The QCD Strong Coupling from Hadronic Tau Decays

A PhD Defense

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The Strong Coupling α_s

$$\mathcal{L}_{QCD}(x) = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x) + \left[\sum_A \frac{i}{2} \overline{q}^A(x) \gamma^\mu \overleftrightarrow{D}_\mu q^A(x) - m \overline{q}^A(x) q^A(x) \right], \tag{1}$$

with $D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} B^a_\mu$

say hello now

The Strong Coupling α_s

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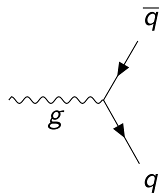
with $D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} B^a_\mu$

$$\mathcal{L}^{QG-Int}_{QCD}(x) = \sqrt{\pi \boldsymbol{\alpha_s}}\, \overline{q}(x) \lambda \gamma_\mu q(x) G(x) \quad \Rightarrow \tag{2}$$



say hello now

The Running of the Strong Coupling

$$\alpha_s(m_\tau^2) \approx 0.33$$
$$\alpha_s(m_Z^2) \approx 1.12 \quad (3)$$

$$m_\tau = 1776.86(12)\,\mathrm{MeV}^1$$
$$m_Z = 91.1876(21)\,\mathrm{GeV}^1 \quad (4)$$

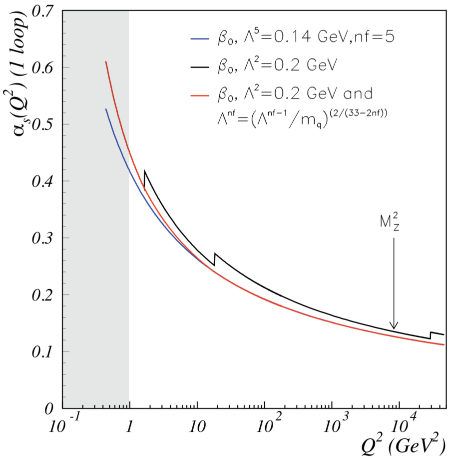


Figure: Taken from Deur, Brodsky, and Teramond, "The QCD Running Coupling", 2016

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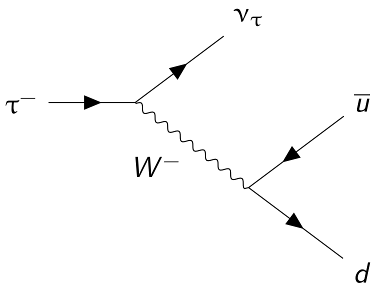
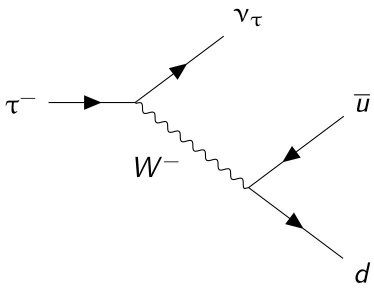
[1] Tanabashi et al., "Review of Particle Physics", 2018

Hadronic τ decays

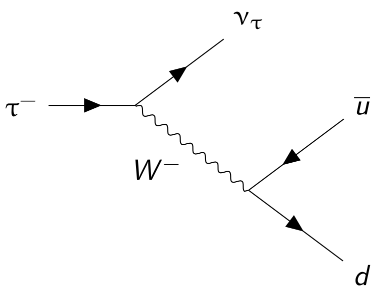


Hadronic τ decays



$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \tag{5}$$

Hadronic τ decays



$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \quad (5)$$

Name	Symbol	Quark content	Rest mass
Pion	π^-	$\overline{u}d$	$139.570\,61(24)$ MeV
Pion	π^0	$(u\overline{u} - d\overline{d})/\sqrt{2}$	$134.9770(5)$ MeV

Duality

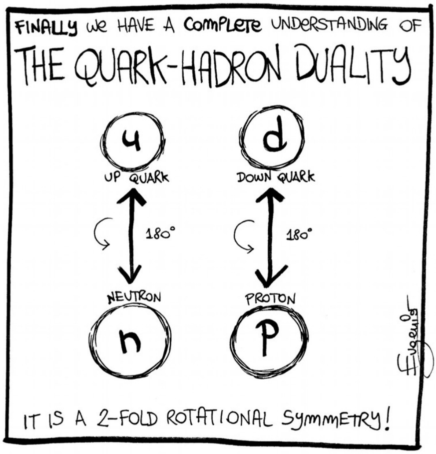

FINALLY WE HAVE A COMPLETE UNDERSTANDING OF
THE QUARK-HADRON DUALITY
u
UP QUARK
d
DOWN QUARK
180°
180°
NEUTRON
n
PROTON
P
IT IS A 2-FOLD ROTATIONAL SYMMETRY!

