VŠB TECHNICKÁ UNIVERZITA OSTRAVA | FAKULTA ELEKTROTECHNIKY A INFORMATIKY

Komponenta výukového serveru TI NP-úplné problémy 2

Component of Learning Server for
Theoretical Computer Science
NP-complete problems 2

Phat Tran Dai

Bakalářská práce

Vedoucí práce: Borivoj Gulas

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Zadání bakalářské práce

Student: **Phat Dai Tran**

Studijní program: B0613A140014 Informatika

Téma: Komponenta výukového serveru TI - NP-úplné problémy 2

Component of Learning Server for Theoretical Computer Science - NP-complete problems 2

Jazyk vypracování: čeština

Zásady pro vypracování:

V rámci diplomových a bakalářských prací vzniká výukový server pro předměty teoretické informatiky. Jedná se o sadu dynamických webových stránek umožňujících studentům pochopení různých typů úloh a problémů tím, že si mohou zadat na stránce libovolné zadání a zobrazí se jim řešení včetně postupu. Cílem této práce je vytvořit komponentu (tedy sadu webových stránek) pro výuku vybraných NP-úplných problémů a převodů mezi nimi.

1. Nastudujte si problematiku tříd složitosti problémů a s tím souvisejícím převodem mezi problémy.
2. Vytvořte dynamické webové stránky umožňující uživateli následující:
a) Nechat si zobrazit postupně po krocích postup algoritmu s polynomiální časovou složitostí převádějícího zadanou instanci jednoho problému na instanci jiného problému (budou implementovány alespoň 3 různé převody mezi problémy).
b) Zadat libovolnou instanci každého z problémů vyskytujících se v těchto převodech.
c) Zobrazit odpověď na otázku daného problému pro zadanou instanci, v případě kladné odpovědi i se zdůvodněním.
3. Není cílem mít co nejefektivněji implementován samotný převod, ale mít jej implementován tak, aby uživateli byla myšlenka tohoto převodu co nejsrozumitelněji ukázána.
4. Vytvořte i ukázkové vstupní instance pro implementované problémy tak, aby uživatel mohl vše vyzkoušet i bez zadávání vlastních vstupů (alespoň 5 instancí pro každý problém).

Studenti řešící toto zadání s rozdílným číslem v názvu mohou (ale nemusí) spolupracovat tak, že výsledek může mít společné uživatelské rozhraní apod. Ale každý bude implementovat jiné 3 převody mezi problémy.

Seznam doporučené odborné literatury:

[1] Sipser, M.: Introduction to the Theory of Computation, PWS Publishing Company, 1997.
[2] Papadimitriou, C.: Computational Complexity, Addison Wesley, 1993.
[3] Sawa, Z.: Prezentace přednášek předmětu Teoretická informatika, dostupné online https://www.cs.vsb.cz/sawa/ti/index.html

Formální náležitosti a rozsah bakalářské práce stanoví pokyny pro vypracování zveřejněné na webových stránkách fakulty.

Vedoucí bakalářské práce: **Ing. Martin Kot, Ph.D.**

Datum zadání:

Datum odevzdání:

Garant studijního programu: doc. Mgr. Miloš Kudělka, Ph.D.

V IS EDISON zadáno:

Abstrakt

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Abstract

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Poděkování

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Seznam použitých zkratek a symbolů

WTF	– What the fuck
DPC	– Do x^2 pice
KDPC	– Kurva do pice $\rightarrow$ aaaah

Obsah

Seznam obrázků

Seznam tabulek

Seznam algoritmů

Seznam výpisů

Úvod

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Kapitola 1

Heading Level 1

QQ *hello* ja fi diofjosdfodsjodslsdlsdlkjflskdjkffffffffffffffffffffffffff kldkjfksdfjlsdfljjjjj.j..jjl jsem k a v o.

