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Global Robust Exponential Stability of Complex-valued Cohen-Grossberg Neural Networks with Mixed Delays

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***Abstract*—The global robust exponential stability of interval complex-valued Cohen-Grossberg neural networks (CGNNs)with mixed delays and distributed delays are considered in this paper. Based on some mild assumptions, we prove the existence, uniqueness and exponential stability of the equilibrium point of the interval complex-valued CGNNs. We introduce two numerical examples in the end.**

***Keywords*—Complex-valued Cohen-Grossberg neural networks; robust exponential stability; Lyapunov function**

I. Introduction

In recent years, complex-valued neural networks attract more and more attention to scholars due to their potential applications for parallel computing, nonlinear programming and associative memory and so on (see [1]–[12]). Actually, complex-valued neural networks are capable of settling some problems which can not be solved with real-valued neural networks, such as the detection of symmetry problem and the XOR problem (see [13]–[15]). As we all know, the stability of the equilibrium point is one of the desirable dynamical properties of the considered neural network. However, in electronic implementations, the stability may be affected by some unavoidable uncertainty and modeling error, time delay, external disturbance and so on (see [16]–[19]).

For the essentiality of CGNNs in many applications, some explores on stability of CGNNs have drawn considerable attention, and a series of significant results have been researched in the literatures. For example, in [20], authors analyzed robust stability of the switched CGNNs with mixed time-varying delays based on a Lyapunov approach and linear matrix inequality (LMI) technique. In [21], authors presented new criteria on the stability of a class of second-order interval Cohen-Grossberg to ensure the equilibrium point of the neural networks exists, unique and the globally stable under some uncertainties. In general, complex-valued neural networks have more and different properties compared with real-valued ones. Recently, there are some papers on complex-valued neural networks. For example, in [22], authors studied the existence and uniqueness of equilibrium point, global exponential stability and global asymptotic stability of a delayed complex-valued neural networks which has two classes of complex-valued activation function.

Actually, every neural networks often have spatial properties that are determined by the existence of plenty of parallel paths with serval axon sizes and length. This fact means the signal propagation cannot be instantaneous. Therefore, it is necessary to introduce distributed delays to make our model closer to reality. In [23], authors obtained some new results in exponential convergence of interval CGNNs with mixed delays. However, as far as we know, few literatures concern the robust stability of complex-valued CGNNs with mixed delays and distributed delays.

According to the above dissertation, we will analyze the global robust exponential stability of complex-valued CGNNs with mixed delays and distributed delays. The following of the paper is consisted of the follow sections. In section II, the neural networks model is introduced, several virtual lemmas and basic definitions are presented. In section III, the new criteria on the existence and global exponential stability of equilibrium point of neural networks are established. In section IV, there are some numerical examples that are provided to demonstrate the applicability and the effectiveness of our proposed stability results.

To make reading easier, the following notations will be used. let $\mathbb{C}$ denote the complex domain. By $\mathbf{i}$, we denote the imaginary unit, that is $\mathbf{i}=\sqrt{-1}$. For complex vector $z\in\mathbb{C}^n$, let $|z|=(|z_1|,|z_2|,\ldots,|z_n|)$ represents the module of the vector z, and $\|z\|=(\sum_{i=1}^n|z_i|^2)^{\frac{1}{2}}$ denotes the norm of the vector z. I is the identity matrix. For any matrix $B\in\mathbb{R}^{n\times n}$, $B>0$(or $B\geq 0$) means B is a positive definite (semi-definite) matrix. $|B|$ is the absolute value matrix of B, i.e. $|B|=(|b_{ij}|)_{n\times n}$. $\|B\|_2=\sqrt{\lambda_{\max}(B^TB)}$, where $\lambda_{\max}(B^TB)$ means the maximum eigenvalue of matrix B^TB.

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II. PROBLEM DESCRIPTION AND PRELIMINARIES

In this paper, we will investigate the following complex-valued CGNNs with mixed delays:

$$\begin{aligned}\dot{z}(t) =& C(z(t))[-Dz(t) + Af(z(t)) + Bf(z(t-\tau(t))) \\ &+Q\int_{t-\beta}^{t} f(z(s))ds + J],\end{aligned} \tag{1}$$

where $z(t) = (z_1(t), \ldots, z_n(t))^T \in \mathbb{C}^n$ denotes the state vector associate with the neurons. $C(z(t)) = (c_1(z(t)), c_2(z(t)), \ldots, c_n(z(t)))^T : \mathbb{C}^n \to \mathbb{R}^n_+$ represents an amplification function, $D = diag\{d_i\}$ is a positive definite diagonal matrix. $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}$, $Q = (q_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$ are connection weight matrices. $J = (J_1, J_2, \ldots, J_n)^T \in \mathbb{C}^n$ is external constant input vector. $f(z(t)) = (f_1(z_1(t)), f_2(z_2(t)), \ldots, f_n(z_n(t)))^T : \mathbb{C}^n \mapsto \mathbb{C}^n$ is the activation function.

Let $z = x + iy$, where $x = Re(z)$, $y = Im(z)$. Then activation function $f(z)$ can also be denoted by separating into its real and imaginary parts as $f(z) = f^R(x, y) + \mathbf{i}f^I(x, y)$. Similarly, matrices A, B, Q can be described as $A = A^R + \mathbf{i}A^I$, $B = B^R + \mathbf{i}B^I$, $Q = Q^R + \mathbf{i}Q^I$. i.e. $a_{ij} = a^R_{ij} + \mathbf{i}a^I_{ij}$, $b_{ij} = b^R_{ij} + \mathbf{i}b^I_{ij}$, $q_{ij} = q^R_{ij} + \mathbf{i}q^I_{ij}$. $i = 1, 2, \ldots, n$. $J^R = (j^R_1, j^R_2, \ldots, j^R_n)^T$, $J^I = (j^I_1, j^I_2, \ldots, j^I_n)^T$ are the real part and the imaginary part of J.

