# A quantitative urban model for transport appraisal

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**Abstract**

Understanding the spatial economic impacts of large-scale transport infrastructure projects is vital for promoting efficient resource allocation in modern economies. However, the current practice of economic evaluation in transport policy is largely confined to partial equilibrium models, which focus solely on transport markets in isolation, ignoring the transformative impact of large-scale interventions. In this paper, we build upon the methodological principles of quantitative spatial economics, an emerging field within the spatial economics literature, to design an appraisal method that captures the interactions between transport supply and the labour, production and real estate markets. Our model features travel time valuations that are micro-founded through an explicit leisure-labour trade-off, making them unique to each residence-workplace pair. The model is invertible and suitable for the theoretically and geographically coherent causal estimation of the dispersion of idiosyncratic location choice preferences and the strength and distance decay of agglomeration forces.

In the numerical implementation of the model we replicate Greater London with 983 distinct spatial units. In a counterfactual scenario, we model the cross-sectoral effects of the commuting cost reduction brought by the Elizabeth Line, a major urban rail project. We compare the welfare estimates in spatial equilibrium to those derived from mainstream partial equilibrium methodologies typically used in cost-benefit analysis. Our approach reveals a rich pattern of local economic outcomes—including shifts in employment, productivity, wages, and real estate prices—triggered by the transport investment. Equipped with the proposed methodology, the dialogue on future transport infrastructure investments can be supported by a solid quantitative tool.

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# 1 Introduction

Transport cost-benefit analysis (CBA), often referred to as benefit-cost analysis (BCA) in the US and transport appraisal in Europe, is one of the most impactful areas of application of microeconomic theory and traffic modelling. In the transport sector, welfare economic models are frequently applied in policy-making to prioritise among a pool of proposed infrastructure investments and justify the societal value of allocating public funding to the construction of transformative megaprojects. In the United Kingdom, transport appraisal results have informed intense public debate over large-scale investments and significantly influenced government decisions on projects such as London's Elizabeth Line and the HS2 high-speed rail line. Since the late 20th century, official appraisal guidelines from the UK, the EU, international development banks, the OECD, and government agencies worldwide have influenced billions of dollars in transport infrastructure spending.

Despite its undeniable impact on policy-making, the mainstream methodology of transport appraisal has remained nearly unchanged over the past two decades. The backbone of the method is what we call direct transport user benefits (DUBs), in which the dominant element is the monetary equivalent of the value of travel time savings that a transport improvement brings to existing and new users in the form of consumer surplus. Since the early 2000s, more research concentrated on additional welfare gains potentially not captured by travellers' willingness to pay for mobility. Wider economic impacts (WEIs) include externalities such as the productivity gain from an increase in access to economic mass (ATEM). The theoretical foundations and the empirical methods of estimating WEIs are summarised by Graham and Gibbons (2019). Note that this classical appraisal framework has remained a partial equilibrium (PE) representation of the economy, i.e. it does not capture explicit interactions between the transport market and potential welfare effects in other sectors of the spatial economy.

Despite its widespread use in practice, the PE appraisal method has been criticised both within and outside academia. One theoretical concern is a potential overlap between the DUB and WEI layers.[1] From the viewpoint of practitioners and the wider public, we often witness scepticism because PE cost-benefit analysis seemingly does not capture a wide range of apparent economic implications of a transport improvement, such as job creation and fluctuations in real estate prices. Although microeconomic theory suggests that partial equilibrium (PE) consumer surplus is a reliable measure of economic gain in the absence of market failures in other sectors, there is a widespread preference for evaluation metrics derived from models that transparently replicate observed economic outcomes, such as wages,

[1] Following the classification introduced by Duranton and Puga (2004), the productivity benefits of increased economic density can be attributed to three Marshallian forces: input sharing, improved matching in the labour market, and knowledge spillovers. Eliasson and Fosgerau (2019) argue that the consumer surplus derived from the partial equilibrium demand function may include a fraction of the labour pooling part of the productivity externality induced by the transport improvement.

housing prices, and residential or workplace relocation, without relying on abstract concepts like consumer surplus or social welfare. Both of these criticisms could be resolved by a move from the partial equilibrium tradition to a suitably designed spatial general equilibrium (SGE) approach.[2]

The quest to produce spatially explicit numerical models that capture the impact of transport policies on location choice, land-use and other socio-economic outcomes has a long history in transport research. Without seeking to cover this large literature comprehensively, let us limit our discussion to two lines of research. Land-use transport interaction (LUTI) models are widely used to support spatial/urban planning in connection with transport policies. From our perspective, LUTI models are less relevant because they are not fully micro-founded models. Thus, they may well be suitable for the prediction of transformations in land-use patterns, but they cannot produce a consistent measure of aggregate welfare changes for economic evaluation. Another important group of models belong to the spatial computable general equilibrium (SCGE) tradition. A more detailed review in Section 2.1 will conclude that SCGE models are the state-of-the-art of spatial modelling in terms of the diversity of theoretical components they feature, but their use in practical appraisal is hindered by the challenge of calibrating a large number of parameters to which the model outcomes may be highly sensitive.

From an empirical point of view, a spatial model is expected to reveal the true *causal* impact of a transport policy on economic outcomes through the mechanisms included in the model. The concept of causality is widely used in statistics and econometrics in the context of single estimating equations.[3] We argue that ensuring causality is equally important in the calibration of more complex economic models as well. Let us illustrate this point in a specific example. A transport improvement as a treatment may coincide with other economic shocks and unrelated policy interventions. For example, the Elizabeth Line opened during the pandemic, and both had an important effect on travel patterns in London. Regular spatial models in the transport literature may be able to capture the impact of a new railway line, but, evidently, they are not designed to handle transformational public health crises. If a naive calibration exercise is based on the *associational* relationship between the Elizabeth Line as a policy treatment and the observed change in the local economy as an outcome of the treatment, one may erroneously confound the impact of the pandemic with that of the new infrastructure. That is, assuming that the former reduces the volume of commuting, there is a threat that an associational calibration attempt would underestimate the role of travel time, and the effect of the Elizabeth Line, in the observed commuting behaviour. Even though the threat of confounding is evident in this case and we expect that researchers would normally overcome it by avoiding the use of data gathered during the pandemic, the possibility of confounding

[2]When market failures do exist in the related markets, this move is not only more convincing for the public but also necessary to derive correct welfare results.

[3]See a comprehensive introduction to causal inference in transport research in Graham (2025).

remains a serious issue as long as model calibration is based on associational instead of causal methods.

Identifying the causal impact of the parameters of an SGE model is not a trivial task, however. First, parameters may enter the general equilibrium model specification nonlinearly and in interaction with other coefficients to be estimated. This prevents the analyst from defining estimating equations that remain coherent with the functional form of the spatial model. Second, many SGE models include variables that cannot be estimated using the locally available data. Thus, it is a common practice to "borrow" parameters from other empirical studies in the literature. The resulting geographical or/and temporal incoherence is another threat to the robustness of parameter identification.

Since the mid-2010s, a new stream of research often referred to as *quantitative spatial economics* (Redding and Rossi-Hansberg, 2017) achieved a major methodological step in model estimation. Quantitative spatial models (QSMs) are SGE models designed specifically to facilitate the use of causal techniques in model estimation. QSMs feature two core empirical properties.

First, they contain a set of variables that the literature often refers to as geographical fundamentals; for example, the attractiveness of each location for residential and workplace activities (i.e. local amenities), a measure of local firm productivity, and a measure of local floorspace density limits. QSMs are designed such that there is a deterministic relationship between the vectors of unknown location-specific characteristics and the observed data. In other words, using the model's equilibrium conditions, one can express the vectors of local fundamentals in function of the observed data algebraically. Thus, at the expense of a set of assumptions about the theoretical structure of the model, these parameters can be quantified without the uncertainty inherent in statistical estimation. These location-specific parameters are in fact structural residuals of the model.

Second, the most important generic parameters of QSMs can be estimated in econometric exercises that remain coherent with the functional forms of the theoretical model. In other words, after transformations that do not violate algebraic equivalence, the equilibrium conditions of the model can be used as econometric estimating equations as well. The most well-known implication of this property is that the equilibrium condition that describes residence-workplace location choice probabilities turn into a gravity equation after log-linearisation. This enables us to estimate the spread of idiosyncratic preferences in location choice by adopting the econometric toolbox that the trade literature developed for the estimation of gravity equations.

The aim of this paper is to bridge the gap between the state-of-the-art of transport analysis and quantitative spatial economics. A gap exists because mainstream QSMs, developed in isolation from the transport research community, are based on numerous simplifying assumptions about travel behaviour and transport supply which make their widespread use in contemporary transport analysis challenging. To name the most pressing examples, (i) temporal and monetary travel costs are not disentangled, and the latter are often entirely ignored, (ii) travel disutility is multiplicative, following the *iceberg* specification, (iii) most of the models are based on commuting only, other trip purposes in passenger transport are ignored, (iv) mode and route choice and congestion are ignored or only partially considered in a simplified manner, (v) transport supply is exogenous, including decisions on pricing as well as capacity provision, and (vi) the welfare predictions of QSMs are not compared against the traditional transport appraisal methodology.

We consider these limitations as a research agenda that opens the door to the widespread use of QSMs in transport policy appraisal. In this paper we address items (i), (ii) and (vi) above. We propose an SGE model in which households face separate temporal and pecuniary budget constraints. Both the time and money cost of travel are additively related to other elements in the respective budget constraints, thus avoiding the need for an iceberg-type specification of travel costs. Our specification is not unique in the transport literature; after a logarithmic transformation, our household utility specification becomes very similar to that of Anas and Liu (2007), for example. However, we encapsulate this specification into a framework that retains the advantageous properties of mainstream QSMs. That is, our model is invertible, so local amenity and productivity levels, and the density limit of floorspace development, can be recovered deterministically. The model also allows us to estimate commuting gravity, agglomeration elasticity and distance decay parameters theoretically and geographically consistently.

Concurrently, the paper delivers a contribution that may be relevant in the wider QSM literature as well. Existing quantitative spatial economic analyses are built on the assumption of inelastic labour supply, thus making the spatial distribution of *employment* identical to the spatial distribution of *aggregate labour supply*. Our approach derives the opportunity cost of time from the leisure-labour trade-off where individual labour supply is an endogenous decision variable of the household problem. Similar to the value of time, we turn individual labour supply into a location-specific outcome of the spatial equilibrium. This modelling approach provides a basis for future labour economic analyses in the QSM framework.

The context of the application of our model is Greater London. Our model has nearly one thousand spatial units in line with the Middle-Layer Super Output Areas (MSOAs) of the UK Office of National Statistics (ONS). This implies that we consider almost a million potential residence-workplace combinations in workers' location choice. This level of granularity is common among QSMs but high in comparison with the previous transport SGE literature. We illustrate the estimated model's application in transport policy appraisal through a case study of the Elizabeth Line (also known as Crossrail), a recently completed urban railway line which connects the Western and Eastern suburbs of London with its central business district. We take advantage of large-scale GTFS timetabling data and a recently published routing algorithm in R for a computationally efficient calculation of the travel time and cost matrices before and after the opening of the Elizabeth Line.

The separation of monetary and temporal budget constraints leads to location-specific travel time valuations in our model, where the value of time is defined as the ratio of the location-specific marginal utilities of time and money. The paper provides a rare insight into the spatial distribution of time valuations, at a granularity that is hard to reproduce in conventional stated or revealed preference empirical approaches. We find that the marginal opportunity cost of travel time is more than 60% higher among Central London residents than in the outskirts, and the model reveals the full spatial distribution of this measure.

The travel time valuations and commuting volumes implied by the model enables us to perform a simplified appraisal of the Elizabeth Line according to the mainstream partial equilibrium CBA methodology, netting out the impact of potential errors in the estimation of the value of time and induced demand in practical CBA. Our PE CBA includes the direct user benefit of travel time savings and the wider economic impact of the productivity externality of improved access to economic mass. The PE welfare result is then benchmarked against the estimated net benefit of the Elizabeth Line in spatial equilibrium, according to our SGE model. Most impartantly, even though the SGE model seems to unlock a much richer set of adjustment mechanisms in response to the intervnetion, the welfare result of the SGE model remains remarkably close to the outcome of the PE CBA approach. We derive four versions of the PE model that differ in whether we allow the value of time to differ between origin-destination pairs whether the new commuting demand induced by residential and workplace relocation is included in the calculation. Under this paper's assumptions on model specification, the SGE welfare result is around 8.5% lower than the highest PE result and than 27.0% higher than the lowest.

Our quantitative benchmarking exercise confirms that transport appraisal in SGE is unlikely to lead to fundamentally different results in policy evaluation and project ranking, compared to an appropriately designed PE CBA with unbiased parameter estimates. However, an SGE model provides much more detail about the impact of a transport project on the spatial economy, including the resulting pattern of urban land use, firm productivity, wages, housing prices and labour supply. These insights may improve trust in the traditional appraisal methodology, and make policy makers and the wider public better informed about the societal impact of future infrastructure projects.

The rest of this paper has the following structure. Section 2 is a literature review focusing on the novelties and commonalities of our method with the surrounding literature. Section 3 introduces the theoretical structure of the model. Section 4 guides the reader through the quantification of this model. In Section 5 we describe the counterfactual simulation of the Elizabeth Line and its evaluation with both partial and general equilibrium methods.

# 2 Links with the surrounding literature

This paper proposes a new approach to transport policy appraisal for which the literature has established methods, both in partial and spatial general equilibrium. The core question we would like to address in this section is therefore whether the proposed QSM approach is truly novel relative to previous contributions, and to what extent QSMs share already acknowledged and so far unexplored commonalities with the previous SGE literature. Section 2.2 then reviews the theory of time valuation in transport models, providing context for a key ingredient of our model.

## 2.1 Spatial general equilibrium

The desire to produce spatially explicit versions of stylised-space general equilibrium models has been recognised and partly satisfied in the past decades. A comprehensive review of transport-relevant spatial economics would describe the literature of land-use and spatial computable general equilibrium models dating back to the 1960s. Land use/transport interaction (LUTI) models, including Hunt and Simmonds (1993), make the spatial impact of transport interventions predictable for a selected geography, following the footpaths of Lowry (1964). The SCGE tradition includes a series of attempts to model the spatial distribution of economic activity with thorough microeconomic foundations. Extensive reviews are available in Paulley and Webster (1991), Wegener (2011), Robson et al. (2018), and a companion review article of this project by Hörcher and Graham (2024) who survey this literature in light of the latest QSM developments.

Quantitative spatial models have shifted the prevailing paradigm in spatial equilibrium modelling since the mid-2010s. Section 1 has already provided a broad definition of what makes an SGE model "quantitative": model inversion to recover the vectors of location-specific fundamentals has not been performed previously, and QSMs are also suitable to causally identify some of the core structural parameters in reduced-form econometric models that remain coherent with the specification of the theoretical model. The pioneering contributions include the trade model of Allen and Arkolakis (2014) and an urban prototype by Ahlfeldt et al. (2015), followed by a series of influential contributions including Donaldson (2018), Monte et al. (2018), Heblich et al. (2020), and Tsivanidis (2023). This literature pays a lot of attention to identifying isomorphism between different sets of assumptions behind the theoretical models. Appendix D of Heblich et al. (2020) provides a valuable methodological overview of the conditions under which five typical SGE specifications, ranging from the canonical urban model to the spatially explicit realisation of New Economic Geography models become isomorphic.

Although transport is a popular subject among the applications of QSMs, this novel literature is still largely disintegrated from the state-of-the-art of transport economic modelling. From one perspective, the authors' experience suggests that QSMs have remained unnoticed by the mainstream transport economics community until recently. From the other perspective, QSMs have several limitations that transport research surpassed a long time ago. We observe new efforts to bridge this gap between the two traditions. For example, Allen and Arkolakis (2022) have incorporated road congestion and stochastic route choice into their previous framework, while Tsivanidis (2023) and Fajgelbaum et al. (2023) integrated mode choice to distinguish car use from other modes of transport. Fajgelbaum and Schaal (2020) applied a QSM to optimise the structure of transport infrastructure networks. Still, several further limitations of the baseline model hinder the adaptation of QSMs in transport policy appraisal. As mentioned in the introduction, the assumption of *iceberg* transport costs is a core limitation that we intend to relax in this work. Our motivation is that QSMs in their current form are not ready for direct adaptation in transport policy appraisal and optimisation. Proost and Thisse (2019) summarised the root cause of this research gap in a rather crude way: "The literature of spatial economics has paid too little attention to what has been accomplished in transportation economics (and vice versa)."

A series of studies by Alex Anas and co-authors deserves special attention in this paper. Their urban models developed since the 1990s show many similarities with today's quantitative urban models in the sense that they were pioneers in the adaptation of random utility discrete location choice specifications in spatial equilibrium modelling.[4] On the basis of the discrete choice framework of Anas and Kim (1996), Anas and Liu (2007) produced the first spatially explicit SGE model and combined the labour, production and construction markets in urban equilibrium with stochastic traffic assignment. Later on, Anas and co-authors developed further implementations of this model for various case study areas including Los Angeles and Paris (Anas, 2020, Anas and Chang, 2023). A common feature of the Anas-type models is the presence of separate time and money constraints in the household problem, in line with the traditions of the transport literature reviewed in Section 2.2. Despite many theoretical similarities between this line of research and QSMs, Anas et al. never exploited the powerful properties of their model in quantitative work (i.e. model estimation). The model presented in this paper is the first attempt to turn the Anas-type SGE approach into a QSM.

At the same time, we acknowledge that the models developed by Anas et al. remain ahead of QSMs including the one in this paper by featuring a full transport mode and route assignment module that Anas et al. nest into the spatial equilibrium model in an iterative process. Nesting traditional transport assignment into a QSM remains an open challenge. The primary reason is that there is no trivial way in which a QSM would remain invertible and suitable for causal parameter estimation when mode and route choice and the congestion levels are endogenous.

[4] Interestingly, Ahlfeldt et al. (2015) may had not been aware of these parallels with the Anas oeuvre as they explain their random utility location choice specification was inspired by the original statistical theory established by Daniel McFadden (1978), and, more specifically, the Eaton and Kortum (2002) approach to modelling heterogeneity in productivity in the trade literature.

Furthermore, this model extension would create significant computational challenges at the spatial granularity of our model: we work with almost 1,000 spatial units as opposed to the few dozens of locations in Anas and co-authors' papers. We believe that future transport-oriented QSMs will capture these specificities in a more aggregate representation of congestion, e.g. by applying a bathtub (i.e. microscopic fundamental diagram) congestion specification (see a related model solution in Koster, 2024).

## 2.2 The theory of time valuation in transport economics

The monetary valuation of travel time is one of the key parameters of the economic evaluation of transport policies. Small (2012), Small and Verhoef (2007) and Jara-Díaz (2007) provide a comprehensive coverage of the evolution of thought on time valuation and its impact on travellers' decisions and optimal policy design. The roots of this literature date back to the general theory of time economics pioneered by Becker (1965) and DeSerpa (1971). Despite the solid theoretical foundations of the theory of time use, in practice, the measurement of the value of time is considered predominantly as an empirical matter; see a recent reflection on the state of the practice in Jara-Díaz (2020).

