

Chapter 1

Introduction

In particle physics we are concerned about small objects and their interactions. Their dynamics are currently best described by the Standard Model (SM).

The SM contains two groups of fermionic, Spin 1/2 particles. The former group, the Leptons consist of: the electron (e), the muon (μ), the tau (τ) and their corresponding neutrinos ν_e , ν_μ and ν_τ . The latter group, the Quarks contain: u , d (up and down, the so called light quarks), s (strange), c (charm), b (beauty or beauty) and t (top or truth). The SM furthermore differentiates between three fundamental forces (and its carriers): the electromagnetic (γ photon), weak (Z- or W-Boson) and strong (g gluon) interactions. The before mentioned Leptons solely interact through the electromagnetic and the weak force (also referred to as electroweak interaction), whereas the quarks additionally interact through the strong force.

The strong force is also referred to as Quantumchromodynamics (QCD). As the name suggests¹ the force is characterized by the color charge. Every quark has next to its type one of the three colors blue, red or green. The color force is mediated through eight gluons, which each being bi-colored², interact with quarks and each other. The strength of the strong force is given by the coupling constant α_s . The coupling constants are a function of energy E and $\alpha_s(E)$ increases with energy³. This is exclusive for QCD and leads to *asymptotic freedom* and *confinement*. The former phenomenon describes the decreasing strong force between quarks and gluons, which become asymptotically free at large energies. The latter expresses the fact, that no isolated quark has been found until today. Quarks appear confined as *Hadrons*, the so called *Mesons*⁴ and *Baryons*⁵. As we measure *Hadrons* in our experiments but calculate with quarks within our theoretical QCD model we have to assume *Quark-Hadron Duality*, which states that QCD is still valid for Hadrons for energies suffi-

¹Chromo is the greek word for color.

²Each gluon carries a color and an anti-color.

³In contrast to the electromagnetic force, where $\alpha(E)$ decreases!

⁴Composite of a quark and an anti-quark.

⁵Composite of three quarks or three anti-quarks.

Flavour	Mass	comment
u	$2.2^{+0.5}_{-0.4}$ MeV	$\overline{\text{MS}}$
d	$4.7^{+0.5}_{-0.3}$ MeV	
s	95^{+9}_{-3} MeV	
c	$1.275^{+0.025}_{-0.035}$ GeV	
b	$4.18^{+0.04}_{-0.03}$ GeV	
t	173.0(40) GeV	

Table 1.1: List of Quarks and their masses[PDG2018].

cently heigh energies. There exist *Duality Violations* (DV), which will be investigated within this work.

1.1 τ -Decays

1.2 Quantumchromodynamics

QCD describes the strong interaction, which occur between *quarks* and are transmitted through *gluons*. A list of quarks can be found in 1.2.

The QCD Lagrange density is similar to that of QED[Jamin2006],

$$\mathcal{L}_{\text{QCD}}(x) = -\frac{1}{4}G_{\mu\nu}^a(x)G^{\mu\nu a}(x) + \sum_A \left[\frac{i}{2}\bar{q}^A(x)\gamma^\mu \overleftrightarrow{D}_\mu q^A(x) - m_A \bar{q}^A(x)q^A(x) \right], \quad (1.1)$$

where $q^A(x)$ represents the quark fields and $G_{\mu\nu}^a$ being the *gluon field strength tensor* given by:

$$G_{\mu\nu}^a(x) \equiv \partial_\mu B_\nu^a(x) - \partial_\nu B_\mu^a(x) + gf^{abc}B_\mu^b(x)B_\nu^c(x), \quad (1.2)$$

where B_μ^a are the *gluon fields*, given in the *adjoint representation* of the $\text{SU}(3)$ gauge group with f^{abc} as *structure constants*. Furthermore we have used A, B, \dots as flavour indeces, a, b, \dots as color indeces and μ, ν, \dots as lorentz indeces.

