Chapter 1

τ decays into hadrons

[Tsai1971]

$$R_\tau = 12\pi \int_0^{m_\tau} = \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)\left[\left(1 + 2\frac{s}{m_\tau^2}\right)\operatorname{Im}\Pi^{(T)}(s) + \operatorname{Im}\Pi^{(L)}\right] \tag{1.1}$$

Cauchy's Theorem

$$\int_{\mathcal{C}} f(z)\,dz = 0 \tag{1.2}$$

$$\begin{aligned}
\oint_{s=m_\tau} \Pi(s) &= \int_0^{m_\tau} \Pi(s+i\epsilon) + \int_{\mathcal{C}_2} \Pi(s)\,ds + \int_{m_\tau}^{0} \Pi(s-i\epsilon)\,ds + \int_{\mathcal{C}_4} \Pi(s)\,ds \\
&= \int_0^{m_\tau} \Pi(s+i\epsilon) - \Pi(s-i\epsilon)\,ds + \int_{\mathcal{C}_2} \Pi(s)\,ds + \int_{\mathcal{C}_4} \Pi(s)\,ds \\
&= \int_0^{m_\tau} \Pi(s+i\epsilon) - \overline{\Pi(s+i\epsilon)} + \int_{\mathcal{C}_2} \Pi(s)\,ds + \int_{\mathcal{C}_4} \Pi(s)\,ds \\
&\overset{\lim \epsilon\to 0}{=} 2i\int_0^{m_\tau} \operatorname{Im}\Pi(s)\,ds + \oint_{s=m_\tau} \Pi(s)\,ds
\end{aligned} \tag{1.3}$$

where $\Pi(z) = \overline{\Pi(\overline{z})}$, because $\Pi(s)$ is analytic and $\Pi(z) - \overline{\Pi(z)} = 2i\operatorname{Im}\Pi(z)$

$$\Pi(s-i) \tag{1.4}$$

$$\int_0^{m_\tau} \Pi(s)\,ds = \frac{i}{2}\oint_{s=m_\tau} \Pi(s)\,ds \tag{1.5}$$

$$R_\tau = 6\pi i \oint_{s=m_\tau} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)\left[\left(1 + 2\frac{s}{m_\tau^2}\right)\Pi^{(T)}(s) + \Pi^{(L)}\right] \tag{1.6}$$