Table of Contents

Outline

Two-Point Function

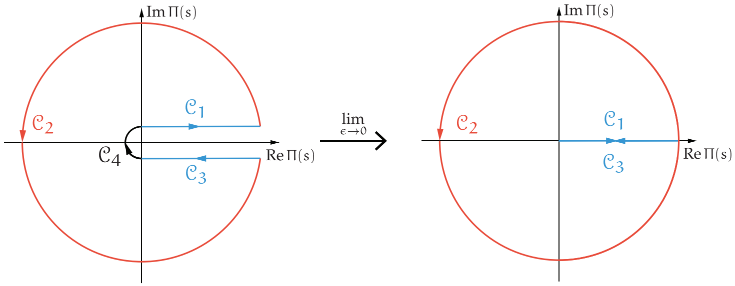
Two-Point Function:

$$\begin{aligned}\Pi^{\mu\nu}_{V/A}(q^2) &\equiv i\int d^4x e^{iqx}\langle 0|T\left\{J^{\mu}_{V/A}(x)J^{\nu}_{V/A}(0)\right\}|0\rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{V/A}(q^2) \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1+0)}_{V/A}(q^2) + q^2 g_{\mu\nu}\Pi^{(0)}(q^2)\end{aligned} \tag{6}$$

$$J^\mu_V = \overline{u}\gamma^\mu d \qquad \text{and} \qquad J^\mu_A = \overline{u}\gamma^\mu\gamma_5 d$$

- Two-point function
- $J^\mu_{V/A}$ is the non-strange V or A current
- superscripts (0) and (1) label spin
- Lorentz decomposed
- $\Pi^{(1+0)}(q^2)q^2\Pi^{(0)}(q^2)$ free of kinematic singularities

Cauchy's Theorem



$$\int_0^{m_\tau} \mathrm{Im}\,\Pi(s)\,\mathrm{d}s = -\frac{1}{2\pi i} \oint_{|s|=m_\tau} \Pi(s)\,\mathrm{d}s$$

- Experimental data only accessible on positive real axis
- Imaginary part of two-point function related to experimental accessible spectral function
- integrate from 0 to m_τ^2 to reproduce $R_\tau(m_\tau^2)$
- Theoretically the positive real axis is not accessible
- Two-point function has poles on positive real axis
- $\Rightarrow$ Cauchy's theorem
- Identify

Finite Energy Sum Rule (FESR)

Spectral Function

$$\rho^{(1+0)}(s) = \frac{1}{\pi} \operatorname{Im} \Pi^{(1+0)}(s) \tag{7}$$

Spectral Moment

$$I^{\omega}_{V/A}(s_0) \equiv \frac{1}{s_0} \int_0^{s_0} \mathrm{d}s\, \omega\left(\frac{s}{s_0}\right) \rho(s) = \frac{-1}{2\pi i s_0} \oint_{|s|=s_0} \mathrm{d}s\, \omega\left(\frac{s}{s_0}\right) \Pi(s) \tag{8}$$

- Spectral function equal to the imaginary part of the correlator
- Spectral function is given by the experiment
- Experiment only valid to certain energy s_0
- $\Rightarrow$ *Finite Energy* Sum Rule
- Define spectral moment
- Introduce weights
- Weights ω have to be analytic

Adler Function:

$$D(s) \equiv s \frac{\mathrm{d}}{\mathrm{d}s} \Pi(s) \tag{9}$$

$$\begin{aligned} R^{\omega}_{\tau,V/A}(s_0) &\equiv \frac{12\pi^2}{s_0} \int_0^{s_0} \mathrm{d}s\, \omega\left(\frac{s}{s_0}\right) \rho(s) \\ &= \frac{6\pi i}{s_0} \oint_{|s|=s_0} \mathrm{d}s\, \omega\left(\frac{s}{s_0}\right) \Pi(s) \\ &= -3\pi i \oint_{|x|=1} \frac{\mathrm{d}x}{x} \omega_D(x) D(xs_0), \end{aligned} \tag{10}$$

where $x \equiv \frac{s}{s_0}$ and $\omega_D \equiv 2 \int_x^1 \mathrm{d}\omega x$

