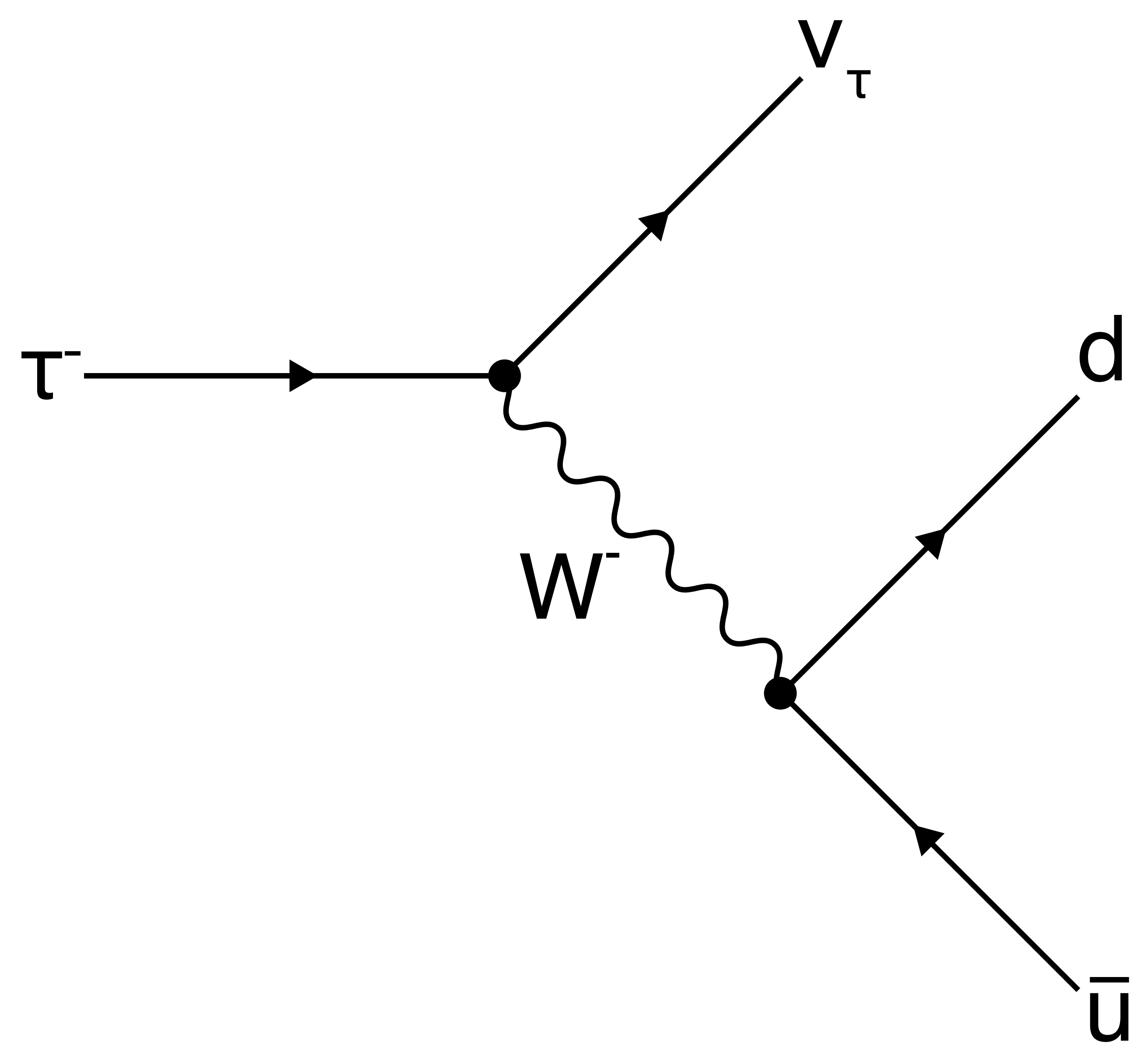
Determination of the QCD Coupling from ALEPH τ Decay Data

