From: Tritton

20

HOMOGENEOUS ISOTROPIC TURBULENCE

20.1 Introduction [44, 45]

Many theoretical investigations of turbulence have been developed around the concept of homogeneous, isotropic turbulence—turbulence of which the statistical properties do not vary with position and have no preferred direction. An approximation to such a motion can be obtained behind a grid, such as the one shown in Fig. 20.1, in a wind-tunnel. The theories have thus been supplemented by observation. However, we postpone consideration of experimentally based ideas about the structure of turbulence to the different contexts of Chapter 21. Here, without detail, we use the context of homogeneous isotropic turbulence to develop some ideas relevant to all turbulent flows.

The energy production term in eqn (19.20) is zero in isotropic turbulence, and so the motion must decay through viscous dissipation. In theoretical work, the turbulence is supposed to be generated at an initial instant and then to decay as time proceeds. Behind a grid, there is strong turbulent energy production for the first ten or so grid mesh-lengths along the tunnel; the turbulence then becomes substantially isotropic and decays with distance down the tunnel. (This, in principle, implies inhomogeneity, but the decay is slow enough for this to be neglected.) We shall consider certain features of the motion applying at any stage of this decay process.

20.2 Space correlations and the closure problem

The assumption of homogeneity and isotropy much simplifies the formulation of space correlations. These depend only on the distance between the two points and not on their location or the orientation of the line joining them. Moreover, it may be shown that the general correlation, as in Fig. 19.4(a), may be expressed in terms of the longitudinal and lateral correlations of Fig. 19.4(b) and (c). Thus only these two functions of r (denoted respectively by $f(r)$ and $g(r)$) are needed for complete specification of the double velocity correlations. When, additionally, the

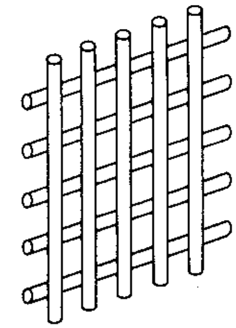

FIG. 20.1 Typical turbulence generating grid.

continuity equation is introduced a relationship between these is found,

$$f + \tfrac{1}{2}r\,\mathrm{d}f/\mathrm{d}r = g. \tag{20.1}$$

Hence, only a single function gives the complete specification.

The mathematics behind these statements runs as follows. The correlation coefficient between velocity component u_i at one point and component u_j at a point $\boldsymbol{r}$ away is a second-order tensor $R_{ij}(\boldsymbol{r})$. When there is isotropy [45, 212]

$$R_{ij}(\boldsymbol{r}) = \zeta(r)r_ir_j + \eta(r)\delta_{ij}. \tag{20.2}$$

For this to take the appropriate forms in the particular cases of longitudinal and lateral correlations,

$$\zeta(r) = (f(r) - g(r))/r^2; \quad \eta(r) = g(r). \tag{20.3}$$

Continuity gives

$$\partial u_j/\partial x_j = 0. \tag{20.4}$$

Substituting this in

$$R_{ij} = \overline{u_i(0)u_j(\boldsymbol{r})}/\overline{u^2} \tag{20.5}$$

gives

$$\partial R_{ij}/\partial r_j = 0, \tag{20.6}$$

which reduces to eqn (20.1).

Figure 20.2 shows an experimental check of eqn (20.1). Direct measurements of g are compared with ones calculated from measurements of f.

No use has been made so far of the dynamical equation. Obviously, one would like to use this to determine f as a function of r. However, one encounters what is known as the closure problem, a consequence of the

