Anomalous-dimension matrices of four-quark operators

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We present one-loop calculations of anomalous-dimension matrices γ for four-quark operators. Flavor-space indexing determines the nature of γ. Four important possibilities are considered for operators with left-left, left-right, and right-right chiral structures.

I. INTRODUCTION

In deriving effective weak nonleptonic Hamiltonians[1–7] for W-mediated low-energy processes the W boson is decoupled in the QCD free-field limit [the SU(2)$\times$U(1) or (2,1) model]. The renormalization group of QCD is employed to establish the form of the effective Hamiltonian that results when W is decoupled in $SU(3)_C\times SU(2)\times U(1)$ [or (3,2,1)].

Decoupling of W in the (2,1) model introduces what we refer to as *fundamental* four-quark operators, i.e., those which are present in the effective Hamiltonian *prior* to QCD renormalization. Before the renormalization-group equation (RGE) can be employed one must calculate the sets of operators closed under renormalization and the associated anomalous-dimension matrices $\underline{\gamma}$, which are generated from fundamental four-quark operators.

In this paper we present calculations of the set of operators closed under renormalization and the associated anomalous-dimension matrices generated by fundamental four-quark operators which result from W and Z decouplings in the nonleptonic sector. These operators have the basic structure of a product of two local color-singlet currents. Three possible chiral structures arise, *L-L*, *L-R*, and *R-R*, where we use L (R) to denote a left- (right-) handed current. In W decoupling three possible flavor structures arise, $(\mathrm{FNC})_1\times(\mathrm{FNC})_2$, $(\mathrm{FNC})_1\times(\mathrm{FC})_2$, and $(\mathrm{FC})_1\times(\mathrm{FC})_2$, where F(N)C represents a flavor-(non) conserving current, and currents 1 and 2 are never the same. In Z decoupling two possible flavor structures can arise, $(\mathrm{FC})_1\times(\mathrm{FC})_2$ where currents 1 and 2 can be the same or different.

We perform the following 12 calculations which are useful in a variety of contexts, but which are sufficiently related to be usefully discussed together. With a flavor-indexed four-quark operator $(\bar{\psi}_{A'}\psi_{B'})(\bar{\psi}_{C'}\psi_{D'})$ we consider for *each chiral case (L-L, L-R, and R-R)*:

(i) $(A'\neq B')\neq(C'\neq D')$,

(ii) $(A'\neq B')\neq(C'=D')$,

(iii) $(A'=B')\neq(C'=D')$,

(iv) $(A'=B')=(C'=D')$.

All calculations are performed in the Landau gauge ($\alpha=0$). We employ dimensional regularization in $n=4+\epsilon$ dimensions and minimal subtraction (MS). This is done because of the simplicity of the resulting renormalization-group equations. In Sec. II the calculational method is described in detail and illustrated with examples. In Sec. III we summarize our findings, while in Sec. IV we make our concluding remarks.

II. THE METHOD

A. Generalities

One will in general find that a given four-quark operator $\mathscr{O}$ will generate a finite, closed QCD renormalization set which we shall write as the components of a vector $\vec{\mathscr{O}}$. The corresponding bare operators $\vec{\mathscr{O}}^{\,0}$ define a matrix renormalization constant $\underline{Z}$,

$$\vec{\mathscr{O}}^{\,0}=\underline{Z}\vec{\mathscr{O}} \ . \tag{1}$$

We shall find that at a one-loop level four-quark operators only generate four-quark operators (why this is so in our calculations will be made clear later). For this reason $\underline{Z}$ may be calculated by evaluation of $G_0^{(4\psi)}(\vec{\mathscr{O}}^{\,0})$, representing an unrenormalized amputated Green's function with four external quarks and an $\vec{\mathscr{O}}^{\,0}$ operator insertion. In other words each component $\mathscr{O}_i^0$ defines a four-quark vertex and $G_0^{(4\psi)}(\mathscr{O}_i^0)$ represents the sum of all Feynman diagrams with four amputated external quark lines, one $\mathscr{O}_i^0$ vertex, and all possible QCD complexities.

Equation (1) implies

$$G_0^{(4\psi)}(\vec{\mathcal{O}}^{\,0}) = Z_\psi^{-2}\underline{Z}G^{(4\psi)}(\vec{\mathcal{O}})\,. \tag{2}$$

At the one-loop level in the Landau gauge, $Z_\psi = 1$. It is not hard to show that in *minimal subtraction* (2) implies, at the one-loop level, the very simple result

$$G_{0(1)}^{(4\psi)}(\mathcal{O}_i^0)_p = Z_{ij}^{(2)}G_{(0)}^{(4\psi)}(\mathcal{O}_j)\,. \tag{3}$$

The left-hand side of (3) refers to the one-loop ϵ^{-1} pole term of $G_0^{(4\psi)}(\mathcal{O}_i^0)$ [hence the subscripts p and (1)]. $G_{(0)}^{(4\psi)}(\mathcal{O}_j)$ is the zero-loop term (hence the vertex factor defined by $\mathcal{O}_j$) of $G^{(4\psi)}(\mathcal{O}_j)$. $\underline{Z}$ has been expanded perturbatively

$$\underline{Z} = \underline{1} + \underline{Z}^{(2)} + O(g^4)\,. \tag{4}$$

The advantages of MS is thus apparent. All we need do is evaluate one-loop ϵ^{-1} pole terms of the operator insertions, and this is easy, particularly since we may use massless QCD for all Z calculations in MS (of course excepting Z_m, the quark mass renormalization constant).

Observe that Eq. (1) in no way *presumes* operator mixing (a 1×1 $\underline{Z}$ is certainly possible). Given a four-quark operator $\mathcal{O}_1^0$ the one-loop pole term is readily evaluated (as is the vertex factor it defines). If it cannot be expressed as a multiple of its own vertex factor we can always express it as a linear combination of vertex factors. From a new vertex factor we can easily reconstruct the operator which gives rise to it and calculate the amputated Green's function which has the new operator as an insertion.

