Name: SN:

Useful Equations:

$$d\omega = \sin\theta d\theta d\phi = -d\mu d\phi \tag{1}$$

$$S = S_0 \cos(\theta) \tag{2}$$

$$a_\lambda + r_\lambda + t_\lambda = 1 \tag{3}$$

$$L_\lambda = \frac{\Delta E}{\Delta\omega\Delta\lambda} \tag{4}$$

$$E_{\lambda\,BB} = \pi B_\lambda \tag{5}$$

where B_λ ($W\,m^{-2}\,sr^{-1}\,\mu m^{-1}$) is the monochromatic blackbody radiance from an infinite flat surface.

$$E_{BB} = \sigma T^4 \tag{6}$$

$$\frac{dT}{dt} = \frac{-1}{\rho c_{pd}} \frac{dE_n}{dz} \tag{7}$$

$$E_\lambda(T) \approx E_{\lambda\,0} + \left.\frac{dE_\lambda}{dT}\right|_{T_0,\lambda} (T - T_0) + \ldots \tag{8}$$

$$\exp(x) = 1 + x + \frac{x^2}{2} + \frac{3^2}{3!} + \ldots \tag{9}$$

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots \tag{10}$$

$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \ldots \tag{11}$$

Beer's law for absorption and scattering (no emission) given slant path $s = dz/\mu$, volume extinction coefficient b, gas density ρ_g (Stull chapter 2)

$$d\tau = n\,b\,dz \tag{12}$$
$$d\tau = \rho_g\,k\,dz \tag{13}$$
$$ds = dz/\mu \tag{14}$$
$$\tau = \int_{z_1}^{z_2} \rho_g k dz \tag{15}$$
$$\frac{dL_\lambda}{L_\lambda} = -\frac{d\tau}{\mu} \tag{16}$$

Hydrostatic equation:

$$dp = -\rho_d\,g\,dz \tag{17}$$

Equation of state

$$p = R_d \rho_d T \tag{18}$$

Slant Transmission

$$\hat{t} \;=\; \exp(-\tau/\mu) \tag{19}$$

Schwarzchild's equation

$$dL = -L\,d\tau/\mu + B_\lambda(z) d\tau/\mu \tag{20}$$

Vertical radiance L_λ with $\mu = 1$

$$L_\lambda(\tau) = B_\lambda(T_{skin})(\exp(-\tau) + \int_0^\tau \exp\left(-(\tau - \tau')\right) B_\lambda(T)\,d\tau' \tag{21}$$

Radiance along a slant path:

$$L_\lambda(\tau/\mu) = B_\lambda(T_{skin})(\exp(-\tau/\mu) + \int_0^\tau \exp\left(-(\tau-\tau')/\mu\right) B_\lambda(T)\, d\tau'/\mu \tag{22}$$

Flux from radiance:

$$E_\lambda = \int_0^{2\pi}\int_0^1 \mu L_\lambda \, d\mu\, d\phi \tag{23}$$

Vertical radiance with change of variables to transmittance:

$$L_\lambda(\tau/\mu) = B_\lambda(T_{skin})\hat{t}_{tot} + \int_0^\tau B_\lambda(T) d\hat{t} \tag{24}$$

For constant temperature:

$$L_\lambda(\tau/\mu) = B_\lambda(T_{skin})\hat{t}_{tot} + (1-\exp(-\tau/\mu))B_\lambda(T) \tag{25}$$

Integrating over a hemisphere for flux, using the simple flux diffusivity approximation:

$$E_\lambda = \pi B_\lambda(T_{skin})\exp(-1.66\tau_{tot}) + \pi B_\lambda(T)(1-\exp(-1.66\tau)) \tag{26}$$

and integrated over all wavelengths:

$$E_\uparrow = \sigma T_{skin}^4 \exp(-1.66\overline{\tau}_{\lambda T}) + \sigma T_{layer}^4(1-\exp(-1.66\overline{\tau}_{\lambda T})) \tag{27}$$

Radar

Rayleigh scattering

$$\frac{L}{L_0} \propto \frac{D^6}{\lambda^4} \tag{28}$$

$$\text{Returned power} \propto |k^2|\frac{Z}{r^2} \tag{29}$$

where $|k^2| \approx 0.2$ for ice and 0.9 for liquid water.

$$Z = \int_0^\infty n(D)D^6 dD \tag{30}$$

Z-R relationship

$$Z = 300RR^{1.4} \tag{31}$$

(rain, RR in mm/hr, Z in mm^6/m^3)

$$MUR = \frac{c}{2\cdot \mathrm{PRF}} \tag{32}$$

Useful constants:

$c_{pd} = 1004 \;\; \mathrm{J\,kg^{-1}\,K^{-1}}$,
$\sigma = 5.67\times 10^{-8} \;\; \mathrm{W\,m^{-2}\,K^{-4}}$
$k_b = 1.381\times 10^{-23} \;\; \mathrm{J\,K^{-1}}$
$c = 3\times 10^{8} \;\; \mathrm{m\,s^{-1}}$
$h = 6.626\times 10^{-34} \;\; \mathrm{J\,s}$
$\pi \approx 3.14159$
$R_d = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$