The QCD Strong Coupling from Hadronic τ decays

A PhD Defense

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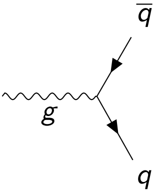
17th July 2019

The Strong Coupling α_s



$$L_{QCD} = \cdots + \sqrt{\pi \boldsymbol{\alpha_s}}\, \overline{q}(x) \lambda \gamma_\mu q(x) G(x) + \ldots \quad (1)$$

$$\begin{aligned} \alpha_s(m_\tau^2) &\approx 0.33 \\ \alpha_s(m_Z^2) &\approx 1.12 \end{aligned} \quad (2)$$

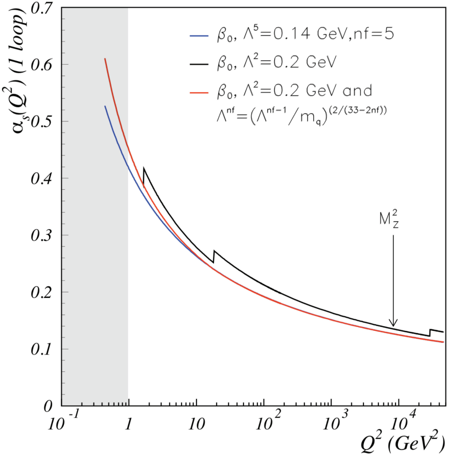
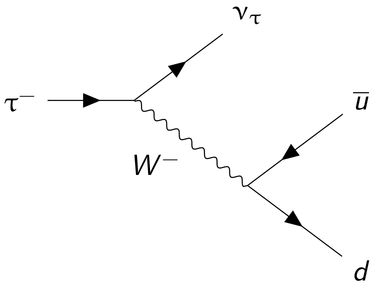


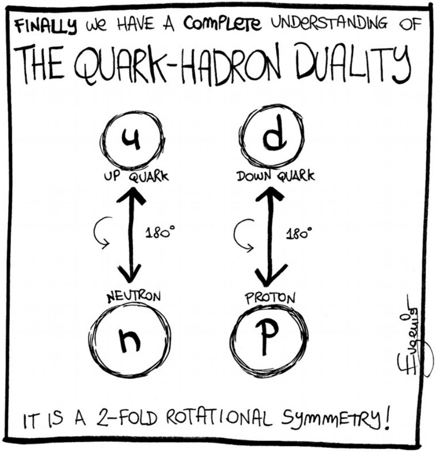
Figure: Taken from Deur, Brodsky, and Teramond, "The QCD Running Coupling"

Exculsive Hadronic τ decays



$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \tag{3}$$

Duality


FINALLY WE HAVE A COMPLETE UNDERSTANDING OF
THE QUARK-HADRON DUALITY
u
UP QUARK
d
DOWN QUARK
180°
180°
NEUTRON
n
PROTON
P
IT IS A 2-FOLD ROTATIONAL SYMMETRY!

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Theoretical Framework

Inclusive Ratio

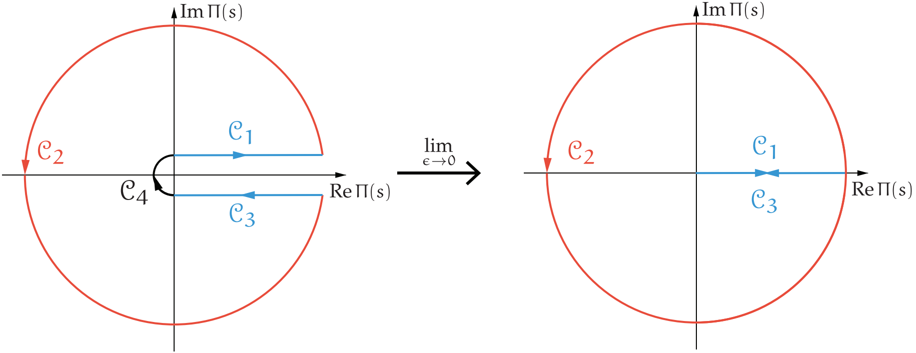
$$\mathrm{R}_\tau = 12\pi |V_{ud}|^2 S_{EW} \int_0^{m_\tau} \frac{\mathrm{d}s}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}_{V/A}(s) + \mathrm{Im}\,\Pi^{(0)}_{V/A}(s)\right] \quad (4)$$