Dirk Hornung



Running of the

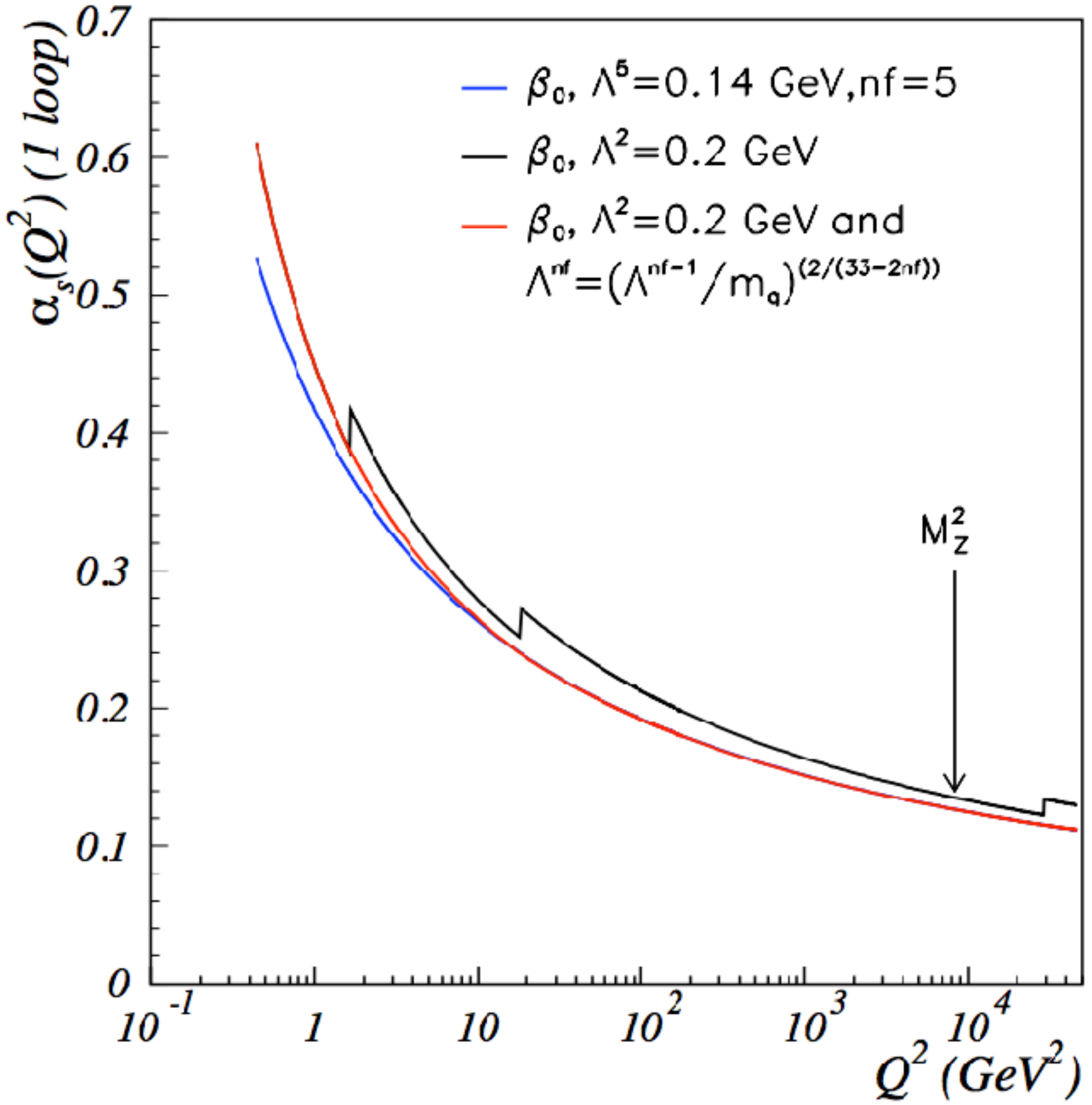
Strong Coupling α_s

$\alpha_s(m_{\tau \cong 3.15 GeV^2}) \cong 0.33$

Exclusive

Hadronic τ-Decay

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]}$$



Contents

Theory

Framework

Inclusive Ratio

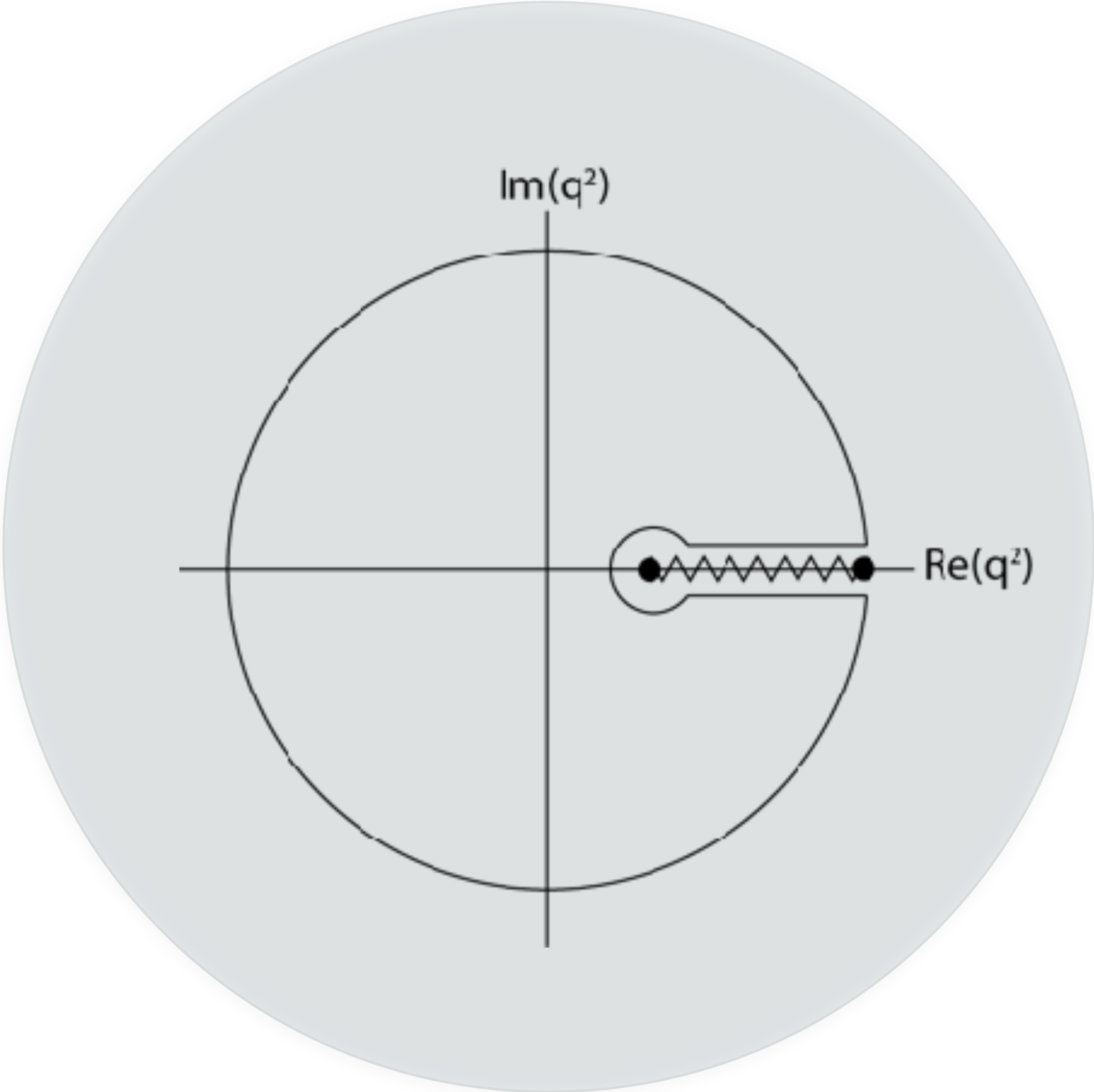
$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{\mathrm{d}s}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv \left|V_{uq}\right|^2\left(\Pi^{(J)}_{ud,V}+\Pi^{(J)}_{ud,A}(s)\right)$$

Two-Point Correlation Function

$$\Pi_{\mu\nu}(q) = i\int \mathrm{d}^4 x e^{iqx}\langle 0|T\left\{\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right\}\rangle$$
$$= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1)}_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij,\mathcal{J}}(q^2)$$

$$(i,j=u,d;\mathcal{J}=V,A) \quad V^\mu_{ij}=\bar{q}_j\gamma^\mu q_i \quad A^\mu_{ij}=\bar{q}_j\gamma^\mu\gamma_5 q_i$$



$$\int_{s_{th}}^{s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\,\mathrm{Im}\,\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\Pi_{V/A}(s)$$

Experiment Theory

Theory

OPE

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

Perturbative

PT

99%

Adler Function:

$$D(s) \equiv -s\frac{\mathrm{d}\Pi^{PT}}{\mathrm{d}s} = \frac{1}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{nk} \ln\left(\frac{-s}{\mu^2}\right)$$

$$A^{\omega,PT} = \frac{i}{2s_0}\oint_{|s|=s_o} \frac{\mathrm{d}s}{s}\left[W(s) - W(s_0)\right] D(s)$$

FOPT or CIPT

Non-Perturbative

NPT

1%

$\mathcal{O}_{2,V/A} \approx 0$ (light quark masses)