In order to characterize the parameter uncertainties completely, the parameters $D = diag\{d_i\}$, $A^R = (a^R_{ij})_{n\times n}$, $B^R = (b^R_{ij})_{n\times n}$, $Q^R = diag(q^R_{ij})$ are shown as follows:

$$\begin{aligned}D_I :=&\{D = diag(d_i) : 0 < \underline{D} \le D \le \overline{D}, i.e., \\ &0 < \underline{d_i} \le d_i \le \overline{d_i}, \forall i = 1, 2, \ldots, n\}, \\ A^R_I :=&\{A^R = (a^R_{ij}) : \underline{A}^R \le A^R \le \overline{A}^R, i.e., \\ &\underline{a}^R_{ij} \le a^R_{ij} \le \overline{a}^R_{ij}, \forall i, j = 1, 2, \ldots, n\}, \\ B^R_I :=&\{B^R = (b^R_{ij}) : \underline{B}^R \le B^R \le \overline{B}^R, i.e., \\ &\underline{b}^R_{ij} \le b^R_{ij} \le \overline{b}^R_{ij}, \forall i, j = 1, 2, \ldots, n\}, \\ Q^R_I :=&\{Q^R = (q)^R_{ij} : \underline{Q}^R \le Q^R \le \overline{Q}^R, i.e., \\ &\underline{q}^R_{ij} \le q^R_{ij} \le \overline{q}^R_{ij}, \forall i, j = 1, 2, \ldots, n\},\end{aligned} \tag{2}$$

A^I_I, B^I_I and Q^I_I are similarly defined.

In this paper, our conclusions are based on the following assumptions.

Assumption 1. *(1) $c_i(z) := c_i(x, y)$ is continuous and it satisfies $0 < \underline{c_i} \le c_i(x, y) \le \overline{c_i}$, where $\underline{c_i}$ and $\overline{c_i}$ are both constants.*
(2) function $\tau_i(t)$ is differentiable, and

$$0 \le \tau_i(t) \le \tau_0, 0 \le \dot{\tau}_i(t) \le \tau_i < 1.$$

We denote $\Gamma = diag(\tau_1, \tau_2, \ldots \tau_n)$

Assumption 2. *The activation function $f^R(x, y)$, $f^I(x, y)$ are both continuous and satisfy the Lipschitz condition, that is to say there exist positive constants k_i, l_i, m_i, n_i, such that*

$$\begin{aligned}|f^R_i(s, t) - f^R_i(s', t')| \le k_i|s - s'| + l_i|t - t'|, \\ |f^I_i(s, t) - f^I_i(s', t')| \le m_i|s - s'| + n_i|t - t'|\end{aligned}$$

for $(s, t), (s', t') \in \mathbb{R}^2, i = 1, 2, \ldots, n$.

For simplicity, we denotes

$$\begin{aligned}K = diag(k_1, k_2, \ldots, k_n), \quad L = diag(l_1, l_2, \ldots, l_n) \\ M = diag(m_1, m_2, \ldots, m_n), N = diag(n_1, n_2, \ldots, n_n)\end{aligned}$$

Definition 1. *A point $z^* = (x^*, y^*)$ is defined to be an equilibrium point of neural network (1), if*

$$\begin{aligned}C(x^*, y^*)[-Dx^* + (A^R + B^R + \beta Q^R)f^R(x^*, y^*) \\ - (A^I + B^I + \beta Q^I)f^I(x^*, y^*) + J^R] = 0 \\ C(x^*, y^*)[-Dy^* + (A^I + B^I + \beta Q^I)f^R(x^*, y^*) \\ - (A^R + B^R + \beta Q^R)f^I(x^*, y^*) + J^I] = 0\end{aligned}$$

equivalently,

$$\begin{aligned}-Dx^* +& (A^R + B^R + \beta Q^R)f^R(x^*, y^*) - (A^I + B^I \\ &+\beta Q^I)f^I(x^*, y^*) + J^R = 0 \\ -Dy^* +& (A^I + B^I + \beta Q^I)f^R(x^*, y^*) - (A^R + B^R \\ &+\beta Q^R)f^I(x^*, y^*) + J^I = 0\end{aligned} \tag{3}$$

Lemma 1. *For any given scalar $\varepsilon > 0$, s, $w \in \mathbb{R}^n$, we can obtain the following inequality :*

$$s^T Aw \le \frac{1}{2\varepsilon}s^T AA^T s + \frac{\varepsilon}{2}w^T w \tag{4}$$

Lemma 2. *[16] If matrix $A \in \mathbb{R}^{n\times n}$ satisfies $A \in [\underline{A}, \overline{A}]$, the following inequality is valid,*

$$\|A\|_2 \le \|\widetilde{A}\|$$

where $\|\widetilde{A}\| = \min\{\sqrt{\|A^*\|^2_2 + \|A_*\|^2_2 + 2\|A^T_*|A^*|\|_2}, \|\hat{A}\|_2$ $\|A^*\|_2 + \|A_*\|_2\}$, $A^* = \frac{1}{2}(\overline{A} + \underline{A})$, $A_* = \frac{1}{2}(\overline{A} - \underline{A})$, $\hat{A} = (\hat{a_{ij}})$ with $\hat{a_{ij}} = \max\{|\underline{a}_{ij}|, \overline{a}_{ij}\}$

Lemma 3. *[24] For scalar $u > 0$, the vector function $g : [0, u] \to \mathbb{R}^n$ and the integrations concerned are well-defined. Considering any constant matrix $A \in \mathbb{R}^{n\times n}$, if $A = A^T$, then*

$$u\int_0^u g^T(s)Ag(s)ds \ge (\int_0^u g(s)ds)^T A(\int_0^u g(s)ds).$$

Lemma 4. *If $F(\theta)$ is a continuous function on $\mathbb{R}^n$ and satisfies the following conditions:*
(i) $F(\theta) \ne F(\beta)$ for all $\theta \ne \beta$.
(ii) $\lim_{\|\theta\|\mapsto\infty} \|F(\theta)\| = \infty$
then $F(\theta)$ is a homeomorphism of $\mathbb{R}^n$.

III. MAIN RESULTS

In this section, we will present some sufficient conditions to demonstrate the existence of a unique equilibrium point of neural network (1). Then, we will discuss the exponential stability of the equilibrium point.

