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## Chapter 1: Primer on Marketing Science

*Keywords for this chapter: Marketing Science, CLV, CLTV, ROI, ROAS, ROMI, CAC*

### What is Marketing Science Anyway?

[Skip to main content](#)

Marketing activities existed for a long time, but some scholars agree that it had its formal start after the end of the Second World War at 1945 in response to the high competition that raised from that time on. Given that scenario, methods and research conducted in other fields like management science and engineering started to get its first applications in marketing. Mathematical methods, psychological studies and fields that were formerly not related to marketing started to get its first applications in optimizing marketing endeavours. At the time, the computational power and data available was nothing when compared with what we have today. Today you were probably already impacted by the efforts of marketing science. For instance, when doing a google search you probably saw "promoted" websites as the top recommended answers to your search. That was not randomly suggested to you, a lot of mathematical methods aligned with business logics were processed in order deliver that exactly promoted websites to you. Also when buying something online, you probably saw a section with other "recommended" products. In a nutshell, this is part of what marketing science is. I like to quote [Meta's marketing science department definition](#):

"Marketing Science transforms marketing efforts so that they are grounded in data and science"

—Meta Marketing Science Department

We agree that science is a broad term, so the first thing that we're going to align from now on is in which scientific part of marketing we're going to focus on: the mathematical part. Hence, we're going to focus on mathematical methods for optimizing, transform and understand marketing efforts. These methods will range from very simple heuristics (like the ones that we'll see in this chapter) to more complex mathematical models and algorithms. With that goal in mind, we'll dive into the first basic concepts on Marketing.

## Basic Concepts for Understanding Marketing Endeavours

There are a couple basic metrics that are very useful when working with any kind of business. In fact that are a lot of them, but for now we'll focus on the ones that are more useful for this session, and dive more in the others as they are necessary. These metrics are:

- Customer Lifetime Value (CLV)
- Customer Acquisition Cost (CAC)

[Skip to main content](#)

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# Customer Lifetime Value

What is the "value" of a customer<sup>[1]</sup>? This is usually the first question that you would like to answer when analyzing any business, mainly subscription-based ones (e.g. SaaS). The word value can have several definitions, but usually when answering this question, we are interested in the net profit that our relationship with the customer will bring to the business. When we have this in mind we are usually talking about the *Customer Lifetime Value* or *CLV*. We can define the CLV as: "The present value of the future cash flows attributed to the customer during his/her entire relationship with the company". So it basically means how much net profit we expect from this customer during his entire relationship with us, or in his relationship with us after a given period of time.

You can find out there a lot of fancy and complex forms for calculating the CLV, for the sake of simplicity we'll use a simpler version. There are a couple factors that we need to take into account when calculating the CLV. The first one is the *Average Customer Revenue* (*Revenue*), which represents the gross revenue that we get on average by each customer in a given period of time  $i$ . The second factor is the *Gross Margin* ( $GM$ ) that basically is the percentage of the gross revenue that is considered profit to the business, after discounting all the operational costs necessary for having that customer (e.g. product production cost, retention cost etc). The last one is the *Churn Rate* ( $CR$ ), which is the percentage of customers that after a given period of time  $i$  are expected to end their relationship with us (or the probability that this customer will churn after a given period of time  $i$ ).

With these factors in mind, we can define a simple version of CLV as:

$$CLV = \frac{\overline{Revenue} \times GM}{CR} \quad (1.0)$$

## Example 1



If we have 10% of churn rate, with 150 of revenue and 70% of gross margin. Then:

$$CLV = \frac{200 \times 0.7}{0.1} = 1050$$

We can notice that our  $CR$  follows a geometric distribution with  $p = CR$  and  $q = 1 - CR$ . So

[Skip to main content](#)

as:

$$CLV_n = \sum_{i=1}^n \frac{\overline{Revenue} \times GM}{(1 - CR)^{i-1} \times CR} \quad (1.1)$$

### Example 2



If we spent 5250 in a marketing campaign that resulted in 642 new customers. Then each customer cost 8.17.

$$CAC = \frac{5250}{642} = 8.17$$

Obviously this is a very naive estimator, fancier and more exact ways to calculate it take into account that our  $\overline{Revenue}$ ,  $GM$ , and  $CR$  are not the same for each period  $i$ . For instance, one can find the assumption that your  $\overline{Revenue}$  by customer for the year 2022 will be equal to the year of 2023 a strong one. Furthermore, mainly for young companies it is not advisable to extrapolate your current results too long over time (i.e. have a high  $n$ ), since you expect, naturally, that the situation of your business will be quite different in  $n \times i$  periods. Another fact that we didn't take into account is the *discount* or *cost of capital* factor. More complex and correct ways to calculate  $CLV$  will be covered as necessary, but for now we must mind our assumptions with this simple model.