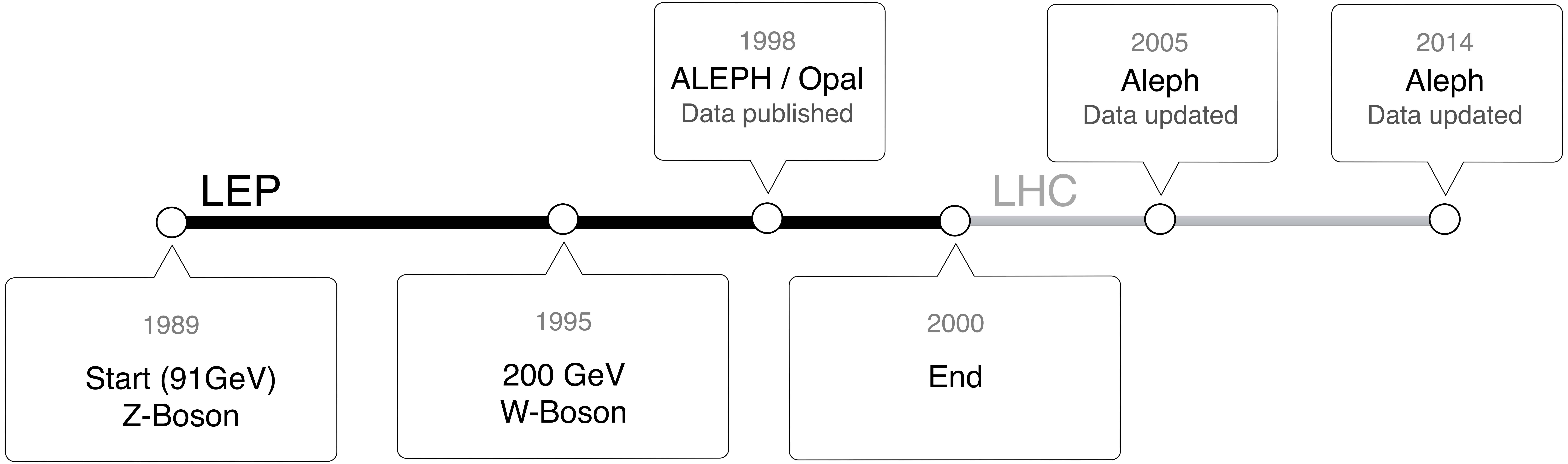
$$\mathcal{O}_{4,V/A} = \frac{1}{12}\left[1 - \frac{11}{18}a_s\right]\langle a_s GG\rangle + \left[1 + \frac{\pm 36 - 23}{27}a_s\right]\langle (m_u + m_d)\bar{q}q\rangle$$

