Solving the Consolidation Problem

Now we mean business

Knowing Your Enemy

Now that we know better, let's look again at the consolidation problem:

$$\arg \min_{x} \; \|z\|_1$$

$$\text{subject to: } Vx = v^*, Ex = e^*$$

$$x \leq Mz$$

$$x \geq 0, z \in \{0, 1\}^n$$

Where do you think most of the complexity will stem from?

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Where do you think most of the complexity will stem from?

- We need to branch only on the integer variables
- So, their number will likely have an impact on complexity

Since we focus on the current Path formulation solution, they won't be many

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What could you say of the impact of using big-Ms?

- Logically, they work just fine
- In practice, they can lead to poor bounds in the LP relaxation

Ideally, they should be avoided. Failing that, use an M as small as possible

The Solution Code

The code for solving the problem is in the `consolidate_paths` function

The function parameter look similar to those of `solve_path_selection_full`

```python
def consolidate_paths(
        tug : ig.Graph,
        paths : list,
        node_counts : dict,
        arc_counts : dict,
        tlim : int = None):
```

However, they are meant to be used differently:

- `paths` should contain those selected by the Path formulation

- `node_counts` should contain the counts from the Path formulation solution

- ...And the same goes for `arc_counts`

`tlim` is a time limit: always use one when dealing with NP-hard problems

The Solution Code

Let's see some relevant code snippets:

We use the CBC solver, via the Google Or-Tools Wrapper

```python
slv = pywraplp.Solver.CreateSolver('CBC')
```

Variables are built using the solver object and stored in lists:

```python
x = [slv.NumVar(0, inf, f'x_{j}') for j in range(npaths)]
z = [slv.IntVar(0, 1, f'z_{j}') for j in range(npaths)]
```

For the big-M constraints ($x \leq Mz$) we use the largest node count

```python
M = max(v for v in node_counts.values())
for j in range(npaths):
    slv.Add(x[j] <= M * z[j])
```

- There no need for a path to use a flow larger than that

The Solution Code

Let's see some relevant code snippets:

Here's the code for the "count matching" constraints, i.e. $Vx = \hat{v}$ and $Ex = \hat{e}$:

```python
for n, p in paths_by_node.items():
    slv.Add(sum(x[j] for j in p) == node_counts[n])
for a, p in paths_by_arc.items():
    slv.Add(sum(x[j] for j in p) == arc_counts[a])
```

■ We rely on a previous step where we grouped path by used node/arc

Here's how we define the objective and optimization direction:

```python
slv.Minimize(sum(z[j] for j in range(npaths)))
```

...And here how to set a time limit:

```python
if tlim is not None: slv.SetTimeLimit(tlim)
```

The Solution Code

Let's see some relevant code snippets:

We trigger the solution process with the `solve` method:

```python
status = slv.Solve()
```

The method returns an integer status code, that should always be checked:

```python
if status in (slv.OPTIMAL, slv.FEASIBLE):
    # Extract the paths in the solution

    ...

    # Return the solution
    if status == slv.OPTIMAL: return sol_flows, sol_paths, True
    else: return sol_flows, sol_paths, False
else:
    return None, None, False
```

Solving the Problem

We can finally solve the consolidation problem for real:

```python
In [2]: node_counts_r, arc_counts_r = util.get_counts(tug, rflows, rpaths)
        cflows, cpaths, cflag = util.consolidate_paths(tug, rpaths, node_counts, arc_counts)
        print('FLOW: PATH')
        util.print_solution(tug, cflows, cpaths, sort='descending')
        print(f'Optimal: {cflag}')

        FLOW: PATH
        8.17: 2,3 > 3,3
        5.47: 0,2 > 1,2 > 2,2 > 3,2
        4.89: 0,1 > 1,1 > 2,0 > 3,0
        3.74: 3,3
        3.32: 1,0 > 2,0 > 3,2
        Optimal: True
```

In our case, the consolidated paths match the ground truth perfectly!

```python
In [3]: print('FLOW: PATH')
        util.print_ground_truth(flows, paths, sort='descending')

        FLOW: PATH
        8.17: 2,3 > 3,3
        5.47: 0,2 > 1,2 > 2,2 > 3,2
        4.89: 0,1 > 1,1 > 2,0 > 3,0
```