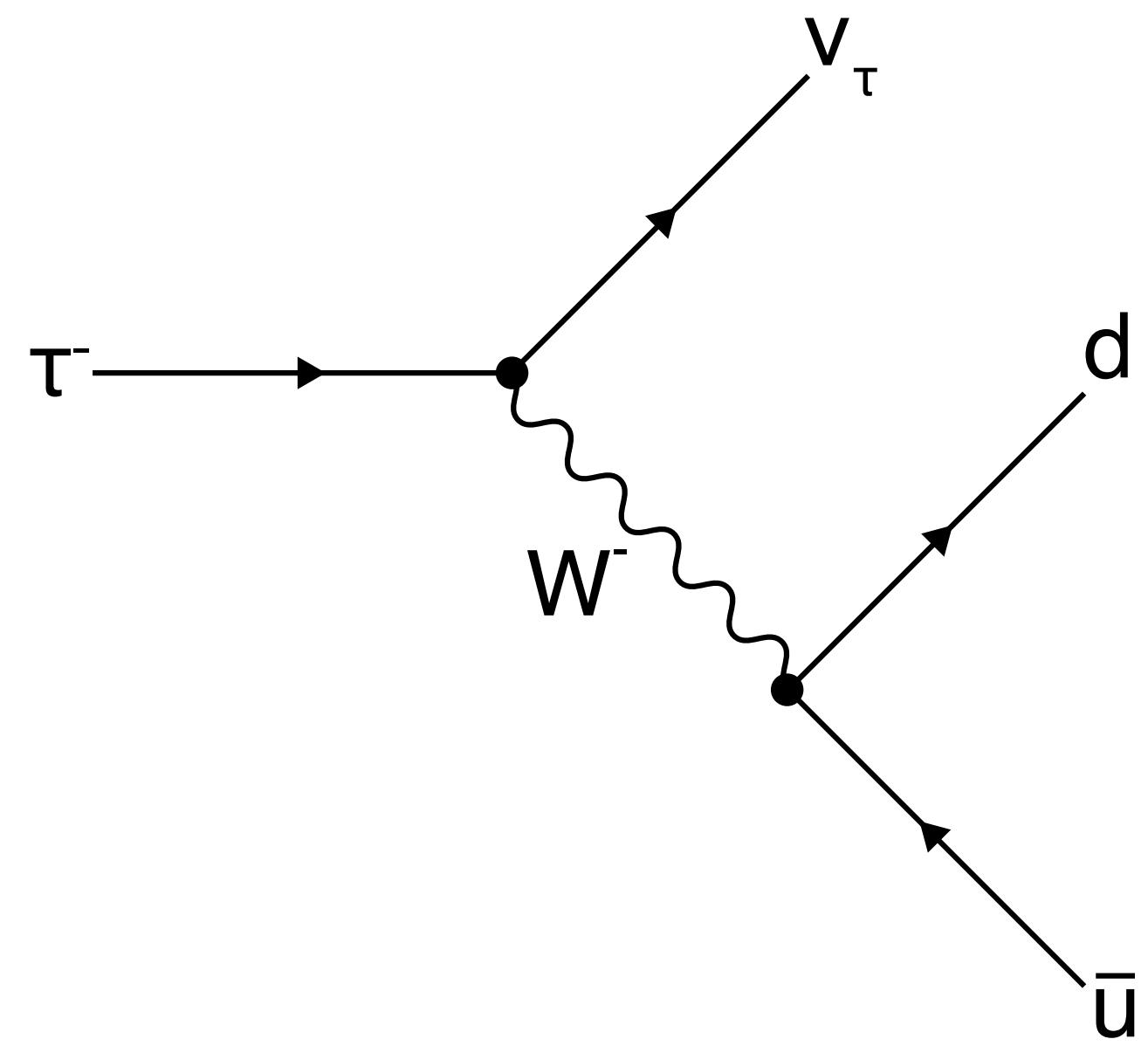


Determination of the QCD Coupling from ALEPH τ Decay Data

Dirk Hornung

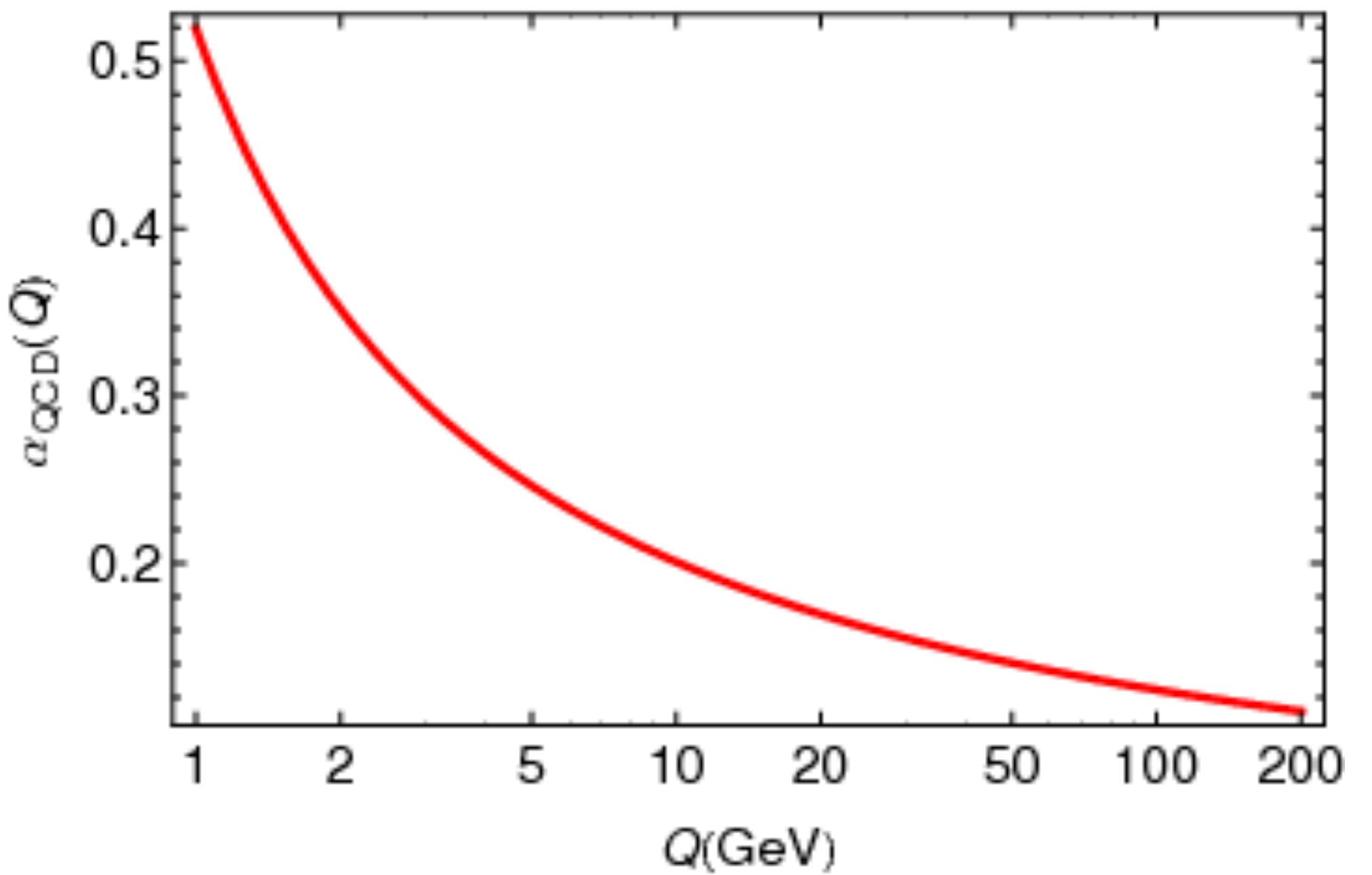


Strong Coupling as

Running of the
 Entrepreneurial activities differ substantially
