

# Measurement of the CKM angle $\gamma$ using $B_s^0 \rightarrow D_s K \pi\pi$ decays

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## Abstract

We present the first measurement of the weak phase  $2\beta + \gamma$  obtained from a time-dependent (amplitude) analysis of  $B_s^0 \rightarrow D_s K \pi\pi$  decays using proton-proton collision data corresponding to an integrated luminosity of **xxx** fb<sup>-1</sup> recorded by the LHCb detector.



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# 1 Introduction

- The weak phase  $\gamma$  is the least well known angle of the CKM unitary triangle. A key channel to measure  $\gamma$  is the time-dependent analysis of  $B_s^0 \rightarrow D_s K$  decays [1], [2].  
 The  $B_s^0 \rightarrow D_s K\pi\pi$  proceeds at tree level via the transitions shown in Fig. 1.1 a) and b).

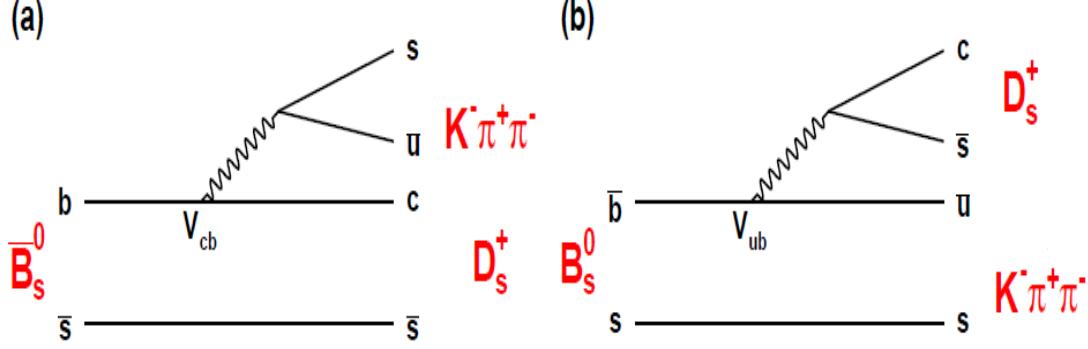


Figure 1.1: Feynman diagram of the  $B_s^0 \rightarrow D_s K\pi\pi$  decay, proceeding via a)  $b \rightarrow c$  transitions or b)  $b \rightarrow u$  transitions.

To measure the weak CKM phase  $\gamma \equiv \arg[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)]$ , a decay with interference between  $b \rightarrow c$  and  $b \rightarrow u$  transitions at tree level is needed [1]. As illustrated in Fig. 1.1, this is the case for the presented decay mode. A measurement of  $\gamma$  using  $B_s^0 \rightarrow D_s K\pi\pi$  decays, where the  $K\pi\pi$  subsystem is dominated by excited kaon states such as the  $K_1(1270)$  and  $K_1(1400)$  resonances, will succeed the branching ratio measurement presented in this note. It is complementary to the above mentioned analysis of  $B_s^0 \rightarrow D_s K$ , making use of a fully charged final state, where every track is detected in the vertex locator. To account for the non-constant strong phase across the Dalitz plot, one can either develop a time-dependent amplitude model or select a suitable phase-space region and introduce a coherence factor as additional hadronic parameter to the fit.  
 This analysis is based on the first observation of the  $B_s^0 \rightarrow D_s K\pi\pi$  decay presented in [3] and [4], where its branching ratio is measured relative to  $B_s^0 \rightarrow D_s \pi\pi\pi$ . The result obtained by the previous analysis is  $0.052 \pm 0.005 \pm 0.003$ , where the uncertainties are statistical and systematical, respectively. The branching ratio measurement is updated, exploiting the full Run 1 data sample, corresponding to  $3 \text{ fb}^{-1}$  of integrated luminosity.

## 2 Sensitivity studies

### 2.1 PDF

First, we define the purely hadronic amplitudes for a given phasespace point  $x$ . The weak phase dependence is written latter explicitly in the pdf.

$$A(B_s^0 \rightarrow D_s^- K^+ \pi\pi) \equiv A(x) = \sum_i a_i A_i(x) \quad (2.1)$$

$$A(B_s^0 \rightarrow D_s^+ K^- \pi\pi) \equiv \bar{A}(\bar{x}) = \sum_i \bar{a}_i \bar{A}_i(\bar{x}) \quad (2.2)$$

$$A(\bar{B}_s^0 \rightarrow D_s^- K^+ \pi\pi) = \bar{A}(x) \text{ (Assuming no direct CPV)} \quad (2.3)$$

$$A(\bar{B}_s^0 \rightarrow D_s^+ K^- \pi\pi) = A(\bar{x}) \text{ (Assuming no direct CPV)} \quad (2.4)$$

The full time-dependent amplitude pdf is given by:

$$\begin{aligned} P(x, t, q_t, q_f) \propto & [(|A(x)|^2 + |\bar{A}(x)|^2) \cosh\left(\frac{\Delta\Gamma t}{2}\right) \\ & + q_t q_f (|A(x)|^2 - |\bar{A}(x)|^2) \cos(m_s t) \\ & - 2\text{Re}(A(x)^* \bar{A}(x) e^{-iq_f(\gamma-2\beta_s)}) \sinh\left(\frac{\Delta\Gamma t}{2}\right) \\ & - 2q_t q_f \text{Im}(A(x)^* \bar{A}(x) e^{-iq_f(\gamma-2\beta_s)}) \sin(m_s t)] e^{-\Gamma t} \end{aligned} \quad (2.5)$$

where  $q_t = +1$  ( $-1$ ) for a  $B_s^0$  ( $\bar{B}_s^0$ ) tag and  $q_f = +1$  ( $-1$ ) for  $D_s^- K^+ \pi\pi$  ( $D_s^+ K^- \pi\pi$ ) final states.

Integrating over the phasespace, we get

$$\begin{aligned} \int P(x, t, q_t, q_f) dx \propto & [\cosh\left(\frac{\Delta\Gamma t}{2}\right) \\ & + q_t q_f \left(\frac{1-r^2}{1+r^2}\right) \cos(m_s t) \\ & - 2 \left(\frac{\kappa r \cos(\delta - q_f(\gamma - 2\beta_s))}{1+r^2}\right) \sinh\left(\frac{\Delta\Gamma t}{2}\right) \\ & - 2q_t q_f \left(\frac{\kappa r \sin(\delta - q_f(\gamma - 2\beta_s))}{1+r^2}\right) \sin(m_s t)] e^{-\Gamma t} \\ = & [\cosh\left(\frac{\Delta\Gamma t}{2}\right) + q_t q_f C \cos(m_s t) - \kappa D_{q_f} \sinh\left(\frac{\Delta\Gamma t}{2}\right) - q_t \kappa S_{q_f} \sin(m_s t)] e^{-\Gamma t} \end{aligned} \quad (2.6)$$

where the  $C, D_{q_f}, S_{q_f}$  are defined exactly as for  $D_s K$ . The coherence factor is defined as :

$$\kappa e^{i\delta} \equiv \frac{\int A(x)^* \bar{A}(x) dx}{\sqrt{\int |A(x)|^2 dx} \sqrt{\int |\bar{A}(x)|^2 dx}} \quad (2.7)$$

$$r \equiv \frac{\sqrt{\int |\bar{A}(x)|^2 dx}}{\sqrt{\int |A(x)|^2 dx}} \quad (2.8)$$

<sup>30</sup> and appears in front of the  $D_{q_f}, S_{q_f}$  terms. This means one additional fit parameter for  
<sup>31</sup> the lifetime fit. In the limit of only one contributing resonance  $\kappa \rightarrow 1$ .

<sup>32</sup>

## 33 2.2 Results

34 Assumptions:

- 35 • Use amplitudes from flavor-averaged, time-integrated fit
- 36 •  $r = 0.4$  (ratio of CKM elements)
- 37 • PDG values for:  $\tau, \Delta m_s, \Delta\Gamma, \beta_s$
- 38 •  $\epsilon(x, t) = \text{const.}$ , perfect resolution
- 39 •  $\epsilon_{Tag} = 0.66, \langle \omega \rangle = 0.4$
- 40 •  $N_{signal} = 3000$  (Run1+15/16 data)

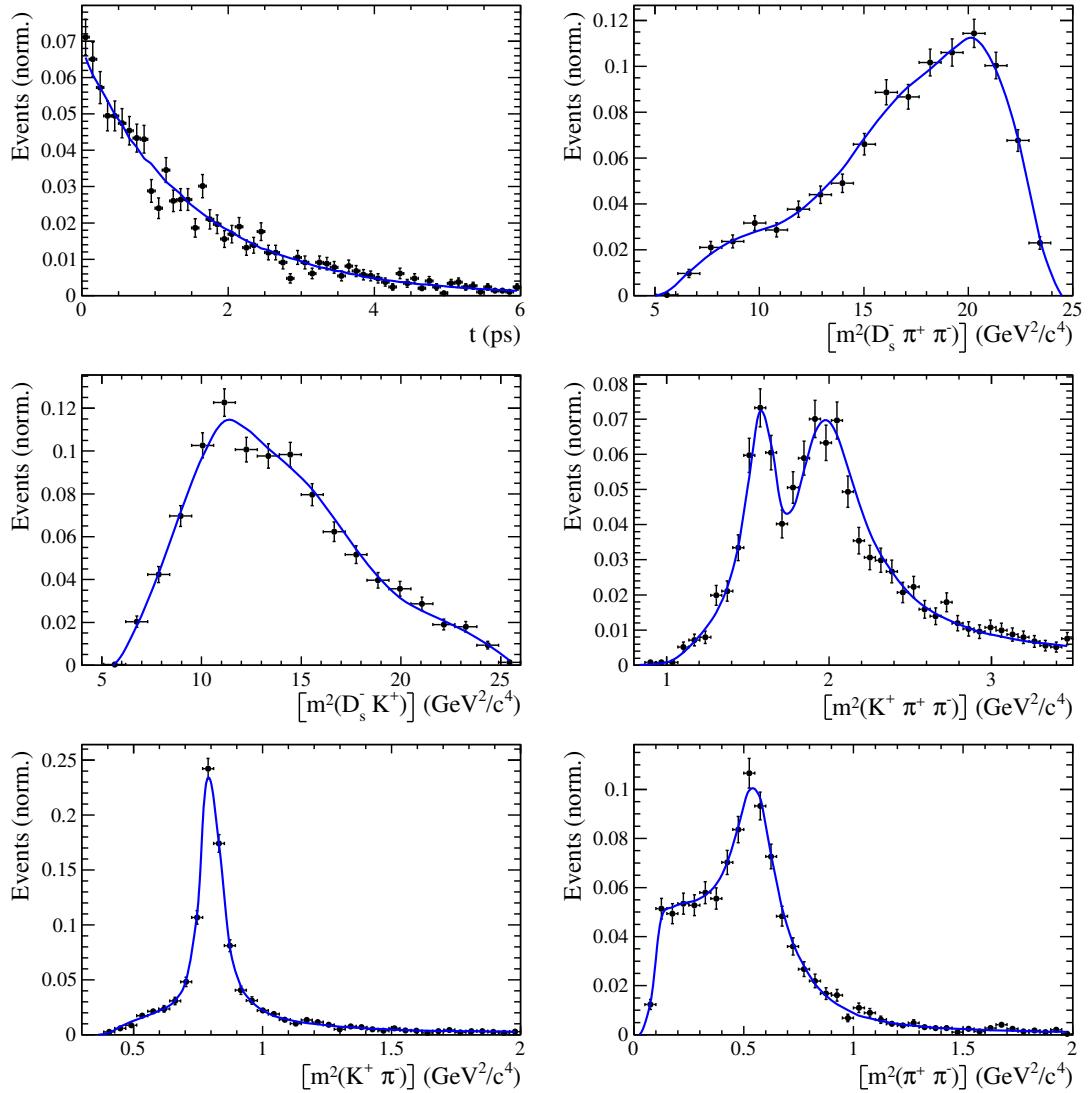
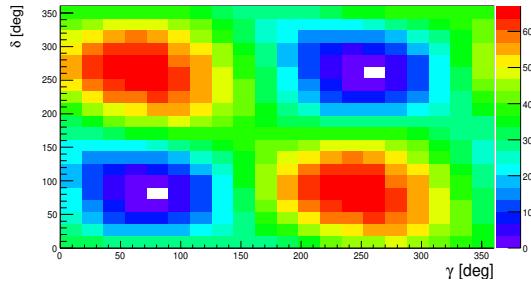


Figure 2.1: Example toy fit



Generated values:

$$\gamma = 70^\circ, \delta = 100^\circ$$

Fit result:

$$\begin{aligned} \gamma &= 74 \pm 15^\circ, \delta = 84 \pm 15^\circ \\ (\gamma &= 254 \pm 15^\circ, \delta = 264 \pm 15^\circ) \end{aligned}$$

Figure 2.2: Likelihood scan

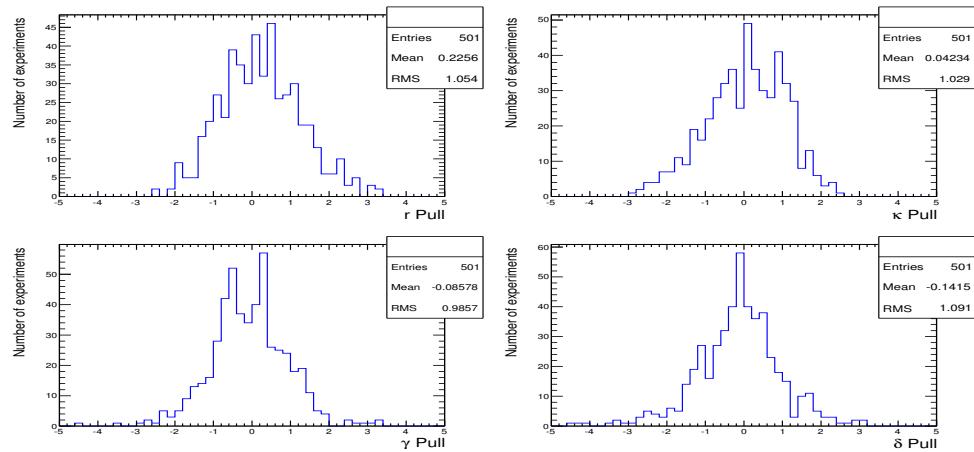


Figure 2.3: Pulls

Table 2.1

	Generated	Full PDF	Phasespace integrated
$r$	0.4	$0.38 \pm 0.06$	unstable
$\kappa$	<b>0.2</b>	$0.23 \pm 0.13$	0.2 (fixed)
$\delta$	100	$99 \pm 22$	unstable
$\gamma$	70	$70 \pm 17$	unstable

	Generated	Full PDF	Phasespace integrated
$r$	0.4	$0.44 \pm 0.07$	$0.43 \pm 0.11$
$\kappa$	<b>0.4</b>	$0.41 \pm 0.14$	0.4 (fixed)
$\delta$	100	$101 \pm 19$	$95 \pm 41$
$\gamma$	70	$69 \pm 16$	$66 \pm 40$

	Generated	Full PDF	Phasespace integrated
$r$	0.4	$0.41 \pm 0.08$	$0.39 \pm 0.11$
$\kappa$	<b>0.6</b>	$0.60 \pm 0.13$	0.6 (fixed)
$\delta$	100	$98 \pm 17$	$92 \pm 25$
$\gamma$	70	$68 \pm 17$	$65 \pm 28$

	Generated	Full PDF	Phasespace integrated
$r$	0.4	$0.42 \pm 0.09$	$0.39 \pm 0.09$
$\kappa$	<b>1.0</b>	$0.96 \pm 0.03$	1.0 (fixed)
$\delta$	100	$100 \pm 17$	$100 \pm 17$
$\gamma$	70	$66 \pm 17$	$67 \pm 17$

41 **3 Selection**

42 For the presented analysis, we reconstruct the  $B_s^0 \rightarrow D_s K\pi\pi$  decay through two different  
 43 final states of the  $D_s$  meson,  $D_s \rightarrow KK\pi$  and  $D_s \rightarrow \pi\pi\pi$ . Of those two final states  
 44  $D_s \rightarrow KK\pi$  is the most prominent one, while  $\mathcal{BR}(D_s \rightarrow \pi\pi\pi) \approx 0.2 \cdot \mathcal{BR}(D_s \rightarrow KK\pi)$   
 45 holds for the other one.

46 A two-fold approach is used to isolate the  $B_s^0 \rightarrow D_s K\pi\pi$  candidates from data passing  
 47 the stripping line. First, further one-dimensional cuts are applied to reduce the level of  
 48 combinatorial background and to veto some specific physical background. This stage is  
 49 specific to the respective final state in which the  $D_s$  meson is reconstructed, since different  
 50 physical backgrounds, depending on the respective final state, have to be taken into  
 51 account. After that, a multivariate classifier is trained which combines the information  
 52 of several input variables, including their correlation, into one powerful discriminator  
 53 between signal and combinatorial background. For this stage, all possible  $D_s$  final states  
 54 are treated equally.

55 **3.1 Cut-based selection**

56 In order to minimize the contribution of combinatorial background to our samples, we  
 57 apply the following cuts to the b hadron:

- 58     • DIRA > 0.99994
- 59     • min IP  $\chi^2 < 20$  to any PV,
- 60     • FD  $\chi^2 > 100$  to any PV,
- 61     • Vertex  $\chi^2/\text{nDoF} < 8$ ,
- 62     •  $(Z_{D_s} - Z_{B_s^0}) > 0$ , where  $Z_M$  is the z-component of the position  $\vec{x}$  of the decay vertex  
 63       for the  $B_s^0/D_s$  meson.

64 Additionally, we veto various physical backgrounds, which have either the same final  
 65 state as our signal decay, or can contribute via a single misidentification of  $K \rightarrow \pi$  or  
 66  $K \rightarrow p$ . In the following, the vetoes are ordered by the reconstructed  $D_s$  final state they  
 67 apply to:

68 1. All:

- 69     (a)  $B_s^0 \rightarrow D_s^+ D_s^-$  :  $|M(K\pi\pi) - m_{D_s}| > 20$  MeV/ $c^2$ .
- 70     (b)  $B_s^0 \rightarrow D_s^- K^+ K^- \pi^+$  : possible with single missID of  $K^- \rightarrow \pi^-$ , rejected by  
 71       requiring  $\pi^-$  to fulfill  $\text{DLL}_{K\pi} < 5$ .

72 2.  $D_s \rightarrow KK\pi$

- 73     (a)  $B^0 \rightarrow D^+(\rightarrow K^+\pi^-\pi^+) K\pi\pi$  : possible with single missID of  $\pi^+ \rightarrow K^+$ , vetoed  
 74       by changing particle hypothesis and recompute  $|M(K^+\pi^-\pi^+) - m_{D_p}| > 30$   
 75       MeV/ $c^2$ , or the  $K^+$  has to fulfill  $\text{DLL}_{K\pi} > 10$ .

- 76           (b)  $\Lambda_b^0 \rightarrow \Lambda_c^+ (\rightarrow pK^-\pi^+)K\pi\pi$  : possible with single missID of  $p \rightarrow K^+$ , vetoed by  
 77           changing particle hypothesis and recompute  $M(pK^-\pi^+) - m_{\Lambda_c^+} > 30 \text{ MeV}/c^2$ ,  
 78           or the  $K^+$  has to fulfill  $(\text{DLL}_{K\pi} - \text{DLL}_{p\pi}) > 5$ .  
 79           (c)  $D^0 \rightarrow KK$  :  $D^0$  combined with a random  $\pi$  can fake a  $D_s \rightarrow KK\pi$  decay and  
 80           be a background to our signal, vetoed by requiring  $M(KK) < 1840 \text{ MeV}/c^2$ .

81       3.  $D_s \rightarrow \pi\pi\pi$

- 82           (a)  $D^0 \rightarrow \pi\pi$  : combined with a random  $\pi$  can fake a  $D_s \rightarrow \pi\pi\pi$  decay and be a  
 83           background to our signal, vetoed by requiring both possible combinations to  
 84           have  $M(\pi\pi) < 1700 \text{ MeV}/c^2$ .

85       The most prominent final state used in this analysis is  $B_s^0 \rightarrow D_s(\rightarrow KK\pi)K\pi\pi$ , where  
 86       the  $D_s$  decay can either proceed via the narrow  $\phi$  resonance, the broader  $K^{*0}$  resonance, or  
 87       non resonant. Depending on the decay process being resonant or not, we apply additional  
 88       PID requirements on this final state:

- 89       • resonant case:

- 90           –  $D_s^+ \rightarrow \phi\pi^+$ , with  $|M(K^+K^-) - m_\phi| < 20 \text{ MeV}/c^2$  : no additional requirements,  
   91           since  $\phi$  is narrow and almost pure  $K^+K^-$ .
- 92           –  $D_s^+ \rightarrow \bar{K}^{*0}K^+$ , with  $|M(K^-\pi^+) - m_{K^{*0}}| < 75 \text{ MeV}/c^2$  :  $\text{DLL}_{K\pi} > 0$  for kaons,  
   93           since this resonance is more than ten times broader than  $\phi$ .

- 94       • non resonant case:  $\text{DLL}_{K\pi} > 5$  for kaons, since the non resonant category has  
 95       significant charmless contributions.

96       For the  $D_s \rightarrow \pi\pi\pi$  final state, we apply global PID requirements:

- 97       •  $\text{DLL}_{K\pi} < 10$  for all pions.  
 98       •  $\text{DLL}_{p\pi} < 10$  for all pions.

99       3.2 Multivariate stage

100      We use TMVA [5] to train a multivariate discriminator, which is used to further improve  
 101      the signal to background ratio. The following variables are used for the training:

- 102       •  $\text{max}(\text{ghostProb})$  over all tracks
- 103       •  $\text{cone}(p_T)$  asymmetry of every track, which is defined to be the difference between the  
   104        $p_T$  of the  $\pi/K$  and the sum of all other  $p_T$  in a cone of radius  $r = \sqrt{(\Delta\Phi)^2 + (\Delta\eta)^2}$   
   105        $< 1 \text{ rad}$  around the signal  $\pi/K$  track.
- 106       •  $\text{min}(\text{IP}\chi^2)$  over the  $X_s$  daughters
- 107       •  $\text{max}(\text{DOCA})$  over all pairs of  $X_s$  daughters
- 108       •  $\text{min}(\text{IP}\chi^2)$  over the  $D_s$  daughters

- $D_s$  and  $B_s^0$  DIRA
- $D_s$  FD significance
- $\max(\cos(D_s h_i))$ , where  $\cos(D_s h_i)$  is the cosine of the angle between the  $D_s$  and another track i in the plane transverse to the beam
- $B_s^0$  IP $\chi^2$ , FD $\chi^2$  and Vertex  $\chi^2$

Various classifiers were investigated in order to select the best performing discriminator. Consequently, a boosted decision tree with gradient boost (BDTG) is chosen as nominal classifier. We use truth-matched MC as signal input. Simulated signal candidates are required to pass the same trigger, stripping and preselection requirements, that were used to select the data samples. For the background we use events from the high mass sideband ( $m_{B_s^0 \text{candidate}} > 5600 \text{ MeV}/c^2$ ) of our data samples. As shown in Fig. 3.1, this mass region is sufficiently far away from signal structures and is expected to be dominantly composed of combinatorial background. For completeness, the mass distribution of preselected  $D_s \rightarrow hh$  candidates (where  $h = \pi$  or  $h = K$ ) is also shown in Fig. 3.1.

123

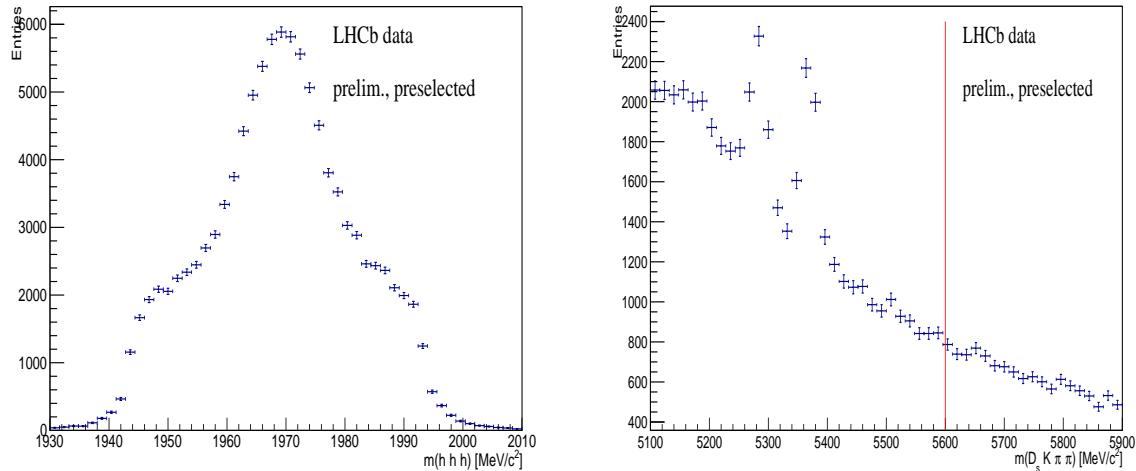


Figure 3.1: Invariant mass distribution of preselected (left)  $D_s \rightarrow hh$  and (right)  $B_s^0 \rightarrow D_s K \pi \pi$  candidates. For the  $B_s^0 \rightarrow D_s K \pi \pi$  candidates, the region right from the red colored line with  $m_{B_s^0 \text{candidate}} > 5600 \text{ MeV}/c^2$  is used as background input for the boosted decision tree.

124 The distributions of the input variables for signal and background and the BDTG  
125 output distribution are shown in the appendix.

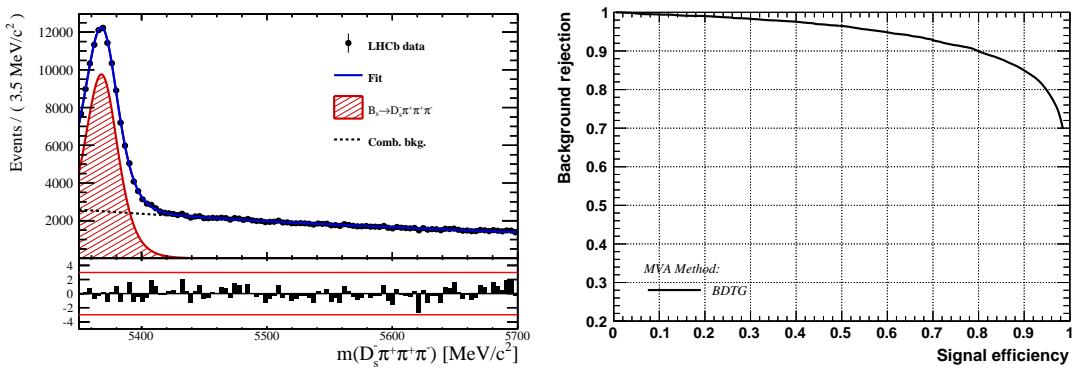


Figure 3.2

126 **4 Fits to invariant mass distributions of signal and**  
 127 **normalization channel**

128 In order to properly model the invariant mass distribution of  $B_s^0 \rightarrow D_s K\pi\pi$  and  $B_s^0 \rightarrow$   
 129  $D_s \pi\pi\pi$  candidates, the expected signal shape, as well as the expected shape for the  
 130 combinatorial and physical background has to be known. This model can then be used to  
 131 fit the distributions and obtain signal sWeights [6], which are employed to suppress the  
 132 residual background that is still left in the sample, for the time-dependent amplitude fit.