Two-Point Function

$$\begin{aligned} \Pi^{\mu\nu}_{V/A}(q^2) &\equiv i \int \mathrm{d}^4 x e^{iqx} \langle 0| T \left\{ J^\mu_{V/A}(x) J^\nu_{V/A}(0) \right\} |0\rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{V/A}(q^2) \end{aligned} \quad (5)$$

where

$$J^\mu_V = \overline{u}\gamma^\mu d \qquad \text{and} \qquad J^\mu_A = \overline{u}\gamma^\mu \gamma_5 d$$



$$\int_0^{m_\tau} \mathrm{Im}\,\Pi(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=m_\tau} \Pi(s)\,ds$$

with $\quad s \equiv -q^2$

$$\int_0^{s_0} \omega(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi_{OPE}(s)\,ds \qquad (6)$$

$$R_\tau(s) = 12\pi \int_0^{m_\tau} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \left[\left(1 + 2\frac{s}{m_\tau^2} \operatorname{Im}\Pi^{(1)}(s)\right) + \operatorname{Im}\Pi^{(0)}(s) \right] \tag{7}$$

$$R_{\tau,V/A} = \frac{N_c}{2} \left(1 + \delta_{pt} + \delta_{npt}\right) \tag{8}$$

PT Contributions

$$D(s) \equiv= s\frac{\mathrm{d}}{\mathrm{d}s}\Pi(s) \tag{9}$$

$$D^{(1+0)}(s) \equiv -s\frac{\mathrm{d}}{\mathrm{d}s}\Pi^{(1+0)}(s), \qquad D^{(0)}(s) \equiv \frac{s}{m_\tau^2}\frac{\mathrm{d}}{\mathrm{d}s}\left(s\Pi^{(0)}(s)\right) \tag{10}$$

$$R_\tau = -\pi i \oint_{|s|=m_\tau^2} \frac{\mathrm{d}x}{x}(1-x)^3\left[3(1+x)D^{(1+0)}(m_\tau^2 x) + 4D^{(0)}(m_\tau^2 x)\right] \tag{11}$$

$$x \equiv \frac{s}{m_\tau^2} \tag{12}$$

$$D_V^{(1+0} = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k} L^{k-1} \quad \text{with} \quad L \equiv \log \frac{-s}{\mu^2} \tag{13}$$

$$c_{0,0} = -\frac{5}{3}, \quad c_{0,1} = 1 \tag{14}$$

$$c_{2,1} = \frac{365}{24} - 11\zeta_3 - \left(\frac{11}{12} - \frac{2}{3}\zeta_3\right) N_f, \tag{15}$$

$$c_{3,1} = \frac{87029}{288} - \frac{1103}{4}\zeta_3 + \frac{275}{6}\zeta_5, \tag{16}$$

$$- \left(\frac{7847}{216} - \frac{262}{9}\zeta_3 + \frac{25}{9}\zeta_5\right) N_f + \left(\frac{151}{162} - \frac{19}{27}\zeta_3\right) N_f^2, \tag{17}$$

$$c_{4,1} = \frac{78631453}{20736} - \frac{1704247}{432}\zeta_3 + \frac{4185}{8}\zeta_3^2 + \frac{34165}{96}\zeta_5 - \frac{1995}{16}\zeta_7, \tag{18}$$

$$c_{5,1} = 283 \tag{19}$$

FOPT

$$\delta_{pt} = \sum_{n=1}^{\infty} a_{\mu}^{n} \sum_{k=1}^{n} k\, c_{n,k} \frac{1}{2\pi i} \oint_{|x|=1} \frac{\mathrm{d}x}{x} (1-x)^3 (1+x) \log\left(\frac{-m_{\tau}^2 x}{\mu^2}\right)^{k-1} \quad (20)$$

$$\delta_{FOPT}^{(0)} = \sum_{n=1}^{\infty} a(m_{\tau}^2)^n \sum_{k=1}^{n} k\, c_{n,k} J_{k-1} \quad (21)$$

$$J_l \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{\mathrm{d}x}{x} (1-x)^3 (1+x) \log^l(-x) \quad (22)$$

