1 Penguin diagram to Q^s_{V+A}

$$
\begin{aligned}
\Gamma \;&=\; -\,g_s^2\!\int \mathrm{d}z\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\,\mathrm{d}x_4\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{e}^{p_1x_1+p_2x_2+p_3x_3+p_4x_4}\,\cdot \\
&\quad\langle T\{q(x_1)\bar q(x_2)(\bar u\Gamma_1 d\bar d\Gamma_2 u)(z)(\bar q\gamma^\lambda t^b B^b_\lambda q)(y_1)(\bar q\gamma^\omega t^c B^c_\omega q)(y_2)q(x_3)\bar q(x_4)\}\rangle \qquad (1)\\
&=\; i\,g_s^2\!\int \mathrm{d}z\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3\,\mathrm{d}x_4\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{e}^{p_1x_1+p_2x_2+p_3x_3+p_4x_4}\,\cdot \\
&\quad[S^u(x_1-z)\Gamma_1 S^d(z-y_1)\gamma^\lambda S^d(y_1-z)\Gamma_2 S^u(z-x_2)t^b]\,\cdot \\
&\quad\sum_q\,[S^q(x_3-y_2)\gamma^\omega S^q(y_2-x_4)t^b]D_{\lambda\omega}(y_1-y_2) \qquad (2)\\
&=\; i\,g_s^2\,\mu^{2\varepsilon}\!\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\,[S^u(p_1)\Gamma_1 S^d(p_1+k)\gamma^\lambda S^d(-p_2+k)\Gamma_2 S^u(-p_2)t^b]\,\cdot \\
&\quad\sum_q\,[S^q(p_3)\gamma^\omega S^q(-p_4)t^b]D_{\lambda\omega}(p_1+p_2) \qquad (3)
\end{aligned}
$$

Green function amputating the external quark propagators:

$$
\begin{aligned}
\Gamma_{\rm amp} \;&=\; i\,g_s^2\,\mu^{2\varepsilon}\!\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\,[\Gamma_1 S^d(p_1+k)\gamma^\lambda S^d(-p_2+k)\Gamma_2 t^b]^{\bar uu}[\gamma^\omega t^b]^{\bar qq}D_{\lambda\omega}(p_1+p_2) \\
&=\; i\,g_s^2\,\mu^{2\varepsilon}\!\int\frac{\mathrm{d}^Ds}{(2\pi)^D}\,[\Gamma_1 S^d(p-s)\gamma^\lambda S^d(-s)\Gamma_2 t^b]^{\bar uu}[\gamma^\omega t^b]^{\bar qq}D_{\lambda\omega}(p) \\
&=\; i\,g_s^2\,\mu^{2\varepsilon}\!\int\frac{\mathrm{d}^Ds}{(2\pi)^D}\,\frac{s_\alpha(p-s)_\beta}{s^2(p-s)^2}\,[\Gamma_1\gamma^\beta\gamma^\lambda\gamma^\alpha\Gamma_2 t^b]^{\bar uu}[\gamma^\omega t^b]^{\bar qq}\Big[g_{\lambda\omega}-(1-a)\frac{p_\lambda p_\omega}{p^2}\Big]\frac{1}{p^2} \quad (4)
\end{aligned}
$$

We have performed the substitution $s\equiv p_2-k$ and set $p\equiv p_1+p_2$, and the sum over $\bar qq$ is always understood.

The momentum integral is given by

$$
\mu^{2\varepsilon}\!\int\frac{\mathrm{d}^Ds}{(2\pi)^D}\,\frac{s_\alpha(p-s)_\beta}{s^2(p-s)^2} \;=\; \frac{i}{(4\pi)^2}\left(\frac{4\pi\mu^2}{-p^2}\right)^{\varepsilon}\frac{\Gamma[2-\varepsilon]^2}{\Gamma[4-2\varepsilon]}\,\Gamma[\varepsilon]\left[\frac{1}{2(1-\varepsilon)}\,g_{\alpha\beta}\,p^2+p_\alpha p_\beta\right]. \quad (5)
$$

