Chapter 1

Constants

In table 1.1 we collect all used constants that we have used in performing our fits.

Quantity	Value	Reference
V_{ud}	0.9742 ± 0.00021	**[PDG2018]**
S_{EW}	1.0198 ± 0.0006	**[Marciano1988]**
B_e	17.815 ± 0.023	?
m_τ	$1.776\,86(12000)$ MeV	**[PDG2018]**
$\langle aGG \rangle_I$	$0.012\,\mathrm{GeV}^2$	**[Shifman1978a]**
$\langle \overline{q}_{u/d} q_{u/d} \rangle (m_\tau)$	$-272(15)$ MeV	**[Jamin2002]**
$\overline{s}\, s / \langle \overline{q}\, q \rangle$	0.8 ±0.3	**[Jamin2002]**

Table 1.1: Numerical values of used constants in our fitting routine.

Chapter 2

Coefficients

2.1 β function

There are several conventions for defining the β coefficients, depending on a minus sign and/or a factor of two (if one substitues $\mu \to \mu^2$) in the β-function **??**. We follow the convention from Pascual and Tarrach (except for the minus sign) and have taken the values from [**Boito2011**]

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f), \tag{2.1.1}$$

$$\beta_2 = \frac{1}{12}(17N_c^2 - 5N_cN_f - 3C_fN_f), \tag{2.1.2}$$

$$\beta_3 = \frac{1}{32}\left(\frac{2857}{54}N_c^3 - \frac{1415}{54}N_c^2N_f + \frac{79}{54}N_cN_f^2 - \frac{205}{18}N_cC_fN_f + \frac{11}{9}C_fN_f^2 + C_f^2N_f\right), \tag{2.1.3}$$

$$\beta_4 = \frac{140599}{2304} + \frac{445}{16}\zeta_3, \tag{2.1.4}$$

where we used $N_f = 3$ and $N_c = 3$ for β_4.

2.2 Anomalous mass dimension

$$\gamma_1 = \frac{3}{2}C_f, \tag{2.2.1}$$

$$\gamma_2 = \frac{C_f}{48}(97N_c + 9C_f - 10N_f), \tag{2.2.2}$$

$$\gamma_3 = \frac{C_f}{32}\left[\frac{11413}{108}N_c^2 - \frac{129}{4}N_cC_f - \left(\frac{278}{27} + 24\zeta_3\right)N_cN_f + \frac{129}{2}C_f^2 - (23 - 24\zeta_3)C_fN_f - \frac{35}{27}N_f^2\right], \tag{2.2.3}$$

$$\gamma_4 = \frac{2977517}{20736} - \frac{9295}{216}\zeta_3 + \frac{135}{8}\zeta_4 - \frac{125}{6}\zeta_5, \tag{2.2.4}$$

where N_c is the number of colours, N_f the number of flavours and $C_f = (N_c^2 - 1)/2N_c$. We fixed furthermore fixed $N_f = 3$ and $N_c = 3$ for γ_4.

2.3 Adler function

The the derivative of the two-point function can be expressed as the Adler function, which can be written in terms of the Adler function coefficients

$$D_V^{(1+0)} = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{n,k} L^{k-1}. \tag{2.3.1}$$

The coefficients are partly dependent on each other via the RGE

$$-\mu\frac{d}{d\mu}D_V^{(1+0)} = \left(2\tfrac{\partial}{\partial L} + \beta\tfrac{\partial}{\partial a_s}\right)D_V^{(1+0)} = 0, \tag{2.3.2}$$

which implies that per order there exists only one coefficient we have to know to describe the Adler function. For completeness we will mention the necessary coefficients up to order $n = 5$ here once again

$$\begin{aligned}
c_{1,1} &= 1 \\
c_{2,1} &= \frac{365}{24} - 11\zeta_3 - \left(\frac{11}{12} - \frac{2}{3}\zeta_3\right)N_f \\
c_{3,1} &= \frac{87029}{288} - \frac{1103}{4}\zeta_3 + \frac{275}{6}\zeta_5 \\
&\quad - \left(\frac{7847}{216} - \frac{262}{9}\zeta_3 + \frac{25}{9}\zeta_5\right)N_f + \left(\frac{151}{162} - \frac{19}{27}\zeta_3\right)N_f^2 \\
c_{4,1} &= \frac{78631453}{20736} - \frac{1704247}{432}\zeta_3 + \frac{4185}{8}\zeta_3^2 + \frac{34165}{96}\zeta_5 - \frac{1995}{16}\zeta_7.
\end{aligned} \tag{2.3.3}$$

The rest of the coefficients are given by

$$\begin{aligned}
c_{2,2} &= -\frac{1}{4}\beta_1 c_{1,1} \\
c_{3,2} &= (-\beta_2 c_{1,1} - 2\beta_1 c_{2,1}), \quad c_{3,3} = \frac{1}{12}\beta_1^2 c_{1,1} \\
c_{4,2} &= \frac{1}{4}(-\beta_3 c_{1,1} - 2\beta_2 c_{2,1} - 3\beta_1 c_{3,1}), \\
c_{4,3} &= \frac{1}{24}(6c_{2,1}\beta_1^2 + 5\beta_2\beta_1 c_{1,1}), c_{4,4} = -\frac{1}{32}\beta_1^3 c_{1,1} \\
c_{5,2} &= \frac{1}{4}(-\beta_4 c_{1,1} - 2\beta_3 c_{2,1} - 3\beta_2 c_{3,1} - 4\beta_1 c_{4,1}), \\
c_{5,3} &= \frac{1}{24}(12c_{3,1}\beta_1^2 + 6\beta_1\beta_3 c_{1,1} + 14\beta_2\beta_1 c_{2,1} + 3\beta_2^2 c_{1,1}), \\
c_{5,4} &= \frac{1}{96}(-12\beta_1^3 c_{2,1} - 13\beta_2\beta_1^2 c_{1,1}), \quad c_{5,5} = \frac{1}{80}\beta_1^4 c_{1,1}
\end{aligned} \tag{2.3.4}$$

and all related to the previous stated Adler function coefficients $c_{n,1}$.