Name and student id:

Instructions: closed book, calculator ok, make sure you put your name on the tephigram and on any blank pages you need. Show all your work, including the tephigram.

1. Wet bulb equation (15 points):

   Starting with the first law in the form of (10), derive the an equation that could be used to find the vapor mixing ratio r_v using a wet bulb thermometer, stating any assumptions. Explain briefly in a couple of sentences how the wetbulb thermometer works – i.e. what it measures, and how you would use those measurements and your equation to find r_v.

2. Tephigram (12 points):

   (a) The attached tephigram shows the temperature (circle) and dewpoint (diamond) for air at 900 hPa and 800 hPa Find (showing your work on the tephigram):

   - The LCL (hPa) of the 900 hPa air
   - The equivalent potential temperature θ_e (K)
   - The entropy ϕ (J/kg/K) or the 900 hPa air
   - The wet bulb potential temperature θ_w (K) of the 900 hPa air

   (b) Is the air at 900 hPa absolutely stable, conditionally unstable, or unstable? Explain

   (c) Is the layer between 900 hPa and 800 hPa convectively unstable? Explain.

3. Mixing (15 points):

   Suppose air at 900 hPa is lifted adiabatically to 800 hPa, where it mixes 50/50 with air at that level.

   (a) Using the tephigram what is the LCL of the resulting mixture? – Clearly label your work, including the tephigram.

   (b) Suppose you were asked to write a python program to find this answer. You have a library that has the following routines used in the course:

      i. find_thetaet(Td, rt, T, p) – calculates θ_e for a parcel
      ii. find_rsat(temp, press) – calculates the saturation mixing ratio
      iii. tinvert_thetae(thetaeVal, rT, press) – find temp, rv, rl given thetae, rT and pressure
      iv. find_interval(the_func, x, *args) – bracket a root for function the_func
      v. fzero(the_func, root_bracket, *args, **parms) – find the root of the_func given a bracket

      Describe using python pseudocode how you would use these routines, plus anything else you needed, to find the LCL (hPa) given the θ_e and r_T of the mixture at 800 hPa.

Equation sheet

$$du = q\,dt - w\,dt = q\,dt - p\,d\alpha \tag{1}$$

$$e = \rho_v\,R_v\,T \tag{2}$$

$$p = \rho\,R_d\,T_v \tag{3}$$

$$w\,dt = p\,d\alpha \tag{4}$$

$$h = u + p\,\alpha \tag{5}$$

$$T_v = T(1 + 0.608 r_v - r_l) \tag{6}$$

$$r_v = \rho_s/\rho_d = \epsilon\frac{e_s}{p - e_s} \tag{7}$$

$$dh = c_{px}\,dT \;(dry\ air\ or\ liquid) \tag{8}$$

$$dh = c_p\,dT\; + l_v\,dr_v \;(air/water\ mixture) \tag{9}$$

$$dh = T\,ds + \alpha\,dp \;(reversible) \tag{10}$$

$$ds = c_p\frac{d\theta}{\theta} = c_p\frac{dT}{T} - R_d\frac{dp}{p} \tag{11}$$

$$ds \geq \frac{q\,dt}{T} \tag{12}$$

$$l_v = h_v - h_l \tag{13}$$

$$dp = -\rho\,g\,dz \tag{14}$$

$$dh_m = c_p\,dT + l_v\,dr_v + g\,dz \tag{15}$$

$$ds = c_p\frac{d\theta_e}{\theta_e} = c_p\frac{d\theta}{\theta} + d\left(\frac{l_v\,r_s}{T}\right) \tag{16}$$

$$ds = c_p\frac{d\theta_l}{\theta_l} = c_p\frac{d\theta}{\theta} - d\left(\frac{l_v\,r_l}{T}\right) \tag{17}$$