CIPT

$$\delta^{(0)}_{CIPT} = \sum_{n=1}^{\infty} c_{n,1}\, J^a_n(m_\tau^2) \tag{23}$$

$$J^a_n(m_\tau^2) \equiv \frac{1}{2\pi i}\oint_{|x|=1} \frac{\mathrm{d}x}{x}(1-x)^3(1+x)a^n(-m_\tau^2 x) \tag{24}$$

FOPT vs CIPT

$$\begin{array}{cccccc} & \alpha_s^2 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 & \alpha_s^5 \end{array}$$

$$\delta^{(0)}_{FOPT} = 0.1082 + 0.0609 + 0.0334 + 0.0174(+0.0088) = 0.2200(0.2288) \tag{25}$$

$$\delta^{(0)}_{CIPT} = 0.1479 + 0.0297 + 0.0122 + 0.0086(+0.0038) = 0.1984(0.2021). \tag{26}$$

Borel Summation

Borel integral

$$A \equiv \int_0^\infty \mathrm{d}t e^{-t} \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n, \tag{27}$$

Borel transform

$$B[A](t) = \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n. \tag{28}$$

$$\frac{12\pi^2}{N_c} D_V^{1+0}(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(\sqrt{(s)})^{n+1}. \tag{29}$$

Borel Model

$$B[\widehat{D}](u) = B[\widehat{D}_1^{UV}](u) + B[\widehat{D}_2^{IR}](u) + B[\widehat{D}_3^{IR}](u) + d_0^{PO} + d_1^{PO} u, \qquad (30)$$

$$B[\widehat{D}_p^{IR}](u) \equiv \frac{d_p^{IR}}{(p-u)^{1+\gamma}} \left[1 + b_1(p-u) + b_2(p-u)^2 + \dots\right] \qquad (31)$$

$$B[\widehat{D}_p^{UV}](u) \equiv \frac{d_p^{UV}}{(p+u)^{1+\gamma}} \left[1 + b_1(p+u) + b_2(p+u)^2\right], \qquad (32)$$

Beneke and Jamin, "α_s and the τ hadronic width: fixed-order, contour-improved and higher-order perturbation theory"

NPT Contributions

OPE

$$\lim_{x\to y} A(x)B(y) = \sum_n C_n(x-y)\mathcal{O}_n(x) \tag{33}$$

$$\Pi_{OPE}(q^2) = -\frac{1}{3q^2}\sum_n \langle\Omega|\mathcal{O}_n(0)|\Omega\rangle \int d^4\, x e^{iqx} C_n(x) \tag{34}$$

$$\Pi^{OPE}_{V/A}(s) = \sum_{D=0,2,4,\dots} \frac{C^{(D)}\langle\Omega|\mathcal{O}^{(D)}(x)|\Omega\rangle}{(-q^2)^{D/2}} \tag{35}$$

Dimension Four Corrections

$$D_{ij}^{(1+0)}(s)\Big|_{D=4} = \frac{1}{s^2}\sum_n \Omega^{(1+0)}(s/\mu^2)a^n, \tag{36}$$

where the $\Omega^{(1+0)}(s/\mu^2)$ is given by

$$\begin{aligned}\Omega_n^{(1+0)}(s/\mu^2) \;=\; & \frac{1}{6}\langle aGG\rangle p_n^{(1+0)}(s/\mu^2) + \sum_k m_k\langle q_k q_k\rangle r_n^{(1+0)}(s/\mu^2)\\ & + 2\langle m_i q_i q_i + m_j q_j q_j\rangle q_n^{(1+0)}(s/\mu^2) \pm \frac{8}{3}\langle m_j q_i q_i + m_i q_j q_j\rangle t_n^{(1+0)}\\ & - \frac{3}{\pi^2}(m_i^4 + m_j^4)h_n^{(1+0)}(s/\mu^2) \mp \frac{5}{\pi^2}m_i m_j(m_i^2 + m_j^2)k_n^{(1+0)}(s/\mu^2)\\ & + \frac{3}{\pi^2}m_i^2 m_j^2 g_n^{(1+0)}(s/\mu^2) + \sum_k m_k^4 j_n^{(1+0)}(s/\mu^2) + 2\sum_{k\neq l} m_k^2 m_l^2 u_n^{(1-}\end{aligned} \tag{37}$$