The calculation proceeds in this well-defined way until no new operators appear. No operator basis is chosen, it is generated (one of course has the freedom to linearly transform or reorder a basis, as this is a matter of preference only).

The anomalous-dimension matrix $\underline{\gamma}$ is defined by

$$\underline{\gamma} = \mu\underline{Z}^{-1}\frac{d\underline{Z}}{d\mu}\,, \tag{5}$$

where μ is the subtraction point. For MS and dimensional regularization in $n = 4 + \epsilon$ dimensions we have the simple consequence that, at the one-loop level,

$$\underline{Z} = \underline{1} + \frac{1}{\epsilon}\underline{\gamma} + O(g^4)\,, \tag{6}$$

hence, from (4), we see that

$$\underline{\gamma} = \epsilon\underline{Z}^{(2)} + O(g^4)\,. \tag{7}$$

To obtain all the results we require we have found that only four generic calculations need be made. These are presented in the following subsection.

B. Four generic one-loop pole terms

Define four typical four-quark operators:

$$\begin{aligned}
T_1 &= (\bar\psi^i_{A'}\psi^i_{B'})_L(\bar\psi^j_{C'}\psi^j_{D'})_L\,,\\
T_2 &= (\bar\psi^j_{A'}\psi^i_{B'})_L(\bar\psi^i_{C'}\psi^j_{D'})_L\,,\\
T_3 &= (\bar\psi^i_{A'}\psi^i_{B'})_L(\bar\psi^j_{C'}\psi^j_{D'})_R\,,\\
T_4 &= (\bar\psi^i_{A'}\psi^j_{B'})_L(\bar\psi^j_{C'}\psi^i_{D'})_R\,,
\end{aligned} \tag{8}$$

where i and j are summed color indices. A', B', C', and D' are flavor indices. A standard shorthand notation is employed where

$$(\bar q q)_{L(R)} \equiv (\bar q\gamma^\mu a(a')q)\,, \tag{9a}$$

$$a(a') = \tfrac{1}{2}(1\mp\gamma_5)\,. \tag{9b}$$

Operators in (8) give zero-loop contributions to $G_0^{(4\psi)}(T_i^0)$, these are just their vertex factors of "Feynman rules" and we denote them by F_i.

Using a completely arbitrary external quark labeling scheme defined in Fig. 1 we find that

$$\begin{aligned}
F_1 &= D_0[\delta^{ij,kl}(e_1+e_2)+\delta^{il,kj}(e_3+e_4)]\,,\\
F_2 &= D_0[\delta^{il,kj}(e_1+e_2)+\delta^{ij,kl}(e_3+e_4)]\,,\\
F_3 &= [\delta^{ij,kl}(e_1D_1+e_2D_2)-\delta^{il,kj}(e_3D_3+e_4D_4)]\,,\\
F_4 &= [\delta^{il,kj}(e_1D_1+e_2D_2)-\delta^{ij,kl}(e_3D_3+e_4D_4)]\,.
\end{aligned} \tag{10}$$

The Dirac factors D_I are defined by

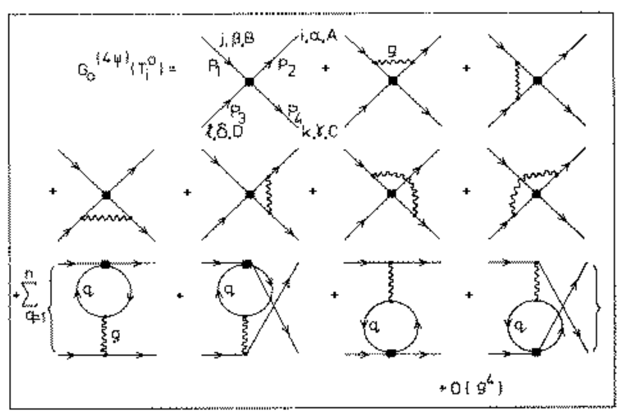

FIG. 1. The bare operator insertion $G_0^{(4\psi)}(T_i^0)$ evaluated to $O(g^2)$. The zero-loop term defines the vertex factor or Feynman rule F_i. The operator T_i^0 has been inserted (represented by ■) into this four-point function with external quarks arbitrarily labeled with color (i,j,k,l), spinor $(\alpha,\beta,\gamma,\delta)$, and flavor (A,B,C,D) indices. p_1, p_2, p_3, and p_4 are momentum labels. All crossed diagrams have been made explicit.

TABLE I. Flavor-space factors e_i, e_{ij}, and h_i. Here $\delta_{AB,CD,EF,\ldots}$ denotes $\delta_{AB}\delta_{CD}\delta_{EF}\cdots$.

$h_1=\delta_{AB}$	$h_2=\delta_{CD}$	$h_3=\delta_{AD}$	$h_4=\delta_{BC}$
$e_1=\delta_{AA',BB',CC',DD'}$	$e_2=\delta_{AC',BD',CA',DB'}$	$e_3=\delta_{AC',BB',CA',DD'}$	$e_4=\delta_{AA',BD',CC',DB'}$
$e_{11}=\delta_{AA',BB',C'D'}$	$e_{21}=\delta_{AC',BD',A'B'}$	$e_{31}=\delta_{AC',BB',A'D'}$	$e_{41}=\delta_{AA',BD',C'B'}$
$e_{12}=\delta_{CC',DD',A'B'}$	$e_{22}=\delta_{CA',DB',C'D'}$	$e_{32}=\delta_{CA',DD',C'B'}$	$e_{42}=\delta_{CC',DB',A'D'}$
$e_{13}=\delta_{BB',CC',A'D'}$	$e_{23}=\delta_{BD',CA',C'B'}$	$e_{33}=\delta_{BB',CA',C'D'}$	$e_{43}=\delta_{BD',CC',A'B'}$
$e_{14}=\delta_{AA',DD',B'C'}$	$e_{24}=\delta_{AC',DB',A'D'}$	$e_{34}=\delta_{AC',DD',A'B'}$	$e_{44}=\delta_{AA',DB',C'D'}$

$$
\begin{aligned}
&D_0=iL_{\alpha\beta}L_{\gamma\delta},\quad D_3=iL_{\gamma\beta}R_{\alpha\delta}\;,\\
&D_1=iL_{\alpha\beta}R_{\gamma\delta},\quad D_4=iL_{\alpha\delta}R_{\gamma\beta}\;,\\
&D_2=iL_{\gamma\delta}R_{\alpha\beta},\quad D_5=iR_{\alpha\beta}R_{\gamma\delta}\;.
\end{aligned}
\tag{11}
$$

