Chapter 1

Introduction

In particle physics we are concerned about small objects and their interactions. The smallest of these objects are referred to as *elemental particles*. Their dynamics are governed by the laws of nature. These laws are organized through symmetries, which can be mathematically described through group theory. Since the 1970 the laws of nature are best described by the *Standard Model* (SM).

The SM contains two groups of fermionic, spin 1/2 particles. The former group, the leptons consist of: the electron (e), the muon (μ), the tau (τ) and their corresponding neutrinos ν_e, ν_μ and ν_τ. The latter group, the quarks contain: u, d (up and down, the so called light quarks), s (strange), c (charm), b (bottom or beauty) and t (top or truth). The SM furthermore differentiates between three fundamental forces (and its carriers): the electromagnetic (γ photon), weak (Z-or W-Boson) and strong (g gluon) interactions. The before mentioned Leptons solely interact through the electromagnetic and the weak force (also referred to as electroweak interaction), whereas the quarks additionally interact through the strong force. A short summary of the taxonomy of the SM can be seen in fig. 1.1

From a more mathematical point of view the SM is a gauge *quantum field theory* (QFT). QFT is the combination of *classical field theory*, *special relativity* and *quantum mechanics*. Its fundamental objects are ruled through its gauge group $SU(3) \times SU(2) \times U(1)$. Each of subgroups introduces a global and a local gauge symmetry. The global symmetry introduces the charges, which fields are carrying. The local symmetry introduce the gauge-fields, which are also referred to as force carriers.

$U(1)$ Is the *abelian* gauge group governs the representation of *quantum electrodynamics* (QED), which is commonly known as the electric force. Its global and local symmetry introduces the electric charge and the photon-field.

$SU(2)$ Is the *non-abelian* symmetry group responsible for the weak-interaction. It introduces the W^+, W^- and Z bosons and the weak charge. The gauge groups $U(1)$ and $SU(2)$ have been combined to the *electroweak interaction*.

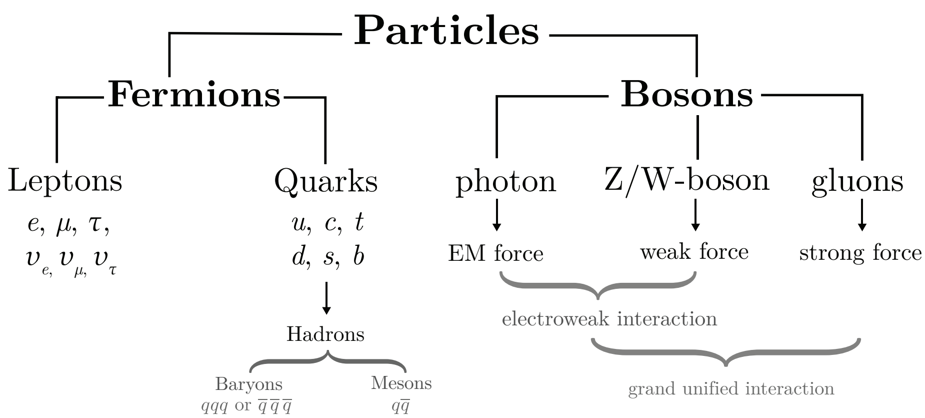

Figure 1.1: Taxonomy of the Standard Model.

***SU*(3)** The $SU(3)$-group is also *non-abelian* and governs the strong interactions, which are summarized in the theory of *quantum chromodynamics* (QCD). The group yields the three colour charges and the gluons. Due to the 8-dimensional adjoint represntation of $SU(3)$ there exist eight different gluons.

All of the particles and forces given in the SM are described by a Lagrangian containing 19 parameters. The parameters are represented by ten masses, four CKM-matrix parameters, the QCD-vacuum angle, the Higgs-vacuum expectation value and three gauge coupling constants. Every single parameter has to be fitted from experimental data. Highly accurate values with low errors are crucial for theoretical calculated predictions. One of the major error inputs of every theoretical output are uncertainties in these parameters. In this work we will focus on one of the parameters, namely the strong coupling α_s, which forms part of the theory of *quantum chromodynamics* (QCD).

