Chapter 1

Measuring the strong coupling

The strong coupling has been measured since many years from hadronic τ decays. An overview of the recently, but different, α_s values can be seen in fig. 1.1. Until today most of the applied QCDSR to τ decays are based on the methodology developed in the early nineties by Braaten, Pich and Narison [**Braaten1991**]. They gathered the at this time available perturbative and NP contributions to extract the strong coupling from comparing their theoretical results to the known inclusive hadronic τ decay ratio R_τ. Pich together with Le Diberger then formulated the fitting strategy of fitting multiple moments of different weights to extract α_s parallel to Wilson coefficients of the OPE [**LeDiberger1992**], which later has been applied as standard in the ALEPH [**Aleph1993**] as well as the OPAL [**Opal1998**] detectors. For the next ten year years the methodology of extracting the strong coupling did not expe-

Table 1.1: Timeline

1991	[**Braaten1991**]: Systematic description, including NP corrections to extract α_s from R_τ.
1992	[**LeDiberder1992**]: Introducing weights and fit methodology
1993	[**Aleph1993**] ALEPH measures the strong coupling constant α_s
1998	[**Opal1998**] OPAL measures the strong coupling constant α_s
2005	[**Aleph2005**] ALEPH improves their data
2011	[**Boito2011a**, **Boito2010**]: Include DV. Discover inconsistencies in ALEPH data.
2014	[**Aleph2014**] ALEPH updates their data.

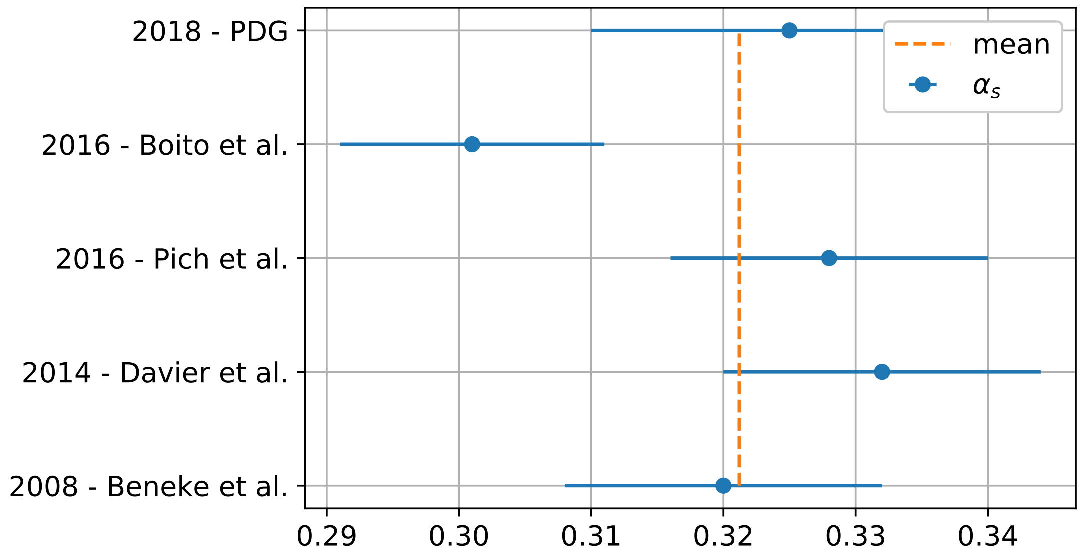

Figure 1.1

rience any major changes until in the year 2011 when Boito, Cata, Golterman, Jamin, Osborn and Peris [**Boito2011a**] applied a duality model to include known DV effects to the QCD analysis of τ decays. The group around Boito and Pich have different opinions on the importance of the newly introduced duality model [**Pich2016**, **Boito2016**] and consequently we want to deliver a third, opinion on the subject, favouring fits without the duality model.

1.1 Fit Strategy

The objective of this work is to extract α_s and argue about the importance of DV. Apart from the two main objectives we want to analyse the contribution of higher order OPE contributions up to dimension ten.

Our fitting strategy will be in choosing weights of lower and higher pinching. Lower pinched weights should be affected by DV, while higher pinched weights should be protected from DV. As a result in comparing different fits of lower and higher pinched weights it should be possible to argue about strength of the DV that are (or are not) present.