The empirical estimation of the opportunity cost of travel time has been in the spotlight since the early days of the field. Advances in discrete choice modelling enable the estimation of the value of time as a ratio of the marginal utilities of travel time and monetary expenditure using either state or revealed preference data. For a methodological overview, see a large-scale collection edited by Hess and Daly (2014) and a meta-analysis by Wardman et al. (2016). Even though the empirical estimates are sometimes differentiated by income groups or other high-level socioeconomic characteristics, the granular spatial pattern of travel time valuations has not been investigated in this literature due the obvious difficulty of data collection.

In this work, we keep the value of time an endogenous outcome of the traveller's utility maximising behaviour. One specific class of models in the transport literature is particularly relevant for this paper. Parry and Bento (2001) and a series of subsequent transport policy appraisal studies built their demand models on the assumptions of (i) endogenous labour supply, (ii) perfect substitution between commuting demand and labour supply, and (iii) the presence of a time constraint in combination with the household's monetary budget constraint. The resulting general equilibrium model provides a transparent expression for the endogenous value of travel time and a tractable link between policy interventions in labour and transport markets. Notable articles in the footsteps of Parry and Bento include De Borger and Van Dender (2003), Arnott (2007), De Borger and Wuyts (2011) and Hörcher et al. (2020). These models are all dimensionless in the spatial sense. Tikoudis et al. (2015) are the first to adapt this framework to the classic Alonso-Muth-Mills monocentric city model, with its obvious limitations in practical applicability.

Our paper intends to advance this literature by capturing the mechanisms behind travel time valuation in a spatial general equilibrium model. The outputs of our model reveal the heterogeneity and disaggregate spatial distribution of travel time valuations in an empirically relevant theoretical framework. Our approach has the potential to make costly stated or revealed preference data collection unnecessary. With spatially differentiated estimates, transport appraisal can be performed with local valuations of travel time savings, thus avoiding the threat of averaging bias.

# 3 The model

Our theoretical model considers three groups of agents: households, a production sector and a floorspace construction sector. Geography is represented by a set of discrete locations connected by the transport network. Floorspace in each location is used for both residential and commercial purposes. A model that includes $n$ locations has $n \times n$ possible residence–workplace combinations indexed by $ij$, where intrazonal commuting $i = j$ is not ruled out. Households' location decisions are modelled in a discrete choice framework. We assume a representative commuter whose location choice probability $\lambda_{ij}$ is interpreted, based on the law of large numbers, as the share of workers in a population of $N$ households that live in $i$ and commute to $j$. At the same time, the representative household makes location-specific decisions on a continuous scale about consumption, residential floorspace use, leisure time, and the intensity of individual labour supply. We present the theoretical model in three steps: Section 3.1 describes the household problem, Section 3.2 derives choice probability equations, and Section 3.3 introduces the production and floorspace construction sides of the economy.

## 3.1 Household preferences

Let us define the utility of a representative worker who resides in location $i$ and commutes to location $j$ as

$$U_{ij} = \left(\frac{L_{ij}}{1-\gamma}\right)^{1-\gamma} \left(\frac{K_{ij}}{\gamma}\right)^{\gamma} z_{ij}; \quad K_{ij} = \left(\frac{C_{ij}}{\beta}\right)^{\beta} \left(\frac{H^R_{ij}}{1-\beta}\right)^{1-\beta}. \tag{1}$$

In this specification $L_{ij}$ is a measure of leisure time, $K_{ij}$ is the composite subutility derived from consumption $C_{ij}$ and residential floorspace use $H^R_{ij}$, $\gamma$ and $\beta$ are structural parameters, and $z_{ij}$ is an idiosyncratic taste shock associated with the combination of locations $i$ and $j$. To keep our notation simple, we suppress the unique identifier of households. Note, however, that $z_{ij}$ takes a different value for each household.

Commuters are confronted by two constraints. First, wage $w_j$ at workplace $j$ and the monetary price of commuting $\tau_{ij}$ times individual labour supply $x_{ij}$ determine the budget available for consumption, given the price of the consumption good, $p_i$, and the price of residential floorspace, $q_i$.

$$x_{ij}\left(w_j-\tau_{ij}\right)=p_iC_{ij}+q_iH_{ij}^R \tag{2}$$

Second, the sum of leisure time $L_{ij}$ and the total time spent at work ($T$) and in commute ($t_{ij}$) cannot exceed $\bar{L}$, the daily time endowment of households.

$$\bar{L}=L_{ij}+x_{ij}\left(T+t_{ij}\right) \tag{3}$$

Again, individual labour supply $x_{ij}\geq 0$ determines how many workdays an individual is willing to provide. The Lagrangian of the constrained optimisation problem is defined as

$$\Lambda=U_{ij}-\kappa_{ij}\left[p_iC_{ij}+q_iH_{ij}^R+x_{ij}\tau_{ij}-x_{ij}w_j\right]-\mu_{ij}\left[L_{ij}+x_{ij}\left(T+t_{ij}\right)-\bar{L}\right], \tag{4}$$

where $\kappa_{ij}$ and $\mu_{ij}$ are Lagrange multipliers that capture the spatially differentiated marginal utilities of money and time, respectively.

The following derivations are detailed in greater depth in Section A.1 of the Appendix. The first-order condition of the optimal choice of individual labour supply implies

$$\kappa_{ij}w_j=\kappa_{ij}\tau_{ij}+\mu_{ij}(T+t_{ij}), \tag{5}$$

which equates the utility associated with the monetary benefit of the marginal trip to work (on the left-hand side), with the disutility of monetary and time resource requirements (on the right-hand side). Rearrangement leads to an expression of the ratio of the marginal utilities of time and money:

$$\frac{\mu_{ij}}{\kappa_{ij}}=\frac{w_j-\tau_{ij}}{T+t_{ij}}=\upsilon_{ij}. \tag{6}$$

We interpret this ratio as a monetary valuation of the incremental relaxation of the worker's time endowment. We call it the (marginal) *value of time* denoted by $\upsilon_{ij}$. As one would expect, the worker's wage is among the determinants of the marginal value of time, as foregone time could always be used to earn income via time reallocation to work. Equation (6) also reveals that the value of time depends on the monetary and time cost of commuting as well. The core consequence from a spatial economic point of view is that the value of time will likely differ between residence–workplace combinations, which is often neglected in mainstream transport policy appraisal. To emphasise this feature of the model, we keep the subscripts of $\upsilon_{ij}$ throughout the forthcoming analysis.

Note that $\upsilon_{ij}$ can also be interpreted as an hourly real wage, were the monetary term in the numerator is the daily wage net of the pecuniary cost of commuting, and the denominator normalises this net daily wage by the gross working hours, including the time spent commuting.

After simple algebraic manipulations, the first-order conditions of (4) lead to the following expression for the optimal time allocation to leisure activities:

$$L_{ij} = (1-\gamma)\bar{L}. \tag{7}$$

That is, $\gamma$ is the share of the total time endowment that households allocate to working and commuting. Plugging this relationship into the time budget constraint, individual labour supply becomes

$$x_{ij} = \frac{\gamma \bar{L}}{T + t_{ij}}. \tag{8}$$

Individual labour supply increases with the relative importance of consumption in household utility ($\gamma$) and decreases with the time requirement of commuting ($t_{ij}$). This highlights a non-trivial channel through which transport supply affects labour markets in the spatial economy.

The household utility maximisation problem yields the following solutions for consumption decisions.

$$\begin{aligned} C_{ij} &= \beta \frac{\gamma \bar{L}\, \upsilon_{ij}}{p_i} \\ H^R_{ij} &= (1-\beta)\frac{\gamma \bar{L}\, \upsilon_{ij}}{q_i} \end{aligned} \tag{9}$$

Naturally, consumption decreases with the unit prices of goods and floorspace at the residential location. More surprisingly, $\upsilon_{ij}$ enters this formula directly. That is, someone with a high value of time is expected to consume more. The net hourly wage interpretation of $\upsilon_{ij}$ is also relevant in the present context. With this interpretation the numerator in both equations, $\gamma \bar{L}\, \upsilon_{ij}$, is the daily net wage which depends on the cost and duration of commuting as well as the time allocated to work-related activities ($\gamma \bar{L}$).

Our solutions yield the following indirect (sub-)utility functions for a given combination of residential and working locations.

$$\begin{aligned} K_{ij} &= \frac{\gamma \bar{L}\, \upsilon_{ij}}{p_i^{\beta} q_i^{1-\beta}} \\ u_{ij} &= \bar{L} \left( \frac{\upsilon_{ij}}{p_i^{\beta} q_i^{1-\beta}} \right)^{\gamma} z_{ij} \end{aligned} \tag{10}$$

The hourly real wage (or in a different interpretation, the marginal value of time), the local price of consumption and residential rents, structural parameters $\gamma$ and $\beta$, and idiosyncratic taste are the only determinants of a residence–workplace combination's attractiveness to households.

### 3.2 Spatial choice probabilities

The idiosyncratic utility shock is specified as an i.i.d. random draw from a Fréchet distribution:

$$F(z_{ij}) = \exp\Big(-X_i E_j z_{ij}^{-\epsilon}\Big), \tag{11}$$

where the average amenity (i.e. the scale parameter) is defined as the product of residence and workplace dependent local fundamentals $X_i$ and $E_j$, and $\epsilon$ governs the spread of individual preferences. These assumptions lead to location choice probabilities that take the form of a commuting gravity equation.[5]

$$\lambda_{ij} = \frac{X_i E_j \left[\frac{\upsilon_{ij}}{p_i^{\beta} q_i^{1-\beta}}\right]^{\gamma\epsilon}}{\sum_r \sum_s X_r E_s \left[\frac{\upsilon_{rs}}{p_r^{\beta} q_r^{1-\beta}}\right]^{\gamma\epsilon}} \tag{12}$$

The term in the numerator depends on the fundamental attractiveness of $i$ and $j$ and the deterministic part of the indirect utility function in (10). The denominator captures multilateral resistance, i.e. the attractiveness of the alternatives of living and working in $i$ and $i$. The summation of $\lambda_{ij}$ over potential workplaces gives the probability of living in $i$ (irrespective of the workplace). Consequently, $\lambda_{ij}$ divided by the probability of living in $i$ gives workplace choice probabilities conditional on residing in $i$.

$$\lambda_{ij|i} = \frac{E_j\left(\upsilon_{ij}\right)^{\gamma\epsilon}}{\sum_s E_s\left(\upsilon_{is}\right)^{\gamma\epsilon}} \tag{13}$$

A key consequence of endogenous labour supply supply is that employment and population measures are not equivalent. Let the total population be denoted by $N$. With this, choice probabilities $\lambda_{ij}$ imply that the residential and workplace populations are

$$\begin{aligned} N_i^R &= N \sum_j \lambda_{ij}; \\ N_j^W &= N \sum_i \lambda_{ij}, \end{aligned} \tag{14}$$

while aggregate labour supply by residence and workplace locations, respectively, are

$$\begin{aligned} M_i^R &= N \sum_j \lambda_{ij} x_{ij}; \\ M_j^W &= N \sum_i \lambda_{ij} x_{ij} \end{aligned} \tag{15}$$

[5]See the derivation of the choice probability expression for a multiplicative indirect utility function and a Fréchet-distributed preference shock in the early contributions of the QSM literature, e.g. Appendix Section S.2.3 of Ahlfeldt et al. (2015), or Appendix Section C2 of Heblich et al. (2020).

Mean individual labour supply in workplace location $j$ is therefore

$$\bar{x}_j = \frac{1}{N_j^W} \sum_i N_i^R x_{ij} = \frac{M_j^W}{N_j^W}. \tag{16}$$

We will return to these features of the model as part of the process of model quantification.

### 3.3 Spatial general equilibrium

To model the production side of the economy we follow the conventional approach of quantitative urban modelling more closely. Production in location $j$ is governed by a Cobb-Douglas function of total labour input $M_j^W$ and commercial floorspace $H_j^W$, with expenditure shares $\alpha$ and $1-\alpha$.

$$Y_j = A_j (M_j^W)^{\alpha} (H_j^W)^{1-\alpha} \tag{17}$$

We assume perfect competition in the goods market combined with zero trade cost within the urban area, which allows us to normalise the unit price of the consumption good to $p_i = 1\,\forall i$. Thus, solving the firm's cost minimisation problem and setting marginal cost equal to unity yields the following factor demand functions.

$$M_j^W = \left(\frac{\alpha A_j}{w_j}\right)^{\frac{1}{1-\alpha}}; \quad H_j^W = \left[\frac{(1-\alpha)A_j}{Q_j}\right]^{1-\alpha} \tag{18}$$

Furthermore, free entry implies zero profits, so that total revenue minus total expenditure on wages and floorspace equals

$$Y_j - w_j M_j^W - Q_j H_j^W = 0. \tag{19}$$

After substituting the production function and (18), straightforward algebraic rearrangement implies that the profit maximising floorspace price is

$$Q_j = (1-\alpha) A_j^{1/(1-\alpha)} \left(\frac{\alpha}{w_j}\right)^{\frac{\alpha}{1-\alpha}}, \tag{20}$$

and the corresponding wage equation is

$$w_j = \alpha A_j^{1/\alpha} \left(\frac{1-\alpha}{Q_j}\right)^{\frac{1-\alpha}{\alpha}}. \tag{21}$$

The general equilibrium setup considers a third group of agents too: the construction sector. Let us define floorspace production $H_i$ in location $i$ as a function of capital input $Z_i$ and land endowment $L_i$:

$$H_i = Z_i^{1-\psi} \left(\phi_i(H_i) L_i\right)^{\psi}, \tag{22}$$

where

$$\phi_i(H_i) = 1 - (H_i/\bar{H}_i). \tag{23}$$

With this specification we follow Delventhal and Parkhomenko (2023) who introduce $\phi_i(H_i)$ to capture that local floorspace supply is often constrained by zoning regulations and geographical characteristics, for example. In the specification above, $\bar{H}_i$ is a theoretical floorspace capacity of each location. When $H_i$ approaches this value, the floorspace requirement of production becomes prohibitively high, so that the development in the in that location cannot grow further.

In each location, total floorspace supply is split into residential and commercial uses, $H_i = H_i^R + H_i^W$. The corresponding unit prices may not be identical, $q_i \neq Q_i$, due to differences in tax policies, for instance. We allow such policies to differ between locations, but we assume that the ratio $\xi_i = Q_i/q_i$ remains constant over time. Construction firms take the average floorspace price

$$\bar{q}_i = q_i \left( \frac{H_i^R}{H_i} + \xi_i \frac{H_i^W}{H_i} \right) \tag{24}$$

as given. Similar to the production sector, we assume profit maximisation and perfect competition in the construction sector. After solving the firm's problem, equilibrium floorspace supply becomes

$$H_i = \frac{[(1-\psi)\bar{q}]^{(1-\psi)/\psi} \cdot L_i}{1 + [(1-\psi)\bar{q}]^{(1-\psi)/\psi} \cdot L_i/\bar{H}}. \tag{25}$$

This terminates our description of the general equilibrium model's three components: household behaviour, goods production and floorspace construction. Given a set of exogenous parameters, the market clearing conditions above determine the values of endogenous variables $\lambda_{ij}$, $N_i^R$, $N_j^W$, $M_i^R$, $M_j^W$, $q_i$, $Q_j$, $w_j$ and $H_i$ in spatial general equilibrium.

# 4 Data and model estimation

The structure of the theoretical model enables the quantification of two types of exogenous parameters. This model features five vectors of location-specific parameters that capture geographical or local regulatory characterisitcs: residential and workplace amenities $X_i$ and $E_j$, local productivity $A_j$, and two vectors capturing local land and real estate regulation, $\xi_i$ and $\bar{H}_i$. Second, the model has a set of spatially undifferentiated structural parameters: $\epsilon$, $\alpha$, $\beta$, $\gamma$, $\psi$, $\bar{L}$, $T$, and two parameters relating to agglomeration economies to be introduced soon. We discuss the estimation/calibration of these two sets of parameters in this section.

The current numerical implementation of the model covers the 983 spatial units identified as middle layer super output areas (MSOA) in Greater London. The paper relies on publicly available data sources only; with access to the Secure Research Service of the UK Office of National Statistics (ONS), the model could be implemented at a higher spatial granularity as well, including the LSOA or even the OA statistical levels.[6] With the current spatial resolution, the model distinguishes $983^2 = 966{,}289$ potential residence–workplace combinations in location choice. Our data sources are enlisted below.

(i) Commuting matrix: ONS 2011 Census. The decennial Census of the UK includes large-scale datasets on the residential and workplace locations of the population. The publicly available version of this dataset is aggregated to the MSOA level. Given that the 2021 Census has been uncoventional in many ways due to the Covid-19 pandemic, the remaining data sources in this paper were collected to complement the 2011 Census.

(ii) Residential and workplace populations: ONS 2011 Census. In principle, the row and column sums of the commuting matrix correspond to the residential and workplace populations of employed individuals for each spatial unit. The total residential and workplace populations, including both active and inactive individuals, are also available online. While we will not use this data in the model implementation, comparing the total population distribution with the commuting matrix offers additional descriptive insights for the analysis.

(iii) Wages: Annual Survey of Hours and Earnings, Table 8. We use the net daily income column of this publicly available dataset, provided at an MSOA level for each year, including 2011, as an approximation of local wages.

(iv) Floorspace prices at an LSOA level (aggregated into MSOAs): Our baseline data source for floorspace prices is the open-source dataset of Ahlfeldt et al. (2023). They match the real estate sales data of the Land Registry with Energy Performance Certificate data, and estimate the floorspace price index using a mix of parametric and non-parametric estimation techniques. Note that this dataset does not include a breakdown of floorspace prices into commercial and residential properties. Therefore, we use two further data sources described as items (v) and (vi) to infer the wedge between commercial and residential floorspace prices ($\xi_i$).

(v) Commercial floorspace prices at Local Authority District level: UK Valuation Office Agency administrative data. This dataset as well as item (vi) are only available at a borough level in London; we apply inverse distance weighting to interpolate this data to the MSOA level.

(vi) Residential floorspace prices at Local Authority District level: Tenant Services Authority RSR.

(vii) Transport infrastructure networks: OpenStreetMap (OSM). We use the entire road and pedestrian path network of Greater London to infer walk times to the nearest public

[6] See a detailed description of statistical geographies on the ONS website: www.ons.gov.uk.

transport stop – see a detailed description of the derivation of travel time matrices below. For visualisation tasks, we extract the shape files of the Elizabeth Line as well.

(viii) Bus network and timetables: Bus Open Data, General Transit Feed Specification (GTFS): Transport for London.

(ix) Railway and metro network and timetables, GTFS: Rail Delivery Group. This dataset contains information on the service provider of each rail and metro line, which enables us to remove certain lines (i.e., the Elizabeth Line) in counterfactual scenarios using the *gtfstools* package in R.

(x) Historic population densities from the online data appendix of Heblich et al. (2020).

The numerical implementation of the model is preceeded by a series of data processing steps. The travel times and monetary travel costs between MSOA centroids are computed via the recently published April 2024 version of the routing package called *r5r*; see Pereira et al. (2021). This library combines infrastructure shapefiles from OSM with public transport timetables stored in GTFS format. The routing algorithm considers walking distance to the nearest stop/station, in-vehicle travel times according to the timetable, and transfer times, to produce a realistic estimate of the travel time and monetary cost matrix along the shortest path. This approach also allowed us to compute both matrices with and without the Elizabeth Line, providing the main input to the counterfactual simulations in Section 5.2.

