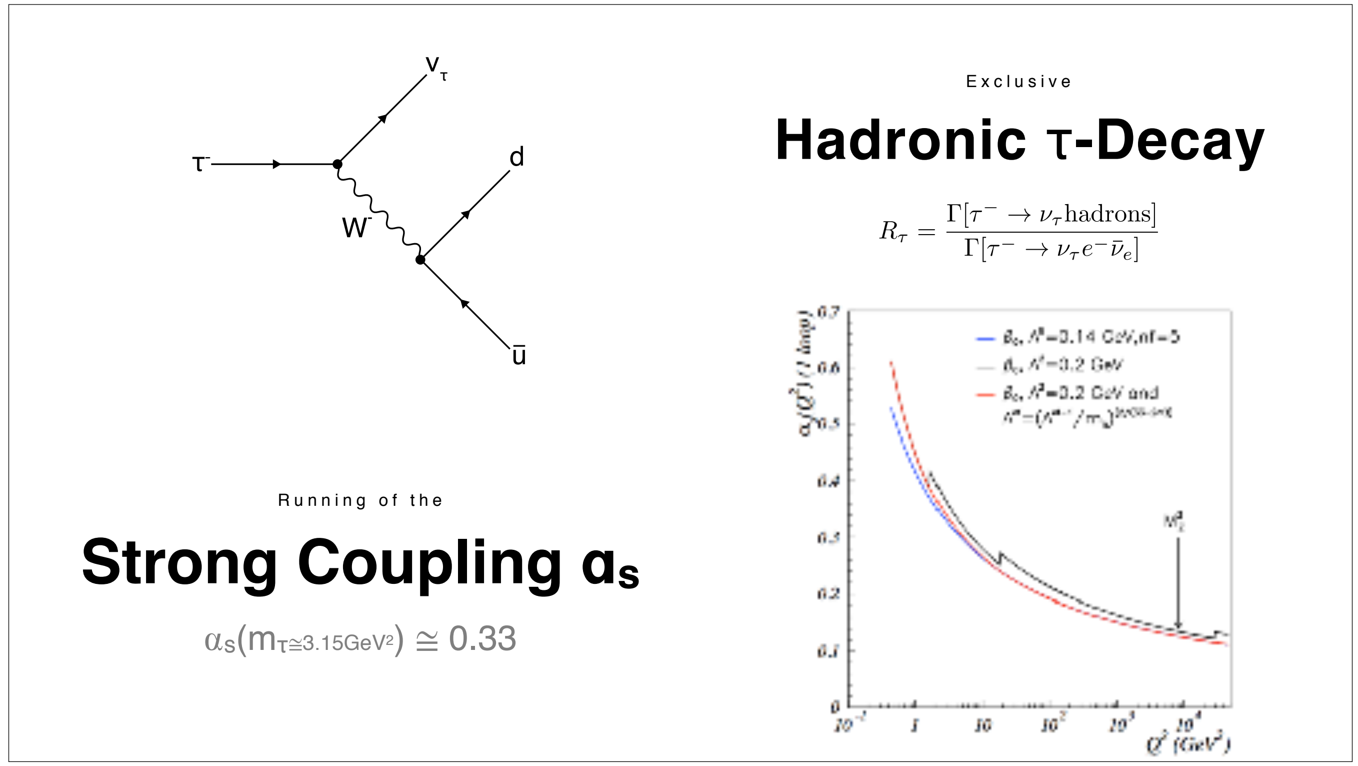
ALEPH
τ Decay

Determination of the QCD Coupling from ALEPH τ Decay Data

Dirk Hornung



- The hadronic decay with of the tau-lepton provides one of the most precise determinations of the strong coupling
- The inclusive hadronic decay ratio can be rigorously calculated within QCD and is furthermore sensitive to input values of alpha_s
- Tau lepton decays into a tau neutrino and a W boson, which decays into lepton-neutrino pairs or quark anti-quark pairs. We are only concerned about the exclusive hadronic (u, d) decay channel
- The strong coupling depends on the invariant mass squared scale. The coupling is strong for low energies and decreases in strength for higher energies. We are interested in the energy regime of up to m_tau (3.15 GeV^2). At m_tau the strong coupling is sizable with a numerical value around 0.33, which makes Rτ sensitive to the strong coupling.
- αs can be determined with a 4% accuracy, but evolving it up to the Mz scale, it implies a 1% precision on αs.