Dimension Six and Eight Corrections

$$
\begin{aligned}
D^{(1+0)}_{ij,V/A}\Big|_{D=8} &= 4\frac{\rho^{(8)}_{V/A}}{s^4} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=10} &= 5\frac{\rho^{(10)}_{V/A}}{s^5} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=12} &= 6\frac{\rho^{(12)}_{V/A}}{s^6}
\end{aligned}
\tag{38}
$$

Duality Violations

$$R^{\omega}_{\tau,V/A} = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left(1 + \delta^{\omega}_{pt} + \delta^{\omega}_{npt} + \delta^{\omega}_{DV}\right) \tag{39}$$

$$\rho^{DV}_{V/A}(s) = e^{-(\delta_{V/A} + \gamma_{V/A} s)} \sin(\alpha_{V/A} + \beta_{V/A} s) \tag{40}$$

$$D_{\omega}(m_\tau^2) = -12\pi^2 \int_{m_\tau^2}^{\infty} \frac{\mathrm{d}s}{m_\tau^2} \omega(s) \rho^{DV}_{V/A} \tag{41}$$

Weights

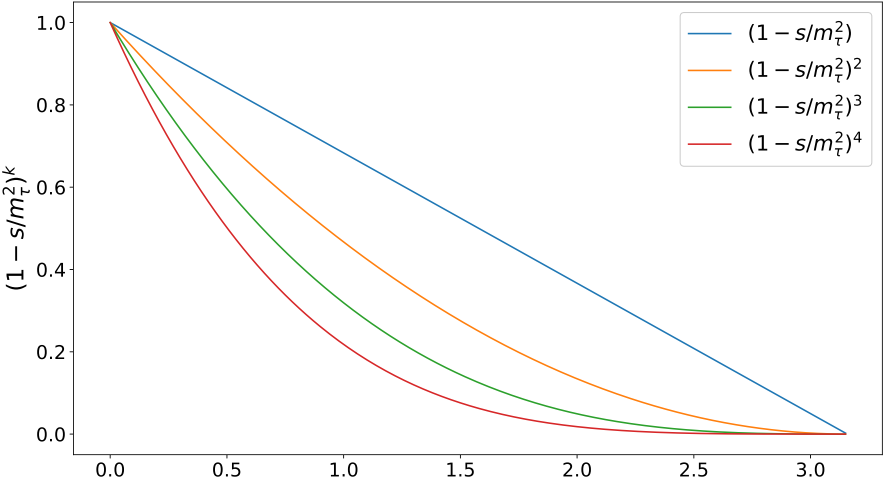
$$\omega(x) \equiv \sum_i a_i x^i \tag{42}$$

kinematic weights

$$\omega_\tau \equiv (1 - \frac{s}{m_\tau^2})^2 (1 + 2\frac{s}{m_\tau^2}) \tag{43}$$

Pinched Weights

$$\omega(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \tag{44}$$



Weighting OPE Contributions

$$\oint_C x^k \, \mathrm{d}x = i \int_0^{2\pi} \left(e^{i\theta}\right)^{k+1} \mathrm{d}\theta = \begin{cases} 2\pi i & \text{if } k = -1, \\ 0 & \text{otherwise} \end{cases}. \tag{45}$$

$$R(x)\Big|_{D=0,2,4,\ldots} = \oint_{|x|=1} \mathrm{d}x \, x^{k-D/2} C^{(D)} \tag{46}$$

