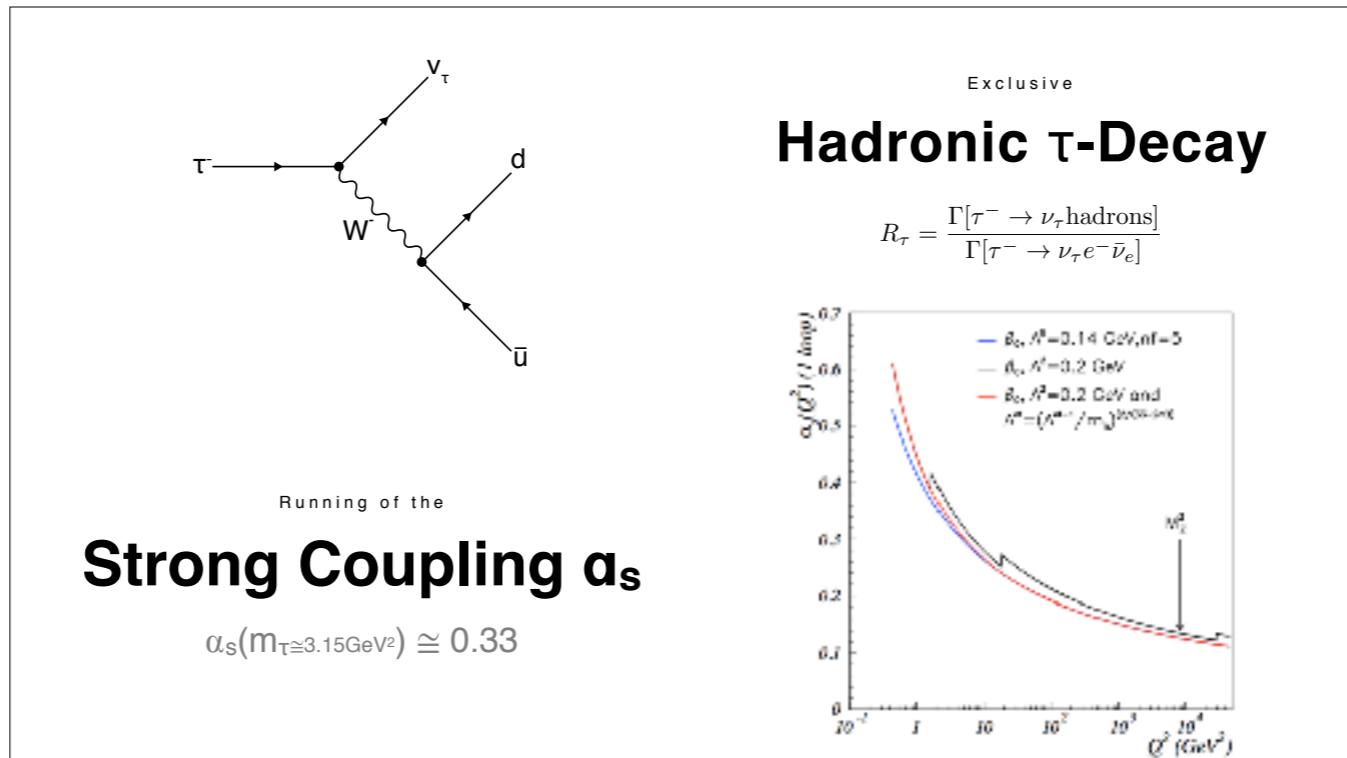


ALEPH

Determination of the QCD Coupling from ALEPH τ Decay Data

Dirk Hornung

τ Decay



- The hadronic decay width of the tau-lepton provides one of the most precise determinations of the strong coupling
- The inclusive hadronic decay ratio can be rigorously calculated within QCD and is furthermore sensitive to input values of α_s
- Tau lepton decays into a tau neutrino and a W boson, which decays into lepton-neutrino pairs or quark anti-quark pairs. We are only concerned about the exclusive hadronic (u, d) decay channel
- The strong coupling depends on the invariant mass squared scale. The coupling is strong for low energies and decreases in strength for higher energies. We are interested in the energy regime of up to m_τ (3.15 GeV 2). At m_τ the strong coupling is sizable with a numerical value around 0.33, which makes R_τ sensitive to the strong coupling.
- α_s can be determined with a 4% accuracy, but evolving it up to the M_Z scale, it implies a 1% precision on α_s .