Our hypothesis is that DV are small enough for fits of the combined vector and axial-vector channel in combination with pinched weights. Consequently

	Symbol	Term	Expansion	OPE Contributions
Pinched	ω_τ	$(1-x)^2(1+2x)$	$1-3x^2+2x^3$	D6, D8
	ω_{cube}	$(1-x)^3(1+3x)$	$1-6x^2+8x^3-3x^4$	D6, D8, D10
	$\omega_{quartic}$	$(1-x)^4(1+3x)$	$1-10x^2+20x^3-15x^4+4x^5$	D6, D8, D10, D12
Monomial	ω_{M2}	$1-x^2$	$1-x^2$	D6
	ω_{M3}	$1-x^3$	$1-x^3$	D8
	ω_{M4}	$1-x^4$	$1-x^4$	D10
Pinched $+x$	ω_{X1}	$(1-x)$	$1-x$	D4
	ω_{X2}	$(1-x)^2$	$1-2x+x^2$	D4, D6
	ω_{X3}	$(1-x)^3$	$1-3x+3x^2-x^3$	D4, D6, D8
	ω_{X4}	$(1-x)^4$	$1-4x+6x^2-4x^3+x^4$	D4, D6, D8, D10

Table 1.2: Displaying three categories of fits, each containing three weights with their corresponding mathematical expression and the OPE contributions the fitted integral momentum will be sensitive to.

we can extract parameters, like the strong coupling α_s from τ decays to high precision without a DV model.

We will perform our analysis in the framework of FOPT. To define a fit we have to choose a weight ω and a momentum s_0. The only restriction from choosing a weight is, that the weight has to be analytic, leaving us with a variety of choices. For our strategy we have chosen three categories of weights, each of them containing fits with three or four different weights. A table with an overview of all used weights is given in table 1.2 To test for the stability of the fitted values and have enough DOF to fit the higher OPE contributions we furthermore fit every weight for various momenta s_0.

1.2 Fits

In the following we will give the results of each of the three previously mentioned fit categories.

The first category contains the *Pinched Weights without Monomial x*. The chosen weights are double (ω_τ), triple (ω_{cube}) and quadruple ($\omega_{quartic}$) pinched and do not contain a monomial term x. An x term would make the fits sensitive to the $D = 4$ OPE contribution, which causes an unreliable perturbative expansion [**Beneke2012**]. The higher the pinching, the higher the suppression of DV. Consequently if we obtain stable values for α_s from the different pinched fits we should expect the DV to have no influence on the value of the strong coupling. The different weights imply an increasing number of active OPE contributions D_6, D_8, D_{10} and D_{12}, which can be used to compare to the stability of higher order OPE contributions and to test for the convergence of the OPE.

The second category contains the *Single Pinched Monomial weights*. In this case all of the weights are only single pinched and, as in the first category, do not carry a monomial in x. Consequently if DV affect the fits we should notice different fitting results in comparison to the fits of the first category. Furthermore the single pinched moments only carry two parameters, the strong coupling and an OPE Wilson coefficients. Thus we can further compare the C_6, C_8 and C_{10} Wilson coefficients and argue about the stability of the fits.

The third and last category contains a similar pinching as the first category, but this time contains a monomial term in x. Consequently these fits are unreliable in the framework of FOPT and we have to apply the *Borel sum* (BS). Following the logic of the second and first category we then can compare the result to analyse the role of DV and compare the Wilson coefficients.

1.2.1 Pinched Weights without a Monomial x

Kinematic weight: $\omega_\tau(x) \equiv (1-x)^2(1+2x)$

We previously encountered the kinematic weight in **??**. It is a polynomial weight function, defined as $\omega_\tau(x) = (1-x)^2(1+2x)$, double pinched, contains the unity and does not contain a term proportional to x. Consequently it is an optimal weight [**Beneke2012**]. As a doubled pinched weight it should have a good suppression of DV contributions and its polynomial contains terms proportional to x^2 and x^3, which makes it sensitive to the dimension six and eight OPE contributions. The fits have been performed within the framework of FOPT for different numbers of s_0. The momentum sets are characterised by its lowest energy s_{min}. We fitted values down to 1.5 GeV. Going to lower energies

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	C_6	C_8	χ^2/dof
BS	2.200	7	0.3274(42)	-0.82(21)	-1.08(40)	0.21
FOPT	2.100	8	0.3256(38)	-0.43(15)	-0.25(28)	1.30
	2.200	7	0.3308(44)	-0.72(20)	-0.85(38)	0.19
	2.300	6	0.3304(52)	-0.69(25)	-0.80(50)	0.25
	2.400	5	0.3339(70)	-0.91(39)	-1.29(83)	0.10
	2.600	4	0.3398(15)	-1.3(1.0)	-2.3(2.5)	0.01

