# Renormalization of 4–Quark Operators and QCD–Sum Rules

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#### Abstract

We compute the renormalization mismatch displayed in 1-loop approximation by classically equivalent 4-quark operators and coming from different possible definitions of the  $\gamma_5$  matrix in dimensional regularization. The result is then employed to study the effect of the various treatments of  $\gamma_5$  upon the size of radiative corrections to 4-quark condensates in the QCD sum rules for  $\rho$  and  $A_1$  mesons. We find that a fully anticommuting  $\gamma_5$  which automatically respects non-anomalous chiral Ward-Slavnov identities leads to considerably smaller corrections and reduces theoretical uncertainty in the QCD prediction for the  $\tau$  hadronic decay rate.

#### 1 Introduction

Dimensional regularization (DR) [1] coupled with the minimal subtraction scheme (MS) [2] is widely acknowledged as theoretically sound and extremely valuable in practice calculational framework. In a theory like QCD with parity conserving amplitudes it respects all the symmetries of the theory such as gauge invariance, Bose symmetry and Ward-Slavnov identities. In addition, dimensionally regularized divergent Feynman integrals are in many respects very similar to the convergent ones, which allowed to develop quite powerful methods of their analytical calculation.

A notorious shortcoming of DR is the absence of a natural generalization of the Dirac matrix  $\gamma_5$  to noninteger values  $D \neq 4$  of the dimension of space-time. Indeed, the well-known relation

$$\operatorname{tr}(\gamma_5 \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4}) = -4i\epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \tag{1}$$

which in fact fixes  $\gamma_5$  in 4 dimensions, is not compatible with another property of  $\gamma_5$  valid at D=4

$$\{\gamma_5, \gamma_\mu\} = 0,\tag{2}$$

The problem is that the totally antisymmetric  $\epsilon$ -tensor is a purely 4-dimensional object and thus can not be self-consistently continued to D dimensions. It may be shown that if (2) is to be held literally for all values of  $\mu = 1 \dots D$  (and the standard properties of the trace operation like cyclycity is respected) then the rhs of (1) should vanish identically [3]. Such a defined  $\gamma_5$  does not, hence, go smoothly at  $D \to 4$  to the 4-dimensional

$$\gamma_5^{(4)} = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \tag{3}$$

in contradiction with general requirements to which every self-consistent regularization must meet [4]. From the same point of view the relation (2) may be freely violated for the values of index  $\mu$  lying outside of the physical 4-dimensional subspace.

The existing solutions of the problem may be divided into two classes. The first one favours a nonvanishing anticommutator  $\{\gamma_5, \gamma_\mu\}$  at  $\mu \neq 0, 1, 2, 3$  and is discussed in general in [5] and [6]. An explicit version of such a scheme was proposed by 't Hooft and Veltman and elaborated by Breitenlohner and Maison [1, 3]. Another class keeps (2) intact at least for amplitudes without traces with odd numbers of  $\gamma_5$  matrices while trying to reproduce the identity (1) either by hand [7] or, in a bit more sophisticated manner, by redefining the trace operation for the traces with odd number of  $\gamma_5$  [8, 9].

Leaving aside a (rather academic) task of finding an explicit formal proof of self-consistency of these solutions, we will study in this work another aspect of the  $\gamma_5$  problem — different possibilities of choosing between classically equivalent 4–quark operators which begin to differ in higher orders due to different treatment of the  $\gamma_5$  matrix. As we shall see below, different choices lead to different theoretical predictions of the QCD sum rule method [10] because of a commonly accepted approximate procedure of the vacuum saturation in estimating the vacuum expectation values of 4-quark operators. On considering a number of particular physical problems, it will be demonstrated that the choice of fully anticommuting  $\gamma_5$  considerably reduces theoretical uncertainty coming from higher order corrections.