Here $L_{\alpha\beta}L_{\gamma\delta}=(\gamma^\tau a)_{\alpha\beta}(\gamma_\tau a)_{\gamma\delta}$, etc. Flavor-space factors $e_i(h_i$ and $e_{ij})$ are defined in Table I. We employ a notation where

$$\delta_{AB,CD,EF,\ldots}=\delta_{AB}\delta_{CD}\delta_{EF}\cdots\;.$$

The arbitrary external quark labeling does not make calculations more difficult and is moreover an aid in identifying the newly generated operators.

It is clear from Fig. 1 that $G_0^{(4\psi)}(T_i^0)$ may be written as the sum of 1PI (one-particle irreducible) and 1PR (one-particle reducible) diagrams. We write this as

$$G_0^{(4\psi)}(T_i^0)=\Gamma_0^{(4\psi)}(T_i^0)+P(T_i^0)\;, \tag{12}$$

since at a one-loop level the 1PR diagrams are the penguin diagrams.

Having fixed the vertex factors F_i it is a straightforward task to evaluate the one-loop pole terms. The quark-gluon vertex factor is modified by inclusion of a flavor-conserving Kronecker δ, $\delta_{FF'}$, so that penguin terms automatically vanish when an inappropriate four-quark insertion occurs [for example, one in which $(A'\neq B')\neq(C'\neq D')$].

We define

$$W=\frac{1}{\epsilon}\frac{\tilde{g}_0{}^2}{8\pi^2}\;, \tag{13}$$

where $\tilde{g}_0$ is the dimensionless bare QCD coupling constant in $4+\epsilon$ dimensions, i.e.,

$$
\begin{aligned}
&\tilde{g}_0{}^2=\mu^\epsilon g_0{}^2\;,\\
&\tilde{g}_0=g+O(g^2)\;.
\end{aligned}
\tag{14}
$$

We then find using MS and the Landau ($\alpha=0$) gauge that

$$
\begin{aligned}
\Gamma_{0(1)}^{(4\psi)}(T_1^0)_p&=W\times 6D_0[(e_1+e_2)T_{ij}^aT_{kl}^a\\
&\qquad+(e_3+e_4)T_{il}^aT_{kj}^a]\;,\\
\Gamma_{0(1)}^{(4\psi)}(T_2^0)_p&=W\times 6D_0[(e_1+e_2)T_{il}^aT_{kj}^a\\
&\qquad+(e_3+e_4)T_{ij}^aT_{kl}^a]\;,\\
\Gamma_{0(1)}^{(4\psi)}(T_3^0)_p&=-W\times 6[(D_1e_1+D_2e_2)T_{ij}^aT_{kl}^a\\
&\qquad-(D_3e_3+D_4e_4)T_{il}^aT_{kj}^a]\;,\\
\Gamma_{0(1)}^{(4\psi)}(T_4^0)_p&=-W\times 8[(D_1e_1+D_2e_2)\delta^{il,kj}\\
&\qquad-(D_3e_3+D_4e_4)\delta^{ij,kl}]\;.
\end{aligned}
\tag{15}
$$

Since

$$T_{ij}^aT_{kl}^a=\tfrac{1}{2}(\delta_{il,kj}-\tfrac{1}{3}\delta_{ij,kl}) \tag{16}$$

we can easily show

$$
\begin{aligned}
\Gamma_{0(1)}^{(4\psi)}(T_1^0)_p&=W[(-1)F_1+(3)F_2]\;,\\
\Gamma_{0(1)}^{(4\psi)}(T_2^0)_p&=W[(3)F_1+(-1)F_2]\;,\\
F_{0(1)}^{(4\psi)}(T_3^0)_p&=W[(-3)F_4+(1)F_3]\;,\\
\Gamma_{0(1)}^{(4\psi)}(T_4^0)_p&=W[(-8)F_4]\;.
\end{aligned}
\tag{17}
$$

The summed penguin graphs $p(T_i^0)$ are found to give the pole terms

$$
\begin{aligned}
P(T_1^0)_p&=W\times\tfrac{2}{3}\{T_{ij}^aT_{kl}^a[h_2(e_{31}+e_{41})(D_0+D_1)+h_1(e_{32}+e_{42})(D_0+D_2)]\\
&\qquad-T_{il}^aT_{kj}^a[h_3(e_{13}+e_{23})(-D_0+D_3)+h_4(e_{14}+e_{24})(-D_0+D_4)]\}\;,\\
P(T_2^0)_p&=W\times\tfrac{2}{3}\{T_{ij}^aT_{kl}^a[h_2(e_{11}+e_{21})(D_0+D_1)+h_1(e_{12}+e_{22})(D_0+D_2)]\\
&\qquad-T_{il}^aT_{kj}^a[h_3(e_{33}+e_{43})(-D_0+D_3)+h_4(e_{34}+e_{44})(-D_0+D_4)]\}\;,\\
P(T_3^0)_p&=0\;,\\
P(T_4^0)_p&=W\times\tfrac{2}{3}(T_{ij}^aT_{kl}^a\{h_2[e_{11}(D_0+D_1)+e_{21}(D_2+D_5)]+h_1[e_{12}(D_1+D_5)+e_{22}(D_0+D_2)]\}\\
&\qquad-T_{il}^aT_{kj}^a\{h_3[e_{33}(-D_0+D_3)+e_{43}(D_4-D_5)]+h_4[e_{34}(D_3-D_5)+e_{44}(-D_0+D_4)]\})\;.
\end{aligned}
\tag{18}
$$

Flavor factors e_{ij} and h_i are given in Table I. Dirac factors in Eq. (11). General results analogous to (17) are not written down easily for $P(T_i^0)_p$. It is much easier to treat cases individually as they arise in our work, the example in Sec. II C will show this.

