The QCD Strong Coupling from Hadronic τ decays

A PhD Defense

Dirk Hornung

Universitat Autònoma de Barcelona
Departamiento de Física

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The Strong Coupling α_s

$$\mathcal{L}_{QCD}(x) = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x) + \left[\sum_A \frac{i}{2} \overline{q}^A(x) \gamma^\mu \overleftrightarrow{D}_\mu q^A(x) - m\overline{q}^A(x) q^A(x) \right], \tag{1}$$

with $D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} B^a_\mu$

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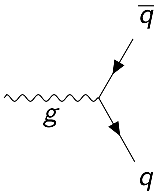
$$\mathcal{L}^{QG-Int}_{QCD}(x) = \sqrt{\pi \alpha_s}\, \overline{q}(x) \lambda \gamma_\mu q(x) G(x) \quad \Rightarrow \tag{2}$$



The Running of the Strong Coupling

$$\alpha_s(m_\tau^2) \approx 0.33$$
$$\alpha_s(m_Z^2) \approx 1.12 \quad (3)$$

$$m_\tau = 1776.86(12)\,\mathrm{MeV}^1$$
$$m_Z = 91.1876(21)\,\mathrm{GeV}^1 \quad (4)$$

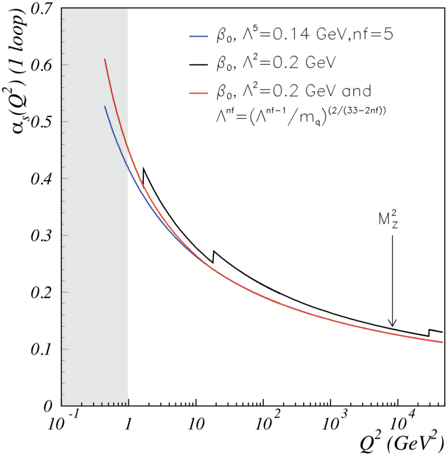
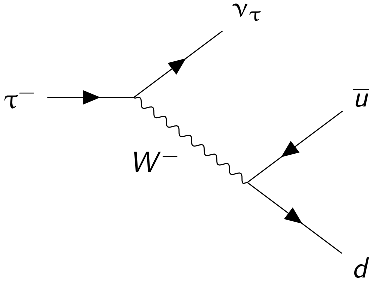


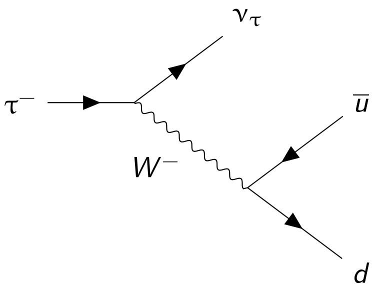
Figure: Taken from Deur, Brodsky, and Teramond, "The QCD Running Coupling", 2016

[1] Tanabashi et al., "Review of Particle Physics", 2018

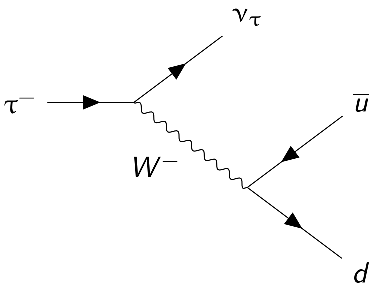
Hadronic τ decays



Hadronic τ decays



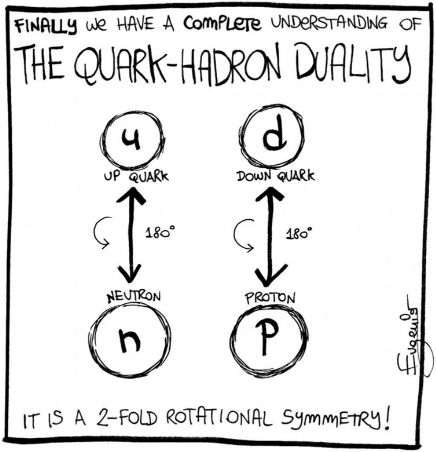
$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \tag{5}$$



$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \tag{5}$$

Name	Symbol	Quark content	Rest mass
Pion	π^-	ud	$139.570\,61(24)\,\mathrm{MeV}$
Pion	π^0	$(uu - dd)/\sqrt{2}$	$134.9770(5)\,\mathrm{MeV}$

Duality


FINALLY WE HAVE A COMPLETE UNDERSTANDING OF
THE QUARK-HADRON DUALITY
u
UP QUARK
d
DOWN QUARK
180°
180°
NEUTRON
n
PROTON
P
IT IS A 2-FOLD ROTATIONAL SYMMETRY!

