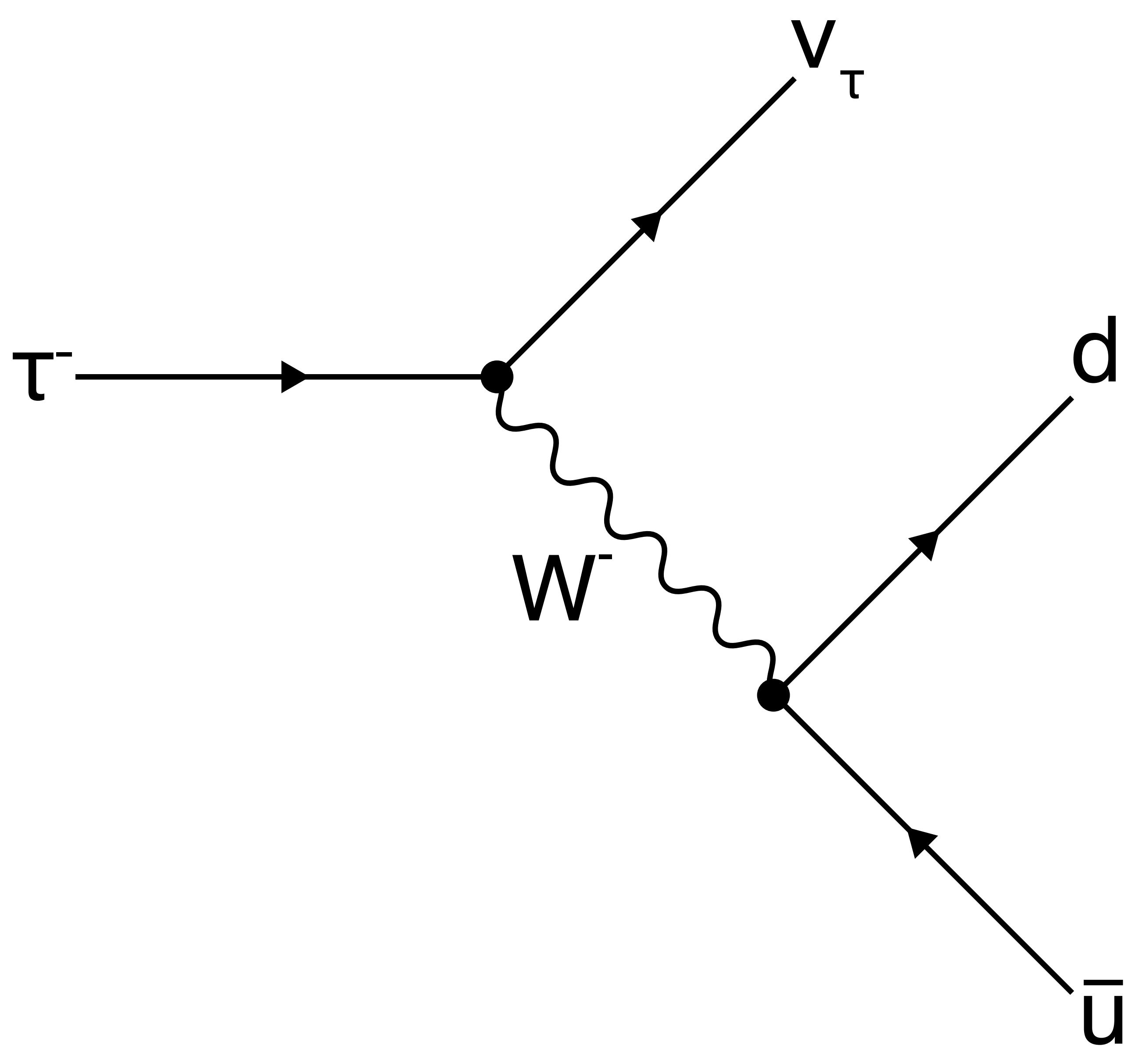
Determination of the QCD Coupling from ALEPH τ Decay Data

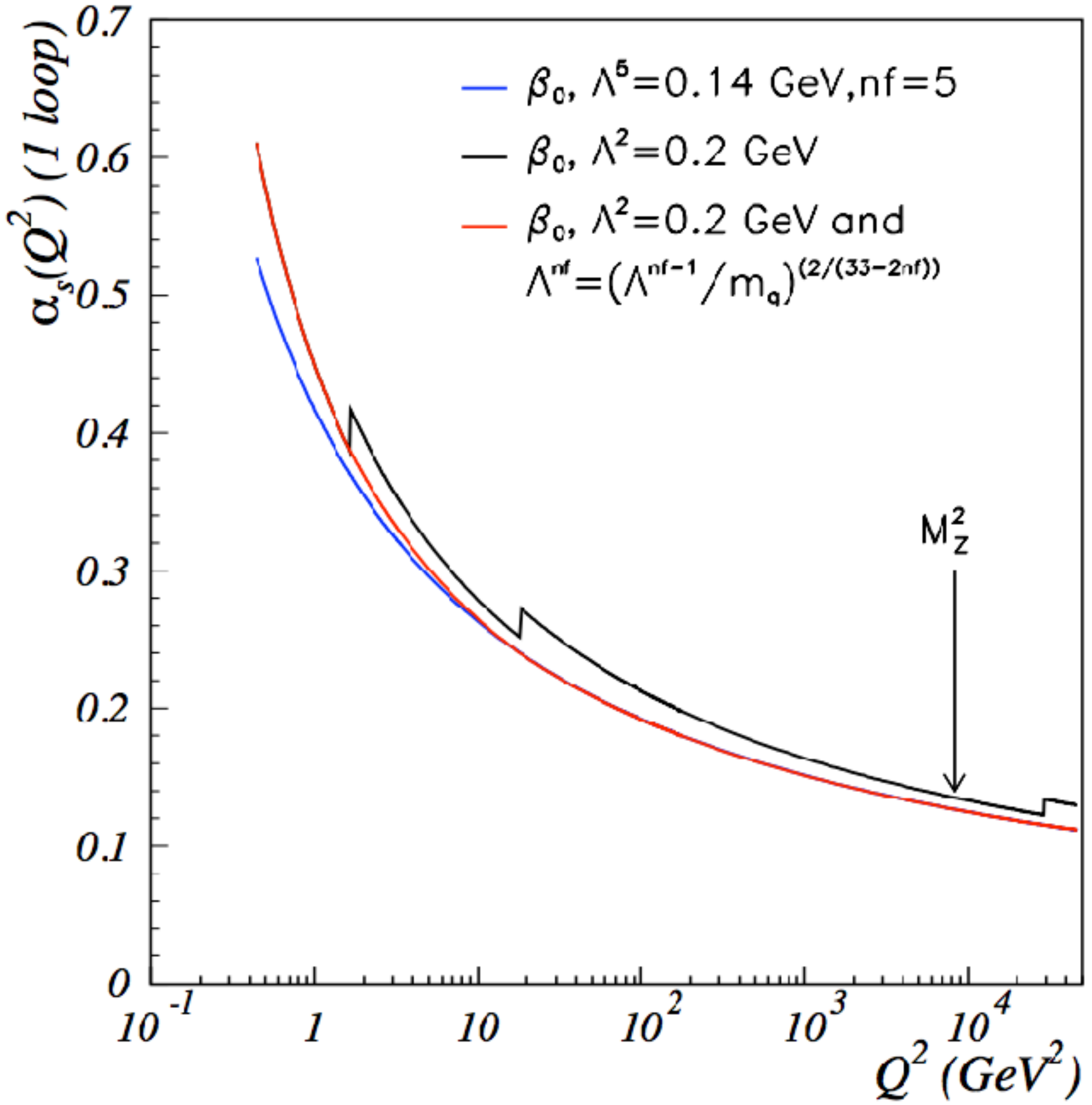
Dirk Hornung



Exclusive

Hadronic τ-Decay

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]}$$



Running of the

Strong Coupling α_s

$\alpha_s(m_{\tau \cong 3.15 GeV^2}) \cong 0.33$

Contents

Theory

Framework

Inclusive Ratio

$$R_\tau = 12\pi S_{EW}\int_0^{m_\tau}\frac{\mathrm{d}s}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