133 **4.1 Signal models for  $m(D_s \pi\pi\pi)$  and  $m(D_s K\pi\pi)$**

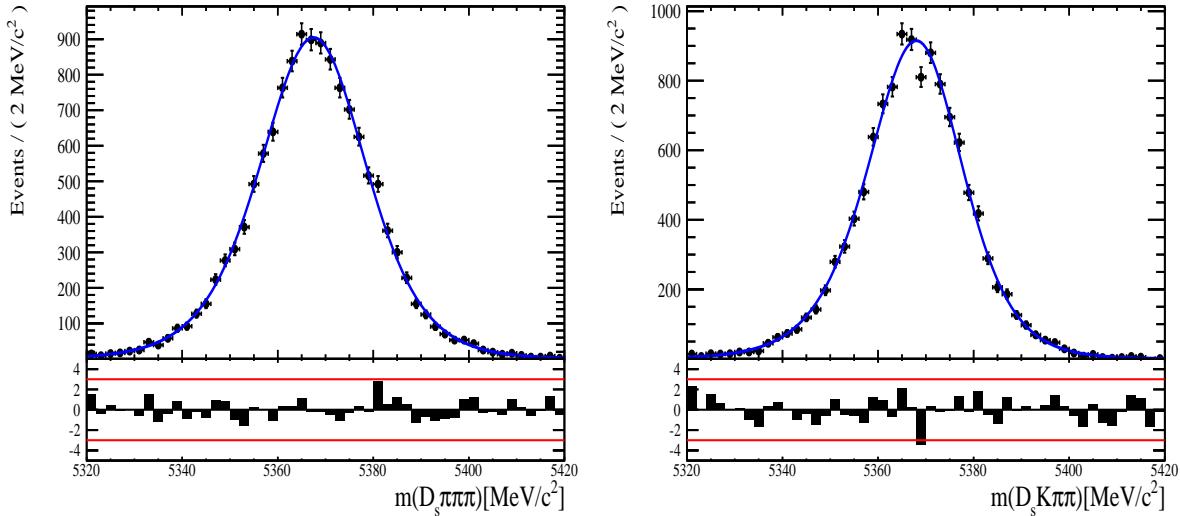


Figure 4.1: Invariant mass distributions of simulated (left)  $B_s^0 \rightarrow D_s \pi\pi\pi$  and (right)  $B_s^0 \rightarrow D_s K\pi\pi$  events. A fit of a RooJohnsonSU function to each distribution is overlaid.

134 The mass distribution of  $B_s^0 \rightarrow D_s K\pi\pi$  signals is modeled using a Johnson SU  
 135 function [7], which is a gaussian function with a Landau-like tail on one side,

$$J(m_{B_s^0}; \mu, \sigma, \gamma, \delta) = \frac{\delta}{\sigma 2\pi \sqrt{1 + (\frac{m_{B_s^0} - \mu}{\sigma})^2}} \exp\left(-\frac{1}{2}[\gamma + \delta \operatorname{Argsh}(\frac{m_{B_s^0} - \mu}{\sigma})]^2\right). \quad (4.1)$$

136 The sign of  $\gamma$  in Eq. 4.1 determines whether the tail is located at lower ( $\gamma > 0$ )  
 137 or higher ( $\gamma < 0$ ) invariant mass values than the mean  $\mu$  of the gaussian function and  
 138  $\delta$  describes the (a)symmetry of the fitted distribution. Higher values of  $\delta$  result in a  
 139 more symmetric, gaussian-like function. Another Johnson SU function function is used  
 140 to account for the contribution of the  $B^0 \rightarrow D_s K\pi\pi$  decay, which is also present in  
 141 the  $m(D_s K\pi\pi)$  spectrum. The width, as well as the tail parameters are fixed to values  
 142 obtained by a fit to the invariant mass distribution of simulated events shown in Fig 4.1.  
 143 A linear scaling factor for the mean  $\mu$  and width  $\sigma$  is floated in the fit to account for  
 144 possible differences between the simulation and real data.

145 The same approach is used to describe the invariant mass distribution of  $B_s^0 \rightarrow D_s \pi\pi\pi$

146 candidates. A Johnson SU function is used to model the signal, the parameters are  
147 determined by a fit to the invariant mass of simulated  $B_s^0 \rightarrow D_s\pi\pi\pi$  decays, shown in  
148 Fig 4.1. A scale factor for the width and the mean is floated to account for differences  
149 between data and MC.

## 150 4.2 Background models for $m(D_s\pi\pi\pi)$

151 Different background sources arise in the invariant mass spectrum of candidates in the  
152 normalization mode.

153 The following backgrounds have to be accounted for:

- 154 • Combinatorial background: This contribution arises from either a real  $D_s$ , which is paired with random tracks to form the  $B_s^0$  candidates, or via real  $X_d$ 's, which are combined with three tracks that fake a  $D_s$  candidate to form a fake  $B_s^0$ .
- 157 • Partially reconstructed  $B^0/B_s^0 \rightarrow D_s^*\pi\pi\pi$  decays, with  $D_s^* \rightarrow D_s\gamma$  or  $D_s^* \rightarrow D_s\pi^0$ , where the  $\gamma/\pi^0$  is not reconstructed in the decay chain.

159 In both cases of combinatorial background, the distribution in the invariant mass of  
160  $B_s^0$  candidates is expected to be smooth and decrease with higher masses. Therefore, one  
161 exponential function is used to model these contributions.

162 The shape of the  $B_s^0 \rightarrow D_s^*\pi\pi\pi$  contribution is expected to be peaking in the  $m(D_s\pi\pi\pi)$   
163 spectrum, with large tails due to the missing momentum, which is carried away by the  $\pi^0$   
164 or  $\gamma$ . The pion or photon from  $D_s^* \rightarrow D_s(\gamma/\pi^0)$  is excluded from the reconstruction. We  
165 model the shape of this contribution using the sum of three bifurcated Gaussian functions.  
166 The shape parameters, as well as the yield of this contribution, are directly determined  
167 on data from a fit to the  $m(D_s\pi\pi\pi)$  invariant mass distribution.

## 168 4.3 Background models for $m(D_sK\pi\pi)$

169 For the signal channel, the following background sources have to be considered:

- 170 • Combinatorial background: same contributions as discussed in Sec. 4.2.
- 171 • Partially reconstructed  $B_s^0 \rightarrow D_s^*K\pi\pi$  decays, with  $D_s^* \rightarrow D_s\gamma$  or  $D_s^* \rightarrow D_s\pi^0$ , where the  $\gamma/\pi^0$  is not reconstructed in the decay chain.
- 173 • Partially reconstructed  $B^0 \rightarrow D_s^*K\pi\pi$  decays, with  $D_s^* \rightarrow D_s\gamma$  or  $D_s^* \rightarrow D_s\pi^0$ , where the  $\gamma/\pi^0$  is not reconstructed in the decay chain.
- 175 • Misidentified  $B_s^0 \rightarrow D_s\pi\pi\pi$  decays, where one of the pions is wrongly identified as a kaon  $\pi \rightarrow K$ .
- 177 • Misidentified, partially reconstructed  $B_s^0 \rightarrow D_s^*\pi\pi\pi$  decays, where one of the pions is wrongly identified as a kaon  $\pi \rightarrow K$  and the  $\gamma/\pi^0$  from  $D_s^* \rightarrow D_s\gamma/\pi^0$  is not reconstructed.

180 The combinatorial background is expected to be non-peaking in the spectrum of the  
181 invariant mass of  $B_s^0 \rightarrow D_sK\pi\pi$  candidates. An exponential function is used to model  
182 this contribution.

183 The shape of the partially reconstructed background without misID is taken from our  
 184 normalization channel, where it can be directly fitted by the sum of three bifurcated  
 185 Gaussian functions as described above. In the signal mass fit, all shape parameters for  
 186 the  $B_s^0 \rightarrow D_s^* K\pi\pi$  background are fixed to the input values from our normalization fit.

187 For the contribution of the  $B^0 \rightarrow D_s^* K\pi\pi$  background, the same shape is used but  
 188 the means  $\mu_i$  of the bifurcated gaussians are shifted down by  $m_{B_s^0} - m_{B^0}$  [?]. The yields  
 189 of both contributions are directly determined in the nominal fit.

190 To determine the shape of misidentified  $B_s^0 \rightarrow D_s \pi\pi\pi$  candidates in the  $m(D_s K\pi\pi)$   
 191 spectrum, we take a truth-matched signal MC sample of our normalization channel. We  
 192 then use the PIDCalib package to determine the  $\pi \rightarrow K$  fake rate. For every candidate  
 193 in our MC sample, a (momentum)  $p$  and (pseudorapidity)  $\eta$ -dependent event weight is  
 194 computed and assigned. We flip the particle hypothesis from pion to kaon for the  $\pi$  with  
 195 the biggest miss-ID weight for each event and recompute the invariant  $B_s^0$  mass. This  
 196 distribution is then modeled using two Crystal Ball functions. The distribution and the  
 197 fit are shown in Fig. 4.2(left).

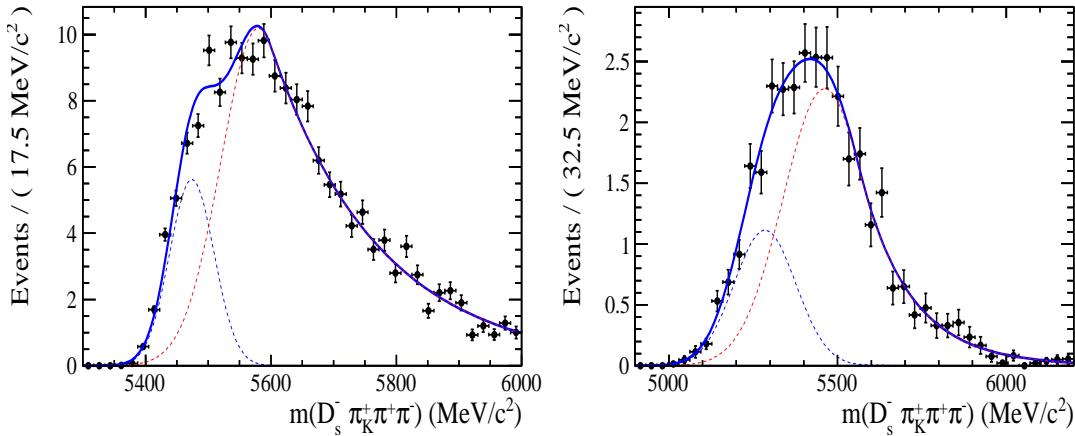


Figure 4.2: Invariant mass distribution of (left) simulated  $B_s^0 \rightarrow D_s \pi\pi\pi$  events, where one of the  $\pi$ 's is reconstructed as a  $K$  and the misID probability for each event is taken into account. The corresponding distribution for simulated  $B_s^0 \rightarrow D_s^* \pi\pi\pi$  events, where the  $\gamma/\pi^0$  from the  $D_s^*$  is excluded from reconstruction, is shown on the right. The solid, black curve on each plot corresponds to the fit consisting of two Crystal Ball functions.

198 The expected yield of misidentified  $B_s^0 \rightarrow D_s \pi\pi\pi$  candidates in the  $m(D_s K\pi\pi)$  spec-  
 199 trum is computed by multiplying the fake probability of  $\propto 3.2\%$ , which is derived from  
 200 PIDCalib, by the yield of  $B_s^0 \rightarrow D_s \pi\pi\pi$  signal candidates, determined in the nominal  
 201 mass fit of our normalization channel.

202 In the same way as mentioned above, we can determine the rate of misidentified, partially  
 203 reconstructed  $B_s^0 \rightarrow D_s^* \pi\pi\pi$  decays in our sample of  $B_s^0 \rightarrow D_s K\pi\pi$  decays using PIDCalib  
 204 and a MC sample of  $B_s^0 \rightarrow D_s^* \pi\pi\pi$  events. The invariant mass distribution we obtain  
 205 when we exclude the  $\gamma/\pi^0$ , flip the the particle hypothesis  $\pi \rightarrow K$  and apply the event  
 206 weights given by the fake rate, is shown in Fig. 4.2 (right). The fit of two Crystal Ball  
 207 functions to this distribution is overlaid. The yield of this contribution is determined  
 208 from the yield of  $B_s^0 \rightarrow D_s^* \pi\pi\pi$  candidates in the nominal mass fit of our normalization  
 209 channel, multiplied by the misID probability of  $\propto 3.6\%$ .

## 210 4.4 Fit to $B_s^0 \rightarrow D_s\pi\pi\pi$ candidates

211 An unbinned maximum likelihood fit is performed simultaneously to the invariant mass  
 212 distribution of  $B_s^0 \rightarrow D_s\pi\pi\pi$  candidates. As discussed in Sec. ??, the fit is given  
 213 as a Johnson SU signal model for the  $B_s^0$  and  $B^0$  signal, the sum of three bifurcated  
 214 Gaussian functions to model the partially reconstructed  $B_s^0 \rightarrow D_s^*\pi\pi\pi$  background and  
 215 an Exponential function to account for combinatorial background. The invariant mass  
 216 distribution and the fit is shown in Fig. 4.3. All simultaneously performed fits to the  
 217  $m(D_s\pi\pi\pi)$  distribution, ordered by the respective  $D_s$  final state, can be found in the  
 218 Appendix ???. The obtained yields are summarized in Table 4.1.

## 219 4.5 Fit to $B_s^0 \rightarrow D_sK\pi\pi$ candidates

220 The shape of the invariant mass distribution of  $B_s^0 \rightarrow D_sK\pi\pi$  candidates is described by  
 221 Johnson SU functions for the  $B^0$  and  $B_s^0$  signal, two sums of three bifurcated Gaussians  
 222 for the  $B_s^0/B^0 \rightarrow D_s^*K\pi\pi$  partially reconstructed background contributions and two  
 223 sums of double Crystal Ball functions for the single misID  $B_s^0 \rightarrow D_s\pi\pi\pi$  and the partially  
 224 reconstructed, misidentified  $B_s^0 \rightarrow D_s^*\pi\pi\pi$  decays. A simultaneous unbinned maximum  
 225 likelihood fit is performed and the result is shown in Fig. 4.3. All simultaneously performed  
 226 fits to the  $m(D_sK\pi\pi)$  distribution, ordered by the respective  $D_s$  final state, can be found  
 227 in the Appendix ???. The obtained yields are summarized in Table 4.1.

## 228 4.6 Extraction of signal weights

229 The sPlot technique [6] is used to extract signal weights from the fits to the invariant  
 230 mass distributions of our signal and normalization channel. This statistical tool assigns  
 231 a weight to every event, according to its position in the respective mass distribution,  
 232 given the fitted signal and background models. The weights can then be used to suppress  
 233 the background components in every other observable distribution of interest. Figure ??  
 234 shows the distribution of weights across the invariant mass spectra of  $B_s^0 \rightarrow D_s\pi\pi\pi$  and  
 235  $B_s^0 \rightarrow D_sK\pi\pi$  candidates.

fit component	yield 2011	yield 2012	yield 2015	yield 2016
$m(D_sK\pi\pi)$				
$B_s^0 \rightarrow D_sK\pi\pi$	$392 \pm 25$	$860 \pm 38$	$309 \pm 21$	$1984 \pm 55$
$B^0 \rightarrow D_sK\pi\pi$	$276 \pm 26$	$692 \pm 41$	$261 \pm 23$	$1385 \pm 58$
$B^0/B_s^0 \rightarrow D_s^*K\pi\pi$	$7 \pm 25$	$171 \pm 75$	$114 \pm 25$	$893 \pm 84$
$B_s^0 \rightarrow D_s^{(*)}\pi\pi\pi$	$63 \pm 0$	$158 \pm 0$	$53 \pm 0$	$314 \pm 0$
combinatorial	$1482 \pm 53$	$2884 \pm 100$	$605 \pm 43$	$4261 \pm 133$
$m(D_s\pi\pi\pi)$				
$B_s^0 \rightarrow D_s\pi\pi\pi$	$9183 \pm 105$	$22083 \pm 166$	$7574 \pm 95$	$43773 \pm 245$
$B^0 \rightarrow D_s\pi\pi\pi$	$289 \pm 58$	$716 \pm 95$	$229 \pm 54$	$968 \pm 147$
$B_s^0 \rightarrow D_s^*\pi\pi\pi$	$3640 \pm 130$	$9086 \pm 232$	$3047 \pm 110$	$17827 \pm 421$
combinatorial	$4991 \pm 154$	$11127 \pm 271$	$3728 \pm 126$	$24589 \pm 500$

Table 4.1: Summary of yields obtained from the fits to Run1 and Run2 data.

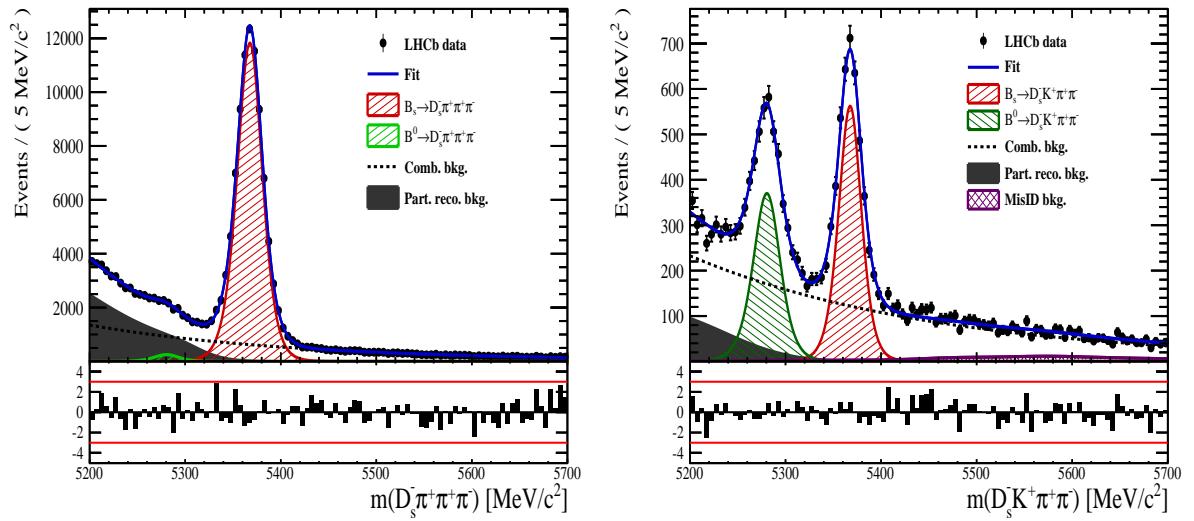


Figure 4.3: Invariant mass distribution of (left)  $B_s^0 \rightarrow D_s \pi \pi \pi$  and (right)  $B_s^0 \rightarrow D_s K \pi \pi$  candidates for Run1 and Run2 data. The respective fit described in the text is overlaid.

## 236 5 Flavour Tagging

237 To successfully perform a time- and amplitude-dependent measurement of  $\gamma$ , the identifi-  
 238 cation of the initial state flavour of the  $B_s^0$  meson is crucial. In the presented analysis,  
 239 a number of flavour tagging algorithms are used that either determine the flavour of  
 240 the non-signal b-hadron produced in the event (opposite site, OS), or they use particles  
 241 produced in the fragmentation of the signal candidate  $B_s^0/\bar{B}_s^0$  (same side, SS).  
 242 For the same side, the algorithm searching for the charge of an additional kaon that accom-  
 243 panies the fragmentation of the signal candidate is used (SS-nnetKaon). For the opposite  
 244 site, four different taggers are chosen: The Two algorithms that use the charge of an  
 245 electron or a muon from semileptonic B decays (OS-  $e,\mu$ ), the tagger that uses the charge  
 246 of a kaon from a  $b \rightarrow c \rightarrow s$  decay chain (OS-nnetKaon) and the algorithm that determines  
 247 the  $B_s^0/\bar{B}_s^0$  candidate flavour from the charge of a secondary vertex, reconstructed from  
 248 the OS b decay product (OS-VtxCharge). All four taggers are then combined into a signal  
 249 OS tagger.  
 250 Every single tagging algorithm is prone to misidentify the signal candidate at a certain  
 251 mistag rate  $\omega = (\text{wrongtags})/(\text{alltags})$ . This might be caused by particle misidentifica-  
 252 tion, flavour oscillation of the neutral opposite site B-meson or by tracks that are wrongly  
 253 picked up from the underlying event. For every signal  $B_s^0/\bar{B}_s^0$  candidate, each tagging  
 254 algorithm predicts a mistag probability  $\eta$ , which is calculated using a combination of  
 255 inputs such as the kinematics of the tagging particles. The inputs are then combined  
 256 to a predicted mistag using neural networks. These are trained on simulated samples  
 257 of  $B_s^0 \rightarrow D_s^- \pi^+$  (SS algorithm) and  $B^+ \rightarrow J/\psi K^+$  (OS algorithms) decays. For the  
 258 presented analysis, the measurable CP-violating coefficients are damped by the tagging  
 259 dilution  $D$ , that depends on the mistag rate:

$$259 \quad D = 1 - 2\omega. \quad (5.1)$$

260 This means that the statistical precision, with which these coefficients can be measured,  
 261 scales as the inverse square root of the effective tagging efficiency,

$$\epsilon_{eff} = \epsilon_{tag}(1 - 2\omega)^2, \quad (5.2)$$

262 where  $\epsilon_{tag}$  is the fraction of events that have a tagging decision. The flavour  
 263 tagging algorithms are optimised for highest  $\epsilon_{eff}$  on data, using the  $B_s^0 \rightarrow D_s^- \pi^+$  and  
 264  $B^+ \rightarrow J/\psi K^+$  samples.

265 Utilizing flavour-specific final states, the predicted mistag  $\eta$  of each tagger has to be  
 266 calibrated to match the observed mistag  $\omega$  on the data sample. For the calibration, a  
 267 linear model of the form

$$\omega(\eta) = p_0 + p_1 \cdot (\eta - \langle \eta \rangle), \quad (5.3)$$

268 where the values of  $p_0$  and  $p_1$  are determined using the  $B_s^0 \rightarrow D_s \pi \pi \pi$  normalization  
 269 mode and  $\langle \eta \rangle$  is the average estimated mistag probability  $\langle \eta \rangle = \sum_{i=1}^{N_{cand}} (\eta_i)/N_{cand}$ .  
 270 Following this model, a perfectly calibrated tagger would lead to  $\omega(\eta) = \eta$  and one would  
 271 expect  $p_1 = 1$  and  $p_0 = \langle \eta \rangle$ . Due to the different interaction cross-sections of oppositely  
 272 charged particles, the tagging calibration parameters depend on the initial state flavour of  
 273 the  $B_s^0$ . Therefore, the flavour asymmetry parameters  $\Delta p_0$ ,  $\Delta p_1$  and  $\Delta \epsilon_{tag}$  are introduced.  
 274 For this analysis, the calibrated mistag is treated as per-event variable, giving a larger

275 weight to events that are less likely to have an incorrect tag. This adds one additional  
 276 observable to the time- and amplitude-dependent fit.

277 The tagging calibration is determined using a time-dependent fit to the full  $B_s^0 \rightarrow D_s\pi\pi\pi$   
 278 sample, where the mixing frequency  $\Delta m_s$  is fixed to the nominal PDG value [8]. The  
 279 calibration procedure for the OS tagging algorithms (Sec.5.1) and the SS kaon tagger  
 280 (Sec.5.2) is applied on the full Run I and 2015 and 2016 Run II  $B_s^0 \rightarrow D_s\pi\pi\pi$  data sample,  
 281 which is selected following the steps described in Sec. 3. The similar selection ensures  
 282 as close as possible agreement between the  $B_s^0 \rightarrow D_s\pi\pi\pi$  and  $B_s^0 \rightarrow D_sK\pi\pi$  samples in  
 283 terms of the decay kinematics, which are crucial for the flavour tagging. Section 5.3 shows  
 284 the compatibility of both samples. After applying the calibration, the response of the OS  
 285 and SS taggers are combined, which is shown in Sec. 5.4.

## 286 5.1 OS tagging calibration

287 The responses of the OS electron, muon, neural net kaon and the secondary vertex charge  
 288 taggers are combined for the mistag calibration. Figure ?? shows the distribution of the  
 289 predicted OS mistag for signal candidates from  $B_s^0 \rightarrow D_s\pi\pi\pi$ . The extracted calibration  
 290 parameters and tagging asymmetries are summarized in Table 5.1 and the measured  
 291 tagging power for the OS combination is  $\epsilon_{eff,OS} = 4.81\%$ .

$p_0$	$p_1$	$<\eta>$	$\epsilon_{tag}$	$\Delta p_o$	$\Delta p_1$	$\epsilon_{eff} [\%]$
$0.025 \pm 0.005$	$0.944 \pm 0.048$	0.347	$0.517 \pm 0.002$	$0.028 \pm 0.005$	$0.037 \pm 0.045$	$4.81 \pm 0.04 \text{ (stat)} \pm 0.37 \text{ (cal)}$

Table 5.1: Calibration parameters and tagging asymmetries of the OS tagger extracted from  $B_s^0 \rightarrow D_s\pi\pi\pi$  decays.

## 292 5.2 SS tagging calibration

293 The SS neural net kaon tagger can be calibrated using the flavour-specific  $B_s^0 \rightarrow D_s\pi\pi\pi$   
 294 decay. It's development, performance and calibration is described in detail in [9]. Figure  
 295 ?? shows the distribution of the predicted mistag of the neural net kaon tagger. The  
 296 extracted calibration parameters and tagging asymmetries are summarized in Table 5.2  
 297 and the measured tagging power for this algorithm is  $\epsilon_{eff,SS} = 3.22\%$ .

