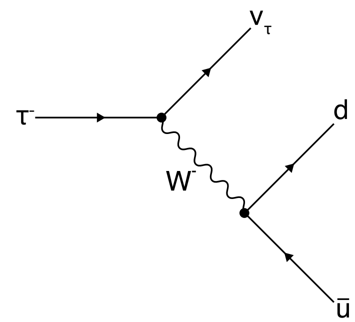
Determination of the QCD Coupling from ALEPH τ Decay Data

Dirk Hornung



Running of the

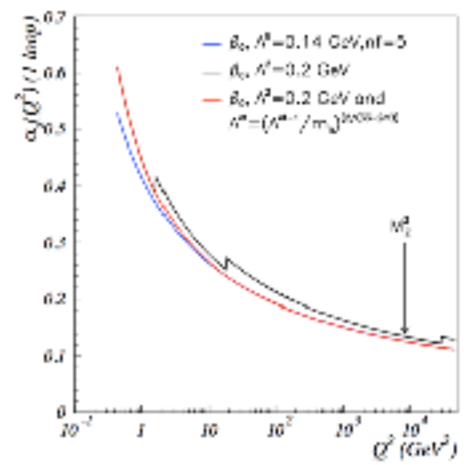
Strong Coupling α_s

$\alpha_s(m_{\tau \cong 3.15 GeV^2}) \cong 0.33$

Exclusive

Hadronic τ-Decay

$$R_\tau = \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e]}$$



- Hadronic decay width of τ most precise determinations of α_s
- R_τ = inclusive hadronic decay ratio can be calculated within QCD & sensitive α_s
- τ decays into a ν_τ & W^- boson, decaying into quark anti-quark (lepton-neutrino) pairs.
- Running αs. $\alpha_s(m_\tau = 3.15\ \text{GeV}^2) = 0.33$ => R_τ sensitive to α_s
- $\alpha_s(m_\tau)$ rel. err: 4%, but $\alpha_s(M_z)$ rel. err. : 1%

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Framework

Inclusive Ratio

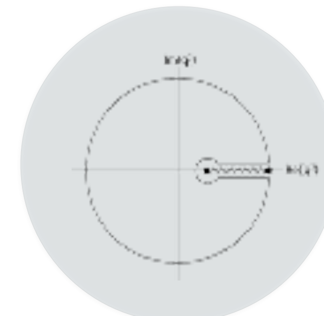
$$R_\tau = 12\pi S_{EW}\int_0^{m_\tau}\frac{\mathrm{d}s}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{uq}|^2\left(\Pi^{(J)}_{ud,V}+\Pi^{(J)}_{ud,A}(s)\right)$$

Two-Point Correlation Function

$$\Pi_{\mu\nu}(q) = i\int \mathrm{d}^4\, x e^{iqx}\langle 0|T\left\{\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right\}\rangle$$
$$= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1)}_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij,\mathcal{J}}(q^2)$$

$$(i,j=u,d;\mathcal{J}=V,A) \quad V^\mu_{ij} = \bar{q}_j\gamma^\mu q_i \quad A^\mu_{ij} = \bar{q}_j\gamma^\mu\gamma_5 q_i$$



$$\int_{s_{th}}^{s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\,\mathrm{Im}\,\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\Pi_{V/A}(s)$$

Experiment Theory

- Main ingredient: R_τ (depends on the two-point function).
- Π split up in transversal (spin 0)/ longitudinal (spin 1) & V/A part.
- Experimentally: measure spectral-function ρ
- Theoretically: employ OPE (not valid on positive real axis, axis of exp. data)
- Use of Cauchy's theorem, connect exp. with th.
- ω(s) is an analytic function, called the weight function

Theory

OPE

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Perturbative

PT

99%

Adler Function:

$$D(s) \equiv -s\frac{\mathrm{d}\Pi^{PT}}{\mathrm{d}s} = \frac{1}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{nk} \ln\left(\frac{-s}{\mu^2}\right)$$

$$A^{\omega,PT} = \frac{i}{2s_0}\oint_{|s|=s_o} \frac{\mathrm{d}s}{s}\left[W(s) - W(s_0)\right] D(s)$$

FOPT or CIPT

Non-Perturbative

NPT

1%

$\mathcal{O}_{2,V/A} \approx 0$ (light quark masses)

$$\mathcal{O}_{4,V/A} = \frac{1}{12}\left[1 - \frac{11}{18}a_s\right]\langle a_s GG\rangle + \left[1 + \frac{\pm 36 - 23}{27}a_s\right]\langle (m_u + m_d)\bar{q}q\rangle$$