Theorem 1. *Assume that the Assumptions 1 and 2 hold, then CGNN (1) has a unique equilibrium point if*

$$\Psi = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix} < 0$$

where,

$$\begin{aligned}
\psi_{11} =& -2\underline{D} + S^R + S^I + 2\beta^2 + 2I + (K^2 + M^2)(p+h) \\
\psi_{12} =& \widetilde{A}^R L + \widetilde{A}^I N + M(\widetilde{A}^R)^T + K(\widetilde{A}^I)^T \\
&+ (MN + KL)(p+h) \\
\psi_{21} =& L(\widetilde{A}^R)^T + N(\widetilde{A}^I)^T + \widetilde{A}^R M + \widetilde{A}^I K \\
&+ (LK + MN)(h+p) \\
\psi_{22} =& -2\underline{D} + U^R + U^I + 2\beta^2 + 2I + (L^2 + N^2)(p+h) \\
p =& \|\widetilde{Q}^I\|^2 + \|\widetilde{B}^I\|^2, h = \|\widetilde{Q}^R\|^2 + \|\widetilde{B}^R\|^2
\end{aligned}$$

Proof. Let $H(x,y) = \begin{pmatrix} H^R(x,y) \\ H^I(x,y) \end{pmatrix}$, where

$$\begin{aligned}
H^R(x,y) =& -Dx + (A^R + B^R + \beta Q^R) f^R(x,y)) \\
&-(A^I + B^I + \beta Q^I) f^I(x,y) + J^R \\
H^I(x,y) =& -Dy + (A^I + B^I + \beta Q^I) f^R(x,y) \\
&+(A^R + B^R + \beta Q^R) f^I(x,y) + J^I
\end{aligned}$$

It is sufficient to demonstrate that $H(x,y)$ is a homeomorphism on $\mathbb{R}^{2n}$ for the proof. According to Lemma 4, in first step, we prove $H(x,y)$ is an injective map. For any $(x_1^T, y_1^T)^T \neq (x_2^T, y_2^T)^T \in \mathbb{R}^{2n}$, we have that

$$\begin{aligned}
&2\begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}^T (H(x_1,y_1) - H(x_2,y_2)) \\
=&2(x_1-x_2)^T(A^R + B^R + \beta Q^R)(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
&-2(x_1-x_2)^T(A^I + B^I + \beta Q^I)(f^I(x_1,y_1) - f^I(x_2,y_2)) \\
&-2(x_1-x_2)^T D(x_1-x_2) \\
&+2(y_1-y_2)^T(A^R + B^R + \beta Q^R)(f^I(x_1,y_1) - f^I(x_2,y_2)) \\
&+2(y_1-y_2)^T(A^I + B^I + \beta Q^I)(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
&-2(y_1-y_2)^T D(y_1-y_2)
\end{aligned}$$

Firstly, it is easy to get the following by inequality using the Assumption 2,

$$\begin{aligned}
&2(x_1-x_2)^T A^R(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
\leq& \sum_{j=1}^{n}\sum_{i=1}^{n} |(x_1)_i - (x_2)_i| \cdot (|a_{ij}^R|k_j + k_i|a_{ji}^R|) \cdot |(x_1)_j - (x_2)_j| \\
&+2\sum_{j=1}^{n}\sum_{i=1}^{n} |(x_1)_i - (x_2)_i| \cdot |(a_{ij}^R l_j)| \cdot |(y_1)_j - (y_1)_j| \\
=&|x_1-x_2|^T S^R |x_1-x_2| + 2|x_1-x_2|^T \widetilde{A}^R L |y_1-y_2|
\end{aligned} \tag{5}$$

Here,

$S^R = (s_{ij}^R)_{m\times n},\ s_{ij}^R = \max\{|\overline{a}_{ij}^R|k_j + k_i|\overline{a}_{ji}^R|, |\underline{a}_{ij}^R|k_j + k_i|\underline{a}_{ij}^R|\}$
$\widetilde{A}^R = (\widetilde{a}_{ij})_{n\times n}^R,\ a_{ij}^R = \max\{|\overline{a}_{ij}^R|, |\underline{a}_{ij}^R|\}$

By means of Lemma1 and Assumption 2, we know that

$$\begin{aligned}
&2(x_1-x_2)^T B^R(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
\leq&\|(B^R)^T B^R\| \cdot \|f^R(x_1,y_2) - f^R(x_2,y_2)\|^2 \\
&+|x_1-x_2|^T|x_1-x_2| \\
\leq&\|\widetilde{B}^R\|^2(|x_1-x_2|^T K^2|x_1-x_2| + |x_1-x_2|^T KL|y_1-y_2|) \\
&+\|\widetilde{B}^R\|^2(|y_1-y_2|^T KL|x_1-x_2| + |y_1-y_2|^T L^2|y_1-y_2|) \\
&+|x_1-x_2|^T|x_1-x_2|
\end{aligned} \tag{6}$$

Moreover,

$$\begin{aligned}
&2(x_1-x_2)^T \beta Q^R(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
\leq&\|(Q^R)^T Q^R\| \cdot \|f^R(x_1,y_1) - f^R(x_2,y_2)\|^2 \\
&+ |x_1-x_2|^T\beta^2|x_1-x_2| \\
\leq&\|\widetilde{Q}^R\|^2(|x_1-x_2|^T K^2|x_1-x_2| + |x_1-x_2|^T KL|y_1-y_2|) \\
&+ \|\widetilde{Q}^R\|^2(|y_1-y_2|^T LK|x_1-x_2| + |y_1-y_2|^T L^2|y_1-y_2|) \\
&+ |x_1-x_2|^T\beta^2|x_1-x_2|
\end{aligned} \tag{7}$$

According to (5, 6, 7), we can obtain that

$$\begin{aligned}
&2(x_1-x_2)(A^R + B^R + \beta Q^R)(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
\leq&|x_1-x_2|^T(S^R + \beta^2 + I + K^2 h)|x_1-x_2| \\
&+ |x_1-x_2|^T(\widetilde{A}^R L + KLh)|y_1-y_2| \\
&+ |y_1-y_2|^T(L(\widetilde{A}^R)^T + LKh)|x_1-x_2| \\
&+ h|y_1-y_2|^T L^2|y_1-y_2|
\end{aligned} \tag{8}$$