Theory

OPE D=6

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

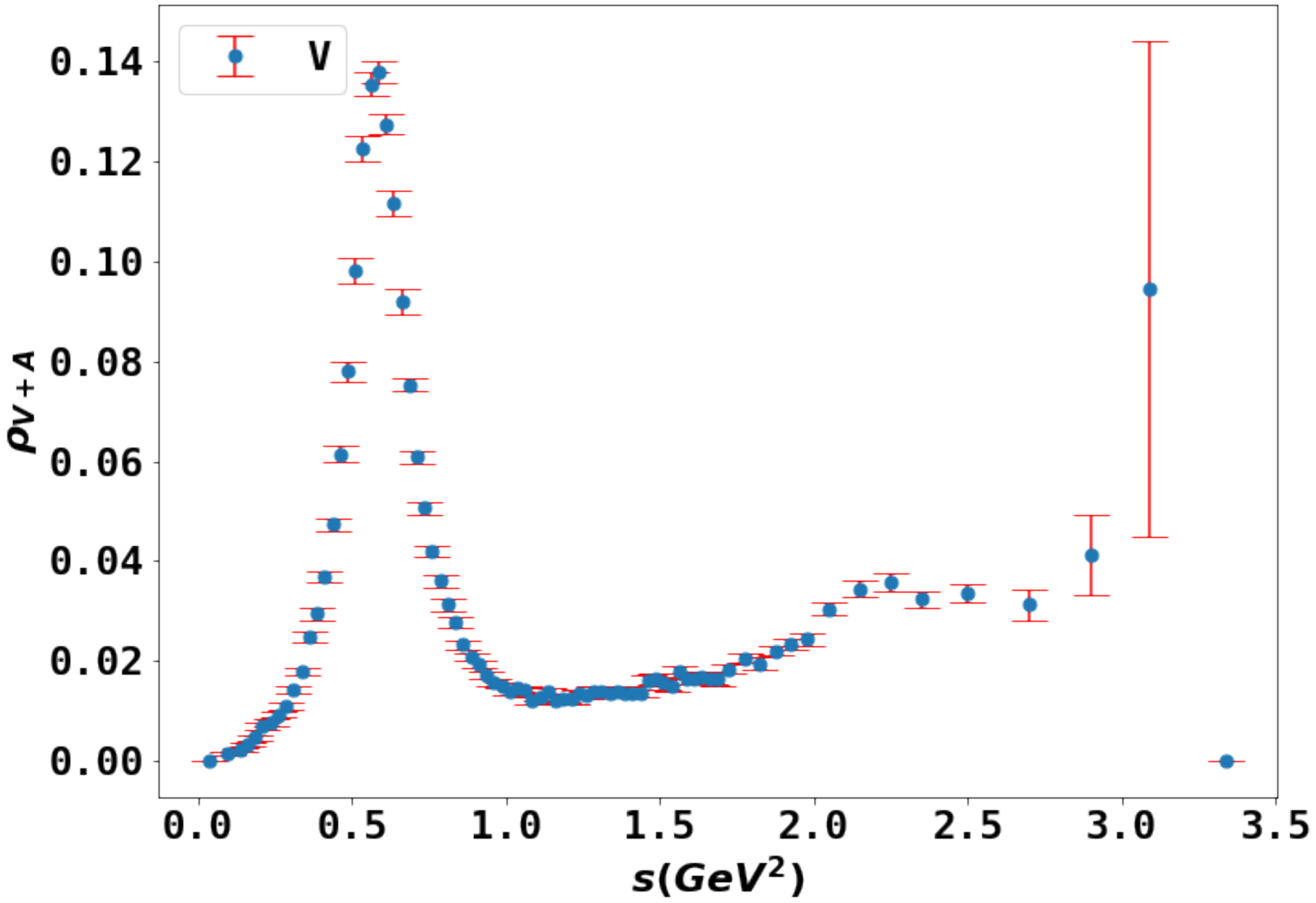
Timeline



K. Ackerstaff et al., OPAL, Eur. Phys. J. C 7, 571, 1999

S. Schael et al., ALEPH, Phys. Rept. 421:191-284, 2005

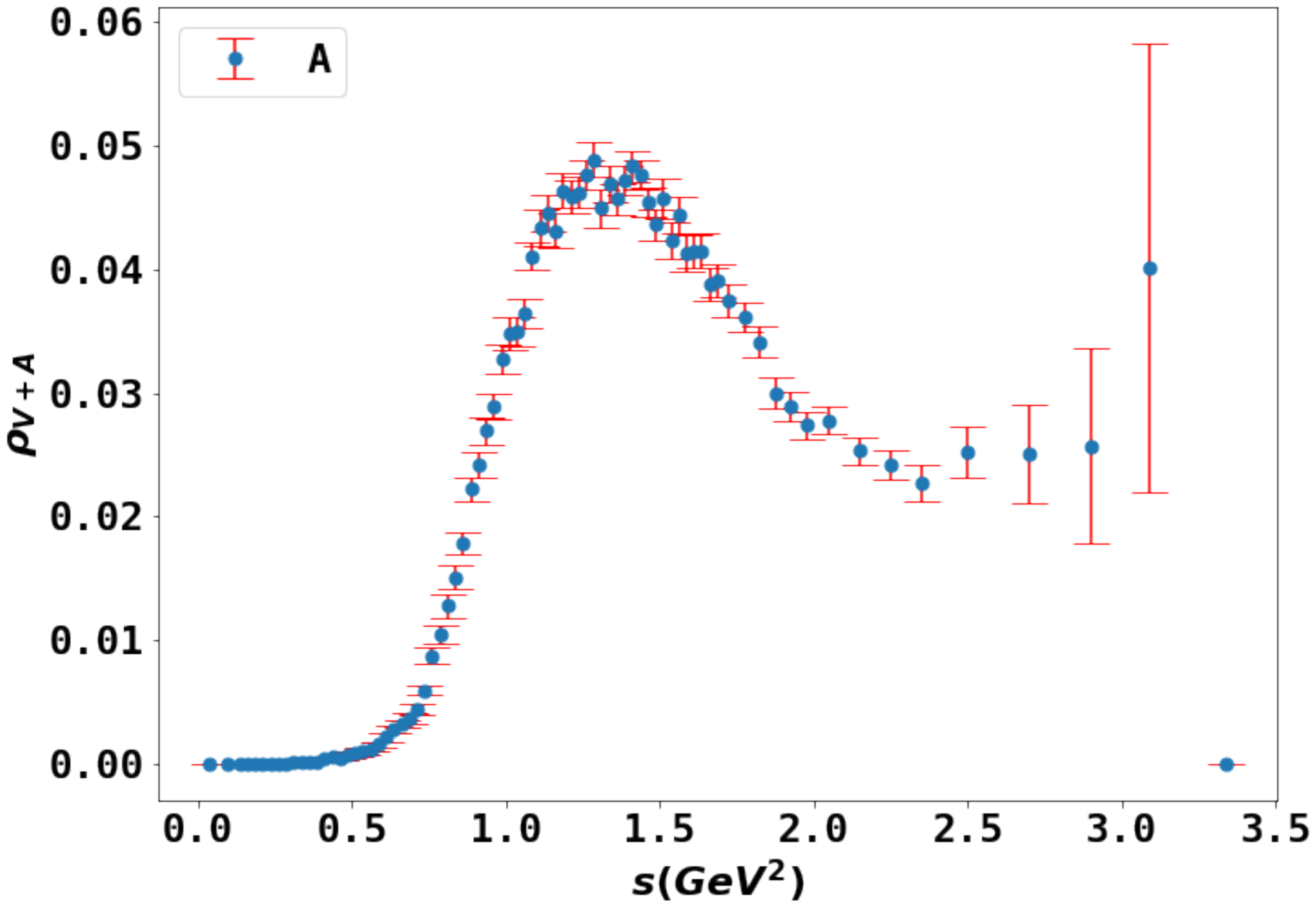
M. Davier, A. Hoecker, B. Malaescu, C. Yuan, Z. Zhang, ALEPH, Eir. Phys. J., C74(3):2803, 2014