As the name suggest[1] QCD is characterized by the color charge. Every quark has next to its type one of the three colors blue, red or green. The color force is mediated through eight gluons, which each being bi-colored[2], interact with quarks and each other. The strength of the strong force is given by the coupling constant α_s. The strong coupling depends on the renormalization scale μ, which is often chosen in a way that the coupling constant $\alpha_s(q)$ depends on the energy q^2. Thus the coupling varies with energy with an exceptional property: it increases for low energies[3]. This is exclusive for QCD and has two main

[1] Chromo is the Greek word for color.
[2] Each gluon carries a color and an anti-color.
[3] In contrast to the electromagnetic force, where $\alpha(q^2)$ decreases!

implications.

The first one states, that for low energies the coupling is too strong for isolate quarks to exist. Until now we have not been able to observe an isolated quark and all experiments can only measure quark compositions. These bound states are called *hadrons* and consist of two or three quarks [4], which are referred to as mesons[5] or baryons [6] respectively. This phenomenon, of quarks sticking together as hadrons is referred to as *confinement*. As the fundamental degrees of freedom of QCD are given by quarks and gluons, but the observed particles are hadrons we need to introduce the assumption of *quark-hadron duality* to match the theory to the experiment. This means that a physical quantity should be similarly describable in the hadronic picture or quark-gluon picture and that both descriptions are equivalent. As we will see in our work quark-hadron duality is violated for low energies. These so-called *duality violations* have an impact on our strong coupling determinations and can be dealt with either suppression or the inclusion of a model [**Pich2006**, **Cata2008**]. Throughout this work we will favor and argument for the former approach.

The second implication concerns *perturbation theory* (PT). The lower the energies we deal with, the higher the value of the strong coupling and the contributions of *non-perturbative* (NP) effects. Currently there are three solutions to deal with NP effects:

- ***Chiral Perturbation Theory*** (ChPT): Introduced by Weinberg [**Weinberg1978**] in the late seventies. ChPT is an effective field theory constructed with a Lagrangian symmetric under chiral transformation in the limit of massless quarks. It's limitations are based in the chiral symmetry, which is only a good approximation for the light quarks u, d and in some cases s.

- ***Lattice QCD*** (LQCD): Is the numerical approach to the strong force. Based on the Wilson Loops [**Wilson1974**] we treat QCD on a finite lattice instead of working with continuous fields. LQCD has already many applications but is limited due to its computational expensive calculations.

- ***QCD Sum Rules*** (QCDSR): Was also introduced in the late seventies by Shifman, Vainstein and Zakharov [**Shifman1978**, **Shifman1978a**]. It relates the observed hadronic picture to quark-gluon parameters through a dispersion relation and the use of the *Operator Product Expansion* (OPE), which treats NP effect through the definition of vacuum expectation values, the so-called *QCD condensates*. It is a precise method for extracting the strong coupling α_s at low energies, although limited to the unknown higher order contributions of the OPE.

In this work we focus on the determination of the strong coupling α_s within the framework of QCD Sum Rules for τ-decays which has been exploited in the beginning of the nineties by Braaten, Narison and Pich [**Braaten1991**]. Within

[4] There exist also so-called *Exotic hadrons*, which have more than three valence quarks.
[5] Composite of a quark and an anti-quark.
[6] Composite of three quarks or three anti-quarks.

this setup we can measure $\alpha_s(m_\tau^2)$ at the m_τ scale. As the strong coupling gets smaller at higher energies, so do the errors. Thus if we obtain the strong coupling at a low scale we will obtain high precision values at the scale of the Z-boson mass m_Z, which is the standard scale to compare α_s values.

The QCDSR for the determination of α_s, from low energies, contain three major issues.