$p_0$	$p_1$	$<\eta>$	$\epsilon_{tag}$	$\Delta p_o$	$\Delta p_1$	$\epsilon_{eff} [\%]$
$0.008 \pm 0.004$	$1.086 \pm 0.059$	0.381	$0.571 \pm 0.002$	$-0.017 \pm 0.004$	$0.135 \pm 0.058$	$3.22 \pm 0.03 \text{ (stat)} \pm 0.26 \text{ (cal)}$

Table 5.2: Calibration parameters and tagging asymmetries of the SS tagger extracted from  $B_s^0 \rightarrow D_s\pi\pi\pi$  decays.

## 298 5.3 Tagging performance comparison between the signal and 299 normalization channel

300 To justify the usage of the tagging calibration, obtained using the  $B_s^0 \rightarrow D_s\pi\pi\pi$  sample,  
 301 for our signal decay, the performance of the taggers in the two decay channels needs to  
 302 be compatible. This is verified using both, simulated signal samples of both decays and

sweighted data, to compare the similarity of the mistag probabilities, tagging decisions and kinematic observables that are correlated with the tagging response, on simulation and data.

The distributions of the predicted mistag probability  $\eta$  for the OS combination and the SS kaon tagger are shown in Fig. ?? (simulation) and Fig. 5.1 (data).

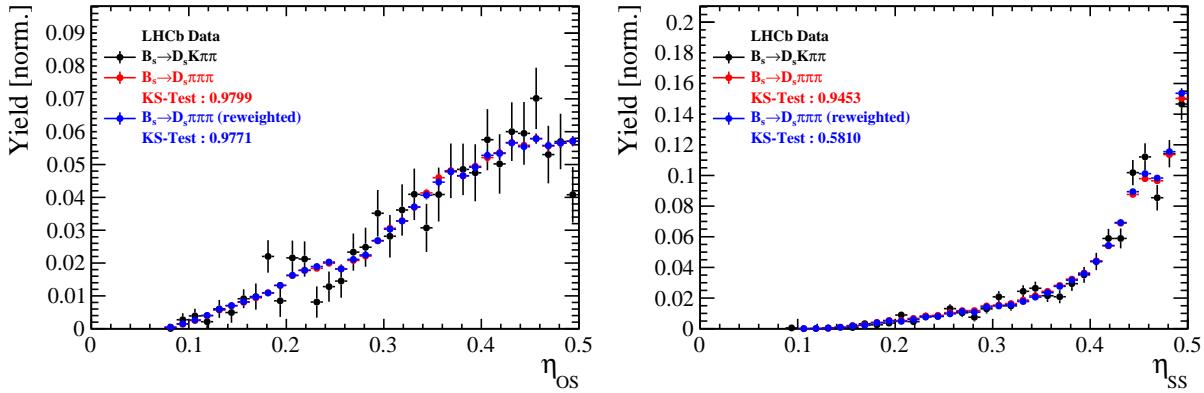


Figure 5.1: Distributions of the predicted mistag  $\eta$  for the OS combination (left) and the SS kaon tagger (right) for signal candidates in the  $B_s^0 \rightarrow D_s K\pi\pi$  (black) and  $B_s^0 \rightarrow D_s \pi\pi\pi$  (red) data samples.

Both, data and simulated samples, show good agreement between the signal and normalization channel. Compatibility is also seen in Fig. ??, which shows the comparison of the tagging decision distributions of the OS and SS tagger for sweighted data.

Fig. ?? shows the signal data distributions of the transverse  $B_s^0$  momentum  $p_T$ , the pseudorapidity  $\eta$  of the signal candidate and the number of reconstructed tracks per event. Sufficient agreement is observed.

To justify the portability of the flavour tagging calibration obtained from  $B_s^0 \rightarrow D_s \pi\pi\pi$  to the  $B_s^0 \rightarrow D_s K\pi\pi$  channel, besides the good agreement of the distributions shown above, the dependence of the measured mistag  $\omega$  on the predicted mistag  $\eta$  has to be compatible in both channel. This dependence is shown in Fig. 5.2 for simulated signal events of both channels, where good agreement is observed.

## 5.4 Combination of OS and SS taggers

In the time- and amplitude-dependent fit to  $B_s^0 \rightarrow D_s K\pi\pi$  data, the obtained tagging responses of the OS and SS tagger will be combined after the calibration described in the previous sections is applied. Events that acquire a mistag probability greater than 0.5 after the calibration will have their tagging decision flipped. For events where only one of the two taggers fired, the combination of the tagging decision is trivial. In those events where both taggers made a decision, we use the standard combination of taggers [10] provided by the flavour tagging group. In the nominal fit, the calibrated mistags  $\omega$  are combined event by event for the OS and SS tagger, thus adding one variable to observable to the fit procedure. This ensures that the uncertainties of the OS and SS tagging calibration parameters are propagated properly to the combined tagging response for each event. The tagging performance for the combined tagger in the categories SS tagged only, OS tagged only and SS+OS tagged, are shown in Tab. ?? for the signal and normalization

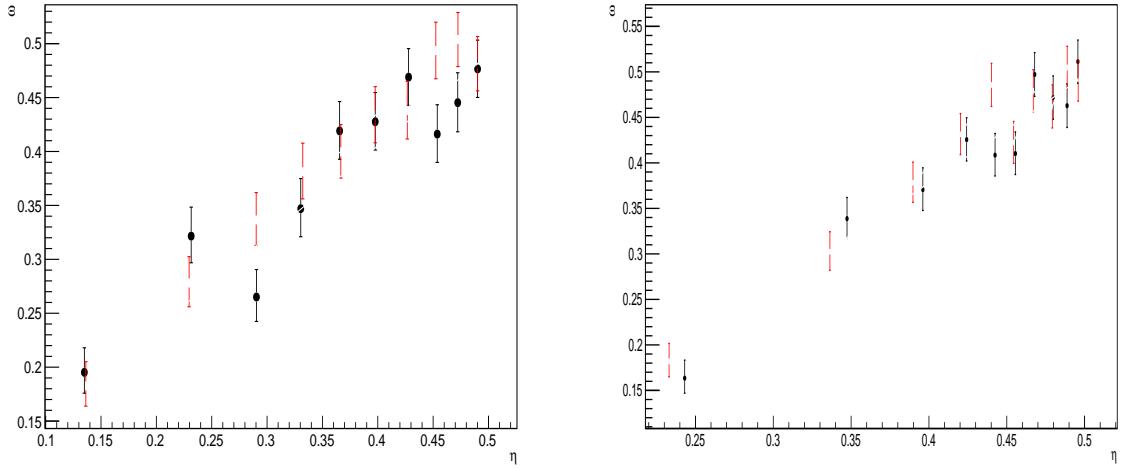


Figure 5.2: Dependence of the observed mistag  $\omega$  on the predicted mistag  $\eta$  for the (left) OS combination and the (right) SS kaon tagger, found in the simulated  $B_s^0 \rightarrow D_s K\pi\pi$  (black) and  $B_s^0 \rightarrow D_s \pi\pi\pi$  (red) signal samples.

channel. The distribution of the observed mistag  $\omega$  as a function of the combined mistag probability  $\eta$  for  $B_s^0 \rightarrow D_s \pi\pi\pi$  decays is shown in Fig. 5.3.

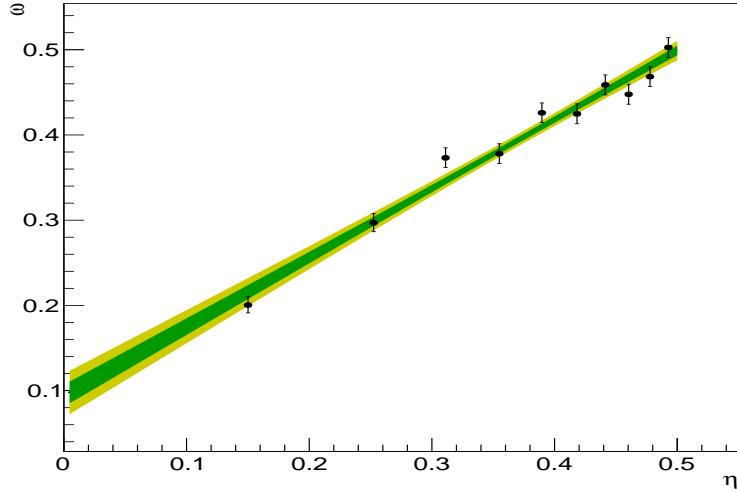


Figure 5.3: Distribution of the predicted combined mistag probability  $\eta$  versus the observed mistag  $\omega$  for  $B_s^0 \rightarrow D_s \pi\pi\pi$  signal candidates. The fit with a linear polynomial, used to determine  $p_0$  and  $p_1$  is overlaid.

$B_s^0 \rightarrow D_s\pi\pi\pi$		$\epsilon_{tag}$	$\epsilon_{eff}$
SS only		$(28.586 \pm 0.165)\%$	$(1.408 \pm 0.018(\text{stat}) \pm 0.082(\text{cal}))\%$
OS only		$(17.221 \pm 0.138)\%$	$(2.027 \pm 0.029(\text{stat}) \pm 0.100(\text{cal}))\%$
SS+OS		$(39.981 \pm 0.179)\%$	$(5.690 \pm 0.047(\text{stat}) \pm 0.196(\text{cal}))\%$
total			
$B_s^0 \rightarrow D_sK\pi\pi$		$\epsilon_{tag}$	$\epsilon_{eff}$
SS only		$(30.094 \pm 0.960)\%$	$(1.379 \pm 0.082(\text{stat}) \pm 0.085(\text{cal}))\%$
OS only		$(18.923 \pm 0.819)\%$	$(1.768 \pm 0.121(\text{stat}) \pm 0.099(\text{cal}))\%$
SS+OS		$(27.277 \pm 0.932)\%$	$(3.914 \pm 0.194(\text{stat}) \pm 0.220(\text{cal}))\%$
total			

Table 5.3: Flavour tagging performances for  $B_s^0 \rightarrow D_s\pi\pi\pi$  and  $B_s^0 \rightarrow D_sK\pi\pi$  events which are only OS tagged, only SS tagged or tagged by both.

## 334 6 Acceptance

### 335 6.1 MC corrections

#### 336 6.1.1 Truth matching of simulated candidates

337 We use the `BackgroundCategory` tool to truth match our simulated candidates. Candidates  
 338 with background category (`BKGCAT`) 20 or 50 are considered to be true signal. Background  
 339 category 60 is more peculiar since it contains both badly reconstructed signal candidates  
 340 and ghost background. This is due to the fact that the classification algorithms identifies  
 341 all tracks for which less than 70% of the reconstructed hits are matched to generated  
 342 truth-level quantities as ghosts. In particular for signal decays involving many tracks (as  
 343 in our case), this arbitrary cutoff is not 100% efficient. Moreover, the efficiency is expected  
 344 to depend on the kinematics which would lead to a biased acceptance determination if  
 345 candidates with `BKGCAT`= 60 would be removed.

346 We therefore include `BKGCAT`= 60 and statistically subtract the ghost background by  
 347 using the `sPlot` technique. The `sWeights` are calculated from a fit to the reconstructed  $B_s$   
 348 mass. The signal contribution is modeled as described in Sec. 4.1 and the background  
 349 with a polynomial. The fit is performed simultaneously in two categories; the first includes  
 350 events with `BKGCAT` = 20 or 50 and the second events with `BKGCAT` = 60. To account  
 351 for the different mass resolution we use a different  $\sigma$  for each category, while the mean  
 352 and the tail parameters are shared between them. The background component is only  
 353 included for the second category.

354 A significant fraction of 8% of the true signal candidates are classified as ghosts, while  
 355 only 20% of the events classified with `BKGCAT`= 60 are indeed genuine ghosts.

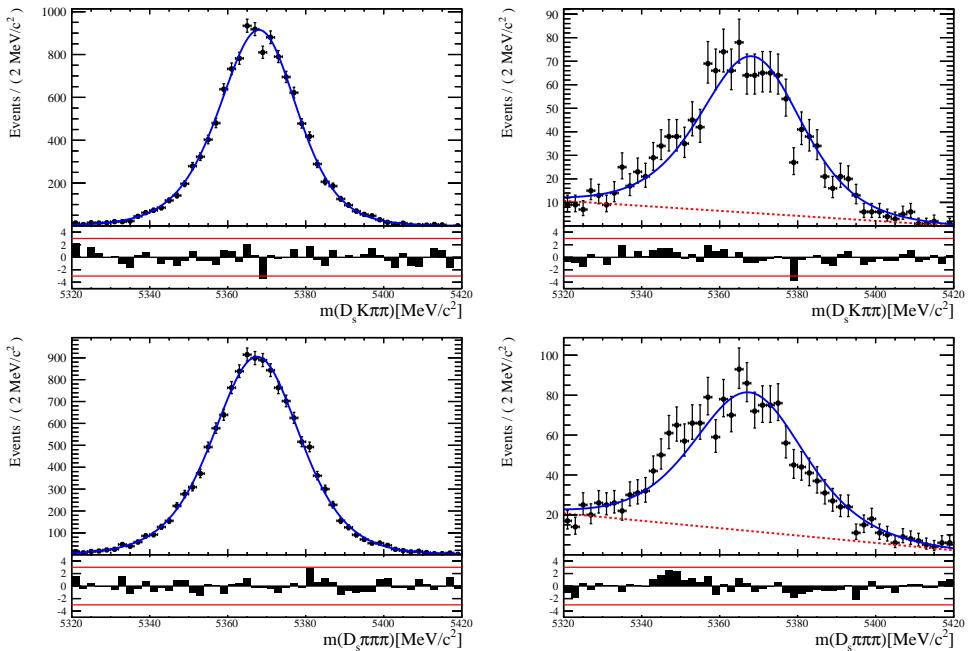


Figure 6.1: The reconstructed  $B_s$  mass distribution for simulated  $B_s \rightarrow D_s K\pi\pi$  (top) and  $B_s \rightarrow D_s \pi\pi\pi$  (bottom) decays classified with `BKGCAT` = 20 or 50 (left) and `BKGCAT` = 60 (right) after the full selection.

356 6.1.2 PID efficiencies

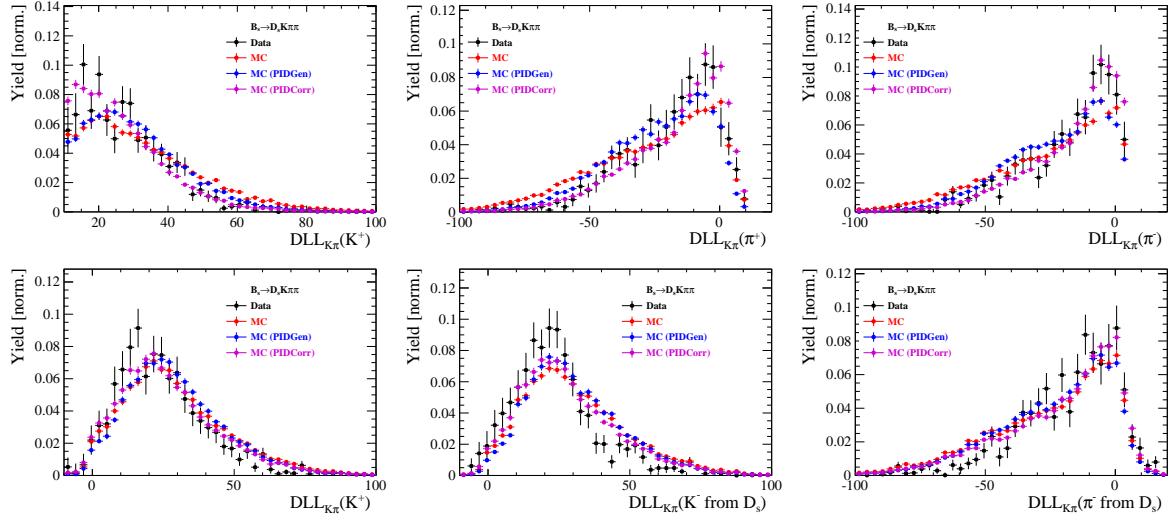


Figure 6.2

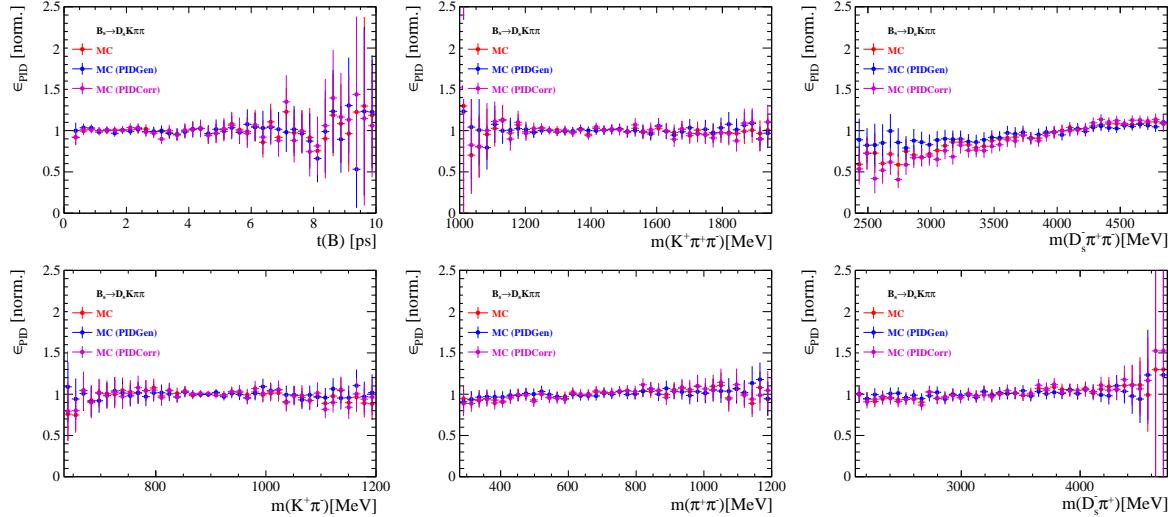


Figure 6.3

357 6.1.3 BDT efficiencies

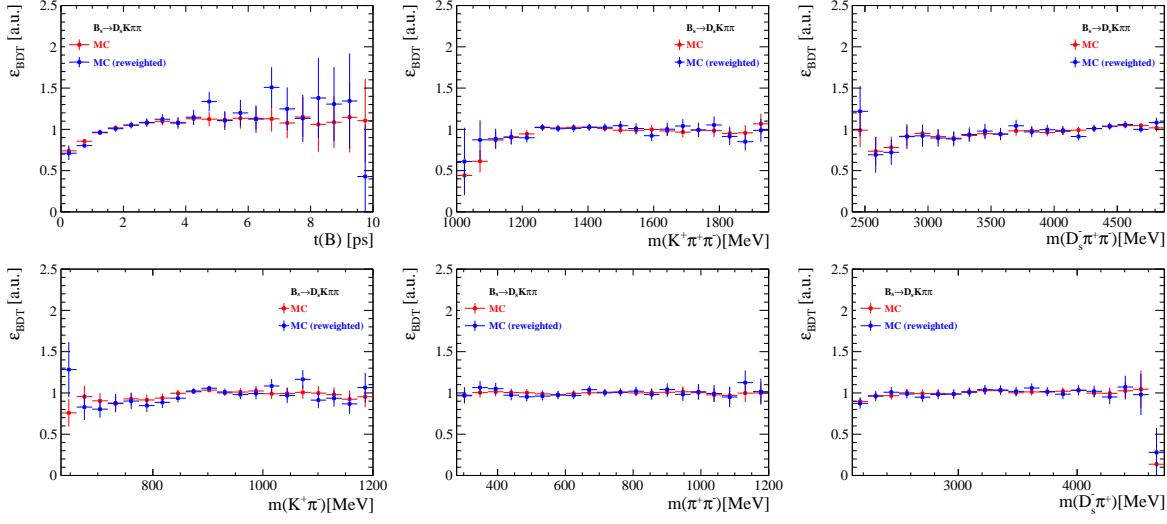


Figure 6.4

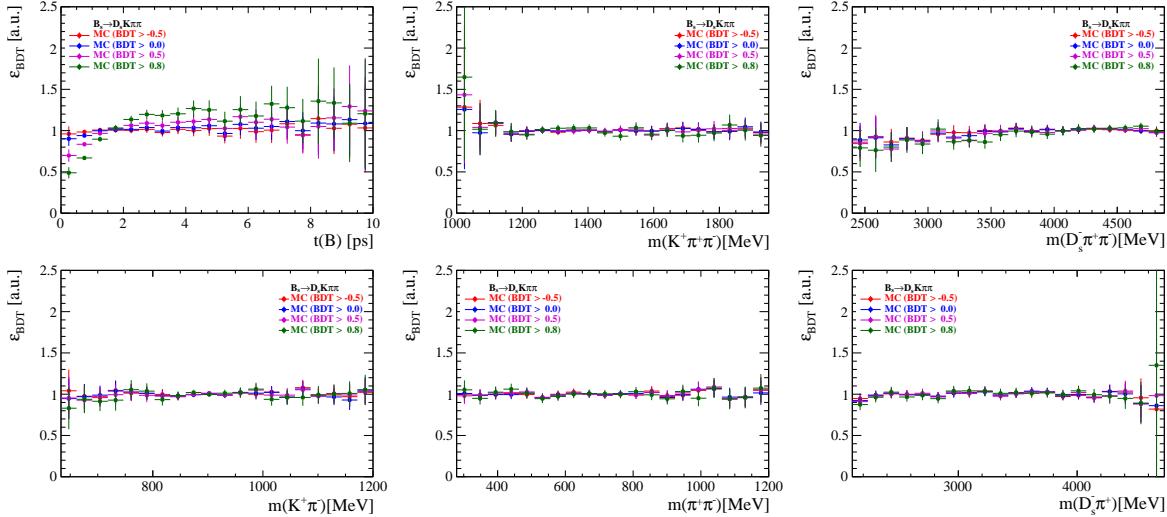


Figure 6.5

<sup>358</sup> 6.1.4 Tracking efficiencies

359 **6.2 Decay-time acceptance**

360 The decay-time distribution of the  $B_s^0$  mesons is sculpted due to the geometry of the LHCb  
 361 detector and the applied selection cuts, which are described in Section 3. In particular,  
 362 any requirement on the flight distance (FD), the impact parameter (IP) or the direction  
 363 angle (DIRA) of the  $B_s^0$  mesons, as well as the direct cut on the lifetime, will lead to a  
 364 decay-time dependent efficiency  $a(t)$ . This efficiency will distort the theoretically expected,  
 365 time-dependent decay rate

$$\frac{\Gamma(t)^{observed}}{dt} = \frac{\Gamma(t)^{theory}}{dt} \cdot a(t), \quad (6.1)$$

366 and has to be modelled correctly, in order to describe the observed decay rate. We  
 367 use our control channel for this measurement, because for  $B_s^0 \rightarrow D_s K\pi\pi$  decays the  
 368 decay-time acceptance is correlated with the CP-observables which we aim to measure.  
 369 Therefore, floating the CP-observables and the acceptance shape at the same time is  
 370 not possible. Hence, a fit to the decay-time distribution of  $B_s^0 \rightarrow D_s \pi\pi\pi$  candidates is  
 371 performed and the obtained acceptance shape is corrected by the difference in shape found  
 372 for the  $B_s^0 \rightarrow D_s K\pi\pi$  and  $B_s^0 \rightarrow D_s \pi\pi\pi$  MC.

373 A PDF of the form

$$\mathcal{P}(t', \vec{\lambda}) = \left[ (e^{\Gamma_s t} \cdot \cosh(\frac{\Delta\Gamma_s t}{2}) \times \mathcal{R}(t - t')) \right] \cdot \epsilon(t', \vec{\lambda}), \quad (6.2)$$

374 is fit to the decay time distribution of  $B_s^0 \rightarrow D_s \pi\pi\pi$  candidates in data. Since the  
 375 fit is performed untagged, the PDF shown in Eq. 6.2 contains no terms proportional  
 376 to  $\Delta m_s$ . The values for  $\Gamma_s$  and  $\Delta\Gamma_s$  are fixed to the latest HFAG results [11]. The  
 377 decay-time acceptance  $\epsilon(t', \vec{\lambda})$  is modelled using the sum of cubic polynomials  $v_i(t)$ , so  
 378 called Splines [12]. The polynomials are parametrised by so-called knots which determine  
 379 their boundaries. Knots can be set across the fitted distribution to account for local  
 380 changes in the acceptance shape. Using more knots is equivalent to using more base  
 381 splines which are defined on a smaller sub-range. In total,  $n + 2$  base splines  $v_i(t)$  are  
 382 needed to describe an acceptance shape which is parametrised using  $n$  knots.

383 For fits shown in the following, the knots have been placed at  $t =$   
 384  $[0.5, 1.0, 1.5, 2.0, 3.0, 9.5] ps$ . To accommodate these 6 knot positions, 8 basic splines  
 385  $v_i$ ,  $i = [1, \dots, 8]$  are used. Since a rapid change of the decay time acceptance at low  
 386 decay times due to the turn-on effect generated by the lifetime and other selection cuts is  
 387 expected, more knots are placed in that regime. At higher decay times we expect linear  
 388 behavior, with a possible small effect due to the VELO reconstruction. Therefore fewer  
 389 knots are used. Furthermore,  $v_7$  is fixed to 1 in order to normalize the overall acceptance  
 390 function. To stabilise the last spline,  $v_8$  is fixed by a linear extrapolation from the two  
 391 previous splines:

$$v_N = v_{N-1} + \frac{v_{N-2} - v_{N-1}}{t_{N-2} - t_{N-1}} \cdot (t_N - t_{N-1}). \quad (6.3)$$

392 Here,  $N = 8$  and  $t_{N-1}$  corresponds to the knot position associated with  $v_{N-1}$ .

### 6.2.1 Comparison of acceptance in subsamples

It is possible that the decay-time dependent efficiency deviates in different subsamples of our data. In particular, the acceptance could differentiate in subsamples with different final state kinematics, such as the run I & run II sample, the various  $D_s$  final states and the ways an event is triggered at the L0 stage. To investigate possible deviations, the full selected  $B_s^0 \rightarrow D_s \pi\pi\pi$  sample is split into subsamples according to the categories mentioned above (run,  $D_s$  state, L0 trigger). For each subsample, the fit procedure described at the beginning of this chapter, using the pdf given by Eq. 6.2, is repeated and the obtained values for the spline coefficients  $v_i$  are compared. Figure 6.6 shows the comparison of the obtained spline coefficients for the different  $D_s$  final states.