Contents

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- ALEPH Data
- Fitting Methodology
- Duality Violation
- Determination of α_s
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Theory

Framework

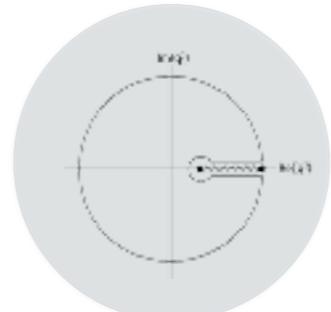
$$\text{Inclusive Ratio} \quad R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im } \Pi^{(1)}(s) + \text{Im } \Pi^{(0)}(s) \right]$$

$$\Pi^{(J)}(s) \equiv |V_{uq}|^2 \left(\Pi_{ud,V}^{(J)} + \Pi_{ud,A}^{(J)}(s) \right)$$

Two-Point Correlation Function

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T \left\{ \mathcal{J}_{ij}^\mu(x) \mathcal{J}_{ij}^{\nu\dagger}(0) \right\} \rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2) \end{aligned}$$

$$(i, j = u, d; \mathcal{J} = V, A) \quad V_{ij}^\mu = \bar{q}_j \gamma^\mu q_i \quad A_{ij}^\mu = \bar{q}_j \gamma^\mu \gamma_5 q_i$$

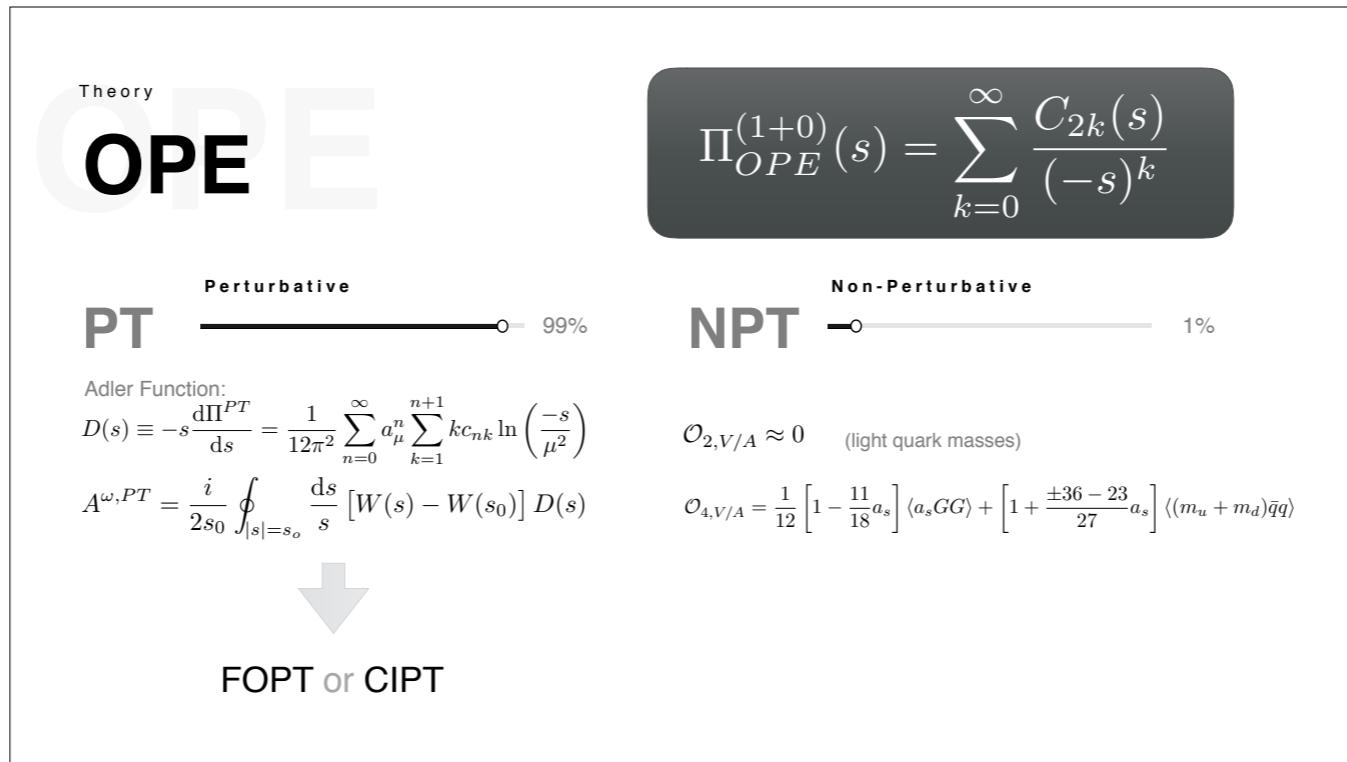


$$\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im } \Pi_{V/A}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{V/A}(s)$$