1.2.1 Renormalisation Group

The perurbations of the QCD Lagrangian 1.1 lead to divergencies, which have to be *renormalized*. There are different approaches to 'make' these divergencies finite. The most popular one is *dimensional regularisation*. In *Dimensional regularisation* we expand the four space-time dimensions to arbitrary dimensions. Consequently the in QCD calculations appearing *Feynman integrals* have to be continued to D-dimensions like

$$\mu^{2\epsilon} \int \frac{d^D p}{(2\pi)^D} \frac{1}{[p^2 - m^2 + i0][(q-p)^2 - m^2 + i0]}, \quad (1.3)$$

where we introduced the scale parameter μ to account for the extra dimensions and conserve the mass dimension of the non continued integral.

In addition *physical quantities*⁶ cannot depend on the renormalisation scale μ . Thus examining a *physical quantity* $R(q, a_s, m)$ that depends on the external momentum q , the renormalised coupling $a_s = \alpha_s/\pi$ and the renormalized quark mass m

$$\mu \frac{d}{d\mu} R(q, a_s, m) = \left[\mu \frac{\partial}{\partial \mu} + \mu \frac{dm}{d\mu} \frac{\partial}{\partial m} \right] R(q, a_s, m) = 0 \quad (1.4)$$

we can define the *renormalisation group functions*:

$$\beta(a_s) \equiv -\mu \frac{da_s}{d\mu} = \beta_1 a_s^2 + \beta_2 a_s^3 + \dots \quad \beta - \text{function} \quad (1.5)$$

$$\gamma(a_s) \equiv -\frac{\mu}{m} \frac{dm}{d\mu} = \gamma_1 a_s + \gamma_2 a_s^2 + \dots \quad \text{anomalous mass dimension.} \quad (1.6)$$

Running gauge coupling

Regarding the β -function we notice, that $a_s(\mu)$ is not a constant, but *runs* by varying the scale μ . Integrating the β -function yields

$$\int_{a_s(\mu_1)}^{a_s(\mu_2)} \frac{da_s}{\beta(a_s)} = - \int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu} = \log \frac{\mu_1}{\mu_2}. \quad (1.7)$$

To analytically evaluate the above integral we can approximate the β -function to first order, with the known coefficient

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f), \quad (1.8)$$

yielding

$$a_s(\mu_2) = \frac{a_s(\mu_1)}{\left(1 - a_s(\mu_1)\beta_1 \log \frac{\mu_1}{\mu_2}\right)}. \quad (1.9)$$

As we have three colours $N_c = 3$ and six flavours $N_f = 6$ the first β -function 1.5 is positive. Thus for $\mu_2 > \mu_1$ $a_s(\mu_2)$ decreases logarithmically and vanishes for $\mu_2 \rightarrow \infty$. This behaviour is known as *asymptotic freedom*. The coefficients of the β -function are currently known up to the 5th order, which are displayed in the appendix ??.

Running quark mass

The properties of the running quark mass can be derived similar to the gauge coupling. Starting from integrating the *anomalous mass dimension* 1.6

$$\log \frac{m(\mu_2)}{m(\mu_1)} = \int_{a_s(\mu_1)}^{a_s(\mu_2)} da_s \frac{\gamma(a_s)}{\beta(a_s)} \quad (1.10)$$

⁶Observables that can be measured.

we can approximate the *anomalous mass dimension* to first order and solve the integral analytically [**Schwab2002**]

$$m(\mu_2) = m(\mu_1) \left(\frac{a(\mu_2)}{a(\mu_1)} \right)^{\frac{\gamma_1}{\beta_1}} (1 + \mathcal{O}(\beta_2, \gamma_2)). \quad (1.11)$$

As β_1 and γ_1 (see ??) are positive the quark mass decreases with increasing μ . The general relation between different scales is given by

$$m(\mu_2) = m(\mu_1) \exp \left(\int_{a_s(\mu_1)}^{a_s(\mu_2)} da_s \frac{\gamma(a_s)}{\beta(a_s)} \right) \quad (1.12)$$

and can be solved numerically to run the quark mass to the needed scale μ_2 .