Table 1.3: Table of our fitting values of $\alpha_s(m_\tau^2), C_6$ and C_8 for the kinematic weight $\omega(x) = (1-x)^2(1+2x)$ using FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

is questionable due to the coupling constant becoming large, which implies a breakdown of PT. Furthermore it bares the risk to be affected by the $\rho(770)$ and a_1 peaks in the vector and axial-vector spectral function, which we cannot model within the framework of the OPE. For the three fitting parameters α_s, C_6 and C_8 we have given the results in table 1.3 and graphically in fig. 1.2.

We only display the fits for s_{min} larger than $2.1\,\mathrm{GeV}$. We noted a jump between the $s_{min} = 2.1\,\mathrm{GeV}$ and $s_{min} = 2.2\,\mathrm{GeV}$ of the χ^2/dof from 0.19 to 1.3. We consequently discarded fits with a $s_{min} < 2.2\,\mathrm{GeV}$, as fits lower s_{min} behave more stable[1]. The values for the less momenta are preferred by us due to two reasons. First below energies of $2.2\,\mathrm{GeV}$ we have to face the problematic influence of increasing resonances. Second, we will see, that the values obtained from the lower moment fits are more compatible with our other fits series. We further discarded the fit with four s_0s momenta, which has very small $\chi^2/\mathrm{dof} = 0.01$. This is due to the fact, that we have four s_0s momenta to fit three parameters, which leaves us with too few DOF.

The selected fits with 8-10 momenta have a small χ^2 per DOF. The fitted parameters, α_s, c_6 and c_8 are in good agreement with each other. For all fits we have a good convergence of the OPE. For later comparisons we will give the means for the strong coupling, $D = 6$ and $D = 8$ contributions:

$$\alpha_s(m_\tau^2) = 0.3317(33), \quad C_6 = -0.77(17) \quad \text{and} \quad C_8 = -0.98(35). \tag{1.2.1}$$

[1] As will be seen by comparing the kinematic weight with the cubic and quartic weight

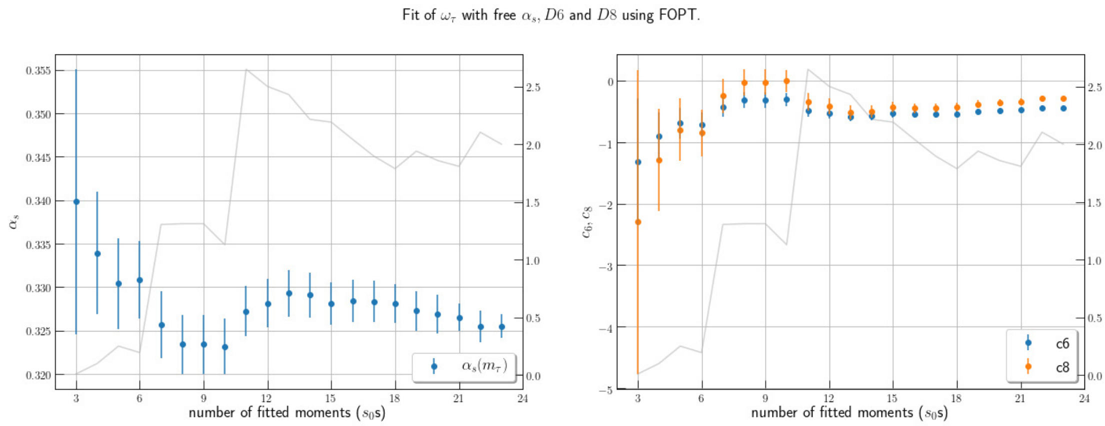

Figure 1.2: Fitting values of $\alpha_s(m_\tau^2)$, C_6 and C_8 for the kinematic weight $\omega(x) = (1-x)^2(1+2x)$ using FOPT for different s_{min}. The left graph plots $\alpha_s(m_\tau^2)$ for different numbers of used s_0s. The right plot contains the dimension six and eight contributions to the OPE. Both plots have in grey the χ^2 per DOF.

