Chapter 1

Theoretical Background

1.1 Quantumchromodynamics

QCD describes the strong nuclear force, which is mediated by *gluons* acting on *quarks*. It is born out of the very successful theory of *quantum electrodynamics* (QED), but has some major differences. QED has two well known constituent fields the fermion and the photon. It is also a so called *abelian* theory, meaning that two consecutive rotations commute $AB = BA$. Due to the abelian nature of the QED gauge group $U(1)$ only fermions carry gauge charge. Photons do not carry charge. In contrast QCD takes quarks[1] and gluons as fields. Additionally QCD is ruled by a *non-abelian* gauge symmetry of $SU(3)$. Thus the group operation of QCD in general do not commute, which leads to gluons carrying charge! This charge is called *color-charge*. The theory requieres three colors and describes interactions between a quark of a color with a quark of another color via gluons. Contrary to QED the force carrier, the gluons, interact with themselve, because they carry color charge. The corresponding Feynman diagrams can be seen in fig. 1.1.

The theory of QCD can be summarized with its *Lagrange density*[**Jamin2006**]:

$$\mathcal{L}_{QCD}(x) = -\frac{1}{4}G^a_{\mu\nu}(x)G^{\mu\nu a}(x) + \sum_A \left[\frac{i}{2}\bar{q}^A(x)\gamma^\mu \overleftrightarrow{D}_\mu q^A(x) - m_A\bar{q}^A(x)a^A(x)\right], \tag{1.1}$$

where $q^A(x)$ represents the quark fields and $G^a_{\mu\nu}$ being the *gluon field strength tensor* given by:

$$G^a_{\mu\nu}(x) \equiv \partial_\mu B^a_\nu(x) - \partial^a_\nu(x) + gf^{abc}B^b_\mu(x)B^c_\nu(x), \tag{1.2}$$

where B^a_μ are the *gluon fields*, given in the *adjoint representation* of the SU(3) gauge group with f^{abc} as *structure constants.* Furthermore we have used $A, B, \ldots = 0, \ldots 5$ as flavour indices, $a, b, \ldots = 0, \ldots, 8$ as color indices and $\mu, \nu, \ldots = 0, \ldots 3$

[1] Quarks are a subset to the fermions.

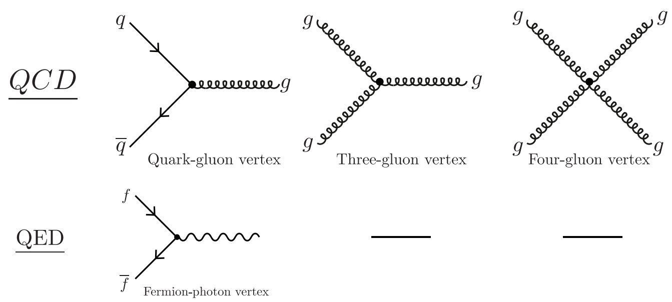

Figure 1.1: Feynman diagrams of the strong interactions with corresponding electromagnetic diagrams. We see that the gluons carry color-charge and thus couple to other gluons, which is not the case for the photons.

Flavour	Mass
u	3.48(24) MeV
d	6.80(29) MeV
s	130.0(18) MeV
c	1.523(18) GeV
b	6.936(57) GeV
t	173.0(40) GeV

Table 1.1: List of Quarks and their masses. The masses of the up, down, strange, charm and bottom quark are the renormalization group invariant (RGI) quark masses and are quoted in the four-flavour theory ($N_f = 2 + 1$) at the scale $\mu = 2\,\mathrm{GeV}$ in the $\overline{MS}$ scheme and are taken from the *Flavour Lattice Averaging Group* [**FLAG2019**]. The mass of the top quark is not disucess in [**FLAG2019**] and has been taken from [**PDG2018**] from direct observations of top events.