- Adler Function for convenience
- In case of vector correlator the derivative (Adler Function) is a physical quantity
- Physical quantities are renormalisation scale invariant
- For the $(1+0)$ and (0) we use a different definition

$$\Pi_{OPE}(q^2) = -\frac{1}{3q^2}\sum_n \langle\Omega|\mathcal{O}_n(0)|\Omega\rangle \int d^4\ e^{iqx} C_n(x) \tag{11}$$

$$\begin{aligned}\Pi_{OPE,V/A}(s) &= \sum_{D=0,2,4,\ldots} \frac{C^{(D)}\langle\Omega|\mathcal{O}^{(D)}(x)|\Omega\rangle}{(s)^{D/2}} \\ &= \underbrace{C_0}_{PT} + \underbrace{\sum_{k=1}^{\infty}\frac{C_{2k}(s)}{s^k}}_{NPT}\end{aligned} \tag{12}$$

$$\Pi_V^{(1+0)}(s) = -\frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=0}^{n+1} c_{n,k} L^k \quad \text{with} \quad L \equiv \log \frac{-s}{\mu^2} \tag{13}$$

$$D_V^{(1+0)} = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k} L^{k-1} \tag{14}$$

Adler Function Coefficients

Renormalisation Group Equation

$$\mu\frac{d}{d\mu}R(q,a_s,m)=\left[\mu\frac{\partial}{\partial\mu}+\mu\frac{da_s}{d\mu}\frac{\partial}{\partial a_s}+\mu\frac{dm}{d\mu}\frac{\partial}{\partial m}\right]R(q,a_s,m)=0 \qquad (15)$$

$$\left(2\frac{\partial}{\partial L}+\beta\frac{\partial}{\partial a_s}\right)D_V^{(1+0)}=0 \qquad (16)$$

$$\begin{aligned}
c_{0,0}&=-\frac{5}{3},\quad c_{0,1}=1,\\
c_{1,1}&=1\\
c_{2,1}&=\frac{365}{24}-11\zeta_3-(\frac{11}{12}-\frac{2}{3}\zeta_3)N_f\\
&\dots
\end{aligned} \qquad (17)$$

$$\begin{aligned}
c_{2,2}&=-\frac{1}{4}\beta_1c_{1,1},\\
c_{3,2}&=\frac{1}{4}(-\beta_2c_{1,1}-2\beta_1c_{2,1}),\\
c_{3,3}&=\frac{1}{12}\beta_1c_{1,1}\\
&\dots
\end{aligned} \qquad (18)$$

- Beta function definition missing

$$\delta_{pt} = \sum_{n=1}^{\infty} a_\mu^n \sum_{k=1}^{n} k\, c_{n,k} \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1-x)^3 (1+x) \log\left(\frac{-m_\tau^2 x}{\mu^2}\right)^{k-1} \tag{19}$$

Fixed-Order Perturbation Theory (FOPT)

Contour-Improved Perturbation Theory (CIPT)

$$\mu \equiv m_\tau^2 \qquad\qquad \mu \equiv -m_\tau^2 x \tag{20}$$

Fixed-Order Perturbation Theory

$$\delta^{(0)}_{FOPT} = \sum_{n=1}^{\infty} a(m_\tau^2)^n \sum_{k=1}^{n} k\, c_{n,k} J_{k-1} \tag{21}$$

$$J_l \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{\mathrm{d}x}{x} (1-x)^3 (1+x) \log^l(-x) \tag{22}$$

CIPT

$$\delta^{(0)}_{CIPT} = \sum_{n=1}^{\infty} c_{n,1}\, J^a_n(m_\tau^2) \tag{23}$$

$$J^a_n(m_\tau^2) \equiv \frac{1}{2\pi i}\oint_{|x|=1} \frac{\mathrm{d}x}{x}(1-x)^3(1+x)\,a^n(-m_\tau^2 x) \tag{24}$$