When quantifying the model, we exploit the recursive structure of the quantitative spatial model and proceed through the following steps: first, we estimate the gravity equation implied by the location choice model, which yields an estimate of $\epsilon$, the Fréchet shape parameter. Second, we use the estimated $\hat{\epsilon}$ to invert the model and recover three sets of location fundamentals: amenities $X_i$ and $E_j$, and the productivity vector $A_j$. Model inversion is a deterministic process: we apply the equilibrium conditions of the model to express one location-specific vector in function of observed and previously identified parameters. In other words, the model structure enables us to establish a one-to-one relationship between the observed data and the unknown vectors of location fundamentals, and quantify the latter as structural residuals of the model. Third, based on the resulting vector of local productivities, we estimate the elasticity of productivity with respect to agglomeration ($\eta$) and the associated distance decay ($\delta$). Fourth, we perform two more deterministic steps to recover the exogenous part of the local productivities and another local fundamental which captures floorspace construction limits due to regulation. Standard errors are calculated using a bootstrap procedure with slightly more than 200 replications of the estimation sequence.

The remaining structural parameters are not separately estimated at the current phase of this research project. We borrow the following values from the literature: $\alpha = 0.8$ from Valentinyi and Herrendorf (2008); $\beta = 0.75$ from Davis and Ortalo-Magné (2011); $\psi = 0.25$ from Combes et al. (2019); we set $\bar{L} = 24$ hours and $T = 8$ hours by intuition.

## 4.1 Commuting gravity

Model quantification begins by estimating $\epsilon$, the spread of the Fréchet-distributed idiosyncratic shock in household's utility function. Following Ahlfeldt et al. (2015) and Heblich et al. (2020), we express the location choice probability equation in (12) in a reduced form as

$$\log N_{ij} = \alpha_0 + \vartheta_i + \vartheta_j + \nu \cdot \log \upsilon_{ij} + \varepsilon_{ij}, \tag{26}$$

In the equation above, $N_{ij}$ is the number of observed travellers in the commuting matrix, $\alpha_0$ is the intercept capturing the denominator (multilateral resistance) in gravity equation (12), $\vartheta_i$ is a residence (origin) fixed effect, $\vartheta_j$ is a workplace (destination) fixed effect, $\nu = \gamma\epsilon$ becomes the coefficient of bilateral resistance $\ln \upsilon_{ij}$, and $\varepsilon_{ij}$ is the error term. The main parameter of interest, $\nu$, is transferable between this estimating equation and the location choice probability function in equation (12).

At the same time, the functional form of (26) is common in the wider empirical literature of gravity estimation in the international trade literature (Head and Mayer, 2014). This enables us to apply one of the most robust estimators in that literature, the Poisson Pseudo-Maximum Likelihood (PPML) method of Santos Silva and Tenreyro (2006). The use of the PPML estimator is motivated by three concerns. First, $N_{ij}$ on the left-hand-side of (26) includes many zeros in the commuting matrix, as the commuting flow is effectively zero between two-thirds of MSOA-pairs in our data. In an OLS log-log model, these observations must be removed, which implies a substantial loss of information. The second concern stems from Jensen's inequality: under heteroskedasticity, the parameters of a log-linearised model estimated by OLS lead to biased estimates, because $E[\log y] \neq \log E[y]$. Third, the identification of $\nu$ in (26) is threatened by endogeneity concerns due to the non-random placement of infrastructure as well as reverse causality due to congestion, which implies that $t_{ij}$ is potentially endogenous. Thus, we instrument the observed travel times by the Euclidean distance between $i$ and $j$, which is independent from infrastructure investment decisions as well as traffic congestion levels.

**Table 1:** Estimating commuting gravity

| | Dependent variable: log commuting flow | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Impedance | real wage | real wage | real wage | real wage | travel time | gen.cost |
| Notation | $\upsilon_{ij}$ | $\upsilon_{ij}$ | $\upsilon_{ij}$ | $\upsilon_{ij}$ | $t_{ij}$ | $\bar{v}t_{ij} + \tau_{ij}$ |
| Method | OLS$^{\dagger}$ | OLS$^{\dagger}$ | PPML | PPML+IV | PPML | PPML |
| Instrument | | | | Eucl.dist | | |
| Estimate | 2.959*** | 12.303*** | 19.035*** | 10.193*** | -0.893*** | -0.855*** |
| | (0.011) | (0.021) | (0.060) | (0.028) | (0.01) | (0.009) |
| Fixed effects | no | yes | yes | yes | yes | yes |
| RMSE | 1.078 | 0.657 | 8.815 | 8.684 | 13.948 | 13.686 |
| AIC | 1,052,942 | 708,040 | 2,767,492 | 2,836,697 | 5,290,953 | 5,524,191 |
| BIC | 1,052,974 | 729,218 | 2,790,654 | 2,859,856 | 5,314,115 | 5,547,353 |
| # of obs. | 352,300 | 352,300 | 966,289 | 965,306 | 966,122 | 966,122 |

$^{\dagger}$: Only origin-destination pairs with positive flows are included.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

The results for six model specifications are summarised in Table 2. Models (1) and (2) are OLS estimates of the model, with and without origin and destination fixed effects. These models rely on a restricted sample because OLS cannot handle zero flows after the log transformation. It is remarkable though that the elasticity estimate in model (2) is close to our most preferred one in (4). Models (3) and (4) are Poisson models with fixed effects. A common endogeneity concern is that impedence between $i$ and $j$ may not be independent from the flows themselves. One potential reason may be the non-random placement of intrastructure. To address this concern, we instrument $\upsilon_{ij}$ by the Euclidian distance in model (4), which gives our preferred estimate.

Assuming that commuters have a fixed travel time budget of 1 hour on average (Kung et al., 2014), such that $\gamma = (T+1)/24$, $\epsilon = \nu/\gamma$ is computed from our preferred empirical estimate of $\hat{\nu} = 10.193$. Note that all four elasticities and the resulting $\hat{\epsilon}$ values are higher in models (2) to (4) than the single-digit estimates in other QSM studies in the literature. In models (5) and (6) we estimate the gravity equation using travel times and generalised travel costs as impedance measures. This exercise confirms that we get lower values when the regular impedance measure is used. The higher Fréchet shape parameter in the main models can be interpreted as less randomness in people's location choice decisions. We attribute this outcome to the fact that $\upsilon_{ij}$ is a more comprehensive measure of the trip generating forces in the commuting context – that is, it has a higher explanatory power in commuting decisions than the pure travel time.

### 4.2 Amenity residuals

One of the most decisive properties of QSMs that distinguish them from previous spatial general equilibrium models is that they are recursive. This means that one can recover some of the fundamental parameters vectors of the model from observed data, assuming that the model's assumptions are correct and the observed state of the economy is an equilibrium of the model. In other words, there is a one-to-one mapping between observed endogenous variables and unobserved parameters. This analytical property is extremely helpful because quantifying some of the fundamental geographical characteristics of locations, such as amenity levels and the fundamentals of firm productivity, are otherwise extremely difficult to quantify with traditional empirical methods. In the forthcoming discussion we show how observed spatial data is utilised to quantify five exogenous variables specific to each location.

Based on the definitions and location choice probabilities defined above, the link between workplace and residential populations can be expressed as

$$N_j^W = \sum_i \lambda_{ij|i} \cdot N_i^R. \tag{27}$$

We can express fundamental amenity levels $E_j$ after substituting (13) into (27):

$$E_j = N_j^W \left( \sum_i \frac{\upsilon_{ij}^{\gamma\epsilon} N_i^R}{\sum_s E_s \upsilon_{is}^{\gamma\epsilon}} \right)^{-1} \tag{28}$$

This implies an equation for each location in function of all other $E_j$'s that we can solve for iteratively to recover the unique vector of workplace amenity levels from the observed distribution of residential and workplace populations and the observed determinants of $\upsilon_{ij}$.

Residential amenities are recoverable in a similar fashion. Let us introduce

$$\tilde{X}_i = X_i \left( q_i^{1-\beta} \right)^{-\gamma\epsilon}, \tag{29}$$

which captures all location-dependent endogenous variables of location choice probability (12). The values of $\tilde{X}_i$ are the solution of

$$\tilde{X}_i = N_i^R \left( \sum_j \frac{\upsilon_{ij}^{\gamma\epsilon} N_j^W}{\sum_r \tilde{X}_r \upsilon_{rj}^{\gamma\epsilon}} \right)^{-1}. \tag{30}$$

From the solutions, $X_i$ is quantified using data on residential floorspace prices $q_i$ and by inverting (29) above.

A visual representation of the residuals in Figure 1 reveal interesting patterns. The results show that the locations in Central London – with the exception of Westminster and the nearby MSOAs – are not particularly pleasant places to work, controlling for the very high

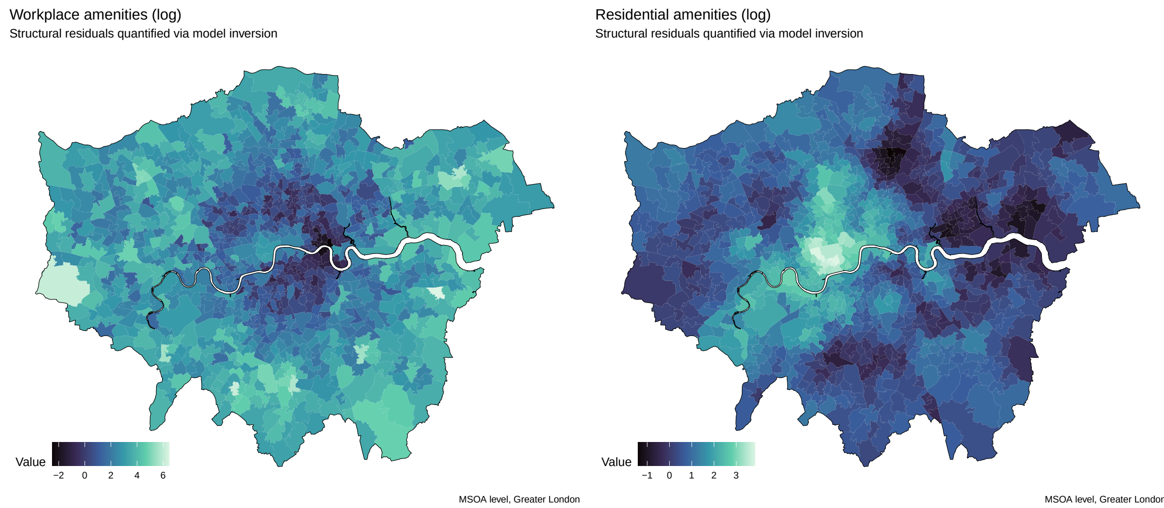


**Figure 1:** Workplace and residential amenities recovered as structural residuals of the model, assuming that the observed data captures a spatial equilibrium

wages offered by these places. An inner ring surrounding the City of London and Canary Wharf has particularly low amenities for working. By contrast, some of the locations in the suburbs are more attractive for working than what their wage levels would justify. Lower density and congestion externalities (i.e. noise, pollution) may explain some of these results. By contrast, the most appealing residential locations are clustered around the central areas of Finchley Road and Swiss Cottage, Kensington and Chelsea, and the low-density residential neighbourhoods of Richmond.

### 4.3 Agglomeration externalities

Local productivity $A_j$, or total factor productivity in production function (19), is measured by rearranging the wage equation in (21):

$$A_j = \left(\frac{w_j}{\alpha}\right)^{\alpha} \left(\frac{1-\alpha}{Q_j}\right)^{\alpha-1}. \tag{31}$$

As we observe both wages ($w_j$) and commercial floorspace prices ($Q_j$), the $A_j$ vector is computed by directly substituting the observed data into (31).

Following the agglomeration literature (Combes et al., 2019, Ahlfeldt et al., 2015) we decompose local firm productivity into an exogenous component and a multiplier dependent on a measure of economic density, i.e. access to economic mass (ATEM), denoted by $\rho_j$.

$$A_j = a_j\, \rho_j^{\eta} \tag{32}$$

Parameter $\eta$ is the agglomeration elasticity of firm productivity. In terms of the specification of ATEM, the literature offers two alternatives. When the spatial units of the model are large enough to assume that only the density of economic activity *within* zone $j$ affects productivity in $j$, it is reasonable to consider

$$\rho_j = \frac{M_j}{\Lambda_j}, \tag{33}$$

where $M_j$ is a measure of economic activity, say employment, and $\Lambda_j$ is the geographical area which may vary between $j$'s. By contrast, when the spatial units are relatively small, it is more likely that productivity spillovers occur between nearby locations. The common specification is then

$$\rho_j = \sum_s \exp(\delta t_{sj})\, M_s, \tag{34}$$

where parameter $\delta < 0$ measures the rate by which agglomeration economies decay over travel time $t_{ij}$. In this paper we test both alternatives. In addition, we select the effective labour supply as the measure of economic activity, to consider that individual labour supply is endogenous in our model, and therefore employment (i.e., workplace population) alone is not a comprehensive measure of economic mass.

$$A_j = a_j \left[ \sum_s \exp(\delta t_{sj})\, N_s^W \bar{x}_s \right]^\eta \tag{35}$$

The general form of our estimating equation based on (32) is

$$\log \hat{A}_j = \eta \log \rho_j(\delta) + \vartheta_{j \in z} + \varepsilon_j, \tag{36}$$

in which $\hat{A}_j$ are the productivity residuals recovered via model inversion in the previous step, $\vartheta_{j \in z}$ are borough fixed effects, and $\varepsilon_j$ is the error term. The core endogeneity concerns well-known in the literature are that (i) ATEM may be correlated with unobserved local characteristics, e.g. natural advantages/endowments, that also affect productivity, and (ii) through the endogenous location choice of firms and competition forces, productivity may also affect the magnitude of agglomeration, fueling reverse causality (Combes et al., 2019, Graham and Gibbons, 2019). To identify the causal effect of agglomeration on productivity, we deploy instrumental variables and control function techniques in three alternative specifications.

The models reported in Table 2 differ in the specification of $\rho_j$, the inclusion of fixed effects, and the estimation method determined by the identification strategy. In models (1) and (2), the general functional form in (32) remains unchanged but we ignore agglomeration spillovers between locations, so $\rho_j$ does not depend on the spatial impedance between MSOAs. More specifically, $\rho_j$ is the density of employment in line with (33). Model (1) is a crude OLS estimate. In model (2) we instrument the log of the employment density by a set of historic and geographical variables. Specifically, we use the log population densities of 99 boroughs in Greater London in 1861 and 1921,[7] which is a common practice in the literature. We also include a third-order polynomial of the logarithm of the distance from the City of London, the central business district (CBD) of London. The underlying assumption is that distance from the CBD is correlated with density, particularly in monocentric cities, due to the fundamental trade-off between commuting costs and competition for land. However, this measure of relative geographical location is assumed not to affect firm productivity through mechanisms other than economic density.

Models (3) and (4) have been estimated in a two-step process which captures the decay in spillover effects between nearby MSOAs, following Koster (2024). In the first step we create 2.5-minute-wide concentric doughnuts around location $j$ denoted by $\mathcal{R}_{jr}$, aggregate the effective labour supply in each ring into and estimate the contribution of each distance ring to the measure of ATEM in $j$ via the parameter vector $\{d_r\}$. Thus, the estimating equation in (36) is specified as

$$\log \hat{A}_j = \eta \log \sum_r \left( d_r \sum_{k \in \mathcal{R}_{jr}} N_k^W \bar{x}_k \right) + \vartheta_{j \in z} + \varepsilon_j. \tag{37}$$

As $\log \hat{A}_j$ depends on $\eta$ and the ring-specific parameters non-linearly, we estimate the model with nonlinear least squares (NLS). Then, in the second step we fit a curve on the coefficient estimates to quantify $\delta$ as the parameter of $\tau_r$, the mean travel time between locations in ring $r$ and location $j$:

$$\log \hat{d}_r = \delta \tau_r + \varepsilon_r. \tag{38}$$

This non-linear specification is no longer suitable to apply instrumental variables, as one would need to instrument a function of a set of unknown parameters. Thus, we follow Koster (2024) again and apply a control function approach. As an initial step, we estimate the nonlinear model in (37) with NLS, ignoring that the resulting $\hat{d}_r$ estimates are likely biased. We use these estimates to compute $\hat{\rho}_j$, the ATEM predicted by the model. Then, in a first-stage regression we regress this outcome using exogenous historical ATEM measures and distance from the CBD. In model (3) of Table 2, the control function is

$$\log \hat{\rho}_j = \sum_{y \in \mathcal{Y}} \log \rho_j^y + f(\text{dist}_j) + \varrho_j, \tag{39}$$

where $\mathcal{Y}$ includes the years 1841, 1861, 1881, 1901 and 1921, $\rho_j^y$ is the population density of location $j$ in year $y$, $f(\text{dist}_j)$ is the third-order polynomial of the distance from the central business district, and $\varrho_j$ is the error term. Model (4) differs from model (3) in the assumed

[7] Our data source, with the corresponding author's permission, is the online appendix of Heblich et al. (2020) who used original data provided by the Cambridge Group for the History of Population and Social Structure (Wrigley, 2011).

control function: we use 30-minute travel time doughnuts and a nonlinear specification similar to the second-stage regression as an instrument of today's ATEM.

$$\log \hat{\rho}_j = \eta^{1861} \log \sum_{r=1}^{3} \left( d_r^{1861} \sum_{k \in \mathcal{R}_{jr}^{1861}} \text{pop}_k^{1861} \right) + \eta^{1921} \log \sum_{r=1}^{3} \left( d_r^{1921} \sum_{k \in \mathcal{R}_{jr}^{1921}} \text{pop}_k^{1921} \right) + \varrho_j, \tag{40}$$

We take the 1861 and 1921 population distributions and travel time matrices from Heblich et al. (2020) to estimate this model with nonlinear least squares.

The residual vectors of (39) and (40), which we denote $\hat{\varrho}_j$, is correlated with the dependent variable and, thus, also correlated with $\varepsilon_j$ in the main regression, because ATEM is endogenous. Thus, when we recover the $\hat{\varrho}_j$ vector and insert it into the second-stage regression

$$\log \hat{A}_j = \eta \log \sum_{r} \left( d_r \sum_{k \in \mathcal{R}_{jr}} N_k^W \bar{x}_k \right) + \vartheta_{j \in z} + f(\hat{\varrho}_j) + \tilde{\varepsilon}_j \tag{41}$$

the original error term is decomposed into $\varepsilon_j = f(\hat{\varrho}_j) + \tilde{\varepsilon}_j$. As $f(\hat{\varrho}_j) = E[\varepsilon_j | \varrho_j]$ is the conditional mean of $\varepsilon_j$, from which $\tilde{\varepsilon}_j$ is independent by construction, we ensure that the remaining error $\tilde{\varepsilon}_j$ is no longer correlated with the endogenous ATEM measure. In other words, we correct for endogeneity and the resulting $\eta$ and $d_r$ estimates are unbiased.[8] In practice, we use the third-order polynomial of $\hat{\varrho}_j$ as $f(\hat{\varrho}_j)$.