$$f(x) = y$$

$$\begin{pmatrix} 1 & 2 & \dots & 10 \\ 2 & 2 & \dots & 10 \\ \vdots & \vdots & \ddots & \vdots \\ 10 & 10 & \dots & 10 \end{pmatrix}$$

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1.1 Heading Level 2

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$$\begin{aligned} \Omega = \{&(2,1),(2,1),(5,1),(5,1), \\ &(2,4),(2,4),(5,4),(5,4), \\ &(2,4),(2,4),(5,4),(5,4), \\ &(2,4),(2,4),(5,4),(5,4)\} \end{aligned}$$

Velikost pravděpodobnostního prostoru je $|\Omega| = 16$. Z rozepsané Ω vidíme, že počet případů, kdy kostka B vyhraje nad A je vyšší (10) než počet, kdy prohraje (6). Pravděpodobnost vypočteme jako:

$$P(B > A) = \frac{2 + 4 \cdot 2}{|\Omega|} = \frac{10}{16} = \underline{\underline{0.625}}$$

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Kapitola 2

OFDM - Orthogonal Frequency Division Multiplexing

2.1 Introduction

In `ref`Kapitola 2.1, we see how to turn Sections into Chapters. And in `ref`kapitole 2.1, it is done manually.

$$\leq \pm$$

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$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n$$

Code snippet in C programming langauge:

```c
#include <stdio.h>

int main() {
  printf("hello, world!\n");
  return 0;
}
```

Výpis 1: Computer program in C language

More simple langauge, for example SQL:

```sql
SELECT
  c.customer_id,
  c.fname,
  c.lname,
  c.email
FROM customer c
WHERE EXISTS (
  SELECT *
  FROM purchase p
  WHERE p.customer_id = c.customer_id
)
```

Výpis 2: Simple SQL query

2.2 Subheading Level 2

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$$\begin{bmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{bmatrix}$$

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2.3 Prokázání netranzitivity

První úkolem je ukázat, že vztahy mezi kostkami nejsou tranzitivní, to znamená, že vztahy mezi kostkami jsou tzv. cyklické[1] Tvrdíme totiž, že platí $B > A$, $C > B$ a současně $A > C$. To znamená, že žádná kostka není „nejlepší“ ve všech případech.

Pro každou dvojici kostek vypočítáme pravděpodobnost vítězství jedné kostky nad druhou, konkrétně $P(B > A)$, $P(C > B)$ a $P(A > C)$, a ověříme, že všechny tyto pravděpodobnosti jsou větší než $\frac{1}{2}$.

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2.4 Subheading Level 2

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```
#show "ArtosFlow": name => box[
  #box(image(
    "logo.svg",
    height: 0.7em,
  ))
  #name
]