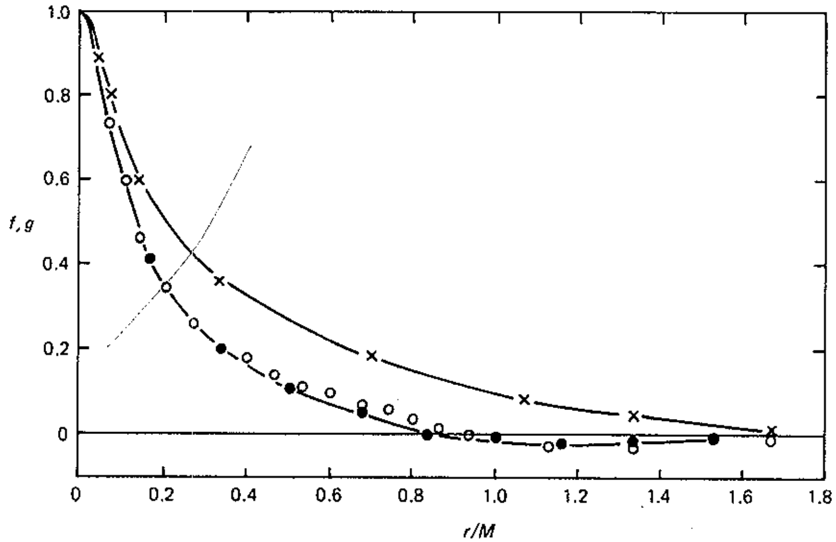

FIG. 20.2 Correlations in grid turbulence: × $f(r)$, measured; ○ $g(r)$, calculated from $f(r)$; ● $g(r)$, directly measured. From Ref. [374].

non-linearity of the equation. The mathematical analysis is somewhat complicated. The point of principle can be illustrated by much simpler equations, such as the Lorenz equations discussed in Chapter 24; the closure problem is therefore analysed in the appendix to that chapter. For turbulence the problem manifests itself as follows: when one formulates an equation for the double correlation function it involves triple correlations, an equation for these involves fourth-order ones; and so on. A variety of suggestions for an additional hypothesis to close the system have been advanced. However, our knowledge of f as a function of r remains primarily experimental (Fig. 20.2).

20.3 Spectra and the energy cascade

An important equation in the theory of homogeneous isotropic turbulence is obtained by considering the Fourier transform of the double velocity correlation. We omit the derivation [20, 44] and quote the result:

$$\partial E(k, t)/\partial t = F(k, t) - 2\nu k^2 E(k, t). \tag{20.7}$$

TKE E is the energy spectrum of eqn (19.32); the dependence on t is shown as a reminder that we are dealing theoretically with a time-dependent situation. The closure problem has come through in the appearance of another function F in the equation for E; F is related to the Fourier transform of the triple correlation. However, it can be given its own physical interpretation through its role in eqn (20.7).

The left-hand side represents the rate of change of the energy associated with wavenumber k. The second term on the right-hand side is a negative term involving the viscosity and is thus the energy dissipation. It can be shown that

$$\int_0^\infty F \, dk = 0 \tag{20.8}$$

and so the first term on the right-hand side of eqn (20.7) represents the transfer of energy between wavenumbers.

Figure 20.3 shows graphs of E and k^2E for a typical situation.† The latter has large values at much higher k than the former. Hence the viscous dissipation is associated with high wavenumbers; i.e. it is brought about by small eddies. This is a consequence of the fact that turbulent flows normally occur at high Reynolds number. The action of viscosity is slight on a length scale of the mean flow (e.g. a grid mesh-length). Yet much more dissipation occurs than in the corresponding laminar flow. This requires the development of local regions of high shear in the turbulence; i.e. the presence of small length scales.

The small dissipative eddies must be generated from larger ones. The effect of this on the energy spectrum is contained in the second term of eqn (20.7); i.e. one expects the transfer of energy to be primarily from low wavenumbers to high. This inference is confirmed experimentally by observations in grid turbulence of changes in the spectrum with distance downstream.

This interpretation of the behaviour of the terms of eqn (20.7) allows the development of a model of turbulence which has relevance not just to homogeneous isotropic turbulence but to most turbulent flows.

Energy fed into the turbulence goes primarily into the larger eddies. (In grid turbulence this happens during the initial generation; in other flows to be considered in Chapter 21 it happens throughout the flow.) From these, smaller eddies are generated, and then still smaller ones. The process continues until the length scale is small enough for viscous action to be important and dissipation to occur. This sequence is called the energy cascade. At high Reynolds numbers (based on $(\overline{u^2})^{1/2}$ and a length scale defined in a way indicated in Section 19.4) the cascade is

† Figure 20.3 is based on experimental data. For homogeneous isotropic turbulence, a relationship can be derived between $E(k)$ and the measurable one-dimensional spectrum, thus overcoming the non-measurability of $E(k)$ mentioned in Section 19.5. However, the conversion involves differentiation of experimental curves and so $E(k)$ is determined with rather poor accuracy.