Note that the estimation of the agglomeration parameters is part of the sequence of empirical work stages detailed in Section 4. There is inherent uncertainty in the data used as input for the present exercise. Consequently, the standard errors produced by the OLS, 2SLS, and NLS estimation algorithms are not informative. The standard errors reported in Table 2 are computed by bootstrapping the entire model inversion and parameter estimation process slightly more than 200 times.

Let us turn to the empirical results in Table 2. The naïve model in column (1) yields an agglomeration elasticity of 12.5%. The 2SLS regression with employment density as the ATEM measure produces a higher result. The two control function specifications lead to estimates in between the OLS and 2SLS results. Our preferred model is (4), with an elasticity

[8] Recall though that the predicted $\hat{\rho}_j$ values in (39) and (40) are based on biased $\hat{d}_r$ values. Thus, we create an iterative process and use the dummy parameters estimated in the second-stage regression in (41) to recalcuate $r\hat{h}o_j$, and repeat the first- and second-stage regressions until convergence is reached. In practice, with a three-digit tolerance, convergence has been reached after just the second iteration. Our understanding is that Koster (2024) used a somewhat different iterative algorithm. He predicted $\hat{\rho}_j$ using an assumed distance decay $\delta$ and the $\rho_j(\delta)$ definition in (34). Then he repeated the two-stage control function regression until he found, numerically, the $\hat{\delta}$ that minimised the root mean squared error of the second stage. Our experience is that this approach is significantly costlier in terms of computation time.

**Table 2:** Agglomeration economies and distance decay

| | Dependent variable: log productivity residual | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| ATEM measure | Emp.density | Emp.density | Total emp.† | Total emp.† |
| Method | OLS | 2SLS | NLS+CF | NLS+CF |
| Productivity elasticity, $\eta$ | 0.125*** | 0.170*** | 0.155*** | 0.149*** |
| | (0.005) | (0.016) | (0.009) | (0.011) |
| Distance decay, $\delta$ | | | -0.082*** | -0.079*** |
| | | | (0.012) | (0.012) |
| Borough fixed effects | no | yes | yes | yes |
| RMSE | 0.08 | 0.06 | 0.06 | 0.06 |
| AIC | -2122.05 | -2573.61 | -2665.81 | -2610.97 |
| BIC | -1950.88 | -2402.44 | -2372.37 | -2317.53 |
| # of obs. | 983 | 983 | 983 | 983 |

†: Total employment is aggregated in 2.5-min travel time bands.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

of $\hat{\eta} = 14.9\%$ and a distance decay of $\hat{\delta} = -0.079$, which implies that spillovers fade quickly after 15 to 20 minutes of travel time. Note that some of the estimated $\hat{d}_r$ coefficients in (41) may not be statistically significantly different from zero. Figure C.1 in the Appendix plots these estimates together with the resulting distance decay curve for model (4); $\delta$ has been estimated using only the $\hat{d}_r$'s that we found statistically significant at the 95% confidence level.

The estimated agglomeration elasticity $\hat{\eta} = 14.9\%$ is at the higher end of the values found in the previous literature (see Graham and Gibbons, 2019). However, our result does not stand out from previous studies focusing on Greater London, specifically. Dericks and Koster (2021) developed a QSM using the same case study context. Exploiting the exogenous variation in density caused by bombings during WW2, the agglomeration elasticity they estimate is somewhat higher than ours, 19.6%. As a sensitivity check, we re-estimate model (3) using MSOAs under and above the median distance from the CBD; see Table C.1 in the Appendix. We find that the elasticity goes up to 19.1% for the central subsample while it reduces to 6.3% for the more perpheral half of London. As we move away from the centre of the city, this clearly non-linear pattern seems to converge close to the UK-wide average of 4.3% recommended by the UK Transport Appraisal Guidance and the 4.7% mean of the global literature reported by Graham and Gibbons (2019).

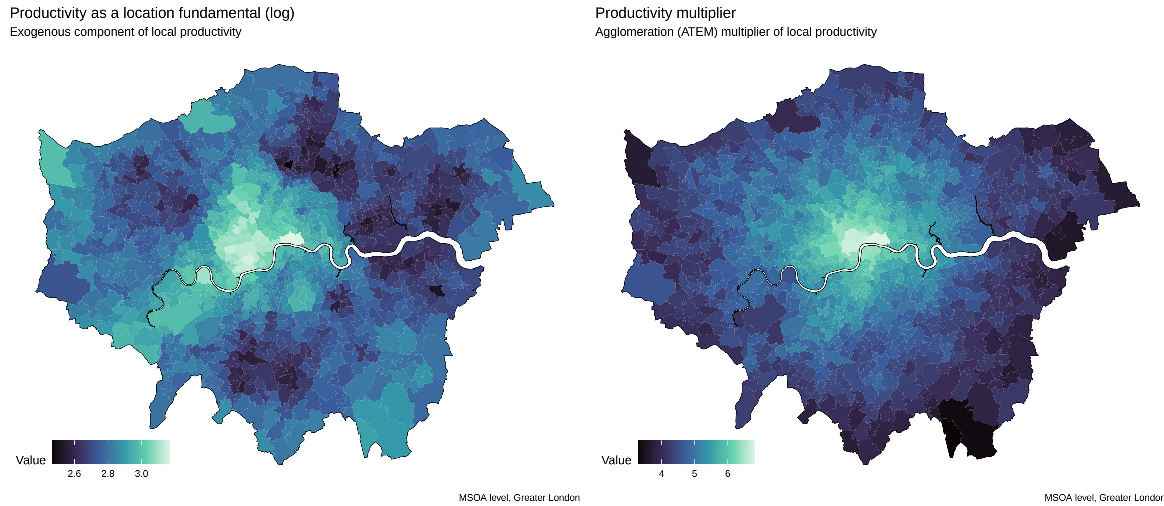


**Figure 2:** Productivity as a location fundamental and the multiplier generated by agglomeration

### 4.4 Productivity fundamentals and density limits

Given the above estimated $\hat{\eta}$ and $\hat{\delta}$ parameters, the observed travel time matrix, and the quantified values of $A_j$, we recover the $\{\hat{a}_j\}$ vector from (35).

$$\hat{a}_j = \hat{A}_j \left[ \sum_s \exp\left(\hat{\delta} t_{sj}\right) N_s^W \bar{x}_s \right]^{1/\hat{\eta}} \tag{42}$$

The $\{\hat{a}_j\}$ vector captures fundamental geographical properties that make firms more productive in certain locations, controlling for the level of access to economic mass.

Figure 2 provides a visual illustration of the decomposition outlined in equation (32). The patterns suggest that, when it comes to the first-nature geographical determinants of productivity, the East and South of Greater London are particularly disadvantaged, and the triangle between the City of London, Camden and Kensington is the most well endowed part of the city. The right hand side of the figure reveals that economic density makes a significant contribution to productivity. Its multiplier effect ranges between 2 and 3, depending on how well connected a location is. It is of course not surprising that places with the highest *exogenous* productivity are also the centrally located ones: the theory of urban spatial structure confirms that cities normally grow around the most productive locations.

Finally, $\xi_i = Q_i/q_i$ is calculated as the ratio of observed commercial and residential floorspace prices while the local density limit, $\bar{H}_i$, is recovered by inverting (25) according to the following equation.

$$\bar{H}_i = \frac{\Omega_i(\bar{q}_i, \xi_i, L_i)}{\Omega_i(\bar{q}_i, \xi_i, L_i) \cdot (H_i^R + H_i^W)^{-1} - 1}, \quad \text{where} \quad \Omega_i = [(1-\psi)\bar{q}_i]^{\frac{1-\psi}{\psi}} L_i. \tag{43}$$

We assume that the local fundamentals remain constant, including the simulation of counterfactual scenarios.

# 5 Policy appraisal

In this section we apply the fully quantified model to perform an illustrative policy appraisal experiment. The infrastructure policy we selected as a case study is the implementation of the Elizabeth Line, a large-scale urban rail investment programme that attracted a lot of attention in the academic literature as well. The counterfactual application of the model assumes that the Elizabeth Line is introduced in the baseline 2011 spatial equilibrium of Greater London; we let the model to converge to a new spatial equilibrium, and assess the economic impact of the new intervention in a comparative statics exercise. Among the main outputs of the simulation, we present predictions on the spatial redistribution of the economic activity, including market outcomes such as the wage and floorspace price distribution (see Section 5.2), and an estimate of the aggregate welfare of the policy in Section 5.3.

Note that this paper does not aim to deliver a comprehensive cost-benefit analysis of the Elizabeth Line. As the concluding section of the paper discusses in more detail, our model does not include a series of important sources of welfare gains and losses, e.g. a potential reduction in congestion and pollution externalities. That said, our aggregate welfare estimates are not comparable with any of the ex-ante and ex-post appraisal reports of the Elizabeth Line. However, in Section 5.3, we do perform a comparative analysis to learn more about how the spatial equilibrium welfare effect relates to the results of a comparable partial equilibrium cost-benefit analysis. This methodology-oriented experiment represent a first step towards reconciling the QSM approach to transport CBA with the mainstream partial equilibrium approach.

## 5.1 The spatial distribution of time valuation

Before turning to policy simulations, let us visualise an outcome of the calibration process that we devote increased attention to. Figure 3 plots the spatial distribution of $v_{ij}$, our theoretical measure of the marginal value of time. Note that each origin–destination pair has a unique $v_{ij}$. In this plot we depict their mean by residential location. As one would anticipate, residential locations in central London have the highest mean value of time, given the concentration of high-wage individuals whose commuting distance is also likely relatively small. Overall, time valuations vary between 12 and 20 pounds per hour. The unweighted mean value of time among all OD-pairs is around 11.6 GBP/hour, while the flow- and distance-weighted means are 14.5 and 14.4 GBP/hour, respectively. These averages are broadly consistent with the empirical estimates of the value of time from the early 2010s.

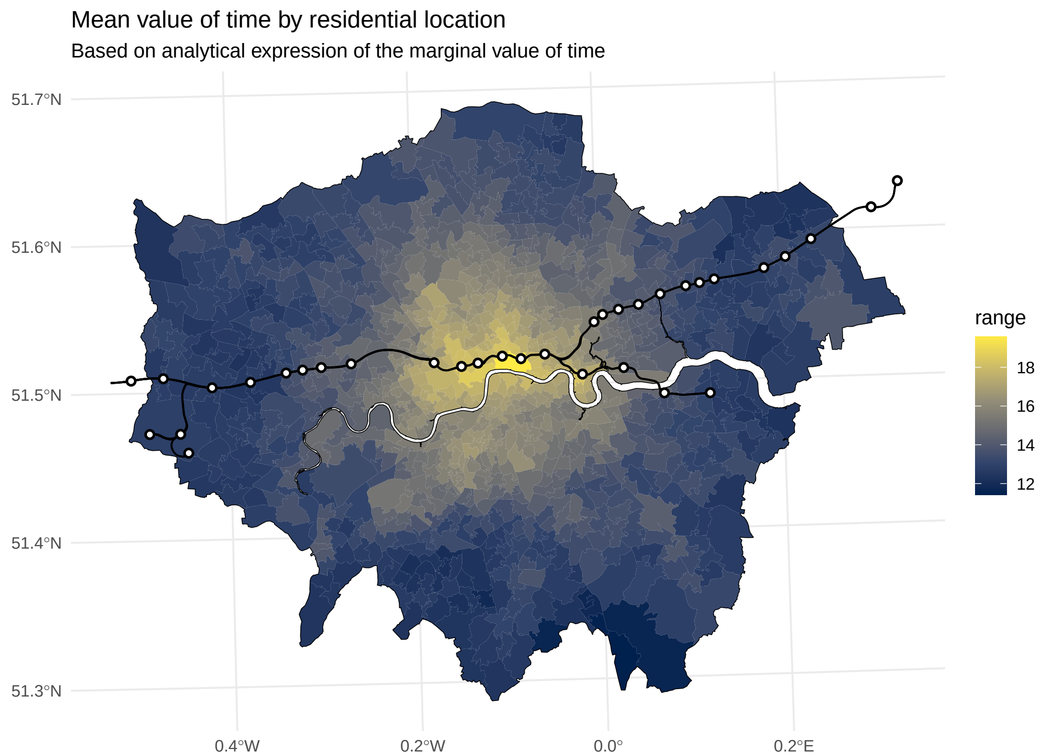


**Figure 3:** Spatial distribution of mean travel time valuations, measured in 2011 GBP/hour and averaged by residential location.

This exercise offers a unique perspective on the spatial distribution of travel time valuations. Such insight is unprecedented, mainly because empirically measuring the value of time using traditional stated or revealed preference data at a comparable level of spatial disaggregation would require an excessively costly data collection effort. By contrast, our value of time distribution is derived from publicly available and easily accessible wage and commuting time/cost data. Of course, the limitation of this measure lies in its reliance on a set of theoretical assumptions without direct empirical validation. In addition, what we derive here is the pure opportunity cost of leisure time, which does not include other components of the value of travel time savings relating to comfort, for example. Nonetheless, we believe that this geographical disaggregation of travel time valuations can prove highly useful, as governments are increasingly eager to conduct transport appraisals with finer spatial granularity.

The fully calibrated QSM is now ready to perform counterfactual simulations; that is, to modify some of the exogenous parameters of the model to mimic a policy intervention and estimate its impact on urban form and general equilibrium outcomes. We perform two simulation exercises. First, counterfactual scenario of a large-scale infrastructure improvement that reduces travel times between a predetermined subset of origin-destination pairs. Then, in the second subsection we investigate the ranking of a series of randomly generated transport improvements to investigate whether welfare analysis in general equilibrium might lead to fundamentally different policy recommendations, in comparison with the most common partial equilibrium appraisal method.

### 5.2 A large-scale infrastructure intervention

In this exercise we assume the hypothetical scenario that the Elizabeth Line (also known as Crossrail) is introduced in the 2011 London economy, and the city transforms into a new spatial equilibrium. The descriptive plot in Figure 4 shows the impact of the policy on the distribution of the mean travel time from individual residential locations (left side) and to individual workplaces (right side). To make the spatial pattern of time savings more visually perceptible, we apply quantile-based colour breaks in these plots, as indicated by the colour spectra in the legend. In the most privileged residential location, the representative commuter gains 12.51 minutes in travel time. Naturally, the gain is the highest along the alignment of the new railway line, but some of the Northern MSOAs, from which residents may transfer to Crossrail, also gain a non-negligible amount. The mean travel time savings that workplaces experience, many of which are located at well-connected places already, vary on a tighter range.

In the counterfactual simulations we assume that the transport improvement is combined with a relaxation of the exogenous constraint on floorspace development in Central London. We increase the value of $\bar{H}$ by 2% within a 1.5 km radius of five stations: Paddington, Bond Street, Tottenham Court Road, Farringdon and Liverpool Street. This assumption is meant to capture the impact of explicit changes in local zoning regulations as well as a tendency to grant discretionary permission for real estate development more readily near Elizabeth Line stations. In Appendix D we test the sensitivity of our results with respect to this ad-hoc assumption.

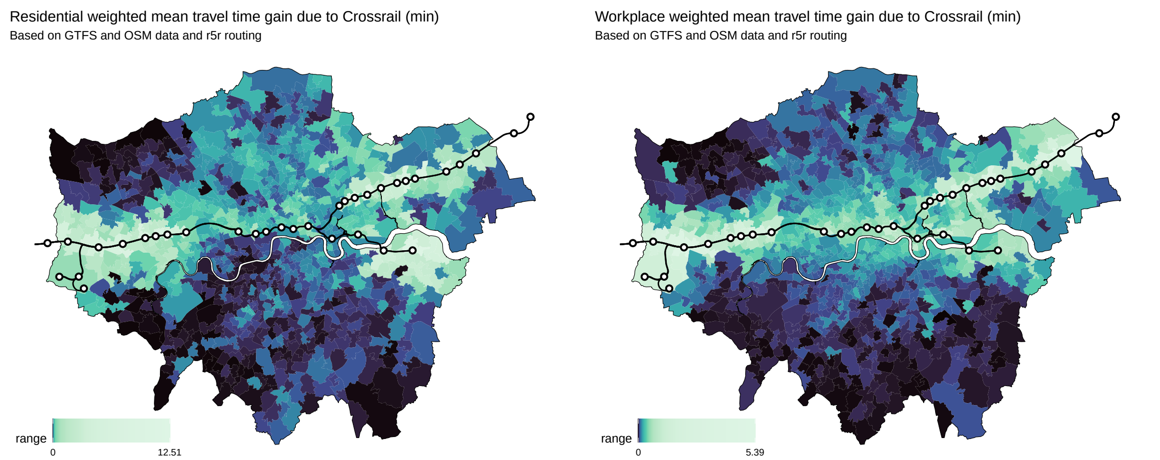


**Figure 4:** Policy simulation: mean travel time savings delivered by the Elizabeth Line for a representative resident (left) and worker (right)

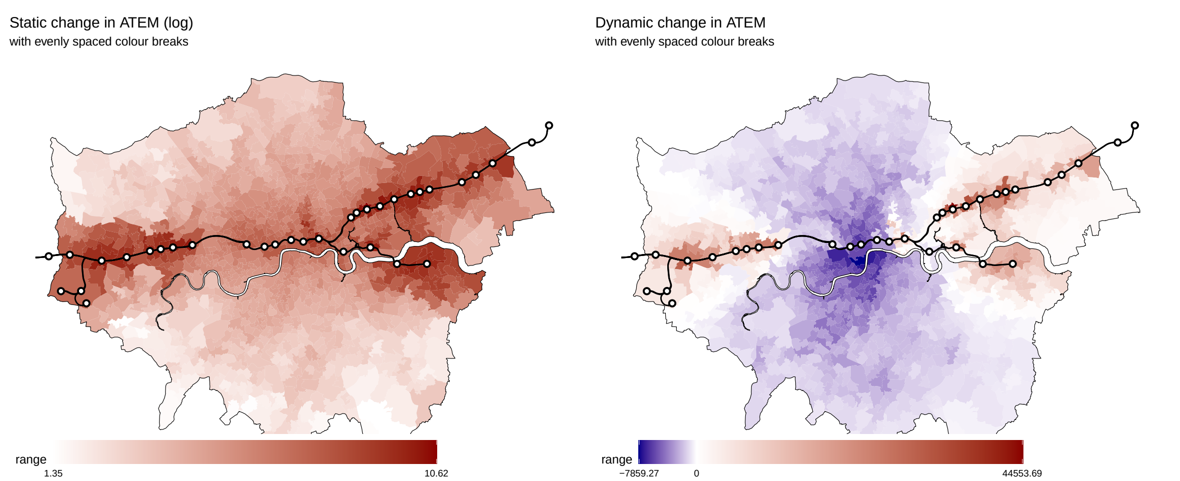


**Figure 5:** Counterfactual outcomes: static and dynamic changes in economic density (access to economic mass)

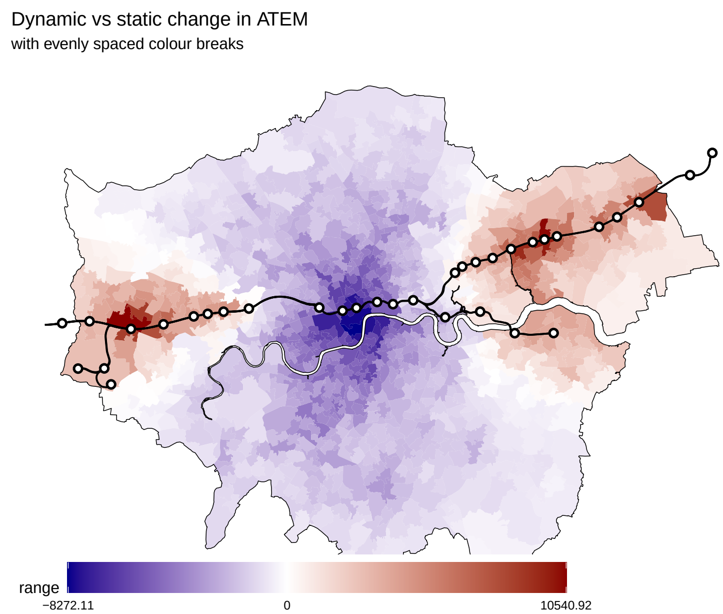


**Figure 6:** The impact of employment relocation on access to economic mass: the difference between the static and dynamic changes plotted in Figure 5

### 5.2.1 Economic density, firm productivity and wages

A key way in which transport improvements influence the urban economy is by increasing the effective economic density of the locations they connect. In other words, it reduces the impedance between firms (business-to-business connectivity) and firms and households (labour pooling), thereby fostering agglomeration economies. Based on our decomposition of local firm productivity in equation (35), let us show the transformation of effective density in two steps.