1. There are two different approaches to treat perturbative and non-perturbative contributions. In particular, there is a significant difference between results obtained using fixed-order (FOPT) or contour improved perturbation theory (CIPT), such that analyses based on CIPT generally arrive at about 7% larger values of $\alpha_s(m_{\tau^2})$ than those based on FOPT [**PDG2018**]. There have been a variety of analyses on the topic been performed [**Pich2013**, **Caprini2009**, **Jamin2005**] and we will favor the FOPT approach, but generously list our results for the CIPT framework.

2. There are several prescriptions to deal with the NP-contributions of higher order OPE condensates. Typically terms of higher dimension have been neglected, even if they knowingly contribute. In this work we will include every necessary OPE term.

3. Finally there are known DV leading to an ongoing discussion of the importance of contributions from DV. Currently there are two main approaches: Either we neglect them, arguing that they are sufficiently suppressed due to *pinched weights* [**Pich2016**] or model DV with sinusoidal exponentially suppressed function [**Cata2008**, **Boito2011**, **Boito2014**] introducing extra fitting parameters. We will argue for the former method, implementing pinched weights that sufficiently suppress DV contributions such as having only a negligible effect on our analysis.

In the first chapter of this work we want to summarize the necessary theoretical background for working with the QCDSR. Starting with the basics of QCD we want to motivate the *renormalization group equation* (RGE), which is responsible for the running of the strong coupling. We then continue with the some aspects of the two-point function and its usage in the dispersion relation, which connects the hadronic picture with the quark-gluon picture. $\cdots$

The τ-particle is an elementary particle with negative electric charge and a spin of 1/2. Together with the lighter electron and muon it forms the *charged Leptons*[7]. Even though it is an elementary particle it decays via the *weak interaction* with a lifetime of $\tau_\tau = 2.9 \times 10^{-13}$ s and has a mass of 1776.86(12) MeV[**PDG2018**]. It is furthermore the only lepton massive enough to decay into Hadrons. The final states of a decay are limited by *conservation laws*. In case of a τ-decay they must conserve the electric charge (-1) and *invariant mass* of the system. Thus, as we can see from the corresponding Feynman diagram (see fig. 1.2)[8]

[7]Leptons do not interact via the strong force.

[8]The τ-particle can also decay into strange quarks or charm quarks, but these decays are rather uncommon due to the heavy masses of s and c.

Name	Symbol	Quark content	Rest mass (MeV)
Pion	π^-	$\bar{u}d$	139.570 61(24) MeV
Pion	π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	134.9770(5) MeV
Kaon	K^-	$\bar{u}s$	493.677(16) MeV
Kaon	K^0	$d\bar{s}$	497.611(13) MeV
Eta	η	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	547.862(17) MeV

Table 1.1: List of mesons produced by a τ-decay. Rare final states with branching Ratios smaller than 0.1 have been omitted. The list is taken from [**Davier2006**] with corresponding rest masses taken from [**PDG2018**].

the τ decays by the emission of a *W boson* and a tau-neutrino ν_τ into pairs of $(e^-, \bar{\nu}_e), (\mu^-, \bar{\nu}_\mu)$ or $(q, \bar{q})$.

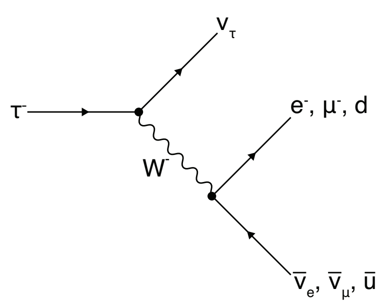

Figure 1.2: Feynman diagram of common decay of a τ-lepton into pairs of lepton-antineutrino or quark-antiquark by the emission of a *W boson*.

taken from tau decay adapt! We are foremost interested into the hadronic decay channels, meaning τ-decays that have quarks in their final states. Unfortunately the quarks have never been measured isolated, but appear always in combination of *mesons* and *baryons*. Due to its mass of $m_\tau \approx 1.8\,\text{GeV}$ the τ-particle decays into light mesons (pions-π, kaons-K, and eta-η, see table 1.1), which can be experimentally detected.

The hadronic $\tau - decay$ provides one of the most precise ways to determine the strong coupling [**Pich2016**] and can be calculated to high precision within the framework of QCD.