This report is embedded in the
ArtosFlow project. ArtosFlow is a
project of the Artos Institute.
```

2.4.1 Subheading Level 3

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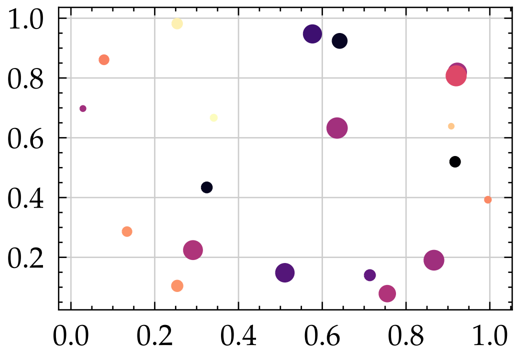
[1]Wikipedia, *Intransitivity*: https://en.wikipedia.org/wiki/Intransitivity

animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut.



Obrázek 1: Some random data visualisation

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Kapitola 3

Heading Level 1

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3.1 Subheading Level 2

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Index	**Value**
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11

Index	Value
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13
1	10
2	11
3	12
4	13

Tabulka 1: A looong table

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3.2 Subheading Level 2

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Shape	**Area**
Circle	πr^2
Square	a^2
Rectangle	ab

Tabulka 2: A simple table

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Definujeme:

$$\phi := \frac{1+\sqrt{5}}{2} \tag{1}$$

Pomocí rovnice 1, dostaneme:

$$F_n = \left\lfloor \frac{1}{\sqrt{5}} \phi^n \right\rceil$$

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$$\begin{aligned} E &= \sqrt{m_0^2 + p^2} \\ &\approx 125 \text{ GeV} \end{aligned}$$

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$$\begin{aligned} E &= mc^2 \qquad (2.1) \\ &= \sqrt{p^2c^2 + m^2c^4} \qquad (2.2) \end{aligned}$$

3.3 Subheading Level 2

Skalární součin dvou vektrorů $\vec{a}$ a $\vec{b}$ je znázorněn rovnicí 3.

$$\begin{aligned} \langle a, b \rangle &= \vec{a} \cdot \vec{b} \\ &= a_1b_1 + a_2b_2 + ...a_nb_n \\ &= \sum_{i=1}^{n} a_ib_i. \qquad (3.1) \end{aligned}$$

Notace sumy v rovnici 3.1 je užitečný způsob zápisu skalárního součinu dvou vektorů.

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Kapitola 4

Heading Level 1

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4.1 Heading Level 2

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4.1.1 Heading Level 2

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4.2 Heading Level 3

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Kapitola 5

Heading 5

Jelikož základ nehomogenní funkce je roven jedné, počet výskytů (násobnost) tohoto základu v multimnožině kořenu charakteristické rovnice je roven jedné a největší exponent n-ka nehomogenní funkce je taktéž roven jedné, je tvar obecného partikulárního řešení rekurentí rovnice p_n roven:

$$
\begin{aligned}
p_n^{(p)} &= n^1(\beta_1 n^1 + \beta_0)1^n \;=\; n(\beta_1 n + \beta_0) \;=\; \beta_1 n^2 + \beta_0 n \\
p_n^{(p)} &= \varphi n^2 + \gamma n \qquad\qquad (\gamma = \beta_0, \varphi = \beta_1).
\end{aligned}
$$

Neznámé γ a φ nalezneme substitucí členů p_n v původní rekurentní rovnici:

$$p_n = 3p_{n-1} - 2p_{n-2} - 10n.$$

partikulárním řešením $p_n^{(p)}$ a vyřesíme soustavu dvou rovnic (dvě kvůli dvěma neznámým), kde si za n zvolíme jakékoliv čísla z $\mathbb{N}_0$.

$p_n \leftarrow p_n^{(p)}$:

$$p_n^{(p)} = 3p_{n-1}^{(p)} - 2p_{n-2}^{(p)} - 10n$$

$$
\begin{aligned}
\varphi n^2 + \gamma n &= 3\big(\varphi(n-1)^2 + \gamma(n-1)\big) \\
&\quad - 2\big(\varphi(n-2)^2 + \gamma(n-2)\big) - 10n \\
\varphi n^2 + \gamma n &= 3\big(\varphi\big(n^2 - 2n + 1\big) + \gamma n - \gamma\big) \\
&\quad - 2\big(\varphi\big(n^2 - 4n + 4\big) + \gamma n - 2\gamma\big) - 10n \\
\varphi n^2 + \gamma n &= 3\big(\varphi n^2 - 2\varphi n + \varphi + \gamma n - \gamma\big) \\
&\quad - 2\big(\varphi n^2 - 4\varphi n + 4\varphi + \gamma n - 2\gamma\big) - 10n \\
\varphi n^2 + \gamma n &= 3\varphi n^2 - 6\varphi n + 3\varphi + 3\gamma n - 3\gamma \\
&\quad - 2\varphi n^2 + 8\varphi n - 8\varphi - 2\gamma n + 4\gamma - 10n
\end{aligned}
$$

$$
\begin{aligned}
\varphi n^2 + \gamma n &= \varphi n^2 + 2\varphi n - 5\varphi + \gamma n + \gamma - 10n \\
0 &= 2\varphi n - 5\varphi + \gamma - 10n
\end{aligned}
$$

Nyní si za n zvolíme například nulu a jedničku.

$$
\begin{aligned}
n = 0&: \quad 0 = 2\varphi \cdot 0 - 5\varphi + \gamma - 10 \cdot 0 \\
n = 1&: \quad 0 = 2\varphi \cdot 1 - 5\varphi + \gamma - 10 \cdot 1
\end{aligned}
$$

$$\begin{cases} 0 = -5\varphi + \gamma \\ 0 = 2\varphi - 5\varphi + \gamma - 10 \end{cases}$$

$$\begin{cases} 0 &= \gamma - 5\varphi \\ 10 &= \gamma - 3\varphi \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & -5 & 0 \\ 1 & -3 & 10 \end{array}\right) \overset{R_2 \leftarrow R_2 - R_1}{\sim} \left(\begin{array}{cc|c} 1 & -5 & 0 \\ 0 & 2 & 10 \end{array}\right) \overset{R_2 \leftarrow \frac{1}{2} R_2}{\sim} \left(\begin{array}{cc|c} 1 & -5 & 0 \\ 0 & 1 & 5 \end{array}\right)$$

$$\varphi = 5$$

$$\gamma - 5 \cdot \varphi = 0 \Leftrightarrow \gamma = 25$$

Tudíž řešení partikulární rovnice $p_n^{(p)}$ je:

$$p_n^{(p)} = \varphi n^2 + \gamma n$$

$$\underline{p_n^{(p)} = 5n^2 + 25n}.$$

Závěr

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Sed iure Mucius. Ego autem mirari satis non queo unde hoc sit tam insolens domesticarum rerum fastidium. Non est omnino hic docendi locus; sed ita prorsus existimo, neque eum Torquatum, qui hoc primus cognomen invenerit, aut torquem illum hosti detraxisse, ut aliquam ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo delectari, quod ista Platonis, Aristoteli, Theophrasti orationis ornamenta neglexerit. Nam illud quidem physici, credere aliquid esse minimum, quod profecto numquam putavisset, si a Polyaeno, familiari suo, geometrica discere maluisset quam illum etiam ipsum dedocere. Sol Democrito magnus videtur, quippe homini erudito in geometriaque perfecto, huic pedalis fortasse; tantum enim esse omnino in nostris poetis aut inertissimae segnitiae est aut fastidii delicatissimi. Mihi quidem videtur, inermis ac nudus est. Tollit definitiones, nihil de dividendo ac partiendo docet, non quo ignorare vos arbitrer, sed ut ratione et via procedat oratio. Quaerimus igitur, quid sit extremum et ultimum bonorum, quod omnium philosophorum sententia tale debet esse.

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Příloha A

Qsort implementation

Implementation is in Haskell.