- Use OPE for scalar Π
- The OPE consist PT (k=0) & NPT part (k>=1)
- NPT-corrections contribute to only 1% (kinematic weight)
- PT -> Adler function (α_s, c_{nk}, log(s)). D(s) “independent” of μ, but higher order => th. err.
- Partial integration, include D(s) => different weights
- NPT terms contain “condensates”.
- Omit D=2 term (proportional to the m_q^2)
- D=4 mainly Gluon Condensate
- FOPT vs CIPT
- Two operations in momentum: integration & sum, which first? sum => FOPT, int. => CIPT
- FOPT: bigger R_{τ} => smaller α_s

Theory

OPE D=6

Wilson coefficients:

$$C_6^{V-A}(Q^2)\,\langle O_6\rangle \;=\; 4\pi^2 a_s \left\{\left[2+\left(\frac{25}{6}-L\right)a_s\right]\langle Q_-^o\rangle - \left(\frac{11}{18}-\frac{2}{3}L\right)a_s\langle Q_-^s\rangle\right\}$$

$$C_6^{V+A}(Q^2)\,\langle O_6\rangle \;=\; -4\pi^2 a_s \left\{\left[2+\left(\frac{155}{24}-\frac{7}{2}L\right)a_s\right]\langle Q_+^o\rangle + \left(\frac{11}{18}-\frac{2}{3}L\right)a_s\langle Q_+^s\rangle + \right.$$

$$\left[\frac{4}{9}+\left(\frac{37}{36}-\frac{95}{162}L\right)a_s\right]\langle Q_3\rangle + \left(\frac{35}{108}-\frac{5}{18}L\right)a_s\langle Q_4\rangle +$$

$$\left(\frac{14}{81}-\frac{4}{27}L\right)a_s\langle Q_6\rangle - \left(\frac{2}{81}+\frac{4}{27}L\right)a_s\langle Q_7\rangle$$

$$a_s \equiv \frac{\alpha_s}{\pi}$$

$$L \equiv \log\frac{Q^2}{\mu^2}$$

L.E. Adam, K.G. Chetyrkin, Phys. Lett. B329, 129 (1994)

- Master (end 2015)
- Anomalous dimension of four-quark operators for D=6 term to 1st order
- Contributions only from 4-quark condensates (3-gluon condensate at 2nd order)
- Contribution to the $\Pi_{\mu\nu}$ computed at next-to-leading order (Adam, Chetyrkin, 1994)
- Results for V-A and V+A (V-A the peguin diagrams cancelled, n_f=3 & n_c=3)

Theory

OPE D=6

Basis

$$
\begin{aligned}
Q_V^O &= (\bar{u}\gamma_\mu t^a d \bar{d}\gamma^\mu t^a u)\,, \quad Q_A^O = (\bar{u}\gamma_\mu\gamma_5 t^a d \bar{d}\gamma^\mu\gamma_5 t^a u)\,, \\
Q_V^S &= (\bar{u}\gamma_\mu d \bar{d}\gamma^\mu u)\,, \quad Q_A^S = (\bar{u}\gamma_\mu\gamma_5 d \bar{d}\gamma^\mu\gamma_5 u)\,, \\
Q_3 &\equiv (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu t^a q)\,, \\
Q_4 &\equiv (\bar{u}\gamma_\mu\gamma_5 t^a u + \bar{d}\gamma_\mu\gamma_5 t^a d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu\gamma_5 t^a q)\,, \\
Q_5 &\equiv (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu q)\,, \\
Q_6 &\equiv (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \sum_{q=u,d,s} (\bar{q}\gamma^\mu\gamma_5 q)\,, \\
Q_7 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu t^a q')\,, \\
Q_8 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu\gamma_5 t^a q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu\gamma_5 t^a q')\,, \\
Q_9 &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu q')\,, \\
Q_{10} &\equiv \sum_{q=u,d,s} (\bar{q}\gamma_\mu\gamma_5 q) \sum_{q'=u,d,s} (\bar{q}'\gamma^\mu\gamma_5 q')\,.
\end{aligned}
$$

Singlet:
$Q_\pm^S \equiv Q_V^S \pm Q_A^S$

Octet:
$Q_\pm^O \equiv Q_V^O \pm Q_A^O$

- The appearing four-quark operators are a subset which belong to the complete basis required for the one-loop renormalisation.
- $Q_{V,A}$: current-current operators
- Q_3 - Q_{10} penguin operators