 depending on the type of organization

Exclusive Hadronic τ -Decay

$$R_\tau = \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e]}$$

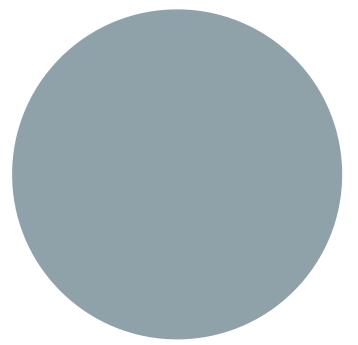
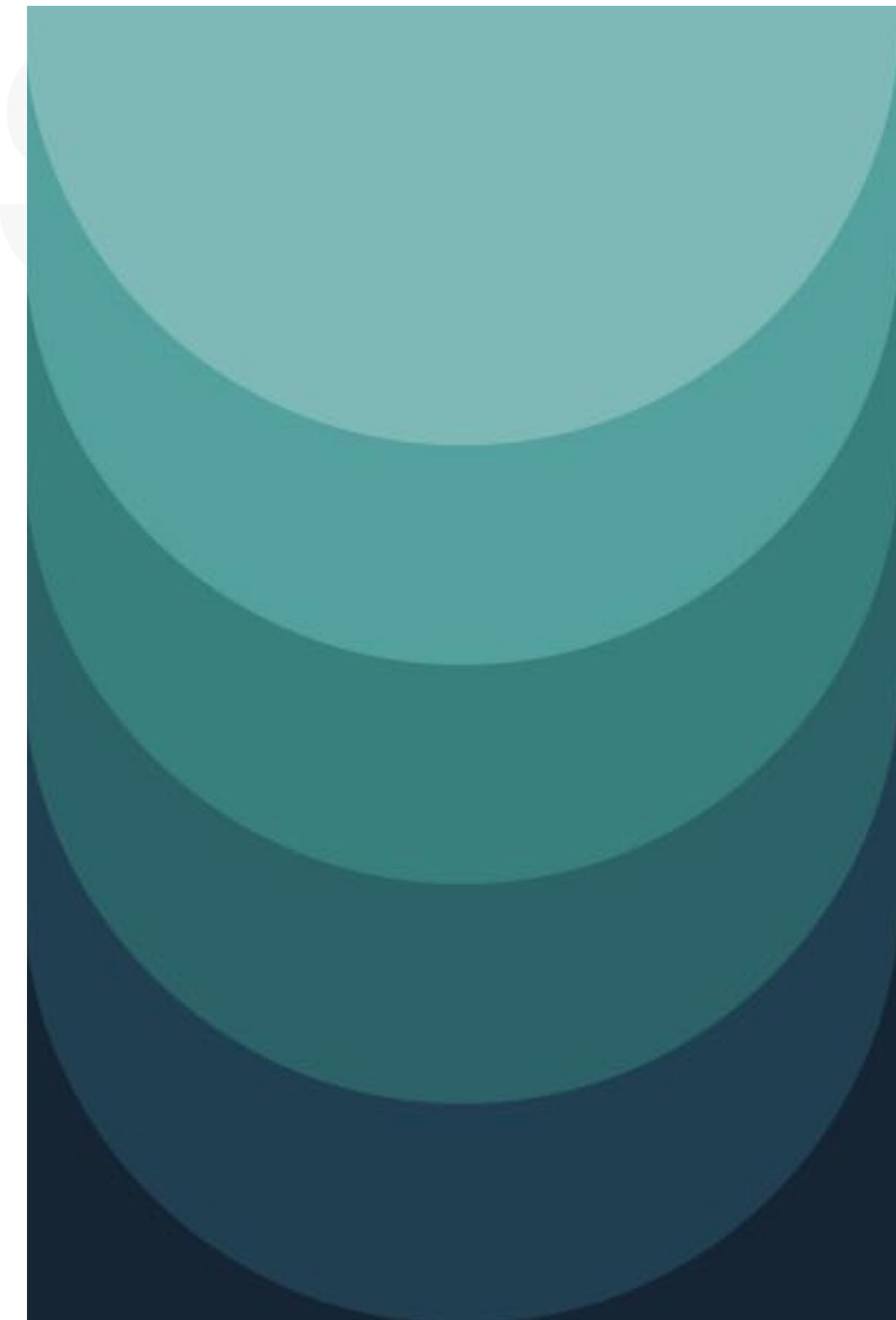