Observe that the penguin pole terms in (18) are momentum independent. Clearly the gluon propagator of the penguin diagram ought to induce momentum dependence in $P(T_i^0)_p$, but this we eliminate by application of the equations of motion of QCD. In our calculation this step is achieved as follows. $P(T_i^0)_p$ is part of an amputated Green's function. We momentarily attach the four external quark spinors, assume they correspond to *free* [hence are on-mass-shell (OMS)] *particles*, apply the Dirac equation which eliminates the momentum dependence, the external spinors are then amputated. This procedure is valid as long as we stipulate that matrix elements of generated operators must eventually be taken between OMS quarks and further should not be evaluated beyond lowest order in the four-quark vertex. Thus, for example, inclusion of wave-function effects in a decay process, strictly speaking, would prohibit the elimination of operators with field derivatives by use of an equation of motion.

The purpose of this last step is to reduce the size of closed renormalization sets. Looking at Eq. (3) one sees that retaining the momentum dependenee in $P(T_i^0)_p$ would have meant the generation of operators with field derivatives such as $(\bar\psi_{A'}\psi_{B'})^a_{L(R)}D_\nu F_a^{\mu\nu}$. These operators can be included, but they are then usually eliminated later. What we have done is to eliminate these operators while calculating $\underline{\gamma}$ rather than allowing them to be generated and then removing them at a later stage by the equation of motion.

Dawson, Hagelin, and Hall[8] have adopted a slightly different approach to calculating anomalous dimensions, yet the same problem arises in their work and the same solution is arrived at. The major conclusion to draw is that four-quark operators only generate four-quark operators upon QCD renormalization, within the stated restrictions.

C. Illustration of a $Z^{(2)}$ calculation

Consider a given operator $\mathscr{O}_1=(\bar\psi^i_{A'}\psi^i_{B'})_L \times(\bar\psi^j_{C'}\psi^j_{D'})_L$, where in this case we set $(A'=B')\neq(C'=D')$. It is sufficient to insert this operator into the diagrams of Fig. 1 with arbitrary external flavor indices such that $A=B, C=D$ (with the possibility of $A=C$ left open). The flavor-space factors h_i, e_i, e_{ij} then simplify. $\mathscr{O}_1$ is like T_1 of Eq. (8). Equation (17) implies that

$$\Gamma^{(4\psi)}_{0(1)}(\mathscr{O}^0_1)_p=W[(-1)F_1+(3)F_2]\ , \tag{19}$$

where $F_1=D_0(\delta_{AA',CC'}+\delta_{AC',CA'})\delta^{ij,kl}$ is the vertex factor of $\mathscr{O}_1$ and a new vertex factor F_2 has been generated, namely,

$$F_2=D_0(\delta_{AA',CC'}+\delta_{AC',CA'})\delta^{il,kj}\ . \tag{20}$$

The operator responsible for F_2 is found to be

$$\mathscr{O}_2=(\bar\psi^i_{A'}\psi^j_{B'})_L(\bar\psi^j_{C'}\psi^i_{D'})_L\ . \tag{21}$$

From (18) we learn that here

$$P(\mathscr{O}^0_1)_p=0\ , \tag{22}$$

and hence (12) implies

$$G^{(4\psi)}_{0(1)}(\mathscr{O}^0_1)=W[(-1)F_1+(3)F_2]\ . \tag{23}$$

Thus from (3) we conclude that

$$\begin{aligned} &Z^{(2)}_{11}=-W\ ,\\ &Z^{(2)}_{12}=3W\ ,\\ &Z^{(2)}_{1j}=0,\ \ j\geq 3\ . \end{aligned} \tag{24}$$

Since $\mathscr{O}_2$ has been generated we must now see how it renormalizes. $\mathscr{O}_2$ is a T_2-type operator [see (8)]. Equation (17) implies

$$\Gamma^{(4\psi)}_{0(1)}(\mathscr{O}^0_2)_p=W[(3)F_1+(-1)F_2]\ , \tag{25}$$

so that $\mathscr{O}^0_2$ has simply regenerated $\mathscr{O}^0_1$ and itself at this stage. However, from (18) we see that

$$P(\mathscr{O}^0_2)_p=W(\tfrac{1}{9})\{(3\delta_{il,kj}-\delta_{ij,kl})[(\delta_{AA'}+\delta_{AC'})(D_0+D_1)+(\delta_{CC'}+\delta_{CA'})(D_0+D_2)]$$
$$+(-3\delta_{ij,kl}+\delta_{il,kj})[(\delta_{AA',CA'}+\delta_{AC',CC'})(-D_0+D_3)+(\delta_{AC',CC'}+\delta_{AA',CA'})(-D_0+D_4)]\}\ . \tag{26}$$

Operators which give the right-hand side of (26) are constructed according to Dirac and color structures. They are

$$\mathscr{O}_3=(\bar{\psi}^i_{A'}\psi^i_{B'})_L\sum_{m=1}^{n}(\bar{\psi}^j_m\psi^j_m)_L\ ,$$

$$\mathscr{O}_4=(\bar{\psi}^i_{A'}\psi^j_{B'})_L\sum_{m=1}^{n}(\bar{\psi}^j_m\psi^i_m)_L\ ,$$

$$\mathscr{O}_5=\sum_{l=1}^{n}(\bar{\psi}^i_l\psi^i_l)_L(\bar{\psi}^j_{C'}\psi^j_{D'})_L\ ,$$

$$\mathscr{O}_6=\sum_{l=1}^{n}(\bar{\psi}^i_l\psi^j_l)_L(\bar{\psi}^j_{C'}\psi^i_{D'})_L\ ,$$

$$\mathscr{O}_7=(\bar{\psi}^i_{A'}\psi^i_{B'})_L\sum_{m=1}^{n}(\bar{\psi}^j_m\psi^j_m)_R\ ,$$

