

$$8x = -4y + 28$$

$$x = \frac{-4y + 28}{8}$$

$$3y^2 + \overset{1}{\cancel{4}}\cdot\left(\frac{-\overset{2}{\cancel{4}}y + \overset{14}{\cancel{28}}}{\underset{2}{\cancel{8}}}\right) + 38y + 51 = 0$$

$$3y^2 - 2y + 14 + 38y + 51 = 0$$

$$3y^2 + 36y + 105 = 0$$

$$y^2 + 12y + 35 = 0$$

$$y_{1,2} = \frac{-12 \pm \sqrt{12^2 - 4\cdot 1\cdot 35}}{2} = \frac{-12 \pm \sqrt{144 - 140}}{2} =$$

$$y_{1,2} = \frac{-12 \pm 2}{2} \begin{cases} -7 \\ -5 \end{cases}$$

$$x_1 = \frac{-4\cdot(-7)+28}{8}$$

$$x_1 = 7$$

$$x_2 = \frac{-4\cdot(-5)+28}{\quad} = \frac{48}{8} = 6$$

8 8

$B_1\ [7, -7]$

$B_2\ [6, -5]$

$f_{xx} = 8$

$f_{xy} = 4$

$f_{yy} = 6y + 38$

B_1:

$$J = \begin{bmatrix} 8 & 4 \\ 4 & 6 \cdot (-7) + 38 \end{bmatrix} = 8 \cdot (-4) - (4 \cdot 4)$$

$-42 + 38$

není

B_2:

$$J = \begin{bmatrix} 8 & 4 \\ 4 & 6\cdot(-5)+38 \end{bmatrix} = 8\cdot 8-(4\cdot 4) =$$

$$= 64-16$$

je lok. min ☺

4.

$y''+2y'-15y=0 \qquad y(0)=0$

$\lambda^2+2\lambda-15=0 \qquad y'(0)=8$

$\Delta = 2^2-4\cdot 1\cdot(-15)$

$\Delta = 64$

$$x_{1,2} = \frac{-2 \pm 8}{2} \quad \begin{cases} 3 \ldots e^{3x} \\ -5 \ldots e^{-5x} \end{cases}$$

$$y = C_1 \cdot e^{3x} + C_2 e^{-5x}$$

$$0 = C_1 \cdot e^{3 \cdot 0} + C_2 \cdot e^{-5 \cdot 0}$$

$$0 = C_1 + C_2$$

$$C_1 = -C_2$$

$$y' = C_1 \cdot e^{3x} \cdot 3 + C_2 \cdot e^{-5x} \cdot (-5)$$

$$8 = C_1 \cdot e^{3 \cdot 0} \cdot 3 + C_2 \cdot e^{-5 \cdot 0} \cdot (-5)$$

$$8 = -3c_2 - 5c_2$$

$$8 = -8c_2$$

$$c_2 = -1$$

$$c_1 = -c_2 = -(-1) = 1$$

$$y = e^{3x} - e^{-5x}$$

5.

$$y' - 3y = x$$

$$e^{\int -3} = e^{-3x}$$

$$e^{-3x} \cdot y' - 3y \cdot e^{-3x} = e^{-3x} \cdot x$$

$$\int (y \cdot e^{-3x})' = \int e^{-3x} x$$

$$y \cdot e^{-3x} = x \cdot \left(-\frac{1}{3}e^{-3x}\right) - \int 1 \cdot \left(-\frac{1}{3}e^{-3x}\right)$$

$$y \cdot e^{-3x} = -\frac{x}{3}e^{-3x} + \frac{1}{3}\int e^{-3x}$$

$u = x \qquad v' = e^{-3x}$

$$y \cdot e^{-3x} = -\frac{x}{3}e^{-3x} + \frac{1}{3} \cdot \left(-\frac{1}{3}e^{-3x}\right)$$

$u' = 1 \qquad v = -\frac{1}{3}e^{-3x}$

$$\int e^{-3x}\,dx = -\frac{1}{3}\int e^{t}\,dt = -\frac{1}{3}e^{-3x}$$

$t = -3x$

$dt = -3\,dx$

$dx = \frac{dt}{-3}$

$$y \cdot e^{-3x} = -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x} + C$$

$$y = -\frac{x}{3} - \frac{1}{9} + \frac{C}{e^{-3x}}$$

6.

$$y' = -ky$$

0	5730	t
1	$\frac{1}{2}$	$\frac{1}{16}$

$y(0) = 1$

$y(5730) = \frac{1}{2}$

$y(t) = \frac{1}{16}$

$$\frac{1}{y} y' = -k$$

$$\int \frac{1}{y} = \int -k \, dt$$

$$\ln|y| = -mt + C$$

$$y = e^{-mt} + e^{C}$$

$$y = e^{-mt} \cdot q_2$$

$$1 = e^{-m \cdot 0} \cdot q_2$$

$$q_2 = 1$$

$$\frac{1}{2} = e^{-5730m} \cdot 1$$

$$\ln\left(\frac{1}{2}\right) = -m5730 + 1$$

$$-\frac{\ln\left(\frac{1}{2}\right)}{\quad} = m \quad ?$$

5730

$$\frac{1}{16} = e^{\frac{\ln\left(\frac{1}{2}\right)}{5730}t}$$

$$\ln\left(\frac{1}{16}\right) = \frac{\ln\left(\frac{1}{2}\right)}{5730}t$$

$$\frac{\ln\left(\frac{1}{16}\right)}{\ln\left(\frac{1}{2}\right)} \cdot 5730 = t$$

$$\frac{-\ln(16)}{-\ln(2)} \cdot 5730 = t$$

$$8 \cdot 5730 = t$$

$$f(x,y) = \operatorname{arctg}\left(\frac{y}{x+y}\right) \qquad (x_0, y_0) = (2,1)$$

$$f_x' = \frac{1}{1+\left(\frac{y}{x+y}\right)^2} \cdot \left(-\frac{y}{(x+y)^2}\right) = \frac{1}{\frac{10}{9}} \cdot \frac{-1}{9} = \left(\frac{1}{\frac{10}{9}}\right) \cdot \left(-\frac{1}{9}\right)$$

$$= \frac{9}{10} \cdot \left(-\frac{1}{9}\right) = -\frac{9}{90} = -\frac{1}{10}$$

$$f_y' = \frac{1}{1+\left(\frac{y}{x+y}\right)^2} \cdot \left(\frac{1 \cdot (x+y) - y \cdot (1)}{(x+y)^2}\right) = \frac{1}{5}$$

$$\operatorname{grad} f \left(-\frac{1}{10}, \frac{1}{5}\right)$$

$$\|\operatorname{grad} f\| = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{5}\right)^2} = \sqrt{\frac{1}{100} + \frac{1}{25}} = \sqrt{\frac{5}{100}}$$

$$= \sqrt{\frac{1}{20}}$$

$$\vec{u} = \frac{\operatorname{grad} f}{\|\operatorname{grad} f\|} = \left(\frac{-\frac{1}{10}}{\frac{1}{\sqrt{20}}} ; \frac{\frac{1}{5}}{\frac{1}{\sqrt{20}}}\right) = \qquad = \frac{1}{\sqrt{20}}$$

$$= \left(-\frac{\sqrt{20}}{10} ; \frac{\sqrt{20}}{5}\right)$$

$$\frac{y}{x+y}\,dx = y\cdot(x+y)^{-1}$$

$$y\cdot\left(-1\cdot(x+y)^{-2}\right)\cdot 1$$

$$\frac{-y}{(x+y)^2}$$