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Inclusive Tau Decay Ratio:

Two-Point Function:

$$\begin{aligned}\Pi^{\mu\nu}_{V/A}(q^2) &\equiv i\int d^4x\, e^{iqx}\langle 0|T\left\{J^{\mu}_{V/A}(x)J^{\nu}_{V/A}(0)\right\}|0\rangle \\ &= (q^{\mu}q^{\nu} - q^2 g^{\mu\nu})\Pi^{(1)}_{V/A}(q^2) + q^{\mu}q^{\nu}\Pi^{(0)}_{V/A}(q^2)\end{aligned} \tag{7}$$

where

$$J^{\mu}_{V} = \overline{u}\gamma^{\mu}d \qquad \text{and} \qquad J^{\mu}_{A} = \overline{u}\gamma^{\mu}\gamma_5 d$$

Inclusive Tau Decay Ratio:

Adler Function

Adler Function:

$$D(s) \equiv= s\frac{\mathrm{d}}{\mathrm{d}s}\Pi(s) \tag{8}$$

$$D^{(1+0)}(s) \equiv -s\frac{\mathrm{d}}{\mathrm{d}s}\Pi^{(1+0)}(s), \qquad D^{(0)}(s) \equiv \frac{s}{m_\tau^2}\frac{\mathrm{d}}{\mathrm{d}s}\left(s\Pi^{(0)}(s)\right) \tag{9}$$

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Inclusive Tau Decay Ratio:

$$R_\tau = -\pi i \oint_{|s|=m_\tau^2} \frac{\mathrm{d}x}{x}(1-x)^3\left[3(1+x)D^{(1+0)}(m_\tau^2 x) + 4D^{(0)}(m_\tau^2 x)\right] \tag{10}$$

with $\quad x \equiv \dfrac{s}{m_\tau^2}$

Adler Function

$$\Pi_V^{(1+0)}(s) = -\frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=0}^{n+1} c_{n,k} L^k \quad \text{with} \quad L \equiv \log \frac{-s}{\mu^2} \tag{11}$$

Adler Function

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$$D_V^{(1+0)} = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k} L^{k-1} \tag{12}$$

Adler Function

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Adler Function Coefficients

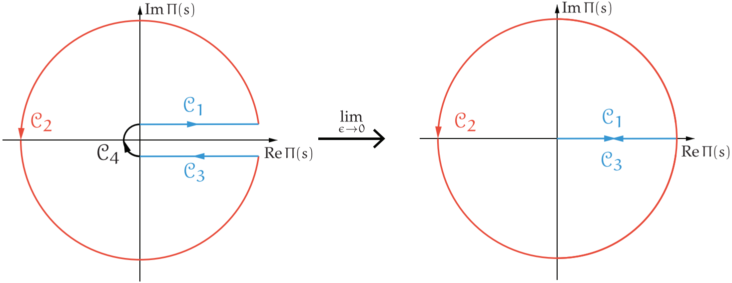
RGE:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}R(q,a_s,m)=\left[\mu\frac{\partial}{\partial\mu}+\mu\frac{\mathrm{d}a_s}{\mathrm{d}\mu}\frac{\partial}{\partial a_s}+\mu\frac{\mathrm{d}m}{\mathrm{d}\mu}\frac{\partial}{\partial m}\right]R(q,a_s,m)=0 \tag{13}$$

$$\left(2\frac{\partial}{\partial L}+\beta\frac{\partial}{\partial a_s}\right)D_V^{(1+0)}=0 \tag{14}$$