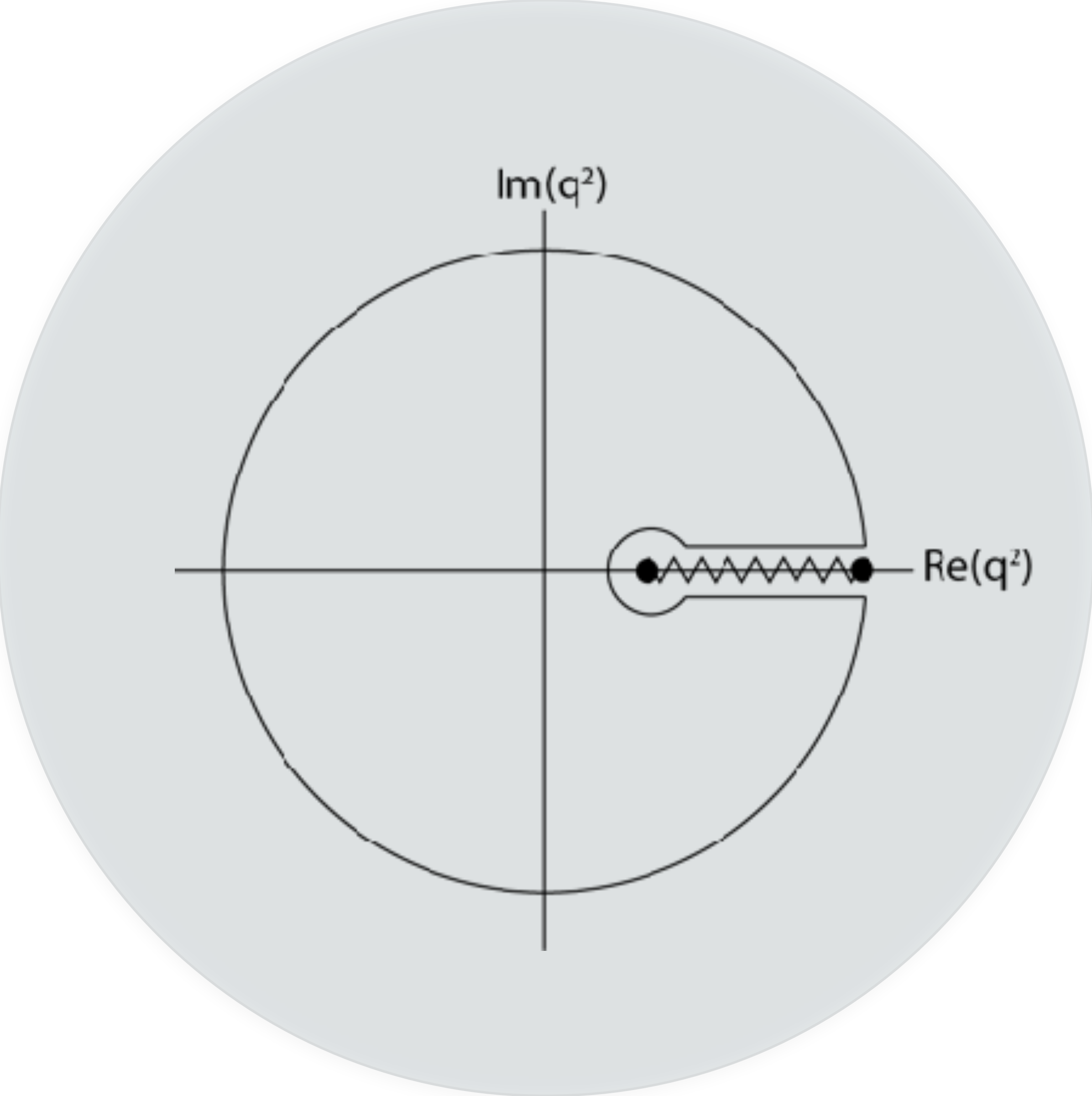
$$\Pi^{(J)}(s)\equiv\left|V_{uq}\right|^2\left(\Pi^{(J)}_{ud,V}+\Pi^{(J)}_{ud,A}(s)\right)$$

Two-Point Correlation Function

$$\Pi_{\mu\nu}(q)=i\int \mathrm{d}^4\,xe^{iqx}\langle 0|T\left\{\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right\}\rangle$$

$$=\left(q_\mu q_\nu-q^2 g_{\mu\nu}\right)\Pi^{(1)}_{ij,\mathcal{J}}(q^2)+q^\mu q^\nu\Pi^{(0)}_{ij,\mathcal{J}}(q^2)$$

$$(i,j=u,d;\mathcal{J}=V,A)\quad V^{\mu}_{ij}=\bar{q}_j\gamma^\mu q_i\quad A^{\mu}_{ij}=\bar{q}_j\gamma^\mu\gamma_5 q_i$$



$$\int_{s_{th}}^{s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\,\mathrm{Im}\,\Pi_{V/A}(s)=\frac{i}{2}\oint_{|s|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\Pi_{V/A}(s)$$

Experiment

Theory

Theory

OPE

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Perturbative

PT

99%

Adler Function:

$$D(s) \equiv -s\frac{\mathrm{d}\Pi^{PT}}{\mathrm{d}s} = \frac{1}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{nk} \ln\left(\frac{-s}{\mu^2}\right)$$

$$A^{\omega,PT} = \frac{i}{2s_0}\oint_{|s|=s_o} \frac{\mathrm{d}s}{s}\left[W(s) - W(s_0)\right] D(s)$$

FOPT or CIPT

Non-Perturbative

NPT

1%

$\mathcal{O}_{2,V/A} \approx 0$ (light quark masses)

$$\mathcal{O}_{4,V/A} = \frac{1}{12}\left[1 - \frac{11}{18}a_s\right]\langle a_s GG\rangle + \left[1 + \frac{\pm 36 - 23}{27}a_s\right]\langle (m_u + m_d)\bar{q}q\rangle$$

OPE D=6

Wilson coefficients:

$$C_6^{V-A}(Q^2)\,\langle O_6\rangle \;=\; 4\pi^2 a_s\left\{\left[2+\left(\frac{25}{6}-L\right)a_s\right]\langle Q_-^{\,o}\rangle-\left(\frac{11}{18}-\frac{2}{3}L\right)a_s\langle Q_-^{\,s}\rangle\right\}$$

$$C_6^{V+A}(Q^2)\,\langle O_6\rangle \;=\; -4\pi^2 a_s\left\{\left[2+\left(\frac{155}{24}-\frac{7}{2}L\right)a_s\right]\langle Q_+^{\,o}\rangle+\left(\frac{11}{18}-\frac{2}{3}L\right)a_s\langle Q_+^{\,s}\rangle+\right.$$

$$\left[\frac{4}{9}+\left(\frac{37}{36}-\frac{95}{162}L\right)a_s\right]\langle Q_3\rangle+\left(\frac{35}{108}-\frac{5}{18}L\right)a_s\langle Q_4\rangle+$$

$$\left.\left(\frac{14}{81}-\frac{4}{27}L\right)a_s\langle Q_6\rangle-\left(\frac{2}{81}+\frac{4}{27}L\right)a_s\langle Q_7\rangle\right.$$

$$a_s\equiv\frac{\alpha_s}{\pi}$$

$$L\equiv\log\frac{Q^2}{\mu^2}$$

L.E. Adam, K.G. Chetyrkin, Phys. Lett. B329, 129 (1994)

