STAT5034_HW5

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Problem 1

For sample variance, we have: $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$.

Thus we can obtain a probability interval of it.

$$\chi^2_{1-\frac{\alpha}{2}} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{\frac{\alpha}{2}}$$

$$\Rightarrow \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

Then we get the lower and upper bounds C.I of σ^2:

$$\Rightarrow \sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}}} \leq \sigma^2 \leq \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}}$$

The bounds for σ is appropriate because the pivot is χ ditributed thus the bounds can not be negtive.

Problem 2

$\overline{Y}$ tends to be normal asymptotically, and we have $\overline{Y} \sim N(\lambda, \frac{\lambda}{n})$

Then we know that $\sqrt{n}\dfrac{\overline{Y}-\lambda}{\sqrt{\lambda}} \sim N(0,1)$, and an appropriate estimator forλ is $\overline{Y}$

Therefore : $P(z_{\alpha_1} \leq \sqrt{n}\dfrac{\overline{Y}-\lambda}{\sqrt{\overline{Y}}} \leq z_{\alpha_2}) = 95\%$

Let $\alpha = 0.05$ The we get our tail probabilities $\alpha_1 = 1 - 0.025 = 0.975$, $\alpha_2 = 0.025$

So $z_{0.975} = -1.96$, $z_{0.025} = 1.96$

The CI for λ is:

$$z_{\alpha_1} \leq \sqrt{n}\frac{\overline{Y}-\lambda}{\sqrt{\overline{Y}}} \leq z_{\alpha_2}$$

$$\Rightarrow \overline{Y} - z_2\frac{\sqrt{\overline{Y}}}{\sqrt{n}} \leq \lambda \leq \overline{Y} - z_1\frac{\sqrt{\overline{Y}}}{\sqrt{n}} \leq \lambda$$

$$\overline{Y} = 7.04, \quad n = 25$$

$$\Rightarrow \text{CI:} \quad 6 \leq \lambda \leq 8.08$$

```
y <- c(11,7,2,7,4,8,13,3,6,6,15,8,2,4,5,11,11,4,9,3,9,8,5,9,6)
length(y)
```

```
## [1] 25
```

```
mean(y)
```

```
## [1] 7.04
```

```
qnorm(0.975,mean = 0,sd = 1, lower.tail = F)
```

```
## [1] -1.959964
```

```
qnorm(0.025,mean = 0,sd = 1, lower.tail = F)
```

```
## [1] 1.959964
```

Problem 3

a

$$\log(f(y,\theta)) = \log(\theta) + (\theta - 1)\log(y)$$

$$\Rightarrow \frac{\partial \log(f(y,\theta))}{\partial\theta} = \tfrac{1}{\theta} + \log(y)$$

Let the partial derivative equals to 0, we get MLE estimator for θ:

$$\hat{\theta} = -\tfrac{1}{\log(y)}$$

b

$$\begin{aligned} f_{Y_1,\ldots,Y_n}(y_1,\ldots,y_n;\theta) &= \prod_{i=1}^{n} f_{Y_i}(y_i;\theta) \\ &= \prod_{i=1}^{n} \theta y_i^{\theta-1} \\ &= \theta^n \left(\prod_{i=1}^{n} y_i\right)^{\theta-1} \end{aligned}$$

$$\Rightarrow \log f = n\log\theta + (\theta-1)\log\left(\prod y_i\right) \Rightarrow \frac{\partial f}{\partial\theta} = n\log\theta + (\theta-1)\log\left(\prod y_i\right)$$

$$\Rightarrow \hat{\theta} = \frac{n}{-\log\left(\prod y_i\right)}$$

Problem 6

(a) CI of the proportion: (0.3129798,0.6870202)

```
f=function(p,alpha,flag=TRUE){
  if(flag==TRUE){difference=alpha-pbinom(15,30,p)}
  if(flag==FALSE){difference=alpha-pbinom(14,30,p,lower.tail=FALSE)}
  return(abs(difference))}

#For alpha_1:
optimize(f,int=c(0,1),maximum=FALSE,alpha=.025,flag=TRUE)
```

```
## $minimum
## [1] 0.6870202
##
## $objective
## [1] 6.871329e-06
```

```
optimize(f,int=c(0,1),maximum=FALSE,alpha=.025,flag=FALSE)
```

```
## $minimum
## [1] 0.3129798
##
## $objective
## [1] 6.871329e-06
```

(b) When sample size is large, binominal distribution weakly converges to normal distribution.

$\overline{X} \sim N(np, np(1-p))$. Then we get $\frac{\overline{X}}{n} \sim N(p, \frac{p(1-p)}{n})$.

$\hat{p} = \frac{\overline{X}}{n} = \frac{15}{30} = 0.5$

Then CI is: $\tilde{p} \pm 1.96\sqrt{\tilde{p}(1-\tilde{p})/n}$. That is (0.3210773,0.6789227). I prefer the large sample one because it is narrower than exact CI.

(c)

The probability is 0.903

```
pbinom(15,30,0.4,lower.tail = T)
```

```
## [1] 0.9029432
```