as lorentz indices. Explicitly the Lagrangian writes:

$$
\begin{aligned}
\mathcal{L}_0(x) = &-\frac{1}{4}\left[\partial_\mu G^a_\nu(x) - \partial_v G^a_\mu(x)\right]\left[\partial^\mu G^\nu_a(x) - \partial^\nu G^\mu_a(x)\right] \\
&+ \frac{i}{2}\overline{q}^{\,A}_{\,\alpha}(x)\gamma^\mu\partial_\mu q^A_\alpha(x) - \frac{i}{2}\left[\partial_\mu\overline{q}^{\,A}_{\,\alpha}(x)\right]\gamma^\mu q^A_\alpha(x) - m_A\overline{q}^{\,A}_{\,\alpha}(x)q^A_\alpha(x) \\
&+ \frac{g_s}{2}\overline{q}^{\,A}_{\,\alpha}(x)\lambda^a_{\alpha\beta}\gamma_\mu q^A_\beta(x)G^\mu_a(x) \\
&- \frac{g_s}{2}f_{abc}\left[\partial_\mu G^a_\nu(x) - \partial_\nu G^a_\mu(x)\right]G^\mu_b(x)G^\nu_c(x) \\
&- \frac{g_s^2}{4}f_{abc}f_{ade}G^b_\mu(x)G^c_\nu(x)G^\mu_d(x)G^\nu_e(x)
\end{aligned}
\tag{1.3}
$$

1.1.1 Renormalisation Group

The perurbations of the QCD Lagrangian 1.1 lead to divergencies, which have to be *renormalized*. There are different aproaches to 'make' these divergencies finite. The most popular one is **dimensional regularisation**.

In *dimensional regularisation* we expand the four space-time dimensions to arbitrary dimensions. Consequently the in QCD calculations appearing *Feyman integrals* have to be continued to D-dimensions like

$$
\mu^{2\epsilon}\int\frac{\mathrm{d}^D p}{(2\pi)^D}\frac{1}{[p^2 - m^2 + i0][(q-p)^2 = m^2 + i0]},
\tag{1.4}
$$

where we introduced the scale parameter μ to account for the extra dimensions and conserve the mass dimension of the non continued integral.

In addition *physical quantities*[2] cannot depend on the renormalisation scale μ. Thus the derivative by μ of a general *physical quantity* $R(q, a_s, m)$ that depends on the external momentum q, the renormalised coupling $a_s \equiv \alpha_s/\pi$ and the renormalized quark mass m has to yield zero

$$
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}R(q, a_s, m) = \left[\mu\tfrac{\partial}{\partial\mu} + \mu\tfrac{\mathrm{d}a_s}{\mathrm{d}\mu}\tfrac{\partial}{\partial m} + \mu\tfrac{\mathrm{d}m}{\mathrm{d}\mu}\tfrac{\partial}{\partial m}\right]R(q, a_s, m) = 0.
\tag{1.5}
$$

eq. (1.5) is referred to as **renormalization group equation** and is the basis for defining the *renormalisation group functions*:

$$
\beta(a_s) \equiv -\mu\tfrac{\mathrm{d}a_s}{\mathrm{d}\mu} = \beta_1 a_s^2 + \beta_2 a_s^3 + \ldots \qquad \beta-\text{function} \tag{1.6}
$$

$$
\gamma(a_s) \equiv -\frac{\mu}{m}\tfrac{\mathrm{d}m}{\mathrm{d}\mu} = \gamma_1 a_s + \gamma_2 a_s^2 + \ldots \qquad \text{anomalous mass dimension.} \tag{1.7}
$$

Running gauge coupling

The β-function and the anomalous mass dimension are responsible for the running of the strong coupling and the running of the quark mass respektively. In this section we will shortly review the β-function and its implications on the

[2]Observables that can be measured.

strong coupling, whereas in the following section we will discuss the anomalous-mass dimension.