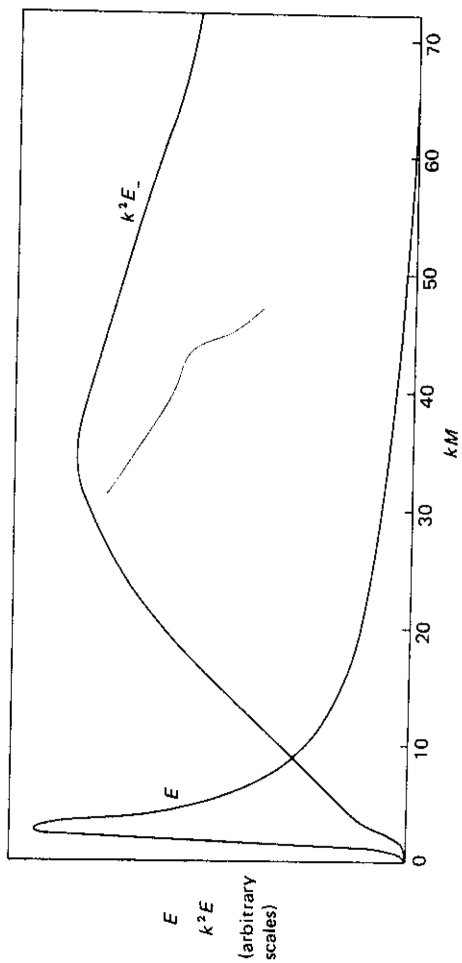

FIG. 20.3 Energy and dissipation spectra in turbulence behind a grid (distance downstream from grid = 48 M, where M = grid mesh-length). Data from Ref. [396].

long; i.e. there is a large difference in the eddy sizes at its ends. There is then little direct interaction between the large eddies governing the energy transfer and the small dissipating eddies. The dissipation is determined by the rate of supply of energy to the cascade by the large eddies and is independent of the dynamics of the small eddies in which the dissipation actually occurs. The rate of dissipation is then independent of the magnitude of the viscosity. An increase in the Reynolds number to a still higher value—conveniently visualized as a change to a fluid of lower viscosity with all else held constant—only extends the cascade at the small eddy end. Still smaller eddies must be generated before the dissipation can occur. (Since, as we shall see, the total energy associated with these small eddies is small, this extension has a negligible effect on the total energy of the turbulence.) All other aspects of the dynamics of the turbulence are unaltered.

This inference, that the structure of the motion is independent of the fluid viscosity once the Reynolds number is high enough, has important implications and will be considered again in Section 21.1. It is given the (somewhat misleading) name of Reynolds number similarity.

The dynamics of the energy cascade and dissipation may be supposed to be governed by the energy per unit time (per unit mass) supplied to it at the large eddy (low wavenumber) end. This is, of course, equal to the energy dissipation, ε. This suggests (but see below) that the spectrum function E is independent of the energy production processes for all wavenumbers large compared with those at which the production occurs. Then E depends only on the wavenumber, the dissipation, and the viscosity,†

$$E = E(k, \varepsilon, \nu). \tag{20.9}$$

If the cascade is long enough, there may be an intermediate range (the inertial sub-range) in which the action of viscosity has not yet come in; that is

$$E = E(k, \varepsilon). \tag{20.10}$$

Dimensional analysis then gives

$$E = A\varepsilon^{2/3}k^{-5/3} \tag{20.11}$$

where A is a numerical constant. This is a famous result, known as the Kolmogorov $-5/3$ law.