First, the left-hand side of Figure 5 shows the change in $A_j$ in response to the new travel time matrix $t_{sj}$ only, keeping the distribution of employment $N_s^W$ and individual labour supply $\bar{x}_s$ constant. Let us call this the *static* change in access to economic mass. As the travel time effect of the transport project is strictly negative,[9] the static change in ATEM is always positive. In line with intuition, this effect is the greatest near the new railway stations, especially in the suburban parts of the line which may have been more poorly connected before the intervention.

Second, the right-hand side of Figure 5 plots the dynamic change in access to economic mass. In this case we allow all endogenous variables to move to the new spatial equilibrium, including

[9] Note that in this model we ignore congestion. In the presence of congestion spillovers in the network, transport costs may we increase on certain origin–destination pairs.

workplace choice to readjust according to the new wage pattern. That is, we allow all variables in (35) to readjust. The primary lesson we learn from the plot is that the dynamic change in ATEM is indeed not necessarily positive. In fact, a significant part of the city, including the suburbs in the North and South as well as the Centre of London experience a reduction in economic density. This happens because the *relative* connectivity of these locations decreases in comparison with the areas near some of the Elizabeth Line stations that gain the most in relative terms. Note from the legend of this figure though that the positive changes in ATEM are much greater in magnitude than the negative changes. Also, the positive changes are highly skewed due to a few outliers that experience extremely high gains in ATEM.

Figure 6 illustrates the difference between the two sides of Figure 5, which reflects the impact of firm relocation (and the readjustment of individual labour supply) on access to economic mass. In other words, it highlights the consequences of shifting from a partial equilibrium model to a spatial general equilibrium framework. The negative and positive deviations are more even in this case. We infer from the resulting pattern that Central London's effective economic density might be overestimated in a partial equilibrium model while the ATEM gain of the suburban workplaces along the Elizabeth Line are underestimated when relocation is ignored in PE.

How do the changes in ATEM translate into effects on firm productivity? Figure 7 depicts the local changes in total factor productivity $A_j$. Note again that we use a quantile-based colour scheme which allows us to make extreme values and subtle differences between small changes visible at the same time. The distribution of the changes we plot here has a long tail in the positive direction. The most affected firms are concentrated at a small number of new stations along the Elizabeth Line: the neighbourhoods that gain the most are Hillingdon and Ealing in West London and Redbridge, Newham, Havering, Bexley, Greenwich and Barking and Dagenham in the East. The rest of the city experienced very little changes in firm productivity. Regarding the main business districts, the model predicts a negligible reduction in productivity in the City of London ($\Delta A_j = -0.056$) and a negligible improvement in Canary Wharf ($\Delta A_j = +0.08$). Note, however, that our urban model does not consider long-distance connectivity. As one of the main purposes of the the Elizabeth Line is to make the City of London and Canary Wharf better connected with Heathrow Airport in the West and City Airport in the East, this urban model may not provide a comprehensive assessment of firm productivity.

### 5.2.2 Relocation, land-use, floorspace prices and relocation

The consequences of the productivity boost in terms of wages and floorspace prices, plotted in Figure 8, are clear and intuitive. The wage rate grows along the entire length of the Elizabeth Line by up to £8 per day at the most privileged locations. By contrast, remote areas in the South and the North may even lose in terms of wage due to the impact of the agglomeration

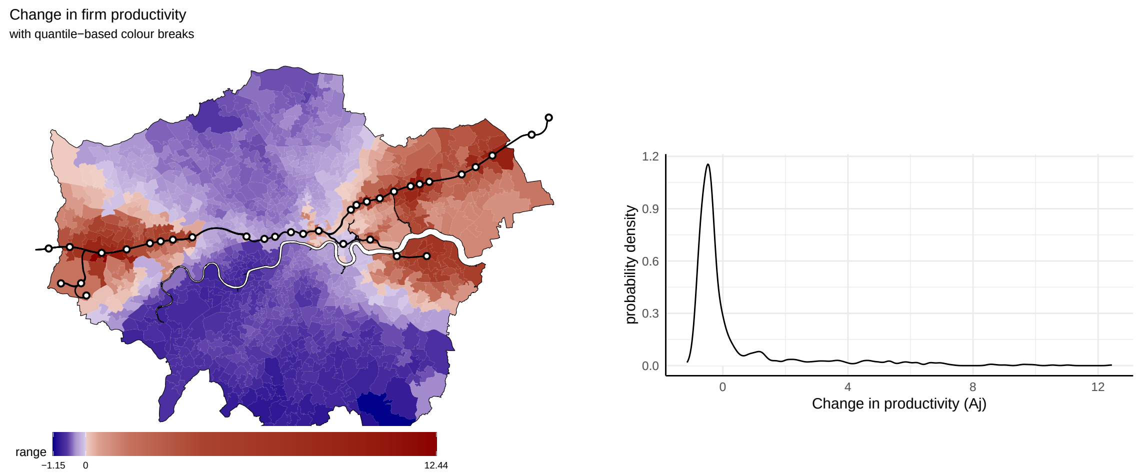


**Figure 7:** Counterfactual outcomes: firm productivity

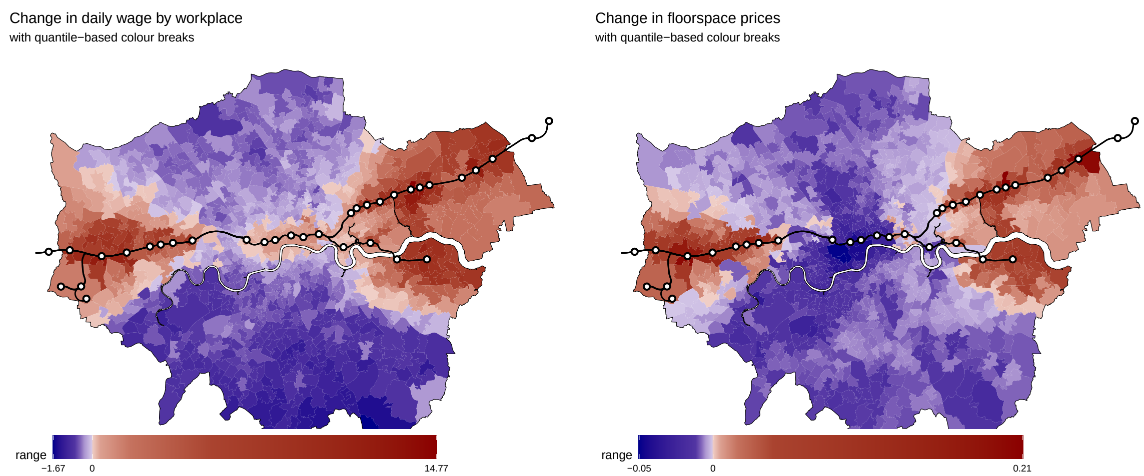


**Figure 8:** Counterfactual outcomes: wages and floorspace prices

shadow. Wage mildly increase in the biggest employment centres: by £0.23 in the City, £0.30 in Canary Wharf, and £0.22 in Westminster. When it comes to floorspace prices, the changes produced by the simulation are almost entirely positive. Surprisingly, we do not observe a significant floorspace price increase in the Centre of London which features the highest density of development already.

The residential and workplace relocations are plotted in Figure 9. By construction of the model, relocation is governed by the new wage and floorspace price distributions as well as commuting times. As both the wage and the floorspace price changes show similar patterns (the correlation between them is 77%), we do not have clear prior expectations about relocation patterns. From Figure 9 we infer that the suburban stations of the Elizabeth Line attract new workplaces mostly, at the expense of the residential population. Figure 10 on

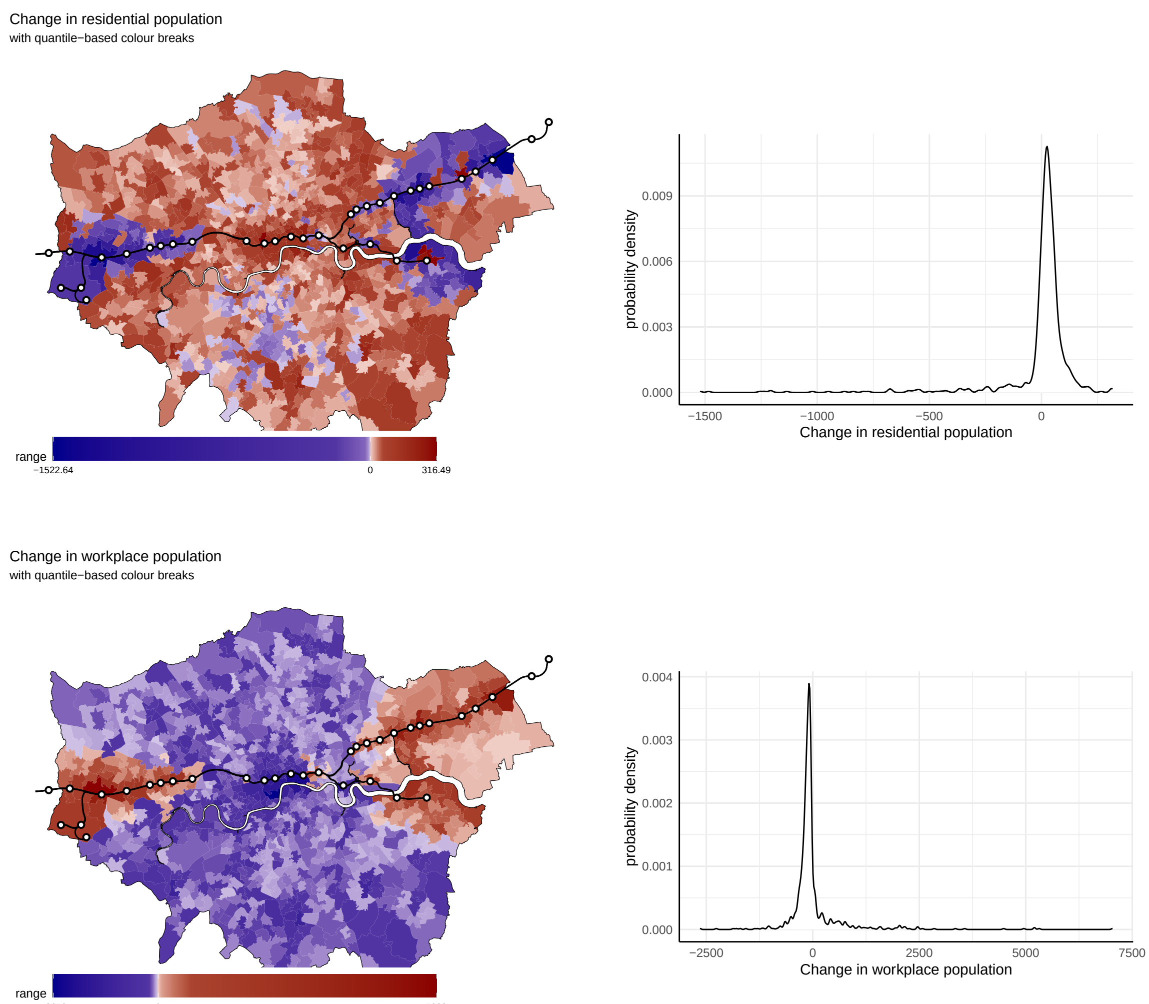


**Figure 9:** Counterfactual outcomes: predicted relocation patterns in response to the Elizabeth Line

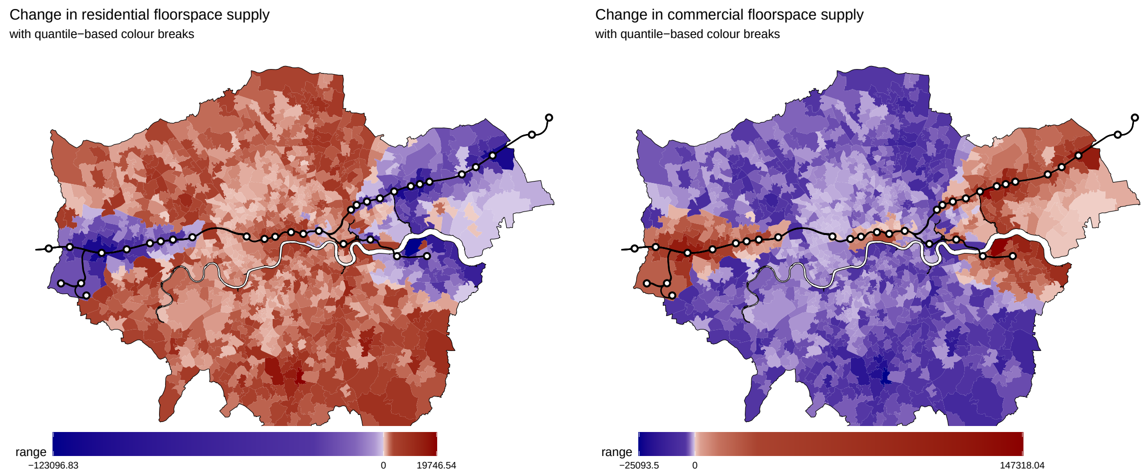


**Figure 10:** Counterfactual outcomes: floorspace supply

the redistribution of floorspace supply confirms that there is tight competition between the residential and commercial uses of land: residents are relocating from the MSOAs where the highest increase in commercial floorspace demand emerges.

Looking more closely at Figure 9, the destination of the residents priced out from the proximity of the Elizabeth Line is less clear. Minor negative and positive changes in the residential population are spread sporadically in North and South London. Interestingly, the dark blue bands near the Elizabeth Line (where residential population decreased significantly) are surrounded by dark red areas. This implies that residents cannot compete with commercial floorspace users at the most attractive locations, but they are trying to move as close as possible to the newly formed employment centres near the Elizabeth Line.

In summary, we observe a pattern of employment decentralisation combined with higher wages, floorspace prices and floorspace density in the CBD of London. The general pattern of residential floorspace supply is pointing towards a tendency of suburbanisation, but not to the immediate proximity of the new infrastructure. Heblich et al. (2020) document in a century-long context that the 19th-century development of London's suburban rail network enabled residents to separate their residential locations and workplaces significantly. That rule continues to hold in the 21st century. Although the mean commuting time decreases by 0.3%, the mean Euclidean commuting distance *increases* by 1.96%, rising from 11.72 to 11.95 kilometres.

### 5.2.3 Further spatial insights

Finally, we would like to draw attention to some of the less conventional spatial outcomes of a transport intervention, plotted in the two panels of Figure 11. These capture the impact of the policy on (i) travel time valuations, which is unique to this paper in the QSM literature, and (ii) on the mean income by workplace locations, taking the wage rate as well individual labour supply into consideration.

The left-hand-side of Figure 11 reveals significant changes in the distribution of travel time valuations, that is, the distribution previously shown in Figure 3. This plot illustrates the fact that the transport improvement itself affects how commuters value travel time. We observe a pattern of increased travel time valuations in suburban locations. This might be explained by relocation of high-wage commuters to these zones, thus increasing $v_{ij}$ in the model. In some cases, the changes predicted by the model are clearly substantial, ranging up to £1.46 per hour. This finding is remarkable in light of the fact that the city-wide mean value of time is nearly unaffected by the policy: it increases by £0.015 or 0.1% only. Apparently, the city-wide average hides significant spatial variation which may have important implications locally. Once again, these are previously unseen insights in the context of transport appraisal that normally keeps the value of time constant, both spatially and temporally.

The right side of the same figure shows the change in the mean by workplace, i.e. the product of the wage rate and the individual labour supply: $w_j \cdot \bar{x}_j$. This outcome is interesting to compare with the changes in productivity (in Figure 7) and the pure wage rate (in Figure 8), which indicated significant heterogeneity. When individual labour supply is taken into account, Figure 11 shows that the resulting income per worker increases almost everywhere in the city. That is, it is unlikely that households would experience a deterioration in their financial state as a result of the new railway line. We believe that these results are convincing in the context of the 2011 economy of London, way before the actual opening of the Elizabeth Line and the major transformation caused by the pandemic shock since 2020. For a more detailed QSM analysis of the pandemic, see early research efforts by Delventhal et al. (2022) and Delventhal and Parkhomenko (2023).

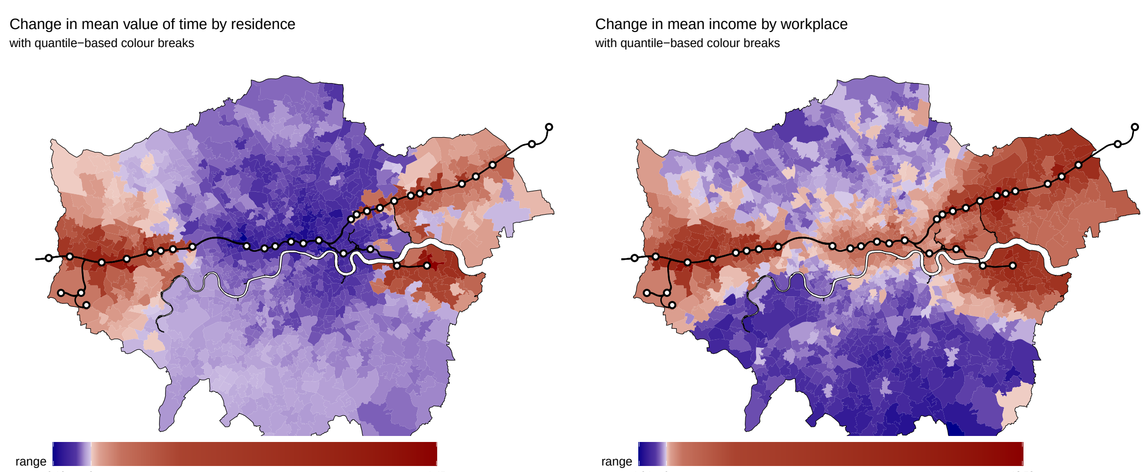


**Figure 11:** Counterfactual outcomes: time valuation and labour supply

### 5.3 Competing evaluation approaches

In the third step of the policy analysis we study the use of the SGE model developed in this paper in a welfare economic cost-benefit analysis, that is, a transport appraisal exercise. The state-of-the-practice in transport appraisal is limited to a partial equilibrium representation of the transport market, and most of the benefits are derived from the monetary valuation of the aggregate travel time savings delivered by an intervention. The core research question of this subsection is whether the QSM approach developed in paper is expected to produce fundamentally different evaluation results in comparison with the mainstream PE methodology.