FOPT vs CIPT

$$
\begin{aligned}
& \qquad\quad\; \alpha_s^2 \qquad\; \alpha_s^2 \qquad\; \alpha_s^3 \qquad\; \alpha_s^4 \qquad\qquad \alpha_s^5 \\
\delta^{(0)}_{FOPT} &= 0.1082 + 0.0609 + 0.0334 + 0.0174 (+0.0088) = 0.2200 (0.2288) \qquad (25) \\
\delta^{(0)}_{CIPT} &= 0.1479 + 0.0297 + 0.0122 + 0.0086 (+0.0038) = 0.1984 (0.2021) \qquad (26)
\end{aligned}
$$

Borel Summation

Borel integral

$$A \equiv \int_0^\infty \mathrm{d}t e^{-t} \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n, \tag{27}$$

Borel transform

$$B[A](t) = \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n. \tag{28}$$

$$\frac{12\pi^2}{N_c} D_V^{1+0}(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(\sqrt{(s)})^{n+1}. \tag{29}$$

Borel Model

$$B[\widehat{D}](u) = B[\widehat{D}_1^{UV}](u) + B[\widehat{D}_2^{IR}](u) + B[\widehat{D}_3^{IR}](u) + d_0^{PO} + d_1^{PO} u, \quad (30)$$

$$B[\widehat{D}_p^{IR}](u) \equiv \frac{d_p^{IR}}{(p-u)^{1+\widetilde{\gamma}}} \left[1 + \widetilde{b}_1 (p-u) + \widetilde{b}_2 (p-u)^2 + \dots\right] \quad (31)$$

$$B[\widehat{D}_p^{UV}](u) \equiv \frac{d_p^{UV}}{(p+u)^{1+\overline{\gamma}}} \left[1 + \overline{b}_1 (p+u) + \overline{b}_2 (p+u)^2\right], \quad (32)$$

Beneke and Jamin, "α_s and the τ hadronic width: fixed-order, contour-improved and higher-order perturbation theory", 2008

NPT Contributions

OPE

$$\lim_{x\to y} A(x)B(y) = \sum_n C_n(x-y)\mathcal{O}_n(x) \tag{33}$$

$$\Pi_{OPE}(q^2) = -\frac{1}{3q^2}\sum_n \langle\Omega|\mathcal{O}_n(0)|\Omega\rangle \int d^4\, x e^{iqx} C_n(x) \tag{34}$$

$$\Pi^{OPE}_{V/A}(s) = \sum_{D=0,2,4,\ldots} \frac{C^{(D)}\langle\Omega|\mathcal{O}^{(D)}(x)|\Omega\rangle}{(-q^2)^{D/2}} \tag{35}$$

Dimension Four Corrections

$$D_{ij}^{(1+0)}(s)\Big|_{D=4} = \frac{1}{s^2}\sum_n \Omega^{(1+0)}(s/\mu^2)a^n, \tag{36}$$

where the $\Omega^{(1+0)}(s/\mu^2)$ is given by

$$\begin{aligned}\Omega_n^{(1+0)}(s/\mu^2) \; = \; & \frac{1}{6}\langle aGG\rangle p_n^{(1+0)}(s/\mu^2) + \sum_k m_k\langle\overline{q}_k q_k\rangle r_n^{(1+0)}(s/\mu^2) \\ & + 2\langle m_i\overline{q}_i q_i + m_j\overline{q}_j q_j\rangle q_n^{(1+0)}(s/\mu^2) \pm \frac{8}{3}\langle m_j\overline{q}_i q_i + m_i\overline{q}_j q_j\rangle t_n^{(1 \\ & - \frac{3}{\pi^2}(m_i^4 + m_j^4)h_n^{(1+0)}(s/\mu^2) \mp \frac{5}{\pi^2}m_i m_j(m_i^2 + m_j^2)k_n^{(1+0)}(s \\ & + \frac{3}{\pi^2}m_i^2 m_j^2 g_n^{(1+0)}(s/\mu^2) + \sum_k m_k^4 j_n^{(1+0)}(s/\mu^2) + 2\sum_{k\neq l} m_k^2 m\end{aligned} \tag{37}$$