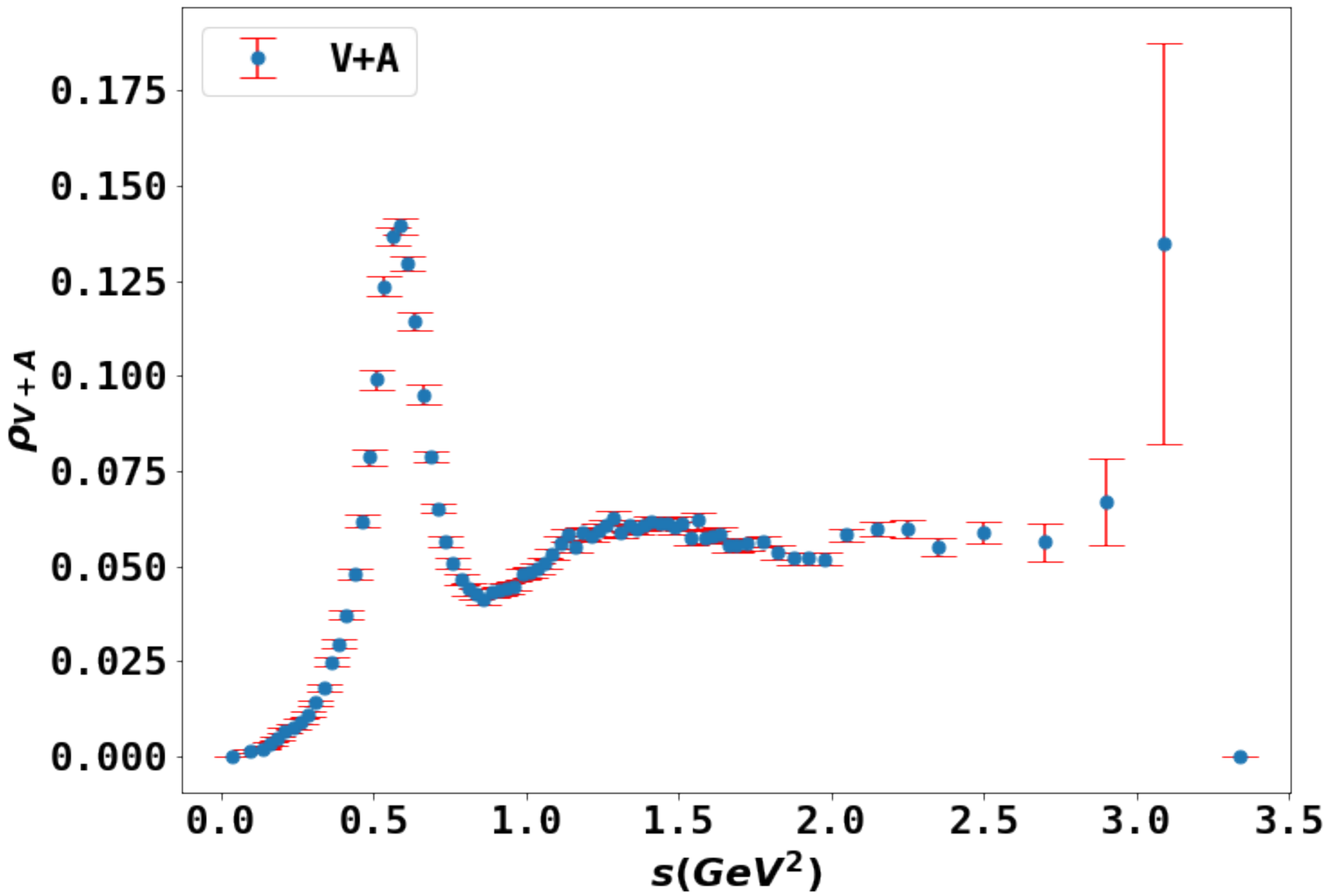


ALEPH

Data

Highly correlated





Data

OPE

ALEPH

$$\rho(s) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

Spectral Fucntion

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,V}(s) = \frac{1}{2\pi} v_1(s)$$