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Contents

- Theoretical Framework
- ALEPH Data
- Fitting Methodology
- Duality Violation
- Determination of a_s
- Program
- Summary



Theory

Framework

Inclusive Ratio

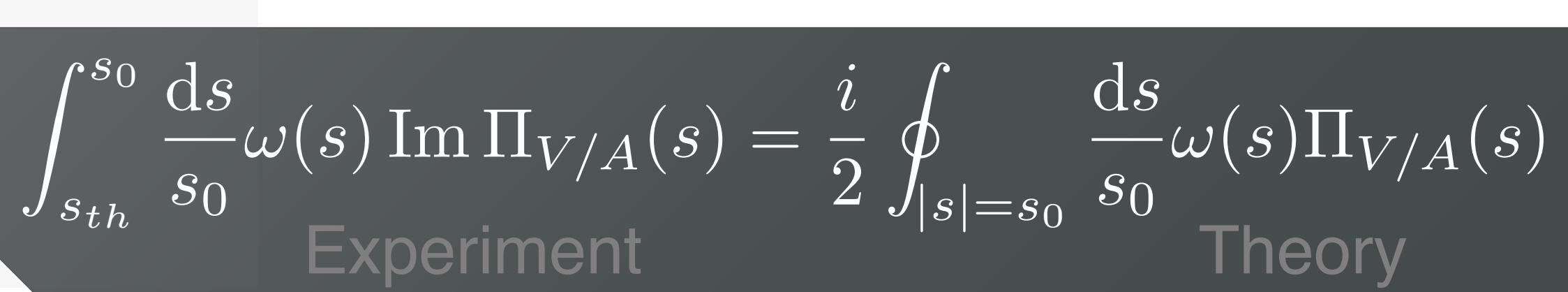
$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im } \Pi^{(1)}(s) + \text{Im } \Pi^{(0)}(s) \right]$$

$$\Pi^{(J)}(s) \equiv |V_{uq}|^2 \left(\Pi_{ud,V}^{(J)} + \Pi_{ud,A}^{(J)}(s) \right)$$

Two-Point Correlation Function

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T \left\{ \mathcal{J}_{ij}^\mu(x) \mathcal{J}_{ij}^{\nu\dagger}(0) \right\} \rangle \\ &= \left(q_\mu q_\nu = q^2 g_{\mu\nu} \right) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2) \\ &= \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi^{(1+0)}(q^2) + q^2 g_{\mu\nu} \Pi^{(0)}(q^2) \end{aligned}$$