Regarding the β-function we notice, that $a_s(\mu)$ is not a constant, but *runs* by varying its scale μ. Lets observe the running of the strong coupling constant by integrating the β-function

$$\int_{a_s(\mu_1)}^{a_s(\mu_2)} \frac{\mathrm{d}a_s}{\beta(a_s)} = -\int_{\mu_1}^{\mu_2} \frac{\mathrm{d}\mu}{\mu} = \log\frac{\mu_1}{\mu_2}. \tag{1.8}$$

To analytically evaluate the above integral we can approximate the β-function to first order, with the known coefficient

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f), \tag{1.9}$$

yielding

$$a_s(\mu_2) = \frac{a_s(\mu_1)}{\left(1 - a_s(\mu_1)\beta_1 \log\frac{\mu_1}{\mu_2}\right)}. \tag{1.10}$$

As we have three colours $N_c = 3$ and six flavours $N_f = 6$ the first β-function 1.6 is positive. Thus for $\mu_2 > \mu_1$ $a_s(\mu_2)$ decreases logarithmically and vanishes for $\mu_2 \to \infty$. This behaviour is known as *asymptotic freedom*. The coefficients of the β-function are currently known up to the 5th order and listed in the appendix **??**.

As QCD we have three colors and six flavours the beta function carries a negative sign (see eq. (1.9)). This implies that the strength of the coupling is **decreasing** for increasing energies. In QED we have a beta-function with a positive sign and thus its coupling strength increases with energy. This behaviour of QCD leads to *confinement* and *asymptotic freedom*.

Color confinement, or simply confinement, means that color charged quarks cannot be isolated. They always appear in composite hadrons. Until today no quark has been measured as single particle. There is no analytic proof of confinement, but until today no single quark has been observed. It is qualitativly explained with the gluon field carrying color charge. These gluons form so-called *flux-tubes* between quarks, which cause a constant strong force between particles regardless of their separation. Consequently the energy needed to separate quarks is proportional to the distance between them and at some point there is enough energy to favour the creation of a new quark pair. Thus before separating two quarks we create a two new quarks and we will probably never be able to observe an isolated quark.

For high energies, or very close quarks, the QCD coupling becomes weak and the quarks and gluons are essentially free. This phenomen is refered to as asymptotic freedom.

Running quark mass

Not only the coupling but also the masses carry an energy dependencies, which is governed by the *anomalou mass dimension* $\gamma(a_s)$.

The properties of the running quark mass can be derived similar to the gauge coupling. Starting from integrating the *anomalous mass dimension* 1.7

$$\log \frac{m(\mu_2)}{m(\mu_1)} = \int_{a_s(\mu_1)}^{a_s(\mu_2)} \mathrm{d}a_s \frac{\gamma(a_s)}{\beta(a_s)} \tag{1.11}$$

we can approximate the *anomalous mass dimension* to first order and solve the integral analytically [**Schwab2002**]

$$m(\mu_2) = m(\mu_1) \left(\frac{a(\mu_2)}{a(\mu_1)} \right)^{\frac{\gamma_1}{\beta_1}} \left(1 + \mathcal{O}(\beta_2, \gamma_2)\right). \tag{1.12}$$

As β_1 and γ_1 (see **??**) are positive the quark mass decreases with increasing μ. The general relation between different scales is given by

$$m(\mu_2) = m(\mu_1) \exp \left(\int_{a_s(\mu_1)}^{a_s(\mu_2)} \mathrm{d}a_s \frac{\gamma(a_s)}{\beta(a_s)} \right) \tag{1.13}$$

and can be solved numerically to run the quark mass to the needed scale μ_2.

QCD in general has a precision problem caused by uncertainties and largness of the strong coupling constant α_s. The fine-structure constant (the coupling constant of QED) is known to eleven digits, whereas the strong coupling is only known to about four. Furthermore for low energies the strong coupling constant is much larger than the fine-structure constant. E.g. at the Z-mass, the standard mass to compare the strong coupling, we have an α_s of 0.11, whereas the fine structure constant would be around 0.007. Consequently to use PT we have to calculate our results to much heigher orders, including tens of thousends of Feynman diagrams, in QCD to achieve a precision equal to QED. For even lower energies, around 1 GeV, the strong coupling reaches a critical value of ≈ 0.5 leading to a break down of PT.

In this work we try to achieve a higher precision in the value of α_s. Our method to measure the strong coupling is called **QCD sum rules**, which by itself is based on a concept called the *two-point function* for which we will devote the following section.

