Chapter 1

Derivation of the used inverse covariance matrix from the Aleph data

While performing a **Generalized least squares** (GLS) we estimate our regression coefficients $\hat{\beta}$ as follows:

$$\hat{\beta} = \underset{b}{\operatorname{argmin}} (\mathbf{y} - \mathbf{X}\mathbf{b})^{\mathsf{T}} \mathbf{\Omega}^{-1} (\mathbf{y} - \mathbf{X}\mathbf{b}), \tag{1.1}$$

with $\mathbf{b}$ being an candidate estimate of β, $\mathbf{X}$ being the design matrix, $\mathbf{y}$ being the response values and $\mathbf{\Omega}^{-1}$ being the **inverse covariance matrix**.

The Aleph data includes the **standard error** (SE), which are equal to the **standard deviation** as per definition. Furthermore Aleph provides the **correlation coefficients** of the errors. We will use these two quantities in combination with **Gaussian error propagation** to derive derive an approximation of the covariance matrix.

1.1 Propagation of experimental errors and correlation

Let $\{f_k(x_1, x_2, \cdots x_n)\}$ be a set of m functions, which a linear combinations of n variables $x_1, x_2, \cdots x_n$ with combination coefficients $A_{k1}, A_{k2}, \cdots A_{kn}$, where $k \in \{1, 2, \cdots, m\}$. Let the covariance matrix of x_n be denoted by

$$\Sigma^x = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{1.2}$$

$\omega_{k,l}$	Polinomial	Contributions
$\omega_{0,0}$	$1-3x^2+2x^3$	D2, D6, D8
$\omega_{0,1}$	$x-3x^3+2x^4$	D4, D8, D10
$\omega_{0,2}$	$x^2-3x^4+2x^5$	D6, D10, D12
$\omega_{1,0}$	$1-x-3x^2+5x^3-2x^4$	D2, D4, D6, D8, D10
$\omega_{1,1}$	$x-x^2-3x^3+5x^4-2x^5$	D4, D6, D8, D10, D12
$\omega_{1,2}$	$x^2-x^3-3x^4+5x^5-2x^6$	D6, D8, D10, D12, D14
$\omega_{1,3}$	$x^3-x^4-3x^5+5x^6-2x^7$	D8, D10, D12, D14, D16
$\omega_{2,0}$	$1-2x-2x^2+8x^3+2x^5$	D2, D4, D6, D8, D12
$\omega_{2,1}$	$x-2x^2-2x^3+8x^4+2x^6$	D4, D6, D8, D10, D14
$\omega_{2,2}$	$x^2-2x^3-2x^4+8x^5+2x^7$	D6, D8, D10, D12, D16
$\omega_{2,3}$	$x^3-2x^4-2x^5+8x^6+2x^8$	D8, D10, D12, D14, D18
$\omega_{3,0}$	$1-3x+10x^3+15x^4+9x^5-2x^6$	D2, D4, D8, D10, D12, D14

ω	Factored	Contributions
$\omega_{kinematic}$	$(1-x)^2(1+2x)$	D2, D6, D8
ω_{cube}	$(1-x)^3(1+3x)$	D2, D6, D8, D10
$\omega_{quartic}$	$(1-x)^4(1+4x)$	D2, D6, D8, D10, D12

Then the covariance matrix of the functions Σ^f is given by

$$\Sigma^f_{ij} = \sum_k^n \sum_l^n A_{ik} \sum_{kl}^x A_{jl}, \quad \Sigma^f = A\Sigma^x A^\intercal. \tag{1.3}$$

In our case we are dealing with non-linear functions, which we will linearized with the help of the **Taylor expansion**

$$f_k \approx f_k^0 + \sum_i^n \frac{\partial f_k}{\partial x_i} x_i, \quad f \approx f^0 + Jx. \tag{1.4}$$

Therefore, the propagation of error follows from the linear case, replacing the Jacobian matrix with the combination coefficients ($J = A$)

c6	c8	χ/dof

Table 1.1: Fits to the kinematic weight $w_\tau = (1-x)^2(1+2x)$. $\alpha_s(m_\tau) = 0.3179$ fixed to PDG(2016) value. $\langle aGG\rangle_{Inv} = 0.021$ also fixed. D6 and D8 free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

α	c6	c8	χ/dof

Table 1.2: Fits to the kinematic weight $w_\tau = (1-x)^2(1+2x)$. $\langle aGG\rangle_{Inv} = 0.021$ also fixed. D6, D8 and α free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

c6	c8

Table 1.3: Fits to the cubic weight $w_{Cubic} = (1-x)^3(1+3x)$. $\alpha_s(m_\tau) = 0.3179$ fixed to PDG(2016) value. $\langle aGG\rangle_{Inv} = 0.021$ also fixed. D6 and D8 free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

α	c6	c8

Table 1.4: Fits to the cubic weight $w_{Cubic} = (1-x)^3(1+3x)$. $\langle aGG\rangle_{Inv} = 0.021$ fixed. α, D6 and D8 free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

c6	c8	c10

Table 1.5: Fits to the cubic weight $w_{Cubic} = (1-x)^3(1+3x)$. $\alpha_s(m_\tau) = 0.3179$ fixed to PDG(2016) value. $\langle aGG\rangle_{Inv} = 0.021$ also fixed. D6, D8 and D10 free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

c6	c8	c10	c12

Table 1.6: Fits to the cubic weight $w_{Cubic} = (1-x)^3(1+3x)$. $\alpha_s(m_\tau) = 0.3179$ fixed to PDG(2016) value. $\langle aGG\rangle_{Inv} = 0.021$ also fixed. D6, D8 and D10 free. Resummation scheme: FOPT. No DV's included. $R_{\tau,V+A}$ uncorrelated rescaled to 3.4718.

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