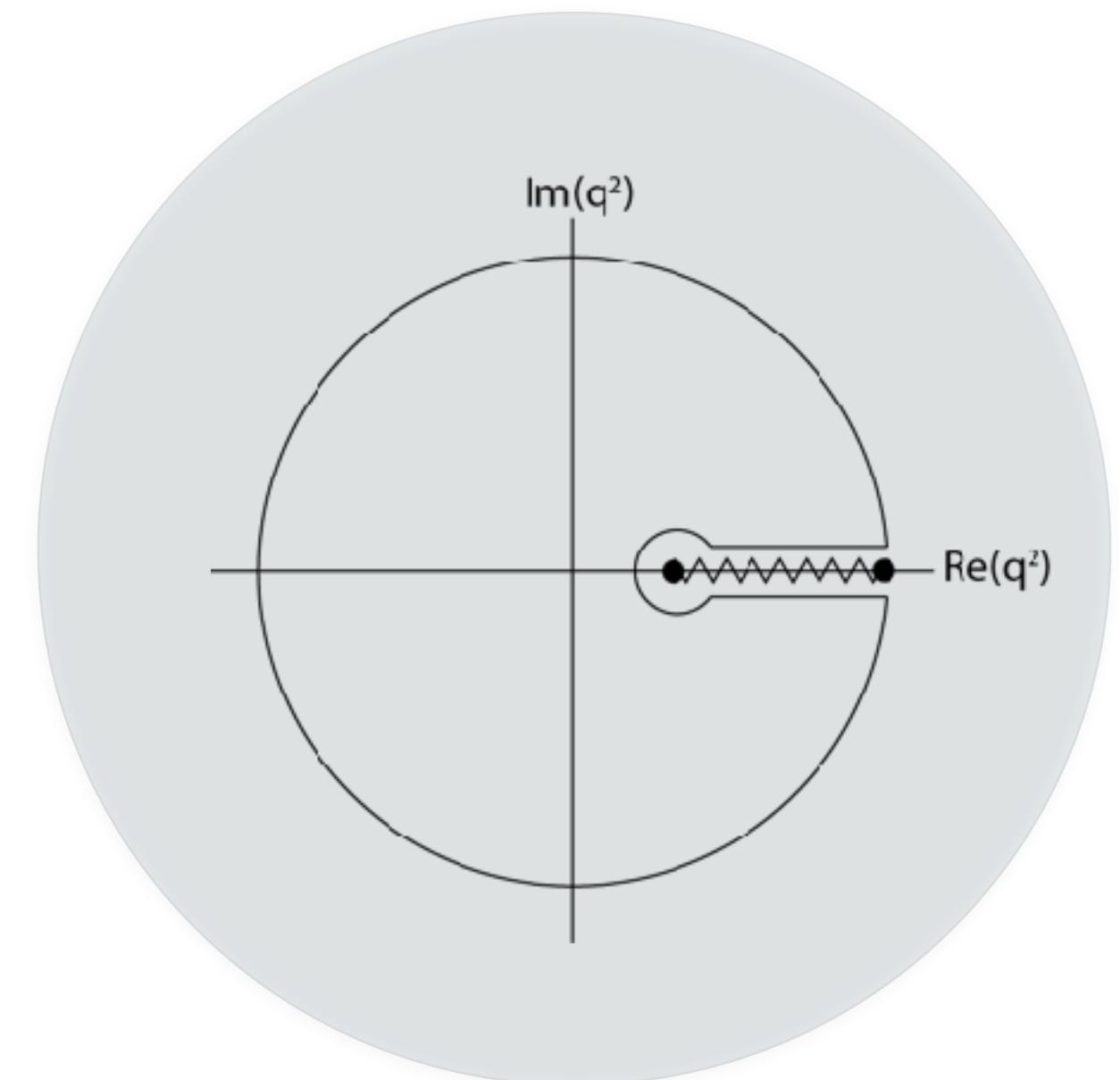
$$(I, j = u, d; \mathcal{J} = V, A) \quad V_{ij}^\mu = \bar{q}_j \gamma^\mu q_i \quad A_{ij}^\mu = \bar{q}_j \gamma^\mu \gamma_5 q_i$$



$$\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im } \Pi_{V/A}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{V/A}(s)$$

Experiment

Theory



Theory

OPE

$$\Pi_{OPE}^{(1+0)}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

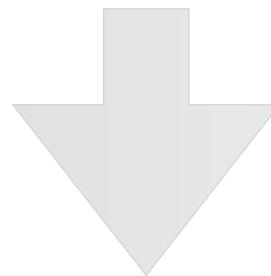
Perturbative

PT



$$D(s) \equiv -s \frac{d\Pi^{PT}}{ds} = \frac{1}{12\pi^2} \sum_{n=0}^{\infty} a_{\mu}^n \sum_{k=1}^{n+1} k c_{nk} \ln \left(\frac{-s}{\mu^2} \right)$$

$$A^{\omega, PT} = \frac{i}{2s_0} \oint_{|s|=s_o} \frac{ds}{s} [W(s) - W(s_0)] D(s)$$