Dimension Six and Eight Corrections

$$
\begin{aligned}
D^{(1+0)}_{ij,V/A}\Big|_{D=8} &= 4\frac{\rho^{(8)}_{V/A}}{s^4} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=10} &= 5\frac{\rho^{(10)}_{V/A}}{s^5} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=12} &= 6\frac{\rho^{(12)}_{V/A}}{s^6}
\end{aligned}
\tag{38}
$$

Duality Violations

$$R^{\omega}_{\tau,V/A} = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left(1 + \delta^{\omega}_{pt} + \delta^{\omega}_{npt} + \delta^{\omega}_{DV}\right) \tag{39}$$

$$\rho^{DV}_{V/A}(s) = e^{-(\delta_{V/A} + \gamma_{V/A} s)} \sin(\alpha_{V/A} + \beta_{V/A} s) \tag{40}$$

$$D_{\omega}(m_\tau^2) = -12\pi^2 \int_{m_\tau^2}^{\infty} \frac{\mathrm{d}s}{m_\tau^2} \omega(s) \rho^{DV}_{V/A} \tag{41}$$

Weights

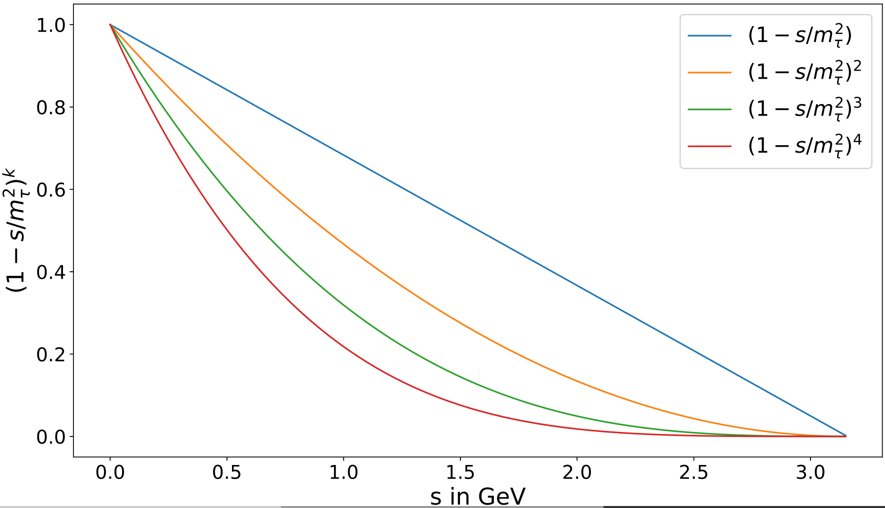
$$\omega(x) \equiv \sum_i a_i x^i \tag{42}$$

kinematic weights

$$\omega_\tau \equiv (1 - \frac{s}{m_\tau^2})^2 (1 + 2\frac{s}{m_\tau^2}) \tag{43}$$

Pinched Weights

$$\omega(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \tag{44}$$



Weighting OPE Contributions

$$\oint_C x^k \, \mathrm{d}x = i \int_0^{2\pi} \left(e^{i\theta}\right)^{k+1} \mathrm{d}\theta = \begin{cases} 2\pi i & \text{if } k = -1, \\ 0 & \text{otherwise} \end{cases}. \tag{45}$$

$$R(x)\Big|_{D=0,2,4,\dots} = \oint_{|x|=1} \mathrm{d}x \, x^{k-D/2} C^{(D)} \tag{46}$$

active dimension

$$D = 2(k+1) \tag{47}$$

monomial:	x^0	x^1	x^2	x^3	x^5	x^6	x^7
dimension:	$D^{(2)}$	$D^{(4)}$	$D^{(6)}$	$D^{(8)}$	$D^{(10)}$	$D^{(12)}$	$D^{(14)}$

Table: List of monomial and their corresponding "active" dimensions in the OPE. Note that the perturbative contributions of the OPE are always present.