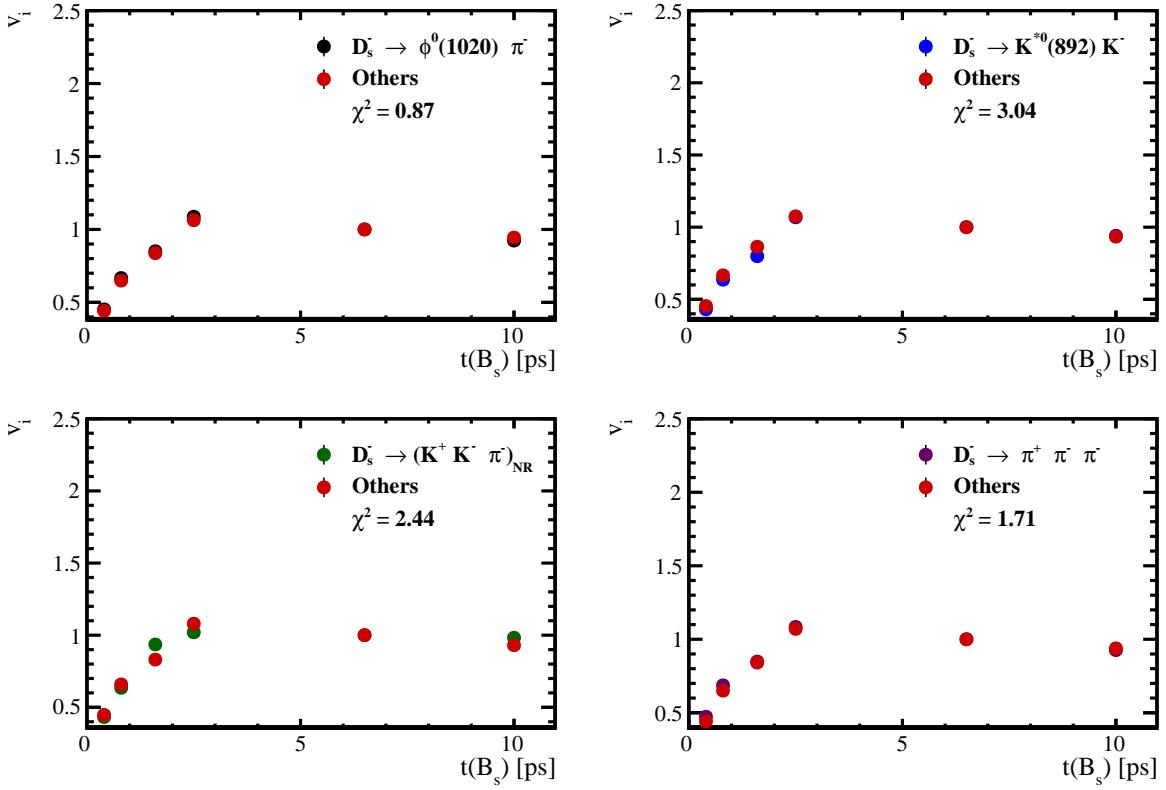


Figure 6.6: Comparison of the spline coefficients obtained from time-dependent fits to the  $B_s^0 \rightarrow D_s \pi\pi\pi$  subsamples of different  $D_s$  final states. The comparison of one particular  $D_s$  state against all other states is shown.

Investigating the obtained spline coefficients from different  $D_s$  final states, good agreement is observed between all four channels and no need to distinguish between different final states in the time-dependent amplitude fit is found. The comparison between spline coefficients for the different runs and L0 trigger categories is shown in Figure 6.7.

Significant deviations between spline coefficients obtained from the two different runs and L0 trigger categories can be observed. The deviations are most pronounced in the  $(0 - 5)$  ps region, where the majority of statistics is found. Therefore, the time-dependent efficiency has to be treated separately for the runs and L0 categories. This is achieved by

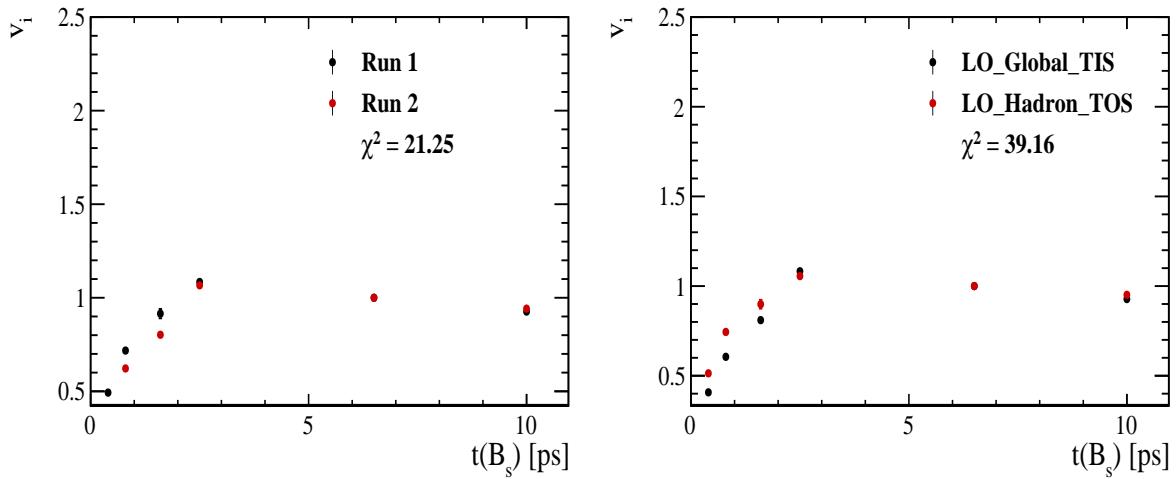


Figure 6.7: Comparison of the spline coefficients obtained from time-dependent fits to the  $B_s^0 \rightarrow D_s \pi \pi \pi$  subsamples of (left) the different runs and (right) L0 trigger categories.

412 implementing a simultaneous fit, where the acceptance description is allowed to vary in  
413 the subsamples.

<sup>414</sup> **6.2.2 Results**

<sup>415</sup> The nominal fit to  $B_s^0 \rightarrow D_s\pi\pi\pi$  data using this configuration is shown in Figure ??.  
<sup>416</sup> Note that the normalization of the splines in the following figures is not in scale. The fit  
<sup>417</sup> parameters obtained from the described fits to data and simulation are summarised in  
<sup>418</sup> Table 6.4.

Table 6.1: Time acceptance parameters for events in category [Run-I,L0-TOS].

Knot position	Coefficient	$B_s^0 \rightarrow D_s K\pi\pi$ data	$B_s^0 \rightarrow D_s K\pi\pi$ MC	Ratio
0.4	$v_0$	$0.561 \pm 0.038$	$0.546 \pm 0.022$	$0.953 \pm 0.060$
0.8	$v_1$	$0.826 \pm 0.059$	$0.785 \pm 0.034$	$0.910 \pm 0.066$
1.6	$v_2$	$0.843 \pm 0.087$	$0.905 \pm 0.056$	$1.055 \pm 0.095$
2.5	$v_3$	$1.154 \pm 0.036$	$1.118 \pm 0.028$	$0.930 \pm 0.045$
6.5	$v_4$	1.0 (fixed)	1.0 (fixed)	1.0 (fixed)
10.0	$v_5$	0.866 (interpolated)	0.897 (interpolated)	1.061 (interpolated)

Table 6.2: Time acceptance parameters for events in category [Run-I,L0-TIS].

Knot position	Coefficient	$B_s^0 \rightarrow D_s K\pi\pi$ data	$B_s^0 \rightarrow D_s K\pi\pi$ MC	Ratio
0.4	$v_0$	$0.368 \pm 0.031$	$0.412 \pm 0.020$	$0.955 \pm 0.077$
0.8	$v_1$	$0.583 \pm 0.050$	$0.648 \pm 0.033$	$0.910 \pm 0.074$
1.6	$v_2$	$0.939 \pm 0.101$	$0.953 \pm 0.061$	$0.947 \pm 0.096$
2.5	$v_3$	$1.052 \pm 0.054$	$1.077 \pm 0.035$	$1.003 \pm 0.051$
6.5	$v_4$	1.0 (fixed)	1.0 (fixed)	1.0 (fixed)
10.0	$v_5$	0.954 (interpolated)	0.932 (interpolated)	0.998 (interpolated)

Table 6.3: Time acceptance parameters for events in category [Run-II,L0-TOS].

Knot position	Coefficient	$B_s^0 \rightarrow D_s K\pi\pi$ data	$B_s^0 \rightarrow D_s K\pi\pi$ MC	Ratio
0.4	$v_0$	$0.486 \pm 0.009$	$0.482 \pm 0.009$	$1.000 \pm 0.000$
0.8	$v_1$	$0.691 \pm 0.014$	$0.707 \pm 0.015$	$1.000 \pm 0.000$
1.6	$v_2$	$0.851 \pm 0.024$	$0.926 \pm 0.026$	$1.000 \pm 0.000$
2.5	$v_3$	$1.061 \pm 0.017$	$1.086 \pm 0.018$	$1.000 \pm 0.000$
6.5	$v_4$	1.0 (fixed)	1.0 (fixed)	1.0 (fixed)
10.0	$v_5$	0.946 (interpolated)	0.925 (interpolated)	1.000 (interpolated)

Table 6.4: Time acceptance parameters for events in category [Run-II,L0-TIS].

Knot position	Coefficient	$B_s^0 \rightarrow D_s K\pi\pi$ data	$B_s^0 \rightarrow D_s K\pi\pi$ MC	Ratio
0.4	$v_0$	$0.300 \pm 0.007$	$0.482 \pm 0.010$	$1.000 \pm 0.000$
0.8	$v_1$	$0.476 \pm 0.012$	$0.707 \pm 0.016$	$1.000 \pm 0.000$
1.6	$v_2$	$0.725 \pm 0.023$	$0.926 \pm 0.026$	$1.000 \pm 0.000$
2.5	$v_3$	$1.064 \pm 0.019$	$1.086 \pm 0.018$	$1.000 \pm 0.000$
6.5	$v_4$	1.0 (fixed)	1.0 (fixed)	1.0 (fixed)
10.0	$v_5$	0.944 (interpolated)	0.925 (interpolated)	1.000 (interpolated)

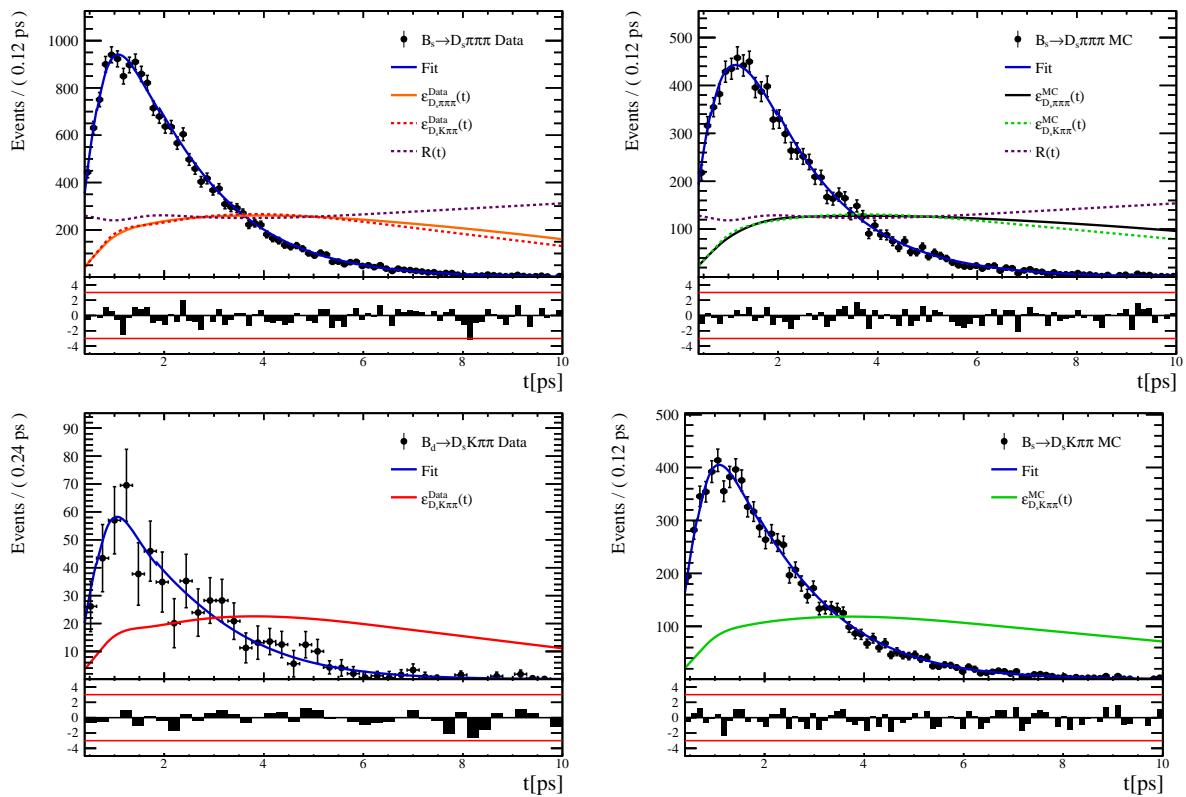


Figure 6.8

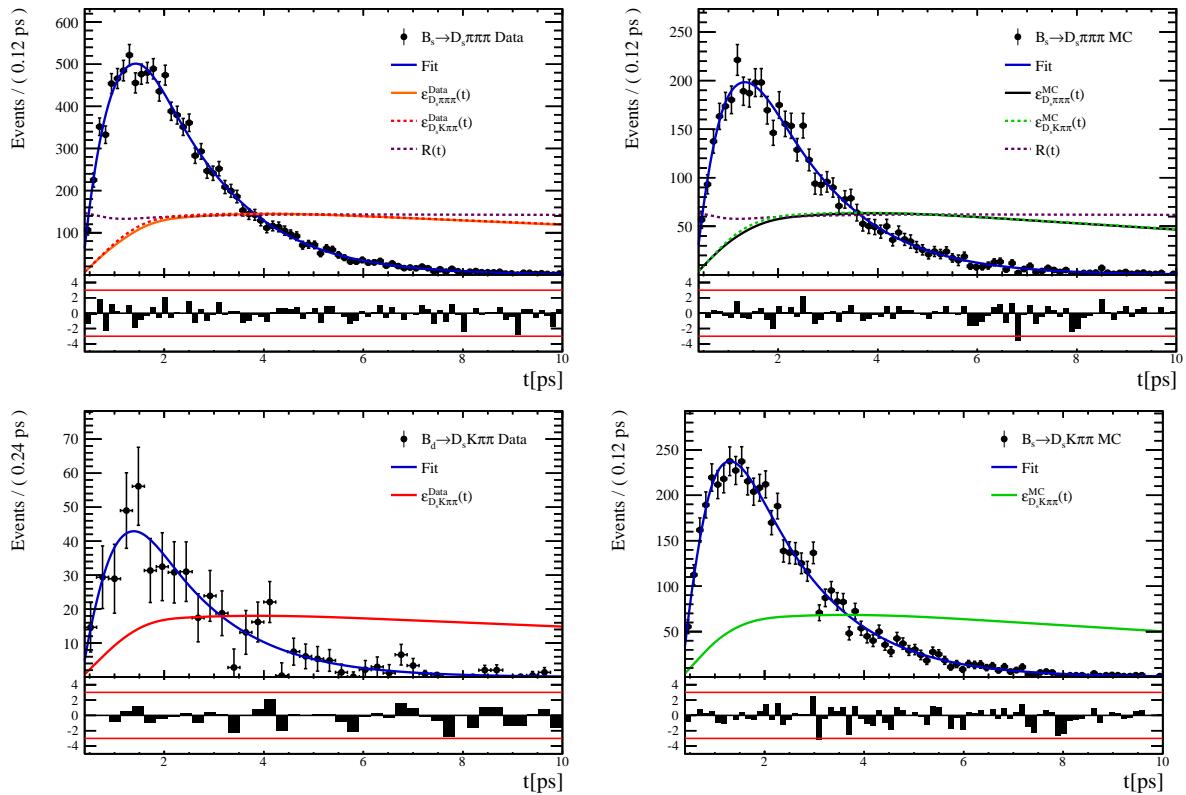


Figure 6.9

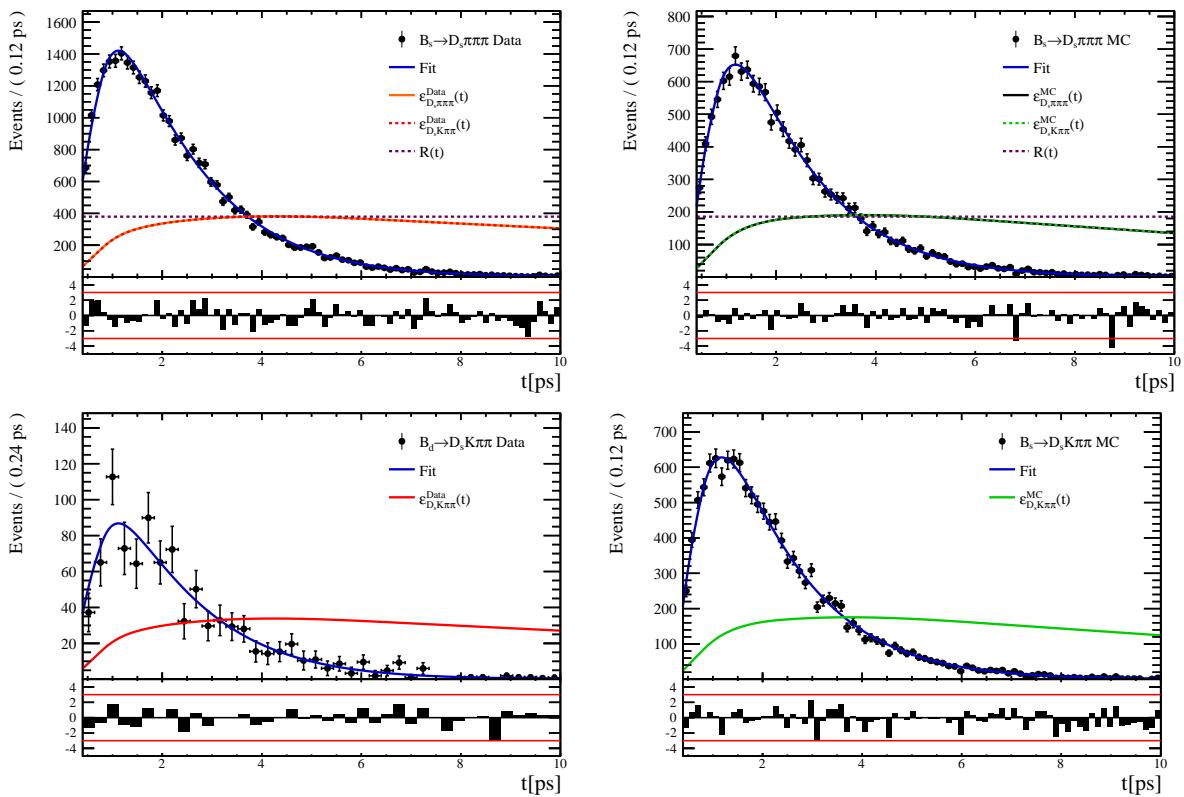


Figure 6.10

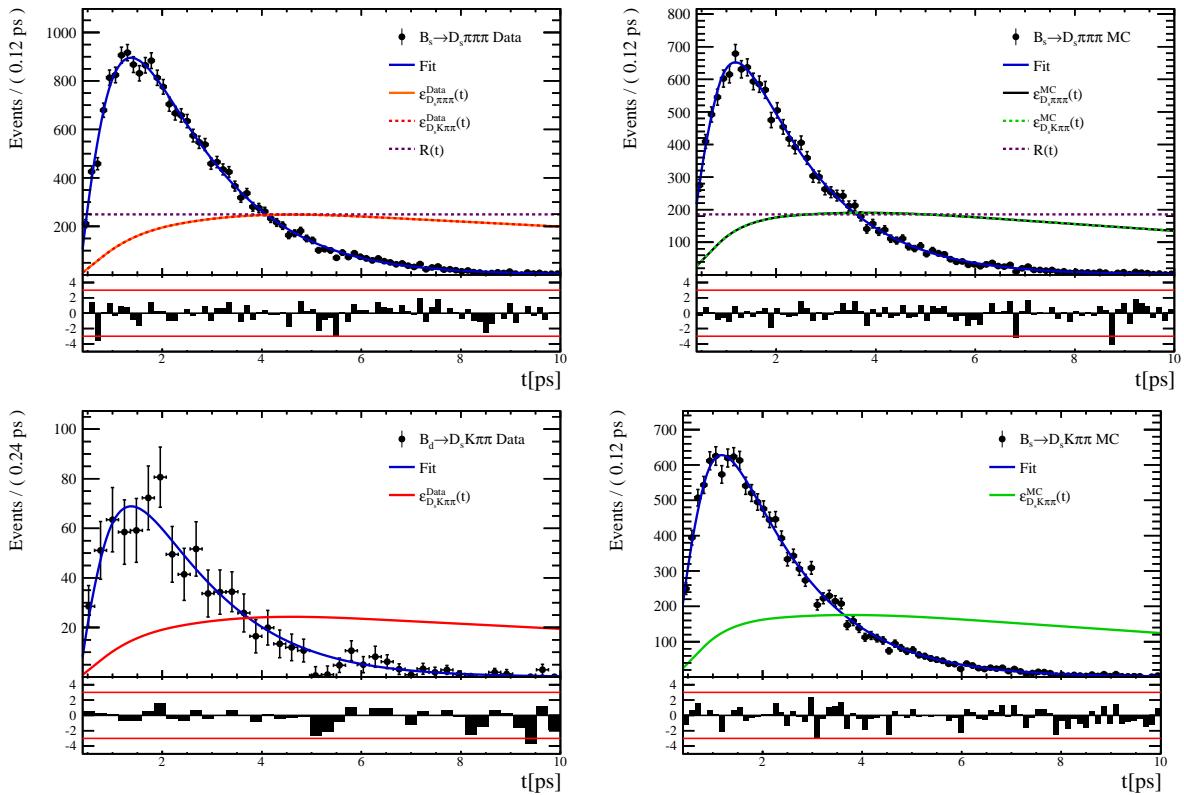


Figure 6.11

419 **6.3 Phasespace acceptance**

## 420 7 Decay-time Resolution

421 The observed oscillation of B mesons is prone to dilution, if the detector resolution is  
 422 of similar magnitude as the oscillation period. In the  $B_s^0$  system, considering that the  
 423 measured oscillation frequency of the  $B_s^0$  [8] and the average LHCb detector resolution [13]  
 424 are both  $\mathcal{O}(50 \text{ fs}^{-1})$ , this is the case. Therefore, it is crucial to correctly describe the  
 425 decay time resolution in order to avoid a bias on the measurement of time dependent CP  
 426 violation. Since the time resolution depends on the particular event, especially the decay  
 427 time itself, the sensitivity on  $\gamma$  can be significantly improved by using an event dependent  
 428 resolution model rather than an average resolution. For this purpose, we use the per-event  
 429 decay time error that is estimated based on the uncertainty obtained from the global  
 430 kinematic fit of the momenta and vertex positions (DTF) with additional constraints on  
 431 the PV position and the  $D_s$  mass. In order to apply it correctly, it has to be calibrated.  
 432 The raw decay time error distributions for  $B_s^0 \rightarrow D_s\pi\pi\pi$  signal candidates are shown in  
 433 Figure 7.1 for Run-I and Run-II data. Significant deviations between the two different  
 434 data taking periods are observed due to the increase in center of mass energy from Run-I  
 435 to Run-II, as well as (among others) new tunings in the pattern and vertex reconstruction.  
 436 The decay time error calibration is consequently performed separately for both data taking  
 437 periods.

438 For Run-I data, we use the calibration from the closely related  $B_s^0 \rightarrow D_s K$  analysis  
 439 which was performed on a data sample of prompt- $D_s$  candidates combined with a random  
 440 pion track from the PV. We verify the portability to our decay channel on MC.

441 For Run-II data, a new lifetime unbiased stripping line (LTUB) has been implemented  
 442 which selects prompt- $D_s$  candidates combined with random  $K\pi\pi$  tracks from the PV.

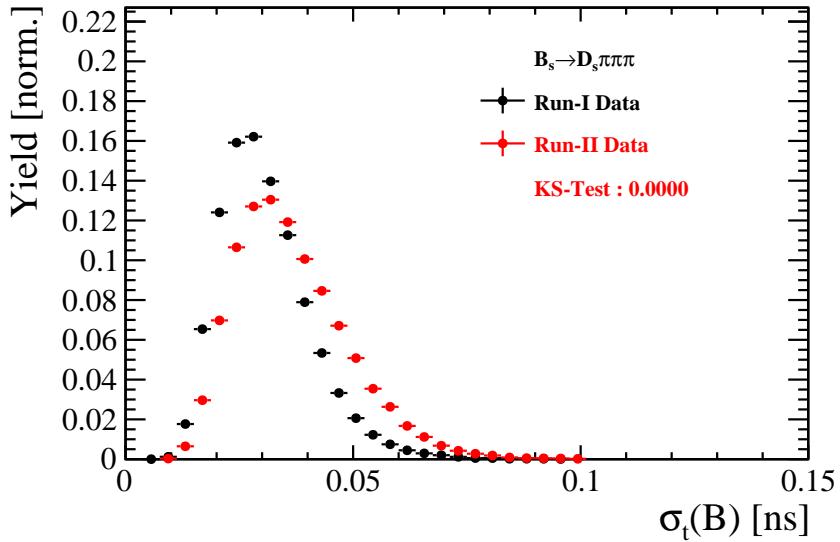


Figure 7.1: Distribution of the decay time error for  $B_s^0 \rightarrow D_s\pi\pi\pi$  signal candidates for Run-I (black) and Run-II (red) data (sWeighted).