FOPT or CIPT

Non-Perturbative

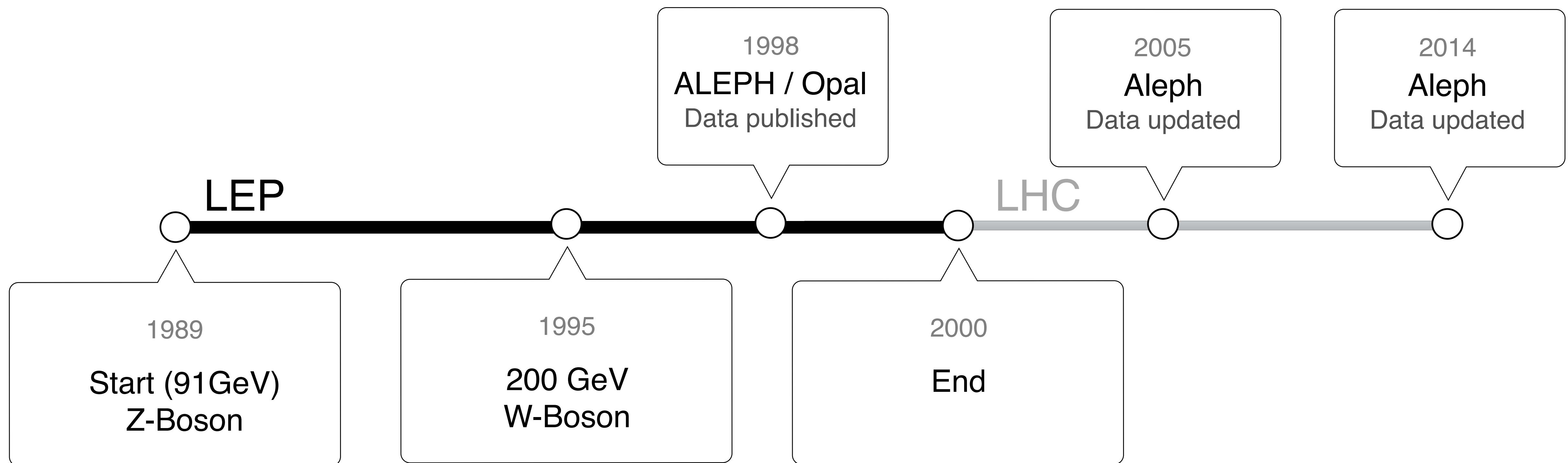
NPT



$$\mathcal{O}_{2,V/A} \approx 0 \quad (\text{light quark masses})$$

$$\mathcal{O}_{4,V/A} = \frac{1}{12} \left[1 - \frac{11}{18} a_s \right] \langle a_s GG \rangle + \left[1 + \frac{\pm 36 - 23}{27} a_s \right] \langle (m_u + m_d) \bar{q}q \rangle$$

