Chapter 1

Constants

In table 1.1 we collect all used constants that we have used in performing our fits.

Quantity	Value	Reference
V_{ud}	0.9742 ± 0.00021	**[PDG2018]**
S_{EW}	1.0198 ± 0.0006	**[Marciano1988]**
B_e	17.815 ± 0.023	?
m_τ	$1.776\,86(12000)$ MeV	**[PDG2018]**
$\langle aGG \rangle_I$	$0.012\ \mathrm{GeV}^2$	**[Shifman1978a]**
$\langle \overline{q}_{u/d} q_{u/d} \rangle (m_\tau)$	$-272(15)$ MeV	**[Jamin2002]**
$\bar{s}\, s / \langle \overline{q}\, q \rangle$	0.8 ±0.3	**[Jamin2002]**

Table 1.1: Numerical values of used constants in our fitting routine.

Chapter 2

Coefficients

2.1 β function

There are several conventions for defining the β coefficients, depending on a minus sign and/or a factor of two (if one substitues $\mu \to \mu^2$) in the β-function **??**. We follow the convention from Pascual and Tarrach (except for the minus sign) and have taken the values from [**Boito2011**]

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f), \tag{2.1.1}$$

$$\beta_2 = \frac{1}{12}(17N_c^2 - 5N_cN_f - 3C_fN_f), \tag{2.1.2}$$

$$\beta_3 = \frac{1}{32}\left(\frac{2857}{54}N_c^3 - \frac{1415}{54}N_c^2N_f + \frac{79}{54}N_cN_f^2 - \frac{205}{18}N_cC_fN_f + \frac{11}{9}C_fN_f^2 + C_f^2N_f\right), \tag{2.1.3}$$

$$\beta_4 = \frac{140599}{2304} + \frac{445}{16}\zeta_3, \tag{2.1.4}$$

where we used $N_f = 3$ and $N_c = 3$ for β_4.

2.2 Anomalous mass dimension

$$\gamma_1 = \frac{3}{2}C_f, \tag{2.2.1}$$

$$\gamma_2 = \frac{C_f}{48}(97N_c + 9C_f - 10N_f), \tag{2.2.2}$$

$$\gamma_3 = \frac{C_f}{32}\left[\frac{11413}{108}N_c^2 - \frac{129}{4}N_cC_f - \left(\frac{278}{27} + 24\zeta_3\right)N_cN_f + \frac{129}{2}C_f^2 - (23 - 24\zeta_3)C_fN_f - \frac{35}{27}N_f^2\right], \tag{2.2.3}$$

$$\gamma_4 = \frac{2977517}{20736} - \frac{9295}{216}\zeta_3 + \frac{135}{8}\zeta_4 - \frac{125}{6}\zeta_5, \tag{2.2.4}$$

where N_c is the number of colours, N_f the number of flavours and $C_f = (N_c^2 - 1)/2N_c$. We fixed furthermore fixed $N_f = 3$ and $N_c = 3$ for γ_4.

2.3 Adler function

The the derivative of the two-point function can be expressed as the Adler function, which can be written in terms of the Adler function coefficients

$$D_V^{(1+0)} = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{n,k} L^{k-1}. \tag{2.3.1}$$

The coefficients are partly dependent on each other via the RGE

$$-\mu\frac{d}{d\mu}D_V^{(1+0)} = \left(2\tfrac{\partial}{\partial L} + \beta\tfrac{\partial}{\partial a_s}\right)D_V^{(1+0)} = 0, \tag{2.3.2}$$

which implies, that for every order, there exists only one coefficient we have to know to describe the Adler function. For completeness we will mention the necessary coefficients up to order $n = 5$ here once again

$$\begin{aligned}
c_{1,1} &= 1\\
c_{2,1} &= \frac{365}{24} - 11\zeta_3 - \left(\frac{11}{12} - \frac{2}{3}\zeta_3\right)N_f\\
c_{3,1} &= \frac{87029}{288} - \frac{1103}{4}\zeta_3 + \frac{275}{6}\zeta_5\\
&\quad - \left(\frac{7847}{216} - \frac{262}{9}\zeta_3 + \frac{25}{9}\zeta_5\right)N_f + \left(\frac{151}{162} - \frac{19}{27}\zeta_3\right)N_f^2\\
c_{4,1} &= \frac{78631453}{20736} - \frac{1704247}{432}\zeta_3 + \frac{4185}{8}\zeta_3^2 + \frac{34165}{96}\zeta_5 - \frac{1995}{16}\zeta_7.
\end{aligned} \tag{2.3.3}$$

The rest of the coefficients are given by

$$
\begin{aligned}
& c_{2,2} = -\frac{1}{4}\beta_1 c_{1,1} \\
& c_{3,2} = (-\beta_2 c_{1,1} - 2\beta_1 c_{2,1}), \quad c_{3,3} = \frac{1}{12}\beta_1^2 c_{1,1} \\
& c_{4,2} = \frac{1}{4}(-\beta_3 c_{1,1} - 2\beta_2 c_{2,1} - 3\beta_1 c_{3,1}), \\
& c_{4,3} = \frac{1}{24}(6c_{2,1}\beta_1^2 + 5\beta_2\beta_1 c_{1,1}), c_{4,4} = -\frac{1}{32}\beta_1^3 c_{1,1} \\
& c_{5,2} = \frac{1}{4}(-\beta_4 c_{1,1} - 2\beta_3 c_{2,1} - 3\beta_2 c_{3,1} - 4\beta_1 c_{4,1}), \\
& c_{5,3} = \frac{1}{24}(12c_{3,1}\beta_1^2 + 6\beta_1\beta_3 c_{1,1} + 14\beta_2\beta_1 c_{2,1} + 3\beta_2^2 c_{1,1}), \\
& c_{5,4} = \frac{1}{96}(-12\beta_1^3 c_{2,1} - 13\beta_2\beta_1^2 c_{1,1}), \quad c_{5,5} = \frac{1}{80}\beta_1^4 c_{1,1}
\end{aligned}
\tag{2.3.4}
$$

and all related to the previous stated Adler function coefficients $c_{n,1}$.