There is, however, a problem over the derivation of the result, for a rather subtle reason connected with the structure of the smaller-scale

† There is no contradiction between (20.9) and the inclusion of a dependence of E on t at the beginning of this section. For time-dependent situations (such as theoretical homogeneous, isotropic turbulence) (20.9) contains this through ε varying with t.

motions. Experiments show clearly that the energy dissipation does not occur roughly uniformly throughout the turbulence; there are patches of intense small eddies involving high dissipation and other patches where there is little dissipation [64, 235]. We shall see in the next section why this should be. The patchiness develops throughout the energy cascade; as the eddies get smaller, so the fraction of the volume in which they are active decreases (though the size of a patch is at each stage large compared with the corresponding eddy size). This implies that *within the patches* the rate of energy transfer per unit mass increases with wave number, thus complicating the derivation of the Kolmogorov law. The fractional volume of activity depends on how far down the cascade one is;

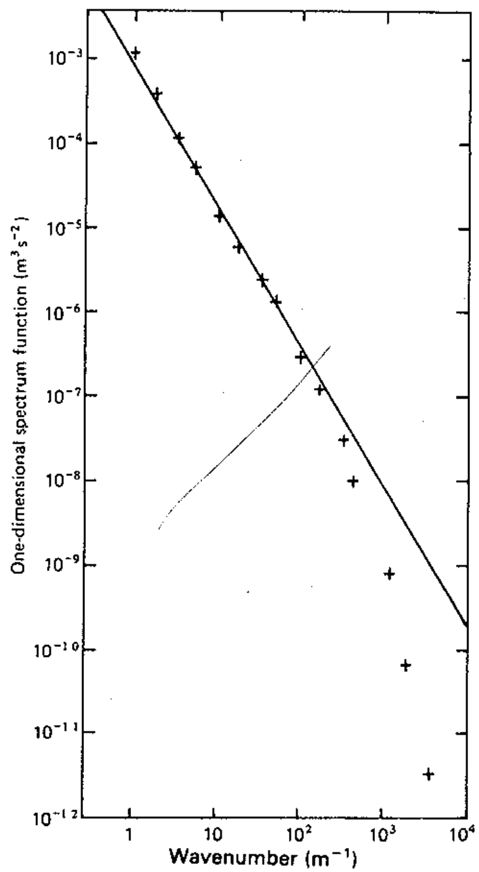

FIG. 20.4 One-dimensional spectrum measured in a tidal channel; line has slope of $-\frac{5}{3}$. Data from Ref. [187].

i.e. on k/k_0, where k_0 is a typical wave number of the turbulence energy production. This indirectly modifies (20.10) to

$$E = E(k, \varepsilon, k_0) \tag{20.12}$$

and so upsets the dimensional argument. There has been extensive theoretical discussion about how this affects the Kolmogorov result [35, 64, 166], suggesting that $E \propto k^{-n}$, with n a little larger than 5/3 but not exactly known.

Any theory which changed n substantially would be inconsistent with the observations. The Kolmogorov law is well supported experimentally. Such experiments have to be performed at very high Reynolds number in order that the inertial sub-range should be of significant length. Some of the best verifications thus come from natural flows rather than laboratory experiments. Figure 20.4 shows a spectrum obtained in an oceanographic channel flow, produced by tides, with a Reynolds number of 4×10^7; the axes are logarithmic and the line has a slope of $-5/3$. (This is actually the measurable one-dimensional spectrum, but it can be shown that if this is proportional to k^{-n}, then so is the energy spectrum.)

20.4 Dynamical processes of the energy cascade

The discussion in Section 20.3 of the energy cascade did not consider the mechanism by which the transfer of energy from large scales to small scales occurs. It is instructive to consider first the related but more readily understood process of the mixing of a scalar contaminant [130]. Suppose a blob of fluid, as shown in Fig. 20.5(1), is marked in some way, for

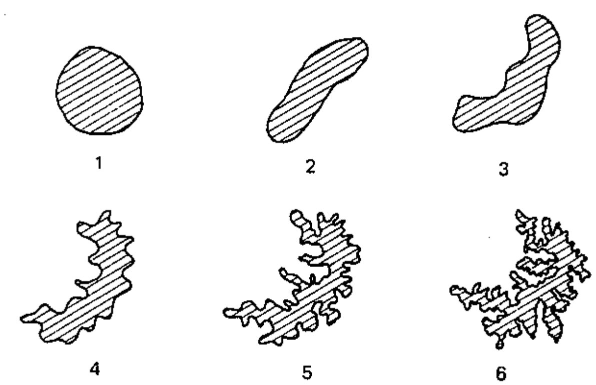

FIG. 20.5 Schematic representation of successive configurations of marked blob of fluid within turbulent motion.