Theory

OPE D=6

Basis

$$Q_V^O = (\bar{u}\gamma_\mu t^a d\bar{d}\gamma^\mu t^a u)\,, \quad Q_A^O = (\bar{u}\gamma_\mu\gamma_5 t^a d\bar{d}\gamma^\mu\gamma_5 t^a u)\,,$$

$$Q_V^S = (\bar{u}\gamma_\mu d\bar{d}\gamma^\mu u)\,, \quad Q_A^S = (\bar{u}\gamma_\mu\gamma_5 d\bar{d}\gamma^\mu\gamma_5 u)\,,$$

$$Q_3 \equiv (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu t^a q)\,,$$

$$Q_4 \equiv (\bar{u}\gamma_\mu\gamma_5 t^a u + \bar{d}\gamma_\mu\gamma_5 t^a d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu\gamma_5 t^a q)\,,$$

$$Q_5 \equiv (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu q)\,,$$

$$Q_6 \equiv (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu\gamma_5 q)\,,$$

$$Q_7 \equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu t^a q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu t^a q')\,,$$

$$Q_8 \equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu\gamma_5 t^a q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu\gamma_5 t^a q')\,,$$

$$Q_9 \equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu q')\,,$$

$$Q_{10} \equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu\gamma_5 q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu\gamma_5 q')\,.$$

Singlet:

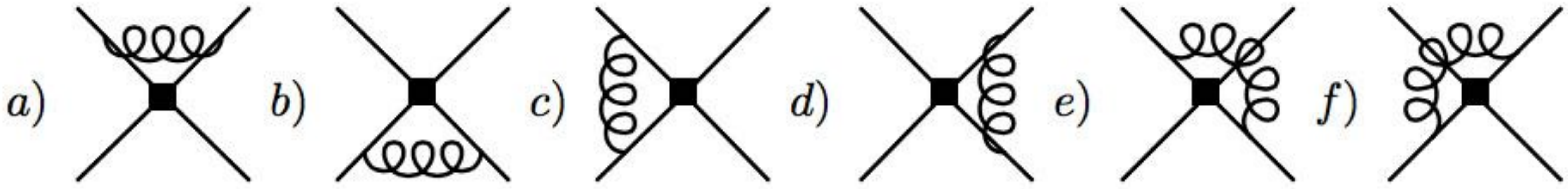
$$Q_\pm^S \equiv Q_V^S \pm Q_A^S$$

Octet:

$$Q_\pm^O \equiv Q_V^O \pm Q_A^O$$

OPE D=6

Current-current diagrams



Penguin diagrams:

Theory

OPE D=6

Flavor non-diagonal

$$
\gamma^{(1)}_{Q_+} = \begin{pmatrix}
-\frac{3}{N_C} & \frac{3C_F}{2N_C} & -\frac{1}{3N_C} & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{N_f}{3} - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & \frac{3C_F}{2N_C} & \frac{2}{3} & 0 & 0 & 0 \\
\frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & \frac{3N_c}{4} + \frac{3}{2} - \frac{11}{6N_c} & -\frac{3N_c}{4} + \frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & -\frac{3}{4} - \frac{3}{4N_c} & -\frac{3}{4} - \frac{3}{4N_c} & \frac{3C_F}{4N_c} & \frac{3C_F}{4N_c} \\
0 & 0 & \frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{N_f}{3} + 1 - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & 0 & \frac{3C_F}{2N_c} \\
0 & 0 & 0 & 0 & 0 & \frac{3N_c}{4} - \frac{10}{3N_c} & -\frac{3N_c}{4} & \frac{3C_F}{2N_c} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{11}{3} & 0 & 0 & 0
\end{pmatrix}
$$

$$
\hat{\gamma}^{(1)}_{Q_-} = \begin{pmatrix} -\frac{3N_C}{2} + \frac{3}{N_C} & -\frac{3C_F}{2N_C} \\ -3 & 0 \end{pmatrix}
$$

Anomalous dimension matrix

$$\hat{\gamma}(a_s) = a_s \hat{\gamma}^{(1)} + a_s^2 \hat{\gamma}^{(2)} + \cdots$$

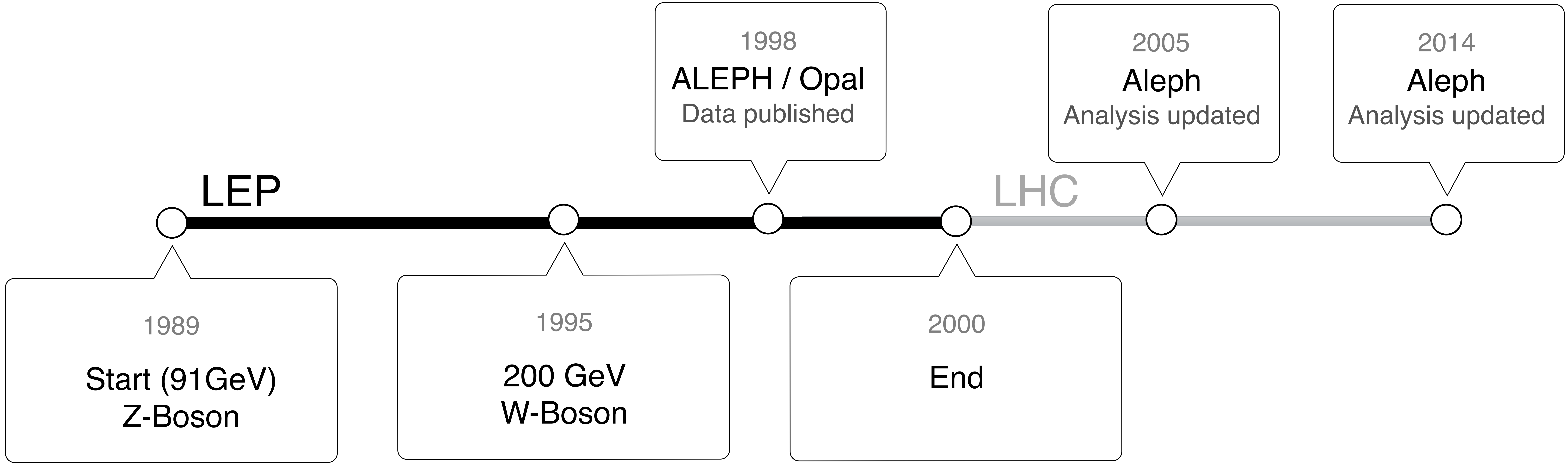
V/A currents:

$$j^V_\mu(x) = (\bar{u}\gamma_\mu d)(x)$$

$$j^A_\mu(x) = (\bar{u}\gamma_\mu \gamma_5 d)(x)$$

Diogo Boito, Dirk Hornung, Matthias Jamin, JHEP 1512 (2015)