# 2 The $\gamma_5$ ambiguity in 4-quark operators

The problem under discussion appears even in cases when initial amplitudes comprise no  $\gamma_5$  matrix at all. Indeed, let us consider the standard derivation of the QCD sum rules for the  $\rho$  and  $A_1$  mesons [10]. It starts from constructing the Wilson expansion [11] for the correlator

$$\Pi_{\mu\nu}(q) = i \int dx \, e^{iqx} \langle T j_{\mu}(x) j_{\nu}^{\dagger}(0) \rangle_{0} = (q_{\mu}q_{\nu} - g_{\mu\nu}q^{2})\Pi(Q^{2})$$
(4)

$$\Pi(Q^2) = C_0 + \frac{C_4 < \mathcal{O}_4 >_0}{Q^4} + \frac{C_6^{i,V/A} < \mathcal{O}_6^i >_0}{Q^6} + \dots$$
 (5)

where  $j_{\mu} = \bar{u} \gamma_{\mu}(\gamma_5)d$  is the current for the  $\rho(A_1)$  meson.

The power corrections of order  $O(1/Q^6)$  to the rhs of the expansion (5) come from 4-quark condensates (that is the vacuum expectation values (VEV) of 4-quark operators) and lead to most important numerically non-perturbative corrections to the sum rule for the  $\rho$  meson. In the lowest order of perturbation theory the contributions of the 4-quark operators are well-known and read [10]

$$\frac{C_6^{i,V}Q_6^i}{g^2} = -\frac{2}{9}(\bar{u} \ \gamma_\mu t^a u + \bar{d} \ \gamma_\mu t^a d) \cdot (\bar{\Psi} \ \gamma^\mu t^a \Psi) - 2\bar{u} \ \gamma_5 \gamma_\mu t^a d \, \bar{d} \ \gamma_5 \gamma_\mu t^a u \tag{6}$$

for the vector correlator and

$$\frac{C_6^{i,A}Q_6^i}{g^2} = -\frac{2}{9}(\bar{u} \ \gamma_\mu t^a u + \bar{d} \ \gamma_\mu t^a d) \cdot (\bar{\Psi} \ \gamma^\mu t^a \Psi) - 2\bar{u}\gamma_\mu t^a d \, \bar{d}\gamma_\mu t^a u \tag{7}$$

for the axial vector one. Here  $t^a$ ,  $a=1,\ldots 8$  are the colour Gell-Mann matrixes normalized as  $Tr(t^at^b)=\frac{1}{2}\delta^{ab}$ ;  $(\bar{\Psi}\Gamma\Psi)$  is a shorthand for  $\sum_{f=u,d,s}(\bar{\Psi}_f\Gamma\Psi_f)$ ; g is the quark gluon coupling constant such that  $\alpha_s=g^2/4\pi$ .

Let us consider in detail the structure of the second term in (6) proportional to the 4-quark operator

$$\mathcal{O}_1' = \bar{u} \, \gamma_5 \gamma_\mu t^a d \, \bar{d} \, \gamma_5 \gamma_\mu t^a u. \tag{8}$$

In fact, the very calculation produces a bit different operator

$$\mathcal{O}_1 = \bar{u} \ \Gamma^{[3]} t^a d \, \bar{d} \ \Gamma^{[3]} t^a u, \quad \Gamma^{[3]} \equiv \frac{1}{2} (\gamma_\mu \gamma_\nu \gamma_\lambda - \gamma_\lambda \gamma_\nu \gamma_\mu) \tag{9}$$

while the result (6) is obtained after a use of the identities

$$\Gamma^{[3]} \bigotimes \Gamma^{[3]} = 6\gamma_5 \gamma_\mu \bigotimes \gamma_5 \gamma_\mu. \tag{10}$$

Similarly, the initial form of the second term in (7)

$$\mathcal{O}_2' = \bar{u}\gamma_\mu t^a d\,\bar{d}\gamma_\mu t^a u. \tag{11}$$

is (with an appropriately changed coefficient function)

$$\mathcal{O}_1 = \bar{u} \ \Gamma^{[3]} \gamma_5 t^a d \, \bar{d} \ \Gamma^{[3]} \gamma_5 t^a u,$$
 (12)

