3D refraction travel times accuracy study in high contrasted media

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Abstract

There is a number of numerical methods to solve eikonal equation. Some of them have clear advantages in speed. The necessary accuracy may change according to desired detail in travel time computation, mainly as a function of scale. We evaluate three eikonal solvers, the classical approach, the fast iterative method and the fast sweeping method. The three methods computes the first arrival of refracted waves in high contrast media and the results are compared to the analytic solution. A large two layer model is used to record refracted waves in long offsets. The absolute errors in travel time are computed among the three methods using different discretization. The comparison shows that the fast sweeping method is the most accurate.

Introduction

The iterative eikonal equation solution via finite difference method was proposed by Vidale (1988). As a nonlinear equation, the eikonal solution follows the consequences of some concepts such Huygens principle, where each wavefront point works as a new wavefront (Cervenỳ, 2001). The motivation of this work is to calculate an accurate refraction travel time first arrival in strongly heterogeneous media. Algorithms was proposed to solve eikonal equation some: Van Trier & Symes (1991); Podvin & Lecomte (1991); Qin et al. (1992); Hole & Zelt (1995). Among all these variations, the algorithm proposed by Podvin & Lecomte (1991) works well even in regions containing large velocity contrasts (Cervenỳ, 2001). After that, Koketsu (2000) developed new finite difference operators to substitute 3D Podvin & Lecomte (1991) operators and Zhao (2005) developed the fast sweeping method (FSM) to calculate all operators per octant, in 3D case, to accelerate the travel times convergence. Noble et al. (2014) shows a FSM methodology using finite difference approximation in eikonal equation, where new operators were developed and a hybrid scheme, between Spherical and Cartesian system, were applied to improve accuracy. On the other hand, new methods of eikonal solution was developed with expanding wavefront philosophy, such as the fast marching method (FMM) (Sethian, 1996) and the fast iterative method (FIM) (Jeong & Whitaker, 2008). Those approaches avoid to visit all points of model to compute travel times because each grid point is calculated using the neighboring points with an active list scheme. Capozzoli et al. (2013) makes a comparison with FMM, FSM and FIM and the last one overcome the two other methods in performance, but nothing was shown about accuracy. In order to find an accurate method to simulate with precision of the first arrival seismic acquisitions, we compare the 3D classic formulation developed by Podvin & Lecomte (1991), the solution of FIM (Jeong & Whitaker, 2008) and the FSM with new finite difference operators Noble et al. (2014) to verify the accuracy of methods using a simple refraction wave analytical solution. The eikonal solvers, analytic solution and the seismic simulation problem are shown. Five shots of eikonal extrapolation and a circular receiver survey is configured to compare travel times computed numerically with travel times computed analytically.

Methodology

Consider the travel time extrapolation based on the eikonal equation for isotropic media

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{v^2(x,y,z)}, \qquad (1)$$

where the solution $T(x,y,z)$ is the travel time volume, which represents the wavefront time propagation according to the velocity model structures, the Cartesian coordinate axes are x,y and z for 3D case and v is the P wave velocity.

Analytic approach

The analytic solution to calculate travel times along ray paths following Figure 1 is the equation below

$$\begin{cases} t = \dfrac{x}{v_2} + \dfrac{2z\cos\theta}{v_1} \\ \theta = \sin^{-1}(v_1/v_2), \end{cases} \qquad (2)$$

where t is the travel time, x is the offset, z thickness of the upper layer, v_1 and v_2 are respectively the P velocity of the upper and lower layer and, θ is the critical angle of incidence. The source and receiver must be in the same elevation (Kearey et al., 2002).

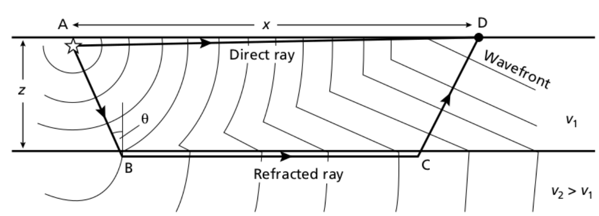

Figure 1 – Wavefront expansion for a two-layer model. Ray paths drawn to represent direct and refracted waves from source A to receiver D (Kearey et al., 2002).