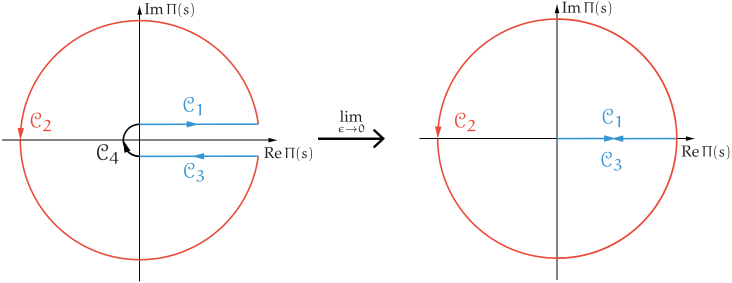
$$\begin{aligned}
c_{0,0}&=-\frac{5}{3},\quad c_{0,1}=1\\
c_{2,1}&=\frac{365}{24}-11\zeta_3-\left(\frac{11}{12}-\frac{2}{3}\zeta_3\right)N_f,\\
c_{3,1}&=\frac{87029}{288}-\frac{1103}{4}\zeta_3+\frac{275}{6}\zeta_5,\\
&\quad-\left(\frac{7847}{216}-\frac{262}{9}\zeta_3+\frac{25}{9}\zeta_5\right)N_f+\left(\frac{151}{162}-\frac{19}{27}\zeta_3\right)N_f^2,\\
c_{4,1}&=\frac{78631453}{20736}-\frac{1704247}{432}\zeta_3+\frac{4185}{8}\zeta_3^2+\frac{34165}{96}\zeta_5-\frac{1995}{16}\zeta_7,\\
c_{5,1}&=283
\end{aligned} \tag{15}$$

QCD Sum Rules



$$\int_0^{m_\tau} \mathrm{Im}\,\Pi(s)\,\mathrm{d}s = -\frac{1}{2\pi i}\oint_{|s|=m_\tau} \Pi(s)\,\mathrm{d}s$$

with $\quad s \equiv -q^2$



$$\int_0^{m_\tau} \mathrm{Im}\,\Pi(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=m_\tau} \Pi(s)\,ds$$

with $\quad s \equiv -q^2$

$$\int_0^{s_0} \omega(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi_{OPE}(s)\,ds \tag{16}$$

$$R_{\tau,V/A} = \frac{N_c}{2}\left(1 + \delta_{pt} + \delta_{npt}\right) \tag{17}$$

FOPT

$$\delta_{pt} = \sum_{n=1}^{\infty} a_{\mu}^{n} \sum_{k=1}^{n} k\, c_{n,k} \frac{1}{2\pi i} \oint_{|x|=1} \frac{\mathrm{d}x}{x} (1-x)^3 (1+x) \log\left(\frac{-m_\tau^2 x}{\mu^2}\right)^{k-1} \tag{18}$$

$$\delta_{FOPT}^{(0)} = \sum_{n=1}^{\infty} a(m_\tau^2)^n \sum_{k=1}^{n} k\, c_{n,k} J_{k-1} \tag{19}$$

$$J_l \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{\mathrm{d}x}{x} (1-x)^3 (1+x) \log^l(-x) \tag{20}$$

CIPT

$$\delta^{(0)}_{CIPT} = \sum_{n=1}^{\infty} c_{n,1}\, J^a_n(m^2_\tau) \tag{21}$$

$$J^a_n(m^2_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1} \frac{\mathrm{d}x}{x}(1-x)^3(1+x)a^n(-m^2_\tau x) \tag{22}$$

FOPT vs CIPT

$$\begin{array}{l} \qquad\qquad\quad \alpha_s^2 \qquad\quad \alpha_s^2 \qquad\quad \alpha_s^3 \qquad\quad \alpha_s^4 \qquad\qquad\quad \alpha_s^5 \\ \delta^{(0)}_{FOPT} = 0.1082 + 0.0609 + 0.0334 + 0.0174(+0.0088) = 0.2200(0.2288) \end{array} \tag{23}$$

$$\delta^{(0)}_{CIPT} = 0.1479 + 0.0297 + 0.0122 + 0.0086(+0.0038) = 0.1984(0.2021). \tag{24}$$

Borel Summation

Borel integral

$$A \equiv \int_0^\infty \mathrm{d}t e^{-t} \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n, \tag{25}$$