We further tested the stability of the dimension six and eight contributions to the OPE within the same fit series but for a fixed value of the strong coupling to our previous averaged result $\alpha_s(m_\tau^2) = 0.3179$. The values for C_6 and C_8 are larger than the values given in our final results from table 1.3. This is explained with a smaller contribution from the strong coupling, which has to be compensated by larger OPE contributions.

Additionally we applied the BS for the fit below the χ^2 threshold containing seven s_0s. Even though we used a different framework than FOPT the results are compatible. This further underlines the good results of the kinematic weight fit and can be seen as an indicator for FOPT being the superior framework as compared to CIPT.

Cubic weight: $\omega_{\text{cube}}(x) \equiv (1-x)^3(1+3x)$

To further consolidate the results from the kinematic weight, we tested a weight of higher pinching, which should suppress DV more than a double pinched weight. Consequently, if we obtain similar results to our previous fits we could exclude DV effects for the kinematic weight. On the other hand, any differences to the previous fit would indicate present DV in the kinematic weight. Our *cubic* weight will be triple pinched and optimal, as it does not contain a x monomial. It is due to its polynomial structure sensitive to the

s_{min}	$\#s_0 s$	$\alpha_s(m_\tau^2)$	C_6	C_8	C_{10}	χ^2/dof
2.000	9	0.3228(26)	-0.196(27)	0.075(28)	0.420(56)	1.96
2.100	8	0.3302(40)	-0.52(11)	-0.58(22)	-1.00(45)	0.43
2.200	7	0.3312(43)	-0.56(12)	-0.68(23)	-1.23(50)	0.55
2.300	6	0.336(11)	-0.78(47)	-1.17(98)	-2.38(22)	0.29
2.400	5	0.3330(96)	-0.63(47)	-0.82(10)	-1.51(26)	0.48

Table 1.4: Table of our fitting values of $\alpha_s(m_\tau^2), C_6, C_8$ and C_{10} for the cubic weight $\omega(x) = (1-x)^3(1+3x)$ using FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

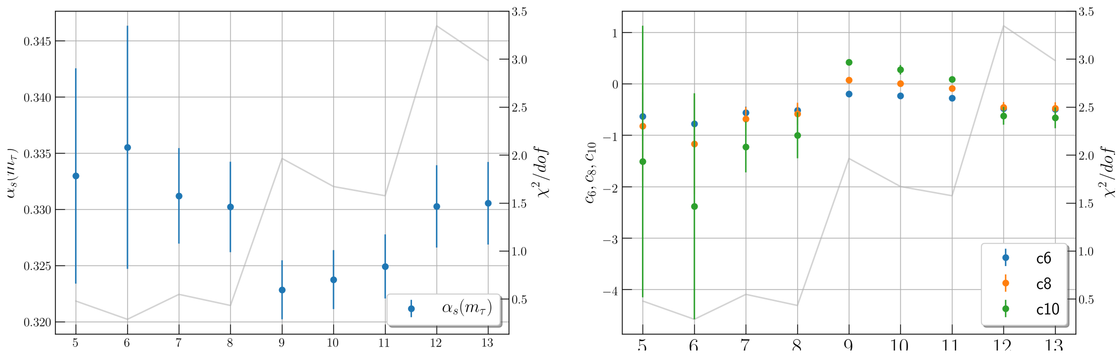

Figure 1.3: Graphic representation of the fitting values of $\alpha_s(m_\tau^2)$ in the left and C_6, C_8 and C_{10} in the right plot for the cubic weight $\omega(x) = (1-x)^3(1+3x)$. The fits have been performed in the FOPT scheme and the data points are given with error bars and are ordered by increasing s_{min}. The grey line in displays the χ^2 function.

dimensions six, eight and ten contributions of the OPE, which yields one more parameter to fit than with the kinematic weight ω_τ. The fitting results can be seen in table 1.4 and graphically in fig. 1.3.

As before we performed fits for $s_0 \leqslant 1.5\,\text{GeV}$, but could only reach convergence for fits with energies larger or equal than $1.8\,\text{GeV}$. As before the χ^2 makes a jump at $s_0 = 2.1\,\text{GeV}$ to values per DOF of almost 2. Consequently we excluded theses fits and focused on fits from five to eight s_0s momenta.

