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Anomalous dimensions of 4-quark operators and renormalon structure of mesonic 2-point correlators

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Abstract: In this work ...

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Contents

1 Introduction

The perturbative expansion in QCD is known to lead to a divergent series which is at best asymptotic. The asymptotic behaviour of the perturbative series manifests itself in the appearance of singularities for its Borel transform which lie on the negative or positive real axis in the Borel variable. Those singularities connected with renormalisation of the theory are termed *renormalons* [1, 2]. More specifically, the ones on the negative real axis are called ultraviolet (UV) renormalons and those on the positive real Borel axis infrared (IR) renormalons.

The presence of IR renormalon poles leads to ambiguities in the definition of the full function which is related to the perturbative series, because the Borel resummation (inverse Borel transform) entails to perform an integral over the positive real Borel axis which naively is not well defined. Associated with the ambiguities in the definition of the Borel integral is the appearance of higher-dimensional operator corrections, the so-called *QCD condensates*, such that the full function is unambiguous. The operators that display renormalon ambiguities are a subset of those that arise in the framework of the operator product expansion (OPE).

Limiting ourselves to correlation functions of vector or axialvector currents with respect to the QCD vacuum, the renormalon pole on the positive real axis closest to the origin of the Borel plane is associated to the vacuum matrix element of one dimension-4 operator, the *gluon consensate*. The next-closest singularity then is found to correspond to the dimension-6 *triple gluon condensate* and a set of dimension-6 *4-quark condensates*. It is these latter 4-quark condensates that we intend to investigate in more detail in the present work.

2 Anomalous dimensions for 4-quark operators

Operator basis:

$$Q_V^o = (\bar{u}\gamma_\mu t^a d \bar{d}\gamma^\mu t^a u)\,, \quad Q_A^o = (\bar{u}\gamma_\mu\gamma_5 t^a d \bar{d}\gamma^\mu\gamma_5 t^a u)\,, \tag{2.1}$$

$$Q_V^s = (\bar{u}\gamma_\mu d \bar{d}\gamma^\mu u)\,, \quad Q_A^s = (\bar{u}\gamma_\mu\gamma_5 d \bar{d}\gamma^\mu\gamma_5 u)\,, \tag{2.2}$$

$$Q_3 \equiv (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu t^a q)\,, \tag{2.3}$$

$$Q_4 \equiv (\bar{u}\gamma_\mu\gamma_5 t^a u + \bar{d}\gamma_\mu\gamma_5 t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu\gamma_5 t^a q)\,, \tag{2.4}$$

$$Q_5 \equiv (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu q)\,, \tag{2.5}$$

$$Q_6 \equiv (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu\gamma_5 q)\,, \tag{2.6}$$

$$Q_7 \equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu t^a q')\,, \tag{2.7}$$

$$Q_8 \equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu\gamma_5 t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu\gamma_5 t^a q')\,, \tag{2.8}$$

$$Q_9 \equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu q')\,, \tag{2.9}$$

$$Q_{10} \equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu\gamma_5 q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu\gamma_5 q')\,. \tag{2.10}$$

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Appendix A: Anomalous dimensions of 4-quark operators

In this Appendix, we present a generalisation of the results of reference [3] to an arbitrary number N_c of colour degrees of freedom. In [3], the leading order anomalous dimension matrix of a complete set of local spin-zero four-quark operators without derivatives was calculated in the case of three quark flavours.

The complete basis consists of 45 four-quark operators which in reference [3] were chosen as follows: with respect to the Dirac-structure, there are five types of operators, namely, scalar, pseudoscalar, vector, axialvector and tensor. They can be expressed as

$$\bar{u}\Gamma u\bar{d}\Gamma d = \left(\bar{u}u\bar{d}d,\ \bar{u}\gamma_5 u\bar{d}\gamma_5 d,\ \bar{u}\gamma_\mu u\bar{d}\gamma^\mu d,\ \bar{u}\gamma_\mu\gamma_5 u\bar{d}\gamma^\mu\gamma_5 d,\ \bar{u}\sigma_{\mu\nu}u\bar{d}\sigma^{\mu\nu}d\right) \tag{A.1}$$