The relation (10) is valid in 4-dimensional space due to the very definition of the  $\gamma_5^{(4)}$  matrix (3). However, it loses any operational sence at generic  $D \neq 4$  within the framework of dimensional regularization. This implies that if both operators  $\mathcal{O}_1$  and  $\mathcal{O}'_1$  are to be minimally renormalized, then the difference

$$\delta \mathcal{O}_1 = \mathcal{O}_1 - 6\mathcal{O}_1' = \mathcal{O}(\alpha_s) \tag{13}$$

is an evanescent operator [14] which vanishes in the tree approximation but typically receives nonzero contributions in higher orders. This also means that higher order corrections to the coefficient functions (CF)  $C_6^i$  also depend on specifying which operators are chosen as basic ones. It should be stressed that this  $\gamma_5$  ambiguity comes from the absence of a unique canonical representation of 4-quark operators in dimensional regularization, which, in turn, does not allow to define the idea of minimal subtraction of UV poles in an unambiguous way.

The above discussion does not imply in any means that the theoretical predictions for the contributions to the rhs of (5) from 4-quark operators are ill-defined in principle: the VEV's of the latter should, of course, also depend on the choice of the renormalization prescription in such a way that the whole combination  $\sum_i C_6^i \mathcal{O}_6^i$  is invariant as it must be.

The real problem lies in our present inability to find the VEV's of 4-quark operators in a sufficiently accurate way. Indeed, the common practice is to use the vacuum saturation (VS) procedure [10]. It states that the VEV of a 4-quark operator  $\mathcal{O} = (\bar{\Psi}\Gamma_1\Psi)(\bar{\Psi}\Gamma_2\Psi)$  may be estimated according to the following formula

$$\langle \mathcal{O} \rangle_0 = \frac{1}{144} \left[ Tr(\Gamma_1) Tr(\Gamma_2) - Tr(\Gamma_1 \Gamma_2) \right] \cdot \langle \bar{q}q \rangle^2$$
 (14)

where  $\bar{q}q = \bar{u}u, \bar{d}d$  or  $\bar{s}s$ .

It is clear that this procedure is too rough and does not feel in any way such fine details as the concrete mode of the renormalization of a 4-quark operator. Hence, if one would like to estimate the effect of higher order corrections to the CF's  $C_6^i$  the numerical results will do depend on specifying 4-quark operators in D dimensions. Unfortunately, these corrections prove to be of considerable size (see [12] and below) at least in the operator basis featuring objects like  $\Gamma^{[3]} \otimes \Gamma^{[3]}$ . Let us try to search for a distinguished operator basis for which the VS would presumably work better.

Of course, from a general point of view there is an infinitely large variety of possible choices of a (different at  $D \neq 4$  and equivalent at D = 4) 4-quark operator. For instance, one could mix operators  $\mathcal{O}_1$  and  $\mathcal{O}'_1$  as follows

$$\mathcal{O}''_1 = \mathcal{O}_1 \cdot \sin^2 \alpha + 6 \,\mathcal{O}'_1 \cdot \cos^2 \alpha \tag{15}$$

with an arbitrary angle  $\alpha \in \{-\pi/2, \pi/2\}$ .

On the other hand a lot of calculational experience invariably demonstrates that if amplitudes under investigation do not comprise traces with an odd number of  $\gamma_5$  at all then the use of a fully anticommuting  $\gamma_5$  is quite self-consistent and leads to great simplifications in doing practical calculations. Much more important, this choice is in a sense unique for the amplitudes from above described class (these will be referred to non-singlet ones in the following) as it respects the chiral  $SU_L(3) \otimes SU_R(3)$  symmetry of

the QCD lagrangian. This is the case since on the purely diagrammatic level the chiral symmetry relies on the possibility of freely anticommutating of  $\gamma_5$  with  $\gamma_{\mu}$  at generic value of the index  $\mu$ . It seems very reasonable to employ identities like (10) in order to maximally simplify kinematical structures appearing. Thus, we will treat the operators  $\mathcal{O}'_1$  and  $\mathcal{O}'_2$  with completely anticommutating  $\gamma_5$  as "natural" ones and reexpress through them the operators  $\mathcal{O}_1$  and  $\mathcal{O}_2$ .