Similarly, we have

$$\begin{aligned}
&-2(x_1-x_2)^T(A^I + B^I + \beta Q^I)(f^I(x_1,y_1) - f^I(x_2,y_2)) \\
\leq&|x_1-x_2|^T(S^I + \beta^2 + I + M^2 p)|x_1-x_2| \\
&+ |x_1-x_2|^T(\widetilde{A}^I N + pMN)|y_1-y_2| \\
&+ |y_1-y_2|^T(N(\widetilde{A}^I)^T + pNM)|x_1-x_2| \\
&+ p|y_1-y_2|^T N^2|y_1-y_2|
\end{aligned} \tag{9}$$

where $S^I = (s_{ij}^I)_{m\times n}, \widetilde{A}^I = (\widetilde{a}_{ij}^I)_{n\times n}$,
$s_{ij}^I = \max\{|\overline{a}_{ij}^I|m_j + m_i|\overline{a}_{ji}^I|, |\underline{a}_{ij}^I|m_j + m_i|\underline{a}_{ij}^I|\}$,
$a_{ij}^I = \max\{|\overline{a}_{ij}^I|, |\underline{a}_{ij}^I|\}$.
Then,

$$\begin{aligned}
&2(x_1-x_2)(A^R + B^R + \beta Q^R)(f^R(x_1,y_1) - f^R(x_2,y_2)) \\
&- 2(x_1-x_2)^T(A^I + B^I + \beta Q^I)(f^I(x_1,y_1) - f^I(x_2,y_2)) \\
&- 2(x_1-x_2)^T D(x_1-x_2) \\
\leq&|x_1-x_2|^T[-2\underline{D} + S^R + S^I + 2I \\
&+ 2\beta^2 + hK^2 + pM^2]|x_1-x_2| \\
&+ |x_1-x_2|^T[hKL + pMN + \widetilde{A}^R L + \widetilde{A}^I N]|y_1-y_2| \\
&+ |y_1-y_2|^T[L(\widetilde{A}^R)^T + N(\widetilde{A}^I)^T + LKh + pNM]|x_1-x_2| \\
&+ |y_1-y_2|^T(hL^2 + pN^2)|y_1-y_2|
\end{aligned} \tag{10}$$

Similarly,

$$
\begin{aligned}
&2(y_1-y_2)^T(A^R+B^R+\beta Q^R)(f^I(x_1,y_1)-f^I(x_2,y_2))\\
&\quad+2(y_1-y_2)^T(A^I+B^I+\beta Q^I)(f^R(x_1,y_1)-f^R(x_2,y_2))\\
&\quad-2(y_1-y_2)^TD(y_1-y_2)\\
\le&|y_1-y_2|^T(-2\underline{D}+U^R+U^I+2\beta^2\\
&\quad+2I+hN^2+pL^2)|y_1-y_2|\\
&\quad+|y_1-y_2|^T(hMN+pKL+\widetilde{A}^RM+\widetilde{A}^IK)|x_1-x_2|\\
&\quad+|x_1-x_2|^T(M(\widetilde{A}^R)^T+K(\widetilde{A}^I)^T+hNM+pLK)|y_1-y_2|\\
&\quad+|x_1-x_2|^T(hM^2+pK^2)|x_1-x_2|
\end{aligned}
\tag{11}
$$

Then we can obtain that

$$
\begin{aligned}
&2\begin{pmatrix} x_1-x_2\\ y_1-y_2\end{pmatrix}^T(H(x_1,y_1)-H(x_2,y_2))\\
\le&\begin{pmatrix} |x_1-x_2|^T, |y_1-y_2|^T\end{pmatrix}\Psi\begin{pmatrix} |x_1-x_2|\\ |y_1-y_2|\end{pmatrix}\\
<&0
\end{aligned}
\tag{12}
$$

So, $H(x_1,y_1)\neq H(x_2,y_2)$ for all $(x_1^T,y_1^T)\neq(x_2^T,y_2^T)$, i.e., H is an injective map. We next prove H is coercive. In fact, by (12),

$$
\begin{aligned}
&2\begin{pmatrix} |x_1-x_2|\\ |y_1-y_2|\end{pmatrix}^T(H(x_1,y_1)-H(x_2,y_2))\\
\le&\lambda_{\max}(\Psi)\|\begin{pmatrix} |x_1-x_2|\\ |y_1-y_2|\end{pmatrix}\|^2<0
\end{aligned}
\tag{13}
$$

Especially, letting $(x_2,y_2)=0$, it is easy to get

$$2(x_1^T,y_1^T)(H(x_1,y_1)-H(0,0))\le\lambda_{\max}(\Psi)\|(x_1^T,y_1^T)\|^2$$

which means that

$$2\|(x_1^T,y_1^T)\|\cdot\|H(x_1,y_1)\|\ge|\lambda_{\max}(\Psi)|\cdot|(x_1^T,y_1^T)|^2$$

Then we have

$$\|H(x_1,y_1)\|+H(0,0)\ge\frac{1}{2}|\lambda_{\max}(\Psi)|\cdot\|H(0,0)\|$$

So,

$$\|H(x_1,y_1)\|\ge\frac{|\lambda_{\max}(\Psi)|}{2}\|H(x_1^T,Y_1^T)\|-H(0,0)$$

Therefore, $\lim\limits_{\|(x,y)\|\to\infty}\|H(x,y)\|=\infty$.
Thus, from Lemma 4, the map $H(x,y):\mathbb{R}^{2n}\to\mathbb{R}^{2n}$ is a homeomorphism. Hence, based on assumpations 1 and 2, if $\Psi<0$, CGNNs(1) has an unique equilibrium point. □