1.1.2 Two-Point function

The vacuum expectation value of the product of the conserved noether current $J_\mu(x)$ at different space-times points x and y is known as the **two-point function** (or simply **correlator**)

$$\Pi_{\mu\nu}(q^2) = \langle 0 | J_\mu(x) J_\nu(y) | 0 \rangle, \tag{1.14}$$

where the noether current is given by

$$J_\mu(x) = \overline{q}\,(x) \Gamma q(y) \tag{1.15}$$

, where Γ stands for one of the dirac matrices $\Gamma \in \{1, i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\}$, specifying the quantum number of the current (S: *scalar*, P: *pseudo-Scalar*, V: *vectorial*, A: *axial-vectorial*, respectively).

The correlator tensor $\Pi_{\mu\nu}(q^2)$ can be lorentz decomposed to a scalar function $\Pi(q^2)$. There are only two possible terms that can reproduce the second order tensor $q_\mu q_\nu$ and $q^2 g_{\mu\nu}$. The sum of both multiplied with two arbitrary functions $A(q^2)$ and $B(q^2)$ yields

$$\Pi_{\mu\nu}(q^2) = q_\mu q_\nu A(q^2) + q^2 g_{\mu\nu} B(q^2). \tag{1.16}$$

By making use of the **Ward-identity** [**Peskin1995**]

$$q^\mu \Pi_{\mu\nu}(q^2) = q^\nu \Pi_{\mu\nu} = 0 \tag{1.17}$$

we can demonstrate, that the two arbitrary functions are related

$$\begin{aligned} q^\mu q^\nu \Pi_{\mu\nu} &= q^4 A(q^2) + q^4 B(q^2) = 0 \\ &\implies A(q^2) = -B(q^2). \end{aligned} \tag{1.18}$$

Thus redefining $A(q^2) \equiv \Pi(q^2)$ we expressed the correlator as a scalar function

$$\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2). \tag{1.19}$$

The scalar QCD two point function can then be related to the spectrum of hadronic states. The correlator is then related to an integral over the **spectral function** $\rho(s)$ via the *Källén-Lehmann spectral representation* [**Kallen1952**, **Lehmann1954**], which is known since the early fities

$$\Pi(q^2) = \int_0^\infty \mathrm{d}s \frac{\rho(s)}{s - q^2 - i\epsilon}. \tag{1.20}$$

Equation 1.20 is refered to as **dispersion relation** analogous to similar relations which arise for example in electrodynamics and defines the **spectral function** (a derivation can be found in [**Rafael1997**])

$$\rho(s) = \frac{1}{\pi} \operatorname{Im} \Pi(s). \tag{1.21}$$

Until know we connected theoretical correlators with the measurable hadronic spectrum. Nevertheless the analytic properties of the correlators have to be discussed as the function has discontinuities.

The main contribution from the spectral function given in eq. (1.20) are the hadronic final states

$$2\pi\rho(m^2) = \sum_n \langle 0|J_\mu(x)|n\rangle\langle n|J_\nu(y)\rangle(2\pi^2)^4\delta^{(4)}(p - p_n), \tag{1.22}$$

which lead to a series of continuous poles on the positive real axis for the two-point function, see Fig. 1.2. These discontinuities can be tackled with *Cauchy's theorem*, which we will apply in **??**.

Until now we exclusively dealt with the perturbative (PT) part of the theory, but QCD is known to have not negligible non-perturbative (NPT) contributions. Thus before continuing with the *Sum Rules* we need a final ingredient the operator product expansion, which implements NPT cotributions to our theory.