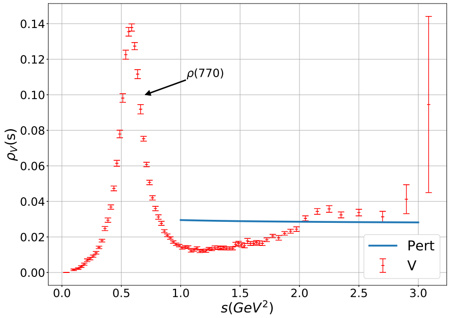
active dimension

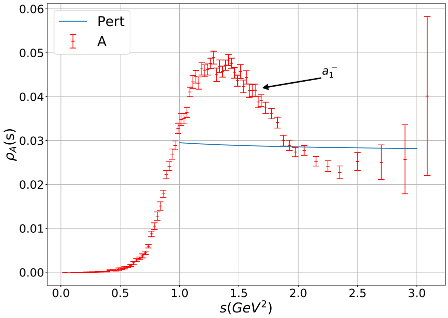
$$D = 2(k+1) \tag{47}$$

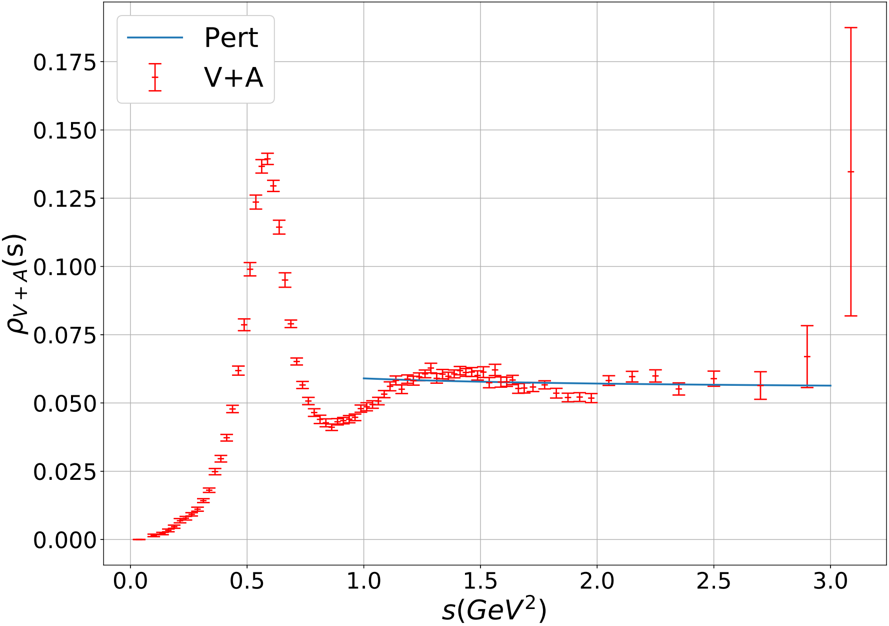
monomial:	x^0	x^1	x^2	x^3	x^5	x^6	x^7
dimension:	$D^{(2)}$	$D^{(4)}$	$D^{(6)}$	$D^{(8)}$	$D^{(10)}$	$D^{(12)}$	$D^{(14)}$

Table: List of monomial and their corresponding "active" dimensions in the OPE. Note that the perturbative contributions of the OPE are always present.

ALEPH data







Pert
V+A
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
$\rho_{V+A}(s)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$s(GeV^2)$

$$R_{\tau,V/A} = \frac{\mathcal{B}_{V/A}}{\mathcal{B}_e} = \int_0^{m_\tau^2} \mathrm{d}s \frac{\mathrm{sfm2}_{V/A}(s)}{100\mathcal{B}_e} \tag{48}$$

$$I^{\omega}_{exp,V/A}(s_0) = \frac{s_\tau}{100\mathcal{B}_e s_0} \sum_{i=1}^{N(s_0)} \frac{\omega\left(\frac{s_i}{s_0}\right)}{\omega_\tau\left(\mathrm{sfm2}_{V/A}(s_i)\right)} \tag{49}$$

$$\chi^2 = (I_i^{exp} - I_i^{th}(\vec{\alpha}))\, C_{ij}^{-1} (I_j^{exp} - I_j^{th}(\vec{\alpha})) \tag{50}$$