Experiment

Theory

- The main ingredients is the inclusive hadronic decay ratio $R\tau$, which depends on the two-point correlation.
- The two-point function is split up in a transversal (spin 0) and a longitudinal (spin 1) part, as well a a vectorial and a axial-vectorial part.
- Experimentally we have access to the spectral functions
- Theoretically we will employ the OPE, which is not valid on the positive real axis, the region where we have experimental data.
- Making use of Cauchy's theorem we can connect the experimental side with the theoretical side.
- $w(s)$ is an analytic function, called the weight function



- We use the OPE expansion for the theoretical two-point function.
- The OPE consist of a perturbative part ($k=0$) and a non-perturbative part ($k>=1$). The NPT-corrections contribute to only 1%
- The perturbative contribution can be represented in form of the Adler function. The Adler function then contains the strong coupling, which will be used for our fit, the c_{nk} coefficients and a logarithmic dependence on s . $D(s)$ is in theory independent of μ , but due to higher order corrections, not completely independent so that we can use μ to estimate the theoretical errors.
- The NPT terms are also referred to as condensates. We will omit the terms of dimension 2, which are proportional to the quark masses and as we are working only with u and d negligible.
- For dimension 4 we are mainly interested in the Gluon Condensate (2nd term is small, due to quark masses)
- Using partial integration one can rewrite our connection between exp and th to A^{ω}
- At the end we are left with two ways of performing the integral: FOPT and CIPT. In FOPT we fix α_s in the integral to s_0 . In CIPT we fix μ^2 to $-s$, so that the log vanished, and run and integrate α_s in the complex plane.

