1-Loop Anomalous Dimensions of 4-Quark Operators

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September 10, 2015

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Introduction

Introduction

Two Point Correlation Function

Two point correlation function:

$$\Pi^{V/A}_{\mu\nu}(q) \equiv i\int \mathrm{d}x\, \mathrm{e}^{iqx}\, \langle\Omega| T\{j^{V/A}_{\mu}(x) j^{V/A}_{\nu}(0)^{\dagger}\}|\Omega\rangle$$
$$= (q_\mu q_\nu - g_{\mu\nu} q^2)\, \Pi^{V/A}(q^2)\,,$$

where

$$j^V_\mu = \bar{u}\gamma_\mu d(x) \qquad \text{and} \qquad j^A_\mu = \bar{u}\gamma_\mu\gamma_5 d(x)$$

Operator product expansion (OPE) :

$$\Pi^{V/A}(Q^2) = C_0(Q^2) + C_4(Q^2)\,\frac{\langle O_4\rangle}{Q^4} + C^{V/A}_6(Q^2)\,\frac{\langle O_6\rangle}{Q^6} + \ldots\,.$$

Wilson-Coefficients

$$C_6^{V-A}(Q^2)\,\langle O_6\rangle = 4\pi^2 a_s \left\{ \left[2 + \left(\tfrac{25}{6} - L\right) a_s \right] \langle Q_-^{\,o}\rangle - \left(\tfrac{11}{18} - \tfrac{2}{3} L\right) a_s \langle Q_-^{\,s}\rangle \right\}$$

$$\begin{aligned} C_6^{V+A}(Q^2)\,\langle O_6\rangle \quad = \quad & -4\pi^2 a_s \left\{ \left[2 + \left(\tfrac{155}{24} - \tfrac{7}{2} L\right) a_s \right] \langle Q_+^{o}\rangle + \left(\tfrac{11}{18} - \tfrac{2}{3} L\right) a_s \langle Q_+^{s}\rangle + \right. \\ & \left[\tfrac{4}{9} + \left(\tfrac{37}{36} - \tfrac{95}{162} L\right) a_s \right] \langle Q_3\rangle + \left(\tfrac{35}{108} - \tfrac{5}{18} L\right) a_s \langle Q_4\rangle + \\ & \left. \left(\tfrac{14}{81} - \tfrac{4}{27} L\right) a_s \langle Q_6\rangle - \left(\tfrac{2}{81} + \tfrac{4}{27} L\right) a_s \langle Q_7\rangle \right. \end{aligned}$$

[L.E. Adam and K.G. Chetyrkin, 1994]

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with

$$a_s \equiv \frac{\alpha_s}{\pi} \qquad \text{and} \qquad L \equiv \ln \frac{Q^2}{\mu^2}$$

Operator Basis

$$
\begin{aligned}
Q_V^O &= (\bar{u}\gamma_\mu t^a d \bar{d}\gamma^\mu t^a u)\,, \quad Q_A^O = (\bar{u}\gamma_\mu\gamma_5 t^a d \bar{d}\gamma^\mu \gamma_5 t^a u)\,,\\
Q_V^S &= (\bar{u}\gamma_\mu d \bar{d}\gamma^\mu u)\,, \quad Q_A^S = (\bar{u}\gamma_\mu\gamma_5 d \bar{d}\gamma^\mu \gamma_5 u)\,,\\
Q_3 &\equiv (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu t^a q)\,,\\
Q_4 &\equiv (\bar{u}\gamma_\mu \gamma_5 t^a u + \bar{d}\gamma_\mu \gamma_5 t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu \gamma_5 t^a q)\,,\\
Q_5 &\equiv (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu q)\,,\\
Q_6 &\equiv (\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu \gamma_5 q)\,,\\
Q_7 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu t^a q')\,,\\
Q_8 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu \gamma_5 t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu \gamma_5 t^a q')\,,\\
Q_9 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu q')\,,\\
Q_{10} &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu \gamma_5 q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu \gamma_5 q')\,.
\end{aligned}
$$