## 443 7.1 Calibration for Run-I data

444 For simulated  $B_s^0 \rightarrow D_s K \pi\pi$  events, the spread of the differences between reconstructed  
 445 decay time and true decay time,  $\Delta t = t - t_{true}$ , is a direct measure of the decay time  
 446 resolution. The sum of two Gaussian pdfs with common mean but different widths is used  
 447 to fit the distribution of the decay time difference  $\Delta t$  as shown in Fig. 7.2. The effective  
 448 damping of the CP amplitudes due to the finite time resolution is described by the dilution  
 449  $\mathcal{D}$ . In the case of infinite precision, there would be no damping and therefore  $\mathcal{D} = 1$  would  
 450 hold, while for a resolution that is much larger than the  $B_s^0$  oscillation frequency,  $\mathcal{D}$  would  
 451 approach 0. For a double-Gaussian resolution model, the dilution is given by [14]

$$452 \quad \mathcal{D} = f_1 e^{-\sigma_1^2 \Delta m_s^2 / 2} + (1 - f_1) e^{-\sigma_2^2 \Delta m_s^2 / 2}, \quad (7.1)$$

452 where  $\sigma_1$  and  $\sigma_2$  are the widths of the Gaussians,  $f_1$  is the relative fraction of events  
 453 described by the first Gaussian relative to the second and  $\Delta m_s$  is the oscillation frequency  
 454 of  $B_s^0$  mesons. An effective single Gaussian width is calculated from the dilution as,

$$455 \quad \sigma_{eff} = \sqrt{(-2/\Delta m_s^2) \ln \mathcal{D}}, \quad (7.2)$$

455 which converts the resolution into a single-Gaussian function with an effective resolution  
 456 that causes the same damping effect on the magnitude of the  $B_s$  oscillation. For the Run-I  
 457  $B_s^0 \rightarrow D_s K \pi\pi$  MC sample the effective average resolution is found to be  $\sigma_{eff} = 39.1 \pm 0.3$  fs.

458 To analyze the relation between the per-event decay time error  $\delta_t$  and the actual  
 459 resolution  $\sigma_t$ , the simulated  $B_s^0 \rightarrow D_s K \pi\pi$  sample is divided into equal-statistics slices of  
 460  $\delta_t$ . For each slice, the effective resolution is determined as described above. Details of the  
 461 fit results in each slice are shown in appendix C. Figure 7.2 shows the obtained values  
 462 for  $\sigma_{eff}$  as a function of the per-event decay time error  $\sigma_t$ . To account for the variable  
 463 binning, the bin values are not placed at the bin center but at the weighted mean of the  
 464 respective per-event-error bin. A linear function is used to parametrize the distribution.  
 465 The obtained values are

$$466 \quad \sigma_{eff}^{MC}(\sigma_t) = (0.0 \pm 0.0) \text{ fs} + (1.232 \pm 0.010) \sigma_t \quad (7.3)$$

466 where the offset is fixed to 0. For comparison, the calibration function found for  $B_s^0 \rightarrow D_s K$   
 467 MC is also shown in Figure 7.2 [14]:

$$468 \quad \sigma_{eff}^{D_s K, MC}(\sigma_t) = (0.0 \pm 0.0) \text{ fs} + (1.201 \pm 0.013) \sigma_t. \quad (7.4)$$

468 Due to the good agreement between the scale factors for  $B_s^0 \rightarrow D_s K$  and  $B_s^0 \rightarrow D_s K \pi\pi$   
 469 MC, we conclude that the resolution calibration for  $B_s^0 \rightarrow D_s K$  data:

$$470 \quad \sigma_{eff}^{D_s K}(\sigma_t) = (10.26 \pm 1.52) \text{ fs} + (1.280 \pm 0.042) \sigma_t \quad (7.5)$$

470 can be used for our analysis. The following calibration functions were used in the  
 471  $B_s^0 \rightarrow D_s K$  analysis to estimate the systematic uncertainty on the decay-time resolution:

$$472 \quad \sigma_{eff}^{D_s K}(\sigma_t) = (-0.568 \pm 1.570) \text{ fs} + (1.243 \pm 0.044) \sigma_t \quad (7.6)$$

$$473 \quad \sigma_{eff}^{D_s K}(\sigma_t) = (0.0 \pm 0.0) \text{ fs} + (1.772 \pm 0.012) \sigma_t \quad (7.7)$$

473 The difference of the scale factors observed on  $B_s^0 \rightarrow D_s K$  and  $B_s^0 \rightarrow D_s K \pi\pi$  MC is  
 474 assigned as additional systematic uncertainty.

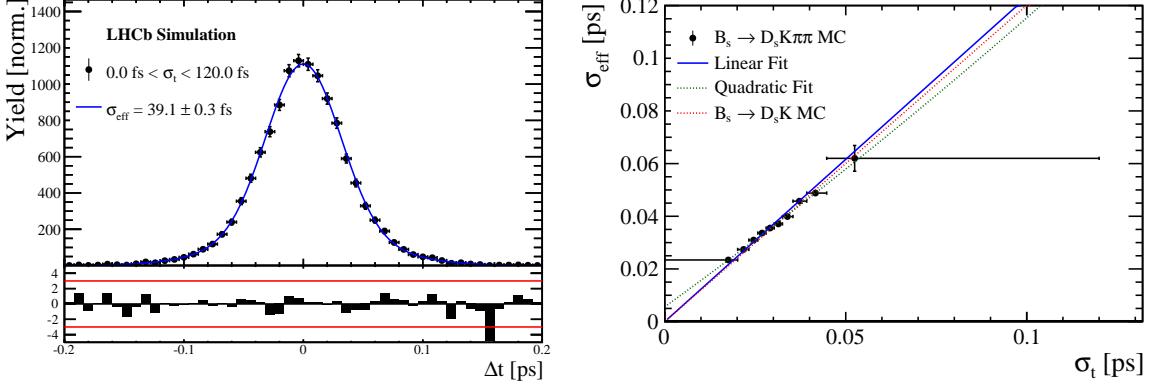


Figure 7.2: (Left) Difference of the true and measured decay time of  $B_s^0 \rightarrow D_s K\pi\pi$  MC candidates. (Right) The measured resolution  $\sigma_{eff}$  as function of the per-event decay time error estimate  $\sigma_t$  for  $B_s \rightarrow D_s K\pi\pi$  MC (Run-I). The fitted calibration curve is shown in blue.

## 475 7.2 Calibration for Run-II data

476 For the resolution calibration of Run-II data, a sample of promptly produced  $D_s$  candidates  
 477 is selected using the B02DsKPiPiLTUBD2HHHBeauty2CharmLine stripping line. This  
 478 lifetime-unbiased stripping line does not apply selection requirements related to lifetime  
 479 or impact parameter, allowing for a study of the resolution. In order to reduce the rate  
 480 of this sample it is pre-scaled in the stripping. Each  $D_s$  candidate is combined with a  
 481 random kaon track and two random pion tracks which originate from the PV to obtain a  
 482 sample of fake  $B_s$  candidates with a known true decay-time of  $t_{true} = 0$ . The difference of  
 483 the measured decay time,  $t$ , of these candidates with respect to the true decay time is  
 484 attributed to the decay time resolution.

485 The offline selection of the fake  $B_s$  candidates is summarized in Tab. 7.1. The selection  
 486 is similar than the one for real data with the important difference that the  $D_s$  candidate  
 487 is required to come from the PV by cutting on the impact parameter significance. Even  
 488 after the full selection, a significant number of multiple candidates is observed. These  
 489 are predominantly fake  $B_s$  candidates that share the same  $D_s$  candidate combined with  
 490 different random tracks from the PV. We select one of these multiple candidates randomly  
 491 which retains approximately 20% of the total candidates. The invariant mass distribution  
 492 of the selected  $D_s$  candidates is shown in Fig. 7.3. To separate true  $D_s$  candidates from  
 493 random combinations, the `sPlot` method is used to statistically subtract combinatorial  
 494 background from the sample.

495 Figure 7.4 shows the `sWeighted` decay-time distribution for fake  $B_s$  candidates. Similar  
 496 as in the previous section, the decay-time distribution is fitted with a double-Gaussian  
 497 resolution model in slices of the per-event decay time error. Since some  $D_s$  candidates  
 498 might actually originate from true  $B_s$  decays, the decay-time distribution of the fake  $B_s$   
 499 candidates might show a bias towards positive decay times. Therefore, we determine the  
 500 decay-time resolution from the negative decay-time distribution only. Details of the fit  
 501 results in each slice are shown in appendix C. The resulting calibration function:

$$\sigma_{eff}^{Data}(\sigma_t) = (9.7 \pm 1.7) \text{ fs} + (0.915 \pm 0.040) \sigma_t \quad (7.8)$$

is in good agreement with the one obtained for the  $B_s \rightarrow J/\psi\phi$  (Run-II) analysis.

$$\sigma_{eff}^{J/\psi\phi}(\sigma_t) = (12.25 \pm 0.33) \text{ fs} + (0.8721 \pm 0.0080) \sigma_t \quad (7.9)$$

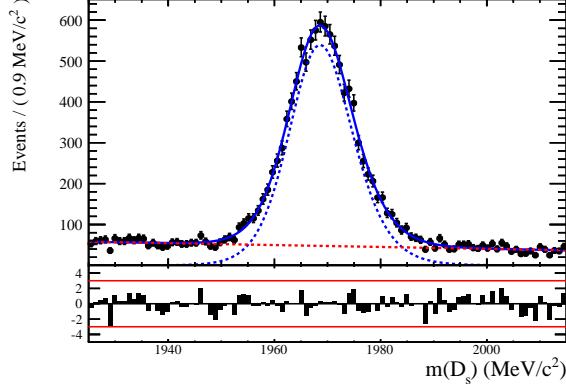


Figure 7.3: The invariant mass distribution for prompt  $D_s$  candidates.

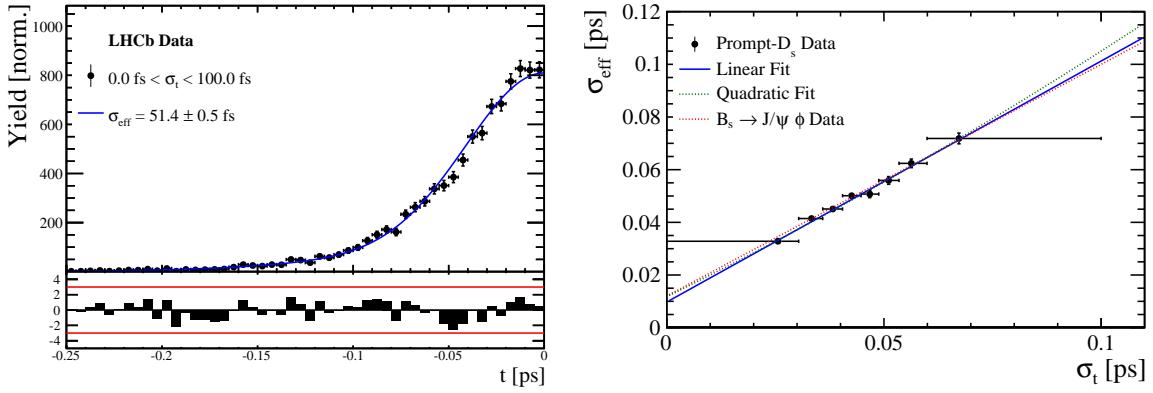


Figure 7.4: (Left) Decay-time distribution for fake  $B_s$  candidates from promptly produced  $D_s$  candidates, combined with random prompt  $K\pi\pi$  bachelor tracks. (Right) The measured resolution  $\sigma_{eff}$  as function of the per-event decay time error estimate  $\sigma_t$  for fake  $B_s$  candidates (Run-II data). The fitted calibration curve is shown in blue.

Table 7.1: Offline selection requirements for fake  $B_s$  candidates from promptly produced  $D_s$  candidates combined with random prompt  $K\pi\pi$  bachelor tracks.

	Description	Requirement
$B_s \rightarrow D_s K\pi\pi$	$\chi^2_{vtx}/\text{ndof}$	< 8
	$\chi^2_{DTF}/\text{ndof}$	< 15
	$t$	< 0 ps
$D_s \rightarrow hhh$	$\chi^2_{vtx}/\text{ndof}$	< 5
	DIRA	> 0.99994
	$\chi^2_{FD}$	> 9
	$p_T$	> 1800 MeV
	$\chi^2_{IP}$	< 9
	$\chi^2_{IP}(h)$	> 5
	Wrong PV veto	$nPV = 1 \parallel \min(\Delta\chi^2_{IP}) > 20$
$D_s^- \rightarrow KK\pi^-$	$D^0$ veto	$m(KK) < 1840$ MeV
	$D^-$ veto	$m(K^+K_\pi^-\pi^-) \neq m(D^-) \pm 30$ MeV
	$\Lambda_c$ veto	$m(K^+K_p^-\pi^-) \neq m(\Lambda_c) \pm 30$ MeV
$D_s^- \rightarrow \phi\pi^-$	$m(KK)$	$= m_\phi \pm 20$ MeV
	PIDK( $K^+$ )	> -10
	PIDK( $K^-$ )	> -10
	PIDK( $\pi^-$ )	< 20
$D_s^- \rightarrow K^*(892)K^-$	$m(KK)$	$\neq m_\phi \pm 20$ MeV
	$m(K^+\pi^-)$	$= m_{K^*(892)} \pm 75$ MeV
	PIDK( $K^+$ )	> -10
	PIDK( $K^-$ )	> -5
	PIDK( $\pi^-$ )	< 20
$D_s^- \rightarrow (KK\pi^-)_{NR}$	$m(KK)$	$\neq m_\phi \pm 20$ MeV
	$m(K^+\pi^-)$	$\neq m_{K^*(892)} \pm 75$ MeV
	PIDK( $K^+$ )	> 5
	PIDK( $K^-$ )	> 5
	PIDK( $\pi^-$ )	< 10
$D_s \rightarrow \pi\pi\pi$	PIDK( $h$ )	< 10
	PIDp( $h$ )	< 10
	$D^0$ veto	$m(\pi^+\pi^-) < 1700$ MeV
$X_s \rightarrow K\pi\pi$	$\chi^2_{IP}(h)$	< 40
	PIDK( $K$ )	> 10
	PIDK( $\pi$ )	< 5
	isMuon( $h$ )	False
All tracks	$p_T$	> 500 MeV

503 **7.3 Cross-checks**

504 **7.3.1 Kinematic dependence**

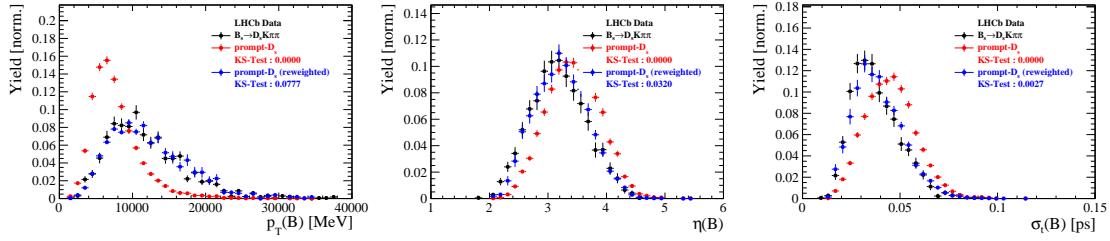


Figure 7.5

505 **7.3.2 DTF constraints**

<sup>506</sup> **8  $B_s$  Production Asymmetry**

Table 8.1:  $B_s$  production asymmetry for 2011 data.

$p_T$ [ GeV/c ]	$y$	$A_P(B_s^0)_{\sqrt{s}=7 \text{ TeV}}$
(2.00, 7.00)	(2.10, 3.00)	$0.0166 \pm 0.0632 \pm 0.0125$
(2.00, 7.00)	(3.00, 3.30)	$0.0311 \pm 0.0773 \pm 0.0151$
(2.00, 7.00)	(3.30, 4.50)	$-0.0833 \pm 0.0558 \pm 0.0132$
(7.00, 9.50)	(2.10, 3.00)	$0.0364 \pm 0.0479 \pm 0.0068$
(7.00, 9.50)	(3.00, 3.30)	$0.0206 \pm 0.0682 \pm 0.0127$
(7.00, 9.50)	(3.30, 4.50)	$0.0058 \pm 0.0584 \pm 0.0089$
(9.50, 12.00)	(2.10, 3.00)	$-0.0039 \pm 0.0456 \pm 0.0121$
(9.50, 12.00)	(3.00, 3.30)	$0.1095 \pm 0.0723 \pm 0.0179$
(9.50, 12.00)	(3.30, 4.50)	$0.1539 \pm 0.0722 \pm 0.0212$
(12.00, 30.00)	(2.10, 3.00)	$-0.0271 \pm 0.0336 \pm 0.0061$
(12.00, 30.00)	(3.00, 3.30)	$-0.0542 \pm 0.0612 \pm 0.0106$
(12.00, 30.00)	(3.30, 4.50)	$-0.0586 \pm 0.0648 \pm 0.0150$

Table 8.2:  $B_s$  production asymmetry for 2012 data.

$p_T$ [ GeV/c ]	$y$	$A_P(B_s^0)_{\sqrt{s}=8 \text{ TeV}}$
(2.00, 7.00)	(2.10, 3.00)	$0.0412 \pm 0.0416 \pm 0.0150$
(2.00, 7.00)	(3.00, 3.30)	$-0.0241 \pm 0.0574 \pm 0.0079$
(2.00, 7.00)	(3.30, 4.50)	$0.0166 \pm 0.0391 \pm 0.0092$
(7.00, 9.50)	(2.10, 3.00)	$0.0482 \pm 0.0320 \pm 0.0067$
(7.00, 9.50)	(3.00, 3.30)	$0.0983 \pm 0.0470 \pm 0.0155$
(7.00, 9.50)	(3.30, 4.50)	$-0.0430 \pm 0.0386 \pm 0.0079$
(9.50, 12.00)	(2.10, 3.00)	$0.0067 \pm 0.0303 \pm 0.0063$
(9.50, 12.00)	(3.00, 3.30)	$-0.1283 \pm 0.0503 \pm 0.0171$
(9.50, 12.00)	(3.30, 4.50)	$-0.0500 \pm 0.0460 \pm 0.0104$
(12.00, 30.00)	(2.10, 3.00)	$-0.0012 \pm 0.0222 \pm 0.0050$
(12.00, 30.00)	(3.00, 3.30)	$0.0421 \pm 0.0416 \pm 0.0162$
(12.00, 30.00)	(3.30, 4.50)	$0.0537 \pm 0.0447 \pm 0.0124$

## 507 9 Time dependent fit

508 This section will cover the phasespace integrated, time dependent fit to  $B_s^0 \rightarrow D_s h\pi\pi$  data,  
 509 which is described by the PDF formulated in Eq. 2.6. For the phasespace integrated fit to  
 510  $B_s^0 \rightarrow D_s K\pi\pi$  data, the sensitivity to the CKM phase  $\gamma$  will depend on the magnitude of  
 511 the coherence factor  $\kappa$ , defined in Eq. 2.7, which is added as an additional fit parameter. In  
 512 order to avoid any pollution of the final data samples by background events, both samples  
 513 are weighted using the sWeights obtained by the fits to the invariant mass distributions,  
 514 described in Sec. 4. This fit approach is commonly known as *sFit*. As additional input to  
 515 the fit, the tagging information (Sec. 5), as well as the decay time acceptance (Sec. 6)  
 516 and resolution (Sec. 7) is used and fixed to the values obtained by the dedicated studies.  
 517 Taking all inputs into account, the final time dependent fit PDF is given by

$$\mathcal{P}\mathcal{D}\mathcal{F}(t, \vec{\lambda}) = \left( \epsilon(t) \cdot \int P(x, t, q_t, q_f) dx \right) \times \mathcal{R}(t - t'), \quad (9.1)$$

518 where  $\int P(x, t, q_t, q_f) dx$  is the PDF given by Eq. 2.6,  $\epsilon(t)$  is the efficiency due to the  
 519 time acceptance effects and  $\mathcal{R}(t - t')$  is the Gaussian time resolution function.

### 520 9.1 sFit to $B_s^0 \rightarrow D_s \pi\pi\pi$ data

521 The phasespace-integrated, time-dependent fit is performed to the full sWeighted sample  
 522 of selected candidates from Run I and 2015+2016 Run II data, containing both possible  
 523 magnet polarities and  $D_s$  final states. In the fit, the values of  $\Gamma_s$  and  $\Delta\Gamma_s$  are fixed to the  
 524 latest PDG report. All tagging parameters are fixed to the central values found in the  
 525 tagging calibration, described in Sec. 5. Due to the fact that the  $B_s^0 \rightarrow D_s \pi\pi\pi$  decay is  
 526 flavour specific, the CP-coefficients can be fixed to  $C = 1$  and  $D_f = D_{\bar{f}} = S_f = S_{\bar{f}} = 0$ ,  
 527 reducing Eq. 2.6 to

$$\int P(x, t, q_t, q_f) dx = [\cosh\left(\frac{\Delta\Gamma t}{2}\right) + q_t q_f C \cos(m_s t)] e^{-\Gamma t}. \quad (9.2)$$

528 Note that in this case, the dependence on the coherence factor  $\kappa$  is dropped and the  
 529 same relation as found for  $B_s^0 \rightarrow D_s \pi$  decays is recovered. Therefore, the only free fit  
 530 parameter left is  $\Delta m_s$ . The data distribution with the overlaid fit is shown in Fig. xXx  
 531 and the obtained value for the mixing frequency is

$$\Delta m_s = xx.xxx \pm 0.yyy. \quad (9.3)$$

### 532 9.2 sFit to $B_s^0 \rightarrow D_s K\pi\pi$ data

### 533 9.3 Results

## A Details of multivariate classifier

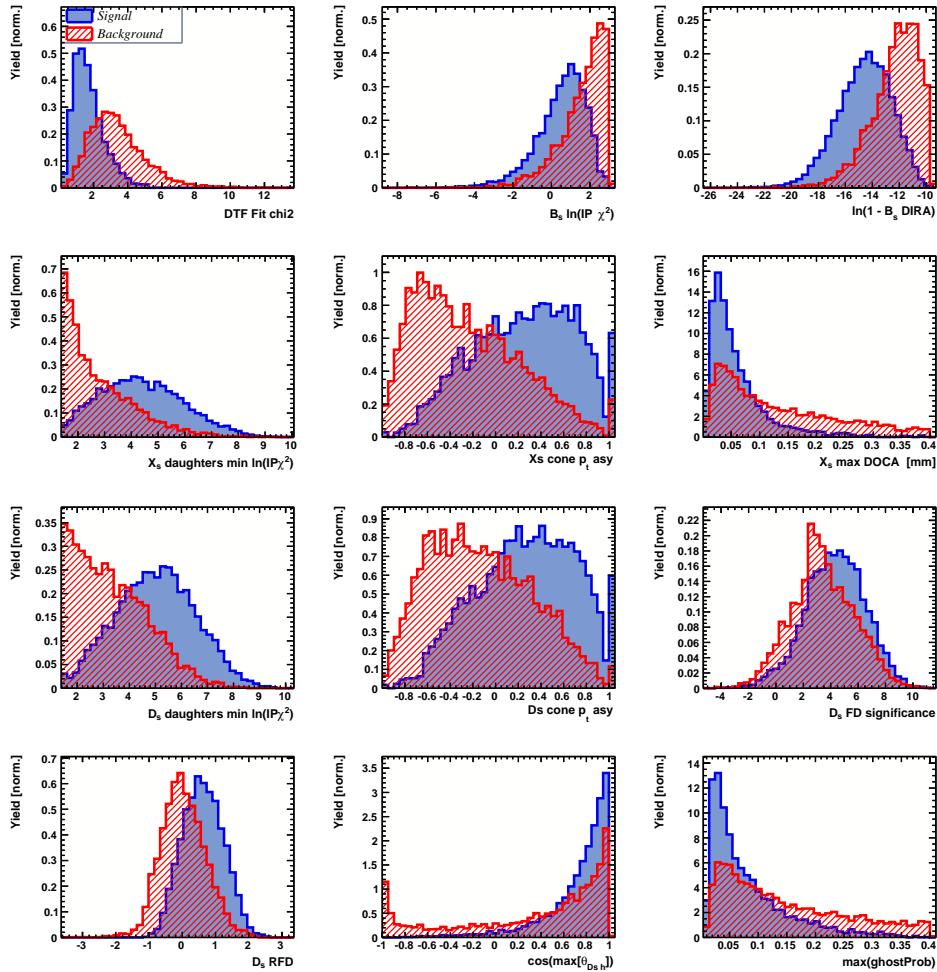


Figure 1.1: Variables used to train the BDTG.

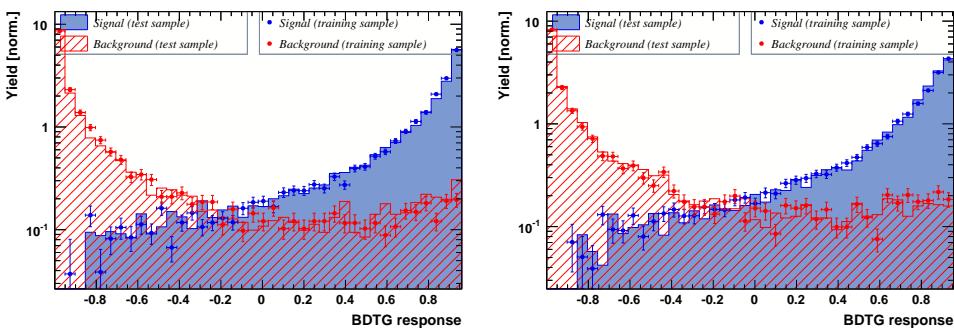


Figure 1.2: Response of the classifier trained on the even (left) and odd (right) sample.

## 535 B Detailed mass fits

536 In this section, all fits to the mass distribution of  $B_s^0 \rightarrow D_s\pi\pi\pi$  and  $B_s^0 \rightarrow D_sK\pi\pi$   
 537 candidates are shown. The fits are performed simultaneously in run period (Run-I, Run-  
 538 II),  $D_s$  final state ( $D_s \rightarrow KK\pi$  non-resonant,  $D_s \rightarrow \phi\pi$ ,  $D_s \rightarrow K^*K$ , or  $D_s \rightarrow \pi\pi\pi$ ) and  
 539 L0 trigger category.

