Chapter 1

Introduction

In particle physics we are concerned about small objects and their interactions. Their dynamics are currently best described by the Standard Model (SM).

The SM contains two groups of fermionic, Spin $1/2$ particles. The former group, the Leptons consist of: the electron (e), the muon (μ), the tau (τ) and their corresponding neutrinos ν_e, ν_μ and ν_τ. The latter group, the Quarks contain: u, d (up and down, the so called light quarks), s (strange), c (charm), b (beauty or beauty) and t (top or truth). The SM furthermore differenciates between three fundamental forces (and its carriers): the electromagnetic (γ photon), weak (Z- or W-Boson) and strong (g gluon) interactions. The before mentioned Leptons solely interact through the electromagnetic and the weak force (also refered to as electroweak interaction), whereas the quarks additionally interact through the strong force.

The strong force is also refered to as Quantumchromodynamics (QCD). As the name suggest[1] the force is characterized by the color charge. Every quark has next to its type one of the three colors blue, red or green. The color force is mediated through eight gluons, which each being bi-colored[2], interact with quarks and each other. The strength of the strong force is given by the coupling constant α_s. The coupling constants are a function of energy E and $\alpha_s(E)$ increases with energy[3]. This is exclusive for QCD and leads to *asymptotic freedom* an *confinement*. The former phenomen describes the decreasing strong force between quarks and gluons, which become asymptotically free at large energies. The latter expresses the fact, that no isolated quark has been found until today. Quarks appear confined as *Hadrons*, the so called *Mesons*[4] and *Baryons*[5]. As we measure *Hadrons* in our experiments but calculate with quarks within our theoretical QCD model we have to assume *Quark-Hadron Duality*, which states that QCD is still valid for Hadrons for energies suffi-

[1] Chromo is the greek word for color.
[2] Each gluon carries a color and an anti-color.
[3] In contrast to the electromagnetic force, where $\alpha(E)$ decreases!
[4] Composite of a quark and an anti-quark.
[5] Composite of three quarks or three anti-quarks.

Flavour	Mass	comment
u	$2.2^{+0.5}_{-0.4}$ MeV	$\overline{\text{MS}}$
d	$4.7^{+0.5}_{-0.3}$ MeV	
s	95^{+9}_{-3} MeV	
c	$1.275^{+0.025}_{-0.035}$ GeV	
b	$4.18^{+0.04}_{-0.03}$ GeV	
t	173.0(40) GeV	

Table 1.1: List of Quarks and their masses[2].

cently heigh energies. There exist *Duality Violations* (DV), which will be investigated within this work.

1.1 τ-Decays

1.2 Quantumchromodynamics

QCD describes the strong interaction. Strong interactions occur between *quarks* and are transmitted through *gluons*. A list of quarks can be found in 1.2.

The QCD Lagrange density is similar to that of QED[1],

$$\mathcal{L}_{QCD}(x) = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu a}(x) + \sum_A \left[\frac{i}{2} \bar{q}^A(x) \gamma^\mu \overleftrightarrow{D}_\mu q^A(x) - m_A \bar{q}^A(x) a^A(x) \right]. \tag{1.1}$$

$$G^a_{\mu\nu}(x) \equiv \partial_\mu B^a_\nu(x) - \partial^a_\nu(x) + g f^{abc} B^b_\mu(x) B^c_\nu(x) \tag{1.2}$$

Chapter 2

Derivation of the used inverse covariance matrix from the Aleph data

While performing a **Generalized least squares** (GLS) we estimate our regression coefficients $\hat{\beta}$ as follows:

$$\hat{\beta} = \underset{b}{\operatorname{argmin}}(\mathbf{y} - \mathbf{X}\mathbf{b})^{T}\mathbf{\Omega}^{-1}(\mathbf{y} - \mathbf{X}\mathbf{b}), \tag{2.1}$$

with **b** being an candidate estimate of β, **X** being the design matrix, **y** being the response values and $\mathbf{\Omega}^{-1}$ being the **inverse covariance matrix**.

The Aleph data includes the **standard error** (SE), which are equal to the **standard deviation** as per definition. Furthermore Aleph provides the **correlation coefficients** of the errors. We will use these two quantities in combination with **Gaussian error propagation** to derive derive an approximation of the covariance matrix.

2.1 Propagation of experimental errors and correlation

Let $\{f_k(x_1, x_2, \cdots x_n)\}$ be a set of m functions, which a linear combinations of n variables $x_1, x_2, \cdots x_n$ with combination coefficients $A_{k1}, A_{k2}, \cdots A_{kn}$, where $k \in \{1, 2, \cdots, m\}$. Let the covariance matrix of x_n be denoted by

$$\Sigma^{x} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2.2}$$

Then the covariance matrix of the functions Σ^f is given by

$$\Sigma^f_{ij} = \sum_k^n \sum_l^n A_{ik} \sum_{kl}^x A_{jl}, \quad \Sigma^f = A\Sigma^x A^T. \tag{2.3}$$

In our case we are dealing with non-linear functions, which we will linearized with the help of the **Taylor expansion**

$$f_k \approx f_k^0 + \sum_i^n \frac{\partial f_k}{\partial x_i} x_i, \quad f \approx f^0 + Jx. \tag{2.4}$$

Therefore, the propagation of error follows from the linear case, replacing the Jacobian matrix with the combination coefficients ($J = A$)

Bibliography

[1] Matthias Jamin. *QCD and Renormalisation Group Methods*. Lecture presented at Herbstschule für Hochenergiephysik Maria Laach. Sept. 2006.

[2] M. Tanabashi et al. "Review of Particle Physics". In: *Phys. Rev.* D98.3 (2018), p. 030001. DOI: 10.1103/PhysRevD.98.030001.