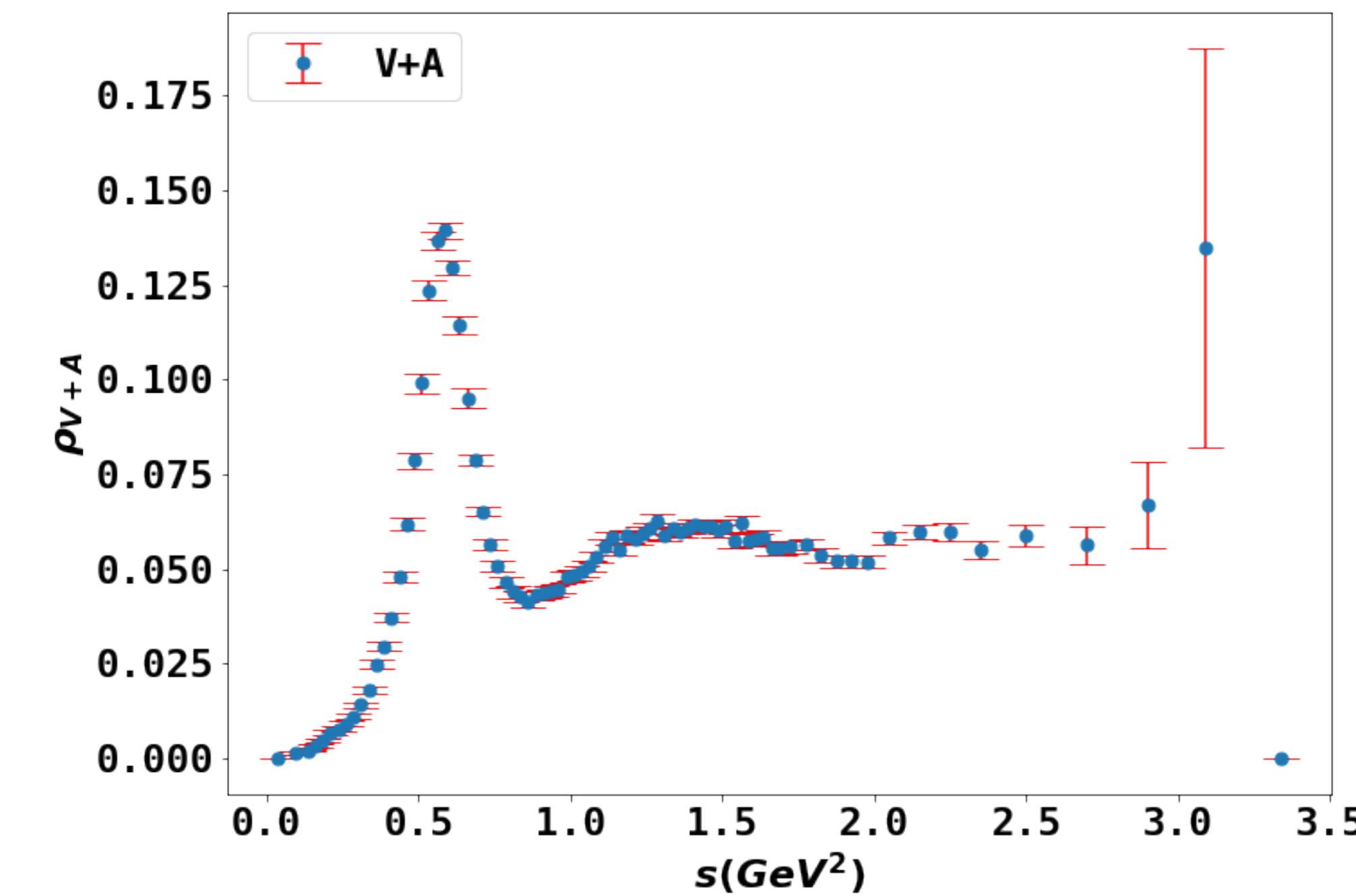
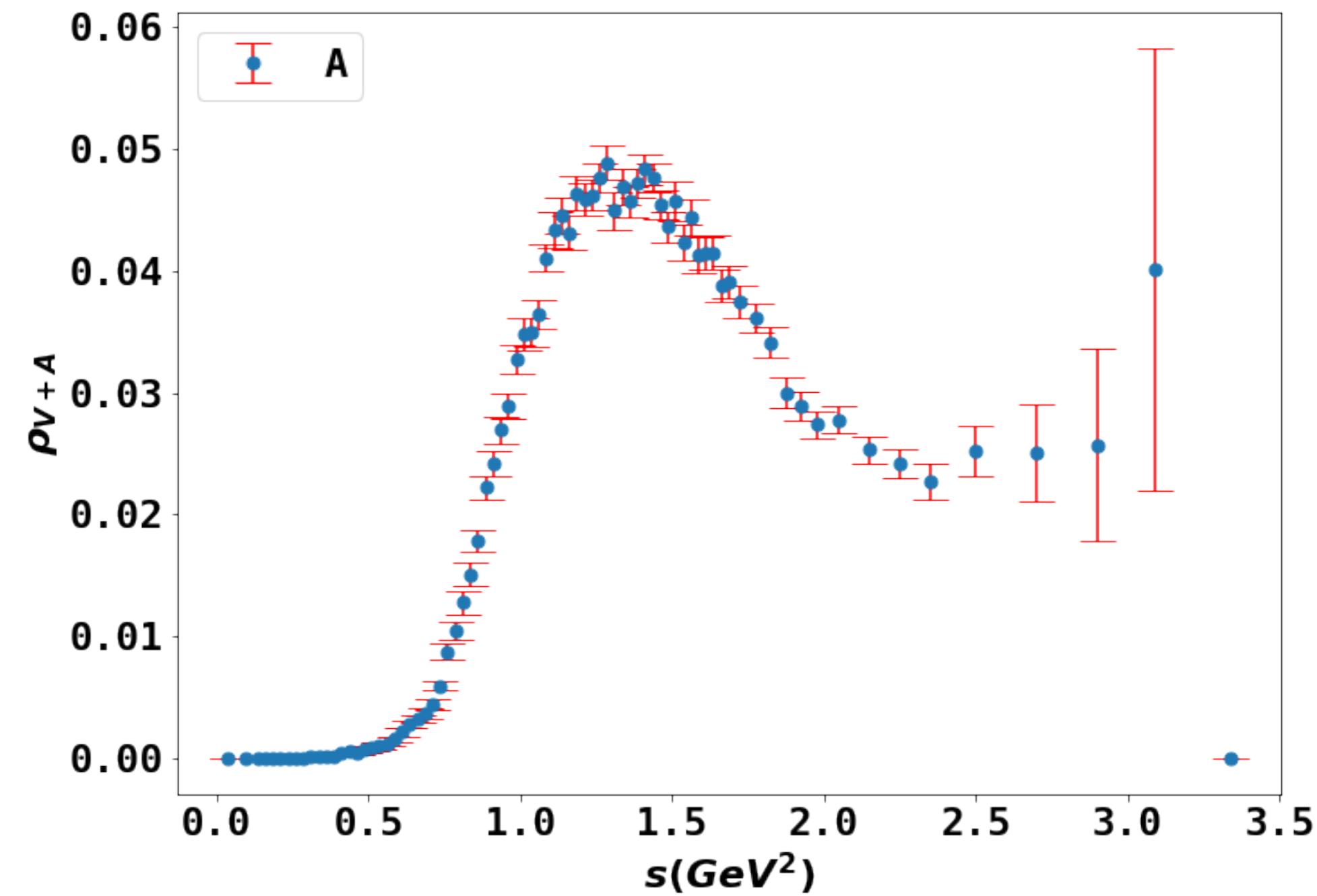
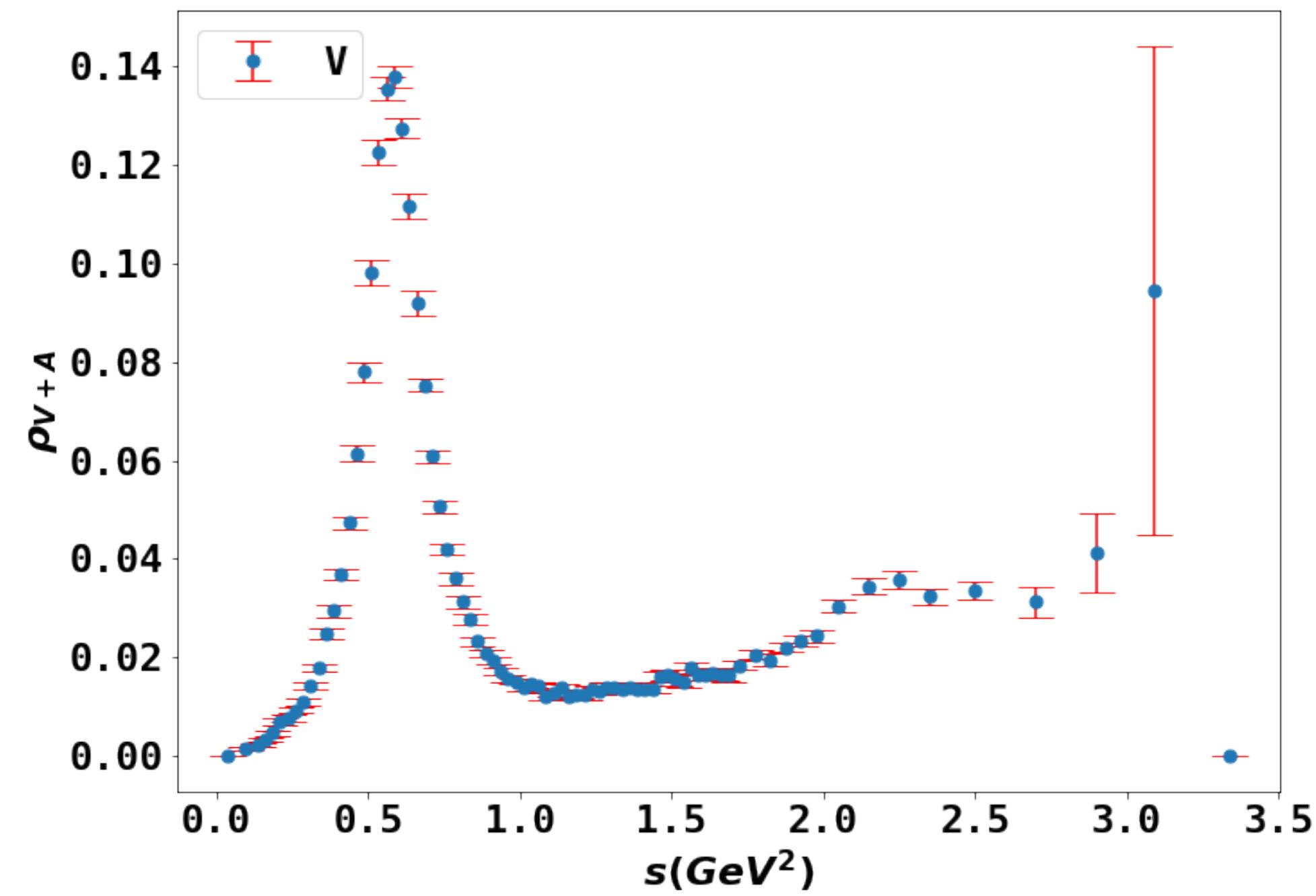
Timeline