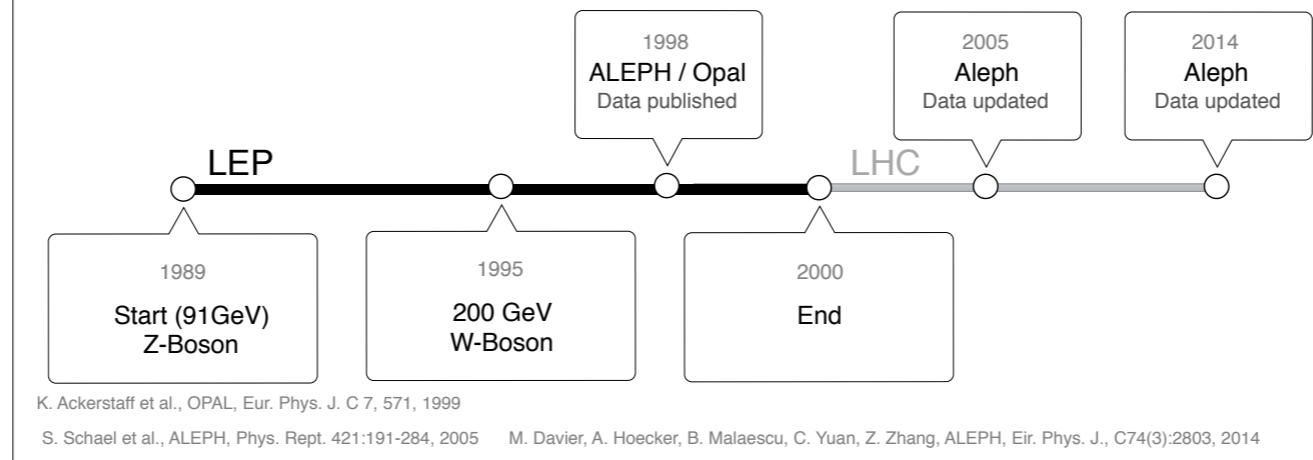
Theory

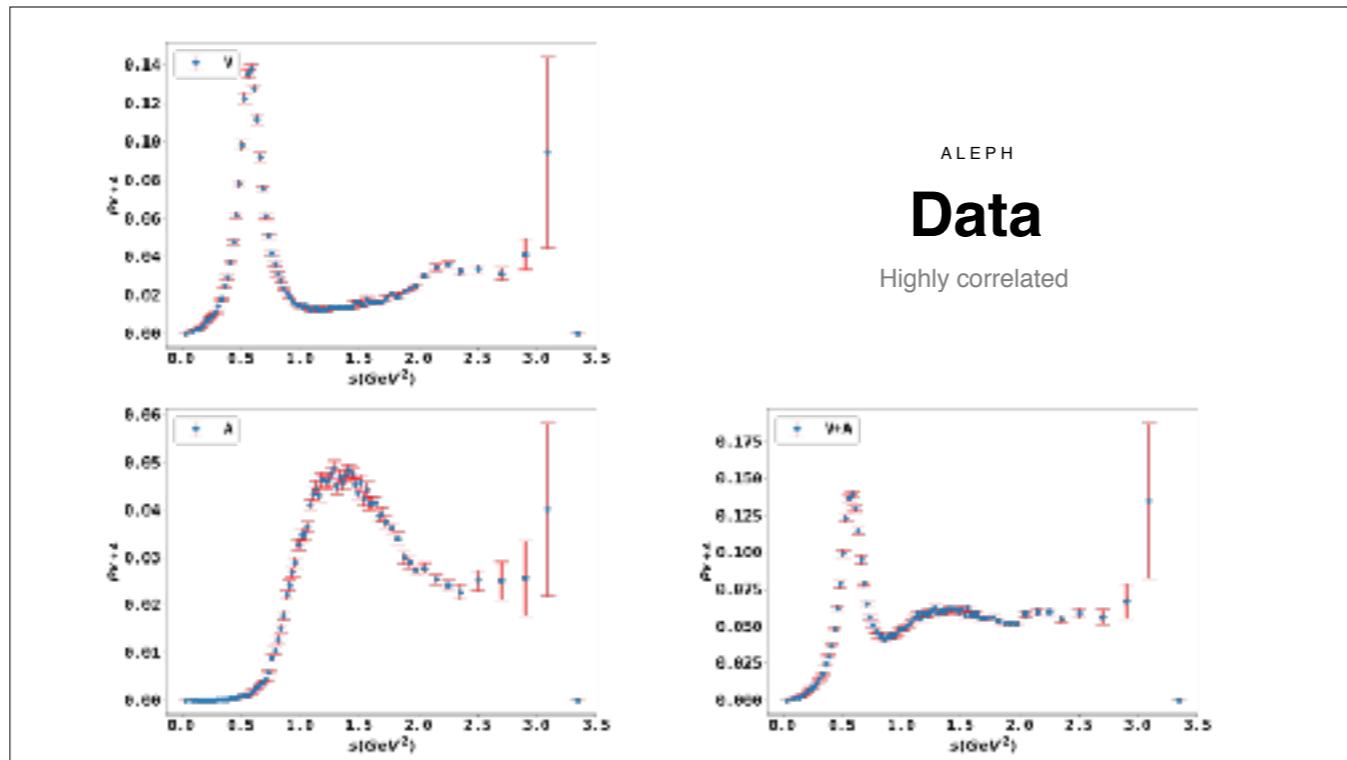
OPE D=6

$$\Pi_{OPE}^{(1+0)}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Duality Violations in Tau Decays

Timeline





- We work with the updated Aleph data (2014).
- The data is given for the different channels: V, A and their combinations. We plotted the spectral function for each channel with the corresponding errors.
- The Aleph group also prepared the correlation matrix between the different data points, which are highly correlated. The correlation matrix is needed for the Chisquared fit.
- In the vectorial diagram one can see the rho resonance and the large errors on the last 2 bins.
- In the axial-vector diagram one can see the a1 resonance and the large error on the last 4 bins.
- In the V+A graph one can see the quick convergence, right after the rho resonance, which is in agreement with the naive Parton model.

Data

ALEPH

$$\rho(s) \equiv \frac{1}{\pi} \operatorname{Im} \Pi(s)$$

Spectral Function

$$\operatorname{Im} \Pi_{\bar{u}d,V}^{(1)}(s) = \frac{1}{2\pi} v_1(s)$$

$$\operatorname{Im} \Pi_{\bar{u}d,A}^{(1)}(s) = \frac{1}{2\pi} a_1(s)$$

$$\operatorname{Im} \Pi_{\bar{u}d,A}^{(0)}(s) = \frac{1}{2\pi} a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow V^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_V}{N_V ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow A^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow \pi^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left(1 - \frac{s}{m_\tau^2}\right)^2$$

The ALEPH Collaboration, Phys. Rept., 421:191-284, 2005

Methodology

Fitting

Chi squared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k, \omega_l) = \int_{s_{th}}^{s_k} \frac{ds}{s_k} \omega_l(s) \operatorname{Im} \Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k, \omega_l) = \frac{i}{2s_k} \oint_{|s|=s_k} \frac{ds}{s} [W_l(s) - W_l(s_k)] D(s)$$

Parameters

$$a_s \quad \langle a_s GG \rangle \quad \mathcal{O}_{6,V+A} \quad \mathcal{O}_{8,V+A}$$

Different s_0

# (k, l)	2 Moments	
1 (1,1)	s_1	w_1
2 (2,1)	s_2	w_1

Different weights

# (k, l)	3 Moments	
1 (1,1)	s_1	w_1
2 (1,2)	s_1	w_2
2 (1,3)	s_1	w_3

E.g.