Contents

Theory

Framework

Inclusive Ratio

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{\mathrm{d}s}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

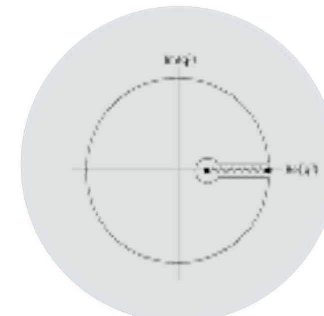
$$\Pi^{(J)}(s) \equiv \left|V_{uq}\right|^2 \left(\Pi^{(J)}_{ud,V} + \Pi^{(J)}_{ud,A}(s)\right)$$

Two-Point Correlation Function

$$\Pi_{\mu\nu}(q) = i\int \mathrm{d}^4\, x e^{iqx} \langle 0|T\left\{\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right\}\rangle$$

$$= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1)}_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij,\mathcal{J}}(q^2)$$

$$(i,j=u,d;\mathcal{J}=V,A) \quad V^\mu_{ij} = \bar{q}_j\gamma^\mu q_i \quad A^\mu_{ij} = \bar{q}_j\gamma^\mu\gamma_5 q_i$$



$$\int_{s_{th}}^{s_0} \frac{\mathrm{d}s}{s_0}\omega(s)\,\mathrm{Im}\,\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0} \frac{\mathrm{d}s}{s_0}\omega(s)\Pi_{V/A}(s)$$

Experiment Theory

- The main ingredients is the inclusive hadronic decay ratio Rτ, which depends on the two-point correlation.
- The two-point function is split up in a transversal (spin 0) and a longitudinal (spin 1) part, as well a a vectorial and a axial-vectorial part.
- Experimentally we have access to the spectral functions
- Theoretically we will employ the OPE, which is not valid on the positive real axis, the region where we have experimental data.
- Making use of Cauchy’s theorem we can connect the experimental side with the theoretical side.
- w(s) is an analytic function, called the weight function

Theory

OPE

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Perturbative

PT

99%

Adler Function:

$$D(s) \equiv -s\frac{\mathrm{d}\Pi^{PT}}{\mathrm{d}s} = \frac{1}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{nk} \ln\left(\frac{-s}{\mu^2}\right)$$

$$A^{\omega,PT} = \frac{i}{2s_0}\oint_{|s|=s_o} \frac{\mathrm{d}s}{s}\left[W(s) - W(s_0)\right] D(s)$$

FOPT or CIPT

Non-Perturbative

NPT

1%

$\mathcal{O}_{2,V/A} \approx 0$ (light quark masses)

$$\mathcal{O}_{4,V/A} = \frac{1}{12}\left[1 - \frac{11}{18}a_s\right]\langle a_s GG\rangle + \left[1 + \frac{\pm 36 - 23}{27}a_s\right]\langle (m_u + m_d)\bar{q}q\rangle$$

- We use the OPE expansion for the theoretical two-point function.
- The OPE consist of a perturbative part (k=0) and a non-perturbative part (k>=1). The NPT-corrections contribute to only 1%
- The perturbative contribution can be represented in form of the Adler function. The Adler function then contains the strong coupling, which will will use for our fit, the c_{nk} coefficients and a logarithmic dependence on s. D(s) is in theory independent of μ, but due to higher order corrections, not completely independeent so that we can use μ to estimate the theoretical errors.
- The NPT terms are also referred to as condensates. We will omit the terms of dimension 2, which are proportional to the quark masses and as we are working only with u and d negligible.
- For dimension 4 we are mainly interested in the Gluon Condensate (2nd term is small, due to quark masses)
- Using partial integration one can rewrite our connection between exp and th to A^{omega…}
- At the end we are left with two ways of performing the integral: FOPT and CIPT. In FOPT we fix alpha_s in the integral to s0. In CIPT we fix μ^2 to -s, so that the log vanished, and run and integrate alpha s in the complex plane.