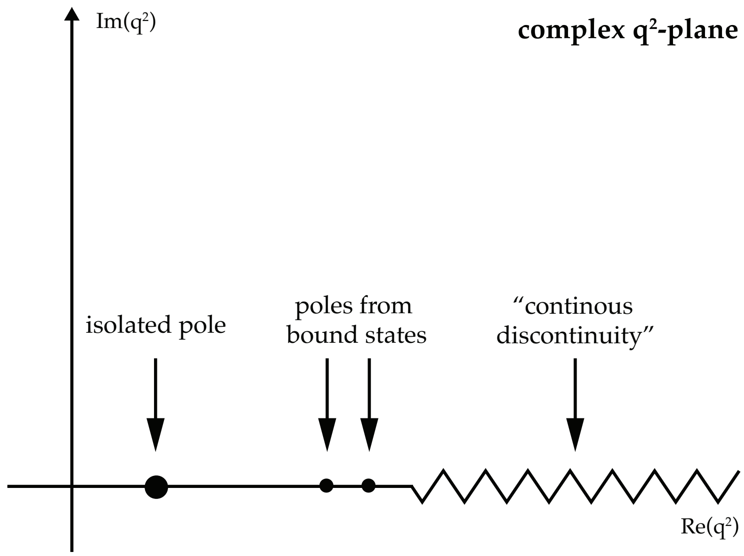

Figure 1.2: Analytic structure in the complex q^2-plane of the Fourier transform of the two-point function. The hadronic final states are responsible for poles appearing on the real-axis. The one-particle states contribute as isolated pole and the multi-particle states contribute as bound-states poles or a continues "discontinuity cut" [**Peskin1995**].

1.1.3 Operator Product Expansion

The **Operator Product Expansion** (OPE) was introduced by Wilson in 1969 [**Wilson1969**]. The expansion states that non-local operators can be rewritten into a sum of composite local operators and their corresponding coefficients:

$$\lim_{x \to y} \mathcal{O}_1(x)\mathcal{O}_2(y) = \sum_n C_n(x-y)\mathcal{O}_n(x), \tag{1.23}$$

where $C_n(x-y)$ are the so-called *Wilson-coefficients*.

The OPE lets us separate *short-distance* from *long-distance* effects. In perturbation theory (PT) we can only amount for *short-distances*, which are equal to hight energies, where the strong-coupling α_s is small. Consequently the OPE decodes the long-distance effects in the higher dimensionsional operators.

The form of the composite operators are dictated by Gauge- and Lorentz symmetry. Thus we can only make use of operators of even dimension. The operators up to dimension six are given by [**Pascual1984**]

$$\begin{array}{ll} \text{Dimension 0:} & \mathbb{1} \\ \text{Dimension 4:} & : m_i \overline{q}\, q : \\ & : G_a^{\mu\nu}(x) G^a_{\mu\nu}(x) : \\ \text{Dimension 6:} & : \overline{q}\, \Gamma q \overline{q}\, \Gamma q : \\ & : \overline{q}\, \Gamma \frac{\lambda^a}{2} q_\beta(x) \overline{q}\, \Gamma \frac{\lambda^a}{2} q : \\ & : m_i \overline{q}\, \frac{\lambda^a}{2} \sigma_{\mu\nu} q G_a^{\mu\nu} : \\ & : f_{abc} G_a^{\mu\nu} G_b^{\nu\delta} G_c^{\delta\mu} :, \end{array} \tag{1.24}$$

where Γ stands for one of the dirac matrices $\Gamma \in \{1, i\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5\}$, specifying the quantum number of the current (S, P, A, respectively). As all the operators appear normal ordered they vanish by definition in PT. Consequently they appear as **Condensates** in Non-perturbative (NPT) QCD like quark-condensate $\langle \overline{q}\, q \rangle$ or the gluon-condensate $\langle aGG \rangle$ (both of dimension four). These non-vanishing condensats characterize the QCD-vacuum.

As we work with dimensionless functions (e.g. Π) in Sum Rules, the r.h.s. of **??** has to be dimensionless. Consequently the Wilson-coefficients have to cancel the dimension of the operator with their inverse mass dimension. To account for the dimensions we can make the inverse momenta explicit

$$\Pi^{OPE}_{V/A}(s) = \sum_{D=0,2,4\ldots} \frac{c^{(D)} \langle \mathcal{O}^{(D)}(x) \rangle}{-s^{D/2}}, \tag{1.25}$$

where we used $C^{(D)} = c/(-s)^{D/2}$ with D being the dimension. Consequently the OPE should converge with increasing dimension for suficienty large momenta s.