Next, we consider the global exponential stability of the equilibrium point of neural networks (1). To simply the process, we first introduce the following denotations,

$$
\begin{aligned}
\grave{h}&=\|(I-\Gamma)^{-1}\|(\|\widetilde{B}^R\|^2+\|\widetilde{Q}^R\|^2)\\
\grave{p}&=\|(I-\Gamma)^{-1}\|(\|\widetilde{B}^I\|^2+\|\widetilde{Q}^I\|^2)\\
\check{h}&=\|e^{\gamma t}(I-\Gamma)^{-1}\|\cdot\|\widetilde{B}^R\|^2+\|\frac{e^{\gamma\beta}-I}{\gamma}\beta^{-1}\|\cdot\|\widetilde{Q}^R\|^2\\
\check{p}&=\|e^{\gamma t}(I-\Gamma)^{-1}\|\cdot\|\widetilde{B}^I\|^2+\|\frac{e^{\gamma\beta}-I}{\gamma}\beta^{-1}\|\cdot\|\widetilde{Q}^I\|^2\\
\phi_{11}&=\psi_{11}+(K^2+M^2)(\grave{p}+\grave{h}-p-h)\\
\phi_{12}&=\psi_{12}+(MN+KL)(\grave{p}+\grave{h}-p-h)\\
\phi_{21}&=\psi_{21}+(NM+LK)(\grave{p}+\grave{h}-p-h)\\
\phi_{22}&=\psi_{22}+(L^2+N^2)(\grave{p}+\grave{h}-p-h)
\end{aligned}
$$

Theorem 2. *Assume that the Assumptions 1 and 2 hold, the equilibrium point of neural network (1) is globally exponentially stabe if*

$$\Phi=\begin{pmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{pmatrix}<0$$

Proof. It is obvious to find $\Psi<\Phi<0$. From Theorem 1, neural network (1) has a unique equilibrium point and it can be denoted by $z^*=(x^*,y^*)$. Making the transformation $s(t)=x(t)-x^*$, $w(t)=y(t)-y^*$, the equilibrium point can be shifted to the origin. Furthermore, we denote $g^R(s(t),w(t))=f^R(x(t),y(t))-f^R(x^*,y^*)$, $g^I(s(t),w(t))=f^I(x(t),y(t))-f^I(x^*,y^*)$, $C(s(t),w(t))=C(x(t)+x^*,y(t)+y^*)$.

According to the condition that $\Phi<0$, we can take a sufficiently small $\gamma>0$ such that

$$\Omega=\begin{pmatrix}\omega_{11} & \omega_{12}\\ \omega_{21} & \omega_{22}\end{pmatrix}<0$$

where,

$$
\begin{aligned}
\omega_{11}=&\psi_{11}+(K^2+M^2)(\check{p}+\check{h}-p-h)+\gamma\underline{C}^{-1}\\
\omega_{12}=&\psi_{12}+(MN+KL)(\check{p}+\check{h}-p-h)\\
\omega_{21}=&\psi_{21}+(NM+LK)(\check{p}+\check{h}-p-h)\\
\omega_{22}=&\psi_{22}+(L^2+N^2)(\check{p}+\check{h}-p-h)+\gamma\underline{C}^{-1}
\end{aligned}
$$

Next we structure a Lyapunov function V as follows,

$$V=G_1+G_2+G_3+G_4+G_5+G_6 \tag{14}$$

where,

$$
\begin{aligned}
G_1 =& 2e^{\gamma t}\sum_{i=1}^{n}\int_0^{s_i(t)}\frac{r}{\hat{c}_i(r,w_i(t))}dr\\
G_2 =& \int_{t-\tau(t)}^{t} e^{\gamma(r+\tau(r))}(g^R(s(r),w(r)))^T(B^R)^T\\
&(I-\Gamma)^{-1}B^Rg^R(s(r),w(r))dr+\int_{t-\tau(t)}^{t}e^{\gamma(r+\tau(r))}\\
&(g^I(s(r),w(r)))^T(B^I)^T(I-\Gamma)^{-1}B^Ig^I(s(r),w(r))dr\\
G_3 =& \sum_{j=1}^{n}\beta_j\int_{-\beta_j}^{0}\int_{t+\theta}^{t}e^{\gamma(r-\theta)}[(\sum_{i=1}^{n}g_i^R(s(r),w(r))q_{ij}^R\beta_j^{-1})^2\\
&+((\sum_{i=1}^{n}g_i^I(s(r),w(r))q_{ij}^I\beta_j^{-1})^2]drd\theta\\
G_4 =& 2e^{\gamma t}\sum_{i=1}^{n}\int_0^{w_i(t)}\frac{r}{\hat{c}_i(s_i(t),r)}dr\\
G_5 =& \int_{t-\tau(t)}^{t} e^{\gamma(r+\tau(r))}(g^R(s(r),w(r)))^T(B^I)^T\\
&(I-\Gamma)^{-1}B^Ig^R(s(r),w(r))dr+\int_{t-\tau(t)}^{t}e^{\gamma(r+\tau(r))}\\
&(g^I(s(r),w(r)))^T(B^R)^T(I-\Gamma)^{-1}B^Rg^I(s(r),w(r))dr\\
G_6 =& \sum_{j=1}^{n}\beta_j\int_{-\beta_j}^{0}\int_{t+\theta}^{t}e^{\gamma(r-\theta)}[(\sum_{i=1}^{n}g_i^R(s(r),w(r))q_{ij}^I\beta_j^{-1})^2\\
&+((\sum_{i=1}^{n}g_i^I(s(r),w(r))q_{ij}^R\beta_j^{-1})^2]drd\theta
\end{aligned}
\tag{15}
$$

It is obvious to see that $V(t,y(t))>0$. Firstly, we calculate the derivative of G_1

$$
\begin{aligned}
\dot{G}_1 =& 2\gamma e^{\gamma t}\sum_{i=1}^{n}\int_0^{s_i(t)}\frac{r}{\hat{c}_i(r,w_i(t))}dr\\
&+2e^{\gamma t}\sum_{i=1}^{n}\frac{s_i(t)}{\hat{c}_i(s_i(t),w_i(t))}\dot{s}_i(t)\\
\le& 2\gamma e^{\gamma t}s(t)^T\underline{C}^{-1}s(t)+2e^{\gamma t}s^T(t)[-Ds(t)+A^Rg(s(t),w(t))\\
&+B^Rg^R(s(t-\tau(t)),w(t-\tau(t)))+Q^R\int_{t-\beta}^{t}g^R(s(r),w(r))dr\\
&-A^Ig^I(s(t),w(t))-B^Ig^I(s(t-\tau(t)),w(t-\tau(t)))\\
&-Q^I\int_{t-\beta}^{t}g^I(s(r),w(r))dr
\end{aligned}
\tag{16}
$$