Extract max. 2 parameters

max. 3 parameters

Fitting

Weights

$$\frac{1}{2\pi i s_0} \oint_{|s|=s_0} ds \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1} \frac{C_{2(n+1)}}{s_0^{n+1}} \delta_{k,n+1}$$

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right)$$

$(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$ Double Pinched

n-th degreee monomial in the weight selects the $D = 2(n+1)$

$$\frac{s}{s_0} + \left(\frac{s}{s_0}\right)^2 + \left(\frac{s}{s_0}\right)^3 + \dots$$

$\downarrow \qquad \downarrow \qquad \downarrow$

$$\mathcal{O}_{4,V/A} \quad \mathcal{O}_{6,V/A} \quad \mathcal{O}_{8,V/A}$$

$$A_{00,V/A}^{ALEPH} = A_{00,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A})$$

$$A_{10,V/A}^{ALEPH} = A_{10,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$

$$A_{11,V/A}^{ALEPH} = A_{11,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A} \mathcal{O}_{12,V/A})$$

$$A_{12,V/A}^{ALEPH} = A_{12,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$

$$A_{13,V/A}^{ALEPH} = A_{13,V/A}^{ALEPH}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

Systematic Errors

- Higher Orders of perturbative term
- Higher orders of non-perturbative term
- FOPT vs CIPT
- Duality Violations

Violations

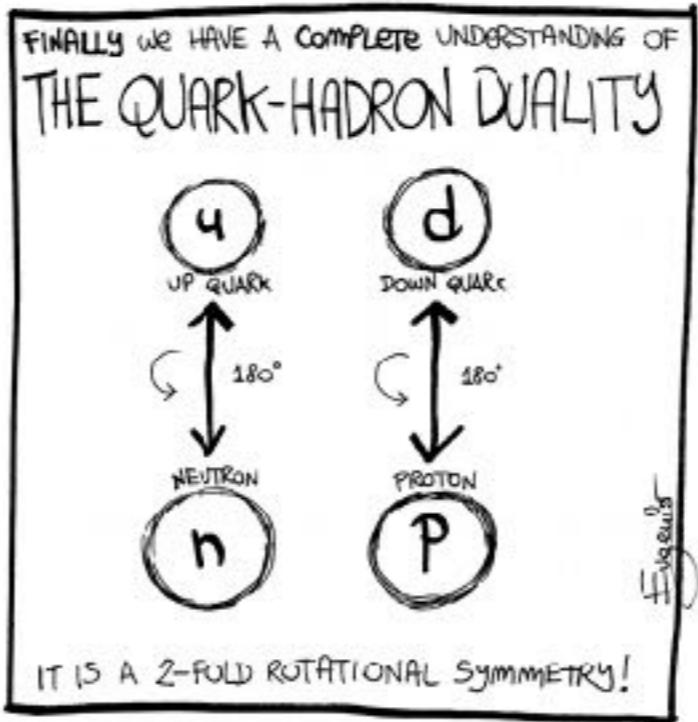
Duality

Experiment
Hadrons Theory
Quark/ Gluons

$$\rho_{\text{exp}} = \rho_{\text{th}}$$

- What energy is high enough for the quark-hadron duality to set in?
- What role play nonperturbative effects?

M. Shifman, hep-th/0009131, 2000



Duality

Model

Model: 8 Parameters

$$\Delta\rho_{V/A}^{DV}(s) = e^{-\delta_{V/A} + \gamma_{V/A}s} \sin(\alpha_{V/A} + \beta_{V/A}s)$$

$$\Delta A_{V/A}^{\omega, DV}(s_0) \equiv \frac{i}{2} \oint_{|s_0|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{OPE}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta\rho_{V/A}^{DV}(s)$$

M. Shiftman, hep-th/0009131, 2000

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman



- A. Pich
- A. Rodríguez-Sánchez



Philosophy

$\alpha_s = 0.32$ (CIPT)

Boito, Goltermann, Peris, Maltman

Pro

- Theoretically well motivated
- Systematic errors have been underestimated
- Assuming Duality a priori leads to worse model

$\alpha_s = 0.35$ (CIPT)

Pich, Rodríguez-Sánchez

Contra

- Cannot be derived from first principles
- Far too many parameters (8)
- Poor statistical quality (low p-value, large uncertainties)
- Sufficiently suppressed (pinched weights)

D. Boito, O. Catà, M. Golterman, M. Jamin, K. Maltman, J. Osborne, S. Peris, Phys. Rev., D84:113006, 2011

Determination of a_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16

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