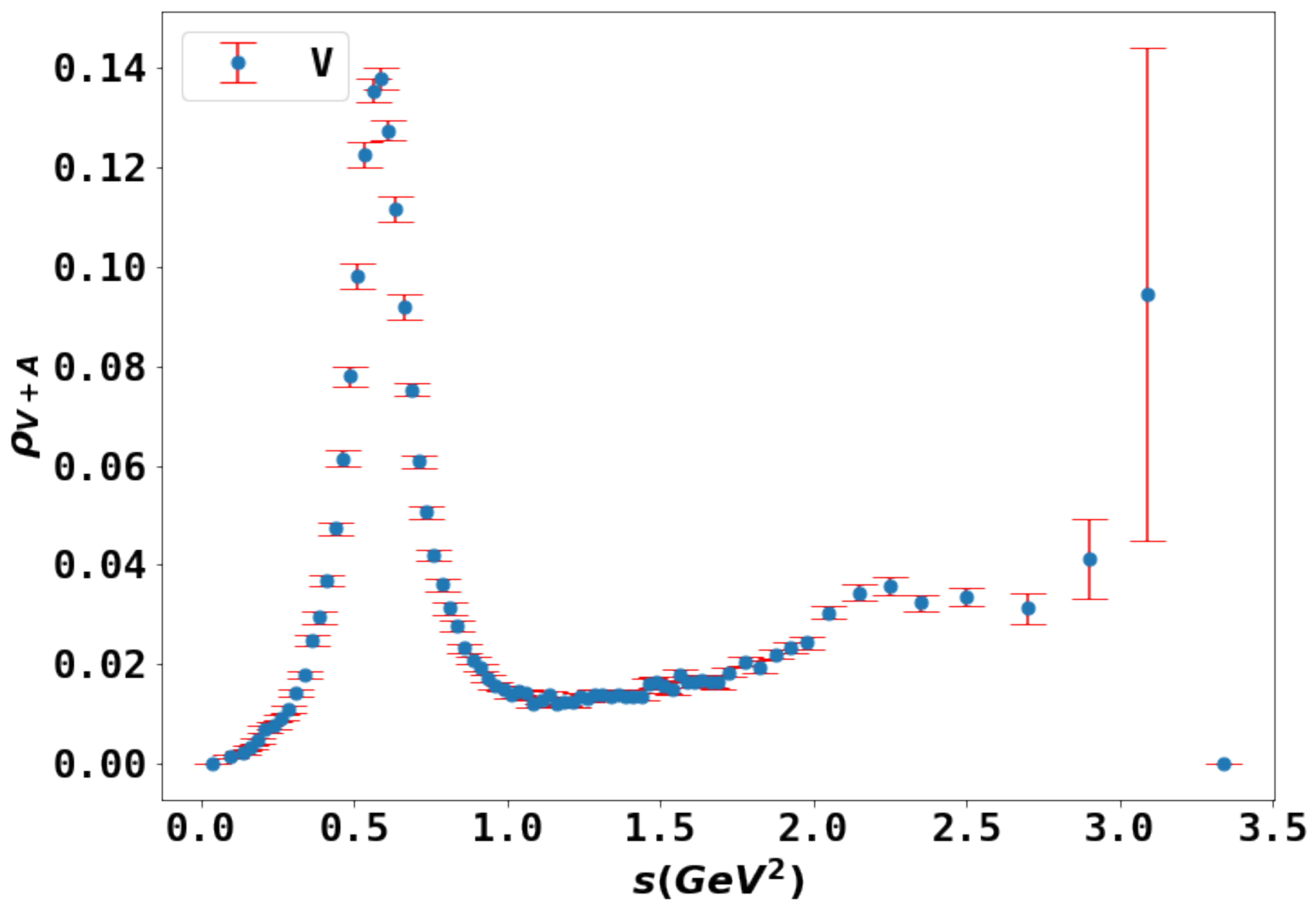
Timeline



K. Ackerstaff et al., OPAL, Eur. Phys. J. C 7, 571, 1999

S. Schael et al., ALEPH, Phys. Rept. 421:191-284, 2005

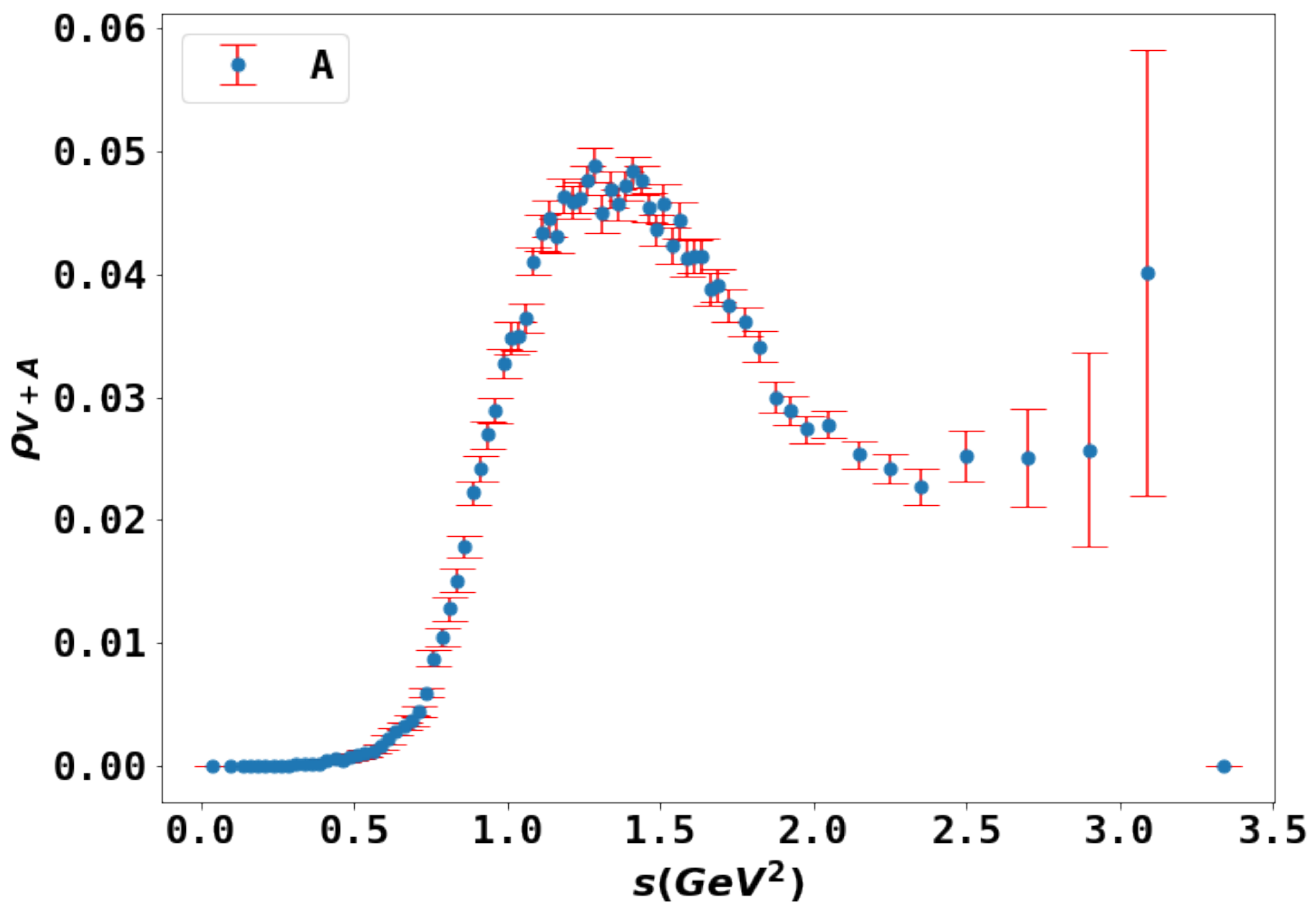
M. Davier, A. Hoecker, B. Malaescu, C. Yuan, Z. Zhang, ALEPH, Eir. Phys. J., C74(3):2803, 2014

