Instructions – Do all questions (note weights) showing all you work. Don't forget to put your name on your tephigram.

1. (10) Cooling: A cloud at a pressure of 600 hPa and temperature of 5 deg C has 12 g/kg of total water. Suppose it cools by 10 K and rains so that at -5 deg C it has 1 g/kg of liquid water remaining. Find

    1. The total precipitated water, in g/kg
    2. The total change in moist static energy h_m in J/kg
    3. The lifting condensation level before and after the cooling, in hPa

2. (8) Taylor series

    Recall that buoyancy for a parcel is defined

$$buoyancy = g\frac{\rho_{env} - \rho_{parcel}}{\rho_{env}} \approx g\frac{T_{vparcel} - T_{venv}}{T_{venv}} \tag{1}$$

    1. Use a Taylor series expansion to derive (1) showing the second order terms you drop. (Hint, expand in the small parameter $\Delta T_v/T_{venv}$, where $\Delta T_v = T_{vparcel} - T_{venv}$)
    2. Suppose a parcel at T_v=281 K maintains a 1 degree temperature difference over an environment at T_{venv}=280 K for an extended period of time. How many minutes would it take for the parcel to reach a velocity of 10 m/s?

3. (8) Suppose you know that cloud drops have a size distribution given by

$$N(r) = N_0 \exp(-\chi r) \tag{2}$$

    where $N(r)$ (m^{-3}) is the number of drops per unit volume with radius $> r$ (for radii 50 μm $< r <$ 200 μm) and N_0 and χ are constants and r is in μm. Suppose also that the drop fall speed $v(r)$ ($\mathrm{m\,s^{-1}}$) depends on linearly on radius:

$$v(r) = Jr \tag{3}$$

    where $J = 6000\ s^{-1}$ and now r is in meters.

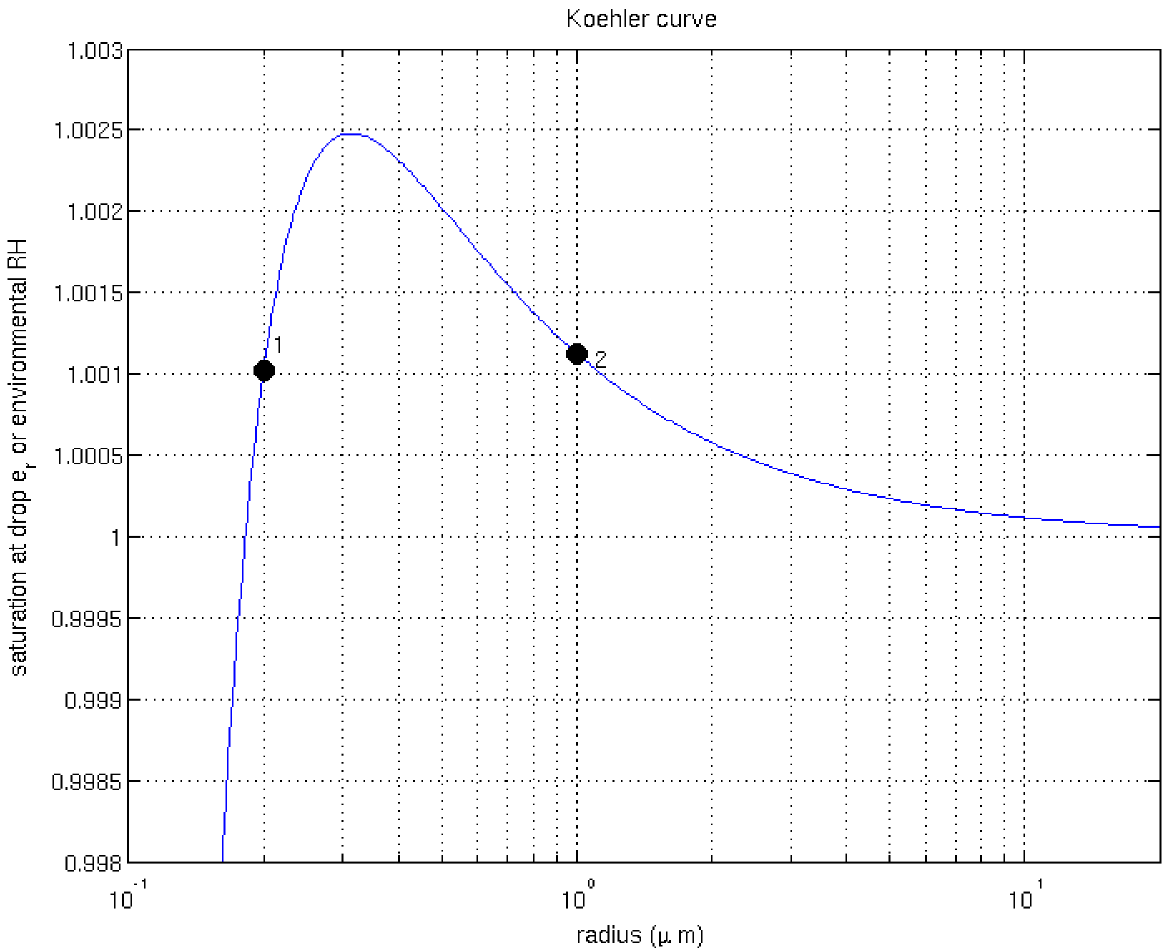
    Derive equations (including unit conversions if necessary) for:

