Chapter 1

Derivation of the used inverse covariance matrix from the Aleph data

While performing a **Generalized least squares** (GLS) we estimate our regression coefficients $\hat{\beta}$ as follows:

$$\hat{\beta} = \underset{b}{\operatorname{argmin}} (\mathbf{y} - \mathbf{X}\mathbf{b})^T \mathbf{\Omega}^{-1} (\mathbf{y} - \mathbf{X}\mathbf{b}), \tag{1.1}$$

with **b** being an candidate estimate of β, **X** being the design matrix, **y** being the response values and $\mathbf{\Omega}^{-1}$ being the **inverse covariance matrix**.

The Aleph data includes the **standard error** (SE), which are equal to the **standard deviation** as per definition. Furthermore Aleph provides the **correlation coefficients** of the errors. We will use these two quantities in combination with **Gaussian error propagation** to derive derive an approximation of the covariance matrix.

1.1 Propagation of experimental errors and correlation

Let $\{f_k(x_1, x_2, \cdots x_n)\}$ be a set of m functions, which a linear combinations of n variables $x_1, x_2, \cdots x_n$ with combination coefficients $A_{k1}, A_{k2}, \cdots A_{kn}$, where $k \in \{1, 2, \cdots, m\}$. Let the covariance matrix of x_n be denoted by

$$\Sigma^x = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{1.2}$$

Then the covariance matrix of the functions Σ^f is given by

$$\Sigma^f_{ij} = \sum_k^n \sum_l^n A_{ik} \sum_{kl}^x A_{jl}, \quad \Sigma^f = A\Sigma^x A^T. \tag{1.3}$$

In our case we are dealing with non-linear functions, which we will linearized with the help of the **Taylor expansion**

$$f_k \approx f_k^0 + \sum_i^n \frac{\partial f_k}{\partial x_i} x_i, \quad f \approx f^0 + Jx. \tag{1.4}$$

Therefore, the propagation of error follows from the linear case, replacing the Jacobian matrix with the combination coefficients ($J = A$)

Chapter 2

Coefficients

2.1 β function

There are several conventions for defining the β coefficients, depending on a minus sign and/or a factor of two (if one substitues $\mu \to \mu^2$) in the β-function **??**. We follow the convention from Pascual and Tarrach (except for the minus sign) and have taken the values from **??**

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f) \tag{2.1}$$

$$\beta_2 = \frac{1}{12}(17N_c^2 - 5N_cN_f - 3C_fN_f) \tag{2.2}$$

$$\beta_3 = \frac{1}{32}\left(\frac{2857}{54}N_c^3 - \frac{1415}{54}N_c^2N_f + \frac{79}{54}N_cN_f^2 - \frac{205}{18}N_cC_fN_f + \frac{11}{9}C_fN_f^2 + C_f^2N_f\right) \tag{2.3}$$

$$\beta_4 = \frac{140599}{2304} + \frac{445}{16}\zeta_3, \tag{2.4}$$

where we used $N_f = 6$ and $N_c = 3$ for β_4.

2.2 Adler function