OPE D=6

Current-current diagrams

Penguin diagrams:

- Lots of penguin diagrams

Theory

OPE D=6

Flavor non-diagonal

$$\gamma^{(1)}_{Q_+} = \begin{pmatrix} -\frac{3}{N_C} & \frac{3C_F}{2N_C} & -\frac{1}{3N_C} & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{N_f}{3} - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & \frac{3C_F}{2N_C} & \frac{2}{3} & 0 & 0 & 0 \\ \frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & \frac{3N_c}{4} + \frac{3}{2} - \frac{11}{6N_c} & -\frac{3N_c}{4} + \frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & -\frac{3}{4} - \frac{3}{4N_c} & -\frac{3}{4} - \frac{3}{4N_c} & \frac{3C_F}{4N_c} & \frac{3C_F}{4N_c} \\ 0 & 0 & \frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{N_f}{3} + 1 - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & 0 & \frac{3C_F}{2N_c} \\ 0 & 0 & 0 & 0 & 0 & \frac{3N_c}{4} - \frac{10}{3N_c} & -\frac{3N_c}{4} & \frac{3C_F}{2N_c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{11}{3} & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\gamma}^{(1)}_{Q_-} = \begin{pmatrix} -\frac{3N_C}{2} + \frac{3}{N_C} & -\frac{3C_F}{2N_C} \\ -3 & 0 \end{pmatrix}$$

Anomalous dimension matrix

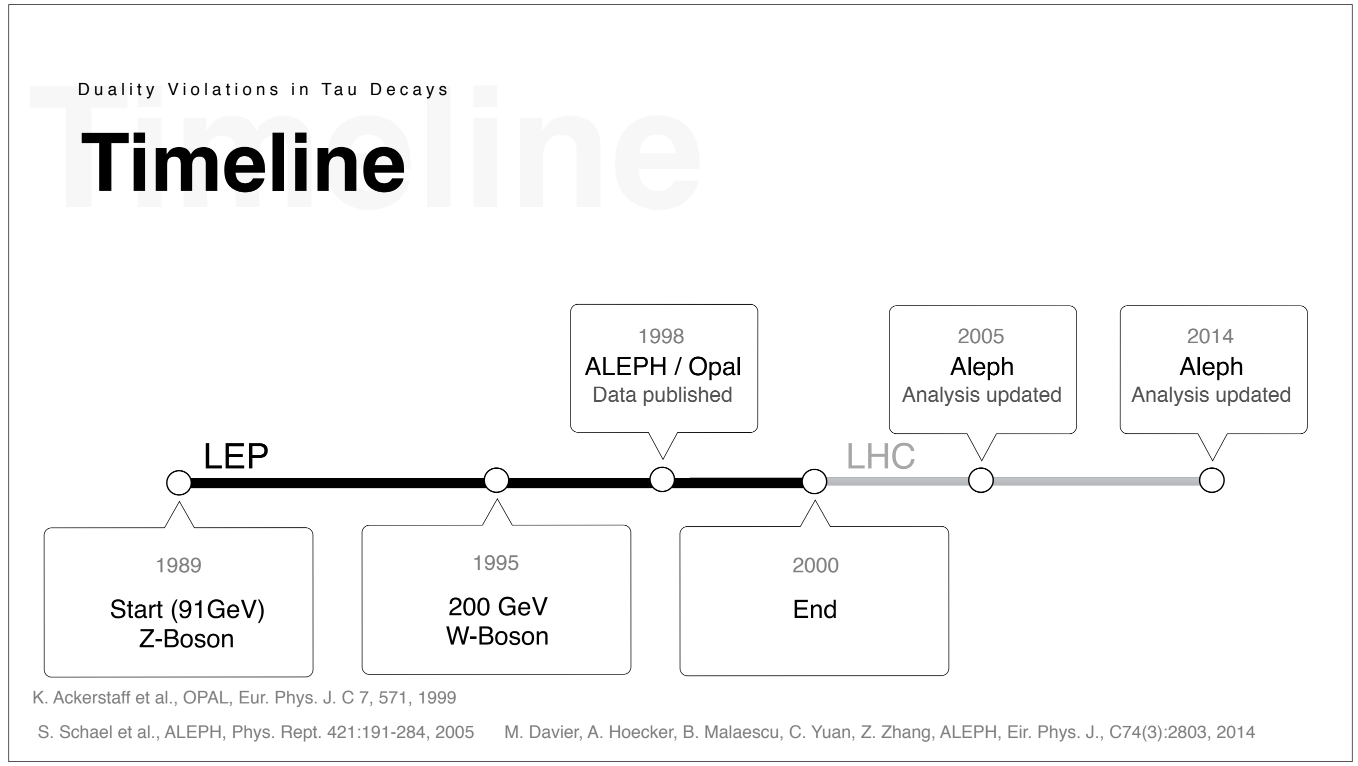
$$\hat{\gamma}(a_s) = a_s \hat{\gamma}^{(1)} + a_s^2 \hat{\gamma}^{(2)} + \cdots$$

