

# Chapter 1

## Introduction

In particle physics we are concerned about small objects and their interactions. Their dynamics are currently best described by the Standard Model (SM).

The SM contains two groups of fermionic, Spin  $1/2$  particles. The former group, the Leptons consist of: the electron ( $e$ ), the muon ( $\mu$ ), the tau ( $\tau$ ) and their corresponding neutrinos  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ . The latter group, the Quarks contain:  $u$ ,  $d$  (up and down, the so called light quarks),  $s$  (strange),  $c$  (charm),  $b$  (beauty or beauty) and  $t$  (top or truth). The SM furthermore differentiates between three fundamental forces (and its carriers): the electromagnetic ( $\gamma$  photon), weak ( $Z$ - or  $W$ -Boson) and strong ( $g$  gluon) interactions. The before mentioned Leptons solely interact through the electromagnetic and the weak force (also referred to as electroweak interaction), whereas the quarks additionally interact through the strong force.

The strong force is also referred to as Quantumchromodynamics (QCD). As the name suggests<sup>1</sup> the force is characterized by the color charge. Every quark has next to its type one of the three colors blue, red or green. The color force is mediated through eight gluons, which each being bi-colored<sup>2</sup>, interact with quarks and each other.

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<sup>1</sup>Chromo is the greek word for color.

<sup>2</sup>Each gluon carries two colors.

## Chapter 2

# Derivation of the used inverse covariance matrix from the Aleph data

While performing a **Generalized least squares** (GLS) we estimate our regression coefficients  $\hat{\beta}$  as follows:

$$\hat{\beta} = \underset{b}{\operatorname{argmin}} (\mathbf{y} - \mathbf{X}\mathbf{b})^T \mathbf{\Sigma}^{-1} (\mathbf{y} - \mathbf{X}\mathbf{b}), \quad (2.1)$$

with  $\mathbf{b}$  being an candidate estimate of  $\beta$ ,  $\mathbf{X}$  being the design matrix,  $\mathbf{y}$  being the response values and  $\mathbf{\Sigma}^{-1}$  being the **inverse covariance matrix**.

The Aleph data includes the **standard error** (SE), which are equal to the **standard deviation** as per definition. Furthermore Aleph provides the **correlation coefficients** of the errors. We will use these two quantities in combination with **Gaussian error propagation** to derive an approximation of the covariance matrix.

### 2.1 Propagation of experimental errors and correlation

Let  $\{f_k(x_1, x_2, \dots, x_n)\}$  be a set of  $m$  functions, which are linear combinations of  $n$  variables  $x_1, x_2, \dots, x_n$  with combination coefficients  $A_{k1}, A_{k2}, \dots, A_{kn}$ , where  $k \in \{1, 2, \dots, m\}$ . Let the covariance matrix of  $x_n$  be denoted by

$$\Sigma^x = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (2.2)$$

Then the covariance matrix of the functions  $\Sigma^f$  is given by

$$\Sigma_{ij}^f = \sum_k^n \sum_l^n A_{ik} \sum_{kl}^x A_{jl}, \quad \Sigma^f = A \Sigma^x A^T. \quad (2.3)$$

In our case we are dealing with non-linear functions, which we will linearized with the help of the **Taylor expansion**

$$f_k \approx f_k^0 + \sum_i^n \frac{\partial f_k}{\partial x_i} x_i, \quad f \approx f^0 + Jx. \quad (2.4)$$

Therefore, the propagation of error follows from the linear case, replacing the Jacobian matrix with the combination coefficients ( $J = A$ )