Podvin & Lecomte approach

This technique is a massively parallel method to compute seismic first arrivals. It follow a systematic application of Huygens principle, where causality is respected in propagation. Seismic travel times are classically computed with ray tracing techniques in smooth velocity models. The eikonal solver allows to compute travel times using high contrasted velocity models (Podvin & Lecomte, 1991). The method uses the equation 1 in a finite difference approximation and derives 1D, 2D and 3D operators for each propagation case. In 3D case computation up to 170 stencils are applicable: six 1D transmitted arrivals, twenty four 2D transmitted arrivals (conditionals), twelve 2D diffracted arrivals, ninety six 3D transmitted plane wavefront (conditionals) and thirty two 3D diffracted arrivals. The conditionals check illumination at each point (Podvin & Lecomte, 1991). High computational effort needs to be applied to compute this solution.

Fast iterative method - FIM

FIM is an iterative method to solve eikonal equation, in other words, a grid point is updated until it converges. This algorithm should not impose a particular update sequence, should not use a heterogeneous data structure for sorting and should be able to update multiple points simultaneously. The main algorithm is formulated empirically based on observations from well know eikonal solvers. The expanding wavefront schemes do not limit the wavefront propagation to any special shape, alternatively, causality is respected in function of the velocity model, that is the wave behavior is preserved anywhere in space (Jeong & Whitaker, 2008). FIM applies an active list for storing the grid points that are being calculated and updates all points in active list simultaneously. This method is broadly parallelizable as shown in Dang & Emad (2014). The processing time comparison is not considered because the list scheme is not implemented in this work.

Fast sweeping method - FSM

Proposed by Zhao (2005), this method is an iterative method which uses upwind difference for discretization and uses Gauss-Seidel iterations with alternating sweeping ordering to solve the discretized system. Zhao (2007) and Detrixhe et al. (2013) developed a parallel version and Noble et al. (2014) proposed new finite difference operators to improve accuracy in fast sweeping method. The Noble et al. (2014) approach uses a hybrid scheme, but only the Cartesian system will be used in this work. The new operators are developed in 2D using 4 points and in 3D using 8 points in finite difference approximation. Those operators, initially proposed by Vidale (1988) in a truncated form, are more accurate because all partial derivatives are computed at the center of the cell (Noble et al., 2014).

Implementation details

The Podvin & Lecomte (1991) and Jeong & Whitaker (2008) algorithms is implemented using the expanding cube strategy with high threaded parallelism. The Noble et al. (2014) is implemented in a serial fast sweeping method just using Cartesian operators. OpenACC directives from NVIDIA HPC SDK package (Farber, 2016) is used to compute first arrivals in parallel using a graphic processing unit. Trilinear interpolation is implemented in source and receivers position to improve travel times accuracy using large grid spacing.

Results

The experiment consists in to calculate travel times in a simple two layer model with common regular discretization parameters. Specifically it is used 100, 50 and 25 meters of spacing between cells in the model. Then the model keep the original dimensions, but the amount of cells changed at each discretization. So the model with 100 m of sample spacing has 221 x 221 x 12 cells in x, y and z direction respectively; the model with 50 m has 441 x 441 x 23 cells and the model with 25 m has 881 x 881 x 45 samples in total. Both models have the same spatial dimensions. The configuration of shots and receivers, in circular survey (Figure 2), is used to verify symmetries in travel times. The velocity model has two velocities: the upper layer has 1500 m/s and the lower layer has 2000 m/s. Only compressional P wave is considered, so the analysis is purely cinematic. Shot 1 position is at $(x,y,z) = (1000,1000,0)$ m, shot 2 is in $(21000,1000,0)$ m, shot 3 is in $(1000,21000,0)$ m, shot 4 is in $(21000,21000,0)$ m and the central shot 5 is in $(11000,11000,0)$ m. The receivers configuration start at point $(21000,11000,0)$ m and rotates along the point $(11000,11000,0)$ m, in XY plane, with 12.5 meters each other. A radial distance calculation is used to implement this kind of geometry.