Borel transform

$$B[A](t) = \sum_{n=0}^{\infty} \frac{a_k}{n!} t^n. \tag{26}$$

$$\frac{12\pi^2}{N_c} D_V^{1+0}(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(\sqrt{(s)})^{n+1}. \tag{27}$$

Borel Model

$$B[\widehat{D}](u) = B[\widehat{D}_1^{UV}](u) + B[\widehat{D}_2^{IR}](u) + B[\widehat{D}_3^{IR}](u) + d_0^{PO} + d_1^{PO} u, \tag{28}$$

$$B[\widehat{D}_p^{IR}](u) \equiv \frac{d_p^{IR}}{(p-u)^{1+\gamma}} \left[1 + b_1(p-u) + b_2(p-u)^2 + \dots\right] \tag{29}$$

$$B[\widehat{D}_p^{UV}](u) \equiv \frac{d_p^{UV}}{(p+u)^{1+\gamma}} \left[1 + b_1(p+u) + b_2(p+u)^2\right], \tag{30}$$

Beneke and Jamin, "α_s and the τ hadronic width: fixed-order, contour-improved and higher-order perturbation theory", 2008

NPT Contributions

OPE

$$\lim_{x \to y} A(x)B(y) = \sum_n C_n(x-y)\mathcal{O}_n(x) \tag{31}$$

$$\Pi_{OPE}(q^2) = -\frac{1}{3q^2}\sum_n \langle\Omega|\mathcal{O}_n(0)|\Omega\rangle \int d^4 x e^{iqx} C_n(x) \tag{32}$$

$$\Pi^{OPE}_{V/A}(s) = \sum_{D=0,2,4,\ldots} \frac{C^{(D)}\langle\Omega|\mathcal{O}^{(D)}(x)|\Omega\rangle}{(-q^2)^{D/2}} \tag{33}$$

Dimension Four Corrections

$$D_{ij}^{(1+0)}(s)\Big|_{D=4} = \frac{1}{s^2}\sum_n \Omega^{(1+0)}(s/\mu^2)a^n, \tag{34}$$

where the $\Omega^{(1+0)}(s/\mu^2)$ is given by

$$\begin{aligned}\Omega_n^{(1+0)}(s/\mu^2) \;=\; & \frac{1}{6}\langle aGG\rangle p_n^{(1+0)}(s/\mu^2) + \sum_k m_k\langle q_k q_k\rangle r_n^{(1+0)}(s/\mu^2) \\ & + 2\langle m_i q_i q_i + m_j q_j q_j\rangle q_n^{(1+0)}(s/\mu^2) \pm \frac{8}{3}\langle m_j q_i q_i + m_i q_j q_j\rangle t_n^{(1+0)} \\ & - \frac{3}{\pi^2}(m_i^4 + m_j^4)h_n^{(1+0)}(s/\mu^2) \mp \frac{5}{\pi^2}m_i m_j(m_i^2 + m_j^2)k_n^{(1+0)}(s/\mu^2) \\ & + \frac{3}{\pi^2}m_i^2 m_j^2 g_n^{(1+0)}(s/\mu^2) + \sum_k m_k^4 j_n^{(1+0)}(s/\mu^2) + 2\sum_{k\neq l} m_k^2 m_l^2 u_n^{(1+\end{aligned} \tag{35}$$

Dimension Six and Eight Corrections

$$
\begin{aligned}
D^{(1+0)}_{ij,V/A}\Big|_{D=8} &= 4\frac{\rho^{(8)}_{V/A}}{s^4} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=10} &= 5\frac{\rho^{(10)}_{V/A}}{s^5} \\
D^{(1+0)}_{ij,V/A}\Big|_{D=12} &= 6\frac{\rho^{(12)}_{V/A}}{s^6}
\end{aligned}
\tag{36}
$$

Duality Violations

$$R^{\omega}_{\tau,V/A} = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left(1 + \delta^{\omega}_{pt} + \delta^{\omega}_{npt} + \delta^{\omega}_{DV}\right) \tag{37}$$

$$\rho^{DV}_{V/A}(s) = e^{-(\delta_{V/A} + \gamma_{V/A} s)} \sin(\alpha_{V/A} + \beta_{V/A} s) \tag{38}$$

$$D_{\omega}(m_\tau^2) = -12\pi^2 \int_{m_\tau^2}^{\infty} \frac{\mathrm{d}s}{m_\tau^2} \omega(s) \rho^{DV}_{V/A} \tag{39}$$