    (a) The number density $n(D)$ ($m^{-3}\,\mu m^{-1}$)
    (b) The liquid water content ($\mathrm{kg\,m^{-3}}$)
    (c) The precipitation rate ($\mathrm{mm\,hr^{-1}}$)

4. (10) Collision/coalescence

    Derive with the help of a sketch the simple model of the growth of rain drops represented by equation (36), defining all your symbols. If the fall speed is given by $J = 6000r\ m\,s^{-1}$ as in (36), roughly how long would it take a 50 μm radius drop to double to 100 μm radius, given $w_l = 0.3\ g\,m^{-3}$? What processes need to be added to (36) to make the description of rain formation more accurate? Explain.

5. (11) Köhler curve



The figure shows two droplets with radii of 0.2 μm and 1 μm at a environmental relative humidity of about 1.001.

(a) (3) Both droplets lie on a curved line called the Koehler curve. What do points on this line represent? i.e. what is it that all points lying on this line have in common?

(b) (4) Suppose the relative humidity increases from 1.001 to 1.002. Describe qualitatively what happens to the two drops.

(c) (4) Quantitatively, what is the initial evaporation/growth rate of droplet 2 in microns/second, assuming a RH of 1.002, temperature T=280, esat(T)=10 hPa.

6. (10) Adiabatic ascent

Suppose someone asks you to provide the temperature, pressure, water vapor mixing ratio and liquid water mixing ratio as a function of height for surface air at press= 1000 hPa, temperature T and relative humidity RH as it ascends adiabatically through the atmosphere. Using pseudo code sketch out a python script that would do this assuming a hydrostatic balance. Assume you can use odeint and the rootfinder as needed. Note: 1) the equations you would need to solve and 2) how you would solve them.

Equation sheet

$$du = q\,dt - w\,dt = q\,dt - p\,d\alpha \tag{4}$$

$$e = \rho_v\,R_v\,T \tag{5}$$

$$p = \rho\,R_d\,T_v \tag{6}$$

$$w\,dt = p\,d\alpha \tag{7}$$

$$h = u + p\,\alpha \tag{8}$$

$$T_v = T(1 + 0.608 r_v - r_l) \tag{9}$$

$$r_{vs} = \rho_s/\rho_d = \epsilon\frac{e_s}{p - e_s} \tag{10}$$

$$dh = c_{px}\,dT\ (dry\ air\ or\ liquid) \tag{11}$$

$$dh = c_p\,dT\ + l_v\,dr_v\ (air/water\ mixture) \tag{12}$$

$$dh = T\,d\phi + \alpha\,dp\ (reversible) \tag{13}$$

$$d\phi = c_p\frac{d\theta}{\theta} = c_p\frac{dT}{T} - R_d\frac{dp}{p} \tag{14}$$

$$d\phi \geq \frac{q\,dt}{T} \tag{15}$$

$$l_v = h_v - h_l \tag{16}$$

$$dp = -\rho\,g\,dz \tag{17}$$

$$dh_m = c_p\,dT + l_v\,dr_v + g\,dz \tag{18}$$

$$d\phi = c_p\frac{d\theta_{es}}{\theta_{es}} = c_p\frac{d\theta}{\theta} + \frac{l_v\,dr_s}{T} \tag{19}$$