K. Ackerstaff et al., OPAL, Eur. Phys. J. C 7, 571, 1999

S. Schael et al., ALEPH, Phys. Rept. 421:191-284, 2005

M. Davier, A. Hoecker, B. Malaescu, C. Yuan, Z. Zhang, ALEPH, Eur. Phys. J., C74(3):2803, 2014



ALEPH

Data

Highly correlated

Data

ALEPH

$$\rho(s) \equiv \frac{1}{\pi} \operatorname{Im} \Pi(s)$$

Spectral Function

$$\operatorname{Im} \Pi_{\bar{u}d,V}^{(1)}(s) = \frac{1}{2\pi} v_1(s)$$

$$\operatorname{Im} \Pi_{\bar{u}d,A}^{(1)}(s) = \frac{1}{2\pi} a_1(s)$$

$$\operatorname{Im} \Pi_{\bar{u}d,A}^{(0)}(s) = \frac{1}{2\pi} a_0(s)$$

$$v_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow V^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_V}{N_V ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1}$$

$$a_1(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow A^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1}$$

$$a_0(s) \equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow \pi^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left(1 - \frac{s}{m_\tau^2}\right)^2$$

Fitting

Chi squared

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha))$$

$$I_{i=kl}^{exp}(s_k, \omega_l) = \int_{s_{th}}^{s_k} \frac{ds}{s_k} \omega_l(s) \operatorname{Im} \Pi_{V/A}(s)$$

$$I_{i=kl}^{th}(s_k, \omega_l) = \frac{i}{2s_k} \oint_{|s|=s_k} \frac{ds}{s} [W_l(s) - W_l(s_k)] D(s)$$

Parameters

$$a_s \quad \langle a_s GG \rangle \quad \mathcal{O}_{6,V+A} \quad \mathcal{O}_{8,V+A}$$

E.g.

# (k, l)	2 Moments	
1 (1,1)	s ₁	w ₁
2 (2,1)	s ₂	w ₁

Different s₀

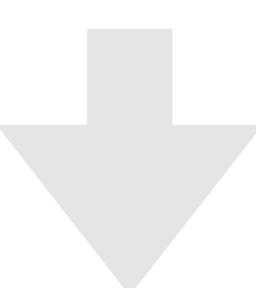
Different weights

# (k, l)	3 Moments	
1 (1,1)	s ₁	w ₁
2 (1,2)	s ₁	w ₂
2 (1,3)	s ₁	w ₃

Extract

max. 2 parameters

max. 3 parameters



Fitting

Weights

$$\frac{1}{2\pi i s_0} \oint_{|s|=s_0} ds \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1} \frac{C_{2(n+1)}}{s_0^{n+1}} \delta_{k,n+1}$$

n-th degree monomial in the weight selects the $D = 2(n+1)$

$$\frac{s}{s_0} + \left(\frac{s}{s_0}\right)^2 + \left(\frac{s}{s_0}\right)^3 + \dots$$

\downarrow \downarrow \downarrow

$\mathcal{O}_{4,V/A}$ $\mathcal{O}_{6,V/A}$ $\mathcal{O}_{8,V/A}$

$$\omega_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right)$$

$(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$ Double Pinched

$$\begin{aligned} A_{00,V/A}^{ALEPH} &= A_{00,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}) \\ A_{10,V/A}^{ALEPH} &= A_{10,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}) \\ A_{11,V/A}^{ALEPH} &= A_{11,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A} \mathcal{O}_{12,V/A}) \\ A_{12,V/A}^{ALEPH} &= A_{12,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}) \\ A_{13,V/A}^{ALEPH} &= A_{13,V/A}^{ALEPH}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A}) \end{aligned}$$