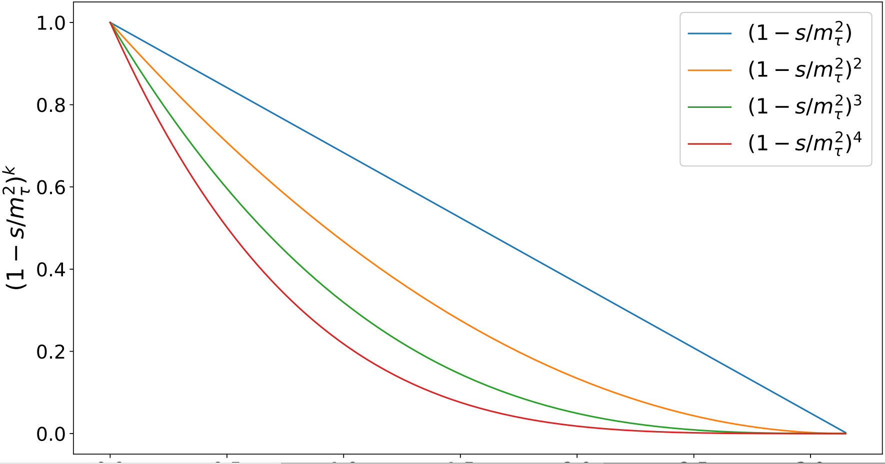
Weights

$$\omega(x) \equiv \sum_i a_i x^i \tag{40}$$

kinematic weights

$$\omega_\tau \equiv (1 - \frac{s}{m_\tau^2})^2 (1 + 2\frac{s}{m_\tau^2}) \tag{41}$$

$$\omega(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \tag{42}$$



Weighting OPE Contributions

$$\oint_C x^k \, \mathrm{d}x = i \int_0^{2\pi} \left(e^{i\theta}\right)^{k+1} \mathrm{d}\theta = \begin{cases} 2\pi i & \text{if } k = -1, \\ 0 & \text{otherwise} \end{cases} . \tag{43}$$

$$R(x)\big|_{D=0,2,4,\dots} = \oint_{|x|=1} \mathrm{d}x \, x^{k-D/2} C^{(D)} \tag{44}$$