It is easy to get that

$$
\begin{aligned}
(i)\quad & 2e^{\gamma t}s^T(t)A^Rg^R(s(t),w(t))\\
&\le e^{\gamma t}(|s^T(t)|S^R|s(t)|+2|s^T(t)|\widetilde{A}^RL|w(t)|),\\
(ii)\quad & 2e^{\gamma t}s^T(t)B^Rg^R(s(t-\tau(t)),w(t-\tau(t)))\\
&\le[g^R(s(t-\tau(t)),w(t-\tau(t)))]^T(B^R)^TB^R\\
&\quad g^R(s(t-\tau(t)),w(t-\tau(t)))+e^{\gamma t}s^T(t)s(t)
\end{aligned}
$$

And according to Lemma 3, we obtain that

$$
\begin{aligned}
&2e^{\gamma t}s^T(t)Q^R\int_{t-\beta}^{t}g^R(s(r),w(r))dr\\
\le& e^{\gamma t}(\int_{t-\beta}^{t}g^R(s(r),w(r)))^T(Q^R)^T(\beta^{-1})^T\beta^{-1}Q^R\int_{t-\beta}^{t}\\
&g^R(s(r),w(r))dr+e^{\gamma t}s^T(t)\beta^2s(t)
\end{aligned}
\tag{17}
$$

As a result, we can get,

$$
\begin{aligned}
\dot{G}_1 \le& e^{\gamma t}|s^T(t)|(-2\underline{D}+\gamma\underline{C}^{-1}+S^R+2\beta^2+2E+S^I)|s(t)|\\
&+2e^{\gamma t}|s^T(t)|\widetilde{A}^RL|w(t)|+2e^{\gamma t}|s^T(t)|\widetilde{A}^IN|w(t)|\\
&+e^{\gamma t}(g^R(s(t-\tau(t)),w(t-\tau(t))))^T(B^R)^TB^R\\
&\quad g^R(s(t-\tau(t)),w(t-\tau(t)))\\
&+e^{\gamma t}(g^I(s(t-\tau(t)),w(t-\tau(t))))^T(B^I)^TB^I\\
&\quad g^I(s(t-\tau(t)),w(t-\tau(t)))\\
&+e^{\gamma t}[\int_{t-\beta}^{t}g^R(s(r),w(r))]^T(Q^R)^T(\beta^{-1})^T\beta^{-1}Q^R\\
&\quad \int_{t-\beta}^{t}g^R(s(r),w(r))dr\\
&+e^{\gamma t}[\int_{t-\beta}^{t}g^I(s(r),w(r))]^T(Q^I)^T(\beta^{-1})^T\beta^{-1}Q^I\\
&\quad \int_{t-\beta}^{t}g^I(s(r),w(r))dr
\end{aligned}
$$

Next, we calculate the derivative of G_2,

$$
\begin{aligned}
&\dot{G}_2 =\\
&e^{\gamma(t+\tau)}[g^R(s(t),w(t))]^T(B^R)^T(I-\Gamma)^{-1}B^Rg^R(s(t),w(t))\\
&-e^{\gamma t}[g^R(s(t-\tau(t)),w(t-\tau(t)))]^T(B^R)^T(I-\Gamma)^{-1}\\
&B^Rg^R(s(t-\tau(t)),w(t-\tau(t)))\\
&+e^{\gamma(t+\tau)}[g^I(s(t),w(t))]^T(B^I)^T(I-\Gamma)^{-1}\\
&B^Ig^I(s(t),w(t))\\
&-e^{\gamma t}[g^I(s(t-\tau(t)),w(t-\tau(t)))]^T(B^I)^T(I-\Gamma)^{-1}\\
&B^Ig^I(s(t-\tau(t)),w(t-\tau(t)))
\end{aligned}
\tag{18}
$$

Making the use of Lemma 3, it yields to

$$
\begin{aligned}
\dot{G}_3 =& \sum_{j=1}^{n}\beta_j\{[(\sum_{i=1}^{n}g_i^R(s(t),w(t))q_{ij}^R\beta_j^{-1})^2\\
&+(\sum_{i=1}^{n}g_i^I(s(t),w(t))q_{ij}^I\beta_j^{-1})]^2\int_{-\beta_j}^{0}e^{\gamma(t-\theta)}d\theta\}\\
&-\sum_{j=1}^{n}\beta_j\{\int_0^{\beta_j}e^{\gamma t}[(\sum_{i=1}^{n}g_i^R(s(t),w(t))q_{ij}^R\beta_j^{-1})^2\\
&+(\sum_{i=1}^{n}g_i^I(s(t),w(t))q_{ij}^I\beta_j^{-1})]^2d\theta\}
\end{aligned}
$$

$$\leq e^{\gamma t}[\|(Q^R)^T \frac{e^{\gamma\beta} - I}{\gamma}\beta^{-1}Q^R\| \cdot \|g^R(s(t), w(t))\|^2$$
$$+\|(Q^I)^T \frac{e^{\gamma\beta} - I}{\gamma}\beta^{-1}Q^I\| \cdot \|g^I(s(t), w(t))\|^2]$$
$$-e^{\gamma t}(\int_{t-\beta}^{t} g^R(s(r), w(r))ds)^T (Q^R)^T \beta^{-2} Q^R$$
$$\int_{t-\beta}^{t} g^R(s(r), w(r))ds$$
$$-e^{\gamma t}(\int_{t-\beta}^{t} g^I(s(r), w(r))ds)^T (Q^I)^T \beta^{-2} Q^I$$
$$\int_{t-\beta}^{t} g^I(s(r), w(r))ds$$