example by being heated or dyed. If this blob is in a turbulent flow, its distribution will change with time in the way indicated schematically by successive configurations in Fig. 20.5 (see also Fig. 21.6). Its distribution in space becomes more and more contorted by the velocity fluctuations, but so long as molecular diffusion plays no role, the marked fluid is always just the same fluid. If the Péclet number is high, diffusion is negligible at first. However, the highly contorted patterns involve very steep gradients of the marker and so ultimately diffusion becomes significant, causing previously unmarked fluid to become marked. The higher the Péclet number, the more contorted the pattern must become before this happens.

This process is analogous to the cascade because it involves the appearance of smaller and smaller length scales until this is limited by molecular effects (diffusion or viscosity). However, the energy cascade is a more complicated matter because it involves the interaction of the velocity field with itself instead of with a quantity not involved in the dynamics. Any brief account of it must involve oversimplification, but three (not wholly independent) processes may be identified.

The first is the process of repeated instability considered in Section 19.1. Each stage may give rise to motions not only of greater complexity but also involving smaller scales than the previous stages. For example, one stage may produce local regions of high shear that can themselves be unstable.

Secondly, turbulence of a smaller scale may extract energy from larger-scale motions in a way analogous to the extraction of energy from a mean flow by the turbulence as a whole (Section 19.3).

Thirdly, there is vortex stretching [375]. The random nature of turbulent motion gives a diffusive action; two fluid particles that happen to be close together at some instant are likely to be much further apart at any later time. The turbulence will have carried them over different paths. This applies to two particles on the same vortex line. The process of vortex stretching, considered in Section 6.6, will thus be strongly present—although occurring in a random fashion. This increases the magnitude of the vorticity, but because of continuity also reduces the cross-section of the vortex tube. There is thus an intensification of the motion on a smaller scale; that is a transfer of energy to smaller eddies.

This process may also be seen as a cause of the patchy distribution of dissipation mentioned in Section 20.3. The vorticity intensification process will be strongest where the vorticity already happens to be large. At any instant the production of small eddies is thus occurring vigorously in some places and only weakly in others.

21

TURBULENT SHEAR FLOWS

21.1 Reynolds number similarity and self-preservation

Shear flows constitute by far the most important class of turbulent flows. We have traced the processes by which they become turbulent in Sections 17.6–17.8 and Chapter 18. With the background of general ideas about turbulence in Chapter 19, we now look at the structure of fully turbulent shear flows. In this and the next section we consider general ideas and illustrate them with various flows (depending largely on the best illustrations available). Subsequently we shall focus particular attention on wakes and boundary layers.

Studies of turbulent flows centre on laboratory experiments. Probably more than in any other branch of fluid dynamics, our knowledge and understanding would be slight without such experiments. As always, it is necessary to know the range of applicability of any measurement—whether it is relevant to other similar flows or whether it is peculiar to the particular situation investigated. Since the number of measurements needed for a reasonably full understanding of any turbulent flow is large, it is highly desirable to make observations of general applicability. Two ideas help here.

The first is the concept of Reynolds number similarity already introduced in Section 20.3. The larger eddies and the mean flow development are independent of the viscosity (so long as this is small enough to make the Reynolds number large). This is true of the mean flow because the last term of eqn (19.14) normally dominates the last-but-one term and because the Reynolds stress is produced by the larger eddies. (The contribution of different length scales to the Reynolds stress may be investigated by applying spectral analysis not only to the energy as in Section 19.5, but also to the Reynolds stress [129, 240]. Figure 21.1 makes a comparison between the energy and Reynolds stress (one-dimensional) spectra in turbulent channel flow; the much more rapid fall-off of the latter at high wavenumbers is apparent.) Hence, experiments at any (sufficiently high) Reynolds number provide information applicable to all values of the Reynolds number. We shall see in Section 21.5 that some qualification of this concept is needed for the motion in the vicinity of a solid boundary.

The second idea is that of 'self-preservation' [46]. This is the counterpart for turbulent flow of the occurrence of similar velocity