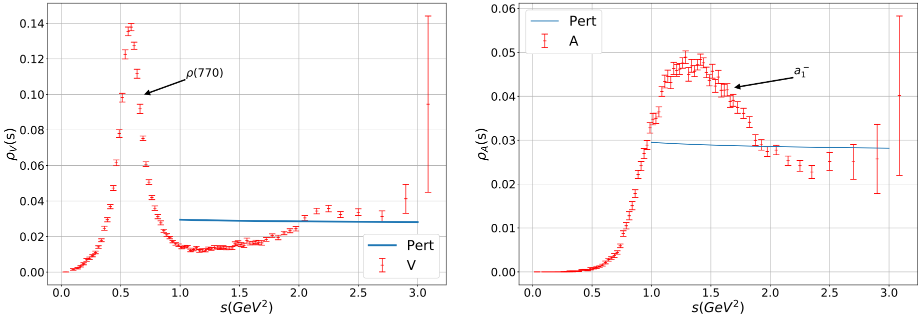
active dimension

$$D = 2(k+1) \tag{45}$$

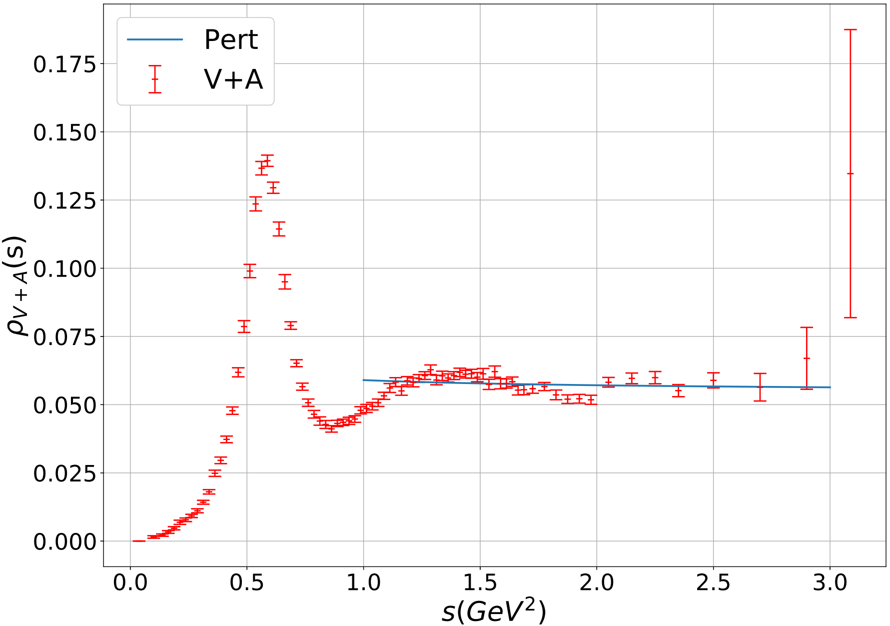
monomial:	x^0	x^1	x^2	x^3	x^5	x^6	x^7
dimension:	$D^{(2)}$	$D^{(4)}$	$D^{(6)}$	$D^{(8)}$	$D^{(10)}$	$D^{(12)}$	$D^{(14)}$

Table: List of monomial and their corresponding "active" dimensions in the OPE. Note that the perturbative contributions of the OPE are always present.

ALEPH data



0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
$\rho_V(s)$
$\rho(770)$
Pert
V
0.0 0.5 1.0 1.5 2.0 2.5 3.0
$s(GeV^2)$
0.06
0.05
0.04
0.03
0.02
0.01
0.00
$\rho_A(s)$
Pert
A
a_1^-
0.0 0.5 1.0 1.5 2.0 2.5 3.0
$s(GeV^2)$


Pert
V+A
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
$\rho_{V+A}(s)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$s(GeV^2)$

$$R_{\tau,V/A} = \frac{\mathcal{B}_{V/A}}{\mathcal{B}_e} = \int_0^{m_\tau^2} \mathrm{d}s \frac{\mathrm{sfm2}_{V/A}(s)}{100\mathcal{B}_e} \tag{46}$$

$$I^{\omega}_{exp,V/A}(s_0) = \frac{s_\tau}{100\mathcal{B}_e s_0} \sum_{i=1}^{N(s_0)} \frac{\omega\left(\frac{s_i}{s_0}\right)}{\omega_\tau\left(\mathrm{sfm2}_{V/A}(s_i)\right)} \tag{47}$$

$$\chi^2 = (I_i^{exp} - I_i^{th}(\vec{\alpha}))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\vec{\alpha})) \tag{48}$$

$$C_{ij} = \text{cov}(I_i^{exp}, I_j^{exp}) \tag{49}$$

$$\chi^2 \approx 1 \tag{50}$$

	Symbol	Term	Expansion	OPE Contributions
Pinched	ω_τ	$(1-x)^2(1+2x)$	$1-3x^2+2x^3$	$D6$, $D8$
	ω_{cube}	$(1-x)^3(1+3x)$	$1-6x^2+8x^3-3x^4$	$D6$, $D8$, $D10$
	$\omega_{quartic}$	$(1-x)^4(1+3x)$	$1-10x^2+20x^3-15x^4+4x^5$	$D6$, $D8$, $D10$, $D12$
Monomial	ω_{M2}	$1-x^2$	$1-x^2$	$D6$
	ω_{M3}	$1-x^3$	$1-x^3$	$D8$
	ω_{M4}	$1-x^4$	$1-x^4$	$D10$
Pinched $+x$	$\omega_{1,0}$	$(1-x)$	$1-x$	$D4$
	$\omega_{2,0}$	$(1-x)^2$	$1-2x+x^2$	$D4$, $D6$
	$\omega_{3,0}$	$(1-x)^3$	$1-3x+3x^2-x^3$	$D4$, $D6$, $D8$
	$\omega_{4,0}$	$(1-x)^4$	$1-4x+6x^2-4x^3+x^4$	$D4$, $D6$, $D8$, $D10$