## Customer Acquisition Cost

This concept is maybe the most straightforward one. In any kind of marketing you aim to prospect new customers. But this comes at a cost, you need to buy the attention of these potential new customers. For instance, let's get back to the example in the beginning of the chapter. A company had to pay in order to appear in the promoted session of your search engine. This *spend* has a crucial role in marketing campaigns, since the more customers you prospect with the lower spend possible the better. In other words, given that  $CA$  is the number of the acquired customers due to the marketing campaign, the lower the ratio  $Spend : CA$  the better. Hence, our *Customer Acquisition Cost* or  $CAC$  can be intuitively defined as:

[Skip to main content](#)



$$CAC = \frac{Spend}{CA} \quad (1.2)$$

### Example 3



If we spent 5250 in a marketing campaign that resulted in 642 new customers. Then each customer cost 8.17.

$$CAC = \frac{5250}{642} = 8.17$$

One can also notice that the ratio of  $CLV$  to  $CAC$  is also important, it basically how much times expected value of that customer is greater than the cost to acquire that customer. The intuition of this relationship naturally bring us to the next basic concept.

## Return on Advertisement Spend

The idea behind the return over some kind of investment has been around for a quite long time. Intuitively, one wants that his investment in a given endeavour return more profit than that given investment. In fact you may already have heard of *Return on Investment* or *ROI* in the business world. Nevertheless, an investment in marketing is intrinsically tied up with some kind of **risk**. One cannot assure a return on a marketing investment, it can be very successful or a total failure. More complex even, something that worked out once may not work if applied twice, see for example what happened to Pepsi Crystal in the 1990's. Hence, it's reasonable that on 1990's terms like *ROMI* (*Return on Marketing Investment*) started to appear more often, since they try to separate the usual investments made by the company with the more risked marketing efforts.

Another import fact is that when we talk about ROMI we are usually talking about the *entire* marketing efforts. What does it mean? It means that if we have several marketing campaigns, we are usually talking about the effectiveness of them as a whole. ROMI is then defined as:

$$ROMI = \frac{\text{Total Marketing Revenue} - \text{Marketing Costs}}{\text{Marketing Costs}} \quad (1.3)$$

[Skip to main content](#)

As you can imagine, marketing costs can cover variety of things, like campaign investments, discounts, and etc. A  $ROMI > 0$  means that the investment is paying off. But which metric we use when we are talking about *individual* marketing campaigns? In that case, we are ideally talking about our *Return on Advertisement Spend* or  $ROAS$ . This metric concerns our specific returns on marketing advertisement (i.e. marketing campaigns). It follows a similar logic, but is a constrained version of  $ROMI$ :

$$ROAS = \frac{\text{Campaign Revenue}}{\text{Campaign Spend}} \quad (1.4)$$

Base on the equation 1.4 we can elaborate more on the formulation of  $ROAS$  given the concepts that we already introduced:

$$\begin{aligned} ROAS &= \frac{\text{Campaign Revenue}}{\text{Campaign Spend}} & (1.5) \\ &= \frac{CA \times CLV}{Spend} \\ &= \frac{Spend \times CLV}{CAC} \times \frac{1}{Spend} \\ &= \frac{CLV}{CAC} \end{aligned}$$

As we can see from equation 1.5, our  $ROAS$  boils down to the concept of the ratio of  $CLV$  to  $CAC$ . This has a series of interesting implications, which we'll analyze a bit further.

Recall that when we defined our  $CLV$ , it was the expected return that we would get from a customer after a given period  $i$ . It implies that to some extent, we expect our  $CLV$  to remain static in our calculations: it's the *expected* or *average* profit of the customer. That means that our  $ROAS$  function will take the form:  $f(x) = \frac{1}{x}$ . This will lead to a couple of interesting corollaries:

#### Corollary 1

Given a  $CLV > 0$  and  $CAC > 0$ , our  $ROAS$  function will never achieve zero.