V/A currents:

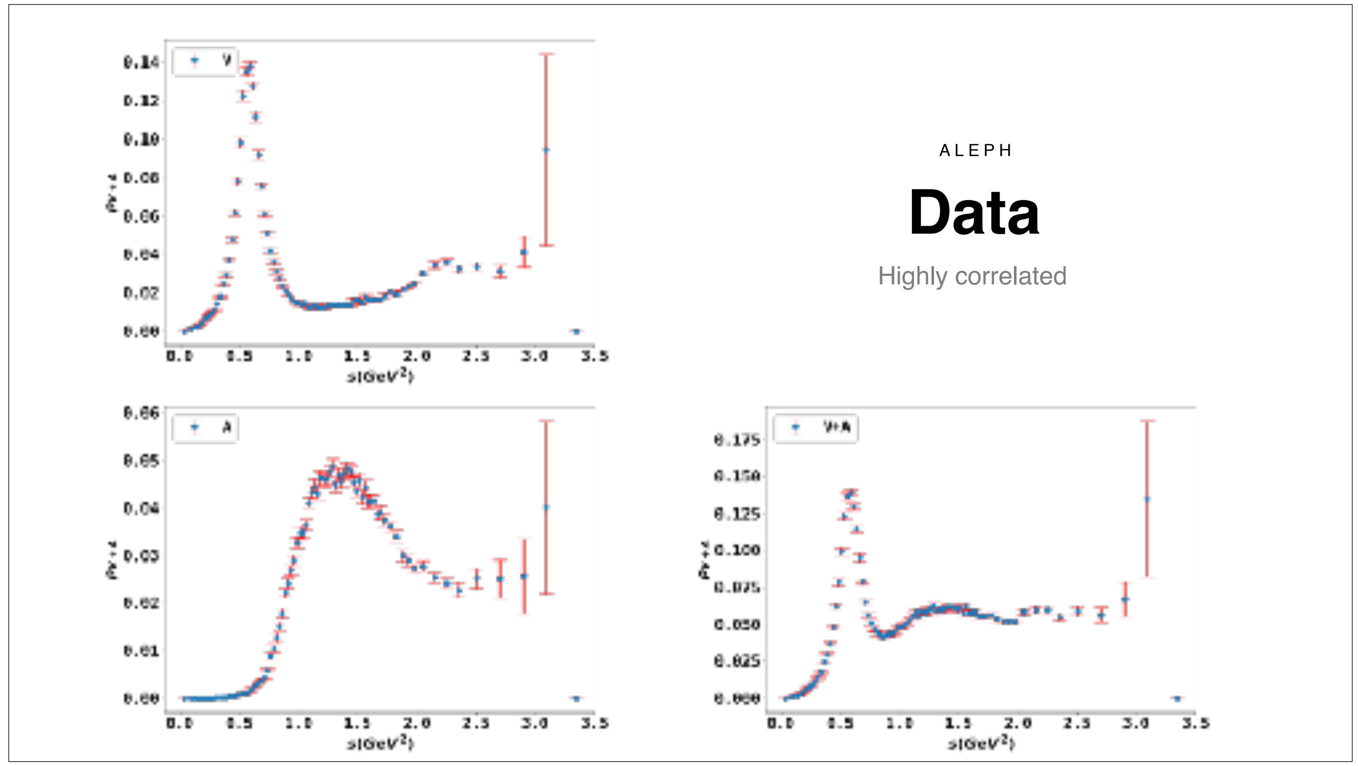
$$j^V_\mu(x) = (\bar{u}\gamma_\mu d)(x)$$
$$j^A_\mu(x) = (\bar{u}\gamma_\mu \gamma_5 d)(x)$$

Diogo Boito, Dirk Hornung, Matthias Jamin, JHEP 1512 (2015)

- The anomalous dimension matrix defines running of the Operators
- D=6 contribution at different s (used as const. in our fits)
- Flavor non-diagonal V/A correlators
- Q_- no penguin diagrams (2x2)
- Same for Flavor diagonal
- used γ to analyse the renormalon structure of dimension 6