The selected fits have a good χ^2/dof and the fitted parameters, α_s, C_6, C_8 and C_{10} are in agreement with each other, except for the fit with six momenta. The fit with a $s_{min} = 2.3\,\text{GeV}$ has the lowest $\chi^2 = 0.29$ and error on α_s, but takes slighlty different values for the OPE Wilson coefficients in comparison to the other selected fits. The means for the strong coupling, $D = 6$ and $D = 8$ contributions:

$$\alpha_s(m_\tau^2) = 0.33, \quad C_6 = -0.62, \quad C_8 = -0.81 \quad \text{and} \quad C_{10} = -1.5. \tag{1.2.2}$$

We furthermore found that the OPE is converging, but not as fast as for the kinematic weight. The values of $\left|\delta^{(8)}\right|$ is only half as large as $\left|\delta^{(8)}\right|$. The values of the lower momentum count are in high agreement with the ones obtained from the kinematic weight. The conclusions that we take from the *cubic weight* are that the kinematic weight, with its double pinching, should sufficiently suppress any contributions from DVs. If DV would have an effect on the kinematic weight, we should have seen an improvement of the fits with the *cubic* weight, due to its triple pinching, which is not the case.

Quartic weight: $\omega(x) \equiv (1-x)^4(1+4x)$

The last fits of the the pinched weights without a monomial term in x uses the *quartic weight* defined as $\omega(x) \equiv (1-x)^4(1+4x)$. It contains five fitting parameters $(\alpha_s, C_6, C_8, C_{10}, C_{12})$ and did only converge for $s_{min} = 2\,\text{GeV}$ (nine s_0s momenta). The results for , with a χ^2 per DOF of 0.67 are given by:

$$\begin{aligned} \alpha_s(m_\tau^2) = 0.3290(11), \quad C_6 = -0.3030(46), \quad C_8 = -0.1874(28), \\ C_{10} = 0.3678(45) \quad \text{and} \quad C_{12} = -0.4071(77) \end{aligned} \tag{1.2.3}$$

Due to the problematic of the fitting routing, which is caused by too many OPE contributions fitted simultaneously, we will discard the fitting results for the quartic weight.

1.2.2 Single Pinched Monomial Weights

To further solidate our hypothesis we want to probe some weights with a single pinching. If DV play a role then we should note deviating results to fits with higher pinchings. The advantage of these weights is that they only let one OPE dimension contribute, thus leaving us with only two parameters per fit.

s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	c_6	χ^2/dof
2.100	8	0.3179(47)	-0.42(17)	1.62
2.200	7	0.3248(52)	-0.77(22)	0.38
2.300	6	0.3260(60)	-0.85(28)	0.43

Table 1.5: Table of our fitting values of $\alpha_s(m_\tau^2)$, and C_6 for the single pinched double power monomial weight $\omega_{M2}(x) = 1 - x^2$ using FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

Second power monomial: $\omega_{M2}(x) \equiv 1 - x^2$

The first weight is defined as $\omega_{M2}(x) \equiv 1 - x^2$. We only have to fit two parameters, the strong coupling α_s and the dimension six OPE contribution. The results can be seen in table 1.5. Note that we have again a jump of the χ^2/dof value between if we fit with a too low s_{min}. The values obtained for fitting six and seven s_0s moments are in agreement with good χ^2 values. The χ^2 per DOF is like in the ω_τ and ω_{cubic} fits good for $s_{min} \leqslant 2.2\,\mathrm{GeV}$, but jumps to values $\chi^2/\mathrm{dof} > 1.6$ for smaller s_{min}. This is explained through resonances that appear in lower energies.

Third power monomial: $\omega_{M3}(x) \equiv 1 - x^3$

The second weight is defined as $\omega_{M3}(x) \equiv 1 - x^3$ and contains a single third power monomial. Consequently it is sensitive to dimension eight contributions from the OPE. Our fitting results can be taken from table 1.6. Due to the good χ^2 and the internally compatible fitting values we averaged over all rows except the last one of table 1.6. The last row, at $s_{min} = 2.8\,\mathrm{GeV}$ has only one DOF and thus high errors. The averaged values are given by

$$\alpha(m_\tau^2) = 0.32382(42) \qquad \text{and} \qquad C_8 = -1.33(67). \tag{1.2.4}$$

We note that the strong coupling is smaller as our expected values from the previous fits of around 3.33, but the dimension eight contribution is in good agreement. This is a sign of appearing DV, although the parameters of the different fits do not vary by huge numbers.

s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	c_8	χ^2/dof
2.100	8	0.3147(44)	-0.27(29)	1.71
2.200	7	0.3214(49)	-1.01(39)	0.41
2.300	6	0.3227(57)	-1.18(54)	0.46
2.400	5	0.3257(67)	-1.58(74)	0.39
2.600	4	0.325(10)	-1.54(1.53)	0.58
2.800	3	0.326(21)	-1.69(4.03)	1.17