Let's show how the OPE contributions are calculated with a the "standard example" (following [**Pascual1986**]), where we compute the perturbative and quark-condensate Wilson-coefficients for the ρ-meson. For the ρ-meson, which is composed of u and d quarks, the current of eq. (1.14) takes the following form

$$j^\mu(x) = \frac{1}{2} \Big(: [\overline{u}\, \gamma^\mu u](x) - \overline{d}\, \gamma^\mu d](x) \Big) . \tag{1.26}$$

In fig. 1.3 we draw the Feynman-diagram, from which we can take the uncontracted mathematical expression for the scalar correlator

$$\begin{aligned} \Pi(q^2) = -\frac{i}{4q^2(D-1)} \int \mathrm{d}^D\, x e^{iqx} \langle \Omega | T\{ : \overline{u}\, (x) \gamma^\mu u(x) - \overline{d}\, (x) \gamma^{mu} d(x) : \\ \times : \overline{u}\, (0 \gamma_\mu u(0) - \overline{d}\, (0) \gamma_\mu d(0) : \} \rangle . \end{aligned} \tag{1.27}$$

Using Wick's theorem we can contract all of the fields and calculate the first term of the OPE ($\mathbb{1}$), which represents the perturbative contribution of the

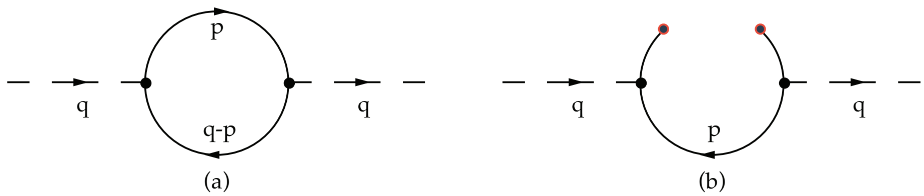

Figure 1.3: Feynman diagrams of the perturbative (a) and the quark-condensate (b) contribution. The upper part of the right diagram is not wick-contracted and responsible for the condensate.

OPE ($\nvdash$)

$$\begin{aligned}\Pi(q^2) &= \frac{i}{4q^2(D-1)}(\gamma^\mu)_{ij}(\gamma_\mu)_{kl}\int \mathrm{d}^D\, x e^{iqx} \\ &\times \quad \left[u_{j\alpha}(x)\overline{u}_{k\beta}(0)\cdot u_{l\beta}(0)\overline{u}_{i\alpha}(x) + (u\to d)\right] \\ &= \frac{3}{8\pi^2}\left[\frac{5}{3} - \log\left(-\frac{q^2}{\nu^2}\right)\right].\end{aligned} \tag{1.28}$$

To calculate the higher dimensional contributions of the OPE we use the same techniques as before, but leave some of the fields uncontracted. For the quark-condensate, which we want to derive for tree-level, we leave two fields uncontracted

$$\begin{aligned}\Pi(q^2) = &\frac{i}{4q^2(D-1)}(\gamma^\mu)_{ij}(\gamma_\mu)_{kl}\int \mathrm{d}^D\, x e^{iqx}\Big[\\ &+ u_{j\alpha}(x)\overline{u}_{k\beta}(0)\cdot\langle\Omega| : \overline{u}_{i\alpha}(x)u_{l\beta}(0) : |\Omega\rangle \\ &+ u_{l\beta}(0)\overline{u}_{i\alpha}(x)\cdot\langle\Omega| : \overline{u}_{k\beta}(0)u_{j\alpha}(x) : |\Omega\rangle + (u\to d)\Big].\end{aligned} \tag{1.29}$$

The non contracted fields can then be expanded in x

$$\begin{aligned}\langle\Omega| : \overline{q}\,(x)q(0) : |\Omega\rangle = &\langle\Omega| : \overline{q}\,(0)q(0) : |\Omega\rangle \\ &+ \langle\Omega| : \left[\partial_\mu\overline{q}\,(0)\right]q(0) : |\Omega\rangle x^\mu + \ldots\end{aligned} \tag{1.30}$$

and redefined to a more elegant notation

$$\langle\overline{q}\,q\rangle \equiv \langle\Omega| : \overline{q}\,(0)q(0) : |\Omega\rangle. \tag{1.31}$$

The finally result can be taken from [**Pascual1984**] and yields

$$\Pi_{(\rho)}(q^2) = \frac{1}{2}\frac{1}{(-q^2)^2}\left[m_u\langle\overline{u}\,u + m_d\langle\overline{d}\,\rangle\right]. \tag{1.32}$$

The usage of the OPE and its validity is far from obvious. We are deriving the OPE from matching the Wilson-coefficients to Feynman-graph analyses. These Feynman-graphs are calculated perturbatively but the coefficients with dimension $D > 0$ correspond to NPT condensates!