- LEP (Large Electron-Positron Collider, same tunnel as LHC)
- Detectors ALEPH & OPAL
- Analysis update (new Parameters, unfolding)
- Opal data have bigger errors



- Updated Aleph data (2014).
- The data channels: V, A & V+A.
- Contains correlation matrix, but highly correlated, needed for χ^2
- V: $\boldsymbol{\rho}$ resonance visible & large errors last bins.
- A: a_1 resonance visible & large errors last 4 bins.
- V+A: quick convergence, after $\boldsymbol{\rho}$ resonance, agreement with naive Parton model.
- V+A: D6 cancels (V,A have oposite signs)

Data

ALEPH

$$\rho(s) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

Spectral Function

Normalized invariant mass-squared distribution

$$\left(\frac{1}{N_{V/A}}\right)\left(\frac{\mathrm{d}N_{V/A}}{\mathrm{d}s}\right)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,V}(s) = \frac{1}{2\pi}v_1(s)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,A}(s) = \frac{1}{2\pi}a_1(s)$$

$$\mathrm{Im}\,\Pi^{(0)}_{\bar{u}d,A}(s) = \frac{1}{2\pi}a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}}\frac{B(\tau^- \to V^-\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}\frac{\mathrm{d}N_V}{N_V\,\mathrm{d}s}\left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}}\frac{B(\tau^- \to A^-\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}\frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s}\left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}}\frac{B(\tau^- \to \pi^-\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}\frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s}\left(1-\frac{s}{m_\tau^2}\right)^2$$

The ALEPH Collaboration, Phys. Rept., 421:191-284, 2005

- Exp. measured Spectral function
- Normalized invariant mass-squared distribution divided by kinematic factor
- No J = 0 contribution (vector current conserved)
- Only contribution to a_0 pion-pole (why do we add this manually?)

Methodology

Fitting

Chisquared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k,\omega_l) = \int_{s_{th}}^{s_k} \frac{\mathrm{d}s}{s_k}\omega_l(s)\,\mathrm{Im}\,\Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k,\omega_l) = \frac{i}{2s_k}\oint_{|s|=s_k}\frac{\mathrm{d}s}{s}\left[W_l(s) - W_l(s_k)\right]D(s)$$

Parameters

a_s $\langle a_s GG\rangle$ $\mathcal{O}_{6,V+A}$ $\mathcal{O}_{8,V+A}$

E.g.

Different s_0

# (k, l)	2 Moments	
1 (1,1)	s_1	w_1
2 (2,1)	s_2	w_1

Different weights

# (k, l)	3 Moments	
1 (1,1)	s_1	w_1
2 (1,2)	s_1	w_2
2 (1,3)	s_1	w_3

Extract

max. 2 parameters

max. 3 parameters

- χ^2: difference exp. th, averaged with correlation matrix inverse
- Integral moments
- Fitted parameters: 4, 12 with DV
- Different combinations => different results
- Different s0
- Different weights (heigh correlations!)
- Different s0 & weights

Fitting

Weights

$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{d}s \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1}$$

n-th degree monomial in the weight selects the D = 2(n+1)

$$\underset{\mathcal{O}_{4,V/A}}{\underset{\big\downarrow}{\frac{s}{s_0}}} + \underset{\mathcal{O}_{6,V/A}}{\underset{\big\downarrow}{\left(\frac{s}{s_0}\right)^2}} + \underset{\mathcal{O}_{8,V/A}}{\underset{\big\downarrow}{\left(\frac{s}{s_0}\right)^3}} + \cdots$$

$$\omega_{kl}(s) = \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^l\left(1+\frac{2s}{m_\tau^2}\right)$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$ Double Pinched

$$A^{ALEPH}_{00,V/A} = A^{ALEPH}_{00,V/A}(a_s, \mathcal{O}_{6,V/A})$$
$$A^{ALEPH}_{10,V/A} = A^{ALEPH}_{10,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$
$$A^{ALEPH}_{11,V/A} = A^{ALEPH}_{11,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}\mathcal{O}_{12,V/A})$$
$$A^{ALEPH}_{12,V/A} = A^{ALEPH}_{12,V/A}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$
$$A^{ALEPH}_{13,V/A} = A^{ALEPH}_{13,V/A}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

- weights used for “activation” of condensates
- Standard definition w_{kl}
- Pinched weight: $1-s/m_\tau$ reduce DV (the closer to mtau the less? Get DV stronger with energy?)
- Some possible weights and their parameters