Theory

OPE D=6

Wilson coefficients:

$$C_6^{V-A}(Q^2)\,\langle O_6\rangle = 4\pi^2 a_s \left\{ \left[2 + \left(\frac{25}{6} - L\right) a_s \right] \langle Q_-^o \rangle - \left(\frac{11}{18} - \frac{2}{3}L\right) a_s \langle Q_-^s \rangle \right\}$$

$$\begin{aligned} C_6^{V+A}(Q^2)\,\langle O_6\rangle = \quad & -4\pi^2 a_s \left\{ \left[2 + \left(\frac{155}{24} - \frac{7}{2}L\right) a_s \right] \langle Q_+^o \rangle + \left(\frac{11}{18} - \frac{2}{3}L\right) a_s \langle Q_+^s \rangle + \right. \\ & \left[\frac{4}{9} + \left(\frac{37}{36} - \frac{95}{162}L\right) a_s \right] \langle Q_3 \rangle + \left(\frac{35}{108} - \frac{5}{18}L\right) a_s \langle Q_4 \rangle + \\ & \left(\frac{14}{81} - \frac{4}{27}L\right) a_s \langle Q_6 \rangle - \left(\frac{2}{81} + \frac{4}{27}L\right) a_s \langle Q_7 \rangle \end{aligned}$$

$$a_s \equiv \frac{\alpha_s}{\pi}$$

$$L \equiv \log \frac{Q^2}{\mu^2}$$

L.E. Adam, K.G. Chetyrkin Phys. Lett. B329, 129 (1994)

- In 2015, within my Master, we calculated the anomalous dimension of the four-quark operators for the dimension-6 term to first order, which only receives contributions from the four-quark condensates. (The three gluon condensate contributes only beginning at 2nd order)
- Their contribution to the two point function has been computed at the next-to-leading order in Adam, Chetyrkin (1994) and we started for the results for V-A and V+A correlation functions, because for V-A the peguin diagrams cancelled.
- For nf=3 and nc=3 we then have …

Theory

OPE D=6

$$
\begin{aligned}
Q_V^O &= (\bar{u}\gamma_\mu t^a d \bar{d}\gamma^\mu t^a u)\,, \quad Q_A^O = (\bar{u}\gamma_\mu\gamma_5 t^a d\bar{d}\gamma^\mu\gamma_5 t^a u)\,,\\
Q_V^S &= (\bar{u}\gamma_\mu d\bar{d}\gamma^\mu u)\,, \quad Q_A^S = (\bar{u}\gamma_\mu\gamma_5 d\bar{d}\gamma^\mu\gamma_5 u)\,,\\
Q_3 &\equiv (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu t^a q)\,,\\
Q_4 &\equiv (\bar{u}\gamma_\mu\gamma_5 t^a u + \bar{d}\gamma_\mu\gamma_5 t^a d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu\gamma_5 t^a q)\,,\\
Q_5 &\equiv (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu q)\,,\\
Q_6 &\equiv (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\sum_{q=u,d,s}(\bar{q}\gamma^\mu\gamma_5 q)\,,\\
Q_7 &\equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu t^a q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu t^a q')\,,\\
Q_8 &\equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu\gamma_5 t^a q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu\gamma_5 t^a q')\,,\\
Q_9 &\equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu q')\,,\\
Q_{10} &\equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu\gamma_5 q)\sum_{q'=u,d,s}(\bar{q}'\gamma^\mu\gamma_5 q')\,.
\end{aligned}
$$

- The appearing four-quark operators are a subset which belong to the complete basis required for the one-loop renormalisation.
- Q_{V,A} are termed current-current operators and Q_3 to Q_10 penguin operators

OPE D=6

Current-current diagrams

Penguin diagrams:

Theory

OPE D=6

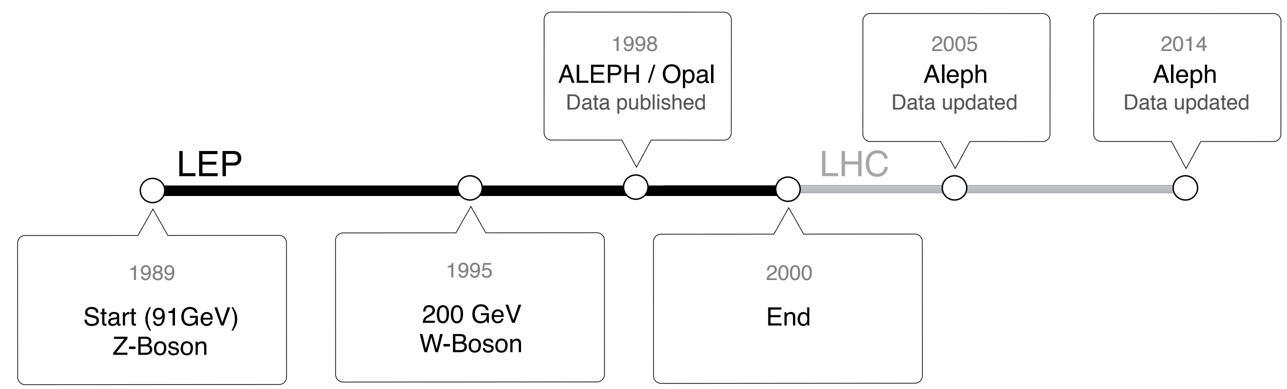
$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Renormalization Constants Z

$$\gamma^{(1)}_{Q_+} = \begin{pmatrix}
-\frac{3}{N_C} & \frac{3C_F}{2N_C} & -\frac{1}{3N_C} & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{N_f}{3} - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & \frac{3C_F}{2N_C} & \frac{2}{3} & 0 & 0 & 0 \\
\frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & \frac{3N_c}{4} + \frac{3}{2} - \frac{11}{6N_c} & -\frac{3N_c}{4} + \frac{3}{2} + \frac{3}{2N_c} & -\frac{3C_F}{2N_c} & -\frac{3}{4} - \frac{3}{4N_c} & -\frac{3}{4} - \frac{3}{4N_c} & \frac{3C_F}{4N_c} & \frac{3C_F}{4N_c} \\
0 & 0 & \frac{11}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{N_f}{3} + 1 - \frac{3N_c}{4} - \frac{1}{3N_c} & \frac{3N_c}{4} - \frac{3}{N_c} & 0 & \frac{3C_F}{2N_c} \\
0 & 0 & 0 & 0 & 0 & \frac{3N_c}{4} - \frac{10}{3N_c} & -\frac{3N_c}{4} & \frac{3C_F}{2N_c} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{11}{3} & 0 & 0 & 0
\end{pmatrix}$$

$$\hat{\gamma}^{(1)}_{O_{V-A}} = \begin{pmatrix} -\frac{3N_C}{2} + \frac{3}{N_C} & -\frac{3C_F}{2N_C} \\ -3 & 0 \end{pmatrix}$$