in the $\bar{u}u\bar{d}d$ flavour case. Employing this notation, the complete basis O of operaptors can be chosen to be:

$$\begin{aligned} O \equiv \big(&\bar{u}\Gamma u\bar{u}\Gamma u,\ \bar{d}\Gamma d\bar{d}\Gamma d,\ \bar{s}\Gamma s\bar{s}\Gamma s,\ \bar{u}\Gamma u\bar{d}\Gamma d,\ \bar{u}\Gamma u\bar{s}\Gamma s,\ \bar{d}\Gamma d\bar{s}\Gamma s,\\ &\bar{u}\Gamma t^a u\bar{d}\Gamma t^a d,\ \bar{u}\Gamma t^a u\bar{s}\Gamma t^a s,\ \bar{d}\Gamma t^a d\bar{s}\Gamma t^a s\big)\,. \end{aligned} \tag{A.2}$$

In this basis, the leading order anomalous dimension matrix takes the form

$$\gamma_O^{(1)} = \begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 & B & B & 0 \\ 0 & A & 0 & 0 & 0 & 0 & B & 0 & B \\ 0 & 0 & A & 0 & 0 & 0 & 0 & B & B \\ 0 & 0 & 0 & C & 0 & 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 & C & 0 & 0 & D & 0 \\ 0 & 0 & 0 & 0 & 0 & C & 0 & 0 & D \\ E & E & 0 & F & 0 & 0 & G & H & H \\ E & 0 & E & 0 & F & 0 & H & G & H \\ 0 & E & E & 0 & 0 & F & H & H & G \end{pmatrix}. \tag{A.3}$$

The submatrices are given by:

$$A = \begin{pmatrix} \frac{11}{12} - 3C_F & \frac{7}{12} & -\frac{1}{12} + \frac{1}{6N_c} & -\frac{1}{12} & -\frac{1}{8} + \frac{1}{4N_c} \\ \frac{7}{12} & \frac{11}{12} - 3C_F & \frac{1}{12} - \frac{1}{6N_c} & \frac{1}{12} & -\frac{1}{8} + \frac{1}{4N_c} \\ \frac{7}{6} & -\frac{7}{6} & \frac{11}{12} - \frac{1}{3N_c} & \frac{11}{12} - \frac{3}{2N_c} & 0 \\ -\frac{11}{6} & \frac{11}{6} & \frac{11}{12} - \frac{11}{6N_c} & \frac{11}{12} & 0 \\ 3 + \frac{6}{N_c} & 3 + \frac{6}{N_c} & 0 & 0 & \frac{3}{2} + C_F \end{pmatrix}, \tag{A.4}$$

$$B = \begin{pmatrix} 0 & 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} -3C_F & 0 & 0 & 0 & 0 \\ 0 & -3C_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_F \end{pmatrix}, \tag{A.5}$$

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ -12 & -12 & 0 & 0 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{12} - \frac{1}{6N_c} & \frac{1}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.6}$$

$$G = \begin{pmatrix} \frac{3}{2N_c} & 0 & 0 & 0 & -\frac{N_c}{8}+\frac{1}{2N_c} \\ 0 & \frac{3}{2N_c} & 0 & 0 & -\frac{N_c}{8}+\frac{1}{2N_c} \\ 0 & 0 & -\frac{3N_c}{4}+\frac{2}{3} & \frac{3N_c}{4}-\frac{3}{N_c} & 0 \\ 0 & 0 & \frac{3N_c}{4}-\frac{3}{N_c} & -\frac{3N_c}{4} & 0 \\ -3N_c+\frac{12}{N_c} & -3N_c+\frac{12}{N_c} & 0 & 0 & C_F-\frac{3N_c}{2} \end{pmatrix}, \qquad (A.7)$$

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (A.8)$$

In contrast to ref. [3], the matrices A, C and G already include the quark self-energy contributions depicted in figure 1c) of [3], such that they are gauge independent. (The corresponding matrices of [3] were given in the Feynman gauge without self-energy contribution.)

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