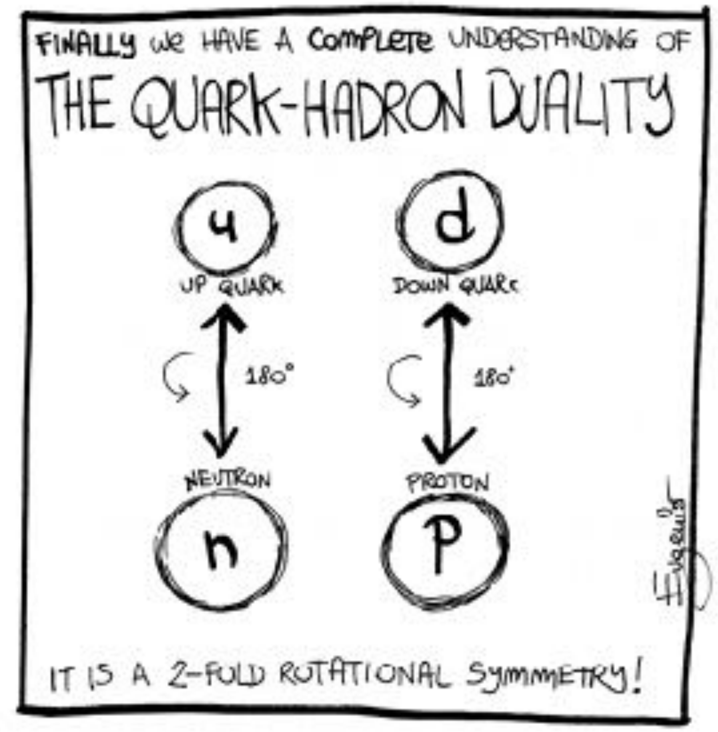
Violations

Duality

Experiment Hadrons ⟷ **Theory** Quark/ Gluons

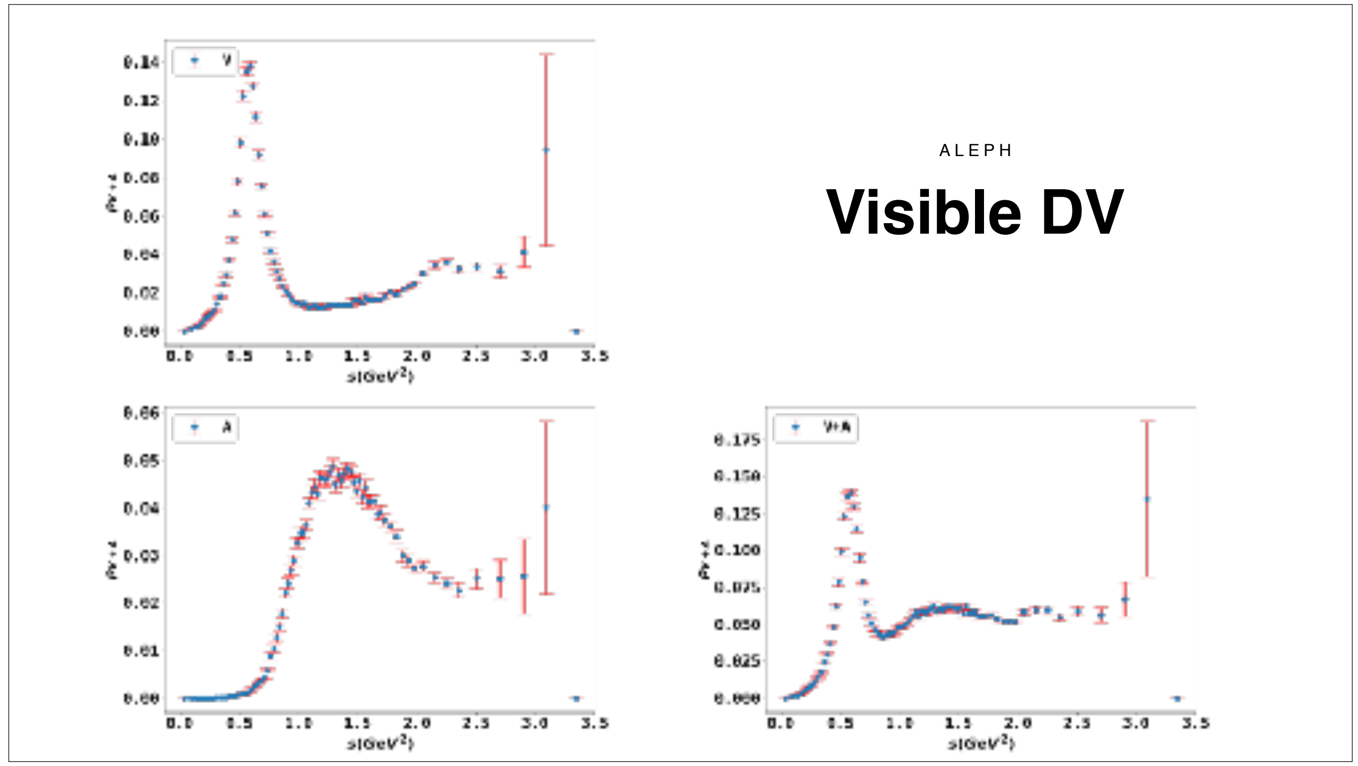
$\rho_{exp} = \rho_{th}$

- What energy is high enough for the quark-hadron duality to set in?



M. Shiftman, hep-th/0009131, 2000

- measure hadrons but calculate with quarks/ gluons?
- Are they really the same?
- Adhoc: $\rho_{exp} = \rho_{th}$
- n, p as Hadrons, flip quarks and they are the same



- V,A: DV clearly visible!
- PT cannot reproduce resonances
- Exponential suppressed
- Sinusoidial
- V+A: less DV

Model

Model: 8 Parameters

$$\Delta\rho^{DV}_{V/A}(s) = e^{-\delta_{V/A}+\gamma_{V/A}s}\sin(\alpha_{V/A}+\beta_{V/A}s)$$