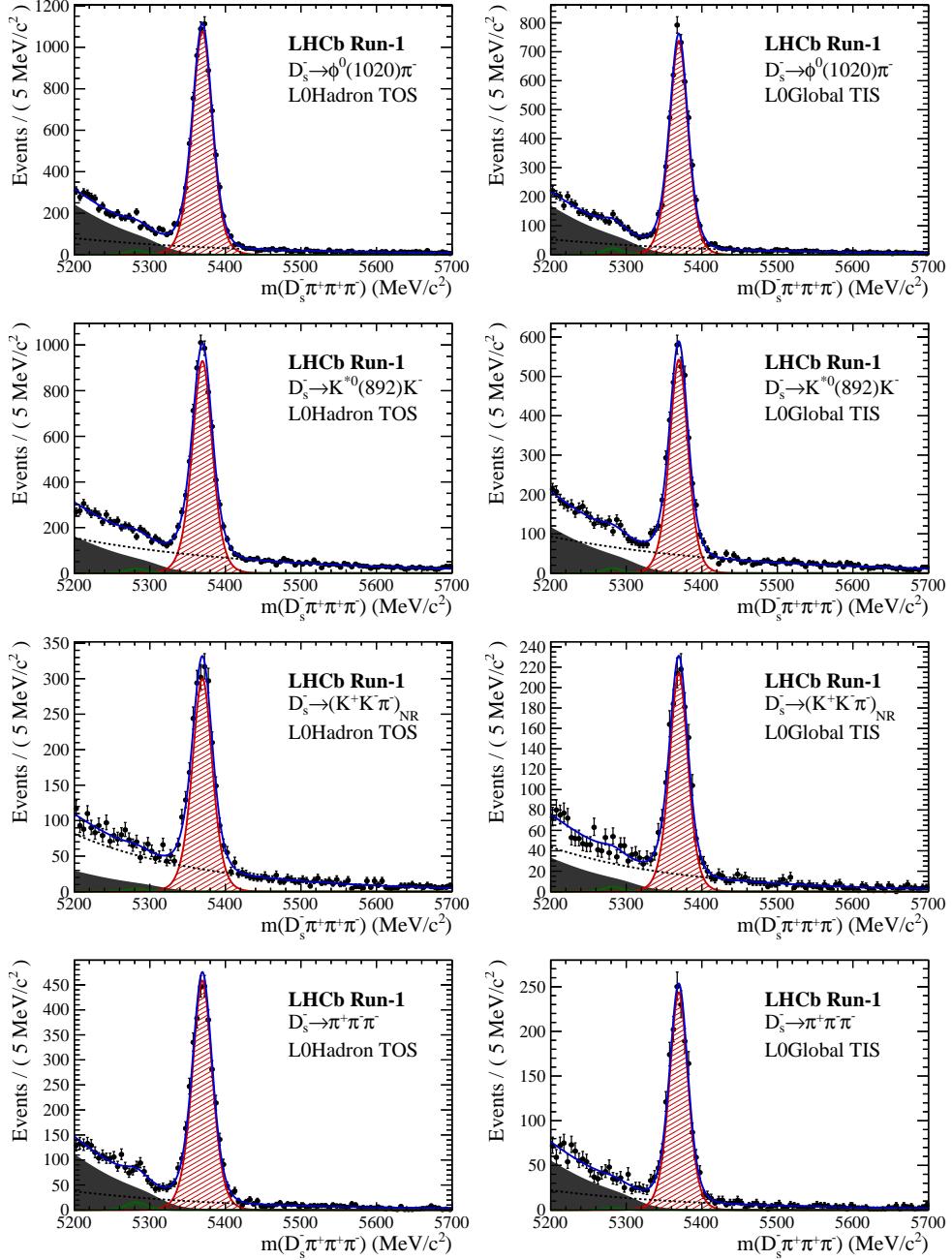


Figure 2.1: Invariant mass distributions of  $B_s^0 \rightarrow D_s\pi\pi\pi$  candidates for Run-I data.

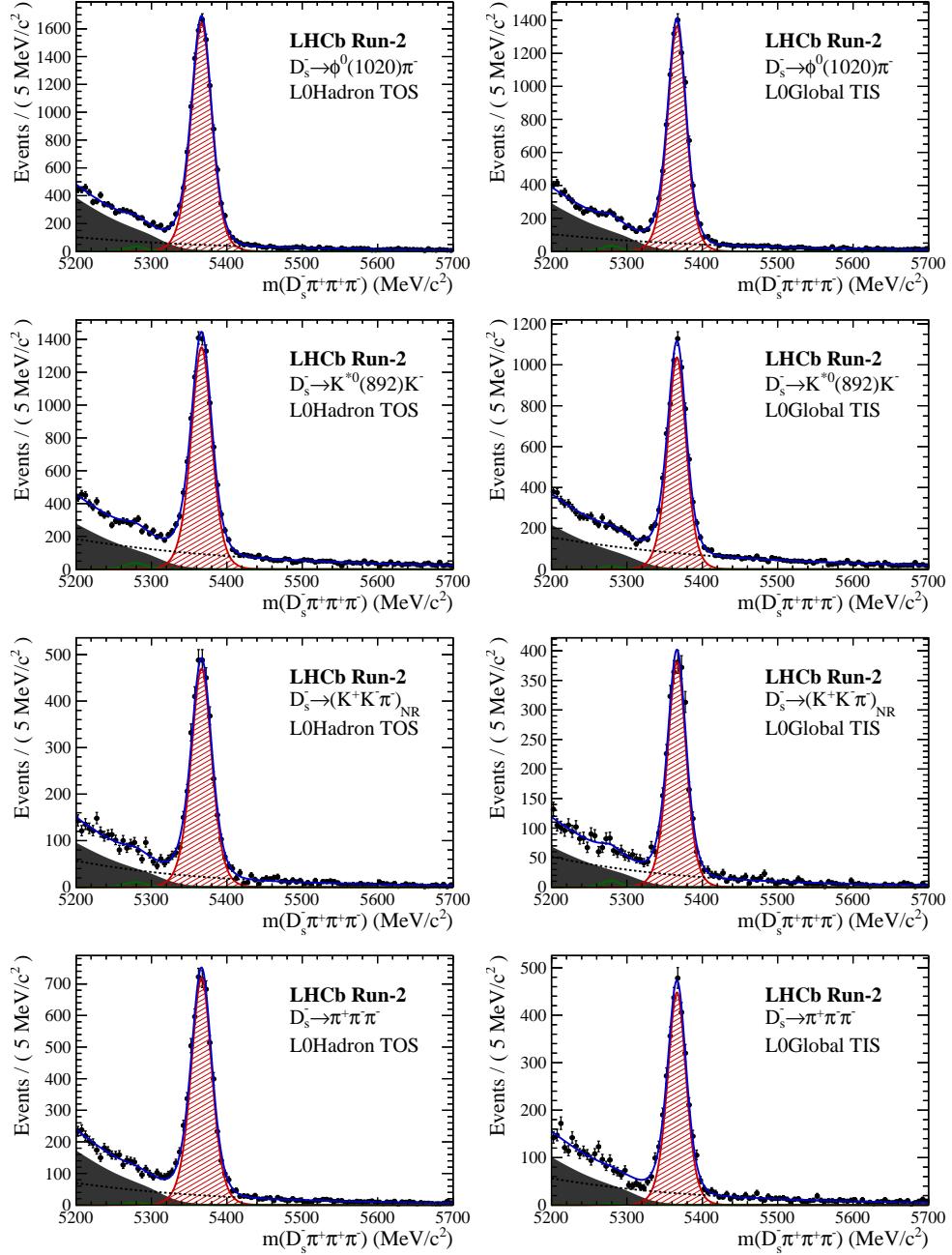


Figure 2.2: Invariant mass distributions of  $B_s^0 \rightarrow D_s\pi\pi\pi$  candidates for Run-II data.

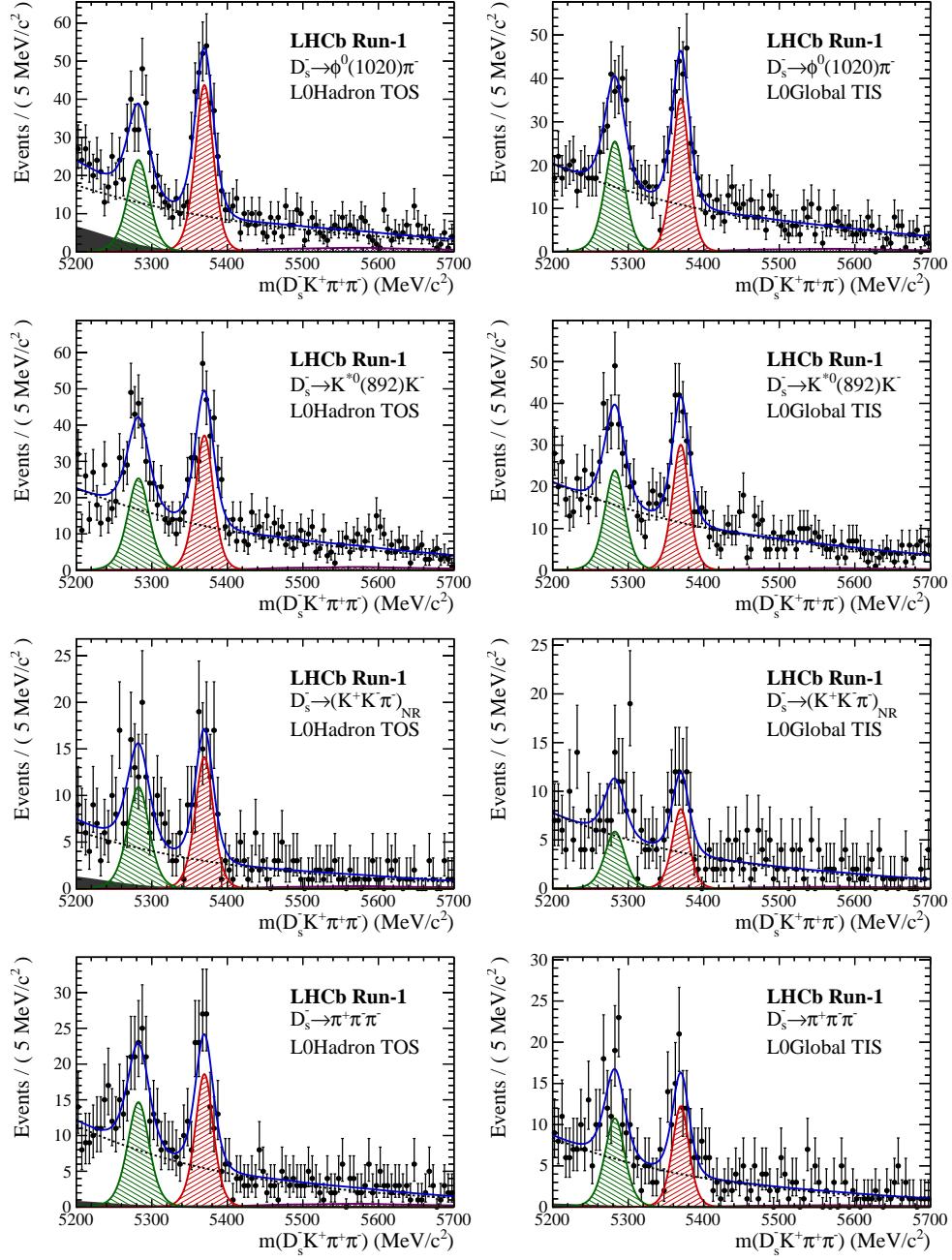


Figure 2.3: Invariant mass distributions of  $B_s^0 \rightarrow D_s K \pi \pi$  candidates for Run-I data.

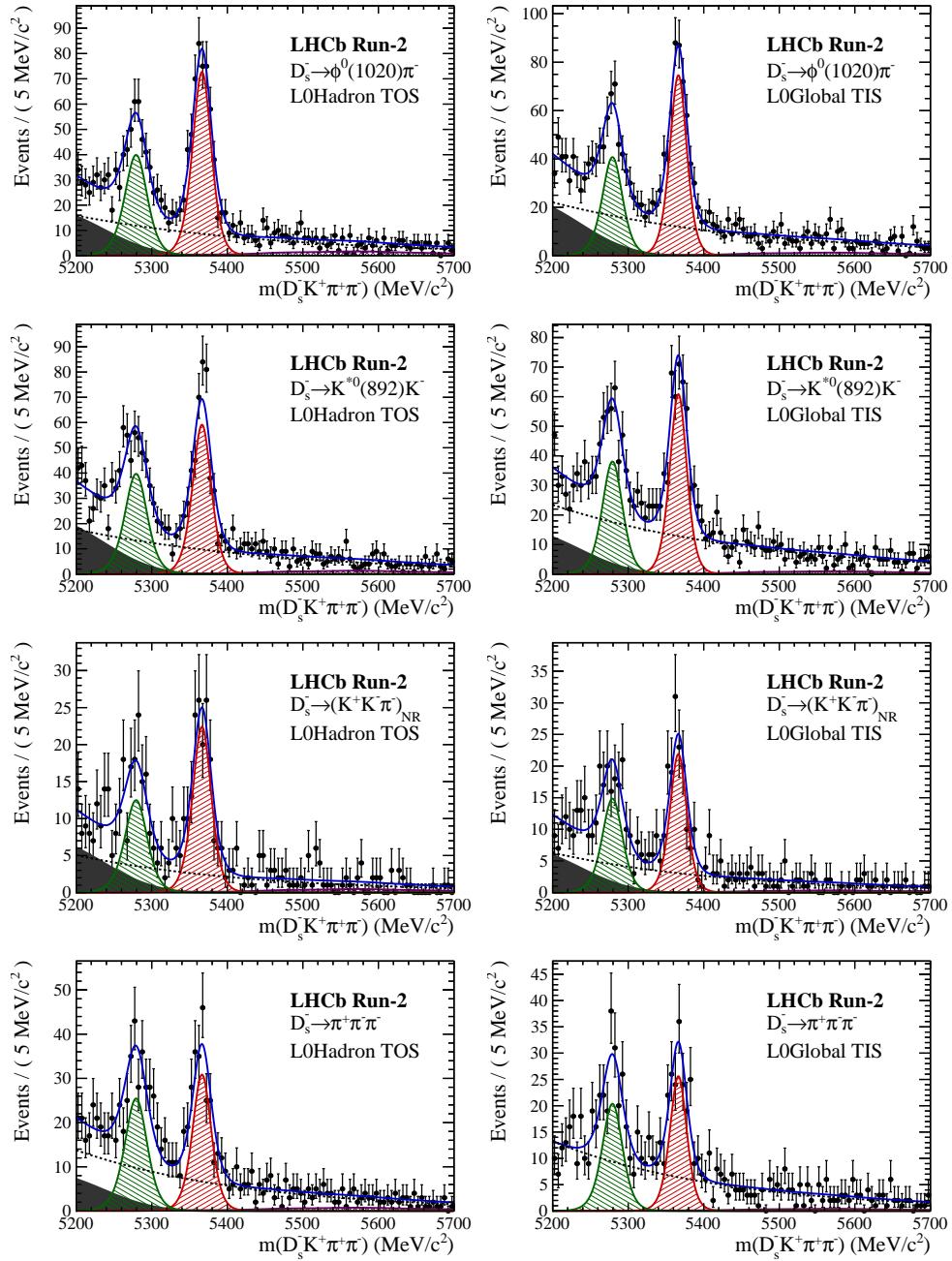


Figure 2.4: Invariant mass distributions of  $B_s^0 \rightarrow D_s K \pi \pi$  candidates for Run-II data.

## 540 C Decay-time Resolution fits

541 This section contains all fits to the distributions of the decay time difference  $\Delta t$  between  
 542 the true and the reconstructed decay time of the truth-matched  $B_s^0$  candidates on MC.  
 543 The fits are performed in bins of the decay time error  $\sigma_t$ , where an adaptive binning  
 544 scheme is used to ensure that approximately the same number of events are found in each  
 545 bin.

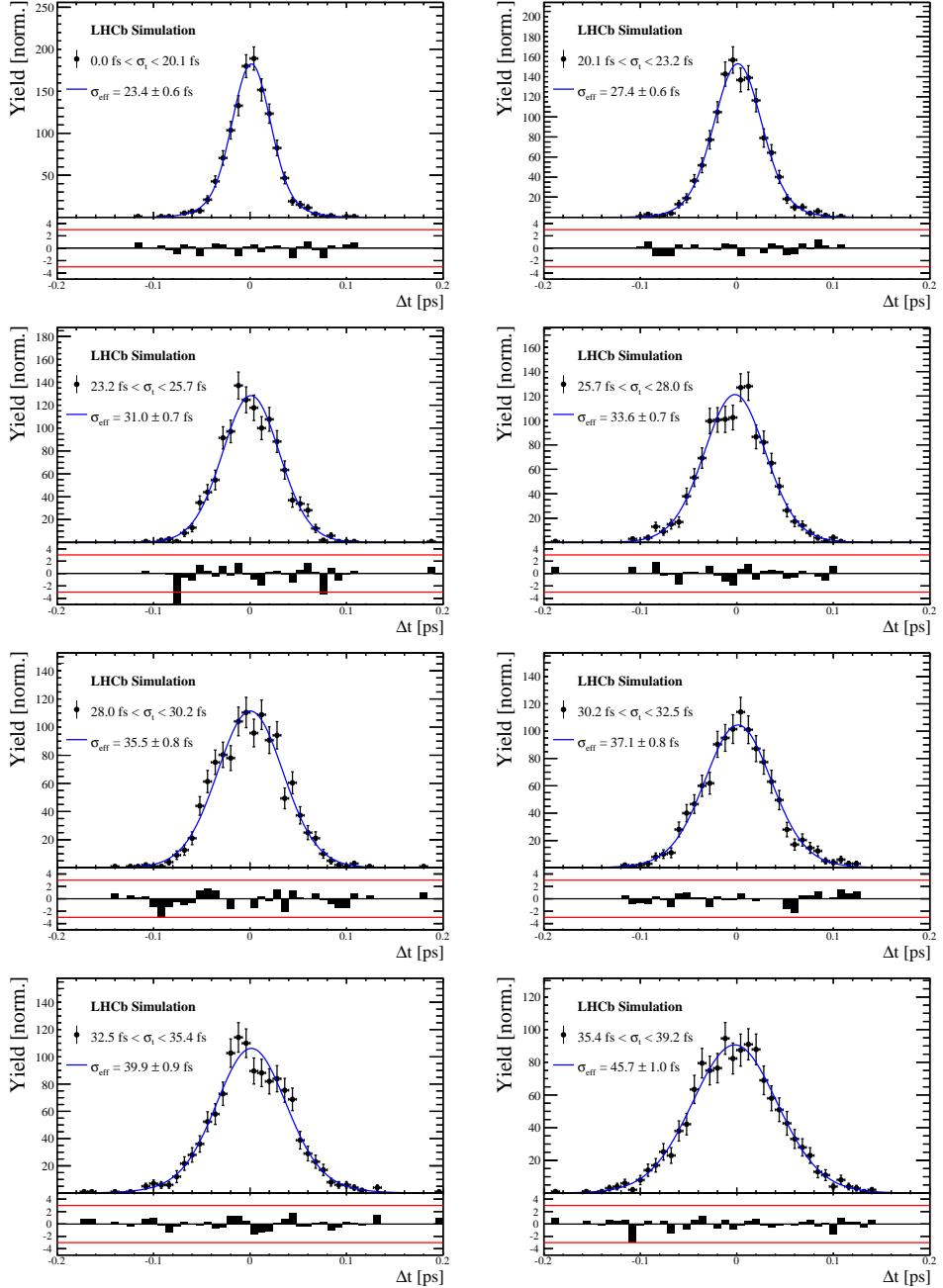


Figure 3.1: Difference of the true and measured decay time of  $B_s^0 \rightarrow D_s K\pi\pi$  MC candidates in bins of the per-event decay time error estimate..

$\sigma_t$ Bin [fs]	$\sigma_1$ [fs]	$\sigma_2$ [fs]	$f_1$	D	$\sigma_{eff}$ [fs]
0.0 - 20.1	$19 \pm 0.675$	$33.8 \pm 1.77$	$0.75 \pm 0$	$0.917 \pm 0.00406$	$23.4 \pm 0.599$
20.1 - 23.2	$23.4 \pm 0.86$	$37.4 \pm 1.95$	$0.75 \pm 0$	$0.888 \pm 0.00477$	$27.4 \pm 0.621$
23.2 - 25.7	$28.1 \pm 1.02$	$38.7 \pm 2.32$	$0.75 \pm 0$	$0.86 \pm 0.00563$	$31 \pm 0.671$
25.7 - 28.0	$30.1 \pm 1.12$	$43.2 \pm 2.56$	$0.75 \pm 0$	$0.837 \pm 0.00651$	$33.6 \pm 0.734$
28.0 - 30.2	$32.4 \pm 1.12$	$44.2 \pm 2.59$	$0.75 \pm 0$	$0.819 \pm 0.00694$	$35.5 \pm 0.756$
30.2 - 32.5	$32.6 \pm 1.38$	$49.2 \pm 3.04$	$0.75 \pm 0$	$0.805 \pm 0.00792$	$37.1 \pm 0.841$
32.5 - 35.4	$34.4 \pm 1.19$	$54.7 \pm 2.85$	$0.75 \pm 0$	$0.778 \pm 0.0086$	$39.9 \pm 0.879$
35.4 - 39.2	$41.9 \pm 1.8$	$56.9 \pm 4.18$	$0.75 \pm 0$	$0.719 \pm 0.00997$	$45.7 \pm 0.962$
39.2 - 44.7	$42.2 \pm 1.56$	$68.1 \pm 4.01$	$0.75 \pm 0$	$0.687 \pm 0.0114$	$48.8 \pm 1.08$
44.7 - 120.0	$55.5 \pm 2.59$	$83 \pm 14.7$	$0.75 \pm 0$	$0.546 \pm 0.0521$	$62 \pm 4.89$

Table 3.1: Measured time resolution for  $B_s \rightarrow D_s K\pi\pi$  MC in bins of the per-event decay time error estimate.

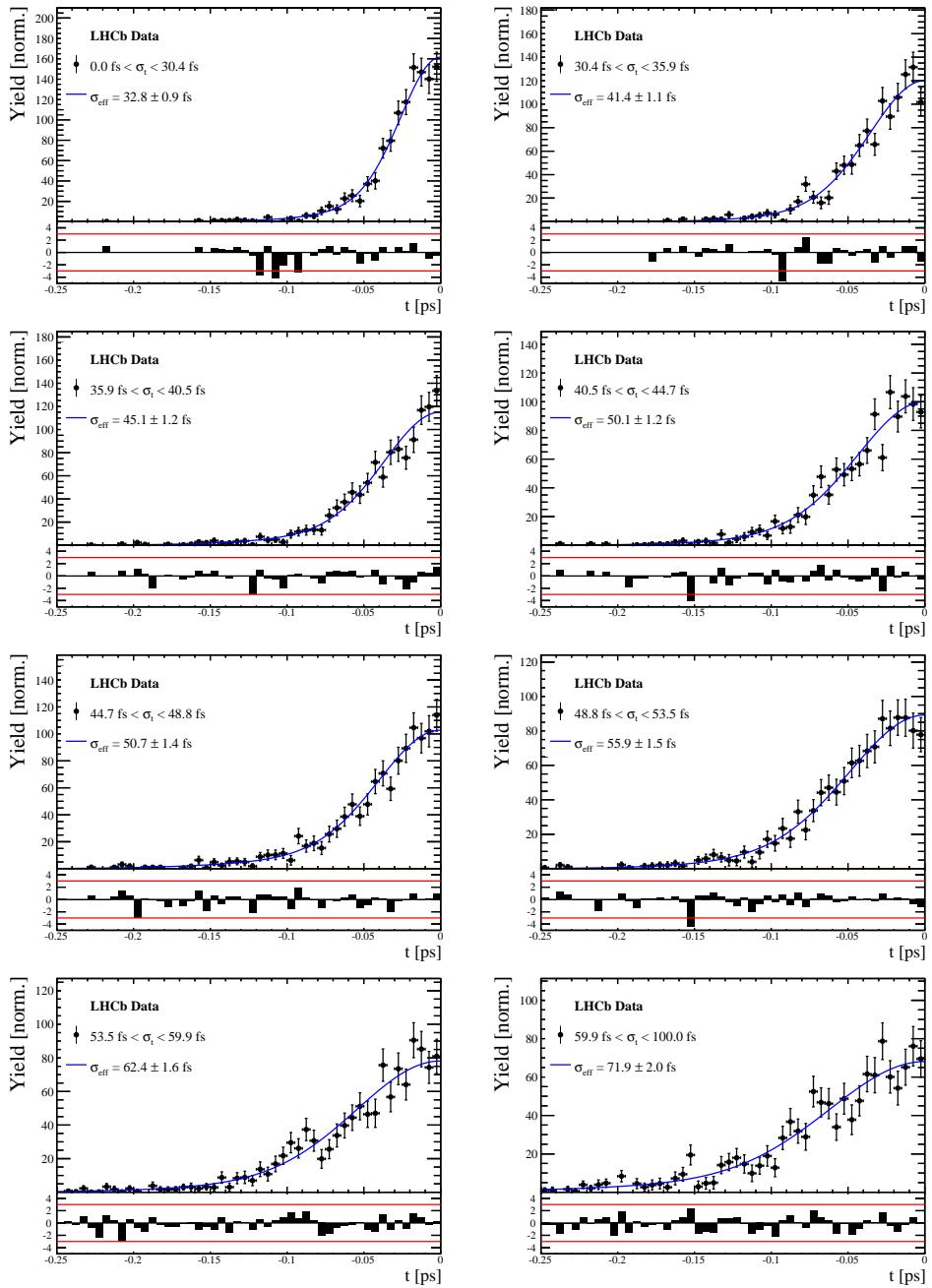


Figure 3.2: Decay-time distribution for fake  $B_s$  candidates from promptly produced  $D_s$  candidates, combined with random prompt  $K\pi\pi$  bachelor tracks, for bins in the per-event decay time error estimate.

$\sigma_t$ Bin [fs]	$\sigma_1$ [fs]	$\sigma_2$ [fs]	$f_1$	D	$\sigma_{eff}$ [fs]
0.0 - 30.4	$25.4 \pm 1.03$	$50.7 \pm 2.77$	$0.75 \pm 0$	$0.844 \pm 0.00822$	$32.8 \pm 0.942$
30.4 - 35.9	$34.5 \pm 1.46$	$60.2 \pm 3.48$	$0.75 \pm 0$	$0.763 \pm 0.0108$	$41.4 \pm 1.08$
35.9 - 40.5	$35.6 \pm 1.35$	$71.3 \pm 3.84$	$0.75 \pm 0$	$0.726 \pm 0.0121$	$45.1 \pm 1.18$
40.5 - 44.7	$42.3 \pm 1.65$	$73.3 \pm 4.21$	$0.75 \pm 0$	$0.673 \pm 0.0132$	$50.1 \pm 1.24$
44.7 - 48.8	$39.6 \pm 1.64$	$84.8 \pm 5.07$	$0.75 \pm 0$	$0.666 \pm 0.0145$	$50.7 \pm 1.36$
48.8 - 53.5	$47.6 \pm 1.94$	$82.4 \pm 5.48$	$0.75 \pm 0$	$0.611 \pm 0.0157$	$55.9 \pm 1.46$
53.5 - 59.9	$53 \pm 2.15$	$95.3 \pm 6.84$	$0.75 \pm 0$	$0.541 \pm 0.0174$	$62.4 \pm 1.63$
59.9 - 100.0	$60.5 \pm 2.8$	$125 \pm 14$	$0.75 \pm 0$	$0.443 \pm 0.0204$	$71.9 \pm 2.03$

Table 3.2: Measured time resolution for prompt- $D_s$  data in bins of the per-event decay time error estimate.