Outline

Inclusive Tau Decay Ratio

Inclusive Tau Decay Ratio

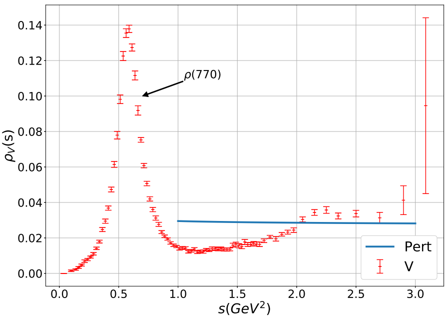
$$R_{V/A,ud}(s_0) = 12\pi |V_{ud}|^2 S_{EW} \frac{1}{s_0} \int_0^{s_0} \mathrm{d}s [\omega^{(1+0)}(s)\rho_{V/A}^{(1+0)}(s) - \omega_L(s)\rho_{V/A}^{(0)}(s)] \tag{48}$$

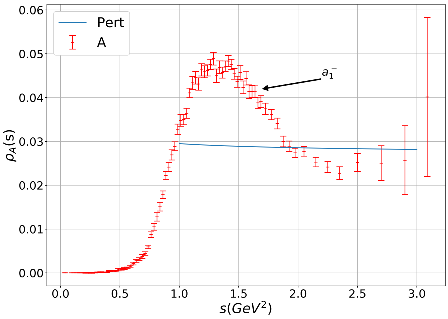
$$R_\tau = -\pi i \oint_{|s|=m_\tau^2} \frac{\mathrm{d}x}{x}(1-x)^3 \left[3(1+x)D^{(1+0)}(m_\tau^2 x) + 4D^{(0)}(m_\tau^2 x)\right] \tag{49}$$

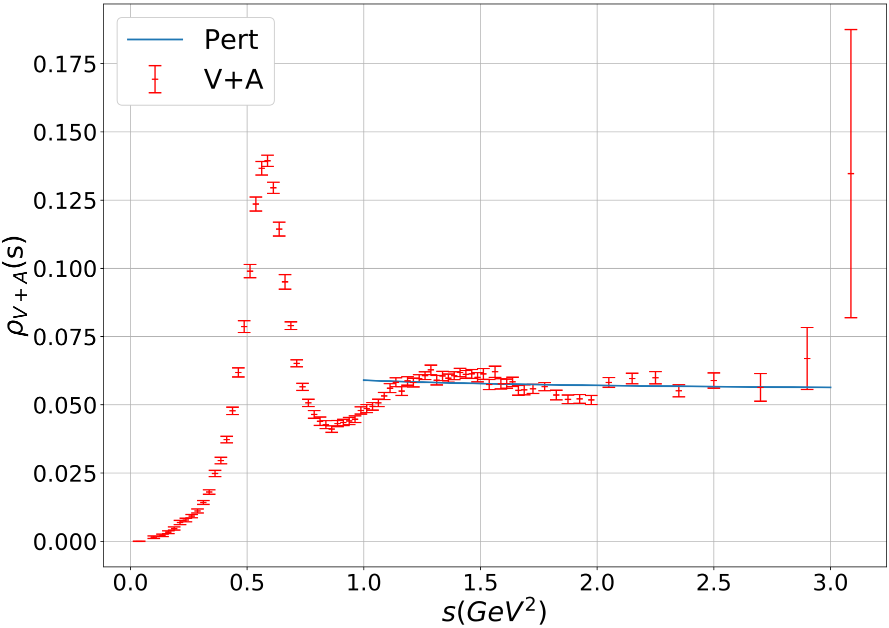
$$(x \equiv \tfrac{s}{m_\tau^2})$$

- Spectral function can be determined via the inclusive tau decay ratio
- We express the spectral function in therms of the Adler function

ALEPH data






Pert
V+A
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
$\rho_{V+A}(s)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$s(GeV^2)$

$$R_{\tau,V/A} = \frac{\mathcal{B}_{V/A}}{\mathcal{B}_e} = \int_0^{m_\tau^2} \mathrm{d}s \frac{\mathrm{sfm2}_{V/A}(s)}{100\mathcal{B}_e} \tag{50}$$

$$I^{\omega}_{exp,V/A}(s_0) = \frac{s_\tau}{100\mathcal{B}_e s_0} \sum_{i=1}^{N(s_0)} \frac{\omega\left(\frac{s_i}{s_0}\right)}{\omega_\tau\left(\mathrm{sfm2}_{V/A}(s_i)\right)} \tag{51}$$

$$\chi^2 = (I_i^{exp} - I_i^{th}(\vec{\alpha}))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\vec{\alpha})) \tag{52}$$