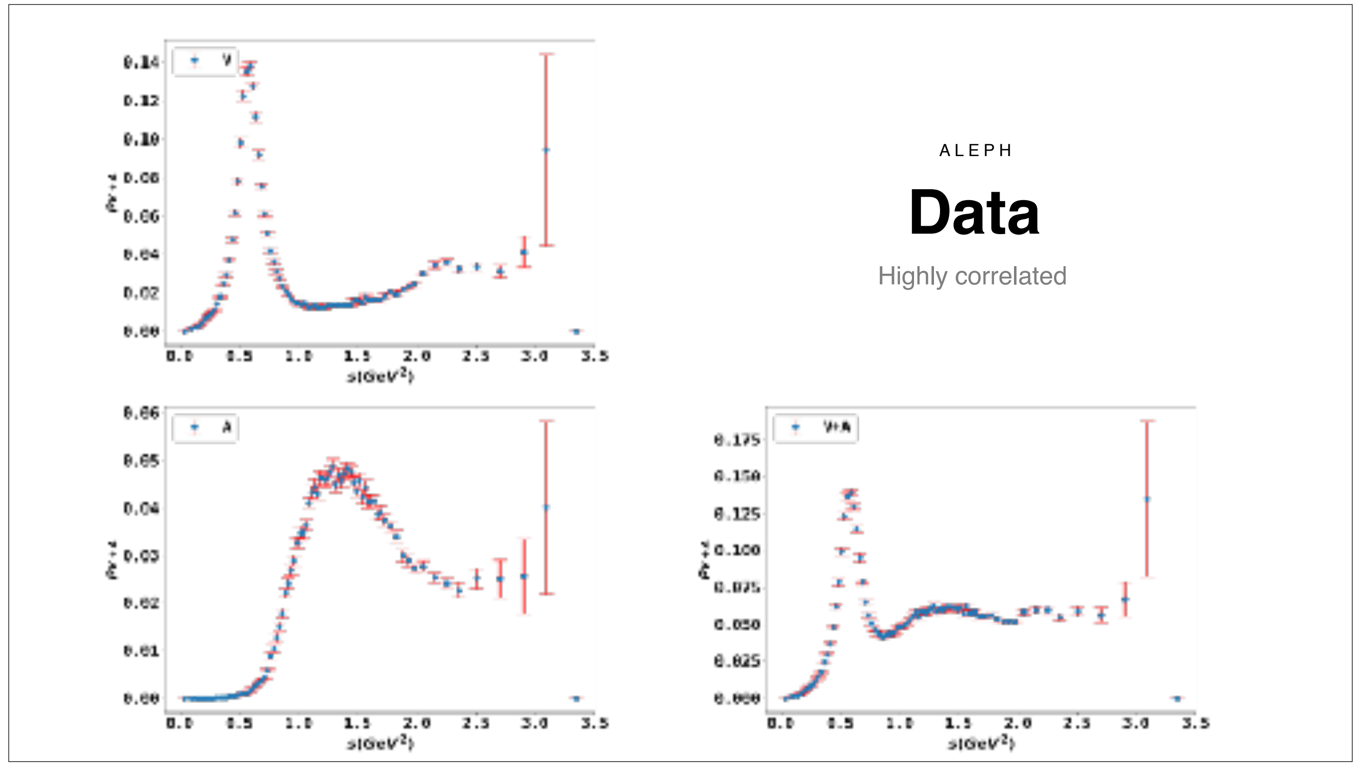
Timeline



K. Ackerstaff et al., OPAL, Eur. Phys. J. C 7, 571, 1999

S. Schael et al., ALEPH, Phys. Rept. 421:191-284, 2005

M. Davier, A. Hoecker, B. Malaescu, C. Yuan, Z. Zhang, ALEPH, Eir. Phys. J., C74(3):2803, 2014



- We work with the updated Aleph data (2014).
- The data Is given for the different channels: V, A and their combinations. We plotted the spectral function for each channel with the corresponding errors.
- The Aleph group also prepared the correlation matrix between the different data points, which are highly correlated. The correlation matrix is needed for the Chisquared fit.
- In the vectorial diagram one can see the rho resonance and the large errors on the last 2 bins.
- In the axial-vector diagram on can see the a1 resonance and the large error on the last 4 bins.
- In the V+A graph one can see the quick convergence, right after the rho resonance, which is in agreement with the naive Parton model.

Data

ALEPH

$$\rho(s) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

Spectral Fucntion

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,V}(s) = \frac{1}{2\pi} v_1(s)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_1(s)$$

$$\mathrm{Im}\,\Pi^{(0)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to V^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_V}{N_V\, \mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to A^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\, \mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to \pi^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\, \mathrm{d}s} \left(1 - \frac{s}{m_\tau^2}\right)^2$$

The ALEPH Collaboration, Phys. Rept., 421:191-284, 2005

Methodology

Fitting

Chisquared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k,\omega_l) = \int_{s_{th}}^{s_k} \frac{\mathrm{d}s}{s_k}\omega_l(s)\,\mathrm{Im}\,\Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k,\omega_l) = \frac{i}{2s_k}\oint_{|s|=s_k}\frac{\mathrm{d}s}{s}\left[W_l(s) - W_l(s_k)\right]D(s)$$

Parameters

$a_s \quad \langle a_s GG\rangle \quad \mathcal{O}_{6,V+A} \quad \mathcal{O}_{8,V+A}$

E.g.