A simple calculation gives:

$$\mathcal{O}_{1} = \left(1 - \frac{\alpha_{s}}{4\pi} \frac{22}{3}\right) \cdot 6 \,\mathcal{O}'_{1} + \frac{25\alpha_{s}}{4\pi} \bar{u} \,\gamma_{\alpha} t^{a} d \,\bar{d} \,\gamma^{\alpha} t^{a} u 
+ \frac{40}{3} \frac{\alpha_{s}}{4\pi} \bar{u} \,\gamma_{\alpha} d \,\bar{d} \,\gamma_{\alpha} u + \frac{2}{9} \frac{\alpha_{s}}{4\pi} (\bar{u} \,\gamma_{\alpha} t^{a} u + \bar{d} \,\gamma^{\alpha} t^{a} d) \,(\bar{\Psi} \gamma^{\alpha} t^{a} \Psi),$$
(16)

$$\mathcal{O}_{2} = \left(1 - \frac{\alpha_{s}}{4\pi} \frac{22}{3}\right) \cdot 6 \,\mathcal{O}'_{2} + \frac{25\alpha_{s}}{4\pi} \bar{u} \,\gamma_{\alpha}\gamma_{5}t^{a}d\,\bar{d}\,\gamma^{\alpha}\gamma_{5}t^{a}u + \frac{40}{3} \frac{\alpha_{s}}{4\pi} \bar{u} \,\gamma_{\alpha}\gamma_{5}d\,\bar{d}\,\gamma_{\alpha}\gamma_{5}u + \frac{2}{9} \frac{\alpha_{s}}{4\pi} (\bar{u} \,\gamma_{\alpha}t^{a}u + \bar{d}\,\gamma^{\alpha}t^{a}d) \,(\bar{\Psi}\gamma^{\alpha}t^{a}\Psi).$$

$$(17)$$

To finish this section, a few technical words about the calculation. It is clear that the very difference  $\mathcal{O}_i - \mathcal{O}_i'$  vanishes in the classical limit and consequently may come only from UV divergences which manifest themselves as poles in  $\epsilon$  in DR [14]. Thus, the calculation is conveniently performed hand in hand with the standard procedure of evaluating one-loop anomalous dimensions of 4-quark operators [10] (see also [15], where similar calculations were performed in the context of evaluating QCD corrections to the effective weak Hamiltonian.

# 3 QCD sum rules for the $\rho$ and $A_1$ mesons

An explicit calculation of one-loop corrections to 4-quark condensates in (6) gives [12] (in the commonly accepted  $\overline{MS}$  scheme [13];  $L = \ln \frac{\mu^2}{Q^2}$ )

$$\frac{C_6^{i,V}Q_6^i}{g^2} = -\frac{2}{9} \left( 1 + \frac{\alpha_s}{\pi} \left[ \frac{95}{72} L + \frac{107}{48} \right] \right) \left( \bar{u} \, \gamma_\mu t^a u + \bar{d} \, \gamma_\mu t^a d \right) \cdot (\bar{\Psi} \, \gamma^\mu t^a \Psi) \qquad (18)$$

$$-\frac{1}{3} \left( 1 + \frac{\alpha_s}{\pi} \left[ \frac{9}{8} L + \frac{431}{96} \right] \right) \bar{u} \, \Gamma^{[3]} t^a d \, \bar{d} \, \Gamma^{[3]} t^a u \qquad (19)$$