<sup>546</sup> D MC corrections

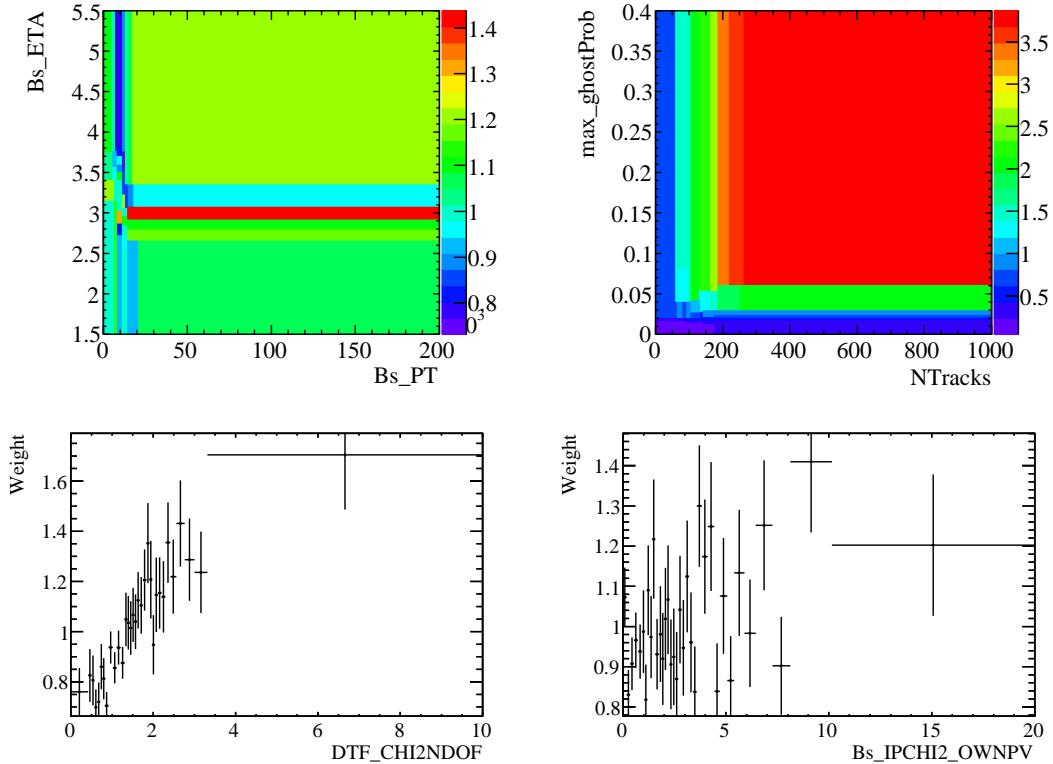


Figure 4.1: Weights applied to correct for Data/MC differences.

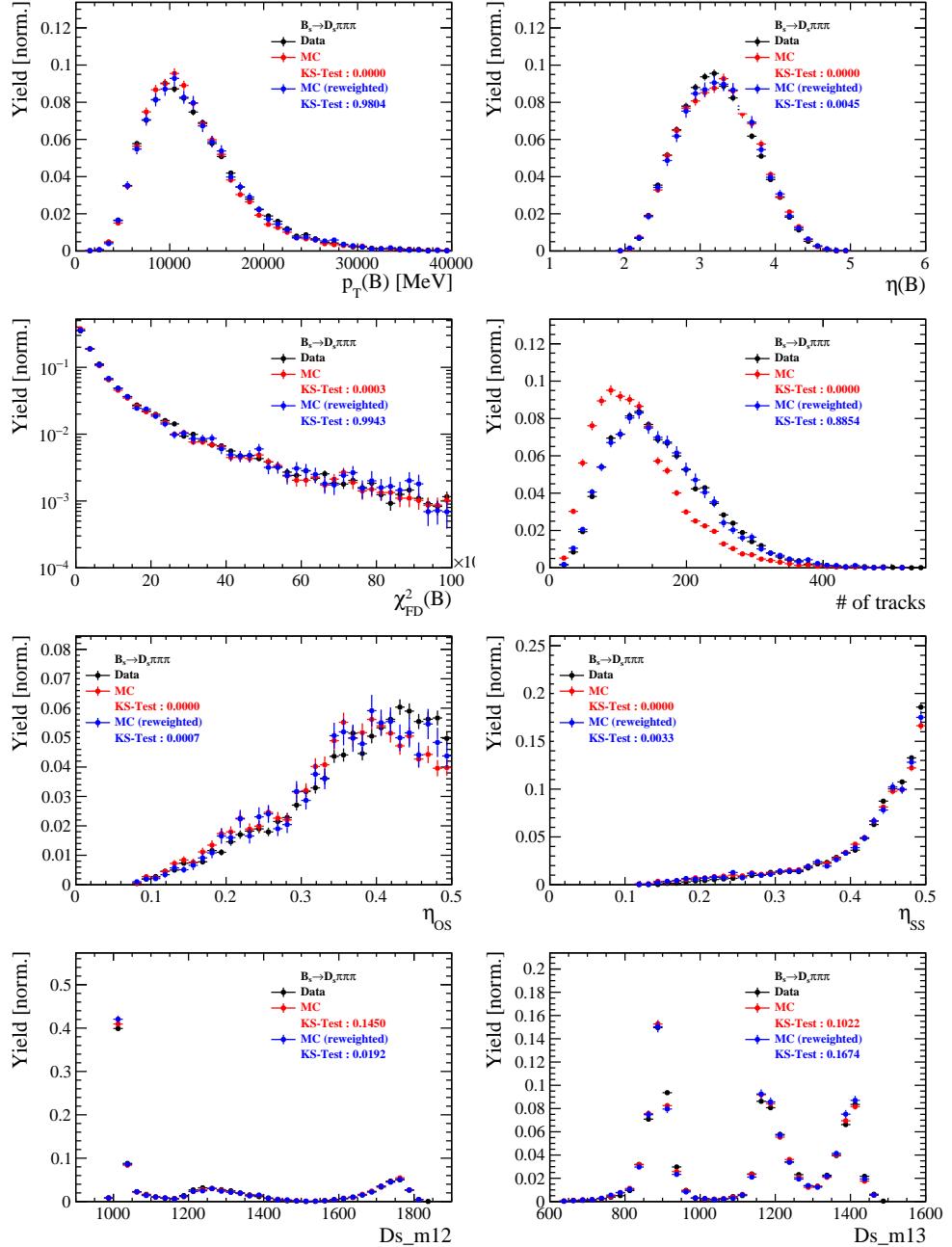


Figure 4.2: Comparison of selected variables for  $B_s \rightarrow D_s \pi\pi\pi$  decays.

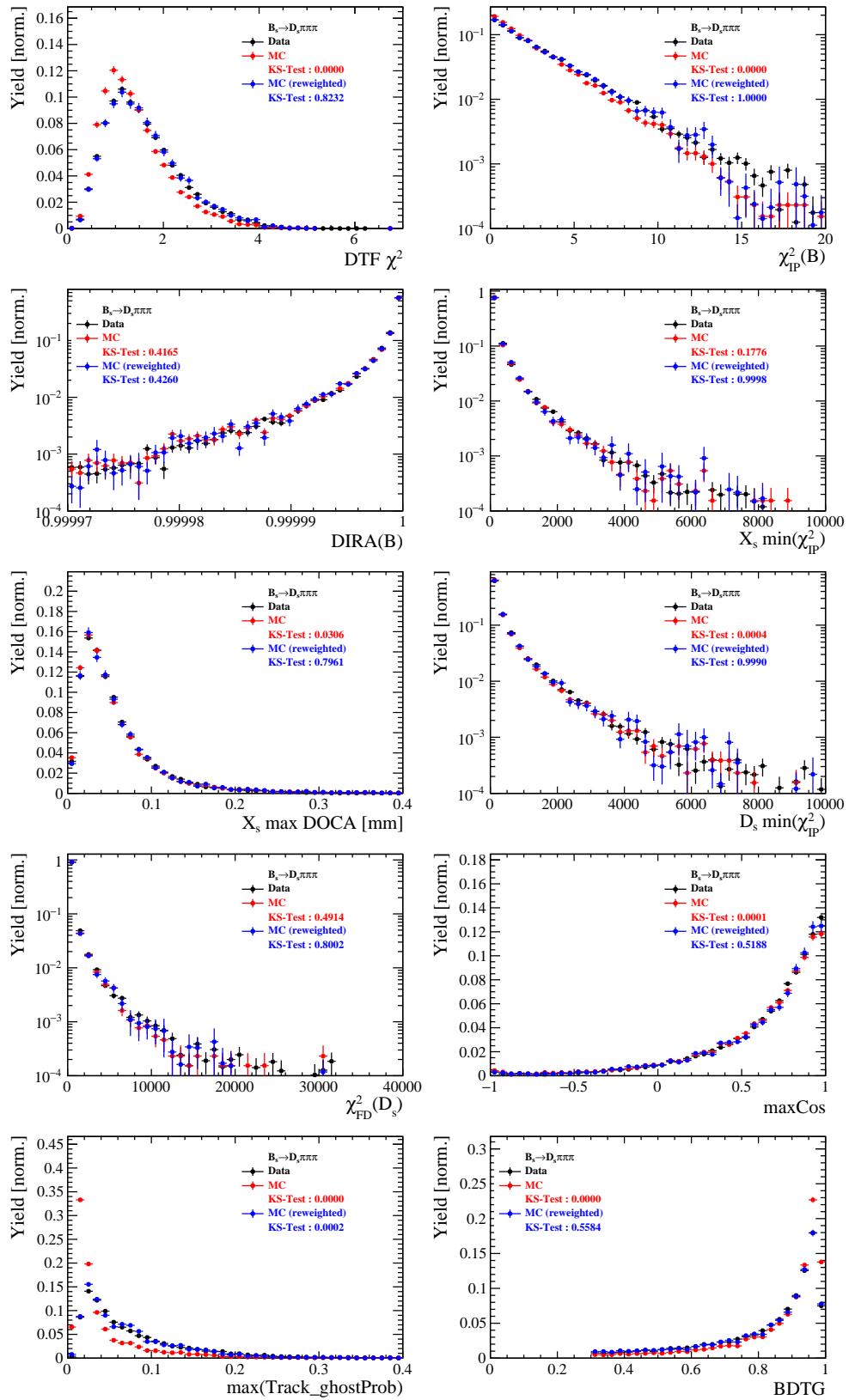


Figure 4.3: Comparison of BDTG input variables and classifier response for  $B_s \rightarrow D_s \pi\pi\pi$  decays.

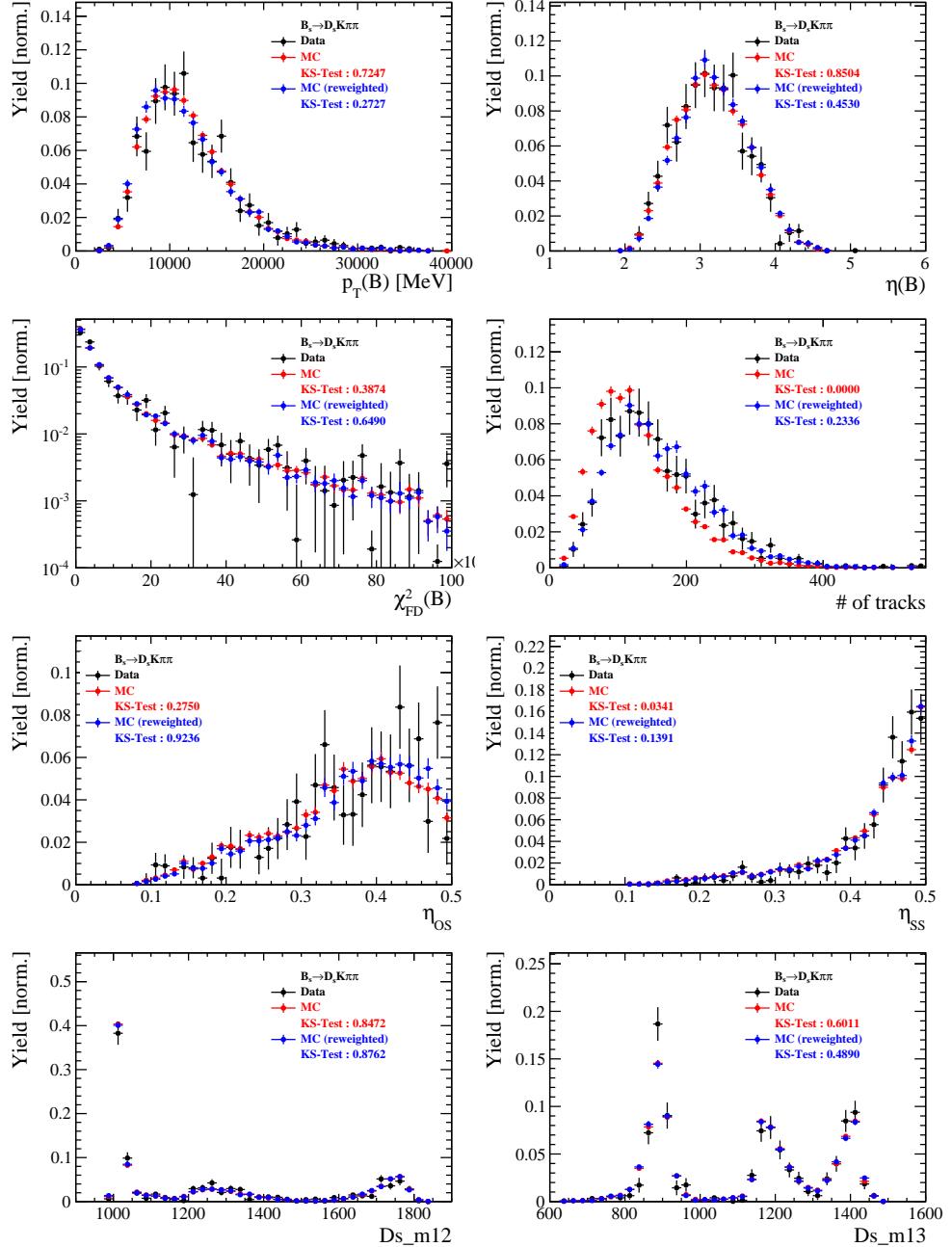


Figure 4.4: Comparison of selected variables for  $B_s \rightarrow D_s K\pi\pi$  decays.

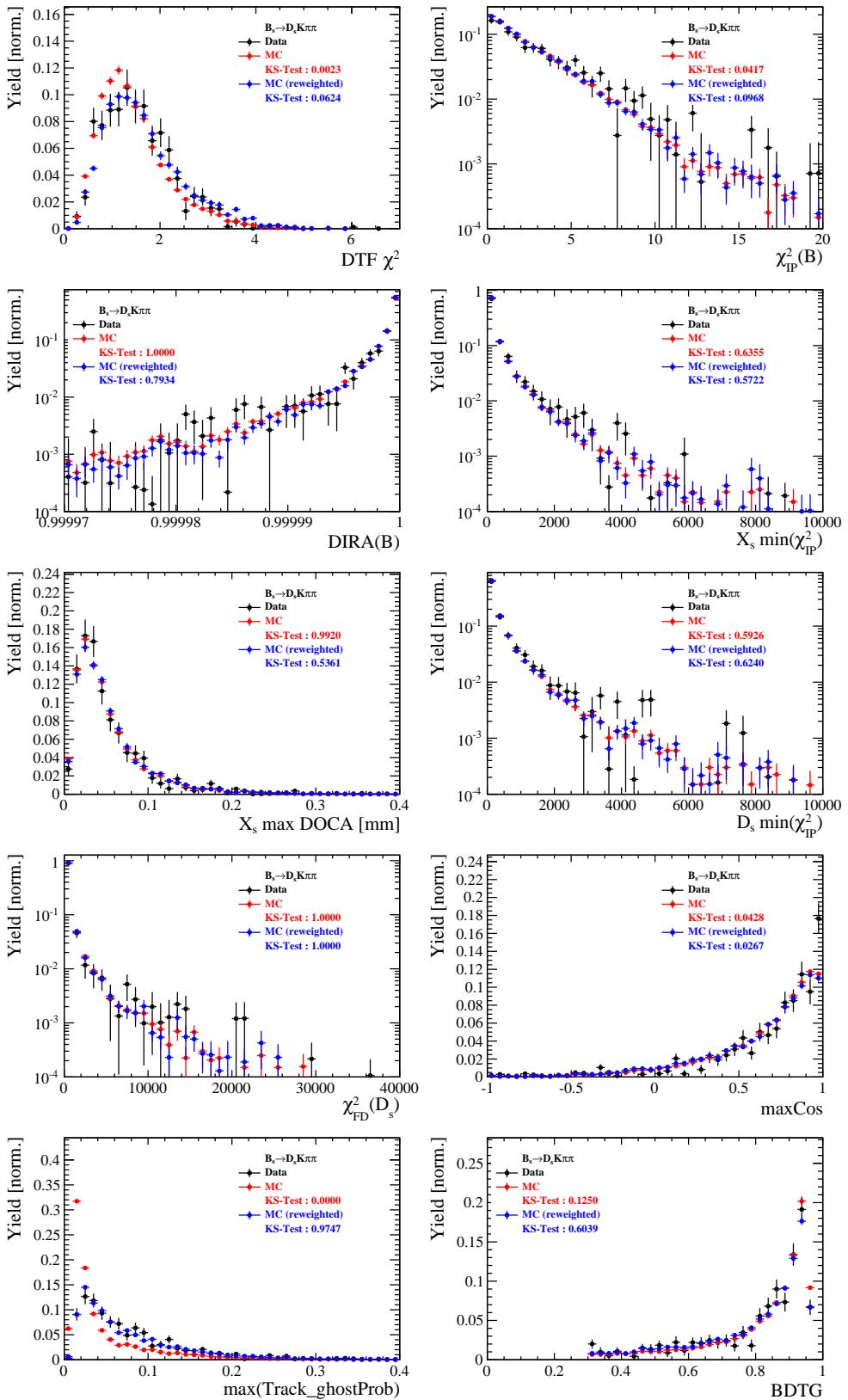


Figure 4.5: Comparison of BDTG input variables and classifier response for  $B_s \rightarrow D_s K\pi\pi$  decays.

547 **E Data distributions**

548 **E.1 Comparison of signal and calibration channel**

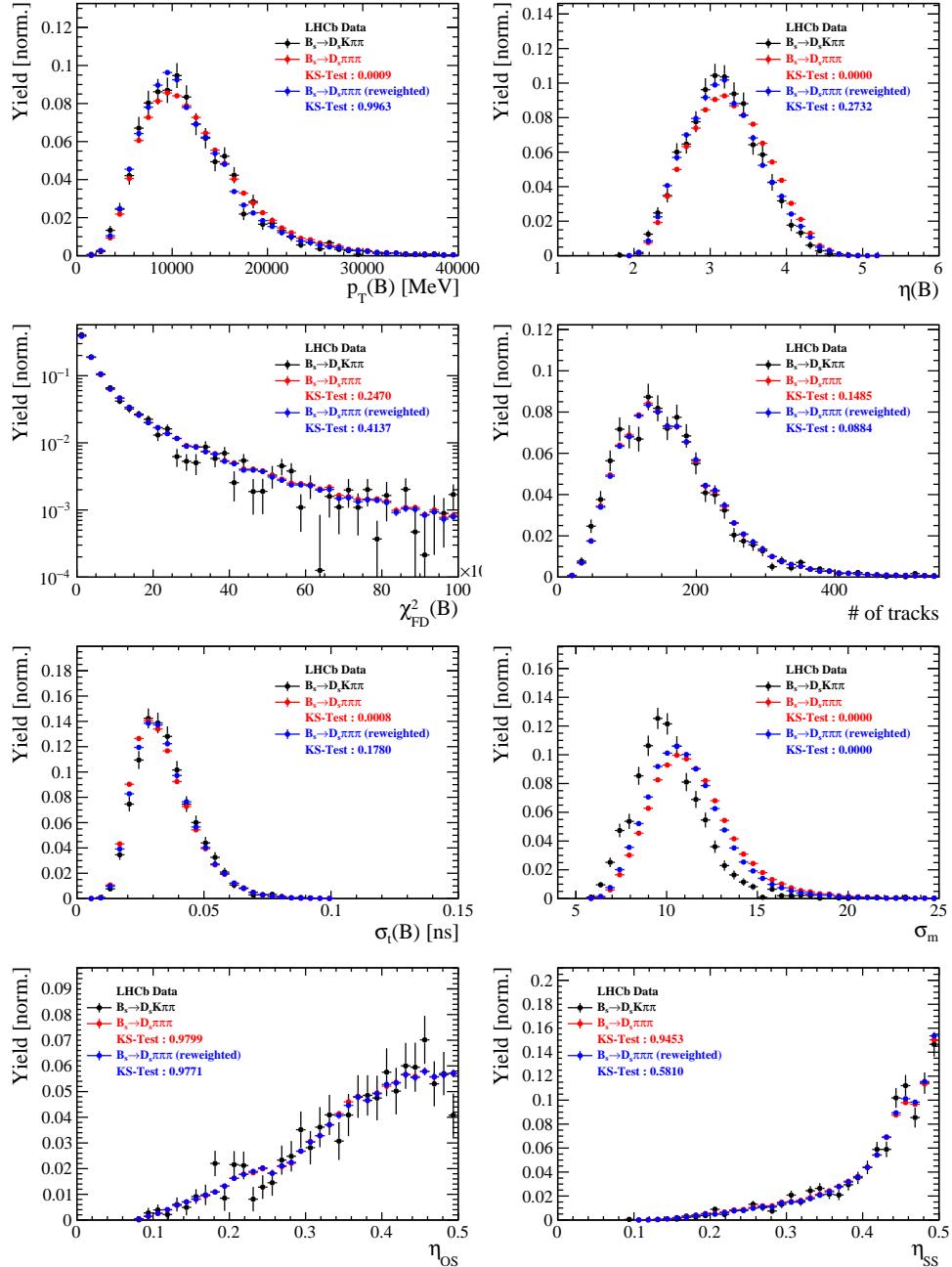


Figure 5.1: Comparison of selected variables.

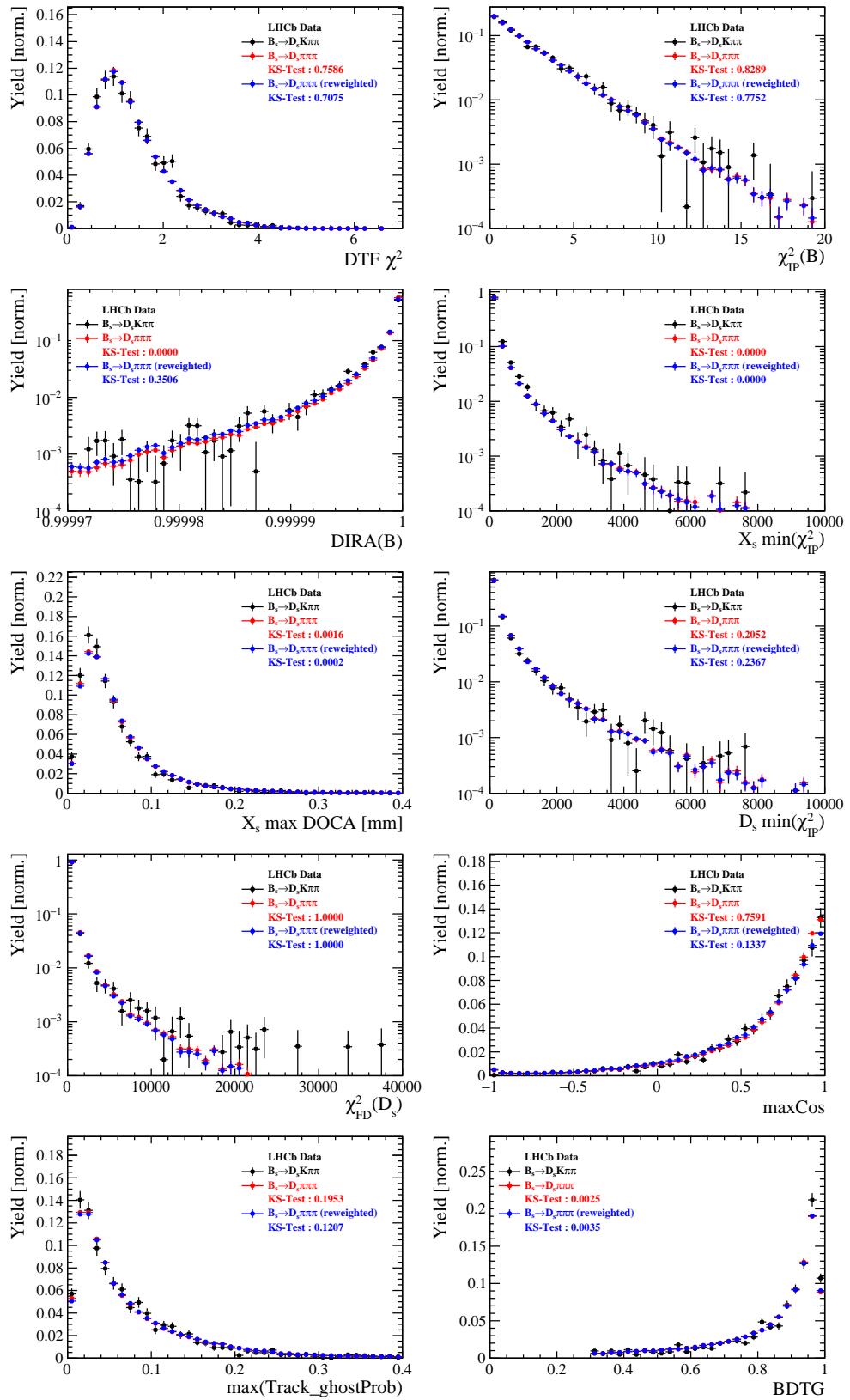


Figure 5.2: Comparison of BDTG input variables and classifier response.

