Chapter 1

Introduction

In particle physics we are concerned about small objects and their interactions. The smallest of these objects are referred to as *elemental particles*. Their dynamics are governed by the laws of nature. These laws are organised through symmetries, which are currently best described by the *Standard Model* (SM).

The SM classifies all known elementary particles and describes three of the four fundamental forces: the electromagnetic, the weak and the strong force. The particles representing matter are contained in two groups of fermionic, spin-1/2 particles. The former group, the leptons consist of: the electron (e), the muon (μ), the tau (τ) and their corresponding neutrinos ν_e, ν_μ and ν_τ. The latter group, the quarks contain: u, d (up and down, the so-called light quarks), s (strange), c (charm), b (bottom or beauty) and t (top or truth). The three fundamental forces, the SM differentiates, are described through their carrier particles, the so-called bosons: the photon (γ) for the electromagnetic, the Z or W boson for the weak and the gluon (g) for the strong interaction. The Leptons solely interact through the electromagnetic and the weak force (also referred to as electroweak interaction), whereas the quarks additionally interact through the strong force. A short summary of the taxonomy of the SM can be seen in fig. 1.1

From a more mathematical point of view the SM is a gauge *quantum field theory* (QFT), which is a combination of *classical field theory*, *special relativity* and *quantum mechanics*. Its fundamental objects are ruled through its gauge group

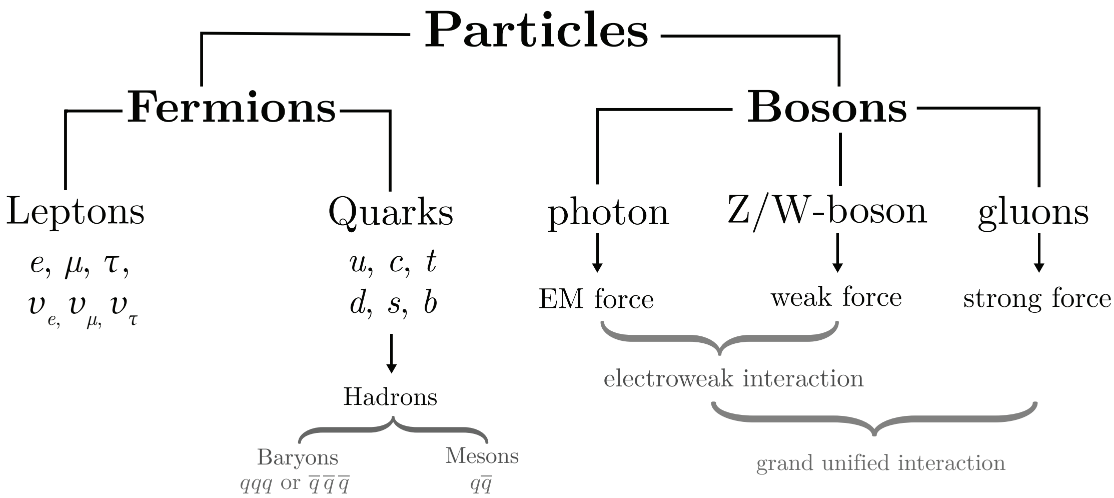

Figure 1.1: Taxonomy of the Standard Model.

$SU(3) \times SU(2) \times U(1)$. Each of its subgroups introduces a global and a local gauge symmetry. The global symmetry introduces the charges, which the fields are carrying. The local symmetry introduce the gauge fields, which represent the previously mentioned force carriers. Naively every subgroup[1] of the gauge group of the standard model is responsible for one of the three forces:

$U(1)$ the *abelian* gauge group governs the representation of *quantum electrodynamics* (QED), which is commonly known as the electric force. Its global and local symmetry introduces the electric charge and the photon field.

$SU(2)$ Is the *non-abelian* symmetry group responsible for the weak interaction. It introduces the W^+, W^- and Z bosons and the weak charge. The gauge groups $U(1)$ and $SU(2)$ have been combined to the *electroweak interaction*.

$SU(3)$ The $SU(3)$ group is also non-abelian and governs the strong interactions, which are summarised in the theory of QCD. The group yields the three colour charges and due to its eight dimensional adjoint representation, eight different gluons.

Unfortunately we are still not able to include gravity, the last of the four forces,

[1] Actually $U(1)$ and $SU(2)$ have to be regarded as combined group to be mapped to the electromagnetic-and weak-force in form of the electroweak interaction.

into the SM. There have been attempts to describe gravity through QFT with the graviton, a spin-2 boson mediator, but there are unsolved problems with the renormalisation of general relativity (GR). Until now GR and quantum mechanics (QM) remain incompatible.

Apart from gravity not being included, the SM has a variety of flaws. One of them is being dependent on many parameters, which have to be measured accurately to perform high-precision physics. In total the Lagrangian of the SM contains 19 parameters. These parameters are represented by ten masses, four CKM-matrix parameters, the QCD-vacuum angle, the Higgs-vacuum expectation value and three gauge coupling constants. Highly accurate values with low errors are crucial for theoretical calculated predictions. One of the major error inputs of every theoretical output are uncertainties in these parameters. In this work we will focus on one of the parameters, namely the strong coupling α_s.