$$\Delta A^{\omega,DV}_{V/A}(s_0) \equiv \frac{i}{2}\oint_{|s_0|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\left\{\Pi_{V/A}(s)-\Pi^{OPE}_{V/A}(s)\right\} = -\pi\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s_0}\omega(s)\Delta\rho^{DV}_{V/A}(s)$$

O. Catà, Maarten Golterman, Santiago Peris, arxiv:0812.2285v2 (2008)

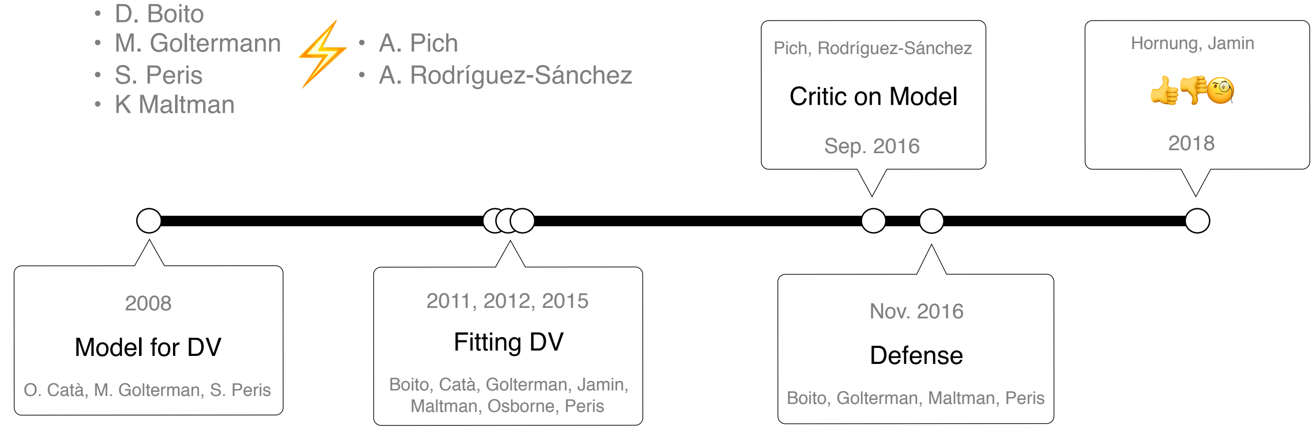
- V: 4 params + A: 4 params A = 8 params
- Add DV to theory side (numerically not easy)

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman

⚡

- A. Pich
- A. Rodríguez-Sánchez



Philosophy

$\alpha_s = 0.322$ (CIPT), $\alpha_s = 0.307$ (FOPT)

Boito, Goltermann, Peris, Maltman

Pro

- Theoretically well motived
- Systematic erros have been underestimated
- Assuming Duality a priori leads to worse model

$\alpha_s = 0.348$ (CIPT), $\alpha_s = 0.324$ (FOPT)