549 E.2 Comparison of Run-I and Run-II data

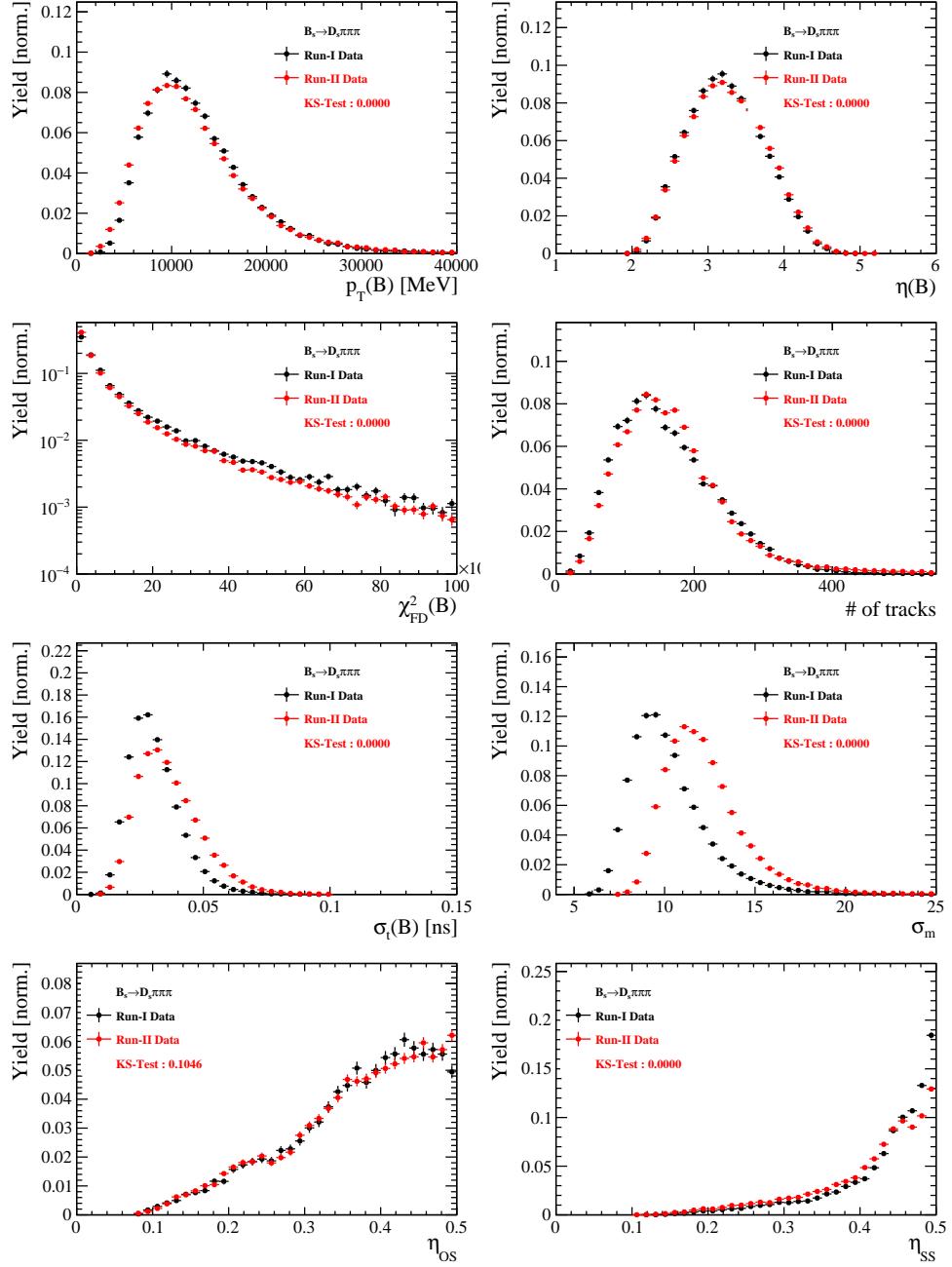


Figure 5.3: Comparison of selected variables.

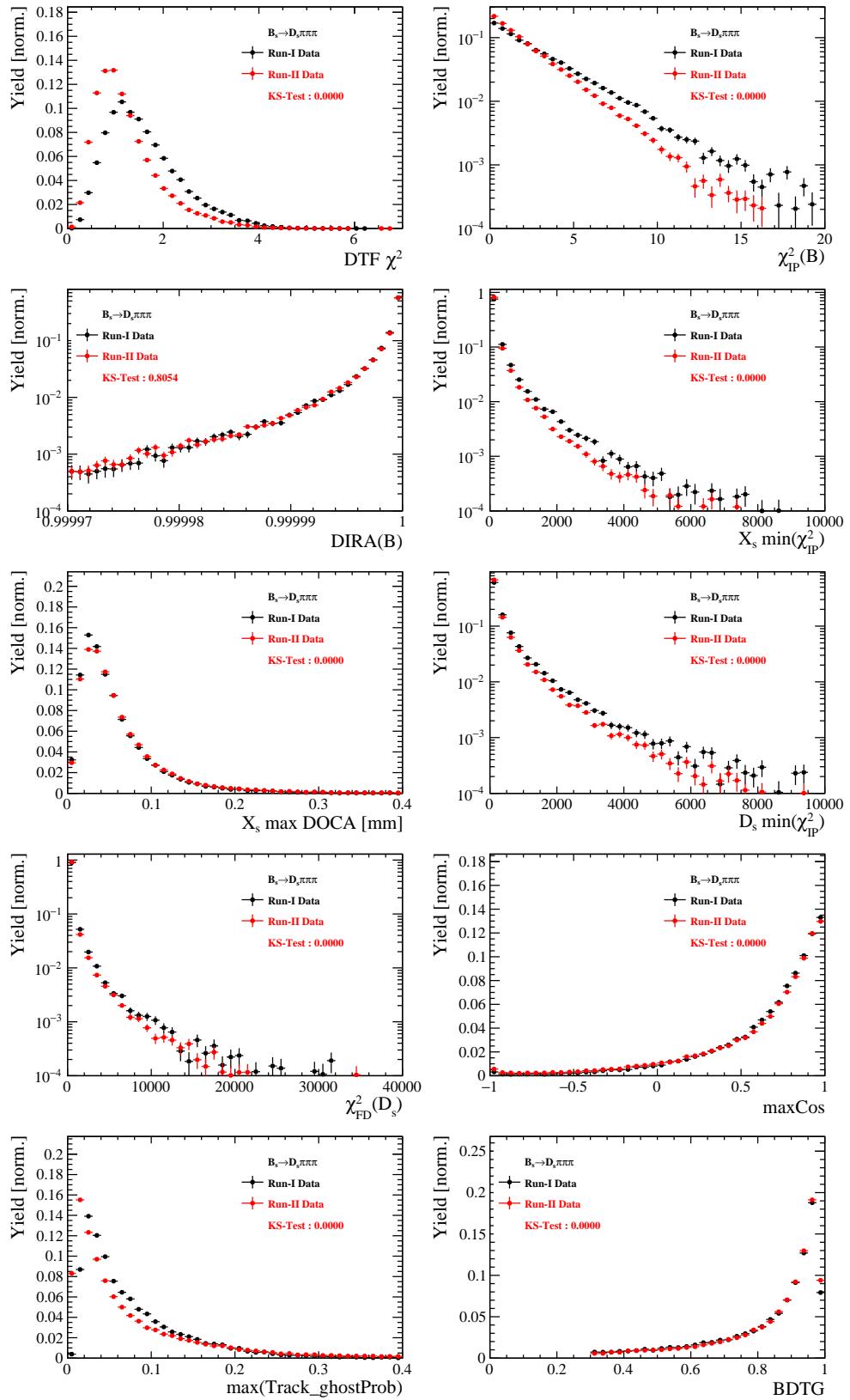


Figure 5.4: Comparison of BDTG input variables and classifier response.

550 E.3 Comparison of  $D_s$  final states

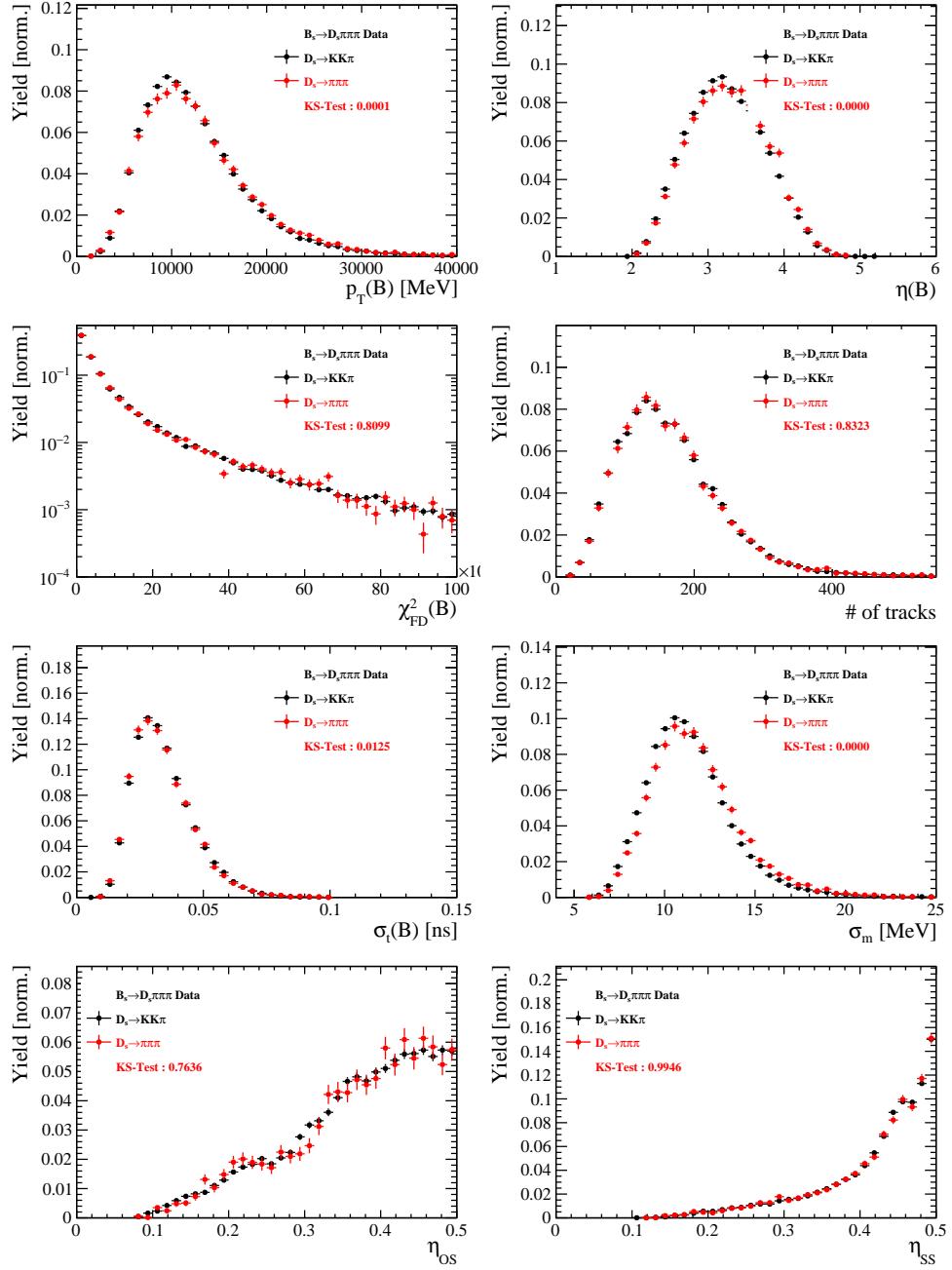


Figure 5.5: Comparison of selected variables.

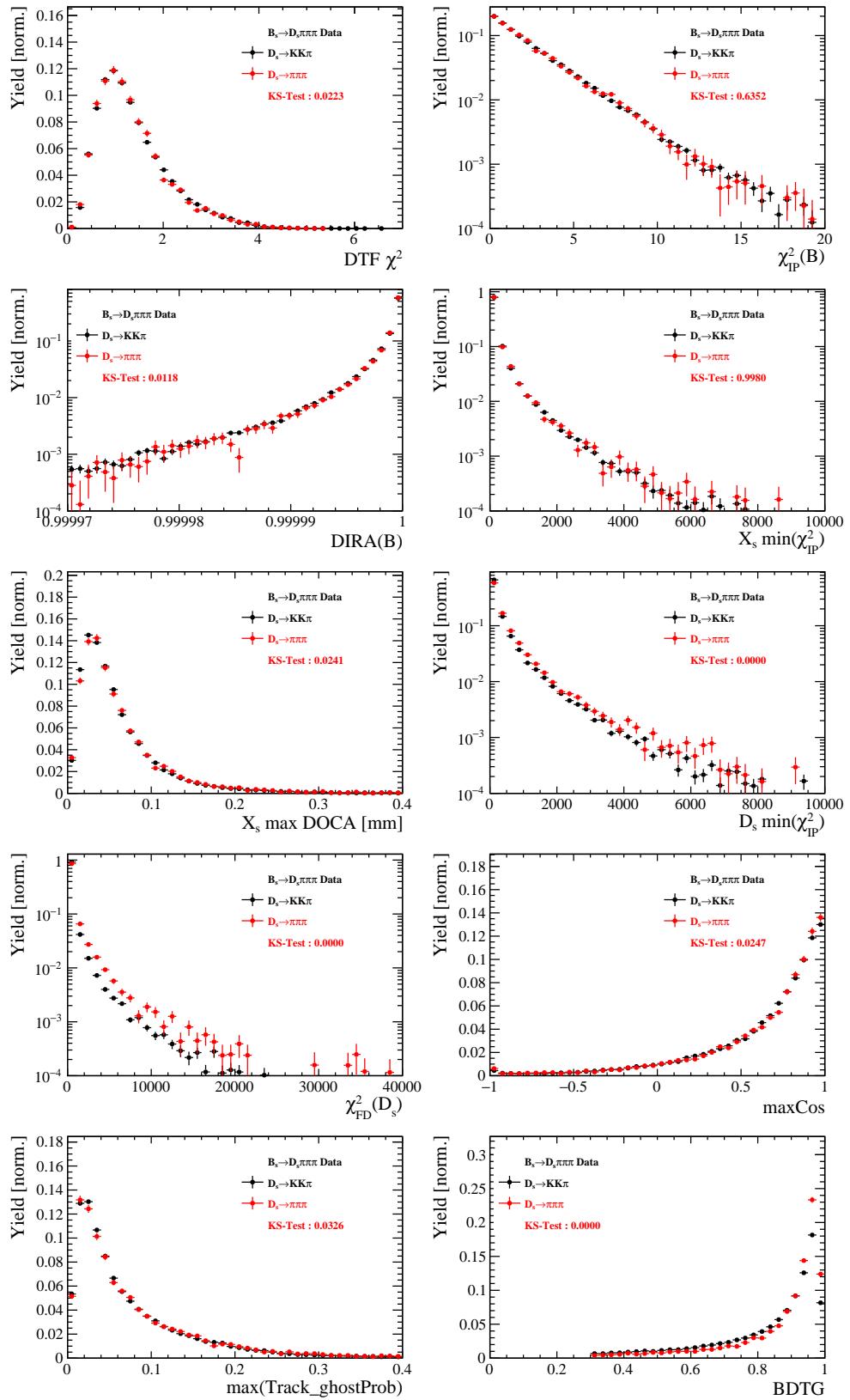


Figure 5.6: Comparison of BDTG input variables and classifier response.

551 E.4 Comparison of trigger categories

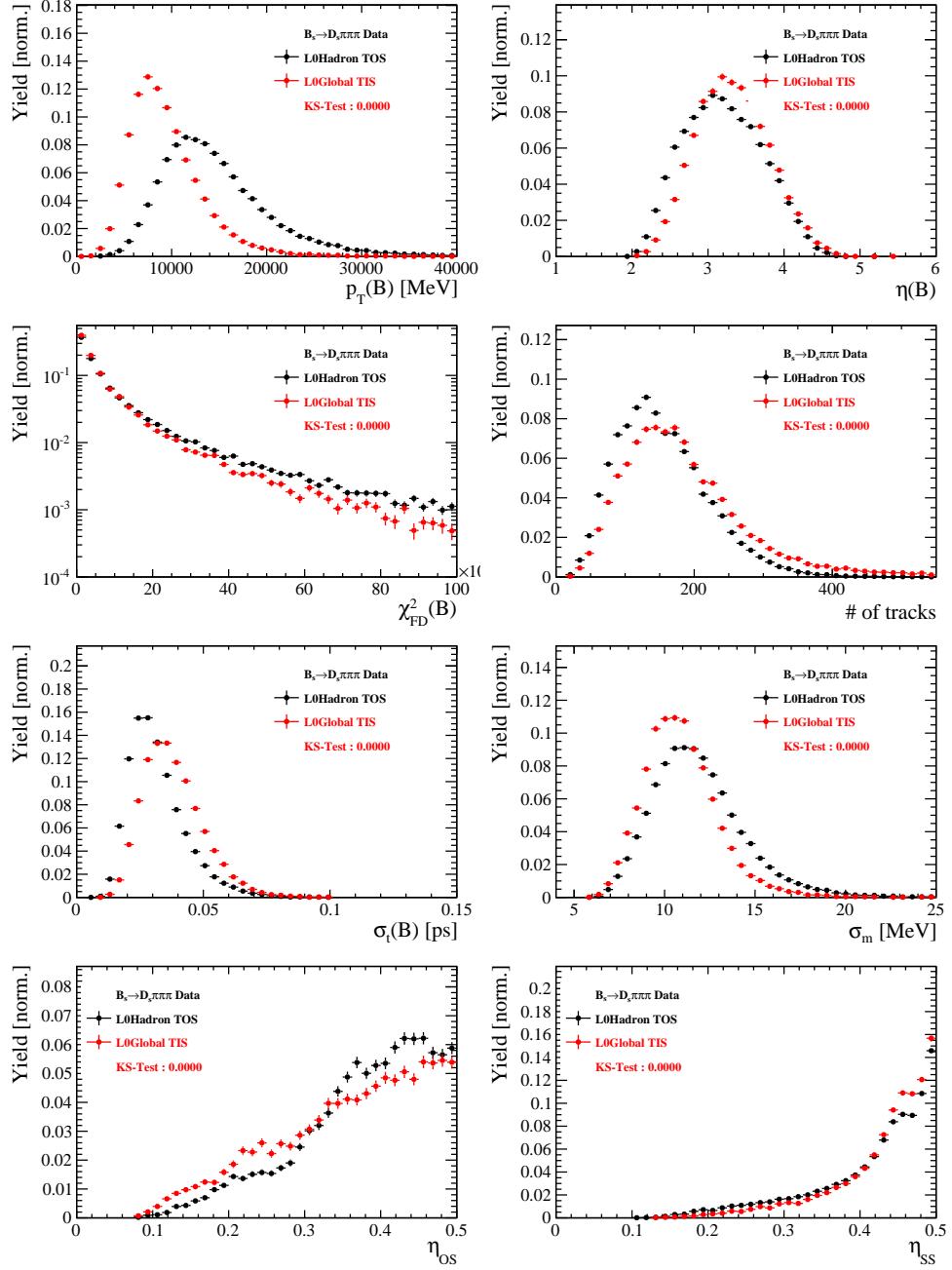


Figure 5.7: Comparison of selected variables.

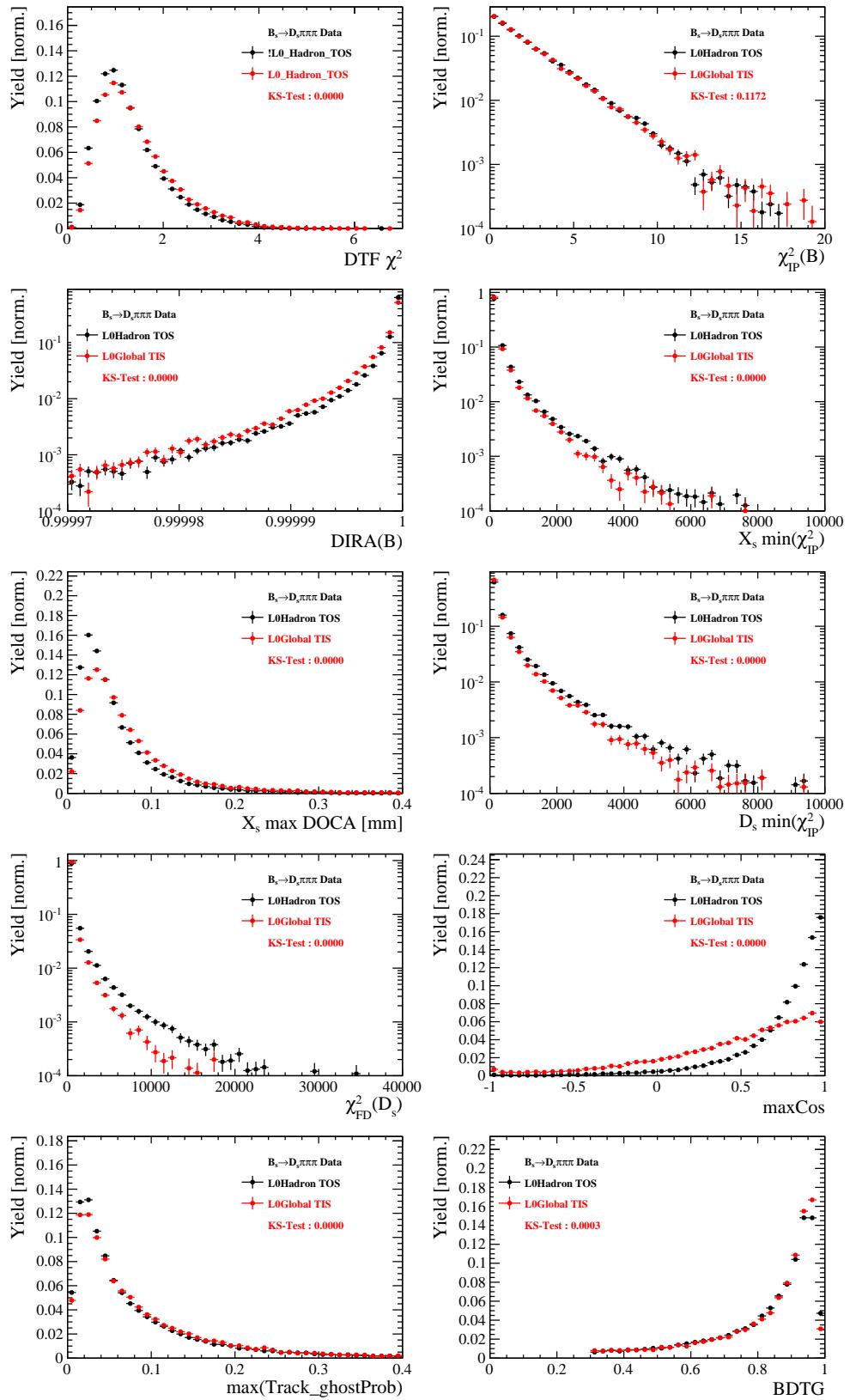


Figure 5.8: Comparison of BDTG input variables and classifier response.

552 E.5 Comparison of  $B_s$  and  $B_d$  decays

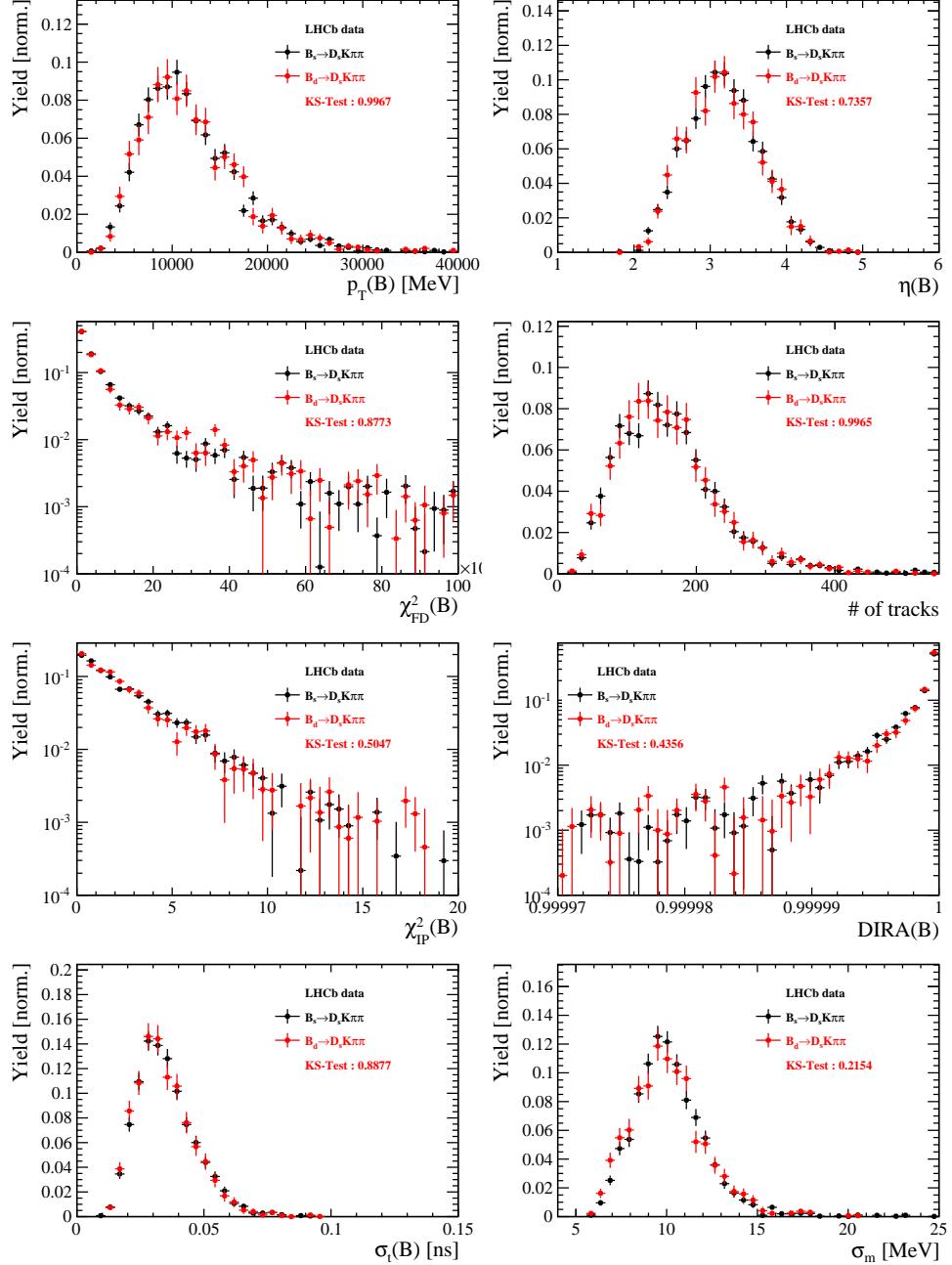


Figure 5.9: Comparison of selected variables.

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