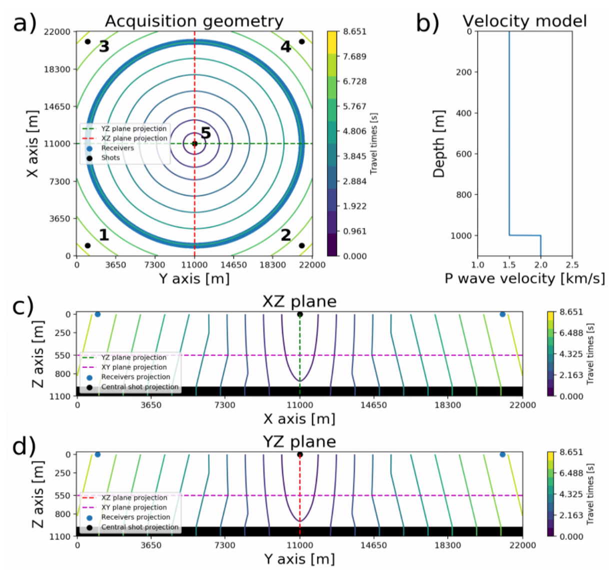

Figure 2 – Seismic simulation scheme. Travel times extrapolation solution are shown in contour plot. a) Geometry acquisition with five numbered shots in black, circular receiver configuration starting in position $(x,y,z) = (21000,11000,0)$ *with 50 m spacing each other in counterclockwise. b) two-layer model considered with interface depth in 1000 m. c) and d) are the XZ and YZ plane projection of model and travel times respectively.*

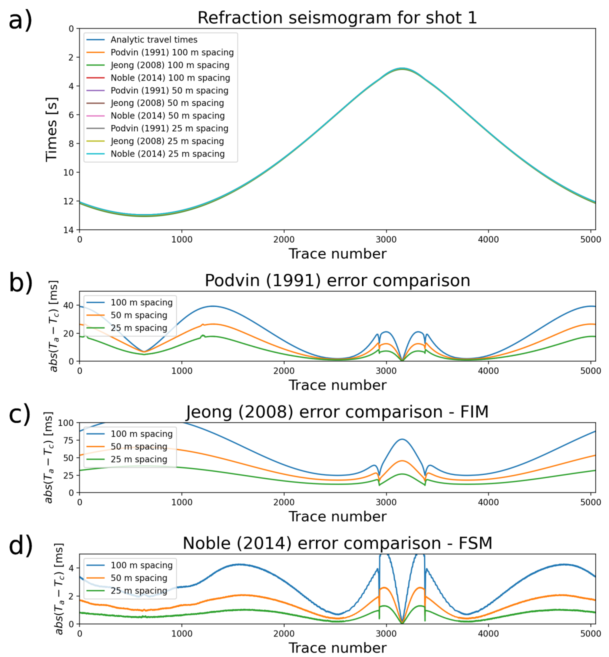

Figure 3 – First shot extrapolation. a) shot domain geometry of first arrivals. b) absolute errors using Podvin & Lecomte (1991) formulation. c) absolute errors using Jeong & Whitaker (2008) formulation. d) absolute errors using Noble et al. (2014) formulation.