$$C_{ij} = \text{cov}(I_i^{exp}, I_j^{exp}) \tag{53}$$

$$\chi^2 \approx 1 \tag{54}$$

Outline

Parameters and Momenta

# (k, l)	2 Moments	
1 (1, 1)	s_1	ω_1
2 (2, 1)	s_2	ω_1

	Symbol	Term	Expansion	OPE Contributions
Pinched	ω_τ	$(1-x)^2(1+2x)$	$1-3x^2+2x^3$	$D6$, $D8$
	ω_{cube}	$(1-x)^3(1+3x)$	$1-6x^2+8x^3-3x^4$	$D6$, $D8$, $D10$
	$\omega_{quartic}$	$(1-x)^4(1+3x)$	$1-10x^2+20x^3-15x^4+4x^5$	$D6$, $D8$, $D10$, $D12$
Monomial	ω_{M2}	$1-x^2$	$1-x^2$	$D6$
	ω_{M3}	$1-x^3$	$1-x^3$	$D8$
	ω_{M4}	$1-x^4$	$1-x^4$	$D10$
Pinched $+x$	$\omega_{1,0}$	$(1-x)$	$1-x$	$D4$
	$\omega_{2,0}$	$(1-x)^2$	$1-2x+x^2$	$D4$, $D6$
	$\omega_{3,0}$	$(1-x)^3$	$1-3x+3x^2-x^3$	$D4$, $D6$, $D8$
	$\omega_{4,0}$	$(1-x)^4$	$1-4x+6x^2-4x^3+x^4$	$D4$, $D6$, $D8$, $D10$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.200	7	0.3274(42)	-0.82(21)	-1.08(40)	0.21
FOPT	2.100	8	0.3256(38)	-0.43(15)	-0.25(28)	1.30
	2.200	7	0.3308(44)	-0.72(20)	-0.85(38)	0.19
	2.300	6	0.3304(52)	-0.69(25)	-0.80(50)	0.25
	2.400	5	0.3339(70)	-0.91(39)	-1.29(83)	0.10
	2.600	4	0.3398(15)	-1.3(1.0)	-2.3(2.5)	0.01

Cubic Weight: $\omega_{cube}(x) \equiv (1-x)^3(1+3x)$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
2.000	9	0.3228(26)	-0.196(27)	0.075(28)	0.420(56)	1.96
2.100	8	0.3302(40)	-0.52(11)	-0.58(22)	-1.00(45)	0.43
2.200	7	0.3312(43)	-0.56(12)	-0.68(23)	-1.23(50)	0.55
2.300	6	0.336(11)	-0.78(47)	-1.17(98)	-2.38(22)	0.29
2.400	5	0.3330(96)	-0.63(47)	-0.82(10)	-1.51(26)	0.48

Quartic Weight: $\omega_{quartic}(x) \equiv (1-x)^4(1+4x)$

$$\alpha_s(m_\tau^2) = 0.3290(11), \quad \rho^{(6)} = -0.3030(46), \quad \rho^{(8)} = -0.1874(28),$$
$$\rho^{(10)} = 0.3678(45) \quad \text{and} \quad \rho_{(12)} = -0.4071(77). \tag{55}$$

$\omega_{M2}(x) \equiv 1 - x^2$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	χ^2/dof
2.100	8	0.3179(47)	-0.42(17)	1.62
2.200	7	0.3248(52)	-0.77(22)	0.38
2.300	6	0.3260(60)	-0.85(28)	0.43

$\omega_{M3}(x) \equiv 1 - x^3$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(8)}$	χ^2/dof
2.100	8	0.3147(44)	-0.27(29)	1.71
2.200	7	0.3214(49)	-1.01(39)	0.41
2.300	6	0.3227(57)	-1.18(54)	0.46
2.400	5	0.3257(67)	-1.58(74)	0.39
2.600	4	0.325(10)	-1.54(1.53)	0.58
2.800	3	0.326(21)	-1.69(4.03)	1.17

Fourth Power Monomial: $\omega_{M4}(x) \equiv 1 - x^4$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(10)}$	χ^2/dof
2.100	8	0.3136(43)	-0.07(54)	1.75
2.200	7	0.3203(48)	-1.64(77)	0.42
2.300	6	0.3216(56)	-2.01(1.13)	0.47
2.400	5	0.3247(66)	-2.98(1.62)	0.39
2.600	4	0.324(10)	-2.86(3.69)	0.58
2.800	3	0.325(20)	-3.43(10.74)	1.17