```haskell
quicksort [] = []
quicksort (p:xs) = (quicksort lesser) ++ [p] ++ (quicksort greater)
  where
    lesser = filter (< p) xs
    greater = filter (>= p) xs
```

Yes. Very cool.

Algorithm 1: Binary Search

procedure BINARY-SEARCH(A, n, v)
    ▷ Initialize the search range
    $l \leftarrow 1$
    $r \leftarrow n$

    while $l \leq r$ **do**
        mid $\leftarrow$ **floor**$(\frac{l+r}{2})$
        if $A[\text{mid}] < v$ **then**
            $l \leftarrow m + 1$
        else if $A[\text{mid}] > v$ **then**
            $r \leftarrow m - 1$
        else
            return m
        end
    end
    return null
end

Příloha B

Shit table

A	B
B	C
C	D

Algorithm 2: Variable Assignment

1: $x \leftarrow y$

Příloha C

OpenGL Shader Compilation

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```cpp
unsigned int OpenGLShader::Compile(
    const std::unordered_map<unsigned int, std::string>& sources
) const
{
    EG_PROFILE_FUNCTION();

    EG_CORE_ASSERT(sources.size() >= 0 && sources.size() <= 3,
        "Can only have three shader sources \
        (vertex, geometry, fragment)!");

    unsigned int program;
    EG_OPENGL_CALL(program = glCreateProgram());

    std::vector<unsigned int> shaders(sources.size());
    for (const auto& [type, source] : sources)
    {
        auto shader = CompileSource(type, source);
        shaders.push_back(shader);
        EG_OPENGL_CALL(glAttachShader(program, shader));
    }

    EG_OPENGL_CALL(glLinkProgram(program));

    int status;
    EG_OPENGL_CALL(glGetProgramiv(program, GL_LINK_STATUS, &status));
    if (status == GL_FALSE)
    {
        int length;
        EG_OPENGL_CALL(glGetProgramiv(
            program, GL_INFO_LOG_LENGTH, &length));

        std::vector<char> message(length);
        EG_OPENGL_CALL(glGetProgramInfoLog(
            m_RendererID, length, &length, message.data()));

        EG_OPENGL_CALL(glDeleteShader(program));
        for (auto shader : shaders)
        {
            EG_OPENGL_CALL(glDeleteShader(shader));
        }
        EG_CORE_ERROR("{}", message.data());
        EG_CORE_ASSERT(false, "Shader compilation failed!");
        return 0;
    }

    EG_OPENGL_CALL(glValidateProgram(program));
    for (auto shader : shaders)
```

```
    {
        EG_OPENGL_CALL(glDeleteShader(shader));
    }

    return program;
}
```