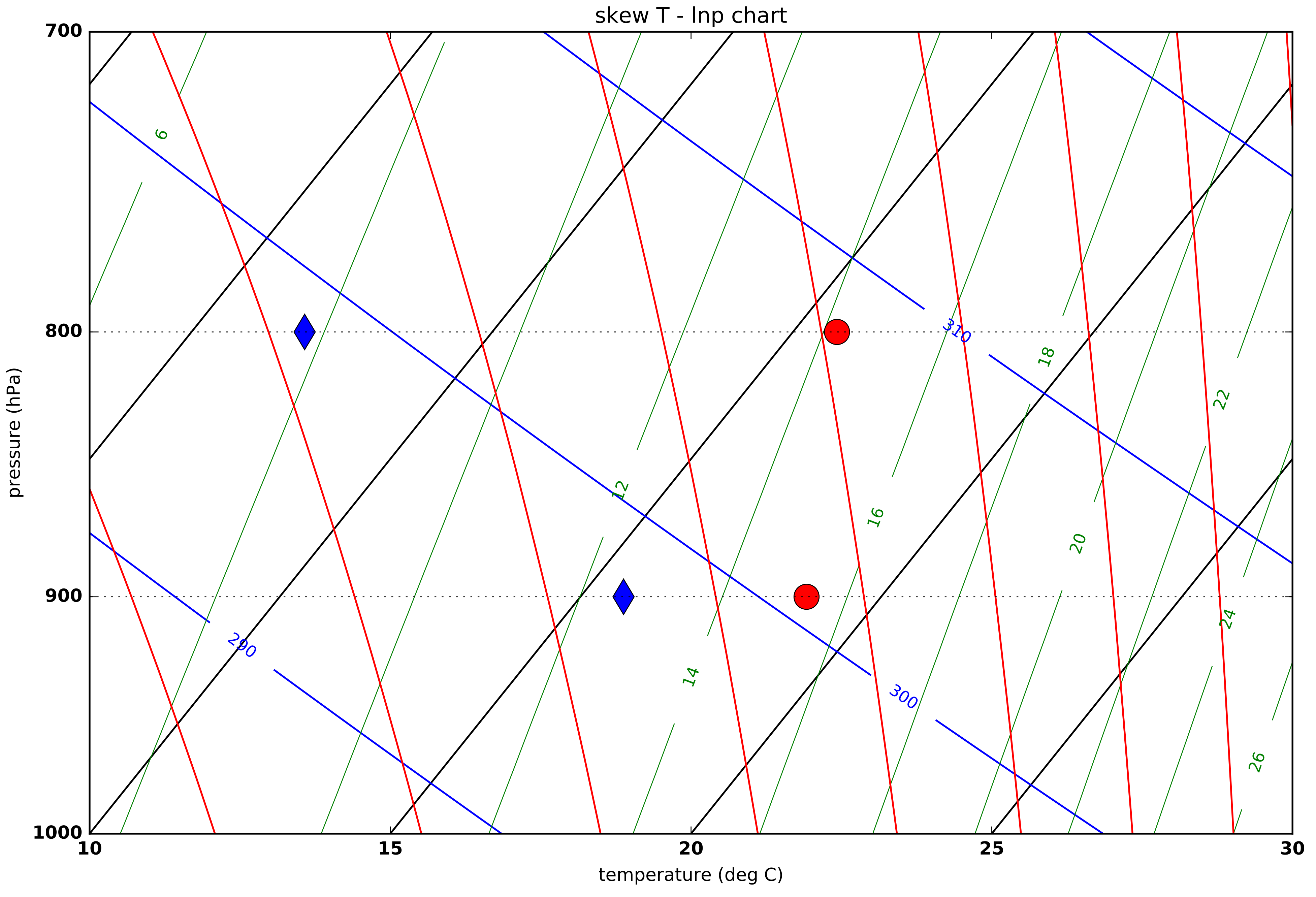
$$\theta_e = \theta\exp\left(\frac{l_v r_v}{c_p T}\right) \tag{18}$$

$$b = g\frac{\rho_v'}{\rho_v} = g\frac{T_v'}{\overline{T_v}} \approx g\frac{T_{vp} - T_{ve}}{T_{ve}} \tag{19}$$

$$dr_v = \frac{w_v}{p - e}\left(\frac{p}{e}de - dp\right) \tag{20}$$

$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x - x_0) \\ &+ \frac{f''(x_0)}{2}(x - x_0)^2 + \ldots \end{aligned} \tag{21}$$

c_{pd}	1006 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_{pv}	1870 $\mathrm{J\,kg^{-1}\,K^{-1}}$
c_l	4190 $\mathrm{J\,kg^{-1}\,K^{-1}}$
R_d	287 $\mathrm{J\,kg^{-1}\,K^{-1}}$
R_v	461 $\mathrm{J\,kg^{-1}\,K^{-1}}$
k	1.381×10^{-23} $\mathrm{J\,K^{-1}}$ molecule^{-1}
l_{v0}	2.5×10^{6} J kg at 0 deg C



skew T - lnp chart
700
800
900
1000
pressure (hPa)
10
15
20
25
30
temperature (deg C)
290
300
310
6
12
14
16
18
20
22
24
26