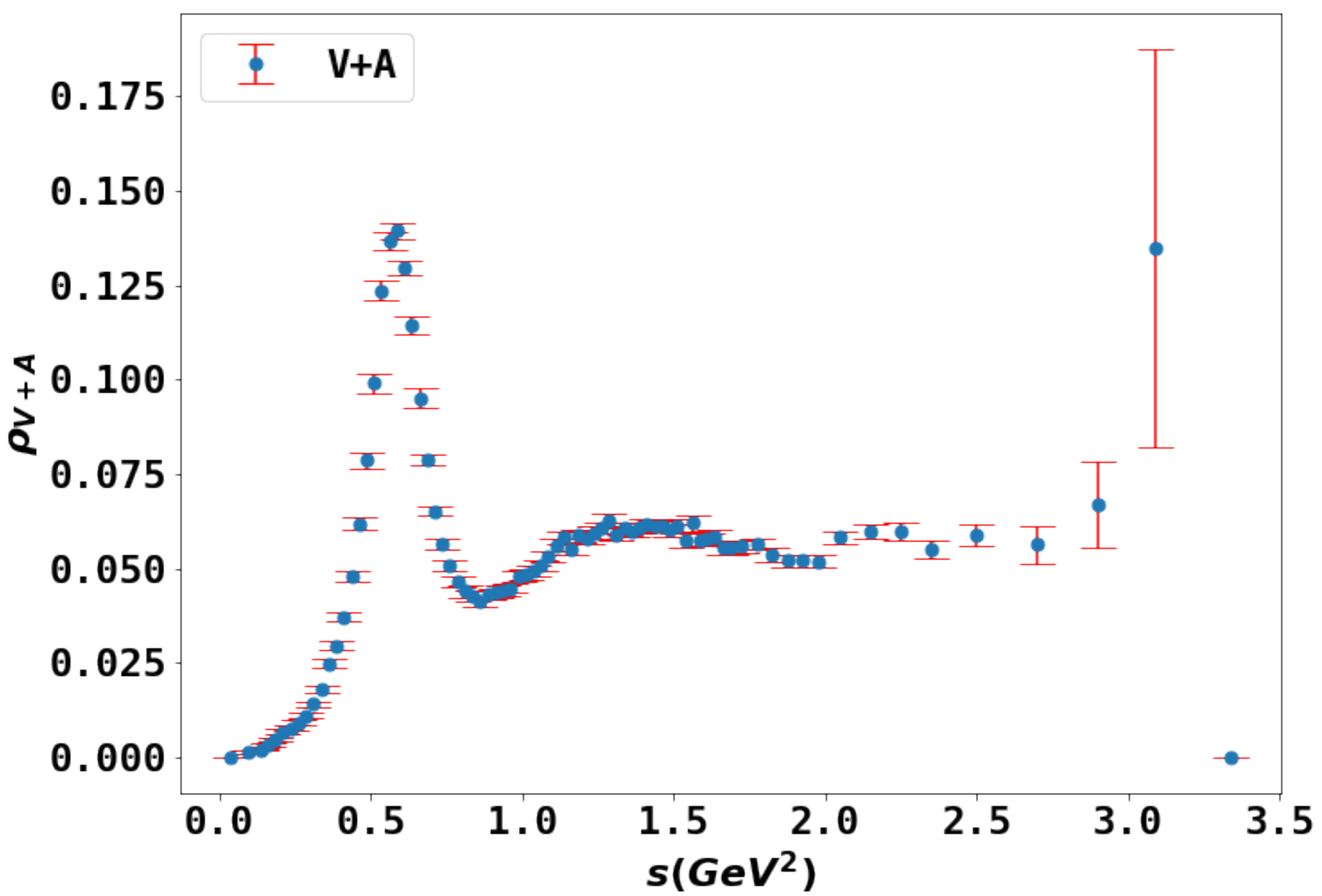


ALEPH

Data

Highly correlated





Data

ALEPH

$$\rho(s) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

Spectral Function

Normalized invariant mass-squared distribution

$$\left(\frac{1}{N_{V/A}}\right)\left(\frac{\mathrm{d}N_{V/A}}{\mathrm{d}s}\right)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,V}(s) = \frac{1}{2\pi} v_1(s)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_1(s)$$

$$\mathrm{Im}\,\Pi^{(0)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to V^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_V}{N_V\,\mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to A^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to \pi^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s} \left(1 - \frac{s}{m_\tau^2}\right)^2$$

The ALEPH Collaboration, Phys. Rept., 421:191-284, 2005

Fitting

Chisquared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k, \omega_l) = \int_{s_{th}}^{s_k} \frac{\mathrm{d}s}{s_k}\omega_l(s)\,\mathrm{Im}\,\Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k, \omega_l) = \frac{i}{2s_k}\oint_{|s|=s_k}\frac{\mathrm{d}s}{s}\left[W_l(s) - W_l(s_k)\right]D(s)$$

Parameters

a_s $\langle a_s GG\rangle$ $\mathcal{O}_{6,V+A}$ $\mathcal{O}_{8,V+A}$

E.g.

Different s_0

# (k, l)	2 Moments	
1 (1,1)	s_1	w_1
2 (2,1)	s_2	w_1

Different weights

# (k, l)	3 Moments	
1 (1,1)	s_1	w_1
2 (1,2)	s_1	w_2
2 (1,3)	s_1	w_3

Extract

max. 2 parameters

max. 3 parameters

Fitting

Weights

$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{d}s \left(\frac{s}{s_0}\right)^{n} \frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1}$$

n-th degree monomial in the weight selects the D = 2(n+1)

$$\begin{array}{ccccccc} \dfrac{s}{s_0} & + & \left(\dfrac{s}{s_0}\right)^2 & + & \left(\dfrac{s}{s_0}\right)^3 & + & \cdots \\ \downarrow & & \downarrow & & \downarrow & & \\ \mathcal{O}_{4,V/A} & & \mathcal{O}_{6,V/A} & & \mathcal{O}_{8,V/A} & & \end{array}$$

$$\omega_{kl}(s) = \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^{l}\left(1+\frac{2s}{m_\tau^2}\right)$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$ Double Pinched

$$A^{ALEPH}_{00,V/A} = A^{ALEPH}_{00,V/A}(a_s, \mathcal{O}_{6,V/A})$$
$$A^{ALEPH}_{10,V/A} = A^{ALEPH}_{10,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$
$$A^{ALEPH}_{11,V/A} = A^{ALEPH}_{11,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}\mathcal{O}_{12,V/A})$$
$$A^{ALEPH}_{12,V/A} = A^{ALEPH}_{12,V/A}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$
$$A^{ALEPH}_{13,V/A} = A^{ALEPH}_{13,V/A}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