Using our paper's previous notation, the measure that the literature and practitioners often call *direct user benefit* is an application of the "rule of one-half" in microeconomics:

$$B = \sum_{ij} \frac{1}{2} N(\lambda_{ij}^0 x_{ij}^0 + \lambda_{ij}^1 x_{ij}^1) \cdot v_{(ij)}(t_{ij}^1 - t_{ij}^0). \tag{44}$$

This is an approximation of consumer surplus in partial equilibrium, assuming a linear inverse demand curve for travel demand that expresses the generalised travel price in function of travel demand.[10] In equation (44), $0.5\, N(\lambda_{ij}^0 x_{ij}^0 + \lambda_{ij}^1 x_{ij}^1)$ is the mean of the commuting volumes in periods 0 (before the intervention) and 1 (after the intervention), and $v_{(ij)}(t_{ij}^1 - t_{ij}^0)$ is the monetary value of the travel time gain/loss between the two periods. That is, this measure includes both the time savings of those who travelled before the policy intervention as well as the surplus associated with newly induced demand. Note that our model enables us to differentiate the value of time, $v_{(ij)}$, between the commuting origin–destination pairs. However, as such differentiation is rarely performed in the state-of-the-practice, we will compute PE results with both homogeneous and heterogeneous time valuations.

In the PE appraisal exercise we would like to consider the agglomeration externality stemming from improved access to economic mass and the associated productivity gain, in line with the transport appraisal literature (Graham and Gibbons, 2019). The common methodology in this area assumes, based on the urban economic theory developed by Venables (2007), that the additional output that firms realise through improved ATEM, keeping all input factors constant, is additional to the direct user benefit of travellers. Using the production function in equation (17) and the decomposition of total factor productivity ($A_j$) in equation (32), the output of $j$ with access to economic mass $\rho_j^t$ is

$$Y_j(\rho_j^t) = a_j \left(\rho_j^t\right)^{\eta} M_j^{\alpha} H_j^{1-\alpha}, \tag{45}$$

and the increase in firm output solely attributed to the growth in economic density from $\rho_j^0$

[10] For the theoretical foundations of consumer surplus derived form generalised travel prices, please refer to Glaister (1974).

to $\rho_j^1$ is

$$\Delta Y_j = Y_j(\rho_j^1) - Y_j(\rho_j^0) = \left[\left(\frac{\rho_j^1}{\rho_j^0}\right)^{\eta} - 1\right] \cdot Y_j(\rho_j^0). \tag{46}$$

We compute this quantity for each location $j$ using the changes in ATEM in response to the Elizabeth Line, as discussed in Section 5.2.1 above, and sum up the resulting $\Delta Y_j$ quantities to get an aggregate measure of the agglomeration externality.

Another methodological dilemma we need to address at this stage is whether we consider relocation in the calculation of PE welfare results. Relocation affects both direct user benefits through the $\lambda_{ij}^0 x_{ij}^0$ and $\lambda_{ij}^1 x_{ij}^1$ measures in (44) as well as the external productivity gain through $\rho_j^1$ in (46). As there is no clear theoretical guidance on whether relocation should be considered in partial equilibrium welfare analysis, we compute both in the following analysis.[11] In the following discussion we call the PE models with and without relocation *dynamic* and *static*.

We use the standard PE welfare measures introduced above to benchmark them against the welfare predictions of the SGE/QSM model developed in the paper. Similar to the logsum formula in logit discrete choice models, there is a closed-form expression for expected value of household utility in the discrete choice framework that features a Fréchet-distributed random utility shock.[12]

$$E[U_{ij}] = \Gamma\left(\frac{\varepsilon - 1}{\varepsilon}\right) \left[\sum_{i,j} X_i E_j \left(\frac{v_{ij}}{p_i^{\beta} q_i^{1-\beta}}\right)^{\gamma\epsilon}\right]^{1/\epsilon} \qquad \forall\, i,j \tag{47}$$

In this expression $\Gamma(\cdot)$ is the Gamma function. Note that in spatial equilibrium, expected utility is the same across all residence–workplace combinations. To derive a monetary measure of the economic gain of households in this model, we compute the compensating wage that achieves the same improvement in expected utility as the transport project. The compensating wage is then multiplied by the number of households. As the production and construction markets are perfectly competitive, economic surplus is not generated there. The only element that we need to add to the monetised change in expected utility is the change in floorspace revenues.

[11] Please note that predicting the pattern of household and employment relocation is a non-trivial challenge in absence of a spatial equilibrium model. In this exercise we assume the partial equilibrium analysis is informed by the SGE model on relocation patterns. The elasticities, ad-hoc gravity models, land-use/transport interaction model that are frequently used in transport appraisal may lead to different results. In this exercise we avoid this empirical source of bias/disagreement between the PE and SGE methods by passing the SGE outputs over to the PE model.

[12] For the detailed derivation of this expression, see Appendix Section S.2.2 of Ahlfeldt et al. (2015) or Appendix Section C1 of Heblich et al. (2020).

**Table 3:** Benchmarking partial and general equilibrium appraisal methods

| Appraisal model | Static partial equilibrium | | Dynamic partial equilbrium | | SGE |
|---|---|---|---|---|---|
| Value of time | homogeneous | heterogeneous | homogeneous | heterogeneous | n.a. |
| Direct user benefit | 831 | 753 | 1,098 | 949 | n.a. |
| Wider economic impact | 1,769 | 1,769 | 775 | 775 | n.a. |
| Total benefit | 2,600 | 2,522 | 1,873 | 1,723 | 2,379 |

Units: Surplus in thousands of GBP per day.

The results of our numerical experiment are provided in Table 3. Some of the outcomes shed light on less known properties of partial equilibrium transport appraisal as well. First, we find that the direct user benefit is significantly higher when the mean of the value of time is applied instead of locally differentiated ones. We find 10 and 16 percent higher direct user benefits in the former case in the static and dynamic models, respectively. Second, the observe a complementary relationship between the DUB and WEI layers: as we move from the static to the dynamic model, the volume of direct user benefits increases due to induced travel demand, but the value of wider economic impacts shrinks (see Figure 6 and the related discussion). These complementary effects partly neutralise each other. On the level of unique locations, we find heterogeneity in terms of whether an MSOA gains (loses) more in the short run (static WEI) relative to the long run. Figure D.1 in the Appendix shows that many locations gain slightly more when the effect of relocation is taken into account, but there several outliers in the negative direction as well.

The final column of Table 3 provides the welfare measure of the SGE model. In this case we are unable to decompose the total benefit into DUB and WEI layers due to the nonlinear multiplicative structure of the model. Our first new insight is that even though SGE may seem like a fundamentally different methodology compared to the more common PE approach, the two groups of welfare estimates are at the same order of magnitude. This may be reassuring from a practical perspective: in general, it is unlikely that the SGE model would disapprove the majority of past policy interventions based on the mainstream PE CBA. Under this paper's assumptions on model specification, the SGE welfare result is around 8.5% lower than the highest PE result and than 27.0% higher than the lowest. Unfortunately, the absence of a clear analytical link between the partial and general equilibrium methods and the approximative nature of the PE approach does not allow us to provide a clear explanation for the observed difference in outcomes. Nevertheless, this result implies that the SGE model is not only more informative as a disaggregate spatial impact assessment tool, but also a source of potentially different benefit-to-cost ratios in practical appraisal.

We have tested the robustness of these findings in two alternative scenarios. First, in Table D.1 we recalculate the appraisal results above with the UK-wide average agglomeration elasticity of $\eta = 0.044$ and a distance decay $\delta = -0.08$. Note that in this scenario we recalucate the vector of productivity fundamentals in line with Section 4.4, so that the observed data remains the pre-intervention spatial equilibrium. In this case we find the SGE model produces significantly higher welfare results, which can be twice as high as the PE measure of benefits. However, this result may well be the consequence of the erroneous use of an elasticity that is incompatible with the dense urban economy of Greater London.

Second, in Table D.2 we check the sensitivity of our results with respect to the assumed magnitude of the relaxation of the floorspace construction restriction, i.e. $\bar{H}$. The static PE model is unaffected by this assumption, as relocation is ignored in this model anyway. The dynamic PE model produces monotonously increasing welfare results. Again, this finding is in line with intuition: as the transport improvement is combined with the possibility of building more floorspace near the new stations, the employment densification unlocks further agglomeration gains. We find an increasing pattern among the SGE results as well, but rate of increase is even more drastical. After a 5% relaction of the floorspace constraint, the welfare gain is more than three times higher than after a pure transport improvement with no land-use intervention. It requires further research to determine whether this remarkable finding is unique to the high density and high agglomeration elasticity of Greater London or if it can be generalised more broadly.

# 6 Conclusions

This paper presents a spatial general equilibrium model that blends the advantageous properties of methods from two influential branches in the literature, namely the empirical properties of quantitative spatial models and the theoretical properties of computable spatial general equilibrium models designed to assess transport policies. The paper makes a contribution to the spatial economics literature by introducing a QSM featuring the leisure-labour trade-off with separate monetary and time budget constraints, endogenous commuting time valuations, and a continuous decision margin on individual labour supply. This also enables us to quantify the marginal opportunity cost of time in monetary terms, that is, a spatially differentiated measure of the value of travel time savings. Taking a model of Greater London with nearly one thousand spatial units as a case study, we use this methodology to uncover the spatial distribution of travel time valuations, thereby providing important insights without a costly data collection effort.

Our study also contributes to the transport literature through a model that shares the theoretical properties of SCGE models, maintaining the desirable empirical properties of QSMs. The model is invertible, so that the impact of exogenous geographic advantages on firm productivity and amenities can be quantified for a large number of locations, and the core structural parameters of the model can be causally estimated using reduced-form estimating equations derived directly from the theoretical model. We emphasise the importance of causality in the estimation of large-scale spatial equilibrium models. We argue that theoretically coherent causal parameter estimation reduces the uncertainty stemming from the ad-hoc parameter selection and associational calibration in previous transport SCGE models.

In a numerical application of the fully calibrated model, we run a counterfactual simulation involving the hypothetical introduction of the Elizabeth Line in the census year of 2011, using historic data to represent the 2011 state of the London economy. The simulation indicates that the model produces intuitive predictions about the reorganisation of economic activity in response to the transport policy. Then, in a second numerical exercise, we benchmark the welfare predictions of the QSM model against traditional partial equilibrium transport appraisal methods.

One of the key messages of this paper is that, in the longer term, quantitative spatial models (QSMs) appropriately adapted for transport analysis have the potential to become a recognised part of the transport appraisal toolbox, acknowledged by policymakers, government agencies, and the wider public. However, it is unlikely that a spatial equilibrium approach would entirely replace the mainstream partial equilibrium (PE) methodology. While spatial equilibrium analyses provide valuable insights into the effects of large-scale transformative interventions, applying them to smaller transport improvements with localised impact areas would involve unnecessary complexity. Even for transformative projects, our preliminary numerical simulation of the Elizabeth Line within the geographical scope of Greater London suggests that spatial equilibrium-based analyses yield aggregate welfare estimates are similar in magnitude to those of an appropriately calibrated PE model combined with an elasticity-based approximation of external agglomeration benefits.

The core advantage of spatial general equilibrium (SGE) models lies in their ability to capture the impact of transport improvements on local economic outcomes with a high degree of spatial granularity. These outcomes include distributions of wages, employment, housing prices, and residential and workplace populations: metrics that policymakers and the public find more intuitive and accessible than abstract indicators such as the net present welfare gain, the benefit-cost ratio, or the internal rate of return. By providing these detailed and comprehensible measures, SGE models enhance the transparency of cost-benefit analysis, ultimately fostering greater trust in transport appraisal processes within the public policy arena.

As briefly acknowledged at the beginning of Section 5, our model, in its current form, is not yet ready to perform a comprehensive cost-benefit analysis or produce results that can be benchmarked against the official CBA reports for the Elizabeth Line or other interventions. In pursuit of this goal, the paper unlocks a range of promising future research directions. One of the most pressing priorities is to enhance the representation of commuting behaviour by incorporating endogenous transport mode and route choice, alongside congestion and crowding externalities. Incorporating mode choice into the current model is feasible, as demonstrated in a successful student project supervised by the lead author and reported in Doffkay (2024).

However, addressing route choice and congestion presents a more significant challenge, both empirically and computationally.

We identify three potential approaches to incorporating congestion into our model. (i) Anas and Liu (2007) integrate a standard stochastic traffic assignment process into their SCGE model. However, we find this approach challenging within our framework, as the present model lacks a clear methodology for causally estimating the parameters of a link-level congestion cost function. Even if this issue were resolved, simulating counterfactual equilibria with one million origin-destination pairs and tens of thousands of route segments would pose a significant computational challenge. (ii) Allen and Arkolakis (2022) have recently addressed both empirical and computational challenges in a different QSM. However, their approach employs a multiplicative iceberg specification for utility with a monetary budget constraint. Consequently, this method is incompatible with the framework of this paper and other models that rely on a leisure-labour trade-off. In our view, this limitation prevents the model's application in a formal transport cost-benefit analysis. (iii) Koster (2024) endogenises travel times as a function of *effective traffic density* at the destination zone of commuting origin-destination pairs. This measure resembles the effective employment density terms commonly used in agglomeration research; see equation (35) in this paper as an example. The concept assumes that congestion delay can be reasonably approximated by the volume of trips directed towards a commuter's workplace, as well as nearby workplaces, weighted by a distance decay term. Koster's empirical approach is theoretically grounded in the 'bathtub' (Arnott, 2013) and 'macroscopic fundamental diagram' (Daganzo and Geroliminis, 2008) models from the transport literature. However, his model overlooks route choice and the zones a commuter may travel through, indicating the need for further refinement to make it suitable for transport-oriented applications.

Another key step towards transforming our model into a comprehensive appraisal methodology is to expand its scope to encompass multiple trip purposes. At present, the model is limited to commuting flows. Future research will focus on developing microfoundations for both leisure and business travel. The empirical significance of non-commuting trips is considerable: commuting typically accounts for less than half of households' total transport consumption, a proportion that has further declined with the rise of remote work since the pandemic (see, e.g., Balbontin et al., 2024). Despite this, spatial models have rarely incorporated non-commuting trips. For instance, Anas and Liu (2007) account for shopping trips through endogenous consumption location choices, while Fajgelbaum et al. (2023) include leisure and business travel components in their long-distance rail model.

One significant challenge of integrating multiple trip purposes is that urban residents often engage in complex trip chains rather than simple, back-and-forth movements between home and a single activity location, as assumed in the current model. This complexity leads to a high-dimensional activity location choice problem. As an early effort to address this issue, Miyauchi et al. (2022) adapt the canonical quantitative urban model (incorporating iceberg utility and travel time costs) and tackle dimensionality with an importance sampling approach. These studies offer valuable insights for future applications of our model in transport appraisal.


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# A Technical appendix of the analytical model

## A.1 Household preferences

The full specification of the Lagrangian function associated with the maximisation of utility in (1) subject to constraints (2) and (3) is

$$\begin{aligned}\Lambda = &\left(\frac{L_{ij}}{1-\gamma}\right)^{1-\gamma}\left[\frac{1}{\gamma}\left(\frac{C_{ij}}{\beta}\right)^{\beta}\left(\frac{H_{ij}^{R}}{1-\beta}\right)^{1-\beta}\right]^{\gamma} z_{ij}\\ &-\kappa_{ij}\left[p_i C_{ij} + q_i H_{ij}^{R} + x_{ij}\tau_{ij} - x_{ij} w_j\right]\\ &-\mu_{ij}\left[L_{ij} + x_{ij}(T_j + t_{ij}) - \bar{L}\right].\end{aligned} \tag{A.1}$$

First-order conditions are

$$\frac{\partial \Lambda}{\partial x_{ij}} = -\kappa_{ij}(\tau_{ij} - w_j) - \mu_{ij}(T_j + t_{ij}) = 0; \tag{A.2}$$

$$\frac{\partial \Lambda}{\partial C_{ij}} = \frac{\partial U_{ij}}{\partial C_{ij}} - \kappa_{ij} p_i = U_{ij}\gamma\left(\frac{C_{ij}}{\beta}\right)^{-1} - \kappa_{ij} p_i = 0; \tag{A.3}$$

$$\frac{\partial \Lambda}{\partial H_{ij}^{R}} = \frac{\partial U_{ij}}{\partial H_{ij}^{R}} - \kappa_{ij} q_i = U_{ij}\gamma\left(\frac{H_{ij}^{R}}{1-\beta}\right)^{-1} - \kappa_{ij} q_i = 0; \tag{A.4}$$

$$\frac{\partial \Lambda}{\partial L_{ij}} = U_{ij}\left(\frac{L_{ij}}{1-\gamma}\right)^{-1} - \mu_{ij} = 0. \tag{A.5}$$

Equations (5) and (6) follow directly from rearranging (A.2). After dividing (A.3) by (A.4), let us express the relationship between $H_{ij}^{R}$ and $C_{ij}$

$$H_{ij}^{R} = \frac{(1-\beta)p_i}{\beta q_i} C_{ij} \tag{A.6}$$

Sum up the consumption-related first-order conditions and then consider (A.5) to isolate the Lagrange multipliers.

$$\begin{aligned}\kappa_{ij} &= U_{ij}\gamma\left(\frac{C_{ij}}{\beta p_i}\right)^{-1}\\ \mu_{ij} &= U_{ij}\left(\frac{L_{ij}}{1-\gamma}\right)^{-1}\end{aligned} \tag{A.7}$$

Substitute these expression into (6) and rearrange for $C_{ij}$.

$$C_{ij} = \frac{\gamma\beta}{1-\gamma}\frac{\upsilon_{ij} L_{ij}}{p_i} \tag{A.8}$$

Finally, express individual labour supply from the monetary budget constraint.

$$x_{ij} = \frac{p_i C_{ij} + q_i H^R_{ij}}{w_j - \tau_{ij}} \tag{A.9}$$

Based on these intermediate steps, we reach the utility-maximising leisure time, consumption and labour supply decisions as follows. Insert (A.9) into the time constraint in (3), substitute (A.6) and (A.8) above, and rearrange for $L_{ij}$ to express the optimal leisure time in (7). Substitute (7) into (A.8) to get the demand for consumer good $C_{ij}$ in equation (9). Plug the resulting $C_{ij}$ function into (A.6) to express the second part of (9), the optimal residential floorspace demand $H^R_{ij}$. As noted in the main text, the utility-maximising individual labour supply (8) follows directly by substituting (7) into the time constraint in (3). Equivalently, one can substitute $C_{ij}$ and $H^R_{ij}$ in (9) into (A.9) to derive individual labour supply $x_{ij}$ in equation (8).

