

$\omega_{k,l}$	Polinomial	Contributions
$\omega_{0,0}$	$1 - 3x^2 + 2x^3$	D2, D6, D8
$\omega_{0,1}$	$x - 3x^3 + 2x^4$	D4, D8, D10
$\omega_{0,2}$	$x^2 - 3x^4 + 2x^5$	D6, D10, D12
$\omega_{1,0}$	$1 - x - 3x^2 + 5x^3 - 2x^4$	D2, D4, D6, D8, D10
$\omega_{1,1}$	$x - x^2 - 3x^3 + 5x^4 - 2x^5$	D4, D6, D8, D10, D12
$\omega_{1,2}$	$x^2 - x^3 - 3x^4 + 5x^5 - 2x^6$	D6, D8, D10, D12, D14
$\omega_{1,3}$	$x^3 - x^4 - 3x^5 + 5x^6 - 2x^7$	D8, D10, D12, D14, D16
$\omega_{2,0}$	$1 - 2x - 2x^2 + 8x^3 + 2x^5$	D2, D4, D6, D8, D12
$\omega_{2,1}$	$x - 2x^2 - 2x^3 + 8x^4 + 2x^6$	D4, D6, D8, D10, D14
$\omega_{2,2}$	$x^2 - 2x^3 - 2x^4 + 8x^5 + 2x^7$	D6, D8, D10, D12, D16
$\omega_{2,3}$	$x^3 - 2x^4 - 2x^5 + 8x^6 + 2x^8$	D8, D10, D12, D14, D18
$\omega_{3,0}$	$1 - 3x + 10x^3 + 15x^4 + 9x^5 - 2x^6$	D2, D4, D8, D10, D12, D14

w	Factored	Polinomial	Contributions
wSquare	$(1 - 2x)^2(1 + 2x)$	$1 - 3x^2 + 2x^3$	D2, D6, D8
wCube	$(1 - 2x)^3(1 + 3x)$	$1 - 6x^2 + 8x^3 - 3x^4$	D2, D6, D8, D10
wQuartic	$(1 - 2x)^4(1 + 4x)$	$1 - 10x^2 + 20x^3 - 15x^4 + 4x^5$	D2, D6, D8, D10, D12

Horrible Chi2/dof values. Same for w03, which has not been displayed here. Seems to be difficult to fit D8 solo.

Good Chi2 values (except 9 and 7). Values from -0.12 to -0.14. Fitted only D6 because D8 is not sensible for w03.

**Remarks on Table 5:** Chi2/dof (0.102) to small?.  $\alpha_s = 0.319$  0.319 is in good agreement.  $\langle aGGInv = -0.0048 - 0.003 \rangle$  is close and within error range.  $C_{V+A}^{(6)} = 1.7$  0.9 is slightly out of error range.  $C_{V+A}^{(6)} = -1.6 - 1.0$  is within error range. All in all, due to the uncertainties in measuring the condensate contributions we have achieved a comparable result.<sup>1</sup>

**Remarks on Table 6:** Excellent Chi2.  $\alpha_s = 0.319$  0.319. No degrees of freedom.  $\alpha_s$  differs b 0.01 and D6 differs slightly to the results obtained by me for Pich Table 1. Errors are smaller on D6 and D8, but not for  $\alpha_s$ .

**Remarks on Table 7:** Fits with 8, 7, 6 or 5 moments have stable D6 and D8 contributions with a good to acceptable  $\chi^2$ .  $c6 \approx 0.248$ ,  $c8 \approx 0.83$ .  $c6$  and  $c8$  have negative sign.

**Remarks on Table 8:**  $\alpha_s$ , D6 and D8 stable.  $\chi^2$  too small.  $\alpha_s \approx 0.323$ .

<sup>1</sup>We are parametrising the D6 and D8 contributions as:  $\rho_{VpA} = -10^2 \mathcal{O}_{6,VpA}$  and  $c_{8,VpA} = 10^2 \mathcal{O}_{8,VpA}$ .

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	-0.144(58)	0.25(12)	1.50
8	2.050	-0.150(65)	0.23(15)	1.74
7	2.150	-0.189(74)	0.12(18)	1.83
6	2.250	-0.131(82)	0.30(21)	1.59
5	2.350	-0.076(97)	0.49(27)	1.76
4	2.500	-0.099(14)	1.14(46)	1.02
3	2.700	-0.45(28)	2.6(1.1)	0.36

Table 1: Fits to kinematic weight  $w_\tau = (1-x)^2(1+2x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
6	2.250	0.487(82)	1.26(21)	4.06
5	2.350	0.601(97)	1.64(27)	3.86
4	2.500	0.87(14)	2.67(46)	1.82

Table 2: Fits to kinematic weight  $w_\tau = (1-x)^2(1+2x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	0.2281(87)	32.71
8	2.050	0.276(10)	27.50
7	2.150	0.330(12)	125.50
6	2.250	0.360(15)	112.55
5	2.350	0.439(20)	19.61
4	2.500	0.559(29)	45.60
3	2.700	0.785(50)	15.08

Table 3: Fits to weight  $w_\tau = (1-x)^2 x(1+2x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $C_{V+A}^{(6)} = -0.15$  fixed.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$\chi^2/dof$
9	1.975	-0.1201(96)	2.34
8	2.050	-0.138(11)	0.44
7	2.150	-0.149(12)	1.83
6	2.250	-0.141(14)	0.33
5	2.350	-0.144(18)	0.39
4	2.500	-0.27(1.14)	0.52
3	2.700	-0.169(38)	0.48