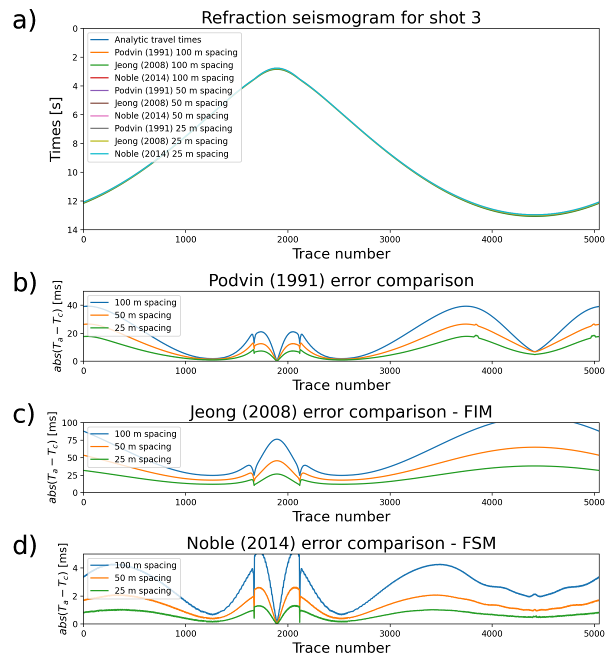

Figure 5 – Third shot extrapolation. a) shot domain geometry of first arrivals. b) absolute errors using Podvin & Lecomte (1991) formulation. c) absolute errors using Jeong & Whitaker (2008) formulation. d) absolute errors using Noble et al. (2014) formulation.

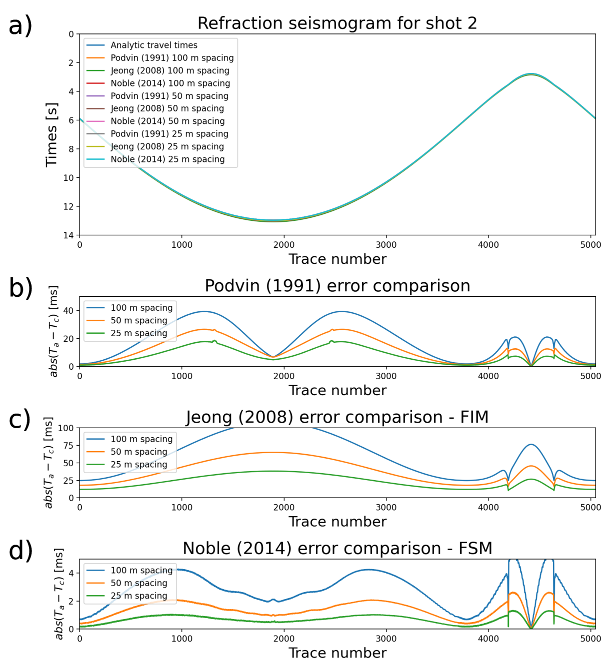

Figure 4 – Second shot extrapolation. a) shot domain geometry of first arrivals. b) absolute errors using Podvin & Lecomte (1991) formulation. c) absolute errors using Jeong & Whitaker (2008) formulation. d) absolute errors using Noble et al. (2014) formulation.

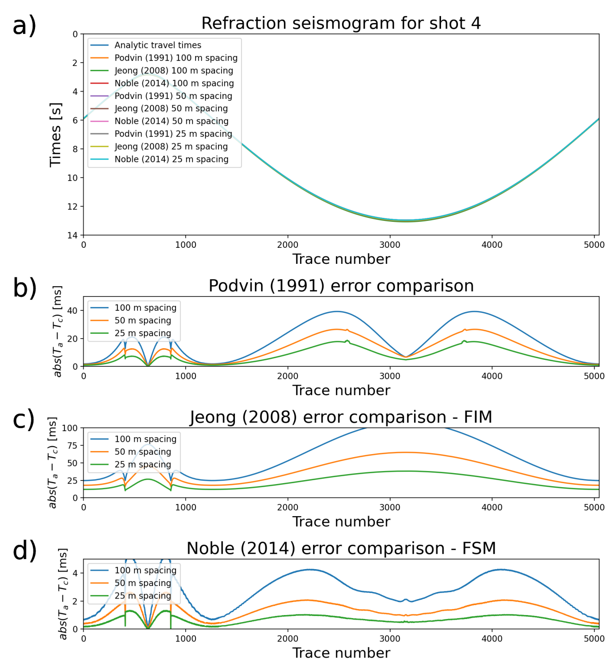

Figure 6 – Fourth shot extrapolation. a) shot domain geometry of first arrivals. b) absolute errors using Podvin & Lecomte (1991) formulation. c) absolute errors using Jeong & Whitaker (2008) formulation. d) absolute errors using Noble et al. (2014) formulation.