$$d\phi = c_p\frac{d\theta_l}{\theta_l} = c_p\frac{d\theta}{\theta} - \frac{l_v\,dr_l}{T} \tag{20}$$

$$dr_v = \frac{r_v}{p - e}\left(\frac{p}{e}de - dp\right) \tag{21}$$

$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x - x_0) \\ &+ \frac{f''(x_0)}{2}(x - x_0)^2 + \ldots \end{aligned} \tag{22}$$

$$l_v = T(\phi_v^* - \phi_l) \tag{23}$$

$$\frac{de_s}{dT} = \frac{l_v e_s}{R_v T^2} \tag{24}$$

$$h_l = c_l(T - T_p) \tag{25a}$$

$$h_v = l_{v0} + c_{pv}(T - T_p) \tag{25b}$$

$$\phi_d = c_{pd}\ln T - R_d \ln p_d \tag{25c}$$

$$\phi_l = c_l \ln\frac{T}{T_p} \tag{25d}$$

$$\phi_v = c_{pv}\ln\frac{T}{T_p} - R_v\ln\frac{e}{e_{s0}} + \frac{l_{v0}}{T_p} \tag{25e}$$

$$g = u + p\alpha - T\phi = h - T\phi \tag{26}$$

$$dg \leq -\phi dT + \alpha dp \tag{27}$$

$$G = m_v g_v + m_l g_l + 4\pi\sigma r^2 \tag{28}$$

$$\int_{a_w e_s}^{e} d(g_l - g_v) \approx -R_v T \ln\left(\frac{S}{a_w}\right) \tag{29}$$

$$\begin{aligned} e_r &= e_s(T)(n_w/(n_w + n_s))\exp(a/r) \\ &= e_s(T)(1 + \frac{a}{r} - \frac{b}{r^3}) \end{aligned} \tag{30}$$

$$s_{crit} = 1 + \left(\frac{4a^3}{27b}\right)^{1/2} \tag{31}$$

$$r_{crit} = \left(\frac{3b}{a}\right)^{1/2} \tag{32}$$

$$\begin{aligned} c_p \frac{d\theta'_e}{\theta'_e} &= \frac{dm}{m}\frac{\Delta h_m}{T'} = \frac{dm}{m}\Delta s \\ &= c_p \frac{dm}{m}\Delta \ln \theta_e \end{aligned} \tag{33}$$

$$\frac{1}{m}\frac{dm}{dt} = \lambda \tag{34}$$

$$\frac{dr'_T}{dz} = (r_T - r'_T)\hat{\lambda} \tag{35}$$

$$\frac{dM}{dt} = \pi R^2 E_c (V(R) - V(r)) r_l \tag{36}$$

where V(r) $\approx$ 0 and V(R) $\approx$ JR (with J=6000 $s^{-1}, R in$ m)

$$\frac{dm}{dt} = 4\pi r D(\rho_{v\infty} - \rho_{vr}) \tag{37}$$

$$\frac{dr}{dt} = \frac{1}{r}\frac{D\rho_{v\infty}}{\rho_l e_\infty}[e_\infty - e_r] \tag{38}$$

$$N(D') = \int_{D'}^{\infty} n(D)dD \tag{39}$$

a	$\frac{2\sigma}{\rho_l R_v T}$
b	$\frac{imM_w}{(4/3)M_s\pi\rho}$
σ	0.075 $\mathrm{J\,m^{-2}}$
ρ_l	1000 $\mathrm{kg\,m^{-3}}$
c_{pd}	1006 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_{vd}	719 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_{pv}	1870 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_{vv}	1408 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_l	4190 $\mathrm{J\,kg^{-1}\,K^{-1}}$
D	2.36 $\times 10^{-5}$ $\mathrm{m^2\,s^{-1}}$
R_d	287 $\mathrm{J\,kg^{-1}\,K^{-1}}$
R_v	461 $\mathrm{J\,kg^{-1}\,K^{-1}}$
k	1.381 $\times 10^{-23}$ $\mathrm{J\,K^{-1}}$ molecule^{-1}
l_{v0}	2.501 $\times 10^{6}$ J kg at 0 deg C