Different s_0

# (k, l)	2 Moments	
1 (1,1)	s_1	w_1
2 (2,1)	s_2	w_1

Different weights

# (k, l)	3 Moments	
1 (1,1)	s_1	w_1
2 (1,2)	s_1	w_2
2 (1,3)	s_1	w_3

Extract

max. 2 parameters

max. 3 parameters

Fitting

Weights

$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{d}s \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1}$$

n-th degree monomial in the weight selects the D = 2(n+1)

$$\begin{array}{ccccccc} \frac{s}{s_0} & + & \left(\frac{s}{s_0}\right)^2 & + & \left(\frac{s}{s_0}\right)^3 & + & \cdots \\ \downarrow & & \downarrow & & \downarrow & & \\ \mathcal{O}_{4,V/A} & & \mathcal{O}_{6,V/A} & & \mathcal{O}_{8,V/A} & & \end{array}$$

$$\omega_{kl}(s) = \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^l\left(1+\frac{2s}{m_\tau^2}\right)$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$ Double Pinched

$$\begin{aligned}
A^{ALEPH}_{00,V/A} &= A^{ALEPH}_{00,V/A}(a_s, \mathcal{O}_{6,V/A}) \\
A^{ALEPH}_{10,V/A} &= A^{ALEPH}_{10,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}) \\
A^{ALEPH}_{11,V/A} &= A^{ALEPH}_{11,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}\mathcal{O}_{12,V/A}) \\
A^{ALEPH}_{12,V/A} &= A^{ALEPH}_{12,V/A}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}) \\
A^{ALEPH}_{13,V/A} &= A^{ALEPH}_{13,V/A}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})
\end{aligned}$$

Systematic Errors

- Higher Orders of perturbative term
- Higher orders of non-perturabtive term
- FOPT vs CIPT
- Duality Violations

Violations

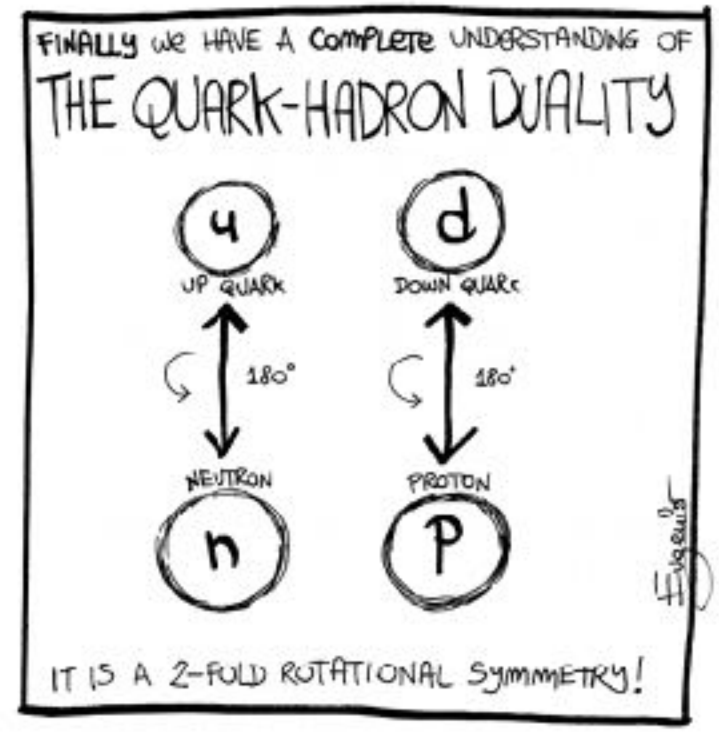
Duality

Experiment
Hadrons

Theory
Quark/ Gluons

$\rho_{\text{exp}} = \rho_{\text{th}}$

- What energy is high enough for the quark-hadron duality to set in?
- What role play nonperturbative effects?