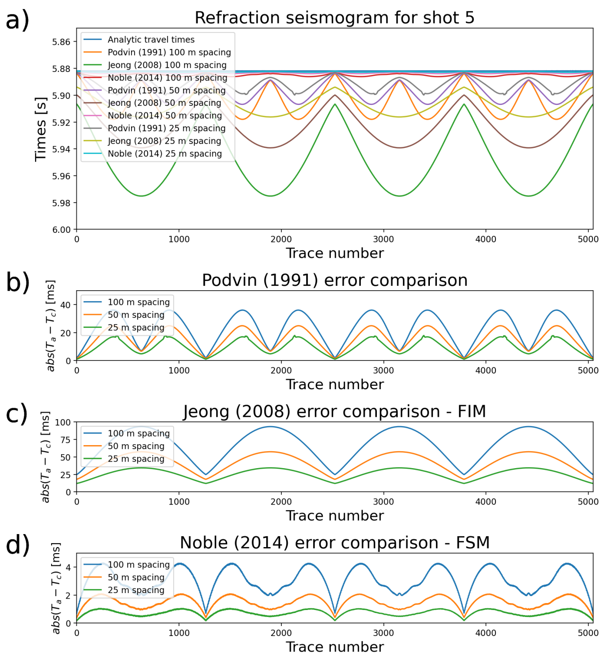

Figure 7 – Fifth shot extrapolation detailed. a) shot domain geometry of first arrivals. b) absolute errors using Podvin & Lecomte (1991) formulation. c) absolute errors using Jeong & Whitaker (2008) formulation. d) absolute errors using Noble et al. (2014) formulation.

All shots are shown separately to highlight the form of refraction travel times. The associated errors for Podvin & Lecomte (1991), Jeong & Whitaker (2008) and Noble et al. (2014) algorithms are calculated for each model discretization. The absolute errors are calculated by the difference of T_a, analytic travel times generated with equation 2, and T_c, the calculated travel times using each eikonal equation solver. The last shot (Figure 7a) shows, with details, the travel times delay in relation of analytic computation for all methods and sample spacing. The errors dimension is in milliseconds.

Discussions

The processing time of algorithms is not considered because the aim of this work is investigate the accuracy of methods. According to our experiments, FIM shows the smallest accuracy than the Podvin & Lecomte (1991) and the Noble et al. (2014) formulation. All figures show that Jeong & Whitaker (2008) errors are too large, reaching more than 25 ms in the smallest grid space used in the experiments. The Noble et al. (2014) formulation overcome other two methods in accuracy because in the coarser grid the errors reach less than 5 ms. Shot 1 (Figure 3) has symmetry with shot 3 (Figure 5) and shot 2 (Figure 4) has symmetry with shot 4 (Figure 6). This reveals that the algorithms azimutal characteristic is working well as shown in Figure 7. Looking at the detailed figure 7a, travel times is higher than it should be making the velocity model looks like slower than it should be. This feature appears more clearly in Jeong & Whitaker (2008) and in Podvin & Lecomte (1991) with a little less difference. The ondulatory characteristics found in error figures is because the Cartesian system does not approximate well the spherical behavior of wavefront. As a consequence of that, hybrid schemes is widely constructed to solve eikonal equation (Alkhalifah & Fomel, 2001; White et al., 2020).

Conclusions

The precision result achieved using Noble et al. (2014) formulation is suitable to simulate seismic acquisition for large offsets. So in seismic applications, such tomography and migration, the fast sweeping method with accurate finite difference operators proves to be more indicated. The complete experiment, from the codes to the figures generation, can be found on the author's GitHub at this url: `https://github.com/pbastosA/seismic_tomography_3D/tree/main/src/eikonal`.

Acknowledgements

I would like to thank the Shell company for sponsoring my master's degree. I thank students, professors, researches and my advisors of *Universidade Federal Fluminense* (UFF) for support my scientific development. Also I would thank the *Grupo de Imageamento Sísmico e Inversão Sísmica* for all knowledge and I thank UFF for the food security provided by the university restaurant.

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