$$-\frac{\alpha_s}{24\pi} \left\{ \begin{array}{l} (16L - 12) \, \bar{u} \Gamma_\mu d \, \bar{d} \, \Gamma_\mu u \\ + (30L - \frac{45}{2}) \, \bar{u} \Gamma_\mu t^a d \, \bar{d} \, \Gamma_\mu t^a u \\ + (\frac{16}{9} \, L - \frac{8}{27}) \, (\bar{\Psi} \, \gamma_\mu t^a \Psi)^2 \\ + (\frac{16}{9} \, L + \frac{56}{27}) \, (\bar{u} \, \gamma_\mu \gamma_5 u + \bar{d} \, \gamma_\mu \gamma_5 d) (\bar{\Psi} \, \gamma^\mu \gamma_5 \Psi) \\ + (\frac{10}{3} \, L + \frac{35}{9}) \, (\bar{u} \, \gamma_\mu \gamma_5 t^a u + \bar{d} \, \gamma_\mu \gamma_5 t^a d) (\bar{\Psi} \, \gamma^\mu \gamma_5 t^a \Psi) \end{array} \right\}. \quad (20)$$

<sup>&</sup>lt;sup>1</sup>Note that we are concerned only with 4-quark operators appearing in the tree approximation; for operators which show up first in 1-looop corrections the subtlety under discussion becomes relevant starting at two loops.

For the axial vector correlator the corrections look exactly the same, only the matrices  $\Gamma^{[3]}$  and  $\Gamma_{\mu}$  should be multiplied by  $\gamma_5$ . After the use of the VS approximation we get for the  $\rho$  meson

$$C_6^{i,V} < \mathcal{O}_6^i >_0 = -\frac{224\pi}{81} \left( 1 + \left( \frac{705}{112} + \frac{13}{252} L \right) \frac{\alpha_s}{\pi} \right) (\alpha_s < \bar{q}q >_0^2)$$
 (21)

and for the  $A_1$  meson

$$C_6^{i,A} < \mathcal{O}_6^i >_0 = \frac{352\pi}{81} \left( 1 + \left( \frac{777}{176} + \frac{149}{396} L \right) \frac{\alpha_s}{\pi} \right) (\alpha_s < \bar{q}q >_0^2)$$
 (22)

At last, a straightforward application of (16),(17) leads us to a new result for (18-20) and consequently for that of (21):

$$C_6^{i,V} < \mathcal{O}_6^i >_0 = -\frac{224\pi}{81} \left( 1 + \left( \frac{685}{336} + \frac{13}{252} L \right) \frac{\alpha_s}{\pi} \right) \left( \alpha_s < \bar{q}q >^2 \right)$$
 (23)

and for the  $A_1$  meson

$$C_6^{i,A} < \mathcal{O}_6^i >_0 = \frac{352\pi}{81} \left( 1 + \left( \frac{917}{528} + \frac{149}{396} L \right) \frac{\alpha_s}{\pi} \right) (\alpha_s < \bar{q}q >_0^2)$$
 (24)

Thus, we observe that uncomfortably large radiative corrections in (21) and (22) are transformed into quite moderate ones after the above described transition to the new basis of 4-quark operators. Moreover, now the  $\alpha_s$  corrections get of approximately the same (relative) size for the vector and axial correlators — a fact that will be of special importance in the next section.

Note also that the terms proportional to L in (23) and (24) are quite small. To our opinion, this fact demonstrates that the effects due to two-loop anomalous dimensions of operators in (20) are presumably small and may be neglected. On the other hand, the combination of operators  $\alpha_s(\mu) < \bar{q}q > (\mu) < \bar{q}q > (\mu)$  has only very weak dependence on  $\mu$  as its anomalous dimensions is fortunately rather small. This means that if the VS approximation is a good one at some scale  $\mu$ , then it should be considered as a reasonable approximation at a diffrent scale  $\mu'$ .

## 4 The tau hadronic width

As was pointed out on [16, 17, 18] some time ago, the methods of perturbative QCD can be applied to estimate the decay rate ratio

$$R_{\tau} = \frac{\Gamma(\tau \to \nu_{\tau} \text{hadrons})}{\Gamma(\tau \to \nu_{\tau} e \bar{\nu}_{e})}.$$
 (25)

An updated theoretical discussion of the ratio (25) within the QCD frameworks was recently given in [20].