A. Pich, A. Rodríguez-Sánchez, arXiv:1605.06830v3, 2016

Violations

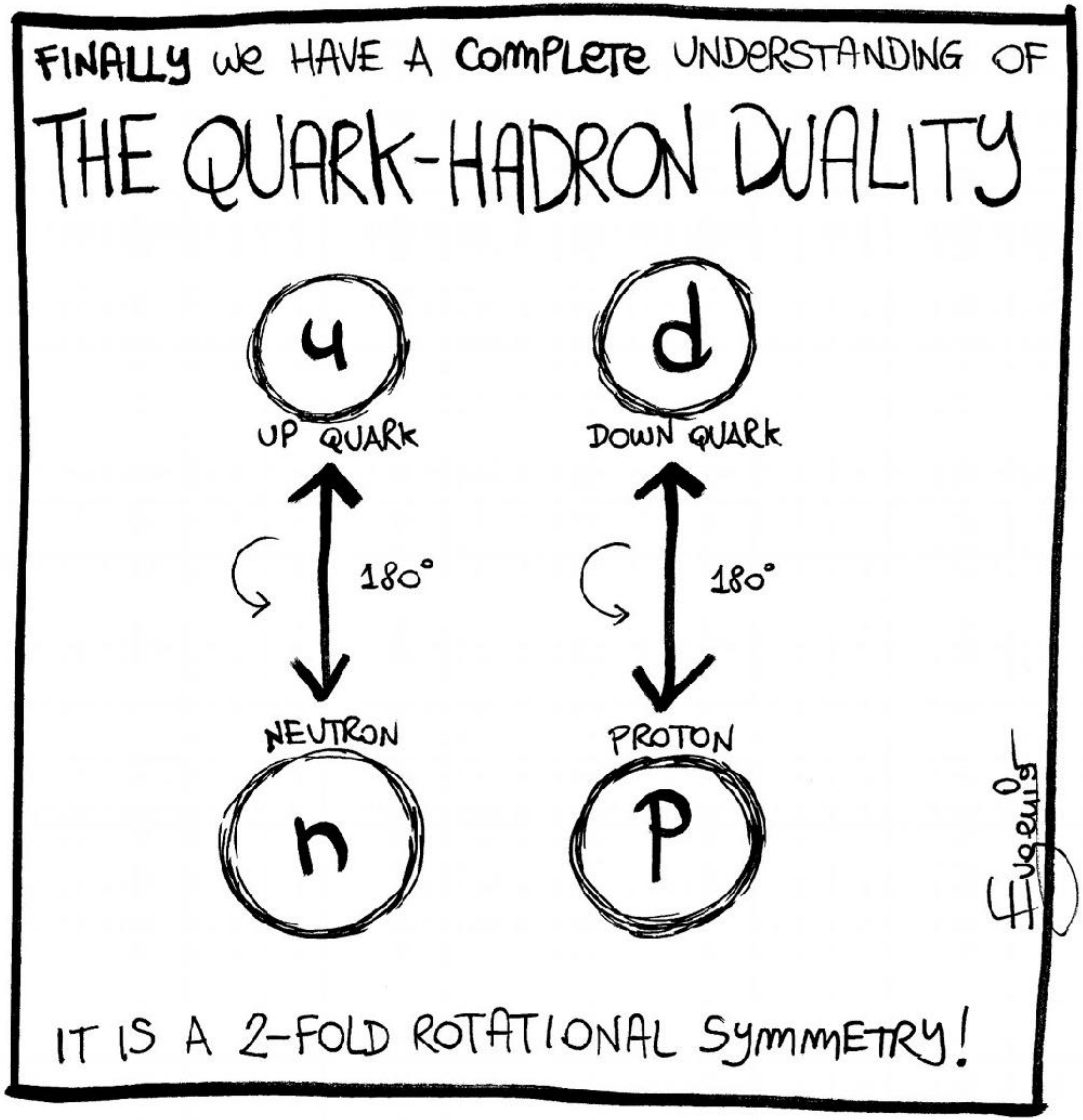
Duality

Experiment
Hadrons

Theory
Quark/ Gluons

$\rho_{\text{exp}} = \rho_{\text{th}}$

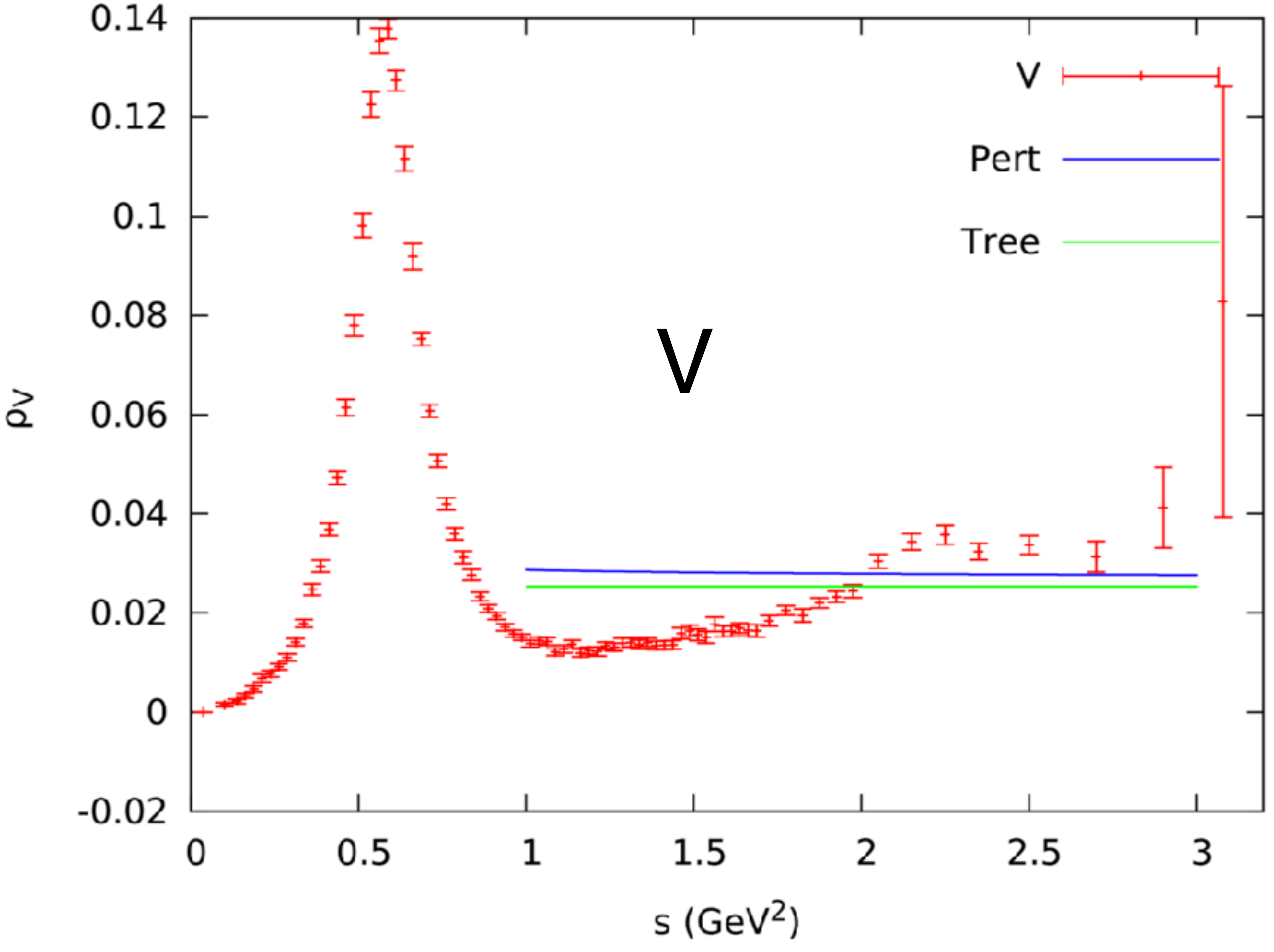
- What energy is high enough for the quark-hadron duality to set in?