Pich, Rodríguez-Sánchez

Contra

- Cannot be derived from first principles
- Far too many parameters (8)
- Poor statistical quality (low p-value, large uncertainties)
- Sufficiently suppressed (pinched weights)

D. Boito, O. Catà, M. Golterman, M. Jamin, K. Maltman, J. Osborne, S. Peris, Phys. Rev., D84:113006, 2011

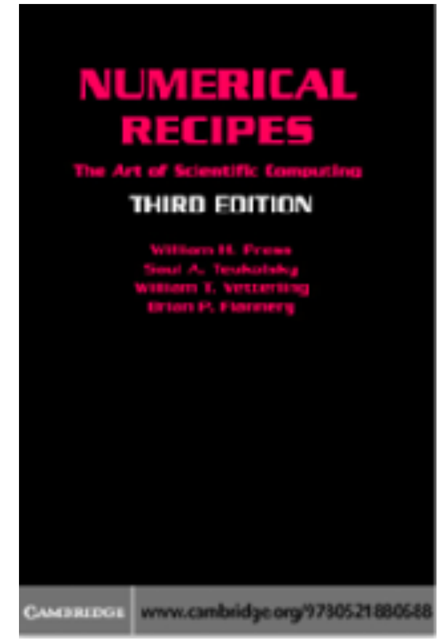
Architecture

Program

Important Frameworks:
- GSL
- BOOST
- ROOT

Infos:
- Editor: Emacs
- Parallelization: use multiple cores
- GitHub

Numerics:
- Integration: GSL - QAG adaptive integration
- Inverse Matrix: BOOST - UBLAS
- Non-linear Equation Solver: GSL - Multidimensional Root-Finding
- Minimization: ROOT - MINUIT2



Determination of α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16

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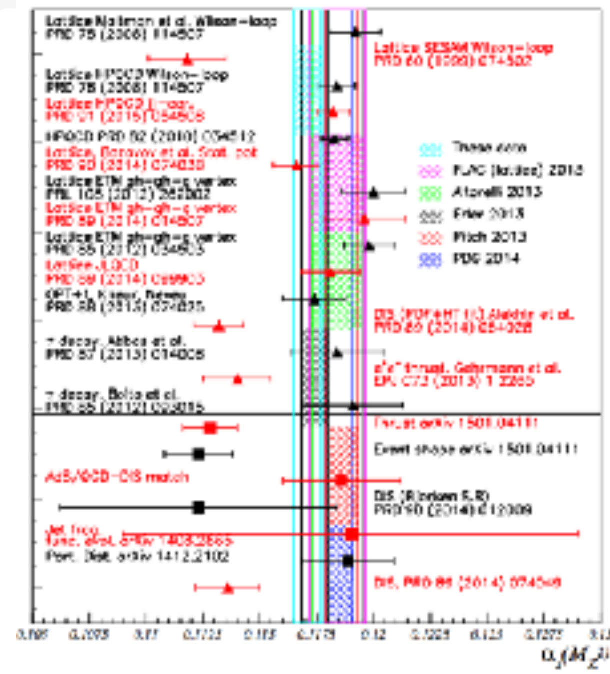

Summary

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG\rangle$
V+A (FOPT)	0.319	-3
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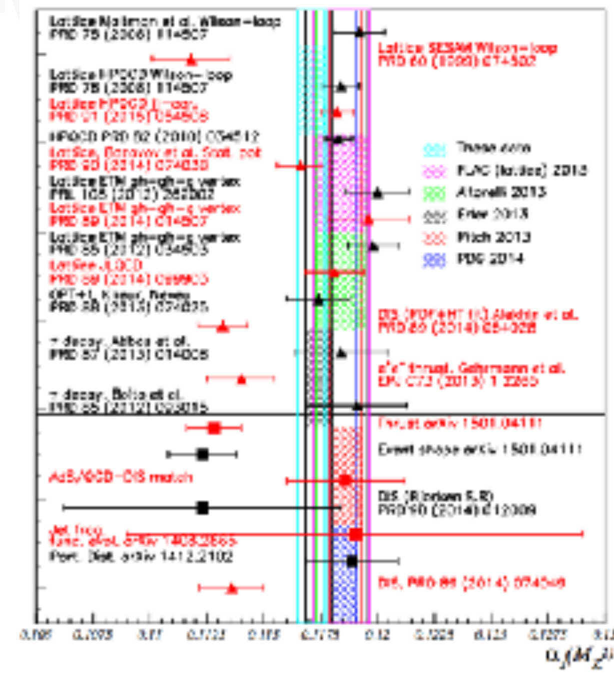

- try to fit without DV