Schematically,  $R_{\tau}$  may represented in the following form (in the chiral limit of the massless u, d and s quarks and neglecting the logarithmic dependence of the Wilson coefficients)

$$R_{\tau} = R_{\tau}^{0} \left\{ 1 + \frac{\alpha_{s}}{\pi} + 5.202 \left(\frac{\alpha_{s}}{\pi}\right)^{2} + 26.37 \left(\frac{\alpha_{s}}{\pi}\right)^{3} + \delta^{D=6} + \ldots \right\}.$$
 (26)

Here the perturbative coefficients are appropriate for the number of flavours f=3 and  $\alpha_s=\alpha_s(M_\tau)$  and the power correction of order  $1/M_\tau^6$  is directly related to the contribution of 4-quark operators to (5), viz.

$$\delta^{D=6} = (\delta_V^{D=6} + \delta_A^{D=6})/2, \quad \delta_{V/A}^{D=6} = 24\pi^2 \frac{-\sum_i (C_6^{i,V/A}) < \mathcal{O} >_i)}{M_\tau^6}$$
(27)

As was argued in [20] the only significant sources of uncertainty in the QCD prediction for  $R_{\tau}$  are due to the uncalculated perturbative QCD correction of order  $\alpha_s^4(M_{\tau})$  and due to the power correction of order  $1/M_{\tau}^6$ . In this section we will concentrate on the latter.

The estimation of the power correction made in [20] amounts essentially to keeping the lowest order contribution to the CF  $\delta^{D=6}$  and using the VS approximation. The result is

$$\delta_{V/A}^{D=6} = \begin{pmatrix} 7 \\ -11 \end{pmatrix} \frac{256\pi^3}{27} \frac{\rho \alpha_s < \bar{\psi}\psi >^2}{M_\tau^6}.$$
 (28)

Here  $\rho\alpha_s < \bar{\psi}\psi >^2 \approx (3.8 \pm 2.0) \times 10^{-4} GeV^6$  is an effective scale invariant matrix element which is determined phenomenologically [21, 22, 23]. It is introduced in order to partially take into account deviations from the VS approximation. An important observation is that when the vector and axial vector terms are averaged to give  $\delta^{D=6}$ , there is a large mutual cancellation between these contributions. It is the assumption — that this cancellation also holds for the uncertainties of separate contributions<sup>2</sup> —, which is crucial in producing a rather accurate prediction for  $\alpha_s$  given in [20], viz.

$$\alpha_s(M_\tau) = 0.34 \pm 0.004. \tag{29}$$

This assumption is certainly quite optimistic and has already been criticized in [24, 25]. From formal point of view it is self-consistent *provided* one may neglect higher order corrections to the coefficient functions involved. However, the above discussion of the leading 1-loop corrections to the CF  $C_6^{i,V/A}$  clearly demonstrates that this neglection is very questionable unless one deals with the primed basis of 4-quark operators and avoids objects like  $\Gamma^{[3]} \otimes \Gamma^{[3]}$  in favour of those featuring the completely anticommuting  $\gamma_5$ .

#### 5 Conclusions

In this work we have studied the problem of the dependence of radiative corrections to the coefficient functions of 4-quark condensates on the exact way of treating the  $\gamma_5$  matrix and related objects specifying a 4-quark operator. It was argued that a choice of a fully anticommutating  $\gamma_5$  matrix which respects the chiral symmetry of the QCD lagrangian is theoretically preferred and leads to better convergent perturbation series. It also gives an extra argument in favour of the stability of (28) and (29) with respect to radiative corrections.

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<sup>&</sup>lt;sup>2</sup>This would become true if the prescription (14) could be made exact by multiplying its rhs with an (operator independent!) correction factor  $\rho$ .

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