Table 4: Fits to weight  $w_\tau = (1-x)^2 x^2 (1+2x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $C_{V+A}^{(8)} = 0.24$  fixed.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

$\alpha_s(m_{tau})$	$\langle aGGInv \rangle$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
0.3194(23)	-0.0048(27)	-0.177(47)	-0.160(65)	0.102

Table 5: Fits to multiple weights  $(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$ . Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

$\alpha_s(m_{tau})$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$
0.3303(52)	-0.26(13)	-0.17(16)

Table 6: Fits to multiple weights  $(k, l) = (0, 0), (0, 1), (0, 2)$ . Fixed  $\langle aGGInv \rangle = 0.021$ . Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	-0.237(14)	-0.1222(16)	2.31
8	2.050	-0.250(21)	-0.084(21)	1.04
7	2.150	-0.250(25)	-0.085(26)	1.25
6	2.250	-0.245(29)	-0.080(33)	1.54
5	2.350	-0.242(36)	-0.074(45)	2.04
4	2.500	-0.190(75)	0.012(75)	2.03
3	2.700	-0.064(81)	0.27(15)	0.00

Table 7: Fits to cube weight  $w_\tau = (1-x)^3(1+3x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$a(m_\tau^2)$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	0.3245(24)	-0.346(30)	-0.160(22)	1.45
8	2.050	0.3224(26)	-0.300(37)	-0.118(29)	0.61
7	2.150	0.3233(27)	-0.319(44)	-0.137(37)	0.61
6	2.250	0.3239(30)	-0.336(54)	-0.155(49)	0.72
5	2.350	0.3264(36)	-0.406(80)	-0.230(81)	0.38
4	2.500	0.3312(65)	-0.56(19)	-0.41(22)	0.00

Table 8: Fits to cubic weight  $w_\tau = (1-x)^3(1+3x)$ .  $\langle aGG \rangle_{Inv} = 0.021$  fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$-10^2 C_{V+A}^{(10)}$	$10^2 C_{V+A}^{(12)}$	$\chi^2/dof$
9	1.975	-0.138(19)	0.208(34)	-0.18(11)	0.435(98)	1.82
8	2.050	-0.199(22)	0.038(44)	-0.08(14)	0.17(12)	1.40
7	2.150	-0.167(25)	0.133(05)	0.17(20)	0.30(16)	1.65
6	2.250	-0.138(31)	0.226(72)	0.30(28)	0.41(24)	2.27
5	2.350	-0.01(13)	0.72(40)	1.0(8.5)	1.0(7.0)	2.05

Table 9: Fits to cubic weight  $w_\tau = (1-x)^3(1+3x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. c10 and c12 variable. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	-0.19644(79)	-0.06017(44)	2.01
8	2.050	-0.2061(77)	-0.0668(46)	1.34
7	2.150	-0.2070(99)	-0.067(63)	1.61
6	2.250	-0.209(12)	-0.0691(84)	1.99
5	2.350	-0.242(36)	-0.074(45)	2.04
4	2.500	-0.213(16)	-0.072(13)	2.62
3	2.700	-0.123(39)	0.038(44)	0.00

Table 10: Fits to quartic weight  $w_\tau = (1-x)^4(1+4x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed to PDG(2016) value.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

c6 and c8 have negative sign.

**Remarks on Table 9:** c8 has changed sign in comparison to fits with fixed D10/D12 contribution. c8 also became unstable.  $\chi^2$  are bigger ( $\approx 2$ ). D10 and D12 are not stable.  $\delta^{(10/12)}$  are null up to e-6?

# points	$s_0^{min}$	$a(m_\tau^2)$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$\chi^2/dof$
9	1.975	0.3201(22)	-0.205(11)	-0.0628(47)	2.17
8	2.050	0.3215(11)	-0.2222(35)	-0.0721(23)	1.08
7	2.150	0.3220(19)	-0.228(12)	-0.0757(62)	1.23
6	2.250	0.3227(24)	-0.239(19)	-0.082(10)	1.37
5	2.350	0.3243(27)	-0.265(28)	-0.101(17)	1.20
4	2.500	0.3281(37)	-0.339(56)	-0.159(42)	0.00

Table 11: Fits to quartic weight  $w_\tau = (1-x)^4(1+4x)$ .  $\langle aGG \rangle_{Inv} = 0.021$  fixed. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.

# points	$s_0^{min}$	$-10^2 C_{V+A}^{(6)}$	$10^2 C_{V+A}^{(8)}$	$-10^2 C_{V+A}^{(10)}$	$10^2 C_{V+A}^{(12)}$	$\chi^2/dof$
9	1.975	-0.2339(68)	-0.1109(76)	-0.1829(75)	0.1169(25)	2.32
8	2.050	-0.2063(76)	-0.0671(88)	-0.097(15)	0.081(13)	2.02
7	2.150	-0.2027(97)	-0.061(12)	-0.092(21)	0.083(17)	2.67
6	2.250	-0.188(12)	-0.336(16)	-0.158(30)	0.167(25)	3.83
5	2.350	-0.128(16)	0.092(25)	0.017(50)	0.171(42)	5.60

Table 12: Fits to quartic weight  $w_\tau = (1-x)^3(1+3x)$ .  $\alpha_s(m_\tau) = 0.3179$  fixed.  $\langle aGG \rangle_{Inv} = 0.021$  also fixed. c10 and c12 variable. Resummation scheme: FOPT. No DV's included.  $R_{\tau,V+A}$  uncorrelated rescaled to 3.4718.