Výpis 3: C++ method for GLSL shader compilation

Příloha D

C# code

```
using System.Diagnostics;
using System.Security.Claims;
using CoworkingApp.Models;
using CoworkingApp.Models.Misc;
using CoworkingApp.Models.ViewModels;
using CoworkingApp.Services.Repositories;
using Microsoft.AspNetCore.Authorization;
using Microsoft.AspNetCore.Mvc;

namespace CoworkingApp.Controllers.ViewControllers;

public class HomeController
    (
        IWorkspaceRepository workspaceRepository,
        ICoworkingCenterRepository coworkingCenterRepository,
        IReservationRepository reservationRepository,
        IUserRepository userRepository
    )
    : Controller
{
    [HttpGet]
    public async Task<IActionResult> Index()
    {
        var workspaces = await workspaceRepository.GetWorkspaces(new ()
        {
            HasPricing = true,
            IncludePricings = true,
            IncludeStatus = true,
        });

        var coworkingCenters = await
coworkingCenterRepository.GetCenters(
            new CoworkingCenterFilter());

        return View(new HomeIndexViewModel()
        {
            Workspaces = workspaces,
            CoworkingCenters = coworkingCenters
        });
    }

    [HttpGet]
    [Authorize]
    public async Task<IActionResult> Dashboard(
        [FromQuery] ReservationSort reservationSort =
ReservationSort.None)
    {
        var userId = User.GetUserId();

        if (userId == null)
        {
```

```
            return Unauthorized(new { message = "User not found" });
        }

        var reservations = await reservationRepository
            .GetReservations(new ReservationsFilter
            {
                CustomerId = userId,
                IsCancelled = false,
                IncludeWorkspace = true,
                Sort = reservationSort,
            });

        var user = (await userRepository.GetUsers(new UserFilter
        {
            UserId = userId
        })).Single();

        return View(new HomeDashboardViewModel
        {
            User = user,
            Reservations = reservations,
            ReservationSort = reservationSort,
        });
    }

    [HttpGet]
    public async Task<IActionResult> Privacy()
    {
        return View();
    }

    [ResponseCache(
        Duration = 0,
        Location = ResponseCacheLocation.None, NoStore = true)]
    public IActionResult Error()
    {
        return View(new ErrorViewModel
        {
            RequestId = Activity.Current?.Id ?? HttpContext.TraceIdentifier
        });
    }
}
```

Výpis 4: Computer program in C# language