$$\mathscr{O}_8=(\bar{\psi}^i_{A'}\psi^j_{B'})_L\sum_{m=1}^{n}(\bar{\psi}^j_m\psi^i_m)_R\ ,$$

$$\mathscr{O}_9=\sum_{l=1}^{n}(\bar{\psi}^i_l\psi^i_l)_R(\bar{\psi}^j_{C'}\psi^j_{D'})_L\ ,$$

$$\mathscr{O}_{10}=\sum_{l=1}^{n}(\bar{\psi}^i_l\psi^j_l)_R(\bar{\psi}^j_{C'}\psi^i_{D'})_L\ , \tag{27}$$

where the summations on l and m extend over all n flavors. The corresponding vertex factors are

$$
\begin{aligned}
F_3&=D_0[(\delta_{AA'}+\delta_{CA'})\delta^{ij,kl}+(2\delta_{AA',CA'})\delta^{il,kj}]\ ,\\
F_4&=D_0[(\delta_{AA'}+\delta_{CA'})\delta^{il,kj}+(2\delta_{AA',CA'})\delta^{ij,kl}]\ ,\\
F_5&=D_0[(\delta_{CC'}+\delta_{AC'})\delta^{ij,kl}+(2\delta_{AC',CC'})\delta^{il,kj}]\ ,\\
F_6&=D_0[(\delta_{CC'}+\delta_{AC'})\delta^{il,kj}+(2\delta_{AC',CC'})\delta^{ij,kl}]\ ,\\
F_7&=[(D_1\delta_{AA'}+D_2\delta_{CA'})\delta^{ij,kl}-(\delta_{AA',CA'})(D_3+D_4)\delta^{il,kj}]\ ,\\
F_8&=[(D_1\delta_{AA'}+D_2\delta_{CA'})\delta^{il,kj}-(\delta_{AA',CA'})(D_3+D_4)\delta^{ij,kl}]\ ,\\
F_9&=[(D_2\delta_{CC'}+D_1\delta_{AC'})\delta^{ij,kl}-(\delta_{AC',CC'})(D_3+D_4)\delta^{il,kj}]\ ,\\
F_{10}&=[(D_2\delta_{CC'}+D_1\delta_{AC'})\delta^{il,kj}-(\delta_{AC',CC'})(D_3+D_4)\delta^{ij,kl}]\ .
\end{aligned}
\tag{28}
$$

In terms of these vertex factors we can rewrite (26) as

$$P(\mathscr{O}_2^0)_p=\frac{W}{9}[(-1)F_3+(3)F_4+(-1)F_5+(3)F_6+(-1)F_7+(3)F_8+(-1)F_9+(3)F_{10}] \tag{29}$$

and with (25) we conclude that

$$
\begin{aligned}
&Z_{21}^{(2)}=3W,\ \ Z_{25}^{(2)}=-\tfrac{1}{9}W,\ \ Z_{29}^{(2)}=-\tfrac{1}{9}W\ ,\\
&Z_{22}^{(2)}=-W,\ \ Z_{26}^{(2)}=\tfrac{1}{3}W,\ \ Z_{2,10}^{(2)}=\tfrac{1}{3}W\ ,\\
&Z_{23}^{(2)}=-\tfrac{1}{9}W,\ \ Z_{27}^{(2)}=-\tfrac{1}{9}W,\ \ Z_{2,j}^{(2)}=0,\ \ j\geq 11\\
&Z_{24}^{(2)}=\tfrac{1}{3}W,\ \ Z_{28}^{(2)}=\tfrac{1}{3}W\ .
\end{aligned}
\tag{30}
$$

The process continues until the set closes upon renormalization. In this case 16 operators will be generated.

The calculations are not as tedious as they appear since one is aided by a great degree of symmetry within the work. However it will unnecessarily expand the paper to present them in step-by-step detail.

III. A SUMMARY OF OUR RESULTS

$\underline{\gamma}$ and closed renormalization sets are presented for L-L, L-R, and R-R four-quark operators with the four types of flavor structures discussed in the Introduction.

n represents the number of flavors assumed in the theory. g is the renormalized gauge coupling constant of QCD (the results can thus be viewed as valid for an effective theory of QCD with n flavors, in which case $g\rightarrow g_n$, the effective coupling).

We begin by listing the 28 four-quark operators we use to describe our results:

$$
\begin{aligned}
&Q_1=(\bar\psi^i_{A'}\psi^i_{B'})_L(\bar\psi^j_{C'}\psi^j_{D'})_L,\quad Q_2=(\bar\psi^i_{A'}\psi^j_{B'})_L(\bar\psi^j_{C'}\psi^i_{D'})_L,\quad Q_3=(\bar\psi^i_{A'}\psi^i_{B'})_L(\bar\psi^j_{C'}\psi^j_{D'})_R\ ,\\
&Q_4=(\bar\psi^i_{A'}\psi^j_{B'})_L(\bar\psi^j_{C'}\psi^i_{D'})_R,\quad Q_5=(\bar\psi^i_{A'}\psi^i_{B'})_R(\bar\psi^j_{C'}\psi^j_{D'})_R,\quad Q_6=(\bar\psi^i_{A'}\psi^j_{B'})_R(\bar\psi^j_{C'}\psi^i_{D'})_R\ ,\\
&Q_7=(\bar\psi^i_{A'}\psi^i_{B'})_L\sum_{m=1}^{n}(\bar\psi^j_m\psi^j_m)_L,\quad Q_8=(\bar\psi^i_{A'}\psi^j_{B'})_L\sum_{m=1}^{n}(\bar\psi^j_m\psi^i_m)_L,\quad Q_9=(\bar\psi^i_{A'}\psi^i_{B'})_L\sum_{m=1}^{n}(\bar\psi^j_m\psi^j_m)_R\ ,\\
&Q_{10}=(\bar\psi^i_{A'}\psi^j_{B'})_L\sum_{m=1}^{n}(\bar\psi^j_m\psi^i_m)_R,\quad Q_{11}=(\bar\psi^i_{A'}\psi^i_{B'})_R\sum_{m=1}^{n}(\bar\psi^j_m\psi^j_m)_L,\quad Q_{12}=(\bar\psi^i_{A'}\psi^j_{B'})_R\sum_{m=1}^{n}(\bar\psi^j_m\psi^i_m)_L\ ,\\
&Q_{13}=(\bar\psi^i_{A'}\psi^i_{B'})_R\sum_{m=1}^{n}(\bar\psi^i_m\psi^j_m)_R,\quad Q_{14}=(\bar\psi^i_{A'}\psi^j_{B'})_R\sum_{m=1}^{n}(\bar\psi^j_m\psi^i_m)_R,\quad Q_{15}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^i_l)_L(\bar\psi^j_{C'}\psi^j_{D'})_L\ ,\\
&Q_{16}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^j_l)_L(\bar\psi^j_{C'}\psi^i_{D'})_L,\quad Q_{17}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^i_l)_L(\bar\psi^j_{C'}\psi^j_{D'})_R,\quad Q_{18}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^j_l)_L(\bar\psi^j_{C'}\psi^i_{D'})_R\ ,\\
&Q_{19}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^i_l)_R(\bar\psi^j_{C'}\psi^j_{D'})_L,\quad Q_{20}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^j_l)_R(\bar\psi^j_{C'}\psi^i_{D'})_L,\quad Q_{21}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^i_l)_R(\bar\psi^j_{C'}\psi^j_{D'})_R\ ,\\
&Q_{22}=\sum_{l=1}^{n}(\bar\psi^i_l\psi^j_l)_R(\bar\psi^j_{C'}\psi^i_{D'})_R,\quad Q_{23}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^i_l)_L(\bar\psi^j_m\psi^j_m)_L,\quad Q_{24}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^j_l)_L(\bar\psi^j_m\psi^i_m)_L\ ,\\
&Q_{25}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^i_l)_L(\bar\psi^j_m\psi^j_m)_R,\quad Q_{26}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^j_l)_L(\bar\psi^j_m\psi^i_m)_R,\quad Q_{27}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^i_l)_R(\bar\psi^j_m\psi^j_m)_R\ ,\\
&Q_{28}=\sum_{l,m=1}^{n}(\bar\psi^i_l\psi^j_l)_R(\bar\psi^j_m\psi^i_m)_R\ .
\end{aligned}
\tag{31}
$$

A. *L-L* four-quark operators

(i) Flavors such that $(A'\neq B')\neq(C'\neq D')$

The set $\vec{\mathcal{O}}$ which closes under QCD renormalization at the one-loop level contains two members. $\vec{\mathcal{O}}$ and its corresponding γ are given by

$$
\begin{bmatrix}\mathcal{O}_1\\ \mathcal{O}_2\end{bmatrix}=\begin{bmatrix}Q_1\\ Q_2\end{bmatrix},\quad \underline{\gamma}=\frac{g^2}{8\pi^2}\begin{array}{cc}\begin{bmatrix}-1 & 3\\ 3 & -1\end{bmatrix} & \begin{matrix}1\\ 2\end{matrix}\\ \begin{matrix}1 & \quad 2\end{matrix} & \end{array}\ .
\tag{32}
$$

(ii) Flavors such that $(A'\neq B')\neq(C'=D')$

The set $\vec{\mathcal{O}}$ which closes under QCD renormalization at a one-loop level contains six numbers. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by

$$
\begin{bmatrix}\mathcal{O}_1\\ \mathcal{O}_2\\ \mathcal{O}_3\\ \mathcal{O}_4\\ \mathcal{O}_5\\ \mathcal{O}_6\end{bmatrix}=\begin{bmatrix}Q_1\\ Q_2\\ Q_7\\ Q_8\\ Q_9\\ Q_{10}\end{bmatrix},\quad \underline{\gamma}=\frac{g^2}{8\pi^2}\begin{array}{cc}\begin{bmatrix}-1 & 3 & 0 & 0 & 0 & 0\\ 3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3}\\ 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3}\\ 0 & 0 & 3-\frac{n}{9} & -1+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3}\\ 0 & 0 & 0 & 0 & 1 & -3\\ 0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{n}{9} & -8+\frac{n}{3}\end{bmatrix} & \begin{matrix}1\\ 2\\ 3\\ 4\\ 5\\ 6\end{matrix}\\ \begin{matrix}1 & 2 & 3 & 4 & 5 & 6\end{matrix} & \end{array}\ .
\tag{33}
$$

(iii) Flavors such that $(A'=B')\neq(C'=D')$

The set $\vec{\mathcal{O}}$ which closes under QCD renormalization at a one-loop level contains 16 members. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by Eq. (34):

$$
\begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \\ \mathcal{O}_6 \\ \mathcal{O}_7 \\ \mathcal{O}_8 \\ \mathcal{O}_9 \\ \mathcal{O}_{10} \\ \mathcal{O}_{11} \\ \mathcal{O}_{12} \\ \mathcal{O}_{13} \\ \mathcal{O}_{14} \\ \mathcal{O}_{15} \\ \mathcal{O}_{16} \end{pmatrix}
=
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{15} \\ Q_{16} \\ Q_{19} \\ Q_{20} \\ Q_{23} \\ Q_{24} \\ Q_{25} \\ Q_{26} \\ Q_{27} \\ Q_{28} \end{pmatrix},
\quad \gamma = \frac{g^2}{8\pi^2}
\begin{array}{c}
\left[\begin{array}{cccccccccccccccc}
-1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3-\frac{n}{9} & -1+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3} & 0 & 0 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{n}{9} & -8+\frac{n}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3-\frac{n}{9} & -1+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{n}{9} & -8+\frac{n}{3} & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3-\frac{2n}{9} & -1+\frac{2n}{3} & -\frac{2n}{9} & \frac{2n}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{2n}{9} & -8+\frac{2n}{3} & -\frac{n}{9} & \frac{n}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{11}{9} & \frac{11}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2n}{9} & \frac{2n}{3} & 3-\frac{2n}{9} & -1+\frac{2n}{3}
\end{array}\right]
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \end{array}
\\
\begin{array}{cccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array}
\end{array}
\qquad (34)
$$