Proof. We can prove this as we take the limit of our *ROAS* function as *CAC* goes towards  $+\infty$ .

$$\lim_{CAC \rightarrow +\infty} \frac{CLV}{CAC} = \frac{CLV}{\infty} = 0 \quad (1.6)$$

Also, this can be seen in the limit of the partial derivative of the *ROAS* function.

$$\frac{\partial}{\partial CAC} \left[ \frac{CLV}{CAC} \right] = -\frac{CLV}{CAC^2} \quad (1.7)$$

$$\lim_{CAC \rightarrow +\infty} -\frac{CLV}{CAC^2} = -\frac{CLV}{\infty^2} = 0$$

This can be seen in the Fig 1, as our *CAC* increases, our *ROAS* function tends towards zero.

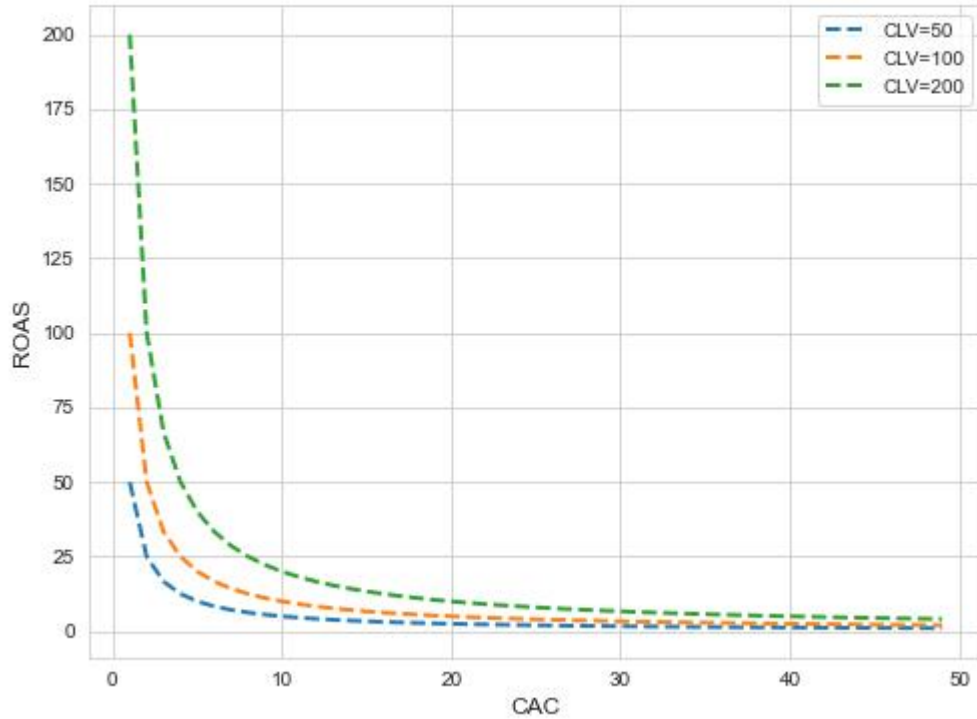


Fig. 1 ROAS for different CAC while maintaining CLV fixed

### Corollary 2

The derivative of  $ROAS$  will be  $-\frac{1}{CLV}$  when  $ROAS = 1$ .

Proof. Notice that when  $ROAS = 1$  that implies that  $CLV = CAC$ . Given that, we can show:

$$-\frac{CLV}{CAC^2} = -\frac{CLV}{CLV^2} = -\frac{1}{CLV} \quad (1.8)$$

## The ROAS Driven Marketing Trap

As we saw from the previous corollaries, our  $ROAS$  will never be equal to zero. However, we want our  $ROAS$  to never be less than 1, that means that we are “receiveing back” as revenue from the customer all the money we spent to acquire him.

**[1]** In this chapter and from here on we'll use the term customer as the “success” event of a company. This can be either making a contract or selling a service, as in SaaS businesses, or making a sale for example. Not subscription-based companies make the calculation of some metrics a bit trickier, but the concepts are usually equally applicable.