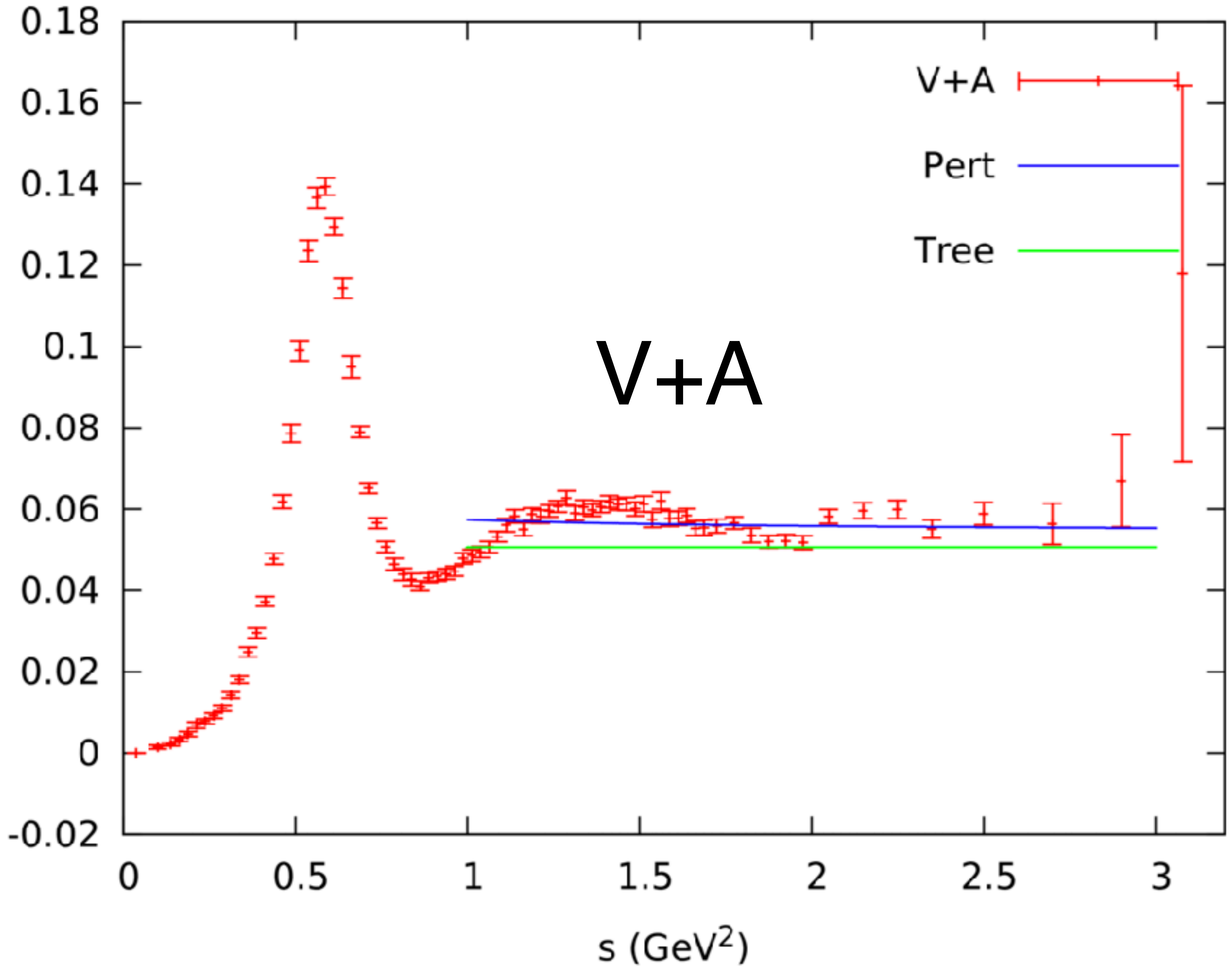


M. Shiftman, hep-th/0009131, 2000

ALEPH

Visible DV





Naive Parton Model | massless perturbative QCD

Model

Model: 8 Parameters

$$\Delta\rho^{DV}_{V/A}(s) = e^{-\delta_{V/A}+\gamma_{V/A}s}\sin(\alpha_{V/A}+\beta_{V/A}s)$$

$$\Delta A^{\omega,DV}_{V/A}(s_0) \equiv \frac{i}{2}\oint_{|s_0|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\left\{\Pi_{V/A}(s)-\Pi^{OPE}_{V/A}(s)\right\} = -\pi\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s_0}\omega(s)\Delta\rho^{DV}_{V/A}(s)$$

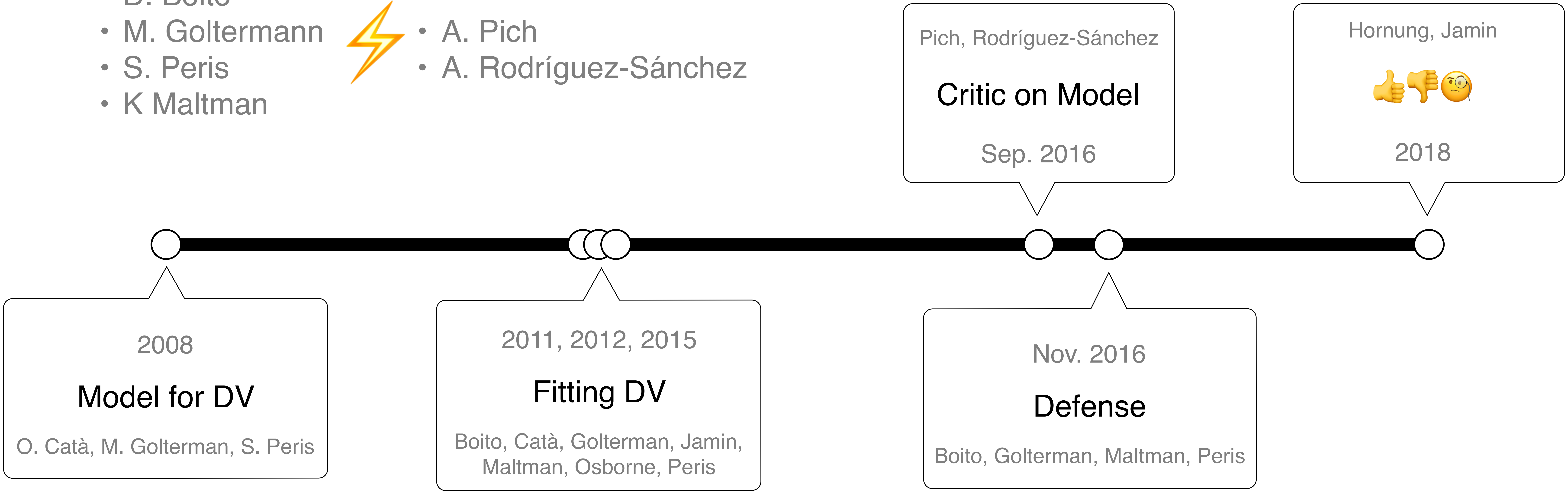
O. Catà, Maarten Golterman, Santiago Peris, arxiv:0812.2285v2 (2008)

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman

⚡

- A. Pich
- A. Rodríguez-Sánchez



Timeline

Philosophy

$\alpha_s = 0.310$ (CIPT), $\alpha_s = 0.296$ (FOPT)

Boito, Goltermann, Peris, Maltman

Pro

- Theoretically well motived
- Systematic erros have been underestimated
- Assuming Duality a priori leads to worse model

$\alpha_s = 0.335$ (CIPT), $\alpha_s = 0.319$ (FOPT)

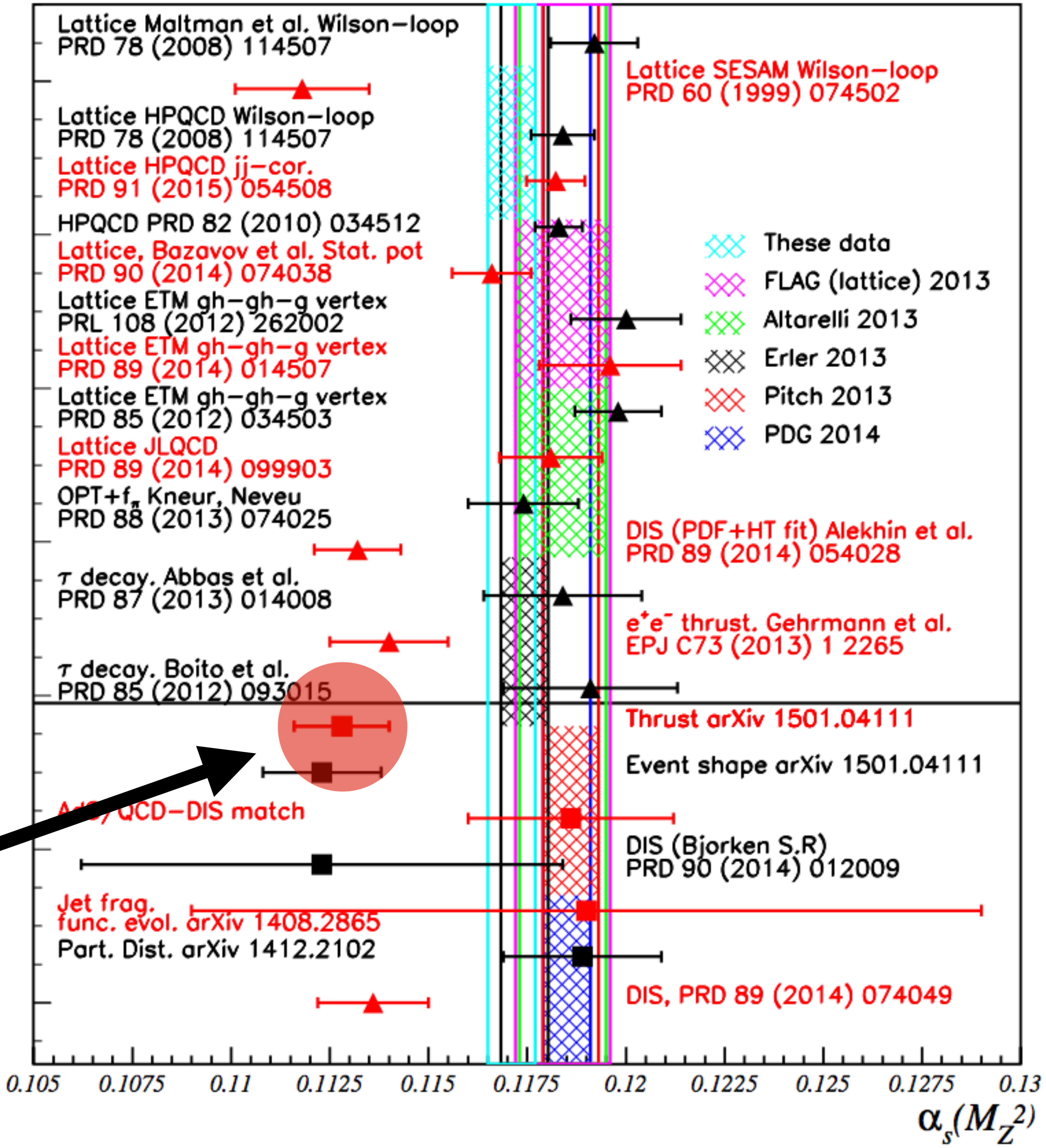
Pich, Rodríguez-Sánchez

Contra

- Cannot be derived from first principles
- Far too many parameters (8)
- Poor statistical quality (low p-value, large uncertainties)
- Sufficiently suppressed (pinched weights)

