wKinematic

January 22, 2019

```
In [55]: exec(open('./initNotebook.py').read())
```

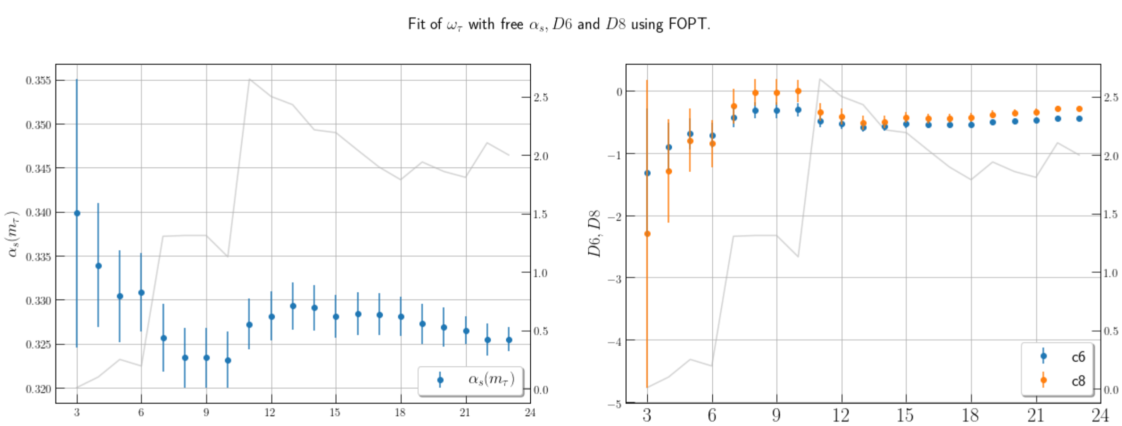
1 Load Data

```
In [5]: kinD6D8 = read_csv('../../FESR/configurations/2019/wKinematicD6D8/fits.csv')
        kinAlD6D8 = read_csv('../../FESR/configurations/2019/wKinematicAlphaD6D8/fits.csv')
```

1.1 Plots with free Alpha

```
In [6]: fig, (axes) = plt.subplots(1, 2, figsize=(21, 7))
        fig.suptitle(r'Fit of $\omega_\tau$ with free $\alpha_s, D6$ and $D8$ using FOPT.')
        plt.xticks(list(sminMap.values()), fontsize=22)
        addAx(axes, 0, ['alpha'], kinAlD6D8, ylabel=r'$\alpha_s(m_\tau)$')
        addAx(axes, 1, ['c6', 'c8'], kinAlD6D8, ylabel='$D6, D8$')

        fig.savefig('./plots/wKinematicAlpha.png', dpi=300)
        plt.show()
```



In the plots we can see the result of fitting $\alpha_s(m_\tau), c_6$ and c_8 using ω_τ in FOPT for 3- to 23 s_0-moments (the lowest s_0 moment is given by $s_{min} = 1.5GeV^2$). The best value (with a $\chi^2/dof = 1.12$) yields $\alpha_s(m_\tau) = 0.3231(32)$ with $c6 = -0.30(11)$ and $c8 = 0.00(18)$ for $s_{min} = 1.925$ ($10 s_0 s$-moments). α_s varies wave-like for inreasing s_0-moments within error ranges and converges around $\alpha_s \approx 0.325$. With an increasing number of $s_0 s$-moments the error decrease, but χ^2/dof increases. The error of α_s is smallest for the biggest s_0s-moment number and would decrease for more s_0s-moments. For c_6 and c_8 we see an inverted wave-like behavior, meaning that their values first increases, then slightly decrease until they converge for a value around $c_6 \approx -0.4$ and $c_8 = -0.3$. This inverted behavior shows that if we increase the α_s contribution to our integral-moment the other contributions, c_6 and c_8, have to decrease. The values of c_6 and c_8 vary also within error ranges, which "vanish" for heigher s_0s-moment numbers. We also plotted the χ^2/dof function as gray line in the background of the plots. Their values can be read off the right y-axis. One notices three plateaus: The first one for $3-6$ s_0s-moments with a $\chi^2/dof \approx 0$ the second one for $7-10$ s_0s-moments with a good $\chi^2/dof \approx 1.3$ and another for $11-23s_0s$-moments with a to big value of $\chi^2/dof > 2$, which tells us that $7-10$ is a good choice for the number of fitted moments. Due to the previous discussion we see that the fit results are solid, with an $\alpha_s(m_\tau) = 0.3231(32)$. Problematic are the heigher values for α_s for lower s_{min} or more s_0s moments. The values of α_s are 2% heigher in the 2nd χ^2 plateau.

1.1.1 Correlation of c_6 and c_8

In [29]:
```python
rAlphaC8, _ = stats.pearsonr(kinAlD6D8['alpha'], kinAlD6D8['c6'])
rC6C8, _ = stats.pearsonr(kinAlD6D8['c6'], kinAlD6D8['c8'])
print(r'Pearsons r for $\alpha_s-c_8 =$: ', rAlphaC8)
print(r'Pearsons r for $c_6-c_8 =$: ', rC6C8)
```

```
Pearsons r for $\alpha_s-c_8 =$:  -0.9914086952783082
Pearsons r for $c_6-c_8 =$:  0.995920629934654
```

We test the correlation of α_s with c_8 and c_6 with c_8 with the help of *Pearsons r*. The former yields a value of $r_{\alpha_s,c_6} = -0.99$, which suggest a strongly antiproportional behavior, whereas the latter $r_{c_6,c_8} \approx 1$ advocates perfect propotionality. The pearson squares explain the inverted wave-like behavior we have seen before.