## A.2 Production sector

Our derivation in this section follows the standard producer problem with a Cobb-Douglas production function. Based on the production function defined in (19), the cost minimisation problem has the following Lagrangian.

$$\Lambda = w_j M^W_j + Q_j H^W_j - \lambda \left[ A_j (M^W_j)^\alpha (H^W_j)^{1-\alpha} - Y_j \right] \tag{A.10}$$

The division of first-order conditions yields the regular equality between the marginal rate of substitution and the price ratio. After substituting this equation back into the production function, we get the following factor demand equations.

$$M^W_j = \left( \frac{\alpha}{1-\alpha} \frac{Q_j}{w_j} \right)^{1-\alpha} \frac{Y_j}{A_j}; \quad H^W_j = \left( \frac{1-\alpha}{\alpha} \frac{w_j}{Q_j} \right)^{\alpha} \frac{Y_j}{A_j} \tag{A.11}$$

The resulting cost function is

$$C_j(Y_j) = \frac{1}{1-\alpha} \left( \frac{1-\alpha}{\alpha} \right)^{\alpha} w_j^\alpha Q_j^{1-\alpha} \frac{Y_j}{A_j}. \tag{A.12}$$

Marginal revenue equals marginal cost at the profit-maximising output. Note that in this urban model we normalise the price of the homogeneous urban product to one, so that $p_i = 1\,\forall i$. Therfore,

$$1 = \frac{1}{1-\alpha} \left( \frac{1-\alpha}{\alpha} \right)^{\alpha} w_j^\alpha Q_j^{1-\alpha} \frac{1}{A_j}. \tag{A.13}$$

Rearranging this equation for $Q_j$ and then $w_j/Q_j$, the profit maximising factor demand expressions in (A.11) become

$$\begin{aligned} M^W_j &= \left( \frac{\alpha A_j}{w_j} \right)^{\frac{1}{1-\alpha}} H^W_j; \\ H^W_j &= \left[ \frac{(1-\alpha) A_j}{Q_j} \right]^{1/\alpha} M^W_j, \end{aligned} \tag{A.14}$$

or equation (18) in the main text. Finally, we consider that under the assumption of perfect competition and free entry to the market, profits drop to zero, so that

$$A_j(M_j^W)^\alpha(H_j^W)^{1-\alpha} - w_jM_j^W - Q_jH_j^W = 0. \tag{A.15}$$

Substitute (A.14) into the zero-profit constraint and rearrange for $Q_j$, the profit-maximising floorspace price under perfect competition, in equation (20) of the main text. Equation (21) follows from (20) after a straightforward rearragement.

## A.3 Construction sector

The production function of the spatially differentiated construction sector in (22) is also Cobb-Douglas. The Lagrangian function corresponding to cost minimisation is

$$\Lambda = Z_i + l_iL_i - \lambda\left[Z_i^{1-\psi}\left(\phi_iL_i\right)^\psi\right], \tag{A.16}$$

where we recall that capital ($Z_i$) is measured in the units of the homogeneous urban product, and thus its price is normalised to one, and $\phi_i$ is the multiplier in (23) that captures the tightness of the regulated local floorspace market. The ratio of the first-order conditions with respect to the imput factors yield the following factor demand equations:

$$\begin{aligned} L_i &= \left(\frac{\psi-1}{\psi}\right)^{1-\psi} l_i^{\psi-1}\phi_i^{-\psi}H_i; \\ Z_i &= \left(\frac{1-\psi}{\psi}\right)^{\psi} l_i^{\psi}\phi_i^{-\psi}H_i. \end{aligned} \tag{A.17}$$

The cost function becomes

$$C_i(H_i) = \psi^{-\psi}\left(\frac{1}{1-\psi}\right)^{1-\psi} l_i^{\psi}\phi_i^{-\psi}H_i. \tag{A.18}$$

The unit price of floorspace supplied in location $i$ is $\bar{q}_i$, the mean of local residential and commercial floorspace prices defined in equation (24). Thus, the first-order condition of profit maximisation in the construction sector is

$$\bar{q}_i = \psi^{-\psi}\left(\frac{1}{1-\psi}\right)^{1-\psi} l_i^{\psi}\phi_i^{-\psi}. \tag{A.19}$$

Using the fact that $\psi$ is the expenditure share spent on land, so that

$$l_i = \psi\frac{\bar{q}_iH_i}{L_i}, \tag{A.20}$$

(A.19) results in the following expression for the profit-maximising floorspace supply.

$$H_i = \phi_i(H_i)\left[(1-\psi)\,\bar{q}_i\right]^{(1-\psi)/\psi} L_i \tag{A.21}$$

Plugging the definition of $\phi_i(H_i)$ into this yields the final form of this expression in (25), which we use as an equilibrium condition.

Finally, the welfare calculations of this study in Section 5.3 requires an explicit expression of land value, using endogenous outcomes in spatial equilibrium. Plugging the definition of $\bar{q}_i$ in (24) into (A.20), we derive

$$l_i = \psi \left( q_i H_i^R + q_i \xi_i H_i^W \right) L_i^{-1}. \tag{A.22}$$

This creates a relationship between floorspace prices and land value. Note that this comes directly from the zero profit constraint in the perfectly competitive floorspace sector: the total expenditure on land ($l_i L_i$) is the $\psi$ fraction of the total revenue from floorspace rents.

# B Spatial equilibrium in counterfactual scenarios

The process of model quantification described in Section 4 ensures that the estimated structural parameters and residuals plugged into the equilibrium conditions reproduce the observed economic outcomes in our data. In other words, the state of the urban economy in which our data have been collected is an equilibrium of the model. We assess the impact of transport interventions in a comparative statics exercise. In other words, we modify some of the exogenous parameters, typically some of the elements of the transport time and monetary cost matrices, and compute the spatial equilibrium determined by the new parameter set.

In this section we provide the steps of the iterative process through which we compute the new equilibrium. Superscripts (0) and (1) denote the initial and the updated values of the variables that we endogenously update in each iteration, while $\hat{x}$ denotes the empirical estimate of any parameter $x$.

**Step 1: Population distribution and labour supply.** First we apply equations (12) and (14) to compute the residential and workplace populations.

$$\begin{aligned}
\lambda_{ij} &= \frac{\hat{X}_i \hat{E}_j \left[ \upsilon_{ij}^{(0)} \left( q_i^{(0)} \right)^{\beta-1} \right]^{\gamma\hat{\epsilon}}}{\sum_r \sum_s \hat{X}_r \hat{E}_s \left[ \upsilon_{rs}^{(0)} \left( q_r^{(0)} \right)^{\beta-1} \right]^{\gamma\hat{\epsilon}}} \\
N_i^R &= N \sum_j \lambda_{ij} \\
N_j^W &= N \sum_i \lambda_{ij}
\end{aligned} \tag{B.1}$$

The counterfactual travel time matrix $t_{ij}$ is substituted into equation (8) to compute $x_{ij}$ for each origin–destination pair. Finally, equation (15) is used to aggregate the effective labour supply $M_j^W$.

**Step 2: Local productivity and wages.** Using the aggregate labour supply $M_j^W$ derived above and the definition of local productivity in (35), we update the workplace-specific wage vector in line with equation (21).

$$
\begin{aligned}
A_j &= \hat{a}_j \left[ \sum_s \exp\left(\hat{\delta} t_{sj}\right) M_j^W \right]^{\hat{\eta}} \\
w_j^{(1)} &= \alpha A_j^{1/\alpha} \left( \frac{1-\alpha}{Q_j^{(0)}} \right)^{\frac{1-\alpha}{\alpha}}
\end{aligned}
\tag{B.2}
$$

This enables us to update the $\upsilon_{ij}$ matrix as well.

$$
\upsilon_{ij}^{(1)} = \frac{w_j^{(1)} - \tau_{ij}}{T_j + t_{ij}} \tag{B.3}
$$

**Step 3: Floorspace prices.** To determine the model's two floorspace price vectors, we first derive the aggregate residential floorspace demand for each location.

$$
H_i^R = N_i^R \sum_j \lambda_{ij|i} H_{ij}^R, \tag{B.4}
$$

which, after substituting (13) for $\lambda_{ij|i}$ and (9) for $H_{ij}^R$, yields

$$
H_i^R = N_i^R (1-\beta) \frac{\gamma \bar{L}}{q_i^{(0)}} \sum_j \frac{\hat{E}_j \left(\upsilon_{ij}^{(1)}\right)^{\gamma\hat{\epsilon}}}{\sum_s \hat{E}_s \left(\upsilon_{is}^{(1)}\right)^{\gamma\hat{\epsilon}}} \upsilon_{ij}^{(1)}. \tag{B.5}
$$

Aggregate commercial floorspace demand comes directly from (A.14),

$$
H_i^W = \left[ \frac{(1-\alpha) A_i}{Q_i^{(0)}} \right]^{1/\alpha} M_i^W, \tag{B.6}
$$

in which we bring $A_i$ and $M_i^W$ from Steps 2 and 1, as detailed above. Total floorspace demand $H_i = H_i^R + H_i^W$ now enables us to update the residential floorspace price vector by substituting (24) into (25) and rearranging the latter for $q_i$.

$$
q_i^{(1)} = \frac{1}{1-\psi} \left( H_i^R + H_i^W \hat{\xi}_i \right)^{-1} \left[ \left( 1 - H_i / \hat{\bar{H}}_i^{-1} \right) L_i \right]^{\psi/(\psi-1)} \tag{B.7}
$$

Finally, commercial floorspace prices are updated via $Q_i^{(1)} = \hat{\xi}_i \, q_i^{(1)}$.

Through the three steps above we update X vectors of location-specific variables, $N_i^R$, $N_j^W$, $M_j^W$, $A_j$, $w_j$, $q_i$ and $Q_j$, ensuring that the equilibrium conditions are met in the labour, production and construction markets as well. Ahlfeldt et al. (2015) prove analytically that a structurally similar general equilibrium model, featuring Fréchet utility shocks in location choice and a multiplicative specification for utility, converges to unique equilibrium. In this project we do not delve into the challenge of an analytical proof of uniqueness; however, our randomised numerical tests did not show any sign of multiplicity in equilibria.

In practice, multiple iterative algorithms can be used to achieve quick convergence. In this project we used a standard adaptive Method of Successive Averages algorithm to update the parameter vectors in $\theta = \{N_i^R, w_j, q_i\}$ as follows:

$$\theta^{(0)} := (1 - \Phi) \cdot \theta^{(0)} + \Phi \cdot \theta^{(1)}, \tag{B.8}$$

where the updating parameter $\Phi$ is adaptive, i.e. it increases by 10% or decreases by 20% depending on whether the sum of the $|\theta^{(1)} - \theta^{(0)}|$ gaps is shrinking or widening between two consecutive iterations. Using this common approach and a standard PC, this model of nearly 1,000 spatial units converges within around 15 minutes.

# C Empirical results on the effects of economic density

This section provides further details on the estimation of the generic parameters that capture the impact of economic density on local firm productivity and amenities.

## C.1 Firm productivity

Our core results on firm productivity are summarised in Table 2 and Section 4.3. Figure C.1 focuses on one specific outcome of the NLS model specified in (41) with control function (40); that is, model (4) in Table 2. It plots the $\hat{d}_r$ estimates and their 95% confidence bands. These coefficients quantify the contribution of the sequence of 2.5-minute travel time doughnuts to our measure of access to economic mass in an increasing order from location $j$, of which the TFP residual is the dependent variable in (41). We fix the value of the first travel time band at one, meaning that employment at location $j$ itself is fully considered in the ATEM measure.

The estimates in Figure C.1 imply that the contribution of employment to access to economic mass is generally decreasing with travel time, confirming the effect of a distance decay. However, this decreasing pattern is not monotonous, and some of the coefficients of the nearest travel time bands are insignificant, which requires further attention. The latter result is not surprising. The low level of statistical significance is caused by the low number of observations at short distances: there are only very few pairs of MSOA centroids located within less than 10 minutes of travel time. The mildly upward sloping pattern between 15 and 30 minutes might indicate, counter-intuitively, that it matters more from a productivity perspective what firms in an MSOA can reach within 30 minutes than in 15 minutes. Based on the confidence bands, we cannot reject though the possibility that this pattern is flat or even very slightly downward sloping. These results remained robust in other specifications we tested during the project. Nevertheless, the pattern we observe here suggests that future research efforts may consider alternative distance decay specifications that differ from the negative exponential function used by the majority of the agglomeration literature.

In the main text we explain that our $\hat{\delta}$ estimate is based on the statistically significant $\hat{d}_r$ values only. The corresponding distance decay function is shown by the solid black line. The solid grey curve in Figure C.1 is the distance decay function that we get by estimating $\delta$ including the non-significant coefficients as well. We find that the agglomeration externality decays more quickly in this case, but the pattern is not materially different.

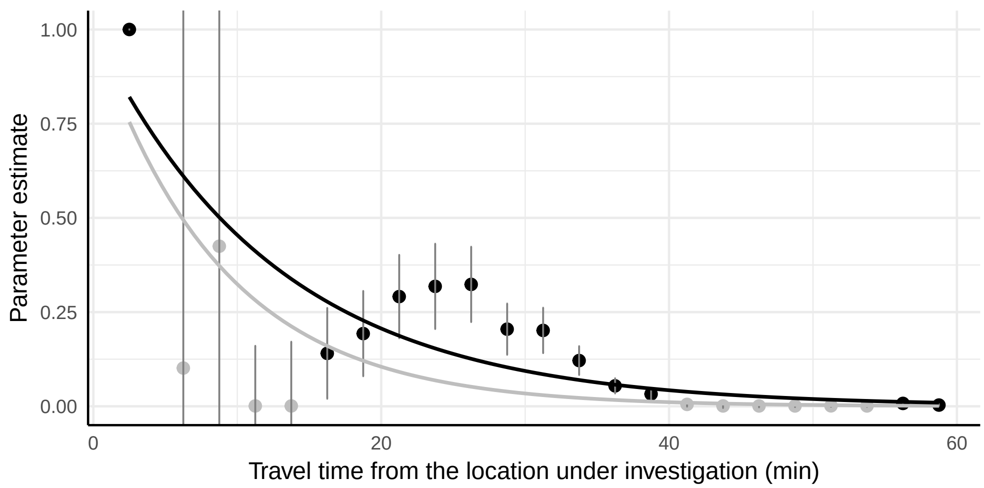


**Figure C.1:** Estimates for travel-time bands in model (4) of Table 2. Black dots represent estimates that are significant at the 95% confidence level and the solid black line is the best-fitting decay function, i.e. $\exp\left(\hat{\delta} t_{rs}\right)$, using significant estimates only. The grey dots and solid line correspond to non-significant estimates and the decay function based on the full sample.

Table C.1 is a senstivity test of our productivity model based on two sub-samples of the full dataset. Model (2) in this table is equivalent to Model (3) in Table 2.[13] The first column re-estimates $\eta$ for the half of the MSOA sample located closer to the CBD. The table shows that the mean employment density is nearly twice as high as in the full sample—however, it is still just a fraction of the density of the City of London. We find that the agglomeration elasticity increases to 19.1% in this case. By contrast, when we consider more distant MSOAs closer to the periphery of the city, where the mean employment density is just a third of the entire city, the elasticity drops to 6.3%.[14] This finding hints that (i) agglomeration economies are highly non-linear, and (ii) the relatively high average agglomeration elasticity we use in this

[13]We use Model (3) in this case because its computational time is significantly shorter than Model (4)'s while the resulting $\eta$ and $\delta$ estimates are similar.

[14]Note that the distance decay parameter seems to remain robust for the city centre, while its value is insignificant in the periphery.

paper converges to the usual range often found in the literature as we move to the moderately dense peripheral parts of Greater London. The non-linear nature of agglomeration is in line with recent findings by Anupriya et al. (2023), but, to the best of our knowledge, it has not been tested in a QSM framework yet.

**Table C.1:** Productivity elasticities in two subsamples based on distance from the CBD

| | Dependent variable: log productivity residual | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Distance from CBD | Below median | *Full sample* | Above median |
| Mean employment density‡ | 4,476 | 2,763 | 1,053 |
| ATEM measure | Total emp.† | Total emp.† | Total emp.† |
| Method | NLS+CF | NLS+CF | NLS+CF |
| Productivity elasticity, $\eta$ | 0.191*** | 0.155*** | 0.063*** |
| | (0.034) | (0.009) | (0.018) |
| Distance decay, $\delta$ | -0.086*** | -0.082*** | -0.037 |
| | (0.028) | (0.012) | (0.026) |
| Borough fixed effects | yes | yes | yes |
| RMSE | 0.06 | 0.06 | 0.05 |
| AIC | -1306.73 | -2665.81 | -1531.46 |
| BIC | -1080.12 | -2372.37 | -1330.03 |
| # of obs. | 491 | 983 | 491 |

‡: Measured in employees per km$^2$. Max value is 84,082 in the City of London.
†: Total employment is aggregated in 2.5-min travel time bands.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

## C.2 Residential and workplace amenities

The rest of this appendix section provides additional experimental results on the dependence of local amenities on the density of residential and employment activities. There is growing interest among researchers and practitioners in understanding agglomeration (more generally, economic density) effects beyond firm productivity (see e.g. Ahlfeldt and Pietrostefani, 2019) and in incorporating such effects in transport appraisal. For example, better connectivity may have external effects on amenities and the attractiveness of urban locations, which is a market failure currently ignored in the official CBA guidelines. From a methodological perspective, replacing the TFP residual ($\hat{A}_j$) with residential or workplace amenity residuals ($\hat{X}_i$ and $\hat{E}_j$) in the empirical models of Section 4.3 is an appealing possibility.

**Table C.2:** The elasticity of residential amenities with respect to population density

| | Dependent variable: log residential amenity | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Mass measure | Population density | | Population in 10 min | |
| Method | 2SLS | CF | 2SLS | CF |
| log mass measure | $0.349^{***}$ | $0.363^{***}$ | $0.419^{***}$ | $0.597^{***}$ |
| | (0.103) | (0.126) | (0.170) | (0.130) |
| Borough fixed effects | yes | yes | yes | yes |
| RMSE | 0.558 | 0.558 | 0.562 | 0.554 |
| AIC | 1713.16 | 1714.46 | 1726.78 | 1700.99 |
| BIC | 1884.33 | 1890.52 | 1897.95 | 1877.05 |
| # of obs. | 983 | 983 | 983 | 983 |

†: Total employment is aggregated in 2.5-min travel time bands.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

In the models reported in Table C.2, the dependent variable is $\log \hat{X}_i$, that is, the logarithm of the residential amenity residual recovered via model inversion (see Section 4.2). We find it natural to use population-related mass measures to explain residential amenities. In models (1) and (2) our choice is the population density, i.e. the ratio of the nighttime population and the area of each MSOA. In models (3) and (4) we broaden this definition to the population of all MSOAs within a travel time of 10 minutes. Unobserved local characteristics and reverse causality raise endogeneity concerns the same way as in the productivity models. Thus, in models (1) and (3) we instrument the mass measure by the 1841, 1861, 1881, 1901 and 1921 population density and a third-order polynomial of the distance from the CBD. In models (2) and (4) we use the control function approach, with the same historical and geographical instruments.