What To Do

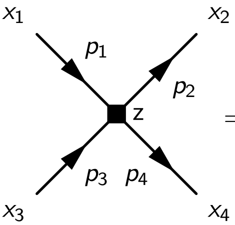
- Compute amplitudes from operator insertions
- Find possible contractions (Feynman diagrams)
- Extract renormalization constants ($\hat{Z}$)
- Calculate Anomalous dimension matrix ($\gamma^{(1)}$)
- Check Renormalization Group Equation (RGE)

0^{th} Order Diagram

4-quark operators:

$$Q^{S}_{V,A} = \left(\bar{q}^{A}\Gamma_1 q^{B}\bar{q}^{B}\Gamma_2 q^{A}\right)(z) \qquad \text{and} \qquad Q^{O}_{V,A} = \left(\bar{q}^{A}t^{a}\Gamma_1 q^{B}\bar{q}^{B}t^{a}\Gamma_2 q^{A}\right)(z)$$

Green function with operator insertion:



$$= q^{i}_{\alpha}(x_1)\bar{q}^{j}_{\beta}(x_2)[\bar{q}^{A}\Gamma_1 q^{B}\bar{q}^{B}\Gamma_2 q^{A}](z)q^{k}_{\delta}(x_3)\bar{q}^{l}_{\gamma}(x_4)$$

$$= \delta^{ij}\delta^{kl}[S^{a}(x_1 - z)\Gamma_1 S^{B}(z - x_2)S^{B}(z - x_2)]_{\alpha\beta}[S^{B}(x_3 - z)\Gamma_2 S^{A}(z - x_4)]_{\delta\gamma}$$

0^{th} Order Diagram

Fourier transform:

$$\delta^{ij}\delta^{kl}[S^A(p_1)\Gamma_1 S^B(-p_2)]_{\alpha\beta}[S^B(p_3)\Gamma_2 S^A(-p_4)]_{\delta\gamma}$$

Amputating external propagators:

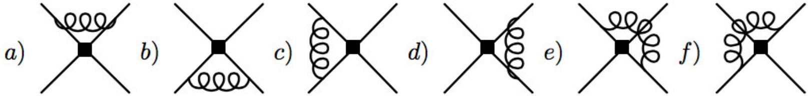
$$\begin{aligned}\Gamma^S_{amp} &: \delta^{ij}\delta^{kl}[\Gamma_1]^{AB}_{\alpha\beta}[\Gamma_2]^{BA}_{\delta\gamma}\\ \Gamma^O_{amp} &: (t^a)^{ij}(t^a)^{kl}[\Gamma_1]^{AB}_{\alpha\beta}[\Gamma_2]^{BA}_{\delta\gamma}\end{aligned}$$

Anomalous Dimension Matrix

Anomalous Dimension Matrix

Possible Diagrams

Current-current diagrams:



Penguin diagrams:

V-A Contribution (singlet)

V-A (singlet)

V-A Contribution (singlet)

$$\Gamma^a = \frac{\alpha_s}{4\pi} C_F \frac{a}{\epsilon} \delta^{ij} \delta^{kl} [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma}$$

$$\Gamma^b = \frac{\alpha_s}{4\pi} C_F \frac{a}{\epsilon} \delta^{ij} \delta^{kl} [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma}$$

$$\Gamma^c = \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} (t^b)^{ij} (t^b)^{kl} \left\{ -\frac{1}{4} [\gamma_\sigma \gamma_\omega \Gamma_1]^{AB}_{\alpha\beta} [\gamma^\sigma \gamma^\nu \Gamma_2]^{BA}_{\delta\gamma} + (1-a) [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma} \right\}$$

$$\Gamma_d = \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} (t^b)^{ij} (t^b)^{kl} \left\{ -\frac{1}{4} [\Gamma_1 \gamma_\sigma \gamma_\omega]^{AB}_{\alpha\beta} [\Gamma_2 \gamma^\omega \gamma^\sigma]^{BA}_{\delta\gamma} + (1-a) [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma} \right\}$$