$$C_{ij} = \mathrm{cov}(I_i^{exp}, I_j^{exp}) \tag{51}$$

$$\chi^2 \approx 1 \tag{52}$$

	Symbol	Term	Expansion	OPE Contributions
Pinched	ω_τ	$(1-x)^2(1+2x)$	$1-3x^2+2x^3$	$D6$, $D8$
	ω_{cube}	$(1-x)^3(1+3x)$	$1-6x^2+8x^3-3x^4$	$D6$, $D8$, $D10$
	$\omega_{quartic}$	$(1-x)^4(1+3x)$	$1-10x^2+20x^3-15x^4+4x^5$	$D6$, $D8$, $D10$, $D12$
Monomial	ω_{M2}	$1-x^2$	$1-x^2$	$D6$
	ω_{M3}	$1-x^3$	$1-x^3$	$D8$
	ω_{M4}	$1-x^4$	$1-x^4$	$D10$
Pinched $+x$	$\omega_{1,0}$	$(1-x)$	$1-x$	$D4$
	$\omega_{2,0}$	$(1-x)^2$	$1-2x+x^2$	$D4$, $D6$
	$\omega_{3,0}$	$(1-x)^3$	$1-3x+3x^2-x^3$	$D4$, $D6$, $D8$
	$\omega_{4,0}$	$(1-x)^4$	$1-4x+6x^2-4x^3+x^4$	$D4$, $D6$, $D8$, $D10$

Kinematic Weight: $\omega_\tau(x) \equiv (1-x)^2(1+2x)$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.200	7	0.3274(42)	-0.82(21)	-1.08(40)	0.21
FOPT	2.100	8	0.3256(38)	-0.43(15)	-0.25(28)	1.30
	2.200	7	0.3308(44)	-0.72(20)	-0.85(38)	0.19
	2.300	6	0.3304(52)	-0.69(25)	-0.80(50)	0.25
	2.400	5	0.3339(70)	-0.91(39)	-1.29(83)	0.10
	2.600	4	0.3398(15)	-1.3(1.0)	-2.3(2.5)	0.01

Cubic Weight: $\omega_{cube}(x) \equiv (1-x)^3(1+3x)$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
2.000	9	0.3228(26)	-0.196(27)	0.075(28)	0.420(56)	1.96
2.100	8	0.3302(40)	-0.52(11)	-0.58(22)	-1.00(45)	0.43
2.200	7	0.3312(43)	-0.56(12)	-0.68(23)	-1.23(50)	0.55
2.300	6	0.336(11)	-0.78(47)	-1.17(98)	-2.38(22)	0.29
2.400	5	0.3330(96)	-0.63(47)	-0.82(10)	-1.51(26)	0.48

$$\alpha_s(m_\tau^2) = 0.3290(11), \quad \rho^{(6)} = -0.3030(46), \quad \rho^{(8)} = -0.1874(28),$$
$$\rho^{(10)} = 0.3678(45) \quad \text{and} \quad \rho_{(12)} = -0.4071(77). \qquad (53$$

$\omega_{M2}(x) \equiv 1 - x^2$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	χ^2/dof
2.100	8	0.3179(47)	-0.42(17)	1.62
2.200	7	0.3248(52)	-0.77(22)	0.38
2.300	6	0.3260(60)	-0.85(28)	0.43

$\omega_{M3}(x) \equiv 1 - x^3$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(8)}$	χ^2/dof
2.100	8	0.3147(44)	-0.27(29)	1.71
2.200	7	0.3214(49)	-1.01(39)	0.41
2.300	6	0.3227(57)	-1.18(54)	0.46
2.400	5	0.3257(67)	-1.58(74)	0.39
2.600	4	0.325(10)	-1.54(1.53)	0.58
2.800	3	0.326(21)	-1.69(4.03)	1.17