$$
\begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \\ \mathcal{O}_6 \\ \mathcal{O}_7 \\ \mathcal{O}_8 \\ \mathcal{O}_9 \\ \mathcal{O}_{10} \\ \mathcal{O}_{11} \end{pmatrix}
=
\begin{pmatrix} Q_1 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{23} \\ Q_{24} \\ Q_{25} \\ Q_{26} \\ Q_{27} \\ Q_{28} \end{pmatrix},
\quad \gamma = \frac{g^2}{8\pi^2}
\begin{array}{c}
\left[\begin{array}{ccccccccccc}
2 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3-\frac{n}{9} & -1+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{n}{9} & \frac{n}{3} & -\frac{n}{9} & -8+\frac{n}{3} & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3-\frac{2n}{9} & -1+\frac{2n}{3} & -\frac{2n}{9} & \frac{2n}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{2n}{9} & -8+\frac{2n}{3} & -\frac{n}{9} & \frac{n}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{11}{9} & \frac{11}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2n}{9} & \frac{2n}{3} & 3-\frac{2n}{9} & -1+\frac{2n}{3}
\end{array}\right]
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{array} \\
\begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array}
\end{array}
\qquad (35)
$$

(iv) Flavors such that $(A'=B')=(C'=D')$

The set $\vec{\mathcal{O}}$ which closes under QCD renormalization at a one-loop level contains 11 members. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by Eq. (35) on the previous page.

B. L-R four-quark operators

(i) Flavors such that $(A'\neq B')\neq(C'\neq D')$

The set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at a one-loop level contains two members. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by

$$\begin{bmatrix}\mathcal{O}_1\\ \mathcal{O}_2\end{bmatrix}=\begin{bmatrix}Q_3\\ Q_4\end{bmatrix},\quad \underline{\gamma}=\frac{g^2}{8\pi^2}\begin{array}{cc}\begin{bmatrix}1&-3\\0&-8\end{bmatrix}&\begin{array}{c}1\\2\end{array}\\ \begin{array}{cc}1&\;\;2\end{array}&\end{array}\;. \tag{36}$$

(ii) Flavors such that $(A'\neq B')\neq(C'=D')$

The set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at a one-loop level contains six members. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by

$$\begin{bmatrix}\mathcal{O}_1\\ \mathcal{O}_2\\ \mathcal{O}_3\\ \mathcal{O}_4\\ \mathcal{O}_5\\ \mathcal{O}_6\end{bmatrix}=\begin{bmatrix}Q_3\\ Q_4\\ Q_7\\ Q_8\\ Q_9\\ Q_{10}\end{bmatrix},\quad \underline{\gamma}=\frac{g^2}{8\pi^2}\begin{array}{cc}\begin{bmatrix}1&-3&0&0&0&0\\ 0&-8&-\frac{1}{9}&\frac{1}{3}&-\frac{1}{9}&\frac{1}{3}\\ 0&0&-\frac{11}{9}&\frac{11}{3}&-\frac{2}{9}&\frac{2}{3}\\ 0&0&3-\frac{n}{9}&-1+\frac{n}{3}&-\frac{n}{9}&\frac{n}{3}\\ 0&0&0&0&1&-3\\ 0&0&-\frac{n}{9}&\frac{n}{3}&-\frac{n}{9}&-8+\frac{n}{3}\end{bmatrix}&\begin{array}{c}1\\2\\3\\4\\5\\6\end{array}\\ \begin{array}{cccccc}1&2&3&4&5&6\end{array}&\end{array}\;. \tag{37}$$

(iii) and (iv) Flavors such that $(A'=B')\neq(C'=D')$ and $(A'=B')=(C'=D')$, respectively

In each of these cases the set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at a one-loop level and contains 16 members. $\vec{\mathcal{O}}$ and its corresponding $\underline{\gamma}$ are given by Eq. (38) on the next page.

C. R-R four-quark operators

Very little work has to be done to extend our results to this case.

Let T be the $L\leftrightarrow R$ inversion operator and Q_i be any local four-quark operator. Define

$$\tilde{Q}_i=TQ_i\;, \tag{39}$$

then it follows that

$$G_0^{(4\psi)}(\tilde{Q}_i^0)=TG_0^{(4\psi)}(Q_i^0)\;. \tag{40}$$

The proof is simple and is given in Fig. 2. In particular (40) implies

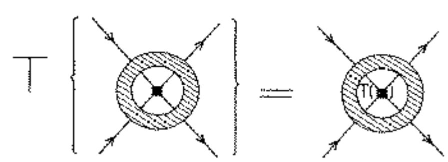

FIG. 2. This figure provides the proof that $TG_0^{(4\psi)}(Q_i^0)=G_0^{(4\psi)}(TQ_i^0)$. The diagram on which T (the $L\leftrightarrow R$ inversion operator) acts represents the unrenormalized four-quark operator insertion. ■ represents the vertex factor defined by the operator. As it only occurs *once* in the diagram it can be isolated as shown. The annulus thus corresponds to a sum over all QCD complexities, i.e., quark-gluon, ghost-gluon, three-gluon, four-gluon vertices and quark, ghost, gluon propagators. These are all invariant under T hence so is the annulus.