$$\Gamma^e = \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} (t^b)^{ij} (t^b)^{kl} \left\{ \frac{1}{4} [\gamma_\sigma \gamma_\omega \Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2 \gamma^\omega \gamma^\sigma]^{BA}_{\delta\gamma} - (1-a) [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma} \right\}$$

$$\Gamma^f = \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} (t^b)^{ij} (t^b)^{kl} \left\{ \frac{1}{4} [\Gamma_1 \gamma_\omega \gamma_\sigma]^{AB}_{\alpha\beta} [\gamma^\sigma \gamma^\omega \Gamma_2]^{BA}_{\delta\gamma} - (1-a) [\Gamma_1]^{AB}_{\alpha\beta} [\Gamma_2]^{BA}_{\delta\gamma} \right\}$$

V-A Contribution (singlet)

Vector ($\Gamma_1 = \Gamma_2 = \gamma_\mu$) contribution:

$$\sum_{i=a,\ldots,f} \Gamma^S_{i,V} = \frac{a_s}{\epsilon}(t^b)^{ij}(t^b)^{kl}\left(-\frac{3}{2}\right)[\gamma_5\gamma_\mu]^{AB}_{\alpha\beta}\,[\gamma^\mu\gamma_5]^{BA}_{\delta\gamma}$$

$$= \frac{a_s}{\epsilon}\left(-\frac{3}{2}\right) Q^O_A$$

Axialvector ($\Gamma_1 = \Gamma_2 = \gamma_\mu\gamma_5$) contribution :

$$\sum_{i=a,\ldots,f} \Gamma^S_{i,A} = \frac{a_s}{\epsilon}(t^b)^{ij}(t^b)^{kl}\left(-\frac{3}{2}\right)[\gamma_\mu]^{AB}_{\alpha\beta}\,[\gamma^\mu]^{BA}_{\delta\gamma}$$

$$= \frac{a_s}{\epsilon}\left(-\frac{3}{2}\right) Q^O_V$$

V-A Contribution (singlet)

Noticing:

$$Q_V^S \to Q_A^O$$
$$Q_A^S \to Q_V^O$$

$$Q_-^S \equiv Q_V^S - Q_A^S \to Q_A^O - Q_V^O = -Q_-^O$$

Total contribution (singlet):

$$\sum_{i=a,\ldots,f}^{S} \Gamma_{i,V-A}^{S} = \frac{a_s}{\epsilon}\frac{3}{2}Q_-^O.$$

V-A Contribution (singlet)

Extract renormalization constant:

$$\langle q\bar{q}(Z^{-1}\vec{O}^{B})q\bar{q}\rangle = \langle q\bar{q}(\mathbb{1} - Z_0^{(1)}\frac{\alpha_s}{\epsilon})\vec{O}^{R}q\bar{q}\rangle$$

V-A renormalization constants:

$$(\hat{Z}_0^{(1)})_{21} = \frac{3}{2} \qquad \text{and} \qquad (\hat{Z}_0^{(1)})_{22} = 0$$

V-A (octet)

V-A (octet)

V-A Contribution (octet)

Colour structures:

$$t^a t^b \otimes t^a t^b = \frac{C_F}{2N_C} \mathbb{1} \otimes \mathbb{1} - \frac{1}{N_c} t^a \otimes t^a$$

$$t^a t^b \otimes t^b t^a = \frac{C_F}{2N_C} \mathbb{1} \otimes \mathbb{1} + \left(\frac{N_c}{2} - \frac{1}{N_c} \right) t^a \otimes t^a$$