Violations

Duality

Experiment
Hadrons

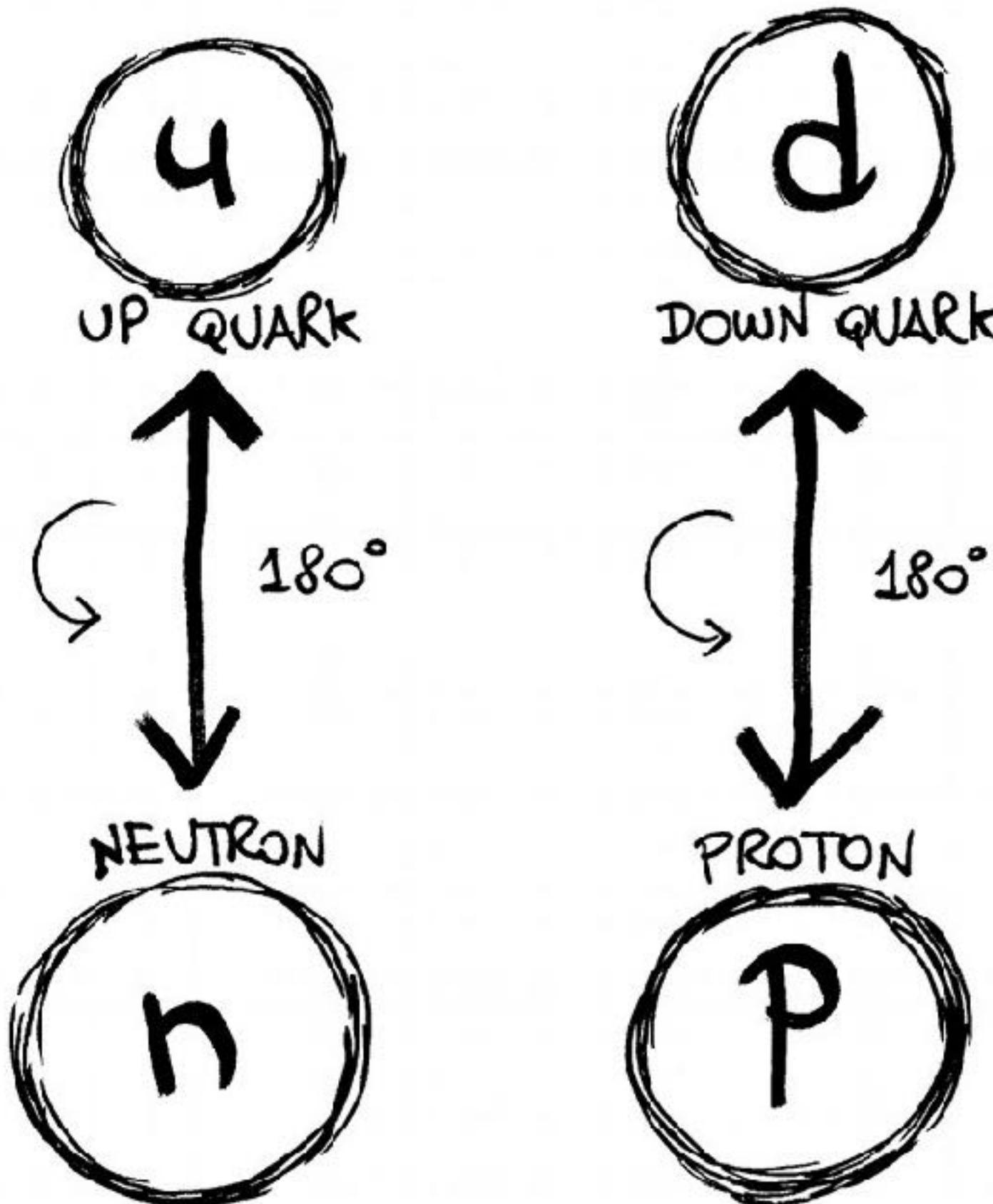


$$\rho_{\text{exp}} = \rho_{\text{th}}$$

Theory
Quark/ Gluons

- What energy is high enough for the quark-hadron duality to set in?
- What role play nonperturbative effects?

FINALLY we HAVE A COMPLETE UNDERSTANDING OF THE QUARK-HADRON DUALITY



IT IS A 2-FOLD ROTATIONAL SYMMETRY!

Model

Model:

8 Parameters

$$\Delta\rho_{V/A}^{DV}(s) = e^{-\delta_{V/A} + \gamma_{V/A}s} \sin(\alpha_{V/A} + \beta_{V/A}s)$$

$$\Delta A_{V/A}^{\omega, DV}(s_0) \equiv \frac{i}{2} \oint_{|s_0|=s_0} \frac{ds}{s_0} \omega(s) \left\{ \Pi_{V/A}(s) - \Pi_{V/A}^{OPE}(s) \right\} = -\pi \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \Delta\rho_{V/A}^{DV}(s)$$

Timeline

- D. Boito
- M. Goltermann
- S. Peris
- K Maltman



- A. Pich
- A. Rodríguez-Sánchez



2008

Model for DV

O. Catà, M. Golterman, S. Peris



2011, 2012, 2015

Fitting DV

Boito, Catà, Golterman, Jamin,
Maltman, Osborne, Peris

Pich, Rodríguez-Sánchez

Critic on Model

Sep. 2016

Hornung, Jamin



2018

Nov. 2016

Defense

Boito, Golterman, Maltman, Peris

Philosophy

$\alpha_s = 0.32$ (CIPT)

Boito, Goltermann, Peris, Maltman

Pro

$\alpha_s = 0.35$ (CIPT)

Pich, Rodríguez-Sánchez

Contra

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Determination of a_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16

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V+A (FOPT)	0.319	-3
V+A (CIPT)	0.339	-16