1.1.2 Test for OPE series convergence

In [56]:
```python
testOPESeriesForConvergence(kinAlD6D8)
```

Out[56]:
```
smin15      True
smin1525    True
smin155     True
smin1575    True
smin16      True
smin1625    True
smin165     True
smin1675    True
smin17      True
smin175     True
smin18      True
smin185     True
smin19      True
smin195     True
smin20      True
```

```
smin21      True
smin22      True
smin23      True
smin24      True
smin26      True
smin28      True
dtype: bool
```

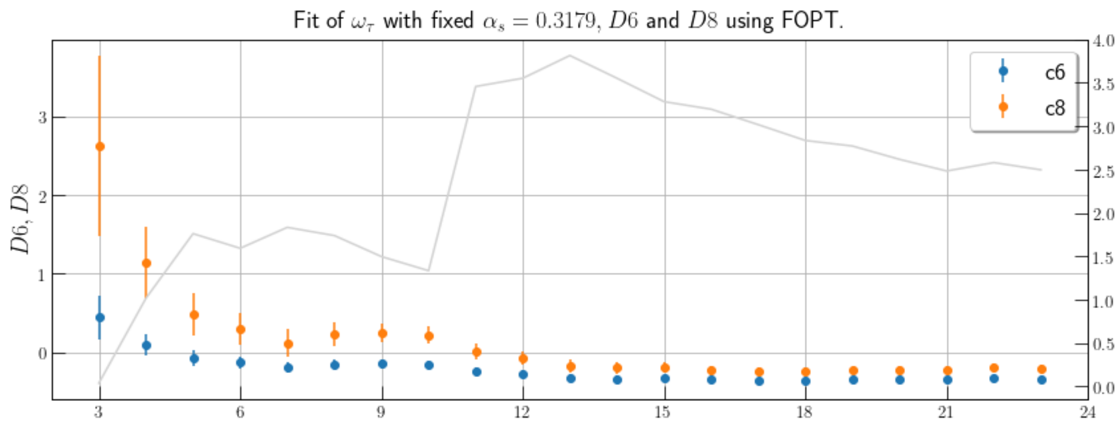
1.2 Plots with fixed Alpha

```
In [72]: s0s = list(map(lambda smin: sminMap[smin], kinD6D8.index))
         plt.title(r'Fit of $\omega_\tau$ with fixed $\alpha_s = 0.3179, D6$ and $D8$ using FOI
         plt.gca().set_ylabel(r'$D6, D8$')
         plt.errorbar(s0s, kinD6D8['c6'], barsabove=True, yerr=kinD6D8['c6Err'], fmt='.', marke
         plt.errorbar(s0s, kinD6D8['c8'], barsabove=True, yerr=kinD6D8['c8Err'], fmt='.', marke
         plt.gca().legend()
         plt.gcf().set_size_inches(14, 5)

         ax_r = plt.gca().twinx()
         ax_r.plot(s0s, kinD6D8['chiDof'], color='lightgrey')
         ax_r.grid(False)

         plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True))

         plt.gcf().savefig('./plots/wKinematicFixedAlpha.png', dpi=300)
         plt.show()
```



In the above plot we investigated the behavior of c_6 and c_8 for a fixed $\alpha_s = 0.3179$. The best $\chi^2/dof = 1.33$ values for $c_6 = -0.15(53)$ and $c_8 = 0.22(11)$ can be found for $10s_0s$ moments for $s_{min} = 1.925GeV^2$. The fit including a free α_s shows very similar behavior to this fit, except the c_6 and c_8 series behavior has been inverted. They decrease, increase and converges to values $c_6 \approx -0.34$ and $c_8 \approx -0.21$. The values of the two OPE coefficients are smaller than the ones from the fit with free α_s, which can be explained due to the lower value for $\alpha_s = 0.3179$ (before we had $\alpha_s \approx$

0.33) compensating the bigger values of c_8 and c_6. One still sees the strong correlation between the two coefficients and also notes that they are getting closer to each other with increasing s_0s-moment number.

```
In [76]: testOPESeriesForConvergence(kinD6D8)
```

```
Out[76]: smin15       True
         smin1525     True
         smin155      True
         smin1575     True
         smin16       True
         smin1625     True
         smin165      True
         smin1675     True
         smin17       True
         smin175      True
         smin18       True
         smin185      True
         smin19       True
         smin195      True
         smin20       True
         smin21       True
         smin22       True
         smin23       True
         smin24       True
         smin26       True
         smin28       True
         dtype: bool
```

OPE series converges for fits with fixed α_s.

2 Conclusion

The fits show no sign of inconsitency and have a good χ^2/dof. Furthermore the variation of α_s for different moments setups is small, which let us raise the question of why additional frameworks than the OPE are needed to measure the strong coupling in τ-fits. Using ω_τ is probably the best weight of measuring α_s. Other weights like the cubic or (worse) the quartic weight generate problems within my fitting routines for heigher numbers of s_0s-moments.