Having gathered all of the necessary concepts we can close the gap between the theory and experiment in the last section of the introduction: QCD Sum Rules.

1.1.4 Sum Rules

To relate the measurable hadronic final states of a QCD process (e.g. τ-decays into Hadrons) to a theoretical calculable **QCD sum rules** have been empliyed by Shifman in the late sevent [**Shifman1978**].

The sum rules are a combination of the two-point function and its analyticity, the OPE, a dispersion relation, the optical theorem and quark hadron duality.

The previously introduced two-point function eq. (1.14) is generally descriped by the OPE to account for NPT effects.

$$\Pi(q^2) = \Pi^{OPE}(q^2). \tag{1.33}$$

Furthermore it is related to the theoretical spectral function $\rho(s)$ via a dispersion relation **??**eq:dispersionRelation). Using QCD we are computing interactions based on quarks and gluons, but due to confinement, we are only able to observe Hadrons. Consequently to connect the theory to the experiment we have to assume **quark-hadron duality**[3], which implies that physical quantities can be described equally good in the hadronic or in the quark-gluon picture. Thus we can rewrite the dispersion relation eq. (1.20) as

$$\Pi_{th}^{OPE}(q^2) = \int_0^\infty \frac{\rho_{exp}(q^2)}{(s - q^2 - i\epsilon)}, \tag{1.34}$$

where we connected the theoretical correlator Π_{th} with the experimental measurable spectral function ρ_{exp}.

We have seen that the theoretical description of the correlator Π_{th} contains poles on the real axis, but the experimental data ρ_{exp} is solely accesible on the positive real axis. Thus we have to make use of Cauchy's theorem to access the theoretical values of the two-point function close to the postive real axis (see section 1.1.4) given by

$$\int_{\mathcal{C}} f(z)\,\mathrm{d}z = 0, \tag{1.35}$$

where $f(z)$ is an analytic function on a closed contour $\mathcal{C}$.

The final ingredient of the QCD sum rules is the *optical theorem*, relating experimental data with the imaginary part of the correlator (the spectral function $\rho(s)$).

[3] Or simply duality.

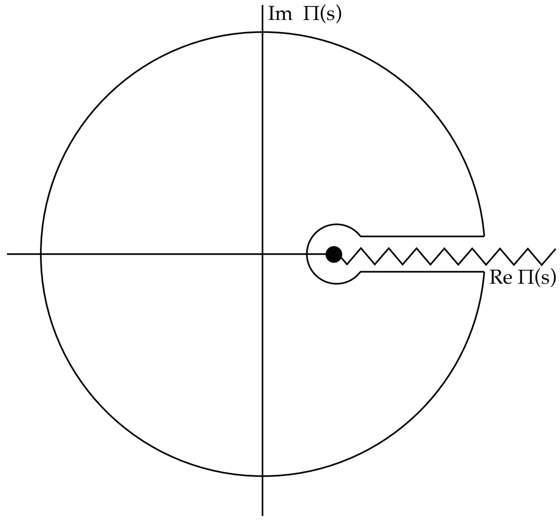

Figure 1.4: Analytical structure of $\Pi(s)$ with the used contour $\mathcal{C}$ for the final QCD Sum Rule expression eq. (1.36).

In total, with the help Cauchy's theorem, the QCD sum rules can be sumed up in the following expression

$$\frac{1}{\pi}\int_0^\infty \frac{\rho_{exp}(t)}{t-s}\,\mathrm{d}t = \frac{1}{\pi}\oint_{\mathcal{C}} \frac{\mathrm{Im}\,\Pi_{OPE}(t)}{t-s}\,\mathrm{d}t, \tag{1.36}$$

where the l.h.s. is given by the experiment and the r.h.s. can be theoretically evaluated with by applying the OPE of the correlator $\Pi_{OPE}(s)$.