Higher population density may affect amenities in various ways: for example, density implies that more services may be available locally and there is higher chance to interact within the local community. At the same time, population density increases the chance of friction between inhabitants, in the form of congestion, for example. Thus, we do not have prior expectations about the sign of the estimated elasticities. Table C.2 reports statistically significant, positive, and relatively high elasticities. In general, the elasticity is higher when the total population within 10 minutes is used as a mass measure. Unfortunately, the estimates reflect substantial sensitivity: they range between 35% and 60%, which the choice of a preferred model specification challenging.

Table C.3 contains estimation results for the same models, except that we replaced the dependent variable with $\log \hat{E}_j$, the logarithm of the workplace amenity residual, and the mass measure is now the employment density (models 1 and 2) and total employment within

10 minutes (models 3 and 4). The latter pair of models did not yield precise estimates even at a 90% confidence level. The elasticities in models (1) and (2) with employment density are significant at the 95% and 90% levels, respectively, which is somewhat below our prior expectations. The elasticities indicate that workplace amenities are negatively affected by employment densities: this result hints that congestion and other nuisance factors are important determinants how attractive a workplace location is. This result is particularly important due to its sign: in an evaluation context, negative workplace amenity externalities may at least partly neutralise the positive productivity externalities that the literature has devoted more attention to.

**Table C.3:** The elasticity of workplace amenities with respect to population density

| | Dependent variable: log workplace amenity | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Mass measure | Employment density | | Employment in 10 min | |
| Method | 2SLS | CF | 2SLS | CF |
| log mass measure | -0.160** | -0.136* | -0.179 | -0.155 |
| | (0.077) | (0.081) | (0.122) | (0.123) |
| Borough fixed effects | yes | yes | yes | yes |
| RMSE | 0.886 | 0.653 | 0.887 | 0.576 |
| AIC | 2622.05 | 2024.28 | 2623.47 | 1777.74 |
| BIC | 2793.22 | 2200.34 | 2794.64 | 1953.8 |
| # of obs. | 983 | 983 | 983 | 983 |

†: Total employment is aggregated in 2.5-min travel time bands.
Standard errors in parentheses, ***: 99%, **: 95%, *: 90%

The experimental results reported in Tables C.2 and C.3 suggest that access to nearby residential and workplace population mass does have a statistically significant causal effect on residential and workplace amenities. Moreover, the magnitude of these effects is generally high, i.e. the estimated amenity elasticities are comparable to, or even higher than, the productivity elasticities in the main text. However, the coefficients and even their statistical significance are highly dependent on the choice of mass measure and the estimation methodology. The literature on amenities and agglomeration is still in its early stages and provides limited guidance for selecting a preferred model among those discussed above. Given the high variability in the estimated coefficients and their significance, we have decided not to incorporate endogenous amenity externalities into the appraisal exercise presented in Section 5.3. This remains a highly relevant subject for future research.

# D Transport appraisal

Figure D.1 breaks down the aggregate wider economic impact measures in Section 5.3 by plotting the external welfare change in equation (46) for each MSOA separately. Specifically, it plots the agglomeration benefit we get in the dynamic partial equilibrium model (enabling firms and workers to relocate) against the static partial equilibrium results. We also add a 45 degree line corresponding to identical values on the two axes to reveal whether dynamic agglomeration economies are greater or smaller than the static benefit on the level of distinct locations. We find a mixed pattern. We observe a cluster of locations that realise relatively high static gains and then their benefits futher increase when relocation is unlocked—this is what common-sense intuition would anticipate. However, there is another group of outliers that showcase reasonably high static gains but then their dynamic agglomeration benefit turns into even higher negative values. This showcases that modelling relocation is crucial in predicting how a transport scheme is expected to affect local economic activity. Limiting the appraisal exercise to static agglomeration gains may severely misinform policy-makers about the presence as well as the spatial distribution of the beneficiaries and victims of place-based policies.

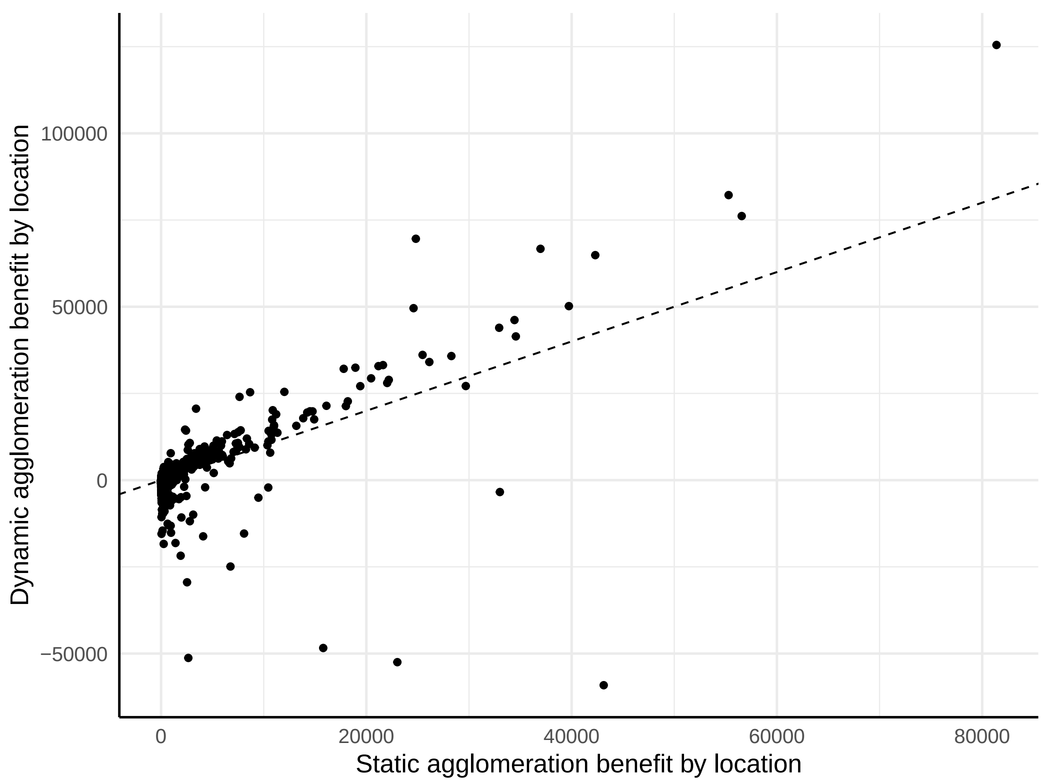


**Figure D.1:** Static and dynamic agglomeration benefits plotted separated for each MSOA of Greater London. The dashed line connects equal values on the two axes.

Tables D.1 and D.2 document the outcomes of two sensitivity tests fo the main appraisal results reported in Table 3. In D.1 we recalculate all appraisal results by assuming an agglomeration elasticity of $\eta = 0.044$ and a distance decay of $\delta = -0.08$. These parameters are often considered as a median of the existing estimates in the literature (see Graham and Gibbons, 2019) as well as approximations of the parameters recommended by the UK Transport Analysis Guidance for the entire country. Table D.2 is a sensitivity test in which we vary the extent to which parameter $\bar{H}_i$, the exogenous floorspace construction constraint in equation (23), is relaxed in the counterfactual scenario of the Elizabeth Line. In this table we include the total benefit only (i.e., the final row of Table 3 in the main text). The results of these senstivity tests are interpreted in the final paragraphs of Section 5.3.

**Table D.1:** Sensitivity test: Figure 3 with lower agglomeration elasticity: $\eta = 0.044$, $\delta = -0.08$.

| Appraisal model | Static partial equilibrium | | Dynamic partial equilbrium | | SGE |
|---|---|---|---|---|---|
| Value of time | homogeneous | heterogeneous | homogeneous | heterogeneous | n.a. |
| Direct user benefit | 831 | 753 | 1,026 | 918 | |
| Wider economic impact | 511 | 511 | 559 | 559 | |
| Total benefit | 1,341 | 1,263 | 1,585 | 1,477 | 2,889 |

Units: Surplus in thousands of GBP per day.

**Table D.2:** Sensitivity test: Total benefit in Figure 3 with different exogenous zoning (floorspace restriction) policies.

| Appraisal model | Static partial equilibrium | | Dynamic partial equilbrium | | SGE |
|---|---|---|---|---|---|
| Value of time | homogeneous | heterogeneous | homogeneous | heterogeneous | n.a. |
| Restriction unchanged | 2,600 | 2,522 | 1,512 | 1,361 | 1,127 |
| 2% relaxation (baseline) | 2,600 | 2,522 | 1,873 | 1,723 | 2,379 |
| 5% relaxation | 2,600 | 2,522 | 2,387 | 2,239 | 4,221 |

Units: Surplus in thousands of GBP per day.

# References

Ahlfeldt, G. M., Carozzi, F. and Makovsky, L. (2023), A micro-geographic house price index for England and Wales, Cep occasional paper, The London School of Economics and Political Science.

Ahlfeldt, G. M. and Pietrostefani, E. (2019), 'The economic effects of density: A synthesis', *Journal of Urban Economics* **111**, 93–107.

Ahlfeldt, G. M., Redding, S. J., Sturm, D. M. and Wolf, N. (2015), 'The economics of density: Evidence from the berlin wall', *Econometrica* **83**(6), 2127–2189.

Allen, T. and Arkolakis, C. (2014), 'Trade and the topography of the spatial economy', *The Quarterly Journal of Economics* **129**(3), 1085–1140.

Allen, T. and Arkolakis, C. (2022), 'The welfare effects of transportation infrastructure improvements', *The Review of Economic Studies* **89**(6), 2911–2957.

Anas, A. (2020), 'The cost of congestion and the benefits of congestion pricing: A general equilibrium analysis', *Transportation Research Part B: Methodological* **136**, 110–137.

Anas, A. and Chang, H. (2023), 'Productivity benefits of urban transportation megaprojects: A general equilibrium analysis of «Grand Paris Express»', *Transportation Research Part B: Methodological* **174**, 102746.

Anas, A. and Kim, I. (1996), 'General equilibrium models of polycentric urban land use with endogenous congestion and job agglomeration', *Journal of Urban Economics* **40**(2), 232–256.

Anas, A. and Liu, Y. (2007), 'A regional economy, land use, and transportation model (RELU-TRAN©): formulation, algorithm design, and testing', *Journal of Regional Science* **47**(3), 415–455.

Anupriya, Graham, D. J. and Bansal, P. (2023), 'Quantification of non-linear effects in agglomeration economies for transport appraisals'. Paper presented at the 16th World Conference on Transport Research, 2023, Montreal, Canada.

Arnott, R. (2007), 'Congestion tolling with agglomeration externalities', *Journal of Urban Economics* **2**(62), 187–203.

Arnott, R. (2013), 'A bathtub model of downtown traffic congestion', *Journal of Urban Economics* **76**, 110–121.

Balbontin, C., Hensher, D. A. and Beck, M. J. (2024), 'The influence of working from home and underlying attitudes on the number of commuting and non-commuting trips by workers during 2020 and 2021 pre-and post-lockdown in australia', *Transportation Research Part A: Policy and Practice* **179**, 103937.

Becker, G. S. (1965), 'A theory of the allocation of time', *The Economic Journal* pp. 493–517.

Combes, P.-P., Duranton, G. and Gobillon, L. (2019), 'The costs of agglomeration: House and land prices in French cities', *The Review of Economic Studies* **86**(4), 1556–1589.

Daganzo, C. F. and Geroliminis, N. (2008), 'An analytical approximation for the macroscopic fundamental diagram of urban traffic', *Transportation Research Part B: Methodological* **42**(9), 771–781.

Davis, M. A. and Ortalo-Magné, F. (2011), 'Household expenditures, wages, rents', *Review of Economic Dynamics* **14**(2), 248–261.

De Borger, B. and Van Dender, K. (2003), 'Transport tax reform, commuting, and endogenous values of time', *Journal of Urban Economics* **53**(3), 510–530.

De Borger, B. and Wuyts, B. (2011), 'The structure of the labor market, telecommuting, and optimal peak period congestion tolls: A numerical optimization model', *Regional Science and Urban Economics* **41**(5), 426–438.

Delventhal, M. J., Kwon, E. and Parkhomenko, A. (2022), 'Jue insight: How do cities change when we work from home?', *Journal of Urban Economics* **127**, 103331.

Delventhal, M. and Parkhomenko, A. (2023), 'Spatial implications of telecommuting', *Available at SSRN 3746555* .

Dericks, G. H. and Koster, H. R. (2021), 'The billion pound drop: the Blitz and agglomeration economies in London', *Journal of Economic Geography* **21**(6), 869–897.

DeSerpa, A. C. (1971), 'A theory of the economics of time', *The Economic Journal* **81**(324), 828–846.

Doffkay, R. (2024), Urban sprawl and public transport: A quantitative spatial model for a railway policy in Budapest, MSc Transport Economics Thesis, Technische Universität Dresden, Dresden, Germany.

Donaldson, D. (2018), 'Railroads of the raj: Estimating the impact of transportation infrastructure', *American Economic Review* **108**(4-5), 899–934.

Duranton, G. and Puga, D. (2004), Micro-foundations of urban agglomeration economies, *in* 'Handbook of regional and urban economics', Vol. 4, Elsevier, pp. 2063–2117.

Eaton, J. and Kortum, S. (2002), 'Technology, geography, and trade', *Econometrica* **70**(5), 1741–1779.

Eliasson, J. and Fosgerau, M. (2019), 'Cost-benefit analysis of transport improvements in the presence of spillovers, matching and an income tax', *Economics of Transportation* **18**, 1–9.

Fajgelbaum, P. D., Gaubert, C., Gorton, N., Morales, E. and Schaal, E. (2023), Political preferences and the spatial distribuion of intrastructure: Evidence from California's High-Speed Rail, Technical report, NBER Working Paper Number 31438.

Fajgelbaum, P. D. and Schaal, E. (2020), 'Optimal transport networks in spatial equilibrium', *Econometrica* **88**(4), 1411–1452.

Glaister, S. (1974), 'Generalised consumer surplus and public transport pricing', *The Economic Journal* **84**(336), 849–867.

Graham, D. J. (2025), 'Causal inference for transport research', *Transportation Research Part A: Policy and Practice* **192**, 104324.

Graham, D. J. and Gibbons, S. (2019), 'Quantifying Wider Economic Impacts of agglomeration for transport appraisal: existing evidence and future directions', *Economics of Transportation* **19**, 100121.

Head, K. and Mayer, T. (2014), Gravity equations: Workhorse, toolkit, and cookbook, *in* 'Handbook of International Economics', Vol. 4, Elsevier, pp. 131–195.

Heblich, S., Redding, S. J. and Sturm, D. M. (2020), 'The making of the modern metropolis: evidence from london', *The Quarterly Journal of Economics* **135**(4), 2059–2133.

Hess, S. and Daly, A. (2014), *Handbook of Choice Modelling*, Edward Elgar Publishing.

Hörcher, D., De Borger, B., Seifu, W. and Graham, D. J. (2020), 'Public transport provision under agglomeration economies', *Regional Science and Urban Economics* **81**, 103503.

Hörcher, D. and Graham, D. J. (2024), 'Quantitative spatial economics and transport research: A review of theoretical and causal econometric model ingredients'. Working Paper, Imperial College London.

Hunt, J. D. and Simmonds, D. C. (1993), 'Theory and application of an integrated land-use and transport modelling framework', *Environment and Planning B: Planning and Design* **20**(2), 221–244.

Jara-Díaz, S. (2007), *Transport Economic Theory*, Emerald Group Publishing Limited.

Jara-Díaz, S. (2020), 'Transport and time use: The values of leisure, work and travel', *Transport Policy* **86**, A7–A13.

Koster, H. R. (2024), 'The welfare effects of greenbelt policy: Evidence from england', *The Economic Journal* **134**(657), 363–401.

Kung, K. S., Greco, K., Sobolevsky, S. and Ratti, C. (2014), 'Exploring universal patterns in human home-work commuting from mobile phone data', *PloS one* **9**(6), e96180.

Lowry, I. S. (1964), A model of metropolis, Technical report, Rand Corporation Santa Monica.

McFadden, D. (1978), Modelling the choice of residential location, *in* A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull, eds, 'Spatial Interaction Theory and Residential Location', North Holland, Amsterdam, p. 75–96.

Miyauchi, Y., Nakajima, K. and Redding, S. J. (2022), The economics and spatial mobility: Theory and evidence using smartphone data, Technical report, NBER Working Paper Number 28497.

Monte, F., Redding, S. J. and Rossi-Hansberg, E. (2018), 'Commuting, migration, and local employment elasticities', *American Economic Review* **108**(12), 3855–90.

Parry, I. W. and Bento, A. (2001), 'Revenue recycling and the welfare effects of road pricing', *Scandinavian Journal of Economics* **103**(4), 645–671.

Paulley, N. J. and Webster, F. V. (1991), 'Overview of an international study to compare models and evaluate land-use and transport policies', *Transport Reviews* **11**(3), 197–222.

Pereira, R. H. M., Saraiva, M., Herszenhut, D., Braga, C. K. V. and Conway, M. W. (2021), 'r5r: Rapid realistic routing on multimodal transport networks with r5 in r', *Findings* .

Proost, S. and Thisse, J.-F. (2019), 'What can be learned from spatial economics?', *Journal of Economic Literature* **57**(3), 575–643.

Redding, S. J. and Rossi-Hansberg, E. (2017), 'Quantitative spatial economics', *Annual Review of Economics* **9**, 21–58.

Robson, E. N., Wijayaratna, K. P. and Dixit, V. V. (2018), 'A review of computable general equilibrium models for transport and their applications in appraisal', *Transportation Research Part A: Policy and Practice* **116**, 31–53.

Santos Silva, J. M. C. and Tenreyro, S. (2006), 'The log of gravity', *The Review of Economics and statistics* **88**(4), 641–658.

Small, K. A. (2012), 'Valuation of travel time', *Economics of Transportation* **1**(1-2), 2–14.

Small, K. A. and Verhoef, E. T. (2007), *The Economics of Urban Transportation*, Routledge.

Tikoudis, I., Verhoef, E. T. and van Ommeren, J. N. (2015), 'On revenue recycling and the welfare effects of second-best congestion pricing in a monocentric city', *Journal of Urban Economics* **89**, 32–47.

Tsivanidis, N. (2023), ‘Evaluating the impact of urban transit infrastructure: Evidence from Bogotá’s TransMilenio’. Working Paper, University of California, Berkeley.

Valentinyi, A. and Herrendorf, B. (2008), ‘Measuring factor income shares at the sectoral level’, *Review of Economic Dynamics* **11**(4), 820–835.

Venables, A. J. (2007), ‘Evaluating urban transport improvements: Cost–benefit analysis in the presence of agglomeration and income taxation’, *Journal of Transport Economics and Policy* **41**(2), 173–188.

Wardman, M., Chintakayala, V. P. K. and de Jong, G. (2016), ‘Values of travel time in europe: Review and meta-analysis’, *Transportation Research Part A: Policy and Practice* **94**, 93–111.

Wegener, M. (2011), Transport in spatial models of economic development, *in* A. de Palma et al., eds, ‘A Handbook on Transport Economics’, Edward Elgar Cheltenham, Glos, pp. 46–66.

Wrigley, E. A. (2011), *The Early English Censuses*, Oxford University Press.

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