Next we can calculate the derivatives of G_4, G_5, G_6 in a similar way. Hence,

$$\dot{V} \leq e^{\gamma t} \begin{pmatrix} |s(t)| \\ |w(t)| \end{pmatrix}^T \Omega \begin{pmatrix} |s(t)|^T, |w(t)|^T \end{pmatrix} < 0$$

As a result of last inequality, $V(s(t), w(t)) \leq V(0, y(0))$, which implies

$$2e^{\gamma t} \sum_{i=1}^{n} \int_0^{s_i(t)} \frac{r}{\hat{c}_i(r, w_i(r))} dr < V(0, 0).$$

By the **Assumption 1**, we have

$$s(t)^T s(t) < \frac{\bar{c}}{2} e^{-\gamma t} V(0, 0),$$

where $\bar{c} = \max\{\bar{c}_i\}$. Similarly, we get

$$w(t)^T w(t) < \frac{\bar{c}}{2} e^{-\gamma t} V(0, 0)$$

For convenience, we denote $N_1 = \frac{V(0,0)\bar{c}}{2}$, $N_2 = \frac{V(0,0)\bar{c}}{2}$. So we have,

$$\|x(t) - x^*\|^2 < N_1 e^{-\gamma t}$$
$$\|y(t) - y^*\|^2 < N_2 e^{-\gamma t}$$

This illustrates the equilibrium point $z^* = (x^*, y^*)$ of the neural networks is globally exponentially stable. □

IV. Numerical Examples

In this section, to show the validity of the established results, several numerical examples with simulations are given.

Example 1. *Consider neural network (1) with the following parameters:* $C = 1$, $\overline{A}^R = 2$, $\underline{A}^R = -2$, $\overline{A}^I = 1$, $\underline{A}^I = -1$, $\overline{B}^R = 1$, $\underline{B}^R = -1$, $\overline{B}^I = 0.5$, $\underline{B}^I = -1$, $\overline{Q}^R = 0.5$, $\underline{Q}^R = -0.5$, $\overline{Q}^I = 1.5$, $\underline{Q}^I = -1.5$, $\overline{D} = 10$, $\underline{D} = 6.5$, $f(x, y) = x + iy$, $\beta = 0.5$, $\tau = 1$, $J = 0$.

It is easy to check that

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 2 & -3 \end{pmatrix} < 0$$

Therefore, we can obtain the global robust exponential stability of the neural network (1) given the above parameters by Theorem 2. In order to verify the conclusion, we take on a simulation that is conducted as shown in the following cases

Case 1: $A^R = \overline{A}^R$, $A^I = \overline{A}^I$, $B^R = \overline{B}^R$, $B^I = \overline{B}^I$, $Q^R = \overline{Q}^R$, $Q^I = \overline{Q}^I$, see Fig.1.

Case 2: $A^R = 1$, $A^I = 0.5$, $B^R = 0.5$, $B^I = -0.5$, $Q^R = 0.2$, $Q^I = 1$, $D = 8$, see Fig.2.

Case 3: $A^R = \underline{A}^R$, $A^I = \underline{A}^I$, $B^R = \underline{B}^R$, $B^I = \underline{B}^I$, $Q^R = \underline{Q}^I$, $Q^I = \underline{Q}^I$, see Fig.3.

It is easy to find the state trajectory converges to an unique equilibrium of the network by Fig.1, Fig.2 and Fig.3, which is accordance with the conclusion of Theorem 2.

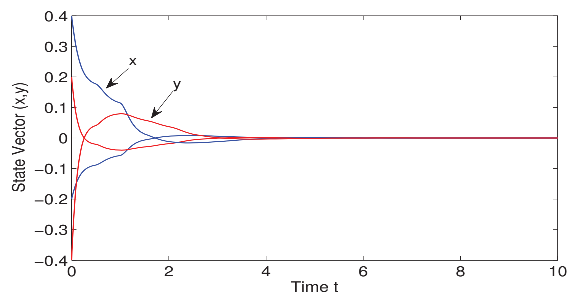

Fig. 1. The state trajectory (x, y) of neural network (1) in Case 1.

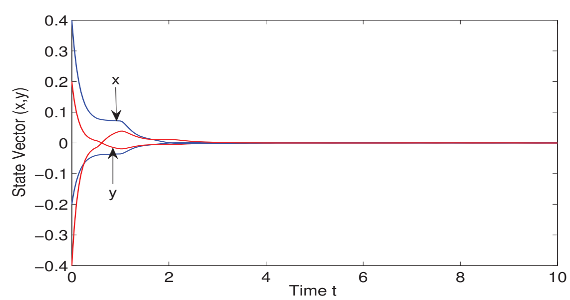

Fig. 2. The state trajectory (x, y) of neural network (1) in Case 2.

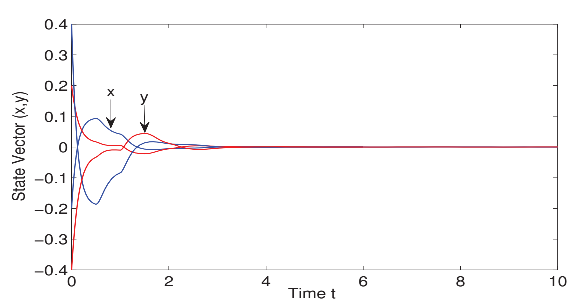

Fig. 3. The state trajectory (x, y) of neural network (1) in Case 3.