The strong coupling is currently measured in six different ways: through τ decays, QCD lattice computations, deep inelastic collider results and electroweak precision fits **[PDG2018]**. We have plotted the values of each of the methods in fig. 1.2. During this work we will focus on the subfield of τ decays to measure the value of the strong coupling $\alpha_s(m_\tau^2)$ at the m_τ^2 scale. We will see that in QCD the value of the coupling "constant" depends upon the scale. The τ is an elementary particle with negative electric charge and a spin of $1/2$. Together with the lighter electron and muon it forms the group of charged leptons[2]. Even though it is an elementary particle it decays via the weak interaction with a lifetime of $\tau_\tau = 2.9 \times 10^{-13}\,\mathrm{s}$ and has a mass of $1776.86(12)\,\mathrm{MeV}$**[PDG2018]**. It is furthermore the only lepton massive enough to decay into hadrons, thus of interest for our QCD anal-

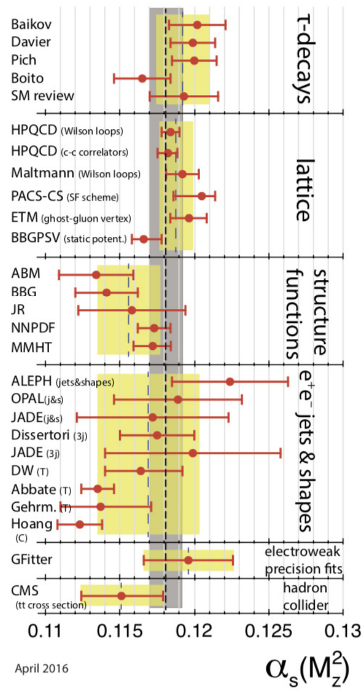

Figure 1.2: The six different subfields and their results for measuring the strong coupling α_s **[PDG2018]**.

[2]Leptons do not interact via the strong force.

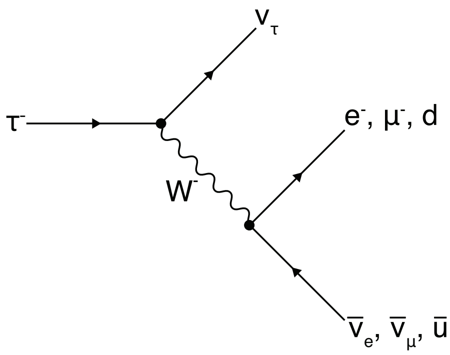

Figure 1.3: Feynman diagram of common decay of a τ-lepton into pairs of lepton-antineutrino or quark-antiquark by the emission of a *W boson*.

ysis. The final states of a decay are limited by conservation laws. In case of the τ decay they must conserve the electric charge ($q_e = -1$) and invariant mass of the system. Thus, we can see from the corresponding Feynman diagram fig. 1.3[3], that the τ decays by the emission of a W boson and a tau-neutrino ν_τ into pairs of $(e^-, \bar{\nu}_e), (\mu^-, \bar{\nu}_\mu)$ or $(q, \bar{q})$. We are foremost interested into the hadronic decay channels, meaning τ decays that have quarks in their final states. Quarks have never been measured isolated. Due to its mass of $m_\tau \approx 1.8\,\text{GeV}$ the τ particle decays into light mesons (pions (π), kaons (K), and eta (η), see table 1.1), which can be experimentally detected. The hadronic τ decay provides one of the most precise ways to determine the strong coupling [**Pich2016**] and is theoretically accessible to high precision within the framework of QCD.

The theory describing strong interactions is QCD. As the name suggest[4] QCD is characterised by the colour charge and is a non-abelian gauge theory with symmetry group $SU(3)$. Consequently every quark has next to its type one of the three colours blue, red or green. The colour force is mediated through eight gluons, which each being bi-coloured[5], interact with quarks and each other. The strength of the strong force is given by the coupling constant α_s,

[3] The τ particle can also decay into strange quarks or charm quarks, but these decays are rather uncommon due to the heavy masses of s and c.
[4] Chromo is the Greek word for colour.
[5] Each gluon carries a colour and an anti-colour.