Fourth Power Monomial: $\omega_{M4}(x) \equiv 1 - x^4$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(10)}$	χ^2/dof
2.100	8	0.3136(43)	-0.07(54)	1.75
2.200	7	0.3203(48)	-1.64(77)	0.42
2.300	6	0.3216(56)	-2.01(1.13)	0.47
2.400	5	0.3247(66)	-2.98(1.62)	0.39
2.600	4	0.324(10)	-2.86(3.69)	0.58
2.800	3	0.325(20)	-3.43(10.74)	1.17

$\omega_{1,0} \equiv (1-x)$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	χ^2/dof
BS	2.100	8	0.3176(47)	-0.0134(48)	1.62
	2.200	7	0.3246(52)	-0.2262(59)	0.38
	2.300	6	0.3260(60)	-0.2453(73)	0.43
FOPT	2.100	8	0.357(12)	-0.072(23)	0.95
	2.200	7	0.3593(97)	-0.079(19)	0.2
	2.300	6	0.3589(99)	-0.078(20)	0.24

$\omega_{2,0} \equiv (1-x)^2$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	χ^2/dof
BS	2.100	8	0.3207(48)	-0.0170(50)	-0.45(17)	1.90
	2.200	7	0.3270(54)	-0.0254(61)	-0.77(21)	0.74
	2.300	6	0.3253(63)	-0.0232(75)	-0.69(27)	0.9
FOPT	2.100	8	0.3331(54)	-0.0108(45)	0.361(76)	1.9
	2.200	7	0.3401(57)	-0.0185(52)	0.220(88)	0.73
	2.300	6	0.3383(68)	-0.0165(67)	0.26(12)	0.89

$\omega_{3,0} \equiv (1-x)^3$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.000	9	0.3169(20)	-0.0123(34)	-0.29(12)	-0.05(24)	2.0
	2.100	8	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	0.46
	2.200	7	0.3251(17)	-0.02283(56)	-0.689(12)	-0.879(33)	0.56
FOPT	2.000	9	0.33985(81)	-0.01124(43)	0.002(10)	-0.242(26)	1.59
	2.100	8	0.3480(47)	-0.0201(36)	-0.264(89)	-1.03(28)	0.31
	2.200	7	0.3483(23)	-0.0204(41)	-0.27(15)	-1.05(40)	0.41

$\omega_{4,0} \equiv (1-x)^4$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$aGGInv$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
BS	1.950	10	0.31711(67)	-0.012432(24)	-0.30013(73)	-0.06785(16)	0.26104(50)	1.09
	2.000	9	0.3206(24)	-0.0167(14)	-0.455(38)	-0.373(67)	-0.36(14)	0.83
	2.100	8	0.3248(21)	-0.02230(47)	-0.6724(63)	-0.834(14)	-1.352(28)	0.23
FOPT	1.950	10	0.3416(14)	-0.01306(83)	-0.050(22)	-0.390(59)	-0.50(19)	1.71
	2.100	8	0.3480(25)	-0.0201(27)	-0.264(91)	-1.02(23)	-339.00(20)	0.41

Comparison

	weight	s_{min}	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
FOPT	ω_τ	2.2	0.3308(44)	-	-0.72(20)	-0.85(38)	-	0.19
	ω_{cube}	2.1	0.3302(40)	-	-0.52(11)	-0.58(22)	-1.00(45)	0.43
	ω_{M2}	2.2	0.3248(52)	-	-0.77(22)	-	-	0.38
	ω_{M3}	2.2	0.3214(49)	-	-	-1.01(39)	-	0.41
BS	$\omega_{1,0}$	2.2	0.3246(52)	-0.2262(59)	-	-	-	0.38
	$\omega_{2,0}$	2.2	0.3270(54)	-0.0254(61)	-0.77(21)	-	-	0.74
	$\omega_{3,0}$	2.1	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	-	0.46

Results

- $\alpha_s(m_\tau^2) = 0.3261 \pm 0.0050$
- $\rho^{(6)} = -0.68 \pm 0.2$
- $\rho^{(8)} = -0.80 \pm 0.38$

Results

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- DV not present if using single pinched weights in the V+A channel
- FOPT more valid than CIPT
- $\alpha_s(m_Z^2) = 0.11940(60)$

Thank you

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