Kinematic Weight: $\omega_\tau(x) \equiv (1-x)^2(1+2x)$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.200	7	0.3274(42)	-0.82(21)	-1.08(40)	0.21
FOPT	2.100	8	0.3256(38)	-0.43(15)	-0.25(28)	1.30
	2.200	7	0.3308(44)	-0.72(20)	-0.85(38)	0.19
	2.300	6	0.3304(52)	-0.69(25)	-0.80(50)	0.25
	2.400	5	0.3339(70)	-0.91(39)	-1.29(83)	0.10
	2.600	4	0.3398(15)	-1.3(1.0)	-2.3(2.5)	0.01

Cubic Weight: $\omega_{cube}(x) \equiv (1-x)^3(1+3x)$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
2.000	9	0.3228(26)	-0.196(27)	0.075(28)	0.420(56)	1.96
2.100	8	0.3302(40)	-0.52(11)	-0.58(22)	-1.00(45)	0.43
2.200	7	0.3312(43)	-0.56(12)	-0.68(23)	-1.23(50)	0.55
2.300	6	0.336(11)	-0.78(47)	-1.17(98)	-2.38(22)	0.29
2.400	5	0.3330(96)	-0.63(47)	-0.82(10)	-1.51(26)	0.48

Quartic Weight: $\omega(x) \equiv (1-x)^4(1+4x)$

$$\alpha_s(m_\tau^2) = 0.3290(11), \quad \rho^{(6)} = -0.3030(46), \quad \rho^{(8)} = -0.1874(28),$$
$$\rho^{(10)} = 0.3678(45) \quad \text{and} \quad \rho_{(12)} = -0.4071(77). \tag{51}$$

$\omega_{M2}(x) \equiv 1 - x^2$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(6)}$	χ^2/dof
2.100	8	0.3179(47)	-0.42(17)	1.62
2.200	7	0.3248(52)	-0.77(22)	0.38
2.300	6	0.3260(60)	-0.85(28)	0.43

$\omega_{M3}(x) \equiv 1 - x^3$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(8)}$	χ^2/dof
2.100	8	0.3147(44)	-0.27(29)	1.71
2.200	7	0.3214(49)	-1.01(39)	0.41
2.300	6	0.3227(57)	-1.18(54)	0.46
2.400	5	0.3257(67)	-1.58(74)	0.39
2.600	4	0.325(10)	-1.54(1.53)	0.58
2.800	3	0.326(21)	-1.69(4.03)	1.17

Fourth Power Monomial: $\omega_{M4}(x) \equiv 1 - x^4$

s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\rho^{(10)}$	χ^2/dof
2.100	8	0.3136(43)	-0.07(54)	1.75
2.200	7	0.3203(48)	-1.64(77)	0.42
2.300	6	0.3216(56)	-2.01(1.13)	0.47
2.400	5	0.3247(66)	-2.98(1.62)	0.39
2.600	4	0.324(10)	-2.86(3.69)	0.58
2.800	3	0.325(20)	-3.43(10.74)	1.17

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	χ^2/dof
BS	2.100	8	0.3176(47)	-0.0134(48)	1.62
	2.200	7	0.3246(52)	-0.2262(59)	0.38
	2.300	6	0.3260(60)	-0.2453(73)	0.43
FOPT	2.100	8	0.357(12)	-0.072(23)	0.95
	2.200	7	0.3593(97)	-0.079(19)	0.2
	2.300	6	0.3589(99)	-0.078(20)	0.24

$\omega_{2,0} \equiv (1-x)^2$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	χ^2/dof
BS	2.100	8	0.3207(48)	-0.0170(50)	-0.45(17)	1.90
	2.200	7	0.3270(54)	-0.0254(61)	-0.77(21)	0.74
	2.300	6	0.3253(63)	-0.0232(75)	-0.69(27)	0.9
FOPT	2.100	8	0.3331(54)	-0.0108(45)	0.361(76)	1.9
	2.200	7	0.3401(57)	-0.0185(52)	0.220(88)	0.73
	2.300	6	0.3383(68)	-0.0165(67)	0.26(12)	0.89