Table 1.6: Table of our fitting values of $\alpha_s(m_\tau^2)$, and C_8 for the single pinched third power monomial weight $\omega_{M3}(x) = 1 - x^3$ using FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	c_{10}	χ^2/dof
2.100	8	0.3136(43)	-0.07(54)	1.75
2.200	7	0.3203(48)	-1.64(77)	0.42
2.300	6	0.3216(56)	-2.01(1.13)	0.47
2.400	5	0.3247(66)	-2.98(1.62)	0.39
2.600	4	0.324(10)	-2.86(3.69)	0.58
2.800	3	0.325(20)	-3.43(10.74)	1.17

Table 1.7: Table of our fitting values of $\alpha_s(m_\tau^2)$ and C_{10} for the single pinched fourth power monomial weight $\omega_{M4}(x) = 1 - x^4$ using FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

Fourth power monomial: $\omega_{m4}(x) \equiv 1 - x^4$

We already analysed the cubic and quartic weights, which depend on the dimension ten OPE contribution, in section 1.2.1 and section 1.2.1 correspondingly. Now, even with the visible DV for fourth power monomial $\omega_{m4} \equiv 1 - x^4$ to study another single pinched moment and the dimension ten OPE contribution. The results of the are given in **??**. The fitting behaviour is very similar to the third power monomial (**??**) and we will directly cite our obtained results:

$$\alpha_s(m_\tau^2) = 0.32277(40) \qquad \text{and} \qquad c_{10} = -2.4(3.6). \tag{1.2.5}$$

As before the values for the strong coupling are lower than the ones obtained by the fit kinematic and cubic weight fits. Furthermore the error on the tenth dimension contribution of the OPE are large. All in all the usage of the single pinched fourth power monomial weight is questionable and does not deliver any additional insights.

1.2.3 Pinched Weights with monomial x

Next to the previously mentioned *optimal weights* from Beneke and Jamin [**Beneke2012**] there exist another type of *optimal' weights*[2] introduced by Pich [**LeDiberder1992**]

$$\omega_{(n,m)}(x) = (1-x)^n \sum_{k=0}^{m} (k+1)x^k, \tag{1.2.6}$$

Combinations of these optimal moments have been widely used by the ALEPH collaboration to perform QCD analysis on the LEP data. The moments fitted in this section include the for FOPT problematic proportional term in x, thus we will perform additional fits using the Borel-sum.

$$\omega_{X1} \equiv (1-x)$$

The first weight is single pinched with only two fitting parameters: α_s and $\chi^2/\mathtt{dof}$. The results for BS and FOPT fits have been displayed in table 1.8. We note that the α_s values of the two frameworks differ, which is most probably due to the problematic of the monomial in x, appearing in the weight function. In general we trust the results of the BS more than those of FOPT. This is further underlined while regarding the higher $\chi^2/\mathtt{dof}$ values of the FOPT fits. Furthermore the values of the BS fits agree with each other, whereas the fits of the FOPT yield inconsistent values. Regarding explicitly the fits from the BS we note that the fits have good $\chi^2/\mathtt{dof}$ values, although a jump from 0.2 to 0.95 between the first two fitted moments. Also note that we had to fit the invariant gluon-condensate for the first time. In the literature $\langle aGG \rangle_I$ should be around 2.1, but here we obtain a smaller, negative value, which could be connected to problems in the fit.

[2] Pich has a different definition of "optimal" moments than Beneke and Jamin. We to differentiate the two definition we marked Pich's optimal' moments with an apostrophe.

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	$\langle aGG\rangle_I$	χ^2/dof
BS	2.100	8	0.357(12)	-0.072(23)	0.95
	2.200	7	0.3593(97)	-0.079(19)	0.2
	2.300	6	0.3589(99)	-0.078(20)	0.24
FOPT	2.100	8	0.3176(47)	-0.0134(48)	1.62
	2.200	7	0.3246(52)	-0.2262(59)	1.91
	2.300	6	0.3260(60)	-0.2453(73)	1.71