Name	Symbol	Quark content	Rest mass (MeV)
Pion	π^-	$\bar{u}d$	$139.570\,61(24)$ MeV
Pion	π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	$134.9770(5)$ MeV
Kaon	K^-	$\bar{u}s$	$493.677(16)$ MeV
Kaon	K^0	$d\bar{s}$	$497.611(13)$ MeV
Eta	η	$(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$	$547.862(17)$ MeV

Table 1.1: List of mesons produced by a τ decay. Rare final states with branching ratios smaller than 0.1 have been omitted. The list is taken from [**Davier2006**] with corresponding rest masses taken from [**PDG2018**].

which depends on the renormalisation scale μ. We often choose the renormalisation scale in a way that the coupling constant $\alpha_s(q^2)$ depends on the energy q^2. Thus the coupling varies with energy. It increases for low and decreases for high energies[6]. This behaviour has two main implications. The first one states, that for low energies the coupling is too strong for isolated quarks to exist. Until now we have not been able to observe an isolated quark and all experiments can only measure quark compositions. These bound states are called *hadrons* and consist of two or three quarks [7], which are referred to as mesons[8] or baryons[9] respectively. This phenomenon, of quarks sticking together as hadrons is referred to as *confinement*. As the fundamental degrees of freedom of QCD are given by quarks and gluons, but the observed particles are hadrons we need to introduce the assumption of *quark-hadron duality* to match the theory to the experiment. This means that a physical quantity should be similarly describable in the hadronic or quark-gluon picture and that both descriptions are equivalent. Quark-hadron duality is in general violated. These so-called DV have an impact on our strong coupling determinations and can be dealt with either suppression or the inclusion of a model [**Cata2008**]. Throughout this work we will favour and argument for the former approach. The second

[6]In contrast to the electromagnetic force, where $\alpha(q^2)$ decreases!
[7]There exist also so-called *exotic hadrons*, which have more than three valence quarks.
[8]Composite of a quark and an anti-quark.
[9]Composite of three quarks or three anti-quarks.

implication, of the running of α_s, concerns . (PT) The lower the energies we deal with, the higher the value of the strong coupling and the contributions of *non-perturbative* (NP) effects. Currently there are three solutions to deal with e (NP)ffects:

- ***Chiral Perturbation Theory*** (CHPT): Introduced by Weinberg [**Weinberg1978**] in the late seventies. CHPT is an effective field theory constructed with a Lagrangian symmetric under a chiral transformation in the limit of massless quarks. It's limitations are based in the chiral symmetry, which is only a good approximation for the light quarks u, d and in some cases s.
- ***Lattice QCD*** (LQCD): Is the numerical approach to the strong force. Based on the Wilson Loops [**Wilson1974**] we treat QCD on a finite lattice instead of working with continuous fields. LQCD has already many applications but is limited due to its computational expensive calculations.
- ***QCD Sum Rules*** (QCDSR): Was also introduced in the late seventies by Shifman, Vainstein and Zakharov [**Shifman1978**, **Shifman1978a**]. It relates the observed hadronic picture to quark-gluon parameters through a dispersion relation and the use of the OPE, which treats NP effect through the definition of vacuum expectation values, the so-called QCD *condensates*. It is a precise method for extracting the strong coupling α_s at low energies, although limited to the unknown higher order contributions of the OPE.

In this work we focus on the determination of the strong coupling α_s within the framework of QCDSR for τ decays which has been exploited in the beginning of the nineties by Braaten, Narison and Pich [**Braaten1991**]. Within this setup we can measure $\alpha_s(m_\tau^2)$ at the m_τ^2 scale. As the strong coupling gets smaller at higher energies, so do the errors. Thus if we obtain the strong coupling at a low scale we will obtain high precision values at the scale of the Z boson mass m_Z, which is the standard scale to compare α_s values.

The QCDSR for the determination of α_s, from low energies, contain three major issues.

1. There are two different approaches to treat perturbative and non-perturbative contributions. In particular, there is a significant difference

between results obtained using FOPT or CIPT, such that analyses based on CIPT generally arrive at about 7% larger values of $\alpha_s(m_{\tau^2})$ than those based on FOPT [**PDG2018**]. There have been a variety of analyses on the topic been performed [**Pich2013**, **Caprini2009**, **Jamin2005**] and we will favour the FOPT approach.

2. There are several prescriptions to deal with the NP contributions of higher order OPE condensates. Typically terms of higher dimension have been neglected, even if they knowingly contribute. In this work we will include every necessary OPE term.

3. Finally there are known DV leading to an ongoing discussion of the importance of contributions from DV. Currently there are two main approaches: Either we neglect DV, arguing that they are sufficiently suppressed due to *pinched weights* [**Pich2016**] or model DV with a sinusoidal exponentially suppressed function [**Cata2008**, **Boito2011a**, **Boito2014**] introducing extra fitting parameters. We will argue for the former method, implementing pinched weights that sufficiently suppress DV contributions such that DV have only a negligible effect on our analysis.

In the following chapter of this work we want to summarise the necessary theoretical background for working with the QCDSR. Starting with the basics of QCD we want to motivate the *renormalisation group equation* (RGE), which is responsible for the running of the strong coupling. We then continue with the two-point function and its usage in the dispersion relation, which connects the hadronic picture with the quark-gluon picture. Then we introduce the OPE to treat the NP part of QCD, before we combine everything to formulate the QCDSR. In the third chapter we will apply the theory gathered in the second chapter to τ decays. In the fourth chapter we will state and interpret our fitting results before concluding in the last chapter.