$$
\begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \\ \mathcal{O}_6 \\ \mathcal{O}_7 \\ \mathcal{O}_8 \\ \mathcal{O}_9 \\ \mathcal{O}_{10} \\ \mathcal{O}_{11} \\ \mathcal{O}_{12} \\ \mathcal{O}_{13} \\ \mathcal{O}_{14} \\ \mathcal{O}_{15} \\ \mathcal{O}_{16} \end{pmatrix}
=
\begin{pmatrix} Q_3 \\ Q_4 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{17} \\ Q_{18} \\ Q_{21} \\ Q_{22} \\ Q_{23} \\ Q_{24} \\ Q_{25} \\ Q_{26} \\ Q_{27} \\ Q_{28} \end{pmatrix},
\quad \gamma = \frac{g^2}{8\pi^2}
\begin{array}{c}
\left(\begin{array}{cccccccccccccccc}
1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -8 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3-\frac{n}{9} & -1+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3} & 0 & 0 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{n}{9} & -8+\frac{n}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & -8+\frac{n}{3} & -\frac{n}{9} & \frac{n}{3} & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{11}{9} & \frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & \frac{n}{3} & 3-\frac{n}{9} & -1+\frac{n}{3} & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{9} & \frac{11}{3} & -\frac{2}{9} & \frac{2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3-\frac{2n}{9} & -1+\frac{2n}{3} & -\frac{2n}{9} & \frac{2n}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{n}{9} & \frac{n}{3} & -\frac{2n}{9} & -8+\frac{2n}{3} & -\frac{n}{9} & \frac{n}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{9} & \frac{2}{3} & -\frac{11}{9} & \frac{11}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2n}{9} & \frac{2n}{3} & 3-\frac{2n}{9} & -1+\frac{2n}{3}
\end{array}\right)
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \end{array}
\\
\begin{array}{cccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array}
\end{array}
\tag{38}
$$

$$G^{(4\psi)}_{0(1)}(\tilde{Q}^0_i)_p = TG^{(4\psi)}_{0(1)}(Q^0_i)_p \tag{41}$$

$$= TZ^{(2)}_{ij}G^{(4\psi)}_{(0)}(Q_j)$$

$$= Z^{(2)}_{ij}G^{(4\psi)}_{(0)}(\tilde{Q}_j)\ , \tag{42}$$

hence

$$\underline{\tilde{Z}}^{(2)} = \underline{Z}^{(2)}\ . \tag{43}$$

In terms of a matrix representation $\underline{\tilde{Z}}^{(2)}$ relative to the ordered basis $\tilde{Q} = TQ$ is given by $\underline{Z}^{(2)}$ relative to the ordered basis Q.

The R-R $\underline{\gamma}$ and closed renormalization sets are thus deducible from the L-L results presented above. We make use of the fact that L-L and R-R operators satisfy the same Fierz identity,

$$L_{\alpha\beta}L_{\gamma\delta} = -L_{\alpha\delta}L_{\gamma\beta}\ ,$$
$$R_{\alpha\beta}R_{\gamma\delta} = -R_{\alpha\delta}R_{\gamma\beta}\ . \tag{44}$$

We have as follows:

(i) Flavors such that $(A' \neq B') \neq (C' \neq D')$

The set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at the one-loop level has two members. $\vec{\mathcal{O}}$ is defined by

$$(\mathcal{O}_1, \mathcal{O}_2) = (Q_5, Q_6) \tag{45}$$

and its anomalous-dimension matrix $\underline{\gamma}$ is given by Eq. (32).

(ii) Flavors such that $(A' \neq B') \neq (C' = D')$

The set $\vec{\mathcal{O}}$ which closes under QCD renormalization at a one-loop level has six members. The set $\vec{\mathcal{O}}$ is defined by

$$(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_6) = (Q_5, Q_6, Q_{13}, Q_{14}, Q_{11}, Q_{12}) \tag{46}$$

and its anomalous-dimension matrix $\underline{\gamma}$ is given by Eq. (33).

(iii) Flavors such that $(A' = B') \neq (C' = D')$

The set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at a one-loop level has 16 members, defined by

$$(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_6, \mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_{12}, \mathcal{O}_{13}, \mathcal{O}_{14}, \mathcal{O}_{15}, \mathcal{O}_{16})$$
$$= (Q_5, Q_6, Q_{13}, Q_{14}, Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{17}, Q_{18}, Q_{27}, Q_{28}, Q_{25}, Q_{26}, Q_{23}, Q_{24})\ . \tag{47}$$

The associated anomalous-dimension matrix $\underline{\gamma}$ is given by Eq. (34).

(iv) Flavors such that $(A' = B') = (C' = D')$

The set $\vec{\mathcal{O}}$ which closes upon QCD renormalization at a one-loop level has 11 members, defined by

$$(\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_{11}) = (Q_5, Q_{13}, Q_{14}, Q_{11}, Q_{12}, Q_{27}, Q_{28}, Q_{25}, Q_{26}, Q_{23}, Q_{24})\ . \tag{48}$$

The associated anomalous-dimension matrix $\underline{\gamma}$ is given by Eq. (35).

IV. CONCLUDING REMARKS

In this paper we have considered one-loop evaluations of anomalous-dimension matrices for a set of 12 distinct types of four-quark operators. Four flavor-space structures [cases (*i*) to (*iv*)] were considered for L-L, L-R, and R-R chiral structures. The MS renormalization scheme and the Landau gauge were used from the point of view of the simplicity of the renormalization group for this choice. We now conclude by indicating in which areas the 12 calculations could prove useful.

$\underline{\gamma}$ for case (*i*) (L-L) is the simplest. It was first calculated by Gaillard and Lee, and Altarelli and Maiani.[3] It may be applied to the flavor-changing penguin-free sector of the W-mediated effective weak nonleptonic Hamiltonian. To the present date it is only this calculation which has been extended to the two-loop level.[6]

$\underline{\gamma}$ for case (*ii*) (L-L) was first encountered in its present form by Gilman and Wise[4] (of which our results are a slight generalization in that it includes n dependence). It has application to the flavor-changing penguin-generating sector of the W-mediated effective weak nonleptonic Hamiltonian.

$\underline{\gamma}$ for case (*iii*) (L-L), (*iii*) (L-R), (*iii*) (R-R), (*iv*) (L-R), and (*iv*) (R-R) proves useful in an investigation of W- and Z-mediated flavor-conserving effective weak nonleptonic Hamiltonians.[9] To the best of our knowledge they are presented for the first time in this paper (despite the fact that such effective Hamiltonians have been studied recently[7]).

The remaining cases were calculated primarily for completeness but could prove useful in discussing interactions generated by right-hand weak or superweak bosons.[10]

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