$$\mathrm{Im}\,\Pi^{(1)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_1(s)$$

$$\mathrm{Im}\,\Pi^{(0)}_{\bar{u}d,A}(s) = \frac{1}{2\pi} a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to V^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_V}{N_V\,\mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to A^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2\, S_{EW}} \frac{B(\tau^- \to \pi^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{\mathrm{d}N_A}{N_A\,\mathrm{d}s} \left(1 - \frac{s}{m_\tau^2}\right)^2$$

The ALEPH Collaboration, Phys. Rept., 421:191-284, 2005

Fitting

Chisquared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k,\omega_l) = \int_{s_{th}}^{s_k} \frac{\mathrm{d}s}{s_k}\omega_l(s)\,\mathrm{Im}\,\Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k,\omega_l) = \frac{i}{2s_k}\oint_{|s|=s_k}\frac{\mathrm{d}s}{s}\left[W_l(s) - W_l(s_k)\right]D(s)$$

Parameters

a_s $\langle a_s GG\rangle$ $\mathcal{O}_{6,V+A}$ $\mathcal{O}_{8,V+A}$

E.g.

Different s_0

# (k, l)	2 Moments	
1 (1,1)	s_1	w_1
2 (2,1)	s_2	w_1

Different weights

# (k, l)	3 Moments	
1 (1,1)	s_1	w_1
2 (1,2)	s_1	w_2
2 (1,3)	s_1	w_3

Extract

max. 2 parameters

max. 3 parameters

Fitting

Weights

$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{d}s \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1}$$

n-th degree monomial in the weight selects the D = 2(n+1)

$$\frac{s}{s_0} + \left(\frac{s}{s_0}\right)^2 + \left(\frac{s}{s_0}\right)^3 + \cdots$$

$$\mathcal{O}_{4,V/A} \quad \mathcal{O}_{6,V/A} \quad \mathcal{O}_{8,V/A}$$

$$\omega_{kl}(s) = \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^l\left(1+\frac{2s}{m_\tau^2}\right)$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$ Double Pinched

$$A^{ALEPH}_{00,V/A} = A^{ALEPH}_{00,V/A}(a_s, \mathcal{O}_{6,V/A})$$
$$A^{ALEPH}_{10,V/A} = A^{ALEPH}_{10,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$
$$A^{ALEPH}_{11,V/A} = A^{ALEPH}_{11,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}\mathcal{O}_{12,V/A})$$
$$A^{ALEPH}_{12,V/A} = A^{ALEPH}_{12,V/A}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$
$$A^{ALEPH}_{13,V/A} = A^{ALEPH}_{13,V/A}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

Methodology

Systematic Errors

- Higher Orders of perturbative term
- Higher orders of non-perturabtive term
- FOPT vs CIPT
- Duality Violations