$\omega_{3,0} \equiv (1-x)^3$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	χ^2/dof
BS	2.000	9	0.3169(20)	-0.0123(34)	-0.29(12)	-0.05(24)	2.0
	2.100	8	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	0.46
	2.200	7	0.3251(17)	-0.02283(56)	-0.689(12)	-0.879(33)	0.56
FOPT	2.000	9	0.33985(81)	-0.01124(43)	0.002(10)	-0.242(26)	1.59
	2.100	8	0.3480(47)	-0.0201(36)	-0.264(89)	-1.03(28)	0.31
	2.200	7	0.3483(23)	-0.0204(41)	-0.27(15)	-1.05(40)	0.41

$\omega_{4,0} \equiv (1-x)^4$

	s_{min}	$\#s_0$s	$\alpha_s(m_\tau^2)$	$aGGInv$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
BS	1.950	10	0.31711(67)	-0.012432(24)	-0.30013(73)	-0.06785(16)	0.26104(50)	1.09
	2.000	9	0.3206(24)	-0.0167(14)	-0.455(38)	-0.373(67)	-0.36(14)	0.83
	2.100	8	0.3248(21)	-0.02230(47)	-0.6724(63)	-0.834(14)	-1.352(28)	0.23
FOPT	1.950	10	0.3416(14)	-0.01306(83)	-0.050(22)	-0.390(59)	-0.50(19)	1.71
	2.100	8	0.3480(25)	-0.0201(27)	-0.264(91)	-1.02(23)	-339.00(20)	0.41

Comparison

	weight	s_{min}	$\alpha_s(m_\tau^2)$	$\langle aGG \rangle_I$	$\rho^{(6)}$	$\rho^{(8)}$	$\rho^{(10)}$	χ^2/dof
FOPT	ω_τ	2.2	0.3308(44)	-	-0.72(20)	-0.85(38)	-	0.19
	ω_{cube}	2.1	0.3302(40)	-	-0.52(11)	-0.58(22)	-1.00(45)	0.43
	ω_{M2}	2.2	0.3248(52)	-	-0.77(22)	-	-	0.38
	ω_{M3}	2.2	0.3214(49)	-	-	-1.01(39)	-	0.41
BS	$\omega_{1,0}$	2.2	0.3246(52)	-0.2262(59)	-	-	-	0.38
	$\omega_{2,0}$	2.2	0.3270(54)	-0.0254(61)	-0.77(21)	-	-	0.74
	$\omega_{3,0}$	2.1	0.3239(40)	-0.0212(42)	-0.63(15)	-0.74(29)	-	0.46

Results

- $\alpha_s(m_\tau^2) = 0.3261 \pm 0.0050$
- $\rho^{(6)} = -0.68 \pm 0.2$
- $\rho^{(8)} = -0.80 \pm 0.38$

Results

- $\alpha_s(m_\tau^2) = 0.3261 \pm 0.0050$
- $\rho^{(6)} = -0.68 \pm 0.2$
- $\rho^{(8)} = -0.80 \pm 0.38$

- DV not present if using single pinched weights in the V+A channel
- FOPT more valid than CIPT
- $\alpha_s(m_Z^2) = 0.11940(60)$

Thank you

Beneke, Martin and Matthias Jamin. "α_s and the τ hadronic width: fixed-order, contour-improved and higher-order perturbation theory". In: *JHEP* 09 (2008), p. 044. DOI: 10.1088/1126-6708/2008/09/044. arXiv: 0806.3156 [hep-ph].

Deur, Alexandre, Stanley J. Brodsky, and Guy F. de Teramond. "The QCD Running Coupling". In: *Prog. Part. Nucl. Phys.* 90 (2016), pp. 1–74. DOI: 10.1016/j.ppnp.2016.04.003. arXiv: 1604.08082 [hep-ph].

Tanabashi, M. et al. "Review of Particle Physics". In: *Phys. Rev.* D98.3 (2018), p. 030001. DOI: 10.1103/PhysRevD.98.030001.