Employing this result, for $\Gamma_1=\Gamma_2=\gamma_\mu$ one obtains:

$$
\begin{aligned}
\Gamma^{Q^s_V}_{\rm amp} \;&=\; -\,\frac{g_s^2}{(4\pi)^2}\left(\frac{4\pi\mu^2}{-p^2}\right)^{\varepsilon}\frac{\Gamma[2-\varepsilon]^2}{\Gamma[4-2\varepsilon]}\,\Gamma[\varepsilon]\,4(1-\varepsilon)\left[[\gamma_\lambda t^a]^{\bar uu}[\gamma^\lambda t^a]^{\bar qq}-[\not p t^a]^{\bar uu}[\not p t^a]^{\bar qq}\frac{1}{p^2}\right] \\
&=\; -\,\frac{a_s}{6}\left\{\frac{1}{\hat\varepsilon}-\ln\frac{-p^2}{\mu^2}+\frac{2}{3}+\mathcal{O}(\varepsilon)\right\}\left[[\gamma_\lambda t^a]^{\bar uu}[\gamma^\lambda t^a]^{\bar qq}-[\not p t^a]^{\bar uu}[\not p t^a]^{\bar qq}\frac{1}{p^2}\right] \qquad (6)
\end{aligned}
$$

The insertion of $\Gamma_1=\Gamma_2=\gamma_\mu\gamma_5$ yields an identical result. This demonstrates that the penguin contributions to Q^s_{V-A} cancel each other and for the singular part of Q^s_{V+A} we find:

$$\Gamma_{\rm amp}^{Q^s_{V+A}} \;=\; -\,\frac{a_s}{3}\,\frac{1}{\hat\varepsilon}\left[[\gamma_\lambda t^a]^{\bar u u}[\gamma^\lambda t^a]^{\bar q q} - [\not{p} t^a]^{\bar u u}[\not{p} t^a]^{\bar q q}\,\frac{1}{p^2}\right] + {\cal O}(1)\,. \tag{7}$$

As we are only interested in the mixing into a local operator, one should just pick the momentum independent term. Thus,

$$\Gamma_{\rm amp}^{Q^s_{V+A}}({\rm local}) \;=\; -\,\frac{1}{3}\,\frac{a_s}{\hat\varepsilon}\,[\gamma_\lambda t^a]^{\bar u u}[\gamma^\lambda t^a]^{\bar q q} + {\cal O}(1)\,. \tag{8}$$

The contraction of the $\bar u u$ pair yields the same structure with $\bar d d$. Hence, this corresponds to a mixing into the operator Q_3^{V+A} defined by

$$Q_3^{V+A} \;\equiv\; (\bar u\gamma_\mu t^a u + \bar d\gamma_\mu t^a d)\sum_{q=u,d,s}(\bar q\gamma^\mu t^a q)\,. \tag{9}$$

If the one-loop renormalisation matrix is defined by

$$\widehat Z_Q \equiv \hat 1 + \widehat Z_Q^{(1)}\,\frac{a_s}{\hat\varepsilon} + {\cal O}(a_s^2)\,, \tag{10}$$

we obtain

$$(\widehat Z_Q^{(1)})_{23} \;=\; -\,\frac{1}{3}\,. \tag{11}$$

Since the relation between the renormalisation matrix and the anomalous dimension matrix takes the form $\hat\gamma_Q^{(1)} = -\,2\,\widehat Z_Q^{(1)}$, the corresponding entry reads

$$(\hat\gamma_Q^{(1)})_{23} \;=\; \frac{2}{3}\,, \tag{12}$$

which agrees with the result of my notes `PiVmA_OPE.pdf` in eq. (54).

For the mixing of the octet operator Q^o_{V+A}, one just has to replace

$$t^a \;\longrightarrow\; t^b t^a t^b \;=\; -\,\frac{1}{2N_c}\,t^a\,. \tag{13}$$

Hence, the corresponding entry of the anomalous dimension matrix reads

$$(\hat\gamma_Q^{(1)})_{13} \;=\; -\,\frac{1}{3N_c}\,, \tag{14}$$

which again agrees with my notes at $N_c = 3$.