Violations

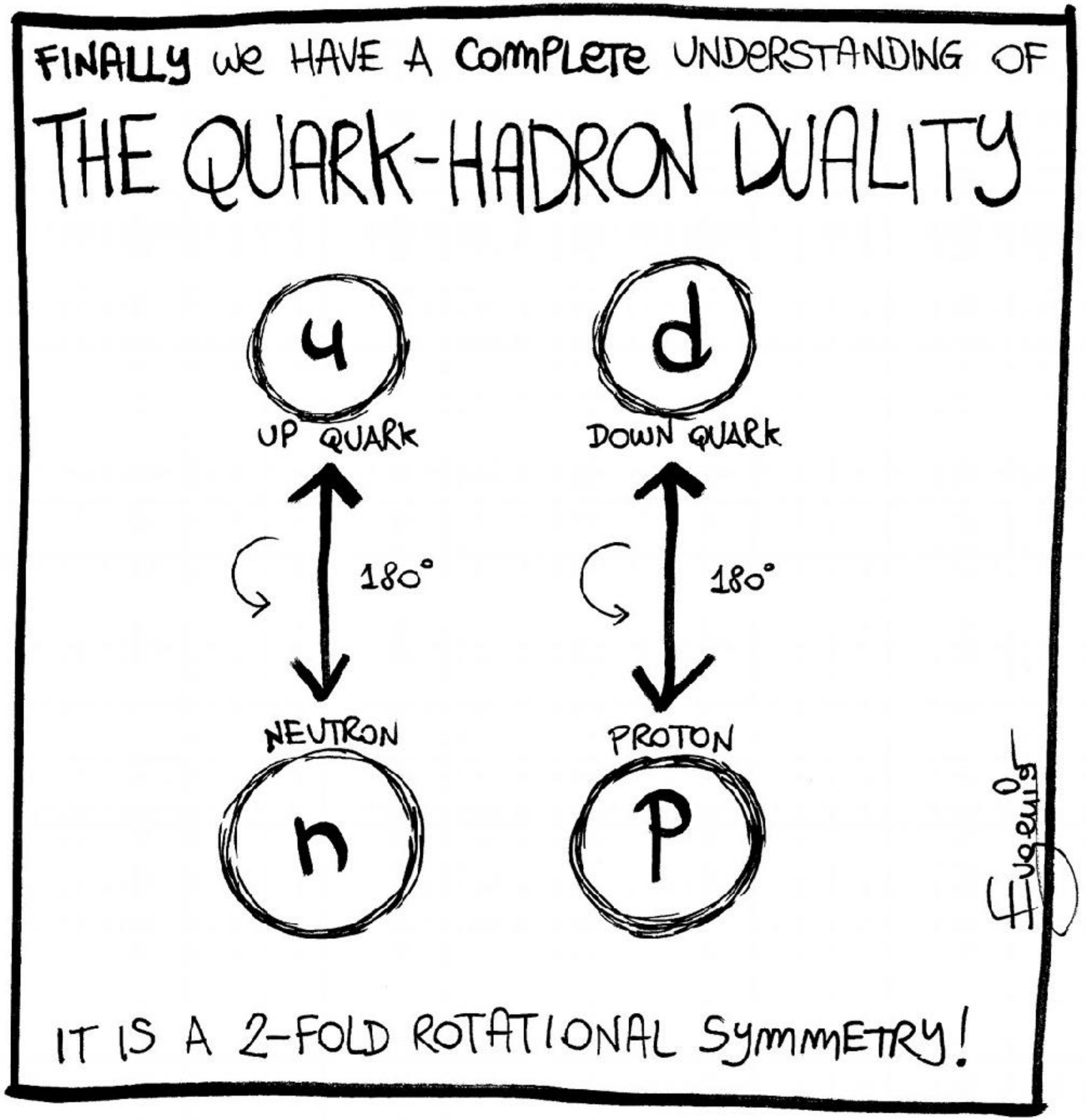
Duality

Experiment
Hadrons

Theory
Quark/ Gluons

$\rho_{\mathrm{exp}} = \rho_{\mathrm{th}}$

- What energy is high enough for the quark-hadron duality to set in?
- What role play nonperturbative effects?



M. Shiftman, hep-th/0009131, 2000

Model

Model: 8 Parameters

$$\Delta\rho^{DV}_{V/A}(s) = e^{-\delta_{V/A}+\gamma_{V/A}s)}\sin(\alpha_{V/A}+\beta_{V/A}s)$$

$$\Delta A^{\omega,DV}_{V/A}(s_0) \equiv \frac{i}{2}\oint_{|s_0|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\left\{\Pi_{V/A}(s)-\Pi^{OPE}_{V/A}(s)\right\} = -\pi\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s_0}\omega(s)\Delta\rho^{DV}_{V/A}(s)$$

M. Shiftman, hep-th/0009131, 2000

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman

⚡

- A. Pich
- A. Rodríguez-Sánchez

2008
Model for DV
O. Catà, M. Golterman, S. Peris

2011, 2012, 2015
Fitting DV
Boito, Catà, Golterman, Jamin, Maltman, Osborne, Peris

Pich, Rodríguez-Sánchez
Critic on Model
Sep. 2016

Nov. 2016
Defense
Boito, Golterman, Maltman, Peris

Hornung, Jamin
👍👎🧐
2018

Timeline

Philosophy

$\alpha_s = 0.32$ (CIPT)

Boito, Goltermann, Peris, Maltman

Pro

- Theoretically well motived
- Systematic erros have been underestimated
- Assuming Duality a priori leads to worse model

$\alpha_s = 0.35$ (CIPT)

Pich, Rodríguez-Sánchez

Contra

- Cannot be derived from first principles
- Far too many parameters (8)
- Poor statistical quality (low p-value, large uncertainties)
- Sufficiently suppressed (pinched weights)

D. Boito, O. Catà, M. Golterman, M. Jamin, K. Maltman, J. Osborne, S. Peris, Phys. Rev., D84:113006, 2011

Determination of α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16

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