V-A Contribution (octet)

$$\Gamma_a^O = \frac{1}{2N_c}\frac{a_s}{4}\frac{a}{\epsilon}Q_{V,A}^O$$

$$\Gamma_b^O = \frac{1}{2N_c}\frac{a_s}{4}\frac{a}{\epsilon}Q_{V,A}^O$$

$$\Gamma_c^O = [t^a t^b \otimes t^a t^b]\left[-\frac{5}{8}Q_{V,A} - \frac{3}{8}Q_{A,V} + (1-a)Q_{V,A}\right]$$

$$\Gamma_d^O = [t^a t^b \otimes t^a t^b]\left[-\frac{5}{8}Q_{V,A} - \frac{3}{8}Q_{A,V} + (1-a)Q_{V,A}\right]$$

$$\Gamma_e^O = \left[t^a t^b \otimes t^b t^a\right]\left[\frac{5}{8}Q_{V,A} - \frac{3}{8}Q_{A,V} - (1-a)Q_{V,A}\right]$$

$$\Gamma_f^O = \left[t^a t^b \otimes t^b t^a\right]\left[\frac{5}{8}Q_{V,A} - \frac{3}{8}Q_{A,V} - (1-a)Q_{V,A}\right]$$

V-A Contribution (octet)

Total contribution (octet):

$$\sum_{i=a,\ldots,f} \Gamma_i^O = \frac{a_s}{\epsilon}\left[\left(\frac{3}{8}N_c - \frac{a}{4}C_F\right) Q^O_{V,A} - \frac{3}{8}\frac{C_F}{N_c} Q^S_{A,V} + \frac{3}{2}\left(\frac{1}{N_c} - \frac{1}{4}N_c\right) Q^O_{A,V}\right]$$

Insertion of Q^O_{V-A} in Landau gauge:

$$\sum_{i=a,\ldots,f} \Gamma_i^O = \frac{a_s}{\epsilon}\left(\frac{3N_c}{4} - \frac{3}{2N_c}\right) Q^O_{V-A} + \frac{3C_F}{4N_C} Q^S_{V-A}$$

Renormalization Constants:

$$(\hat{Z}_0^{(1)})_{12} = \frac{3N_C}{4} - \frac{3}{2N_C} \qquad \text{and} \qquad (\hat{Z}_0^{(1)})_{12} = \frac{3C_F}{4N_C}$$

V-A Anomalous Dimension Matrix

Definition:

$$\hat{\gamma}_O(a_\mu) \equiv Z_O^{-1}(\mu)\mu\frac{d}{d\mu}\hat{Z}_O(\mu)$$

$$\hat{\gamma}_O^{(1)}(a_\mu) = -2\hat{Z}_O^{(1)}$$

Anomalous Dimension Matrix (V-A):

$$\hat{\gamma}^{(1)}_{O_{V-A}} = \begin{pmatrix} -\frac{3N_C}{2} + \frac{3}{N_C} & -\frac{3C_F}{2N_C} \\ -3 & 0 \end{pmatrix}$$

V+A Contribution

V+A Contribution

V+A Contribution

Total contribution (singlet):

$$\sum_{i=a,\ldots,f} \Gamma^{S}_{i,(V,A)} = \frac{a_s}{\epsilon} \frac{3}{2} Q^{O}_{A,V}$$

Total contribution (octet):

$$\sum_{a,\ldots,f} \Gamma_i^{Q_+^O} = \frac{a_s}{\epsilon} \left[\frac{3N_c}{8} Q^{O}_{V,A} - \frac{3C_F}{4N_c} Q^{S}_{A,V} - \frac{3}{4} \left(\frac{N_c}{2} - 2N_c \right) Q^{O}_{A,V} \right]$$

V+A Contribution (Q_+^O)

Q_+^O mixing:

$$Q_+^O = \bar{u}\gamma_\mu t^a d \bar{d}\gamma^\mu u + \bar{u}\gamma_\mu\gamma_5 t^a \bar{d}\gamma^\mu\gamma_5 t^a u$$