M. Shiftman, hep-th/0009131, 2000

Model

Model: 8 Parameters

$$\Delta\rho^{DV}_{V/A}(s) = e^{-\delta_{V/A}+\gamma_{V/A}s)}\sin(\alpha_{V/A}+\beta_{V/A}s)$$

$$\Delta A^{\omega,DV}_{V/A}(s_0) \equiv \frac{i}{2}\oint_{|s_0|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\left\{\Pi_{V/A}(s)-\Pi^{OPE}_{V/A}(s)\right\} = -\pi\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s_0}\omega(s)\Delta\rho^{DV}_{V/A}(s)$$

M. Shiftman, hep-th/0009131, 2000

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman

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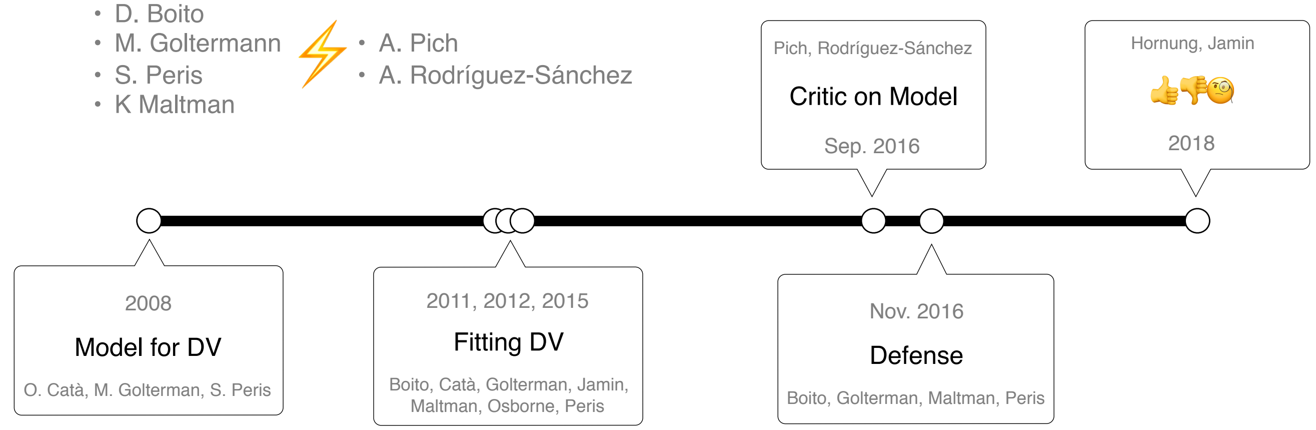
- A. Pich
- A. Rodríguez-Sánchez



Philosophy

Timeline

$\alpha_s = 0.32$ (CIPT)

Boito, Goltermann, Peris, Maltman

Pro

- Theoretically well motived
- Systematic erros have been underestimated
- Assuming Duality a priori leads to worse model

$\alpha_s = 0.35$ (CIPT)

Pich, Rodríguez-Sánchez

Contra

- Cannot be derived from first principles
- Far too many parameters (8)
- Poor statistical quality (low p-value, large uncertainties)
- Sufficiently suppressed (pinched weights)

D. Boito, O. Catà, M. Golterman, M. Jamin, K. Maltman, J. Osborne, S. Peris, Phys. Rev., D84:113006, 2011

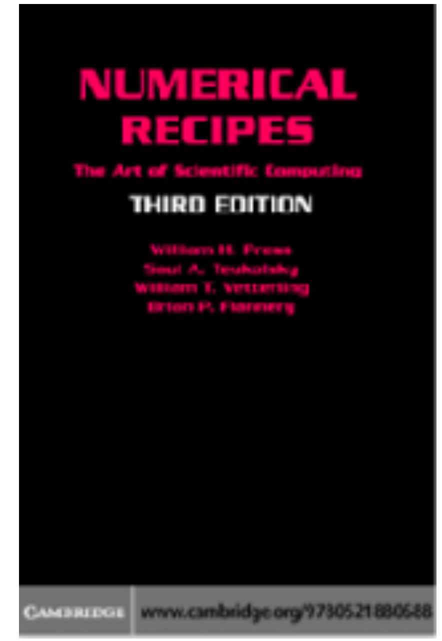
Architecture

Program

Important Frameworks:
- GSL
- BOOST
- ROOT

Infos:
- Editor: Emacs
- Parallelization: use multiple cores
- GitHub

Numerics:
- Integration: GSL - QAG adaptive integration
- Inverse Matrix: BOOST - UBLAS
- Non-linear Equation Solver: GSL - Multidimensional Root-Finding
- Minimization: ROOT - MINUIT2



Determination of α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16

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