Table 1.8: Table of our fitting values of $\alpha_s(m_\tau^2)$ and $langleaGG\rangle_I$ for the single pinched optimal weight $\omega_{X1}(x) = (1-x)$ using the FOPT and BS ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

$$\omega_{X2} \equiv (1-x)^2$$

The next weight is double pinched. Additionally to the strong coupling and the invariant gluon-condensate we also had to fit the dimension six OPE contribution. Our fits can be seen in table 1.9. If we compare the BS with the FOPT fits we note, next to the before mentioned incompatibilities, a sign difference for the $D = 6$ contribution. From now we will skip the FOPT discussion for weights containing monomial x, and trust in the BS fits. In comparison to the previous fit with the single pinched weight we have higher χ^2/dof values, a lower α_s value and an $\langle aGG\rangle_I$ numeric value similar to the value from the literature around 0.21, but with opposite sign. Consequently we note some tension between the single pinched weight and the double pinched weight, which could be caused by DV being not sufficiently suppressed by a single pinched weight containing a monomial x.

$$\omega_{X3} \equiv (1-x)^3 \textbf{ and } \omega_{X4} \equiv (1-x)^4$$

The fits with a triple and quadruple pinched weight do not give any further insights. We give the results in table 1.10 and table 1.11. Both of the weights include similar values to the double pinched weights, which affirms the problems of the single pinched weights. The quadruple pinched weight contains five fitting parameters and has notable convergence problems.

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	C_6	χ^2/dof
BS	2.100	8	0.3207(48)	-0.0170(50)	-0.45(17)	1.90
	2.200	7	0.3270(54)	-0.0254(61)	-0.77(21)	0.74
	2.300	6	0.3253(63)	-0.0232(75)	-0.69(27)	0.9
FOPT	2.100	8	0.3331(54)	-0.0108(45)	0.361(76)	1.9
	2.200	7	0.3401(57)	-0.0185(52)	0.220(88)	0.73
	2.300	6	0.3383(68)	-0.0165(67)	0.26(12)	0.89

Table 1.9: Table of our fitting values of $\alpha_s(m_\tau^2), \langle aGG \rangle_I$ and C_6 for the double pinched optimal weight $\omega_{\chi 2}(x) = (1-x)^2$ using the BD or FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	C_6	C_8	χ^2/dof
BS	2.000	9	0.3169(20)	-0.0123(34)	-0.29(12)	-0.05(24)	2.0
	2.100	8	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	0.46
	2.200	7	0.3251(17)	-0.02283(56)	-0.689(12)	-0.879(33)	0.56
FOPT	2.000	9	0.33985(81)	-0.01124(43)	0.002(10)	-0.242(26)	1.59
	2.100	8	0.3480(47)	-0.0201(36)	-0.264(89)	-1.03(28)	0.31
	2.200	7	0.3483(23)	-0.0204(41)	-0.27(15)	-1.05(40)	0.41

Table 1.10: Table of our fitting values of $\alpha_s(m_\tau^2), \langle aGG \rangle_I, C_6$ and C_8 for the optimal weight $\omega_{\chi 3}(x) = (1-x)^3$ using the BS or FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	aGGInv	C_6	C_8	C_{10}	χ^2/dof
BS	1.950	10	0.31711(67)	-0.012432(24)	-0.30013(73)	-0.06785(16)	0.26104(50)	1.09
	2.000	9	0.3206(24)	-0.0167(14)	-0.455(38)	-0.373(67)	-0.36(14)	0.83
	2.100	8	0.3248(21)	-0.02230(47)	-0.6724(63)	-0.834(14)	-1.352(28)	0.23
FOPT	1.950	10	0.3416(14)	-0.01306(83)	-0.050(22)	-0.390(59)	-0.50(19)	1.71
	2.100	8	0.3480(25)	-0.0201(27)	-0.264(91)	-1.02(23)	-339.00(20)	0.41

Table 1.11: Table of our fitting values of $\alpha_s(m_\tau^2), \langle aGG \rangle_I, C_6, C_8$ and C_{10} for the optimal weight $\omega_{\chi 4}(x) = (1-x)^4$ using the BS or FOPT ordered by increasing s_{min}. The errors are given in parenthesis after the observed value.

	weight	s_{min}	$\alpha_s(m_\tau^2)$	$\langle aGG\rangle_I$	C_6	C_8	C_{10}	χ^2/dof
FOPT	ω_τ	2.2	0.3308(44)	-	-0.72(20)	-0.85(38)	-	0.19
	ω_{cube}	2.1	0.3302(40)	-	-0.52(11)	-0.58(22)	-1.00(45)	0.43
	ω_{M2}	2.2	0.3248(52)	-	-0.77(22)	-	-	0.38
	ω_{M3}	2.2	0.3214(49)	-	-	-1.01(39)	-	0.41
BS	ω_{X2}	2.2	0.3270(54)	-0.0254(61)	-0.77(21)	-	-	0.74
	ω_{X3}	2.1	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	-	0.46