Current-current contribution:

$$\sum_{a,\ldots,f} \Gamma_i^{Q_+^O} = \frac{a_s}{\epsilon}\left[\frac{3}{2N_c} Q_+^O - \frac{3C_F}{4N_c} Q_+^S\right]$$

$$Z_{11} = \frac{3}{2N_c} \qquad \text{and} \qquad Z_{12} = -\frac{3C_F}{4N_c}.$$

V+A Contribution (Q_+^O)

Penguin contribution (singlet case):

$$\Gamma_{amp}^{Q_V^S} = -\frac{a_s}{6}\left[\frac{1}{\hat{\epsilon}} - ln\left(\frac{-p^2}{\mu^2}\right) + \frac{2}{3} + \mathcal{O}(\epsilon)\right] \cdot \left\{\left[\gamma^\lambda t^b\right]^{\bar{u}u} \sum_q \left[\gamma_\lambda t^b\right]^{\bar{q}q} + \frac{\left[\not{p} t^b\right]^{\bar{u}u}\left[\not{p} t^b\right]^{\bar{q}q}}{p^2}\right\}$$

Same for vector (γ_μ) and axialvector ($\gamma_\mu\gamma_5$):

$$\Gamma_{amp}^{Q_{V+A}^S}(local) = -\frac{a_s}{3}\frac{1}{\hat{\epsilon}}\left[[\gamma_\lambda t^a]^{\bar{u}u}[\gamma^\lambda t^a]^{\bar{q}q}\right] + \mathcal{O}(1).$$

Mixing into:

$$Q_3 = (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a) \sum_{q=u,d,s} (\bar{q}\gamma^\mu t^a q).$$

V+A Contribution (Q_+^O)

Penguin contribution (octet case):

$$t^a \to t^a t^b t^a = -\frac{1}{2N_c},$$

$$Z_{13} = \frac{1}{6N_c}.$$

Anomalous dimension:

$$\gamma_{Q_+^O} = \begin{pmatrix} -\frac{3}{N_c} & \frac{3C_F}{2N_c} & -\frac{1}{3N_c} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

V+A Contribution (Q_+^S)

Q_+^S mixing:

$$Q_+^S = \bar{u}\gamma_\mu d \bar{d}\gamma_\mu u + \bar{u}\gamma_\mu\gamma_5 d \bar{d}\gamma_\mu\gamma_5 u$$

$$\Gamma^{Q_+^S} = -\frac{a_s}{\epsilon}\frac{3}{2}Q_+^O \qquad \text{and} \qquad \Gamma^{Q_+^S}_{pen} = -\frac{1}{3}\frac{a_s}{\epsilon}$$

Renormalization constants:

$$Z_{21} = -\frac{3}{2} \qquad \text{and} \qquad Z_{23} = -\frac{1}{3}$$

Anomalous dimension:

$$\gamma_{Q_+^S} = \begin{pmatrix} 3 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

V+A Contribution (Q_3)

Q_3 mixing:

$$Q_3 = (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a) \sum_{q=u,d,s} (\bar{q}\gamma^\mu t^a q).$$

Current-current contribution:

$$\sum_{a,\ldots,f} \Gamma_i = \frac{a_s}{\epsilon} \left[\frac{3N_c}{8} Q_3 - \frac{3C_F}{4N_c} Q_6 - \frac{3}{4} \left(\frac{N_c}{2} - \frac{2}{N_c} \right) Q_4 \right]$$

V+A Contribution (Q_3)

$$\Gamma^{(1)}_{pen} = q(x_1)\bar{q}(x_2)[\bar{q}q\bar{q}q](z)q(x_3)\bar{q}(x_4)[B\bar{q}q](y_1)[B\bar{q}q](y_2),$$

$$\Gamma^{(2)}_{pen} = q(x_1)\bar{q}(x_2)[\bar{q}q\bar{q}q](z)q(x_3)\bar{q}(x_4)[B\bar{q}q](y_1)[B\bar{q}q](y_2),$$