Example 2. *Assume that parameters of neural network (1) are shown as follows :*

$$\overline{A}^R = \begin{pmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{pmatrix}, \underline{A}^R = \begin{pmatrix} -0.1 & -0.1 \\ -0.1 & -0.1 \end{pmatrix}$$

$$\overline{B}^R = \begin{pmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{pmatrix}, \underline{B}^R = \begin{pmatrix} -0.3 & -0.1 \\ -0.1 & -0.3 \end{pmatrix}$$

$$\overline{B}^I = \begin{pmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{pmatrix}, \underline{B}^I = \begin{pmatrix} -0.2 & -0.2 \\ -0.2 & -0.2 \end{pmatrix}$$

$$\overline{Q}^R = \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, \underline{Q}^R = \begin{pmatrix} -0.4 & -0.2 \\ -0.2 & -0.4 \end{pmatrix}$$

$$\overline{Q}^I = \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{pmatrix}, \underline{Q}^I = \begin{pmatrix} -0.3 & -0.2 \\ -0.2 & -0.3 \end{pmatrix}$$

$$\overline{D} = \begin{pmatrix} 6 & 0 \\ 0 & 5 \end{pmatrix}, \underline{D} = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$$

$$f(x,y) = \begin{pmatrix} \frac{1}{4}x_1 + e^{-\frac{1}{5}y_1} + (e^{-\frac{1}{4}x_1} + \frac{1}{3}y_1)i \\ e^{-\frac{1}{6}x_2} + \frac{1}{4}y_2 + (\frac{1}{5}x_2 + \frac{1}{3}y_2)i \end{pmatrix}$$

$C(x,y) = \begin{pmatrix} 3.5 + sinx_1 + cosy_1 \\ 3 + 1.5cosx_2 + sin2y_2 \end{pmatrix}$, $\beta = \mathbf{1}$, $\tau = \mathbf{0.2}$, $J = \mathbf{0}$.

According to **Assumption 2**, we can set matrices K, L, M, N as follow,

$$K = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{6} \end{pmatrix}, L = \begin{pmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{5} \end{pmatrix}, K = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

and after simple calculation, we have

$$\phi_{11} = \begin{pmatrix} -3.625 & 0.132 \\ 0.132 & -1.765 \end{pmatrix}, \phi_{12} = \begin{pmatrix} 0.401 & 0.167 \\ 0.14 & 0.254 \end{pmatrix}$$

$$\phi_{22} = \begin{pmatrix} -3.583 & 0.157 \\ 0.157 & -1.522 \end{pmatrix}, \phi_{21} = \begin{pmatrix} 0.401 & 0.14 \\ 0.167 & 0.254 \end{pmatrix}$$

so we can obtain matrix Φ is negative defined. It can be demonstrated Neural networks (1) with above parameters is globally exponentially stable, Combining with the Theorem 2. To test and verbify the theoretical conclusion, we make a simulation that is conducted as shown in the following cases:

Case 4 : $A^R = \overline{A}^R$, $A^I = \overline{A}^I$, $B^R = \overline{B}^R$, $B^I = \overline{B}^I$, $Q^R = \overline{Q}^R$, $Q^I = \overline{Q}^I$, $D = \overline{D}$. See Fig. 4.

Case 5 :

$A^R = \begin{pmatrix} -0.05 & -0.05 \\ -0.05 & -0.05 \end{pmatrix}$, $A^I = \begin{pmatrix} -0.1 & -0.1 \\ -0.1 & -0.1 \end{pmatrix}$

$B^R = \begin{pmatrix} -0.15 & -0.05 \\ -0.05 & -0.15 \end{pmatrix}$, $B^I = \begin{pmatrix} -0.1 & -0.05 \\ -0.05 & -0.1 \end{pmatrix}$

$Q^R = \begin{pmatrix} -0.2 & -0.1 \\ -0.1 & -0.2 \end{pmatrix}$, $Q^I = \begin{pmatrix} -0.15 & -0.1 \\ -0.1 & -0.15 \end{pmatrix}$

$D = \begin{pmatrix} 6 & 0 \\ 0 & 3 \end{pmatrix}$. See Fig.5.

Case 6 : $A^R = \underline{A}^R$, $A^I = \underline{A}^I$, $B^R = \underline{B}^R$, $B^I = \underline{B}^I$, $Q^R = \underline{Q}^R$, $Q^I = \underline{Q}^I$, $D = \underline{D}$. See Fig.6.

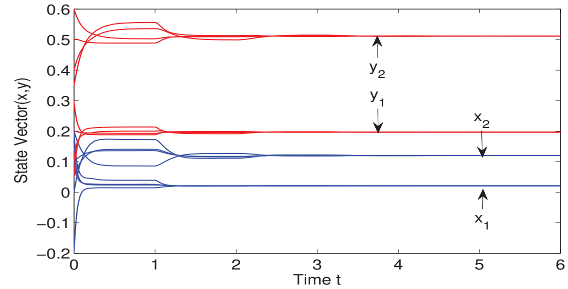

Fig. 4. The state trajectory (x, y) of neural network (1) in Case 4.

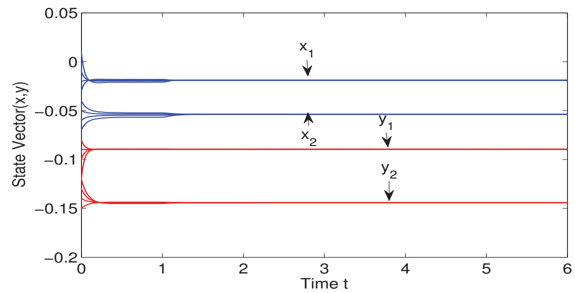

Fig. 5. The state trajectory (x, y) of neural network (1) in Case 5.

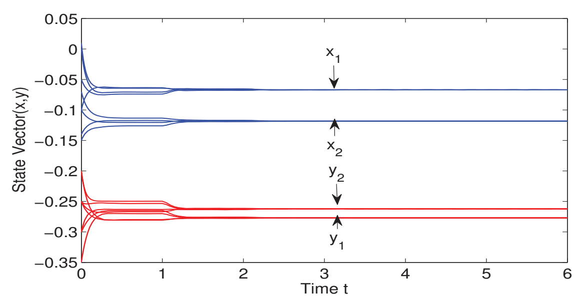

Fig. 6. The state trajectory (x, y) of neural network (1) in Case 6.

V. Conclusion

In this paper, there presents several new sufficient criteria for global robust stability of the complex-valued CGNNs with mixed delays. The existence of a unique equilibrium point of the neural network is derived by homomorphic theorem. And we proved the robust stability by employing Lyapunov function and by application of robust stability theory. We give some numerical examples and simulations to illustrate the obtained result.

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