Moreover, atmospheres have no
away.
the radiation field everywhere,
oile. All mathematical models
ation fields at each point in an
n, we need a model closer than
uct (on paper or in our minds)
only one direction. These media
nated as *plane-parallel*. If there
(i.e., scattering and absorption
media exist. An atmosphere, as
roperties vary in all directions;
we are careful to recognize the
media (i.e., unreal media) they

t are such that emission within
idiation such as sunlight) is as-

ate, which is why time is absent
enter is in the time-dependence
dary illumination changes with
neously throughout the medium
ances and times of interest. For
times of order 10^2 s, whereas
imes of order 10^{-5} s. What this
here in a cloud or in air adjusts

a cloud illuminated by a short
y, then turned off suddenly after
cattered photons are received by
photon has traveled a different
relative to that of the transmitted
4.2).
the consequences of interference
from different sources. This is
riving the equations of transfer
ch implies that all quantities are
ous we could not apply to them a
d waves taking account of phase
s a glass of water, which comes
e water molecules are separated
it and their positions are strongly
ules getting out of its way). Such
rrelated molecules is a multiple-
ifference between them lies only

in the theories applicable to them. The radiative transfer theory in this book applied to a glass of water does not yield the laws of specular reflection and refraction, not even approximately. To determine scattering by this coherent array of molecules we would have to find the waves scattered by all of them, add all these waves taking account of phase differences, then square the resultant to obtain the power. We therefore assume continuity for mathematical purposes when we derive the equation of transfer but apply it to discrete media because only for such media can we ignore phase differences. We can get away with this for air because even though its molecules are separated by distances small compared with the wavelengths of sunlight, they are in constant motion and their positions are essentially uncorrelated (see Sec. 3.4.9). And we can get away with this for clouds because the constantly changing separation between droplets is large compared with the wavelengths of solar and terrestrial radiation and also with their lateral coherence lengths (see Sec. 3.4.2).

Because we assumed incoherent scattering, the photon language is the natural one for discussing the radiative transfer theory considered here. We look upon photons as discrete blobs of energy without phases, so that the energy transported by two photons traveling in the same direction is the sum of the energy transported by each one separately. Our radiative transfer theory is a theory of multiple scattering of photons rather than of waves.

5.1.3 Mean Free Path

What happens to photons as we imagine them launched from a particular point (call it the origin of the x-axis) in a medium? Photons obey statistical laws. We cannot determine what happens to a particular photon (photons are indistinguishable so the concept of a "particular photon" is meaningless) but we can determine what happens in a statistical sense to an ensemble of many photons. For example, what is the probability that a photon propagating along the x-axis beginning at $x = 0$ is absorbed between x and $x + \Delta x$? This is given by the integral of the probability distribution function $p(x)$

$$\int_x^{x+\Delta x} p(x)\, dx. \tag{5.28}$$

From Eq. (2.7) for exponential attenuation by absorption it follows that the probability a photon is *not* absorbed over a distance x from the origin is $\exp\{-\kappa x\}$, the probability it is *not* absorbed over a distance $x+\Delta x$ is $\exp\{-\kappa(x+\Delta x)\}$, and hence the probability it *is* absorbed in the interval Δx is the difference

$$\int_x^{x+\Delta x} p(x)\, dx = \exp\{-\kappa x\} - \exp\{-\kappa(x+\Delta x)\}. \tag{5.29}$$

If $p(x)$ is continuous and bounded then according to the mean value theorem of integral calculus the integral in Eq. (5.29) is

$$p(\overline{x})\Delta x = \exp\{-\kappa x\} - \exp\{-\kappa(x+\Delta x)\}, \tag{5.30}$$

where $\overline{x}$ lies between x and $x + \Delta x$. Divide both sides of this equation by Δx and take the limit as $\Delta x \to 0$:

$$p(x) = -\frac{d}{dx}\exp(-\kappa x) = \kappa \exp(-\kappa x). \tag{5.31}$$

With this probability distribution, the integral of which over all x is 1, we can find the mean distance a photon propagates before being absorbed:

$$\langle x \rangle = \int_0^\infty x\kappa \exp(-\kappa x)\, dx = \frac{1}{\kappa} = \ell_a. \tag{5.32}$$

Thus the physical interpretation of κ is that its inverse is the mean free path for absorption ℓ_a. Attenuation by scattering also is exponential provided that a photon is removed from a beam of radiation if it is scattered once in any direction whatsoever. If we denote by β the scattering coefficient in the expression $\exp(-\beta x)$, the mean free path for scattering ℓ_s is $1/\beta$, and the total mean free path is

$$\ell_t = \frac{1}{\kappa + \beta}, \tag{5.33}$$

where

$$\frac{1}{\ell_t} = \frac{1}{\ell_a} + \frac{1}{\ell_s}. \tag{5.34}$$

We need one more result for the analysis that follows. What is the probability that given that a photon reaches x without being absorbed, it is absorbed in Δx? What we are after here is a *conditional probability*, the probability of absorption in Δx given that a photon has reached x, which is related to, but not the same as Eq. (5.30). The probability that a photon propagates a distance x from the origin without absorption is $\exp(-\kappa x)$. The probability that a photon is absorbed in Δx is given by Eq. (5.30). So this probability divided by the previous one is the conditional probability we are after:

$$1 - \exp(-\kappa \Delta x). \tag{5.35}$$

This quantity is the probability that if a photon finds itself at x it is absorbed as it propagates a further distance Δx. If $\kappa \Delta x \ll 1$, this probability is approximately $\kappa \Delta x$. And similarly, the probability of scattering is $\beta \Delta x$ provided that $\beta \Delta x \ll 1$.

Although the absorption coefficient κ and scattering coefficient β are fundamental quantities in the theory of radiative transfer, determining their values lies outside this theory. We either have to obtain them from measurements or appeal to a theory that comes to grips with the discreteness of molecules and particles.

5.2 Two-Stream Theory of Radiative Transfer

The preceding sections were fairly general, applicable to all theories of radiative transfer. To proceed further we have to make some specific assumptions, and the simplest one is that the radiation field consists of irradiances F in two and only two directions (streams), denoted as upward and downward. This is an idealization given that strictly monodirectional irradiances do not exist; even a laser beam has a finite angular spread. Scattering can therefore occur in only these two directions: a photon directed downward can be scattered only downward or upward, and similarly for a photon directed upward. We also ignore the polarization state of the