V+A Contribution (Q_3)

$$\Gamma_{\bar{q}q} = q(x_1)\bar{q}(x_2)[(\bar{u}u + \bar{d}d)\sum \bar{q}q](z)q(x_3)\bar{q}(x_4)[\sum \bar{q}q](y_1)[\sum \bar{q}q](y_2).$$

$$\Gamma_{\bar{u}u} = q(x_1)\bar{q}(x_2)[(\bar{u}u + \bar{d}d)\sum \bar{q}q](z)q(x_3)\bar{q}(x_4)[\sum \bar{q}q](y_1)[\sum \bar{q}q](y_2).$$

$$\Gamma_{cross} = q(x_1)\bar{q}(x_2)[(\bar{u}u + \bar{d}d)(\bar{u}u + \bar{d}d + \bar{s}s)](z)q(x_3)\bar{q}(x_4)[\sum \bar{q}q](y_1)[\sum \bar{q}q](y_2),$$

$$\Gamma_{\bar{q}q} = -\frac{a_s}{\epsilon}\frac{N_f}{6}Q_3 \qquad \Gamma_{\bar{d}d} = -\frac{a_s}{\epsilon}\frac{1}{3}Q_7 \qquad \Gamma_{cross} = -\frac{a_s}{\epsilon}\frac{1}{N_c}\frac{1}{6}Q_3$$

$$\gamma_{Q3} = \begin{pmatrix} 0 & 0 & -\frac{3N_c}{4} + \frac{N_f}{3} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & \frac{3C_F}{2N_c} & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$

V+A Contribution (Q_4)

Q_4 mixing:

$$Q_4 = (\bar{u}\gamma_\mu\gamma_5 t^a u + \bar{d}\gamma_\mu\gamma_5 d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu\gamma_5 t^a q)$$

Current-current diagrams:

$$\sum_{i=a,\dots,f} \Gamma_i = \frac{a_s}{\epsilon}\left[\frac{3N_c}{8}Q_4 - \frac{3C_F}{4N_c}Q_5 - \frac{3}{4}\left(\frac{N_c}{2} - \frac{2}{N_c}\right)Q_3\right]$$

$$(1 - \frac{1}{N_c})Q_2 = 2Q_1 + 2Q_3 + 2Q_4 - \left(1 - \frac{1}{N_c}\right)(Q_5 + Q_6) - Q_7 - Q_8 - \left(1 - \frac{1}{N_c}\right)\left(\frac{Q_9 + Q_{10}}{2}\right)$$

$$\gamma_{Q4} = \begin{pmatrix} 0 & 0 & \frac{N_f}{3} - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & -\frac{3C_F}{2N_c} & -\frac{3}{4} - \frac{3}{4N_c} & -\frac{3}{4} - \frac{3}{4N_c} & \frac{3C_F}{4N_c} & \frac{3C_F}{4N_c} \end{pmatrix}$$

V+A Contribution ($Q_5, \ldots, Q_{10}$)

- Q_5 linear dependent
- $Q_6, \ldots, Q_{10}$ same calculation method

Anomalous Dimension Matrix (V+A)

$$
\gamma^{(1)}_{Q_+} = \begin{pmatrix}
-\frac{3}{N_C} & \frac{3C_F}{2N_C} & -\frac{1}{3N_C} & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{N_f}{3} - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & \frac{3C_F}{2N_C} & \frac{2}{3} & 0 & 0 & 0 \\
\frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & \frac{3N_c}{4} + \frac{3}{2} - \frac{11}{6N_c} & -\frac{3N_c}{4} + \frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & -\frac{3}{4} - \frac{3}{4N_c} & -\frac{3}{4} - \frac{3}{4N_c} & \frac{3C_F}{4N_c} & \frac{3C_F}{4N_c} \\
0 & 0 & \frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{N_f}{3} + 1 - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & 0 & \frac{3C_F}{2N_c} \\
0 & 0 & 0 & 0 & 0 & \frac{3N_c}{4} - \frac{10}{3N_c} & -\frac{3N_c}{4} & \frac{3C_F}{2N_c} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{11}{3} & 0 & 0 & 0
\end{pmatrix}
$$