$\omega_{1,0} \equiv (1-x)$

	s_{min}	$\#s_0s$	$\alpha_s(m_\tau^2)$	$\langle aGG\rangle_I$	χ^2/dof
BS	2.100	8	0.3176(47)	-0.0134(48)	1.62
	2.200	7	0.3246(52)	-0.2262(59)	0.38
	2.300	6	0.3260(60)	-0.2453(73)	0.43
FOPT	2.100	8	0.357(12)	-0.072(23)	0.95
	2.200	7	0.3593(97)	-0.079(19)	0.2
	2.300	6	0.3589(99)	-0.078(20)	0.24

$\omega_{2,0} \equiv (1-x)^2$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	χ^2/dof
BS	2.100	8	0.3207(48)	-0.0170(50)	-0.45(17)	1.90
	2.200	7	0.3270(54)	-0.0254(61)	-0.77(21)	0.74
	2.300	6	0.3253(63)	-0.0232(75)	-0.69(27)	0.9
FOPT	2.100	8	0.3331(54)	-0.0108(45)	0.361(76)	1.9
	2.200	7	0.3401(57)	-0.0185(52)	0.220(88)	0.73
	2.300	6	0.3383(68)	-0.0165(67)	0.26(12)	0.89

$\omega_{3,0} \equiv (1-x)^3$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.000	9	0.3169(20)	-0.0123(34)	-0.29(12)	-0.05(24)	2.0
	2.100	8	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	0.46
	2.200	7	0.3251(17)	-0.02283(56)	-0.689(12)	-0.879(33)	0.56
FOPT	2.000	9	0.33985(81)	-0.01124(43)	0.002(10)	-0.242(26)	1.59
	2.100	8	0.3480(47)	-0.0201(36)	-0.264(89)	-1.03(28)	0.31
	2.200	7	0.3483(23)	-0.0204(41)	-0.27(15)	-1.05(40)	0.41

$\omega_{4,0} \equiv (1-x)^4$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$aGGInv$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
BS	1.950	10	0.31711(67)	-0.012432(24)	-0.30013(73)	-0.06785(16)	0.26104(50)	1.09
	2.000	9	0.3206(24)	-0.0167(14)	-0.455(38)	-0.373(67)	-0.36(14)	0.83
	2.100	8	0.3248(21)	-0.02230(47)	-0.6724(63)	-0.834(14)	-1.352(28)	0.23
FOPT	1.950	10	0.3416(14)	-0.01306(83)	-0.050(22)	-0.390(59)	-0.50(19)	1.71
	2.100	8	0.3480(25)	-0.0201(27)	-0.264(91)	-1.02(23)	-339.00(20)	0.41

Comparison

	weight	s_{min}	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
FOPT	ω_τ	2.2	0.3308(44)	-	-0.72(20)	-0.85(38)	-	0.19
	ω_{cube}	2.1	0.3302(40)	-	-0.52(11)	-0.58(22)	-1.00(45)	0.43
	ω_{M2}	2.2	0.3248(52)	-	-0.77(22)	-	-	0.38
	ω_{M3}	2.2	0.3214(49)	-	-	-1.01(39)	-	0.41
BS	$\omega_{1,0}$	2.2	0.3246(52)	-0.2262(59)	-	-	-	0.38
	$\omega_{2,0}$	2.2	0.3270(54)	-0.0254(61)	-0.77(21)	-	-	0.74
	$\omega_{3,0}$	2.1	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	-	0.46

Outline

Conclusions

- $\alpha_s(m_\tau^2) = 0.3261 \pm 0.0050$
- $\rho^{(6)} = -0.68 \pm 0.2$
- $\rho^{(8)} = -0.80 \pm 0.38$

Conclusions

- $\alpha_s(m_\tau^2) = 0.3261 \pm 0.0050$
- $\rho^{(6)} = -0.68 \pm 0.2$
- $\rho^{(8)} = -0.80 \pm 0.38$

- DV not present if using single pinched weights in the V+A channel
- FOPT more valid than CIPT
- $\alpha_s(m_Z^2) = 0.11940(60)$

Questions

Appendix

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