Table 1.12: Table of the best fits (selected by χ^2/dof and compatibility of the fitting values) for each weight including at least the strong coupling $\alpha_s(m_\tau^2)$ as a fitting variable. All fits have been performed using FOPT, except weights marked with a star ω^*, which have been fitted using the *Borel sum*.

1.3 Comparison

To create an overview of our previous results we have gathered the most compatible rows by hand. As in we noted in every weight a threshold for the χ^2/dof value we selected always the fits with the most s_0s momenta included with the condition, that the fit is below the χ^2 threshold. These are shown in table 1.12, which is composed of two parts. The upper four rows are fits using FOPT and the lower two rows are fits using BS. The fits are in great agreement with each others. The strong coupling as the OPE contributions up to dimension eight are compatible within small error ranges, which we have visualised in fig. 1.4. The fits furthermore all have a good χ^2/dof.

As the weights ω_{M2} and ω_{M3} are only single pinched, but still in high agreement with the other higher pinched weights we conclude that DV only play a minor role, even for single pinched weights!

As a result for α_s we state the outcome of the ω_τ fit. The kinematic weight is double pinched, does not contain a monomial x term and has only three parameters to fit. The final value α_s then reads

$$\alpha_s(m_\tau^2) = 0.3308 \pm 0.0044. \tag{1.3.1}$$

The dimension six OPE contribution is as the strong coupling parameter very stable. Stating the result from the kinetic weight fit yields

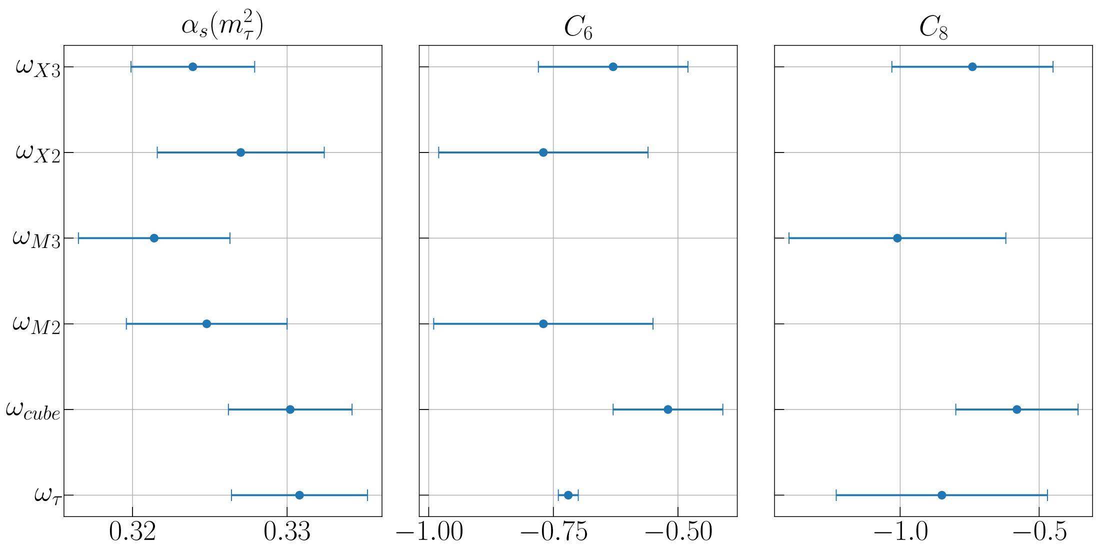

Figure 1.4: dies das

$$C_6 = -0.72 \pm 0.2. \tag{1.3.2}$$

Note that the C_6 values from the cubic weight are slightly different. This is due to the fact, that the cubic weight includes a fourth fitting parameter, which contribution needs to be compensated by the other parameters.

The value of higher dimension OPE parameters are still compatible, but have a higher variation than the previous two parameters. Beginning from the $D = 10$ contribution we do not have enough good fits to compare the parameter with and cannot determine the goodness of the variable. Consequently we do not want to state a single value for OPE parameters of dimension six and higher.