Renormalization Group Equation

Renormalization Group Equation
(RGE)

Wilson coefficients

$$C_6^{V-A}(Q^2)\,\langle O_6\rangle \;=\; 4\pi^2 a_s \left\{ \left[2 + \left(\tfrac{25}{6} - L \right) a_s \right] \langle Q_-^o \rangle - \left(\tfrac{11}{18} - \tfrac{2}{3} L \right) a_s \langle Q_-^s \rangle \right\},$$

$$\begin{aligned} C_6^{V+A}(Q^2)\,\langle O_6\rangle \quad = \quad & -4\pi^2 a_s \left\{ \left[2 + \left(\tfrac{155}{24} - \tfrac{7}{2} L \right) a_s \right] \langle Q_+^o \rangle + \left(\tfrac{11}{18} - \tfrac{2}{3} L \right) a_s \langle Q_+^s \rangle + \right. \\ & \left[\tfrac{4}{9} + \left(\tfrac{37}{36} - \tfrac{95}{162} L \right) a_s \right] \langle Q_3 \rangle + \left(\tfrac{35}{108} - \tfrac{5}{18} L \right) a_s \langle Q_4 \rangle + \\ & \left(\tfrac{14}{81} - \tfrac{4}{27} L \right) a_s \langle Q_6 \rangle - \left(\tfrac{2}{81} + \tfrac{4}{27} L \right) a_s \langle Q_7 \rangle , \end{aligned}$$

[L.E. Adam and K.G. Chetyrkin, 1994]

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with

$$a_s \equiv \frac{\alpha_s}{\pi} \qquad \text{and} \qquad L \equiv \ln \frac{Q^2}{\mu^2}$$

Renormalization Group Equation

General term:

$$R_O = \vec{C}^T(\mu)\langle\vec{O}(\mu)\rangle,$$

Scale independent (μ):

$$\left[\mu\frac{d}{d\mu}\vec{C}^T(\mu)\right]\langle\vec{O}(\mu)\rangle = -C^T(\mu)\left[\mu\frac{d}{d\mu}\langle\vec{O}(\mu)\rangle\right]$$

Anomalous dimension definition:

$$-\mu\frac{d}{d\mu}\langle\vec{O}(\mu)\rangle \equiv \hat{\gamma}_O(a_\mu)\langle\vec{O}(\mu)\rangle$$

$\Rightarrow$

$$\mu\frac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}_O^T(a_\mu)\vec{C}(\mu)$$

with

$$\mu\frac{d}{d\mu} = \mu\frac{\partial}{\partial\mu} - \beta_1 a_s^2\frac{\partial}{\partial a_s}$$

Renormalization Group Equation

RGE equation:

$$\mu \frac{d}{d\mu} \vec{C}(\mu) = \hat{\gamma}_O^T(a_\mu) \vec{C}(\mu).$$

V-A case:

$$6 a_s^2 \pi^2 \begin{pmatrix} \frac{4}{N_c} - 2N_c \\ -1 + N_c^{-2} \end{pmatrix}$$

V+A case:

$$a_s^2 \pi^2 \begin{pmatrix} \frac{24}{N_c} & -6 & \frac{6}{N_c^2} & \frac{88}{27N_c} + \frac{4N_c}{3} - \frac{16N_f}{27} & \frac{16}{3N_c} - \frac{4N_c}{3} & -\frac{4}{3} + \frac{4}{3N_c^2} & -\frac{32}{27} & 0 & 0 & 0 \end{pmatrix}$$

Outlook

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- Ambiguities in the definition of OPE terms correspond to exponentially suppressed pertubative higher orders.
- Those ambiguities are reflected in singularities of the Borel transform of the correlator on the real axis (Renormalons).
- Renormalon structure can be inferred from OPE structure, which includes the anomalous dimensions.
- Leading order anomalous dimension influence strength of renormalon pole.

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