A. Pich, A. Rodríguez-Sánchez, arXiv:1605.06830v3, 2016
D. Boito, M. Golterman, K. Maltman, S. Peris, arXiv:1611.03457v2, 2016

Determination of α_s



Determination of

Determination of α_s

Five different weights:

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$ $(10^{-3} GeV^4)$	$\mathcal{O}_6$ $(10^{-3} GeV^6)$	$\mathcal{O}_8$ $(10^{-3} GeV^8)$
V+A (FOPT)	$0.319^{+0.010}_{-0.006}$	-3^{+6}_{-11}	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
V+A (CIPT)	$0.339^{+0.011}_{-0.009}$	-16^{+5}_{-5}	$0.9^{+0.3}_{-0.4}$	$-1.0^{0.5}_{-0.7}$

A. Pich, A. Rodríguez-Sánchez, arXiv:1605.06830v3, 2016

Three different weights, multiple s0s:

Channel	$s_{min}(GeV^2)$	$\alpha_s(m_\tau^2)$	$\mathcal{O}_{6V,A}$	$\mathcal{O}_{8V,A}$	$\delta_{V,A}$	$\gamma_{V,A}$	$\alpha_{V,A}$	$\beta_{V,A}$
V+A (FOPT)	1.550	0.292(9)	−0.90(13)	1.57(22)	3.19(51)	0.80(30)	−2.65(79)	4.42(41)
			−0.63(61)	3.0(2.2)	1.53(56)	1.42(24)	5.73(84)	1.84(43)
V+A (CIPT)	1.550	0.312(13)	−0.90(13)	1.48(25)	3.35(49)	0.70(29)	−2.28(81)	4.23(42)
			1.59(55)	1.44(25)	5.37(89)	2.03(46)	−0.33(56)	2.0(1.8)

D. Boito, M. Golterman, K. Maltman, J. Osborne, S. Peris, arXiv:1410.3528v2, 2015

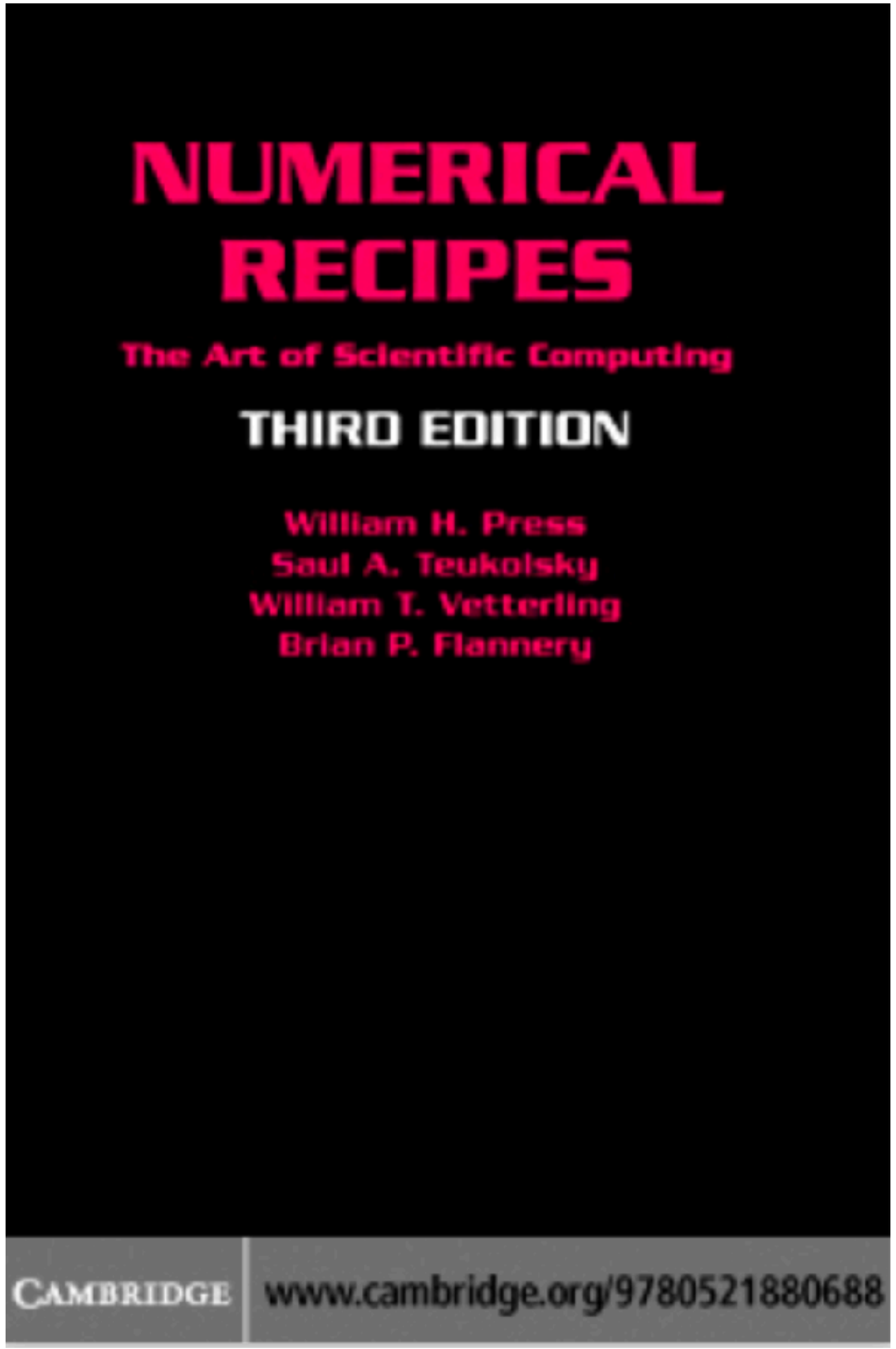
Architecture

Program (C++)

Important Frameworks:
- GSL
- BOOST
- ROOT

Infos:
- Editor: Emacs
- Parallelization: use multiple cores
- GitHub

Numerics:
- Integration: GSL - QAG adaptive integration
- Inverse Matrix: BOOST - UBLAS
- Non-linear Equation Solver: GSL - Multidimensional Root-Finding
- Minimization: ROOT - MINUIT2



Summary

- τ decays lead to precise determinations of α_s
- Duality Violations exist
- There are two ways of extracting α_s from τ decays: DV-model & truncated-OPE-model
- Two very different values: $\alpha_s = 0.319$ (Pich) I $\alpha_s = 0.296$ (Boito)

Pich:

- "… completely dominated by the perturbative contribution"
- "… needs an improved understanding of higher-order perturbative corrections."

Boito:

- "… truncated-OPE-modal … should no longer be used for a precision determination of α_s"

Third party analysis needed

- **Do we need to include DV?**
- **If so is the proposed model the right approach?**